Cutting of cortical bone tissue: analysis of deformation and fracture process

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Abstract

Cortical bone tissue – one of the most intriguing materials found in nature – demonstrate some fascinating behaviours that have attracted great attention of many researchers from all over the world. In contrast to engineering materials, bone has its unique characters: it is a material that has both sufficient stiffness and toughness to provide physical support and protection to internal organs and yet adaptively balanced for its weight and functional requirements. Its structure and mechanical properties are of great importance to the physiological functioning of the body. Still, our understanding on the mechanical deformation processes of cortical bone tissue is rather limited.

Penetration into a bone tissue is an intrinsic part of many clinical procedures, such as orthopaedic surgery, bone implant and repair operations. The success of bone-cutting surgery depends largely on precision of the operation and the extent of damage it causes to the surrounding tissues. The anisotropic behaviour of cortical bone acts as a distinctive protective mechanism and increases the difficulty during cutting process. A comprehensive understanding of deformation and damage mechanisms during the cutting process is necessary for improving the operational accuracy and postoperative recovery of patients. However, the current literature on experimental results provides limited information about processes in the vicinity of the cutting tool-bone interaction zone; while; numerical models cannot fully describe the material anisotropy and the effect of damage mechanisms of cortical bone tissue. In addition, a conventional finite-element scheme faces numerical challenges due to large deformation and highly localised distortion in the process zone.

This PhD project is aimed at bridging the gap in current lack of understanding on cutting-induced deformation and fracture processes in the cortical bone tissue through experimental and numerical approaches.

A number of experimental studies were accomplished to characterise the mechanical behaviour of bovine cortical bone tissue and to analyse deformation and damage mechanisms associated with the cutting process.
along different bone axes in four anatomic cortices, namely, anterior, posterior, medial and lateral. These experiments included: (1) a Vickers hardness test to provide initial assessments on deformation and damage processes in the cortical bone tissue under a concentrated compressive load; (2) uniaxial tension and compression tests, performed to understand the effect of orientation and local variability of microstructure constituents on the macroscopic material properties of cortical bone; (3) fracture toughness tests, aimed at elucidating the anisotropic character of fracture toughness of cortical bone and its various fracture toughness mechanisms in relation to different orientations; (4) penetration tests, conducted to evaluate and validate mechanisms involved in bone cutting as well as orientation associated anisotropic deformation and damage processes at various different cortex positions. Information obtained in these experimental studies was used to assist the development of advanced finite-element models: (1) the effective homogenised XFEM models developed in conjunction with three-point bending test to represent a macroscopically, anisotropic elastic-plastic fracture behaviour of cortical bone tissue; (2) three microstructured XFEM models to further investigate the effect of the randomly distributed microstructural constituents on the local fracture process and the variability of fracture toughness of cortical bone; (3) a novel finite-element modelling approach encompassing both conventional and SPH elements, incorporating anisotropic elastic-plastic material properties and progressive damage criteria to simulate large deformation and damage processes of cortical bone under penetration. The established models can adequately and accurately reflect large deformations and damage processes during the penetration in bone cutting.

The results of this study made valuable contributions to our existing understanding of the mechanics of cortical bone tissue and most importantly to the understanding of its mechanical behaviours during the cutting process.

**Key words:** Cortical bone tissue, Cutting, Anisotropy, Variability, Microstructured model, Crack propagation, XFEM, SPH.
Acknowledgement

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## Symbols

- **a**: Crack length of three-point bending specimen
- **A**: Surface area of indentation imprint
- **a, b, c, d and e**: Material-dependent constants in CDM-based fatigue model
- **A, m**: Dimensionless plasticity constants in elastic-visco-plastic constitutive model
- **a₀**: Average original crack length of three-point bending specimen
- **aᵢ, bᵢ<sup>a</sup>**: Enriched degree-of-freedom vectors
- **b, m**: Scale and shape parameters of the Weibull’s distribution function
- **B**: Specimen thickness
- **d̅**: Average diagonal length of d₁ and d₂
- **D**: Damage variable
- **E₁**: Elastic modulus for interstitial bone
- **Eᵢⱼ**: Individual components for Elastic modulus
- **Eₙ, εₚₙn**: Elastic modulus and permanent strain after n loading cycles
- **E₀**: Elastic modulus for osteonal bone
- **Eₚ**: Elastic modulus for plexiform bone
- **Eₜ**: Tangential modulus
- **E<sub>total</sub>**: Total effective elastic modulus
- **F**: Applied force
- **Fₐ(x)**: Asymptotic crack-tip function
- **Fₗ**: Cutting force
- **Fₖ**: Kilograms-force
- **Fₘₐₓ**: Maximum force measured during fracture toughness test
- **F₉**: Force measured for calculating K₉ in fracture toughness test
- **GᵢC**: Mode-I critical fracture energy release rate
- **Gᵢⱼ**: Individual components for shear modulus
- **h**: Smoothing length
- **H(x)**: Discontinuous jump function
- **J**: J-integral value
- **JₐC**: Mode-I critical value of J-integral
- **Jₚ**: Plastic component of critical value of J-integral
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<tr>
<td>$k$</td>
<td>Shear yield strength</td>
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<td>$K_{(I,II,III)}$</td>
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<td>$p$</td>
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</tr>
<tr>
<td>$R$</td>
<td>Specific work of surface separation</td>
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<tr>
<td>$S_0$</td>
<td>Plasticity constant in visco-elastic-plastic relationship</td>
</tr>
<tr>
<td>$t$</td>
<td>Offcut thickness</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Displacement of discontinuity</td>
</tr>
<tr>
<td>$\mathbf{u}$</td>
<td>Nodal displacement vector function</td>
</tr>
<tr>
<td>$U_p$</td>
<td>Plastic component of area under the load-line displacement curve for three-point bending test</td>
</tr>
<tr>
<td>$V_1$</td>
<td>Volumetric (area in 2D) fraction for interstitial bone</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Volumetric (area in 2D) fraction for osteonal bone</td>
</tr>
<tr>
<td>$V_P$</td>
<td>Volumetric (area in 2D) fraction for plexiform bone</td>
</tr>
<tr>
<td>$V_{P0}$</td>
<td>Volumetric (area in 2D) fraction for porosity</td>
</tr>
<tr>
<td>$w$</td>
<td>Cutting width</td>
</tr>
<tr>
<td>$W$</td>
<td>Specimen’s width</td>
</tr>
<tr>
<td>$W_f$</td>
<td>Work of fracture</td>
</tr>
<tr>
<td>$W_{brid}$</td>
<td>Fracture energy produced from wake bridging</td>
</tr>
<tr>
<td>$W_{tip}$</td>
<td>Fracture energy produced from crack-tip toughening mechanisms</td>
</tr>
<tr>
<td>$W_{total}$</td>
<td>Total fracture energy</td>
</tr>
<tr>
<td>$x$</td>
<td>Sample (Gauss) point</td>
</tr>
</tbody>
</table>
\( \alpha \) Significance level used in statistical analysis
\( \alpha_r \) Rake angle
\( \beta_f \) Friction angle
\( \beta, \gamma \) Parameters governing exponential decay in stiffness in visco-elastic-plastic damage model
\( \Gamma \) Work done for plastic deformation
\( \delta \) Cutting distance of cutting tool
\( \varepsilon_d, \varepsilon_s, \varepsilon_{sl} \) Strain components for dashpot, spring and sliding element in visco-elastic-plastic constitutive model
\( \varepsilon^e \) Elastic strain component
\( \varepsilon^p \) Plastic strain component
\( \varepsilon_u \) Ultimate strain
\( \varepsilon_y \) Yield strain
\( \dot{\varepsilon}^{\text{VE}} \) Strain rate component for visco-elastic deformation
\( \dot{\varepsilon}^{\text{VP}} \) Strain rate component for visco-plastic deformation
\( \dot{\varepsilon} \) Strain rate
\( \Lambda \) Elastic strain energy
\( \nu_{ij} \) Individual components of Poisson’s ratio
\( \sigma_d, \sigma_s, \sigma_{sl} \) Stress components for dashpot, spring and sliding element in visco-elastic-plastic constitutive model
\( \sigma_u \) Ultimate stress
\( \sigma_y \) Yield stress
\( \sigma_{YS} \) 0.2% proof strength in fracture toughness test
\( \tau_u \) Shear strength
\( \phi \) Shear angle between material’s shear plane and cutting direction
### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>Two-dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three-dimensional</td>
</tr>
<tr>
<td>A_C</td>
<td>Anterior specimen with circumferential crack</td>
</tr>
<tr>
<td>A_L</td>
<td>Anterior specimen with longitudinal crack</td>
</tr>
<tr>
<td>A_R</td>
<td>Anterior specimen with radial crack</td>
</tr>
<tr>
<td>ALE</td>
<td>Arbitrary Lagrangian-Eulerian</td>
</tr>
<tr>
<td>ANOVA</td>
<td>Analysis of variance</td>
</tr>
<tr>
<td>BMD</td>
<td>Bone mineral density</td>
</tr>
<tr>
<td>C</td>
<td>Circumferential direction</td>
</tr>
<tr>
<td>C1 &amp; C2</td>
<td>Circumferential surfaces of specimen used in Vickers hardness test</td>
</tr>
<tr>
<td>CDM</td>
<td>Continuum damage mechanics</td>
</tr>
<tr>
<td>CEL</td>
<td>Coupled-Eulerian-Lagrangian</td>
</tr>
<tr>
<td>C-L</td>
<td>Penetration direction perpendicular to circumferential axis and in parallel with longitudinal axis</td>
</tr>
<tr>
<td>CM/CZM</td>
<td>Cohesive model/ Cohesive zone method</td>
</tr>
<tr>
<td>Comp</td>
<td>Compression</td>
</tr>
<tr>
<td>C-R</td>
<td>Penetration direction perpendicular to circumferential axis and in parallel with radial axis</td>
</tr>
<tr>
<td>CT</td>
<td>Compact tension specimen</td>
</tr>
<tr>
<td>CTOD</td>
<td>Crack tip opening displacement</td>
</tr>
<tr>
<td>DMA</td>
<td>Dynamic mechanical analysis</td>
</tr>
<tr>
<td>EPFM</td>
<td>Elastic-plastic fracture mechanics</td>
</tr>
<tr>
<td>fps</td>
<td>Frames per second</td>
</tr>
<tr>
<td>GFEM</td>
<td>Generalized finite-element method</td>
</tr>
<tr>
<td>H-Bond</td>
<td>Hydrogen-bond</td>
</tr>
<tr>
<td>HV</td>
<td>Vickers hardness value</td>
</tr>
<tr>
<td>In</td>
<td>Inner surface of specimen used in Vickers hardness test</td>
</tr>
<tr>
<td>kgf</td>
<td>Kilograms-force</td>
</tr>
<tr>
<td>L</td>
<td>Longitudinal direction</td>
</tr>
<tr>
<td>L_C</td>
<td>Lateral specimen with circumferential crack</td>
</tr>
<tr>
<td>L_L</td>
<td>Lateral specimen with longitudinal crack</td>
</tr>
<tr>
<td>L_R</td>
<td>Lateral specimen with radial crack</td>
</tr>
<tr>
<td>L-C</td>
<td>Penetration direction perpendicular to longitudinal axis and in parallel with circumferential axis</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
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</tr>
<tr>
<td>LEFM</td>
<td>Linear-elastic fracture mechanics</td>
</tr>
<tr>
<td>L-R</td>
<td>Penetration direction perpendicular to longitudinal axis and in parallel with radial axis</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear variable differential transducer</td>
</tr>
<tr>
<td>M_C</td>
<td>Medial specimen with circumferential crack</td>
</tr>
<tr>
<td>M_L</td>
<td>Medial specimen with longitudinal crack</td>
</tr>
<tr>
<td>M_R</td>
<td>Medial specimen with radial crack</td>
</tr>
<tr>
<td>M_0</td>
<td>Outer surface of specimen used in Vickers hardness test</td>
</tr>
<tr>
<td>P_C</td>
<td>Posterior specimen with circumferential crack</td>
</tr>
<tr>
<td>P_L</td>
<td>Posterior specimen with longitudinal crack</td>
</tr>
<tr>
<td>P_R</td>
<td>Posterior specimen with radial crack</td>
</tr>
<tr>
<td>PEEQ</td>
<td>Equivalent plastic strain</td>
</tr>
<tr>
<td>R</td>
<td>Radial direction</td>
</tr>
<tr>
<td>RVE</td>
<td>Representative volume elements</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning electron microscopy</td>
</tr>
<tr>
<td>SEN</td>
<td>Single-edged-notched specimen</td>
</tr>
<tr>
<td>SHPB</td>
<td>Split-Hopkinson pressure bar</td>
</tr>
<tr>
<td>SIF</td>
<td>Stress intensity factor</td>
</tr>
<tr>
<td>SPH</td>
<td>Smoothed particle hydrodynamics</td>
</tr>
<tr>
<td>STATUS</td>
<td>Damage scale factor for XFEM</td>
</tr>
<tr>
<td>T</td>
<td>Transverse directions</td>
</tr>
<tr>
<td>Ten</td>
<td>Tension</td>
</tr>
<tr>
<td>Tukey_HSD</td>
<td>Honestly significant difference</td>
</tr>
<tr>
<td>VCCT</td>
<td>Virtual crack closer technique</td>
</tr>
<tr>
<td>XFEM</td>
<td>Extended finite element method</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Research background

Research on biomaterials, and, more specifically, cortical bone tissue has attracted significant attention of both academy and industry over the past few decades. In contrast to other engineering materials, bone has its unique features both in biological and engineering fields. From the biological point of view, bone is the main component that builds up our skeletal system. It has the ability of self-repair and adaptation to load and environmental changes. From the engineering point of view, bone has a complex hierarchical structure and heterogeneous mechanical properties. Therefore, studies on bone tissue not only serve as a natural inspiration source for the development of advanced engineering and bionics materials, but also help to prevent bone diseases and trauma.

Bone-related surgeries are common in several medical fields, such as orthopaedics, dentistry and osteotomy. The demand of bone surgery is increasing dramatically in recent decades. In 2007, statistics from Goodfellow et al. [1] showed that there were nearly a million knee replacements (or arthroplasties) performed worldwide every year. Despite this large demand for the bone surgery, research on bone cutting is still limited. The technologies employed in bone surgical operations, especially cutting tools, remain nearly unchanged for decades. Traditional cutting tools, such as electrical saws, burrs and manual chisels, lack cutting precision, leave large regions of damaged tissues and yield rough surface finish. Poorly controlled cutting forces can also cause significant deformation and fracture in the vicinity of a cutting path and, consequently, lead to bad surgical results and longer postoperative recovery time for patients [2]. Therefore, it is necessary to further comprehend the deformation and fracture processes involved in bone cutting to minimize such damage.
Finite-element (FE) simulations are a powerful approach to provide insight into, and understanding of, the interaction between the cutting tool and the bone tissue. However, FE models largely rely on the accuracy of the material properties; they are only reliable when there is an adequate understanding of underpinning material behaviours. Modelling of bone cutting is a challenging task that there are only a few studies available so far in literature and none of them can fully represent the complicated behaviour of cortical bone. The cutting process involves an anisotropic elastic-plastic deformation of cortical bone and subsequent damage evolution as cutting continues. A heterogeneous distribution of bone's microstructural constituents can also greatly affect its deformation and fracture processes.

To the author’s knowledge, there is still a lack of comprehensive understanding of the anisotropic deformation and fracture behaviour of cortical bone tissue. Adequate FE models are yet to be developed to incorporate these complex, multi-scale processes involved in bone cutting. Therefore, the aim and objectives of this study are formulated in the next section.

1.2 Aim and objectives

Aim:

The aim of this PhD project is to advance our understanding of mechanics involved in penetration of cortical bone tissue through both experimental and numerical procedures. Accordingly, the experimental part of the studies focuses on acquiring a more precise material database of bovine cortical bone tissue which can be used to characterise its complex deformation and fracture behaviours. The knowledge obtained from experiments is used to assist the development of realistic and adequate finite-element models for further investigation of the deformation and fracture processes in the vicinity of the cutting tool-bone interaction zone.

Objectives:
To accomplish the aim set for this project, the following objectives are identified:

1. To obtain and characterise the mechanical behaviour of cortical bone tissue under different loading conditions (tensile and compressive loading) and orientations (parallel and perpendicular to osteons) at different cortex positions.

2. To evaluate the effect of orientation and local variability of microstructure constituents on the macroscopic material properties of cortical bone.

3. To analyse the distinct deformation mechanisms and damage regimes of cortical bone in response to tensile and compressive loading conditions for different orientations.

4. To quantify the fracture toughness of cortical bone and analyse fracture toughening mechanisms and their changes with respect to different bone axes.

5. To develop advanced finite-element models to evaluate the non-linear fracture process of cortical bone tissue with respect to different bone axes and cortices.

6. To develop statistical realizations of random microstructure for cortical bone models to investigate the effect of local heterogeneity of microstructural constituents on the fracture propagation process.

7. To develop adequate and accurate finite-element models based on obtained material properties to capture and evaluate the deformation and damage processes of cortical bone in the cutting-tool penetration process.

8. To quantify and validate the developed FE models using penetration experiments and further comprehend the deformation and damage mechanisms associated with penetration of cortical bone for various bone’s axes and cortices.
1.3 Thesis outline and key contributions

The thesis is divided into 8 chapters, an outline and key contributions of the remaining chapters are summarised in the following sections.

Chapter 2: Bone Mechanics

A literature review chapter focuses on discussing the mechanical aspect of cortical bone tissue including its various structural hierarchy and mechanical behaviour of cortical bone tissue. This chapter identifies current gaps between existing material properties available in literature and our lack of understanding on the variability and anisotropy deformation of cortical bone tissue.

Chapter 3: Modelling of Bone and Bone Cutting

A broad review of various bone and bone-cutting models are discussed and summarised in this chapter. Different modelling techniques such as XFEM, CZM are also presented in this chapter along with discussions of their novelty and drawbacks.

Chapter 4: Variability and Anisotropy of Mechanical Behaviour of Bovine Cortical Bone Tissue

This is an experimental chapter which focuses on studying the mechanical behaviour of cortical bone that has not been fully addressed previously in literature. In this chapter, the experimental results, for the first time, fully characterise the anisotropic elastic-plastic material properties of bovine femoral cortical bone for both tensile and compressive loadings using specimens acquired from the same animal. Furthermore, the statistical and microstructure analyses reveal a strong correlation between the variability of the mechanical properties of cortical bone and mechanical induced bone adaptation process which is evidenced by the transitions of microstructural constituents across different cortex.

Chapter 5: Analysis of Anisotropic Fracture in Cortical Bone
This chapter focuses on characterising and evaluating the anisotropic fracture behaviour of bovine femoral cortical bone using both experimental and numerical approaches based on a three-point bending test. The results of the experimental works demonstrate a significant anisotropy and non-linear fracture behaviours which cannot be characterised by the LEFM approach. Instead, the non-linear fracture toughness parameter: $J$-integral is calculated and is in good agreement with the results obtained from the developed fracture models.

**Chapter 6: Numerical Analysis of Fracture Process in Microstructured Cortical Bone Tissue**

Based on the previous experimental study, an extended numerical study of the non-linear fracture process of cortical bone at micro-scale is presented in this chapter. The developed models, which incorporate a submodelling technique, for the first time, demonstrate crack propagation within the four-phase random distributed microstructured cortical bone tissue. Quantitative analyses of the developed models also demonstrate a strong influence of the microstructural constituents on the variability of the macroscopic fracture toughness and the crack propagation trajectory.

**Chapter 7: Experimental and Numerical Investigations of Penetration of Cutting Tool into Cortical Bone**

In this chapter, both experimental and numerical studies are conducted to provide comprehensive understanding of deformation and fracture processes of the cortical bone tissue under penetration of a sharp cutting tool. The designed penetration tests adequately reveal the anisotropic deformation behaviour of bovine cortical bone in the vicinity of the cutting zone through a high-speed camera. The novel SPH based cutting models, for the first time, realize the variability and anisotropy of the deformation and damage behaviours of cortical bone during sharp penetration process. The developed models provide useful information and powerful tools for further studies that can underpin improvement of the accuracy of orthopaedic surgical procedures.
Chapter 8: Conclusions and Future Work

The chapter summarises current research outcomes and achievements as well as suggestions for future studies.
Chapter 2

Bone Mechanics

2.1 Introduction

The complexity and hierarchical architecture of bone tissue have resulted in some interesting mechanical behaviour, a material that has both sufficient stiffness and toughness to provide physical support and protection to internal organs and yet adaptively sophisticated for its weight and functionality. Its structure and mechanical properties are of great importance to the physiological functioning of the body and have been extensively studied for centuries. Still, our understanding of its mechanical deformation is only a fraction of the whole picture and advances day-by-day thanks to technology improvement brought into both experimental and computational fields. This knowledge contributes further to the wider development of techniques to prevent bone failure in accidents and improve treatments of bone disease and post-recovery time for bone surgical operations.

In this chapter, an in-depth review of the bone’s mechanical behaviour, particularly, the outer cortex, cortical bone tissue, is studied.

2.2 Bone as a living composite material

The common term “bone” is a type of biological hard tissue covering a wide range of calcified materials that can be found in all vertebrates. Depending on the species and anatomical position, bones can have rather different structures and mechanical functionalities. Despite great dissimilarities, all bones, regarded as a material, share the same basic building components: minerals, organic phases and water. The mechanical properties of bone are largely determined by the volume fraction and arrangement of these building blocks.
Mineral phase consists of mainly plate-shaped nanoparticles in the form of hydroxyapatite-like crystals (calcium phosphate-based apatite mineral) with average size of 50 nm × 25 nm × 3 nm [3]. These high stiffness (114 GPa [4]) carbonated apatite nanocrystals make up approximately 65% of total bone mass [5] and 33% – 43% of bone volume [6], and are the main load bearing components within the bone tissue.

Organic phase comprises mainly type-I collagen (90% of total proteins is type-I collagen, the other 10% is made up by non-collagenous proteins) and takes up to 35% of total bone mass and 32% – 44% of bone volume [6]. With measured Young’s modulus of only a few giga-Pascals [7], this relatively soft organic matrix enhances its overall strength by enabling nucleation and growth of the carbonated mineral crystals inside and on itself to form a much stiff and yet, tough nano-composite structure [8].

Water is another major component of bone tissue and indeed, for all biological materials. It fills about 15% – 25% of bone volume [6] and provides essential fluidity for both intracellular and extracellular environments. Water is also acting as one of the key ‘gluing’ agents along with non-collagenous proteins that offer supplementary bonding support between mineral/collagen interfaces in addition to those of strong covalent bonds and ionic bonds [5]. These additional weak bonds may well facilitate some moderate ductility and viscosity during deformation processes [8].

From the mechanical point of view, the combination of low modulus organic matrix and high modulus mineralized nano-crystals reflects millions of years’ fine-tuning and adaptation of bone’s structure and function. This results a sophisticated solution for bone that is capable of enduring different types of loadings during an animal’s life [3, 8].

As a living organic tissue, bone provides not only the rigidity and the ductility required as part of the musculo-skeleton system, but also a dynamic physiological environment for carrying out various metabolic processes such as transfer and storage of some key ingredients like minerals, fat, acid. The newly formed blood cells are also synthesized here inside bone (marrow) and
then transferred into the surrounding circulatory system. All these processes require dedicated spaces and passages which are interconnected with each other both in and outside bone structures for the exchange of nutrition, information and substances. As a result, a hollowed, composite-like structure is created with integrated multi-functional networks. To further complicate matters, there are processes in bone that constantly remove and rebuild its existing structure for the purpose of mineral metabolism (homeostasis), adaptation and damage repair, called bone remodelling process [5]. Wolff’s law [9] was among the earliest hypotheses proposed by Julius Wolff in the early 19th century that outlined this process. However inappropriate the law may be [8], the remodelling process is now a widely accepted phenomena which explains the adaptation process of cortical bone and it determinants the fact that bones are different from the common fabricated engineering materials [10].

2.3 Morphology of bone tissue

Generally, bones can be classified into variety of categories based on their shape, location and function. For example, a comprehensive review carried out by Currey [11] demonstrated some ‘not-too-familiar’ bones that are specially developed for some specialized usages. It is true that the structures and morphologies of those unusual bones presented in [11] are completely different from what people commonly understood. However, in this chapter, the focus is only drawn to those of ‘standard’ bones, such as limb bone of mammals, for their clinical importance and the purpose of this research project. Based on the anatomic structure, bones can be classified into five different types in the human body: long bone (length is much longer than its diameter), short bone (similar dimensions in length, width, depth or diameter), flat bone (relatively thin with large surface to volume ratio), sesamoid bone (special developed bones normally find in tendon, acting as mechanical enhanced protective structure) and irregular bone (bones having complicated shapes that do not fall into all other categories).
As a material being continuously refined through natural evolution, bone tissue certainly has not only a complex morphology involved at its exterior appearance, but also a deeply ingrained internal structure owing to millions of years’ adaptation process. Across different length-scales, there are some intricate structures (such as those discussed below) that are cleverly assembled together to achieve different functions.

2.3.1 Macroscopic and microscopic structures of bone tissue

The reason to put the macroscopic and microscopic structures together in this section is to classify bone’s structure at different length-scales as the definitions in literature can be somewhat ambiguous and vague, and often, there are overlaps between different levels: Currey [8] in his recent review outlined four different levels of bone structure, while Weiner et al. [3] and Rho et al. [12] used seven intricate levels. These judgements are purely based on different perspectives either physically or histologically. None of these classifications are wrong because each of these levels is closely correlated with and sometimes defined by their upper or lower level structures. In general, beyond the microscopic length-scale, bone can be distinguished as cortical bone and trabecular bone, or woven bone and lamellar bone [8].

Cortical bone and trabecular bone

Based on the anatomical structure, a typical matured long bone consists of both cortical bone and trabecular bone regions defined by their porosity. These two different types of bones are distinguishable to the naked eye (Figure 2–1). Cortical bone (also called compact bone) has a fairly dense structure with maximum porosity ratio normally below 25%, whereas trabecular bone (also called spongy or cancellous bone) has a complex three-dimensional (3D) interconnected sponge-like network structure with porosity up to 90% [5]. As a result, their roles in a skeleton system are quite different. In a long bone, cortical bone is mainly formed at the exterior surface and the diaphysis acting as a hard, load bearing structure, while, trabecular bone is predominantly located at the interior and two ends of the long bone, epiphysis. Comparing with cortical bone, trabecular bone offers larger
compliance and serves as a soft, supporting structure with a large weight to volume ratio for shock absorption and joint connection.

**Figure 2–1** Schematic illustration of the structures of bone tissue [13]

*Woven bone and lamellar bone*

From the histological point of view, bone can also be divided into woven bone and lamellar bone. The difference between these two types of bone can be determined by the way the fibril bundles arrange in terms of their orientation (Figure 2–2).

Woven bone has disorientated fibril bundles (Figure 2–2b), rather than its misleading-name suggested, and is often found in new bone tissues of mammals that undergo either embryonic growth or healing process from a fracture [5]. It can also be traced in the fibrolamellar bone structure which is commonly found in some fast growing large animals [8]. Woven bone has a faster ossification process and both of its collagen and minerals components are deposited very quickly in a disordered manner compared with other types of bones [3]. Due to this rapid growth rate, woven bone usually has poor mechanical properties, but, provides necessary support to sustain the fast growth demand of the animal in a short period of time.
Lamellar bone on the other hand, consists of multiple layers of lamella, has same oriented fibril bundles running parallel within one ply (Figure 2–2a), but slightly disorientated between adjacent plies [3, 5]. Each ply has a thickness around 3 to 7 µm and a slightly rotational degree to the neighbouring ply according to an earlier research summarised in [3]. The result of this plywood-like structure is asymmetry of lamellar bone. It is indicated that this asymmetric structure, unlike other artificially made composite materials, is specifically tailored to accommodate loads in various directions [3]. Therefore, the lamellar bone has a much better mechanical performance than that of the woven bone and is the most common bone structure present in adult humans [3].

Depending on the manner in which they are organized, the lamellae bones can be further divided into three major patterns (Figure 2–3) [5]:

- Lamellae bone that forms in a circular configuration with concentric layers of lamella is called the osteonal bone or Haversian system. Within each osteon, there is a central vascular canal passes through;
• Lamellae bone that forms at the outer and sometimes the inner layer of the circumference axis of bone tissue is called circumferential lamellae;

• The remaining fragments of what were formerly osteonal or circumferential lamellae and intercepted by the newly formed Haversian system are called the interstitial lamellae.

Figure 2–3 Illustration of various lamellae bone structures [14]

Similar to the lamellae bone, fibrolamellar bone (also known as plexiform bone) is another type of bone structure found in some large and fast-growing animals, such as bovine, ovine and canine. However, it is rarely seen in humans [5, 15]. It has a sandwich structure consisting of consecutive lamellar bone, woven bone, and lamellar bone (Figure. 2–4). Like the woven bone, fibrolamellar bone is laid down rapidly. However, it contains a more dense system, the vascular plexuses which makes the plexiform bone a much tougher and stiffer structure compared with that of lamellae bone [8] when it is loaded along the bone main axis (in compression). The residual of the vascular spaces results a brick-like appearance of the plexiform bone [15]. Comparing to other types of bone structure, it offers much larger surface area [16]. These rapid increases in both bone mass and bone stiffness in a short period of time are the primary concerns for large and fast growing animals.
2.3.2 Hierarchical architecture of bone tissue

Nature always has some complex designs in materials to fulfil various functional requirements. These design complexities directly relate to the formation of structural hierarchies in biological materials. There is no exception for bone as well [17]. As a naturally formed composite material, a cortical bone tissue is formed by heterogeneously distributed microstructural constituents that could be categorised into several hierarchical organizations all the way from nano- to macro-scale levels (Figure 2–5) [3, 8, 12, 18].

At nano-scale, bone is composed of mineralized collagen fibril and extrafibrillar mineral particles known as carbonated hydroxyapatite [19, 20]. At the micro-scale, cortical bone is laid down in layers of lamellar structure (typically 3-7 µm in thickness) that is similar to that of plywood composite: parallel with each other within layer, but having a staggered arrangement between the adjacent layers [21]. Across a bone section, concentric layers of lamellae together with hollowed vascular channels form the most observable structure under microscope, a Haversian system (containing osteon and a Haversian canal) embedded into the remnants of a bone’s remodelling
process called interstitial matrix. Osteons are, on average, 200 µm in diameter and up to 10 mm long cylindrical structure parallel to the bone’s longitudinal axis [22]. In addition, a network of canals and channels formed across the bone’s section accommodates blood vessels, a nervous system and bone cells; those large canals, on average 50-90 µm in diameter, parallel to the bone’s main axis are called Haversian canals; those channels running perpendicular to Haversian canals, interconnects adjacent osteons and Haversian canals are called Volkmann canals (Figure 2–3). As a living tissue, bone also houses living cells such as osteocytes that live within the oblong space called lacunae which is connected to the surrounding region by a star-like network of miniature-channels called canaliculi. The latter are responsible for exchange of nutrients and waste between osteocytes [22]. The interface between osteons and interstitial matrix is called cement line; it is 2-5 µm in thickness. It plays a key role in the bone’s mechanical behaviour, especially its fracture behaviour. However, opinions on the mechanical properties of cement line in literature are rather divided. Different experimental observations reported that the cement line can act either as a toughening mechanism deflecting a crack from osteons or as a weakening path that facilitates the crack initiation [8, 18]. Together, the Haversian system (including osteons and Haversian canals), interstitial matrix and cement lines form the core constituents of the cortical bone tissue at microscopic level and attract a great deal of research interests in both experimental and computational modelling fields. At the millimetre length scale, the dense and thick outer layer of cortical bone and the porous sponge-like trabecular bone make up the tissue-level bone structures as discussed in previous section.

