Minimisation of inverter-fed induction-motor losses by optimisation of PWM voltage waveforms

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MINIMISATION OF INVERTER-FED INDUCTION-MOTOR
LOSSES BY OPTIMISATION OF PWM VOLTAGE WAVEFORMS

BY

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Thesis submitted in partial fulfilment
of the requirements for the award of the
degree of Ph.D. of the
Loughborough University of Technology

October, 1977

Supervisor: G. K. CREIGHTON Ph.D.

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SUMMARY

This thesis describes a method of minimising the total losses of an inverter-fed 3-phase squirrel-cage induction motor when the motor is subjected to a pulse-width modulated (PWM) voltage waveform. The inverter is supplied from a d.c. link and operates at variable frequency to provide speed control of the motor. Appropriate triggering of the inverter's six main thyristors generates pulse-width modulated voltage waveforms for application to the induction motor.

The operation of an induction motor with nonsinusoidal voltage applied results in a reduction of the motor's efficiency due to the harmonics present in the waveform. The aim of the project is to minimise the total losses by obtaining optimum PWM voltage waveforms rather than by improving the design of the motor. This requires a thorough examination of motor losses.

The determination of a PWM voltage waveform which may be produced by the described inverter is subjected to constraints which characterise the operation of the drive system. The motor operates with constant airgap flux density throughout its speed range to obtain maximum output power at fixed per unit slip. In addition the switching frequency of the thyristors must not exceed a specified limit to avoid short circuiting of the inverter.

The calculation of the motor's steady-state performances for both sine wave and PWM supplies is incorporated in a computer program. The detail of the experimental and theoretical
performances are given and comparison is made between sinusoidal and PWM voltage wave systems. Good agreement is obtained between test and calculated results on an inverter-fed 7.5 kW squirrel-cage induction motor.

It is concluded that the degradation of motor efficiency due to the applied PWM voltage waves is mainly the result of increased copper losses, which are produced by harmonic currents. The minimisation of the losses for continuous, constant-flux, operation of the induction motor is achieved for the given constraints. It is found that the total losses can be further minimised if the d.c. link voltage is variable. This permits improved motor performance but adds complexity and cost to the d.c. link voltage supply.
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LIST OF SYMBOLS

Capital Letters

$A_B$ = Area of rotor bar
$A_{er}$ = Area of end-ring
$A_n$ = Fourier coefficient for cosine terms
$A_0$ = Constant term of Fourier series
$B_n$ = Fourier coefficient for sine terms
$B_g$ = Flux density in air-gap
$B_s$ = Flux density in stator core
$B_{st}$ = Flux density in stator tooth
$B_{rt}$ = Flux density in rotor tooth
$B_r$ = Flux density in rotor core
$C_{ss}$ = Surface loss coefficient of stator
$C_{sr}$ = Surface loss coefficient of rotor
$D$ = Diameter of air-gap
$D_a$ = Average diameter of end-ring
$E$ = Induced voltage
$F(t)$ = Fourier expansion
$F$ = Ampere-turns
$HP$ = Amplitude of harmonic voltage expressed as a percentage of the fundamental
$I_{s}, I_r$ = R.M.S. values of stator and rotor phase currents, respectively.
$I_n$ = $n$th order harmonic current
$I_{o}, I_{sh}$ = No-load and blocked-rotor test currents of induction motor, per phase, respectively.
$I_g$ = Load current of d.c. generator
$K$ = Stray loss factor in terms of rotor resistance
$K'$ = Iron-loss coefficient
$L_b$ = Length of rotor bar
$L'$ = Axial length of stator slot
\( L'_a \) = Axial gross-length of stator
\( L'_n \) = Axial net-length of stator
\( L_s, L_r \) = Leakage inductances of stator and rotor, respectively.
\( L_{ss}, L_{rr} \) = Self inductances of stator and rotor, per phase, respectively.
\( L_{ssm}, L_{rrm} \) = Mutual inductances between stator phases and rotor phases, respectively.
\( L_{app} \) = Apparent self inductance
\( N \) = Number of series connected turns per phase
\( N_{PWM} \) = Modulation number of PWM voltage waveform
\( P_i, P_{ig} \) = Power input of induction motor and generator, respectively.
\( P_o \) = No-load power input of induction motor
\( P_{og} \) = No-load loss of d.c. generator
\( P_{op} \) = Power output of the induction motor
\( P_{sh} \) = Power input of induction motor at block-rotor test
\( P_w, P_{wa} \) = Pulse-width in milliseconds and degrees, respectively.
\( Q_s, Q_r \) = Number of stator and rotor slots of induction motor, respectively.
\( R_s, R_r \) = Stator and rotor resistances per phase, respectively.
\( R_a \) = Armature resistance of d.c. generator
\( R_m \) = Fictitious resistance that represents iron losses in the induction machine
\( T_{sp} \) = Half of the space between two consecutive pulses, (ms)
\( T_{spm} \) = Minimum value of \( T_{sp} \)
\( T \) = Period of voltage waveform, (ms)
\( T_{on}, T_{off} \) = Turn-on and turn-off times of a thyristor (ms)
\( T_q \) = Torque on the shaft of induction motor
\( V_{AB} \) = Line to line voltage between phases A and B of inverter
\( V_{A-0}, V_{B-0} \) = Phase voltages of inverter with respect to reference point
\( V_1, V_n \) = Magnitudes of fundamental and nth harmonic voltages
\( V_{dc} \) = Mean value of d.c. link voltage
Maximum values of reference and carrier voltage waves.

Output voltage of generator.

Volume of iron (cm$^3$).

Weighting factor for PWM waveforms.

Copper losses.

End winding losses.

Losses due to high frequency pulsations on rotor.

Skew losses.

Surface losses on stator and rotor, respectively.

Friction and windage loss.

Iron losses.

Total losses for two different PWM waveforms.

Leakage reactances of stator and rotor of induction motor.

Magnetising reactance of induction motor.

Skew leakage reactance.

Stator and rotor differential leakage reactance, respectively.

Stator and rotor slot leakage reactance, respectively.

Impedance.

Equivalent impedance of induction motor's nth harmonic.

Number of parallel paths of generator's armature winding.

Width tooth.

Width of stator tooth calculated by Simpson's rule.

Width of rotor tooth calculated by Simpson's rule.

Order of space harmonic.

Frequency of voltage waveform.

Frequency in the stator.

Frequency in the rotor.
\( f' \) = Slot frequency

\( f(t) \) = Nonsinusoidal periodic function

\( g \) = Length of air-gap

\( h \) = Height of slot

\( k_{d}, k_{p}, k_{w} \) = Distribution, pitch and winding factors of stator winding

\( k_{sk} \) = Skew factor

\( k_{dfs}'k_{dfr} \) = Damping factor of stator and rotor, respectively.

\( k_{r} \) = Skin effect factor for resistance

\( k_{z} \) = Skin effect factor for reactance

\( m \) = Number of phases

\( m_{f} \) = Slope of linear magnetisation characteristic

\( n \) = Order of time harmonics

\( n_{s} \) = Synchronous speed of induction motor (rev/min)

\( n_{r} \) = Speed of rotor rotation (rev/min)

\( n_{br} \) = Number of brushes of d.c. generator

\( p \) = Pairs of poles

\( p' \) = Ratio of coil-span to pole-pitch

\( q \) = Number of phase bands per pole

\( s \) = Slip

\( s_{s} \) = Number of slots per phase belt

\( t \) = Time

\( u \) = Turn ratio of induction motor

\( v \) = Temperature °C

\( Y_{a}', Y_{b}', Y_{c} \) = Turns distribution of phases A, B and C, respectively.

Greek Letters

\( \alpha \) = Increment of pulse-width on the right

\( \alpha \) = Increment of pulse-width on the left

\( \alpha_{s} \) = Skew angle in stator slot-pitch

\( \alpha_{b} \) = Phase belt in electrical angles
\( \beta_x \) = Increment of pulse-width on the right
\( \beta_y \) = Increment of pulse-width on the left
\( \beta_f \) = Loss factor of material
\( \beta_w \) = Loss ratio of minimisation
\( \delta \) = Classical depth of penetration
\( \delta' \) = Lamination thickness
\( \phi \) = Flux per pole
\( \phi_s \) = Phase difference between stator and rotor
\( \theta \) = Angle
\( \Omega \) = Mechanical angular synchronous speed
\( \lambda \) = Permeance
\( \gamma_c \) = Chording angle
\( \gamma_r \) = Phase difference between rotor bars
\( \psi \) = Flux linkage
\( \Lambda_s, \Lambda_r \) = Stator and rotor slot pitch
\( \omega \) = Angular frequency
\( \omega_s \) = Synchronous angular speed
\( \mu_0 \) = Permeability of free space
\( \sigma \) = Ratio of skew to one stator slot pitch
\( \delta' \) = Density of material
\( \Delta V_b \) = Voltage drop at each brush
CHAPTER I

INTRODUCTION
1. **GENERAL**

The speed of an induction motor can be varied smoothly from standstill to full speed in either the forward or reverse directions by control of the supply frequency. The supply frequency control is achieved by many methods that can be found in the literature, (1, 2, 3, 4, 5, 6, 7). It has been found that the speed control of an induction motor by applying a sinusoidal voltage waveform whose frequency can be varied is an expensive method due to equipment required. The overall efficiency of the drive system is poor due to use of more than two machines. Therefore, static frequency changers, that supply variable frequency nonsinusoidal voltage waveforms with higher efficiency than that of the machines, have been brought to use recently for the speed control of an induction motor. Since the sinusoidal voltage waveform with variable frequency is not obtainable from the static frequency changer, research is directed towards the calculation of an induction motor's total losses when nonsinusoidal voltage waveforms are in use, (8, 9, 10, 11, 12, 13). These authors, who have calculated the performance of the induction motor using Alger's method (14, 15, 16), have found that the losses are increased and the efficiency therefore reduced due to effects of harmonic voltages embodied in nonsinusoidal voltage waveform.

An inverter is a static frequency changer that generates alternating voltage at a required frequency from a direct voltage. A three-phase inverter comprises six main electric valves which can be thyristors or transistors. The choice is made according to the amount of power required. Thyristors are used for large power
installations for which transistors are unsuitable. The maximum ratings of thyristors for inverter use at present are given as 940 amperes r.m.s. and 1500 volts r.m.s., while for transistors 30 amperes r.m.s. and 700 volts r.m.s., (17, 18). Since the input voltage of the inverter has constant voltage magnitude, the output voltage waveform of an inverter can be obtained either in a stepped waveform or a train of pulses of fixed magnitude. The nonsinusoidal output voltage waveforms of inverters have been examined by various authors. The paper published by C.W. Flairty, (1), shows that if four separate 3-phase inverters are employed the elimination of 5th, 7th, 11th and 13th harmonics is possible. In reference (19), a method is introduced for the cancellation of a voltage harmonic component by shifting the phase of gate signals of thyristors which belong to two different phases in inverter. This inverter cannot generate a pulse-width modulated, (PWM), voltage waveform and comprises twelve thyristors. The principles of PWM voltage waveform generated by stepped envelope and sine envelope modulation techniques are discussed in reference (5) where a method employing a transformer enables 5th and 7th harmonics to be cancelled.

The voltage waveforms that can be produced by an inverter may be examined in two groups. The first group comprises waveforms with fixed pulse-duration irrespective of the period. That is, the conduction time of the thyristor is fixed. Fundamental voltage control of this kind of waveform is not possible unless the frequency of the waveform or the d.c. link voltage magnitude is varied. The second group comprises voltage waveforms whose pulse-duration can be varied by changing the conduction time of the
thyristors. For this reason the magnitude of the output voltage waveform can be varied independently of the frequency of voltage waveform, whereas the output voltage magnitude obtained by former method is related to the frequency. The PWM voltage waveforms provide wider voltage control than that of former group. The other advantage of PWM is to introduce more freedom for alteration of the harmonic content of the nonsinusoidal voltage waveform by variation of pulse-duration. This speciality of the waveform makes the PWM unrivalled at low frequencies where the other waveforms obtained by the former method have increased harmonic content, (5, 20).

The operation of an induction motor with nonsinusoidal voltage waveform differs from that with sinusoidal voltage waveform. The harmonic voltages cause harmonic currents to flow in the induction motor at their own frequencies which are at integral multiples of the fundamental frequency. These harmonic currents cause additional copper losses in the stator and rotor windings. Copper losses are proportional to winding resistance which changes with the frequency of currents, due to skin effect, (7, 14). The skin effect is an important phenomenon for squirrel-cage motors which have deep rotor bars. The deeper the bar, the stronger the skin effect obtained in the rotor. This results in a large resistance and, consequently, copper losses for fixed current. The remainder of the total losses is yielded by friction and windage, stray losses and iron losses. Although, the stray losses are a very small part of the total losses in a sinusoidal voltage waveform application, they grow considerably large for nonsinusoidal
voltage applications. Iron losses are also affected by harmonics which increase eddy-current losses depending upon the magnitudes of the harmonic voltages that are present in the applied nonsinusoidal voltage waveform. The additional iron losses are caused by harmonic fluxes of corresponding harmonic voltages.

The common assumptions made by all mentioned authors are the neglect of space harmonics and the saturation of the motor. Also the induction motor is assumed to have balanced stator windings and to be excited by an inverter which can generate PWM voltage waveforms independent of load. The deep-bar effect of the rotor bars and stray losses are neglected in reference (8). In reference (12) G.B. Kliman neglects the stray and core losses and calculates the motor performance by superposing the harmonic effects. References (9), (10) and (11) show that the calculation of induction motor losses has been extended step by step. In reference (10) the effects of space harmonics are considered and the importances of some minor losses are shown experimentally and analytically, whereas these are not considered in reference (9). Reference (11) shows the calculation of stray losses of an induction motor with a six-stepped voltage waveform over a wide frequency range, and describes the induction motor operation by control of the d.c. link voltage while the harmonic content of the waveform remains fixed for the whole speed range. The equivalent circuits and parameter determination for space harmonics are given in reference (21) together with an explanation of space harmonic losses and their deteriorating effects on the torque of the induction machine.
In this thesis, PWM voltage waveforms are examined with the view to minimizing their harmonic content for given constraints, (chapter 3, section 1). All mentioned authors have achieved the cancellation of harmonics by employing additional equipment (inverters, transformers), besides the main inverter and increased the cost of the drive system. In order to avoid additional equipment and to determine waveforms with low harmonic contents, the voltage waveforms generated by the inverter are analysed in time and angle domains. The examinations show that the use of angle domain is restricted to inverters with variable d.c. link voltage, since the voltage waveform is fixed over the frequency range. The PWM waveforms with even numbers and odd numbers of pulses are examined separately in the time domain for various frequency ranges until their harmonic contents exceed the given limits. The time domain is used for the inverters which are supplied with a fixed d.c. link voltage. The method which shows the determination of a PWM waveform is given when two of its harmonics are specified as a percentage of the fundamental. Since the variation of fundamental voltage with frequency is a nonlinear function for PWM waveforms, another method is given to determine two frequencies between which the slope of the fundamental harmonic voltage is approximately equal to that of a linear function that passes through these points. The voltage waveforms determined by harmonic analysis are used to form a route between two frequencies which correspond to the induction motor's maximum and minimum speeds. The steady-state operation of induction motor by these voltage waveforms is accomplished with maximum efficiency for PWM voltage waveform application.
On the induction motor side, effects of major time harmonics and their major space harmonics are considered with the assumptions that the induction motor has a balanced three-phase stator winding and rotor bars well insulated from the iron. It is assumed that the output voltage of the inverter is not affected by the inductive load (induction motor), that the friction and windage of the motor are not affected by the nonsinusoidal voltage waveform. Further, it is assumed that for both sinusoidal and PWM voltage waveforms applications the shape of the hysteresis loop of the magnetic core is unaffected by the frequency.

The total losses of a squirrel-cage induction motor for PWM voltage waveform applications are calculated and compared with experimental results. The comparisons show that the theory for prediction of harmonic losses can be used with confidence. The calculation of the motor's steady-state performance and the total losses for PWM voltage waveform supplies are incorporated in a computer program. The method of calculation of total losses is related with nature of PWM voltage waveforms so that the total harmonic losses are determined in terms of pulse-duration. This method is further developed to include the PWM waveforms with variable voltage amplitude. The minimisation of motor losses is obtained by considering the copper and stray losses as the main part of the total additional losses caused by harmonics. It is also shown that, in this case, the harmonic losses are further reduced by 65% with respect to losses caused by the best waveform that is produced by an inverter with fixed voltage amplitude. Both procedures are incorporated in separate computer programs.
2. **THE OBJECT OF THE INVESTIGATION**

The main object of the project is to ensure variable speed operation of the inverter-fed induction motor with minimised total losses between its maximum and minimum speeds for constant air-gap flux. There are two methods available to improve this efficiency: the first is from an improvement of the induction motor design; the second is from an optimisation of the applied voltage waveform's harmonic content. The latter method is considered here. The minimisation of the total losses in variable speed operation of an induction machine requires a method of reducing the harmonic effects by decreasing their magnitude relative to that of the fundamental. This objective can be accomplished either by elimination or optimisation of harmonics of the nonsinusoidal voltage waveforms. The voltage waveforms must be generated by an inverter which has six main thyristors.

3. **MATERIALS OF THE INVESTIGATION**

A three-phase, diode bridge-connected rectifier supplied from an a.c. source was used together with a thyristorised bridge-connected inverter. The 50 kVA inverter supplied a 7.5 kW (10 HP) three-phase squirrel-cage induction motor coupled to 15 BHP separately excited d.c. generator. All equipment used in the experiments was supplied by Brush Electrical Engineering Company of Loughborough. Measured values of induction motor input and d.c. generator output powers, speed of the drive system and other quantities are given in tables from 5A to 14B of chapter 8.
4. **METHOD OF APPROACH**

The method of speed control, by changing supply frequency, provides the use of torque-slip characteristic of the motor, which corresponds to the rated voltage and rated frequency, at any applied frequency where the voltage to frequency ratio is equal to that of rated voltage and rated frequency. The minimisation of total losses of an induction motor by optimisation of PWM voltage waveform requires a thorough examination of losses. The induction motor is assumed to operate outside the saturation area so that the superposition theorem can be used for calculations. The total losses of the induction motor when the motor is supplied by a PWM voltage waveform are calculated and compared with test results. The differences between the calculations and tests show the accuracy of the method. The experiments are performed for both sinusoidal and PWM waveforms. The constant air-gap flux operation is obtained, by neglecting the voltage drop due to impedance of the stator winding, at frequencies above 10 Hz where voltage to frequency ratio can be obtained at the specified value. The frequencies below 10 Hz do not satisfy the given voltage to frequency ratio. The magnitude of the voltage for frequencies below 10 Hz is higher than that obtained by V/f ratio of higher frequencies. The reason is that the flux produced at low frequency by low voltage is not enough to induce large currents for the development of rated torque. The voltage drop in the stator increases as the motor draws more current. In order to compensate the voltage drop the magnitude of the applied voltage is to be higher than that obtained for frequencies above 10 Hz with constant V/f.
To obtain the total losses of the induction motor, those due to the fundamental voltage harmonic are calculated and added to the harmonic losses. To check the validity of the theory, predicted values of total losses are compared with those obtained by experiment. The pulse-duration consistent with minimum harmonic content of the copper and stray losses caused by harmonic currents. PWM voltage waveforms of various modulation numbers from 1 to 8 are examined for their harmonic contents and their applications to wide frequency range. While constructing the voltage waveforms the number of PWM voltage waveforms has been kept to a minimum to avoid complexity in the control circuitry of the inverter. Furthermore a high number of voltage waveforms causes torque discontinuities in the motor. This arises because the fundamental voltage magnitude of two waveforms with different pulse durations cannot have equal magnitudes at a frequency, and the load current of the inverter possesses step changes as the voltage waveform is changed. The variation of harmonic losses with pulse-width and frequency is presented to find the optimum waveform.
CHAPTER II

PWM VOLTAGE WAVEFORMS
1. **GENERATION OF PWM WAVEFORMS**

PWM waveforms are generated by an inverter which accomplishes the reverse process of rectification. The thyristors of the inverter must be able to hold off forward voltage and the cyclic conducting period of each thyristor must be controllable.

Most inverters require a means for controlling the output voltage. This control may be required because of variations in inverter source voltage or to provide a stepless adjustment of inverter output voltage. The variation of thyristor's conduction time provides the voltage control. The operation of a single-phase inverter will be first examined as it leads to an easy understanding of a polyphase inverter. In most cases the number of phases of an inverter is three, unless it is used for harmonic cancellation as in reference (1) where 24 phases are used to eliminate four harmonics.

In figure 1 a single-phase inverter is represented with thyristors and their feed-back diodes. Feedback diodes provide an alternative route for currents going into the lead when a thyristor, that is in conduction, is turned off by means of a commutation circuit which is not shown in figure 1. The sudden turn-off of a thyristor by reversed bias causes the current to decrease rapidly. This could result in large negative voltages across an inductive load but is avoided by using feed-back diodes that carry current from the opposite polarity to the load. Generally, the operation of the system is such that the polarity of the voltage across the load is reversed cyclicly by allowing the conduction of thyristors two at a time which are either T1-T4 or T3-T2. The simultaneous conductions of thyristors on one phase, such as T1 and T2, are not permitted to happen, since it causes short circuiting of the d.c. supply due to very low internal resistance of thyristors. If the thyristors T1
and T4 are considered in conduction for the duration of one half of a period which is 180 degrees, the other thyristors must provide the remainder of the waveform with reversed polarity, as shown in figure 2. Suppose an inductive load is connected between points A and B and the thyristors T1 and T4 are conducting. The voltages at points A and B are shown in figure 2. The current flows from A to B. If T2 is turned on after T1 is turned off, the current due to stored energy in the load flows through D2 until T2 is turned on to reverse the polarity of load voltage. When T4 is turned off in addition to T1 the stored energy flows through D2 and D3 and charges capacitor C.

A different type of voltage can be obtained across the load if the phase shift between the gating signals of T1, T4 and T2, T3 is changed from the value that is given in figure 2, which is 180°, to a new one which is retarded by an angle $\beta_p$. Thus, the voltage waveform shown on figure 3 is obtained across the load. This process provides stepless voltage control of the inverter output waveform.

$L_1$, the coil, reduces any current surge that may tend to occur as the thyristors are turning on and off. This is necessary to provide time for the thyristors to recover their forward blocking characteristics, reference (26).

The operation of a 3-phase inverter is similar to that of a single-phase inverter. It can be considered as three identical single-phase inverters supplying voltages with 120 degrees phase shift to a load. In this case the polyphase inverters provide the opportunity to minimise the harmonics in the output voltage waveform. Generally, since three-phase output is required from most high power inverters, the third harmonic and its multiples are not present in the output waveform. Further harmonic cancellation is achieved in the inverter.
output-waveform when a large number of phases is used in the inverter output as in reference (1).

2. TYPES OF PFM WAVEFORMS

2.1 General
The choice of a particular method or mode of pulse-width modulation lies on the requirements of the application. It involves the balancing of losses in the inverter incurred in using high switching frequency against the improved performance and reduced losses in the motor. The cost of the inverter could be a problem for a waveform which is complex. The elimination of harmonics in PWM waveforms can be achieved depending upon the design of the inverter's control circuitry. This circuitry becomes more costly as the requirements increase. Generally, a pulse-width modulated waveform is obtainable by two different methods. The method that will be discussed first is called modulation with a sine wave envelope, while the other is a simple pulse-width modulation technique and is called stepped envelope modulation.

2.2 Modulation with Sine Wave Envelope
The PWM waveform is generated from a combination of sine and triangular waveforms. The method of construction is shown in figure 4, where the PWM voltage waveform is formed by the interaction of these two waveforms. The result is a train of pulses. The output voltage waves are shifted 120° with respect to one another for three-phase inverter operation. The triangular waveform is called the carrier with its frequency fixed. The reference waveform has a variable frequency which determines the frequency of the PWM waveform. The amplitude of the reference wave is also variable and its frequency may be varied independently. The variation of the
amplitude of the reference wave provides voltage control for PWM inverters.

The principle of PWM waveform generation is shown in figure 4 for one whole cycle. The lower the frequency of the reference waveform, the greater the number of pulses produced for a fixed frequency of the carrier wave. The pulses of the output waveform are reduced in width as the frequency of the reference wave is decreased, while the amplitude of the reference wave is unchanged, reference (2). This process makes the harmonics smaller. For the opposite case in which the frequency of the reference wave is near to that of carrier wave, the pulse widths become larger. The harmonic content of the output voltage waveform causes difficulties for motor operation due to high amplitudes. Therefore, it is better choice for this kind of modulation to work at low output frequencies of the inverter. The peak fundamental voltage $V_1$ whose frequency is the same as the output frequency of the inverter for sine wave envelope modulation is given in terms of the mean d.c. link voltage, reference amplitude voltage and carrier amplitude as follows,

$$V_1 = \frac{V_{dc}}{2V_{CW}} \frac{V_{RW}}{2V_{CW}}$$

(1)

Where $V_{dc}$, $V_{RW}$, $V_{CW}$ are respectively, mean d.c. link voltage, peak to peak values of reference and carrier wave voltages, reference (3).

The ratio of reference wave voltage to carrier wave voltage cannot be greater than unity for the conservation of modulation. Choosing the ratio as unity the fundamental voltage becomes:

$$V_1 = \frac{V_{dc}}{2}$$

(2)

while the fundamental voltage of stepped envelope modulation is

$$V_1 = \frac{2}{\pi} V_{dc}$$

(3)
Theoretically, the ratio between the frequencies of carrier and reference waves is to be larger than 9 with the condition that $\frac{V_{RW}}{V_{CW}}$ is between zero and one to obtain maximum fundamental voltage amplitude, reference (3).

Finally, another drawback of this method is the lack of phase relationship between reference and carrier waves. As a result of this the pulse pattern fails to repeat itself identically from cycle to cycle, but small changes continually occur at a rate related to the difference between the carrier waveform frequency and some multiples of the reference frequency. This operation is called free running. The high number of pulses per output cycle and low magnitude of output fundamental has no detrimental effect on motor operation. For higher frequencies the system becomes more sensitive due to the small number of pulses and large pulse-widths. This causes low rate fluctuations in the machine flux, current, torque and speed beats.

When the amplitude of the reference waveform exceeds a given critical percentage of the carrier waveform peak level, minimum conduction time is ensured. The minimum conduction time causes two drawbacks. First, the output level ceases to be a linear function of the reference level, which results in even lower maximum output than equation (2). Secondly, the most important consequence is that the free-running pulse pattern becomes disturbed in correspondence with the crests of the reference. As a result of these, to increase the amplitude of reference wave over a given percentage of carrier voltage peak level is avoided, reference (3).
2.3 Stepped Envelope Modulation

This method of pulse-width modulation is simpler and yields less drawbacks compared to the one discussed in the previous section. The generation of PWM voltage waveforms does not require the use of carrier and reference waveforms. The principle lies in the arrangement of the conduction times of the thyristors. Once this is determined all possible waveforms that are obtainable by this method can be generated.

The firing order of the thyristors is very important to avoid short circuit. It will be assumed that the thyristors that belong to separate phases in the inverter conduct for one half of a cycle. That means that one of the two thyristors in one phase conducts while the other is turned off. This kind of operation results in voltages shown in figure 2. If the voltage of the phase B shifted by $\beta_p$ then the line to line voltage $V_{AB}$ is obtained as shown in figure 3. But the harmonic content of such a waveform is high and is unsuitable for motor operation.

Different conduction times result in different line to line voltages. The total conduction time is to be 180 degrees of a cycle. This length of conduction time can be completed with or without the pause of the conducting thyristor. For example the thyristors, T1 and T4, of figure 5 complete a whole cycle of voltage by conducting 180 degrees each.

The waveforms used in the project are made up by a series of pulses of different widths unequally spaced from each other to minimise the harmonic content. The gating signal patterns of the thyristors for the generation of PWM line voltages can be determined by the method explained in section 2.32 of this chapter. In pulse-width
modulation the important feature is to lower the harmonic content as much as possible by arranging the pulses in such a way that the waveform should approach a sine wave by having wider pulses in the middle of each half cycle. Meanwhile, it is necessary to avoid short-circuiting the bridge. The three-phase inverter bridge given in figure 5 has the following firing sequence for its six thyristors that are marked as T1, T2, T3, T4, T5, T6.

- Phase A: T1 T1 T1 T4 T4 T4
- Phase B: T6 T6 T3 T3 T3 T6
- Phase C: T5 T2 T2 T2 T5 T5

The duration of conduction of the thyristors is set such that the sum of voltages, at any instant, on three phases, must equal zero, otherwise short-circuiting is caused by conduction of two thyristors in a phase at the same time. If no voltage drop occurs in the inverter, the amplitudes of the pulses equal the d.c. link voltage. The line to line voltages are determined such that two of the three phases have voltages of opposite polarity, while the third has zero voltage.

In addition to these requirements there are two more conditions to be satisfied which are related with the pulse-width to avoid misfiring, overlapping and as a result short circuiting.

A. Waveforms made up by odd number of pulses, such as 3, have a middle pulse that is wider than the others. The duration of middle pulse-width must be equal to sum of the other pulse-widths. This will be further explained in section 5. To have good operation of the thyristor bridge, semiconductors must conduct in such a way that the space between the two pulses of line voltage must coincide with the pulse of second line voltage, whilst the third line voltage has a pulse covering all this region.
B. The waveforms made up by an even number of pulses, such as 2, 4, 6, 8 are examined in two groups.

B1. Pulse-widths are equal to each other. Each pulse must be followed by a space which must not be shorter than the time required for a thyristor to be turned off and on.

B2. Pulse-widths are not equal to one another. In both cases the summation of the three line to line voltages at any instant of time must be zero.

2.3.1 The Method for Determining the Gating Sequence of Thyristors in One Phase Leg

Since the harmonic content of a nonsinusoidal periodic waveform is determined with Fourier series, the conduction sequence and conduction time of the thyristors on all phases are very important in order to obtain the line to line voltage waveform that is required for minimum harmonic content. Initially, the line to line voltage is examined for lowest harmonic content. Then, the conduction sequence and conduction time of the thyristors are found by the process described in this section. Therefore, it is necessary to give explanations here, before proceeding to other sections. Assuming that 2 pulse-width modulated waveform is given as line to line voltage to be impressed across the motor terminals, the phase voltages of the inverter must be such that the subtraction of these voltages gives line to line voltage. The amplitude of line to line voltage is twice the voltage level of phase to reference point.

\[ V_{A-B} = V_{A-0} - V_{B-0} \quad (4) \]

For a given waveform of \( V_{AB} \), in figure 6, \( V_{A-0} \) and \( V_{B-0} \) are obtained as follows. If \( V_{AB} \) is positive, \( V_{A-0} \) and \( V_{B-0} \) are drawn on positive and negative sides of the zero voltage axis, respectively.
If \( V_{AB} \) is negative, \( V_{A-0} \) and \( V_{B-0} \) are drawn, respectively, on the negative and the positive sides of the zero voltage axis. If the remaining part of the voltage waveform has zero voltage then \( V_{A-0} \) and \( V_{B-0} \) must have the same polarity to cancel each other when they are subtracted. But the remaining process is to be performed such that both thyristors must conduct equal length of time and the waveform must be periodic. In figure 6 the dotted lines represent the above mentioned description.

Similarly conduction times of any number of pulse-width modulated waveforms can be obtained, irrespective of the position of the pulses with respect to each other.

2.3.2 Three-Phase System

In the previous section it was shown that the gating sequence of thyristors in one phase of a 3-phase inverter can be determined if the voltage waveform across the load is provided. The same principle is used to find the gating sequences of thyristors in a 3-phase inverter by introducing 120 degree phase shift between three gating signal patterns. Figure 7 represents the given line to line voltage with two pulses in one half of a period with the gating sequences of inverter's thyristors. As it is seen the gating signal of a thyristor in a phase does not last continuously for 180 degrees, but in three different steps. The sum of these steps is equal to 180 degrees. The aim is to determine the conduction pattern and duration of gating signals of thyristors.

The intervals corresponding to pulses of the line voltages are marked by \( t_1, t_2, t_3, t_4, t_5, t_6 \) milliseconds, figure 7. In these intervals, only one of three line to line voltages does not have a pulse. The sign of voltage amplitude of a line voltage shows that
at interval $\tau_1$ which thyristor of what phase is gated. In this case, two of the gated thyristors, 1, 6, of two phases are found by referring to the signs of the pulses and the polarities of the bus bars to which the thyristors are connected. The remaining thyristor must be gated such that line voltage $V_{CA}$ is to have no pulse as the algebraic sum of other line voltages is zero. To determine which thyristor of phase C is to be gated, the figure 8 is used, where the letters of three phase signals are shown on a circle. The arrow shows the positive direction of the rotation of the line voltages. The gated thyristor of phase C is found as follows: since $V_{CA}$ is followed by $V_{AB}$ in figure 8, which has positive pulse at interval $\tau_1$, the gated thyristor is to be $T_5$ which is connected to positive bus bar, otherwise the gating pattern of phase C becomes different from those of phase A and phase B.

Example: The polarity of the gating signal for one of two thyristors of phase B is required together with the thyristor number at interval $\tau_5$. In figure 7a line voltage $V_{BC} = 0$. In figure 8, $V_{BC}$ is followed by $V_{CA}$ which has a positive pulse in figure 7a. Hence, the polarity of the gating signal is positive for this interval as shown by the wavy line for distinction. Therefore, the thyristor number is 3 since $V_{CA}$ has a positive pulse.

3. INTRODUCTION OF ANGLES IN WAVEFORMS

3.1 General

As mentioned before the PWM waveform can have low harmonic content, if it approaches a sine wave. It is easy for sine envelope methods, since the pulses gradually become wider as the time progresses to the quarter of the period. But the harmonic content of the waveform cannot be improved for stepped envelope modulation technique unless
some changes are made on the duration of pulses. Changes that are
introduced to improve the waveforms depend upon the number of pulses
and are called angle variations. The introduction of angles must be
made without changing the cyclic variation of the waveform.

The balance of the voltages of three phases being the major drawback,
any change in any pulse-width needs essential adjustments of conduction
of other thyristors. Therefore, it is impossible for a waveform to have
gradually increasing pulse-width in a three phase system. The reason
for this is that each pulse-width of a phase depends upon the places
and widths of the other pulses in different phases. The better use
of angle variation is obtained if the system is single phase. The
pulses are independent of non-existent pulses of other two phases. It
is, therefore, easy to obtain any waveform required. In fact, there is
a possibility of creating a required waveform as in single phase
inverter, that is to employ auxiliary thyristors which are not to be
accounted for in this project, reference (20).

The variation of angle is arranged in different forms, depending
on the reference point chosen on the waveform. The reference point
is normally selected on the pulse which it is intended to vary. The
maximum of two angle variations can be used for a pulse, since the
width is changed either from the left or the right of the pulse. The
enlargement of the pulse is the replacement of space by pulse, so
that the width of the pulse increases by the amount it is made.
Opposite to this case is the replacement of pulse-width by space,
which results in a narrower pulse than before. The polarity of
increment of pulse-width is a matter of choice. If it is chosen to
be positive, then the counter case corresponding to reduction of
pulse-width in terms of angle is negative. The figure 9 represents
a pulse-width with its associate angles in degrees.
By using different angles it is possible to obtain plenty of different waveforms out of one and analyse them for harmonic content whether they are suitable for the given limits or not. The use of two different angles on a pulse avoids confusion when the pulses of 3-phase system are changed for high modulation numbers.

3.2 Choice of Axes for Waveforms

It is thought to place this section in the thesis to avoid confusion when a given waveform is to be represented in different axes. The voltage waveforms are represented on three different reference axes.

(i) For angles, from 0 degree to 360 degrees, (ii) For radians, from 0 to $2\pi$ radians, (iii) Waveforms given in millisecond, from 0 millisecond to $T$ milliseconds, where $T$ represents the period of the waveform.

If a waveform is represented on the time axis and its pulse-width is of constant duration, the angle value of the pulse-width changes with respect to whole cycle as the frequency is varied.

The use of axes for radians and degrees does not make any difference but the radian axis mentioned above makes the points to be worked out easier for computer programming.

The example given in figure 10 will help in understanding the nature of this section easily. Let us suppose that a 2 pulse-width modulated waveform with its pulses each of 1 ms duration is given at 40 Hz. It is required to represent the waveform at 80 Hz; (i) maintaining pulse-width at 1 millisecond, (ii) maintaining the pulse-width in terms of angle as it is at 40 Hz.

Figure 10a shows the 2 pulse-width modulated waveform at 40 Hz, pulse-width = 1 ms, in which the first pulse begins at 52.8 degree.
In figure 10b pulse-width maintained constant at 1 ms and frequency increased to 80 Hz. The position of the pulses with respect to one another is still the same. By doing this, the first pulse starts at 45.6 degrees because 1 ms pulse-width covers $\frac{1}{12.5}$ of all period while it was $\frac{1}{25}$ in figure 10a. Thus, a different waveform is obtained. In order to keep the waveform as it is in figure 10a at $\frac{1}{25}$ of period the pulse-width is reduced to 0.5 ms which gives the ratio of $0.5/12.5$ which is, in fact, equal to $\frac{1}{25}$.

It is now quite clear that a waveform can be identified as unchanged with varying frequency if it is given only on angle axis. The harmonic content of a waveform given on the time axis varies with frequency. Therefore, if the harmonic content of a waveform, at a frequency is required with respect to the fundamental magnitude, the waveform must have that frequency as fundamental frequency.

4. WAVEFORMS WITH EVEN NUMBER OF PULSES

The waveforms that have even number of pulses in one half a cycle are called by the number which is the order of modulation. Therefore, a 2 PWM waveform has four, a 4 PWM waveform has eight pulses and so on in one cycle. The width of the pulses may not be equal throughout the cycle, but for some advantages they have to occupy certain places in the cycle of waveform. If it is realised that the harmonic analysis of any kind of nonsinusoidal, periodic waveform is achieved by Fourier analysis, for simplicity and in order to reduce the number of harmonic terms, it is essential to construct the waveform to diminish some of its harmonic terms by setting the waveform properly on its axis. As it is very well known that Fourier series of a periodic function is given by
where \( A_n \) and \( B_n \) are Fourier coefficients, \( A_0 \) is the d.c. component of waveform.

\[
\begin{align*}
F(t) &= A_0 + \sum_{n=1,2,3}^{\infty} \left( A_n \cos \left( \frac{2\pi}{T} nt \right) + B_n \sin \left( \frac{2\pi}{T} nt \right) \right) \\
A_n &= \frac{2}{T} \int_{0}^{T} f(t) \cos \left( \frac{2\pi}{T} nt \right) dt \\
B_n &= \frac{2}{T} \int_{0}^{T} f(t) \sin \left( \frac{2\pi}{T} nt \right) dt \\
A_0 &= \frac{1}{T} \int_{0}^{T} f(t) dt
\end{align*}
\]  

Where \( T, n, t \) are respectively period of the waveform, order of harmonic and time.

In order to diminish certain harmonics of a waveform it should satisfy the following conditions:

(i) \( A_0 = 0 \), the waveform has to have equal positive and negative areas.

(ii) Waveforms, symmetrical about \( \pi \), do not contain even harmonics, \( f(\theta + \pi) = -f(\theta) \).

(iii) Waveforms symmetrical about the origin have only cosine terms, i.e. \( B_n = 0 \).

(iv) If \( f(-\theta) = -f(\theta) \), waveform contains only sine terms, i.e. \( A_n = 0 \).

Constructions of waveforms are achieved by considering these conditions for low harmonic content. The waveforms with an even number of pulses have the advantage that they can satisfy all of the four conditions given above, whether their pulses are equal to each other in width or not. As a result of this, waveforms with an even pulse number consist of only odd harmonics of sine terms.
In order to obtain only sine term harmonics for a low harmonic content in a waveform with even number of pulses and for the sake of symmetry in the waveform, it is essential to place one quarter of the pulses between 30 and 90 degrees and the second quarter between 90 and 150 degrees. Leaving a large space as wide as at least 60 degrees between the last pulse of a cycle and the first pulse of the following cycle. Symmetry is obtained in this case. The time needed between two successive pulses must not be shorter than the total time needed for one thyristor to be turned off and the other to be turned on. This total time is given by equation (9).

\[ 2T_{sp} \geq T_{on} + T_{off} \]  \hspace{1cm} (9)

where \( T_{off} \) and \( T_{on} \) are, respectively, turn-off and turn-on times of a thyristor.

The reason for choosing this total time as \( 2T_{sp} \) is to bring simplicity into the algebra.

4.1 Analysis of Waveforms with Constant Pulse-Duration

An example of the WENPs (waveforms with even number of pulses) is one whose pulse-widths are the same. The number of pulses varies from two to infinity. The number of pulses is determined as follows,

\[ N_{pw} = \frac{T}{\frac{1}{2}(P_w + 2T_{sp})} \]  \hspace{1cm} (10)

where \( N_{pw} \) represents the number of pulses in one half cycle.

The figure expected from equation 10 for \( N_{pw} \) is an even integer number by ignoring the decimals and choosing the nearest even number. It is determined quickly. The equation (10) is deduced from the fact that two-thirds of the period can be used for the pulses, in order to satisfy the conditions mentioned in the previous section. The relationship between voltage, frequency and pulse-width
can be determined by using Fourier series for a particular harmonic that is required. The Fourier expansion of a 2 PWM waveform illustrated in figure 12 is given below.

\[ V_{NT} = \frac{4V_{dc}}{\pi n \sqrt{2}} \left[ \cos \left( \frac{T}{12} + T_{sp} \right) \omega n \right] - \cos \left( \frac{T}{12} + T_{sp} + P_w \right) \right] \omega n \right) \right] \right] (11) \]

where \( \omega = \frac{2\pi}{T} \)

\[ T = \frac{10^{-3}}{F} \quad T \text{ is in millisecond.} \quad (12) \]

\[ T_{sp} = \left( \frac{T}{6} - P_w \right) / 2 \quad (13) \]

Replacing equations (13) and (14) in equation (11) we get,

\[ V_n = \frac{8V_{dc}}{\sqrt{2} \pi n} \sin \left( \frac{\pi n}{3} \right) \sin \left( \frac{n \pi f P_w}{10^3} \right) \quad (15) \]

\[ P_w = \frac{10^{-3}}{\pi f P_w} \arcsin \left\{ \frac{V_n \sqrt{2} \pi n}{8V_{dc} \sin \left( \frac{\pi n}{3} \right)} \right\} \quad (16) \]

Equation (16) enables one to calculate the pulse-width in terms of harmonic voltage and frequency. The r.m.s. value of required harmonic is calculated from equation (15). The variation of pulse-width with respect to frequency at constant voltage is a hyperbolic function. Equation (16) derived for 2 PWM modulated waveform is used only if the pulses all have the equal width. The same equation for waveforms with a high number of pulses becomes more complicated due to increase of cosine terms. The number of cosine terms becomes twice that of the pulse number.

The direct solution to the determination of harmonic, once the pulse-width is found by equation (16), is to use equation (17) which gives the percentage of harmonic amplitude with respect to fundamental.

\[ HP_{n} = \frac{100}{n} \frac{\sin \left( \frac{\pi n}{3} \right) \sin \left( n \pi f P_w / 1000 \right)}{\sin \left( \pi / 3 \right) \sin \left( \pi f P_w / 1000 \right)} \quad (17) \]
The derivations of similar formulas for 4, 6 and 8 pulse-width modulated waveforms are given in Appendix 1.

\[ v_n = \frac{16V_{dc}}{\sqrt{2} \pi n} \sin \left( \frac{\pi n}{3} \right) \cos \left( \frac{\pi n}{12} \right) \sin \left( \frac{n \cdot \pi f \cdot P_w}{10^3} \right) \]  

(18)

for 4 PWM waveform

\[ v_n = \frac{8V_{dc}}{\sqrt{2} \pi n} \left[ 2 \sin \left( \frac{5\pi n}{18} \right) \cos \left( \frac{\pi n}{18} \right) + \sin \left( \frac{4\pi n}{9} \right) \right] \sin \left( \frac{n \cdot \pi f \cdot P_w}{10^3} \right) \]  

(19)

for 6 PWM waveform

\[ v_n = \frac{32V_{dc}}{\sqrt{2} \pi n} \cos \left( \frac{\pi n}{12} \right) \cos \left( \frac{\pi n}{24} \right) \sin \left( \frac{2\pi n}{3} \right) \sin \left( \frac{n \cdot \pi f \cdot P_w}{10^3} \right) \]  

(20)

for 8 PWM waveform

The minimum time needed between two successive pulses, \( T_{sp} \), is

\[ T_{sp} = \frac{T}{24} - \frac{P_w}{2} \]  

(21)

for 4 PWM waveform,

\[ T_{sp} = \frac{T}{36} - \frac{P_w}{2} \]  

(22)

for 6 PWM waveform,

\[ T_{sp} = \frac{T}{48} - \frac{P_w}{2} \]  

(23)

for 8 PWM waveform.

These equations of (15), (18), (19) and (20) show that the waveforms having 2, 4, 6, 8 pulses with equal pulse-duration throughout the cycle have increasing harmonic content with decreasing frequency. It is due to reduction of mark to space ratio. If the pulses were given in terms of angles rather than in milliseconds, the variation of frequency would not make any difference to the harmonic content.
5. WAVEFORMS WITH ODD NUMBER OF PULSES (WONP)

Waveforms with an odd number of pulses in a half cycle are constructed differently from those of WENP. They always have an odd number of pulses and the middle one has to be divided into two by symmetry axis at $\sqrt{2}$. The harmonic content of the waveform, in this case, like in WENP has only odd harmonics of sine terms. This is a good advantage for WONP whose pulse arrangement looks as if it can be obtained from sine envelope modulation technique. In fact the difficulties are caused by the firing order and the conduction times of thyristors. They do not permit the waveform to have a form that can be obtained by sine envelope method. The variation of pulses, nevertheless, is accomplished as far as the incorporation of thyristors allow.

The formation of WONP has certain conditions for stepped envelope operation. If these conditions are not met, short circuiting is caused by misfiring of the thyristors or harmonic content contains cosine terms in addition to sine terms. The width of the middle pulse must be equal to sum of widths of other pulses in one half a cycle. In a 3-phase system the phase shift is 120 degrees between the voltages of two phases. This phase shift, 120 degrees, is covered by the narrow pulses of the first and second halves of the whole cycle together with some part of the middle pulse. The order of the pulses must follow a particular pattern which is given in section 6 to ensure inverter operation. There is no restriction for WONP that its pulses must be located as those of WENP. The whole of the axis can be used, if it is required. That does not change the content of harmonics as long as the symmetry about the axis of $\pi$ is not broken.

In this section 1, 3 and 5, 7 pulse-width modulated waveforms will be discussed and their figures will be given.
The one pulse-width modulated waveform, as commonly called, quasi-square wave can be laid between 30 and 150 degree points in its positive half cycle. It is positioned like WEMP and has all kinds of symmetries that makes the waveform independent of cosine terms and even harmonics. If this waveform is represented on the angle axis, the variation of frequency does not alter the harmonic content of the waveform. But representing the waveform on the time axis and changing the frequency, while the pulse-duration is maintained at a fixed value, which is obtained when the pulse is laid between 30 and 150 degrees, at a frequency the harmonic content of the waveform changes. If the frequency is increased continuously the pulse will tend to cover whole of the one half cycle, since its duration is kept constant. Hence, it will eventually cover 180 degrees. But this is a practically impossible case because thyristors will be forced to conduct one after another without having a time to be turned off and on. This will result in short circuiting of the bridge. Therefore, maximum possible pulse-width is to be calculated on time axis rather than angle axis, since the turn-on and turn-off times of thyristors are given in time rather than in degrees at a particular frequency by the manufacturers, (26, 27). Therefore, the conversion of time to angle is not required while dealing with constant pulse-duration for a frequency range.

Any required harmonic voltage can be determined from equation (24) given below. For further information see appendix 2.

\[
V_n = \frac{8V_{dc}}{\pi Z_n} \sin \left( \frac{n \pi P_w \cdot f}{1000} \right)
\]  

(24)

The pulse modulated waveforms are not like quasi-square waveforms. They have to satisfy above mentioned conditions. For this waveform only, the width of the middle pulse does not have to be equal to the sum of the remaining pulse widths. The side pulses on both sides of the middle pulse must be symmetrical about \( \pi/2 \) and can make the start
immediately after the origin, leaving enough time for the thyristors to recover their new states. As in all waveforms, this waveform which consists of three pulses has a tendency to possess richer content of harmonic when the frequency is reduced, while its duration of pulses is maintained at a fixed value.

The waveforms with five and seven pulses have the general characters of WONPs. They are not like quasi-square waveform or three-pulse-width modulated waveform. Due to high number of pulses they possess high number of angle variations to change harmonic contents of waveforms. Further explanations for the waveforms will be given in section 6. The relationships, between frequency and pulse-width of all the waveforms shown in figures 13, 14, 15 and 16, are given by equations (25), (26) and (27) for three, five and seven pulse-width modulated waveforms.

