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DIGITAL DATA TRANSMISSION OVER

VOICE CHANNELS

by

H.Y. NAJDI, B.Sc., M.Sc.

A Doctoral Thesis
Submitted in Partial Fulfilment of the Requirements
for the Award of the Ph.D. Degree
of the Loughborough University of Technology
May, 1982.

SUPERVISOR: Dr. A.P. Clark
Department of Electronic and Electrical Engineering

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ABSTRACT

The thesis is concerned with the detection of digital data transmitted over voice frequency channels such as telephone circuits and HF radio links, where the main impairment is additive noise and intersymbol interference, and the latter may be time-invariant or may vary slowly with time. The characteristics of these channels are briefly reviewed and a survey of the most important known detection techniques is presented.

The thesis includes also a detailed study of quadrature amplitude modulated (QAM) signals transmitted over voice-channels, both, when the transmission path has time-invariant characteristics and when it introduces Rayleigh fading into the transmitted data signal. Based on this study, baseband models of QAM systems are suggested for use when these are to be computer simulated. A systematic study of channel models is carried out here.

The transmission and detection of baseband signals over telephone circuits in the presence of frequency offset is investigated and a baseband signal generated by means of Hilbert transform pairs is suggested for this purpose. It is shown that this signal exhibits theoretical and experimental equivalence to a QAM signal.

Several near-maximum likelihood detection techniques have been developed for the detection of digital data signals serially transmitted at 19200 bit/s over telephone lines and at 9600 bit/s over HF radio links. The performance of the detection systems has been evaluated by computer simulation and is given in terms of their tolerance to additive white Gaussian noise.
The author is deeply indebted to his supervisor, Dr. A.P. Clark, for his continuous support and encouragement, without which this work could not have been completed.

The author also wishes to thank the Government of the Syrian Arab Republic for the generous financial support.

The patience of Sabah, Yassin and Iyad will never be forgotten.
## LIST OF PRINCIPAL SYMBOLS

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<th>Description</th>
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<tr>
<td>$a(t)$</td>
<td>Impulse response of a filter</td>
</tr>
<tr>
<td>$a^*$</td>
<td>Complex conjugate of $a(t)$</td>
</tr>
<tr>
<td>$\hat{a}(t)$</td>
<td>Hilbert transform of $a(t)$</td>
</tr>
<tr>
<td>$</td>
<td>a(t)</td>
</tr>
<tr>
<td>$\text{F.T.}[a(t)]$</td>
<td>Fourier transform of $a(t)$</td>
</tr>
<tr>
<td>$\text{H.T.}[a(t)]$</td>
<td>Hilbert transform of $a(t)$</td>
</tr>
<tr>
<td>$A(f)$</td>
<td>Fourier transform of $a(t)$. Transfer function of the filter whose impulse response is $a(t)$</td>
</tr>
<tr>
<td>$A^*(f)$</td>
<td>Complex conjugate of $A(f)$</td>
</tr>
<tr>
<td>$</td>
<td>A(f)</td>
</tr>
<tr>
<td>$\hat{A}(f)$</td>
<td>Fourier transform of the Hilbert transform of $a(t)$</td>
</tr>
<tr>
<td>$b(t)$</td>
<td>Impulse response of a filter</td>
</tr>
<tr>
<td>$B(f)$</td>
<td>Fourier transform of $b(t)$</td>
</tr>
<tr>
<td>$</td>
<td>B(f)</td>
</tr>
<tr>
<td>$c(t)$</td>
<td>Impulse response of a bandpass filter (receiver filter)</td>
</tr>
<tr>
<td>$\hat{c}(t)$</td>
<td>Hilbert transform of $c(t)$</td>
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<tr>
<td>$C(f)$</td>
<td>Fourier transform of $c(t)$</td>
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<tr>
<td>$</td>
<td>C(f)</td>
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<tr>
<td>$d_{\min}$</td>
<td>The minimum distance between noise-free signal-vectors in the signal vector space</td>
</tr>
<tr>
<td>$d_k$</td>
<td>Cost of vector $P_k$ (Section 7.5)</td>
</tr>
<tr>
<td>$d_k^*$</td>
<td>Alternative cost of vector $P_k$ (Section 7.5)</td>
</tr>
<tr>
<td>$d(t)$</td>
<td>Impulse response of a filter</td>
</tr>
<tr>
<td>$D(f)$</td>
<td>Fourier transform of $d(t)$</td>
</tr>
<tr>
<td>$e_i$</td>
<td>Error signal (Sections 3.9 and 7)</td>
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\( e_i^2 \): Mean-square error
\( e(k) \): The location of the largest component among the first \( f(k) \) components of the sampled impulse response \( Y_k \) (Section 7.5)
\( e'(k) \): Alternative value of \( e(k) \) (Section 7.5)
\( E[.] \): The expected value of [ . ]
\( E_0 \): Average transmitted energy per bit
\( E_r \): Average energy per signal element in the signal \( r(t) \)
\( f & f(k) \): Location of first significant component of \( Y_k \) (Section 7.5)
\( f'(k) \): Alternative value of \( f(k) \)
\( f_c \): Carrier frequency in QAM systems
\( f_0 \): Carrier frequency in a carrier link or HF radio link
\( f_3 \): Cut-off frequency at -3 db
\( f_{r.m.s.} \): Mean-square root frequency of a Gaussian filter
\( f_{sp} \): Frequency spread \( f_{sp} = 2 f_{r.m.s.} \)
\( f(t) \): Impulse response of a Hilbert transform
\( F(f) \): Transfer function of a Hilbert transform
\( g+1 \): Number of components in a sampled impulse response \( Y \)
\( g(t) \): Impulse response of a filter
\( \hat{g}(t) \): Hilbert transform of \( g(t) \)
\( G(f) \): Fourier transform of \( g(t) \)
\( |G(f)| \): Absolute value of \( G(f) \)
\( h(t) \): Impulse response of a voice-channel
\( H(f) \): Fourier transform of \( h(t) \)
\( |H(f)| \): Absolute value of \( H(f) \)
\( j \): When not used as a subscript, \( j=\sqrt{-1} \)
\( \varepsilon \): Number of sampling intervals delay in the early detection (Section 7)
m: Number of levels of a real-valued data-symbol $s_i$

$m^2$: Number of levels of a complex-valued data-symbol $s_i$

n: Number of sampling intervals delay in detection

n(t): White Gaussian noise process with zero mean and two-sided power spectral density $\frac{1}{2}N_0$

$\frac{1}{2}N_0$: Power spectral density of n(t)

N: Length of a transmitted data message in data-symbols

p: Frequency offset (Sections 5 and 6). Also the largest possible value of $f$ (Section 7.5)

$p'$: Estimated value of frequency offset (Sections 5 and 6)

$p(R|S)$: The conditional probability density of $R$ given $S$

p(t): Signal at the output of the linear baseband channel (Section 6)

$\{p_i\}$: Samples of p(t) at $t=iT$, $i=1,2,...$ (Section 6)

$P_k$: A vector given by the last $(n+1)$ components of a vector $X_k$, and is obtained when a stored vector $Q_{k-1}$ is expanded by increasing the number of its components by 1 (Sections 6.4 and 7.5). A component of $P_k$ may have one of the possible values of $s_i$.

$P(S=X|R)$: The a posteriori probability that $S=X$ given $R$

q(t): A random process with zero mean Gaussian probability density and a power spectrum that is Gaussian in shape with a r.m.s. frequency $f_{\text{r.m.s.}}$ (Section 7.2)

$q_1(t), q_2(t), q_3(t)$ and $q_4(t)$: Statistically independent random processes each with the same characteristics as q(t)

$\{q_i\}$: Samples of q(t) at a rate of 4800 samples/second

$\{q_{1,i}\}, \{q_{2,i}\}, \{q_{3,i}\}$ and $\{q_{4,i}\}$: Sequences obtained by sampling $q_1(t)$, $q_2(t)$, $q_3(t)$ and $q_4(t)$, respectively, at a rate of 4800 samples/second (Section 7)
$Q_k$: A stored vector held by a near maximum-likelihood detector and is given by the last $n$ components of a vector $X_k$, and each of its components may have one of the different possible values of $s_i$

$r(t)$: Received signal

$\{r_i\}$: Sequence of received signal samples that is obtained by sampling $r(t)$ at $t=iT$, $i=1,2,...$

$R_k$: A $k$-component vector whose $i$th component is $r_i$

$R_u(t)$: Auto-correlation function of the random process $u(t)$

$R_{u_i,u_i,k}$: Auto-correlation function of the random sequence $\{u_i\}$

$R(z)$: $z$-transform of the sequence given by the vector $R_k$

$s_i$: The $i$th transmitted data symbol. It may have $m$ possible real values (Sections 3, 5.4.1 and 6.2.1) or $m^2$ possible complex-values (Sections 4, 5, 6 and 7)

$s_i^l$: The detected value of $s_i$

$s_i''$: The "early" detected value of $s_i$ (Section 7)

$\overline{s_i^2}$: Mean-square value of $s_i$

$S_k$: A $k$-component vector whose $i$th component is $s_i$ and represents the transmitted data message up to time $t=kT$.

$S(z)$: $z$-transform of the sequence given by the vector $S_k$

$T$: Sampling period

$\frac{1}{T}$: Signal-element rate in bauds or sampling rate in samples per second

$\frac{1}{T_1}$: Signal-element rate in bauds or sampling rate in samples per second (Section 5.4.1)

$u(t)$: A Gaussian random process with zero mean

$\{u_i\}$: A sequence obtained by sampling $u(t)$ at $t=iT$, $i=1,2,...$
\( U_k \): A k-component vector whose ith component is \( u_i \) (Sections 6 and 7)

\( U(f) \): The step function of \( f \) (Section 5, eqn. 5.27)

\( w(t) \): A Gaussian random process with zero mean

\( \{w_i\} \): A sequence obtained by sampling \( w(t) \) at time \( t = iT, i = 1, 2, \ldots \)

\( w_k \): A k-component vector whose ith component is \( w_i \)

\( |w_k|^2 \): Cost of a vector \( X_k \), \( P_k \) or \( Q_k \) (Sections 3, 6 and 7)

\( x_i \): The ith component of a vector \( X_k \) and may have one of the different possible values of \( s_i \)

\( X_k \): A k-component vector whose ith component is \( x_i \) and represents an estimate of the transmitted data sequence \( S_k \)

\( y(t) \): Impulse response of a time-invariant linear baseband channel

\( y_i(t-iT) \): Impulse response of a time-varying linear baseband channel at time \( t = iT \) (Section 7)

\( Y = y_0, y_1, \ldots, y_g \)

\( Y_k = y_k, 0, y_k, 1, \ldots, y_k, g \)

\( Y(z) \): z-transform of the sequence given by vector \( Y \)

\( Y'_k \): An estimate of \( Y_k \)

\( Y'_k, k-\xi-1 \): The prediction of \( Y_k \) over the prediction interval \( \xi T \) seconds (Section 7)

\( z_i \): The data-signal component in \( r_i \)

\( Z_k \): A k-component vector whose ith component is \( z_i \) and represents the noise-free received signal sequence

\( \alpha_k \): The kth transmitted binary digit (with possible values 0 or 1)

\( \{\alpha_k\} \): Transmitted sequence of binary digits
\(a_k^t\): Detected value of \(a_k\)

\(\delta(t)\): Dirac function (unit impulse)

\(\lambda\): Number of vectors \(\{P_k\}\) into which a vector \(Q_{k-1}\) is expanded (eqn. 6.109, Section 6.4.4)

\(\lambda(k)\): A variable with possible values 0 or 1 and controls the mode of operation of the detector (Section 7.5.3)

\(\mu\): Number of stored vectors \(\{Q_k\}\) held by a near-maximum-likelihood detector

\(\sigma_u^2\): Variance of the random process \(u(t)\) or the random sequence \(\{u_i\}\)

\(\tau\): Differential multipath propagation delay (Section 7)

\(\psi\): Signal-to-noise ratio in db \((\psi = 10 \log_{10} \frac{E_0}{N_0})\)

---

\[\text{Adder}\]

\[\text{Multiplier}\]

\[\text{Accumulator}\]

\[\text{Delay of one sampling period}\]

\[\text{Sampler (at a rate of } \frac{1}{T}\text{)}\]

\[\text{A linear system (a filter or a transmission path) with an impulse response } a(t)\]
1. INTRODUCTION

1.1 BACKGROUND

No doubt, communication has always been and will be an integral part of human organisation and the degree of sophistication of communication systems is a reflection of the need for this in different aspects of man's life. As a response to the rapid development in various fields of technology these days, communication systems have grown dramatically, both in technical sophistication and usage, and one of the most striking of recent developments has been the rapid growth of digital data communication systems.\(^{(81)}\)

The basic feature of a digital transmission system is that it transmits a series of separate data elements which carry the information to be transmitted.\(^{(81)}\) Different media are used for digital data transmission, but the most important of these are the voice-frequency channels over the telephone network and HF radio links.\(^{(4-51)}\) Although telephone circuits were originally designed for the transmission of speech,\(^{(5,15)}\) they play now an important role in data transmission, because of their world-wide existence on one hand and because data transmission became, with the increased use of computers and digital data systems, a daily requirement of business in various fields. HF radio links are also important for data communication, because they represent the most cost effective communication channels, in some applications, such as long distance communications.\(^{(23)}\)

Data transmission is as old as telegraphy, but the transmission speed has only recently become relatively high,\(^{(20-26,46-51)}\) under the pressure of the increased demand for fast and reliable data communication. In 1928, Nyquist\(^{(1)}\) formulated the filtering requirements in order to detect successive transmitted data symbols independently (without interference from
one data symbol into another), where he showed that the maximum symbol rate over a bandlimited channel whose passband extends over the frequency range 0 to $W$ Hz, is $2W$ symbols per second. When a data symbol may have two possible values only, as in binary transmission, it carries up to 1 bit of information, and the maximum information rate over the given bandlimited channel becomes $2W$ bit/s. As a data symbol may be taken to have any number of possible values, it may consequently carry any number of bits of information, and therefore the maximum information rate over the bandlimited channel is unlimited. This, of course, assumes the absence of noise, which, in practice, is likely to be added by the channel to the transmitted signal. The effect of additive noise on the maximum information rate over a bandlimited channel was studied by Shannon in 1948, when he showed that the information rate is limited only by the signal to noise power-ratio at the output of the channel regardless of the intersymbol interference, and as the noise power goes to zero, the possible information rate increases without limit. Over a typical telephone circuit, and at a signal-to-noise ratio of the order of 30 db, the maximum information rate is about 23000 bit/s, when both the information source and additive noise are Gaussian. Theoretically, this rate should be achievable with an arbitrarily small error rate, so long as a sufficiently involved encoding scheme is used, as Shannon suggests; however, the precise method of implementing such a scheme was not determined by Shannon.

Early last decade, the transmission of 2400 bit/s over the switched telephone network became common practice, and later, techniques for the transmission of data at 9600 bit/s over the telephone network started to appear. Yet, more than three decades have gone after the Shannon limit has been determined and the maximum transmission rate which has been achieved over telephone circuits in practice is still below half that limit.
The reason behind this delay seems to be the problem of finding suitable detection techniques, which must be, not only reliable, but also simple, cheap and easy to implement. Over HF radio links, satisfactory transmission at a rate of 2400 bit/s has been achieved only very recently\(^{(51)}\) this being well below what can, at present, be achieved over the telephone network.

The complexity of a detector, if it is to operate reliably, depends, to a large extent, upon the transmission rate as well as on the nature of the signal impairments likely to be encountered over the channel. The most important of these impairments are amplitude and phase distortions, both of which introduce intersymbol interference\(^{(81)}\) and which are the main factors to be taken account of in the detection process\(^{(52-119)}\).

Detection processes may be classified into two separate groups\(^{(81)}\), depending upon the way in which the detector tackles intersymbol interference. In the first of these, an equalizer\(^{(63-83)}\) is used to remove the intersymbol interference from the received signal, such that the individual data symbols are detected independently from the resultant signal. The process of removing the intersymbol interference by the equalizer often results in using only a part of the transmitted signal energy in the detection of a data symbol, with a consequent reduction in tolerance to additive noise\(^{(81)}\). The other groups of detection techniques, instead of removing the intersymbol interference, take full account of it, thus using the entire transmitted energy in the detection process. These techniques are known as the maximum or near-maximum likelihood detectors\(^{(90-119)}\). The detected data sequence, which may involve the entire or part of the transmitted message, is here determined after some delay (ideally at the end of the message). Unlike the equalizer, the maximum likelihood detector is optimum, in the sense that under the appropriate conditions it minimizes the probability of error in the detection of the whole message, and in many cases, its performance is as good as if there were no intersymbol interference\(^{(90)}\).
of the maximum-likelihood detector, compared with the equalizer, is its high complexity, which in most cases prevents its usage in practice. As a compromise, the near-maximum likelihood detector combines the simplicity of the equalizer with the optimality of the maximum-likelihood detector to give a system which is closer to the former in its simplicity and to the latter in its performance.

The main feature of voice-frequency channels is that they possess band-pass frequency characteristics. But since data signals to be transmitted are basically baseband signals (whose spectrum extends to zero frequency), it is necessary to adjust the data-signal spectrum to match that of the channel. This is usually achieved by carrier modulation techniques known as amplitude shift keying (ASK), phase shift keying (PSK) and frequency shift keying (FSK). With the introduction of partial response techniques, signal spectrum shaping became possible without the need for carrier modulation, and the transmission of baseband signals should have become possible, but it did not. Over telephone circuits, the presence of frequency offset, which may have values up to 5Hz, causes the frequency characteristics of the channel to vary (very slightly) with time and may prevent entirely the correct detection of the transmitted baseband signals. When a carrier-modulated signal is transmitted, the receiver uses, in the coherent demodulation of the signal, a reference carrier which has the same instantaneous frequency as the received signal carrier, thus removing the effect of any frequency offset that may be present in the received signal. A reference carrier, with the same instantaneous frequency as the received signal carrier, may be generated at the receiver quite accurately, using well known phase-locked-loop techniques. Over HF radio links, time-variations in the channel characteristics are caused by fading, whose rate may be up to 15 fades per minute or more. Whereas the relatively
small time-variations in the telephone circuit characteristics may be
removed, so that the detector has to handle a basically time-invariant inter-
symbol interference, this can not be done over HF radio links, and the
detector here must be adaptive to be able to deal with time-varying inter-
symbol interference. (115, 119)

1.2 OUTLINE OF THE INVESTIGATIONS

With the appearance of techniques for the transmission of 9600 bit/s
over telephone circuits and 2400 bit/s over HF radio links, the question of
transmitting at higher rates arises. Over telephone circuits, a feasibility
study has been carried out in this work about the possibility of the trans-
mission at a rate closer to the Shannon limit, i.e., at 19200 bit/s. The
main concern here has been the development of various near-maximum likelihood
detectors which can operate satisfactorily over models of typical telephone
circuits. The possibility of transmitting baseband signals over telephone
circuits has also been investigated, and it is shown that the frequency offset
can be corrected for at the receiver and dealt with as in modulated-carrier
systems. The transmission of baseband signals in Hilbert transform pairs is
shown to be the counterpart of the quadrature amplitude modulation (QAM),
and the equivalence between baseband signals and QAM signals is shown
analytically and experimentally. Both signal formats have been used in
testing the detectors developed for the detection of data transmitted at
19200 bit/s. Over HF radio links, the transmission of 9600 bit/s is
investigated. A near-maximum likelihood detector is developed for the
detection of data signals transmitted at 9600 bit/s over a model of an HF
radio link in the presence of Rayleigh fading and multipath propagation.
The developed detector is then tested jointly with a channel estimator,
which has been developed independently,(183) where it is found that trans-
mission of the 9600 bit/s over HF radio links is possible and quite
promising.

In Section 2 of this thesis, a brief description of the most important
properties of voice-frequency channels is presented. Section 3 is basically
cconcerned with well known techniques for digital data detection. Equalization
 techniques are briefly considered here, followed by a detailed study of the
Viterbi algorithm detector and various detection techniques developed from
this. In Section 4, the analysis of a QAM system is presented. In Section 5
a model is developed to represent the effects of frequency offset and a
technique to correct for it when a baseband signal is transmitted is
described. The transmission of baseband signals is then analysed, and an
analytical comparison with the QAM signal is presented. Section 6 is
exclusively concerned with the detection of digital data at 19200 bit/s.
Four models of synchronous serial digital data transmission systems, two of
which transmit baseband signals and the other two transmit QAM signals, are
described together with four near-maximum likelihood detectors, which have
been tested, by computer simulation, over models of four telephone circuits.
Results of computer simulation tests are then presented and analysed.
Section 7 is concerned with the detection of data transmitted at 9600 bit/s
over HF radio links. A model of a two Rayleigh fading sky-wave HF
radio link is described, followed by an analysis of the QAM system over this
model. A model of a synchronous serial digital data transmission system,
which includes the HF link model, is then presented together with a near-
maximum-likelihood detector, which has been developed particularly for use
here. The system is computer simulated, first assuming perfect knowledge
of the channel at the receiver and then using a channel estimator, which
provides the detector with an estimate of the time-varying intersymbol
interference caused by the HF radio link. The results of the computer
simulation tests are finally analysed.
2. VOICE FREQUENCY CHANNELS

2.1 INTRODUCTION

Voice frequency channels are those which ideally have a negligible attenuation over the frequency band 300-3000Hz, but do not necessarily pass any frequencies outside this band.\(^\text{(23)}\) The importance of these channels lies in their very wide existence in the form of the world-wide telephone network and the many HF radio links. As the original function of these channels has been the satisfactory transmission of speech, they have been designed and optimized to achieve this objective most economically.\(^\text{(15)}\) Whereas it is important for satisfactory speech transmission to reduce the amplitude distortion and noise level introduced by the channel into the transmitted signal, phase distortion, for example, has been largely ignored\(^\text{(15)}\) as the human ear is very tolerant to it.\(^\text{(197)}\) Digital data signals are much more sensitive than are speech signals to all kinds of impairments likely to be encountered over voice-channels.\(^\text{(4-45)}\) Therefore a data transmission system operating over these channels is usually much more complicated than is an ordinary speech communication system.

The transmission characteristics of voice channels, in practice, vary over a wide range, because of the variety and number of different links forming a typical channel.\(^\text{(5)}\) In this section, a brief survey of voice channels will be presented.
2.2 **Definitions** *(23)*

The attenuation characteristic of a channel is the variation of attenuation with frequency, when a constant level sine-wave, with adjustable frequency, is fed over the channel.

The attenuation distortion in a given frequency band is the variation of attenuation over that frequency band.

The bandwidth of a channel is the range of frequencies (frequency band) over which the attenuation does not exceed some specified value, usually 3, 6 or 20 db above its lowest level.

A voice-frequency channel has ideally a negligible attenuation over the frequency band 300-3000Hz, but does not necessarily pass any frequencies outside this band.

The group-delay characteristic of a channel is the variation of group delay (envelope delay) with frequency. The group delay at a given frequency is the delay in transmission of energy at that frequency. It is measured as the delay in transmission of a slowly varying envelope (peak value) of a sine wave having the given frequency.  

\[ \text{(of } f_0 \text{ bandwidth, a plane response)} \]

The delay distortion in a given frequency band is the variation of group delay over the frequency band.

The signal distortion is the change in the shape of the transmitted signal, resulting from the attenuation and group delay characteristics of the channel.

Let \( h(t) \) be the impulse response of the channel (\( h(t) \) is the signal at the output of the channel when the signal at the input is \( \delta(t) \), where \( \delta(t) \) is the Dirac function), then \( H(f) \), the transfer function of this

*The contents of Sections 2.2, 2.3 and 2.4, other than otherwise referenced, have been mostly copied from reference (23), with the permission of its author, Dr. A.P. Clark. These sections have been presented here in fact for completeness, and their content is actually beyond the scope of the research conducted in this work.*
channel in the frequency domain is given by the Fourier transform of \( h(t) \)

\[
H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt
\]  

(2.1)

Let \( A_t(f) \) and \( \theta_t(f) \) be the attenuation and group-delay characteristics, respectively, of the channel. \( A_t(f) \) and \( \theta_t(f) \) are related to \( H(f) \) by (199)

\[
A_t(f) = -10 \log_{10} |H(f)|^2 \quad \text{[in db]}
\]  

(2.2)

and

\[
\theta_t(f) = \frac{1}{2\pi} \frac{d\phi(f)}{df} \quad \text{[in sec.]} \]

(2.3)

where \( |H(f)| \) is the absolute value of \( H(f) \), and \( \phi(f) \) is the phase of \( H(f) \), at the frequency \( f \), and is given by

\[
\phi(f) = \tan^{-1} \left( \frac{\text{Imaginary part of } H(f)}{\text{Real part of } H(f)} \right)
\]  

(2.4)

2.3 CHARACTERISTICS OF AND SIGNAL IMPAIRMENTS OVER TELEPHONE CIRCUITS (23)

A telephone circuit connecting one subscriber to another is normally made up of two or more links connected in tandem (end-to-end). These links are usually of three distinct types: unloaded audio, loaded audio or carrier. Microwave links are often used, and for long distance telephone circuits, satellite links and sometimes HF radio links.

A long telephone circuit of several hundred miles in length, may, for example, contain the following arrangement of individual links: unloaded audio, loaded audio, carrier, carrier, loaded audio, unloaded audio. Clearly, the type of distortion to be expected over a telephone circuit will usually be a combination of the distortions introduced by the different individual links.

Unloaded audio links are generally very short (not more than two or three miles) and comprise a pair of wires with impedance 600Ω. Because
of their short length, they have a good frequency response, with some attenuation distortion and negligible delay distortion over the voice-frequency band. The attenuation increases as the square root of the frequency, over the voice-frequency band, and is typically about 2\frac{1}{2} \text{ db per mile}, in the centre of the band, with a variation of about 2 db over the band. The delay distortion per mile, over the voice frequency band, is of the order of 20 \mu\text{s} and is quite negligible. It is the high attenuation per mile that prevents the use of long unloaded audio links.

Loaded audio links may be very much longer (up to about a hundred miles). They comprise a pair of wires with impedance 600\Omega and with inductances (often 44 or 88 mH) inserted at regular intervals (typically 200 yds.). Their attenuation characteristics resemble that of a low-pass filter, such that up to a certain frequency (often below 3000 Hz) there is only small attenuation (less than 1 db per mile) and above this frequency the attenuation increases rapidly with frequency. Thus the attenuation over the centre of the voice-frequency band is reduced by the loading coils, in return for a greatly increased attenuation over the higher frequencies. Not only is the response of the link restricted at high frequencies, but the group delay increases towards the high frequency end of the band, to give a delay distortion of the order of 200 \mu\text{s} per mile over the range of frequencies for which there is only small attenuation. The attenuation and delay distortions introduced by a loaded audio link increase with its length, and the longer links introduce considerable attenuation and delay distortions at the high frequency end of the band.

Carrier links may be very much longer than loaded audio links. A process of single sideband suppressed carrier amplitude modulation is used here to shift the signal frequency band to higher frequencies, for transmission over a wideband channel which may be a coaxial cable or open wire line.
This channel carries several signals, each using a different frequency band, in an arrangement of frequency-division multiplexing (FDM). At the other end of the carrier link, each of the multiplexed signals is first isolated from the others by means of a band-pass filter, and a process of linear demodulation is then used to restore the signal to the voice-frequency band.

The modulation process at the transmitter first multiplies the voice frequency signal by a sine wave with the appropriate frequency, to give a double sideband signal centred on this frequency. The sideband required for transmission is then isolated by means of a filter. The process of linear demodulation first multiplies the received single-sideband signal by a sine wave with ideally the same frequency as that used at the transmitter, and the required voice-frequency signal is then isolated by means of a filter.

The filters involved in the processes of modulation and demodulation in a carrier link have an effect on the demodulated voice-frequency signal, at the output of the link, equivalent to that of a high-pass filter with a cut-off frequency in the range 100-300 Hz. They also restrict the high-frequency components of the demodulated signal, but to a much less significant degree. Thus a carrier link has a restricted low frequency response, and appreciable delay distortion is introduced at the low-frequency end of the band before the attenuation becomes large. The distortion introduced in a carrier link originates in the terminal equipment and does not depend on the length of the link.

If the sine wave carriers, used for modulating and demodulating the signal in a carrier link, do not have exactly the same frequency, the frequency spectrum of the signal at the output of the carrier link is shifted by an amount equal to the difference between the two carrier frequencies, but the shape of the spectrum remains otherwise unchanged. This is known as frequency offset. The frequency offset over carrier links
may have a value up to \( \pm 5 \) Hz, but usually lies within the range \( \pm 2 \) Hz. A detailed study of the effect of the frequency offset on baseband data signals is given in Sections 5.2 and 5.3.

Microwave, satellite and PCM links are in general engineered to much tighter specifications than are loaded audio and carrier links. Their attenuation and delay characteristics, as well as their noise properties are usually less harmful to data-transmission systems than are those of the poorer loaded audio and carrier links, nor do they introduce any significant effects of a different nature to those obtained over loaded audio and carrier links. In other words, a data-transmission systems capable of satisfactory operation over a poor telephone circuit, containing both loaded audio and carrier links, should also give satisfactory operation over the corresponding telephone circuit in which one or more of the loaded audio and carrier links are replaced by microwave, satellite or PCM links.

Loaded audio links of more than a few miles in length require repeaters (amplifiers) to offset the attenuation introduced by the line. Over a long loaded-audio link, the repeaters are spaced at regular intervals along the line. Since the amplifier only passes the signal in one direction, two amplifiers are in fact required in a repeater, one for each direction of transmission. Thus a 4-wire line is needed, two wires for each direction. Carrier circuits also operate with separate go and return channels, using separate processes of modulation and demodulation for the two channels, so that carrier circuits also require 4-wire lines (at least at their inputs and outputs).

Most subscribers, however, are connected to the local exchange via a single pair of wires, carrying signals in both directions. Thus where a telephone circuit contains repeaters or carrier links, arrangements must be made to couple 2-wire lines to 4-wire lines. This is achieved by means of hybrid transformers.
Telephone circuits may be divided into two distinct groups: private and switched lines. A private line is one which is rented permanently or on a part-time basis by a subscriber. It is not connected through any of the automatic switches in the exchange and it is also disconnected from the exchange battery supplies which are used for d.c. signalling and various other purposes. The private line is also checked for its overall attenuation-frequency characteristic to ensure a reasonable frequency response. A switched line, that is a line on the public network, is the circuit obtained when using an ordinary telephone to set up a call, either by dialling a number or through the local exchange operator. The line is connected through a number of switches and to the exchange battery supplies. It is connected through transmission bridges, which increase the attenuation over the voice-frequency channel and further reduce the low frequency response. In addition, a switched line is made up of a number of separate links, each chosen at random from sometimes quite a large number. It is therefore obviously not possible to check the frequency characteristics of complete circuits, since far too many possible combinations of the individual links are involved. Instead only the frequency characteristics of the individual links are checked. One of the results of this is that it is possible over any such telephone circuit to obtain serious mismatches between individual links.

Each mismatch reflects a portion of the signal reaching it, with the result that when there are two or more mismatches along one line, a portion of a signal, reflected back from a mismatch near a receiving end of the line, will be reflected forward again by a mismatch near the transmitting end and a portion of this signal will eventually reach the receiver as an echo of the main signal. Thus the received signal comprises the main component of the transmitted signal (received via direct transmission over the line) followed by several echoes which are attenuated and delayed with respect to
the main component. Each echo is of course the component of the received
signal which has travelled via a different combination of the mismatches
along the line. Similar effect occurs also when the telephone circuit
includes a pair of mismatched hybrid transformers. These are both examples
of multipath propagation, where the transmitted signal can reach its
destination over two or more different paths, usually having different
attenuations and delays.

Any given set of echoes in the received signal corresponds to (or
indicates the presence of) the appropriate combination of attenuation and
delay distortions. When the echoes are not small compared with the main
signal component, they indicate the presence of severe attenuation and delay
distortions. A single large echo, for instance, corresponds to a large
sinusoidal ripple superimposed on both the attenuation and group-delay
characteristics, where the wave length of the ripple (measured along the
frequency scale) is inversely proportional to the delay of the echo.

On account of the reflections caused by mismatching, the attenuation and
delay distortions experienced over switched lines may be appreciably more
severe than those over private lines. The echoes caused by mismatched hybrid
transformers can of course occur on both private and switched lines.

Fig. 2.1 shows the attenuation-frequency characteristic of an ideal
voice-frequency channel. Frequencies below 300 Hz and above 3000 Hz are not
required for the intelligible reception of speech and the attenuation of the
channel may therefore increase rapidly at frequencies below 300 Hz and above
3000 Hz. Figs. 2.2 and 2.3 show a set of samples of actually measured
attenuation and group-delay characteristics of a number of different
circuits in the British public switched telephone network. As can be
seen from Fig. 2.2, a typical telephone circuit has a relatively small
FIGURE 2.1: Attenuation Characteristic of an Ideal Voice Channel

FIGURE 2.2: Samples of Actually Measured Attenuation Characteristics of a Number of Telephone Circuits in the U.K. (taken from Ref. 19)

FIGURE 2.3: Samples of Actually Measured Group Delay Characteristics of a Number of Telephone Circuits in the U.K. (taken from Ref. 19)
attenuation distortion over the frequency band 500 to 1500 Hz. Over the frequency band 1500 to 3000 Hz, the attenuation distortion is significantly higher, and below 300 Hz, the attenuation may be very high. The group-delay (Fig. 2.3) increases towards the lower and upper ends of the frequency band.

As mentioned before, private telephone circuits are designed to have attenuation and group-delay characteristics which fall within specific limits. Figs. 2.4 and 2.5 show these limits as defined by the CCITT and the British Post Office (15).

The attenuation and group delay characteristics shown in Figs. 2.1-2.5 cause time dispersion of received signal and this time dispersion usually increases with the attentionation and delay distortions in the signal frequency band. As a result, the impulse response of the channel (the signal at the channel output when the input signal is the Dirac function) is a continuous rounded waveform of duration not less than 1/3 ms. When the input signal to the voice-frequency channel is a sequence of impulses \( \delta(t-iT) \), where \( i=1,2,\ldots \), and \( T \) is the time interval separating two successive impulses, the output signal is a sequence of continuous rounded waveforms, each being given by an appropriately delayed version of the channel impulse response. When the time interval \( T \) is shorter than the duration of the channel impulse response, then the consecutive output rounded pulses overlap producing what is known as the intersymbol interference.

The noise obtained on telephone circuits may be classified into two distinct groups: additive noise in which a waveform is added to the transmitted signal, and multiplicative noise in which the transmitted signal is modulated by the interfering waveform. When the interfering waveform reaches a sufficient level, its effect is to cause the receiver to interpret incorrectly the received signal and so of course to introduce errors at the
FIGURE 2.4: Limits for Attenuation Characteristics of Private Telephone Circuits for Non-Speech Applications

FIGURE 2.5: Limits for Group Delay Characteristics of Private Telephone Circuits for Non-Speech Applications
receiver output. Additive noise becomes less effective in producing errors as the signal level is increased, whereas multiplicative noise has the same effect in producing errors regardless of the signal level.

The different types of noise are as listed below:

1. Additive noise: Impulsive noise
   Speech and signalling tone cross-talk
   White noise.

2. Multiplicative noise:
   a. Amplitude modulation effects: Modulation noise
      Transient interruptions
      Sudden level changes
   b. Frequency modulation effects: Frequency offset
      Sudden phase changes.

Impulsive noise comprises short bursts of random additive noise, often with a duration of 20 ms, over typical telephone circuits. Individual noise pulses (bursts) frequently occur in groups, sometimes in quick succession but more often relatively widely scattered. Impulsive noise is the predominant type of noise over switched lines where its effects will often swamp those of other types of noise. It is also sometimes important over private lines. Impulsive noise on switched lines is usually introduced by both direct pick up from the electromechanical switches in an exchange, and through common impedance coupling via the exchange batteries to the switches, other equipment and telephone circuits fed from these. The common impedance coupling via the batteries may often be the most important source of impulsive noise. Since private lines are not connected either to the switches or to the battery supplies, the level of impulsive noise over switched lines is in general very much higher than that over private lines.
Speech and signalling tone cross-talk results from stray coupling with other telephone circuits. They are not normally important in causing errors except at usually low signal levels or under definite line fault conditions.

