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Spatio-Temporal Correlations of Jets
using High-Speed
Particle Image Velocimetry

by

Christopher David Pokora

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of
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Department of Aeronautical and Automotive Engineering
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Abstract

The major source of aircraft noise at take-off is jet noise. If jet noise is not adequately addressed environmental impact concerns will constrain the planned growth of the air transport system. A considerable amount of research worldwide has therefore been aimed at identifying ways to reduce jet noise including development of a predictive tool that can estimate the noise generated by new nozzle designs. Current noise prediction techniques, however, still require the input of empirically calibrated noise source models and their performance is still inadequate. In addition, development of detailed noise source identification measurements and the associated understanding of how to control (and reduce) the noise at the source has been limited.

The fundamental turbulence property which acts as the source of propagating noise in shear layers is the two-point space-time velocity correlation \( R_{ijkl} \). Very few measurements exist for this property to guide model development. It is therefore the aim of the work reported in this thesis to provide new experimental data that helps identify the turbulence sources located within the shear layer of jets. The technique of Particle Imaging Velocimetry (PIV) is used to capture directly the flowfield and all relevant turbulent statistics. Both single and coaxial axisymmetric jets are considered in a water flow experiment appropriately scaled to reproduce similarity with free ambient subsonic air flow test conditions. Stereoscopic Particle Imaging Velocimetry (SPIV) tests have been performed to investigate the spatio-temporal correlations. Measurements were taken in a water flow experiment where, for the same Reynolds number, the range of dynamically important turbulent structures is at much lower frequencies than in an air flow experiment, but the turbulent structures, when suitably normalised, are shown to be identical. (Measured Lagrangian and Eulerian lengthscales and timescales are consistent with previous air flow measurements). By using a global technique time-resolved proper orthogonal decomposition (POD) of the jet has been possible. This enables the identification of the different scales of coherent structures and their associated energy content, resulting in its development as the basis of a spatio-temporal filtering procedure suitable for removing noise from 3-component PIV data. The results show that, by using SPIV with a repetition rate of 1kHz, given the correct application of the method (e.g. sufficiently small PIV interrogation cell size in relation to the local turbulent length scales), even 4th order correlations can be captured, and were demonstrated to reproduce the quality of those captured by point-based probe techniques such as Constant Temperature
Anemometry (CTA). These measurements deliver new insights into the characteristics of the 4th order correlation $R_{ijkl}$, for example in a round jet only 5, of the 21 possible independent components, are significant. This level of detail is valuable for aeroacoustic prediction methods which need to assume a model for $R_{ijkl}$.

Although Reynolds Averaged Navier Stokes (RANS) CFD still forms the basis for most jet noise prediction procedures, the models for $R_{ijkl}$ that have been proposed are rather crude, and the measurements discussed above should help improve the modelling of $R_{ijkl}$ considerably. There is an outstanding question whether Large Eddy Simulation (LES) CFD can provide an alternative means to direct measurement for predicting $R_{ijkl}$. Work to address this issue has also been part of the present project, and a comparison between LES-predicted $R_{ijkl}$ and measured data has been carried out, with encouraging results for future simulations of $R_{ijkl}$ in more complex nozzle geometries.

**Keywords:** Particle Image Velocimetry, Large Eddy Simulation, Jet Noise, Aeroacoustics, Spatio-temporal Correlations, Sub-cell Filtering, Proper Orthogonal Decomposition.
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<tbody>
<tr>
<td>( a_o )</td>
<td>Velocity of Sound in fluid at rest</td>
</tr>
<tr>
<td>( c )</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>( D_j )</td>
<td>Jet nozzle exit diameter</td>
</tr>
<tr>
<td>( E )</td>
<td>Energy; Frequency Spectra</td>
</tr>
<tr>
<td>( i,j,k )</td>
<td>Cartesian directions; Tensor notations</td>
</tr>
<tr>
<td>( kL_{ij} )</td>
<td>Integral lengthscale defined by ( R_{ij} ) in ( k ) co-ordinate direction</td>
</tr>
<tr>
<td>( kL_{ij}^L )</td>
<td>Lagrangian lengthscale defined by ( R_{ij} ) in ( k ) co-ordinate direction</td>
</tr>
<tr>
<td>( N_{buf} )</td>
<td>Number of buffer samples</td>
</tr>
<tr>
<td>( N_t )</td>
<td>Number of time steps</td>
</tr>
<tr>
<td>( r )</td>
<td>Radial co-ordinate</td>
</tr>
<tr>
<td>( Re_{D_j} )</td>
<td>Jet exit Reynolds number ( U_{jet}D_j/v )</td>
</tr>
<tr>
<td>( R_{ij} )</td>
<td>2\textsuperscript{nd} order 2-point 2-time correlation</td>
</tr>
<tr>
<td>( R_{ijkl} )</td>
<td>4\textsuperscript{th} order 2-point 2-time correlation</td>
</tr>
<tr>
<td>( t )</td>
<td>Time</td>
</tr>
<tr>
<td>( T_{ij} )</td>
<td>Integral timescale</td>
</tr>
<tr>
<td>( T_{ij}^L )</td>
<td>Integral timescale</td>
</tr>
<tr>
<td>( U_{jet} )</td>
<td>Peak average velocity at nozzle exit</td>
</tr>
<tr>
<td>( u_i )</td>
<td>Instantaneous velocity in ( i ) direction</td>
</tr>
<tr>
<td>( U_i )</td>
<td>Time mean velocity in ( i ) direction</td>
</tr>
<tr>
<td>( u_i^{rms} )</td>
<td>Root mean square fluctuating velocity in ( i ) direction</td>
</tr>
<tr>
<td>( u,v,w )</td>
<td>Cartesian velocity</td>
</tr>
<tr>
<td>( x, y, z )</td>
<td>Cartesian distance</td>
</tr>
<tr>
<td>( x, r, \theta )</td>
<td>Polar distance</td>
</tr>
<tr>
<td>( \bar{e} )</td>
<td>Cartesian co-ordinate vector</td>
</tr>
</tbody>
</table>
### Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Oblique Viewing Angle</td>
</tr>
<tr>
<td>$\delta*$</td>
<td>Displacement thickness</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Filter width</td>
</tr>
<tr>
<td>$\vec{\eta}$</td>
<td>Displacement vector ($\eta_1=$axial, $\eta_2=$radial, $\eta_3=$circumferential)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Frequency</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\tau$</td>
<td>SGS stress tensor; Temporal separation</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Circumferential co-ordinate</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>2C</td>
<td>Two Component</td>
</tr>
<tr>
<td>3C</td>
<td>Three Component</td>
</tr>
<tr>
<td>ACF</td>
<td>Autocorrelation Function</td>
</tr>
<tr>
<td>CAA</td>
<td>Computational Aeroacoustics</td>
</tr>
<tr>
<td>CAD</td>
<td>Computer Aided Design</td>
</tr>
<tr>
<td>CCD</td>
<td>Charged Coupled Device</td>
</tr>
<tr>
<td>CCF</td>
<td>Cross Correlation Function</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CFL</td>
<td>Courant-Friedrichs-Lewy condition</td>
</tr>
<tr>
<td>CPU</td>
<td>Central Processing Unit</td>
</tr>
<tr>
<td>CTA</td>
<td>Constant Temperature Anemometry</td>
</tr>
<tr>
<td>CV</td>
<td>Control Volume</td>
</tr>
<tr>
<td>DFG</td>
<td>Digital Filtering-based Generation</td>
</tr>
<tr>
<td>DNS</td>
<td>Direct Numerical Simulation</td>
</tr>
<tr>
<td>FoV</td>
<td>Field of View</td>
</tr>
<tr>
<td>FVM</td>
<td>Finite Volume Method</td>
</tr>
<tr>
<td>HMN</td>
<td>Hoest-Madsen and Nielsen SGF compensation method</td>
</tr>
<tr>
<td>ICAO</td>
<td>International Civil Aviation Organisation</td>
</tr>
<tr>
<td>LDA</td>
<td>Laser Doppler Anemometry</td>
</tr>
<tr>
<td>LEE</td>
<td>Linearised Euler Equations</td>
</tr>
<tr>
<td>LES</td>
<td>Large Eddy Simulation</td>
</tr>
<tr>
<td>MGB</td>
<td>Mani-Gliebe-Balsa</td>
</tr>
<tr>
<td>MGBK</td>
<td>Mani-Gliebe-Balsa-Khavaran</td>
</tr>
<tr>
<td>MPI</td>
<td>Multiple Processor Interface</td>
</tr>
<tr>
<td>Nd-YAG</td>
<td>Neodym Yttrium Aluminium Garnet (laser)</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Imaging Velocimetry</td>
</tr>
<tr>
<td>PLIF</td>
<td>Planar Laser Induced Fluorescence</td>
</tr>
<tr>
<td>POD</td>
<td>Proper Orthogonal Decomposition</td>
</tr>
<tr>
<td>RANS</td>
<td>Reynolds-Averaged Navier-Stokes</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>SGS</td>
<td>Sub-Grid Scale</td>
</tr>
<tr>
<td>SVC</td>
<td>Spatial Velocity Correlation</td>
</tr>
<tr>
<td>TKE</td>
<td>Turbulent Kinetic Energy</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

The first commercial jet-powered aircraft, a De Havilland Comet operating between London and Johannesburg, entered service in May 1952. During the 1960's and 70's the economic vitality of jet service triggered explosive growth in the air transportation industry[1]. This growth increased the global impact aircraft had on the environment (NOx emissions, CO2, Radiative forcing / global warming) in addition to local impacts near airports. Aviation noise is of most concern when considering the local impacts near airports, as sound generated during cruise is not a global environmental issue. It is aviation noise which is of interest in this study.

1.1 Background

As airports grew in size and in importance, the areas they impacted expanded. When the number of jet operations reached a level where the noise interfered with daily life for the public community near these growing airports, many countries were forced to address aviation noise. These local environmental issues have led to the development of certification standards for measuring and for limiting aircraft noise. These certification standards, which paralleled technological improvements in aircraft engine designs, are contained in the International Civil Aviation Organisation (ICAO) Annex 16[2]. It was decided by NASA[3] in 1990 that the natural growth of the industry was being limited by airports imposing operating restrictions, as well as financial penalties on airlines breaching noise limits. This has led to aircraft and engine manufacturers jointly developing new jet engine technologies, with government and industry sponsorship, that produce lower noise levels in addition to improving other environmental areas of concern, such as, lower NOx emissions and better fuel-efficiency[1]. It is not surprising therefore that new noise prediction methods as well as improvements in design practices to achieve aviation noise reduction have also been required. This research and development has already provided significant improvement
Introduction

although research into ways to resolve the growing noise problem is still needed. Constant development in both these areas is important in order to meet upcoming noise reduction requirements and to increase efficiency of the design process cycle.

This chapter presents an overview of the industrial, commercial and environmental drivers behind the increase in jet noise research and a comprehensive review of the more pertinent experimental and computational studies. The chapter is organised as follows. Section 1.2 describes the growth of the aviation industry and the associated growth in public awareness towards the environment. This is followed in Section 1.3 by an overview of jet engine noise sources and developments in engine design particularly with regard to jet exhaust noise. This is followed by Sections 1.4, 1.5 and 1.6, where there is a comprehensive review of the more pertinent analytical, numerical, and experimental studies which have been conducted over the recent years, including discussion as to the development in measurement techniques and analysis and how the results have impacted the way we attempt to model aerodynamically generated sound. Within Section 1.4 some current ideas regarding the dominant source mechanisms in jets are outlined. This Chapter ends with Sections 1.7 and 1.8, which identify the Project objectives and research strategy to be adopted, and the structure of this thesis respectively.

1.2 Civil Aviation Industry

Over the last 50 years, civil aviation has seen a five-fold increase in volume, including a doubling in the amount of air freight since 1990. Currently more than one half of the UK population makes at least one flight each year. With this growth has come a significant increase in public interest in the environmental impact of aircraft flights and greater sensitivity to noise levels. Although new jet transport airplanes in today's fleet are considerably quieter than the first jet transports introduced about 50 years ago, noise levels around airport communities continue to be an important national and international issue. A white paper [3] stated an anticipated 3-8% growth in passenger and cargo operations well into the 21st Century and with the slow introduction of new noise reduction technology into the fleet, world aircraft noise levels will remain essentially constant until about 2020 to 2030 and thereafter begin to rise unless further noise reducing measures are introduced.

Although noise reduction at the source is the most efficient way to mitigate community noise, the only long-term solution that produces the full results will be when current airplanes are retired and replaced, some 20 to 25 years from now. The ever more challenging international aircraft noise standards set by the ICAO and the Committee on Aviation Environmental Protection (CAEP)[2] are only currently achieved by incorporating a wide variety of other mitigation measures to protect the community from aviation noise. As well as engine design and develop-
Introduction

ment, operational measures are an example of these other measures which play a large role in airport noise abatement programs via curfews, noise budgets and slot restrictions. The initial standards for jet-powered aircraft designed before 1977 were included in Stage 2 of ICAO Annex 16. The Boeing 727 and the Douglas DC-9 are examples of aircraft covered by Stage 2. Subsequently, newer aircraft have been required to meet the stricter standards contained in Stage 3 of ICAO Annex 16. The Boeing 737-300/400, Boeing 767 and Airbus A319 are examples of Stage 3 aircraft types. In June 2001, on the basis of recommendations made by the fifth meeting of the Committee on Aviation Environmental Protection (CAEP/5), the Council adopted a new Stage 4 noise standard, more stringent than that contained in Stage 3. These standards are mandated to apply from 2006 and apply to newly certificated airplanes and to Stage 3 aeroplanes for which re-certification to Stage 4 is required. Figure 1.1 shows ICAO stages and relative noise levels of commercial jet aircraft.

In addition to limiting general noise levels specific night flight noise restrictions have also been imposed. Night flights are a particularly controversial aspect of aviation. Studies have shown that sleep can be disturbed at a relatively low Equivalent Continuous Sound Level (Leq dBA) of just 30. The first restrictions on night flights were imposed at Heathrow in 1962. Reviews have taken place since then in 1988, 1993 and 1998. Ten UK airports are now subject to night noise controls under the Aerodromes Noise Restrictions (Rules and Procedures) Regulations 2003. The Government undertook to consult on a new night noise regime in 2004, and decided that the existing limits on night flights should remain until 2012.

The 1995 White House National Science and Technology Council report[4] stated that “Environmental issues are likely to impose the fundamental limitation on air transportation growth in the 21st century”. The noise issues mentioned above are also inhibiting expansion or construction of new airport facilities. Furthermore, the Environmental Protection Agency (EPA) has established that a Day-Night Average Level of 55 decibels is ‘requisite to protect the public health and welfare with an adequate margin of safety’. If the noise issue is not adequately addressed these restrictions will continue to limit capacity and constrain the natural growth of the air transport system.

Today’s new jet transport airplanes are about 20 decibels (dB) quieter than those introduced in the 1950’s. This is perceived by people as being 75% quieter. This reduction in aviation noise levels has been achieved by major engine cycle advances including high bypass ratios (reducing exhaust noise), fan and engine inlet design (reducing fan noise) and acoustic liners (reducing core and fan stream noise). Additional advances such as serrations have shown promise to aid in future noise reductions as seen in Figure 1.2 (discussed in detail later).
Introduction

This section has highlighted the standards imposed on the aircraft industry, and the potential for these to limit its natural growth. The section has also mentioned that with the ever increasing international aircraft noise standards further advances to the current aircraft and engine design must be made. The following section distinguishes different areas of an aircraft which produce noise and their relative importance to total aircraft noise.

1.3 Jet Engine Noise Sources

Aircraft noise is generally divided into two sources: that due to the engines, and that associated with the airframe itself. As higher bypass ratio engines have become more common and aircraft have become larger, interest in airframe-related noise has grown, but engine noise still accounts for most of the aircraft external noise. The engine contains a variety of noise sources all with various relative importance. A breakdown of these noise components of a typical engine during takeoff and approach to landing can be seen in Figure 1.3. The largest sources of aircraft noise (particularly during take-off where engine thrust requirements are at their greatest and aircraft are in closest proximity to densely populated areas) are those produced by the jet exhausts (consisting of the fan stream and the hot core / turbine stream). It is this exhaust associated noise, more commonly called ‘jet noise’, which is the area of interest in this research.

Jet noise is a strong function of the exhaust velocity and hence efforts have been made to increase the bypass ratio and therefore decrease the exhaust velocity and reduce jet noise. The introduction of the bypass turbofan engine reduced the two principle noise generating components which are associated with the engine noise although these components remain high noise generating components (Figure 1.3). The maximum benefit has now been derived from this method and new techniques need to be found. One of these is through careful control of the way in which the jet plume develops, as it is believed to be possible to reduce noise production through changing the plume. Research efforts to date have been directed at influencing the way in which the near-field region of the jet plume develops. Flow control is divided into two general categories: passive and active.

Passive control does not add energy to the flow and is often accomplished by geometric modifications of the nozzle trailing edge using tabs, chevrons, and lobed nozzles [5, 6, 7, 8]. Tabs generate pairs of streamwise vortices in the jet and provide a reduction in the far-field overall sound pressure level (OASPL). A draw back of tabs is that they protrude into the jet and thus provide a thrust loss. For this reason devices like chevrons, which have significantly less area blockage than tabs, provide a better alternative. Mixing of the jet core and bypass exhausts with the surrounding atmosphere produces high noise levels represented by a very broadband, haystack shaped sound frequency spectrum as seen in Figure 1.2. The curves show the poten-
tial reduction in noise levels of 5dB over a range of frequencies between 50Hz and 400Hz when chevrons have been placed on the inner and outer nozzles. Although it is true that low frequency noise travels much greater distances than high frequency noise, since it is the local environmental impacted by aviation noise, both high and low frequencies are important. In addition the shape of this spectrum reflects the fact that the eddies that comprise the turbulent mixing process vary considerably, increasing in size progressively downstream of the exhaust nozzle and decaying in intensity as the average velocity falls and the mixing becomes complete.

Active control adds energy to the flow. A few examples include either steady or unsteady (pulsed) fluidic injection through microjets [9, 10, 11] and fluidic chevrons [12]. Active control can be further divided into open loop and closed loop systems. In open loop systems, actuations take place based on a predetermined input. In a closed loop system, information from a sensor or sensors in the flow, along with a flow model, guides the actuation process [13].

As mentioned earlier, a jet exhaust consists of the fan stream and the core stream. The core flow stream is typically at a higher speed than the fan stream. As the two flow streams mix with each other, noise is created in the surrounding air. The most significant issue with attempting to reduce the jet exhaust noise is that it is created after the exhaust has left the engine. This means that physical measures cannot be applied to reduce the jet noise where it is actually created. Any reduction method must therefore work by removing the causes of the noise creation. This therefore requires extensive understanding of the noise source mechanisms and is discussed in the next section. Although the primary focus of this thesis is the development of a new experimental technique to aid steps to reduce jet noise it is important to start with a clear understanding of the current analytical and numerical methods used to predict the sources and development of jet noise.

1.4 Analytical Methods used for the Prediction of Jet Noise

In the 1950's Lighthill [14, 15], whilst working on identifying sources of sound in turbulent flow, observed that the exact equations of fluid motion can be recast in the form of an inhomogeneous wave equation, whose inhomogeneity comprises all the non-linearities of the Navier-Stokes equations. This formed an equation which described the freely propagating linear disturbances (an acoustic field) in terms of the characteristics of the source terms on the right hand-side. The resulting wave equation was shown by Lighthill to be:

$$\frac{\partial^2 \rho}{\partial t^2} - c^2 \frac{\partial^2 \rho}{\partial x_j \partial x_j} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

(1.1)
Introduction

where $\rho$ is the density, $c_o$ is the ambient speed of sound and $T_{ij}$ is the Lighthill stress tensor containing all non-linearities.

Expressing Equation 1.1 in terms of pressure allows a more physical interpretation relating the pressure fluctuation to the development of sound / noise. If it is assumed that $p - p_o = c_o^2(\rho - \rho_o)$, where $p_o$ and $\rho_o$ are pressure and density of the fluid in its equilibrium state, then Lighthill's wave equation can be expressed as:

$$\frac{1}{c_o^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x_j \partial x_j} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$  \hspace{1cm} (1.2)

where the Lighthill stress tensor on the right hand side is given by:

$$T_{ij} = \rho \nu_i \nu_j + (p - \rho c_o^2) \delta_{ij} - \tau_{ij}$$  \hspace{1cm} (1.3)

where $\nu_i$, $p$, $\tau_{ij}$ are the velocity, pressure and viscous stress tensors respectively and $\delta_{ij}$ is the Kronecker delta.

It is customary to neglect the effects of viscosity of the fluid (e.g. $\tau_{ij} = 0$) because it is generally accepted that the effect of the latter on noise generation are orders of magnitudes smaller than those due to the other terms in high Reynolds number flows. An in-depth discussion of this is provided by Lighthill[14].

It is important to note that the wave equation itself is not the acoustic analogy. The acoustic analogy comes when the right hand side is replaced by simple sources (e.g. dipole, quadrupole sources). In the case of all analogies there is an implicit linearisation about some base flow. The difference between this base-flow and the full compressible Navier-Stokes equations, is used to define the source, which is then considered to drive the base-flow system. Lighthill assumed propagation in a medium at rest, resulting in Equation 1.1 representing the acoustic (fluctuating) contribution not the underlying mean flow. The problem with this approach is that the source terms must be known in full (to result in the full Navier-Stokes solution), whilst the simpler the base-flow, the greater the complexity of the source term. It is therefore apparent that differences in the linearisation procedure results in differences within the definition of the source. This leads to questions over what is meant by the 'source' mechanism, and what can / should be included in the base flow.

Increased understanding of jet plume development through experimental testing has generated information about the mechanisms underlying the production of sound by turbulence. The jet was known to comprise nearly random turbulent fluctuations, correlated over some time and some
Introduction

spatial extent defined by the integral scales of the turbulence. The right hand side of Equation 1.1 involves the second spatial derivatives of the products of velocities resulting in the famous quadrupole source distribution of turbulent noise[14, 16] and so the sources of jet noise came to be understood as quadrupole elemental deformations associated with these correlated turbulent eddies.

Lighthill’s solution to the wave equation (Equation 1.1) assumed that the acoustic propagation takes place in a homogeneous medium at rest which means no solid bodies are present and that refraction of sound due to shear is not taken into account unless these effects are inherent in the source field. This means that any effect of inhomogeneity in the flow must be represented by the source field which is used to evaluate the source terms. Extentions of Lighthill’s analogy to incorporate the effects of solid surfaces in the flow were published by Curle[17] in 1955 and later by Ffowcs Williams and Hawkins[18] among others[19, 20]. Lilley[21] modified the wave equation whereby refractive ‘flow-acoustic’ interaction effects were effectively separated from the ‘production’ mechanisms and incorporated into a third order Pidmore-Brown wave operator. Lilley’s analogy changed our vision of the mechanism by which the free jet produces sound. The aeroacoustic system was now considered to comprise compact, convected sources, whose sound fields are modified by the sheared mean-flow into which they radiate. Massive amounts of work detailing changes and developments to the acoustic analogy, based around the linearisation and the empirical definition of the source terms, has been carried out, but is not covered here. For a detailed review of the developments the reader is referred to work by Goldstein[22] and Hubbard[23].

Lilley also derived an alternative form of Lighthill’s equation using momentum and energy conservation equations that explicitly allowed for enthalpy fluctuations (i.e. hot jets) as dipoles to be combined with Lighthill’s original quadrupole. These conclusions have recently been re-discussed in depth by Viswanathan[24]. For the purpose of the present work the discussion has been restricted to cold jets; for more information the reader is referred to work by Tanna et al[25], Tanna[26] and Viswanathan[24].

1.4.1 Current Source Mechanisms

Following Goldstein[27] an expression for the far-field acoustic intensity spectrum \(I_\omega\) can be obtained from Lighthill’s equation as follows:

\[
I_\omega(\vec{x}) = \frac{1}{32\pi^3\rho_0 c_0^2} \omega^4 \int \int \mathcal{R}(\vec{x}, \vec{\eta}, \omega) e^{-i\omega \frac{\eta}{c_0}} d^3\eta d^3y
\]

where \(\rho_0\) and \(c_0\) are the ambient density and speed of sound respectively.
Introduction

Using overbars to denote time averages, $R$ is given by:

$$R(\bar{x}, \bar{\eta}, \omega) = \int_{-\infty}^{+\infty} \mathcal{T}_{xx}(\bar{x}, t) \mathcal{T}_{xx}(\bar{x} + \bar{\eta}, t + \tau) e^{-i\omega \tau} d\tau = \int_{-\infty}^{+\infty} R(\bar{x}, \bar{\eta}, \tau) e^{i\omega \tau} d\tau \quad (1.5)$$

Performing a Fourier transform of the two-point correlation of the Lighthill stress tensor components in the direction of the far-field observer[28] gives:

$$R(\bar{x}, \bar{\eta}, \tau) = \mathcal{T}_{xx}(\bar{x}, t) \mathcal{T}_{xx}(\bar{x} + \bar{\eta}, t + \tau) \quad (1.6)$$

For an isothermal jet, as discussed by Morris and Farassat[29], it is reasonable to approximate the Lighthill stress tensor components in Equation 1.6 by the Reynolds stress, e.g:

$$\mathcal{T}_{xx} = \rho_s u_x u_x \quad (1.7)$$

where $\rho_s$ is the mean density in the source region (which in cold jets can be taken as equal to the ambient density $\rho_0$) and $u_x$ is the turbulent velocity fluctuation in the direction of the far-field observer. It is usual to assume that the two-point space-time correlation function takes the form:

$$R(\bar{x}, \bar{\eta}, \tau) = u^4 \hat{R}(\bar{x}, \bar{\eta}, \tau) \quad (1.8)$$

where $u$ is a velocity characteristic of the turbulence and $\hat{R}(y, \eta, \tau)$ is a normalised 'shape function'. The latter is often taken to have a form similar to that of the normalised velocity correlation as measured, for instance, by Fisher and Davies[30].

To increase further the understanding of the jet plume development and the noise production, experimentalists must therefore try to understand how to relate what is measured to something which can be meaningfully considered to describe the sound production mechanisms. A summary of studies which have tried to address this are detailed later. The following section outlines how the idea of base flow and source addition has been used in numerical predictions carried out over the past 20 years.

1.5 Numerical Methods for the Prediction of Jet Noise

The numerical method for describing the noise radiation from an aeroacoustic source or the sound generation and propagation by unsteady flows in an inhomogeneous flow field is called Computational Aeroacoustics (CAA). CAA as a process uses a form of numerical computation to
Introduction

produce acoustical information for aerodynamic phenomena. This includes all types of acoustical propagation techniques (Lighthill's acoustic analogy, the Kirchhoff method, Ffowcs William-Hawkins (FW-H) equations), as well as Linearised Euler approaches, and combined / Hybrid procedures with CFD.

Most of the CAA tools in use are hybrid types in which sound generation due to aerodynamics is more or less decoupled from the acoustic transport process to the far field. This decoupling allows for tailored algorithms to be used for the sound 'generation' method and sound 'transport' method.

Sound transport methods include both computational or analytical transport methods. The computational methods are similar to CFD computation in the sense they solve some partial differential equations in the entire field up to the observer but solve in the acoustic domain (solving equations such as Linearised Euler equation LEE, or the wave equation), while the analytical methods employ an integral form of the relevant acoustic propagation equations (Kirchhoff surface integral / FW-H equations), where the sound pressure at an observer at a specific point in time is computed by an integration of source terms along a surface. The sound transport methods are not of interest in this study and will not be discussed further. For more information about these methods the reader is referred to work by Wagner et al[31]

The area of most interest to this study is the sound generation methods in which the sound sources are identified in the aerodynamic active area. These generation methods include a CFD 'resolved' sources method and a semi-empirical 'reconstructed' sources method. The CFD resolved method uses a form of CFD (Direct Numerical Simulation - DNS, Large Eddy Simulation - LES, or Detached Eddy Simulation - DES) to model the turbulence and unsteady fluctuations in the aerodynamic domain. The generation of the sound source is therefore sensitive to dispersion and diffusion errors, which must therefore be keep low (usually at great computational expense). However, these demands are not as high as for direct methods because only the aerodynamic domain is solved for and then coupled via a transport method to the much larger far-field acoustic domain. The semi-empirical 'reconstructed' source methods use a straightforward RANS CFD prediction which provides information about the turbulent length and time scales that translate via empirical relations into sound source spectra. These spectra are then radiated by one of the transport methods. Of course this process depends heavily on the soundness of the empirical inputs and validation data used to calibrate them.

All these different techniques form a general map of noise prediction methods generated by Wagner et al[31] and shown in Figure 1.6 and modified to emphasise the part of interest to this project via the bold circles. Further discussion behind the difference between CAA and CFD and the use of CFD within CAA is given by Tam[32, 33] Lele[34] and Wells and Renaut[35].
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A particular challenge to CAA, due to its dependence upon CFD, is that CFD itself faces significant challenges with regards to spatial resolution for high Reynolds number (Re) flows and boundary condition sensitivity. For example, if the fluid mechanics are incorrect within the CFD results, the sources of the propagation are incorrect, and when incorporated into CAA calculations the farfield sound pressure levels will also be incorrect. In addition, integrated with the need to resolve the smallest turbulent scales with high accuracy is the need to, at the same time, give the largest acoustic scales room to propagate without numerical dispersion or dissipation errors. Identification of these issues has led to the two main computational approaches mentioned above. These are discussed further in the following subsections.

1.5.1 RANS-based noise prediction

One approach to the resolution of these difficulties has been to use a relatively fast running CFD code, such as a RANS scheme to generate input data for acoustic source and propagation models. Coupling an acoustic source model to a steady flow prediction is not new and was considered as long ago as 1977 by Balsa and Gliebe\cite{36} and Mani et al\cite{37}. Their scheme is generally referred to as the MGB method and has been extended by Khavaran\cite{38,39} (MGBK) to use a RANS solution based on a $k - \varepsilon$ turbulence model (where $k$ denotes the time-averaged kinetic-energy associated with the local fluctuating (turbulent) component of flow, and $\varepsilon$ denotes the time-averaged rate of viscous dissipation of this turbulent kinetic energy into internal energy).

Recently Tam and Auriault\cite{40} have also used a $k - \varepsilon$ turbulence model with a RANS solver to provide parameters for a semi-empirically based space-time correlation function of the fluctuation turbulent kinetic energy which, in contrast to the source term used in the MGBK code, is not based on Lighthill's analogy. Instead they postulate a relationship between the turbulence kinetic energy and fluctuating pressure. A transport model is then used to project the near-field source onto the far-field and the subsequent propagation of sound was described by solving the Linearised Euler Equations (LEE), with the anisotropy of turbulence being incorporated into the source model via an axisymmetric turbulence submodel. The Tam and Auriault scheme has achieved a higher agreement with measured data than other methods. Morris and Farassat\cite{29} argued, however, that this improved match was not due to any fundamental flaw in the standard Lighthill acoustic analogy, but because of the difference in prediction models used for statistical description of the turbulent noise sources. Of course this approach depends heavily on the soundness of the empirical inputs and validation data used to calibrate the form of the cross-correlation function (defined in section 1.4.1), which is central to obtaining an accurate prediction of the radiated noise spectrum. Morris and Farassat provided further proof of this, by showing that the model of Tam and Auriault and a model based on the acoustic analogy gave identical noise prediction given a consistent statistical description of the turbulence was used.
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The quick convergence time and simplicity of RANS schemes makes this form of noise prediction very appealing, and, although acceptable results have been produced, the fundamental weakness of the RANS approach up to now has been not so much the empirical modelling of unsteadiness but the extreme level of simplification in the way this was done (e.g. isotropic turbulence) with the (consequential) limitation that the RANS model based approaches are generally found only able to predict the noise at 90° to the jet axis.

1.5.2 DNS/LES based noise prediction

Another possible solution to source definition and increased accuracy in sound propagation has been to solve directly for the flow unsteadiness. This approach then generates the acoustic sources numerically directly rather than relying on semi-empirical assumptions of correlation functional form / shape. As mentioned in Section 1.5 DNS, given sufficient grid nodes, solves the full Navier-Stokes equations accurately. In recent years the increased demand for simulations of more industrially relevant high Reynolds number jets, where the nozzle geometry can be included, and which are currently beyond the capability of DNS has led to the development and introduction of LES. Within LES, the governing equations are low-pass filtered and only the largest scales of turbulence are captured by the simulation, resulting in the use of relatively coarser space-time grids. The effects of the small scales, called subgrid scales as their lengthscale is not resolvable on the grids, are modelled (see Chapter 2). LES is therefore able to predict unsteady turbulent fields, however, the subgrid scale model (SGS) must be able to include correct effects of the high-frequency content on the flow. For more information on the problem of evaluating and modelling the contribution of the unresolved scales to the radiated noise production the reader is referred to Seror et al[41], while a fuller description of LES is given in Chapter 2 below and by Sagaut [42] and Wagner et al[31].

The prediction of high-speed, high Reynolds number jet noise is one important problem that benefits from recent developments in LES. Early acoustic predictions by Choi et al[43] in 1999 were limited to very low frequencies and did not capture the peak frequency. Boersma and Lele[44] performed investigations into the suitability of LES within aeroacoustic applications, focusing on the subgrid scale modelling. They found that the compressible subgrid scale model of Moin et al[45] performed reasonably well in capturing the mean and RMS turbulent fields, but noted some dependence on nozzle exit boundary conditions. This dependence on boundary conditions has remained an important issue, even for highly spatially and temporally resolved simulations. The ability of LES to simulate the extremely thin nozzle exit turbulent boundary layer and to predict the transition to the thin initial free shear layer which itself becomes self-similar fairly quickly (and within which the source mechanisms occur) requires a prohibitively large number of mesh nodes. Calculations of a Mach 0.9 jet by Bogey et al[46, 47] and Bogey and...
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Bailly[48, 49, 50] using a rectangular grid solver and various subgrid scales closures, compared favourably to the available experimental turbulence data in the self-similar region of the jet plume (see Figure 1.5). The region at the end of the potential core was, however, identified by Bogey and Bailly[51] as an important, intermittent producer of sound, which agreed well with the experimental findings of Juvé et al[52], Guj et al[53] and Panda et al[54].

With regard to the source mechanism itself, the predicted two-point space-time correlations of Andersson et al[55] were validated using the experimental data of Jordan and Gervais[56]. Andersson et al stated that this provided solid evidence that the ability does exist for LES to simulate the flow fluctuations which generate the sound. This being said, the LES simulation predicted a potential core length of 5.45Dj in comparison to the experimental results of 6.50Dj and the LES assessment was only obtained using 2nd order correlations not the 4th order correlations needed for noise source modelling. In addition, the LES one point auto-correlation results $R(x,0,t+\tau)$ possessed a Gaussian shape, whereas exponential shapes are more appropriate to match experimentally obtained data(real flow). This suggests that although usable results were obtained, the high-frequency content was (unsurprisingly) being limited in the numerical simulations. A second issue remains of how robust the numerical models are to changes in nozzle design, and how many parameters have had to be tuned in order to achieve the good match to the experimental results. The fundamental question remains that if an experimental data set is required for every nozzle design in order to calibrate the numerical simulations then this requirement alone eradicates the need for such simulations.

It is also critical that more emphasis should be placed on the ability of the model to produce the 4th order correlations since it is these which are most important. It appears that, thus far, no 'direct' comparison of the ability of LES to predict this correlation accurately (for all components) has been carried out. In addition, there are very few direct measurements of the 4th order correlation terms, with most studies opting to use a quasi-Gaussian joint probability assumption to obtain these via a product of the 2nd order correlation terms (see Millionshchikov [57], and discussed in more detail in Chapter 4). This approximation has, however, never been proven as appropriate / accurate. Due to these issues, the necessity to capture the real flow physics through accurate measurements of the fluctuating field remains a valuable part of both noise generation understanding and also for use in validating computational simulations. Experimental techniques able to provide such measurements are discussed in the following section.
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1.6 Experimental Techniques Focussed on Sound Production Mechanisms

Correct analysis of flow behaviour is essential if we are to arrive at a complete understanding of the sound production problem and develop methods to find the needle (the source) in the haystack (the whole turbulent field). Goldstein [22] states that, from a fundamental point of view, the physics of the sound-generation process is actually quite simple to understand (pressure fluctuations must occur in the flow in order to balance the fluctuations in momentum and since the fluid medium is compressible, pressure will propagate away as sound). However, only a small fraction of these fluctuations actually radiate as sound, which makes the identification of the noise-generating structures very difficult. The ability of the experimental approach to identify the true ‘source’ dynamics has been limited, not by a lack of computing power like numerical simulation, but by the inability to measure the fluctuating pressure in the flow field and the limited instrumentation available. In this section an account is provided of previous work, including the developments in instrumentation which accompanied the advancement in our understanding, which in turn improved the statistical methods used to analyse the data.

Since the work of Lighthill in 1952 it is accepted that the sound power radiated from a jet has been directly connected to the fourth order, spatio-temporal velocity correlation tensor (see Section 1.4). By performing a Reynolds decomposition of the velocity field, this can be shown to comprise, second, third and fourth order terms, although the third order terms are generally neglected as they integrate to zero in homogeneous, isotropic turbulence. However, the turbulence in a round jet is neither homogeneous nor isotropic and this assumption leads to an over-simplification of the flowfield. Lighthill[14] and Ribner[16] introduced the so-called ‘shear’ and ‘self-noise’ mechanisms related to the linear (second order) and quadratic (fourth order) pressure production mechanisms.

Laurence[58], Davies et al[59], Fisher and Davies[30], Bradshaw et al[60] and Harper-Bourne[61] were the first to target their experimental work at capturing the source mechanism by directly measuring the two-point velocity correlations. These experiments were carried out within moderate speed jet airflow experiments and used Constant Temperature (Hotwire) Anemometry (CTA). As well as the problems caused by its physical presence within the flow the instrumentation was limited to resolving only one of the nine 2nd order components of the correlation tensor, usually chosen to be the axial component with axial and radial separations. These problems were compounded by then invoking incorrect assumptions of isotropy and homogeneity in order to model the other terms. With further developments in instrumentation Davies et al[59] included temporal measurements and found that the Lagrangian timescale of $< u'u' >$ within the shear
layer was approximated well by a fraction (roughly 0.2) of the local shear rate \((\frac{\partial u}{\partial y})^{-1}\). Along the lipline, the eddy convection velocity \((U_c)\) was measured as 0.65 times the local centreline velocity; this factor has later ranged between 0.58 and 0.65 in other studies.

With the introduction of non-intrusive point measurement techniques with high temporal resolution such as Laser Doppler Velocimetry (LDV) more detailed data on unsteady flow were published by Lau et al[62] and Lau[63]. These improvements have allowed the modelling focus to include more of the statistics of the flow dynamics. A recent development by Jordan and Gervais[64] combined an axisymmetric turbulence model with a technique developed by Devenport et al[65] in order to deal with the inhomogeneity of the jet structure, and a direction-dependent lengthscale was proposed in order to deal with the anisotropy of the turbulence. Jet noise predictions were then made using data obtained from two-point LDV measurements. Another recent development in the statistical modelling has been the use of frequency-dependent space-time scales[66, 67]. As mentioned earlier, traditionally a Gaussian form has been adopted for the shape of the temporal part of the two-point velocity correlation. Khavaran and Bridges[68] have shown how the shape of this temporal part can be fundamental to the accuracy of predictions, and that exponential forms are more appropriate than the traditionally used Gaussian form. Jordan et al[69] suggest that it is also important to model the curvature of the correlation function close to zero. They therefore proposed a function obtained via convolution of exponential and Gaussian forms, characterised respectively by the integral and Taylor scales of the flow. Again emphasis has been on the inclusion of physical flow quantities, rather than the use of empirical constants to get the right answer.

A Laser Induced Fluorescence (LIF) visualisation by Dimotakis et al[70] indicated the existence of both axisymmetric and helical structures and their transitional forms, which was in agreement with Tso and Hussain[71]. Yoda[72] however, disagreed with the large-scale helical structure and instead suggested a sinusoidal structure from her LIF experiments. The helical structure form was also doubted by Ninomiya[73], who extracted the organised structure by applying linear stochastic estimation to the velocity field, which was obtained by three-dimensional particle tracking velocimetry (PTV). Recent numerical studies suggests the existence of a group of hairpin-shaped vortices inclined downstream, which might explain the characteristics of the statistical properties reported in earlier research. With developments in experimental techniques it has been increasingly possible to capture the flow in detail. The latest development has led to a planar velocity measurement technique called Particle Imaging Velocimetry. This technique is in principle capable of resolving all 3 velocity components within a predetermined planar Field of View (FoV), allowing 2D visualisation of the turbulence as well as quantitative data within the near-field of the jet as produced by Bridges and Wernet[74] and Bridges and Brown[75]. The use of PIV is very attractive in modern aerodynamics because it can capture global characteristics
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of unsteady flow phenomena such as jet shear layer development. PIV enables spatially resolved measurements of the instantaneous flow velocity field within a very short time and allows the detection of large and small scale spatial structures in the velocity field. As previously mentioned, a large volume of high quality experimental data is required for validation of numerical simulations. For this, carefully designed experiments have to be performed for the experimental data to possess high resolution in time and space. The PIV technique is an appropriate experimental tool for this task, especially since it is the instantaneous velocity field information which is required. Recent PIV data collected at the NASA/Glenn Research Center AeroAcoustic Propulsion Laboratory (AAPL) for a series of turbulent subsonic round jets [74, 76] provides useful information to assist in turbulence model validation for jet noise prediction. In particular, mean and fluctuating velocity measurements have been collected over a diverse range of exit velocities and temperature ratios at locations up to twenty-five jet diameters downstream of the nozzle exit.

However, the application of PIV in large industrial facilities poses a number of special problems and limitations. Examples are the large observation areas, long distances between the observation area and the light source and the recording camera, high speed / high frequency information of interest, necessity of high fluid seeding (discussed further in Chapter 2), restricted time for the measurement, and high operational cost of the facility are some of the challenges faced. While some of these problems can be solved by developments in high zoom lenses, high power lasers, and larger budgets, many problems have as yet not been solved (e.g. still only relatively low frequency cameras available in relation to the frequencies of interest in airflow experiments, difficulties in fluid seeding given the high velocities). These problems have led to alternative approaches being considered such as application within water flows, where the technical problems are similar but usually much less severe than in air flows. The main advantage of this new technology is that it allows the modelling of jet noise to be based more on the physical flow dynamics rather than the use of empirical constants to get the right answer, thereby making the process more versatile to testing new nozzle shapes and configuration. More details on PIV will be explained in Chapter 2.

One final experimental approach is simultaneous flow-acoustic measurement, which allows determination of the causal relationship between the flow/source dynamics and the acoustic effects (the sound field) via an appropriate signal processing technique. One such technique was performed by Hileman et al[77], who implemented a methodology similar to Juvé et al[52] and Guj et al[53] using combined high-speed PIV data and farfield acoustic data. The farfield acoustic signal was used to sort the flow images into noisy and quiet ensembles, Proper Orthogonal Decomposition (POD) was then applied to both ensembles in order to understand the characteristic features comprised by the images associated with periods of high noise and periods of low noise.
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A detailed description of the work done within this area of causality is presented by Jordan and Gervais[78]. However, no further reference will be made to this technique as it is not relevant to this study and only mentioned briefly for completeness.

1.7 Project Objectives / Research Plan

This chapter has detailed the fundamental principles underpinning jet noise, and has emphasised that the fundamental turbulence property which acts as the source of propagating noise in shear layers is the 4th order two-point space-time velocity correlation. It has also been explained how these acoustic source terms are typically determined either from semi-empirical models or directly from numerical simulations which aim to predict the unsteadiness within the simulation (DNS / LES). Outputs from both of these methods are then typically fed into some form of transport model (LEE / Kirchhoff surface) to obtain the far-field acoustic pressure levels. Recent research has found that the combination of turbulence statistics deduced from LES predictions and an acoustic propagation method can be fairly successful in predicting far-field jet noise, although so far only in simple flows and with empirical constants and scalings. This raises the question of how accurately the 4th order correlations are predicted by LES, which has so far not received detailed attention. Therefore, the necessity of direct measurements of the 4th order correlation in representative experiments for use as validation data has been revealed.

Given the instrumentation developments explained in Section 1.6, the primary aim of this project was therefore to develop, quantify and validate a methodology for providing unsteady, spatially and temporally resolved velocity fields in jet shear layers by utilising the full potential of the PIV technique.

A detailed list of project objectives may thus be stated as:

- To select one or more suitable test cases, relevant to jet noise studies of aeroengine exhaust configurations, and to develop experimental nozzle test facilities, equipment and practices in order to produce numerical databases of turbulent flowfield measurements (including two-point space-time correlations) with the aid of PIV.

- To explore the possible use of water tunnel facilities and PIV instrumentation to capture flow field data representative of subsonic isothermal airflow jet turbulence.

