Variable frequency performance of the single phase capacitor run induction motor

This item was submitted to Loughborough University's Institutional Repository by the/an author.

Additional Information:

- A Doctoral Thesis. Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University.

Metadata Record: [https://dspace.lboro.ac.uk/2134/13762](https://dspace.lboro.ac.uk/2134/13762)

Publisher: © Abdul Sattar Khadum Muslih

Please cite the published version.
This item was submitted to Loughborough University as a PhD thesis by the author and is made available in the Institutional Repository (https://dspace.lboro.ac.uk/) under the following Creative Commons Licence conditions.

![Creative Commons License](https://creativecommons.org/licenses/by-nc-nd/2.5/)

**Attribution-NonCommercial-NoDerivs 2.5**

**You are free:**
- to copy, distribute, display, and perform the work

**Under the following conditions:**

**Attribution.** You must attribute the work in the manner specified by the author or licensor.

**Noncommercial.** You may not use this work for commercial purposes.

**No Derivative Works.** You may not alter, transform, or build upon this work.

- For any reuse or distribution, you must make clear to others the license terms of this work.
- Any of these conditions can be waived if you get permission from the copyright holder.

Your fair use and other rights are in no way affected by the above.

This is a human-readable summary of the Legal Code (the full license).

Disclaimer:

For the full text of this licence, please go to:
http://creativecommons.org/licenses/by-nc-nd/2.5/
VARIABLE FREQUENCY PERFORMANCE OF THE
SINGLE-PHASE CAPACITOR-RUN INDUCTION MOTOR
by
ABDUL SATTAR KHADUM MUSLIH, M.Sc.

A Doctoral Thesis
Submitted in partial fulfilment of the
Requirements for the Award of the Degree of
Doctor of Philosophy
of
Loughborough University of Technology

Supervisor: Professor I R Smith, B.Sc., Ph.D.,
D.Sc., C.Eng., F.I.E.E.

© by Abdul Sattar Khadum Muslih
ACKNOWLEDGEMENTS

I would like to express my gratitude to my supervisor, Professor I R Smith for his advice and counsel throughout the period of my research and for the thoughtful guidance he has supplied during the preparation of this thesis.

I wish to thank the Head and Staff of the Electronic and Electrical Engineering Department at Loughborough University of Technology for the facilities provided throughout this work.

My acknowledgements go to the Electrical Industries Company in Iraq for the financial support received while this project was undertaken.

I am greatly indebted to my brother J Muslih and his family and my wife for their support and help through the period of study.

I also wish to thank Mrs J Smith and Mr P LeGood for their considerable help in the typing and preparing copies of the characteristics in this thesis.
SYNOPSIS

A capacitor-run motor is often necessary where silent running, low starting current, high efficiency and power factor and overload capacity are required. When a capacitor motor is used in control or similar applications a knowledge of the variation in performance over a range of supply frequency is often important, and this has been investigated by previous workers using analyses of varying validity. For example, in early studies it was common to base performance calculations on the revolving-field theory, when a general lack of agreement was found between calculated and test values. This occurred particularly in the starting winding under running conditions, and was attributed usually to saturation and to space harmonics of mmf. The cross-field theory provides a useful alternative to the revolving-field theory for analyses of single-phase motors, and when applied to the capacitor motor it is found that a much more accurate prediction of the machine performance is obtained.

This thesis investigates the behaviour of a small single-phase capacitor-run induction motor with relation to variable frequency supplies and different capacitor values. The performance of the capacitor motor is calculated, using an equivalent circuit developed from basic cross-field theory considerations. However, before calculating the performance of the motor, the experimental determination of the parameters involved is necessary. From the equivalent circuit for no-load and locked rotor conditions, a set of voltage/current equations was obtained. Use of these
equations requires a very complicated set of mathematical procedures to determine the required parameters, and no solution could be found. This difficulty was overcome by the extensive use of a digital computer, enabling the parameters to be obtained by the Newton-Raphson method of numerical iteration. Using these parameters, the predicted performance of the machine was compared with that obtained from an extensive experimental investigation. The agreement which was found to exist confirms both the validity of the theoretical analysis and the accuracy of the measurement from which the parameters were derived.
LIST OF PRINCIPAL SYMBOLS

\( V_M \) = Voltage applied to the M-phase stator winding, \( V \)
\( V_S \) = Voltage applied to the S-phase stator winding, \( V \)
\( I_M \) = M-phase stator winding current, A
\( I_S \) = S-phase stator winding current, A
\( I_L \) = Line current = \( I_M + I_S \), A
\( E_M \) = Counter emf of the M-phase, V
\( E_S \) = Counter emf of the S-phase, V
\( E_{FM} \) = emf induced in the M-phase stator winding by its forward rotating flux, V
\( E_{BM} \) = emf induced in the M-phase stator winding by its backward rotating flux, V
\( E_{FS} \) = emf induced in the S-phase stator winding by its forward rotating flux, V
\( E_{BS} \) = emf induced in the S-phase stator winding by its backward rotating flux, V
\( E_{MFS} \) = emf induced in the M-phase stator winding by the forward rotating flux of the S-phase winding, V
\( E_{MBS} \) = emf induced in the M-phase stator winding by the backward rotating flux of the S-phase stator winding, V
\( E_{SFM} \) = emf induced in the S-phase stator winding by the forward rotating flux of the M-phase stator winding, V
\( E_{SBM} \) = emf induced in the S-phase stator winding by the backward rotating flux of the M-phase stator winding, V
\( R_2 \) = rotor resistance referred to the M-phase stator winding, \( \Omega \)
\( R_{1M} \) = stator resistance of the M-phase stator winding, \( \Omega \)
\( a^2 R_{1S} \) = stator resistance of the S-phase stator winding, \( \Omega \)
\( X_{IM} \) = stator leakage reactance of the M-phase stator winding, \( \Omega \)
\( a^2 X_{IS} \) = stator leakage reactance of the S-phase stator winding, \( \Omega \)
\( X_2 \) = rotor leakage reactance referred to the M-phase stator winding, \( \Omega \)
\( X_c \) = capacitor reactance in series with S-phase stator winding, \( \Omega \)
\( X_m \) = magnetizing reactance of the main axis, \( \Omega \)
\( X_s \) = magnetizing reactance of the field axis, \( \Omega \)
\( R_b \) = effective resistance of stator winding to the M-phase backward rotating flux, \( \Omega \)
\( R_f \) = effective resistance of stator winding to the M-phase forward rotating flux, \( \Omega \)
\( X_b \) = effective reactance of stator winding to the M-phase backward rotating flux, \( \Omega \)
\( X_f \) = effective reactance of stator winding to the M-phase forward rotating flux, \( \Omega \)
\( T_{ins} \) = instantaneous torque
\( T_{av} \) = average electromagnetic torque developed
\( w_o \) = output power of motor, w
\( w_i \) = input power to the motor, w
\( n \) = motor efficiency
\( \cos \theta \) = motor power factor
\( \phi \) = phase angle between \( I_M \) and \( I_S \) elec. rad.
\( V \) = line voltage, V
\( I_{mM} \) = magnetizing current of the main axis, A
\( I_{mS} \) = magnetizing current of the field axis, A
\( I_{2M} \) = rotor current of the main axis, A
\( I_{2S} \) = rotor current of the field axis, A
\( \phi_{nM} = \) mutual flux linking the M-phase and rotor windings of the main axis, Wb

\( \phi_M = \) leakage flux of the M-phase stator winding, Wb

\( \phi_S = \) leakage flux of the S-phase stator winding, Wb

\( \phi_{mS} = \) mutual flux linking the S-phase stator winding and the rotor winding of the field axis, Wb

\( \phi_{2M} = \) leakage flux of the rotor winding of the main axis, Wb

\( \phi_{2S} = \) leakage flux of the rotor winding of the field axis, Wb

\( \theta_M = \) phase angle between \( V \) and \( I_M \), elec. rad.

\( \theta_S = \) phase angle between \( V \) and \( I_S \), elec. rad.

\( \theta_1 = \) phase angle between \( E_M \) and \( I_{2S} \), elec. rad.

\( \theta_2 = \) phase angle between \( E_S \) and \( I_{2M} \), elec. rad.

\( \psi = \) phase angle between \( I_{2S} \) and \( I_{2M} \), elec. rad.

\( \Theta = \) phase angle between \( V \) and \( I_L \), elec. rad.

\( V_M^+ = \) positive-sequence component of \( V_M \), V

\( V_M^- = \) negative-sequence component of \( V_M \), V

\( V_S^+ = \) positive-sequence component of \( V_S \), V

\( V_S^- = \) negative-sequence component of \( V_S \), V

\( I_M^+ = \) positive-sequence component of \( I_M \), A

\( I_M^- = \) negative-sequence component of \( I_M \), A

\( I_S^+ = \) positive-sequence component of \( I_S \), A

\( I_S^- = \) negative-sequence component of \( I_S \), A

\( I_{2M}^+ = \) positive-sequence component of \( I_{2M} \), A

\( I_{2M}^- = \) negative-sequence component of \( I_{2M} \), A

\( I_{2S}^+ = \) positive-sequence component of \( I_{2S} \), A

\( I_{2S}^- = \) negative-sequence component of \( I_{2S} \), A

\( Z_{2}^+ = \) negative-sequence rotor impedance referred to the M-phase, \( \Omega \)

\( Z_{2}^- = \) effective rotor and magnetizing positive-sequence impedance, \( \Omega \)
\[ Z_2^{-1} = \text{effective negative-sequence rotor and magnetizing impedance, } \Omega \]
\[ Y_m = \text{magnetizing admittance of M-phase winding} = \frac{1}{Z_m}, \frac{1}{\Omega} \]
\[ I_{OM} = \text{short-circuit current through the virtual load resistor } R \]
\[ \text{from M-phase winding alone, } A \]
\[ I_{eM} = \text{values of } I_M \text{ when the virtual load resistor } R \text{ is short-circuited, } A \]
\[ I_{OS} = \text{short-circuit current through the virtual load resistor } R \]
\[ \text{from S-phase winding alone, } A \]
\[ I_{eS} = \text{values of } I_S \text{ when the virtual load resistor is short-circuited, } A \]
\[ I_o = \text{short-circuit current through the virtual load resistor} \]
\[ R = I_{OM} + I_{OS}, A \]
\[ I = \text{current through the virtual load } R, A \]
\[ E = \text{back emf across the virtual load } R, \text{ Volts} \]
\[ E_{OM} = \text{M-phase open-circuit voltage} = I_{OM}/Y_{OM}, \text{ Volts} \]
\[ E_{OS} = \text{S-phase open-circuit voltage} = I_{OS}/Y_{OS}, \text{ Volts} \]
\[ I_S' = \text{S-phase winding current in M-phase winding terms, with } 90^\circ \]
\[ \text{backward phase shift} = -jaI_s, A \]
\[ I_{2S}' = \text{field axis rotor current in M-phase winding terms with } 90^\circ \]
\[ \text{backward phase shift} = -jaI_{2S}, A \]
\[ T_{\text{max}} = \text{maximum torque of the motor, Kg.cm} \]
\[ T_{\text{st}} = \text{starting torque of the motor, Kg.cm} \]
\[ T = \text{heating time constant, min} \]
\[ t = \text{time, min} \]

Other symbols are defined as they occur.
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page Nos</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>i</td>
</tr>
<tr>
<td>SYNOPSIS</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF PRINCIPAL SYMBOLS</td>
<td>iv</td>
</tr>
<tr>
<td>CONTENTS</td>
<td>viii</td>
</tr>
<tr>
<td>CHAPTER I: INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>CHAPTER II: THEORY OF THE SINGLE-PHASE CAPACITOR-RUN INDUCTION MOTOR</td>
<td>5</td>
</tr>
<tr>
<td>2.1 The Revolving-Field Theory of the Capacitor Motor</td>
<td>5</td>
</tr>
<tr>
<td>2.2 The Cross-Field Theory of the Capacitor Motor</td>
<td>14</td>
</tr>
<tr>
<td>2.3 The Method of Symmetrical Components</td>
<td>22</td>
</tr>
<tr>
<td>2.4 Performance Calculations of the Capacitor Motor</td>
<td>35</td>
</tr>
<tr>
<td>2.4.1 The equivalent circuit of the capacitor motor</td>
<td>35</td>
</tr>
<tr>
<td>2.4.2 Performance determination of the capacitor-run induction motor</td>
<td>39</td>
</tr>
<tr>
<td>2.4.3 The maximum torque of the motor</td>
<td>48</td>
</tr>
<tr>
<td>2.5 Conclusion</td>
<td>50</td>
</tr>
<tr>
<td>CHAPTER III: DETERMINATION OF CONSTANTS FOR THE EQUIVALENT CIRCUIT OF THE CAPACITOR MOTOR</td>
<td>51</td>
</tr>
<tr>
<td>3.1 Determination of Parameters using the Newton-Raphson Iterative Method</td>
<td>58</td>
</tr>
<tr>
<td>3.2 Conclusion</td>
<td>64</td>
</tr>
<tr>
<td>CHAPTER IV: BALANCED AND UNBALANCED OPERATIONS OF A CAPACITOR MOTOR</td>
<td>67</td>
</tr>
<tr>
<td>4.1 Balanced Operation</td>
<td>67</td>
</tr>
<tr>
<td>4.2 Unbalanced Operation</td>
<td>72</td>
</tr>
<tr>
<td>4.3 Conclusion</td>
<td>78</td>
</tr>
<tr>
<td>CHAPTER V: EXPERIMENTAL PROCEDURE USED WITH THE CAPACITOR MOTOR AND RESULTS</td>
<td>80</td>
</tr>
<tr>
<td>5.1: The Experimental Equipment</td>
<td>80</td>
</tr>
<tr>
<td>Section Description</td>
<td>Page Nos</td>
</tr>
<tr>
<td>------------------------------------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>5.2 No-Load Test</td>
<td>80</td>
</tr>
<tr>
<td>5.3 Separation of Iron Losses into Hysteresis and Eddy-Current Losses</td>
<td>83</td>
</tr>
<tr>
<td>5.4 Speed-Torque Characteristics</td>
<td>84</td>
</tr>
<tr>
<td>5.5 Stator Winding-Ratio Test</td>
<td>86</td>
</tr>
<tr>
<td>5.6 Load Test</td>
<td>87</td>
</tr>
<tr>
<td>5.7 Calculated Results</td>
<td>87</td>
</tr>
<tr>
<td>5.8 Temperature Measurements</td>
<td>88</td>
</tr>
<tr>
<td>5.9 Conclusion</td>
<td>90</td>
</tr>
<tr>
<td>CHAPTER VI: EFFECT OF FREQUENCY ON THE MOTOR PERFORMANCE</td>
<td>107</td>
</tr>
<tr>
<td>6.1 Relation Between Voltage and Frequency</td>
<td>107</td>
</tr>
<tr>
<td>6.2 Effect of Variation of Voltage and Frequency on the Performance of the Motor</td>
<td>108</td>
</tr>
<tr>
<td>6.3 Relationship Between Torque, Speed and Output Power</td>
<td>109</td>
</tr>
<tr>
<td>6.4 Frequency Effect on the Motor Performance for a Constant Magnetic Field</td>
<td>110</td>
</tr>
<tr>
<td>6.5 Effect of the Running Capacitor on the Motor Performance at Different Frequencies</td>
<td>112</td>
</tr>
<tr>
<td>6.5.1 Effect of the Running Capacitor on the Motor Performance</td>
<td>113</td>
</tr>
<tr>
<td>6.5.2 Effect of the Running Capacitor on the Maximum and Starting Torque</td>
<td>114</td>
</tr>
<tr>
<td>6.6 Conclusion</td>
<td>116</td>
</tr>
<tr>
<td>CHAPTER VII: COMMENTS AND CONCLUSION</td>
<td>122</td>
</tr>
<tr>
<td>7.1 Conclusion</td>
<td>122</td>
</tr>
<tr>
<td>7.2 Comments</td>
<td>125</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>126</td>
</tr>
<tr>
<td>APPENDICES</td>
<td>130</td>
</tr>
<tr>
<td>Appendix I: DERIVATION OF THE CURRENT EQUATIONS IN THE REVOLVING-FIELD THEORY</td>
<td>130</td>
</tr>
<tr>
<td>Appendix II: DERIVATION OF THE SYMMETRICAL COMPONENTS EQUATIONS OF THE CAPACITOR MOTOR</td>
<td>134</td>
</tr>
<tr>
<td>Appendix III: COMPUTER PROGRAMS</td>
<td>143</td>
</tr>
<tr>
<td>A. Computer program used to determine the parameters of the motor</td>
<td>143</td>
</tr>
<tr>
<td>B. Computer program used to calculate the motor performance characteristics</td>
<td>161</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION

The necessity for operating motors at a frequency different from that for which they were originally designed, may result either from changing the frequency of the power supply (thereby affecting a number of motors in an installation), or from the use of repurchased motors in new circuits. A change in the frequency or the voltage from that for which a machine is rated will obviously cause the performance to depart from that specified by the manufacturer. Voltage variations of 10% are allowed for in most commissioning rules, but it must be noted that the variations will have the effect of changing both the power-factor and the efficiency. A small variation in frequency, with the voltage held constant, will yield a torque variation almost inversely proportional to the square of the frequency. If both voltage and frequency are varied together, in the same direction and over a small range, the torque will remain substantially constant.

The main objectives of this thesis are:

a) to investigate the behaviour with variable frequency of a small single-phase capacitor-run induction motor.

b) to also consider how the behaviour varies with capacitor values slightly different from that normally connected in series with the auxiliary (S)-phase stator winding.

The presence of a running capacitor has a large effect on the performance of the motor\(^{(1,2,3)}\), generally:
i) increasing the maximum torque by between 5% and 30%.

ii) improving slightly the full-load efficiency and improving substantially the full-load power-factor.

iii) reducing the full-load running current.

iv) reducing the noise produced under full-load operating conditions.

v) increasing the starting torque by between 5% and 20%.

The mechanism by which the running capacitor improves the performance of a capacitor motor (4,5) becomes clear when the true rotating field produced by the stator is considered as composed of two component pulsating fields, spaced 90° (electrical) apart in space and differing in time phase by 90°. When one of these components is missing, its effect has to be provided by magnetizing current flowing in the rotor. When the S-phase stator winding is connected to the same mains supply as the main (M)-phase stator winding, but is in series with a running capacitor of a specific value, the current drawn by the S-phase stator winding leads that drawn by the M-phase stator winding by approximately 90°. These two currents produce a rotating field substantially as in a genuine 2-phase motor, and eliminate or very much reduce the magnetizing current in the rotor and any accompanying rotor copper losses. At best, however, a perfect balance is achieved at only one load, although the capacitor will contribute a substantial effect at nearly all normal running loads. The effect of the running capacitor is thus to make the motor performance approach more nearly to that of a genuine 2-phase motor, particularly at one value of load. The point is developed later in Chapter IV.
This thesis contains in Chapter II an explanation of the several different ways by which an analytical approach can be developed for the single-phase capacitor-run induction motor. The different techniques used all have their own individual merits, and the choice of any given method depends partly on what is most desired: e.g. speed, torque, accuracy, etc. The capacitor motor can thus be analysed using either the revolving-field theory\(^6\), the cross-field theory\(^7,8\) or the symmetrical component method\(^9\). Some phenomena are more easily understood by applying the revolving-field theory, while others are made more clear by the use of the cross-field theory. To obtain the fullest appreciation of the performance of a motor, neither the revolving-field nor the cross-field theory nor the symmetrical component method should be used exclusively. Theoretical results obtained using the cross-field theory are normally more accurate than those obtained using the revolving-field theory\(^8,10\). This is because the hysteresis lag between the flux and the mmf may be included in the cross-field theory by assuming a complex impedance \(Z_m = R_m + jX_m\) for the magnetizing impedance, instead of a pure reactance \(jX_m\). The inclusion of hysteresis effects in the revolving-field theory is found to complicate excessively the equations involved, and such effects are therefore usually neglected. To assist with this type of situation, the motor performance is calculated using an equivalent circuit developed from basic cross-field theory considerations, and the motor characteristics are determined at different frequencies ranging between 40 Hz to 60 Hz in 5 Hz steps, and also at a range of different capacitor values.
The results obtained are compared with corresponding experimental results, and it is found that good agreement generally exists.

However, before calculating the performance of the motor, the experimental determination of the individual parameters must be made from no-load and locked-rotor tests made at the motor terminals\(^{(11,12)}\). From the equivalent circuits for these two conditions, a set of voltage/current equations is obtained. Use of these equations requires a very complicated set of mathematical procedures to determine the required parameters, and no explicit solution could be found by the present author. This difficulty was overcome by the extensive use of a digital computer, enabling the parameters to be obtained by the Newton-Raphson method of numerical iteration\(^{(13,14)}\) as explained in Chapter III. The parameters were determined for different operational frequencies, and the performance of the machine predicted from these parameters was compared with that obtained from an extensive experimental investigation. The agreement which was found to exist confirms both the validity of the theoretical analysis and the accuracy of the measurements from which the parameters were derived.
CHAPTER II
THEORY OF THE SINGLE-PHASE CAPACITOR-RUN
INDUCTION MOTOR

The single-phase capacitor-run induction motor may be analyzed by many techniques\(^1,2,3\), including the revolving-field theory\(^6\), the cross-field theory\(^7,8\) and the use of symmetrical components\(^9\). Some phenomena are more easily understood from a study based on the revolving-field theory, while others are more clearly demonstrated through the cross-field approach. For the routine calculation of performance characteristics, many graphical and analytical solutions based on either the revolving-field or the cross-field have been developed\(^{15,16,17,18}\).

2.1 The Revolving-Field Theory of the Capacitor Motor

The revolving-field theory of the capacitor motor\(^6\) is an extension of the general method of analysis of the unbalanced 2-phase motor\(^{19}\). In the revolving-field theory of the single-phase motor, the pulsating and sinusoidally distributed mmf produced by the stator winding is resolved into two equal sinusoidally distributed mmf waves of equal and constant amplitude, gliding at synchronous speed and in opposite directions round the periphery of the air-gap. The effects on the stator winding of each of these revolving-fields, and also of the field of the rotor, can be represented by the parallel circuit with two branches shown in Figure (2.1). One branch of each half of this circuit contains the magne-
tizing reactance $X_m$, while the other comprises the secondary leakage reactance $X_2$ in series with the secondary resistance $R_2$, divided by the slip $S$ for the forward rotating field and by 2-$S$ for the backward rotating field. With the suffix $M$ denoting the $M$-phase winding, the stator impedance is shown in Figure (2.1) as $(R_{1M} + j X_{1M})$: where:

$$R_{1M} = \text{resistance of the } M\text{-phase winding}$$
$$X_{1M} = \text{leakage reactance of the } M\text{-phase winding}$$

The emf induced by the forward revolving flux is $E_{fM}$, and that induced by the backward revolving flux is $E_{bM}$. If, instead of the $M$-phase, a further phase (the $S$-phase) is considered, displaced backward spatially by 90° electrical from the $M$-phase winding and with a different number of turns, the corresponding equivalent circuit is as shown in Figure (2.2). The ratio (effective $S$-phase turns) (effective $M$-phase turns) is denoted by $a$, so that the primary impedance of the $S$-phase stator winding is now $(a^2 R_{1S} + j a^2 X_{1S})$ and the magnetizing reactance is $a^2 X_m$, but the capacitor series impedance remains as $-j X_c$: where:

$$a^2 R_{1S} = \text{resistance of } S\text{-phase stator winding}$$
$$a^2 X_{1S} = \text{leakage reactance of } S\text{-phase stator winding}.$$  

The emf induced by the forward revolving flux is $E_{fS}$, and that induced by the backward revolving flux is $E_{bS}$.

If the $M$ and $S$-phases are excited simultaneously, their mmf's superimpose without distortion. By combining the magnetizing
FIG. 2.1. EQUIVALENT CIRCUIT FOR THE MAIN WINDING (THE M-PHASE).

