Frequency converter current-compounding excitation systems

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FREQUENCY CONVERTER CURRENT-COMPOUNDING
EXCITATION SYSTEMS

By

TIONG CHEE KOK, B.Sc.

A Doctoral Thesis
Submitted in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy of the University of Technology, Loughborough

July, 1983

Supervisor: Professor I.R. Smith
Department of Electronic and Electrical Engineering

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To my mother
I would like to express my deepest gratitude to Professor I.R. Smith for his guidance throughout my research and for his help in many other respects. His vast knowledge in the field of Electrical Power Engineering and his unceasing enthusiasm for the research have been the major driving force that saw me through this seemingly impossible task. Without him, the study would not have been possible.

I am indebted to the committee of Vice-Chancellors and Principals for assistance through their Fees Support Scheme and to The Lima Electric Co. (Ohio, USA) for financial support during the course of my study. I also gratefully acknowledge the latter organisation for providing the information, test units and test results used in some parts of the thesis. I wish also to thank the Computer Centre and the Department of Electronic and Electrical Engineering (both of Loughborough University) for their excellent computing and research facilities, the technicians of the Department for constructing the test units (EMEU-1, EMEU-2 and the test unit discussed in Chapter 9), Miss Sue Smith for typing several reports for The Lima Electric Co., Dr G.K. Creighton for performing some of the initial analysis on the CONMAC unit and Mr D.A. Gordon for re-winding the exciter unit of Chapter 9.

My colleagues in the Power System Research Group have been
responsible for creating a conducive research environment, and I thank them for the numerous technically rewarding discussions we had. I have learnt much from my supervision of undergraduates in the Electrical Machine Laboratory, and that I owe to Dr J.K. Hall, Mr J.G. Kettleborough, Dr S. Williams and Mr R.C. Cattell.

I appreciate all the efforts of my friends in Loughborough, especially Dr C.G. Kang, Mr F.S.C. Yeoh, Miss Karol Tong and Dr K.J. Blois and family, to make my stay here a pleasant experience.

Finally, a very special note of thanks to my family, my cousin and Miss Patricia Chin for their constant encouragement and for their valuable and unique contributions. In particular, my utmost gratitude to my long-suffering mother.
SYNOPSIS

Static current-compounding excitation schemes that provide an isolated synchronous generator with a field current that increases both with increasing load and with worsening power factor have been in existence for many years. Such schemes, while capable of maintaining the on-load terminal voltage substantially constant, require brushes to carry the excitation current to the generator field winding and in some practical applications these brushes may present serious maintenance problems. In a recent development, a new brushless frequency-converter type of excitation system was introduced. This system, while using only passive components, can furnish a loaded synchronous generator with a compounded excitation current such that its terminal voltage is maintained to within a reasonably close tolerance, irrespective of the load power factor. The central unit of the system is a special purpose exciter, the rather unconventional stator windings of which make the concept of an automatically-regulated and brushless generator a practical reality.

In this thesis, a detailed study of the exciter is presented. Equations are developed to allow a computer simulation of the arrangement, with the validity of the model being established by a comparison between predicted performance and experimental results. Methods for calculating the exciter parameters, either from open-circuit test results or from
basic machine constants, are included, with attention being devoted particularly to certain important design aspects. An optimisation study is undertaken on the exciter with the aid of a digital computer. The results from such a study indicate that the present arrangement is incapable of achieving an optimal design, and a new arrangement that uses capacitors in series with one stator winding of the exciter is therefore proposed. Detailed investigations conducted on the new arrangement reveal its many advantages. A major disadvantage of both the original and the modified forms of the exciter is that the nominal output voltage of the generator is not readily changed. To overcome this difficulty, while still providing the advantages of the current-compounding principle, a novel arrangement is proposed for the exciter in which both the stator and the rotor contain windings having different pole numbers.

The thesis also includes an examination of a proposal for a current-compounded excitation system which, if successful, would provide the basis for a variable-speed constant-frequency generator unit, by connecting it both mechanically and electrically to a wound-rotor induction machine. This novel concept is considered in some detail and, as is the practice throughout the thesis, considerable experimental evidence is provided in support of the accompanying theoretical investigation.
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<td>a, b</td>
<td>slot and tooth widths (Chapter 5)</td>
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<td>$B_g$</td>
<td>mean airgap flux density</td>
</tr>
<tr>
<td>$C_1$</td>
<td>phase transformation matrix</td>
</tr>
<tr>
<td>$C_2$</td>
<td>commutator transformation matrix</td>
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</table>
| $C_{I2}, C_{V1}$ | current and voltage constants of exciter  
(a bar placed above a symbol denotes a normalised value) |
| D       | mean airgap diameter                                                        |
| E       | excitation voltage of generator                                             |
| $E_{4r}$ | excitation voltage of 4-pole machine (Chapter 9)                          |
| f       | stator supply frequency                                                     |
| $f_i, f_o$ | input and output frequencies of CONMAC unit                             |
| $F_a, F_f, F_g$ | armature, field and resultant airgap m.m.f.'s                             |
| $F_c, F_g, F_t$ | m.m.f.'s required for the core, airgap and toothed part of magnetic circuit |
| g       | actual airgap length                                                        |
| $g_e$   | equivalent airgap length                                                    |
| $i_r, i_y, i_b$ | instantaneous values of phase currents                                   |
| $I_1, I_2, I_3$ | exciter shunt, series and rotor currents  
(a bar placed above a symbol denotes a normalised value) |
| $I_{com}$ | current in common section of stator winding                                  |
| $I_{dc}$ | rectified rotor current (during experimental investigation)                |
\( I_f \)  
field current

\( I_{fo}, I_{f1} \)  
no-load and rated-load field currents

\( I_{f4} \)  
d.c. current in winding SH (Chapter 9)

\( I_o, I_i, I_r \)  
input, output and rotor currents of CONMAC unit

\( I_L \)  
load current of generator

\( I_{se}, I_{sh}, I_r \)  
measured currents in series, shunt and rotor windings of exciter

\( I_t, I_{ct} \)  
current on the a.c. side of bridge rectifier and secondary current of current transformer

\( j \)  
\( \sqrt{-1} \)

\( k \)  
negative of the ratio of input electrical power to mechanical power developed (Chapter 8)

\( K_a \)  
specific coil area (area per radian)

\( K_d \)  
winding distribution factor

\( K_{dc} \)  
\( |K_i/Z_{et}| \)

\( K_i \)  
current transfer ratio

\( K_{mn} \)  
mutual reactance between windings \( m \) and \( n \), when each winding consists of one effective turn

\( K_{mm} \)  
self reactance of a one-turn winding \( m \)

\( K_p, K_s \)  
coil pitch and saturation factors

\( K_w \)  
winding factor of winding \( m \)

\( L \)  
stack length or per phase self inductance of winding as denoted by subscript
<table>
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<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>( L_{mm} )</td>
<td>self inductance of winding ( m )</td>
</tr>
<tr>
<td>( M )</td>
<td>per phase mutual inductance of winding as denoted by subscript (or subscripts)</td>
</tr>
<tr>
<td>( M_{mn} )</td>
<td>mutual inductance between windings ( m ) and ( n )</td>
</tr>
<tr>
<td>( n )</td>
<td>turns ratio of compounding transformer (Chapter 2)</td>
</tr>
<tr>
<td>( n_v )</td>
<td>ratio of shunt winding voltage to terminal voltage of generator</td>
</tr>
<tr>
<td>( n_r )</td>
<td>rotor speed of CONMAC unit (Chapter 8)</td>
</tr>
<tr>
<td>( N_1, N_2, N_3 )</td>
<td>number of series turns per phase in the shunt, series and rotor windings of the exciter</td>
</tr>
<tr>
<td>( N_{c1}, N_{c2} )</td>
<td>turns per coil in the 2-pole and 4-pole windings</td>
</tr>
<tr>
<td>( p )</td>
<td>differential operator ( \frac{d}{dt} ) or number of pole-pairs</td>
</tr>
<tr>
<td>( P_1, P_2 )</td>
<td>number of pole-pairs for machines ( M_1 ) and ( M_2 ) (Chapter 8)</td>
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<tr>
<td>( P_e, P_g )</td>
<td>pole-pairs for exciter and generator</td>
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<td>( P_e )</td>
<td>input electrical power</td>
</tr>
<tr>
<td>( P_{eo} )</td>
<td>( P_e ) during open-circuit test</td>
</tr>
<tr>
<td>( P_L )</td>
<td>power loss in CONMAC unit</td>
</tr>
<tr>
<td>( P_m )</td>
<td>mechanical power developed</td>
</tr>
<tr>
<td>( P_{ms}, P_{mo} )</td>
<td>( P_m ) during short-circuit and open-circuit tests</td>
</tr>
<tr>
<td>( P_o )</td>
<td>output power or no-load field power (Chapter 7)</td>
</tr>
<tr>
<td>( r )</td>
<td>combined rotor resistance or resistance of circuit element as denoted by subscript (Chapter 8)</td>
</tr>
</tbody>
</table>
\( R_{11} \)  resistance of shunt winding

\( R_{22}, R_{33} \)  resistances of series and rotor windings or resistances of windings SE and RSE (Chapter 9)

\( R_{4r} \)  armature resistance of 4-pole machine

\( R_{a} \)  armature resistance of generator

\( R_{c} \)  commutating resistance

\( R_{e} \)  effective resistance of rectifier/field-winding

\( R_{f} \)  field winding resistance

\( R_{r} \)  \( R_{33} + R_{4r} \) (Chapter 9)

\( s \)  slip

\( s_{o} \)  slip at \( P_e = 0 \)

\( s_{k} \)  slip at which \( P_e / P_m = -k \)

\( s_{\alpha} \)  slip at which \( P_e / P_o = \alpha \)

\( S_1, S_2, S_3 \)  volt-ampere requirements of exciter shunt, series and rotor windings (a bar placed above a symbol denotes a normalised value)

\( S_i \)  input volt-ampere requirement

\( S_t, S_w \)  exciter total and winding volt-ampere requirements (a bar placed above a symbol denotes a normalised value)

\( t_b, s_b \)  widths of tooth and slot at back of slot

\( v_r, v_y, v_b \)  instantaneous values of phase voltages

\( V \)  terminal voltage

\( V_1, V_2, V_3 \)  exciter shunt, series and rotor voltages

\( V_2 \)  normalised value of \( V_2 \)

\( V_{3o} \)  open-circuit rotor voltage
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$V_c$</td>
<td>voltage across Boucherot capacitor</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>d.c. field voltage</td>
</tr>
<tr>
<td>$V_i', V_o', V_r$</td>
<td>input, output and rotor voltages of CONMAC unit</td>
</tr>
<tr>
<td>$V_{me}', V_{mg}$</td>
<td>voltages across the magnetising branches of exciter and generator (Chapter 8)</td>
</tr>
<tr>
<td>$V_{mr}$</td>
<td>magnetising rotor voltage</td>
</tr>
<tr>
<td>$V_{se}', V_{sh}', V_r$</td>
<td>voltages across the series, shunt and rotor windings of exciter</td>
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<tr>
<td>$x$</td>
<td>reactance of circuit element as denoted by appropriate subscript (Chapter 8)</td>
</tr>
<tr>
<td>$x_{mm}$</td>
<td>resistance to reactance ratio of winding m</td>
</tr>
<tr>
<td>$X$</td>
<td>combined stator leakage reactance</td>
</tr>
<tr>
<td>$X_{22}', X_{33}, X_{23}$</td>
<td>reactances of windings SE and RSE and the mutual reactance between windings SE and RSE (only for Chapter 9)</td>
</tr>
<tr>
<td>$X_{4r}$</td>
<td>armature reactance of 4-pole machine</td>
</tr>
<tr>
<td>$X_a$</td>
<td>armature reactance of generator</td>
</tr>
<tr>
<td>$X_{al}$</td>
<td>armature leakage reactance</td>
</tr>
<tr>
<td>$X_B$</td>
<td>capacitive reactance of Boucherot capacitor</td>
</tr>
<tr>
<td>$X_c, \overline{X_c}$</td>
<td>commutating reactance and its normalised value</td>
</tr>
<tr>
<td>$X_e$</td>
<td>effective reactance of rectifier/field-winding</td>
</tr>
<tr>
<td>$X_m$</td>
<td>magnetising reactance or reactance of current transformer when referred to the secondary side</td>
</tr>
<tr>
<td>$X_{mn}$</td>
<td>mutual reactance between windings m and n</td>
</tr>
<tr>
<td>$X_{mm}$</td>
<td>self reactance of winding m (a bar placed above a symbol denotes a normalised value)</td>
</tr>
</tbody>
</table>
\( X_r \) compounding reactance (Chapter 2) or \( X_{33} + X_{4r} \) (Chapter 9)

\( X_s, \overline{X}_s \) synchronous reactance and its normalised value

\( Z_c \) commutating impedance

\( Z_{et} \) effective exciter rotor impedance

\( Z_{mn} \) mutual impedance between windings \( m \) and \( n \)

\( Z_{mm} \) self impedance of winding \( m \)

\( B \) amount of short-chording (in number of slots)

\( Q \) slots/pole/phase

\( S \) slots/pole

\( \alpha \) ratio of input electrical to output power (Chapter 8), phase difference between voltage sources (Chapter 9)

\( \alpha_c, \alpha_A \) coil pitch, coil pitch of coil \( A \) (or coil in winding \( A \))

\( \phi \) lagging phase angle of load current or flux per pole

\( \overline{\phi_m} \) flux in the exciter airgap produced by the resultant m.m.f. of all windings

\( \theta \) angular displacement

\( \delta \) load angle

\( \sigma \) Carter's coefficient

\( \psi_{mn} \) mutual flux linkage between windings \( m \) and \( n \)

\( \psi_{mm} \) self flux linkage of winding \( m \)

\( \omega \) angular frequency

\( \omega_r \) \( p \theta \)

\( \zeta \) phase lead of the current constant over the voltage constant
\( \mu_0 \) permittivity of free space

\( \gamma \) slot angle

\( \lambda \) effective phase displacement between the exciter shunt and series windings

**Subscripts**

1, 2, 3 exciter shunt, series and rotor windings (except for Chapters 8 and 9)

1, 2 stator and rotor quantities of machine \( M_1 \)

a, b stator and rotor quantities of machine \( M_2 \)

d, q d-axis and q-axis quantities

D differential component of leakage reactance

f fundamental component of flux linkage

m magnetising reactance due to fundamental component of flux linkage

r, y, b three a.c. phases

r, se, sh exciter rotor, series and shunt windings

r, s rotor and stator quantities

R, Y, B \((R,Y,B)\) representation of machine

t transpose of a matrix

a, \( \beta \), 0 \((\alpha,\beta,0)\) representation of machine

**Abbreviations**

a.c. alternating current

AVR automatic voltage regulator

CONMAC unit derived from interconnecting two induction machines
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<tr>
<td>d.c.</td>
<td>direct current</td>
</tr>
<tr>
<td>e.m.f.</td>
<td>electromotive force</td>
</tr>
<tr>
<td>ECU</td>
<td>experimental CONMAC unit</td>
</tr>
<tr>
<td>EMEU</td>
<td>experimental MAC exciter unit</td>
</tr>
<tr>
<td>FCE</td>
<td>frequency converter excitation</td>
</tr>
<tr>
<td>m.m.f.</td>
<td>magnetomotive force</td>
</tr>
<tr>
<td>MAC</td>
<td>trade-name for exciter of FCE system</td>
</tr>
<tr>
<td>ph</td>
<td>phase</td>
</tr>
<tr>
<td>p.u.</td>
<td>per unit</td>
</tr>
<tr>
<td>U.p.f.</td>
<td>unity power factor</td>
</tr>
<tr>
<td>Z.p.f.</td>
<td>zero power factor</td>
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CHAPTER 1

INTRODUCTION

The terminal voltage of an isolated synchronous generator supplied with constant excitation current falls considerably as load is applied, and in almost all practical situations it is essential to provide some form of excitation control if a substantially constant terminal voltage is required. Over the years, the need for such control has seen the development of many different excitation schemes. However varied these schemes may appear, the underlying principles that govern their operation are based essentially on one of two well-known techniques. The first, which relies upon the action of an automatic voltage regulator (AVR), must be clearly distinguished from the second, which uses the so-called self-regulating technique.

In excitation schemes that incorporate an AVR, the output voltage of the generator is sensed and compared with a reference voltage, to produce an error signal that is used to initiate a corrective action through an amplifying device, and hence to provide closed-loop error-actuated control of the output voltage. In the past electromechanical regulators (such as those of the rolling-contact or carbon-pile types) have been in common use, but such devices are comparatively bulky and offer only a limited accuracy of control. Although they were largely superseded by regulators based on magnetic
amplifiers, both forms are now virtually obsolete, having been almost entirely replaced by new designs employing solid-state components. Apart from offering the advantages of a smaller size and an improved accuracy of control, the performance of these modern regulators is continuously being enhanced in response to advances made in the field of semiconductor technology.

In contrast to an AVR-based scheme, a self-regulating excitation system is at no time aware of the actual output voltage of the generator it controls. If, by previous experience, it is established that a certain increase in load requires a specific correction in excitation, in order to maintain a constant output voltage, a self-regulating scheme is designed to bring this about when the situation arises. Within the realm of self-regulation, there are however two quite separate approaches to the concept. The first of these is empirical in nature, and relies upon certain on-load phenomena in the synchronous generator being used to modify the main flux of the machine. A typical example, which found wide application for many years, had a d.c. generator using the same slots and magnetic circuits as the a.c. machine. The field of the a.c. machine was fed from the commutator of the self-excited d.c. machine and it was arranged that the armature reaction effect in the a.c. machine increased the field of the d.c. machine such that the output voltage of the a.c. machine was, in turn, maintained
substantially constant. The second approach follows a more
direct concept, and relies on an ideal matching of the
generator characteristic to the characteristic of an
excitation circuit which provides a load-current feedback
from the output of the generator.

For machine of medium and large output, say from 200kVA
upwards, the need for a separate AVR presents no serious
technical or economic difficulties. Such machines, being
housed in suitable buildings, can be properly maintained, and
the cost of the AVR is only a small proportion of the total
capital outlay. Moreover, these generators are used mainly
for central generation, where the stringent demands on
voltage accuracy can only be met with the use of AVR control.
The situation for smaller machines is however often very
different, since these may be located where expert attention
is not readily available but where reliability is often
essential. With the reduced cost of such smaller machines
the contribution of the AVR now becomes a major consideration,
whereas the high degree of accuracy provided by a regulator
is not usually required. Furthermore, the development
during the last thirty or so years of the relatively small
and high speed diesel engine has led to an almost ideal
prime mover for small generating sets, and the demand for
'gensets' up to about 200kW in output has received a
tremendous boost. Some obvious applications include
generators for isolated situation such as farms, estates and
plantations, for standby sets in hospitals, factories or even private households, for numerous marine applications and for transportable sets used on isolated civil engineering and similar constructional sites. To cater for the needs of these markets, self-regulating generators have also undergone an intense phase of development. Consequently, load current feedback (more widely known as current compounding) has become a well-established technique for regulating synchronous generators.

This thesis begins with, in Chapter 2, a detailed examination of the various techniques of current compounding. After considering the excitation characteristic of an isolated synchronous generator, some early current-compounding schemes are presented, to illustrate the main features of typical systems. Although these early schemes were crude and highly inaccurate by modern standards, they are clearly the forerunners of the now widely-used and phase-sensitive schemes capable of providing a field current increasing both with load current and with worsening load power factor. There are many different practical arrangements for static current-compounding schemes, but the underlying principles are explained in the thesis in terms of the two basic arrangements, known commonly as the shunt and series schemes. Static schemes suffer inevitably from the disadvantage of requiring a brush/slip-ring assembly to carry the excitation current to the field winding of the generator, and the unit may be
unsuitable for application in which maintenance-free operation is important.

In a recent development of considerable novelty, Sparrow introduced a frequency-converter type of excitation system, which made the concept of a brushless and current-compounding generator unit a practical reality. The central unit of this system is a special purpose induction type exciter, and this thesis is concerned to a large extent with a detailed study of this machine. In Chapter 3, a theoretical model is developed and it is shown that, for balanced 3-phase operation, the exciter may be represented by a simple per phase 3-coil equivalent circuit. The use of this circuit indicates that the exciter is indeed capable of providing current-compounded excitation in a manner similar to that of a static scheme. Attention is then focused on this important feature of the exciter, with particular emphasis being placed on those parameters that affect the compounding characteristic. To verify the theoretical model, a series of tests was undertaken on two experimental units and a detailed account of these tests and the results obtained is presented in Chapter 4. Some results from an operational unit are also included to provide further confirmation of the model. The next three chapters (5, 6 and 7) consider certain design and optimisation aspects of the exciter. In particular, it is shown that an optimal exciter design is only possible with the inclusion of capacitors in series with one of the stator windings. Following on from this, a
thorough theoretical and experimental investigation is conducted on the modified excitation arrangement, in order that its potentially numerous advantages may be fully considered.

By connecting the exciter both mechanically and electrically to a wound-rotor induction machine, initial considerations as presented in Chapter 8 show that the resulting arrangement appears to provide the basis for a variable-speed constant-frequency generator unit. Furthermore, since the exciter is capable of providing current-compounded excitation, it may also be used to regulate the on-load output voltage of the unit. Although the proposal was conceived on an apparently sound theoretical basis, the performance of any practical unit must conform with certain requirements, such as a low power level in the exciter and a high operating efficiency, if it is to be regarded as a serious contender to a conventional 'genset'. Bearing in mind that the proposed unit may essentially be regarded as a 2-induction machine unit with interconnected stator and rotor windings, an assessment of its performance was made through a detailed theoretical and experimental study, with ample evidences being presented to indicate the feasibility, or otherwise, of the proposal.

In Chapter 9, an alternative arrangement is proposed for the exciter, in which both stator and rotor contain windings having different pole numbers. Apart from offering
the advantages of current-compounded excitation, this combination allows the level of the voltage at which the scheme exercises control to be more readily adjusted. The underlying operating principles for the exciter are explained and an equivalent circuit is developed. It is shown that the exciter is capable of providing phase-sensitive current compounding and this is confirmed by results obtained from an experimental unit. The chapter also contains an investigation into the possibility of using a single combined rotor winding, to respond simultaneously to the m.m.f.'s of two different pole numbers created by the stator, and which is capable of performing the functions of the otherwise two separate rotor windings.

Finally, a summary of all the important findings in the thesis is presented in Chapter 10.
CHAPTER 2
CURRENT COMPOUNDING

The problem of the voltage regulation of synchronous generators has received a great deal of attention since the very day these machines came into existence. The corresponding problem is, of course, also present in d.c. generators, although in this case the inherent regulation is comparatively small since the action of the commutator confines the armature m.m.f. to the quadrature axis and thereby prevents it from having any first-order effect on the generator output voltage. If desired, the inherent voltage regulation of a d.c. generator can be reduced by passing the generator output current through a few compounding turns wound round the field poles, so boosting the excitation as the generator is loaded. This is perhaps the most widely known instance where current compounding is used to maintain the output voltage of a loaded generator substantially constant. Although naturally more complicated, current-compounding excitation schemes for synchronous generator have also been developed, and detailed discussions of the two basic schemes are presented in this chapter in order to explain their underlying principles.

2.1 Excitation characteristics of synchronous generators

When a synchronous generator is loaded, the terminal voltage
decreases considerably from the no-load value as the load current is increased, unless its excitation is altered correspondingly. Furthermore, the exact excitation needed to maintain a particular output voltage is affected by the power factor of the load. The task of providing the correct excitation for accurate voltage regulation is accordingly more onerous in a synchronous generator than in a compounded d.c. generator.

The internal voltage drop of a synchronous generator is due to the effects of load current in the armature resistance $R_a$ and the leakage reactance $X_{al}$, together with the effect of the armature reaction. Except in the very smallest machines, the effect of armature resistance is negligible, whereas the effect of the armature reaction is always a major contributing factor. In the analysis of a cylindrical-rotor machine, it is usual to regard the resultant airgap m.m.f. $\mathbf{F}_g$ as the phasor sum of the armature m.m.f. $\mathbf{F}_a$ and the field m.m.f. $\mathbf{F}_f$, as shown in the space phasor diagram of Fig. 2.1. The relationship between these three phasors may then be expressed by

$$\mathbf{F}_g = \mathbf{F}_a + \mathbf{F}_f \quad \ldots \quad (2.1)$$

The armature m.m.f. $\mathbf{F}_a$ is the fundamental component of the m.m.f. established by the currents flowing in the armature conductors and its magnitude is therefore proportional to the load current. In the usual circuit representation, the
effect of the armature reaction is allowed for by the circuit element $X_a$, the reactance associated with the armature reaction. Noting that the excitation voltage $E$ is proportional to $\dot{F}_f$, the induced armature voltage $V'$ to $\dot{F}_g$ and the armature voltage $i_LX_a$ to $\dot{F}_a$, a voltage phasor diagram corresponding to the m.m.f. phasor diagram may be drawn, as also shown in Fig. 2.1. The terminal voltage $V$ is then obtained after the volt-drops across $X_{al}$ and $R_a$ are deducted from $V'$. If $R_a$ is neglected and $X_{al}$ and $X_a$ are as usual lumped to give the synchronous reactance $X_s$, the excitation voltage $E$ may be expressed as

$$E = V + j i_LX_s$$

... (2.2)

or

$$E^2 = V^2 + (i_LX_s)^2 + 2VI_LX_s \sin \phi$$

where $\phi$ is the lagging phase angle of the load current. If the terminal voltage $V$ is to be kept constant, eqns. (2.2) shows clearly that the excitation must increase both with load current and with worsening power factor, as is illustrated by the excitation characteristic presented in Fig. 2.2 for a typical generator.

The excitation requirement of a salient-pole synchronous generator is somewhat more difficult to assess. Following the usual 2-axis approach a phasor diagram can be derived as in Fig. 2.3, showing the relationship between the various voltages and currents involved. In the diagram subscripts $d$
and \( q \) denote respectively the appropriate axis quantities and \( \delta \) is the load angle. Inspection of the diagram indicates again the need to increase the excitation as the load conditions change if it is required to maintain a constant terminal voltage, and the typical excitation characteristics are, in fact, very similar to those of Fig. 2.2.

2.2 Line current compounding

The problem of controlling the excitation current of a synchronous generator so as to regulate the output voltage is evidently a more difficult proposition than in a similar d.c. generator. One obvious method of voltage control, visualised readily by analogy with the d.c. situation, is to increase the exciting m.m.f. on load by the addition to the field system of a winding carrying a current proportional to the load current. A schematic arrangement for such a system\(^1\) using a d.c. exciter is shown in Fig. 2.4. The separate series winding on the exciter is supplied from the d.c. side of a rectifier, which is in turn fed from a series transformer in an output line of the generator. A single-phase transformer is sufficient for a moderately balanced load, although a 3-phase arrangement is desirable for an unbalanced load. The shunt field is designed to produce from the exciter the excitation current required by the generator for normal voltage on no-load, and a trimming regulator is included to allow adjustment for speed and temperature.
variations. When the generator is supplying load, the additional excitation from the series winding increases the exciter output and thus maintains the output voltage of the generator. This very simple arrangement provides an overall regulation of ±5% to ±10% in cases when the load power factor is relatively constant.

A similar compounding arrangement that uses only static rectifiers is shown in Fig. 2.5. The d.c. exciter is now omitted, with the entire excitation power being derived from the generator output. There are two field windings on the generator and the m.m.f.'s of these are added to give the resultant field m.m.f. The no-load magnetising m.m.f. is provided by the shunt field, while the series field, by suitable design, provides the additional m.m.f. required to maintain the on-load voltage at any one given power factor. As with the previous arrangement, a voltage regulation of ±5% to ±10% can be achieved if the power factor remains substantially constant.

In both the above schemes, the addition of the series m.m.f. to the shunt m.m.f. is performed on the d.c. side of the system. The excitation current is therefore not responsive to the phase angle of the line current, which severely limits the application of the schemes to a near-constant power factor load if the resulting regulation is to be acceptable. Nevertheless, the voltage regulation can be significantly improved by feeding into the excitation system a component
that is proportional to the load current, and this application of load current feedback is usually known as current compounding in the literature.

2.3 Phase sensitive current compounding

For applications with load of a wide-ranging power factor, the next technically logical step is the development of current-compounding schemes that are sensitive to the load power factor, as well as to the load current. Numerous practical current-compounding circuits have been developed in recent years and a sample of these can be found in references (2-16). To explain the underlying principles of phase-sensitive current compounding, it is only necessary to consider the two basic forms, i.e. the basic shunt and series arrangements, and an appropriate reference should be consulted for further details on any particular practical design.

Fig. 2.6a shows the basic shunt-compounding circuit as applied to a 3-phase generator. The no-load field current $I_{fo}$ is obtained via the reactor $X_r$ and the full-wave bridge rectifier $BR$, with the additional current required on load being introduced by the current transformer $CT$. The reactor ensures that the field current components due to the generator output voltage $V$ and the load current $I_L$ are added with the correct phase relationship, and the turns ratio of the current transformer is adjusted to give the correct
excitation. If the rectifier/field-winding combination is represented by a resistance $R_e$ in series with a reactance $X_e$, the per phase equivalent circuit for the exciter (shown in Fig. 2.6b) may be derived. It can be shown from this circuit that the a.c. supply current to the rectifier $I_t$ is

$$I_t = \frac{V + jI_{ct}X_r}{R_e + j(X_e + X_r)} \quad \ldots (2.3)$$

where $I_{ct}$ is the current on the secondary side of the current transformer. If the bridge rectifier has a current transfer ratio of $K_1$ and the turns ratio of the current transformer is $n$, the excitation current $I_f$ is given by

$$I_f = K_1 \frac{|V + jnI_LX_r|}{|R_e + j(X_e + X_r)|} \quad \ldots (2.4)$$

in which the values of $K_1$, $R_e$ and $X_e$ are constant for a particular field winding and reactor. Thus, if the reactor is designed to provide the required no-load field current, comparison of eqns. (2.4) and (2.2) indicates that the scheme will provide ideal current compounding at all conditions of load and power factor when the turns ratio of the transformer is $(X_s/X_r)$. The turns ratio for a practical design must however depart slightly from this ideal value to give the best overall performance, taking into account non-linear effects such as saturation. Typically, a practical design can achieve a voltage regulation accuracy of ±3% over the full range of load current and with a power factor ranging from zero lagging to unity.
Although the effect of saturation is undesirable in an ideal theory of current compounding, consideration of this effect is indeed necessary in an explanation of how a synchronous generator provides a particular output voltage on no-load. Fig. 2.7 shows typical open-circuit characteristics for both a static exciter and a generator, and while the generator open-circuit characteristic is non-linear due to the effect of saturation, that of the exciter is linear with a slope of $|R_e + j(X_e + X_r)|/K_i$. Consideration of Fig. 2.7 indicates how build-up from the low residual voltage will occur in a manner comparable to that of a d.c. generator, with the final output voltage being determined by the intersection of the two characteristics. It is evident that the point of intersection may be shifted by altering the slope of the exciter characteristic, which implies that the required no-load voltage may be selected by using an appropriate value for $X_r$. However, just as in the case of a d.c. shunt generator, sufficient residual magnetism must be present in the synchronous generator to ensure that the process of positive feedback is initiated.

The basic arrangement of a series compounding scheme is shown in Fig. 2.8a, and the corresponding per phase equivalent circuit in Fig. 2.8b. In this circuit the phasor addition of the secondary circuit voltage of the current transformer to the supply voltage provides the required field excitation, with the necessary compounding reactance being provided by an
airgap introduced deliberately into the magnetic circuit of the current transformer. If that reactance referred to the transformer secondary is \(X_m\), it can be shown with the aid of Fig. 2.8b that

\[
I_f = \frac{K_i}{|R_e + j(X_e + X_m)|} |V + jnI_L X_m| \quad \ldots \quad (2.5)
\]

an expression very similar to the corresponding relationship for the basic shunt compounding scheme. In fact, the behaviour of both schemes is identical if \(X_m = X_r\) and all previous comments pertaining to the shunt scheme are therefore applicable in the present case.

The above considerations have demonstrated that the two basic current-compounding circuits can, in theory, provide a cylindrical-rotor generator with a very accurate voltage regulation, regardless of the conditions of load and power factor. The output voltage is, however, not under closed-loop control and the accuracy of regulation is therefore limited to typically \(\pm 3\%\) by such practical consideration as the non-linearity of the circuit elements and the generators. When finer control of voltage is required, simple closed-loop control circuitry may be added to supplement the rapid but relatively coarse regulation provided by the current compounding. This approach, which offers the advantages of current compounding, together with a high accuracy of voltage control, is adopted in a number of practical circuits. In
one case, an accuracy of ±~% was claimed using a rather simple regulator.

It is worth noting that the application of the basic current-compounding excitation schemes is by no means restricted to cylindrical-rotor generators. The fact that the schemes provide an excitation increasing with both the magnitude and the lagging phase angle of load makes them also suitable for salient-pole generators. Although the provision of ideal current compounding is now theoretically impossible, the schemes are still capable of maintaining the output voltage of such generators substantially constant over a wide range of load current and power factor. If the accuracy of the control requires improvement, a supplementary closed-loop control circuit can always be introduced to add the advantage of accurate voltage control to the rapid response associated with current compounding.

2.4 Boucherot capacitors

In normal operating circumstances, a synchronous generator will be called upon to operate over a range of temperature, with the actual values depending on its loading and the ambient conditions. Variations in temperature change the resistance of the field winding, which consequently affects the resistance $R_e$ and the reactance $X_e$ in the equivalent circuits of Figs. 2.6b and 2.8b. As is evident from inspection of eqns. (2.4) and (2.5), the resulting effect of
a temperature variation is to cause the excitation current and hence the terminal voltage of the generator to drift unduly, unless $X_m$ or $X_r$ provides a sufficient swamping effect. It is, of course, possible to design exciters having large reactances, but such an approach invariably results in an inefficient design. Alternatively, the problem may be solved by making use of the constant current properties of a Boucherot circuit\(^{18}\).

The basic shunt current-compounding arrangement with Boucherot capacitors is shown in Fig. 2.9a. At the operating frequency, the Boucherot capacitors have a capacitive reactance of $X_B$ and if the capacitors are tuned to resonate with the reactors at that frequency (i.e. $X_B = X_r$) a simple analysis of the per phase equivalent circuit of Fig. 2.9b establishes that the generator exciting current of

$$I_f = \frac{K_i}{X_r} |V + jnI_L X_r|$$  \hspace{1cm} \ldots (2.6)

is entirely independent of the field winding resistance. The field winding of the generator appears therefore to be fed from a current source, so that the terminal voltage remains unaltered as the machine temperature changes. Furthermore, since eqn. (2.6) has the same form as eqn. (2.4), the ability of the circuit to provide current-compounded excitation is not in any way affected by the inclusion of the Boucherot capacitors.
In the basic series-compounding circuit, just as with the shunt scheme, Boucherot capacitors may be included to prevent undue drift in the generator voltage, with Figs. 2.10a-b showing respectively the circuit diagram and equivalent circuit for this arrangement. Noting that the capacitors are chosen to resonate with the secondary magnetising reactance of the current transformer, the equation derived for $I_f$ from the equivalent circuit is the same as eqn. (2.6), but with $X_r$ replaced by $X_m$. Again, the excitation current is independent of the field winding resistance.

Boucherot capacitors are widely used in many practical circuits. Especially where a single-phase current-compounding circuit is used, a reactor alone is much less effective as a swamping reactance and the Boucherot capacitor is always necessary to prevent undue drift in the generator voltage.

2.5 Brushless current compounding

In the practical implementation of a basic current-compounding arrangement, a brush/slip-ring assembly is required to carry the excitation current from the static rectifier system to the rotating field winding. Although brushes bearing on slip rings do not present as serious a problem as when they bear on a commutator, the presence of sliding contacts is a major disadvantage that may prevent the use of current-compounded excitation in some situations where it is required to keep routine maintenance to a minimum. This consideration is
especially important when generating sets are intended for use as standby supplies, since the prolonged periods of non-running may lead to the formation of oxide layers on the slip rings, unless the sets are maintained at regular intervals. The need to economise on the operating cost is perhaps the reason why many modern synchronous machine installations are brushless, a trend made possible by the advent of reliable semi-conductor rectifiers capable of withstanding large centrifugal stress.

In a typical conventional brushless installation, a signal proportional to the output voltage of a synchronous generator is fed via an AVR to a d.c. field winding on the stator of an a.c. exciter, which is itself another small generator but with a rotating armature winding and a stationary field winding. This signal induces in turn a 3-phase voltage in the rotor winding of the exciter, and after rectification by a shaft-mounted rectifier the corresponding current is fed to the rotating field winding of the main generator. The armature windings of this machine are, of course, on the stator. With a modern regulator, an extremely accurate control of the terminal voltage to well beyond full-load current is obtained, for a wide range of power factors. However, as already noted in Chapter 1, the regulators are relatively expensive for use with small and medium size generators, and the situations in which such machines are employed often do not require a very precise voltage control.
This is one reason why many small and medium size generators are still equipped with the simpler, cheaper and more robust static current-compounding excitation circuits, even though the resulting units are not brushless. In addition, current-compounded systems offer very good short-circuit performance, since they can easily provide 3 to 3.5 times full-load current into a terminal short circuit, thereby making the systems particularly useful for on-line starting of induction motors. It should be noted that a similar short-circuit performance for a conventional brushless unit can only be achieved by incorporating an additional, and costly, current-feedback circuitry in the AVR. One further reason for the continuing popularity of the static current-compounding scheme is its inherently fast response, due primarily to the effect of load current feedback and the absence of the a.c. exciter and its associated time constant. Clearly, it is desirable to have a current-compounding arrangement which is also brushless, for situations where simplicity and ruggedness are of prime importance.

In a recent development, such a scheme was introduced. This new frequency-converter type of excitation, while using only passive components, can furnish a loaded synchronous generator with a current-compounded exciting current such that its terminal voltage is maintained to within a reasonably close tolerance, irrespective of the load power factor. The system thus offers the excellent short-circuit performance...
and rapid response familiar to any static current-compounding scheme together with the advantages of brushless operation. The central unit of the system is a special purpose exciter, and a detailed study of this type of exciter constitutes a major later portion of this thesis.
Fig. 2.1 Voltage and m.m.f. phasor diagram for a typical cylindrical-rotor synchronous generator

Fig. 2.2 Constant voltage excitation characteristic
Fig. 2.3 Phasor diagram of salient-pole generator

Fig. 2.4 Circuit diagram for line-current compounding
Fig. 2.5 Circuit diagram of static current-compounded excitation (Not sensitive to phase angle)

Fig. 2.6a Basic shunt compounding scheme
Fig. 2.6b Equivalent circuit for basic shunt compounding scheme

Fig. 2.7 Determination of the no-load voltage of a self-excited generator

Fig. 2.8a Basic series compounding scheme
Fig. 2.8b Equivalent circuit for basic series compounding scheme

Fig. 2.9a Basic shunt compounding scheme with Boucherot capacitors

Fig. 2.9b Equivalent circuit for shunt compounding scheme with Boucherot capacitors
Fig. 2.10a Basic series compounding scheme with Boucherot capacitors

Fig. 2.10b Equivalent circuit for basic series compounding scheme with Boucherot capacitors
CHAPTER 3
A THEORETICAL MODEL FOR MAC EXCITER

In reference (19), an experimental study on a newly developed frequency-converter excitation system (FCE) for synchronous generators was presented. The central unit of this system is a special purpose exciter (hereafter called a MAC exciter), the rather unconventional stator windings arrangement of which makes the concept of a brushless and self-regulated generator a practical reality. Ample experimental evidence was included in the reference to demonstrate the ability of the exciter to furnish an excitation current to a loaded generator such that its terminal voltage is maintained substantially constant, irrespective of the load current and power factor. There was however no satisfactory explanation on the peculiar, although highly desirable, action of the exciter. With this omission in mind, the present chapter presents a rigorous theoretical analysis of the exciter and a simple 3-coil equivalent circuit (for each phase), very similar to that for a 3-winding transformer, is developed. It is shown that the exciter is, in fact, providing a current-compounded excitation similar to that of a static current-compounding scheme. Detailed attention is therefore given to this important feature of the exciter, with particular emphasis placed on those parameters that affect the compounding characteristics.
3.1 System arrangement and operation

Essentially, a MAC exciter is (say) a 4-pole special purpose induction machine, with two sets of 3-phase windings on the stator and a conventional 3-phase wound rotor. For reasons that will become obvious later, the two sets of stator windings are separately referred to as the shunt and the series windings. The coils belonging to each of these two windings are wound on alternate poles, as shown in the stator connection diagram of Fig. 3.1, such that their magnetic axes are mutually displaced by $180^\circ$. The exciter rotor contains a conventional 3-phase double-layer winding, with the rotor connection diagram being given in Fig. 3.2.

In its normal application when feeding a 4-pole synchronous generator, the exciter and the generator stators are housed in the same external frame, as shown in Fig. 3.3, with the two rotors mounted rigidly on the same shaft. The exciter and generator are also connected electrically, as shown in Fig. 3.4. The shunt winding is connected across a small portion of the armature winding, while the series winding is connected between the load and the generator such that it is carrying the load current. The generator field winding is fed by the exciter via a shaft-mounted 3-phase bridge rectifier. On no-load, the voltage across the shunt winding establishes an airgap field that rotates in the reverse direction to the exciter rotor, thereby inducing in it a double-frequency voltage, which in turn establishes the
excitation current in the generator field winding through a bridge rectifier. A small permanent magnet is incorporated into the field structure of the generator to ensure a positive build-up of the no-load voltage from the low residual level. When the generator is supplying load, the flow of current through the series winding reinforces the stator m.m.f. of the exciter, causing in turn the field current to increase in such a manner that the terminal voltage of the generator is maintained substantially constant. At this stage, it is instructive to note that the exciter is operating in a generating mode with a slip of 2, so that one-half of the generator field power is derived directly from the mechanical shaft power. Further considerations of this power amplification ability of the system are given in Section 3.5, which also includes an analysis on the operation of a MAC exciter with a different pole number from that of the generator.

3.2 General voltage equations and equivalent circuits

Fig. 3.5 presents a diagrammatic representation of the 3-phase windings of a MAC exciter, with a coil such as R_sh representing a group of coils which individually produce similar magnetic and electric effects. Three similar coils are grouped, as indicated to form a winding, and the three windings so formed will be referred to respectively as windings (1), (2) and (3) to facilitate the use of a
subscript notation in the following analysis. Winding (1) is fed from a voltage source to provide the no-load excitation and may alternatively be termed the shunt winding; winding (2) is connected in series with the load and may alternatively be termed the series winding, while the corresponding alternative term for winding (3) is the rotor winding.

The matrix equation that governs the operation of the exciter is, in compound form,

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} \\
Z_{21} & Z_{22} & Z_{23} \\
Z_{31} & Z_{32} & Z_{33}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\] ... (3.1)

or \( V = Z I \). Matrices \( V_1, V_2 \) and \( V_3 \) are all \((3 \times 1)\) column matrices and contain respectively the three voltages of windings (1), (2) and (3), while the \((3 \times 1)\) current matrices \( I_1, I_2 \) and \( I_3 \) similarly define all the nine currents of the exciter. It is thus apparent that each element on the leading diagonal of the impedance matrix \( Z \) is itself a \((3 \times 3)\) matrix, containing the usual resistance, self-inductance and mutual inductance terms that relate the voltages and the currents of the 3-phase winding. The remaining elements of \( Z \) are also \((3 \times 3)\) matrices, but these contain only mutual inductance terms relating the voltages and currents of the two windings denoted by the subscripts. For example, \( Z_{21} \) relates the voltage of winding (2) to the current in winding
It is, of course, possible to analyse the exciter with the aid of eqn. (3.1), but the fact that there are \((9 \times 9)\) non-zero terms in the impedance matrix \(Z\) makes this approach extremely tedious. Furthermore, the angle-varying terms in \(Z_{13}, Z_{23}, Z_{31}\) and \(Z_{32}\) can only further complicate the analysis. A useful alternative is to use a technique similar to that of reference (20), and to reduce the basic phase-variable model to a 2-axis representation. It is shown later that this transformation introduces a significant number of zero elements into the transformed impedance matrix, and hence provides a considerable simplification in the analysis. In the case of balanced 3-phase operation, it leads to the derivation of a simple voltage/current relationship, very similar in form to that for a conventional 3-winding transformer.