- To ensure excellent spatial (which can usually be assured) and temporal (for which even with 'high-speed' PIV is still limited) resolution can be achieved and to consider the affect of these on the flow field statistics.
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- To define an optimal method by which the available PIV instrumentation may be used in stereoscopic configuration to provide the three velocity components necessary for all the 4th order correlations terms to be identified.

- To develop methods for analysis of both experimentally measured and numerical simulated (LES) data sets to provide any derived statistics that may be necessary.

- To evaluate the limitations of the technique via analysis of error identification and correction.

- To produce computational RANS and LES predictions from which to assess the accuracy and acceptability of LES to capture the correct turbulence statistics used within most acoustic analogies.

- To apply the newly developed PIV/LES methods to a more complex shear flow, such as coaxial jets.

- To increase knowledge into the fundamental turbulent sources of noise within unsteady shear layers.
1.8 Thesis Structure

The remainder of this thesis is dedicated to describing how the objectives outlined in the previous section have been addressed and completed.

Chapter 2 will detail the fundamental background behind the experimental approach that was used in this research - PIV - including image acquisition, processing, validation, and analysis and the computational approach adopted - LES - including governing equations, sub-grid scale models, grid generation and boundary conditions and some details of the particular code deployed in this study. This chapter will also detail the post-processing techniques adopted.

Chapter 3 will comprehensively discuss the experimental facilities, and the technical specification of the PIV system including justification and evaluation of the chosen experiment and a detailed technical specification of the instrumentation system, post-processing methodology, and an evaluation of the statistical accuracy of the PIV system. This chapter concludes with details of the process by which the computational simulations of the test geometries defined earlier in the chapter were defined.

Chapter 4 will present experimental results for two single round jet nozzle designs (LU40 and LU40P). This includes confirmation of the ability to produce a free jet plume development within an enclosed environment, the design of the nozzle and the ability of the PIV technique to capture accurately the spatial and temporal characteristics of the turbulence.

Chapter 5 will assess the RANS and LES predictions and their ability to capture mean velocity and turbulence statistics of a single round jet flowfield. The nature of an LES calculation allows the capture of time dependant turbulence statistics. This chapter will therefore also describe these unsteady characteristics in detail in relation to the accuracy of directly predicting $R_{ijkl}$.

Chapter 6 will detail the investigation into the turbulence statistics of the shear layers of a coaxial jet as well as the spatio-temporal correlations and associated eddy convection velocities. This chapter will include both experimental measurement and numerical simulation results.

Chapter 7 concludes the thesis with a summary of the major achievements of the present research and suggestions for future work.
Figure 1.1: Diagram showing ICAO stages and relative noise level of commercial jet aircraft. Image adopted from Huff [79]

Figure 1.2: Examples of baseline and noise reduction concept separate flow nozzles and noise spectra [3]
Figure 1.3: Breakdown of noise generating components during takeoff and approach [80]

Figure 1.4: Comparison of master spectra [24]
Far-field measurement point

Figure 1.5: Schematic of the structure of a single round turbulent jet and co-ordinate system used for calculations

Figure 1.6: Structure of CAA
Chapter 2

Experimental and Numerical Methods

The application of new experimental and CAA numerical modeling techniques to the problem of jet noise has been outlined in Chapter 1. This chapter now details the background behind the experimental approach used in the present research - PIV - covering image acquisition, processing, validation, and analysis in Section 2.1. Section 2.2 serves the same purpose for the computational approaches adopted - RANS and LES - covering governing equations, sub-grid scale models, grid generation and boundary conditions and some details of the particular code deployed in this study. Finally, Section 2.3 details the generalised post-processing tool ‘Xact’ [81], which provides an easy and efficient method of analysing multiple monoscopic 2D PIV data sets via Matlab 7.1 software. This section also outlines some further development to enable analysis of stereoscopic PIV data sets, implementation for analysis of numerical data sets, and the efficient extraction of two-point two-time correlations at 4th order level.

2.1 Particle Image Velocimetry

Particle Image Velocimetry (PIV) provides quantitative flow information with high spatial resolution through the acquisition of a series of instantaneous flow images which contain displacement information of seeding particles in the flow, which are converted into a vector field. A history of the development of PIV is not given here, but can be found in Hollis [82] and Grant [83, 84]. This section will only detail information about PIV relative to this study. If further general PIV information is required the reader is referred to Raffel et al[85].
Experimental and Numerical Methods

In this study a 2D planar PIV technique has been used as illustrated in Figure 2.1. The technique uses laser light which is first formed into a sheet and then fired through the flow being measured. The flow is seeded with an appropriate particulate material, which is illuminated by two short duration laser pulses (of the order 6-10ns to freeze the particle motion), while the pulse separation (which is a function of flow velocity) provides the two images from which the velocity vector can be calculated. The sheet plane is either viewed orthogonally by a single camera, in the case of monoscopic PIV (which allows the resolution of the two in-plane velocity components, 2C-PIV), or obliquely using two cameras, for stereoscopic PIV (which allows the resolution of all three velocity components at points with the 2D plane, 3C-PIV). Two images (best considered as an image pair or double image) are recorded. The Field of View (FoV) captured in the double image is sub-divided into a series of interrogation cells (1 cell is normally 32×32 pixels) where a cross-correlation is performed between the two images in order to determine the modal particle motion within the cell, and hence the velocity vector (the cell size sets the spatial resolution of the captured data). This double image capture is then repeated a number of times, dependent on the storage capabilities of the camera, to produce a series of instantaneous vector fields (this sets the temporal resolution of the captured data).

2.1.1 Image Acquisition

The acquisition of the double frame images (raw images) is the most critical stage in achieving high levels of accuracy. An example of the captured double frame image taken in a jet shear layer with a FoV of 60mm×60mm is shown in Figure 2.2. The acquisition stage involves camera and laser setup (which are unique to each study and depend upon the fluid used, geometry of interest, geometry restrictions and available methods of camera calibration), flow seeding, and laser illumination. Each of these areas will be discussed below.

Determining camera and laser locations can be very difficult depending on the experimental facility and model test geometry. Optical access may be restricted, requiring the use of mirrors. It is necessary to know the physical size of the image area seen by the camera (i.e. the FoV). This process is called 'Image Calibration' and can be achieved using physical model test geometry visible in the FoV, or by temporarily placing a calibration plate in the FoV. Image calibration may be problematic due to a lack of visible geometry or the difficulty inherent in using a calibration plate. The presence of visible geometry in the FoV may also introduce practical problems such as glare. A few important features to remember during the setup are that the camera lens and location need to be such that the FoV should cover the minimum area of interest to produce the desired spatial resolution. An excessively large FoV may produce results with insufficient particle size and spatial resolution. Previous investigations by Raffel et al[85] have shown suitable seeding can be obtained by the use of 20μm Polyamid particles in water, or atomised 2μm Shell Ondina
Experimental and Numerical Methods

oil in air. Given these particle sizes, Robinson[81] states that a field of view of approximately 150mm$^2$ in water and 80mm$^2$ in air are the upper limits of what should be attempted to avoid insufficient particle size with a $1024 \times 1024$ pixel FoV. Another requirement on FoV size, if spatial correlations / lengthscales are of interest, is that the FoV needs to be large enough to capture these adequately. This can create a conflict between accuracy and global size, and the balance between the two should be considered carefully. This is important in the application of PIV for the current study and will be discussed in Section 3.1.4.

Once the camera and laser locations have been determined, an accurate image calibration is essential to ensure that the later step of resolving true particle shift can be achieved. A FoV plane is ideally defined using 3 coordinate locations. Calibration plates or physical model geometry can be used to fix the area of interest. Calibration plates enable higher accuracy and confidence in oblique angle correction. The calibration plates used in this study have on average 100 marks per FoV; the LaVision software states a requirement of 40 – 100 marks on the complete image to determine calibration to sufficient accuracy[86]. The calibration plate must be located within the flow medium at the desired object distance (i.e located at a plane in the flow where velocity measurements are required). Translation of the calibrated camera together with the laser is possible as long as the object distance and its components within each different medium (fluid and test section materials) are kept constant. Any changes will result in defocusing of the image with recalibration being required. Single camera monoscopic PIV requires the camera to be perpendicular to the plane of measurement. A slight distortion at the rim of the recorded image is always present, since, when the camera angle is 90° in the middle of the image it may be only 89° at the rim for example, depending on the distance to the object plane and the magnification factor. The effect of this unintentional oblique viewing on the measured velocity is a function of the camera viewing angle and the magnitude of the out-of-plane velocity component. Correction for this is not possible on an instantaneous basis as the out-of-plane displacement remains unknown. However, can be applied to time-mean data at measurement locations at which approximations to both in-plane and out-of-plane displacements are known. The correction can be calculated as follows:

$$
\alpha = \tan^{-1}\left(\frac{\text{FoV}/2}{\text{Camera to Object Distance}}\right) \tag{2.1}
$$

$$
U_{i,\text{true}} = \frac{U_{i,\text{measured}}}{\cos \alpha} - U_k \tan \alpha \tag{2.2}
$$

where $U_{i,\text{true}}$ is the actual real-world in-plane velocity, $U_{i,\text{measured}}$ is the measured velocity which are contaminated by perspective error, $U_k$ is the out-of-plane velocity, and $\alpha$ is the camera viewing angle in the measurement plane of interest.
If distortion is present, or it is not possible to view perpendicular to the FoV, a calibration plate like that shown in Figure 2.3a, must be employed from which the PIV software should be able to reconstruct a corrected image. If it is necessary to view at a significant angle (< 80°) errors will be introduced due to pixel location and size distortion which are not correctable. If this is the case the use of stereoscopic PIV should be adopted, including a Scheimpflug lens, which will allow focus to be maintained over an inclined field. Normally, the lens and image (CCD Sensor) planes of a camera are parallel, and the plane of focus (PoF) is parallel to the lens and image planes. If the FoV is also parallel to the image plane, it can coincide with the PoF, and the entire subject can be rendered sharply. If the FoV plane is not parallel to the image plane, it will be in focus only along a line where it intersects the PoF, as illustrated in Figure 2.4a. When an oblique tangent is extended from the image plane, and another is extended from the lens plane, they meet at a line through which the PoF also passes, as illustrated in Figure 2.5. With this condition, a FoV that is not parallel to the image plane can be completely in focus as shown in Figure 2.4b. For details on the Scheimpflug principle see Merklinger [87]. Stereoscopic PIV resolves the out of plane motion, therefore calibration at two or more locations in the through-plane direction is necessary. This may be performed by translation of a single level calibration plate, although the implementation of a two-level calibration plate (see Figure 2.3b) removes any ambiguity in the translation of the plate. As such, a two-level calibration plate has been used for all stereoscopic datasets in this study.

In order for the FoV to be transformed between the camera co-ordinate system and the world co-ordinate system, two common mapping functions can be used. The first is a camera pinhole model, (Willert [88]) which uses the calibration image to calculate parameters that define the rotation, R, and translation, T, between the world co-ordinates, \( X_W \), and camera co-ordinates, \( X_C \), for a given focal length to 'pinhole' point, f, as shown in Figure 2.6, such that:

\[
X_C = RX_W + T
\]  

(2.3)

The second mapping function fits a 3rd order polynomial to the distortion in both directions in the plane of the image. The coefficients of the polynomial are determined by a non-linear least-squares fit to the calibration image.

The polynomial fit approach may be beneficial if the image contains strong distortions, which could stop the pinhole model converging. However, extrapolation of the polynomial fit is poor and requires calibration marks covering the whole image. The pinhole model has the added advantages that it can take into account the thickness of the calibration plate when using a set-up that requires the cameras to view from opposite sides, and can also take advantage of the self-calibration technique proposed by Wieneke[89]. The nature of the experimental facility used
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here (detailed in Chapter 3) produces very little distortion, reducing the need for a polynomial fit and therefore the higher accuracy of the pinhole model means this method has been used throughout this study.

The second critical area is the choice and delivery of seeding within the fluid. Seed particles must be small enough to follow the flow (in particular unsteady flow fluctuations) over a wide frequency range, but large enough to be effective at light scattering, and of sufficient density to provide an accurate calculation of the modal particle motion within each interrogation cell. Investigations by Raffel et al[85] have shown suitable seeding can be obtained by the use of 20μm Polyamid particles in water and therefore they will be used within this study. Raffel et al[85] also states that during processing to detect the particle motion accurate to 5% there should be a minimum of 5 particles per interrogation cell. This should give a particle image diameter of around the optimum 2 pixels for standard FoV's. This is important as for particle images less that 2 pixels the phenomena of peak locking can occur, whereby particle image displacements are biased towards integer values. This has been avoided throughout this current work. Delivery of the seeding to obtain a uniform distribution of seeding over the FoV is very important, especially if spatial (2-point) information is required as in this study. Delivery methods are specific to each experimental facility and experimental geometry. By using a water tunnel of recirculatory design (detailed in Section 3.1) complete seeding saturation was possible and produced uniform seeding distribution over the FoV.

The third critical area to consider is flow illumination. Laser illumination is used to give high density light that can easily be formed into a light-sheet. A divergent lens must be used to ensure all areas of the FoV are illuminated without wasting any laser power to surrounding areas. For 2C PIV the particle shift should ideally be in the in-plane directions only. Therefore the light sheet should be as thin as possible to eliminate out of plane motion error as identified in Figure 2.7. In the case where out of plane motion is large, the particle can be present in the first frame of the double frame image but be missing in the second frame, in other words the particle moves from within to outside the light sheet during the interframe time. To ensure the particles are present in both of the double frame images the thickness can be increased. However, it is important to remember the error in the resultant velocity is increased. Similarly, the interframe time can be reduced, however, this reduces the pixel shift of the particles between the double frame images and increases inaccuracies in peak detection (discussed below). It is important to avoid blurred images and this is one reason laser pulses are used. The pulses have a duration of only 6-10 ns and consequently freeze any particle motion for flow speeds of relevance here.

The final key critical area is the image acquisition sequence and timing for which a High Speed Controller (HSC) with highly accurate electronics is used. The HSC controls the sample time (the
time between the first frames of successive double frame images) and the inter-frame time (the
time between each frame of the double frame image). It is important to understand these variables
and their various effect on the final results. The sample time obviously determines the temporal
resolution. However, due to limited memory storage capacity (the current camera is limited to 8
Gbyte, which implies 3072 double frame images), the higher the temporal resolution the shorter
the overall sample period becomes (50Hz ≈ 60 secs, 1kHz ≈ 3 secs). The PIV equipment available
for use within this project had a maximum temporal resolution of 1kHz. The inter-frame time is
dependent upon the flow rate, optical setup and the post processing software used. LaVision’s
DaVis 7.2 software[86] suggests that the pulse separation ‘dt’ has to be adjusted such that the
particle image shift ‘ds’ is in the interval between the resolution of the system (Typically 0.1
Pixel) and the maximum allowable particle shift, approximately one quarter of the interrogation
window size (1/4 of 32 pixel interrogation window = 8 Pixels). However, as the calculated
displacement error increases with decrease in displacement, a maximum displacement of 8 pixels
was set.

2.1.2 Processing

In PIV the term processing refers to the conversion of raw images into vector fields. It is
these vector fields which are required for analysis. The following section explains the processing
methods implemented in this study.

As explained before, the image is discretised into interrogation cells in which the modal dis-
placement of the particles is calculated using the spatial correlation between the double image via
Fast Fourier Transform. This results in a correlation map with the peak intensity at the modal
displacement. A typical correlation map is shown in Figure 2.8. A Gaussian fitting scheme is
generally employed to determine particle location together with the assumption that all particles
within the interrogation cell move homogeneously. This of course is not always the case, (e.g.
in large interrogation cells or areas of high dynamic range) in which case dynamic averaging
will occur whereby the real particle motion is averaged. A correction for this is available and is
discussed further by LaVision[86].

Since the interrogation cell size is a changeable variable, a measure of the quality of the
correlation peak is the so-called Q-factor. This is defined as the ratio of the highest peak (the
assumed correct displacement) to the next highest (deemed to be noise). Thus for an intensity,
P, LaVision[86] defines:

\[ Q = \frac{P_1 - P_{\min}}{P_2 - P_{\min}} \]  

(2.4)
This quantifies the signal to noise ratio: A Q-ratio of greater than 2 indicates strong confidence that the vector is valid, whereas values close to 1 indicate that the vector is probably determined by instrument noise.

The size of the interrogation cell is governed by two criteria. The cell should in theory be small enough that the homogeneity assumption is valid (i.e. ideally of the order of the Kolmogorov length scale) and it should contain sufficient particles to obtain a valid correlation (typically at least 5 particles). Very commonly the cell size must be increased to satisfy the minimum number of particles criterion. For almost all practical situations a cell size of $32 \times 32$ pixels is sufficient. Vector grid density may be increased by overlapping the interrogation cells, hence parts of the image are used more than once. The standard overlap is 50% of cell size, thereby increasing the number of vectors by a factor of 4.

As outlined by Lecerf et al.[90], correlations performed on the image pairs of each camera in a 3C-PIV system provides the projected particle displacement for that camera. Knowledge of the relative calibrated positions of the cameras can then be used to extract the in-plane components (and through-plane component in stereoscopic PIV).

In a similar way to the loss of information and need for SGS modelling in LES CFD mentioned within Section 1.5.2, discretising a PIV image into interrogation cells where the cell sizes are in fact usually significantly larger than the Kolmogorov scale results in sub-cell filtering. The sub-cell filtering leads to individual particle displacements caused by eddies smaller than the cell size being ignored, producing possibly significant effects on the flow statistics. The effect of sub-cell filtering would be minimised if cell sizes close to the Kolmogorov scales could be used. However, due to other issues such as seeding size and density, cell sizes are usually increased, introducing sub-cell filtering. Quantification and correction for this effect is critical for an accurate representation of turbulent statistics, and one means of achieving this will be discussed in Section 2.3.3. In a similar way, the discrete temporal separation between frames of an image pair produces dynamic averaging, since any non-linear motion of the particle is averaged over the temporal separation as shown in Figure 2.9. The temporal separation ideally should again be less than the Kolmogorov time scale, although this criterion usually has to be relaxed due to the requirement to optimise the pixel shift.

When performing stereoscopic PIV and using the camera pinhole model for calibration (mentioned in Section 2.1.1) a self-calibration may be performed prior to calculation of the vector fields. The self-calibration calculates a disparity vector map on the real particle images by cross-correlation of the images from cameras 1 and 2 to determine if the calibration plate coincides with the light sheet. By triangulation of the disparity vector map, the true position of the light sheet in space is fixed and the mapping functions are corrected accordingly. It is shown by Wienke[89]
that it is possible to derive accurate mapping functions, even if the calibration plate is quite far from the light sheet, making the calibration procedure much easier.

A number of techniques exist, where one or all can be used, to enhance the quality of the calculated vector data, and the number of valid vectors One method is the Second Order Correlation technique. This technique calculates the correlation between frame A and frame B in each cell twice on slightly shifted interrogation windows. The two calculated correlation functions are then multiplied resulting in an increased correlation peak magnitude, and suppressed noise. A second method is the Multi-Pass technique. This technique uses an interrogation cell to calculate the velocity vector. This vector is then used in the next pass to shift subsequent cells, before the velocity vector is recalculated. This helps to minimise loss of in plane particle pairs (where the particle is present in the interrogation cell in the first frame, but has moved beyond the cells spatial limit by the second frame). This technique can be extended to become an Adaptive Multi-Pass technique. This technique initially uses larger interrogation cell sizes to calculate the velocity vector. This vector is then used to shift subsequent smaller cells, before the velocity vector is recalculated on the smaller cells. This method can be used to process smaller than normal cell sizes, thus enhancing spatial resolution, but one still has to consider the problems associated with low particle seeding density.

2.1.3 Validation

To ensure a high level of confidence in the results the quality of the vector map is usually assessed as part of the data processing step. This assessment of the vector map produces a quantification of its quality. A good validation procedure requires the identification of a set of criteria that a valid vector should meet, the subsequent removal from the results of spurious vectors, and the replacement of the data voids with a suitable scheme as necessary. This is an acceptable procedure providing the raw data is of the highest possible quality from the outset, however, should not be used to re-invent large quantities of idealised, smoothed data in any given frame of data.

There are a number of ways in which validation can take place. The most reliable method is to compare a given vector to its neighbours. This method was proposed first by Westerweel et al.\[91\], who stated that if a vector deviates substantially in direction or magnitude compared to its neighbouring vectors, flow continuity is not satisfied and the vector must be spurious (although this cannot be strictly true for a 3D vector field if captured in a 2D frame). A technique was proposed whereby at each vector location, the average magnitude and standard deviation of the surrounding velocities (usually 8 vectors) were calculated. The vector in question is then compared to the average of its neighbours, plus or minus a factor (defined by the user) of the
calculated standard deviation. If the vector in question falls outside the defined range the vector is deemed spurious and removed. The factor chosen which is applied to the standard deviation depends on the turbulence of the flow, but is typically 1.3.

The vector may also be evaluated via the Q-factor given in Equation 2.4. A threshold value may be set below which the vector is rejected. This approach is applicable for good quality data, but for data sets with high noise there is the possibility of removing genuine vectors. In such cases the threshold should be set fairly low, typically 1.3.

It is important that if data is removed from a vector field because it is deemed to be spurious, it is then replaced to maintain a continuous field. LaVision's[86] replacement technique and the one used throughout this study is for the replacement value to be determined by considering the next highest peak in the correlation map. The associated vector is checked against the validation criteria; if found also to be invalid, the process is repeated for the 3rd and 4th peaks. If the vector associated with the 4th peak does not match the criteria then linear interpolation is employed (any further peaks are deemed to be real noise). Linear interpolation calculates the average magnitude of the surrounding vectors and replaces the missing vector(s) with the appropriate value. This ensures flow continuity but the danger is that it can 'smooth' the flow field if too many spurious vector are removed. Any vectors replaced via interpolation are identified to the user.

Another method is to impart some pre-determined limits to the processing. This approach has two main options, geometry masking, and allowable velocity limits. Masking out certain regions where it is known no valid vectors exist, such as walls, eliminates the risk of neighbouring vectors being corrupted. Masking can be used with confidence and has the added benefit of reducing the computational time. The technique of applying pre-defined knowledge about the flow is undertaken by excluding any velocity vectors outside the pre-defined velocity limits and requires some previous knowledge of the flow. Caution should be taken when setting limitations on the flow as it is possible that real vector data will be excluded, perhaps due to a burst of energetic flow e.g. unexpected separation. Therefore the limits should only be set after other validation methods have been exhausted. This final method has been not used within the current work.

2.1.4 Analysis

Each instantaneous vector component field, \( u(\vec{r}, t) \), contains quantitative information regarding structures within the flow, while the whole series (potentially 3072 vector fields for the camera memory storage capacity used in the present work) provides information regarding spatial and temporal turbulence data. A distinct challenge is to be able to analyse and present the data in
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a compact and descriptive manner. As well as point-based first and second order statistics (as common with CTA/LDA instrumentation) PIV data has the ability to obtain numerous single and two-point statistics from the same FoV (velocity correlations; length/time scales) and to provide data for turbulent structure identification techniques (Proper Orthogonal Decomposition; Linear Stochastic Estimation). For this reason Section 2.3 later presents in detail the development of the analysis software used in this study.

2.2 Numerical Simulations

The numerical predictions in the present study use an existing CFD code called 'DELTA' developed within the Department of Aeronautical and Automotive Engineering at Loughborough University[92]. The DELTA code has been used to produce RANS predictions (mean velocity and time-averaged turbulence levels) and LES predictions (time dependant flow predictions from which time-dependant turbulence statistics can be extracted). Section 2.2.1 below details the mathematical formulation of the equations solved, while Sections 2.2.2 and 2.2.3 detail the turbulence (for RANS) and sub-grid scale (for LES) models, respectively. Section 2.2.4 provides a summary of the basic discretisation / numerical algorithms used for the RANS/LES modes of DELTA. Section 2.2.5 describes the geometry definition used, including grids, mesh files, and topology files. Finally, Section 2.2.6 describes the boundary conditions available and implemented for the chosen computational domains.

2.2.1 Governing Equations

The Navier-Stokes equations include the Conservation of Mass (often referred to as continuity), Conservation of Momentum (Newton's Second Law of Motion) and Conservation of Energy (1st law of thermodynamics) when applied to a fluid continuum. The Navier-Stokes equations may be obtained by using an infinitesimal or finite control volume approach and the governing equations can be expressed in integral or differential form. For a detailed discussion of the derivation of these equations the reader is referred to Anderson[93]. Incompressible isothermal forms (no energy equation required) of these equations are used and presented in this study since the experimental measurements presented are for incompressible constant fluid property (constant \(\rho, \mu, T\)) water flows. The conservative forms of the Navier-Stokes equations written in differential form for an incompressible constant property flow can be written in Cartesian tensor notation as:
Conservation of mass (continuity):

\[ \frac{\partial u_i}{\partial x_j} = 0 \]  \hspace{1cm} (2.5)

Conservation of momentum:

\[ \rho \frac{\partial u_i}{\partial t} + \rho \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \]  \hspace{1cm} (2.6)

where \( \tau_{ij} \) represents the viscous stresses, and \( p \) and \( u_i \) represent instantaneous pressure and velocity vector.

For a constant density Newtonian viscous fluid, the viscous stress tensor is given by:

\[ \tau_{ij} = 2\mu S_{ij} \]  \hspace{1cm} (2.7)

where \( \mu \) is the fluid molecular viscosity and \( S_{ij} \) is given by:

\[ S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]  \hspace{1cm} (2.8)

In terms of Equation 2.7, the momentum equation can be expressed as:

\[ \rho \frac{\partial u_i}{\partial t} + \rho \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] \]  \hspace{1cm} (2.9)

2.2.2 Reynolds-Averaged Navier-Stokes (RANS) CFD Approach and Turbulence Modelling

The Reynolds-averaged Navier-Stokes (RANS) version of the above equations are time-averaged equations. They are used when only statistics of the turbulence are chosen for modelling to reduce the computational effort rather than solving for (at least partly) the turbulence unsteadiness directly (e.g. with LES). In order to obtain the RANS equations the above instantaneous quantities are replaced by the sum of their mean and fluctuating parts (Reynolds Decomposition), e.g:

\[ u_i = \overline{u_i} + u'_i \]  \hspace{1cm} (2.10)
where

\[ \overline{u_t} = \lim_{T \to \infty} \frac{1}{T} \int_{t_0}^{t_0+T} u_t dt \]  

(2.11)

where the overbar denotes a time-averaged quantity.

The RANS equations for incompressible flow can be written (for statistically stationary flows where time-averaged properties do not vary with time) as:

\[ \frac{\partial \overline{u_i}}{\partial x_j} = 0 \]  

(2.12)

\[ \rho \frac{\partial \overline{u_i}}{\partial t} + \rho \frac{\partial}{\partial x_j} (\overline{u_i} \overline{u_j}) = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right) \right] - \rho \frac{\partial}{\partial x_j} \left( \overline{u_i} \overline{u_j} \right) \]  

(2.13)

The additional unknown correlation terms, the turbulent Reynolds stresses \( \overline{u_i u_j} \) must be modelled. In the current work turbulence closure is achieved using the \( k - \epsilon \) model, one of the most common eddy viscosity RANS turbulence models. Through Boussinesq's hypothesis, the turbulent stresses may be related to the strain rate introducing the Eddy Viscosity [94] as:

\[ -\rho \overline{u_i u_j} = 2 \mu_t \overline{S_{ij}} + \frac{2}{3} \rho k \delta_{ij} \]  

(2.14)

where \( \mu_t \) is the eddy or turbulent viscosity and where the time-averaged strain rate is given by:

\[ \overline{S_{ij}} = \frac{1}{2} \left( \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right) \]  

(2.15)

The momentum equation (Equation 2.13) can therefore be written as:

\[ \rho \frac{\partial \overline{u_i}}{\partial t} + \rho \frac{\partial}{\partial x_j} (\overline{u_i} \overline{u_j}) = -\frac{\partial \overline{p_t}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu_e \left( \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right) \right] \]  

(2.16)

where \( \mu_e = \mu + \mu_t \) and the second component of the turbulent stress in Equation 2.14 has been absorbed into a modified pressure:

\[ \overline{p^*} = \overline{p} + \frac{2}{3} \rho k \]  

(2.17)

In the \( k - \epsilon \) turbulence model, the turbulent viscosity is written as:
\[ \mu_t = \rho C_{\mu} \frac{k^2}{\epsilon} \]  

(2.18)

which requires two additional modelled transport equations to be solved for \( k \) and \( \epsilon \), which are (for high Re flow):

\[
\rho \frac{\partial (k)}{\partial t} + \rho \frac{\partial}{\partial x_j} (\overline{u_j} k) = \frac{\partial}{\partial x_j} \left( \left( \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right) + P_k - \rho \epsilon
\]  

(2.19)

\[
\rho \frac{\partial (\epsilon)}{\partial t} + \rho \frac{\partial}{\partial x_j} (\overline{u_j} \epsilon) = \frac{\partial}{\partial x_j} \left( \left( \frac{\mu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_j} \right) + \frac{\epsilon}{k} (C_{\epsilon 1} P_k - C_{\epsilon 2} \rho \epsilon)
\]  

(2.20)

where the production of turbulence kinetic energy \( (P_k) \) is given by:

\[
P_k = -\rho \overline{\omega_i} \frac{\partial \omega_i}{\partial x_j} \left( \frac{\partial \overline{u_i}}{\partial x_j} \right)
\]  

(2.21)

and \( C_{\mu}, C_{\epsilon 1}, C_{\epsilon 2}, \sigma_k \) and \( \sigma_\epsilon \) are empirical constants as defined by Launder and Spalding[95] and given in Table 2.1

<table>
<thead>
<tr>
<th>( C_{\mu} )</th>
<th>( C_{\epsilon 1} )</th>
<th>( C_{\epsilon 2} )</th>
<th>( \sigma_k )</th>
<th>( \sigma_\epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>1.44</td>
<td>1.92</td>
<td>1.0</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Table 2.1: \( k - \epsilon \) turbulence model coefficients

2.2.3 Large Eddy Simulation (LES) CFD Approach and Sub-Grid Scale (SGS) Modelling

High Re turbulent flows are always 3D, highly unsteady and contain a wide range of length and time scales. Large scale motions are generally flow-dependent and carry the majority of the fluctuating energy, whilst small-scale motions are more universal, tending towards isotropy. The philosophy behind LES is therefore to perform a spatial (not a temporal like a RANS) filtering of the instantaneous velocity field. ‘Filtering’ here means that any motions whose length scales are greater than a specified filter width will be numerically resolved, while any smaller than the filter width will be removed. This separates the instantaneous field into a ‘resolved’, large-scale part, and a ‘residual’, sub-grid scale (SGS) part. The resolved part will be computed directly using spatially filtered versions of the Navier-Stokes equations (2.5 and 2.9) and the residual part will be modelled using a suitable SGS model. The underlying premise of LES to be a superior CFD approach than RANS is that, whilst it will be considerably more expensive than RANS computationally, the SGS modelling of just a range of smaller scales not containing a lot of
fluctuating energy should be substantially easier than the RANS approach, where the turbulence models has to represent the correct ranges of all scales of motion present.

The general form of the spatial filter, as presented by Pope [94], can be written as:

\[
\tilde{u}_i(x', t) = \int G(\vec{r}', x') u_i(x' - \vec{r}', t) \, d\vec{r}' \tag{2.22}
\]

where \( \tilde{ } \) denotes a spatially filtered quantity; \( G(\vec{r}', x') \) the filter kernel, is a local function and has a length scale filter width \( \Delta \) associated with it. Eddies of size larger than \( \Delta \) are kept within the numerically resolved flow, whilst those smaller than \( \Delta \) are filtered out, and require modelling. The most commonly used filter function is the top hat filter:

\[
G(x) = \begin{cases} \frac{1}{\Delta} & \text{if } |x| \leq \frac{\Delta}{2} \\ 0 & \text{otherwise} \end{cases} \tag{2.23}
\]

As a consequence of the filtering, the velocity field is now defined by the decomposition:

\[
u_i = \tilde{u}_i + u'_i \tag{2.24}
\]

where the \( \tilde{ } \) denotes a spatially filtered quantity.

Although similar to a Reynolds decomposition, \( \tilde{u}_i \) is still a time-dependent field. The filtered forms of the Navier-Stokes equations are thus given as:

\[
\frac{\partial \tilde{u}_i}{\partial x_i} = 0 \tag{2.25}
\]

\[
\rho \frac{\partial \tilde{u}_i}{\partial t} + \rho \frac{\partial}{\partial x_j} (\tilde{u}_i \tilde{u}_j) = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \right] - \frac{\partial}{\partial x_j} (\rho \tilde{u}_i u_j - \rho \tilde{u}_i \tilde{u}_j) \tag{2.26}
\]

where \( (\rho \tilde{u}_i u_j - \rho \tilde{u}_i \tilde{u}_j) \) is the SGS stress tensor ASGS model is required for this term. In DELTA the standard Smagorinsky model is used:

\[
(\rho \tilde{u}_i u_j - \rho \tilde{u}_i \tilde{u}_j) = \mu_{sgs} \frac{\partial}{\partial x_j} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) + \frac{2}{3} \rho k_{sgs} \delta_{ij} \tag{2.27}
\]

where \( \mu_{sgs} \) is the SGS eddy viscosity, modelled as:

\[
\mu_{sgs} = \rho l_{sgs}^2 \bar{S} \tag{2.28}
\]
where $\tilde{S}$ is the characteristic filtered rate of strain given as:

$$\tilde{S} = \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}} \quad (2.29)$$

The Smagorinsky length scale $l_{sgs}$, is set proportional to a measure of the local grid spacing given by:

$$\Delta_{sgs} = (\Delta x \Delta y \Delta z)^{\frac{1}{3}} \quad (2.30)$$

where the constant of proportionality is the Smagorinsky coefficient $C_s$, which is typically between 0.1 → 0.2. In this study, the value 0.15 was used for $C_s$. The final form of the eddy viscosity is thus:

$$\mu_{sgs} = \rho (C_s \Delta_{sgs})^2 \tilde{S} \quad (2.31)$$

The standard Smagorinsky model eddy viscosity is modified by adding a Van Driest damping treatment to limit the length scale in the high aspect ratio cells in the near wall viscous affected region. The Van Driest corrected Smagorinsky constant as defined by Pope[94] is given as:

$$C_s^{corr} = C_s \left(1 - e^{-\frac{\gamma^+}{A^+}}\right) \quad (2.32)$$

where $\gamma^+$ is the distance from the wall in viscous wall units and $A^+$ is a constant (25).

2.2.4 DELTA Code Features

The DELTA code (for a detailed description see [92, 96]) implements a finite volume method (FVM) to discretise the governing equations. The FVM recasts the conservation form of the Partial Differential Equations (PDE's) into algebraic form. This guarantees the conservation of fluxes through a particular control volume.

$$\frac{\partial}{\partial t} \int Q \, dV + \int F \, dA = S \quad (2.33)$$

where $Q$ is the vector of conserved variables ($\rho, \rho u_i$), $F$ is the vector of fluxes, $S$ is the vector of source terms, $V$ is the cell volume, and $A$ is the cell surface area. DELTA uses a co-located flow variables arrangement, on a structured curvilinear mesh, in combination with Rhie-Chow smoothing, to avoid pressure-velocity decoupling. DELTA adopts the SIMPLE pressure correc-
tion method, designed to handle both incompressible and compressible flows.

In the pressure correction method, the code initialises a flow field and then solves the momentum equations to obtain intermediate velocity values. Because these will not necessarily satisfy the continuity equation, the guessed pressure and intermediate velocity are corrected using a pressure correction, dP. dP is calculated from a pressure correction equation, which is derived by a combination of the momentum and continuity equations. When the correct value for pressure and velocity have been obtained (i.e. simultaneous satisfaction of continuity and momentum (and for RANS-turbulence model) equations), the procedure is repeated for the next time step.

DELTA's spatial discretisation is implemented as a family of schemes ranging from central differencing through first order upwind to high order upwind, with a single controlling parameter. In the present study a second order upwind scheme has been used as a compromise between numerical robustness and acceptable dispersion and dissipation characteristics. Momentum equations are solved in a sequential manner with a spatially implicit scheme. Temporal advancement is by a first order backward Euler implicit method.

As in most LES codes, DELTA has a 'parallel' version which makes use of OpenMP or Message Passing Interface, MPI, libraries to enable parallel processing based on a domain decomposition approach (using the multi-block grid topology as detailed below).

### 2.2.5 Geometry Definition and Grid Generation

This section outlines the process by which a description of the solution domain geometry is converted into a computational grid and imported into DELTA. In structured grids used by DELTA, each cell volume is bounded by 6 faces, and each face defined by 4 corners. Cell volumes can be internal to the flow domain or be so-called 'halo-cells' used to implement boundary conditions or multiblock connectivity. At the domain boundaries, a single finite volume face will be aligned with the boundary. For identification of various cell locations, a convention is used whereby uppercase I,J,K indicates a cell centre, whilst lower case i,j,k indicates a corner or cell vortex. This is illustrated for two-dimensions in Figure 2.10a. Note that this means that the original (primary) grid that is defined by the grid generation process is used to define the co-ordinates (x,y,z) values) for the cell vertices, and cell centre co-ordinates are determined by interpolation. Complex geometries are handled by using the multiblock approach, where individual structured blocks are linked together in an unstructured manner. The internal size of each block is defined by the start and finish indices of the cell centres in each co-ordinate direction (see Figure 2.10b). DELTA does not incorporate grid generation or visualisation components,
instead communication interfaces within the DELTA code read 'foreign' grid files (e.g. generated using other software: Fluent, Plot3d, ICEM CFD Hexa). Throughout this thesis ICEM CFD Hexa version 11 has been used for mesh generation.

Grid generation starts with a 3D CAD representation of the flow geometry and its surrounding flow domain. An initial block is generated around the entire geometry which can then be subdivided into smaller blocks and adjusted to fit the detailed geometry (e.g. the nozzle). Hexa adopts a top-down approach whereby each time a block is modified all other connected blocks are also modified. Once a suitable blocking strategy has been established, the user can attach the computational geometry (block edges) to the physical geometry (CAD representation) through a process of association (assigning vertices, edges and faces of the blocks with the relevant parts of the physical geometry). Mesh generation can then be initiated via input of edge parameters (e.g. the number of nodes and their distribution along each block edge is defined).

Associated with the block and grid generation is a topology file which is required by DELTA to define linkages between blocks. Many codes that use a structured multiblock approach assume that linkages between blocks occurs at a complete face level. This type of topology is illustrated in Figure 2.11a. DELTA, however, incorporates a more flexible definition of linkages, where any sub-set of a single block face can be linked to any other block surface. This is illustrated in Figure 2.11b. This enables a drastic reduction in the number of blocks required to model a given configuration.

Splitting the domain into blocks enables each block (or a selected collection of blocks to improve load balancing) to be allocated to an individual processor on a multi-processor cluster. This is achieved using a message-passing interface called MPI, which is incorporated into the DELTA code. All computations presented in this thesis have been carried out on Loughborough University's 160-processor 64-bit Itanium LYNX cluster. LYNX consists of 20 compute nodes, each having four dual-core Itanium 1.6 GHz CPUs and 16GB of memory. The compute nodes are connected by a Quadrics network for message passing and data transfer. There is approximately 10TB of disc storage available.

The grid density used for a given flow solution must be selected with an eye on the trade-off between numerical accuracy and affordability. The radically different run-times of RANS and LES, and the different criteria governing accuracy means different approaches must be adopted for grid density selection. The particular ones selected are explained below in Chapter 3.
2.2.6 Boundary Conditions

Several boundary condition types are provided by the DELTA code. Four were used within this thesis and are explained in this section. The boundary conditions used were:

- Fixed velocity at flow inlets. This involves fixing a specific value to all velocity components (and also turbulent kinetic energy and dissipation rate for RANS solutions) at each grid node in the inlet plane. Since the state and shape of the boundary layer at the inlet to the nozzle are important in this study, experimental inlet profiles were interpolated onto the computational mesh based on wall normal distance using a cubic spline method to determine values at cell centres.
- Centreline boundaries were set using a symmetry boundary condition (only relevant to RANS) - zero velocity normal to body and zero gradient closure.
- Wall boundary conditions were implemented on the nozzle walls and the tunnel sidewalls. A viscous wall (no-slip) condition was implemented to enable boundary layer modelling and interaction, to nozzle exit flow conditions. This included use of a wall-function approach.
- Outlet boundary conditions use a zero gradient and convective outlet approach for RANS and LES respectively. The convective outlet is widely used in LES because it ensures the convection of the flow through the outlet plane, with a constant velocity defined by the bulk velocity at outlet $U_B$, without the generation of disturbance wave reflection\[97, 98\].

2.3 PIV and LES Post-Processing Tools

Currently, LaVision's DaVis 7 PIV software, as used in this study, only produces mean and RMS data, in addition to the raw vector files, for each data set. A Matlab program was therefore written by Robinson\[81\] ('Xact') to provide an easy and efficient method of more detailed analysis of multiple monographic 2D PIV data sets via Matlab 7.1 software. This program has been extended as part of the present study to enable analysis of stereoscopic PIV data sets and the efficient extraction of two-point two-time correlations at 4th order level. Similarly, to analyse 3D unsteady LES data in a compact and descriptive manner is a significant challenge. A typical LES solution can generate up to 1GB of information per time step, producing an LES time series (taken to be, for example, of equal length in time to a PIV dataset) of approximately 3.1TB ($3.1 \times 10^{12}$ bytes). To enable similar processing of the LES data to that carried out for the PIV data requires extraction of 2D planar information from the LES solution. DELTA was thus modified with the addition of a specific subroutine which extracted information from the time-varying predictions.
within the whole computational domain given user defined inputs (for example plane location and the size of the 2D plane of interest - smaller than the whole 2D plane covering the CFD solution domain, but large enough to evaluate required 2-point correlations). This information was then converted into an identical structured format to that produced from the PIV equipment using a specially written MATLAB-based filter code. This then enabled identical processing to be carried out for both LES and PIV data by the Xact code. The following section gives an overview of the Xact code; this is followed by a summary description of the single point and two point statistical analysis methods used throughout this study. A detailed review of Xact is provided by Robinson[99].

2.3.1 Program Overview

Xact is run by launching a master routine also called 'Xact'. This routine reads data files as required, either through an input file or via user commands which control the reading of a series of new files, or a previously read .mat file (this step is also used to input LES data for processing, via the converted DELTA files mentioned above). The master routine can accommodate requests for any or all of 6 analysis routines listed below by calling the functions associated with each specific method. These methods are:

- Calculate and plot the mean and RMS of the velocity components, meanRMS.m
- Calculate and plot velocity correlations and integral length/timescales, correl8.m.
- Calculate and plot the Reynolds stresses, rstress.m.
- Calculate and plot the velocity time history at a given point(s), pnthist.m.
- Calculate and plot the modes of a Proper Orthogonal Decomposition, POD.m.
- Calculate and plot properties of the Power Spectra, spectra.m.
- Create velocity field animations, anim8.m.

The user subsequently has the option to save calculated data to text files in a Tecplot-readable format. Figure 2.12 shows the graphical interface of the above functions.

2.3.2 Single Point Statistics

Complex turbulent flowfields contain a vast amount of information with spatial and temporal point information being correlated with other areas of the flowfield. Single point statistics are the first simple, but still informative, method of analysis.
Mean and RMS

Statistical quantities such as time average velocities, $U_i(x)$, and root mean square (RMS) fluctuating velocity data, $u_i^{rms}(x)$, are calculable via the Reynolds decomposition of the instantaneous velocity field, $u_i$:

$$u_i(x, t) = U_i(x) + u'_i(x, t) \quad (2.34)$$

Since each data set contains a FoV spatial range, $x$, and a temporal range made up of a finite number of samples, $N_{samp}$, at discrete time intervals, the mean velocity is calculated from:

$$U_i(x) = \frac{1}{N_{samp}} \sum_{n=1}^{N_{samp}} u_i(x, t_n) \quad (2.35)$$

Likewise, the RMS velocity, $u_i^{rms}$, is calculated from:

$$u_i^{rms}(x) = \sqrt{\frac{1}{N_{samp}} \sum_{n=1}^{N_{samp}} u_i^2(x, t_n)} \quad (2.36)$$

while the turbulent kinetic energy (TKE), $k$, is defined as half the sum of the square of the RMS values:

$$k(x) = \frac{1}{2} (u_i^{rms}(x))^2 \quad (2.37)$$

Such point based information is usually the start-point for comparison between CFD predictions and experimental data, and the ability to visualise and quantify numerous points at the same time means this method of PIV data presentation is widely used. Examination of the fluctuating fields is traditionally used as a preliminary method by which turbulent eddies may be identified. This relatively quick method can be very informative for exploratory visualisation of a flowfield. By calculating the TKE, a first attempt at identifying the significant energy containing structures can be made. It is important to note that sub-cell filtering (mentioned in Section 2.1.2) can affect both the RMS and the TKE due to the unresolved turbulence scales. This could result in an under-estimation of the true turbulence energy levels. Correction of these properties is not possible via single point analysis but can be undertaken once correlation information is available, and this will be discussed in Section 2.3.3.
Reynolds Stresses

Monoscopic PIV allows for the two in-plane Reynolds normal stresses \((i=j=1,2)\) and one shear stress \((i\neq j)\) to be obtained. With the inclusion of the out of plane velocity component (stereoscopic PIV) all six Reynolds stresses are obtainable, allowing also the turbulence anisotropy to be estimated. Individual Reynolds stresses are obtained by taking the product of any two fluctuating velocities.

\[
\overline{u'_i(x)u'_j(x)} = \frac{1}{N_{samp}} \sum_{n=1}^{N_{samp}} u'_i(x,t_n)u'_j(x,t_n)
\]  

(2.38)

Point Histories

Through the use of point histories, the temporal evaluation at a given point can be visualised and assessed for periodic content. Identification of the periodic nature of the flow enables the number of independent samples can be estimated. This gives a sense of how statistically converged are the mean values.

