Autopilot design for ship control

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AUTOPilot DESIGN FOR SHIP CONTROL

BY

CHENG CHEW LIM, B.Sc.

A DOCTORAL THESIS

Submitted in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy of the University of Technology, Loughborough.

November 1980

Supervisor: Mr. W. Forsythe, M.Sc.

Department of Electronic and Electrical Engineering

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SYNOPSIS

The advent of high fuel costs and the increasing crowding of shipping lanes have initiated considerable interest in ship automatic pilot systems, that not only hold the potential for reducing propulsion losses due to steering, but also maintain tight control when operating in confined waterways.

Since the two requirements differ significantly in terms of control specification it is natural to consider two separate operating modes. Conventional autopilots cannot be used efficiently here, partly because the original design catered for good gyrocompass heading control only, and partly because the requirement of reducing propulsion losses cannot be easily translated into control action in such schemes.

Linear quadratic control can be used to design a dual mode autopilot. The performance criterion to be minimised can readily be related to either the propulsion losses while course-keeping, or to the change of heading while manoeuvring, and therefore, the same controller can be used for both functions. The designed control system is shown, from the computer simulation study, to perform satisfactorily in disturbed seas. However, the need for detailed knowledge of the ship dynamics in the controller design implies that time-consuming ship trials may be required. Hence an alternative method of design using adaptive self-tuning control is studied.

Because the self-tuning approach combines controller design and coefficient identification in such a way that the two processes proceed simultaneously, only the structure of the equation of ship motion is needed. As in the case of quadratic control, a well specified performance criterion is firmly linked to the design so that a closely controlled optimal performance results.
SYMBOLS, NOMENCLATURE AND ABBREVIATIONS

I. GENERAL CONVENTIONS

1. Formulae, figures and tables are numbered sectionwise. They are identified by a number of the form 4.3-5 where 4.3 is the section number and 5 the item number.

2. Laplace transformed qualities are denoted by a bar: \( \bar{y}(s) \) is the Laplace transform of \( y(t) \), and \( s \) is used as the Laplace operator.

3. Matrices are denoted by uppercase letters and vectors by lowercase letters.

4. Superscript \( T \) denotes the transpose of a matrix or vector.

5. Superscript \(-1\) denotes the inverse of a matrix.

6. \( \equiv \) means 'equals by definition'.

7. For derivatives with respect to time, the dot notation is used, e.g. \( \dot{x} = \frac{dx}{dt} \).
II SHIP MANOEUVRING

1. The sign convention used is illustrated below

\[ O \quad x \quad y \quad z \]
\[ +x, u, \dot{u} \]
\[ v \quad -\beta \quad U \]
\[ +N \quad r \quad \dot{r} \]
\[ +y \quad +y \quad v \]
\[ +5 \]

2. \( O \quad x \quad y \quad z \) : moving axes of the ship.
   Right-hand system fixed with respect to the ship,
   z-axis vertically down, x-axis forward.

\( \bar{O} \quad \bar{x} \quad \bar{y} \quad \bar{z} \) : fixed axes relative to the earth.
   Right-hand orthogonal system normally fixed in
   space, \( \bar{z} \)-axis vertically down, \( \bar{x} \)-axis in the
   general direction of the initial motion.

3. \( B \) beam
   \( C_B \) block coefficient
   \( D \) draught
   \( g \) acceleration due to gravity
   \( h \) significant wave height
   \( I_z \) moment of inertia with respect to z-axis through
   amidships
L  
length of ship - generally between perpendiculars

m  
mass of ship

N  
moment component on body relative to z-axis

r  
yaw rate

S(\omega)  
wave energy spectral density

K, T_1, T_2, T_3, T_a  
gain and time constants of ship equations

T_R  
inertia lag time constant of steering gear

u  
forward velocity in x-axis

v  
drift velocity in y-axis

U  
speed of ship

x_g  
distance from amidships to centre of gravity measured along x-axis

X, Y  
force components on body relative to x and y-axes

\beta  
drift angle

\delta  
rudder angle

\rho  
mass density

\psi  
yaw angle

\omega  
circular frequency of wave

V  
displacement volume

4. When a quantity is to be expressed in non-dimensional form, it is denoted by the use of the prime '. Unless otherwise specified, the non-dimensionalising factor is a function of \rho, L, U.

\text{e.g. } m' = m/\rho L^3, \quad L' = L/\rho U^2 L^3

5. A lower case subscript is used to denote the denominator of a partial derivative, e.g. \text{ } Y_v = \partial Y/\partial v
III LINEAR QUADRATIC OPTIMAL CONTROL

$A(t), A$  plant matrix of a finite-dimensional linear differential system

$B(t), B$  input matrix of a finite-dimensional linear differential system

$C(t), C$  output matrix of a finite-dimensional linear differential system

$I$  performance criterion

$K(t), K$  solution of the matrix Riccati equation

$P$  weighting matrix of the state

$Q$  weighting matrix of the controller output

$x(t), x$  state variables

$u(t), u$  controller output variables
IV  SELF-TUNING CONTROL

1. $E(\cdot)$ denotes unconditional expectation: the ensemble average is over all possible realisations of the random processes affecting the argument.

2. Cov means 'covariance': if the column vectors $x_1$ and $x_2$ are random variables, then $\text{cov}(x_1, x_2) \triangleq E\{(x_1 - E_x)(x_2 - E_x)^T\}$

3. A slanting bar (/) in connection with $E$ indicates that the expectation is conditional on the quantities written behind that bar.

4. $z$ denotes shift operator, e.g. $z^{-1}u(t) = u(t-h)$

5. $A(z^{-1})$ polynomial of order $n_a$ corresponding to system output; $a_0 = 1$

   $B(z^{-1})$ polynomial of order $n_b$ corresponding to system input

   $C(z^{-1})$ polynomial of order $n_c$ corresponding to an uncorrelated zero mean random sequence; $c_0 = 1$

   $d$ constant disturbance

   $J(z^{-1})$ polynomial corresponding to system control law

   $e(t)$ uncorrelated zero mean noise at sample instant $t$

   $h$ sampling interval

   $I$ performance criterion

   $P(t)$ matrix proportional to the covariance of the estimated parameter at sample instant $t$

   $P(z^{-1}), Q(z^{-1}), S(z^{-1})$ costing polynomials

   $t$ time in sample instants
$w(t)$, $u(t)$, $y(t)$ system set point, input and output at sample instant $t$, respectively

$x(t)$ vector containing measured data at sample instant $t$.

$\beta$ exponential forgetting factor

$\theta$ vector of controller coefficients

$\lambda, \lambda_1, \lambda_2$ weighting factors

$\phi(t)$ performance criterion related output function at sample instant $t$. 
CHAPTER 1

INTRODUCTION

Up to the present time, almost all commercially available ship automatic pilots are based on the proportional, integral and derivative (PID) type control. Typically, the measured heading angle is compared with the desired course, and the difference is used as the input to the controller. The output of the controller is then fed to the rudder servomechanism interface which generates suitable control signals to drive the ship's rudder. Some parameters of the autopilot need to be specified, and this may be most conveniently carried out during sea trials, based on certain rules of thumb to obtain a good quality steering for the vessel under consideration.

However, the judgement of steering quality is itself a highly subjective matter. The common tendency of assuming good steering to be that which holds a tight course is understandable, because for most ships, the only means of measuring steering performance is the heading error from the course recorder. It thus becomes necessary to define the efficiency of steering in quantitative terms, in order that a meaningful performance evaluation can be established, and when necessary, a new autopilot can be designed to optimise the defined efficiency.

The functions of an autopilot can be divided into those of course-changing and course-keeping. Course-changing demands a fast and accurate response especially when manoeuvring in crowded and confined waterways, while course-keeping requires a control that maintains a reasonable course with minimum rudder activity. Obviously, the difficulty of defining good steering is greater in course-keeping than in course-changing. Consequently, several attempts have been made to derive a performance index based on propulsion losses due to rudder and hull motions for an open seas...
course-keeping system. However, it is generally difficult to relate these losses directly to the parameters of the PID autopilot, in such a way that an optimum control system using such an autopilot may be readily implemented. As a result, an alternative design method using linear quadratic optimal control theory is investigated.

In the linear quadratic control approach, a ship's motions are described in the form of state equations, and the loss function to be optimised is cast into an integral quadratic form. From such a formulation, the coefficients of an optimal controller may be systematically derived. Since the optimal controller utilizes all the state variables to determine a control action, it not only possesses the property of optimality, but also tolerates nonlinearity better than the ordinary PID controller. However, not all the state variables are conveniently available from direct measurement, and those that are inaccessible may have to be reconstructed for the controller from those measurable state variables and from a knowledge of the plant. Nevertheless, it has been shown that satisfactory control from such an output-feedback controller is possible. In the matter of practical realisation of the design, the need for state reconstruction increases the complexity drastically.

Although the implementation may present no major difficulty using the present electronic technology, a further investigation of a reduced-order controller may be useful. As it turns out, the simplified controller can achieve a performance similar to the full order design, as well as offers the simplicity of the PID type configuration.

As for the course-changing operation, it is shown in the thesis that an identical design procedure to that of the course-keeping function
can be used. The criterion for optimisation in this case is minimisation of the heading error, with constraint on the controller output, so that a fast and accurate heading response without overshoot can be achieved and that the controller output is within the saturation limit of the steering gear.

Since the derivation of controller coefficients relies heavily on a priori knowledge of the dynamics of the controlled plant, the numerical values of each of the individual coefficients appearing in the chosen mathematical model must be known. Unfortunately, in most ships, such data are not always available completely, mainly because ship trials are expensive and time is often too short for adequate information to be obtained. Scale model tests\textsuperscript{14,15} may be used to obtain all the data but usually, they are conducted only for the ship of new design in the scale model basin, for propulsive performance only. Theoretical estimation based on the hull geometrical data using wing and flow theories\textsuperscript{16,17} or some semi-empirical equations\textsuperscript{18} is possible, but the correlation between the estimated and the measured results is not always satisfactory\textsuperscript{19}. Consequently, the concept of designing an adaptive controller that is capable of identifying its own system coefficients and then working out an appropriate control necessary to fulfil certain prespecified performance functions becomes an attractive alternative. There is another factor that establishes further the need for an adaptive autopilot. It is found that for a given sea-state, the constant coefficient controller that produces minimum steering propulsion losses at a particular wave-to-ship encounter angle, may not necessarily produce minimum propulsion losses when the encounter angle changes. Thus the maintenance of minimum propulsion losses in a
changing operating environment can only be possible when the coefficients of the controller are adjusted continuously.

Numerous adaptive control strategies such as model reference, adaptive filtering, PID controller with gains varying on-line, and self-tuning regulator to minimise output error variance have been employed in autopilot research in recent years. Nevertheless, work towards improved schemes using new adaptive control theories is still going on. The approach chosen in this thesis is developed from the self-tuning control strategy of Clarke and Gawthrop. This technique differs from the self-tuning regulator mentioned above in that the controller output is explicitly incorporated in the performance criterion. This is in harmony with the course-keeping philosophy of minimising propulsion losses caused by hull and rudder drags. With minor modification of the criterion, it is possible to achieve a fast system response such as that demanded by manoeuvring operations. The self-tuning regulator, on the other hand, cannot be made to perform this manoeuvring function efficiently. Further comparison of the adopted approach with the model-reference method reveals that the latter, although it works well during manoeuvring, is not suitable for course-keeping because the external disturbances are not accounted for in the controller design. The self-tuning autopilot, in contrast, is explicitly designed to suppress the disturbances. Nevertheless, an extension of the method to eliminate steady-state error caused by nonzero mean disturbances is still necessary, and is reported in this work.

The thesis is organised as follows. The different tasks of the automatic pilot are specified in Chapter 2, with formulation of performance criteria suitable for use in both optimal and adaptive control
designs. Chapter 3 considers the mathematical description of the ship simulation model including its steering gear. The nature of external stochastic disturbances is discussed and the mathematical model for generating nonlinear disturbing motions is also derived in this chapter. Chapter 4 gives detailed design procedures of the full order feedback controller, the observer for reconstructing unmeasurable states, and the reduced order controller. Chapter 5 deals with the comparative performance evaluation of these systems, and Chapter 6 considers the design of the self-tuning controller. A detailed account is given of the extension of the self-tuning control to eliminate steady-state output errors. Chapter 7 discusses the simulation results of the self-tuning autopilot system, highlighting their practical significance. Finally, Chapter 8 offers general conclusions and suggests areas for further investigation.
2.1 INTRODUCTION

As we have seen, the automatic pilot of a surface ship is required to perform the functions of course-keeping and course-changing which in most cases, demand different control characteristics for efficient steering. When changing course, accurate and fast responses with minimum overshoot are desired, particularly when manoeuvring in congested waters. In the open sea during straight course keeping in a long and uninterrupted voyage, the system then requires a control that minimises propulsion losses caused by rudder and hull motions. Therefore, a dual mode autopilot is a better means of efficient steering control than the conventional approach that is a compromise between those two functions in a single mode autopilot.

To design the dual mode autopilot, it is necessary to specify each mode of operation in an explicit manner. The following sections state performance requirements and derive criteria for these two operations.

2.2 COURSE-KEEPING

A straight course keeping vessel is usually expected to keep the ship's heading deviation small, despite the external disturbances. Tight control is not recommended because the rudder would be too active in correcting zero-mean heading errors, resulting in additional rudder drag, and unnecessary wear and tear on the steering gear. On the other
hand, allowing the ship to yaw freely over long periods of time by introducing a dead-band mechanism (known as the yaw gap) to reduce rms rudder demand is not necessarily a better approach, because there would then be prolonged periodic yawing motions and extra hull resistance, which increase the journey time and running cost. Consequently, the control of the rudder application and the heading accuracy for this operation should be carried out in such a way that the resultant yawing and rudder motions produce as little propulsion losses due to steering activity as possible.

Work to express these propulsion losses analytically has been done in the past and it is recognised that in general, propulsion losses arise from many different sources. The main ones, summarised by Norrbin, are:

1. Wind resistance of superstructures,
2. Hull resistance due to incident wave reflection,
3. Added hull resistance due to drift in waves and wind,
4. Varying hull resistance due to rolling and pitching motions,
5. Added hull resistance due to coupled yaw and sway motions on ship hull, and
6. Induced rudder resistance due to steering.

Among these losses, only those that are affected by steering control action concern us here. Therefore, the induced rudder drag and the added hull resistance due to coupled sway and yaw motions are the two main losses to be considered, and are referred to collectively as the added propulsion losses due to steering.

In mathematical terms, the instantaneous added propulsion losses may be
expressed as

\[ x = (m + x_{vr}) \cdot v \cdot r + X_{\delta \delta} \cdot \delta^2 \]  \hspace{1cm} 2.2-1

where \( m + x_{vr} \) is the virtual mass of the ship

\[ X_{\delta \delta} \] force coefficient due to rudder angle,

\[ v, r \] and \( \delta \), the sway velocity, yaw rate and rudder angle respectively.

Normally, the sway velocity is inaccessible for measurement, therefore, an approximate expression:

\[ v = -\beta \cdot U \]

\[ v = -\frac{OP}{L} \cdot L \cdot r \]

is used, under the assumption that the ship is operating on straight course and yawing at low frequency with small amplitude around a steady state pivot point. The constant \( OP \) is the distance between amidships and the pivot point which, for most ship forms, is somewhat abaft the bow. \( L \) is the ship length between perpendiculats.

Substituting the approximated \( v \) into eqn. 2.2-1 yields:

\[ x = -(m + x_{vr}) \cdot L \cdot \frac{OP}{L} \cdot r^2 + X_{\delta \delta} \cdot \delta^2 \]

The total added resistance time-average may then be

\[ \overline{x} = \frac{1}{T} \int_{0}^{T} \left[-(m + x_{vr}) \cdot L \cdot \frac{OP}{L} \cdot r^2 + X_{\delta \delta} \cdot \delta^2 \right] dt \]

\[ \overline{x} = -(m + x_{vr}) \cdot L \cdot \frac{OP}{L} \cdot r^2 + X_{\delta \delta} \cdot \delta^2 \]  \hspace{1cm} 2.2-2

With a further assumption that the yaw motion is of sinusoidal nature, the added resistance time-average may be expressed in terms of heading angle instead of yaw rate as

\[ \overline{x} = -(m + x_{vr}) \cdot L \cdot \frac{OP}{L} \cdot \omega^2 \cdot \psi^2 + X_{\delta \delta} \cdot \delta^2 \]  \hspace{1cm} 2.2-3
where $\omega$ may be regarded as the natural frequency of the closed-loop steering control system. Eqn. 2.2-2 or 2.2-3 can therefore be used as a basis for design and evaluation of autopilots for steady-state course-keeping.

2.3 COURSE-CHANGING

Since safety is the main factor to be concerned with during manoeuvring, there is a need for a responsive and stringent control, possibly at the expense of rudder motions. A fast manoeuvre requires that the steady turning rate be as high as compatible with ship and steering engine dynamics, and that the ship settles without oscillation to the desired course.

The control should therefore minimise the difference between the desired and the actual course, in some optimal fashion, with a constraint on the demanded rudder angle.