All these microstructural constituents work in different ways and complement each other at different hierarchic levels to achieve enhanced macroscopic mechanical properties with adequate, if not optimum, functionalities under various loading conditions [3].
Figure 2–5 Hierarchical structure of cortical bone at various length scales [10]

2.4 Mechanical behaviour of cortical bone tissue at macro-scale

The results of having such complex heterogeneous and hierarchical architectures of the cortical bone tissue directly affect the mechanical behaviour and load bearing capacities of cortical bone. In fact, bone’s mechanical properties are well adapted to cope with the required mechanical and physiological functionalities, and vary at different loading conditions and
directions. As a living tissue, bone’s properties are compromised by many factors such as material properties of its building components, the requirement of metabolic process. Increase bone’s size and mineral concentration or remove its metabolic network (no voids, channels or blood vessels) would certainly make bone a stronger material. However, it would also increase the time (life cycles) and cost (the amount of materials and energy required) to build and may not be practical at all from the physiological point of view [8]. Therefore, it is not the purpose of cortical bone to be the best or strongest material, but a material which can sufficiently manage itself with various requirements, in another words, ‘living dangerously’, as it was suggested by Currey [8].

Underlying by this building principle, the builder (the individual bone cells), has to work efficiently in terms of the material usage and structural arrangement. Hence, it can be concluded that the cortical bone tissue, with regard to its mechanical behaviour, is a preferentially enhanced material that tuned to fit for its particular functional requirements, such as orientation-dependent properties, loading dependent properties, variable material properties across different locations of bone.

2.4.1 Anisotropic elastic-plastic behaviour

**Anisotropy**

It is well established that cortical bone tissue has an orientation-dependent material behaviour thanks to its preferentially aligned hierarchical structures. Figure 2–6 demonstrates the schematic representation of three mutual perpendicular directions of a femur cortical bone: longitudinal, circumferential and radial directions. As results of the preferential alignment of the nano-scale components such as mineral crystal and collagen fibril as well as the upper level structural features like osteons and lamellae, the elastic and post-yield deformation behaviours of cortical bone tissue vary considerably between different bone’s axes at macro-scale [23]. The anisotropic mechanical behaviours of cortical bone tissue (a general term refers to non-isotropic material properties of cortical bone, rather than the meaning of fully
anisotropic material properties), were widely confirmed in a number of studies [3, 8, 23-29]. Like wood, the mechanical behaviours of cortical bone differ along its three axes (Figure 2–6). However, the difference between the circumferential and radial directions is so little that most researchers believe that cortical bone, in general, can be considered as a transversely-isotropic material (the isotropic plane is perpendicular to the main axis of bone) [23, 24, 27, 30, 31], a special case of the orthotropic material.

![Diagram of three mutual perpendicular directions of a femur cortical bone](image)

**Figure 2–6** Schematic representation of three mutual perpendicular directions of a femur cortical bone

This orientation dependency has been long known from experiments at either macro-scale [23, 32] or micro-scale [33] by means of mechanical or non-mechanical testing techniques [27, 31]. Bonfield and Grynpas [29] developed an ultrasonic method to measure the effect of orientation on elastic moduli by varying the angle of specimen from 0° to 90°. A maximum ratio (anisotropic ratios between elastic moduli measured at different angles) of two was reported between 0° (longitudinal) and 90° (transverse) of a mutual bovine femur; this ratio changes for different loading modes. Reilly and Burstein [23] reported a reduction in the anisotropic ratio for the elastic modulus measured in compression.

The origin of the anisotropic mechanical properties of cortical bone may lie on many facts, such as the restriction due to the formation of substance
materials (mineral particles), genetic or functional requirement, which are yet to be thoroughly studied [8]. However, there are variances of loading imposed on individual bone due to the particular musculoskeletal arrangement of the animal, as so, it is essential to prioritise enhancement of the material properties in particular direction where it requires the most. Nevertheless, in particular cases, like the one of canine mandible (irregular bone), bone tissue can be isotropic [34], by having a homogenized rather than highly order arrangement of the hierarchical structure.

**Elastic behaviour**

The elastic moduli of cortical bone are the most fundamental, yet prominent material properties for this biological tissue. They are the decisive factors for the overall stiffness and structural integrity of the cortical bone tissue. For an orthotropic material, nine independent elastic constants in total are required to completely describe the elastic material properties, while five for the transversely isotropic material. A list of the elastic moduli of cortical bone measured from experiments for both human and bovine cortical bones is summarised in Table 2–1, where, the mechanical testing typically supports the assumption of transverse isotropic material but ultrasonic testing yield some anomalies. In this study, the transverse isotropic assumption will be considered based on the discussion at the beginning of section 2.4.1.

In general, human and bovine cortical bones have very similar values of Young’s moduli ranging from 10 GPa to 20 GPa, depending on the type of bone and material orientations. However, bovine bone usually has a higher anisotropic ratio of the Young’s modulus (higher modulus in longitudinal direction and lower modulus in radial and circumferential directions) compared with the one that of the human due to existing differences between the two species. Firstly, bovine is heavier than human, hence, its bones are obliged to sustain a larger mechanical load along the main axis than its counterpart. Secondly, apart from the difference in size, bovine cortical bones (especially in long bones) develop a special structure called fibrolamellar bone (plexiform bone) with increased stiffness and strength in longitudinal direction but weak in other directions, therefore have higher anisotropic ratio
compared with that of human bones [8]. Furthermore, the behaviours of human are much more active, and the loading conditions of human bone are more complex and change more frequently than that of bovine bones. Thus, a more sufficient support in all directions is required [3].

**Table 2–1** Elastic constants for human and bovine cortical bone ($E_{ij}$, $\nu_{ij}$ and $G_{ij}$ denote corresponding components for Young’s modulus, Poisson’s ratio and shear modulus, respectively; subscript 1, 2 and 3 represent longitudinal, radial and circumferential direction)

<table>
<thead>
<tr>
<th>Bone type</th>
<th>Human Femur</th>
<th>Human Femur</th>
<th>Bovine Femur</th>
<th>Bovine Phalanx</th>
<th>Bovine Femur</th>
</tr>
</thead>
<tbody>
<tr>
<td>Testing method</td>
<td>Mechanical</td>
<td>Ultrasonic</td>
<td>Mechanical</td>
<td>Ultrasonic</td>
<td>Ultrasonic</td>
</tr>
<tr>
<td>Reference</td>
<td>[23]</td>
<td>[27]</td>
<td>[23]</td>
<td>[35]</td>
<td>[36]</td>
</tr>
<tr>
<td>$E_{11}$ (GPa)</td>
<td>17.0</td>
<td>20.0</td>
<td>22.6</td>
<td>22.0</td>
<td>21.9</td>
</tr>
<tr>
<td>$E_{22}$ (GPa)</td>
<td>11.5</td>
<td>12.0</td>
<td>10.2</td>
<td>11.3</td>
<td>11.6</td>
</tr>
<tr>
<td>$E_{33}$ (GPa)</td>
<td>11.5</td>
<td>13.4</td>
<td>10.2</td>
<td>11.3</td>
<td>14.6</td>
</tr>
<tr>
<td>$G_{12}$ (GPa)</td>
<td>3.3</td>
<td>5.6</td>
<td>3.6</td>
<td>5.4</td>
<td>6.3</td>
</tr>
<tr>
<td>$G_{13}$ (GPa)</td>
<td>3.3</td>
<td>6.2</td>
<td>3.6</td>
<td>5.4</td>
<td>7.0</td>
</tr>
<tr>
<td>$G_{23}$ (GPa)</td>
<td>3.6</td>
<td>4.5</td>
<td>3.4</td>
<td>3.8</td>
<td>5.3</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.31</td>
<td>0.22</td>
<td>0.16</td>
<td>0.20</td>
<td>0.11</td>
</tr>
<tr>
<td>$\nu_{13}$</td>
<td>0.31</td>
<td>0.24</td>
<td>0.16</td>
<td>0.20</td>
<td>0.21</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>0.58</td>
<td>0.38</td>
<td>0.51</td>
<td>0.48</td>
<td>0.30</td>
</tr>
<tr>
<td>$\nu_{21}$</td>
<td>0.46</td>
<td>0.37</td>
<td>0.36</td>
<td>0.4</td>
<td>0.21</td>
</tr>
<tr>
<td>$\nu_{31}$</td>
<td>0.46</td>
<td>0.35</td>
<td>0.36</td>
<td>0.4</td>
<td>0.31</td>
</tr>
<tr>
<td>$\nu_{32}$</td>
<td>0.58</td>
<td>0.42</td>
<td>0.51</td>
<td>0.48</td>
<td>0.38</td>
</tr>
</tbody>
</table>

**Post-yield behaviour and properties**

Upon reaching the elastic limit, bones would undergo permanent plastic deformation followed by stiffness reduction and damage accumulation. As part of the skeleton system, bones are, in general, designed to maintain the rigidity and shape of the body rather than undergo permanent plastic deformation. The remodelling process can also help to gradually remove and
repair damage regions induced by excessive loading or fracture. As a result, the post-yield deformation of cortical bone is lack of refinement throughout natural evolution process in comparison with its elastic properties. Early experiments done by Reilly and Burstein [23] revealed some of the post-yield properties for both human and bovine femoral cortical bone using a series of tests including tension, compression and torsion (summarised in Table 2–2). From these data, it can be concluded that the yield point is quite close to that of the ultimate failure point under tensile loading, that is, a large proportional elastic deformation region followed by a narrow post-yield section caused by the poor plastic deformation tolerance.

Mercer and his co-authors further investigated the inelastic deformation of bovine cortical bone in both tension and compression simultaneously using four-point bending tests and extracted consistent results [37]. The yield strains were reported in the range of 0.6% for tension and 1.1% for compression, whereas, 2.4% and 1.6% for the ultimate strain in tension and compression, respectively [37, 38]. However, the material properties for the transverse directions were missing in their report. Bayraktar et al. [39] in another research testified the yield properties of human cortical bone for both tensile and compressive loading using uniaxial tests. Still, there is no data available for the transverse directions. Ebacher and Wang [40] characterised the inelastic deformation behaviour on both longitudinal and transverse specimens and a similar value of yield strain was reported for compressive loading. To date, there is no comprehensive material data available to fully describe the anisotropic post-yield mechanical behaviour of cortical bone.

The deformation mechanisms of cortical bone are strongly linked to its material constituents and their hierarchical arrangement. Studies performed by different authors [23, 40-42] have all indicated observations of cross-hatched deformation pattern induced by the shear deformation mechanisms during compression, possibly due to the interactions between the interfaces across hierarchical levels (Figure 2–7) [37]. Novitskaya et al. [14] studied the relationship between the mineral particle and the collagen fibril by comparing the mechanical behaviour of untreated, demineralized and deproteinized
cortical bone specimens. Their findings suggested that the mineral phase was the main load bearing component within the cortical bone structure, while a strong interaction holds between the mineral and collagen phases, which characterises the post-yield deformation behaviour. As for the post-yield stress-strain response, the hardening slope (Tangential modulus $E_T$) varies from one to the other, with Reilly et al. [23] reported in a ranging from 0.63 to 1 GPa for human femur and 0.34 to 0.77 GPa for bovine femur. Mercer et al. [37] stated a value of 1 GPa for the longitudinal direction in tension. Whereas Natali and Meld, in an earlier review, summed up the range from 0.2 to 1.34 GPa [43].

**Table 2–2** Post-yield material properties for human and bovine cortical bone (Ten and Comp denote tension and compression, respectively; notation L, C and R represent Longitudinal, circumferential and radial directions, respectively) [23]

<table>
<thead>
<tr>
<th>Species</th>
<th>Human Femur</th>
<th>Bovine Femur</th>
</tr>
</thead>
<tbody>
<tr>
<td>Types</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orientations</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>C &amp; R</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>C</td>
</tr>
<tr>
<td>$E_T$ (MPa)</td>
<td>0.812 ± 0.15</td>
<td>0.34 ± 0.27</td>
</tr>
<tr>
<td>$\sigma_y$ (MPa)</td>
<td>114 ± 3.1</td>
<td>140 ± 6.8</td>
</tr>
<tr>
<td>$\varepsilon_y$ (%)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_u$ (MPa)</td>
<td>133 ± 15.6</td>
<td>144 ± 6.2</td>
</tr>
<tr>
<td>$\varepsilon_u$ (%)</td>
<td>3.80 ± 0.70</td>
<td>1.60 ± 0.7</td>
</tr>
<tr>
<td>$\sigma_y$ (MPa)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\varepsilon_y$ (%)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_u$ (MPa)</td>
<td>205 ± 17.3</td>
<td>272 ± 6.8</td>
</tr>
<tr>
<td>$\varepsilon_u$ (%)</td>
<td>4.2 ± 0.3</td>
<td>4.2 ± 0.15</td>
</tr>
<tr>
<td>Shear $\tau_u$ (MPa)</td>
<td>67</td>
<td>70</td>
</tr>
</tbody>
</table>
2.4.2 Dissimilarity in tension and compression

It has been noticed for a long time that the mechanical properties of cortical bone are far better in compression than that in tension for the same material orientation (Figure 2–8). This diversity between tension and compression behaviours can be explained that bones (especially long bones), in life, are predominantly loaded in compression, and vary occasionally, in tension, torsion or bending [8]. Therefore, the mechanical and structural properties of cortical bones are adequately adjusted through the lengthy period of natural evolution to fit well for the most important task: to be able to withstand continuously and repeatedly compressive load [3].

Again, Reilly et al. [23] are among the earliest to investigate these differences by carrying out experiments separately in both tension and compression. The results showed in considerable difference in terms of ultimate strength. The longitudinal compressive strength of human cortical bone was reported nearly two times higher than that in tension and four times higher than the
transverse tensile strength [23]. However, a fully completed data set (for the same type of bone in all directions) was not demonstrated in that study (Table 2–2). In terms of the Young’s module, Mercer et al. [37] suggested in their flexure test (4-point bending) paper that cortical bone has the same value of Young’s module under tension and compression. It is also believed by many others [23, 38]. However, Barak et al. [44], raised further the discussion on this matter with a detailed investigation on the Young’s modulus of the equine cortical bone using an experimental setup similar to that of Mercer. But, the system equipped a water chamber and digital image correlation system instead of the cross-head strain gauges used in previous study. The result from Barak et al. indicated a surprisingly higher Young’s modulus (6%) in tension than that in compression based on the bending theory and statistical analysis. The yield strain was also reported higher in compression than in tension [44], whereas, a lower ultimate strain was reported in compression compared with that in tension [37, 38].

**Figure 2–8** Tensile and compressive stress/strain curves for cortical bone along the axis of a long bone [37]

Despite the significant difference, there is still very limited knowledge available in literature regarding the mechanical properties of cortical bone in compression. It can be partially explained by the fact that it is usually very common and easier to perform a test in tension or bending rather than in
compression [8]. Furthermore, the moment of failure in compression is sometimes difficult to clearly identify. Unlike tensile test, where failure occurs as soon as the ultimate strength is attained, a bone specimen in compression test can still carry load even after severe deformation [8]. Distinct stress-strain responses observed in tension and compression indicate different deformation mechanisms are activated at different loading conditions [37, 42]. Reilly and Burstein [24] pointed out in their review that a localized shear deformation mechanism is acting in compression. Mercer et al. [37] further studied the damage behaviour of a bovine cortical bone and suggested the deformation is likely caused by a slippage interaction (irreversible deformation) between collagen and mineral phase. Nyman et al. [42] investigated damage mechanisms for both tension and compression in their work and found a diffuse damage pattern in tension, whereas in compression, a less hazardous, crosshatch type of damage which allows more energy to be dissipated was observed (Figure 2–9).

![Figure 2–9](image-url) Different damage patterns developed in cortical bone: (Left) diffused micro-crack pattern observed in tensile specimens [45]; (Right) crosshatched damage pattern observed in compressive specimens [46]

### 2.4.3 Variability of mechanical properties

Unlike other engineering materials which are fabricated precisely in accordance, hence, consistent in material properties (variability introduced by manufacturing process is usually very small and statistically insignificant), the composition and microstructural distribution of cortical bone tissue may vary
from one bone to another. In fact, it is impossible to find two bones with identical morphology and composition. Therefore, the mechanical properties of cortical bones are more likely to have wider ranges of values and may well be influenced by many other factors such as species, age, gender [47] and even the testing technique which is employed [28]. A comprehensive review carried out by Reilly and Burstein [24] demonstrated that the Young’s modulus of bovine femur, which was tested in the longitudinal direction at low strain rate, ranges from 8.5 GPa to 24.5 GPa. The ultimate stress and strain tested in the same condition vary from 219 MPa to 283 MPa and 1.6% to 3.3%, respectively [23, 24]. Currey [48] described this variability as incompatible mechanical properties in his work. A list of mechanical properties for ‘normal’ bone was presented in the same work with extreme cases. Indeed, the research subject regarding the relationships between material properties and other measurable factors such as bone mineral density (BMD), porosity, age, weight (Figure 2–10) are hot and are frequently debated in literature. Over the years, more than dozens of empirical studies have shown different formulae that represent the various relationships between the properties of cortical bones and influencing factors [49], but none of them work well universally. Clearly, our knowledge of understanding this variability of cortical bone is still very limited.

Figure 2–10 Illustration of relationships between mineral content, Young’s modulus and ultimate strain of a collection of boney tissues in [47]
Apart from the variations due to origin, age, health state and some other secondary factors, the variability of mechanical properties exists even at the level of individual bone. Studies carried out by Su et al. [50], Lai et al. [51], Skedros et al., [52], Abdel-Wahab et al. [53], Bonney et al. [54] and Li et al. [55] have all confirmed correlations between mechanical properties and local anatomic position for both artiodactyl and human bone. The findings in these researches suggested that these correlations could be attributed to various factors such as the variation of mineral content [51, 56]; adaptive transition of microstructure [57]; and local stress-strain conditions [50]. At different anatomic positions, the microstructural constituents of cortical bone can vary considerably either in terms of proportion or the way in which they distributed. However, there is still a lack of quantitative understanding with regard to the distributions of microstructural constituents in literature.

Other factors such as time [58-63], temperature [64-66] and hydration level [18, 23, 67-70] also have profound effects on the mechanical behaviours of bone tissue. However, they are outside the scope of this study, hence, not discussed here in detail.

2.5 Heterogeneity of cortical bone tissue at micro-scale

At microscopic level, there are as many as a dozen of microstructural features of cortical bone. However, only four of them are commonly considered in literature due to mechanical importance of these microstructural constituents during the deformation and damage processes [18]. The four-phase composite material consists of osteon, Haversian canal, cement line and interstitial matrix. With all the requirements of metabolism, modelling, remodelling and repairing processes, the distribution of these microstructural constituents can become highly heterogeneous and varies dramatically at local region.

2.5.1 Random distribution of microstructural constituents

The random distribution (i.e., non-periodical) of the microstructural constituents of cortical bone tissue has for decades attracted many
researches to investigate its structure-mechanical relations and its novel design features that may aspire the next generation of bionics. At the nano-scale, the constituents of cortical bone seem to have rather organized arrangement that stagger together to form single orientated structure – mineralized collagen fibril [3]. Moving towards upper scales, the degree of well-organized matter diminishes as smaller components joint together to form larger ones. Ideally, a hexagon shaped osteon array arranged in the transverse plane could result a perfect isotropic transverse plane that can expend in any direction continually without interfering the isotropic nature of the plane [27]. In reality, arrangement of osteons in such a way is rarely achieved. Osteons can be any irregular round shape within a any diameter around of 100 µm to 200 µm, and in fact at any position no matter what its predecessor was. On the other hand, this lacking of symmetry and patterns of the microstructural organisation provides rather beneficial freedom and fast response for cortical bone tissue to adapt to its internal biological demands.

2.5.2 Effective modulus and rule of mixture

The effect of microstructure has always been an important focus point in the research area [71]. At micro-scale level, there are mainly three types of bone found in large vertebrates animals [16, 18]: (i) plexiform (or fibrolamellar) bone is formed more rapidly to allow a fast increase in the load-bearing capacity; (ii) osteonal bone (including primary and secondary osteons) consists of concentric lamellae structures hollowed by a vessel channel or a Haversian canal depending on the type of osteon; (iii) interstitial bone is usually the remaining structure surrounding the osteonal bone. Due to different formation periods and methods, the mechanical behaviours of individual constituents vary slightly.

Earlier attempts to predict the bone’s effective elastic modulus were more concentrated on the influence of individual constituents such as osteon, mineral contents, density and porosity, or feature on the overall elastic properties [72, 73]. More advanced models, accounting for shape and orientation effects of individual constituents [74, 75] as well as the interaction
between collagen and mineral contents [76] were reported. Despite this variety, most have one common feature: all the approaches were related to a fundamental concept of the theory of composite materials - the rule of mixture.

Sevostianov and Kachanov [74] calculated the elastic modulus using a modified formula that is commonly employed for high-elastic-contrast composite-like materials and achieved good agreement with experimental result. On the other hand, McKittrick and co-authors [14, 76] calculated the effective elastic modulus using a modified Voigt scheme for both elk antlers and bovine cortical bone. However, their results showed a large disparity with the experimental data. Then, they further suggested that the effective elastic constant of cortical bone is a complex combination effect of individual constituents in terms of orientation, shape, volume fraction and their interactions at several hierarchy levels.

Some recent studies [7, 77] proposed a comprehensive approach that involved a multi-scale realization to estimate an effective elastic constant by constructing the stiffness tensor for each constituent at multiple levels using a Mori-Tanaka scheme based on micromechanics and structural arrangement. Their model indeed provided a better understanding of the influence of each constituent at various levels. However, the questions of local variability and widely ranged mechanical properties of cortical bone in the literature still remain unsolved.

### 2.6 Fracture mechanics of cortical bone tissue

As one of the principal structural components of a skeleton; bones provide the body with unique roles. Their structural integrity is vital for the quality of life. Unfortunately, bones can only sustain loads until a certain limit, beyond which they fail. Understanding the mechanical aspects of the fracture behaviours of cortical bone will help to reduce the chance of bone fracture event in real life.
2.6.1 Factors affecting bone toughness

In real life, fracture of cortical bone can happen at fairly small scale where micro-cracks and defects may further coalesce into or initiate bigger cracks and cause catastrophic failure event. Therefore, from a material design prospective, the capability to resist fracture is very crucial for cortical bone tissue [8, 19]. In theory, it is best to have a material that is both stiff and tough at the same time. In the reality, it is a well-known fact that material’s stiffness and toughness are often contrary to each other [8, 48, 78], the stiffer a material is, the less tough it will be. For instance, Currey [48, 79] in a number of his works demonstrated the correlation between stiffness and the fracture toughness of cortical bone using specimens from various species: an uprising tendency of bone’s stiffness always accompanies with the decline of bone’s toughness (Figure 2–11). The author took further investigation on relationship between bone mineral density (BMD) and its Young’s modulus and found out there is a more potent connection between these two factors. It is therefore naturally to assume that the BMD is directly related with the toughness of bone because of the high stiffness of mineral itself [8, 48].

Figure 2–11 Correlation between Total workd and Young's modulus of cortical bone form various animal species [48]
In early works done by Rogers and Zioupos [80] and subsequently by Currey \textit{et al.} [81], and lately re-emphasised in [8] by Currey, several types of bones, including human and bovine femur cortical bones and some extremely mineralized bone-like tissues such as those of the mesoplodon rostrum (96\% mineralization) and mother of pearl (nacre) (98\%), were investigated for a possible correlation between their mechanical properties and the level of mineralization. The results indicated that brittle fracture behaviour is usually accompanied by significant reduction on fracture toughness and the increase of mineral content, except that of nacre. Despite having higher mineralization, the fracture toughness of nacre was higher than the rostrum of mesoplodon. The authors further indicated that a well-organized organic-mineral structure like that observed in nacre was the key factor for this higher toughness [81].

For those less mineralized bones, Currey [8] suggested that a dangerous crack once initiated, will propagate in favour of a denser mineral platelet than a less dense one, based on the model proposed in [82, 83]. Consequently, the post-yield deformation of collagen in less mineralized bone is inhibited by the staggered arrangement of collagen-mineral phase. Evidences from experiments done by the same author [84], based on the fracture surfaces of a less mineralized wet antler bone from a red deer and a highly mineralized wet tympanic bulla bone from a fin whale, further conformed this hypothesis. Two different fracture characters were observed in that study: an extremely rough fracture surface formed by the antler bone required a work of fracture of 6200 J m$^{-2}$, while a smooth fracture surface formed by the tympanic bulla required only 20 J m$^{-2}$ fracture energy. This reduction in mineralization associated with increase in fracture energy was considered as a strong indication of the transition from a brittle fracture mode of the highly mineralized bone tissue to a quasi-ductile fracture mode of the less mineralized bone [85].

Other studies on relationships between various factors of cortical bone and its material properties have become a wider topic and been discussed over and over again, such as those studies in [49], [86] and [45] on the relationship between animal age and toughness of cortical bone; the
investigation between porosity and toughness in [87] and the influence of water content on fracture toughness in [69] have all been debated. It seems that the correlation between the fracture toughness of cortical bone and its underlying mechanisms of cortical bone are not always governed by the effect from a single factor. On the contrary, different factors can mix together having collective effects which contribute either positively or negatively to the overall fracture toughness of cortical bone [10].

**Figure 2–12** Complex toughening mechanisms of cortical bone: extrinsic (shielding) toughening mechanisms and intrinsic (energy dissipation) toughening mechanisms [10]
In order to comprehend the fracture mechanics of cortical bone and to elucidate the roles of the readily observable factors and their underpinning principles, one has to look broadly from a multi-scale point of view and accept the fact that the fracture resistance of cortical bone is deeply rooted within each level of structural hierarchy [10]. It combines both inter- and intra-level interactions from different micro-constituents across various length scales to form the optimized fracture resistance that are well adapted to the loading environment (Figure 2–12).

**2.6.2 Origin of fracture toughness**

Like any other material properties, the origin of toughness of cortical bone tissue is underpinned by its three fundamental building blocks: the stiffness and strength of the mineralized nano-particles, the toughness and ductility of the organic-phases (mainly, the triple helix protein structure of collagen molecule) and the viscous effects from water compound. The combination of these three elements provides the basic requirements of the rigidity and ductility to resist mechanical loadings induced deformation and fracture.

In addition to this, the interfaces between them also act as bonding agents to further sustain the structural integrity of cortical bone tissue. The strength of these bonding properties were continuously evolved over the thousands of years adaptation process, so that it is not too strong to endanger the compliance of the fibres and fibrils, nor too week to compromise the integrity of the whole system [88]. Depending on the nature of the interaction, the interfacial bonding can be categorised into three different types [88]: mechanical (including the molecular entanglement and mechanical interlocking), chemical (including covalent bond and Hydrogen bond (H-bond)) and electrostatic attraction (such as ionic bond). These interfacial bonding properties are particularly important in terms of determinants of the elastic, post-yield and indeed, fracture behaviour of cortical bone. Based on their geometric and physicochemical characteristics, interfacial bonding can be either very strong like those of the intra-molecular forces such as ionic bond or covalent bone or very weak like those of the inter-molecular forces such as van der Waals force or H-bond [89].
Being a self-organized natural composite material, the toughening mechanisms come not only from cortical bone’s inherent material compositions, but also from the hierarchical architectures and the non-uniform distribution of constituents across different regions, which in turn, form complex but distinct fracture resistant mechanisms at various levels (Figure 2–12). Depending on the nature of the toughening mechanisms, they can be summarized into intrinsic and extrinsic toughening mechanisms.

2.6.3 Intrinsic toughening mechanisms

Interacting primarily at the length scale below 1 µm, there are several specially designed intrinsic deformation mechanisms [10] ahead of the crack tip that are able to undergo significant plastic deformation and some limited damage at multiple length scales to retard crack propagation. Traditional thinking of fracture toughness is the material’s ability to dissipate energy without furthering crack propagation. However, studies [10] suggested that the abilities of a material to initiate permanent deformation (plasticity) as well as micro-damage ahead of the crack tip at the local stress-concentration area are critical for dissipation of excessive energy that would otherwise cause cracking. Launey et al. [10] summarised these intrinsic toughening mechanisms as follow:

- **Molecular uncoiling and H-bond breakage:** At molecular level, the soft, entangled tropocollagen molecular has the ability to deform up to 50% [90] under tensile loading without catastrophic brittle failure. This highly deformable structure provides the fundamental plastic deformation mechanism; In addition, the intermolecular cross-links between collagen molecules, Hydrogen-bonds, provide additional energetic mechanisms that discharge energy at 10% to 20% strain.

- **Fibrillar sliding of mineralized collagen fibrils:** The staggered arrangement of the mineralized collagen fibrils combines both the stiff mineral-platelet and the ductile collagen fibrils and is vital for sustaining large deformation at higher stress level. It offers a great
deal of energy dissipation under deformation due to the repeated intermolecular slippage [91].

- Fibrillar sliding of collagen fibre arrays: At submicro-scale, bundles of mineralized collagen fibrils are grouped together and further embedded into a thin layer of extrafibrillar matrix to form the lamellae structure. This thin layer of extrafibrillar matrix is acting as a ‘glue’ agent [10], once loaded, evolves into tensile and shear deformations between mineralized fibrils. The deformation mechanisms include breaking of sacrificial bonds [92, 93] and subsequent ductile deformation of noncollagenous proteins (hidden length) which act as energy dissipation mechanism (see Figure 2–13).

- Micro-cracking: One of the well-known mechanisms of cortical bone at micro-scale [94-96], micro-cracking process considerably diminishes stress-concentration ahead of the crack tip and hence provides essential protection of cortical bone from catastrophic failure.

