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Modeling & experimentation of vibration transmission through an angled joint

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ABSTRACT
Analysis of vibration transmission and reflection in beam-like engineering structures requires better predictive models in order to further optimize structural behavior. Various studies have used flexural and longitudinal structural wave motion to model the vibrational response of angled junctions in beam-like structures, in order to better understand the transmission and reflection properties. This study considers a model of variable angle joint which joins two semi-infinite rectangular cross-section beams. In a novel approach, the model allows for the joint to expand in size as the angle between the two beams is increased. Thus, making the model a good representative in wide range of angles. Predicted results are compared to an existing model of a joint between two semi-infinite beams where the joint was modeled as a fixed inertia regardless of the angle between the beams, thus limiting its physical representation, especially at the extremes of angle. Results from experimentation were also compared to the modeling, which is in good agreement for the range of angles investigated. Optimum angles for minimum vibrational power transmission are identified in terms of the frequency of the incoming flexural or longitudinal wave. Analysis of the effect of changing the joint material properties is also reported.

Keywords: Angled \& Variable Joint, Vibration Power Measurement

1. INTRODUCTION
Recent progress and trends in aeronautical and automotive vehicles have shown an exponential growth of demands for power, speed and reliability. This has also introduced several risks and attempts to reduce those risks, requires investigation and research. As an example, aircraft wing structures experience a large variation in their frictional and stiffness parameters as function of velocity. These changes occur in proportion to the aircraft velocity and velocity invariant characteristics which are entirely dependent upon the structural properties of wing assembly, in terms of its mass-inertia, natural damping and stiffness [1-2]. This has drawn interest to predict these vibration characteristics before the damage occurs. It is also observed and understood that any of these built-up structures needs a model of a joint for a complete assembly, and a vibration level study for the damping and transmission coefficients at the joints are still a great area of unknown. Thus, this paper will look at an advanced joint model in order to predict the vibration level and thus better

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understand the transmission and reflection. A comparison of the predicted results with experimental beam structures is also presented.

2. LITERATURE

An initial understanding of reflection and transmission of flexural and longitudinal wave propagation at a joint were derived and illustrated in Cremer et al [3], and further investigated through various approaches by Doyle et al [4] and Mace [5]. J L Horner, [6-7] was concerned with the prediction of vibrational power transmission through bends or joints in beam-like structures, impinged on by either flexural or longitudinal waves. Models were developed which determined the wave type which carried most power in each section of the system. The work estimated vibration transmission of an angled joint for various combinations of beam arrangement and ratio dimensions using a fixed mass in the joint. Predictions of vibrational power transmission were made for beam-like structures with jointed beam, as well as estimating wave transmission & reflection. Measurement of vibrational power flow emphasized the approach for its clear physical interpretations and the advantage of giving source and direction to the transmission paths. Transmitted power in curved beams was developed in relation to time-average power transmission to the travelling wave amplitudes [8-9].

Vibrational energy transmission at low frequency was investigated theoretically and experimentally for an L-junction of square rods where thin rod theory was employed. Bending vibration in the plane of that system was coupled with compressional waves and the bending vibration normal to the plane of the system was coupled to torsional waves [10]. Lower to mid-frequency were considered from the understanding of the limitation in Euler-Bernoulli beam theory that neglects rotary inertia and shear distortion of the beam. C.MeI et al. [11] considers Timoshenko’s beam theory for wave reflection and transmission in beam analysis. The main consideration is on discontinuities caused by general point supports, boundaries and changes in cross section for straight or 90° degree bent beams.

Calculation of reflection and transmission coefficients of joints using a hybrid finite element/wave and finite element approach provides more recent knowledge to better predict wave behavior at a joint and the force response of the whole structure [12]. The work presented a fixed joint for analysis with consideration of finite dimensions and reflection at the boundary. Several orientations of the structures are investigated in parallel or perpendicular positions such as L-frame and lap-jointed beams. Other angles of beam orientations are still a great area of unknown for investigations and understanding.

3. THEORY

3.1 Beam model

Flexural and Longitudinal waves are considered in this analysis of vibrational power transmission. Each of these two waves types are represented as follows [3] for time average power in longitudinal($P_l$) and flexural($P_f$):

$$
\langle P_l \rangle = \frac{1}{T} \int_0^T X_l \, dt = \left[ \frac{0.5EAk_{l}v_{l}^2}{\omega} \right], \\
\langle P_f \rangle = \frac{1}{T} \int_0^T X_f \, dt = \left[ \frac{EIk_{f}v_{f}^2}{A_f \omega} \right],
$$

Where $X_l$ is rate of work done for longitudinal and $X_f$ is rate of work done for flexural waves $E$ is Young’s Modulus, $A$ is cross section of the beam, $I$ is the rotational inertia at cross section midpoint, $k_l$ and $k_f$ are the wave numbers for longitudinal and flexural waves, $A_l$ and $A_f$are the wave amplitudes of longitudinal and flexural waves, and $\omega$ is the radial frequency.