Mathematically, the criterion may be written

$$I = \int_0^T \left[ \psi_{error}^2 + \lambda \cdot u^2 \right] dt$$

where $\lambda$ limits the rudder demand to within its saturation level, and thus affects the transient response. An additional constraint on yaw rate may also be incorporated into the above criterion, i.e.

$$I = \int_0^T \left[ \psi_{error}^2 + \lambda_1 \cdot \dot{\psi}^2 + \lambda_2 \cdot u^2 \right] dt$$

to provide a better heading control. (See Chapters 5 and 7.)
3.1 INTRODUCTION

In the simulation studies of automatic ship steering systems, it is essential that a relatively accurate model of the system dynamics affecting steering performance be formulated.

The system in question can be regarded as the ship with steering actuator, subject to external disturbances as shown in Fig. 3.1-1.

![Block diagram of an autopilot system](image)

Fig. 3.1-1: Block diagram of an autopilot system
3.2 SHIP STEERING DYNAMICS

The ship may be modelled as a rigid body with six degrees of freedom. However, for the study of a surface ship autopilot design, the horizontal motions of the hull, driven by its propeller and controlled by its rudder, are the main concerns. Therefore, only the horizontal motions are considered.

3.2.1 Basic Equations of Ship Motion

Defining $\mathbf{O} x y z$ as the fixed axes relative to the earth and $\mathbf{O} x y z$ as the moving axes fixed with respect to the moving ship as shown in Fig. 3.2-1, the equations of motion may be written:

$$\ddot{X} = m \cdot x_{OG}$$

$$\ddot{Y} = m \cdot y_{OG}$$

$$\ddot{N} = I_z \cdot r$$

where $m$ is the mass

$x_{OG}$, $y_{OG}$, the distances,

$I_z$, the moment of inertia, and

$r$, the yaw rate.

Fig. 3.2-1: Coordinate system and symbols used in the surface ship kinematics
These forces and moments can be expressed in the moving axes as:

\[
\begin{align*}
X &= m.(\dot{u} - v.r) \quad \text{surge equation} \\
Y &= m.(\dot{v} + ur) \quad \text{sway equation} \\
N &= Iz.\dot{r} \quad \text{yaw equation}
\end{align*}
\]

where

\[
\begin{align*}
X &= X_\cos \psi + Y_\sin \psi \\
Y &= Y_\cos \psi - X_\sin \psi
\end{align*}
\]

Equations 3.2-1 are known as the manoeuvrability equations where the left-hand sides are hydrodynamic forces and moments generated by the rudder force, the propeller thrust, the resultants of pressures acting on the ship hull and rudder, etc. External disturbances such as wind and waves are not considered at this stage and waters are assumed calm. It is further assumed that the hydrodynamic forces and moment are functions of the rudder oscillation and of the ship motion cruising at moderate speed in open and deep waters relative to the ship draught at that instant\textsuperscript{18-20}.

\[
\begin{bmatrix}
X \\
Y \\
N
\end{bmatrix} = f(u,v,r,\dot{u},\dot{v},\dot{r},\delta,\dot{\delta},\ddot{\delta})
\]

These forces and moments can then be expanded in a Taylor series about the initial state of motions. However, because the complete Taylor
expansion is totally out of the question, truncation is necessary. Abkowitz retained the series up to the third order terms and after taking some important physical factors into account, developed a fairly complete nonlinear ship model. In most cases, Abkowitz's nonlinear model is considered too complex for practical purposes because there are many nonlinear terms whose coefficients cannot be determined easily through model tests, ship trials, or theoretical calculations. Consequently, a simplified model that retains only the linear terms of the series is often used in ship manoeuvring and control studies. That is

\[
-u (u-U) + (mX_U)u = 0 \quad 3.2-2a
\]

\[
-v (-v - (Y_1 - mU)r - (Y_2 - mXg)\dot{r} = Y_\delta \quad 3.2-2b
\]

\[
-v (-v (N_1 - mXg)\dot{v} - (N_2 - mXgU)r + (T_1 - N_1)\dot{r} = N_\delta \quad 3.2-2c
\]

As the surge equation is independent of the other two equations, it is often handled separately. Thus only the coupled sway and yaw motions are considered when investigating the manoeuvrability and steering of the ship, and its autopilot design.

3.2.2 Nonlinear Simulation Model of Ship

By eliminating \( v \) and \( \dot{v} \) and replacing \( r, \dot{r} \) and \( \ddot{r} \) by \( \psi, \dot{\psi} \) and \( \ddot{\psi} \) respectively, eqns. 3.2-2b and 3.2-2c can be reduced to a single differential equation describing rudder-to-yaw response:

\[
\dddot{\psi} + \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \ddot{\psi} + \left( \frac{1}{T_1^2 \cdot T_2} \right) \dot{\psi} = \left( \frac{K \cdot T_3}{T_1 \cdot T_2} \right) \dot{\delta} + \left( \frac{K \cdot T}{T_1 \cdot T_2} \right) \delta \quad 3.2-3
\]

where its coefficients are related to the hydrodynamic derivatives of eqn. 3.2-2 and are given in Table 5.2-3 of Chapter 5.

When a ship is directionally stable and operating in either straight course-keeping or moderately active manoeuvres, eqn. 3.2-3 may be used to
describe the motions adequately. However, when the ship is directionally unstable, the linear equations are no longer valid unless the range of operation is small.

Bech and Wagner Smith considered that the reasons for the invalidity were that the coefficient \( \frac{1}{T_1 T_2} \) varied significantly during large manoeuvring while other coefficients remained fairly constant. Therefore, a term

\[
\frac{K}{T_1 T_2} \dot{\psi} + \frac{1}{T_1} \ddot{\psi} + \frac{K}{T_1 T_2} H(\dot{\psi}) = \frac{K}{T_1 T_2} (T_3 \delta + \delta)
\]

where the nonlinear manoeuvring characteristics of the ship were accounted for by the introduction of a non linear function \( H(\dot{\psi}) \) while other coefficients are invariant. \( H(\dot{\psi}) \) is commonly referred to as the steering characteristic function and may be derived by curve fitting to spiral test results for directionally stable ships, or reverse spiral test results for directionally unstable ships.

In general, the steering function \( H(\dot{\psi}) \) may have the characteristics shown in Fig. 3.2-2 and may be approximated by the polynomial

\[
H(\dot{\psi}) = a \dot{\psi}^3 + b \dot{\psi} + c
\]

where \( a \) is a negative constant

\[
b
\]

a negative value when the steering is directionally stable, and positive if unstable, and

\[
c
\]

a constant to account for the small residual helm caused by the effect of single-screw propeller action on a straight course.
3.3 STEERING GEAR MODEL

The ship's rudder is actuated by an electrohydraulic power steering gear consisting of two hydraulic rams, one on either side of the tiller arm connected to the rudder post. The hydraulic rams are connected by pipes to the steering pump. When the floating lever receives a demanded helm from the bridge, it compares the demand with the actual rudder position through the connecting linkage known as the hunting rod. The difference is transmitted to the constant speed hydraulic pump which provides hydraulic pressure to the steering gear and so rotates the rudder. As the rudder achieves the required helm angle, the hunting rod moves back the floating lever which takes the stroke off the pump and so stops the rudder rotation. This motion may be approximated

Fig. 3.2-2: Nonlinear steering characteristic function \( H(\psi) \)
by a first order differential equation:

\[ \dot{\delta} = \frac{1}{T_R} (\delta_i - \delta) \] 3.3-la

where \( \delta_i \) is the demanded helm angle.

However, when a large helm angle is demanded, the hydraulic pump is then at its full stroke and discharging the hydraulic fluid to the rams at an approximately constant rate which in turn, limits the rudder rotating speed to a constant rate. Under these circumstances, the model should be represented as:

\[ \dot{\delta} = \frac{1}{T_R} . e_{\text{max}} . \text{sign}(\delta_i - \delta) \] 3.3-1b

where \( e_{\text{max}} \) is the maximum effective input error. A typical value for \( e_{\text{max}} \) is 6 degrees while maximum rudder slew rate is around 3 degrees per second. In addition, the maximum rudder deflection should be limited to its useful range of \( \pm 30 \) degrees from amidships.

Fig. 3.3-1 is the block diagram of the nonlinear hydraulic steering system model.

---

![Block diagram](image_url)
3.4 EXTERNAL DISTURBANCES

Ocean waves are in general considered to be the most important external disturbance affecting ship motion in a seaway. The fundamental assumption in the linearised ship motion theory is that the response of the ship to any individual sinusoidal wave, described as a regular sea, is a linear function of its amplitude with frequency equal to the encounter wave frequency. However, in the horizontal plane of motion, the ship experiences steady drift motion in addition to the periodic motion. This steady motion is of engineering significance because there is no restoring hydrostatic force or moment in this plane. An irregular sea is regarded as the sum of a large number of sinusoidal waves, each component having a particular frequency, amplitude, direction and randomly distributed phase angle. A ship is therefore affected similarly in such a sea by the nonzero mean slowly varying sway and yaw motions, together with other oscillatory motions having frequency components equal to the encounter frequency of the individual wave components.

A suitable representation of the wave force and moment in irregular waves, expressed in the form of Volterra power series, has been developed by researchers in the field, and may be described briefly as follows.

Let the drift force or yaw moment be represented by $y$, and the irregular wave excitation by $x$, then

$$y(t) = \int_{-\infty}^{\infty} h_1(\tau).x(t-\tau).d\tau + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1,\tau_2).x(t-\tau_1).x(t-\tau_2).d\tau_1.d\tau_2 + ...$$

3.4-1

where the kernel function $h_1(\tau)$ in the first term is the familiar impulse
response of the linear system, while \( h_2(t_1, t_2) \) is the second-order two dimensional function. It is assumed that the series converges and that sufficiently accurate sway and yaw motion generation may be represented by the first and second-order terms only.

As for the irregular wave excitation \( x(t) \), it may be taken as

\[
x(t) = \int_0^\infty \cos[\omega t - \epsilon(\omega)] \cdot \sqrt{2} S(\omega) \, d\omega
\]

where \( \sqrt{2} S(\omega) \, d\omega \) is the encounter wave amplitude, obtaining from a continuous encounter wave energy density spectrum \( S(\omega) \), and \( \epsilon(\omega) \) is the random phase angle uniformly distributed over \( 0 \) to \( 2\pi \) radians.

By substituting the representation into the first-order term of the series:

\[
y_1(t) = \int_0^\infty h_1(\tau) \, d\tau \cdot \int_0^\infty \cos[\omega(t-\tau) - \epsilon(\omega)] \cdot \sqrt{2} S(\omega) \, d\omega
\]

\[
= \int_0^\infty \cos[\omega t - \epsilon(\omega) - \omega \tau] \cdot \int_0^\infty h_1(\tau) \cdot \sqrt{2} S(\omega) \, d\omega \, d\tau
\]

Let

\[
H(\omega) \triangleq \int_{-\infty}^\infty h_1(\tau) \, e^{-j\omega \tau} \, d\tau
\]

\[
= |H(\omega)| \cdot e^{j\phi(\omega)}
\]

then

\[
y_1(t) = \int_0^\infty \cos \left[ \omega t - \epsilon(\omega) + \phi(\omega) \right] \cdot \sqrt{2} |H(\omega)| \cdot S(\omega) \, d\omega
\]

which represents the first-order wave force or moment acting on the ship hull, it contains many high and low frequency components and is a zero mean process.

Similarly, for the second-order wave motions,

\[
y_2(t) = \int_{-\infty}^\infty \int_{-\infty}^\infty h_2(\tau_1, \tau_2) \, d\tau_1 \, d\tau_2
\]
\[ \int_0^\infty \int_0^\infty \{ \cos[\omega_1 t + \omega_2 t - \omega_1 \tau_1 - \omega_2 \tau_2 - \varepsilon(\omega_1) - \varepsilon(\omega_2)] \\
\cos[\omega_1 t - \omega_2 t - \omega_1 \tau_1 + \omega_2 \tau_2 - \varepsilon(\omega_1) + \varepsilon(\omega_2)] \} \]

\[ = \sqrt{S(\omega_1) S(\omega_2)} d\omega_1 d\omega_2 \]

\[ \int_0^\infty \int_0^\infty \{ (\omega_1 + \omega_2) t + \phi(\omega_1, \omega_2) - \varepsilon(\omega_1) - \varepsilon(\omega_2) \} \]

\[ + \sqrt{|H(\omega_1, \omega_2)|^2 S(\omega_1) S(\omega_2)} d\omega_1 d\omega_2 \]

where

\[ H(\omega_1, \omega_2) = \int_0^\infty \int_0^\infty h_2(\tau_1, \tau_2) e^{-j(\omega_1 \tau_1 + \omega_2 \tau_2)} d\tau_1 d\tau_2 \]

\[ = |H(\omega_1, \omega_2)| e^{j\phi(\omega_1, \omega_2)} \]

and

\[ H(\omega_1, -\omega_2) = \int_0^\infty \int_0^\infty h_2(\tau_1, \tau_2) e^{-j(\omega_1 \tau_1 - \omega_2 \tau_2)} d\tau_1 d\tau_2 \]

\[ = |H(\omega_1, -\omega_2)| e^{-j\phi(\omega_1, -\omega_2)} \]

The first term of eqn. 3.4-4 represents the contribution of wave frequency pair sums to the second order wave force excitation, and the second term represents the wave frequency pair differences. However,
knowledge of the two dimensional transfer function is limited, therefore an approximated but practical approach suggested by Newman\textsuperscript{46} is often used\textsuperscript{47}.

Newman considered that the rapidly varying second-order, being high frequency and zero mean, had a smaller effect on the system than its slowly varying nonzero mean counterpart, therefore, it could be discarded. The remaining second-order transfer function $H(\omega_1, -\omega_2)$ could be approximated throughout the bi-frequency plane by its diagonal value $H(\omega_1, -\omega_1)$ which might be interpreted physically as the second-order steady force or moment acting on the vessel in regular waves of unit amplitude and encounter frequency $\omega_1$. This diagonal value may then be obtained from regular wave model tests or theoretical estimation from a given ship hull\textsuperscript{15, 48-50}.

In general, the error resulting from the approximation cannot be determined rigorously, but nevertheless, the approach offers a practical way of synthesising the slowly varying force and moment, and is therefore adopted in the simulation.

3.5 EQUATIONS OF SHIP MOTION IN DISTURBED SEAS

The ship motion in calm water represented by eqns 3.2-2b and 3.2-2c may be modified to account for the effects of sea waves by regarding the sway force and yaw moment as additive terms:

$$
\begin{bmatrix}
-Y_v + m & -(Y_x - m_x g) \\
-N_v + m_x g & I z - N_x \\
\end{bmatrix}
\begin{bmatrix}
\dot{v} \\
\dot{r} \\
\end{bmatrix}
+ \begin{bmatrix}
-Y_v & -(Y_x - m_x U) \\
-N_v & -(N_x - m_x g U) \\
\end{bmatrix}
\begin{bmatrix}
v \\
r \\
\end{bmatrix}
= \begin{bmatrix}
Y_\delta \\
N_\delta \\
\end{bmatrix}
+ \begin{bmatrix}
Y_w \\
N_w \\
\end{bmatrix}
$$

3.5-1
where \( Y_w \) and \( N_w \) are the wave force and moment acting on the ship hull.

Recasting the equation into a single differential equation by eliminating \( v \) and replacing \( r \) by \( \dot{\psi} \), it becomes:

\[
\ddot{\psi} + \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \dot{\psi} + \left( \frac{1}{T_1 T_2} \right) \psi = \frac{K T_3}{T_1 T_2} \ddot{\psi} + \frac{K}{T_1 T_2} \dot{\psi} + D_1 \dot{Y}_w + D_2 Y_w + D_3 N_w + D_4 N_w
\]

Table 5.2-3 of Chapter 5 shows the relationship between the coefficients of this equation and those of eqn. 3.5-1.

Modification to accommodate the description of nonlinear effects during large manoeuvre may be carried out by replacing

\[
\left( \frac{1}{T_1 T_2} \right) \dot{\psi} \quad \text{by} \quad \left( \frac{K}{T_1 T_2} \right) H(\dot{\psi}) \quad \text{as in Sect. 3.2.2, rendering}
\]

\[
\ddot{\psi} + \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \dot{\psi} + \left( \frac{K}{T_1 T_2} \right) \psi = \frac{K T_3}{T_1 T_2} \ddot{\psi} + \frac{K}{T_1 T_2} \dot{\psi} + \dot{D}_1 \dot{Y}_w
\]

\[+ D_2 \dot{Y}_w + D_3 N_w + D_4 N_w \quad 3.5-2\]
4.1 INTRODUCTION

In the commonly used PID autopilots the determination of the three-term gains to obtain an efficient steering is always difficult. This is because the relationship between the ship dynamics and the gains with a given performance requirement is implicit with the result that the tuning of the system usually relies heavily on empirically based knowledge. The need is therefore established to develop a more systematic and rigorous approach to computing the parameters of the autopilot, which can either be of the existing PID type or an entirely new configuration.