Figure 2–13 Schematic illustration of possible kinds of sacrificial bonds involved in the glue between the mineralized collagen fibrils [92]

2.6.4 Extrinsic toughening mechanisms

Different from that of the intrinsic toughening mechanisms, at the length scale above 1 µm, the toughness of cortical bone is extrinsically controlled primarily by the arrangement of microstructural features within the sizes range of 1-100 µm, such as osteons [97]. Unlike the intrinsic toughening
mechanisms operating during the crack initiation stage, these microstructural constituents act as a source of toughening mechanisms during crack propagation [10]. At the centre, the extrinsic fracture toughening mechanisms are formed by different characters of each microstructural constituent, for instance, the preferentially aligned osteons and Haversian canals produce a source of anisotropic deformation mechanism which results in dissimilar fracture resistances with respect to different bone axes [18, 85, 98, 99], a weak crack path induced by inhomogeneous distribution of microstructural constituents and discontinuity at the interfaces acts as a crack deflection mechanism to stall crack from advancing straight and therefore, increases the total fracture energy [100]. According to Launey et al. [10], the existence of extrinsic toughening mechanisms of cortical bone can be categorised into the following:

- **Constrained micro-cracking:** The concept of micro-cracking’s contribution to the extrinsic toughening mechanisms is probably not as visible as that of the intrinsic ones. Earlier studies [18, 94-96] showed that the development of micro-cracking around the growing crack was believed to be responsible for the rising toughness behaviour (R-curve) in bone. However, recent research [100, 101] revealed that it is in fact the indirect contribution of the micro-cracking mechanism which facilitates the formation of crack bridging and deflection mechanisms.

- **Crack deflection/twist:** In theory, crack trajectory follows the same direction as the maximum strain energy release rate or where the mode-II stress intensity is zero ($K_{II} = 0$) [102] in a homogeneous material. However, in anisotropic composite materials like bone where material discontinuity and imperfection at interfaces may create potential risks (weak paths) for crack propagation, the trajectory of a crack is not solely determined by the maximum strain energy release rate. In fact, it is a competition between the projected direction of the maximum strain energy release rate and the path along the weakest microstructural resistance [10]. When these two directions are not aligned with each other, higher fracture toughness is usually observed.
as a result of crack deflection and twist (Figure 2–12). In bone, the deflection and twist mechanisms are the result of preferentially aligned microstructures which act as crack barriers to cause crack deflection and extreme rough fracture surfaces, consequently, dissipate energy significantly.

- Crack bridging: Crack bridging effect can be observed in many composite materials [78]. Depending on the orientation of underlying microstructures, bridging mechanisms can happen at either side of the crack (Figure 2–12): bridging behind the crack tip is called fibre and fibril bridging and usually happens when the crack growth direction is perpendicular to the fibre direction. Intact fibres and fibrils act as springs which carry load between the two fracture surfaces [101, 103, 104]; bridging ahead of the main crack tip is called uncracked-ligament-bridging according to Ritchie [18], it usually happens when crack propagation direction is parallel to the fibre direction, where coalescence of micro-cracking leads to the formation of uncracked regions which in turn, reduce the crack-driving force along the crack propagation direction [18, 105, 106].

### 2.6.5 Fracture toughness

One of the main purposes of studying the fracture mechanics of cortical bone is to quantify its fracture resistance values, viz., fracture toughness. Commonly, fracture toughness can be measured by means of different techniques, such as work of fracture, which calculates toughness based on the area under the load-displacement curve during test; or fracture mechanics based methods like linear-elastic fracture mechanics (LEFM). The work of fracture approach was widely used in the past, but suffered from its dependence from specimen size and geometry, therefore, is not preferred [10, 18]. The LEFM approach assumes that materials undergo a linear-elastic fracture process with only limited plastic deformation at a small region so that the stress field near the crack tip can still be described under linear-elastic framework by the stress intensity factor (SIF), $K$. Depending on the applied loading conditions, $K$ can be described based on three fracture modes:
Cutting of Cortical Bone Tissue: Analysis of Deformation and Fracture Process

mode-I (tensile mode), mode-II (in plane shear mode) and mode-III (anti-plane shear mode) [107]. The fracture toughness is then defined based on particular mode of the loading as the critical value of SIF, i.e., $K_C$.

Alternatively, the fracture toughness can also be described in terms of the change in potential energy such as the critical strain energy release rate, $G_C$, or its non-linear-elastic equivalent form $J_C$, a critical value of $J$-integral [107]. One of the differences between $G_C$ and $J_C$ is the validity under non-linear fracture process: the concept of $G_C$ is based on the LEFM framework using linear-elastic material assumption which is only valid for elastic materials or when plasticity is negligible, while $J_C$ accounts for the non-linear behaviour of the material and is valid for wider range of materials.

### Table 2–3 Fracture toughness of bovine femur measured using various testing methods for longitudinal and transverse directions (CT and SEN, stands for compact tension test and single-edged-notch test, respectively)

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Specimen type</th>
<th>$K_{IC}$ (MPa√m)</th>
<th>$G_{IC}$ J/m$^2$</th>
<th>$J_{IC}$ J/m$^2$</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal</td>
<td>CT</td>
<td>2.4 – 5.2</td>
<td>920 – 2780</td>
<td></td>
<td>[18]</td>
</tr>
<tr>
<td></td>
<td>CT</td>
<td>3.0 ±0.24</td>
<td>644 ±102</td>
<td></td>
<td>[99]</td>
</tr>
<tr>
<td></td>
<td>SEN</td>
<td>3.21 ±0.43</td>
<td>1383 – 2557</td>
<td></td>
<td>[108]</td>
</tr>
<tr>
<td></td>
<td>SEN</td>
<td>2.4 – 3.0</td>
<td>2000 – 2800</td>
<td></td>
<td>[109]</td>
</tr>
<tr>
<td></td>
<td>SEN</td>
<td>5.7 ±1.4</td>
<td></td>
<td></td>
<td>[18]</td>
</tr>
<tr>
<td>Transverse</td>
<td>CT</td>
<td>6.0 ±0.41</td>
<td>1374 ±183</td>
<td></td>
<td>[99]</td>
</tr>
<tr>
<td></td>
<td>SEN</td>
<td>5.58 ±0.52</td>
<td>3135 – 5534</td>
<td></td>
<td>[108]</td>
</tr>
<tr>
<td></td>
<td>SEN</td>
<td>4.3 – 5.7</td>
<td>5900 – 7600</td>
<td></td>
<td>[109]</td>
</tr>
</tbody>
</table>

Throughout the literature, a lot of efforts have been focused on quantifying the fracture toughness of cortical bone tissue using various testing methods and different fracture toughness parameters (Table 2–3). Similar to that of other material parameters described earlier in this chapter, the fracture
toughness of cortical bone has a significant wider range of values (Table 2–3). However, there is still a lack of understanding of the underpinning fracture behaviour of cortical bone in relation to the variation of the toughness values.

2.7 Summary

- Cortical bone tissue is a highly heterogeneous and hierarchical composite-like material.
- Bones exhibit significant anisotropic elastic-plastic behaviour with poor ductility when the load is perpendicular to the longitudinal bone axis.
- The mechanical behaviour of cortical bone is different in tension and compression.
- Bones are designed to work within its comfort zone, which is at compressive elastic loading condition predominantly in longitudinal direction; out of this comfort zone, bones exhibit poor post-yield behaviour and significant reduction in terms of strength and toughness.
- The mechanical properties of cortical bone vary dramatically compared with other engineering materials and even at single bone level. Previous studies focused on the variations attributed to other external factors such as species, age and gender. There is no previous study available to quantitatively address the reasons behind this large variability.
- The diversity of the mechanical properties of cortical bone and the random distribution of its microstructure constituents reflect the fine-tuning and adaptation of the bone tissue to its functional requirement. However, the relationship between variability of the microstructural constituents and variability of the mechanical properties is yet to be studied.
- Cortical bones exhibit an anisotropic non-linear fracture process which can be affected by many factors including BMD, age, porosity and microstructural distribution, in various combinations. But, there is no
statistical analysis available to systematically study the variability of the fracture toughness.

- Across different length scales, the heterogeneous arrangements of the microstructural constituents of cortical bone form complex intrinsic and extrinsic toughening mechanisms which are yet to be fully understood.

- Therefore, it is very important to build up a comprehensive understanding of mechanical behaviours of cortical bone tissue which can be used to further assist the development of accurate and adequate computational models.
Chapter 3
Modelling of Bone and Bone Cutting

3.1 Introduction

Benefit from the modern computational power and numerical interpretations of mechanical behaviour of cortical bone, computational models of bone tissue can provide additional insights, visualizations and detailed analyses which otherwise cannot be achieved through the experimental measurements. As a result, large and vast numbers of cortical bone models which have been implemented throughout literature are continuously pushing the new frontier of our knowledge of mechanical behaviour of cortical bone at different levels. Depending on the type of material behaviour or the interest of the length-scale, there are many different theoretical approaches or modelling techniques can be used to model cortical bone tissue. However, their accuracies largely depend on the understanding of the underpinning mechanics of the material and the ability of particular computational technique which are employed. Above all, FE models are only tools which are designed to (partially) represent specific material response of the material that are adequate within the boundaries of limitation. Therefore, it is useful to summarise current available finite-element modes of cortical bone systematically for study and comparison purposes.

In this chapter, an overview of various kinds of bone models was studied in order to gain insights of current modelling techniques and their limitation for further development of adequate cortical bone models.

3.2 Macro-scale bone models

At macroscopic scale, cortical bone tissue can be considered as homogeneous material with continuous material medium. Based on the
classical continuum mechanics theory, models at this level generally focus on the overall response of cortical bone as a structural material.

### 3.2.1 Elastic and elastic-plastic models

#### Elastic models

Some of the earliest structural bone models in the early 70s [110, 111], used a simple two-dimensional (2D) beam theory to calculate the stress distribution in femur within the whole structure. Then, a 3D model was reported later on [112]. Limited by the development of the finite-element method and the measurement techniques at that time, the early models are limited to ideal elastic, isotropic material and then a transversely isotropic model was introduced in the 80s by Vichnin and Batterman [113]. Since the 90s, the employment of micro-computed tomography (microtomography or micro-CT) technique significantly improved the accuracy of the model geometry [114, 115] which is big concern for bones, as their shapes are unique to each individual. Until now, there are still many elastic cortical bone models in the literature due to their specific research interests.

#### Elastic-plastic models

An elastic-plastic model is more practical in the sense of accompanying of the post-yield material behaviour. For instance, Kotha and Guzelsu [116] used an elastic-perfectly plastic continuum model to evaluate the effect of mineral content on the tensile behaviour of cortical bone of bovine femur. Models neglecting the hardening effect are also found in [117] and [118] as a result of relatively less prominent hardening effect of cortical bone tissue [119].

Models which account for the tangential (hardening) modulus tend to diverge on the different mathematic formulations of the flow rule and their range of parameters. Hight and Brandeau [120] proposed a Ramberg-Osgood relationship to describe the elastic-plastic behaviour of cortical bone using different data sets from literatures. Its generalized form can be written as
\[ \varepsilon = \varepsilon^e + \varepsilon^p = \frac{\sigma}{E} + K \left(\frac{\sigma}{E}\right)^n, \quad (3-1) \]

Here, \( \varepsilon, \sigma \) and \( E \) are strain, stress and the Young’s modulus, respectively, and superscripts \( e \) and \( p \) denote elastic and plastic component, respectively. \( K \) and \( n \) are plasticity constants that depend on the type of material. The first term on the right-hand side of the equation represents the elastic strain component, while the second term is the plastic strain component. This power-law relationship implies the presence of plastic strain even at low levels of stress [120]. In recent studies carried by Zheng et al. and Johnson et al. [63, 121], the Ramberg-Osgood equation (3–1) was employed successfully in both models which accurately reproduced their experimental data. Known to the difference in terms of tensile and compressive strength, cortical bone can be modelled using pressure-dependent yield models such as the Mohr-Coulomb model [122] or Drucker-Prager model [123] which captured different tensile and compressive yield stresses and, separately, a pressure-independent anisotropic yield model in [124]. However, one of the deficiencies of using these empirical fitting parameters is that they are always lack of consistency: fitting of different experimental data using the same model in [120] resulted in three different sets of parameters.

### 3.2.2 Visco-elastic-plastic models

A more versatile model would be the recognition of the time-dependent feature of cortical bone that has been well documented in many previous studies [125-128]. Although, the time-dependent material behaviour happens at both elastic regime and post-yield regime, it can be mathematically decomposed into visco-elastic constitutive model and visco-plastic constitutive model.

**Visco-elasticity**

The linear visco-elastic behaviour could be described as a combination of elasticity and viscosity characteristics. The elastic component can be represented as linear spring using Hooke’s law, while the viscous components can be described as dashpots, with the stress-strain rate
relationship given as \( \sigma = \eta \frac{d\varepsilon}{dt} \). Here, \( \frac{d\varepsilon}{dt} \) is the time derivative of strain (strain rate) and \( \eta \) is the proportionality constant (viscosity). Depending on the type of behaviour required, visco-elastic constitutive models can have various arrangements: Maxwell model (Figure 3–1) put the elastic and viscous components in a series arrangement, exhibiting a linear creep reaction with an instantaneous deformation and an exponential relaxation under irregular strain; while, Kelvin-Voigt model (Figure 3–1) places the two in a parallel position, therefore, having an asymptotical creep characteristic without any relaxation; by having different arrangement of both Maxwell model and Kelvin-Voigt model, the generalized Maxwell model combines both the advantage of the two models and can be implemented in various combination to suit for the characteristic of the material.

![Maxwell model](image1)

![Kelvin-Voigt model](image2)

**Figure 3–1** Creep and relaxation behaviour of Maxwell and Kelvin-Voigt models: (a) creep under constant stress, (b) relaxation under constant strain

*Visco-plasticity*
The visco-plasticity describes the inelastic deformation response of materials in relation to the loading rate. In a pure elastic-visco-plastic material, the constitutive formulation involves three components: the non-linear dashpot component represents the rate-dependent viscous term; the sliding element represents the plasticity term and the linear spring element account for the elastic term. Such models used to model bone behaviour can be found in a number of works [127, 136]. Hight and Brandeau [120] proposed a modified version of the Ramberg-Osgood equation to capture the rate-dependent plasticity characteristics of cortical bone. However, the developed five parameter model exhibited some inconsistency across different strain rates. The Johnson–Cook model is another flow stress based models implemented in [129] by Alam et al., describing the response of the flow stress in relation to the applied deformation strain, strain rate and temperature.

**Visco-elastic-plastic models**

Further development of identifying the visco-elastic-plastic behaviour of cortical bone was carried out by Johnson et al. who developed a constitutive model consisting of two branches of Maxwell-Weichert model along with a modified Ramberg-Osgood equation [63]. The system shown in Figure 3–2a has a total of 6 elements and 7 parameters consisting of one spring element parallel with two Maxwell branches and a non-linear dashpot element in series. Based on literature data, the authors hypothesised that cortical bone behaved in a non-linear visco-elastic manner, requiring two time constants (one response a lower rates lever and one at a higher rate level) to describe the material’s initial visco-elastic behaviour [63]. For a constant-strain-rate loading history, the proposed one-dimensional visco-elastic model can be written as follows:

$$
\sigma(t) = E_0 \dot{\varepsilon}_V t + \eta_1 \dot{\varepsilon}_V \left(1 - e^{t/\eta_1}\right) + \eta_2 \dot{\varepsilon}_V \left(1 - e^{t/\eta_2}\right), \quad (3–2)
$$

where $\eta$ is the proportionality constant (viscosity) and the superscript $VE$ is used to distinguish the visco-elastic strain rate from the total strain rate; the visco-plastic component is introduced with the Ramberg-Osgood equation:
\[ \dot{\varepsilon}_{VP} = \frac{\sigma}{|\sigma|} \left( \frac{|\sigma|}{S_0} \right)^m. \]  

(3–3)

Where \( m \) and \( S_0 \) are constants which are determined based on post-yield stress-strain curves. The total strain rate is the sum of the two parts. \( \dot{\varepsilon} = \dot{\varepsilon}_{VE} + \dot{\varepsilon}_{VP} \).

**Figure 3–2** (a) Schematic of two-branches Maxwell-Weichert model after [63]; and (b) a four-parameter constitutive model after [121]

Unlike the previous model, which has many parameters and components in the system, Zhang et al. [121] proposed a four-parameters constitutive model using a modified Kelvin-Voigt equation incorporating a linear dashpot in parallel with elastic-plastic spring with a Ramberg-Osgood relationship. The schematic of the system is shown in Figure 3–2b with four parameters: \( E \) denotes Young’s modulus, \( A \) and \( m \) are two dimensionless plasticity constants, \( \eta \) is the viscosity of the dashpot. The numeric relationship of the model can be described as

\[ \varepsilon = \varepsilon_d = \varepsilon_s + \varepsilon_{sl}, \]  

(3–4)

\[ \sigma = \sigma_s + \sigma_d, \]  

(3–5)

\[ \sigma_s = \sigma_{sl}. \]  

(3–6)
The subscripts $d$, $s$ and $sl$ denote dashpot, spring and sliding element. From the arrangement of the system, it is easy to predict that the model exhibits an asymptotic creep under constant stress with no instantaneous deformation. The relaxation behaviour is similar to that of the Maxwell model. The model was used to simulate an indentation test with various loading, unloading and holding time. The results achieved a close fit of their experimental data [121].

3.3 Micro-scale bone models

At micro-scale level, bone models can be categorised into two different types: continuum mechanics based models which assume that cortical bone is still a continuum material medium, but usually in a much smaller scale: within the level of microstructural constituents; and the microstructure based models which account for the heterogeneity of the microstructural constituents. Models at this level normally focus on the material response at localised region and the effects of the microstructural constituents rather than the structure response.

3.3.1 Models based on homogenised material

Like the models at macro-scale, micro-scale homogenised material models bypass the heterogeneity due to variations of microstructure and are interest in simulating only a limited region within microstructural constituent. Therefore, model geometry is still maintained in a simple form while research is focused on the constitutive formulation of the material deformation behaviour of cortical bone such as Nanoindentation modelling. Tai et al. [117] studied the effect of mineralization on the nano-scale deformation mechanisms of cortical bone using 3D elastic-plastic constitutive model. The model demonstrated significant variation of local material properties due to partial and complete demineralization process and showed a good agreement with experimental results. The previously proposed and verified constitutive model by Zhang et al. [121] at macro-scale level was also implemented at micro-scale level and the model was in good agreement with experiments.
3.3.2 Microstructure based models

The hierarchical anisotropic behaviours of cortical bone are always an important subject in bone research community and affect greatly of its mechanical behaviour. For instance, at microscopic level, the fracture properties of cortical bone strongly depend on the position of local microstructural constituents and their material properties rather than the bulk material behaviour. Experimental results provide only limited knowledge regarding the fracture processes and the interactions between each microstructural constituent due to the limitations of the experimental techniques. The microstructured bone model can provide a deep inside understanding of the interaction between them and help to reveal underpinning mechanisms that govern the overall bone’s behaviour.

Finite-element modelling of the microstructural cortical bone can be broadly categorised into three different approaches. One is based on the idealized geometry of the microstructural features of the cortical bone tissue. Another type however, uses various different imaging or scanning techniques to digitalize and discretize the real-world microstructure of cortical bone. The third type of models only generates a numerical representation rather than the real bone microstructure based on the analysed images.

An early stage model developed by Hogan [130] employed simple osteons unit cell model with idealized circular geometry to predict the effective elastic modulus as a function of porosity (Haversian canal). The model produced similar results compared with experimental ones. Similarly, Prendergast et al. [131] developed a five-phase single osteon model including the little empty space called lacunae. It was suggested that lacunae increased micro-damage accumulation that in turn change the local deformation behaviour. In the study carried out by Najafi et al. [132], an idealized interstitial bone model containing a matrix, osteons and cement line was developed. The model was implemented using Franc 2D software using remeshing based criterion in a plane-strain condition. Osteons were considered as homogenous and isotropic fibres with approximately 70% fraction of the total area (Figure 3–3). Cement line was presented as interface tissue modelled with both low and
high elastic modulus. Different crack position and cement line properties were considered during the analysis and the authors concluded a strong influence of microstructure morphology on the crack trajectories. No matter how promising, their results were established only based on idealized geometry which is lack of representation of the real world.

**Figure 3–3** Microstructured cortical bone model: (a) a single microcrack within the interstitial tissue and (b) a quarter of the model [132]

On the other hand, the microstructured models were all having or partially having numerical representation of the actual microstructure of cortical bone through various techniques. For example, the Microtomography-based models like those developed by Pistoia *et al.* [133] and Akhtar *et al.* [134] were all based on reconstructed 3D micro-CT images as the prototype for their finite-element models. The 3D model in [133] was first scanned and then discretized into a so called micro-finite element model, hence, not only the unique shape of the particular bone specimen was recreated, but also its internal microstructures including trabecular structure and voids and canals within cortical bone. The model was used to evaluate the mechanical behaviour due to changes of microstructural contents. The mesh was generated using 3D 8-nodes brick elements in the bone model, which consisted of both cortical and trabecular bone structures (Figure 3–4). The respective elastic constants were applied to the structure individually. Another type of microstructure based models like the one done by Mischinski and Ural [135] and the one in the work of Budyn and Hoc [136], in which
microscopic images taken from the transverse section of cortical bone were duplicated and discretized into 2D finite-element meshes according to the distribution and shape of individual Haversian system. The former model taken the overall shape of each osteon into account and idealize them into oval or circular inclusions omitting the present of Haversian canals, while the latter model preserved the contour profile of each microstructure entities at high fidelity (Figure 3–5). These models were developed in the effort to investigate the effect of microstructural contents and their interactions under various loading conditions which can provide detailed quantitative measure and predictions in additional to their experimental observations. However, models involving real image or structure scanning techniques are usually very time consuming as they require large amount of image processing and analysing work and are tied to the specific specimen geometry which they use and consequently, their results are not necessary generally applicable to others.

**Figure 3–4** Micro-finite-element model of original distal radius: (a) single image created from the micro-CT scanner; (b) reconstructed finite-element model [133]
Budyn and Hoc [137] also developed another type of microstructure based cortical bone model with a four-phase composite-like microstructure which composed of an interstitial matrix and osteons surrounded by thin layer of cement lines and hollowed by concentric Haversian canals assuming to have cytoplasmic fluid inside (Figure 3–6). Different to what has been discussed so far, the four-phase microstructured model neither used the real microstructure image nor based on the idealized microstructural distribution. Instead, the model was regularized into a finite element model using statistical algorithms. The geometrical parameters such as volumetric fractions, densities, diameter distributions were statistically measured from several microscopic observations. The anisotropic material properties such as the Young’s modulus and Poisson ratio were measured locally and randomly distributed using the Gaussian fit. The osteons were then randomly populated as isotropic material into a 2D model or as transversally isotropic tubular structure in the case of a 3D model according to the statistical
parameters. The cement line was considered as having an isotropic elastic behaviour with a 25% lower Young’s modulus. The matrix was considered homogeneous and transversally isotropic with an elastic modulus about 10% stronger than the mean value for the osteons [137].

To sum up, all techniques used for constructing the microstructured models have their advantages and drawbacks. The unit cell method concentrated on microstructural features of cortical bone but with an idealized geometry and distributions, while the image-based model reflected to the actual structure of the cortical bone, but has limited usability. The statistical analysis based models can be used to characterise more generalized results, but may not accurately preserve detailed microstructure features. Still, they are models developed with great amount of computational and analytical effort to deal particular research tasks and cannot be used regardless of the circumstances.

**Figure 3–6** Three microscopic images taken of human cortical bone with different age groups and realizations of 2D finite-element models based on statistical analysis of microstructure distribution [137, 138]
3.4 Other non-damage based models

Apart from constitutive models and microstructured models discussed early, there are other types of cortical bone models which aimed at specific modelling tasks.

Figure 3–7 Schematic of multi-scale modelling approach used in [7]

Fritsch and Hellmich [77] and Hamed and his co-authors in various works [7, 139] have developed various multi-scale analytical models to predict the macroscopic elastic properties of cortical bone tissue based on the
hierarchical microstructural constituents. The general idea of the models was based on the concept of rule of mixture in composite materials. By utilizing different representative volume elements (RVE) across different length-scales (Figure 3–7) along with a Mori–Tanaka homogenization scheme, the models were capable to estimate the effective stiffness tensor for numerous hierarchical organizations based on their volume fraction and shape of each RVE. The models' predictions of elastic modulus for both standard and demineralized cortical bone tissue were verified successfully by comparing with experimental results [7, 139].

Other types of non-damage based bone models including models for structural optimization based on bone remodelling concept developed by Jang and Kim [140] at macro-scale and Gao [141] at nano-scale; collagen-mineral interaction models [141-143] at nano-scale are all having their own research interests and less concerned to the scope of this study, therefore, will not be discussed in detail.

### 3.5 Modelling of damage and fracture of cortical bone

Damage and fracture modelling of cortical bone material are equally important, at least no less than the modelling of pre-damage behaviour. Successful modelling and consequently, being able to predict bone failures are at the centre of bone research as it is important to know when bones are likely to fail in real life and therefore, be able to develop procedures that can help to reduce the chance of bone failure events. The number of models available in the literature is even larger than those of non-damage models. In a broader sense, modelling of damage and fracture can be considered into two categories: implicit modelling of damage degradation using continuum damage mechanics and the explicit modelling damage and fracture process using fracture mechanics theory in conjunction of a number of modelling techniques.
3.5.1 Continuum damage mechanics

Continuum damage mechanics (CDM) describes the material failure behaviour based on the continuum mechanics framework by introducing a damage variable $D$. The simplest form of this parameter is a scalar, which represents the void volume fraction of damage material (void or cavities) [144]. The advantage of this method is its implicit modelling of damage behaviour that bypasses the cumbersome calculation process involving crack initiation and propagation. Instead, the model assumes the damage accumulation is an evolution of the loss of material integrity. The degradation of the material stiffness may be represented by increasing the fraction of the damage variable. Then, the value $D$ ranges from 0 to 1, with 0 corresponding to undamaged state while 1 indicating a complete loss of material integrity. The model hypothesizes that a degradation of a material responding to damage in terms of the stress-strain relationship is governed by constitutive equations of the undamaged material in addition of which the stiffness is simply replaced by its effective magnitude [144]. For example, one-dimensional linear elasticity is given in Table 3–1.

<table>
<thead>
<tr>
<th>Table 3–1 Effective stress-strain relationship of one-dimensional linear-elastic material subjected to damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon = \frac{\sigma}{E}$</td>
</tr>
<tr>
<td>$\varepsilon = \frac{\sigma}{(1-D)E}$</td>
</tr>
</tbody>
</table>

In a study by Taylor and his colleagues [145], the fatigue behaviour of human cortical bone was studied using a CMD-based model. The degradation of stiffness and accumulation of permanent strain were defined as non-linear functions of damage:

\[
E_n = a(D)^3 + b(D)^2 + c(D) + E_0, \quad (3-7)
\]

\[
\varepsilon_{pn} = d(D)^e, \quad (3-8)
\]
where $E_n$ is the modulus after $n$ loading cycles, $\varepsilon_{pn}$ is the permanent strain after $n$ loading cycles and $a$, $b$, $c$, $d$ and $e$ are material-dependent constants. Then the coefficients were fitted with experimental data using linear regression analysis. Another CDM model was introduced by Moreo et al. [146] who developed a variant of damage that may become negative due to remodelling process. Zheng et al. [147] also introduced a plastic damage model into their previously developed visco-elastic-plastic bone model [121]. They assumed damage attributed exclusively to a compressive loading and the damage-evolution relationship was defined as a scalar function of equivalent plastic strains in an exponential relation:

$$D(k_c) = \gamma(1 - e^{-\beta k_c}),$$  \hspace{1cm} (3–9)

where $D$ and $k_c$ are the scalar damage variable and critical equivalent plastic strain, respectively, and $\gamma$ and $\beta$ are additional parameters governing the exponential decay in stiffness. Although the simulation results showed a good agreement with the experiment data (Figure 3–8), a validation of the model at different strain-rate is still missing [147].

![Figure 3–8](image)

Comparison of a four-parameter visco-elastic-plastic model with and without improved plastic damage model [147]

3.5.2 Fracture mechanics based crack propagation models

*Linear elastic fracture mechanics*
The theory of fracture mechanics provides the foundation of the quantitative understanding of cracks and crack propagations inside materials. Models based on the fracture mechanics are normally expected to explicitly reproduce the discontinuity and singularity induced by the presents of cracks within the continuous finite-element medium. Linear elastic fracture mechanics (LEFM) is one of the fundamental theories within the fracture mechanics framework which enables the mathematical formulation of the resistance of a material to crack propagation using the linear elastic theory. Since the development of Griffith’s criterion [148] and later modification by Irwin [148], the calculation of fracture properties (toughness) has become possible and the number of fracture models is growing and exclusively using LEFM based analysis [149].

A LEFM-based cortical bone micro-crack propagation model was evaluated by Najafi et al. [132]. The simulation was performed using Franc 2D [132] with remeshing ability. The crack initiation and propagation was determined using a simple maximum-stress criterion and the crack trajectory is determined around the crack tip using maximum hoop stress. Subsequently, similar models [150, 151] were developed by the same research group to compute Stress Intensity Factor (SIF) near the micro-cracks tips in vicinity of a osteon. Their results suggested a possible stress amplification and stress shielding effect due to different micro-cracks morphology and configurations. For a valid LEFM hypnosis, the deformation region around the crack is assumed to undergo only elastic deformation with no or very limited plastic deformation, viz., small scale plastic yielding during fracture process [107]. To accomplish the increasing effect of plasticity, the Dugdale-BCS model was introduced based on the LEFM hypnosis, in which the plastic zone is assumed to spread out at the two ends of a crack tip. In recent research, Gao [141] employed a Dugdale-BCS model to predict the critical size for a strip of width $2h$ to tolerate a central crack of length $2a$ in bone and bone-like material. The author claimed that by designing the mineral crystals at nano-scale, the normally brittle material become insensitive to crack-like flaw which leads to a robust fracture resistance.
**Non-linear fracture mechanics**

However, due to the existence of post-yield deformation in cortical bone and its fracture toughening mechanisms, non-linear effects dominate the fracture process and the hypnosis based on LEFM is not valid any more. Therefore, the number of models incorporating non-linear fracture mechanics concepts grows dramatically in recent years. From an engineering perspective, the judgement on the applicability of LEFM or non-linear fracture theory is determined (according to testing standards: BS 7448-1 [152]) purely by the load-displacement curves based on the material behaviour during fracture tests [149]. A common requirement for a non-linear fracture mechanics based model is to be able to characterise the non-linear effect over the load-displacement curves. Cohesive element method is one of the prominent models that capable of replicate the non-linear fracture processes through the implementation of a cohesive-like, traction-separation behaviour at the crack tip during crack opening process.