### 3.2.1 Phase transformation

When applied to a 3-phase \((R,Y,B)\) winding, a phase transformation \(C_1\) transforms this to an equivalent \((0,\alpha,\beta)\) winding. The \(\alpha\) and \(\beta\) coils form a balanced 2-phase winding and the 0 coil accounts for any zero-sequence effects. The elements of \(C_1\) are defined by

\[
C_1 = \begin{bmatrix} 0 & \alpha & \beta \\ R & \begin{bmatrix} 1 \\ \sqrt{2} \\ \sqrt{2} \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ Y & \begin{bmatrix} 1 \\ \sqrt{2} \\ -\sqrt{2} \end{bmatrix} & \begin{bmatrix} -1 \\ \sqrt{2} \\ -\sqrt{2} \end{bmatrix} \\ B & \begin{bmatrix} \sqrt{2} \\ 1 \\ \sqrt{2} \end{bmatrix} & \begin{bmatrix} 1 \\ \sqrt{2} \\ \sqrt{2} \end{bmatrix} \end{bmatrix} \]

...(3.2)
and the complete transformation matrix required for eqn. (3.1) is, in compound form,

\[
C = \begin{bmatrix}
C_1 & 0 & 0 \\
0 & C_1 & 0 \\
0 & 0 & C_1
\end{bmatrix}
\] ... (3.3)

The transformed impedance matrix \( Z' \) is given by

\[
Z' = C_t Z C
= \begin{bmatrix}
C_{1t}Z_{11}C_1 & C_{1t}Z_{12}C_1 & C_{1t}Z_{13}C_1 \\
C_{1t}Z_{21}C_1 & C_{1t}Z_{22}C_1 & C_{1t}Z_{23}C_1 \\
C_{1t}Z_{31}C_1 & C_{1t}Z_{32}C_1 & C_{1t}Z_{33}C_1
\end{bmatrix}
\] ... (3.4)

where the subscript \( t \) denotes the transpose of a matrix. In accordance with that for a single 3-phase winding, the complete phase transformation has the effect of replacing windings (1), (2) and (3) with their respective equivalent windings \((0_1, \alpha_1, \beta_1)\), \((0_2, \alpha_2, \beta_2)\) and \((0_3, \alpha_3, \beta_3)\). The elements of eqn. (3.4) define the exact nature of these windings.

3.2.1.1 Transformation of shunt, series and shunt-series impedances

Since the exciter slots are uniformly distributed round the periphery of the airgap, it is reasonable to assume that there is no appreciable saliency and that the impedance matrices for the shunt and series windings do not contain any angle-dependent terms. For example, the impedance matrix
of the shunt winding is given by

\[
\begin{align*}
Z_{11} &= Y_{sh} \begin{bmatrix}
R_{sh} & Y_{sh} & B_{sh} \\
R_{ll} + L_{sh} & M_{sh} & M_{sh} \\
M_{sh} & R_{ll} + L_{sh} & M_{sh}
\end{bmatrix} \\
&= Y_{sh} \begin{bmatrix}
R_{ll} + L_{sh} & M_{sh} & M_{sh} \\
M_{sh} & R_{ll} + L_{sh} & M_{sh} \\
M_{sh} & M_{sh} & R_{ll} + L_{sh}
\end{bmatrix} \quad \ldots (3.5)
\end{align*}
\]

where \( R_{ll} \) is the per phase resistance, \( L_{sh} \) the per phase inductance, \( M_{sh} \) the mutual inductance between any two phases and \( p \) the differential operator \( d/dt \). Note that, by assigning a single value \( L_{sh} \) to the self inductances, it is assumed that the three self inductances are equal. Similar assumptions are made for the six mutual inductances and the three winding resistances. After performing the necessary matrix multiplication, it can be shown that

\[
Z'_{11} = C_1^t Z_{11} C_1
\]

\[
= \begin{bmatrix}
R_{ll} + L_{10} & 0 & 0 \\
0 & R_{ll} + L_{11} & 0 \\
0 & 0 & R_{ll} + L_{11}
\end{bmatrix} \quad \ldots (3.6)
\]

where \( L_{10} = L_{sh} + 2M_{sh} \) and \( L_{11} = L_{sh} - M_{sh} \). Similarly, the transformed impedance for the series winding is

\[
Z'_{22} = C_1^t Z_{22} C_1
\]

\[
= \begin{bmatrix}
R_{22} + L_{20} & 0 & 0 \\
0 & R_{22} + L_{22} & 0 \\
0 & 0 & R_{22} + L_{22}
\end{bmatrix} \quad \ldots (3.7)
\]
where \( L_{20} = L_{se} + 2M_{se} \) and \( L_{22} = L_{se} - M_{se} \). \( R_{22}, L_{se}, \) and \( M_{se} \) are respectively the resistance, self inductance and mutual inductance terms for the series winding, corresponding to the similar terms for the shunt winding.

The shunt-series mutual reactance matrix \( Z_{12} \) may be written as

\[
Z_{12} = Y_{sh} \begin{pmatrix}
R_{se} & Y_{se} & B_{se} \\
R_{sh} & -M_{RRP} & M_{RYP} \\
M_{RYP} & -M_{RRP} & M_{RYP} \\
B_{sh} & M_{RYP} & -M_{RRP}
\end{pmatrix}
\] ...

\[ (3.8) \]

where \( M_{RR} \) is the mutual inductance between the R-phase of the shunt and series windings and \( M_{RY} \) the mutual inductance between the R-phase of the shunt winding and the Y-phase of the series winding. Because of the geometric arrangement of the windings, as evident from inspection of Fig. 3.5, all the leading diagonal of \( Z_{12} \) have the same mutual inductance \( M_{RR} \) and all the remaining elements have the same value \( M_{RY} \). After performing the necessary matrix multiplication, it follows that

\[
Z'_{12} = C_{1t} Z_{12} C_{1} \begin{pmatrix}
-M_{120P} & 0 & 0 \\
0 & -M_{12P} & 0 \\
0 & 0 & -M_{12P}
\end{pmatrix}
\] ...

\[ (3.9) \]
where $M_{120} = M_{RR} + 2M_{RY}$ and $M_{12} = M_{RR} - M_{RY}$. It is apparent that $Z_{21}'$, which is given by $C_1 Z_{21} C_1$, contains the same elements as $Z_{12}'$.

3.2.1.2 **Transformation of rotor impedances**

Based on the same simplifying assumptions as in the previous section, regarding the resistances and the self and mutual inductances, the impedance matrix of the rotor winding is given by

$$Z_{33} = \begin{bmatrix}
R_{33} + L_{r}P & M_{r}P & M_{r}P \\
M_{r}P & R_{33} + L_{r}P & M_{r}P \\
M_{r}P & M_{r}P & R_{33} + L_{r}P \\
\end{bmatrix} \quad \ldots (3.10)$$

where $R_{33}$, $L_{r}$ and $M_{r}$ are defined as for the shunt, or series, winding. The transformed impedance matrix $Z_{33}'$ is then given by

$$Z_{33}' = \begin{bmatrix}
R_{33} + L_{30}P & 0 & 0 \\
0 & R_{33} + L_{33}P & 0 \\
0 & 0 & R_{33} + L_{33}P \\
\end{bmatrix} \quad \ldots (3.11)$$

where $L_{30} = L_{r} + 2M_{r}$ and $L_{33} = L_{r} - M_{r}$.

It is apparent from Fig. 3.5 that the mutual inductance between a rotor phase and a stator phase varies with the angular position $\theta$ of the rotor. For instance, the mutual inductance the $R$-phase of the rotor winding and the $R$-phase of the shunt winding may be expressed as
where $M_{sh1}$, $M_{sh3}$ and $M_{sh5}$ are the magnitudes of the respective Fourier components. The expression contains only odd harmonics, since the variation of $M^{RR}_{13}$ is symmetrical about both the polar and interpolar axes. Inspection of Fig. 3.5 shows that the equations for the mutual inductance between the R-phase of the rotor winding and the other two stator phases of the shunt winding, i.e. $M^{RY}_{13}$ and $M^{RB}_{13}$, can similarly be obtained by replacing $\theta$ in eqn. (3.12) with $(\theta + 120^o)$ and with $(\theta + 240^o)$ respectively. The mutual inductances between the other phases of the rotor and shunt windings can then be determined by using the following equations:

$$M^{YY}_{13} = M^{BB}_{13} = M^{RR}_{13} \quad \ldots \ (3.13)$$

$$M^{YB}_{13} = M^{BR}_{13} = M^{RY}_{13} \quad \ldots \ (3.14)$$

$$M^{YR}_{13} = M^{BY}_{13} = M^{RB}_{13} \quad \ldots \ (3.15)$$

thereby defining all the elements of $Z_{13}$ by

$$Z_{13} = \begin{bmatrix}
M^{RR}_{13} & M^{RY}_{13} & M^{RB}_{13} \\
M^{YR}_{13} & M^{YY}_{13} & M^{YB}_{13} \\
M^{BR}_{13} & M^{BY}_{13} & M^{BB}_{13}
\end{bmatrix} \quad \ldots \ (3.16)$$

When harmonics of the fifth order and above are neglected,
it follows, after suitable matrix multiplication, that the transformed rotor impedance is

\[
Z_{13} = \mathbf{C}_1^T \mathbf{Z}_{13} \mathbf{C}_1
\]

\[
= \begin{bmatrix}
M_{130} \cos 3\theta & 0 & 0 \\
0 & M_{13} \cos \theta & M_{13} \sin \theta \\
0 & -M_{13} \sin \theta & M_{13} \cos \theta
\end{bmatrix}
\]

... (3.17)

where \( M_{13} = \frac{3}{2} M_{\text{shl}} \) and \( M_{130} = 3M_{\text{sh3}} \). Transformation of \( Z_{31} \) yields a result similar to that of eqn. (3.17). By replacing \( \theta \) in eqn. (3.12) by \( (\theta - \pi) \), the mutual inductance between the \( R \)-phases of the rotor and series winding is

\[
M_{23}^{RR} = M_{\text{se1}} \cos (\theta - \pi) + M_{\text{se3}} \cos 3(\theta - \pi) + M_{\text{se5}} \cos 5(\theta - \pi) + ...
\]

... (3.18)

where \( M_{\text{se1}}, M_{\text{se3}} \) and \( M_{\text{se5}} \) are the respective magnitudes of the Fourier component, and the other related coefficients can similarly be derived. Using the previous assumption concerning the harmonics, it can be shown that

\[
Z_{23} = \mathbf{C}_1^T \mathbf{Z}_{23} \mathbf{C}_1
\]

\[
= \begin{bmatrix}
M_{130} \cos 3\theta & 0 & 0 \\
0 & M_{23} \cos \theta & -M_{23} \sin \theta \\
0 & M_{23} \sin \theta & -M_{23} \cos \theta
\end{bmatrix}
\]

... (3.19)
where \( M_{23} = \frac{3}{2} M_{s1} \) and \( M_{230} = 3 M_{s3} \). By performing the transformation on \( Z_{32} \), the relationship that \( Z'_{32} = Z'_{23} \) can be verified.

3.2.1.3 The complete matrix \( Z' \)

Using the results of the two preceding sections, the complete transformed matrix \( Z' \) can now readily be obtained. It is noted that \( Z' \) can be rearranged to form the compound matrix

\[
Z' = \begin{bmatrix}
Z'_{00} & 0 \\
0 & Z'_{a\beta} \\
\end{bmatrix}
\]

such that

\[
V'_{0} = Z'_{00} I'_{0} \tag{3.20}
\]

\[
V'_{a\beta} = Z'_{a\beta} I'_{a\beta} \tag{3.21}
\]

where

\[
Z'_{00} = 0_{1}
\begin{bmatrix}
0_{1} & 0_{2} & 0_{3} \\
0_{2} & M_{120}p & M_{130}p \cos 3\theta \\
0_{3} & M_{130}p \cos 3\theta & M_{230}p \cos 3\theta & R_{33} + L_{30}p
\end{bmatrix}
\]

and
\[
Z_{\alpha\beta}' = \begin{pmatrix}
\alpha_1 & \beta_1 & \alpha_2 & \beta_2 & \alpha_3 & \beta_3 \\
R_{11} + L_{11} & 0 & -M_{12} & 0 & M_{13} \cos \theta & -M_{13} \sin \theta \\
0 & R_{11} + L_{11} & 0 & -M_{12} & M_{13} \sin \theta & M_{13} \cos \theta \\
-M_{12} & 0 & R_{22} + L_{22} & 0 & -M_{23} \cos \theta & M_{23} \sin \theta \\
0 & -M_{12} & 0 & R_{22} + L_{22} & -M_{23} \sin \theta & -M_{23} \cos \theta \\
M_{13} \cos \theta & -M_{13} \sin \theta & -M_{23} \cos \theta & -M_{23} \sin \theta & R_{33} + L_{33} & 0 \\
-M_{13} \sin \theta & M_{13} \cos \theta & M_{23} \sin \theta & -M_{23} \cos \theta & 0 & R_{33} + L_{33}
\end{pmatrix}
\]

\ldots \ (3.24)
A diagrammatic representation of the \((0, \alpha, \beta)\) windings is given in Fig. 3.6. It is clear from eqns. (3.21) and (3.22) that the voltages and currents of the \(\alpha\) and \(\beta\) coils are independent of those of the 0 coils, and it follows that the \((\alpha, \beta)\) winding and the \((0)\) winding can be treated separately. In the case of balanced 3-phase operation no zero-sequence current flows, \(I'_0 = 0\), and a similar situation arises when there is no 4th-wire connection. For such cases only the \((\alpha, \beta)\) quantities need to be considered, so that the 3-phase \((R,Y,B)\) windings have thus been replaced by an equivalent 2-phase \((\alpha, \beta)\) windings.

### 3.2.2 Commutator transformation

A commutator transformation \(C_2\) is used to eliminate the angle-varying terms in the machine inductances, and hence to replace a \((\alpha, \beta)\) winding pair with an equivalent pseudo-stationary \((d,q)\) winding. It can be shown that the required transformation matrix is

\[
C_2 = \begin{bmatrix}
\sin \theta & \cos \theta \\
\cos \theta & -\sin \theta \\
\end{bmatrix}
\] ...

(3.25)

Obviously, only the rotor quantities of eqn. (3.24) need to be so transformed, and the overall transformation matrix in compound form is

\[
C = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & C_2 \\
\end{bmatrix}
\] ...

(3.26)
The transformed impedance matrix $Z''_{\alpha\beta}$ is then given by

$$Z''_{\alpha\beta} = C_t Z'_{\alpha\beta} C$$

$$= a_1 \begin{bmatrix} R_{11} + L_{11} p & 0 & -M_{12} p & 0 & 0 & M_{13} p \\ 0 & R_{11} + L_{11} p & 0 & -M_{12} p & M_{13} p & 0 \\ -M_{12} p & 0 & R_{22} + L_{22} p & 0 & 0 & -M_{23} p \\ 0 & -M_{12} p & 0 & R_{22} + L_{22} p & -M_{23} p & 0 \\ -M_{13} \omega_r & M_{13} p & M_{23} \omega_r & -M_{23} p & R_{33} + L_{33} p & -L_{33} \omega_r \\ M_{13} p & M_{13} \omega_r & -M_{23} p & -M_{23} \omega_r & -L_{33} \omega_r & R_{33} + L_{33} p \end{bmatrix}$$

... (3.27)
where \( \omega_r = p\theta \). The voltage equation is

\[
V_{a\beta}^{\prime} = Z_{a\beta}^{\prime} I_{a\beta}^{\prime} 
\]

... (3.28)

where the \((6 \times 1)\) column matrices \( V_{a\beta}^{\prime} \) and \( I_{a\beta}^{\prime} \) respectively define the currents and voltages associated with the \((\alpha, \beta)\) coils of Fig. 3.6.

3.2.3 Balanced 3-phase operation

If \( I \) is the r.m.s. phase current, then the instantaneous currents in the three phases of a 3-phase winding can be expressed respectively as

\[
i_r = \sqrt{2} I /\omega t \quad i_y = \sqrt{2} I /\omega t + 120^\circ \quad i_b = \sqrt{2} I /\omega t + 240^\circ
\]

... (3.29)

where \( \omega \) is the angular frequency of the supply. Using the transformation

\[
I(0, \alpha, \beta) = Clt I(R, Y, B)
\]

... (3.30)

it follows that

\[
\begin{bmatrix}
I_0 \\
I_\alpha \\
I_\beta
\end{bmatrix} = \sqrt{3} I /\omega t \begin{bmatrix}
0 \\
1 \\
-j
\end{bmatrix}
\]

... (3.31)

Similarly,

\[
\begin{bmatrix}
V_0 \\
V_\alpha \\
V_\beta
\end{bmatrix} = \sqrt{3} V /\omega t \begin{bmatrix}
0 \\
1 \\
-j
\end{bmatrix}
\]

... (3.32)
where $V$ is the r.m.s. phase voltage. Using the transformation

$$I(q,d) = C_2 t I(\alpha,\beta)$$

it can be shown that

$$\begin{bmatrix} I_q \\ I_d \end{bmatrix} = \sqrt{3} I/\omega t + \theta \begin{bmatrix} -j \\ 1 \end{bmatrix}$$

... (3.34)

If $\omega_1$ is now the stator frequency and $\theta = \omega_r t$, it follows that $\omega = \omega_2 = \omega_1$ and that $\omega_r = (\omega_1 - \omega_2) = \omega_1 (1-s)$, where $s$ is the slip. Thus

$$\begin{bmatrix} I_q \\ I_d \end{bmatrix} = \sqrt{3} I/\omega_1 t \begin{bmatrix} -j \\ 1 \end{bmatrix}$$

... (3.35)

which shows that the transformation has brought about a change in the frequency of the rotor currents from $\omega_2$ to $\omega_1$. Using similar arguments, it can be shown that

$$\begin{bmatrix} V_q \\ V_d \end{bmatrix} = \sqrt{3} V/\omega_1 t \begin{bmatrix} -j \\ 1 \end{bmatrix}$$

... (3.36)

During steady-state operation, the operator $p$ in eqn. (3.28) is replaced by $j\omega$, where $\omega$ is the supply frequency. If the phase voltage and current of the shunt winding are respectively $V_1$ and $I_1$, those of the series winding $V_2$ and $I_2$ and those of the rotor winding $V_3$ and $I_3$, then the following three equations can be obtained from eqn. (3.28), after applying the appropriate voltage and current transformations.
\[
V_1 = (R_{11} + jX_{11})I_1 - jX_{12}I_2 + jX_{13}I_3 \quad \ldots (3.37)
\]
\[
V_2 = -jX_{12}I_1 + (R_{22} + jX_{22})I_2 - jX_{23}I_3 \quad \ldots (3.38)
\]
\[
\frac{V_3}{s} = jX_{13}I_1 - jX_{23}I_2 + \left(\frac{R_{33}}{s} + jX_{33}\right)I_3 \quad \ldots (3.39)
\]

where \( s = (\omega - \omega_r) / \omega \) and the various reactances are obtained using the general results \( X_{aa} = \omega L_{aa} \) and \( X_{ab} = \omega M_{ab} \).

Since the currents in the \((\alpha_1, \beta_1)\) winding have an opposite magnetic effect to those in the \((\alpha_2, \beta_2)\) winding (due to the \(180^\circ\) winding displacement), negative terms are present in eqns. (3.37-3.39). The sign of these terms however need to be reversed, in order to allow a simple circuit representation of the equations, and this can most readily achieved by redefining the currents in the windings. Thus, when positive currents in the \((\alpha_2, \beta_2)\) winding are now defined as currents which have the same magnetic effect as positive currents in the \((\alpha_1, \beta_1)\) winding, it follows that

\[
V_1 = (R_{11} + jX_{11})I_1 + jX_{12}I_2 + jX_{13}I_3 \quad \ldots (3.40)
\]
\[
V_2 = jX_{12}I_1 + (R_{22} + jX_{22})I_2 + jX_{23}I_3 \quad \ldots (3.41)
\]
\[
\frac{V_3}{s} = jX_{13}I_1 + jX_{23}I_2 + \left(\frac{R_{33}}{s} + jX_{33}\right)I_3 \quad \ldots (3.42)
\]

Eqns. (3.40-3.42) may be represented by the set of three mutually coupled coils shown in Fig. 3.7. It should be noted that the rotor rotation has been allowed for by the slip \( s \) and that all the reactances in Fig. 3.7 are derived quantities, the values of which have already been defined.
For normal balanced operation, a MAC exciter can thus be regarded as an arrangement of three stationary coils with the appropriate self and mutual reactances.

3.3 Current compounding with a MAC exciter

When a MAC exciter is used to regulate the voltage of a synchronous generator, the various windings in the exciter are connected to the generator as shown in Fig. 3.4 (note that a detailed description of the arrangement has already been given in Section 3.1). In this section, equations describing the performance of an exciter connected as such are developed from the use of eqns. (3.40-3.42), or the equivalent circuit of Fig. 3.7.

3.3.1 The compounding equations

If the exciter rotor is on open-circuit, it can be shown from eqns. (3.40-3.42) that

\[
\frac{V_{30}}{s} = V_1 C_{V1} + I_2 C_{I2} \quad \ldots \quad (3.43)
\]

\[
I_1 = \frac{V_1 - jX_{12} I_2}{R_{ll} + jX_{ll}} \quad \ldots \quad (3.44)
\]

\[
V_2 = jX_{12} I_1 + (R_{22} + jX_{22}) I_2 \quad \ldots \quad (3.45)
\]

where \( V_{30} \) is the open-circuit rotor phase voltage,

\[
C_{V1} = \frac{jX_{12}}{R_{ll} + jX_{ll}} \quad \ldots \quad (3.46)
\]
\[ C_{I2} = jX_{23} + \frac{X_{13}X_{12}}{R_{11} + jX_{11}} \] ... (3.47)

and all the other quantities are as previously defined.

Since \( C_{V1} \) and \( C_{I2} \) appear respectively as multiplying factors for the shunt voltage \( V_1 \) and the series current \( I_2 \), it is appropriate to term them the voltage and current constants of the exciter. Eqns. (3.46) and (3.47) may be rearranged to show that

\[ C_{V1} = \frac{X_{13}}{X_{11}(1 + x_{11}^2)} (1 + jx_{11}) \] ... (3.48)

\[ C_{I2} = \frac{X_{13}X_{12}x_{11}}{X_{11}(1 + x_{11}^2)} + j\{X_{23} - \frac{X_{13}X_{12}}{X_{11}(1 + x_{11}^2)}\} \] ... (3.49)

where \( x_{11} = R_{11}/X_{11} \). It will become clear later that it is these two constants that determine the compounding characteristic of the exciter.

If the rectifier/field-winding combination is represented\(^{17}\) by a resistor \( R_e \) in series with a reactor \( X_e \), the d.c. excitation current \( I_r \) is \( K_i |I_r| \), where \( I_r \) is the rotor current on the a.c. side of the rectifier. Using eqns. (3.40-3.42), together with the appropriate substitutions \( I_r = -I_3 \) and \( \frac{V_3}{s} = I_r \left( \frac{R_e}{s} + jX_e \right) \), it can be shown that

\[ I_3 = -\frac{V_1 C_{V1} + I_2 C_{I2}}{Z_{et}} \] ... (3.50)
\[ I_1 = \frac{V_1 - jX_{12}I_2 - jX_{13}I_3}{R_{11} + jX_{11}} \] ... (3.51)

\[ V_2 = jX_{12}I_1 + (R_{22} + jX_{22})I_2 + jX_{23}I_3 \] ... (3.52)

where \( Z_{et} = \frac{R_e}{s} + R_c + j(X_e + X_c) \) ... (3.53)

\[ Z_c = R_c + jX_c = \frac{R_{22}}{s} + \frac{X_{13}^2}{R_{11} + jX_{11}} + jX_{33} \] ... (3.54)

Inspection of the equations above reveals that \( Z_{et} \) is the effective impedance presented to the exciter rotor and \( Z_c \) the effective impedance on the a.c. side of the rectifier. It is demonstrated in Appendix A1 that both the real part \( R_c \) and the imaginary part \( X_c \) of \( Z_c \) affect the commutation of the rectifier and they are referred to respectively as the commutating resistance and the commutating reactance, while \( Z_c \) may correspondingly be termed the commutating impedance. Since the rectifier is located on the rotor, the values of \( R_c \) and \( X_c \) must first be evaluated at the actual rotor frequency, and these values when referred back to the supply frequency of the exciter are used in the above equations. If the rectifier diodes are assumed to be ideal and the commutating effect of \( R_c \) is neglected, then both \( Z_{et} \) and \( K_i \) are constant. Hence,

\[ I_f = K_{dc} |V_1 C_{V1} + I_2 C_{I2}| \] ... (3.55)

where \( K_{dc} \), as given by \( |K_i/Z_{et}| \), is just another constant. The excitation current provided by the exciter is thus the phasor sum of two components, which for convenience can be
termed respectively the voltage component \( K_{dc} C V_1 V_1 \) and the current component \( K_{dc} C I_2 I_2 \). Noting that the series current \( I_2 \) is actually the same as the load current \( I_L \), the two component are respectively proportional to the shunt voltage \( V_1 \) and the load current \( I_L \). The voltage component can therefore be arranged to provide the no-load excitation of the generator, with the current component providing some additional on-load excitation. The actual field current is, of course, dependent on the magnitude of the current constant and its phase lead or lag with respect to the voltage constant. By judicious design, it will be shown that the exciter can furnish an ideal current-compounded excitation to a loaded cylindrical-rotor generator, in precisely the same way as that provided by a static current-compounding excitation scheme.

3.3.2 Ideal current compounding

It is reasonable to assume that in a properly designed exciter \( R_{ll} \ll X_{ll} \), whence eqns. (3.48) and (3.49) may be used to arrive at the expressions

\[
C_{V1} = \frac{X_{13}}{X_{ll}} \quad \ldots \quad (3.56)
\]

\[
C_{I2} = j(X_{23} - \frac{X_{12} X_{12}}{X_{ll}}) \quad \ldots \quad (3.57)
\]

Later considerations in Section 3.5 indicate that the bracketed term in eqn. (3.57) is always positive and finite,
provided that the stator coil pitch is less than 180°. The current constant of a resistance-free exciter thus has a 90° phase lead over the voltage constant, a necessary precondition if the exciter is to provide the correct phase sensitive current compounding to a cylindrical-rotor generator.

Suppose that the exciter is used with a cylindrical-rotor generator with an output voltage of \( V \), such that the voltage across the shunt winding is \( n_v V \). In a similar way to generators equipped with static current-compounding schemes, the no-load voltage can be determined from the intersection of the exciter and the generator characteristics. The slope of the exciter characteristic is now given by \( 1/K_{dc} n_v C V l \), which can be altered by appropriate design to a suitable value to give the required no-load output voltage. If the generator has a synchronous reactance \( X_s \) and the exciter is designed such that \( n_v X_s = \frac{X_{11}}{X_{13}} (X_{23} - \frac{X_{13} X_{12}}{X_{11}}) \), then the current-compounding characteristic of the exciter is matched to the generator excitation characteristic. A resistance-free MAC exciter will accordingly provide a loaded cylindrical-rotor generator with precisely the excitation current required to maintain a constant terminal voltage, irrespective of the load power factor. Obviously, the equality has to be altered slightly to give the best overall results when non-linear effects such as saturation are taken into account. This is
also necessary when the generator is a salient-pole machine, though in such a case the exciter can also maintain the terminal voltage substantially constant at the usual conditions of load and lagging power factor.

When the resistance of the shunt winding is allowed for, the phase lead of the current constant deviates from the ideal 90° and the stator coils must be short-chorded by a considerable amount for the exciter to provide a reasonably phase sensitive excitation current. The full significance of this is considered in Section 3.4.

3.3.3 The compounding equations allowing for iron loss

In the normal circuit-theory analysis of a flux-creating winding, where a magnetic core is involved, iron loss in the core is accounted for by the inclusion of a parallel resistor in the equivalent circuit to dissipate an amount of power equals to the core loss. On this basis the equivalent circuit for the MAC exciter may be revised to include the iron loss, as shown in Fig. 3.8. Based on that figure, the following equations apply when the rotor is on open circuit.

\[
\frac{V_{20}}{s} = V_1 C_{v1} + I_2 C_{i2} \tag{3.58}
\]

\[
I_1 = \frac{V_1 - jX_{12} a_2 I_2}{R_{11} + ja_1 Y_{11}} \tag{3.59}
\]

\[
V_2 = jX_{12} a_1 I_1 + I_2 (R_{22} + ja_2 X_{22}) \tag{3.60}
\]

where \( a_1 = \frac{R_{10}}{R_{10} + jX_{11}} \), \( C_{v1} = \frac{ja_1 Y_{11}}{R_{11} + ja_1 Y_{11}} \)
\[ a_2 = \frac{R_{20}}{(R_{20} + jX_{22})} \]
\[ C_{12} = ja_2 X_{23} + (a_1 a_2 X_{13} X_{12}/(R_{11} + ja_1 X_{11})) \]

Using the same method as in Section 3.3.1 to represent a rectifier load, the new compounding equations are

\[ I_3 = \frac{V_1 C_{V1} + I_2 C_{I2}}{Z_{et}} \quad \ldots (3.61) \]
\[ I_1 = \frac{V_1 - jX_{12} a_2 I_2 - jX_{13} I_3}{R_{11} + ja_1 X_{11}} \quad \ldots (3.62) \]
\[ V_2 = jX_{12} a_1 I_1 + (R_{22} + ja_2 X_{22}) I_2 + jX_{23} I_3 \quad \ldots (6.63) \]

where

\[ Z_{et} = \frac{R_e}{s} + R_c + j(X_e + X_c) \]
\[ Z_c = R_c + jX_c = \frac{R_{22}}{s} + jX_{33} + \frac{a_1 X_{13}^2}{R_{11} + ja_1 X_{11}} \]

and the remaining variables are as defined in eqns. (3.58) to (3.60).

Apart from altering the magnitudes of the voltage and current constants, the introduction of iron loss into the analysis affects the phase difference between the two constants. However, with both the stator and the rotor of the exciter laminated to minimise iron loss, the current compounding performance of the exciter is not expected to be seriously affected.

3.4 The voltage and current constants
To facilitate a closer examination on the effect of these two constants, it is useful to define a ratio $K_{mn}$ such that

$$K_{mn} = \frac{X_{mn}}{N_m N_n} \quad \ldots (3.64)$$

where $X_{mn}$ is the reactance between the $m$ and $n$ windings and $N_m$ and $N_n$ are respectively the effective number of turns of these two windings. If $m = n$, then $X_{mn}$ is a self reactance, whereas if $m \neq n$ it is a mutual reactance. The ratio $K_{mn}$ can also be visualised as the reactance between the $m$ and $n$ windings, when each winding consists of one effective turn.

The shunt and series windings of a MAC exciter are similarly arranged in the stator slots but with $180^\circ$ displacement. Since the only other difference between these two windings is their number of effective turns, it follows intuitively that $K_{13} = K_{23}$, a fact confirmed during a later experimental investigation. It can be shown, using eqns. (3.48) and (3.49), that

$$|C_{vl}| = \frac{K_{13} N_2}{K_{11} N_1} \frac{1}{\sqrt{1 + x_{11}^2}} \quad \ldots (3.65)$$

$$|C_{i2}| = \frac{K_{13} N_2 N_3}{\sqrt{1 + x_{11}^2}} \{(1 - \frac{K_{12}}{K_{11}})^2 + x_{11}^2\}^{\frac{1}{2}} \quad \ldots (3.66)$$

$$\zeta = \tan^{-1}\left(\frac{1}{x_{11}}(1 - \frac{K_{12}}{K_{11}})\right) \quad \ldots (3.67)$$

where $\zeta$ is the phase lead of the current constant over the voltage constant. The previous considerations of
Section 3.3.2 established $\zeta = 90^\circ$ as the basis for an ideal compounding scheme for a cylindrical-rotor generator. When $\zeta$ is any general angle the voltage and the current components of the excitation are added as indicated in Fig. 3.9, from which it is evident that the maximum compounding occurs when the two components are co-phasal. This occurs when the load current has a lagging power factor of $\cos \zeta$, since the mutual reactance between two windings of one effective turn each is always less than the self reactance, i.e. $K_{12} < K_{11}$. Thus, $\zeta$ must not deviate too much from $90^\circ$ if the compounding effect is to correspond to changes in the load power factor.

Eqns. (3.66-3.67) show the direct effect of the ratio $K_{12}/K_{11}$ on $\zeta$ and $|C_{12}|$, and hence on the whole compounding characteristic. This ratio is considered in more detail in the following sections.

3.4.1 The ratio $K_{12}/K_{11}$ and the stator coil pitch

The ratio of $K_{12}/K_{11}$ can be obtained from the division of the mutual reactance between the shunt and series winding by the self reactance of the shunt winding, when both windings contain only one effective turn each. To obtain an expression for $K_{12}/K_{11}$, it is therefore necessary to consider the flux linkages between the different stator coil groups, so that $X_{12}$ and $X_{11}$ can be determined. In the considerations which follows it is assumed that the stator and rotor slots are of negligible width, and that the m.m.f. waveforms have therefore a vertically stepped distribution.
3.4.1.1 Flux linkage between coil group A and coil B

Consider a stator coil group A of M coils, with \( N_A \) turns in each coil and a coil pitch of \( \alpha_A \). The coils are distributed in \( M \) adjacent stator slots separated by a slot angle \( \gamma \).

With the near side of the first coil defined as the origin, the m.m.f. distribution of the coil group when carrying a current \( I_A \) and for the particular case of \( M = 3 \) is given in Fig. 3.10. If it is recognised that the relative permeability of the stator and rotor iron is not infinite, then the resulting flux density distribution is that of Fig. 3.11. Appendix A2 provides an explanation for this distribution and also the definition of \( h \). If coil B has \( N_B \) turns and pitch of \( \alpha_B \) and the near side of the coil is at angle \( \theta_B \) from the origin, then the usual techniques of Fourier analysis and integration show that the flux linkage of coil B is

\[
\psi_{AB} = \frac{4I_A \mu_0 K}{\pi g} \sum_{n=1}^{\infty} A_n (B_n - x_n C_n) \quad \ldots \quad (3.68)
\]

where

\[
x_n = \frac{2h}{N_A I_A} \quad \ldots \quad (3.69)
\]

\[
A_n = \frac{1}{n^2} \sin\left(\frac{n \alpha_B}{2}\right) \cos\left(\frac{\alpha_A}{2} - \frac{\alpha_B}{2} + \gamma - \theta_A\right) \quad \ldots \quad (3.70)
\]

\[
B_n = \frac{\sin\left(\frac{Mn\gamma}{2}\right)}{M \sin\left(\frac{n\gamma}{2}\right)} \sin\left(\frac{n \alpha_A}{2}\right) \quad \ldots \quad (3.71)
\]

\[
C_n = \frac{1}{n} (1 - \cos nn) \quad \ldots \quad (3.72)
\]
$K_a$ is the specific coil area, i.e. area per radian, and $g$ the airgap length.

### 3.4.1.2 Flux linkage between coil group A and another coil group C

Suppose that a coil group C consists of $J$ coils distributed in $J$ adjacent slots, with a coil pitch of $\alpha_C$ and containing $N_C$ turns per coil, and that the near side of the first coil is an angle $\theta_C$ from the origin. When coil group A carries a d.c. current $I_A$, the mutual flux linkage of coil groups A and C is given by

$$\psi_{AC} = \frac{4I_A \mu_0 K_a N_A N_C}{\pi g} f(\theta_C)$$

... (3.73)

where $f(\theta_C) = \sum_{n=1}^{\infty} \left\{ \sin(\frac{n\alpha_C}{2}) JK_{dn} \cos(n\theta_C) \left( \frac{MK_{dn}}{n^2} \sin(\frac{n\alpha_A}{2}) \right) - \frac{xh}{n^3}(1 - \cos n\pi) \right\}$

... (3.74)

$$K_{dn}^M = \frac{\sin(\frac{MnY}{2})}{M \sin(\frac{NY}{2})}$$

... (3.75)

$$K_{dn}^J = \frac{\sin(\frac{JnY}{2})}{J \sin(\frac{NY}{2})}$$

... (3.76)

If $\omega$ is the angular frequency, then

$$X_{AC} = \frac{4\omega \mu_0 K_a N_A N_C}{\pi g} f(\theta_C)$$

... (3.77)
3.4.1.3 Expression for $K_{12}/K_{11}$

By reference to Section 3.2.1 for the phase transformation, and using eqn. (3.77), it can readily be shown that

$$\frac{K_{12}}{K_{11}} = \frac{f(300^\circ) - f(180^\circ)}{f(0^\circ) - f(120^\circ)} \quad \ldots (3.78)$$

where eqn. (3.74) defines the function $f$. Thus, with a given stator, the ratio $K_{12}/K_{11}$ can be changed most easily by using different values for the stator coil pitch. Variations in the ratio $K_{12}/K_{11}$ with the coil pitch for the MAC exciter are given in Fig. 3.12, for a range of $x_h$. It should be noted that both shunt and series windings of the exciter considered have 3 slots/pole/phase. Inspection of Fig. 3.12 shows that the stator coil pitch has a very direct effect on the ratio $K_{12}/K_{11}$. Consider, for example, the case when it is assumed that the iron is infinitely permeable, i.e. $x_h = 0$, so that the ratio varies almost linearly between a very small value when the coil pitch is $20^\circ$ to unity when it is $180^\circ$. The variations of this ratio at the other values of $x_h$ show a similar trend.

3.4.2 Effect of $K_{12}/K_{11}$ on the compounding characteristic

The stator coil pitch exerts a major influence on the ratio of $K_{12}/K_{11}$, which in turn has a dominating effect on the compounding characteristics. For the case of $x_{11} = 0$, the phase lead is always $90^\circ$. The magnitude of the current
constant however decreases as the coil pitch increases, and it approaches zero when the coil pitch nears 180°. This suggest that the compounding, as indicated by \(|C_{12}|\), may be changed by altering the stator coil pitch. If the fact that \(x_{11}\) is non-zero is accounted for, then the reduction in the magnitude of the current constant is accompanied by a decrease in the phase lead \(\zeta\), as shown in Fig. 3.13. In deriving Fig. 3.13 the value of \(x_{11}\) is assumed to be 0.15, a value that can readily be achieved in most practical designs. The stator coils of a MAC exciter must always therefore be appreciably short-chorded, to preserve an adequate phase lead, so that the compounding can correspond reasonably well with the variations in load power factor. Alternatively, the ratio \(x_{11}\) may be made as small as practicable, to reduce the effect of \(K_{12}/K_{11}\) on the phase lead.

3.5 Slip and power amplification

In the general case the pole number of the MAC exciter need not be the same as that of the generator, implying that the exciter can operate at a slip other than 2. If the exciter and generator have respectively \(p_e\) and \(p_g\) pole-pairs, the slip of the exciter is then

\[
s = 1 \pm \frac{p_g}{p_e} \quad \ldots \text{(3.79)}
\]

where the positive sign is taken if the airgap m.m.f. of the exciter rotates in the reverse direction to the shaft, and
vice versa. When both the generator and the exciter have the same pole number, eqn. (3.79) indicates that it is only meaningful to operate the exciter with a slip of 2. (The alternative slip of zero corresponds to the situation where the rotor rotates in synchronism with the airgap m.m.f., and no voltage is induced in the rotor winding). If the pole numbers are different, then the exciter may operate at one of the two alternative slips. Normally, the larger value is preferred, since a smaller value of \( Z_{et} \) (see eqns. (3.50) and (3.53)) allows more efficient use of the exciter windings. It also renders the field current less dependent on the resistive term, and hence less sensitive to the variation in the ambient temperature. In addition, the following considerations on power amplification provide yet another reason in favour of the higher slip operation, although it should be noted that such operation entails higher losses.

Using the compounding equations (eqns. (3.50-3.52)), for the MAC exciter, it can be shown that the field power required is \( 3I_3^2R_e \) and that the amount supplied electrically is \( 3I_3^2(\Re_e/s) \), with the remainder coming directly from the prime mover. If all iron and copper losses in the exciter are neglected, the ratio of these two quantities is \( s \), which is the power amplification of the exciter. The combined power input to the shunt and series windings is amplified by this factor to give the field power, and it is this feature which is introduced by the change in frequency between the stator
and the rotor of a MAC exciter that differentiates it clearly from corresponding static current-compounding schemes. The generator connected to the exciter therefore needs to provide only a portion of its own excitation power if the slip is greater than unity, an operating condition that can always be achieved in practice.
Fig. 3.1 Stator connection diagram
(SE-Series, SH-Shunt)
Fig. 3.2 Rotor connection diagram
Fig. 3.3 Layout of FCE synchronous generator

Fig. 3.4 Circuit diagram for current-compounding an a.c. generator with MAC exciter
Fig. 3.5 Phase-variable representation of MAC exciter

Fig. 3.6 (α, β, 0) representation of MAC exciter
Fig. 3.7 3-coil equivalent circuit of MAC exciter

Fig. 3.8 3-coil equivalent circuit of MAC exciter
(Allowing for iron loss)
Fig. 3.9 Phasor diagram of current compounding

Fig. 3.10 m.m.f. distribution for coil group A with $M = 3$

Fig. 3.11 Flux distribution resulting from m.m.f. of Fig. 3.10
Fig. 3.12 Ratio $K_{12}/K_{11}$ at different stator coil pitch
Fig. 3.13 Phase lead at different stator coil pitch
(For $x_h = 0$ and $x_{11} = 0.15$)
CHAPTER 4
EXPERIMENTAL INVESTIGATION

The theoretical considerations of Chapter 3 have shown that a MAC exciter can be represented by a simple 3-coil equivalent circuit, from which certain important parameters that determine its current-compounding performance can be identified. To verify experimentally that the 3-coil model is adequate, a number of tests were performed on two experimental MAC exciter units (EMEU), hereafter referred to as EMEU-1 and EMEU-2. The essential data for EMEU-1 is summarised in Figs. 3.1 and 3.2 and in Table 4.1. Except for the stator coil pitch, which is 180° rather than 120°, the data for EMEU-2 is similar to that for EMEU-1. It is perhaps worthy to note that the coil pitches have been selected to give the maximum illustration of the factors that determine the compounding performance.

In this chapter, a detailed account of these tests is presented, under the headings of open-circuit tests, open-circuit load tests (without the rectifier load) and rectifier-load tests. The open-circuit test enables the parameters of an EMEU to be determined, while the various load tests yield results that can be compared with predicted results to establish the validity of the model. Computer programs for automatic computation of the exciter parameters from the results of the open-circuit test and for the computation of the exciter performance are developed, and the
computational techniques employed in these programs are discussed in detail. The difference in the compounding behaviour of the two experimental units is explained. Finally, some results from an operational exciter unit are included to give further confirmation of the computer model.

4.1 Open-circuit test and parameters of EMEU†

The rotor of the EMEU on which the tests were to be performed was locked securely into a supporting frame. One of the exciter windings was then supplied with an increasing and balanced 3-phase voltage, with the two other windings open-circuited, until the current level in the winding being supplied reached about 125% of its rated level. The voltage across the three windings, together with the supply current and the input power, were all measured at intervals of about 2A of supply current, and the process was repeated with the other two windings of the exciter supplied in turn.

Figs. 4.1a, c and e show the full open-circuit characteristics for EMEU-1, with the corresponding variations in input powers being given in Figs. 4.1b, d and f. The corresponding characteristics for EMEU-2 are given in Figs. 4.2a-f. Actual data points are indicated by the various symbols in the figures, with the full lines being obtained using a computer curve-fitting subroutine, which implements the technique of fitting a rational fraction polynomial to a set of experimental data. Further details

† The test exciters were originally intended for 60Hz operation. However, for reason of availability, both open-circuit and load tests were performed using a 50Hz supply. It should also be noted that the open-circuit test was performed with a locked rotor, the term 'open-circuit' referring to the state of the rotor terminal connection.
of the method are provided in Appendix A3. It should be noted that all voltages and currents are phase quantities, and that they are obtained as the average of the three phase quantities.

Since the machines under investigation have a very small airgap, the effect of saturation renders the open-circuit characteristics extremely non-linear. Inspection of Figs. 4.1a, c and e and Figs. 4.2a, c and e shows that the curve-fitting technique of Appendix A3 is ideally suited to fitting curves to this type of data. Even for the input power variations, the method is found to be satisfactory.

In Figs. 4.1 and 4.2, the subscripts denoting the various power and voltage quantities follow the usual convention. For example, $V_{13}$ represents the rotor voltage when the supply is to the shunt winding and $V_{32}$ represents the series winding voltage when the supply is to the rotor winding.