Due to the high number of pulses with different durations in 5 and 7 PWM waveforms, pulse-durations are related with one another, in order to simplify the equation that is used for calculation of harmonic voltage magnitudes. Further information is available in appendix 2. Equations become more complicated as the number of pulses increases. The formulas that give the pulse-width at the required frequency and voltage are given below:

For three PWM

$$P_{wl} = \frac{10^3}{2\pi f} \arccos \left[ \frac{\sqrt{2V_n}n + \cos\left(\frac{2\pi n}{3}\right) \cos\left(\frac{\pi n}{3}\right)}{\cos\left(\frac{\pi n}{6}\right)} \right]$$

(25)

For five PWM

$$P_{wl} = \frac{10^3}{2\pi f} \left\{ \frac{\pi n}{6} - \arccos \left[ \frac{\sqrt{2V_n}n + \cos\left(\frac{7\pi n}{24}\right) \cos\left(\frac{5\pi n}{24}\right) - \sin\left(\frac{5\pi n}{24}\right) \sin\left(\frac{\pi n}{24}\right)}{\cos\left(\frac{\pi n}{12}\right)} \right] \right\}$$

(26)
for seven PWM

\[
P_{wl} = \frac{10^3}{2\pi f} \left\{ \frac{\pi n}{8} - \arccos \left[ \frac{nV_n}{8V_{dc}\cos\left(\frac{\pi n}{6}\right)} \right] + \frac{\sin\left(\frac{10\pi n}{96}\right)\sin\left(\frac{\pi n}{16}\right) - \cos\left(\frac{\pi n}{8}\right) + \sin\left(\frac{3\pi n}{16}\right)\sin\left(\frac{\pi n}{16}\right)}{\cos\left(\frac{\pi n}{6}\right)} \right\}
\]

(27)

6. ANALYSIS OF WAVEFORMS WITH PULSE-WIDTH THAT VARIES WITH ANGLE

6.1 General

So far WENPs and WONPs have been considered for constant duration pulse-widths. It is possible to change the width of the pulse and to obtain different waveforms. The variation of pulse-duration changes the harmonic content. In this section, the aim is to determine a PWM waveform which has minimum harmonic content to cause minimum total losses in an induction motor over a frequency range, that is defined as the band of frequency in which only one waveform is applied to the motor.

If the number of pole-pairs is fixed in an induction machine by means of stator winding, the minimum and the maximum speeds are defined, respectively, by the lowest and highest frequencies between which the motor is to operate continuously. Therefore, the wider the frequency range of a waveform, the smaller the number of different waveforms required for the motor operation between the minimum and the maximum defined frequencies. The complexity of the inverter's control circuitry is related to the number of waveforms employed for the variable speed drive. This number is kept small in order to avoid complexity.
The frequency analysis of a waveform, whose pulse-duration is maintained constant, is made by starting from the highest frequency of its frequency range. The purpose of choosing the highest frequency as a starting point is to obtain the length of the waveform's frequency range for minimum harmonic content towards lower frequencies. The waveform, which has minimum harmonic content at the highest frequency, possesses minimum harmonic content over a frequency band when it is compared with those of other waveforms. This is because the waveform's harmonic content is the function of frequency. The lower the frequency, the wider the spaces yielded between two consecutive pulses.

There are two methods to determine a waveform which covers a frequency band without its harmonic exceeding the given limits, chapter 3, section 1. One of the two methods is the analysis of various waveforms in such a frequency band and to obtain the best by comparison of their harmonic contents. This method may not give the waveform that possesses minimum harmonic content at the highest frequency, since the choice of the examined waveforms is random. In the second method, the waveforms are examined in angles to ensure minimum harmonic content at the highest frequency. This is followed by an analysis of the waveform, which has minimum harmonics, in the frequency range for the variation of its harmonic content against frequency. This second method is a precise and less time-consuming process.

The determination of a PWM voltage waveform requires safety for the thyristor bridge. The angles that are to be introduced for the change of the pulse-width are given in degrees. A small change of time may result in large angle differences at two different frequencies. For example, 0.1 ms at 40 Hz corresponds to 1.44 degrees while at 80 Hz to 2.88 degrees. The space between the pulses must not be shorter
than $2 T_{\text{spm}}$ converted to degrees. Therefore, the process is initiated from the top frequency where the safe operation of the inverter is guaranteed by necessary conversion of $2 T_{\text{spm}}$. Once $2 T_{\text{spm}}$ is obtained in degrees and the determination of harmonic content is carried out with respect to frequency, it is a matter of choice of the inverter's designer to maintain the pulse-duration constant in time or in degrees. The former choice results in higher harmonic content with decreasing fundamental voltage, while the second does not have any change either in harmonic content or in amplitude of the fundamental and requires variable voltage input for the voltage control of inverter output. But the waveform required must have both low harmonic content and decreasing fundamental voltage amplitude by decreasing frequency. In addition, the fundamental voltage to frequency ratio is to be constant at a required value. This will be analysed in detail in sections 9 and 10.

The variation of pulse-width by angle causes different constructional changes on WENP and WONP waveforms. The number of angle variations is limited by the number of pulses in one half of a cycle. The variation of a pulse-width is achieved by the method already described in section 3 of this chapter. It is not possible, here, to mention all the angle variations of waveforms which have 1, 2, 3, 4, 5, 6, 7 and 8 pulses in one half of a cycle. But as an example for WONP and WENP, 5 and 6 PWM waveforms will be discussed in this section.

6.2 6 PWM Waveform

In order to determine the angle variations of a waveform with any number of pulses, three phases will always be considered. The line voltages of a 6 PWM waveform which has equal pulse-widths are illustrated on figure 17. The change of a pulse distinctly shows its
effect on the pulses of the other phases. Since the waveform is symmetrical about $\pi/2$ and $\pi$, changes caused by the angle variations on the pulses of a quarter cycle are repeated on the other pulses of the cycle. A 6 PWM waveform has three pulses in a quarter of a cycle. Therefore, the maximum number of angle variations that can be used is not more than the number of pulses in half of a cycle.

In figure 18 the pulses of interest are shown by $a_1$, $b_1$, $c_1$, $a_2$, $b_2$ and $c_2$. It is obvious that the changes caused by angle variations on pulse $a_1$ is moved to $a_1^1$ and $a_2$ because of the symmetry about $\pi/2$ and $\pi$. The sum of $c_1$ and $a_2$ and the sum of $c_2$ and $a_1$ make zero. This means that $c_1$ and $c_2$ must have the same changes as $a_1$ and $a_2$ not to cause short circuiting of the inverter. The angle variations of $b_1$ and $b_2$ are the same. They are not affected by the changes caused on other pulses. Thus, the shape of the waveform cannot be made to have wider pulses in the middle as in the waveform that is generated by the sine envelope method. The waveform that is obtained from the sine envelope method requires the pulses of the 6 PWM waveform to have longer durations towards the centre of the half cycle in the directions of $\beta_x$ and $\gamma_y$. Although these angle variations cannot widen the pulses like those of a waveform generated by the sine envelope method, they help to reduce the harmonic content. To achieve this, $\gamma_y$ must be greater than $\beta_x$. The changes of $\beta_x$ and $\gamma_y$ cannot be made continuous until 6 pulse-width modulated wave becomes 4 PWM, 2 PWM and finally quasi-square wave. Because when the space between two consecutive pulses becomes shorter than the sum of a thyristor's turn-on and turn-off time, the inverter is short-circuited. The same fact is also valid for the reduction of pulse-width which must not be shorter than the sum that is mentioned above to avoid such danger.
6.3 5 PWM Waveform

The 5 PWM waveform is a good example for showing the derivation of permitted angle variations. The three line voltages for this waveform are shown in figure 18. The method of derivation of the angle variation is the same as that applied for the 6 PWM waveform. The relationship between the pulses of two separate phases of a 5 PWM waveform for a given angle variation is not the same as that of a 6 PWM. This difference is caused because of the different number of pulses. When a 5 PWM voltage waveform is required for an illustration in three phases, the middle pulse is set equal to the summation of the remaining four narrow pulses. The reason for such arrangement is to make the spaces between two consecutive pulses equal. Thus, the angle variations of these pulses in one of these spaces become equal. The change of the middle pulse affects the pulses which are at both ends of the half cycle. This effect is caused by 120 degrees phase shift between the phases. The arrangement of the pulses must be made such that the space between the last pulse of the positive half cycle and the first pulse of the negative half cycle is not less than 2 $T_{spm}$. This space is equally shared by these two pulses. Because of these reductions made on these two end pulses, the middle pulse-width, therefore, becomes narrower by an amount which is equal to the reduction of one end pulse on both sides. This process is illustrated in figure 18 in which the cross-shaded area represents the emptiness of $c_1$ that is caused by the reduction of $a_1$. These reductions in pulse-widths are made, respectively, by $\alpha_y$ and $\gamma_y$ on $a_1$ and $c_1$. The emptiness caused by $\alpha_y$ on pulse $a_1$ is shown by finely cross-shaded area. If $a_1$ is increased by $\alpha_x$, $b_1$ has to be decreased by $\beta_y$ which is equal to $\alpha_x$. Thus the space, between $a_1$ and $b_1$, is preserved for $\alpha_x$ variation. The second angle variation of $\beta_y$ on pulse $b_1$ increases the harmonic content. The symmetry about $\pi/2$ is broken due to
increment of $\beta_x$ on pulse $b'_1$. The state of the waveform caused by this kind of angle variation is not shown in figure 18.

The third type angle variation is obtained if only $b_1$ and $b'_1$ are considered independently of the other pulses. The increase of $\beta_y$ causes corresponding decrease on the other side of the pulse by $\beta_x$. Thus $\beta_x$ and $\beta_y$ are equal. These angle variations are represented by minus and plus signs, respectively, corresponding to decrement and increment of the width of $b_1$.

7. **ANALYSIS OF WAVEFORMS WITH PULSE-DURATION THAT VARIES WITH FREQUENCY**

In the previous section the analysis of PWM waveforms with angle variations has been explained for a frequency range. In this section the subject will be the determination of a PWM waveform which has equal durations for its pulses. The analysis will be considered for a frequency range. Therefore, the waveform is represented on time axis, since the analysis is based on the frequency.

The larger the pulse-duration, the lower the harmonics obtained in a PWM waveform. Therefore, the harmonic content can be reduced by increasing the duration of pulses at a fixed frequency. Because, if a waveform with equal pulse-widths has its frequency decreased while its pulses are maintained at constant duration, the harmonic content increases due to reduction of mark-space ratio. Hence the minimum harmonic content of a PWM waveform is obtained at a frequency by setting the duration of the pulses to the maximum permissible value which is related to the sum of a thyristor's turn-on and turn-off times and the d.c. link voltage of the inverter. The magnitude of the fundamental voltage increases with the pulse-duration. Therefore, the voltage-frequency ratio cannot be kept at a required value. In order to keep it constant, the duration of the pulse is to be selected such that the $V/f$ is satisfied.
The calculation of pulse-width is made by two different methods. The first comprises the stepless variation of pulse-width against frequency. This is achieved by the determination of frequency against pulse-width characteristic, from appendix 1. The second method introduces small step changes in frequency and the pulse-duration is calculated for every new frequency. The determined value of the pulse-duration is maintained constant until the next value of frequency, where the duration of the pulse is recalculated. This method causes voltage differences between the magnitudes of fundamentals of voltage waveforms due to different pulse-durations at a frequency. The difference between the change-over voltages of two separate waveforms is required to be a minimum to avoid additional pulsations on current waveform of the load as the frequency is changed. The first method ensures minimum difference in change-over voltages, since the step of the frequency is infinitely small, and practically does not cause current pulsations.

The harmonics of the waveform can be determined by equation (18). The minimum and maximum pulse-durations are calculated, respectively, from equations (9) and (21). This method is to be incorporated in a computer program if the frequency range is wide and the number of steps of the frequency for calculation of the pulse duration is very small.

8. WAVEFORMS FOR A FREQUENCY BAND WHERE V/f IS NOT CONSTANT

PWM waveforms generated by stepped envelope modulation technique are unlike those obtained by sine envelope method. Waveforms obtained by sine envelope modulation possess low harmonic content, but they cannot provide voltage control so wide as it is accomplished by stepped envelope method, when the supply frequency is, generally, below 10 Hz. This is because the durations of its pulses are related to the frequencies of carrier and reference waves, references(2,3,4), and
the magnitude of reference wave is fixed. Whereas, the mentioned control is achieved by the variation of pulse-duration in waveforms generated by stepped envelope method. The change of frequency by 1 Hz, in latter method, causes 11.11\% increment in period of the waveform. This increment widens the spaces between two consecutive pulses resulting in high harmonic content and in small fundamental voltage magnitude as the frequency is decreased. But the level of harmonic content and the magnitude of the fundamental voltage can be rearranged by appropriate setting of the pulse-duration. Thus, the stepped envelope modulation becomes superior to sine envelope modulation for frequencies smaller than 10 Hz.

It is known that if the pulse-duration is constant in angle, the magnitude of the fundamental voltage harmonic and harmonic content of the waveform do not change with frequency variation. Whereas, the fundamental voltage magnitude change with frequency, although the duration of pulses are constant in time. The magnitude of the fundamental voltage is to be higher for a frequency below 10 Hz than that which corresponds to higher frequency than 10 Hz. The high voltage is required in order to maintain constant output power of the motor. The flux produced in the motor due to small voltage-frequency ratio is not high enough to induce large currents which in turn produce required output power. Therefore, the voltage magnitude at low frequencies is kept higher than it is calculated by the ratio of \( V/f \). In addition the harmonic content is required minimum.

These requirements are best obtained, when the pulses of the waveform considered for low frequency use have two different parts on their pulse-widths; (i) constant duration, (ii) constant angle. Time is chosen as the independent variable on the voltage waveforms. The determination of a PWM waveform is achieved by obtaining minimum
harmonic content and required fundamental voltage magnitude at highest frequency, for further information see last two sections of this chapter. The arrangement of the pulse-width is made such that, its main part which satisfies these requirements is maintained constant in time against the frequency change. The remaining part of the pulse is changed in angle, in order to provide variable part on the pulse so that the voltage control can be achieved irrespective of frequency change. Since the part that is defined by degrees is narrower than that kept in constant duration, the fundamental magnitude is not affected by frequency change as it would be in other types of PWM waveforms with constant pulse-duration.

The Fourier series of the waveform is given in time axis by making conversion of pulse-width, that is given in degrees, into milliseconds for every frequency. When the total mark-space ratio and frequency are fixed, the waveform which has small number of pulses has higher fundamental voltage magnitude than that of a waveform with high number of pulses. The reason is the number of trigonometric terms involved in the Fourier series. Therefore, the choice of the waveform is made according to the magnitude of the fundamental voltage required and the number of pulses of the waveform.

9. THE MINIMISATION OF HARMONICS

So far in this chapter waveforms of all kinds have been examined both analytically and constructively. It was shown theoretically that every waveform contains different levels of odd harmonics. The existence of harmonics reduces the efficiency of an induction motor by causing extra losses. Therefore, the improvement in the efficiency of an induction motor needs applied harmonics to be reduced as much as possible, in order to approach the efficiency obtainable by applied pure sine wave voltage.
The method that minimises the harmonics is given for a 4 PWM waveform which has equal widths for all pulses as shown in figure 19. The choice of this waveform is merely to avoid unnecessary complexity in trigonometric representations of waveforms by Fourier series. This method is applicable to all kinds of PWM waveforms. The Fourier series of this waveform is given by equation (28).

\[ V_n = \frac{4V_{dc}}{\pi n^2} \left[ \cos \left( \frac{T - 4P_W}{8} \frac{2\pi n}{T} \right) - \cos \left( \frac{T + 4P_W}{8} \frac{2\pi n}{T} \right) + \cos \left( \frac{5T - 12P_W}{24} \frac{2\pi n}{T} \right) \right] \]

\[ - \cos \left( \frac{5T + 12P_W}{24} \frac{2\pi n}{T} \right) \]  \hspace{1cm} (28)

and \[ T_{sp} = \frac{T}{24} - \frac{P_W}{2} \]

The fundamental harmonic of such a waveform from equation (28) is given by replacing \( n \) with 1, while the 5th and 7th harmonics with 5 and 7, respectively.

The percentage of a harmonic with respect to fundamental is called \( HP_n \) and given by

\[ HP_n = \frac{V_n}{V_1} \times 100 \]  \hspace{1cm} (29)

hence

\[ V_n - V_1 \times HP_n /100 = 0 \]  \hspace{1cm} (30)

Since the derivative of equation (28) with respect to pulse-duration yields an equation in which the pulse-duration cannot be left alone on one side of the equality, the maximum and minimum points of \( V_n \) against \( P_w \) cannot be obtained unless a numerical method is used. Therefore, equation (28) is represented in a different form by replacing cosine terms with their equivalents of Taylor series. It is, of course, impossible to use an infinite number of terms to represent only one of the cosine terms, but very small error is introduced by using the first three terms. The use of first three
terms is only possible if waveform is symmetrical about \( \pi/2 \), otherwise first six terms have to be used, because the error caused by substituting larger angles than \( \pi/2 \) in the first three terms of Taylor series is considerably large so that it cannot be ignored. Whereas, the first three terms of the series provide satisfactory results for narrower angles than \( \pi/2 \). Taylor series for \( \cos(x) \) is represented by

\[
\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \ldots
\]  

(31)
calling

\[
S' = \frac{4V_{dc}}{\pi \sqrt{2}}
\]

and

\[
\gamma_1 = \frac{2\pi}{8T}
\]

the fundamental component of the waveform is given by

\[
V_1 = S' \left\{ 1 - \frac{\gamma_1^2 (T-4P_w)^2}{2} + \gamma_1^4 (T-4P_w)^4 - \frac{\gamma_1^2 (T+4P_w)^2}{2} - \frac{\gamma_1^4 (T+4P_w)^4}{24} + 1 - \frac{\gamma_1^2 (5T-12P_w)^2}{18} + \gamma_1^4 (5T-12P_w)^4 - 1 + \frac{\gamma_1^2 (5T+12P_w)^2}{18} - \frac{\gamma_1^4 (5T+12P_w)^4}{1944} \right\}
\]

(32c)

After simplification

\[
V_1 = 8P_w \gamma_1^2 S'T \left\{ \frac{8}{3} - \frac{76}{81} \gamma_1^2 T^2 - \frac{64}{9} \gamma_1^2 P_w^2 \right\}
\]

(33)
The 5th and 7th harmonic voltages are determined in the same way, by replacing \( n \)th with 5 and 7, and are given by equations (34) and (35), respectively.

\[
V_5 = \frac{81}{5} \left\{ - \frac{64}{3} \gamma_5^4 F_w^4 - \frac{160}{3} \gamma_5^3 P_w^3 \left( \frac{2}{3} \gamma_5 T \pi \right) - 8 \gamma_5^2 P_w^2 (2.5 \pi^2 - 3 \pi T) \right\}
\]

(32c)
where
\[ y_5 = \frac{\pi}{20T} \]
\[ y_7 = \frac{\pi}{28T} \]

Showing the percentage of 5th and 7th harmonics by HP_5 and HP_7 and substituting equations (33) and (34) in (30), the 5th harmonic voltage is thus expressed in terms of period, pulse-width and its percentage. By setting the period and percentage to required figures pulse-width can be obtained if the equation is solved.

\[
\frac{S'}{5}
\left[
- \frac{2}{3} y_5 P \left( -23 \pi^3 + 49 \pi^2 y_5 T - 109 \pi y_5^2 T^2 + \frac{250}{27} y_5^3 T^3 + 30 \pi - 20 y_5 T \right) - 2.4 \pi^2
\right]
\]

Similarly 7th harmonic is calculated by substituting (33), (35) in (30)

\[
\frac{S'}{7}
\left[
\left( \frac{8}{3} y_7 P \left( \frac{4}{3} y_7 \pi^- y_7 - \pi \right) \right) + \frac{136 \pi^3 y_7 T^3 (2 \pi - \frac{7}{3}) - 2 \pi^2 y_7 T}{81}
\right]
\]
If equations (38) and (39) are arranged for $P_w$,

\[
S' \left\{ - \frac{64}{15} h^4 P_w^4 + P_w^3 \left[ H P Y Y_5 T \frac{512}{9} \left( \frac{2}{3} \right) \right] - \frac{1.6}{P_w} (22.674 - 3 \pi Y_5 T + Y_5 T^2) - P_w \left[ \frac{2}{15} Y_5 (-618.89 + 49 \pi Y_5 T - 199 \pi Y_5 T + \frac{250}{27} Y_5 T - 20 Y_5 T^2) + 8 H P Y_5 T \right] + \frac{8.76}{3} Y_5 T^2 \right\} = 0
\]

(40a)

calling $Y_o = Y_5 T^2 + 4 \pi Y_5 T - 3 \pi^2$

similar equation for seventh harmonic is given by

\[
0.02369 Y_7 T^2 + P_w^3 - 1.5238 Y_7 T^2 + P_w (1.1968 Y_7 T^2 - 0.4353 H P Y_7 T + 0.00312 H P T) + \frac{1.3}{7} Y_7 T \left[ \frac{136}{81} Y_7 T^3 \left( 2 \pi Y_7 T^2 - \frac{2}{9} \pi Y_7 T \left( 3 \pi Y_7 T - 35 \pi Y_7 T \right) + \frac{10}{3} \pi + \frac{4}{9} \right) \right] = 0
\]

(41)

By equations (40) and (41) pulse-width can be determined for 5th and 7th harmonics. Since these equations are long, each constant term of $P_w$ in both equations will be represented by $K_{i,j}$ where "i" indicates the order of harmonic while "j" the power of $P_w$. Constant terms which do not contain $P_w$ are represented merely by $K_{i,0}$. Thus, equations (40) and (41) are given as follows by two different functions,

\[
F_n(5) = K_{5,4} P_w^4 + K_{5,3} P_w^3 + K_{5,2} P_w^2 + K_{5,1} P_w + K_{5,0}
\]

(42a)

\[
F_n(7) = K_{7,4} P_w^4 + K_{7,3} P_w^3 + K_{7,2} P_w^2 + K_{7,1} P_w + K_{7,0}
\]

(42b)

where $F_n(5)$ and $F_n(7)$ show, respectively, functions of 5th and 7th harmonics.
When the minimisation of these harmonics is required by changing the pulse-duration, section 11 of this chapter is to be referred for the explained method. In order to achieve this with a mathematical view, the following procedure is to be performed.

If the sum of 5th and 7th harmonics are not to be greater than a figure, \( A_m \), which is maximum permissible limit for these harmonics, equation (43), the values of \( P_w \) which satisfy this equation are determined by substituting equations (42a) and (42b) in (43).

\[
F_n(5) + F_n(7) \leq A_m \quad (43)
\]

thus

\[
K_{54} P_{w}^4 + (K_{55} + K_{73}) P_{w}^2 + (K_{52} + K_{72}) P_{w}^2 + (K_{51} + K_{71}) P_{w} + (K_{58} + K_{78} - A_m) = 0 \quad (44)
\]

If the real roots of equation (44) are used in equation (28), the waveform contains 5th and 7th harmonics as the percentage given by equation (29).

It has been shown in this section that the formulas which are used to obtain the pulse-duration to reduce any harmonic, are long. Therefore, the development of formulas for higher number of pulses; such as 5, 6, 7, 8 ... etc., is avoided due to complication of equations.

Another method of solving the problem is to represent equation (28) by a shorter form and work out the harmonics separately as in the form of equations (25), (26) and (27). But the use of these formulas cannot establish a control over the harmonic content of the waveform. Although, equation (44) is long and difficult to obtain, it provides the control of two harmonics by pulse-width variation.

The waveforms of small numbers such as quasi-square wave, two pulse waveform can be examined separately. But it is not possible to solve
A trigonometric function which has a trigonometric term on both sides of equality and one of them is multiplied by a constant. The solution to the problem is obtained by either tabulation or plotting of the unknown in terms of other quantities. A 2 PWM waveform with equal duration on all pulses is shown in figure 12.

Equation (11) represents the coefficients of Fourier series. Since the coefficients are the magnitudes of the harmonics, it is necessary to reduce the coefficient in order to decrease the harmonic.

Replacing equations (12) and (14) in (11)

\[ V_n = \frac{4V_{dc}}{\pi \sqrt{2n}} \left[ \cos\left(\frac{2\pi}{T}n\left(\frac{T}{2} - \frac{P_w}{2}\right)\right) - \cos\left(\frac{2\pi}{T}n\left(\frac{T}{2} + \frac{P_w}{2}\right)\right) \right] \]  

(45)

\[ V_n = \frac{8V_{dc}}{\pi \sqrt{2n}} \sin\left(\frac{\pi nP_w}{T}\right) \sin\left(\frac{\pi nP_w}{T}\right) \]  

(46)

The derivative of (11) with respect to pulse-width, \( P_w \), shows what value of pulse-duration is needed for the minimum and maximum points of \( V_n \).

\[ \frac{\partial V_n}{\partial P_w} = 0 \]  

(47)

The derivative of (11) with respect to \( P_w \) produces the following equation

\[ \cos\left(\frac{\pi nP_w}{T}\right) = 0 \]  

(48)

hence

\[ P_w = \frac{(2k + 1)T}{2n} \]  

(49)

where \( k = 0, 1, 2, 3 \)

Using the equation (49) in (45) for different values of harmonics it is possible to obtain all harmonic voltage coefficients. But as it was mentioned before, it may not satisfy the harmonic voltage magnitudes for the given limits.
It can be concluded, that the waveforms which have low number of pulses can be examined to reduce their harmonic content by the use of latter method, while waveforms which have 4 or 5 pulses in one half cycle are analysed by first method. A higher number of pulses than 6 in a waveform makes both solution methods unsuitable due to large number of terms. Therefore, the waveforms with number of pulses higher than 6 are examined by plotting the variation of harmonic voltage magnitudes against angle.

10. **INTRODUCTION TO V/f OPERATIONAL LINE**

V/f represents the ratio of a voltage magnitude to its frequency. This ratio denotes the slope of a characteristic between two different frequencies. The variation of V/f in a frequency band depends upon the relationship between the voltage and the frequency. If the voltage magnitude is varied independently of the frequency, the variation of V/f is obtained as required; constant, with a slope or step increase, ... etc. But in some methods of voltage generation the voltage magnitude changes with frequency regarding to a relationship between them. This change of voltage magnitude is inherited from the nature of generation. It cannot be changed unless additional voltage control is impressed upon the generation system. If V/f is to be maintained at a constant value in a frequency band, the voltage magnitude, therefore, changes as the frequency varied. As a result of frequency change the variation of voltage magnitude between two frequencies becomes a straight line for a fixed V/f. For example, the harmonic voltage magnitudes of a PWM waveform are related to their corresponding frequencies. The harmonic voltage magnitudes of a PWM waveform vary independently of the magnitudes of other harmonics. The variation of voltage magnitude produces a non
linear characteristic against frequency for a fixed pulse-duration. The additional voltage control is achieved by the variation of pulse-duration in order to alter the value of voltage at a frequency. Thus, V/f ratio is set free to have other values beside which is inherited from the generation.

The speed control of an induction motor with the variation of supply frequency requires constant V/f over the whole frequency range in which the motor operates continuously. Therefore, the voltage magnitude must be variable such that the V/f is to be constant. The reason for maintaining constant V/f ratio is to obtain maximum output power from the motor by keeping the air-gap flux constant at all frequencies. Although, the air-gap flux of the motor is proportional to the induced voltage, if the voltage drop in the stator is ignored, due to its small value, the impressed voltage to the motor can be assumed equal to the induced voltage. Thus, the maximum efficiency of the motor which corresponds to rated running conditions can be obtained at every frequency. The torque of the motor is shown in equation (51b) that it is proportional to voltage magnitude for a fixed value of slip and V/f.

The applied voltage is given by

\[ V = E + I_s Z_s \]  \hspace{1cm} (50a)

where \( I_s Z_s \) is assumed negligible with respect to V and E, hence,

\[ V = E \]  \hspace{1cm} (50b)

whereas,

\[ E = K'\phi f \]  \hspace{1cm} (51a)

where \( K' = 4.44 \, kN \)

The torque is given by
These equations show that the induction motor can operate to supply the rated torque-slip characteristic at every frequency. Somehow, as the frequency is decreased below 10 Hz to reduce the motor speed, while \( V/f \) is at a fixed value, the motor draws larger current to provide constant torque. The larger the current, the larger the voltage drop caused in the stator winding and hence less flux, resulting in poorer efficiency with respect to that obtained at rated running conditions. In order to maintain the flux at rated value, the applied voltage is increased together with \( V/f \).

The control of the value of \( V/f \) is achieved by additional sources which are directly related to the magnitude of the voltage waveform. The control of \( V/f \) is accomplished in two different methods for two different supply systems. One of the two refers to the variation of alternator's field current, when it is driven by a prime mover at a constant speed. The second method of control is employed to change the magnitude of the fundamental harmonic voltage of a PWM waveform by changing the duration of its pulses. Thus, \( V/f \) is kept constant over a frequency range. The variation of voltage magnitude with respect to its associated frequency is called the operational line on which the induction motor operates between its stationary state and maximum speed. Although, the operational line is required to possess constant \( V/f \), this requirement cannot be yielded by the use of only one voltage waveform, since the magnitude of the fundamental voltage would not follow a straight line from 0-50 Hz. Additionally, the harmonic content of the waveform could be a drawback for this frequency range and it may exceed the given limits. Therefore, the
whole frequency range of the motor is divided into narrower frequency bands to satisfy the conditions for harmonic content and fundamental voltage magnitude. Further information is available in next section.

11. **ESSENTIAL DATA FOR OPERATIONAL LINE**

The construction of an operational line with optimisation is difficult due to large number of waveforms available for a frequency band. The choice of the waveform is made to cause lowest total losses in an induction motor among those which satisfy the required limits. Therefore, a measure is introduced for the comparison of waveforms. This measure is called "weighting". The idea behind that is explained as follows.

The harmonic frequencies in a nonsinusoidal voltage waveform are given by the product of fundamental frequency and the order of harmonic. Thus, the higher the order of harmonic, the higher the harmonic frequencies are obtained. If harmonics are applied to an induction motor separately, figure 20, the machine reactances associated with harmonics increase by harmonic order. If the stator and rotor resistances are ignored, the impedances are proportional to the frequency. Thus, the harmonic impedance becomes proportional to the harmonic order. With this assumption the current going into the machine can be found as follows:

\[
V_1 = I_1 Z_1
\]

\[
V_n = I_n Z_n
\]  \hspace{1cm} (52a)

where

\[
Z_n = Z_1 n
\]  \hspace{1cm} (53)

After substituting (53) in (52a) and setting (52a) equal to (52),

\[
V_1 = I_n Z_1 n
\]  \hspace{1cm} (54a)
It is seen that the magnitude of nth harmonic current is one-nth of fundamental current. In addition, the ratio between the harmonic voltages is given by,

\[ \frac{V_n}{V_1} = \frac{I_{nZ_1}}{I_{Z_1 \text{HP}}} \]  

then substituting (55) in (52)

\[ V_n = I_{Z_1 \text{HP}} \]  

replacing \( V_{nT} \) in (52a), and substitute (53) in (52a)

\[ I_{Z_1 \text{HP}} = I_{Z_1 n} \]  

\[ I_n = I_{Z_1 n} \text{HP} \]  

where \( \text{HP} \leq 1 \)

This analysis shows that the harmonic currents flowing in the machine increase by magnitudes of harmonic voltages. But they are inversely proportional to their corresponding harmonic orders. Since the copper losses are proportional to the square of current, the current must be small in order to cause less losses in the machine. The summation of absolute values of harmonic currents can be considered as the representative of copper losses due to the above mentioned fact. Thus, the higher the sum of the harmonic currents, the larger the losses caused in the machine. Since the limits of harmonic voltages are given in ratio with respect to fundamental voltage, the harmonic current is determined by dividing the voltage ratio to order of harmonic. Hence, it is written for 5th and 7th harmonics that
The essential information required to make a constant flux induction motor operate between its stationary state and maximum speed is given in following sections;

1. Fundamental frequency range in terms of minimum and maximum frequency.
2. Fundamental voltage magnitudes corresponding to these two frequencies.
4. Maximum permissible limit of switching frequency of thyristors.
5. Minimum and maximum permissible turn-off time of thyristors.
6. The number of main thyristors to be used in the inverter to generate waveforms.
7. Number of phases of the inverter.
8. Magnitude of constant direct voltage input to the inverter.

It is useful to explain the data mentioned above for their importance. The minimum and maximum frequencies and their corresponding fundamental voltage magnitudes determine the slope of ideal constant-flux motor operation line. The voltage to frequency ratio should not be either greater or smaller than this figure. In this case only, the motor can provide the required, rated torque-slip characteristic at all frequencies as mentioned in section 10.

The limits given in percentage for those harmonics to be reduced must not be exceeded. The switching frequency determines the number of pulses of a waveform that can be used in a frequency band. The maximum frequency of a selected frequency band multiplied by the number of pulses must not exceed the permitted and maximum switching frequency. The minimum turn-off time of a thyristor must always be
smaller than the space between two consecutive pulses. Otherwise two of the thyristors of a phase will be in conduction, to result in short-circuiting. The number of main thyristors used in the inverter is important from the harmonic cancellation point of view. If the number of thyristors is doubled, the harmonic content can be reduced further, reference (5). The number of phases of the inverter determines the conduction pattern and the conduction duration of the thyristors used, (refer Chap.2, Sec.23). The magnitude of the direct voltage determines the magnitude of fundamental at any pulse-width. If the magnitude of the direct voltage were variable by means of a voltage control system, the fundamental voltage amplitude could have been varied without changing the pulse-width and hence harmonic content of the waveform. Thus, the minimisation of the harmonic content of a PWM waveform would be further improved.

Some stages have to be followed in order to form a constant flux line for an induction motor. Firstly, the whole frequency range is divided into frequency bands, depending upon the switching frequency and the number of pulses that are being considered for use in a waveform. The switching frequency has a constant value and is determined by the product of the modulation number of a PWM waveform and the operation frequency. Thus, the number of frequency bands are determined and they become equal to the number of waveforms that intended to be utilised. Secondly, the waveforms, which satisfy the switching frequency in the frequency range, with different modulation numbers, are analysed by Fourier series on the angle axis to determine the harmonic content. As explained in section 32 the representation and the analysis of a waveform on the angle axis produces a result that can be used at any frequency required, providing that the waveform which has low harmonic content on the angle axis, must have
the same harmonic content at a frequency for which the pulse-width is converted to time from angle. Thus, the selection of the waveforms, which have minimum weighting factor, is achieved for the operational line by this method. The important feature of this analysis is that the variation of angles is to be selected in such a way that the minimum width of the pulse, in time, must not be shorter than the time needed for a thyristor to be fully in conduction.

In the next stage, these selected waveforms are analysed by Fourier series on the time axis, to determine the variation of harmonics against frequency for the confirmation of minimum harmonic content in a frequency band. This process is accomplished, after converting the pulse-widths into time from degrees, by maintaining the pulse-duration constant for a frequency band in which the frequency is decreased in small steps. The choice of the selected waveforms for a frequency band is made regarding to the magnitudes of the fundamental voltages the values of the weighting factors. The magnitude of fundamental voltage reduces with decreasing frequency, due to increase of the space in the period. All of the waveforms have a tendency to increase their harmonic content. The change of harmonic magnitudes with frequency causes the weighting factor, which is the summation of absolute values of harmonic magnitudes, to change. Therefore, the variation of the weighting factor against frequency produces a useful characteristic by which the harmonic effects on the motor can be obtained approximately. If the weighting factor against angle variation is plotted, the minimum point of this characteristic shows the minimum effect of the harmonics caused on the motor. The maximum permitted value of the weighting factor is dependent on the number of harmonics considered to be affecting the motor. If the magnitudes of 5th and 7th harmonics are given with respect to that of the
fundamental as 30% and 40% respectively, the weighting factor is calculated from equation (59) as 11.714 which has not a dimension.

The establishment of the operational-line for an induction motor, which is to operate with constant air-gap flux over a wide frequency range, is accomplished in two stages. Firstly, the minimum and the maximum frequencies of the frequency range are determined, respectively, from the minimum and the maximum speeds of the motor. The voltage magnitude at the maximum frequency determines the voltage-frequency ratio which remains constant between maximum frequency and 10 Hz. The \( V/f \) is changed at this frequency until the minimum frequency reached as explained in section 10. The voltage magnitude is referred to that of the fundamental harmonic of the waveform. Therefore, the variation of fundamental voltage magnitude against frequency represents the planned form of the operational-line. Secondly, the waveforms, whose fundamental magnitudes satisfy the voltage variation of operational-line against frequency, are examined to obtain minimum weighting factor. The procedure to find optimum waveform was explained in previous section. Although, the planned operational-line is given as a straight line, its establishment is not possible with the use of one waveform. Because the harmonic voltage magnitude of a PWM waveform does not change linearly with frequency. Therefore, more than one waveform is used in the construction of the operational-line. The reconciliation of two different waveforms at a frequency is important, since they would have completely different structure from each other. The fundamental voltage magnitudes must be equal to provide smooth operation of the inverter. In addition, the weighting factor of the new waveform must be smaller than that of other waveform.
The decision to change a waveform is taken under two circumstances. First, either the maximum permissible weighting factor is reached or the fundamental voltage magnitude fails to satisfy \( V/f \) ratio. Thus, the construction of the operational-line is achieved with minimum number of different waveforms. Secondly, the number of waveforms is disregarded so that the weighting factor is to be minimum over the whole frequency range, which is defined by the frequencies corresponding to the stationary state and the maximum speed of the motor. The large number of waveforms increases the cost of the control circuitry in the inverter.

The construction of the operational-line is commenced from both ends of the whole frequency range to avoid unnecessary turn backs due to the disreconciliation of two different waveforms at a change-over frequency. The waveform, which provides the motor operation in the first frequency band that is commenced with maximum frequency, continues until its weighting factor is beaten by that of another waveform. In the last frequency band the \( V/f \) ratio is not important, since it cannot be satisfied due to the nature of the motor operation. But the terminal voltage of the waveform is to be equal to the minimum voltage that is required to start up the motor. The number of waveforms that can be used in the last frequency band has no limit. If the slope of the operational-line is known between 10 Hz and the maximum frequency, the pulse-duration in a PWM waveform which has equal pulses can be calculated as given below.

In figure 21, the variation of quasi-square wave is given by \( V_{An} \), while the straight line which represents constant flux operation is given by \( V_B \). The equation of \( V_{An} \) is given in equation (24) as
\[ V_{An} = \frac{8V_{dc}}{n} \sin \left(\frac{n}{2}\right) \sin \left(\frac{n}{2} f_p w\right) \]

while,

\[ V_B = m_B \cdot f + c \]  \hspace{1cm} (60)

for a frequency \( f_o \)

\[ V_{Ano} = \frac{8V_{dc}}{n} \sin \left(\frac{n}{2}\right) \sin \left(\frac{n}{2} f_o w\right) \]  \hspace{1cm} (61)

\[ V_{Bo} = m_B f_o + c \]  \hspace{1cm} (62)

\[ \Delta V' = V_{Ano} - V_{Bo} \]  \hspace{1cm} (63)

and to make \( V_{Ano} = V_{Bo} \), \( \Delta V' \) must be zero, substituting (61) and (62) in (63) and setting equal to zero, pulse-width is obtained as follows

\[ P_w = \frac{10^3}{n f_o} \arcsin \left( \frac{m_B f_o + c}{8V_{dc} \sin \left(\frac{n}{2}\right)} \right) \]  \hspace{1cm} (64)

Thus, variation of a harmonic voltage can be made to coincide with the straight line by the control of the pulse-duration. The fundamental voltage magnitudes of the intermediate frequency bands, between the terminal frequency of first band and 10 Hz, can be determined from equation (64).
CHAPTER III

THE RESULTS OF PWM WAVEFORM CALCULATIONS
1. **GENERAL**

The theory for the analysis of various PWM waveforms has been given in Chapter 2. The application of the theory, in this chapter, will be given for a practical case which is required by the Brush Electrical Engineering Company of Loughborough. They would like to control the speed of an induction motor, between its stationary state and maximum permissible speed by changing the supply frequency and ensuring minimum total motor losses. The variation of supply frequency is achieved by controlling the frequency of triggering the thyristors. The inverter used generates PWM waveforms from a fixed d.c. link voltage and has six main thyristors. The induction motor operation is required for constant flux to ensure maximum output power at every frequency.

In order to achieve this aim, they have used 2, 4 and 8 PWM voltage waveforms in different parts of the whole frequency region. These waveforms comprise equal pulse-durations in their cycles. Hence, the reduction of frequency causes the fundamental voltage to decrease and percentages of harmonic voltages with respect to that of fundamental to increase. Therefore, the total motor losses at small frequencies, below 15 Hz, increase sharply due to the harmonics. Meanwhile, the 2 PWM waveform at high frequencies, between 40 and 50 Hz, also causes high losses due to the same fact. The frequency band, between 15 and 40 Hz, is covered by 4 PWM waveforms which fail to maintain the voltage to frequency ratio at a constant value. The reduction of frequency causes different voltage magnitudes at the frequency where the waveform is changed from one to another. The following data was supplied by the Brush Electrical Engineering Company and was used in an attempt to avoid the above mentioned drawbacks.
1. Fundamental frequency range 0-50 Hz.
2. Fundamental voltage 28 V at frequency 0.1 Hz and 400 V at 50 Hz.
3. The fifth and seventh harmonic voltages must be less than 30% and 40% of the fundamental, respectively.
4. The maximum switching frequency must not be greater than 200 Hz.
5. The minimum time allowed for a thyristor to be turned off is 50 μs.
6. The pulse waveforms must be generated by a 3-phase bridge.
7. The magnitude of direct voltage supplied to the inverter is 525 volts.

This chapter will continue with the explanation of the graphs that are related with PWM waveforms. Waveforms will be examined in two groups. Firstly, the WENPs and secondly the WONPs will be given with their associated graphs and figures.

2. EXPLANATIONS TO ENABLE USE OF GRAPHS

The graphs shown between figures from 21 to 107 cover some of the waveforms analysed with Fourier series to determine their harmonic voltage magnitudes. In order to obtain better comparison of harmonic contents harmonic voltage magnitudes, except the fundamental harmonic, are given in percentage with respect to fundamental voltage magnitude. The variation of fundamental voltage magnitude of a PWM waveform is always given in r.m.s. values against angle or frequency, depending upon the base of the analysis. The 5th and the 7th harmonic voltages are also represented in the same dimension as the fundamental on a separate horizontal axis. The vertical axes are always given in percentage of their magnitudes. Hence, a PWM waveform is given with three different figures.
The analyses of the waveforms based upon the frequency may produce such results that differences between these waveforms cannot be distinctly seen on the graphs due to scaling and characters of the waveforms. Therefore, analysed waveforms are presented under the figure in descending order, such that the waveform which has highest magnitude is given on the top of the list which shows the specifications of the used waveforms.

The figures plotted against the angle vary in quantity which is dependent on number of angle variations. Since the number of angle variations of a PWM waveform can be more than two, the variations of harmonic voltage magnitudes would have complicated presentation in multi dimensions rather than two. Therefore, when the number of dimensions exceeds two, the additional dimensions are considered as parameters. Consequently, the number of figures is increased for this kind of waveform. If there are two angle variations as in 4 PWM, the representation of harmonic content with respect to these angles is simple and completed in a few figures. One of these angles is given on the horizontal axis, while other is given as a parameter under the figure. Hence, the variation of the harmonic voltage magnitudes are obtained against one angle variation while the other is fixed at a value. If the number of angle variations is three, the third angle variation is shown on the horizontal axis while the other two are fixed at predetermined values.

The weighting factor is represented against angle and frequency for various waveforms. Since the numerical figures of the weighting factor usually varies in small decimals, the scales cannot be made standard to fit all. The minimum total losses in the machine is
obtained when the characteristic of the weighting factor is minimum. But the choice of weighting factor characteristic depends upon the magnitudes of fundamental and harmonics. If there is more than one waveform satisfying the given restrictions for a frequency band, the choice of waveforms is achieved by using weighting factor graphs. The lower the numeric value of weighting factor at a frequency or at an angle, the better the motor performance obtained. Weighting factor characteristics of waveforms which are not used in the construction of the C.F.O.L. (constant flux operational-line) are not shown in the figures. The waveforms that form the C.F.O.L. are compared with one another and their characteristics of weighting factors are given separately in figures from 44 to 47.

3. **WAVES WITH EVEN NUMBER OF PULSES**

The waveforms that have 2, 4, 6 and 8 pulses are investigated in this section with Fourier series. The variations of fundamental voltage and harmonics are represented against angle and pulse-duration in separate groups of graphs. The analyses based upon angle of pulse-duration are made with Fourier series.

The Brush Electrical Engineering Company have supplied 2, 4 and 8 PWM voltage waveforms whose pulses were in equal duration. The frequency analysis of these waveforms is given in figures 22, 23 and 24 in which fundamental magnitudes, 5th and 7th harmonic voltages are shown, respectively. The pulse-duration of the waveforms with 4 and 8 pulses per half a cycle are chosen such that the pulse-duration of the 4 PWM wave is half of that of 2PWM and it is quarter of that of 2 PWM in 8 PWM waveform. In figure 22 the magnitudes of the voltage waveforms cannot have 400 volts at 50 Hz frequency and fail to satisfy
the required V/f ratio, which is 8. Although, 5th and 7th harmonics are below the specified limits for 4 and 8 PWM waves, the 5th harmonic of 2 PWM is higher than the limit and the seventh harmonic reaches 40% when the frequency is 38 Hz, as shown in figures 23-24. Harmonics of 2, 4 and 8 PWM waves increase with decreasing frequency.

2, 4, 6 and 8 PWM line to line voltage waveforms are given in figures 25, 26, 27 and 28, respectively. The angles that change the pulse-widths are shown by \( \alpha_x, \alpha_y, \beta_x \) and \( \beta_y \). The angle, \( \beta_x \) indicates the enlargement of middle pulses in 6 and 8 PWM waveforms, as shown in figures 27 and 28. The middle pulses cannot be as free as side pulses, because their places are in the middle of quarter cycle. Therefore, any increase or decrease in a middle pulse of a phase is moved by corresponding pulse of other phase. The changes that cannot be followed by the pulses of other phases are not permitted.

The origins of the angles shown in figures 25, 26, 27 and 28 begin at the each pulse edge. The arrangement of the waveform is made to have equal pulse-widths so that the spaces between the pulses are also equal. Thus, the angles shown by \( \alpha_x, \alpha_y, \beta_x \) and \( \beta_y \) have positive values. The direction of arrow shows the positive changes that its associated angle can have. The negative changes are made into the pulse body, except figure 26 where the direction of \( \alpha_x \) shown by its associated arrow is positive.

The maximum value of a pulse-width is obtained when the space between two consecutive pulses is a minimum equal to \( 2 T_{spm} \). The general expression by which the space is calculated in degrees is given below,
\[ 2T_{spa} = \frac{2}{N_{pw}} \left( 60^\circ - \frac{N_{pw} P_{wa}}{2} \right) \]  

(65)

where \( 2T_{spa} \) is obtained in degrees.

The minimum angle for a space is

\[ T_{spma} = T_{spm} \frac{360}{T} \]  

(66)

Hence, subtracting equation (66) from equation (65) and dividing the result by 2

\[ \alpha = \frac{1}{2} \left[ \frac{2}{N_{pw}} (60 - \frac{N_{pw} P_{wa}}{2} - T_{spma}) \right] \]  

(67)

Since the other angles are equal to \( \alpha \)

\[ \alpha_x = \alpha_y = \beta_x = \beta_y \]  

(67a)

The negative values of the angles are obtained simply by dividing the pulse-width into 2, since every pulse has two sides from which the angles are originated. Hence, the largest negative value of an angle is

\[ \alpha_x = \frac{1}{2} (P_{wa} - T_{spma}) \]  

(68)

It is an advantage to calculate all angles by two formulae quickly and be able to set three phase voltage waveforms on the angle axis irrespective of the number of pulses. Equations (67) and (68) show, respectively, that the larger the pulse-width, the smaller the positive angle and the larger the negative angle obtained in a PWM waveform.