**Cohesive models**

In contrast with LEFM, the cohesive model (CM) was originated from the same concept of Dugdale-BCS model with the considering of the non-linear fracture effect, namely, bridging and diffuse micro-crack tip at damage zone (Figure 3–9). The model defines a single dominant crack consisting of a traction-free area and a bridging or cohesive zone area for a plane symmetry condition. In recent studies, Yang et al. [103, 104, 149] employed a cohesive model to characterise the fracture properties of human cortical bone. The model employed the linear-spring idealisation at the frontier of crack tip to represent non-linear fracture process (Figure 3–9). The nonlinear damaged-material behaviour is represented by the traction separation force introduced at the damage zone by the cohesive elements. When the crack opening displacement reaches to a critical value, traction force reduces to zero and the crack surfaces become traction-free.
The model in [103, 104, 149] was implemented in Abaqus software in a plane stress condition. The cohesive traction law was arbitrarily chosen as a bilinear format [103] (Figure 3–10a), for which the fracture energy can be formulated as:

\[ W_{\text{total}} = W_{\text{tip}} + W_{\text{brid}}, \]  

\[ W_{\text{tip}} = 2 \int_{u_0}^{u_e} p_3(u_3) du_3, \]  

\[ W_{\text{brid}} = 2 \int_{u_0}^{u_e} p_3(u_3) du_3, \]  

where tip and brid refers to crack tip and wake bridging, respectively; \( u_i \) and \( p_i \) represent displacement of discontinuity and traction fields, respectively; the subscripts 1 and 3 imply coordinates \( x_1 \) and \( x_3 \) oriented as shown in Figure 3–9. The bone material was modelled as orthotropic with \( E_1 = 16 \) GPa and \( E_3 = 12 \) GPa for a compact-tension test. Then parameters of the bilinear traction law were fitted using the experimental data. The result showed a better agreement with other experimental data, compared with the LEFM model (Figure 3–10b).
Figure 3–10 Hypothetical bilinear traction-separation law for mode-I cohesive zone model, the vertical and horizontal axes are traction-separation force and crack opening displacement used in cohezive zone model, respectively (a); and Comparison of load-displacement between cohesive law model, LEFM model and experimental results (b) [149].

Morais et al. [153] developed a double cantilever beam model using proposed cohesive zone method to simulate the crack initiation and evolution under pure mode-I loading condition. The introduced model along with a new data reduction scheme was verified by the experimental tests and proofed to be efficient on fracture characterisation. A cohesive finite-element method developed by Tomar [154] pioneered the cohesive traction-separation law on a microstructured trabecular bone model featuring irreversible bilinear traction-separation laws for both tensile and compressive conditions. The developed model promoted the importance of the structural arrangement of the trabecular bone in its unique fracture resistance properties. Ural et al. [155, 156] studied the effect of porosity and strain rate on the fracture toughness of cortical bone tissue using micro-CT based CT-specimen model incorporated with cohesive zone methods. The results indicated strong negative correlations of the fracture toughness value of cortical bone with respect to increasing of porosity and strain rate.
The cohesive model described above has significant advantages over the usability and predictability of non-linear fracture processes of materials compared with traditional LEFM method. Still, the cohesive method itself has many limitations. First of all, CM relies on the pre-defined crack path to simulate the crack propagation, whereas in reality, crack does not necessary follow the pre-defined route. Secondly, current method can only cope with a single-crack issue at a time, while the multiple cracks are present in most of fracture events. Thirdly, the artificial high-stiffness of the cohesive elements introduced by the traction-separation law could cause changing of the overall material behaviour and subsequently, produces inaccurate result. In an attempt of resolving some of the problems facing by the cohesive models, Ural and Mischinski [157] and Mischinski and Ural [135] in a series studies have proposed a novel approach using the cohesive element method to overcome the crack path dependency problem. Instead of defining cohesive element only at a pre-defined crack path, the proposed models superimposed cohesive elements onto edges of all the existing continuum

Figure 3–11 Traction separation law defining the behaviour of cohesive zone elements (a); configuration of cohesive zone elements with their corresponding continuum finite-elements (b); and cracks propagating through microstructured cortical bone model with different cement line properties (c) [157]
elements (Figure 3–11). By doing so, crack initiation and propagation were no longer limited to restricted crack path, but along all the element boundaries. Therefore, a relative path independent crack process was ensured. Still crack propagation is limited to the boundary of each element and cannot go inside an element, hence, has an element size dependency.

*Extended finite element method (XFEM)*

In terms of crack propagation techniques, there are several other algorithms which also can be applied to accomplish the crack propagation requirement within a continuum finite-element medium, such as element de-bonding, visual crack closure technique (VCCT) and remeshing, all of which require the elements near crack-front to match with the numerical discontinuity of the crack geometry as the crack propagates. Sometimes, a pre-defined crack path or remeshing [158] has to be made in order to facilitate the crack formation. As a result, the propagation of cracks may not be necessary solution dependent. A newly emerged technique that features a solution dependent crack propagation capability has been largely implemented in recent years. This technique uses the concept of local element enrichment to deal with the crack initiation and propagation associated numerical discontinuity and singularity (Figure 3–12). Cracks propagation inside elements are permitted by allowing these extended element enrichment and therefore, its name is called eXtended-Finite-Element-Method or XFEM by Belytschko and Black in 1999 [159].

*Figure 3–12* Representation of enriched finite-element approximation and the presence of crack inside elements, circular nodes are enriched by step
A multiple scale based model for modelling multiple-crack growth in cortical bone was presented by Bundyn and her co-authors in series research papers [137, 138, 160]. The XFEM based models were used to investigate many different aspects of mechanical and fracture behaviours of cortical bone such as aging or porosity effects on structure and mechanical properties of cortical bone or crack propagation characteristic within microstructured cortical bone model. The models were developed as a four-phase composite, with each microstructural constituents been statistically measured and regularized with a Gaussian norm and random positioned inside the model. A crack propagation criterion was introduced based on an elastic-damage strain driven criterion. And, location of the initial crack was determined by using the critical maximum principal strain.

Liu et al. [161] developed a homogenised XFEM model to predict fracture of a proximal femur due to impact. The material model was assumed to be simple isotropic linear elastic with different bone density distribution. In addition, parameters used for crack initiation and propagation were also defined as a function of bone density distribution. The results of the model demonstrated a good agreement with their experimental observation. In another recent attempt, Feerick et al. [162] developed a homogenised XFEM model incorporating with an anisotropic damage criterion implemented in user-subroutine (UDMGINI) in Abaqus to evaluate the anisotropic fracture behaviour due to a pull out motion of screw based implant. The model demonstrated dissimilarity damage patterns due to the anisotropic fracture behaviour of cortical bone which has been widely observed experimentally. The result of this study also emphasized the necessity on further development of anisotropic damage models of the bone tissue. To date, the existing XFEM based cortical bone models only focus on linear elastic material properties prior to fracture, whereas cortical bone demonstrates noticeable post-yield deformation. Furthermore, there is no model available to address the large deviation of the fracture toughness value observed in the
literature. Last but not least, current three-phase or four-phase composite bone models are only validated at their respective microscopic length scales, whereas models at micro-scale do not necessarily represent the macroscopic deformation scenarios and are yet to be validated.

3.6 Modelling of bone cutting

Cutting has been an important skill of mankind that dates back to the origins of civilization. In the early age of human race, cutting skills helped our ancestors making tools for food hunting, defence, construction and even writing [163]. In the modern world, cutting is inevitably covering every aspect of people’s life from kitchen to factory, from farm to hospital, involving almost every kind of materials from food technology to medicine as well as the traditional engineering materials [163]. With respect of bone cutting, it is of great importance in bone surgical operations involved in several medical fields, such as orthopaedics, dentistry and osteotomy. Cutting of bones differs from that of engineering materials since bone is living tissue with hierarchical microstructure. Large cutting force and high temperature generated at cut site cause damage and thermal necrosis of bone during cutting process leading to bad postoperative result and longer recovering time for patient [2]. Numerical simulations of cutting of cortical bone can provide broad insight in the vicinity of tool-bone interaction zone. However, modelling of bone cutting can be a cumbersome task due to the complex material behaviour of cortical bone and its anisotropic damage mechanisms involved during the cutting process. Understand the cutting mechanisms and modelling techniques are crucial for furtherance development of comprehensive bone cutting models.

Some previous experiments [164, 165] were performed predominantly with regard to clinical outcomes and provided little information to quantify the cutting parameters. The obtained qualitative results indicated that the cortical bone tissue is sensitive to changes of cutting techniques and parameters. Measurement of forces generated during bone drilling reported in [166-168] suggested that they depended greatly on a drilling direction with respect to
the bone’s main axis due to high anisotropy of cortical bone tissue. Work carried by Lucas and her groups in various researches [169-171] have shown the potential benefit of using an ultrasonically assisted tool on cutting of food and biological tissues. Subsequently, a tool geometry optimization study [172] was conducted to investigate the influence of the blade tip profile on the temperature, cutting speed and static loading parameters in cutting of bovine cortical bone. A thermal necrosis reduction scheme was proposed latterly by the same research group [173] featuring series of experimental validations using the developed ultrasonic cutting blade. Further research done by Mitsuishi and co-authors [174-176] focused on the effect of microstructure on various cutting parameters using a developed bone cutting device; they proposed a new cutting method based on the characteristics of crack propagation. Still, information provided using experimental methods were very limited due to inadequate measurement techniques and the limitations of bone’s natural geometry.

### 3.6.1 Analytical models

The mechanism underpinning the cutting process has been discussed for centuries. Despite a large amount of research efforts in this area, there are still many unknown features of the cutting process due to low fidelity of the machining models [177, 178]. Throughout history, cutting models and theories were predominately developed based on metal cutting processes. There are only a few models available that are dedicated to modelling of cutting of cortical bone. In a recent book of Atkins [163], he revisited some of the classical metal cutting theories and provided some rethinking and amendment on current existing models in which he suggested the classical metal cutting theories lack the expression of the work done for new surface generation during cutting process. Fracture may not seem to be one of the most important phenomena when cutting a piece of ductile metal. However, when the workpiece material behaves like a brittle material such as cortical bone, the classical models do not fit very well with what has been observed during the experiments: crack propagates ahead of the cutting tip as cutting continues [176]. Atkins suggested that cutting could be regarded as a branch
of fracture mechanics where larger strain is often accompanied with fracture present at various scales [179, 180]. Even for cutting of ductile materials, the energy required to separate the material is not negligible [181]. In his modified model, the incremental work done for separating the material is attributed to four parts:

$$F_Cd\delta = Rwd\delta + d(f\text{riction}) + d\Lambda + d\Gamma,$$

(3–13)

from the right hand side of the equation 3-13, the first to the fourth terms of are: incremental work done for generating new surfaces, frictional work increments, incremental strain energy stored due to elastic deformation and incremental work due to plastic deformation, respectively; \(F_C\), \(\delta\), \(R\), \(w\), \(\Lambda\) and \(\Gamma\) represent cutting force, cutting distance of the cutting tool, specific work of surface separation, cutting width, elastic strain energy and work done for plastic deformation, respectively. The difference between Atkins’ suggestion and the classical metal cutting theories lies between whether or not the contribution of the fracture component in a ductile material cutting process is considered. From equation 3-13, the cutting force which considers the surface separation can be written in the format:

$$F_C = (wkt) \left[ \frac{\cos(\beta_t - \alpha_r)}{\sin \phi \cos(\phi + \beta_t - \alpha_r)} \right] \left[ 1 + \frac{R \cos(\alpha_r - \phi) \sin \phi}{kt \cos \alpha_r} \right],$$

(3–14)

comparing the one without considering of surface separation:

$$F_C = (wkt) \left[ \frac{\cos(\beta_t - \alpha_r)}{\sin \phi \cos(\phi + \beta_t - \alpha_r)} \right],$$

(3–15)

where \(k\) is the shear yield strength of the material and \(t\) is the offcut thickness with \(\alpha_r\), \(\beta_t\) and \(\phi\) denote the rake angle, friction angle and shear angle, respectively. This model provides new improved view on the original thought of the mechanics of cutting that is suitable for not only metal materials.

In a recent analytical development on modelling of bone cutting, Lee et al. [182] developed a mechanistic model for the prediction of thrust forces and
torques experienced during drilling of cortical bone. The forces were calculated analytically incorporating radially varying drill-bit geometry and cutting conditions. The forces were decomposed based on the drill geometry into an oblique cutting configuration. Three stage drilling experiment was also setup with a pre-drilled pilot hole to assist the model calibration and validation. Due to the uncertainty of the coefficient of friction and the inaccurate material properties at higher strain rate, the authors did not use the material properties available in literature. Instead, they took a mechanistic method to empirically calibrate their model’s parameters using specific energy formulation. The results of the model demonstrated good prediction at various low speed and low cutting depth with experiments. However, their results did not match well when the speed and cutting depth were large.

The limitations of analytical models are that they can only provide very limited information regarding the full-field understanding of the interaction mechanisms between tool and workpiece, and often, models have to be calibrated carefully through experimental test before implementation, which makes the whole model partially subjected to specific experimental configuration and less transferable to other applications without significant rework.

### 3.6.2 Finite-element models

From the modelling perspective, modern computational power allows numerical simulations to obtain comprehensive information about material’s behaviour in a much more convenient way that is usually difficult or impossible to achieve otherwise, e.g. through experiments. However, only a handful of models available in the literature demonstrate that less attention was paid to address issues related to the deep bone penetration than the focus of simulations on image visualization for medical training purposes [183]. An early research by Davidson and James [184] and later in [185, 186] focused on studying the influence of drilling parameters on temperature distribution within cortical bone tissue. The material models were based on simply isotropic elastic-plastic assumption, while material removal and heat generation were computed according to traditional metal cutting theory [183].
Though all models achieved good agreement with their experimental measurements, they provided neither analysis of cutting forces nor deformation contour of cortical bone tissue. There is no information of the contribution of each force components to the overall effect of temperature gradients. With drilling speed vary from 100 to 200,000 rpm in the first model, and 200 to 1200 rpm in the second one, the comparability between each of them is really low.

Sugita et al. [176] proposed a new cutting method which was aimed to minimize the risk of crack propagation in the remaining of uncut bone material based on their finite-element study. With no explicit explanation of the model configuration, the authors predicted an upwards crack propagation path when the feeding direction of the cutting tool was above 45º upwards (Figure 3–13). Their model also predicted a higher cutting force when the tool cut through osteons transversely compared with other directions.

Recently, in a study by Alam et al. [129], a new finite-element approach incorporating a Johnson-Cook material model was developed. The study, for the first time, analysed the effect of different cutting parameters on cutting forces. The implemented empirical based Johnson-Cook material model, featuring strain-rate and temperature-dependent material properties in conjunction with the remeshing capability, successfully captured the visco-plasticity material behaviour of cortical bone and the temperature gradient in the vicinity of the tool-bone interaction zone (Figure 3–14). Although no damage criterion and material anisotropy were utilized, the model showed consistency with the obtained experimental results.
Another FE model using the theory of critical distances as a fracture criterion was developed by Kasiri and co-authors [187], who evaluated indentation forces for various directions and tool configurations; the results showed considerable deviations for different cutting parameters. In spite of a simplified isotropic material model used, simulation results accomplished a good agreement with their experiment results. To date, the challenge of developing an adequate finite-element model to describe the complex mechanical behaviour of cortical bone under cutting tool still remains.

**3.7 Summary**

- Various kinds of cortical bone models were studied and summarised in this chapter.
- At macro-scale, cortical bone can be modelled as elastic, elastic-plastic, visco-elastic, visco-plastic and visco-elastic-plastic constitutive models in various combinations either isotropically or anisotropically.
- At micro-scale, cortical bone can be modelled as either homogenised material or microstructured composite material.
- Modelling of damage and fracture of cortical bone can be performed with implicit modelling approach using continuum damage mechanics.
or explicit modelling approach using various crack initiation and propagation techniques such as cohesive zone method or XFEM. However, both techniques face certain restrictions due to their intrinsic limitation of the algorithms.

- The complexity encountered with modelling of mechanical behaviour of cortical bone means there is only a handful models available that directly deal with modelling of bone cutting. Development of adequate finite-element models to describe the complex mechanical behaviour of cortical bone under cutting tool still remains a challenge.

- Amongst the existing cortical bone models, some are capable of representing the anisotropic nature of cortical bone tissue, whereas the damage and fracture phenomenon are omitted; others focus on the characterisation of the anisotropic fracture behaviour but lose the fidelity of the pre-damage material behaviour. From the structural point of view, some models address the microstructural heterogeneity of the material in detail; others focus on the accurate characterisation of the material constitutive equation based on the assumption that the material is homogeneous. There is an urgent need to develop a comprehensive finite-element model to adequately capture both the anisotropic deformation and its damage evolution processes of cortical bone tissue during the cutting process.
4.1 Introduction

The deformation mechanisms of bones differ from those of metals, polymers and composites since bones consist of a living tissue with a continuously evolving hierarchical microstructure. Mechanical properties of cortical bone vary not only from bone to bone; they demonstrate a spatial viability even within the same bone due to changes of the underlying microstructure [53, 188]. They also depend considerably on a loading mode and orientation. Mechanical properties of cortical bone, such as the Young’s modulus, yield strain and ultimate stress, in a longitudinal direction, i.e., parallel to osteons, were reported to be significantly higher than those in a transverse direction, i.e., perpendicular to osteons [23]. Dissimilar mechanical properties measured by nano-indentation for each tissue’s constituent provide information on its heterogeneity and anisotropy [33]. Furthermore, deformation of cortical bone was reported to have different tensile and compressive behaviours by Reilly et al. [24]. This phenomenon may be the direct result of distinct yield-stress values in tension and compression and was further investigated by Thompson et al. and Yeni and Fyhrie [189, 190]. Mercer et al. however, suggested that the material’s porous structure might affect directly the yield mechanism under tension and compression [37]. Boyce and his co-authors reported two very different deformation patterns in regions under different loading modes [41]. To complicate the matter even more, the results by Abdel-Wahab et al. and Bonney et al. pointed out a statistically significant difference of mechanical properties for four anatomical sites of the bone [53, 54]. Considering the wide spectrum of material properties of cortical bone and its intricate deformation processes associated with various loading modes and orientations, a further investigation is needed.
for comprehending variations of material properties in relation to the local regions and underpinning microstructural constituents.

To the author’s knowledge, currently, there is no study available in the literature that provides a full comparison of deformation behaviours of bovine cortical bone both for different loading conditions and orientations. Also, there is no systematic study on the variability of the mechanical properties of cortical bone. Hence, in this chapter, a Vickers hardness test (the deformation mechanism is similar to that of the initiation stage of a cutting process) was conducted initially to evaluate the anisotropic deformation behaviour of bone under concentrated compressive load at various sites. And then, the second part of this study provided a direct comparison of anisotropy and variability of mechanical properties of bovine cortical bone under both tensile and compressive loads, from which, the complete set of material properties can be extracted and used to further assist the development of the accurate finite-element models. The third part of the chapter analysed statistical variability of the mechanical properties across different cortices and transition of the microstructure by means of optical microscopy. Other types of variability – caused, for instance, by different age, health state, nutrition – are outside the remit of this study, and, therefore, not discussed here.

4.2 Specimen preparation

Specimens used for the entire period of study were all obtained from the mid-diaphysis of fresh bovine femoral bones aged around 1.5 to 2 years from a local butchery shop soon after slaughter. Soft tissue and bone marrow were carefully removed, and the femurs were thoroughly cleaned using cold water. Procedures for cutting, grinding and preservation processes followed generally adopted methods, widely accepted and performed by various researchers [53, 70, 191].

4.2.1 Vickers hardness tests

For the Vickers indentation, specimens were excised from one femur along its four anatomical positions using a low-speed band saw, and then grinded
using standard ANSI silicon carbide paper (Grinder-Polisher machine, Buehler) to the desired dimension (brick shape, approximately 5 mm × 5 mm × 60 mm). Finally, all surfaces were polished using a series of fine-grade silicon carbide papers to avoid any defects and improve surface finish. The cutting process was kept under constant irrigation (tap water). The specimens were carefully examined under a microscope and stored in 0.9% saline solution at -20˚C prior to the test. It was previously concluded that this storage procedure had no effect on the mechanical properties of the cortical bone tissue [192].

4.2.2 Uniaxial tension and compression tests

Two bovine femurs obtained from the same animal were excised into rectangular-shaped samples according to four anatomic quadrants (anterior, medial, posterior and lateral) (Figure 4–1) from the mid-diaphysis using a low-speed band saw and subsequently a diamond blade (Isomet Low-Speed Saw, Buehler). The specimens were cut for two orientations: longitudinal (parallel to the main axis of the femur) and transverse (perpendicular to it). They were then polished to improve their surface finish and eliminate surface damage induced during the cutting process. After that, the specimens were divided into two groups for different tests – tension and compression. For a tension test, the specimens were further machined into dumb-bell shape (15 mm in gauge length × 5 mm × 2 mm) using a diamond blade and a milling machine. For a compression test, a precise lathe and drilling machine were used to turn the rectangular specimens into cylindrical-shape specimens (Ø5 mm × 5 mm). All specimens for testing were finally polished with a fine polishing cloth with diamond-dust suspension. All cutting procedures were kept under continuous water irrigation to avoid specimen’s dehydration and temperature rise. A total of over 100 specimens were acquired for both tension and compression tests and were stored in 0.9% saline solution at room temperature prior to tests. All mechanical experiments were finished within 72 h after sample preparation.
Figure 4–1 Schematic illustration of specimen preparation process for: (a) Vickers indentation test, C1 and C2 denote two circumferential directions; (b) uniaxial tension and compression tests

4.3 Experimental procedures

4.3.1 Vickers Indentation

Indentation hardness test is a simple and yet fundamental experiment used to determine one of the basic material properties – hardness, an extension of material’s resistance to deformation/penetration due to a compressive load from an indenter [193]. The hardness (measured in Pascals) is normally calculated based on the applied load and the area of the permanent imprint.
in a material; the definition and value of hardness can be different, depending on the shape of the indenter. This convenient approach offers an easy-to-operate quantification of material property for the engineering and scientific community. Nowadays, featured with displacement sensors, indenters are state-of-art, multi-functional testing machines that not only measure the hardness value of materials, but are also capable of determining other material properties or behaviours such as elastic modulus, stress-strain curves, viscous behaviour, fracture properties at various length scales from macro-scale to micro-scale and nano-scale.

Experiments carried out in this study used simple indentation techniques to evaluate the variation of material property – hardness – in adjacent regions of a cortical bone specimen. Such information is very important to gain preliminary understanding towards the variability of the material properties within the same piece of cortical bone and, therefore, allow realistic modelling of surgical operations. The Vickers indenter was chosen due to the symmetry of its tip, which is preferable for distinguishing the deformation patterns for orthotropic materials. A constant deformation gradient along its four surfaces also provides a stable strain rate, which might otherwise affect the consistency of indentation.

Load controlled indentation tests were applied transversely (radial and circumferential directions) to the surface of bovine femur specimens using a standard Vickers hardness testing machine (Vickers-Armstrong, UK) to determine the regional variations of the hardness values. The Vickers tip has a symmetrical square-base pyramid diamond tip with an angle of 136° between each pair of opposite faces. The indentation tests were conducted using a standard indentation machine on each side (outside (out, in Figure 4–1), inside (in) and two circumferential directions (C1 & C2)) of separated specimens for four anatomical positions as shown in Figure 4–1. The indenter was positioned with each of its two diagonals parallel and perpendicular to the bone’s longitudinal axis, respectively, for the benefit of fracture assessment along different bone axes (Figure 4–1). Each face was subjected to a series of load of 1, 5, 10, 30, 50 and 100 kgf (kilograms-force).
The spacing between two adjacent indentation imprints was kept at least 5 mm to avoid their interaction. The Vickers hardness was calculated using the following equation:

\[
HV = \frac{F_k}{A} = \frac{2F_k \sin \left(\frac{136^\circ}{2}\right)}{d^2}, \quad (4–1)
\]

where \(F_k\) is the kilogram force, \(A\) is the surface area of the imprint and \(d\) is the average diagonal length of \(d_1\) and \(d_2\) (Figure 4–1). In post-processing, an optical 3-D microscope (Infinite focus, Alicona) was used to measure the imprint mark caused by the indenter.

### 4.3.2 Uniaxial tension and compression

Both uniaxial tensile and compressive tests were conducted using a universal testing machine (Instron 3366 bench-top dual column system, Instron, USA) with a 10 kN load cell. Each type of test was performed for four different quadrants in both longitudinal and transverse directions. The specimens were selected to provide an equal distribution across the femur’s cross-section for a more accurate representation of population. Seven tests were conducted for each cortical position. Each specimen was loaded under displacement control until failure at a strain rate of \(1 \times 10^{-3} \text{ s}^{-1}\). The level of displacement was measured using an extensometer with a gage length of 10 mm (2630 Series, Instron) and a linear variable differential transducer (LVDT) sensor with a travel length of ± 2.5 mm (2601 Series, Instron), respectively in tensile and compressive tests. Experiments were carried out at ambient room temperature (20°C), and the specimens were kept hydrated using saline spray. A thin layer of lubricant was applied to the contact surfaces between end-plates and specimens prior to compression tests to reduce the frictional end effect. The Young’s modulus was determined using the tangential modulus of the initial slope of the stress-strain curve. The yield point was determined by means of 0.2% strain offset.
4.3.3 Statistical analysis

For a more robust comparison of various properties within this study and subsequently in other chapters, several types of statistical analysis were carried out.

The ANOVA (analysis of variance) analyses were utilized to determine the statistical variation between the means of multiple (usually more than two) testing groups. The method provides a statistical conclusion on whether or not the means of several testing groups are equal (statistically insignificant). Depending on the number of variables within the designed experiment, ANOVA analysis can have one-way (one variable), two-way (two variables) or multi-factor (multiple variables) analysis. For Vickers hardness tests, there was only one variable within the experiments, namely individual cortices. Therefore, one-way ANOVA analysis was conducted to evaluate the statistical significance between different cortices. For the uniaxial tension and compression tests, since the tests were designed with regard to two variables: individual cortices (Factor A) as well as loading modes (Factor B), two-way ANOVA analyses were performed to validate the significant differences between tested groups as well as loading modes. A significance level ($\alpha$) of 0.05, viz., 5%, was used by convention [194] for all the analyses in this study. The significant difference is reported by rejecting the null hypothesis (for one-way ANOVA, the null hypothesis is the means form each group are equal; for two-way ANOVA, the null hypotheses are the means of testing samples grouped by each factor are the same, and the two factors are independent variables with no interaction) when the $p$-value (the probability value of obtaining a test statistics at extreme cases where null hypothesis is true) is smaller than the $\alpha$ value. Following the ANOVA analysis, the post-hoc test was conducted after each successful rejection of the null hypnosis. The Tukey HSD (honestly significant difference) multiple comparison test was chosen as the testing method for the balance between over conservative and liberal estimation which might lead to Type I error [195]. All data were imported to and calculated using statistics software package – SPSS (IBM SPSS Statistics, IBM Corporation, USA).
The Weibull survival analysis [196] was employed to analyse the probability distributions of the events when failure is likely to happen. The Weibull distribution function is given as

\[ F(x, b, m) = 1 - \exp\left(\frac{x}{b}\right)^m, \]  
\( (4–2) \)

where \( b \) and \( m \) are the scale and shape parameters of the distribution function. \( m \) is also called the Weibull modulus; it determines the variability of a given distribution. \( x = b \) is the characteristic corresponding to failure of 63.21\% of the specimens, according to Weibull [196].

### 4.3.4 Microstructure analysis

To quantify the variation of microstructure across the cross-section of bovine femur, two cross-sectional rings from the top and bottom of the mid-diaphysis of one femur were excised (Figure 4–1), polished and analyzed with optical microscopy (Olympus BX60M, Japan). 16 tiling images were taken for each ring section and then analyzed using Image-Pro software (Image-Pro 7.0, Media Cybernetics, USA). The images were evenly distributed across the ring (four images for one cortex) and each image consisted of a series of tiling images across the thickness (radius direction) of cortical bone. The microstructure analysis was carried out by distinguishing the following constituents: osteons (including both primary and secondary osteons), plexiform, interstitial and porosity areas. The following procedure was implemented to calculate the area fractions of each part:

- The image was trimmed to remove dark edges and then porosity (including Haversian canal, resorption cavity and lacunae) was calculated by counting the remaining dark spots of the image.
- The plexiform area was calculated first by drawing the outline of the corresponding region and then subtracting the porosity area and areas of other structures from the region.
- Determination between primary and secondary osteons can be somewhat difficult and cumbersome, therefore, in this study, all
osteonal structure were counted. The osteonal area was calculated by capturing the outline of individual osteons (excluding the areas occupied by Haversian canal, resorption cavity and lacunae) and summing up inputs from individual osteons. The procedure to distinguish the osteonal bone was based on the criteria employed by Saha and Hayes [72] and the identification principles are as follow:

a. Osteons, especially secondary osteons are normally bounded by the cement lines abruptly within surrounding area; for primary osteons, there is no such demarcation line, the surrounding structures follow the curvature much like the streamlines around a smooth body.

b. The most noticeable feature for osteons at the transversal section plane is the central Haversian system, which exists in most osteonal structures, unless it is encroached by a newer osteon.

c. The concentric layers of lamellar structure and lacunae which can often been observed in osteons are additional features which could help to identify the position of an osteon structure.

d. Fragments from previously old generation of osteonal structure which had been replaced by resorption cavity or newer generation osteons are also accounted as part of the area fraction of osteons, while those that have large, irregular and convoluted microstructure are not.

- The interstitial area was calculated as the area of the remaining part.

The method used to capture the irregular shapes of microstructure employed a combination of image processing techniques and image auto-recognition features to increase the contrast of the border line between constituents. However, manual adjustment was still required to compensate for deficiencies of the computer algorithm. The area fractions were calculated using a pixel-based planimetry measuring technique. The boundaries between primary osteons and interstitial area as well as plexiform to
interstitial interface are not easy to distinguish. Occasionally, the border line between them was not clear. As a result, the reproducibility of the measurement using this method was tested to be within ± 4%.

Based on a modified Voigt-Reuss-Hill (VRH) averaging scheme [197, 198] which is preferable for composites materials having lower contrast of moduli between different phases [199, 200], the effective Young’s modulus was calculated according to the following equation:

$$E_{\text{total}} = (E_O V_O + E_I V_I + E_P V_P)(1 - V_{PO})^3.$$  \hfill (4–3)

The subscripts $O$, $I$ and $P$ denote osteonal, interstitial and plexiform areas, respectively; $E$ and $V$ represent the Young’s modulus and volumetric (area in 2D) fractions of respective parts; $V_{PO}$ is the fraction of porosity. The Young’s moduli used in the calculation are listed in Table 4–1.