The winding resistance per phase of the various EMEU windings was measured using a Kelvin double bridge, and the results obtained are presented in Table 4.2. Using a method which allows a fairly accurate estimate to be made of the a.c. resistance of an iron-cored coil, the 50Hz resistances of the winding were found to be 1.17 times the d.c. resistances. To allow for possible experimental error and the effects of temperature rise when the EMEU is loaded, a ratio of 1.2 was assumed in all subsequent calculations where the a.c. resistances were required.
By putting $s = 1$ in the equivalent circuit of Fig. 3.7, and by using the appropriate set of open-circuit characteristics, nine secant reactances (three self and six mutual reactances) can readily be obtained at any level of magnetisation. In view of the non-linearity exhibited by the open-circuit characteristics, it is important to ensure that all the reactance calculation for any selected level of magnetisation are performed at that level throughout the full set of calculations. The six mutual reactances can then be paired without incurring any excessive error to form the three mutual reactances of Fig. 3.7. A computer subroutine was written to perform these reactance evaluations and pairing operations at any given level of magnetisation, as defined by the open-circuit rotor voltage. After the specification of a particular rotor open-circuit voltage, the subroutine makes use of the two open-circuit voltages (when any one of the three exciter windings is being supplied) to determine the appropriate current level at which the reactance evaluation is to be performed. For any set of calculations within the range of data points on the open-circuit characteristics, the two reactance to be paired were found to differ from each other by less than 0.1%. Outside this range the accuracy deteriorates only slightly, demonstrating that the methods employed in both the curve fitting and the reactance calculations are extremely accurate.

The equivalent circuit of Fig. 4.3 can be used in the calculations when the supply is to the shunt winding and it
is required to account for iron loss. Only one core-loss resistor \( R_{10} \) is needed in this circuit, since the shunt winding is the only flux-creating winding. The same principle applies if the supply is to either of the other two windings. Using the technique described above, and making appropriate use of the input power curves, six reactances and three core-loss resistances can be determined for any level of core magnetisation, again with the aid of a computer subroutine. To use these results, together with the equivalent circuit of Fig. 3.8, it is assumed that \( R_{10} = 2R_{10}' \), thus distributing the average iron loss equally between the two flux-creating windings.

4.2 Method of computation

As already mentioned, a knowledge of the magnetisation level in the exciter core (as indicated by the open-circuit rotor voltage), is required to determine the appropriate parameters of the EMEU for use in the performance calculations. However, the problem arises that this voltage is itself unknown until a solution is obtained, and it is clear that an iterative method must be used to yield the final solution from an assumed initial value. Obviously, the solution to such a problem is most readily obtained with the aid of a digital computer.

The general arrangement of the computer program for simulating the MAC exciter, and written for implementation on
a Prime 400 computer, is given in the flow chart of Fig. 4.4. The program is written in modular form, so that the segments performing certain identifiable functions closely associated with each other are grouped together, wherever convenient, to form subroutines. The three main subroutines formed in this manner are for curve fitting, parameter evaluation and rectifier simulation, as shown in Fig. 4.4. Although the other blocks of the flow chart are mostly embodied in the main program, some of their functions are implemented with small auxiliary subroutines. The purpose of the curve-fitting subroutine and the techniques employed in the parameter-evaluation subroutine have already been considered in the preceding section. However, it should be stressed that the six exciter reactances are obtained from the nine curves fitted to the set of open-circuit characteristics, with the magnetisation level specified by the open-circuit rotor voltage. Such an approach appears to be not very useful when a rectifier load, or any other type of load, is connected to the rotor, as is encountered in most normal operations. In such a situation, the magnetisation level can be defined in terms of the rotor magnetising voltage $V_{mr}$ such that

$$V_{mr} = (I_1' + I_2' + I_3') X_{33}$$

(4.1)

where $I_1'$ and $I_2'$ are respectively the shunt and the series currents when referred to the rotor winding. Since this
particular quantity gives an indication of the resulting flux level existing in an exciter with currents in all its windings, it defines the magnetisation level in the same way as does the open-circuit rotor voltage on the open-circuit characteristics. It will be noted that the rotor quantities have been selected to reflect the magnetisation level, since the normal double-layer rotor winding is expected to have very small flux leakage.

To initiate the iterative process, an arbitrary magnetising voltage is assumed, usually within the range of the open-circuit characteristic. Based on this value, the evaluation of the reactances and the other necessary computations were performed in accordance with the flow chart of Fig. 4.4. Before proceeding further, the resulting magnetising voltage is checked against the assumed value. If these differ by more than 1%, then the computations above are repeated, using reactances evaluated at the newly obtained value of the magnetising voltage. The procedure of computation and checking are repeated until the most recent value of magnetising voltage differs from the previous one by not more than 1%. Due to the smooth nature of the open-circuit characteristic, the solution converges rapidly, and only two or three iterations are required in most cases. Depending on the model selected, the calculations of the exciter performance are based on the compounding equations either with iron loss neglected or with iron loss included.
In normal operation, the rotor of a MAC exciter supplies the generator field winding via a 3-phase bridge rectifier. This particular arrangement was in fact investigated in the rectifier load test. In order to avoid the introduction of a significant error, unrelated to the 3-coil model of the exciter, it is therefore necessary to represent the rectifier and its load to a high degree of accuracy. With this in mind, the simple methods described in some recent studies \(^{17,21-24}\) were judged to be inadequate. Although the method in reference \(^{25}\) is significantly better than the others, it can further be improved by allowing for the imperfections of practical diodes. The required modifications, together with a full theoretical analysis, are presented in Appendix A1, which also includes an account of the computer simulation program. For the sake of completeness, a single-phase bridge rectifier is also considered in the same appendix.

4.3 **Load test**

To assess the validity of the theoretical model, a series of load test was undertaken on the two EMEUs, to provide results for comparison with results obtained as described in the preceding section. These tests were performed on the experimental units, with either a stationary or a rotating rotor. Some salient differences between the compounding characteristics of the two units can be deduced from the experimental and computed results.
4.3.1 Experimental procedures

A variable 3-phase supply used to simulate the armature winding of the generator of Fig. 3.4 was fed through an EMEU, as shown in Fig. 4.5, to a 3-phase loading bank of variable power factor. The star-connected rotor was fed through a 3-phase bridge rectifier to a resistive-inductive load drawing a fully smoothed current, and representing the field winding of the generator of Fig. 3.4. Tests were performed with switch SW1 both closed and opened, with the latter being referred to as an open-circuit load test and the former as a rectifier load test.

With the rotor of the EMEU firmly locked, open-circuit and rectifier load tests were carried out on both EMEU-1 and EMEU-2. In the open-circuit load tests, the variable a.c. voltage was adjusted to give a shunt current $I_{sh}$ of 18A for a load current $I_L$ of zero. The value of the shunt voltage was noted as 6.23V. With the power factor of the loading bank set at unity, and the shunt voltage maintained constant, the load current was increased in approximately 5A steps up to about 30A. At each step all circuit voltages and currents were recorded, and the above procedure of increasing the load current and noting the various circuit quantities was repeated for power factor of the loading bank at 0.8 lagging and zero lagging. Similar tests were carried out with switch SW1 closed, to obtain the rectifier load characteristics.

The effective resistance of the resistive-inductive load,
which simulates the field winding, was 1.75Ω.

The rotor of the EMEU-1 was then coupled mechanically to a d.c. shunt motor and driven at 1500 rpm in the reverse direction to the synchronous field established by the stator windings. With the shunt voltage maintained at about 4.5V, the previous procedures were repeated to obtain the rectifier load characteristics. A higher field resistance of 2.55Ω was used, to ensure that the d.c. current was within a convenient range.

Results from the above tests are presented in the next subsection. It should be noted that all the voltages and currents are the average of the three phase quantities, with the obvious exceptions of the d.c. current and voltage.

4.3.2 Results and discussions

Experimental results and computer predictions for the open-circuit load test on EMEU-1 at the different power factors, and with the rotor firmly locked, are presented in Figs. 4.6a-4.11b. Eqns. (3.43-3.47) with \( s = 1 \) are used to provide the computed results in Figs. 4.6a-4.8b (iron loss neglected), whereas eqns. (3.58-3.60) with \( s = 1 \) give the computed results of Figs. 4.9a-4.11b (iron loss included). Experimental results and computer predictions for the open-circuit load test on EMEU-2 at the different power factor are similarly presented in Figs. 4.12a-4.17b. It is evident from both sets of figures that the predicted results
generally agree well with the experimental results, except for the currents $I_{sh}$ and $I_{com}$ at the power factors of unity and 0.8 lagging. Although the inclusion of iron loss in the computation makes a slight improvement here, it was found that a very much more significant improvement can be achieved by re-computing results at power factor slightly different from the assumed values of 0.8 lagging and unity (as indicated on the loading bank). The two currents are clearly very sensitive to power factor changes near unity and the uncertainty in the assumed values of the power factor is therefore a main source of experimental error.

Computed and experimental results for the rectifier load test on the stationary EMEU-1 are shown in Figs. 4.18a-4.23b. The computed results in the first six of these figures, with iron loss neglected, are obtained using eqns. (3.50-3.54) while those with iron loss included are obtained using eqns. (3.61-3.63), and substituting $s=1$ in all the relevant equations. Corresponding results for the EMEU-2 are presented in a similar manner in Figs. 4.24a-4.29b. Inspection of these figures shows an excellent accuracy of prediction, again with the exception of the shunt and common currents and with the inclusion of iron loss having a slight, but perceptable, effect. These two currents are in fact very sensitive to slight variations in the power factors, indicating once again that the slight uncertainty in the assumed value of power factors is a main source of error.
Experimental and computed results for the rectifier load test on EMEU-1, with the rotor turning at 1500 rpm in the reverse direction to the synchronous field, are presented in Figs. 4.30a-4.32b. In this mode of operation the slip of the exciter is 2, and this value is used in the appropriate equations (eqns. (3.50-3.54)) to yield the computed results. These figures only show results with iron loss neglected, since the effects of including iron loss in the calculations are already familiar. In general, the computed results agree well with experimentally obtained values.

In conclusion it can be stated that the results presented here have demonstrated that the 3-coil equivalent circuit developed in Chapter 3 is basically a very good model for the EMEU and hence more generally the MAC exciter. Provided that the non-linear nature of the magnetisation characteristic is accounted for, very good predictions of the exciter performance can be obtained. It has been shown that this task can most readily be accomplished by the use of experimentally obtained open-circuit characteristics. Moreover, these characteristics, together with the winding resistances, provided sufficient input for the computation of all the required parameters. Although the inclusion of iron loss can somewhat improve the accuracy of the computed results, it is not really necessary since computations not allowing for this loss were shown to produce satisfactory results, with a consequent saving in computing time.
There are, however, several important features of both the experimental and predicted results that have so far not been discussed, and these are considered in the following section.

4.4 Current compounding characteristics

Inspection of the results for the rectifier load test shows that the d.c. current supplied to the resistive-inductive load (which represents the field winding), generally increases with an increasing load current. Since the increase in the field current is fairly linear, a good measure of the amount of compounding provided by the experimental units in each situation is given by the slope of the straight line that fits the relevant set of data points. On this basis, the results recorded in Table 4.3 were obtained, for the two EMEUs with stationary rotor. As indicated by this Table, the main difference between the two experimental units is that, while EMEU-1 provides a field current that also increases with worsening power factor, the field current of EMEU-2 varies in a reverse manner with respect to power factor. Furthermore, the field current provided by EMEU-1 changes more rapidly as the load is increased. This difference in behaviour can be explained by considering the effects the stator coil pitch have on the current constant and the phase lead of the exciter, both of which have profound implications on the compounding performance. Indeed, in Chapter 3, it was shown that a
resistance-free exciter with a stator coil pitch of $180^\circ$ will not provide a significant compounding effect. Additionally, when the effect of shunt winding resistance is included, the compounding is small and is in incorrect sense. These points have now been confirmed by experimental observations on EMEU-2.

Table 4.4 records some typical values of compounding constants for the two experimental units (with stationary rotor). Bearing in mind that the only constructional difference between the two units is the pitch of their stator windings, the values recorded in this Table illustrate the close inter-relationship between the quantity and the magnitudes of the current constant and phase lead. The small phase lead of only $17.2^\circ$ for EMEU-2 clearly indicates that the compounding effect of this exciter is far from ideal. Hence, to ensure that a resulting design will furnish a field current increasing both with load current and with worsening power factor, it is necessary to short-chord the stator coils so that sufficient phase lead is preserved.

4.5 Results for an operational MAC exciter

In the operational MAC unit on which results were available, a type WC 0077 exciter was coupled to a type WC 876 generator rated to deliver 15.7A at 0.8 power factor lagging and a terminal voltage of 230V (line). The generator field winding resistance is 1.053Ω and the shunt winding voltage
of the exciter changes from 4.15V at no load to 4.17V at rated load. Test and calculated results for the unit are compared in Table 4.5. The type WC 0077 exciter is in fact similar in all respects to the experimental unit EMEU-1, and the open-circuit characteristic of the latter is therefore used in the calculations after making suitable allowances for the now 60Hz operation. Calculated results are obtained from the computer model of the exciter, by solving the compounding equations (neglecting iron loss) for a slip of two and using the correct operating conditions for the bridge rectifier. The good agreement evident in Table 4.5 further confirms the validity of the exciter model and enables theoretical results obtained in the later studies to be treated with confidence.
### Exciter Rotor

**Wound Assy. No. 791089-0A**

**Date:** 9.3.75

**Frame:** 280  
**K.W.:** 5

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### Exciter Stator

**Wound Assy. No. 789183-0A**

**E.M.C**

**DATE:** 9.3.75

**Wound Assy. No. 789183-0A**

**Frame:** 280  
**K.W.:** 5

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<td>LAM. MAT. CRS</td>
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<td>SLOT WEDGE</td>
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<td>99 Duroid or Nomex</td>
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<td>PHASE INSUL.</td>
<td>.010</td>
<td>Varnished Glass Cloth</td>
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*Changed 3/15/76  
Was 7 and 10  
**Revised 11/2/78 was 8

---

**Table 4.1 Specification of EMEU-1**
EMEU-1 (coil pitch = 120°)
\[ R_{11} = 0.045455 \Omega \quad R_{22} = 0.052778 \Omega \quad R_{33} = 0.142815 \Omega \]

EMEU-2 (coil pitch = 180°)
\[ R_{11} = 0.052261 \Omega \quad R_{22} = 0.062925 \Omega \quad R_{33} = 0.142815 \Omega \]

Table 4.2 Winding resistances of experimental units

<table>
<thead>
<tr>
<th>Power factor</th>
<th>Average rate of change of field current</th>
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<tr>
<td>Unity</td>
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</tr>
<tr>
<td>0.8 lagging</td>
<td>0.265</td>
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<tr>
<td>zero lagging</td>
<td>0.272</td>
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Table 4.3 Average rate of change of field current with respect to load current

EMEU-1 (coil pitch = 120°)
\[ |C_{V1}| = 1.675 \]
\[ |C_{I2}| = 0.3129 \Omega \quad R_{11}/X_{11} = 0.163 \]
\[ \zeta = 69.2^\circ \ (\text{computed at } V_{mr} = 15.3V) \]

EMEU-2 (coil pitch = 180°)
\[ |C_{V1}| = 1.822 \]
\[ |C_{I2}| = 0.1463 \Omega \quad R_{11}/X_{11} = 0.1743 \]
\[ \zeta = 17.24^\circ \ (\text{computed at } V_{mr} = 14.8V) \]

Table 4.4 Some typical constants
<table>
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<th>Test results (A)</th>
<th>Predicted results (A)</th>
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<tr>
<td><strong>Generator field current</strong></td>
<td></td>
<td></td>
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<tr>
<td>No load</td>
<td>10.0</td>
<td>10.16</td>
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<tr>
<td>Rated load</td>
<td>18.0</td>
<td>18.21</td>
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<td>80% rated load at u.p.f.</td>
<td>12.7</td>
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<td><strong>Shunt winding current</strong></td>
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<td>No load</td>
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<td>20.70</td>
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<td>Rated load</td>
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Table 4.5 Comparison of test and predicted results for an operating unit (* at 60% rated load the predicted current is 12.74A)
Fig. 4.1a Open-circuit characteristics of EMEU-1 with the supply on the shunt winding 
(Full lines and symbols denote computed and experimental results respectively) 
Stator coil pitch = 120 Deg.

Fig. 4.1b Input power for the open-circuit test on EMEU-1 with the supply on the shunt winding 
(Full line and symbol denote computed and experimental results respectively) 
Stator coil pitch = 120 Deg.
Fig. 1.1c Open-circuit characteristics of EMEU-1 with the supply on the series winding
(Full lines and symbols denote computed and experimental results respectively)
Stator coil pitch = 120 Deg.

Fig. 1.1d Input power for the open-circuit test on EMEU-1 with the supply on the series winding
(Full line and symbol denote computed and experimental results respectively)
Stator coil pitch = 120 Deg.
Fig. 4.1e Open-circuit characteristics of EMEU-1 with the supply on the rotor winding
(Full lines and symbols denote computed and experimental results respectively)
Stator coil pitch = 120 Deg.

Fig. 4.1f Input power for the open-circuit test on EMEU-1 with the supply on the rotor winding
(Full line and symbol denote computed and experimental results respectively)
Stator coil pitch = 120 Deg.
Fig. 1.2a Open-circuit characteristics of EMEU-2 with the supply on the shunt winding
(Full lines and symbols denote computed and experimental results respectively)
Stator coil pitch = 180 Deg.

Fig. 1.2b Input power for the open-circuit test on EMEU-2 with the supply on the shunt winding
(Full line and symbol denote computed and experimental results respectively)
Stator coil pitch = 180 Deg.
Fig. 4.2c Open-circuit characteristics of EMEU-2 with the supply on the series winding
(Full lines and symbols denote computed and experimental results respectively)
Stator coil pitch = 180 Deg.

Fig. 4.2d Input power for the open-circuit test on EMEU-2 with the supply on the series winding
(Full line and symbol denote computed and experimental results respectively)
Stator coil pitch = 180 Deg.
Fig. 4.2e Open-circuit characteristics of EMEU-2 with the supply on the rotor winding
(Full lines and symbols denote computed and experimental results respectively)
Stator coil pitch = 180 Deg.

Fig. 4.2f Input power for the open-circuit test on EMEU-2 with the supply on the rotor winding
(Full line and symbol denote computed and experimental results respectively)
Stator coil pitch = 180 Deg.
Fig. 4.3 Definition of currents and voltages for open-circuit test when the supply is on the shunt winding

\[ X'_{13} = \frac{V_3}{I'_1} \]
\[ X'_{12} = \frac{V_2}{I'_1} \]

Fig. 4.4 Flow chart for computer simulation of MAC exciter
Fig. 4.5 Circuit diagram for load test (Shown for open-circuit load test, close switch SW1 for rectifier-load test)
Fig. 1.6a Circuit currents for open-circuit load test on EMEU-1 at unity power factor.
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1

Fig. 1.6b Circuit voltages for open-circuit load test on EMEU-1 at unity power factor.
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1
Fig. 4.7a Circuit currents for open-circuit load test on EMEU-1 at 0.8 power factor (lagging) 
(Full lines and symbols denote computed and experimental results respectively) 
Neglecting iron loss, Slip = 1

Fig. 4.7b Circuit voltages for open-circuit load test on EMEU-1 at 0.8 power factor (lagging) 
(Full lines and symbols denote computed and experimental results respectively) 
Neglecting iron loss, Slip = 1
Fig. 4.8a Circuit currents for open-circuit load test on EMEU-1 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1

Fig. 4.8b Circuit voltages for open-circuit load test on EMEU-1 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1
Fig. 4.9a Circuit currents for open-circuit load test on EMEU-1 at unity power factor (Full lines and symbols denote computed and experimental results respectively) Including iron loss, Slip = 1

Fig. 4.9b Circuit voltages for open-circuit load test on EMEU-1 at unity power factor (Full lines and symbols denote computed and experimental results respectively) Including iron loss, Slip = 1
Fig. 4.10a Circuit currents for open-circuit load test on EMEU-1 at 0.8 power factor (lagging) (Full lines and symbols denote computed and experimental results respectively) Including iron loss, Slip = 1

Fig. 4.10b Circuit voltages for open-circuit load test on EMEU-1 at 0.8 power factor (lagging) (Full lines and symbols denote computed and experimental results respectively) Including iron loss, Slip = 1
Fig. 4.11a Circuit currents for open-circuit load test on EMEU-1 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1

Fig. 4.11b Circuit voltages for open-circuit load test on EMEU-1 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1
Fig. 4.12a Circuit currents for open-circuit load test on EMEU-2 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1

Fig. 4.12b Circuit voltages for open-circuit load test on EMEU-2 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1
Fig. 4.13a Circuit currents for open-circuit load test on EMEU-2 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1

Fig. 4.13b Circuit voltages for open-circuit load test on EMEU-2 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1
Fig. 4.11a Circuit currents for open-circuit load test on EMEU-2 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1

Fig. 4.11b Circuit voltages for open-circuit load test on EMEU-2 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1
Fig. 4.15a Circuit currents for open-circuit load test on EMEU-2 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1

Fig. 4.15b Circuit voltages for open-circuit load test on EMEU-2 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1
Fig. 1.16a Circuit currents for open-circuit load test on EMEU-2 at 0.8 power factor (lagging) (Full lines and symbols denote computed and experimental results respectively) Including iron loss, Slip = 1

Fig. 1.16b Circuit voltages for open-circuit load test on EMEU-2 at 0.8 power factor (lagging) (Full lines and symbols denote computed and experimental results respectively) Including iron loss, Slip = 1
Fig. 4.17a Circuit currents for open-circuit load test on EMEU-2 at zero power factor (lagging) (Full lines and symbols denote computed and experimental results respectively) Including iron loss, Slip = 1

Fig. 4.17b Circuit voltages for open-circuit load test on EMEU-2 at zero power factor (lagging) (Full lines and symbols denote computed and experimental results respectively) Including iron loss, Slip = 1
Fig. 4.18a Circuit currents for rectifier load test on EMEU-1 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1

Fig. 4.18b Circuit voltages for rectifier load test on EMEU-1 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1
Fig. 4.19a Circuit currents for rectifier load test on EMEU-1 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1

Fig. 4.19b Circuit voltages for rectifier load test on EMEU-1 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1
Fig. 4.20a Circuit currents for rectifier load test on EMEU-1 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1

Fig. 4.20b Circuit voltages for rectifier load test on EMEU-1 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1
Fig. 1.21a Circuit currents for rectifier load test on EMEU-1 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1

Fig. 1.21b Circuit voltages for rectifier load test on EMEU-1 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1
Fig. 4.22a Circuit currents for rectifier load test on EMEU-1 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1

Fig. 4.22b Circuit voltages for rectifier load test on EMEU-1 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1
Fig. 4.23a Circuit currents for rectifier load test on EMEU-1 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1

Fig. 4.23b Circuit voltages for rectifier load test on EMEU-1 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1
Fig. 4.24a Circuit currents for rectifier load test on EMEU-2 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1

Fig. 4.24b Circuit voltages for rectifier load test on EMEU-2 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1
Fig. 4.25a Circuit currents for rectifier load test on EMEU-2 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1

Fig. 4.25b Circuit voltages for rectifier load test on EMEU-2 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1
Fig. 4.26a Circuit currents for rectifier load test on EMEU-2 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1

Fig. 4.26b Circuit voltages for rectifier load test on EMEU-2 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 1
Fig. 1.27a Circuit currents for rectifier load test on EMEU-2 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1

Fig. 1.27b Circuit voltages for rectifier load test on EMEU-2 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1
Fig. 1.28a Circuit currents for rectifier load test on EMEU-2 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1

Fig. 1.28b Circuit voltages for rectifier load test on EMEU-2 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1
Fig. 1.29a Circuit currents for rectifier load test on EMEU-2 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1

Fig. 1.29b Circuit voltages for rectifier load test on EMEU-2 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Including iron loss, Slip = 1
Fig. 1.30a Circuit currents for rectifier load test on EMEU-1 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 2

Fig. 1.30b Circuit voltages for rectifier load test on EMEU-1 at unity power factor
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 2
Fig. 1.31a Circuit currents for rectifier load test on EMEU-1 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 2

Fig. 1.31b Circuit voltages for rectifier load test on EMEU-1 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 2
Fig. 4.32a Circuit currents for rectifier load test on EMEU-1 at zero power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 2

Fig. 4.32b Circuit voltages for rectifier load test on EMEU-1 at 0.8 power factor (lagging)
(Full lines and symbols denote computed and experimental results respectively)
Neglecting iron loss, Slip = 2
CHAPTER 5
SOME DESIGN CONSIDERATIONS

A mathematical model for the MAC exciter, based on a simple 3-coil representation of the windings, was developed in Chapter 3. Use of this model, together with parameters derived from the results of open-circuit tests on the various windings, allows accurate predictions to be made of the on-load performance of the exciter. The experimental studies conducted on two experimental units and on an actual operating unit have indeed verified the validity of the theoretical model. However, open-circuit characteristics are not always available and this is of course the case when the design of a new line of exciter is being considered. In this type of situation, it is clearly necessary to evaluate the exciter parameters by some analytical approach. Such a method is presented in this chapter, whereby the various exciter parameters are expressed in terms of basic machine constants, like the dimensions of the stator and rotor punchings, the stack length, the number of turns in each winding, etc. A possible design strategy is discussed, with attention being extended to several important aspects of the design.

5.1 Parameters of the MAC exciter

The main parameters of the MAC exciter are the three self and
the three mutual reactances associated with the various windings. Once these reactances are known, together with the corresponding winding resistances, the various derived parameters of the exciter can be determined and a prediction made of the compounding characteristics of the exciter. In the present section, attention is focused therefore on the derivation of the winding reactances.

5.1.1 Background to the method

Although a MAC exciter is basically a wound-rotor induction motor, there is a major difference in that the stator contains two discrete sets of windings. Numerous papers have presented methods by which the reactances of the windings of an induction machine can be determined, with the calculations involving normally the separate determination of the magnetising and leakage reactances. Differential, slot and end-winding leakage components are normally considered, with a division of the differential leakage into its zig-zag and belt-leakage components frequently being made. In the method described below for the MAC exciter, the same procedure of separating the calculation into two main parts is followed. However, although the magnetising reactance is obtained by a technique similar to that described in the literature, a rather different emphasis is placed on the leakage components. The differential leakage of an induction motor winding with more than 1 slot/pole/phase is a very small percentage of the magnetising reactance, and the
overall calculation of the other leakage reactances assumes therefore a major importance. On the other hand, the rather unconventional stator windings of the MAC exciter have a comparatively large differential leakage, and in order to simplify the following analysis it is assumed that it is reasonable to neglect all but this component. Nevertheless, the analysis is structured in a form that permits ready introduction of the other leakage components should this necessity arise.

5.1.2 Magnetising reactance

The magnetising reactance attributed to the fundamental flux linkage of a 3-phase winding can be expressed\(^27\) as

\[
X_m = \frac{24\pi f N^2 K_p^2 K_d^2 DL}{10^7 p^2 g_e K_s} \quad \ldots (5.1)
\]

where \(f\) is the operating frequency (Hz), \(N\) the number of series turns per phase, \(K_d\) the distribution factor, \(K_p\) the coil pitch factor, \(D\) the mean airgap diameter (m), \(L\) the stack length (m), \(p\) the number of pole-pairs, \(g_e\) the equivalent airgap length (m) and \(K_s\) the saturation factor.

The slotted surfaces on one or both side of the airgap distort the airgap flux, effectively extending the flux path and resulting in the airgap having an equivalent length \(g_e\) greater than its actual length \(g\). If only one surface is slotted, then\(^{33,34}\)
\[ g_e = g \left( \frac{1}{1 - \frac{ab}{a+b}} \right) \] ... (5.2)

where \( \sigma = \frac{1}{90} \tan^{-1} \frac{b}{2g} - 1.466 \frac{g}{b} \log \left( 1 + \frac{b^2}{4g^2} \right) \) ... (5.3)

and \( a \) and \( b \) are respectively the tooth width and the slot width at the surface. The term \( \sigma \) is commonly referred to as Carter's coefficient and it is introduced to account for the effect of flux fringing. When the stator and rotor are both slotted, the equivalent airgap is

\[ g_e = g \left( \frac{1}{1 - \frac{\sigma_s b_s}{a_s + b_s}} \right) \left( \frac{1}{1 - \frac{\sigma_r b_r}{a_r + b_r}} \right) \] ... (5.4)

where subscripts \( s \) and \( r \) denote respectively the appropriate stator and rotor quantities as defined in eqns. (5.2) and (5.3). The multiplying factor in eqn. (5.4) is also known as the gap extension factor, and a more rigorous evaluation of this factor for the doubly slotted armature surfaces \(^{35}\) shows that eqn. (5.4) always provides a over-estimate of the true value. In the case where the tooth and slot are of equal width, the over-estimate is about 4%, although it should be noted that the effects of saturation in small machines are difficult to quantify and that the associated error may render this figure insignificant.

The most difficult term to establish numerically in eqn. (5.1) is the saturation factor \( K_s \), and the problem can only be resolved when a certain level of airgap flux density is
assumed. Under this assumption, reference (27) gives the saturation factor as

\[ K_s = 1 + \frac{F_t}{F_c} \frac{F_c + F_g}{F_g} \] ...

(5.5)

where \( F_t \), \( F_c \) and \( F_g \) are the respective m.m.f.'s required by the tooth, core and airgap portions of the magnetic circuit to establish the required airgap flux density. Since the flux density in a MAC exciter depends on the loading conditions of the generator, the difficulty of selecting the level at which it is appropriate to calculate \( K_s \) is obvious. Perhaps the most reasonable approach is to perform the calculation at the level appropriate to the rated load conditions.

5.1.3 Self and mutual reactances

The differential flux linkage of a phase winding is defined as the difference between the total flux linkage and that due to the fundamental component of the flux density distribution. If the total flux linkage of the shunt winding of the MAC exciter is \( \psi_{1l} \) and that due to the fundamental component is \( \psi_{1lf} \), the differential flux linkage is \( (\psi_{1l} - \psi_{1lf}) \). If the magnetising reactance of the winding is \( X_{1lm} \), the differential leakage reactance is given by

\[ X_{1ld} = \left( \frac{\psi_{1l} - \psi_{1lf}}{\psi_{1lf}} \right) X_{1lm} \] ...

(5.6)
and with all leakage, other than differential leakage, disregarded, the total self reactance of the winding is

\[ X_{11} = \frac{\psi_{11}}{\psi_{11f}} X_{11m} \]  (5.7)

The self reactances of the series and rotor windings may similarly be expressed as

\[ X_{22} = \frac{\psi_{22}}{\psi_{22f}} X_{22m} \]  (5.8)

and

\[ X_{33} = \frac{\psi_{33}}{\psi_{33f}} X_{33m} \]  (5.9)

respectively.

When expressed in terms of the magnetising reactances, the three mutual reactances of a MAC exciter are

\[ X_{12} = \frac{\psi_{12}}{\psi_{11f}} X_{11m} \]  (5.10)

\[ X_{13} = \frac{\psi_{13f}}{\psi_{11f}} X_{11m} \]  (5.11)

\[ X_{23} = \frac{\psi_{23f}}{\psi_{22f}} X_{22m} \]  (5.12)

Inspection of eqns. (5.10-5.12) shows that the total flux linkages are used in the determination of \( X_{12} \), in contrast to the fundamental components used when evaluating \( X_{13} \) and \( X_{23} \). This apparent discrepancy can, however, be justified if
further consideration is given to the harmonic content of the flux waveform associated with each winding. Furthermore, the use of fundamental mutual flux linkages can bring considerable simplification (as evident in Section 5.1.4) to the calculation and it is therefore often adopted when this is not expected to introduce any significant error.

It is apparent that the accuracy of the proposed method depends primarily on the accuracy with which the various magnetising reactances and flux linkages are determined and iron saturation may be a major factor limiting this accuracy. Whilst the effect saturation has on the magnetising reactances is satisfactorily accounted for by the introduction of a saturation factor, its effect on the various flux linkages is more difficult to predict and will be neglected in this initial analysis.

5.1.4 Flux linkages

Under the assumptions that the slots have negligible width and that the iron of the magnetic circuit is infinitely permeable, it can be shown from eqn. (3.73) that the fundamental flux linkage of a full-pitch $S$ slots/pole 3-phase winding carrying unit current is

$$\psi_f = \frac{6SN^2K_w^2}{\pi^2} \quad \ldots (5.13)$$

where $N$ is the number of series turns per phase and $K_w$ the
winding factor. The unit for $\psi_f$ is chosen so that a single-turn full-pitch coil produces a total flux linkage of $S$ Wb-t, and this is represented by the area under the flux density distribution of Fig. 5.1. It will become clear later that this particular choice of unit greatly simplifies the calculation of the total and mutual flux linkages.

5.1.4.1 $\psi_{11}$, $\psi_{22}$ and $\psi_{12}$

A graphical approach may conveniently be adopted for the evaluation of $\psi_{11}$, $\psi_{22}$ and $\psi_{12}$, and the technique involved is illustrated by consideration of a 36-slot stator containing two sets of 4-pole windings each short-pitched by 3 slots. In the usual MAC terminology these are the shunt and series windings, and they are accordingly denoted by the subscripts sh and se. If the slots are numbered from 1 to 36, the two sets of windings are distributed as shown in Fig. 5.2a, with the top layer containing one set of coil sides and the bottom layer the other set. On the basis of Fig. 5.1, and with one turn per coil, the flux density distribution due to unit current in the series winding is as shown in Fig. 5.2b. If the total flux linkage this distribution produces with the series coils in the a phase is $A_{se}$ and the corresponding quantity for the b phase is $B_{se}$, then

$$\psi_{22} = A_{se} - B_{se} \quad \ldots (5.14)$$

The two flux linkages in the above equation can most readily
be evaluated by determining the area beneath the flux density distribution, between the limits determined by the two coil sides of the appropriate coil. The total flux linkage for the shunt winding $\psi_{11}$ can be determined in a similar manner, with the expression for the mutual flux linkage of the shunt winding being

$$\psi_{12} = B_{sh} - A_{sh} \quad \ldots (5.15)$$

where $A_{sh}$ and $B_{sh}$ have meanings which correspond to those for $A_{se}$ and $B_{se}$. It should be noted that, if coils from the same winding do not all have the same number of turns, the steps in the flux-density distribution must be altered accordingly and an appropriate allowance made when calculating the flux linkages in eqns. (5.14) and (5.15).

Since the flux linkages in eqns. (5.14) and (5.15) are all expressed in the same unit as the fundamental flux linkage in eqn. (5.13), the ratios $\psi_{11}/\psi_{11f}$, etc. are obtained by the appropriate division. Table 5.1 gives the ratio $\psi_{11}/\psi_{11f}$ as determined on this basis for various values of the coil pitch of a MAC exciter winding, and also for a normal 3-phase double-layer winding. Inspection of the Table shows that, with the exception of the case when the coil pitch equals 180°, the differential leakage forms a major part of the stator leakage of a MAC exciter. As it is undesirable† to use a stator coil pitch of 180°, the present approach in which major importance is attributed to the differential leakage.

† See Sections 3.4.2 and 4.4.
is fully justified.

To facilitate automatic computation, the graphical method described in the preceding paragraphs may be formulated into expressions suitable for implementation on a digital computer. If the $S$ slots/pole 3-phase stator windings of the exciter have $Q$ slots/pole/phase (i.e. $Q = S/3$), stator coils short-chorded by $B$ slots, and if it is further assumed that all stator coils contain only one turn each, then

$$\psi_{11}^I = \frac{6SQ^2K_w^2}{\pi^2} \quad \ldots \quad (5.16)$$

$$\psi_{11} = Q^2(S - B) + \frac{Q}{3}(1 - Q^2) - \sum_{k=0}^{Q} \sum_{j=1}^{Q} f_1(j,k) \quad \ldots \quad (5.17)$$

where $f_1(j,k) = (S - Q - B - j - k)$ when $j + k < S - Q - B$

$$= 0 \quad \text{otherwise}$$

$$\psi_{12} = \sum_{k=0}^{Q} \sum_{j=1}^{Q} f_2(j,k) - 2 \sum_{k=0}^{Q} \sum_{j=1}^{Q} f_3(j,k) \quad \ldots \quad (5.18)$$

where $f_2(j,k) = (S - B - j - k)$ when $j + k < S - B$

$$= 0 \quad \text{otherwise}$$

and $f_3(j,k) = (Q - B - j - k)$ when $j + k < Q - B$

$$= 0 \quad \text{otherwise}$$

In the above equations, the primed quantities denote that the various flux linkages have been computed on a per-turn basis. When the stator windings have different number of turns, the required ratios are given by
\[
\frac{\psi_{11}}{\psi_{11f}} = \frac{\psi_1}{\psi_{11f}} 
\] ...
(5.19)

\[
\frac{\psi_{12}}{\psi_{11f}} = \frac{N_2 \cdot \psi_{12}}{N_1 \cdot \psi_{11f}}
\] ...
(5.20)

where \(N_1\) and \(N_2\) are respectively the number of series turns per phase of the shunt and series windings.

5.1.4.2 \(\psi_{33}, \psi_{13}\) and \(\psi_{23}\)

Since the rotor winding of a MAC exciter is of a conventional double-layer construction, the per unit differential leakage can be expressed in the familiar form \((\psi_{33} - \psi_{33f})/\psi_{33f}\), as given in reference (27), and the required ratio \(\psi_{33}/\psi_{33f}\) can then be expressed as \(1 + (\psi_{33} - \psi_{33f})/\psi_{33f}\).

When only the fundamental mutual flux linkages are considered, it can be shown that

\[
\frac{\psi_{13f}}{\psi_{11f}} = \frac{K_w N_3}{K_w N_1}
\] ...
(5.21)

\[
\frac{\psi_{23f}}{\psi_{22f}} = \frac{K_w N_3}{K_w N_2}
\] ...
(5.22)

Thus, the required ratios can be expressed merely in terms of the appropriate winding factors and winding turns.

5.1.5 Comparison of results and discussions

Table 5.2 presents the parameters for exciter unit WC 0077,
as obtained using methods above and the results of open-circuit tests. The values presented correspond to rated load and power factor conditions (15.7A at 0.8 lagging), and the value of the saturation factor $K_s$ used in the calculated results is in fact obtained by test. Inspection of the Table shows generally good agreement between the two sets of results, with any discrepancies due largely to the fact that only differential leakage is accounted for and that various saturation effects are neglected. When the calculated parameters are used to estimate the generator field current for no-load and for rated load and power conditions, the values of 10.3A and 16.7A compares well with corresponding test figures of 10A and 18A. Since errors in the predicted values of both $|C_V|$ and $X_c$ have opposing effects on the calculation of field current, the predicted currents are evidently more accurate than the parameters values. In conclusion, the simple analytical approach presented in this section is capable of providing results of sufficient accuracy to enable it to be used for many important investigations of the exciter.

5.2 Design considerations

The main objective of the design process is to produce an exciter with parameters that provide the compounding characteristic required to establish the correct level of excitation in the generator under all loading conditions. However, since ideal compounding at all levels is not possible
with a salient-pole machine, it is normal to design an exciter to provide the correct field current at no load and at rated load conditions. Minor design changes may then be made to obtain an improved overall compounding characteristic.

There are basically two approaches possible to the design of electrical machines, which may be termed respectively design by analysis and design by synthesis. In the first of these, the basic machine parameters (dimensions of stator and rotor punchings, slot details, winding arrangement, etc.) must be expressed explicitly in terms of the parameters that describe the desired performance, and this is the most difficult and potentially the most hazardous part of the approach. Design by synthesis is in essence a cut-and-try method, and is far more practical to implement. In the design process the performance is computed for a set of basic machine parameters, with appropriate adjustments being made to these until the desired performance is achieved. It is therefore a laborious approach, although a digital computer may be utilised to aid a rapid search for an ideal design.

In common with the approach followed for other types of electrical machine, the design by synthesis approach is clearly the most suitable method for tackling the design problems posed by a MAC exciter. Since the exciter uses standard induction motor type punchings for both rotor and stator laminations, the problem is to determine suitable laminations and stack lengths and to select windings which
give the required characteristic. Some of the major considerations involved are discussed below in the present section.

5.2.1 Generator data

Since a MAC exciter does not function in the manner of an error-actuated closed-loop control scheme, a given design will be intended for use with a generator requiring a given terminal-voltage/load-current characteristic. Before the design process can be initiated it is therefore necessary for the following generator data to be provided:

(a) Rating and field resistance.

(b) Field current at no load and at rated load and power factor. The synchronous reactances may naturally be provided as an alternative to the field current at rated load.

(c) Voltage available for the shunt winding of the exciter.

Although all the above information can be obtained from the generator design data, the use of test data wherever possible will account for practical imperfections not normally part of the paper considerations.

5.2.2 Flux levels

A MAC exciter should be designed to operate always at a reasonable level of flux density, although this will of course vary widely between the no load and rated load
conditions. Too high a flux density will lead to problems associated with saturation and excessive iron loss, while too low a level will not make economic use of the iron. A good measure of the magnetic loading in the exciter is the mean airgap flux density $\bar{B}_g$, given by

$$\bar{B}_g = \frac{\phi}{\pi D L} (2p) \quad \ldots \ (5.23)$$

where $\phi$ is the fundamental flux per pole and the other symbols have the same significances as in eqn. (5.1). For a stator supply frequency of $f$, the fundamental flux/pole of a 3-phase winding is

$$\phi = \frac{V_{mr}}{\sqrt{2\pi N_3 f K_w}} \quad \ldots \ (5.24)$$

where $V_{mr}$ is defined by eqn. (4.1). The maximum allowable mean airgap flux density must be kept below the level which causes the back iron to saturate excessively. Assuming that a flux density of 1.4T represents an acceptable maximum for iron, then

$$\bar{B}_g < 1.4\left(\frac{t_b}{t_b + s_b}\right) \quad \ldots \ (5.25)$$

where $t_b$ and $s_b$ are respectively the widths of the tooth and the slot at the back of the slot. Working backwards from this operating flux density enables the corresponding no-load level to be determined.
One particularly desirable feature of a MAC exciter is its ability to boost the generator field current such that the generator can supply 5 to 10 times full-load current into a terminal short circuit. This ability is however only possible if the exciter is designed to operate on normal loading with a flux density much below that given by eqn. (5.25). Available data on an operational unit shows that the required performance is achieved by keeping $B_g$ under normal full load conditions to about 0.13T. Although this may seem uneconomic, it is the penalty that must be paid to provide a generator/exciter unit ideally suited for situations in which relatively large induction motors have to be started directly across the generator terminals.

5.2.3 Shunt, series and rotor windings

In the present arrangement, as shown in Fig. 3.4, the shunt winding is connected across a portion of the armature winding such that the winding can be designed to carry the same level of current as that of the series winding, and thus to allow conductors of similar size to be used for both sets of windings. Although this approach is advantageous from the manufacturing point of view (by reducing the range of required wire stock), it is by no means a necessity since the shunt winding may alternatively be designed for direct connection across the whole of the armature winding, resulting in the arrangement of Fig. 5.3. With the number of shunt turns adjusted to reflect the voltage difference between the two
connections, the performance of both exciters is in all respects precisely the same. Connecting the shunt winding across the whole of the armature winding has the advantage of not requiring additional voltage tapping points on the armature, although this must be weighed against the inconvenience of using conductors of greatly different sizes in the stator windings of the exciter.

As already mentioned, the shunt and series windings on a MAC stator are arranged with their pole axes $180^\circ$ (electrical) apart. The choice of the coil pitch to be adopted for these windings is not straightforward, since together with the number of turns used it controls the compounding characteristic, and a considerable short-chording is employed to provide the characteristic required. In Chapter 7, it will be shown that an economic design is only possible with a stator coil pitch ranging between $60^\circ$ and $120^\circ$. Other factors which may influence the choice are indicated later in Section 5.2.5.

A MAC rotor contains a conventional double-layer winding, with a coil pitch selected to minimise the differential leakage. For a particular selection of stator and rotor coil pitch, the compounding equations, together with the generator data, can be used to determine the number of turns in each winding, with the winding reactances being evaluated by the methods described in the preceding section. Winding resistances can be calculated from the cross-sectional area
and the length of wire required, with an estimate of the wire length being provided by

\[ L_w = \left(2L + 1.5 \frac{\pi D_s \alpha_c}{2p\alpha_p} \right) N \]

... (5.26)

where \( N \) is the number of turns per phase, \( L \) the stack length, \( D_s \) the mean diameter of either the rotor or stator at the slot center, \( \alpha_c \) the coil pitch, \( \alpha_p \) the pole pitch and \( p \) the number of pole-pairs. To determine the number of turns for each of the three windings, it is most convenient to start with the shunt and rotor windings. Appropriate numbers of turns are chosen to provide the required generator field current on no load, with due consideration given to the level of both flux and current densities. For a given number of rotor turns, a reduction in the number of shunt turns has the effect of increasing the voltage constant \( C_{V1} \) and hence the no-load field current. However, it also has the effect of increasing the shunt current and the no-load flux density. Particular attention must therefore be given to both these quantities in any attempt to increase the no-load field current by reducing the number of shunt turns.

