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Additional Information:

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Metadata Record: https://dspace.lboro.ac.uk/2134/13737

Version: Published

Publisher: © CRC Press (Taylor & Francis Group)

Please cite the published version.
Transient effects of high speed trains crossing soft soil
Les déplacement dynamique sous des trains de grande vitesse sur sol mou

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Keywords: high speed trains, dynamic magnification

ABSTRACT: To reduce the environmental impact (in terms of visibility and noise) it is desirable to construct inter-urban high-speed rail lines with small embankments. However, these small embankments tend to be flexible and on soft ground track-soil bending waves may result in significant transient train-induced soil deflections. These deflections in the permanent way could, in turn, have a major effect on ride quality and also on maintenance costs. A variable frequency inertial vibrator and a series of geophones have been used to examine the response of soil both with and without a rail. The measured soil responses have been used to predict soil model parameters, which are introduced into analytical models in order to predict bending waves in the track/embankment system. The consequent displacements are highly dependent on the speed of the train. This maximum deflection was also found to be dependent on the amount of damping in the system. For all reasonable assumptions the amount of damping present is insufficient to limit deflection to tolerable magnitudes. Thus, the theoretical models can indicate suitable restrictions on train speeds for particular track conditions.

RÉSUMÉ: Au fin de réduire le dommage à l'environnement (visibilité et bruit), on veut construire les chemins de fer de grande vitesse inter-urbain sur les petits remblais. Ces remblais ont tendance à être flexibles et, donc, sur sol mou, les ondes de flexion peuvent avoir pour résultat des déflexions significantes verticales sous l'action du train. Ces déplacements de la voie ferrée peuvent à tour rôles avoir un effet profond sur le confort du voyage et également sur le coût d'entretien. Un vibrateur inertiel à fréquence variable et une ligne de géophones ont été utilisés pour examiner la réponse d'une couche de sol avec et sans un chemin de fer. La réponse mesurée du sol a été utilisée pour prédire les paramètres modèles du sol, qui sont fait entrés dans des modèles analytique à fin de prédire ondes de flexion dans le système de la voie/remblai. Les déplacements qui suivent dépendent fortement du fait que la vitesse du train atteigne un maximum à une vitesse critique semblable à celle des trains à grande vitesse typiques. Cette déflexion maximum se trouve également à être dépendante sur la quantité d’amortissement dans le système. Pour toutes assumptions raisonnables, la quantité d’amortissement présente est suffisante pour limiter la déflexion aux valeurs tolérables. Donc, les modèles théoriques peuvent indiquer des restrictions convenables sur la vitesse des trains par rapport aux conditions ferroviaires particulières.

1 INTRODUCTION

Due to the increasing congestion in the skies over the World’s major conurbations, passenger travel by airplane is increasingly seen as a non-environmentally friendly method of mass transit. The alternative of high speed railways has been identified Worldwide as the form of transit more appro-
plate and cost efficient for medium distance journeys (< 1000 km). The Channel Tunnel Rail Link in the United Kingdom, the T.G.V. in France, the I.C.E in Germany and the Shinkansen (‘Bullet’) trains of Japan are examples. The building of the track structure itself is a significant cost to such a project and so recently there has been an increase in research in this area. Many of the design problems are caused by the tendency for modern railroad embankments to be flexible because the embankment height is limited by the two requirements that visibility and noise pollution should be reduced as much as possible. This flexibility is exacerbated as, in many localities, the only undeveloped land available for construction is comprised of soft clays and peats which were avoided by previous constructions. On ground with a low soil stiffness, track-soil bending waves may result in significant transient train-induced soil deflections while ride quality and maintenance costs also have a direct relationship to the magnitude of these deflections (Sharpe 1998). In this paper a model of a Euler beam on a damped Winkler foundation (a series of discrete springs) is studied and used to demonstrate how the magnitude of the resulting displacements are dependent on train speed and foundation damping. In-situ tests have been performed and the results are presented in this paper with reference to the proposed model.