Linear quadratic optimal control may be used for this purpose, being a class of modern control theory which provides an analytical design procedure that shifts the load of the design task from the designer's ingenuity onto his mathematical ability and the computer used in carrying out the design. The main characteristics of the approach are the state-space description of the controlled plant, and the optimisation of a specified performance criterion.
4.2 FUNDAMENTAL OPTIMUM CONTROL

4.2.1 State-variable Representation

The linear equations of ship motion are shown in Sec. 3.2 to be

\[
\begin{bmatrix}
-m \dot{Y}_v & m \dot{x}_g - Y_r \\
-m \dot{x}_g + N_r & \dot{m} x_g - N_r
\end{bmatrix}
\begin{bmatrix}
\dot{v} \\
\dot{r}
\end{bmatrix}
+
\begin{bmatrix}
-Y_v & -Y_r + m \cdot U \\
-N_v & -N_r + m \cdot x_g \cdot U
\end{bmatrix}
\begin{bmatrix}
v \\
r
\end{bmatrix}
=
\begin{bmatrix}
Y_\delta \\
N_\delta
\end{bmatrix}
\]

or

\[
\begin{bmatrix}
\dot{v} \\
\dot{r}
\end{bmatrix}
=rac{1}{\Delta}
\begin{bmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{bmatrix}
\begin{bmatrix}
v \\
r
\end{bmatrix}
+rac{1}{\Delta}
\begin{bmatrix}
n_1 \\
n_2
\end{bmatrix}
\begin{bmatrix}
\delta
\end{bmatrix}
\]

where \( \Delta = (m \cdot Y_v) (\dot{I}z - N_r) - (m \cdot x_g - Y_r) (m \cdot x_g - N_r) \)

\[
m_{11} = (\dot{I}z - N_r) \cdot Y_v + (Y_r - m \cdot x_g) \cdot N_v
\]

\[
m_{12} = (\dot{I}z - N_r) \cdot (-m \cdot U + Y_r) + (-Y_r + m \cdot x_g) \cdot (m \cdot x_g - N_r)
\]

\[
m_{21} = (m \cdot Y_r) \cdot N_v + Y_v \cdot (N_r - m \cdot x_g)
\]

\[
m_{22} = (Y_v - m) \cdot (m \cdot x_g - N_r) + (m \cdot U - Y_r) \cdot (\dot{N}r + m \cdot x_g)
\]

\[
n_1 = (\dot{I}z - N_r) \cdot Y_\delta + (Y_r - m \cdot x_g) \cdot N_\delta
\]

\[
n_2 = (m \cdot Y_v) \cdot N_\delta + (N_v - m \cdot x_g) \cdot Y_\delta
\]

and the linear equation of the rudder servo, eqn. 3.3-1

\[
\delta = \frac{1}{\tau_R} \cdot (\delta_1 - \delta)
\]

may be transformed into the state equations

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{m_{22}}{\Delta} \cdot x_2 + \frac{m_{21}}{\Delta} \cdot x_3 + \frac{n_2}{\Delta} \cdot x_4
\end{align*}
\]
\[ x_3 = \frac{m_{12}}{\Delta} x_2 + \frac{m_{11}}{\Delta} x_3 + \frac{n_1}{\Delta} x_4 \]

\[ \dot{x}_4 = \frac{1}{T_R} (-x_4 + u_1) \]

Here \( x_1 = \psi \), heading angle  
\( x_2 = r = \dot{\psi} \), yaw rate  
\( x_3 = v \), sway velocity  
\( x_4 = \delta \), rudder angle

and \( u_1 = \delta_1 \), demanded rudder angle

If the heading demand \( x_5 \) is taken as a step for mathematical convenience, then

\[ \dot{x}_5 = 0 \]

is another state equation. Including \( x_5 \) in the system equations yields:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 
\end{bmatrix} =
\begin{bmatrix}
0 & a_{12} & 0 & 0 & 0 \\
0 & a_{22} & a_{23} & a_{24} & 0 \\
0 & a_{32} & a_{33} & a_{34} & 0 \\
0 & 0 & 0 & a_{44} & 0 \\
0 & 0 & 0 & 0 & 0 
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 
\end{bmatrix} +
\begin{bmatrix}
0 \\
o \\
o \\
o \\
o 
\end{bmatrix}
\begin{bmatrix}
u_1 
\end{bmatrix}
\]

or \( x(t) = A x(t) + B u(t) \)

where

\[ a_{12} = \frac{m_{11}}{\Delta} \quad a_{22} = -\frac{m_{22}}{\Delta} \quad a_{23} = \frac{m_{23}}{\Delta} \]

\[ a_{24} = -\frac{n_2}{\Delta} \quad a_{32} = \frac{m_{12}}{\Delta} \quad a_{33} = -\frac{m_{33}}{\Delta} \]

\[ a_{34} = \frac{n_1}{\Delta} \quad a_{44} = -\frac{1}{T_R} \quad b_4 = \frac{1}{T_R} \]
4.2.2 Quadratic Performance Equations and Basic Controller Design

The performance requirement for a course-changing manoeuvre is shown in Chapter 2 to be

\[ \int_{0}^{T} [(\text{course deviation})^2 + \lambda \cdot (\text{demanded rudder angle})^2] \, dt \]

which is to be minimised by choice of the demanded rudder angle, i.e.

\[ I = \min_{u_1(t)} \int_{0}^{T} [e(t)^2 + \lambda \cdot u_1(t)^2] \, dt \]

where \( e(t) \) is the heading error, and \( u_1(t) \), the rudder demand

In more general terms, the quadratic equation may be written as;

\[ I = \min_{u(t)} \int_{0}^{T} [x(t)^T \cdot P \cdot x(t) + u(t)^T \cdot Q \cdot u(t)] \, dt \]

where \( x \) and \( u \) are column vectors, while \( P \) and \( Q \) are at least positive semidefinite matrices, so that the integrand may never be negative.

As for keeping a straight course, the required performance criterion is to minimise propulsion losses due to steering, and is approximated as

\[ I = \min_{u_1(t)} \int_{0}^{T} [r(t)^2 + \lambda \cdot u_1(t)^2] \, dt \]

with \( r \), the yaw rate, and

\[ \lambda = \frac{X_{\delta \delta}}{-(m+X_{vr}) \cdot \frac{\delta B}{L} \cdot L} \]

Here the demanded rudder angle \( u_1(t) \) replaces the achieved rudder angle \( \delta(t) \) of eqn.2.2-2 for mathematical convenience. This approximation is made on the grounds that the steering gear time constant is relatively short, and that the operating range of rudder angle is small and hence linear in operation.
Eqn. 4.2-4 can further be transformed to the standard quadratic form of eqn. 4.2-3 which is used for the relative ease with which it can be handled mathematically and for the fact that it results in linear feedback control.\textsuperscript{7-10}

\[ u(t) = G \cdot x(t) \]

where \( G = -Q^{-1}B^T \cdot K \),

with \( K \), the symmetric matrix, satisfying the matrix Riccati differential equation

\[ K = K \cdot B \cdot Q^{-1} \cdot B^T \cdot K - A^T \cdot K \cdot A - P \]

One further restriction on the \( Q \) matrix, that it must be positive definite rather than semidefinite, becomes necessary at this point because inverse \( Q \) is used in both the feedback law and the Riccati equation.

If the final time \( T \) of eqn. 4.2-3 is taken as infinite, based on the fact that the period of time over which the ship would operate is lengthy when compared to the main time constant associated with the system, then \( K \) becomes time invariant and is the unique positive definite solution of the algebraic matrix Riccati equation

\[ K \cdot B \cdot Q^{-1} \cdot B^T \cdot K - A^T \cdot K \cdot A - P = 0 \]

The engineering significance of the alteration is that the realisation of eqn. 4.2-5, the control algorithm with time invariant \( K \) avoids the time consuming computation of the solution of the Riccati differential equation on-line, thus reducing drastically the cost of implementation.

Fig. 4.2-1 depicts the state feedback control scheme where all the states are assumed accessible for measurement at all time. Obviously, this assumption is unrealistic because state variables \( x_2 \), the yaw rate and \( x_3 \), the sway velocity, are not often available from measurement, and therefore,
some artifice to get around this difficulty is considered in the next section.

Fig.4.2-1: State feedback autopilot

To summarise, the design procedure of an optimal controller can be arranged as:

1. Represent the plant dynamics using the state-space matrix equation \( \dot{x} = Ax + Bu \)
2. Formulate the quadratic integral performance criterion
   \[ I = \int_0^\infty [x^T P x + u^T Q u] dt \]

3. Compute the \( K \) matrix from the algebraic matrix Riccati equation
   \[ K B Q^{-1} B^T K - A^T K K A - P = 0 \]
   and

4. Calculate the coefficients of the control algorithm:
   \[ G = -Q^{-1} B^T K \]
   and then form the control law
   \[ u(t) = G x(t) \]

4.2.3 State Reconstruction

Since the entire state vector is required by the controller to generate
system control, it is essential that all the state variables are
available from measurement. However, the yaw rate and the sway velocity
are often inaccessible in most steering systems, and therefore, state
reconstruction becomes necessary. One technique often referred to for
this purpose was first proposed by Luenberger\textsuperscript{11} and developed by
Gopinath\textsuperscript{12,13} as follows.

To design the state observer, the matrix state equations of the con-
trolled plant are arranged and partitioned as:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & a_{12} & 0 \\
0 & a_{24} & a_{22} & a_{23} \\
0 & a_{34} & a_{32} & a_{33} \\
0 & 0 & a_{44} & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} +
\begin{bmatrix}
0 \\
b_4
\end{bmatrix}
\begin{bmatrix}
u_1
\end{bmatrix}
\]

which can further be written as
\begin{align*}
\dot{y} &= B_{11}y + B_{12}w + C_1u \\
\dot{w} &= B_{21}y + B_{22}w + C_2u
\end{align*} \tag{4.2-8a}
\tag{4.2-8b}

where \( y \triangleq [x_1 \ x_4]^T \), the measurable states
\( w \triangleq [x_2 \ x_3]^T \), the inaccessible variables

\[
\begin{align*}
B_{11} &= \begin{bmatrix} 0 & 0 \\ 0 & a_{44} \end{bmatrix} & B_{12} &= \begin{bmatrix} a_{12} & 0 \\ 0 & 0 \end{bmatrix} \\
B_{21} &= \begin{bmatrix} 0 & a_{24} \\ 0 & a_{34} \end{bmatrix} & B_{22} &= \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \\
C_1 &= \begin{bmatrix} 0 \\ b_4 \end{bmatrix}^T \text{ and } C_2 &= \begin{bmatrix} 0 & 0 \end{bmatrix}^T
\end{align*}
\]

Initially, the observer equation is defined in a closed form as (cf eqn.4.2-8b):

\[
\dot{\hat{w}} = B_{21}y + B_{22}w + L.B_{12}(w-\hat{w}) + C_2u
\tag{4.2-9}
\]

where \( \hat{w} \) is the estimated state vector, 
\( w-\hat{w} \), the estimation error, and
\( L \), a constant matrix whose elements, chosen by the designer, affect the convergence rate of estimation.

The term \( B_{12}w \) of the above equation can be replaced, from eqn.4.2-8 by

\[
\dot{y} = B_{11}y + C_1u
\]

resulting in

\[
\dot{\hat{w}} = (B_{22} - L.B_{12})\hat{w} + L(\dot{y} - (B_{11}y + C_1u)) + C_2u + B_{21}y
\]

Finally, collecting the differential terms together and defining \( z = \hat{w} - L.y \) yields
The observer equations for the multivariable feedback autopilot can now be obtained by substituting the matrices in eqn. 4.2-8 into eqn. 4.2-10, rendering

\[
\begin{align*}
\dot{z}_1 &= \begin{bmatrix}
 a_{22} - L_{11} \cdot a_{12} & a_{23}
\end{bmatrix} \begin{bmatrix}
 z_1 \\
 z_2
\end{bmatrix} + \begin{bmatrix}
 -L_{12} \cdot b_{4}
\end{bmatrix} + \begin{bmatrix}
 u_1
\end{bmatrix} \\
\dot{z}_2 &= \begin{bmatrix}
 a_{32} - L_{21} \cdot a_{12} & a_{33}
\end{bmatrix} \begin{bmatrix}
 z_1 \\
 z_2
\end{bmatrix} + \begin{bmatrix}
 -L_{22} \cdot b_{4}
\end{bmatrix}
\end{align*}
\]

4.2-11

Therefore, \( \hat{x}_2 = z_1 + L_{11} \cdot x_1 + L_{12} \cdot x_4 \)

\( \hat{x}_3 = z_2 + L_{21} \cdot x_1 + L_{22} \cdot x_4 \)

Because the observer contains a feedback loop as can be seen from Figure 4.2-2, the question of stability arises. It has been shown that the observer is always stable\(^{11,13}\) but the choice of design parameters must be such that the estimates converge to their actual values in a sufficiently short duration. To do that, the eigenvalues of the matrix \([ B_{22} - L \cdot B_{12} ]\) of eqn. 4.2-10 must be made somewhat more negative than the roots of the observed system by choice of L so that convergence is faster than the system response.
Fig. 4.2-2: The state observer
4.3 CONSTANT DISTURBANCES AND INTEGRAL CONTROL

When a ship is keeping a straight course in open seas, it is necessary that any steady state heading error be relatively small, if not absent totally. The error is caused mainly by nonzero mean sustained side forces and yaw moment due to constant disturbances such as waves and ocean cross currents exerted on the ship hull.

Elimination of such an error can be achieved through the use of integral control, but the problem is to incorporate it in the most efficient manner. In the case of quadratic optimal control design, the incorporation may be carried out as follows,

Let \( x_6 = \int \psi_{\text{error}} \, dt \)
and includes this integral state in the system state equations.

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix} = 
\begin{bmatrix}
0 & a_{12} & 0 & 0 & 0 & 0 \\
0 & a_{22} & a_{23} & a_{24} & 0 & 0 \\
0 & a_{32} & a_{33} & a_{34} & 0 & 0 \\
0 & 0 & 0 & a_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix} + 
\begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
u_4 \\
0 \\
0
\end{bmatrix}
\]

We can now consider the problem of minimising a criterion of the form

\[
\int_0^T r(t)^2 + \lambda_1 u_1(t)^2 + \beta x_6(t)^2 \, dt
\]

where the term \( \beta x_6(t)^2 \) is added to the integrand so that the steady state heading error may be forced to zero by the control:

\[
u_1(t) = \sum_{i=1}^{6} G_i x_1(t)
\]

Realisation of the control algorithm presents no particular problem here because \( x_1 \) and \( x_5 \) can be measured, \( x_2 \) and \( x_3 \) can be reconstructed while \( x_6 \) can be obtained by integrating the heading error.
4.4.1 The Reduced Dimension Controller

When a ship is operating within the range of straight course-keeping to moderately active manoeuvres with a very stable steering characteristic, the motion may be described by

\[ \ddot{\psi} + \frac{1}{T_a} \dot{\psi} = \frac{K}{T_a} \delta \]  

4.4-1

which can be regarded as the simplified version of eqn. 3.2-3, and can be used as the plant model in the reduced order controller design.

The system state equations may be written as:

\[
\begin{bmatrix}
\dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & -1/T_a & K/T_a & 0 & 0 \\
0 & 0 & 0 & -1/T_R & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 
\end{bmatrix}
\begin{bmatrix}
x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 
\end{bmatrix} +
\begin{bmatrix}
0 \\ 0 \\ 1/T_R \\ 0 \\ 0 
\end{bmatrix}
\begin{bmatrix}
u_1 
\end{bmatrix}
\]
where $x_1$, is the heading angle
$x_2$, the yaw rate
$x_3$, the achieved rudder angle
$x_4$, the desired heading angle, and
$x_5$, the integral of heading error.

The control algorithm for this design, with the performance criterion as

$$ I = \min_{u_1(t)} \int_0^\infty \left[ x_2(t)^2 + \lambda_1 u_1(t)^2 + \beta_1 x_5(t)^2 \right] dt $$

or

$$ = \min_{u(t)} \int_0^\infty \left[ x^T . P . x + u^T . Q . u \right] dt $$

is

$$ u_1 = G_1 . x_1 + G_2 . x_2 + G_3 . x_3 + G_4 . x_4 + G_5 . x_5 $$

which is one dimension lower than the full order integral control of Sec.4.3.

As the abandoned state variable is the inaccessible sway velocity, the state observer therefore reduces to perform only a single state reconstruction of yaw rate. In fact, if a well engineered differentiator is incorporated, the autopilot can be realised without the state observer altogether.

4.4.2 The PID Autopilot

Since the control coefficients of heading angle $x_1$ and desired course $x_3$ are always equal but opposite in sign, the control law can be recast as

$$ u_1 = G_1 \cdot \left( \psi - \psi_{\text{demand}} \right) + G_2 \cdot \dot{\psi} + G_4 \cdot \delta + G_5 \cdot \int \left( \psi - \psi_{\text{demand}} \right) dt $$

which is the same as the PID type control except for the term $G_4 \cdot \delta$.  

Hence if we introduce a further simplification, that the demanded rudder angle equals the achieved rudder angle, on the grounds that the inertial time lag of the steering gear is negligible when compared to the inertial lag of the ship $T_a$, then the system equations become,

$$
\begin{bmatrix}
\dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_4 \\ \dot{x}_5
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -1/T_a & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\ x_2 \\ x_4 \\ x_5
\end{bmatrix} +
\begin{bmatrix}
0 \\ k/T_a \\ 0 \\ 0
\end{bmatrix}
\begin{bmatrix}
u_1
\end{bmatrix}
$$

and the control law becomes:

$$
u_1 = G_1(\psi - \psi_{\text{demand}}) + G_2\dot{\psi} + G_3(\psi - \psi_{\text{demand}})\,dt$$

which is of the PID form. This design approach can therefore be used to determine the gains of a PID autopilot with a firm link to a well specified criterion and as such gives a closely controlled performance.
CHAPTER 5

CONTROL SYNTHESIS AND SYSTEM SIMULATION STUDIES

5.1 SIMULATION MODELS

The overall simulation system may be represented by the block diagram shown in Fig. 5.1-1 with two main subsystems: the controlled plant and the controller. The plant consists of the steering gear, the ship and the external wave disturbances acting on the hull, while the controller is made up of the state observer and the control synthesiser.