When the number of shunt turns is kept constant, the effect of varying the number of rotor turns on the no-load field current of the generator is somewhat less direct. Increasing the rotor turns increases, proportionally, the voltage constant, and hence the no-load rotor voltage. However, the
commutation reactance is also increased, and if the windings are of zero resistance this increase is proportional to the square of the increase in the turns. The no-load field current may therefore be either increased or decreased, depending on the initial value of the commutating reactance in relation to the generator field resistance. This topic forms the subject of a further study in Section 6.2, but for the moment it is sufficient to note that increasing the rotor turns does not necessarily increase the no-load generator field current.

The number of series turns in a MAC exciter is determined on the basis of providing the field current required at the rated load and power factor conditions, $I_L$ and $\cos \phi$ respectively. When this condition applies, then

$$|C_{I2}| = \frac{V_1 |C_{V1}|}{I_L} \left\{ \left( \frac{I_{fl}}{I_{f0}} \right)^2 - 1 + \sin^2 \phi \right\}^{\frac{1}{2}} - \sin \phi \right\} \quad ... \ (5.27)$$

where $I_{f0}$ and $I_{fl}$ are respectively the no-load and rated-load field currents. Using the appropriate expression for the current constant enables the number of series turns to be determined.

5.2.4 On-load variation of various voltages and currents

To allow a quick visual inspection of the variations with load of the series winding voltage, the shunt winding current, the rotor winding current and the current in the common
section between the exciter and the generator armature, it may be useful to sketch these characteristics. From the load characteristics presented in the preceding chapter, it is evident that the series voltage and the rotor current both increase gradually with load, whereas the shunt and common current variations may exhibit minimum points which need to be located.

Application of the familiar compounding equations shows that the shunt current $I_1$ may be expressed as

$$I_1 = \frac{1}{R_{11} + jX_{11}} \left( V_1 (1 + j\frac{X_{13}C_{V1}}{Z_{et}}) - jX_{12}I_2 + j\frac{X_{13}I_2C_{I2}}{Z_{et}} \right)$$

... (5.28)

and the common current $I_{com}$ as

$$I_{com} = \frac{1}{R_{11} + jX_{11}} \left( V_1 (1 + j\frac{X_{13}C_{V1}}{Z_{et}}) - jX_{12}I_2 + j\frac{X_{13}I_2C_{I2}}{Z_{et}} \right) - I_1 (R_{11} + jX_{11})$$

... (5.29)

If $V_1$ is taken as the reference phasor, the phase angle of $I_2$ is $-\phi$, and if it is further assumed that the arguments of $Z_{et}$, $C_{V1}$ and $C_{I2}$ are $\theta$, $\theta_V$ and $\theta_I$ respectively, then analysis with the aid of a phasor diagram shows that $|I_1|$ is a minimum when

$$|I_2| = \left( a \sin(\theta_I - \phi + \Theta) - b \sin(90^\circ - \theta_I + \theta_V - \phi - \beta) \right) / x$$

... (5.30)

where
a = |V_1|  

b = \frac{X_{12} |G_{V1}|}{|Z_{et}|} a 

c = \frac{X_{12} |G_{I2}|}{|Z_{et}|}  

d = X_{12} 

x = (d^2 + c^2 - 2dc \cos(\theta_I - \theta))^\frac{1}{2}  

\beta = \cos^{-1}\left(\frac{c^2 + x^2 - d^2}{2cx}\right) 

and that the minimum shunt current is

\[ |I_1|_{\text{min}} = \frac{1}{|R_{11} + jX_{11}|} \left( a \cos(\theta_I - \phi - \theta + \beta) 
+ b \cos(90^\circ - \theta_I + \phi - \beta) \right) \]  ... (5.31)

A similar approach shows that the minimum common current occurs when

\[ |I_2| = \left\{ a \sin(\theta_I - \phi - \theta + \beta + \beta') \right. 
- b \sin(90^\circ - \theta_I + \theta + \phi - \beta - \beta') \left. \right\}/x' \]  ... (5.32)

where a, b, c, x and \(\beta\) are as defined above, and

\[ d = X_{11} + X_{12} \quad x' = (R_{11}^2 + x^2 - 2xR_{11} \cos(90^\circ + \theta_I - \theta + \beta))^\frac{1}{2} \]

\[ \beta' = \cos^{-1}\left(\frac{x^2 + x'^2 - R_{11}^2}{2x x'}\right) \]

The minimum value of \( |I_{\text{com}}| \) is

\[ |I_{\text{com}}|_{\text{min}} = \frac{1}{|R_{11} + jX_{11}|} \left( a \cos(\theta_I - \phi - \theta + \beta + \beta') \right. 
+ b \cos(90^\circ - \theta_I + \theta + \phi - \beta - \beta') \left. \right) \]  ... (5.33)

and in both eqns. (5.32) and (5.33) a positive sign for \(\beta'\) is
5.2.5 Effect of stator coil pitch on some important parameters

As already mentioned, it is difficult to formulate a definite criteria to govern the choice of an appropriate coil pitch for the stator windings of a MAC exciter. Apart from its known effect on the current constant and the phase lead, it also influences, amongst other factors, the shunt reactance, the voltage constant and the commutating reactance. These parameters, in turn, naturally affect the overall performance of the exciter. A knowledge of how the various quantities are affected by the stator coil pitch can therefore assist the selection process.

Using the results of Section 5.1, and the corresponding equations, it can be shown that for a resistance-less exciter

\[ |C_{V1}| = \frac{\psi_{11} K w_3 N_2}{\psi_{11} K w_1 N_1} \]  \hspace{1cm} \ldots (5.34)

\[ |C_{I2}| = X_{23} (1 - \frac{N_1 K w_1 \psi_{12}}{N_2 K w_2 \psi_{11}}) \]  \hspace{1cm} \ldots (5.35)

and \[ X_c = X_{33m} \left( \frac{\psi_{23}}{\psi_{33f}} - \frac{\psi_{11}}{\psi_{11}} \right) \]  \hspace{1cm} \ldots (5.36)

Eqns. (5.7) and (5.34-5.36) indicate that \( X_{11}, C_{V1}, C_{I2} \) and \( X_c \) are all affected by the flux linkages and winding factors, and it will be recalled that these are both functions of the stator coil pitch. As an example, the variation of the four
parameters with coil pitch, for a winding with 3 slots/pole/phase, is shown in Fig. 5.4. It should be noted in the figure that the values for $X_{11}$ and $C_{V1}$ are normalised to unity when the coil pitch is $180^\circ$ and that $X_c$ and $C_{I2}$ are normalised to unity when the coil pitch is $60^\circ$, since these two later quantities are expected to be very small at the coil pitch of $180^\circ$. A normalised quantity is denoted by a bar placed on top of the appropriate symbol.

As the coil pitch is increased from $60^\circ$ to $180^\circ$, $\overline{C_{I2}}$ increases initially from unity to 1.05 at $80^\circ$ before reducing to zero at $180^\circ$, and with all else kept constant the current constant for a particular series winding varies in the same manner. The ability of a MAC to provide the additional field current required by a loaded generator is largely influenced by this constant, and because of this very direct effect attention must be paid to the careful selection of a suitable coil pitch. To provide a given amount of compounding, the use of an $80^\circ$ coil pitch results in a series winding having the least number of turns. Obviously, this arrangement is the most efficient as far as the size of the series winding is concerned. Although the degree of current compounding is reduced by the use of a larger coil pitch, some of the additional effects which accompany this may be undesirable. Firstly, the shunt winding resistance has an increasingly important effect on the current constant, which in turn reduces the phase lead of the current constant over the voltage constant. Indeed, in the extreme case when the coil
pitch is $180^\circ$, the experimental results presented in the preceding chapter demonstrated that a completely incorrect compounding effect was produced. Secondly, the accompanying reduction in the commutating reactance causes the generator field current to become more temperature sensitive. This aspect is considered in the following paragraph.

Inspection of Fig. 5.4 shows that $X_c$ reduces from unity at $60^\circ$ to about 0.1 at $180^\circ$. For a given number of rotor turns, an increased field current can therefore be obtained by the use of a greater coil pitch. Although this may make more efficient use of the rotor copper, the field current becomes more temperature sensitive due to the reduced commutating reactance. (The swamping effect provided by the commutating reactance is the subject of further considerations in Section 6.3.1). On the other hand, if the maximum swamping effect of the commutating reactance is to be provided by the least number of rotor turns, then a smaller coil pitch must be adopted.

The normalised voltage constant $\bar{C}_{V1}$ reduces from about 1.2 when the stator coil pitch is $60^\circ$ to a minimum of 0.95 at $120^\circ$ and then increases to unity at $180^\circ$. The effect of the stator coil pitch on the normalised voltage constant is apparently less than its effect on the normalised shunt reactance, in which a similar variation in the coil pitch brings about an increase from 0.4 to unity. Noting that the shunt current is in general universally proportional to $X_{II}$
indicates that an increased shunt current will flow when the shunt pitch is small. However, the larger value of $\bar{C}_{V1}$ which permits more shunt turns to be used allows the shunt current to be brought down slightly.

Although the above discussion has considered individually the effects resulting from a variation in the coil pitch, such isolation is clearly not possible in practice and it is the combined effects which eventually determine the performance of any exciter design.

5.2.6 Reducing the compounding effect

Examination of the compounding equations shows that the no-load generator excitation is provided by the voltage component $K_{dc}V_{1}C_{V1}$, with the current component $K_{dc}I_{2}C_{I2}$ boosting the excitation in such a manner that the terminal voltage is maintained substantially constant. The relationship between $C_{V1}$ and $C_{I2}$ to achieve this objective is defined by eqn. (5.27). For a given voltage constant, the current constant can be adjusted merely by a change in the number of series turns, which provides the simplest means of satisfying eqn. (5.27). However, since experience indicates that with larger generators even a one-turn series winding can produce excessive compounding, the alternative methods detailed below for producing a reduced compounding effect may sometimes need to be considered.

(a) Delta-connected series winding: When the series winding
is connected in delta, the current flowing through each phase of the series winding is reduced by a factor of $\sqrt{3}$, resulting in the compounding effect being reduced by a similar amount. The shunt winding needs to be reconnected to preserve the proper phase relationship, and the whole arrangement is shown in Fig. 5.5. One main disadvantage of this arrangement is the absence of a star point in the generator armature winding.

(b) Increased coil pitch for stator coils: It has already been shown that the value of $C_{I2}$ is very small when the coil pitch is near $180^\circ$. However, many other important considerations arise if this approach is contemplated. In particular, sufficient phase lead must be preserved to ensure a correct phase sensitive compounding effect.

(c) Increased rotor turns: Even though the shunt voltage is fixed, different values of $C_{Vl}$ may be used to provide the required no-load excitation current by appropriate adjustment to the number of turns in both the shunt and rotor windings. When the number of rotor turns is increased above a certain value (see a later consideration in the following chapter), the correct no-load excitation can only be provided by reducing the number of shunt turns. Since these actions both have the effect of increasing $C_{Vl}$, eqn. (5.27) indicates that a larger value of $C_{I2}$ is required to produce the correct level of compounding. It must be noted that this approach can
result in a greater shunt winding current, an increased flux density and an inefficient design. However, the extra commutating reactance introduced by the additional rotor turns renders the field current less sensitive to temperature variations.

(d) Increased airgap: If the above-mentioned methods all fail to produce a satisfactory design, the only remaining variable is the airgap length, although to increase this will require the provision of either stator or rotor punchings of different diameter from that normally adopted.

5.3 General comments.

In this chapter an analytical method for parameter evaluation and a systematic approach to the design of a MAC exciter have been proposed. The systematic approach is basically a trial-and-error method, which attempts to match the characteristic of the outline design to that required by the generator. Although the method may appear somewhat tedious, a knowledge of how certain objectives can be achieved will greatly reduce the number of trials necessary. The discussions in Section 5.2 are thought to be helpful in this respect, as are the further considerations detailed in the following chapter.
Coil Pitch | $\psi_{11}/\psi_{11f}$ | MAC exciter | Normal winding
--- | --- | --- | ---
180° | 1.014 | 1.0140
160° | 1.046 | 1.0115
140° | 1.124 | 1.0111
120° | 1.234 | 1.0140
100° | 1.352 | 1.0143
80° | 1.494 | 1.0137
60° | 1.695 | 1.0140

Table 5.1 Ratio of $\psi_{11}/\psi_{11f}$ for windings with 3 slots/pole/phase

<table>
<thead>
<tr>
<th>Calculated results ($K_s = 1.18$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{11} = 0.380\Omega$</td>
</tr>
<tr>
<td>$X_{12} = 0.299\Omega$</td>
</tr>
<tr>
<td>$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{11} = 0.356\Omega$</td>
</tr>
<tr>
<td>$X_{12} = 0.225\Omega$</td>
</tr>
<tr>
<td>$</td>
</tr>
</tbody>
</table>

Table 5.2 Parameters of exciter WC 0077
Fig. 5.1 Flux density distribution of single full pitch coil

Fig. 5.2a Winding distribution of MAC exciter

Fig. 5.2b Flux density distribution due to unit current in a phase of series winding
Fig. 5.3 MAC exciter design with shunt winding connected across the whole of the generator armature
Fig. 5.1 Effect of stator coil pitch on some important constants
Fig. 5.5 MAC exciter design with delta-connected series winding
CHAPTER 6
OPTIMISATION STUDY

As there are three distinct windings on a MAC exciter it is evident that various alternative designs may be produced, with the different numbers of turns in the windings still producing almost identical compounding characteristics. The performance of the exciters will however be different in other respects, and detailed considerations given to these differences may identify a particular design offering an overall performance significantly better than the others. A optimal design of this kind is clearly desirable.

In this chapter, an optimisation study is undertaken on a type WC 0077 MAC exciter with the aid of a digital computer. Detailed considerations are given to the results obtained from the study, and a selection criteria for an optimal exciter is formulated on the basis of these considerations. To aid a successful search for an optimal design, it is shown that certain conflicting performance requirements cannot be specified concurrently.

6.1 Program modification

The optimisation study investigates the performance of different exciter designs providing the same compounding characteristics, but with a particular stator and rotor frame containing different combinations of the various
winding turns. This is most easily accomplished by suitably modifying the experimental open-circuit characteristics of an original design to reflect the changes made in the numbers of turns of the different windings, while maintaining magnetic conditions such that the resulting characteristics are still applicable for the given frame size. With reference to the set of open-circuit characteristics, Figs. 4.1a, c and e, the modified currents and voltages with the supply to the shunt winding are

\[ I_1' = I_1 \frac{N_1}{N_1} \quad \ldots (6.1) \]

\[ V_{11}' = V_{11} \frac{N_1}{N_1} \quad \ldots (6.2) \]

\[ V_{12}' = V_{12} \frac{N_1 N_2}{N_1 N_2} \quad \ldots (6.3) \]

\[ V_{13}' = V_{13} \frac{N_1 N_3}{N_1 N_3} \quad \ldots (6.4) \]

where a prime denotes the new quantities, and the currents and voltages expressions may be correspondingly modified when the supply is to either the series or the rotor winding. Such modifications have been incorporated into the curve-fitting subroutine of the computer model for the exciter, thereby providing it with the capability to compute the performance of new exciter designs with any combination of winding turns. All results presented in the following
studies are obtained from the revised program.

6.2 Variation of no-load field current with rotor turns

It was mentioned in Section 5.2.3 that an increase in the number of rotor turns may bring about either an increase or a decrease in the no-load field current, depending on the initial number of turns. This is in fact evident in Fig. 6.1, which shows the resulting no-load field current produced by a variation in the number of rotor turns between 8 and 128 on the exciter/generator unit WC 0077/WC 876. Initially, the no-load field current is proportional to the number of turns, but after reaching a maximum at about 48 turns the no-load field current then shows an almost inverse variation with the number of rotor turns.

The no-load field current of the exciter is given by

\[ I_{fo} = K_1 \left| \frac{V_1 C V_1}{R + j (X_e + X_c)} \right| \quad \ldots (6.5) \]

Generally, for a given number of shunt turns \( C_{V1} \propto N_3 \), \( X_c \propto N_3^2 \) and \( R_c \propto N_3^2 \), and although \( K_1 \), \( R_e \) and \( X_e \) are mainly determined by the field resistance \( R_f \), they are also affected to a lesser extent by \( X_c \) and \( R_c \). Noting that \( R_c \ll X_c \), differentiation of eqn. (6.5) with respect to \( N_3 \) shows that the maximum of \( I_{fo} \) occurs when
\[
N_3 = \left\{ \frac{(R_e/s + R_c)^2 + X_e^2}{K_c^2} \right\}^{1/2}
\]

where \(K_c\) is the commutating reactance per turn of the rotor winding. For the unit under investigation \(K_c\) is approximately \(3.55 \times 10^{-4} \Omega\), and the corresponding value of \(N_3\) of 37 agrees reasonably well with that given by Fig. 6.1.

The form of relationship between \(I_{f_0}\) and \(N_3\) has far reaching implications when considerations are given to possible alternative designs. For example, exciters with the same stator windings but with rotor windings having 32 and 76 turns will give the same no-load field current. After making appropriate adjustment to the series winding, both designs will provide identical compounding characteristics although their performance in other respects will be very different. In the following optimisation study, the performances of these alternative exciter designs with different number of rotor turns are examined in detail.

6.3 Alternative exciters

The original design of exciter WC 0077 provides a field current of 10.2A on no-load and 18.2A at rated load current of 15.7A and a power factor of 0.8 lagging, with \(N_1 = 27\), \(N_2 = 33\) and \(N_3 = 56\). Table 6.1 shows different combinations of \(N_1\), \(N_2\) and \(N_3\) all of which provide the required no-load and rated-load field excitations. Each of these combinations
represents an alternative design which will hereafter be referred to by the number of turns contained in its rotor winding.

In Table 6.1, the same current density is maintained throughout the windings of the alternative designs and the winding resistances need therefore to be adjusted such that $R_{11} \propto N_1^2$, $R_{22} \propto N_2$ and $R_{33} \propto N_3$. All the alternative shunt windings then require the same volume of copper, whereas the alternative series and rotor windings require a volume proportional to their respective number of turns.

6.3.1 Temperature sensitivity

In common with static current-compounding schemes, changes in the temperature of the present exciter/generator unit, and in particular the generator field winding, may cause a drift in the terminal voltage. A good measure of the temperature stability of a unit is given by the variation in the field current resulting from some specific change in the field resistance, and it is convenient to consider a 20% change since this represents about a 50°C change in the temperature. For the alternative designs of Table 6.1, Fig. 6.2 shows the percentage change in field current brought about by such a variation in the field resistance, with the exciter operating at rated load and power factor. Assuming that the generator is operated in a reasonably saturated condition, the percentage drift in terminal voltage
is about 0.7 of that in the field current, which is about 4% in the case of the original design. It is worth noting that the manufacturer of the actual operating unit regards this value as being reasonably in agreement with the drift experienced in operational units.

When the number of rotor turns is increased, $X_c$ increases in proportion to $N^2$ and provides an increasingly large swamping effect. The ratio $sX_c/R_f$ gives a measure of the swamping effect provided by the commutating reactance over the field resistance, and the variation of the ratio for the different exciter designs is also shown in Fig. 6.2. While this ratio is small the field current is sensitive to changes in the temperature, but the use of more rotor turns raises the ratio and results in the field current becoming more stable. If the drift in the terminal voltage of about 2% is regarded as acceptable, then Fig. 6.2 requires the ratio to exceed two, which can only be achieved by designs with 80 or more rotor turns.

6.3.2 Flux levels

Fig. 6.3 shows how the open-circuit flux density of the different design of MAC exciter varies at different loading conditions. There is clearly a close relationship between these curves and the no-load field current curve of Fig. 6.1. The voltage induced in the open-circuit rotor winding is given by the expression $(V_1 C_{V1} + I_2 C_{I2})$, showing that the two compounding constants have a direct effect on the open-
circuit flux density. To maintain a given no-load field current, $C_{V_1}$ needs to bear an inverse relationship to the curve of Fig. 6.1. Since $C_{I_2}$ and $C_{V_1}$ are directly related (see eqn. (5.27)), the open-circuit flux density necessary to provide any given field current also therefore has an inverse relationship to the curve of Fig. 6.1, with the maximum of this curve at $N_3 = 48$ coinciding with the minimum of curves in Fig. 6.3.

It should be noted that the open-circuit flux density curves of Fig. 6.3 are derived from the open-circuit rotor voltage and the demagnetising effect of the rotor current is therefore not included. As is evident from Fig. 6.4, which shows the magnetising flux density of the different exciters at the various loading conditions, the demagnetising effect reduces the levels of flux density, an effect which becomes increasingly more significant as the number of rotor turns is increased. Since the magnetising flux density is derived from the magnetising rotor voltage (eqn. (4.1)), it represents the combined magnetising or demagnetising effects of all the currents in the exciter and the figure thereby obtained gives a good measure of the flux density actually existing in the airgap of the exciter.

In the event of a terminal short circuit, the field current provided by the exciter will be limited mainly by saturation effects within the magnetic core. Exciters with a low operating level of flux density thus provide a better short-
circuit performance. If such a performance is required, Fig. 6.4 indicates that exciter designs with a larger number of rotor turns are advantageous.

6.3.3 Current levels

For any given MAC exciter, the current in the series winding is determined by the rating of the generator. The rated current in the rotor winding is fixed mainly by the field current required but also partly by the mode in which the rectifier bridge is operating, with the current transfer ratio $K_1$ exerting a small influence on the actual value. Since, for the range of $N_3$ investigated, $K_1$ varies only between 1.27 and 1.37, the rotor currents do not show a great variation for the different designs investigated. There is, however, a considerable variation in the shunt current required in the different designs.

The predicted shunt winding currents of the different exciter designs are presented in Fig. 6.5, both for no load and for rated load current at power factors of unity, 0.8 lagging and zero lagging. Inspection of this figure shows that, with the exception of the zero power factor result, a definite minimum point occurs, although these are at a different number of rotor turns for the different load conditions.

Examination of eqn. (3.51) shows that the no-load shunt current contains three components; that to establish a
reactance voltage $I_1X_{11}$, that to balance the m.m.f. of the series winding and that to balance the m.m.f. of the rotor winding. Since the shunt current is in general inversely proportional to $X_{11}$, exciters with a smaller number of shunt turns have a higher level of shunt current, as is evident from comparison of Table 6.1 and Fig. 6.5. The correspondence is however not exact, due to the shunt current also having to provide a component to balance the rotor m.m.f. and another, when appropriate, to balance the series m.m.f. Furthermore, the on-load shunt current may either decrease or increase from the no-load figure, depending on the load power factor and the number of rotor turns, which both affect the rotor and series m.m.f.'s. For instance, when $N_3 = 16$ the shunt current is smallest at zero power factor, followed in increasing order by 0.8 power factor, unity power factor and lastly the no load condition. However, when $N_3 = 128$ the least shunt current is at unity power factor, followed by no load, 0.8 power factor and zero power factor.

At rated load and power factor, Fig. 6.5 indicates that it is advantageous to use an exciter with $N_3 = 24$, when the low level of shunt current means either that the shunt winding loss is low or that smaller diameter conductors may be used. However, the large variation in the shunt current between no load and rated load (when it is only 50% of the no-load level) cannot be overlooked. It will be noted that although
a variation of this magnitude can be reduced by using an exciter design with an increased number of rotor turns, the shunt winding current will now generally be of a higher level.

Since the shunt winding of the exciter is connected across a portion of the armature winding of the generator, which must therefore carry the shunt winding current, exciters with shunt currents significantly above the rated generator current are clearly not permissible if the same size conductors are to be used throughout the armature winding. On this basis and allowing for a 25% overload, Fig. 6.5 shows that exciters with more than 60 rotor turns cannot be used.

Because the position of the minima in the curves of shunt current against rotor turns vary with power factor, it is not possible on this basis to choose a single best exciter. In general, as far as the level of shunt current is concerned, exciter designs with rotor turns within the range 24 to 48 appear to be equally good.

6.3.4 Series voltages

The terminal voltage $V_t$ of the generator/exciter unit is

$$V_t = V_{gt} - V_2$$

where $V_{gt}$ is the phase voltage at the generator terminals and $V_2$ the voltage across the series winding. For the different exciter designs, the magnitude and phase angle of
the voltage across the series winding at no load, unity power factor, 0.8 power factor lagging and zero power factor lagging rated load are presented in Figs. 6.6a and b, with \( V_{gt} \) taken as the reference. It will be noted that these voltages are small in comparison with the rated generator phase voltage of 133V.

The compounding equations for the MAC exciter show that the series voltage may be expressed by

\[
V_2 = K_V V_1 + K_Z I_2
\]  

... (6.8)

where

\[
K_V = \frac{jX_{12}}{R_{11} + jX_{11}} - \frac{C_{12} V_1}{Z_{et}}
\]  

... (6.9)

and

\[
K_Z = R_{22} + jX_{22} + \frac{X_{12}^2}{R_{11} + jX_{11}} - \frac{C_{12}^2}{Z_{et}}
\]  

... (6.10)

Thus, in general, the series winding voltage has two components, one of these being a reflected voltage due to the resultant airgap flux and the other an impressed voltage due to the flow of load current in the equivalent impedance \( K_Z \). The series voltage on no load is therefore a wholly reflected voltage, with a magnitude proportional to the number of series turns and the airgap flux density. Hence, after allowing for the fact that a larger number of series turns causes the series voltage to show an increasing trend when the number of rotor turns is large, there is a fairly direct correspondence between the variation in the no-load series winding voltage and the no-load flux density curve of
Fig. 6.4. The on-load variation of the series winding voltage also follows a pattern closely similar to the curves of Fig. 6.4, with any differences being explained by the change in the number of series turns and the effect of the load current on the resistance and the leakage reactance of the winding. Close examination of eqn. (6.9) shows that the phase angle of the no-load series voltage is zero when \( N_3 = 0 \), becoming progressively more lagging as \( N_3 \) increases and tending toward \( 180^\circ \) when \( N_3 \) is large. This tendency is confirmed by examination of Fig. 6.6b. The on-load phase angle of the series winding voltage is however less predictable, since it is affected by the relative magnitude of the two voltage components in eqn. (6.8).

Although the series winding voltage is subtracted from the generator phase voltage to give the actual output voltage, the output voltage is not necessarily smaller since the subtraction is performed vectorially. If the phase angle of the series winding voltage exceeds \( 90^\circ \), either leading or lagging, the output voltage will actually be greater than the generator voltage. For a given series winding voltage, the reduction in the terminal voltage becomes greater as the phase angle is reduced, and it is rather unfortunate that this is precisely the condition encountered at around rated power factor conditions. To prevent this from posing a serious problem, the series winding voltage, and therefore the operating flux level and the number of series turns,
must be kept low. It appears that exciters with $N_3$ in the range from 40 to 88 turns will perform reasonably well in this respect.

6.3.5 **Power consumption, losses and efficiency**

In the process of supplying the generator field excitation the MAC exciter naturally consumes power, some of which reappears as generator field power but some of which is dissipated internally in the form of losses. In view of the differing combination of turns in the different designs, there will also be noticeable differences in the total input power and in the distribution of the internal losses.

The total exciter input power (both electrical and mechanical) when the different exciter designs operate at the usual four load conditions considered is shown in Fig. 6.7. In a typical comparison, the generator field power is 109W at no load, 246W at rated unity power factor load, 349W at rated 0.8 power factor load and 379W at rated zero power factor load. It is clear that, for each condition, the total input power of the exciter generally increases with the number of rotor turns, indicating an increasing level of loss in the exciter. There are however definite minimum points to the variations, and if minimising the input power at rated load and power factor conditions is the only consideration, an exciter design with $N_3 = 24$ is clearly the best.
Apart from affecting the input power, the losses within the exciter also influence the efficiency and the relative inflow of the electrical and mechanical power. The variation of the various exciter and rectifier losses (see Appendix A1) and the power drawn from the generator with the number of rotor turns is given in Fig. 6.8a. It should be noted that the curves apply only to the rated load and power factor conditions. Since the series winding current is constant and the rotor winding current varies only slightly with \( N_3 \), the copper loss in these two windings can be seen as proportional to the number of turns in the respective windings. The rectifier loss remains almost constant, with a slight reduction at a large number of rotor turns due to the marginally falling rotor current that represents a more efficient mode of operation for the rectifier. On the other hand, the shunt winding copper loss varies widely, changing from the least important component when the number of rotor turns is small to become eventually the major component. The dominating influence that the shunt loss has on the variation of the total system loss is illustrated by the shapes of the two curves in Fig. 6.8a. Evidently, the total loss is a minimum for an exciter design with \( N_3 = 24 \).

Conventional induction motor theory shows that the power drawn from the generator has to supply the copper loss in the shunt and series windings, plus one-half of the rotor loss, the rectifier loss and the generator field power. If the overall
loss, and in particular that associated with the stator, is low, then less power is required from the generator. This point is well illustrated by Fig. 6.8a.

Since the rotor copper loss is a major component of the total loss for exciter designs with a few rotor turns, the minimum total loss can be further reduced by using rotor conductors with an increased cross-sectional area. With the variation in the rotor turns performed so as to maintain the same total volume of copper as in the original exciter \((N_3 = 56)\), the total loss for the different exciters vary as shown in Fig. 6.8b. It is significant that the lowest total exciter loss is now only 39% of that of the original exciter, and while this may not be important for small units it certainly represents an important potential saving for units of higher rating.

Fig. 6.9 presents the efficiencies of the various exciter designs, when the generator operates at 0.8 power factor lagging and delivers rated current of 15.7A. The copper efficiency allows only for the loss in the three machine windings, whereas since the system efficiency accounts also for the additional loss in the rectifier it bears a closer relationship to the total loss variation of Fig. 6.8a. Since minimising the total loss has the effect of maximising the efficiency, the maximum copper and system efficiencies, of 79% and 70% respectively, both occur when \(N_3 = 24\), which corresponds well with the point of minimum loss. Evidently,
the increasing level of exciter loss forces the efficiency figures to deteriorate as the number of rotor turns is increased.

6.3.6 Mechanical power

The variation in the mechanical input power to the MAC exciter for the different exciter designs, when operating at rated load and power factor conditions, is shown in Fig. 6.10a. As already mentioned in Chapter 3, the mechanical input power provides one half of the combined field power, the rotor copper loss and rectifier loss. Since the power required by the field winding is constant and the rotor copper loss is proportional to $N_3^2$, the mechanical input power shows a gradual increase with the number of rotor turns used in the exciter design. The ratio of mechanical input power to the total power input to the exciter for the same load conditions as Fig. 6.10a is shown in Fig. 6.10b. In the absence of stator loss this ratio would be 0.5, but the inclusion of stator loss causes the ratio to be always below this figure and to reach a maximum of only 0.44 when $N_3 = 32$.

6.3.7 Reactive power and total volt-amperes

The reactive power requirement of the different exciter designs is given in Fig. 6.11, for the same load conditions considered previously. For designs with a few rotor turns, the reactive power is expended mostly on establishing the high operating flux level. Although the operating flux level
decreases as the number of rotor turns is increased, an increasing reactive power is required to balance the lagging component of the rotor m.m.f. and, as is evident in Fig. 6.11, this eventually causes the reactive power requirement of the exciter to increase. With the reactive power requirement being necessarily provided by the generator, it must be limited to a reasonable level not too far above the minimum at \( N_3 = 48 \).

Fig. 6.12 shows the volt-ampere demand that the different exciter designs impose on the generator at rated load and power factor. With the demand being mainly reactive, the shape of the curve is the same as the corresponding curve in Fig. 6.11, but with a minimum occurring at slightly fewer rotor turns (\( N_3 = 40 \)). The manner in which the shunt and series windings share the power and the volt-amperes absorbed by the exciter is also illustrated by Fig. 6.12. These curves show that it is possible for a situation to arise in which there is a net outflow of power from the shunt winding.

A good indication of the physical size of the exciter is given by its total volt-ampere requirement \( S_t \), defined as

\[
S_t = I_1^2X_{11} + I_2^2X_{22} + I_3^2X_{33}
\]  

... (6.11)

Using the no-load field power as a base, it is common practice to normalise \( S_t \) so that the size of the exciter can be visualised in terms of the field winding that it supplies. The variations of the normalised total volt-ampere
requirement for the various exciter designs at the familiar four loading conditions are presented in Fig. 6.13. The no-load field power is in fact 109W. As evident in the figure, the curves all exhibit minimum points at either $N_3 = 24$ or $N_3 = 32$. But since considerations are made normally at rated load and power factor, the 32-rotor-turn exciter design must be regarded as the smallest, and with a volt-ampere requirement of 6.5 p.u. it is only about one-half the size of the original exciter design.

6.4 A basic contradiction

Investigations on the alternative exciters have produced several important findings that can assist in the search for an optimum MAC exciter design. There are however some objectives that are mutually contradictory, and which cannot be specified concurrently when the criteria for an optimum exciter are defined. The contradiction arises mainly because the drift in the generator voltage with temperature is reduced by using a large number of turns on the rotor, whereas most other major considerations, such as loss and normalised volt-ampere requirements, indicate the undesirability of such a design. For instance, although the WC 0077 exciter will operate with a minimum copper loss when the rotor winding contains 24 turns, an unacceptable change in field current of about 12% is produced by a 20% change in the field resistance. An increase in the rotor turns to 48 reduces the drift to 7.7%, but this is at the expense of an
increase in the total copper loss in the exciter from 101W to 158W. A further increase to 56 turns (as used in the existing design) reduces the current drift to 6% but increases the copper loss to 189W, and if 64 turns are used the drift is 4.9% and the copper loss 226W. This close inter-relationship is an inherent feature of the MAC exciter, and the existing design is at best only a working compromise between the level of copper loss that can be tolerated and the drift in the field current that will produce an acceptable change in the generator voltage. Similar conclusion may also be drawn when the normalised volt-ampere requirement is considered instead.

6.5 Conclusions

An optimisation study was conducted on the exciter type WC 0077 by investigating the performance characteristics of a range of alternative exciter designs that give compounding characteristics similar to those of the original design. It is interesting to note that the method employed in obtaining these alternative designs may be used to assist the design process detailed in the preceding chapter. Based on the technique presented in that chapter, a prototype exciter may be constructed to provide the required open-circuit characteristic. A similar method to that used for generating the alternative exciters can then give the required exciter design.
An analysis of the results in Section 6.3 indicates that conflicting performance requirements may prevent the use of a design which is in some senses optimal. In particular, the need for an exciter which produces only a low drift in the generator field current as the generator temperature changes is opposed to almost all the other major considerations. Since the size of the exciter and the stability of its field current are the two most important design considerations, it is rather unfortunate that they are in direct conflict with one another. Furthermore, the study indicates that there is always a direct relationship between the size of the exciter and the level of its operating loss. Thus, the MAC exciter as presently arranged is not capable of achieving an optimal design, and the best design is only a good engineering compromise between the two commonly specified features.
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Table 6.1 Alternative winding arrangements for exciter WC 0077
Fig. 6.1 No-load field current for different number of rotor turns (the number of shunt turns is constant)

Fig. 6.2 Percentage change in field current and the ratio of commutating reactance to field resistance at rated load conditions for different exciter designs (as specified by the number of rotor turns)

Variation of field current computed for 20% change in field resistance.
Fig. 6.3 Open-circuit airgap flux density at different load conditions for different exciter designs (as specified by the number of rotor turns)

Fig. 6.4 Magnetising flux density at different load conditions for different exciter designs (as specified by the number of rotor turns)
Fig. 6.5 Magnitude of shunt current at different load conditions for different exciter designs (as specified by the number of rotor turns)

Fig. 6.6a Magnitude of series voltage at different load conditions for different exciter designs (as specified by the number of rotor turns)
Fig. 6.6b Phase angle of series voltage at different load conditions for different exciter designs (as specified by the number of rotor turns)

Fig. 6.7 Total power consumption at different load conditions for different exciter designs (as specified by the number of rotor turns)
**Fig. 6.8a** Losses and demand on generator power at rated load conditions for different exciter designs (as specified by the number of rotor turns).
Field power at rated conditions = 310 W

**Fig. 6.8b** Total losses at rated load conditions for different exciter designs (as specified by the number of rotor turns).
Design such that the original amount of rotor copper is used.
Fig. 6.9 Copper and system efficiencies at rated load conditions for different exciter designs (as specified by the number of rotor turns)

Fig. 6.10a Input mechanical power at rated load conditions for different exciter designs (as specified by the number of rotor turns)
Fig. 6.10b Ratio of input mechanical power to total power required at rated load conditions for different exciter designs (as specified by the number of rotor turns).

Fig. 6.11 Reactive volt-ampere requirement at different load conditions for exciter of different designs (as specified by the number of rotor turns).
Fig. 6.12 Demand on generator volt-ampere and shunt and series volt-ampere and power at rated load conditions for different exciter designs (as specified by the number of rotor turns)

Fig. 6.13 Normalised total volt-ampere requirement at different load conditions for exciter of different designs (as specified by the number of rotor turns)
CHAPTER 7
MODIFIED MAC EXCITER AND GENERALISED OPTIMISATION

The studies in the preceding chapter have demonstrated that conflicting performance requirements may prevent the design of a MAC exciter that is in some senses optimal. In particular, the need for an exciter which produces only a small drift in the generator field current as the temperature changes is opposed to almost all the other major considerations, and although undesirable this is an inherent feature of a MAC exciter. A closely associated problem also exists in the static current-compounding scheme (see Chapter 2) where it is common practice to employ Boucherot capacitors to ensure a substantially drift-free generator field current, by making the field winding effectively fed from a constant-current rather than a constant-voltage source.

In the present chapter, it is shown that an effect similar to that produced by Boucherot capacitors can be obtained in a MAC exciter, by connecting appropriate capacitors in series with the shunt winding on the stator. This modified MAC exciter arrangement will hereafter be referred to as the new arrangement, in contrast with the conventional arrangement of the existing design. An extensive experimental investigations was conducted on the new arrangement, providing results for comparison with predicted values to
establish the validity of the theoretical exciter model and which demonstrates clearly the practicality of the new arrangement. Using a computer-based analysis, similar to that presented in the preceding chapter, an optimisation study of the new scheme is undertaken on exciter type WC 0077. Amongst the several advantageous effects of incorporating shunt capacitors, it is now feasible to design an optimal exciter. Some practical design considerations of the new arrangement are also considered.

A generalised approach to the optimisation study is presented in the final part of this chapter. Apart from extending the scope of the study, the approach is particularly useful when considering the effect stator coil pitch has on the performance characteristics of the exciter.

7.1 Modified MAC exciter

7.1.1 Analysis

The axis of the shunt winding of a MAC exciter has a spatial displacement of 180° from that of the series winding, and the connections to either set of windings are reversed to give the correct compounding effect (see Chapter 3). An effective winding displacement of 0° is therefore necessary for the MAC exciter to operate correctly in the conventional arrangement. If the effective winding displacement, taking into account any phase shift introduced by the differing terminal connections, is λ, the compounding equations are
\[ I_3 = \frac{C_{v1}V_1 + C_{I2}I_2}{Z_{et}} \] ... (7.1)

\[ I_1 = \frac{V_1 - jX_{12}e^{j\lambda I_2} - jX_{13}I_3}{R_{11} + jX_{11}} \] ... (7.2)

\[ V_2 = jX_{12}e^{-j\lambda I_1} + (R_{22} + jX_{22})I_2 + jX_{23}e^{-j\lambda I_3} \] ... (7.3)

where 

\[ C_{v1} = \frac{jX_{13}}{R_{11} + jX_{11}} \] ... (7.4)

\[ C_{I2} = (jX_{23} + \frac{X_{13}X_{12}}{R_{11} + jX_{11}})e^{j\lambda} \] ... (7.5)

\[ Z_{et} = \frac{R_e}{s} + R_c + j(X_e + X_c) \] ... (7.6)

\[ Z_c = R_c + jX_c = \frac{R_{33}}{s} + jX_{33} + \frac{X_{13}^2}{R_{11} + jX_{11}} \] ... (7.7)

and the derivations of the above equations are presented in Appendix A4.

In the new arrangement, identical capacitors are connected in series with each of the three phases of the shunt winding. If the capacitive reactance at the operating frequency is \( X_B \), the relevant compounding equations are obtained simply by replacing \( X_{11} \) in eqns. (7.1-7.7) by \( (X_{11} - X_B) \), with \( V_1 \) now being the supply voltage to the shunt circuit.

7.1.1.1 Resistance-free exciter

When the capacitors of the new arrangement are selected such that \( X_B = X_{11} \), the rotor current \( I_3 \) of a resistance-free exciter is
while the shunt current $I_1$ and the series winding voltage are respectively

\[
I_1 = \frac{V_1 A}{X_{L1^2}} - I_2 e^{jX_{L1^2}}(\frac{X_{L2^2}}{X_{L1^2}} - j\frac{AX_{L2^2}}{X_{L1^2}}) \quad \cdots (7.9)
\]

and

\[
V_2 = V_1 e^{-jX_{L1^2}} + I_2 \left\{ j\left(2X_{L2^2}X_{L3^2} + AX_{L2^2}^2\right) \right\} \quad \cdots (7.10)
\]

where $A = \frac{R}{s} + jX_{33}$ \quad \cdots (7.11)

Noting that the current transfer ratio of the 3-phase bridge rectifier in this limiting mode of operation $17 (R_c + \infty)$ is $\sqrt{2}$, the field current furnished by the exciter of

\[
I_f = \sqrt{2}\left| \frac{V_1}{X_{L1^2}} - jI_2 \frac{X_{L2^2}}{X_{L1^2}} e^{jX_{L1^2}} \right| \quad \cdots (7.12)
\]

is affected only by the reactance parameters of the exciter. Thus, under these ideal conditions, the exciter will provide a drift-free current to the field winding of the generator.

The two components of the field current in eqn. (7.12) may, as usual, be termed the voltage and current components, since they are determined respectively by the shunt winding supply voltage $V_1$ and the load current $I_2$. It is their vector sum that determines the field current, and comparison of eqn. (7.12) with eqn. (2.1) indicates that $\lambda$ must be $180^\circ$ if
ideal compounding is to be provided for a cylindrical-rotor
generator at all levels of load current and power factor.
For a salient-pole generator, although the provision of ideal
compounding is theoretically impossible, it is still
essential that the current constant \((-jX_{12}e^{j\lambda/X_{13}})\) should
have a near 90⁰ phase lead over the voltage constant \((1/X_{13})\)
to provide a correct phase sensitive compounding, which also
requires \(\lambda\) to be 180⁰. This 180⁰ phase shift is indeed
provided by the spatial displacement between the axes of the
shunt and the series windings, in contrast to the
conventional MAC arrangement where it has been shown that
the connections to one of the two stator windings need to be
reversed to achieve the correct phase sensitive compounding.

If the generator field resistance is \(R_f\), it can be shown,
from the per-turn reactance ratio defined in Section 3.4 and
eqns. (7.9-7.11), that the shunt current and series voltage
may alternatively be expressed as

\[
I_1 = \frac{V_1}{X_{11}} K_{23} K_{11} \left(\frac{2}{3} \frac{R_f}{sX_{33}} + j\right) - I_2 e^{j\lambda} \frac{N_2}{N_1} \left\{ (1 + \frac{K_{33} K_{12}}{K_{13}^2}) \right. \\
\left. - j\left(\frac{2}{3} \frac{R_f}{sX_{33}} \frac{K_{23} K_{12}}{K_{13}^2}\right) \right\} \quad \ldots \ (7.13)
\]

and

\[
V_2 = V_1 \frac{N_2}{N_1} \left( 1 - \frac{K_{12} K_{33}}{K_{13}^2} \right) + j \left( \frac{K_{12} K_{33}}{K_{13}^2} \left( \frac{2}{3} \frac{R_f}{sX_{33}} \right) \right) \\
+ I_2 \frac{K_{33} K_{12}^2}{K_{22} K_{13}^2} + j\left( 1 + \frac{K_{33} K_{12}^2}{K_{22} K_{13}^2} - \frac{2K_{12}}{K_{22}} \right) \quad \ldots \ (7.14)
\]

respectively.
Thus, although the field current in unaffected by the field resistance, eqns. (7.13-7.14) indicate that it has an effect on both the shunt current and series voltage. For a given stator coil pitch, the different ratios (for example, $K_{12}$, $K_{13}$, etc.) can be determined using the method described in Chapter 3 and substitution of these values into the two equations allows $I_1$ and $V_2$ to be estimated. It must be noted that both the equations above are arranged in a manner which permits evaluation of both $I_1$ and $V_2$ (for different windings turns) with less effort than using eqns. (7.9) and (7.10), thus making them useful when calculating trial designs.