2 ANALYTICAL ANALYSIS

The springs in the Winkler foundation model are discrete in that there is no shear coupling in the ‘soil’ model along the axis of the beam. Although this is not the case for real soils, it does provide a model that can be solved relatively simply in closed form (Fryba 1972). A Kern/Pasternak (1961) or a similar foundation model could include shear interaction. However, Kneifati (1985) showed that the Winkler model is sufficient to predict deflections away from any end effects of the beam. The railway track under consideration is modeled as an infinite beam and, therefore at least initially, the Winkler model is an appropriate idealization. The defining differential equation of this problem is:

\[ EI \frac{d^4 y(x,t)}{dx^4} + \mu \frac{d^2 y(x,t)}{dt^2} + C \frac{dy(x,t)}{dt} + k y(x,t) = P \delta(x - vt) \quad (1) \]

where \( EI \) is the flexural rigidity of the beam resting on the foundation, \( y \) is the vertical displacement, \( x \) is the distance along the beam, \( t \) is time, \( C \) is the damping coefficient, \( k \) is the stiffness coefficient of the Winkler foundation, \( \mu \) is the mass per unit length of the beam, \( P \) is the load and \( \delta(x - vt) \) is the Dirac function of a point unit load moving with velocity \( v \).

Equation 1 can be solved in closed form (Fryba 1972). However, in this paper, a more general convolution based numerical solution is used to include the possibility of incorporating experimental and Finite element (FE) results. This method is explained briefly in the following.

It can be shown that the vertical displacement response, \( y \) at position, \( x \) and time, \( t \) due to an applied load, \( P \) moving at a speed \( v \) can be given by (Hardy 1990):}

\[ y(x,t) = \int_{-\infty}^{t} h(x - (d + v \tau),t - \tau) P(\tau) d\tau \quad (2) \]

where \( d \) is the position of the load at time \( t = 0 \). \( h(x,t) \) is the vertical displacement impulse response function at position \( x \) and time \( t \) due to a unit load applied at the origin at time \( t = 0 \). In the following analysis the force applied by the axle at time \( t \), \( P(t) \), is assumed to be constant with respect to time and can be taken outside the integration. The analysis assumes a linear system and the response from multiple axle loads can be obtained by the superposition of the responses of the individual axle loads (Hardy 1990).

For an elastic beam on a damped Winkler foundation the impulse response function can be obtained by taking the inverse two-dimensional Fourier Transform of the frequency response function \( H(\xi,\omega) \), where \( \omega \) is the angular frequency and \( \xi \) is the wavenumber, given by (Hardy 1990):

\[ H(\xi,\omega) = \frac{1}{(2\pi)^2 \left( EI\xi^4 - \mu \omega^2 + j \omega C + k \right)} \quad (3) \]
Alternatively, the impulse response function can be obtained from other more complicated models (for example, Finite Element Models), or directly from in-situ measurements on the rail track.

The Winkler/Euler system is divided into a beam, with bending stiffness, and the supporting foundation, with a spring stiffness and damping (Equation 1). However, there is disagreement in the literature regarding what should be considered to be the beam for the Winkler-Euler model. If a track on embankment is considered then the approach suggested by Fortin (1983) is that only the rail is considered to provide the bending stiffness of the beam whereas the mass of the beam includes the mass of the rails, sleepers and ballast. The ballast is ignored for the beam stiffness because it would be on the bottom of the ‘beam’ and, therefore, in tension and the granular materials normally used for ballast do not have a significant bulk tensile strength (the suction in such a coarse material being almost zero). The foundation stiffness is comprised of both the embankment and the underlying subgrade. Conversely, Hunt (1994) takes the whole embankment to be part of the beam. In this case the bending stiffness of the rail is not significant compared to the bending stiffness of the embankment. The foundation is comprised of the subgrade stiffness only. The validity of these assumptions will be discussed with the aid of results from in-situ testing.

3 IN-SITU TESTING

Tests, using a 90 kg variable frequency vibrator (supplied by G.D.S. Ltd Egham) were performed on top of an embankment approximately 7 m high. The embankment was formed using a relatively stiff clay with a shear modulus of 200-300 MPa. The underlying subgrade had a significantly lower shear stiffness (Heelis et al 1999). Testing was carried out between the rails with geophones placed on or between the sleepers and on a neighbouring site from which the sleepers and rails had been removed. Using results from these sites the effects of the rails and sleepers on the measured wave velocity will be investigated.