It is possible to extend the point history signal analysis to identify the grouping of fluctuations via calculation of a Probability Density Function (PDF). The velocity range is divided into a series of 'bins', \(\bar{a}\). The PDF is the probability that a fluctuation in the time series is greater than a lower limit, \(\bar{a}_1 = \bar{a} - \delta a\), and less than an upper limit, \(\bar{a}_2 = \bar{a} + \delta a\) surrounding a particular bin \(\bar{a}\):

\[
P(\bar{a}) = P(\{u_i > \bar{a}_1\} \cap \{u_i < \bar{a}_2\})
\]  

(2.39)

Thus, for a given discrete signal, the PDF is calculated from:

\[
P(\bar{a}) = \lim_{(\bar{a}_2-\bar{a}_1)\to 0} \frac{1}{(\bar{a}_2 - \bar{a}_1)} \left[ \lim_{N_{samp} \to \infty} \frac{N_{\bar{a}}}{N_{samp}} \right]
\]  

(2.40)

where \(N_{\bar{a}}\) denotes the number of samples in the bin \(\bar{a}\). For a finite number of samples as in a PIV data set the PDF can be adequately represented by setting the bin width to one twentieth of the observed velocity range (i.e. \((\bar{a}_2 - \bar{a}_1) = 0.05 \times (u_{max} - u_{min})\)).
2.3.3 Two Point Statistics

PIV data, because of its planar and time-sampled nature, has the ability to capture the variation of a velocity in space or time at a selected point in the FoV, with respect to the variation in space or time at another point within the same FoV. The spatio-temporal nature of turbulence can provide valuable insight into the make-up of the fluctuations in terms of the temporal and spatial lengthscales that are present. This interpretation gives rise to the 'eddy' description of turbulence. Goldstein[22] has clearly stated that, when considering the acoustic properties of turbulent flows, knowing the space-time behaviour is crucial since the ability of a turbulent motion to generate sound is directly related to its space-time correlation (detailed earlier in Chapter 1).

A major objective of the present project was, therefore, to measure directly the inter-dependencies in the unsteady velocities within the jet shear layer, in particular the fourth order spatio-temporal velocity correlations commonly associated with acoustic analogies as discussed in Section 1.4.1. Analysis of all the 81 possible components of the correlations would allow for identification of the dominant spatial and temporal coherence. At the commencement of this study it was anticipated that shear layer turbulence had several strong dominant coherent structures related to sound generation. The expectation was that, once identified in both temporal and spatial domains, it would then be easier to judge the effectiveness of sound reduction technologies by examining the extent to which these modified the all-important 4th order correlations. As described in Chapter 1, current analytical models either generate shape functions for the $R_{1111}(\vec{x}, \eta, \tau)$ correlation based on assumptions of homogeneity and isotropy, or attempt to use DNS to fully resolve the whole flowfield. The possibility therefore exists that if accurate prediction of the far field sound was achievable by modelling, for example, the top 10 dominant correlations, increased accuracy would be achievable whilst generating a large time saving compared to DNS modelling methods[100].

Correlations

A correlation is defined as a mutual relationship between events separated in space and/or time, thereby quantifying flow coupling effects and characterising the nature of the turbulence. The definitions for the (normalised) 2nd and 4th order spatio-temporal cross correlations, $R$, between two points, separated by the vector, $\vec{\eta}$, in space and by $\tau$ in time, are as follows:

Let A, B and C identify particular spatial / temporal co-ordinate pairs:

\[ A = (\vec{x}, t) \quad B = (\vec{x} + \vec{\eta}, t) \quad C = (\vec{x} + \vec{\eta}, t + \tau) \] (2.41)
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Then the non-dimensional, normalised 2\textsuperscript{nd} order correlation is given as:

\[ R_{ij} (\bar{x}, \bar{\eta}, \tau) = \frac{u_i' (A) u_j' (C)}{u_i' (A)^2 u_j' (B)^2} \] (2.42)

and the non-dimensional, normalised 4\textsuperscript{th} order correlation is given as:

\[ R_{ijkl} (\bar{x}, \bar{\eta}, \tau) = \frac{\frac{u_i' (A) u_j' (A) u_k' (C) u_l' (C)}{u_i' (A)^4 u_j' (B)^4} - \left(\frac{u_i' (A) u_j' (A)}{u_i' (A)^2}\right)\left(\frac{u_k' (B) u_l' (B)}{u_k' (B)^2}\right)}{\left(\frac{u_i' (A)^4}{u_i' (A)^4}\right)^{\frac{1}{4}} \left(\frac{u_j' (B)^4}{u_j' (B)^4}\right)^{\frac{1}{4}} \left(\frac{u_k' (B)^4}{u_k' (B)^4}\right)^{\frac{1}{4}} - \frac{u_i' (A)^2 u_j' (A)^2 u_k' (B)^2 u_l' (B)^2}{u_i' (A)^4 u_j' (B)^4} \frac{u_i' (A)^4}{u_i' (A)^4} \frac{u_j' (B)^4}{u_j' (B)^4} \frac{u_k' (B)^4}{u_k' (B)^4} \frac{u_l' (B)^4}{u_l' (B)^4}} \] (2.43)

These particular definitions are adopted so that the peak value of the correlation (at zero time/space separations) is unity, and to ensure the correlation decays to zero as the separation increases to infinity. Simpler correlations of interest may be obtained by selecting specific values of \( \bar{\eta} \) and \( \tau \). For example, the auto-correlation function is obtained by setting \( \bar{\eta} = 0 \) and the spatial correlation function is obtained by setting \( \tau = 0 \). Traditionally, the calculated correlation coefficients are restricted to an Eulerian frame basis; i.e. using fixed frame of reference, time varying quantities. Although spatial velocity correlations are performed relatively easily, general spatio-temporal correlations are difficult and time consuming to perform with single point anemometers. The multipoint nature of PIV enables the spatial velocity correlations to be calculated, whilst also enabling a Lagrangian approach to be used which identifies a fluid particle (or group of particles) and follows these in time. This is known as a Moving Frame autocorrelation or Moving Frame spatial velocity correlation. To achieve a Lagrangian approach the speed of convection \( U_c \) of the fluid element being tracked must be calculated. Plotting equi-correlation levels of (the Eulerian) \( R_{ij} \) with respect to \( \eta \) and \( \tau \), as illustrated in Figure 2.13, enables the convection velocity to be identified. The angle \( \phi \) of the ellipses to the \( \eta \)-axis yields the convection speed:

\[ U_c = \frac{1}{\tan(\pi - \phi)} \] (2.44)

The projection of \( R_{ij} \) onto the \( \tau = 0 \) (spatial correlation) and \( \eta = 0 \) (autocorrelation) axes correspond to the Eulerian approach, while projection of \( R_{ij} \) along the line of convection onto the \( \tau \) and \( \eta \) axes corresponds to the Lagrangian approach.
Lagrangian and Eulerian Integral Lengthscales

A lengthscale, by definition, gives quantitative information about the size of the turbulent structures present in the flow. Turbulence is characterised by motions over a wide range of scales. The energy cascade, introduced by Richardson[101] and described in detail by Pope[94], explains how turbulent energy is generated in the larger scales and transferred through progressively smaller scales until the smallest scales present (Kolmogorov microscale), where the viscosity acts to dissipate the kinetic energy. Eulerian Integral lengthscales may be extracted from the spatial velocity correlation, e.g at 2nd order level:

\[ kL_{ij}(\vec{x}) = \int_{0}^{\infty} R_{ij}(\vec{x}, \eta, 0) \, d\eta \]  (2.45)

where \( k \) indicates the component of the separation vector along which the integration is carried out. For example, the streamwise integral scale is evaluated as \( ^1L_{11} \) and the transverse integral scale as \( ^2L_{11} \). Integral lengthscales are only considered for correlations where \( i = j \) and due to the planar nature of PIV \( k = 1, 2 \). Therefore four or six lengthscales may be defined from monoscopic and stereoscopic PIV respectively.

Calculation of the integral lengthscale is carried out for limits from \( 0 \rightarrow \infty \) according to the ideal definition. For finite FoV's it is not possible to integrate to infinity and hence it is standard practice to integrate up to the first zero crossing of the separation axis, as in Fleury et al[102]. It has been shown by Hollis[82] that this is generally a very close approximation to the true fully-integrated lengthscale. For accuracy it is wise to ensure that the evaluation point \( (\vec{x}) \) is sufficiently far from the edge of the FoV that the correlation decreases to zero well within the FoV in the direction of integration. Hollis[82] also showed that data can adequately be approximated by an exponential form (Equation 2.46 below) if the distribution is curtailed and never reaches the zero crossing point within the FoV. Figure 2.14 illustrates the fully captured lateral (blue line) correlations distribution, while the longitudinal (black line) correlation distribution requires the additional exponential model (red line) to close the curtailed distribution.

\[ R_{ij} = \left( \frac{R_{ij,\text{curtailed}}}{e^{-\Delta \zeta_{\text{curtailed}}}} \right) e^{-\Delta \xi} \]  (2.46)

where the 'curtailed' subscript denotes the values at the curtailment location, i.e. the last values in the spatial velocity correlation distribution.

When the flow is assumed to be statistical stationary, properties such as the mean, the variance, autocovariance, autocorrelation, etc do not depend on \( t \). The autocorrelation is the correlation...
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coefficient between the process at times \( t \) and \( t + \tau \) and will always produce a symmetrical distribution about zero-time delay, whereas this is not always the case for the spatial correlation. As a result, analysis for both positive and negative values of the separation distance of the integral is usually necessary. The identification of a difference between the positive and negative sides can indicate strong inhomogeneity (e.g. obstacles in the flow) causing the eddy to undergo significant change while passing through that region. It is, however, possible that the curtailment problem mentioned above may affect the positive and negative separation directions differently. In order to achieve an accurate lengthscale evaluation, a confidence-weighted average of the positive and negative separation sides contribution to the integral has been implemented, as developed by Hollis[82]. The approach is based on a 'confidence level' related to the amount of real (calculated) spatial correlation data available in the integral from the respective parts of the separation axis, and takes the form:

\[
\begin{align*}
C &= 0 \\
C &= 1.125 - (1.25 \times R_{ij} (\bar{x}, 0, \tau)_{\text{curtailed}}) \\
C &= 1
\end{align*}
\]

where \( C \) denotes the confidence coefficient. This means that if the function is curtailed at anything greater than a value of \( R_{ij} (\bar{x}, \eta, 0) = 0.9 \), no confidence can be attributed to the integral for that separation direction because it relies too heavily on the estimated curve. If the function is curtailed at less than \( R_{ij} (\bar{x}, \eta, 0) = 0.1 \), entire confidence can be placed on the integral, because the estimated portion is so small. Between the two values a linear relationship between confidence and the curtailed value is assumed. The actual lengthscale is calculated from a confidence-weighted average:

\[
k_{L_{ij}} = \frac{(C_- k_{L_{ij},-} + C_+ k_{L_{ij},+})}{C_- + C_+}
\]

Table 2.2: Confidence coefficient

where the + and - subscripts refer to the positive and negative separation contributions to the lengthscale integral. As mentioned earlier, physically correct reasons can underpin an unsymmetrical spatial correlation distribution, therefore the user must be aware of this when interpreting the confidence-weighted averaged lengthscale.

The Lagrangian approach provides a moving frame autocorrelation distribution (Figure 2.13). The Lagrangian lengthscale is obtained by tracking the distance travelled by the turbulent structure (motion of the peak correlation at the convection speed) until the autocorrelation value is equal to a predetermined value. This Lagrangian lengthscale, \( k_{L_{ij}} \), provides information about
the distance a turbulent structure identified at one location interacts with points in other regions. To ensure accuracy of this lengthscale the domain requires information from a large number of points in order to resolve the location at which the Lagrangian autocorrelation reaches the defined value, typically $\frac{1}{\epsilon}$. Due to the potential asymmetrical spatial correlation, the same confidence weighting averaged method as mentioned above is implemented within the Lagrangian approach. In addition, the same exponential form as used in the Eulerian approach can be included to estimate missing data.

**Sub-cell Correction**

Extracting an accurate length scales (and also an accurate RMS or $k$ value), even given the considerations taken above, is often hampered by the spatially discretised nature of PIV. Discretising the image into interrogation cells can result in sub-cell filtering, as explained in Section 2.1.2. Eddies smaller than the cell size are essentially filtered out. These unresolved scales must be considered if accurate turbulence statistics are to be acquired. The discretisation in Section 2.1.2 has indicated that correcting for sub-cell filtering requires knowledge of the 'true' integral lengthscale, so a correction to the raw measured lengthscale extracted from PIV is also needed. Saarenrinne et al[103] have highlighted how this problem is analogous to unresolved LES scales, where the velocity field is filtered according to a filter function associated with the cell size, $G(r)$, integrated across the interrogation cell area, $D$, such that:

$$U(x,t) = \int_D G(r,x)U(x-r,t)dr$$ (2.48)

A detailed description of sub-cell filtering within PIV and suggested techniques to account for its effects on derived turbulence statistics is given in Hollis[82]. The major approach presented (and further developed and validated) within Hollis[82] follows the original proposal of Hoest-Madsen and Nielsen[104] (the HMN method), who gave a theoretical examination of the problem of sub cell filtering in PIV, presenting a method by which the effect of filtering on the RMS velocity could be related to the local integral length scale and cell size assuming homogenous isotropic turbulence. Such simple turbulence does not exist in high shear flows[105] but many high Re flows approach nearly isotropic turbulence as the eddy size gets smaller. HMN state that the assumption of homogeneity and isotropy need only be true inside individual cells. The analysis approach adopted within Xact is the Hollis extended version of the HMN sub-cell filtering correction and is based on empirically established correlation function shapes rather than the theoretical HMN curves. This empirical curve, shown in Figure 2.15, describes the relationship between true and measured RMS values.
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The ratio of the measured RMS, $u_{meas}^{rms}$, to the true RMS, $u_{true}^{rms}$ is approximated by the following exponential relationship (see [104] and [82] for more details):

$$\frac{u_{meas}^{rms}}{u_{true}^{rms}} = \begin{cases} 
\frac{e^{-0.3235(\Delta X/L_{true})}}{-0.2181\ln(\Delta X/L_{true}) + 0.7501} & \text{if } (\Delta X/L_{true}) < 1.0 \\
0.2181\ln(\Delta X/L_{true}) + 0.7501 & \text{if } (\Delta X/L_{true}) \geq 1.0 
\end{cases} (2.49)$$

where $\Delta X$ is the interrogation cell size.

As this correction requires the true local integral lengthscale, Hollis[82] developed another empirical curve (Figure 2.16) whereby the true lengthscale, $L_{true}$, and its ratio to the measured value is assumed to be given by the following equation:

$$\frac{L_{true}}{L_{meas}} = \begin{cases} 
e^{-0.5141(\Delta X/L_{true})} & \text{if } (\Delta X/L_{true}) < 0.65 \\
0.2300\ln(\Delta X/L_{true}) + 0.6230 & \text{if } (\Delta X/L_{true}) \geq 0.65 
\end{cases} (2.50)$$

Equations 2.49 and 2.50 must be solved iteratively (given input values of the measured $u_{meas}^{rms}$ and $L_{meas}$ extracted at any point in the FoV from measured PIV data) to provide a correction to the RMS and lengthscale calculated at each PIV interrogation cell location. Initial studies[82] which compared PIV and LDA data (which does not suffer from sub-cell filtering errors) suggested that the HMN and Hollis method as extended to include the correction based on the integral lengthscale (Equation 2.50) provided a reliable means of correcting raw PIV turbulence statistics for the effect of sub-cell filtering.

Lagrangian and Eulerian Integral Timescales

Timescales provide a measure of the lifetime of turbulent structures in the flow. The Eulerian autocorrelation is obtained when $\tau = 0$ and its shape can be of particular use in identifying periodicity within the flow, as well providing a timescale estimate. The Eulerian integral timescale is defined as:

$$T_{ij} (\bar{X}) = \int_0^\infty R_{ij} (\bar{X}, 0, \tau) d\tau (2.51)$$

The autocorrelation is a one-sided integral about zero-time delay. This eliminates the necessity for any confidence-weighted averaging. As previously mentioned, only correlations where $i=j$ are considered for timescale calculations and as all integrations are taken along the time axis there is no ‘third spatial dimension’ as appears in the lengthscale definition. Two timescales are obtainable from monoscopic PIV while three are obtainable from stereoscopic PIV. It should be noted that the temporal resolution of the measurement equipment must be sufficient such that
the autocorrelation decay is well resolved to ensure accuracy in the resultant timescale. However, due to an unavoidably finite size sample data set, the precise characteristics in time of a flowfield can have a significant effect on statistical convergence.

The Lagrangian autocorrelation, due to its moving frame nature, can display differences between the positive and negative parts of the distribution (similar to the spatial correlation distribution, due to the eddy undergoing significant change while passing through that region). The Lagrangian timescale does not have an integral definition, but instead is defined by the time taken for the correlation (in the moving frame) to drop to a predefined value, typically \( \frac{1}{e} \). This is analogous to the Lagrangian lengthscale and its relationship can thus be defined by:

\[
\frac{k T_{ij}^L}{k L_{ij}^L} = \frac{U_c}{U_c}
\]

(2.52)

Spectral Information

By considering the energy spectrum function \( E(k) \), it is possible to determine how the turbulent kinetic energy is distributed among eddies of different sizes. Lynn[106] shows that the energy spectrum (or Power Spectral Density) may be obtained from a Fourier transform of the Eulerian autocorrelation function such that:

\[
E_{ij}(x, \omega) = 2 \int_0^\infty R_{ij}(x, 0, \tau) \cos(\omega \tau) \, d\tau
\]

(2.53)

The Power Spectral Density (PSD) quantifies the distribution of energy in the frequency domain at a given point in space. If \( i=j \) \( E_{ij} \) represents the PSD of the \( i^{th} \) velocity component. If \( i \neq j \) then the result is a coherence function. If the spatial correlation function is used the result is known as a cross-spectral density.

Noisy and poorly resolved data sets due to low sample rates can produce noisy spectra making interpretation difficult. Within this study (as implemented into Xact by Robinson[81]) spectra have been calculated as the mean of the spectra of the 9 PIV cell points surrounding a given location. Spectra may also be calculated as the mean of multiple spectra obtained by down-sampling of a point signal (i.e. reducing the sample rate by considering only every \( N \) samples). This however requires a large time sample and as such is best suited to higher temporal resolution measurement techniques (such as CTA or LDV) than PIV.
2.3.4 Turbulent Structure Identification

Locally coherent motions (eddies) are usually associated in flow visualisation with local vortex motions but preclude precise definition. However, a description of a vortex that is generally accepted is that of Kline and Robertson[107] who state that: “A vortex exists when instantaneous streamlines mapped onto a plane normal to the core exhibit a roughly circular or spiral pattern”. Identifying the coherent structures which are present in the flows studies here are also of significant interest. Three methods of turbulent structure identification are described below.

Conditional Averaging

To visualise particular turbulent structures that have contributed to a given correlation map the technique of conditional averaging may be adopted and has been incorporated into the Xact program. Conditional averaging calculates the ensemble average of those velocities at a point \((x,y)\) in the data field which has been conditioned using the fact that the velocity at a point \((x_0, y_0)\), is a factor (K) greater than the RMS at \((x,y)\). This highlights the fluctuations that lie in the ‘tails’ of the PDF distribution at the point \((x,y)\), as shown in Figure 2.17.

Mathematically, the conditionally averaged fluctuations for a given velocity component from the positive, \(\langle u_{ca+} \rangle\), and negative, \(\langle u_{ca-} \rangle\), regions of the PDF may be described as:

\[
\langle U_{ca+}(x) \rangle = \frac{1}{N_{ca+}} \sum_{k=1}^{N_{buf}} a_{i+}(t_k) u(x, t_k) \tag{2.54}
\]

where \(a_{i+} = \frac{\nu(x_0, y_0, t_k) < K u'(x_0, y_0)}{1 u(x_0, y_0, t_k) \geq K u'(x_0, y_0)}\) and \(N_{ca+} = \sum_{k=1}^{N_{buf}} a_{i+}\)

\[
\langle U_{ca-}(x) \rangle = \frac{1}{N_{ca-}} \sum_{k=1}^{N_{buf}} a_{i-}(t_k) u(x, t_k) \tag{2.55}
\]

where \(a_{i-} = \frac{\nu(x_0, y_0, t_k) > K u'(x_0, y_0)}{1 u(x_0, y_0, t_k) \leq K u'(x_0, y_0)}\) and \(N_{ca-} = \sum_{k=1}^{N_{buf}} a_{i-}\)

It was found that using an ensemble average of both components of the velocity field (i.e \(u_i\) and \(u_j\)), conditionally averaged by the velocity component and location of the particular correlation in question, gave a good visualisation of the large scale structures contributing to that correlation. Hence, when correlations are plotted for \(ij = 11\) or \(22\), and \(Nt = 0\), conditional average vectors may be overlaid on the contours of \(R_{ij}(x, \bar{y}, z)\).
Proper Orthogonal Decomposition

Proper Orthogonal Decomposition (POD) is a technique which can be used to identify the modally decomposed motions which, on average, contain the most energy, and is a well-known technique for determining a basis for the reconstruction of a data set (Karhunen[108], Loeve[109]). It is also known as Principle Component Analysis or Karhunen-Loeve Decomposition. It is important to understand how the method can be applied in order to separate spatial and temporal characteristics of the modes and, most importantly, how this allows for the representation of the most energetic mode shapes pertaining to a set of fluctuating velocity data.

POD extracts a series of time-independent spatial basis functions, \( \varphi_k(x) \) (or POD modes), and the associated time-dependent uncorrelated temporal coefficients, \( a_k(t_i) \) from a velocity fluctuation field, such that the fluctuation field may then be reconstructed via:

\[
u'(x,t_i) = \Sigma_{k=1}^{N_{\text{samp}}} a_k(t_i) \varphi^k(x), \quad i = 1,2,...,N_{\text{samp}} \quad (2.56)
\]

where \( k \) denotes the mode number. The \( a_k \) coefficients and \( \varphi_k \) basis functions are identified by an optimal filtering to the energy in the fluctuation field. \( u'(x,t_i) \) is, for example, a single velocity component fluctuation and can be views as the column vectors of the matrix \( A \), shown below:

\[
A = \begin{bmatrix}
u'_{x1,t1} & \cdots & \nu'_{x1,tN_{\text{samp}}} \\
\vdots & \ddots & \vdots \\
u'_{xm,t1} & \cdots & \nu'_{xm,tN_{\text{samp}}}
\end{bmatrix} \quad (2.57)
\]

Thus, the spatial points at which information is available sets the row structure and the \( N_{\text{samp}} \) temporal instants sets the column structure. For a single PIV measured velocity component in the current data \( A \) is a \( (64\times64 =) \) 4096 row \( \times \) 3072 column matrix. The spatial correlation matrix, \( R \), between all points in the field is obtained from:

\[
R = A.A^T \quad (2.58)
\]

Sirovich[110] shows that the optimally energetic spatial modes \( \varphi_k(x) \) result from the eigenvectors of \( R \). It was shown by Chatterje[111] that these may be obtained from the Matlab function for Singular Value Decomposition (SVD) giving:

\[
R = U\Sigma\Phi^T \quad (2.59)
\]
where $U$ is an $M \times M$ matrix, $\Sigma$ is an $M \times N_{\text{amp}}$ matrix, and $\Phi$ is an $N_{\text{amp}} \times N_{\text{amp}}$ matrix with all zero elements except on the leading diagonal, where the elements are the singular values arranged in descending order, see Figure 2.18. The columns of $\Phi$ are the eigenvectors. Hence, by taking the $k^{\text{th}}$ column of $\varphi$ across the field gives the $k^{\text{th}}$ spatial mode. Unfortunately the $R$ matrix is of size $M \times M$ and calculating all $M$ POD modes would be computationally very expensive. Therefore, the ‘snap-shot’ method of Sirovich[110] is adopted as an efficient method where the first $N_{\text{amp}}$ eigenvectors can be calculated by converting the $M \times M$ matrix to an $N_{\text{amp}} \times N_{\text{amp}}$ matrix. Rather than considering the correlations, the matrix $C$ is generated such that:

$$C = (A^T A)$$

Performing an SVD on $C$ now produces the coefficients $\kappa$ as the eigenvectors. Linear combination of the $k^{\text{th}}$ column of eigenvectors with the fluctuation matrix then yields the $k^{\text{th}}$ spatial POD mode:

$$\varphi_k(x) = \sum_{i=1}^{N_{\text{amp}}} \kappa_k(t_i) A(x, t_i)$$

Now $N_{\text{amp}}$ rather than $m$ POD modes are obtained. Although $N_{\text{amp}}$ modes may be calculated, it has been found by Robinson[81] that around 500 independent samples are sufficient to give convergence of lower order modes. It is necessary not just to determine the mode itself but also to identify the amount of energy contained within each mode. In Equation 2.59 the singular values on the diagonal of the $\Sigma$ matrix are the square root of the eigenvalues, $\lambda$, of the $A$ matrix. It is these eigenvalues which are representative of the energy contained within each mode. The cumulative proportion of energy contained within each mode, $E_k$, can thus be defined as:

$$E_k = \frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{N_{\text{amp}}} \lambda_i}$$

The cumulative distribution of the eigenvalues gives a representation of the turbulent kinetic energy contained across all POD modes. The amount of total energy contained within the first POD mode is often a good measure of whether the flow is dominated by any distinct periodic structures or whether their energy is spread over a broadband range of structures. This should also be identifiable from the energy spectrum.

Whereas a full reconstruction using Equation 2.56 returns the instantaneous velocity field, a reconstruction using less than $N_{\text{amp}}$ modes gives a new velocity field containing only contributions from those modes used in the reconstruction, $N_{\text{modes}}$. In order to obtain a reconstruction
from Equation 2.56, rearrangement is required to give the temporal coefficients, $a_k$. However, since the 'snap-shot' method is used to calculate $\varphi_k$, the matrix is no longer square, and thus cannot be directly inverted. Therefore $a_k$ may be obtained from:

$$a = u \varphi^T (\varphi \varphi^T)^{-1}$$  \hspace{1cm} (2.63)

and may subsequently be applied to Equation 2.64 in order to reconstruct the instantaneous velocity field.

$$u'(x, t_i) = \sum_{k=1}^{N_{\text{modes}}} a_k (t_i) \varphi_k (x) \hspace{1cm} i = 1, 2, ..., N_{\text{modes}}$$  \hspace{1cm} (2.64)

From these reconstructed velocity fields, statistics can be recalculated using just a certain proportion of the total energy. This can allow evaluation of how a change in energy content within the flowfield affects certain statistics.
2.4 Closure

This chapter has detailed the background to the experimental and numerical methods utilised during this study. The philosophy behind the selection and implementation of the LaVision high-speed PIV system was detailed, including best practices for image acquisition, processing, validation, and analysis. The background to the RANS and LES CFD approaches has also been detailed including the equation features of the DELTA code used for all numerical calculations in this study.

The processing methods available to both PIV and LES unsteady time series datasets with the in-house postprocessing tool ‘Xact’ has been discussed and ensures confidence in the statistical results from both methods. The extension of Xact to enable analysis of stereoscopic PIV together with the efficient extraction of two-point two-time correlations at 4\textsuperscript{th} order level has also been detailed.

The design, operation and modifications to the Loughborough University Water Tunnel Test Facility used during the measurement of jet plume and shear layer development in the current work, together with the implementation of the PIV technique and optimisation of the various parameters will be assessed in the next chapter. The next chapter will also detail the computational costs and impact, and the resulting computational domains and CFD data sampling procedures.
Experimental and Numerical Methods

Figure 2.1: PIV system arrangement

Figure 2.2: Example of two frames used for velocity vectors computation [112]

Figure 2.3: Types of calibration plate

(a) Single level calibration plate

(b) Two level calibration plate
Figure 2.4: Stereoscopic FoV focusing

Figure 2.5: Scheimpflug arrangement for stereoscopic PIV
Figure 2.6: Camera pinhole model [86]

Figure 2.7: Inherent error using monoscopic PIV
Correlation Peak

Noise

dx
dy

Figure 2.8: Typical correlation map (LaVision[86])

Low Dyn Averaging
(low \( \Delta t \) gives small \( ds \))

Moderate Dyn Averaging
(med \( \Delta t \) gives med \( ds \))

High Dyn Averaging
(high \( \Delta t \) gives high \( ds \))

Real Particle Trajectory

Recorded Particle Trajectory

Figure 2.9: Illustration of dynamic averaging (for an arbitrary flow velocity)[82]
Experimental and Numerical Methods

(a) Finite-volume cell nomenclature
(b) Domain bounds notation

Figure 2.10: Cell identification and notation

(a) Typical structured multiblock
(b) DELTA complex multiblock

Figure 2.11: Multiblock approach

Welcome to Xact
An input file has been detected.
You may either read the input file or select operations manually:

- Read new data files into Matlab
- Calculate mean and RMS values
- Calculate correlations and length/time scales
- Calculate Reynolds stresses
- Calculate point time histories and PDF's
- Perform Proper Orthogonal Decomposition
- Calculate Power Spectra
- Create velocity field animations

Use input file

Go to options

Help

Figure 2.12: Graphical interface of Xact user options
Figure 2.13: Illustration of spatio-temporal correlation with notation

Figure 2.14: Illustration of lengthscale distributions for $R_{11}$ including exponential form
Figure 2.15: Comparison of synthetic data with HMN curve[82]

Figure 2.16: Trend for integral lengthscales calculated from synthetic data[82]
Figure 2.17: Conditional averaging of a PDF distribution

\[ R = U \Sigma \Phi^T \]

Figure 2.18: Graphical depiction of SVD of a matrix R
Chapter 3

Experimental Facilities and Numerical Modelling Details

Most of the previous experimental studies of jet and shear layer turbulence related to far field jet noise have been conducted in air flow facilities, some at high speed \(63, 74, 76, 102\) (e.g. Mach No > 0.4, containing compressible flow effects) and some at low speed \(61, 63, 113\) (Mach No < 0.4 - essentially incompressible flow). Since it is the spatial and temporal resolution that is of prime interest in experiments aimed at capturing the turbulent statistics of the noise source, the range of dynamically important turbulent frequencies in air flow studies presents numerous difficulties, which can be avoided if water is used as the working fluid. Chatellier et al\([113]\) state that, in water flow experiments, velocities of the order 1m/s can be dynamically resolved using PIV up to frequencies of order 1kHz. In order for a jet with bulk velocities of order 1m/s in a nozzle of reasonable laboratory scale to still maintain a sufficiently high Reynolds number involves increasing the fluid viscosity above that of air. A water flow experiment was therefore selected for this study as it has the advantage of achieving a high Reynolds number at reasonable scale and low velocity, thereby creating lower characteristic frequencies and, most importantly, bringing the frequency range to be resolved within the range achievable at reasonable cost with currently available PIV systems.

In this chapter the experimental facilities, and the technical specification of the PIV system used are presented. Section 3.1 provides a justification and evaluation of the chosen experiment, including an assessment to ensure acceptable jet flow characteristics, a detailed explanation of nozzle sizing considerations, a description of the chosen nozzle configurations, and an explanation of the chosen measurement locations and associated test matrix.
PIV is a complex measurement technique, with many user adjustable parameters which must be carefully selected to ensure optimal results are achieved, as detailed in Chapter 2. Section 3.2 therefore provides a technical specification of the instrumentation system, the postprocessing methodology, and an evaluation of the statistical accuracy of the PIV system for the selected measurement locations and test matrix conditions.

This chapter then concludes with details of the process by which computational simulations of the test geometries were conducted. Section 3.3 covers the definition of the computational grid, the choices and modifications to facilitate use of in-house postprocessing software (Xact), the inlet conditions, and finally the computational costs and their impact on the grid design and CFD data sampling procedures.

3.1 Experimental Facility

A schematic of the water tunnel is shown in Figure 3.1. The rig is of re-circulating design and is very adaptable to varying configurations. The re-circulating design results in complete seeding saturation of the 20\(\mu\)m polyamid particle seeding, which is of great importance to the accuracy of the PIV process as mentioned in Section 2.1.1. The test section is 2010mm long, 375mm wide and 300mm high. Perspex sides enable non-intrusive measuring instrumentation to be used to monitor the flow from two orthogonal directions. A slow flow through the test section co-flowing with the jet discharge from the jet unit is provided by a primary pump which can deliver a maximum of 24 litres per second. The reasons for including this co-flow stream are given below.

After leaving the reservoir tank the water passes along a 4 inch diameter plastic duct, through control valves and flow meters, and into an inlet settling chamber. From here the water passes through a turbulence management system, illustrated in Figure 3.2, in order to remove the turbulence and straighten the flow. Finally the flow passes through a contraction stage (with horizontal and vertical contraction ratios of 2.5:1 and 2:1 respectively) before reaching the test section. Good uniformity of the co-flow delivered to the test section is invaluable when trying to measure and analyse the mean flow and turbulence generated by additional flow sources introduced into the test section. The facility has previously been successfully applied to the study of coaxial jets[114, 115], impinging jets in a crossflow[116, 117], and simulated combustion chamber flow[81]). Evaluation of a 'clean' (i.e additional flow geometry free) test section was therefore performed first to ensure acceptable co-flow characteristics for use within this study; details of this are given in the following section.
Experimental Facilities and Numerical Modelling Details

3.1.1 Water Tunnel Flow Evaluation

Evaluation of the 'clean' (no jet) flow through the test section ensures acceptable baseline flow characteristics in the experimental facility and allows a high level of confidence to be derived from the turbulence measurements generated from the additional single and coaxial jet flow sources to be introduced during the present study. As part of initial rig evaluation and commissioning, the methods previously used for alignment and operation of the facility instrumentation were assessed and found to require improvement. High resolution alignment of the instrumentation and accurate and repeatable traversing equipment is essential for the present study since there is no geometry downstream of the nozzle exit (visible in the PIV FoV) to fix accurately the measurement locations. To allow proper alignment and repeatable positioning of measurement planes downstream of the nozzle exit at precisely the same locations required the implementation of a Dantec Dynamics 3 axis linear traverse with a resolution of 6.25μm, and for this to be orientated parallel to the nozzle centreline to within ±0.2mm. The traverse used is able to accommodate a variety of instrumentation including two cameras and a laser as required for stereoscopic PIV. In the experience of the author the traverse allows a FoV accuracy to within ±0.25mm to be achievable at locations up to 1.5m downstream of any calibration plate or physical geometry. Figure 3.3 illustrates the relative positions of the water tunnel cross-section, traverse, PIV system set up, and nozzle exit location.

In order to assess the flow characteristics within the 'clean' test section, monoscopic PIV was used to measure the axial and radial velocities. Chapter 2 has outlined how the accuracy of the PIV process is sensitive to the experimental setup, flow field and vector computation parameters. However, even after the optimisation of these parameters, any PIV system will still give rise to errors that largely arise due to the discretisation from a continuous field to digitised samples in time and space. Section 2.1.1 has outlined how error due to sub-pixel particle displacement accuracy defines an unavoidable low level error limit to any data captured. In general for the cross-correlation procedure used in the image processing the particle shift (ds pixels) should be larger than the accuracy of the peak detection (0.1 pixels) and smaller than a quarter of the selected interrogation window size (Δx) to ensure accurate results as shown in:

\[ 0.1 < ds < \frac{1}{4} \Delta x \text{ (in pixels)} \]  

(3.1)

Throughout this study the interframe time has been set such that the maximum expected velocity in the selected FoV produces a particle shift at the upper limit of the optimal particle shift (Equation 3.1). Given a typical interrogation cell size of 32×32 pixels, and hence an upper particle shift, ds, limit of 8 pixels, an uncertainty of at least ±1.25% is present in each vector
Experimental Facilities and Numerical Modelling Details

Computation (i.e. $\frac{41}{8}$). Given the maximum velocity in the 'clean' water tunnel test section is approximately 0.18 m/s it follows there will be a constant uncertainty of $\pm0.0023$m/s. Figures 3.4a-b present profiles of the measured axial and vertical mean velocities. The co-ordinate frame used here has the x-direction horizontal and orientated in the tunnel main flow direction, $z$ is horizontal spanwise across the tunnel (increasing right to left when looking against the tunnel flow) and $y$ is measured vertically across the tunnel (increasing from bottom to top). These profiles are shown at three different heights ($y$). Both the axial and vertical mean velocity show spatial variations of approximately 0.002 m/s about an average velocity of 0.178 m/s and 0 m/s respectively. As stated above, anything lower than 0.0023 m/s produces a particle shift smaller than the accuracy of the peak detection and hence can only be considered as noise. The nominal vertical motion within the tunnel (fluctuations about 0 m/s) indicates good flow characteristics and gives the test section a resultant mean velocity accurate to at least 1.25%. Comparisons to a newly constructed research water tunnel at R. H. Research Co.[118] which displayed a uniform flow with only 2% of mean velocity variation indicates that the water tunnel used in this research is comparable in its flow characteristics.

3.1.2 Nozzle Confinement Considerations

Confined flow experimental facilities for testing jets, which claim to be representative of a 'free jet in infinite surroundings' flow field, often undergo detailed investigations to identify and quantify any effects induced by the test section confinement on the development of the jet flow. A confined jet can develop in a manner which differs considerable from a free jet. Turbulent entrainment causes the jet to increase its mass flux while spreading as it develops downstream of nozzle exit. With confinement, it is essential to ensure this process is not influenced by the side walls of the tunnel. The mixing of the jet and 'ambient' flow (i.e. the fluid surrounding the jet inside the water tunnel) sets up a pressure rise in the downstream tunnel direction. Adverse pressure gradients are therefore established which, in turn, can affect the evolution of the flow. For example, the adverse pressure gradient can create a reverse flow opposing the jet direction near the tunnel walls. Much further downstream, the flow completely loses its jet-like characteristics and would develop eventually into a duct flow regime if the duct is long enough. Depending on the ratio of jet to ambient (co-flow) velocities and also nozzle to tunnel size, two different flow regimes occur. If both ratios are small, the jet does not entrain all the co-flow and does not spread to reach the tunnel walls (before exiting the test-section), so that the outer co-flow remains unseparated. Under some circumstances, the opposite happens and the entrainment of the jet causes reverse flow (recirculation) within the test-section outer regions[119]. These flow features, involving two interacting flows of differing velocities and different sizes can be found in many engineering applications such as in combustion chambers and ejectors. The work of Craya and Curtet [120] and Curtet [121] produced an empirical method to assess the strength of
confinement effects and more specifically to predict the onset of recirculation. Craya and Curtet [120] proposed a dimensionless parameter called the Craya-Curtet number $C_t$ which is a function of the integrals of mass and momentum fluxes across the inlet plane of the confined jet/ambient flows. Equation 3.2 shows the definition of the Craya-Curtet number:

$$C_t = \frac{U_m}{\left[(U_j^2 - U_a^2)(\frac{D_o}{D_j})^2 + \frac{(U_j^2 - U_a^2)}{2}\right]^{\frac{1}{2}}}$$  \hspace{1cm} (3.2)$$

where $D_o$ is the diameter of the confinement (assumed of circular cross-section in [120]), $D_j$ is the diameter of the jet, $U_j$ and $U_a$ are jet and co-flow velocities and $U_m$ is the fully mixed mean velocity defined as:

$$U_m = (U_j - U_a)(\frac{D_j}{D_o})^2 + U_a$$  \hspace{1cm} (3.3)$$

Experimental studies by Becker et al [122] have shown that the flow and mixing characteristics of an isothermal confined jet are unique functions of $C_t$. Curtet and others [121, 123, 124] have shown that, for the fully turbulent case, if $C_t < 0.75$ recirculation will occur downstream of the jet exit near the duct wall. It has also been stated by Kang [123] that "the critical value of $C_t$ for which recirculation appears ranges from 0.75 to 0.976". The conclusion is therefore that recirculation can occur to some degree or another when $C_t \leq 0.98$, and this restraint must be considered when deciding on nozzle / jet conditions for the present experiment.

In addition to confinement issues, nozzle sizing is also limited by the camera resolution and FoV sizing constraints (particle sizes, number of particles per interrogation cell, particle shift, dynamic and spatial filtering) which must be considered relative to the temporal and spatial turbulent scales present within the jet plume. Figure 3.5 illustrates the evolution of the integral lengthscales in the jet shear layer captured in previous studies as presented by Fleury et al [102]. To ensure a 95% accuracy in the PIV measured turbulence energy (TKE) (see Figure 2.15) the relationship between the PIV interrogation cell size and the integral lengthscale should correspond to an interrogation cell size smaller than 10% of the integral scale. Based on the results illustrated in Figure 3.5 a range of acceptable FoV sizes for axial and radial scales at various axial locations can be estimated using Equation 3.4, based on an interrogation cell size of 32x32 pixels (discussed further in Section 3.2):

$$\frac{FoV}{D_j} = \Delta x \times 0.1 \frac{L}{D_j} \hspace{1cm} \text{(where } \Delta x = 32 \text{ pixels)}$$  \hspace{1cm} (3.4)$$
Experimental Facilities and Numerical Modelling Details

These FoV sizes are shown in Table 3.1.

<table>
<thead>
<tr>
<th>Axial Location</th>
<th>( \frac{L_{1}^{(1)}}{D_j} )</th>
<th>( \frac{L_{1}^{(2)}}{D_j} )</th>
<th>( \frac{L_{2}^{(2)}}{D_j} )</th>
<th>( \frac{L_{2}^{(3)}}{D_j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.36</td>
<td>0.27</td>
<td>0.14</td>
<td>0.54</td>
</tr>
<tr>
<td>4.0</td>
<td>0.72</td>
<td>0.37</td>
<td>0.36</td>
<td>0.57</td>
</tr>
<tr>
<td>6.5</td>
<td>0.97</td>
<td>0.42</td>
<td>0.48</td>
<td>0.71</td>
</tr>
<tr>
<td>10.0</td>
<td>1.19</td>
<td>0.56</td>
<td>0.60</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Table 3.1: Theoretical \( \frac{FoV}{D_j} \) to achieve 95% accuracy in axial and radial scales

Table 3.1 shows that FoV sizes smaller than 0.14\(D_j\) are needed close to the jet and less than 0.5\(D_j\) further downstream for measurement accuracy. In contrast, it is necessary to maintain a sufficiently large FoV to capture the spatial shift of the correlations in time necessary for the Lagrangian lengthscale. Chapter 2 discussed the Lagrangian length and time scales, and, from previous literature[61], the decay to a \( \frac{1}{2} \) level requires a FoV ranging from 0.8\(D_j\) to 1.2\(D_j\) depending on downstream axial position. Seeding density must also be accounted for when considering FoV sizes. It is the author's opinion that taking the available lens limitations, and physical constraints on camera location (at least half tunnel width 187.5mm away) into account, the smallest FoV possible is of approximately 25mm \( \times \) 25mm. Taking all of these considerations into account, the nozzle diameter was fixed at \( D_j = 40 \)mm.