<table>
<thead>
<tr>
<th>Table 4–1 Magnitudes of Young’s modulus (GPa) used in the calculations based on Equation 4–3 (a- [23]; b- [70]; c- [31]; d- [137])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Osteonal</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>Longitudinal</td>
</tr>
<tr>
<td>22.7 $^a$</td>
</tr>
<tr>
<td>Transverse</td>
</tr>
<tr>
<td>12.85 $^c$</td>
</tr>
</tbody>
</table>

4.4 Results and analysis

4.4.1 Vickers hardness values

The results from the Vickers hardness test showed that the bovine cortical bone exhibited significant plastic deformation under sharp indentation. Inelastic deformation was observed even at the smallest level of load confirming the presence of plastic deformation at the early stage of
deformation. The Vickers hardness value was higher at lower levels of load due to dominance of elastic deformation and reduced as load increased as a result of the increased proportions of the plastic deformation and damage. With regard to the four anatomical positions, the experimental results indicated the existence of dissimilarity (Figure 4–2): the highest mean value of Vickers hardness of 0.624 GPa was found at the anterior position, while the lateral position had the lowest (0.526 GPa). The values of Vickers hardness at medial and posterior positions were 0.587 GPa and 0.562 GPa, respectively. The results coincide with previous literature data reported in [201]. A one-way ANOVA statistical analysis showed a significant difference of Vickers hardness value among four anatomic cortices. The post-hoc analysis revealed significant difference existing between anterior to medial ($p = 0.023$), medial to posterior ($p = 0.029$) and posterior to lateral ($p = 0.04$) data. In terms of local variations in each specimen, the inside bone material had highest hardness value, while the outside had the lowest one. Significant differences were observed between internal surface and two circumferential surfaces as well as two circumferential surfaces and external surface for three out of the four anatomical positions (Figure 4–2): posterior ($p = 0.04$, $p = 0.015$), medial ($p = 0.014$, $p = 0.018$), lateral ($p = 0.041$, $p = 0.016$), respectively. However, no significant difference was found for the anterior position. Post-test crack assessments of cortical bone specimens demonstrated strong anisotropic crack propagation character. However, no significant difference was observed among the crack lengths of the four anatomic cortices. Images taken from optical microscopy confirmed that majority cracks were formed along the longitudinal direction of cortical bone (see Figure 4–3), while only minor cracks were found along the edge of the imprint and no observation of transverse cracks were evident.
4.4.2 Stress-strain relations in tension and compression

Following the indentation tests, uniaxial tension and compression tests were conducted to further investigate the variability and anisotropy of material properties of cortical bone tissue across different cortices. The results
indicated that the mechanical behaviour of cortical bone under different loading conditions diverged dramatically (Table 4–2 and Figures 4–4 and 4–5), both in terms of stress-strain relation and damage mechanism.

**Table 4–2** Average and standard deviation of mechanical properties of bovine cortical bone for four cortices (including anterior, medial, posterior and lateral) at different loading conditions

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Loading type</th>
<th>$E$ (GPa)</th>
<th>$\sigma_y$ (MPa)</th>
<th>$\varepsilon_y$ (%)</th>
<th>$\sigma_u$ (MPa)</th>
<th>$\varepsilon_u$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal</td>
<td>Compression</td>
<td>19.09</td>
<td>184.62</td>
<td>1.20</td>
<td>214.39</td>
<td>2.37</td>
</tr>
<tr>
<td></td>
<td>±2.84</td>
<td>±22.51</td>
<td>±0.09</td>
<td>±27.57</td>
<td>±0.38</td>
<td></td>
</tr>
<tr>
<td>Tension</td>
<td></td>
<td>20.22</td>
<td>75.85</td>
<td>0.61</td>
<td>97.41</td>
<td>1.85</td>
</tr>
<tr>
<td></td>
<td>±3.12</td>
<td>±13.98</td>
<td>±0.11</td>
<td>±19.88</td>
<td>±0.39</td>
<td></td>
</tr>
<tr>
<td>Transverse</td>
<td>Compression</td>
<td>11.62</td>
<td>112.78</td>
<td>1.06</td>
<td>131.16</td>
<td>2.40</td>
</tr>
<tr>
<td></td>
<td>±2.40</td>
<td>±19.61</td>
<td>±0.21</td>
<td>±22.02</td>
<td>±0.73</td>
<td></td>
</tr>
<tr>
<td>Tension</td>
<td></td>
<td>12.43</td>
<td>32.92</td>
<td>0.27</td>
<td>40.18</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>±2.37</td>
<td>±7.85</td>
<td>±0.09</td>
<td>±9.00</td>
<td>±0.18</td>
<td></td>
</tr>
</tbody>
</table>

At the initial elastic stage, similar linear stress-strain relationships were observed for both loading modes. The averaged elastic modulus $E$ in tension and compression were found to be 20.22 GPa ± 3.12 GPa and 19.09 GPa ± 2.84 GPa for a longitudinal direction, 12.43 GPa ± 2.37 GPa and 11.62 GPa ± 2.4 GPa for a transverse one, respectively. The results are compatible with those reported in literature [23]. Although, there was a difference of some 6% between the tension and compression moduli, statistical analysis discussed in the following section (Section 4.4.4) revealed a detailed statistical significance across all testing groups, and the results were not consistent for longitudinal and transverse directions. A small toe region (a non-linear initial portion of the stress-strain curve) that appeared at the beginning of the deformation process was also observed in other studies that employed
different testing configurations: bending tests performed by Currey [68]; tension tests by Kotha and Guzelsu [202] and compression tests by Hamed et al. [14]. The results suggest that this initial deformation process might be evoked by the native deformation mechanisms across multiple levels, such as stretching and sliding at interfaces due to the weak bond and opening/closure of voids and porous space.

![Stress-strain curves](image)

**Figure 4–4** Typical stress-strain curves for longitudinal specimen in tension and compression (anterior quadrant); Inserts show strengthening portions beyond the elastic region, the divergence starts. Tensile specimens sustain much lower deformation and stress levels than compressive specimens. Yield happens at a relative early stage. A bi-linear stress-strain relationship was observed in both longitudinal and transverse orientations [53]. However, in compression, yield commences at a much higher stress. The average levels of yield stress $\sigma_y$ and strain $\varepsilon_y$ were 184.62 MPa ± 22.51 MPa and 1.20% ± 0.09% for longitudinal specimens, 112.78 MPa ± 19.61 MPa and 1.06% ± 0.21% for transverse ones, respectively. That was more than two times higher than for tension both in terms of yield stress and yield strain (see Table 4–2). The representative post-yield stress-strain curves from anterior quadrant for both tension and compression are shown in Figures 4–4 and 4–
5. In tension, the flow stress-strain curves between individual specimens are much closer to each other (shaded area) and comparable (similar tangential modulus). Failure in tension occurs as soon as damage starts, whereas in compression, damage propagation is a relatively slow process and is inhibited by the compressive state, therefore, combining both damage softening and strain hardening effects. As a result, the flow curves in compression are more spread out and their tangential moduli vary significantly.

![Figure 4-5 Typical stress-strain curves for transverse specimen in tension and compression (anterior quadrant); Inserts show strengthening portions](image)

After reaching the maximum stress \( \sigma_u \), the damage mechanism becomes dominant and material starts disintegrating and losing its load-bearing capacity. However, the failure occurred in different ways for different loading modes. In tension, cortical bone fails in a brittle manner just after the stress-strain curves pass the ultimate point [203]. On the contrary, due to a shear damage mechanism observed in compression (Figure 4-6), the character of stress-strain curve is determined by combined effects of local stress hardening and shear sliding. Higher local stress hardening and lower shear sliding lead to steeper stress-strain curves. Generally, there are three types
of stress-strain relations as shown in Figure 4–6: (a) a sudden reduction of stress at lower strain when normally a major fracture throughout the specimen occurs (Figure 4–6a); (b) a plateau usually accompanied by multiple fractures (Figure 4–6b); (c) a gradual reduction corresponds to an intermediate case between the previous two, with several cracks propagating through the specimen (Figure 4–6c). As a result, failure in compression is less predictable and could happen well before reaching the average ultimate stress. With data recording up to 6% of strain, the maximum stress reduction observed during the experiments was about 50% and 60% in longitudinal and transverse direction, respectively.

**Figure 4–6** Schematic illustration of different damage behaviors (a, b and c, see descriptions in text) for compressive loading: first row represents character of stress-strain curves, second and third lines represent typical damage patterns with regard to osteons (shown in blue tubular)

The possible failure envelopes for cortical bone specimens for different loading conditions are demonstrated in Figure 4–7, determined by positions of the ultimate stress. In general, compressive specimens have larger failure envelopes compared with tensile specimens, especially for the transverse orientation, where resistance to shear sliding is significantly lower due to the microstructural alignment of its constituents.
Figure 4–7 Representative failure regions determined by ultimate stress levels (anterior quadrant)

Figure 4–8 Weibull probability distribution plots for: (a) ultimate stress and (b) ultimate strain (number of specimens: 28; Comp: compression; Ten: tension; L: longitudinal; T: transverse)
The Weibull distribution revealed the probability distribution of failure in terms of ultimate stress (Figure 4–8a) and ultimate strain (Figure 4–8b). As obvious from Figure 4–8a, distributions of failure strengths have no overlaps, while the distributions of failure strains for specimens in compression have overlapping ranges. Although, the mean ultimate strain for compressive transverse specimens is similar to that for the longitudinal direction; a lower Weibull modulus (3.85 compared with 7.17) indicates lower reliability in the transverse direction, which means that the specimens are more likely to fail at lower strain (Figure 4–8b).

4.4.3 Orientation anisotropy

Figure 4–9 Representative stress-strain curves for different loading conditions for longitudinal and transverse directions in anterior cortex

The diverged mechanical behaviours were also observed with respect to orientation (Figure 4–9). Transverse specimens loaded in tension appear to be rather brittle and fail at much lower strains compared with those for the longitudinal direction, but the difference for compression is less prominent. Regardless of the loading mode, specimens loaded in longitudinal direction always demonstrate a higher stiffness (higher Young's modulus) and strength (higher ultimate stress) than those in the transverse direction. An anisotropy
ratio is introduced here as the parametrical ratio of material properties obtained for the longitudinal and transverse orientations. The summarised data for these ratios are shown in Table 4–3. The ratio for the Young’s modulus ranges from 1.47 to 1.79; this closely agrees with the previous research [53]. Generally, the stress ratios (yield and ultimate) in tension (over 2) are higher than in compression (1 to 2). Still, the anisotropy ratios for strain are quite different for different loading modes. In tension, the ratios for yield and ultimate strains are in the range from 2 to 4, but in compression, they are almost orientational independent (i.e., nearly isotropic).

**Table 4–3** Anisotropy ratio for both tension and compression for various anatomic quadrants

<table>
<thead>
<tr>
<th>Anisotropy ratio</th>
<th>Young’s modulus</th>
<th>Yield stress</th>
<th>Yield strain</th>
<th>Ultimate stress</th>
<th>Ultimate strain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ten</td>
<td>Comp</td>
<td>Ten</td>
<td>Comp</td>
<td>Ten</td>
</tr>
<tr>
<td>Anterior</td>
<td>1.73</td>
<td>1.70</td>
<td>2.69</td>
<td>1.69</td>
<td>2.33</td>
</tr>
<tr>
<td>Medial</td>
<td>1.52</td>
<td>1.47</td>
<td>2.10</td>
<td>1.51</td>
<td>2.48</td>
</tr>
<tr>
<td>Posterior</td>
<td>1.76</td>
<td>1.79</td>
<td>2.58</td>
<td>1.83</td>
<td>2.52</td>
</tr>
<tr>
<td>Lateral</td>
<td>1.52</td>
<td>1.65</td>
<td>2.11</td>
<td>1.52</td>
<td>1.82</td>
</tr>
</tbody>
</table>

4.4.4 Variability of cortical bone and statistical analysis

Material properties from both tension and compression experiments for four anatomic quadrants (cortices) are presented in Figure 4–10. Apparently, the anterior quadrant had the highest Young’s modulus in longitudinal direction, while the medial quadrant had the highest one in transverse direction. The lowest values were for lateral and posterior quadrants for longitudinal and transverse directions, respectively. The difference between the highest and lowest values of the Young’s modulus was around 5 GPa in longitudinal and 4 GPa in transverse orientations, i.e., more than 20%. The relations across different quadrants were compared in terms of significance of variances using the two-way ANOVA analysis. Though the results showed a statistical significance for factor A (between cortices) (Table 4–4), the significance level
for factor B (between loading modes) did not result in a uniform conclusion. The interaction between the two factors appeared to be negative, which means that loading modes do not have effective contribution to the variability across cortices and vice versa. Results of detailed Tukey HSD tests together with pairwise comparisons between factors are summarised in Table 4–4. According to the statistical analysis, no significant variances were found between anterior to medial and posterior to lateral quadrants in all analyses, which suggests a strong linkage between each of the two pairs. On the other hand, the differences between the opposite quadrants were consistently significant ($p < 0.05$ for all the comparisons); there, dissimilar microstructures were also observed with microstructure analysis that is discussed below. Less-consistent values were found between anterior to lateral and medial to posterior quadrants, where the transition of the microstructure happened to be the most severe. A schematic illustration of the significant difference between the four cortices is presented in Figure 4–11.

![Figure 4–10 Variability of Young's modulus across cortices for longitudinal and transverse directions](image_url)
Table 4–4 Result of two-way ANOVA analysis at significant level of 0.05 using Tukey HSD multiple comparison with pairwise comparison between factors

<table>
<thead>
<tr>
<th></th>
<th>Anterior</th>
<th>Medial</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Medial</td>
<td>Posterior</td>
<td>Lateral</td>
</tr>
<tr>
<td><strong>Factor A (Between cortices)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal</td>
<td>0.487</td>
<td><strong>2E-05</strong></td>
<td>0.002</td>
</tr>
<tr>
<td>Transverse</td>
<td>0.506</td>
<td><strong>0.001</strong></td>
<td>8E-06</td>
</tr>
<tr>
<td><strong>Factor A within Factor B (Between cortices)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comp</td>
<td>0.743</td>
<td><strong>0.029</strong></td>
<td>0.228</td>
</tr>
<tr>
<td>Ten</td>
<td>0.522</td>
<td><strong>4E-4</strong></td>
<td><strong>0.018</strong></td>
</tr>
<tr>
<td>Transverse</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comp</td>
<td>0.672</td>
<td><strong>0.043</strong></td>
<td><strong>0.003</strong></td>
</tr>
<tr>
<td>Ten</td>
<td>0.562</td>
<td><strong>0.036</strong></td>
<td><strong>0.001</strong></td>
</tr>
</tbody>
</table>

Although a significant difference was found for the longitudinal direction within factor B ($p = 0.048$), to state that there is a significant difference between the loading modes may not be sensible. Firstly, the $p$ value was just under the critical value of 0.05. Secondly, an opposite result was observed for the transverse direction. Last but not least, pairwise comparisons between each quadrant also provided the contrasting results (Table 4–4).
The results of ANOVA analysis for yield stress, yield strain and ultimate strain showed no significant difference between cortices. Still, for ultimate stress, the significant difference only existed for the opposite cortices \((p = 0.003)\). This result implies that the post-yield behavior does not have a stable correlation with the variation of the microstructure. This could be due to the complexity and additional statistical variables induced by randomised microstructure and various damage mechanisms \([204, 205]\) of cortical bone, not accounted for in this statistical analysis.

### 4.4.5 Microstructure analysis

The results from the optical microscopic analysis confirmed regional differences between different quadrants. Generally, the anterior quadrant was dominated by plexiform bone (over 50%) and posterior quadrant had more osteonal bone (over 50%). Medial and lateral quadrants were the transition sections between the two. This transition could also be evidenced by a relatively large standard deviation for porosity for these two quadrants, \(\pm 2.21\%\) and \(\pm 3.94\%\), respectively (Table 4–5). Within each quadrant, plexiform bone was usually located at the outer surface and there was a
transition into a mixture of osteonal and interstitial bone with a gradual increase of osteonal structure throughout the thickness (Figure 4–12). Therefore, it is not difficult to presume that higher hardness values at outer surface of cortical bone observed in the Vickers indentation tests were partially attributed to the stiffer plexiform bone structure.

Table 4–5 Microstructure analysis of average and standard deviation of volumetric area fractions for constituents for four cortices (Note: the maximum and minimum values instead of standard deviations were used for the volumetric fraction of plexiform for posterior and lateral cortices due to large fluctuation of the data)

<table>
<thead>
<tr>
<th>Volumetric fraction (%)</th>
<th>Osteonal</th>
<th>Interstitial</th>
<th>Plexiform</th>
<th>Porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anterior</td>
<td>13.13 (±10.16)</td>
<td>17.66 (±16.01)</td>
<td>65.69 (±24.79)</td>
<td>3.53 (±0.73)</td>
</tr>
<tr>
<td>Medial</td>
<td>16.4 (±14.49)</td>
<td>25.51 (±19.56)</td>
<td>53.4 (±34.52)</td>
<td>4.69 (±2.21)</td>
</tr>
<tr>
<td>Posterior</td>
<td>54.01 (±17.58)</td>
<td>30.4 (±16.96)</td>
<td>7.58 (25.6)</td>
<td>8.02 (±1.89)</td>
</tr>
<tr>
<td>Lateral</td>
<td>44.43 (±16.41)</td>
<td>35.81 (±9.51)</td>
<td>10.26 (35.1)</td>
<td>9.51 (±3.94)</td>
</tr>
</tbody>
</table>

As expected, the standard deviations of each constituent based on the results of optical microstructure analysis (Table 4–5) are large as the microstructure changes considerably within and between cortices. A colour-coded microstructure image across the thickness of the medial quadrant is demonstrated in Figure 4–12. Substantial variations were also observed for upper to lower mid-diaphysis. The average area fraction of osteonal bone was between 13% and 54%; interstitial bone ranged from 17% to 35%; plexiform bone from 10% to 56% and porosity around 3% to 9.51%. Comparing the Young’s moduli and the area fractions of microstructural constituents for four cortices (see Figure 4–13), a possible linkage emerges between the transition of microstructural constituents and the variation of the Young’s moduli.

The values of the effective Young’s modulus calculated using Equation (4–3) are compared with experimental results in Table 4–6. Apparently, both for the
longitudinal and transverse Young’s moduli for four anatomic quadrants, the theoretical predictions closely agree with the experimental results. A larger error for the transverse direction could be due to the fact that the volumetric fractions were measured at the surface parallel to the longitudinal loading plane, and may not be the same for the transverse loading plane.

Figure 4–12 (a) Microstructure transition across thickness at medial quadrant; (b) colour-coded image for image analysis

Figure 4–13 Correlation between Young’s modulus and volumetric fraction for four cortices
Table 4–6 Comparison between theoretical prediction of effective elastic moduli and experimental data, experimental data are based on average for compression and tension specimens

<table>
<thead>
<tr>
<th>Young’s modulus (GPa)</th>
<th>Theoretical prediction</th>
<th>Experimental data</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Anterior</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal</td>
<td>22.29</td>
<td>22.40</td>
<td>0.52</td>
</tr>
<tr>
<td>Transverse</td>
<td>13.00</td>
<td>13.06</td>
<td>0.52</td>
</tr>
<tr>
<td><strong>Medial</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal</td>
<td>21.02</td>
<td>21.17</td>
<td>0.71</td>
</tr>
<tr>
<td>Transverse</td>
<td>12.19</td>
<td>14.11</td>
<td>13.6</td>
</tr>
<tr>
<td><strong>Posterior</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal</td>
<td>17.04</td>
<td>17.93</td>
<td>4.94</td>
</tr>
<tr>
<td>Transverse</td>
<td>9.67</td>
<td>10.08</td>
<td>4.13</td>
</tr>
<tr>
<td><strong>Lateral</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal</td>
<td>16.15</td>
<td>17.14</td>
<td>5.75</td>
</tr>
<tr>
<td>Transverse</td>
<td>9.17</td>
<td>10.85</td>
<td>15.5</td>
</tr>
</tbody>
</table>

4.5 Discussion

The obtained experimental results for mechanical properties of cortical bone tissue in this study are well correlated with the literature data, see e.g. [23]. Still, they differ from those in [37], who reported a failure strain of 2.4% in tension and 1.6% in compression measured with a strain gauge in a four-point bending test. This difference could possibly ascribe to the effect of various mechanical testing methods or testing conditions [28]. Other factors such as dissimilarity of species and differences in their age and gender can contribute to variability of the mechanical properties; still, quantification of these differences is beyond the scope of this study and, therefore, not discussed here. The variability of mechanical properties across different cortices is consistent with previous research [53]. In summary, the mechanical properties of cortical bone depend on both loading modes and orientations. When bone is loaded in its natural mode – along the longitudinal direction in compression, it offers the best load-bearing capacity both in terms of strength and toughness. Outside this natural loading mode, mechanical performance of cortical bone reduces dramatically: only half of its highest stiffness and one fifth of its highest strength were demonstrated for
tension in transverse direction. The slight but not significant variance observed between the elastic moduli of tension and compression could be explained as the result of closure of porous spaces in compression, which leads to a softening effect on overall stiffness. The effect of compliance of the testing rig is also more noticeable in compression than in tension as the maximum strength in compression was double of that in tension. Still, high stiffness of the used load cell makes this effect rather small.

The distinct post-yield regions in tensile and compressive stress-strain curves indicated complex damage and strain hardening effects in compression. With micro-crack initiated at low stress levels [206], damage could start well before the material yields. At the same time, stiffness reduction was suppressed in compressive conditions due to the crack closure [187]. The overall result is a mixture of a progressive damage process and local strain hardening in compression. Different damage mechanisms associated with tension and compression also affected the load-bearing capacity. In tension, a diffused micro-crack character was reported in the literature, while linear, cross-hatched micro-cracks were found in the region under compression [41]. With more energy dissipated in cross-hatch damage than in a case of diffuse damage [42], more energy is needed in compression to generate new crack surfaces than in tension. That, in turn, results in higher stress and strain values in compression and a larger area underneath the stress-strain curve.

As a result of a random, heterogeneous nature of cortical bone’s microstructure, the progressive damage behaviour is less predictable in compression. The magnitude of stress reduction is inversely correlated to the amount of on-site damage and interaction of friction as well as strain hardening. The Weibull analysis reveals the distinctive failure regions, with the longitudinal compressive specimens having the highest reliability both in terms of ultimate stress and ultimate strain.

The findings for anisotropy linked to orientations of bone constituents coincides with those reported in [53] and can be explained by anisotropic alignment of microstructural elements within the hierarchic layout of the material. From micro to nano scale, individual constituents such as collagen
fibres, mineral platelets, osteons and lamellae structures are aligned predominantly parallel to the longitudinal axis [28] resulting in the highest strength in this direction.

Variability of the elastic modulus across different quadrants of bone is largely related to its changing microstructure, and this result was also corroborated by previous research [53]. Significant differences were found for various quadrants but not for different loading modes. The results of statistical analysis revealed a possible histological linkage between the anterior and medial quadrants and the posterior and lateral quadrants. Skedros and co-authors [50, 207] tested the calcaneus of a mule deer using *in vitro* loading method, and their results concluded that anterior and medial quadrants were predominately exposed to longitudinal compressive loading, while posterior and lateral quadrants were subjected to longitudinal tensile loading. The position of the neutral loading axis reported in the same study was similar to that shown in Figure 4–11. Although the analysis for *in vitro* loading conditions is a more complicated process, which includes the interaction between tendon and muscles as well as a centre of gravity, and is beyond the scope of this study, results from current analysis, as well as in [50, 207], demonstrate a possible linkage between the bone’s mechanical properties and mechanical-induced bone adaptation. However, it is too early to draw the conclusion from these results. Riggs *et al.* [208] analysed the collagen fibre orientation of the equine radius and revealed the contrasting relation between four quadrants. Potentially, this implies that the variation of material properties or microstructure may differ between species or bone types.

Microstructural analysis produced details of changes in the microstructure between and within different quadrants. The anterior quadrant was dominated by plexiform bone and had the lowest porosity area fraction, while the posterior quadrant was extensively remodelled, with the largest volume fraction of secondary osteons. The medial and lateral quadrants are transitions from the anterior to posterior quadrant. For the volume fractions of each individual constituent, porosity [73, 74] and secondary osteons [72] were reported to be inversely correlated with the Young’s modulus, which
corroborates current findings. The result of theoretical predictions of the effective elastic modulus based on the VRH-scheme proved to be accurate. The overall elastic behaviour of cortical bone depends strongly on the microstructure, i.e., distribution and orientation of constituents [7]; hence, over simplification of it can cause inaccurate results [14].

4.6 Conclusions

The variability and anisotropy of mechanical parameters of cortices of cortical bone were observed, quantified and compared in this study both for different loading modes and orientations. The deformation and damage behaviours differ, and they depend on the type of loading applied. A compressive longitudinal mode results in the best load-bearing capacity, whereas tensile transverse loading provides very poor results. Stress reduction after the damage onset is less predictable in compression due to the involvement of various damage mechanisms. The random arrangement of the microstructure contributes to a wide range of mechanical properties observed in this study as well as in the literature. The use of statistical analysis tools demonstrated the correlations between the variation of elastic modulus and histological quadrants and a possible linkage between mechanical properties and mechanically induced bone adaptation. Beyond the linear-elastic working conditions, mechanical properties of cortical bone demonstrates fewer correlations because of the involvement of damage mechanisms, different spatio-temporal realizations of which affect the overall stress-strain relationship. The microstructural analysis additionally confirmed this transition at microscopic level between anatomic quadrants. The theoretical calculations of the effective Young’s modulus accurately reproduced the experimental results, which provide another evidence of the strong relationship between microstructure and elastic modulus.
Chapter 5
Analysis of Anisotropic Fracture in Cortical Bone

5.1 Introduction
Understanding of underpinning mechanisms of, and processes associated with, damage and fracture is key to adequate modelling of the deformation and fracture processes of cortical bone under cutting conditions. In order to do so, the entire mechanical behaviour, which includes both pre-damage and onset-of-damage behaviours, have to be characterised and quantified experimentally. This is especially important for subsequent implementation of numerical models, incorporating material properties that depend fully on the accuracy and understanding of the experimental results. Therefore, in this chapter, the anisotropic fracture behaviour of cortical bone is studied both experimentally and numerically.

5.2 Fracture toughness tests
Fracture toughness is one of the most important material properties for many engineering applications. It defines the material's behaviour due to internal defects and its ability to resist fracture. Experimental characterisation of fracture toughness involves quantification of material's resistance to the presence of a flaw in terms of the maximum load required to cause a brittle or ductile crack extension with a standard specimen geometry containing a fatigue pre-crack. The final result is then calculated in terms of fracture toughness parameters such as $K_{IC}$, critical values of $J$ or CTOD according to the material’s behaviour or the experimental techniques chosen. Experiments can be implemented in various configurations including three-point bending, four-point bending or compact tension [152]. Depending on the experimental
setup, specimens can also be prepared in different ways: single-notch bending specimen or compact tension specimen. In this study, a three-point bending configuration was chosen due to simplicity and accuracy of the experimental setup as well as concerns for specimen geometry: a smaller specimen size is required for three-point bending tests than in four-point bending or compact tension tests.

5.2.1 Specimen preparation

Specimen preparation procedures and storage methods followed the same principles as those for Vickers hardness tests and uniaxial tension and compression tests described in Sections 4.2.1 and 4.2.2. Experimental results discussed earlier have all indicated variability of mechanical properties of cortical bone. Therefore, in this study specimens extracted from three fresh bovine femurs were also separated into four groups according to their anatomic positions – anterior, posterior, medial and lateral (Figure 5–1). Fifteen specimens cut from each cortex were notched to allow crack growth along three different orientations relative to the bone axis – longitudinal, transverse and radial as shown in Figure 5–1. After cutting, specimens were polished and then checked under microscope to insure that surfaces are free from scratches and damage. Specimens were kept hydrated in a 0.9% physiological saline solution prior to tests. All specimens were prepared with the same dimensions for comparison according to British Standard: BS 7448-1 [152]: 25 mm x 2.72 mm x 5.43 mm (total length x width x thickness). Also, a very fine slit of 2.7 mm was produced using a low-speed diamond blade for all specimens according to [152]. Distinguished from previous chapters, specimens in this chapter are labelled based on the crack propagation directions rather than their geometrical orientations: longitudinal, transverse and radial. Hence, specimens with a crack propagating parallel to the bone’s main axis are called longitudinal, perpendicular to it transverse and in the radial direction radial, see Figure 5–1. Due to dimensional constraints of the cortical bone tissue, and in order to provide comparability, specimens with the same length ($L = 25$ mm) were used for all cortices and crack directions. Hence, the span ($S$), width ($W$), thickness ($B$) of specimens and crack length
(a) were chosen based on the full length of 25 mm and proportions for dimensions defined in [152]. The used dimension proportions are $L = 4.6W$, $S = 4W$, $a/W = 0.5$, and $B = W/2$.

![Diagram of bone and specimen positions](image)

**Figure 5–1** (a) Schematic illustration of bovine femur; (b) cortex positions in cortical bone; (c) specimens with different crack propagation directions: longitudinal, transverse and radial. Arrows show crack propagation directions.

### 5.2.2 Experimental procedures

**Fracture toughness measurements**

The fracture toughness tests were performed according to British Standard – BS 7448-1 [152] on an Instron 3345 single column bench-top machine (Instron, USA) using single-edge-notch specimens for bending. All specimens were loaded quasi-statically up to failure with displacement controlled loading rate at 1 mm/min. Due to dimensional constrain and material’s brittleness of the cortical bone tissue, as well as the consideration of experimental simplicity and accuracy, experiments were performed with a three-point bending fixture using single-edge-notch bending specimens to minimise potential damage caused by specimen preparation prior to the tests.
The deflection load was measured using the machine’s 5 kN load cell and the corresponding load-line displacement was simultaneously measured using a LVDT sensor (2601 Series, Instron, USA), see Figure 5–2. The obtained load-displacement curves were then analysed according to the classification described in [152].