In figures from 29 to 56, 4, 6 and 8 PWM waveforms are represented against angle variation. Figures 29 - 31, 32 - 47, 48 - 56, respectively, represent 4, 6 and 8 PWM waveforms. The reason for fewer graphs for 4 PWM is that the number of angles is only two, while it is three for 6 and 8 PWM waveforms.
The 4 PWM voltage waveform of figure 26 is represented in figures 29, 30 and 31 for its three main harmonics. The characteristics are shown for the variation of $\alpha_x$ while $\alpha_y$ is fixed at various values. The pulse-width is selected at 25.2 degrees in order to have high mark-space ratio in the waveform. The maximum values of $\alpha_y$ and $\alpha_x$ are calculated, respectively, from equations (67) and (68). Hence, they are found that the maximum values of $\alpha_y$ and $\alpha_x$ are, respectively, 4.35 and 12.375 degrees when $2T_{SPM}$ is equal to 50 $\mu$s. Figure 29 shows the variation of fundamental magnitude against $\alpha_x$ which causes reduction in pulse-width. It is seen that the magnitude reaches 400 volts when $\alpha_x$ is zero. Thus, the waveform which has 5th and 7th harmonics smaller than given limits and its weighting factor smallest when it is compared with those of the waveforms whose fundamentals are 400 volts is obtained.

The figures 30 and 31 show, respectively, the variation of 7th and 5th harmonics against $\alpha_x$. The waveform that it is shown by 1, has 5% 5th and 29% 7th harmonic when $\alpha_x$ is zero. Although, these harmonics are below their specified limits, the analysis of this waveform based upon the frequency shows that the 7th harmonics reaches the 40% limit quickly. Therefore, this waveform is not chosen for use in C.F.O.L. The figure which shows the frequency analysis of this waveform is not given, since the frequency band of this waveform, which is between 50 Hz and the frequency that 7th harmonics exceeds the 40% limit is narrow. Since the other waveforms fail to have a fundamental magnitude of 400 volts they are not considered for C.F.O.L. at high frequencies.

The above examination of a 4 PWM waveform has shown that in order to
obtain 400 volts at 50 Hz and to accomplish 8 for V/f is not possible due to the harmonic content of the waveform. Since the waveforms with higher number of pulses than 4 cannot be used due to the limit given on switching frequency, 2 PWM and 3 PWM voltage waveforms are to be examined for high frequencies. The examination of 2PWM waveform is given later in this section, but 3 PWM waveform will be discussed in section 4 of this chapter.

The figures from 32 to 47 show the analysis of a 6 PWM voltage waveform whose diagram is given in figure 27. In this figure three different angle variations are shown as $\alpha_x, \alpha_y$ and $\beta_x$, and width of the pulse is given in degrees. The maximum permitted frequency for a 6 PWM is determined as 337 Hz from the switching frequency. Therefore, a 6 PWM waveform cannot be used for frequencies above this value.

It is known that the maximum and the minimum values of the angles are related to the pulse-width. Therefore, a 6 PWM is analysed for different pulse-widths at various angles. The variation of harmonic content is given against $\beta_x$ while $\alpha_x$ and $\alpha_y$ are maintained at fixed and predetermined values. Figures from 32 to 37 show the variation of fundamental magnitude, when the pulse-width is chosen as 9, 10, 11.2 and 13 degrees. The reason of selecting these values as pulse-width is to obtain fundamental voltage magnitude to satisfy V/f as 8 between frequencies 20 Hz and 30 Hz. In order to achieve this, equation (19) is rearranged such that the pulse-width is calculated for a given fundamental magnitude. Angles $\alpha_x, \alpha_y$ and $\beta_x$ are determined from equations (67) and (68) for their positive and negative values. If the space between two consecutive pulses is a
minimum the pulses have their maximum width, hence the angle variation in positive direction becomes zero and in the negative direction rises to maximum. But, if the pulses are made in equal width to supply fundamental magnitude at about 200 volts, there will be enough space available to change the pulse-width by use of angles. This method of analysis of the waveforms provides an infinite number of possibilities for waveform construction. In figures 32, 34 and 35 the change of fundamental magnitude with pulse-width is given when $\alpha_y$ and $\alpha_x$ are selected for -3 degrees. This shows that the wider the pulse, the higher the fundamental magnitude obtained. 5th and 7th harmonic voltages are given, respectively, in percentage of fundamental magnitude, in figures 36, 38, 39 and 40, 42, 43. The weighting factors are illustrated in figures 44, 45, 46 and 47. From these characteristics it is seen that the weighting factor of a waveform decreases with increasing pulse-width.

The figures from 48 to 50 illustrate the variation of fundamental voltage harmonic of an 8 PWM waveform against $\beta_x$ at various values of $\alpha_x$ and $\alpha_y$. Since the number of characteristics is large, they are separated into three groups to avoid complexity. The groups are formed with regard to values of $\alpha_y$. The figures 48, 49 and 50 show the characteristics for $\alpha_y$ whose values are given as 0, 1.5 and 3 degrees. The diagram of the 8PWM waveform whose pulses are in equal width of 12.6 degrees is given in figure 28. Figures from 51 to 53 and 54 to 56 show the variations of the 5th and 7th harmonics, respectively, for the same $\alpha_y$ and $\alpha_x$ as given in figures 48, 49 and 50. In these figures it is shown that the waveforms for given $\alpha_y$ and $\alpha_x$ at any $\beta_x$ have either a 5th or 7th harmonic that is too high for a
wide range of frequency operation. This is because when the waveform is analysed on time axis, the higher harmonic exceeds the limits which were given in section 1 of this chapter. For example, 200 volt magnitude is satisfied by the fundamentals of the following waveforms in figure 48: (i) $\alpha_x = 7.5^\circ$, $\alpha_y = 0^\circ$, $\beta_x = 3^\circ$, (ii) $\alpha_x = 6^\circ$, $\alpha_y = 0^\circ$, $\beta_x = 4.5^\circ$, (iii) $\alpha_x = 4.5^\circ$, $\alpha_y = 0^\circ$, $\beta_x = 5.8^\circ$. Their major harmonics are given in the same order; (i) 5th 22%, 7th 6%, (ii) 5th 9.5%, 7th 25%, (iii) 5th 2%, 7th 41%. Hence, the weighting factors for these waveforms are (i) 5.257, (ii) 5.471, (iii) 6.257. This shows that the larger the pulse, the smaller the weighting factor obtained.

So far in this section the examinations of PWM waveforms with an even number of pulses have been given with their characteristics and diagrams. The following figures from 57 to 71 show 2, 4, 6 and 8 PWM voltage waveforms whose pulse-durations are equal maintained constant in a frequency range, except figures 63, 64 and 65. Figures 57, 58, 59 show, respectively, the variations of fundamental, 5th and 7th harmonic voltages against frequency at different pulse-durations. Since the magnitudes of 5th and 7th harmonics are too high, the utilisation of a 2 PWM waveform is not possible for even a small frequency band. Therefore, it is left out of consideration. The figures of 60, 61 and 62 show the variation of a 4 PWM wave for various pulse-durations from 50 Hz to 0 Hz. Although, the 4 PWM waveform cannot be used for pulse-durations longer than 1.614 ms at 50 Hz, the $P_w = 1.62$ ms and $P_w = 1.632$ ms waveforms are extended to 50 Hz to show the variation of harmonic voltages if the values of $T_{spm}$ were, respectively, 0.0233 and 0.01733 ms. These figures are calculated from equation (21).
Figures 63, 64 and 65 represent the variation of a 6 PWM, whose pulse-widths are determined from figures 32 to 47 for use in C.F.O.L. and maintained at a duration that corresponds to the conversion of angles into time at 25 Hz, between the frequencies of 25 Hz and 0 Hz. The selection of these particular waveforms from figures 32 to 47 is made such that the waveform is to have fundamental voltage magnitudes equal to those of the waveforms, which are used in C.F.O.L., at changeover frequencies and smallest weighting factor. Since the 5th and 7th harmonics are related with the pulse-duration of the PWM waveform, as well as fundamental voltage magnitude; in order to obtain a constant V/f at all frequencies of the whole frequency range of the drive system, the priority in selection has to be given to magnitude of the fundamental voltage. Otherwise the weighting factor is selected first, then the fundamental voltage magnitude. The priority of the weighting factor over the magnitude of fundamental produces unequal change-over voltages when the waveforms are changed at a frequency. The unequal change-over voltages may cause current pulsations in the line. Whereas, if the fundamental voltage is selected before the weighting factor, the above mentioned shortcoming would not be caused. Therefore, the choice of the waveform is made among those, whose fundamental magnitudes satisfy above mentioned condition. For example, in figure 25, 6 PWM waveform is selected, with its angles α_y = 3, α_x = -6 and \( P_{wa} = 13 \) degrees, to give a fundamental voltage magnitude of 230 volts, at \( \beta_x = 0.58 \) degrees. Figure 47 shows that the weighting factor of this waveform is a minimum at \( \beta_x = 0.58 \) degrees. The 5th and 7th harmonics are given, respectively, in figures 39 and 43. The values of these harmonics are, in percentage of fundamental voltage, 9 and 19. If the angles, α_x and α_y, of a waveform are not shown on figures from 32 to 47, its characteristic can be approximated by using the others.
Figures from 66 to 68 show the variations of 6 PWM waveforms. These waveforms comprise pulses with equal durations. The 5th and 7th harmonics are shown, respectively, in figures 67 and 68 where the variation of the harmonics show that, somehow, the amount of changes made in the pulse-durations do not affect the harmonic content of the waveform below 30 Hz. Similar results are shown in figures from 69 to 71 for a 8 PWM waveform when its pulses have equal durations.

The examination of waveforms for frequencies below 10 Hz are illustrated in figures from 72 to 85. The 4 PWM waveform is found to satisfy the conditions given in section 1 of this chapter. The pulse-duration, that it is maintained at 1.4 ms, is determined from figure 60, although this figure does not include this particular duration. The reason for choosing this duration is to obtain a fundamental magnitude, which is little smaller than the maximum voltage of this waveform's frequency band. Thus, freedom is provided for use of angles in order to obtain the required fundamental magnitude at a frequency. The harmonic content of the waveform, therefore, can be improved in conjunction with the angle variation.

The pulses of the waveform are calculated from an equation that can be derived in the same way as equation (16) from equation (18) for a required voltage and a frequency. The spaces between these pulses are then evaluated from equation (21). Although, the 4 PWM waveform is analysed for a wide angle variation, satisfactory results are obtained only when \( \alpha_y \) is changed from 1.7 degrees to 2.2 degrees. Meanwhile, \( \alpha_x \) is also changed from -1 degrees to +1 degrees. Figures from 72 to 85 show the variations of harmonics of a 4 PWM waveform for these angles. Figures 72, 73 and 74 show, respectively, the variations of fundamental, 5th and 7th harmonic voltages against \( \alpha_y \) at frequencies 1 Hz and 9 Hz. Figure 72 shows that the fundamental voltage magnitude
changes little by this angle. The 5th is illustrated in figure 73 where the analysis shows that the difference between the magnitudes of this harmonic at frequencies 9 Hz and 1 Hz is, somehow, practically indistinguishable. It may be due to small mark-space ratio.

The remaining figures from 75 to 83 show the variations of fundamental, 5th and 7th harmonic voltages, separately, against frequency for various angle combinations. Figures 78, 79, 80 and 81, 82, 83 exhibit, respectively, the variations of 5th and 7th harmonic which remain practically constant throughout the frequency band. Figures 84 and 85 show, respectively, the variations of weighting factors with frequency and $\alpha$. Using these graphs the low frequency region of a C.F.O.L. can possess a waveform which has the least harmonic content when compared with other waveforms that could be used instead. The selection of the optimum waveform will be given in section 5 of this chapter.

4. WAVEFORMS WITH ODD NUMBER OF PULSES

The waveforms with an odd number of pulses are examined on angle axis and time axis. Two separate 3 PWM voltage waveforms which are constructively different from one another are shown in figures 86 and 90 with their angle variations. The 3 PWM voltage waveform which is shown in figure 86 has only one angle variation that it is given by $\alpha_y$. The figures 87, 88 and 89 show how the fundamental 5th and 7th harmonic voltages of this waveform vary with increasing angle. The fundamental voltage increases with increasing $\alpha_y$ due to widening of the middle pulse. Figures 91, 92 and 93 correspond to the 3 PWM voltage waveform illustrated in figure 90 with the angle variation of $\alpha_x$. The 5th and 7th harmonic voltages decrease as $\alpha_x$ increases. The waveform becomes quasi-square wave when $\alpha_x = 30^\circ$. But in practice, the space between the side pulse and the middle pulse must not be shorter than $2T_{spm}$ in order to avoid short circuiting. In figures
91, 92 and 93 the waveform whose $\alpha$ is 30 degrees is exhibited for comparison purposes with other characteristics. This waveform is commonly known as quasi-square wave. The frequency range is not extended to cover frequencies between 25 Hz and 0 Hz due to the high harmonic content.

The location of pulses is selected to give the same amount of angle variation to side and middle pulses. The increase of side pulse can be achieved in either direction, towards middle pulse or origin. In the former case, the width of the middle pulse is always twice the side pulse and the maximum width of expansion per pulse is $(30^\circ - T_{spa})$. In the latter case, the expansion of middle pulse-width is equal to the same amount of reduction of side pulse-width from the origin side. As a result of this, the width of the middle pulse becomes wider than twice the width of a side pulse in a quarter cycle. The angle variation of a side pulse is equal to that of the middle pulse and therefore, the fixed point for 3 PWM waveform is chosen at 30 degrees. A 3 PWM waveform constructed as in figure 86 cannot be used for inverter operation due to overlappings caused by the conduction times of the thyristors.

The line voltage diagrams of a 5 PWM waveform are given in figures 94 and 95. It is shown that it has two different angle variations. Figure 94 exhibits the waveform in which the middle pulse is changed. This change affects only the side pulse, and either increase or decrease in the middle pulse always reduces the width of the side pulse from the origin side. The second pulse in the quarter of the cycle does not affect the middle pulse by its change. An increase or a decrease of the second pulse-width towards the middle pulse creates the same amount of change on the first pulse. While an increase of the second pulse towards the first pulse results in reduction of
pulse-width on the other side. This sort of angle variation is given in figure 95.

The location of the pulses is important. The second pulse always begins at $30^\circ$. The symmetry axis of the waveform is at $90^\circ$. There is no symmetry about the $60^\circ$ axis. The space that follows the second pulse must be equal to the width of the first pulse. If the middle pulse-width is extended beyond $60^\circ$, towards a smaller angle, then the first pulse-width reduces by the same amount.

The figures 96, 97 and 98 represent the change of harmonics of 5 PWM waveform, shown in figures 94 and 95, against angles, where characteristics indicated by VMI and VMD correspond to figure 94 while WSI and VSD to figure 95. The VMI and the VMD abbreviate, respectively, the increase and the decrease of the middle pulse-widths of a 5 PWM waveform, while the VSI and the VSD for the increase and decrease of the second pulse, respectively. The harmonic contents of these waveforms are given in figures 96, 97 and 98 and exhibits that VSI and VMD have high harmonic content with increasing $\alpha$.

The line voltage of a 7 PWM waveform is given in figure 99, 100 and 101 with different variations of angles. In figure 99 pulses are described by $P_{wa1}$, $P_{wa2}$, $P_{wa3}$ and $P_{wa4}$ for easy identification. The half of the middle pulse is equal to sum of the other pulses in a quarter cycle. This equality is never broken for 7 PWM. The figure 99 shows the change of first pulse-width affects the middle pulse in the same manner. The width of middle pulse decreases as the first pulse reduces its width from the origin side. The location of the pulses does not produce a symmetry axis at $\pi/3$, but the starting point of the first pulse can be determined by subtracting the width of the middle pulse from the point at which it starts. Thus, the middle
pulse-width becomes equal to twice the width of other three pulses in a quarter cycle. The larger the period (lower the frequency), the larger the space between the origin and the first pulse. The whole waveform is compiled towards the centre of the period, if the pulses have constant widths. The variations of the other pulse-widths introduced in figures 100 and 101 are merely to make the waveform having larger pulses as the \( \theta \) axis is traversed towards the middle of the period. But the construction of the waveform prevents smooth enlargement of consecutive pulses along the axis, \( \theta \). That is due to variation of second pulse. It can only be enlarged towards the first pulse which in turn reduces the third pulse as in figure 101. This feature of the second pulse does not permit the smooth enlargement of the pulses as progressed from the first pulse to the middle pulse. Meanwhile, the figure 100 shows the variation of the third pulse which effects the first pulse to increase its width towards the second pulse. This also deteriorates the harmonic content of the waveform.

The order of the pulses with spaces between them is shown in figure 99. This order must be satisfied to bring half of the middle pulse-width in equality with the sum of the other pulses in a quarter cycle.

Figures 102, 103 and 103a show the variations of fundamental, 5th and 7th harmonics against the angles of the waveforms illustrated in figures 99, 100, and 101, respectively.

The 3 PWM waveform can be constructed by one of two methods. In the first, the side pulse occupies the region from the origin to 30 degrees and is shown in figure 86. This cannot be used for different frequencies because a change of frequency affects the width of the space between the side and middle pulses. This is true even though the pulse-width is kept constant. Also in this method, the three phase operation of an inverter is not permitted. The voltage pulses
on different phases fail to cancel each other, hence one of two thyristors of a phase conducts while none of the thyristors of the other phases are in conduction. Figures 87, 88 and 89 show, respectively, the variations of fundamental, 5th and 7th harmonic voltages, although this waveform cannot be used for different frequencies by maintaining the durations of the pulses at fixed values. The variations of pulses are given against the angle $\alpha_y$ that it is shown in figure 86.

In the second method, 3 PWM waveform can be used for different frequencies by keeping the pulse-duration constant, because its side pulse begins at 30 degrees and ends at an angle that is smaller than 60 degrees. Thus, the overlapping of the thyristors' conduction times is avoided. A 3 PWM waveform that it is constructed with this principle is illustrated in figure 90 with its associated angle variation. The following figures, 91, 92 and 93 show, respectively, the variations of fundamental, 5th and 7th harmonic voltages against frequency for fixed values of $\alpha_x$. The analysis of this waveform on the angle axis shows that the higher the value of $\alpha_x$, the smaller the weighting factor obtained. Therefore, $\alpha_x$ is given for four different values of which 29.5 degrees corresponds to maximum pulse-duration for given $T_{spm}$ at the frequency 50 Hz.

The line voltages of 5 PWM and 7 PWM waveforms are given, respectively, in figures 94, 95 and 99, 100, 101 with their associated angle variations. The application of these waveforms for a frequency band, while their pulse-durations are maintained at fixed values, causes short circuiting in inverter operation, because none of these waveforms set at a frequency can be used at another frequency with pulse-durations unchanged.
5. **THE WAVEFORMS SELECTED FOR CONSTANT-FLUX OPERATIONAL LINE (C.F.O.L.)**

In sections (3) and (4) waveforms with even and odd number of pulses have been examined and their analysis with respect to angle and frequency represented by graphs. The selection of the waveforms which are used for constant-flux operation is achieved by use of these graphs. The weighting factor of the waveforms is considered for those which satisfy the required limits. The difficulty of representing the weighting factors of waveforms arises from the large number of waveforms. It is, therefore, better to show the variations of waveform weighting factors with respect to angle and frequency only for those which are used in the constant flux operation when necessary.

The maximum speed region of the induction motor which corresponds to high fundamental frequencies of PWM waveforms allows only 2, 3 and 4 PWM waveforms to be used due to the limit of the switching frequency. Other waveforms such as 5 and 6 PWM waveforms cannot be used higher than 40 Hz and \(35\frac{1}{2}\) Hz, respectively. Therefore, 2, 3 and 4 PWM waveforms are considered only for the high frequency region of C.F.O.L. Since the 2 PWM waveform has high harmonic content as shown in figures 57, 58 and 59 for various pulse-durations, it is not used in the construction of C.F.O.L. The remaining waveforms, 3 and 4 PWM, have been analysed on angle and frequency axes. Eventually, the results of the analysis show that the 4 PWM waveform cannot be used for frequencies above 48 Hz, since its fundamental magnitude fails to satisfy \(V/f\) for value of 8. The 4 PWM waveform cannot provide 400 volts r.m.s. at 50 Hz, unless its pulse-duration is less than 1.62 ms. The limit for the pulse-width of 4 PWM waveform at 50 Hz is 1.614 ms, which does not cause short circuiting because the space between two consecutive pulses is 0.052 ms. But, if the pulse-width is 1.62 ms, this space reduces to 0.046 ms and the thyristors cannot have enough
time to recover their new state and the result is short circuiting of the inverter bridge. As a result of elimination of 2 PWM and 4 PWM waveforms, only the 3 PWM wave can be used to give 400 volts at 50 Hz. The examination of the 3 PWM waveform is given in section 4. The waveform which has $x$ equal to 29.5 degrees possesses minimum harmonic content. The examination of this waveform with respect to frequency while its side pulse-duration is maintained at 1.6388 ms, which is obtained by converting 29.5 degrees into milliseconds at 50 Hz, shows that the magnitude of its fundamental voltage becomes 385 and 326.5 volts at 48 Hz and 41 Hz, respectively, and corresponding voltage-frequency ratios are 8.02 and 7.96. Although, this waveform is analysed in the 25 to 50 Hz region, it exceeds the specified limit for 7th harmonic at 41 Hz. Therefore, it is necessary to change the waveform for frequencies smaller than 41 Hz. The important feature is to find a waveform which has a fundamental voltage magnitude equal to that of the 3 PWM wave at the change-over frequency. The analysis of the 4 PWM shows that the pulse-duration of 1.632 ms at 48 Hz has the same magnitude of fundamental as the 3 PWM waveform, and its weighting factor is a minimum. When the 4 PWM waveform with a pulse-duration of 1.632 ms, which is kept constant, is analysed for frequencies smaller than 48 Hz, the variation of fundamental magnitude does not follow a straight line. In order to keep V/f at 8 all the time it is necessary to vary the pulse-duration with a step or to provide a stepless variation of the pulse-duration with frequency. Since the stepless variation requires infinitely small steps, a computer program is prepared in order to determine the pulse-durations. The computation time increases with decreasing step length. But the stepless variation reduces the difference, between two voltage magnitudes at a change-over frequency, to practically zero. If the step length is chosen 1 Hz, as in figure 105, this difference becomes
bigger, but does not exceed 0.5% of the fundamental magnitude of the
highest frequency. The increase of the pulse-duration with
decreasing frequency improves the harmonic content of the waveform
with respect to the waveform that has constant pulse-duration for all
frequencies. The 4 PWM waveform determined in this way has minimum
harmonic content when it is compared with other waveforms whose pulses
have equal durations. The fundamental voltage magnitudes of waveforms,
whose pulse-durations are different from one another at a particular
frequency, are calculated separately for every step. The difference
of these magnitudes against frequency is given by ∆V in figure 104.
In the same figure the variations of pulse-durations and the space
between two successive pulses are given with respect to frequency.
Although ∆V of 5 PWM waveform has smaller harmonic content than 4 PWM
at 40 Hz, figures 97, 98 and 105, this waveform is not permitted to
take part in the construction of C.F.O.L. It would cause short
circuiting when the frequency is changed as explained in previous
section. For this reason 4 PWM waveform is continued until the 6 PWM
waveform can take over the operation, at \( \frac{33\frac{1}{2}}{2} \) Hz. The restrictions on
change-over voltage and harmonic content do not allow the change of
waveform at this frequency.

The 6 PWM waveform is examined in a frequency band, from \( 33\frac{1}{2} \) to 0 Hz,
for various pulse-durations in order to satisfy the change-over voltage
with 4 PWM wave and the limits on the harmonic content. From this
analysis of the 6 PWM and 4 PWM voltage waveforms it is found that
their change-over voltages become equal at the frequency 25 Hz when
all of the pulse-durations of 6 PWM waveform are 1.243 ms. The 5th
and 7th harmonics of the waveform are below the limits and almost
constant over the whole frequency band, as shown in figures 66, 67,
68. A further attempt is made to reduce the harmonic content. The
waveform is examined on the angle axis for different angle variations, as shown in figures 32 to 47. In the end of this analysis, the waveforms which satisfy the change-over voltage magnitude and have smaller harmonic content are analysed in the frequency band, from 25 Hz to 0 Hz, as shown in figures 63, 64, 65. The fundamental magnitudes of these waveforms are almost equal at every frequency of the frequency band. But their harmonic contents are different. The weighting factors of these waveforms are compared with one another at every frequency, regarding the fact that the difference between the fundamental voltage magnitudes are negligible. Hence, the waveform which has minimum harmonic content is selected for the given conditions. The frequency band of this waveform is continued until the change-over voltages of this waveform and of some other waveform, whose harmonic content is smaller than the currently used waveform, are equal.

Although, the frequency band, which is from 25 Hz to 0 Hz, is acceptable for use of the 8 PWM waveform, it is not used due to the fact that the weighting factor of the 6 PWM waveform is smaller than that of the 8 PWM waveform. The weighting factors of 6 PWM and 8 PWM waveforms are available on calculation, respectively, from figures 64, 65 and 67, 68. In addition, this waveform fails to provide large V/f ratios for frequencies smaller than 10 Hz. It should achieve 28.14 volts, which is 7% of 400 volts, at 1 Hz or even lower frequency. To overcome this drawback the 8 PWM waveform is to be considered like the 4 PWM waveform that is used from 48 Hz to 25 Hz. But this cannot help to improve the harmonic content of the waveform, since it is seen on figures 69, 70 and 71 that the increase of pulse-duration does not change the harmonic content significantly.
Since the 6 PWM waveform has small harmonic content, it can be used until 3 Hz, to reach the 30% limit for its 5th harmonic, as shown in figure 105. But its fundamental voltage magnitude undesirably reduces to values below 28 volts. Therefore, some other voltage waveform is required to start C.F.O.L. with 28 volts at 0.1 Hz which is considered to be the minimum frequency. The relationship between the fundamental voltage magnitude and its frequency is not to be linear. The V/f is to be reduced as the frequency is increased until this ratio becomes equal to that of the 6 PWM waveform. This means that the waveform which is to be used between the origin and the 6 PWM waveform has to be constructed such that its fundamental voltage magnitude must be 28 volts at 0.1 Hz and these waveforms must have equal magnitudes at a frequency where its harmonic content is smaller than that of 6 PWM waveform.

4 PWM and 8 PWM waveforms are examined for various pulse-durations in the frequency band, from 15 Hz to 0.1 Hz. The examination of these waveforms shows them to be unsatisfactory due to the fact that the fundamental voltage magnitudes do not satisfy the voltage values at 0.1 Hz and at any frequency where the voltages of 6 PWM waveform and examined waveform are equal. If the waveform is arranged to have 28 volts at 0.1 Hz, its fundamental magnitude exceeds that of the 6 PWM wave at any frequency between 0.1 Hz and $32\frac{1}{2}$ Hz. Alternatively, if it is arranged to satisfy the magnitude of the 6 PWM at $32\frac{1}{2}$ Hz, the waveform possesses smaller voltages than 28 volts at 0.1 Hz.

Additionally, the comparison between 4 PWM and 8 PWM waveforms show that the fundamental voltage magnitude of the 4 PWM wave is higher than that of the 8 PWM wave at any frequency for a fixed mark-space ratio. Therefore, the 4 PWM wave is chosen for use in the low frequency band.
As mentioned in section 8 of Chapter 2 and section 3 of this chapter, angles are added to pulses in order to vary the harmonic content and the fundamental voltage of the 4 PWM waveform. The conditions given in section 1 of this chapter are met when the angles and the pulse-duration of the 4 PWM waveform are set, respectively, to, $\alpha_x = 0$ degrees, $\alpha_y = 1.8$ degrees and $P_w = 1.4$ ms. The harmonic variation of this waveform is shown in figure 105. Although, the 7th harmonic is smaller than its specified limit, it remains almost constant from 9 Hz to 0.1 Hz.

In figure 105 the final state of C.F.O.L. is illustrated with 5th and 7th harmonics of the used waveforms. The C.F.O.L. is divided into four frequency bands which are defined by their associated frequencies, as shown on the figure. The variations of fundamental voltage against frequency in every band is linear. But the C.F.O.L., as a whole, from 50 Hz to 0.1 Hz, is not linear due to the requirement of 28 volts at the smallest frequency. This requirement causes high voltages in frequency band 4. The waveforms in the other bands are to follow one another in order to provide smooth changes of waveforms at the change-over frequencies. Therefore, in some parts of the C.F.O.L. the V/f is little higher than 9 volts/Hz. The advantage of high fundamental voltage is that the percentage harmonic content of the waveform becomes small. Thus, the wider the pulse, the higher the fundamental voltage magnitude and the smaller the harmonic content obtained. Therefore, the choice is to be made between the value of V/f and the harmonic content. The waveforms used in the construction of this C.F.O.L. are given, respectively, in figures 105a, 105b, 105c and 105d for frequency bands 1, 2, 3 and 4.

In figure 106 another C.F.O.L. is represented by four different waveforms. One of these is the same as used in frequency band 1 of
figure 105. As seen the C.F.O.L. is linear from 50 Hz to 9 Hz. But the linearity is broken at 9 Hz. Nevertheless, the waveform used in the frequency band 4 still provides linear fundamental voltage variation with frequency. The voltage obtained at 0.1 Hz is 13 volts with the smallest 5th harmonic in the four frequency bands of this figure. Although, the waveforms of figures 105 and 106, in corresponding frequency bands, are different and have different harmonic contents from one another, the scaling is not big enough to show these differences. The waveforms used in the frequency bands 2, 3 of figure 106 have constant pulse-durations. These pulse-durations are shown on figures 106a and 106b. In band 4, the 4 PWM waveform is employed when its pulse-duration and the angles associated with one of its pulses are, respectively, \( P_w = 1.4 \) ms, \( \alpha_y = 1.8 \) degrees and \( \alpha_x = 1 \) degrees. This waveform is shown in figure 106c where its pulse-durations are given in terms of period.

The comparison of C.F.O.L.s shown in figures 105 and 106 shows that the higher the slope of the voltage-frequency ratio smaller the harmonic content obtained.
CHAPTER IV

THEORY OF SQUIRREL-CAGE INDUCTION MOTOR
1. **INTRODUCTION**

The squirrel-cage induction motor is a widely used a.c. motor. Because of its ruggedness and simplicity, the squirrel-cage motor is utilised substantially for constant speed industrial applications. A d.c. motor or a wound rotor induction motor is commonly employed when the requirement is for an adjustable speed. The squirrel-cage motor cannot be used for continuously variable speed operation over a wide range unless the supply frequency is variable. The change of applied frequency is achieved by two methods. One is to utilise a motor-alternator set to change the frequency of the sinusoidally varying alternator's output voltage. But this method has low efficiency due to the number of machines used. The other is to produce a non-sinusoidal voltage waveform, whose frequency is controllable by the generating system, to cause minimum total loss in a cage motor. Therefore, it is necessary to look into the theory of the squirrel-cage induction motor, and determine the equivalent circuit parameters of the machine due to harmonic voltages embodied in an applied voltage waveform. All of the machine parameters are derived by using the design data. The change of these parameters with voltage harmonics, the variation of airgap at any point on the stator periphery and determination of line and phase currents will be given in this chapter.

2. **EQUIVALENT CIRCUIT OF AN INDUCTION MOTOR**

2.1 **The Equivalent Circuit for Sinusoidal Voltage Application**

The performance of an induction motor with a sine wave impressed on its terminals can be obtained by using the well known equivalent circuit of a phase, figure 107. In this figure $R_s$ and $X_s$ are, respectively, the resistance and the reactance of the stator winding.
\( R_r \) and \( X_r \) are the resistance and the reactance of the rotor winding, which are referred to the stator. \( X_m \) stands for the magnetising reactance. \( R_m \) is the resistance that accounts for iron losses when it is multiplied by the square of the magnetising current.

The equivalent circuit of an induction motor differs from that of a transformer because of the rotation of the rotor. Therefore, the stator magnetic field and the rotor magnetic field which is produced by the induced currents in the rotor travel at the same speed. The sum of the speed of the magnetic field relative to the rotor with the speed of rotation of the rotor is equal to the synchronous speed which is determined by the applied frequency and number of pole pairs of the machine. Hence, the relationship between synchronous speed and the speed of the rotor rotation is expressed by the ratio which is known as slip and is denoted by \( s \).

\[
S = \frac{n_s - n_r}{n_s} \quad (69)
\]

Thus, the frequency corresponding to the rotor rotation is given by:

\[
f_{rt} = \frac{p \cdot n_r}{60} \quad (70)
\]

the difference between the synchronous and rotor rotation frequency determines the induced frequency in the rotor. Substituting (70) in equation (69) and replacing \( n_s \) by its equivalent, \( 60f_s/p \)

\[
s = \frac{f_s - f_{rt}}{f_s} \quad (71)
\]

If the reactances in the equivalent circuit are given in terms of leakage inductances, their values are obtained by multiplying these inductances with the angular frequency of the supply, as shown in equation (72). The leakage reactance of the stator is given by

\[
X_s = 2\pi f_s L_s \quad (72)
\]

The leakage reactance of the rotor is determined similarly.
The induced voltages in the stator and rotor are given by equations (73) and (74) at slip \( s \).

\[
E_s = \sqrt{2} \pi k_{ds} k_{ps} f_s N_s \phi 
\]

\[
E_r = \sqrt{2} \pi k_{dr} k_{pr} f_s N_r \phi 
\]

Each of these e.m.f.s will lag behind \( \phi \) by 90 electrical degrees and since the rotor winding constitutes a closed circuit, the magnitude of the rotor current is given by

\[
I_r = E_r \sqrt{R_r^2 + s \chi_r^2} 
\]

If the rotor of an induction motor is assumed stationary and the slip is unity, the voltages induced in stator and rotor windings are determined by transformer action. Replacing \( s \) by unity in equation (74) and dividing (73) by (74), the turn ratio of an induction motor is given as:

\[
U = k_{ds} k_{ps} N_s / (k_{dr} k_{pr} N_r) 
\]

If the actual rotor current is known and is required for the equivalent circuit, the current transformation ratio which will convert actual rotor current to the equivalent stator current is obtained from ampere-turns of the motor, reference (7). It is given by

\[
U_c = (m k_{ds} k_{ps} N_s) / (m k_{dr} k_{pr} N_r) 
\]

In a squirrel-cage motor the number of phases is equal to the number of bars, and the winding factor of the cage rotor which is the product of distribution and pitch factors, is unity. Hence the turns ratio of a squirrel-cage motor is given by

\[
U = 2m k_{ws} N_s / N_r 
\]

If complex impedances of stator, rotor and magnetising branch are
represented by $Z_s$, $Z_r$, and $Z_m$, respectively, the stator and rotor currents of an induction motor are determined by using figure 107 and are given by the following equations.

$$\begin{align*}
I_s &= \frac{V(Z_r + Z_m)}{Z_s Z_r + Z_m(Z_r + Z_s)} \\
I_r &= \frac{VZ_m}{Z_s Z_r + Z_m(Z_r + Z_s)} \\
I_m &= \frac{VZ_m}{Z_s Z_r + Z_m(Z_r + Z_s)}
\end{align*}$$

(79) \hspace{1cm} (80) \hspace{1cm} (81)

In the equivalent circuit given in figure 107, the whole effect of the motor load is represented by the term $R_{r/s}$. But, since

$$R_{r/s} = R_r + R_r(1-s)$$

(82)

the second term shows the electrical analogue of the mechanical load; or the induction motor can be considered as a generator, feeding a fictitious resistance. The power dissipated in this resistance is equal to the power output.

$$P_{op} = m_r I_r^2 R_r(1-s)$$

(87)

hence the torque,

$$T_q = 30P_{op}/\pi n_r$$

(88)

The equations given in this section are derived on the basis of following assumptions:

(i) Applied voltage is sinusoidal

(ii) Space harmonics are neglected

(iii) Uniform airgap between the stator and rotor

(iv) No stray losses

(v) Magnetic circuit of the motor is unsaturated

2.2 Equivalent Circuit for Space Harmonics

The change of the second assumption, made in previous section, has deteriorating effect on the machine performance overall. Since the
winding distribution of the machine cannot be made sinusoidal due to the slotting of the stator, the product of the current, that flows in the winding, and the winding distribution yields the stator ampere-turns of the induction motor. If the flows of sinusoidal currents are assumed in the stator winding and the winding distribution is known, the magnetomotive force, (mmf), can be determined. Since the product of sinusoidal and nonsinusoidal functions results in nonsinusoidal function, the mmf in the machine is therefore always nonsinusoidal.

Fourier analysis of this function produces the harmonics called SPACE HARMONICS. They can be considered independent of the fundamental harmonic, as submachines operating in series with the fundamental. They have their own rotor circuits, but common stator. Therefore, the harmonics created by mmf behave separately from the fundamental at different frequencies causing their own losses which are called extra losses. The superposition of all kinds of losses is achieved only if magnetic saturation is not obtained.

Assuming a three-phase single-layer full-pitch distributed winding of which the phase band is \( \alpha \) electrical degrees and \( N \) is the number of series turns per phase. The distribution of the turns is approximately a trapezoidal wave which is shown in figure 108.

Fourier series of this waveform is given as

\[
y = \frac{4h'}{\pi \alpha} \sum_{e=1,3,5\ldots}^{\infty} \frac{\sin(e\alpha) \sin(e\alpha)}{e^2}
\]

where

\[
h' = \frac{N}{2p}
\]

and assuming three-phase sinusoidal currents flowing in the phases, the total airgap ampere-turns is given by
\[ F = I_a y_a + I_b y_b + I_c y_c \]  \hspace{1cm} (90)

where

\[ I_a = I \cos \omega t, \quad I_b = I \cos (\omega t - 2\pi/3) \quad \text{and} \quad I_c = I \cos (\omega t - 4\pi/3) \]

Substituting equation (89) and currents in equation (90), the total ampere-turns is obtained

\[ F = \frac{3}{2} \frac{4h^1}{\pi \alpha} I \left[ \sin(\alpha \sin(\theta - \omega t)) + \frac{1}{5} \sin(5\alpha \sin(5\theta + \omega t)) + \frac{1}{7} \sin(7\alpha \sin(7\theta - \omega t)) + \ldots \right] \]

(91)

Since the distribution factor is:

\[ k_d = \sin(\alpha)/\alpha \]  \hspace{1cm} (92)

\[ k_p = \sin(\pi p'/2) \]  \hspace{1cm} (93)

Since the pitch-factor is unity in the full-pitch distributed winding,

\[ F = \frac{6h^1 I}{\pi} \left[ k_w l \sin(\theta - \omega t) + \frac{k_w^5}{5} \sin(5\theta + \omega t) + \frac{k_w^7}{7} \sin(7\theta - \omega t) + \ldots \right] \]

(94)

Consequently, the ratio of fundamental to any harmonic is

\[ \frac{F_1}{F_e} = \frac{k_{wns}}{k_{wl}} \frac{1}{e} \]  \hspace{1cm} (95)

In order to obtain a comparison between the fundamental and harmonic quantities which are comprised of the airgap mmf, it is assumed that the current in the winding is the same as for fundamental.

Since r.m.s. values of the induced voltages have

\[ \frac{E_1}{E_e} = \frac{\phi_1}{\phi_e} \frac{k_{wl}}{k_{we}} \]  \hspace{1cm} (96)

and

\[ \frac{F_1}{F_e} = \frac{B_1}{B_e} \]  \hspace{1cm} (97)

\[ \frac{\phi_1}{\phi_e} = \frac{B_1}{B_e} \frac{L D \pi/(2 \pi)}{L D \pi/(2p \pi)} \]  \hspace{1cm} (98)
\[ \frac{\phi_1}{\phi_e} = e^{2\frac{k \omega_1}{k \omega_e}} \]  
\[ \frac{X_1}{X_e} = e^{2\frac{k \omega_1}{k \omega_e}} \]  

(99)  
(100)

The equivalent circuit of an induction motor can be represented as in figure 109. As shown by equation (100), the magnetising reactances of harmonics are very much smaller than that of the fundamental. The harmonic voltage is also small in comparison with \( V_1 \). Therefore, the harmonic voltage does not significantly affect the current \( I_s \) determined by the fundamental alone. Hence, it may be assumed that \( I_s \) in figure 109 is the same as that in figure 107.

The number of the harmonic circuits is infinite, but there are only a few of importance. The order of a harmonic is given by
\[ e = 2kq + 1 \]  
(101)

where
\[ k = 1, 2, 3 \ldots \]

Therefore \( e \) is 1, 5, 7, 11, 13, 17, 19 \ldots etc. missing the triplen harmonics. When \( 2kq \) is equal to the number of slots, the pair of corresponding harmonics are called slot harmonics whose winding factors are equal to that of the fundamental. Therefore, the equation (100) is changed into
\[ \frac{X_1}{X_e} = e^{2} \]  
(102)

When \( e \) has the value of twice the number of slots, the slot harmonics are called second-order slot harmonics, but since they are very small in relation to the fundamental they are neglected. The remainder are called belt harmonics since the order is associated with the number of phase belts.
The secondary reactances for harmonics are calculated in a similar manner:

\[
\frac{X_{r1}}{X_{re}} = \left( \frac{k_{wl}}{k_{we}} \right)^2 \tag{103}
\]

The secondary leakage reactance consists of secondary slot leakage, differential leakage and skew leakage reactances. The calculation of all these will be given in the next section. The secondary resistance in squirrel-cage motors is divided into two parts: 1. The d.c. resistance of the rotor bars, and 2. The d.c. resistance of the end-ring. Since the rotor bar resistance is subjected to skin effect due to high frequencies induced in the rotor, skin-effect factor must be included to evaluate the correct resistance. The end-ring resistance, in practice, can be ignored since it is small in comparison with bar resistance for high frequencies.

The space harmonics, as presented by equation (94), travel at subsynchronous speeds around the periphery depending on the order of harmonic. The higher the order, the lower the speed of rotation obtained. The (+) sign in this equation denotes the backward rotating field, while the (-) the forward rotating field. Since the harmonics travel around the periphery of the airgap at subsynchronous speeds, for the same fundamental frequency, their pole numbers increase by the same amount as their speeds decrease. In equation (94) the speed of the 5th harmonic is \( \frac{1}{5} \)th of the fundamental and hence, the number of poles produced by this harmonic increases five times. The rotation of 5th harmonic is opposite to that of fundamental. Therefore, the relative speed of 5th harmonic with respect to rotor is \( 6n_s/5 \) when the rotor rotates at the synchronous speed. As a result of this speed, six times the fundamental frequency is induced in the rotor. Table 1 shows the induced frequencies in the rotor.
The slot harmonics are given by \(2kq_s\) where \(k\) is an integer and determines the order of slot harmonic. In table 1, frequencies induced in the rotor due to the major space harmonics are shown for 50 Hz of stator frequency. Table 2 shows the variation of slot harmonic frequency with number of slots per pole per phase belt. The calculation of induced frequencies in the rotor due to space harmonics while the induction motor is running at slip, \(s\), is similarly obtained as in table 1. Table 3 shows the frequencies induced in the rotor when the slip is "s".

The slip for space harmonics is derived from the same principle as for fundamental

\[
s_e = \frac{\tilde{7}n_e/(2kq + 1) - n_s(1 - s)}{\tilde{7}n_s/(2kq + 1)}
\]

\[
s_e = 1 + (2kq + 1) \tilde{7} (2kq + 1)s
\]

\[
s_e = 1 + e \tilde{7} se
\]

where \(e = 2kq + 1\)

From the equivalent circuit of the fundamental shown in figure 107, the total power transmitted across the air-gap is given by \(I_{re}^2 R_{re}/s\), where \(R_{re}/s\) is the apparent resistance in the fundamental rotor circuit. This is the total input power of the rotor due to the fundamental, and hence is equal to the sum of the mechanical power developed by the rotor, the copper losses of the rotor bars, the friction and windage losses, the high frequency iron losses and the parasitic torques of the harmonic fields. The power transmitted across the air-gap for a space harmonic is given similarly to that of the fundamental by equation (107), where \(I_{re}\) and \(R_{re}\), respectively, represent the magnitude of harmonic rotor current and the rotor resistance which is obtained by considering the skin effect in the rotor bars due to the harmonic frequency induced in the rotor.
Although, the equivalent circuit is illustrated as in figure 109, the harmonic fields due to the rotor winding are neglected to avoid complexity. In fact every stator harmonic field produces a series of backward and forward revolving harmonic fields in the rotor. But since the effects of all these harmonics are very small, they are not taken into consideration. The procedure is the same used in calculating the rotor impedances for the stator harmonic fields, simply interchanging winding coefficients of stator and rotor. For further information see reference (14).

3. DETERMINATION OF INDUCTION MOTOR PARAMETERS

In the previous section the equivalent circuit of an induction motor was explained by considering all the space harmonics produced by the mmf distribution in the air-gap. The parameters shown on figures 107 and 109 will be calculated in this section by using the design data of a squirrel-cage induction motor. But, since the shape of stator and rotor slots are different, the slot leakage reactances will be shown twice and the calculation of these leakage reactances will refer to the squirrel-cage motor used in the tests. The principle behind these calculations is fixed for any kind of slot shape that can be used. In this section the determination of stator resistance will not be shown since its measurement is easily assessed by using d.c. power source.

3.1 Leakage Reactances

The equivalent circuit of an induction motor has three important elements which are namely, magnetising reactance, stator and rotor leakage reactances. The magnetising reactance represents the
useful air-gap flux, while the other two represent leakage fluxes that occur in several parts of the machine. The sources of the leakage fluxes are associated with the designs of the motor. Therefore, it is necessary to look into the determination of these reactances in order to match those obtained by experiments. The leakage reactances of an induction motor vary with the operating conditions. The cause of this variation is the magnetic saturation of the leakage paths, that increases with current and decreases the leakage reactance, reference (6). The definition of leakage flux is the difference between the flux produced by one of two windings and the total flux linking the second. The total leakage flux can be resolved into components to be determined. Although the nonlinearity occurs when magnetic saturation is attained, the calculations, here, will be given without considering the saturation. Therefore, superposition can be applied to obtain the total leakage reactances of each member of the induction motor.

3.1.1 Stator Slot Leakage Reactance

In general the distribution of the current is assumed to be uniform over the cross section of the conductor, and the leakage flux produced by the current is considered to have a straight path across the slot. The reluctance of the path is assumed to be concentrated in the slot portion. After making these assumptions the leakage inductance of a winding, in general, is

\[ L_s = \frac{N\phi}{I} \]  \hspace{1cm} (108)

and the flux that crosses the reluctance path is

\[ \phi = IN\lambda \]  \hspace{1cm} (109)

consequently

\[ L_s = N^2\lambda \]  \hspace{1cm} (120)
The permeance is given by
\[ \lambda = \mu_0 \mu_R A / L' \]  \hspace{1cm} (121)

where \( \mu_R \) is the relative permeability of media and \( \mu_0 \) is the permeability of the free space.

Therefore, the leakage inductance is proportional to the permeance of the slot. In figure 110 an ordinary stator slot is illustrated with a double-layer winding.

The flux linkages are calculated for every height shown by \( h_1 \) to \( h_5 \) including the mutual flux linkage.

The permeance of strip shown by \( dx \) is given by
\[ \frac{d\lambda}{dx} = \mu_0 \frac{L'}{b_2} \]  \hspace{1cm} (122)

and the number of conductors is in height \( X \)
\[ N_x = \left( \frac{X}{h} \right) N \]  \hspace{1cm} (123)

hence the leakage inductance of the bottom layer is given by
\[ L_{sB} = \int_0^{h_5} \left( \frac{X}{h_5} \right)^2 N^2 \frac{\mu_0 L'}{b_2} \]  \hspace{1cm} (124)

Since the flux produced by this coil links all the heights shown in the figure 110, the total leakage inductance is therefore equal to the sum of all, that is
\[ L_{sB} = \frac{2}{b_1 L'} \left( \frac{h_1}{3b_2} + \frac{h_2}{3b_2} + \frac{2h_2}{3(b_1+b_2)} + \frac{h_1}{3b_1} \right) \]  \hspace{1cm} (125)

while the total leakage inductance for the top coil side is
\[ L_{sT} = \frac{2}{b_1 L'} \left( \frac{h_2}{3b_2} + \frac{2h_2}{3(b_1+b_2)} + \frac{h_1}{3b_1} \right) \]  \hspace{1cm} (126)

The following leakage inductance corresponds to the flux linkage associated with the top layer due to the bottom layer flux.
If the leakage inductances of bottom and top layers are equal to each other, the gross total of leakage inductance for a slot is given by

\[ L_{sMB} = N_B N_T \frac{\mu_0 L^{'dx}}{b_2} + \int_0^{h_2} N_B \frac{X}{h_2} N_T \frac{\mu_0 L^{'dx}}{(b_1 + b_2)/2} + \int_0^{h_1} N_B \frac{X}{h_1} N_T \frac{\mu_0 L^{'dx}}{b_1} \]  

(127)

that is

\[ L_{sMB} = N_B N_T \left( \frac{h_2}{2b_2} \left( \frac{h_2}{b_2} + \frac{h_1}{b_1} \right) \right) \]  

(128)

If the winding is full-pitched, the total slot permeance is known, the stator leakage reactance is determined by:

\[ X_{SL} = \frac{2 \pi f L_s}{N_T} \]  

(131)

\[ X_{SL} = 0.203f \, m \, L' \, N^2 \lambda_5 \, 10^{-7}/Q_s \text{ ohms per phase} \]  

(132)

If the total slot permeance is known, the stator leakage reactance is determined by equation (132).
3.1.2 Rotor Slot Leakage Reactance

The principle of calculating a leakage reactance was given in section (3.1.1) and stator slot leakage reactance was calculated. The determination of rotor slot leakage reactance is similarly achieved, but the difference is that the number of coil sides is only one. Therefore, there will be no term for mutual flux linkage. The slot permeance of the rotor for a given slot as shown in figure 111 is calculated as follows.