White noise is a steady background noise of low level and relatively wide bandwidth. It only produces errors at very low signal levels and is not normally a significant cause of errors.

Over many private lines containing carrier links and particularly at the higher signal levels modulation noise probably causes a large number of the errors. This appears as amplitude modulation of the signal by band-limited white noise and it normally occurs in short bursts, each lasting up to one or two seconds and causing a scattered group of errors over its duration. Modulation noise may be caused both by microphonic effects when equipment is knocked or shaken, and possibly also by inter-modulation effects when common amplifiers in the carrier link are temporarily overloaded.

Transient interruptions appear as complete breaks in transmission lasting usually from 1 to 100 ms. Over some of the longer and more complex private line telephone circuits, these transient interruptions could be responsible for a considerable number of the errors obtained, although over the shorter telephone circuits they are not likely to be important.

Sudden signal level changes, usually of the order of 1 or 2 db but sometimes even exceeding 5 or 6 db, may occur several times a day. Again this effect will tend to become more frequent and serious over the longer and more complex telephone circuits, and to be less important over the shorter circuits.

Frequency modulation effects occur only over telephone circuits containing carrier links. As mentioned before, the frequency offset may have a value up to ±5 Hz but usually lies in the range ±2Hz, and appears in the
output signal as though the impulse response of the channel varies in a cyclic or repetitive manner. Sudden phase changes may occur quite regularly and many involve a large change of phase, normally occurring almost instantaneously.

Switched lines have the same amplitude and frequency modulation effects and additive noise as private lines, but they have in addition a high level of impulsive noise which may often mask the other types of noise present.

2.4 CHARACTERISTICS OF AND SIGNAL IMPAIRMENTS OVER HF RADIO LINKS (23)

The voice-frequency signal fed to the radio transmitter for transmission over an HF radio link, modulates a sine-wave carrier in the frequency band 3-30 MHz, using a process of single sideband suppressed carrier modulation. This shifts the signal spectrum, which has a bandwidth of about 3 kHz, to the required band in the HF spectrum. At the other end of the radio link the received signal is multiplied by a sine-wave carrier with nominally the same frequency as that used at the transmitter, and the resultant signal is filtered to remove the high-frequency components. This is a process of linear demodulation that shifts the signal spectrum back to the voice-frequency band, to give a signal which is ideally the same as that fed to the radio transmitter at the other end of the link. The process of shifting the signal spectrum to the HF band and back again (using linear modulation and demodulation) are basically the same as the corresponding process used in a telephone carrier link. Since the two carrier frequencies, used for modulating and demodulating the signal, do not necessarily have exactly the same frequency, the spectrum of the voice-frequency signal obtained at the output of the radio receiver may be shifted by 1 or 2 Hz relative to the
original signal spectrum at the input to the radio transmitter. In other words, the HF radio link may introduce a frequency offset of 1 or 2 Hz into the voice-frequency signal.

Several voice frequency signals may be transmitted over a single HF radio link, the signals being frequency division multiplexed in a manner similar to that used over telephone carrier circuits.

Whereas the attenuation and delay characteristics of telephone circuits do not vary much with time, so that these are essentially time-invariant channels, the attenuation and delay characteristics of HF radio links may vary considerably with time in an effect known as frequency selective fading. This is caused by multipath propagation of the transmitted HF radio wave, when it travels from the transmitter to the receiver via two or more different paths with appreciably different transmission delays. For example, the transmitted radio signal may travel from the transmitter to the receiver via both one and two hops, that is, being reflected either once or twice from the ionosphere, there being a difference here of one or two milliseconds between the two transmission delays. Variations in the effective height of the ionosphere cause variations in the relative time delays of the different transmission paths, with sometimes large changes in the attenuation and delay characteristics of the voice-frequency channel. Considerable attenuation and delay distortions can be obtained, and these often appear as sharp peaks moving across the corresponding frequency characteristics.

The time dispersion of a transmitted signal element caused by frequency selective fading, does not often exceed 6 ms and is more often less than 3 ms. Not only can this distortion be considerably more severe than that normally experienced over telephone circuits, but it is time-
varying, with fading rates most often in the range 4-15 fades per minute.

Different types of additive and multiplicative noise are also obtained on HF radio links. These are listed below:

1. Additive noise:
   - Atmospheric noise
   - Impulsive noise

2. Multiplicative noise:
   - Amplitude modulation effects in the form of flat fading
   - Frequency modulation effects in the form of frequency offset.

The main source of additive noise is atmospheric noise caused by lightening discharges. This occupies a frequency band from very low frequencies to around 30 MHz, at the input of the radio receiver. The radio receiver band-limits the received signal to a bandwidth of about 3 kHz, in an essentially linear process. It follows therefore that at any instant the band-limited signal contains components originating from several lightening discharges, which can be assumed to be statistically independent sources. From the central-limit theorem, the band-limited noise signal is approximately equivalent to a Gaussian signal, at least over the range of values in the neighbourhood of the mean (that is, near zero). At values away from zero, the probability density of the noise signal begins to depart from the Gaussian probability density, the discrepancy becoming appreciable along the tails. Although the additive noise signal at the input to the receiver of the data modem is not truly Gaussian, it is normally of a sufficiently random nature and sufficiently close to band-limited Gaussian noise, that a data transmission system having a better tolerance to additive Gaussian noise than another, will almost certainly have a better tolerance to atmospheric noise.
Impulsive noise in HF radio links is usually man-made interference and therefore only becomes really important in built-up areas or where the radio receiving equipment is close to a source of electrical interference.

Flat fading is the variation with time of the received signal level. It occurs when there is multipath propagation, with a total spread in transmission delay which is small compared with the signal-element period. The spread in transmission delay is of course, the time dispersion of the received signal due to the presence of the corresponding echoes. The transmitted radio wave is hence reflected at two or more different heights in the ionosphere. The variation in the level of the received signal due to flat fading is typically up to 40 dB (sometimes more), and fading rates are usually in the range 4-15 fades per minute.

HF radio links also introduce time modulation effects which are known as Doppler shifts. A steadily increasing path length, caused by a steady increase in the height of the ionosphere, has the effect of stretching out (slowing down) the received signal in time. Similarly, a steadily reducing path length, caused by a steady decrease in the height of the ionosphere, has the effect of compressing (speeding up) the received signal in time. Random variations in the effective height of the ionosphere produce the corresponding random variations in the rate of arrival of the received signal waveform, which is therefore modulated in time. Since the transmitted HF radio signals has a bandwidth of 3 kHz and a carrier frequency in the range 3-30 MHz, it is a narrow-band signal. It can be shown that the Doppler shifts are now approximately equivalent to frequency modulation effects. The combination of these with the small frequency offset introduced by the radio equipment, results in a frequency offset of the received voice-frequency signal, which varies slowly with time and usually has a value in the range ±5 Hz.
Whereas the predominant form of additive noise over telephone circuits is impulsive noise, over HF radio links it is atmospheric noise which resembles additive white Gaussian noise. Again, HF radio links do not often introduce such rapid changes in amplitude or phase as occur over telephone circuits. Although the amplitude and phase changes over an HF radio link may at times be very large, they normally take place relatively slowly. The main exception to this is during very deep fades, when the signal is in any case very seriously attenuated.

2.5 THE TOLERANCE OF DATA TRANSMISSION SYSTEMS TO NOISE

Experimental and theoretical considerations have shown that although the tolerance of a data transmission system to additive white Gaussian noise is not necessarily an accurate measure of its actual tolerance to the additive noise over the voice-frequency channels, the relative tolerances to white Gaussian noise of different data transmission systems are nevertheless a good measure of their relative tolerances to this additive noise. Since Gaussian noise lends itself well to theoretical calculations and is also easily produced in the laboratory over the required frequency range, it may be used in the evaluation of the relative tolerances of different data transmission systems to additive noise.
3. DETECTION TECHNIQUES FOR DISTORTED DIGITAL SIGNALS

3.1 INTRODUCTION AND BASIC ASSUMPTIONS

A communication system, can be regarded, from the technical point of view as consisting of three basic parts: \(2-3,81\) the transmitter, the transmission path and the receiver (Fig. 3.1). The signal \(s(t)\), at the transmitter input, carries the information to be transmitted. The transmitter operates on \(s(t)\) to produce the signal \(x(t)\) which is most suitable for transmission over the given transmission path. This path may typically be a telephone circuit, an HF radio link, a satellite link or a microwave system. Unfortunately, practical transmission paths introduce a number of different types of distortion and add to the transmitted signal unwanted interfering signals, i.e. noise, so that the signal \(p(t)\) at the output of the transmission path may look completely different from \(x(t)\), the signal at its input. The receiver has the task of extracting, from \(p(t)\), the signal \(z(t)\) which ideally should be identical to \(s(t)\).

A digital communication system is that in which the transmitted signal \(s(t)\) is itself composed of separate signal-elements (often referred to as symbols, digits, bits or pulses) and these signal elements carry the data (information) which is required to transmit (81). The data carried by an individual signal-element form a symbol, which is often one of the numerals \(0,1,2,\ldots\), and the data carried by the entire transmitted group of signals elements form a message. Here and throughout the thesis, a distinction will be made between a signal-element, which is the actual waveform transmitted
FIGURE 3.1: A General Communication System
and the corresponding symbol which is the data represented by that waveform. As the transmission path in Fig. 3.1 may distort the shape of the signal-element, it is the task of the receiver to determine the symbol corresponding to a received signal-element, although now that symbol may not be related in a simple manner to the distorted received signal-element.\(^{(81)}\)

In general, the data communication system may be a serial or parallel system. A serial system is one in which the transmitted signal comprises a sequential stream of data elements whose frequency spectrum occupies the whole of the available bandwidth of the transmission path. A parallel system, is one in which two or more sequential streams of data elements are transmitted simultaneously and the spectrum of one individual data stream occupies a part of the available bandwidth.\(^{(23)}\)

Most often in a serial system the signal-elements are transmitted at a steady rate of a given number of elements per second (bauds), the receiver being held in time synchronism with the received signal. Such a system is known as a synchronous serial system and, in applications where a relatively high transmission rate is required over a given channel, it is the most commonly used system.\(^{(81)}\) Therefore it will be assumed throughout this work. The system is shown in Fig. 3.2. The signal \(s(t)\) at the input of the transmitter filter is a sequence of regularly spaced impulses at intervals of \(T\) seconds,

\[
s(t) = \sum_{i} s_i \delta(t-iT) \tag{3.1}
\]

where \(\delta(t)\) is the Dirac function and \(s_i\) is the \(i\)th transmitted data symbol which may have one of \(m\) possible values,

\[
s_i = 2\ell - m + 1, \quad \ell = 0, 1, \ldots, m-1. \tag{3.2}
\]

Clearly, the baud rate is \(\frac{1}{T}\). Impulses \(\{s_i \delta(t-iT)\}\) are assumed here (eqn. 3.1) to simplify the theoretical analysis of the system. In practice,
FIGURE 3.2: Model of Data Transmission System
rectangular or rounded waveforms would be used instead with the appropriate change in the transmitter filter to give the same signal $x(t)$ at the input of the transmission path.\(^{(81)}\) Also, the $\{s_i\}$ are assumed to be statistically independent and equally likely to have any of their $m$ possible values. Where this condition is not satisfied by the $\{s_i\}$, it can normally be achieved by scrambling the transmitted sequence of data symbols and appropriately descrambling the corresponding detected data symbols at the receiver.\(^{(81)}\) The transmitter filter, with the impulse response $a(t)$, is employed to shape the spectrum of the signal fed to the transmission path to match its available bandwidth and consequently to maximise the signal power at the receiver input for a given transmitted signal power.\(^{(23,81)}\) The transmission path could be a lowpass or a bandpass channel with an impulse response $h(t)$ which, for practical purposes, will be assumed to be of finite duration, and could be time-invariant or varies slowly with time. White Gaussian noise $n(t)$, with zero mean and two-sided power spectral density of $\frac{1}{2}N_0$, is added to the signal at the transmission path output. Although the transmission path, in practice, introduces different kinds of additive noise, the relative tolerances of different data-transmission systems to additive white Gaussian noise is a good measure of their relative tolerances to most practical types of additive noise.\(^{(23)}\) The receiver filter, with the impulse response $b(t)$, removes the noise frequencies outside the signal frequency-band without excessively bandlimiting the signal itself. It will, in fact, be assumed here that $|B(f)|$, the absolute value of the transfer function of the receiver filter, has the rectangular shape as shown in Fig. 3.3.

The transmitter filter, the transmission path and the receiver filter in cascade are assumed here to form a linear baseband channel with the impulse response $y(t)$. The linear baseband channel is defined in this work
FIGURE 3.3: Transfer Function of the Receiver Filter
as the linear channel whose output signal does not contain any carrier component, apart from the frequency offset, introduced by any part of the channel. In other words every modulation process carried out at any stage in the transmission is followed by a coherent demodulation process, the two processes in cascade having only linear effects on the transmitted signal. The resultant output signal may have a lowpass or bandpass spectrum.

The impulse response of the baseband channel (Fig. 3.2), $y(t)$, is for practical purposes of finite duration. When the transmission path has a time-invariant impulse response, so will be $y(t)$, which is then given by

$$y(t) = a(t) * h(t) * b(t)$$

(3.3)

where * represents the operation of convolution. When $h(t)$ is time-varying, then $y(t)$ will vary with time and the relationship between $y(t)$ and $a(t)$, $h(t)$ and $b(t)$ will depend on the type of the variation of $h(t)$.

The received signal at the receiver filter output is

$$r(t) = \sum_i s_i y(t-iT) + w(t)$$

(3.4)

where $w(t)$ is the noise component in $r(t)$ and is given by

$$w(t) = n(t) * b(t)$$

(3.5)

The autocorrelation function of $w(t)$ is

$$R_w(\tau) = \frac{1}{N_0} \int_{-1/2T}^{1/2T} |B(f)|^2 e^{i2\pi f\tau} df .$$

(3.6)

For $B(f)$ as given in Fig. 3.3, eqn.3.6 gives

$$R_w(\tau) = \frac{1}{N_0} \frac{\sin \frac{\tau}{T}}{\frac{\tau}{T}}$$

(3.7)

so that $w(t)$ now is a band-limited Gaussian noise with zero mean and variance $R_w(0) = \frac{1}{N_0}$ and autocorrelation function $R_w(\tau)$ as given by eqn.3.7.

The signal $r(t)$ is sampled once per signal-element at times $t=iT$, where $i$ takes on all positive integer values. The signal sample at time
instant \( jT \) is (eqn. 3.4)

\[
r(jT) = \sum_{i} s_i y((j-i)T) + w(jT)
\]

which gives the \( j^{th} \) received sample as (81)

\[
r_j = \sum_{h=0}^{g} s_{j-h} y_h + w_j
\]

where

\[
y_h = y(hT)
\]

and \( y(hT) = 0 \) for \( h < 0 \) and \( h > g \). The samples \( y_h, h = 0, \ldots, g \), form the \( (g+1) \)-component vector \( Y \) which is given by

\[
Y = y_0, y_1, \ldots, y_g
\]

The vector \( Y \) is called the sampled impulse response of the baseband channel, and it is assumed here to be known to the receiver.

The \( \{w_i\} \), in eqn. 3.9 are the samples of \( w(t) \) at times \( t = iT \), and therefore they are Gaussian random variables with zero mean and variance \( \sigma_0 \), and according to eqn. 3.7, \( w_k \) and \( w_l \), for all \( k \neq l \) are uncorrelated and therefore the \( \{w_i\} \) are statistically independent. This point is essential for the analysis to come, and the transfer function, in Fig. 3.3, has been chosen deliberately in that form to achieve the statistical independence of the \( \{w_i\} \). Of course, that objective could have been achieved by a more practical form of the transfer function of the receiver filter such as the square root of the raised-cosine spectrum, but, as will be shown in Section 3.5, the chosen transfer function here is optimum when the sampling rate of \( r(t) \) is not less than the Nyquist rate, which is usually the case for high speed data transmission.

The detector, in Fig. 3.2, operates on the \( \{r_i\} \) to produce the detected data sequence \( \{s_i\} \), which should be the same as the sequence \( \{s_i\} \) if the detection is correct.

Suppose that, on the receipt of the signal sample \( r_j \), the detector
determines the value of \( s_j \) from \( r_j \). Now, if \( y_h = 0 \) for \( h = 1, \ldots, g \), then eqn. 3.9 becomes,
\[
r_j = s_j y_0 + w_j
\]  (3.12)
and in the absence of the noise sample \( w_j \), the detector simply decides that \( s_j = r_j / y_0 \) which is a correct decision. Even in the presence of the noise sample \( w_j \), a correct decision can be made so long as \( |w_j / y_0| < 1 \), where \( s_j' \) (the detected value of \( s_j \)) takes on the nearest value, in eqn. 3.2, to \( r_j / y_0 \).

Let \( y_h \) now have a non-zero value for \( h = 0, 1, \ldots, g \), then eqn. 3.9 may be re-written as
\[
r_j = s_j y_0 + \sum_{h=1}^{g} s_{j-h} y_h + w_j
\]  (3.13)
The term \( \sum_{h=1}^{g} s_{j-h} y_h \) represents intersymbol interference, and unless the inequality
\[
|\sum_{h=1}^{g} s_{j-h} y_h + w_j| < |y_0|
\]  (3.14)
is true, the correct decision will not be obtained.

As can be seen from eqns. 3.4 and 3.9, intersymbol interference arises whenever one transmitted signal-element, at the receiver input, does not die away completely before the arrival of the next one. As can be seen from eqn. 3.14, the intersymbol interference is handled by the detector, which detects \( s_j \) as its nearest possible value to \( r_j / y_0 \), as if it were a noise, and, depending on its value, the correct decision may not be obtained even in the absence of the noise.

Several techniques are available to handle the detection of the digital data signals in the presence of intersymbol interference. These techniques may be classified into two separate groups. In the first of these an equalizer is employed to remove the intersymbol interference from \( r_j \).
in eqn. 3.13, such that the signal at the detector input becomes,

\[ e_j = s_j + u_j \]  \hspace{1cm} (3.15)

and \( u_j \) relates to \( w_j \) according to some formula depending on the equalizer.

In the second group of detection techniques, the decision process itself is modified to take account of the signal distortion (intersymbol interference), and often no attempt is made to remove or even reduce the signal distortion prior to the actual decision process.

This section is devoted to the most important detection techniques for use in the presence of intersymbol interference. A quick look will be made at the basic equalization techniques followed by a detailed description of the optimum non-linear equalizer. The optimum detector will then be defined and its most practical form, i.e., the Viterbi algorithm detector, will be presented. Finally, near-optimum but simpler detection processes will be described, and in particular, the near-maximum likelihood detection processes, which form the basis of the techniques developed here.

3.2 EQUALIZATION TECHNIQUES

3.2.1 Linear Equalizers

Let the vectors \( S_{n_1} \) and \( R_{n_2} \) be:

\[ S_{n_1} = s_1, s_2, \ldots, s_{n_1} \]  \hspace{1cm} (3.16)

\[ R_{n_2} = r_1, r_2, \ldots, r_{n_2} \]  \hspace{1cm} (3.17)

where \( s_i \) and \( r_i \) are given by eqns. 3.2 and 3.9, respectively, and the vector \( S_{n_1} \) represents the transmitted data sequence whereas the vector \( R_{n_2} \) represents the received signal sequence (at the sampler output, Fig. 3.2). As is clear from eqn. 3.9, the vector \( R_{n_2} \), in the absence of noise, is obtained by convolving the vector (sequence) \( S_{n_1} \) with the vector (sequence) \( Y \)
in eqn. 3.11. Consequently, the $z$-transform of the received sequence $R_n$ in the absence of noise is \(81\)

$$R(z) = S(z)Y(z)$$ \hspace{1cm} (3.18)

where

$$R(z) = \sum_{i=1}^{n_2} r_i z^{-i}$$ \hspace{1cm} (3.18a)

$$S(z) = \sum_{i=1}^{n_1} s_i z^{-i}$$ \hspace{1cm} (3.18b)

and

$$Y(z) = \sum_{i=0}^{g} y_i z^{-i}$$ \hspace{1cm} (3.18c)

As eqn. 3.18 suggests, the intersymbol interference is removeable by passing the received sequence $R_n$ through a linear filter with $z$-transform $Y^{-1}(z)$. This gives,

$$R(z)Y^{-1}(z)z^{-h} = S(z)z^{-h}$$ \hspace{1cm} (3.19)

where the factor $z^{-h}$ allows for the delay likely to be introduced by the linear filter and has no effect, apart from the delay, on the signal detection. The linear filter with the $z$-transform $Y^{-1}(z)$ may be implemented as a feedforward transversal equalizer and under the appropriate conditions as a feedback equalizer. \(81\)

Fig. 3.4 shows the linear feedback transversal equalizer for the signal sequence $\{r_i\}$ and the sampled impulse response $Y$. The signal sample at the output of the equalizer at time $t=jT$ is:

$$\epsilon_j = \frac{r_j}{y_0} - \sum_{h=1}^{g} \frac{y_h}{y_0} \epsilon_{j-h} \hspace{1cm} (3.20)$$

assuming, of course, that $y_0 \neq 0$. From eqn. 3.13, and in the absence of the noise, $\epsilon_j$ will equal $s_j$ (when $e_i = s_i$ for $j-gx<i$), so that the linear feedback equalizer in cascade with the baseband channel and sampler in Fig. 3.2 has an overall sampled impulse response of the form,

$$1.0 \hspace{0.5cm} 0.0 \hspace{0.5cm} 0.0 \hspace{0.5cm} 0.0 \ldots$$
\[ e_j = \frac{1}{y_0} (r_j - \sum_{h=1}^{g} e_{j-h} y_h) \]

**FIGURE 3.4:** Linear Feedback Transversal Equalizer

\[ \frac{1}{y_0} \sum_{h=1}^{g} e_{j-h} y_h \]

**FIGURE 3.5:** Linear Feedforward Transversal Equalizer
i.e. there is no intersymbol interference.

The problem with the linear feedback equalizer is that it may become unstable in the presence of noise or of any inaccuracies in the tap gains if $Y(z)$ (eqn.3.18c) has roots on or outside the unit circle in the $z$-plane. (81)

When all the roots of $Y(z)$ lie inside the unit circle, the linear feedback equalizer is always stable, and gives exact equalization of the linear distortion introduced by the baseband channel in Fig. 3.2.

Approximate equalization of the channel with the $z$-transform $Y(z)$ may be obtained using a linear feedforward equalizer, whether the roots of $Y(z)$ lie inside or outside, but not on, the unit circle in the $z$-plane. (81) The equalization may be made as accurate as required. Let the sampled impulse response of the linear feedforward equalizer in Fig. 3.5 be:

$$C = c_0, c_1, \ldots, c_R \quad (3.21)$$

with the $z$-transform,

$$C(z) = \sum_{i=0}^{L} c_i z^{-i} \quad (3.22)$$

The signal sample at the output of the filter at time $t=jT$ is:

$$e_j = \sum_{h=0}^{L} r_{j-h} c_h \quad (3.23)$$

so that the $z$-transform of the sequence $\{e_j\}, E(z)$ is (81)

$$E(z) = R(z)C(z) \quad (3.24)$$

where $R(z)$ is given by eqn. 3.18a. From eqn. 3.18, in the absence of noise, eqn. 3.24 becomes,

$$E(z) = S(z)Y(z)C(z) \quad (3.25)$$

or

$$E(z) = S(z)E(z) \quad (3.26)$$

where

$$E = e_0, e_1, \ldots, e_{g+2} \quad (3.27)$$

is the sampled impulse response of the baseband channel and sampler in Fig. 3.2 in cascade with the linear equalizer, and $E(z)$ is the $z$-transform
of the sequence $E$. Clearly,

$$ E(z) = Y(z)C(z) \quad (3.28) $$

For the exact equalization of the channel, $E$ must have only one non-zero component, with value 1.0. Let that component be $e_h$. This implies that, in eqn. 3.28, $C(z)$ must be such that:

$$ C(z) = z^{-h}Y^{-1}(z) \quad (3.29) $$

Since the equalizer now has a non-recursive nature, and to satisfy eqn. 3.29, the number of taps in the equalizer, i.e. $(t+1)$ in eqn. 3.21, is in general, infinite. In practice, this number must be restricted to a finite value, and consequently $C(z)$ can not satisfy eqn. 3.29 exactly, but it can be chosen to do so approximately. Consequently, eqn. 3.29 becomes,

$$ C(z) = z^{-h}Y^{-1}(z) \quad (3.30) $$

where \( \approx \) means approximately equal.

Now, for the chosen $C(z)$, $E$ in eqn. 3.27 may have components with non-zero amplitude beside the main component whose value is 1.0. The presence of these components represents residual distortion in the equalized signal.

Various techniques are known to determine $C(z)$, i.e. the tap gains of the linear feedforward equalizer, and different criteria are used to minimize the residual distortion in the equalized signal. Let $e_h$, in eqn. 3.27, be the main component of $E$ with any non-zero value. The remaining components of $E$ may or may not be zero. The peak distortion in the equalized signal, when a binary signal is transmitted ($m=2$), is defined to be,

$$ D_p = \frac{1}{|e_h|} \sum_{i=0}^{g+t} |e_i| \quad , \quad (3.31) $$

the mean-square distortion is:

$$ D_m = \frac{1}{e_h^2} \sum_{i=0}^{g+t} e_i^2 \quad (3.32) $$

and the mean-square error (due to intersymbol interference) is:
In Ref. (81), different methods of designing a linear feedforward equalizer are given, where the minimization of one of the three quantities just defined is the design criterion.

From eqns. 3.26, 3.28 and 3.30 we have

\[ \mathbb{S}(z) = S(z)z^{-h} \]  

which gives at time \( t = (j+h)T \), at the input of the detector,

\[ \varepsilon_{j+h}^j = s_j \]  

In the presence of noise, from eqns. 3.9 and 3.23, the noise component in \( \varepsilon_j \) is \( u_j \), where

\[ u_j = \sum_{i=0}^{l} w_{j-i} c_i \]  

and consequently, eqn. 3.35 becomes in the presence of noise,

\[ \varepsilon_{j+h}^j = s_j + u_{j+h} \]  

Since the \( \{w_i\} \) in eqn. 3.36 are statistically independent Gaussian random variables with zero mean and variance \( \sigma^2 \), the \( \{u_i\} \) are Gaussian random variables with zero mean and variance (81)

\[ n^2 = \sigma^2 \sum_{i=0}^{l} c_i^2 = \sigma^2 |C|^2 \]  

where \( |C| \) is the length of the vector \( C \). Eqn. 3.38 shows that the linear equalizer changes the variance of the noise and enhances it if \( |C|^2 > 1 \), the latter being the case when the channel introduces amplitude distortion into the transmitted signal. (81)

According to eqn. 3.37, the detector takes, as the detected value of \( s_j \), its possible value (eqn. 3.2) that is nearest to \( \varepsilon_{j+h}^j \). An error occurs in the detection of \( s_j \) whenever the noise component \( u_{j+h} \) is such that \( \varepsilon_{j+h}^j \) is closer to a possible value of \( s_j \) different from its correct value, that is, whenever the noise component \( u_{j+h} \) carries \( \varepsilon_{j+h}^j \) onto the opposite side of
a decision threshold with respect to the transmitted \( s_j \). 

Since \( u_{j+h} \) (eqn. 3.36) is a sample value of a Gaussian random variable with zero mean and variance \( \eta^2 \) (eqn. 3.38), its probability density function is given by

\[
p(u) = \frac{1}{(2\pi)^{\frac{1}{2}} \eta} \exp\left(-\frac{u^2}{2\eta^2}\right) \tag{3.39}
\]

Now, when \( s_j = m-1 \) and \( u_{j+h} < -1 \), an error occurs in the detection of \( s_j \) with probability

\[
P_{e_1} = \int_{-\infty}^{-1} \frac{1}{(2\pi)^{\frac{1}{2}} \eta} \exp\left(-\frac{u^2}{2\eta^2}\right) \, du = \int_{-\infty}^{-1} \frac{1}{(2\pi)^{\frac{1}{2}}} \exp\left(-\frac{u^2}{2\eta^2}\right) \, du = Q\left(\frac{1}{\eta}\right),
\tag{3.40a}
\]

and when \( s_j = -(m-1) \) and \( u_{j+h} > 1 \), an error occurs in the detection of \( s_j \) with probability

\[
P_{e_2} = \int_{1}^{\infty} \frac{1}{(2\pi)^{\frac{1}{2}} \eta} \exp\left(-\frac{u^2}{2\eta^2}\right) \, du = \int_{1}^{\infty} \frac{1}{(2\pi)^{\frac{1}{2}}} \exp\left(-\frac{u^2}{2\eta^2}\right) \, du = Q\left(\frac{1}{\eta}\right).
\tag{3.40b}
\]

When \( s_j \) has one of its possible values other than \((m-1)\) or \(-(m-1)\) an error occurs whenever \(|u_{j+h}| > 1\), whether \( u_{j+h} \) is positive or negative. Thus, the probability of an error in the detection of \( s_j \) is now

\[
P_{e_3} = \int_{-\infty}^{-1} \frac{1}{(2\pi)^{\frac{1}{2}} \eta} \exp\left(-\frac{u^2}{2\eta^2}\right) \, du + \int_{1}^{\infty} \frac{1}{(2\pi)^{\frac{1}{2}}} \exp\left(-\frac{u^2}{2\eta^2}\right) \, du = 2Q\left(\frac{1}{\eta}\right).
\tag{3.40c}
\]

Since \( s_j \) is equally likely to have any of its \( m \) possible values, the average probability of error in the detection of \( s_j \) is

\[
P_e = \frac{1}{m} (P_{e_1} + P_{e_2} + (m-2)P_{e_3})
= \frac{2(m-1)}{m} Q\left(\frac{1}{\eta}\right)
= \frac{2(m-1)}{m} Q\left(\frac{1}{\sigma |C|}\right).
\tag{3.41}
\]
where \( \eta \) has been replaced by \( \sigma |C| \), according to eqn. 3.38. In eqns. 3.40a-3.41, \( Q(.) \) is given by

\[
Q(x) = \int_{-\infty}^{\infty} \frac{1}{x} \exp\left(-\frac{u^2}{2}\right) du
\]

From eqn. 3.41, it is clear that \( P_e \) increases with \( |C| \). When the channel introduces pure phase distortion, the linear equalizer performs pure phase equalization, where \( |C|=1 \), and the linear equalizer achieves the best available tolerance to additive white Gaussian noise.\(^{(81)}\)

### 3.2.2 Non-Linear Equalizers

Unlike linear equalizers, non-linear equalizers use the detected data symbols \( \{s_i\} \) to remove some of the intersymbol interference from the received signal in a process of decision directed cancellation, and because of their non-linear structure they may equalize channels whose z-transforms \( \{Y(z)\} \) may have their roots anywhere in the z-plane, and sometimes may achieve a much better tolerance to additive noise than linear equalizers.\(^{(81)}\)

Fig. 3.6a shows the pure non-linear (decision feedback) equalizer for the signal sequence \( \{r_i\} \) and the intersymbol interference defined by eqns. 3.13 and 3.11, respectively. On the assumption that the \( \{s_i\} \), for \( i=j-g, \ldots, j-1 \), have been correctly detected and from eqn. 3.13, the equalized signal sample \( e_j \) at time \( t=jT \) at the detector input is

\[
e_j = \frac{1}{y_0} \left[ r_j - \sum_{h=1}^{g} e_j^{h} y_h \right] = s_j + \frac{w_j}{y_0}
\]

assuming that \( y_0 \neq 0 \). The detector in Fig. 3.6a operates on \( e_j \) and takes as \( s_j \), the detected value of \( s_j \), its possible value (eqn.3.2) closest to \( e_j \). In eqn. 3.43, the noise component is now \( \frac{w_j}{y_0} \), and since the \( \{w_i\} \) are Gaussian random variables with zero mean and variance \( \sigma^2 \), the \( \{w_i/y_0\} \) are the same but with variance \( \sigma^2/y_0^2 \). By following the same argument in
(a) A pure non-linear equalizer for the detection of $s_j$ from $r_j$

(b) A pure non-linear equalizer for the detection of $s_j$ from $r_{j+n}$

**FIGURE 3.6:** Pure Non-Linear Equalizer
determining the probability of error $P_e$ in eqn. 3.41, it may be shown that the probability of an error in the detection of $s_j$ from $e_j$ (eqn. 3.43) (following the correct detection of all $\{s_{j-1}\}$, $i=1,2,\ldots,g$) is now given by

$$P_e = \frac{2(m-1)}{m} \int_{1}^{\infty} \frac{1}{(2\pi)^{\frac{1}{2}}|y_0|^\frac{1}{2}} \exp\left(-\frac{w^2}{2y_0}\right) dw$$

$$= \frac{2(m-1)}{m} Q\left(\frac{|y_0|}{\sigma}\right) \quad (3.44)$$

where $Q(.)$ is given by eqn. 3.42.

When a received signal element is incorrectly detected, its inter-symbol interference in the following elements, instead of being eliminated, is enhanced, and this greatly increases the probability of error in their detection. Errors therefore tend to occur in bursts, and the system suffers from error-extension effects.

Depending on the type of distortion introduced by the channel (for a given average transmitted signal energy) the value of $y_0$ may vary and it may very well be that $|y_0|<|y_n|$ for $i=0,1,\ldots,n-1$. In such a case the tolerance of the equalizer to additive Gaussian noise (eqn. 3.44) is highly degraded, and it may be much better now to detect $s_j$ from $r_{j+n}$ instead of $r_j$.

The arrangement of the equalizer in this case is shown in Fig. 3.6b.

Following the correct evaluation of $s_{j-1}$, $i=1,\ldots,g-n$, the signal sample at the detector input is:

$$e_{j+n} = s_j + \frac{1}{y_n} \sum_{h=0}^{n-1} s_{j+n-h} y_h + w_{j+n}/y_n \quad (3.45)$$

Now, the term $\frac{1}{y_n} \sum_{h=0}^{n-1} s_{j+n-h} y_h$ is considered as a noise, but since $|y_j|<|y_n|$ for $i=0,1,\ldots,n-1$, the detection of $s_j$ from $r_{j+n}$ gives a better tolerance to additive noise.

Eqn. 3.45 suggests that even a further improvement in the tolerance to noise may be obtained with this arrangement by adding a linear filter at
the input to the decision feedback equalizer, to remove the intersymbol interference component in $e_{j+n}$ that is not eliminated by the non-linear equalizer. The arrangement then looks as shown in Fig. 3.7, and represents a conventional non-linear equalizer which now contains a linear filter in cascade with a pure nonlinear equalizer (decision feedback equalizer). The channel is now equalized partly by the linear filter and partly by the non-linear filter, the two filters together achieving the accurate equalization of the channel. Under these conditions, the equalization of the channel may be achieved by an infinite number of different combinations of linear and non-linear filters. Thus there must be one or more particular combinations of the two filters that give the best tolerance to additive white Gaussian noise, and consequently give the optimum non-linear equalizer.

\subsection*{3.2.3 The Optimum Non-Linear Equalizer

Let the linear filter in Fig.3.7 have $k$ taps and a sampled impulse response

$$D = d_0, d_1, \ldots, d_{k-1} \quad (3.46)$$

with the z-transform

$$D(z) = \sum_{i=0}^{k-1} d_i z^{-i} \quad (3.47)$$

Also let

$$D(z) = C(z)E(z) \quad (3.48)$$

where

$$E(z) = \sum_{i=0}^{k-k} e_i z^{-i} \quad (3.49)$$

and $C(z)$ is the z-transform of the k-tap linear equalizer for the baseband channel with z-transform $Y(z)$, such that

$$C(z) = z^{-h_0} Y^{-1}(z) \quad (3.50)$$

The z-transform of the channel and linear filter is:

$$Y(z)D(z) = Y(z)C(z)E(z) \quad (3.51)$$

and from eqn. 3.50:
Figure 3.7: Non-Linear Equalizer
\[ Y(z)D(z) = z^{-h}E(z) \]  
(3.52)

so that the sampled impulse response of the channel and linear filter, neglecting the delay of \(hT\) seconds, is given approximately by the \((\ell-k+1)\)-component row vector

\[ E = e_0, e_1, \ldots, e_{\ell-k} \]  
(3.53)

where

\[ e_0 = 1 \]  
(3.53a)

and the remaining \(\{e_i\}\) may be determined in the optimization process.