7.1.1.2 Some practical considerations

If the new arrangement is used with an exciter in which the shunt winding is designed for connection across only a portion of the armature winding, the required capacitance can be extremely large (a rough estimate, using reactances appropriate for a type WC 0077 exciter, suggests a figure of about 10,000 μF). Although the required voltage rating is low, such capacitors are less common than those with a low capacitance and a high voltage rating. However, if the shunt winding is designed for connection across the entire armature winding, the capacitance required falls to the very practical level of a very few microfarads.

A practical excitation scheme must provide a field current that increases with load current at unity and all lagging
power factors. As is evident from the inspection of eqns. (7.4) and (7.5), a finite shunt winding resistance causes the current constant to lead the voltage constant by an angle somewhat greater than 90°, with the actual value depending on the ratio \( \frac{R_{ll}}{X_{ll}} \). At unity or near-unity power factor, this effect may result in a field current which initially decreases with load current, although this behaviour is eliminated if the three phases of the shunt circuit are connected across the line terminals in a manner that reduces the phase lead by 30° (i.e. \( \lambda = 150° \)). For normally designed shunt windings, the 30° reduction will reduce the phase lead of the current constant to less than 90°. (Note that in a conventional arrangement the resistance of the shunt winding inherently reduces the phase lead).

Eqn. (7.12) was derived under the assumption that \( \frac{R_{ll}}{X_{ll}} = 0 \), and it remains a good approximation provided that the ratio is appreciably less than unity. To ensure this, and so maintain a field current which is largely unaffected by temperature variation, the shunt winding must be designed such that \( \frac{R_{ll}}{X_{ll}} \leq 0.15 \), or thereabouts. Such a design requirement can readily be achieved in practice.

7.1.2 Experimental investigation of the new arrangement

An experimental study of the new arrangement with capacitors in series with the shunt winding was undertaken on the experimental unit EMEU-2 (stator coil pitch = 180°). The
characteristics of this particular unit in a conventional arrangement were investigated in Chapter 4, which also contains full details of the unit. The present study is divided into two areas. In the first the no-load performance of the exciter is considered with different values of shunt capacitance, while the second investigates the exciter load characteristics using an ideal value of shunt capacitance selected from the results of the no-load test. Both tests were conducted with a rectifier load on the rotor circuit.

7.1.2.1 Experimental procedures

A 3-phase 240V laboratory supply fed a variable power factor load bank via a 3-phase variac (to simulate the generator), variable shunt capacitors and the exciter unit EMEU-2, as shown in Fig. 7.1a. The rotor of the exciter was locked firmly to the stator frame, with the star-connected rotor winding feeding a smooth current to a resistive-inductive load (simulating the generator field winding) through a 3-phase bridge rectifier. The variable shunt capacitors of Fig. 7.1a were obtained from the combination of fixed 21μF capacitors and a variac, as shown in Fig. 7.1b. The effective field resistance was 2.1Ω, and provision was made to increase this by up to 20%.

With the shunt supply voltage maintained at 6V, and the connections to the 3-phase loading bank removed, the variac setting of Fig. 7.1b was varied from 0% to 7% in steps of 1%. (Note that the variac setting provides a convenient measure
of the capacitive reactance introduced into the shunt circuit by the capacitor/variac combination of Fig. 7.1b, see Table 7.1 for the reactance value corresponding to a particular setting). At each step, all the circuit voltages and currents were noted, together with the power dissipated in the capacitor/variac combination. The percentage change in field current due to a 20% change in the field resistance was also noted at each step.

Results from the above test indicate that the change in field current due to a 20% increase in field resistance is a minimum at the 5% variac setting. With the setting maintained at this value and the loading bank reconnected, a load test was performed for load power factors of unity, 0.8 lagging and zero lagging, with the effect on the field current of a 20% increase of field resistance also being noted.

7.1.2.2 Results and discussions

The presence of the variac introduces a significant extra resistance into the exciter shunt circuit, and it is necessary therefore to represent the capacitor/variac combination by a capacitor \( X_B \) in series with a resistor \( R_B \). The values of \( X_B \) and \( R_B \), as derived from the no-load test results, at the various variac settings are recorded in Table 7.1. The evidence of Table 7.1 is that, even at the lowest variac setting, the additional resistance introduced into the shunt circuit exceeds the shunt winding resistance
(0.0627Ω), so that although the arrangement is convenient for experimental purpose, it is certain to provide results inferior to those produced by the inclusion of an appropriate capacitor alone.

No load test results at the various variac settings are presented in Figs. 7.2a-c, and it is evident that these agree well with the corresponding theoretical results also given in the figures. Note that the theoretical results are obtained from the computer model of the exciter (see Chapter 4), which uses eqns. (7.1-7.7) and the open-circuit characteristics of EMEU-2. The resonant nature of the circuit formed by the shunt capacitors and the shunt winding inductance is well illustrated by the shape of the voltage and current characteristics of both Figs. 7.2a-b. However, the presence of the large shunt-circuit resistance (due mostly to \( R_B \)) causes the resonant point, which occurs at a 3% variac setting, to shift from the point where the commutating impedance \( Z_c \) reaches a maximum. This latter point, occurring at a variac setting of 5%, is identified in Fig. 7.2c as the setting at which the field current is most stable. If the shunt circuit resistance is negligible, then the resonant point at which the field current reaches a maximum will also correspond to the one at which it is most stable, with both occurring at \( X_B = X_{11} \). The experimental arrangement is therefore some way away from an ideal new arrangement, but, as the results indicate, it is quite capable of demonstrating the advantage of using series
capacitors in the shunt circuit of the exciter. In particular, their advantageous effect on stabilizing the field current is clearly demonstrated.

Both experimental and theoretical results showing the on-load compounding characteristics of the new arrangement at the load power factors of unity, 0.8 lagging and zero lagging are given in Figs. 7.3a-7.5b. Inspection of these figures shows generally good agreement between experimental and predicted results, with any errors involved being associated with the difficulty in predicting precisely the voltages and currents existing in a practical near-resonant circuit. Fig. 7.3a shows an initial fall in the field current as the unity power factor load is applied, a phenomenon discussed previously in Section 7.2.2 where a remedy was also proposed. Bearing this in mind, the load test results indicate that the new arrangement is providing the correct compounding effect, with the field current increasing with either an increase in the load current or a worsening of the power factor. It is very significant that, as was shown in Chapter 4, without the shunt capacitors the same exciter is incapable of providing the correct compounding effect.

The percentage variation in field current due to the 20% change in the field resistance value remains fairly constant at about 6% throughout the load test, implying that the loading conditions on the exciter do not have an important effect on the stability of the field current.
7.1.2.3 Conclusions from experimental investigation

The experimental studies described above have demonstrated clearly that it is possible to reduce considerably the temperature drift in the field current by the use of suitable series capacitors in the shunt circuit of the exciter. Furthermore, the ability of the exciter to provide a load and phase sensitive current compounding is unaffected by the inclusions of these capacitors. In particular, an experimental unit originally incapable of furnishing a phase-sensitive field current, did so after the inclusions of shunt capacitors. Despite the presence of additional shunt circuit resistance, the drift in the generator field current was reduced from 11% to 6%. Thus, with the shunt winding designed for the direct connection of series capacitors, it can be anticipated that the resulting arrangement will provide a practically drift-free field current.

7.2 Optimal exciter and practical designs

Since the new MAC arrangement employs capacitors in series with the shunt circuit, alternative exciter/capacitor combinations may be designed to produce a range of exciters with compounding characteristics identical to those of an original unit. In the discussions that follow, these exciters (referring to both the machine and the capacitors of the new arrangement) are specified in terms of the number of winding turns and their associated capacitance. It is
shown that their performance in other respects is different, and there is an optimal design when the exciters are assessed on the basis of these differences. Some practical designs are also proposed and their performance is compared with those of conventional MAC exciters.

7.2.1 Alternative exciters

Using the same technique as in the preceding chapter, and the rotor/stator frame of exciter WC 0077, alternative exciters were designed to provide the correct compounding effect for generator WC 876. Table 7.2 records the number of winding turns and the required capacitances of these exciters. (Note that the required capacitance can also be obtained from Fig. 7.6). The exciters in Table 7.2 are designed with each phase of the shunt winding connected in series with a capacitor of appropriate size across the line voltage of the star-connected generator, so that the phase lead is reduced to a value slightly less than 90°. Although exciters with stator coil pitches of both 120° and 180° are designed, the discussions which follow concentrate on the larger of these pitches. The performance characteristics of the other exciters are substantially similar. Although this may at first seem to limit the scope of the present discussion, it is rectified in the later generalised optimisation study (see Section 7.3), where exciters with a stator coil pitch of 180° are shown to be only marginally better.
7.2.1.1 Temperature sensitivity

In the conventional arrangement, the major stumbling block that prevents the design of an optimal exciter is the close inter-relationship between the drift of the field current and the number of rotor turns. The computed results in Fig. 7.7 show that although this drift is still exhibited by the new arrangement, the effect is comparatively small, such that for a 56-turn rotor winding the drift falls from 6% to a practically negligible figure of 1.1%. If a drift of 3% is regarded as acceptable, all designs with at least 24 rotor turns may be used. Since it is most likely that an optimal design will have a low number of rotor turns, this extension to the lower limit of the range is significant. Furthermore, the drift of field current can be further reduced by using a lower ratio of $R_{ll}/X_{ll}$ in the design, and thus removing a major constraint in the optimisation process.

Although the commutating impedance $Z_c$ is responsible for stabilising the field current in both the new and the conventional arrangements, the relative magnitudes of the commutating resistance $R_c$ and the commutating reactance $X_c$ are different in the two cases. In the conventional arrangement, $X_c$ is mainly responsible for producing a stabilising effect on the field current. However, in the new arrangement $R_c$ is much larger than $X_c$ and it is therefore the former that is responsible for producing the swamping effect. In fact, for an ideal and resistance-free exciter,
the value of $X_c$ is zero while that of $R_c$ is infinitely large. It may appear surprising that the commutating resistance provides the swamping effect which stabilises the field current against variations due to temperature effects on another resistance (the generator field winding), but it should be noted that since $R_c$ is determined mainly by the reactance parameters of the exciter, temperature variations will have no significant effect on its value.

7.2.1.2 Flux levels

Fig. 7.8 shows the variation on the exciter flux density for the various designs at no load and at rated load and power factors of unity, 0.8 lagging and zero lagging. The flux density is computed from the magnetising rotor voltage as defined in eqn. (4.1), and it provides a good measure of the airgap flux. Evidently, exciters with a large number of rotor turns operate at lower flux levels, a rather unfortunate feature since better short-circuit performance is obtained from exciters with lower operating flux levels. Thus, with the original MAC exciter having a magnetising flux density of 0.129T at rated conditions, a design with 64 rotor turns is the only one to offer a better short-circuit performance. It must, however, be noted that the precise effect on the short-circuit performance of operating the exciter at a moderately increased flux density is hard to quantify. Assuming a normal magnetisation characteristic for iron, it is possible that there may not be any perceivable
deterioration in the short-circuit performance even if the flux level reaches 0.4T. Furthermore, it is reasonably easy to change the flux level in the exciter by minor changes to its geometry. For instance, doubling the effective length of the airgap and the stack length simultaneously reduce the flux density by half, but everything else remain unchanged. The operating flux level thus does not impose an intractable problem in an optimisation process.

7.2.1.3 Capacitor and exciter volt-amperes

The capacitance required by all the new exciter arrangements considered is shown in Fig. 7.6, which confirms that, since the number of shunt turns decreases with any increase in the number of rotor turns, the capacitance required also increases. The corresponding voltage rating, given in Fig. 7.9, is obviously never very high. Thus, commonly available industrial type capacitors can be used with the new arrangement, and the scheme is practically feasible.

A good indication of the physical size and cost of the capacitors is provided by their volt-ampere ratings, given in Fig. 7.10a for no load and for various rated load conditions, with the ratings being normalised to a base of the no-load generator field power of 109W. Since the capacitors are tuned to resonate with the shunt winding reactance, the volt-ampere requirements of the shunt winding at the various no load and load conditions all follow similar variations as evident in Fig. 7.10a. For comparative purposes, the
normalised volt-ampere requirements for the exciter windings at the same load conditions as previously considered are given in Fig. 7.10b. At rated load the 32-turn rotor winding has the lowest rated capacitor, while the winding volt-ampere requirement is lowest for a 24-turn rotor winding. (It should be noted that this latter requirement gives a good indication of the loss level of the exciters). A rough measure of the overall size of the exciters (including the associated capacitors) is provided by the total volt-ampere requirement, shown in Fig. 7.10c. It is apparent that, on this basis, the design with 32-turn rotor represents the best choice and, furthermore, since this particular design is also very close to the one that provides minimum loss ($N_3 = 24$), it is also very efficient.

7.2.1.4 Alternative exciters - conclusions

The foregoing considerations of the exciter volt-ampere requirements have indicated the desirability of using a 32-rotor-turn design. Although the drift of field current due to temperature variations can be further reduced by an increased number of rotor turns, such an attempt is not necessary since the drift of that exciter design is only some 2.1%, a figure regarded in practice as acceptable. Moreover, if the intending application requires the field current drift to be lower than this value, the same objective can be achieved more efficiently by the use of shunt conductors with larger cross-sectional area, instead of
varying the rotor turns. The other major considerations which can influence the choice of exciter is the operating flux density of the design, and this is perhaps the only consideration that may prevent the use of a 32-turn rotor. However, with a flux density of only 0.21T at the rated load and power factor conditions, this exciter design would be expected to provide a reasonably good short-circuit performance. Thus, this particular design can be regarded as an optimal exciter.

7.2.2 Some practical designs

In designing a practical exciter, the fact that there are preferred values of capacitance may need to be considered, and it is perhaps convenient to design exciters using capacitors of 2µF, 3µF and 4µF. Designs for both 180° and 120° stator coil pitch windings, together with those for conventional MAC arrangements (included for comparison purposes) are all given in Table 7.3, which also records the major performance characteristics of each design.

It is evident from the Table that, at any given flux level, the characteristics for a 180° pitch exciter are substantially similar to those for a corresponding 120° pitch exciter (compare 3µF, 180° pitch column with 2µF, 120° pitch column), with the 180° pitch being preferred since it has a marginally smaller volt-ampere rating. Bearing this in mind, it is therefore not really fruitful to compare designs for the different coil pitches.
The most prominent feature of all the new designs is their comparatively low drift of field current with temperature, with the worst figure recorded of 2.2% only being matched in a conventional exciter by using a 112-turn rotor. However, such a design has a volt-ampere requirement of 42.8 p.u. and an efficiency of only 39.9% and is clearly not of any practical significance. If an increased flux density of 0.207T is acceptable, the 180° pitch 32-turn rotor represents the best design achievable, with its total volt-ampere requirement of 11.5 p.u. being lower even than that of the original \( N_3 = 56 \) MAC design.

7.2.3 Variation of field current with frequency

When the speed of the isolated generator/exciter unit changes, both the frequency and the magnitude of the output voltage are affected. Although the change in frequency is proportional to the speed variation, the actual variation in the terminal voltage is more difficult to predict since, while both the variations in terminal voltage and output frequency affect the field current provided by the exciter, the effect this has in turn on the terminal voltage can only be assessed with a detailed knowledge of the magnetisation characteristic of the generator. For the purpose of comparison between the different exciter designs, it is therefore convenient to assume a constant terminal voltage and to investigate the effect frequency variation has on the field current.
Assuming a constant terminal voltage, and hence a constant shunt supply voltage, the percentage change in the exciter field current brought about by a 5\% reduction in frequency for the different exciter designs previously considered (in both new and conventional arrangements) are recorded in Table 7.4. Since the new arrangement contains series resonant circuits formed by the shunt winding reactances and the shunt capacitors, it is not surprising that the field current in this configuration shows a greater variation as the frequency change causes the resonant circuits to become detuned. In contrast, the frequency effect on the field current of the conventional designs is comparatively small. Furthermore, some conventional designs (i.e. $N > 48$) even show an increase in field current, and with the terminal voltage falling due to the reduction in speed, this feature is particularly desirable. The above results therefore indicate that the performance of the designs in the new arrangement in inferior in this respect.

7.3 Generalised optimisation

Optimisation studies on both the conventional and the new MAC arrangements have hitherto been undertaken only on a specific size exciter, with a given stator/rotor frame and pre-determined coil pitches. Although the study has produced several important findings, these are only applicable to the range of exciters under consideration. In particular, the variation of the performance characteristics
with respect to stator coil pitch has not been considered. To allow for this and also several other factors, an optimisation study that is more generalised in nature is presented in this section.

7.3.1 Analysis

Consider a conventional MAC arrangement which provides the correct field excitation for a cylindrical-rotor synchronous generator having a per unit synchronous reactance \( X_s \) and a field resistance \( R_f \). Using the compounding equations of Section 7.1.1 and the per-turn reactance ratios defined in Section 3.4, the currents and voltages of the exciter windings may alternatively be expressed as

\[
I_1 = \frac{V_1}{X_{11}} \bar{I}_1 \quad \text{... (7.15)}
\]

\[
I_2 = \frac{V_1 N_1}{X_{11} N_2} \bar{I}_2 \quad \text{... (7.16)}
\]

\[
I_3 = \frac{V_1 N_1}{X_{11} N_3} \bar{I}_3 \quad \text{... (7.17)}
\]

\[
V_2 = \frac{V_1 N_2}{N_1} \bar{V}_2 \quad \text{... (7.18)}
\]

where

\[
\bar{I}_1 = \frac{1}{1 - jx_{11}} \left( -j + \frac{K_{13}^2}{K_{11} K_{33}} \cdot \frac{1}{1 - jx_{11}} \cdot \frac{X_{33}}{Z_{et}} + \frac{X_s e^{j\alpha}}{1 - jx_{11}} \cdot \frac{1}{1 - jx_{11} - \frac{K_{12}}{K_{11}}} \right) \]

\[
\left( -\frac{K_{12}}{K_{11}} + j \frac{K_{13}^2}{K_{11} K_{33}} \cdot \frac{X_{33}}{Z_{et}} \cdot \frac{1 - jx_{11} - \frac{K_{12}}{K_{11}}}{1 - jx_{11}} \right) \quad \text{... (7.19)}
\]
and \( x_{11}, x_{22} \) and \( x_{33} \) are respectively the resistance to reactance ratios for the shunt, series, and rotor windings. For a given coil pitch, the various fractions associated with the per-turn reactance ratios (for example, \( K_{12}/K_{11} \), \( K_{13}/K_{11} \) and \( K_{12}/K_{33} \)) remain constant, regardless of the physical size of the stator/rotor frame. Furthermore, since the ratio \( X_{33}/Z_{et} \) is determined mainly by the ratio of \( X_{33} \) to \( R_f \), with their actual values having negligible effect, the various quantities in eqns. (7.19-7.22) are therefore affected only by the stator coil pitch and the ratio \( sX_{33}/R_f \). It is thus useful to regard \( \bar{I}_1, \bar{I}_2, \bar{I}_3 \) and \( \bar{V}_2 \) as being respectively the normalised shunt current, series current, rotor current and series voltage, with their actual values being obtained by appropriate multiplication. It should be
noted that the normalisation process allows exciters of different physical size to be compared directly. By following a similar approach, the normalised magnetising flux per pole of the exciter $\Phi_m$ may be expressed as

$$\Phi_n = \left(\frac{K_{w1}}{K_{w3}} I_1 + \frac{K_{w2}}{K_{w3}} I_2 + \frac{K_{33}}{K_{11}} I_3\right) \ldots (7.25)$$

where the base value of flux is $\left(V_1/(\sqrt{2} \pi N_1 f_1)\right)$.

It can be shown that the per phase no-load field power is

$$P_o = \frac{V_1^2}{X_{11}} P_n \ldots (7.26)$$

where

$$P_n = \frac{K_{13}^2}{K_{11} K_{33}} \frac{1}{|1 - j X_{11}|^2} \frac{X_{33} R_e}{|Z_{et}|^2} \ldots (7.27)$$

Using this figure as a base, the various exciter volt-ampere requirements are

$$S_1 = \frac{|I_1|^2}{P_n} \ldots (7.28)$$

$$S_2 = \frac{|I_2|^2}{P_n} \ldots (7.29)$$

$$S_3 = \frac{|I_3|^2}{P_n} \ldots (7.30)$$

where $S_1$, $S_2$ and $S_3$ are respectively the normalised requirements of the exciter shunt, series and rotor windings.

It then follows that the total normalised volt-ampere
requirement of the exciter is

\[ \bar{S}_l = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 \]  \hspace{1cm} (7.31)

Noting that the ratio \((X_{33}R_e/|Z_{et}|^2)\) of eqn. (7.27) is also only affected by the ratio of \(sX_{33}\) to \(R_f\), the value of the latter therefore represents an important consideration in the design process. In fact, the present optimisation study will investigate the performance characteristics of exciters with different values of \(sX_{33}/R_f\) and of the stator coil pitch.

Since eqns. (7.15-7.31) were derived for a conventionally arranged exciter, \(\lambda\) must be \(0^\circ\). However, in the new arrangement the same equations are still applicable, but with the term \((1 - jx_{11})\) replaced by \(-jx_{11}\), and \(\lambda\) is now \(150^\circ\) (instead of \(180^\circ\) for reasons already considered in Section 7.1.1.2). Furthermore, the normalised volt-ampere requirements of the exciter winding \(\bar{S}_w\) and the total requirement \(\bar{S}_t\) are now given by

\[ \bar{S}_w = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 \]  \hspace{1cm} (7.32)

and

\[ \bar{S}_t = 2\bar{S}_1 + \bar{S}_2 + \bar{S}_3 \]  \hspace{1cm} (7.33)

respectively.

In situations where the exciter is used with a salient-pole generator, it is necessary to define a value for \(X_s\). If the no-load field current of the generator is \(I_{f0}\) and the field current at rated load (at a lagging power factor of \(\cos \phi\)) is
I_{f1}, then it is perhaps most sensible to use eqn. (7.34) for determining $\overline{X}_s$, since that definition corresponds to providing the generator with the correct level of compounding at the rated load and power factor condition. Thus

$$\overline{X}_s = \left( \frac{I_{f1}}{I_{fo}} \right)^2 - 1 + \sin^2 \phi \right) \frac{1}{2} - \sin \phi \quad \ldots \quad (7.34)$$

In the following investigations of the exciter characteristics, it is assumed that the winding resistance to reactance ratios are 0.15 for both the shunt and series windings and 0.13 for the rotor winding. These values are selected so that they correspond roughly to those adopted in the existing MAC design. The fractions associated with per-turn reactance ratios such as $K_{12}/K_{11}$ and $K_{13}/K_{33}$, are obtained by the flux linkage method described in Section 3.4.1.3. and although $x_h$ in eqn. (3.74) is taken as 0.1 to make some allowance for saturation effects it must be noted that such effects cannot be fully accounted for in this generalised analysis.

7.3.2 Exciter performance characteristics at different effective reactance ratios

Previous studies have shown that different performance characteristics are obtained from exciter designs using different number of rotor turns. Since a variation in the rotor turns effectively alters the rotor reactance $X_{33}$, and
hence the ratio $sX_{33}/R_f$, it is instructive to consider the
effect this ratio has on the performance characteristics
with various coil pitches. In view of its importance, the
ratio will be termed the effective reactance ratio.

Figs. 7.1la-f give the performance characteristics of a 30°
coil pitch exciter connected to a generator in the
conventional current-compounding arrangement, with the
generator supplying rated load at a power factor of 0.8
lagging. Fig. 7.1la shows the percentage change in field
current due to a 20% change in the field resistance, at
different effective reactance ratios. When this ratio is
low, the field current is unstable due to the small swamping
effect of $X_c$. As the ratio is increased, the drift of field
current is reduced. In Figs. 7.1lb-c, the variation of the
various normalised volt-ampere requirements with the
effective reactance ratio are shown respectively for
$\bar{X}_s = 0.8$ p.u. (this figure corresponds to the $\bar{X}_S$ required to
provide generator WC 876 with the correct level of
compounding at rated load and power factor factor condition,
as determined by eqn. (7.34)) and $\bar{X}_S = 2.0$ p.u. Generally
speaking, the rotor winding requirement increases linearly
with the ratio while the shunt and the series winding
requirements both show an initial fall to a minimum followed
by an increasing trend as the ratio is further increased.
For both $\bar{X}_S = 0.8$ p.u. and $\bar{X}_S = 2.0$ p.u., the occurrence of
minimum points in the variations of the total requirement are
clearly evident, demonstrating the fact that there is always a most efficient arrangement. Apart from increasing the total requirement, the larger value of $\overline{X}_s$ also increases the size of both the series and the rotor windings in comparison to that required for the shunt winding. The variation of the normalised shunt current, series voltage and magnetising flux for $\overline{X}_s = 0.8$ p.u. and $\overline{X}_s = 2.0$ p.u. are shown respectively in Figs. 7.11d, e and f. The shunt current increases with the effective reactance ratio, to reach a constant level when the ratio is large, while both the series voltage and the magnetising flux are reduced as the ratio is increased. The effect of $\overline{X}_s$ is clearly evident in these figures.

When the stator windings of a conventional MAC exciter have a coil pitch of $140^\circ$, the corresponding performance characteristics are shown in Figs. 7.12a-f, where the results are presented in a similar pattern to those of Figs. 7.11. In comparison to exciters with a coil pitch of $80^\circ$, the effectiveness of lowering the drift of field current by increasing the reactance ratio is reduced. Except for the shunt current at $\overline{X}_s = 0.8$ p.u., the various volt-ampere requirements, series voltage and magnetising flux are now all at a higher level. Although the variation in the total requirements still exhibit minimum points, these minima are significantly larger. It must also be noted that the effects of increasing $\overline{X}_s$ are now more pronounced. Thus the $140^\circ$ coil pitch designs may, in general, be regarded as inferior.
Similar performance characteristics for exciters in the new arrangement are given in Figs. 7.13a-f for exciters with a 180° coil pitch and in Figs. 7.14a-f for exciters with a 60° coil pitch. In general, the drift of field current, shunt current, series voltage and magnetising flux all decrease with an increasing effective reactance ratio. Except for the series volt-ampere requirement, which now increases with this ratio, all the other requirements show similar variations to those described for the conventional arrangement. While the effects of increasing $X_s$ is still the same, it should be noted that the drift in field current is now at a very much reduced level, an important feature of the new arrangement. As evident from the figures, the 180° coil pitch design gives a significantly better overall performance.

One important finding from the preceding investigations is the widely different effect the stator coil pitch has on the performance characteristic, depending on the arrangement adopted. In the conventional arrangement, designs with a smaller coil pitch give a better overall performance, whereas the converse is true for the new arrangement. This point is considered further in the following section.

7.3.3 Minimum volt-ampere requirements

To illustrate the effects of the stator coil pitch, it is perhaps most useful to consider the minimum volt-ampere
requirement at the various coil pitches and for different load conditions. Depending on the arrangement, the minimum volt-ampere requirement may be obtained by minimising $S_t$ in either eqn. (7.31) or eqn. (7.33). A computer program was written to perform this minimisation on the LUT Prime 400 computer and the result obtained are presented in Figs. 7.15a-7.18d.

With the exciters used in the conventional arrangement and a per unit synchronous reactance for the generator of 0.8, the variations of minimum exciter volt-ampere requirement with stator coil pitch, at the four usually considered load conditions, are given in Fig. 7.15a. The effective reactance ratios at which these minima occur are provided by Fig. 7.15b, while Figs. 7.15c and d show respectively the corresponding percentage change in field current due to a 20% change in field resistance and the normalised magnetising flux. Corresponding results for $X_s = 2.0$ p.u. are shown, in a similar order, in Figs. 7.16a-d. As the coil pitch is increased from 60° to 160°, the minimum no-load requirement decreases due to the improved coupling between the shunt and rotor windings. In contrast, the on-load minimum requirement for similar variation in the coil pitch shows an initial decline, to reach a minimum at a coil pitch of 80°, before increasing for any further increase in coil pitch. Since the on-load requirement gives an indicative measure of the exciter size, a stator coil pitch of 80° should be adopted whenever
possible. As evident from a comparison between Figs. 7.15a and 7.16a, this becomes increasingly important when the exciter is intended for use on generators with a large value of $\bar{X}_s$. If other considerations require the use of other coil pitches, these should preferably be in the range from $60^0$ to $120^0$. Although the flux density can be adjusted to a suitable level by an appropriate design of the exciter frame, the main objection to the use of these exciters is their excessive drift in field current (over 10% for a 20% change in field resistance). It is rather unfortunate that the drift can only be reduced by designing an exciter for a larger effective reactance ratio than given in Figs. 7.15b or 7.16b (depending on the value of $\bar{X}_s$), but such a design is no longer optimal and a heavy penalty is paid in terms of the increased volt-ampere requirement.

For the new arrangement, results similar to those presented in Figs. 7.15a-7.16d are given in Figs. 7.17a-7.18d. In contrast to those of the conventional arrangement, the minimum volt-ampere requirement of the exciters decreases with an increase in the stator coil pitch. A $180^0$ stator coil pitch must be used when an exciter design in this configuration is contemplated. For situations where this is not possible, a slightly smaller coil pitch may be adopted, but it should not be less than $120^0$, a particularly important consideration when $\bar{X}_s$ is large. In the range of interest ($120^0 \leq$ stator coil pitch $\leq 180^0$), the drift in field
current due to a 20% change in field resistance is around 3%, an acceptable figure in practice. Since the flux density can be adjusted by a correct selection of the exciter frame, the designs are therefore optimal.

When a direct comparison is made between the two arrangements, it is evident that the minimum volt-ampere requirement of the exciter in the new arrangement is higher than that of the corresponding exciter in the conventional arrangement. Bearing in mind that an exciter with the minimum requirement in the conventional arrangement is not practical due to its comparatively unstable field current, it is therefore meaningless to compare the two arrangements in this manner. In fact, as the results recorded in Table 7.3 show, a design in the conventional arrangement will have a prohibitively large requirement when an attempt is made to reduced its associated field current drift to a value comparable to that of the new arrangement.
### Table 7.1 Values of $R_B$ and $X_B$ at different variac settings

<table>
<thead>
<tr>
<th>Variac Setting (%)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_B$ (Ω)</td>
<td>0.01</td>
<td>0.03</td>
<td>0.10</td>
<td>0.21</td>
<td>0.35</td>
<td>0.52</td>
<td>0.75</td>
<td>1.02</td>
</tr>
<tr>
<td>$R_B$ (Ω)</td>
<td>0.07</td>
<td>0.07</td>
<td>0.08</td>
<td>0.09</td>
<td>0.11</td>
<td>0.14</td>
<td>0.17</td>
<td>0.21</td>
</tr>
</tbody>
</table>

### Table 7.2 Alternative exciter designs for the new arrangement

<table>
<thead>
<tr>
<th>Stator coil pitch = 120°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_3$</td>
</tr>
<tr>
<td>$N_2$</td>
</tr>
<tr>
<td>$N_1$</td>
</tr>
<tr>
<td>Capacitance (μF)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stator coil pitch = 180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_3$</td>
</tr>
<tr>
<td>$N_2$</td>
</tr>
<tr>
<td>$N_1$</td>
</tr>
<tr>
<td>Capacitance (μF)</td>
</tr>
<tr>
<td>Coil pitch</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>Capacitance (μF)</td>
</tr>
<tr>
<td>N₁</td>
</tr>
<tr>
<td>N₂</td>
</tr>
<tr>
<td>N₃</td>
</tr>
<tr>
<td>Magnetising flux density (T)</td>
</tr>
<tr>
<td>Capacitor Voltage (V)</td>
</tr>
<tr>
<td>Capacitor volt-ampere (p.u.)</td>
</tr>
<tr>
<td>Winding volt-ampere (p.u.)</td>
</tr>
<tr>
<td>Total volt-ampere (p.u.)</td>
</tr>
<tr>
<td>Efficiency (%)</td>
</tr>
<tr>
<td>Variation in field current (%)</td>
</tr>
</tbody>
</table>

Table 7.3 Some practical exciter designs (base power = 109W)

† conventional arrangement

* due to a 20% change in field resistance

(volt-ampere requirements, voltage and flux density are computed for rated generator load, 15.7A at line voltage of 230V and power factor of 0.8 lagging)
<table>
<thead>
<tr>
<th>$N_3$</th>
<th>24</th>
<th>40</th>
<th>48</th>
<th>56</th>
<th>72</th>
<th>88</th>
<th>104</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variation (%)</td>
<td>-1.0</td>
<td>-0.3</td>
<td>+0.1</td>
<td>+0.5</td>
<td>+1.0</td>
<td>+1.2</td>
<td>+1.4</td>
<td>+1.0</td>
</tr>
</tbody>
</table>

**New arrangement (stator coil pitch = 180°)**

<table>
<thead>
<tr>
<th>$N_3$</th>
<th>14</th>
<th>16</th>
<th>24</th>
<th>32</th>
<th>40</th>
<th>48</th>
<th>56</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variation (%)</td>
<td>-6.3</td>
<td>-5.7</td>
<td>-4.2</td>
<td>-3.5</td>
<td>-3.0</td>
<td>-2.9</td>
<td>-2.9</td>
<td>-3.1</td>
</tr>
</tbody>
</table>

Table 7.4 Effects on exciter field current due to 5% reduction in frequency
Fig. 7.1a Circuit diagram for no-load and load tests

Fig. 7.1b Variable shunt capacitors
Fig. 7.2a Circuit currents for no-load test at different variac settings
(Full lines and symbols denote computed and experimental results respectively)

Fig. 7.2b Circuit voltages for no-load test at different variac settings
(Full lines and symbols denote computed and experimental results respectively)
Fig. 7.2c Percentage change in field current due to 20% change in the field resistance value at different variac settings (Full line and symbol denote computed and experimental results respectively)

Fig. 7.3a Circuit currents for load test at unity power factor and variac setting of 5% (Full lines and symbols denote computed and experimental results respectively)
Fig. 7.3b Circuit voltages for load test at unity power factor and variac setting of 5%
(Full lines and symbols denote computed and experimental results respectively)

Fig. 7.4a Circuit currents for load test at 0.8 power factor (lagging) and variac setting of 5%
(Full lines and symbols denote computed and experimental results respectively)
Fig. 7.1b Circuit voltages for load test at 0.8 power factor (lagging) and variac setting of 5%.
(Full lines and symbols denote computed and experimental results respectively.)

Fig. 7.5a Circuit currents for load test at zero power (lagging) and variac setting of 5%.
(Full lines and symbols denote computed and experimental results respectively.)
Fig. 7.5b Circuit voltages for load test at zero power factor (lagging) and variac setting of 5% (Full lines and symbols denote computed and experimental results respectively)

Fig. 7.6 Required capacitance for different exciter designs (as specified by the number of rotor turns)
Fig. 7.7 Variation in field current due to 20% change in field resistance value for different exciter designs (as specified by the number of rotor turns).

Fig. 7.8 Magnetising flux density at different load conditions for different exciter designs (as specified by the number of rotor turns).
Stator coil pitch = 180 deg.
Fig. 7.9 Capacitor voltage at different load conditions for different exciter designs (as specified by the number of rotor turns).
Stator coil pitch = 180 deg.

Fig. 7.10a Normalised capacitor volt-ampere at different load conditions for different exciter designs (as specified by the number of rotor turns).
Stator coil pitch = 180 deg.
Fig. 7.10b Normalised windings volt-ampere at different load for different exciter designs (as specified by the number of rotor turns)
Stator coil pitch = 180 deg.

Fig. 7.10c Normalised total exciter volt-ampere at different load conditions for different exciter designs (as specified by the number of rotor turns)
Stator coil pitch = 180 deg.
Fig. 7.11a Percentage change in field current due to a 20% change in the field resistance. Conventional arrangement, Stator coil pitch = 80 deg.

Fig. 7.11b Normalised volt-ampere requirements at a synchronous reactance of 0.8 p.u. Conventional arrangement, Stator coil pitch = 80 deg.
Fig. 7.11c Normalised volt-ampere requirements at a synchronous reactance of 2.0 p.u.
Conventional arrangement, Stator coil pitch = 80 deg.

Fig. 7.11d Normalised shunt current at synchronous reactances of 0.8 p.u. and 2.0 p.u.
Conventional arrangement, Stator coil pitch = 80 deg.
Fig. 7.11e Normalised series voltage at synchronous reactances of 0.8 p.u. and 2.0 p.u.
Conventional arrangement, Stator coil pitch = 80 deg.

Fig. 7.11f Normalised magnetisation level at synchronous reactances of 0.8 p.u. and 2.0 p.u.
Conventional arrangement, Stator coil pitch = 80 deg.
Fig. 7.12a Percentage change in field current due to a 20% change in the Field resistance.
Conventional arrangement, Stator coil pitch = 110 deg.

Fig. 7.12b Normalised volt-ampere requirements at a synchronous reactance of 0.8 p.u.
Conventional arrangement, Stator coil pitch = 110 deg.
Fig. 7.12c Normalised volt-ampere requirements at a synchronous reactance of 2.0 p.u.
Conventional arrangement, Stator coil pitch = 140 deg.

Fig. 7.12d Normalised shunt current at synchronous reactances of 0.8 p.u. and 2.0 p.u.
Conventional arrangement, Stator coil pitch = 140 deg.
Fig. 7.12e Normalised series voltage at synchronous reactances of 0.8 p.u. and 2.0 p.u.
Conventional arrangement, Stator coil pitch = 110 deg.

Fig. 7.12f Normalised magnetisation level at synchronous reactances of 0.8 p.u. and 2.0 p.u.
Conventional arrangement, Stator coil pitch = 110 deg.
Fig. 7.13a Percentage change in field current due to a 20% change in the field resistance.
New arrangement, Stator coil pitch = 180 deg.

Fig. 7.13b Normalised volt-ampere requirements at a synchronous reactance of 0.8 p.u.
New arrangement, Stator coil pitch = 180 deg.
Fig. 7.13c Normalised volt-ampere requirements at a synchronous reactance of 2.0 p.u.
New arrangement, Stator coil pitch = 180 deg.

Fig. 7.13d Normalised shunt current at synchronous reactances of 0.8 p.u. and 2.0 p.u.
New arrangement, Stator coil pitch = 180 deg.
Fig. 7.13e Normalised series voltage at synchronous reactances of 0.8 p.u. and 2.0 p.u.
New arrangement, Stator coil pitch = 180 deg.

Fig. 7.13f Normalised magnetisation level at synchronous reactances of 0.8 p.u. and 2.0 p.u.
New arrangement, Stator coil pitch = 180 deg.
Fig. 7.14a Percentage change in field current due to a 20% change in the field resistance.
New arrangement, Stator coil pitch = 60 deg.

Fig. 7.14b Normalised volt-ampere requirements at a synchronous reactance of 0.8 p.u.
New arrangement, Stator coil pitch = 60 deg.
Fig. 7.11c Normalised volt-ampere requirements at a synchronous reactance of 2.0 p.u.
New arrangement, Stator coil pitch = 60 deg.

Fig. 7.11d Normalised shunt current at synchronous reactances of 0.8 p.u. and 2.0 p.u.
New arrangement, Stator coil pitch = 60 deg.
Fig. 7.11e Normalised series voltage at synchronous reactances of 0.8 p.u. and 2.0 p.u.
New arrangement, Stator coil pitch = 60 deg.

Fig. 7.11f Normalised magnetisation level at synchronous reactances of 0.8 p.u. and 2.0 p.u.
New arrangement, Stator coil pitch = 60 deg.
Fig. 7.15a Minimum volt-ampere requirements for various load conditions at a synchronous reactance of 0.8 p.u. Conventional arrangement

Fig. 7.15b Effective reactance ratio for minimum volt-ampere requirement at a synchronous reactance of 0.8 p.u. Conventional arrangement
Fig. 7.15c Percentage change in field current due to 20% change in field resistance at a synchronous reactance of 0.8 p.u.
Conventional arrangement

Fig. 7.15d Magnetisation level for the various load conditions at a synchronous reactance of 0.8 p.u.
Conventional arrangement
Fig. 7.16a Minimum volt-ampere requirements for various load conditions at a synchronous reactance of 2.0 p.u. Conventional arrangement

Fig. 7.16b Effective reactance ratio for minimum volt-ampere requirement at a synchronous reactance of 2.0 p.u. Conventional arrangement
Fig. 7.16c Percentage change in field current due to 20% change in field resistance at a synchronous reactance of 2.0 p.u.
Conventional arrangement

Fig. 7.16d Magnetisation level for the various load conditions at a synchronous reactance of 2.0 p.u.
Conventional arrangement
Fig. 7.17a Minimum volt-ampere requirements for various load conditions at a synchronous reactance of 0.8 p.u.
New arrangement

Fig. 7.17b Effective reactance ratio for minimum volt-ampere requirement at a synchronous reactance of 0.8 p.u.
New arrangement
Fig. 7.17c Percentage change in field current due to 20\% change in field resistance at a synchronous reactance of 0.8 p.u.
New arrangement

Fig. 7.17d Magnetisation level for the various load conditions at a synchronous reactance of 0.8 p.u.
New arrangement
Fig. 7.18a Minimum volt-ampere requirements for various load conditions at a synchronous reactance of 2.0 p.u.
New arrangement

Fig. 7.18b Effective reactance ratio for minimum volt-ampere requirement at a synchronous reactance of 2.0 p.u.
New arrangement
Fig. 7.18c Percentage change in field current due to 20% change in field resistance at a synchronous reactance of 2.0 p.u.
New arrangement

Fig. 7.18d Magnetisation level for the various load conditions at a synchronous reactance of 2.0 p.u.
New arrangement
CHAPTER 8
CONSTANT FREQUENCY GENERATOR

Conventional generators are clearly unsuitable for situations in which the generation of constant-frequency electrical power from a variable-speed drive is desired. This problem of generating constant-frequency power when the shaft speed of the prime mover is variable is of long standing and is particularly pronounced when the prime mover is, for example, an aircraft engine. Previous attempts at solving the problem have concentrated on two basic approaches. The first of these involved the development of constant-speed drives capable of converting the variable shaft speed of, for example, turbomachinery to a constant shaft speed for the generator drive. Such systems are normally hydraulic but may also be pneumatic or mechanical. Regardless of the type, the systems are complex, generally lacking in reliability as a result of their complexity, and rely upon external equipment such as pressurized fluid sources for proper operation. The use of cycloconverters or static converters to change the variable generator frequency to a constant output frequency provides the basis for the second approach, but such systems are normally limited to a low power range and are unsuited for application in which the environment is hostile. Despite the drawbacks involved, the lack of other commercially viable systems means that all present-day constant frequency generating units are based on one of the two basic techniques.
The studies described in the previous chapters have demonstrated that the use of MAC exciter with a synchronous generator provides a brushless and self-regulated generator/exciter unit, in which the current-compounded nature of the excitation control gives the unit a very fast response characteristic. It will be recalled that the exciter is energised by alternating currents in the stator windings, which induce corresponding alternating currents in a rotor winding that is coupled to the generator rotor through a rectifier, so that it delivers direct current to the field winding. Since the field current is unidirectional, the rotational speed of the exciter does not influence the frequency of the output from the generator stator, which is a function only of the rotor speed and the number of poles on the generator. However, if a MAC exciter is used with a wound-rotor induction machine (instead of the synchronous generator), with the two rotors linked both mechanically and electrically, the resulting unit will provide a constant-frequency output irrespective of the rotor speed. No external connections through brushgear and slip rings are made to the rotor circuit, and the absence of a d.c. link between the rotors of the exciter and the generator may lead to a brushless constant-frequency generator with significant advantages over conventional arrangements.

In this chapter, this possibility of employing a MAC exciter with an induction machine is examined. The theoretical basis
for the arrangement is considered and a possible current-compounding constant-frequency generator unit is proposed. In particular, a feasibility study is conducted and the findings from this study are substantiated by test results from experimental units.

8.1 Background

As indicated in reference (37), the concept of combining two induction machines on a common shaft may be attributed to Boucherot who, in 1900, proposed a composite motor with two stators and a common rotor. This double-induction machine has since received consideration as a possible means for supplying constant-frequency a.c. power from a variable speed drive, and two recent attempts are described in references (38) and (39). The system introduced by reference (38) employs two polyphase wound-rotor induction machines mounted on the same shaft, with their rotor windings connected in a reversed phase sequence and their stator windings interconnected via a direct-current link type static frequency changer, as shown in Fig. 8.1. However, the frequency changer required is a complex and expensive piece of equipment, which makes the system unsuitable for most normal applications. In reference (39), a simpler and more rugged scheme is proposed, and the present study is in fact mainly concerned with this system.