The total permeance of the slot given in figure 111A is obtained by considering the permeances of four different areas separately. The total is the sum of these four different permeances.

\[ P = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \]  

(133a)

\( \lambda_2 \) and \( \lambda_4 \) are semi-circular portions; one of them is shown in figure 111B.

The permeance strip is

\[ d\lambda_x = \mu_0 N I_b \frac{dx}{AB} \]  

(133b)

where

\[ AB = 2R \cos \theta \]

\[ X = R \sin \theta \]

\[ dx = R \cos \theta d\theta \]

Hence substituting \( dx \) and \( AB \) in (133) and integrating from 0 to \( \pi/2 \)

\[ \lambda = \mu_0 N I_b \frac{\pi}{4} \]  

(134)

Consequently the total inductance of the slot is

\[ L_r = \mu_0 N I_b \left( \frac{h_2}{2} + \frac{\pi}{2} + \frac{2h_1}{3(b_1+b_2)} \right) \]  

(135)

The slot leakage inductance of the rotor as referred to for the stator is given by
where $L_b$ is given in meters and leakage reactance is calculated in ohms per phase.

As a result of the determination of leakage reactances of stator and rotor the total leakage reactance in the motor can be represented as in equation (137)

$$X_o = 0.203 f m^2 10^{-7} (L_s/Q_s + k^2_{ws} L_b/(k^2_{wr} Q_r))$$  \hspace{1cm} \text{(137)}$$

For a squirrel-cage winding, $k_{wr}$ is unity and each bar is taken as half a turn. Therefore, the $k^2_{wr}$ term in equation (137) is not shown for squirrel-cage motors.

3.1.3 Differential Leakage Reactance

Owing to the distribution of the stator winding in a discrete number of slots, the flow of sinusoidal currents in such a winding produces a series of flux harmonics in the air-gap. These harmonic waves of air-gap field revolve around the periphery at subsynchronous speeds which are shown in Table 1. All of the harmonics induce voltages at fundamental frequency in the stator winding. If these voltages are divided by their corresponding sinusoidal currents, the reactance for each harmonic field is obtained and is called differential leakage reactance. The superposition of these reactances is possible only if saturation is not attained in the magnetic circuit of the motor. The differential reactance is considered to be the sum of two leakage components, namely zigzag and belt leakages. The zigzag leakage is caused by all air-gap harmonics that would be produced if the winding had one slot per pole per phase. This means that if each slot made a complete phase belt and
each slot carried the same r.m.s. current, equally spaced apart in time as well as space phase. The belt leakage is an additional reactance due to actual phase belts, which may be several slots wide, or, in fractional-pitch windings, may have varying widths. The order of belt leakage harmonics varies with the number of phase belts per pole, while the zigzag leakage with the number of slots per pole. The orders of belt leakage harmonics were given in section (2.2). The order of zigzag leakage is given by \((k/Q_p/p+1)\), where \(k\) is an integer. Cage rotors carry harmonic currents in addition to the fundamental harmonic current. These harmonic currents are caused by the harmonic voltages which are produced by the stator harmonic fields. Thus, each of these harmonic currents set up their own mmf wave in the air-gap and contain their own harmonics.

The differential leakage can be considered in two parts, 1. rotor differential leakage reactance, 2. stator differential leakage reactance. The former is obtained by dividing the sum of the voltages induced in the bar by fundamental bar current. The eddy currents that are induced on the rotor surface cause losses as well as reducing the differential harmonic leakage-flux, and result in lower differential leakage reactance. Consequently, the harmonic currents flowing in the rotor bars increase and produce more copper losses. Therefore, the correction of differential leakage reactance is necessary by multiplying it by a factor called the reduction factor, reference(22). Hence, the differential leakage reactance of the rotor is given by

\[
X_{dlre} = X_{me}(1/\zeta^2 de - 1)k_{dlre}
\]

where

\[
\zeta_{de} = \sin(\pi e P/Q_p)/(e P/\pi Q_p)
\]

and
The second part of the total differential leakage reactance is obtained from the stator harmonic fluxes which link the rotor-tooth surface and penetrate to the rotor-tooth body. The part which passes to the tooth body links the conductor in the slot as well and produces a damping effect due to currents induced in the conductor. The relative magnitudes of the two flux components depend on the rotor tooth-pitch and wavelength of the harmonic flux. For belt harmonics since the wavelength is large compared with the pitch of rotor slots, it is convenient to consider these harmonics in terms of submachines where the currents produced in the rotor tend to cancel the flux. These currents flowing in the bars have the frequencies calculated as shown in table 3 of section (2.2). As for the slot harmonics the wavelength is as large as a slot-pitch and, therefore, can be considered in two different subsections.

1. If the number of stator slots is equal to that of rotor, the harmonic flux does not link the rotor bars. Therefore, all the losses associated with this harmonic are surface losses.

2. When the number of rotor slots is twice that of stator, the harmonic fluxes close from pole to pole causing no loss.

The damping factor $k_{dfse}$ for a flux which goes down a rotor-tooth is the ratio of the damped flux in the tooth to the flux which would exist if there were no damping, reference (22). Hence, the stator differential leakage reactance is the summation of the contributions of each stator harmonic including the damping factors.

$$X_{dse} = \sum \left[ X_{me} k_{dfse} \right]$$  \hspace{1cm} (139)

where

$$k_{dfse} = 1 - \frac{(X_{me} + X_{dre})}{\left( X_{me} + X_{dre} + X_{me} (1/k^{2}_{ske} - 1) + X_{r1} \right)}$$  \hspace{1cm} (140)

The factor $k_{ske}$ will be explained in the next subsection.
3.1.4 Skew Leakage Reactance

The air-gap flux density of an induction motor for an unskewed rotor does not change the axial direction, as the currents induced in the rotor bars flow axially. If the rotor bars are skewed by some electrical angle, the coupling between the stator and the rotor windings decreases together with magnetising reactance. Since the produced flux should be constant, the leakage flux increases. Therefore, the induced e.m.f.s and currents of fundamental frequency are reduced by the skew factor which is given as, reference (6),

$$k_{ske} = \sin\left(\frac{2\pi \alpha_s}{2Q_s}\right) / \left(2\pi \frac{\pi r}{2Q_s}\right)$$  \hspace{1cm} (141)

where $\alpha_s$ is the skew in stator slot-pitches.

Hence the skew leakage reactance is

$$x_{ske} = x_{me} \left(\frac{1}{k_{ske}^2} - 1\right)$$  \hspace{1cm} (142)

In this equation the order of the space harmonic is represented by "e". Fundamental skew leakage reactance is obtained by setting "e" to unity. Since the skewing reduces the induced voltages and currents, the transformation ratios are to be corrected regarding this fact. For the voltage it is divided, and for the current multiplied, by the skewing factor. In equation (141) the numerator of the sine function is multiplied by the order of the harmonic, since the number of pairs of poles increases together with the harmonic order.

3.2 Squirrel-Cage Resistance

A squirrel-cage can be considered as a polyphase winding whose number of poles is equal to the number of poles of the rotating field. The bars are solidly connected to the end rings. It is assumed that the currents induced in the bars are sinusoidally distributed around the periphery by angle difference of $2\pi p/Q_r$. The determination of cage resistance is achieved by two methods. One is to calculate it from
the design data as in reference (14). The other is to determine the
total resistance, if the resistance values of a bar and a ring
element are known separately. The determination of either ring or
bar resistances needs design data. But using the second method it is
possible to obtain ring and bar currents separately. In figure 111A
the shape of the rotor slot is illustrated. The cross section of the
end-ring is shown in figure 112. By using the dimensions given on
these figures the area of ring and bar can be determined. Thus, the
area of a bar which covers the whole slot up to the tooth tip is given
by
\[ A_B = \frac{\pi}{2}(b_1^2 + b_2^2) + (b_1 + b_2)\left\{h_1 - (b_1 + b_2)/2\right\} \]  
(143)

The area of the end-ring is determined by taking the average diameter
of the inner and outer diameters of the end-ring and also accounting
for the curvatures of the edges. Hence, the area of the end-ring is
given by
\[ A_{er} = b_e h_e - 2(r_o^2 - \pi r_o^2/4) \]  
(144)

The total cage resistance of the rotor, which is referred to for the
stator, is given by equation (145). If the algebraic procedure is
performed, the first term gives the rotor-bar resistance, while the
second represents the end-ring resistance.
\[ R_r = 6.78 m_k^2 \omega_N^2 (\frac{L_B}{A_B} + \frac{D_e}{\pi h_e} p^2) / 10^4 \]  
(145)

The constant is to be increased approximately by 20% due to the
aluminium used in cast windings containing a small amount of iron and
silicon.

The currents in the bars and end-rings are determined by considering
two adjacent ring elements at a junction. Hence,
\[ I_{\text{bar}} = 2I_{\text{ring}} \sin(\frac{\pi p}{Q_r}) \]  \hspace{1cm} (146)

Since the squirrel-cage rotor is considered to have \( Q_r \) phases, each bar element can be taken as one half of a turn and the winding factor becomes unity. Under these conditions bar current and voltage become the current and voltage of the rotor. Since the currents in a bar and in a ring are different, \( r'_{\text{ring}} \) is considered as the ring element resistance as referred to the current \( I_{\text{bar}} \). Using the equality of the copper losses, it is found that

\[ r'_{\text{ring}} = r_{\text{ring}} \left( \frac{I_{\text{ring}}}{I_{\text{bar}}} \right)^2 \]  \hspace{1cm} (147)

where \( r_{\text{ring}} \) is the actual resistance of a ring element, and hence

\[ r'_{\text{ring}} = r_{\text{ring}} \frac{4 \sin^2(\frac{\pi p}{Q_r})}{(\frac{\pi p}{Q_r})} \]  \hspace{1cm} (148)

4. EQUIVALENT CIRCUIT FOR TIME HARMONICS

The equivalent circuit of an induction motor was given in section (2.1) including space harmonics for a sinusoidally varying applied voltage. When the applied voltage waveform is changed from sinusoidal to nonsinusoidal, the equivalent circuit of figure 109 cannot be used due to the different voltage variations that are applied to the machine terminals. Fourier analysis of a nonsinusoidal periodic voltage waveform yields a series of harmonic voltages together with the fundamental harmonic. The effect of these harmonics causes the machine performance to deteriorate by causing extra copper losses, extra stray losses and negative torques. The harmonics that exist in the applied voltage waveform are called time harmonics, whose frequencies are multiples of fundamental frequency, and are described in Chapter 2. It is assumed that the parameters of the equivalent circuit shown in figure 107 do not change with the variation of current such as leakage inductances and resistances.
The magnetic saturation is also assumed as unattained. In addition, the secondary effects of the space harmonics are ignored for the sake of simplicity. The variation of leakage reactance is assumed proportional to the applied frequency. Thus, the impedance of the equivalent circuit of an induction motor varies approximately in direct proportion to harmonic frequency. The effect of high frequency on the machine inductance and the resistance will be explained in the following section, accounting for the skin effect of the conductors in the rotor bars. Although, the leakage reactance of the rotor increases with frequency, it is reduced by the skin-effect factor which is smaller than unity. The skin effect will be described in the next section. The investigation of machine performance is conducted by considering each harmonic component applied to the machine separately. This method of calculation is easier than determining the new value of a reactance for nonsinusoidal voltage waveform. The other method that describes the determination of an inductive impedance for a nonsinusoidal voltage waveform is also based on the superposition of the harmonics by applying them separately to the induction motor. The major drawback in this method is the complexity of the equivalent impedance of the induction machine, which makes the equations very long. It is assumed that a nonsinusoidal voltage waveform, which has only odd harmonics of sine terms, for simplicity, is applied across the terminals of an inductive reactance which is connected in series with a resistance. The impedance offered to the fundamental and to the various harmonics is different. Assuming that the resistance and inductance remain constant for the variations of current, the reactance is proportional to the frequency, and resistance remains unaltered. The r.m.s. value of the applied voltage magnitude is represented by
\[ |V| = \sqrt{|V_1|^2 + |V_3|^2 + |V_5|^2 + \ldots} \quad (149) \]

and the r.m.s. value of the current by

\[ |I| = \sqrt{|I_1|^2 + |I_3|^2 + |I_5|^2 + \ldots} \quad (150) \]

The impedance, \( Z \), is then equal to

\[ |Z| = \sqrt{|V_1|^2 + |V_3|^2 + |V_5|^2 + \ldots} / \sqrt{|I_1|^2 + |I_3|^2 + |I_5|^2 + \ldots} \quad (151) \]

Since the value of impedance changes with frequency,

\[ |I_1| = \left| \frac{V_1}{Z_1} \right|, \quad |I_3| = \left| \frac{V_3}{Z_3} \right| \ldots \text{ can be used.} \]

In addition the harmonic impedance is represented in terms of fundamental impedance by introducing \( C_n \), given as

\[ |Z_1| = \sqrt{R^2 + X_L^2} \quad (152a) \]

\[ |Z_n| = \sqrt{R^2 + n^2X_L^2} \quad (152b) \]

\[ C_n = \sqrt{(R^2 + n^2X_L^2)/(R^2 + X_L^2)} \quad (152c) \]

substituting \( C_n \) and currents in equation (151)

\[ |Z| = |Z_1| \left( \sqrt{|V_1|^2 + |V_3|^2 + |V_5|^2 + \ldots} \right) / \left( |V_1|^2 + |V_3|^2 + |V_5|^2 + \ldots \right) \quad (153a) \]

\[ |Z| = |Z_1| \cdot C_0 \quad (153b) \]

Thus, the impedance shown depends upon the waveform and also upon the relative magnitudes of resistance and inductance of the circuit.

In the first method the impedance of the equivalent circuit of an induction motor can be solved as though the harmonic voltages were applied to the terminals separately. For this method the equivalent circuit is given by figure 113.
In figure 113 the value of the reactances increases in proportion to harmonic order, therefore the magnetising reactance becomes very large in comparison to the other reactances and limits the current flowing on this branch. In fact the low value of voltage magnitude also limits the flux produced by harmonic voltage and causes a small amount of iron losses in the machine in respect to the fundamental component. The slip of the machine for harmonic components can be determined by assuming that the shaft is running at a constant speed close to the fundamental speed. The synchronous speed of any harmonic is greater than that of the fundamental in the order of the harmonic because of its frequency. Therefore, the slip of any time harmonic is given as

\[ s_n = \frac{(n_n - n_r)}{n_n} \]  (154)

where \( n_r \) is the speed of the rotor and equal to \( (1-s)n_s \)

\[ s_n = \frac{1}{n}(1-s)/n \]  (155)

The plus sign denotes the harmonics that produce counter rotating fields in respect to the fundamental field, while the minus sign, the same direction of rotation. The value of slip for harmonics is almost unity, especially for small slips. Therefore, the induction motor can be assumed to operate as if its rotor is blocked and the difference between stator and rotor current is very small. Thus, the circuit can be rearranged by removing the magnetising reactance to the terminals. The error caused by this approximation is very small due to the large value of reactance. The torque produced by harmonic currents is calculated similarly to that of the fundamental. But the direction of harmonic field rotation determines whether the harmonic torque acts as a brake or not. The harmonic slips, greater than unity, cause braking torques, while those smaller than unity produce the torques in the same direction as that of the fundamental.
The mmf harmonics of each time harmonic can be ignored as they would have little effect on the machine's performance due to low winding factors and the low magnitudes of time harmonic voltages.

5. DEEP-BAR EFFECT IN SQUIRREL-CAGE WINDING

Since the induction motor can be assumed to be a moving secondary transformer, the change of frequency in the rotor conductors with slip affects the resistance and inductance. This phenomenon is called skin-effect which takes place when the frequency of a current in a conductor is changed. The current density in the bar is pushed nearer towards the rotor surface as the frequency increases. This results in larger resistance due to concentration of the current in a narrow strip of conductor. The distribution of the current is assumed uniform in the bar and is independent of the skewing effect in the axial direction, reference (22). Meanwhile the slot leakage inductance behaves in exactly the opposite way to bar resistance. The lower the frequency, the deeper the penetration of current obtained. Hence, the flux produced by this current links more of the bar causing more leakage inductance. When the induction motor is at a standstill it operates like a transformer. The rotor frequency is the same as that of the stator and, therefore, the current density is larger in the top of the bar. As the rotor is freed for the rotation, the frequency of the rotor reduces as the slip decreases. As a result of this the current density in the bar decreases, since the current flows in a larger area of the bar than it did before. At a speed near to synchronism, the rotor frequency is very low and the skin-effect diminishes. The current becomes more uniformly distributed over the conductor cross section. The variation of bar impedance with frequency can be obtained if a fundamental Maxwell
From Faraday's Law
\[ \oint \mathbf{E} \cdot \mathbf{d}a = \frac{d\Phi}{dt} \] (156)

where \( \mathbf{E} \) is the electric field intensity. Dividing both sides by area equation (156) is converted into point form.

\[ \nabla \times \mathbf{E} = \frac{\mathbf{B}}{\mathbf{t}} \] (157)

Using Ampere's law and integrating the magnetic field intensity around a closed loop the total current which links the loop is obtained as

\[ \oint \mathbf{H} \cdot \mathbf{d}a = \mathcal{I} \] (158)

\[ \nabla \times \mathbf{H} = \mathbf{U} + \frac{\partial \mathbf{D}}{\partial t} \] (159)

where \( \partial \mathbf{D}/\partial t \) is displacement current density and is normally negligible in a conductor.

Equation (160) is obtained from the replacement of \( \mathbf{U} \), current density, by its equivalent \( \sigma \mathbf{E} \), where \( \sigma \) is conductivity, and from the elimination of \( \mathbf{E} \) in equations (157) and (159).

\[ \nabla \times \mathbf{H} = \mu_0 \sigma \frac{\partial \mathbf{D}}{\partial t} \] (160)

since \( \nabla \) is a vector operator \( \mathbf{H} \) can be represented in rectangular co-ordinates to fit the actual representation of the rotor. It is assumed that the permeability of iron is very large compared with that of the conductor so that the magnetic field intensity in the conductor is very much greater than that in the iron for the same magnetic flux density. Hence, all mmf is concentrated across the conductor. Also it is assumed that the flux lines enter and leave the iron at right angles to the surface. In figure 114 a cage bar of an induction motor with open slots is shown. The variation of the current density is only in the \( Z \) direction and its direction of flow is in the \( y \) direction. The direction of the magnetic field intensity is in \( x \) and its variation is in \( Z \) direction. Hence, the
vectors of \( \mathbf{H} \) and \( \mathbf{U} \) can be represented as
\[
\mathbf{H} = (H_x(Z), 0, 0) \quad \text{and} \quad \mathbf{U} = (0, U_y(Z), 0)
\]  
(161)

Replacing \( \mathbf{H} \) by current density in equation (160)
\[
d^2U/dZ^2 - j\omega^2 U = 0
\]  
(162)

where \( \omega = \sqrt{\mu \sigma_{w}} \)

the solution to this differential equation is
\[
\mathbf{U} = A' \cosh \left( j \frac{3}{4} \omega \right) Z + B' \sinh \left( j \frac{3}{4} \omega \right) Z
\]  
(163)

where \( A' \) and \( B' \) are the constants. In figure 114 the boundary

conditions at the bottom of the slot are given as \( Z = 0 \) and \( H = 0 \).
This shows that \( H \) decreases as the depth is increased to the bottom

of the slot until \( H = 0 \) when \( Z = 0 \), because the current is equal to

zero due to the integral given below.

\[
I = \int_{x'}^{x''} H \, da
\]  
(164a)

i.e. the total current is given by
\[
I = H \, b
\]  
(164b)

To represent this result in terms of current density \( \mathbf{U} \) at \( Z = 0 \),
\( H = 0 \) and, therefore, \( B = 0 \) the equation (157) becomes
\[
\text{Curl} \, \mathbf{U} = 0 \text{ at } Z = 0
\]  
(165)
\[
\frac{\partial U}{\partial Z} = 0 \quad \text{when } Z = 0
\]  
(166)

The derivative of equation (163) with respect to \( Z \) is given by
\[
\frac{\partial U}{\partial Z} = A'' \sinh \left( j \frac{3}{4} \omega \right) Z + B'' \cosh \left( j \frac{3}{4} \omega \right) Z = 0
\]  
(167)

In order to satisfy the above equation \( B'' \) must be zero because \( \cosh \) term cannot be zero. Therefore current density is given by
\[
\mathbf{U} = A' \cosh \left( \frac{1+1}{2} \omega \right) Z
\]  
(168)

where \( J = \frac{1+1}{2} \)
Equation (168) can be represented by

$$\vec{U} = A' \{ \cosh \left( \frac{m'Z}{\sqrt{2}} \right) \cos \left( \frac{m'Z}{\sqrt{2}} \right) + \sinh \left( \frac{m'Z}{\sqrt{2}} \right) \sin \left( \frac{m'Z}{\sqrt{2}} \right) \} \quad (169)$$

Thus, the magnitude and the phase of current density can be obtained at any depth of \(Z\) above the bottom of the slot. \(|U| = A'\) when \(Z = 0\).

To find the impedance per unit length the electric field intensity at the surface of the conductor is divided by the whole current which is the integration of current density over the bar area. Hence,

$$\vec{E} = \frac{A'}{b} \cosh \left( j\frac{1}{2}m'h \right) \quad (170)$$

$$\vec{I} = \int_{0}^{b} A' \cosh \left( j\frac{1}{2}m'Z \right) b \; dZ \quad (171)$$

The impedance is determined from equations (170) and (171) as

$$\vec{Z} = \frac{(1+\jmath)m'/\sqrt{2}}{\frac{b}{\sigma_0} \cosh \left[ (1+\jmath)(m'h/\sqrt{2}) \right]} \quad (172)$$

The imaginary part of equation (172) corresponds to reactance while the real part gives the value of resistance. In order to determine real and imaginary parts the numerator and the denominator are multiplied by the conjugate of the denominator. The multiplication yields the following result:

$$\vec{Z} = \frac{\nu}{\sigma_0 b} \cosh (2\epsilon) + \frac{\jmath \sinh (2\epsilon)}{\cosh (2\epsilon) - \cos (2\epsilon)} + \frac{\jmath \sinh (2\epsilon)}{\cosh (2\epsilon) - \cos (2\epsilon)} \quad (173a)$$

where

$$\epsilon = \nu h \quad (173b)$$

and

$$\nu = \sqrt{\nu_0 \mu_0} \quad (173c)$$

Thus, the ratios given by trigonometric functions represent the factors by which the resistance and the inductance vary.

Separately \(k_r\) and \(k_x\) stand for these factors as given below. For further information see appendix 3.
6. AIRGAP VARIATION IN INDUCTION MACHINE

In order to determine the instantaneous variation of flux crossing the air-gap in terms of angle around the periphery of the rotor, a change of air-gap length is required to find its permeance variation. The determination of the air-gap variation would be simpler if the rotor were unslotted. In this case, the length of air-gap would vary between the values of the bottom of a stator slot to the rotor surface, and a stator tooth surface to the rotor surface. In general, since the rotor has slots the problem becomes more complex than the previous case. The variation of air-gap length is deduced with respect to angle which is defined as the displacement angle between the pole axis and the point at which the air-gap length is to be calculated. The flux crossing the air-gap can then be determined by multiplying the permeance of the air-gap by an mmf equation.

The calculation of air-gap length is performed by assuming the open slots for stator and rotor to avoid complexity in the equations represented here. The process is performed in two different sections. Firstly, the stator and the rotor slots are considered separately with respect to a reference line, which divides the air-gap into two between the facing teeth of the stator and the rotor. Hence, the stator and the rotor air-gap variations are obtained with respect to a reference line. Secondly, these air-gap variations are added to one another at an angle to determine the air-gap length between the stator and the rotor.
In figure 115 one pole-pitch of the induction motor is illustrated with 9 slots on the stator and 8 slots on the rotor, and the reference line in the middle. Fourier series is used to examine the variation of air-gap between the stator and the reference line against the angle. The examination shows that Fourier series of the air-gap contains all sine and cosine terms with a constant term. In general, the Fourier series is given by

\[ F(\theta) = A_0 + \sum_{n=1}^{\infty} \left( A_n \cos(n\theta) + B_n \sin(n\theta) \right) \]  

(176)

where \( A_0 = \frac{4}{\pi} \int_0^{\pi} f(\theta) \, d\theta \), \( A_n = \frac{2}{\pi} \int_0^{\pi} f(\theta) \cos(n\theta) \, d\theta \)

and \( B_n = \frac{2}{\pi} \int_0^{\pi} f(\theta) \sin(n\theta) \, d\theta \)

The slot pitch of stator is the sum of the width of the tooth tip and slot so

\[ t_s + S_s = \frac{\pi}{Q_s / 2P} \]  

(177)

where \( \pi \) is the pole pitch in electrical radians.

The function represented by \( f(\theta) \) need not be expressed over the whole pole pitch, since it is repetitive for every slot pitch. Therefore, one slot pitch is represented by \( 2\pi \) as a whole period and the variation of the air-gap is given accordingly. Since the slot is occupied by conductors whose permeability is almost unity, the conductors in the slots can be assumed to be like air. While using Fourier series it is important that the slot pitch must be multiplied by the number of slots to obtain a series in terms of mechanical angle for the whole stator circumference. Thus, the air-gap variation of the stator is

\[ G_s(\theta) = A_{0s} + \sum_{n=1}^{\infty} \left( A_{ns} \cos(n\theta Q_s) + B_{ns} \sin(n\theta Q_s) \right) \]  

(178)
similarly for the rotor,

\[ q_r(\theta) = A_r + \sum_{n=1}^{\infty} \left\{ A_{nr} \cos(n\alpha_{nr}) + B_{nr} \sin(n\alpha_{nr}) \right\} \]  

(179)

If the rotor would remain stationary relative to the stator, the summation of equations (178) and (179) would yield the variation of air-gap length. But in actual fact, the rotor rotates relative to the stator at a speed depending upon the supply frequency and the applied load. The stator and rotor waves do not remain stationary when the speed is introduced. The rotor-teeth move in the direction of rotation. If the speed of the rotor is \( n_r \) revolutions per minute, the time needed for a point on the rotor to cross the slot-pitch of the stator from one end to the other is given by

\[ t = \frac{(2\pi/\alpha_s)}{(2\pi n_r/60)} \]  

(180)

Hence the frequency of this action is given by

\[ f' = \frac{\alpha_s n_r}{60} \]  

(181)

and it is called slot frequency. Replacing \( \alpha_s \) by \( \alpha_r \), rotor slot frequency is determined. A point on the rotor will progress in the direction of rotation as time passes, therefore, equation (179) is to be modified by adding an extra term to account for the speed of the rotor.

\[ q_r(\theta) = A_r + \sum_{n=1}^{\infty} \left\{ A_{nr} \cos(n\alpha_{nr} + 2\pi n_r t/60) + B_{nr} \sin(n\alpha_{nr} + 2\pi n_r t/60) \right\} \]  

(182)

where \( 0 \leq t \leq \frac{60}{\alpha_s n_r} \)

The integration limits for Fourier coefficients are given by dividing slot-pitch by slot-width. Thus, for stator

\[ \theta_s = \left( S + t_s \right)/S_s \]  

(183)

and using figure 115
Stator ampere-turns are obtained by the product of the stator current and the turns-distribution of the stator winding. The stator current changes with time, while the turns-distribution with the angle around the air-gap periphery. Therefore, the stator ampere-turns have two simultaneous variables which are time and angle. The variation of the stator ampere-turns changes as the current waveform is changed. The turns-distribution of the stator is fixed by the winding diagram. It varies with angle around the air-gap periphery. If the winding has more than one phase, the total ampere-turns at a point on the air-gap is the sum of the ampere-turns of the separate phases. If it is assumed that the stator winding has balanced phases, the turns-distribution of one phase is analysed alone.

The turns-distribution of the winding is obtained by considering the number of conductors in each slot. This produces step waveform. If it is assumed that the conductors of a phase are uniformly distributed in a phase belt, without considering slotting, the turns-distribution is approximated trapezoid. Thus, it is determined in terms of angle around the air-gap periphery. If the stator winding comprises two layers whose numbers of turns and coil spans are identical, one of them is considered. For windings with fractional-pitch, the chording angle is accounted for by the calculations as shown later in this section. Figure 117 shows the turns distribution of a double-layer fractional-pitch winding which is chored by an angle, $\gamma_c$. 

$$
A_{os} = \frac{4}{\pi} \left( \int_{0}^{\theta_s} g_{ss} d\theta + \int_{\theta_s}^{2\pi} g_{st} d\theta \right), \text{the other coefficients are obtained}
$$

in a similar way.

### 7.1 Determination of Stator Ampere-Turns

Stator ampere-turns are obtained by the product of the stator current and the turns-distribution of the stator winding. The stator current changes with time, while the turns-distribution with the angle around the air-gap periphery. Therefore, the stator ampere-turns have two simultaneous variables which are time and angle. The variation of the stator ampere-turns changes as the current waveform is changed. The turns-distribution of the stator is fixed by the winding diagram. It varies with angle around the air-gap periphery. If the winding has more than one phase, the total ampere-turns at a point on the air-gap is the sum of the ampere-turns of the separate phases. If it is assumed that the stator winding has balanced phases, the turns-distribution of one phase is analysed alone.

The turns-distribution of the winding is obtained by considering the number of conductors in each slot. This produces step waveform. If it is assumed that the conductors of a phase are uniformly distributed in a phase belt, without considering slotting, the turns-distribution is approximated trapezoid. Thus, it is determined in terms of angle around the air-gap periphery. If the stator winding comprises two layers whose numbers of turns and coil spans are identical, one of them is considered. For windings with fractional-pitch, the chording angle is accounted for by the calculations as shown later in this section. Figure 117 shows the turns distribution of a double-layer fractional-pitch winding which is chored by an angle, $\gamma_c$. 

$$
A_{os} = \frac{4}{\pi} \left( \int_{0}^{\theta_s} g_{ss} d\theta + \int_{\theta_s}^{2\pi} g_{st} d\theta \right), \text{the other coefficients are obtained}
$$

in a similar way.
The height of a trapezoidal wave is given by the number of turns in one phase belt. The angle shown by $\alpha_b$ represents half the width of the phase belt. Representing the distribution in terms of angle around the air-gap periphery, Fourier expansion of this trapezoidal wave is given by

$$Y_t = \sum_{e=1,3,5}^{\infty} \left( \frac{4h'}{\pi e^2} \right) \sin\left(\frac{2\pi}{2}\right) \sin\left(e(\alpha_b + \gamma_c/2)\right) \sin(e\theta)$$  \hspace{1cm} (184)

where $h'$ was defined in equation (89a).

It is seen that the turns-distribution of top layer does not contain odd and even harmonics of cosine terms. The turns-distribution of the bottom layer is obtained similarly but accounting for the chording angle.

$$Y_B = \sum_{e=1,3,5}^{\infty} \left( \frac{4h'}{\pi e^2} \right) \sin\left(e\frac{\pi}{2}\right) \sin\left(e(\alpha_b + \gamma_c/2)\right) \sin(e\theta)$$  \hspace{1cm} (185)

Consequently, the total ampere-turns is given by

$$Y_{res} = \frac{8h'}{\pi e^2} \sum_{e=1,3,5}^{\infty} \left( \frac{1}{e^2} \right) \sin\left(e\frac{\pi}{2}\right) \sin(e\alpha_b) \cos\left(e\frac{\gamma_c}{2}\right) \sin(e\theta)$$  \hspace{1cm} (186)

The ampere-turns produced in the stator are determined by assuming the balanced currents flowing in the winding which has identical phases. The total ampere-turns were given in equation (90), supposing 3-phase currents in the winding phases. In a three phase balanced system the phase difference between the currents is 120 degrees. The harmonic content of the current waveform is determined by using Fourier analysis. If the time is frozen at an instant and all time harmonics are multiplied by the turns-distribution at a value of $\theta$ and are added up, the total ampere-turns produced by the stator winding is obtained. The repetition of the process at the same frozen time for various $\theta$ values provides the variation of total ampere-turns of the stator winding. If the process is performed by
freezing the value of \( \theta \) and changing the time, in this case the variation of stator ampere-turns is obtained against time. This variation is more important than the previous one which is expressed in terms of \( \theta \). The peak value of the ampere-turns which effects the saturation of an iron member of the machine can be determined. The use of Fourier series in determination of the ampere-turns is to find the time harmonic components of the ampere-turns which results in time harmonic fluxes to increase the saturation level of the iron member. The application of any kind of nonsinusoidal voltage waveform to the machine terminals causes nonsinusoidal current flows in the windings. The mathematical representation of the current comprises solutions of differential equations which make the determination of ampere-turns difficult to obtain. The method described above is used here.

Finally the ampere-turns of a stator phase are given as

\[
F'_{sa} = \left[ I'_{ao} + \sum_{n=1,3,5,...}^{\infty} \left( I'_{aA} \cos(n \omega t) + I'_{aB} \sin(n \omega t) \right) \right] Y'_{res} \sin(e \theta) \\
\]

(187)

where

\[
Y'_{res} = Y_{res}/\sin(e \theta),
\]

(188)

\( I'_{ao}, I'_{an} \) and \( I'_{aB} \) are the Fourier coefficients of the current in phase "a".

The ampere-turns produced by other phases are

\[
F'_{sb} = \left[ I'_{ao} + \sum_{e=1,3,5,...}^{\infty} \left( I'_{aA} \cos(n \omega t - 2\pi/3) \right) + I'_{aB} \sin(n \omega t - 2\pi/3) \right] Y'_{res} \sin(e(\theta - 2\pi/3))
\]

(189)

\[
F'_{sc} = \left[ I'_{ao} + \sum_{n=1,3,5,...}^{\infty} \left( I'_{aA} \cos(n \omega t - 4\pi/3) \right) + I'_{aB} \sin(n \omega t - 4\pi/3) \right] Y'_{res} \sin(e(\theta - 4\pi/3))
\]

(190)
The total ampere-turns is
\[ F'_s = F'_{sa} + F'_{sb} + F'_{sc} \]  \hspace{1cm} (191)

7.2 Rotor Ampere-Turns

The determination of rotor ampere-turns in an induction motor is by the same principle that is used for stator ampere-turns. Although, the same principle is used, the squirrel-cage rotor requires further assumptions for the calculation of ampere-turns. The assumptions are mainly due to current distribution in several parts of the cage. The difference in resistances of a bar and a ring segment is not negligible. Each bar is considered as a phase in the cage rotor. The field that induces the rotor e.m.f. which causes rotor currents is not sinusoidal. Thus, the currents induced in the bars cannot be sinusoidal. This fact complicates the subject since it comprises time harmonics. Nevertheless, the solution of the problem is still possible even if the waveform of the current is nonsinusoidal.

Fourier analysis of the current waveform reveals time harmonics that can be considered separately for superposition purpose. In this section the method will be developed for fundamental current harmonic which is represented by a sine function. The ampere-turns produced by the remaining time harmonics will be represented in general form by referring to the field of fundamental.

The method to determine the turns-distribution of the rotor is similar to that of the stator. The only difference is that every bar is a single conductor and the end-ring is disregarded. In figure 118A one half of the turns-distribution of 32 slots, 4-pole squirrel-cage induction motor is illustrated. The phase difference between two successive bars is
in an electrical angle.

Fourier series of the wave shown in figure 118A without being approximated to trapezoid does not contain cosine terms and even harmonics of sine terms. Hence, it is given by

\[
\gamma_{tr} = \sum_{n=1,3,5}^{\infty} \frac{16N}{\pi e} \cos\beta_0 \cos(\beta_0/2) \cos(\beta_0/4) \sin(e \theta)
\]

where \( \beta_0 = \pi e/4 \)

If it is assumed that the fundamental current in every conductor is sinusoidally varying with maximum amplitude of \( I_m \), the current in the bar which is under the pole-axis is given by

\[
I_r = \frac{I_m}{\sqrt{2}} \sin(\omega t - \phi_s)
\]

The number of bars half way from the pole axis is equal \( (Qr/p) \).

The current in the following bar is given by

\[
I_r = \frac{I_m}{\sqrt{2}} \sin(\omega t - \phi_s + \pi/4)
\]

The current in the bar which is half way from the pole axis is given by

\[
I_r = \frac{I_m}{\sqrt{2}} \sin\{\omega t - \phi_s + \pi/2\}
\]

In general, \( v^{\text{th}} \) bar current is given by

\[
I_r = \frac{I_m}{\sqrt{2}} \sin\{\omega t - \phi_s + (2v-1)p\pi/4\}
\]

In figure 118B the magnetic axis and bars are shown with phase difference in the electrical angle. The rotor mmf is the product of current and turns-distribution. The end-ring resistance will be assumed negligible to make the bar currents equal to those of ring segments. Thus, the bar currents are supposed to flow in ring
segments as well as in the bars. If the rotor is prevented from rotating, the induction motor will operate like a secondary short-circuited transformer. The number of poles of the stator winding is exactly created on the rotor by dividing it into equal areas. These areas come under the stator poles. The bar currents under the poles N and S are in opposite directions and close their loops through end-ring segments. This means that the number of bars under each N pole is equal to that under the S pole, or the number of bars is even. The odd number of rotor bars is generally avoided due to disadvantages in respect to even numbers. The harmonic losses, noise and crawling effect in cage rotors with an odd number of bars, are greater than those with an even number. It is shown that in figure 119 half the number of bars of each pole close their loops with those of neighbouring poles. Therefore, the end-ring segments on the magnetic axis have maximum current density, while those under the pole axis have minimum current density. Considering the end-ring on the magnetic axis, there are four currents at different instantaneous amplitudes and phase angles from one another. The total current on this end-ring is the vector sum of these four currents. Two of the four are produced by the voltages induced in the bars, 2 and 2', while the remainder are by 1 and 1'. The rings shown by A carry the sum of the currents of 1 and 1'. These currents are separately induced in their own bars and close their loops as shown in figure 119. Hence, the total current in the end-ring A is the vector sum of 1 and 1'.

If the fundamental harmonic current is assumed sinusoidal, the phase differences of the rotor-bar currents and end-ring currents are obtained by taking the pole axis of the S-pole as a reference. The currents in various parts of the cage are determined as follows:
The current in bar 1 is given by

\[ i_1 = \frac{i_{ml}}{\sqrt{2}} \sin(sw - \phi_s + \pi p/Q_r) \]  \hspace{1cm} (198)

while in bar 1' is given by

\[ i_1' = \frac{i_{ml}'}{\sqrt{2}} \sin(sw - \phi_s + (\pi - \pi p/Q_r)) \]  \hspace{1cm} (199)

Since \( \sin x = \sin (\pi - x) \)

\[ i_1 = i_1' \]  \hspace{1cm} (200)

Therefore, the instantaneous current in ring-segment A is given by

\[ i_A = \frac{1}{\sqrt{2}} (i_{ml} + i_{ml}') \sin(sw - \phi_s + \pi p/Q_r) \]  \hspace{1cm} (201)

The currents in bars 2 and 2' are given by

\[ i_2 = \frac{i_{m2}}{\sqrt{2}} \sin(sw - \phi_s + (\pi/2 - \pi p/Q_r)) \]  \hspace{1cm} (202)

\[ i_2' = \frac{i_{m2}'}{\sqrt{2}} \sin(sw - \phi_s + (\pi/2 + \pi p/Q_r)) \]  \hspace{1cm} (203)

Since \( \sin(\pi/2 - x) = \sin(\pi/2 + x) \), the current in ring-segment B is given by

\[ i_B = \frac{1}{\sqrt{2}} \left[ (i_{ml} + i_{ml}') \sin(sw - \phi_s + \pi p/Q_r) + (i_{m2} + i_{m2}') \sin(sw - \phi_s + \pi/2 - \pi p/Q_r) \right] \]

The fields produced by these currents are in opposite directions about the magnetic axis. The resultant flux crossing the air-gap at both ends of the rotor is reduced by fluxes caused by end-ring currents.

The currents induced under the pole-N flow in opposite direction to those of pole S at the corresponding end-rings. Their phase angles are 180° lagging those of pole-S. Therefore, the current induced under pole-N has a negative sign.

The rotor ampere-turns is the product of rotor current and the turns-distribution.
Thus, the resultant ampere-turns in the air-gap are given by the vectorial sum of equations (191) and (204).

\[
\mathbf{F}'_{\text{res}} = \mathbf{F}'_{s} + \mathbf{F}'_{r}
\]  

(205)

Since the stator and rotor ampere-turns can be obtained numerically for a given \( \theta \), the algebraic sum of these two produces the resultant ampere-turns. The phase difference between stator and rotor currents is given in equation (194).

8. DETERMINATION OF LINE CURRENT

The determination of line current of an inverter-fed induction motor requires the solution of differential equations. The motor equations are derived with the assumptions that the losses of magnetic core, effects of saturation, and space harmonics are neglected. Also the stator windings are perfectly coupled to one another, but not to the rotor windings. Meanwhile, the rotor winding of the squirrel-cage is replaced by its three phase equivalent winding. The position of the rotor in respect to the stator is denoted by \( \theta \) electrical degrees.

The equivalent circuit of an induction motor gives the self-inductance of stator and rotor by summing the appropriate leakage inductances with mutual inductance of one phase of the stator and the rotor. This is the apparent self-inductance of the motor and is given by

\[
L_{\text{app}} = L_{s} + M
\]  

(206)

where \( L_{s} \) and \( M \) are, respectively, leakage inductance of one stator phase and mutual inductance between one stator and rotor phase. If the stator winding is considered alone without having a coupling with the rotor winding, the currents flowing in a three-phase winding would
produce following flux linkage for the phase "A", figure 120.

\[ \psi_a = I_a L_{ss} - \frac{1}{2} I_b L_{ss} - \frac{1}{2} I_c L_{ss} \]  

(207)

where \( I_a, I_b \) and \( I_c \) represent the maximum amplitudes of the currents and the phases are assumed symmetrical and have identical impedances.

If the currents in phase-B and in phase-C are given in terms of \( I_a \),

the flux linkage associated with phase A is given as

\[ \psi_a = I_a L_{ss} \left( \frac{1}{2} \cos(120) - \frac{1}{2} \cos(240) \right) \]  

(208)

Hence, the flux linkage is given by

\[ \psi_a = \frac{3}{2} I_a L_{ss} \]  

(209)

and

\[ L_{app} = \frac{3}{2} L_{ss} \]  

(210)

This shows that the flux linkage of a phase is increased 50\% by the neighbouring phases, through mutual inductances. The self-inductance of phase-A alone becomes two-thirds of the apparent inductance of the 3-phase system. The self-inductance of the rotor is obtained similarly to equation (210). From equation (210),

\[ L_{ss} = \frac{2}{3} L_{app} \]  

(211)

If the following assumptions are made; the stator winding is excited, the rotor is rotating and the angle between the stator phase-A and the rotor phase-A is zero, the flux linkage associated with phase-A is given by

\[ \psi_a = \frac{3}{2} L_{ss} I_a + \frac{3}{2} M_{sr} I_{sr} \]  

(212)

where the first term shows the flux linkage of the stator phase-A caused by the other stator phases, while the second represents the flux linkage caused by the mutual inductance, \( M_{sr} \), between the stator phase-A and the rotor phases. Therefore, the mutual inductance of the equivalent circuit, \( M \), is given by
\[ M = \frac{3}{2} M_{sr} \]  \hspace{1cm} (213)

If equations (211) and (213) are substituted in equation (206), the leakage inductance is obtained as

\[ L_s = \frac{3}{2} (L_{ss} - M_{sr}) \]  \hspace{1cm} (214)

The mutual inductance between the stator phases is given by

\[ L_{sm} = L_{ss} \cos(120) \]  \hspace{1cm} (215)

The induction motor is represented by its stator and rotor windings as in figure 121 with the consideration of the rotor position with respect to the stator. The voltage equations are written in instantaneous values considering the couplings between the phases as well as those of stator and rotor. Hence, six differential equations are established to represent the steady-state operation of the motor. The solution of these, six differential, equations with six unknowns are solved by the numerical method in the computer. The equations of the motor are given by

\[ V_{ab} = I_{ab} R + L_{ss} \frac{dI_{sa}}{ds} + L_{sb} \frac{dI_{sb}}{ds} + L_{sc} \frac{dI_{sc}}{ds} + M_{ra} \frac{dI_{ra}}{dt} + M_{rb} \frac{dI_{rb}}{dt} + M_{rc} \frac{dI_{rc}}{dt} \]  \hspace{1cm} (216a)

It is considered that the windings of the stator and the rotor are balanced, the mutual inductance between the stator phases is obtained as

\[ L_{ssm} = (M_{aa} + L_{sa}) \cos 120^\circ \]  \hspace{1cm} (216b)

where \( M_{aa} \) and \( L_{sa} \) are the mutual inductance and the leakage inductance of the equivalent circuit for phase-A, respectively. The mutual inductance between the stator phase-A and the rotor phase-B is given by

\[ M_{ab} = M_{aa} \cos(120^\circ) \]  \hspace{1cm} (216c)
where $M_{aa}$ is the mutual inductance between the stator phase-A and the rotor phase-A.

Hence, the voltage equations are given, when $\theta$ is equal to zero, as

$$V_{ab} = I_{sa} R_s + L_s \left[ \frac{dI_{sa}}{dt} - \frac{1}{2} \left( \frac{dI_{sb}}{dt} + \frac{dI_{sc}}{dt} \right) \right] + M_{aa} \left[ \frac{dI_{ra}}{dt} - \frac{1}{2} \left( \frac{dI_{rb}}{dt} + \frac{dI_{rc}}{dt} \right) \right]$$

(217)

Similarly the voltage equations for other phases

$$V_{ca} = I_{sc} R_s + L_s \left[ \frac{dI_{sc}}{dt} - \frac{1}{2} \left( \frac{dI_{sa}}{dt} + \frac{dI_{sb}}{dt} \right) \right] + M_{aa} \left[ \frac{dI_{rc}}{dt} - \frac{1}{2} \left( \frac{dI_{ra}}{dt} + \frac{dI_{rb}}{dt} \right) \right]$$

(218)

$$V_{bc} = I_{sb} R_s + L_s \left[ \frac{dI_{sb}}{dt} - \frac{1}{2} \left( \frac{dI_{sa}}{dt} + \frac{dI_{sc}}{dt} \right) \right] + M_{aa} \left[ \frac{dI_{rb}}{dt} - \frac{1}{2} \left( \frac{dI_{ra}}{dt} + \frac{dI_{rc}}{dt} \right) \right]$$

(219)

The mutual coupling between stator and rotor phases varies with the position of rotor as it rotates. Therefore, the mutual inductance, $M_{aa}$, must be introduced as a function of $\theta$ which shows the position of rotor in regard to stator. Equation (217a) can be re-established by introducing $\theta$ into equation (217c)

$$V_{ab} = I_{sa} R_s + L_s \left[ \frac{dI_{sa}}{dt} + M_{aa} \cos(\theta + 240) \frac{dI_{ra}}{dt} \right]$$

(220)

Similarly $V_{ca}$ and $V_{bc}$ can be represented in this form. The rotor equations are obtained by the same method but the voltages are zero.

$$\frac{dI_{ra}}{dt} + M_{aa} \cos(\theta + 240) \frac{dI_{ra}}{dt}$$

(221)
The equations for phases b and c are obtained by the same method. The representation of these six equations by matrices is given as follows

\[
\begin{bmatrix}
V \\
\end{bmatrix} = \begin{bmatrix}
R \\
\end{bmatrix} \begin{bmatrix}
I \\
\end{bmatrix} + \begin{bmatrix}
L \\
\end{bmatrix} \frac{dI}{dt}
\]

(222)

It must be noted that the value of the determinant of matrix \(L\) is a small number because of the self and mutual inductances of the induction motor. Therefore, the numerical accuracy of the inverted matrix is to be preserved to avoid the over-flow in the computation.

The determinant of matrix \(L\) is given by

\[
\text{Det} \begin{bmatrix} L \end{bmatrix} = (L_{ss} + 2L_{sm})(L_{rr} + 2L_{rm}) \left( (L_{ss} - L_{sm})(L_{rr} - L_{rm}) - (1.5M_{aa})^2 \right)^2
\]

(223)

The determinant of \(L\) should not be zero unless the leakage inductances between the stator and the rotor is zero.