**Figure 5–2** Three-point bending setup with single-edge-notch cortical bone specimen and LVDT mounted on Instron 3345 machine

*Calculation procedures*

Depending on the experimental setup and specimen dimensions as well as the predefined criterion (Figure 5–3), the analysis can determine either plane strain fracture toughness $K_{IC}$, crack tip opening displacement (CTOD), or $J$ integral value based on the specimen dimensions, depth of notch, material properties (e.g. 0.2% proof strength ($\sigma_{YS}$)) and specific data from the force-displacement curve of the fracture test. When the fracture follows elastic-plastic conditions, it is not possible to determine a valid $K_{IC}$ value as a measure of fracture toughness of a material; however, either critical CTOD or critical $J$ values can be calculated in this case. Validity of $K_{IC}$ value depends on the shape of the force-displacement diagram, the specimen’s size and form, and the 0.2% proof strength and toughness of the material at the temperature of interest. For a valid measurement of $K_{IC}$, the specimen’s
dimensions (average original crack length \(a_0\), thickness \(B\) and the ligament \((W - a_0)\)) each must not to be less than \(2.5 \left(\frac{K_Q}{\sigma_{YS}}\right)^2\), where \(K_Q\) is the provisional value of \(K_{IC}\). Additionally, the ratio of maximum force and particular force determined by the standard \((F_{max}/F_Q)\) must not be greater than 1.1 [152]. Otherwise, either CTOD or the value of \(J\)-integral will be calculated. In this study, the behaviour of all specimens was predominantly non-elastic, and all specimens failed to satisfy the validation criterion of \(K_{IC}\). Therefore, the elastic-plastic fracture mechanics (EPFM) parameter, \(J\)-integral, was calculated based on [152] using the following equation:

\[
J = \left[\frac{FS}{BW^{1.5}} \times f\left(\frac{a_0}{W}\right)\right]^2 \left(1 - v^2\right) + \frac{2U_p}{B(W - a_0)},
\]

where \(S\) is the bending span, \(F\) is the applied force, the Poisson’s ratio \((v)\) was obtained previously in [209], \(E\) is the elastic modulus for a respective cortex, based on previous results in Chapter 4, \(U_p\) is the plastic component of area under the load-line displacement curve for the tested specimen, \(B\) is the specimen’s thickness, \(W\) is the effective width of the test specimen, \(a_0\) is the average original crack length and \(f\left(\frac{a_0}{W}\right)\) is a mathematical function of \(\left(\frac{a_0}{W}\right)\) defined as:

\[
f\left(\frac{a_0}{W}\right) = \frac{3 \left(\frac{a_0}{W}\right)^{0.5}}{2 \left[1 + 2\left(\frac{a_0}{W}\right)\right]} \left[1.99 - \left(\frac{a_0}{W}\right)\left(1 - \frac{a_0}{W}\right)\left(2.15 - \frac{3.93a_0}{W^2} + \frac{2.7a_0^2}{W^4}\right)\right].
\]
5.2.3 Scanning electron microscopy

After fracture tests, fracture surfaces of all the specimens were investigated using scanning electron microscopy (SEM) (Stereoscan 360, Carl Zeiss, Germany). The principle of SEM is that the focused beam of electrons is projected towards the target surfaces and interacts with atoms within the target. The detectors receive the backscattered electrons, secondary electrons and X-ray spectra and produce various signals to compute high magnification images of the surface topology of a specimen. SEM offers a fast imaging process with a high depth field compared with conventional
optical microscopy and, therefore, is ideal for analysing complex surface structures such as fracture surfaces. Due to the nature of electrons, SEM usually requires the electron beam source to be operated within a vacuum chamber to reduce the effect of ambient electric and magnetic fields. Specimens used in SEM must be conductive in order to interact with the electrons. Since cortical bone is a non-conductive material, a thin layer of gold/palladium coating was applied to the specimens using a low-vacuum sputter coater (Emitech SC7640 Sputter Coater, Polaron, Quorum Technologies Ltd, UK) before investigation. The thickness of applied coating is typically within a few nanometres range, therefore, does not have a noticeable effect on the general state of original fracture surfaces.

5.3 Numerical modelling of three-point bending

Finite-element analysis of the actual crack initiation and growth is another powerful tool, which enables studying of the fracture and damage processes of cortical bone in the very vicinity of the fracture zone. This is hard to achieve using approaches such as element de-bonding, cohesive zone method (CZM) or a virtual crack closer technique (VCCT) due to a well-known fact that the crack path has to be well defined in advance. However, with the Extended Finite-Element Method (XFEM), a crack propagation process can be modelled based on a solution-dependent criterion without introducing a predefined crack path. Thus, the aim of this part of the study was to develop and validate numerical models using XFEM for analysis of deformation and fracture behaviours of the cortical bone tissue under a quasi-static loading regime of three-point bending.

5.3.1 Extended Finite Element Method

The extended finite-element method (XFEM), also called enriched finite element method or generalized finite element method (GFEM) is an extended interpretation of the classical finite-element method based on the concept of partition of unity developed by Melenk and Babuška [210] and introduced by Belytschko and Black [159]. The method incorporates local enrichment
functions into the finite-element approximation to model discontinuities within a continuum domain, such as holes, interfaces or cracks, without conforming the meshes to geometrical discontinuities. Therefore, it is generally applied very effectively in fracture analysis. Benefited from the additional degree of freedom thanks to enrichment function, XFEM enables analysis of crack initiation and propagation without a pre-defined crack trajectory or considerable remeshing and mesh refinement around the crack tip as the crack propagates.

*Enrichment function*

The enrichment function is typically introduced in terms of a displacement vector with two additional enrichment functions, which include an asymptotic tip function approximating the singularity around the crack tip and a displacement jump function which describes the displacement discontinuity across the crack interfaces. Therefore, the overall approximation for the nodal displacement vector function \( \mathbf{u} \) including the geometrical discontinuity is formulated as [211]:

\[
\mathbf{u} = \sum_{i=1}^{N} N_i(x) \left[ \mathbf{u}_i + H(x) \mathbf{a}_i + \sum_{a=1}^{4} F_a(x) \mathbf{b}_i^a \right],
\]

(5–3)

where \( N_i(x) \) defines the usual nodal shape functions; within the square bracket, the first term represents the normal nodal displacement vector that is the same for a non-enriched finite-element solution; the second and third terms are the two enriched degree-of-freedom vectors \( \mathbf{a}_i \) and \( \mathbf{b}_i^a \), with their associated discontinuous jump function \( H(x) \) across the crack surfaces and the elastic asymptotic crack-tip functions \( F_a(x) \), respectively. Different usage of the second and third terms of the equation (5–3) enables differentiation between the enrichment functions applied to different nodes: second term is only applicable to nodes whose shape function support is cut by crack interior, while the third term is for those nodes whose shape function support is cut by crack tip [211].
The respective discontinuous jump function and asymptotic crack-tip functions for an isotropic material are formulated as [211]:

\[
H(x) = \begin{cases} 
1 & \text{if } (x - x^*).n \geq 0, \\
-1 & \text{otherwise},
\end{cases} \tag{5-4}
\]

where \(x\) is a sample (Gauss) point, \(x^*\) denotes the point on the crack that is the closest to point \(x\), and \(n\) is the unit outward normal to the crack at \(x^*\) (Figure 5–4a);

and

\[
F_{\alpha}(x) = \left[ \sqrt{r} \sin \frac{\theta}{2}, \sqrt{r} \cos \frac{\theta}{2}, \sqrt{r} \sin \theta \sin \frac{\theta}{2}, \sqrt{r} \sin \theta \cos \frac{\theta}{2} \right], \tag{5-5}
\]

where \((r, \theta)\) represents a polar coordinate system with its origin at the crack tip, while the crack-tip tangent coincides with \(\theta = 0\) (Figure 5–4a).

**Figure 5–4** (a) Schematic illustration of discontinuous jump function across the crack surfaces; (b) principle of separation of cracked element using phantom-nodes method [211]
Crack initiation and propagation

To track continuously the crack propagation direction, accurate modelling of the crack-tip singularity is required, which can be significantly cumbersome, especially in a non-isotropic material. To overcome the deficiency of the computational power, a crack is assumed to propagate though the entire element at a time, and the needs of the calculation of asymptotic function is therefore only required for stationary crack analysis. Instead, the crack initiation criteria are used in a propagating crack analysis as an effective engineering approximation. For modelling of crack propagation and growth within the XFEM framework, two distinct damage modelling techniques can be employed [211]: a cohesive segment method, which uses the traction-separation law – an extension from the general framework of a surface-based cohesive behaviour; and the LEFM based approach that utilizes the similar principle as in that of VCCT for interfacial de-bonding. Although different, both approaches share the same crack initiation criteria – either stress-based or strain-based damage initiation criteria depending on the material’s behaviour. One of the differences between them is that the LEFM-based method can be only implemented when no initial crack is presented in the model. Due to the original formulation of the method, VCCT approach is only suitable for modelling of brittle fracture, while, the CZM-based approach can be used in modelling both brittle and ductile fracture analysis.

Current limitations of XFEM approach

Being a powerful and attractive numerical tool, the current form of XFEM (in Abaqus v6.11 [211]) does have its limitations:

- It cannot produce crack bifurcation or crack intersection due to the limitation of enrichment function;
- A crack growing at the boundary of a model does not perform well;
- Crack initiation is limited only to tension states;
- Crack propagates through one element at a time introducing some element dependence;
• A crack is not allowed to turn more than 90° per increment during the analysis.

### 5.3.2 Model specification

In the simulation part, two groups with a total of eight finite-element models were developed to evaluate the fracture behaviour of cortical bone under quasi-static loading in the three-point bending test setup: Group A and Group B for longitudinal and transverse cracks, respectively. Simulations were performed using XFEM implemented in the finite-element software Abaqus 6.11/Implicit. The geometry and dimensions of specimens used in simulations are shown in Figure 5–5. The diameters of pin holders were modelled as 10 mm based on the experimental setup. The following model assumptions were made: (1) plane-strain conditions of the specimen; (2) elastic transversely isotropic material properties for the bone specimens (see Table 5–1); (3) a friction coefficient of 0.3 was assumed between the pins and the specimen based on [212, 213].

![Figure 5–5](image)

**Figure 5–5** (a) Schematic of three-point bending setup, distance between fixed grips is $S = 4W = 21.72$ mm; (b) mesh used for cortical bone specimen; (c) geometry and dimensions of cortical bone specimens used in tests and simulations
In these simulations, damage initiation and evolution criteria employed a surface-based cohesive traction-separation law based on the elastic-plastic fracture mechanics. The model determined damage based on a chosen fracture strain, which corresponded to maximum principal strain of 0.6% [39, 138, 214] in this case. When the fracture strain was reached, damage initiation started, and then, damage evolution took place. The evolution criterion was defined in terms of fracture energy, and a linear damage softening response was chosen for the analysis. The crack follows an arbitrary, solution-dependent path in the bulk material, and the path is independent of the element boundaries in the mesh. The magnitude of fracture toughness obtained from the experimental part of this study was introduced into the developed XFEM models as fracture energy as shown in the results section. The initial notch was introduced as a 2.7 mm-long straight line in the model, and the whole specimen was chosen as XFEM enrichment area.

For Models A and B, a total number of 8600 linear quadrilateral (CPE4R) plane strain elements were used in each model to generate a mesh for the simulated bone specimen to minimized shear-lock and hour-glass effects. The fixtures of three-point bending were modelled as 2D analytical rigid shell, planar wire. A general contact with a penalty-friction formulation was defined between the bone specimen and these fixtures.

Table 5–1 2D Material properties of cortical bone [30, 53] used in FE model (subscripts denote axial orientation: 1. longitudinal; 2. transverse)

<table>
<thead>
<tr>
<th></th>
<th>$E_{11}$ (GPa) [53]</th>
<th>$E_{22}$ (GPa) [53]</th>
<th>$\nu$ [30]</th>
<th>$G_{12}$ (GPa) [53]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anterior</td>
<td>23.15</td>
<td>13.20</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Posterior</td>
<td>18.00</td>
<td>10.20</td>
<td>0.29</td>
<td>3.0</td>
</tr>
<tr>
<td>Medial</td>
<td>21.13</td>
<td>14.67</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Lateral</td>
<td>15.14</td>
<td>11.18</td>
<td>0.29</td>
<td></td>
</tr>
</tbody>
</table>
5.4 Results and analysis

5.4.1 Elastic-plastic fracture parameters

Critical values of fracture toughness $J_C$ of the studied cortical bone tissue were calculated with respect to three crack-growth directions: longitudinal, radial, and transverse; in addition, anisotropy ratios of the fracture toughness values were analysed. The obtained experimental data demonstrated that all specimens exhibited a non-linear elastic-plastic fracture process; hence, based on British Standard: BS 7448-1 [152], the $J$-integral was used to quantify the fracture toughness. Table 5–2 lists the average levels and standard deviations for the critical values of $J$-integral and for all crack growth directions and for four cortices.

<table>
<thead>
<tr>
<th>Units: N/m</th>
<th>Anterior</th>
<th>Medial</th>
<th>Posterior</th>
<th>Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>Long</td>
<td>1033.9 ±254.5</td>
<td>1768.5 ±98.8</td>
<td>1165.7 ±340.1</td>
<td>2034.3 ±509.9</td>
</tr>
<tr>
<td>Radial</td>
<td>1199.1 ±153.1</td>
<td>1418.2 ±97.2</td>
<td>983.0 ±369.5</td>
<td>2664.2 ±554.4</td>
</tr>
<tr>
<td>Trans</td>
<td>4509.1 ±422.1</td>
<td>5925.5 ±802.9</td>
<td>3876.7 ±847.3</td>
<td>5661.6 ±452.7</td>
</tr>
</tbody>
</table>

It can be noticed from these results that the fracture-toughness values for specimens cut from different cortices of bovine femoral cortical bone are significantly different. In general, cortical bone shows higher resistance to fracture when a crack grows perpendicular to the osteon direction (see Figure 5–1) and lower resistance for the radial and longitudinal directions (i.e., with the fracture surfaces parallel to osteons). For a crack growing in transverse direction, specimens from the medial quadrant had the highest critical value of $J$-integral while those for posterior specimens were the lowest. The Tukey HSD test ($\alpha = 0.05$) found statistically significant differences between medial to posterior ($p = 0.035$) and posterior to lateral ($p = 0.028$).
cortices. On the other hand, specimens with radially extended cracks were found to have the highest fracture toughness in case of the lateral quadrant and the lowest for the posterior one. The calculated critical values of $J$-integral for the radial cracks, ranging from 983 N/m to 2664 N/m, were significantly lower compared with those for specimens having transverse cracks. Significant differences were found between anterior to lateral ($p = 0.027$) and posterior to lateral ($p = 0.015$) quadrants. Finally, for specimens with cracks extending along the direction parallel to osteons (longitudinal cracks), the critical $J$-integral values were comparable with those for radial cracks, and their highest value was found for the lateral quadrant whereas the lowest was in anterior specimens. Statistically significant differences in this case were found between anterior to medial ($p = 0.043$) and anterior to lateral ($p = 0.02$) quadrants. Generally, comparing the date for all four cortices, higher fracture toughness was demonstrated by specimens cut from the medial and lateral quadrants. The disparity between these two groups ranges from as low as 18.3% up to 171%.

This non-uniform fracture resistance across different cortices of the bovine femur implies that the variation of microstructure has a great impact on the localised fracture toughness values. Optical-microscopy images presented in Figure 5–6 demonstrate distinct features of microstructure with respect to anatomic cortices. Anterior and posterior quadrants are predominantly occupied by primary and secondary osteons, respectively, whereas medial and lateral quadrants have a mixture of both primary and secondary osteons together with a large proportion of interstitial matrix. Previous research [53, 188] showed that a change in the volume fraction of constituents at microstructure level largely affected the local material properties, such as elastic modulus, yield stress, ultimate strength, which, in turn, influenced fracture properties. The effect of microstructural orientation also has an important effect on anisotropy of fracture toughness values. Higher resistance to fracture was found where the cracks propagated perpendicular to osteons orientation, while lower resistance when cracks extended parallel to osteons direction. The anisotropy ratios (calculated as ratios of respective values of $J_C$) between transversely-orientated cracks and longitudinally- or
radially orientated cracks are presented in Table 5–3. Apparently, the anisotropy ratios also vary for different cortices ranging from 2.13 to 4.36, with the lowest ratio found for the lateral quadrant and the highest ratio for the anterior quadrant.

Table 5–3 Anisotropy ratios of fracture toughness values compared for different crack growth directions for various cortex positions

<table>
<thead>
<tr>
<th></th>
<th>Anterior</th>
<th>Medial</th>
<th>Posterior</th>
<th>Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse/Longitudinal</td>
<td>4.36</td>
<td>3.35</td>
<td>3.33</td>
<td>2.78</td>
</tr>
<tr>
<td>Transverse/Radial</td>
<td>3.76</td>
<td>4.18</td>
<td>3.94</td>
<td>2.13</td>
</tr>
</tbody>
</table>

Figure 5–6 Representative microstructural features of different cortex positions: (a) anterior; (b) medial; (c) posterior; (d) lateral (refer to section 4.3.4 for the procedure to distinguish primary and secondary osteon)

5.4.2 SEM analysis

Fracture surfaces were analysed for all the tests using SEM. The results obtained for different crack-extension directions and cortex positions are
grouped in Figure 5–7. Dissimilar characters of roughness of fracture surfaces were evident among the four cortex positions; an indication of a variety of fracture toughening mechanisms acting in different cortex positions. The transition of the underlying microstructure from one type to another could be the reason for these differences. As shown in Figure 5–7, the fracture surfaces from the anterior and posterior quadrants are relatively smooth compared with those for the medial and lateral quadrants. Empirical evidence [8] suggested that the surface roughness is associated with the amount of energy required to generate the fracture surface: lower level of fracture energy indicates smoother fracture surface.

Additionally, a combination of microstructural changes and different crack-extension directions triggered complicated toughening mechanisms, which, in turn, were reflected in different fracture-toughness values and levels of surface roughness. Generally, for the longitudinal fracture specimens, with crack fronts propagating along the direction parallel to the axis of osteons, the fracture toughening mechanism was dominated by uncracked-ligament bridging during the process of osteons splitting, rupture, interface failure and fibre delamination (see Figure 5–8 L_a, L_b). Similarly, for cracks propagating in the radial direction, the toughening mechanism was still governed by uncracked-ligament bridging as a result of osteon splitting or fibre delamination. However, a slight difference in this case was the existence of interface areas or empty spaces such as cement line or Haversian canals that had a larger contribution towards cracks arresting at these regions [215]. As a result, twists and kinks of osteons were observed in current analysis (see Figure 5–8 R_a, R_b). In contrast to the previous two cases, cracks growing along the transverse direction required a larger traction force for the crack front to penetrate and cross the osteons as longitudinal strength of osteons was much higher than transverse one. Cracks were therefore more likely to be deflected due to imperfections and heterogeneity of the microstructure or complete pull outs of osteons (see Figure 5–8 T_a). Consequently, higher values of fracture toughness were obtained and rougher crack surfaces were observed. In the elastic-plastic fracture regime, the tensional field at the back of the crack tip also promoted
a multi-scale bridging effect through shear sliding between interface regions at different levels (see Figure 5–8c).

**Figure 5–7** SEM images of fracture surfaces for various cortex positions and crack propagation directions: A, M, P, L denote anterior, medial, posterior and lateral; _L, _R, _T denote crack propagation directions for longitudinal, radial and transverse ones, respectively, white arrows indicate crack growing direction.
Figure 5–8 Schematic illustrations and SEM images of various toughening mechanisms for longitudinal (a), radial (b) and transverse (c) cracks-growth directions, Labels at the bottom of each image indicate the corresponding magnified areas in Figure 5–7

5.4.3 Numerical simulations

The simulation part of the study was focused on the crack initiation and propagation processes in the cortical bone specimens under different loading configurations. The simulations were performed at quasi-static conditions using the Abaqus/implicit solver at a constant loading rate until complete fracture of the specimen. The employed damage evolution criterion was based on fracture energy calculated using the obtained experimental results. Results of finite-element simulations are compared with the experimental data in Figure 5–9 for different cortices and crack orientations; this comparison shows very good agreement for force-displacement curves. The developed finite-element models successfully reproduced the variability of material responses across four cortices for both longitudinal and transverse crack directions. The results indicate that the fracture-toughness values are largely affected by the local anisotropic material properties linked to the
variation of the microstructure [188]. The models also predicted an early-stage damage initiation (Figure 5–9, horizontal dotted lines), followed by a non-linear progressive damage-evolution process. By using a surface-based cohesive traction criterion based on the experiment result, these complex non-linear damage propagation processes were captured reasonably well. Both initial curvature of the graphs and the peak-force levels were determined very close to the obtained experiment results. It was also observed that damage initiation for transverse-crack specimens in medial quadrant is lower than for longitudinal-crack specimens. The lower damage initiation combined with a higher ultimate fracture force indicates the existence of a strong toughening mechanism for medial transverse-crack specimens (Figure 5–9). On the other hand, a high damage-initiation load with a low peak force in longitudinal crack specimens from the anterior quadrant is an indication of a weak toughening mechanism.

Figure 5–9 Comparison of experimental and calculated force-displacement curves: A, M, P and L denote anterior, medial, posterior and lateral specimens; _L and _T denote longitudinal and transverse crack propagation directions; dotted lines indicate damage initiation position.
5.5 Discussion

The experimental study of the deformation and fracture processes in specimens of bovine femoral cortical bone demonstrated non-uniformity and anisotropy of fracture toughness across various cortex positions and for different crack orientations. The calculated critical values of $J$-integral range from 983 N/m to 5661 N/m. That is in good agreement with the literature data [18]. This wide spectrum of fracture-toughness values could be interpreted as a result of the material anisotropy due to the microstructure orientation as well as changes in the character of distribution of microstructural constituents at various anatomic positions. Large anisotropy ratios of the material properties for three mutual perpendicular bone axes lead to significantly higher fracture resistance of transverse-crack specimens than that of longitudinal- and radial-crack specimens (Figure 5–10a). Changes in the microstructure between cortex positions result in different levels of fracture toughness at different cortices (i.e., non-uniform distribution of this parameter of a bone’s cross-section). Due to a natural loading regime exerted by animal’s weight and muscle forces, long bones are normally exposed to combined loading conditions that are spatially non-uniform. As it is well known from literature, bone is a dynamic tissue that reacts to mechanical loading by adapting its shape, internal microstructure and material properties to meet external loading environment [8]. The differences in value of fracture toughness (critical $J$-integral) could be the outcome of bone adaptation to its natural non-uniform loading conditions, where lateral to medial axis may require higher fracture resistance to sustain the loading condition (Figure 5–10b).

From another point of view, the stronger toughening mechanisms at medial and lateral quadrants could be another reason to cause higher fracture toughness. A good proportion of hard and soft materials usually results in a tougher combination as toughening mechanisms at interfaces usually enhance the overall fracture resistance. In other words, combining the stiff interstitial matrix with soft secondary osteons may facilitate formation process of toughening mechanism [216]. Yet, excessive primary or secondary
osteons could unbalance the formation process and result in a decline of fracture toughness. Determining the individual fracture toughness of each microstructure constituent or evaluating the natural loading condition of cortical bone will certainly help to gain further understanding of fracture process in the cortical bone tissue. However, they are well beyond the scope of this study and are not discussed here.

![Illustration of variability of mean critical J-integral values: (a) a bar chart indicates mean and standard deviation (error bars) for different crack-propagation directions; (b) a radar chart indicates fracture toughness along the anatomic positions](image)

**Figure 5–10** Illustration of variability of mean critical $J$-integral values: (a) a bar chart indicates mean and standard deviation (error bars) for different crack-propagation directions; (b) a radar chart indicates fracture toughness along the anatomic positions

### 5.6 Conclusions

In the present study, fracture toughness of bovine femoral cortical bone was evaluated, and the effect of its local microstructural changes on fracture toughness values was examined. Based on this study, the following conclusions were made:

- Bovine femoral cortical bone demonstrated non-uniform elastic-plastic fracture behaviour for different cortices. The mean values of critical $J$-integral covers a range from $983 \text{ N/m}$ to $5661 \text{ N/m}$ with an anisotropic ratio ranging from 2 to 4 depending on the anatomic position and crack propagation direction.
• Variation and anisotropy of the underlying microstructure plays an important role in variability of fracture resistance.

• Fracture toughening mechanisms varied for different fracture propagation directions. Longitudinal and radial cracks specimens were dominated by uncracked-ligaments, while transverse crack specimens were governed by crack deflection and multi-scale bridging.

• Employing the full advantage of non-linear fracture mechanics, the developed XFEM models successfully reproduced the macroscopic variability and anisotropy of the non-linear fracture process in cortical bone under three-point bending configurations, which confirmed a strong link between fracture toughness values to the localised material properties.
Chapter 6
Numerical Analysis of Fracture Process in Microstructured Cortical Bone Tissue

6.1 Introduction

Bones tissues are heterogeneous materials that consist of various microstructural features at different length scales. The fracture process in cortical bone is affected significantly by its microstructural constituents and their non-uniform distribution. At the micro-scale level, an osteon or a Haversian system is the most recognizable constituent. Generally, oriented along the long axis of bones, osteons are composed of a Haversian canal surrounded by concentric rings of lamellae (3–7 µm), embedded into the remnants of a bone’s remodelling process called interstitial matrix. The existence of a thin layer of interface, cement line, plays an important role in the bone’s mechanical behaviour, especially its fracture. This heterogeneous architecture of cortical bone has a significant effect on its mechanical and fracture properties. Moreover, preferential alignment of both collagen fibrils and mineral crystals at nano-scale as well as of osteons and Haversian canals at micro-scale results in a highly anisotropic mechanical and fracture behaviour of the tissue [217]. The anisotropy ratio of fracture toughness for different crack propagation directions can be significantly large – from 2 to 4 as observed in Chapter 5 – depending on interaction of a propagating crack with microstructural features, activation of various toughening mechanisms affecting fracture resistance: formation of micro-cracks in the vicinity of the main crack due to stress concentrations ahead of its tip [94, 218, 219] and crack deflection and blunting at cement lines that create discontinuity at the boundary layers [220]. Recently, it was reported that ligament bridging of crack in the wake zone is a dominant toughening mechanism in cortical bone as it reduces a driving force at the crack tip [98, 101, 221]. Several authors reported that toughening mechanisms are highly dependent on a crack
propagation direction; therefore, fracture toughness of long bones is significantly higher in transverse and radial directions compared with the longitudinal one [221-223].

Being a physiological living tissue, bone has the ability of continuously remodelling, repairing and adapting itself to the surrounding environment. Due to this inherent dynamics, both the microstructure of cortical bone and its mechanical behaviour vary dramatically from one part to another. Considering differences introduced by various testing methods and specimen’s sizes, the level of fracture energy of cortical bone varies from 920 N/m to 2780 N/m [18] for the same type of bone tested at same orientation. This variability is significant even for different cortices of a single bone, as was verified both in previous research (Chapter 4) and in the literature [54]. Unlike other engineered composite materials, which have a pre-defined average volume fraction of their constituents and, consequently, a limited range of their mechanical properties, the local volume fraction of each constituent of cortical bone is not unique and changes during the bone remodelling process. As a result, randomly distributed elements of microstructure in local regions have a significant impact on variability of the mechanical behaviour of cortical bone at macro-scale level. However, little has been done to unveil a correlation between the variation of microstructure and variability of the mechanical behaviour of cortical bone.

Finite-element simulations provide a powerful tool to analyse the fracture behaviour of materials at different length scales. The cohesive-zone method (CZM) accounts for a non-linear fracture mechanism and describes the non-linear fracture process in terms of a traction-separation law; it was broadly used in the literature [103, 156] to investigate fracture of cortical bone. However, it has an inherent drawback: the crack extension has to follow a predefined path around elements of the mesh. Obviously, in the case of fracture in heterogeneous material such a crack path is hard to predict. The extended finite-element method (XFEM) was also used in a small number of papers on fracture in bones, as discussed in previous section (Section 3.5.2). Despite many attempts by various researchers, the model development for
fracture of cortical bone is still limited to simplified formulations: simplified material properties [137]; a full-size bone model but with an assumption of homogenised continuum [161]. There is still a need in a comprehensive XFEM model that can reflect adequately the main features characteristic to the bone fracture process.

Therefore, in this chapter, a microstructured model of cortical bone is proposed to further investigate both the non-linear fracture process of cortical bone tissue at micro-scale and the effect of its microstructure on variability of fracture toughness for the case of three-point bending.

6.2 Model specification

6.2.1 Submodelling approach

To investigate the variability of fracture toughness and various toughening mechanisms induced by random microstructure, three 2D models of microstructured cortical bone were developed in commercial finite-element software Abaqus [224] based on configuration of previous three-point-bending experiments detailed in (Chapter 5). Specimens with radially extended cracks were chosen to be modelled because of the prominent interactions between the crack and microstructural constituents at this direction. These models were constructed using a submodelling technique that focuses the computational power at the crack-propagation region while maintaining the full-scale approach of the model.

The submodelling technique allows development of multiple models based on the same modelling object and extends the level of interest into a pre-defined region (usually with a finer mesh or more local geometric details, even with different material properties) to achieve adequate and accurate results. The computational cost of this technique is usually much lower compared with the full-size model having the same level of accuracy. The use of ‘submodel’ is in comparison with the phase ‘global model’, whose solution is interpolated onto the relevant parts of the boundary of the submodel [224]. This approach can be, in generally applied many times (a
submodel can be used in another analysis as a global model) in various types of simulations: an explicit submodelling analysis can be driven by that of a global model performed using an implicit solver and vice versa. The procedure of submodelling employs two separate analyses [224]: one for the coarser global model and one for the finer submodel, of which the time-dependent values of variables saved from the global analysis are transferred to the relevant boundary nodes or surfaces. Based on the driven variable used, submodelling can be classified into two different types [224]: a node-based submodelling technique, which uses the degrees of freedom from nodal results (including displacement, temperature, or pressure) to interpolate global results onto the submodel nodes; or a surfaced-based submodelling technique, which uses components of stress tensor at the integration points of element faces to interpolate results of the global model onto the submodel integration points on the driven element-based surface facets. The differences in terms of use between the two applied techniques are compared in Table 6–1. In this study, the node-based submodelling technique was chosen owing to large deformations of the three-point bending specimen associated with the global model.

**Table 6–1 Preferences of node-based and surface-based submodelling techniques summarised from [224]**

<table>
<thead>
<tr>
<th>Influence factors</th>
<th>Node-based submodelling</th>
<th>Surface-based submodelling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capability</td>
<td>Available for most types of analyses</td>
<td>Only available for solid models and static analyses</td>
</tr>
<tr>
<td>Stiffness</td>
<td>Preferred when stiffness between global and submodel are comparable</td>
<td>Preferred stress field results stiffness between global and submodel are different</td>
</tr>
<tr>
<td>Locomotion</td>
<td>Preferred when model has large deformation/rotation</td>
<td>Preferred when model involves rigid-body motion</td>
</tr>
<tr>
<td>Displacement response</td>
<td>Preferred when global and submodel have same response</td>
<td>Preferred when global and submodel have different responses</td>
</tr>
<tr>
<td>Date transmission</td>
<td>Preferred when displacement filed is transmitted</td>
<td>Preferred when stress field is transmitted</td>
</tr>
<tr>
<td>between global and submodel</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The developed approach employed two different levels of modelling of the bone tissue: a full-size global model for the macroscopic response of the entire specimen under three-point bending and three submodels reflecting microscopic responses of different cases of localised microstructures during the crack propagation process. The boundary conditions in the submodel were derived for the correspondent region from the results of the global model using the displacement-control criterion based on the nodal field variables as described above.