FIG. 2.2. EQUIVALENT CIRCUIT FOR THE AUXILIARY WINDING (THE S-PHASE).
FIG. 2.3. REDUCED EQUIVALENT CIRCUITS OF THE CAPACITOR MOTOR.
impedance and the rotor impedance as a single series resistive-inductive circuit, the equivalent circuits of Figures (2.1) and (2.2) become as shown in Figure (2.3), where, in addition to the forward and backward rotating self induced emf's in each phase, there are also corresponding forward and backward rotating emf's ($E_{MFS}$, $E_{MbS}$, $E_{SFM}$ and $E_{SbM}$) due to the other phase (i.e. $E_{MFS}$ and $E_{MbS}$ = emf's induced in the M-phase winding by the forward and backward rotating flux of the S-phase winding respectively, $E_{SFM}$ and $E_{SbM}$ = emf's induced in the S-phase winding by the forward and backward rotating flux of the M-phase winding respectively). Since the M-phase winding is displaced forward by $90^\circ$ electrical from the S-phase winding, the emf induced in the M-phase winding by the S-phase forward rotating flux must lag in time phase by $90^\circ$ on the emf which the same flux produces in the S-phase, and because of the different number of turns in the M and S-phase the magnitude of the emf in the M-phase is $\frac{1}{\alpha}$ times that in the S-phase. From these considerations, the impressed emf equivalent to the emf produced in the M-phase by the S-phase forward rotating flux is:

$$E_{MFS} = -j \frac{1}{\alpha} E_{FS}$$

(2.1)

By following a similar line of reasoning, the impressed emf equivalent to the other mutually generated emf's can be shown to be as given in the equivalent circuit of Figure (2.3). The equations for the total voltages impressed on each of the stator phases then following from Figure (2.3) as:
The simultaneous solution of (2.2) and (2.3) gives for the M-phase current of the capacitor motor:

\[ V_M = T_M \left[ (R_{1M} + R_f + R_b) + j(X_{1M} + X_f + X_b) \right] - j T_S a \left[ (R_f - R_b) + j(X_f - X_b) \right] \]  

\[ (2.2) \]

\[ V_S = T_S \left[ a^2 (R_{1S} + R_f + R_b) + j X_c + j a^2 (X_{1S} + X_f + X_b) \right] + j T_M a \left[ (R_f - R_b) + j(X_f - X_b) \right] \]  

\[ (2.3) \]

The simultaneous solution of (2.2) and (2.3) gives for the M-phase current of the capacitor motor:

\[ T_M = \frac{V_M \left[ a^2(R_{1S} + R_f + R_b) + j [X_c + a^2 (X_{1S} + X_f + X_b)] \right]}{\left[ \frac{a^2(R_{1S} + R_f + R_b) + j [X_c + a^2 (X_{1S} + X_f + X_b)]}{\left[ (R_{1M} + R_f + R_b) + j (X_{1M} + X_f + X_b) \right] - a^2[(R_f - R_b) + j(X_f - X_b)]^2} \right] + ja \left[ (R_f - R_b) + j(X_f - X_b) \right]} \]  

\[ (2.4) \]

and for the S-phase current:

\[ T_S = \frac{V_M \left[ (R_{1M} + R_f + R_b) + j (X_{1M} + X_f + X_b) \right] - ja\left[ (R_f - R_b) + j (X_f - X_b) \right]}{\left[ \frac{a^2 (R_{1S} + R_f + R_b)}{\left[ (R_{1M} + R_f + R_b) + j (X_{1M} + X_f + X_b) \right] - a^2 [(R_f - R_b) + j (X_f - X_b)]^2} \right] + R_f + R_b} \]  

\[ (2.5) \]
where:

\[ R_f = \frac{X_m^2 R_2}{\left(\frac{R_2}{S} \right)^2 + (X_2 + X_m)^2} \]

\[ X_f = \frac{X_m \left[ \left( \frac{R_2}{S} \right)^2 + X_2 (X_2 + X_m) \right]}{\left(\frac{R_2}{S} \right)^2 + (X_2 + X_m)^2} \]

\[ R_b = \frac{X_m \left( \frac{R_2}{S} \right)}{(X_2 + X_m)^2} \]

\[ X_b = \frac{X_m \left[ \left( \frac{R_2}{S} \right)^2 + X_2 (X_2 + X_m) \right]}{(X_2 + X_m)^2} \]

When both windings are connected in parallel \( V_M = V_S \), and the total line current \( I_L \) is:

\[ I_L = \frac{V_M}{\left[ a^2 (R_{1S} + R_f + R_b) + R_{1M} + R_f + R_b \right] + j[X_c + a^2 (X_{1S} + X_f + X_b)]} \]

\[ + \frac{j [X_c + a^2 (X_{1S} + X_f + X_b) + X_{1M} + X_f + X_b]}{[R_{1M} + R_f + R_b] + j (X_{1M} + X_f + X_b)] - a^2 [(R_f - R_b) + j(X_f - X_b)]^2} \]

\[ (2.6) \]

The general torque expression for a 2-phase unbalanced motor is:

\[ T_{ins} = I_m^2 \left[ (R_f - R_b) + (R_f - R_b) \cos 2\omega t - (X_f - X_b) \sin 2\omega t \right] \]

\[ + I_s^2 a^2 \left[ (R_f - R_b) + (R_f - R_b) \cos 2(\phi + \omega t) - (X_f - X_b) \sin (\phi + \omega t) \right] + 2 I_m I_s a \left( R_f + R_b \right) \sin \phi \]

\[ (2.7) \]
In which the average is:

\[ T_{av} = (I_M^2 + a^2 I_S^2)(R_f - R_b) + 2a (I_M I_S \sin \phi)(R_f + R_b) \]  \hspace{1cm} (2.8)

The output power of the motor is

\[ W_o = T_{av} (1 - S) - W_f \]  \hspace{1cm} (2.9)

where:

- \( S \) = fractional slip
- \( W_f \) = power lost in friction.

It follows therefore that:

\[ W_o = [(I_M^2 + a I_S^2)(R_f - R_b) + 2I_M I_S a (R_f + R_b) \sin \phi].(1 - S) - W_f \]  \hspace{1cm} (2.10)

The motor input power is\(^{(6)}\);

\[ W_I = \text{real part of the line current } I_L \times V_M + w_i \]  \hspace{1cm} (2.11)

where:

- \( w_i \) = iron losses

In the revolving-field theory outlined above, the iron losses are neglected in the equations and the performance calculations show a general lack of agreement with test values\(^{(8,10)}\), particularly in the auxiliary (S-phase) winding quantities and under running condi-
tions, and this is usually attributed to saturation and to the space harmonics of mmf.

The derivation of the basic equations of the revolving-field theory are given in Appendix I.
2.2 The Cross-Field Theory of the Capacitor Motor

In the cross-field theory of the single-phase induction motor\(^{(7,8)}\), the main flux of the motor and the flux due to the rotor currents are considered separately along both the main and the field axes (the axis of the M-phase stator winding may be called the main-axis, and of the S-phase winding the field axis). The two axes use the same magnetic structure and at standstill are magnetically independent, but when the motor is running the flux along one axis induces a voltage proportional to speed in the rotor winding on the other axis. The mutual fluxes and the leakage fluxes, together with their paths for both the M and S-phases, are shown in Figure 2.4(a) and (b) respectively. Mutual fluxes of \(\Phi_{mM}\) and \(\Phi_{mS}\) respectively link the stator and the rotor windings along the two axes, and there are also leakage fluxes \(\Phi_{2M}, \Phi_{2M}', \Phi_{2S}\) and \(\Phi_{2S}'\) to be associated with the stator and the rotor windings on these axes. All the mutual and leakage fluxes are stationary in space but variable with time. The current and voltage relationship for the stator windings are shown in the phasor diagrams of Figure (2.5), where that for the field axis is drawn with the mutual flux \(\Phi_{mS}\) in space quadrature with the main axis mutual flux \(\Phi_{mM}\). In the separate rotor circuits there are internal impedance drops \(I_{2M} (R_2 + jX_2)\) and \(I_{2S} (R_2 + jX_2)\) [where \(I_{2M}\) and \(I_{2S}\) are respectively the rotor currents in the main-axis circuit and in the field-axis circuit], a generated emf induced by cutting of the air-gap flux of the other axis, and a generated emf resulting from cutting of the rotor leakage flux of the other axis. In the M and S-phase stator windings, there are impedance drops \(I_M (R_{1M} + jX_{1M})\) and \(I_S (\frac{R_{1S} + jX_{1S}}{a^2})\) respectively, and a component of impressed voltage.
FIG. 2.4 (a,b). MUTUAL AND LEAKAGE FLUXES FOR THE
(a) MAIN WINDING.
(b) AUXILIARY WINDING.
FIG. 2.5. PHASE DIAGRAM SHOWING CURRENT AND VOLTAGE RELATIONSHIPS FOR THE STATOR AND ROTOR WINDINGS.
equal and opposite to the rotor induced voltage. For the main-axis of the stator winding, Figure (2.5a) shows how the voltage applied to the stator terminals must overcome the resistance drop and the mutual and leakage reactance drops due to $\phi_{mM}$ and to $\phi_M$, so that:

$$V_M = I_M (R_{1M} + jX_{1M}) + (I_M - I_{2M}) \left( \frac{jR_{mM}X_m}{R_m + jX_m} \right)$$  \hspace{1cm} (2.12)

Along the main-axis rotor circuit, the sum of the voltage induced by the transformer action of $\phi_{mM}$ and $\phi_{2M}$, and by rotation through the flux $\phi_{mS}$, plus the resistance drop $R_2 I_{2M}$ must equal zero. Thus:

$$I_{2M} (R_2 + jX_2) - (I_M - I_{2M}) \left( \frac{jR_{mM}X_m}{R_m + jX_m} \right) + jS \left[ - \frac{jR_SX_S}{R_S + jX_S} \cdot \frac{1}{a^2} \right]$$

$$(I_S - I_{2S}) + ja I_{2S} X_2 = 0$$  \hspace{1cm} (2.13)

where:

$$S = \frac{\text{actual speed}}{\text{synchronous speed}}$$

Similarly, for Figure (2.5b)

$$V_M = I_S \left( \frac{R_{1S} + jX_{1S} - jX_c}{a^2} \right) + (I_S - I_{2S}) \frac{jR_SX_S}{R_S + jX_S} \cdot \frac{1}{a^2}$$  \hspace{1cm} (2.14)
and for the field-axis rotor circuit, the sum of the emf's induced by the transformer action of \( \Phi_{ms} \) and \( \Phi_{2S} \), and by rotation through the flux \( \Phi_{mM} \), plus the resistance drop \( R_2 I_{2S} \) must equal zero.

\[
a I_{2S} (R_2 + jX_2) - (I_S - I_{2S}) \cdot \frac{1}{a} \left( \frac{R_{xS} X_S}{R_s + jX_s} \cdot \frac{1}{a^2} \right) - j S \left[ - (I_M - I_{2M}) \right].
\]

\[
\frac{j R_{mM} X_m}{R_m + J X_m} + j I_{2M} X_2 = 0
\]

(2.15)

solving these four equations simultaneously gives the stator currents as:

\[
I_M = \frac{C_1 B_2 - C_2 B_1}{A_1 B_2 - A_2 B_1}
\]

(2.16)

and

\[
I_S = \frac{A_1 C_2 - A_2 C_1}{A_1 B_2 - A_2 B_1}
\]

(2.17)

where:

\[
A_1 = S \left[ (R_{1M} + jX_{1M}) - X_2 (1 + \frac{R_{1M} + jX_{1M}}{R_m + jX_m}) \right]
\]

(2.18)
\[ B_1 = -[a (R_2 + j X_2) + (\frac{R_{1S} + j X_{1S} - j X_{c}}{a^2} \frac{1}{a} + \frac{a (R_2 + j X_2)}{R_s + j X_s} \frac{1}{a^2})] \] (2.19)

\[ C_1 = V \left[ S \left( j - \frac{X_2}{R_m X_m} \right) - \left( \frac{1}{a} + \frac{a (R_2 + j X_2)}{R_s X_s a^2} \right) \right] \] (2.20)

\[ B_2 = S \left[ -a X_2 + \left( \frac{R_{1S} + j X_{1S} - j X_{c}}{a^2} \frac{1}{a} - \frac{a X_2}{R_s X_s a^2} \right) \right] \] (2.21)

\[ A_2 = [(R_{1M} + j X_{1M})(1 + \frac{R_2 + j X_2}{R_m X_m}) + (R_2 + j X_2)] \] (2.22)

\[ C_2 = V \left[ [(1 + \frac{R_2 + j X_2}{R_m X_m}) + S \left( \frac{1}{a} - \frac{a X_2}{R_s X_s a^2} \right) \right] \] (2.23)

From equations (2.12) and (2.14), the rotor currents may be obtained as:

\[ I_{2M} = \frac{I_M [j R_m X_m + (R_{1M} + j X_{1M})(R_m + j X_m)] - V_M (R_m + j X_m)}{j R_m X_m} \] (2.24)
For locked-rotor conditions, equations (2.16) and (2.17) reduce to:

\[
I_2S = \frac{I_S [j R_S X_S + (R_{1S} + j X_{1S} - j X_C)(R_S + j X_S)] - V_M a^2 (R_S + j X_S)}{j R_S X_S}
\]  
(2.25)

and

\[
I_M = \frac{V_M [j R_m X_m + (R_2 + j X_2)(R_m + j X_m)]}{(R_{1M} + j X_{1M}) [j R_m X_m + (R_2 + j X_2)(R_m + j X_m)] + (R_2 + j X_2)}
\]  
(2.26)

and

\[
I_S = \frac{V_M a^2 [j R_S X_S + a^2 (R_2 + j X_2)(R_S + j X_S)]}{(R_{1S} + j X_{1S} - j X_C) [j R_S X_S + a^2 (R_2 + j X_2)(R_S + j X_S)] + a^2 j R_S X_S (R_2 + j X_2)}
\]  
(2.27)

The counter induced emf's in the M and S-phase windings are respectively:

\[
E_M = V_M - I_M (R_{1M} + j X_{1M})
\]  
(2.28)

and

\[
E_S = V_M - I_S \left( \frac{R_{1S} + j X_{1S} + j X_C}{a^2} \right)
\]  
(2.29)
The rotor input $W_{Ir}$, or the power transferred across the air-gap\((8)\), is

$$W_{Ir} = E_M I_{2M} \cos \theta_M + E_S I_{2S} \cos \theta_S$$ \hspace{1cm} (2.30)

The power converted to mechanical form\((8)\) equals the power transferred across the air-gap, given by equation (2.30), minus the rotor copper losses \((I_{2M}^2 + a^2 I_{2S}^2)R_2\).

The net output is therefore:

$$W_{\text{onet}} = \text{power converted to mechanical form} - w_f$$

where: $w_f$ is the combined friction and windage loss.

The electromagnetic torque\((8)\) is:

$$T = a E_M I_{2S} \sin \theta_1 + \frac{E_S}{a} I_{2M} \sin \theta_2$$ \hspace{1cm} (2.31)

where $\theta_1$ is the angle between the counter emf of the main-axis and the rotor current in the field-axis and $\theta_2$ is the angle between the counter emf in the field-axis and the rotor current in the main-axis.

The stator input power is:

$$W_I = V_M I_M \cos \theta_M + V_M I_S \cos \theta_S$$ \hspace{1cm} (2.32)

Performance calculations on the capacitor motor using the cross-field analysis developed above show good agreement with the actual motor performance\((7,8)\).
2.3 The Method of Symmetrical Components

In general, the solution of power and current flow problems in polyphase circuits when the applied voltages are not balanced, or when the load impedances are not equal, are readily accomplished by the method of symmetrical components\(^{(9)}\). Since the single-phase motor may be considered as a special case of an unbalanced 2-phase motor, the method of symmetrical components may be used for its analysis. The unbalanced supply voltages are converted into two systems of balanced 2-phase voltages of opposite phase sequence, termed respectively the positive-phase sequence and the negative-phase sequence systems. Positive-phase sequence voltages produce only positive-sequence currents and torque and negative-phase sequence voltages produce only negative-phase sequence currents and torque. The total current and torque are then found by the principle of superposition.

The derivation of the symmetrical component equations for the single-phase motor is given in Appendix II, where it is shown that the positive and negative sequence components of current in the M and S-phase stator windings are respectively:

\[
I^+_M = V_M \frac{Z^-_{tS} - ja Z^-_{tM}}{Z^+_{tM} Z^-_{tS} + Z^-_{tM} Z^+_{tS}} \tag{2.33}
\]

\[
I^-_M = V_M \left( \frac{Z^+_{tS} + j \frac{Z^+_{tS}}{a}}{Z^+_{tM} Z^-_{tS} + Z^-_{tM} Z^+_{tS}} \right) \tag{2.34}
\]
where, by definition, the M- and the S-phase currents are:

\[ I_M = I_M^+ + I_M^- \]  \hspace{1cm} (2.37)

and

\[ I_S = I_S^+ + I_S^- \]  \hspace{1cm} (2.38)

respectively. In the above equations \( Z_{tM}^+, Z_{tM}^-, Z_{tS}^+, Z_{tS}^- \) are the total positive and negative-sequence impedances for the M- and S-phases, respectively, and in the single-phase motor the same voltage is applied to both the M- and the S-phases, so that \( V_M = V_S \).

From equations 2.33, 2.34 and 2.37, we obtained:

\[ I_M = V_M \left[ \frac{(Z_{tS}^+ + Z_{tS}^-) + ja (Z_{tM}^+ - Z_{tM}^-)}{Z_{tM}^+ Z_{tS}^- + Z_{tM}^- Z_{tS}^+} \right] \]  \hspace{1cm} (2.39)
and from equations 2.35, 2.36 and 2.38:

\[
I_S = V_M \left[ \frac{(Z_{tM}^+ + Z_{tM}^-) - j/a (Z_{tS}^+ - Z_{tS}^-)}{Z_{tM}^+ Z_{tS}^- + Z_{tM}^- Z_{tS}^+} \right]
\]  \hspace{1cm} (2.40)

The line current of the motor is:

\[
I_L = I_M + I_S = V_M \left[ \frac{(Z_{tM}^+ + Z_{tM}^- + Z_{tS}^+ + Z_{tS}^-) + j(a Z_{tM}^+ - a Z_{tS}^- - Z_{tS}^+ Z_{tM}^- Z_{tS}^+ Z_{tM}^-)^{-1}}{Z_{tM}^+ Z_{tS}^- + Z_{tM}^- Z_{tS}^+} \right]
\]  \hspace{1cm} (2.41)

These equations can be applied to the capacitor motor, and the equivalent circuits thus derived for the M- and S-phases are shown in Figures (2.6) and (2.7) respectively.

The rotor impedance to positive-sequence current is:

\[
Z_2^+ = \frac{R_2}{S} + j X_2
\]  \hspace{1cm} (2.42)

The impedance \(Z_2^{+1}\) obtained by combining the magnetizing impedance \((R_m + j X_m)\) with the rotor impedance \(Z_2^+\), is

\[
Z_2^{+1} = \frac{1}{\frac{R_2}{S} + j X_2} + \frac{1}{R_m + j X_m}
\]
FIG. 2.6(a,b) EQUIVALENT CIRCUITS OF THE M-PHASE STATOR WINDING.
FIG 2.7 (a, b) EQUIVALENT CIRCUIT OF THE S-PHASE STATOR WINDING
\[
Z^+ = \frac{R_2^+ + X_2^+}{(S^{-1} R_m - X_2 X_m)(S^{-1} R_m + R_m) + (S^{-1} R_m - X_2 X_m)(R_2 + X_m)} + \frac{R_2^+ + X_2^+}{(S^{-1} R_m + R_m)^2 + (X_2 + X_m)^2}
\]

\[
\frac{R_2}{S} X_m + X_2 R_m (S^{-1} R_m + R_m) - \frac{R_2}{S} R_m - X_2 X_m (X_2 + X_m)
\]

\[
\frac{R_2}{S} + (X_2 + X_m)^2
\]

or

\[
Z_2^+ = R_2^+ + j X_2^+ \quad (2.43)
\]

where \( R_2^+ \) and \( j X_2^+ \) are respectively the positive-sequence resistance and reactance.

Similarly, since

\[
Z_2^- = \frac{R_2}{S - 5} + j X_2 \quad (2.44)
\]

The impedance \( Z_2^- \) to negative-sequence current is:

\[
Z_2^- = \frac{R_2}{S - 5} R_m - X_2 X_m \frac{R_2}{S - 5} + R_m \frac{R_2}{S - 5} X_m + X_2 R_m (X_2 + X_m)}{(S^{-1} R_m + R_m)^2 + (X_2 + X_m)^2}
\]

\[
(\frac{R_2}{S - 5} + R_m)^2 + (X_2 + X_m)^2
\]
\[
\begin{align*}
\frac{R_2}{(2-S) X_m + X_2 R_m} + \frac{R_2}{(2-S) R_m - X_2 X_m} (X_2 + X_m) \\
j \left( \frac{R_2}{(2-S) R_m} \right)^2 + (X_2 + X_m)^2
\end{align*}
\]

or

\[Z_2^{-1} = R_2^{-1} + j X_2^{-1}\] \hspace{1cm} (2.45)

where \(R_2^{-1}\) and \(j X_2^{-1}\) are respectively the negative-sequence resistance and reactance.

The stator impedance of the M-phase is

\[Z_{1M}^+ = Z_{1M}^- = R_{1M} + j X_{1M}\] \hspace{1cm} (2.46)

and that of the S-phase is

\[Z_{1S}^+ = Z_{1S}^- = [a^2 R_{1S} + j (a^2 X_{1S} - X_c)]\] \hspace{1cm} (2.47)

The total positive and negative-sequence impedances for the M and S-phases respectively, are therefore:

\[Z_{tM}^+ = (R_{1M} + R_2^{+1}) + j (X_{1M} + X_2^{+1})\] \hspace{1cm} (2.48)

\[Z_{tM}^- = (R_{1M} + R_2^{-1}) + j (X_{1M} + X_2^{-1})\] \hspace{1cm} (2.49)

\[Z_{tS}^+ = (a^2 R_{1S} + a^2 R_2^{+1}) + j (-X_c + a^2 X_{1S} + a^2 X_2^{+1})\] \hspace{1cm} (2.50)
\[ Z_{rs}^{-1} = (a^2 R_{1s} + a^2 R_{2s}^{-1}) + j(-X_c + a^2 X_{1s} + a^2 X_{2s}^{-1}) \]  

(2.51)

If equation (2.33) is combined with equations (2.48), (2.49), (2.50) and (2.51),

\[ I^+_M = V_M \frac{(a^2 R_{1s} + a^2 R_{2s}^{-1}) + j(a^2 X_{1s} - X_c + a^2 X_{2s}^{-1}) - ja(R_{1M} + R_{2M}^{-1}) + j(X_{1m} + X_{2m}^{-1})}{[(R_{1M} + R_{2M}^{-1}) + j(X_{1m} + X_{2m}^{-1})][(a^2 R_{1s} + a^2 R_{2s}^{-1}) + j(a^2 X_{1s} - X_c + a^2 X_{2s}^{-1})]} \]

\[ + \frac{(a^2 R_{1s} + a^2 R_{2s}^{-1}) + j(a^2 X_{1s} - X_c + a^2 X_{2s}^{-1})}{[(R_{1M} + R_{2M}^{-1}) + j(X_{1m} + X_{2m}^{-1})][(a^2 R_{1s} + a^2 R_{2s}^{-1}) + j(a^2 X_{1s} - X_c + a^2 X_{2s}^{-1})]} \]

\[ = V_M \frac{[a^2 R_{1s} + a^2 R_{2s}^{-1}] + j[a^2 X_{1s} - X_c + a^2 X_{2s}^{-1}] - a(R_{1M} + R_{2M}^{-1})}{[(R_{1M} + R_{2M}^{-1}) + j(X_{1m} + X_{2m}^{-1})][(a^2 R_{1s} + a^2 R_{2s}^{-1}) + j(a^2 X_{1s} - X_c + a^2 X_{2s}^{-1})]} \]

\[ + [(R_{1M} + R_{2M}^{-1}) + j(X_{1m} + X_{2m}^{-1})][(a^2 R_{1s} + a^2 R_{2s}^{-1}) + j(a^2 X_{1s} - X_c + a^2 X_{2s}^{-1})] \]

(2.52)

and similarly from equation (2.34).
and from equation (2.39):

\[
I_M = V_M \left[ \frac{\left[2a^2 R_1^S + a^2 (R_2^+ + R_2^-)\right] + j \left[2a^2 X_1^S - 2 X_c + a^2 (X_1^+ + X_1^-)\right] + j [a^2 X_1^+ X_2^- + a^2 X_1^- X_2^+]}{\left[R_1^M + R_2^M\right] + j (X_1^M + X_2^M)} \right] \left[\left(R_1^M + R_2^M\right) + j (X_1^M + X_2^M)\right][\left(a^2 R_1^S + a^2 R_2^S\right) + j (a^2 X_1^S - X_c + a^2 X_2^S)] + \]

\[
\frac{j a \left[\left(R_2^+ - R_2^-\right) + j (X_2^+ + X_2^-)\right]}{\left[R_1^M + R_2^M\right] + j (X_1^M + X_2^M)} \left[\left(a^2 R_1^S + a^2 R_2^S\right) + j (a^2 X_1^S - X_c + a^2 X_2^S)\right] + \]

(2.54)

From equation (2.35):

\[
I_S^+ = V_M \left[ \frac{\left[R_1^M + R_2^M\right] - \frac{1}{a} (a^2 X_1^S - X_c + a^2 X_2^S)}{\left[R_1^M + R_2^M\right] + j (X_1^M + X_2^M)} \right] \left[\left(a^2 R_1^S + a^2 R_2^S\right) + j (a^2 X_1^S - X_c + a^2 X_2^S)\right] + \]

\[
\frac{1}{a} (a^2 R_1^S + a^2 R_2^S) \left[\left[R_1^M + R_2^M\right] + j (X_1^M + X_2^M)\right][\left(a^2 R_1^S + a^2 R_2^S\right) + j (a^2 X_1^S - X_c + a^2 X_2^S)] + \]

(2.55)
From equation (2.36):

\[
I_s^* = \frac{V_M}{\left[\left(\frac{R_{1M}+R_{2}^+}{R_{1M}+R_{2}^+} + \frac{1}{a} (a^2 x_{1S} - x_c + a^2 x_{2}^+)\right) + j\left(x_{1M}+x_{2}^+\right)\right]} \quad (2.56)
\]

\[
\frac{1}{a} (a^2 R_{1S} + a^2 R_{2}^+)\]

\[
+ \left[\left(\frac{R_{1M}+R_{2}^+}{R_{1M}+R_{2}^+} + j\left(x_{1M}+x_{2}^+\right)\right)\right] \quad (2.57)
\]

\[
(a^2 R_{1S} + a^2 R_{2}^+) + \left(a^2 x_{1S} - x_c + a^2 x_{2}^+\right)\]

In the case of a balanced 2-phase motor, the gross rotor output power is:

\[
31
\]
since under these conditions only positive-sequence currents exist. The net rotor output is the gross output less friction and windage losses, iron loss and copper loss.