The matrix of \(L\) is given by

\[
\begin{bmatrix}
L_{ss} & L_{srm} & L_{ssm} & M_{aa} \cos(\theta) & M_{aa} \cos(\theta + 120) & M_{aa} \cos(\theta + 240) \\
L_{srm} & L_{ss} & L_{ssm} & M_{aa} \cos(\theta + 240) & M_{aa} \cos(\theta) & M_{aa} \cos(\theta + 120) \\
L_{ssm} & L_{ss} & L_{sm} & M_{aa} \cos(\theta + 120) & M_{aa} \cos(\theta + 240) & M_{aa} \cos(\theta) \\
M_{aa} & M_{aa} \cos(\theta) & M_{aa} \cos(\theta + 240) & L_{rr} & L_{rrm} & L_{rrm} \\
M_{aa} & M_{aa} \cos(\theta + 120) & (\theta + 240) & L_{rrm} & L_{rr} & L_{rrm} \\
M_{aa} & M_{aa} \cos(\theta + 240) & (\theta + 120) & L_{rrm} & L_{rr} & L_{rrm}
\end{bmatrix}
\]

(224)

\(\theta\), the position of rotor can be expressed in terms of mechanical speed and slip. The rotor covers \((1-s)\omega_s\) radians per second hence the position of rotor at any time, \(t\), is given by

\[
\theta = 180t(1-s)\omega_s / \pi
\]

(225)
The process for digital computation is to calculate the flux linkage in the beginning and then the current after the integration of flux linkage.

\[
\frac{d\psi}{dt} = V - RI
\]  

(226)

and

\[
I = [L]^{-1}\psi
\]  

(227)

This method can be used for balanced and unbalanced operating conditions of the induction motor. The variables used are the physical quantities of the motor which are obtained from the equivalent circuit. The variation of machine parameters can be directly used due to any change of operating condition.
CHAPTER V

LOSES OF AN INDUCTION MOTOR
1. **GENERAL**

The difference between the input and the output powers is known as the losses. In an electrical machine the losses are separated into two main groups. They are electrical and mechanical losses. The losses caused either by electrical or mechanical energy are dissipated as heating energy. The mechanical energy loss is very small in comparison with losses caused by electrical energy so that it can be considered of secondary importance. The electrical losses are caused by currents flowing in conductors and the flux density in the magnetic circuit. The electrical losses can be resolved into three major components. They are copper losses, iron losses and stray losses. The largest parts comprise copper and iron losses. The sinusoidal voltage waveform applied to the terminals of an induction motor provides the conventional operation. The losses obtained from this operation are stated above. The determination of these losses is given in references (14) and (15). The application of nonsinusoidal voltage waveform instead of sinusoidal voltage waveform changes the motor performance by increasing the total losses due to time harmonic voltages. The aim of this chapter is to examine these losses separately and to determine the extra losses caused by harmonic voltages and currents.

2. **COPPER LOSSES**

The copper losses are the unavoidable part of the losses for electrical devices. They are the consequence of the electrical resistance of the conductor to the electrical current. The copper losses in an induction machine can be considered separately in the stator and rotor. The stator winding is made with conductors of small diameters and, therefore, the skin-effect phenomenon can be neglected for high
frequencies. The only factor for stator resistance is the temperature effect that changes the resistivity of the conductor. The new value of the resistance is given by

\[ R_v = R_{20} \left( \frac{234.4 + v}{254.4} \right) \]  

(228)

where 234.4 is the inferred absolute zero, \( R_v \) and \( R_{20} \) represent the resistance of the conductor at \( v \) and 20°C, respectively, reference (23). The resistance at \( v \) °C is calculated in respect to that at 20°C which is normal room temperature. In the calculations of equivalent circuit the stator resistance is assumed to be constant for all the frequencies. The copper losses in the rotor of an induction motor with squirrel-cage winding cannot be considered as they are in the stator. The rotor current that produces the copper losses is dependent on the slip. In addition, the deep-bar effect changes the value of resistance by driving the current density nearer to the surface of the conductor. Therefore, the rotor resistance has to be calculated for different frequencies of operation, especially for harmonics. The skin-effect factors given in chapter 4, section 4 have to be multiplied by the d.c. resistance value of the rotor to determine the new resistance. The temperature correction of the machine is made by equation (228). Thus, the new value of rotor resistance which is corrected for temperature and frequency changes is given by

\[ R_r = R_{dc} R_r \left( \frac{234.4 + v}{254.4} \right) \]  

(229)

Since the theory to calculate the induction-motor performance is based on the separate application of harmonic voltages, the total copper losses, therefore, are obtained by summation of those associated with every harmonic. The solution of the equivalent circuit for every harmonic provides its stator and rotor currents, chapter 4, section 1. Thus, the copper losses associated with time harmonics for the fundamental space harmonic are given by
The copper losses that occur due to space harmonics will be considered later in section 4 of this chapter.

3. **IRON LOSSES**

The magnetic circuit of the induction motor is another loss source. The determination of loss in the magnetic circuit is a complicated problem. The behaviour of a magnetic material in a magnetic field changes as the magnetic field intensity, $H$, is varied. The magnetic flux density, as a result of increase of $H$, increases until the saturation of the material is reached. The precise calculation of magnetic-core losses of a saturated electrical machine is an unsolved problem due to the nonlinear variation of the material's permeability.

In an induction motor the resultant field produces different flux densities in various parts, which consequently cause losses in the iron. These losses are known as hysteresis and eddy-current losses. The determination of the sum of these losses is given by

$$W_{Fe} = \beta_f \cdot f \cdot V' \cdot \rho' \cdot 10^{-3}$$

(231)

where $\beta_f$ is the loss factor of the material in terms of Watts/kg and can be obtained from magnetisation characteristic of the material, reference (24). The iron losses caused by time harmonics are calculated by using the same formula, with the assumption that the magnetic circuit of the machine is unsaturated. Therefore, the magnetisation curve is linear and the losses in the magnetic material increase in proportion to frequency. The value of the iron losses is dependent of the flux density which changes in several parts of the motor. Therefore, the losses which occur in the stator core are
different from those in the rotor teeth, stator teeth and rotor core. The losses of all these members have to be calculated separately. But a good approximation can be made by taking the stator volume as a whole if the flux densities of stator core and teeth are almost equal. This approximation cannot be used for the losses which occur in the rotor core and teeth, since the flux density of the rotor teeth is almost twice that of the rotor core. The iron losses of a tooth are calculated by considering the maximum flux density due to rapid saturation of tooth-tip and body. In fact, the saturation of tooth body does not occur as rapid as in a tooth-tip. It is because the area of the body is larger than that of the tooth-tip.

The flux densities in several parts of the motor are calculated, considering that there is no leakage flux in the stator and rotor teeth. The induced voltage in the stator winding is given by

\[ E_s = 4.44 \times \frac{k_{ws}}{N_f} \phi \]  

hence, the air-gap flux density is obtained as

\[ B_g = \frac{2\phi}{\pi DL'} \]  

The flux density on the stator teeth is, reference (24)

\[ B_{st} = \frac{\pi}{2} \frac{B_s L'_a}{N_L n} \]  

while on the rotor

\[ B_{rt} = \frac{\pi}{2} \frac{B_r L'_a}{N_L n} \]  

In the stator and rotor core, respectively,

\[ B_s = \frac{\phi}{d_{cs} L'} \]  

\[ B_r = \frac{\phi}{d_{cr} L'} \]  

where \( d_{cs} \) and \( d_{cr} \) are the mean net-length of flux-path in stator and rotor cores, respectively. These lengths are determined by excluding the strip of teeth of the appropriate member of the motor.
In equation (234) and (235) the widths of the stator and rotor teeth are determined by Simpson's rule which approximates the flux density in a tooth-body by taking one-third of the summation of minimum flux-density with twice the maximum flux-density. In other words, the width of the tooth is taken as one-third of the summation of maximum width with twice the minimum width.

The superposition of iron losses for time harmonics is possible if the magnetic core of the induction motor is not saturated. The sum of the iron losses in such a case is given by

\[ W_{Fe} = \sum_{n=1,5,7}^{\infty} n.f.p'10^{-3}(\beta_{fnsc}'V'_{so} + \beta_{fnrt}'V'_{rt} + \beta_{fnrc}'V'_{rc}) \]  

(238)

where \( \beta_{fnsc}' \), \( \beta_{fnrt}' \) and \( \beta_{fnrc}' \) respectively, represent the loss factors of iron for the flux densities of stator core, rotor teeth and rotor core. \( V'_{so}, V'_{rt}, V'_{rc} \) represent respectively, the volumes of stator core, rotor teeth and rotor core.

The low magnitudes of applied harmonic voltages produce low fluxes in the air-gap. Therefore, their loss factors are determined by considering the linear variation of magnetisation characteristics for the low magnetic flux densities. The effects of these low harmonic fluxes on saturation are neglected. It is assumed that the saturation of magnetic circuit is not affected by them, but they produce iron losses.

The effects of space harmonics on iron losses are neglected because of the very low harmonic voltage magnitudes. In chapter 4, the voltage magnitudes of these harmonics are calculated with respect to that of the fundamental space harmonic. It was found that the voltage magnitude of a space harmonic was inversely proportional to the square of harmonic order. Therefore, the losses associated with these harmonics are not considered in the calculations. If they are considered for
5th time harmonic, the voltage magnitude of any space harmonic is given by
\[ v_{5e} = \frac{V^2}{5} \left( \frac{k_2^2}{e} \frac{2}{\pi} \frac{k_1^2}{w_0} V_1 \right) \]  \hfill (239)

4. **STRAY LOSSES**

These losses constitute the minor part of the total losses which occur in an induction motor. They are caused by additional eddy-currents and core losses, by the increase of air-gap leakage fluxes with load, and by high frequency pulsations of these fluxes. These flux pulsations produce high-frequency counter currents in the rotor which will produce, in turn, flux pulsations on the stator. The iron losses occur, in an induction machine, apart from those calculated in previous section, are due to high frequency pulsation of fluxes in the steel and on the surface of stator and rotor teeth. The effect of these losses on the machine performance can be ignored with respect to the total losses constituted by copper, iron and friction and windage when a sinusoidal voltage waveform is applied to the induction machine. But the application of a nonsinusoidal voltage wave with its time harmonics produce extra stray losses as they produce copper and iron losses. These losses occur at high frequencies and cause over heating in the machine.

4.1 **Surface Losses**

Surface losses are produced by eddy currents induced in the laminations due to the high-frequency pulsations in the air-gap flux. These pulsations are caused by the slot openings and also steps in the mmf distribution because of the concentration of current in the slots. The slot harmonic field is due to permeance variation caused by the slot openings and the steps in mmf wave at each tooth. The surface
losses are given, reference(15), for the fundamental time harmonic on the stator as

\[ W_{ss} = 1290 \frac{D.L(I_s)^2 B_s^2}{Q_{ss} I_0} C_{ss} \Lambda_r \]  \hspace{1cm} (240)

for the losses which occur on the rotor

\[ W_{sr} = 1290 \frac{D.L(I_s/Q_{sr})^2 B_{sr}^2}{Q_{sr} I_0} C_{sr} \Lambda_s \]  \hspace{1cm} (241)

where \( C_{ss} \) and \( C_{sr} \) can be determined from the characteristic given in reference (15). Although this figure is in watt/in\(^3\), it can be used in equations (240) and (241) without the need for conversion to the metric system. If \( C_{ss} \) and \( C_{sr} \) are given as watt/m\(^3\) the constant is replaced by \( 7.89 \times 10^7 \). The slot frequencies of stator and rotor are given for nth time harmonic, respectively,

\[ f'_s = n f_s Q_s/p \]  \hspace{1cm} (242a)

\[ f'_r = n f_s Q_r/p \]  \hspace{1cm} (242b)

The constants, \( C_{ss} \) and \( C_{sr} \), are determined by using the frequencies which are given by equations (242a) and (242b) as in reference (15). Equations (240) and (241) are also used to evaluate harmonic losses, if load and no-load currents of time harmonics are calculated as they are given in chapter 4. Since the equivalent circuit is considered to be a secondary short-circuited transformer and the magnetising branch is shifted to the terminals the required quantities can be determined.

4.2 High Frequency Rotor Pulsation Losses

These losses occur due to the penetration of high frequency fluxes into the rotor teeth. These fluxes cause circulating currents in the rotor bars. If the rotor and stator have an equal number of slots, no current is induced in the rotor winding. In this case since the rotor slot-pitch is equal to 360 degrees of the stator slot harmonics, no
harmonic field would penetrate to the rotor teeth, figure 121. Thus, surface losses would result in high frequency rotor losses. When the number of rotor slots is different from that of stator, the harmonic fluxes penetrate to the rotor teeth to induce voltage in the bars. The current produced tends to cancel the inducing flux. Thus, the rotor $I^2R$ and teeth pulsation-losses increase considerably. Therefore, the number of rotor slots which determines the rotor bar resistance is important. As it is given in reference (15) the losses can be obtained by

$$W_{hf} = C_{hf} \cdot m I_s^2 k_r R_b$$  \hspace{1cm} (243)

where $C_{hf}$ represents the loss factor and is obtainable from reference (14). The frequency of the current flow in the rotor bar is equal to that given by equation (242a).

### 4.3 Losses Due to Belt Harmonics

The existence of space harmonics together with time harmonics which are supplied by the air-gap mmf and the applied voltage complicate the flux density distribution. The calculation of losses due to phase-belt harmonics is achieved by the superposition method. The losses associated with phase-belt harmonics are obtained by applying the formula given below for every time harmonic. It is possible to assume the induction motor as a secondary short-circuited transformer, since the secondary impedance is small in comparison with the magnetising reactance for the same order of space harmonic. By this approximation, as suggested in reference (15), the losses are calculated by

$$W_B = m I_s^2 k_r R_b (k_{w_5}^2 + k_{w_7}^2)/k_{w_1}^2$$  \hspace{1cm} (244)

The frequencies of the currents that cause $I^2R$ loss in the rotor winding are determined as in table 2 of chapter 4, i.e.
\( f_r = f_s \{(6k + 1) s - 6k\} \) \( (245) \)

where \( k \) is an integer.

The effect of end-ring resistance is omitted because its value is inversely proportional to the square of the number of pole pairs. The number of pole-pairs produced by the space harmonics is proportional to the order of the harmonic. Thus, the determination of losses caused by phase-belt harmonics for different time harmonics is given by

\[
W_B = m R_b \left( k_x^2 + k_y^2 \right) \left\{ \sum_{n=1}^{\infty} \frac{I_{sn}^2}{k_w^2} \right\} \quad (246)
\]

### 4.4 Losses Due to End Winding

The losses in the end structure of the machine are caused due to leakage fluxes. The flux in the end part of the machine has two components: peripheral and axial. The losses associated with this part of the motor are caused mainly by the peripheral component, reference (15). These leakage fluxes enter the laminations in the axial directions and also penetrate the other metal parts. They are larger when the coil overhang is long. The formula for the calculation of losses due to end winding is given by

\[
W_E = 1.89 \times 10^{-6} m^2 N^2 D f_s \log(1 + \frac{U_0^2 Y_1 Y_2}{p^2}) \quad (247)
\]

where \( U_0, Y_1 \) and \( Y_2 \) are shown in figure 122 and \( U_0 \) can be calculated from the design data of the machine, reference (14).

Total losses due to various time harmonics are obtained by superposing the end losses for each harmonic, as if the machine were considered to be excited by a series of voltage sources. Thus, equation (247) is given by

\[
W_E = 1.89 \times 10^{-6} m^2 N^2 D \log(1 + \frac{U_0^2 Y_1 Y_2}{p^2}) \left\{ \sum_{n=1}^{\infty} \frac{I_{sn}^2}{k_w^2} n \right\} \quad (248)
\]
4.5 Skew Leakage Losses

If the rotor slots are skewed, there is a phase displacement between the fundamental mmf waves of the stator and the rotor at the two ends of the core, reference (16). The phase difference between the stator and rotor mmf increases as the distance is increased from the core centre. Since the load current component is nearly 90 degrees displaced from the magnetising component, the increase in flux due to the skew is greatest along the axial line where the fundamental flux is zero. Therefore, the skew fluxes can be accounted as independent of no-load flux. Representing skew by $\sigma$ in terms of stator slot-pitch, the losses due to the skew leakage are given by

$$W_{sk} = \frac{n^2}{3} \left( \frac{\sigma I'}{Q_s I_{om}} \right)^2 (W_{Fe})$$  \quad (249a)

where $I'$ is the vector difference between the load current and the magnetising component of this current, $I_{om}$, reference (15).

The total skew-losses caused by skewing for various harmonics are calculated similarly,

$$W_{sk} = \frac{n^2}{3} \left( \frac{Q_s}{Q_s} \right)^2 \left\{ \sum_{n=1}^{\infty} \left( \frac{I'}{I_{omn}} \right)^2 \left( W_{Fe} + W_{sro} \right) \right\}$$  \quad (249b)

where $W_{sro}$ can be obtained from equation (241) by setting the bracketed term equal to unity, reference (15).

5. EXPERIMENTS NEEDED TO EXAMINE MACHINE PERFORMANCE

The experiments needed to determine the performance of an induction motor are well known tests from which its parameters can be obtained. They are no-load, blocked-rotor and load tests. In addition, the reversed rotation and rotor removed tests are useful, if the losses are to be segregated. The comparison of induction motor performances between the applications of PWM waveform and sinusoidal voltage
waveform is obtained, if the motor is run under identical conditions. The tests which are described in this section will give insight into the evaluation of machine parameters for both sinusoidal and PWM voltage waveform applications.

5.1 No-Load Test

The aim of the test is to determine the machine parameters together with the blocked-rotor test. The no-load test of the machine provides iron and copper losses in the stator. It is assumed that the rotor current is very small, even negligible due to very low slip. Hence, the motor can be considered as secondary open circuited. The current drawn from the supply provides the flux in the air-gap which produces iron losses in the stator. The iron losses of the rotor can be disregarded because of the very low induced rotor frequency, which is almost equal to 1 or 2 Hz. Therefore, the iron losses in the no-load test of the machine are assumed to belong to stator only. The no-load slip for fundamental harmonic is small, but it is almost equal to unity for other harmonics. Therefore, the operation of the induction motor is regarded as if it were at a standstill for time harmonics. Since the impedance of a magnetising branch is very large in comparison with those of the stator and the rotor, the current on this branch does not affect the magnitudes of the stator and the rotor currents, which are almost equal to one another. Thus, the effect of time-harmonics on a no-load operation is the increase of copper losses. The change of the iron losses due to nonsinusoidal voltage waveforms is given in appendix 4. It is shown that the increase of the iron losses depends upon the applied voltage waveform. The friction and windage losses of the motor are assumed unchanged for the speeds which correspond to the voltage changes made during the course of the tests. The friction and windage losses are obtained from the
extrapolation of the characteristic of power input against the phase voltage, when the motor is supplied with a sinusoidal voltage waveform.

5.2 Blocked-Rotor Test

The aim of this test is to avoid iron losses as much as possible and to obtain the losses due to dissipation in the copper. The operation of an induction motor when its rotor is prevented from rotation is the same as a secondary short-circuited transformer. The motor draws a large current from the supply, if the stator is connected to rated voltage, which burns the insulation of the winding. To avoid this dangerous case the voltage applied is adjusted so that the rated current flows in the winding. The low magnitude of the applied voltage is not enough to produce the rated flux which would be produced if the applied voltage were equal to the rated value. Hence, the iron losses in this test can be ignored and the total losses of the motor is caused by dissipation in the conductors. The equivalent circuit for all harmonics is represented by removing the magnetising branch to the terminals of the machine, since the slip is always near to unity for all the harmonics. Thus, the ratio of phase voltage to phase current gives the sum of stator and rotor impedances. The segregation of harmonic losses in no-load and blocked-rotor tests is not possible unless the motor is run separately with applied sinusoidal voltage waveform, whose magnitude is equal to that of the fundamental harmonic of nonsinusoidal voltage waveform, under the same operating conditions. These conditions are determined, in general, by the temperature of the winding, load and the frequency of the supply.

5.3 Load Test

The aim of this test is to determine the performance of an induction machine for sinusoidal and nonsinusoidal voltage waveform applications.
The comparison of performances for two different waveform-applications reveals the losses produced by time harmonics in the machine. Since the sinusoidal-voltage waveform application is the conventional operation of the machine and does not permit continuous speed control, unless a motor-alternator set is employed, the variation of supply frequency is achieved by an inverter, which provides pulse-width modulated voltage-waveform. This waveform inherently consists of time harmonic voltages which lead to deterioration of motor efficiency when compared with identical running conditions of sinusoidal-voltage waveform application. A separately excited d.c. generator which is connected to a d.c. circuit is used to load the induction motor. The difference between input and output powers of the induction motor is the losses which occur in several parts of the machine. The methods to calculate these losses have been given in this chapter. The harmonic equivalent circuits of the induction motor are represented like those in blocked-rotor tests because of the large values of slip. The load test of an induction motor is performed at various frequencies and with different PWM waveforms. The comparison of an induction motor performance between two different PWM-voltage waveform applications is reached when these waveforms are used at the same fundamental-voltage magnitude and frequency. Hence, the losses caused in the machine are used to determine the better waveform.

The efficiency of the motor under load condition is given by substituting equations (230), (238), (240), (243), (246), (248) and (249b) in equation (250a).

\[ \eta = \frac{P_i - (W_{cu} + W_{Fe} + W_{ss} + W_{sr} + W_{mf} + W_B + W_{Fe} + W_{sk} + W_{fr})}{P_i} \times 100 / P_i \]  

(250a)

The efficiency is given by

\[ \eta = \frac{P_i - W_L}{100 / P_i} \]  

(250b)
where
\[ W_L = W_{cu} + W_{Fe} + W_{ss} + W_{sr} + W_B + W_{hf} + W_E + W_{sk} + W_{FW} \] (250c)

The efficiency of the induction motor is, thus, given as a percentage.

The output power of the motor is expressed as the sum of all harmonic contributions, which is given by
\[ P_{op} = P_1 - W_L \] (250d)
which is also determined by equation (87) for each harmonic component. But the skin-effect factor and harmonic slip are to be considered for calculations. Hence, the total output-power is
\[ P_{op} = \sum_{n=1,5,7,\ldots}^{\infty} \left\{ I_{rn}^2 R_r k_r \frac{(1-s_n)/s_n}{s_n} \right\} \] (250e)
where \( k_r \) is calculated from equation (172b) and (174). \( s_n \) is given by equation (155).
CHAPTER VI

EXPERIMENTS CONDUCTED ON INDUCTION MACHINE
1. **THE EXPERIMENTAL EQUIPMENT**

The tests for PWM and sinusoidal voltage waveform applications are performed by using a separately excited d.c. generator mechanically coupled to the induction motor. The d.c. generator is connected to a d.c. circuit where the applied load is variable. This simple arrangement is enough to obtain the losses of an induction motor with good accuracy for any kind of voltage waveform applications. Although, the segregation of losses in the machine is not of interest, the total losses and the stray losses which contribute to a small part of the total losses, can be calculated by given formulas, chapter 5. The input-power of the induction motor is supplied from a motor-alternator set for sinusoidal voltage waveform operation, whereas it is obtained from inverter for PWM voltage waveform operation. The former set is able to supply power to the machine at frequencies higher than 15 Hz. The output voltage of the alternator is controlled by changing the field excitation. Therefore, the applied voltage to the induction motor can be adjusted to any magnitude as desired. An inverter supplied by constant d.c. link voltage cannot have such variation of output voltage. The width of the pulse determines the magnitude of the harmonic voltages. The word voltage will represent the fundamental voltage magnitude in the remainder of the thesis. The variation of output voltage is obtained if the pulse-width is changed. But this results in different harmonic content of the supplied voltage waveform. The possibility of obtaining a variable d.c. input voltage is achieved, if the d.c. voltage of the inverter input is obtained from an a.c. power source through a rectifier bridge. By employing a transformer which has variable output-voltage before the rectifier bridge, it is possible to change the d.c. link voltage of the inverter. This system is useful if the voltage magnitude of the a.c. line does not drop below a certain value at which the inverter must be able to
produce the PWM voltage waveform having a voltage equal to that of sinusoidal voltage waveform supplied by a motor-alternator set. In practice this sort of application may not be possible due to variations of a.c. voltage magnitude produced by the connections of large loads to the same circuit. The input voltage of the inverter, therefore, decreases due to large voltage drops in the supply, causing voltage to decrease below the required value. Therefore, the desired input voltage is obtained by the use of a variable output transformer.

The output leads of the motor-alternator set and the inverter are connected to a double-throw switch. Hence, the operation of the induction motor can be selected for either the inverter or motor-alternator set. An extra three-phase breaker is placed between the induction motor terminals and a double-throw switch to make sure that the induction motor is not connected to the supply, while the essential adjustments are being made on one of two systems. This breaker is particularly useful, while the adjustment of pulse-width is necessary for the waveforms which have high harmonic content and cause rapid heating in the machine. Two wattmeters are connected to the secondaries of the current transformers to measure the input power to the induction motor. The shunt resistance with no inductive effect is series connected to the motor to obtain the waveform of the current by an oscilloscope. This shunt resistance is also used to create a voltage drop across its terminals to measure harmonic currents by a wave-analyser for PWM voltage waveform applications. The ammeters connected to the motor cannot be used for PWM waveform operations, since they would show the total rms value of the current flowing into the machine. The losses associated with this current could not be calculated due to the skin-effect phenomenon in
The measurement of frequency is another important subject for PWM waveforms. It is very easy for sinusoidal voltage waveform application. Employing a digital counter across the shunt resistance the frequency of the voltage waveform can be measured. For
nonsinusoidal waveforms, whose currents may have several peaks in a period, produce a voltage drop across the terminals of the shunt resistance, which has the same waveform as the current, and trigger the digital counter as many times as its number of peaks. Thus, it results in a wrong display of the applied frequency. Another method of measurement is to use an oscilloscope at the output of the inverter and measure the period. But this method is bound to introduce more errors than expected. The error is the combination of inaccurate reading, which takes the major part, and inaccurate display of the oscilloscope. Therefore, the best place for the measurement of the inverter's output frequency is to connect the digital counter to the voltage to frequency converter of the inverter which produces a series of pulses. Thus, the frequency of the output voltage waveform is obtained accurately by reading the time between two consecutive pulses.

The measurement of induction motor speed is achieved by many methods. The method used in the experiments is to employ a stroboscope. The frequency of the flashes on the instrument cannot be read accurately by eye, when the high frequency is required. The amount of error made in reading the scale results in wrong determination of motor speed. A digital counter is connected to the stroboscope to count the interval of two successive flashes produced by the stroboscope to avoid such reading error. But the consistency of the displayed figure by digital counter is bound to be poor because of the variation of mains frequency even by a small amount. Therefore, the speed of the motor can be determined by taking the average of the most frequently displayed readings of the digital counter.
On the d.c. generator side, all the instruments and the method used to measure the output power of the generator are conventional. One voltmeter and ammeter are used to measure output power delivered to the variable resistance which is used to load the generator. The excitation of the field winding is supplied by a.c. lines via variac and a rectifier bridge to obtain d.c. current. This system enables us to change the field current without using a rheostat on which a certain amount of power is dissipated. The power to the field winding of the alternator is also supplied by a separate, but similar circuit arrangement mentioned, for the excitation of the d.c. generator.

The induction motor is a squirrel-cage type and has a delta-wound double-layer winding on the stator. It is rated at 10 HP. Three thermo-couples are placed on the stator winding with 120 degrees between them to measure the temperature of the winding. Although, this measurement cannot give the accurate temperature of the stator winding because of the effect of ventilation, it provides an approximate reading. Hence, the corrections of the stator and the rotor resistances due to high temperatures can be achieved by using the formula given in chapter 5, section 2.

In figure 123 the block diagram of the experimental arrangement is illustrated. The variacs shown by VR1 and VR2 are used to perform the no-load and the blocked-rotor tests of the induction motor. The breakers SW4 and SW5 serve for the utilisation of variacs.

2. THE TESTS

2.1 No-Load Test

The induction motor is supplied with sinusoidal-voltage waveform by the motor-alternator set at frequencies below the rated value. The
d.c. generator is separated from the induction motor by taking off the belts on the pulley. Thus, the effects of the d.c. generator and the belts are diminished on the operation of the induction motor.

The output of the motor-alternator set is connected to a variac to obtain variable a.c. voltage for the terminals of the motor. The instruments, ammeter and wattmeters, are connected to the secondaries of the current transformers. Since the voltmeter and ammeters are moving-iron type instruments, they show the r.m.s. values of the line voltages and the line currents. The test frequencies are chosen below the rated frequency, since the applications of 4 and 8 pulse-width modulated waveforms do not give the required fundamental voltage magnitude to satisfy \( V_{1/f} \) to be equal 8. The recovery time of the thyristors used in the inverter for generating PWM voltage waveforms is 500 \( \mu s \). Therefore, the maximum frequency of the 8 PWM voltage waveform is determined at 33 Hz and the tests are performed at 32.5 Hz in order not to force the inverter to operate at its limit. Two more test frequencies are selected which are 40 and 25 Hz having 32.5 Hz as the centre. The frequencies less than 25 Hz are avoided because of the difficulty arising from the setting of the supply frequency of the induction motor from the motor-alternator set.

Since the alternator is driven by a Schrage motor which is capable of reducing its speed to one-third of the maximum speed by changing the position of the brushes, the minimum output frequency of the motor-alternator set is obtained as 16.66 Hz.

The voltage applied to the induction motor terminals is set to obtain 8 for value of \( V_{1/f} \) at all test frequencies. The magnitude of the applied voltage is then reduced to 25\% of the voltage which satisfies \( V_{1/f} \) to be 8. The power input, the voltage, the line current and the speed are measured.
The no-load tests with PWM voltage waveforms are performed by using the variac before the rectifier bridge to reduce the input voltage to the motor. Thus, the variation of voltage is obtained while maintaining the pulse-width unchanged, hence the harmonic content of the voltage waveform. The amount of change allowed in the voltage is restricted by the rated value of voltages of the thyristors and the diodes used in the inverter and rectifier. Therefore, the voltage is changed within 25% of rated voltage of the thyristors.

The tests show that the power input to the induction motor is increased for PWM voltage waveform application compared to sinusoidal voltage waveform application. The power input also increases with increasing harmonic content. The harmonic-frequency currents are almost equal to their equivalents of blocked-rotor currents. The high value of slip causes large stator and rotor currents to flow in the machine. The r.m.s. values of the harmonic currents do not change with the variation of slip, since its effect is negligible at small values. Therefore, the no-load losses of the machine increase mainly because of the copper losses produced by harmonic currents. The iron losses caused by nonsinusoidal-voltage waveforms are dependent on the shape of the voltage wave. The contribution of eddy-current losses is not smaller than 10% of those produced by a sinusoidal-voltage waveform whose magnitude is equal to that of a PWM waveform's fundamental. Hysteresis losses are assumed unchanged as explained in appendix 4.

The purpose of the no-load test is to determine the total impedance of the equivalent circuit for harmonics. Since the slip of the motor is almost zero, the no-load harmonic impedance is smaller than that which would be obtained from a blocked-rotor test. This impedance can be used as a reference to check the value of the
equivalent impedance of the motor which is calculated by considering the deep-bar effect. The no-load test performed with a nonsinusoidal voltage-waveform cannot be used as it is for sinusoidal voltage waveform application. The iron losses and no-load stray losses for harmonics cannot be separated from the total no-load loss. But the sum of these two components can be obtained by subtracting the sum of harmonic copper losses with friction and windage loss from the total no-load loss.

The calculation of the equivalent-circuit parameters from the no-load test is obtained by plotting the no-load characteristic for the input power against the phase voltage. The characteristic is extrapolated to zero voltage. The intersection of the characteristic and the axis which represents the power input gives the friction and windage loss of the induction motor. The remainder of the total losses is the sum of copper, iron and no-load stray losses. If no-load stray losses are assumed to be part of the iron losses, the difference between the total no-load losses and the sum of copper losses together with friction and windage losses gives the iron losses. The calculation of copper losses is obtained, if the stator current and d.c. resistance of the stator winding are measured. Hence,

\[ W_{cu} = 3 I_o^2 R_s \]  \hspace{1cm} (251)

where \( I_o \) is the r.m.s. magnitude of the no-load phase-current.

The iron losses with no-load stray losses are calculated as

\[ W_{Fe} = P_o - W_{cu} - W_{FW} \]  \hspace{1cm} (252)
The resistance that represents the iron losses in the motor and connected in series with the magnetising reactance is given by

\[ R_m = \frac{W_F}{3I_o^2} \]  

(253)

The ratio of the applied voltage to the current determines the no-load equivalent impedance of the motor,

\[ \frac{V}{I_o} = \left( R_s + R_m \right)^2 + \left( X_s + X_m \right)^2 \]  

(254)

Hence, the total no-load reactance is obtained as

\[ X_s + X_m = \sqrt{\left( \frac{V}{I_o} \right)^2 - \left( R_s + R_m \right)^2} \]  

(255)

The solution of equation (255) is not possible since \( X_s \) and \( X_m \) are, both, unknowns. Therefore, the blocked-rotor test is performed to obtain second equation to determine \( X_s \).

2.2 Blocked-Rotor Test

This test is used in conjunction with a no-load test to determine the parameters of the induction motor. It is performed with reduced applied voltage to avoid large currents and excessive temperature rise. The induction motor operates as a secondary short-circuited transformer.

The rotor of the induction motor is blocked by means of an iron bar which is screwed to the fly-wheel on the shaft, and the other end based on the ground. Thus, the rotor is prevented from rotation. A variac is connected to the primary winding to change the applied voltage to the motor. The sinusoidal-voltage fed tests are performed with this circuit, while those with PWM waveforms require a step down transformer to change the magnitude
of the applied voltage of the inverter input. The variac used in a sine-wave test is transferred to the input of the rectifier bridge. The step-down transformer is connected between the output of the inverter and the input of the induction motor. Here, the use of a variac is to change the magnitude of the applied voltage of the rectifier so that the magnitude of the inverter output voltage is made controllable without changing the pulse-width at a frequency. Hence, the harmonic content of the PWM voltage waveform is maintained unchanged. But the control of the input-voltage magnitude of the rectifier is limited, by the rated values of semiconductors used in the inverter, to 25% of nominal voltage magnitude. This reduction of 25% in the voltage applied to the motor is not enough, if the pulse-width is to be maintained at the value which makes V/f equal to 8 as in the load-test in high frequency operations. The 25% reduced voltage for large pulse-widths can easily destroy the winding of the motor by causing large currents to flow. Therefore, a step-down transformer is used to reduce this high voltage to 50 Volts so that the machine is safely protected. The voltage variation is obtained by a variac.

In the blocked-rotor operation with reduced voltage the iron losses are negligible because of the low applied voltage. The friction and windage losses vanish due to the blocked rotor. Therefore, the losses which occur in the machine are accounted for by dissipation due to copper and aluminium conductors. The magnetising reactance can be transferred to the motor terminals in the equivalent circuit. This transfer can be made only if the iron losses are ignored. The magnetising current is very small for
low voltage, since the magnetising reactance is very large when it is compared with the leakage reactances. Therefore, the current measured by an ammeter is considered to be equal in the stator and in the rotor. Hence, the total resistance of the machine is given by

\[ R_s + R_r = \frac{P_{sh}}{3I_{sh}} \]  

(256)

The rotor resistance is evaluated from equation (256), if \( R_s \) is known.

The blocked-rotor impedance of the machine provides the sum of the stator and the rotor leakage reactances, as it is shown in equation (257).

\[ X_s + X_r = \sqrt{\left(\frac{V_{sh}}{I_{sh}}\right)^2 - (R_s + R_r)^2} \]  

(257)

The equation (257) is not enough to obtain \( X_r \) and \( X_s \) separately, therefore the table given in reference (25) is used to determine these reactances. The magnetising reactance is then obtained from equation (255) by substituting \( X_s \). The method described here is not applicable to nonsinusoidal-voltage waveforms, since the value of the rotor resistance is dependent on skin effect. In this case, the separation of harmonic currents is not possible, even if the current is measured with an ammeter which can measure the r.m.s. value of the sum of the first five harmonics. The measurement of the applied voltage with a voltmeter of the same kind is also far from the solution. Therefore, the wave-analyser is used to overcome the difficulty arising in the measurement of the harmonic currents and voltages. In fact the losses which occur in the machine are mainly caused by the fundamental current.
The fundamental-harmonic equivalent-circuit has highest voltage and lowest circuit parameters, while in the other harmonic equivalent-circuits harmonic voltages are lower than that of the fundamental and their reactances are greater than those of the fundamental. Consequently, the substantial part of the total losses in blocked-rotor operation is caused by the fundamental component of the current. The effects of other harmonics on the total losses in the blocked-rotor case may be neglected depending upon the harmonic content. If the percentage of 5th harmonic is above 50% of fundamental magnitude, the contribution of this harmonic to the total losses cannot be neglected, especially for the motors that have high rotor resistance.

2.3 Load Test

The purpose of the test is to find the performance of the induction motor under various load conditions with different PWM voltage waveforms applied. The estimation of total losses under these circumstances reveals the performance of the induction motor. The parts in which the losses occur can be reconsidered to minimise the waste to increase the motor efficiency. It is a fact that the nonsinusoidal voltage-waveform application results in loss of efficiency of the machine with respect to its sinusoidal voltage waveform operation. The difference in efficiencies of two operations reveals the causes of extra losses due to nonsinusoidal voltage-waveforms harmonics. The tests are performed for sinusoidal and nonsinusoidal voltage waveforms under identical conditions, such as temperature of the motor windings, instruments to measure various quantities of the motor and the same voltage to frequency ratio.
The arrangement of the equipment used in the tests is the same for both sine and PWM voltage-waveform applications, except for the supplies and measuring technique of the current and voltage. The conventional ammeter and voltmeter are used in the sinusoidal voltage waveform application, while the wave-analyser in PWM voltage waveform is used for the measurement of the current and the voltage. The induction motor-d.c. generator set is directly connected either to the inverter or to the motor-alternator set, depending upon the type of voltage waveform to be used. The load of the induction motor is increased by means of drawing more power from the separately excited d.c. generator. The output power of the d.c. generator is obtained by the voltmeter and the ammeter. Its input power is calculated from the following formula:

\[
P_{ig} = \frac{V I_g}{g} + R_{a_g} I_g^2 + \frac{\Delta V I N_b}{g/2a} + P_{og} + 0.01 \frac{V I_g}{g} \tag{258}
\]

where \( \frac{V I_g}{g} \) represents the output power of d.c. generator and \( R_{a_g} I_g^2 \) is the power dissipated in the armature winding. The third term from the right gives the power loss due to carbon brushes. \( \Delta V_b \) is taken as 2 volts, reference (27). The iron losses and friction and windage losses are included in the no-load losses of the generator. In the equation the first term from the right represents the additional losses which are due to non-uniform current density in conductors and on the brush contact surfaces with commutator, eddy currents. The input power of the generator is equal to the output power of the induction motor. The input power of the induction motor is measured by one wattmeter. This wattmeter is connected to two of three phases of the motor separately, without disconnecting the motor from the supply. The
connection is achieved by using two contactors on appropriate phases. Figure 124 shows the wiring diagram of the wattmeter. The reason for using this method is the lack of wattmeters with response to high frequencies which have current coils with high current ratings. The connection of the wattmeter's current coil to the secondary of a current transformer is avoided because of the rated frequency of the transformer. The frequency of the supply voltage waveform is measured by a digital counter which is arranged to display the period. Consistency of measurement of the frequency or the period is very difficult to obtain. As it was mentioned before, the frequency of the supply voltage is obtained by varying the speed of Schrage motor for sinusoidal voltage, while in the inverter it is achieved by the control of a potentiometer which commands the triggering pulses of the thyristors. Therefore, the period adjustment of the waveform cannot be consistent for two different tests. These results in different synchronous speeds of the induction motor and leads to large errors in the calculation of motor performance for small slips.

The temperature of the induction motor winding is also as important as the measurement of the period of the supply voltage waveform. The increase in temperature increases the resistance of the stator and the rotor. The latter causes more power loss in the machine and leads to smaller output-power for a fixed slip compared with the cold value of the rotor resistance. Therefore, the temperature of the windings must be measured in a part of the machine where the effect of the fan is minimal. The best accuracy of temperature measurement is obtained, if the thermo-couples are placed in the centre of the rotor in a slot. But practically this is not
possible since the rotor is made of stampings. The coefficient of temperature can be calculated by the formula given in equation (228). This shows that when the temperature of the motor increases to $40^\circ$C, the amount of change in the resistance is $7.3\%$ of its normal value which corresponds to $20^\circ$ C. The performance of all the tests are begun while the motor is at normal temperature and the temperature rise is recorded as the tests are carried out. Therefore, the rotor resistance calculated with equation (145) which is given for $75^\circ$C is corrected for the mean value of the temperature calculated at the end of the tests. This leads to an error in the calculation of motor performance. It will be discussed in the following chapter.

The comparison of sinusoidal and PWM voltage waveform applications for a motor operation shows that the losses of the induction motor, when PWM waveform is used, increase to very large values and cause the loss of efficiency. The effects of harmonics on the losses are obtained and are given in Chapter 5. The applications of these formulas require the unsaturated operation of an induction motor.
CHAPTER VII

THE METHOD TO MINIMISE INDUCTION MOTOR LOSSES
The method of calculation of the total losses of a squirrel-cage induction motor was introduced in chapter 5 for sinusoidal and nonsinusoidal voltage-waveform applications. It is seen that the total loss of the machine is increased due to the harmonics present in the nonsinusoidal voltage waveform. Therefore, it is intended to determine a method of solution to reduce the extra losses caused by harmonics. This method is given in this chapter.

The total losses of an induction motor, when it is fed by an inverter which can generate PWM voltage waveforms, varies with the variation of the pulse-duration and the magnitude of the mean value of the d.c. link voltage. The variation of the pulse-duration in a PWM waveform changes the harmonic content and the magnitude of the fundamental. Whereas, when the mean value of the d.c. link voltage is varied, magnitudes of the harmonics and the fundamental change in direct proportion to the change made in the d.c. link voltage. But this variation does not affect the harmonic content of the waveform. For further information, please see chapter 2.

There are two types of inverters which generate PWM voltage waveforms. The common feature of these inverters is that they must have a pulse-duration which must be controllable irrespective of the frequency. The difference between the inverters lies in the d.c. link voltage which can be made either constant or variable. The variable d.c. link voltage can be obtained from a controlled rectifier, and the inverter which has constant d.c. link voltage may be supplied from a diode bridge.

The minimisation of the total losses of an induction machine when it is subjected to a PWM voltage waveform is obtained, if the losses are related to the pulse-duration of the waveform. This is achieved, if the total losses of the machine is obtained in terms of the harmonic
voltages. This means that the losses caused by the currents which flow in various parts of the motor are to be represented separately as functions of the applied voltage. It is known that if the load torque is constant, the speed of the motor and, thus, the fundamental slip are determined by the fundamental voltage and this is held constant. Harmonics are assumed to produce no effect on the motor's average speed, although, they cause speed pulsations. Furthermore, if the temperatures of the stator and the rotor windings are assumed equal to one another, the equivalent impedance of the induction-motor's equivalent circuit does not change as long as the slip, the frequency and the temperatures of the windings are unaltered. Thus, the amount of change made in the pulse-duration affects the voltage magnitude which in turn causes variation in the total losses of the motor.

Each loss component of the total losses is calculated for every harmonic voltage considered with the method given later in this chapter. The variation of the total losses is, thus, obtained in terms of pulse-duration at a fixed frequency. This variation shows which value of pulse-duration causes minimum losses. This pulse-duration is compared with the permissible value. If it is smaller than the permissible value, it is used in an inverter which has constant input-voltage, otherwise with the one which has a controlled rectifier supply. The next step is to use this pulse-duration in a computer program which calculates the total losses for a frequency band in which the motor is assumed to operate at a constant slip and at a constant winding temperature so that the total losses can be compared with one another.
For the minimisation of the total losses, the motor is considered to operate with two different inverters which are mentioned above. In order to concentrate on the losses caused by the harmonics, those caused by the fundamental are subtracted from the total losses. The variation of the total harmonic losses with the harmonic content is obtained, if the motor is operated on different voltage waveforms whose fundamental voltage-magnitudes are equal to one another, but their harmonic contents are different. This type of motor operation can be achieved by using either of the two inverters. One, which has constant d.c. link voltage is used, if the position of the pulses can be changed with respect to each other and approximate changes are made to the pulse-duration, in order to keep the magnitude of the fundamental at a constant value. But an inverter with control of the pulse-position was not available, and therefore, tests could not be made in this case, but a theoretical analysis has been performed.

The second type of the inverter, which has a variable input-voltage is considered to possess a pulse-duration independent of the control of the variable d.c. link voltage. This inverter becomes the modified version of the inverter mentioned above by having a variable d.c. link voltage. Thus, any desire to keep the fundamental voltage-magnitude at a constant value becomes possible, even though the harmonic content of the PWM voltage waveform is changed by variation of the pulse-duration. The control of the harmonic content is obtained from the variation of the pulse-duration which also changes the fundamental magnitude. But the compensation needed to set the fundamental to a required value is achieved by the control of the d.c. link voltage. In this case, the positions of the pulses do not change with respect to one another, since they are equally spaced one after the other. In this case, theoretical and experimental results are compared.
In general, the principle of the method to reduce the total losses is explained with the following formula:

\[ \beta_w = \frac{\sum_{n=1,5,7,\ldots}^{\infty} W_{an}}{\sum_{n=1,5,7,\ldots}^{\infty} W_{bn}} \]  

(259)

where \( W_{bn} \) is the total losses of the induction motor for a PWM voltage waveform which is taken as a reference and is required to be reduced. \( \Sigma W_{an} \) is the total losses of the same induction motor which is caused by a different PWM voltage waveform. This ratio becomes less than unity if the reduction in the total losses is achieved by means of varying the harmonic content of the applied voltage waveform. Equation (259) can be given as shown in equation (260) by resolving the numerator and the denominator into their components.

\[ \beta_w = \frac{(W_{a1} + W_{a5} + W_{a7} + \ldots)}{(W_{b1} + W_{b5} + W_{b7} + \ldots)} \]  

(260)

Equation (260) is examined in two ways for different kinds of inverter-motor operation. The first, which comprises variable-voltage input to the inverter, has unchanged fundamental-harmonic losses. Therefore, the comparison is made only between the harmonic-component losses. \( W_{a1} \) and \( W_{b1} \) become equal to one another for different pulse -durations and for different inverter input voltage, if the fundamental voltage-magnitudes of the applied waveforms are equal and the operating conditions are identical. Thus, the sum of all harmonic losses given by equation (260) and \( \beta_w \) are written as

\[ \beta_w = \frac{(W_{a5} + W_{a7} + W_{a11} + \ldots)}{(W_{b5} + W_{b7} + W_{b11} + \ldots)} \]  

(261)

The losses caused by the fundamental harmonics are excluded from the total losses of both tests, since they are unchanged.

The second case, in which the possibility of holding the magnitude of fundamental voltage at a value cannot be achieved experimentally due to change of pulse-width and constant input-voltage magnitude of the inverter, does not have the same fundamental losses for two different
waveforms. Therefore, equation (260) is used for this kind of operation.

The approach to the solution of the problem is to minimise each harmonic loss component so that the $\beta^w$ is less than unity. Since the numerator of equation (261) is a function which is the sum of an infinite number of components, the total losses of test "a" is represented by $W_L^a$. For the sake of simplicity, only three harmonic components are shown in the calculations of this chapter. Losses of each harmonic component are represented by a function whose variable is the harmonic voltage-magnitude. The reason for choosing the voltage, as a variable, is to obtain a direct relationship with the inverter output. Once the value of voltage magnitude is determined the pulse-width is calculated according to formulas given in chapter 2. To obtain the relationship between the total losses and the output voltage of the inverter, the equivalent circuit of the motor is considered for a constant slip and for a constant temperature. The secondary impedance of the machine remains unchanged as long as these operating conditions are not changed. Hence, the currents of the stator and of the rotor decrease when the applied voltage is reduced, and increase when it is increased.

Figure 125 shows the equivalent circuit of an induction motor. In this figure, the change of the stator, the rotor and the magnetising currents are given by $\Delta I_s$, $\Delta I_r$ and $\Delta I_m$, respectively, while the amount of change of the voltage is by $\Delta V$. Hence, the change in the current is calculated in terms of the voltage change. Since the losses in most cases are caused by the currents; losses can be shown by a function of the applied voltage, if the currents are represented in terms of voltage. Thus, in general equation (261) is given as below
\[ \beta_w = W_{La}(V_5', V_7', V_{11}) / W_{Lb}(V_5', V_7', V_{11}) \]  

where \( W_{La} \) and \( W_{Lb} \) are given as functions of harmonic voltages.