A semi-infinite structure is considered which allows for a joint to be included. The impinging flexural waves will result in reflected and transmitted near field and propagating flexural waves, while impinging compressive waves will result in reflected and transmitted compressive waves. Figure 1 shows both the longitudinal $U(x,t)$ and flexural $W(x,t)$ motion of displacement. Assume a beam is bent through an angle $\theta$, and both incident side of displacement $W_c(x,t)$ and $U_c(x,t)$ are denoted by suffix minus symbol. As for the transmitted side of the bend, $W_t(x,t)$ and $U_t(x,t)$,
are denoted with suffix plus symbol. Relationship of $\psi = x \cos \theta$, is for relative displacement in transmitted from incident side.

![Figure 1 – Illustration of joint model and notations](image)

To further illustrate the notation and model [4-5] as in Figure 1, ‘A’ represents waves on the incident side and ‘B’ for the transmitted side. Each odd suffix number is representing waves travelling to the left, with an even number for waves travelling to the right side. Here $A_3, A_4, B_4$, are the flexural travelling waves amplitudes, $A_1, B_2$, are the near field flexural waves amplitudes and $A_7, A_8, B_8$, are the longitudinal travelling waves amplitudes. The solution will later consider $A_1, A_4$ as input waves amplitudes (both or either one) to the governing equations.

### 3.2 Rigid joint

In the analysis of this joint, torsional vibration was ignored and only planar excitations are considered. The axial force, shear force and bending moments are considered for the summation of forces and the continuity of the system.

![Figure 2 – Model of rigid joint and notations](image)

The length ‘L’ is the thickness and ‘w’ is the width of joint as well as beam into the figure plane a parameter characterizing the size of the joint with a rigid mass and inertia. This arrangement shown in Figure 2 of joint model illustrates that the joint represented a rigid quarter of a cylinder which remains unchanged regardless of decrease or increment of angle $\theta$. Such an assumption implies that it is not physically representative of large angles as the transmitting side of the beam would effectively be “folded into” the incident side.
3.3 Variable joint

The variable joint is a sector of rigid circular cylinder lying between the beams. Rotation of the joint occurs about the center point of its mid-line as for elements in Euler-Bernoulli beam theory. Unlike the rigid joint, this joint would consider variable sizes of the joint that are proportional to the angle between the two beams. This would represent a more realistic joint, where the mass of the joint increases with the angle of the jointed beam as in Figure 3. The rigid joint previously, could not consider the physical orientation at 0° where no joint mathematically exists, or to the extreme at 180° where the arm 2 would be physically coincided with arm 1.

Three equations of continuity are representing displacement in axial, perpendicular and angular distances were derived using Euler-Bernoulli beam theory:-

\[
\begin{align*}
U_+ &= U_+ \cos \theta - W_+ \sin \theta + \frac{L_0}{2} [1 - \cos \theta] \\
W_- &= U_+ \sin \theta + W_+ \cos \theta - \frac{L_0}{2} \sin \theta \\
\frac{\partial W_-}{\partial x} &= \frac{\partial W_+}{\partial \psi} = 0
\end{align*}
\]

Those equations above are similar solution derived for rigid joint in literature [6-7], but for this advanced model, it assures definite limit check at every angle of \(\theta = 0, \pi/2, \pi\).

\[I_N = \frac{mL^2}{12}\]

Figure 3 – Variable joint model considering a change of mass and inertia

Figure 4 – Forces and moments in the variable joint model
Referring to Figure 4, there were also additional terms introduced in the equilibrium equation for compressive and shear force, as well as a dynamic relationship of joint increments observed in the bending moment equation:

\[
\begin{align*}
\dot{m} = F_x \cos \theta + V_x \sin \theta - F_-
\end{align*}
\]  
(6)

\[
\begin{align*}
\dot{m} = V_+ + F_x \sin \theta - V_+ \cos \theta
\end{align*}
\]  
(7)

\[
\begin{align*}
\dot{\phi} = M_+ - M_- - V_+ e - V_- e - F_+ f + F_- f
\end{align*}
\]  
(8)

Where \( m \) is the variable joint mass, \( \dot{m} \) and \( \dot{m} \) are the corresponding accelerations for longitudinal and flexural waves for the joint mass, \( V \) is the shear force, \( F \) is the compressive force, and \( M \) is the moment. Lengths \( e \) and \( f \) are perpendicular to the shear and compressive forces about the geometrical center of the joint beam. The accelerations and forces around the variable joint mass are acted in proportion to the change in angle. These are not considered in the rigid joint model. Solution involves 6X6 matrix from all six equations above, (for time averaged vibrational power of flexural and longitudinal vibrations) as well as the cross coupling terms between wave types. MATLAB was used for code development.