To illustrate the operating principle of the proposal of
reference (39), two interconnected induction machines are considered, as shown in Fig. 8.2. The machines are driven at a common shaft speed \( n_r \) by a separate prime mover, and since the rotor circuits are linked the currents in the two rotors are of the same frequency. If machine \( M_1 \) has \( 2p_1 \) poles and is fed from a supply of frequency \( f_1 \), then it has a synchronous speed \( n_1 \) of \( f_1/p_1 \) and a slip \( s \) of \( (n_1 - n_r)/n_1 \). The rotor currents are at a frequency \( sf_1 \), and if machine \( M_2 \) has \( p_2 \) poles these currents set up a field which rotates at a speed \( n_2 = sf_1/p_2 \) with respect to the rotor surface. This, in turn, induces an e.m.f. in the stator of \( M_2 \) at a frequency

\[
f_0 = f_1 - n_r(p_2 + p_1)
\]

... (8.1)

where the negative sign is adopted if the rotor circuits are connected in the same phase sequence. If the two machines have the same pole number (\( p_1 = p_2 \)), eqn. (8.1) shows that the frequency of the output voltage is independent of the rotor speed and is equal to the supply frequency \( f_1 \). When machine \( M_1 \) acts as an exciter for the main generator formed by the second machine \( M_2 \), the basis for a constant-frequency variable-speed generator unit is established. A direct electrical link between the two stator windings provides closed-loop frequency control (capable of controlling the frequency at some predetermined value) and some desired feedback effects.

The basic arrangement of the constant-frequency variable-
speed generator unit of reference (39) is given in Fig. 8.3, which shows a wound-rotor induction machine supplying a load via the MAC exciter and a change-over switch. The two machines have the same pole numbers and their rotors are coupled both mechanically and electrically. Bearing in mind that the MAC exciter is essentially an induction machine, the scheme is basically similar to that of Fig. 8.2. During start-up of the unit, the change-over switch is switched to the 'start' position, to provide an excitation signal to the 3-phase series winding (acting as the starting winding) of the exciter. With the rotor turning at the required speed a voltage of the same frequency as that of the control voltage appears in the stator winding of the induction machine and this, in turn, supplies the shunt winding of the exciter. The output voltage will thus build up in a manner similar to that of the usual MAC arrangement. After successful build-up, the change-over switch is switched to the 'run' position which couples the load to the unit, with the series winding carrying the load current, and thereby provides a current-compounded effect capable of maintaining a substantially constant terminal voltage irrespective of the load and power factor conditions. Since the shunt and series windings of the exciter and the stator winding of the induction generator are linked, a closed frequency loop is established and the output frequency of the unit will remain constant. The self-regulated and brushless generator/exciter unit of Fig. 8.3 thus provides a constant-frequency characteristic.
In common with other conventional generator units, the power level in the exciter of Fig. 8.3 must be only a small fraction of the total output power if the unit is to be feasible. An investigation into this, and many other practical aspects, can most readily be conducted using the 2-induction machine arrangement of Fig. 8.2. For easy reference, such an arrangement is referred to as the CONMAC scheme.

8.2 Basic considerations on the CONMAC scheme

As already mentioned, the CONMAC scheme consists of two induction machines interconnected as shown in Fig. 8.2, with the two machines having the same number of poles and representing respectively an induction generator and an exciter, and the well-known equivalent circuit of an induction motor leads to an overall equivalent circuit for the scheme as in Fig. 8.4a. Subscripts 1 and 2 refer respectively to the stator and rotor quantities of machine $M_1$ and subscripts $a$ and $b$ have corresponding meanings for machine $M_2$. Note that all the circuit quantities in the figure are referred values.

In most normal induction machines, the impedances of the parallel magnetising branches are much greater than the series leakage impedances, and the currents flowing in these branches are accordingly much smaller than the full-load current of the machines. To obtain an initial assessment of
the way in which the CONMAC scheme will function as a generator unit, the two magnetising branches are therefore removed from the equivalent circuit. Although some error is thereby introduced into any equations which are subsequently derived, this is one of accuracy only and not one of principle. If all the circuit elements of the same kind are combined into a single parameter, a fully simplified equivalent circuit may then be drawn as in Fig. 8.4b, in which R is the combined stator resistances \((r_1 + r_a)\), r the combined rotor resistance \((r_2 + r_b)\) and X the combined leakage reactance \((x_1 + x_2 + x_a + x_b)\) of the two machines.

To assist in the following discussions, test results from an experimental CONMAC unit (ECU) are included whenever applicable. The ECU is a 4-pole unit rated to deliver 20kVA at 420V (line to line) at an operating frequency of 60Hz. From test results on the unit (see Appendix A5), the parameters of the equivalent circuit of Fig. 8.4b were obtained as: \(R = 1.03\Omega, X = 4.02\Omega\) (at 60Hz) and \(r = 1.05\Omega\). Note that a more detailed experimental investigation undertaken on a second ECU is presented in Section 8.3†.

8.2.1 The simplified analysis

If a load impedance \(Z/j\phi\) is connected to each phase of the generator output, the following per-phase relationships can be derived directly from Fig. 8.4b:

\[
P_o = V_o I_o \cos \phi
\]

\[\text{... (8.2)}\]

† The operating frequency of the second ECU is also 60Hz.
\[ P_m = I_o^2 \frac{1-s}{s} \] ...
\[ P_L = I_o^2 (R + r) \] ...
\[ P_e = I_o^2 (R + \frac{r}{s}) + V_o I_o \cos \phi \] ...
\[ V_i = V_o + I_o (R + \frac{r}{s}) + jI_o X \] ...

where \( P_o \) is the output power, \( P_m \) the mechanical power developed, \( P_L \) the copper loss in the windings, \( P_e \) the electrical input power to the exciter and \( V_i \) the required input voltage.

In any practical implementation of a CONMAC scheme, the electrical power input to the exciter must be only a small fraction of the power output to the load, with the major portion of the output being derived from the mechanical power provided by the prime mover. To achieve this, the mechanical power developed and the term \((r/s)\) in eqn. (8.5) must both be negative, which can only be obtained when the slip is negative and the speed of the CONMAC unit exceeds the synchronous speed, as given by \((f_1/p_1)\). It is worth noting that a negative value for \( P_m \) denotes a flow of mechanical power from the shaft into the rotor circuit.

The limiting case of zero electrical power input to the exciter is clearly an ideal design objective, and it follows from eqn. (8.5) that this is achieved at only the single slip of
\[ s_o = \frac{-I_o r}{I_o R + V_o \cos \phi} \] ...

which is obviously dependent on both the load conditions and the winding resistances. If the unit speed is such that the slip is more negative than this value, power will flow into the exciter stator and the unit will function as a 2-stage generator. If the slip is less negative than that given by eqn. (8.7), the mode of operation of the exciter reverses and electrical power flows out of the stators of both machines.

Central to the consideration of any generating unit is the efficiency of power conversion, and it can be shown from eqns. (8.2) and (8.4) that the efficiency of the CONMAC scheme is given by

\[ \eta = \frac{V_o \cos \phi}{V_o \cos \phi + I_o (R + r)} \] ...

or alternatively

\[ \eta = \frac{r + s_o R}{r - s_o R} \] ...

There is obviously a close inter-relation between \( \eta \) and \( s_o \), and the more negative is \( s_o \) the smaller is the efficiency. Assuming that the stator resistances are negligible, an upper limit to the efficiency can be established, given by

\[ \eta_{max} = \frac{1}{1 - s_o} \] ...

(8.10)
Thus, with the exciter operating on a null exciter input, a reduction in the efficiency must be accepted as the slip is increased. For example, if $s_0$ is $-1/3$ (2400 rpm), the maximum efficiency is 75%, whereas if $s_0$ is $-2/3$ (3000 rpm) this falls to only 60%. It must be emphasized that this reduction is a consequence of changing the parameters of the CONMAC unit, so as to raise the slip at which the power input to the exciter is zero. If the slip is increased while the parameters remain unchanged, then, as eqn. (8.8) shows, the efficiency remains unaltered. However, as will become clear later, the input power to the exciter may become excessive.

Table 8.1 presents some typical calculations for the ECU. It is evident from the table that in all the cases considered $s_0$ is low, emphasizing that the most useful operating range of the experimental unit is only slightly above synchronous speed. Operation at near synchronous speed also has a high maximum theoretical efficiency, and hence a high actual efficiency. Efficiency figures calculated from the simplified model are also compared with corresponding test results in Table 8.1, and bearing in mind the approximations inherent in the simplified analysis and the limitations in the method by which the measured efficiency was obtained, these show an unexpectedly good measure of agreement.

Further experimental results show that when the unit was run at 1900 rpm the power flow to the exciter was zero for an output current and power of 15.7A and 3.8kW at rated unity power factor. The simplified analysis predicts that the zero
power flow condition is achieved when $I_o$ is 15.3A and $P_o$ is 3.7kW, results again in remarkably good agreement with measured values.

8.2.2 **Operation at high slip**

Inspection of eqns. (8.2-8.5) shows that when a CONMAC unit operates at a large negative slip the mechanical input power supplies only the rotor copper loss, with the electrical input to the exciter stator having to meet the load demand and provide the stator copper loss. Such a situation is clearly undesirable, and it is rather unfortunate that the above statement is largely true even at a moderately large negative slip. Thus, the range of speed over which the CONMAC principle can be exploited in practice is extremely restricted. For example, if the experimental unit is run at 3000 rpm ($s = -2/3$), then for a unity power factor load of 6.67kW/ph, $P_m = -1.99$kW, $P_e = 6.25$kW and $P_o = 1.57$kW, showing that even in the speed range for which the CONMAC unit is intended almost all the output power is derived from the exciter input, with only a small contribution coming from the mechanical input. As the unit stands, operation at this slip is clearly impracticable, and to reduce the exciter input to an acceptable level it is necessary to make the value of $s_o$ more negative. The following sub-sections will discuss ways by which this may be achieved and the practical limitations and implications of each.
8.2.2.1 Reduction of $R$

It is apparent from eqn. (8.7) that the effect of reducing $R$ is to raise $s_0$, which has the advantage of increasing the efficiency. However, since the conductor size of the stator windings must have some upper limit the amount by which $s_0$ can be raised is small. For the ECU, even in the limit when $R = 0\Omega$, $s_0$ for a rated unity power factor load is only changed from -0.107 to -0.119. It follows that reducing $R$ has too limited an effect for it to be of any practical value.

8.2.2.2 Additional rotor resistance

If the output conditions are maintained constant, eqn. (8.7) indicates that $s_0$ is directly proportional to $r$, and if additional resistance can readily be included in the rotor circuit a very convenient control is provided over $s_0$. The direct effect of $r$ on $s_0$ is illustrated theoretically in Fig. 8.5, for rated unity power factor load. However, increasing the rotor resistance also has the undesirable effect of increasing the loss in the CONMAC unit and as Fig. 8.5 also shows, the accompanying reduction in efficiency is very substantial. For example, to operate the CONMAC unit with zero exciter input at a slip of -2/3 requires an increase in the total rotor circuit resistance to 6.56\Omega, which reduces the maximum theoretical efficiency to 60\% (note that it is 53.7\% when stator copper loss is accounted for and only 37\% when iron loss is also included). The clear implication from Fig. 8.5 is that if a CONMAC unit is to be run at a high speed and with
an acceptable power input to the exciter, an efficiency lower than that achievable at low (near-synchronous) speeds and much lower than that obtainable with conventional machines must be accepted.

The idealised theoretical variations of electrical and mechanical inputs with slip at a constant output power was mentioned previously, and is shown in Fig. 8.6a for the basic experimental unit and in Fig. 8.6b for a unit with the rotor resistance raised from the original $1.05\Omega$ to $6.56\Omega$ (the value for $P_e = 0$ at $s_o = -2/3$). Note that, for both figures, the total output power is $20\text{kW}$ at unity power factor and $420\text{V}$ (line). Fig. 8.6a emphasizes yet again the impracticability of operating the basic CONMAC unit at a high slip, with the exciter input rising rapidly until it exceeds the mechanical power input and becomes very nearly equal to the output power. Furthermore, since the power level in the exciter must be restricted to a reasonable level, the range of speed over which the CONMAC unit is useful is very limited. Comparison of Figs. 8.6a and 8.6b shows that the increased rotor resistance has the effect of moving the exciter input power characteristic downwards, resulting in a much reduced exciter requirement when the slip is in the region at which a practical unit would be required to operate. But this reduction in exciter input is only achieved at the expense of increasing the total rotor copper loss to $14.9\text{kW}$, in comparison to the original value of $2.38\text{kW}$. Bearing in mind
that the total output from the unit is only 20kW, such a high level of rotor copper loss must be regarded as totally unacceptable.

8.2.3 Operation at varying power factor

When the CONMAC generator is called upon to supply a zero power factor load, a very unexpected situation develops. The exciter input power, as given by $I_o^2(R + r/s)$, is proportional to the square of the load current, and if optimal conditions of operation for a given load current are determined at a power factor other than zero, eqn. (8.5) requires $(R + r/s)$ to be negative. This implies an increasing outflow of power from the exciter stator as the load current is increased. Since a zero power factor load is incapable of accepting real power, another sink must be provided to re-establish a power equilibrium and it is difficult to foresee how this can be implemented practically.

If the performance of the CONMAC generator at a slip $s_o$ is optimised for zero exciter input power at zero power factor, the required rotor resistance is given by $r = -s_o R$. Inspection of eqns. (8.3-8.5) shows that at any other load conditions the exciter input power is then actually equal to the load power, with the mechanical input supplying only the copper loss of the two machines. It is therefore apparent that operation at this particular setting is not in any way a practical proposition.
In any practical situation, the CONMAC unit is expected to supply loads with lagging power factor. Since the input volt-amperes $S_i$ of the basic unit are given by

$$S_i = V_o I_o + I_o^2 (R + \frac{R}{s}) + jI_o^2 X$$ \hspace{1cm} (8.11)

al all the reactive load demand has to be provided by an external source. Clearly, the arrangement of such a source would be extremely complicated if it was to function properly at all load conditions and speeds.

8.2.4 Maximum efficiency for non-zero exciter power

Previous considerations have been related to the maximum theoretical efficiency $\eta_{\text{max}}$ which can be achieved when the excitation power is required to be zero at a given slip $s_0$. If this restriction is relaxed, it can be shown that the maximum efficiency becomes

$$\eta_{\text{max}} = \frac{1 + k(1 - s_k)}{(1 - s_k)(1 + k)}$$ \hspace{1cm} (8.12)

where $s_k$ is the slip at which the ratio of the exciter power to mechanical power is negative $k$. Although this expression indicates that $\eta_{\text{max}}$ approaches unity if $-k$ is large, practical constraints restrict $-k$ to a range probably between 0 and 0.1. The variation of $\eta_{\text{max}}$ with $s_k$ for different values of $k$ is shown in Fig. 8.7. Even if these results do indicate an increase in the maximum efficiency, it is only
from 75% to 77.3%, when the slip is -1/3, and from 60% to 63.6% when it is -2/3.

8.2.5 Basic considerations - conclusions

The preliminary analysis outlined in this section has revealed several major limitations associated with the CONMAC proposal for a brushless constant-frequency generator based on a 2-induction machine arrangement. Perhaps the most fundamental of these relates to the efficiency, which will unavoidably be low if the exciter power input is kept to a manageable level by appropriate increase in the rotor resistance. To maintain this condition in a generator working over a range of speed, it is further necessary for the resistance to be variable. Even if this is possible, operation at power factors away from unity is still not practicable. Unlike conventional generators, the reactive power loading on the output of a CONMAC unit is supplied by the exciter input, and although operation may be tuned to minimise the power requirements, a low power factor condition (such as during motor starting) will require the exciter and the excitation control equipment, if any, to be of comparable rating to the main generator.

In summing up, it is clear that, although the CONMAC concept will in principle provide a constant-frequency variable-speed generator, there are too many inherent disadvantages for it to be successfully exploited in the speed range from 2400 rpm to 3000 rpm, originally envisaged for a 4-pole unit.
to take advantage of the favourable characteristics of a diesel-engine prime mover. Only when the speed is marginally above synchronous and the load is substantially constant will its potential be fully realised with a high efficiency. Even then the need for the exciter and the excitation circuit to handle the reactive component of the output presents a formidable stumbling block.

To substantiate the findings of this initial analysis, further theoretical work and a full experimental study was undertaken on an experimental unit, with the details being presented in the following section. Although this study is primarily concerned with the more practical aspects of the CONMAC unit, proposals for improving the performance of the unit are also considered.

8.3 Experimental investigations

The use of a simplified model in the preceding section has predicted, in principle, the existence of certain inherent limitations of the proposed CONMAC scheme, particularly the adverse effect on efficiency when operating at a large and negative slip. However, it is not to be expected that this simplified model will, in general, provides results in good agreement with those obtained from a detailed experimental study on a second ECU, which is also a 4-pole unit but rated to deliver 20kVA at 440V (line). Thus, an improved equivalent circuit for the ECU needs to be developed to assist with the
present study. Once established, this model will allow a further assessment to be made of many practical aspects of the CONMAC scheme.

8.3.1 Improved equivalent circuit of ECU

A complete equivalent circuit for a CONMAC unit is given in Fig. 8.4a. Since this is based on the well-known equivalent circuit for an induction motor, the stator core loss and rotor core loss in each machine are lumped together and represented by a single loss resistor (r_{ml} or r_{ma} in Fig. 8.4a) that is independent of slip and incapable of developing mechanical power. However, open-circuit and short-circuit tests (Section 8.3.1.1) on the ECU, established that a significant amount of mechanical power is developed due to the rotor core loss, and the inclusion of a rotor core-loss resistor with each magnetising branch is therefore necessary. The equivalent circuit as improved by this addition is given in Fig. 8.8a. It will be noted in Fig. 8.8a that the equivalent rotor core-loss resistances are divided by s, which is consistent with the treatment accorded to all other resistive rotor-circuit quantities. This dependence on slip will allow the improved model to be used to represent the CONMAC unit over a wide range of slip.

8.3.1.1 Derivation of circuit parameters

A series of locked-rotor, short-circuit and open-circuit tests was performed on both the exciter (M_1) and the main
generator \((M_2)\) of the ECU. The terms short-circuit and open-circuit refer to the state of the rotor terminals, and tests were performed at speeds of 1800 rpm, 2400 rpm, 2700 rpm and 3000 rpm, corresponding respectively to slips of 0, -1/6, -1/3, -1/2 and -2/3 (see Appendix A5 for further details and test results). The following discussions indicate how the various circuit elements of Fig. 8.8a are derived from this programme of tests. Where it is not reasonable to represent a circuit element by a constant number, numerical approximation with aid of a digital computer is used. Although the discussion is confined to the exciter, precisely the same methods apply to the main generator.

(a) Leakage reactances: Using results from the locked-rotor test, the leakage reactances \(x_1\) and \(x_2\) can be evaluated by methods outlined in many standard texts. It is worth noting that the numerical value of the leakage reactance depends on the saturation level of the leakage flux paths, and some errors must necessarily be incurred at high currents since the reactances are treated as linear elements.

(b) Stator and rotor resistances: When obtaining \(x_1\) and \(x_2\) from the locked-rotor test, the sum of the stator and rotor resistances \((r_1 + r_2)\) can also be determined. Direct measurement of winding resistances with a Kelvin double bridge allows the d.c. values of \(r_1\) and \(r_2\) to be determined individually, and making suitable use of \((r_1 + r_2)\) as derived from locked-rotor results enables the a.c. values of \(r_1\) and
$r_2$ to be determined. As the CONMAC scheme operates with a constant 60Hz stator frequency $r_1$ will be constant, but because the rotor frequency varies directly with slip $r_2$ is dependent on the speed at which the unit is driven. Using results obtained from a short-circuit test, $r_2$ at a given slip and rotor current $I_r$ may be evaluated from

$$P_{ms} = I_r^2 \left( \frac{1-s}{s} \right) r_2 \quad \ldots \text{(8.13)}$$

where $P_{ms}$ is the mechanical power per phase developed under short-circuit conditions. Note that the expression for $P_{ms}$ neglects the contribution made by $r_{m2}$, since the short-circuit test is conducted at a low voltage level and reduces such a contribution to a negligible proportion. Test data from the ECU (Fig. A5.3b) showed that $r_2$ could be regarded as constant at any given slip, with its value being given by the slope of a graph of $P_{ms}$ against $I_r^2$ at that slip. It will be noted that obtaining $r_2$ this way accounts inherently for any uncharacteristic behaviour due to the rubbing contacts (introduced only for purpose of instrumentation) in the ECU.

(c) Rotor and stator core-loss resistances: Considering only the exciter portion of the equivalent circuit in Fig. 8.8a, the following relationships can be established under the open-circuit conditions:

$$P_{mo} = \frac{V_{me}^2}{r_{2m}} (1-s)s \quad \ldots \text{(8.14)}$$
where $P_{mo}$ and $P_{eo}$ are respectively the mechanical power developed and the stator input electrical power in the open-circuit test. An analysis of test data obtained on the ECU indicated that the rotor core-loss resistance certainly increased with slip, but in a manner which is not readily amenable to analytical representation. However, a computer subroutine written to implement eqn. (8.14) enables the value of $r_{m2}$ to be readily obtained. Then, with appropriate inputs covering the range of interest, the subroutine can be written to give exact values for $r_{m2}$ at given data points, with linear interpolation between data points being used to give intermediate values of $r_{m2}$.

After $r_{m2}$ has been determined, use of eqn. (8.15) enables the value of $r_{ml}$ to be obtained. Within the limits of experimental error, analysis of the open-circuit test data for the ECU shows that $r_{ml}$ can be approximated as a constant resistance. This reflects the fact that the stator core-loss is dependent on the input frequency (which is constant) and varies almost linearly with $V_{me}^2$. (Note that $V_{me}$ is the voltage across the magnetising branch).

(d) Magnetising reactance: Saturation of the main flux path results in the magnetising reactance being a non-linear element in the equivalent circuit of any electrical machine. An open-circuit test at 1800 rpm (the synchronous speed) will
provide data relating \( V_{me} \) to \( I_{mex} \), the current flowing in \( X_{m1} \). Using the curve-fitting technique of Appendix A3 enables the non-linearity to be expressed as the ratio of two rational polynomial functions of \( V_{me} \). Thus,

\[
I_{mex} = \frac{a_0 + a_1 V_{me} + a_2 V_{me}^2}{1 + b_1 V_{me} + b_2 V_{me}^2} V_{me}
\]...

(8.16)

where \( a_0, a_1, a_2, b_1 \) and \( b_2 \) are constant coefficients. At any voltage \( V_{me} \), the magnetising reactance is given by

\[
X_{m1} = \frac{V_{me}}{I_{mex}}.
\]

8.3.1.2 Circuit parameters for ECU

Using the methods detailed above, the various constants required to model the ECU were obtained as in Table 8.2. Note that the circuit elements involved are defined in Fig. 8.8a.

8.3.1.3 Notes on calculation using model of ECU

Assigning appropriate values to each circuit element of Fig. 8.8a enables the performance characteristic of the ECU to be predicted under no-load and varying load conditions. Since the output conditions are normally specified, it is convenient to work in stages toward the input terminals. By this means, the non-linearities of the various circuit elements can be accounted for in a simple but effective manner. For obvious reasons, a digital computer based analysis is adopted. In cases where the input conditions are
specified, notably those associated with the no-load test, an iterative procedure is used to locate the exact solution. Applying Kirchhoff's voltage and current laws enables the various currents and voltages defined in Fig. 8.8b to be evaluated for the individual branches of the circuit of Fig. 8.8a. The various powers, together with the overall efficiency are then given by

\[ P_o = V_o I_o \cos \phi \]  \hspace{1cm} (8.17)

\[ P_\ell = I_o^2 r_a + I_r^2 (r_2 + r_b) + I_i^2 r_l + \frac{V_{mg}^2}{r_{mg}} + \frac{V_{me}^2}{r_{me}} \]

\[ + \frac{V_{mg}^2}{r_{mb}} + \frac{V_{me}^2}{r_{m2}} \]  \hspace{1cm} (8.18)

\[ P_m = I_r^2 \left( \frac{1-s}{s} \right) (r_2 + r_b) + \left( \frac{V_{me}^2}{r_{m2}} + \frac{V_{mg}^2}{r_{mb}} \right) (1-s)s \]  \hspace{1cm} (8.19)

\[ P_e = P_o + P_\ell + P_m \]  \hspace{1cm} (8.20)

\[ \eta = \frac{P_o}{P_o + P_\ell} \]  \hspace{1cm} (8.21)

with all currents and voltages being identified in Fig. 8.8b.

8.3.2 Characteristics of the basic ECU

All the computed results in this and the next two sections are calculated using the model for the ECU established in Section 8.3.1. Most results are presented in graphical form and, whenever possible, actual test data are included for comparative purposes. All voltages and currents quoted are line quantities, while all powers and volt-amperes are the
total for the unit. The currents and voltages labels on the graphs may be identified by reference to Fig. 8.8b.

8.3.2.1 Open-circuit characteristics

Figs. 8.9a-d show the overall (generator-output/exciter-input) open-circuit or no-load characteristics of the ECU at a slip of -1/6 (2100 rpm), with the corresponding characteristics for a slip of -2/3 (3000 rpm) being given in Figs. 8.10a-d. Visual examination of these graphs leaves little doubt of the good agreement between experimental results and theoretical predictions provided by the model.

It is also evident from the figures that although the exciter and rotor currents and the rotor and output voltages all show the effect of saturation, this is most prominent in the current results. Since the open-circuit current and output voltage characteristics are dominated mainly by currents flowing in the magnetising reactances, with values which remain unchanged at different slips, the two sets of characteristics are therefore largely similar. The rotor voltage, being proportional to the rotor speed, is evidently higher at the larger negative slip.

Due to the increased rotor loss, both copper and iron, the total loss is appreciably higher at the higher negative slip. In accordance with terms containing (1 - s)/s the mechanical power developed should decrease at the higher negative slip, but the results show in fact an increase at the larger slip, indicating that the reverse effect of the increased rotor
loss is more significant during no-load tests. On no load, the input volt-amperes supply only the magnetising branches of the ECU, so that about 10kVA is required to sustain an output voltage of 400V and the slip has very little influence on this requirement. In comparison with the nominal 20kW output expected of each machine of the experimental unit, the 10kVA magnetisation demand is rather severe.

8.3.2.2 Load characteristics at unity power factor

The characteristics of the ECU when loaded at unity power factor are shown in Figs. 8.11a-12d, for a constant terminal voltage of 400V and slips of -1/6 and -2/3 respectively. It is again apparent that results predicted on the basis of the model developed for the ECU agree well with experimentally obtained results.

As the load current increases, the voltage across the leakage reactances, which are in quadrature with the output voltage, and those across the stator resistances, which are cophasal with the output voltage, both have the effect of requiring an increased input voltage, whereas the voltages across the effectively negative rotor resistances have the opposite effect. Evidently, the latter effect is most prominent at a small negative slip, where it results in a reducing input voltage being required as the output load current is increased. At a slip of -2/3, the two opposing effects cancel and the input voltage remains almost constant at 480V as the load current is varied.
The rotor current $I_r$ is the phasor sum of the output current and the magnetising current of the main generator. Since the terminal voltage is maintained constant at 400V, curves giving the changes in $I_r$ with load are therefore almost independent of slip. The input current to the exciter $I_i$ is the phasor sum of the rotor current and the magnetising current of the exciter. At a small slip, the decreasing input voltage requirement results in a smaller magnetising current in the exciter, so that the input current increases at a slower rate than when the slip is $-2/3$. On the other hand, due to the high voltage and current levels the input volt-amperes are higher at the larger negative slip. Thus at the full load current of 26.2A, the input volt-amperes are even higher than the output volt-ampere, rising, for example, from 22kVA when $s = -1/6$ to 30kVA when $s = -2/3$. Such an excessive requirement is clearly an undesirable feature of the scheme.

The mechanical power input to the unit is made up of a no-load component and a load component. As the load current increases so too does the mechanical power, but at a rate which falls as the slip becomes increasingly negative, due again to the reduced magnitude of the term $(1 - s)/s$. Close inspection of the graphical results of Figs. 8.12a-d reveals that at $s = -2/3$ only the loss in the unit is supplied by the mechanical input, leaving the electrical input to the exciter to supply entirely the output power. This undesirable phenomenon was anticipated in the simplified analysis of Section 8.2. At the larger slip, the total loss is slightly higher than that
for the lower slip, since the rotor loss has increased. Its
effect on efficiency is not however serious, as illustrated
by the fact that at full load current the efficiency decreases
by only 4% over a very wide range of slip, falling from 62%
at $s = -1/6$ to 58% at $s = -2/3$.

8.3.2.3 Lagging load: 0.8 power factor

Load characteristics for the ECU at slips of $-1/6$ and
$-2/3$ for 0.8 lagging power factor 400V load are presented in
Figs. 8.13a-14d. Comparison of experimental and calculated
results shows that, although there are some slight
discrepancies, satisfactory predictions are again provided
by the unit model. For reasons that will become clear
later, it is some slight uncertainty over the values of the
leakage reactances and power factor which is mainly
responsible for the reduced accuracy.

While the active component of the load current has the same
effect as discussed previously for a unity power factor load,
the lagging reactive component develops voltages across the
leakage reactances cophasal with the output voltage, thereby
having a direct effect in increasing the required input
voltage. Voltage drops in the rotor resistances due to this
component are in quadrature with the output voltage, but since
they are in phase opposition to the leakage reactances drops
due to the active current component, their effects depend on
the relative magnitudes. As is evident from the figures, the
input voltage required increases with the output current but
shows little variation with slip. This indicates that the inphase voltage drops have a dominant effect on the input voltage, so that it is the lagging reactive component of the load current which is mainly responsible for the more rapid increase in the input voltage.

As a consequence of the higher voltage level throughout the unit, the currents in the two machines increase at a faster rate than when the load is at unity power factor. Since the input voltage exhibits only a little variation with slip, it follows that the current variations are necessarily independent of slip.

The variation of the mechanical power with slip shows the same trend as in the unity power factor case, although both this power and the total loss in the unit are higher due to the increased currents. At $s = -2/3$ the mechanical input apparently provides only the loss, leaving the electrical input to meet the output demand. Since the input volt-amperes are the product of the input voltage and the input current, they show little variation with slip but increase rapidly with load current. It should be noted that most of the volt-ampere input is used to meet the magnetising demand and the reactive load demand, so that when supplying a load of 18.2kVA (26.2A at 400V) a totally unacceptable input of 50kVA is required.

In view of the lower output power and the higher loss at any particular level of output current, the efficiency of the ECU
is lower at 0.8 power factor lagging than at unity power factor. At rated load current, the efficiency falls from about 48% when \( s = -1/6 \) to about 44% when \( s = -2/3 \).

### 8.3.2.4 Lagging load: lower power factor

It is difficult in practice to obtain a zero power factor lagging load and the nearest that could be obtained to this had a power factor of about 0.1. Figs. 8.15a-16d show the ECU characteristics under these conditions, at a terminal voltage of 400V and for slips of -1/6 and -2/3. Comparison between predicted and experimental results generally shows satisfactory agreement, with any discrepancy being attributed to uncertainty in the values of both leakage reactances and power factor.

Now that the output current is primarily reactive, it is obvious that the input voltage, input current and the volt-ampere requirement all increase at a much faster rate than for a near-unity power factor load. Furthermore, in view of the very small influence of the active current component, the input voltage, and hence the input current and volt-ampere, are higher at the smaller negative slip. Thus when the slip is -2/3 and the output current 26.2A, an input of over 70kVA is required to supply a load taking merely 18.2kVA, but when the slip is -1/6 the input requirement reaches 110kVA.

Because of the higher rotor current, the mechanical power
and the total loss are greater than at the power factors considered previously. In accordance with the smaller output powers the input power is also reduced.

8.3.2.5 Input volt-ampere and efficiency

Based on the performance of the ECU, it is now possible to review some important features of a possible CONMAC scheme when supplying loads of varying power factor. The inability of a CONMAC unit to generate reactive power is a clear shortcoming, and the situation is made worse by the increase in magnetising demand caused by the reactive loading. The input volt-amperes are much greater than those demanded by the load, and in the specific instance of a 0.8 lagging power factor load the input can be three times the output. To handle this will require an extremely large exciter and an excitation system capable of generating the necessary volt-amperes. It is difficult to foresee how this could be justified on any economic basis.

Apart from the excessively high input volt-amperes, reactive loading also brings about a high loss and a considerably reduced efficiency. In the experimental unit, the efficiency at full load current fell from about 60% to about 46%, at all slips in the range of -1/6 to -2/3, as the power factor changed from unity to 0.8 lagging. Such low figures are clearly unacceptable in any practical scheme.
8.3.3 Characteristics of ECU with added rotor resistance

Due to the limitation imposed by the d.c. driving motor of the experimental unit, the highest speed at which a load test could be performed was 2700 rpm (s = -1/2). Thus, to allow comparison between experimental and predicted results, only results at that slip and a slip of -1/6 are presented in graphical form. Computed results for a slip of -2/3 are however quoted, for comparative purpose, in the following discussions. A similar problem also limited the added rotor resistance to 0.5Ω.

Figs. 8.17a-18d show the open-circuit characteristics of the ECU at slips of -1/6 and -1/2, and with added rotor resistance of 0.5Ω per phase. The good correlation between experimental and predicted results provides further proof of the validity of the model used. Because of the dominant effect of the magnetising circuit of the main generator on the open-circuit characteristic, the small addition in the rotor resistance causes very little change. Perhaps the only noticeable difference is in the slightly higher levels of both the mechanical input power and the loss, but even these are rather small and insignificant.

8.3.3.1 Load characteristics: unity power factor

Load characteristics of the ECU with 0.5Ω/phase added rotor resistance are shown in Figs. 8.19a-20d, for a terminal voltage of 400V and a purely resistive load. The generally
good pattern of agreement between theoretical and practical results is repeated here again.

As mentioned previously, the voltage drops across the effectively negative rotor resistance causes the exciter input to decrease as the output is increased. Plainly, this effect should be most noticeable at low slip and with the added rotor resistance. Although other inphase voltage drops mask this effect somewhat (in particular those due to the magnetising currents flowing in the leakage reactances), it is nevertheless clearly observable. Because of the lower input voltage, a lower exciter input follows from the increase in the rotor resistance, with the rotor current remaining unaffected. There is also a slight fall in the input volt-amperes, so that when supplying full load current of 26.2A at 400V and a slip of -2/3, the input is 31kVA with no added resistance and 29kVA when 0.5Ω/phase is added.

The most notable feature arising from the addition of rotor resistance is the redistribution of the power flow through the two machines, bringing about an increase in the total loss and the mechanical input power, and a reduction in the electrical power input. It is clear that any reduction in the input electrical power is obtained only at the expense of a higher rotor loss and therefore of a reduced efficiency. At all levels of slip considered, the efficiency drops by about 4% for the added rotor resistance of 0.5Ω/phase, and since it is envisaged that about 5Ω/phase is required to
bring the electrical input down to a practical level at a slip of -2/3 the consequent drastic fall in the efficiency is not difficult to imagine.

8.3.3.2 Load characteristics: lagging power factors

Figs. 8.21a-22d show the characteristics of the ECU with an added rotor resistance of 0.5Ω/phase and a lagging load power factor of 0.8. The terminal voltage of the unit is 400V. Again, experimental results show generally good agreement with theoretical results derived from the model.

Since the increase in the rotor resistance considered here is small, the input current and voltage characteristics at the larger slip show comparatively little variation when compared with the case of no added resistance. But the large quadrature voltage drops across the effectively larger negative rotor resistance at $s = -1/6$ causes a more noticeable difference in the input characteristics at this slip. With full-load current of 26.2A at 400V, giving an output of 18.2kVA, the input volt-amperes are about 70kVA at $s = -1/6$, but remain at about 50kVA at the more negative slip.

As expected, the introduction of additional rotor resistance moves the entire input power characteristic downwards, resulting in a substantial outflow of power from the exciter of, typically, 10kW at a slip of -1/6 and full-load current. In fact, this results in an additional demand on the mechanical power, which is already high due to the increased
rotor loss. The added resistance also degrades the performance of the unit by a 4% reduction in its efficiency.

Figs. 8.23a-24d present the load characteristics when the load power factor is 0.1 lagging. The increase in the input voltage, current and volt-amperes brought about by the increased rotor resistance are more evident here than at unity power factor, and in fact when the slip is -1/6 an input of 120kVA is required for only a 18.2kVA output. At the lower slip and full-load current, the outflow of exciter power is about 24kW, which is higher even than the rated output of the unit. Furthermore, the mechanical power at the larger slip, which goes largely to supply the rotor loss, also exceeds the rated output.

8.3.4 The ECU with added capacitance

The CONMAC equivalent circuit shown in Fig. 8.8a is essentially inductive in nature. The current required by the magnetising branch of the main generator causes an increased rotor current, which has an adverse effect on both the loss in and the efficiency of the unit. Moreover, the lagging magnetising current raises substantially the voltage required at the exciter input and causes a corresponding increase in the input volt-amperes. These considerations lead to the conclusion that, by connecting capacitors across the output terminals of the ECU, it should be possible to compensate for the generator magnetising current and to reduce the
rotor current. This possibility is investigated in detail below.

8.3.4.1 Open-circuit characteristics

The open-circuit characteristics of the ECU with an added capacitive reactance of 46.08Ω/phase, 5kVAR (all capacitive kVAR figures are the totals for the three phases) at 480V, are given for slips of -1/6 and -2/3 in Figs. 8.25a-26d. Except when the slip is -1/6, the calculated and experimental results do not agree well when the input voltage is low. Experimental results at the higher slip show that under these conditions the ECU may self-excite, to give an output voltage higher than that existing in the absence of the capacitors and which cannot be controlled by the input voltage. It was found that with no input voltage and a slip of -1/3, the output voltage rose excessively for added capacitive reactance of 5kVAR, reached stable values of 300V with 15kVAR, 260V with 20kVAR, 210V with 30kVAR, 180V with 40kVAR, 143V with 50kVAR and did not self excite with 60kVAR. It appears that the resonant circuit formed by the capacitors and the magnetising branch of the generator is mainly responsible for the self-excitation at a low input voltage, but a full understanding of the phenomenon requires further study. As is evident from the experimental results, self-excitation does not occur at a high input voltage, when the fundamental frequency component is dominant.
8.3.4.2 Load characteristics

The load characteristics of the ECU with 5kVAR capacitive reactance connected across the output terminals are given in Figs. 8.27a-28d, for an output voltage of 440V and at unity power factor. Evidently, an accurate prediction of the actual performance is again obtained.

When compared with the characteristics given in Figs. 8.29a-30d for similar load conditions but without the output capacitors, a marked reduction in the rotor current is evident at all slips. The rotor current characteristics are shifted downwards by about 10A at a slip of -1/6 and about 5A at a slip of -2/3. Due to the reduced lagging component in the rotor current, the input voltage characteristics are also shifted downwards, and together with the reduced rotor current this accounts for the reduction evident in both the input current and the input volt-ampere of the exciter. Thus at full-load current and a slip of -2/3, the rotor current, exciter input current, exciter input voltage and exciter input volt-ampere are respectively 30A, 36A, 510V and 31.8kVA, which show a noticeable reduction from the corresponding values of 34A, 41A, 560V and 39.8kVA for the unit without capacitors.

It is evident that, as a result of the reduced rotor current, a lower mechanical input, lower loss and a higher electrical input are all experienced. In particular, when the slip is
-2/3 the increase in the electrical input is barely noticeable, while the significant reductions which occur in both the mechanical input and the loss further illustrate the close relation between these two variables when the slip is large and negative. General comments made previously concerning the variation of all three quantities with slip are again applicable, and a comparison of the results shows that perhaps the most notable achievement brought about by the addition of capacitors is an improvement in the full-load efficiency of, typically, about 4%.

8.4 CONMAC unit with added rotor resistance and capacitance

The analysis in Section 8.2 has shows that the only way in which the CONMAC principle can be successfully exploited at a high negative slip is by the introduction of considerable extra rotor resistance. However, although this effectively reduces the electrical input power to a manageable level, it is at the expense of a large reduction in the efficiency. Use of the ECU model established in Section 8.3, together with the experimental studies, has provided a further indication that this is a major limiting feature of the concept. The study in Section 8.3 has also demonstrated the beneficial effects of incorporating output capacitors in the CONMAC scheme. Thus, it is appropriate at this stage to examine more carefully the performance of a CONMAC unit with optimum values of rotor resistance and output capacitance.
This will be a theoretical study only, based on the ECU model by now thoroughly substantiated.

8.4.1 Variation of efficiency with slip

In deriving eqn. (8.12), k was defined as the negative of the ratio of the electrical input power to the mechanical input power. It will be noted that adopting this definition leads to a useful expression that is independent of the machine parameters. However, since we are now concerned with varying load conditions, it is more appropriate to introduce a new ratio $\alpha$, defined as the ratio of the electrical power input to the exciter to the electrical power output from the generator. Under this definition, the efficiency of the simplified CONMAC model is

$$\eta = \frac{1}{\alpha - \{I_r(\frac{s_a}{s_a})/V_o \cos \phi\}} \quad \ldots (8.22)$$

where the slip $s_a$ for a given $\alpha$ is

$$s_a = \frac{-rI_o}{(1 - \alpha)V_o \cos \phi + I_o R} \quad \ldots (8.23)$$

By neglecting both the stator resistances, it can be shown that the maximum efficiency is given by

$$\eta_{\text{max}} = \frac{1}{1 + s_a(\alpha - 1)} \quad \ldots (8.24)$$

which is again independent of the machine parameters.
Although $\eta_{\text{max}}$ approaches 100% as $\alpha$ tends to unity, the unit will only be expected to operate properly if $\alpha$ is within the range of 0 to 0.1. The variation of $\eta_{\text{max}}$ with $s_\alpha$ at different values of $\alpha$ is depicted in Fig. 8.31a, which shows that a low maximum theoretical efficiency must be accepted if the unit is to operate with a reasonable level of exciter power and at a large negative slip. For instance, at a slip of -2/3, the maximum efficiency is 60% at $\alpha = 0$, and improves only slightly to 62.5% when $\alpha = 0.1$.

8.4.2 Efficiency of the ECU

At rated unity power factor output conditions, i.e. 20kW at 440V, the variation of the predicted efficiency with operating slip at different levels of exciter electrical input is as shown in Fig. 8.31b. The variations in the rotor resistance required to maintain a given $\alpha$ as the slip changes are also shown. (Note that the various graphs with $X_c$ added are considered later). Comparison of Fig. 8.31b with the variation of $\eta_{\text{max}}$ given in Fig. 8.31a indicates that a drastic reduction in efficiency is caused by the stator copper loss and the core loss of the two machines, not accounted for in the expression for $\eta_{\text{max}}$. The effect of this addition is to reduce the efficiency at a given $\alpha$ by about 17%, so that at the operating slip of -2/3 the efficiency is 43% when $\alpha = 0$ and improves only slightly to 46% when $\alpha = 0.2$. Evidently, any generating unit with this level of efficiency is unlikely to be a serious economic proposition.
For a particular output condition, $s_a$ is directly proportional to $r$. Inspection of Fig. 8.31b leads to a similar conclusion, and emphasises again the importance of the rotor resistance in the determination of the exciter electrical input. At $s = -2/3$, the rotor resistance of about $5\Omega$ required to reduce the electrical input to zero represents an approximate five fold increase from the original parameters. Apart from causing excessive loss, the need to vary the rotor resistance such that a given level of electrical input is maintained is exceedingly difficult for a brushless unit, if not technically impossible.

8.4.3 The ECU with added capacitance and rotor resistance

The study of Section 8.3.4 demonstrated that the efficiency of the ECU can be improved by the addition of capacitance at the output terminals. Further considerations, based on the model established in Section 8.3, indicate that there is a given capacitance which maximise the efficiency at a given slip and a given ratio of exciter electrical input to output power ($\alpha$). However, the same considerations also show that the efficiency is insensitive to slip around the maximum region, and a capacitive reactance $X_C$ of $13\Omega$/phase can reasonably be assumed to be the optimum value. The variation of efficiency with slip using this capacitance for a ratio of input electrical to output power of 0, 0.1 and 0.2 is shown in Fig. 8.31b, together with the value of rotor resistance required to achieve these ratios.
It follows from Fig. 8.31b that the much reduced rotor current requires an increased rotor resistance to maintain any particular level of $\alpha$, such that with $\alpha = 0$ and a slip of -2/3 the required rotor resistance is 6.7Ω with added capacitance instead of the 5Ω without. For a given $\alpha$, the efficiency curve is shifted upwards by about 6%, and although this takes the efficiency at $s = -2/3$ to 49% it remains some way below the theoretical limit of 60% at this slip and is still too low for any possible practical implementation.