The non-linear filter in Fig. 3.7 now equalizes \(E\). The details of the system are shown in Fig. 3.8, where the signals are given at time \(t=(j+h)T\).

A delay of \(hT\) seconds is assumed and at time \(t=(j+h)T\), the data symbol \(s_j\) is detected. Clearly, the linear feedforward transversal filter that forms part of the non-linear filter has \(\ell-k\) taps whose gains are equal respectively to the \(\{e_i\}\) in eqn. 3.53, for \(i=1,2,\ldots,\ell-k\). The signal sample at the output of the linear filter at time \(t=(j+h)T\) is:

\[ r_{j+h}^i = \sum_{i=0}^{\ell-1} r_{j+h-i}d_i \]  
(3.54)

which, according to eqns. 3.9, 3.48, 3.50 and 3.52, can be written as:

\[ r_{j+h}^i = s_j + \sum_{i=1}^{\ell-k} s_j-i e_i + u_{j+h} \]  
(3.55)

where \(e_0=1\) (eqn. 3.53a) and

\[ u_{j+h} = \sum_{i=0}^{\ell-1} w_{j+h-i}d_i \]  
(3.56)

is the noise component in \(r_{j+h}^i\). Since the \(\{w_i\}\) are statistically independent Gaussian random variables with zero mean and variance \(\sigma^2\), the \(\{u_i\}\) are Gaussian random variables with zero mean and variance \(\eta^2\)

\[ \eta^2 = \sigma^2 \sum_{i=0}^{\ell-1} d_i^2 = \sigma^2 |D|^2 \]  
(3.57)

where \(|D|\) is the length of the vector \(D\).

Following the correct detection of \(\{s_{j-i}^i\}, i=1,2,\ldots,(\ell-k)\), the signal
at the detector input at time $t=(j+h)T$ is (Fig. 3.8 and eqn. 3.55)

$$
\varepsilon_{j+h} = r_{j+h}^{r} - \sum_{i=1}^{k} s_{j-i}^{s} e_{i}^{e}
$$

$$
= s_{j}^{s} + u_{j+h}^{u}
$$

(3.58)

Now, the detector takes as the detected data-symbol $s_{j}^{S}$ the possible value of $s_{j}$ (eqn. 3.2) nearest to $\varepsilon_{j+h}$. Thus, the probability of error $\varepsilon_{j}$ in the detection of a received data-symbol, following the correct detection of the preceding $i-k$ data-symbols, may be shown to be

$$
p_{e} = \frac{2(m-1)}{m} Q\left(\frac{1}{\sigma|D|}\right)
$$

(3.59)

where $Q(.)$ is given by eqn. 3.42.

Neglecting the error extension effects, it is clear from eqn. 3.59 that to minimise the error rate in the detected element values $\{s_{i}^{S}\}$, it is necessary to minimise $|D|$. But since eqn. 3.59 has been given under the constraint imposed by eqns. 3.48, 3.52, 3.53 and 3.58, which imply the accurate equalization of the channel, the minimization of $|D|$ must, of course, be subject to that constraint.

Let the $z$-transform of the channel be factorized into two factors, $X_{1}(z)$ and $X_{2}(z)$:

$$
Y(z) = X_{1}(z)X_{2}(z)
$$

(3.60)

where all the roots of $X_{1}(z)$ lie inside or on the unit circle in the $z$-plane and all the roots of $X_{2}(z)$ lie outside the unit circle. Let also $X_{3}(z)$ be such that its roots are the complex conjugates of the reciprocals of those of $X_{2}(z)$, i.e. all the roots of $X_{3}(z)$ lie inside the unit circle.

It is shown in Ref. (81) that the optimum choice of $D$, which minimizes its length $|D|$ subject to the accurate equalization of the channel is:

$$
D(z) = az^{-h}X_{2}^{-1}(z)X_{3}(z)
$$

(3.61)
FIGURE 3.8: The Non-Linear Equalizer
where a is a constant and its value is such that eqn. 3.53a is satisfied.

The z-transform of the channel and linear filter is now:

\[ Y(z)D(z) = z^{-h}aY_1(z)Y_3(z) \]  

(3.62)

Comparing eqn. 3.62 with eqn. 3.52 results in

\[ E(z) = aY_1(z)Y_3(z) \]  

(3.63)

Eqn. 3.63 simply says that the optimum choice of the linear filter is that which replaces all the roots of the z-transform of the channel which lie outside the unit circle by the complex conjugate of their reciprocals.

The resultant sampled impulse response of the linear baseband channel, sampler and linear filter is given by the vector E, which represents now a minimum-phase sequence.

In Ref. (81), it is also shown that D(z) in eqn. 3.61 has the following properties:

1. It performs a pure phase equalization with the result that the noise samples \[ \{u_i\} \] at its output are statistically independent if the samples \[ \{w_i\} \] at its input are statistically independent.

2. The length of D is equal to \( |a| \):

\[ |D| = |a| \]  

(3.64)

with the result that the variance of the noise samples \[ \{u_i\} \] is \( \sigma^2a^2 \), according to eqn. 3.57.

3. Since this linear filter performs a pure phase equalization, i.e. orthogonal transformation, it does not change the signal-to-noise ratio, i.e. the signal-to-noise ratio at its output is equal to that at its input. But since \( |D| \) now is minimized, subject to the accurate equalization of the channel, and according to eqns. 3.58 and 3.59, the signal-to-noise ratio at the detector input is maximised. This means that the first non-
zero component of $E$, the sampled impulse response of the channel and the linear filter, is maximised relative to the other components in $E$, which implies that the energy of the transmitted signal-element, which is spread over the components of $Y$, is now shifted towards the first component of $E$. The very important subsequent result of this is that $E$ now contains less intersymbol interference than $Y$.

When the baseband channel introduces pure phase distortion, the optimum non-linear equalizer degenerates into a pure linear equalizer, which now achieves the best available tolerance to additive Gaussian noise. Again, when all the roots of $Y(z)$ lie on or inside the unit circle, the optimum non-linear equalizer becomes a pure non-linear equalizer, as shown in Fig. 3.6.

3.3 THE OPTIMUM DETECTOR

From the method of operation of the non-linear equalizer, it can be seen that, except in the particular case where the channel introduces pure phase distortion, only a portion of the received signal-element energy is used in the detection of that element, the remaining part being eliminated by the intersymbol interference cancellation process. Furthermore, the non-linear equalizer, because of the process of decision directed cancellation of intersymbol interference, suffers from the error extension effects which become more serious both as the number of the large components of $E$ and the number of signal levels $m$ (eqn. 3.2) increase.

Since the successive received signal samples $\{r_j\}$ contain intersymbol interference from successive data symbols $\{s_i\}$, i.e. $r_j$ contains information about $s_j$ as well as $s_{j-h}, h=1,2,\ldots,g$ (eqn. 3.9), a detector
which operates simultaneously on a group of the \{r_i\} in such a way that the intersymbol interference is involved in the joint detection of the corresponding \{s_i\} instead of being eliminated, may achieve a better tolerance to additive white Gaussian noise than the optimum equalizer.\(^{(81)}\)

Let the discrete-time model of the baseband channel and sampler in Fig. 3.2 be represented by the feedforward transversal filter whose tap gains are given by the vector \(Y\):

\[
Y = y_0, y_1, \ldots, y_g
\]  

(3.65)

as shown in Fig. 3.9, and let the \(N\)-component vector \(S_N\) represent the whole sequence of transmitted signal elements in one message,

\[
S_N = s_1, s_2, \ldots, s_N
\]  

(3.66)

where, as before, \(s_i\) may have \(m\) possible values:

\[
s_i = 2^{\ell-m} + 1, \quad \ell = 0, 1, \ldots, m-1
\]  

(3.67)

Also, let the vector \(R_N\) represent the whole sequence of received signal samples corresponding to the same message:

\[
R_N = r_1, r_2, \ldots, r_N
\]  

(3.68)

where

\[
r_j = \sum_{h=0}^{g} s_j y_{h} + w_j
\]  

(3.69)

and the \(\{w_j\}\) are random, but not necessarily Gaussian, variables and represent the noise components in the \(\{r_j\}\).

Given that the whole message has been received, i.e., the vector \(R_N\) is known to the detector, before the start of the detection process, i.e., determining the vector \(S_N\), two main definitions of the optimum detector may be encountered in the literature: \(^{(81, 84-90)}\)

1. Minimum error probability; the optimum detector is defined here as that which minimizes the probability of error in the detection of the whole received message.\(^{(85)}\) Let \(X_{N,i}\) be the \(N\)-component vector given by the \(i^{th}\)
FIGURE 3.9: Discrete-time Model of the Data Transmission System in Fig. 3.2.
of the $m^N$ possible values of $S_N$ (eqn. 3.66). The optimum detector computes the $m^N$ a posteriori probabilities:

$$P(S_N = X_{N,i} | R_N) \quad i=1,2,\ldots,m^N$$

corresponding to the $m^N$ different possible values $\{X_{N,i}\}$ of $S_N$, and accepts the hypothesis $S_N = X_{N,j}$ if

$$P(S_N = X_{N,j} | R_N) > P(S_N = X_{N,i} | R_N), \quad i=1,2,\ldots,m^N, \quad i \neq j. \quad (3.70)$$

Using Bayes' theorem, eqn. 3.70 may be written as

$$P(S_N = X_{N,j} | R_N) p(R_N | S_N = X_{N,j}) > P(S_N = X_{N,i} | R_N) p(R_N | S_N = X_{N,i})$$

$$i=1,2,\ldots,m^N, \quad i \neq j \quad (3.71)$$

where $P(S_N = X_{N,i} | R_N)$ is the a priori probability that $S_N = X_{N,i}$, $p(R_N | S_N = X_{N,i})$ is the conditional probability density function of $R_N$ (at the given value $R_N$), given that the vector $X_{N,i}$ has been transmitted. When all the $m^N$ possible values of $S_N$ (eqn. 3.66) are equally likely, then eqn. 3.71 becomes

$$p(R_N | S_N = X_{N,j}) > p(R_N | S_N = X_{N,i}), \quad i=1,2,\ldots,m^N, \quad i \neq j \quad (3.72)$$

and the detector now accepts the hypothesis $S_N = X_{N,j}$ for which the probability density function of $R_N$ is maximum. This is a process of maximum likelihood detection.

2. Minimum probability of error; the optimum detector is defined here as that which minimizes the probability of error in the detection of an individual received data-symbol (87-89) and hence minimizes the average digit-error rate over all received data-symbols (81) when these are statistically independent. The optimum detector now computes, for every component $s_i$ of $S_N$ the $m$ a posteriori probabilities $P(s_i = b_j | R_N)$ for $j=1,2,\ldots,m$, where $b_j$ is one of the $m$ possible values of $s_i$ (eqn. 3.67) and accepts the hypothesis $s_i = b_j$ if
Based on either of the two criteria, several optimum and sub-optimum detection systems have been described,(85-89) but the complexity of such systems is very high and the analytical evaluation of their performance is very difficult.(90) A relatively simple and realizable detector is the maximum-likelihood detector, which, using a recursive algorithm known as the Viterbi algorithm,(90) gives a maximum-likelihood estimate of $S_N$ without need to evaluate the total of $m^N$ conditional probability densities as suggested by eqn. 3.72. Of course, maximum likelihood detection (eqn. 3.72) is not designed to minimize the probability of error in the detection of a data-symbol. However, at high signal-to-noise ratios and with white Gaussian noise, maximum-likelihood detection approximates to this and minimizes the average probability of error in the detection of an individual data-symbol.(81)

### 3.4 THE VITERBI ALGORITHM \(^{(90,93)}\)

Let $S_N, R_N$ and $W_N$ be the $N$-component vectors

$$S_N = s_1, s_2, \ldots, s_N \tag{3.74}$$

$$R_N = r_1, r_2, \ldots, r_N \tag{3.75}$$

and

$$W_N = w_1, w_2, \ldots, w_N \tag{3.76}$$

where $s_i$ may have one of the $m$ possible values given by eqn. 3.2,

$$r_i = \sum_{h=0}^{g} s_i - h y_h + w_i \tag{3.77}$$

and the $\{w_i\}$ are statistically independent random variables (not necessarily Gaussian) with zero mean and a fixed variance. The $\{y_h\}$
(eqn. 3.77), given by the \((g+1)\)-component vector,

\[
y = y_0, y_1, y_2, \ldots, y_g
\]  \hspace{1cm} (3.78)

represent the sampled impulse response of the linear baseband channel (Fig. 3.2) whose discrete-time model is shown in Fig. 3.9.

Let also \(X_N\) and \(Z_N\) be the \(N\)-component vectors

\[
X_N = x_1, x_2, \ldots, x_N
\]  \hspace{1cm} (3.79)

and

\[
Z_N = z_1, z_2, \ldots, z_N
\]  \hspace{1cm} (3.80)

where \(x_i\) may have one of the \(m\) possible values of \(s_i\) (eqn. 3.74) and

\[
z_i = \sum_{h=0}^{g} x_{i-h} y_h
\]  \hspace{1cm} (3.81)

When \(X_N\) is taken at the receiver as an estimate of the vector \(S_N\) (the transmitted data sequence) then \(z_i\) is an estimate of the signal component in \(r_i\) (eqn. 3.77). Clearly there are \(m^N\) possible vectors \(\{X_N\}\), and the one of these for which \(p(R_N|S_N = X_N)\) (eqn. 3.72) is maximum, is the maximum likelihood estimate of \(S_N\). \(p(R_N|S_N = X_N)\) is the conditional probability density function of \(R_N\) (at the given value \(R_N\)), given that \(S_N\) is equal to the corresponding \(X_N\).

Now, and according to eqn. 3.81, every vector \(X_N\) is associated with a vector \(Z_N\). The vector \(Z_N\), corresponding to the maximum-likelihood vector \(X_N\), is also a maximum likelihood estimate of the signal component in \(R_N\). Thus, the maximization of \(p(R_N|S_N = X_N)\) is equivalent to the maximization of the corresponding \(p(R_N|Z_N)\), which is the conditional probability density function of \(R_N\) given that \(Z_N\) is equal to the signal component in \(R_N\). Under these conditions,

\[
p(R_N|S_N = X_N) = p(R_N|Z_N)
\]

\[
= p(R_N-Z_N)
\]

\[
= p(W_N)
\]  \hspace{1cm} (3.82)

where \((R_N-Z_N)\) is the possible value of the noise component in \(R_N\), given
$Z_N$, i.e. a possible value of the vector $W_N$, according to eqns. 3.75-3.77.

Since the $\{w_i\}$, the components of $W_N$ are assumed to be statistically independent, $p(W_N)$ is given by

$$p(W_N) = p(w_1, w_2, \ldots, w_N) = p_w(w_1)p_w(w_2)\cdots p_w(w_N)$$

$$= \prod_{i=1}^{N} p_w(w_i) \tag{3.83}$$

where $p_w(w_i)$ is the probability density function of $w_i$. Thus eqn. 3.82 becomes

$$p(R_N | Z_N) = \prod_{i=1}^{N} p_w(r_i - z_i) \tag{3.84}$$

or equivalently,

$$p(R_N | S_N = X_N) = \prod_{i=1}^{N} p_w(r_i - \frac{g}{h} \sum_{h=0}^{i} x_{i-h} y_h) \tag{3.85}$$

where $z_i$ has been replaced by its value in eqn. 3.81.

Let the $k$-component vector $X_k$ be an initial segment of the vector $X_N$, where $k < N$, and let the $g$-component vector $Q'_k$ be given by the last $g$ components of $X_k$,

$$Q'_k = x_{k-g+1}, x_{k-g+2}, \ldots, x_k \tag{3.86}$$

$X_N, X_k$ and $Q'_k$ are shown on the following diagram:

```
  \[X_N\]
   \[X_k\]
  i=1
\[Q'_k\]
   \[i=k\]
```

Now, eqn. 3.85 may be written as

$$p(R_N | S_N = X_N) = \prod_{i=1}^{k} p_w(r_i - \frac{g}{h} \sum_{h=0}^{i} x_{i-h} y_h) \prod_{i=k+1}^{N} p_w(r_i - \frac{g}{h} \sum_{h=0}^{i} x_{i-h} y_h) \tag{3.87}$$

where

$$p(k_1, k_2) = \prod_{i=k_1}^{k_2} p_w(r_i - \frac{g}{h} \sum_{h=0}^{i} x_{i-h} y_h) \tag{3.88}$$
Supposing now that $Q_k'$ were known to the receiver, then the maximization of $p(R_N | S_N = X_N)$ over all vectors $\{X_N\}$ in which $Q_k'$ is a common part, may be achieved by maximising both of $p(1,k)$ and $p(k+1,N)$ independently. That is because the common $\{x_i\}$ to both $p(1,k)$ and $p(k+1,N)$ are given by $Q_k'$ and therefore fixed. Thus, the vector $X_k$ for which $p(1,k)$ is maximum, given the vector $Q_k'$, is definitely a part of the maximum likelihood vector $X_N$, the detected value of $S_N$.

The detector of course does not know $Q_k'$, but it knows that $Q_k'$ may definitely have only $m^g$ different values. Thus, for any value of $k$, it can be decided that there are only $m^g$ vectors $\{X_k\}$ which may be candidates to form a part of the maximum likelihood vector $X_N$, each of these $\{X_k\}$ is of course associated with a maximized value of $p(1,k)$, given the corresponding $Q_k'$. These vectors $\{X_k\}$ are called survivors, and each of them has a different vector $Q_k'$ as an end segment. In other words, a vector of the $m^g$ possible vectors $\{Q_k'\}$ can not be a common part in two survivors $\{X_k\}$.

Assume now, that the detector is able to obtain the $m^g$ survivors $\{X_k\}$, each being associated with the corresponding $p(1,k)$. For the $m$ different possible values of $x_{k+1}$, every $X_k$ may give $m$ vectors $\{X_{k+1}\}$, each having a conditional probability density

$$p(1,k+1) = \prod_{i=1}^{k+1} p_w (r_i - \sum_{h=0}^{g} x_{i-h} y_h)$$

$$= p(1,k)p_w (r_{k+1} - \sum_{h=0}^{g} x_{k+1-h} y_h)$$

(3.89)

using of course the corresponding value of $p(1,k)$. Clearly, there are now $m^{g+1}$ vectors $\{X_{k+1}\}$. But as mentioned before, only $m^g$ of these $\{X_{k+1}\}$ may be candidates to form a part of the maximum likelihood vector $X_N$, each of the candidates having, of course, a different vector $Q_{k+1}'$ (out of $m^g$ possible vectors $\{Q_{k+1}'\}$) as an end segment. Now, as a result of
expanding the $m^g$ vectors $\{x_k\}$ into the $m^{g+1}$ vectors $\{x_{k+1}\}$, there are $m^{g+1}$ vectors $Q'_{k+1}$, each being the end segment of a different vector $x_{k+1}$. But since there are only $m^g$ different possible vectors $\{Q'_{k+1}\}$, then there are $m^g$ groups of vectors $\{x_{k+1}\}$, all vectors in a group having the same $Q'_{k+1}$ as an end segment. Of course a group contains $m$ vectors $\{x_{k+1}\}$. Consequently, the required $m^g$ vectors $\{x_{k+1}\}$ may be obtained by selecting one vector from each of the $m^g$ groups (of $m$ vectors $\{x_{k+1}\}$), the selected vector having the maximum value of $p(1,k+1)$ in the corresponding group.

Having obtained the $m^g$ vectors $\{x_{k+1}\}$, these are now used together with their corresponding quantities $\{p(1,k+1)\}$ to determine the $m^g$ vectors $\{x_{k+2}\}$ by following the same steps, and the recursion continues in that manner. Clearly, to start the process it is not necessary to wait until the whole message has been received. In other words, the recursion may start as soon as the first signal element is received. The process may now be summarised as follows. At time $t=kT$, the receiver holds in store $m^g$ vectors $\{x_k\}$ (eqn. 3.79) each being associated with a probability density $p(1,k)$ (eqn. 3.88). On the receipt of the signal sample $r_{k+1}$, the receiver expands each vector $x_k$ into $m$ vectors $\{x_{k+1}\}$, where the first $k$ components in a vector $x_{k+1}$ are given by the corresponding $x_k$, and $x_{k+1}$ in the $m$ vectors $\{x_{k+1}\}$ takes on its $m$ different possible values. For each of the $m^{g+1}$ resultant vectors $\{x_{k+1}\}$, the detector determines the value of $p(1,k+1)$, according to eqn. 3.89, using of course, the value of $p(1,k)$ corresponding to $x_k$, from which the given $x_{k+1}$ has originated. Now, the detector selects from each group of $m$ vectors $\{x_{k+1}\}$, whose last $g$ components are given by the same vector $Q'_{k+1}$, the vector with the maximum value of $p(1,k+1)$, giving a total of $m^g$ selected vectors $\{x_{k+1}\}$, which are stored together with their conditional probability densities $\{p(1,k+1)\}$ in preparation for the next
recursion cycle on the receipt of the signal sample \( r_{k+2} \). At the end of
the message, the detector holds in store \( m^g \) vectors \( \{X_N\} \), each being
associated with a probability density \( p(1,N) \). The vector \( X_N \) with the
maximum value of \( p(1,N) \) is the maximum likelihood vector and represents
the detected value of the transmitted message given by the vector \( S_N \).

To start the recursion in practice, the first \( g \) data-symbols in the
transmitted data sequence \( S_N \) may be made known to the receiver (as a training
sequence). These \( g \) components now form one of the \( m^g \) initial vectors \( \{X_g\} \),
and the probability density \( p(1,g) \) of this vector is put to \( 1.0 \). The
remaining \((m^g-1)\) vectors \( \{X_g\} \) may be chosen arbitrarily, but each is then
assigned a probability density \( p(1,g)=0 \). After \( g \) recursion cycles, all
these arbitrarily chosen vectors \( \{X_g\} \) will have been discarded and all the
\( m^g \) survivors \( \{X_{2g}\} \) have the same initial segment \( X_g \) which has been used as
a training sequence.

The Viterbi algorithm detector now computes \( m^{g+1} \) probability densities
per transmitted data-symbol, so that for a \( N \)-data-symbol message only
\( N m^{g+1} \) of these are to be computed. This is to be compared with \( m^N \)
probability densities required according to eqn. 3.72 if the recursive
algorithm is not used. Of course \( N \gg g \).

It is clear that the length of any of the \( m^g \) survivors, and therefore
the amount of storage required by the detector, grows linearly with time.
Suppose now, that at time \( t=kT \), all survivors \( \{X_k\} \) have the same initial
segment \( X_{k-n} \), where \( n<k \). Thus, this segment is definitely a part of the
maximum-likelihood vector \( X_N \), and therefore, a decision about the detected
value of \( S_{k-n} \) can be made at time \( t=kT \) without need for waiting for the
whole message to be received. The time interval \( nT \) is then called "delay
in detection." Now, once a decision is made about the initial segment \( S_{k-n} \),
the detector does not need to remember this segment any longer, thus enabling
the length of the survivors to be reduced. Of course, there may be
difficulty in deciding whether there is a segment \( S_{k-n} \) which forms a common
part to all survivors \( \{X_k\} \), and even when this is possible, the value of \( n \)
may not be constant. In practice, and at the expense of some reduction in
the detector's optimality, one can fix \( n \) to some given value. Normally
\( n \geq g + 1 \).

Now, the \( m^g \) survivors \( \{X_k\} \) held by the detector are replaced by
the \( m^g \) vectors \( \{Q_k\} \), where \( Q_k \) is given by the last \( n \) components of the
corresponding \( X_k \),

\[
Q_k = x_{k-n+1} x_{k-n+2} \ldots x_k
\]

On the receipt of the signal sample \( r_{k+1} \), the \( m^g \) vector \( \{Q_{k+1}\} \) with their
corresponding \( \{p(1,k)\} \) are obtained as before, and the detected value of
the data symbol \( s_{k-n+1} \) is taken as the value of \( x_{k-n+1} \) in the vector \( Q_k \)
from which the vector \( \{Q_{k+1}\} \) with the largest value \( p(1,k) \) has originated.

Of course, the delay in detection is \( n \) sampling intervals.

Clearly, the size of the store is now fixed and does not grow with
time nor does it depend on the length of the transmitted signal message.
Therefore, this length need not be restricted.

Notice that the algorithm assumes that the noise components \( \{w_i\} \) are
statistically independent random variables, but not necessarily Gaussian.
In the special case, when the \( \{w_i\} \) are Gaussian random variables with zero
mean and variance \( \sigma^2 \), i.e. with the probability density,

\[
p_w(w_i) = \frac{1}{(2\pi)^{\frac{1}{2}} \sigma} \exp\left(-\frac{w_i^2}{2\sigma^2}\right)
\]

then according to eqn. 3.88, the probability density corresponding to a
vector \( X_k \) is given by

\[
p(1,k) = \prod_{i=1}^{k} \frac{1}{(2\pi)^{\frac{1}{2}} \sigma} \exp \left[ \frac{-(r_i - \sum_{h=0}^{g} x_{i-h} y_h)^2}{2\sigma^2} \right]
\]

\[ (3.92) \]
and the maximization of \( p(l,k) \) becomes equivalent to maximizing the quantity,

\[
\ln[p(l,k)] = -\frac{k}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{k} \sum_{h=0}^{L} (r_i - \frac{g}{h} x_{i-h} y_h)^2.
\] (3.93)

Since the first term on the right hand side of eqn. 3.93 is the same for all survivors and their expansions at any time, the maximization of \( \ln[p(l,k)] \) may be accomplished by the maximization of the second term, i.e. the minimization of the function:

\[
|\mathbf{w}_k|^2 = \sum_{i=1}^{k} \left( r_i - \frac{g}{h} x_{i-h} y_h \right)^2
\] (3.94)

which geometrically represents the square of the distance between the received signal vector \( \mathbf{r}_k \) and the possible signal vector \( \mathbf{z}_k \) (eqn. 3.80) resulting from the corresponding survivor. The recursive formula in eqn. 3.89 becomes now:

\[
|\mathbf{w}_{k+1}|^2 = |\mathbf{w}_k|^2 + (r_{k+1} - \frac{g}{h} x_{k+1-h} y_h)^2
\] (3.95)

According to eqn. 3.72, the Viterbi detector minimizes the probability of error in the entire transmitted sequence, provided that an unbounded delay in detection is allowed, and in this sense it is optimum.\(^{(90)}\) In fact, the algorithm is not designed to minimise the symbol-error probability \( P(e) \), but at moderate-to-high signal-to-noise ratios, the detector that minimizes \( P(e) \) can not improve substantially on the performance of this detector.\(^{(90)}\)

A detailed study of \( P(e) \) in Ref. (90) shows that, when the noise components \( \{\mathbf{w}_i\} \) are Gaussian random variables with zero mean and variance \( \sigma^2 \), it is lower- and upper- bounded as

\[
d_{\min} Q\left(\frac{d_{\min}}{2\sigma}\right) \leq P(e) \leq k_1 Q\left(\frac{d_{\min}}{2\sigma}\right)
\] (3.96)

where \( Q(\cdot) \) is given by eqn. 3.42, \( d_{\min} \) is the minimum distance between
any two possible signal vectors \( \{Z_n\} \) (eqn. 3.80), \( N \) is the message length, and \( k_0 \) and \( k_1 \) are constants. In Ref.(60), an alternative proof of eqn.3.96 to that given in Ref.(90) is presented, and in Ref.(94), an upper-bound for \( k_1 \), in eqn. 3.96, is given as

\[
k_1 < \sum_{h=1}^{\infty} 2h\left(\frac{m-1}{m}\right)^h = 2m(m-1)
\]

Also in Ref.(94), it is shown that \( d_{\min} \) is a function of the energy of the sampled impulse response \( \sum_{h=0}^{g} y_h^2 \) (eqn. 3.78) and the number of its components \((g+1)\) so that the performance of the Viterbi algorithm detector over a given channel may be predicted if these quantities are known. Similarly, in Ref.(100), the aperiodic autocorrelation function of the sampled impulse response \( y \) is used to predict the value of \( d_{\min} \) for a given channel. In Ref.(95), a geometric interpretation of \( d_{\min} \) is given with an iterative technique to compute it.

Since the performance of the Viterbi algorithm detector is dependent on \( d_{\min} \) and \( \sigma \) (eqn. 3.96), the effective signal-to-noise ratio, as seen by the detector, may be defined as (90)

\[
\text{SNR}_{\text{eff}} = \frac{s_i^2 d_{\min}^2}{\sigma^2}
\]

(3.97)

whereas the conventional definition of the signal-to-noise ratio is (90)

\[
\text{SNR} = \frac{s_i^2 \sum_{h=0}^{g} y_h^2}{\sigma^2}
\]

(3.97a)

where \( s_i^2 \) is the mean square value of \( s_i \). Clearly, two channels which may have the same SNR, because both have the same \( \sum_{h=0}^{g} y_h^2 \), may have different \( \text{SNR}_{\text{eff}} \), because \( d_{\min} \) is dependent on the type of distortion introduced by each of the channels. (60,94,95,100)
3.5 **THE WHITENED MATCHED FILTER**

The data transmission system assumed so far (Fig. 3.2) uses a receiver filter with a transfer function shown in Fig. 3.3. Because of this assumption, the noise components \( \{w_i\} \) are statistically independent (eqn. 3.7), the fact which has been used throughout Sections 3.2 and 3.4.

Let the transmitter filter and the transmission path in Fig. 3.2, be combined together and have the real-valued impulse response \( g(t) \), where

\[
g(t) = a(t) * h(t)
\]

and let the signal at the input to the receiver filter be

\[
z(t) = q(t) + n(t)
\]

where

\[
q(t) = \sum_{i=1}^{\infty} s_i g(t-iT)
\]

and \( n(t) \) is white Gaussian noise process with zero mean and two-sided power spectral density \( N_0 \). It may be shown that, for finite \( g(t) \), a receiver filter which is matched to \( g(t) \), i.e. which has an impulse response \( g(-t) \), maximizes the signal-to-noise ratio at its output, and the samples of the output, at time \( t=iT, i=1,2,\ldots \), form a set of sufficient statistics for the estimation of the transmitted data sequence \( \{s_i\} \). Furthermore, and for any criterion of optimality, the optimum linear receiver filter can be expressed as a cascade of a matched filter and a transversal filter.

When the optimization criterion of the combination of the matched filter and the transversal filter is the minimization of the mean-square-error resulting from both the noise and the intersymbol interference, at the transversal filter output, it may be shown that the matched filter enhances the signal-to-noise ratio whereas the transversal filter reduces the amount of the intersymbol interference in the output signal.

Now, consider the data-transmission system in Fig. 3.10 where \( g(t) \) represents the impulse response of the transmitter filter and transmission.
path in cascade and \( n(t) \) is a white Gaussian noise with zero mean and power spectral density of \( \frac{1}{2} N_0 \). Let the receiver filter be matched to \( g(t) \) so that its impulse response (not physically realisable) is given by

\[
b(t) = g(-t)
\]

From Fig. 3.10, the sample \( r'_i \) of the signal \( r'(t) \) at the output of the matched filter at time \( t=iT \) is given by

\[
r'_i = \sum_{h=0}^{2k} s_{i-h} y'_h + u'_i
\]

where

\[
y'_h = \int_{-\infty}^{\infty} g(\tau) b(t-\tau)d\tau|_{t=(h-k)T}
\]

\[
= \int_{-\infty}^{\infty} g(\tau) g(h-k)T+\tau d\tau
\]

is the \((h+1)\)th component of the sampled impulse response of the channel and receiver filter which is such that \( y'_h=0 \) for \( h<0 \) and \( h>2k \). In fact the \( \{y'_h\} \) are obtained by sampling the autocorrelation function of \( g(t) \) which is symmetric about zero and extends, here, from \(-kT\) to \(kT\), giving thus \(2k+1\) components \( \{y'_h\} \). \( u'_i \) in eqn. 3.99, which is given by

\[
u'_i = \int_{-\infty}^{\infty} n(\tau) g(iT+\tau) d\tau
\]

is the noise component in \( r'_i \). The autocorrelation function of the \( \{u'_i\} \) is

\[
E[u'_i u'_{i+h}] = E[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} n(t)g(iT+t)n(\tau)g((i+h)T+\tau) dt d\tau]
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E[n(t)n(\tau)]g(iT+t)g((i+h)T+\tau) dt d\tau
\]

\[
= \frac{1}{2} N_0 \delta_{h+0}
\]

where eqn. 3.100 has been used together with the fact

\[
E[n(t)n(t+\tau)] = \int_{-\infty}^{\infty} \frac{1}{2} N_0 e^{j2\pi\tau f} df
\]

\[
= \frac{1}{2} N_0 \delta(\tau)
\]
FIGURE 3.10: Model of Data Transmission System with a Whitened Matched Filter
or equivalently,

\[ E[n(t)n(t')] = \delta(t-t'). \]

Thus, the noise components \( \{u_i\} \) in the received signal samples \( \{r_i\} \) are correlated, their autocorrelation function being proportional to the sampled impulse response of the channel and receiver filter, which, in the same time, is the autocorrelation function of \( g(t) \) as can be seen from the definition of the \( \{y_h\} \) (eqn. 3.100). Of course, noise correlation now prevents the Viterbi algorithm detector from being a true maximum likelihood detector and it becomes necessary to decorrelate the noise before feeding the received signal samples to the detector.

Let the z-transform of the sampled impulse response \( \{y_h\} \) of the channel and receiver filter be \( Y'(z) \). Since the \( \{y_h\} \) are obtained by sampling the autocorrelation function of \( g(t) \), and since the autocorrelation function is always symmetric, the \( \{y_h\} \) are symmetric about their middle component (namely about \( y_k \)) and therefore \( Y'(z) \) has the following properties. First, it has \( 2k \) complex roots (\( Y'(z) \) is a polynomial in \( z \) with \( 2k+1 \) terms). Second, \( Y'(z)=Y'(z^{-1}) \) which results in the fact that the roots of \( Y'(z) \) occur in complex-conjugate reciprocal pairs, i.e. every root of \( Y'(z) \) is accompanied by a root at the complex conjugate of its reciprocal.

Consequently, \( Y'(z) \) may be expressed as

\[ Y'(z) = Y_1(z)Y_2(z) \]

where \( Y_1(z) \) is any polynomial of degree \( k \) whose roots consist of one root from each complex conjugate reciprocal pair of roots of \( Y'(z) \), and \( Y_2(z) \) is the polynomial of degree \( k \) whose roots are the complex conjugate reciprocals of the roots of \( Y_1(z) \). In fact, \( Y_2(z) \) may be obtained from \( Y_1(z) \) by first reversing the order of its coefficients and then replacing these by their complex conjugates. (81)

It is shown in Ref. (90) that if the sampler in Fig. 3.10 is followed
by a linear feedforward transversal filter, whose tap gains are given by the coefficients of the polynomial

$$F(z) = Y_1^{-1}(z),$$

(3.104)

then the noise components in the signal samples at the output of this transversal filter will be uncorrelated. In the case when $Y_1(z)$ contains no roots on the unit circle, then the transversal filter is physically realizable. The cascade of the matched filter, as a receiver filter, the sampler and the linear transversal filter is called a "whitened matched filter". While this terminology is restricted to the case where $Y_1(z)$ has no roots on or inside the unit circle, details of the general case, when $Y_1(z)$ and $Y_2(z)$ represent any factorization of $Y'(z)$ (within the conditions assumed for eqn. 3.103), are also given in Ref. (90).