Figure 1 shows the wave velocity plotted against frequency for the two sites. There is a general tendency for the wave velocity to reduce as the frequency increases. For a homogeneous half-space, the speed of Rayleigh waves can be related to the shear stiffness (and, hence the Young’s modulus) and density of a elastic half-space using the following equation (Miller and Pursey 1952),

\[ v_r = \frac{G}{\rho} = \frac{E_s}{2(1+\nu_s)\rho} \]

For a typical clay \( \nu_s = 0.4 \) \( \alpha = 0.94 \) from Miller and Pursey (1952). If the stiffness of the medium increases with depth then the higher frequency waves, which have shorter wavelengths and, therefore, tend to penetrate to a lesser depth, tend to travel with slower velocities (Matthews et al 1996). This effect can be identified in Figure 1 and it is possible to conclude that the soil stiffness increases with depth.

It can be seen from Figure 1, that the addition of the rails and embankment into the system under testing reduces the speed of the transmitted waves significantly between 9 and 17 Hz. For example, at 11 Hz the wave velocity has dropped from 300 m/s to 200 m/s. This reduction in wave speed contradicts the assumption that the bending stiffness of the rails (typically 10 MNm²) is not sufficient to make such a large difference in wave velocity when compared to the bending stiffness of the whole embankment.

If the speed of Rayleigh waves is taken to be 300 m/s then the effective Young’s Modulus of the equivalent half-space foundation, \( E_a \) (embankment and subgrade) calculated using Equation 4, assuming a Poisson’s ratio of 0.4 and a density of 2000 kg/m³ will be 570 MN/m².

In order to use the Euler beam-Winkler foundation model it is necessary to find a relationship between the Young’s modulus of the foundation and the Winkler spring. According to Vesic (1963) the Young’s modulus of an elastic half-space can be related to the coefficient of subgrade reaction, k, by:
\[ k = \frac{0.65E}{\left(1 - v^2\right)^{1/2}} \sqrt{\frac{E_s B^4}{EI}} \]  

where B is the width of the beam, taken as the length of the sleepers in this case (B = 2.5 m).

![Graph showing wave velocity vs frequency with markers indicating off track (on embankment) and between track and sleepers.

Figure 1. Continuous Surface wave data for test site.

The critical speed of bending waves for the Winkler-Euler foundation system is given by (see Equation 1 for the definition of the terms),

\[ v_{crit} = \left(\frac{4kEI}{\mu^2}\right)^{1/2} \]

Hence,

\[ v_{crit} = \left(\frac{4EI}{\mu^2 (1 - v^2)} \right)^{1/2} \left(\frac{E_s B^4}{EI}\right)^{1/4} = \left(\frac{2.6EI}{\mu^2 (1 - v^2)} \right)^{1/4} \left(\frac{E_s B^4}{EI}\right)^{1/4} \]  

The typical bending stiffness of rails in the United Kingdom is 10 MNm². Taking, the mass per unit length of the rail and sleepers as 550 kg/m and the mass per unit length of the ballast to be 2250 kg/m, the overall mass of the beam is 2800 kg/m. This gives a critical velocity of 255 m/s, which approximates to the measured value (see Figure 1). This site was constructed on stiff clay and the predicted speed is in excess of those achievable by current trains. However, on a soft peat site, with a Young's modulus of 10 MN/m², the critical speed would reduce to 85.6 m/s (300 kph). This speed can be exceeded by current high speed trains. The problems that arise from approaching the critical speed will now be outlined with reference to the model previously introduced.