From the equivalent circuit given in figure 125 the complex currents are determined as follows:

\[ I_s = V / Z_e \]  
\[ I_r = V / Z' \]

where

\[ Z_e = \left[ Z_r Z_s + Z_m (Z_r + Z_s) / (Z_s + Z_m) \right] \]  
\[ Z'_e = Z_e / Z_m \]

It is assumed that the resistance of the stator winding is not affected by the skin-effect. Thus, it is neglected when the stator impedance is calculated for time harmonics, since the leakage reactance is many times greater than the winding resistance. The absolute value of the stator impedance is given by

\[ Z_s = \sqrt{R_s^2 + (nX_s)^2} \]  

This equation is given as below, if \( R_s \) is neglected

\[ Z_s \approx nX_s \]  

Considering the skin-effect factors for the resistance and the inductance of the rotor due to the deep-bar effect, the absolute value of the rotor impedance is given by

\[ Z_r = \sqrt{(k R_{dc}/s_n)^2 + (n X_r k_n / k_{xf})^2} \]

and calling

\[ k_x = k_{xn} / k_{xf} \]

where \( k_{xn} \) and \( k_{xf} \) are, respectively, the skin effect factors of the \( n \)th and the fundamental harmonics for an inductance. Since the harmonic
slip is nearly unity and the rotor leakage reactance is greater than the rotor resistance for time harmonics, the rotor resistance can be neglected. In this case, the error in the rotor impedance becomes about 5% for 5th and 7th harmonics and decreases below 5% for higher-order harmonics. Hence, equation (267) can be given as

\[ Z_r \approx n \frac{k}{X_r} \quad (271) \]

The impedance of the magnetising branch comprises only magnetising reactance, since the iron losses due to time harmonics are extremely small and the magnetising reactance increases with harmonic order in direct proportion. Therefore, it is given by

\[ Z_m \approx n X_m \quad (272) \]

If equations (269), (271) and (272) are substituted in equations (265) and (266), equations (273) and (274) are obtained as follow:

\[ Z_e = n \frac{X_r X_m X_k + X_m (X_s + X_{s/k})}{(X_r X_m + X_s)} \quad (273) \]

and

\[ Z' = n \frac{X_r X_m X_k + X_m (X_s + X_{s/k})}{X_m} \quad (274) \]

If it is supposed that the harmonic voltages of all harmonics are increased by \( \Delta V \), the corresponding increases in the magnitudes of the stator and the rotor currents are obtained, respectively, from equation (263) and (264). These currents are given by equations (275) and (276).

\[ I_s + \Delta I_s = \frac{V + \Delta V}{Z_e} \quad (275) \]

and

\[ I_r + \Delta I_r = \frac{V + \Delta V}{Z'} \quad (276) \]

Thus, the corresponding increases in currents are given by
The total losses caused by the new values of the currents and the voltage magnitude of nth harmonic is given by

\[ W_{snl} = (I_{snl} + \Delta I_{snl})^2 R_{snl} + (I_{rnl} + \Delta I_{rnl})^2 R_{rnl} + R_{bnl} K_{nnl} (I_{snl} + \Delta I_{snl})^2 + K'_{nnl} (V_{nl} + \Delta V_{nl}) \]  

where \( R_{bnl} \) is the value of bar resistance and \( K_{nnl} \) is the stray-loss constant for \( n \)th time harmonic

and

\[ K_{nnl} = \frac{(W_{sknl} + W_{fnl} + W_{snnl} + W_{srnl} + W_{E_{nnl}} + W_{B_{nnl}})}{R_{nnl} (I_{snl} + \Delta I_{snl})^2} \]  

This formula can be written as above, since all the components of the stray-losses can be made proportional to the square of the current, as they are shown later in this chapter. The magnetic characteristic of the material has a linear variation for the unsaturated operation of the motor. If it is assumed that the voltage drop in the stator winding is neglected, the airgap flux-density in the machine becomes proportional to the applied voltage. Hence,

\[ \phi + \Delta \phi = \frac{(V + \Delta V)}{(4.44 f_k N)} \]  

\[ B_g + \Delta B_g = \frac{(V + \Delta V) 2 p}{(13.948 D L' f_k N)} \]  

Iron losses per unit mass vary linearly against the frequency for unsaturated operation of the motor. The total losses in the motor for only one harmonic at a fixed frequency is given by equation (231). Since the iron loss-factor is proportional to the magnetic flux density when the magnetic part of the machine is unsaturated,

\[ \beta_f = B/m_f \]  

and the loss-factor is given for a linear characteristic by substituting equation (281) in equation (282)

\[ \beta = \frac{2pV}{(13.948 DL' k_f N)m_f} \]
where \( m_f \) is the slope of the characteristic. Hence, equation (231) becomes
\[
W_{Fe} = 2\rho V'\rho' 10^{-3}/(13.948 m_f D L'k_w N)
\] (284)
and
\[
K' = 2\rho V'\rho' 10^{-3}/(13.948 m_f D L'k_w N)
\] (285)
Equation (285) becomes as below,
\[
W_{Fe} = VK'
\] (286)
Consequently, the total losses caused by the change of harmonic voltage the \( n_1 \)th due to the change of the pulse-duration is obtained by substituting equations (275) and (276) in equation (278)
\[
W_{La_{n_1}} = (V_{n_1 + \Delta V_{n_1}})^2 \frac{3R_s}{Z_{en_1}^2} + (V_{n_1 + \Delta V_{n_1}})^2 \frac{3R_{nn_1}}{Z_{n_1}^2} + \frac{R_{bn_1} K_{nn_1}}{Z_{en_1}^2} (V_{n_1 + \Delta V_{n_1}})^2 + K'_{n_1} (V_{n_1 + \Delta V_{n_1}})
\] (287)
hence,
\[
W_{La_{n_1}} = (V_{n_1 + \Delta V_{n_1}})^2 \left( \frac{3R_s}{Z_{en_1}^2} + \frac{3R_{nn_1}}{Z_{n_1}^2} + K_{nn_1} \frac{R_{bn_1}}{Z_{en_1}^2} \right) + (V_{n_1 + \Delta V_{n_1}})K'_{n_1}
\] (288)
calling,
\[
G_{n_1} = \left\{ \frac{3R_s}{Z_{en_1}^2} + \frac{3R_{nn_1}}{Z_{n_1}^2} + K_{nn_1} \frac{R_{bn_1}}{Z_{en_1}^2} \right\}
\] (289)
equation (288) is given by
\[
W_{La_{n_1}} = (V_{n_1 + \Delta V_{n_1}})^2 G_{n_1} + (V_{n_1 + \Delta V_{n_1}})K'_{n_1}
\] (290)
Similarly the losses associated with \( n_2 \)th harmonic are given by
\[
W_{La_{n_2}} = (V_{n_2 + \Delta V_{n_2}})^2 G_{n_2} + (V_{n_2 + \Delta V_{n_2}})K'_{n_2}
\] (291)
The summation of two positive figures, \( W_{La_{n_1}} \) and \( W_{La_{n_2}} \), is the total losses associated with these two harmonics.
The total losses before the increase of pulse-duration is given as the sum of \( W_{Lb}^{n1} \) and \( W_{Lb}^{n2} \), where they are given by

\[
W_{Lb}^{n1} = v_{n1}^2 g_{n1} + v_{n1} k_{n1}'
\]

and

\[
W_{Lb}^{n2} = v_{n2}^2 g_{n2} + v_{n2} k_{n2}'
\]

If equations (293) and (294) are substituted in equation (292)

\[
W_{Ln1} + W_{Ln2} = W_{Lbn1} + W_{Lbn2} + \Delta W_{n1} + \Delta W_{n2}
\]

where

\[
\Delta W_{n1} = g_{n1} \Delta v_{n1}^2 + (2v_{n1} g_{n1} + k_{n1}') \Delta v_{n1}
\]

Similarly,

\[
\Delta W_{n2} = g_{n2} \Delta v_{n2}^2 + (2v_{n2} g_{n2} + k_{n2}') \Delta v_{n2}
\]

If the following substitutions are made

\[
J_{n1} = 2v_{n1} g_{n1} + k_{n1}'
\]

\[
W_{La} = W_{Ln1} + W_{Ln2}
\]

The total harmonic losses caused by the \( n_1 \)th and the \( n_2 \)th harmonics are obtained as

\[
W_{Lb} = W_{Lb}^{n1} + W_{Lb}^{n2}
\]

Thus, by substituting equations (298b) and (298c) in equation (295)

\[
W_{La} - W_{Lb} = \Delta W_{n1} + \Delta W_{n2}
\]

At this stage the sign, (+ or -), of equation (299) shows whether the losses in the case of "a" are increased, decreased or unchanged. If \((W_{La} - W_{Lb}) > 0\), i.e. the total losses associated with these harmonics is increased, for \((W_{La} - W_{Lb}) < 0\) it is decreased. If the
left hand side of the equation is zero, the losses remain unchanged. Since minimisation is required, the value of this loss-difference must be as low as possible. This equation comprises two independent variables, \( \Delta W_{n1} \) and \( \Delta W_{n2} \). The absolute minimum of this equation is obtained when both functions are at minimum. But the absolute minima of these functions, separately, give negative losses which are not obtainable. This is because they are quadratic functions as they are given by equations (296) and (297) whose constants are positive numbers. Therefore, they will be considered for the voltage values which give positive \( \Delta W \). The variation of losses produced by a harmonic voltage are represented as in figure 126. In this figure, the voltage increase is assumed due to the change of pulse-width. But the harmonic voltage-amplitude does not always increase with the increase of a pulse-width. It varies according to the formulas given in chapter 2 when the pulse-duration is changed. Its frequency is determined by the harmonic order. Therefore, the linear variation of a voltage magnitude against the pulse-width cannot be obtained for a harmonic. As a result of this, the equations which relate losses to the pulse-width are nonlinear.

When the pulse-width is changed, the amount of change of the voltage magnitude for a harmonic is determined from the equations which is given for 2 PWM voltage waveform as

\[
V_{n1} = \frac{4V_{p}d_{o}}{\pi \cdot n_{1}} \left[ \cos(n_{1}\theta) - \cos(n_{1}(\theta + \omega_{w})) \right]
\]

(300)

where \( \theta \) is a constant, since it is the starting point of the waveform. Introducing \( \Delta P_{w} \) as an increase to \( P_{w} \) in equation (300), the voltage magnitude of \( n_{1} \)th harmonic is given by
\[ V_{n1} + \Delta V_{n1} = \frac{g_o}{n_1} \left[ \cos(n_1\theta) - \cos(n_1(\theta + \Delta P_w)) \right] \]  \hfill (301)

where
\[ g_o = \frac{4V_{dc}}{n\sqrt{2}} \]  \hfill (302)

The difference between equations (300) and (301) gives \( \Delta V_{n1} \). The sine of small angles is assumed to be equal to its argument in the solution. Therefore, \( \sin(n_1\Delta P_w/2) \) is replaced by \( (n_1\Delta P_w/2) \).

\[ \Delta V_{n1} = g_o \Delta P_w \sin(n_1(\theta + \Delta P_w/2)) \]  \hfill (303)

It is seen that the constant term of the voltage change is independent of harmonic order, therefore, the voltage changes can be represented as

\[ \frac{\Delta V_{n1}}{\Delta V_{n2}} = \frac{\sin(n_2(\theta + \Delta P_w/2))}{\sin(n_1(\theta + \Delta P_w/2))} \]  \hfill (304)

Hence,

\[ \Delta V_{n2} = \Delta V_{n1} \frac{\sin(n_2(\theta + \Delta P_w/2))}{\sin(n_1(\theta + \Delta P_w/2))} \]  \hfill (305)

If equation (305) is substituted in equation (297), \( \Delta W_{n2} \) is expressed in terms of \( \Delta V_{n1} \).

\[ \Delta W_{n2} = g \Delta V_{n1}^2 \frac{\sin^2(n_2\alpha)}{\sin^2(n_1\alpha)} + j \Delta V_{n1} \frac{\sin(n_1\alpha)}{n_2 \sin(n_2\alpha)} \]  \hfill (306)

where \( \alpha = \theta + P_w + \Delta P_w/2 \)

Hence, the total loss-change due to \( n_1 \)th and \( n_2 \)th harmonics is given by

\[ W_{La} - W_{Lb} = \Delta V_{n1}^2 \left( g_{n1} + g_{n2} \frac{\sin^2(n_1\alpha)}{\sin^2(n_2\alpha)} + j \Delta V_{n1} \frac{\sin(n_1\alpha)}{n_1 \sin(n_2\alpha)} \right) \]  \hfill (307)
Equation (307) represents the total loss-change against voltage-change of \( n_1 \)th harmonic. In order to obtain negative values for \( (W_{La} - W_{Lb}) \), it is necessary to determine the values of the roots of this equation. Therefore, \( W_{La} \) is assumed equal to \( W_{Lb} \). The analytical solution of this equation with respect to \( \alpha \), which is multiplied by harmonic orders and, thus, constitutes two unknowns, is not possible. Nevertheless, the number of unknowns can be reduced to one, if one of the harmonic orders is assumed twice the value of the other, i.e. \( 2l = 4l/2 \). Thus, all the sine terms in equation (307) can be given in terms of one harmonic order. The solution of the equation without this assumption provides two roots. One of them is zero while the other is given as below.

\[
\Delta V_{nl} = - \left( J_{n1} + J_{n2} \frac{\sin(n_1 \alpha)}{\sin(n_2 \alpha)} \right) \sqrt{G_{n1} + G_{n2} \frac{\sin^2(n_1 \alpha)}{\sin^2(n_2 \alpha)}}
\]

After rearrangement

\[
\Delta V_{nl} = - \frac{J_{n1} \sin^2(n_2 \alpha) + J_{n2} \sin(n_1 \alpha) \sin(n_2 \alpha)}{G_{n1} \sin^2(n_2 \alpha) + G_{n2} \sin^2(n_1 \alpha)}
\]

The solution of this equation to determine \( \alpha \) is not possible, except by a numerical method. But with the above mentioned assumption, a solution is obtainable. If \( n_2 = 2n_1 \),

\[
\Delta V_{nl} = - \frac{J_{n1} \sin^2(2n_1 \alpha) + J_{n2} \sin(n_1 \alpha) \sin(2n_1 \alpha)}{G_{n1} \sin^2(2n_1 \alpha) + G_{n2} \sin^2(n_1 \alpha)}
\]

After some algebraic manipulation and calling \( U_o = \cos(n_1 \alpha) \), the following equation is obtained

\[
U_o^2(4J_{n1} \Delta V_{nl} G_{n1}) + 2 J_{n2} U_o - \Delta V_{nl} G_{n2} = 0
\]
The roots of this equation give the required relationship between \( \Delta P_w \) and the losses.

\[
U_o = \frac{-J_n n_2 \sqrt{J_n^2 + \Delta V n_1 G_n n_2} (4J_n n_1 - 4 \Delta V n_1 G_n n_1)}{4(J_n n_1 - \Delta V n_1 G_n n_1)}
\]  

(312)

Since

\[
U_o = \cos\{n_1 (\theta + pw + \Delta pw)\}
\]

(313)

\[
\Delta P_w = \left[\frac{1}{n_1} \arccos(U_o) - \theta - P_w\right]_2
\]

(314)

where \( \theta, P_w \) and \( \Delta P_w \) are given in radians. If \( \Delta P_w \) has to be shown in milliseconds which is always used in practice, \( \theta \) and \( P_w \) are changed into milliseconds. Thus, the above equation is

\[
\Delta P_w = \left[\frac{2\pi f}{n_1 10^{-3}} \arccos(U_o) - \theta - P_w\right]_2
\]

(315)

where \( f \) represents the frequency in Hz and \( \theta \) is the starting point of the waveform in milliseconds. Equation (303) is also arranged for pulse-width if given in milliseconds.

\[
\Delta v_{n_1} = 2\pi 10^{-3} G_o \Delta P_w \sin\{2\pi 10^{-3} n_1 (\theta + P_w + \Delta P_w / 2)\}
\]

(316)

This method of solution leads to an error which is greater than expected, because the amplitude of \( n_2 \)th harmonic cannot be determined with a small error. The amplitudes of harmonics, whose orders are \( n_2 = 2n_1 \) and \( n_2 = 2n_1 + 1 \), are different from one another due to harmonic orders and harmonic frequencies. The difference between the amplitudes of these harmonics is a maximum when the angle is 90° in equation (316). Therefore, unless the harmonic order is high, such as 17, this method should not be used.
The difficulties that occurred in the solution of the equation leads the problem to the more simplified version which is obtained by the following assumption. In equation (296) the value \( G_{nl} \) is a small quantity, since each term of equation (289) is divided by large figures. Meanwhile, \( J_{nl} \) is a very large quantity when it is compared with \( G_{nl} \), since it comprises \( V_{nl} \) and \( K'_{nl} \). Therefore, \( G_{nl} \) can be omitted and thus, equation (291) is given by

\[
\Delta W_{nl} = (2V_{nl}G_{nl} + K'_{nl})\Delta V_{nl}
\]  

(317)

Hence, the total change of loss is given by

\[
\delta W = J_{nl} \Delta V_{nl} + J_{n2} \Delta V_{n2}
\]  

(318)

The change of pulse-width by \( \Delta P_w \) causes the voltage magnitude to change. If equation (300) is subtracted from equation (301), the voltage change of \( n_{1\text{th}} \) harmonic, \( \Delta V_{nl} \), is determined as given below.

\[
\Delta V_{nl} = \frac{2G_o}{n_{1\text{th}}^2} \sin(n_{1\text{th}}(\theta + P_w + \Delta P_w/2)) \sin(n_{1\text{th}}(\theta + P_w + \Delta P_w/2))
\]  

(319)

If this equation is substituted in equation (318) and it is differentiated with respect to \( \Delta P_w \), the result is

\[
\frac{\partial (\delta W)}{\partial (\Delta P_w)} = G_o \left[ J_{nl} \sin(n_{1\text{th}}(\theta + P_w + \Delta P_w)) + J_{n2} \sin(n_{2\text{nd}}(\theta + P_w + \Delta P_w)) \right]
\]  

(320)

The solution of equation (320) is obtained by plotting the variation of \( \frac{\partial (\delta W)}{\partial (\Delta P_w)} \) with respect to \( \Delta P_w \). Intersection points of the characteristic with the \( \Delta P_w \) axis give the extremum points of the real function which is given as

\[
\delta W = G_o \left[ \frac{J_{nl}}{n_{1\text{th}}} \cos(n_{1\text{th}}(\theta + P_w)) - \cos(n_{1\text{th}}(\theta + P_w + \Delta P_w)) \right] + \frac{J_{n2}}{n_{2\text{nd}}} \left[ \cos(n_{2\text{nd}}(\theta + P_w)) - \cos(n_{2\text{nd}}(\theta + P_w + \Delta P_w)) \right]
\]  

(321)

In equation (321) the value of \( P_w \) is used to determine the change of losses. Thus, the ratio given by equation (260) can be made smaller than unity. But sometimes, the value of \( \Delta P_w \) may become so large that the change of the fundamental voltage-magnitude cannot be ignored.
Therefore, the solution of the problem is obtained by comparing more than one waveform for the same running conditions of the motor. These waveforms must have the same fundamental voltage-amplitude, so that the losses due to the fundamental harmonic remain unchanged. If the voltage amplitude of the rectifier input is controllable, the change of pulse-width which affects the amplitude of the fundamental can be compensated for by a corresponding change made in the rectifier input-voltage. The problem, without voltage control, becomes complicated, since various waveforms have to be searched to obtain the minimum losses. Although, the procedure given in this chapter only deals with two of the whole harmonics, excluding the fundamental, a realistic view of the machine performance can be obtained by including another pair of harmonics. Thus, the total number of harmonics rises to four. They are 5th-11th and 7th-13th. As a result of this increase, the equation given by (321) becomes more complicated, having extra $J_{n3}$ and $J_{n4}$ with their cosine products.

The method presented so far requires thorough calculation of motor performance which requires a computer program. In order to avoid programming, the coefficients of the harmonic losses can be determined from the formulas already given in chapter 5. Since most of these formulas are given in terms of frequency and design data, the evaluation of the coefficient of each stray-loss component for a harmonic is possible. It may be assumed that the amount of change of stray-load losses with slip is zero, or they remain constant. Thus, the total of the coefficients of each stray-loss component is not affected by the slip. The reason is obvious; because all harmonic slips are almost unity. The purpose of this process is to represent all of the stray-losses as copper losses, in terms of bar resistance. From equations (240) and (241) for the stator and the rotor surface losses the coefficients are, respectively:
\[ \Lambda_{ss} = 1290 \cdot D \cdot L \cdot B \cdot C_{ss} \cdot \frac{I^2}{R_{ss} \cdot I_0} \] (322)

\[ \Lambda_{sr} = 1290 \cdot D \cdot L \cdot B \cdot C_{sr} \cdot \frac{I^2}{R_{sr} \cdot I_0} \] (323)

where \( C_{ss} \) and \( C_{sr} \) are frequency dependent loss-factors. But, since the stator and the rotor slot frequencies are very high and the harmonic slip does not change a great deal, these factors may be assumed constant. Thus, the total surface-losses on both members of the motor is given by

\[ W_{ss} + W_{sr} = \left( \frac{\Lambda_{ss}}{R_{ss}} + \frac{\Lambda_{sr}}{R_{sr}} \right) I^2 \] (324)

The losses due to high frequency pulsations are given by equation (243).

\[ \Lambda_{hf} = m \cdot C_{hf} \] (325)

where \( C_{hf} \) is a constant and depends on the ratio of slot-width to airgap. Hence, these losses are given as

\[ W_{hf} = \Lambda_{hf} \cdot R_{hf} \cdot I^2 \] (326)

The coefficient of belt leakage losses is given as follows

\[ \Lambda_{b} = m \left( \frac{k^2_{w_5} + k^2_{w_7}}{k^2_{w_1}} \right) \] (327)

and the losses associated with this leakage are

\[ W_{B} = \Lambda_{b} \cdot R_{b} \cdot I^2 \] (328)

The end leakage losses are a function of the frequency and the square of the stator current. Thus, from equation (247) it is given as

\[ W_{E} = \Lambda_{e} \cdot I^2 \cdot R_{b} \] (329a)

where

\[ \Lambda_{e} = 1.89 \cdot 10^{-6} \cdot m^2 \cdot N^2 \cdot D \cdot \log(1+U^2/4Y_1Y_2)/P^2R_b \] (329b)

In equation (329b) \( N^2 \) includes the square of the winding factor. The skew-leakage loss coefficient is given by
Thus, skew losses are

$$W_{sk} = A_{sk} \frac{I_{s n l}^2}{R_b} k_{rsk}$$

(331)

The total of these losses is given as follows

$$W_{ST} = I_{s n l}^2 R_b \left( \frac{A_{ss}}{k_{rs}} + \frac{A_{sr}}{k_{rr}} + A_{hf} k_{rh} + A_{hr} k_{hb} + A_{fs e} + A_{sk k_{rsk}} \right)$$

(332)

In this equation $k_{rs}$ have different values for each term given in the brackets. But once these frequencies are calculated for determinations of skin-effect factors, they are assumed almost unchanged for the change of the harmonic slips. Thus, the coefficients are found for each stray-loss component. If this method were not introduced, the calculation of the stray losses would be tedious.

The easiest method to determine the optimum PWM voltage waveform is to consider 5th, 7th, 11th and 13th harmonics and to obtain the corresponding loss factors, $G_5$, $G_7$, $G_{11}$ and $G_{13}$. If it is assumed that the iron losses caused by these harmonics are negligible compared with the whole loss, since the airgap flux for harmonics is small with respect to that of the fundamental, equation (293) is given by

$$W_{Lb_{n1}} = V_{n1}^2 G_{n1}$$

(333)

where $n_1$th harmonic voltage is given from equation (300) as

$$V_{n1} = \frac{8v_{dc}}{\pi n_1} \sin \{n_1 (\theta + P/2)\} \sin (n_1 P/2)$$

(334)

Thus, the total losses due to all major harmonics is obtained by substituting the harmonic voltage magnitudes of these harmonics, first in equation (333) and then in equation (335) which is given below.

$$W_L = W_{L_5} + W_{L_7} + W_{L_{11}} + W_{L_{13}}$$

(335)
By plotting the $W_L$ against pulse-width the variation of the total losses is obtained for a fixed value of d.c. link voltage. The minimum $W_L$ corresponds to a pulse-width. The latter is compared with the permissible pulse-width which is determined as in chapter 2 for a fixed frequency. If the pulse-width, which gives minimum power loss, is shorter than both the permitted value and that of the previous experiment represented by suffix "a", the d.c. link voltage is increased and the pulse-width is reduced to this new value. In the contrary case, $V_{dc}$ is reduced when the pulse-width found by plotting is longer than that of reference case, "a". This method is only used when the d.c. link voltage of the inverter is variable. The variation of d.c. link voltage in relation to the pulse-width makes the magnitude of the fundamental-harmonic voltage unchanged. Meanwhile, the harmonic content changes with pulse-width.

The second case that mentioned in the beginning of this chapter, concerns the d.c. link voltage when it is not variable. If the pulse-width is changed according to the plotting, which is obtained between $W_L$ and $P_w'$, the magnitude of the fundamental changes as well as those of harmonics. This is an undesired case. But to avoid this drawback, various waveforms of equal fundamental magnitudes are compared with each other and one which gives lowest loss, is selected for use. The use of per unit values of harmonic voltages instead of voltage magnitudes are unnecessary, since the slip of the motor is assumed not to be affected by the little variations of the fundamental magnitude. Hence, the effects of harmonics, merely, increase the total losses. The increase or the decrease of the harmonic voltages in any level does not cause a change in the speed of the motor.
The following example proves that the theory is correct.

A 4 PWM waveform with 1.562 ms pulse-width and 525 Volts d.c. link voltage is applied to the induction motor. The motor is run at 1% slip. The equivalent circuit whose parameters are given at the fundamental frequency, 40 Hz, is illustrated in figure 127. From equations (155), (268), (273) and (274) the values of rotor resistance, (although it is neglected) slip, skin-effect factor, $Z_e$ and $Z'$ for 5th, 7th, 11th and 13th harmonics are calculated as shown in table 4.

Substituting calculated $G_n$s in equation (335) for corresponding harmonic orders, the variation of total losses in the induction motor is obtained in terms of the pulse-width. The analytical solution of the problem is not possible, therefore, the minimum point of this function is determined by the numerical method. The important point is that the maximum permissible value of the pulse-duration which is determined by the turn-on time of the thyristors must not be overrun in this calculation. If the numerical solution shows that the minimum losses are obtainable within the permissible limits of the pulse-width, the d.c. link voltage is set to such a value that the required fundamental voltage-magnitude is obtained with the calculated pulse-width.

For this example a computer program is prepared to find the pulse-duration which gives minimum losses in the motor. However, the pulse-duration is found to be 2.101 ms. which is longer than the permissible duration that can be evaluated by equation (21). The permissible duration is found to be 1.5833 ms which is well under the figure determined by the program. Therefore, the pulse-duration is set to 1.5833 ms. The d.c. link voltage is reduced to maintain the fundamental voltage at the fixed magnitude. The new value of d.c.
link voltage for 309 volts magnitude of fundamental is 519 volts. This program and its associated characteristics are given in appendix 6.

If the pulse-duration which is determined as 2.101 ms is required for the application to the waveform, the permissible maximum operating frequency of the inverter for this pulse-width and the given number of pulses has to be determined by considering the minimum turn-off and turn-on times of the thyristors. This frequency is found to be 38.56 Hz when 60 μs time is allowed between two successive pulses. Hence, the fundamental voltage-magnitude, in r.m.s., which satisfies 8 for \( V_{l}/f \) is found to be 308.5 Volts to which corresponds the d.c. link voltage calculated from equation (18) as 406.7 Volts. The application of this waveform can only begin at 38.56 Hz. Therefore, in figure A64 of appendix 6 the characteristic numbered 2 is not defined between 40 Hz and 38.56 Hz. Figure A63 and A64 show the effect of pulse-duration on the total harmonic losses. These characteristics show that if the slip of the induction motor is maintained unchanged at a per unit slip, irrespective of frequency, the total losses due to harmonics are reduced, at some frequencies, by about 65%.
CHAPTER VIII

COMPARISON BETWEEN THE EXPERIMENTAL AND THE THEORETICAL PERFORMANCES OF THE INDUCTION MOTOR
1. **GENERAL**

In chapters 4 and 5 the performance of an induction motor with a squirrel-cage rotor has been described theoretically for nonsinusoidal voltage waveform applications. The experimental rig has been presented as a separate chapter, but the results of experiments and theory have not been shown and compared. Therefore, this chapter is prepared for comparison purposes. The data obtained from the experiments are applied to the theory of the squirrel-cage induction motor. The assumptions presented during the process of the theory cause differences between the experimental and the theoretical results. But they should not lead to large differences.

In this chapter, the load-test results of the squirrel-cage induction motor with sinusoidal and PWM voltage waveforms of different modulation numbers, the photographs of the line-currents at no-load operation for separate PWM waveforms which are distinguished by their modulation numbers, at 40 Hz, 32.5 Hz and 25 Hz frequencies, are represented, respectively. Finally, the comparison is made between the sinusoidal and the nonsinusoidal voltage waveform applications.

2. **LOAD TESTS**

The results of experiments and theoretical calculations are shown by figures from 128 to 137 and tables from 5A to 14B. Each experiment is shown by one figure and a pair of tables. The table that has label "A" indicates the experimental results, while "B" theoretical results. The order of the tables is arranged as that of the figures. The characteristics of the motor in figures from 128 to 137 are plotted for experimental results only. The theoretical results are avoided for simplicity in the figures.
The difference between the calculated and measured results can be explained as follows; although, the difference is generally not more than 10% of measured value at most experimental points except for small slips. The deterioration of accuracy at small slips is caused by the inaccurate reading of the rotor speed with the stroboscope. The display of the digital counter which shows the period between two successive flashes produced by the stroboscope, changes arbitrarily depending upon the triggering level. Therefore, the speed of the motor cannot be determined with 100% accuracy. Especially, the small difference in low slip affects the motor output a great deal.

The second possible source of error is the inaccurate setting of the output frequency of the inverter or of the motor-alternator set. Even, a very small amount of mis-setting of the frequency causes large relative errors in the slip. Consequently, the calculation of the output power which affects the input power is affected by almost the same amount. If it is supposed that the motor speed is measured 100% accurately and the frequency is set to 50 Hz by a difference of 0.02 Hz, the error caused by this approximation is given as follows in the slip of the motor. By derivation of equation (69) with respect to \( n_s \)

\[
\Delta_s = \frac{n_r}{n_s} \Delta n_s
\]  

(336)

where \( \Delta_s \) and \( \Delta n_s \) are absolute errors in slip and synchronous speed, respectively. If the frequencies are given as 50 Hz and 49.98 Hz, and the rotor speed is as 1490 r.p.m., from equation (336) \( \Delta s \) is determined as \( 3.97 \times 10^{-4} \) as absolute error. But the relative error in slip is obtained as 5.95% as follows. The value of slip at 50 Hz is given by

\[
s = \frac{(1500 - 1490)}{1500}
\]

where \( s = 0.006666 \)
At 49.98 Hz

\[ s = \frac{(1499.4 - 1490)}{1499.4} \]

i.e. \( s = 0.006269 \)

Thus, the relative error is \( \Delta s = \frac{(0.006666 - 0.006269)}{0.006666} \) and \( \Delta s = 5.95\% \).

The mis-setting of the frequency also affects the output-voltage waveform of the inverter. The lower the frequency, the higher the voltage magnitude obtained for a fixed pulse-duration. This causes the induction motor to operate at a speed and a voltage different from what they should be. In the theoretical calculations the voltage drop in the inverter is ignored and the ideal generation of PWM voltage waveform is considered. In tables 5, 6 and 7 sinusoidal, 2 and 4 PWM voltage waveform applications at 40 Hz are shown, respectively.

The tables 6A and 6B show that the large values of harmonic voltages leads to saturation of the motor. This effect may also be seen on the harmonic currents whose calculated and measured values are shown in these tables. The measured values of the line-current are bigger than those calculated.

The effect of temperature is highly important, since the amount of change of the output power is proportional to that of the rotor resistance. Thus, a small amount of temperature change causes different output power. The temperature of the motor rises until its continuous rating when it is initially switched on, cold. The rise continues until the machine reaches its final temperature. Therefore, the characteristic of the induction motor changes continuously as it is warmed and as its rotor resistance is being increased. Hence, the slip of the motor increases for a fixed output power with its increasing rotor resistance. The effect of the temperature rise cannot be compensated accurately by averaging the initial and the final temperatures.
The tables from 8A to 11B show the calculations and measurements of induction motor's load test with sinusoidal, 2, 4 and 8 PWM voltage waveforms. The sinusoidal voltage wave application shows very good agreement between the theoretical and the experimental results. The 2 PWM voltage waveform has a larger harmonic content than it has at 40 Hz. Therefore, the effect of saturation is more distinct at 32.5 Hz. However, the calculated PWM voltage waveform has a fundamental whose magnitude is 235 volts, whereas it is recorded as 269 volts in the experiment, though the magnitude of the 7th harmonic is almost equal to its calculated value. This shows that the readings taken by the wave-analyser have more than 5% error. This error may rise to 10% as seen in tables 9A and 9B. The origin of the error does not only belong to the wave-analyser but to the person who reads the deflection. This is repeated in tables 10A and 10B where 4 PWM is represented, the measured quantities of the 5th and 7th harmonics are greater than those obtained by calculation. This also proves the incorrect deflection of the wave analyzer. The 8 PWM waveform at the same frequency also shows that the calculated figures of the voltage harmonics are somehow greater than those obtained by the measurements. The causes of these may be the incorrect deflection of the wave-analyser. The oscilloscope by which the pulse-duration is determined can be another source of error. This error may become very important for high d.c. link voltage amplitudes which determines the magnitudes of the harmonics.

The calculation of losses in the theory incorporates 10 harmonics, from 1st to 29th. Since the copper losses can be calculated with a good accuracy and contribute the major part of the total losses in both cases, sinusoidal and PWM wave applications, the calculation of the total losses cannot be greatly changed by the assumptions made to obtain iron losses in chapter 5.
3. COMPARISON OF INSTANTANEOUS CURRENT WAVEFORMS

The line current of the induction motor is calculated as explained in chapter 4. The photographs of the current waveforms caused by various PWM voltage waveforms are shown in figures 138A-144A. These figures are displayed by giving priority to the highest frequency and the lowest modulation number. The calculated current waveforms are shown by figures 138B-144B. The figures which have labels "C", the bottom figures, show the line to line voltage waveforms applied to the induction motor. The photographs of voltage waveforms are not presented, since the difference in amplitude between the measured and the calculated values would not be distinguishable. The 2 PWM modulated voltage waveform application is shown by 138A and 138B. It is seen that satisfactory agreement is achieved if spikes are excluded. The difference between the two figures is less than 10% which is the result of inaccurate readings of slip, omission of saturation, space harmonics, etc. The photograph and the calculated line current of the induction motor due to the 4 PWM voltage waveform are illustrated by figures 139A and 139B, respectively. The agreement between the figures is excellent excluding the spikes which appear in the photograph. The spike in the middle of two groups of zig-zag shaped spikes has a magnitude of 26.6 A if it is measured, while the magnitude of the corresponding pulse in figure 139B is 26.2 A. The group of zig-zag pulses which is on the left of this centre pulse has 23 A and 14 A, while on the photograph they are, respectively, 24 and 12 A.

At 32.5 Hz frequency the 4 PWM which is given by figures 141A and 141B show perfect agreement with the photograph. The application of 8 PWM voltage waveform to the induction motor produces the current waveforms of figures 142A and 142B. Although, the spikes have equal amplitudes in these figures, the spike which corresponds to the
fourth pulse is greater than that which corresponds to the first pulse of the following cycle. These two pulses seem to be symmetrical about the pulse which stands out on its own in every half cycle. The symmetry of these spikes is broken when the motor is loaded. The spike on the left has lower amplitude than that which is on the right. When the load is removed, the symmetry between the pulses is re-established. The predicted values given in figure 142B shows that the speed of the motor has been measured greater than it should have been. Therefore, the spike on the left has a smaller magnitude than that which is on the right.

The current waveforms of 4 and 8 PWM voltage waveforms at 25 Hz frequency are given by figures 143A - 144B. The photograph of the current waveform for 4 PWM waveform is illustrated by figure 143A in which each square corresponds to 13.33 A/cm vertically and 5 ms/cm horizontally. Thus, the pulse which stands out at the centre of the photograph has a magnitude of 33 A, while it is obtained as 30 A from the calculation. The groups of spikes on both sides of the centre spike, which consist of two spikes per group, have the following amplitudes. The group on the left hand side; the first spike has 24 A measured, 28 A calculated; the second spike, the highest, has 36.6 A measured and 37.6 calculated. The group on the right hand side; the first spike has 30.6 A measured and 33.3 A calculated; the second pulse which is shorter than the first, has 20 A measured and 23.6 A calculated. The small spikes between these groups and the centre spike have magnitudes as follows; 10 A measured, 12.25 A calculated. This shows that the calculated figures are larger than those on the photographs. The reason can be the misreading of the slip, greater than it should be and inaccuracy of the oscilloscope which can be as high as 3% as given in its manual. The major reason
can be the temperature of the winding. Since the average value of the winding temperature is used for the calculation of the induction-motor performance, this value of temperature at which the photograph is taken may be different from the average value by 10°C or 15°C so that the change of the rotor resistance becomes 4 or 6%, respectively.

The line-current waveform which is produced by the 8 PWM voltage waveform is illustrated by its photograph as in figure 144A, while the respective calculation of waveform by 144B. The calculated and the experimental results are in good agreement, although the value of the slip has been measured a little smaller than it should be. This can be observed on the second group of zig-zag spikes which are on the right of the centre pulse. The amplitude of the highest spike which belongs to the group on the right is smaller than the amplitude of that which belongs to the group on the left. The difference between these two spikes increases, however, as the slip is increased.

It can be concluded that the method of determination of the induction motor's line current, when a nonsinusoidal voltage waveform is applied, produced satisfactory results which agree with all the presented tests and can be used with confidence. The determination of the current waveform is important, since it reveals the ampere-turns of its associated member of the motor when it is multiplied by the turns-distribution of this member.

4. COMPARISON OF INDUCTION MOTOR OPERATION FOR SINUSOIDAL AND PWM VOLTAGE WAVEFORMS

Two different modes of operation of a squirrel-cage induction motor are obtained by applying PWM and sinusoidal voltages. The figures, from 128 to 137, show the variations of the input and output powers, of the line-current and the efficiency of the motor under different excitations
at various frequencies. Generally, the difference of the motor performances is related to the harmonic content of the applied voltage waveform. If the operation of the induction motor is considered at 40 Hz, where 2, 4 PWM and sinusoidal voltage waveforms are used in the tests, the highest efficiency of these three operations is observed as belonging to that of the sine wave which is assumed to be pure sinusoidal voltage wave and does not contain harmonics of any order or any kind. While the 2 and 4 PWM waveforms have odd order of harmonics whose effects on the motor performance are undesirable. Every harmonic voltage has its own frequency to which the motor responds differently and causes losses. Consequently, the whole losses of the machine are the contributions of each individual harmonic.

In figures 128, 129 and 130 the power input of the machine for a fixed output power increases when the voltage waveform is changed to nonsinusoidal waveform. Efficiency drops due to the increase of the total losses. The total r.m.s. current, that can be calculated as mentioned in chapter 2, rises because of the harmonic currents. Thus, the higher the amplitudes of the harmonic voltages, the larger the currents obtained. The fundamental current remains unchanged for both modes of operation. The reason of having higher loss in PWM operation is the existence of harmonics which do not improve the performance, on the contrary, they worsen it. Their large slips cause the harmonic equivalent circuit of the motor to behave as a secondary shorted transformer. Therefore, the stator and the rotor currents become almost equal which lead to large copper losses. The effects of harmonic voltages become distinct as the frequency of the voltage waveform is reduced from 40 Hz to 32.5 Hz and to 25 Hz. Figures 131 and 137 show the effect of harmonics with increased magnitudes.
CONCLUSION

The minimisation of total losses in an induction machine, when it is subjected to PWM voltage waveforms, for the unsaturated motor operation, has been analysed by optimisation of harmonic contents of the PWM waveforms. The designs of these waveforms and their applications, for a continuous motor operation between the stationary state and maximum speed at a constant flux, have been analysed by introducing methods to overcome constraints of the drive system. The results of the investigation presented here reveal the following.

a) The PWM voltage waveforms, whose pulse-durations or pulse-widths are predetermined, cannot possess more than one fundamental voltage magnitude at a particular frequency, when the inverter input voltage has a constant magnitude. The harmonic contents of these waveforms change as the positions of the pulses are changed with respect to one another. Therefore, only one fundamental-voltage magnitude is obtained for a fixed harmonic content at a frequency.

b) The waveform, whose pulse-duration is maintained at a particular value, possesses higher harmonic content as the frequency is decreased due to the increase of space between two consecutive pulses. The percentage of the harmonics, with respect to the fundamental, becomes significantly high for frequencies below 10 Hz. The harmonic content decreases as the fundamental frequency is increased with its voltage magnitude.

c) The nonlinear mathematical equations of the harmonic voltages, when they are obtained from Fourier series, do not allow a
single PWM waveform to cover the complete speed range of the motor for a fixed V/f. The variation of fundamental-voltage magnitude to frequency ratio is greater at higher frequencies, (40 Hz – 50 Hz), than that in low frequencies, (below 40 Hz). It is due to the rapid change of mark-space ratio. This is overcome by the change of the pulse duration at each step of frequency.

d) The operation of the induction motor was achieved between 0.1 Hz and 50 Hz without exceeding the limits of 5th and 7th harmonics which had been defined as 30% and 40% respectively.

e) The method of selecting a PWM voltage waveform is given to minimise the total losses of the motor for application to a constant V/f ratio.

f) The use of PWM voltage waveforms improves the motor operation for a wide speed range, especially at low frequencies when compared with 1 PWM wave which is initially arranged as a quasi-square wave. The constant pulse-duration of 1 PWM wave causes high harmonic content for low frequencies, when the inverter has a fixed input voltage. The harmonic content is reduced by the use of PWM waveform.

g) The effect of the modulation number is determined experimentally and theoretically. It is found that the higher the number of modulation, the lower the harmonics obtained. But the fundamental voltage to frequency ratio decreases when it is compared with that of a waveform, which has a small number of pulses, for a fixed value of total pulse-duration in half a cycle.
h) An alternative method to reduce the harmonic content of a PWM voltage waveform has been given for a variable d.c. link voltage application. The comparison between the PWM waveforms generated by two different inverters, which have constant and variable input-voltage magnitudes, for a fixed value of fundamental voltage, shows that the use of a variable d.c. link voltage together with the pulse-width modulation technique enables a waveform to have its harmonic content further improved and provides wider voltage control than the use of constant input voltage over a frequency range. Consequently, the total additional-losses of the machine are reduced by 64\% of the total additional-losses caused by the waveform which is generated by an inverter which has constant input-voltage magnitude. The smaller the space between two consecutive pulses, the wider the pulse duration required for a fixed frequency. Therefore, the fundamental voltage magnitude increases with pulse-duration for a constant inverter input-voltage. The harmonic content becomes a minimum when the pulse-duration is maximum. In order to maintain the harmonic content at lowest value and to change the fundamental voltage magnitude with frequency, the inverter’s input-voltage magnitude is to be controllable. The control of the harmonic content is achieved by the variation of pulse-width. The major harmonics of a PWM waveform are minimised to weaken their effects on the machine performance.

1) The experiments and the calculations have shown that the nonsinusoidal voltage waveform application to an induction motor causes extra losses whose major part is constituted by the copper losses. These occur due to harmonic currents in the windings of the stator and of the rotor. The rotor resistance changes with the harmonic frequency according to skin effect. The magnitudes of
the harmonic currents must be reduced to decrease the losses associated with them. The further reduction of the total losses can be achieved beyond the minimisation of harmonic content of the PWM waveform by introducing some improvements in the design of the motor. For example, to reduce the depth of the rotor bars weakens the skin effect.

j) In order to reduce the high harmonic copper-losses due to the major harmonics, 5th, 7th, 11th and 13th, the slot configuration is to be designed so that the harmonic rotor currents encounter low resistance but high reactance. This is accomplished by increasing the width of the slot-part which is below the tooth-tip. Thus, the slot leakage reactance increases and a larger path is provided for high frequency harmonic currents.

k) The losses associated with end winding leakage are the main component of the total stray-losses of the induction motor. Therefore, the overhangs of the stator must be made as short as possible.

l) The hysteresis losses caused by a PWM voltage waveform in an induction motor are almost equal to that of a sinusoidal voltage waveform, since the amplitude of the maximum flux produced in the machine by a PWM waveform is almost equal to that produced by a sine waveform. The number of flux reversals per cycle is also maintained at the value that it is produced by the sine wave.

m) The relationship obtained between the PWM voltage waveforms and eddy-current losses of the induction motor show that the higher the number of pulses, the smaller the losses produced. Furthermore, the waveforms with short duration of pulses increase the eddy current losses.
n) The harmonics, that produce magnetic fields which revolve in the opposite direction to the rotation of the fundamental, have slips larger than unity. They cause braking effects on the machine performance and induce large currents to increase the copper losses in the stator winding and in the rotor winding.

e) The theoretical and the experimental results are shown in tables 5A to 14B for sinusoidal and PWM voltage waveform applications. Satisfactory agreement has been achieved for the predictions of an induction motor performance under load conditions.
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APPENDIX 1

RELATIONSHIP BETWEEN A PULSE DURATION AND A HARMONIC VOLTAGE MAGNITUDE IN WENPs

The relationship between a harmonic voltage magnitude and parameters of a PWM waveform or a nonsinusoidal voltage waveform is obtained from Fourier series. If the waveform considered comprises pulses of equal duration, the solution of the problem is achieved as follows. Consider a waveform which has an even number of pulses, such as four pulses, whose one-half cycle is shown in figure A1. Since the pulses have equal durations, the space between two consecutive pulses is given by

\[ T_{sp} = T/24 - P_w/2 \]  \hfill (1)

Fourier series of the waveform is given for a harmonic voltage as

\[ V_{nm} = \frac{16V_{dc}}{\pi n} \sin\left(\frac{\pi}{3} + \frac{8T_{sp}}{2} + \frac{4P_w}{\pi n/2T}\right) \cos\left(\frac{4P_w}{\pi n/2T}\right) \sin\left(\pi n P_w/T\right) \]  \hfill (2)

If equation (1) is substituted in equation (2) and \( T \) is replaced by its equivalent, \( 1000/f \)

\[ V_{nm} = \frac{16V_{dc}}{n} \sin\left(\pi n/3\right) \cos\left(\pi n/12\right) \sin\left(\pi n P_w/1000\right) \]  \hfill (3)

The r.m.s. value of nth harmonic voltage magnitude for a 4 PWM voltage waveform is given by

\[ V_n = \frac{16V_{dc}}{\sqrt{2} \pi n} \sin\left(\pi n/3\right) \cos\left(\pi n/12\right) \sin\left(\pi n P_w/1000\right) \]  \hfill (4)

Similarly, the same relationship for a 6 PWM voltage waveform which has equal pulse-durations is determined and given as follows.

\[ V_n = \frac{8V_{dc}}{\sqrt{2} \pi n} \left\{ \sin(\omega nT/9) + \sin(\omega nT/6) + \sin(2\omega nT/9) \right\} \sin(\omega n P_w) \]  \hfill (5)
where

\[ \omega = \frac{2\pi}{T} \quad (6) \]

\[ V_n = \frac{8V_{dc}}{\sqrt{2}\pi} \left\{ 2 \sin\left(\frac{5\omega n T}{36}\right) \cos\left(\frac{\omega n T}{36}\right) + \sin\left(\frac{2\omega n T}{9}\right) \right\} \sin\left(\frac{\omega n T}{2}\right) \quad (7) \]

if equation (6) is substituted in equation (7) and \( T \) is replaced by \( \frac{\text{1000}}{f} \)

\[ V_n = \frac{8V_{dc}}{\sqrt{2}\pi} \left\{ 2 \sin\left(\frac{\pi n}{18}\right) \cos\left(\frac{\pi n}{18}\right) + \sin\left(\frac{4\pi n}{9}\right) \right\} \sin\left(\frac{\pi n f}{1000}\right) \quad (8) \]

This method is also applicable to 2 PWM and 8 PWM voltage waveforms, if their pulses have equal durations. When the pulses of a cycle have different durations from one another, this method cannot be used due to the number variables which is more than one. The method given in appendix 2 is used instead.
APPENDIX 2

RELATIONSHIPS BETWEEN A PULSE-DURATION AND A HARMONIC VOLTAGE MAGNITUDE IN WONPs

A.S.1 GENERAL

The relationship between the harmonic voltage magnitude and the pulse-duration is given for WONPs whose pulses have equal durations in appendix 1. When the number of pulses in one-half a cycle is odd, the durations of the pulses cannot be held equal to one another in order to avoid short-circuiting of the thyristor bridge. Therefore, there must be some conditions by which PWM waveforms with an odd number of pulses can be constructed. They are given in chapter 2. In this case, the representation of a harmonic voltage magnitude comprises more than one variable which are the durations of various pulses in one-half a cycle. The duration of a pulse can be dependent on that of another pulse so that they vary together. The other feature to be accounted for is the differences between the pulse-durations. In order to determine a harmonic voltage magnitude in terms of a pulse-duration, there must be a relationship between the pulse-durations of the waveform. Thus, the number of the unknowns can be reduced to one. Waveforms with 1, 3, 5 and 7 pulses have different characters from one another. Therefore, each of them will be given here.