An interesting property of the whitened matched filter is observed in Ref. (116). When the channel is used at the Nyquist rate, which is the usual case in high speed data transmission, then the whitened matched filter, as defined above, degenerates into a lowpass filter having a flat amplitude frequency response and linear phase characteristic over the passband and a cut-off frequency (in Hz) at half the signal element rate, this filter being followed by a sampler and the linear transversal filter which forms the first part of the conventional non-linear equalizer, considered in Section 3.2.3. This linear transversal filter (linear phase equalizer) replaces now the roots of the $z$-transform of the sampled impulse response of the channel, which lie outside the unit circle by the complex-conjugate of their reciprocals, so that the $z$-transform of the channel, lowpass filter, sampler and linear transversal filter has no roots outside the unit circle.

A more general class of whitened matched filters are considered in Ref. (91). Here, the whitened matched filter is defined as the linear filter
whose output samples are a sufficient statistic for the estimation of the transmitted data sequence, with the further property that the noise components in these output samples are uncorrelated. This definition corresponds to the general case of factorizing $Y'(z)$ in eqn. 3.103. Furthermore, the problem is handled here (91) in the frequency domain.

Let $G(f)$ represent the Fourier transform of $g(t)$ in Fig. 3.10 and define the periodic function $P(f)$, with a period $\frac{1}{T}$, to be

$$P(f) = \sum_{h} |G(f + \frac{h}{T})|^2$$

(3.105)

where $h$ takes on all negative and positive integer values. Also let the transfer function of the whitened matched filter be denoted $W(f)$.

It may be shown that, for any channel impulse response $g(t)$ of practical interest, the transfer function of the whitened matched filter for that channel satisfies (91)

$$\sum_{h} |W(f + \frac{h}{T})|^2 = T$$

$-\infty < f < \infty$

(3.106)

and at frequencies where $P(f) \neq 0$, is given by:

$$W(f) = G^*(f) \sqrt{\frac{T}{P(f)}} \exp(-j\beta(f))$$

(3.107)

where $\beta(f)$ is an arbitrary periodic function of $f$ with period $\frac{1}{T}$ and $G(f)$ is the complex conjugate of $G(f)$. $G(f)$ is, of course, the transfer function of the transmitter filter and transmission path in cascade (Fig. 3.10). When $P(f)$ contains a zero, say at $f=f_0$, then $W(f_0)$ must be chosen to satisfy eqn. 3.106 (91).

As can be seen from eqn. 3.107, the whitened matched filter, is formed, in the general case, by a linear filter matched to the channel and given by $G(f)$, followed by a transversal filter given by $[(T/P(f))^{1/2}\exp(-j\beta(f))]$.

When the channel is bandlimited to $-\frac{1}{2T}$ to $\frac{1}{2T}$, i.e. used at the Nyquist rate, the individual terms under the summation in eqn. 3.105 do not overlap,
and over the channel bandwidth, i.e. for $h=0$ in eqn. 3.105, $P(f)$ satisfies:

$$\sqrt{P(f)} = |G(f)| \cdot \frac{1}{2T} \leq f \leq \frac{1}{2T}$$  \hspace{1cm} (3.108)

Now, writing $G(f)$ as:

$$G(f) = |G(f)| \cdot \exp(j\alpha(f))$$  \hspace{1cm} (3.109)

where $|G(f)|$ is the absolute value of $G(f)$ and $\alpha(f)$ is its phase, and substituting eqn. 3.108 in eqn. 3.107 gives,

$$W(f) = \sqrt{T} \cdot \exp(j\gamma(f)) \cdot \frac{1}{2T} \leq f \leq \frac{1}{2T}$$  \hspace{1cm} (3.110)

where

$$\gamma(f) = \alpha(f) + \beta(f)$$  \hspace{1cm} (3.111)

Since $\beta(f)$ is an arbitrary function, then for any $\alpha(f)$, $\gamma(f)$ is an arbitrary function.

The result in eqn. 3.110 is merely that, when the channel is strictly band-limited to $-\frac{1}{2T}$ to $\frac{1}{2T}$, then the whitened matched filter, for that channel, degenerates, in the general case, into a lowpass filter with an amplitude frequency response

$$|W(f)| = \sqrt{T} \cdot \frac{1}{2T} \leq f \leq \frac{1}{2T}$$

$$= 0 \quad \text{elsewhere}$$  \hspace{1cm} (3.112)

and an arbitrary phase characteristic. The noise components in the signal samples (obtained at rate of $\frac{1}{T}$) at the output of this filter are, of course, statistically independent. It is this filter which has been assumed as a receiver filter, in fact, in Figs. 3.2 and 3.3.

$\gamma(f)$ in eqn. 3.110 represents a phase characteristic and has no effect on the minimum distance $d_{\text{min}}$ corresponding to the sampled impulse response of the channel and whitened matched filter in cascade. Therefore, the performance of the Viterbi algorithm detector (eqn. 3.96) is independent of $\gamma(f)$. But if this is chosen such that the resultant sampled impulse response of the transmitter filter, transmission path and receiver filter
is minimum phase, then the number of components of this sampled impulse response becomes minimum, leading thus to a considerable reduction in the Viterbi algorithm complexity which is proportional to \( m^6 \). In fact, the arrangement here may be realized by using a receiver filter which satisfies eqn. 3.112 but with a linear phase, followed by a sampler and a linear feed-forward transversal filter which forms the first part of the optimum non-linear equalizer \(^{(81)}\) (Section 3.2.3). In this case, the arrangement corresponds exactly to that described in Ref. (116) as mentioned earlier.

When the channel is not used at or above the Nyquist rate (so that the element rate is less than the Nyquist rate of the channel), eqns 3.108 to 3.112 are no longer valid. However, in Ref. (99), it is suggested that to avoid the complications of using the whitened matched filter, the receiver filter becomes a lowpass filter with a rectangular transfer function and a cut-off frequency of \( \frac{1}{T} \) Hz. The technique of double sampling and a Viterbi detector which operates at twice the element rate are now used. The double sampling rate is equal or greater than the Nyquist rate of the channel but equal to the Nyquist rate of the lowpass filter. Under these conditions, the signal samples at the detector input form a set of sufficient statistics for the data signal estimation, while the noise components in these samples are statistically independent.

### 3.6 THE VITERBI ALGORITHM DETECTOR WITH A DESIRED IMPULSE RESPONSE

Although the Viterbi algorithm detector represents an implementable optimum detector by virtue of its recursive nature, it is still unacceptably complicated for most realistic channels. This is because its storage and computation requirements grow exponentially with the number of components
in the channel sampled impulse response.

One approach to reduce the complexity of the Viterbi algorithm detector is to shorten the channel impulse response to a desired length by pre-filtering the received signal before feeding it to the Viterbi detector. The sequence of received signal samples \( \{r_i\} \) is fed to a linear transversal filter whose tap gains are given by the vector \( C \), where:

\[
C = c_0, c_1, \ldots, c_L
\]  

(3.113)

The signal sample at the filter input is given by

\[
r_i = \sum_{h=0}^{L} s_{i-h} h + w_i
\]  

(3.114)

where the sampled impulse response of the channel is given by the vector \( Y \) (eqn. 3.78). Let the desired sampled impulse response be represented by the vector \( D \)

\[
D = d_0, d_1, \ldots, d_{p-1}
\]  

(3.115)

where \( p \) is the desired number of components of \( D \) and is typically 2 or 3.

The Viterbi detector now receives the signal sequence \( \{p_i\} \) instead of \( \{r_i\} \), where

\[
p_i = \sum_{h=0}^{L} s_{i-h} d_h
\]  

(3.116)

and the linear filter should, ideally, give at its output the sequence \( \{p_i\} \). But since the input signal \( \{r_i\} \) is a noisy one, the \( \{p_i\} \) can not be obtained exactly. Instead the linear filter gives the sequence \( \{r_i'\} \), where

\[
r_i' = \sum_{k=0}^{L} r_{i-k} c_k
\]  

(3.117)

The error between what the Viterbi detector should ideally receive and what it actually receives, is:

\[
e_i = r_i' - p_i
\]

\[
e_i = \sum_{k=0}^{L} r_{i-k} c_k - \sum_{h=0}^{p-1} s_{i-h} d_h
\]  

(3.118)
Several techniques are presented in the published literature to determine the tap gains of the linear filter \( \{c_k\} \) and the desired sampled impulse response \( D \), under the constraint of fixing the number of components in \( D \). \( \rho \) consecutive samples from the channel sampled impulse response \( \{y_n\} \) may be chosen to be the desired impulse response, and the linear filter tap gains are then determined by minimizing the mean-square-error \( e_i^2 \)

where, \( e_i \) is given by eqn. 3.118. Alternatively, both the \( \{c_k\} \) and \( \{d_k\} \) may be determined jointly by minimizing \( e_i^2 \), while constraining \( \sum_{k=0}^{\rho-1} d_k^2 \) (eqn. 3.115) to some fixed value. In Ref. (107), instead of minimizing the mean-square-error \( e_i^2 \), the desired impulse response and the linear filter tap gains are determined by maximizing the effective signal-to-noise ratio (eqn. 3.97) at the filter's output. Here, the desired impulse response has its amplitude frequency response as close (in the mean square sense) to that of the original channel as is possible under the constraint of the length of the desired impulse response.

In all these techniques, the linear filter performs some amplitude equalization of the channel which results in correlating the noise and enhancing its variance \((81, 107, 109, 115-116)\), in which case the Viterbi detector becomes non-optimum. To avoid noise correlation, the use of a non-linear equalizer in place of the linear filter is suggested to reduce the number of components in the sampled impulse response. The non-linear equalizer makes tentative decisions and uses them to partially equalize the channel (removes some of the intersymbol interference) leaving in its output signal the intersymbol interference of the desired impulse response \((108, 109)\). Clearly, this technique suffers from the error extension effects in the non-linear equalizer. Also, such a technique,
although not enhancing the noise variance, eliminates some of the signal energy by the intersymbol interference cancellation process, which implies a reduction in the effective signal-to-noise ratio.

In conclusion, all these techniques which attempt to reduce the complexity of the Viterbi detector by pre-processing the received signal linearly or non-linearly, while constraining the length of the desired impulse response, do so at the expense of the optimality of the detector.

3.7 THE REDUCED STATE VITERBI DETECTOR

An alternative approach is to modify the algorithm of the detector itself. As shown in Section 3.4, when the noise components in the signal samples are statistically independent Gaussian random variables with zero mean and fixed variance, the Viterbi detector retains, at any time \( t=kT \), only \( m^e \) survivors, each being associated with a "cost" \( |W_k|^2 \) (eqn. 3.94), where \( |W_k| \) is the distance, in the \( k \)-dimensional space, between the received signal vector and the signal vector resulting from the corresponding survivor. At high signal-to-noise ratios, it may be shown (111-112) that the detector needs to retain only those survivors associated with those\( |W_k|^2 \) satisfying

\[
|W_k| < \frac{d_{min}}{2} \quad (3.119)
\]

where \( d_{min} \) is as defined for eqn. 3.96. Such a detector is known as the reduced state Viterbi detector. When the noise variance approaches zero, the performance of this detector approaches that of the optimum detector in the sense that it minimizes the probability of a symbol-error.

The algorithm of the reduced state Viterbi detector is given in Ref. (112) as follows: The detector processes the signal exactly as for the Viterbi detector (as explained in Section 3.4) except that at each time \( t=kT \) it
retains precisely those survivors which originate from the survivors at
time \( t=(k-1)T \) and satisfy eqn. 3.119.

At finite signal-to-noise ratios, the reduced state Viterbi detector
becomes sub-optimum, because it does not retain all the \( m^g \) survivors
required, at every time \( t=kT \), by the maximum likelihood detector. The
degradation is obviously dependent on the ratio of the number of the
retained survivors to \( m^g \), and when \( m^g \) becomes smaller the degradation will
be less.

3.8. NEAR-MAXIMUM-LIKELIHOOD DETECTORS (113-117)

These are other forms of the reduced state Viterbi detector. The
structure and the operation of the detector here have been modified to
greatly simplify the system without excessive reduction in the tolerance
to noise relative to that of the Viterbi detector. A near-maximum
likelihood detector holds in store only a limited and predetermined number
of vectors (as survivors). The length of each of these vectors is also
fixed, contrary to the case of the Viterbi detector survivors whose length
grows linearly with time.

The data transmission system is as shown in Fig. 3.9, where the
detector is a near-maximum likelihood detector. The linear baseband
channel, which includes the transmission path and all equipment filters,
has a sampled impulse response which is given by

\[
Y = y_0, y_1, \ldots, y_g
\]

(3.120)

As before, the data symbols \( \{s_i\} \) are statistically independent and
equally likely to have any of \( m \) possible values. The additive noise is
represented by the sequence \( \{w_i\} \) where these are statistically independent
Gaussian random variables with zero mean and fixed variance. The received
signal sample at time \( t = kT \) is given by

\[
    r_k = \sum_{h=0}^{g} s_{k-h} y_h + w_k
\]

(3.121)

The near-maximum likelihood detector operates on the \( \{r_i\} \) to give at
time \( t = kT \) an estimate \( s'_{k-n} \) of the transmitted data symbol \( s_{k-n} \), where \( n \)
is an integer and represents the delay in detection (in sampling intervals).

Just prior to the receipt of the signal sample \( r_k \) at time \( t = kT \), the
detector holds in store \( \{Q_{k-1}\} \), where

\[
    Q_{k-1} = x_{k-n} x_{k-n+1} \ldots x_{k-1}
\]

(3.122)

and \( x_i \) is a possible value of \( s_i \). Each stored vector is associated with

a cost \( |w_{k-1}|^2 \), where

\[
    |w_{k-1}|^2 = \sum_{i=0}^{k-1} (r_i - \sum_{h=0}^{g} x_{i-h} y_h)^2
\]

\[
= |w_{k-2}|^2 + (r_{k-1} - \sum_{h=0}^{g} x_{k-1-h} y_h)^2
\]

(3.123)

Eqn. 3.123 has been derived in Section 3.4 (eqn.3.94).

On the receipt of the signal sample \( r_k \), the detector expands every
vector \( Q_{k-1} \) into \( m (n+1) \)-component vectors \( \{P_k\} \),

\[
    P_k = x_{k-n} x_{k-n+1} \ldots x_k
\]

(3.124)

The first \( n \) components of \( P_k \) are as in the original vector \( Q_{k-1} \) and the
last component \( x_k \) takes on the \( m \) possible values of \( s_i \). Then the detector
evaluates for each expanded vector \( P_k \) its cost

\[
    |w_k|^2 = |w_{k-1}|^2 + (r_k - \sum_{h=0}^{g} x_{k-h} y_h)^2
\]

(3.125)

and takes the value of \( x_{k-n} \) of the vector \( P_k \) with the smallest cost \( |w_k|^2 \)
as the detected data symbol \( s'_{k-n} \). \( n \) is usually greater than \( g \). Afterwards,
the detector selects from the \( \mu m \) vectors \( \{P_k\} \) just \( \mu \) vectors \( \{P_k\} \)
according to some criterion to be considered later. It then removes from
each selected vector $P_k$ its first component $x_{k-n}$ to give the corresponding $\mu$ vectors $\{Q_k\}$ which are stored with their costs $|W_k|^2$ in preparation for the next detection cycle.

The number of vectors $\mu$, retained by the near-maximum likelihood detector at any time $t=1T$, is usually very small compared with $m^g$ survivors required to be retained by the Viterbi detector, which leads to a possible reduction in the detector tolerance to additive noise compared with the Viterbi detector. Therefore the criterion employed in the choice of the $\mu$ vectors $\{Q_k\}$ must be such that the degradation in tolerance to noise is kept to the minimum under the given conditions. Several criteria have been suggested for choosing these vectors,\(^{113-117}\) and the most representative of which are the following.

**SYSTEM 1 [System 1 in Ref. (113)]:**

The detector here selects the $\mu$ vectors associated with the smallest costs $|W_k|^2$ out of the $\mu m$ expanded vectors $\{P_k\}$. The major drawback of this system is the tendency for some of the stored vectors and their associated costs to become the same which reduces the effective number of the stored vectors and consequently degrades the system's performance. To overcome this drawback, the system is modified by what is called an anti-merging procedure to give the following system.

**SYSTEM 2 [System 3 in Ref. (115)]:**

After expanding the $\mu$ vectors $\{Q_{k-1}\}$ to give $\mu m$ vectors $\{P_k\}$ and then evaluating their costs, as described before, System 2 starts by selecting the expanded vector $P_k$ that is associated with the smallest cost $|W_k|^2$ and taking the value of $x_{k-n}$ in this vector as the detected value $s_{k-n}$ of the data-symbol $s_{k-n}$. The system then discards all vectors
\( \{P_k\} \) for which \( x_{k-n} \neq s_{k-n}' \) and from the remaining vectors \( \{P_k\} \) (including of course the one originally chosen) it selects the \( u \) vectors associated with the smallest costs \( |W_k|^2 \).

This system differs from System 1 only by discarding all vectors for which \( x_{k-n} \neq s_{k-n}' \), and this is what prevents the vectors from merging. The effect of this process can be explained as follows: suppose that, at time \( t=kT \), all the stored vectors \( \{Q_{k-1}\} \) differ in, at least, one component \( x_{k-j} \), \( j=1,2,\ldots,n \). When the vectors \( \{Q_{k-1}\} \) are expanded into the \( um \) vectors \( \{P_k\} \), it is definite that all of these vectors must differ in one component at least, and when the detector discards all vectors for which \( x_{k-n} \neq s_{k-n}' \) it is effectively eliminating the possibility that two vectors, which differ only by \( x_{k-n} \) are selected, because after removing \( x_{k-n} \) from two such vectors, two identical vectors \( \{Q_k\} \) are obtained. Thus the selected vectors \( \{Q_k\} \) do not become the same providing that all of them were different at the start.

SYSTEM 3 [System 5A in Ref. (114)]:

After having expanded the \( u \) vectors \( \{Q_{k-1}\} \) into \( um \) vectors \( \{P_k\} \) and evaluated their costs, the detector starts by selecting, for each of the \( m \) different values of \( x_{k-h+2} \), where \( h=u/m \), the vector \( P_k \) with the given value of \( x_{k-h+2} \) and associated with the smallest cost \( |W_k|^2 \). The process is then repeated, in turn, for \( x_{k-h+3}, x_{k-h+4}, \ldots, x_{k} \), to give a total of \( u-m \) selected vectors. A vector once selected is not available for selection a second time. A further \( m \) vectors \( \{P_k\} \) are finally selected from the remaining (non-selected) vectors \( \{P_k\} \) as those with the smallest costs \( |W_k|^2 \) with no constraints as to the values of any \( \{x_i\} \), giving a total of \( u \) selected vectors. The first component \( x_{k-n} \) of the vector with the smallest cost \( |W_k|^2 \) gives the detected value \( s_{k-n}' \) of the data-symbol.
Now the detector removes the first component of each of the selected vectors \( \{P_k\} \) to give the corresponding \( \mu \) vectors \( \{Q_k\} \) which are stored with their costs.

It is clear that the near-maximum likelihood detectors are essentially aimed to be a practical, simple and efficient substitute to the maximum likelihood detector. But since these detectors retain in memory at each recursion step only a limited number (\( \mu \)) of vectors which is usually very small compared with \( m^g \) survivors required to be retained by the Viterbi detector, the performance of these detectors may show some degradation relative to what can be achieved by the Viterbi detector, when \( m \) and \( g \) are both large.

Since the near-maximum likelihood detectors are simplified versions of the Viterbi detector, the mechanism of their operation should be understood in the light of the method of operation of that detector itself. Therefore, the whitened matched filter, which is required by the Viterbi detector to achieve optimum performance, is required also by the near-maximum likelihood detector. When the channel is used at the Nyquist rate, the whitened matched filter degenerates into a lowpass filter with a rectangular transfer function, followed by a sampler and a linear transversal filter which forms the first part of the optimum non-linear equalizer (section 3.2.3), as has been shown in Section 3.5. Such an arrangement is used in Ref. (116) in the detection of digital data transmitted at 9600 bit/s over telephone circuits.

3.9 **ADAPTIVE MAXIMUM AND NEAR-MAXIMUM LIKELIHOOD DETECTION**

So far, it has been assumed that the detector, whatever it is, has an exact knowledge of the sampled impulse response of the linear baseband
channel. In practice, the data transmission system is expected to operate on different transmission paths which may have considerably different characteristics. Therefore, a practical data transmission system must be capable of adjusting itself to match the characteristics of the channel. Also, the channel itself may vary slowly with time, and the receiver must track the channel variations to ensure correct operation. Therefore an estimator is used to form an estimate of the sampled impulse response of the linear baseband channel, sampler and linear filter (Fig. 3.11). The linear filter must, of course, also be adaptive in order to adjust itself to the requirements described in Sections 3.5 and 3.6.

First, consider the method of operation of the estimator (Fig. 3.12). The received signal sequence at the detector input is \( \{r_i\} \), where

\[
r_i = \sum_{h=0}^{g} s_{i-h} y_i, h + w_i
\]

Here, the sampled impulse response to be estimated is given by

\[
Y_i = y_{i,0}, y_{i,1}, \ldots, y_{i,g}
\]

The time index \( i \) used for \( Y_i \) here is to indicate the possibility that \( Y_i \) may be different from \( Y_k \) for any \( k \neq i \). The estimator now uses the received signal sequence \( \{r_i\} \) and the detected data sequence \( \{s'_i\} \) to give an estimate \( Y'_i \) of the sampled impulse response \( Y_i \). Here, every \( T \) seconds the estimator forms an estimate \( r'_{i-n} \) of the received signal sample \( r_{i-n} \) using the previous estimate of \( Y_{i-n-1} \), and the detected data sequence \( \{s'_{i-k}\} \), \( k=n,n+1,\ldots,n+g \), where

\[
r'_{i-n} = \sum_{h=0}^{g} s'_{i-n-h} y'_{i-n-1}, h
\]

and then it forms the error

\[
e_{i-n} = r_{i-n} - r'_{i-n}
\]
FIGURE 3.11: Model of Data Transmission System with an Adaptive Linear Filter and a Channel Estimator.
FIGURE 3.12: Channel Estimator
This error is scaled by a factor $\delta_1$, which is a small positive constant. The new estimate of the $h^{th}$ component of $Y_{i-n}$ is now given by (92,101-103, 121-122)

$$y'_{i-n,h} = y_{i-n-1,h}' + \delta_1 e_{i-n,h}$$

(3.130)

for $h=0,1, \ldots, g$. $y'_{i-n,h}$ is known as the updated estimate of $y_{i-n,h}$.

Eqn. 3.130 is a steepest-descent algorithm, and it may be shown (101) that when this arrangement is used with a sufficiently slowly time-varying channel, the mean-square-error $e_i^2$ is minimized. When the channel is time invariant, the detected data sequence is free of errors (i.e. identical to the transmitted data sequence), and the data-symbols are statistically independent, the mean values of the estimates $\{y_{i,h}'\}$ become equal to the correct values $\{y_{i,h}\}$ and the mean-square-error is proportional to the variance of the noise components in the received signal sequence. (92)

For starting up, one may send a short training sequence of data-symbols, whose values are known to the receiver, to perform the initial adjustment of the estimated sampled impulse response.

Since the type of detector assumed here determines the value of a detected data-symbol after a delay of $n$ sampling intervals, the resultant estimates (eqn. 3.130) obtained after the receipt of $r_i$ are in fact for the $\{y_{i-n,h}\}$ and not for the $\{y_{i+1,h}\}$, the latter being required by the detector on the receipt of the signal sample $r_{i+1}$. However, when the sampled impulse response is time-invariant or varies very slowly with time, the $\{y_{i+1,h}'\}$ can be taken as the $\{y_{i-n,h}'\}$ with insignificant error, which is the case assumed in Fig. 3.11. When the channel varies considerably with time, such as with HF radio channels, this arrangement becomes unsuitable to determine the $\{y_{i+1,h}'\}$. Instead, the $\{y_{i+1,h}\}$ are obtained from the $\{y_{i-n,h}'\}$ by a prediction process (122,183).
estimation and prediction process of Ref. (183) will be described in Section 7.6.

The adaptive linear filter (Fig. 3.11) is adjusted as shown in Fig. 3.13. The tap gains of this filter are given by the vector,

\[ \mathbf{c} = c_{0}, c_{1}, \ldots, c_{L} \]

whose \( h \)th component is updated using the error signal \( e_{i-n} \) given by eqn. 3.129 (formed by the estimator, Fig. 3.12) and the received signal sequence \( \{p_i\} \) and is given by (81, 103, 116)

\[ c_{i+h} = c_{i+h} - \delta_{2} e_{i-n} p_{i-n+h} \]  

for \( h = 0, 1, \ldots, L \), where \( \delta_{2} \) is a small positive constant. Notice here that the signal samples \( \{p_{i-n-h}\} \) (Fig. 3.13) are delayed by \( n \) sampling intervals to account for the delay in the detector.

When the channel is used at the Nyquist rate, and when there is no constraint imposed on the estimated sampled impulse response, the linear filter will be adjusted, at high signal-to-noise ratios, to give the linear transversal filter which replaces the roots of the z-transform of the sampled impulse response of the linear baseband channel, which lie outside the unit circle by the complex conjugates of their reciprocals (81) and when the receiver filter (Fig. 3.11) is a lowpass filter, with a rectangular transfer function and cut-off frequency \( 1/2T \) Hz, then the receiver filter, sampler and the adaptive linear filter will be equivalent to the whitened matched filter considered in Section 3.5.

\[ \text{3.10} \quad \text{NEAR MAXIMUM LIKELIHOOD DETECTION OF COMPLEX-VALUED SIGNALS} \]

Throughout the preceding sections, real-valued signals have been assumed, but all the described techniques can also be used with complex-
FIGURE 3.13: Adaptive Linear Filter
valued signals such as those encountered in quadrature amplitude modulation (QAM) systems \(^{80,106,115-116,127-129}\). The discrete-time model of the data transmission system is exactly as shown in Fig. 3.9, where now all signals are complex-valued. The data to be transmitted is carried on the data symbols \( \{s_i\} \), where

\[
s_i = s_{1,i} + j s_{2,i}
\]

\( i = 1 \ldots m \)

\( j = \sqrt{-1} \) and \( m^2 \) is the total number of signal levels (possible symbol values). Clearly, in eqn. 3.134, each \( s_{1,i} \) and \( s_{2,i} \) have \( m \) possible values, and therefore is an \( m \)-level symbol. The sampled impulse response of the linear baseband channel is given now by the complex vector:

\[
Y = y_0, y_1, \ldots, y_g
\]

and the complex-valued received signal sample is

\[
r_i = \sum_{h=0}^{g} s_{i-h} y_h + w_i
\]

where the \{\( w_i \)\} are complex-valued samples each of their real and imaginary parts are Gaussian random variables with zero mean and the same variance.

In a near maximum likelihood detector (Section 3.8), the vectors \( \{Q_k\} \) or \( \{P_k\} \) (eqn. 3.122 and 3.124 respectively) are now complex valued vectors and the component \( x_i \) may have any of the \( m^2 \) possible complex values of \( s_{i} \). The cost \( |W_k|^2 \) (eqn. 3.125) now becomes:

\[
|W_k|^2 = |W_{k-1}|^2 + |r_k - \sum_{h=0}^{g} x_{k-h} y_h|^2
\]

where the term \( |r_k - \sum_{h=0}^{g} x_{k-h} y_h|^2 \) is used instead of the term \( (r_k - \sum_{h=0}^{g} x_{k-h} y_h)^2 \).

Recalling eqn. 3.94 and its interpretation, \( |W_k|^2 \) represents now the square of the unitary distance in the \( k \)-dimensional complex vector space between
the received signal vector $r_k$ and the noise-free signal vector $Z_k$ (eqn. 3.80) resulting from the corresponding survivor (115,116) when the vector $Q_k$ is given by the last segment of that survivor. Now let

$$r_k = \sum_{h=0}^{H} \Delta x_{k-h} \Delta y_{k-h} a_k + j b_k$$

(3.138)

then, eqn. 3.137 becomes,

$$|W_k|^2 = |W_{k-1}|^2 + a_k^2 + b_k^2$$

(3.139)

which clearly involves two squaring operations ($a_k^2$ and $b_k^2$) and two additions. But multiplication, including squaring operations, are in general undesirable in practice because of the complexity of the hardware required to perform such operations. An approach to avoid the two squaring operations in eqn. 3.139 is to replace them by some other simpler arithmetic such as addition or multiplication by $2^i$, $i$ integer, (which is just a shift in binary arithmetic), without introducing significant error in the value of $|W_k|^2$. In Ref.(120) several measures are suggested as an alternative to $|W_k|^2$, the best of which is given by

$$|W_k| = |W_{k-1}| + E_k$$

(3.140)

where

$$E_k = 2c_k + |c_k - c_{k-1}|$$

(3.141)

and

$$c_k = 2(|a_k| + |b_k|) + |a_k| - |b_k|$$

(3.142)

$a_k$ and $b_k$ are given by eqn. 3.138. Clearly, eqns. 3.140-3.142 need much simpler hardware and much less computation time than does eqn. 3.139. In the complex plane, the distance measure of eqn. 3.142 maps the unit circle $(a_k^2 + b_k^2) = 1$ into an octagon

$$2(|a_k| + |b_k|) + |a_k| - |b_k| = 3$$

as shown in Fig. 3.14(120)
FIGURE 3.14: The Mapping of the Unit Circle into an Octagon According to Eqn. 3.142.
4. QUADRATURE AMPLITUDE MODULATION (QAM)

4.1 INTRODUCTION

In digital-data transmission systems, the information to be transmitted is usually carried by means of baseband signals. The baseband data signal is defined as one whose spectrum extends to zero frequency (d.c.) or to very low frequencies, and which carries information (data) in terms of its values at certain points. Since most of the voice-frequency channels have very poor transmission characteristics in the neighbourhood of zero frequency, they are regarded as passband channels and therefore unsuitable for the transmission of baseband signals, because of the possible loss in the signal power at low frequencies and the severe distortion likely to be introduced by these channels into the transmitted signal. In order to obtain satisfactory transmission over typical voice-frequency channels the baseband signal spectrum is shifted into the channel frequency band by some kind of modulation such as amplitude modulation (AM), phase modulation (PM) or frequency modulation (FM). Amongst the different types of modulation techniques, quadrature amplitude modulation (QAM) is frequently used for high-speed data transmission.

A QAM signal is the sum of two double-sideband suppressed carrier AM signals in phase quadrature. The two AM signals are in element synchronism and their two carriers have the same frequency but at a phase angle of 90°. The advantages of this modulation technique can be summarized as follows:

1. Efficient bandwidth utilization.

2. Ability to track phase jitter unaided by any auxiliary transmitted pilot tone.
3. No particular phase relationship need be maintained between the reference carriers used for the coherent demodulation of the two AM signals and the suppressed carriers of the AM signals themselves, just so long as the rate of change of the relative phase angles remains fairly small.\(^{129}\)

4. High immunity to Gaussian noise.\(^{124-126}\)

5. It is a linear modulation method which greatly simplifies the implementation of equalizers and maximum-likelihood detectors.

Because of its advantages over other modulation techniques, a QAM system is used here, both in its own right and as a reference in evaluating the relative performances of data transmission systems which transmit baseband signals. The latter systems are described and analysed in Section 5.

The QAM system is also used in an investigation of the transmission of data at 19.2 kb/s over telephone lines and 9.6 kb/s over HF radio channels which will be presented in Sections 6 and 7, respectively. QAM systems are best studied as the corresponding baseband system and the basic theory of the baseband model of the QAM system will now be presented.

4.2 THE QAM SYSTEM

The quadrature amplitude modulation system is shown in Fig. 4.1. The information to be transmitted is carried by the two statistically independent streams of data-symbols \(\{s_{1,i}\}\) and \(\{s_{2,i}\}\), which are in symbol-synchronism. The \(\{s_{1,i}\}\) and the \(\{s_{2,i}\}\) are statistically independent and equally likely to have any of \(m\) possible values,

\[
s_{1,i}, s_{2,i} = 2^{m} - 1, \quad i = 0, 1, \ldots, m-1,
\]  \(4.1\)
FIGURE 4.1: Model of QAM Data Transmission System
so that any two data-symbols \((s_1, s_2)\) may have one of \(m^2\) possible combinations. The two data streams are fed separately to two transmitter filters (Filters \(A_1\) and \(A_2\)), the real-valued impulse response of each is \(a(t)\). The output signals of these two filters are modulated (multiplied) by two carriers in phase quadrature but with the same frequency \(f_c\). The output signals of the two linear modulators are added together to form the QAM signal \(x(t)\) which is fed to the transmission path whose impulse response \(h(t)\) is real-valued. The value of the carrier frequency \(f_c\) is such that \(|X(f)|\), the amplitude spectrum of the QAM signal, fits into the frequency characteristics of the transmission path appropriately with no significant loss in the transmitted signal energy. \(|X(f)|\) (Fig. 4.3) is obtained by shifting the absolute value of the transfer function of the transmitter filter \(|A(f)|\) (Fig. 4.2) appropriately by \(f_c\) Hz. For \(|X(f)|\) to have a bandpass shape, \(f_c\) must be not less than \(\frac{1}{2T}\) where \(|A(f)|\) is assumed band-limited to \(-\frac{1}{2T}\) to \(\frac{1}{2T}\) Hz, i.e. the transmission baud rate is at the Nyquist rate. The only additive noise assumed here is a stationery white Gaussian noise \(n(t)\), with zero mean and two-sided power spectral density \(\frac{1}{2} N_0\), which is added to the signal at the output of the transmission path. At the receiver, the receiver filter (Filter C), whose real-valued impulse response is \(c(t)\), removes as much as possible of the noise outside the signal frequency-band without excessively distorting the signal itself. The absolute value of the transfer function of this filter, \(|C(f)|\), is therefore band-limited, over positive frequencies to a frequency band from \(f_c - \frac{1}{2T}\) to about \(f_c + \frac{1}{2T}\) Hz, as shown in Fig. 4.4. \(f_c - \frac{1}{2T} \approx 300\). The removal of the frequencies below 300 Hz is necessary to avoid the presence of the mains harmonics (hum) in the received signal. The output signal of the receiver filter is now coherently demodulated by two reference carriers which have the same frequency \(f_c\) and are in phase quadrature. The outputs of the two demodulators are then lowpass filtered to
FIGURE 4.2: Absolute Value of the Transfer Function of Transmitter Filters A1 and A2

FIGURE 4.3: Amplitude Spectrum of the QAM Signal at the Transmission Path Input
FIGURE 4.4: Absolute Value of the Transfer Function of Receiver Filter C

FIGURE 4.5: Transfer Function of the L.P. Filters $B_1$ and $B_2$ in the Coherent Demodulators
remove the high frequency components resulting from the demodulation process.

The impulse response of each of the lowpass filters $B_1$ and $B_2$ (Fig. 4.1) is $b(t)$. $b(t)$ is real-valued and its Fourier transform $B(f)$ has a rectangular shape and is bandlimited to $-\frac{1}{2T}$ to $\frac{1}{2T}$ Hz (as $|A(f)|$) as shown in Fig. 4.5.

From Fig. 4.1, the QAM signal at the transmission path input is:

$$x(t) = \sqrt{2} \left[ \sum_{i_1} s_{1,i_1} a(t-iT) \cos 2\pi f_c t - \sum_{i_2} s_{2,i_2} a(t-iT) \sin 2\pi f_c t \right]$$

(4.2)

where $f_c$ is assumed here to have the value

$$f_c = \frac{1}{2T} + f_L$$

(4.3)

and $f_L$ is the lower cut-off frequency of the receiver filter $C$ ($f_L = 300$ Hz in Fig. 4.4). Defining the complex data-symbol $s_i$ as

$$s_i = s_{1,i} + js_{2,i}$$

(4.4)

where $j = \sqrt{-1}$, eqn. 4.2 may be written as

$$x(t) = \sqrt{2} \Re \left[ \sum_{i} s_i a(t-iT) e^{j2\pi f_c t} \right]$$

(4.5)

or equivalently,

$$x(t) = \frac{1}{\sqrt{2}} \left[ \sum_{i} (s_i e^{j2\pi f_c t} + s_i^* e^{-j2\pi f_c t}) a(t-iT) \right]$$

(4.6)

where $(s_i^* e^{-j2\pi f_c t})$ is the complex conjugate of $(s_i e^{j2\pi f_c t})$.