6.2.2 Model geometry

Figure 6–1 Schematic illustration of model configurations for three-point-bending setup using global model and microstructured sub-model: global model simulates the full scale response of the cortical bone under three-point bending, while submodels only model the region of interest (blue region); submodels are driven by the displacement field of the global model at the same location; red box highlights the difference between global mesh and sub-model mesh.
The model geometry corresponds to that of the specimens used in previous experiment: 25 mm × 2.72 mm × 5.43 mm (total length × width × thickness). Cylindrical loading pins at the three-point setup were modelled as analytical rigid bodies with a radius of 5 mm. The span (length between the centres of two holding pins, see Figure 6–1) and the pre-crack length, a, were chosen according to the experimental setup: 21.72 mm and 2.72 mm, respectively. Then, the submodel was defined at the central un-cracked region of the global model with dimension of 2.72 mm × 2.72 mm (Figure 6–1). The pre-crack is mostly outside the submodel with only one element of its bottom middle surface cut by it.

6.2.3 Generation of random microstructures

Three microstructured submodels for specimens with radially extended cracks were constructed based on the randomly distributed four-phase composite structures consisting: (i) interstitial bones and Haversian systems that include (ii) osteons, (iii) Haversian canals and (iv) cement lines.

Statistical parameters

Geometrical parameters of each model were defined based on statistical analysis of microstructure constituents obtained from [225]. The statistical distributions of diameters of osteons and Haversian canals were measured using optical microscopic images and regularized with a hypersecant distribution function

\[
f_0(x) = \frac{\text{sech}\left(\frac{\pi(x - 35.3)}{199.8}\right)}{199.8},
\]

(6–1)

and the Dagum (4P) distribution function,

\[
f_H(x) = \frac{4.1\left(\frac{x - 3.3}{12.9}\right)^{3.1}}{12.9\left(1 + \left(\frac{x - 2.7}{12.9}\right)^{2.7}\right)^{13.5}}.
\]

(6–2)
respectively, as described in [225]. The measured average diameters for osteons and Haversian canals were 99.89 µm and 23.1 µm, respectively [225]. The average width of cement line was close to 5 µm [225]. The volumetric fractions of osteons were measured within the range from 13% to 55% (Section 4.4.5), while the porosity ratio was between 3.53% and 9.51% (Section 4.4.5).

Random microstructure

**Figure 6–2** Schematic illustration of bone specimen, image of real microstructure, Cumulative distribution functions and three statistical realizations of random microstructure with different fractions of constituents for three submodels
The algorithm to generate random microstructures in the submodels was first programmed in a custom-developed Matlab code (developed in MoAM Research Group in conjunction with a MSc project, Wolfson School of Mechanical and Manufacturing Engineering, Loughborough University) according to the statistical data for real bone specimens, and then all the geometrical parameters were encoded into a Python script to construct the microstructural models in Abaqus (Figure 6–2). Three models of cortical bone with different microstructures: Models A, B and C were developed and employed in this study based on the statistical measurements for each constituent: osteons volume fraction varies from 30% to 51%, while porosity changes from 5% to around 8% (Figure 6–2). The full data on the volume fractions of microstructure constituents used in the models are listed in Table 6–2.

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Model A</th>
<th>Model B</th>
<th>Model C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Osteon</td>
<td>30%</td>
<td>44.5%</td>
<td>51.2%</td>
</tr>
<tr>
<td>Porosity</td>
<td>5.01%</td>
<td>5.02%</td>
<td>8.14%</td>
</tr>
<tr>
<td>Interstitial matrix</td>
<td>58.77%</td>
<td>41.25%</td>
<td>30.04%</td>
</tr>
<tr>
<td>Cement line</td>
<td>6.22%</td>
<td>9.23%</td>
<td>10.62%</td>
</tr>
</tbody>
</table>

6.2.4 Material properties

In this study, the mechanical behaviour of cortical bone was introduced using an elastic-plastic transversely isotropic material formulation with regard to the radial-transverse section as the isotropic plane of cortical bone (see Figure 6–1). At the macroscopic level, the effective homogeneous material was used in the global model neglecting microscopic heterogeneity based on previous experiments in Chapter 4. The effective elastic-plastic material properties obtained from the same study were applied in the global model. On the other hand, at the microscopic level, microstructural constituents play an important role in the localised fracture process and formation of
toughening mechanisms. Consequently, the four-phase microstructured models of cortical bone were employed in the submodel, and individual material properties based on nano-indentation results [225] obtained from previous research within my research group were assigned to the constituents (characterising the material properties for individual microstructural constituents is outside the remit of this study). The elastic modulus of cement lines was initially set to be 25% lower than that of osteons based on the findings in [137, 226], and two other levels, i.e. equal to that of osteons and 25% higher, were also used to investigate the effect of cement line’s properties on the fracture process in cortical bone. A strain-based yield criterion was implemented both in the global model and submodels, and a yield strain of 0.6% was chosen based on [53]. The post-yield material behaviours in both global and sub-models were based on flow stress-strain curves obtained experimentally [53, 225]. A summary of material properties used in this study is given in Tables 6–3 and 6–4.

<table>
<thead>
<tr>
<th>Table 6–3 Material properties used in global model and microstructured submodels ([137, 204, 225, 227])</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Effective homogenised material</strong></td>
</tr>
<tr>
<td><strong>Elastic modulus (GPa)</strong></td>
</tr>
<tr>
<td><strong>Poisson’s ratio</strong></td>
</tr>
<tr>
<td><strong>Yield strain</strong></td>
</tr>
<tr>
<td><strong>Fracture initiation strain</strong></td>
</tr>
<tr>
<td><strong>Fracture energy release rate (N/m)</strong></td>
</tr>
</tbody>
</table>
Damage and crack propagation in this study were modelled using the XFEM technique in Abaqus [224] that allows a crack to initiate and propagate through an arbitrary, solution-dependent path subject to the local material’s response. Hence, the XFEM enrichment was applied to the whole model for all the cases. The local crack initiation and evolution were evaluated continuously based on the chosen criteria. Crack initiation in a hard biological tissue (cortical bone) was commonly described with a strain-driven criterion [98]. Therefore, a strain-based crack-initiation criterion was set up both in global and microstructured models. It assumes that crack initiates when the maximum principal strain reaches its critical value and the newly defined crack direction is orthogonal to that of the maximum principal strain. A crack initiation strain of 0.65% was chosen based on observations of material’s response for the radial-transverse direction detailed in [53]. Once initiated, the crack conforms to the energy-based damage evolution criterion, and the
cracked element starts degradation and eventually fails. The governing formulation for the onset of crack utilizes a cohesive traction-separation constitutive behaviour to define the damage evolution of the cracked surface. It describes the rate, at which the cohesive stiffness of the cracked surface degrades once the crack-initiation criterion is fulfilled for the particular element. The energy dissipated (fracture energy) as a result of damage progress is equal to the area under the traction-separation curve at the point of complete damage. The fracture energy in the developed models (Table 6–3) was defined according to the previous results [18, 204, 225].

6.2.5 Mesh-convergence analysis

The global model was discretised into 14,000 four-node bilinear plane-strain quadrilateral elements and ran on an eight-processor (quad-core Intel I7 970 CPU) PC while the submodels were meshed using 150,000 to 200,000 elements of the same type and ran on a 60-processors (five six-core Intel Westmere Xeon X5650 CPUs) high-performance cluster. The Abaqus implicit solver was used in both types of simulations. A mesh-convergence study was carried out for the global model using six different element sizes, and the obtained results were analysed in terms of peak reaction forces as demonstrated in Figure 6–3. Apparently, the reaction force converged when the minimum element size reduced to 100 µm or below. Therefore, the minimum element size for the global model was chosen to be 50 µm and the minimum element size for the submodel was around 5 µm. The size of the random generated microstructural domain (2.72 mm × 2.72 mm) is decided based on the consideration of computational cost and the size of the potential crack path. Therefore, there is no direct analysis of the domain convergence in this study. However, study can be achieved to verify domain convergence in the future.
6.3 Results and discussion

6.3.1 Effect of microstructure on variability of fracture toughness

As discussed, three microstructured models of cortical bone were analysed in this study. Their results are compared with that of the effective homogeneous model as well as experimental data in terms of force – displacement diagram in Figure 6–4. Dissimilar fracture-resistance behaviours were evident for three different microstructured models. The change in the microstructure at microscopic level has a significant impact on the macroscopic fracture toughness of cortical bone. Among the three models studied, Model A demonstrated the highest critical value of $J$ -integral – 2503 N/m, while Models B and C resulted in 2369 N/m and 2212 N/m, respectively. This decreasing trend in the fracture resistance is apparently linked to the increasing volume fractions of osteons and porosity (Haversian canals in this case). From the morphological point of view, the bone remodelling process generates new Haversian systems (each including an osteon, a Haversian canal and a cement line) to replace the old, damaged regions as an adaptive process. The newly formed bone cell is usually less mineralized than its surrounding area due to the fact that mineral-concentration period lasts longer than the remodelling process [8]. As a result, a large fraction of less
mineralized osteons associated with the bone-mass and stiffness reduction has a negative impact on the overall fracture resistance of cortical bone. Still, benefiting from their low stiffness but high fracture toughness, osteons demonstrate a higher failure strain compared with interstitial matrix and, in general, offer a positive effect on fracture toughness. On the other hand, the increasing fraction of Haversian systems leads to the increase in structural compliance as a result of cavitation, hence, to an increased overall fracture strain (Figure 6–4). These mutual effects of microstructural constituents result in the variation of macroscopic fracture toughness. Significant non-linearity observed at the initial loading stage during the experiments was successfully captured using the microstructured model. Comparing the contribution of the plastic component \( J_p \) to the critical value of the \( J \)-integral \( J_c \) in each model, an increased tendency for the energy associated with plastic deformation is observed for the increase of volume fractions of osteons and porosity (Figure 6–5). Based on the above findings, it can be concluded that the bone remodelling process related with the increasing fraction of osteons and porosity changes the bone’s fracture resistance from a stress-based mode to a more strain-based mode: less tolerance in stress than in strain.
6.3.2 Microstructure-related heterogeneity of fracture process

At the global level, the effective homogeneous material model was able to capture a macroscopic response in terms of a force – displacement curve (Figure 6–4). However, the detailed fracture evolution process, especially the localised damage zone was neglected at this level. On the contrary, the heterogeneous models with random microstructures, operating within the framework of the global model with direct displacement-controlled boundary conditions, emphasise the effect of the local non-uniform stress-strain field on the crack propagation process at the microscopic level. Figure 6–6 presents contour plots for von Mises stress, maximum principal strain, equivalent plastic strain (PEEQ) and a damage scale factor for XFEM (STATUS) for Model B when the crack is approaching the state of the maximum reaction force. As evidenced from the figure, a diffused stress pattern is characteristic for the von Mises contour, while a cross-hatched strain pattern for the contour of maximum principal strain is located ahead of the crack tip (in the compressive region of the specimen) with a diffused strain pattern near it (in the tensile region). These dissimilar stress and strain patterns around the crack tip coincide with results of the previous experimental studies [40, 42, 228], in which the authors indicated that such distinctive stress and strain

![Image](image-url)
fields in tension and compressive regions could lead to realization of different damage and fracture mechanisms. Equivalent plastic strain illustrated in Figure 6–6c indicates that the area undergoes plastic deformation during the crack propagation process. The identified plastic zone around the crack tip is within 1 – 2 osteons’ radius, i.e., approximately 100 µm in length. The value seems to be higher than 17 µm reported in the experimental work [229]. One possible reason for a larger plastic zone size predicted in the developed model is the lack of multiple cracks formation in the current model, while, in reality, micro-cracks and natural imperfections (e.g. inclusions) in front of the crack tip may develop into mini cracks that can release local stress concentration, thus reducing the plastic-zone size.

Figure 6–6 (a) Contour plots for von Mises stress, (b) maximum principal strain, (c) equivalent plastic strain (PEEQ) and (d) damage scale factor for XFEM (STATUS: red colour means fully damaged material, dark blue means non-damaged material) for Model B for crack propagation at maximum reaction force ((a) and (b) represent full Model B)
6.3.3 Microstructure-related difference in toughening mechanisms

The damage scale factor denoted as STATUS (red colour means fully damaged material, dark blue means non-damaged material) in Figure 6–6d indicates that approx. 1/3 of the crack surface is still under traction force and acts as a toughening mechanism that contributes to the non-linear fracture process. The toughening mechanisms active in a radial-transverse crack specimen can be divided predominantly into three types [18]: (i) interfacial debonding as a result of the material's discontinuity at the interface between osteons and interstitial matrix – formation of the weak path by a cement line; (ii) crack diversion due to microstructural heterogeneity and material imperfections, at which a crack is redirected along the weakest part producing a twisted and deflected fracture path; (iii) uncracked-ligament bridging caused by osteon splitting and rupture acting as a post-crack toughening mechanism behind the crack tip. In this study, the microstructured models were able to capture these main features of the toughening mechanisms as shown in Figure 6–7. Figure 6–7d demonstrates...
an interface failure predicted by the model as the crack bends away from the osteon due to the discontinuity in the cement-line region. Figure 6–7e reveals the crack-diversion mechanism as crack deviates from its central line towards a weak part resulting in a twisted and deflected crack path. The uncracked-ligament bridging behind the crack tip is presented as a cohesive traction force between the damaged elements along the crack path (Figure 6–7f).

6.3.4 Analysis of crack lengths

To investigate the effect of microstructural constituents on the crack propagation process, the crack length is plotted in Figure 6–8 as a function of displacement of loading pin for the global model and three different sub-models. Their respective crack propagation paths are demonstrated in Figure 6–9, row a. The total crack length was measured until the maximum reaction force was reached. It is clear from Figure 6–8 that Model A has the longest overall crack length, while Model C has the shortest one. Comparing the respective crack trajectories, the higher crack length related to Model A is largely defined by significant crack deflections observed in Figure 6–9a. As a result of increase in the fractions of osteons and porosity from Model A to Model C, the effect of crack-deflection mechanism gradually reduces (Figure 6–9a, b and c). On the other hand, the crack-propagation rate (with respect to the loading-pin displacement) in Model C is higher at initial stage, but gradually reduces as the crack propagates through more Haversian systems, whereas Model B shows a moderate linear evolution process and Model A demonstrates an increased crack-propagation rate. It seems that an increased fraction of Haversian systems has a negative effect on the crack-propagation rate and constrains the crack-diversion magnitude. This finding is consistent with experimental observation in [230], where the authors concluded that age-related changes in morphology of microstructure as a result of remodelling process may lead to suppression of the crack-deflection mechanism and reduction of the total crack length. In comparison with the global model, introduction of heterogeneity (microstructure randomness) in the developed models evidenced a fluctuating, but increasing of total crack length compare with the homogenised model (Figure 6–8). This coincides with the findings in [231-233], where a shift (increase) of the mean crack
length was observed as a result of a mixture of both weakening and toughening components within the system.

![Figure 6–8 Crack length – displacement diagram for global and microstructured models (total crack length is measured until reaching maximum reaction force)](image)

### 6.3.5 Effect of cement line

The effect of cement line was studied by changing the magnitude of its elastic modulus within the range from 25% below and to 25% above that of osteons. The respective results for the crack propagation trajectory are compared in Figure 6–9 for three different microstructural models. The result indicates that an increase in cement line’s modulus to the levels equal to, or 25% higher than that of the osteon results in similar crack trajectories, which differ from the initial ones (i.e., for 25% lower modulus) for both Models A and B. This higher stiffness of cement line leads to some rise of crack propagation in the regions with low fracture toughness – interstitial areas (Figure 6–10). Moreover, higher stiffness also results in a higher rate of interface debonding in Models A and B (Figure 6–9b and c) where cement lines facilitate crack propagation around osteons. However, no substantial difference was found between the two groups (equal to and 25% higher). On the other hand, the lower cement modulus increases the chance of osteonal fracture and penetration into Haversian canal in Models A and B (Figure 6–10), where high fracture toughness and high compliance regions could potentially increase the overall fracture resistance and may lead to more
crack deflections and arrests. As the osteonal and porosity density increases in Model C, the effect of the local heterogeneity becomes more dominant. Cracks are likely to grow along the path of lowest resistance, and the effect of cement line diminishes. Hence, the influence on the crack-propagation trajectory is less pronounced than in two other models (Figure 6–10).

Figure 6–9 Crack propagation trajectories for various elastic moduli of cement line for three microstructured models: row (a): 25% lower than that of osteon; row (b): equal to that of osteon; row (c): 25% higher than that of osteon

In summary, the cement line plays an important role in the crack-propagation process in cortical bone. Variation of its mechanical properties can considerably affect the shape of the local crack trajectory. Both scenarios
demonstrated in the developed models have been widely discussed in previous review papers [8, 18]. Considering the fact that bone is a dynamic living tissue, the mechanical properties of cement line are likely to vary with time and locations. It is thus sensible that a 25% differences in the cement line’s modulus within the local area can cause both toughening and weakening mechanisms as observed in experiments [234].

Figure 6–10 Fractions of crack path in microstructure constituents for various magnitudes of cement line’s modulus

6.4 Conclusions

The fracture process in cortical bone was evaluated in this study based on the developed XFEM models. Three models with different random microstructures were developed and embedded into a homogeneous global model to investigate the effect of microstructural changes and the related
varying local mechanical behaviours on the fracture propagation process in cortical bone.

- The results obtained in this study indicate that local changes in volume fractions of microstructural constituents have a significant effect on variability of the macroscopic fracture toughness.

- The developed microstructured models of cortical bone are able to represent accurately the non-uniform plastic deformation associated with the non-linear fracture process as well as realization of distinct damage and fracture toughening mechanisms observed in experiments.

- The use of different statistical realizations of random microstructure demonstrated importance of the local heterogeneity on the fracture propagation process.

- Various crack propagation trajectories and crack lengths were observed with different microstructured bone models.

- The changes in the underlying microstructure of cortical bone and its mechanical properties resulted in different toughening mechanisms, which, in turn, affected the crack propagation process in a dissimilar manner.

- High volume fractions of osteons and porosity resulted in a smoother fracture surface as a result of lacking crack-diversion mechanisms; the higher stiffness of cement line supresses osteonal cracks and facilitates the interstitial damage and interface debonding.

- The knowledge obtained through the development of these microstructured XFEM models provides an additional insight into the micro-scale fracture process in cortical bone and might be used in the future to provide support and guidance for treatments against bone fracture.
Chapter 7
Experimental and Numerical Investigations of Penetration of Cutting Tool into Cortical Bone

7.1 Introduction

Cutting of cortical bone is a very complicated process involving many different aspects of understandings on the mechanical behaviour of cortical bone. Prior to the separation of bone tissue in the vicinity of the cutting zone, cortical bone deforms significantly. The anisotropic deformation behaviour of cortical bone and its intrinsic hierarchical microstructure significantly affect the stress, strain distributions around the cutting region. Upon the initiation of damage and propagation of cracks, there are various toughening mechanisms acting either as a crack barrier to deflect the crack or as a protective mechanism to increase the amount of energy required to furthering the crack. Therefore, comprehensive understanding of the anisotropic deformation and damage mechanisms involved in the bone penetration process is essential for the development of accurate modelling of bone cutting. Based on knowledge that acquired in previous chapters, it is now possible to propose and develop an advanced bone cutting model in this chapter that can fully represent the macroscopic behaviours of cortical bone during the cutting process.

To the author’s knowledge, there is no experimental data available in literature to provide enough information regarding the processes in the vicinity of the interaction zone, while numerical models do not account for material anisotropy or the effect of damage mechanisms. In addition, a conventional finite-element scheme faces numerical challenges due to large deformation (more than 20% strain) and highly localised distortion in the process zone.
In this Chapter, both experimental and numerical studies were conducted to tackle these issues and to elucidate the anisotropic mechanical behaviour of cortical bone tissue under conditions of cutting-tool penetration. Penetration tests were performed using a standard razor blade at different cutting directions with respect to the bone axis at quasi-static loading condition. The process was recorded using a micro-lens high-speed camera. A new smoothed particle hydrodynamic model incorporating anisotropic mechanical properties and damage mechanisms was developed to provide an insight into development of deformation and damage in the vicinity of the tool-bone interaction zone. Although the development of more detailed microstructured bone cutting model is theoretically achievable, it requires significant amount of additional work and is well outside the scope of this study, hence, is not discussed in this chapter.

### 7.2 Experimental procedures

Specimen preparation procedures and storage methods for this study followed the same principles as those for Vickers hardness tests and uniaxial tension and compression tests described in Sections 4.2.1 and 4.2.2. A total number of 40 rectangular specimens with dimensions of 30 mm × 3 mm × 3 mm (length × width × thickness) were prepared. The specimens were extracted for both longitudinal and transverse directions (Figure 7–1) and further categorised into four groups according to their anatomic quadrants, namely, anterior, posterior, medial and lateral, in order to reduce inconsistency caused by material’s variability across different regions observed in previous Chapters (Chapter 4).

Penetration tests were performed on an Instron MicroTester 5848 with a 2 kN load cell. The specimens were kept hydrated in saline solution prior to the experiments and then super-glued to the testing base. Five penetrations were made at each cutting directions: perpendicular to osteons (L-C and L-R planes, Figure 7–1) and along them (C-L and C-R planes) using a standard single-edge razor blade at quasi-static loading conditions with displacement-controlled loading rate at 1.8 mm/min. A high-speed camera (Fastcam SA-3,
Photron) equipped with micro-lens (AF Micro-Nikkor 105 mm f/2.8D, Nikon) was employed to capture the deformation process in cortical bone at micro-scale. The camera is capable to provide 2000 fps (frames per second) at 1024 by 1024 pixel resolution, and up to 120000 fps at reduced resolution operations. With eight-Gigabytes onboard memory, the camera was set at continuous-loop acquisition at 5000-7500 fps for optimized recording duration and signalled to stop with an event trigger when penetration force dropped by 20%.

Figure 7–1 (a) Setup for cutting experiments mounted on Instron MicroTester 5848; (b) schematic of cutting configurations and specimen dimensions, notation according ASTM E399 standard (first letter refers to normal direction, while second letter refers to the plane of motion, i.e., C-L means direction perpendicular to circumferential axis and in parallel with longitudinal axis); (c) superimposed image of razor and cortical-bone specimen taken with high-speed camera (Fastcam SA-3, Photron)
7.3 Model challenges

7.3.1 Theoretical gaps

Penetration into a bone tissue is an intrinsic part of many clinical procedures, such as orthopaedic surgery, bone implant and repair operations. The success of bone-cutting surgery depends largely on precision of the operation and the extent of damage it causes to the surrounding tissues. An excessive force generated by a sharp surgical tool or an implant device can lead to formation of micro-cracks and fracture [46, 235], and, ultimately, cause permanent damage to the adjacent area of cortical bone tissue that, in turn, can delay postoperative recovery of patients [236]. Therefore, information on deformation behaviour of cortical bone under penetration of a sharp tool is essential to understand the interaction process at tool-bone interface; this can improve control of a surgical instrument to minimize damage caused to surrounding bone tissues.

Earlier experimental research on penetration of cortical bone provided very little and rather limited qualitative results due to inadequate access to bone samples in addition to natural geometrical limitations of the tissues [164-168, 174-176] (detailed in Section 3.6). From the modelling perspective, there are only a few finite-element models [129, 176, 183-187] available in the literature trying to address issues associated with bone cutting, but more from medical or biological point of view (detailed in Section 3.6.2). Consequently, there is still no adequate model that can fully describe the material's anisotropy and damage behaviour of cortical bone under conditions of the tool-penetration process.

This lack of sophisticated models in the literature can be partially ascribes to two reasons: non-trivial material properties of cortical bone and numerical difficulties linked to simulation of the penetration process due to large deformations and element distortion. From the modelling point of view, the mechanical behaviour of cortical bone is highly anisotropic, with higher values of the Young’s modulus, yield and ultimate stresses reported for the main bone’s axis (parallel to the osteons) and lower for the transverse
direction (perpendicular to the osteons) [14, 24, 76, 139, 237]. Its mechanical behaviour also varies markedly across different anatomic quadrants [53, 188] as a result of random heterogeneous arrangement of its microstructural constituents [205] and the on-going remodelling process. Being a natural composite material (mainly consisting of mineralized nano-particles and collagen fibres), its deformation behaviour combines both brittle and ductile features. The inelastic deformation region spans from 0.27% up to 2.5% depending on the loading condition and material orientations [37, 188], while failure could well start at strain of 0.4% as reported in [138]. At microscopic level, preferentially aligned microstructural constituents and variations of their material properties activate distinct deformation and toughening mechanisms according to the applied load and its orientation, causing anisotropy both in mechanical behaviour and fracture resistance [204, 238].

7.3.2 Numerical difficulties

Modelling of large deformations induced by a penetration process has always been a challenging subject. Numerical difficulties arise when severe deformation and generation of new surfaces becomes a dominant feature in the cutting region so that conventional mesh-based finite-element methods are prone to increased errors due to large mesh distortion and convergence problems associated with topological changes. Various approaches to tackle such numerical challenges involving Eulerian and Lagrangian analysis schemes as well as unzipping, re-meshing and element-deletion techniques are usually employed separately or in various combinations.

The Eulerian approach describes the material’s deformation within a fixed-mesh domain, where material flows in and out the element mesh depending on the deformation process. Because there is no mesh distortion involved, it is preferable for modelling extreme deformation processes, but in a steady state condition where the initial deformation-transition process can be neglected [239].

An arbitrary Lagrangian-Eulerian (ALE) approach combining the advantages of both Eulerian and Lagrangian schemes was introduced to handle large
distortions within the Lagrangian mesh domain. Mesh distortion is controlled by remapping adaptively an internal mesh framework while maintaining its original topology. As a result, simulations of large deformations involving topology changes or generation of free surfaces (e.g. in modelling of cutting) can only be realized in conjunction with a pre-defined surface-separation criterion [240], such as element unzipping.

Remeshing and element-deletion are other modelling approaches broadly used to deal with large element distortion. However, they all suffer from some round-off errors: errors occur in re-meshing due to transformation of the associated mesh domain, with the stress field calculated in previous increment usually neglected and accuracy of results affected by a number of increments and a mesh size; numerical instability as a result of material loses due to element-deletion and consequently unbalanced equilibrium accumulation, which can diminish accuracy of the results [241]. Hence, it is necessary to develop a new modelling approach to solve the element-distortion problem without compromising accuracy of simulations.

7.4 Modelling specification

7.4.1 SPH approach

In recent years, the development of new meshless FE algorithms has greatly assisted modelling on large-deformation processes [239]. Smoothed particle hydrodynamics (SPH), first proposed by Gingold and Monaghan [242], is one of meshless techniques that have been implemented to simulate machining processes [239, 243]. It was initially implemented for complex physics such as astrophysical or geophysical problems, where discrete particles could be used in a relatively inexpensive way to model the interactions between physical entities such as planets [242]. The interpretation of the relationship between particles utilizes the kernel estimation (based on the Monte Carlo method) to represent processes of hydrodynamics or continuum mechanics derived from equations of motion, where the properties of particles could be calculated [242]. The energy equilibrium state of the system is achieved by
using a Lagrangian method for conserved linear and angular momenta. The improvement over the past decades resulted in an expansion of the algorithm to continuum mechanics [242].

Similar to other particle-based interpretations, such as discrete element method [244] and material point method [241], the advantage of SPH is in its combination features of meshless methods and those of the Lagrangian formulation. Here, traditional elements and nodes as well as a superposed mesh domain are substituted by discrete particle elements. Adjacent particles are interconnected with the central point within the predefined radius called smoothing length \((h)\). Interactions between them are defined through a gradient influence dependent on the relative distance defined by the chosen kernel function (Figure 7–2). Benefiting from its mesh-free discretization method, the shortcomings associated with traditional finite-element schemes, e.g. in cases of large deformations and generation of free surface, could be resolved in a natural way. Each integration point in SPH not only has more than one neighbouring element but also interchangeable neighbours during each interpolation of field variables, which provides a smoother interpolation results than conventional FE methods. Absence of predefined neighbouring elements also means that it is suitable for applications such as impact, explosion, splitting, brittle fracture and fragmentation that involve generation of free surfaces. Comparing with the Coupled-Eulerian-Lagrangian (CEL) method and pure Lagrangian method, SPH is less expensive (computational cost) than CEL but more accurate than the Lagrangian method in high deformation scenario [211].

In this chapter, a new 3D finite-element modelling approach – encompassing both conventional and SPH elements – was developed in Abaqus [211]. Benefiting from its Lagrangian formulation and the absence of a fixed mesh grid, the SPH elements were implemented directly along the penetration path to analyse large deformations which cannot be computed accurately using conventional elements, while conventional elements were employed elsewhere to diminish the computational cost and enhance accuracy at small deformations. With the SPH formulation, spatial derivatives calculated based
on a kernel-discretization method, which interpolates properties and state variables directly at a discrete set of particles located within a given distance ($2h$) without pre-defined neighbouring elements; the radius $h$ of this interaction area is defined as the smoothing length. Within this interaction zone, the influence of each particle is weighted by the kernel function according to their relative distance to the point of interest.

**Figure 7–2** Interpolation of gradient influence of neighbouring particles through kernel function $W(x)$ within a circle of $2h$ [245]

In this study, a cubic-spline kernel function was selected for its good properties of regularity that suit most of the application demands [211]. The smoothing length was chosen to be 2.2 times of the characteristic length (i.e., imaginary particle radius) of the associated particle volume. The selected smoothing length allows for some 30 connections of particle elements for each element at the beginning of the simulation for a balanced computational efficiency and accuracy. The maximum allowable connections for each element were set to be 140 as by default [211].

### 7.4.2 Model geometry

The FE models were configured in accordance with the experimental setup. A plane-strain condition was assumed throughout the thickness of the specimen, and, therefore, for the computational efficiency, the cortical bone specimen was modelled with the following dimensions: $6 \text{ mm} \times 3 \text{ mm} \times 0.02 \text{ mm}$ (length $\times$ width $\times$ thickness), with symmetric boundary conditions applied
to both front and back sides in the x-y plane as shown in Figure 7–3. The bottom surface of the specimen was fully constrained, while two lateral edges were constrained in the y-z plane. Particle elements (PC3D) were implemented in the middle section of the specimen with a width of 0.4 mm in the x-y plane (Figure 7–3b). The remaining two sections were modelled using continuum elements (C3D8R) with a single-bias mesh transition. Tie constrains were applied at the boundaries between continuum and particle elements. To maximize computational efficiency, the razor was modelled as an analytical rigid body at current stage. Its geometry was measured using a 3D scanning optical microscope (OGP Smartscope Flash 200): the radius of cutting tip was 4 μm; the wedge angle and vertical edge height were 18° and 0.7 mm, respectively, see Figure 7–3. A total number of 455,106 equally spaced particle elements were used with a fixed spacing of 4 μm. Each simulation with the model running on an eight-processor (quad-core Intel I7 970 CPU) PC using Abaqus explicit solver lasted for 12 days. Prior to the main simulations, model reliability tests were conducted to evaluate sensitivity of the model to changes in the mesh size and geometrical features. Geometrical analysis indicated that variation caused by geometrical dissimilarity (the size of SPH particle section in this case) remained insignificant (Figure 7–4a). Mesh sensitivity studies were performed for three different mesh sizes: 2 μm, 4 μm and 10 μm, and a mesh size of 4 μm was chosen for both computational efficiency and accuracy. Tensile instability of particle elements that is one of SPH problems was less profound in the developed models since the deformed area was mainly under compressive load.
Figure 7–3 (a) Geometry of razor-penetration model using SPH method (grey colour presents shape of experimental specimens, red is used for FE model); (b) x-y plane of FE model with SPH domain (represented by red region).