The positive-sequence currents in the rotor winding produce torque in the normal direction of rotation of the motor, and when negative-sequence currents exist they produce a counter torque tending to cause reverse rotation. From the equivalent circuit of Figure (2.6), the positive-sequence rotor current \( I_{2M}^{+} \) is:

\[
I_{2M}^{+} = \frac{R_m + j X_m}{R_2 + j X_2 + (R_m + j X_m)} I_M^{+} \tag{2.59}
\]

and the negative-sequence rotor current \( I_{2M}^{-} \) is

\[
I_{2M}^{-} = \frac{R_m + j X_m}{R_2 + j X_2 + (R_m + j X_m)} I_M^{-} \tag{2.60}
\]

The gross rotor output power is therefore:

\[
W_{o\text{gross}} = \left[ |I_{2M}^{+}|^2 \left( \frac{R_2}{Z-S} \right) + |I_{2S}^{+}|^2 \left( \frac{a^2 R_2}{Z-S} \right) - |I_{2M}^{-}|^2 \left( \frac{R_2}{Z-S} \right) - |I_{2S}^{-}|^2 \left( a^2 \frac{R_2}{Z-S} \right) \right] (1-S) \tag{2.61}
\]
where:

\[ |I^+_{2S}| = \frac{I^+_{2M}}{a} \]

\[ |I^-_{2S}| = \frac{I^-_{2M}}{a} \]

Substituting in equation (2.61):

\[ W_{\text{gross}}^2 [ |I^+_{2M}|^2 \left( \frac{R_2}{S} \right) - |I^-_{2M}|^2 \left( \frac{R_2}{2-S} \right) ] (1-S) \]  

(2.62)

and from equations (2.59) and (2.43):

\[ |I^-_{2M}|^2 \frac{R_2}{2-S} = \frac{(R_m^2 + X_m^2) \frac{R_2}{S}}{- \frac{R_2}{S} + R_m^2 + (X_2 + X_m)^2} \left( |I^+_{m}|^2 \right) \]

\[ = R_2^* |I^+_{m}|^2 \]  

(2.63)

and from equations (2.60) and (2.45):

\[ |I^-_{2M}|^2 \frac{R_2}{2-S} = \frac{(R_m^2 + X_m^2) \frac{R_2}{2-S}}{- \frac{R_2}{S} + R_m^2 + (X_2 + X_m)^2} \left( |I^-_{m}|^2 \right) \]

\[ = R_2^{-1} |I^-_{m}|^2 \]  

(2.64)
so that equation (2.62) reduces to:

\[
W_{\text{ogross}} = 2[|I_M^+|^2 R_2^+ - |I_M^-|^2 R_2^-] (1-S) \tag{2.65}
\]

By adding \( I_M \) (equation (2.54)) and \( I_S \) (equation (2.57)), the line current \( I_L \) is obtained. The in-phase component of \( I_L \) multiplied by the applied voltage \( V_M \) gives the input power \( W_I \), while \( V_M \) times \( I_L \) gives the VA input.

The power factor \( \cos \theta \) is:

\[
\cos \theta = \frac{W_I}{\text{VA input}} \tag{2.66}
\]

Subtracting the friction and windage losses together with the iron loss and the copper loss from equation (2.65) gives the net output power \( W_{\text{onet}} \).

The efficiency is then:

\[
\eta = \frac{W_{\text{onet}}}{W_I} \tag{2.67}
\]
2.4 Performance Calculations of the Capacitor Motor

2.4.1 The equivalent circuit of the capacitor motor

From equations (2.12 - 2.15) and equation (2.31), an equivalent circuit (10) for the capacitor motor can be obtained. The equations can be rewritten as:

\[ E_M + I_M (R_{1M} + jX_{1M}) = V_M \]  \hspace{1cm} (2.68)

\[ E_S + I_S (R_{1S} + jX_{1S} - jX_C) = V_M \]  \hspace{1cm} (2.69)

\[ \frac{E_S}{a} + jS(-E_M + jI_{2M}X_2) = aI_{2S} (R_2 + jX_2) \]  \hspace{1cm} (2.70)

\[-E_M + jS(- \frac{E_S}{a} + jI_{2S}X_2) = -I_M (R_2 + jX_2) \]  \hspace{1cm} (2.71)

\[ E_M = (I_M - I_{2M}) \left( \frac{jR_mX_m}{R_m + jX_m} \right) \]  \hspace{1cm} (2.72)

\[ E_S = (I_S - I_{2S}) \left( \frac{jR_sX_s}{R_s + jX_s} \right) \]  \hspace{1cm} (2.73)

and

\[ T = aE_M I_{2S} \sin \theta_1 + \frac{E_S}{a} I_{2M} \sin \theta_2 \]  \hspace{1cm} (2.74)

In writing equations (2.68 - 2.74), the supply voltage \( V_M \) is taken as the reference phasor for all the other voltages and currents, so that its phase angle is therefore zero.
When the scalar equation (2.74) is converted to vector form it becomes:

\[
T = \text{Real part of } [-j \left( I_{2S} \text{ Conj } E_M - I_{2M} \text{ Conj } \frac{E_S}{a} \right)]
\] (2.75)

When equations (2.70) and (2.71) are solved for \( E_S \) and \( E_M \), they yield:

\[
\frac{E_S}{a} = j \frac{S R_2}{1-S^2} I_{2M} + j a X_2 I_{2S} + \frac{R_2}{1-S^2} a I_{2S}
\] (2.76)

\[
E_M = \frac{-j S R_2}{1 - S^2} a I_{2S} + j X_2 I_{2M} + \frac{R_2}{1-S^2} I_{2M}
\] (2.77)

respectively, and with the introduction of the simplifying substitutions:

\[
R = \frac{S R_2}{1-S^2}
\]

\[
Z_2' = \frac{R_2}{1+S} + j X_2
\]

\[
E'_S = -\frac{j E_S}{a}
\]

\[
I_{2S}' = -j a I_{2S}
\]
Equations (2.76), (2.77) and (2.75) become:

\[ E_S' = (Z_S' + R) I_{2S}' + R I_{2M}' \]  \hspace{1cm} (2.78)

\[ E_M' = (Z_M' + R) I_{2M}' + R I_{2S}' \]  \hspace{1cm} (2.79)

and

\[ T = \text{Real part of} \ (E_M' \ \text{conj.} E_S' + I_{2M}' \ \text{conj.} E_S') \]  \hspace{1cm} (2.80)

(where \( E_S' = \frac{j \ \text{conj.} E_S}{a} \))

respectively.

Substituting equations (2.78) and (2.79) into equation (2.80) yields:

\[ T = \text{Real part of} \ [I_{2M}' (\text{conj} \ Z_S' + R) \ \text{times conj} \ I_{2S}' + R I_{2M}' \ \ \text{conj} \ I_{2S}' + I_{2S}' \ \ \text{conj} \ I_{2M}' + \ \text{conj} \ I_{2S}' + R I_{2S}' \ \ \text{conj} \ I_{2S}' + \ \text{conj} \ I_{2M}'] \]

\[ = \text{Real part of} \ [\ \text{conj} \ Z_S' (I_{2M}' \ \text{conj} \ I_{2S}' + I_{2S}' \ \text{conj} \ I_{2M}') + R (I_{2M}' + I_{2S}') \ \text{conj} \ (I_{2M}' + I_{2S}')] \]  \hspace{1cm} (2.81)
where

\[ I_{2M} \text{ conj } I_{2S}^t + I_{2S}^t \text{ conj } I_{2M} = 2 I_{2S}^t I_{2M} \cos \psi \]

Only the real part of the expression on the right hand side of equation (2.81) contributes to the torque, so that:

\[ T = \frac{2 R_2}{1 + S} I_{2S}^t I_{2M} \cos \psi + R |I_{2M}^t + I_{2S}^t|^2 \quad (2.82) \]

Equations (2.78) and (2.79) are readily recognized as the mesh equations for the circuit shown in Figure (2.8).

Multiplying equations (2.69) and (2.73) by -j/a and noting that:

\[ E_S^t = -\frac{j E_S}{a} \]

\[ I_S^t = -ja I_S \]

gives

\[ E_S + I_S^t \left( \frac{R_1S + j X_1S - j X_c}{a^2} \right) = -\frac{jV}{a} \quad (2.83) \]

\[ E_S^t = \frac{j R_S X_S}{R_S + j X_S} \cdot \frac{1}{a^2} (I_S^t - I_{2S}^t) \quad (2.84) \]
Equations (2.83), (2.84), (2.68) and (2.72) can be used to add stator parameters to the rotor circuit of Figure (2.8a), to provide the full equivalent circuit of Figure (2.9).

Figure (2.9) is the equivalent circuit of a capacitor motor in which the capacitor \( \frac{X}{a^2} \) is in series with the S-phase winding, with the combination being in parallel with the M-phase winding and both ends of the circuit being supplied from the same voltage source. A simplified equivalent circuit, in which currents from the M- and S-phases feed into the rotor branches is shown in Figure (2.9). The rotor impedances referred to both the M-phase and S-phase windings are the same, so that the two rotor currents are cophasal\(^{10}\). In order that these currents correspond to the actual motor condition of being in time quadrature, it is necessary for the operator \( j \) to be applied to the S-phase stator winding voltage of the equivalent circuit, even though in practice the M-phase and S-phase windings are connected to the same source. Further, since the M-phase and S-phase stator windings have different numbers of turns, the voltages applied to the S-phase winding must also be divided by \( a \), so that the resulting S-phase winding voltage is \(-\frac{j}{a}\) times the M-phase winding voltage.

2.4.2 Performance determination of the capacitor-run induction motor using the equivalent circuit

In this section, the performance of the capacitor motor is calculated using the equivalent circuit shown in Figure (2.9). If the voltage across the virtual load resistor \( R \) (where \( R = \frac{S}{1-S^2}R_2 \)) is \( E \), then by applying Kirchhoff's voltage law to the M and
FIG. 2.8. (a, b). SIMPLIFIED EQUIVALENT CIRCUIT VIEWED FROM THE ROTOR.

FIG. 2.9. EQUIVALENT CIRCUIT FOR THE CAPACITOR MOTOR WHERE 
\[ R = \frac{SR_2}{1-S^2} \]
FIG. 2.10. APPROXIMATE EQUIVALENT CIRCUIT AT MAXIMUM TORQUE.
S-phase windings separately the following equations can be written:

For the M-phase stator winding:

The voltage across the magnetizing branch is:

\[ E_M = E + I_{2M} Z^r_2 \]  \hspace{1cm} (2.85)

where \( Z^r_2 \) = rotor impedance = \( \frac{R_2}{1+\delta} + j X_2 \)

The current in the magnetizing branch is:

\[ (I_M - I_{2M}) = E_M Y_m \]  \hspace{1cm} (2.86)

The applied voltage is:

\[ V_M = E_M + I_M Z_{1M} \]  \hspace{1cm} (2.87)

Substituting equation (2.85) in equation (2.86) yields:

\[ I_M = I_{2M} (1 - Y_m Z^r_2) + E Y_m \]  \hspace{1cm} (2.88)

The current in the M-phase rotor winding can be obtained from equations (2.87) and (2.88) as:

\[ I_{2M} = \frac{V_M (Z^r_2 + \frac{1}{Z_{1M}} + \frac{Z_{1M} Y_m}{Z^r_2 + \frac{1}{Z_{1M}} + \frac{Z^r_2}{Z_{1M} Y_m}}) - E \left( \frac{1 + Z_{1M} Y_m}{Z^r_2 + Z_{1M} + \frac{Z^r_2}{Z_{1M} Y_m}} \right)}{Y_{LM} - E Y_{OM}} \]

\[ = \frac{V_M Y_{LM} - E Y_{OM}}{Y_{OM}} \]  \hspace{1cm} (2.89)
and the M-phase stator current follows from equations (2.88) and (2.89) as:

\[
I = V_M \left( \frac{1 + Z_1^2 Y_m}{Z_2^2 + Z_{1M} + Z_2^1 Z_{1M} Y_m} \right) - E \left( \frac{1}{Z_2^2 + Z_{1M} + Z_2^1 Z_{1M} Y_m} \right)
\]

\[
= V_M Y_{eM} - E Y_{tM}
\]

\[
= I_{eM} - E Y_{tM}
\]

(2.90)

where:

\[
Y_{tM} = \frac{1}{Z_2^2 + Z_{1M} + Z_2^1 Z_{1M} Y_m}
\]

(2.91)

\[
Y_{OM} = (1 + Z_{1M} Y_m) Y_{tM}
\]

\[
= \frac{1 + Z_{1M} Y_m}{Z_2^2 + Z_{1M} + Z_2^1 Z_{1M} Y_m}
\]

(2.92)

\[
I_{OM} = E Y_{tM}
\]

(2.93)

\[
I_{eM} = E Y_{eM}
\]

(2.94)

For the S-phase stator winding:

The voltage across the magnetizing branch is:

\[
E_S = E + I_{2S} Z_2^1
\]

(2.95)
The current in the magnetizing branch is:

\[(I_1' - I_2') = EY_s a^2\]  \hspace{1cm} (2.96)

where \(Y_s = \text{magnetizing admittance} = \frac{R_s + jX_s}{JRSX_s}\)

The applied voltage is:

\[-j\frac{V_M}{a} = E_s + I_1' Z_{1s}^*\]  \hspace{1cm} (2.97)

where \(Z_{1s}^* = \frac{1}{a^2}(R_{1s} + jX_{1s} - jX_c)\)

Substituting equation (2.96) in equation (2.97) yields:

\[I_1' = I_2' (1 + Y_s a^2 Z_2^*) + E a^2 Y_s\]  \hspace{1cm} (2.98)

The field-axis rotor current in the M-phase winding terms with a 90° backward phase shift can be obtained from equations (2.97) and (2.98) as:

\[I_2' = (-j\frac{V_M}{a}) \left(\frac{1}{Z_2^* + Z_{1s}^* + Z_2^* Z_{1s}^* a^2 Y_s}\right) - E \left(\frac{1 + Z_{1s}^* a^2 Y_s}{Z_2^* + Z_{1s}^* + Z_2^* Z_{1s}^* a^2 Y_s}\right)\]

\[= (-j\frac{V_M}{a}) Y_{1s} - E Y_{0s}\]

\[= I_{0s} - E Y_{0s}\]  \hspace{1cm} (2.99)
and the S-phase winding current in the M-phase winding terms
with a 90° backward phase shift from equations (2.98) and (2.99)
as:

\[
I_S^I = (-j \frac{V_M}{a}) \left( \frac{1 + \frac{Z_2}{Z_2} a^2 \gamma_S}{Z_2 + Z_{1S} + Z_2 Z_{1S} a^2 \gamma_S} \right) - E \left( \frac{1}{Z_2 + Z_{1S} + Z_2 Z_{1S} a^2 \gamma_S} \right)
\]

\[
= (-j \frac{V_M}{a}) \gamma_{eS} - E \gamma_{1S}
\]

\[
= I_{eS} - E \gamma_{1S}
\]

where:

\[
\gamma_{1S} = \frac{1}{Z_2 + Z_{1S} + Z_2 Z_{1S} a^2 \gamma_S}
\]

\[
\gamma_{0S} = \frac{1 + \frac{Z_1}{Z_2} a^2 \gamma_S}{Z_2 + Z_{1S} + Z_2 Z_{1S} a^2 \gamma_S}
\]

\[
\gamma_{eS} = \frac{1 + \frac{Z_2}{Z_2} a^2 \gamma_S}{Z_2 + Z_{1S} + Z_2 Z_{1S} a^2 \gamma_S}
\]

\[
I_{0S} = (-j \frac{V_M}{a}) \gamma_{1S}
\]

\[
I_{eS} = (-j \frac{V_M}{a}) \gamma_{eS}
\]
The current $I$ through the virtual load resistor $R$ is obtained by adding equations (2.89) and (2.99) as:

$$I = I_{2M} + I_{2S}$$

$$= (I_{OM} + I_{OS}) - E (Y_{OS} + Y_{OM})$$

$$= I_0 - E Y_0$$  \hspace{1cm} (2.106)

where:

$$I_0 = I_{OS} + I_{OM}$$  \hspace{1cm} (2.107)

$$Y_0 = Y_{OS} + Y_{OM}$$  \hspace{1cm} (2.108)

The line current $I_L$ is given from equations (2.90) and (2.100) as:

$$I_L = I_M + I_S$$

$$= I_M + j \frac{I_S}{a}$$

$$= (I_{eM} + j \frac{I_{eS}}{a}) - E (Y_{tM} + j \frac{Y_{1S}}{a})$$  \hspace{1cm} (2.109)

where: $I_S = -j \frac{I_{S}}{a}$
Substituting $E = IR$ in equation (2.106) we obtain:

$$I = \frac{I_0}{1 + R \gamma_0} \quad (2.110)$$

At any given speed, $I_0$ and $\gamma_0$ can be calculated using equations (2.91 - 2.94), (2.101 - 2.105) and (2.107, 2.108) and from equation (2.110) $I$ and $E$ then follow. From equations (2.89), (2.90), (2.99) and (2.100) $I_{2M}$, $I_M$, $I_{2S}$ and $I_S$, may be determined and the performance of the motor (torque, line current, efficiency and power factor) consequently specified.

The output power ($W_0$) of the motor is:

$$W_0 = I^2 R S \quad (2.111)$$

The input power ($W_1$) of the motor is:

$$W_1 = V_M \times \text{Real part of the line current (I}_L) \quad (2.112)$$

The input power-factor of the motor is:

$$\cos \theta = \text{cosine of angle between the applied voltage and the line current.}$$

The motor torque is:

$$T = \frac{2R_2}{1 + S} I_{2M} I_{2S} \cos \psi + I^2 R \quad (2.113)$$

where $\psi$ = angle between the rotor currents of the main axis and the field axis.
2.4.3 The maximum torque of the motor

Any attempt to derive an expression for the maximum torque of a capacitor motor is bound to lead to long and tedious calculations. The equivalent circuit theory is certainly no exception to this if the circuit of Figure (2.9) and the torque equation (2.113) are used without approximation. However, many unaccountable factors may introduce an error far larger than that caused by suitable approximations of Figure (2.9) and equation (2.113). Typical of these are the temperature variation of the stator and rotor resistances, the effect of the space harmonics, the variation of the air-gap length and skin effect in the rotor conductors. Taken together these factors may introduce a far more appreciable error, and thus, the maximum torque may be difficult to be calculated accurately. The maximum torque of the motor can be evaluated from an approximate procedure as follows:

1. Since $Z'_2$ is dependent on $S(=\frac{\text{actual speed}}{\text{synchronous speed}})$ and the maximum variation of $S$ is between 0.7 to 1, then $(\frac{R_2}{1+S})$ varies between 0.59 $R_2$ and 0.5 $R_2$. We may therefore assume that:

$$Z'_2 = 0.55 R_2 + j X_2$$

(2.114)

2. The variation \( (\frac{2R_2}{1+S} I^2 M \frac{I'_2}{2S} \cos\psi) \) of equation (2.113) is very small compared with the first term, and since it amounts to not more than 5% at the speed for maximum torque and introduces an error below that introduced by the factors mentioned previously, it can reasonably be neglected\(^{(10)}\). It can therefore be assumed that the maximum torque occurs when $RI^2$ is a maximum.
From equation (2.110), it is evident that $RI^2$ is a maximum when

$$R = \frac{1}{Y_0},$$

whence:

$$T_{\text{max}} = \frac{I_0^2}{Y_0} \left| \frac{1}{1 + \frac{Y_0}{Y_0}} \right|^2 + \frac{2R_2^2}{1 + \frac{S}{2}} I_{2M} I_{2S} \cos \psi$$

(2.115)

(where $Y_0$ is the magnitude of $Y_o$).

Since the second term of equation (2.113) is neglected, the maximum torque is now given by:

$$T_{\text{max}} = \frac{I_0^2}{Y_0} \left| \frac{1}{1 + \frac{Y_0}{Y_0}} \right|^2$$
$$= K_t \frac{I_0^2}{Y_0}$$

(2.116)

where the coefficient $K_t$ depends only on the phase of $Y_0$.

If $\theta_1$ is the phase angle of $Y_0$, then:

$$K_t = \left| \frac{1}{1 + \frac{Y_0}{Y_0}} \right|^2 = \left| \frac{1}{1 + e^{j\theta}} \right|^2 = \left( \frac{1}{2 \cos \frac{\theta_1}{2}} \right)$$

(2.117)

The starting torque of the capacitor motor is obtained by substituting $S = 0$ into equation (2.113), to give:

$$T_{\text{st}} = 2 R_2 I_{2M} I_{2S} \cos \psi$$

(2.118)
where $I_{2M_0}$ and $I_{2S_0}$ are respectively the values of $I_{2M}$ and $I_{2S}$ for $S = 0$ and $Z_2' = R_2 + jX_2$.

2.5 Conclusion

The revolving-field(6) and the cross-field(7,8) theories are the two main methods of analysis for the capacitor motor. Both theories explain most of the known and demonstrable phenomena and have been used to develop quantitative methods of analysis. The symmetrical component method(9) may be used in situations where the voltages or impedances are not balanced.

In this thesis, the equivalent circuit(10) of the capacitor motor based on the cross-field theory will be used in calculating the performance of the capacitor motor, since it offers the following advantages.