Numerical calculations of confined axisymmetric turbulent jets have been reported by Gosman et al [125], Habib and Whitelaw [126], Jones and Marquis [127], and Khalil et al [128]. In these calculations, turbulence effects were represented either by the \( k-\varepsilon \) model or by second-moment closures. Concerns over the ability of RANS CFD to predict free jet flows correctly exists as mentioned in Section 1.5 and the simulation will be no better for confined jets. However, the ability of RANS to predict flow features with acceptable accuracy at low costs makes it a valid method for initial nozzle design and flow condition assessment. The DELTA code detailed within Section 2.2.4 has therefore been used with a \( k-\varepsilon \) turbulence model to evaluate the confinement issues related to this study. With the nozzle exit diameter and tunnel size defined, analysis of the confinement issues and associated flow conditions necessary to avoid flow features such as the outer wall recirculations mentioned above and indicated in Figure 3.6 were undertaken. Table 3.2 presents the test matrix used for this CFD analysis. The tunnel (assumed axisymmetric for convenience) and LU40 nozzle (detailed in Section 3.1.3) were modelled with a sector angle of 15° and a mesh of 525,000 nodes as illustrated in Figure 3.7, to minimise storage requirements and aid computational time.
Experimental Facilities and Numerical Modelling Details

<table>
<thead>
<tr>
<th>$U_j$ (m/s)</th>
<th>$U_a$ (m/s)</th>
<th>$D_j$ (m)</th>
<th>$D_o$ (m)</th>
<th>$C_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.18</td>
<td>0.04</td>
<td>0.30</td>
<td>1.626</td>
</tr>
<tr>
<td>1.5</td>
<td>0.18</td>
<td>0.04</td>
<td>0.30</td>
<td>1.102</td>
</tr>
<tr>
<td>2.0</td>
<td>0.18</td>
<td>0.04</td>
<td>0.30</td>
<td>0.838</td>
</tr>
</tbody>
</table>

Table 3.2: Numerical Test Matrix

Figures 3.8a-c illustrate the predicted jet flow fields using jet exit velocities of 1m/s, 1.5m/s and 2m/s respectively. The contour levels are set to velocities above $0.5U_j$ and three isosurfaces are defined: 0.16m/s, 0.14m/s and 0.12m/s. The contours show that the potential core length of the jet in all three cases is essentially identical. The isosurfaces show the effect jet entrainment has on the coflow due to the increased jet exit velocity. Figure 3.8c identifies that $U_j = 2$m/s is the only condition which shows clear signs of recirculation. This is consistent with expectations, since this is the condition where the Craya-Curtet number is less than 0.98 as shown in Table 3.2. Additional streamlines have been used to identify this recirculation. For both jet exit velocities of $U_j = 1$m/s and $U_j = 1.5$m/s no recirculation was observed. However, comparison between the isosurfaces in Figure 3.8a and 3.8b show a distinct increase in the entrainment of the outer flow with the increase in jet exit velocity from 1.0m/s to 1.5m/s. This is clearly illustrated by the coflow isosurface of 0.12m/s, which has moved much closer to the jet centreline in the higher 1.5m/s jet exit velocity case in comparison to the 1m/s case.

Data produced by NASA[129] (amongst other investigations) has shown that for free jets where no external effects influence jet development, the jet velocity profiles, when normalised by the centreline velocity, collapse onto each other. Radial profiles of predicted mean axial velocity appropriately normalised using the external co-flow velocity and the centreline velocity at various axial locations are presented in Figures 3.9a-d. Comparison between the profiles produced by the 1.5m/s jet case and even the recirculation producing 2m/s jet case show very little difference. However, there is a noticeable difference between these two jet cases and the low speed 1m/s jet case. Figures 3.10a-d present radial profiles of the predicted TKE (normalised) at the same axial locations. These TKE results show again a similar relationship between the 1.5m/s and 2m/s jet cases and a small difference for the 1m/s jet case. The increased turbulence (mixing) and growth of the shear layer at the two higher jet velocities results in a small reduction in potential core length as shown in Figure 3.11. This provides evidence that the presence or absence of an outer recirculation zone cannot be the sole criteria to assess the effects of confinement.

By comparing the 1m/s confined jet numerical prediction with previous experimental data [130, 26, 74] for free jets any effects due to the outer co-flow or confinement can be identified. Figures 3.12a-b present centreline and radial profiles of normalised axial velocity. The similarity
between the predicted profiles in Figure 3.12a-b is considered good enough to suggest that for \( U_j = 1.0 \text{m/s} \) and \( U_e = 0.18 \text{m/s} \) there are no significant effects on jet development caused by confinement or outer co-flow (Note, for example the scatter even in the experimental data). The 1m/s jet case was therefore selected as the flow condition to be investigated in study. The experimental test matrix and arrangement of measurement locations are discussed further in Section 3.1.4 below.

3.1.3 Test Nozzles and Associated Water Supply System

Within this study, two single round jet and one coaxial round jet configuration have been used. The primary geometry considered was a convergent conical nozzle design. One geometry was based on the JEAN [131] nozzle with an internal contraction half angle of 6° which continues to the nozzle exit as shown in Figures 3.13 and 3.14. The second geometry was based on the primary nozzle of the CoJeN coplanar coaxial jet, which has been used in previous publications[132, 133] and whose co-ordinates are documented by Mead[134]. This nozzle had an internal contraction half angle of 11° and included a short parallel wall section of approximately 0.9\( D_j \) in length beyond the contraction as shown in Figures 3.15 and 3.16. Both nozzles have the same inlet diameter of 0.054m, nozzle exit diameter of 0.040m, and lip thickness of 1.2mm, although they have different contraction lengths of 0.066m and 0.049m respectively. The contraction only nozzle (JEAN) is hereafter designated as LU40 and the parallel extension nozzle (CoJeN) is designated LU40P.

Nozzles with parallel wall exits have been shown to remove the presence of a vena-contracta[135] in comparison with contraction only nozzles. Their effect on jet noise sources however, is unknown. Within this study the effects of such single round nozzles of different design on shear layer turbulent noise sources will be assessed.

The other geometry considered was a coaxial round jet nozzle based on the CoJeN coplanar coaxial jet[134], as shown in Figures 3.17 and 3.18. The primary and secondary nozzle exit diameters were 0.040m and 0.080m respectively (hereafter designated as LU80C). The geometry of the contraction region for the primary nozzle is identical to the LU40P nozzle described above. The secondary nozzle has an internal contraction half angle of 14° over a contraction length of 0.069m. The nozzles were manufactured from stainless steel and were polished to produce a hydraulically smooth surface. Due to manufacturing limitations, it was not possible to machine a sharp corner where the primary nozzle passage begins to contract. It is estimated that the transition consists of a fillet radius of approximately 5mm.
It is important that all test nozzle configurations should be located in the centre of the tunnel test section. In order to achieve this a supporting cross-shaped structure must be located upstream of the nozzle pipe feed. This 'nozzle feed' structure is an existing design [136]. Figure 3.19 shows a schematic of the nozzle feed structure showing the nozzle supply ducts used for primary and secondary jets. Two pumps (similar to that used for the coflow) independently feed the primary and secondary jet flow circuits allowing for both single and coaxial jet configurations. The nozzle feed structure is supplied from the reservoir tank via control valves and flow meters. Feeding the jet from the same reservoir as the co-flow ensures seeding particle saturation and uniform seeding density. The nozzle feed structure is located directly after the turbulence management system but before the contraction. Due to the bends required in the nozzle feed circuit, turbulence management systems are located within the supply ducts. Various test nozzles can be mounted via a series of grub screws distributed equally around the circumference of the upstream end of the nozzle. The supply ducts are approximately 1m long. The primary and secondary supply duct diameters are 0.057m and 0.126m respectively. Two outer 'spiders' were fabricated in order to preserve coaxial alignment in single and coaxial jet configurations respectively. These spiders were located at the inlet to the test section and are approximately 0.5m before the supply duct / nozzle attachment point. This ensures the nozzle is aligned to within 1° with respect to the tunnel walls. When in the coaxial configuration, an internal spider was also placed at the end of the primary supply duct directly before the nozzle to maintain the 0.0235m distance between the primary and secondary nozzles. The internal spider locates into 3 slots, as illustrated in Figure 3.20 and ensures the coaxial jet is axisymmetrically located accurate to within 0.2mm.

3.1.4 Experimental Test Matrix and Selection of Measurement Locations

In order to present evidence to support the use of a jet in a water flow facility a detailed validation of the experimental results obtained from the current PIV measurements against previous airflow experimental results obtained using LDV/CTA were conducted. To allow for this comparison some of the measurement locations chosen must match locations tested in previous experiments, but must also include locations of interest in relation to the high noise generating regions mentioned in Chapter 1. The FoV size at each location must also be carefully considered. If a constant FoV size were selected, as the flow develops and spreads, the integral lengthscales increase causing a decrease in the level of sub-cell filtering. In addition, the timescales reduce with downstream distance causing differences in the number of independent samples in each data set. Careful thought needs to be given to both of these issues in order to produce PIV measured 2nd and 4th order spatio-temporal correlations to at least the same level of accuracy achieved with CTA and LDV in airflow experiments.
Thus, to allow proper analysis of lengthscale evaluation and hence the level of sub-cell filtering at each location of interest, measurements were conducted using various FoV sizes. In order to achieve an accurate estimate of the lengthscale at selected test points the FoV's were (for the larger FoV's) centred about the point of interest allowing the spatial velocity correlation (SVC) to decay to zero within the FoV in both in-plane directions (desirable to yield accurate lengthscales from integration of this SVC distribution). For smaller FoV's where this was not possible, the FoV was shifted to allow for at least the downstream spatial correlation to decay to zero within the FoV. The confidence weighted approach (described in Section 2.3.3) was used to account for this one-sided information. The proposed FoV sizes and relative positions are shown in Figure 3.21. The selected layout of the PIV FoV’s cover six regions of interest as shown in Figure 3.22a. Of these six regions, two are within the shear layer (on the nozzle lipline) and are upstream of the potential core end; two are at the end of the potential core (one on the lipline and the other on the centreline); and finally, two regions were selected downstream of the potential core end (where the jet has a fully-mixed profile rather that a shear-layer profile), again one on the lipline and the other on the centreline. At each location a 50Hz and a 1kHz data set (both containing 3072 samples) was taken. The former was chosen to obtain better statistical convergence (due to its longer overall sampling time) and the latter for maximal temporal resolution. The proposed layout of the PIV FoV’s for the coaxial jet are shown in Figure 3.22c and cover an axial traverse of the upper shear layer, in addition to both shear layers being captured with smaller FoV’s at two axial locations.

The FoV’s described in Figures 3.21 and 3.22 are categorised as \( \tau \tau \)-planes and were captured using both monoscopic and stereoscopic PIV. FoV’s parallel to the nozzle exit plane were also used and are categorised as \( \tau \theta \)-planes. Since viewing normal to the \( \tau \theta \)-plane (necessary for monoscopic PIV) is not possible in this test rig without placing mirrors within the flow (producing a blockage to the flow and potentially affecting the jet development), data in this plane could only be obtained via stereoscopic PIV. Most importantly, the use of stereoscopic PIV in both \( \tau \tau \) and \( \tau \theta \) planes allowed for all three velocity components to be captured simultaneously, thereby enabling all of the correlation components to be calculated. Stereoscopic data was gathered at the same \( \tau \tau \)-plane locations as in Figure 3.22 and at \( \tau \theta \)-planes along the lipline at \( x/D_j = 1.5, 4, 6.5, \) and 10, and along the centreline at \( x/D_j = 6.5 \) and 10. The FoV’s for stereoscopic PIV were fixed in size in the \( r \)-direction to match the monoscopic PIV data sets, however, elongation due to perspective effects gives a FoV width of approximately 1.2x the height. Calibration was conducted using a two-sided two-level calibration plate (Figure 2.3b) of known thickness placed at the location of interest. The pinhole model, as detailed in Section 2.1.1, and subsequently self-calibration as detailed in section 2.1.2, determined the effective image correction. Scheimpflug mounts were employed for both cameras, with uniform focus achieved at an angle of 30°.
Modification of the standard rig setup was required to allow imaging using stereoscopic PIV. Due to the cameras now viewing obliquely at the air-perspex-water tunnel side interface, distortion of the image would be caused in the direction normal to the interface. The solution was to mount two removable, triangular, water-filled prisms onto the side of the rig, with sides angled at 45°. This allows the cameras to view normal to the first interface, as used previously by Lang et al [137] and Parker et al [138]. The various experimental setups for the monoscopic and stereoscopic PIV are illustrated in Figure 3.23 for both the \( x\tau \) and \( r\theta \)-planes.

3.2 Instrumentation Specification

All the experimental measurements within this project use a high-speed PIV system. The system is a commercially available package supplied by LaVision GmbH. PIV is a relatively new technique with new developments to the accuracy and versatility of the instrument being introduced all the time. The technical specification of the system used in the present work is shown in Table 3.3.
**Experimental Facilities and Numerical Modelling Details**

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<thead>
<tr>
<th>Camera</th>
<th>1 or 2 x Photon High Speed Star 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution</td>
<td>1024 x 1024 Pixels @ 10bit dynamic range</td>
</tr>
<tr>
<td>Frame Rate</td>
<td>1000Hz (double frame) @ full resolution</td>
</tr>
<tr>
<td>Min Inter-frame Time</td>
<td>2µs</td>
</tr>
<tr>
<td>N° of Samples</td>
<td>3072 @ full resolution</td>
</tr>
<tr>
<td>CCD</td>
<td>10bit monochrome</td>
</tr>
<tr>
<td>CCD Size</td>
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</tr>
<tr>
<td>Memory</td>
<td>8 Gbyte</td>
</tr>
</tbody>
</table>

<table>
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<th>Nikon Nikkor Macro</th>
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</thead>
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<td>24mm, 50mm, 60mm, 105mm, x2 converters</td>
</tr>
<tr>
<td>Fnumber F</td>
<td>2.8 - 32</td>
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<td>Magnification, M</td>
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<table>
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</tr>
<tr>
<td>Max Repetition Rate</td>
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<tr>
<td>Output Wavelength</td>
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</tr>
<tr>
<td>Pulse Width</td>
<td>135ns</td>
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<table>
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<th>Light Sheet Optics</th>
<th>Selectable 6°, 12°, 30°, 60°</th>
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</thead>
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<tr>
<td>Cylindrical Lens Focal Length</td>
<td>Selectable -6mm, -10mm, -20mm</td>
</tr>
<tr>
<td>Beam Thickness</td>
<td>0.5mm to 2.5mm</td>
</tr>
<tr>
<td>Working Distance</td>
<td>300mm to 2000mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System Computer</th>
<th>Pentium Dual Board</th>
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<td>Processor</td>
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<tr>
<td>Memory</td>
<td>4 Gbyte RAM</td>
</tr>
<tr>
<td>Internal Storage</td>
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<td></td>
<td>1 x CD-RW</td>
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<tr>
<td>External Storage</td>
<td>1 x 1.58 Tbyte filestore</td>
</tr>
<tr>
<td></td>
<td>1 x 2.58 Tbyte filestore</td>
</tr>
<tr>
<td>Software</td>
<td>DaVis v7.2, Matlab 2008, Tecplot 360 2008</td>
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</table>

Table 3.3: PIV System Specification
3.2.1 Vector Calculation and Validation Parameters

Chapter 2 referred to the practical aspects of PIV, and user-determined parameters which must be optimised. The following looks at the parameters used in the present study for calculation and validation of the vector field. There are options to 'pre-process' the image prior to vector calculation, but this was avoided altogether for processing monoscopic PIV data because the raw images provided excellent quality data. The only pre-processing option that was utilised was the subtraction of a sliding background image to reduce the effect of reflections upon the data quality for stereoscopic PIV measurements. The 'subtract sliding background' function computes an average local background intensity of the image. The user may define a scale length (in pixels) for the sub-area over which the local background intensity is calculated. This function works like a local low pass filter (large fluctuations are filtered out and the small fluctuations (particle signal) can pass through). Subtracting the local average background from the original image eliminates varying non-zero backgrounds, so that only the desired peaks are visible. The scale length is an important parameter and should be slightly bigger than the mean particle size [86] (typically 7 pixels).

Vector Calculation

As previously explained the raw PIV image is discretised into finite size interrogation cells. From previous water flow experimental studies [82, 81] it has been shown that for most practical magnification levels 32×32 pixels provide the optimum cell size and hence this level was utilised in this study. To increase the data density cell overlap has been used (defined as a percentage of the cell size). Examples in the literature exist where up to 92.5% overlap has been used, resulting in a data yield increase of 178! (Scarano and Reithmuller [139]). However 50% is the most commonly adopted value and has been used throughout this study.

A number of techniques exist, where one or all can be used, to enhance the quality of the calculated vector data, and the number of valid vectors. Both the Second Order Correlations and Adaptive multi-pass methods (discussed in Section 2.1.2) were used extensively in the project to increase data quality. Results presented in this thesis used a single pass using an interrogation cell size of 64×64 pixels followed by two subsequent passes using an interrogation cell size of 32×32 pixels.

Vector Validation

Validation is the process of evaluating the quality and reliability of the vector field. There are a number of ways in which validation can take place. One method is to utilise pre-defined limits.
Experimental Facilities and Numerical Modelling Details

Masking was utilised in this project to define the nozzle area, if present within the FoV. Specifying global allowable velocity limits was not used in this study.

In terms of quantifying the quality of the data calculated within the vector field, results presented in this thesis have a Q-ratio (defined in Section 2.1.2) of greater than 2. The most common local flow validation method considers neighbouring vectors was used in this project. The allowable variation from the neighbours depends on the turbulence of the flow, and was set to 1.3. If data is removed from a vector field because it is deemed to be spurious, that it is then replaced. A method used was by which if the highest peak is deemed invalid, then the next highest peak is assessed for its validity, and so on. On all monoscopic PIV the number of first choice vectors (those deemed valid from the highest peak) was generally always above 98%, for stereoscopic PIV this measure fell to 80% although the number of first and second choice vectors was ≈95%.

A summary of the parameter settings used within this study are given in Table 3.4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Processing</strong></td>
<td></td>
</tr>
<tr>
<td>Correlation Iteration</td>
<td>Adaptive Multi-Pass Grids</td>
</tr>
<tr>
<td>Initial Cell Size (pixels) / Passes</td>
<td>64 / 1</td>
</tr>
<tr>
<td>Final Cell Size (pixels) / Passes</td>
<td>32 / 2</td>
</tr>
<tr>
<td>Cell Overlap</td>
<td>50%</td>
</tr>
<tr>
<td>Correlation Function</td>
<td>Second Order Correlations</td>
</tr>
<tr>
<td><strong>Validation</strong></td>
<td></td>
</tr>
<tr>
<td>Remove vectors with Q-ratio</td>
<td>&lt;2</td>
</tr>
<tr>
<td>Remove vectors outside range</td>
<td>&lt;1.3</td>
</tr>
<tr>
<td>Replace removed vectors</td>
<td>With 2nd, 3rd or 4th choice peaks or interpolated data where no peak fits surrounding fluid dynamic behaviour.</td>
</tr>
</tbody>
</table>

Table 3.4: Parameters for vector processing, calculation, and validation

3.2.2 Statistical Accuracy

As with any measurement method which evaluates statistical data from an ensemble of individual samples, the accuracy of statistical averages extracted from PIV is dependent on the number of independent samples. It is well known that statistical averages converge with a sufficiently large number of statistically independent samples. However, with a technique which has a limitation
on the sample size it is important to recognise and quantify the errors in the calculated statistical data. It was with this in mind that statistical convergence was studied utilising the LU40 nozzle configuration. It is tempting when gathering data that will also be used to identify correlations with a range of temporal separations to use the highest possible sample rate (1kHz for present instrumentation). However, if we use the common definition of statistically-independent samples (Westerweel et al [91], Weisgraber and Liepman [140]), it is the number of samples separated by at least one integral timescale that defines the number of statistically independent samples within a data set. Thus, a high sample rate may be detrimental to the resultant statistical accuracy. The highest number of independent samples are more likely to be gathered using a lower sampling frequency (e.g. 50Hz).

The convergence of time-averaged first and second moment statistics from PIV and LDA data has been shown by Hollis [82] to be well represented by normalised standard error estimates, such as presented by Montgomery and Runger[141]:

\[
\varepsilon_u = \frac{z u'}{U_{rej}} \sqrt{\frac{1}{N_{I-sample}}} \tag{3.5}
\]

\[
\epsilon_u' = \frac{z u'}{U_{rej}} \sqrt{\frac{1}{2N_{I-sample}}} \tag{3.6}
\]

where \(U_{rej}\) is a reference velocity (in this study the jet exit velocity), \(N_{I-sample}\) is the number of statistically independent samples used. \(u'\) represents the true RMS value, although this is adequately represented by using a value calculated from all available samples, and \(z\) relates to the confidence band. If the error is assumed to be normally distributed, then \(z = 2.576\) for a 99% confidence band and \(z = 1.96\) for a 95% confidence band.

For high-speed PIV, the definition of confidence limits is clouded by the uncertainty of the true number of independent samples, not only due to the spatial variation of timescales across the domain but because each vector is itself a modal average of a number of particle displacements. This requires the convergence error used to be based on the 'effective' number of independent samples, \(N_{I-sample}^E\). It was shown by Robinson [81] that sub-cell averaging can increase the number of independent samples by a factor of 2.6 \((N_{I-sample}^E = 2.6 \times N_{I-sample})\) although there is no mathematical calculation for this value. Figures 3.24 and 3.25 show the standard error estimates evaluated within the shear layer and on the jet centreline respectively from both 1kHz and 50Hz data. The error analysis in Figures 3.24 and 3.25 suggests that the 3072 sample data set effectively contains 280 and 162 independent samples, respectively. These value are obtained from a timescale of 0.011s at \(x/D_j = 1.5r/D_j = 0.5\) and 0.019s at \(x/D_j = 10r/D_j = 0\)
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(deduced in Section 4.2.2). Note, these values take no account of the sub-cell averaging of the (approximately) 5 particles per interrogation cell as this cannot be accounted for mathematically.

Figures 3.24 and 3.25 typify the convergence of the mean velocity and second order moments. The convergence can be seen to compare extremely well with standard error estimate curves. Table 3.5 tabulates the errors for both 50Hz and 1kHz data sets assuming a 95% confidence band. Data gathered from previous studies [61, 63, 59] allow the absolute errors to be presented in terms of approximate percentages error (shown in brackets). However, caution should be applied if near zero velocities are evident because percentage errors are not necessarily always the most appropriate form. In these cases only the absolute errors are presented.

<table>
<thead>
<tr>
<th>Flow Statistics</th>
<th>Absolute Error m/s (Percentage Error)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(x/D_j = 1.5, r/D_j = 0.5)</td>
</tr>
<tr>
<td></td>
<td>50Hz</td>
</tr>
<tr>
<td>Umean</td>
<td>0.0047 (0.47%)</td>
</tr>
<tr>
<td>Vmean</td>
<td>0.0034</td>
</tr>
<tr>
<td>Urms</td>
<td>0.0033 (2.2%)</td>
</tr>
<tr>
<td>Vrms</td>
<td>0.0024 (2.5%)</td>
</tr>
</tbody>
</table>

Table 3.5: Statistical convergence errors for two locations within the flowfield for 50Hz and 1kHz data

From Figures 3.24 and 3.25 it can be seen that the 1kHz data falls well within the standard error lines. In addition to the statistical convergence errors there are additional errors arising from the sub-pixel particle location accuracy detailed previously. Hollis [82] provides a Pythagorean method by which these errors may be combined:

\[
\epsilon_u = \sqrt{(\epsilon_{ds})^2 + (\epsilon_u)^2}
\]

(3.7)

where \(\epsilon_{ds}\) is the relative particle shift (where \(\epsilon_{ds}\) is the accuracy of the peak detection), and \(\epsilon_u\) is the statistical error (for N samples) described earlier. Therefore, if it is assumed the average particle displacement of the measurements to calculate the average is 6 pixels:

\[
\frac{\epsilon_{ds}}{\epsilon_d} = \frac{0.1}{6} = \pm 1.67\%
\]

Therefore the total error at \(x/D_j = 1.5\) for the 50Hz axial mean and RMS velocity increases to \(\approx \pm 1.7\%\) and \(\approx \pm 2.8\%\) respectively, while the total error for the 1kHz data for the axial mean and RMS velocity increases to \(\approx \pm 2.3\%\) and \(\approx \pm 7.6\%\) respectively.
3.3 Numerical Modelling Details

When conducting CFD studies, there is a need to compare computational with experimental results. For the comparison to be valid and accurate, there are a few critical areas which must be considered. Firstly, it is necessary to perform the computations (either RANS or LES CFD) in a comparable computational domain to that of the experiments. The grid resolution within the domain must also be considered, which is a function of the numerical discretisation, and the spatial and temporal gradients to be numerically resolved. The ability of the SGS model to perform adequately is also relevant. Furthermore, it is essential as far as possible to ensure matched inflow conditions are used, given the sensitive nature of initial shear layer development to boundary layer conditions at nozzle exit. The temporal resolution required in LES calculations can lead to huge data storage. Finally, the sampling frequency from the time-varying CFD solution must be sufficient to resolve desired frequencies, similar to the experimental measurement, although this latter issue does not directly influence the CFD solution itself, it does affect subsequent data processing.

3.3.1 Computational Domain

The LU40 and LU80C nozzles were chosen for the computational study since each illustrates the fundamental noise source generation mechanisms in a single and a more aeroengine representative coaxial jet. For both nozzle simulations, a 360° sector has been used, as required by the 3D nature of LES calculations. For all simulations several versions of the numerical meshes were first explored in order to reduce the results mesh dependency. Obviously for the present project, it was beyond the scope (both time and cost) to refine the mesh until the results were completely mesh independent. Particular attention was paid to near-wall resolution, ensuring that the first cell centre was located at a wall normal distance of \( y \approx 0.2 \text{mm} \), whilst increasing gradually from the wall following an exponential distribution to ensure good boundary layer resolution. The refined grid near the nozzle exit produced a first cell \( y^+ \) of \( \approx 7 \). An axial exponential grid contraction was also used in the upstream direction (starting at solution domain exit) to allow increased mesh density at the nozzle exit (boundary layer to shear layer transition) whilst ensuring near wall cells around the nozzle exit had cell aspect ratios of \( \approx 1 \). Figures 3.26a-b and 3.27a-b show two-dimensional slices through the meshes used for the single and coaxial jet nozzles respectively (to obtain the 3D meshes, these slices are revolved through 360°).

To ensure good distribution of non-skewed cells a polar computational domain with an overall radial dimension of 0.15m was used to give a confinement equal to that of the narrowest width of the (rectangular) experimental test section. Both meshes utilise the ability to cluster mesh nodes
around given areas to increase resolution. Figures 3.26b and 3.27b indicates the mesh design used to provide high resolution in the jet shear layer region.

The computational domain inlet starts 3 nozzle exit diameters (3 primary nozzle exit diameters for the coaxial configuration) upstream of nozzle exit. This location corresponds to the location of previous experimental inlet boundary layer measurements recorded across the primary nozzle supply duct using LDV [142, 136]. It is this experimental data which provides the nozzle inlet conditions in the simulations (see Section 3.3.2). The axial extent of the coaxial jet computational domain downstream is 1.5m, which is the full length of the experimental test section, while for the single round jet this was reduced to 0.7m. During grid refinement tests it was observed that flow development was not influenced by this reduction in axial domain size whilst a significant time saving was made.

For simulations of the single round nozzle, the final mesh comprised of $398 \times 88 \times 360$ cells in the axial, radial and azimuthal directions (Total mesh size = 12.6 million) as shown in Figures 3.26a-b. This was split into 17 blocks which were distributed over 16 processors. The final coaxial round jet mesh comprised $502 \times 136 \times 360$ cells in the axial, radial and azimuthal directions (Total mesh size = 24.6 million) shown in Figures 3.27a-b. This was split into 33 blocks which were distributed over 32 processors. The multiblock approach was used to provide better control of mesh quality around nozzle exit and shear layer spread, whilst allowing for multiprocessor solution and selective block postprocessing.

3.3.2 Inlet Conditions

An experimental study was previously carried out by Behrouzi et al [136] to investigate the effect of coaxial jet flow parameters on near field plume development. As part of this study nozzle inlet axial mean and RMS fluctuation profiles had been measured using LDA in the same facility as used in the present study. These LDA profiles have therefore been used as the inlet conditions for the computational calculations performed. In addition to these profiles, flat profiles were also initially implemented in order to evaluate the sensitivity of the predicted nozzle exit profiles to inlet conditions specifications $3D_j$ upstream of the nozzle exit. Figures 3.28a-c show comparisons between flat inlet profile conditions and experimental inlet profile conditions for both LES and RANS predictions, in comparison to PIV measurements downstream of the nozzle exit at $x/D_j = 0.5, 2, \text{ and } 4$. It can be clearly seen that the numerical predictions using the experimental inlet profile generate flowfields downstream of the nozzle exit which are in better agreement with the experimental results.
In addition to the prescribed mean velocity inlet profiles, 10% white noise perturbation was added in order to generate unsteady conditions for use as LES inlet data. Although 10% seems a large disturbance because it is an uncorrelated disturbance it decays quickly. Also acceleration in the nozzle contraction reduces turbulence levels. Digital filtering is known to produce more realistic LES inlet conditions [81, 97], but this was not attempted in the present work.

### 3.3.3 Data Storage Considerations

Whenever LES computations are undertaken and post processing of the unsteady data is to be undertaken to produce correlation information, the available storage and processing capabilities available must be considered. Storage considerations influence sampling decisions while processing capabilities directly influence the computational time and expense of both the simulations themselves, and their subsequent analysis.

Storage limitations which are used to restrict the mesh sizes to such a degree that the grid resolution is too coarse are obviously totally detrimental. This should therefore be avoided, usually at the expense of the temporal resolution of sampled data. Although some statistics can be gathered 'on the fly', requiring reduced storage, spatio-temporal correlation statistics require a complete time series to be explicitly stored. In this study, in order to apply identical analysis to both PIV and LES data, it was desirable that the LES CFD solution should be downscaled to produce a sample of equal size to the PIV results, which implies a restricted LES temporal resolution of 1kHz. (NB. the LES time step is dictated purely by numerical stability constraints (CFL number \( \approx 0.2 \)) and hence is very small (\( \approx 6 \times 10^{-5} \) secs) and would in principle allow higher frequency visualisation).

All the computational calculations throughout this study have been performed on the Loughborough University 160-node 64-bit Itanium lynx cluster detailed in Section 2.2.5. Storage restrictions implied a disc area of 3.5TB was available for this study. Given that the same temporal sample size was desired to enable identical processing to be carried out for both LES and PIV data, this storage limit lead to the meshes detailed above in Section 3.3.1. Table 3.6 summarises the computational mesh size, each individual data file size and the full storage requirements for a full sample.
Modification of the DELTA code has enabled the extraction of 'zoomed-in' 2D planar velocity information from the whole 3D computational domain to allow the LES data to be post-processed using the same software (Xact) as used for PIV data post-processing. Through using the same post-processing software confidence can be gained in the results while removing any uncertainties about differing processing routines. The plane of velocity data can be of any size up to the full domain size, although typically a size of approximately the same size as the PIV measurement FoV was used. Since the data rate of the LES predictions is much higher than that achievable by the current PIV instrumentation, a plane of velocity data was exported from each solution at a downsampled rate of 1kHz. Before LES data sampling began, each simulation was run for a 'start-up' period, which allowed the flow to forget its initial state and become statistically stationary. During this period the volume integral of turbulent kinetic energy present in the whole solution domain was monitored. The start-up period was judged to be complete when this volume integral reached a time-independent level (typically 7 flow through times or ≈ 100,000 time steps for the single jet case, and ≈ 200,000 time steps for the coaxial jet case). This downsampling also enabled a large data reduction in the storage requirements. Table 3.7 shows the file size reduction due to this process.
3.4 Closure

The experimental facilities, and the technical specification of the PIV system have been comprehensively discussed. Justification and preliminary evaluation have been given of the chosen experiment, including assessment to ensure acceptable flow characteristics, explanation of nozzle sizing considerations, explanation of the particular nozzle configurations, the associated water supply system, and an explanation of the chosen measurement locations and associated test matrix. The PIV process has been discussed in detail, together with the approach to optimisation of the set-up parameters. By ensuring a good quality basic experimental set up, and routinely checking the quality of the data (signal to noise ratio, peak locking, number of first choice vectors) during the tuning of the timing parameters, the PIV results should be of the highest possible standard. This chapter has also detailed the process by which computational simulations of the test geometries have been conducted, including the computational grid and the necessary specification of boundary conditions.

The following three chapters detail the results of the experimental and computational studies carried out. Single and two-point statistics will be discussed and validated, together with the ability to resolve spatio-temporal correlations.
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Figure 3.1: Schematic diagram of the water tunnel

Figure 3.2: Diagram of the turbulence management system

Figure 3.3: Image of water tunnel, traverse, PIV camera and jet nozzle
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Chapter 4

Experimental Results

The main objective of this thesis as stated in Chapter 1 is to investigate, using experimental techniques, the high-order spatio-temporal turbulence correlations which are fundamental to the development of improved noise source modelling. This chapter presents experimental results for both single round jet nozzle designs (LU40 and LU40P). These results were gathered from tests undertaken in the water tunnel facility described in Section 3.1, using the PIV measurement equipment and techniques described in Section 2.1.

The chapter is organised as follows:

Section 4.1 presents single point statistics for the LU40 nozzle to demonstrate and assess the ability of the current PIV set-up to capture the flow statistics. The section also compares these statistics to other free jet airflow experimental results. These comparisons provide information regarding the effects of confinement as well as the sensitivity of the data to the PIV implementation.

Section 4.2 presents two-point statistics, including the shape and distribution of important spatial correlations, and the resultant Eulerian lengthscale information. Monoscopic PIV was used at this stage resulting in an evaluation of four lengthscales. Comparison of these lengthscales to previous experimental studies are shown in order to present further evidence to support the argument that the fundamental jet flow turbulence structure is not affected when water is used as the fluid medium. A detailed examination of the FoV size in relation to the interrogation cell size and associated sub-cell scale filtering, together with the appropriate correction methods is given for lengthscale (Section 4.2.1) and timescale (Section 4.2.2) characteristics. Power spectra are presented in Section 4.2.3. The power spectra also enable identification of any high frequency energy lost through sub-cell filtering and any high frequency spurious energy (noise) present in
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the signal. Section 4.2.4 concentrates on the spatio-temporal correlations, and the ability of the PIV technique (with its limited temporal resolution and sample size) to capture a true and accurate representation of such correlations in both shape and magnitude. This section includes evidence on the statistical convergence of the PIV data with increasing sample size and the spatial limitations to capturing accurate Lagrangian length and time scale information. Finally a detailed look is taken at how correlation maps (achievable with global measurement techniques such as PIV) can capture accurate representation of eddy convection velocities and how these compare to previous airflow experimental results.

Section 4.3 examines the significance of nozzle design details with regard to the shear layer development, potential core length, turbulence levels and ultimately the two-point spatio-temporal correlations. The significance of the findings of Trumper[143] with respect to the precise nozzle design (specifically a short parallel extension at the convergent nozzle exit) and the effect this has on the exit profiles are investigated with respect to turbulent noise sources.

Section 4.4 presents stereoscopic PIV results for the LU40 nozzle for both $x\tau$ and $r\theta$ planes, and these results are compared with monoscopic PIV data. Both single and two point statistics are presented. This section highlights the effect of the highly sensitive stereoscopic optical configuration on the results and illustrates how statistics such as the 4th order spatio-temporal correlations are affected. Section 4.4.3 presents a new method for correction of stereoscopic PIV data via the use of Proper Orthogonal Decomposition (POD) as a frequency filter. A major advantage of the new technique is that the data does not suffer the typical deterioration of reduction in sample sizes (i.e downsampling). This correction procedure allows all 81 components of the 4th order spatio-temporal correlation to be captured accurately. Section 4.4.4 presents these 4th order spatio-temporal correlations, including identification of which of the 81 components are the largest contributors to the far-field noise source and hence must be included in noise source models and more importantly which components can be ignored, in order to reduce computational cost significantly.

Finally, Section 4.5 presents a summary and evaluation of the current method, in which 4th order spatio-temporal correlations are obtained from direct measurements, by comparing this to the often used method which obtains 4th order quantities via the quasi-Gaussian approximation using products of 2nd order spatio-temporal correlations.

4.1 Single Point Statistics - LU40 Nozzle

Single point measurements were made to provide initial confirmatory evidence to support the use of PIV data gathered in a water flow experiment as being representative of subsonic isothermal
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airflow jet turbulence. For the purpose of comparison with previous data, and numerical results (presented later Chapter 5), all data is presented in cylindrical polar co-ordinate form, although the PIV data was taken in a Cartesian reference frame, such that, when $\theta = 90^\circ$, Cartesian decomposed velocity components can be interpreted as cylindrical polar, by taking $V = V_r$, and $W = V_\theta$ ($\theta = 0^\circ$, $V = V_r$, $W = -V_r$, or along $\theta = 180^\circ$, $V = -V_\theta$, $W = V_\theta$). The accuracy of a velocity field measured via PIV is highly dependent upon the implementation of the system. FoV and interrogation cell size optimisation are necessary to capture accurate fluctuation data, as mentioned in Section 2.1. This is particularly difficult in flows with dominant flow directions and high dynamic ranges requiring high spatial resolution, whilst also needing to maintain the necessary spatial domain size to track the motion of turbulent eddies for correlation mapping and decay rate information. Flowfield measurements were conducted using various FoV sizes (discussed in Section 3.1.4) in monoscopic mode in the $x\tau$ plane orientation of the jet plume. Measurements were taken at four locations within the shear layer (i.e. at $r/D_j = 0.5$) at $x/D_j = 1.5, 4, 6.5, \text{ and } 10$, and at two locations on the jet centreline (i.e. at $r/D_j = 0$) at $x/D_j = 6.5$ and 10. A visual representation of the FoV sizes and locations is given in Figure 3.22. Measurements were conducted for the LU40 nozzle only as it was felt necessary to validate the experimental practice for a single nozzle prior to investigating the effects of nozzle geometry, presented in Section 4.3. The analysis undertaken and discussed in Section 3.1.2 explained the issue of confinement and presented a test condition sufficient to produce flowfields representative of free jets. This test condition is shown in Table 4.1

<table>
<thead>
<tr>
<th>$U_j$(m/s)</th>
<th>$U_m$(m/s)</th>
<th>$D_j$(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.18</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 4.1: Test Conditions

The following subsections detail the plume development at this condition via axial and radial profiles. Although single point first moment statistics are the simplest form of data reduction, they provide a useful starting point in PIV analysis. The concept of identifying single point first and second statistical moments of the turbulence (mean and RMS) is based on the principle of Reynolds decomposition of the instantaneous velocity field (detailed in Section 2.3)

4.1.1 Axial Profiles

Figures 4.1 - 4.3 illustrate the development of the axial mean and RMS velocity along the centreline $r/D_j = 0$ and nozzle lipline $r/D_j = 0.5$ (where the turbulence levels are expected to be largest). Data from the largest 100mm×100mm FoV are presented as lines while the smaller
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FoV data are presented as symbols at single locations ($x/D_j = 1.5, 4, 6.5, \text{ and } 10$). Figure 4.1a shows that the mean axial velocity measurements are insensitive to FoV size, as expected since it is the second moment RMS that is subject to filtering effects from unresolved sub-cell turbulence scales. Previous experimental airflow data are presented in Figure 4.2 and compared to the results from the 100mm×100mm FoV. This comparison provides valuable information to show that the jet centreline development of the current experiment (in an enclosed environment) is in very good agreement with the previous data of Tanna[26] and Bridges and Wernet[74] (sp7 $M_j=0.9, \frac{T_e}{T_a}=0.835, \text{ NPR}=1.861$) both in terms of potential core length (taken as the point at which $U_{pc} = 0.98U_j$, which corresponds to $x/D_j = 6.6$) and the decay of the centreline velocity downstream of the potential core. The comparison with the data of Crow and Champagne[130] and Bridges and Wernet[74] (sp3 - $M_j=0.5, \frac{T_e}{T_a}=0.95, \text{ NPR}=1.197$) shows slight differences in potential core length (known to be sensitive to nozzle design / test rig conditions), however, the centreline decay rate downstream of the potential core is in good agreement. This decay rate agreement indirectly indicates that the behaviour of the shear layer (whose growth is the cause of the closure of the potential core) and fully merged jet near-field flow are not strongly affected by the current experimental enclosed environment and use of water compared to air (no identifiable effects of Re or Mach number). Figure 4.1b shows the measured variation of the axial RMS velocity along the centreline. It displays unusual behaviour in the region $1.5 < x/D_j < 6.5$ (within the potential core). Within this region there is no change in the mean axial velocity, hence no axial or radial gradients of axial velocity exist on the centreline. Thus there are no turbulence production terms active, so the RMS level should remain the same as nozzle exit (or even decrease slightly). Figure 4.1b however, shows a linear increase in the RMS but at a smaller gradient than is observed when the annular shear layer meets the jet axis at the potential core end. The change in gradient denotes the end of the potential core, and as illustrated, occurs at $x/D_j = 6.4$ in close agreement with potential core length deduced from the mean velocity. Similar results were identified by Trumper [143] and Power et al[144]. Power et al revealed through measurement of the energy spectrum of the axial RMS velocity a low frequency narrow band peak on the centreline within the potential core. They suggested this was the result of local irrotational unsteadiness induced by the streamwise passage of large turbulent structures in the initial region of the annular shear layer surrounding the jet core. Note also in Figure 4.1b that for $x/D_j < 3$, there seems to be little effect of the FoV size on measured RMS; at $x/D_j = 6.5$ and 10 however, the indications are that only the smaller FoV sizes produce consistent RMS levels, although there is still some variation at $x/D_j = 10$.

The lipline profiles of axial mean and RMS velocities are shown in Figures 4.3a-b. The mean velocity shows lower values within the first 2-3 jet exit diameters, which are dependant upon the FoV used, so here only FoV's smaller than 60mm seem reliable even for mean velocity. The lipline velocity peaks at a non-dimensional value of 0.5 as expected since this is in the middle
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of the shear layer and therefore halfway through the jet to coflow velocity gradient. A slight decrease is then seen downstream of $x/D_j \approx 5$. This behaviour is attributed to reaching the end of the potential core and the transition beginning to a fully developed jet flow rather than an annular shear layer. The large range of 'raw' PIV measured RMS velocity values shown in Figure 4.3b shows that the FoV is very important in obtaining accurate turbulence information. As the location increases downstream the variation between the RMS values obtained from different FoV's reduces. This is due to an increase in the local integral lengthscale and hence a reduction in the sub-cell scale filtering, therefore reducing the error and increasing the accuracy of the 'raw' RMS velocity captured by the larger FoV's.

4.1.2 Radial Profiles

To provide a detailed evaluation of the shear layer development radial profiles were examined. Figures 4.4 and 4.5 show the development of the radial profiles of axial mean and RMS velocity respectively. At all four axial locations the mean velocity profiles (non normalised) fall on top of each other regardless of the FoV used. This provides confidence that the experimental procedures implemented result in a highly repeatable experiment. Measurements at $x/D_j = 1.5$ and $x/D_j = 4$ shown in Figures 4.4a and 4.4b contain a region of constant peak velocity near the centreline within the potential core while the plateau region reduces in size and the gradient region becomes shallower and wider (representing the spreading of the shear layer). Measurements at $x/D_j = 6.5$ shown in Figure 4.4c illustrate no plateau region and a drop in centreline velocity to $U \approx 0.98U_j$ corresponding to the closure of the potential core. The dashed line in Figure 4.4c represents profiles from the centreline FoV while the solid lines represent profiles from the lipline FoV. The peak velocity continues to decrease reaching $U \approx 0.85U_j$ by $x/D_j = 10$.

Figures 4.5a-d show the RMS axial velocity corresponding to the same locations shown in Figures 4.4a-d. The growth and development of the shear layer is illustrated by the widening of the high turbulence regions of the profiles. All the FoV sizes capture similar shear layer thicknesses although they produce different peak magnitudes. As expected the smaller FoV's capture higher turbulence levels due to less sub-cell scale filtering. The range in the peak magnitude values from different FoV's reduces as the axial location increases and as the sub-cell scale filtering reduces due to increasing local lengthscale. At $x/D_j = 10$ the range of peak magnitude values produced by the different FoV's is small, while the smoothness of the profiles has reduced. The most likely explanation for this is that it is caused by the increase in lengthscale and related reduction in the turbulent timescale, resulting in a smaller number of independent samples and a lower level of statistical convergence given the same sample size is used as at upstream locations. It is likely therefore, that the smallest FoV gives the most accurate measurement irrespective of location. Figures 4.6a-b show a comparison of the radial profiles measured with the 25mm×25mm and
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40mm×40mm FoV with previous airflow experimental data at $x/D_j = 1.5$ and 4. For both FoV's the peak RMS underpredicts that measured by Lau et al[63] but is in good agreement with the Harper-Bourne[61] data. Unfortunately due to the lack of experimental data at other axial locations no conclusions can be made about the levels of underprediction over the length of the jet plume. One would, however, expect this underprediction to reduce as the level of sub-cell scale filtering reduces. The underprediction of the raw RMS velocity from PIV measurements using the smallest possible FoV (given seeding density and associated particles per interrogation cell restriction mentioned in Section 2.1.1) demonstrates the sensitivity of the turbulence levels captured to the interrogation cell method. Error from this method (even present in 25mm² FoV) has been considered and assessed together with the implementation of the correction method proposed by Spencer and Hollis [145, 82] based on the HMN method to provide a correction for the low-pass filtering effects of sub-cell filtering.