The ship motion is characterised by eqn. 3.5-2

\[
\ddot{\psi} + \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \dot{\psi} + \frac{K}{T_1T_2} H(\psi) = \frac{K}{T_1T_2} (T_3 \dot{\delta} + \delta) + D_1 \dot{Y}_w + D_2 Y_w \\
+ D_3 N_w + D_4 N_w
\]

where the wave drift force \( Y_w \) and yaw moment \( N_w \) are generated by the wave excitation of a given sea state acting on the hull at a certain wave-to-ship encounter angle using eqns. 3.3-3 and 3.3-4.

The steering gear is modelled as

\[
\dot{\delta} = \frac{1}{T_R} (\delta_i - \delta) \quad ; \quad |\delta_i - \delta| < e_{\text{max}}
\]

\[
= \frac{1}{T_R} e_{\text{max}} \cdot \text{sign}(\delta_i - \delta) \quad ; \quad |\delta_i - \delta| \geq e_{\text{max}}
\]

where \( e_{\text{max}} \) is the maximum effective input error and has a typical value of 6 degrees.

Other consideration in the system simulation such as the use of full or reduced order controller will be discussed in due course.
5.2 NUMERICAL DATA

The main particulars and hydrodynamic coefficients of the ship used in the simulation study are given in Tables 5.2-1 and 5.2-2. These coefficients are determined experimentally for a Mariner class vessel.
which can be regarded as representative of the modern fast single-screw cargo ship.

Conversion from hydrodynamic coefficients to constants suitable for eqns. 3.5-2 and 4.2-1 is required and results in Table 5.2-3.

When generating the wave force and moment acting on the ship hull, it is required to specify a wave energy spectrum. The adopted spectrum is the general propose ITTC wave spectrum

\[ S(\omega) = \frac{A}{\omega^5} \cdot e^{-B/\omega^4} \]

where \( \omega \) is the circular frequency of the waves in radians per sec.

\[ A = 8.1 \times 10^{-3} \times 9.807^2, \text{ and} \]
\[ B = 3.11/\bar{h}^2 \]

The significant wave height \( \bar{h} \) is taken as between 2.5m to 4.5m. Other information required for wave force generation is the wave-to-ship encounter angle which is assumed to be 120 degrees for most of the studies, together with other angles such as 60 and 150 degrees for comparative evaluation of system performance.
Table 5.2-1: Particulars of model and ship of the Mariner type

<table>
<thead>
<tr>
<th>Model scale</th>
<th>Model</th>
<th>Ship</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length overall</td>
<td>6.748</td>
<td>171.80</td>
<td>m</td>
</tr>
<tr>
<td>Length between perpendiculars</td>
<td>6.437</td>
<td>160.93</td>
<td>m</td>
</tr>
<tr>
<td>Design waterline length</td>
<td>6.349</td>
<td>158.72</td>
<td>m</td>
</tr>
<tr>
<td>Maximum beam</td>
<td>0.927</td>
<td>23.17</td>
<td>m</td>
</tr>
<tr>
<td>Draught For'd</td>
<td>0.274</td>
<td>6.86</td>
<td>m</td>
</tr>
<tr>
<td>Draught Aft</td>
<td>0.323</td>
<td>8.06</td>
<td>m</td>
</tr>
<tr>
<td>Volume of displacement</td>
<td>1.0674</td>
<td>16622.</td>
<td>m³</td>
</tr>
<tr>
<td>Speed</td>
<td>1.544</td>
<td>7.716</td>
<td>m.s⁻¹</td>
</tr>
<tr>
<td>Block coefficient</td>
<td>0.60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rudder area</td>
<td>0.048</td>
<td>30.012</td>
<td>m²</td>
</tr>
<tr>
<td>Rudder aspect ratio</td>
<td>1.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Propulsion plant type</td>
<td>Turbine</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Propeller diameter</td>
<td>0.27</td>
<td>6.706</td>
<td>m</td>
</tr>
<tr>
<td>Direction of rotation</td>
<td></td>
<td>right hand</td>
<td></td>
</tr>
<tr>
<td>Propeller revolutions</td>
<td>345</td>
<td>69.0</td>
<td>rpm</td>
</tr>
</tbody>
</table>
Table 5.2-2: Hydrodynamic coefficients

<table>
<thead>
<tr>
<th></th>
<th>Nondimensional $x \times 10^{-5}$</th>
<th>Conversion factor</th>
<th>Dimensional $x \times 10^6$</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_v$</td>
<td>-1160</td>
<td>$\frac{\rho}{2} L^2 U$</td>
<td>-1.188</td>
<td>kg s$^{-1}$</td>
</tr>
<tr>
<td>$N_v$</td>
<td>-264</td>
<td>$\frac{\rho}{2} L^3 U$</td>
<td>-43.534</td>
<td>kg m s$^{-1}$</td>
</tr>
<tr>
<td>$Y_{r-m}U$</td>
<td>-499</td>
<td>$\frac{\rho}{2} L^3 U$</td>
<td>-82.286</td>
<td>kg m s$^{-1}$</td>
</tr>
<tr>
<td>$N_{r-m}xgU$</td>
<td>-166</td>
<td>$\frac{\rho}{2} L^4 U$</td>
<td>-4407.1</td>
<td>kg m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$Y_{r-m}$</td>
<td>-1546</td>
<td>$\frac{\rho}{2} L^3$</td>
<td>-33.066</td>
<td>kg</td>
</tr>
<tr>
<td>$N_{r-m}xg$</td>
<td>23</td>
<td>$\frac{\rho}{2} L^4$</td>
<td>79.2</td>
<td>kg m</td>
</tr>
<tr>
<td>$N_r-Iz$</td>
<td>-82.9</td>
<td>$\frac{\rho}{2} L^5$</td>
<td>-45959</td>
<td>kg m$^2$</td>
</tr>
<tr>
<td>$Y_{r-m}xg$</td>
<td>9.</td>
<td>$\frac{\rho}{2} L^4$</td>
<td>30.991</td>
<td>kg m</td>
</tr>
<tr>
<td>$Y_\delta$</td>
<td>278</td>
<td>$\frac{\rho}{2} L^2 U^2$</td>
<td>2.195</td>
<td>kg m s$^{-2}$</td>
</tr>
<tr>
<td>$N_\delta$</td>
<td>-139</td>
<td>$\frac{\rho}{2} L^3 U^2$</td>
<td>-176.72</td>
<td>kg m$^2$ s$^{-2}$</td>
</tr>
<tr>
<td>$m$</td>
<td>798</td>
<td>$\frac{\rho}{2} L^3$</td>
<td>17.045</td>
<td>kg</td>
</tr>
<tr>
<td>$Iz$</td>
<td>39.2</td>
<td>$\frac{\rho}{2} L^5$</td>
<td>21685.2</td>
<td>kg m$^2$</td>
</tr>
</tbody>
</table>
Table 5.2-3: Coefficients for eqns. 3.5-2 and 4.2-1

<table>
<thead>
<tr>
<th>Derived from</th>
<th>Numerical value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$</td>
<td>$(y_v-m)(N_x-Iz)-(y_v-m.xg)(N_x-m.xg)$</td>
<td>$151722 \times 10^{12}$</td>
</tr>
<tr>
<td>$a_{22}$</td>
<td>$\frac{(y_v-m)(m.xg.U-N_v)+(m.U-y_v)(m.xg-N_v)}{\Delta}$</td>
<td>$-0.10034$</td>
</tr>
<tr>
<td>$a_{23}$</td>
<td>$\frac{(m-y_v)N_v + (N_v-m.xg)y_v}{\Delta}$</td>
<td>$-0.10107 \times 10^{-2}$</td>
</tr>
<tr>
<td>$a_{24}$</td>
<td>$\frac{(m-y_v)N_\delta + (N_v-m.xg)y_\delta}{\Delta}$</td>
<td>$-0.37368 \times 10^{-2}$</td>
</tr>
<tr>
<td>$a_{32}$</td>
<td>$\frac{(Iz-N_x)(y_v-m.U)+(m.xg-y_v)(m.xg.U-N_v)}{\Delta}$</td>
<td>$-2.582584$</td>
</tr>
<tr>
<td>$a_{33}$</td>
<td>$\frac{(Iz-N_x)y_v + (y_v-m.xg)N_v}{\Delta}$</td>
<td>$-0.36875 \times 10^{-1}$</td>
</tr>
<tr>
<td>$a_{34}$</td>
<td>$\frac{(Iz-N_x)y_\delta + (y_v-m.xg)N_\delta}{\Delta}$</td>
<td>$0.6288 \times 10^{-1}$</td>
</tr>
<tr>
<td>$a_{44}$</td>
<td>$-\frac{1}{T_R}$</td>
<td>$-0.50$</td>
</tr>
<tr>
<td>$\frac{1}{T_1} + \frac{1}{T_2}$</td>
<td>$(y_v-m)(N_x-m.xg.U)+(N_x-Iz)y_v-(y_v-m.xg)N_x-(N_x-m.xg)(y_v-mU)$</td>
<td>$0.1372$</td>
</tr>
<tr>
<td>$K$</td>
<td>$\frac{N_v y_\delta - y_v N_\delta}{\Delta}$</td>
<td>$-0.20135 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\frac{K.T_3}{T_1.T_2}$</td>
<td>$(N_v-m.xg)y_\delta - (y_v-m)N_\delta$</td>
<td>$-0.37368 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\frac{1}{T_1.T_2}$</td>
<td>$(N_x-m.xg.U)y_v - (y_v-mU)N_x$</td>
<td>$0.10897 \times 10^{-2}$</td>
</tr>
<tr>
<td>$H(\psi)$</td>
<td>$\dot{\psi} - \delta$ curve</td>
<td>$-30.0 \psi^3 \pm 5.6 \dot{\psi}$</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
D_1 & = \frac{N_v - m \cdot xg}{\Delta} & 52.20 \times 10^{-12} \text{ kg} \cdot \text{m}^{-1} \\
D_2 & = \frac{N_v}{\Delta} & -28.69 \times 10^{-12} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1} \\
D_3 & = \frac{m - Y_v}{\Delta} & 21.79 \times 10^{-12} \text{ kg} \cdot \text{m}^{-2} \\
D_4 & = \frac{-Y_v}{\Delta} & 0.783 \times 10^{-12} \text{ kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}
\end{align*}
\]
5.3 COURSE-KEEPING AUTOPILOT SIMULATION RESULTS

For the purposes of minimising propulsion losses due to steering and eliminating steady state heading error, the performance criterion used in the course-keeping autopilot design is

\[ I = \min_{u_1(t)} \int_0^{\infty} \left[ r(t)^2 + \lambda u_1(t)^2 + \beta x_6(t)^2 \right] dt \]

where

\[ x_6 = \int \psi_{error} dt \]

as explained previously. However, the selection of weighting factors \( \lambda \) and \( \beta \) may pose some problems here because the choice of \( \lambda \) according to expression (see Sec. 4.2.2)

\[ \frac{x_{\delta\delta}}{-(m+X_{\delta V}) \cdot \frac{LP}{L}} \]

may need modification due to the inclusion of \( \beta x_6(t)^2 \) in the criterion. Nevertheless, it still provides us with a good initial value for \( \lambda \) for the simulation study. As for \( \beta \), a relatively small value with respect to \( \lambda \) is used so that the destabilising effect of the integrating action of heading error may be kept to an acceptable level. Therefore, \( \lambda \) is set to \( 1.0 \times 10^{-3} \) and \( \beta \) to \( 0.5 \times 10^{-7} \) for the chosen ship.

5.3.1 Full Order Output Feedback Controller

To investigate the effects of different values of \( \lambda \) on the hull and rudder drags, the rate of change of rudder movement, and the steady state heading error, a system consisting of the multi-output feedback
controller and the controlled plant is simulated. The characteristics of the ship and its steering gear are shown in Table 5.2-3, while the seaway is assumed to comprise irregular waves, as discussed in Sec.3.4, with significant wave height taken as 4.0 metres and encounter angle 120 degrees. The controller is the full order six states control where two inaccessible states are reconstructed by the state observer.

The observer design parameters, the L-matrix elements of eqns. 4.2-11, must be specified during the simulation. In general, the choice is not critical as long as the eigenvalues of the matrix in eqn.4.2-11:

\[
\begin{bmatrix}
  a_{22} - L_{11}a_{12} & a_{23} \\
  a_{22} - L_{21}a_{12} & a_{33}
\end{bmatrix}
\]

are more negative than the roots of the closed-loop system, i.e. plant and controller. It can be seen from the above matrix that only \( L_{11} \) and \( L_{21} \) need to be specified and they are taken as unity and remain unchanged.

Fig.5.3-1 shows the simulation results where the values of \( \lambda \) vary from \( 1.0 \times 10^{-4} \) to 1.0 and \( \beta \) is adjusted in proportion from \( 0.5 \times 10^{-7} \) to \( 0.5 \times 10^{-2} \) in order to maintain the same value of \( G_6 \) in eqn.4.3-1 throughout the comparison. As can be seen from the results, when \( \lambda \) increases, implying a looser control, the hull drag also increases but the rudder drag decreases. If the two drags are summed, it is clear that there is a region where minimum drag can be achieved. However, because both heading error and rudder rate \( \delta \) are relatively high when \( \lambda \) is at this lower drag region, it is reasonable to settle \( \lambda \) at a slightly higher value say \( 1.0 \times 10^{-3} \) so that \( \delta \) may be reduced and hence lower wear and tear on the steering servo may result.
It is noteworthy that the value for $\lambda$ finally chosen is in fact the initial value, derived from the ratio of coefficients of rudder drag to hull drag, i.e.

$$\frac{X_{\delta\delta}}{-(m + X_{\nu_T}) L \frac{\delta}{L}}$$
Fig. 5.3-1: Simulation results of the full order output feedback integral system during course-keeping
5.3.2 Comparison With An Existing Autopilot

In order to assess the performance of the full order output feedback system, a comparative performance evaluation is carried out. The autopilot to be compared with is the phase advance integral autopilot of the form 1.51

\[
\frac{\bar{u}_1(s)}{\bar{\psi}_e(s)} = G\cdot \frac{1 + s.a}{1 + s.b + \frac{1}{s.c}}
\]

where a, b, c and G are constants derived from the ship length, speed and rules of thumb as follows:

a is the ratio of ship length to cruising speed, multiplied by an arbitrary constant, typically 1,

b can be taken as one tenth of a,

c ranges from 60 to 200, typically 80, and

G is between 0.5 to 3.0

To allow a meaningful comparison, the above design constants must be chosen properly, and therefore, various values have been tested for the two important constants a and G in the hope of finding the most suitable ones.

Fig.5.3-2 shows the results, from which it can be seen that when G increases, while a is at its nominal value, b = a/10 and c = 180, the rudder drag also increases but the hull drag decreases. When the two drags are summed, as in the case of output feedback control, it becomes obvious that the suitable value for G is 0.75. Further
improvement can be achieved when $a$ is taken as $1.5 \times \frac{L}{U}$ rather than the nominal $1.0 \times \frac{L}{U}$. Consequently, the design parameters for the phase advance integral autopilot are:

$$G = 0.75, \quad a = 1.50 \times \frac{L}{U}, \quad b = \frac{a}{10}, \quad c = 180$$

Table 5.3-1 compares the performance of the output feedback integral controller with $\lambda = 0.001$, $\beta = 5 \times 10^{-7}$ and the phase advance integral autopilot under the same wave disturbances. It can be seen (column 1 and 2) that a constant rudder deflection having about $\frac{1}{4}$ degree mean angle is necessary to compensate for the nonzero mean wave motion and so maintain the course requirement of small mean heading error. The final column indicates that the propulsion losses due to steering of the output feedback system are about 3.5% smaller than the existing phase-advance autopilot which according to some authors $6, 27$ corresponds to a speed gain of approximately 2%. A further saving is to be expected from the lower figures for rudder rate (column 3) both from the point of view of fuel consumption and wear and tear.
Fig. 5.3-2: Simulation results of the phase advance integral system during course-keeping.
Table 5.3-1: Performance comparison of the full order output feedback and the phase advance integral systems during course-keeping

<table>
<thead>
<tr>
<th></th>
<th>heading error mean deg</th>
<th>heading error mean square deg $^2$</th>
<th>rudder angle mean deg</th>
<th>rudder rate mean square [deg/sec] $^2$</th>
<th>hull drag mean N</th>
<th>rudder drag mean N</th>
<th>total drag mean N</th>
<th>relative propulsion losses due to steering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase advance integral system</td>
<td>0.0129 0.0274</td>
<td>0.74</td>
<td>0.0013</td>
<td>77.6</td>
<td>226.7</td>
<td>304.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full order output feedback integral system</td>
<td>0.0371 0.0650</td>
<td>0.74</td>
<td>0.0010</td>
<td>75.1</td>
<td>218.9</td>
<td>294.0</td>
<td>$\frac{294.0 - 304.3}{304.3} \times 100$</td>
<td>$\approx -3.4%$</td>
</tr>
</tbody>
</table>


5.3.3 Reduced Order Output Feedback Controller

As the full order controller can perform somewhat more efficiently than the phase advance integral autopilot, it is of practical interest to investigate the course-keeping performance of the PID form reduced order system of Sec. 4.4.2.