1. The iron loss is included in the calculation of the parameters.
2. The equivalent circuit can be used to calculate the performance of other types of motor by modifying the constants of the 5-phase stator winding.
3. The equation for the maximum torque of the capacitor motor is accurate and comparatively simple.
4. It is claimed(10) that the results are more accurate than those obtained by other methods.
CHAPTER III
DETERMINATION OF CONSTANTS FOR THE EQUIVALENT CIRCUIT OF THE CAPACITOR MOTOR

The parameters of a capacitor motor can be obtained experimentally in a variety of ways, using measurements of the current, voltage and the input power to the stator. To determine these parameters, as defined in the equivalent circuit [10] of Figure (2.9) of Chapter II, no-load and locked-rotor tests were made in a manner similar to that followed for a true single-phase machine [11,12], with each winding being excited independently in turn [8]. Since the two windings of the experimental machine are identical, the tests were performed on the M-phase winding only. For no-load \( S = 1 \) and \( R = \infty \), when the equivalent circuit of Figure (2.9) reduces to that of Figure (3.1). The input impedance \( Z_o \) of Figure (3.1) when the input power factor angle is \( \theta_o \) is:

\[
Z_o = R_o + jX_o
\]

\[
= \frac{V_o}{10 (\cos \theta_o + j \sin \theta_o)}
\]

\[
\frac{(Z_2 + Z_m) Z_m}{Z_2 + 2Z_m} \frac{(R_2 + j2X_2) + (\frac{jX_m R_m}{R_m + jX_m}) (\frac{jX_m R_m}{R_m + jX_m})}{(R_2 + j2X_2) + (\frac{j2X_m R_m}{R_m + jX_m})} + (R_{1M} + jX_2)
\]

(3.1)
FIG. 3.1. APPROXIMATE EQUIVALENT CIRCUIT FOR NO-LOAD CONDITIONS, WITH \( S = \frac{\text{ACTUAL SPEED}}{\text{SYNCHRONOUS SPEED}} = 1 \)
Since as given in Chapter II:

\[ Z_{1M} = R_{1M} + j X_{1M} \]

\[ Z_2 = R_2 + j X_2 \]

and

\[ Z_m = \frac{j R_m X_m}{R_m + j X_m} \]

Since \( Z_s = a^2 Z_m \), and the two windings of the experimental machine are identical (i.e. \( a = 1 \)), then:

\[ Z_s = Z_m \]

In most cases, the actual distribution of leakage reactance between the primary and the secondary of an induction motor are not accurately known since there is no method by which this distribution can be determined from test results. It is therefore justifiable, for many purposes, to assume that \( X_{1M} = X_2 \) (5):

then:

\[ Z_{1M} = R_{1M} + j X_2 \]

\[ Z_m = \frac{j R_s X_s}{R_s + j X_s} \]

For a locked-rotor test \( S = 0 \) and \( R = 0 \) when the equivalent circuit of Figure (2.9) reduces to that of Figure (3.2). If the input power
FIG. 3.2. APPROXIMATE EQUIVALENT CIRCUIT FOR LOCKED ROTOR CONDITIONS WITH S = 0.
factor angle under these conditions is $\theta_L$, the corresponding input impedance $Z_L$ is:

$$Z_L = R_L + jX_L$$

$$= \frac{V}{I_{1L} (\cos \theta_L + j \sin \theta_L)}$$

$$= \frac{Z_x Z_m}{Z_x + Z_m} + Z_{1M}$$

$$= \frac{(R_2 + jX_2) (\frac{jX_m R_m}{R_m + jX_m})}{(R_2 + jX_2) + (R_{1M} + jX_2)} \quad \text{(3.2)}$$

The effective resistances and reactances of the equivalent circuits representing both no-load and locked-rotor conditions may be obtained from equations (3.1) and (3.2).

Thus:

$$R_o = \frac{[- R_2 R_m X_m^2 - 2 R_m^2 X_2 X_m - R_m^2 X_m^2)(R_2 R_m^2 - 4 X_2 X_m R_m - R_2 X_m^2)}{[(R_2 R_m^2 - 4 X_2 X_m R_m - R_2 X_m^2)}$$

$$\frac{4 X_2 X_m R_m - R_2 X_m^2 - 2 R_m X_m^2 + (R_m^2 R_2 X_m - 2 R_m X_m^2 X_2)}{-(2 R_m X_m^2)^2 + (2 R_2 R_m X_m + 2 R_m^2 X_2 +}$$

$$\frac{(2 R_2 R_m X_m + 2 R_m^2 X_2 + 2 R_m^2 X_m - 2 X_2 X_m^2)}{2 R_m^2 X_m - 2 X_2 X_m^2)^2] + R_{1M}} \quad \text{(3.3)}$$
\[X_o = \frac{[(R_2 R_m X_m^2 + 2 R_m^2 X_2 X_m + R_m^2 X_m^2)]}{[(R_2 R_m^2 - 4 X_2 X_m R_m - R_m^2 X_m^2)] + (2 R_2 R_m X_m + 2 R_m^2 X_2)\sqrt{2 R_2 R_m X_m (2 R_m^2 X_2 + 2 R_m^2 X_2 + 2 R_m^2 X_m - 2 X_2 X_m^2) + (2 R_m^2 X_m^2)^2}}\]

\[R_2 X_m^2 - 2 R_m X_m^2\] + \[X_2\] \hfill (3.4)

\[R_L = \frac{[(- X_2 R_m X_m) (R_2 R_m - X_2 X_m) +]}{[(R_2 R_m^2 - X_2 X_m)^2 + \frac{(R_2 R_m X_m) (R_m X_2 + R_2 X_m + R_m X_m)]}{(R_m X_2 + R_2 X_m + R_m X_m)^2}} + R_{1M}\]

\[X_L = \frac{[(X_2 R_m X_m) (R_m X_2 + R_2 X_m + R_m X_m)]}{[(R_2 R_m^2 - X_2 X_m)^2 + \frac{(R_2 R_m X_m) (R_2 R_m - X_2 X_m)]}{(R_m X_2 + R_2 X_m + R_m X_m)^2}} + \[X_2\] \hfill (3.6)\]
These four equations (3.3), (3.4), (3.5) and (3.6), include the iron losses in the form of the term $R_m$. However, the direct determination of the machine parameters from these equations requires a very complicated set of mathematical procedures, and presents a difficulty that can only be overcome by the use of a digital computer to obtain the parameters through an iterative process.
3.1 Determination of Parameters Using the Newton-Raphson Iterative Method

The resistive and reactive components of the input impedances may be arranged in the form of a matrix \([Z]\) as follows

\[
[Z] = \begin{bmatrix}
R_0 \\
X_0 \\
R_L \\
X_L \\
\end{bmatrix}
\]  

(3.7)

where the elements of the input impedances are numbers obtained from no-load and locked-rotor tests. Assumed initial estimates for \(R_2, X_2, R_m\) and \(X_m\) may be obtained from manufacturers specifications of similarly rated machines. Corresponding values for \(Z\) can be determined using equations (3.3) to (3.6) inclusive, and an assumed impedance \([Z']\) can be written as:

\[
[Z'] = \begin{bmatrix}
R'_0 \\
X'_0 \\
R'_L \\
X'_L \\
\end{bmatrix}
\]  

(3.8)

The errors between the assumed and the experimental parameters are then:

\[
[e] = \begin{bmatrix}
R_0 - R'_0 \\
X_0 - X'_0 \\
R_L - R'_L \\
X_L - X'_L \\
\end{bmatrix} = \begin{bmatrix}
e_1 \\
e_2 \\
e_3 \\
e_4 \\
\end{bmatrix}
\]  

(3.9)
Assuming $X_2$, $R_m$ and $X_m$ to be constant, but $R_2$ to change such that $R_2 = R_2 + \Delta R_2$, we can determine a further impedance $Z''$ as:

$$
\begin{bmatrix}
R''_0 \\
X''_0 \\
R''_L \\
X''_L
\end{bmatrix}
= \begin{bmatrix}
Z''
\end{bmatrix}
$$

(3.10)

where $R''_0$, $X''_0$, $R''_L$ and $X''_L$ are the assumed machine parameters as changed by the change in $R_2$.

The errors between the changed values and the experimental values are:

$$
\begin{bmatrix}
R_0 - R''_0 \\
X_0 - X''_0 \\
R_L - R''_L \\
X_L - X''_L
\end{bmatrix}
= \begin{bmatrix}
e_1 \\
e_2 \\
e_3 \\
e_4
\end{bmatrix}
$$

(3.11)

and the differences in the errors in equations (3.9) and (3.11) are:

$$
\begin{bmatrix}
e_1' - e_1 \\
e_2' - e_2 \\
e_3' - e_3 \\
e_4' - e_4
\end{bmatrix}
= \begin{bmatrix}
\Delta e_1 \\
\Delta e_2 \\
\Delta e_3 \\
\Delta e_4
\end{bmatrix}
$$

(3.12)

From equation (3.12), the error rate with respect to $R_2$, with $X_2$,
Rm and Xm constant, can be defined as:

\[
[D_1] = \begin{bmatrix}
\frac{\Delta e_1}{R_2} \\
\frac{\Delta e_2}{R_2} \\
\frac{\Delta e_3}{R_2} \\
\frac{\Delta e_4}{R_2}
\end{bmatrix}
\]

(3.13)

In a similar way to that followed above, the error rate can be determined for variations in X2, i.e. R2, Rm and Xm are constant and \(X_2 = X_2 + \Delta X_2\), when the impedance matrix using equations (3.9) to (3.12) is:

\[
[Z''] = \begin{bmatrix}
R_{01}'' \\
X_{01}'' \\
R_{L1}'' \\
X_{L1}''
\end{bmatrix}
\]

(3.14)

The error between these values and the experimental values is:

\[
[e''] = \begin{bmatrix}
R_0 - R_{10}'' \\
X_0 - X_{10}'' \\
R_L - R_{L1}'' \\
X_L - X_{L1}''
\end{bmatrix}
= \begin{bmatrix}
e_1'' \\
e_2'' \\
e_3'' \\
e_4''
\end{bmatrix}
\]

(3.15)
and the difference in the errors between equations (3.9) and (3.15) is:

\[
[\Delta e'] = \begin{bmatrix}
  e_1'' - e_1 \\
  e_2'' - e_2 \\
  e_3'' - e_3 \\
  e_4'' - e_4
\end{bmatrix} = \begin{bmatrix}
  \Delta e_1' \\
  \Delta e_2' \\
  \Delta e_3' \\
  \Delta e_4'
\end{bmatrix}
\] (3.16)

From equation (3.16), we can define the error rate with respect to \( X_2 \), with \( R_2 \), \( R_m \) and \( X_m \) constant, as:

\[
[D_2] = \begin{bmatrix}
  \frac{\Delta e_1'}{\Delta X_2} \\
  \frac{\Delta e_2'}{\Delta X_2} \\
  \frac{\Delta e_3'}{\Delta X_2} \\
  \frac{\Delta e_4'}{\Delta X_2}
\end{bmatrix}
\] (3.17)

Similarly, the error rates for variations of \( R_m \) and \( X_m \) are:

\[
[D_3] = \begin{bmatrix}
  \frac{\Delta e_1'}{\Delta R_m} \\
  \frac{\Delta e_2'}{\Delta R_m} \\
  \frac{\Delta e_3'}{\Delta R_m} \\
  \frac{\Delta e_4'}{\Delta R_m}
\end{bmatrix}
\] (3.18)
and:

\[
[D_4] = \begin{bmatrix}
\frac{\Delta e_1}{\Delta x_1} & \frac{\Delta e_2}{\Delta x_1} & \frac{\Delta e_3}{\Delta x_1} & \frac{\Delta e_4}{\Delta x_1} \\
\frac{\Delta e_1}{\Delta x_2} & \frac{\Delta e_2}{\Delta x_2} & \frac{\Delta e_3}{\Delta x_2} & \frac{\Delta e_4}{\Delta x_2} \\
\frac{\Delta e_1}{\Delta x_3} & \frac{\Delta e_2}{\Delta x_3} & \frac{\Delta e_3}{\Delta x_3} & \frac{\Delta e_4}{\Delta x_3} \\
\frac{\Delta e_1}{\Delta x_m} & \frac{\Delta e_2}{\Delta x_m} & \frac{\Delta e_3}{\Delta x_m} & \frac{\Delta e_4}{\Delta x_m}
\end{bmatrix}
\]

respectively. From the above equations, we can assemble the finite difference matrix:

\[
[D] = [D_1 \ D_2 \ D_3 \ D_4]
\]

\[
\begin{bmatrix}
\frac{\Delta e_1}{\Delta x_2} & \frac{\Delta e_1}{\Delta x_2} & \frac{\Delta e_2}{\Delta x_2} & \frac{\Delta e_2}{\Delta x_2} \\
\frac{\Delta e_1}{\Delta x_3} & \frac{\Delta e_2}{\Delta x_3} & \frac{\Delta e_3}{\Delta x_3} & \frac{\Delta e_3}{\Delta x_3} \\
\frac{\Delta e_1}{\Delta x_m} & \frac{\Delta e_2}{\Delta x_m} & \frac{\Delta e_3}{\Delta x_m} & \frac{\Delta e_4}{\Delta x_m}
\end{bmatrix}
\]

By inverting \([D]\) and using the relationship\(^{(13,14)}\):
\[ [\Delta X] = - [D]^{-1} [\Delta e] \]  

(3.21)

where:

\[
[\Delta X] = \begin{bmatrix}
\Delta R_2 \\
\Delta X_2 \\
\Delta R_m \\
\Delta X_m
\end{bmatrix}
\]

We can obtain modified values for \( R_2', X_2', R_m' \) and \( X_m' \), as given by:

\[
\begin{align*}
R_2' &= R_2 + \Delta R_2 \\
X_2' &= X_2 + \Delta X_2 \\
R_m' &= R_m + \Delta R_m \\
X_m' &= X_m + \Delta X_m
\end{align*}
\]

If the values of \( \Delta X \) are within appropriate limits, the modified values of \( R_2', X_2', R_m' \) and \( X_m' \) are assumed to be correct. If, however, they lie outside these limits, the procedure is repeated from equation (3.7) onwards, until the solution converges to the required accuracy. The values for \( R_2, X_2, R_m \) and \( X_m \) are obtained by using the computer program described in Appendix III(A), and the results are shown in Table (3.2). Table (3.1) shows the M-phase stator winding resistance for no-load and full load conditions at different frequencies.
3.2 **Conclusion**

From the equivalent circuits of Figures (3.1) and (3.2) for the no-load and locked-rotor tests, a set of non-linear voltage/current equations were obtained. These equations include the iron losses in the form of the term $R_m$. However, the solution of these equations requires a very complicated set of mathematical procedures; although this difficulty can readily be solved on a digital computer, using the Newton-Raphson method of numerical iteration\(^{(13,14)}\) to give the machine parameters for the no-load and locked-rotor conditions for a variation in frequency between 40 Hz and 60 Hz. The parameters were found to increase with frequency, due to the increase in temperature caused by the increased iron losses and an increase in resistance due to skin effect.
<table>
<thead>
<tr>
<th>$R_{1M}$ ohm</th>
<th>$f$ Hz</th>
<th>At ambient temperature 20°C</th>
<th>At no-load; (winding temperature obtained by resistance measurement)</th>
<th>At full-load (winding temperature obtained by resistance measurement)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
<td>115</td>
<td>121 (24°C)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>115</td>
<td>121 (24°C)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>115</td>
<td>121 (24°C)</td>
<td>132 (42°C)</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>115</td>
<td>121 (24°C)</td>
<td>133 (45°C)</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>115</td>
<td>122 (24°C)</td>
<td>137 (50°C)</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>115</td>
<td>123 (24°C)</td>
<td>142 (64°C)</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>115</td>
<td>124 (24°C)</td>
<td>150 (86°C)</td>
</tr>
</tbody>
</table>

**TABLE 3.1**

M-phase stator winding resistance for no-load and full-load conditions at different frequencies.
### Table 3.2

<table>
<thead>
<tr>
<th>$f$ Hz</th>
<th>$R_2$ ohm</th>
<th>$X_2$ ohm</th>
<th>$R_m$ ohm</th>
<th>$X_m$ ohm</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>197</td>
<td>115</td>
<td>19841</td>
<td>1920</td>
</tr>
<tr>
<td>55</td>
<td>194</td>
<td>113</td>
<td>18215</td>
<td>1600</td>
</tr>
<tr>
<td>50</td>
<td>190</td>
<td>107</td>
<td>17668</td>
<td>1356</td>
</tr>
<tr>
<td>45</td>
<td>188</td>
<td>94</td>
<td>15773</td>
<td>1240</td>
</tr>
<tr>
<td>40</td>
<td>186</td>
<td>89</td>
<td>15552</td>
<td>1200</td>
</tr>
</tbody>
</table>

Machine parameters obtained from the no-load and the locked-rotor tests
CHAPTER IV
BALANCED AND UNBALANCED OPERATIONS
OF A CAPACITOR MOTOR

4.1 Balanced Operation

To obtain a purely rotating field at full load, a capacitor motor must operate under balanced conditions, i.e. the M and S-phase stator windings must function as a symmetrical 2-phase arrangement (22,23). In order to achieve this, two conditions must be satisfied.

1. The mmf's produced by the two windings must be equal in magnitude although separated both in time phase and in space phase by 90°.

2. The constants of the S-phase stator winding when referred to the M-phase stator winding must equal those of the M-phase stator winding.

The first of the above conditions is satisfied when:

\[-j\frac{\alpha}{\omega} I_s = I_M\]  \hspace{1cm} (4.1)

The emf's of the two windings are proportional to their number of effective turns and are also displaced in time phase by 90° i.e.

\[a E_s = j E_m\]  \hspace{1cm} (4.2)
where the turns ratio \( a \) for the test motor was equal to unity. If both windings are similarly distributed, the second condition is satisfied with respect to the leakage reactances by:

\[ a^2 X_{1S} = X_{1M} \]  \hspace{1cm} (4.3)

However, the second condition is satisfied with respect to the resistances only when the ratio of the cross-sectional area of the conductors of the S-phase stator winding to that of those of the M-phase stator winding is equal to \( a \), so that

\[ R_{1M} = a \, R_{1S} \quad R_{1S} = a^2 \, R_{1S} \]  \hspace{1cm} (4.4)

where: \( K_a = \) ratio of the cross-sectional area of the conductors of the two windings

and the weight of the copper and the current densities in the two windings are both equal. Figure (4.1) shows a phasor diagram for balanced operation of the test motor, where

\[ X_C = \frac{1}{\omega C} \]

is the reactance of the capacitor. It will be noted that \( I_S \) and \( -E_S \) are displaced by \( 90^\circ \) with respect to \( I_M \) and to \( -E_M \) respectively. Figure 4.1 can be used to determine the winding ratio \( a \) and the capacitance \( C \) required for balanced operation, when the constants of the M-phase stator winding are given. For this, it is assumed that the current of the M-phase stator winding \( I_M \) and the phase \( \theta \) between this current and the applied voltage \( V_M \) are known. This determines that part
FIG. 4.1. - VOLTAGE AND CURRENT PHASOR DIAGRAM FOR BALANCED OPERATION OF A CAPACITOR MOTOR.

SCALE: 1 mm = 1 volt

1 mm = 1 amp
of the phasor diagram which refers to the M-phase stator winding, and also the directions and positions of \(-E_S, I_S\) and \(I_S \times C\), the last two phasors being mutually perpendicular. From equations (4.1), (4.3) and (4.4)

\[
I_S R_{IS} = \frac{1}{a} I_M R_{IM} \tag{4.5}
\]

\[
I_S X_{IS} = \frac{1}{a} I_M X_{IM} \tag{4.6}
\]

If a value is assumed for the turns ratio \(a\), this then determines immediately the two phasors of equations (4.5) and (4.6), and enables a check to be made on whether or not they lie between \(-E_S\) and \(I_S \times C\). The turns ratio \(a\) and the capacitance \(C\) for balanced operation can be found approximately from the phasor diagram of Figure 4.1, in which it is assumed that \(V_S\) is displaced by \(90^\circ\) with respect to \(V_M\).

It follows from Figure 4.1 that,

\[
a = \frac{V_M}{V_S} = \cot \theta \tag{4.7}
\]

and that \(I_S \times C \cos \theta = a I_M \times C \cos \theta = V_M \tag{4.8}\)

From equation (4.8) we obtain:

\[
X_C = \frac{1}{\omega C} = \frac{V_M}{a I_M \cos \theta} \tag{4.9}
\]
The VA rating of the capacitor $W_c$, is

$$W_c = \frac{V_M}{\cos \theta} \cdot I_M \quad (4.10)$$

The line current $I_L$ is

$$I_L = I_M + i_S = I_M (1 + ja)$$

or

$$I_L = I_M \sqrt{1 + a^2} \quad (4.11)$$

The input power of the motor is

$$W_I = V_M I_L \cos \theta_L = 2 V_M I_M \cos \theta \quad (4.12)$$

The line power factor may be obtained from equations (4.11) and (4.12) as:

$$\cos \theta_L = \frac{2 V_M I_M \cos \theta}{V_M I_L}$$

$$= \frac{2 I_M \cos \theta}{I_M \sqrt{1 + a^2}} = \frac{2 \cos \theta}{\sqrt{1 + \cot^2 \theta}}$$

$$= 2 \cos \theta \sin \theta = \sin 2\theta \quad (4.13)$$
The phasor diagram of Figure 4.1 shows that the angle between the two stator currents is not $90^\circ$ electrical as was expected, indicating that the turn ratio between the motor windings is not in fact unity. However, since the turn ratio has been shown previously to be unity, this discrepancy must arise from the actual capacitor value differing from the nominal value, as might follow from the use of a commercially available capacitor.

4.2 Unbalanced Operation

If the turn ratio $a$ and the capacitor are chosen to provide balanced operation at a certain load and slip, the capacitor motor will naturally become unbalanced at other loads and slips (25). To determine the currents in both windings, the torque, etc. at these other loads, the method of symmetrical components can be used, with the unsymmetrical 2-phase currents of the machine being resolved into two symmetrical 2-phase current systems of different phase sequence. If it is assumed that the two stator windings have the same weight of copper and the same winding distribution, the resistances and reactances of the S-phase winding referred to the M-phase winding are the same as those of the M-phase stator winding. This implies that the reduction factor is $\frac{1}{a}$ for the current $I_S$, $a$ for the voltage of the S-phase stator winding, and $a^2$ for the resistance and reactance of the S-phase stator winding.
Thus:

\[ R'_{IS} = a^2 R_{IS} \]

\[ X'_{IS} = a^2 X_{IS} = X_{IM} \]

\[ X'_c = a^2 X_c \]

The identities

\[ T_M = \frac{1}{2} (T_M - j T_S') + \frac{1}{2} (T_M + j T_S') = T_f + T_b \]  \hspace{1cm} (4.14)

\[ T_S' = \frac{1}{2} (j T_M + T_S') + \frac{1}{2} (-j T_M + T_S') = j T_f - j T_b \]  \hspace{1cm} (4.15)

yield, for the unsymmetrical 2-phase current system, \( I_M \) and \( I_S' \), two symmetrical 2-phase current systems of opposite phase sequence, one being \( I_f \) and \( +jI_f \) and the other \( I_b \) and \( -jI_b \).

These two current systems produce rotating fields travelling in opposite directions. The unbalanced capacitor motor is thus in effect replaced by two balanced 2-phase machines, one having in its stator windings the currents \( I_f \) and \( +jI_f \) and the other the currents \( I_b \) and \( -jI_b \), where:

\[ T_f = \frac{1}{2} (T_M - j T_S') \]  \hspace{1cm} (4.16)

\[ T_b = \frac{1}{2} (T_M + j T_S') \]  \hspace{1cm} (4.17)
If the rotor rotates in the direction of the rotating field produced by the current system \( I_f \) and \( j I_f \), its impedance relative to the forward rotating field is different from that relative to the rotating field produced by the current system \( I_b \), \(-j I_b\), since the rotor has slips of \( S \) and \( 2-S \) relative to the two fields respectively. If the total impedance of the capacitor motor relative to the forward rotating system is \( Z_f \).

Then

\[
Z_f = \frac{Z_{1M} + Z_2^* + \overline{Z}_{1M} Z_2^* V_m}{1 + V_m Z_2^*} \tag{4.18}
\]

where:

\[
Z_{1M} = R_{1M} + j X_{1M}
\]

\[
Z_2 = \left( -\frac{R_2}{S} \right) + j X_2^*
\]

\[
V_m = \frac{1}{Z_m} = \frac{1}{R_m + j X_m}
\]

\[
\overline{Z}_{1S} = R_{1S} + j X_{1S}^*
\]

\[
\overline{Z}_c = -j X_c^*
\]

The impedance relative to the backward rotating field \( \overline{Z}_b \) is obtained by substituting \((2-S)\) for \( S \) in equation (4.18).
The voltage equation for the M-phase stator winding is:

\[ V_M = I_f \bar{Z}_f + I_b \bar{Z}_b \]  \hspace{1cm} (4.19)

and that for the S-phase stator winding is

\[ V_S = a \bar{V}_M = j I_f (\bar{Z}_f + \bar{Z}'_c) - j I_b (\bar{Z}_b + \bar{Z}'_c) \]  \hspace{1cm} (4.20)

From equations (4.19) and (4.20), the currents of the forward and backward systems are

\[ I_f = \frac{\bar{V}_M}{2} \left( \frac{(1 - ja) \bar{Z}_b + \bar{Z}'_c}{2 \bar{Z}_f \bar{Z}_b + \frac{1}{2} \bar{Z}'_c (\bar{Z}_f + \bar{Z}_b)} \right) \]  \hspace{1cm} (4.21)

and

\[ I_b = \frac{\bar{V}_M}{2} \left( \frac{(1 + ja) \bar{Z}_f + \bar{Z}'_c}{2 \bar{Z}_f \bar{Z}_b + \frac{1}{2} \bar{Z}'_c (\bar{Z}_f + \bar{Z}_b)} \right) \]  \hspace{1cm} (4.22)

respectively.