The signal at the coherent demodulator input in Fig. 4.1 is given by

$$z(t) = x(t) * h(t) * c(t) + n(t) * c(t)$$

(4.7)

and the two signals at the coherent demodulator outputs are,

$$r_1(t) = [\sqrt{2} z(t) \cos (2\pi f_c t + \theta)]*b(t)$$

(4.8)

$$r_2(t) = [-\sqrt{2} z(t) \sin (2\pi f_c t + \theta)]*b(t)$$

(4.9)

where $\theta$ is the phase of the reference carrier relative to the signal carrier.

Combining $r_1(t)$ and $r_2(t)$ together in a complex form gives the output
complex signal \( r(t) \),

\[
r(t) = r_1(t) + j r_2(t)
\]  

(4.10)

From eqns. 4.8, 4.9 and 4.10 results:

\[
r(t) = \sqrt{2} \left[ z(t) \ e^{-j(2\pi f_c t + \theta)} \right] * b(t)
\]  

(4.11)

and eqns. 4.6, 4.7 and 4.11 give,

\[
r(t) = \{ \left[ \sum_i s_i e^{j2\pi f_c t} + s_i^* e^{-j2\pi f_c t} a(t-iT) \right] * h(t) * c(t) \} e^{-j(2\pi f_c t + \theta)} \} * b(t) + \sqrt{2} \left[ \left[ n(t) * c(t) \right] e^{-j(2\pi f_c t + \theta)} \right] * b(t)
\]  

(4.12)

Now, noting that

\[
[f_1(t) * f_2(t)] e^{-j2\pi f_c t} = (f_1(t) e^{-j2\pi f_c t}) * (f_2(t) e^{-j2\pi f_c t})
\]  

(4.13)

which results by the direct application of the convolution integral, eqn. 4.12 becomes,

\[
r(t) = \sum_i s_i \alpha(t-iT) * [(h(t) * c(t)) e^{-j2\pi f_c t}] e^{-j\theta} * b(t) + \\
+ \sum_i \beta_i e^{-j4\pi f_c t} a(t-iT) * [(h(t) * c(t)) e^{-j2\pi f_c t}] e^{-j\theta} * b(t) + \\
+ \sqrt{2} \left[ \left[ n(t) * c(t) \right] e^{-j(2\pi f_c t + \theta)} \right] * b(t)
\]  

(4.14)

The term under the second summation in eqn. 4.14 has a zero value because the spectrum of \( e^{-j4\pi f_c t} a(t) \), which represents the demodulated signal second harmonic, lies outside the frequency band of the lowpass filters \( B_1 \) and \( B_2 \) according to eqn. 4.3 and Figs. 4.2 and 4.5. Therefore eqn. 4.14 reduces to

\[
r(t) = \sum_i s_i y(t-iT) + u(t)
\]  

(4.15)

where

\[
y(t) = \left\{ \alpha(t) * [(h(t) * c(t)) e^{-j2\pi f_c t}] \right\} e^{-j\theta} * b(t)
\]  

(4.16)

is the overall system impulse response, and

\[
u(t) = \sqrt{2} \left[ \left[ n(t) * c(t) \right] e^{-j(2\pi f_c t + \theta)} \right] * b(t)
\]  

(4.17)

is the resultant complex-valued noise component in the received signal \( r(t) \).

In eqn. 4.16, the impulse response \( (h(t) * c(t)) \) has a bandpass transfer
function whose frequency band is limited by \(|C(f)|\) (Fig. 4.4). Let
\[
h_1(t) = h(t) \ast c(t) \tag{4.18}
\]
have the Fourier transform
\[
H_1(f) = H(f)C(f) \tag{4.19}
\]
where \(H(f)\) and \(C(f)\) are the transfer functions of the transmission path and receiver filter \(C\), respectively. The real-valued bandpass impulse response \(h_1(t)\) may be expressed in terms of a lowpass complex-valued impulse response \(m(t)\) and a carrier as
\[
h_1(t) = 2 \text{Re}[m(t)e^{j2\pi fc_t}] \tag{4.20}
\]
where the factor 2 is used here for convenience. The Fourier transform of \(m(t)\), \(M(f)\), is related to \(H_1(f)\) by
\[
H_1(f) = M(f-f_c) + M^*(-f-f_c) \tag{4.21}
\]
Now, eqn. 4.20 may be written as
\[
h_1(t) = m(t)e^{j2\pi fc_t} + m^*(t)e^{-j2\pi fc_t} \tag{4.22}
\]
where \(m^*(t)\) is the complex conjugate of \(m(t)\). Replacing \(h_1(t)\) (i.e. \(h(t) \ast c(t)\)) in eqn. 4.16 by its value in eqn. 4.22 gives
\[
y(t) = [(a(t) \ast m(t))e^{-j\theta}] \ast b(t) + \\
+[(a(t) \ast m^*(t)e^{-j4\pi fc_t}e^{-j\theta}] \ast b(t) \tag{4.23}
\]
The Fourier transform of the waveform \(m^*(t)e^{-j4\pi fc_t}\) is given by \(M^*(-f-2f_c)\) and when this is zero over the passband of \(A(f)\) (Fig. 4.2), i.e. when
\[
M^*(-f-2f_c) = 0 \text{ for } -\frac{1}{2T} < f < \frac{1}{2T} \tag{4.24}
\]
then \(a(t) \ast m^*(t)e^{-j4\pi fc_t}\) (eqn. 4.23) becomes zero and eqn. 4.23 becomes,
\[
y(t) = [(a(t) \ast m(t))e^{-j\theta}] \ast b(t) \tag{4.25}
\]
which represents the impulse response of the baseband system that is equivalent to the QAM system in Fig. 4.1.
The representation in eqn. 4.25 requires in fact the knowledge of \( m(t) \) which depends on both \( h_1(t) \) and \( f_c \), according to eqns. 4.20-4.21. Furthermore, eqn. 4.25 assumes that eqn. 4.24 is satisfied, and the latter can only be true with ideal filters. On the other hand, although \( y(t) \) in eqn. 4.16 contains a carrier component \( e^{-j2\pi f_c t} \) so that it does not appear to be a baseband waveform, it is in fact so, because \( (h(t) \ast c(t)) \) is a bandpass waveform which is converted into a lowpass impulse response through multiplying it by \( e^{-j2\pi f_c t} \), which results in a frequency shift of \(-f_c \) Hz in the transfer function, together with the bandlimiting by \( A(f) \) and \( B(f) \), the Fourier transforms of \( a(t) \) and \( b(t) \), respectively. Furthermore, all the quantities \( \{a(t), h(t), c(t), b(t) \) and \( f_c \} \) in eqn. 4.16 are those that are known or can most easily be determined from the given information (e.g. see Figs. 2.2 and 2.3 for the characteristics of the transmission path). Also, the representation in eqn. 4.16 does not require any bandlimiting conditions to be imposed on \( (h(t) \ast c(t)) \), these being assumed in eqn. 4.25, according to eqns. 4.21 and 4.24. Therefore it is more convenient to use the channel model in eqn. 4.16, particularly, when the system is to be computer-simulated. With this in view, eqn. 4.16 will be taken from now on to represent the impulse response of the baseband system that is equivalent to the QAM system in Fig. 4.1. By a similar argument, eqn. 4.17 will be taken to represent the noise component in the received signal \( r(t) \), without referring to the lowpass representation of \( c(t) \).

In deriving eqn. 4.15, perfect knowledge is assumed by the receiver about the suppressed carrier frequency of the transmitted signal, whose value may be offset, by a carrier link included in the transmission path (typically by less than \( \pm 5 \) Hz\(^{(23)} \)). In practice, however, the receiver tracks the signal carrier variations by employing the well known phase locked loop technique\(^{(135-152)} \). In the recovery process of the carrier frequency at
the receiver, an error is normally present between the actual carrier frequency and its estimated value. This error appears in fact as slow and random variations in the value of the reference phase $\theta$. These slow variations will be neglected here and $\theta$ will be assumed constant, so that eqn. 4.16 becomes,

$$y(t) = e^{-j\theta}[a(t) \star h(t) \star c(t)]e^{-j2\pi f_c t} \star u(t)$$  \hspace{1cm} (4.26)

Clearly, in the detection of the data symbols $\{a_i\}$ from the received signal $r(t)$, the detector requires prior knowledge about $y(t)$ and therefore about $\theta$. Usually, this can be obtained quite accurately using the mentioned phase locked loop techniques.\(^{(150)}\)

Eqn. 4.15 represents the baseband model of the QAM system assumed in Fig. 4.1. The baseband model is shown in Fig. 4.6. Here, $y(t)$ is in general complex-valued and represents the impulse response of a linear baseband channel formed by the baseband signal shaping filters $A_1$ and $A_2$, the two linear modulators, the transmission path, the receiver bandpass filter $C$ and the two coherent demodulators. The input data signal is given by the stream of the complex-valued data-symbols $\{s_i\}$, where $s_i$ is given by eqns. 4.1 and 4.4 and may have $m^2$ possible values. The additive noise is now represented by $u(t)$ which, in general, is complex-valued (eqn. 4.17).

Using eqn. 4.13, eqn. 4.17 may be written as

$$u(t) = \sqrt{2} \{[(n(t)e^{-j2\pi f_c t}) \star (c(t)e^{-j2\pi f_c t})]e^{-j\theta} \star u(t)$$  \hspace{1cm} (4.27)

Now, since $n(t)$ is a white Gaussian noise, i.e. with a flat spectrum over all frequencies, with a two-sided power spectral density of $\frac{1}{2}N_0$, and since the factor $e^{-j2\pi f_c t}$ in $(n(t)e^{-j2\pi f_c t})$ represents a pure shift of the spectrum of $n(t)$ in the frequency domain without affecting its power density, the power spectral density of $n(t)e^{-j2\pi f_c t}$ will be the same as that of $n(t)$ and therefore it is $\frac{1}{2}N_0$. Thus, the power spectral density of $u(t)$ in eqn. 4.27 is given by...
INPUT COMPLEX-VALUED DATA SIGNAL

\[ \sum s_i \delta(t-iT) \]

LINEAR BASEBAND CHANNEL WITH IMPULSE RESPONSE \[ y(t) \] (Eqn. 4.16)

\[ r(t) \] (Eqn. 4.15)

\[ u(t) \] NOISE (Eqn. 4.17)

**FIGURE 4.6**: Baseband Model of the QAM System in Fig. 4.1
where $C(f+f_c)$ is the Fourier transform of $c(t)e^{-j2\pi fc t}$. Eqn. 4.28 shows clearly that $u(t)$ is a lowpass random process. From eqn. 4.28 and according to the Weiner-Khintchine theorem, the auto-correlation function of $u(t)$ is given by

$$R_u(\tau) = N_0 \int_{-\infty}^{\infty} |C(f+f_c)|^2 |B(f)|^2 e^{j2\pi \tau f} df$$

(4.29)

For $B(f)$ as shown in Fig. 4.5, eqn. 4.29 becomes,

$$R_u(\tau) = N_0 \int_{-1/2T}^{1/2T} |C(f+f_c)|^2 e^{j2\pi \tau f} df$$

(4.30)

$R_u(\tau)$ in eqn. 4.30 is in general complex-valued. In Appendix A, an alternative but rather lengthy derivation of eqn. 4.30 is given, where it is also shown that the auto-correlation function of each of the real and imaginary parts of $u(t)$ is given by half the real part of $R_u(\tau)$, whereas the cross-correlation function of those parts is an odd function of $(\tau)$ and is given by half the imaginary part of $R_u(\tau)$, and when $R_u(\tau)$ is purely real, then the real and imaginary parts of $u(t)$ are uncorrelated with each other. Clearly, this may be achieved when the transfer function of the receiver filter $C$ over the positive frequencies is symmetric about $f_c$, i.e., when $C(f+f_c)$ for $-1/2T < f < 1/2T$ is symmetric about zero. In the special case when $|C(f)|$ is such that

$$|C(f)| = 1 \quad \text{for} \quad \frac{1}{2T} < |f| < \frac{1}{2T}$$

$$= 0 \quad \text{elsewhere}$$

(4.31)

which implies that

$$|C(f+f_c)| = 1 \quad \text{for} \quad \frac{1}{2T} < f < \frac{1}{2T}$$

and

$$\frac{1}{2T} < f < 2f_c \quad \frac{1}{2T} \quad \text{elsewhere}$$

(4.32)

then eqn. 4.30, gives,
\[ R_u(\tau) = N_0 \int_{-1/2T}^{1/2T} e^{j2\pi f \tau} df \]
where
\[ R_u(0) = N_0 \int_{-1/2T}^{1/2T} |C(f + f_c)|^2 df \]
which is real-valued and therefore the real and imaginary parts of \( u(t) \) are uncorrelated.

From eqn. 4.30, the variance of \( u(t) \) is given by
\[ \sigma_u^2 = R_u(0) = N_0 \int_{-1/2T}^{1/2T} |C(f + f_c)|^2 df \]  \[ (4.34) \]
and for the special case in eqn. 4.31,
\[ \sigma_u^2 = \frac{N_0}{T} \]  \[ (4.35) \]

From eqn. 4.15, the average energy per signal element in \( r(t) \) is
\[ \bar{\varepsilon}_r = E[|s_1|^2] \int_{-\infty}^{\infty} |y(t)|^2 dt \]  \[ (4.36) \]
where \( E[.] \) denotes the expected value. When the \( \{s_1\} \) are statistically independent and with zero mean eqn. 4.36 becomes,
\[ \bar{\varepsilon}_r = \frac{1}{s_i} \int_{-\infty}^{\infty} |y(t)|^2 dt \]  \[ (4.37) \]
where \( \frac{1}{s_i} \) is the expected value of \( |s_1|^2 \). From eqn. 4.16 and using the Parseval's theorem,
\[ \bar{\varepsilon}_r = \frac{1}{s_i} \int_{-1/2T}^{1/2T} |A(f)|^2 |H(f + f_c)|^2 |C(f + f_c)|^2 |B(f)|^2 df \]
\[ = \frac{1}{s_i} \int_{-1/2T}^{1/2T} |A(f)|^2 |H(f + f_c)|^2 |C(f + f_c)|^2 df \]  \[ (4.38) \]
where \( |B(f)| = 1 \) over the integration range.

To see the effect of the coherent demodulator on the signal energy, let the average energy per signal element in the noisy signal \( z(t) \), at the demodulator's input (Fig. 4.1), be determined. From eqns. 4.6 and 4.7, the
average energy per signal element in \( z(t) \) is given by

\[
\mathbb{E}_z = E \left[ \frac{1}{T} \int_{-\infty}^{\infty} \left| s_i e^{j2\pi f_c t} + s_i^* e^{-j2\pi f_c t} \right| a(t-iT) * h(t) * c(t) \right|^2 dt \]

which, by applying the Parseval's theorem, becomes,

\[
\mathbb{E}_z = E \left[ \frac{1}{T} \int_{-\infty}^{\infty} \left| s_i A(f-f_c) + s_i^* A(f+f_c) \right|^2 |H(f)|^2 |C(f)|^2 df \]

(4.40)

where \( H(f) \) is the Fourier transform of \( h(t) \). According to eqn. 4.3 and Fig. 4.2, \( |A(f-f_c)| \cdot |A(f+f_c)| = 0 \), and therefore eqn. 4.40 becomes,

\[
\mathbb{E}_z = \frac{1}{T} s_i^2 \int_{-1/2T}^{1/2T} \left( A(f-f_c) \right|^2 |H(f)|^2 |C(f)|^2 \right] df +
\]

\[
\int_{-1/2T}^{1/2T} \left( A(f+f_c) \right|^2 |H(f)|^2 |C(f)|^2 \right] df \]

(4.41)

which, by variable change gives,

\[
\mathbb{E}_z = \frac{1}{T} s_i^2 \int_{-1/2T}^{1/2T} \left( A(f) \right|^2 |H(f-f_c)|^2 |C(f+f_c)|^2 \right] df +
\]

\[
\int_{-1/2T}^{1/2T} \left( A(f) \right|^2 |H(f+f_c)|^2 |C(f-f_c)|^2 \right] df \]

(4.42)

The two terms on the right-hand side of eqn. 4.42 have equal values because each of \( |A(f)|, |H(f)| \) and \( |C(f)| \) are symmetric about zero. Thus,

\[
\mathbb{E}_z = s_i^2 \int_{-1/2T}^{1/2T} \left( A(f) \right|^2 |H(f+f_c)|^2 |C(f+f_c)|^2 \right] df \]

(4.43)

which is similar to eqn. 4.38 and therefore the coherent demodulator in Fig. 4.1 does not change the signal energy.

Now, since the data symbol \( s_i \) in eqn. 4.4 may have \( m \) possible values, it carries \( 2 \log_2 m \) bits of information. On the other hand, since \( s_{1,i} \) and \( s_{2,i} \) in eqn. 4.4 are statistically independent and both are given by eqn. 4.1, then,

\[
\frac{2}{s_i^2} = \frac{2}{s_{1,i}^2} + \frac{2}{s_{2,i}^2} = \frac{2}{m} \sum_{k=0}^{m-1} (2k-m+1)^2 \]

(4.44)
and \[ \frac{s_1}{2} = \frac{2}{s_2} \] (4.45)

Thus, from eqn. 4.38, the average energy per bit in \( r(t) \) is
\[
\mathcal{E}_0 \approx \frac{m^{-1} \sum_{k=0}^{m-1} (2k-m+1)^2}{m \log_2 m} \int_{-1/2T}^{1/2T} |A(f)|^2 |H(f+f_c)|^2 |C(f+f_c)|^2 df \] (4.46)

From eqn. 4.2, the average transmitted energy per signal element at the input of the transmission path (Fig. 4.1) is given by,
\[
\mathcal{E}_x = (\frac{s_1}{2}^2 + \frac{s_2}{2}^2) \int_{-\infty}^{\infty} a^2(t) dt \] (4.47)

from which, and after applying Parseval's theorem, the average transmitted energy per bit at the transmission path input is
\[
\mathcal{E}_0 = \frac{m^{-1} \sum_{k=0}^{m-1} (2k-m+1)^2}{m \log_2 m} \int_{-1/2T}^{1/2T} |A(f)|^2 df \] (4.48)

In one other approach to coherently demodulate the signal at the receiver is to use a 90° phase splitter as shown in Fig. 4.7. The signal \( z(t) \) at the output of the receiver filter \( C \) (Fig. 4.1) is fed to a Hilbert transform (Fig. 4.7) and the resultant signal \( \hat{z}(t) \) is combined with \( z(t) \) in a complex form to give the signal
\[
\hat{r}_1(t) = z(t) + j\hat{z}(t) \] (4.49)

where
\[
\hat{z}(t) = \text{H.T.}[z(t)] = z(t) \ast f(t) \] (4.50)

is the Hilbert transform of \( z(t) \), and \( f(t) \) is given by (186)
\[
f(t) = \frac{1}{\pi t} \] (4.51)

The Fourier transform of \( f(t) \), \( F(f) \) is defined as (186)
\[
F(f) = j \quad f < 0 \\
= 0 \quad f = 0 \\
= -j \quad f > 0 \] (4.52)

Now, from the property of the Hilbert transform (186)
\[ H.T[p(t)q(t)] = p(t)H.T[q(t)] \] (4.53)

and from eqn. 4.7
\[ \hat{z}(t) = \hat{x}(t)h(t)*c(t) + n(t)\hat{c}(t) \] (4.54)

where \( \hat{x}(t) \) and \( \hat{c}(t) \) are the Hilbert transforms of \( x(t) \) and \( c(t) \), respectively.

Thus, from eqns. 4.7, 4.49 and 4.54
\[ r_1(t) = (x(t)+j\hat{x}(t))h(t)*c(t)+n(t)\hat{c}(t) \] (4.55)

From the properties of Hilbert transforms
\[ H.T\{g(t)e^{j2\pi f_ct}\} = -jg(t)e^{j2\pi f_ct} \] (4.56)
where \( g(t) \) is bandlimited to \( |f| < f_c \), and from eqns. 4.3 and 4.6 and Fig. 4.2,
\[ (x(t)+j\hat{x}(t)) = \frac{1}{\sqrt{2}} \sum_i (s_i e^{j2\pi f_c t} + s_i e^{-j2\pi f_c t} + s_i e^{j2\pi f_c t} - s_i e^{-j2\pi f_c t})a(t-iT) \]
\[ = \sqrt{2} \sum_i s_i e^{j2\pi f_c t} a(t-iT) \] (4.57)
so that eqn. 4.55 becomes,
\[ r_1(t) = \sqrt{2} \sum_i s_i (e^{j2\pi f_c t}a(t-iT)*h(t)*c(t)) + n(t)*(c(t)+j\hat{c}(t)) \] (4.58)

The demodulated signal \( r(t) \) (Fig. 4.7) is now given by
\[ r(t) = \frac{1}{\sqrt{2}} r_1(t)e^{-j(2\pi f_c t+\theta)} \]
\[ = \sum_i [a(t-iT)*[h(t)*c(t)]e^{-j2\pi f_c t}]e^{-j\theta} + \]
\[ + \frac{1}{\sqrt{2}}[n(t)*(c(t)+j\hat{c}(t))]e^{-j(2\pi f_c t+\theta)} \] (4.59)
where eqn. 4.13 is used. Now, since the filters \( A_1, A_2, B_1 \) and \( B_2 \) in Fig. 4.1 have the same bandwidth (Figs. 4.2 and 4.5), then
\[ \{a(t-iT)*[h(t)*c(t)]e^{-j2\pi f_c t}\}e^{-j\theta} = \{a(t-iT)*[h(t)*c(t)]e^{-j2\pi f_c t}\}e^{-j\theta}*b(t) \] (4.60)
Also, from eqn. 4.52, the Fourier transform of \( (c(t)+jc(t')) \) is given by

\[
C(f)+j\hat{C}(f) = 2C(f) \quad f > 0
\]
\[
= 0 \quad f < 0
\] (4.61)

which, according to Figs. 4.4 and 4.5, gives,

\[
\frac{1}{\sqrt{2}} [n(t) * (c(t)+jc'(t))] e^{-j(2\pi f_c t+\theta)} = \sqrt{2} [(n(t)*c(t)) e^{-j(2\pi f_c t+\theta)}] * b(t)
\] (4.62)

so that, eqn. 4.59 becomes

\[
\begin{align*}
\mathcal{R}(t) &= \mathcal{R}_1(t) + \mathcal{R}_2(t) \\
\mathcal{R}_1(t) &= \sum_{1} a(t-iT) * [h(t)*c(t)] e^{-j2\pi f_c t} e^{-j\theta} * b(t) + \\
&\quad + \sqrt{2} [(n(t)*c(t)) e^{-j(2\pi f_c t+\theta)}] * b(t)
\end{align*}
\] (4.63)

which is exactly as eqns. 4.15 - 4.17. As a result, the demodulation process which uses the Hilbert transform (Fig. 4.7) is equivalent to that shown in Fig. 4.1.

Notice that the spectrum of the signal \( r_1(t) \) (eqn. 4.49) is given by

\[
R_1(f) = Z(f) + jZ(f)F(f)
\] (4.64)

where \( Z(f) \) is the spectrum of \( z(t) \) and \( Z(f)F(f) \) is the spectrum of \( \hat{z}(t) \).

\( F(f) \) is given by eqn. 4.52. From eqns. 4.64 and 4.52,

\[
R_1(f) = 2Z(f) \quad f > 0
\]
\[
= 0 \quad f < 0
\] (4.65)

When \( r_1(t) \) is multiplied by \( \frac{1}{\sqrt{2}} e^{-j(2\pi f_c t+\theta)} \) (Fig. 4.7), \( R_1(f) \) is shifted by \(-f_c \) Hz to give \( R(f) \), the spectrum of \( r(t) \). \( R(f) \) now contains no higher frequency components (no components outside the frequency band \(-\frac{1}{2T} \) to \( \frac{1}{2T} \) Hz, since \( Z(f) \) is originally bandlimited by filter \( C \), Fig. 4.4). Therefore, the lowpass filters \( B_1 \) and \( B_2 \) used in Fig. 4.1 to remove the higher frequency components of the demodulated signal (the second term in eqn. 4.14) are no longer required when the Hilbert transform technique (Fig. 4.7) is used.
In fact, the second demodulation method is easier to implement, since a tenth order lowpass filter is required to suppress the high frequencies in Fig. 4.1.(131)

To detect the transmitted data message \( \{ s_i \} \) at the receiver, the signal \( r(t) \) in eqn. 4.15 is sampled once per signal element, i.e. at a rate of \( \frac{1}{T} \) samples/second as shown in Fig. 4.8, to give the sequence \( \{ r_i \} \) which is fed to the detector. The latter, then, gives at its output the detected sequence \( \{ s'_i \} \), which should ideally be identical to the transmitted sequence \( \{ s_i \} \). The detector here must be capable of handling complex signals and it may be one of the types considered in Section 3.

From eqn. 4.15, the signal sample \( r_k \) at time \( t=kT \) is

\[
r_k = r(kT) = \sum_{i} s_i y(k-i) + u(kT)
\]

or

\[
r_k = \sum_{i} s_i y_{k-i} + u_k
\]

where \( y_{k-i} = y(k-i)T \) and \( u_k = u(kT) \). When the impulse response \( y(t) \) is of finite duration so that \( y_h = 0 \) for \( h<0 \) and \( h>g \), then \( r_k \) becomes,

\[
r_k = \sum_{h=0}^{g} s_{k-h} y_h + u_k
\]

The sequence \( \{ u_k \} \) represents the noise samples, and in the special case of eqns. 4.31 and 4.32, the autocorrelation function \( R_u(kT) \) in eqn. 4.33 becomes zero for all \( k \neq 0 \) with the result that the \( \{ u_k \} \) are uncorrelated. The special case of the receiver filter in eqn. 4.31 corresponds to the optimum receiving filter, under the assumed conditions, as has been shown in Section 3.5.
FIGURE 4.7: Model of the Coherent Demodulation of the QAM Signal Using a Phase Splitter

FIGURE 4.8: The Detection of the Transmitted Data from $r(t)$
5. THE TRANSMISSION OF BASEBAND SIGNALS OVER TELEPHONE LINES

5.1 INTRODUCTION

As has been mentioned in Section 4.1, most voice-frequency channels are regarded as bandpass channels and therefore unsuitable for the transmission of baseband signals. The conventional approach in using these channels is to let a baseband signal modulate a carrier so that the spectrum of the modulated signal matches the channel passband. With the introduction of partial-response signalling techniques,\(^{153-161}\) it became possible to shape the spectrum of the baseband signal to match the channel passband without modulation, but the existence of frequency modulation effects (frequency offset) over many of these channels prevents the implementation of such a system by conventional means.\(^{23}\) In fact, the effect of the frequency offset on the transmitted signal is such that the impulse response of the channel now varies with time in a cyclic or repetitive manner, and unless the frequency offset is determined and compensated for, it prevents the correct detection of the baseband signal altogether.\(^{23}\) Of course, carrier frequency-and-phase recovery techniques\(^{135-152}\) used in modulated-carrier systems take account of the frequency offset and compensate for it automatically, and this is probably the most compelling single reason for the transmission of modulated-carrier signals over voice-frequency channels.\(^{23}\)

The present section is devoted to the analysis of the frequency offset effects on baseband signals. Two schemes for the transmission of these signals without modulation are then analysed and compared with the QAM system.
5.2 THE FREQUENCY OFFSET MODEL

A telephone circuit between two subscribers may normally consist of two or more links connected in tandem, and one of these may be a carrier link which employs as the transmission path a coaxial cable, satellite link, microwave link or even HF-radio link. The transmission path itself here is a wideband channel and carries several signals, each using a different frequency band, in an arrangement of frequency-division multiplexing (FDM). A process of single sideband suppressed carrier amplitude modulation (S.S.B.) is used here to shift the signal spectrum to the required band in the carrier link.\(^{(23)}\) At the receiver, the multiplexed signals are isolated by band-pass filtering, and each is shifted back to its frequency band by a process of linear demodulation.

Let \(x(t)\) be the signal at the input of the carrier link. The signal at the S.S.B. linear modulator output is given by\(^{(3)}\)

\[
\rho(t) = \sqrt{2}[x(t)\cos(2\pi f_0 t) - \hat{x}(t)\sin(2\pi f_0 t)]
\]  

where \(f_0\) is the carrier frequency and \(\hat{x}(t)\) is the Hilbert transform of \(x(t)\). The Fourier transform of \(\hat{x}(t)\) is related to the Fourier transform of \(x(t)\) by

\[
\hat{X}(f) = X(f)F(f)
\]  

where \(F(f)\) is given by eqn. 4.52. At the other end of the carrier link, the signal \(\rho(t)\) is coherently demodulated by a reference carrier whose frequency should ideally be \(f_0\). In practice the reference carrier may differ from \(f_0\) by, say \(p\) Hz, where \(p\) may have a value up to \(\pm 5\) Hz.\(^{(23)}\)

By multiplying the signal \(\rho(t)\) by \(\sqrt{2}\cos(2\pi (f_0 - p)t)\) and removing the high frequency component of the resultant signal by lowpass filtering, the carrier link output signal will be given by\(^{(162)}\)
\[ m(t) = x(t)\cos(2\pi pt) - x(t)\sin(2\pi pt) \quad (5.3) \]

Eqn. 5.3 assumes that the carrier link does not introduce into the signal any distortion other than the frequency offset. Clearly, when \( p=0 \), \( m(t) \) becomes equal to \( x(t) \).

Let now \( x(t) \) be a baseband signal composed of a series of signal-elements (data pulses) transmitted at a rate of \( \frac{1}{T} \) bauds, such that
\[ x(t) = \sum_i s_i g(t-iT) \]
where the \( \{s_i\} \) are the data symbols and \( g(t) \) is the impulse response of the transmitter filter in the data equipment (the modem). Thus, eqn. 5.3, becomes,
\[ m(t) = \sum_i s_i [g(t-iT)\cos(2\pi pt) - \hat{g}(t-iT)\sin(2\pi pt)] \quad (5.4) \]
where \( \hat{g}(t) \) is the Hilbert transform of \( g(t) \). In a compact form, eqn. 5.4 may be written as
\[ m(t) = \sum_i s_i y(t-iT) \quad (5.5) \]
where \( y(t-iT) \) is the impulse response of the channel including the carrier link at time \( t=iT \), and is given by
\[ y(t-iT) = g(t-iT)\cos(2\pi pt) - \hat{g}(t-iT)\sin(2\pi pt) \quad (5.6) \]

Now, the product \([g(t-iT)\cos(2\pi pt)]\) in eqn. 5.6 represents a waveform that is dependent on the instant \( iT \) at which the data symbol \( s_i \) is transmitted, unlike the product \([g(t-iT)\cos(2\pi p(t-iT))]\), which represents a waveform that is the same for any \( i \). Let \( G(f) \) be the Fourier transform of \( g(t) \), then the Fourier transform of \([g(t-iT)\cos(2\pi pt)]\) is
\[ \{iG(f-p)+iG(f+p)e^{-j2\pi fiT}\}, \]
which apart from a linear phase shift represented by \( e^{-j2\pi fiT} \), is independent of \( i \). The Fourier transform of \([g(t-iT)\cos(2\pi pt)]\) is
\[ \{iG(f-p)e^{j2\pi fiT}+iG(f+p)e^{-j2\pi fiT}\}e^{-j2\pi fiT} \] which is
now dependent on the value of \( i \). The factor \( e^{-j2\pi fiT} \) is a linear phase shift (pure delay), but \( e^{j2\pi iT} \) and \( e^{-j2\pi iT} \) represent a change of \( 2\pi pT \) rad. in the phase of the frequency offset component in \( \{g(t-iT)\cos(2\pi pt)\} \) in every data element relative to the phase of the corresponding component in the immediately preceding data element. The same applies to the product \( \{\hat{g}(t-iT)\sin(2\pi pt)\} \), with the result that \( y(t-iT) \), in eqn. 5.6, is now a function of \( i \).

When \( m(t) \) in eqn. 5.4 is sampled once per signal-element, the \( k^{th} \) sample at time \( t=kT \) is given by

\[
m_k = \sum_{i=1}^{L} s_i [g(kT-iT)\cos(2\pi pkT) - \hat{g}(kT-iT)\sin(2\pi pkT)]
\]

For practical purposes, \( g(t) \) and \( \hat{g}(t) \) are of finite duration and are such that \( g(hT) = \hat{g}(hT) = 0 \) for \( h < 0 \) and \( h > g \). Thus, eqn. 5.7 becomes,

\[
m_k = \sum_{i=k-g}^{k} s_i [g((k-i)T)\cos(2\pi pkT) - \hat{g}(k-i)T)\sin(2\pi pkT)]
\]

where

\[
g_h = g(hT)
\]

and

\[
\hat{g}_h = \hat{g}(hT)
\]

Now, putting

\[
y_{k,h} = g_h \cos(2\pi pkT) - \hat{g}_h \sin(2\pi pkT)
\]

then eqn. 5.8 becomes,

\[
m_k = \sum_{h=0}^{g} s_{k-h} y_{k,h}
\]

where subscripting \( y \) by \( k \) indicates the time-varying nature of the \( \{y_{k,h}\} \).

The sampled impulse response of the time-varying channel is now given by the \((g+1)\)-component vector

\[
y_k = y_{k,0}, y_{k,1}, \cdots, y_{k,g}
\]
Notice in eqn. 5.9 that the factors \(\cos(2\pi pkT)\) and \(\sin(2\pi pkT)\) are independent of \(h\), so that \(Y_k\) in eqn. 5.11 relates to the time-invariant sampled impulse responses

\[
G = g_0, g_1, \ldots, g_N
\]

and

\[
\hat{G} = \hat{g}_0, \hat{g}_1, \ldots, \hat{g}_N
\]

by the equation

\[
Y_k = \cos(2\pi pkT)G - \sin(2\pi pkT)\hat{G}
\]

i.e. the sampled impulse response concerning one signal sample \(m_k\) at any time \(t=kT\) is a linear combination of two time-invariant sampled impulse responses. Of course, it is assumed here that the frequency offset is introduced after generating the baseband signal \(x(t)\) (eqns. 5.3-5.4).

5.3 THE COMPENSATION FOR FREQUENCY OFFSET IN BASEBAND SIGNALS

Consider the baseband data transmission system in Fig. 5.1. The sequence of real-valued data symbols \(\{s_i\}\) is fed to the transmitter filter \(G\) whose impulse response is \(g(t)\). The transmission path is a voice-frequency channel (with bandwidth 300 to 3000 Hz) with an overall impulse response \(h(t)\). A frequency offset of \(p\) Hz is assumed to be introduced into the transmitted signal at the input of the transmission path. The receiver filter \(C\) has the impulse response \(c(t)\), and the signal \(z(t)\) at its output is fed to a Hilbert transformer (filter \(F\)) with the impulse response \(f(t)\) (eqn. 4.51) whose Fourier transform is given by eqn. 4.52. The complex-valued signal \(r(t)\) is composed of the signal \(z(t)\) and its Hilbert transform \(\hat{z}(t)\) as shown in Fig. 5.1.

In the absence of frequency offset and additive noise, the signal \(r(t)\) is given by,

\[
r(t) = z(t) + j\hat{z}(t)
\]
FIGURE 5.1: Frequency Offset Correction by Means of a Phase Splitter
and \( z(t) \) is given by
\[
z(t) = \sum_i s_i y(t-iT) \tag{5.16}
\]
where
\[
y(t) = g(t)h(t)c(t) \tag{5.17}
\]
Thus,
\[
r(t) = \sum_i s_i [y(t-iT) + j\hat{y}(t-iT)] \tag{5.18}
\]
where \( \hat{y}(t) \) is the Hilbert transform of \( y(t) \).