In the simulation, weighting factors $\lambda$ and $\beta$ are chosen to be the same as the full order system, and with an identical operating environment. Results shown in Fig. 5.3-3 reveal that the minimum propulsion losses due to steering occur when $\lambda = 0.2 \times 10^{-2}$ but it is sensible, as in the case of full order controller, to choose a slightly higher $\lambda$ value, in the region between $0.5 \times 10^{-2}$ to $1.0 \times 10^{-2}$ for the sake of reducing the wear on the steering engine.

By and large, the performance of the reduced order controller, shown in Table 5.3-2, compares favourably with the phase advance integral autopilot although the full order controller is the best. Therefore, the design procedure of the reduced order controller given in Sec. 4.4 offers a useful alteration to the rules of thumb commonly used in designing the existing phase advance integral autopilot.
Fig.5.3-3: Results of the reduced order output feedback system during course-keeping
<table>
<thead>
<tr>
<th></th>
<th>heading error</th>
<th>rudder angle</th>
<th>rudder rate</th>
<th>hull drag</th>
<th>rudder drag</th>
<th>total drag</th>
<th>relative propulsion losses due to steering</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean deg</td>
<td>mean deg</td>
<td>mean [deg/sec]²</td>
<td>mean N</td>
<td>mean N</td>
<td>mean N</td>
<td></td>
</tr>
<tr>
<td>Phase advance integral system</td>
<td>0.0129</td>
<td>0.74</td>
<td>0.0013</td>
<td>77.6</td>
<td>226.7</td>
<td>304.3</td>
<td></td>
</tr>
<tr>
<td>Reduced order output feedback</td>
<td>0.0108</td>
<td>0.74</td>
<td>0.0019</td>
<td>73.1</td>
<td>224.8</td>
<td>297.9</td>
<td>$\frac{297.9-304.3}{304.3} \times 100 \approx -2.5%$</td>
</tr>
</tbody>
</table>

Table 5.3-2: Performance comparison of the reduced order output feedback integral and the phase advance integral systems during course-keeping.
5.4 THE COURSE-CHANGING AND THE DUAL MODE SYSTEMS

The performance criterion for the course-changing operation is shown in Sec.2.3 to be

\[ I = \int \left[ \psi^2 + \lambda u^2 \right] \, dt \]

so that heading error during course alteration can be minimised. However, as the external disturbances are usually of a nonzero mean character, the control resulting from such a criterion may not guarantee a zero mean heading error. Nevertheless, heading error correction will only be dealt with when the controller switches to the course-keeping mode, where some integral action is incorporated. This is because the resultant nonzero mean heading error, although depending on the mean of external disturbances, is usually small and does not warrant the use of the integrator which causes excessive heading overshoot, especially when large course alteration is involved.

5.4.1 Full Order System

As the integral control is not used during course alteration, the controller requires only five state variables, of which two are to be reconstructed. Fig.5.4-1 shows the heading responses of the course-changing control system, with values of \( \lambda \) ranging from 0.05 to 1.0, and subject to ±5 degree periodic change in heading demand. The ship has, as in other previous simulations, a nonlinear but directionally stable steering characteristic, and is operating at 15 knots in moderate seas of 3 meter significant wave height, with 120 degree wave-to-ship encounter angle. The results indicate that when a tight control, i.e. small \( \lambda \) value, is used, the overshoot in heading and the nonzero mean
heading error is relatively small. It thus seems that a smaller $\lambda$ is preferable, but there are other factors to take into account. Performance is limited by the slew rate of the hydraulic pumps, while the maximum rudder deflection decreases with increasing $\lambda$, all of which suggests a value for $\lambda$ of about 0.1, the value used in the subsequent studies.

Because the ship settles to its new heading, usually well within three minutes, it is reasonable to change the operating mode from course-changing to course-keeping after this point of time. Fig.5.4-2 shows the heading rudder angle and yaw rate responses of such a dual mode system when $\pm 5$ degree periodic demand is made. The weighting factor of the course-change operation is, as mentioned, 0.1 while for course-keeping, $\lambda = 1.0 \times 10^{-3}$ and $\beta = 0.5 \times 10^{-7}$. The system behaves excellently during the change-over and the steady state heading error vanishes after the switching-in of the course-keeping operation. When large course alteration of $\pm 15$ degrees is requested, the ship, as shown in Fig.5.4-3, responds reasonably well.

As the steering characteristic $H(\dot{\psi})$ of the ship varies considerably during operation, an autopilot system capable of tolerating the variation, possibly from stable to unstable, is desirable. However, it is realised that the designed course-changing controller produces a fairly large heading overshoot, as shown in Fig.5.4-4, when the ship is directionally unstable which, in this study, is simulated by altering the steering characteristic to

$$H(\dot{\psi}) = -30.0 \dot{\psi}^3 + 5.6 \dot{\psi}$$

Although a tight control resulting from a smaller $\lambda$ value in the performance criterion may be used to reduce the mentioned overshoot it is found that a more invariant control can be expected if the
performance criterion of the course-changing control is recast as

\[ I = \int [\psi_{\text{error}}^2 + \lambda_1 \dot{\psi}^2 + 0.1 u_1^2] \, dt \]

where a constraint is imposed on the yaw rate. The value \( \lambda_1 \) may be chosen using Merriam's guide \(^52\) by relating \( \dot{\psi} \) and \( \psi_{\text{error}} \) as

weighting factor \( x (\dot{\psi}_{\text{max}})^2 = (\psi_{\text{error}})^2 \)

If \( \psi_{\text{error}} \) is taken as 5 degrees and \( \dot{\psi}_{\text{max}} \) to be 0.20 for this heading error, then,

weighting factor \( \approx 600.0 \)

which is found to be a good estimate as the control resulting is satisfactory in this case.

Fig.5.4-4 compares the heading and yaw rate responses of various systems where the same course-keeping control as in the previous simulations is used, but with different course-changing controls. It can be seen that the system heading responses are less sensitive to variation of the steering characteristic when the yaw rate constraint is imposed.
Fig. 5.4-1: Heading demands and responses of the course-changing systems with various weighting factors.
Fig. 5.4-2: Responses of the dual mode autopilot system subject to ±5 degree course alteration demands.
Fig. 5.4-3. Responses of the dual mode autopilot system subject to $\pm 15$ degree course alteration demands.
Fig. 5.4-4: Heading and yaw rate responses of various dual mode full order autopilot systems.
5.4.2 Reduced Order Controller

For a non-integral reduced order course-changing control, there are only three state variables involved. For these three, namely yaw rate, achieved heading and course demand, only two controller coefficients need to be computed because the coefficients for course demand and achieved heading are always equal but opposite in sign. Consequently, the controller coefficients are fairly easily obtained and it has been shown\(^\text{10}\) that for achieved heading, the coefficient has a direct relationship to the weighting factor \(\lambda\), i.e.

\[
G_1 = \frac{1}{\sqrt{\lambda}}
\]

while the coefficient for yaw rate may be obtained from the expression

\[
G_2 = -\frac{1}{K}\left[-1+\sqrt{\frac{2K.T_a}{\lambda}}\right]
\]

where \(K\) and \(T_a\) are the gain and time constant of the simplified ship equation, eqn. 4.4-1.

Fig. 5.4-5 shows the responses of the reduced order course-changing system when the weighting factor \(\lambda\) values are taken as 0.05, 0.1, 0.5 and 1.0. It is obvious that the tendency to larger steady state heading error with a looser control, as in the case of full order system, remains the same.

The responses of the reduced order dual mode system are illustrated in Fig. 5.4-6 where the weighting factor \(\lambda\) and \(\beta\) during course-keeping are 0.01 and 0.5 \(\times 10^{-7}\) respectively, while \(\lambda = 0.1\) with \(\beta\) excluded for course-changing operation. As can be seen from both Figs. 5.4-6 and 5.4-7 the change-over from course alteration to course-keeping or vice versa is fairly smooth.

As for the effect of the steering characteristic, when it becomes unstable, it can be seen from Fig. 5.4-8 that the reduced order system can tolerate such a change with minor heading overshoot.
Fig. 5.4-5: Heading demands and responses of the reduced order course-changing autopilot systems.
Fig. 5.4-5: Responses of the reduced order dual mode system subject to ±5 degree course alteration commands.
Fig. 5.4-7: Responses of the reduced order dual mode system subject to ±15 degree course alteration demands.
Fig. 5.4-6: Responses of the directionally unstable ship under reduced order dual mode autopilot control.
5.5 CONSIDERATION OF THE HYDRODYNAMIC CHARACTERISTIC

5.5.1 Estimation of Hydrodynamic Coefficients

The hydrodynamic coefficients of the equations of ship motion, eqn.4.2-1, obtained from model tests are used in the controller design. However, the whole set of coefficients may not be available in practice, as full model tests are seldom conducted while full scale steering trials are expensive and time consuming. One solution considered here is to reduce the number of coefficients to manageable proportions by making some approximations and then to determine the remaining coefficients either by standard ship tests or by semi-empirical estimation using available hull data.

The motion of a ship in calm water is shown in Sec.4.2.1 to be described by

\[
\begin{bmatrix}
\dot{\psi} \\
\dot{\mathcal{r}} \\
\dot{v}
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 \\
0 & a_{22} & a_{23} \\
0 & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
\psi \\
r \\
v
\end{bmatrix}
+ \begin{bmatrix}
0 \\
a_{24} \\
a_{34}
\end{bmatrix}
\cdot \delta
\]

The relationship between the above coefficients and the individual hydrodynamic coefficients are given in Table 5.2-3 and may be simplified if the following approximations and assumptions are made:

(i) For ships with large length to beam ratio, i.e. \( \frac{L}{B} \geq 5 \),

the magnitude of \( Y_v \) is approximately that of the ship mass, or

\[
Y_v = -m
\]

(ii) The magnitude of \( N_r \) is almost as large as \( I_z \). Theoretical
calculation\textsuperscript{16,17} shows that
\[ N_r = -0.7 \text{IZ} \quad \text{for} \quad L/B = 5 \]
\[ = -0.8 \text{IZ} \quad \text{= 8.5} \]
\[ = -\text{IZ} \quad \text{= \infty} \]

In addition IZ may be expressed in terms of m and L:
\[ IZ = m(L/4)^2 \]

(iii) \( x_g \), the distance from amidships to cg of the ship, may be taken as zero.

(iv) As \( |(Y_v - m)(N_r - IZ)| >> |Y_v - N_v| \)
\[ \Delta \] of Table 5.2-3 may be written as
\[ \Delta = (Y_v - m)(N_r - IZ) \]
\[ = 4.m.IZ \]

(v) A similar approximation allows:
\[ a_{24} = \frac{(m - Y_v)N_\delta}{4.IZ.m} \]
\[ = \frac{N_\delta}{2.IZ} \]
\[ a_{22} = \frac{N_r}{2.IZ} \]
\[ a_{23} = \frac{N_v}{2.IZ} \]
\[ a_{32} = \frac{Y_r - m.U}{2.m} \quad \text{and} \]
\[ a_{34} = \frac{Y_\delta}{2.m} \]
(vi) Coefficients for eqns 4.2-1 and 4.4-1 may be approximated as

\[ \frac{1}{T_1} + \frac{1}{T_2} = -\frac{N_r}{2.1z} - \frac{Y_v}{2.m} \]

\[ \frac{K}{T_1.T_2} = -\frac{Y_v.N_\delta - Y_\delta N_v}{4m.1z} \]

\[ \frac{K.T_3}{T_1.T_2} = \frac{N_\delta}{2.1z} \]

\[ \frac{1}{T_1.T_2} = \frac{Y_v.N_r - (Y_r - mU)N_v}{4.m.1z} \]

and \( T_3 = T_1 + T_2 - T_3 \)

Further, \( T_3 \) may be obtained from \( 53 \)

\[ T_3 = \frac{2}{\left( \frac{1}{T_1} + \frac{1}{T_2} \right)} \]

The coefficients required are now reduced to \( Y_v, Y_r, Y_\delta, N_v, N_r \) and \( N_\delta \).

These derivatives can be obtained by performing some standard scale model tests such as captive and free steering tests, or estimated using the following semi-empirical formulae \( 18 \):

\[ Y_v = \frac{\rho}{2} \cdot L^2 \cdot U \cdot [-1.69 \cdot \pi \cdot \left( \frac{D}{L} \right)^2 - 0.08 \cdot \frac{V}{L^3}] \]

\[ Y_r = \frac{\rho}{2} \cdot L^3 \cdot U \cdot [1.29 \cdot \pi \cdot \left( \frac{D}{L} \right)^2 - 0.36 \cdot \frac{V}{L^3}] \]

\[ N_v = \frac{\rho}{2} \cdot L^3 \cdot U \cdot [-1.28 \cdot \pi \cdot \left( \frac{D}{L} \right)^2 + 0.04 \cdot \frac{V}{L^3}] \]

\[ N_r = \frac{\rho}{2} \cdot L^4 \cdot U \cdot [-1.88 \cdot \pi \cdot \left( \frac{D}{L} \right)^2 + 0.18 \cdot \frac{V}{L^3}] \]
\[ \gamma_\delta = \frac{\rho}{2} \cdot L^2 \cdot u^2 \cdot \frac{A_R}{L \cdot D} \cdot \frac{\partial C}{\partial \beta} \]

\[ N_\delta = -\frac{1}{2} \cdot L \cdot \gamma_\delta \]

where the slope of the lift-curve, \( \frac{\partial C}{\partial \beta} \) may be obtained from Fig. 41 of Mandel\(^{19}\), and \( A_R \) is the rudder area.

Other estimation techniques such as those using complex flow and wind theories\(^{16,17}\) are also available but in practice, the estimates do not always agree with the measured values which themselves are subject to scaling effects when model tests are used, or external disturbances if full scale sea trials are conducted.

It thus appears that an alternative approach to controller design should be considered, that needs no a priori knowledge of the ship dynamics but identifies and computes the required coefficients during operation.
5.5.2 Effects of Variation in External Disturbance Characteristics

In the course-keeping simulation study, it is found that the propulsion losses due to steering for a particular sea state may have a minimum value for a carefully chosen \( \lambda \) value. However, because the sea condition varies frequently, it is of practical interest to investigate its effects on the system performance in terms of added propulsion losses.

As the encounter angle between incident wave and ship is the dominant factor in deciding the magnitude and frequency of external disturbing force and moment acting on the hull, different angles are used in the study. Fig. 5.5-1 gives the results of the reduced order output feedback system with encounter angles of 150 and 120 degrees, representing the bow seas, and of 60 degrees for the quartering seas. The significant wave height is taken as 4.5 metres and the ship operates at 15 knots. Comparing these three sets of results, the propulsion losses are seen to increase as the angle of encounter reduces, and the \( \lambda \) value for minimum mean losses to occur in the quartering seas tends to shift towards a lower value, implying a tighter heading control but heavier rudder activity.

Therefore, to operate an efficient course-keeping control, the characteristics of the controller should be adjustable, either manually or automatically, depending on the sea condition. In the next chapter, adaptive, self-tuning control is considered, in an attempt to find a better means of course-keeping while at the same time, maintaining an effective manoeuvring system.
Fig. 5.5-1: Added propulsion losses of the reduced order system with various encounter angles during course-keeping.
6.1 INTRODUCTION

In recent years, various adaptive control strategies for autopilots have been proposed: model reference; self-tuning regulator which minimises heading error variances; adjusting heuristically the parameters of a PID autopilot to reduce the value of some loss function approximately representing the increase in propulsion losses due to steering; etc.

The model reference autopilot works well for course-changing but because the external disturbances are not explicitly incorporated in the reference model, course-keeping operation cannot be dealt with effectively. The heuristic adaptation and the self-tuning regulator methods, on the other hand, are well suited to course-keeping but not to course manoeuvring. Although modifications in terms of control schemes have been proposed in these systems to deal with both course-keeping and course-changing, it is considered, from the practical point of view, that a more unified approach capable of performing both functions using only a single adaptive strategy, with perhaps some minor modification, would be useful.

The work described in the subsequent sections is intended to achieve control on this basis.

6.2 SELF-TUNER - ITS DEVELOPMENT

The idea of combining least squares estimation with feedback control to produce an adaptive, or self-adjusting algorithm for a
plant with known structure but unknown parameters was first proposed more than two decades ago by Kalman. However, because of inadequate theory and inappropriate technology, little was done in the control field until Peteka revived and strengthened the theory and Astrom, Wittenmark et al undertook a series of excellent theoretical and applied research projects. Since then there has been widespread interest in the approach resulting in various significant developments.

Basically, a self-tuning control scheme can be represented diagrammatically as in Fig.6.2-1 where a parameter estimator, usually of recursive least square error nature is used to identify the parameters of a suitable plant model. At each sample interval, the latest parameter estimates are passed to a controller which synthesises the appropriate control efficiencies according to some prespecified design strategy. The updated controller coefficients are then used to compute the next system input.