Figure 7–4 (a) Contour plots of vertical displacement of two models with different sizes of particle sections indicate there is no significant variation due to geometrical change; (b) comparison of evolution of energy ratio (kinetic energy/internal energy) with penetration depth for different loading rates.
To ensure that the simulation results are within the confined quasi-static conditions, verification tests with different loading speeds were conducted. The effect of inertia is compared in Figure 7–4b in terms of the ratio of kinetic energy to internal one. Results shown in Figure 7–4b demonstrate that the overall kinetic energy does not constitute a significant part – it is less than 5% – of the entire internal energy in the model when the speed is below 10 m/s. Hence, the level of loading speed was chosen at 1 m/s throughout simulations. Friction between the razor and the bone specimen was modelled using a classical Coulomb’s model with a coefficient of friction of 0.3 following previous results [246].

### 7.4.3 Material properties

The mechanical behaviour of specimens of bovine cortical bone was modelled as that of a macroscopically homogenized media with transversely isotropic elastic-plastic material properties (see Table 7–1) employing a Hill’s anisotropic yield criterion and progressive material degradation. The material properties used in the models were mostly obtained from previous experiments and results within the research group (MoAM) [53, 188, 204]. The anisotropic yield behaviour was described in terms of the Hill’s yield criteria in quadratic form:

\[
F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 = 1 , \tag{7 - 1}
\]

where \( \sigma_{ij} \) are components of the stresses tensor, and parameters \( F, G, H, L, M \) and \( N \) are constants that can be determined by tests. Subscripts 1, 2, and 3 denote here the three perpendicular bone axes: longitudinal (parallel to the main axis of femur), radial and circumferential, respectively (see Figure 7–1). The six Hill’s constants in Table 7–2 were calculated based on the literature data in [23]. Isotropic strain hardening for different directions was defined using previous experimental data (Table 7–3) [53, 188]. Criteria for damage initiation and evolution were chosen based on experimental observation in [37]. Damage initiation in a hard biological tissue was commonly introduced
as strain-driven criteria [98]; therefore, the onset of damage was assumed when a fracture strain of 2% was reached [188]. Once initiated, the damage evolution process can be described employing an energy-based approach. The evolution criterion defines progressive degradation of the material in two forms: decrease in the yield stress and stiffness degradation. Based on the continuum-damage-mechanics theory, the criterion assumes that material damage increases gradually up to its complete failure when the energy dissipated at the onset of damage is equal to a critical energy release per unit area. A summary of material properties used in the numerical models is given in Tables 7–1, 2 and 3.

<table>
<thead>
<tr>
<th>Table 7–1 List of material properties used in simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Property</strong></td>
</tr>
<tr>
<td>Density (kg/m³)</td>
</tr>
<tr>
<td>$E_{11}$ (GPa)</td>
</tr>
<tr>
<td>$E_{22}, E_{33}$ (GPa)</td>
</tr>
<tr>
<td>$\nu_{12}, \nu_{13}$</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
</tr>
<tr>
<td>$G_{12}, G_{13}$ (GPa)</td>
</tr>
<tr>
<td>$G_{23}$ (GPa)</td>
</tr>
<tr>
<td>$\varepsilon_{y11}$ (%)</td>
</tr>
<tr>
<td>$\varepsilon_{f11}, \varepsilon_{f22}$ (%)</td>
</tr>
<tr>
<td>$J_{IC11}$ (N/m)</td>
</tr>
<tr>
<td>$J_{IC22}$ (N/m)</td>
</tr>
</tbody>
</table>
Table 7–2  Hill’s parameters for cortical bone calculated using experimental data from [23]

<table>
<thead>
<tr>
<th></th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>L</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.8</td>
<td>0.5</td>
<td>0.5</td>
<td>3</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 7–3  Post-yield stress-strain relationships for different cortices

<table>
<thead>
<tr>
<th>Flow Strain (%)</th>
<th>Flow stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Anterior</td>
</tr>
<tr>
<td>0.00%</td>
<td>215.9</td>
</tr>
<tr>
<td>0.08%</td>
<td>226.73</td>
</tr>
<tr>
<td>0.23%</td>
<td>237.29</td>
</tr>
<tr>
<td>0.47%</td>
<td>244.67</td>
</tr>
<tr>
<td>0.72%</td>
<td>252.51</td>
</tr>
<tr>
<td>0.98%</td>
<td>258.25</td>
</tr>
<tr>
<td>1.23%</td>
<td>263.75</td>
</tr>
<tr>
<td>1.47%</td>
<td>269.49</td>
</tr>
<tr>
<td>1.72%</td>
<td>275.16</td>
</tr>
</tbody>
</table>

7.5  Results and analysis

7.5.1  Experimental results

Penetrations with the razor were made for different penetration directions: perpendicular to the osteons (L-C and L-R planes) and parallel to them (C-L and C-R planes), and the respective results are compared in terms of the maximum cutting force per unit thickness for the four anatomic quadrants in Table 7–4. The obtained experimental data demonstrated significantly anisotropic behaviour for different penetration directions and considerable variations across different cortices. Generally, cortical bone exhibited a
higher resistance force when the razor cut perpendicular through osteons (L-C and L-R), and a significantly lower force was obtained when the cut was parallel to the osteons (C-L and C-R). Among different cortices, specimens obtained from the medial cortex had the highest average penetration forces while those from the posterior cortex were the lowest. This disparity in the level of maximum penetration force across different cortices of the bovine femur indicates the influence of the microstructural changes along the bone’s circumference on the local material deformation process. However, statistical analysis using Tukey HSD tests ($\alpha = 0.05$) indicated significant difference only between the posterior and lateral cortices ($p = 0.03$) for penetration parallel to the osteons and no statistical significance was found for penetration made along the direction perpendicular to the osteons. This result implies that the post-yield behaviour does not have a stable correlation with the variation of the microstructure [188]. At large deformations, a material behaviour can be complex as a result of a combined effect of stiffness, toughness and localised damage mechanisms, which is an additional randomising factor diminishing the extent of generality of the analysis.

<table>
<thead>
<tr>
<th>Force (N/mm)</th>
<th>Anterior</th>
<th>Posterior</th>
<th>Medial</th>
<th>Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Perpendicular to osteons</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L-C</td>
<td>188.8±33.5</td>
<td>152.1±45.5</td>
<td>208.6±34.4</td>
<td>170.4±39.9</td>
</tr>
<tr>
<td>L-R</td>
<td>192.1±23.3</td>
<td>159.6±32.3</td>
<td>214.7±14.0</td>
<td>175.9±40.2</td>
</tr>
<tr>
<td><strong>Parallel to osteons</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-L</td>
<td>108.8±15.9</td>
<td>73.6±17.4</td>
<td>104.8±30.5</td>
<td>124.6±28.0</td>
</tr>
<tr>
<td>C-R</td>
<td>103.1±21.8</td>
<td>71.2±26.2</td>
<td>110.3±31.6</td>
<td>117.9±38.3</td>
</tr>
</tbody>
</table>

The effect of orientation of penetration with regard to microstructural elements also plays an important role in terms of anisotropy of cutting forces.
In general, the energy required to break an osteon is higher than that used to split it. As a result, higher cutting forces were measured when the cutting direction was perpendicular to osteons; while lower cutting forces for penetration parallel to direction of osteons. The anisotropic ratios for two cutting directions are demonstrated in Table 7–5. Apparently, the anisotropy ratio varied from cortex to cortex in the range from 1.43 to 2.15, where the lowest ratio was found in the lateral quadrant and the highest ratio in the posterior one.

<table>
<thead>
<tr>
<th>Anisotropic ratio perpendicular /parallel</th>
<th>Anterior</th>
<th>Posterior</th>
<th>Medial</th>
<th>Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.80</td>
<td>2.15</td>
<td>1.95</td>
<td>1.43</td>
</tr>
</tbody>
</table>

Images taken with the high-speed camera also demonstrated distinct damage phenomena that were closely attributed to the underlying microstructure and its orientation (see Figures 7–5 and 7–6). Various microstructure-related toughening mechanisms were activated for different penetration directions. Generally, for penetration along the longitudinal axis (C-L direction), damage was well ahead of the cutting tip and mainly caused by splitting and separation. As a result, the toughening mechanism was dominated by the uncracked-ligament bridging during the process of osteons’ splitting, rupture and interface failure (Figure 7–5 a-c). Similarly, for penetration along the radial direction (C-R), there was still the uncracked bridging effect due to splitting of osteons or fibres/fibrils. However, microstructural constituents had more influence on the toughening mechanisms than in the previous case. Crack deflection as a result of material’s heterogeneity towards, and arrest at, cavities such as Haversian canals were observed (Figure 7–5 d-f). In contrast to the previous two cases, penetration perpendicular to the osteons (L-C and L-R) required higher energy to propagate through the region due to its higher fracture toughness. Cracks were therefore more likely to be deflected transversely to the cutting
direction as a result of material’s imperfections and heterogeneity (Figure 7–6). Consequently, higher penetration forces were the cause of larger deformations and damaged areas. There were two types of damage mechanisms observed during current experiments: a more brittle damage pattern involving fragmentation and material’s peeling off was predominantly observed at the plexiform bone region (Figure 7–6 g-i); a more diffused ductile damage pattern was associated with large deformation of the osteonal structure (Figure 7–6 j-l).

**Figure 7–5** High-speed camera images captured distinct damage processes during cutting parallel to osteons’ direction: (a-c) C-L direction; (d-f) C-R direction; white lines designate profile of razor blade, red dotted lines indicate crack path and white arrows point at positions of osteons
Figure 7–6 High-speed camera images captured distinct damage processes during cutting perpendicular to osteons’ direction (L-C and L-R): (g-i) brittle damage pattern that was predominantly observed at plexiform-bone region; (j-l) diffused ductile damage pattern associated with large deformation of osteonal structure; white lines designate profile of razor blade and arrows point at positions of cracks

7.5.2 Numerical results

The current implementation of SPH (in Abaqus [211]) suffers from inconsistencies when symmetric boundary condition is applied to the centre layer of the SPH domain directly underneath the cutting tool. Therefore, no symmetric boundary is applied at this position. To gain further detailed understanding of this anisotropic deformation behaviour of cortical bone in the vicinity of the cutting tip, numerical simulations were conducted using the developed SPH-based finite-element approach. A total of eight models were developed for two penetration directions (perpendicular and parallel to osteons) at four different cortices. Contour plots demonstrated in Figures 7–7 to 7–9 reveal the detailed character of localised stresses and strains caused by material anisotropy around the cutting tip for cutting perpendicular to osteons. The evolution of deformation process in cortical bone escalated as the penetration depth increased. It started in a relatively small region
localised around the cutting tip with a high concentration of stresses (Figure 7–7a). As penetration continued, deformation accumulated and the material in front of the cutting tip underwent an increased plastic deformation (Figure 7–9b). Formation of a large shear band can also be observed at the forefront of the cutting blade as a result of combined effect of the wedge angle and material’s anisotropy (Figure 7–8b). This deformation pattern continued to expand around the cutting blade until a saturation state was reached, at which the cutting force attained its maximum value. After that point, plastic deformation and damage mechanisms became increasingly dominant, and the material started to deform plastically towards the upper free edge with more damage brought to the surrounding region (Figure 7–9d). Material softening accompanied with high plastic deformation caused the material disintegration and stiffness reduction (Figures 7–7e to 7–9e). Finally, reduction of the cutting force was observed as a result of complete failure of the material.

![Figure 7–7 Distribution of equivalent stress (in MPa) for different levels of penetration depth D, white box represents SPH domain](image-url)
Figure 7–8 Distribution of shear stress component ($\sigma_{12}$; in MPa) at different penetration stages, white box represents SPH domain.

Figure 7–9 Distribution of equivalent plastic strain component at different penetration stages, white box represents SPH domain.
It is clear that the progressive damage mechanisms strongly affected the deformation process. It is especially true for those quasi-brittle or low-ductility materials which have no or limited plastic deformation [163], such as cortical bone tissue. Therefore, damage should be considered when simulations deal with large-deformation processes in such materials [163]; neglecting such behaviour may lead to inadequate or erroneous solutions. A comparison study for two models – with and without account for the progressive damage mechanism – is illustrated in Figure 7–10. The models were compared in terms of force-displacement curves: the model that did not take into account damage showed an unrealistic continuous growth of the cutting force (indicated by the dotted line, Figure 7–10); on the contrary, when damage was accounted for, after a peak, the cutting force degraded progressively with displacement in a way similar to that observed in experiments.

![Figure 7–10](image)

**Figure 7–10** Evolution of cutting force (per unit width) with penetration depth in cases with and without account for damage.

A comparison between simulations and experimental data for relationships between the levels of specific force (per unit width) and displacement (i.e., penetration depth D) demonstrates that the obtained simulation results are well within the range of experimental data (presented by error bars in Figure 7–11) for different cutting directions and cortices. Apparently, the relationships between the cutting force and the penetration depth were linearly correlated up to somewhat below the maximum cutting force, and
their initial slopes for the both cutting directions were similar. However, the maximum levels both of force and displacement for specimens been cut parallel to osteons’ directions (C-L or C-R) were much lower than those for other directions. This lower resistance could be attributed to the distinct microstructure-dependent deformation and damage mechanisms that were observed in current experiments (Figures 7–5 and 7–6). By incorporating this orientation-dependent material formulation and damage mechanisms, the developed models were able to reproduce the anisotropic character of failure with both forces and displacements predicted adequately for various cutting directions and cortices. Dissimilar damage and fracture processes captured by the developed models reproduced experimental observations (see Figure 7–12). A comparison of simulation results with images taken with the high-speed camera and with optical microscopy for damage induced by cutting both directions are demonstrated in Figure 7–12.

**Figure 7–11** Force (per unit width) – displacement diagrams for cutting of cortical bone in different orientations across four cortices; ⊥ and // denote cutting perpendicular and parallel to osteons’ direction, respectively
At the maximum cutting forces, the level of damage, presented by a damage scalar index SDEG in simulations in Figure 7–12, was much higher for cutting parallel to the osteons (Figure 7–12b) than perpendicular to them (Figure 7–12a). Lower fracture resistance for the C-L and C-R directions also facilitated damage propagation in front of the cutting tip as observed both in simulations and experiments (Figure 7–12b, d, f), while higher fracture toughness (L-C and L-R directions) resulted in mainly lateral damage (Figure 7–12a, c, e). Consequently, lower cutting forces were the result of higher material degradations when cutting parallel to osteons (Figure 7–11).

Figure 7–12 Comparison of simulation results (a, b) with images taken with high-speed camera (c, d) and optical microscopy (e, f) of damage induced by cutting along different directions: (a, c, e) perpendicular to bone axis, lateral damage propagation was observed; (b, d, f) parallel to bone axis, damage was in front of cutting tip. White lines designate profile of razor blade and red dotted lines indicate crack lines and arrows point at crack path.
Another way to quantify the distinct damage evolution processes for two different cutting directions is to use the ratio of energy dissipation for damage (ALLDMD in Abaqus) and plastic deformation (ALLPD) as shown in Figure 7–13. Apparently, both cutting directions share some similar features in evolution of this ratio. At the beginning, the energy ratio was similar for both directions, followed by a sudden decrease and a plateau, and then by a stage with a rapid growth in damage energy dissipation due to higher material degradation. However, this increase in the damage energy dissipation happened rather earlier when cutting parallel to the osteons’ direction; the overall damage ratio in this case was also higher compared with that for cutting perpendicular to the osteons’ direction. As a result, considerably lower cutting forces were obtained when cutting parallel to osteons.

![Figure 7–13](image)

**Figure 7–13** Evolution of energy dissipation ratio for damage (ALLDMD) and plastic deformation (ALLPD) with penetration depth for different cutting directions

### 7.6 Discussion

Cortical bone tissue has a unique ability to adapt itself to the loading environment through the remodelling process. Previous research (Chapters 4, 5, and 6) on its mechanical behaviour showed that the changes in the volume...
fraction of constituents at microstructural level largely affected the local material properties, such as the elastic modulus, yield, fracture toughness, which, in turn, influenced the macroscopic mechanical behaviour [53, 188, 205]. Microstructure of cortices that changes from a plexiform bone-dominant region to an extensively remodelled osteonal bone affected the mechanical properties of cortical bone [14] and altered its deformation character from a quasi-brittle damage pattern to a more diffused ductile damage. The statistical insignificance for the cutting forces across different cortices reported in this study could be explained as a result of the combined effects of multiple factors attributed to the complex deformation process of cortical bone that diminished the statistical power of the analysis. Additionally, the remodelling process in cortical bone does not favour any particular quadrant. As a result of natural adaptation process, cortical bone tissue continuously relocates its vital material components to the most needed region to cope with mechanical requirements through the remodelling process. Therefore, some cortices, like anterior and medial, might have higher stiffness [188] to sustain high-stress environment, while the lateral and posterior quadrants are modified into a strain-tolerant state [204, 205] to withstand large deformations. This self-adaptation process enhances the response of particular regions to the local requirements and also improves the performance of bone with regard to catastrophic failure.

The developed numerical model successfully captured the non-uniformity and anisotropy of deformation and damage processes in cortical bone under penetration across varies cortices, and the obtained results agreed well with current experimental findings. The anisotropic damage criteria employed in the models were able to reproduce the orientation-dependent damage characteristics observed both in the current experimental tests and literature [248]. However, microstructure-related toughening mechanisms observed in the experiments such as uncracked-ligament bridging, crack deflection due to microstructural heterogeneity and orientation were not accounted in the current development since the microstructural constituents were not introduced directly into the homogenized model. Characterisation of the deformation behaviour linked to the local microstructure would certainly
assist further understanding of the main processes; however, a microstructural modelling approach for the current case would be computationally impractical; it is well beyond the scope of this study and not discussed here.

7.7 Conclusions

In this chapter, the anisotropic deformation and damage mechanisms arising in a process of penetration into cortical bone were evaluated using a combination of experimental and numerical approaches. Both the anisotropic character and variability of deformation processes along with orientation-dependent damage mechanisms were observed in the experiments.

- The penetration forces were largely dependent on orientation of the underlying microstructure: higher forces were measured for cutting perpendicular to osteons and considerably lower cutting forces in the case of penetration parallel to osteons.

- Distinct toughening mechanisms were activated for different penetration trajectories with respect to microstructure orientation.

- The newly developed SPH-based finite-element model demonstrated a good agreement compared with experimental observations for various cutting directions and cortices.

- The results clearly demonstrated the necessity for incorporating the anisotropic material behaviour and progressive damage mechanisms into the modelling scheme in order to characterise the mechanical behaviour of the cortical bone tissue for various loading conditions and orientations.

- The model provided a useful insight into the deformation behaviour of, and damage mechanisms in, the anisotropic material under such loading conditions.
• The developed model provided a powerful analytical tool for further studies that can underpin improvement of the accuracy of orthopaedic surgical procedures and optimization of surgical cutting tools.
Chapter 8
Conclusions and Future Work

8.1 Conclusions

The main purpose of this research project was to bridge gaps in current knowledge on penetration of a cutting tool into a cortical bone tissue and respective deformation and fracture processes in the vicinity of tool-bone interaction zone. This was achieved, firstly, by conducting an extensive literature review on mechanics of bone tissue and its various modelling schemes involving deformation, fracture and cutting processes, then followed by both experimental and numerical studies. The experimental part of this research focused mainly on two aspects: to characterise the material properties of cortical bone that can be used in FE models to adequately and accurately represent the cortical-bone’s behaviour during the cutting process, and to validate the developed finite-element models through experimental procedures and analyse the effect of underlying microstructural orientation and distribution on the deformation and fracture processes of cortical bone. On the other hand, several numerical models were created based on the experimental findings of this study to provide detailed insights on these anisotropic deformation and fracture processes in the cortical bone tissue. The results of this study made valuable contributions to our existing understanding of the mechanics of cortical bone tissue, and most importantly, of its mechanical behaviours during the cutting process. Main parts of the study have been published in peer-reviewed scientific journals as listed between pages V–VII. Based on the objectives identified above, the current research brought upon the following conclusions.

8.1.1 Experimentation

The experiments were designed and conducted as part of continuation and extension on the existing research at Loughborough University. The new
experiments performed in this study included: the Vickers hardness test, uniaxial compression test, fracture toughness test and penetration test.

**Vickers hardness and uniaxial tests:**

The Vickers hardness tests were performed to provide initial assessments of the deformation and damage processes in the cortical bone tissue under a concentrated compressive load, similar to the initial stage of the cutting process. The experimental results demonstrated a strong variation of hardness values across different cortices, and a significantly anisotropic fracture character was also observed.

In order to further comprehend the mechanical behaviours of cortical bone and also to provide the accurate material properties for the development of finite-element models, the uniaxial tension and compression tests were conducted and their results were compared within the four anatomic cortices. The undertaken experiments, for the first time, characterised the full anisotropic elastic-plastic stress-strain responses for both tensile and compressive loadings using specimens acquired from same animal. Still, significant variability of the material properties of cortical bone was observed. The experimental results provided valuable insight of the distinct deformation mechanisms of cortical bone under various loading conditions and also demonstrated the necessity to address the variability of material properties in the development of FE models. Based on the results obtained, the following conclusions could be drawn:

- The deformation and damage behaviours of cortical bone depended significantly on the applied load.
- When bone is loaded in its natural mode i.e., along the longitudinal direction in compression, it offers the best load-bearing capacity both in terms of ultimate stress (average: 214.39 MPa) and strain (average: 2.37%). Outside this natural loading mode, mechanical performance of cortical bone reduces dramatically: only a half of its highest stiffness and one fifth of its highest strength were demonstrated for tension in
transverse direction. Therefore, it is important to take the anisotropic behaviour of cortical bone into account in FE models.

- The heterogeneous distribution of microstructural constituents contributed significantly to the variability of mechanical properties of cortical bone observed in this study as well as in the literature [53]. Statistical analysis revealed the strong correlations between the variation of elastic modulus and histological quadrants; this finding coincided with that in [50, 207], where the authors concluded a possible link between the bone’s mechanical properties and the mechanically induced bone adaptation process. Therefore, to simulate cortical bone’s material behaviour more accurately, the variations of its material properties should be addressed appropriately in FE models.

- The microstructural analysis additionally confirmed this transition at microscopic level between anatomic quadrants. The calculations of the effective Young’s modulus adequately reproduced the experimental results, which provided another evidence of the strong relationship between microstructure and elastic modulus. This finding can be used as additional tool to further assist the development of adequate bone models.

Fracture toughness test

In order to quantify the anisotropic fracture toughness of cortical bone for implementations into the finite-element models and to understand the underpinning mechanisms of, and processes associated with, damage and fracture, fracture toughness tests were performed using single-edge-notched specimens on a three-point bending test rig. For the first time, the non-linear elastic-plastic fracture toughness parameter $J$-integral was characterised in three different directions. Distinct fracture surfaces were observed at different crack propagation directions and cortices positions using SEM images. Based on the results obtained from this experimental analysis, the following conclusions can be summarised:
• A non-uniform, elastic-plastic fracture behaviour of bovine femoral cortical bone tissue was characterised. The determined mean values of the fracture toughness parameter: $J$ -integral, were in a good agreement with the existing literature data [18]; the range was from 983 N/m to 5661 N/m. However, the large variation of the fracture toughness of cortical bone is not only related to the crack propagation directions, but also due to the variation of the microstructural distribution.

• The fracture toughness of cortical bone demonstrated a significant orientation dependency: higher resistance when a crack grows perpendicular to the osteons’ direction, while a lower fracture resistance for a crack growing parallel to that of osteons.

• Among different cortices, medial and lateral quadrants had the highest fracture toughness values for crack propagating perpendicular to and parallel to osteons, respectively. This variation and anisotropy of the fracture toughness were underpinned by the preferential alignment of microstructural constituents and their transition along the cross-section of cortical bone.

• It is therefore, necessary to address the orientation dependent damage and fracture behaviour in FE models.

**Penetration tests**

The penetration tests were performed in order to: (a) evaluate the cutting induced anisotropic deformation and damage patterns of cortical bone tissue; (b) provide quantitative results for validation of the developed FE models. Based on the obtained experimental results, the following conclusions are made:

• The obtained experimental data demonstrated a significantly anisotropic behaviour, and the cutting forces were largely dependent on the orientation of the underlying microstructure.
• Generally, higher forces were measured for cutting perpendicular to osteons (152.1 N – 214.7 N) and considerably lower ones in the case of penetration parallel to osteons (71.2 N – 124.6 N).

• Among different cortices, the specimens from the medial cortex had the highest average penetration forces as a result of combination of higher stiffness and toughness (as measured in the previous tests), while those form the posterior cortex were the lowest due to poor stiffness and toughness. However, no significant statistical difference was found between different cortices. This result suggested that the post-yield behaviour did not have a stable correlation with the variation of the microstructure as a result of the combined effect of stiffness, toughness and localised damage mechanisms.

• Various microstructure-related toughening mechanisms were activated for different penetration directions. Generally, for penetration along the longitudinal and radial axes (C-L and C-R directions), damage was well ahead of the cutting tip, and the toughening mechanism was dominated mainly by the uncracked-ligament bridging. On the contrary, for penetration perpendicular to the osteons (L-C and L-R), cracks were more likely to be deflected transversely to the cutting direction due to higher fracture toughness along the cutting plane. The results also indicated damage patterns were directly linked to the underline microstructure: a brittle damage pattern involving fragmentation and material's peeling off was predominantly observed at the plexiform bone region, while a more diffused ductile damage pattern was associated with large deformation of the osteonal structure (Figure 7–6).

8.1.2 Modelling

*Effective homogenised XFEM models:*

The developed homogenised XFEM models successfully captured the non-uniform, anisotropic elastic-plastic fracture behaviour of the cortical bone tissue and were validated through fracture toughness testing data obtained
from the previous experimental tests. The results of finite-element simulations demonstrated a very good agreement with the obtained experimental data. The developed finite-element models successfully realised the variability of material responses across four cortices for both longitudinal and transverse crack directions. It strongly indicated that it is necessary to consider the cortical bone tissue as an anisotropic material in the future development. The results also indicated that the fracture toughness values were largely affected by the local anisotropic material properties linked to the variation of microstructure constituents.

Microstructured XFEM model

In order to capture the effect of microstructure constituents on the local fracture process of cortical bone, three random microstructured cortical bone models with different volume fractions of microstructural constituents were developed as an extension to the previous homogenised model in order to investigate both the non-linear fracture process at micro-scale and the microstructural effect on the variability of fracture toughness at macro-scale. Summarising analysis of the developed models and their results, the following conclusions are made:

- The results obtained from the developed models indicated that local changes in the volume fraction of microstructural constituents had a significant effect on variability of the macroscopic fracture toughness.
- The developed microstructured models of cortical bone, for the first time, were able to represent accurately the non-uniform plastic deformation associated with the non-linear fracture process as well as realization of distinct damage and fracture toughening mechanisms observed in experiments.
- The use of different statistical realizations of random microstructure demonstrated importance of the local heterogeneity on the fracture propagation process.
- Various crack propagation trajectories and crack lengths were observed in different microstructured models of bone. The changes in
the underlying microstructure of cortical bone and its mechanical properties resulted in different toughening mechanisms, which, in turn, affected the crack propagation process in a dissimilar manner.

- High volume fractions of osteons and porosity resulted in a smoother fracture surface as a result of lacking crack-diversion mechanisms; the higher stiffness of cement line suppressed osteonal cracks and facilitated the interstitial damage and interface debonding.

- The knowledge obtained through the development of these microstructured XFEM models provided an additional insight into the micro-scale fracture process in cortical bone.

**SPH modelling of penetration of cutting tool into cortical bone**

A new 3D finite-element modelling approach – encompassing both conventional and SPH elements – was developed. The developed models combined the advantages of accuracy of the conventional finite-element method and of the mesh-free Lagrangian formulation of the SPH approach and, therefore, were adequate to simulate large deformations and damage processes linked to penetration of bone, which has been a challenge for many years. Based on the obtained results from the developed models, the following conclusions are drawn:

- The new developed SPH-based models successfully captured the non-uniformity and anisotropy of deformation and damage processes in cortical bone under penetration across varies cortices, and the obtained results agreed well with the experimental findings.

- It was clear that the progressive damage mechanisms affected strongly the deformation process. The anisotropic damage criteria employed in the models were able to reproduce the orientation-dependent damage characteristics observed both in the current experimental tests and literature [248].

- The results clearly demonstrated the necessity to incorporate the anisotropic material behaviour and progressive damage mechanisms
into the modelling scheme in order to characterise the complex response of the cortical bone tissue to various loading conditions and for different orientations.

- However, the microscopic heterogeneity is not preserved in this model due to the time and computational limitation within this study. It is possible to further develop the existing models for the implementation of the microstructural cutting models.

- The model gave a useful insight into the deformation behaviour of, and damage mechanisms in, the anisotropic material under such loading conditions. It provided a powerful analytical tool for further studies that can underpin improvement of the accuracy of orthopaedic surgical procedures and optimization of surgical cutting tools.

### 8.2 Recommendations for further work

The results that were highlighted in this research project achieved a number of new findings; still, some new topics arose during the course of the study which could benefit further research.

#### 8.2.1 Experimentation

- Characterisation of the anisotropic mechanical properties of each microstructural constituent of the studied cortical bone tissue.
- Characterisation of the anisotropic visco-elastic-plastic material properties of cortical bone tissue.
- Study on the mechanical behaviour of cortical bone tissue at various strain rates would be another future area, especially the damage and fracture process under dynamic loading.
- Further development on understanding the R-curve behaviour.
- Further development on 3D imaging technique in order to differentiate microstructural constituents in 3D that can be used in future FE models.
8.2.2 Modelling

- Develop 3D microstructured cortical bone models could help to create a more realistic numerical representation of cortical bone tissue.
- Further improvement of the XFEM-based models to include crack branching and intersecting would improve the prediction and accuracy of the current models.
- Further development of SPH based microstructural bone model based on existing penetration model by incorporating material heterogeneity.
- Development of the comprehensive constitutive models of the cortical bone tissue including anisotropic visco-elastic-plastic behaviours and its progressive damage criteria is a long-term process worth of continuous investment.
- Further development of material constitutive model to realise the asymmetric tension/compression behaviour of cortical bone in conjunction with the existing anisotropic material model will further enhance the fidelity of the FE models in much broader applications.
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