In the presence of frequency offset, \( z(t) \) is given by (Fig. 5.1)
\[
z(t) = m(t)h(t)c(t) \tag{5.19}
\]
which, with eqn. 5.3, gives
\[
z(t) = [x(t)\cos(2\pi pt) - \hat{x}(t)\sin(2\pi pt)]*d(t) \tag{5.20}
\]
where
\[
x(t) = \sum_i s_i g(t-iT) \tag{5.21}
\]
and
\[
d(t) = h(t)c(t) \tag{5.22}
\]
From eqns. 5.19 and 5.22,
\[
z(t) = m(t)d(t) \tag{5.23}
\]
and \( r(t) \) in eqn. 5.15 becomes,
\[
r(t) = [m(t)+j\hat{m}(t)]d(t) \tag{5.24}
\]
where eqn. 4.53 has been used. \( \hat{m}(t) \) is the Hilbert transform of \( m(t) \).

In the frequency domain, eqn. 5.24 becomes,
\[
R(f) = [M(f)+j\hat{M}(f)]D(f) \tag{5.25}
\]
where \( R(f), M(f), \hat{M}(f) \) and \( D(f) \), are the Fourier transforms of \( r(t), m(t), \)
\( \hat{m}(t) \) and \( d(t) \), respectively. From eqn. 5.3, \( M(f) \) is given by
\[
M(f) = X(f)[\frac{-1}{2} \delta(f-p)+\frac{i}{2} \delta(f+p)] - X(f)[2j(1-U(f))]* \]
\[
* [\frac{-1}{2} \delta(f-p)+\frac{i}{2} \delta(f+p)] \tag{5.26}
\]
where \( X(f) \) is the Fourier transform of \( x(t) \) and \( U(f) \) is the step function of \( f \).
\[ U(f) = 1, \quad f \geq 0 \]
\[ = 0, \quad f < 0 \]

In eqn. 5.26, the Fourier transform of the Hilbert transform given by eqn. 4.52 has been expressed as

\[ F(f) = 2j[I - U(f)] \quad (5.28) \]

so that the Fourier transform of \( \dot{x}(t) \) is given by \( X(f)[2j(I - U(f))] \). The representation in eqn. 5.28 is not quite correct because it assumes non-zero amplitude of the zero frequency (d.c.) which is in contrast with eqn. 4.52. However, since the concern here is with voice-frequency channels, the transmitted signal is shaped to match these channels, and the transmitter filter \( G \) has therefore a transfer function \( G(f) \) such that

\[ G(f) = 0 \quad f < 300 \text{ Hz} \quad (5.29) \]

Consequently, \( X(0) = 0 \) (eqn. 5.21), and the use of eqn. 5.28 in eqn. 5.26 is justified. Now, re-arranging eqn. 5.26 gives

\[ M(f) = X(f + p)[1 - U(f + p)] + X(f - p)U(f - p) \quad (5.30) \]

On the other hand, and since \( \dot{M}(f) = M(f)F(f) \), where \( F(f) \) is given by eqn. 5.28, eqn. 5.25 becomes,

\[ R(f) = M(f)[1 + jF(f)]D(f) \]

Substituting for \( M(f) \) from eqn. 5.30 gives,

\[ R(f) = \{2X(f + p)[1 - U(f + p)]U(f) + 2X(f - p)U(f - p)U(f)\}D(f) \quad (5.32) \]

Now, when \( p > 0 \), eqn. 5.32 becomes,

\[ R(f) = 2X(f - p)U(f - p)D(f) \quad (5.33) \]

and when \( p < 0 \), and noting that \( X(f) \) is bandlimited as \( G(f) \) in eqn. 5.29, eqn. 5.32 becomes,

\[ R(f) = 2X(f - p)U(f)D(f) \quad (5.34) \]
Again, because of eqn. 5.29, eqn. 5.34 may be written as eqn. 5.33, bearing in mind that \( p < 300 \) Hz. Now, since \( p \) mostly has values less than 5 Hz, and since \( D(f) \) extends over the frequency band 300 to about 3000 Hz it can be replaced by \( D(f-p) \) without introducing a significant error. Thus, eqn. 5.33 becomes,

\[
R(f) = 2X(f-p)U(f-p)D(f-p)
\]

Clearly, \( R(f) = 0 \) for \( f < 0 \). The additive noise is ignored here.

Notice that \( X(f-p) \) may not be replaced by \( X(f) \) in the same fashion applied to \( D(f) \). In fact, \( D(f) \) is the transfer function of a time-invariant system and replacing \( D(f) \) by \( D(f-p) \) represents a negligible fixed shift by \( p \) Hz of the transfer function in the frequency domain. Although the same argument applies to \( X(f) \), which represents the spectrum of a group of data-elements \( \{s_i g(t-iT)\} \) (eqn. 5.21), it does not apply to the individual spectra of these data-elements. This may be shown as follows. From eqn. 5.21,

\[
X(f) = \sum_i s_i G(f)e^{-j2\pi fiT}
\]

which gives

\[
X(f-p) = \sum_i s_i G(f-p)e^{-j2\pi (f-p)iT}e^{j2\pi piT}
\]

The spectrum of the \( i \)th data-element is now given by \( [s_i G(f-p)e^{-j2\pi fiT}e^{j2\pi piT}] \), where it is clear that the effect of the frequency offset is not only the negligible fixed shift of \( G(f) \) in the frequency domain, but also the multiplication by \( e^{j2\pi piT} \) which takes different values for the different \( \{s_i\} \). Clearly, replacing \( X(f-p) \) by \( X(f) \) implies ignoring the components \( \{e^{j2\pi piT}\} \) in the different data-elements.

Now, from eqn. 5.28, \( U(f) = i[1+jF(f)] \), so that \( R(f) \) in eqn. 5.35 becomes,

\[
R(f) = 2X(f-p)i[1+jF(f-p)]D(f-p)
\]

\[
= [X(f-p)+jX(f-p)F(f-p)]D(f-p)
\]

\[
= [X(f-p)+j\tilde{X}(f-p)]D(f-p)
\]

(5.36)
where $\hat{X}(f) = X(f)F(f)$ is the Fourier transform of $x(t)$, the Hilbert transform of $x(t)$. The inverse Fourier transform of $R(f)$ is now

$$r(t) = \{[x(t)+j\hat{x}(t)]d(t)e^{j2\pi pt}\}
= \sum_i s_i [g(t-iT)+j\hat{g}(t-iT)]h(t)c(t)e^{j2\pi pt} \quad (5.37)$$

where $x(t)$ and $d(t)$ are replaced by their values in eqns. 5.21 and 5.22, respectively. Of course, $\hat{g}(t)$ is the Hilbert transform of $g(t)$. From eqn. 5.17, eqn. 5.37 becomes

$$r(t) = \sum_i s_i [y(t-iT)+j\hat{y}(t-iT)]e^{j2\pi pt} \quad (5.38)$$

The property of the Hilbert transform in eqn. 4.53 has been used in obtaining $\hat{y}(t)$, the Hilbert transform of $y(t)$. Of course, the additive noise is ignored in eqn. 5.38. It should be noted here that eqn. 5.38 has been derived under the conditions that eqn. 5.29 is true, $p<300$ Hz and $D(f)$ may be replaced by $D(f-p)$ (eqns. 5.34 and 5.35) without introducing a significant error.

By comparing eqn. 5.38 with eqn. 5.18, which represents the case of no frequency offset, it follows that to remove the frequency offset from the received signal it is sufficient to multiply the latter by $e^{j\theta(t)}$, where

$$\theta(t) = -2\pi pt \quad (5.39)$$

The value of $\theta(t)$ may be estimated quite accurately, using the techniques of carrier frequency-and-phase recovery employed in carrier-modulated signals systems. (135-152)

Having removed the frequency offset, the resultant received signal will have the form in eqn. 5.18. The data symbols $\{s_i\}$ may now be detected from the real part of $r_1(t)$ (Fig. 5.1). The $\{s_i\}$ are assumed to have real values. In Section 5.4.2, the case when the $\{s_i\}$ are complex-valued is considered.
5.4 **THE TRANSMISSION OF BASEBAND SIGNALS OVER TELEPHONE LINES**

As has been shown, the frequency offset may be removed from the received baseband signal, so that it is no longer a reason to avoid the use of such signals over telephone lines. Furthermore, bandpass shaping of baseband signals can easily be achieved using the partial-response technique. (153-161) So then, why should not the transmission of baseband signals be considered again? And, could such signals have any advantage in tolerance to additive noise over carrier-modulated signals?

In fact, it may be shown (23) that, for a transmission path which does not introduce any signal distortion and for the same average transmitted energy per bit, a quaternary phase-shift keyed (PSK) signal (of which the 4-point QAM signal is a special case) has the same tolerance to additive white Gaussian noise as does a binary polar baseband signal. In the presence of signal distortion, i.e. intersymbol interference, the relative tolerance to additive Gaussian noise becomes, unfortunately, difficult to evaluate directly. Assuming that the optimum detector (defined here as the maximum likelihood detector) is employed at the receiver, the evaluation of the relative tolerances of different signal formats to additive white Gaussian noise involves the evaluation of the minimum distance $d_{\text{min}}$ (eqn. 3.96) for the given intersymbol interference, which is a difficult thing to do.

Taking the QAM signal as a reference for the purpose of comparison, the different aspects of the signal energy and noise statistics will be discussed here for two different baseband signals (Sections 5.4.1 and 5.4.2). It will then be shown in Section 5.5 that for the same attenuation distortion (defined in Section 2.2), these signals have the same tolerance to additive white Gaussian noise as does the QAM signal.
5.4.1 The One-Dimensional Baseband Data Transmission System

An m-level baseband polar signal is transmitted here. The data transmission system is shown in Fig. 5.2. The information to be transmitted is carried over the real-valued statistically independent data symbols \{s_i\}, where \(s_i\) is equally likely to have any of \(m\) possible values,

\[ s_i = 2^i - m + 1, \quad i=0,1,...,m-1 \]  

(5.40)

The \{s_i\} are fed to Filter G whose real-valued impulse response is \(\frac{1}{\sqrt{2}} g(t)\). The factor \(\frac{1}{\sqrt{2}}\) is used for the purpose of comparison later with the QAM system. The absolute value of the transfer function of Filter G, which is \(\frac{1}{\sqrt{2}} |G(f)|\), is shown in Fig. 5.3, and the passband of this filter lies within the passband of the transmission path. It is assumed here that the signal element rate is \(\frac{1}{T_1}\) bauds, so that the transmitter filter (Filter G) is used at the Nyquist rate. The transmission path has a real-valued impulse response \(h(t)\) whose Fourier transform is \(H(f)\), and it introduces into the transmitted signal a frequency offset of \(p\) Hz in the manner discussed in Section 5.2. Also, the transmission path is assumed here to add to the signal white Gaussian noise \(n(t)\) with zero mean and two-sided power spectral density of \(\frac{1}{2}N_0\). The receiver filter (Filter C, Fig. 5.2) has a real-valued impulse response \(c(t)\) and a flat amplitude response \(|C(f)|\) which extends over the signal frequency band as shown in Fig. 5.4.

The signal \(z(t)\) (Fig. 5.2) and its Hilbert transform \(\hat{z}(t)\) form the complex-valued signal \(p(t)\). \(\hat{z}(t)\) is obtained by passing \(z(t)\) into a Hilbert transformer (Filter F, Fig. 5.2). The resultant signal \(p(t)\) is now multiplied by \(e^{-j2\pi p't}\) to remove the frequency offset and give the complex-valued signal \(p_1(t)\). \(p'\) is an estimate of the frequency offset \(p\) and is produced by a phase-locked loop (not shown on Fig. 5.2) which is normally used in the carrier frequency-and-phase recovery in carrier-modulated signals systems.
FIGURE 5.2: Model of the One-Dimensional Baseband Data Transmission System
FIGURE 5.3: Absolute Value of the Transfer Function of Transmitter Filter $G$ in Fig. 5.2

FIGURE 5.4: Absolute Value of the Transfer Function of Receiver Filter $C$ in Fig. 5.2
It is assumed here that the estimate $p'$ is accurate enough so that $p' = p$. Now, the real part of the complex-valued and frequency-offset-free signal $p_1(t)$ is taken as the received signal $r(t)$ which is sampled once per signal element to give the received real-valued signal sequence $\{r_i\}$. These are fed to the detector to give at its output the detected data sequence $\{s'_i\}$, which, in the absence of errors is identical to the input sequence $\{s_i\}$.

In a manner similar to the derivation of eqn. 5.38, it may be shown that the signal component in $p(t)$ (Fig. 5.2) is given by

$$\sum_i s_i [y(t-iT_1) + j\hat{y}(t-iT_1)]e^{j2\pi pt}$$

where

$$y(t) = \frac{1}{\sqrt{2}} g(t) * h(t) * c(t)$$

and $\hat{y}(t)$ is the Hilbert transform of $y(t)$. Also, from Fig. 5.2 and eqn. 4.53, the noise component in $p(t)$ is given by

$$n(t) [* [c(t) + j\hat{c}(t)]$$

where $\hat{c}(t)$ is the Hilbert transform of $c(t)$. Thus,

$$p(t) = \sum_i s_i [y(t-iT_1) + j\hat{y}(t-iT_1)]e^{j2\pi pt} +$$

$$+ n(t) [* [c(t) + j\hat{c}(t)]$$

(5.42)

After removing the frequency offset, i.e., after multiplying $p(t)$ by $e^{-j2\pi p't}$, where as assumed before $p' = p$, the signal at the output of the frequency offset corrector (Fig. 5.2) is given by

$$p_1(t) = \sum_i s_i [y(t-iT_1) + j\hat{y}(t-iT_1)] +$$

$$+ n(t) [* [c(t) + j\hat{c}(t)]e^{-j2\pi pt}$$

(5.43)

Now, the frequency-offset-free signal at the output of the linear baseband channel (Fig. 5.2) is given by the real part of $p_1(t)$, i.e.,

$$r(t) = \text{Real}[p_1(t)]$$

$$= \sum_i s_i y(t-iT_1) + u(t)$$

(5.44)
where $y(t)$ is given by eqn. 5.41 and is real-valued, and $u(t)$ is the real-valued noise component in $r(t)$ and is given by

$$u(t) = \text{Real} \{ n(t) * [c(t)+j\hat{c}(t)] e^{-j2\pi ft} \}$$  \hspace{1cm} (5.45)$$

Notice in eqn. 5.44 that the element rate is $\frac{1}{T_1}$ bauds and that both $|G(f)|$ and $|C(f)|$ in Figs. 5.3 and 5.4, respectively, are used at the Nyquist rate (the symbol $\frac{1}{T_1}$ is used in this section instead of the symbol $\frac{1}{T}$, the latter being used with other systems which transmit at a different element rate and which will be compared with this system in Section 5.5).

The signal $r(t)$ is now sampled once per signal element to give the received signal sequence $\{r_i\}$, where

$$r_i = \sum_{h=0}^{g} s_{i-h} y_h + u_i$$  \hspace{1cm} (5.46)$$

$$y_h = y(hT_1)$$

$$u_i = u(iT_1)$$

and $y_h = 0$ for $h < 0$ and $h > g$. The $(g+1)$-component vector

$$\mathbf{Y} = [y_0, y_1, \ldots, y_g]$$  \hspace{1cm} (5.47)$$

represents the sampled impulse response of the linear baseband channel in Fig. 5.2.

From eqn. 5.44, the average energy per signal element in $r(t)$ is

$$E_r = E[\int_{-\infty}^{\infty} s_i^2 y^2(t) dt]$$  \hspace{1cm} (5.48)$$

where $E[.]$ denotes the expected value. From eqn. 5.41, Parseval's theorem and Figs. 5.3 and 5.4,

$$E_r = \overline{s_i^2} \int_{-1/2T_1}^{1/2T_1} \frac{1}{2} |G(f)|^2 |H(f)|^2 |C(f)|^2 df$$  \hspace{1cm} (5.48a)$$

where $\overline{s_i^2}$ is the expected value of $s_i^2$. Notice here that $E_r$ is also the average energy per signal element in $z(t)$ at the receiver filter output (Fig. 5.2). From eqn. 5.40,
\[ \overline{s_i^2} = \frac{1}{m} \sum_{j=0}^{m-1} (2^j \cdot m+1)^2 \]  

Also, since a data symbol \( s_i \) may have \( m \) equally likely possible values, it carries \( \log_2 m \) bits of information. Thus, from eqns. 5.48a and 5.49, the average energy per bit in \( r(t) \) is:

\[
E_{r_0} = \frac{1}{m \log_2 m} \sum_{j=0}^{m-1} \frac{(2^j \cdot m+1)^2}{\int_{-1/2T_1}^{1/2T_1} \frac{1}{2} |G(f)|^2 |H(f)|^2 |C(f)|^2 df} \quad (5.50)
\]

and will be used in the comparison with the QAM system later on.

From Fig. 5.2, the average energy per signal element in \( x(t) \) at the input of the transmission path is

\[
E_x = E[\int_{-\infty}^{\infty} \overline{s_i^2} \overline{g^2(t)} dt] = \overline{s_i^2} \int_{-1/2T_1}^{1/2T_1} \frac{1}{2} |G(f)|^2 df \quad (5.51)
\]

where Parseval's theorem is used. \( |G(f)| \) is shown in Fig. 5.3. From eqns. 5.49 and 5.51, the average transmitted energy per bit at the input of the transmission path is

\[
E_0 = \frac{1}{m \log_2 m} \sum_{j=0}^{m-1} \frac{(2^j \cdot m+1)^2}{\int_{-1/2T_1}^{1/2T_1} \frac{1}{2} |G(f)|^2 df} \quad (5.52)
\]

The autocorrelation function of \( u(t) \) in eqn. 5.45 is shown in Appendix B (eqn. B.24) to be given by

\[
R_u(\tau) = N_0 \text{Re} \left[ \int_{-\infty}^{\infty} |C(f)|^2 e^{j2\pi(f-p)\tau} df \right] \quad (5.53)
\]

where, of course, \( N_0 \) is the power spectral density of \( n(t) \) (eqn. 5.45). The integration limits in eqn. 5.53 should not be confused with the fact
that $|C(f)|$ has non-zero values over the negative frequencies as well as over the positive frequencies (Fig. 5.4). $|C(f)|$ is in fact symmetric about zero, because $c(t)$ is real-valued. When $p=0$, it may be shown that eqn. 5.53 becomes

$$R_u(\tau)|_{p=0} = iN_0 \int_{-\infty}^{\infty} |C(f)|^2 e^{j2\pi f \tau} df$$

(5.54)

which is, in fact, the autocorrelation function of the noise component in the signal $z(t)$ at the input of the phase splitter both in the presence and absence of the frequency offset. The integration limits in eqn. 5.53 and 5.54 are in fact for the general case where $C(f)$ may have any spectral shaping including of course the given case in Fig. 5.4.

The variance of $u(t)$ is given by $R_u(0)$, so that from eqn. 5.53 (or equivalently eqn. 5.54) results

$$\sigma_u^2 = R_u(0) = N_0 \int_{-\infty}^{\infty} |C(f)|^2 df$$

$$= iN_0 \int_{-\infty}^{\infty} |C(f)|^2 df$$

(5.55)

From eqn. 5.55, it is clear that the variance of $u(t)$ is independent of the frequency offset and not affected by the arrangement of frequency offset correction.

Since the interest here is in the samples of $u(t)$ at times $t=iT_1$, $i=1,2,...$, (eqn. 5.46), two noise components $u_i$ and $u_{i+k}$, separated by $\tau=kT_1$ seconds, are correlated according to $R_{u_i,u_{i+k}}$. Thus, from eqn. 5.53,

$$R_{u_i,u_{i+k}} = N_0 \text{Real} \left[ e^{-j2\pi pkT_1} \int_{-\infty}^{\infty} |C(f)|^2 e^{j2\pi f kT_1} df \right]$$

(5.56)

In the special case when Filter C (Fig. 5.2) has a rectangular amplitude response

$$|C(f)| = 1$$

$$f_\omega < |f| < \frac{1}{2T_1}$$

$$= 0 \text{ elsewhere}$$

(5.57)

where $f_\omega \approx 300$ Hz is the lower cut-off frequency of Filter C, eqn. 5.56 gives
\[ \mathcal{R}_{u_1, u_{i+k}} = N_0 \text{Real} \left[ e^{-j2\pi pk T_1} e^{j2\pi f_e k T_1} \frac{e^{j\pi}}{j2\pi k T_1} \right] \] (5.58)

and according to eqn. 5.55, the variance of the \( \{u_i\} \) is
\[ \sigma_u^2 = N_0 \left[ \frac{1}{2T_1} - f_e \right] \] (5.59)

For a voice-frequency channel, \( f_e = 300 \) Hz and \( \frac{1}{2T_1} = 3000 \) Hz, so that \( \sigma_u^2 \) in eqn. 5.59 may be taken as
\[ \sigma_u^2 = \frac{N_0}{2T_1} \] (5.60)
with an error of less than 0.5 dB.

5.4.2 The Two-Dimensional Baseband Data Transmission System

Here two \( m \)-level baseband signals in phase quadrature (analogous to the QAM signal) are transmitted. The \( 90^\circ \) phase difference between the two signal components is achieved here by means of the Hilbert transform pairs. The digital data transmission system is shown in Fig. 5.5. The information to be transmitted is carried over the data-symbols \( \{s_i\} \), where \( s_i \) is now complex-valued (two-dimensional) and may have any of \( m^2 \) possible complex values given by,
\[ s_i = s_{1,i} + js_{2,i} \]
where
\[ s_{1,i}, s_{2,i} = 2^k-m+1, k=0,1,\ldots,(m-1) \] (5.61)
so that each of the real and imaginary parts of \( s_i \) is an \( m \)-level data-symbol. It is assumed here that the \( \{s_i\} \) are statistically independent and equally likely to have any of their \( m^2 \) possible values. The real part of \( s_i, s_{1,i} \), is fed to the transmitter filter \( G_1 \) whose impulse response is \( \frac{1}{\sqrt{2}} g(t) \), whereas the imaginary part of \( s_i, s_{2,i} \), is fed to the transmitter filter \( G_2 \) whose impulse response is \( (- \frac{1}{\sqrt{2}} \hat{g}(t)) \), where \( \hat{g}(t) \) is the Hilbert transform of \( g(t) \). The amplitude response of each of Filters \( G_1 \) and \( G_2 \) is \( \frac{1}{\sqrt{2}}|G(f)| \) (Fig. 5.6), which is arranged such that it lies within the passband of the transmission path. The signal-element rate is \( \frac{1}{T} \) bauds, and this is half the Nyquist rate, for Filters \( G_1 \) and \( G_2 \). The output signals of Filters \( G_1 \) and \( G_2 \) are added together to form a real-valued Hilbert transform pair baseband signal which is fed to the transmission
FIGURE 5.5: Model of the Two-Dimensional Baseband Data Transmission System
FIGURE 5.6: Absolute Value of the Transfer Function of Transmitter Filters $G_1$ and $G_2$ in Fig. 5.5.

FIGURE 5.7: Absolute Value of the Transfer Function of Receiver Filter C in Fig. 5.5.
path. Clearly, this signal consists now of two m-level signals in phase quadrature. The transmission path introduces into the signal a frequency offset of p Hz. The only additive noise assumed here is a white Gaussian noise n(t) with zero mean and two-sided power spectral density $\frac{1}{2}N_0$, which is added to the data signal at the output of the transmission path. The receiver filter C has a transfer function with an absolute value $|C(f)|$ which is shown in Fig. 5.7. The signal at the output of Filter C is fed to the phase splitter whose output signal p(t) is complex-valued. p(t) is then multiplied by $\frac{1}{\sqrt{2}} e^{-j2\pi p't}$ to remove the frequency offset. p' is an estimate of p and is produced by a phase-locked loop (not shown on Fig. 5.5) which is normally used in the carrier frequency-and-phase recovery in carrier-modulated signals systems. It is assumed here that p' is accurate enough so that p'=p. The factor $\frac{1}{\sqrt{2}}$ in $\frac{1}{\sqrt{2}} e^{-j2\pi p't}$ is used for convenience and has no effect on the signal-to-noise ratio, since it multiplies the data component as well as the noise component in p(t). The complex-valued signal r(t) (Fig.5.5), in which the frequency offset has been removed, is now sampled once per signal element, i.e. at a rate of $\frac{1}{T}$ samples per second, and the resultant signal sequence $\{r_i\}$ is fed to the detector. It should be noted here that the $\{r_i\}$ are complex-valued, and not real-valued as in the one-dimensional baseband data system (Fig. 5.2).

In the same manner as that used for the derivation of eqn. 5.38, it may be shown that the signal p(t) (Fig. 5.5) is given by

$$p(t) = \{[x(t)+j\hat{x}(t)]*h(t)*c(t)\}e^{j2\pi pt} + n(t)*[c(t)+j\hat{c}(t)]$$

(5.62)

where $\hat{x}(t)$ and $\hat{c}(t)$ are the Hilbert transforms of x(t) and c(t), respectively. Also, from Fig. 5.5,

$$x(t) = \frac{1}{\sqrt{2}} \text{ } \sum \{s_1, i g(t-iT)-s_2, i \hat{g}(t-iT)\}$$

(5.63)
Now noting that $H.T.\{\hat{g}(t)\} = -g(t)$ (186), and from eqn. 5.63,

$$\hat{x}(t) = \frac{1}{\sqrt{2}} \sum_{i} [s_{1,i} \hat{g}(t-iT) + s_{2,i} \hat{g}(t-iT)]$$  \hspace{1cm} (5.64)

From eqns. 5.63 and 5.64

$$x(t) + j\hat{x}(t) = \frac{1}{\sqrt{2}} \sum_{i} [s_{1,i} \{g(t-iT) + j \hat{g}(t-iT)\} -$$

$$- s_{2,i} \{\hat{g}(t-iT) - jg(t-iT)\}]$$

$$= \frac{1}{\sqrt{2}} \sum_{i} (s_{1,i} + js_{2,i}) [g(t-iT) + j \hat{g}(t-iT)]$$

$$= \frac{1}{\sqrt{2}} \sum_{i} s_{1,i} [g(t-iT) + j \hat{g}(t-iT)]$$  \hspace{1cm} (5.65)

where eqn. 5.61 has been used. Eqns. 5.62 and 5.65 give

$$p(t) = \frac{1}{\sqrt{2}} \sum_{i} s_{1,i} \{[g(t-iT) + j \hat{g}(t-iT)] \hat{h}(t) * c(t)\} e^{j2\pi pt} +$$

$$+ n(t) *[c(t) + j \hat{c}(t)]$$  \hspace{1cm} (5.66)

The signal $p(t)$ is now multiplied by $\frac{1}{\sqrt{2}} e^{-j2\pi p't}$ (Fig. 5.5) to remove the frequency offset. With $p' = p$, as has been assumed, the signal at the output of the frequency offset corrector is

$$r(t) = \frac{1}{\sqrt{2}} \sum_{i} s_{1,i} \{[g(t-iT) + j \hat{g}(t-iT)] \hat{h}(t) * c(t)\} +$$

$$+ \frac{1}{\sqrt{2}} \{n(t) *[c(t) + j \hat{c}(t)]\} e^{-j2\pi pt}$$  \hspace{1cm} (5.67)

or in a compact form

$$r(t) = \sum_{i} s_{1,i} y(t-iT) + u(t)$$  \hspace{1cm} (5.68)

where

$$y(t) = \frac{1}{2} \{[g(t) + j \hat{g}(t)] \hat{h}(t) * c(t)\}$$  \hspace{1cm} (5.69)

is the complex-valued impulse response of the linear baseband channel in Fig. 5.5, and

$$u(t) = \frac{1}{\sqrt{2}} \{n(t) *[c(t) + j \hat{c}(t)]\} e^{-j2\pi pt}$$  \hspace{1cm} (5.70)

is the complex-valued noise component in $r(t)$. The received signal sample $r_k$
at time $t=kT$, is given by

\[ r_k = \sum_i s_i y(k-i)T + u(kT) \]  

(5.71)

For practical purposes, $y(t)$ is of finite duration and is such that $y(hT)=0$ for $h<0$ and $h>g$. Thus,

\[ r_k = \sum_{h=0}^{g} s_{k-h} y_h + u_k \]  

(5.72)

where $y_h=y(hT)$ and $u_k=u(kT)$. The sampled impulse response of the linear baseband channel (Fig. 5.5) is given by the complex-valued vector

\[ Y = y_0, y_1, \ldots, y_g \]  

(5.73)

From eqn. 5.69, the Fourier transform of $y(t)$ is given by

\[ Y(f) = \frac{1}{2}[G(f)+jG(f)F(f)]H(f)C(f) \]  

(5.74)

where $G(f)F(f)$ is the Fourier transform of $g(t)$ and $F(f)$ is the transfer function of the Hilbert transformer and is given by eqn. 4.52. Thus, from eqns. 4.52 and 5.74,

\[ Y(f) = G(f)H(f)C(f) \quad f > 0 \]

\[ = 0 \quad f < 0 \]  

(5.75)

Also, in eqn. 5.70, the Fourier transform of $c(t)+jc(t)$ is given by

\[ F.T. [c(t)+jc(t)] = C(f)+jC(f)F(f) \]  

(5.76)

which with eqn. 4.52 gives

\[ F.T. [c(t)+jc(t)] = 2C(f) \quad f > 0 \]

\[ = 0 \quad f < 0 \]  

(5.76a)

The fact that $Y(f)$ (eqn. 5.75) or $F.T. [c(t)+jc(t)]$ being zero over the negative frequencies should not be confused with the fact that each of $G(f)$, $H(f)$ and $C(f)$ have non-zero values over the negative as well as over the positive frequencies. $g(t)$, $h(t)$ and $c(t)$ are in fact real-valued impulse responses. Now, from eqns. 5.70 and 5.76a, it is clear that the
spectral density of \( u(t) \) is zero over the negative frequencies, bearing in mind that \(|C(f)|\) (Fig. 5.7) is zero for \(|f|<300 \text{ Hz}\) and that \(p<<300 \text{ Hz}\). Therefore, and since \(Y(f)=0\) for \(f<0\) (eqn. 5.75), the spectrum of \( r(t) \) in eqn. 5.68 is zero for \(f<0\), i.e.,

\[
R(f) = 0 \quad \text{for } f < 0 \tag{5.77}
\]

Now, the average energy per signal-element in \( r(t) \) (eqn. 5.68) is given by,

\[
\mathbb{E}_r = \text{E}[|s_i|^2 \int_{-\infty}^{\infty} |y(t)|^2 dt] \tag{5.78}
\]

which by applying Parseval's theorem gives,

\[
\mathbb{E}_r = \frac{s_i^2}{T} \int_{-\infty}^{\infty} |Y(f)|^2 df
\]

\[
= \frac{s_i^2}{T} \int_{0}^{1/T} |G(f)|^2 |H(f)|^2 |C(f)|^2 df
\]

\[
= \frac{s_i^2}{T} \int_{-1/T}^{1/T} \frac{1}{2} |G(f)|^2 |H(f)|^2 |C(f)|^2 df \tag{5.79}
\]

where \(Y(f)\) has been replaced from eqn. 5.75 and the fact that \(|G(f)|,|H(f)|\) and \(|C(f)|\) are symmetric about zero has been used. The integration limits \((-\frac{1}{T}, \frac{1}{T})\) are for \(|C(f)|\) as shown in Fig. 5.7. \(s_i^2\) is the expected value of \(|s_i|^2\).

From Fig. 5.5, the signal component in \(z(t)\) at the output of the receiver filter \(C\) is given by (in the absence of the frequency offset),

\[
\frac{1}{\sqrt{2}} \sum_{i} [s_{1,i} g(t-iT) - s_{2,i} \hat{g}(t-iT)] * h(t) * c(t)
\]

and the average energy per signal element in \(z(t)\) is given by

\[
\mathbb{E}_z = \frac{1}{2} \text{E}[s_{1,i}^2 \int_{-\infty}^{\infty} (g(t) * h(t) * c(t))^2 dt + \int_{-\infty}^{\infty} (\hat{g}(t) * h(t) * c(t))^2 dt - 2s_{1,i} s_{2,i} \int_{-\infty}^{\infty} (g(t) * h(t) * c(t)) (\hat{g}(t) * h(t) * c(t)) dt] \tag{5.80}
\]
But, for the \( \{s_i\} \) as defined by eqn. 5.61

\[
\overline{s_i^2} = s_{1,i}^2 + s_{2,i}^2 = \frac{2}{m} \sum_{t=0}^{m-1} (2t-m+1)^2
\]  

(5.81)

and

\[
E[s_1^i s_2^i] = 0
\]  

(5.81a)

since the \( \{s_i\} \) are statistically independent and equally likely to have any of the \( m^2 \) possible values in eqn. 5.61. Thus, eqn. 5.80 becomes,

\[
\mathbb{R}_z = \frac{1}{2} \int_{-\infty}^{\infty} \left[ g(t) \cdot h(t) \cdot c(t) \right]^2 dt + \frac{1}{2} \int_{-\infty}^{\infty} \left[ \hat{g}(t) \cdot h(t) \cdot c(t) \right]^2 dt
\]  

(5.82)

which by applying Parseval's theorem gives,

\[
\mathbb{R}_z = \frac{1}{2} \int_{-\infty}^{\infty} |G(f)|^2 |H(f)|^2 |C(f)|^2 df
\]

\[
= \frac{1}{2} \int_{-1/T}^{1/T} |G(f)|^2 |H(f)|^2 |C(f)|^2 df
\]  

(5.83)

since \( |G(f)| \) is the amplitude response of both \( g(t) \) and \( \hat{g}(t) \). The integration limits in eqn. 5.83 are again for \( |C(f)| \) as shown in Fig. 5.7. From eqns. 5.79 and 5.83, \( \mathbb{R}_z = \mathbb{R}_r \), which implies that the arrangement of removing the frequency offset, including of course the phase splitter (Fig. 5.5) does not change the signal energy.

Since the data symbol in eqn. 5.61 may have \( m^2 \) equally likely possible values, it carries \( 2 \log_2 m \) bits of information. Thus, from eqns. 5.79 and 5.81, the average energy per bit in \( r(t) \) is

\[
\mathbb{E}_0 = \frac{\mathbb{R}_0}{m \log_2 m} \int_{-1/T}^{1/T} \frac{1}{2} |G(f)|^2 |H(f)|^2 |C(f)|^2 df
\]  

(5.84)

\( \mathbb{E}_0 \) will be used in the comparison with the QAM system later on. In the same manner, it may be shown that the average transmitted energy per bit in \( x(t) \) at the input of the transmission path (Fig. 5.5), is given by
The noise component $u(t)$ (eqn. 5.70) in the received signal $r(t)$ (eqn. 5.68) is complex-valued. It is shown in Appendix C that the complex-valued auto-correlation function $R_u(\tau)$ of $u(t)$ is given by

$$R_u(\tau) = N_0 e^{-j2\pi \tau} \int_0^\infty |C(\xi)|^2 e^{j2\pi \xi \tau} d\xi$$  \hspace{1cm} (5.86)$$

Also, it is shown in Appendix C that the auto-correlation function of each of the real and imaginary parts of $u(t)$ is given by $\{\text{Real}[R_u(\tau)]\}$, whereas the cross-correlation function of these parts is an odd function of $\tau$ and is given by half the imaginary part of $R_u(\tau)$, and when $R_u(\tau)$ is purely real, then the real and imaginary parts of $u(t)$ are uncorrelated.

Since the interest here is in the samples of $u(t)$ at time instants $t=it$, $i=1,2,\ldots$, i.e. in the $\{u_i\}$ (eqn. 5.72), then the auto-correlation function of two samples $u_i$ and $u_{i+k}$, which are separated by $kT$ seconds is given by $R_u(kT)$. Thus,

$$R_{u_i,u_{i+k}} = N_0 e^{-j2\pi pkT} \int_0^\infty |C(\xi)|^2 e^{j2\pi k\xi T} d\xi.$$  \hspace{1cm} (5.87)$$

When $p=0$ and when $|C(\xi)|$ over the positive frequencies is symmetric about $\xi=\frac{1}{2T}$, it may be shown that $R_{u_i,u_{i+k}}$ becomes purely real and the real and imaginary parts of the $\{u_i\}$ become uncorrelated.

The variance of $u(t)$, and therefore the variance of the $\{u_i\}$ is given by

$$\sigma_u^2 = R_u(0)$$

$$= N_0 \int_0^\infty |C(\xi)|^2 d\xi$$

$$= \frac{1}{N_0} \int_{-1/T}^{1/T} |C(\xi)|^2 d\xi$$  \hspace{1cm} (5.88)$$
bearing in mind that \(|C(f)|\) is symmetric about zero (Fig. 5.7).