3.4 Comparison of predicted results between the rigid and variable joints

Results were compared at 1500Hz using Perspex beam material properties (\( \rho = 1170 \text{ kg/m}^3 \) and \( E = 1.75 \times 10^9 \text{ N/m}^2 \)) with 100mm x 20mm beam cross sections. The same material will later be used in experimental measurements. The percentage power normalization is the ratio of reflected and transmitted power to the power of the impinging wave. A unity value of 1 was used for both impinging amplitude for flexural and longitudinal. For the incident side, the cross coupling waves of beam 1 were defined as,

- incident flexural wave with flexural reflected wave, denoted with FF
- incident flexural wave with longitudinal reflected wave, denoted with FL
- incident longitudinal wave with longitudinal reflected wave, denoted with LL
- incident longitudinal wave with flexural reflected wave, denoted with LF

Similarly with beam 2 for transmitted waves.

Result in Figure 5 show significant differences for longitudinal power LL at the extreme of angle for both incident and transmitted sides. The variable joint predicts more reflected power in beam 1 and reduced transmitted power for beam 2 from angles 110° to 180°. The flexural power of the variable joint at 0° angles (no joint exist) shows no reflected power in beam 1 and 100% transmitted to beam 2. This is physically agreed with the real straight beam. The rigid joint observed some reflected power since the mass is remains in the governing equations.

![Figure 5 – Comparison between a rigid joint (dotted lines) and a variable joint (bold lines) for incident and transmitted sides of the beam. (Red-FF, Blue-LL, Green-FL&LF)](image)

The difference at extreme angles was contributed by true cross coupling effects from both FL and LF due to the increase of mass with increasing angle of the joint. The inertia \( I \), used in the variable joint model was implemented by considering a geometrical centroid which gives better realistic
result compared to fixed inertia in the rigid joint. More power observed in variable joint due to the cross coupling power which reduces pure flexural and increases longitudinal power in beam 1, and it is reversely at beam 2 for both longitudinal and flexural power. A similar pattern in the vibrational response was observed at other frequencies.

### 3.5 Change of joint material properties

Using the variable angle codes developed using MATLAB, further investigation of changing the joint material properties would benefit additional knowledge on the vibration power transmitted and reflected at the joint. A range of densities from 910 kg/m³ (polyethylene), 1170 kg/m³ (Perspex beam material), 2700 kg/m³ (aluminium) and 4500 kg/m³ (titanium) were taken as comparisons at 1500Hz in Figure 6. Percentage power were plotted for total flexural (FF+LF) and total longitudinal (LL+FL), to have better comparison in terms to each longitudinal and flexural input as in Figure 6.

![Figure 6](image)

Figure 6 – Percentage power changes of (FF+LF) & (LL+FL) for joint material density change (Low density [thinner lines] to high density [thicker lines]).

Lower density of joint gives longitudinal waves effectively increase in power (reflected) after 100° angle for incident side. Hence more power transmitted (in beam 2) longitudinally with increase of joint density at the extreme of angle. Flexural waves have similar effect for angles 145° to 180° and between 30° to 90° of angle, but reverse effect observed in range 100° to 145° of angle. There were also equal power level observed regardless of joint density change at about 95° for flexural and at about 110° for longitudinal.

### 3.6 Modeling of Force excitation into the system

Further analysis is required to examine the model with load or force which represents a real application in engineering as in Figure 7.

![Figure 7](image)

Figure 7 – Excitation of force into collinear beam with variable angle joint
The solution was derived for 12x12 matrixes where 6 additional terms of continuity, forces and moment equations are considered at the excitation position, \(x_f\). Predicted results are illustrated together with experimental results in Figures 9 and 10.