4.1.3 Sub-Cell Scale Filtering Correction

The effects of sub-cell filtering are tangible and can be quantified using the Hollis correction method through Equation 2.49. By simple manipulation of this equation the amount of TKE that is unresolved at any given location in a PIV flowfield is obtained. Figures 4.7 and 4.8 illustrate the ratio between the raw measured RMS velocity and the corrected RMS velocity estimated from the correction method. Figure 4.7a-b shows the axial RMS error produced by the 100mm×100mm FoV and the 60mm×60mm FoV respectively. It can clearly be seen that just after the nozzle exit the levels of filtering are substantial for the 100mm×100mm FoV ($\approx 30\%$) but decrease as the axial location increases until at $x/D_j = 10$ the error has reduced to $\approx 4\%$. The same trend is seen for the 60mm×60mm FoV (Figure 4.7b) although there are considerably lower levels of error over the whole flowfield. Around the nozzle exit the error has reduced to $\approx 10\%$, while by $x/D_j = 10$ the error has reduced to $\approx 2\%$. Figures 4.8a-b show the radial RMS error produced by the same FoV's. Once again the error reduces as the axial location increases and the smaller FoV has lower levels of error at each location. It should also be noticed that the error is lower for the radial RMS velocity than the axial RMS velocity shown in Figure 4.7. To enable an accurate comparison of the error levels for all the FoV sizes for both axial and radial fluctuations the errors within the centre of the shear layer at various axial stations are shown in Tables 4.2 and 4.3.
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<table>
<thead>
<tr>
<th>FoV Size</th>
<th>Axial Station</th>
<th>x/Dj = 1.5</th>
<th>x/Dj = 4</th>
<th>x/Dj = 6.5</th>
<th>x/Dj = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>25mm×25mm</td>
<td>3.9%</td>
<td>1.9%</td>
<td>1.6%</td>
<td>1.2%</td>
<td></td>
</tr>
<tr>
<td>40mm×40mm</td>
<td>5.9%</td>
<td>2.3%</td>
<td>1.9%</td>
<td>2.0%</td>
<td></td>
</tr>
<tr>
<td>60mm×60mm</td>
<td>8.0%</td>
<td>4.0%</td>
<td>2.9%</td>
<td>2.2%</td>
<td></td>
</tr>
<tr>
<td>80mm×80mm</td>
<td>9.8%</td>
<td>3.8%</td>
<td>2.7%</td>
<td>2.4%</td>
<td></td>
</tr>
<tr>
<td>100mm×100mm</td>
<td>11.6%</td>
<td>6.0%</td>
<td>4.4%</td>
<td>3.4%</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Error in axial RMS

<table>
<thead>
<tr>
<th>FoV Size</th>
<th>Axial Station</th>
<th>x/Dj = 1.5</th>
<th>x/Dj = 4</th>
<th>x/Dj = 6.5</th>
<th>x/Dj = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>25mm×25mm</td>
<td>3.2%</td>
<td>2.7%</td>
<td>2.8%</td>
<td>2.1%</td>
<td></td>
</tr>
<tr>
<td>40mm×40mm</td>
<td>4.5%</td>
<td>3.5%</td>
<td>3.4%</td>
<td>3.0%</td>
<td></td>
</tr>
<tr>
<td>60mm×60mm</td>
<td>11.4%</td>
<td>3.8%</td>
<td>4.4%</td>
<td>3.8%</td>
<td></td>
</tr>
<tr>
<td>80mm×80mm</td>
<td>16.2%</td>
<td>6.9%</td>
<td>5.4%</td>
<td>5.5%</td>
<td></td>
</tr>
<tr>
<td>100mm×100mm</td>
<td>19.2%</td>
<td>7.3%</td>
<td>6.2%</td>
<td>5.6%</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: Error in radial RMS

In summary, it is clear that in order to capture as much of the turbulence field as possible data should be acquired using the smallest FoV size possible. Axial and radial profiles along two radial and at four axial locations have shown that, although the jet was discharged within an enclosed environment, the jet plume development of the water flow experiment is very similar to that of an isothermal airflow jet, given the correct test conditions and scaling. In addition Tables 4.2 and 4.3 show that, given the correct FoV, there is in fact very little sub-cell filtering occurring with ≈ 3% correction being applied to the RMS velocities. The radial errors are significantly reduced by using the correct FoV size, reducing from errors of ≈ 20% to ≈ 3.2% at x/Dj = 1.5 and from ≈ 12.5% to ≈ 2.1% at x/Dj = 10. However, if the effects of sub-cell filtering can at least partially be corrected by the method of Hollis [82], the ability to gather instantaneous data over larger areas does allow for extended tracking and correlation of flowfield structures. In terms of the spatio-temporal correlation, it should be noted that, the Hollis correction method can not be extended to recreate corrected instantaneous velocity values, so only ‘raw’ measured data is used here. Hence, the larger FoV’s will still be used for data processing of 2-point quantities.

The spatial resolution used with the smallest (lowest error) FoV corresponds to an interrogation cell size of 0.78mm×0.78mm, and, with a interrogation cell overlap of 50% there is a vector every 0.39mm. This compares very favourably with the spatial resolution of the Fleury[102] data.
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(2mm), the Wernet[76] data (at least 4mm), and the Chatellier et al[113] data (0.89mm) as shown in Table 4.4. Each of these studies has also used an interrogation cell overlap of 50%. It is believed therefore that there is lower spatial filtering in the current PIV data than in these other studies.

<table>
<thead>
<tr>
<th>Authors</th>
<th>$D_j$ (mm)</th>
<th>FoV (Pixels)</th>
<th>FoV ($D_j$)</th>
<th>Cell Size (Pixels)</th>
<th>Resolution (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wernet[76] (PIV)</td>
<td>50.8</td>
<td>1024 x 144</td>
<td>2.95 x 0.37</td>
<td>32 x 32</td>
<td>4.69 x 4.22</td>
</tr>
<tr>
<td>Fleury et al[102] (PIV)</td>
<td>38</td>
<td>1280 x 1024</td>
<td>2.2 x 1.8</td>
<td>32 x 32</td>
<td>2.09 x 2.14</td>
</tr>
<tr>
<td>Chatellier et al[113] (PIV/LDV)</td>
<td>50.8</td>
<td>1280 x 256</td>
<td>1.4 x 0.28</td>
<td>16 x 16</td>
<td>0.89 x 0.89</td>
</tr>
<tr>
<td>Pokora (PIV)</td>
<td>40</td>
<td>1024 x 1024</td>
<td>0.63 x 0.63</td>
<td>32 x 32</td>
<td>0.78 x 0.78</td>
</tr>
</tbody>
</table>

Table 4.4: Resolution achieved in various studies using PIV

The following section details the two-point statistics acquired through the ‘best practice’ approach of two-point PIV analysis developed by Hollis[82] and Robinson[81], detailed in Section 2.3.3, and summarised below:

- **Sub-cell scale correction**
  - Best practice is initially to reduce the correction level, via FoV sizing, as much as possible.
  - Application of the proposed Hollis correction method for sub-cell filtering is employed where needed.

- **Integral lengthscales**
  - It is not possible to integrate to infinity, and hence integration is performed up to the first crossing of the separation distance axis.
  - If the distribution is curtailed (never reaches zero crossing) an exponential form is used to approximate the ‘missing’ data.
  - Where curtailment is present, a confidence weighting approach, based on the relative proportion of data curtailment in the distribution, is used to calculate the lengthscales

- **Integral timescales**
  - Necessity to select a sample frequency such that the local timescale and total sample size ensure a large enough number of independent samples to achieve statistical convergence to as high a level as possible

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- PSD

- Spectra obtained from PIV data may be noisy or poorly resolved (especially data at low sample rates), therefore the spectra at a given point are calculated as the mean of the spectra of the 9 points surrounding it.

4.2 Two Point Statistics - LU40 Nozzle

Investigation into the nature of the turbulence should include examination of the temporal and spatial correlation scales, which in turn gives rise to the 'eddy' description of turbulence, is presented in this section. The concept of the energy cascade, introduced by Richardson[101] and described in some detail by Pope[94], explains how turbulent energy is generated in the larger scales and is transferred by vortex stretching through progressively smaller scales. At the Kolmogorov scale \( \eta \) the viscosity acts to dissipate the energy and this is thus the smallest scale present. It follows therefore that there is always some finite distance and time over which flow coherence is observed. In order successfully to capture the spatio-temporal correlations which relate to sound source modelling, not only the correct magnitude of turbulence energy but its distribution over space and time need to be captured to examine its spatial and temporal coherence. This section therefore presents these velocity correlations, and deduced length and time scales, in order to provide some quantification of the size and mutual relationship of turbulent regions within a FoV.

Figures 4.9 and 4.10 show the spatial correlations of axial and radial velocity fluctuations with respect to axial and radial separations at four locations along the lipline. The differences between the various FoV plots match the trends mentioned earlier. The larger FoV (which has larger unresolved scales) produces larger correlations at a given spatial separation than the smaller FoV's, in particular at larger \( x/D_j \). This results in a larger lengthscale as will be seen later. The difference between the FoV's reduces as the axial location increases. Figure 4.9d for example illustrates that, since at \( x/D_j = 10 \) the local lengthscale is larger, even the larger FoV's provide a correlation distribution virtually identical to that seen in the smaller FoV's. This trend was shown in Table 4.2, which presented the difference between the 100mm×100mm FoV and the 25mm×25mm FoV at \( x/D_j = 1.5 \) being 7.7%, but decreasing to 2.2% at \( x/D_j = 10 \). Figure 4.10 indicates that the radial velocity length scales are smaller than for the axial velocity, so that eddies in general are non-circular. Figure 4.10 also highlights a sensitivity to applying an exponential curve for the closure of the correlation distribution when there is no definite crossing of the zero correlation axis. The larger FoV's tend to plateau at \( R_{22} \approx 0.2 \) resulting in larger FoV's producing a larger area under the profile before the end of the FoV is reached and the exponential curve is fitted (implying larger lengthscale).
4.2.1 Lengthscales

The whole field jet plume lengthscales extracted from the above 2-point correlations can be seen in Figures 4.11 and 4.12 for corrected axial and radial lengthscale of axial and radial velocity respectively. Figure 4.11a shows the presence of larger lengthscales in the potential core (determined by the turbulent lengthscales associated with the nozzle exit flow) and on the outer edges of the shear layer (water tunnel co-flow characteristics). The radial lengthscale illustrated in Figure 4.11b shows that large radial lengthscales develop towards the end of the potential core caused by movement of the merging annular shear layer across the symmetry axis, essentially a flapping motion at the potential core end. Figure 4.12a shows low radial velocity correlation regions with axial separation occurring in the predominantly axial potential core and co-flow regions, resulting in high lengthscales. Meanwhile any radial motion within the shear layer is uncorrelated with locations upstream and downstream yielding low lengthscale regions. Figure 4.12b shows the opposite effect when radial separation is considered. Large lengthscale values identify the edges of the shear layer with a region of increased lengthscales when the inner edges of the annular shear layer meet thereby closing the potential core. From both Figures 4.11 and 4.12 one can see that the axial lengthscale of axial velocity $L_{11}$ is the most dominant structure (as expected due to the axial convection of the structures).

When considering derived statistics such as lengthscales it is important to recall there is a portion of the 'true' instantaneous experimental flow field that has not been captured in the PIV measurements, and to consider what impact this might have had on the presented statistics. As explained in Section 2.3.3 the approach developed by Hollis[82], following the initial proposals of Hoest-Madsen and Nielsen[104] for the correction of the effects of sub-cell filtering on the derived statistics has been used in this thesis, and the lengthscale data presented in Figures 4.11 and 4.12 are corrected lengthscales. The estimate for corrected to measured lengthscales deduced for this analysis is by Equation 2.50, reproduced here:

$$\frac{L_{\text{corrected}}}{L_{\text{meas}}} = \begin{cases} 
e^{-0.5141(\Delta X/L_{\text{true}})} & \text{if } (\Delta X/L_{\text{true}}) < 0.65 \\ -0.2300\ln(\Delta X/L_{\text{true}}) + 0.6230 & \text{if } (\Delta X/L_{\text{true}}) \geq 0.65 \end{cases}$$

Thus the $L_{\text{meas}}$ deduced from the uncorrected 2-point correlation measurements and shown in Figures 4.9 and 4.10 have been used with the above equation to deduce the corrected lengthscales shown in Figures 4.11 and 4.12. If the equation for $L_{\text{corrected}}$ was completely valid then the correction would result in lengthscale components from 'any' FoV producing the same 'corrected' value. Figures 4.13 and 4.14 show this is not always the case. The overall shape of the lengthscale distributions remains similar, however the smaller FoV's still produce smaller lengthscale values. In general the two FoV's selected above as producing the best velocity statistics produce curves
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which are fairly close together, and lengthscales deduced from the radial velocity collapse better. Figure 4.13 shows profiles of the axial separation lengthscale based on axial velocity fluctuations. Inspection of the larger FoV's shows that, due the the larger levels of filtering and hence high levels of correction, there is not a monotonic change in lengthscale between various FoV's which might imply a well behaved convergence process. On occasions the 60mm×60mm FoV produces the largest lengthscales. This is attributed to the larger levels of correction needed. For the smaller 25mm×25mm and 40mm×40mm FoV's this does not occur. The non-convergence of the deduced lengthscale values after correction shows that the assumptions that the correction method is based upon are not wholly valid; there are clearly other factors causing the 'lost' energy (e.g inaccuracy in the exponential closures to approximate the 'missing' parts of the correlation distribution). Figure 4.13 illustrates a growth in the lengthscale at the centre of the shear layer ($r/D_j = 0.5$) from $\approx 4\text{mm}$ at $x/D_j = 1.5$ (Figure 4.13a) to $\approx 15\text{mm}$ at $x/D_j = 10$ (Figure 4.13d), which reflects the shear layer width growth seen in the velocity and RMS profiles. Figure 4.14 shows profiles of axial separation lengthscale based on radial velocity fluctuations. Once again the smaller FoV's produce a smaller lengthscale in all of the profiles apart from at $x/D_j = 1.5$. The reason for this is assumed to be inaccuracies with the correction of the larger FoV's and not with the quality of the smaller FoV. It is also possible to notice the better level of convergence of these (generally smaller) lengthscales compared to those in Figure 4.13. Without further information, and noticing that the convergence is better for smaller FoV's, the most accurate measurements are adopted from the 25mm×25mm FoV.

Several airflow experiments over a range of subsonic jet Mach numbers (and at jet Re numbers considerably higher than the present experiment) have provided a variety of lengthscale information at locations within jet shear layers [59, 61, 102]. Lengthscales evaluated from the current 25mm$^2$ FoV 2nd order correlations of both axial and radial velocity fluctuations, and with respect to both axial and radial separation vectors are shown in Figure 4.15 compared to the range of data presented in Fleury et al [102]. It can be seen that the values obtained from the present water flow experiments are in excellent agreement for all four lengthscales with the airflow data obtained using a variety of experimental techniques and for Mach numbers from 0.1 to 0.9. This provides strong support to the argument that the turbulence structures present in low speed water flow experiments, when suitably normalised, are identical to subsonic airflow characteristics.

4.2.2 Timescales

Although not technically a two-point statistic it is appropriate at this point also to analyse the local integral turbulent timescales measured within the jet plume and the relationship between this single point quantity and the FoV size used. Firstly the distribution of integral timescales
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within the jet plume deduced from both axial and radial velocities are shown in Figures 4.16a-b. The axial growth of the axial timescale can be seen to increase on the outer edges of the shear layer from $\approx 0.025\text{seconds}$ at $x/D_j = 2$ to $\approx 0.075\text{seconds}$ by $x/D_j = 10$. This increase reduces the number of independent samples on the outer edge of the shear layer from 123 at $x/D_j = 2$ to 40 at $x/D_j = 10$. Comparing these to the timescales measured along the lipline, $\approx 0.012\text{seconds}$ at $x/D_j = 2$, and $\approx 0.03\text{seconds}$ by $x/D_j = 10$, and the resulting number of independent samples, 256 at $x/D_j = 2$ and 102 at $x/D_j = 10$, it is clear to see that the location of interest plays a fundamental part in the level of convergence one can expect from a fixed sample size and fixed sampling frequency. The radial timescales seen in Figure 4.16b are approximately half the size of the axial timescales, resulting in a doubling of the number of independent samples based on the radial velocity. The smaller the timescale, the fewer samples there are present within one timescale, causing its accuracy to reduce. i.e at $x/D_j = 2$, $T_{22} = 0.007$ which results in 439 independent samples, although there are only 70 samples within that integral timescale.

The effects of the independent samples present within the current study have been discussed in Section 3.2.2 and will be discussed further in Section 4.2.4.

Figure 4.17 shows the effect of FoV size, and associated dynamic filtering, on deduced turbulent timescales. Radial profiles of the timescale based on axial velocity fluctuations are shown. As expected the smaller FoV's with their higher spatial resolution produce profiles without the erratic jumps seen in the larger FoV's. In general, apart from the 100mm×100mm FoV, the profiles collapse well. Figure 4.17a shows that within the region on the inner side of the shear layer the same timescale magnitudes are produced independent of FoV. However, on the outer edge of the shear layer, where the velocity drops and the timescales increase, there is a noticeable difference.

4.2.3 Power Spectra

Measurements taken with various FoV's have shown that raw measured turbulence levels can be influenced through sub-cell scale filtering and dynamic filtering. The causes are changes in the local integral lengthscale and timescale as compared to the local interrogation cell size. The local integral timescale also affects the convergence of the statistics given the sample size and the duration of sampling time. In order to provide further evaluation of the effects of these in the current measurements, Power Spectral Densities (PSD) are plotted (Figure 4.18). Note these PSD are raw, uncorrected PIV data. Loss of energy in the measured data is statistically not likely to occur in the high frequency part of the spectra. Thus, 1kHz data has been used here, resulting in potentially unconverged results below $\approx 20\text{Hz}$, but it is the high frequency part of the spectrum that is of interest. Figures 4.18a-d show the PSD of axial and radial velocity. From these figures it can be seen that the smallest 25mm×25mm FoV shows a small increase in energy contained
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at high frequencies raising the spectra above the -5/3 slope. The -5/3 slope represents the expected natural decay rate of turbulence at high Re, any increase above this slope indicates the presence of additional (unphysical) high frequency energy in the signal (typically instrumentation noise). Both larger FoV's show slight presence of this 'high frequency lift' but remain lower than the 25mm×25mm FoV line. The 60mm×60mm also shows a reduction in energy levels earlier and by a larger amount than the 40mm×40mm data. Since the local integral lengthscale increases downstream, the sub-cell scale filtering reduces, resulting in the 40mm×40mm FoV departing from the 25mm×25mm (and the -5/3 slope) profile at approximately 200Hz at \( x/Dj = 10 \) (Figure 4.18c) compared to 80Hz at \( x/Dj = 1.5 \) (Figure 4.18a). The PSD results for the radial fluctuations re-emphasise the reduction in energy in the larger FoV's at higher frequencies; meanwhile the smallest 25mm×25mm FoV seems able to capture the natural turbulence decay (-5/3 slope) very well up to \( \approx 250 \)Hz and have much lower (nearly no) noise at the highest frequency (500Hz). The significance of these results is that although there are doubts over the correction methods used to correct for sub-cell scale filtering and the use of the exponential curve necessary to close the integral, confidence can be gained that, given a sufficiently small FoV, correction effects are small, very little high frequency energy is lost and only a small amount of noise is present. The 25mm×25mm FoV is identified as the best FoV choice for the present experiment. This allows an increased level of confidence in the measured flow statistics, while the concern over the exponential integral closure method only affects lengthscale values and does not influence the two-point spatio-temporal correlations, which are of primary interest in the present study and are detailed within the following section.

4.2.4 Spatio-Temporal Correlations

Most experimental studies to date have measured only the second order tensor correlations and then only for axial velocity fluctuations with axial spatial separation, with very few studies using spatial separations in the radial and azimuthal directions. Even fewer experimental observations[61, 74] have 'directly' measured the fourth order tensor correlations (those required for noise source models) usually opting to approximate this by products of second order correlations[74]. An investigation into the adequacy of this approximation is provided in Section 4.5. In this study direct measurement and evaluation of all 9 2nd order and 81 4th order correlation components is possible. The symmetry between the tensor indices, implies the 81 4th order components, only possess 36 independent components (i.e. \( U_A U_B U_V V_B = U_A U_B V_B V_B \), where A and B represent \( (x, t) \) and \( (x + \eta, t + \tau) \) respectively). When the correlation peak amplitudes are of interest spatial and temporal separations are zero (\( \eta = 0 \) and \( \tau = 0 \)) reducing the number of independent components and resulting in 6 2nd order and 21 4th order components, as shown in Table 4.5. (Note: bold font represents those requiring stereoscopic PIV).
Table 4.5: 2nd and 4th order correlation components

<table>
<thead>
<tr>
<th>Independent Correlation Components</th>
<th>Repeated Correlation Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>UU</td>
<td></td>
</tr>
<tr>
<td>UV</td>
<td>VU</td>
</tr>
<tr>
<td>UW</td>
<td>WU</td>
</tr>
<tr>
<td>VV</td>
<td></td>
</tr>
<tr>
<td>VW</td>
<td>WV</td>
</tr>
<tr>
<td>WW</td>
<td></td>
</tr>
<tr>
<td>UUUU</td>
<td></td>
</tr>
<tr>
<td>UUUV</td>
<td>VUUU</td>
</tr>
<tr>
<td>UUWW</td>
<td>WUUU</td>
</tr>
<tr>
<td>UUWV</td>
<td>VUWV</td>
</tr>
<tr>
<td>UUUW</td>
<td>WUUU</td>
</tr>
<tr>
<td>UUVV</td>
<td>VVVV</td>
</tr>
<tr>
<td>UUVW</td>
<td>UUWV</td>
</tr>
<tr>
<td>UUWW</td>
<td>WUWW</td>
</tr>
<tr>
<td>UVUW</td>
<td>VUUW</td>
</tr>
<tr>
<td>UVVW</td>
<td>UVWV</td>
</tr>
<tr>
<td>UVWW</td>
<td>UWWV</td>
</tr>
<tr>
<td>UWUW</td>
<td>WUUW</td>
</tr>
<tr>
<td>UWVW</td>
<td>UUWW</td>
</tr>
<tr>
<td>UWWW</td>
<td>WUUW</td>
</tr>
<tr>
<td>VWUW</td>
<td>VUWW</td>
</tr>
<tr>
<td>VWWW</td>
<td>UWVW</td>
</tr>
<tr>
<td>VWWW</td>
<td>WVVW</td>
</tr>
<tr>
<td>WWW</td>
<td>VWWW</td>
</tr>
</tbody>
</table>
Experimental Results

mean and RMS fluctuation statistics namely:

\[ \varepsilon_{<u>} = \frac{z u'}{U_{\text{ref}}} \sqrt{\frac{1}{N_{I-\text{samp}}}} \] \hspace{1cm} (4.1)

\[ \varepsilon_{u'} = \frac{z u'}{U_{\text{ref}}} \sqrt{\frac{1}{2N_{I-\text{samp}}}} \] \hspace{1cm} (4.2)

where \( z \) relates to the confidence band and \( N_{I-\text{samp}} \) is the number of statistically independent samples. If the error is assumed to be normally distributed, then \( z = 2.576 \) for a 99% confidence band.

Given that the current experimental facility produces the same flow features as an equivalent isothermal airflow experiment, the currently adopted sample size (i.e. \( \approx 3 \times 10^3 \)), although shown by Hollis to be large enough to produce convergence of 1st and 2nd moment single point turbulence statistics, could be a limiting factor in capturing accurately converged 2nd and 4th order correlations. To examine this further, 2nd and 4th order averages were produced using the 25mm\( \times \)25mm FoV measurements from different sample sizes. For this exercise, two 3072 sample data sets were combined. Figures 4.19a-b show the convergence of the non-normalised 2nd order \( R_{ij} \) correlations \( ij=11, 12 \) and 22 and Figures 4.19c-f show the convergence of the non-normalised 4th order \( R_{ijkl} \) correlations \( ijk1 = 1111, 1112, 1122, 1212, 1222, \) and 2222 at \( x/D_j=1.5 \) and 10. \( R_{ij}^{\text{mean}} \) and \( R_{ijkl}^{\text{mean}} \) were obtained from the merged 2\( \times \)3072 sample data set. Good convergence is achieved with \( N_{I-\text{samp}} = 3072 \) for both the 2nd and 4th order terms at both locations.

The current monoscopic PIV measurements have been processed to produce correlation maps of axial and radial fluctuations with axial separation (e.g. \( R_{11}(x', \eta_1, \tau) \) etc) in points along the shear layer and the jet centreline (Figures 4.20 and 4.21). It should be noted that the time axis in such maps is usually non-dimensionalised by using a Strouhal number based on the time separation (\( \tau \)), the convection velocity (\( U_c \)) (evaluated from these plots) and the nozzle diameter (\( D_j \)), allowing for direct comparison to previous CTA data. However, Figures 4.20 and 4.21 are presented here with the correlation maps plotted in terms of dimensional axial and temporal separation, to emphasise the physical distance the correlations covers, the physical time taken to decay, and also to aid the visualisation of the eddy convection velocity (gradient). It can be seen that the convection velocity is virtually constant for all locations and for both 2nd and 4th order correlations (Figures 4.22). Table 4.6 gives a numerical comparison of the 2nd and 4th order convection velocity deduced from the gradients seen in Figures 4.20 - 4.22.
Experimental Results

\[ \frac{U_c}{U_j} \]

<table>
<thead>
<tr>
<th>Axial Station</th>
<th>( \frac{r}{D_j} )</th>
<th>( \frac{r}{D_j} )</th>
<th>( \frac{r}{D_j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x}{D_j} = 1.5 )</td>
<td>0.63</td>
<td>0.62</td>
<td>0.65</td>
</tr>
<tr>
<td>( \frac{x}{D_j} = 4 )</td>
<td>0.65</td>
<td>0.63</td>
<td>0.64</td>
</tr>
<tr>
<td>( \frac{x}{D_j} = 6.5 )</td>
<td>0.67</td>
<td>0.65</td>
<td>0.68</td>
</tr>
<tr>
<td>( \frac{x}{D_j} = 10 )</td>
<td>0.65</td>
<td>0.61</td>
<td>0.67</td>
</tr>
<tr>
<td>( \frac{x}{D_j} = 6.5 )</td>
<td>0.96</td>
<td>0.94</td>
<td>0.92</td>
</tr>
<tr>
<td>( \frac{x}{D_j} = 10 )</td>
<td>0.80</td>
<td>0.78</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Table 4.6: Convection velocity \( \frac{U_c}{U_j} \) for LU40

In most previous experiments the convection velocity has been measured for low speed unheated jets \([60, 61]\) on the lipline both upstream and downstream of the close of the potential core. Such experiments show the convection velocity to be approximately given by \( U_c \approx 0.65U_j \). Current results (Table 4.6) are consistent with these experimental findings. Note, however, that values on the centreline differ from those in the shear layer. It is noteworthy that there is significant change in the convection velocity with radial location across the shear layer, as would be expected given the large mean shear. Table 4.7 shows the convection velocity at \( \frac{x}{D_j} = 4 \) for varying radial locations across the shear layer.

<table>
<thead>
<tr>
<th>Location</th>
<th>( \frac{r}{D_j} )</th>
<th>( \frac{r}{D_j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{r}{D_j} = 0.4 )</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>( \frac{r}{D_j} = 0.45 )</td>
<td>0.69</td>
<td>0.70</td>
</tr>
<tr>
<td>( \frac{r}{D_j} = 0.5 )</td>
<td>0.65</td>
<td>0.63</td>
</tr>
<tr>
<td>( \frac{r}{D_j} = 0.55 )</td>
<td>0.55</td>
<td>0.57</td>
</tr>
<tr>
<td>( \frac{r}{D_j} = 0.6 )</td>
<td>0.48</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Table 4.7: Convection velocity \( \frac{U_c}{U_j} \) for LU40 at \( \frac{x}{D_j} = 4 \)

From Figures 4.20a-d, a gradual spread of the correlation map can be seen, with an associated increase in lengthscale (horizontal axis through \( \tau = 0 \)). and in the time taken for the correlated area to pass the reference point and hence an increase in the timescale (vertical axis slice through \( \eta_1 = 0 \)). Figure 4.21 for the radial fluctuation correlation \( R_{22} \) illustrates the recurrence of highly correlated areas within the same correlation map. This reflects the movement of correlated structures past the same location at different temporal separations. This is more likely to be seen closer to the nozzle exit as illustrated in Figure 4.21b due to the presence of vortex ring structures, and becomes less likely as the jet plume becomes fully developed and self similar.
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The correlation maps for radial fluctuations (Figures 4.21e-f) on the centreline generally indicate a narrow region of correlation. Comparisons between Figures 4.20a-f and 4.21a-f show the dominance of the axial fluctuations compared to the small and short lived radial structures. A visual assessment of the quality (low spurious noise levels) of the PIV data correlation maps shows well defined edges to the correlation contours and a constant zero background correlation level, indicating low noise levels and high quality. This low noise level trend continues when evaluation of the 4th order correlations of axial fluctuation with axial separation (e.g. \( R_{1111}(\overline{x}, \eta_1, r) \)) is performed, as shown in Figure 4.22. The natural spread in lengthscale and timescale of this component can be seen to be significantly smaller and shorter lived than the 2nd order correlations. The convection velocity for \( R_{1111} \) is also in good agreement with the 2nd order values. It is however, the excellent clarity in the distribution of the 4th order correlation maps which is extremely striking. To illustrate the quality of this directly measured 4th order correlation a comparison between the current PIV data and previous PIV data of Bridges et al[74] is shown in Figures 4.23 and 4.24 for the \( R_{1111} \) and \( R_{2222} \) correlations respectively. The contrast in noise levels between the two directly measured PIV datasets provides confidence in the current experimental practices and the ability to capture accurately the 4th order correlations is very promising for providing information to aid noise sound acoustic modelling.

4.2.5 Lagrangian Scales

The Lagrangian lengthscale is a convenient choice of lengthscale for the correlation volume. This lengthscale is defined by the spatial separation at which the peak auto-correlation value decays to \( \frac{1}{e} \) of its initial value in a moving frame of reference (i.e. along the eddy convection velocity axis seen in the spatial correlation maps). The lengthscales obtained in this way are shown in Table 4.8.

<table>
<thead>
<tr>
<th>Location</th>
<th>( L_{11}/D_j )</th>
<th>( L_{22}/D_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x/D_j = 1.5 r/D_j = 0.5 )</td>
<td>0.72</td>
<td>0.46</td>
</tr>
<tr>
<td>( x/D_j = 4 r/D_j = 0.5 )</td>
<td>1.15</td>
<td>0.55</td>
</tr>
<tr>
<td>( x/D_j = 6.5 r/D_j = 0.5 )</td>
<td>1.27</td>
<td>0.62</td>
</tr>
<tr>
<td>( x/D_j = 10 r/D_j = 0.5 )</td>
<td>1.35</td>
<td>0.72</td>
</tr>
<tr>
<td>( x/D_j = 6.5 r/D_j = 0 )</td>
<td>2.07</td>
<td>1.65</td>
</tr>
<tr>
<td>( x/D_j = 10 r/D_j = 0 )</td>
<td>1.86</td>
<td>1.19</td>
</tr>
</tbody>
</table>

Table 4.8: Lagrangian lengthscale at all stations

These Lagrangian lengthscales are plotted against the axial location in Figure 4.25, where it is seen that the Lagrangian lengthscales based on both axial and radial velocity along the lipline

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increase as the axial distance increases. At \( \frac{x}{D_j} = 4 \) the lengthscales settle to a constant growth rate of \( 0.053 \frac{1.1L_1}{x} \) and \( 0.016 \frac{1.2L_2}{x} \) up to \( \frac{x}{D_j} = 10 \) (which is beyond the end of the potential core). The growth rate of the Lagrangian lengthscales along the centreline are seen to decrease as the axial distance increases at a rate of \( 0.053 \frac{1.1L_1}{x} \) and \( 0.016 \frac{1.2L_2}{x} \). Figure 4.25 also shows that the axial Lagrangian lengthscales based on axial velocity are about one half of the Lagrangian lengthscales based on radial velocity.

Comparison with data from Harper-Bourne[61] for the 2\(^{nd}\) and 4\(^{th}\) order Lagrangian lengthscales at \( \frac{x}{D_j} = 4 \) for all 3 directional lengthscales when based on axial velocity are shown below in Tables 4.9 and 4.10:

<table>
<thead>
<tr>
<th></th>
<th>(1L_1^L/D_j)</th>
<th>(2L_1^L/D_j)</th>
<th>(3L_1^L/D_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harper-Bourne's CTA data[61]</td>
<td>1.14</td>
<td>0.114</td>
<td>0.101</td>
</tr>
<tr>
<td>Current PIV data</td>
<td>1.15</td>
<td>0.118</td>
<td>0.099</td>
</tr>
</tbody>
</table>

Table 4.9: Comparison of 2\(^{nd}\) order Lagrangian lengthscales of axial velocity between CTA data[61] and PIV data at \( \frac{x}{D_j} = 4, \frac{r}{D_j} = 0.5 \)

<table>
<thead>
<tr>
<th></th>
<th>(1L_{1111}^L/D_j)</th>
<th>(2L_{1111}^L/D_j)</th>
<th>(3L_{1111}^L/D_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harper-Bourne's CTA data[61]</td>
<td>0.515</td>
<td>0.071</td>
<td>0.073</td>
</tr>
<tr>
<td>Current PIV data</td>
<td>0.426</td>
<td>0.058</td>
<td>0.066</td>
</tr>
</tbody>
</table>

Table 4.10: Comparison of 4\(^{th}\) order Lagrangian lengthscales of axial velocity between CTA data[61] and PIV data at \( \frac{x}{D_j} = 4, \frac{r}{D_j} = 0.5 \)

The Lagrangian lengthscales of Harper-Bourne's[61] CTA data compares excellently with the measured lengthscales of the current PIV measurements for both 2\(^{nd}\) and 4\(^{th}\) order statistics.

The Lagrangian timescale is the time taken for the peak auto-correlation value to decay to \( \frac{1}{e} \) of its initial value in the moving frame of reference. The Lagrangian timescales are obtained directly from the correlation data (via extrapolation identical to that used for the Lagrangian lengthscales) and through calculation of the convection velocities (shown in Table 4.6) and the Lagrangian lengthscales. Both of these methods produced virtually identical results illustrating the accuracy of the data extrapolation employed. Table 4.11 shows the timescales at all stations.

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<table>
<thead>
<tr>
<th>Location</th>
<th>$^1T_{11}^L$</th>
<th>$^1T_{22}^L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x/D_j = 1.5r/D_j = 0.5$</td>
<td>0.046</td>
<td>0.030</td>
</tr>
<tr>
<td>$x/D_j = 4r/D_j = 0.5$</td>
<td>0.071</td>
<td>0.035</td>
</tr>
<tr>
<td>$x/D_j = 6.5r/D_j = 0.5$</td>
<td>0.076</td>
<td>0.038</td>
</tr>
<tr>
<td>$x/D_j = 10r/D_j = 0.5$</td>
<td>0.083</td>
<td>0.047</td>
</tr>
<tr>
<td>$x/D_j = 6.5r/D_j = 0$</td>
<td>0.086</td>
<td>0.070</td>
</tr>
<tr>
<td>$x/D_j = 10r/D_j = 0$</td>
<td>0.093</td>
<td>0.061</td>
</tr>
</tbody>
</table>

Table 4.11: Lagrangian timescale at all stations

These timescales show the duration of the turbulence in a moving frame of reference. Similar to the integral timescales, within the shear layer, both the axial ($^1T_{11}^L$) and radial ($^1T_{22}^L$) velocity based correlation decay time increases as axial distance increases, while the $^1T_{11}^L$ is twice that of $^1T_{22}^L$ at all locations. Along the centreline however, the axial velocity based correlation decay increases with increased axial distance whilst the radial velocity based correlation decay actually reduces.

4.3 LU40 versus LU40P

The primary objective of this thesis was to investigate the spatio-temporal correlations used to drive noise source modelling. This included the design of two round nozzles to explore the sensitivity of the near nozzle exit shear layer to small nozzle lip changes. Measurements were therefore taken to examine the spatio-temporal correlations for there different nozzle lip changes. A comparison between a pure contraction nozzle (LU40) and a nozzle with a short parallel-walled section at the end of the nozzle after the contraction (LU40P) was performed and is detailed within this section. The addition of the parallel-walled section provides a region at the end of the nozzle in which acceleration was removed. This allows some distance over which the nozzle wall boundary layer could recover (perhaps from a nearly laminarised state due to the strong effects of favourable pressure gradient) prior to leaving the nozzle to form the initial jet free shear layer. The LU40P nozzle featured a parallel extension of $\approx 36$mm compared to the 40mm nozzle exit diameter. From the findings of Sections 4.1 and 4.2 it was decided that only 40mm$x$40mm and 25mm$x$25mm FoV's would be used. Results are again presented in terms of single point and two point statistics.
4.3.1 Single Point Statistics

Figure 4.26 shows radial profiles of the mean axial velocity at four axial stations along the jet axis. At $x/D_j = 1.5$ and $x/D_j = 4$ any effects of the parallel extension on the mean profiles are non evident. The profiles are nearly identical with the growth of the shear layer (gradient region) increasing equally in both LU40 and LU40P measurements. The profiles are not non-dimensionalised in order to provide evidence to show that both jet centreline velocities are equal. At $x/D_j = 6.5$ the reduction in centreline velocity is slightly larger in the LU40P measurements indicating the closure of the potential core has occurred slightly earlier than in the LU40 nozzle. The LU40 nozzle shows a centreline velocity of $U \approx 0.98U_j$, whereas the LU40P nozzle indicates $U \approx 0.97U_j$. By $x/D_j = 10$ this difference has increased to 0.86m/s (LU40) vs 0.82m/s (LU40P), (3%). The slightly earlier and enhanced decay of the LU40P nozzle indicates a slight faster growing shear layer, consistent with a more turbulent state at nozzle exit for LU40P.

Figure 4.27 shows the axial development of the axial RMS velocity. As before, these give a good insight into any difference in jet plume development. Profiles at $x/D_j = 1.5$ within the shear layer indicate already noticeable variations. A peak value of 0.140 is observed for LU40 compared to the larger peak of 0.145 for the LU40P nozzle. The LU40P profile is also slightly wider and displays signs of higher turbulence levels in the coflow region, possibly due to the change in the boundary layer on the external walls of the nozzle due to the extension. As the downstream axial distance increases the difference in the peak magnitude decreases. This indicates the effect of the extension is restricted mainly to the transition zone between the two boundary layers on the internal / external nozzle walls merging to become a free shear layer. Meanwhile the centreline magnitude increases slightly in accordance with the shortening of the potential core. By $x/D_j = 6.5$ the peak magnitude is virtually identical but the centreline turbulence levels are significantly increased ($U_{rms} \approx 0.06$) for LU40P compared to the LU40 nozzle ($U_{rms} \approx 0.05$). This is confirmed in Figure 4.27e focussing on the centreline area. Finally, at $x/D_j = 10$ there is some deterioration in the statistical convergence of the data causing further conclusions to be hard to determine.

The thickness and growth of the jet shear layer in the initial region was considered next. Fondse et al [146] and Hussain and Zedan [147] used the growth of the shear layer thickness $\frac{dB}{D(x/D)}$ to investigate the influence of the nozzle exit conditions. The shear layer thickness is defined as:

$$B = r_{0.05} - r_{0.95}$$

Where $r_{0.05}$ and $r_{0.95}$ denote the points at which the axial velocity has decreased to 0.05 and 0.95 of centreline velocity. Figure 4.28 shows the growth of the shear layer in the first 6.5 nozzle
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exit diameters for both nozzle designs. Results from Trumper [143] for his 48mm nozzle are also shown (denoted LU48 and LU48P). For both sets of data the shear layer growth rate, based on a linear fit of the shear layer thickness, is slightly greater for the nozzle with the parallel extension compared to the contraction only nozzle, with an increase of 7.1% between the LU40 and LU40P nozzles and an increase of 8.6% for Trumper's [143] data. The slight difference between the current measurements and those of Trumper's [143] data is probably due to different conditions at the nozzle exit (boundary layer thickness) given the shear layer sensitivity to this.

4.3.2 Two Point Statistics

The variation of the $L_{11}$ and $L_{22}$ lengthscales for the two nozzles is presented in the form of radial profiles in Figures 4.29 and 4.30. The small differences between lengthscales extracted from different FoV sizes is not discussed here as this has previously been addressed. Inspection of the radial profiles of $L_{11}$ shows that there is very little variation across the jet plume, particularly for the smaller (more accurate) 25mm×25mm FoV. A decrease in lengthscale is evident towards the shear layer outer edge (Figure 4.29b). Close inspection shows an increase within the high shear zone as the end of the potential core is approached. The same strong similarity between the radial profiles of axial lengthscale of radial velocity ($L_{22}$) can be seen in Figure 4.30. Note also that this lengthscale increases towards the axis for $x/D_j$ less than the potential core length, since within the potential core the lengthscales are determined by the nozzle exit conditions rather than the shear layer conditions. This illustrates that even the smaller features of the flow (generally more sensitive to change) are unchanged by the presence of a parallel extension nozzle compared to a contraction only nozzle.

For completeness the axial timescale ($T_{11}$) is shown in Figure 4.31. Once again the similarity between the two nozzles is undoubtedly strong. Figure 4.31b shows a noticeable difference between the 40mm×40mm FoV profiles for the LU40 and LU40P nozzles on the outer edge of the shear layer. With the use of smaller FoV's (with less sub-cell scale filtering) the agreement between the 25mm×25mm FoV profiles is much better.

To provide insight into the sensitivity of the two-point spatio-temporal correlations to the nozzle geometry and hence the initial boundary layer to free shear layer transition, $2^{nd}$ and $4^{th}$ order axial velocity correlations with axial separation ($R_{11}(\eta_1, \tau)$ and $R_{1111}(\eta_1, \tau)$) have been calculated and are shown in Figures 4.32 and 4.32. The match for the $2^{nd}$ order correlation in space and time at all location in the shear layer, and on the centreline is excellent, even considering the slightly earlier closure of the potential core for the LU40P nozzle. The $4^{th}$ order correlations are also in good agreement, although on close inspection the correlations produced from the LU40P nozzle are slightly smaller in both space and time compared to the LU40 nozzle.
at each location. This is however, not surprising considering the slightly different growth rates of the shear layer as seen in Figure 4.28. As discussed previously, the gradient of the correlation map plots yield the convection velocity. Table 4.12 presents the convection velocities at various locations for both LU40 and LU40P nozzles. The agreement in velocities along the shear layer is excellent. The agreement along the centreline shows a lower convection velocity for the LU40P nozzle, although once again this is due to the shorter potential core length.

<table>
<thead>
<tr>
<th>Location</th>
<th>LU40</th>
<th>LU40P</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x/D_j = 1.5 \ r/D_j = 0.5)</td>
<td>0.63</td>
<td>0.65</td>
</tr>
<tr>
<td>(x/D_j = 4 \ r/D_j = 0.5)</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>(x/D_j = 6.5 \ r/D_j = 0.5)</td>
<td>0.67</td>
<td>0.65</td>
</tr>
<tr>
<td>(x/D_j = 10 \ r/D_j = 0.5)</td>
<td>0.65</td>
<td>0.63</td>
</tr>
<tr>
<td>(x/D_j = 6.5 \ r/D_j = 0)</td>
<td>0.96</td>
<td>0.92</td>
</tr>
<tr>
<td>(x/D_j = 10 \ r/D_j = 0)</td>
<td>0.80</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Table 4.12: Convection velocity for LU40 and LU40P

4.4 Monoscopic 2C-PIV versus Stereoscopic 3C-PIV

The technique of stereoscopic PIV provides the capability simultaneously to capture all three velocity components. In turn this allows for all correlation components and subsequently all lengthscales and timescales to be calculated. Advantages of stereoscopic PIV also include the ability to view planes previously impossible via monoscopic PIV. Stereoscopic PIV data has been gathered in six \(x\theta\) planes (as in the monoscopic PIV data) as well as six \(r\theta\) planes at axial locations of interest along the shear layer (planes centred on points \((x/D_j = 1.5, r/D_j = 0.5), (x/D_j = 4, r/D_j = 0.5), (x/D_j = 6.5, r/D_j = 0.5), (x/D_j = 10, r/D_j = 0.5), (x/D_j = 6.5, r/D_j = 0)\) and \((x/D_j = 10, r/D_j = 0))

The PIV set-up parameters (such as size of FoV) were optimised as described in Sections 4.1 and 4.2 using monoscopic 2C-PIV of the \(x\theta\) plane velocity field. With the introduction of a stereoscopic 3C-PIV set-up, initial measurements were conducted to judge the quality of the 3C-PIV data in comparison with the 2C-PIV data. It was expected that there would be some deterioration and an associated increase in measurement error but to what degree was of interest.
4.4.1 Single Point Statistics

Figure 4.34 shows radial profiles of the mean axial velocity for 3C-PIV data extracted from both $xr$ and $r\theta$ planes at all axial stations along the shear layer compared with 2C-PIV data. In general the 3C-PIV data taken with the laser sheet in the $xr$ plane are in slightly better agreement with the 2C-PIV measurements although overall the similarity is excellent. On close inspection Figure 4.34b shows slightly lower measurements for the mean velocity on the outer edge of the shear layer, although by $x/D_j = 6.5$ this is no longer present. When the laser sheet is in the $r\theta$ plane configuration the streamwise velocity is the out-of-plane component. In order to capture this large out-of-plane velocity the interframe time must be reduced and this change increases the inaccuracies in particle shift identification. The mean profiles of this out-of-plane velocity are however, still in very good agreement with the 2C-PIV measurements.