There are three main controller design strategies available to-date. The minimum output variance control algorithm of Astrom, concerned principally with the regulation of the process output in the presence of random external disturbances, is the simplest of the three; it's algorithm coefficients are obtained directly from the parameter estimates of the least squares estimator, without any manipulation. However, it has the drawback of not being able to track a reference signal efficiently. Clarke and Gawthrop overcome this defect by incorporating explicitly system output, control variables and set point variation into a generalised cost
function and then derives the control algorithm to minimise the function. This approach, commonly known as the generalised minimum variance method, increases significantly the flexibility of the self-tuner. The pole-shifting technique of Wellstead et al.\textsuperscript{59,60} is another useful design approach which differs from the previous two in that it's control objective is to move the closed-loop system poles to prespecified positions rather than to optimise some functions. This regulator is considered more robust and can be applied with more ease to non-minimum phase systems or to systems with varying transport lags. However, tracking the reference input
is still a problem although introducing a servo-compensator in the loop may overcome the difficulty, at the cost of a significant increase in computational effort.

In this study of automatic steering systems, the control strategy of Clarke is chosen as the basis of design mainly because of its ability to track a reference signal.

6.3 SYSTEM EQUATION FOR DIGITAL CONTROLLER DESIGN

Given a continuous-time system, the object is to design a digital controller which will regulate fluctuations in the output variable while tracking the input reference signal. Therefore, it is necessary to express the continuous-time plant in a discrete form suitable for controller design:

\[
(1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{n_a} z^{-n_a}) \cdot \psi(t)
\]

\[
= z^{-k} \cdot (b_0 + b_1 z^{-1} + \ldots + b_{n_b} z^{-n_b}) \cdot \delta(t)
+ (1 + c_1 z^{-1} + \ldots + c_{n_c} z^{-n_c}) \cdot e(t)
\]

or \[
A(z^{-1}) \cdot \psi(t) = z^{-k} \cdot B(z^{-1}) \cdot \delta(t) + C(z^{-1}) \cdot e(t)
\]

where \( z^{-1} \) is the backward shift operator,

\( k \), the time delay expressed as an integral number of sample interval,

\( \{\psi(t), \delta(t)\} \), sequence of heading and rudder demand measured at the sample instants,

\( \{e(t)\} \), a zero mean white noise disturbance sequence.

In the automatic steering controller design, the ship motion is represented by the simplified model for computational efficiency as
\[
\frac{\psi(s)}{\delta(s)} = \frac{K}{s(l+sT_a)} e^{-st}
\]

in which the ship is represented by a second order equation together with a transport lag. This model can be transformed into its discrete-time equivalent, eqn.6.3-1, by various means such as numerical integration, bilinear transformation, Taylor series expansion, pole-zero mapping, etc.

Since the coefficients of the discrete-time model are identified implicitly on-line by the estimator incorporated in the self-tuner, the values of coefficients are not relevant here. The orders of the A and B polynomials, i.e. na and nb, determine the controller order and in this study, na = nb = 2.

As for the sea disturbances, a first order model together with a constant term, d, reflecting the nonzero mean nature is adopted. Consequently, the discrete-time system equation for the self-tuner design may be written as

\[
(1 + a_1z^{-1} + a_2z^{-2})y(t) = z^{-k}(b_0 + b_1z^{-1} + b_2z^{-2})u(t) + (1 + c_1z^{-1})e(t) + d
\]

where \( y(t) \equiv \psi(t) \)
\( u(t) \equiv \delta(t) \) and

\( k \), the time delay, may be taken as 1 for most practical systems.
6.4 SELF-TUNER CONTROLLER DESIGN

The design of a self-tuning system is a two stage process

(i) design a controller to meet a suitable performance criterion, assuming that plant parameters have the nominal values

(ii) tune the parameters of the control algorithm automatically to ensure the optimum performance as specified by the criterion, despite the absence of detailed knowledge of external disturbances, plant parameters and their variation with time.

6.4.1 Performance Criterion Formulation

It has been established that an efficient autopilot should have different control characteristics for course-keeping and course-changing functions, and that they may be obtained by formulating the criterion differently according to the requirements.

In the self-tuning design, the same criteria used in the quadratic optimal controller can be carried forward, except that all the variables in the criterion are now in the discrete sampled form, and their expected values are to be minimised. This is because the self-tuner can be most conveniently designed using discrete-time methods and the plant model in the controller is assumed stochastic. Consequently the performance criterion for course-changing is:

\[ I = \lim_{N \to \infty} \frac{1}{N} \mathbb{E} \left\{ \sum_{t=0}^{N} \left[ (y(t+h) - w(t))^2 + (\lambda_1 r_e(t+h))^2 + (\lambda_2 u(t))^2 \right] \right\} \]

Where \( E \) is the expectation operator

\( h \), the sampling interval

\( \lambda_1, \lambda_2 \), the weighting factors
y(t), w(t), u(t), \( r_e(t) \), the heading angle, the set course, the control variable, the rate of heading error at sample instant \( t \), respectively.

If the heading error is assumed zero mean and \( r_e(t+h) \) is approximated by

\[
\left[ y(t+h) - w(t) \right] - \left[ y(t) - w(t-h) \right]
\]

then

\[
I = \lim_{N \to \infty} \frac{1}{N} \mathbb{E} \left\{ \sum_{i=0}^{N} \left[ (y(t+h) - w(t) + \lambda_1 r_e(t+h))^2 + (\lambda_2 u(t))^2 \right] \right\}
\]

which may be written

\[
I = \lim_{N \to \infty} \frac{1}{N} \mathbb{E} \left\{ \sum_{i=0}^{N} \left[ (P (y(t+h) - w(t)))^2 + (S u(t))^2 \right] \right\}
\]

where

\[
P = 1 - z^{-1} \cdot \frac{\lambda_1}{\lambda_1 + h} \quad \text{and} \quad S = \frac{\lambda_2 h}{\lambda_1 + h}
\]

For course-keeping

\[
I = \lim_{N \to \infty} \frac{1}{N} \mathbb{E} \left\{ \sum_{i=0}^{N} \left[ r(t+h)^2 + (\lambda u(t))^2 \right] \right\}
\]

Here \( r(t+h) \) is the yaw rate and if it is taken as

\[
\frac{y(t+h) - y(t)}{h}
\]

the performance criterion becomes
\[ I = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N} \left( \frac{(P.y(t+h))^2 + (S.u(t))^2}{y(t), \ldots, u(t), \ldots, w(t), \ldots} \right) \]

where \( P = 1 - z^{-1} \) and \( S = \lambda.h \)

However, adopting these criteria in the design of a self-tuning autopilot will involve, at each sample step, a time consuming computation of the steady state solution of the Riccati equation. Hence the functions are modified to more convenient one-step criteria in which only the first sample of each variable is included with the expectations conditional upon system output and input sample acquired up to the time \( t \), i.e. for course-changing.

\[ I = E \left\{ \left[ \frac{(P.(y(t+h) - w(t)))^2}{y(t), \ldots, u(t), \ldots, w(t), \ldots} \right] \right\} \]

and for course-keeping

\[ I = E \left\{ \left[ \frac{(P.y(t+h))^2}{y(t), \ldots, u(t), \ldots} \right] \right\} \]
6.4.2 System with Known Parameters

As shown previously, the ship motion under disturbed sea conditions may be modelled as

\[ y(t) = -a_1 y(t-h) - a_2 y(t-2h) \]

\[ + b_0 u(t-h) + b_1 u(t-2h) + b_2 u(t-3h) \]

\[ + e(t) + c_1 e(t-h) \]

\[ + d \]

for the purpose of automatic steering controller design, while the performance criterion has the general form:

\[ I = E \left\{ [P.(y(t+h) - w(t))]^2 + [S.u(t)]^2 / y(t), \ldots, u(t) \ldots \right\} \]

The optimal control algorithm can then be obtained as:

\[ \frac{\partial I}{\partial u(t)} = 2P.\left[ y(t+h) - w(t) \right] \cdot b_0 + 2S_0 S u(t) \]

\[ = 0 \]

so

\[ u(t) = \frac{-b_0}{S_0 S} \cdot P.\left[ y(t+h) - w(t) \right] \]

where \( y(t+h) \) can be obtained by prediction using the past and current values of system output \( y \) and input \( u \) in some optimum fashion.
The one-step ahead predictor

The system equation may be expressed as

\[ y(t+h) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} u(t) \]

\[ + \frac{1 + c_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} e(t) \]

\[ + \frac{d}{1 + a_1 z^{-1} + a_2 z^{-2}} \]

or

\[ y(t+h) = \frac{B}{A} u(t) + z. \frac{C}{A} e(t) + \frac{d}{A} \] 6.4-5

where the noise term may be expressed as the sum of

(i) disturbances that have occurred

(ii) future disturbances

Hence by long division,

\[ z \cdot \frac{1 + c_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} e(t) = \frac{(c_1 - a_1) - z^{-1} a_2}{1 + a_1 z^{-1} + a_2 z^{-2}} e(t) + e(t+h) \]

or

\[ z. \frac{C}{A} e(t+h) = \frac{F}{A} e(t) + e(t+h) \] 6.4-5
The system equation can now be written as

\[ y(t+h) = \frac{B}{A} \cdot u(t) + \frac{F}{A} \cdot e(t) + e(t+h) + \frac{d}{A} \]

Defining the least squares prediction as

\[ y^*(t+h/t) = y(t+h) - \varepsilon(t+h) \]

with \( \varepsilon \), the prediction error,

then \[ y^*(t+h/t) = \frac{B}{A} \cdot u(t) + \frac{F}{A} \cdot e(t) + \frac{d}{A} + e(t+h) - \varepsilon(t+h) \]

Here \( \varepsilon(t+h) \) may be taken as \( e(t+h) \) because the error in the least squares prediction is entirely due to system disturbances. It follows that

\[ y^*(t+h/t) = \frac{B}{A} \cdot u(t) + \frac{F}{A} \cdot \dot{e}(t) + \frac{d}{A} \]

\[ y^*(t+h/t) \cdot [1 + z^{-1} \cdot \frac{F}{A}] = \frac{B}{A} \cdot u(t) + \frac{F}{A} \cdot y(t) + \frac{d}{A} \]

\[ y^*(t+h/t) \cdot [A + F \cdot z^{-1}] = B \cdot u(t) + F \cdot y(t) + d \]

substituting the polynomial identity of eqn. 6.4-5, i.e. \( C = A + z^{-1} \cdot F \) into the expression above, the final form of the predictor becomes

\[ y^*(t+h/t) = \frac{B}{C} \cdot u(t) + \frac{F}{C} \cdot y(t) + \frac{d}{C} \]

Substituting this expression into eqn. 6.4-3 and taking \( P = 1 + p_{1} z^{-1} \)

\[ u(t) = \frac{-b_{0}}{s_{0}} \cdot \{ P[y^*(t+h/t) - w(t)] \} \]
\[ (y^*(t+h/t)-w(t)) + p_1[y(t)-w(t-h)] \]

\[ = - \frac{b_0}{s_0^2s} \cdot \{ F.y(t) + B.u(t) - C.w(t) + d \}
+ p_1.C. [y(t) - w(t-h)] \]

\[ B.u(t) + J.y(t) + C.\gamma(t) + d = 0 \]

where \( \gamma(t) = Q. u(t) - P.w(t) \);

\[ Q = \frac{s_0^2s}{b_0}, \text{ and} \]

\[ J = F + p_1.C \]

This is the control law and it may be represented schematically
as shown in Fig. 6.4-1 where the plant in its discrete form is
also included.
Fig. 6.4-1: Structure of the known parameter control system
6.4.3 Unknown Parameter System

As the dynamics of ship and ocean waves are not known with any reasonable degree of accuracy and they vary with speed, sea states etc., some on-line identification techniques should therefore be incorporated. The self-tuning approach combines the controller design and the parameter identification in such a way that the two processes proceed simultaneously, i.e. the controller parameters B, C, J and d are identified and then the control algorithm uses these estimates at each instant as if they were the true ones.

The identification technique employed here is the recursive least square algorithm, which requires the estimated parameters and the measurable variables to be expressed in the form

\[ \phi(t) = \theta \cdot x(t) + \xi(t) \]

where \( \theta \) is the set of parameters to be estimated (in this case, B, C, J and d)

\( x \), the vector containing measurable variables \( y, u \) and \( r \)
\( \xi \), the random noise and
\( \phi \), the output function measurable or computable from the system variables

To achieve a relationship of this form, define

\[ \phi(t+h) = P [y(t+h) - w(t)] + Q \cdot u(t) \] \hspace{1cm} 6.4-10

and note the similarity to the performance criterion of eqn. 6.4-2.
If eqn. 6.4-6 i.e. 
\[ y(t+h) = y^*(t+h/t) + \epsilon(t+h) \]
is substituted into eqn. 6.4-10, \( \phi \) can be written as;
\[ \phi(t+h) = P \cdot y^*(t+h/t) - P \cdot w(t) + Q \cdot u(t) + \epsilon(t+h) \]  
6.4-11

where \( \epsilon(t+h) \) and \( \xi(t+h) \) are the same.
The optimal least squares prediction of \( \phi \) is therefore
\[ \phi^*(t+h/t) = P \cdot y^*(t+h/t) - P \cdot w(t) + Q \cdot u(t) \]  
6.4-12

where \( y^*(t+h/t) = \frac{F^C}{C} \cdot y(t) + \frac{B}{C} \cdot u(t) + \frac{1}{C} \cdot d \)

If taking \( P = 1 + P_1 z^{-1} \)
then \[ \phi^*(t+h/t) = \left( \frac{F^C}{C} + P_1 \right) \cdot y(t) + \frac{B}{C} \cdot u(t) + Q \cdot u(t) - P \cdot w(t) + \frac{d}{C} \]
or \[ C \cdot \phi^*(t+h/t) = J \cdot y(t) + B \cdot u(t) + C \cdot \gamma(t) + d \]  
6.4-13a

and because the control algorithm eqn. 6.4-9 is
\[ J \cdot y(t) + B \cdot u(t) + C \cdot \gamma(t) + d = 0 \]

therefore \[ C \cdot \phi^*(t+h/t) = 0 \]  
6.4-13b

From eqns. 6.4-11, 6.4-12 and 6.4-13a
\[ \phi(t+h) = \frac{1}{C} \cdot [ \theta \cdot x(t) ] + \xi(t+h) \]
where
\[ \theta = [j_0, j_1, \ldots, b_0, b_1, \ldots, c_0, c_1, \ldots, d]^T \]
\[ x(t) = [y(t), y(t-h), \ldots, u(t), u(t-h), \ldots, w(t), \ldots, 1] \]

If \( C = 1 \), then the form required by the least squares technique can be obtained:
\[ \phi(t+h) = \theta \cdot x(t) + \xi(t+h) \]  
6.4-14
where the unknown coefficient vector \( \theta \) can now be estimated on-line by the recursive least squares algorithm (which will be considered in the following section) allowing the control algorithm, eqn.6.4-9, to be implemented adaptively.

When \( C \) is a polynomial other than 1, i.e.
\[
C = 1 + z^{-1}c_1 + ...
\]
eqn.6.4-14 becomes;
\[
\phi(t+h) = \theta \cdot x(t) + \xi(t+h) + (1-C) \cdot \phi^*(t+h/t)
\]
where the extra term \((1-C) \cdot \phi^*(t+h/t)\) will create some biased estimate when using the least squares algorithm. However, this is not serious because the control law
\[
B \cdot u(t) + J \cdot y(t) + C \cdot y(t) + d = 0
\]
sets \( \phi^* \) to zero at each stage of calculation thus eliminating the offending term.32

Recursive least squares algorithm

The essential component of the self-tuning control is the recursive least squares algorithm 65-68:

\[
\hat{\theta}(t) = \hat{\theta}(t-h) + K(t) \cdot [\phi(t) - x(t-h)^T \hat{\theta}(t-h)]
\]

6.4-15

with \( K(t) = P(t-h) \cdot x(t-h) \cdot [1 + x(t-h)^T \cdot P(t-h) \cdot x(t-h)]^{-1} \)

and \( P(t) = P(t-h) - K(t) \cdot [1 + x(t-h)^T \cdot P(t-h) \cdot x(t-h)] \cdot K(t)^T \)

These recursive equations can be explained intuitively as follows. The new estimate \( \theta(t) \) (designated \( \hat{\theta} \)) is formed by adding to the previous estimate a correction which is proportional to (see Fig.6.4-2)

\[
\phi(t) - x(t-h)^T \cdot \hat{\theta}(t-h)
\]
The term \([x^T \hat{\theta}]\) would be the value of \(\phi\) at time \(t\) if there were no disturbances. The correction term is therefore proportional to the difference between the measured value of \(\phi(t)\) and the prediction of \(\phi(t)\) based on the previous parameters:

\[
\hat{\theta}(t) = \hat{\theta}(t-h) + \text{Gain} \cdot [\text{new measurement} - \text{predicted new measurement}]
\]

The components of the gain matrix \(K(t)\) are weighting factors indicating how the previous estimation of \(\theta\) should be weighted, and the covariance matrix \(P(t)\) can be interpreted as an indication
of the accuracy of the estimates of \( \theta \). A decrease in the size of the elements of \( P(t) \) reflects the fact that the parameter estimates are converging and becoming more steady which is desirable for a constant parameter system. However, because the characteristics of the ship may change appreciably in a short time, (for example when entering or leaving port) it is necessary to prevent the elements of \( P(t) \) from getting so small that continuous correction to the estimates of \( \theta \) becomes inefficient. Unfortunately, preventing the covariance matrix from decreasing will also increase the random estimation errors in \( \hat{\theta} \), therefore, a compromise is sought between the parameter adaptive capability which requires large values in covariance matrix elements, and the error filtering feature of the estimation algorithm, requiring small values.