In the special case when \(|C(f)|\) is such that

\[
|C(f)| = 1 \quad \text{for} \quad f_{\ell} \leq |f| \leq \frac{1}{T}
\]

\[= 0 \quad \text{elsewhere}
\]

eqn. 5.87 becomes,

\[
R_{u_1, u_{i+k}} = N_0 e^{-j2\pi pfkT} \int_{f_{\ell}}^{1/T} e^{j2\pi fkT} df
\]

\[
= \frac{N_0 (1-f_{\ell} T)}{T} e^{jk\pi (1+f_{\ell} T-2pT)} \frac{\sin[k\pi (1-f_{\ell} T)]}{k\pi (1-f_{\ell} T)}
\]

(5.90)

and the variance of the \(\{u_i\}\) becomes

\[
\sigma_u^2 = R_{u_1, u_1} = \frac{N_0 (1-f_{\ell} T)}{T}
\]

(5.91)

For a voice-frequency channel, \(f_{\ell} \approx 300\) Hz and \(\frac{1}{T} \approx 3000\) Hz (Fig. 5.7), so that \(\sigma_u^2\) may be taken as

\[
\sigma_u^2 = \frac{N_0}{T}
\]

(5.92)

with an error of less than 0.5 dB. Notice that eqn. 5.92 gives the exact value of \(\sigma_u^2\) when \(f_{\ell} = 0\). Also, when \(f_{\ell} = 0\), the \(\{u_i\}\) become uncorrelated, as can be seen from eqn. 5.90, regardless of the value of the frequency offset. \(f_{\ell} = 0\) corresponds to the case when Filter C (eqn. 5.89) is a lowpass rectangular filter.

As mentioned before, \(r(t)\) is sampled once per signal element, i.e. at a rate of \(\frac{1}{T}\) samples per second. This rate, although it is half the Nyquist rate for the transmitter and receiver filters (Figs. 5.6 and 5.7), does not cause any aliasing in the spectrum of the resultant sampled signal \(\{r_i\}\), because the spectrum of \(r(t)\) (eqn. 5.77), is zero over the negative frequencies.
In practice it may be more convenient to perform the Hilbert transform at the receiver in a digital form. In this case, a sampler may be placed at the output of Filter \( C \) (Fig. 5.5). It should be noted here that the signal \( z(t) \) in Fig. 5.5 has a spectrum which is bandlimited by Filter \( C \) (Fig. 5.7), and to avoid aliasing and ensure the correct result at the Hilbert transform output, the sampling rate of \( z(t) \) must be not less than the Nyquist rate, i.e. \( z(t) \) must be sampled twice per signal element (at a rate of \( \frac{2}{T} \) samples per second). Now, the sequence of signal samples at the output of the digitally-implemented phase-splitter is given at a rate of \( \frac{2}{T} \) samples per second. The required signal samples are obtained by taking every second sample of that sequence. Again, if the transmitter filters \( G_1 \) and \( G_2 \) (Fig. 5.5) are to be digitally implemented, care must be taken in feeding the input data signal into these filters. In this case, the tap gains of Filters \( G_1 \) and \( G_2 \) must be obtained by sampling the impulse responses \( \{ \frac{1}{\sqrt{2}} g(t) \} \) and \( \{ \frac{1}{\sqrt{2}} \hat{g}(t) \} \), respectively, at the Nyquist rate (at \( \frac{2}{T} \) samples per second), in order to ensure correct spectrum shaping. The sequence of data-elements \( \{ s, \delta(t-iT) \} \) (Fig. 5.5) which is to be transmitted should now be injected by zeros to give the sequence \( \{ \delta_k \delta(t-k\frac{T}{2}) \} \) which is fed to the digitally-implemented transmitter filters. The sequence \( \{ \delta_k \} \) is now given by

\[
\delta_k = \frac{s}{2}(k+1)/2 \quad k \text{ odd}
\]
\[
= 0 \quad k \text{ even}
\]  

(5.93)

Of course, \( \delta_k = 0 \) for \( k \leq 0 \) when \( s_i = 0 \) for \( i \leq 0 \).

5.5 PERFORMANCE EVALUATION OF THE ONE- AND TWO-DIMENSIONAL BASEBAND DATA TRANSMISSION SYSTEMS

The two data transmission systems presented in Sections 5.4.1 and 5.4.2
will be compared with the QAM system which is considered here as an optimum modulation scheme (Section 4.1). The comparison will be made on the basis of the tolerance of the systems to additive white Gaussian noise. Although practical voice-frequency channels do not introduce a significant amount of white Gaussian noise into the signal, the relative tolerances of different data transmission systems to this type of noise are a good measure of their relative performances in the presence of other types of additive noise.

The comparison will be made under the following conditions. Firstly, all systems must use the same transmission path which introduces the same level of white Gaussian noise. Therefore, the transmission paths in Figs. 4.1, 5.2 and 5.5 all have the same real-valued impulse response \( h(t) \) and the spectral density of the additive white noise is \( \frac{1}{2} N_0 \). Secondly, the signal fed to the transmission path must have the same amplitude-spectral shaping in the three systems so that identical linear distortion is introduced into the signal at the output of the transmission path. As shown in Fig. 4.1, the shape of the QAM-signal spectrum is given by shifting the amplitude spectrum \( |A(f)| \) in Fig. 4.2 upwards (for positive frequencies) and downwards (for negative frequencies) by the value of the carrier frequency \( f_c \) which will be assumed here to be given by

\[
f_c = \frac{1}{2T}
\]  

(5.94)

Eqn. 5.94 may be obtained from eqn. 4.3 by putting

\[
f_c = 0
\]  

(5.95)

To simplify the following analysis, eqn. 5.95 will be assumed to be true throughout this section. Now, and for the signal to have the same amplitude spectrum at the input of the transmission path in the three systems, the transmitter filters \( G \) and \( G_1 \) (Figs. 5.2 and 5.5) each must
have a transfer function \( \frac{1}{\sqrt{2}} G(f) \) such that

\[
G(f) = A(f+\frac{1}{2T}) + A(f-\frac{1}{2T})
\]  

(5.96)

where \( A(f) \) is the transfer function of each of Filters \( A_1 \) and \( A_2 \) in the QAM system (Fig. 4.1) and is bandlimited to \(-\frac{1}{2T}\) to \(\frac{1}{2T}\) Hz (Fig. 4.2). Filter \( G_2 \) (Fig. 5.5) has, of course, the same amplitude response \( \frac{1}{\sqrt{2}} |G(f)| \) as does filter \( G_1 \) (Fig. 5.6).

In the three systems, the received signal \( r(t) \) is sampled once per signal element, i.e. at the signal-element rate. In order that the linear distortion in the received signal sequence \( \{r_i\} \) is the same in the three systems, \( r(t) \) is assumed to be sampled at the lowest rate which does not cause spectrum overlapping (aliasing). This rate is the Nyquist rate in both the QAM and the one-dimensional baseband systems, and half the Nyquist rate in the two-dimensional baseband system. From Figs. 4.2 and 5.3 and eqn. 5.96, the one-dimensional baseband system must have a signal-element rate which is twice that of the QAM system, i.e.,

\[
\frac{1}{T_1} = \frac{2}{T}
\]  

(5.97)

where, of course, \( \frac{1}{T} \) and \( \frac{1}{T_1} \) are the signal-element rates of the QAM system and the one-dimensional baseband system, respectively. From Figs. 4.2 and 5.6 and eqn. 5.96, the two-dimensional baseband system operates at the same element rate as does the QAM-system.

Thirdly, of course, the same information rate will be assumed in the three systems. As can be seen from eqns. 4.1 and 4.4, the signal element in the QAM-system carries \( 2\log_2 m \) bits of information. Thus, at an element rate of \( \frac{1}{T} \) bauds, the bit rate is given by

\[
B = \frac{2}{T} \log_2 m \text{ bits/s}
\]  

(5.98)
In the one-dimensional baseband system, the signal element (eqn. 5.40) carries \( \log_2 m \) bits of information; but since the element rate is \( \frac{1}{T} \) bauds, and from eqn. 5.97, the bit rate is now equal to that in eqn. 5.98. In the two-dimensional baseband system, the signal element (eqn. 5.61) carries \( 2\log_2 m \) bits of information, whereas the element rate is \( \frac{1}{T} \) bauds. Therefore, the bit rate is again given by eqn. 5.98.

Fourthly, the same signal power, i.e. the same average energy per bit is assumed at the input of the transmission path. From eqn. 4.48, the average transmitted energy per bit in the QAM signal is

\[
\varepsilon_0 = \frac{m-1}{m \log_2 m} \left( \sum_{\ell=0}^{m-1} (2\ell-m+1)^2 \right)^{1/2T} \int_{-1/2T}^{1/2T} |A(f)|^2 df \quad (5.99)
\]

It can be seen from eqns. 5.96 and 5.97 that, \( \varepsilon_0 \) in eqns. 5.52 and 5.85, giving the average transmitted energy per bit in the one- and two-dimensional baseband systems, respectively, has the same value as that in eqn. 5.99.

Finally, the receiver Filter \( C \) is assumed here to satisfy eqns. 4.31, 5.57 and 5.89, for the QAM, the one-dimensional and the two-dimensional baseband systems, respectively. This with eqns. 5.94, 5.95 and 5.97, give

\[
|C(f)| = \begin{cases} 
1 & -\frac{1}{T} < f < \frac{1}{T} \\
0 & \text{elsewhere}
\end{cases} \quad (5.100)
\]

for the three systems. The noise components in the received signal samples in both the QAM and the two-dimensional baseband systems are now uncorrelated, as can be seen from eqns. 4.33 and 5.90 (\( f = 0 \), eqn. 5.95). In the one-dimensional baseband system, the noise components become uncorrelated when \( p \), the frequency offset, is zero, as can be seen from eqn. 5.58. This will be assumed here. Consequently, the noise components in the noisy signal samples \( \{x_i\} \) at the output of the linear baseband
channel of each of the three systems (Figs. 4.1, 5.2 and 5.5) are statistically independent Gaussian random variables with zero mean and variances given by eqns. 4.35, 5.59 and 5.91, for the QAM, one- and two-dimensional baseband systems respectively. From eqns. 5.95 and 5.97, the three variances are equal and have the value,

\[ \sigma^2_u = \frac{N_0}{T} \]  

(5.101)

Under the assumed conditions, the average energy per bit in the received signal \( r(t) \), in the three systems, has the same value as can be seen from eqns. 4.46, 5.50 and 5.84, and therefore the signal-to-noise ratio in \( r(t) \) is the same. The impulse responses of the linear baseband channels in the QAM, one- and two-dimensional baseband systems are given by,

\[ y_Q(t) = a(t) \ast \left[ (h(t) \ast c(t)) e^{-j2\pi f_c t} \right] \]  

(5.102)

\[ y_B(t) = \frac{1}{\sqrt{2}} g(t) \ast h(t) \ast c(t) \]  

(5.103)

and

\[ y_H(t) = \frac{1}{2} [g(t) + jg(t)] \ast h(t) \ast c(t) \]  

(5.104)

from eqns. 4.16, 5.41 and 5.69, where \( \theta \), the phase error in the estimate of the carrier frequency (eqn. 4.16) is assumed here to be zero, for the purpose of comparison. In eqn. 5.102, \( b(t) \) is omitted because it has a rectangular transfer function with value unity over the passband of \( a(t) \) (Figs. 4.2 and 4.5), so that it has no effect on \( y_Q(t) \) (eqn. 5.102) itself.

The received signal sequence \( \{r_i\} \) is given by eqns. 4.68, 5.46 and 5.72 for the three systems, respectively, as follows,

\[ r_{Q,k} = \frac{g_1}{L} \sum_{h=0}^{L} s_{k-h} y_{Q,h} + u_{Q,k} \]  

(5.105)

\[ r_{B,k} = \frac{g_2}{L} \sum_{h=0}^{L} y_{k-h} y_{B,h} + u_{B,k} \]  

\[ r_{H,k} = \frac{g_3}{L} \sum_{h=0}^{L} s_{k-h} y_{H,h} + u_{H,k} \]
where the \( \{s_k\} \) and \( \{\gamma_k\} \) are given by

\[
s_k = s_{1,k} + js_{2,k} \quad (5.106a)
\]

and

\[
\gamma_k = s_{1,k} + js_{2,k} = 2^z_m + 1, \quad z=0,1,\ldots,(m-1) \quad (5.106b)
\]

The \( \{\gamma_k\} \) have \( m \) possible values, all real, whereas the \( \{s_k\} \) have \( m^2 \) possible complex values. The \( \{y_{Q,h}\} \) and \( \{y_{H,h}\} \) are obtained by sampling \( y_Q(t) \) and \( y_H(t) \) (eqns. 5.102 and 5.104) at a rate of \( \frac{1}{T} \) samples per second, and the \( \{y_{B,h}\} \) are obtained by sampling \( y_B(t) \) (eqn. 5.103) at a rate of \( \frac{1}{T_1} = \frac{2}{T} \) samples per second. The resultant sampled impulse responses are given by

\[
Y_Q = y_{Q,0}y_{Q,1},\ldots,y_{Q,g_1} \quad (5.107)
\]

\[
Y_B = y_{B,0}y_{B,1},\ldots,y_{B,g_2} \quad (5.108)
\]

\[
Y_H = y_{H,0}y_{H,1},\ldots,y_{H,g_3} \quad (5.109)
\]

Next, the relative tolerances of the three systems to additive white Gaussian noise will be evaluated both in the absence and presence of inter-symbol interference.

5.5.1 Relative Performances in the Absence of Intersymbol Interference

For this purpose, assume that each of the transmitter filters \( A_1 \) and \( A_2 \) (Fig. 4.1) has a rectangular transfer function such that

\[
A(f) = T, \quad -\frac{1}{2T} \leq f \leq \frac{1}{2T} \quad (5.110)
\]

\[
= 0, \quad \text{elsewhere}
\]

Consequently, and from eqns. 5.96,

\[
G(f) = T, \quad -\frac{1}{T} \leq f \leq \frac{1}{T} \quad (5.111)
\]

Also let

\[
h(t) = \delta(t) \quad (5.112)
\]

which represents a perfect transmission path. \( \delta(t) \) is the Dirac function.

The receiver filter \( C \) in all systems has been assumed previously to have
a rectangular amplitude response (eqn. 5.100). In this section (Section 5.5.1) Filter C is assumed to have the transfer function

\[
C(f) = \begin{cases} 
1 & -\frac{1}{T} \leq f \leq \frac{1}{T} \\
0 & \text{elsewhere}
\end{cases}
\]  

(5.113)

The assumed transfer functions in eqns. 5.110-5.113 do not in fact represent bandpass characteristics and have been used here deliberately to give no intersymbol interference in the \(\{r_i\}\) in any of the three systems.

Now, the Fourier transforms of \(Y_Q(t)\), \(Y_B(t)\) and \(Y_H(t)\) (eqns. 5.102-5.104) are given by

\[
\begin{align*}
Y_Q(f) &= T \delta \left( f - \frac{1}{2T} \right) \\
Y_B(f) &= \frac{T}{\sqrt{2}} \delta \left( f - \frac{1}{T} \right) \\
Y_H(f) &= T \delta \left( f - \frac{1}{T} \right)
\end{align*}
\]  

(5.114)

where eqns. 5.110 to 5.113 have been taken into account. The inverse Fourier transforms of the \(Y(f)\)'s give

\[ y_Q(t) = \frac{\sin \frac{\pi t}{T}}{\frac{\pi t}{T}} \]  

\[ y_B(t) = \frac{\sin \frac{2\pi t}{T}}{\frac{2\pi t}{T}} \]  

\[ y_H(t) = e^{j\frac{\pi t}{T}} \frac{\sin \frac{\pi t}{T}}{\frac{\pi t}{T}} \]  

(5.115)

Sampling \(y_Q(t)\) and \(y_H(t)\) at \(\frac{1}{T}\) samples per second, and neglecting delay in transmission, eqns. 5.107 to 5.109 become,

\[ Y_Q = 1.0, 0.0, 0.0 \]  

\[ Y_B = \sqrt{2}, 0.0, 0.0 \]  

\[ Y_H = 1.0, 0.0, 0.0 \]  

(5.116)

and eqns. 5.105 become

\[ r_{Q,k} = s_k + u_{Q,k} \]  

(5.117)
\[
\begin{align*}
  r_{B,k} &= \sqrt{2} \gamma_k + u_{B,k} \\
  r_{H,k} &= s_k + u_{H,k}
\end{align*}
\]

so that the \( \{s_k\} \) and \( \{\gamma_k\} \) may be detected from the \( \{r_k\} \) by the simple threshold detection technique. In eqn. 5.117, \( r_{Q,k} \) is complex-valued and may be written as

\[
  r_{Q,k} = (s_{1,k} + j s_{2,k}) + (u_{Q1,k} + j u_{Q2,k})
\]

\[
  = s_{1,k} + u_{Q1,k} + j (s_{2,k} + u_{Q2,k})
\]

(5.120)

Since the real and imaginary parts of \( s_k \) and \( u_{Q,k} \) are statistically independent, \( s_k \) can be detected by detecting \( s_{1,k} \) and \( s_{2,k} \) independently from the real and imaginary parts, respectively, of \( r_{Q,k} \). Noting that each of \( s_{1,k} \) and \( s_{2,k} \) are given by eqn. 5.106b and that the variance of each of \( \{u_{Q1,k}\} \) and \( \{u_{Q2,k}\} \) is half the variance of \( \{u_{Q,k}\} \), i.e.,

\[
  \sigma_{u_1}^2 = \sigma_{u_2}^2 = \frac{1}{2} \sigma_u^2
\]

the error probability in the detection of \( s_{1,k} \) or \( s_{2,k} \) is given by (23)

\[
  P_{Q}^e = \frac{2(m-1)}{m} Q\left(\frac{1}{\sigma_u/\sqrt{2}}\right)
\]

(5.121)

where \( Q(.) \) is given by,

\[
  Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{v^2}{2}\right) dv
\]

(5.122)

Now, an error in the detection of \( s_k \) occurs whenever \( s_{1,k}, s_{2,k} \) or both are incorrectly detected. In general, the error events in the detected values of the \( \{s_{1,k}\} \) and the \( \{s_{2,k}\} \) are not mutually exclusive, but at high signal-to-noise ratios, when the error probability in the detection of each of \( s_{1,k} \) and \( s_{2,k} \) is low, the probability of error in the detection of \( s_k \) is approximately equal to the sum of the probabilities of error in the detection of \( s_{1,k} \) and \( s_{2,k} \). Hence
When \( P_Q \) (eqn. 5.121) is high, then the resultant probability of error in the detection of a data symbol \( s_k \) is slightly higher than \( P'_Q \), but considerably less than \( 2P'_Q \).

The same argument applies to the detection of \( s_k \) from \( r_{H,k} \) (eqn. 5.119) where the \( \{s_k\} \) and the \( \{u_{H,k}\} \) have the same properties as before, so that the probability of error in the detection of a data symbol at high signal-to-noise ratios is approximately given by

\[
P_H = \frac{4(m-1)}{m} Q\left(\frac{\sqrt{2}}{\sigma_u}\right)
\]

In eqn. 5.118, \( r_{B,k} \) is real-valued, \( \gamma_k \) is given by eqn. 5.106b, and the \( \{u_{B,k}\} \) are real-valued with variance \( \sigma_u^2 \). Thus, the probability of error in the detection of \( \gamma_k \) from \( r_{B,k} \) is given by (23)

\[
P_B = \frac{2(m-1)}{m} Q\left(\frac{\sqrt{2}}{\sigma_u}\right)
\]

Now, from eqns. 5.99 and 5.110,

\[
\frac{\sigma_0}{\sigma_u} = T \frac{\sum_{k=0}^{m-1} (2k-m+1)^2}{m \log_2 m}
\]

and from eqns. 5.101 and 5.126,

\[
\frac{\sqrt{2}}{\sigma_u} = \sqrt{T} \frac{\sum_{k=0}^{m-1} (2k-m+1)^2}{\sqrt{4N_0} \sum_{k=0}^{m-1} (2k-m+1)^2}
\]

Eqns. 5.123-5.127 give

\[
P_Q = \frac{4(m-1)}{m} Q\left(\frac{\sqrt{8O}}{4N_0} \frac{m \log_2 m}{\sum_{k=0}^{m-1} (2k-m+1)^2}\right)
\]

\[
P_H = \frac{4(m-1)}{m} Q\left(\frac{\sqrt{8O}}{4N_0} \frac{m \log_2 m}{\sum_{k=0}^{m-1} (2k-m+1)^2}\right)
\]
and

\[ P_B = \frac{2(m-1)}{m} Q\left( \sqrt{\sum_{l=0}^{m-1} \frac{m \log_2 m}{N_0 (2l-m+1)^2}} \right) \]  \hspace{1cm} (5.130)

Eqns. 5.128 to 5.130 give the probability of error in the detection of a data-symbol. The bit error probability is related to the symbol-error probability in a manner dependent on the type of coding applied to obtain the data-symbols from the binary data. When possible, the data-symbols are coded (in terms of the binary digits) such that two adjacent symbol values correspond to a change in only one bit (Gray code). Non-adjacent data-symbol values may correspond to a change in more than one bit. Assuming this type of coding is used here, one symbol-error produces only one bit-error if the data symbol is detected as a possible value adjacent to its correct value. When the data symbol is detected as a possible value that is not adjacent to its correct value, the single symbol-error causes more than one detected binary digit to be in error.

At high signal-to-noise ratios, i.e. when the noise variance tends to be very small, the noise is most likely to cause the data-symbol to be detected as an adjacent possible value rather than as a non-adjacent possible value. Let the average number of bit-errors caused by a single symbol-error be \( N_Q \), \( N_B \) and \( N_H \) for the three systems, respectively. By noting that a data-symbol carries \( 2 \log_2 m \) bits of information in the QAM and the two-dimensional baseband systems and \( \log_2 m \) bits of information in the one-dimensional baseband system, the average bit-error probabilities (from eqns. 5.128-5.130) are given by

\[ P_{Q0} = \frac{2N_Q(m-1)}{m \log_2 m} Q\left( \sqrt{\sum_{l=0}^{m-1} \frac{m \log_2 m}{N_0 (2l-m+1)^2}} \right) \]  \hspace{1cm} (5.131)
Clearly, $N_H = N_Q$ when the same coding scheme is used in both, the QAM and the two-dimensional baseband systems and consequently, the two systems always have the same performance. At high signal-to-noise ratios, which is the case of practical interest, each of $N_Q, N_B$ and $N_H$ approaches the value 1, and the three systems will have the same tolerance to additive white Gaussian noise, as can be seen from eqns. 5.131-5.133.

5.5.2 Relative Performances in the Presence of Intersymbol Interference

So far, it has been assumed that there is no intersymbol interference. Unfortunately, when the received signal suffers from intersymbol interference, the simple threshold detector can no longer be employed, and more sophisticated detection processes must be used. For the purpose of comparison, the optimum detection process, i.e. the maximum-likelihood detection algorithm (Section 3.4) will be assumed here. The probability of error in the detection of a data-symbol at high signal-to-noise ratios and when this detector is used, is given by eqn. 3.96, which is of the form,

$$P_B(e) = k_e Q\left(\frac{d_{\text{min}}}{2\sigma_u}\right)$$  \hspace{1cm} (5.134)

where $k_e$ is a constant. Eqn. 5.134 has originally been derived\(^{90}\) for the case when real signals are used, where $d_{\text{min}}$ is the minimum Euclidean distance between any two possible real-valued signal vectors $\{Z_N\}$ (eqn. 3.80) that lie in the $N$-dimensional Euclidean vector space which contains
all possible vectors \( \{Z_N\} \). Therefore eqn. 5.134 may be used directly with the one-dimensional baseband system. But when the signal vectors \( \{Z_N\} \) are complex-valued, as is the case with the QAM and the two-dimensional baseband systems, \( d_{\text{min}} \) is then the minimum unitary distance between any two possible vectors \( \{Z_N\} \) in the \( N \)-dimensional complex vector space which contains all possible complex-valued vectors \( \{Z_N\} \). Here, the noise components in the received signal are also complex-valued (eqns. 4.68 and 5.72). In the same manner as that used in Reference (90) to derive eqn. 5.134, it may be shown that the error probability here is a function of the individual variances of the real and imaginary parts of the noise components. Since these variances here are equal and each is given by \( \frac{\sigma^2}{2} \) (eqn. 5.101), the probability of error in the detection of a data-symbol, when the maximum likelihood detector is used with the QAM or the two-dimensional baseband systems, is given by

\[
P_{Q,H}(e) = k_e' Q\left(\frac{d_{\text{min}}}{2\sigma_u/\sqrt{2}}\right)
\]

(5.134a)

where \( k_e' \) is a constant of the same order as \( k_e \) (eqn. 5.134).

The evaluation of the tolerance to additive white Gaussian noise of any system requires a knowledge of \( d_{\text{min}} \) for the intersymbol interference in the given system. Unfortunately, there is in general no way to relate \( d_{\text{min}} \) to the distortion introduced into the signal, but for a given intersymbol interference, \( d_{\text{min}} \) may be determined computationally. (95) Even here, considerable computation may be required. In the present analysis, the minimum distance \( d_{\text{min}} \), in the three systems, will be indirectly compared without actually evaluating it.

First, let the impulse responses of the one- and two-dimensional baseband systems be determined in terms of the impulse response of the
QAM system, under the conditions assumed earlier (except, of course, those given by eqns. 5.110-5.113). From eqn. 5.102, the impulse response \( y_Q(t) \) of the linear baseband channel in the QAM system, is given by

\[
y_Q(t) = \int \frac{1}{\sqrt{2T}} A(f) H(f+\frac{1}{2T}) C(f+\frac{1}{2T}) e^{j2\pi ft} df
\]

(5.135)

where, as previously assumed, \( f_c = \frac{1}{2T} \), \( A(f) \), the Fourier transform of \( a(t) \), is strictly bandlimited to \( -\frac{1}{2T} \) to \( \frac{1}{2T} \) Hz, and \( H(f) \) and \( C(f) \) are the Fourier transforms of \( h(t) \) and \( c(t) \) respectively. From eqns. 5.96 and 5.103, the impulse response of the linear baseband channel in the one-dimensional baseband system is

\[
y_B(t) = \int \frac{1}{\sqrt{T}} H(f) C(f) e^{j2\pi ft} df
\]

(5.136)

which may be rewritten as

\[
y_B(t) = \int \frac{1}{\sqrt{T}} \left[ A(f+\frac{1}{2T}) + A(f-\frac{1}{2T}) \right] H(f) C(f) e^{j2\pi ft} df
\]

(5.137)

since \( A(f+\frac{1}{2T}) \) is zero over positive frequencies whereas \( A(f-\frac{1}{2T}) \) is zero over negative frequencies. By a change of variable, eqn. 5.137 becomes,

\[
y_B(t) = \int \frac{1}{\sqrt{T}} \left[ e^{j\pi t} A(f) H(f-\frac{1}{2T}) C(f-\frac{1}{2T}) e^{j2\pi ft} df + e^{-j\pi t} A(f) H(f+\frac{1}{2T}) C(f+\frac{1}{2T}) e^{j2\pi ft} df \right]
\]

(5.138)

Now, by changing the sign of \( f \) in the first integrand in eqn. 5.138 and noting that

\[
A(-f)H(-f-\frac{1}{2T})C(-f-\frac{1}{2T}) = [A(f)H(f+\frac{1}{2T})C(f+\frac{1}{2T})]^*
\]

(5.139)
because \( a(t) \), \( h(t) \) and \( c(t) \) are all real-valued, eqn. 5.138 becomes,

\[
\begin{align*}
y_B(t) &= \frac{1}{\sqrt{2}} e^{-j\pi t} \int_{-1/2T}^{1/2T} \left[ A(f)H(f+\frac{1}{2T})C(f+\frac{1}{2T}) \right]^* e^{-j2\pi ft} df + \\
&\quad + \frac{1}{\sqrt{2}} e^{j\pi t} \int_{-1/2T}^{1/2T} A(f)H(f+\frac{1}{2T})C(f+\frac{1}{2T}) e^{j2\pi ft} df
\end{align*}
\]

which if compared with eqn. 5.135 gives,

\[
y_B(t) = \frac{1}{\sqrt{2}} e^{-j\pi t} y_Q^*(t) + \frac{1}{\sqrt{2}} e^{j\pi t} y_Q(t)
\]

where \( y_Q^*(t) \) is the complex conjugate of \( y_Q(t) \). Of course eqn. 5.141 may be written as

\[
y_B(t) = \sqrt{2} \text{Real}[e^{j\pi t} y_Q(t)]
\]

From eqns. 5.96 and 5.104, the impulse response of the linear baseband channel in the two-dimensional baseband system is given by

\[
y_H(t) = \int_0^{1/T} A(f-\frac{1}{2T})H(f)C(f) e^{j2\pi ft} df
\]

where the Fourier transform of \((g(t)+j\hat{g}(t))\) is given by

\[
\text{F.T.}[g(t)+j\hat{g}(t)] = 2G(f) = 2A(f-\frac{1}{2T}), \quad f > 0
\]

\[
= 0, \quad f < 0
\]

Eqn. 5.143 may now, by a change of variable, be written as

\[
y_H(t) = e^{j\pi t} \int_{-1/2T}^{1/2T} A(f)H(f+\frac{1}{2T})C(f+\frac{1}{2T}) e^{j2\pi ft} df
\]

which, again, when compared with eqn. 5.135 gives,
The tolerance to noise of the two-dimensional baseband system will now be considered. The sampled impulse response of the linear baseband channel of this system is obtained by sampling \( y_H(t) \) at a rate of \( \frac{1}{T} \). Thus, putting \( t = hT \) in eqn. 5.146 gives the \( h \)th sample as,

\[
y_{H,H}(t) = e^{j\pi} y_Q(hT) = \cos(h\pi) y_{Q,h}(t)
\]

where \( h \) takes all negative and positive integer values for which \( y_Q(hT) \) has significant amplitude (\( y_Q(t) \) is truncated to zero for values of \( t < t_1 \) and \( t > t_2 \), where its amplitude is very small compared with its peak). Clearly, the \( \{y_{Q,h} \} \) form the sampled impulse response of the linear baseband channel in the QAM system where the sampling rate is also \( \frac{1}{T} \). As can be seen from eqn. 5.147, the sampled impulse response in the two-dimensional baseband system is obtained by changing the sign of every second component in the sampled impulse response of the linear baseband channel in the QAM system. In Appendix D, it is shown that changing the sign of every second sample of the sampled impulse response does not change the minimum distance between possible signal vectors \( \{Z_N \} \) (eqn. 3.80) in the appropriate unitary vector space. The \( \{Z_N \} \) are of course functions of the data symbol values and the sampled impulse response of the channel. Hence, the two-dimensional baseband system should perform exactly as does the QAM system when a maximum-likelihood detector is employed.
Next, let the minimum distance in the one-dimensional baseband system be compared with the minimum distance in the QAM-system. First, assume that the sampled impulse response of the linear baseband channel of the QAM system is given by the complex-valued vector

\[ Y = y_0, y_1, \ldots, y_g \]  

where

\[ y_h = y_{1,h} + jy_{2,h} = y_Q(t_1 + hT) \]

and \( y_Q(t) = 0 \) for \( t < t_1 \) and \( t > t_1 + gT \). Clearly, the \( \{ y_{1,h} \} \) and the \( \{ y_{2,h} \} \) are obtained by sampling the real and imaginary parts of \( y_Q(t) \), respectively, at a rate of \( \frac{1}{T} \). Let \( S_k, Z_k, S'_k \) and \( Z'_k \) be the \( k \)-dimensional vectors whose \( i \)-th components are \( s_i, z_i, s'_i \) and \( z'_i \), respectively, where \( s_i \) and \( s'_i \) are both complex-valued and drawn from the same set of data-symbol values defined by eqns. 5.106a-b,

\[
z_i = \sum_{h=0}^{g} s_{i-h}y_h
\]

\[
z'_i = \sum_{h=0}^{g} s'_{i-h}y_h
\]

\[ s_i = s_{1,i} + js_{2,i} \]

\[ s'_i = s'_{1,i} + js'_{2,i} \]

Now, in the \( k \)-dimensional signal vector space, the unitary distance between the two vectors \( Z_k \) and \( Z'_k \) is given by the square root of

\[
d^2 = |Z_k - Z'_k|^2
\]

\[
= \sum_{i=1}^{k} (z_i - z'_i)(z_i - z'_i)^*
\]

where \(|.|\) denotes the length of the vector and \((z_i - z'_i)^*\) is the complex-
conjugate of \((z_i - z'_i)\). From eqns. 5.150 to 5.153, eqn. 5.154 gives,

\[
d^2 = d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2
\]  
(5.155)

where

\[
d_1^2 = \sum_{i=1}^{k} \sum_{h=0}^{g} (s_i, i-h - s'_i, i-h) y_{1, h}^2
\]
\[
d_2^2 = \sum_{i=1}^{k} \sum_{h=0}^{g} (s_i, i-h - s'_i, i-h) y_{2, h}^2
\]
\[
d_3^2 = \sum_{i=1}^{k} \sum_{h=0}^{g} (s_i, i-h - s'_i, i-h) y_{3, h}^2
\]
\[
d_4^2 = \sum_{i=1}^{k} \sum_{h=0}^{g} (s_i, i-h - s'_i, i-h) y_{4, h}^2
\]
\[
d_5^2 = 2 \sum_{i=1}^{k} \sum_{h=0}^{g} y_{1, h} y_{2, h} \left[ (s_i, i-h - s'_i, i-h) (s_i, i-h - s'_i, i-h) - (s_i, i-h - s'_i, i-h) (s_i, i-h - s'_i, i-h) \right]
\]  
(5.156)

It is assumed here that \(k \gg g\). The minimum value of \(d\), over all possible sequences \(S_k \neq S'_k\), represents the minimum distance \(d_{\text{min}}\) corresponding to the intersymbol interference resulting from the sampled impulse response in eqn. 5.148.

Now, the sampled impulse response of the linear baseband channel in the one-dimensional baseband system is obtained by sampling \(y_B(t)\) (eqn. 5.142) at a rate of \(\frac{1}{T_1} = \frac{2}{T}\) (eqn. 5.97), so that the sampling interval is \(\frac{1}{T}\). Let this sampled impulse response be represented by the \((2g+1)\)-component real-valued vector \(Y^*_B\)

\[
Y^*_B = y^*_B, 0, y^*_B, 1, \ldots, y^*_B, 2g
\]  
(5.157)

where

\[
y^*_B, \ell = y^*_B(t_2 + \ell T_1)
\]  
(5.158)

and \(y^*_B(t) = 0\) for \(t < t_2^*\) and \(t > t_2^* + 2gT_1\). Now, according to equations 5.142 and 5.158, and when the sampling phase is adjusted such that \(t_2^* = \rho T_1\), then

\[
y^*_B, \ell = \sqrt{2} \left[ \cos \left( \frac{\rho + \ell}{2} \right) y_1((\rho + \ell)T_1) - \sin \left( \frac{\rho + \ell}{2} \right) y_2((\rho + \ell)T_1) \right]
\]  
(5.159)
which gives,
\[
\begin{align*}
\psi_{Bz} &= \sqrt{2} \left( -1 \right)^{\frac{p+z}{2}} y_1 \left( \frac{p+z}{2} T \right), \quad (p+z) \text{ even} \\
\psi_{Bz+1} &= \sqrt{2} \left( -1 \right)^{\frac{p+z+1}{2}} y_2 \left( \frac{p+z+1}{2} T \right), \quad (p+z) \text{ odd}
\end{align*}
\]
where \( p \) is a constant and
\[
y_1(t) + jy_2(t) = y_Q(t)
\]

Now, comparing eqns. 5.160 with eqns. 5.148 and 5.149, \( Y_B \) in eqn. 5.157 may be written as
\[
Y_B = \sqrt{2} \left[ y_1 \delta - y'_2, y_1' \right]
\]
where the \( \{y_{1,h}\} \) are the real parts of the \( \{y_h\} \) in eqn. 5.148 and the \( \{y'_{2,h}\} \) are similar to the imaginary parts of the \( \{y_h\} \) (eqn. 5.148) but taken at a sampling phase which differ by \( s \). Clearly, the \( \{y_{1,h}\} \) and \( \{y'_{2,h}\} \) are obtained by sampling the real and imaginary parts of \( y_Q(t) \) at a rate of \( \frac{1}{T} \), which is the Nyquist rate in both cases, as can be seen from eqn. 5.102, since \( a(t) \) is real-valued and has a Fourier transform \( A(f) \) that is bandlimited to \( -\frac{1}{2T} \) to \( \frac{1}{2T} \) Hz. This point will be exploited later on.