Figures 4.35a-d show comparison of radial profiles of axial RMS velocity. The peak magnitude produced by the $xr$ 3C-PIV measurements shows an excess above the 2C-PIV data by $\approx 13\%$ at $x/D_j = 1.5$. The over-estimate decreases slightly as the plume develops although always remains larger than the 2C-PIV results. In spite of the increase in the RMS magnitude, the shape and radial distribution of the $xr$ 3C-PIV data is in good agreement with the 2C-PIV data. This indicates that the shear layer thickness is captured accurately by the $xr$ 3C-PIV measurements, implying that the increase in RMS magnitude is caused by an additional and approximately constant source of additional energy. This is perhaps most likely caused by some form of additional instrumentation / experimental noise.

The $r\theta$ 3C-PIV data is in better agreement with the $xr$ 2C-PIV data in both shape and magnitude than the $xr$ 3C-PIV results. Surprisingly, this uniform increase in turbulence level is not apparent in the $r\theta$ 3C-PIV measurements. Performing 3C-PIV in the $r\theta$ plane leads to the out-of-plane motion being the largest velocity component present. This requires a short interframe time to capture the particles moving through the thin light sheet. The particle shift is only accurate to 0.1 pixels, and with the short movement of particles across the light sheet, the error levels are increased. The agreement between the $r\theta$ 3C-PIV data and the $xr$ 2C-PIV data looks good. This could contradict the 'additional noise in 3C-PIV data' argument, however, it is assumed that this argument is still true and that the increase error levels in the $r\theta$ 3C-PIV data (due to small particle shift through the light sheet) causes a cancelling of errors (not seen in the $xr$ 3C-PIV data).
4.4.2 Two Point Statistics

Turning attention to two-point statistics, radial profiles of the $^2L_{11}$ and $^2L_{22}$ lengthscales are shown in Figures 4.36 and 4.37. The axial lengthscales of axial velocity are the larger scale (Section 4.2), however, to allow for comparison of the $r\theta$ plane 3C-PIV data (where no axial separation is captured) only radial separation lengthscales for axial and radial velocity are presented. Figure 4.36 shows $^2L_{11}$. The match between $xr$ 2C-PIV and the corresponding $xr$ 3C-PIV profiles show a fairly constant underestimate ($\approx 17\%$) in the stereoscopic configuration. The 3C-PIV data captured in the $r\theta$ orientation show a much stronger match to the 2C-PIV results as perhaps expected from the RMS results. This being said, the variation of the $r\theta$ results from the 2C-PIV data show no constant shift, instead a range of variation with both over and underestimates. Examination of the radial lengthscales of radial velocity indicates a very strong agreement between the $xr$ 2C-PIV results and the $r\theta$ 3C-PIV result (with the exception of the inner edge of the shear layer, $r/D_j = 0.2$, at $x/D_j = 1/5$). The match between the $xr$ 2C-PIV and the $xr$ 3C-PIV is much better than seen for $^2L_{11}$, but still slightly under-estimating the 2C-PIV results.

As mentioned previously, the identification of both spatial and temporal scales is of great importance. The temporal scales can be affected by dynamic filtering, and although similar FoV sizes were adopted in both 2C and 3C-PIV measurements, it is important to assess the ability of the 3C-PIV results to capture the same temporal scales as the 2C-PIV results. Radial profiles of axial timescales are therefore examined and presented in Figures 4.38a-f. The agreement between the 2C-PIV and both the $xr$ and $r\theta$ 3C-PIV results is excellent at all axial locations.

To gain a better insight into the overestimate of the axial RMS in the 3C-PIV measurements, Power Spectral Density results are shown in Figure 4.39. At all stations the notable deviation at frequencies greater than $\approx 100\text{Hz}$ is evidence of the increased contamination by noise in the 3C-PIV data. Although not shown here, repeated tests produced very similar spectra giving high confidence in the data. There does seem evidence, however, of some systematic / experimental noise present in the 3C-PIV data at high frequencies, probably caused by the increased complexity of calibration. It is therefore necessary, if the stereoscopic PIV data is to be used fully in the present study, for the stereoscopic data to be corrected for these high noise levels, whilst maintaining the accuracy of the raw data. This correction is detailed in the following section.
4.4.3 3 Component Correction

A typical visual representation of the fluctuation velocity vector field at $x/D_j = 6.5$ is shown in Figures 4.40a-b. Figure 4.40a shows the original raw vector field of the $x$-3C-PIV. It is possible to notice the presence of spurious vectors which have not been removed during the normal vector post processing. These spurious vectors could be the cause of the high frequency noise in the spectra. Modification of the postprocessing parameters (described in Section 3.2) to increase the removal of spurious vectors was conducted and the data was postprocessed. The new postprocessing parameters are shown in Table 4.13 (compared with Table 3.4). Care must of course be taken when applying very strong parameters to avoid smoothing the data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Validation</td>
<td></td>
</tr>
<tr>
<td>Remove vectors with Q-ratio</td>
<td>$&lt;2$</td>
</tr>
<tr>
<td>Remove vectors outside range</td>
<td>$&lt;1.1$</td>
</tr>
<tr>
<td>Replace removed vectors</td>
<td>With 2nd, 3rd or 4th choice peaks or interpolated data where no peak fits surrounding fluid dynamic behaviour.</td>
</tr>
</tbody>
</table>

Table 4.13: Refined parameters for vector processing, calculation, and validation

Figure 4.40b shows the fluctuation velocity vector field at $x/D_j = 6.5$ postprocessed using the parameters in Table 4.13. A reduction in the number of spurious vectors can be seen, although even with this refined postprocessing it is possible to identify some remaining spurious vectors. Figures 4.41a-f show the effects of this stronger postprocessing on the energy content of the PSD curves. Although there is some reduction in the high frequency noise, there is still a noticeable deviation of the 3C-PIV data above the 2C-PIV data at high frequency. In order to reduce this high frequency noise a different method to simple spurious vector removal is necessary.

It is of great importance that any correction method used does not affect the accuracy of the raw data in regions which are not contaminated by high frequency noise. The easiest and widely understood form of high frequency noise filtering is downsampling. Due to the limited sample size (3072 time slices) of the current PIV data, any downsampling would worsen the convergence level of statistics evaluated from the dataset and reduce the accuracy of the results. Hence another form of filtering is needed in order to remove the presence of the identified spurious high frequency energy. It is suggested here that the method of Proper Orthogonal Decomposition (POD) can be used for this purpose. POD is first used to analyse the velocity field and identify a hierarchy of spatial modes, ordering them in terms of decreasing contribution to the overall energy. From this
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A series of velocity fields can be reconstructed containing only the most energetic structures in, for example, only the first $N$ modes. An example of the effect on the PSD of such reconstruction of the flowfield from different numbers of modes (corresponding to certain fractions of the total energy contained in the raw 3C-PIV data) is shown in Figure 4.42. This figure shows that if a large amount of energy is filtered out, for example by selecting a small number of modes $N$ (e.g. leads to just 20, 40 or 60% of the total energy in the reconstructed field), then the original PSD is altered not only in the high frequency region where the spurious high frequency noise lift is present but also genuine energy at frequencies in the spectrum are also affected. However, if a larger number of modes is selected, say $N=313$, which filters only 20% of the raw energy, leaving 80% in the reconstructed field, then the resulting PSD is very close to the 2C-PIV PSD over practically the whole range of temporally resolved frequencies. For example, the raw 3C-PIV with all 3072 modes departs on the high side away from the 2C-PIV data at $\approx 50$Hz. If the 80% POD filtered 3C-PIV spectrum is used, this remains similar to the 2C spectrum up to $\approx 300$Hz, a six fold improvement.

A method was therefore developed whereby a reconstruction using a ‘corrected’ number of modes may be identified. It is proposed that the ‘corrected’ number of modes ($N^*$) can be defined via an evaluation of the cumulative energy content from $N^*$ modes such that it is equal to the total energy content of the raw data (all modes) minus an estimated amount of energy in the high frequency portion of the spectra with is assumed to be spurious. The ‘corrected’ number of modes ($N^*$) may vary from point to point across the FoV and from location to location in the jet plume. It is however, the opinion of the author that this method should only be considered in cases where the fraction of energy attributed to noise contribution is lower than $\approx 30\%$ of the overall energy content. For the sake of clarity it is important to state that when this method is undertaken, the energy content of the reconstructed field is considered to be a best estimate of the true energy present within the flow field, since the percentage removed is considered as spurious noise superimposed onto a correct PSD shape.

An illustration of this method is given in Figure 4.43 and further explained here. The amount of spurious energy identified for removal is sensitive to the limits of integration selected, as illustrated in Figure 4.43. For this study the upper limit has been set to the maximum resolved frequency (500Hz). The lower limit has been set at the location at which it is considered high frequency spurious noise has started to enter the signal. In the example shown here (and all data presented), the departure from the 2C-PIV spectrum has been used to identify the lower limit. It should be pointed out that in general this will of course not be available, but the departure from the -5/3 spectrum could just as well (with similar results) have been used.
Using this method, Table 4.14 shows for varying points, the amount of spurious energy identified within the $xr$ plane 3C-PIV measurements which needs to be removed in order to correct the 3C-PIV data spectra.

<table>
<thead>
<tr>
<th>Location</th>
<th>Spurious energy $\Delta E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x/D_j = 1.5, r/D_j = 0.5$</td>
<td>29.3 %</td>
</tr>
<tr>
<td>$x/D_j = 4, r/D_j = 0.5$</td>
<td>20.5 %</td>
</tr>
<tr>
<td>$x/D_j = 6.5, r/D_j = 0.5$</td>
<td>14.0 %</td>
</tr>
<tr>
<td>$x/D_j = 10, r/D_j = 0.5$</td>
<td>22.3 %</td>
</tr>
<tr>
<td>$x/D_j = 6.5, r/D_j = 0$</td>
<td>10.7 %</td>
</tr>
<tr>
<td>$x/D_j = 10, r/D_j = 0$</td>
<td>12.1 %</td>
</tr>
</tbody>
</table>

Table 4.14: Energy to be removed from $xr$ 3C-PIV measurements

Once this spurious energy level has been identified, the POD mode analysis is then used to see at what value of $N^* (<3072)$ the energy to be removed is identified (see Figure 4.43).

Figures 4.44a-b show the original fluctuation velocity vector field at $x/D_j = 6.5$ of the raw $xr$ 3C-PIV, and the fluctuation velocity vector field following the method defined above. Figure 4.44b illustrates that the POD-based filtering method has removed the spurious vectors as well as the high frequency spurious noise, while the larger coherent structures have become more defined. The filtered fluctuation velocity vectors removed from the FoV are shown in Figure 4.45. Excluding the large spurious vectors, the average change to the vector field is small in comparison to the original velocity magnitude and corresponds very clearly to incoherent energy modes. Figures 4.46a-f show the effects of this POD-based filtering method on the resulting PSD. The reduction in the high frequency noise is considerable and the comparison between the 2C-PIV spectra and the filtered 3C-PIV spectra is now excellent for all locations.

All 3C $xr$ and $r\theta$ plane data have been filtered using this method with the exception of the $r\theta$ 3C-PIV data for $x/D_j = 1.5$ and $x/D_j = 4$ as these required no filtering (as illustrated in Figure 4.39). Following the filtering procedure outlined above for the 3C-PIV measurements, radial profiles of axial RMS velocity were examined again at shear layer locations. Figures 4.47a-d show the original $xr$ 3C-PIV and $xr$ 2C-PIV compared to the new filtered $xr$ 3C-PIV (denoted as $xr$ 3C-PIV*). The improvement in the agreement between the 2C and 3C filtered $xr$ plane data is extremely promising and provides evidence to support the POD methodology derived as a frequency filter which also improves the usually evaluated statistics. At $x/D_j = 1.5$ the profile has dropped almost exactly onto the 2C-PIV profile producing the same distribution and peak magnitude. Moving downstream the improved match is very noticeable. The presence of
increased statistical noise on the profiles is not surprising considering that the removal of the
spurious noise results in an increased prominence of the energetic modes. Whilst it is acknowl-
edged that this approach results in fewer independent samples and less statistically converged
solutions, it is believed that the effects of this method are minimal compared to the reduction in
independent samples if down sampling had been used.

2nd order spatio-temporal plots for streamwise separations were produced for all measurement
stations. Comparisons between $x_r$ 2C-PIV and $x_r$ 3C-PIV as well as with filtered $x_r$ 3C-PIV* are shown in Figures 4.48a-f. The agreement for the 2nd order quantity is very good. The peak
amplitude decay and shift seen in the axial component shows the convection velocity at this
point is in good agreement between all PIV data sets. There is, however, a large underestimate
of the peak correlation magnitude value for the original 3C-PIV measurements (as expected).
The correlations produced from the filtered 3C-PIV* data show the stereoscopic technique with
the addition of the POD filter can produce the same correlation curves as the 2C-PIV data. On
the basis of this evidence, this filtering methodology has been used for all further 3C PIV data
presented.

4.4.4 Spatio-Temporal Correlations

It is now appropriate to return to the main aim of this study, namely to investigate the spatio-
temporal correlations which are fundamental to the development of jet noise source modelling.
Up to this point, single and two point statistics have been examined from PIV data obtained from
monoscopic and stereoscopic configurations. They have provided evidence that the captured flow
field is representative of airflow experiments of free jets, and is both spatially and temporally
well resolved. This section will thus now present, what are believed to be the first detailed and
comprehensive evaluation of the 2nd and 4th order spatio-temporal correlations at various loca-
tions within a jet plume. Comparison is conducted against previous hot-wire experimental data
of Davies et al[30] and Harper-Bourne[61] and LDV data of Chatellier et al[113] where available,
while examination of all 6 2nd order and 21 4th order independent correlation components within
the shear layer, and for the first time, along the jet centreline is presented.

4.4.5 Correlation Distributions

A full comparison of 2nd and 4th order correlation at $x/D_j = 4, r/D_j = 0.5$ has been produced
with CTA data by Harper-Bourne[61], while 2nd order correlation data is also available at $x/D_j =
1.5$ from Davies et al[30] and at $x/D_j = 4$ from Chatellier et al[113]. Figure 4.49 presents
correlation distributions at $x/D_j = 1.5$ for axial velocity with axial separation, while Figures
4.50 - 4.52 present the axial velocity correlation distributions at $(4D_j, 0.5D_j, 0°)$ for axial, radial
and circumferential separations respectively. Both monoscopic PIV and filtered stereoscopic PIV results are shown. For all figures, the horizontal axis has been plotted in non-dimensional form \( \tau U_c/D_j \), where the convection velocity has been set to \( U_c = 0.6U_j \) for all the data presented. By applying a fixed convection velocity value, any difference between the local convection velocity in each experimental flowfield can be identified.

Figure 4.49a illustrates the comparison between the 2C and 3C PIV data and the CTA results of Davies et al[30] at \( x/D_j = 1.5 \). The solid 2C-PIV lines represent data from the more accurate 25mm×25mm FoV, while the dashed line represents data from the 40mm×40mm FoV. The increase in peak magnitudes due to increased sub-cell filtering is shown here to illustrate the sensitivity of the level of sub-cell filtering on the comparison to previous data, which itself is subject to filtering. The agreement between the convection velocity is seen by the match in peak correlation locations at all separation values. The PIV data does however, show a lower peak correlation value than is produced by the CTA data. The similarity in the correlation shapes for each separation provides strong evidence that the PIV results contain turbulent scales and behaviour as captured in airflow experiments of free jets. Figure 4.49b shows the 4\( \text{th} \) order correlation plots for the PIV data, however, no additional experimental data is available for comparison. The smoothness of the curves implies reasonable convergence of the statistics, while the strong comparison between the 2C and filtered 3C PIV illustrates the accuracy of the data. The faster reduction of the 4\( \text{th} \) order correlation indicates the shorter life span of the correlation, which is consistent with a greater high frequency content.

Figure 4.50a-d presents the 2\( \text{nd} \) and 4\( \text{th} \) order correlation for axial separation \( x/D_j = 4 \). The comparison shows good agreement between the PIV and the CTA results, while the combined LDA/PIV results shows the superiority of the 'pure' PIV method. Close inspection between PIV and CTA data shows a slightly higher convection velocity for the PIV results, while the CTA data once again predicts slightly higher 2\( \text{nd} \) order correlation magnitudes at all separation values, although this difference reduces for the 4\( \text{th} \) order correlations. It is fair to conclude, that both methods will contain some error; the CTA has very high temporal resolution, however, due to anti-aliasing methods during acquisition, some high frequency content is lost (this causes increased correlation peak magnitudes - as shown later in Section 5.2). The PIV technique, although subject to sub-cell filtering (which has been shown to be minimal for the 25mm×25mm FoV), is also subject to spurious vector / instrumentation noise (even after the correction method used here has been applied) which causes a reduction in the correlation peak magnitudes.

In order to produce radial separation correlations from CTA data, a constant axial separation of 0.1\( D_j \) was necessary to avoid probe interference[61]. This axial separation was therefore also included in the PIV data analysis. Figure 4.51a-d presents comparison of the 2\( \text{nd} \) and 4\( \text{th} \) order
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correlation with radial separation. The agreement for both 2\textsuperscript{nd} and 4\textsuperscript{th} order correlations is good. The auto-correlation ($\eta_2 = 0$) is essentially the same as the axial separation correlations ($\eta_1 = 0.1D_j$) and hence again shows an underestimate of peak value relative to CTA data. The spatial decay (correlation decay from one separation value to another) of the 2C PIV data is shown to be slower than the CTA data; this causes the underestimate at $\eta_2 = 0$ to become an overestimate by $\eta_2 = 0.098D_j$. The likely cause of this difference is a differing shear layer thickness between the two experiments. This would result in the absolute separation values being a smaller percentage of the local shear layer thickness in the PIV measurements. The high sensitivity of the shear layer to the nozzle design and boundary layer conditions makes this possible. With this in mind the agreement between the 2C PIV and the CTA is very good. The 2C and 3C PIV results are also in very good agreement for $\eta_2 = 0$ and $\eta_2 = 0.059D_j$, although deteriorate by $\eta_2 = 0.098D_j$. The decay of the correlation peaks can be seen to move towards the left as the radial separation increases.

Finally, in order to compare the circumferential correlations a constant axial separation of $0.05D_j$ was necessary\cite{61}, once again to avoid probe interference. However, due to the planar nature of the PIV technique, it was not possible to include this in the PIV data process and it was assumed negligible and set to $0D_j$. The 2C and 3C PIV results are in excellent agreement with the CTA results as shown in Figures 4.52a-b. The most noticeable difference is the auto-correlation $\eta_3 = 0$ curve, while all the other separation values for both 2\textsuperscript{nd} and 4\textsuperscript{th} order correlations are very similar. The most likely reason for the Gaussian shape distribution for the CTA data auto-correlations is the axial separation ($\eta_2 = 0$ is essentially the same as the axial separation correlations $\eta_1 = 0.05D_j$).

It is worth noting that the planar stereoscopic PIV technique removes the need to introduce any separations to avoid probe interference and also provides the opportunity to produce the true correlation components. The whole set of 4\textsuperscript{th} order velocity correlations for the most significant correlation components (discussed more in Section 4.4.6) for axial, radial and circumferential separations at $x/D_j = 4$ are included in Appendix A for completeness.

The high levels of agreement with previous experimental data shown in this section provides strong support that PIV measurements at 1kHz, with the use of a water flow experiment, have managed to achieve adequate temporal correlation information to be comparable to the high temporal resolution CTA method in airflow experiments. The planar optical measurement techniques provides another advantage over point based techniques by enabling the visualisation of correlation function maps. Figures 4.53 and 4.54 display 2\textsuperscript{nd} and 4\textsuperscript{th} order correlation maps in the $xz$ plane at various time delays for all 3 velocity components. For 2\textsuperscript{nd} order quantities (Figure 4.53) the elongated elliptical shape of the axial component compared to the more circular shape
of the radial component is clear to see, as has also been observed by Fleury et al[102]. Figure 4.53 also provides the $R_{33}$ component not measured by Fleury et al[102], which is initially similar to the $R_{11}$ component, (although narrower), but becomes more elliptical as time proceeds. The close to axial trajectory of the peak value for all 3 components is very noticeable. These results do not seem to be unduly contaminated by noise for the whole time duration shown. The 4\textsuperscript{th} order results (Figure 4.54) are certainly more affected by noise, while the elliptical tendency is weaker for the higher order components and the decay rate has increased. However, Figure 4.54 shows these to still be sufficiently accurate (as supported by the comparison to Harper-Bourne’s CTA data [61]) to act as good validation data for noise source models for at least the first two time levels shown.

4.4.6 Correlation Component Amplitudes

To address the question of relative amplitudes of correlation components, the peak magnitude (numerator) of all 6 2\textsuperscript{nd} order and 21 4\textsuperscript{th} order independent correlation components for zero time and space separations normalised by the axial component peak magnitude at $(4,0.5,0^\circ)$ are examined. Figures 4.55 and 4.56 present these 2\textsuperscript{nd} and 4\textsuperscript{th} order component amplitudes. The agreement between the 2C-PIV and 3C-PIV data is good, as it is between measurements made viewing in the $xr$ and $r\theta$ plane. The relative magnitude of $R_{22}$ and $R_{33}$ is a reflection of the anisotropy of the single point Reynolds stresses. Table 4.15 shows a comparison between the averaged PIV results and the CTA results from Harper-Bourne[61]. The non-isotropic 2D turbulence can be clearly seen in both results in the difference between $R_{22}$ and $R_{33}$.

<table>
<thead>
<tr>
<th></th>
<th>$R_{11}$</th>
<th>$R_{12}$</th>
<th>$R_{13}$</th>
<th>$R_{22}$</th>
<th>$R_{23}$</th>
<th>$R_{33}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average PIV</td>
<td>1</td>
<td>0.30</td>
<td>-0.04</td>
<td>0.42</td>
<td>0.03</td>
<td>0.67</td>
</tr>
<tr>
<td>Harper-Bourne’s CTA data [61]</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>0.61</td>
<td>-</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 4.15: Comparison of 2\textsuperscript{nd} order correlation component amplitudes as a ratio of $R_{11}$ between average PIV results and CTA results

For the 4\textsuperscript{th} order components, the most significant result of the present measurements is that it highlights that, for turbulent jet shear layers (data in other regions is given later), the largest components are $R_{1111}$, $R_{1112}$, $R_{1212}$, $R_{1313}$, $R_{2222}$ and $R_{3333}$ with all other components small by comparison. Such data is extremely useful for calibration of noise source models.

Assumptions about the absolute correlation amplitudes have to be made when used in noise prediction models in order to determine empirical constants in the source description and enable absolute predictions for the sound field[100]. One assumption used by Karabasov et al[100]
was that the amplitude of the correlation components for zero spatial and temporal separations \((\eta = 0 \text{ and } \tau = 0)\) was independent of axial position along the jet shear layer. Figure 4.57 shows the amplitude of the 4th order correlation components for the 2C-PIV measurements at several axial locations along the middle of the shear layer \((\tau/D_j = 0.5)\). This assumption seems to be acceptable and accurate, with all correlation components showing strong independence of axial location from nozzle exit to the end of the potential core (slight reduction in \(R_{111}\) near the nozzle exit). These findings allow for examination of the correlation component amplitudes at locations downstream of the nozzle exit, where the local lengthscales are larger, resulting in lower levels of sub-cell filtering and higher accuracy of results. It also shows that the PIV measurements made within this study show little signs of reduced accuracy near the nozzle exit.

4.4.7 Shear Layer vs Centreline Correlation Components

Stereoscopic PIV measurements have been conducted along the centreline and lipline downstream of potential core closure. Results of the axial velocity correlation with axial separation at \(x/D_j=6.5\) and 10 have been shown previously in Figure 4.48. The most noticeable difference between the centreline and lipline correlations is the much steeper increase and decrease of the correlation magnitude for each separation of \(\eta 1\). The convection velocity has also been shown in Table 4.6, and is much larger than the shear layer convection velocity and is approximately equal to the local axial velocity.

Figures 4.58 - 4.60 show the 2nd order correlation maps in the \(xr\) plane at various time delays for all 3 velocity components. The elliptical shape of the axial velocity correlations, seen earlier in Figure 4.53, has increased in size along the lipline by \(x/D_j = 10\). The centreline correlation shows a similar elliptical shape, although this is orientated along the centreline rather than at an angle to it. Figure 4.59 illustrates that the more circular shape of the radial correlation \(R_{22}\) also still exists along the centreline, although the spatial distribution of the correlation is smaller when compared to the radial correlations on the lipline. The circumferential velocity correlation \(R_{33}\), shown in Figure 4.60, highlights the most noticeable difference in shape between the lipline and centreline. The elliptical shape seen along the lipline upstream of the potential core closure is still present at \(x/D_j = 10, \tau/D_j = 0.5\), however, along the centreline there is no elliptical shape and it is instead a circular shape, similar to the radial velocity correlation \(R_{22}\). The increased convection velocity along the centreline can be identified in all these figures, by the peak magnitude location occurring further downstream for a given temporal separation.

At \(x/D_j = 10\) the lengthscales and the timescales are larger resulting in fewer independent samples. However, none of these results seem to be unduly contaminated by noise or low convergence levels for the whole of the time duration, and give a clear visualisation of the shape and
passage of the turbulence. Evidence also suggests that although similar there are differences in
the nature of the turbulence along the lipline and centreline and as such these ought to be taken
into account in noise source modelling.

To address the question of relative amplitudes of the correlation components, the peak values
of all independent components of \( R_{ijkl} \) for zero time and space separation relative to the axial
component peak magnitude are presented at \( x/Df = 10 \) for both lipline and centreline locations.
These are shown in Figure 4.61. The overall agreement between radial locations is good. The
relative magnitudes of \( R_{1111}, R_{1212}, R_{1313}, R_{2222}, R_{3333} \) remain significant components and are
of approximately equal magnitude. The component \( R_{1112} \) identified early as a significant term
along the lipline, is nearly zero at the centreline location. In comparison, the \( R_{1113} \) term, which
has a relative magnitude of nearly zero at the lipline, becomes a significant term at the centreline.
On close inspection, there is also a slight shift in the difference between \( R_{2222} \) and \( R_{3333} \), which
appear to become similar in magnitude at the centreline, in comparison to the lipline.

4.5 Approximation of fourth order correlations via products of
the second order correlations

The 4\textsuperscript{th} order two-point space-time correlation lies at the heart of conventional acoustic analo-
gies and is a quantity which until recently has defied attempts to measure directly and for all
components. Previously attempts to document high-order, multi-point statistics (with many
components in general) from experimental data were prone to large errors and uncertainties.
Work by Millionshchikov [57J showed that a relationship between 4\textsuperscript{th} order and 2\textsuperscript{nd} order
velocity correlations could be deduced on the assumption of a ‘quasi-Gaussian’ (nearly normal)
probability density function for the velocity fluctuations. Early experiments [59, 30J, were
limited in their measurement capabilities, and were therefore designed to extract information related
to the source mechanisms via direct turbulence measurements of the 2\textsuperscript{nd} order velocity correla-
tion (and for only one of the nine components of the correlation tensor, the axial component).
Assumptions of isotropy and homogeneity were then invoked in order to model the remaining
terms. Aeroacoustic noise prediction schemes have followed a similar route, by assuming that the
4\textsuperscript{th} order correlation of velocity can be approximated (using the findings of Millionshchikov [57])
by a product of 2\textsuperscript{nd} order correlations. Based on the assumption and analysis of Millionshchikov
[57], this yields:

\[
\overline{u_i u_j u_k u_\ell} \approx \overline{u_i u_j} \overline{u_k u_\ell} + \overline{u_i u_k} \overline{u_j u_\ell} + \overline{u_i u_\ell} \overline{u_j u_k} \tag{4.4}
\]
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The relation used here for the normalised 4th order spatio-temporal cross correlation can be written using the quasi-Gaussian approximated (Equation 4.4) in terms of the product of 2nd order correlations between two points, separated by the vector, \( \vec{\eta} \), in space and by \( \tau \) in time, as follows:

Let A, B and C identify particular spatial / temporal co-ordinate pairs:

\[
A = (\vec{x}, t) \quad B = (\vec{x} + \vec{\eta}, t) \quad C = (\vec{x} + \vec{\eta}, t + \tau)
\]  

(4.5)

Then the 4th order correlation appearing in \( R_{ijkl} \) is:

\[
I_{ijkl} = u'_i(A) u'_j(A) u'_k(C) u'_l(C)
\]

This may be approximated using the quasi-Gaussian relation as:

\[
R_{ijkl}^{QG} \approx I_{ijkl} - \left( \frac{u'_i(A) u'_j(A) u'_k(B) u'_l(B)}{(u'_i(A)^2)^{\frac{1}{4}} (u'_j(A)^2)^{\frac{1}{4}} (u'_k(B)^2)^{\frac{1}{4}} (u'_l(B)^2)^{\frac{1}{4}}} \right)
\]

(4.6)

and the normalised 4th order correlation is then given as:

\[
R_{ijkl}^{QG}(\vec{x}, \vec{\eta}, \tau) = \frac{R_{ijkl}^{QG} - \left( \frac{u'_i(A) u'_j(A) u'_k(B) u'_l(B)}{(u'_i(A)^2)^{\frac{1}{4}} (u'_j(A)^2)^{\frac{1}{4}} (u'_k(B)^2)^{\frac{1}{4}} (u'_l(B)^2)^{\frac{1}{4}}} \right)}{- \left( \frac{\sqrt{u'_i(A)^2} \sqrt{u'_j(A)^2} \sqrt{u'_k(B)^2} \sqrt{u'_l(B)^2}}{u'_i(A)^2} \right) - \left( \frac{\sqrt{u'_i(A)^2} \sqrt{u'_j(A)^2} \sqrt{u'_k(B)^2} \sqrt{u'_l(B)^2}}{u'_j(A)^2} \right) - \left( \frac{\sqrt{u'_i(A)^2} \sqrt{u'_j(A)^2} \sqrt{u'_k(B)^2} \sqrt{u'_l(B)^2}}{u'_k(B)^2} \right) - \left( \frac{\sqrt{u'_i(A)^2} \sqrt{u'_j(A)^2} \sqrt{u'_k(B)^2} \sqrt{u'_l(B)^2}}{u'_l(B)^2} \right)}
\]

(4.7)

Note, that the normalisation is performed using the measured velocity fluctuations, and hence if \( I_{ijkl} \neq I_{ijkl}^{QG} \) then unity will not be achieved. Simpler correlations of interest may of course again be obtained by selecting specific values of \( \vec{\eta} \) and \( \tau \) (as mentioned in Section 2.3.3).

Approximation of the 4th order correlation via products of 2nd order correlations has, as yet, never been thoroughly analysed. Bridges and Wernet[74] presented results of measured \( R_{1111} \) and \( R_{2222} \) components and compared these to the approximated components \( R_{1111}^{QG} \) and \( R_{2222}^{QG} \) as shown in Figure 4.62. They found overall that “the two statistics agreed within the 10% or so error band that seemed to characterise the measurements”. It is also very noticeable that there was considerably more scatter in the measured \( R_{1111} \) than \( R_{1111}^{QG} \) and that the measured \( R_{2222} \) was slightly larger than \( R_{2222}^{QG} \). It has previous been established that, from the point of view of turbulence modelling, an undesirable consequence of the quasi-Gaussian assumption is the possibility that the turbulent energy and/or its individual components may become negative[148].
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Although the work of Bridges and Wernet\cite{74} showed no effects of this, this may occur in the other components. From the point of view of acoustic source modelling, it is clear that it would be much better if direct measurements of $R_{ijkl}$ were available that were not so affected by experimental noise, and which covered more than two components. Only then can the adequacy of the quasi-Gaussian approximation be judged as to whether it is sufficiently accurate. This has strongly motivated the direction of the work reported in this thesis.

Figure 4.63 shows a repeat of Figure 4.62, but now using the current PIV data for $R_{1111}$ vs $R_{1111}^{QG}$ and $R_{2222}$ vs $R_{2222}^{QG}$. Firstly it is obvious that a large reduction in scatter of the directly measured 4\textsuperscript{th} order components compared to the directly measured data of Bridges and Wernet\cite{74} has been achieved. From Figure 4.63 the adequacy of the Q-G approximation seems good, at least for these two components. The approximated results show not only a reduction in the scatter, but a complete removal of any background correlation fluctuation outside of the main correlation region. The edges of the correlation region are also smooth, and hence show a reduction in the size and decay of the correlation which would therefore yield smaller length and timescale information. An increase in the magnitude of the approximated correlation (also present in the results of Bridges et al\cite{74}) can clearly be seen in Figures 4.64a-d. These plots present the measured and approximated correlations for $R_{1111}$ at the 4 axial stations measured. Upon close inspection of the correlation values, Figures 4.64b and 4.64c illustrate that the approximated correlations have values above 1, when normalised by the directly measured denominator (if the approximation were perfect, a unity peak should be achieved). This stronger correlation was also observed by Bridges and Wernet\cite{74} for the approximated 4\textsuperscript{th} order correlation. Over the whole axial distance the reduction in background correlation remains essentially constant and although the correlation peak does appear high at $x/D_j = 4$ and 6.5, the overall agreement between the results is still good.

As presented in Section 4.4.4 it is no longer appropriate only to consider $R_{1111}$ within noise models. The accuracy of this approximation for all correlation functions (not just the dominant $R_{1111}$ term) must therefore be conducted to ensure this gives the proper form for the correlation model in space and time. Figures 4.65 and 4.66 show the dimensional values of the correlations for zero spatial and temporal separation available from 2C-PIV data, for the measured and approximated 4\textsuperscript{th} order correlations at individual axial locations. Excluding the previously discussed over approximation of the $R_{1111}$ terms, the agreement in relative magnitudes of the different correlation components is very good, providing evidence for the first time, that this approximation is able to produce a variety of correlation components with acceptable accuracy.

Finally, explained and present in Section 4.4.4, the correlation components show a constant amplitude independent of axial location. If the approximation is to be accurate, the approximated
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correlations must also be independent of axial location. Figure 4.67 presents further evidence that the approximated correlations do an acceptable job of matching the directly measured correlations.

4.6 Closure

Single jet configurations have been designed and investigated to provide turbulence information to aid the development of accurate and usable noise source models. This chapter has detailed the development, quantification and validation of a generalised and widely applicable PIV-based methodology for providing unsteady, spatially and temporally resolved velocity fields. An experimental approach using a water tunnel facility has demonstrated that the gathered flowfield data is representative of subsonic airflow jet turbulence. The technique recognises and works with the high frequency, small scale nature of the unsteady velocity field within jet plumes, and PIV instrumentation limitations.

Through the use of a global technique, a time-resolved proper orthogonal decomposition (POD) of the jet velocity field has been possible. This has resulted in development of a POD-based approach for a spatio-temporal filtering procedure suitable for removing ‘spurious’ noise from 3C-PIV data. The results show that, by using stereoscopic PIV with a repetition rate of 1kHz, and given the correct application of the method (e.g. sufficiently small PIV interrogation cell size in relation to the local turbulent length scales), even 4th order correlations can accurately be captured, and were here demonstrated to reproduce the quality of those captured by point-based probe techniques such as CTA. The measurements deliver new insights into the characteristics of the 4th order correlation $R_{ijkl}$, for example that, of the 21 independent correlation components, in a round jet only 6 are significant. This level of detail is invaluable for aeroacoustic prediction methods which need to assume a model for $R_{ijkl}$. Finally, the assumption of approximating the 4th order correlations ($R_{ijkl}$) via the quasi-Gaussian approach (using products of the second order correlations) has been shown to provide an acceptable level of accuracy in comparison to those obtained from direct measurements of $R_{ijkl}$. Small discrepancies, such as increased peak magnitudes, and increased resultant length and time scales do exist, but on the whole the quasi-Gaussian assumption has been thoroughly validated.

The following chapter details RANS and LES numerical calculations of the single round jet carried out and analysed to assess their ability to capture mean velocity and turbulence statistics, in addition to the extended assessment of the LES calculations to assess the two-point spatio-temporal information, which characterises the nature of the flowfield.
Figure 4.1: Axial development of the mean and RMS of the axial velocity along the centreline for the LU40 nozzle.
Experimental Results

Figure 4.2: Axial development of the mean and RMS of the axial velocity along the centreline for the LU40 nozzle comparison to other data.
Experimental Results

Figure 4.3: Axial development of the mean and RMS of the axial velocity along the lipline for the LU40 nozzle
Figure 4.4: Radial profiles of axial mean velocity for the LU40 nozzle (m/s)
Figure 4.5: Radial profiles of axial RMS velocity for the LU40 nozzle.
Experimental Results

Figure 4.6: Radial profiles of axial RMS velocity for the LU40 nozzle at $x/D_j = 4$ compared to other previous data.
Experimental Results

Figure 4.7: Axial RMS error maps
Experimental Results

Figure 4.8: Radial RMS error maps
Figure 4.9: Two-point correlations of axial velocity fluctuations with axial separation (all measurements at $r/D_j = 0.5$)
Figure 4.10: Two-point correlations of radial velocity fluctuations with radial separation (all measurements at $r/D_j = 0.5$)
Figure 4.11: Contour maps of corrected axial and radial length scales of axial velocity (dimensions in mm)
Figure 4.12: Contour maps of corrected axial and radial lengthscales of radial velocity (dimensions in mm)
Experimental Results

Figure 4.13: Radial profiles of $^1L_{11}$ at all stations
Experimental Results

Figure 4.14: Radial profiles of $^1L_{22}$ at all stations
Experimental Results

(a) Axial lengthscales

(b) Radial lengthscales

Figure 4.15: Axial and radial lengthscales at $r/D_j = 0.5$. The red and blue lines represent the range of data for lengthscales obtained from experimental results presented in Fleury et al[102]
Figure 4.16: Contour maps of timescales of axial and radial velocity ($T_{11}$ and $T_{22}$) (dimensions in seconds)
Figure 4.17: Radial profiles of axial timescales ($T_{11}$) at various axial stations
Experimental Results

Figure 4.18: Power spectra of axial velocity at all locations
Figure 4.19: Statistical convergence of spatio-temporal correlations $R_{ij}$ and $R_{ijkl}$ on the lipline at $x/D_j=1.5$ and 10
Experimental Results

Figure 4.20: 2nd order correlation maps of $R_{11}$ with axial separation at various stations

(a) $x/D_j = 1.5, r/D_j = 0.5$

(b) $x/D_j = 4, r/D_j = 0.5$

(c) $x/D_j = 6.5, r/D_j = 0.5$

(d) $x/D_j = 10, r/D_j = 0.5$

(e) $x/D_j = 6.5, r/D_j = 0$

(f) $x/D_j = 10, r/D_j = 0$
Experimental Results

Figure 4.21: 2\textsuperscript{nd} order correlation maps of $R_{22}$ with axial separation at various stations
Figure 4.22: 4th order correlation maps of $R_{1111}$ with axial separation at various stations
Experimental Results

Figure 4.23: 4th order correlation maps of $R_{1111}$ and $R_{2222}$ with axial separation at $x/Dj = 10$ compared to results of Bridges
Figure 4.24: 4th order correlation maps of $R_{1111}$ and $R_{2222}$ with axial separation at $x/Dj = 10$ compared to results of Bridges
Figure 4.25: Growth rate of the Lagrangian lengthscales along the shear layer (solid lines) and the centreline (dashed lines)
Figure 4.26: Radial profiles of axial mean velocity for the LU40 and LU40P nozzles (m/s)
Figure 4.27: Radial profiles of axial RMS velocity for the LU40 and LU40P nozzles
Figure 4.28: Growth of Shear Layer for both LU40 and LU40P nozzles
Experimental Results

Figure 4.29: Radial profiles of $^{1}L_{11}$ at all stations
Experimental Results

Figure 4.30: Radial profiles of $^1L_{22}$ at all stations
Experimental Results

Figure 4.31: Radial profiles of axial timescale $T_{11}$ at all stations
Figure 4.32: Comparison between LU40 and LU40P 2nd order correlation maps of $R_{11}$ with axial separation
Experimental Results

Figure 4.33: Comparison between LU40 and LU40P 4th order correlation maps of $R_{1111}$ with axial separation

(a) LU40 $x/D_j = 1.5$, $r/D_j = 0.5$

(b) LU40P $x/D_j = 1.5$, $r/D_j = 0.5$

(c) LU40 $x/D_j = 10$, $r/D_j = 0.5$

(d) LU40P $x/D_j = 10$, $r/D_j = 0.5$

(e) LU40 $x/D_j = 10$, $r/D_j = 0$

(f) LU40P $x/D_j = 10$, $r/D_j = 0$
Figure 4.34: Radial profiles of axial mean velocity for the LU40 nozzle (m/s)
Figure 4.35: Radial profiles of axial RMS velocity for the LU40 nozzle
Experimental Results

Figure 4.36: Radial profiles of $L_{11}$ at all stations
Figure 4.37: Radial profiles of $^{2}L_{22}$ at all stations
Experimental Results

Figure 4.38: Radial profiles of $T_{11}$ at all stations
Experimental Results

Figure 4.39: Power spectra of axial velocity at all locations

(a) $x/D_j = 1.5, r/D_j = 0.5$

(b) $x/D_j = 4, r/D_j = 0.5$

(c) $x/D_j = 6.5, r/D_j = 0.5$

(d) $x/D_j = 10, r/D_j = 0.5$

(e) $x/D_j = 6.5, r/D_j = 0$

(f) $x/D_j = 10, r/D_j = 0$
Experimental Results

Figure 4.40: Vectors of fluctuation velocity at $x/D_j = 6.5$ (spurious vectors depicted via circles)

(a) Raw data

(b) After postprocessing with modified parameters to remove spurious vectors
Experimental Results

Figure 4.41: Power spectra of axial velocity at all locations for stronger postprocessed measurements
Figure 4.42: Effects of POD mode reconstruction on PSD for various energy level reconstructions
Experimental Results

\[ E \propto C \cdot PIV \]

Limits of Integration

\[ \Delta E \]

\[ \text{Energy Content} \]

\[ \text{Number of Modes needed for POD mode reconstruction to produce flowfield with} \]

\[ E = E_{\text{TOTAL}} - \Delta E \]

Figure 4.43: Illustration of POD implementation methodology
Figure 4.44: Vectors of fluctuation velocity at $x/D_j = 6.5$
Experimental Results

Figure 4.45: Vectors of the difference between the original and filtered fluctuation velocities at $x/D_j = 6.5$
Experimental Results

Figure 4.46: Power spectra of axial velocity at all locations (* indicates filtered results).
Figure 4.47: Radial profiles of axial RMS velocity for the LU40 nozzle (* indicates filtered results)
Figure 4.48: Correlation plots of axial velocity with axial separations (* indicates filtered results)
Experimental Results

Figure 4.49: Spatio-temporal correlation functions of axial velocity with axial separations at $x/D_j = 1.5$, $r/D_j = 0.5$ (Solid lines = 25mm×25mm FoV, Dashed lines = 40mm×40mm FoV)
Figure 4.50: Spatio-temporal correlation functions of axial velocity with axial separations at $x/D_j = 4$, $r/D_j = 0.5$
Experimental Results

Figure 4.51: Spatio-temporal correlation functions of axial velocity with radial separations at $x/D_j = 4$, $r/D_j = 0.5$
Experimental Results

Figure 4.52: Spatio-temporal correlation functions of axial velocity with circumferential separations at $x/D_j = 4$, $r/D_j = 0.5$
Experimental Results

Figure 4.53: 2nd order spatio-temporal correlation functions at $x/D_j = 4$, $r/D_j = 0.5$. The correlations levels are white = 0 and black = 1.
Experimental Results

Figure 4.54: 4\textsuperscript{th} order spatio-temporal correlation functions at $x/D_j = 4$, $r/D_j = 0.5$. The correlations levels are white = 0 and black = 1
Figure 4.55: Comparison between 2\textsuperscript{nd} order correlation peak magnitudes as a ratio of $R_{11}$ for PIV measurements and CTA results.
Experimental Results

Figure 4.56: Comparison between 4\textsuperscript{th} order correlation peak magnitudes as a ratio of $R_{1111}$ for PIV measurements
Experimental Results

Figure 4.57: Comparison between axial development of PIV measured 4th order correlation amplitudes at $r/D_j = 0.5$
Experimental Results

Figure 4.58: 2nd order spatio-temporal correlation functions of $R_{11}$ at $x/D_j = 10$. The correlations levels are white = 0 and black = 1.
Experimental Results

Figure 4.59: 2nd order spatio-temporal correlation functions of $R_{22}$ at $x/D_j = 10$. The correlations levels are white = 0 and black = 1
Experimental Results

Figure 4.60: 2nd order spatio-temporal correlation functions of $R_{33}$ at $x/D_j = 10$. The correlations levels are white = 0 and black = 1
Figure 4.61: Comparison between 4th order correlation peak magnitudes as a ratio of $R_{1111}$ at $x/D_j = 10$ for PIV measurements at $r/D_j = 0$ and $r/D_j = 0.5$
Figure 4.62: Comparison between Bridges data[74] of measured 4th order correlation and approximated 4th order correlation at $x/D_j = 10$, $r/D_j = 0.5$
Figure 4.63: Comparison between current PIV data of measured 4\textsuperscript{th} order correlation and approximated 4\textsuperscript{th} order correlation at $x/D_j = 10$, $r/D_j = 0.5$.
Figure 4.64: Comparison between measured and approximated 4th order correlations plots of axial velocity with axial separations (all measurements at \( r/D_j = 0.5 \))
**Experimental Results**

Figure 4.65: Comparison between measured and approximated 4th order correlation amplitudes at $x/D_j = 1.5$ and $x/D_j = 4$ (all measurements at $r/D_j = 0.5$)
Experimental Results

Figure 4.66: Comparison between measured and approximated $4^{th}$ order correlation amplitudes at $x/D_j = 6.5$ and $x/D_j = 10$ (all measurements at $r/D_j = 0.5$)
Experimental Results

Figure 4.67: Comparison between axial development of measured (solid lines) and approximated (dashed lines) 4th order correlation amplitudes at $r/D_j = 0.5$. 
Chapter 5

Computational Results

In order to solve directly the Navier-Stokes equations a massive amount of computational power is required for high Re number flows of engineering relevance. Due to recent developments in computing power it is now possible to use two different computational approaches to the modelling of jet noise. These are a RANS based approach and a first principles / LES based approach. The quick convergence time and simplicity of RANS schemes makes this form of noise prediction very appealing, and although acceptable results have been produced, the technique is still found to have drawbacks, e.g. it is possible to predict sideline noise accurately, but not the peak directivity noise levels with the same model. The fundamental limitation of this approach is that the flow unsteadiness, which is the source of noise, has to be modelled semi-empirically. The functional form of the noise model used is normally taken from observations of the two-point space-time correlation \( \langle u'(x, t)u'(x + \eta, t + \tau) \rangle \), as a function of \( \eta \) and \( \tau \) and the form of this is usually chosen from drastically simplified theoretical assumptions such as Gaussian correlation shapes, isotropic turbulence etc. The second more recently investigated solution to noise source definition and increased accuracy in sound prediction has been to solve directly for the flow unsteadiness via LES predictions. This approach solves for the sources numerically rather than relying on semi-empirical modelling. A well performed LES simulation has the potential to be a valuable tool in understanding noise generation via turbulence. However, within any LES calculation, the governing equations are low-pass filtered and only the largest scales of turbulence (relative to the chosen grid size) are captured by the simulation, resulting in filtered turbulence information. The effects of the small scales, called sub-grid scales as their length scales are not resolvable on the chosen grids, on the large (resolved) scales are modelled (see Chapter 2). LES calculations are therefore able to predict more details of the turbulent fields, however the sub-grid scale model (SGS) must be able to represent adequately the effects of the high-frequency content of the flow.
In both of the above mentioned numerical approaches the dependence on boundary conditions remains an important issue, particularly for highly spatially and temporally resolved LES simulations. In addition it is important to recognise that, for the present flow problem, the transition from an extremely thin boundary layer at nozzle exit to a thin initial free shear layer is a particularly daunting task. The development of turbulent structures within the shear layer is the crucial factor which drives the turbulent acoustic source mechanisms. To calculate this process correctly will require a very large number of mesh nodes, particularly at high Re number. It could therefore be argued that the results of any CFD investigation of this problem, will require perhaps 100's of millions of nodes.