8.4.4 Load characteristics with optimum rotor resistance and output capacitance

Figs. 8.32a-32d present the unity power factor characteristics of the ECU with optimum values of rotor resistance, as defined in Fig. 8.31b, and output capacitance of 13Ω/phase at slips of -1/6 and -2/3 and a terminal voltage of 440V. The input exciter power is zero when the unit is supplying the rated load current of 26.2A. It is clear from the graphs that the input current, voltage, volt-amperes and power show similar variations at both slips. As the load current increases, the input voltage decreases to a minimum of about 120V, before rising to the full-load value of 140V, while the rotor and exciter input currents are both reduced to a low level approximately equal to the output current. The output capacitance is thus very effective in compensating for the lagging magnetising current, and the input volt-
amperes are consequently also low, with only about 8kVA being required to sustain a load of 20kW. The variations of loss and mechanical power follow the now familiar pattern, with the comparatively high level of both resulting in a low efficiency when the slip is large. For example, at full-load current and a slip of -2/3, 41kW of mechanical output supplies 21kW of loss and 20kW of output, representing an efficiency of just 49%.

Figs. 8.34a-35d show load characteristics for the ECU, under conditions similar to those above but with a lagging power factor of 0.8. The input current, voltage, volt-ampere and power characteristics are again identical at different slips, with the input voltage this time decreasing to about 310V before rising to 460V at full-load current. Due to the higher level of input voltage, the exciter input current is generally higher than the rotor current, which has roughly the same magnitude as the output current. At full-load current there is a small outflow of power of about 15kW from the exciter, which is then handling about 20kVA. The mechanical input power and loss characteristics follow a trend similar to that at unity power factor. However, lower efficiencies are experienced, with, typically, the full load efficiency at a slip of -2/3 of 44% being 5% down on the unity power factor figure.

The zero power factor characteristics of the modified ECU are given in Figs. 8.36a-37d. The comments given above are
again generally applicable, although the input voltage does not now fall to a minimum, but rather increases continuously as the output current is increased and reaches 770V at the full-load current. As a consequence the exciter input current shows a rapid rate of increase, raising the exciter loading to about 70kVA at full load with an outflow of 7kW of power at the slip of -2/3.

8.5 Conclusions

It is clear from the analysis presented in this chapter that the total rotor circuit resistance is the dominant parameter controlling the power flow through the two machines in a CONMAC arrangement. To run the unit at the large values of slip for which it was intended requires a very large rotor resistance indeed, if the electrical input power is to be brought down to a realistic level. The highly undesirable effect this has on the efficiency has been emphasised several times, but it is a fundamental aspect of the arrangement that power from the mechanical drive can only be converted into electrical power through the agency of the rotor resistance. A further inherent feature is that, unlike a conventional synchronous generator, the CONMAC arrangement cannot generate reactive power, and the reactive demand of the load must be supplied at the exciter input. This would impose a very arduous duty indeed on any excitation control circuitry, which would naturally need to be rated for the most severe
loading conditions the unit is expected to encounter.

An interesting finding in the study is the beneficial effect which may be brought about by the use of output capacitors, although this introduces the possibility of self-excitation of the unit with the output voltage being uncontrollable at the input. Even with capacitors, the already low theoretical maximum efficiency cannot be approached with the ECU, and it is difficult to see how any improvements in design can lead to a significant increase in the actual efficiency. Furthermore, a given rotor resistance will only optimise the performance at a given slip and a given load, and although input capacitors could be added to minimise the volt-ampere of an excitation circuit at this load, they, like the rotor resistance would need to be changed with any change in load.

Due to its several and very severe shortcomings, it does appear that the CONMAC scheme cannot be a practical proposition, except in the unlikely case where the load does not vary and the low efficiency is not an important criterion.
Table 8.1 Some typical results

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<th>6.67</th>
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<tbody>
<tr>
<td>$P_o$ (kW/phase)</td>
<td>Unity</td>
<td>Unity</td>
<td>0.8 lagging</td>
</tr>
<tr>
<td>$s_o$ (p.u.)</td>
<td>-0.107</td>
<td>-0.075</td>
<td>-0.112</td>
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<tr>
<td>$\eta_{max}$ (%)</td>
<td>90.3</td>
<td>93.0</td>
<td>89.9</td>
</tr>
<tr>
<td>$n$ from simplified model (%)</td>
<td>80.9</td>
<td>86.2</td>
<td>80.1</td>
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<td>$n$ - measured (%)</td>
<td>-</td>
<td>80.0</td>
<td>81.0</td>
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Table 8.2 Measured per phase parameters of ECU

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<td>-</td>
<td>80.0</td>
<td>81.0</td>
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Table 8.2 Measured per phase parameters of ECU

Exciter

\[
\begin{align*}
    x_1 &= 1.151\Omega & r_2 &= 0.7362\Omega \text{ at } s = -1/6 \\
    x_2 &= 1.151\Omega & r_2 &= 0.7938\Omega \text{ at } s = -1/3 \\
    r_1 &= 0.9262\Omega & r_2 &= 0.9278\Omega \text{ at } s = -1/2 \\
    r_{ml} &= 116\Omega & r_2 &= 1.022\Omega \text{ at } s = -2/3 \\
    a_0 &= 2.5543 \times 10^{-2} & a_1 &= -1.5728 \times 10^{-4} & a_2 &= 3.0228 \times 10^{-7} \\
    b_1 &= -5.1214 \times 10^{-3} & b_2 &= 7.2035 \times 10^{-6} \\
\end{align*}
\]

Generator

\[
\begin{align*}
    x_a &= 0.5612\Omega & r_b &= 0.2981\Omega \text{ at } s = -1/6 \\
    x_b &= 0.5612\Omega & r_b &= 0.3288\Omega \text{ at } s = -1/3 \\
    r_a &= 0.3833\Omega & r_b &= 0.3563\Omega \text{ at } s = -1/2 \\
    r_{ma} &= 180\Omega & r_b &= 0.3864\Omega \text{ at } s = -2/3 \\
    a_0 &= 3.6183 \times 10^{-2} & a_1 &= -2.1593 \times 10^{-4} & a_2 &= 4.5370 \times 10^{-7} \\
    b_1 &= 5.3661 \times 10^{-3} & b_2 &= 8.0673 \times 10^{-6} \\
\end{align*}
\]
Fig. 8.1 Constant frequency output 2-stage induction machine system

Fig. 8.2 Two interconnected induction motors
Fig. 8.3 Constant-frequency generating unit incorporating a MAC exciter and an induction generator.
Fig. 8.4a Equivalent circuit for interconnected induction motors

Fig. 8.4b Fully simplified equivalent circuit for interconnected induction motors
Fig. 8.5 Variation of efficiency and $s_o$ with $r$. 

- $s_o$
- $\eta_{max}$
- $\eta$ (including stator copper loss)
- $\eta$ (including stator copper loss and rotor and stator iron loss)
Fig. 8.6a Variation of $P_m$ and $P_e$ with $s$, $r = 1.052\Omega$, output = 20kW (total) at unity power factor 420V (line)
Fig. 8.6b Variation of $P_m$ and $P_e$ with $s$, $r = 6.565\Omega$, output = 20kW (total) at unity power factor 420V (line)
Fig. 8.7 Variation of $\eta_{max}$ with slip, for different values of $k$
Fig. 8.8a Improved equivalent circuit for the CONMAC generator

Fig. 8.8b Definition of currents and voltages in improved equivalent circuit
Open-circuit characteristics of the ECU

(Full lines and symbols denote predicted and experimental results respectively)

Slip = -1/6
Open-circuit characteristics of the ECU
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -2/3
Load characteristics of the ECU, unity power factor, 100V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/6
Load characteristics of the ECU, unity power factor, 400V terminal voltage.

(Full lines and symbols denote predicted and experimental results respectively.)

Slip = -2/3
Load characteristics of the ECU, 0.8 power factor (lag), 100V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/6
Load characteristics of the ECU, 0.8 power factor (lag), 100V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -2/3
Load characteristics of the ECU, 0.1 power factor (lag), 100V terminal voltage (full lines and symbols denote predicted and experimental results respectively)
Slip = -1/6
Load characteristics of the ECU, 0.1 power factor (lag), 400V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -2/3
Open-circuit characteristics of the ECU, 0.5 Ohm/phase added rotor resistance
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/6
Open-circuit characteristics of the ECU, 0.5 Ohm/phase added rotor resistance
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/2
Load characteristics of the ECU, unity power factor, 400V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/6, with 0.5 Ohm/phase added rotor resistance
Load characteristics of the ECU, unity power factor, 400V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/2, with 0.5 Ohm/phase added rotor resistance
Load characteristics of the ECU, 0.8 power factor (lag), 400V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/6, with 0.5 Ohm/phase added rotor resistance
Load characteristics of the ECU, 0.8 power factor (lag), 400V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/2, with 0.5 Ohm/phase added rotor resistance
Load characteristics of the ECU, 0.1 power factor (lag), 480V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/6, with 0.5 Ohm/phase added rotor resistance
Load characteristics of the ECU, 0.1 power factor (lag), 100V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/2, with 0.5 Ohm/phase added rotor resistance
Open-circuit characteristics of the ECU, capacitive reactance = 46.08 Ohm/phase

(Full lines and symbols denote predicted and experimental results respectively)

Slip = -1/6
Open-circuit characteristics of the ECU, capacitive reactance = 16.08 Ohm/phase
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -2/3
Load characteristics of the ECU, unity power factor, 440V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/6, output capacitive reactance = 46.08 Ohm/phase
Load characteristics of the ECU, unity power factor, 110V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -2/3, output capacitive reactance = 16.00 Ohm/phase
Load characteristics of the ECU, unity power factor, 440V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -1/6
Load characteristics of the ECU, unity power factor, 110V terminal voltage
(Full lines and symbols denote predicted and experimental results respectively)
Slip = -2/3
Fig. 8.31a Variation of $\eta_{\text{max}}$ with $s_\alpha$ at different values of $\alpha$
Fig. 8.31b Variation of total rotor resistance $r$ for a particular $\alpha$ and efficiency $\eta$ with slip.
Load characteristics of the ECU, unity power factor, 440V terminal voltage
Slip = -1/6, with optimum rotor resistance and output capacitance
Load characteristics of the ECU, unity power factor, 110V terminal voltage

Slip = -2/3, with optimum rotor resistance and output capacitance
Load characteristics of the ECU, 0.8 power factor (lag), 440V terminal voltage
Slip = \(-1/6\), with optimum rotor resistance and output capacitance
Load characteristics of the ECU, 0.8 power factor (lag), 440V terminal voltage
Slip = -2/3, with optimum rotor resistance and output capacitance
Load characteristics of the ECU, zero power factor (lag), 140V terminal voltage
Slip = -1/6, with optimum rotor resistance and output capacitance
Load characteristics of the ECU, zero power factor (lag), 440V terminal voltage
Slip = -2/3, with optimum rotor resistance and output capacitance
CHAPTER 9
AN ALTERNATIVE TO THE MAC EXCITER

Since it is relatively difficult to exercise control over the voltage across the shunt circuit of a MAC exciter (in either the modified or the original form), the nominal output voltage of the generator cannot be readily altered. This difficulty can however be overcome by a novel exciter proposal, in which both the stator and the rotor contain windings having different pole numbers. In this chapter the principles that underly this proposal are discussed and an equivalent circuit is developed to assist in explaining its compounding characteristic. In view of the rather complicated nature of the rotor windings, a study on the possibility of replacing these with a single combined winding is also undertaken.

9.1 Background

When a single-phase synchronous generator is loaded, a pulsating m.m.f. is established in the airgap as a result of current flowing in the armature winding. This m.m.f. can be resolved into two counter-rotating components; firstly, a forward-rotating component that is stationary with respect to the rotor and contributes to the widely-known synchronous reactance effect as with a 3-phase generator, and secondly a backward-rotating component which, apart from the extra
losses it produces, plays no major role in the performance of a normal generator. However, it is possible\textsuperscript{40-42} to utilise this backward-rotating component as the basis for a single-phase self-regulated generator, in which the double-frequency e.m.f. induced in a suitably placed and additional rotor winding by the action of the backward-rotating m.m.f. is fed to the field winding through a rotating rectifier unit to increase the on-load excitation. In particular, reference (40) shows that the principle can be successfully exploited to give a single-phase self-regulating generator (see Fig. 9.1) which is also self-exciting.

In Fig. 9.1, A is the main armature winding having X poles and B is an auxiliary 2X-pole stator winding. On the rotor, F is the X-pole main field winding, with RSE and RSH being respectively X-pole and 2X-pole windings. Winding B establishes a 2X-pole field in the airgap, which induces an e.m.f. at double the output frequency of the generator in winding RSH, which is subsequently rectified by the bridge rectifier BR to provide the no-load excitation. When on load, the double-frequency e.m.f. induced in winding RSE by the action of the backwards rotating armature m.m.f. is added to the e.m.f. in winding RSH to boost the excitation, and by suitable design of windings B, RSE and RSH can be expected to maintain the output voltage to within the limits normally required for single-phase generator\textsuperscript{40}. Since windings B, RSE and RSH are all housed in the same stator/rotor frame as the
main generator windings A and F, and share a common magnetic circuit, the scheme provides a very cost-effective way of implementing a single-phase self-regulating generator. Furthermore, the otherwise unwanted reverse-rotating component of m.m.f. is now usefully employed to overcome the voltage regulation problem, which of course is due in the first case to its forward-rotating counterpart.

For a 3-phase synchronous generator supplying a balanced load, the reverse-rotating m.m.f. component due to current in the armature winding does not exist, and a scheme similar to the single-phase case cannot therefore be implemented directly. However, the basic principle of that scheme can be exploited by means of a special-purpose 3-phase exciter that contains windings of two different pole numbers.

9.2 System arrangement

The proposed exciter is connected to a X-pole synchronous generator as shown in Fig. 9.2. On the exciter stator there are two sets of windings: an X-pole series winding SE and a 2X-pole shunt winding SH. The exciter rotor, which is also coupled mechanically to the generator, contains the rotor series winding RSE and the rotor shunt winding RSH, having X and 2X poles respectively. As indicated in the figure, each phase of the two rotor windings is connected in series to supply the field winding of the generator via a rotating bridge rectifier RBR.
When winding SH is supplied with d.c. current through the bridge rectifier BR, a double-frequency e.m.f. (taking the generator output frequency as reference) is induced in winding RSH and this is subsequently rectified to provide the no-load excitation. Thus, with sufficient residual magnetism incorporated into the field system of the generator, the terminal voltage will build up in a manner very similar to that of a d.c. shunt generator. When the generator is on load, the current flowing in winding SE establishes a reverse-rotating (with respect to the rotor rotation) magnetic field in the exciter airgap, which in turn induces an e.m.f. of double the main frequency in winding RSE. The addition of this e.m.f. to that of winding RSH boosts the on-load excitation, and with a judiciously designed exciter it may be possible to maintain the terminal voltage of the generator to an accuracy approaching that of the MAC excitation scheme. However, since the d.c. current in winding SH may be adjusted readily by means of a trimming regulator VR, the proposal offers a very simple means of altering the nominal output voltage of the generator. If, as shown in Fig. 9.3, the bridge rectifier BR is replaced by a thyristor controller TC, together with an AVR to control its firing angle, closed-loop control over the output voltage can be achieved.
9.3 Analysis

In the analysis of electrical machines it is usual to consider only a 2-pole unit, since the flux-density distribution is repeated after every two poles. On this basis, the present analysis will consider a 2-pole synchronous generator and an exciter containing groups of 2-pole (SE and RSE) and 4-pole (SH and RSH) windings. Bearing in mind that there is no first-order magnetic interaction between the 2-pole and 4-pole groups of windings, the exciter may be regarded as two separate machines sharing a common magnetic circuit, with windings SE and RSE forming a 2-pole machine and windings SH and RSH a 4-pole machine. A phase-variable representation of the two machines and the generator is shown in Fig. 9.4, in which the angles and angular velocities are all defined in terms of the 2-pole field and the axis of the R-phase of the synchronous generator is taken as the reference position. It is important to note in Fig. 9.4 that the stator winding of the 2-pole machine has a reverse phase sequence and that operation of the 4-pole machine is accounted for by doubling all the appropriate angles and the angular velocity.

If it is recognised that the 2-pole machine operates as an induction machine with a slip of 2 and that the 4-pole machine is basically a small synchronous generator with a rotating armature, a steady-state equivalent circuit for the exciter can be developed by combining the well-known
equivalent circuits of these two machines. Thus, with the rectifier/field winding represented by a resistance $R_e$ and a reactance $X_e$, the per phase equivalent circuit for the exciter takes the form of Fig. 9.5a, where the circuit elements associated with either the 2-pole or 4-pole machine are clearly identified. In this circuit, the load current $I_L$ flows in winding SE with $R_{22}$ and $X_{22}$ being respectively the resistance and reactance of this winding, $R_{33}$ and $X_{33}$ the resistance and reactance of winding RSE, $X_{23}$ the mutual reactance between the windings SE and RSE and $R_{4r}$, $X_{4r}$ and $E_{4r}$ the armature resistance, synchronous reactance and excitation voltage for the 4-pole machine, with all the reactances being computed at the rotor frequency (twice the main frequency). When rotor circuit elements of the same kind are lumped, this part of the circuit may be simplified as shown in Fig. 9.5b.

To account for the phase angle of the load current and the spatial displacements of the windings axes from the reference position (i.e. $\theta_1$, $\theta_2$ and $\theta_3$), a phase angle $\alpha$ between the two voltage sources of the exciter rotor (i.e. $|jX_{23}I_L|$ and $E_{4r}$) is introduced into the equivalent circuits of Fig. 9.5a-b. The relationship between the latter angle and the former angles can most readily be derived by considering an open-circuited exciter rotor.

During balanced 3-phase operation, the instantaneous phase
voltages of the synchronous generator may be defined as

\[ v_r = \sqrt{2}|V| \sin(\omega t) \]
\[ v_y = \sqrt{2}|V| \sin(\omega t - \frac{2\pi}{3}) \quad \ldots (9.1) \]
\[ v_b = \sqrt{2}|V| \sin(\omega t - \frac{4\pi}{3}) \]

and when a load current \( I_L \) is supplied at a lagging phase angle \( \phi \), the phase currents are

\[ i_r = \sqrt{2}|I_L| \sin(\omega t - \phi) \]
\[ i_y = \sqrt{2}|I_L| \sin(\omega t - \frac{2\pi}{3} - \phi) \quad \ldots (9.2) \]
\[ i_b = \sqrt{2}|I_L| \sin(\omega t - \frac{4\pi}{3} - \phi) \]

If the load angle of the generator is \( \delta \), then at time \( t \)

\[ \theta = \omega t + \delta \quad \ldots (9.3) \]

and it follows from the flux linkage relationship that the open-circuit voltage induced in the R-phase of winding RSE and RSH are respectively

\[ v_{r2} = 3\sqrt{2}|I_L| \hat{M}_2 \cos(2\omega t + \delta - \theta_1 - \phi) \quad \ldots (9.4) \]

and

\[ v_{r4} = 2I_{f4} \hat{M}_4 \sin(2\omega t + 2\delta - 2\theta_3 - 2\theta_2) \quad \ldots (9.5) \]

where \( \hat{M}_2 \) is the peak mutual inductance between the R-phase of windings SE and RSE, \( \hat{M}_4 \) the corresponding value between the winding SH and the R-phase of winding RSH and \( I_{f4} \) the d.c. current in winding SH. Hence, the phase difference \( \alpha \)
between the voltage sources is

\[ \alpha = \frac{\pi}{2} - \delta - \phi + 2(\theta_3 + \theta_2) - \theta_1 \]  \quad \ldots (9.6)

and if the windings axes are arranged such that

\[ 2(\theta_3 + \theta_2) - \theta_1 = 0, \]

then

\[ \alpha = \frac{\pi}{2} - \delta - \phi \]  \quad \ldots (9.7)

Although the expression \( 2(\theta_3 + \theta_2) - \theta_1 = 0 \) can be satisfied in a number of ways, it is convenient from considerations of the combined rotor winding (see Section 9.5) that \( \theta_3 \) should be \(-45^\circ\), which requires that \( \theta_1 = \theta_2 = 90^\circ \). Physically, these requirements are achieved when, firstly, the axis of winding SE is in alignment with that of winding SH and both axes are displaced by \( 90^\circ \) from the reference position (R-phase of the generator) and, secondly, the axis of the R-phase of winding SH is \( 45^\circ \) behind that of the R-phase of winding RSE.

After substituting from eqn. (9.7) into the equivalent circuit of Fig. 9.5b, the exciting field current can be derived as

\[ I_f = \frac{K_i}{|R_r + R_e + j(X_r + X_e)|} |E_4 - jI_r X_2 e^{-j\delta}| \]  \quad \ldots (9.8)

which indicates that the exciter is providing a current-compounded excitation to the generator. However, due to the present of \( \delta \) in eqn. (9.8), the exciter will not provide the
correct excitation at power factors other than zero, although it must be noted that the excitation will still increase as the power factor becomes more lagging. Since the compounding characteristic is now affected by the generator parameters (δ depends on both the reactances of the generator and on its loading conditions), a sensible discussion on the characteristic is only possible when reference is made to some specific example, and such a case is considered later in Section 9.4.3.

9.4 Experimental study

To enable a full assessment to be made of the potential of the present proposal, the experimental set-up requires a synchronous generator as well as prototype exciter. Since the manufacturer of generator WC 876 has agreed to undertake the experimental study, an exciter prototype was designed for use with that generator. The details on the prototype are given in Table 9.1 and Figs. 9.6a-d, and a discussion on the design procedures is provided in Appendix A6.1. However, at the time when this thesis was compiled the manufacturer had not provided the author with any experimental results, and the following preliminary experimental investigation was therefore conducted in the Department.

9.4.1 Experimental procedures

The experimental exciter was prepared from an existing frame
of the EMEU (EMEU-1 in Chapter 4). After stripping down the existing windings, the unit was rewound to the specification given in Table 9.1 and Figs. 9.6a-d, with the following minor modifications: gauge-17 wire for the rotor windings (for reason of availability) and 6 turns per coil for the series winding (to compensate for the shorter stack length and lower operating frequency).

A 3-phase loading bank of variable power factor was fed from the laboratory supply via the experimental exciter unit, as shown in Fig. 9.7. The exciter shunt winding was supplied from a d.c. source through a variable resistor VR, with the exciter rotor being fed through a 3-phase bridge rectifier BR to a resistive-inductive load (representing the field winding of the generator) drawing a fully smoothed current. A synchronous motor M connected to the supply through a phase shifter provided the synchronous drive for the exciter rotor.

Before conducting a load test on the exciter, the winding axis of winding SE was adjusted so that \( \theta_1 \) (and hence \( \theta_2 \)) in Fig. 9.4 was displaced by 90\(^\circ\) from its reference position (now the R-phase axis of the motor). The adjustment was accomplished with the aid of the phase shifter in the following manner: with the exciter rotor driven at synchronous speed (1500 rpm), the winding SH energised and a zero lagging power factor current passed through winding SE, the setting of the phase shifter was altered until the field
current $I_{dc}$ was maximised (note that it can be shown from Fig. 9.5b and eqn. (9.6) that the condition $2(\theta_3 + \theta_2) - \theta_1 = 0$ is satisfied when the setting of the phase shifter is adjusted as described).

With the phase shifter maintained at the above setting and a direct current of 1.81A in winding SH, the field current $I_{dc}$ was noted at 5A steps of load current when this was increased from zero to 25A. The test was conducted for power factors of unity, 0.8 lagging and zero lagging, and the results obtained are presented in the next sub-section. It should be noted that the measured field resistance $R_f$ is 1.06Ω.

9.4.2 Results and discussions

Experimental and computed results for the load test are shown in Fig. 9.8. The computed results are obtained from eqn. (9.8), with $\delta = 0$, together with the design and test parameters of the exciter (see Appendix A6.2). It is worth noting that the assumption $\delta = 0$ is made on the basis that the exciter presents only a small load to the synchronous motor, and the generally good agreement between the computed and test results indicates the validity of this assumption. Although the effect of $\delta$ on the compounding characteristic cannot be investigated with the present experimental set-up, the results obtained do indicate that the proposed unit is capable of providing phase-sensitive compounding. It will be noted that the effect $\delta$ has on the compounding
characteristic is considered in the following section.

9.4.3 Current compounding of generator WC 876

It is instructive now to consider the compounding characteristic of the proposed exciter when used with generator WC 876. If it is assumed that the exciter is designed to provide the correct excitation for the zero lagging power factor condition, the excitation that will be provided at other load and power factor conditions can be determined from eqn. (9.8), together with the generator phasor diagram given in Chapter 2. Results obtained on this basis for power factors of 0.8 lagging and unity are presented in Fig. 9.9, which also shows the excitation required to maintain a constant generator terminal voltage and the excitation provided by an exciter whose compounding is not affected by the generator load angle (for example, the MAC exciter). Evidently, due to the effect the load angle $\delta$ has on the compounding behaviour, the generator is over-compounded by a considerable amount at the near-unity power factors. Thus, the proposed scheme is less sensitive to variations in power factor and without an AVR the exciter can only be expected to achieve satisfactory regulation for a narrow range of power factors.

9.5 Combined rotor winding

Since the two rotor windings (RSE and RSH) of the exciter
carry the same current, it may be possible to replace them with a single winding responsive to m.m.f.'s of both pole numbers, and resulting in a considerable saving in the volume of copper. In the single-phase case, such a winding has been in existence\textsuperscript{40,43} for a number of years and it is convenient therefore to use this as a starting point for the present study.

Fig. 9.10a shows the m.m.f. distribution due to unit current flowing in a 2-pole single-phase 1 slots/pole full-pitch winding with $N_{c1}$ turns per coil, while the corresponding distribution for a similar 4-pole winding with $N_{c2}$ turns per coil is given in Fig. 9.10b. It should be noted that the axis of the 4-pole m.m.f. distribution is displaced from that of the 2-pole distribution by $-45^\circ$ (angle defined in terms of the 2-pole field). By combining the 2-pole and 4-pole m.m.f.'s, the resultant m.m.f. distribution of Fig. 9.10c is obtained. Evidently, this resultant can also be obtained with a combined winding using coils of $(N_{c1} + N_{c2})$ and $(N_{c1} - N_{c2})$ turns. If the ratio of the volume of copper required for the combined winding to that required for the two separate 2-pole and 4-pole windings is $r_v$, then

$$r_v = \frac{2}{\frac{2N_{c2}}{L_{c2}} + \frac{N_{c1}}{L_{c1}}}$$  \text{ for } N_{c1} > N_{c2}$$

$$= \frac{2N_{c1}L_{c1}}{N_{c2}L_{c2} + 2}$$  \text{ otherwise}$$

...(9.9)
where $L_{c1}$ and $L_{c2}$ are the lengths of copper conductor per turn of the 2-pole and 4-pole windings respectively. By selecting a rotor winding with $N_{c1} = N_{c2}$, the two middle coils of the combined winding can be omitted, since $N_{c1} - N_{c2} = 0$, which results in a maximum saving in the volume of copper, seen to be in the region of 33% to 50%, when it is noted that $1 < L_{c1}/L_{c2} < 2$. This approach therefore provides the basis for an efficient single-phase combined winding design. If such a winding has $N$ series turns and coils distributed in $J_s$ consecutive slots of slot angle $\gamma$, it follows from the usual Fourier analysis that

$$T_n = \frac{N}{n} \sin \left( \frac{n\alpha}{2} \right) \sum_{J=1}^{J_s} \sin \left( \frac{n\alpha}{2} + nJ - 1 \gamma + n\gamma_0 \right) / J_s \quad \ldots \quad (9.10)$$

where $T_n$ is the nth-harmonics turns, $\alpha_c$ the coil pitch and $\gamma_0$ the angle at which the first coil starts (see illustration in Fig. 9.10d for a 3 slots/pole winding). It will be noted that $nT_n$ is the effective number of series turns that will respond to the nth harmonics of the m.m.f. field. For example, if the fundamental pole number is 4, then $T_1$ gives the effective series turns for the 4-pole field with $2T_2$ the corresponding quantity for the 8-pole field. An exciter prototype with single-phase combined rotor winding was designed for application with generator WC 876 (Table 9.2 and Figs. 9.11a-b), and further details of this are provided in Appendix A6.3.
Although a single-phase rotor winding may be used for the exciter, its use is not encouraged since it makes less efficient use of the available rotor slots while, furthermore, the flow of single-phase rotor current induces odd-order time harmonics of voltage in the exciter series winding. (Note that the latter problem is a well-known phenomenon of a single-phase generator and a pole-face damper winding is usually employed to improve the waveform of the output voltage). A 3-phase combined winding is therefore desirable, but it must be noted that there is a major design difficulty for such a winding. If each phase of the 3-phase combined winding has a similar arrangement to that of the single-phase combined winding, all even harmonics in the m.m.f. distribution will have a reversed phase sequence. Clearly, the three phases of the combined winding cannot be identically arranged, thus presenting problem from the manufacturing point of view. However, a combined 3-phase winding can still be obtained on the same basis as with the single-phase case.

For example, consider a 1 slot/pole/phase 3-phase 2-pole and 4-pole windings with \( N_{c1} = N_{c2} \) and with each carrying unit current. The resultant m.m.f. due to both windings is given in Fig. 9.12 for all three phases. It is evident that the m.m.f. distribution is dissimilar between the phases and the same distribution can only be provided by a combined winding containing coils having different coil pitches, as shown in
Fig. 9.13b. The details of a prototype exciter using such a combined 3-phase winding are included in Table 9.3 and Figs. 9.13a-b, with further information being given in Appendix A6.4.

9.6 Conclusions

It has been demonstrated in this chapter that the proposed exciter is capable of providing a phase-sensitive current-compounded field excitation. However, since the phase difference between the current component and the voltage component of the field current is also influenced by the load angle of the generator, the proposal is less sensitive to variations in power factor than, say, the MAC excitation scheme. Despite this shortcoming, the new system has the distinctive advantages that its nominal level of field current can be readily altered and that the implementation of an AVR for additional closed-loop control is a relatively simple procedure. Finally, the possibility of using a combined rotor winding to achieve an improved efficiency has also been indicated.
### Table 9.1 Specification of a prototype exciter having two different pole numbers

(All dimensions in ins.)

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<tr>
<td>NO. OF GROUPS/SET</td>
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<tr>
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<td>1-8</td>
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<tr>
<td>POLES</td>
<td>8</td>
<td>4</td>
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</tbody>
</table>

|                      | 8-POLE | 4-POLE |                      |
| WIRE SIZE            | 2 no. 20 | 2 no. 20 | O.D. 5.770 BORE 1.774 GA. 24 |
| TURNS PER COIL       | 5     | 3      | STACK LENGTH 2       |
| COILS PER GROUP      | 1     | 2      | SLOT TYPE NO. T114 SLOTS 24 |
| NO. OF GROUPS/SET    | 24    | 12     |                      |
| COIL THROW           | 1-4   | 1-6    |                      |
| POLES                | 8     | 4      |                      |

EXCITER STATOR

EXCITER ROTOR
### EXCITER STATOR

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<th>Series</th>
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### EXCITER ROTOR

|                | 2 no. 20        |                 | O.D. 5.770 BORE 1.774 GA. 24 |
| **WIRE SIZE**  |                 |                 |                             |
| **TURNS PER COIL** | 20             |                 | STACK LENGTH 2               |
| **COILS PER GROUP** | 2              |                 | SLOT TYPE NO. T114 SLOTS 24  |
| **NO. OF GROUPS/SET** | 4              |                 |                             |
| **COIL THROW**  | 2-4             |                 |                             |
| **POLES**       |                 |                 |                             |

Table 9.2 Specification of single-phase combined rotor winding  
(All dimensions in ins.)
### EXCITER STATOR

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<td><strong>POLES</strong></td>
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<td>4</td>
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### EXCITER ROTOR

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<td><strong>POLES</strong></td>
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Table 9.3 Specification of 3-phase combined rotor winding
(All dimensions in ins.)
Fig. 9.1 Single-phase self-regulating generator

Fig. 9.2 Current compounding with the new exciter (Basic system)
Fig. 9.3 Current compounding with the new exciter (System incorporating an AVR)

Fig. 9.4 Phase-variable representation of the generator and exciter (consist of 2-pole and 4-pole machines)
Fig. 9.5a Equivalent circuit for the new exciter

Fig. 9.5b Simplified equivalent circuit for the rotor of the new exciter \((R_r = R_{33} + R_{4r}, \ X_r = X_{33} + X_{4r})\)
Fig. 9.6a Stator connection diagram
Fig. 9.6b Rotor connection diagram
Fig. 9.6c Developed diagram on 3-phase stator windings
(Only a-phase and d.c. field shown)
Fig. 9.6d Developed diagram on 3-phase rotor windings (Shown for a-phase only)
Fig. 9.7 Circuit diagram for load test
Fig. 9.8 Variation of field current with load (Comparison of results)

Fig. 9.9 Compounding characteristic for generator WC 876
Fig. 9.10a m.m.f. distribution of 2-pole winding

Fig. 9.10b m.m.f. distribution of 4-pole winding

Fig. 9.10c Combined m.m.f. distribution of 2-pole and 4-pole field

Fig. 9.10d m.m.f. distribution of a single-phase combined winding
Fig. 9.11a Rotor connection diagram for single-phase combined winding

Fig. 9.11b Developed diagram of single-phase combined rotor winding
Fig. 9.12 Resultant m.m.f. distribution for a 3-phase rotor winding with $N_{c1} = N_{c2}$ (i.e. both the 2-pole and 4-pole windings have equal number of turns per coil)
Fig. 9.13a Rotor connection diagram for 3-phase combined winding
Fig. 9.13b Developed diagram of 3-phase combined rotor winding
CHAPTER 10
CONCLUSIONS

This thesis has demonstrated that a frequency-converter excitation (FCE) system provides current-compounded excitation in a manner very similar to that of the familiar and phase-sensitive static current-compounding schemes. Apart from having the fast transient response normally associated with these schemes, the FCE system offers two additional features that distinguish its operation from that of the static schemes. Firstly, the use of a special-purpose induction exciter allows the complete elimination of brush gear and slip rings from the excitation system, resulting in a brushless and current-compounded generator unit. Secondly, the generator in an FCE system needs to supply only a portion (given by $l/s$) of the excitation power from the output terminals, and not the full excitation as must be the case with all static schemes.

A theoretical model for the MAC exciter that constitutes the central unit of the FCE system has been developed. It was shown that for balanced 3-phase operation the exciter can be represented by a 3-coil equivalent circuit per phase, very similar to that for a 3-winding transformer. Provided that experimental open-circuit characteristics are available, very accurate no-load and load performance predictions can be obtained by the use of this simple circuit.
Both theoretical and experimental investigations of the compounding behaviour of the MAC exciter have produced many important findings, with perhaps the most significant of these being the direct effect that the stator coil pitch has on the compounding characteristic. Thus, it has been demonstrated that an exciter with a stator coil pitch of $180^\circ$ is incapable of providing any significant compounding effect. To overcome this problem, the stator coils must be short-chorded by a quite considerable amount, so that sufficient phase lead (the angle between the current and voltage constants) is preserved to ensure that the field current is increased with both load current and worsening power factor.

It is interesting to note that the same conclusion can also be reached from the physical point of view. Consider, for example, a resistance-free exciter with a stator coil pitch of $180^\circ$, in which the coupling between the shunt and series windings is perfect when the small slot and end-winding components of leakage flux are disregarded. Since the voltage across the shunt winding is fixed by the shunt voltage, the airgap flux, which links both stator windings, must remain constant irrespective of the load current flowing in the series winding. The resistance-free exciter with a stator coil pitch of $180^\circ$ cannot thus provide any compounding effect. However, when the coils of the stator windings are short-chorded, the coupling between the two windings is no longer perfect and there are leakage paths for the flux arising from the load current flowing in the series winding.
The addition of the series to the shunt flux increases the exciter airgap flux and hence the on-load excitation. Since the two fluxes are added vectorially, the field current provided by the exciter is also sensitive to load power factor.

The exciter parameters required for performance predictions can be obtained by two different methods. The first of these requires open-circuit tests to be performed on all the windings of a stationary exciter, with the parameters required being calculated from the results of these tests. Although this method allows very accurate performance predictions to be made, it is obviously only applicable in cases where open-circuit test results are available, or where only small changes in the number of winding turns from an existing design are contemplated. In the second method, the exciter parameters are formulated in terms of the basic machine constants, with the resulting analytical expression being particularly useful for design purpose.

The optimisation considerations undertaken on a conventional MAC arrangement have demonstrated that conflicting performance requirements can prevent the design of an optimal exciter. In particular, the need for an exciter which produces only a low drift in the generator field current as the generator temperature changes can only be achieved with the use of more rotor turns, while other major considerations, such as the size of the exciter, indicate the adverse effects
of such an approach. For instance, although the type WC 0077 exciter has a minimum volt-ampere requirement (which is a convenient size indicator) of 6.5 p.u.\(^\dagger\) when designed with 32 rotor turns, the change in field current due to a 20\% change in field resistance is at an unacceptably high 11\%. The drift is reduced to a more practical 5.5\% by the use of a 56-rotor-turn design, but such a reduction is achieved only at the expense of doubling the size of the exciter. The direct relationship between these two quantities is an inherent feature of the conventional MAC arrangement, with the clear implication that the best design can only be a good engineering compromise between what level of drift in field current is tolerable on the one hand and what exciter size may be regarded as acceptable on the other. However, the use of a new arrangement containing shunt capacitors can overcome this optimisation difficulty. It was shown that with capacitors of suitable size connected in series with the shunt winding of MAC exciter, the new arrangement is capable of providing a practically drift-free field current if the resistance to reactance ratio of the shunt winding is less than 0.15, a figure that is readily achieved in most practical designs. The effectiveness of shunt capacitors in reducing the drift in field current is well illustrated by the following example: With a total volt-ampere requirement (including that for the capacitors) of 11.5 p.u., the 32-rotor-turn exciter design of the new arrangement has a negligible drift of 2.2\%, whereas in the conventional

\(^\dagger\) This is about 10\% of the rated generator output volt-ampere.
arrangement the same degree of field current stability is only achieved with a 112-rotor-turn design and a prohibitively large volt-ampere requirement of 42.8 p.u.

The fact that in the new MAC arrangement a $180^\circ$ stator coil pitch exciter is capable of producing the correct compounding effect indicates that the provision of leakage flux paths is unnecessary, in contrast to the conventional arrangement where these paths must be provided by short-chording the stator coils. Physically, this is possible since the voltage across the shunt winding is not fixed, and the flux linking this winding can therefore alter to enable the additional m.m.f. produced by load current flowing in the series winding to be added vectorially to the no-load shunt m.m.f. in the exciter airgap to give the required compounding effect. A further basic difference between the new and conventional arrangements was illustrated in a generalised optimisation study, in which it was shown that although the new arrangement is most efficient with a $180^\circ$ stator coil pitch, the conventional arrangement is most efficient with a $80^\circ$ stator coil pitch.

When a MAC exciter is connected both electrically and mechanically to a wound-rotor induction machine, the preliminary study described in the thesis indicated that the resulting system might provide the basis for a novel variable-speed constant-frequency generating unit. However, further investigations, both theoretical and experimental, revealed
several major shortcomings of the proposal. It was established that, during operation at the intended high slip, the exciter input power can only be brought down to a practical level by the addition of significant resistance to the rotor circuit, which will inevitably result in a drastic reduction in the system efficiency. At the intended slip of \(-2/3\), the theoretical efficiency is only 60\%, a figure which makes it virtually impossible to justify the proposed unit on any economic basis. A technical stumbling block is presented by the need to vary the rotor resistance of the brushless unit so as to maintain a consistently low exciter input throughout a range of operating speeds. Furthermore, the unit is unable to provide the reactive load demand, which needs to be supplied from an external source such as a static capacitor bank or an over-excited synchronous motor. The arrangement is thus clearly unsuitable for any practical implementation.

A novel compounding arrangement, in which both the stator and the rotor contain windings having different pole numbers, was proposed in the thesis. Theoretical considerations, substantiated by results obtained from an experimental unit, indicated that the arrangement is capable of providing a field current increasing with both the magnitude and phase angle of the load current. Bearing in mind that the nominal field current of this arrangement can be altered more readily, it thus appears to provide a promising alternative to the MAC
exciter in which the terminal voltage may be readily controlled. It must however be noted that the investigation of this arrangement is still only at a preliminary stage, and that it will form the basis for more extensive work in the Department.
REFERENCES


7. Eugen Renz: 'Self-regulating synchronous generators for private power plants', ibid., 1957, 2, pp. 73-76.


38. United States Patent 3571693


APPENDIX A1

FULL-WAVE BRIDGE RECTIFIER

A1.1 3-phase bridge rectifier

Fig. A1.1 shows a 3-phase full-wave bridge rectifier fed from a balanced voltage source of reactance $X_c$ and resistance $R_c$. The bridge supplies an inductive load of resistance $R_f$, with an inductance $L_f$ sufficiently large to ensure that the rectifier direct current $I_f$ is smooth. Neglecting the effect of $R_c$ on commutation, three different modes of operation (usually referred to as mode-1, mode-2 and mode-3) may be identified, with each mode having a distinct pattern of diode conduction. In reference (25), the effect of $R_c$ on commutation was included and it was shown that the condition of changeover from one mode to another was thus altered significantly. It should be noted that all the above studies assumed an ideal diode characteristics, with the diode regarded as a perfect conductor when forward biased and a perfect insulator when reverse biased. However, the characteristics of any practical diode indicate that this assumption is far from accurate, and a more realistic representation is to regard the diode as having a forward voltage drop $V_d$ and a slope resistance $R_d$ when in the conduction mode (for the diodes of Chapter 4, $V_d$ and $R_d$ are respectively 0.77V and 0.0276Ω) but an infinite reverse impedance. The analysis which follows takes into account the effect of this non-ideal diode representation.
The balanced voltage source of Fig. Al.1 is defined by

\[ v_a = \sqrt{2}V \sin \omega t \quad \ldots \quad (A1.1) \]
\[ v_b = \sqrt{2}V \sin (\omega t - 2\pi/3) \quad \ldots \quad (A1.2) \]
\[ v_c = \sqrt{2}V \sin (\omega t - 4\pi/3) \quad \ldots \quad (A1.3) \]

where \( V \) is the r.m.s. phase voltage.

Al.1.1 Mode-1 operation

During the commutation interval \( u \) (see Fig. Al.2) in this mode, three diodes are conducting and two phases are effectively short-circuited. If the current transfer between the a-phase and the c-phase is considered, the equivalent circuit is as given in Fig. Al.3. Based on this equivalent circuit, it follows that

\[ v_a - v_c = 2L_c \frac{di}{dt} + 2R_i a - I_f R \quad \ldots \quad (A1.4) \]

where \( R = R_c + R_d \) and \( L_c = X_c / \omega \).

The transfer of current from the c-phase to the a-phase begins when

\[ v_c - I_f R - V_d = v_a \quad \ldots \quad (A1.5) \]

and if this occurs when \( \omega t = \pi/6 - \phi \), then

\[ \phi = \sin^{-1}\left(\frac{I_f R + V_d}{\sqrt{6V}}\right) \quad \ldots \quad (A1.6) \]
Applying the technique of Laplace Transforms to the solution of eqn. (Al.4), with the appropriate initial conditions, the current $i_a$ during the commutation interval may be determined as given in Table Al.1, which in fact defines the a-phase current throughout half a period of mode-1 operation.