The displacement under the applied load and the peak displacements have been plotted in Figures 2a and 2b respectively for varying speeds and degrees of system damping. The displacements, calculated using Equation 1, have been normalised by the response due to a static point load, obtained from the Fryba (1972) solution to equation 1 for the special case of x = 0 and v = 0:

\[ v_{static} = \frac{P}{2\sqrt{2\pi} \sqrt{k^3 EI}} \]  

The damping and the speed also have been non-dimensionalised and are represent by the two parameters, \( \alpha \) and \( \beta \), which are given by:

\[ \alpha = c \left(\frac{\mu^2}{4 k EI}\right)^{1/4} \]
Figure 2a. Non-dimensional plot of vertical displacement under a single axle.
Figure 2b. Non-dimensional plot of maximum vertical displacement.

\[ \beta = C \sqrt{\frac{1}{4k\mu}} \]  

(10)

The displacement directly under the load is a measure of the ride comfort of the system. The maximum displacement experienced by the foundation (which does not necessarily occur directly under the moving load) will affect the maintenance cycle required for the section of track. For the case of soft organic clays and peats, which often form the ground on which construction will take place, the damping, expressed in the non-dimensional form, \( \beta \), due to the subgrade is expected to be in the range of 5-10\%. This range was calculated using results from resonant column testing (see Heelis et al 1998 for more details). Note that the damping from the rails and ballast is minimal. For 5\% damping the displacement experienced by both the train and the track system are considerably magnified compared to the static deflections as the critical speed is approached. This has been observed in practice on the East Coast Mainline in the United Kingdom, where the problem of large transient deflections have already been identified as causing excessive maintenance requirements. A track speed limit of 45 m/s (160 kph), has been imposed specifically due to the magnification effects of high speed trains.

4 FUTURE WORK

To use the Euler/Winkler model presented above the soil stiffness has to be converted into an equivalent spring stiffness. In reality the soil stiffness often varies with depth and in this paper only an effective soil modulus has been used. The use of other models to relate the soil stiffness to the foundation spring modulus is discussed in Heelis et al (1999). It is also possible to use more complicated layered models (for example, Hardy 1990). Finite Element Modelling (FEM) also allows the modelling of multiple soil layers and three dimensional wave effects. It is also possible to assess the inertia effect of the subgrade more accurately. Such refinements to the method outlined here are currently being investigated. However, it can be seen that field measurements are confirming some of the attributes of the simple model used here.

In the current model, the inertia of the subgrade can only be modelled by adding an effective mass to the beam. This is only an approximation, as is the magnitude of the ballast mass which is part of the beam. Dynamic finite element modelling is seen as a way of providing more accurate models which reflect the physical movements actually taking place in the soil-foundation system. This modelling should lead to additional refinements in the proposed technique as well as the possibility of modelling and testing proposed designs.
5 IMPLICATIONS

There are several practical ways to limit the magnitude of track deflection in the railway environment. Firstly, it is possible to limit the speed of the trains. However, this has an economic impact on the viability of railway use compared to alternative forms of travel. Therefore, the alternative is to increase the stiffness of the subgrade or the beam rigidity. The second of these is not straightforward as most methods of increasing the subgrade stiffness also lead to an increase in the effective mass of the beam. This has the opposite effect to increasing the beam stiffness and largely negates the benefits. However, subgrade stiffness may be increased by many means - physical and chemical. Thus, piling and deep ground treatment, for example lime columns or wick drainage, could be effective solutions. The need to carry horizontal (braking forces) will require significant longitudinal stiffening indicating the need for raked piles or panels to carry the shear forces.

6 CONCLUSIONS

A model of a moving load on a beam on a Winkler foundation has been analyzed using a moving convolution technique. This simple model has predicted the reduction in wave propagation speeds in an embankment as a consequence of the addition of a rail track. Such a reduction in wave speed has been observed in practice using a geophysical testing technique. The wave speeds that are predicted are too high to affect trains, but this is only for the particular site reported in this paper. The model predicts that on soft peat/clay sites the wave speed will approximate the speed of high speed trains. In this case, large vertical displacements are also predicted which will affect both the ride quality and the maintenance costs associated with the site. The work is on-going and proposed refinements in the modelling technique, taking into account three dimensional and inertia effects of the subgrade, are identified. Finally, some methods of reducing the flexibility of the foundation have been suggested.

REFERENCES