Since such a large scale CFD study was not intended as part of the present project, it is necessary to make clear the purpose of the CFD calculations presented in this chapter. The previous chapter has presented what are believed to be the first detailed and comprehensive datasets for the 4th order spatio-temporal correlations. LES CFD is, in principle, capable of predicting this quantity. Hence, following the lines indicated earlier (which have discussed how the available data storage sets a limit on the maximum LES grid size (≈ 30 million cells) that can be consequently post-processed to extract the 4th order correlations) the CFD predictions conducted in the present study have been carried out using the maximum possible grid to perform LES calculations of the present experiments. The question to be answered is whether the 4th order correlation quantities, that can be extracted from such an LES prediction, compare well with those observed in the current measurements.

This chapter thus details the RANS and LES predictions carried out to assess their ability to capture mean velocity and turbulence statistics of a single round jet flowfield. The nature of the LES calculation means that this assessment should also extend to two-point spatio-temporal information. This chapter will describe the unsteady characteristics from the LES predictions in detail and the accuracy with which they predict $R_{ijkl}$. The numerical predictions use the DELTA code (discussed in detail in Chapter 2) which implements a finite volume method (FVM) to solve the filtered Navier Stokes equations as described in Section 2.2.2. DELTA uses the standard Smagorinsky formulation for the sub-grid scale model with a Van Driest near-wall damping treatment, detailed in Section 2.2.3. The same grid generation and multi-block treatment is followed in both RANS and LES calculations carried out with the DELTA code.

This chapter is organised as follows:

Section 5.1 details the RANS and LES predictions generated from DELTA using the single round jet nozzle geometry and mesh shown in Figure 3.26. This section includes evaluation of single point statistics, flowfield visualisation, and axial and radial profiles. The profiles are
examined at the same locations as the experimental PIV measurements detailed in Section 3.1.4 and illustrated in Figure 3.22.

Section 5.2 details the two point statistics obtained from the LES prediction. A full 3072 sample time series at 1kHz has been gathered in order to enable identical samples sizes from which the LES and PIV statistics are compared. This section includes the shape and distribution of the spatial correlations as well as their convection and decay. The associated Eulerian and Lagrangian spatial and temporal scale information is compared with the experimental PIV results. Comparison of the turbulent scales to previous experimental studies is shown in order to present further evidence to support the arguments presented for the assessment of the LES predictions and PIV measurements. Throughout this section 2C-PIV measurements will be used as far as possible as the benchmark experimental PIV results to avoid any uncertainties regarding the necessary filtering required for the 3C-PIV data. However, where the third velocity component is required the filtered 3C-PIV data will be used.

Finally, Section 5.3 presents a full evaluation of the 2\textsuperscript{nd} and 4\textsuperscript{th} order spatio-temporal correlations obtained from the LES predictions against PIV measurements and previous experimental results\cite{61}. This includes identification of which of the 21 independent components of $R_{ijkl}$ are predicted to be the largest contributors to the far-field noise and whether these findings agree with the experimental findings. This section allows conclusions to be drawn as to the current ability of LES to be used as a truly predictive method for the 4\textsuperscript{th} order spatio-temporal correlation $R_{ijkl}$.

### 5.1 Single Point Statistics - RANS vs LES vs PIV

Currently RANS calculations provide the basis for most jet noise prediction procedures, while LES predictions are being used to help improve the rather crude proposed models for $R_{ijkl}$. The accuracy of the velocity field predicted by the RANS and LES calculations is therefore important. Single point statistics from the RANS and LES predictions have therefore been produced to examine the ability of the numerical simulations to produce results similar to the experimental results.

#### 5.1.1 Flowfield Visualisation

Flowfield visualisation allows an early assessment of the overall pattern predicted in the jet plume as well as allowing the behaviour and nature of the unsteady turbulence field of the jet to be analysed. Figures 5.1a-b present a comparison of the near-field mean axial velocity between RANS and LES predictions and the PIV measurements via an $xz$ plane contour plot through
Computational Results

the centreline of the jet. The comparison clearly shows the time-averaged flow predicted directly by the RANS approach and after time-averaging of the LES predictions, produces a spreading shear layer. The 50Hz PIV results are considered to be statistically converged, and show strong similarity with the RANS prediction. Upon close inspection the RANS prediction shows a region of reduced velocity on the outer edge of the shear layer at nozzle exit. This is associated with entrainment of ambient (co-flow) fluid and is also present within the PIV measurements but to a much smaller degree. In addition it can be seen that the RANS prediction slightly underestimates the mixing rate and therefore produces a longer potential core length. The comparison between the LES prediction and the PIV measurements shown in Figure 5.1b illustrates the lack of flowfield samples (only 3072 samples at 1kHz) identified by the non-smooth nature of the shear layer edge. The gradients at the nozzle exit are well resolved although at downstream locations (where the timescales are slower and the number of independent samples are fewer) spatial gradients are observed to be oscillatory in comparison to the PIV results. At the nozzle exit, the same region of reduced velocity on the outer edge of the shear layer exists, as in the RANS calculation, however, the overall thickness of the shear layer at nozzle exit appears to be much thinner in the LES prediction than in the PIV measurements and the potential core length of the LES prediction is also noticeably shorter than the PIV results. Both of these aspects are related to the ability of the internal nozzle flow calculation in the LES prediction to capture the turbulent boundary layer at nozzle exit, and will be discussed further below.

Figures 5.2 and 5.3 illustrate the time-resolved nature of the shear layer development in the LES predictions via contour plots of the instantaneous axial velocity in $x\tau$ and $r\theta$ planes respectively. The contours are cropped at $0.3m/s$ to help clarify the shear layer behaviour. Both figures illustrate that the LES predicted shear layer between nozzle exit and $x/D_j = 1.5$ is not fully turbulent and is rather more laminar/transitional in nature. In contrast, further downstream of $x/D_j = 2$, large scale highly energetic turbulent eddies are clearly visible in the shear layer. These unsteady motions can be seen to penetrate the potential core, through the ‘bursting’ of strong vortex ring structures (discussed below). The presence of this ‘bursting’ is seen at its clearest in Figure 5.2b, and this initiates the increase in turbulence, which shortens the predicted potential core length.

The unsteady motions in the turbulent jet that are captured by the LES are illustrated further in Figures 5.4a-b. These figures display an $x\tau$ plane view of instantaneous vorticity magnitude and an iso-surface of vorticity magnitude coloured by the axial velocity respectively. Both figures show that the Gaussian noise prescribed at nozzle inlet, and subsequent boundary layer growth prediction inside the nozzle, has not provided a fully turbulent condition at the nozzle exit lip for initial shear layer development. Immediately downstream of the nozzle exit the developing vortices have a ring like structure which means that the development of three-dimensional tur-
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Bulence is delayed. Finer 3D structures are visible downstream of $x/D_j = 3$. The reasons for this behaviour are thought to be as follows.

The LES flow visualisation pictures shown indicate that nearly no grid-resolved turbulence is emerging from the nozzle. The lack of turbulence at the nozzle exit is due to (i) the uncorrelated nature of the inlet white noise disturbance, (ii) the acceleration in the nozzle causing any resolved turbulence rapidly to decay to very low levels and (iii) lack of grid resolution in the near wall region inside the nozzle, relative to the boundary layer thickness, and its decreasing size as it approaches nozzle exit. These reasons result in a conflict with the experiments where near wall turbulence clearly exists in spite of the nozzle acceleration and adds excitation to the natural free shear layer instabilities. These features of high Re number jet flows from nozzles have been observed before in LES predictions ([149, 100, 130, 150]), which have very often shown underprediction of potential core lengths in comparison to measurements. The solution primarily lies in adopting an improved method for synthetic turbulence generation in the nozzle inlet boundary conditions and also use of a finer grid. Neither of these were followed in the present work, as it was not primarily aimed at improving the LES prediction methods for jets. The approach that has been adopted here was to use, as indicated above, the finest mesh that could be afforded and, in spite of the problems indicated above in the region very close to the nozzle, to assume that the shear layer turbulence does develop eventually in a realistic manner (although slightly further downstream than in the experiments), i.e. that the shear layer turbulence eventually ‘forgets’ its near nozzle exit plane origins. The LES predictions may then be analysed to examine the turbulence structure in this further downstream region and assess the extent to which they predict the turbulent correlations measured in the PIV experiments.

5.1.2 Axial and Radial Profiles

To compare quantitatively numerical predictions and experimental measurements a number of profiles have been examined. Comparison between the RANS and LES predictions and PIV measurements of single point velocity statistics are presented in Figures 5.5 and 5.6. Figure 5.5 shows comparison of the centreline decay of the mean axial velocity. The start of reduction in the centreline velocity indicates the closure of the potential core. The LES predicts this to occur sooner than in the PIV measurements and predicts a slightly faster rate of decay downstream of the potential core in comparison to the PIV measurements. The closure of the potential core in the LES prediction occurs at $\approx 5D_j$ (23% shorter than the experimental PIV measurements). The RANS calculations predicts a potential core length larger than both the LES and PIV results, although the centreline decay rate is a similar gradient to the LES predictions. The closure of the potential core in the RANS prediction occurs at $\approx 7.8D_j$ (20% longer than the experimental PIV measurements). One could argue that the flowfield from each prediction should be scaled
in terms of the potential core length so that further comparisons between the methods is done on a scaled frame of reference. This scaling procedure has previously been used[100] on the basis that the closure of the potential core is the meeting of the annular shear layer on the centreline, and therefore any comparison between methods is best carried out at a location relative to this point. The use of a scaling procedure would certainly improve the similarity between the different methods. However, if these methods are to be used as truly ‘predictive’ methods of jet noise, either to obtain the base flow defined by RANS calculations, or to obtain the noise source models of $R_{ijkl}$ via LES calculations, then these methods should require no scaling (possible only when experimental data is available). Therefore, based on this belief, none of the results below have been presented on a scaled frame of reference and instead have used the original absolute frame of reference.

Figures 5.6a-d show radial profiles of the mean axial velocity at four axial locations. The agreement between the RANS predictions and the PIV measurements is very good, with the gradients across the shear layer matching well at all locations. The longer potential core generated in the RANS calculations affects the comparison slightly as illustrated in Figure 5.6c where the RANS profile shows a constant velocity of 1m/s at $x/D_j = 6.5$ while the PIV data has already reduced to 0.98m/s. The RANS centreline velocity also decays faster downstream of the potential core as illustrated in Figure 5.6d where at $x/D_j = 10$ the RANS predicts a centreline velocity of 0.81m/s in comparison to the 0.86m/s seen in the PIV measurements. The LES predictions in comparison do not have such strong similarity with the PIV measurements. The gradient at $x/D_j = 1.5$ is steeper, which is assumed to be due to the presence of the laminar / transitional initial shear layer in comparison to the naturally occurring fully turbulent shear layer in the experiments. At $x/D_j = 4$ the velocity gradient across the shear layer matches well with the PIV measurements. Unfortunately by $x/D_j = 6.5$ (at the end of the experimental potential core) the LES prediction shows a profile well past the end of its predicted potential core closure. This is repeated at $x/D_j = 10$. As previously mentioned, no scaling has been implemented in these comparisons. An indication of the effect scaling based on potential core length does to such comparisons is presented in Figure 5.7. At $x/L_{pc} = 0.23$ the laminar nature of the shear layer predicted by the LES calculation has become much more defined, while both the RANS prediction and the experimental results remain in close agreement. However, further downstream the LES prediction shows good agreement with the experimental results where previously (in the absolute frame) this was not seen. The good agreement at large downstream distances ($x/L_{pc} = 1.0$ and $x/L_{pc} = 1.54$) provides confidence in the assumption that the shear layer turbulence does develop eventually in a realistic manner and ‘forgets’ its origin.

In addition to the velocity gradients across the shear layer and potential core length it is also important to examine predicted turbulence levels within the jet plume. Since 2C-PIV does
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not resolve the out of plane velocity (and hence turbulence), the total kinetic energy of 3C-PIV data was compared to the RANS and LES predictions. Figures 5.8a-d show the comparison of the radial profiles of total turbulence levels. The RANS predictions show good agreement in the turbulence magnitude when compared to the experimental 3C-PIV measurements for all locations. The widths of the profiles also correspond well. At \( x/D_j = 6.5 \) the extended potential core length in the RANS predictions starts to affect the comparison. This can be identified by the zero predicted turbulence on the centreline while there is a small increase in the measured turbulence at the same location. The RANS also slightly over estimates the turbulence magnitude by 12%. By \( x/D_j = 10 \) the rise in the RANS centreline turbulence magnitude matches the 3C-PIV results. In contrast the LES prediction shows variable levels of agreement with the 3C-PIV measurements dependent on the axial location. At \( x/D_j = 1.5 \) the turbulence level is slightly less than observed in the experimental measurements, which is most likely due to the laminar / transitional shear layer behaviour, but is in good agreement with the RANS prediction. The presence of a laminar / transitional initial shear layer instead of a fully turbulent shear layer causes a violent transition period which causes increased mixing and results in high turbulence levels and a reduction in potential core length. This increased mixing is evident at \( x/D_j = 4 \), predicting turbulence levels 60% higher than the measurements, whilst the predicted centreline turbulence level has increased from zero (evidence of the ‘bursting’ discussed previously). The early onset of this centreline line turbulence also shortens the potential core, and by \( x/D_j = 6.5 \), the LES profile shows a high level of centreline turbulence, whilst the lipline peak turbulence level has decayed to a level comparable to the experimental results. It is important however, to remember that the nature of the flowfield at this absolute location is very different between the LES prediction and the experiment. The LES is at \( x/L_{pc} = 1.3 \) while the 3C-PIV measurements are at \( x/L_{pc} = 1 \). At \( x/D_j = 10 \) the results are in fair agreement since the flow has reached a fully turbulent condition in both the predictions and the experiment.

A comparison of the axial profile along the centreline (\( r/D_j = 0 \)) and radial profiles at \( x/D_j = 1.5 \) and \( x/D_j = 4 \) across the shear layer are presented in Figures 5.9 and 5.10a-b against previous numerical[151, 47] and experimental[26, 130, 61] studies. The development of the axial velocity along the centreline (Figure 5.9) shows a wide range of profiles highlighting the difficulties previous studies have had in matching what appears to be a relatively simple statistic but is actually highly sensitive to the complex boundary layer / shear layer transition at the nozzle exit. The previous numerical study by Freund[151] also produced short potential core lengths. This is typically attributed to a laminar initial shear layer followed by an increased violent transition to fully turbulent conditions. This results in an increased growth rate of the shear layer thickness (shown in Figure 5.11 and discussed later). The match between the current LES predictions and the numerical DNS predictions of Freund[151] shows a good agreement. This agreement is irrespective of the presence of the tunnel walls modelled within the current LES prediction.
match the enclosed environment of the experiment facilities. This provides supporting evidence that both the experimental and numerical confined flow cases are still representative of a free jet. Meanwhile, Figure 5.9 illustrates the extended potential core length predicted by the RANS calculations, with only the results of Bogey et al[26] showing some agreement.

Figure 5.10 shows a good match between the Davies et al[59] data and the RANS predictions, while the LES predictions, with the steeper gradient (thinner shear layer thickness), agrees less well. The match improves by $x/D_j = 4$ for all predictions with the experimental profiles of Lau et al[63], while the results of Harper-Bourne[61] show a slightly thicker shear layer.

The thickness and growth of the jet shear layer are now considered. As detailed in Section 4.3.1 and defined in Equation 4.3 the shear layer thickness can be given as:

$$B = r_{0.05} - r_{0.95}$$

where $r_{0.05}$ and $r_{0.95}$ denote the points at which the axial velocity has decreased to 0.05 and 0.95 times the local centreline velocity respectively.

Figure 5.11 shows the growth of the shear layer in the first 7 nozzle exit diameters of the jet plume for both RANS and LES predictions and PIV measurements. The RANS prediction and PIV measurements match very well, with the extended potential core predicted in the RANS calculation resulting in a very slightly shallower gradient. The LES prediction illustrates the thin initial shear layer thickness (50% thinner than RANS and 40% thinner than PIV) which represents the laminar / transitional nature of the shear layer. This then changes to a larger growth rate in shear layer thickness, illustrated by the steeper gradient, resulting in the shorter potential core length. At $x/D_j = 4$ the growth rate becomes similar to the RANS prediction and PIV measurement, although the shear layer thickness itself is approximately 17% thicker than the experimental results. This match in growth rate supports the assumption that the turbulence does eventually develop in a realistic manner.

5.2 Two Point Statistics - LES vs PIV

The nature of LES calculations allows the capture of higher order turbulent statistics (two-point spatio-temporal information), which allows for prediction of $R_{ijkl}$. Two point statistics from the full 3072 sample 1kHz datasets of the LES prediction and PIV measurements are discussed in this section. This includes velocity correlations, extracted length and time scales and a full analysis of the spatio-temporal correlations from which the sound source is modelled.
5.2.1 Velocity Correlations

Figures 5.12 and 5.13 show profiles at four locations along the lip line of 2\textsuperscript{nd} order axial velocity correlations with axial and radial separation respectively. The lack of sub-grid scale high frequency content in the LES is evident by the 'fuller' shape of the 'whole' auto-correlation distribution of the LES in contrast to the experimental distributions, i.e the exponential form of the distribution seen in the experimental measurements has been replaced by a Gaussian distribution in the LES predictions due to the lack of unresolved scales (excluding at zero separation where the distribution will always have Gaussian (zero gradient) initial curve). At $x/D_j = 1.5$ the significant oscillation of the LES correlation illustrates the presence of a periodic component produced by the vortex rings. The negative region is related to the separation between the vortex rings. By $x/D_j = 4$ and onwards the oscillating shape does not exist, however, the Gaussian shape still remains.

The level of unresolved, high frequency, small scale turbulence can be identified from the PSD plots presented in Figures 5.14a-d. The significant drop in the energy content reflects the cut-off frequency between the resolved and SGS modelled turbulence. The only way of increasing the cut-off frequency would be to increase the spatial resolution of the mesh. Through the use of the POD filtering (detailed in Section 4.4.3) evidence can be provided to suggest that the Gaussian shape of the predicted correlation is a function of the resolved scales, more particularly, the amount of unresolved scales and their energy content. By using POD to filter a 3C-PIV dataset so that 100%, 80%, 60%, 40% and 20% of the TKE is present within the reconstructed velocity field, and then subsequently extracting the spatio-temporal correlations from the filtered fields it is possible to see the effect removal of high frequency energy has on the auto-correlation distribution. Figures 5.15a-b illustrate the 2\textsuperscript{nd} and 4\textsuperscript{th} order auto-correlations of axial velocity and illustrate the transition from an exponential shape function (3C-PIV TKE=100%) to a Gaussian shape as the level of filtering increases. These figures also illustrate the amount of unresolved small scale information in the LES predictions, only showing good agreement to the PIV data when approximately 40% of the high frequency energy has been filtered. In the opinion of the author, any LES prediction which produces a Gaussian distribution can be classified as under resolved and therefore its use in determining the shape functions for sound source modelling may be questioned.

5.2.2 Lengthscales

Figures 5.16 and 5.17 show the axial lengthscales of axial and radial velocity deduced from the correlation distributions for both experimental and numerical data. As expected both lengthscales produced by the LES prediction are larger than the experimental measurements. The
lengthscale of axial velocity (Figure 5.16) overpredicts the experimental results, however, given the increased growth rate of the shear layer, the lengthscales downstream of $x/D_j = 3$ are in fair agreement given the local shear layer thickness. The lengthscales of radial velocity (Figure 5.17) again overpredict the experimental results, but to a larger degree than the axial velocity based lengthscale. This is attributed to the highly 2 dimensional nature of the vortex rings, introducing large radial velocity components, which do not exist in the experiments.

5.2.3 Timescales

The temporal behaviour of the turbulence was assessed in order to give confidence in the temporal discretisation scheme used within DELTA (first order backward Euler implicit model). Dynamic filtering is a function of this temporal discretisation scheme as well as the time steps chosen in relation to the local grid size and local velocity magnitude (CFL number). As with the PIV measurements this has to be minimised and it is important that the LES does not suffer from high levels of dynamic filtering. Figure 5.18 shows the timescales at every axial station are in strong agreement with the experimental PIV results across the whole of the shear layer.

5.3 Spatio-temporal Correlations

The LES predicted spatio-temporal correlations are discussed in detail in this section. To obtain the spatio-temporal correlation functions from the LES predictions, the extracted flow samples (3072) were processed using the same PIV processing software Xact, via a custom written subroutine within DELTA.

Initial comparison of the LES predicted 2nd order spatio-temporal correlations of axial velocity for axial separation with the monoscopic PIV and filtered stereoscopic PIV are shown in Figure 5.19. There is good agreement shown between LES predictions and PIV measurements, with the peak correlation for each axial separation value occurring at the same position at all axial locations. This indicates that the eddy convection velocity is well predicted. One noticeable difference between experimental and numerical results is the magnitude of the correlation peaks for each separation value of $\eta_1$, although the error is not very large. The Gaussian shape of the auto-correlation ($R_{ij}(x,0,0)$) also remains visible at all locations. The correlations lose the oscillating shape caused by the vortex rings from $x/D_j = 4$ onwards, producing the fully turbulent shape observed in the measurements. The agreement with PIV data is also observed to improve with downstream distance.

Further comparison between the numerical and experimental results for the 4th order correlation of axial velocity for axial separation ($R_{1111}$) is presented in Figure 5.20. The difference
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between the results is more pronounced as would be expected given the shorter life span of the correlation, consistent with a greater high frequency content. In addition there is a noticeable change in the difference between the LES predictions and the PIV measurements as the axial location varies. This change (unsurprisingly) matches the change in the shear layer thickness and behaviour between the LES predictions and the PIV measurements (Figure 5.11). At $x/D_j = 1.5$ the shear layer is very thin and grows at a very slow rate due to its laminar / transitional (including vortex rings) nature; this is portrayed in Figure 5.20a by a strong sinusoidal nature. After this axial location the LES predicted shear layer undergoes a violent transition which causes an increase in the shear layer thickness with, in addition, a large increase in its growth rate. At $x/D_j = 4$ the LES predicted shear layer thickness slightly overpredicts that of the PIV measurements, although is of a similar fully turbulent nature. It is at this location that the correlation magnitudes shown in Figure 5.20b match exceptionally well. Due to the increased growth rate of the shear layer in the LES prediction and hence the shorter potential core by $x/D_j = 6.5$ the shear layers have merged. This results in increased relative lengthscales within the LES prediction in comparison with PIV measurements, causing an increased correlation peak magnitude as shown in Figure 5.20c. By $x/D_j = 10$ the experimental and numerical results are downstream of the end of the potential core and have become fully turbulent. This similarity in jet plume behaviour is the cause of improved agreement (although still with too high peak values) shown in Figure 5.20d.

The 2nd and 4th order two-point two-time correlations of the axial velocity for an axial, radial and circumferential separation vector (extracted from the LES at a point in the middle of the shear layer at $x/D_j = 4$) are compared with the current PIV measurements and the hot-wire experimental data of Harper-Bourne[61] in Figures 5.21, 5.22 and 5.23. This location was chosen as it allows full comparison with Harper-Bourne’s[61] CTA data.

For the axial separation the reduction of the peak correlation (emphasising the non-frozen nature of the turbulence) predicted by the LES is in good agreement with the PIV measurements and CTA data for both 2nd and 4th order correlations, although the LES prediction of the auto-correlation displays again the Gaussian shape. The peak magnitudes at different axial separations in the 2nd and 4th order correlations occur at the same temporal separations, which shows that the eddy convection times of the LES predictions match those of Harper-Bourne’s[61] CTA data and the PIV measurements. The peak magnitudes however, vary slightly between the different results. The LES overpredicts the PIV measurements but match the results of Harper-Bourne extremely well. It must be remembered that this location ($x/D_j = 4$) is the location of best match between the LES prediction and PIV measurements with respect to the shear layer thickness. It must also be noted however, that neither the results of the CTA data nor the PIV measurements should be assumed to be 100% correct. The CTA data has very high spatial resolution although both
CTA and PIV suffer from high frequency filtering at some level due to the anti-aliasing methods and the sub-cell filtering, causing a slight overprediction of the true peak magnitude. The true answer is therefore somewhere between the two. The results of the LES prediction, do exhibit levels of filtered energy through unresolved scales, however, the comparison to experimental data is highly encouraging. The 4th order correlations decay more rapidly in space and time, being consistent with a greater high frequency content, and match excellently between LES predictions, PIV measurements and Harper-Bourne's[61] CTA results.

In order to compare the radial correlation a constant axial separation of 0.1Dj was included. This had been necessary in the CTA data to avoid probe interference, and hence was also used for the PIV and LES data analysis. The comparisons are shown in Figures 5.22a-b. The agreement for the 2nd order quantities is very good between the CTA and PIV measurements; the LES prediction agrees well for the autocorrelation (η2 = 0Dj) but again overpredicts the peak magnitudes for all other η2 separations. The levels of overprediction seen in the radial correlations are much larger than those seen in the axial or circumferential correlations. It is believed that this is related to the presence of the vortex rings which have only just decayed by this location, but whose highly radial motions are still present (larger lengthscales \( L_{22} \)). The separation value η2 is also smaller in relation to the shear layer thickness and hence has a stronger correlation.

Finally, in order to compare the circumferential correlation a constant axial separation of 0.05Dj was necessary once again in the CTA data to avoid probe interference. However, due to the planar nature of the PIV technique, and also the planar method of LES processing, this axial separation was assumed to be negligible and set to 0Dj. The agreement is excellent between the LES predictions and the CTA data. Both of these do however, due show slight signs of high frequency filtering due to their Gaussian auto-correlation shape in comparison to the PIV measurements. At all separation values the match between all three results is very good.

Figures 5.24 and 5.25 address the question of relative amplitudes of correlation components. The peak magnitude of all components of \( R_{ij} \) and \( R_{ijkl} \) for zero time and space separations, relative to the axial component peak magnitude, are examined at \( x/D_j = 4, r/D_j = 0.5 \). The agreement between the 2C-PIV and 3C-PIV data has been discussed in Section 4.4. The relative magnitudes of \( R_{22} \) and \( R_{33} \) is a reflection of the anisotropy of the single point Reynolds stresses. The LES predictions show fair agreement with the experimental results for both 2nd and 4th order correlations. Table 5.1 shows the 2nd order correlation component amplitudes for the averaged PIV results against the LES results. Apart from the high correlation for \( R_{22} \), the ability of the LES prediction to capture the amplitudes is good. For the 4th order components, the most significant result that can be observed from Figure 5.25 is that, for turbulent jet shear layers, the largest components predicted from the LES calculations match those identified by the PIV
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measurements as $R_{1111}$, $R_{1112}$, $R_{1212}$, $R_{1313}$, $R_{2222}$ and $R_{3333}$ with all other components small by comparison. Such data is extremely useful for calibration of noise source models.

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<th>$R_{11}$</th>
<th>$R_{12}$</th>
<th>$R_{13}$</th>
<th>$R_{22}$</th>
<th>$R_{23}$</th>
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<td>0.49</td>
<td>-0.04</td>
<td>0.68</td>
<td>0.01</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Table 5.1: Comparison of 2nd order correlation component amplitudes as a ratio of $R_{11}$ between average PIV results and LES results

Assumptions about the nature of the absolute correlation amplitudes have to be made when used in noise prediction models in order to determine empirical constants in the source description and enable absolute predictions for the sound field[100]. One assumption used by Karabasov et al[100] was that the amplitude of the correlation components for zero spatial and temporal separations ($\eta = 0$ and $\tau = 0$) was independent of axial position along the jet shear layer. Earlier results from PIV measurements (Section 4.4.4) found this to be an accurate assumption. Figure 5.26 shows the amplitude of the 4th order correlation components for the 2C-PIV measurements and the LES predictions at several axial locations along the middle of the shear layer ($r/D_j = 0.5$). The effect of the much discussed vortex ring structures, and the violent transition to fully turbulent flow are clearly visible. The most dominant correlation component close to the nozzle exit ($x/D_j = 1.5$) is predicted by the LES to be $R_{2222}$ in contrast to the experimental $R_{1111}$ component. This is followed by a larger increase and reordering of the correlation components, both in terms of amplitude and dominance. This rearrangement does not meet the assumption of independence of axial distance. However, by $x/D_j = 10$ both the order of correlation dominance and the amplitudes are in excellent agreement with the PIV results. This further supports the belief that the shear layer turbulence does develop eventually in a realistic manner in the current LES predictions in spite of the near nozzle exit problems discussed above.

It has been seen that the LES predictions have shown good correlation shape functions for $R_{ijkl}$ and, although not quite to the same level of accuracy as the PIV measurements, the LES prediction produces good peak locations and magnitudes for the axial and circumferential components, and fair predictions of the radial correlations. However, the LES prediction has shown a tendency to produce Gaussian shape auto-correlation distributions which is a sign of an under resolved solution. The good similarity with experimental data has shown that LES predictions can provide a useful tool in jet noise studies, especially considering the global correlation capabilities of LES which far exceed that of the PIV measurements currently achievable. Its ability to maintain high levels of spatial resolution in all three dimensions allows LES to be used for exploratory work in guiding experimental measurements into sound source locations and distributions. The
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biggest problem that needs to be resolved is how to prevent the laminar / transitional region near nozzle exit in the LES predictions which is not observed in the experiments at high values of the jet Reynolds number.

5.4 Closure

The use of RANS CFD within jet noise propagation models, and the potential of LES CFD to predict the sound source models has led to the comparison of the PIV approach with RANS and LES numerical techniques being very beneficial. Therefore, numerical RANS and LES simulations of a single round jet using 12.6 million nodes were performed. The approach that has been adopted here was to use the finest mesh on which subsequent postprocessing of spatio-temporal correlations could be afforded and, in spite of the problems indicated above in the region very close to the nozzle, it is believed that useful information has been extracted from the current LES. The shear layer turbulence does develop eventually in a realistic manner (although slightly further downstream than in the experiments), i.e. the shear layer turbulence eventually 'forgets' its near nozzle exit plane origins. Results indicate that the use of RANS as a base-flow for noise transport models is certainly possible. The ability of LES predictions to provide detailed descriptions of the noise sources with any confidence is currently achievable but with some notable differences due to under-resolution of the mesh. The ability of LES to predict the correct correlation amplitudes relative to $R_{1111}$ is a very significant outcome, whilst a large increase in the ability of the correct absolute amplitudes to be obtained once the turbulence has 'forgotten' the 'spurious' laminar / turbulent transition region also exists.

The following chapter provides application of PIV and LES to a more complex coaxial jet configuration.
Figure 5.1: Comparisons between numerical predictions and experimental PIV measurements of the $x \tau$ plane mean axial velocity
Computational Results

Figure 5.2: LES predicted contours of the $xr$ plane axial velocity
Figure 5.3: LES predicted contours of the rθ plane axial velocity
Computational Results

Figure 5.4: LES predicted contours and isosurfaces of vorticity magnitude
Figure 5.5: Comparison of axial centreline velocity between RANS and LES predictions and PIV measurements
Figure 5.6: Radial profiles of axial velocity at all stations (m/s)
Figure 5.7: Radial profiles of axial velocity at all stations scaled based on potential core length (where $L_{pc}$ is $6.5D_j$ in PIV data, $5D_j$ in LES data, and $7.8D_j$ in RANS data) (m/s)
Figure 5.8: Radial profiles of $k$ at all stations
Figure 5.9: Comparison of axial centreline velocity between PIV, RANS, LES and previous air flow data\cite{151, 47, 26, 130, 74}

Figure 5.10: Comparison of radial profiles of axial velocity between PIV, RANS, LES and previous air flow data\cite{63, 61}
Figure 5.11: Comparison of shear layer growth against axial location between RANS and LES predictions and PIV measurements
**Computational Results**

Figure 5.12: Radial profiles of $R_{11}$ for axial separation at all stations

(a) $x/D_j = 1.5$, $r/D_j = 0.5$

(b) $x/D_j = 4$, $r/D_j = 0.5$

(c) $x/D_j = 6.5$, $r/D_j = 0.5$

(d) $x/D_j = 10$, $r/D_j = 0.5$
Computational Results

Figure 5.13: Radial profiles of $R_{22}$ for axial separation at all stations
Computational Results

Figure 5.14: Comparison of PSD between LES prediction and PIV measurements at all stations along the lipline
Figure 5.15: Autocorrelation comparison for various levels of POD filtering vs LES
Figure 5.16: Radial profiles of $L_{11}$ for axial separation at all stations
Figure 5.17: Radial profiles of $L_{22}$ for axial separation at all stations
Computational Results

Figure 5.18: Radial profiles of $T_{11}$ at all stations
Computational Results

Figure 5.19: 2\textsuperscript{nd} order correlation plots of axial velocity with axial separations

(a) $x/D_j = 1.5$, $r/D_j = 0.5$
(b) $x/D_j = 4$, $r/D_j = 0.5$
(c) $x/D_j = 6.5$, $r/D_j = 0.5$
(d) $x/D_j = 10$, $r/D_j = 0.5$
Computational Results

Figure 5.20: 4th order correlation plots of axial velocity with axial separations

(a) $x/D_j = 1.5$, $r/D_j = 0.5$

(b) $x/D_j = 4$, $r/D_j = 0.5$

(c) $x/D_j = 6.5$, $r/D_j = 0.5$

(d) $x/D_j = 10$, $r/D_j = 0.5$
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Figure 5.21: Spatio-temporal correlation functions of axial velocity with axial separations ($x/D_j = 4$, $r/D_j = 0.5$)
Figure 5.22: Spatio-temporal correlation functions of axial velocity with radial separations \((x/D_j = 4, r/D_j = 0.5)\)
Figure 5.23: Spatio-temporal correlation functions of axial velocity with circumferential separations ($x/D_j = 4, r/D_j = 0.5$)
Figure 5.24: Comparison between 2nd order correlation peak magnitudes as a ratio of $R_{11}$ for PIV measurements and LES predictions.
Figure 5.25: Comparison between 4\textsuperscript{th} order correlation peak magnitudes as a ratio of $R_{1111}$ for PIV measurements and LES predictions
Figure 5.26: Comparison between axial development of PIV measured and LES predicted 4th order correlation amplitudes at $r/D_j = 0.5$ (Solid lines = LES predictions, Dashed Lines = PIV measurements)
Chapter 6

Experimental and Computational Coaxial Jet Results

So far in this thesis the focus has been on the turbulent noise producing regions of a single round jet with statistical descriptions of aeroacoustic noise sources being produced from both experiments and LES computations. While many commercial and even military propulsion systems comprise of approximately axisymmetric exhaust jet geometries, most of these constitute separate-flow exhaust nozzles. The exhaust constitutes a coaxial nozzle with core (primary) and bypass (secondary) exhaust streams. There is, however, surprisingly little archived material (Ko and Kwan[152], Balsa and Gliebe[36], Fisher et al[153], Viswanathan[24] and finally the European program CoJeN[154, 155]) describing the turbulence and acoustic characteristics of coaxial jet flows. Given the prevalence of coaxial jets in use today it was important to determine whether the experimental and computational approaches developed in this thesis for single round jets were also capable of successful application to coaxial jets.

In deciding to include coaxial jets it was recognised that many different configurations of primary and secondary nozzles are possible; these include: eccentric nozzle configurations (in which the primary nozzle is translated vertically up relative to the bypass nozzle centreline), coplanar and short cowl nozzles, and nozzle exit plane treatments (i.e serrations, or vanes and tabs installed near the exit planes). This chapter presents experimental and computational results from the more traditional coaxial arrangement, whereby the primary nozzle is fixed in the centre of the secondary nozzle and both nozzle exit planes are at the same axial location (as detailed in Section 3.1.3). The nozzle design is essentially that of the coplanar CoJeN project nozzle as illustrated in Figures 3.17 and 3.18 and denoted here as LU80C. The primary nozzle is identical to the LU40P nozzle used within Section 4.3 and has an exit diameter of $D_p = 40mm$ and lip thickness of 1.2mm. The secondary nozzle has an exit diameter of $D_s = 80mm$ and lip
thickness of 0.9mm. Since the principal noise generation regions are located where the mixing processes are most significant, interest has principally been focused on both inner and outer shear layers \( (r/D_j = 0.5 \text{ and } r/D_j = 1.0) \). A schematic of the measurement locations is given in Figure 3.1.4.

The turbulence statistics of the coaxial jet have been measured in detail to provide information on the spatial and temporal scales present within the shear layers, together with the spatio-temporal correlations and associated eddy convection velocities. The experimental measurements were acquired using monoscopic PIV in order to ensure the highest levels of confidence in the results. LES numerical predictions were again conducted using the DELTA code and a 24.6 million node mesh split into 33 blocks and distributed over 32 processors, as detailed in Chapter 2.

This chapter is organised as follows:

Section 6.1 lists the experimental and computational operating parameters. This section discusses the range of possible operational conditions of practical relevance, and the transformation from engine relevant to isothermal conditions necessary to make experimental measurements within a water flow facility as representative as possible.

Section 6.2 details the PIV measurements and the RANS and LES predictions generated from DELTA. This section includes evaluation of single point statistics, flowfield visualisation, as well as axial and radial profiles. Profiles from the numerical predictions were taken at the same locations as the experimental PIV measurements detailed in Section 3.1.4 and illustrated in Figure 3.22.

Section 6.3 details the two point statistics obtained from both PIV measurements and LES predictions. Once again a full 3072 sample time series at 1kHz has been gathered (3.38TB) in order to enable identical samples sizes from which LES and PIV statistics are presented and compared. This section includes the shape and distribution of the spatial correlations as well as their convection velocities and decay, as well as Eulerian and Lagrangian spatial and temporal scale information from both experimental and numerical approaches.

Finally, Section 6.4 presents a full evaluation of the 2nd and 4th order spatio-temporal correlations obtained from PIV and LES for the coaxial jet configuration. This again includes identification of which of the 21 independent correlations are the largest contributors to far field sound within each of the shear layers and, in addition, whether the findings from the outer shear layer produce similarity with the results from the single round jet configuration. This section also considers differences and similarities between experimental and numerical datasets, allowing
a conclusion to be drawn as to the 4th order spatio-temporal correlations ($R_{ijkl}$) found within a coaxial jet configuration, and also the current ability of LES to be used as a truly predictive method of obtaining spatio-temporal correlations.

6.1 Coaxial Jet Operating Conditions

Chapters 4 and 5 have shown that the turbulence, and more specifically, the spatio-temporal velocity fluctuation correlations are fairly insensitive to mean flowfield parameters when suitably normalised (e.g. results comparing excellently with data from flows at a wide range of Mach and Re numbers). This allows for several possibilities for the flow conditions for the coaxial jet tests. However, given that the CoJeN nozzle has been selected, it was decided to run it at a condition which was as close as possible to one of the operating points used in the CoJeN project[134]. In that project the experimental data had been acquired at hot core conditions so some method of appropriate scaling in this project was necessary to derive an ‘equivalent’ isothermal condition.

Munk and Prim[156] developed a similarity principle based on consideration of inviscid isentropic flows, but which were not homoenthalpic (i.e the entropy, or, effectively, the stagnation enthalpy or temperature, was constant along streamlines but could vary from streamline to streamline). Their analysis showed that the streamline pattern, and all pressures and Mach numbers in such flows, were unchanged if along each streamline the values of density and velocity were multiplied by a factor $m$ and $m^{-\frac{4}{5}}$ respectively (which again could vary between streamlines). This idea was later adopted and applied (as the ‘approximate Munk and Prim substitution principle’) by Greitzer et al[157] to flows which involved significant heat and momentum transfer due to viscous mixing processes. The substitution principle was shown to work extremely well on the evidence of experimental data taken from aeroengine and jet exhaust mixing measurements. This (perhaps surprising) result has since been used extensively in model testing of jet exhaust mixers and ejectors by Presz et al[158, 159, 160] and Barankiewicz et al[161] and even turbine Nozzle Guide Vane (NGV) cooling applications by Povey et al[162]. The strong implication is that it allows jet-mixing problems with large temperature differences in their inlet conditions to be simulated experimentally at suitably scaled conditions, where the isothermal results can then be scaled back with confidence to the temperature conditions of interest. This principle has therefore been used in the present work to convert the hot / cold CoJeN coaxial jet mixing experiments to an isothermal condition.