The average d.c. voltage $V_f$ is given by

$$V_f = \frac{3N}{\sqrt{6}} (\cos \phi + \cos (u - \phi)) - 2V_d - I_f R (2 - \frac{3}{2\pi} u)$$

...(Al.7)

which may alternatively be defined as

$$V_f = I_f R_f$$

...(Al.8)

On using the results of Table Al.1, it can readily be shown that

$$I_f = \frac{\sqrt{6} V}{X_c (1 + r^2)(1 + e^{-ru})} (r \sin (u - \phi) - \cos (u - \phi)

+ (r \sin \phi + \cos \phi) e^{-ru})$$

...(Al.9)

Al.1.2 Mode-2 operation

Mode-1 operation terminates when the commutation angle $u$ becomes equal to $60^\circ$. After that current commutation between two phases is delayed, and a continuous 2-phase short circuit is switched between pairs of the input lines to the rectifier. When the transfer of current from the c-phase to the a-phase is considered, with the diodes labelled as in Fig. Al.4, the
transfer of current from diode $c^-$ to diode $a^+$ cannot begin until commutation from diode $a^-$ to diode $b^-$ is completed. Thus, throughout the interval $\beta$ shown in Fig. Al.5, the short circuit condition involves the a-phase and the b-phase, and this is followed immediately by a short circuit between the a-phase and the c-phase. The angle $\beta$ by which the current commutation is delayed is usually referred to as the mode-2 delay angle.

Using a technique similar to that adopted for mode-1 operation, the a-phase current $i_a$ over half a period is derived and the results are recorded in Table Al.2. The average d.c. voltage $V_f$ is

$$V_f = \frac{9V}{\sqrt{2}\pi} \cos (\beta - \phi + \frac{\pi}{6}) - 2V_d - \frac{3I_fR}{2} \quad \ldots \ (Al.10)$$

while the d.c. current $I_f$ is

$$I_f = \frac{\sqrt{3}V}{\sqrt{2}(1 + r^2)X_C} \left[ r \sin \left( \frac{\pi}{3} + \beta - \phi \right) - \cos \left( \frac{\pi}{3} + \beta - \phi \right) \right. \\
\left. - e^{-\pi r/3} \left( r \sin (\beta - \phi) - \cos (\beta - \phi) \right) \right] \quad \ldots \ (Al.11)$$

Mode-2 operation terminates when the instantaneous value of $V_f$ is zero. At that point, $\beta = \pi/6 + \phi - \phi'$ where

$$\phi' = \sin^{-1} \left( \frac{I_fR + V_d}{\sqrt{2Vd}} \right) \quad \ldots \ (Al.12)$$
Al.1.3 Mode-3 operation

The analysis of the rectifier in this mode of operation is the most difficult. Commutation begins with a 3-phase short circuit, followed by a 2-phase short circuit and terminated with another 3-phase short circuit. When commutation from diode $c^+$ to $a^+$ is considered, it is convenient to divide the analysis into the following three intervals:

(i) $\frac{\pi}{3} - \phi' < \omega t < \frac{\pi}{3} - \phi' + \varepsilon$

(ii) $\frac{\pi}{3} - \phi' + \varepsilon < \omega t < \frac{2\pi}{3} - \phi'$

(iii) $\frac{2\pi}{3} - \phi' < \omega t < \frac{2\pi}{3} - \phi' + \varepsilon$

Since the occurrence of the 2-phase short circuit is delayed from $\omega t = \frac{\pi}{3} - \phi'$ by an angle $\varepsilon$, as shown in Fig. Al.6, $\varepsilon$ is referred to as the mode-3 delay angle.

(i) Interval $\frac{\pi}{3} - \phi' < \omega t < \frac{\pi}{3} - \phi' + \varepsilon$

The equivalent circuit for this interval is shown in Fig. Al.7. Using this circuit, a pair of differential equations involving $i_{a1}$ and $i_{a2}$ may be derived.

\[
L_c \frac{di_{a1}}{dt} + R_s i_{a1} - \frac{2}{3} R d i_{a2} = \frac{v_a + v_b - 2v_c}{3} + I_f R \quad \ldots \quad (A1.13)
\]

\[
L_c \frac{di_{a2}}{dt} + R_s i_{a2} - \frac{2}{3} R d i_{a1} = \frac{v_a - 2v_b + v_c}{3} - I_f R \quad \ldots \quad (A1.14)
\]

where $R_s = R_c + \frac{4}{3} R_d$ and $R = R_c + R_d$. 
Inserting the appropriate initial conditions and applying the technique of Laplace Transforms to the solution of the coupled differential equations, it can be shown that the currents \( i_{a1} \) and \( i_{a2} \) are respectively

\[
i_{a1} = \sqrt{2} V \left( A(\omega T_1) \cos \phi' - B(\omega T_1) \sin \phi' - H(\omega T_1) \sin \left( \frac{\pi}{6} - \phi' \right) \right) + G(\omega T_1) \cos \left( \frac{\pi}{6} - \phi' \right) + I_f \left( C(\omega T_1) - F(\omega T_1) \right) + E(\omega T_1) i_{a2}(0) \quad \ldots \quad (A1.15)
\]

and \( i_{a2} = \sqrt{2} V \left( -A(\omega T_1) \sin \left( \frac{\pi}{6} - \phi' \right) + B(\omega T_1) \cos \left( \frac{\pi}{6} - \phi' \right) \right) + H(\omega T_1) \cos \phi' - G(\omega T_1) \sin \phi' + I_f \left( F(\omega T_1) - C(\omega T_1) \right) + D(\omega T_1) i_{a2}(0) \quad \ldots \quad (A1.16)

where

\[
A(\omega T_1) = \frac{1}{2X_c} \left( \frac{e}{1+r_1^2} + \frac{e}{1+r_2^2} \right) + \frac{1}{X_c(1+r_1^2)(1+r_2^2)} \left[ \left( (r_1 r_2 - 1) - r_s (r_1 + r_2) \right) \cos (\omega T_1) \right. \\
\left. + (r_s (r_1 r_2 - 1) + (r_1 + r_2)) \sin (\omega T_1) \right] \quad \ldots \quad (A1.17)
\]

\[
B(\omega T_1) = \frac{1}{2X_c} \left( \frac{e}{1+r_1^2} - r_1 \omega T_1 \right) + \frac{r_2}{1+r_2^2} e^{-r_2 \omega T_1} + \frac{1}{X_c(1+r_1^2)(1+r_2^2)} \left[ \left( r_s (r_1 r_2 - 1) + (r_1 + r_2) \right) \cos (\omega T_1) + (r_s (r_1 r_2 - 1) - (r_1 r_2 - 1)) \sin (\omega T_1) \right] \quad \ldots \quad (A1.18)
\]

\[
C(\omega T_1) = \frac{R}{2} \left( \frac{r_1 \omega T_1}{s_1} + \frac{r_2 \omega T_1}{s_2} \right) + \frac{R S}{s_1 s_2} \quad \ldots \quad (A1.19)
\]
\[D(\omega_1) = \frac{1}{2} \left( e^{-r_1 \omega_1} + e^{-r_2 \omega_1} \right) \quad \ldots \ (A1.20)\]

\[E(\omega_1) = \frac{1}{2} \left( e^{-r_1 \omega_1} - e^{-r_2 \omega_1} \right) \quad \ldots \ (A1.21)\]

\[F(\omega_1) = \frac{R}{2} \left( e^{-r_2 \omega_1} - e^{-r_1 \omega_1} \right) + \frac{2R_d R}{3s_1 s_2} \quad \ldots \ (A1.22)\]

\[G(\omega_1) = \frac{1}{2X_c} \left( \frac{-r_1}{1 + r_1^2} e^{-r_1 \omega_1} + \frac{r_2}{1 + r_2^2} e^{-r_2 \omega_1} \right) + \frac{2R_d \left( r_1 r_2 - 1 \right) \cos(\omega_1) + \left( r_1 + r_2 \right) \sin(\omega_1)}{3X_c^2 (1 + r_1^2)(1 + r_2^2)} \quad \ldots \ (A1.23)\]

\[H(\omega_1) = \frac{1}{2X_c} \left( \frac{1}{1 + r_1^2} e^{-r_1 \omega_1} - \frac{1}{1 + r_2^2} e^{-r_2 \omega_1} \right) + \frac{2R_d \left( r_1 r_2 - 1 \right) \sin(\omega_1)}{3X_c^2 (1 + r_1^2)(1 + r_2^2)} (\pi - (r_1 + r_2) \cos(\omega_1)) \quad \ldots \ (A1.24)\]

\[s_1 = R_c \left( 1 + \frac{2}{3} R_d \right) \quad \ldots \ (A1.25)\]

\[s_2 = R_c + 2R_d \quad \ldots \ (A1.26)\]

\[r_1 = \frac{s_1}{X_c} \quad \ldots \ (A1.27)\]

\[r_2 = \frac{s_2}{X_c} \quad \ldots \ (A1.28)\]

\[r_s = \frac{R_s}{X_c} \quad \ldots \ (A1.29)\]

\[\omega_1 = \omega t - \frac{\pi}{3} + \phi' \quad \ldots \ (A1.30)\]
Since \( i_{a2} = 0 \) when \( \omega t_1 = \epsilon \),

\[
i_{a2}(0) = \left[ I_f(C(\epsilon) - F(\epsilon)) - \sqrt{2}V(- \sin \left( \frac{\pi}{6} - \phi' \right) A(\epsilon) + \cos \left( \frac{\pi}{6} - \phi' \right) B(\epsilon) + \cos \phi' H(\epsilon) - \sin \phi' G(\epsilon) \right] / D(\epsilon)
\]

... (Al. 31)

During this interval, the a-phase current is given by

\[
i_a(\omega t_1) = i_{a1}(\omega t_1) + i_{a2}(\omega t_1)
\]

... (Al. 32)

(ii) \underline{Interval} \( \frac{\pi}{3} - \phi' + \epsilon < \omega t < \frac{2\pi}{3} - \phi' \)

During this interval, a 2-phase short circuit occurs. An analysis very similar to that for model operation shows that the a-phase current \( i_a \) is

\[
i_a(\omega t_2) = \frac{\sqrt{3}V}{\sqrt{2}X_c(1 + r^2)} \left[ \cos \left( \frac{\pi}{6} + \epsilon - \phi' \right) e^{-r\omega t_2} - \cos (\omega t_2)
\right.

+ r \sin (\omega t_2) + \sin \left( \frac{\pi}{6} + \epsilon - \phi' \right) \left[ -r e^{-r\omega t_2}
\right.

\[
+ r \cos (\omega t_2) + \sin (\omega t_2) \right] + \frac{I_f}{2} \left( 1 - e^{-r\omega t_2} \right)
\]

\[
+ i_a(0) e^{-r\omega t_2}
\]

... (Al. 33)

where \( \omega t_2 = \omega t - (\frac{\pi}{3} - \phi' + \epsilon) \) and \( i_a(0) \) is given by eqn. (Al. 32) with \( \omega t_1 = \epsilon \).
(iii) Interval \( \frac{2\pi}{3} - \phi' < \omega t < \frac{2\pi}{3} - \phi + \epsilon \)

In this interval, the a-phase current is the same as \(-i_b\) of interval (i). Since

\[
i_b + i_{a2} = -I_f
\]

it follows that

\[
i_a(\omega_3) = I_f + i_{a2}(\omega_3)
\]

where \(\omega_3 = \omega t - (\frac{2\pi}{3} - \phi')\).

On the basis of the three basic intervals, the current \(i_a\) over half a period may be derived and the result is given in Table Al.3. The average d.c. voltage is

\[
V_f = \frac{9V}{\sqrt{2}\pi} \left\{ \cos \left( \frac{2\pi}{3} + \epsilon - \phi' \right) + \cos \phi' \right\} - \frac{9I_fR}{2\pi} \left( \frac{\pi}{3} - \epsilon \right) - \frac{6V_d}{\pi} \left( \frac{\pi}{3} - \epsilon \right)
\]

and when \(\epsilon = 60^\circ\), a continuous 3-phase short circuit occurs.

Al.1.4 Computer simulation of 3-phase rectifier

The simulation of the bridge rectifier with its three modes of operation presents a fairly difficult problem, since the value of \(I_f\) is not known until the commutation angle or the delay angle is determined. But, without a knowledge of \(I_f\), these angles cannot be solved and the mode of operation of the rectifier is also not defined. It is clear that a
solution can only be found by implementing an iterative method with the aid of the digital computer.

By assuming a value for $I_f$, a numerical method can be used to solve for the commutation angle $\alpha$ from eqn. (Al.9), or the delay angle $\beta$ from eqn. (Al.11), or the delay angle $\epsilon$ from eqn. (Al.37).

$$i_{a2}(0) + I_f = i_a(\omega \tau_2) |_{\omega \tau_2 = \frac{\pi}{3} - \epsilon} \quad \text{... (Al.37)}$$

In the search for that solution, a dummy angle $\alpha'$ is defined as in Table Al.4, to assist the computation and to update the mode of operation at each stage of the calculation.

Depending on the mode of operation, the value of $I_f$ and that of either $\alpha$ or $\beta$ or $\epsilon$ are substituted into the appropriate equation (either eqn. (Al.7) or (Al.10) or (Al.36)) to obtain a value for $V_f$. This value is checked against that given by eqn. (Al.8), to see if they agree with each other. The whole process is repeated until a unique value of $V_f$ is obtained.

Having obtained $V_f$, $I_f$ and either $\alpha$, or $\beta$ or $\epsilon$, the r.m.s. value of $i_a$ ($I_a$), the fundamental Fourier component of $i_a$ ($i_{a1}$) and its phase lag $\phi_{a1}$ are readily obtained from results in one of Tables Al.1-Al.3, depending on the mode of operation. The effective reactance $X_e$, resistance $R_e$ and the current transfer ratio $K_i$ of the rectifier can be determined as follows:
The rectifier loss is given by \((3I_a^2R_e - I_f V_f)\).

### Al.2 Single-phase bridge rectifier

Fig. Al.8 shows a single-phase bridge rectifier fed from an ideal voltage source \(v = \sqrt{2}V \sin \omega t\), via a source resistance \(R_c\) and reactance \(X_c\). During the commutation interval \(u\), Fig. Al.9, all four diodes are conducting, and it follows from using the equivalent circuit of Fig. Al.10 that

\[
v = L_c \frac{di}{dt} + iR
\]

\[
\text{where } R = R_c + R_d \text{ and } L_c = X_c / \omega.
\]

Noting that commutation begins when

\[
\sqrt{2}V \sin \omega t - I_f (R_c + 2R_d) - 2V_d = 0
\]

the angle \(\phi\) is given by

\[
\phi = \sin^{-1}\left(\frac{I_f (R_c + 2R_d) + 2V_d}{\sqrt{2}V}\right)
\]

Using the technique of Laplace Transforms to assist in the
solution of eqn. (AI.41), the variation of $i$ over a half period can be determined and the results obtained are recorded in Table Al.5. The direct current $I_f$ and voltage $V_f$ are respectively given by

$$I_f = \frac{\sqrt{2}V}{X_c(1+r^2)(1+e^{-ru})} \{ \cos \phi (r \sin u - \cos u + e^{-ru})$$

$$- \sin \phi (\sin u + r \cos u - re^{-ru}) \} \quad \ldots \quad (AI.44)$$

and

$$V_f = \frac{\sqrt{2}V}{\pi} \{ \cos \phi + \cos (u - \phi) \} - \left( I_f (R_c + 2R_d) + 2V_d \right) \frac{\pi - u}{\pi} \quad \ldots \quad (AI.45)$$

where $r = R/X_c$.

The computer simulation of the single-phase rectifier can be performed in a way similar to that for the 3-phase case.
\[ \frac{\pi}{6} - \phi < \omega t < \frac{\pi}{6} - \phi + u \]

\[ i_a(\omega \tau_1) = \frac{\sqrt{3}V}{\sqrt{2}X_c(1+r^2)} \left\{ r \sin (\omega \tau_1 - \phi) - \cos (\omega \tau_1 - \phi) \right\} + (r \sin \phi + \cos \phi)e^{-r\omega \tau_1} + \frac{I_f}{2}(1 - e^{-r\omega \tau_1}) \]

\[ \frac{\pi}{6} - \phi + u < \omega t < \frac{5\pi}{6} - \phi \]

\[ i_a(\omega t) = I_f \]

\[ \frac{5\pi}{6} - \phi < \omega t < \frac{5\pi}{6} - \phi + u \]

\[ i_a(\omega \tau_2) = \frac{\sqrt{3}V}{\sqrt{2}X_c(1+r^2)} \left\{ -r \sin (\omega \tau_2 - \phi) + \cos (\omega \tau_2 - \phi) \right\} - (r \sin \phi + \cos \phi)e^{-r\omega \tau_2} + \frac{I_f}{2}(1 + e^{-r\omega \tau_2}) \]

\[ \frac{5\pi}{6} - \phi + u < \omega t < \frac{7\pi}{6} - \phi \]

\[ i_a(\omega t) = 0 \]

where \( \omega \tau_1 = \omega t - \left( \frac{\pi}{6} - \phi \right) \), \( \omega \tau_2 = \omega t - \left( \frac{5\pi}{6} - \phi \right) \) and \( r = \frac{R}{X_c} \)

Table A1.1 a-phase current over a half period of mode-1 operation
\[
\begin{align*}
\frac{\pi}{6} + \beta - \phi < \omega t < \frac{\pi}{2} + \beta - \phi \\
&\quad \Rightarrow
\begin{align*}
    i_a(\omega t_1) &= \frac{\sqrt{3}V}{\sqrt{2}(1 + r^2)X_C} \left[ r \sin(\omega t_1 + \beta - \phi) - \cos(\omega t_1 + \beta - \phi) \\
    &\quad - e^{-r\omega t_1} \{ r \sin(\beta - \phi) - \cos(\beta - \phi) \} \right] + \frac{I_f}{2}(1 - e^{-r\omega t_1}) \\
\frac{\pi}{2} + \beta - \phi < \omega t < \frac{5\pi}{6} + \beta - \phi \\
&\quad \Rightarrow
\begin{align*}
    i_a(\omega t) &= I_f \\
\frac{5\pi}{6} + \beta - \phi < \omega t < \frac{7\pi}{6} + \beta - \phi \\
&\quad \Rightarrow
\begin{align*}
    i_a(\omega t_2) &= \frac{I_f}{2}(1 + e^{-r\omega t_2}) - \frac{\sqrt{3}V}{\sqrt{2}(1 + r^2)X_C} \left[ r \sin(\omega t_2 + \beta - \phi) \\
    &\quad - \cos(\omega t_2 + \beta - \phi) - e^{-r\omega t_2} \{ r \sin(\beta - \phi) - \cos(\beta - \phi) \} \right]
\end{align*}
\end{align*}
\end{align*}
\]

Table Al.2 a-phase current over a half period of mode-2 operation

where \( \omega t_1 = \omega t - (\frac{\pi}{6} + \beta - \phi) \), \( \omega t_2 = \omega t - (\frac{5\pi}{6} + \beta - \phi) \) and \( r = \frac{R}{X_C} \)
\[
\frac{\pi}{3} - \phi' < \omega t < \frac{\pi}{3} - \phi' + \varepsilon
\]
As specified by eqn. (Al.32)

\[
\frac{\pi}{3} - \phi' + \varepsilon < \omega t < \frac{2\pi}{3} - \phi'
\]
As specified by eqn. (Al.33)

\[
\frac{2\pi}{3} - \phi' < \omega t < \frac{2\pi}{3} - \phi' + \varepsilon
\]
As specified by eqn. (Al.35)

\[
\frac{2\pi}{3} - \phi' + \varepsilon < \omega t < \pi - \phi'
\]

\[
i_a(\omega t) = I_f
\]

\[
\pi - \phi' < \omega t < \pi - \phi' + \varepsilon
\]

\[
i_a(\omega T_5) = I_f - i_{a1}(\omega T_5)
\]

\[
\pi - \phi' + \varepsilon < \omega t < \frac{4\pi}{3} - \phi'
\]

\[
i_a(\omega T_6) = I_f - i_a(\omega T_2)|_{\omega T_2 = \omega T_6}
\]

Where \(\omega T_5 = \omega t - (\pi - \phi')\) and \(\omega T_6 = \omega t - (\pi - \phi' + \varepsilon)\)

Table Al.3 a-phase current over a half period of mode-3 operation
$0 < u' < \frac{\pi}{3}$

$u = u'$, set rectifier to mode-1 operation

$\frac{\pi}{3} < u' < (\frac{\pi}{2} + \phi - \phi')$

$\beta = u' - \frac{\pi}{3}$, set rectifier to mode-2 operation

$(\frac{\pi}{2} + \phi - \phi') < u' \leq (\pi + \phi - \phi')$

$\epsilon = u' - (\frac{\pi}{2} + \phi - \phi')$, set rectifier to mode-3 operation

<table>
<thead>
<tr>
<th>Table Al.4 Definition of u'</th>
</tr>
</thead>
</table>

$-\phi < \omega T < u - \phi$

$$i(\omega T_1) = \frac{\sqrt{2}V}{X_c(1+r^2)} \left[ \cos \phi (r \sin \omega T_1 - \cos \omega T_1 + e^{-r\omega T_1}) \\
- \sin \phi (\sin \omega T_1 + r \cos \omega T_1 - r e^{-r\omega T_1}) \right] - I_f e^{-r\omega T_1}$$

$u - \phi < \omega T < \pi - \phi$

$$i(\omega T) = I_f$$

where $\omega T_1 = \omega T + \phi$

| Table Al.5 Supply current over a half period for single-phase rectifier |
Fig. A1.1 Circuit diagram of a 3-phase bridge rectifier

Fig. A1.2 Voltage waveforms of 3-phase bridge rectifier in mode-1 operation
Fig. A1.3 Equivalent circuit during commutation interval between a-phase and c-phase

Fig. A1.4 3-phase bridge rectifier (with each diode appropriately labelled)
Fig. A1.5 Voltage waveforms of 3-phase bridge rectifier in mode-2 operation

Fig. A1.6 Voltage waveforms of 3-phase bridge rectifier in mode-3 operation
Fig. A1.7 Equivalent circuit for interval \( \frac{\pi}{3} - \phi' < \omega t < \frac{\pi}{3} - \phi' + \varepsilon \) in mode-3 operation

Fig. A1.8 Circuit diagram of a single-phase bridge rectifier
Fig. A1.9 Voltage waveforms of single-phase bridge rectifier

Fig. A1.10 Equivalent circuit for commutation interval of single-phase bridge rectifier
Consider a conductor in a slot carrying a current I which flows into the plane of the paper, as shown in Fig. A2.1. The length of the airgap is \( g \) and the iron core on either side has a relative permeability of \( \mu_r \). If the flux path is rectangular in shape, as indicated in Fig. A2.1, then

\[
I = 2gH_g + 2H_i(x+y)
\]  

where \( H_g \) is the magnetising force in the airgap and \( H_i \) in the iron. For an incremental width \( \delta x \) carrying a flux \( \delta \phi \),

\[
\frac{\delta \phi}{L\delta x} = \frac{\mu_0 H_g}{\mu_r g} = \frac{\mu_0 H_i}{\mu_r g}
\]  

where \( L \) is the effective length of the iron core. It can then be shown that the airgap flux density \( B_g \) is given by

\[
B_g = \frac{\mu_0 I}{2g} \left( 1 - \frac{2y}{\mu_r g^2} \right) \frac{\mu_0 I}{\mu_r g^2 x}
\]  

The flux density distribution therefore decreases linearly (if \( \mu_r \) is assumed to be constant) and the slope \( h \) is given by \( (\mu_0 I)/(\mu_r g^2) \). In a multi-conductor system the principle of superposition may be used to obtain the resultant \( B_g \), as shown in Fig. 3.11.

Fig. A2.1 Flux path of a conductor embedded in slot
APPENDIX A3
CURVE FITTING

Experimental data relating the magnetising voltage $V_m$ to the magnetising current $I_m$ of an electrical machine suggest that an analytical functional relationship can be obtained between these two quantities in the region of interest. Moreover, $I_m$ is linearly related to $V_m$, both when $V_m$ is small and when $V_m$ is large, and it is therefore reasonable to represent $I_m$ by

$$I_m = \frac{a_0 + a_1 V_m + a_2 V_m^2 + \ldots + a_n V_m^n}{1 + b_1 V_m + b_2 V_m^2 + \ldots + b_n V_m^n} V_m$$

(A3.1)

where $a_0$, $a_1$, $a_2$, \ldots $a_n$ and $b_1$, $b_2$, \ldots $b_n$ are constant coefficients and $n$ is the exponent of the last term of the polynomials. When $V_m$ is small, the ratio of the polynomials is $a_0$ and when $V_m$ is large it is $a_n/b_n$, giving the two distinct slopes of the magnetising relationship.

Using a method that minimises the sum of absolute square of error at all the data points, the required coefficients for eqn. (A3.1) can be derived from the equation.
\[
\begin{bmatrix}
B_2 & B_3 & B_4 & \ldots & B_{n+2} & -H_2 & -H_3 & \ldots & -H_{n+1} \\
B_3 & B_4 & B_5 & \ldots & B_{n+3} & -H_3 & -H_4 & \ldots & -H_{n+2} \\
B_4 & B_5 & B_6 & \ldots & B_{n+4} & -H_4 & -H_5 & \ldots & -H_{n+3} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
B_{n+2} & B_{n+3} & B_{n+4} & \ldots & B_{2n+2} & -H_{n+2} & -H_{n+3} & \ldots & -H_{2n+1} \\
-H_2 & -H_3 & -H_4 & \ldots & -H_{n+2} & T_2 & T_3 & \ldots & T_{n+1} \\
-H_3 & -H_4 & -H_5 & \ldots & -H_{n+3} & T_3 & T_4 & \ldots & T_{n+2} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
-H_{n+1} & -H_{n+2} & -H_{n+3} & \ldots & -H_{2n+1} & T_{n+1} & T_{n+2} & \ldots & T_{2n}
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
a_2 \\
\vdots \\
a_n
\end{bmatrix}
= \begin{bmatrix}
H_1 \\
H_2 \\
H_3 \\
\vdots \\
H_{n+1}
\end{bmatrix}
\]
\hspace{1cm} \text{... (A3.2)}

where
\[
B_\ell = \sum_{i=1}^{k} P_{Li} V_{mi}^{\ell} \hspace{1cm} \text{... (A3.3)}
\]
\[
H_\ell = \sum_{i=1}^{k} P_{Li} V_{mi}^{\ell} I_{mi} \hspace{1cm} \text{... (A3.4)}
\]
\[
T_\ell = \sum_{i=1}^{k} P_{Li} V_{mi}^{\ell} I_{mi}^{2} \hspace{1cm} \text{... (A3.5)}
\]
\[
P_{Li} = 1/(1 + b_1 I_{mi}^{1} + b_2 I_{mi}^{2} + \ldots + b_n I_{mi}^{n})^2 \hspace{1cm} \text{... (A3.6)}
\]

Note that in eqn. (A3.2), k data points are considered, with \(V_{mi}\) and \(I_{mi}\) being the respective values of \(V_m\) and \(I_m\) at the \(i\)th data point. Furthermore, \(P_{Li}\) is evaluated using \(b_1, b_2, \ldots, b_n\) obtained in the \((L-1)\)th iteration. To initiate the iteration process, \(b_1, b_2, \ldots, b_n\) are all set to zero, which is equivalent to weighting the errors at the different data points by \(1/P_{Li} \big|_{L=\infty}\), and the iterative process progressively reduces the weightings.
In practice, the larger are $n$ and $L$ the more time is consumed in the computation involving eqn. (A3.2), and for the experimental data considered in this thesis, $n = 2$ and $L = 10$ were found to provide sufficient accuracy.
APPENDIX A4

GENERALISED MAC EXCITER

The MAC exciter described in Chapter 3 has its shunt and series windings arranged in such a manner that their magnetic axes are displaced mutually by 180°. It must, however, be noted that other winding arrangements giving different winding displacements are practically feasible. If, for example, the coils in the alternate phase belt of the double-layer winding are used to form respectively the shunt and series windings, the two winding axes are displaced from each other by 60°. In addition, the winding displacement can also be affected by the terminal connections of the windings, and an example of this is when one stator winding is star-connected while the other remains in delta, thus introducing a further 30° winding displacement. To take these situations into account, it is necessary to extend the analysis presented in Chapter 3 and this is most conveniently accomplished by considering a generalised MAC exciter, with an arbitrary winding displacement of \( \lambda \).

Fig. A4.1 shows the phase-variable representation of such an exciter. Applying the phase transformation of Section 3.3.1 to the impedance matrix obtained from Fig. A4.1, it can be shown that the exciter may be represented by the \((a, b, 0)\) windings of Fig. A4.2, with the transformed impedance matrix \(Z_{\alpha \beta}'\) given by
\[
Z_{\alpha \delta}^{j} = \alpha_{1} \left[
\begin{array}{cccc}
\alpha_{1} & \beta_{1} & \alpha_{2} & \beta_{2} & \alpha_{3} & \beta_{3} \\
R_{11} + L_{11} & 0 & M_{12} \cos(\lambda) & -M_{12} \sin(\lambda) & M_{13} \cos(\theta) & -M_{13} \sin(\theta) \\
0 & R_{11} + L_{11} & M_{12} \sin(\lambda) & M_{12} \cos(\lambda) & M_{13} \sin(\theta) & M_{13} \cos(\theta) \\
M_{12} \cos(\lambda) & M_{12} \sin(\lambda) & R_{22} + L_{22} & 0 & M_{23} \cos(\theta - \lambda) & -M_{23} \sin(\theta - \lambda) \\
-M_{12} \sin(\lambda) & M_{12} \cos(\lambda) & 0 & R_{22} + L_{22} & M_{23} \sin(\theta - \lambda) & M_{23} \cos(\theta - \lambda) \\
M_{13} \cos(\theta) & M_{13} \sin(\theta) & M_{23} \cos(\theta - \lambda) & M_{23} \sin(\theta - \lambda) & R_{33} + L_{33} & 0 \\
-M_{13} \sin(\theta) & M_{13} \cos(\theta) & -M_{23} \sin(\theta - \lambda) & M_{23} \cos(\theta - \lambda) & 0 & R_{33} + L_{33} \\
\end{array}
\right]
\]

\[... (A4.1)\]
Note that all the variables in eqn. (A4.1) have the same meanings as those in eqn. (3.24). When the \((a_3, b_3)\) winding is replaced with an equivalent pseudo-stationary \((d, q)\) winding, it can be shown that the transformed impedance matrix is given by \(Z''_{a\beta} = C_t Z'_{a\beta} C\), and with \(C\) and \(C_t\) defined as in eqn. (3.26) this leads to the solution
\[ Z_{\alpha \beta} = \begin{bmatrix} \alpha_1 & \beta_1 & \alpha_2 & \beta_2 & q & d \\ R_{11} + L_{11}p & 0 & M_{12} \cos(\lambda) p & -M_{12} \sin(\lambda) p & 0 & M_{13}p \\ 0 & R_{11} + L_{11}p & M_{12} \sin(\lambda) p & M_{12} \cos(\lambda) p & M_{13}p & 0 \\ M_{12} \cos(\lambda) p & M_{12} \sin(\lambda) p & R_{22} + L_{22}p & 0 & M_{23} \sin(\lambda) p & M_{23} \cos(\lambda) p \\ -M_{12} \sin(\lambda) p & M_{12} \cos(\lambda) p & 0 & R_{22} + L_{22}p & M_{23} \cos(\lambda) p & -M_{23} \sin(\lambda) p \\ -M_{13} \omega_r & M_{13} p & M_{23} \{\sin(\lambda) p - \cos(\lambda) \omega_r\} & M_{23} \{\cos(\lambda) p + \sin(\lambda) \omega_r\} & R_{33} + L_{33}p & -L_{33} \omega_r \\ M_{13} p & M_{13} \omega_r & M_{23} \{\cos(\lambda) p + \sin(\lambda) \omega_r\} & M_{23} \{-\sin(\lambda) p + \cos(\lambda) \omega_r\} & L_{33} \omega_r & R_{33} + L_{33}p \\ \end{bmatrix} \]

\[ \ldots (A4.2) \]
Following the same steps as detailed in Section 3.2.3, the voltage/current relationship of the generalised MAC exciter for 3-phase balanced operation can be derived as

\[ V_1 = (R_{11} + jX_{11})I_1 + jX_{12}e^{j\lambda}I_2 + jX_{13}I_3 \quad \text{... (A4.3)} \]

\[ V_2 = jX_{12}e^{-j\lambda}I_1 + (R_{22} + jX_{22})I_2 + jX_{23}e^{-j\lambda}I_3 \quad \text{... (A4.4)} \]

\[ \frac{V_3}{s} = jX_{13}I_1 + jX_{23}e^{j\lambda}I_2 + \left(\frac{R_{33}}{s} + jX_{33}\right)I_3 \quad \text{... (A4.5)} \]

and the \( \lambda^o \) winding displacement has manifested itself as a \( \lambda^o \) phase advance in the series current. The use of these equations gives the compounding equations of Chapter 7.
Fig. A4.1 Phase-variable representation of generalised MAC exciter

Fig. A4.2 \((a, \beta, \theta)\) representation of generalised MAC exciter
APPENDIX A5

TEST DATA ON ECU

To provide experimental results that substantiate the theoretical findings of Chapter 8, a series of load test was performed on two separate ECUs. The first of these provides the results recorded in Section 8.2, and the second unit those of Section 8.3. In this appendix an account is presented of the tests performed to determine the circuit parameters of these two experimental units.

A5.1 The first ECU

The study conducted in Section 8.2 was based on the fully simplified equivalent circuit of Fig. 8.4b, the parameters of which can readily be derived from those of Fig. 8.4a. To determine the parameters for the latter circuit, the usual no-load and locked-rotor tests were performed on both the exciter and the output generator of the unit, and the results obtained are shown in Figs. A5.1a-A5.2b. The measured stator winding resistances of the exciter and the generator are respectively 0.174Ω and 0.301Ω. Using these results, the unsaturated parameters of both machines are determined, as shown in Table A5.1.

When the exciter and the generator were connected to form the ECU, it was necessary (due to the lack of a suitable high current supply) to re-connect the exciter.
stator winding in series-star, instead of the parallel-star connection used in the above no-load and locked-rotor test. Bearing this in mind and taking into account the turns ratios of the two machines (1.1 for the exciter and 1.05 for the generator) the parameters (referred to the exciter stator) for the equivalent circuit of Fig. 8.4a can be evaluated as recorded in Table A5.2.

A5.2 The second ECU

Since the improved equivalent circuit of Fig. 8.8a is used to model the second ECU, the tests required to obtain its parameters are necessarily more involved and include:

(a) Locked-rotor test - The test was carried out in the usual manner, with the results obtained being presented in Fig. A5.3a.

(b) Short circuit test - With the rotor short-circuited and driven at a given super-synchronous speed, an increasing voltage was applied to the stator while the rotor current and input mechanical power were noted at regular interval. To cover the range of interest, the test was performed at speeds of 2100 rpm, 2400 rpm, 2700 rpm and 3000 rpm, and the results obtained are shown in Fig. A5.3b. Note that the results are presented in a manner that allows them to be used directly in Section 8.3.1.1.

(c) Open-circuit test - For this test the rotor was open-circuited and driven at speeds of 1800 rpm, 2100 rpm,
2700 rpm and 3000 rpm. At each speed an increasing voltage was applied to the stator, with the stator current, input electrical power and mechanical power being noted at regular interval. Figs. A5.3c-g show the results obtained from this test. It was noted that both the exciter and the generator have unity turns ratio.
| Table A5.1 Parameters of the exciter and the generator of the first ECU  
| (Values referred to the stator of each machine and with the exciter stator connected in parallel-star) |

<table>
<thead>
<tr>
<th></th>
<th>Exciter</th>
<th>Generator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>0.174Ω</td>
<td></td>
</tr>
<tr>
<td>$x_1 = x_2$</td>
<td>0.324Ω</td>
<td></td>
</tr>
<tr>
<td>$r_2$</td>
<td>0.183Ω</td>
<td></td>
</tr>
<tr>
<td>$r_{m1}$</td>
<td>76.2Ω</td>
<td></td>
</tr>
<tr>
<td>$x_{m1}$</td>
<td>11.9Ω</td>
<td></td>
</tr>
<tr>
<td>$r_a$</td>
<td>0.301Ω</td>
<td></td>
</tr>
<tr>
<td>$x_a = x_b$</td>
<td>0.646Ω</td>
<td></td>
</tr>
<tr>
<td>$r_b$</td>
<td>0.290Ω</td>
<td></td>
</tr>
<tr>
<td>$r_{ma}$</td>
<td>241Ω</td>
<td></td>
</tr>
<tr>
<td>$x_{ma}$</td>
<td>27.7Ω</td>
<td></td>
</tr>
</tbody>
</table>

| Table A5.2 Parameters of the first ECU for use with Fig. 8.4a |

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>0.696Ω</td>
<td></td>
</tr>
<tr>
<td>$x_1 = x_2$</td>
<td>1.296Ω</td>
<td></td>
</tr>
<tr>
<td>$r_2$</td>
<td>0.732Ω</td>
<td></td>
</tr>
<tr>
<td>$r_{m1}$</td>
<td>304.8Ω</td>
<td></td>
</tr>
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<td>$x_{m1}$</td>
<td>47.6Ω</td>
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</tr>
<tr>
<td>$r_a$</td>
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<td></td>
</tr>
<tr>
<td>$x_a = x_b$</td>
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</tr>
<tr>
<td>$r_b$</td>
<td>0.32Ω</td>
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</tr>
<tr>
<td>$r_{ma}$</td>
<td>265.7Ω</td>
<td></td>
</tr>
<tr>
<td>$x_{ma}$</td>
<td>30.5Ω</td>
<td></td>
</tr>
</tbody>
</table>
Fig. A5.1a No-load test data for the exciter of the first ECU
Fig. A5.1b Locked-rotor test data for the exciter of the first ECU
Fig. A5.2a No-load test data for the generator of the first ECU
Fig. A5.2b Locked-rotor test data for the generator of the first ECU
Fig. A5.3a Locked-rotor test data for the second ECU
Fig. A5.3b Short-circuit test data for the second ECU
Fig. A5.3c Open-circuit test data for the second ECU
(s = 0)
Fig. A5.3d Open-circuit test data for the second ECU
\( (s = -1/6) \)
Fig. A5.3e Open-circuit test data for the second ECU
\((s = -1/3)\)
Fig. A5.3f Open-circuit test data for the second ECU
\( s = -1/2 \)
Fig. A5.3g Open-circuit test data for the second ECU
(s = -2/3)
APPENDIX A6
EXCITER DESIGNS FOR GENERATOR WC 876

A6.1 Design with separate 4-pole and 8-pole rotor windings

The synchronous generator is similar to the unit used in Section 4.5. To supplement the information provided in that section, the synchronous reactances of the generator are $X_{sd} = 1.0 \text{ p.u.}$ and $X_{sq} = 0.5 \text{ p.u.}$ It will be noted that the operating frequency of the generator is 60Hz, and the rotor frequency of the exciter is therefore 120Hz. Since the generator is a 4-pole unit, the exciter windings SE and RSE must have 4 poles and the exciter windings SH and RSH 8 poles.

It is proposed that the design will be based on the stator/rotor punchings of MAC exciter WC 0077†. The design process is thus to determine for a given excitation requirement (as specified by the generator parameters) the appropriate stack length and the number of turns needed in the various exciter windings.

Although it is convenient to separate the two machines (which comprise the exciter) in deriving the equivalent circuit of Fig. 9.5b, such an approach is not feasible in the design process, since the field current cannot be determined until the whole rotor design is complete. A possible design strategy, and the one adopted by the author, is as follows:

(a) Winding RSH - Before commencing the design process, the

† Slot diagrams for the exciter is given in Fig. A6.1.
exciter no-load flux density $B_{go}$ is fixed at some acceptable level ($B_{go} < 0.3T$, in the author's experience). From an assumed stack length, together with the dimensions of the stator/rotor stampings, the fundamental flux per pole $\phi_f$ can be readily evaluated. If it is further assumed that the required exciter open-circuit rotor voltage is $E_{4r}$, then the series turns per phase of winding $RSH (N_s)$ can be determined from the usual e.m.f. equation

$$E_{4r} = \sqrt{2}\pi N_s \phi_f K_w$$

... (A6.1)

and the synchronous reactance of the winding $X_{4r}$ from eqn. (5.1) (note that Carter's coefficient for the stator/rotor frame under consideration is 1.31).

(b) Winding $SE$ and $RSE$ - There is considerable flexibility in the selection of the number of turns for these two windings. If the number of series turns in winding $RSE$ is $N_3$ and the corresponding quantity in winding $SE$ is $N_2$, a simple analysis indicate that for a given compounding effect the total loss in the two windings is a minimum when $(R_{11}/R_{22}) = (I_{r}^2/I_{L}^2)$. This consideration requires $N_2 = N_3$ for the present design. In addition, for ideal compounding at zero power factor, $N_2$ and $N_3$ must be selected such that

$$X_{23} = \frac{\bar{X}_{sd} E_{4r}}{I_L}$$

... (A6.2)

Design values for $X_{22}$, $X_{33}$ and $X_{23}$ may then be obtained using the
method already described in Chapter 5. Trial designs were computed for different values of $N_3$, with eqn. (9.8) and the 3-phase rectifier model of Appendix A1 being used to check the no-load field current. At this stage, it is often necessary to repeat procedure (a) with a different $E_{4r}$ and/or stack length until the required no-load field current is obtained.

(c) Winding SH - This winding is designed to give the ampere-turn that will establish the no-load airgap flux density $B_{go}$. After allowing for the winding factor and the slotting effect (Carter's coefficient), the ampere-turn required for the airgap can readily be obtained. Since the magnetisation characteristic of the iron core was not available, the core magnetisation was assumed to require 30% of the airgap ampere-turns. The sum of these two ampere-turns then gives the total that the winding needs to provide, with the number of turns being independently selected to conform with design specification.

The prototype exciter detailed in Table 9.1 and Figs. 9.6a-d was designed following the above procedures. A stack length of 2" in used to allow the operating no-load flux density to be 0.27T, and the design has the following parameters:

$$X_{4r} = 0.23\Omega, \quad X_{22} = 0.91\Omega, \quad X_{33} = 0.33\Omega \quad \text{and} \quad X_{23} = 0.55\Omega$$

(unsaturated values at 120Hz). Furthermore, to establish the 480AT/pole to give the no-load flux density, winding SH
requires a d.c. current of about 0.9A for the design.

A6.2 Design and test parameters of the experimental unit

When the ampere-turns required for the core magnetisation are not allowed for, the exciter open-circuit voltage $E_{4r}$ with 1.81A flowing in winding $SH$ is computed as 9.77V. The corresponding experimental value is 7.4V, which indicates that about 25% of the total ampere-turn is in fact expended in core magnetisation. The design values of the reactances are

$X_{4r} = 0.12\Omega$, $X_{33} = 0.15\Omega$, $X_{22} = 0.35\Omega$ and $X_{23} = 0.23\Omega$

(unsaturated values at 100Hz). By supplying only winding SE and measuring the open-circuit rotor voltage (magnitude given by $|I_LX_{23}|$), $X_{23}$ was obtained as 0.164Ω. Thus, allowing for the effect of saturation, the reactances of the test unit are $X_{4r} = 0.086\Omega$, $X_{33} = 0.11\Omega$, $X_{22} = 0.25\Omega$ and $X_{23} = 0.164\Omega$.

The measured resistances are $R_r = 0.14\Omega$, $R_{22} = 0.0406\Omega$ and that of the shunt winding is 40.22Ω.

A6.3 Design with single-phase combined rotor winding

The design strategy is the same as that described in Appendix A6.1. However, in the implementation of those procedures, it should be noted that the reactances associated with a single-phase winding are only 1/3 of the corresponding values for a 3-phase winding with the same number of turns per phase. Using eqn. (9.10) it follows that for a winding
distribution of Fig. 9.11; \( T_1 \) = 0.418N and \( T_2 \) = 0.325N, where \( N \) is the series turns per phase. (This particular distribution is adopted to minimise the m.m.f. harmonics of the third and higher order). Thus, for a combined winding of \( N \) series turns per phase, the equivalent turns number in windings RSE and RSH (i.e. \( N_3 \) and \( N_s \)) are respectively 0.418N and 0.63N. The design process is therefore concerned mainly with finding the value of \( N \) that will provide the required excitation, and the resulting design obtained as such is given in Table 9.2 and Figs. 9.11 Figs. 9.11a-b. The design parameters of the exciter are \( X_{4r} \) = 0.52Ω, \( X_{33} \) = 0.86Ω, \( X_{22} \) = 0.43Ω and \( X_{23} \) = 0.61Ω.

A6.4 Design with 3-phase combined rotor winding

Inspection of the data for the exciter prototype with two separate rotor windings (Table 9.1) indicates that the design has \( N_{c2} \) equal to 5 and \( N_{c1} \) approximately equal to \( N_{c2} \) (i.e. two 3-turn coil distributed in two adjacent slots). Thus, the combined rotor winding with 10 turns per coil and arranged as shown in Fig. 9.13a-b will effectively replace the two separate rotor windings.
Fig. A6.1 Slot diagrams for exciter WC 0077
(All dimensions in ins.)