4. EXPERIMENTAL APPARATUS AND MEASUREMENT METHOD

This research experiment will look at the behavior of vibration power transmitted and reflected comparing to the mathematical model derived earlier. Angled joints have been fabricated as illustrated in Figure 8. The 2-beam system was taken for the measurement on each angle of 10°, 40°, 60°, 90°, 120°, 140°, 160° and 175°. The support conditions at the joint should be well defined if the results of the dynamic measurements are to reflect the properties of the structure without undue influence from the support. As the nature of this research requires observation of transmission and reflection of vibration power, it is understood that the best applied support would be a free boundary condition where the two beam jointed structure of the test set-up will be suspended. Both ends of the beam system were buried in sand boxes to comply with the assumption of semi-infinite conditions, where both ends are treated as infinite and thus only waves around the joint and excitation location will be considered for analysis.

A twelve channel analyzer, a shaker, power amplifier, a force transducer connected to digital amplifier, and an accelerometer with three sets of paired transducers (30mm distance apart) were used to measure transmitted power at regions of \(\gamma\) (between excitation and left sand box), \(\alpha\) (between excitation and joint) and \(\beta\) (between joint and right sand box) as with subscripts in Figure 7 earlier. The method of transmitted power measurement from Verheij [12] and the wave reflection contribution by Linjama et.al [13] were used. Both ends of the beam were buried for 0.35m, leaving about 1.5m length exposed for each beam. Joints were assembled using chemically mixed glue for a firm adhesion between the beam and both sides of the joint.

Initial measurements for mobility and estimation of damping value were carried out from one of the beam. A shaker was placed at end of the beam with accelerometer and force transducer, the other end buried in sand box for semi-infinite boundary condition. The value obtained was used to simulate the modeling algorithm. A continuous straight beam were also tested by joining two beam above without any joint to represents result of power for the basis of comparing damping effect by the glue material. The lower frequency limit was around 250Hz and upper frequency limit at around 2250Hz due to anechoic termination and transducer’s distance, respectively.
5. RESULTS

Pure flexural power excitation only was considered for both the model and the experiment. Ratio of power result was corresponding to earlier variable joint model.

![Graph of Ratio Power over Input at 0 degree joint (straight beam)](image)

Figure 9 – Predicted Model (bold lines) vs Experiment (thin lines) results at 0° degree (no joint), (blue-power at beta, red-power at alpha, green-power at gamma, black-sum of beta+gamma).

A damping ratio of 0.06, obtained from the frequency bandwidth method, was used throughout the model simulation and can be seen in the ratio power plot for 0 degree (straight) beam of beta section in Figure 9. The ratio power for beta+gamma or beta+alpha will be at 1 throughout the frequency range if an ideal beam with no damping was considered in the model simulation. Input power excited on point $x_f$ (between gamma and alpha section) as in Figure 7, was only for the Flexural direction. The experimental measurements picks up total flexural vibrations at each section, however for the modeling, it considers both flexural and cross coupling effects.

Comparisons were made between the frequency ranges mentioned in section 4. Lower angles (below 90 degree) were examined to ensure good correspondence between the model and experiment results. Higher resonances were observed in all experiments due to the imperfection of sand box performance. Nevertheless, the power level, noted good agreement for range of frequency investigated. The predicted power ratio for alpha (red) is identical to the predicted power in gamma (green) at this angle.

Figure 10 compares the predicted and measurement results for a 90° angle where significant resonant behavior is apparent due to wave reflections between the force location and the joint position. More power is now reflected back from the joint when compared to the 0° case.

Figure 11 compares results from the higher angle of 175 degrees. The limitation of the experiment to 175 degree of angle was due to accelerometer positions that will touch between beams and also due to the anechoic terminations in the sand boxes where the beam ends will be almost touching each
other. It can be seen in Figure 11 that the resonant behavior is now more pronounced than in Figure 10.

Figure 10 – Predicted Model (bold lines) vs Experiment (thin lines) results at 175° degree of angle joint. (blue-power at beta, red-power at alpha, green-power at gamma, black-sum of beta+gamma).

Figure 11 – Predicted Model (bold lines) vs Experiment (thin lines) results at 175° degree of angle joint.
6. CONCLUSIONS

The variable joint model derived above provides a more accurate response with respect to the physical changes of angle than the previously reported rigid joint model. In particular, the variable joint model shows more realistic behavior of the vibration reflected and transmitted in the joint compared to the rigid/fixed joint, especially above 100° of angle. Experimental results show a plausible relationship to the predicted model behavior and this will enables wider research with regards to various cross-sections and geometrical complexity.

A table of coefficients for reflection and transmission could also be established based upon material type, cross sections and the angles occurred. Such information will further assist the understanding of vibration power at structural joints and becomes a platform for more complex or advanced connections in structures of interest.

REFERENCES