The rotor currents which correspond to the stator currents \( I_f \) and \( I_b \) can be found from the equation derived for the poly-phase machine (25)

\[ I_1 = \frac{1 + \bar{V}_m \bar{Z}_2'}{\bar{Z}_{1M} + \bar{Z}'_2 + \bar{Z}_{1M} \bar{Z}'_2 \bar{V}_m} \]  \hspace{1cm} (4.23)
From equations (4.23) and (4.24) the ratio of the secondary to the primary currents of the polyphase motor is

\[ \frac{T_2'}{T_1} = -\frac{1}{Y_m Z_2' + 1} = \frac{j X_m}{(\frac{R_{2'}}{S}) + j (1 + \frac{X_2'}{X_m}) X_m} \]

Thus the forward-rotating field induces in the rotor a current

\[ T_{2f}' = -I_f \frac{j X_m}{(\frac{R_{2'}}{S}) + j (1 + \frac{X_2'}{X_m}) X_m} \]

and the backward-rotating field a current

\[ T_{2b}' = -I_b \frac{j X_m}{(\frac{R_{2'}}{S}) + j (1 + \frac{X_2'}{X_m}) X_m} \]

The torques produced by the forward and backward-rotating fields can be determined respectively from
FIG. 4.2.—PHASE ANGLE $\psi$ CURVE BETWEEN $I_m$ AND $I_S$ STATOR WINDINGS CURRENT AGAINST LOAD.
\[ T_f = 2 I_{2f}^2 \left( \frac{R_s}{S} \right) \cdot \frac{95.4}{n_s} \text{ kg.cm} \]  
\[ T_b = 2 I_{2b}^2 \left( \frac{R_s}{2-S} \right) \cdot \frac{95.4}{n_s} \text{ kg.cm} \]  

where the resultant torque is

\[ T = T_f + T_b \]  

Figure 4.2 shows the variation with load of the phase angle \( \phi \) between the stator currents \( I_m \) and \( I_s \) of the test motor. It can be seen from Figure 4.2 that for frequencies above 50 Hz, the stator windings current \( I_m \) and \( I_s \) are unbalanced and it is impossible to achieve the ideal condition of 90° electrical. For frequencies 50 Hz and below, it is possible to obtain balanced operation of some given at only one load condition.

4.3 Conclusion

It is noteworthy that the requirement of balanced operation fixed both the winding turns ratio \( a \) and the capacitance \( c \). Thus, to obtain perfectly balanced operation at a number of load points, it is necessary to vary the turn ratio \( a \), and/or the capacitance at each point. It is not possible to obtain perfectly balanced operation merely by varying the capacitance.
of the capacitor(24,25). It may often be necessary to adjust the value of the turn ratio a, in order to use a commercially available capacitor.
CHAPTER V

EXPERIMENTAL PROCEDURE USED WITH THE CAPACITOR MOTOR AND RESULTS

The experimental techniques needed to determine the performance of a single-phase capacitor-run induction motor are the well known no-load, locked-rotor and load-tests. These tests were carried out at different frequencies, with the supply voltage adjusted so that the magnetic field within the motor was kept constant.

5.1 The Experimental Equipment

The tests were carried out using the experimental arrangement of Figure (5.1). The input power to the test motor was derived from a 5 KVA synchronous generator, driven by a d.c. motor. The test motor was a single-phase capacitor-run induction motor with the following nameplate data:

\[
\begin{align*}
\text{Supply voltage} &= 240 \text{ V} \\
\text{Output power} &= 30 \text{ W} \\
\text{Synchronous speed} &= 3000 \text{ rpm} \\
\text{Rated frequency} &= 50 \text{ Hz} \\
\text{Running capacitor} &= 2 \mu\text{F}
\end{align*}
\]

The tests were all taken with the 2 \(\mu\text{F}\) capacitor in circuit.
5.2 No-load Test

The aim of this test, together with the locked-rotor test considered later, is to determine the motor parameters (resistance and reactances), together with the iron and copper losses in the stator, and the associated friction and windage losses. In the stator, the frequency of the magnetization is equal to the line frequency \( f \), but in the rotor it varies with the relative speed between the rotating flux wave and that of the rotor. The frequency of both the rotor voltages and currents is

\[ f_2 = s f_1 \]

and since at rated speed the slip is small, the rotor iron losses can usually be neglected without introducing any appreciable errors. The test was performed with an applied test voltage initially slightly higher than the rated voltage. The applied voltage was gradually reduced, and the no-load current and input power were recorded up to the voltage at which the no-load current began to increase rapidly. The iron loss \( w_i \) and friction windage losses \( w_f \) were determined from \((28, 29, 30, 31)\):

\[
w_i + w_f = w_i - I_0^2 R_1 M - \frac{1}{2} I_0^2 R_2
\]

(5.1)

With a supply of 240 V and 50 Hz, the input current and power were 0.237 A and 21 W respectively. Substituting in the above equation, the corresponding value of \( w_i + w_f \) is obtained from equation (5.1) as:

\[
w_i + w_f = 21 - (0.237)^2 \times 122 - \frac{1}{2} \times (0.237)^2 \times 190 = 8.52 \text{ W}
\]
Changing the test voltage causes variations in \( w_i \) and \( I_o \) and different values of \( w_i + w_f \) are hence obtained. These are plotted against the square of the corresponding test voltage in Figure (5.2). It will be seen from this figure that \( w_i \) and \( w_f \) at rated voltage are equal to 6.52 W and 2 W respectively. Similar tests were performed to determine the variations in \( w_i \) and \( w_f \) with supply frequency between 30 - 60 Hz. The measured values of the input power \( w_i \) and current \( I_o \) for the same range of frequency are given in Table (5.1). To check the saturation level of the motor, values of the magnetizing current \( I_{OM} \) were calculated from the relationship:

\[
I_{OM} = I_o \sin \theta
\]

where:
\[
\theta = \cos^{-1} \frac{w_i}{V/I_o}
\]

The variation of \( I_{OM} \) with \( \frac{V}{V_r} \) is shown in Figure (5.3), for \( f = 50 \) Hz. To evaluate the saturation factor \( K_\mu \) \((31,32)\), a tangent to the curve passing through the origin is drawn. The intersection of a vertical line at a particular voltage with the tangent and the curve gives a measure of the saturation factor of that voltage. For instance, for \( \frac{V}{V_r} = 1 \),

\[
K_\mu = \frac{CB}{CA} = 0.227
\]

Figure (5.4) shows the effect of frequency variations on the iron losses, friction and windage losses, and saturation factor. It is
confirmed by this figure that the saturation present during the tests is unaffected by the frequency changes, as would be expected since the magnetic flux of the motor was maintained constant. The friction and windage, and iron losses are however plainly affected by frequency changes.

5.3 Separation of Iron Losses into Hysteresis and Eddy-Current Losses

The total iron loss obtained in Section (5.2) is the sum of the hysteresis and eddy-current losses, and can be expressed as (29,31,32):

\[ w_i = \sigma_h f B_m^\alpha + \sigma_e f^2 B_m^2 \] (5.2)

where:

- \( \alpha \) = exponent of maximum flux density \( B_m \) ranging between 1.5 to 2.5, with a value of 2 often used at the flux densities common in machines (32).
- \( \sigma_h \) = constant depending on the quality of the iron
- \( \sigma_e \) = constant depending on the resistivity of the iron and the thickness of the laminations.

and \( \sigma_h f B_m^\alpha \) and \( \sigma_e f^2 B_m^2 \) are respectively the hysteresis and eddy-current losses.

If values of \( \sigma_h \) and \( \sigma_e \) are determined, the total iron loss \( w_i \) can be separated into its hysteresis and eddy-current components. This is achieved by solving two simultaneous equations, obtained
by substituting in equation (5.2) values of the iron loss at
two different frequencies and the same maximum flux density.
On dividing equation (5.2) by the frequency $f$, we obtain:

$$\frac{W_i}{f} = \sigma_h B_m^\alpha + \sigma_e fB_m^2$$

(5.3)

which, since $\sigma_h B_m^\alpha$ and $\sigma_e fB_m^2$ are constants, is the equation of
a straight line. If $\frac{W_i}{f}$ is plotted against $f$, the intercept of
the resulting straight line with the vertical axis (OA) provides
$\sigma_h B_m^\alpha$, as shown in Figure (5.5). At a particular frequency $f$,
$\sigma_e fB_m^2$ is the ordinate at that frequency minus $\sigma_h B_m^\alpha$. The
hysteresis and eddy-current losses corresponding to that frequency
are then $\sigma_h B_m^\alpha$ and $\sigma_e fB_m^2$ multiplied by the frequency. Values of
the hysteresis and eddy-current losses thus found at 50 Hz are
2.09 W and 4.35 W respectively, and the variation of these quanti­
ties with frequency is shown in Figure (5.6).

5.4 **Speed-Torque Characteristics**

Initially, locked-rotor tests were performed to determine
the starting torque and starting current of the motor, and
also to investigate how these both vary with frequency. To
obtain the starting torque of the motor, two spring balances were
suspended on a thin string wrapped once round the shaft in a
direction opposed to that of the rotation, with each end of the
string attached to a spring balance, as shown in Figure (5.1).
The starting torque is determined from the equation:

\[ T_{st} = A (F_a - F_b) \]  

(5.4)

where \( F_a \) and \( F_b \) are the spring tensions as measured by the spring balances and \( A \) is the radius of the pulley.

The motor was supplied with a reduced voltage \((V_{sh})\) of about 25 - 30% of the rated voltage \(V\), and the current in both the S- and M-phase stator windings, the tensions \( F_a \) and \( F_b \) and the reduced voltage \( V_{sh} \) were all recorded. The starting torque \((T_{sh.st})\) for a reduced voltage \((V_{sh})\) was determined using equation (5.4), and the starting current and the starting torque at rated voltage were obtained from (5.12):

\[ I_{r.st} = I_{sh.st} \frac{V}{V_{sh}} \]

\[ T_{r.st} = T_{sh.st} \left(\frac{V}{V_{sh}}\right)^2 \]

In a subsequent test, the maximum torque the motor can deliver was measured by increasing the load torque until the speed began to fall very rapidly. Measurements were taken for frequencies between 40 Hz and 60 Hz. This data for the complete set of speed/torque curves plotted in Figure (5.7). The starting torque, maximum torque and rated torque for the rated condition of 240 V,
f = 50 Hz are 0.988 Kg.cm, 2.2 Kg.cm and 1.047 Kg.cm respectively. Table (5.2) shows some significant figures from these curves, together with the ratio of the maximum and starting torques to the rated torque, and the starting current to the rated current for the complete range of frequency. Due to the variations of motor reactances with changes in the rotor frequency and their relative predominance in determining the torque at any given slip, the torque for high values of slip is directly proportional to the slip and inversely proportional for low values of slip. Therefore the family of characteristics takes the form evident in Figure (5.7).

5.5 Stator Winding-Ratio Test

The motor was run on no-load, and rated voltage V was applied to the M-phase stator winding only. The voltage $V'_s$ appearing across the S-phase stator winding was measured. The motor was run with a voltage $V_s$ applied to the S-phase stator winding, and the voltage $V'_M$ appearing across the M-phase stator winding was measured.

A close approximation to the winding ratio of the stator is then (25):

$$a = \sqrt{\frac{V}{V'_s} \frac{V'_M}{V'_s}}$$

$$= \sqrt{\frac{240}{147} \frac{147}{240}} = 1$$

86
which is the expected result since the two stator windings are identical.

5.6 Load Test

This was performed to determine the characteristics of the capacitor-run induction motor at different frequencies under load conditions. Measurements were carried out by increasing the mechanical load, and noting the corresponding values of the input power ($w_I$), line current ($I_L$), M and S-phase stator windings current, speed and torque. The torque and speed were used to evaluate the output power ($w_o$) and from the measured values of $w_I$, $I_L$ and $V$ the power-factor $\cos \theta$ was determined. The performance characteristics of the motor are shown in Figures (5.8 - 5.12) for the range of different frequencies at which the motor was investigated.

5.7 Calculated Results

Using the parameters described in Chapter III and the computer programs of Appendix 3(B), the motor performance was determined for a 2 µF capacitor over the complete range of frequency. Predicted results for the speed-torque characteristics are compared in Figure 5.7 with the experimental results obtained previously, and the predicted and measured load characteristics of the motor are given in Figures (5.8 - 5.12). In all these figures a quite accep-
table accuracy of prediction is evident. The calculations were then repeated for a range of capacitor values from 0.5 to 4 μF for the same range of frequencies, giving results which will be discussed in the next chapter.

5.8 Temperature Measurements

The temperature-rise of the motor with a 2 μF running capacitor was measured using a thermocouple attached to the stator core. The results shown in Figure (5.13) are for the motor supplied at 50 Hz, and the results at the other frequencies considered are given in Table (5.3). Referring to Figure (5.13), the heating time constant \(T^{(26,28,30)}\) is determined by drawing a tangent to the heating curve (1) at point A, and measurement of the intersection of this with the line BCM (corresponding to the final temperature-rise), gives the heating time constant as 30 min. The final temperature-rise of the motor at other frequencies can be determined by use of the heat capacity \(Gc^{(30,31)}\):

\[
Gc = \frac{T \times \Sigma W}{\theta_{fin}}
\]

(5.5)

where:

- \(G\) = weight
- \(c\) = specific heat
- \(T\) = heating time constant
- \(\Sigma W\) = sum of motor losses
From equation (5.5), \( G_c \) is obtained at 50 Hz as:

\[
G_c = \frac{30 \times 23.88}{50}
\]

\[= 14.8 \text{ min. } ^{\circ C} \text{Kg}
\]

Since \( G_c \) remains constant, the final temperature-rise at any other frequency is:

\[
\theta_{\text{fin}} = \frac{T \times \Sigma \omega}{14.8}
\]

where \( \Sigma \omega \) is the sum of the losses at that frequency.

The results calculated for the complete range of frequency are also shown in Table (5.5).

The temperature-rise as a function of time is\((30,31)\):

\[
\theta = \theta_{\text{fin}} (1 - e^{\frac{-t}{T}}) + \tau_{s.m} e^{\frac{-t}{T}}
\]

(5.6)

and if at \( t = 0 \) the temperature-rise \( \tau_{s.m} = 0 \), equation (5.6) gives:

\[
\theta = \theta_{\text{fin}} (1 - e^{\frac{-t}{T}})
\]

(5.7)

In accordance with equation (5.7), for time periods \( t = T, 2T, 3T \ldots \ldots \) the quantity \( \frac{\theta}{\theta_{\text{fin}}} \) is

89
\[ t = \ldots \quad T \quad 2T \quad 3T \quad 4T \]
\[ \frac{\theta}{\theta_{\text{fin}}} = \ldots \quad 0.632 \quad 0.865 \quad 0.95 \quad 0.982 \]

From these values and the final temperature-rise obtained experimentally, the heating time curve for equation (5.7) may be obtained as shown by curve (2) of Figure (5.13). From this, it can be seen that the motor attains a steady-state temperature-rise \( \theta_{\text{fin}} \) during a time interval practically equal to four heating time constants.

From the results shown in Table (5.5), it is clear that up to 55 Hz, the temperature-rise is within the thermal limits\(^{(11,12)}\) specified for the insulation. At higher frequencies the temperature-rise exceeds these limits due to the large copper losses and the increased iron losses. It will be seen that the heating time constant is independent of the frequency.

5.9 Conclusions

Experimental and calculated results were obtained using an auxiliary-winding capacitor of 2 \( \mu \)F throughout the full range of frequency and Figures (5.8 - 5.12) show the load characteristics of the single-phase capacitor-run induction motor. It can be seen that the experimental results agree well with the calculated results.
<table>
<thead>
<tr>
<th>Frequency f Hz</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input power $w_{I}$ (w)</td>
<td>12.8</td>
<td>15.47</td>
<td>17.34</td>
<td>19.6</td>
<td>21</td>
<td>22.7</td>
<td>24.4</td>
</tr>
<tr>
<td>Current $I_{o}$ (A)</td>
<td>0.207</td>
<td>0.226</td>
<td>0.23</td>
<td>0.234</td>
<td>0.237</td>
<td>0.242</td>
<td>0.245</td>
</tr>
<tr>
<td>Voltage $V$ (V)</td>
<td>144</td>
<td>168</td>
<td>192</td>
<td>216</td>
<td>240</td>
<td>264</td>
<td>288</td>
</tr>
</tbody>
</table>

**TABLE 5.1**

Measured values of $w_{I}$, $I_{o}$ and $V$ for different frequencies at no-load condition
<table>
<thead>
<tr>
<th>Frequency $f$ (Hz)</th>
<th>60</th>
<th>55</th>
<th>50</th>
<th>45</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated torque $T$ (Kg.cm)</td>
<td>1.014</td>
<td>1.028</td>
<td>1.047</td>
<td>1.0714</td>
<td>1.114</td>
</tr>
<tr>
<td>Maximum torque $T_{max}$ (Kg.cm)</td>
<td>2.6</td>
<td>2.41</td>
<td>2.2</td>
<td>1.81</td>
<td>1.6</td>
</tr>
<tr>
<td>Starting torque $T_{st}$ (Kg.cm)</td>
<td>1.6</td>
<td>1.172</td>
<td>0.988</td>
<td>0.78</td>
<td>0.65</td>
</tr>
<tr>
<td>Rated current $I_L$ (A)</td>
<td>0.327</td>
<td>0.26</td>
<td>0.226</td>
<td>0.223</td>
<td>0.225</td>
</tr>
<tr>
<td>Starting current $I_s$ (A)</td>
<td>0.72</td>
<td>0.65</td>
<td>0.63</td>
<td>0.6</td>
<td>0.564</td>
</tr>
<tr>
<td>Ratio $\frac{T_{max}}{I}$</td>
<td>2.56</td>
<td>2.344</td>
<td>2.1</td>
<td>1.689</td>
<td>1.436</td>
</tr>
<tr>
<td>Ratio $\frac{T_{st}}{I}$</td>
<td>1.578</td>
<td>1.14</td>
<td>0.943</td>
<td>0.728</td>
<td>0.583</td>
</tr>
<tr>
<td>Ratio $\frac{I_{st}}{I_L}$</td>
<td>2.2</td>
<td>2.5</td>
<td>2.78</td>
<td>2.59</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**TABLE 5.2**

Maximum and starting torques test results at 2 µF
<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Voltage (V)</th>
<th>$\Delta W$ (W)</th>
<th>Temperature rise test $\theta$ (°C)</th>
<th>Calculated $\theta$ (°C)</th>
<th>Heating time con. $T_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>192</td>
<td>20.2</td>
<td>42.0</td>
<td>42.3</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>216</td>
<td>21.57</td>
<td>45</td>
<td>45</td>
<td>30</td>
</tr>
<tr>
<td>50</td>
<td>240</td>
<td>23.88</td>
<td>50</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>264</td>
<td>31</td>
<td>67</td>
<td>64.9</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>288</td>
<td>42</td>
<td>86</td>
<td>87.9</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE (5.3)**

Temperature-rise results of the capacitor motor at different frequencies
FIG. 5.1. MEASUREMENT OF TORQUE/SPEED CHARACTERISTICS OF SINGLE-PHASE CAPACITOR-RUN INDUCTION MOTOR.

\[ T = A(F_Q - F_b) \]

WHERE \( F_Q \) AND \( F_b \) ARE THE SPRING TENSIONS AS MEASURED BY THE SPRING BALANCE, AND

\( A \) IS THE RADIUS OF THE PULLEY
FIG 5.2. SEPARATION OF NO-LOAD LOSSES AT 50 Hz
FIG. 5.3. DETERMINATION OF THE SATURATION FACTOR AT 50Hz.
FIG. 5.4. EFFECT OF FREQUENCY ON THE IRON LOSS, FRICTION AND WINDING LOSSES AND SATURATION FACTOR
HYSTERESIS LOSSES  $W_h = \delta_h f B_m^x = 2.09 \, \text{W}$

EDDY CURRENT LOSSES  $W_e = \delta_e f^2 B_m^2 = 4.35 \, \text{W}$

![Graph showing the separation of iron losses into hysteresis and eddy-current losses.](image-url)
FIG. 5.6. HYSTERESIS AND EDDY CURRENT LOSS: RESULTS FOR THE MOTOR
FIG. 5.7. SPEED-TORQUE CHARACTERISTICS OF THE MOTOR AT DIFFERENT FREQUENCIES AND AT 2μF
FIG. 5.8. PERFORMANCE CHARACTERISTICS OF THE SINGLE-PHASE CAPACITOR-RUN INDUCTION MOTOR AT 40Hz AND 2μF
FIG. 5.9. PERFORMANCE CHARACTERISTICS OF THE SINGLE-PHASE CAPACITOR-RUN INDUCTION MOTOR AT 45Hz & 2μF.
FIG. 5.10. PERFORMANCE CHARACTERISTICS OF THE SINGLE-PHASE CAPACITOR-RUN INDUCTION MOTOR AT 50Hz & c=2μF
FIG. 5.11. PERFORMANCE CHARACTERISTICS OF THE SINGLE-PHASE CAPACITOR-RUN INDUCTION MOTOR AT 55 Hz AND 2 μF.
FIG. 5.12. PERFORMANCE CHARACTERISTICS OF THE SINGLE-PHASE CAPACITOR-RUN INDUCTION MOTOR AT 60Hz AND 2\mu F
FIG. 5.13. TEMPERATURE-RISE CURVE OF THE CAPACITOR MOTOR
AT FREQUENCY $f = 50\,Hz$ AND CAPACITOR $C = 2\,\mu F$
CHAPTER VI

EFFECT OF FREQUENCY ON THE MOTOR PERFORMANCE

6.1 Relation between Voltage and Frequency

The most important and immediately noticeable change which occurs when the supply frequency is changed is that a motor operates at a different speed. The change in synchronous speed is directly proportional to the change in frequency, as shown by the expression:

\[
\text{synchronous speed} = \frac{\text{frequency}}{\text{number of pair of poles}} \quad (6.1)
\]

If the supply frequency is raised the flux density distribution rotates more quickly and, with more conductors cut in a given time, a greater counter emf is generated in the stator for a given level of flux density distribution, according to the relationship:

\[
\text{emf} = 4.44 \ K_1 \ f_1 \ N_1 \ \phi \quad (6.2)
\]

where:

- \( K_1 \) = winding factor
- \( f_1 \) = supply frequency
- \( N_1 \) = number of stator windings turns
- \( \phi \) = flux per pole
In order to maintain the same magnetic loading in the motor, it is therefore necessary to raise the voltage approximately in proportion to the frequency and conversely to lower the voltage if the frequency is lowered.

6.2 Effect of Variation of Voltage and Frequency on the Performance of the Motor

Many motors are used at voltages and/or frequencies other than those for which they are designed, and under such conditions the performance will vary from that stated by the manufacturers. Voltage variations of 10% are allowed for in most commissioning rules, but it must be noted that the variations also have the effect of changing the power-factor and the efficiency. When a motor is running under constant load torque a small increase in voltage will usually result in a slight decrease in the efficiency and a decided lowering in the power-factor, whereas a small decrease in the voltage usually gives a slight improvement in the efficiency and an increase in the power-factor. The effect of variations of voltage on efficiency and power-factor can be deduced from the following equation:

\[ T = K_1 \phi I_2 \cos \phi \]  \hspace{1cm} (6.3)

When the torque \( T \) is kept constant, an increase in voltage introduces a strengthening of the magnetic-field and a consequent increase in the reactive component of the rotor current \( I_2 \).
At the same time the active component of the current decreases, and these two effects together cause a decrease in the power-factor. Moreover, the increase in the magnetic field increases both the iron losses and the leakage flux, which in turn causes an increase in the temperature of the motor and a reduction in its efficiency. Similarly, it can be shown that a decrease in voltage increases both efficiency and power-factor. A small variation in frequency, with the voltage held constant, will yield a torque variation nearly inversely proportional to the square of the frequency. If both voltage and frequency are varied together, in the same direction and over a small range, the torque will remain substantially constant.

6.3 Relationship Between Torque, Speed and Output Power

If the frequency is increased while the voltage is kept constant, the magnetic field decreases such that the new field, rotating at the new synchronous speed, generates the same back emf as the original field rotating at the original synchronous speed. The result of a decrease in the magnetic field is a decrease in the torque, and this would result in a reduced output power were it not for the speed increase. If however the torque is constant, the output power varies directly as the speed. That is, a higher speed produces a larger output power and a lower speed produces a smaller output power. If the magnetic field and the winding currents are kept constant, irrespective of the frequency variations, both the speed and the available
output power vary directly with changes in voltage and frequency, but the heating effect varies somewhat more due to the variation in the iron loss with frequency and in the ventilating effect with speed. Just as raising the frequency may result in an increased output power, so reducing the frequency may decrease the output to a level below the rated power.