A.S.2 PWM WAVEFORM

If the pulse is laid between T/12 and 5T/12 the waveform is called a quasi-square wave. A change of frequency changes the mark-space ratio of the waveform, therefore, the starting and the ending points of the waveform change with the frequency. The harmonic content of the waveform increases as the mark-space ratio is decreased. Harmonic voltage magnitudes increase with respect to the fundamental. Therefore,
it is essential to have a relationship between the harmonic voltage magnitude and the pulse-duration. The waveform shown in figure 21 is given by Fourier series as follows.

\[ v_n = \frac{4V_{dc}}{\pi n^2} \left[ \cos \left( \frac{2\pi m}{T} \left( \frac{T}{4} - \frac{P}{2} \right) \right) - \cos \left( \frac{2\pi m}{T} \left( \frac{T}{4} + \frac{P}{2} \right) \right) \right] \]  \hspace{1cm} (1)

If \( T \) is replaced by its equivalent \( 10^3/f \),

\[ v_n = \frac{8V_{dc}}{\pi n^2} \sin(\pi n/2) \sin(\pi n P/10^3) \] \hspace{1cm} (2)

hence,

\[ P_w = \frac{1000}{\pi n^2} \arcsin \left( \frac{\sqrt{2}V_n}{8V_{dc}} \right) \] \hspace{1cm} (3)

**A.5.3 3 PWM WAVEFORM**

A 3 PWM voltage waveform is given in figure A22. The waveform has two different pulses as shown by \( P_{w1} \) and \( P_{w2} \). In general, a harmonic voltage magnitude is given by

\[ v_n = \frac{4V_{dc}}{\pi n^2} \left[ \cos \left( \frac{2\pi n}{T} \left( \frac{T}{12} - P_{w1} \right) \right) - \cos(\pi n/6) + \cos \left( \frac{2\pi m}{T} \left( \frac{T}{4} - P_{w2} \right) \right) \right] \] \hspace{1cm} (4)

Meanwhile, from figure A22

\[ \frac{P_{w2}}{2} = \frac{T}{12} + T_{sp} \] \hspace{1cm} (5)

and

\[ \frac{T}{12} = T_{sp} + P_{w1} \] \hspace{1cm} (6)

Hence, if equation (6) is substituted in equation (5)

\[ P_{w2} = \frac{T}{3} - 2P_{w1} \] \hspace{1cm} (7a)

\[ T = \frac{1000}{f} \] \hspace{1cm} (7b)

If equations (7a) and (7b) are substituted in equation (4), the duration of the side pulse is obtained as
A 5 PWM waveform comprises five pulses with durations that are different. The conditions for the construction of such a waveform are given in chapter 2. If the following assumptions are made a relationship between a harmonic voltage magnitude and a pulse-duration is determined by the method given for 1 PWM and 3 PWM waveforms. The assumptions are made on the durations of the pulses as shown in figure A23 and are given by

\[ P_{wl} = \frac{T}{24} - T_{sp} \]  

and

\[ P_{w2} = \frac{T}{24} \]  

From figure A23 the middle pulse-duration is given by

\[ P_{w3} = \frac{T}{12} + T_{sp} \]  

and calling

\[ \omega = \frac{2\pi f}{T} \]  

a harmonic voltage magnitude is given by

\[ V_n = \frac{8V_{dc}}{\pi n \sqrt{2}} \left[ \cos\left(\omega\left(\frac{T}{24} - P_{wl}\right)\right) - \cos\left(\omega\frac{T}{24}\right) + \cos\left(\omega\frac{T}{12}\right) - \cos\left(\omega\frac{T}{12} + P_{w2}\right) \right] \]

\[ + \cos\left(\omega\left(\frac{T}{4} - P_{w3}\right)\right) - \cos\left(\omega\frac{T}{4}\right) \]  

If equations (10) and (11) are replaced in equation (13)

\[ V_n = \frac{8V_{dc}}{\pi n \sqrt{2}} \left[ \cos\left(\omega\left(\frac{T}{12} - P_{wl}\right)\right) \cos\left(\omega\frac{T}{24}\right) - \cos\left(\omega\frac{T}{48}\right) \cos\left(\omega\frac{T}{48}\right) \right] \]

\[ + \sin\left(\omega\frac{T}{48}\right) \sin\left(\omega\frac{T}{48}\right) \]  

and replacing T by its equivalent of 1000/f, the duration of the first pulse is obtained as

\[ P_{wl} = \left(\frac{\pi n}{6} - \arccos(A')\right) \frac{1000}{2\pi f} \]
where

\[ A' = \left\{ \frac{\pi n \sqrt{2V}}{8V_{dc}} + \cos(\pi n/24) \cos(\pi n/24) - \sin(\pi n/24) \sin(\pi n/24) \right\} \]

\[ \{1/\cos(\pi n/12)\} \]  \hspace{1cm} (16)

All the durations are given in milliseconds.

A.S.5 7 PWM WAVEFORM

One quarter of a cycle of a 7 PWM voltage waveform is shown in figure A24. The middle pulse which is symmetrical about T/4 is shown in half, since the waveform is also symmetrical about T/4. The durations of the four different pulses are represented by \( P_{w1} \), \( P_{w2} \), \( P_{w3} \) and \( P_{w4} \). The positions of the pulses with respect to one another or to some special points, such as T/12, T/6, T/4 ... etc., and the variations of their durations are discussed in chapter 2. The relationship between a harmonic voltage magnitude and a pulse-duration is determined similarly to those of the waveforms described in the previous sections, if the duration of one of the four pulses can be given in terms of the others. It is assumed that the relationship between the pulse-durations as given in figure A24 is as follows:

\[ P_{w4} = P_{w1} + P_{w2} + P_{w3} \]  \hspace{1cm} (17)

\[ P_{w3} = (T/12 - P_{w2}) - (T/48 + P_{w1}) \]  \hspace{1cm} (18)

If the duration \( P_{w2} \) is given as

\[ P_{w2} = T/48 \]  \hspace{1cm} (19)

by substituting equation (19) in equation (18), the duration of the third pulse is obtained as

\[ P_{w3} = T/24 - P_{w1} \]  \hspace{1cm} (20)

The duration of the fourth pulse is obtained in terms of \( P_{w1} \) if equations (19) and (20) are substituted in equation (17), thus it is found that
$P_{w4} = \frac{3T}{48}$ (21)

$P_{w4}$ is independent of other pulse-durations. A harmonic voltage magnitude is given by Fourier series as

$$V_n = \frac{8V_{dc}}{\pi n v^2} \left[ \cos(\omega T/12) \cos \left(\frac{n(T-P_{w1})}{6}\right) \right] - \frac{\sin(5\omega T/96) \sin(\omega T/32)}{\sin(\omega T/16)}$$

$$+ \cos(\omega T/16) - \sin(3\omega T/32) \sin(\omega T/96) \right] \tag{22}$$

From this equation duration of the first pulse is obtained as

$$P_{w1} = \frac{1000}{2\pi m} \left[ \frac{\pi n V_n}{8V_{dc} \cos(\pi n/6)} + B' \right] \tag{23}$$

where

$$B' = \left\{ \frac{1}{\cos(\pi n/6)} \right\} \left\{ \sin(5\pi n/48) \sin(\pi n/16) - \cos(\pi n/8) + \sin(3\pi n/16) \sin(\pi n/48) \right\} \tag{24}$$
APPENDIX 3

SKIN EFFECT COEFFICIENTS FOR BAR RESISTANCE AND INDUCTANCE

Although skin-effect coefficients are defined in reference (7) and (14), the results described in them are not satisfactory. In reference (14) the skin-effect coefficient is not defined for $1.5 \leq \varepsilon \leq 2$, while in reference (7) there is assumed to be a printing mistake in the formula which is shown by equation number (23.18) in page 552. The formula is printed without the brackets around the ratio of trigonometric functions which must be multiplied by the ratio of $3/2\varepsilon$ as derived below.

In chapter 4 of this thesis, equation (173) represents the bar impedance of which the real and imaginary parts give the values of its resistance and reactance, respectively. But the solution to the problem, when $\varepsilon$ tends to either zero or infinity, produces undefined results. Therefore, each term is examined separately here.

The real part of equation (173) is given by

$$Z_{Re} = \frac{v}{\sigma_0 b} \frac{\sinh(2\varepsilon) + \sin(2\varepsilon)}{\cosh(2\varepsilon) - \cos(2\varepsilon)} \tag{1}$$

multiplying (1) by $h/h$ and setting $\varepsilon$ to a very small value, so that the values of the hyperbolic sine and the sine can be taken as their angles, then

$$Z_{Re} = \frac{vh}{\sigma_0 bh} \frac{(2\varepsilon + 2\varepsilon)}{1+2\varepsilon^2-1+2\varepsilon^2} \tag{2}$$

and substituting $vh$ by $\varepsilon$

$$Z_{Re} = \frac{1}{\sigma_0 bh} \tag{3}$$

Equation (3) gives the d.c. resistance, since $\varepsilon$ is very small and, consequently, the frequency is almost zero. Hence, equation (1) can
be written as

$$Z_{Re} = R_{dc} \cdot \varepsilon \frac{\text{Sinh}(2\varepsilon) + \text{Sin}(2\varepsilon)}{\text{Cosh}(2\varepsilon) - \text{Cos}(2\varepsilon)}$$  \hspace{1cm} (4)$$

when $\varepsilon$ tends to infinity the Sine and Cosine terms become negligible beside the large values of the hyperbolic sine and hyperbolic cosine which are nearly equal to each other. Therefore, equation (4) is given by

$$Z_{Re} = \varepsilon R_{dc}$$  \hspace{1cm} (5)$$

Hence, the factor by which the resistance changes is given by

$$k_R = \varepsilon$$  \hspace{1cm} (6)$$

The imaginary part of equation (173) is given as below

$$Z_{im} = \frac{\nu h}{\sigma_{o bh} \cdot \text{Sinh}(2\varepsilon) - \text{Sin}(2\varepsilon)}$$  \hspace{1cm} (7)$$

This formula can be represented by its equivalent if the trigonometric terms are expanded into Taylor series and the necessary algebra is performed without involving the first term.

$$Z_{im} = \frac{\nu h}{\sigma_{o bh}} \frac{(2\varepsilon)^3}{3!} \frac{1 + (2\varepsilon)^4}{4.5.6.7} + \frac{(2\varepsilon)^8}{4.5.6.7...11} + \cdots$$  \hspace{1cm} (8)$$

substituting big brackets by $k_x$ and $\nu h$ by $\varepsilon$, equation (8) is reduced to

$$Z_{im} = R_{dc} \frac{2\varepsilon^2}{3} (k_x)$$  \hspace{1cm} (9)$$

where

$$k_x = 1 - \frac{8\varepsilon^4}{315} + \frac{32\varepsilon^8}{31185} + \cdots$$  \hspace{1cm} (10)$$

From equation (7) and (9) the skin-effect coefficient for inductances is obtained as
\[ k_x = \frac{3}{2\varepsilon} \left( \frac{\text{Sinh}(2\varepsilon) - \text{Sin}(2\varepsilon)}{\text{Cosh}(2\varepsilon) - \text{Cos}(2\varepsilon)} \right) \]  

(11)

Hence the factor defined by equation (11) gives required value of the skin-effect coefficient for any value of \( \varepsilon \). This formula can be simplified for values of \( \varepsilon > 3.2 \) as

\[ k_x = 1.5\varepsilon \]  

(12)

The large values of \( \varepsilon \) make Sine and Cosine terms negligible with respect to those obtained from hyperbolic sine and the hyperbolic cosine. Therefore, the skin-effect coefficient for a reactance is represented generally by equation (11). Figures associated with the formulas given in references (7) and (14), and those calculated by equation (11) are obtainable from the program illustrated, FM301, for various values of \( \varepsilon \).
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VARIATION OF IRON LOSSES WITH VOLTAGE WAVE SHAPE

The variation of the iron losses with the applied voltage waveform is an important subject that is not introduced in the major part of the thesis. The analysis of the variation of iron losses is considered in two groups. One of the two which is proportional to magnetic flux, is the hysteresis loss, while the other which is proportional to the square of flux density, is the eddy-current loss. The procedure of the analysis is achieved in three separate steps in which three different voltage-waveform applications are discussed. The iron losses caused by the application of a sinusoidal-voltage waveform is held as a base for comparison purposes with nonsinusoidal-voltage waveforms which are, namely, the quasi-square wave and the PWM voltage waveforms.

It is assumed that a sinusoidally varying line-voltage waveform with a peak amplitude, $V$, produces sinusoidal time variation of flux linkage with the coils of one phase in the airgap of the induction motor, then by neglecting the voltage drop in the stator winding, the flux is given in terms of the applied voltage as below,

$$\phi = \frac{VT}{2\pi} \sin(\frac{2\pi t}{T})$$  \hspace{1cm} (1)

where

$$V(t) = V \cos(\frac{2\pi t}{T})$$  \hspace{1cm} (2)

Thus, the maximum value of flux linkage due to the sinusoidal voltage wave is obtained as $VT/2\pi$. Hysteresis and eddy-current losses are determined, respectively, from the following formulas, reference (24).

$$W_{\text{Fe h}} = k_h \phi$$  \hspace{1cm} (3)
\[ W'_{\text{Fee}} = k_e \phi^2 \]  
(4a)

where

\[ k_h = k_{hc} \cdot f \]  
(4b)

\[ k_e = k_{ec} \cdot f \]  
(4c)

\( k_{ec} \) and \( k_{hc} \) are machine constants.

If the peak value of \( \phi \) in equation (1) is replaced by \( \phi \) in equations (3) and (4a),

\[ W'_{\text{Feh}} = k_h \frac{\sqrt{V^2/2\pi}} {2/3} \]  
(5)

\[ W'_{\text{Fee}} = k_e \left( \frac{\sqrt{V^2/2\pi}}{2/3} \right)^2 \]  
(6)

The quasi-square line-voltage wave is given by figure A81a, whose r.m.s. value is given by

\[ V_{\text{rms}}^{\text{qw}} = \sqrt{\frac{2}{3}} \]  
(7)

and its r.m.s. value of fundamental is given by

\[ V_{\text{rms}}^{\text{fqw}} = \frac{2}{\sqrt{2\pi}} \int_{\pi/6}^{5\pi/6} V \sin(\theta) d\theta \]  
(8)

\[ V_{\text{rms}}^{\text{fqw}} = \frac{\sqrt{6}}{\pi} \]  
(9)

If it is assumed that the quasi-square wave and the sine wave have the same fundamental voltage, \( E \), the maximum amplitude of the fundamental is given by

\[ \frac{2\sqrt{6}}{\pi} = E \sqrt{2} \]  
(10)

\[ V_{\text{peak}}^{\text{qw}} = \frac{E \pi}{\sqrt{6}} \]  
(11)

while for the sine wave

\[ V_{\text{peak}}^{\text{sw}} = E \sqrt{2} \]  
(12)
The flux due to the quasi-square line-voltage wave is given by

\[ \phi = - \int_0^{T/3} V dt \]  

(12a)

\[ \phi = \frac{VT}{3} \]  

(12b)

In figure A41b the variation of flux is shown against time

\[ \phi_{qW} = \frac{VT}{6} \]  

(13)

Replacing equation (11) in equation (13)

\[ \phi_{qW} = \frac{EVT}{6/6} \]  

(14)

If equation (14) is divided by the magnitude of flux given in equation (1a),

\[ \frac{W''_{Feh}}{W'_{Feh}} = \frac{\pi^2}{6/3} = 0.95 \]  

(15)

This shows that hysteresis losses due to a quasi-square wave are 5% smaller than those caused by a sine wave. The eddy-current losses are determined similarly.

\[ W''_{Fee} = k_e \int_0^T \left( \frac{d}{dt} \phi(t) \right)^2 dt \]  

(16)

\[ W''_{Fee} = k \frac{2TV^2}{e^3} \]  

(17)

Eddy-current losses due to sine wave are

\[ W'_{Fee} = k_e \int_0^T \left( \frac{W_0}{\pi} \sqrt{2} \right)^2 \sin^2 \left( \frac{2\pi}{T} t \right) dt \]  

(18)

\[ W'_{Fee} = \frac{6V^2 T}{\pi^2} k_e \]  

(19)

If equation (17) is divided by equation (19)
\[ \frac{W_{\text{Fee}n}}{W_{\text{Fee}}'} = \frac{\pi}{9} \approx 1.1 \quad (20) \]

Equation (20) shows that the eddy-current losses increase by 10% with respect to those caused by the sine wave.

The calculation of eddy-current and hysteresis-losses due to a PWM voltage waveform is achieved by similar method used for the quasi-square voltage wave. But the PWM waveform is different from quasi-square waveform, QS, since it is made by a train of pulses. Therefore, the single pulse of a QS wave is divided into smaller pulses and some spaces, reducing the total pulse-width to values smaller than \( T/3 \). The maximum total pulse-width is decreased as the number of modulation is increased. The duration of a pulse, \( P_w \), for a PWM waveform which has \( N_{\text{pw}} \) pulses in one half a cycle is given by

\[ P_w = cT \frac{N}{N_{\text{pw}}} \quad (21) \]

where \( c \leq 1 \), a factor which determines the pulse-duration.

Thus, the r.m.s. value of a 4 PWM voltage waveform which is illustrated in figure A82a with equal pulse-durations is given by

\[ V_{\text{pwm}}^{\text{rms}} = \sqrt{\frac{2c}{3}} \quad (22) \]

The r.m.s. value of PWM waveform's fundamental is

\[ V_{\text{fpwm}}^{\text{rms}} = \frac{\sqrt{6}}{\pi} (\sqrt{6} + \sqrt{2}) \sin\left(\frac{\pi c}{12}\right) \quad (23) \]

The ratio between r.m.s. value of the fundamental and that of the whole waveform is

\[ \frac{V_{\text{fpwm}}^{\text{rms}}}{V_{\text{pwm}}^{\text{rms}}} = \frac{3}{\pi} \frac{\sqrt{6} + \sqrt{2}}{\sqrt{c}} \sin\left(\frac{\pi c}{12}\right) \quad (24) \]
When \( c \) and \( \sin(\pi/12) \) are substituted by unity and \( (\sqrt{6} - \sqrt{2})/4 \), respectively, the ratio is obtained as

\[
\frac{V_{\text{rms}}}{V_{\text{pwm}}} = \frac{3}{\pi} \tag{25}
\]

Supposing PWM and sinusoidal waveforms have the same fundamental voltage, \( E \), the maximum magnitude of PWM waveform's fundamental is obtained from equation (23) as

\[
\frac{V\sqrt{6}}{\pi} \left( \sqrt{6} + \sqrt{2} \right) \sin\left(\frac{\pi a}{12}\right) \sqrt{2} = E \sqrt{2}
\]

and therefore

\[
V = \frac{E\pi}{\sqrt{6}(\sqrt{6} + \sqrt{2}) \sin\left(\frac{\pi a}{12}\right)} \tag{26}
\]

The peak value of the flux due to PWM voltage waveform, as in figure A42b, is determined by using equation (1) and substituting equations (21) and (26) in it.

\[
\phi_{\text{pwm}} = -V \left[ \int_{t_1}^{t_2} dt + \int_{t_3}^{t_4} dt + \int_{t_5}^{t_6} dt + \int_{t_7}^{t_8} dt \right] \tag{27}
\]

where

\[
\begin{align}
t_1 &= \frac{3T_{\text{PWM}}}{24} - \frac{P_W}{2}, \\
t_2 &= \frac{3T_{\text{PWM}}}{24} + \frac{P_W}{2}, \\
t_3 &= \frac{5T_{\text{PWM}}}{24} - \frac{P_W}{2}, \\
t_4 &= \frac{5T_{\text{PWM}}}{24} + \frac{P_W}{2}, \\
t_5 &= \frac{7T_{\text{PWM}}}{24} - \frac{P_W}{2}, \\
t_6 &= \frac{7T_{\text{PWM}}}{24} + \frac{P_W}{2}, \\
t_7 &= \frac{9T_{\text{PWM}}}{24} - \frac{P_W}{2}, \\
t_8 &= \frac{9T_{\text{PWM}}}{24} + \frac{P_W}{2}
\end{align}
\]

\[
\phi_{\text{pwm}} = \frac{VCT}{3} \tag{28}
\]

Since this magnitude corresponds to the height between the maximum and the minimum points of the characteristic given in figure A42b, the magnitude of one half-cycle is given by

\[
\frac{\phi_{\text{pwm}}}{6} = \frac{VCT}{6} \tag{29}
\]
in which the equation (26) is substituted, thus

\[
\phi_{\text{pwm}} = \frac{EV}{\sqrt{6}(\sqrt{6} + \sqrt{2}) \sin(\pi c/12)} \cdot \frac{CT}{6}
\]  

(30)

While the flux due to sinusoidal voltage waveform is

\[
\phi_{\text{sine}} = \frac{\sqrt{2} E T}{2\pi}
\]  

(31)

thus the ratio between the fluxes of PWM and sinusoidal waveforms is given by

\[
\frac{\phi_{\text{pwm}}}{\phi_{\text{sine}}} = \frac{\pi^2 c}{6 \sqrt{3}(\sqrt{6} + \sqrt{2}) \sin(\pi c/12)}
\]  

(32)

If \(\sin(\pi c/12)\) and \(c\) are replaced by \((\sqrt{6} - \sqrt{2})/4\) and 1, the PWM voltage waveform becomes the quasi-square voltage waveform. This ratio also represents the change of hysteresis losses due to nonsinusoidal voltage waveform.

Determination of eddy-current losses due to a PWM voltage waveform is obtained similarly to that of QS voltage waveform. These losses are calculated by equation (16). In order to have a comparison between the eddy-current losses that are caused by sinusoidal and PWM voltage waveforms, the r.m.s. value of a sinusoidal voltage wave is to be equal to that of PWM voltage waveform's fundamental. Suppose that the sine wave has a voltage magnitude, \(E'\).

\[
\frac{E'}{\sqrt{2}} = \frac{\sqrt{6}(\sqrt{6} + \sqrt{2})}{\pi} V \sin(\pi c/12)
\]

\[
E' = \frac{\sqrt{2}}{\pi} V(\sqrt{6} + \sqrt{2}) \sin(\pi c/12)
\]  

(33)

Using equations (1) and (16) the eddy-current losses due to sine wave with amplitude of \(E'\) is given by

\[
W'_{\text{Fee}} = k_e \int_0^T \left[ \frac{2\sqrt{3}}{\pi} V(\sqrt{6} + \sqrt{2}) \sin(\pi c/12) \right]^2 \sin^2(\frac{2\pi}{T}t) dt
\]  

(34)
The eddy-current losses for PWM voltage waveform are determined by using equations (1), (16) and (22).

\[ W'_{\text{Fee}} = k_e \frac{2\sqrt{3}}{\pi^2} \left(\sqrt{6} + \sqrt{2}\right)^2 v^2 \sin^2\left(\frac{\pi c}{12}\right) \]  

(35)

Dividing equation (36) by equation (35)

\[ \frac{W''''_{\text{Fee}}}{W'_{\text{Fee}}} = \frac{\pi^2}{9} \frac{c}{\left(\sqrt{6} + \sqrt{2}\right)^2 \sin^2\left(\frac{\pi c}{12}\right)} \]  

(37)

If \( c \) is equal to unity, this implies that the waveform is quasi-square wave. Hence the eddy-current losses caused by such a waveform are obtained as shown by equation (20).

\[ \frac{W''''_{\text{Fee}}}{W'_{\text{Fee}}} = 1.1 \]

Equation (37) shows the amount of change of eddy-current losses with respect to those produced by a sinusoidal voltage waveform in terms of pulse-width. Although the pulse-width is not directly shown in the above equation, it can be included by using the equation (21).
APPENDIX 5

DETAILS OF EQUIPMENT USED

NO-LOAD AND BLOCKED-ROTOR TESTS OF THE INDUCTION MACHINE

1. Squirrel Cage Induction Motor

Manufacturer: Brook Motors

Continuous operation voltage: 380 - 420 volts

Full load current: 14.6 A

Horse-power: 10 HP (7.5 kw)

Number of phases and connection of stator winding: 3 phase,

Full load speed = 1430 rev/min

Frequency = 50 Hz

2. D.C. Generator

Manufacturer: Veritys, Birmingham, England

Continuous operation voltage: 460 volts

Continuous full load current: 27.7 Amperes

Speed: 1500 rev/min

Horse-power: 15 BHP (11.2 kw)

Serial number: 067832

3. Inverter

Manufacturer: Brush Electrical Engineering, Loughborough, Leics.

D.C. output voltage from rectifier bridge: 550 volts (mean value)

Full load current: 76 Amperes

Output power of the inverter = 50 kVA

Number of thyristors: 6 main and 6 auxiliary
No-load and blocked-rotor tests have been carried out on the 3-phase squirrel-cage induction motor at 40, 32.5 and 25 Hz frequencies. Tables T51 and T52 show the measured input-power, the phase current and the phase voltage in the no-load and in the blocked-rotor tests, respectively. The calculations of the equivalent circuit parameters are carried out with regard to formulas shown in chapter 6. The resistance of the stator winding is determined by the ammeter-voltmeter method which is given in table T53. Thus, the equivalent circuit parameters from these tests are determined as shown in table T54.

Friction and windage losses of the motor are obtained as a result of extrapolation of the input-power against the r.m.s. value of the phase voltage. It is assumed that the friction and windage losses of the motor do not change due to an incorrect setting of the supply frequency which will produce an incorrect speed or an incorrect reading of the rotor speed from the stroboscope-digital counter system.
APPENDIX 6

COMPUTER PROGRAM FOR THE MINIMISATION OF INDUCTION MOTOR LOSSES

The calculations of the components of an induction motor's total losses are given in chapter 5. Each loss component is given in terms of the applied voltage magnitude so that it can be related directly to the pulse-duration of a nonsinusoidal waveform. Thus, the total losses of an induction motor are determined in terms of pulse duration. If the losses caused by the fundamental voltage component are assumed unchanged by controlling the variation of the inverter's input-voltage, additional losses caused by the harmonics of the waveform change with the magnitudes of these harmonics which can be controlled by the variation of the pulse-duration. Further information about the relationship between the pulse-duration and the total additional-losses of an induction motor is given in chapter 7. The relationship is a non-linear function. Its solution, in terms of pulse duration, is not analytically possible. Therefore, it is obtained by the numerical method. Thus, the variation of the total additional losses is determined by changing the pulse-duration in small steps. A computer program is prepared for this purpose. Its flow-chart is shown in figure A61. The output of the plotter is shown in figure A62 where the minimum total losses of the motor is obtained when the pulse-duration is 2.101 ms. Although, this pulse-duration does not give the absolute minimum which is obtained at 4.3 ms, the thyristor turn-on and turn-off times limit the maximum permissible pulse-duration for a frequency. This is explained in chapter 7. In figures A63 and A64 variations of total additional-losses are shown against the frequency. The numbers of the characteristics shown in figures A63 and A64 denote the duration of the pulses (1): 1.562 ms, (2): 1.5833 ms and (1): 1.562 ms, (2): 2.101 ms, respectively.
APPENDIX 7

COMPUTER PROGRAM FOR THE CALCULATION OF TOTAL LOSSES OF AN INDUCTION MOTOR

A computer program is prepared to calculate the total losses of an unsaturated induction motor caused by the harmonics of a nonsinusoidal voltage waveform. Its flow-chart is given in figure A7. The method of calculation of the equivalent circuit parameters are obtained as given in chapter 4. In figure A7 NSP and NHT represent respectively the order of space harmonic and the order of time harmonics. Each component of the stray losses is calculated as described in chapter 5. Constants given in the equations which are used to calculate the stray-load losses are obtained from the given references. The constants are functions of the frequency for surface losses and for high-frequency rotor-pulsation losses. The characteristics of these constants are given in the references. Each characteristic is approximated by straight line segments.

Input to the program comprises the design data of the induction motor, the slip and the variation of the voltage waveform with time. The efficiency, input-power, output-power and the total additional losses caused by the harmonics are printed out in the output of the program. In the computer program two different equivalent circuits are used to calculate the machine performance. One is used for the fundamental harmonic while the other for harmonics. The difference between them is that the magnetising branch is taken to the terminals of the equivalent circuit when the machine performance is calculated for harmonics. The reason for this approximation is described in chapter 4.
APPENDIX 8

COMPUTER PROGRAM TO DETERMINE THE LINE CURRENT WAVEFORM OF AN INDUCTION MOTOR

The determination of the line current waveform of an induction motor is necessary, if the applied voltage has a nonsinusoidal waveform. The level of saturation of the machine's magnetic material changes with the value of the peak current-magnitude. Therefore, the variation of the line-current or the phase-current against time is important, if the performance of a saturated motor is to be examined. The flowchart of a computer program is given in figure AS, to calculate and to plot the waveforms of the line-current and the phase-current of the motor against time. The input-data comprise the equivalent-circuit parameters and the instantaneous variation of the applied voltage.

After setting the L matrix of the motor, the phase-currents of the stator and the rotor are obtained from the corresponding flux linkages of each phase. Time is changed from zero to a final value, which is equal to five periods of the applied voltage-waveform, in steps. The choice of the step-length determines the accuracy of the peak value of the instantaneous current and the time of the computation. Therefore, the higher the accuracy, the longer the computation time needed to determine the instantaneous current variation of the motor.
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<td>( \frac{n_s - n_s}{5} )</td>
<td>5( \frac{n_s}{5} = 6f_s )</td>
<td>300</td>
</tr>
<tr>
<td>7</td>
<td>7P</td>
<td>( \frac{n_s}{7} )</td>
<td>( f_s )</td>
<td>( \frac{n_s - n_s}{7} )</td>
<td>7( \frac{n_s}{7} = 6f_s )</td>
<td>300</td>
</tr>
<tr>
<td>11</td>
<td>11P</td>
<td>(-\frac{n_s}{11})</td>
<td>( f_s )</td>
<td>( \frac{n_s - n_s}{11} )</td>
<td>11( \frac{n_s}{11} = 12f_s )</td>
<td>600</td>
</tr>
<tr>
<td>13</td>
<td>13P</td>
<td>( \frac{n_s}{13} )</td>
<td>( f_s )</td>
<td>( \frac{n_s - n_s}{13} )</td>
<td>13( \frac{n_s}{13} = 12f_s )</td>
<td>600</td>
</tr>
<tr>
<td>( 2k+1 )</td>
<td>( (2k+1)P )</td>
<td>( \frac{n_s}{(2k+1)} )</td>
<td>( f_s )</td>
<td>( \frac{n_s - n_s}{(2k+1)} )</td>
<td>( (2k+1) \frac{n_s}{(2k+1)} = \frac{2kqns}{(2k+1)} )</td>
<td>( 100 ) ( kq )</td>
</tr>
</tbody>
</table>

The frequencies induced in the rotor due to space harmonics...
### TABLE 2

<table>
<thead>
<tr>
<th>$S_s$</th>
<th>Order of slot harmonic</th>
<th>Frequency induced in the rotor</th>
<th>Slot frequency for 50 Hz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>17 and 19</td>
<td>18 $fs$</td>
<td>900</td>
</tr>
<tr>
<td>4</td>
<td>23 and 25</td>
<td>24 $fs$</td>
<td>1200</td>
</tr>
<tr>
<td>5</td>
<td>29 and 31</td>
<td>30 $fs$</td>
<td>1500</td>
</tr>
<tr>
<td>6</td>
<td>35 and 37</td>
<td>36 $fs$</td>
<td>1800</td>
</tr>
<tr>
<td>$S_s$</td>
<td>$(2qS_s + 1)$</td>
<td>$2qS_s$ $fs$</td>
<td>$100qS_s$</td>
</tr>
</tbody>
</table>

The relationship between the slot harmonic frequency induced in the rotor and the number of slots.

### TABLE 3

<table>
<thead>
<tr>
<th>Order of space harmonics</th>
<th>Speed with respect to rotor</th>
<th>Frequency induced in the rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$n_s - n_S (1 - S)$</td>
<td>$Sfs$</td>
</tr>
<tr>
<td>5</td>
<td>$n_S - n_S (1 - S)$</td>
<td>$(5S - 6)fs$</td>
</tr>
<tr>
<td>7</td>
<td>$n_S - n_S (1 - S)$</td>
<td>$(7S - 6)fs$</td>
</tr>
<tr>
<td>11</td>
<td>$n_S - n_S (1 - S)$</td>
<td>$(11S - 12)fs$</td>
</tr>
<tr>
<td>13</td>
<td>$n_S - n_S (1 - S)$</td>
<td>$(13S - 12)fs$</td>
</tr>
<tr>
<td>$(2kq + 1)$</td>
<td>$-(2kq + 1)n(1 - S)$</td>
<td>$(2kq + 1)S - 2kq$ $fs$</td>
</tr>
</tbody>
</table>

The harmonic frequencies induced in the rotor at slip $S$. 
Table 4 The comparison of two different $G_{n1}$s, which are determined from direct calculation of harmonic losses and from the method introduced in chapter 7, for different time harmonics.
**TABLE: 5a: LOAD TEST by sinusoidal voltage waveform**

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>0.191</th>
<th>0.3416</th>
<th>0.75</th>
<th>1.1</th>
<th>1.391</th>
<th>1.866</th>
<th>2.416</th>
<th>3.03</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>680</td>
<td>880</td>
<td>1412</td>
<td>1890</td>
<td>2344</td>
<td>2960</td>
<td>3696</td>
<td>4560</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>340</td>
<td>540</td>
<td>1090</td>
<td>1435</td>
<td>1905</td>
<td>2400</td>
<td>3080</td>
<td>3770</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>340</td>
<td>340</td>
<td>322</td>
<td>455</td>
<td>439</td>
<td>560</td>
<td>616</td>
<td>790</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>50</td>
<td>61.3</td>
<td>77.3</td>
<td>76</td>
<td>81.3</td>
<td>81</td>
<td>83.3</td>
<td>82.6</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_1$ (A)</td>
<td>3.54</td>
<td>3.56</td>
<td>3.74</td>
<td>3.89</td>
<td>4.13</td>
<td>4.48</td>
<td>5.02</td>
<td>5.73</td>
</tr>
<tr>
<td>5th Harmonic current (phase) $I_5$ (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7th Harmonic current (phase) $I_7$ (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Frequency = 40 Hz  D.C. Link Voltage = —— Volts, The pulse-width = —— ms., Average temperature = 21°C

Fundamental voltage = 320 volts  5th Harmonic Voltage = —— Volts, 7th Harmonic Voltage = —— Volts
TABLE: 5b: Calculated Load Test when sinusoidal voltage applied.

<table>
<thead>
<tr>
<th>Slip (%) s</th>
<th>0.191</th>
<th>0.3416</th>
<th>0.75</th>
<th>1.1</th>
<th>1.391</th>
<th>1.866</th>
<th>2.416</th>
<th>3.03</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>654</td>
<td>860</td>
<td>1421</td>
<td>1899</td>
<td>2296</td>
<td>2904</td>
<td>3673</td>
<td>4487</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>263</td>
<td>467</td>
<td>1012</td>
<td>1469</td>
<td>1842</td>
<td>2403</td>
<td>3095</td>
<td>3806</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>391</td>
<td>393</td>
<td>409</td>
<td>430</td>
<td>454</td>
<td>501</td>
<td>578</td>
<td>681</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>40.2</td>
<td>54.2</td>
<td>71.2</td>
<td>77.3</td>
<td>80.2</td>
<td>82.7</td>
<td>84.2</td>
<td>84.8</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_1$ (A)</td>
<td>3.67</td>
<td>3.71</td>
<td>3.88</td>
<td>4.0</td>
<td>4.3</td>
<td>4.7</td>
<td>5.2</td>
<td>5.9</td>
</tr>
<tr>
<td>5th Harmonic cur. (phase) $I_5$ (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7th Harmonic cur. (phase) $I_7$ (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Frequency = 40 Hz., D.C. Link Voltage = --- Volts, The pulse-width = --- ms., Average temperature = 21 °C

Fundamental Voltage = 320 Volts, 5th Harmonic Voltage = --- Volts, 7th Harmonic Voltage = --- Volts
TABLE: 6a: LOAD TEST 2 PWM voltage waveform

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>0.375</th>
<th>0.425</th>
<th>0.525</th>
<th>0.716</th>
<th>0.95</th>
<th>1.283</th>
<th>1.675</th>
<th>2.158</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>1880</td>
<td>1960</td>
<td>2100</td>
<td>2320</td>
<td>2660</td>
<td>3100</td>
<td>3540</td>
<td>4160</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>450</td>
<td>510</td>
<td>650</td>
<td>860</td>
<td>1200</td>
<td>1550</td>
<td>2010</td>
<td>2510</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>1430</td>
<td>1450</td>
<td>1450</td>
<td>1460</td>
<td>1460</td>
<td>1550</td>
<td>1530</td>
<td>1650</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>24</td>
<td>26</td>
<td>31</td>
<td>37</td>
<td>45.1</td>
<td>50</td>
<td>57</td>
<td>60</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_1$ (A)</td>
<td>4.89</td>
<td>4.92</td>
<td>4.92</td>
<td>4.93</td>
<td>4.98</td>
<td>5.01</td>
<td>5.08</td>
<td>5.26</td>
</tr>
<tr>
<td>5th Harmonic current (phase) $I_5$ (A)</td>
<td>6.89</td>
<td>6.95</td>
<td>6.97</td>
<td>6.89</td>
<td>6.7</td>
<td>6.7</td>
<td>6.74</td>
<td>6.78</td>
</tr>
<tr>
<td>7th Harmonic current (phase) $I_7$ (A)</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Frequency = 40 Hz., D.C. Link Voltage = 560 Volts, The pulse-width = 3.0 ms., Average temperature = 24°C
Fundamental Voltage = 321 volts, 5th Harmonic Voltage = 174 Volts, 7th Harmonic Voltage = 33 volts
### TABLE: 6b: Calculated Load test for 2 PWM voltage waveform

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>0.375</th>
<th>0.425</th>
<th>0.525</th>
<th>0.716</th>
<th>0.95</th>
<th>1.283</th>
<th>1.675</th>
<th>2.158</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>1718</td>
<td>1786</td>
<td>1923</td>
<td>2184</td>
<td>2502</td>
<td>2953</td>
<td>3480</td>
<td>4126</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>454</td>
<td>521</td>
<td>654</td>
<td>908</td>
<td>1213</td>
<td>1641</td>
<td>2131</td>
<td>2719</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>1264</td>
<td>1265</td>
<td>1268</td>
<td>1276</td>
<td>1288</td>
<td>1312</td>
<td>1348</td>
<td>1406</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>26.4</td>
<td>29.1</td>
<td>34</td>
<td>41.6</td>
<td>48.5</td>
<td>55.5</td>
<td>61.2</td>
<td>65.9</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_1$ (A)</td>
<td>3.73</td>
<td>3.75</td>
<td>3.79</td>
<td>3.87</td>
<td>4.0</td>
<td>5.23</td>
<td>4.55</td>
<td>5.01</td>
</tr>
<tr>
<td>5$^{th}$ Harmonic cur. (phase) $I_5$ (A)</td>
<td>5.88</td>
<td>5.88</td>
<td>5.88</td>
<td>5.88</td>
<td>5.88</td>
<td>5.88</td>
<td>5.88</td>
<td>5.88</td>
</tr>
<tr>
<td>7$^{th}$ Harmonic cur. (phase) $I_7$ (A)</td>
<td>1.52</td>
<td>1.52</td>
<td>1.52</td>
<td>1.52</td>
<td>1.52</td>
<td>1.52</td>
<td>1.52</td>
<td>1.52</td>
</tr>
</tbody>
</table>

Frequency = 40 Hz., D.C. Link Voltage = 560 Volts, The pulse-width = 3.0 ms., Average temperature = $24^\circ$C

Fundamental voltage = 321 Volts, 5th Harmonic Voltage = 166 Volts, 7th Harmonic Voltage = 60 Volts
### TABLE: 7a: Load Test by 4 PFM voltage waveform

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>Power input Pᵢ (W)</th>
<th>Power output Pₒ (W)</th>
<th>Total losses W_L (W)</th>
<th>Efficiency η (%)</th>
<th>Fundamental current (phase) I₁ (A)</th>
<th>5⁰ Harmonic cur. (phase) I₅ (A)</th>
<th>7⁰ Harmonic cur. (phase) I₇ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>0.43</td>
<td>0.475</td>
<td>0.65</td>
<td>0.76</td>
<td>1.0</td>
<td>1.315</td>
<td>1.74</td>
</tr>
<tr>
<td></td>
<td>1312</td>
<td>1380</td>
<td>1540</td>
<td>1740</td>
<td>2050</td>
<td>2440</td>
<td>2960</td>
</tr>
<tr>
<td></td>
<td>520</td>
<td>580</td>
<td>720</td>
<td>930</td>
<td>1280</td>
<td>1630</td>
<td>2080</td>
</tr>
<tr>
<td></td>
<td>790</td>
<td>800</td>
<td>820</td>
<td>810</td>
<td>770</td>
<td>810</td>
<td>980</td>
</tr>
<tr>
<td></td>
<td>39.6</td>
<td>42</td>
<td>46.7</td>
<td>53.4</td>
<td>62.4</td>
<td>66.8</td>
<td>70.3</td>
</tr>
<tr>
<td></td>
<td>3.57</td>
<td>3.59</td>
<td>3.65</td>
<td>3.74</td>
<td>4.85</td>
<td>4.08</td>
<td>4.43</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Frequency = 40 Hz., D.C. Link Voltage = 525 Volts, The pulse-width = 1.562 ms., Average temperature = 23°C

Fundamental Voltage = 309 Volts, 5th Harmonic Voltage = 79.5 Volts, 7th Harmonic Voltage = 66.6 Volts
**TABLE:** 7b: Calculated Load test by 4 FWM voltage waveform

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>Power input $P_i$ (W)</th>
<th>Power output $P_o$ (W)</th>
<th>Total losses $W_L$ (W)</th>
<th>Efficiency $\eta$ (%)</th>
<th>Fundamental current (phase) $I_1$ (A)</th>
<th>5th Harmonic current (phase) $I_5$ (A)</th>
<th>7th Harmonic current (phase) $I_7$ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.43</td>
<td>1291</td>
<td>529</td>
<td>762</td>
<td>40.9</td>
<td>3.60</td>
<td>2.5</td>
<td>1.51</td>
</tr>
<tr>
<td>0.475</td>
<td>1.343</td>
<td>580</td>
<td>763</td>
<td>43.2</td>
<td>3.61</td>
<td>2.5</td>
<td>1.51</td>
</tr>
<tr>
<td>0.65</td>
<td>1565</td>
<td>794</td>
<td>769</td>
<td>50.8</td>
<td>3.68</td>
<td>2.5</td>
<td>1.51</td>
</tr>
<tr>
<td>0.76</td>
<td>1700</td>
<td>925</td>
<td>783</td>
<td>54.4</td>
<td>3.74</td>
<td>2.5</td>
<td>1.51</td>
</tr>
<tr>
<td>1.0</td>
<td>2001</td>
<td>1215</td>
<td>786</td>
<td>60.7</td>
<td>3.87</td>
<td>2.5</td>
<td>1.51</td>
</tr>
<tr>
<td>1.315</td>
<td>2394</td>
<td>1586</td>
<td>807</td>
<td>66.2</td>
<td>4.08</td>
<td>2.5</td>
<td>1.51</td>
</tr>
<tr>
<td>1.74</td>
<td>2926</td>
<td>2080</td>
<td>845</td>
<td>71</td>
<td>4.43</td>
<td>2.5</td>
<td>1.51</td>
</tr>
<tr>
<td>2.25</td>
<td>3553</td>
<td>2648</td>
<td>904</td>
<td>74.5</td>
<td>4.9</td>
<td>2.5</td>
<td>1.51</td>
</tr>
</tbody>
</table>

Frequency = 40 Hz., D.C. Link Voltage = 525 Volts, The pulse-width = 1.562 ms., Average temperature = 23 °C

Fundamental Voltage = 308 Volts, 5th Harmonic Voltage = 70.3 Volts, 7th Harmonic Voltage = 59.4 Volts
### TABLE: 8a: LOAD TEST by sinusoidal voltage waveform

<table>
<thead>
<tr>
<th>Slip (%) ( s )</th>
<th>0.205</th>
<th>0.4615</th>
<th>0.748</th>
<th>1.507</th>
<th>2.041</th>
<th>2.676</th>
<th>3.712</th>
<th>4.82</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input ( P_i ) (W)</td>
<td>560</td>
<td>720</td>
<td>1000</td>
<td>1720</td>
<td>2260</td>
<td>2840</td>
<td>3560</td>
<td>4460</td>
</tr>
<tr>
<td>Power output ( P_o ) (W)</td>
<td>220</td>
<td>320</td>
<td>630</td>
<td>1310</td>
<td>1680</td>
<td>2280</td>
<td>2960</td>
<td>3640</td>
</tr>
<tr>
<td>Total losses ( W_L ) (W)</td>
<td>340</td>
<td>400</td>
<td>370</td>
<td>410</td>
<td>580</td>
<td>500</td>
<td>600</td>
<td>820</td>
</tr>
<tr>
<td>Efficiency ( \eta ) (%)</td>
<td>39</td>
<td>44</td>
<td>43</td>
<td>76.1</td>
<td>74.3</td>
<td>80.3</td>
<td>83.1</td>
<td>81.6</td>
</tr>
<tr>
<td>Fundamental current (phase) ( I_1 ) (A)</td>
<td>3.67</td>
<td>3.67</td>
<td>3.74</td>
<td>3.98</td>
<td>4.34</td>
<td>4.86</td>
<td>5.61</td>
<td>6.66</td>
</tr>
<tr>
<td>5\textsuperscript{th} Harmonic current (phase) ( I_5 ) (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7\textsuperscript{th} Harmonic current (phase) ( I_7 ) (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Frequency = 32.5 Hz., D.C. Link Voltage = —— Volts, The pulse-width = —— ms., Average temperature = 21 °C

Fundamental voltage = 260 Volts, 5\textsuperscript{th} Harmonic Voltage = —— Volts, 7\textsuperscript{th} Harmonic Voltage = —— Volts

222.
TABLE: Calculated Load Test when sinusoidal voltage applied

<table>
<thead>
<tr>
<th>Slip (%) s</th>
<th>0.205</th>
<th>0.4615</th>
<th>0.748</th>
<th>1.507</th>
<th>2.041</th>
<th>2.676</th>
<th>3.712</th>
<th>4.82</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>517</td>
<td>749</td>
<td>1008</td>
<td>1688</td>
<td>2160</td>
<td>2718</td>
<td>3611</td>
<td>4543</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>185</td>
<td>413</td>
<td>665</td>
<td>1309</td>
<td>1743</td>
<td>2240</td>
<td>3005</td>
<td>3761</td>
</tr>
<tr>
<td>Total losses $W_l$ (W)</td>
<td>332</td>
<td>336</td>
<td>343</td>
<td>379</td>
<td>417</td>
<td>577</td>
<td>605</td>
<td>781</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>357</td>
<td>55</td>
<td>65.9</td>
<td>77.5</td>
<td>80.6</td>
<td>82.4</td>
<td>83.2</td>
<td>82.8</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_i$ (A)</td>
<td>3.66</td>
<td>3.71</td>
<td>3.8</td>
<td>4.17</td>
<td>4.52</td>
<td>5.0</td>
<td>5.9</td>
<td>6.94</td>
</tr>
<tr>
<td>$5^{th}$ Harmonic current (phase) $I_5$ (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$7^{th}$ Harmonic current (phase) $I_7$ (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Frequency = 32.5 Hz, D.C. Link Voltage = —— Volts, The pulse-width = —— ms., Average temperature = 21°C
Fundamental Voltage = 260 Volts, 5th Harmonic Voltage = —— Volts, 7th Harmonic Voltage = ——
TABLE: 9a: LOAD TEST 2 PWM voltage waveform

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>0.482</th>
<th>0.554</th>
<th>0.831</th>
<th>1.12</th>
<th>1.734</th>
<th>2.329</th>
<th>3.181</th>
<th>4.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>2500</td>
<td>2560</td>
<td>2680</td>
<td>2960</td>
<td>3260</td>
<td>3680</td>
<td>4240</td>
<td>4920</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>220</td>
<td>280</td>
<td>420</td>
<td>630</td>
<td>970</td>
<td>1320</td>
<td>1780</td>
<td>2280</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>2280</td>
<td>2280</td>
<td>2260</td>
<td>2330</td>
<td>2300</td>
<td>2360</td>
<td>2460</td>
<td>2640</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>8.8</td>
<td>11</td>
<td>15.6</td>
<td>21.2</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>46</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_1$ (A)</td>
<td>4.03</td>
<td>4.05</td>
<td>4.08</td>
<td>4.1</td>
<td>4.15</td>
<td>4.23</td>
<td>4.49</td>
<td>5.0</td>
</tr>
<tr>
<td>5(^{th}) Harmonic cur. (phase) $I_5$ (A)</td>
<td>8.6</td>
<td>8.6</td>
<td>8.6</td>
<td>8.6</td>
<td>8.5</td>
<td>8.5</td>
<td>8.46</td>
<td>8.46</td>
</tr>
<tr>
<td>7(^{th}) Harmonic cur. (phase) $I_7$ (A)</td>
<td>4.18</td>
<td>4.18</td>
<td>4.18</td>
<td>4.18</td>
<td>4.18</td>
<td>4.18</td>
<td>4.18</td>
<td>4.18</td>
</tr>
</tbody>
</table>

Frequency = 32.5 Hz., D.C. Link Voltage = 570 Volts, The pulse-width = 2.63 ms., Average temperature = 55°C
Fundamental Voltage = 261 Volts, 5th Harmonic Voltage = 190 Volts, 7th Harmonic Voltage = 120 Volts
TABLE: 9b: Calculated Load test for 2 PWM voltage waveform

<table>
<thead>
<tr>
<th>Slip (%) s</th>
<th>0.482</th>
<th>0.554</th>
<th>0.831</th>
<th>1.12</th>
<th>1.734</th>
<th>2.329</th>
<th>3.181</th>
<th>4.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>1981</td>
<td>2028</td>
<td>2209</td>
<td>2402</td>
<td>2794</td>
<td>3175</td>
<td>3715</td>
<td>4411</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>245</td>
<td>291</td>
<td>467</td>
<td>652</td>
<td>1020</td>
<td>1369</td>
<td>1846</td>
<td>2434</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>1736</td>
<td>1737</td>
<td>1748</td>
<td>1749</td>
<td>1783</td>
<td>1806</td>
<td>1868</td>
<td>1976</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>12.4</td>
<td>14.4</td>
<td>21.1</td>
<td>27.1</td>
<td>36.5</td>
<td>43</td>
<td>49.7</td>
<td>55.2</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_f$ (A)</td>
<td>3.4</td>
<td>3.4</td>
<td>3.45</td>
<td>3.55</td>
<td>3.81</td>
<td>4.13</td>
<td>4.66</td>
<td>5.45</td>
</tr>
<tr>
<td>5th Harmonic cur. (phase) $I_5$ (A)</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
<td>7.3</td>
</tr>
<tr>
<td>7th Harmonic cur. (phase) $I_7$ (A)</td>
<td>3.7</td>
<td>3.7</td>
<td>3.7</td>
<td>3.7</td>
<td>3.7</td>
<td>3.7</td>
<td>3.7</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Frequency = 32.5 Hz., D.C. Link Voltage = 570 Volts, The pulse-width = 2.63 ms., Average temperature = 55 °C

Fundamental Voltage = 235 Volts, 5th Harmonic Voltage = 173 Volts, 7th Harmonic Voltage = 121 Volts
### TABLE: 10a: LOAD TEST by 4 PWM voltage waveform

<table>
<thead>
<tr>
<th>Slip (%) (s)</th>
<th>0.654</th>
<th>0.714</th>
<th>0.88</th>
<th>1.14</th>
<th>1.5</th>
<th>1.94</th>
<th>2.45</th>
<th>3.28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input (P_i) (W)</td>
<td>1480</td>
<td>1540</td>
<td>1700</td>
<td>1940</td>
<td>2260</td>
<td>2620</td>
<td>3160</td>
<td>3800</td>
</tr>
<tr>
<td>Power output (P_o) (W)</td>
<td>500</td>
<td>570</td>
<td>750</td>
<td>930</td>
<td>1270</td>
<td>1600</td>
<td>2000</td>
<td>2350</td>
</tr>
<tr>
<td>Total losses (W_L) (W)</td>
<td>980</td>
<td>970</td>
<td>950</td>
<td>1010</td>
<td>1000</td>
<td>1020</td>
<td>1260</td>
<td>1450</td>
</tr>
<tr>
<td>Efficiency (\eta) (%)</td>
<td>33.8</td>
<td>37.0</td>
<td>44.1</td>
<td>47.9</td>
<td>56.2</td>
<td>61.0</td>
<td>60.0</td>
<td>62.0</td>
</tr>
<tr>
<td>Fundamental current (phase) (I_1) (A)</td>
<td>4.6</td>
<td>4.6</td>
<td>4.6</td>
<td>4.6</td>
<td>4.6</td>
<td>4.65</td>
<td>4.91</td>
<td>5.23</td>
</tr>
<tr>
<td>5(^{th}) Harmonic current (phase) (I_5) (A)</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.23</td>
<td>3.20</td>
<td>3.20</td>
</tr>
<tr>
<td>7(^{th}) Harmonic current (phase) (I_7) (A)</td>
<td>1.93</td>
<td>1.96</td>
<td>1.89</td>
<td>1.89</td>
<td>1.89</td>
<td>1.88</td>
<td>1.87</td>
<td>1.88</td>
</tr>
</tbody>
</table>

Frequency = 32.6 Hz., D.C. Link Voltage = 525 Volts, The pulse-width = 1.625 ms., Average temperature = 32°C.