Essentially, the Munk and Prim principle requires momentum to be maintained along streamlines. Hence on any streamline, e.g within the primary stream:

$$\frac{(\rho V^2)_{\text{Primary}}}{\text{hot}} = \frac{(\rho V^2)_{\text{Primary}}}{\text{iso}}$$

(6.1)
or as a ratio between the primary stream and the secondary stream:

\[
\left( \frac{\rho V^2}{\rho V^2} \right)_{\text{hot}}^{\text{P}} = \left( \frac{\rho V^2}{\rho V^2} \right)_{\text{iso}}^{\text{S}}
\]

(6.2)

given \( \rho \) is constant in the isothermal experiment, this implies:

\[
\left( \frac{V_P}{V_S} \right)_{\text{iso}} = \left( \frac{V_P}{V_S} \right)_{\text{hot}} \sqrt{\frac{T_S}{T_P}}
\]

(6.3)

The CoJeN operational conditions which have been chosen for this study correspond to the OP1-2 test case[134] and are detailed in Table 6.1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Operational Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secondary Total Temperature (K)</td>
<td>334.8</td>
</tr>
<tr>
<td>Primary Total Temperature (K)</td>
<td>849.1</td>
</tr>
<tr>
<td>Secondary Velocity (m/s)</td>
<td>306.8</td>
</tr>
<tr>
<td>Primary Velocity (m/s)</td>
<td>404.5</td>
</tr>
<tr>
<td>Primary / Secondary Momentum Ratio</td>
<td>0.646</td>
</tr>
<tr>
<td>Primary / Secondary Velocity Ratio</td>
<td>1.318</td>
</tr>
<tr>
<td>Outer Flight Stream (m/s)</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.1: CoJeN OP1-2 Operating Conditions

The application of Equation 6.3 yields an isothermal primary / secondary momentum ratio of 0.646 and a primary / secondary velocity ratio of 0.804. Note that the primary / secondary ratio is inverted in the isothermal case relative to the hot / cold operating condition, but this is forced by the constant momentum ratio, required by the Munk and Prim principle, as it was in the applications reported in [158, 159, 160]. Given the confinement considerations discussed in Section 3.1.2 for the single round jet, it is again important to ensure that the placing of the coaxial jet within the water tunnel enclosure does not influence the development of the jet plume. In addition, by basing the secondary flow velocity on the single round jet exit velocity, the velocity gradient and shear between the co-flow stream and the secondary stream will be identical to the single round jet. This will allow conclusions to be drawn about the presence of the primary jet stream on the outer shear layer behaviour. By using the Craya-Curtet number \( Ct \), (discussed in Section 3.1.2) it is possible to assess the level of recirculation present. Basing the \( Ct \) number on the bulk exit velocity of the coaxial jet (0.92m/s) yields \( Ct=1.07 \). Although slightly lower than the \( Ct \) number used for the single round jet case, it is still above the limit of \( Ct=0.98 \) for recirculation to occur and allows for comparable co-flow stream / jet stream ratio between the
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single and coaxial jet cases. Given the isothermal test condition ratios, and the confinement considerations, the isothermal jet conditions are given in Table 6.2. These conditions were used for PIV testing, but were also the basis of all CFD predictions performed, so that predictions may be compared directly with measurements, without any further scaling.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Operational Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secondary Velocity (m/s)</td>
<td>1.0</td>
</tr>
<tr>
<td>Primary Velocity (m/s)</td>
<td>0.804</td>
</tr>
<tr>
<td>Primary / Secondary Velocity Ratio</td>
<td>0.804</td>
</tr>
<tr>
<td>Flight Stream (m/s)</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 6.2: Isothermal Coaxial Jet Test Conditions

6.2 Single Point Statistics - RANS vs LES vs PIV

Single point statistics were gathered from the PIV measurements and the RANS and LES numerical predictions to provide information about the flowfield and its turbulent structure. Comparison between experimental and numerical single point statistics also provides evidence to allow the ability to predict the flowfield given the increased complexity in a coaxial jet configuration to be assessed. The accuracy of the experimental measurements must pay due attention to the implementation of the PIV system (as discussed in depth in Chapter 4). Flowfield measurements have been conducted using three FoV sizes, 80mm×80mm, 50mm×50mm and 30mm×30mm. For the coaxial jets, these were restricted to monoscopic mode and xz plane orientation. Measurements have been taken at 4 locations, 2 within each shear layer (i.e. r/D_j=0.5 and 1.0) at 2 axial locations (z/D_j=3 and 8).

One initial comment on coaxial jet flows that has been reported in the literature is that the noise produced by the primary jet in such a coaxial configuration could be reduced by the shielding effect due to the secondary (bypass) flow. However, it has been shown [163, 164] that the primary jet potential core in coaxial arrangements is lengthened, while the secondary stream becomes mixed out quickly and usually well upstream of the end of the primary stream potential core. As a result, a significant portion of the noise may still be emitted by the primary jet stream. It is therefore important to recognise that if RANS calculations are used as the base-flow in sound propagation models that this secondary to primary stream mixing is correctly achieved. In addition, if the sound source terms are to be derived from LES calculations, the same accuracy in secondary to primary stream mixing must also be achieved.
6.2.1 Flowfield Visualisation

Flowfield visualisation allows an assessment to be undertaken of the quality of mean statistics as well as allowing the behaviour and nature of the unsteady turbulent motions of the jet to be analysed. Figures 6.1a-c show comparisons between current RANS and CoJeN RANS[165], RANS and PIV, and PIV and LES respectively. Figure 6.1a provides strong evidence to support the use of the approximate Munk and Prim substitution principle. The CoJeN RANS simulation was carried out for the hot / cold boundary conditions of the experiment as indicated in Table 6.1 and using a compressible CFD formulation. The predicted solution was converted using Equation 6.1 to an 'equivalent' isothermal velocity field, which is shown in Figure 6.1a. The match between the current RANS (Table 6.2 conditions) and the scaled CoJeN RANS predictions is very good, with the shear layer development and potential core lengths being in excellent agreement. On close inspection, one can see a slight difference in the thickness of the shear layer between the primary and secondary streams (inner shear layer) at the nozzle exit. The CoJeN RANS prediction shows a very thin shear layer which is short lived while the current RANS prediction shows a thicker shear layer which is present until \( z/D_j = 4 \). This difference is attributed to the difference in lip thickness. In the CoJeN simulation the lip thickness was \( r/D_j = 0.01 \) in contrast to the current RANS simulation (which is based on the nozzle tested in the current experiments with its manufacturing limitations) which has a lip thickness of \( r/D_j = 0.03 \). The shear layer between the secondary and flight streams (outer shear layer) shows similar trends but to a lesser degree. The differences are in agreement with a smaller difference in lip thickness, with the CoJeN outer lip thickness being \( r/D_j = 0.01 \) while the current nozzle lip thickness is \( r/D_j = 0.0225 \).

Figure 6.1b shows a comparison between RANS predicted and PIV measured flowfields. For this visualisation the PIV measurements were conducted using the larger 80mm×80mm FoV (as the turbulence levels were not required). The comparison is promising with the outer potential core showing a good match in length, although slightly thicker in the experimental measurements. The inner shear layer between the primary and secondary streams is of similar thickness (due to identical lip thickness in CFD and experiments) although the downstream penetration does not seem to be well captured by the PIV measurements. The failure to capture the full penetration is attributed to the high dynamic ranges present in the shear layer region and the low spatial resolution of the larger 80mm×80mm FoV.

Figure 6.1c shows the comparison between LES prediction and the PIV measurements. As for the single round jet the LES predicts potential core lengths significantly shorter, while the outer shear layer is very thin at the nozzle exit. This is similar to the conclusions made in Section 5.1 illustrating the maintained trend accuracy for a more complex coaxial jet flowfield given the same spatial resolution. The nature of the thin shear layers at nozzle exit in the
predictions is illustrated by the instantaneous axial velocity contour plots shown in Figures 6.2 and 6.3 and by the vorticity plots shown in Figures 6.4a-b. Figure 6.2 illustrates that the outer shear layer between the nozzle exit and \( x/D_j = 2.5 \) is clearly not fully turbulent but instead undergoes transition from a laminar like shear layer to a turbulent layer via large vortex ring structures. The inner shear layer also goes through this transition, although the laminar region is not present and is instead replaced by a large number of small vortex ring structures propagating from the nozzle lip. The unsteady motions in the turbulent jet that are predicted by the LES are illustrated in Figures 6.4a-b via \( xr \) plane contours of vorticity magnitude and an isosurface of vorticity magnitude coloured by axial velocity respectively. It can clearly be seen that the vortex ring structures are present at the inner nozzle lip and from approximately \( x/D_j = 0.5 \) downstream of the outer nozzle lip. These strong two-dimensional structures indicate the delayed development of three-dimensional turbulence. As discussed in Chapter 5, this is attributed to the inadequate treatment of the nozzle inlet conditions (which contain only uncorrelated turbulence disturbances) and the failure of the mesh to resolve the physics of the nozzle wall boundary layer profiles.

### 6.2.2 Axial and Radial Profiles

To quantify the differences identified in the previous section, axial and radial profiles have been plotted. The development of the axial mean velocity along the centrelines of both primary and secondary streams \((r/D_j = 0 \text{ and } r/D_j = 0.75)\) is shown in Figure 6.5. The largest 80mm×80mm FoV, which covers the whole axial range, is compared to the RANS and LES predictions and the Munk and Prim scaled CoJeN RANS predictions. The data from both RANS predictions compare well along both stream centrelines. The only slight difference is that the current RANS prediction has a larger potential core length in the primary stream, but produces a shorter secondary stream potential core length. This is not surprising, however, since in the current RANS prediction there is an additional (ambient co-flow) stream mixing with the outer shear layer in contrast to the zero outer co-flow stream of the CoJeN RANS prediction. The presence of a tunnel outer stream leads to a different level of shear in the outer shear layer but a similar level of shear in the inner shear layer. A noticeable difference can be seen between both RANS predictions and the LES prediction. Both LES prediction and PIV measurements have managed to capture a slight increase in the potential core velocity in both primary and secondary streams. This increase occurs at the same axial location for both streams and from Figure 6.1 appears to be where the primary and secondary streams become fully mixed. The axial location of the joining of primary and secondary streams occurs at approximately \( x/D_j = 12 \) and \( x/D_j = 16.5 \) for the LES predictions and PIV measurements respectively. The LES prediction underestimates the mixing location by 27%. This is very similar to the underprediction of 23% in the potential core length of the single round jet predictions (Section 5.1). The experimental decay of the
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Secondary stream centreline velocity downstream of the potential core closure is, however, well matched by the LES prediction.

Figures 6.6a-b present radial profiles of the mean axial velocity, which allows the movement of the jet streams to be identified and quantified. The most obvious difference is the location of the peak velocity for the secondary stream. Firstly, both experimental and numerical results show that the middle of the peak region moves radially inwards of the secondary stream centreline \((r/D_j = 0.75)\). Secondly, the experimental peak location occurs at approximately \(r/D_j = 0.68\). This is in contrast to both RANS predictions and the LES prediction which show this to occur at approximately \(r/D_j = 0.72\). This is in all probability due to the presence of the wake-like behaviour of the inner shear layer in the numerical predictions. On close inspection, the presence of a drop in the inner shear layer velocity to less than either of the streams is characteristic of a wake-region. This therefore leads to a movement of the location of greatest shear (steepest gradient) radially outwards (as seen later in Figure 6.7). By \(x/D_j = 8\) both RANS predictions and the experimental measurements locate the peak velocity at the same radial location of \(r/D_j = 0.61\). This continued movement of the peak velocity location radially inwards illustrates the difference in the rate of mixing between the inner and outer shear layer. The LES predictions, however, illustrate no defined peak in the velocity due to the increased mixing of the streams associated with the reduction in potential core length.

The turbulence levels across the shear layers are presented in Figures 6.7a-b. The location of the outer shear layer for all results and at both axial locations can be seen to occur at the same radial location (along the lipline \(r/D_j = 1.0\)). The inner peak turbulence location representing the centre of the inner shear layer from the numerical calculations can be seen to predict the peak at a larger radial distance than is present in the experimental results (where the peak turbulence is located along the inner lipline at \(r/D_j = 0.5\)). This is attributed to the wake-like shear layer and the presence of the low velocity region between the two jet streams. As for the turbulence levels, the experimental results and the RANS prediction match well across both shear layers and at both axial locations. The LES prediction has much larger turbulence level which is unsurprising given the problems with the shear layer (spurious) laminar / turbulent transition and the level of turbulence energy associated with it.

6.3 Two Point Statistics - LES vs PIV

The ability of both PIV measurements and LES predictions to capture time dependent turbulent statistics allows for investigation into the temporal and spatial scales present, both of which are analysed and discussed in the following section. This also includes evaluation of the level of high frequency, small scale, turbulence energy lost due to the sub-cell filtering in the PIV data. The
Eulerian length and time scales, as well as the Lagrangian length and time scales and the eddy convection velocity are presented.

6.3.1 Spatial filtering levels and their effects

The effects of spatial filtering can be assessed by reference to the levels of correction necessary to the turbulent intensities (RMS) and the integral lengthscale quantities. The correction has been performed using the Hollis correction method as discussed in Section 2.3.3. Figures 6.8 and 6.9 show the PIV measurements of the axial and radial RMS correction levels respectively for the 80mm×80mm and 30mm×30mm FoV's used. As observed in previous results (Section 4.1.3), the smaller FoV with lower levels of sub-cell filtering requires less correction. Experimental results from the coaxial jet configuration shows that the axial RMS levels obtained from the 30mm×30mm FoV data are ≈98% of the true value downstream of x/Dj = 3 (Figure 6.8b), while the radial RMS levels are ≈96% of the true value downstream of x/Dj = 3 (Figure 6.9b). As it is the instantaneous velocity values which are required in order to obtain the spatio-temporal correlations, and these can not be corrected for, it is critical to obtain the most accurate flowfield representation possible. The very low correction level needed for the turbulence levels measured by the 30mm×30mm FoV in this experimental setup give confidence in the accuracy of the spatio-temporal correlations.

Figures 6.10a-d present the power spectral density at both axial locations (x/Dj = 3 and x/Dj = 8) in both shear layers. These figures illustrate the lack of high frequency content in the LES predictions in comparison to the PIV measurements (which themselves are subject to some spatial filtering). As discussed in Section 5.2 this lack of small scale information will have a detrimental effect on the spatio-temporal correlations, resulting in larger peak correlation magnitudes, and larger deduced integral length and time scales. This will subsequently increase the Lagrangian lengthscales and reduce the levels of dissipation (increase Lagrangian timescales). The lack of high frequency content at x/Dj = 3 can also be attributed to the strong two-dimensional turbulence due to the vortex ring structures which can be identified by the strong peak at ≈10Hz (lacking in the predictions at x/Dj = 8).

6.3.2 Lengthscales

Figure 6.11 shows the Eulerian integral lengthscales for PIV measurements from the smaller (lower error) FoV's (50mm×50mm, 30mm×30mm) and the LES predictions. Figures 6.11a-b show the axial lengthscale of axial velocity. The 50mm×50mm FoV measurements and LES predictions overpredict the lengthscale at the centreline (r/Dj = 0) in comparison to the 30mm×30mm FoV results. This overprediction reduces as the radial location increases. Further downstream at
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$x/D_j = 8$, the LES continues to overpredict the lengthscale while the 50mm x 50mm FoV measurements show good agreement with the 30mm x 30mm FoV measurements. The axial lengthscale of radial velocity shown in Figures 6.11c-d shows similar trends.

6.3.3 Timescales

The agreement in deduced Eulerian integral timescales between PIV measurements and LES predictions are acceptable, as illustrated in Figures 6.12a-d. There are a few regions, particularly downstream at $x/D_j = 8$, where the timescales have reduced due to the natural growth of the scales present. In these regions the limited number of samples reduces the convergence of the statistics and hence discrepancies between the results are identifiable. In general the results are similar to observations made for the simpler single jet flow.

6.3.4 Convection Velocities and Lagrangian Statistics

Through the production of spatio-temporal correlation maps it is possible to analyse the Lagrangian statistics of the turbulence as well as gain information about eddy convection velocities. Figures 6.13 and 6.14 illustrate a selected sample of 2nd and 4th order spatio-temporal correlations at $x/D_j = 3$ within the outer shear layer for both axial and radial velocity correlations (a full set at all locations is given in Appendix B). A common trend with all of these correlations is the larger highly correlated area, in both space and time, predicted by the LES calculation in comparison to the PIV measurements. This is consistent with the absence of high frequency energy within the LES calculation. The large vortex ring structures present at $x/D_j = 3$ can also be seen to produce secondary areas of high correlation which are not present in the experimental correlations. The experimental trend that the radial correlations are significantly thinner than the axial is reproduced well by the LES data. As expected the 4th order correlations are noticeably smaller in both space and time in comparison to the 2nd order correlations. The LES, however, still overestimates the correlation magnitude and underestimates the spatial and temporal decay of the correlation. The over and underestimates of the LES in comparison to the PIV measurements have stayed the same in the coaxial results as were shown in the single jet, especially for the 2nd order correlations, illustrating a maintained level of trend accuracy prediction for the more complex flow.

The gradient of these correlation maps yields the eddy convection velocity. This velocity deduced from each of the axial 2nd and 4th order correlations is presented in Table 6.3. For the single round jet shear layer the convection velocity was measured to be $\approx 0.63$ m/s. In comparison the shear layer between the ambient flow and secondary jet stream ($r/D_j = 1.0$) produces a convection velocity obtained from the 2nd order correlations measured by the PIV technique of
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0.65m/s at \(x/D_j = 3\) rising slightly to 0.68m/s by \(x/D_j = 8\). The convection velocity between the primary and secondary streams is significantly higher at \(\approx 0.85\)m/s for both axial locations. The LES prediction does capture this difference in the convection velocity between the two shear layers, however, it is \(\approx 18\%\) slower for the inner shear layer and \(\approx 13\%\) slower for the outer shear layer at both axial locations. The convection velocities obtained from the 4\(^{th}\) order correlations should be identical to those obtained through the 2\(^{nd}\) order correlations. As seen in Table 6.3 they do produce very similar results although the difference between the PIV measured and LES prediction results is still present.

<table>
<thead>
<tr>
<th>Location</th>
<th>(R_{11}) PIV</th>
<th>(R_{11}) LES</th>
<th>(R_{1111}) PIV</th>
<th>(R_{1111}) LES</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x/D_j = 3 r/D_j = 0.5)</td>
<td>0.89</td>
<td>0.73</td>
<td>0.85</td>
<td>0.74</td>
</tr>
<tr>
<td>(x/D_j = 3 r/D_j = 1.0)</td>
<td>0.65</td>
<td>0.56</td>
<td>0.64</td>
<td>0.55</td>
</tr>
<tr>
<td>(x/D_j = 8 r/D_j = 0.5)</td>
<td>0.87</td>
<td>0.74</td>
<td>0.88</td>
<td>0.74</td>
</tr>
<tr>
<td>(x/D_j = 8 r/D_j = 1.0)</td>
<td>0.68</td>
<td>0.59</td>
<td>0.69</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Table 6.3: Convection velocity for LU80

The decay of the correlation magnitude in a moving frame of reference (at the convection speed) yields the Lagrangian statistics. The Lagrangian lengthscale is defined as the distance taken in the moving frame of reference for the correlation magnitude to reduce to \(\frac{1}{6}\) of its original value (as detailed in Section 2.3.3). To maintain the accuracy of the turbulence and hence the correlation measurements, the smallest 30mm×30mm FoV was used to obtain the Lagrangian scales. Due to this small FoV the 2\(^{nd}\) order correlation magnitude did not decay to \(\frac{1}{6}\) within the FoV. Exponential curve fitting and extrapolation were therefore used to obtain the scale. The faster decaying 4\(^{th}\) order correlations did not decay within the FoV for all correlation functions, although the amount of curve fitting and extrapolation was significantly less than for the 2\(^{nd}\) order correlations. The full flowfield nature of the LES predictions meant that no such data extrapolation was required. The 2\(^{nd}\) and 4\(^{th}\) order Lagrangian axial lengthscales of axial velocity for both PIV measurements and LES predictions are presented in Table 6.4.

<table>
<thead>
<tr>
<th>Location</th>
<th>(L_{11}^{1}/D_j) PIV</th>
<th>(L_{11}^{1}/D_j) LES</th>
<th>(L_{1111}^{1}/D_j) PIV</th>
<th>(L_{1111}^{1}/D_j) LES</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x/D_j = 3 r/D_j = 0.5)</td>
<td>2.70</td>
<td>1.43</td>
<td>1.11</td>
<td>0.96</td>
</tr>
<tr>
<td>(x/D_j = 3 r/D_j = 1.0)</td>
<td>1.00</td>
<td>1.46</td>
<td>0.43</td>
<td>1.20</td>
</tr>
<tr>
<td>(x/D_j = 8 r/D_j = 0.5)</td>
<td>2.18</td>
<td>2.93</td>
<td>1.36</td>
<td>1.45</td>
</tr>
<tr>
<td>(x/D_j = 8 r/D_j = 1.0)</td>
<td>1.76</td>
<td>1.83</td>
<td>0.64</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Table 6.4: Lagrangian axial lengthscales of axial velocity

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6.4 Spatio-temporal Correlations

So far the unsteady statistics obtainable from the LES predictions have been shown to under-predict potential core length and overpredict mixing rates, energy levels and lengthscales due to strong vortex ring structures in the initial shear layer regions. It is still, however, believed to be valuable to examine the ability of the 24.6 million node LES simulations to predict the spatio-temporal correlation function shapes and evolution. This section will address the comparison with experimental data of both spatio-temporal correlation distributions and the identification of the most significant correlation terms. The final part of this section is an investigation into the behaviour of the shear layer of the single round jet and the secondary (outer) shear layer of the coaxial jet. Both of these shear layers are generated through the mixing of a 1m/s jet flow and a 0.18m/s ambient co-flow, hence allowing the differences due to the presence of the primary jet to be assessed.

6.4.1 Spatio-temporal Correlation Distributions

Comparisons of the 2nd order spatio temporal correlations of axial velocity for axial separations are shown in Figure 6.15. The slight reduction in the convection velocity (identified in Table 6.3) predicted by the LES simulation can be identified with the peak locations for each axial separation distance ($\eta$) occurring to the right of the PIV peak locations. The most noticeable difference is the magnitude of the correlation peak and the Gaussian shape of the zero separation correlation (both observed also in the single jet analysis). Comparison of the 4th order correlations of axial velocity and axial separation is presented in Figure 6.16. As expected the variation between the two results is far more pronounced for the 4th order statistics, particularly at $x/D_j = 3$, although the same trends are identified.

The numerical predictions do, however, offer one large advantage over the experimental approach. This advantage is the ability to maintain the appropriate grid resolution over the whole domain and not be restricted by local FoV sizes. This ability enables low frequency / high spatially separated correlation areas to be identified and investigated. This is illustrated in Figure 6.17 and Figure 6.18. These figures show the 2nd and 4th order correlations of axial and radial velocity with axial separation at $x/D_j = 8$. The FoV size has been increased by a factor of 4 to illustrate what is captured in a FoV size dictated by experimental limitations and more importantly what is missed. Both Figures 6.17b and d and 6.18b and d identify the recurrence of low frequency / highly spatially separated regions of high correlation which were not seen in the smaller FoV correlation plots. Through the larger FoV size available in LES predictions, visualisation and analysis of spectral peak frequencies can be produced (e.g the 10Hz peak mentioned...
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in Section 6.3.1 can be matched to the passage of certain highly correlated regions).

6.4.2 Spatio-temporal Correlation Function Amplitudes

It has been clearly identified that the accuracy of the inlet conditions and the current computing power limitation in terms of the number of nodes realistically usable in an LES simulation reduces the ability of the current LES prediction to predict accurately the shape and distribution (both spatially and temporally) of the 4th order velocity correlations critical for accurate noise source modelling. However, it would be wrong to dismiss completely the LES data as it does have the ability to aid in the sound source identification and the selective inclusion of those sound sources into propagation models. Figure 6.19 illustrates the comparison of the 2nd order correlation magnitudes (as a ratio of $R_{11}$) between PIV measurements and LES predictions at $x/D_j = 3$ and $x/D_j = 8$. Figure 6.19a shows the comparison at $r/D_j = 0.5$ whilst Figure 6.19b shows the comparison at $r/D_j = 1.0$. Generally this comparison shows a strong trend match between LES predictions and experimental results. The overpredicted $R_{22}$ term at $(x/D_j = 3, r/D_j = 1.0)$ is attributed to the vortex ring structures present in the LES prediction at this location which are not present at $(x/D_j = 8, r/D_j = 1.0)$ where better agreement with the experimental results is observed. This vortex ring structure, due to its two-dimensional nature, can also be seen to influence the $R_{33}$ correlation term, producing very little correlation in comparison to $x/D_j = 8$, where no vortex ring structure is present.

Figures 6.20 and 6.21 illustrate the comparison of the 4th order correlation magnitudes as a ratio of $R_{1111}$ between PIV measurements and LES predictions at both axial stations within the inner and outer shear layers respectively. The comparison between the PIV measurements and the LES predictions is good for all terms at $x/D_j = 8$ for both inner and outer shear layers. However, at $x/D_j = 3$, the ability of the LES to predict any correlation incorporating the tangential velocity ($U_3$) can be seen to be significantly underpredicted (approaching zero), while any correlation incorporating the radial velocity ($U_2$) is underpredicted in the inner shear layer but overpredicted in the outer shear layer (dominated by the vortex ring structures). Analysis of the experimental results does, however, show that there is very little change in the amplitude (as a ratio of $R_{1111}$) as the axial location is changed, although there can be seen to be a slightly larger amplitude of the correlation terms incorporating the radial velocity in the outer shear layer than in the inner shear layer. This implies that, all the difficulties associated with the nozzle exit boundary layer / shear layer transition have been 'lost' in the LES, and the physics of the 4th order correlations are well represented.
6.4.3 Single Round Jet vs Coaxial Round Jet

Scaling the coaxial jet secondary stream velocity to be identical to the single round jet velocity makes it possible to analyse and compare the correlation functions (shapes, magnitudes, and relative amplitudes) from the shear layer incorporating the outer co-flow stream with the single round jet shear layer. The comparisons were performed using data captured at two axial locations \(z/D_j = 1.5\) and \(4\) based on the corresponding shear layer jet diameter (i.e. Single round jet \(D_j = 40\,\text{mm}\), Coaxial round jet \(D_j = 80\,\text{mm}\)).

Figures 6.22a-b show the 2nd order axial velocity correlation distributions for axial separations at \(z/D_j = 1.5\) and \(z/D_j = 4\) respectively. At \(z/D_j = 1.5\) Figure 6.22a shows that there is a strong similarity (in peak magnitudes, convection velocity) between both of the PIV datasets and between both of the LES datasets, independent of nozzle configuration. The main noticeable difference is the level of negative correlation. This could be due to the presence of the primary stream, even close to the nozzle exit, however, is more likely to be due to (in the LES) the differing impact of the (spurious) laminar / turbulent transition, and due to the difference in azimuthal effects due to the larger jet diameter. At \(z/D_j = 4\) some difference in the experimental results due to jet configuration is present, with the coaxial jet producing a reduced peak magnitude compared to the single jet, due to the mixing between primary and secondary streams. The same can be seen for the numerical results and although larger than in the experimental results, the reduced peak magnitude of the coaxial jet is produced which illustrates the ability of the LES calculations to produce the same trends.

Figures 6.23a-b show the 4th order axial velocity correlation distributions for axial separations at the same two axial stations. At \(z/D_j = 1.5\) the experimental results show a good match in peak correlation magnitude and convection velocity although the coaxial jet does produce a much narrower correlation distribution. This is not seen in the LES predictions, with the autocorrelation being very similar, and a slightly higher peak correlation magnitude for the coaxial jet results. At \(z/D_j = 4\) the difference seen in Figure 6.22b is repeated, with the same difference between the single and coaxial configurations being once again present in both PIV and LES results.

Finally, it is possible to assess the most significant correlation terms in relation to the nozzle geometry. Figures 6.24 and 6.25 illustrate the amplitudes of the 2nd and 4th order correlations. These figures provide evidence to support the conclusion that the most significant correlation functions are independent of the nozzle geometry and hence are independent of the presence of the primary stream. It can be concluded that the most significant correlations terms are \(R_{1111}\), \(R_{1112}\), \(R_{1212}\), \(R_{1313}\), \(R_{2222}\) and \(R_{3333}\) with all other components small by comparison. This is not
to say that the amplitude itself is identical, but that the information provided here can be used to guide the noise source terms used in farfield noise level prediction models irrespective of the nozzle geometry of interest, at least between single and coaxial configurations.

6.5 Closure

A coaxial jet configuration has been scaled using the Munk and Prim principle to derive an 'equivalent' isothermal condition for one hot / cold condition set within the CoJeN experiments. As for the single round jet, the coaxial jet RANS and LES predictions showed good trend accuracy with the PIV measurements, while the LES prediction produces the same 'spurious' laminar / turbulent transition, once again reiterating the necessity for accurate inlet boundary conditions and boundary layer / shear layer transition. The measured and predicted turbulence statistics of the coaxial jet configuration to provide information on the spatial and temporal scales present within the outer and inner shear layers have been detailed, and deliver new insights into the characteristics of the $4^{th}$ order correlation $R_{ijkl}$, for example that in a coaxial jet, the same 6 correlation components as the single round jet are significant. Once again this provides valuable information for aeroacoustic predictions.
Figure 6.1: Comparisons between numerical and experimental mean axial velocity contours.
Figure 6.2: LES contours of the $x\tau$ plane axial velocity
Figure 6.3: LES contours of the $r\theta$ plane axial velocity
Figure 6.4: LES contours and isosurfaces of vorticity
Figure 6.5: Comparison of axial centreline velocity between RANS and LES predictions and PIV measurements \((m/s)\)
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Figure 6.6: Radial profiles of axial velocity at all stations (m/s)

Figure 6.7: Radial profiles of $k$ at all stations
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Figure 6.8: Axial RMS error maps

(a) FoV = 80mm×80mm

(b) FoV = 30mm×30mm
Figure 6.9: Radial RMS error maps
Figure 6.10: Comparison of PSD between LES prediction and PIV measurements at all stations along the inner and outer lipline
Figure 6.11: Radial profiles of $L_{11}$ and $L_{22}$ for axial separation at all stations
Figure 6.12: Radial profiles of $T_{11}$ and $T_{22}$ at all stations
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Figure 6.13: 2nd order correlation maps of axial and radial velocity with axial separations at \( x/D_J = 3, r/D_J = 1.0 \)
Figure 6.14: 4th order correlation maps of axial and radial velocity with axial separations at $x/D_j = 3$, $r/D_j = 1.0$
Figure 6.15: 2\textsuperscript{nd} order correlation plots of axial velocity with axial separations
Figure 6.16: 4th order correlation plots of axial velocity with axial separations.
Figure 6.17: 2nd order correlation maps of axial and radial velocity with axial separations at $x/D_j = 8$, $r/D_j = 1.0$
Figure 6.18: 4\textsuperscript{th} order correlation maps of axial and radial velocity with axial separations at $x/D_j = 8, r/D_j = 1.0$
Figure 6.19: Comparison of 2nd order correlation peak magnitudes as a ratio of $R_{11}$ between PIV measurements and LES predictions
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Figure 6.20: Comparison of 4th order correlation peak magnitudes as a ratio of $R_{1111}$ between PIV measurements and LES predictions at $r/D_j = 0.5$.
Figure 6.21: Comparison of 4th order correlation peak magnitudes as a ratio of $R_{1111}$ between PIV measurements and LES predictions at $r/D_j = 1.0$
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Figure 6.22: Comparison of 2nd order correlation plots of axial velocity with axial separations between single round jet (solid lines) and coaxial round jet (dashed lines) in the outer shear layer. ($D_j$ based on outer jet diameter)

(a) $x/D_j = 1.5$

(b) $x/D_j = 4$
Figure 6.23: Comparison of 4th order correlation plots of axial velocity with axial separations between single round jet (solid lines) and coaxial round jet (dashed lines) in the outer shear layer. ($D_j$ based on outer jet diameter)
Figure 6.24: Comparison of 2\textsuperscript{nd} order correlation peak magnitudes as a ratio of $R_{11}$ between PIV measurements and LES predictions at $x/D_j = 4$ for both single and coaxial round jets.
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Figure 6.25: Comparison of 4th order correlation peak magnitudes as a ratio of $R_{1111}$ between PIV measurements and LES predictions at $x/D_j = 4$ for both single and coaxial round jets.
Chapter 7

Conclusions and Recommendations

The current level of noise generated by propulsive jets at take-off is one of the major factors affecting the development of air travel. Any measures that can be applied to engine design that reduce the noise level will secure many benefits. To date a number of efforts have been made to develop predictive tools that can estimate the noise generated by new nozzle designs, however, all of these tools require the input of empirically calibrated noise source models and their performance is still inadequate.

The main objectives of this project have therefore been to develop experimental nozzle facilities, instrumentation, and test practices using a water tunnel facility, in order to provide unsteady turbulence information to improve the accuracy of noise source models. Jet nozzle configurations relevant to jet noise studies were used. In addition, it was important to use instrumentation that captured the flowfield data representative of full scale subsonic airflow jet turbulence. Due to the detailed nature of the unsteady turbulent statistics required it was recognised that any such methods would have to possess excellent spatial and temporal resolution, whilst also being able to capture full 3 component velocity flowfield data over large enough areas to track the turbulent motions. A subsidiary objective of this project was to assess the accuracy and acceptability of numerical RANS and LES calculations to predict the mean flowfield statistics, and the ability of LES to capture the unsteady turbulent correlations.

Single and coaxial jet configurations were therefore designed and investigated to provide turbulence information to aid the development of more accurate and usable noise source models. The objectives mentioned above were delivered through further development, quantification and validation of a generalised and widely applicable PIV-based methodology for providing unsteady, spatially and temporally resolved velocity field. The result has been the development of an experimental technique using a water tunnel facility in which the gathered flowfield data has been
Conclusions and Recommendations

demonstrated to be representative of subsonic airflow jet turbulence. The technique recognises and works with the high frequency nature of the unsteady velocity field within jet plumes, and PIV instrumentation limitations (frequency resolution of 1kHz).

Through the use of a global technique a time-resolved proper orthogonal decomposition (POD) of the jet velocity field was carried out, enabling the identification of different modes of coherent structures and their associated energy content. This has resulted in the technique's development as the basis of a spatio-temporal filtering procedure suitable for removing 'spurious' noise from 3-component PIV data. The results show that, by using stereoscopic PIV with a repetition rate of 1kHz, and given the correct application of the method (e.g. sufficiently small PIV interrogation cell size in relation to the local turbulent lengthscales), even 4th order correlations can be captured accurately, and were demonstrated to reproduce the quality of those captured by point-based techniques such as Constant Temperature Anemometry (CTA). These measurements deliver new insights into the characteristics of the 4th order correlation $R_{ijkl}$, for example that, of the 21 possible independent components, in a round jet only 6 are significant. This level of detail is valuable for aeroacoustic prediction methods which need to construct a model for $R_{ijkl}$.

Currently RANS CFD is widely used in jet noise predictions, while LES CFD has the potential to predict the sound source correlations and aid jet noise prediction. Therefore, in addition to the PIV studies, data was also gathered by performing numerical RANS and LES simulations of a single and a coaxial round jet using 12.6 million and 24.6 million node meshes respectively. The use of RANS as a base-flow for noise transport models was judged adequate. For LES CFD the identification of a problem related to a rather simplistic treatment of nozzle inflow conditions, restricted the ability of the current LES predictions to provide detailed descriptions of the noise sources to a very precise level, although many features were reproduced. The approach that was adopted here was to use the finest mesh that could be afforded and, in spite of the problems indicated above in the region very close to the nozzle, to assume that the shear layer turbulence does develop eventually in a realistic manner (although slightly further downstream than in the experiments), i.e. that the shear layer turbulence eventually 'forgets' its near nozzle exit plane origins. The LES predictions were then analysed to examine the turbulence structure in this further downstream region and assess the extent to which it predicted the turbulent spatio-temporal correlations measured in the PIV experiments. In general it was observed that the LES did predict several important features of the high order correlations, for example the correlation amplitudes relative to $R_{1111}$. The further development of the LES technique is covered in the Recommendations section below.
In conclusion the single most important finding of this research is that water based experimental testing using high speed PIV instrumentation as applied in this thesis is a valuable tool in jet noise predictive studies to identify and describe the 4th order correlation sound source characteristics.

The following sections detail the conclusions of the experimental and numerical studies undertaken in this thesis.

7.1 Experimental Conclusions

- The major sources of intrinsic error in monoscopic PIV processing in this project originate from the sub-pixel particle location accuracy and statistical convergence of the data sample. These introduce an uncertainty of $\approx 0.5\%$ and $\approx 2.0\%$ in the resultant velocity field for 50Hz and 1kHz data respectively.

- Spatial filtering and the associated loss of accuracy due to the relative size of FoV and local integral scale of turbulence has been reduced by use of a much smaller FoV ($25\text{mm} \times 25\text{mm}$) than in previously published experiments. This has led to a sub-cell filtering RMS error of less than 2% for the axial RMS and less than 3% for the radial RMS.

- Comparison of the turbulent structures produced by the proposed enclosed water flow configuration with single point data from other studies is shown to be excellent, with turbulent integral length scales and time scales estimated from the present water flow PIV measurements matching well those obtained from airflow experiments.

- Stereoscopic PIV measurements have been conducted for the axisymmetric single jet within the water tunnel facility. Obtaining successful stereoscopic PIV data is more challenging than for a monoscopic arrangement. Care must be taken to account for the effect of oblique viewing, camera calibration and insufficient seeding, which can all lead to a reduction in the data quality. This being said, stereoscopic data has been gathered and provided strong evidence that the measured turbulence statistics are closely representative of high subsonic jet shear layers. Spectral analysis did however, highlight a significant increase in high frequency energy contamination (classified as high frequency noise).

- Implementation of a time-resolved POD analysis and its use as a low pass filter has been shown to be a valid method of filtering stereoscopic PIV high frequency noise, with limited sample sizes. The low energy (i.e high frequency) POD filter acts as a more intelligent filter than simple downsampling whilst not reducing the total sample size, allowing for no reduction in statistical convergence and hence maintaining accuracy of results.
Conclusions and Recommendations

- 2nd and 4th rank tensor spatio-temporal correlations have been evaluated from the present measurements and are shown to be of comparable accuracy to previous CTA results both near the nozzle exit (where scales are small) and further downstream. This includes turbulent Lagrangian length scales, time scales and eddy convection velocities estimated from the present water flow PIV measurements matching well those obtained from airflow experiments.

- In addition to validating the stereoscopic 3C-PIV against CTA results (it is believed for the first time for the 4th order correlations), whole field visualisation of the correlations of all 3 velocity components has been possible via spatio-temporal correlation maps.

- The individual components of 2nd and 4th order correlations most likely to contribute to the magnitude of the local shear layer noise source in a single round jet have been identified as $R_{1111}, R_{1112}, R_{1212}, R_{1313}, R_{2222}$ and $R_{3333}$ with all other components small by comparison. Such information is proposed as extremely useful for construction and calibration of noise source models and should improve jet noise simulations.

- Finally, the assumption of approximating the 4th order correlations ($R_{ijkl}$) via the quasi-Gaussian assumption (i.e. via products of the second order correlations) has been shown to provide an acceptable level of accuracy in comparison to the data obtained from direct measurements of $R_{ijkl}$. It was shown that the approximated $R_{ijkl}^{QG}$ had increased strength of the peak correlation magnitudes (as already mentioned by Bridges et al [74]) and hence the overestimated Lagrangian length and timescales. In addition the absolute peak values for the correlations at zero spatial and temporal separation were larger for the approximated 4th order correlations $R_{ijkl}^{QG}$ at $x/D_j = 4$ and 6.5. This being said, overall the approximated correlations were in good agreement and showed the ability to identify the same 4th order components likely to contribute to noise sources.

7.2 Numerical Conclusions

- An existing multi-block finite volume RANS and LES code has been modified to allow the export of planar samples of velocity data for direct postprocessing using the same postprocessing tools developed for PIV data. This allowed total confidence in comparisons between simulation and measured data.

- The numerical results from the single round jet and coaxial round jet test cases show the simulated mean flow including important flowfield features such as potential core length and jet spreading rate are in fair agreement with experimental results for RANS calculations, although are less accurate for LES calculations, primarily due to inadequate treatment of nozzle exit boundary layer / shear layer transition.
Conclusions and Recommendations

- Accepting the limitation that RANS predictions can only provide time-averaged turbulent information, the RANS results show good capability to capture the trends exhibited by experiments when altering the geometry (single to coaxial jet configurations). The RANS solutions are also computationally inexpensive, meaning that RANS methods certainly have a role to play in nozzle design and as the base-flow for hybrid aeroacoustic predictions.

- The ability of LES to produce unsteady flow statistics makes it a potentially crucial tool for sound source modelling. The high grid resolution and storage requirements and associated computational expense must however, be considered along with the accuracy of the predictions.

- The important issue in current jet LES predictions is the specification of the nozzle inflow conditions and the related boundary layer / shear layer transition at nozzle exit. The approach used in this study namely to assume that the shear layer turbulence does develop eventually in a realistic manner, was correct. However, correctly specifying the initial shear layer and removing the laminar / transitional nature close to the nozzle exit would substantially improve the flow development and increase the accuracy of the spatio-temporal correlations in this region. Downstream of these 'spurious' flow transitions, the LES results provide reassurance that the turbulence is realistic by producing relative correlation magnitudes which are in very good agreement with the experimental data.

- Given the hybrid approach to CAA, the ability of LES to surpass RANS as the method of choice for exhaust nozzle and jet plume base-flow generation is close, although expensive in time and cost. In addition, LES currently does an acceptable, but not yet sufficiently accurate, job of predicting the noise source characteristics, but experimental data to validate any numerical results is still required. LES, however, does offer the potential for much larger amounts of flowfield information over much larger spatial domains than experimental approaches.

7.3 Recommendations

This thesis has demonstrated the potential of water flow based testing using high speed PIV instrumentation to identify and describe the sound source functions for $4^{th}$ order correlation terms. The test cases here have shown the ability of this method to resolve the spatio-temporal correlations needed for farfield sound source modelling to the same levels of accuracy as previous CTA data. Further use of this approach for testing more realistic nozzle designs such as including the inclusion of tabs, cheverons, microjets, and pylons within a typical coaxial jet configuration, as well as novel nozzle designs should be undertaken.
Conclusions and Recommendations

Although not performed in this thesis for the coaxial jet configuration, the implementation of the stereoscopic PIV technique together with the newly developed filtering methodology should also be utilised in order to gain full identification of all independent 4th order correlation terms in the coaxial jet. An opportunity for database generation for validation of the numerical approaches being developed also exists and is critical for accurate analysis of numerical models.

The majority of jet noise studies that have used LES driven sound source models have used a Gaussian shape function for the autocorrelation. As this project has found, the true shape is a combination of exponential and Gaussian functions, and such models should be developed. Although not technically a continuation of this study, the inclusion of such information on the shape function within aeroacoustic models would provide valuable information on the sensitivity of the far-field noise to the prescription of this function.

It is the strong opinion of the author that more progress is needed towards an understanding of how different noise reduction techniques change the shape, magnitude, and evolution of the correlations functions, and how 'delta' changes in the correlation functions affect and give rise to 'delta' changes in the far-field noise levels. If we are to meet future noise level restrictions, it is the identification of how these turbulent structures can be affected and by how much which is required for novel nozzle designs to be developed.

Finally, although the exact answers from the numerical predictions have been shown to lack the levels of accuracy required for absolute correlation function identification, further numerical simulations should be performed to support the developments of sound reduction techniques which are extremely hard to undertake using experimental facilities. For example, the investigation of the inclusion of microjets around the circumference of the nozzle exit and their operation as pulsed jets in various azimuthal modes. The spatial and temporal resolution of LES will allow examination of pulsations frequencies difficult to achieve in experiments.
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Bibliography


Appendix A

It is worth noting that the planar stereoscopic PIV technique removes the need to introduce any separations to avoid probe interference and also provides the opportunity to produce the true correlation components. The whole set of $4^{th}$ order velocity correlations for the most significant correlation components (discussed more in Section 4.4.6) for axial, radial and circumferential separations at $x/D_j = 4$ are included here for completeness.
Figure A-1: Spatio-temporal correlation functions of $R_{1111}$ and $R_{1112}$ with axial, radial and circumferential separations at $x/D_j = 4$, $r/D_j = 0.5$
Figure A-2: Spatio-temporal correlation functions of $R_{1212}$ and $R_{1313}$ with axial, radial and circumferential separations at $x/D_j = 4$, $r/D_j = 0.5$
Figure A-3: Spatio-temporal correlation functions of $R_{2222}$ and $R_{3333}$ with axial, radial and circumferential separations at $x/D_j = 4$, $r/D_j = 0.5$
Appendix B

Through the production of spatio-temporal correlation maps it is possible to analyse the Lagrangian statistics of the turbulence as well as gain information about eddy convection velocities. A full sample of $2^{nd}$ and $4^{th}$ order spatio-temporal correlations at $x/D_j=3$ and 8 within the outer and inner shear layers for both axial and radial velocity correlations are illustrated here. A common trend with all of these correlations is the larger highly correlated area, in both space and time, predicted by the LES calculation in comparison to the PIV measurements. This is consistent with the absence of high frequency energy within the LES calculation.
Appendix B

Figure B-1: 2nd order correlation maps of axial velocity with axial separations at $x/D_j = 3$
Figure B-2: 2nd order correlation maps of axial velocity with axial separations at $x/D_j = 8$
Figure B-3: 2nd order correlation maps of radial velocity with axial separations at $x/D_J = 3$
Figure B-4: 2nd order correlation maps of radial velocity with axial separations at \( x/D_j = 8 \)
Figure B-5: 4th order correlation maps of axial velocity with axial separations at $x/D_j = 3$
Figure B-6: 4th order correlation maps of axial velocity with axial separations at $x/D_j = 8$
Figure B-7: 4th order correlation maps of radial velocity with axial separations at $x/D_j = 3$
Appendix B

Figure B-8: 4th order correlation maps of radial velocity with axial separations at $x/D_j = 8$