6.4 Frequency Effect on the Motor Performance for a Constant Magnetic-Field

In order to keep the magnetic field of the motor constant, the ratio \( V/f \) was kept constant throughout a series of performance tests conducted on the experimental machine. The frequency was varied over a range from 40 Hz to 60 Hz in 5 Hz steps and the operating voltage was correspondingly varied to maintain the \( V/F \) ratio at the rated value of \( \frac{240}{50} = 4.8 \) (26). The magnetic field was checked as constant by employing the no-load test shown in Figure (5.4) Chapter V. Calculated and experimental performance characteristics, with the 2 \( \mu \)F running capacitor, are plotted against frequency in Figures (6.1) to (6.8). Figure (6.1) shows the variation in efficiency of the motor with the variation in frequency. The curve clearly shows that with a 2 \( \mu \)F capacitor the efficiency is a maximum at the rated frequency of 50 Hz. It decreases with increasing frequency, due to the increase in both the iron and copper losses, until at 60 Hz it has fallen by 26% of the rated value, whereas at 40 Hz, the reduction is only about 1% of this value. The variation in the full-load power-factor of
the motor with frequency is shown in Figure (6.2). It is evident that the power-factor decreases with increasing frequency, due to the increases in reactive stator winding current and voltage. At a frequency of 60 Hz, the reduction in the power factor is 20% of the rated value, whereas at 40 Hz, it is only 3% of this value. Figures (6.3) and (6.4) show the effect of frequency on the S- and M-phase stator winding currents. It will be seen that the S-phase current increases with frequency, whereas the M-phase current decreases with increasing frequency. The line current of the motor is the vector sum of $I_S$ and $I_M$, and it is clear from Figure (6.5) that frequency has only a relatively small effect on this current, due to the differing directions of the variation in the S- and M-phase stator winding currents.

Figure (6.6) shows the effect of frequency on the maximum torque, which is influenced by variations in voltage and frequency. The effects of variations in the voltage can be nullified by introducing proportionate variations in frequency. In this case the maximum torque is supposed to assume a constant value, but it actually decreases with a decrease in frequency and increase when the frequency is increased, due to the variation in the reactances with frequency. This is illustrated in Figure (6.6). At 60 Hz the maximum torque is increased by 24% over the rated value and at 40 Hz, it is decreased by 31% of this value. Similar variations are obtained when the starting torque is studied, as in Figure (6.7). At 60 Hz the increase is now 60% of the rated value and at 40 Hz the reduction is 66% of this value. Figure (6.8) shows the variation in temperature-rise with variations in frequency. It is clear that the temperature-rise
increases only slowly with frequency up to the rated frequency of 50 Hz, but that after this it rapidly increases due to the high copper losses and the increased iron losses.

6.5 Effect of the Running Capacitor on the Motor Performance at Different Frequencies

The presence of a running capacitor has a large effect on the performance of the motor and the power-factor, efficiency, and torque pulsations can all be improved by 5-20%\(^{(4,5)}\). For example, the capacitor and the S-phase stator winding could be designed to ensure perfect 2-phase operation at any desired load, although a very large capacitance is in fact necessary to permit sufficient current to flow to yield a perfectly balanced starting condition. As more capacitance is added to the circuit the benefits increase up to a point, but then, although the starting and accelerating torques continue to increase, the running performance becomes very poor and excessive noise, or reduced speed, and overheating all result. A permanent-capacitor motor is in fact always a compromise in design between the large capacitor needed for starting and the small capacitor needed for balanced running. However, the presence in the air-gap of a flux system from the S-phase stator winding can greatly reduce the pulsating flux of double frequency which is a major problem with all single-phase motors. It is as a result of this reduction that the capacitor motor can operate comparatively quietly and smoothly.
6.5.1 Effect of the Running Capacitor on the Motor Performance

The motor performance for a range of running capacitors between 0.5 and 4 μF was predicted theoretically, and the calculated performance characteristics are added to Figures (6.1-6.7). Thus it will be seen in Figure (6.1) that the effect of an increasing frequency is to cause the efficiency to increase with frequency for 1 and 2 μF capacitors, but to decrease for capacitors larger than 2 μF, due to the increase in both iron and copper losses. Figure (6.2) demonstrates that the full-load power-factor is a maximum at 50 Hz for only a 2 μF capacitor and that it rapidly departs from this ideal situation. It will be seen from Figure (6.3) that the S-phase stator winding current increases slightly with frequency for 1 and 1.5 μF capacitors, and that it increases to an increasing extent with frequency for capacitance values exceeding 1.5 μF. Figure (6.4) shows the effect of frequency on the M-phase stator winding current for different capacitor values. It is seen that the current decreases noticeably with frequency for 1.0, 1.5, 2.0 and 2.5 μF capacitors, but that it increases to an even greater extent with frequency for capacitance values exceeding 2.5 μF. The variation of the motor line current with frequency for different capacitor values is shown in Figure (6.5). The curves clearly show that the line current decreases slightly with frequency for capacitor values less than 1.5 μF, due to the decrease in the M-phase stator winding current. For a capacitance of 2 μF, the line current increases gradually with frequency, and for capacitor values exceeding 2 μF, it increases rapidly due to the large increase in both the S- and M-phase stator winding currents.
6.5.2 Effect of the Running Capacitor on the Maximum and Starting Torques

A capacitor connected in series with the S-phase stator winding is necessary to permit sufficient current to flow in this winding to provide an acceptable starting torque. The accelerating torque will also be increased, but with the maximum torque increasing only slightly over that developed by the M-phase winding. Although the starting and accelerating torques both initially increase with the addition of more capacitance, a stage is reached after which the performance of the motor starts to deteriorate. An optimum design requires a high value of capacitance to ensure a high starting torque and a low-value for balanced running on full load. Figure (6.6) shows the effect of capacitance on the maximum torque for a range of frequency from 40 to 60 Hz. It will be seen here that the maximum torque increases with both capacitance and frequency. Thus at 50 Hz, the maximum torque with a 4 μF capacitor is 38% above the rated value, while for a 0.5 μF capacitor it is only by 50% of the rated value. Similarly at 60 Hz a 4 μF capacitor will give an increase of 38% in the maximum torque, while a 0.5 μF capacitor causes an increase of only 21%. At 40 Hz a 4 μF capacitor will result in a reduction in the maximum torque of 38%, while for a 0.5 μF capacitor the reduction will be 17%. Figure (6.7) illustrates the effect of capacitance on the starting torque for the same range of frequency. The starting torque increases with both capacitance and frequency, with 50 Hz a 4 μF capacitor resulting in an increase in the starting torque of 40% above the figure for 2 μF, whereas for a 0.5 μF capacitor that is a reduction of 45% of this value. At 60 Hz, a 4 μF capacitor will produce an increase in the starting torque of 50%.
of the rated value, and for a 0.5 μF capacitor the corresponding increase will be 70%. At 40 Hz, a 4 μF capacitor will reduce the starting torque by 60%, and a 0.5 μF capacitor by 80%.
6.6 Conclusion

The experimental and calculated results are shown in Figures (6.1 - 6.8). It can be concluded for a 2 μF capacitor that both the efficiency and the power-factor are seen to be maxima at 50 Hz, this indicating clearly the optimum condition for which the machine is designed.

From the calculated results it can be concluded that:

1. The efficiency curves shown in Figure (6.1) indicate a decrease with both increased capacitance and increased frequency and a very poor efficiency at frequencies above 50 Hz and a capacitance greater than 2 μF.

2. Figure (6.2) shows a decreasing power-factor with increasing frequency and capacitance, while the corresponding increases in the S-phase stator winding current Figure (6.3).

3. The variation of the M-phase stator winding current shown in Figure (6.4) leads to the conclusion from Figure (6.5) that the line current decreases with both frequency and capacitance for a small capacitance, but increases for a capacitance above the normal value.

4. It is clear from Figures (6.6) and (6.7) that both the maximum torque and the starting torque increase with both frequency and capacitance.

5. Figure (6.8) shows that the steady-state temperature-rise of the motor also increases with frequency.
Fig. 6.1 EFFECT OF FREQUENCY ON THE MOTOR EFFICIENCY FOR DIFFERENT CAPACITOR VALUES.

Fig. 6.2 EFFECT OF FREQUENCY ON THE FULL-LOAD POWER-FACTOR OF THE MOTOR FOR DIFFERENT CAPACITOR VALUES.
**Fig. 6.3** Effect of frequency on the S-phase stator winding current for different capacitor values.

**Fig. 6.4** Effect of frequency on the M-phase stator winding current for different capacitor values.
Fig. 6.5 EFFECT OF FREQUENCY ON THE LINE CURRENT OF THE MOTOR FOR DIFFERENT CAPACITOR VALUES
Fig. 6.6 EFFECT OF FREQUENCY ON THE MAXIMUM TORQUE OF THE MOTOR FOR DIFFERENT CAPACITOR VALUES

Fig. 6.7 EFFECT OF FREQUENCY ON THE STARTING TORQUE OF THE MOTOR FOR DIFFERENT CAPACITOR VALUES
**FIG. 6.8. EFFECT OF FREQUENCY ON THE TEMPERATURE-RISE OF THE MOTOR WITH C = 2 uF**
7.1 Conclusion

Continual investigations over many years into the behaviour of the capacitor motor have yielded new analytical methods for improving the overall performance of the motor. These methods are usually based on either the revolving-field theory or the cross-field theory. When using the revolving-field theory in performance calculations, a general lack of agreement between the calculated and test values has been observed (8,10). The cross-field theory has been found to be the most accurate, since the iron loss in the motor may be included in the model. The inclusion of iron loss into the revolving-field theory was found to excessively complicate the equations involved.

Capacitor motors sometimes operate in circuits of voltage or frequency other than those for which they are rated and, under these conditions, the performance of the motor will vary from the standard rating. To cater for this type of situation, this thesis has been concerned with:

a) motor performance calculations using an equivalent circuit developed from basic cross-field theory.

b) investigating the motor behaviour with a variable frequency ranging between 40 Hz and 60 Hz in 5 Hz steps.

c) considering how the behaviour varies with capacitor values
ranging (from the nominal 2 µF) between 0.5 µF and 4 µF, connected in series with the auxiliary S-phase stator winding.

Before calculating the performance of the motor, the experimental determination of the individual parameters needs to be obtained from no-load and locked-rotor tests made at the motor terminals. From the equivalent circuits of the capacitor motor for these two conditions, a set of voltage/current equations is obtained. The solution of these equations to determine the equivalent circuit parameters is complicated, because the no load branch is represented in the equations by a complex impedance $Z_m = \frac{jR_m X_m}{R_m + jX_m}$ instead of merely a pure reactance $X_m$. This difficulty was overcome by using a digital computer to obtain the parameters using the Newton-Raphson method of numerical iteration.

The parameters of the motor were determined for different operational frequencies, and the performance of the motor predicted from these parameters was compared with that obtained from extensive experimental investigations, throughout all the range of frequency considered and with the nominal value of capacitance of 2 µF. The correspondence between the predicted and experimental performance was very close, indicating that the calculation of the parameters is accurate.

From experimental and calculated results, over a wide range of frequency and capacitor values, it can be concluded that for a 2 µF capacitor both the power-factor and the efficiency are a
maximum at 50 Hz. This indicated clearly the optimum condition for which the motor is designed. The motor performance for a range of running capacitor values between 0.5 μF and 4 μF was predicted theoretically, and from these results, with the capacitance greater than 2 μF, the induced voltage in the S-phase stator winding will be increased and will reduce the current and power in the M-phase stator winding. The S-phase stator winding therefore takes both more current and power. The line current and input power will naturally increase, resulting in a lower efficiency and a correspondingly increased heating of the motor.
7.2 Comments

From the results obtained in this thesis, the following comments can be made:

1) The efficiency of the motor is poor for frequencies above 50 Hz and capacitance greater than 2 \mu F.

2) The full load power-factor of the motor is a maximum only at 50 Hz and 2 \mu F capacitance.

3) The maximum torque and starting torque increasing with both frequency and capacitance.

4) For frequencies of 50 Hz and below, it is possible to obtain balanced operation at some given load, but for frequencies above 50 Hz the stator windings current are always unbalanced, and it is impossible to achieve the ideal phase shift of 90^\circ between the two stator windings current.

5) The steady-state temperature-rise of the motor increases with frequency and is excessively high temperature by the time the frequency reaches 60 Hz.

Summarising the results, it is seen that the motor can be used only with a supply frequency between 45 Hz and 50 Hz, and that a 2 \mu F capacitor gives the best performance characteristic results.
REFERENCES


<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>23.</td>
<td>P H Trickey.</td>
</tr>
</tbody>
</table>

127

Fractional horsepower motors. (Book). 1951.
p 155.

40. M A Strain

Mathematical methods for technologists. (Book).

41. E M Lapukina and

G S Samikina.

1961, p 218.

42. A T Dover and

F T Chapman.

Electrical engineering (Book). p 321.

43. R R Lawrence

Principles of alternating-current machinery.

44. A Covo and

L E Lingo.

Graphical determination of starting performance
of capacitor motors. AIEE Power Apparatus and


Electrical machines (Book) 1973.
The total voltage applied to the M-phase stator winding is the sum:

\[ E_M = E_{1M} + E_{fM} + E_{bM} + E_{M,fS} + E_{M,bS} \]  \hspace{1cm} (1)

Similarly, the total voltage applied to the S-phase winding is the sum:

\[ E_S = E_{1S} + E_{fS} + E_{bS} + E_{S,fM} + E_{S,bM} \]  \hspace{1cm} (2)

where:

- \( E_{fM} \) and \( E_{bM} \) = the forward and backward emf's induced in the M-phase winding by its forward and backward rotating fluxes.

- \( E_{M,fS} \) and \( E_{M,bS} \) = the forward and backward emf's induced in the M-phase winding by the forward and backward rotating fluxes of the S-phase winding.

- \( E_{fS} \) and \( E_{bS} \) = the forward and backward emf's induced in the S-phase winding by its forward and backward rotating fluxes.

- \( E_{S,fM} \) and \( E_{S,bM} \) = the forward and backward emf's induced in the S-phase winding by the forward and backward rotating fluxes of the M-phase winding.
The various components of the M-phase voltage are in phasor form:

\[ E_{1M} = \overline{I}_M (R_{1M} + jX_{1M}) \quad (3) \]

\[ E_{fM} = \overline{I}_M (R_f + jX_f) \quad (4) \]

\[ E_{bM} = \overline{I}_M (R_b + jX_b) \quad (5) \]

\[ E_{M,fS} = a\overline{I}_S (X_f - jR_f) \quad (6) \]

\[ E_{M,bS} = a\overline{I}_S (-X_b + jR_b) \quad (7) \]

while those of the S-phase are:

\[ E_{1S} = \overline{I}_S (a^2 R_{1S} + ja^2 X_{1S} + jX_c) \quad (8) \]

\[ E_{fS} = a^2 \overline{I}_S (R_f + jX_f) \quad (9) \]

\[ E_{bS} = a^2 \overline{I}_S (R_b + jX_b) \quad (10) \]

\[ E_{S,fM} = a\overline{I}_M (-X_f + jR_f) \quad (11) \]

\[ E_{S,bM} = a\overline{I}_M (X_b - jR_b) \quad (12) \]

On summing equations (2) to (7):
\[
E_M = I_M \left( (R_{1M} + R_f + R_b) + j(X_{1M} + X_f + X_b) \right) + I_S a [(X_f - X_b) - j(R_f - R_b)]
\]

(13)

and on summing equations (8) to (12):

\[
E_S = I_S \left\{ a^2(R_{1S} + R_f + R_b) + j[X_c + a^2(X_{1S} + X_f + X_b)] \right\} -
\]

\[
I_M a [(X_f - X_b) - j(R_f - R_b)]
\]

(14)

Since both phases are connected to the same supply, \( E_M = E_S \), and equations (13) and (14) may be solved to give:

\[
T_M = \frac{a^2(R_{1S} + R_f + R_b) + j[X_c + a^2(X_{1S} + X_f + X_b)]}{\{a^2(R_{1S} + R_f + R_b) + j[X_c + a^2(X_{1S} + X_f + X_b)]\}}
\]

\[
+ \frac{ja[(R_f - R_b) + j(X_f - X_b)]}{\{(R_{1M} + R_f + R_b) + j(X_{1M} + X_f + X_b)\} - a^2[(R_f - R_b) + j(X_f - X_b)]}
\]

(15)
The line current of the motor is:

\[
T_L = T_M + T_S
\]

\[
E_M = \frac{\left[ a^2(R_{1S} + R_f + R_b) + R_{1M} + R_f + R_b \right] + j[X_c + a^2(X_{1S} + X_f + X_b)]}{a^2(R_{1S} + R_f + R_b) + j[X_c + a^2(X_{1S} + X_f + X_b)]}
\]

\[
\frac{(X_{1S} + X_f + X_b) + X_{1M} + X_f + X_b}{\{(R_{1M} + R_f + R_b) + j(X_{1M} + X_f + X_b)\} - a^2[(R_f - R_b) + j(X_f - X_b)]^2}
\]
APPENDIX II
DERIVATION OF THE SYMMETRICAL COMPONENTS
EQUATIONS OF THE CAPACITOR MOTOR

By definition the positive and negative sequence components
of \( V_M \) are:

\[ V_M = V_M^+ + V_M^- \] (1)

the corresponding components of \( V_S \) are:

\[ V_S = V_S^+ + V_S^- \] (2)

where:

\[ V_S^+ = V_M^+ \sqrt{90^\circ} = + j V_M^+ \] (3)

\[ V_S^- = V_M^- \sqrt{90^\circ} = - j V_M^- \] (4)

Since the \( S \)-phase voltage leads the \( M \)-phase voltage in time phase,
from equations (2), (3) and (4):

\[ V_S = j (V_M^+ - V_M^-) \] (5)

Since the magnitudes and time phases of \( V_M \) and \( V_S \) are all known,
the symmetrical components of each can be calculated by adding
equation (1) to \(-j\) times equation (5), giving:

\[
V_M^+ = \frac{V_M - j V_S}{2}
\]  
(6)

By subtracting \(-j\) times equation (5) from equation (1):

\[
V_M^- = \frac{V_M + j V_S}{2}
\]  
(7)

Similarly:

\[
I_M^+ = \frac{I_M - j I_S}{2}
\]  
(8)

and

\[
I_M^- = \frac{I_M + j I_S}{2}
\]  
(9)

In a general 2-phase system, with \(Z_M\) and \(Z_S\) being the load impedances of the M- and the S-phases respectively.

\[
V_M = I_M Z_M
\]  
(10)

and

\[
V_S = I_S Z_S
\]  
(11)

From equations (10), (1):
\[ V_M^+ + V_M^- = (I_M^+ + I_M^-) Z_M \] (12)

from equations (11), (2):

\[ V_S^+ + V_S^- = (I_S^+ + I_S^-) Z_S \] (13)

From equations (3) and (4) and (13):

\[ j V_M^+ - j V_M^- = (j I_M^+ - j I_M^-) Z_S \] (14)

or

\[ V_M^+ - V_M^- = (I_M^+ - I_M^-) Z_S \] (15)

Adding equations (12) and (15):

\[ V_M = \frac{1}{2} \left[ I_M^+ (Z_M + Z_S) + I_M^- (Z_M - Z_S) \right] \] (16)

Subtracting equation (15) from equation (12) and clearing:

\[ V_M^- = \frac{1}{2} \left[ I_M^+ (Z_M - Z_S) + I_M^- (Z_M + Z_S) \right] \] (17)

In the case of a different number of turns in the two windings:

\[ |I_S^+| = \frac{|I_M^+|}{a} \] (18)

\[ |I_S^-| = \frac{|I_M^-|}{a} \] (19)
and

\[ V_S^+ = a |V_M^+| \]  \hspace{1cm} (20)

\[ V_S^- = a |V_M^-| \]  \hspace{1cm} (21)

where:

\[ a = \text{effective turns of the S-phase winding} \]
\[ \text{effective turns of the M-phase winding} \]

so that:

\[ V_S^+ = j a V_M^+ \]  \hspace{1cm} (22)

\[ V_S^- = -j a V_M^- \]  \hspace{1cm} (23)

\[ V_S = j a (V_M^+ - V_M^-) \]  \hspace{1cm} (24)

Similarly:

\[ I_S = \frac{j}{a} (I_M^+ - I_M^-) \]  \hspace{1cm} (25)

If the windings are fed from the same single phase supply,

\[ V_S \equiv V_M \]  \hspace{1cm} (26)
and equation (24) becomes:

\[ V_M = j a (V^+_M - V^-_M) \]  

(27)

Let

\[ Z^+_M = \text{positive sequence impedance to the positive sequence current } I^+_M \]
\[ Z^-_M = \text{negative sequence impedance to the negative sequence current } I^-_M \]
\[ Z^+_S = \text{positive sequence impedance to the positive sequence current } I^+_S \]
\[ Z^-_S = \text{negative sequence impedance to the negative sequence current } I^-_S \]

Multiplying equation (27) by \(- \frac{j}{a}\) and adding to equation (1):

\[ \frac{V^+_M}{Z} = \frac{V^-_M}{Z} \left( 1 - \frac{j}{a} \right) \]  

(28)

or by subtraction from equation (1):

\[ \frac{V^+_M}{Z} = \frac{V^-_M}{Z} \left( 1 + \frac{j}{a} \right) \]  

(29)

From equations (22) and (28):

\[ \frac{V^+_S}{Z} = \frac{V^+_M}{Z} \left( 1 + ja \right) \]  

(30)
and from equations (23) and (29):

\[ V_S^- = \frac{V_M}{Z} \left(1 - \frac{j}{a}\right) \]  \hspace{1cm} (31)

The voltage applied to the M-phase is the sum of the positive and the negative sequence voltage drops in this phase, or:

\[ V_M = I_M^+ Z_M^+ + I_M^- Z_M^- \]  \hspace{1cm} (32)

\[ V_M = \frac{j}{a} \left(I_M^+ Z_S^+ - I_M^- Z_S^-\right) \]  \hspace{1cm} (33)

To solve for \( I_M^- \), multiply equation (32) by \( Z_S^+ /a^2 \) and equation (33) by \(- j/a \) \( Z_M^+ \):

\[ V_M \frac{Z_S^+}{a^2} = I_M^+ Z_M^+ \frac{Z_S^+}{a^2} + I_M^- Z_M^- \frac{Z_S^+}{a^2} \]  \hspace{1cm} (34)

\[ -j V_M \frac{Z_M^+}{a^2} = I_M^+ \frac{Z_S^+}{a^2} Z_M^+ - I_M^- Z_M^- \frac{Z_S^+}{a^2} \]  \hspace{1cm} (35)

and on subtracting equation (35) from equation (34):

\[ V_M \left(\frac{Z_S^+}{a^2} + j \frac{Z_M^+}{a^2}\right) = I_M^- \left(\frac{Z_M^- Z_S^+}{a^2} + \frac{Z_M^+ Z_S^-}{a^2}\right) \]

or:
\[ I_M^- = V_M \left( \frac{Z_S^+ + j a Z_M^+}{Z_M^- Z_S^- + Z_M^+ Z_S^+} \right) \] (36)

from equations (19) and (35):

\[ I_S^- = V_M \left( \frac{Z_M^- - j \frac{Z_S^+}{a}}{Z_M^- Z_S^- + Z_M^+ Z_S^+} \right) \] (37)

wherever:

\[ I_S^- = -j I_M^- \]

To solve for \( I_M^+ \), multiply equation (32) by \( \frac{Z_S^-}{a^2} \) and equation (33) by \(-j/a Z_M^+\),

\[ \frac{Z_S^-}{V_M a^2} = I_M^+ Z_M^+ \frac{Z_S^+}{a^2} + I_M^- Z_M^- \frac{Z_S^+}{a^2} \] (38)

\[ j \frac{Z_M^-}{V_M a} = -I_M^+ Z_S^- \frac{Z_M^+}{a^2} + I_M^- Z_S^- \frac{Z_M^-}{a^2} \] (39)

and an subtracting equation (39) from equation (38):

140
\[ V_M \left( \frac{Z_S}{a^2} - j \frac{Z_M}{a^2} \right) = I_M^+ \left( Z_M^+ \frac{Z_S^-}{a^2} + Z_M^- \frac{Z_S^+}{a^2} \right) \]

or

\[ I_M^+ = V_M \left( \frac{Z_S^- - j a Z_M^-}{Z_M^+ Z_S^- + Z_M^- Z_S^+} \right) \quad (40) \]

from equations (18) and (40)

\[ I_S^+ = V_M \left( \frac{Z_S^- + j \frac{Z_S^+}{a}}{Z_M^+ Z_S^- + Z_M^- Z_S^+} \right) \quad (41) \]

where:

\[ I_S^+ = j I_M^+ \]

It follows from equations (36) and (4) that:

\[ I_M = V_M \left[ \frac{\left( Z_S^+ + Z_S^- \right) + j a \left( Z_M^+ - Z_M^- \right)}{Z_M^+ Z_S^- + Z_M^- Z_S^+} \right] \quad (42) \]

where:

\[ I_M = I_M^+ + I_M^- \]
and from equations (37) and (41) that:

\[ I_S = V_M \left[ \frac{(Z^+ + Z^-) - j/a (Z^+_S - Z^-_S)}{Z^+_M Z^-_S + Z^-_M Z^+_S} \right] \]  

where:

\[ I_S = I^+_S + I^-_S \]

The line current taken by the motor is the sum of the currents of the main and the auxiliary phases:

\[ I_L = I^+_M + I^-_S \]

\[ = V_M \left[ \frac{(Z^+_M + Z^-_M + Z^+_S + Z^-_S) + j(a Z^+_M - a Z^-_M + \frac{Z^-_S}{a} - \frac{Z^+_S}{a})}{Z^+_M Z^-_S + Z^-_M Z^+_S} \right] \]

(44)
A) A computer program written in Fortran IV was used to determine the parameters of the single-phase capacitor-run induction motor over the range of frequencies considered. Figure (A) shows a flow chart of the program (Motor 1), followed by the program listing, where the variables used are as follows:

\[
\begin{align*}
Z_1 &= R_0 \\
Z_2 &= X_0 \\
Z_3 &= R_L \\
Z_4 &= X_L \\
D_{Z1} &= e_1 \\
D_{Z2} &= e_2 \\
D_{Z3} &= e_3 \\
D_{Z4} &= e_4 \\
P_{Z1} &= e'_1 \\
P_{Z2} &= e'_2 \\
P_{Z3} &= e'_3 \\
P_{Z4} &= e'_4 \\
D_{F1} &= \Delta e_1 \\
D_{F2} &= \Delta e_2 \\
D_{F3} &= \Delta e_3 \\
D_{F4} &= \Delta e_4
\end{align*}
\]
Fig (A)

Flow Chart of the Digital Computer Program (Motor 1) Used to Determine the Parameter of the Capacitor Motor.