Fundamental Voltage = 261 Volts, 5th Harmonic Voltage = 72 Volts, 7th Harmonic Voltage = 63 volts.
### TABLE: Calculated Load test for 4 PWM voltage waveform

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>0.654</th>
<th>0.715</th>
<th>0.88</th>
<th>1.14</th>
<th>1.5</th>
<th>1.94</th>
<th>2.45</th>
<th>3.28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>1512</td>
<td>1566</td>
<td>1708</td>
<td>1931</td>
<td>2250</td>
<td>2628</td>
<td>3192</td>
<td>3763</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>552</td>
<td>604</td>
<td>742</td>
<td>954</td>
<td>1254</td>
<td>1603</td>
<td>2110</td>
<td>2607</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>960</td>
<td>961</td>
<td>966</td>
<td>976</td>
<td>995</td>
<td>1024</td>
<td>1081</td>
<td>1155</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>36.5</td>
<td>38.6</td>
<td>43.4</td>
<td>49.4</td>
<td>55.7</td>
<td>61</td>
<td>66.1</td>
<td>69.3</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_1$ (A)</td>
<td>3.78</td>
<td>3.8</td>
<td>3.86</td>
<td>3.98</td>
<td>4.17</td>
<td>4.44</td>
<td>4.8</td>
<td>5.46</td>
</tr>
<tr>
<td>5th Harmonic cur. (phase) $I_5$ (A)</td>
<td>2.65</td>
<td>2.65</td>
<td>2.65</td>
<td>2.65</td>
<td>2.65</td>
<td>2.65</td>
<td>2.65</td>
<td>2.65</td>
</tr>
<tr>
<td>7th Harmonic cur. (phase) $I_7$ (A)</td>
<td>1.69</td>
<td>1.69</td>
<td>1.69</td>
<td>1.69</td>
<td>1.69</td>
<td>1.69</td>
<td>1.69</td>
<td>1.69</td>
</tr>
</tbody>
</table>

Frequency = 32.6 Hz., D.C. Link Voltage = 525 Volts, The pulse-width = 1.625 ms., Average temperature = 32°C

Fundamental voltage = 262 Volts, 5th Harmonic Voltage = 62.7 Volts, 7th Harmonic Voltage = 55.6 Volts
TABLE: 11a: LOAD TEST 8 PWM voltage wave

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>0.266</th>
<th>0.286</th>
<th>0.43</th>
<th>0.706</th>
<th>1.0</th>
<th>1.596</th>
<th>2.31</th>
<th>3.21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>900</td>
<td>920</td>
<td>1110</td>
<td>1320</td>
<td>1580</td>
<td>2080</td>
<td>2620</td>
<td>3260</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>220</td>
<td>280</td>
<td>420</td>
<td>630</td>
<td>970</td>
<td>1320</td>
<td>1780</td>
<td>2280</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>680</td>
<td>640</td>
<td>690</td>
<td>690</td>
<td>610</td>
<td>760</td>
<td>840</td>
<td>980</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>24.5</td>
<td>30.4</td>
<td>38</td>
<td>48</td>
<td>61.4</td>
<td>63.4</td>
<td>68</td>
<td>70</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_{f1}$ (A)</td>
<td>2.66</td>
<td>3.66</td>
<td>3.66</td>
<td>3.7</td>
<td>3.72</td>
<td>3.94</td>
<td>4.35</td>
<td>5.00</td>
</tr>
<tr>
<td>5th Harmonic cur. (phase) $I_5$ (A)</td>
<td>2.3</td>
<td>2.3</td>
<td>2.3</td>
<td>2.3</td>
<td>2.3</td>
<td>2.3</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>7th Harmonic cur. (phase) $I_7$ (A)</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Frequency = 32.58 Hz., D.C. Link Voltage = 565 Volts, The pulse-width = 0.74 ms., Average temperature 30 °C

Fundamental voltage = 261 Volts, 5th Harmonic Voltage = 54.6 Volts, 7th Harmonic Voltage = 12.6 Volts
TABLE: 11b: Calculated Load Test 8 PWM voltage waveform

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>0.266</th>
<th>0.286</th>
<th>0.43</th>
<th>0.706</th>
<th>1.0</th>
<th>1.596</th>
<th>2.31</th>
<th>3.21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>940</td>
<td>958</td>
<td>1077</td>
<td>1306</td>
<td>1637</td>
<td>2040</td>
<td>2624</td>
<td>3347</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>214</td>
<td>231</td>
<td>348</td>
<td>572</td>
<td>889</td>
<td>1268</td>
<td>1800</td>
<td>2435</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>726</td>
<td>726</td>
<td>728</td>
<td>734</td>
<td>748</td>
<td>772</td>
<td>823</td>
<td>912</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>22.8</td>
<td>24.1</td>
<td>32.3</td>
<td>43.8</td>
<td>54.3</td>
<td>62.1</td>
<td>68.6</td>
<td>72.7</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_1$ (A)</td>
<td>3.59</td>
<td>3.6</td>
<td>3.63</td>
<td>3.71</td>
<td>3.86</td>
<td>4.12</td>
<td>4.58</td>
<td>5.27</td>
</tr>
<tr>
<td>5th Harmonic curr. (phase) $I_5$ (A)</td>
<td>2.26</td>
<td>2.26</td>
<td>2.26</td>
<td>2.26</td>
<td>2.26</td>
<td>2.26</td>
<td>2.26</td>
<td>2.26</td>
</tr>
<tr>
<td>7th Harmonic curr. (phase) $I_7$ (A)</td>
<td>1.21</td>
<td>1.21</td>
<td>1.21</td>
<td>1.21</td>
<td>1.21</td>
<td>1.21</td>
<td>1.21</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Frequency = 32.58 Hz., D.C. Link Voltage = 565 Volts, The pulse-width = 0.74 ms., Average temperature = 30 °C
Fundamental voltage = 255 Volts, 5th Harmonic Voltage = 53.5 Volts, 7th Harmonic Voltage = 40 Volts
## TABLE: 12a: Load Test by Sinusoidal Waveform

<table>
<thead>
<tr>
<th>Slip (%) ( s )</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
<th>1.7</th>
<th>2.4</th>
<th>3.4</th>
<th>4.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input ( P_1 ) (W)</td>
<td>432</td>
<td>480</td>
<td>600</td>
<td>840</td>
<td>1160</td>
<td>1560</td>
<td>2080</td>
<td>2780</td>
</tr>
<tr>
<td>Power output ( P_2 ) (W)</td>
<td>160</td>
<td>220</td>
<td>360</td>
<td>570</td>
<td>910</td>
<td>1250</td>
<td>1720</td>
<td>2220</td>
</tr>
<tr>
<td>Total losses ( W_L ) (W)</td>
<td>270</td>
<td>260</td>
<td>240</td>
<td>270</td>
<td>250</td>
<td>310</td>
<td>360</td>
<td>560</td>
</tr>
<tr>
<td>Efficiency ( \eta ) (%)</td>
<td>37</td>
<td>46</td>
<td>60</td>
<td>67.8</td>
<td>78.4</td>
<td>80.1</td>
<td>82.7</td>
<td>79.8</td>
</tr>
<tr>
<td>Fundamental current (phase) ( I_1 ) (A)</td>
<td>3.6</td>
<td>3.6</td>
<td>3.63</td>
<td>3.63</td>
<td>3.81</td>
<td>4.13</td>
<td>4.67</td>
<td>5.54</td>
</tr>
<tr>
<td>5th Harmonic curr. (phase) ( I_5 ) (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7th Harmonic curr. (phase) ( I_7 ) (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Frequency = 25 Hz., D.C. Link Voltage = —— Volts, The pulse-width = —— ms., Average temperature = 22 °C

Fundamental voltage = 200 Volts, 5th Harmonic Voltage = —— Volts, 7th Harmonic Voltage = —— Volts
### TABLE: Calculated Load test when Sinusoidal Voltage applied

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
<th>1.7</th>
<th>2.4</th>
<th>3.4</th>
<th>4.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>457</td>
<td>522</td>
<td>649</td>
<td>741</td>
<td>1154</td>
<td>1535</td>
<td>2060</td>
<td>2641</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>183</td>
<td>246</td>
<td>370</td>
<td>458</td>
<td>848</td>
<td>1195</td>
<td>1653</td>
<td>2134</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>276</td>
<td>278</td>
<td>279</td>
<td>283</td>
<td>306</td>
<td>340</td>
<td>407</td>
<td>507</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>40</td>
<td>47</td>
<td>57</td>
<td>61.8</td>
<td>73.5</td>
<td>77.8</td>
<td>80.2</td>
<td>80.7</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_1$ (A)</td>
<td>3.66</td>
<td>3.68</td>
<td>3.72</td>
<td>3.76</td>
<td>4.02</td>
<td>4.35</td>
<td>4.93</td>
<td>5.67</td>
</tr>
<tr>
<td>5th Harmonic current (phase) $I_5$ (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7th Harmonic current (phase) $I_7$ (A)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Frequency = 25 Hz., D.C. Link Voltage = --- Volts, The pulse-width = --- ms., Average temperature = 22 °C

Fundamental voltage = 200 Volts, 5th Harmonic Voltage = --- Volts, 7th Harmonic Voltage = --- Volts
### TABLE: 13a: Load Test by 4 PWM voltage waveform

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>0.26</th>
<th>0.48</th>
<th>0.65</th>
<th>1.2</th>
<th>1.86</th>
<th>2.93</th>
<th>4.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>1380</td>
<td>1420</td>
<td>1560</td>
<td>1772</td>
<td>2064</td>
<td>2600</td>
<td>3140</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>120</td>
<td>180</td>
<td>310</td>
<td>530</td>
<td>870</td>
<td>1220</td>
<td>1680</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>1260</td>
<td>1240</td>
<td>1250</td>
<td>1240</td>
<td>1194</td>
<td>1380</td>
<td>1460</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>8.6</td>
<td>12.6</td>
<td>20</td>
<td>30</td>
<td>43</td>
<td>47</td>
<td>53.5</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_1$ (A)</td>
<td>3.66</td>
<td>3.54</td>
<td>3.57</td>
<td>3.64</td>
<td>3.8</td>
<td>4.31</td>
<td>5.08</td>
</tr>
<tr>
<td>5th Harmonic cur. (phase) $I_5$ (A)</td>
<td>2.92</td>
<td>2.9</td>
<td>2.91</td>
<td>2.92</td>
<td>2.92</td>
<td>2.9</td>
<td>2.9</td>
</tr>
<tr>
<td>7th Harmonic cur. (phase) $I_7$ (A)</td>
<td>1.77</td>
<td>1.77</td>
<td>1.77</td>
<td>1.77</td>
<td>1.77</td>
<td>1.77</td>
<td>1.77</td>
</tr>
</tbody>
</table>

Frequency = 25 Hz, D.C. Link Voltage = 550 Volts, The pulse-width = 1.54 ms, Average temperature = 42 °C

Fundamental voltage = 192 Volts, 5th Harmonic Voltage = 46.5 Volts, 7th Harmonic Voltage = 45 Volts
TABLE: 13b: Calculated Load test for 4 FWM Voltage Waveform

<table>
<thead>
<tr>
<th>Slip (%) ( % )</th>
<th>0.26</th>
<th>0.48</th>
<th>0.65</th>
<th>1.2</th>
<th>1.86</th>
<th>2.93</th>
<th>4.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input ( P_i ) (W)</td>
<td>1283</td>
<td>1391</td>
<td>1474</td>
<td>1742</td>
<td>2060</td>
<td>2570</td>
<td>3166</td>
</tr>
<tr>
<td>Power output ( P_o ) (W)</td>
<td>114</td>
<td>220</td>
<td>301</td>
<td>558</td>
<td>856</td>
<td>1314</td>
<td>1822</td>
</tr>
<tr>
<td>Total losses ( W_L ) (W)</td>
<td>1169</td>
<td>1171</td>
<td>1173</td>
<td>1184</td>
<td>1204</td>
<td>1256</td>
<td>1344</td>
</tr>
<tr>
<td>Efficiency ( \eta ) (%)</td>
<td>8.9</td>
<td>15.8</td>
<td>20.4</td>
<td>32</td>
<td>41.5</td>
<td>51.1</td>
<td>57.5</td>
</tr>
<tr>
<td>Fundamental current (phase) ( I_1 ) (A)</td>
<td>3.64</td>
<td>3.65</td>
<td>3.68</td>
<td>3.85</td>
<td>4.08</td>
<td>4.58</td>
<td>5.37</td>
</tr>
<tr>
<td>5th Harmonic cur. (phase) ( I_5 ) (A)</td>
<td>2.69</td>
<td>2.69</td>
<td>2.69</td>
<td>2.69</td>
<td>2.69</td>
<td>2.69</td>
<td>2.69</td>
</tr>
<tr>
<td>7th Harmonic cur. (phase) ( I_7 ) (A)</td>
<td>1.82</td>
<td>1.82</td>
<td>1.82</td>
<td>1.82</td>
<td>1.82</td>
<td>1.82</td>
<td>1.82</td>
</tr>
</tbody>
</table>

Frequency = 25 Hz., D.C. Link Voltage = 550 Volts, The pulse-width = 1.54 ms., Average temperature = 42 °C
Fundamental Voltage = 200 Volts, 5th Harmonic Voltage = 50.5 Volts, 7th Harmonic Voltage = 47.4 Volts
TABLE: 14a: LOAD TEST 8 FWM

<table>
<thead>
<tr>
<th>Slip (%)</th>
<th>0.4</th>
<th>0.42</th>
<th>0.66</th>
<th>1.2</th>
<th>2.05</th>
<th>2.93</th>
<th>4.41</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_1$ (W)</td>
<td>880</td>
<td>936</td>
<td>1080</td>
<td>1280</td>
<td>1640</td>
<td>2040</td>
<td>2660</td>
</tr>
<tr>
<td>Power output $P_2$ (W)</td>
<td>90</td>
<td>150</td>
<td>290</td>
<td>500</td>
<td>840</td>
<td>1190</td>
<td>1650</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>790</td>
<td>790</td>
<td>790</td>
<td>780</td>
<td>800</td>
<td>850</td>
<td>1010</td>
</tr>
<tr>
<td>Efficiency $\eta$ (%)</td>
<td>10</td>
<td>16</td>
<td>26.8</td>
<td>39</td>
<td>51.2</td>
<td>58.3</td>
<td>62</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_1$ (A)</td>
<td>3.46</td>
<td>3.57</td>
<td>3.63</td>
<td>3.66</td>
<td>3.84</td>
<td>4.0</td>
<td>4.88</td>
</tr>
<tr>
<td>5th Harmonic cur. (phase) $I_5$ (A)</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
</tr>
<tr>
<td>7th Harmonic cur. (phase) $I_7$ (A)</td>
<td>1.23</td>
<td>1.23</td>
<td>1.23</td>
<td>1.23</td>
<td>1.17</td>
<td>1.17</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Frequency = 25 Hz., D.C. Link Voltage = 540 Volts, The pulse-width = 0.775 ms., Average temperature = 75 °C
Fundamental voltage = 195 Volts, 5th Harmonic Voltage = 40.5 Volts, 7th Harmonic Voltage = 29.7 Volts
## TABLE: 14b: Calculated Load test for 8 FWM voltage waveform

<table>
<thead>
<tr>
<th>Slip ($%$)</th>
<th>0.4</th>
<th>0.42</th>
<th>0.66</th>
<th>1.2</th>
<th>2.05</th>
<th>2.93</th>
<th>4.41</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power input $P_i$ (W)</td>
<td>884</td>
<td>945</td>
<td>1045</td>
<td>1270</td>
<td>1601</td>
<td>1981</td>
<td>2575</td>
</tr>
<tr>
<td>Power output $P_o$ (W)</td>
<td>108</td>
<td>168</td>
<td>266</td>
<td>483</td>
<td>793</td>
<td>1136</td>
<td>1645</td>
</tr>
<tr>
<td>Total losses $W_L$ (W)</td>
<td>776</td>
<td>777</td>
<td>779</td>
<td>787</td>
<td>808</td>
<td>845</td>
<td>930</td>
</tr>
<tr>
<td>Efficiency $\eta$ ($%$)</td>
<td>12.2</td>
<td>17.8</td>
<td>25.5</td>
<td>38</td>
<td>49.5</td>
<td>57.3</td>
<td>63.8</td>
</tr>
<tr>
<td>Fundamental current (phase) $I_1$ (A)</td>
<td>3.58</td>
<td>3.59</td>
<td>3.62</td>
<td>3.73</td>
<td>3.97</td>
<td>4.33</td>
<td>4.94</td>
</tr>
<tr>
<td>5th Harmonic cur. (phase) $I_5$ (A)</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
</tr>
<tr>
<td>7th Harmonic cur. (phase) $I_7$ (A)</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Frequency = 25 Hz., D.C. Link Voltage = 540 Volts, The pulse-width = 0.775 ms., Average temperature = 75°C

Fundamental voltage = 200 Volts, 5th Harmonic Voltage = 41.5 Volts, 7th Harmonic Voltage = 31.3 Volts
### Table T51

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>P_i (W)</th>
<th>I_p (A)</th>
<th>V_{AB} (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>400</td>
<td>3.63</td>
<td>320</td>
</tr>
<tr>
<td>32.5</td>
<td>320</td>
<td>3.73</td>
<td>260</td>
</tr>
<tr>
<td>25</td>
<td>240</td>
<td>3.55</td>
<td>220</td>
</tr>
</tbody>
</table>

### Table T52

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>P_i (W)</th>
<th>I_s (A)</th>
<th>V_{AB} (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>800</td>
<td>7.67</td>
<td>59</td>
</tr>
<tr>
<td>32.5</td>
<td>390</td>
<td>5.54</td>
<td>37.2</td>
</tr>
<tr>
<td>25</td>
<td>453</td>
<td>5.97</td>
<td>34.9</td>
</tr>
</tbody>
</table>

### Table T53

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.6</td>
<td>4</td>
</tr>
<tr>
<td>10.6</td>
<td>5</td>
</tr>
<tr>
<td>12.8</td>
<td>6</td>
</tr>
</tbody>
</table>

### Table T54

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>R_f [ohm]</th>
<th>X_s [ohm]</th>
<th>X_r [ohm]</th>
<th>X_M [ohm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>2.55</td>
<td>3.36</td>
<td>2.83</td>
<td>84</td>
</tr>
<tr>
<td>32.5</td>
<td>2.28</td>
<td>2.82</td>
<td>2.37</td>
<td>66.55</td>
</tr>
<tr>
<td>25</td>
<td>2.26</td>
<td>2.18</td>
<td>1.83</td>
<td>59.7</td>
</tr>
</tbody>
</table>
FIGURE 1: Single Phase of a 3-Phase Inverter

FIGURE 2: The Phase and the Load Voltages Generated by 180° Conduction of Thyristors in a Single-Phase Inverter

FIGURE 3: The Phase and the Load Voltages Generated by 120° Conduction of Thyristors in a Single-Phase Inverter
FIGURE 4: The principle for the Generation of PWM Waveform with Sine Wave Envelope Modulation

Input

Carrier wave

Reference wave

Output

PWM wave

FIGURE 5: 3-Phase Bridge-Connected Inverter

FIGURE 6: The Conduction Pattern of the Thyristors of a Single-Phase Inverter for a 2 PWM Waveform
FIGURE 7:
(a) Line Voltages of a 2PWM Waveform Generated by a 3-Phase Inverter
(b) Conduction Pattern of the Thyristors to Produce 2 PWM Waveform
FIGURE 8: The Determination of Line Voltages with a Circle

[Diagram of a circle with labeled points A, B, and C, indicating direction of rotation]

FIGURE 9: The Application of Angles to a Pulse

[Diagram showing angles and pulse with labeled components]
FIGURE 10: The Representation of a 2 PWM Waveform; (a) $P_w = 1\text{ ms}, f = 40\text{ Hz}$, (b) $P_w = 1\text{ ms}, f = 80\text{ Hz}$, (c) $P_w = 0.5\text{ ms}, f = 80\text{ Hz}$
FIGURE 11: The Representation of $T_{sp}$ and $P_w$ on Time Axis

FIGURE 12: A 2 PWM Voltage Waveform Whose Pulses are in Equal Duration is Represented on Time Axis
FIGURE 13: The Quasi-Square Wave Shown on Angle Axis

FIGURE 14: The Representation of a PWM Waveform Which is Symmetrical about 90° and 180°
FIGURE 15: 5 PWM Waveform on Angle Axis

FIGURE 16: 7 PWM Waveform on Angle Axis
FIGURE 17: The Angle Variations Associated with the Pulses of a 6 PWM waveform.
FIGURE 18: The Angle Variations Developed for the Pulses of a 5 PWM Waveform
FIGURE 19: A 4 PWM Voltage Waveform with Equal Pulse Durations on Time Axis
FIGURE 20: The Equivalent Circuit of an Induction Motor for Blocked-Rotor State

FIGURE 21: The Illustrations for a Harmonic Voltage Variation of a PWM Voltage Waveform, $V_{An_0}$, and a Straight Line, $V_B$, whose Slope is Determined by $V/f$, on the Frequency Axis.
FIGURE 22: The Comparison of the Fundamental Voltages of 2, 4 and 8 PWM Waveform with Equal Duration of Pulses in their Periods. Pulse Durations of 2, 4 and 8 PWM Waveforms are, Respectively, 2.4, 1.2, 0.6 ms.
FIGURE 23: The Comparison of the 5th Harmonics of 2, 4, 8 PWM Waveforms whose Pulse Durations are Respectively 2.4, 1.2, 0.6 ms
FIGURE 24: The Comparison of the 7th Harmonics of 2, 4, 8 PWM Waveforms Whose Pulse Durations are Respectively 2.4, 1.2, 0.6 ms

HA (%) vs Frequency (Hz)
FIGURE 25: 2 PWM Voltage Waveform with Equal Duration of Pulses

FIGURE 26: 4 PWM Voltage Waveform with Variation of Angles
FIGURE 27: Half a Cycle of 6 PWM Waveform with Angle Variations

FIGURE 28: Half a Cycle of 8 PWM Waveform with Angle Variations
FIGURE 29: The Variations of Fundamental Voltage Magnitudes of 4 PWM Waveforms against Angle where $1 - \alpha = 6^\circ$, $2 - \alpha = 4.5^\circ$, $3 - \alpha = 1.5^\circ$, $4 - \alpha = 0^\circ$ and $P_{wa} = 25.2^\circ$ at 50 Hz.
FIGURE 30: The Variations of 7th Harmonic Voltages of the 4 PWM Waveforms, whose Fundamental Voltage Variations are shown in Figure 29, against Angle

FIGURE 31: The Variations of 5th Harmonic Voltages of the 4 PWM Waveforms, whose Fundamental Voltage Variations are shown in Figure 29, against Angle
FIGURE 32: The Variations of Fundamental Voltage Magnitudes of 6 PMN Waveforms against Angle, where
1) $\alpha_y = 4.5^\circ$, $\alpha_x = 4.5^\circ$, 2) $\alpha_y = 2^\circ$, $\alpha_x = 4.5^\circ$
3) $\alpha_y = 2^\circ$, $\alpha_x = 2^\circ$, 4) $\alpha_y = 4.5^\circ$, $\alpha_x = -2^\circ$
5) $\alpha_y = -2^\circ$, $\alpha_x = 2^\circ$, 6) $\alpha_y = 2^\circ$, $\alpha_x = -4.5^\circ$
7) $\alpha_y = 2^\circ$, $\alpha_x = -2^\circ$, 8) $\alpha_y = -2^\circ$, $\alpha_x = -4.5^\circ$ and $\beta_{wa} = 10^\circ$

Fundam. volt. magn.
(volts)

$\beta_x$ (Degrees)
FIGURE 33: The Variations of Fundamental Voltage Magnitudes of 6 Pam Waveforms against Angle, where 1) $\alpha = 5^\circ, \alpha = 5^\circ$, 2) $\alpha = 1^\circ, \alpha = 5^\circ$
3) $\alpha = 5^\circ, \alpha = 1^\circ$ 4) $\alpha = 3^\circ, \alpha = 5^\circ$ 5) $\alpha = 1^\circ, \alpha = 1^\circ$ 6) $\alpha = 5^\circ, \alpha = -4^\circ$
7) $\alpha = 3^\circ, \alpha = 2^\circ$ 8) $\alpha = 1^\circ, \alpha = -4^\circ$ 9) $\alpha = -3^\circ, \alpha = 3^\circ$ and $P_{wa} = 90^\circ$

Fundam. volt. magn. (volts)

$\beta_x$ (Degrees)
Figure 34 The Variations of Fundamental Voltage Magnitudes of 6 PWM Waveforms Against Angle, Where

1) \( \alpha_y = 3^\circ, \alpha_x = 2^\circ \), 2) \( \alpha_y = 1^\circ, \alpha_x = 3^\circ \)
3) \( \alpha_y = 1^\circ, \alpha_x = 2^\circ \), 4) \( \alpha_y = -3^\circ, \alpha_x = 3^\circ \), 5) \( \alpha_y = 3^\circ, \alpha_x = -3^\circ \)
6) \( \alpha_y = -3^\circ, \alpha_x = 2^\circ \), 7) \( \alpha_y = 1^\circ, \alpha_x = -3^\circ \), 8) \( \alpha_y = 3^\circ, \alpha_x = -6^\circ \)
9) \( \alpha_y = -3^\circ, \alpha_x = -3^\circ \) and \( \beta_{wa} = 13^\circ \).
Figure 35 The Variations of Fundamental Voltage Magnitudes of 6 PWM Waveforms Against Angle, Where 1) $\alpha = 4^\circ, \beta = 4^\circ$, 2) $\alpha = 1^\circ, \beta = 0^\circ$, 3) $\alpha = 4^\circ, \beta = 1^\circ$, 4) $\alpha = 1^\circ, \beta = 1^\circ$, 5) $\alpha = 2^\circ, \beta = 4^\circ$, 6) $\alpha = 4^\circ, \beta = 2^\circ$, 7) $\alpha = 3^\circ, \beta = 3^\circ$, 8) $\alpha = 1^\circ, \beta = 3^\circ$, 9) $\alpha = 2^\circ, \beta = 3^\circ$ and $P_{wa} = 11.2$
FIGURE 36: The Variations of 5th Harmonic Voltages of the 6 PWM Voltage Waveforms, whose Fundamental Voltage Magnitudes are Shown in Figure 32, against Angle

FIGURE 37: The Variations of 5th Harmonic Voltages of the 6 PWM Voltage Waveforms, whose Fundamental Voltage Magnitudes are Shown in Figure 33, against Angle
Figure 38 The Variations of 5th Harmonic Voltages of the
6 PWM Voltage Waveforms, Whose Fundamental Voltage
Magnitudes are Shown in Figure 34, Against Angle.

Figure 39 The Variations of 5th Harmonic Voltages of the
6 PWM Voltage Waveforms, Whose Fundamental Voltage
Magnitudes are Shown in Figure 35, Against Angle.
Figure 40: The Variations of 7th Harmonic Voltages of the 6 PWM Voltage Waveforms, Whose Fundamental Voltage Magnitudes are Shown in Figure 32, Against Angle.

Figure 41: The Variations of 7th Harmonic Voltages of the 6 PWM Voltage Waveforms, Whose Fundamental Voltage Magnitudes are shown in Figure 33, Against Angle.
Figure 42: The Variations of 7th Harmonic Voltages of the 6 PWM Voltage Waveforms, Whose Fundamental Voltage Magnitudes are Shown in Figure 34, Against Angle.

Figure 43: The Variations of 7th Harmonic Voltages of the 6 PWM Voltage Waveforms, Whose Fundamental Voltage Magnitudes are Shown in Figure 35, Against Angle.
Figure 44  The Weighting Factors of the Waveforms Whose Fundamental Voltage Magnitudes are Shown in Figure 32.
Figure 45 The Weighting Factors of the Waveforms Whose Fundamental Voltage Magnitudes are Shown in Figure 33.
Figure 46 The Weighting Factors of the Waveforms Whose Fundamental Voltage Magnitudes are Shown in Figure 34.
Figure 47 The Weighting Factors of the Waveforms Whose Fundamental Voltage Magnitudes are Shown in Figure 35.
Figure 48  The Variations of the Fundamental Voltage Magnitudes of the 8 PwM Waveforms Against Angle. The Angles Associated with the Waveforms are Given on the Left of the Vertical Axis.

1) \( \alpha_x = 0^\circ \), 2) \( \alpha_x = 1.5^\circ \), 3) \( \alpha_x = 3^\circ \), 4) \( \alpha_x = 4.5^\circ \), 5) \( \alpha_x = 6^\circ \), 6) \( \alpha_x = 7.5^\circ \) and \( \alpha_y \) is maintained at \( 0^\circ \).
Figure 49. The Variations of the Fundamental Voltage Magnitudes of the SPWM Waveforms Against Angle. The Angles Associated with the Waveforms are Given on the Left of the Vertical Axis.

1) $\alpha = 0^\circ$, 2) $\alpha = 1.5^\circ$, 3) $\alpha = 3.0^\circ$, 4) $\alpha = 4.5^\circ$, 5) $\alpha = 6^\circ$

6) $\alpha = 7.5^\circ$ and $\alpha_y$ is maintained at $1.5^\circ$. 
Figure 50 The Variation of the Fundamental Voltage Magnitudes of the 8 PWM Waveforms Against Angle. The Angles Associated with the Waveforms are Given on the Left of Vertical Axis.
1) $\alpha_x = 0^\circ$, 2) $\alpha_x = 1.5^\circ$, 3) $\alpha_x = 3.0^\circ$, 4) $\alpha_x = 4.5^\circ$, 5) $\alpha_x = 6^\circ$, 6) $\alpha_x = 7.5^\circ$, and $\alpha_y$ is maintained at $3^\circ$. 

Fundam. volt. magn. (volts)

\[\beta_x\] (Degr.)

0 1.5 3.0 4.5 6.0 7.5
Figure 51 The Variations of 5th Harmonic Voltages of the 8 FwM Waveforms, Whose Fundamental Voltage Magnitudes are Shown in Figure 48.
Figure 52 The Variations of 5th Harmonic Voltages of the 8 PWM Waveforms, Whose Fundamental Voltage Magnitudes are Shown in Figure 49.
Figure 53. The Variations of 5th Harmonic Voltages of the 8 PWM Waveforms, Whose Fundamental Voltage Magnitudes are Shown in Figure 50.
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Figure 55 The Variations of 7th Harmonic Voltages of the 8 PWM Waveforms, Whose Fundamental Voltage Magnitudes are Shown in Figure 49.
Figure 56 The Variations of 7th Harmonic Voltages of the 8 PWM Waveforms, Whose Fundamental Voltage Magnitudes are Shown in Figure 50.
Figure 57: The Fundamental Voltage Magnitude Variations of 2 PWM Waveforms Against Frequency. The Numbers Denote Following Information:
1) $P_w = 3.2 \text{ ms}$, 2) $P_w = 3 \text{ ms}$, 3) $P_w = 2.5 \text{ ms}$, 4) $P_w = 1.5 \text{ ms}$, 5) $P_w = 0.9 \text{ ms}$.
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Figure 59 The Variations of 7th Harmonic Voltages of the Waveforms Whose Fundamentals are Shown in Figure 57.
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1) $P_w = 1.632$, 2) $P_w = 1.620$ ms, 3) $P_w = 1.5$ ms,
4) $P_w = 1.25$ ms, 5) $P_w = 0.9$ ms.
Figure 61 The Variations of 5th Harmonic Voltages of the Waveforms Whose Fundamentals are Shown in Figure 60.
Figure 62: The Variations of 7th Harmonic Voltages of the Waveforms Whose Fundamentals are Shown in Figure 60.
Figure 63 The 6 PWM Voltage Waveforms Whose Fundamentals Have the Same Variation Against Frequency
Figure 64  The 5th Harmonic Variations of the Waveforms Whose Variations of Fundamentals are Shown in Figure 63. The Numbers denote  
1) \( \alpha_x = -4.5^\circ, \alpha_y = 4^\circ \), \( \beta_x = 2.46^\circ \) and \( P_{wa} = 10^\circ \),  
2) \( \alpha_x = -4.5^\circ, \alpha_y = 4.5^\circ \), \( \beta_x = 2.019^\circ \) and \( P_{wa} = 10^\circ \),  
3) \( \alpha_x = -6^\circ, \alpha_y = 3^\circ \), \( \beta_x = 0.52^\circ \) and \( P_{wa} = 13^\circ \),  
4) \( \alpha_x = -3^\circ, \alpha_y = 5^\circ \), \( \beta_x = 1.57^\circ \) and \( P_{wa} = 9^\circ \).

Figure 65  The 7th Harmonic Variations of the Waveforms Whose Variations of 5th Harmonics are Shown in Figure 64.
Figure 66 The Fundamental Voltage Variation of 6 PWM Waveform Against Frequency by Maintaining Pulse Durations as 1) $P_w = 1.245$ ms, 2) $P_w = 1.094$ ms, 3) $P_w = 1.091$ ms, 4) $P_w = 0.75$ ms.
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$HP_5$ (%)
Figure 68 The 7th Harmonics of the Waveforms Shown in Figure 66
Figure 69 The Fundamental Voltage Variations of 8 PWM Voltage Waveforms Against Frequency. The Numbers Denote the Following Information: 1) $P_w = 0.8$ ms, 2) $P_w = 0.75$ ms, 3) $P_w = 0.6$ ms, 4) $P_w = 0.45$ ms, 5) $P_w = 0.3$ ms
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1) $\alpha_x = -1^\circ$, 2) $\alpha_x = 0^\circ$, 3) $\alpha_x = 1^\circ$.

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Figure 76 The Fundamental Voltage Variations of 4 PWM Waveforms in Small Frequencies. The Variable is $\alpha_y$ which is Assigned as 1) 2.2°, 2) 2.1°, 3) 2.0°, 4) 1.9°, 5) 1.8°, 6) 1.7° while $\alpha_x$ is Maintained at 0°. $P_w = 1.4$ ms
Figure 77 The Fundamental Voltage Variations of 4 PWM Waveforms in Small Frequencies. The Variable is \( \alpha \) which is assigned as 1) 2.2°, 2) 2.1°, 3) 2.0°, 4) 1.9°, 5) 1.8°, 6) 1.7°, while \( \alpha_x \) is maintained at \(-1.6\), \( P_w = 1.4 \) ms.
Figure 78 The 5th Harmonic Variations of the Waveforms Shown in Figure 75.
Figure 79 The 5th Harmonic Variations of the Waveforms Shown in Figure 76.
Figure 80  The 5th Harmonic Variations of the Waveforms Shown in Figure 77.
Figure 81 The 7th Harmonic Variations of the Waveforms Shown in Figure 75.
Figure 82 The 7th Harmonic Variations of the Waveforms Shown in Figure 76.
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Figure 85 The Weighting Factors of the Same 4 PWM Waveforms Against $\alpha_y$ at Three Different Frequencies. The Waveforms are Distinguished by Using the Following Keys: A) $f = 1$ Hz B) $x \alpha = 0^\circ$

- $f = 5$ Hz $x \alpha = 1^\circ$
- $f = 9$ Hz $x \alpha = 1^\circ$
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Figure 88  The 5th Harmonic Variation of the Waveform Shown in Figure 86 Against Angle

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Figure 91  The Fundamental Voltage Variations of the Waveform Shown in Figure 90. The Numbers From 1 to 4 Indicate the Value of $\alpha$: 1) $\alpha = 29.5^\circ$  2) $\alpha = 29^\circ$  3) $\alpha = 28.5^\circ$  4) $\alpha = 28^\circ$
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Figure 100 Quarter Cycle of a 7 PWM Waveform Whose Third Pulse Varies by $\alpha_x$. The Positive Variation of the Angle is from Left to Right.

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Fundam. volt. magn. (volts)

HP$_5$, HP$_7$ (%)
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Figure 107 The Equivalent Circuit of an Induction Motor for Sinusoidal Voltage Waveform when Space Harmonics are Neglected.

Figure 108 The MMF Waveform Produced by Single-Layer Full-Pitch Winding.
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![Cross Section of the End Ring](image1)

Figure 113 The Equivalent Circuit of an Induction Motor of a Time Harmonic, Excluding the Effects of Space Harmonics

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[Diagram showing delta connections and rotor phase position]
Figure 121 The Illustration of High Frequency Flux Linkage with Rotor, A) $Q_s = Q_r$, B) $Q_s = Q_r/2$

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Figure 124 The Measurement of Power with One Wattmeter on a 3-Phase System Without Disconnecting the Lines

Yellow Push button

Red Push button

Yellow Contactor

Red Contactor

Wattmeter's Current Coil

Wattmeter's Voltage Coil
Figure 125 Changes caused in the Currents of the Stator and the Rotor due to the Change of Applied Voltage in the Equivalent Circuit of an Induction Motor

\[ Z_s \]
\[ I_s + \Delta I_s \]
\[ V + \Delta V \]
\[ I_m + \Delta I_m \]
\[ Z_m \]
\[ R + \Delta R \]

Figure 126 The Variation of Losses of an Induction Motor with Voltage

Figure 127 The Equivalent Circuit Parameters of the Induction Motor Used

\[ X_{s1} = 336 \text{ ohm} \]
\[ X_{r1} = 2.83 \text{ ohm} \]
\[ X_{m1} = 84 \text{ ohm} \]
Figure 128 The Performance Characteristics of the Induction Motor with Applied Sinusoidal Voltage Waveform whose Magnitude is 320 Volts at 40 Hz.
Figure 129 The Performance Characteristics of the Induction Motor with Applied 2 PWM Waveform whose Pulse-Duration is 3.0 ms at 40 Hz
FIGURE 130  The performance characteristics of the induction motor with applied 4 PWM waveform whose pulse-duration is 1.562 ms at 40 Hz.
FIGURE 131  The performance characteristics of the induction motor with applied sinusoidal voltage waveform whose magnitude is 260 volts at 32.5 Hz.
The performance characteristics of the induction motor with applied 2 PWM waveform whose pulse-duration is 2.63 ms at 32.5 Hz.
FIGURE 133 The performance characteristics of the induction motor with applied PWM waveform whose pulse-duration is 1.625 ms at 32.6 Hz.
FIGURE 134 The performance characteristics of the induction motor with applied 8 PWM waveform whose pulse-duration is 0.740 ms at 32.58 Hz.
FIGURE 135 The performance characteristic of the induction motor with applied sinusoidal voltage waveform whose magnitude is 200 volts at 25 Hz.
FIGURE 136 The performance characteristics of the induction motor with applied PWM waveform whose pulse-duration is 1.540 ms at 25 Hz.
FIGURE 137 The performance characteristics of the induction motor with applied 8 PWM waveform whose pulse-duration is 0.775 ms at 25 Hz.
FIGURE 138A  The induction motor's no-load line-current for 2 PWM voltage-waveform application at 40 Hz.

FIGURE 139A  The induction motor's no-load line-current for 4 PWM voltage-waveform application at 40 Hz.
Figure 138(b) Calculated no-load line-current for 2 PWM voltage-waveform application at 40 Hz.

Figure 138(c) Applied 2 PWM voltage-waveform, $P_w = 3.0$ ms., $V_{dc} = 560$ Volts, $f = 40$ Hz.
FIGURE 139(b) Calculated no-load line-current for 4 PWM voltage-waveform application at 40 Hz.

FIGURE 139(c) Applied 4 PWM voltage-waveform, $P_w = 1.562$ ms, $V_{dc} = 525$ volts, $f = 40$ Hz.
FIGURE 140(b) Calculated no-load line-current for 2 PWM voltage-waveform application at 32.5 Hz.

FIGURE 140(c) Applied 2 PWM voltage-waveform, \( P_W = 2.63 \) ms, \( V_{dc} = 570 \) volts \( f = 32.5 \) Hz
FIGURE 141A  The induction motor's no-load line current for 4 PWM voltage-waveform application at 32.6 Hz. Horizontal scale 5.1 ms/cm, 50 mV/cm

FIGURE 142A  The induction motor's no-load line-current for 8 PWM voltage-waveform application at 32.58 Hz. Horizontal scale 5.1 ms/cm, vertical scale 50 mV/cm
FIGURE 141(b) The calculated no-load current for 4 PWM voltage-waveform application at 32.6 Hz.

FIGURE 141(c) Applied 4 PWM voltage-waveform, \( P_w = 1.625 \text{ ms} \), \( V_{dc} = 525 \text{ volts} \), \( f = 32.6 \text{ Hz} \).
FIGURE 142(b) The calculated no-load current for 8 PWM voltage-waveform application at 32.57 Hz.

FIGURE 142(c) Applied 8 PWM voltage-waveform, \( P_w = 0.74 \), \( V_{dc} = 565 \) volts. \( f = 32.58 \) Hz
FIGURE 143A: The induction motor's no-load line-current for 4 PWM voltage-waveform application at 25 Hz. Horizontal scale 5 ms/cm, vertical scale 50 mV/cm.

FIGURE 144A: The induction motor's no-load line-current for 8 PWM voltage-waveform application at 25 Hz. Horizontal scale 5 ms/cm, vertical scale 60 mV/cm.
The calculated no-load line-current for 4 PWM voltage-waveform application at 25 Hz.

Applied 4 PWM voltage-waveform, \( P_W = 1.54 \) ms, \( V_{dc} = 550 \) volts, \( f = 25 \) Hz.
FIGURE 144(b) The calculated no-load line-current for 8 PWM voltage-waveform application at 25 Hz.

FIGURE 144(c) Applied 8 PWM voltage-waveform, $P_W = 0.775$ ms, $V_{dc} = 540$ volts, $f = 25$ Hz.
FIGURE A1
One half cycle of a \( \frac{1}{4} \) PWM waveform which has equal pulse durations on the time axis.

FIGURE A2.1
One half cycle of a quasi-square wave on the time axis.

FIGURE A2.2
One half cycle of a 3 PWM waveform on the time axis.

FIGURE A2.3
One half cycle of a 5 PWM waveform on the time axis.

FIGURE A2.4
One quarter cycle of a 7 PWM waveform on the time axis.
FIGURE A4.1 a) The quasi-square voltage-waveform on the time axis

b) The airgap flux due to the quasi-square voltage waveform

FIGURE A4.2 a) 4 PWM voltage-waveform on time axis

b) The airgap flux due to the 4 PWM waveform
The flow chart of the computer program used to determine the pulse width for the minimum losses of the induction motor. In the flow chart NHT, PFI and PW denote, respectively, order of time harmonic, final value of pulse duration and pulse duration.
FIGURE A6.2  The variation of total harmonic losses against pulse duration
FIGURE A6.3  The variations of power losses caused by 4 FWM waveforms whose pulse durations are 1.562 ms and 1.5833 ms. The numbers of the characteristics denote the following information; (1) 1.562 ms, (2) 1.5833 ms.
The variations of power losses caused by 500W waveforms whose pulse durations are 1.562 ms and 2.101 ms. The numbers of the characteristics denote the following information; (1) 1.562 ms and (2) 2.101 ms.
The flow-chart of the computer program that calculates the total losses of the induction motor, where NSP and NHT denote respectively, the order of space harmonics and the order of time harmonics.
Appendix 8

The flow-chart of the computer program that calculates the line-current for the transient period and for the steady-state of the induction motor. TFI, L represent, respectively, the final time and L matrix of the motor.