One well known approach to manipulating the magnitude of the covariance matrix elements is the exponential 'forgetting factor' method, which introduces a degree of 'forgetting' into the algorithm so that emphasis is placed on more recent data. This is done by dividing \( P(t) \) at each stage by a factor \( \beta \), normally just less than unity, so that the matrix elements are increased slightly at each recursion, allowing the algorithm to forget old parameter values. In practice, a low forgetting factor (say 0.97) is chosen at the beginning of the identification control cycle to phase out poor information supplied to the estimator. The forgetting factor should then be increased gradually with time, as the initial coarse-tuning stage of the estimation passes, but maintained at a certain level low enough to allow retuning when necessary.
6.4.4 Steady State Output Error

When a ship is course-keeping, the steady state heading error must be very small, if not totally absent. The control algorithm, eqn. 6.4-9

\[ u(t) = -\frac{1}{B} \cdot [J \cdot y(t) + C \cdot y(t) + d] \]

is found to be incapable of eliminating steady state heading error although it contains a constant term \( d \) representing the nonzero mean disturbances.

To investigate the ineffectiveness of \( d \), the closed-loop relationship of the system is formulated by substituting control equation 6.4-9 into system equation 6.4-1, i.e.

\[ A \cdot y(t) = B \cdot u(t-h) + C \cdot e(t) + d \]

\[ y(t) = \frac{B \cdot P}{Q \cdot A + B \cdot P} \cdot w(t-h) + \frac{Q}{C \cdot (Q \cdot A + B \cdot P)} \cdot d + \frac{B \cdot Q \cdot C}{Q \cdot A + B \cdot P} \cdot e(t) \]

Zero constant output error requires that in the steady state, i.e. when \( z = 1 \), both

(a) \[ \frac{B \cdot P}{Q \cdot A + B \cdot P} = 1 \] and

(b) \[ \frac{Q}{C \cdot (Q \cdot A + B \cdot P)} = 0 \]

(Hereafter, steady state value of \( A, B, P \) etc shall be represented by \( A(1), B(1), P(1) \), etc)

Condition (a) can be met in the case of automatic steering control because the ship equation of motion contains an integral term which will set \( A(1) \) to zero, hence

\[ \frac{B(1) \cdot P(1)}{A(1) \cdot Q(1) + B(1) \cdot P(1)} = 1 \]
Condition (b), however, cannot be satisfied easily because Q is the weighting factor on \( u(t) \) in eqn. 6.4-10 and zero Q will usually result in an unstable closed-loop system especially when the plant is nonminimum phase (i.e. the roots of B outside unit circle) which is common in digital systems. Therefore, some artifice to get around this difficulty is proposed.

**Steady state output error elimination algorithm**

If a constant R is included in the output function \( \phi \) so that it now reads:

\[
\phi(t-h) = P[y(t+h) - w(t)] + Q.u(t) + R \tag{6.4-17}
\]

the optimal control law can then be obtained as:

\[
\frac{\partial \phi(t+h)}{\partial u(t)} = 2[P.y(t+h) - P.w(t) + Q.u(t) + R].[b_0 + q_0]
\]

then

\[
P.y(t+h) - P.w(t) + Q.u(t) + R = 0
\]

Taking \( P \) as \( 1 + z^{-1}.p_1 \) which is fairly general for most applications, and replacing \( y(t+h) \) by its predicted value (eqn.6.4-8) yields

\[
[F + p_1.C].y(t) + B.u(t) + C.[Q.u(t) - P.w(t) + R] + d = 0
\]

Defining \( J = F + p_1.C \) as before, and

\[
v(t) = Q.u(t) - P.w(t) + R
\]

then

\[
J.y(t) + B.u(t) + C.v(t) + d = 0 \tag{6.4-18}
\]

which is the new control law.
The new closed-loop system equation then becomes

\[
y(t) = \frac{B.P}{Q.A+B.P} \cdot w(t-h) + \frac{Q}{Q.A+B.P} \cdot d + \frac{Q.C+B}{Q.A+B.P} \cdot e(t) - \frac{B.R}{Q.A+B.P}
\]

and zero steady state output error requires

(a) \( \frac{B(1).P(1)}{Q(1).A(1)+B(1).P(1)} = 1 \) and

(b) \( \frac{Q(1).d}{Q(1).A(1)+B(1).P(1)} - \frac{B(1).R}{Q(1).A(1)+B(1).P(1)} = 0 \)

Condition (a) can, as shown previously, be satisfied, and condition (b) can now be fulfilled when

\( Q(1).d - B(1).R = 0 \)

which means that by substituting

\( R = \frac{Q(1)}{B(1)} \cdot d \)

into eqn. 6.4-18, the steady state output error can be made zero.

To summarise, the operating sequence is arranged as follows:

(i) sample current values of \( y \) and \( w \),

(ii) form the state vector for the recursive least squares algorithm

\[
x(t-h) = [y(t-h), y(t-2h), \ldots, u(t-h), \ldots, v(t-h), \ldots, 1]
\]

with \( v(t) = Q.u(t) - P.w(t) + R \)

and \( R \) may be assumed zero initially.
(iii) calculate the output function
\[ \phi(t) = P[y(t) - w(t-h)] + Q.u(t-h) + R \]
which is a scalar

(iv) use the recursive least squares estimation technique operating
on \( x(t-h) \) and \( \phi(t) \) above to identify \( J, B, d \) and \( C \), but
excluding \( C_0 \) which is fixed at 1.0

(v) calculate \( R \) from
\[ R = \frac{Q(l).d}{B(l)} \]
and update \( v \) which contains the new \( R \)

(vi) compute \( u(t) \) from the control law
\[ B.u(t) + J.y(t) + C.v(t) + d = 0 \]

(vii) repeat from (i) for the next value of \( t \).
CHAPTER 7

SIMULATION STUDIES OF THE SELF-TUNING AUTOPILOT

7.1 INTRODUCTION

Computer simulation is used to study course-keeping and course-
manoeuvring under various sea conditions. The dynamics of the ship,
the steering gear and the seas are modelled as described in Section 5.1.
The control law, as shown in Chapter 6, is

\[ u(t) = - \frac{1}{B} \cdot (J_y(t) + C_v(t) + d) \]

and the recursive least squares estimation technique is used to
identify all the control law coefficients except \( c_0 \) which is fixed at
its value of 1.0.

7.2 CONTROLLER CONSTANTS

Some specified constants are needed when designing the self-tuner.

(i) The sampling interval \( h \).

It is considered that a single \( h \) should be sufficient for both
course-keeping and course-changing operations, and \( h = 1 \) second
is chosen.

(ii) The weighting polynomials \( P \) and \( Q \).

The system responses depend heavily on the chosen values of \( P \) and \( Q \)
and so, detailed consideration is given to the choice in Section 7.3.
(iii) Limit on the control signal $u(t)$.

During the initial tuning-in transient, excessive control demand may occur, therefore, amplitude limits that fall within the steering actuator saturation limits are imposed on the control signal and in this study: $\pm 20$ degree limits are used.

(iv) Initial coefficient estimates $\theta(0)$.

When no previous knowledge of the coefficients is available, they may be taken as zero.

(v) Initial covariance matrix $P(0)$.

This is used in the recursive least squares identification algorithm to reflect the level of confidence in the initial coefficient estimates. When little is known a priori about these coefficients, $P(0)$ should be made a diagonal matrix with large diagonal elements, of the order of $10^3$ to $10^5$ indicating little confidence in the initial estimates and no knowledge of the cross-variance properties of the estimates.

(vi) Exponential forgetting factor $\beta$.

This constant is included in the identification algorithm to allow poor information supplied to it in the form of the guesses of initial conditions be 'forgotten'. Typically the value of 0.98 may be used at the beginning and increased gradually to unity as time progresses.
7.3 COURSE-CHANGING SIMULATION

Initially, the performance criterion of the course-changing system was chosen to minimise heading error, with a constraint on demanded rudder angle.

\[ I = E \{[y(t+h)-w(t)]^2 + [\lambda_1 u(t)]^2/y(t), \ldots, u(t), \ldots w(t), \ldots\} \]

but the resulting system response in this case is found to be oscillating and heavily under-damped. To improve the response, a longer sampling interval is sometimes recommended, but for course-changing a large sampling interval may lead to an unacceptably long delay in responding to a change of heading. Therefore, an alternative approach is used that introduces a term in the criterion to penalise excessive rate of change of heading error and dampen the yawing motion. The performance criterion is then in the form given in Section 6.4-1, i.e.

\[ I = E \{[y(t+h)-w(t)]^2 + [\lambda_1 r_e(t+h)]^2 + [\lambda_2 u(t)]^2/y(t), \ldots, u(t), \ldots, w(t), \ldots\} \]

or in the polynomial expression,

\[ I = E \{[p. (y(t+h)-w(t))]^2 + [s. u(t)]^2/y(t), \ldots w(t), \ldots, u(t), \ldots\} \]

with \( p = 1 - z^{-1} \frac{\lambda_1}{h+\lambda_1} \) and

\[ s = \frac{\lambda_2}{h+\lambda_1} \]

The related output function \( \phi \) used in the recursive identification algorithm then becomes
\[ \psi(t+h) = P \left[ y(t+h) - w(t) \right] + Q \cdot u(t) \] \hspace{1cm} 7.3-3

where

\[ Q = \left[ \frac{\lambda_2}{h + \lambda_1} \right]^2 / b_0 \]

Fig. 7.3-1 shows the heading and rudder responses of the self-tuning course-changing systems using eqns. 7.3-1 and 7.3-2 as their performance criteria. The chosen values for weighting factors are:

\[ \lambda = 0.03 \]
\[ \lambda_1 = 20.0 \] and \[ \lambda_2 = 0.10 \]

It should be noted that these values are selected in the light of simulation results given in Section 5.4, and that the Merriam guide may still be used for providing initial guess values. As for \( b_0 \) which appears in \( Q \) of eqn. 7.3-3, a value \(-1.0 \times 10^{-3}\) is assumed.

The results indicate clearly that the inclusion of the extra term to dampen the oscillating output response is essential for course-changing self-tuning autopilot design. However, because the wave disturbing force and moment are usually nonzero mean, a steady state heading error is therefore unavoidable, as explained in Section 6.4-4. Consequently, the proposed algorithm to offset the steady state error on-line is now brought in.

The performance criterion now incorporates a constant \( R \) which is adjusted adaptively,

\[ I = E\left[ \left( P \left( y(t+h) - w(t) \right) \right)^2 + \left( S \cdot u(t) \right)^2 + R^2 / y(t), \ldots, u(t), \ldots, w(t), \ldots \right] \]

and Fig. 7.3-2 shows the responses of this system.

It can be observed that the undesirable heading error is eliminated completely and that the usual excessive overshoot of the integral approach does not occur.
Fig. 7.3-1: Responses of systems without constraint (TOP) & with constraint on heading error changing rate
Fig. 7.3-2: Responses of the course-changing self-tuning autopilot system with $\lambda_1=20.0$ and $\lambda_2=0.10$
Because choice of weighting factors is important in the determination of system behaviours, it is now examined in some detail.

As shown, the polynomials $P$ and $Q$ have the relationship with $\lambda_1$ and $\lambda_2$:

$$P = 1 - z^{-1} \cdot \frac{\lambda_1}{h + \lambda_1}$$

$$Q = \left[ \frac{\lambda_2}{h + \lambda_1} \right]^2$$

These polynomials appear in the system closed-loop equation eqn.6.4-19

$$y(t) = \frac{B.P}{Q.A + B.P} \cdot w(t-h) + \frac{Q}{Q.A + B.P} \cdot u + \frac{Q.C+B}{Q.A + B.P} \cdot e(t) - \frac{B}{Q.A + B.P} \cdot R$$

in which the closed-loop poles depend on the location of the roots of the polynomial equation

$$Q.A + B.P = 0$$

Therefore, there are various possible combinations of $\lambda_1$ and $\lambda_2$ which determine the pole locations and these are now considered.

**Case I** : $\lambda_1 = \lambda_2 = 0$

This means $P = 1.0$ and $Q = 0$ which is the minimum output error variance self-tuning regulator of Astrom\textsuperscript{56}.

**Case II** : $\lambda_1 \neq 0$, $\lambda_2 = 0$

Hence $P = 1 + z^{-1} \cdot P_1$ and $Q = 0$. This is unsatisfactory because the closed-loop performance depends greatly on the choice of sampling interval $h$ and the plant dynamics.
Case III: \( \lambda_1 = 0 \), \( \lambda_2 \neq 0 \)

\[ P = 1.0 \text{ and } Q = \frac{\lambda_2^2}{b_0} \]. As shown in Fig.7.3-1, a sufficiently damped output response cannot be achieved if \( h \) is to remain very much shorter than the dominant open-loop system time constant.

Case IV: Both \( \lambda_1 \) and \( \lambda_2 \) are nonzero. Various observations may be made regarding the choice of these two parameters, and their effects on the system performance.

a. Selection of weighting factor values

To choose the values for \( \lambda_1 \) and \( \lambda_2 \), let us consider the guide suggested by Merriam again.

Taking the heading error \( = 5 \) degrees so that the system operating point is well within its linear region and the maximum rate of change for this heading error to be \( 0.25 \) degrees per second, then

\[ (\lambda_1) \cdot (0.25) = 5 \]

or

\[ \lambda_1 = 20 \]

For \( \lambda_2 \), if the maximum demanded rudder angle is set to 30 degrees, then

\[ (\lambda_2) \cdot (30) = 5 \]

\[ \lambda_2 = 0.15 \]

Obviously, this method provides only an initial estimate of the weighting factors and the derived values depend heavily on the assumed maximum yaw rate and rudder angle. Therefore, a wide
range of values for both \( \lambda_1 \) and \( \lambda_2 \) are used in the simulation for purposes of comparison.

Fig. 7.3-3 shows the heading responses of the system as the value for \( \lambda_2 \) ranges from 0.05 to 0.2 with \( \lambda_1 \) fixed at 20. i.e. \( P = 1 - 0.95 \, z^{-1} \) and \( Q \) from -0.0057 to -0.091. It can be seen that as the value of \( \lambda_2 \) increases, the heading control becomes looser with larger overshoots, and that a lower \( \lambda_2 \) value than the initial guess of 0.15 may produce a better heading response but at the expense of an increase in rudder activity.

Fig. 7.3-4 displays the results of the system with \( \lambda_1 \) varying from 5.0 to 30.0, or \( P = 1 - 0.83 \, z^{-1} \) to \( P = 1 - 0.968 \, z^{-1} \), while \( Q = -0.02 \). It is clear that some damping on the system output can be obtained by increasing \( \lambda_1 \).

b. Large heading demand

To observe how the system with \( \lambda_1 = 20.0 \) and \( \lambda_2 = 0.1 \) responds to large input demands, a periodic course alteration of \( \pm 15 \) degrees is demanded. Fig. 7.3-5 shows the system responses and it is clear that this input presents no particular problem here.

c. Coefficient estimates

Fig. 7.3-6 gives the time evolution of the control law coefficient estimates of the system whose responses are shown in Fig. 7.3-2. It is obvious that even at the beginning of the simulation during the tuning-in transient when the estimates are far from their desired values, a reasonable control is still maintained.
Fig. 7.3-3: Heading demands and responses of the STC course changing systems with $\lambda_2 = 20.0$ but different $\lambda_2$. 
Fig. 7.3-4: Heading demands and responses of the STC course changing systems with different values of $\lambda_1$ & $\lambda_2$.
Fig. 7.3-5: Responses of the self-tuning course-changing autopilot system with $\lambda_1 = 20.0$ and $\lambda_2 = 0.10$
Fig. 7.3-6: Evolution of the STC coefficients during course-changing with $\lambda_1 = 20$, $\lambda_2 = 0.1$ and $\beta = 0.96$ initially.
7.4 COURSE-KEEPING SIMULATION

It has been shown that propulsion losses due to steering are proportional to yaw rate squared and rudder deflection squared. Therefore, the performance criterion for this mode of operation may be written, as given in Section 6.4.1

I = E \{ r(t+h)^2 + \left[ \lambda u(t) \right]^2 / y(t),...,u(t),... \}

If the rate of change of achieved heading angle \( r \) is replaced by the heading error changing rate, the criterion becomes more general because it leads to the design of a servo controller rather than a regulator. Hence

I = E \{ r_e(t+h)^2 + \left[ \lambda u(t) \right]^2 / y(t),...,u(t),...,w(t),... \}

where \( r_e(t+h) \) may be approximated by

\[
\frac{y(t+h) - w(t)}{h} - \frac{y(t) - w(t-h)}{h}
\]

resulting in

I = E \{ [P.(y(t+h) - w(t))]^2 + [S.u(t)]^2 / y(t),...,u(t),...,w(t),... \}

with \( P = 1 - z^{-1} \) and \( S = \lambda / h \)

However, the above criterion, either with \( r \) or with \( r_e \), is unsatisfactory because a closed-loop pole located on the unit circle is created, leading to a poor system performance. To explain, the closed-loop relationship of the system, eqn.6.4-16 is restated,
and the roots of the polynomial equation

\[ Q.A + B.P = 0 \]

determine the poles. Because the open-loop plant equation contains an integrator, the A polynomial then includes a term \( 1 - z^{-1} \). If \( P \) is taken, as suggested, to be \( 1 - z^{-1} \), the undesirable pole then results.