![Flow Chart Diagram]

- Calculate the initial values of the equivalent circuit impedances using $R_2$, $X_2$, $R_m$, $X_m$

  - $R_2 = R_2 + \Delta R_2$
  - Calculate $Z_1 \rightarrow Z_4$
  - And $D(1,2) \rightarrow D(4,2)$

- $R_2 = R_2 - \Delta R_2$
  - $X_2 = X_2 + \Delta X_2$
  - Calculate $Z_1 \rightarrow Z_4$
  - And $D(1,1) \rightarrow D(4,1)$

- $X_2 = X_2 - \Delta X_2$
  - $R_m = R_m + \Delta R_m$
  - Calculate $Z_1 \rightarrow Z_4$
  - And $D(1,3) \rightarrow D(4,3)$
\[ R_m = R_m - \Delta R_m \]
\[ X_m = X_m + \Delta X_m \]

**CALCULATE** \[ Z_i \rightarrow Z^* \]
**AND** \[ D(1,4) \rightarrow D(4,4) \]**

**CALL NAG**
**F01AAF** **TO** **CALCULATE**
**THE** **INVERSE** **OF** **D**

**CALCULATE** **THE** **MATRIX** **[D \_\_]**

\[ 0 < DX(I) < 0.03 \]

\[ I = I + 1 \]

**YES**

**WRITE** **THE** **REQUIRED** **RESULTS**

**STOP**
MASTER MOTOR

This program is used to calculate the motor parameters using the Newton-Raphson method at 40Hz.

Dimension D(4,4), Z(4), Di(4,4), AINT(4), DX(4), DZZ(4), Z0(4)

Data R2, X2, RM, XM, DR2, DX2, DRM, DXM/186, 89, 15552, 1995.

10, 03, 0.03, 0.03, 0.03/0.03/0.03/0.03

Data Z10, Z20, Z30, Z40/205, 5, -768, 7, 157, 5, -179, 7/

Z1(R2, X2, RM, XM) = ((XM**2*RM*R2+2*XM*RM**2*X2=XM**2*R2+RM**2)*(
1*RM**2+R2-4*XH+X2*RM=XM**2+2*R2+2*RM**2*RM)+(XM*RM**2*R2-2*X2=XM**2
1*X2+RM) + (2*XM*RM*R2+2*RM**2*X2=2*XM*RM**2-2=2*XM**2+X2))/
(1*(RM**2+R2-4*XH+X2=RM**2+2*R2+2*RM**2*R2)=2+(2*XM*RM*R2+2*R2+RM**2
1*X2+RM)**2)

Z2(R2, X2, RM, XM) = ((XM**2+RM+R2+2*XM+XM**2+RM**2=RM**2)*((2*XH*
2*RM**2+R2+2*XM**2*RM)**2+(2*XM+R2+2*RM**2+X2=2*XM**2+X2**2))
2*(RM**2+R2-4*XH+X2=RM**2+2*R2+2=2*XM**2+X2))/

Z3(R2, X2, RM, XM) = (2*XM+RM+XH+X2=2*XH+X2=RM**2+2=2)**

Z4(R2, X2, RM, XM) = (2*XM+RM+XH+X2=2*XH+X2=RM**2+2=2) *

Z10=Z1(P,2,X2+RM, XM)

Z20=Z2(R2, X2, RM, XM)

Z30=Z3(R2, X2, RM, XM)

Z40=Z4(R2, X2, RM, XM)

PZ1=210-Z1(R2, X2, RM, XM)

PZ2=Z20-Z2(R2, X2, RM, XM)

PZ3=Z30-Z3(R2, X2, RM, XM)

PZ4=Z40-Z4(R2, X2, RM, XM)

Z20+PZ1-Z20=Z21

Z20+PZ2-Z20=Z23

DZ2=Z20-Z2(R2, X2, RM, XM)

DZ3=Z30-Z3(R2, X2, RM, XM)

DZ4=Z40-Z4(R2, X2, RM, XM)

DZ1=Z10-Z1(R2, X2, RM, XM)

PZ1=PZ1-DZ1

PZ2=PZ2-DZ2

PZ3=PZ3-DZ3

PZ4=PZ4-DZ4

D(1,2)=DZ1/DR2

D(2,2)=DZ2/DR2

D(3,2)=DZ3/DR2

D(4,2)=DZ4/DR2

R2=R2+DR2

X2=X2+DX2
Z(1) = Z1(R2, X2, RM, XM)
Z(2) = Z2(R2, X2, RM, XM)
Z(3) = Z3(R2, X2, RM, XM)
Z(4) = Z4(R2, X2, RM, XM)
PZ11 = Z10 - Z1(R2, X2, RM, XM)
PZ12 = Z20 - Z2(R2, X2, RM, XM)
PZ13 = Z30 - Z3(R2, X2, RM, XM)
PZ14 = Z40 - Z4(R2, X2, RM, XM)
DF11 = PZ11 - DZ1
DF12 = PZ12 - DZ2
DF13 = PZ13 - DZ3
DF14 = PZ14 - DZ4
D(1, 1) = DF11 / DX2
D(2, 1) = DF12 / DX2
D(3, 1) = DF13 / DX2
D(4, 1) = DF14 / DX2
X2 = X2 - DX2
RM = RM + DRM
Z(1) = Z1(R2, X2, RM, XM)
Z(2) = Z2(R2, X2, RM, XM)
Z(3) = Z3(R2, X2, RM, XM)
Z(4) = Z4(R2, X2, RM, XM)
PZ21 = Z10 - Z1(R2, X2, RM, XM)
PZ22 = Z20 - Z2(R2, X2, RM, XM)
PZ23 = Z30 - Z3(R2, X2, RM, XM)
PZ24 = Z40 - Z4(R2, X2, RM, XM)
DF21 = PZ21 - DZ1
DF22 = PZ22 - DZ2
DF23 = PZ23 - DZ3
DF24 = PZ24 - DZ4
D(1, 3) = DF21 / DRM
D(2, 3) = DF22 / DRM
D(3, 3) = DF23 / DRM
D(4, 3) = DF24 / DRM
RM = RM - DRM
XM = XM + DXM
Z(1) = Z1(R2, X2, RM, XM)
Z(2) = Z2(R2, X2, RM, XM)
Z(3) = Z3(R2, X2, RM, XM)
Z(4) = Z4(R2, X2, RM, XM)
PZ31 = Z10 - Z1(R2, X2, RM, XM)
PZ32 = Z20 - Z2(R2, X2, RM, XM)
PZ33 = Z30 - Z3(R2, X2, RM, XM)
PZ34 = Z40 - Z4(R2, X2, RM, XM)
DF31 = PZ31 - DZ1
DF32 = PZ32 - DZ2
DF33 = PZ33 - DZ3
DF34 = PZ34 - DZ4
D(1, 4) = DF31 / DXM
D(2, 4) = DF32 / DXM
D(3, 4) = DF33 / DXM
D(4, 4) = DF34 / DXM
IFAIL = 0
CALL F01AAF(D, 6, 4, D1, 4, AINT, IFAIL)
DO 10 I = 1, 4

147
DX(I)=0
DO 10 J=1,4
DX(I)=-DX(I)+DI(I,J)*DZZ(J)
10 CONTINUE
WRITE(3,2)(R2,X2,RM,XM,(DX(K),K=1,4))
DO 11 I=1,4
IF(DX(I).GT.0.0.AND.DX(I).LT.0.03)GO TO 11
GO TO 100
11 CONTINUE
WRITE(3,2) (DX(I),I=1,4)
WRITE(3,3) ((D(I,J),J=1,4),I=1,4)
3 FORMAT (4F20.5)
WRITE(3,2) (Z(I),I=1,4)
2 FORMAT(1X,8(G 11.4,3X))
STOP
END
MASTER PROGRAM

THIS PROGRAM IS USED TO CALCULATE THE MOTOR PARAMETERS USING
THE NEWTON-RAPHSON METHOD AT 45 Hz.

DIMENSIONS: D(4,4) Z(4), D1(4,4), A(1)4), DX(4), DZ(4), R(4), 70(4)
DATA R2,X2, RM, XRM, DR2, DX2, DRM, DRM, DXM, 180, .94, 15773, 1284, 10.03, 0.73, 0.03, 0.03, 0.03
DATA Z1=Z20, Z40/35.5, -850, 177.8, -178.7
Z1(R2, X2, RM, XM) = (-XMX2 + RM-R2-2*XM+RM*2+X2-XM*2+RM*2) * 1 + RM*2 + R2-2*XM+RM*2+X2-XM*2+RM*2)
1 + RM*2 + R2-2*XM+RM*2+X2-XM*2+RM*2)
Z2(R2, X2, RM, XM) = (Z2(R2, X2, RM, XM))
Z3(R2, X2, RM, XM) = ((XMX2 + RM-R2+2*XM+2*RM+2*RM+2*RM+2) * (Z2(R2, X2, RM, XM))
Z4(R2, X2, RM, XM) = (Z4(R2, X2, RM, XM))

100 R2=2+D2
Z1=Z10-Z11K2 • X2 • XM)
DZ1=Z20-72(~2X2.RM.XM)
DZ2=Z20-72(~2X2.RM.XM)
DZ3=Z30-73(~2X2.RM.XM)
DZ4=Z40-74(~2X2.RM.XM)
DZ(1) = Z1
DZ(2) = Z2
DZ(3) = Z3
DZ(4) = Z4
Z0(1) = Z10
Z0(2) = Z20
Z0(3) = Z30
Z0(4) = Z40
WRITE(3,2) (D771, 1=1, 4)
Z(1) = Z1(R2, X2, RM, XM)
Z(2) = Z2(R2, X2, RM, XM)
Z(3) = Z3(R2, X2, RM, XM)
Z(4) = Z4(R2, X2, RM, XM)
PZ11 = Z10 - Z1(R2, X2, RM, XM)
PZ12 = Z20 - Z2(R2, X2, RM, XM)
PZ13 = Z30 - Z3(R2, X2, RM, XM)
PZ14 = Z40 - Z4(R2, X2, RM, XM)
DF11 = PZ11 - D21
DF12 = PZ12 - D22
DF13 = PZ13 - D23
DF14 = PZ14 - D24
D(1, 1) = D11 / nx2
D(2, 1) = D12 / nx2
D(3, 1) = D13 / nx2
D(4, 1) = D14 / nx2
x2 = x2 - D2
RM = RM + D; n
Z(1) = Z1(R2, X2, RM, XM)
Z(2) = Z2(R2, X2, RM, XM)
Z(3) = Z3(R2, X2, RM, XM)
Z(4) = Z4(R2, X2, RM, XM)
PZ21 = Z10 - Z1(R2, X2, RM, XM)
PZ22 = Z20 - Z2(R2, X2, RM, XM)
PZ23 = Z30 - Z3(R2, X2, RM, XM)
PZ24 = Z40 - Z4(R2, X2, RM, XM)
DF21 = PZ21 - D21
DF22 = PZ22 - D22
DF23 = PZ23 - D23
DF24 = PZ24 - D24
D(1, 3) = D21 / nRM
D(2, 3) = D22 / nRM
D(3, 3) = D23 / nRM
D(4, 3) = D24 / nRM
XM = XM + D; XM
Z(1) = Z1(R2, X2, RM, XM)
Z(2) = Z2(R2, X2, RM, XM)
Z(3) = Z3(R2, X2, RM, XM)
Z(4) = Z4(R2, X2, RM, XM)
PZ31 = Z10 - Z1(R2, X2, RM, XM)
PZ32 = Z20 - Z2(R2, X2, RM, XM)
PZ33 = Z30 - Z3(R2, X2, RM, XM)
PZ34 = Z40 - Z4(R2, X2, RM, XM)
DF31 = PZ31 - D31
DF32 = PZ32 - D32
DF33 = PZ33 - D33
DF34 = PZ34 - D34
D(1, 4) = D31 / nXM
D(2, 4) = D32 / nXM
D(3, 4) = D33 / nXM
D(4, 4) = D34 / nXM
FAIL = 0
CALL F01AA(NT, 4, 4, D1, 4, AINT, FAIL)
DO 10 1 = 1, 4
DX(1) = 0
DO 10 J = 1, 4
DX(I) = -x(I)*D1(I,J)*DZZ(J)
10 CONTINUE
WRITE (3, 2) (X(2), X(3), X(5), (DX(K), K=1, 4))
DO 11 I = 1, 4
IF (DX(I), GT, 0.0, AND. DX(I), LT, 0.03 ) GO TO 11
GO TO 130
11 CONTINUE
WRITE (3, 2) (DX(I), I=1, 4)
WRITE (3, 3) ((0(I,J), J=1, 4), I=1, 4)
3 FORMAT (4F20.5)
WRITE (3, 2) (T(I), I=1, 4)
2 FORMAT (1X, 8(G 11.4, 3X))
END
This program is used to calculate the motor parameters using the Newton-Raphson method at 50 Hz.

\[ \begin{align*}
\text{DATA} \quad & R_2, X_2, \text{DR}_2, \text{DX}_2, \text{RM}, \text{DX}_H/180, \pm 107.17668, 1353, 10.03, 0.03, 0.03/ \\
\text{DATA} \quad & Z_1, Z_2, Z_3, Z_4 /240/251, -941, 147, -215, / \\
Z_1 (R_2, X_2, \text{RM}, \text{AX}) = & (-X_2, R_2-2*X_2, \text{RM})*2+X_2-2*X_2, \text{RM})*2+(X_2-2*X_2, \text{RM})*2 \\
1 (R_2-2*X_2-2*X_2, \text{RM})*2+X_2-2*X_2, \text{RM})*2+(X_2-2*X_2, \text{RM})*2 \\
Z_2 (R_2, X_2, \text{RM}, \text{VM}) = & ((X_2-2*X_2, \text{RM})*2-R_2-2*X_2, \text{RM})*2+X_2-2*X_2, \text{RM})*2 \\
2 (R_2-2*X_2-2*X_2, \text{RM})*2+X_2-2*X_2, \text{RM})*2+(X_2-2*X_2, \text{RM})*2 \\
Z_3 (X_2, X_2, \text{RM}, \text{VM}) = & ((X_2-2*X_2, \text{RM})*2+X_2-2*X_2, \text{RM})*2+(X_2-2*X_2, \text{RM})*2 \\
Z_4 (X_2, X_2, \text{RM}, \text{VM}) = & ((X_2-2*X_2, \text{RM})*2+X_2-2*X_2, \text{RM})*2+(X_2-2*X_2, \text{RM})*2 \\
R_Z (1) = & 71 \\
R_Z (2) = & 72 \\
R_Z (3) = & 73 \\
R_Z (4) = & 74 \\
Z_0 (1) = & 710 \\
Z_0 (2) = & 720 \\
Z_0 (3) = & 730 \\
Z_0 (4) = & 740 \\
\text{WRITE} (3, 2) / D2Z (1), 1 = 1, 4/ \\
\end{align*} \]
\[
\begin{align*}
Z(1) &= Z1 : x2 : x2 : Rm : Xm) \\
Z(2) &= Z2 : x2 : x2 : Rm : Xm) \\
Z(3) &= Z3 : x2 : x2 : Rm : Xm) \\
Z(4) &= Z4 : x2 : x2 : Rm : Xm) \\
PZ11 &= Z1 - 21 (x2 , x2 , Rm , Xm) \\
PZ12 &= Z2 - 22 (x2 , x2 , Rm , Xm) \\
PZ13 &= Z3 - 23 (x2 , x2 , Rm , Xm) \\
PZ14 &= Z4 - 24 (x2 , x2 , Rm , Xm) \\
DF1 &= PZ11 - Z1 \\
DF2 &= PZ12 - Z2 \\
DF3 &= PZ13 - Z3 \\
DF4 &= PZ14 - Z4 \\
D(1, 1) &= \text{.Failure}(1) \\
D(2, 1) &= \text{.Failure}(2) \\
D(3, 1) &= \text{.Failure}(3) \\
D(4, 1) &= \text{.Failure}(4) \\
X2 &= x2 - Z2 \\
Rm &= Rm + Dv \\
Z(1) &= Z1 : x2 : x2 : Rm : Xm) \\
Z(2) &= Z2 : x2 : x2 : Rm : Xm) \\
Z(3) &= Z3 : x2 : x2 : Rm : Xm) \\
Z(4) &= Z4 : x2 : x2 : Rm : Xm) \\
PZ21 &= Z1 - 21 (x2 , x2 , Rm , Xm) \\
PZ22 &= Z2 - 22 (x2 , x2 , Rm , Xm) \\
PZ23 &= Z3 - 23 (x2 , x2 , Rm , Xm) \\
PZ24 &= Z4 - 24 (x2 , x2 , Rm , Xm) \\
DF21 &= PZ21 - Z1 \\
DF22 &= PZ22 - Z2 \\
DF23 &= PZ23 - Z3 \\
DF24 &= PZ24 - Z4 \\
D(1, 3) &= \text{.Failure}(1) \\
D(2, 3) &= \text{.Failure}(2) \\
D(3, 3) &= \text{.Failure}(3) \\
D(4, 3) &= \text{.Failure}(4) \\
Rm &= Rm + Dv \\
Xm &= Xm + Dv \\
Z(1) &= Z1 : x2 : x2 : Rm : Xm) \\
Z(2) &= Z2 : x2 : x2 : Rm : Xm) \\
Z(3) &= Z3 : x2 : x2 : Rm : Xm) \\
Z(4) &= Z4 : x2 : x2 : Rm : Xm) \\
PZ31 &= Z1 - 21 (x2 , x2 , Rm , Xm) \\
PZ32 &= Z2 - 22 (x2 , x2 , Rm , Xm) \\
PZ33 &= Z3 - 23 (x2 , x2 , Rm , Xm) \\
PZ34 &= Z4 - 24 (x2 , x2 , Rm , Xm) \\
DF31 &= PZ31 - Z1 \\
DF32 &= PZ32 - Z2 \\
DF33 &= PZ33 - Z3 \\
DF34 &= PZ34 - Z4 \\
D(1, 4) &= \text{.Failure}(1) \\
D(2, 4) &= \text{.Failure}(2) \\
D(3, 4) &= \text{.Failure}(3) \\
D(4, 4) &= \text{.Failure}(4) \\
\text{IF} &= 0 \\
\text{CALL} &= \text{Failure}(1, 4, 4, 4, \text{.AINT, TFAIL}) \\
D = 0.01 \text{.1} = 1.4
\end{align*}
\]
DO 10 J=1,4
DX(1) = DX(1) + DI(1, J)*DZ(J)
10 CONTINUE
WRITE(3,7) (R, X2, RM, XM, (DX(K), K=1,4))
DO 11 I=1,4
IF (DX(I).GT.0.0.AND.DX(I).LT.0.03) GO TO 11
GO TO 10
11 CONTINUE
WRITE(3,2) (DX(I), I=1,4)
WRITE (3, 3) ((DI(I, J), J=1,4), I=1,4)
3 FORMAT (4F20.5)
WRITE(3, 2) (7(I), I=1,4)
2 FORMAT (1X, 8G 11.4,3X)
STOP
END
MASTER MOTOR

This program is used to calculate the motor parameters using the Newton-Raphson method at 55 Hz.

DIMENSION D(4), Z(4), D(4), A(4), DX(4), DZ(4), Z(4)
DATA R, X, R, M, DR, D, DX, DRM, DXM, 194., 115., 0.215., 159., 10.03, 0.03, 0.03, 0.03
DATA Z1, Z2, Z3, Z4, D1, D2, D3, D4, Z1, Z2, Z3, Z4
DATA Z(1), Z(2), Z(3), Z(4)

WRITE(3, Z) (07, Z(1), Z(2), Z(3), Z(4))

100 R1 = R2 + D1
Z(1) = Z1 + Z(2) + Z(3) + Z(4)
D1 = P1 - P2
D2 = P2 - P3
D3 = P3 - P4
D4 = P4 - P5

D(1, 2) = D(1, 2) + D(2, 2)
D(2, 2) = D(2, 2) + D(3, 2)
D(3, 2) = D(3, 2) + D(4, 2)
D(4, 2) = D(4, 2) + D(5, 2)
R2 = R2 - D1
X2 = X2 + D1

155
Z(1) = Z1(R2, X2, RM, XM)
Z(2) = Z2(R2, X2, RM, XM)
Z(3) = Z3(R2, X2, RM, XM)
Z(4) = Z4(R2, X2, RM, XM)
PZ11 = Z1(R2, X2, RM, XM)
PZ12 = Z2(R2, X2, RM, XM)
PZ13 = Z3(R2, X2, RM, XM)
PZ14 = Z4(R2, X2, RM, XM)
DF11 = PZ11
DF12 = PZ12
DF13 = PZ13
DF14 = PZ14
DF11 = F11
DF12 = F12
DF13 = F13
DF14 = F14
X2 = X2 - DF
RM = RM + D
XM = XM + DM
Z(1) = Z1(R2, X2, RM, XM)
Z(2) = Z2(R2, X2, RM, XM)
Z(3) = Z3(R2, X2, RM, XM)
Z(4) = Z4(R2, X2, RM, XM)
PZ21 = Z2(R2, X2, RM, XM)
PZ22 = Z2(R2, X2, RM, XM)
PZ23 = Z3(R2, X2, RM, XM)
PZ24 = Z4(R2, X2, RM, XM)
DF21 = PZ21
DF22 = PZ22
DF23 = PZ23
DF24 = PZ24
D(1, 3) = F21
D(2, 3) = F22
D(3, 3) = F23
D(4, 3) = F24
RM = RM + DRM
XM = XM + DM
Z(1) = Z1(R2, X2, RM, XM)
Z(2) = Z2(R2, X2, RM, XM)
Z(3) = Z3(R2, X2, RM, XM)
Z(4) = Z4(R2, X2, RM, XM)
PZ31 = Z1(R2, X2, RM, XM)
PZ32 = Z2(R2, X2, RM, XM)
PZ33 = Z3(R2, X2, RM, XM)
PZ34 = Z4(R2, X2, RM, XM)
DF31 = PZ31
DF32 = PZ32
DF33 = PZ33
DF34 = PZ34
D(1, 4) = F31
D(2, 4) = F32
D(3, 4) = F33
D(4, 4) = F34
IFAIL = 0
CALL F014E(0, 4, 4, IN, IAINT, IFAIL)
DO 10 I = 1, 4
DX(I) = 0
DO 10 J = 1, 4
DX(I) = DX(I) + D(I,J)*DZ(Z(J)
10 CONTINUE
WRITE(3,2) (R2, X2, RM, XM, (DX(K), K = 1, 4))
DO 11 I = 1, 4
IF(DX(I) .GT. 0.0 .AND. DX(I) .LT. 0.03) GO TO 11
GO TO 10
11 CONTINUE
WRITE(3, 2) (DX(I), I = 1, 4)
WRITE(3, 3) ((D(I,J), J = 1, 4), I = 1, 4)
3 FORMAT (4F20.5)
WRITE(3, 2) (X(I), I = 1, 4)
2 FORMAT(1X,8(F11.4,3X))
STOP
END
MASTER MOTOR

This program is used to calculate the motor parameters using the Newton-Raphson method at 60Hz.

Dimension D(4,4); Z(4), D1(4,4), A1NT(4), D2(4), Z0(4)

Data R2, X2, RM, DR2, X2, DRM, XM/197., 115., 19841., 1917.,
10.03, 0.03, 0.03/

Data Z10, Z20, Z30, Z40/281.9, -1103.7, 196.2, -200.2/

Z1(R2, X2, RM, XM) = ((-XM*R2~2*RM*R2-2*X2-XM*2*RM**2)*Z1(R2, X2, RM, XM))

Z2(R2, X2, RM, XM) = ((-XM*R2~2*RM*R2+2*X2-XM*2*RM**2)*Z2(R2, X2, RM, XM))

Z3(R2, X2, RM, XM) = ((-XM*R2~2*RM*R2+2*X2-XM*2*RM**2)*Z3(R2, X2, RM, XM))

Z4(R2, X2, RM, XM) = ((-XM*R2~2*RM*R2+2*X2-XM*2*RM**2)*Z4(R2, X2, RM, XM))

Write(3,2)(O7Z(I), I=1,4)

R2=R2+DR2

Z(1)=Z1(R2, X2, RM, XM)
Z(2)=Z2(R2, X2, RM, XM)
Z(3)=Z3(R2, X2, RM, XM)
Z(4)=Z4(R2, X2, RM, XM)

PZ1=Z10-Z1(R2, X2, RM, XM)
PZ2=Z20-Z2(R2, X2, RM, XM)
PZ3=Z30-Z3(R2, X2, RM, XM)
PZ4=Z40-Z4(R2, X2, RM, XM)

DF1=DF1+DF2
DF2=DF2+DF3
DF3=DF3+DF4
DF4=DF4+DF5

D(1,2)=DF1/DR2
D(2,2)=DF2/DR2
D(3,2)=DF3/DR2
D(4,2)=DF4/DR2
R2=R2-DR2
X2=X2+DX2
Z(1) = Z1(r2, x2, rm, xm)
Z(2) = Z2(r2, x2, rm, xm)
Z(3) = Z3(r2, x2, rm, xm)
Z(4) = Z4(r2, x2, rm, xm)
PZ11 = Z1 - Z1(r2, x2, rm, xm)
PZ12 = Z2 - Z2(r2, x2, rm, xm)
PZ13 = Z3 - Z3(r2, x2, rm, xm)
PZ14 = Z4 - Z4(r2, x2, rm, xm)

DF11 = PZ11 - 0z1
DF12 = PZ12 - 0z2
DF13 = PZ13 - 0z3
DF14 = PZ14 - 0z4

D(1, 1) = DF11 / dx2
D(2, 1) = DF12 / dx2
D(3, 1) = DF13 / dx2
D(4, 1) = DF14 / dx2

X2 = X2 - dx2

RM = RM + DRM
XM = XM + DRM

Z(1) = Z1(r2, x2, rm, xm)
Z(2) = Z2(r2, x2, rm, xm)
Z(3) = Z3(r2, x2, rm, xm)
Z(4) = Z4(r2, x2, rm, xm)
PZ21 = Z1 - Z1(r2, x2, rm, xm)
PZ22 = Z2 - Z2(r2, x2, rm, xm)
PZ23 = Z3 - Z3(r2, x2, rm, xm)
PZ24 = Z4 - Z4(r2, x2, rm, xm)

DF21 = PZ21 - 0z1
DF22 = PZ22 - 0z2
DF23 = PZ23 - 0z3
DF24 = PZ24 - 0z4

D(1, 3) = DF21 / DRM
D(2, 3) = DF22 / DRM
D(3, 3) = DF23 / DRM
D(4, 3) = DF24 / DRM

RM = RM - DRM
XM = XM - DRM

Z(1) = Z1(r2, x2, rm, xm)
Z(2) = Z2(r2, x2, rm, xm)
Z(3) = Z3(r2, x2, rm, xm)
Z(4) = Z4(r2, x2, rm, xm)
PZ31 = Z1 - Z1(r2, x2, rm, xm)
PZ32 = Z2 - Z2(r2, x2, rm, xm)
PZ33 = Z3 - Z3(r2, x2, rm, xm)
PZ34 = Z4 - Z4(r2, x2, rm, xm)

DF31 = PZ31 - 0z1
DF32 = PZ32 - 0z2
DF33 = PZ33 - 0z3
DF34 = PZ34 - 0z4

D(1, 4) = DF31 / nxm
D(2, 4) = DF32 / nxm
D(3, 4) = DF33 / nxm
D(4, 4) = DF34 / nxm

IFA11 = 0

CALL F01AAF(D, 4, 4, DI, 4, AIK, IFAIL)
DO 10 I = 1, 4
DX(I) = 0
DO 10 J = 1, 4
DX(I) = DX(I) + DI(I,J) * DZZ(J)
10 CONTINUE
WRITE(3,2)(R7,X2,RM,XM,(DX(K),K=1,4))
DO 11 I = 1, 4
IF(DX(I).GT.0.0 .AND. DX(I).LT.0.03) GO TO 11
GO TO 100
11 CONTINUE
WRITE(3,2) (DX(I),I=1,4)
WRITE(3,3) ((D(I,J),J=1,4),I=1,4)
3 FORMAT(4F20.5)
WRITE(3,2) (7(I),I=1,4)
2 FORMAT(1X,8(6 11.4,3X))
STOP
END
B) A digital computer program was also used to calculate the motor performance characteristics at the different operational frequencies and the different capacitor values considered. Figure (B) shows the flow chart of the program (Motor 2), followed by the program listing.
FIG. (B). FLOW CHART OF THE DIGITAL COMPUTER PROGRAM USED TO CALCULATE THE MOTOR PERFORMANCE
MASTER MOTOR 2

This program is used to calculate the load characteristics of the single-phase capacitor-run induction motor at 40Hz and at capacitor value of 2 microfarad.

Complex Zs, Ym, Y1m, Yem, Aem, Y1s, Yos, Yes, Aos, Ae, Ao, Ye
Complex cur, am, curj, as, alin, yi, xi1, si, y1

Complex z21, z1m, z1s
z1m = complex(132., 89.)
ym = complex(0.0000643, -0.0008333)
j = complex(0.0, 1.0)
ak = 1.0
r2 = 186.
v = 192.
z1s = complex(132., -1900.4)
y1s = complex(0.0000643, -0.0008333)
y1m = 1.0 / (z21 + z1m + z1s + z21 * ym)
yom = (1.0 + z1m * ym) * y1m
yem = (1.0 + z21 * ym) * y1m
y1s = 1.0 / (z21 + z1s + z1s + z21 * ak * y2 * y1s)
yos = (1.0 + z1s * ak * y2 * y1s) * y1s
yes = (1.0 + z21 * ak * y2 * y1s) * y1s
aem = yem + v
aom = y1m - v
aos = y1s + (-j * v)
aes = yes + (-j * v)
ag = aos + aom
yo = yos + yom
write(2, 1000) ym, y1m, yom, yem, y1s, yos, yes, yo
write(2, 1000) aem, aom, aos, aes, ao, a

1000 format(1x, 2f15.8)
7 read(1, 5, end=6) x,y, z
write(2, 1000) x,y, z
5 format(0, 0)
r = r + z**2 / (1.0 - s**2)
write(2, 1000) r
write(2, 1000) f, am, as, a
a = ao / (1.0 + r * yo)
p0 = ((cabs(a))**2) * r
pow = po + s
out = pow - 1.2
e = ar
cur = y1m - e
am = aem - cur
curj = je * y1s * e
as = j * aes - curj
alin = am + as
put = real(alin) * v
cos = real(alin) / (cabs(alin))

163
100 FORMAT(50X,5HSLIP=,F15.10)
WRITE(2,1001)OUT
1001 FORMAT(1X,15HOUTPUTPOWER=,F15.8/)
WRITE(2,1002)ALIN
1002 FORMAT(1X,15HLINECURRENT=,2F15.8/)
WRITE(2,1003)PUT
1003 FORMAT(1X,15HINPUTPOWER=,F15.8/)
WRITE(2,1004)COP
1004 FORMAT(1X,15HPowerFactor=,F15.8/)
WRITE(2,1005)EFF
1005 FORMAT(1X,15HEfficiency=,F15.8/)
WRITE(2,1006)T
1006 FORMAT(1X,15HTorque=,F15.8/)
WRITE(2,1007)Y1
1007 FORMAT(1X,15HCURRENT1=,2F15.8/)
WRITE(2,1008)XI
1008 FORMAT(1X,15HMotorCurrent2=,2F15.8/)
WRITE(2,1009)SI
1009 FORMAT(1X,20HCapacitorCurrent=,2F15.8/)
GO TO 7
6 STOP
END
MASTER MOTOR2

C THIS PROGRAM IS USED TO CALCULATE THE LOAD CHARACTERISTICS OF
C THE SINGLE-PHASE CAPACITOR-RUN INDUCTION MOTOR AT 45HZ AND AT
C CAPACITOR VALUE OF 2 MICROFARAD.
C
COMPLEX ZS, J, YM, Y1M, YOM, YEM, AOM, AEM, Y1S, YOS, AES, AO, AO, A
C
COMPLEX CUR, AM, CUR, AS, ALIN, VI, XI1, SI, YS
COMPLEX Z21, Z1M, Z1S
Z21=CMPLX(103.4, 94.)
Z1M=CMPLX(133., 94.)
YM=CMPLX(0.00006339, -0.00080645)
J=CMPLX(0.0, 1.0)
AK=1.0
R2=188.
V=216.
Z1S=CMPLX(133., -1674.38)
YS=CMPLX(0.00006339, -0.00080645)
Y1M=1.0/(Z21+Z1M+Z1M*Z21*YM)
YOM=(1.0+Z1M*YM)*Y1M
YFM=(1.0+Z21+YM)*Y1M
Y1S=1.0/(Z21+Z1S+Z1S*Z21*AK**2*YS)
YOS=(1.0+Y1S*AK**2*YS)*Y1S
YES=(1.0+Z21*AK**2*YS)*Y1S
AEM=VEM+V
AOM=V1M+V
AES=Y1S*(-J*V)
AO=AO+S+AOM
YO=YOS+YOM
WRITE(2,1000)YM, Y1M, YOM, YEM, Y1S, YOS, YES, V0
WRITE(2, 1000)AEM, AOM, AES, AO, A
1000 FORMAT(1X, 2F15.8)
7 READ(1, 2F15.8)
5 FORMAT(15, END=6) S
WRITE(2, 1000)
R=S*R2/(1.0-S**2)
WRITE(2,1000) R
WRITE(2,1000)E, AM, AS, A
A=A0/(1.0+R*YO)
PO=((CABS(A))**2)*R
POW=PO*S
OUT=POW-1.76
E=A*R
CUR=Y1M*E
AM=AEM-CUR
CUR=J*Y1S*E
AS=J*AES-CUR
ALIN=AM-AS
PUT=REAL(ALIN)*V
COP=REAL(ALIN)/(CABS(ALIN))
EFF=OUT/PUT
Y1=Y1M+V-YOM+E
XI1=A0S-YOS+E
SI=AES-Y1S+E
T=PO+2.0*R2/(1.0+S)*CABS(XI1)*CABS(Y1)*COS(ATAN(AIMAG(XI1)/REAL
1(XI1)) - ATAN(AIMAG(Y1)/REAL(Y1))) -1.76

165
100 FORMAT(50X,5H SLIP= ,F15.10)
WRITE(2,1001) OUT
1001 FORMAT(1X,15H OUTPUT POWER= ,F15.8/)
WRITE(2,1002) ALIN
1002 FORMAT(1X,15H LINE CURRENT= ,2F15.8/)
WRITE(2,1003) PUT
1003 FORMAT(1X,15H INPUT POWER= ,F15.8/)
WRITE(2,1004) COP
1004 FORMAT(1X,15H POWER FACTOR= ,F15.8/)
WRITE(2,1005) FFF
1005 FORMAT(1X,15H EFFICIENCY= ,F15.8/)
WRITE(2,1006) T
1006 FORMAT(1X,15H TORQUE= ,F15.8/)
WRITE(2,1007) VI
1007 FORMAT(1X,15H CURRENT1= ,2F15.8/)
WRITE(2,1008) XI1
1008 FORMAT(1X,15H ROTOR CURRENT1= ,2F15.8/)
WRITE(2,1009) SI
1009 FORMAT(1X,20H CAPACITOR CURRENT= ,2F15.8/)
GO TO 7
6 STOP
END
MASTER MOTOR

THIS PROGRAM IS USED TO CALCULATE THE LOAD CHARACTERISTICS OF
THE SINGLE-PHASE CAPACITOR-RUN INDUCTION MOTOR AT 50Hz AND AT
CAPACITOR VALUE OF 2 MICROFarAD.

COMPLEX ZS,J,VM,Y1M,YOM,YEM,AOM,AEM,Y1S,YOS,YES,AOS,AES,AO,YO,A,E
COMPLEX CUR,AM,CURJ,AS,ALIN,YI,XI1,SI,YS
COMPLEX Z21,Y1M,Z1S
Z21=CMPLX(104.5,107.)
Z1M=CMPLX(137.,107.)
YM=CMPLX(0.000056599,-0.00073746)
J=CMPLX(0.0,1.0)
AK=1.0
R2=190.
V=240.
Z1S=CMPLX(137.,-1484.5)
YS=CMPLX(0.000056599,-0.00073746)
Y1M=1.0/(Z21+Z1M*Z1M*Z21*YM)
YOM=(1.0+Z1M*VM)*Y1M
YEM=(1.0+Z21*YM)*Y1M
Y1S=1.0/(Z21+Z1S+Z1S*Z21*AK**2*YS)
YOS=(1.0+Y1S*AK**2*YS)*Y1S
YES=(1.0+Z21*AK**2*YS)*Y1S
AFM=V*Y1M
AOM=Y1M+V
AOS=Y1S*(-J*V)
AES=YES*(-J*V)
AO=AOS+10M
YO=YOS+YO
WRITE(2,1000)YM,Y1M,YOM,YFM,Y1S,YOS,YES,YO
WRITE(2,1000)AEM,AOM,AOS,AES,AO,A
1000 FORMAT(1X,2F15.8)
7 READ (15,FND=6)S
WRITE(2,1000)<
5 FORMAT(F0.0)
R=5+R2/(1.0-S**2)
WRITE(2,1000)R
WRITE(2,1000)E,AM,AS,A
A=AO/(1.0+R*YO)
PO=(CABS(A)**2)*R
PO=PO*R
OUT=PO*-2.
E=A*R
CUR=Y1M+F
AM=AEM-CUR
CURJ=J*Y1S+F
AS=j*AES-CURJ
ALIN=AM+AS
PUT=REAL(ALIN)*V
COP=REAL(ALIN)/(CABS(ALIN))
EFF=OUT/PUT
YI=Y1M+V-YOM+F
XI1=AOS+YOS+F
SI=AES-Y1S+F
T=D*2.0+R2/(1.0+S)*CABS(XI1)*CABS(YI)*COS(ATAN(AIMAG(XI1)/REAL(1(XI1)))-ATAN(AIMAG(YI)/REAL(YI))) = 2.

167
100 FORMAT(50X,5H SLIP=,F15.10)
   WRITE(2,1001) SLIP
1001 FORMAT(1X,15H OUTPUT POWER= ,F15.8/)  
   WRITE(2,1002) ALIN
1002 FORMAT(1X,15H LINE CURRENT= ,2F15.8/)  
   WRITE(2,1003) PUT
1003 FORMAT(1X,15H INPUT POWER= ,F15.8/)  
   WRITE(2,1004) COP
1004 FORMAT(1X,15H POWER FACTOR= ,F15.8/)  
   WRITE(2,1005) EFF
1005 FORMAT(1X,15H EFFICIENCY= ,F15.8/)  
   WRITE(2,1006) T
1006 FORMAT(1X,15H TORQUE= ,F15.8/)  
   WRITE(2,1007) YI
1007 FORMAT(1X,15H CURRENT1= ,2F15.8/)  
   WRITE(2,1008) XI1
1008 FORMAT(1X,15H MOTOR CURRENT2= ,2F15.8/)  
   WRITE(2,1009) SI
1009 FORMAT(1X,20H CAPACITOR CURRENT= ,2F15.8/)  
   GO TO 7
6 STOP
   END
MASTER MOTOR 2

This program is used to calculate the load characteristics of the single-phase capacitor-run induction motor at 55 Hz and at capacitor value of 2 microfarad.

Complex Zs, j*Ym, Y1M, YOM, YEM, AOM, AEM, Y1S, YCS, YES, AOS, AES, AO, Y0, A, E
Complex CUR, AM, CUR, AS, ALIN, VI, XI1, SI, YES

Complex Z21, Z1M, Z1S
Z21 = CMPLX(106, 5, 113.)
Z1M = CMPLX(142, -113.)
Ym = CMPLX(0.00005489, -0.0000625)
J = CMPLX(0.0, 1.0)
AK = 1.0
R2 = 194.
V = 264.
Z1S = CMPLX(142, -1333.8)
YS = CMPLX(0.00005489, -0.0000625)
Y1M = 1.0/(Z21 + 21M + Z1M + Z21 + Z1M + Y1M)
YOM = (1.0 + Z1M + YM) + Y1M
YEM = (1.0 + Z21 + YM) + Y1M
Y1S = 1.0/(Z21 + Z1S + Z1S + Z21 + AK*2*YS)
YOS = (1.0 + Z1S + AK*2*YS)**2
YES = (1.0 + Z21 + AK*2*YS)**2
AEM = YEM + V
AOM = Y1M + V
AOS = Y1S + (J*V)
AES = YES + (J*V)
AO = AOS + AOM
YO = YOS + YOM

WRITE(2, 1000) VM, Y1M, YOM, YEM, Y1S, YOS, YES, YO
WRITE(2, 1000) AEM, AOM, AOS, AES, AO, A

1000 FORMAT(1X, 2F15.8, /)
7 READ(1, 5, END=6) S
WRITE(2, 1000) S
5 FORMAT(F0.0)
R = S*A2/(1.0 - S2)
WRITE(2, 1000) R
WRITE(2, 1000) E, AM, AS, A
A = AO/(1.0 + R*VO)
PO = (CABS(A)**2)*R
POU = PO**S
OUT = POW - 2.2
E = A + R
CUR = Y1M + E
AM = AEM - CUR
CUR = J*Y1S + E
AS = J*AES - CUR
ALIN = AM + AS
PUT = REAL(ALIN)*V
CP = REAL(ALIN)/(CABS(ALIN))
EFF = OUT/PUT
VI = Y1M - YOM + E
XI1 = AOS - YOS + F
SI = AES - 15 + E
T = PO + 2.0*R2/(1.0 + S)*CABS(XI1)*CABS(YI)*COS(ATAN(AIMAG(XI1)/REAL(1(XI1)) - ATAN(AIMAG(YI)/REAL(YI))) + 2.2

169
100 FORMAT(50X,5HSLIP=,F15.10)
WRITE(2,1001) OUT
1001 FORMAT(1X,15HOUTPUTPOWER=,F15.8/
WRITE(2,1002) ALIN
1002 FORMAT(1X,15HLINECURRENT=,2F15.8/
WRITE(2,1003) PUT
1003 FORMAT(1X,15HINPUTPOWER=,F15.8/
WRITE(2,1004) COP
1004 FORMAT(1X,15HPowerFactor=,F15.8/
WRITE(2,1005) FFF
1005 FORMAT(1X,15HEfficiency=,F15.8/
WRITE(2,1006) T
1006 FORMAT(1X,15HTorque=,F15.8/
WRITE(2,1007) VI
1007 FORMAT(1X,15HCURRENT1=,2F15.8/
WRITE(2,1008) XI1
1008 FORMAT(1X,15HROTORCURRENT2=,2F15.8/
WRITE(2,1009) SI
1009 FORMAT(1X,20HcapacitorCurrent=,2F15.8/
GO TO 7
6 STOP
END
MASTER MOTOR 2

THIS PROGRAM IS USED TO CALCULATE THE LOAD CHARACTERISTICS OF
THE SINGLE-PHASE CAPACITOR-RUN INDUCTION MOTOR AT 60Hz AND AT
CAPACITOR VALUE OF 2 MICROFARAD.

COMPLEX ZS, J, YM, Y1M, YOM, AOM, AEM, Y1S, YOS, YES, AOS, AES, AO, YO, A,

COMPLEX CMP, AM, CUR, AS, ALIN, Y1, XI1, SI, YS

COMPLEX Z21, Z1M, Z1S

Z21=CMPLX(108.35, 115.)
Z1M=CMPLX(150., 115.)
YM=CMPLX(0.0000504, -0.0005208)
J=CMPLX(0.0, 1.0)
AK=1.0
R2=197.
V=288.
Z1S=CMPLX(150., -1211.)
YS=CMPLX(0.0000504, -0.0005208)
Y1M=1.0/(Z21+Z1M+Z1M*Z21*YM)
YOM=(1.0+Z1M*YM)*Y1M
YEM=(1.0+Z21*YM)*Y1M
Y1S=1.0/(Z21+Z1S+Z1S*Z21*AK**2*YS)
YOS=(1.0+Z1S*AK**2*YS)*Y1S
YES=(1.0+Z21*AK**2*YS)*Y1S
AEM=YE M*V
AOM=YM*V
AOS=YS*(-J*V)
AES=YS*(-J*V)
AO=AOS+AOM
YO=YS+YO
WRITE(2,1000) YM, Y1M, YOM, YEM, Y1S, YOS, YES, YO
WRITE(2, 1000) AEM, AOH, AOS, AES, AO, A

1000 FORMAT(1X, 2F15.8)

7 READ(1,5,END=6) S
WRITE(2, 1000) S

5 FORMAT(F0.0)
R=R*S/2/(1.0-S**2)
WRITE(2, 1000) R
WRITE(2, 1000) E, AM, AS, A
A=AN/(1.0+R*Y0)
POW=((CABS(A))**2)*R
POW=POW-2.6
OUT=POW-2.6
E=A*R
CUR=Y1M+E
AM=AM+CUR
CUR=J*Y1S+E
AS=AS+CUR
ALIN=OM-AS
PUT=REAL(ALIN)*V
CP=REAL(ALIN)/(CABS(ALIN))
EFF=OUT/PUT
YI=Y1M*V-YOM*F
XI1=AOS-YES*F
SI=AES-YI5*E
T=PO+2.0*W/(1.0+S)*CABS(XI1)*CABS(YI)*COS(ATAN(AIMAG(XI1))/REAL
1(XI1))-ATAN(AIMAG(YI)/REAL(YI))) -2.6
100 FORMAT(50X,5HSLIP=,F15.10)
WRITE(2,1001) OUT
1001 FORMAT(1X,15HOUTPUTPOWER=,F15.8/
WRITE(2,1002) ALIN
1002 FORMAT(1X,15HLINECURRENT=,2F15.8/
WRITE(2,1003) PUT
1003 FORMAT(1X,15HINPUTPOWER=,F15.8/
WRITE(2,1004) COP
1004 FORMAT(1X,15HPOWERFACTOR=,F15.8/
WRITE(2,1005) EFF
1005 FORMAT(1X,15HEFFICIENCY=,F15.8/
WRITE(2,1006) T
1006 FORMAT(1X,15HTORQUE=,F15.8/
WRITE(2,1007) YI
1007 FORMAT(1X,15HCURRENT1=,2F15.8/
WRITE(2,1008) XI1
1008 FORMAT(1X,15HROTORCURRENT2=,2F15.8/
WRITE(2,1009) SI
1009 FORMAT(1X,20HCAPACITORCURRENT=,2F15.8/
GO TO 7
6 STOP
END