One possible alternative means of forming the criterion to avoid such a pole is to use eqn. 2.2-3 as the formulation basis. As mentioned in Section 2.2, yaw frequency \( \omega \) and yaw angle \( \psi \) may replace yaw rate \( \dot{\psi} \) of eqn. 2.2-2, if the yaw motion is approximately sinusoidal; hence, the criterion becomes

\[ I = E( [y(t+h) - w(t)]^2 + [u(t)]^2/y(t),...,u(t),...,w(t),...) \]

An extra term to restrict the heading error changing rate may also be included, allowing a greater flexibility in the design for good control, and, because the control law resulting from such a criterion is identical in form to that for course-changing, it therefore has the added advantage of reducing the complexity of the controller implementation. Consequently, the criterion is now:

\[ I = E( [y(t+h) - w(t)]^2 + [\lambda_1 \dot{\psi}(t+h)]^2 + [\lambda_2 u(t)]^2/y(t),...,w(t),...,u(t),...) \]

or \( I = E( [P(y(t+h) - w(t))]^2 + [S u(t)]^2/y(t),...,w(t),...,u(t),...) \)

with \( P = 1 + z^{-1} \cdot \frac{-\lambda_1}{h+\lambda_1} \) and

\[ S = \frac{\lambda_2}{h+\lambda_1} \]
It should be noted that the selection of weighting factor values for course-keeping will differ from that for course-manoeuvring in order to achieve maximum steering efficiency.

In addition, as zero mean steady state heading error is essential during course-keeping, the performance criterion above is to include an adaptive term $R$, so that the controller can offset the possible steady state error caused by nonzero mean external disturbances, as explained in Section 6.4.4.

Figure 7.4-1 gives the simulation results showing the increases in propulsion losses due to steering when the performance criterion

$$I = E[(P.(y(t+h)-w(t))]^2+[S.u(t)]^2+R^2/y(t),...,w(t),...]$$

is used. The weighting factor $\lambda_1$ is fixed at 30.0 and $\lambda_2$ varies from 0.1 to 0.5, implying that $P = 1-0.968 z^{-1}$ and $S$ ranges from 0.003 to 0.016. The external disturbances acting on the ship hull are simulated as before, by the nonzero mean wave force and moment which are generated using 4.5m significant wave height, with 120 degree wave-to-ship encounter angle and 15 knot cruising speed.

It is obvious from the results that when $S$ is smaller than 0.006, the control is too tight for course-keeping purposes, because the rudder drag and the rate of change of rudder movement are relatively high. However, when $S$ is larger than 0.015, the yawing motion and the mean heading error become unacceptably large. Therefore, when $P = 1-0.968 z^{-1}$ a suitable value for $S$ is 0.01 and this is used in future studies.

Fig.7.4-2 shows the results when $p_1$, the second term of $P$ polynomial, varies from 0.90 to 0.98 with $S = 0.01$, and it indicates that the variation of $p_1$ in general, has smaller effects on system behaviours than the variation of $S$. 
Fig. 7.4-1: Results of the STC course-keeping system with P fixed at $1 - 0.968 \lambda_2^{-1}$, and S from 0.325 to 1.61.
Fig. 7.4-2: Results of the STC course-keeping system with $S$ fixed at 0.00975, and $\lambda_1$ from 10 to 40
Table 7.4-1 compares the results of the proposed self-tuner system with $P = 1 - 0.968 z^{-1}$, $S = 0.01$, to those of the phase-advance integral and the full order output feedback systems. It can be seen that the rate of change of rudder motion and the rudder drag of the self-tuner are the smallest but its hull drag is marginally the highest. As a result, the total drag is the lowest among them. In addition, as the wear and tear on the steering gear are proportional to the speed squared of the rudder movement, it is likely that the self-tuner causes the least wear.
<table>
<thead>
<tr>
<th></th>
<th>heading error</th>
<th>rudder angle</th>
<th>rudder rate</th>
<th>hull drag</th>
<th>rudder drag</th>
<th>total drag</th>
<th>relative propulsion losses due to steering</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean deg</td>
<td>mean deg^2</td>
<td>mean deg</td>
<td>mean [deg/sec]^2</td>
<td>mean N</td>
<td>mean N</td>
<td></td>
</tr>
<tr>
<td>Phase advance integral system</td>
<td>0.0129</td>
<td>0.0274</td>
<td>-0.747</td>
<td>0.0013</td>
<td>77.6</td>
<td>226.7</td>
<td>304.3</td>
</tr>
<tr>
<td>Full order output feedback</td>
<td>0.0371</td>
<td>0.0650</td>
<td>-0.744</td>
<td>0.0010</td>
<td>75.1</td>
<td>218.9</td>
<td>294.0</td>
</tr>
<tr>
<td>integral system</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-tuning system</td>
<td>0.0357</td>
<td>0.0603</td>
<td>-0.745</td>
<td>0.0004</td>
<td>79.5</td>
<td>211.7</td>
<td>291.1</td>
</tr>
</tbody>
</table>

Table 7.4-1: Performance comparison of the self-tuning, the phase advance integral and the output feedback integral systems during course-keeping.
7.5 DUAL MODE SYSTEM

The means of changing operating mode from course-manoeuvring to course-keeping, or vice-versa, in the self-tuning system is to set values of P and S (or Q) for the required mode. However, it is found from the simulation that when assigning new values for both P and S simultaneously on-line, there is a transient effect on the system responses which is unacceptably large for any practical purpose, but satisfactory responses can be obtained if only S, the more dominant parameter of the two, is adjusted while P remains untouched.

In the present study, P is therefore fixed at $1 - 0.968 \ z^{-1}$ by setting $\lambda_1 = 30$ while S has a value of 0.0048 for tight manoeuvring control and 0.008 for course-keeping operation, corresponding to $\lambda_2 = 0.015$ and 0.25.

Fig. 7.5-1 shows the system responses of the dual mode self-tuner, operating in disturbed seas of 4.5 m wave height and 120 degrees encounter angle, as in previous studies. The change-over from course-keeping to manoeuvring occurs immediately after application of the turn, and then switches back to keeping course three minutes later. As can be seen, the change-over presents no problem, and when course-changing, there is tighter control on heading angle than when course-keeping. Note that rudder movement is significantly more frequent during tight control.

The results therefore indicate clearly that the idea of dual mode control can be implemented efficiently.
Fig. 7.5-1: Dual mode system with $\lambda_1$ fixed at 30, $\lambda_2 = .15$ for manoeuvring, but for course-keeping $\lambda_2 = .25$
7.6 SPEED ADAPTIVITY

Speed variations have a significant influence on ship dynamics which, in the simulation, are accounted for approximately by making the hydrodynamic coefficients of the ship equation:

\[ \ddot{\psi} + \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \dot{\psi} + \frac{K}{T_1 T_2} \psi = \frac{K}{T_1 T_2} \left( T_3 \delta + \delta \right) + D_1 \dot{Y}_w + D_2 Y_w + D_3 \dot{N}_w + D_4 N_w \]

speed dependent. The wave drift force \( Y_w \) and yaw moment \( N_w \) acting on the hull are also functions of speed.

Because the steering behaviour changes with speed, the control characteristics of the autopilot designed for a nominal speed must vary in order to maintain a reasonably speed invariant performance. The self-tuner, in this respect, can be made speed adaptive by having \( Q \) proportional to speed squared, i.e.

\[ Q = \left[ \frac{\lambda_2}{h \lambda_1} \cdot \frac{U}{U_0} \right]^2 / D_0 \]

Here \( U_0 \) is the nominal speed while \( U \) is the current operating speed and it may be, in practice, obtained from the speed log.

Figs. 7.6-la and 7.6-lb show the heading and rudder responses of the dual mode system operating at full, half-full and quarter-full nominal speeds with speed dependent \( Q \). The increase in sluggishness of the ship response due to decrease in speed is compensated for efficiently by the decrease in \( Q \), as can be seen from the relatively speed independent heading responses. However, the rudder deflection increases as the speed reduces, therefore, it becomes necessary to change the demanded rudder angle limits to \( \pm 30 \) degrees when at quarter-full nominal speed so that a good control can still be maintained.
Fig. 7.6-1c: Heading responses of the duct mode system at full, half-full & quarter-full nominal speeds. The weighting factors remain the same for these three different operating speeds.
Fig. 7.6-1b: Rudder responses of the dual mode system at full, half-full & quarter-full nominal speeds.
To avoid the need for a speed log, it is possible to utilise the estimated coefficients of the self-tuner to make $Q$ speed dependent. One of the estimated polynomials is $B$ whose coefficients $b_0$, $b_1$ and $b_2$ are affected strongly by ship dynamics. If the ship motion is described by

$$\ddot{\psi} + \frac{1}{T_a} \dot{\psi} = \frac{K}{T_a} \delta(t-t_0)$$

then these coefficients are directly proportional to $K/T_a$. Since $K$ is proportional to speed and $T_a$ is inversely proportional to speed, then $K/T_a$ is a function of speed squared and the $B$ coefficients are also proportional to speed squared. Thus, the expression

$$\hat{B}(\xi)/B(1)$$

where $B(1)$ is the steady-state value at nominal speed, and

$$\hat{B}(\xi)$$

is the sum of estimated coefficients $b_0$, $b_1$ and $b_2$ at operating speed,

is a function of speed squared and $Q$ can now be modified to

$$Q' = Q \cdot \frac{\hat{B}(\xi)}{B(1)}$$

where $Q$ is the nominal speed value.

Figs. 7.6-2a and 7.6-2b show the heading and rudder responses of the dual mode system operating at full, half-full and quarter-full nominal speeds with the incorporation of the modified $Q$ in the controller,
and Fig. 7.6-2c gives the time evolution of the $B(\xi)$ at these three speeds. Because the estimates are far from their final steady values during the initial tuning-in transient, the nominal $Q$ values are used during the first three minutes. Using the steady-state values shown in Fig. 7.6-2c, $B(\xi)$ ratios of full speed to half-full, and half-full speed to quarter-full, are found to be in the region of $(0.56)^2$ which is slightly larger than the expected value of $(0.50)^2$. Nevertheless, the shown heading responses are still fairly speed invariant, indicating therefore that the proposed method may be used for this purpose.
Fig. 7.6-2a: Full, half-full and quarter-full nominal speed heading responses with speed dependent $Q$, but the weighting factors remain the same for these three different speeds.
Fig. 7.5–2b: Full, half-full and quarter-full nominal speed rudder responses with speed dependent 0.
Fig. 7.6-2c: Evolution of the \( \hat{B}(\Sigma) \) of the self-tuning system at full, half-full & quarter full nominal speeds.
VARIATION OF STEERING CHARACTERISTICS

It has been found that the influence of speed on ship handling can be largely eliminated by making Q vary according to speed. When analysing the effect of changes in steering characteristics due to depth of water, loading of the ship etc., the forward speed is therefore fixed at the nominal 15 knots first, but the expression for speed dependent Q using control coefficient estimates is still included.

Fig. 7.7-1 displays the responses of the self-tuner system steering characteristics $H(\psi)$ taken as

$$H(\psi) = -30 \dot{\psi}^3 + 5.6 \dot{\psi}$$

representing a directionally unstable ship. The mode of operation changes from course-keeping to course-changing immediately after a demand is made, and then back to course-keeping three minutes later. It can be seen that with the same weighting polynomial P and Q as those of the directionally stable ship, the heading response is quite acceptable, indicating the tolerance of the self-tuner to the nonlinear and unstable variations of steering characteristics during manoeuvring. Further investigation of its operating performance at half-full nominal speed is carried out and Fig. 7.7-2 shows the results where the heading response is fairly invariant to both speed and steering characteristic changes.
\[ \lambda_2 = 0.15 \] for manoeuvring
\[ \lambda_1 = 0.25 \] for course-keeping
\[ \lambda_1 = 30.0 \] for both functions

Fig. 7.7-1: Responses of the directionally unstable ship at full nominal speed under self-tuning control.
\[ \lambda_2 = 0.15 \text{ for manoeuvring} \]
\[ \lambda_3 = 0.25 \text{ for course-keeping} \]
\[ \lambda_1 = 30.0 \text{ for both functions} \]

Fig. 7.7-2: Responses of the directionally unstable ship operating at half-full nominal speed.
7.8 EFFECT OF ENCOUNTER ANGLE

Since the wave-to-ship encounter angle is the important factor in determining the magnitude and frequency of wave drift force and yaw moment exerted on the ship hull, different encounter angles are now used to simulate various operating environments. The cruising speed of the ship is fixed at the nominal 15 knots and the proposed expression for $Q$, i.e.

$$Q = \frac{\lambda_2^2}{h+\lambda_1} \cdot \frac{B(\Sigma)}{B(L)}$$

is still incorporated, as in previous sections.

Fig.7.8-1 gives the simulation results when the encounter angles are 150 and 120 degrees, representing the bow seas, and 60 degrees for the quartering seas. The minimum mean added propulsion losses due to steering are seen to occur when $\lambda_2$ is in the region of 0.2 for bow waves, and 0.15 for quartering seas. Judging from the narrow region of 'optimal' $\lambda_2$ values over this range of encounter angles, it is considered that a fixed but properly chosen value of $\lambda_2$ may be used, although it would be possible to vary $\lambda_2$ within a specified interval of, say [0.15 0.25], systematically on-line.
Fig. 7.8-1: Added propulsion losses of the self-tuning system with various wave-to-ship encounter angles during course-keeping
In an attempt to optimise ship operations from the point of view of economy and safety, a control scheme using linear quadratic optimal control is studied. This approach departs from the conventional in that empirically based knowledge in determining controller coefficients is no longer relied on. Instead, a more systematic means is used, firmly linked to the performance criterion specified by the operating personnel. It thus offers a convenient way of mode changing between stringent manoeuvre and less responsive course-keeping by altering, within certain constraints, the performance criterion.

In the course-keeping mode, the criterion can be formulated so as to minimise the yaw rate squared, the rudder demand squared and the heading error integral squared. The first two quantities representing approximately the added propulsion losses due to steering, contributed by hull and rudder drags respectively. The third quantity of heading error integral is needed to produce an integral control for steady state heading error elimination. This error is caused by nonzero mean external disturbances. In the course-changing mode, the heading error squared is to be minimised with a constraint on demanded rudder squared.

Because the directional stability of the ship is influenced by factors such as trim and loading of ship, depth of water etc., it is beneficial to include an extra term restricting yaw rate squared in the course-changing criterion so as to provide sufficient damping for ships whose stability may change from stable to unstable during operation.
Further development of the quadratic controller from its full order to reduced order by utilising the simplified ship model in the controller design can produce a PID configuration, providing an attractive means of deriving the coefficients of the existing autopilots whose structure is mostly in PID form.

However, quadratic control design requires a priori knowledge of ship dynamics and must be obtained from time-consuming model tests or sea trials, or approximated from unreliable semi-empirical formulae. Therefore, we study the technique of self-tuning, which has the advantages that it can identify the required control parameters and provide optimum control simultaneously.

In self-tuning controller design, similar performance criteria to those in quadratic control can be used but with some modification for mathematical convenience. The mode changing presents no problem and propulsion losses during course-keeping are found to be less sensitive to variation of disturbance dynamics. The proposed algorithm for steady state heading error elimination based on estimated control coefficients is shown to be efficient in terms of both complete error elimination and minimum heading overshoot. The speed variation effect on the ship, causing excessive heading overshoot, can be dealt with successfully by varying the appropriate weighting factor in the performance criterion according to speed. The necessary information on operating speed can either be obtained from a speed log or be derived on-line from certain speed dependent control coefficient estimates.

It has been demonstrated that a self-tuning strategy can be used successfully in the design of an efficient autopilot to satisfy modern steering requirements.
Nevertheless there are areas of work to be pursued further. It is recognised that the control scheme should, for practical convenience, be realisable using current 8 or 16 bit microprocessor systems. The standard recursive least squares algorithm used in identifying control parameters requires a long word length of 24 bit plus exponent for number representation in order to avoid possible numerical difficulty caused by rounding error. It is therefore necessary to modify the algorithm using perhaps the square-root algorithm $68,69$, or the U-D (Upper triangular Diagonal matrix) covariance factorisation $71,72$ so that a lower precision number representation can be used.

The sampling interval of one second used in the simulation study should, in general, be well outside the execution time required for the whole self-tuning algorithm using an 8 or 16 bit digital system, but it is of practical interest to explore the minimum execution time, and to relate the system performance to measurement resolution, word length and sampling interval.

In the modern ship, the tendency is to design a digital steering system of which the autopilot is only a part and therefore, the hardware and software of the self-tuner should be designed in a manner compatible with the other sub-systems. For the least squares algorithm, which is invariant from ship to ship and consumes a lot of processor time, a low level language would be appropriate, while the remaining control activities, which must be adapted to suit the individual ship, warrant the use of a high level language which also facilitates software maintenance.
As the Mariner cargo ship is the only vessel used in the work reported, it would be useful to include different types of ships in future simulations to build up a more general picture of ship control problems. Finally, sea trials covering many different vessels will undoubtedly be necessary to provide a full evaluation of the autopilot performance.
REFERENCES


29. Proceeding of the IEEE, special issue on Adaptive Systems, 64, No.8, August 1976.