Let now the 2k-component real-valued vector \( \mathbf{r}_k \) be obtained from the sequence \( \mathbf{s}_k \) by interleaving the imaginary parts of its components with the real parts, such that the (2i)th and (2i+1)th components of \( \mathbf{r}_k \) are given by
\[
\begin{align*}
y_{2i} &= s_{2,i} \\
y_{2i+1} &= s_{1,i}
\end{align*}
\]
where \( s_{1,i} \) and \( s_{2,i} \) are the real and imaginary parts of \( s_i \) in eqn. 5.152. Thus, transmitting the components of the vector \( \mathbf{r}_k \) at a rate of \( \frac{1}{T_1} = \frac{2}{T} \) in the one-dimensional baseband system gives the same information rate as transmitting the components of the vector \( \mathbf{s}_k \) at a rate of \( \frac{1}{T} \), in the QAM system.
Let the vector $E_k$ be a real-valued $2k$-component vector whose $(2i)$th and $(2i+1)$th components are given by,

$$e_{2i} = \frac{2g}{h=0} \gamma_{2i-h} Y_{B,h}$$  \hspace{1cm} (5.165)

and

$$e_{2i+1} = \frac{2g}{h=0} \gamma_{2i+1-h} Y_{B,h}$$  \hspace{1cm} (5.166)

where the $\{y_{B,h}\}$ are given by the components of the vector $Y_B$ in eqn. 5.162. The summation, on the right-hand side of eqn. 5.165 may be broken into two parts such that

$$e_{2i} = \frac{g}{h=0} \gamma_{2i-2h} Y_{B,2h} + \frac{g}{h=1} \gamma_{2i-2h+1} Y_{B,2h-1}$$  \hspace{1cm} (5.167)

which by using eqns. 5.162 to 5.164 becomes

$$e_{2i} = \sqrt{2} \left[ \frac{g}{h=0} (-1)^h (s_{2,i-h} Y_{1,h} + s_{1,i-h} Y_{2,h}) \right]$$  \hspace{1cm} (5.168)

where $y_{2,h} = 0$ for $h<0$. In the same way, it may be shown that $e_{2i+1}$ in eqn. 5.166 may be written as

$$e_{2i+1} = \sqrt{2} \left[ \frac{g}{h=0} (-1)^h (s_{1,i-h} Y_{1,h} - s_{2,i-h} Y_{2,h}) \right]$$  \hspace{1cm} (5.169)

Defining the vector $\Gamma'_k$ which is obtained from $S'_k$ in the same way as $\Gamma_k$ is obtained from $S_k$, and the vector $E'_k$ which is related to $\Gamma'_k$ as $E_k$ is to $\Gamma_k$ (eqns. 5.165 and 5.166), the Euclidean distance between the two vectors $E_k$ and $E'_k$ in the $2k$-dimensional signal vector space is given by the square root of

$$d'^2 = \|E_k - E'_k\|^2$$

$$= \frac{k}{i=1} \left[ (e_{2i} - e'_{2i})^2 + (e_{2i+1} - e'_{2i+1})^2 \right]$$  \hspace{1cm} (5.170)

where $|\cdot|$ represents the length of the vector. Eqns. 5.168-5.170 give
\[ d'^2 = 2(d_1'^2 + d_2'^2 + d_3'^2 + d_4'^2 + d_5') \]  \hspace{1cm} (5.171)

where

\[
\begin{align*}
d_1' &= \sum_{i=1}^{k} \left( \sum_{h=0}^{g} (-1)^h (s_1, i-h - s_1, i-h) y_{1, h} \right)^2 \\
d_2' &= \sum_{i=1}^{k} \left( \sum_{h=0}^{g} (-1)^h (s_1, i-h - s_1, i-h) y_{2, h-1} \right)^2 \\
d_3' &= \sum_{i=1}^{k} \left( \sum_{h=0}^{g} (-1)^h (s_2, i-h - s_2, i-h) y_{1, h} \right)^2 \\
d_4' &= \sum_{i=1}^{k} \left( \sum_{h=0}^{g} (-1)^h (s_2, i-h - s_2, i-h) y_{2, h} \right)^2 \\
d_5' &= 2 \sum_{i=1}^{k} \left( \sum_{h=0}^{g} \sum_{z=0}^{g} [y_{1, h} y_{1, z} (s_1, i-h - s_1, i-h) (s_1, i-z - s_1, i-z)] - y_{1, h} y_{2, z} (s_2, i-h - s_2, i-h) (s_2, i-z - s_2, i-z)] \right) \cdot (-1)^{h+z} \\
\end{align*}
\]  \hspace{1cm} (5.172)

Now from eqns. 5.134 and 5.134a, and at high signal-to-noise ratios where both \( P_B(e) \) and \( P_{Q,H}(e) \) are low, a change of less than two times in the probability of an error corresponds to a change of only a small fraction of 1 db in the signal-to-noise ratio which can be neglected here.\(^{(23)}\) Therefore, and since \( k_e \) (eqn. 5.134) and \( k'_e \) (eqn. 5.134a) are of the same order, and for \( P_B(e) \) (the probability of error in the one-dimensional baseband system) and \( P_{Q,H}(e) \) (the probability of error in the QAM and two-dimensional baseband systems) to be equal, \( d_{\text{min}} \) in the one-dimensional baseband system must be equal to \( \sqrt{2} d_{\text{min}} \) in the QAM system, when \( \sigma \) has the same value in both, eqn.5.134 and eqn. 5.134a, which is assumed here (eqn.5.101). In other words, the minimum value of \( d'^2 \) (eqn. 5.171) must be equal to twice the minimum value of \( d^2 \) (eqn. 5.155), if the QAM and the one-dimensional baseband systems have the same tolerance to additive white Gaussian noise. Of course, the same average transmitted energy per bit (eqn. 5.99) and the same noise power spectral density (\( \frac{1}{2} N_0 \)) are assumed in both systems. Thus, to
show that the two systems have the same performance, it is sufficient to show that the \( \{d_1^i\} \) in eqns. 5.172 are equal to the \( \{d_1^j\} \) in eqns. 5.156. This will be shown as follows.

The \( \{y_{2,h}^i\} \) in eqn. 5.172 are obtained by sampling the imaginary part of \( y_Q(t) \), as can be seen from eqn. 5.162, the sampling phase of the \( \{y_{2,h}^i\} \) being delayed by \( \frac{T}{2} \) relative to the sampling phase of the \( \{y_{2,h}^j\} \), the imaginary components of the sampled impulse response of the QAM system (eqn. 5.148). But since the sampling rate in both cases is at the Nyquist rate, as mentioned earlier, replacing the \( \{y_{2,h}^i\} \) by the \( \{y_{2,h}^j\} \) in eqns. 5.172 should not change the values of \( d_2^i \) and \( d_4^i \). This may be explained as follows. In the signal vector space, the relative distances between any two vectors in that space is independent of the phase distortion present in the signal and is determined only by the amplitude distortion in that signal. But since the sampling phase represents a linear phase shift (i.e. represents a pure orthogonal transformation) when the sampling rate is not less than the Nyquist rate, as is shown in Appendix G, the relative distances in the vector space are independent of the sampling phase. Of course, \( d_2^i \) is the square of the Euclidean distance between the two signal vectors formed by the \( \{s_{1,i}\} \), the \( \{s_{1,i}'\} \) and the \( \{y_{2,h}^i\} \). Similarly, \( d_4^i \) is the square of the Euclidean distance between the two signal vectors formed by the \( \{s_{2,i}\} \), the \( \{s_{2,i}'\} \) and the \( \{y_{2,h}^i\} \).

Now, if the factor \( (-1)^h \) is used in eqns. 5.156, in the same manner as it appears in eqns. 5.172, the value of \( d^2 \) in eqn. 5.155 will not change. This is because the factor \( (-1)^h \) represents a change in the sign of every second component in the sampled impulse response of the linear baseband channel of the QAM system, and this has no effect on distances between the different vectors in the signal vector.
space, as is shown in Appendix D. Thus, replacing the \( Y_i \) by the \( Y_{i,h} \) in \( d_i^2 \) and \( d_i' \) (eqn. 5.172) and using the factor \((-1)^h\) in eqn. 5.156 as just described gives,

\[
d_i^2 = d_{i,1}^2 \quad i=1,2,3 \text{ and } 4
\]  

(5.173)

It remains now to determine the relationship between \( d_5 \) (eqn.5.156) and \( d_5' \) (eqn. 5.172). Since we here are concerned with the minimum distance only, it will be assumed that the vectors \( S_k \) and \( S_k' \) are such that \( d^2 \) in eqn.5.155 is at its minimum value. Although the particular vector \( S_k' \) assumed here is not necessarily at the minimum distance from \( S_k \), it seems reasonable to assume that it differs from \( S_k \) only by a small number of components, i.e., there are a limited number of non-zero quantities \( (s_{1,i} - s_{1,i}) \) and \( (s_{2,i} - s_{2,i}) \). With this in view, it is highly likely that most of the cross products \( (s_{2,i} - s_{2,i})(s_{1,i} - s_{1,i}) \), over all values of \( i=1,\ldots,k \), are zero, and consequently, both \( d_5 \) (eqn.5.156) and \( d_5' \) (eqn.5.172) may very well have very small values compared with the other \( d_i^2 \) and \( d_i' \). In other words, it seems very likely that \( d_5 \) and \( d_5' \) have a negligible effect on the values of \( d^2 \) and \( d_i^2 \), respectively. This, together with eqns.5.155, 5.171 and 5.173, give

\[
d_{i,\text{min}}^2 = 2d_{\text{min}}^2
\]  

(5.174)

which, as mentioned before, implies that the one-dimensional baseband system has a similar tolerance to additive white Gaussian noise to that of the QAM system, when the maximum-likelihood detector is employed.

A point worth mentioning here is that when the Fourier transform of \((h(t) * c(t))\) is conjugate symmetric about \( \frac{1}{2T} \) over the positive frequencies, then \( y_Q(t) \) (eqn.5.135) becomes purely real and both the \( Y_{2,h} \) and the \( Y_{2,h}' \) and therefore each of \( d_2^2, d_4^2, d_5, d_2'^2, d_4'^2 \) and \( d_5' \) (eqns. 5.156 and 5.172) become zero. In this case, eqn.5.174 becomes exactly correct without any further assumptions.
As a result, the one- and two-dimensional baseband systems have, theoretically, the same tolerance to additive white Gaussian noise as does the QAM system, when a maximum likelihood detector is used at the receiver.
6. THE DETECTION OF DIGITAL DATA SIGNALS
TRANSMITTED AT 19200 BIT/S OVER
TELEPHONE LINES

6.1 INTRODUCTION

The ever increasing use of computers and data processing systems in commercial, industrial and military areas has substantially increased the demand for higher speed digital-data transmission. The public switched telephone network is one of the most important media for data transmission, and achieving higher transmission speeds over it is a major task facing workers in this field. The data transmission rate is undoubtedly limited over telephone circuits by the different impairments likely to be encountered over these circuits. In the absence of all other kinds of impairments, additive noise, on its own, limits the reliable transmission speed to a value dependent on the relative level of this additive noise compared with the transmitted signal level. Shannon\(^{(2)}\) has shown that, for a channel that introduces no attenuation over the frequency band \(-W\) to \(W\) Hz, and infinite attenuation elsewhere, when this channel is fed at the Nyquist rate of \(2W\) signal-elements per second from a Gaussian source of data (where the data-symbols are drawn out of a predetermined finite set of statistically independent Gaussian random variables), the maximum speed for reliable transmission (i.e. vanishingly small error rate) in the presence of white Gaussian noise is given by

\[
C = W \log_2(1+\text{SNR}) \text{ bit/s}
\]  

(6.1)

where SNR is the signal-to-noise power ratio. As an example when \(W=3000\) Hz i.e. an ideal telephone circuit\(^{(23)}\) and for a signal-to-noise ratio of 30 dB,
C in eqn. 6.1 is equal to 29900 bit/s. In this example, the channel introduces no linear distortion other than being band-limited. In the presence of linear amplitude distortion, i.e. when the channel attenuation characteristic is not constant over the channel bandwidth, the maximum speed, for the same value of the signal-to-noise ratio, becomes even lower. In Ref. (3), a typical telephone circuit is used as an example where the signal-to-noise ratio is in the order of 30 db. There, it is shown that the maximum transmission rate over that channel is 23500 bit/s when the data source is Gaussian and 19700 bit/s when the data source is binary with equal probability for its possible values.

As mentioned in section 2, a telephone circuit in the switched telephone network may have any of a very wide range of attenuation and group delay characteristics. Furthermore, arbitrary but usually high values of the signal-to-noise ratio may be encountered over telephone circuits (19). Therefore, the maximum speed of data transmission varies considerably from one telephone circuit to another and no fixed absolute value for all channels can be assumed. However, as the figures given above have been determined for a typical telephone circuit, they may be considered as a target to be reached. This section, is concerned with the detection of digital data signals transmitted at 19200 bit/s, this being the highest of the CCITT standard rates.

For satisfactory transmission of data at rates as high as 19200 bit/s over the switched telephone network, the receiver must be able to take full account of the distortion introduced by the channel into the transmitted signal. At the lower rates, the conventional adaptive non-linear equalizer, that is adjusted to minimise the mean-square-error in the equalized signal, is known to operate satisfactorily over telephone circuits, but at rates
equal or greater than 9600 bit/s satisfactory operation of that equalizer is not always obtainable over the poorer telephone circuits.\(^\text{(116)}\) As the transmission rate increases, the performance of the adaptive non-linear equalizer deteriorates, and more sophisticated techniques are required for reliable operation.

The alternative approach to the non-linear equalizer is to use the Viterbi-algorithm detector\(^\text{(90,93)}\) details of which are considered in Section 3.4, where it is shown that this detector, under certain conditions, is optimum. When the additive Gaussian noise samples at the detector input are statistically independent, the detector performs a process of maximum-likely-detection, and when, in addition, the transmitted data symbols are equally likely to have any of their possible values, the detection process minimizes the probability of error in the received message. Unfortunately when the sampled impulse response of the channel contains more than a very few components and when the number of signal levels is high (as it is in the present application), the Viterbi-detector becomes very complicated. Both, the computing and storage requirements grow rapidly as either of the two quantities is increased. To simplify the Viterbi-detector, the "desired impulse response" technique considered in Section 3.6 may be used. In this technique, a linear adaptive feed-forward transversal filter is used ahead of the detector to force the overall sampled impulse response of the channel and filter together to have a small number of components, e.g. 2 components. Now the Viterbi-detector may be implemented very simply, but unfortunately, again, the linear adaptive filter is here likely to perform a process of amplitude equalization of the channel characteristics, which causes noise enhancement\(^\text{(81)}\) and therefore leads to an inferior performance.\(^\text{(116)}\)

As has been shown in Section 3.5, the ideal arrangement for the Viterbi
detector is to be preceded by a whitened matched filter. When the channel is used at the Nyquist rate (the signal-element rate in bauds is equal to the Nyquist rate of the channel), this filter degenerates into the following arrangement. First, there is a lowpass filter whose transfer function is rectangular and is bandlimited to $-\frac{1}{2T}$ to $\frac{1}{2T}$ Hz, where $\frac{1}{T}$ is the element rate. This is followed by a sampler and then a linear feedforward transversal filter that forms the first part of the optimum non-linear equalizer for the given channel (Section 3.2.3). In practice, the latter filter is implemented adaptively and its function, when correctly adjusted, is to replace the roots of the z-transform of the sampled impulse response of the channel and receiver filter, which lie outside the unit circle in the z-plane by the complex conjugates of their reciprocals. The lowpass filter and sampler, without the linear transversal filter, are under the assumed conditions equivalent to a whitened matched filter in the sense that the output of the sampler forms a sufficient statistic for the estimation of the sequence of data-symbols (see Section 3.5), but when the linear transversal filter is also used, as just described, the Viterbi-algorithm detector takes its simplest form for the given channel, because of the reduced number of components of the sampled impulse response of the channel and whitened matched filter, where this sampled impulse response is now a minimum phase sequence (Section 3.2.3). Again, and for most channels of interest, the sampled impulse response obtainable by this arrangement still consists of more than a few non-zero components and therefore the required Viterbi-detector is still unacceptably complicated.

A practical and near-optimum alternative to the Viterbi detector is the near-maximum likelihood detector (Section 3.8). This is a form of a reduced-state Viterbi detector and requires much less complex equipment.
while attaining a performance very close to the optimum. While the Viterbi detector retains \( m^g \) survivors at each recursion cycle, where \( m \) and \( g+1 \) are the number of the possible signal levels and the number of components in the sampled impulse response of the channel, respectively, the near-maximum likelihood detector holds in store only \( \mu \) survivors, where \( \mu \ll m^g \), so that the performance of this detector becomes progressively degraded, relative to the optimum, as the ratio \( \mu/m^g \) decreases. Furthermore, the performance of the near-maximum likelihood detector, when used without an adaptive linear filter ahead of the detector, is seriously affected by phase distortion introduced by the channel into the data signal, since the channel impulse response now tends to rise slowly towards its peak, so that the first few components of the resultant sampled impulse response have steadily increasing magnitudes. Under these conditions, an adaptive linear filter, which converts the sampled impulse response of the channel into a minimum phase sequence, without changing the amplitude distortion in the signal, should improve the performance of the near-maximum likelihood detector, when used ahead of this detector. This is because it concentrates the energy of the sampled impulse response in its first few components and thereby effectively reduces the number of components in this sampled impulse response. This linear filter is in fact the first part of the optimum non-linear equalizer, and also forms a part of the arrangement into which the whitened matched filter degenerates when the channel is used at the Nyquist rate. The near-maximum likelihood detector, with the adaptive linear filter, is used in Ref. (116), in the detection of digital data signals transmitted at 9600 bit/s over a range of typically good to typically very poor telephone lines. It is shown that the performance is quite satisfactory here. In the present work, the same approach is followed
in the development of near-maximum likelihood detectors for the detection of digital data signals transmitted at 19200 bit/s over telephone lines.

Although the present section is mainly concerned with the detection of data signals transmitted at 19200 bit/s over telephone lines, the systems used here employ the same basic techniques as those described in Sections 4 and 5. Four data transmission systems have been studied here. In system A, a real (one-dimensional) 8-level baseband signal (Section 5.4.1) is transmitted at an element rate of 6400 bauds. In system B, a complex (two-dimensional) 64-level baseband signal (Section 5.4.2) is transmitted at an element rate of 3200 bauds. In systems C and D, 64- and 256-level QAM-signals (Section 4) are used at signal-element rates of 3200 and 2400 bauds, respectively. The performances of these systems are compared by computer simulation.

The values of the signal-element rates given above are close to the Nyquist rates for the corresponding channels (whose characteristics are to be given later), so that a receiver filter with a lowpass rectangular transfer function, followed by a sampler and a linear transversal filter, may be used instead of the whitened matched filter. When a QAM signal is transmitted the lowpass filter, of course, operates on the demodulated signal waveform. Unfortunately, a true rectangular lowpass filter is not realizable in practice. Instead, a filter with very sharp transitions from the passband to the stopband may be used. Three different equipment (transmitter and receiver) filters are used in the tests; one of which has a $\sin^2$ transfer function (23) whereas the other two filters have transfer functions that are closer to the rectangular.

In all cases, the systems are compared on the basis of their tolerances to additive white Gaussian noise. The latter is assumed to be
added to the data signal at the output of the telephone line. The various
types of additive and multiplicative noise normally introduced by telephone
circuits into the transmitted signal are neglected here. Although telephone
lines do not normally introduce a significant level of white Gaussian noise,
the relative tolerances of different systems to this type of noise are good
measures of their relative tolerances to additive noise actually introduced
by telephone lines.\(^{(23,81)}\) Non-linear distortion which is likely to be
introduced by telephone circuits into the transmitted signal is also ignored
here because it is not usually very important.\(^{(3)}\)

6.2 MODELS OF DATA-TRANSMISSION SYSTEMS

6.2.1 System A

The model of data transmission system A is shown in Figure 6.1. It is
a synchronous serial system in which an 8-level baseband signal is transmitted
at an element rate of 6400 bauds. The theoretical analysis of this system
is given in Section 5.4.1. In Figure 6.1, a stream of binary digits \(\{a_k\}\),
\[ a_k = 0 \text{ or } 1 \]  \(\text{(6.2)}\)
is fed at a rate of 19200 bit/s into the encoder where the \(\{a_k\}\) are first
differentially encoded and then recoded into the data-symbols \(\{s_i\}\),
\[ s_i = \pm 1, \pm 3, \pm 5, \pm 7 \]  \(\text{(6.3)}\)
at a rate of 6400 bauds. The coding of the \(\{a_k\}\) into the \(\{s_i\}\) is
considered in Section 6.2.4. It is assumed here that the \(\{a_k\}\) are
statistically independent and equally likely to have either of the binary
values, which results in the \(\{s_i\}\) being also statistically independent and
equally likely to have any of the eight possible values in eqn. 6.3.
Whenever this condition is not satisfied, the \(\{a_k\}\) must be appropriately
FIGURE 6.1: Model of System A
scrambled before being coded into the \( \{s_i\} \). Also it is assumed here that the \( \{a_k\} \) are such that \( s_i = 0 \) for \( i < 0 \). The \( \{s_i\} \) are fed to the transmitter-filter \( G \) which has a bandpass transfer function chosen to approximately fit the passband of an ideal voice channel (Figure 2.1). The signal at the output of this filter is fed to the telephone circuit. The spectral shaping of this signal is performed entirely by the filter \( G \) and no any linear modulation process is involved. The telephone circuit introduces into the data signal linear amplitude and phase distortions and a frequency offset of \( p \) Hz, where \( p \) is in the order of a few Hz (up to \( \pm 5 \) Hz\(^{(23)} \)). At the output of the telephone circuit is added a white Gaussian noise \( n(t) \) with zero mean and two-sided power spectral density of \( \frac{1}{2} N_0 \).

The receiver filter \( C \) has a bandpass transfer function and is taken here to be the same as the transmitter filter \( G \), so that it removes as much of the noise as possible without excessively distorting the received signal. Now, the signal at the receiver filter output is a linearly-distorted and noisy version of the transmitted signal and has a time-varying nature which results from the presence of the frequency offset, (Section 5.3). This signal is fed to the filter \( F \) whose output signal is the Hilbert-transform of its input signal. The output signal of the filter \( F \) is considered (in the theoretical analysis) to be multiplied by \( j^{\omega T} \) and combined, with an appropriately delayed version of the (real-valued) signal at the receiver filter output to give the complex-valued signal \( p(t) \). The latter appears at the output of the linear baseband channel formed by the transmitter filter \( G \), telephone circuit, receiver filter \( C \) and phase splitter (the delay line and filter \( F \)). \( p(t) \) is then sampled once per signal element at times \( t = iT, i = 1, 2, \ldots \), to give the corresponding signal samples, \( \{p_i\} \), where

\[
p_i = \sum_{h=0}^{\infty} s_{i-h}(d_{h} + jd_{h})e^{j\phi_{i}}w_{i}
\]

(6.4)
\[ d_h = d(hT) \]
\[ \hat{d}_h = \hat{d}(hT) \]
\[ \phi_i = \phi(iT) \]
\[ w_i = w(iT) \]

\( d(t) \) is the impulse response of filter \( G \), telephone circuit and filter \( C \) in cascade, \( \hat{d}(t) \) is the Hilbert transform of \( d(t) \), and \( (d(t)+j\hat{d}(t)) \) is the impulse response of the linear baseband channel (Figure 6.1) and is assumed here, for practical purposes, to be of finite duration so that \( (d_h+j\hat{d}_h)=0 \) for \( h<0 \) and \( h>p \). \( \phi(t) \) represents the time-varying phase in \( p(t) \) due to the frequency offset which is introduced by the telephone circuit. The \( \{w_i\} \) are complex-valued Gaussian random variables with zero mean and variance \( \sigma_w^2 \), and are slightly correlated by the filter \( C \) because of the departure of its transfer function from the ideal lowpass rectangular form. Each of the real and imaginary parts of the \( \{w_i\} \) have the same variance which is given by \( \frac{\sigma_w^2}{2} \).

The samples \( \{p_i\} \) are now fed to an adaptive linear transversal filter which performs a process of pure phase transformation on the \( \{p_i\} \). This filter, when correctly adjusted and at high signal-to-noise ratios, replaces the roots of the z-transform of the sampled impulse response \( \{d_h+j\hat{d}_h\} \) of the linear baseband channel which lie outside the unit circle in the z-plane by the complex-conjugates of their reciprocals. Correct adjustment of this filter is assumed here, so that test results, which will be given later, represent the best that can be achieved, as far as the linear adaptive filter is concerned. The properties and the adaptive structure of this filter are considered in detail in Sections 3.2.3, 3.5 and 3.9.

The signal sample \( r_i \), at time \( t=iT \), at the output of the adaptive linear filter (Figure 6.1) is now given by
where the \( \{y_h + j\hat{y}_h\} \) are the components of the sampled impulse response of the linear baseband channel, sampler and linear adaptive filter in cascade. Of course, the z-transform of the \( \{y_h + j\hat{y}_h\} \) has now no roots outside the unit circle. It is assumed in eqn. 6.5 that \( (y_h + j\hat{y}_h) = 0 \) for \( h < 0 \) and \( h > l \).

The \( \{v_i\} \) are complex-valued Gaussian random variables with the same statistics as those of the \( \{w_i\} \), since the linear adaptive filter (when correctly adjusted) performs a pure orthogonal transformation on its input signal. When this filter introduces into the signal samples any gain or attenuation (scaling), then this applies to the signal components as well as to the noise components so that no change in the overall signal-to-noise ratio is introduced.

The frequency offset corrector (Figure 6.1) multiplies now the signal samples \( \{r'_i\} \) by the corresponding \( e^{-j\phi_i} \) to remove the time-variation from the data-signal components in the \( \{r'_i\} \) due to the frequency offset. The frequency offset corrector includes a phase-locked loop estimator which produces an estimate \( \hat{\phi}_i \) of \( \phi_i \) using the received signal samples \( \{p_i\} \) and the detected data-symbols \( \{s_i\} \). It is assumed here that \( \hat{\phi}_i = \phi_i \) so that the signal sample at the frequency offset corrector output is given by

\[
\begin{align*}
r''_i &= \sum_{h=0}^{g} s_{i-h}(y_h + j\hat{y}_h) + v_i e^{-j\phi_i} 
\end{align*}
\]  

Finally, the sequence \( \{r'_i\} \) is obtained by taking the real part of the \( \{r''_i\} \). The resultant signal sample \( r_i \), at time \( t = iT \) at the detector input is

\[
\begin{align*}
r_i &= \sum_{h=0}^{g} s_{i-h}y_h + u_i 
\end{align*}
\]  

where \( y_h = 0 \) for \( h < 0 \) and \( h > g \). The parameters \( f \) and \( g \) in eqns. 6.6 and 6.7, are, in general, not the same since usually the \( \{\hat{y}_h\} \) consist of more non-zero
components than the \( \{y_i\} \). Therefore, the subscript \( i \) in eqn. 6.7 has a delayed or advanced value relative to that in eqn. 6.6. In every case it was taken to be such that the first non-zero component of the \( i \)th transmitted data-element appears in the \( i \)th sample \( r_i' \) or \( r_i \). The sampled impulse response of the overall system, as seen by the detector is given by the real-valued vector,

\[
Y = y_0, y_1, \ldots, y_g
\]

which, when every step previously described is correctly performed, corresponds to the sampled impulse response of the transmitter filter \( G \), telephone circuit, receiver filter \( C \), sampler and adaptive linear filter in cascade. The \( \{u_i\} \) in eqn. 6.7 are the real-valued noise components in the \( \{r_i\} \), where (eqns. 6.6 and 6.7)

\[
u_i = \text{Real}[v_i e^{-j\phi_i}]
\]

\[
= v_{1,i} \cos \phi_i + v_{2,i} \sin \phi_i
\]

\( v_{1,i} \) and \( v_{2,i} \) being the real and imaginary parts of \( v_i \), respectively. Since the adaptive linear filter (Figure 6.1) does not change the noise statistics, the \( \{u_i\} \) are correlated (as given by eqn. 5.56) by the receiver filter \( C \) and the effect of the frequency offset as well. Here the correlation due to the frequency offset will be neglected since it is normally very small, and the only correlation in the \( \{u_i\} \) is that introduced by the receiver filter \( C \).

The detector operates on the \( \{r_i\} \) to give the detected data-symbols \( \{s_i'\} \) after a delay of \( nT \) seconds. The detector has a prior knowledge of the possible values of the \( \{s_i\} \), and it uses this together with an estimate \( Y' \) of \( Y \) (eqn. 6.8) in the detection of the \( \{s_i'\} \). The estimate \( Y' \) is produced by the channel estimator (92, 101-103, 121-122) that operates on the \( \{s_i'\} \) and an appropriately delayed version of the \( \{r_i\} \) (Figure 6.1). The basic method of
operation of the channel estimator is outlined in Section 3.9. Since channel estimation is beyond the scope of this work, perfect estimation is assumed, so that $Y' = Y$, and the test results given later represent the upper bound to the system performance when actual estimation is involved.

Finally, the detected data-symbols $\{s'_i\}$ are decoded into the $\{a'_k\}$ which are the detected values of the transmitted binary digits $\{a_k\}$. The decoding process is considered in Section 6.2.4.

6.2.2 System B

The data transmission system B is shown in Figure 6.2. It is a synchronous serial system in which a 64-point 2-dimensional baseband signal is transmitted at an element rate of 3200 bauds. The theoretical analysis of the system is given in Section 5.4.2.

The information to be transmitted is carried by a stream of binary digits $\{a_k\}$ ($a_k = 0$ or 1) at a rate of 19200 bit/s. The $\{a_k\}$ are fed to the encoder which first differentially codes the $\{a_k\}$ and then recodes them into the 64-level complex-valued data-symbols $\{s_i\}$, where

$$ s_i = s_{1,i} + js_{2,i} $$

(6.10)

and $j = \sqrt{-1}$, the coding process being described in Section 6.2.4. The $\{a_k\}$ are such that $s_i = 0$ for $i \leq 0$ so that $s_i$ is the $i^{th}$ transmitted data-symbol. Also, the $\{a_k\}$ are assumed here to be statistically independent and equally likely to have either binary value, which results in the $\{s_i\}$ being statistically independent and equally likely to have any of their 64 possible values. Whenever the $\{a_k\}$ are significantly correlated, they must be appropriately scrambled before being code into the $\{s_i\}$.
FIGURE 6.2: Model of System B
The \( s_{1,i} \) which represent the real part of the \( s_i \) are now fed to the transmitter filter \( G \), whereas the \( s_{2,i} \), the imaginary parts of the \( s_i \), are fed to the transmitter filter \( G_1 \). The transmitter filter \( G_1 \) has an impulse response which is the negative of the Hilbert transform of the impulse response of the filter \( G \), so that the two signals at the outputs of these two filters are in phase-quadrature. The absolute values of the transfer function of both filters \( G \) and \( G_1 \) are the same and these are chosen to approximately fit the passband characteristics of an ideal voice channel (Figure 2.1). It should be noted here that since the signal-element rate is 3200 bauds, i.e. about the highest frequency in the passband of the voice frequency channel, both filters \( G \) and \( G_1 \) are used at approximately half the Nyquist rate.

The two orthogonal baseband signals at the outputs of the filters \( G \) and \( G_1 \) are then added to each other and fed to the telephone circuit which introduces linear amplitude and phase distortion into the transmitted baseband signal. Also, the telephone circuit introduces into the transmitted signal a frequency offset of \( p \) Hz, where \( p \) is of the order of one or two Hz (up to \( \pm 5 \) Hz\(^{(23)} \)). At the output of the telephone circuit, a white Gaussian noise, \( n(t) \), with zero mean and two-sided power spectral density \( \frac{1}{2} N_0 \), is added to the transmitted baseband signal. The noisy signal is now fed to the receiver filter \( C \) which is taken here to have a transfer function with the same absolute value as that of the transmitter filters \( G \) and \( G_1 \), so that the filter \( C \) removes all noise components outside the signal frequency band, without excessively distorting the transmitted signal. The signal at the receiver filter output is now a real-valued linearly-distorted and noisy version of the transmitted
signal. It has a time-varying nature which results from the presence of
the frequency offset (Section 5.3). This signal is then fed to the
filter F whose output signal is the Hilbert transform of its input signal.
For the purpose of theoretical analysis, the output signal from the filter
F is considered to be multiplied by \( j = \sqrt{-1} \) and added to an appropriately
delayed version of the real-valued signal at the input to the filter F.
This gives the complex-valued signal \( p(t) \) at the output of the linear
baseband channel, formed by the transmitter filters \( G \) and \( G_1 \), the
telephone circuit, the receiver filter \( C \) and the phase splitter (the delay
line and filter F). Next, \( p(t) \) is sampled once per signal-element at times
t=\( iT \), \( i = 1, 2, \ldots \), to give the corresponding signal samples \( \{ p_i \} \), where
\[
p_i = \sum_{h=0}^{\rho} s_i - h \cdot d_h \cdot e^{j \phi_i} + w_i \tag{6.11}
\]
and
\[
\begin{align*}
d_h &= d(ht) \\
\phi_i &= \phi(iT) \\
w_i &= w(iT)
\end{align*} \tag{6.12}
\]
d(\( t \)) is the complex-valued impulse response of the linear baseband channel
whose amplitude response is zero over the negative frequencies (Section
5.4.2). \( d(\cdot) \) is for practical purposes assumed to have a finite duration,
so that \( d_h = 0 \) for \( h < 0 \) and \( h > \rho \). Any delay in transmission is neglected here,
so that the sample \( p_i \) contains the first non-zero component of the ith
transmitted data-symbol \( s_i \). \( \phi(t) \) represents the time-varying phase in
\( p(t) \) due to the frequency offset introduced by the channel. The \( \{ w_i \} \)
are complex-valued Gaussian random variables with zero mean and variance
\( \sigma_w^2 \). Also, they are slightly correlated by the filter C because of the
departure of its transfer function from the ideal lowpass rectangular form.
The samples \{p_i\} are fed to an adaptive linear filter which, when correctly adjusted, replaces the roots of the z-transform of the \{d_h\} which lie outside the unit circle in the z-plane by the complex conjugates of their reciprocals. Correct adjustment of this filter is assumed in all tests so that these show the best performance that can be achieved with an adaptive linear filter. The properties and the adaptive structure of this filter are considered in Sections 3.2.3, 3.5 and 3.9. The signal samples \{r'_i\} at the output of the adaptive linear filter are such that

\[ r'_i = \sum_{h=0}^{\infty} s_{i-h} y_h e^{j\phi_i} + v_i \]  

(6.13)

and the \{y_h\} are given by the complex-valued vector

\[ Y = y_0, y_1, \ldots, y_g \]  

(6.14)

which represents the sampled impulse response of the linear baseband channel, sampler and adaptive linear filter in cascade. The \{v_i\} are the complex-valued noise components in the \{r'_i\} and have the same statistics as the \{w_i\}, since the adaptive linear filter does not change these statistics. Furthermore, any gain or attenuation introduced by this filter will affect the data signal as well as the noise, so that the signal-to-noise ratio in the \{r'_i\} is still the same as it is in the \{p_i\}.

Next, the samples \{r'_i\} are fed to the frequency offset corrector which multiplies them by the corresponding \{e^{-j\phi_i}\} to remove the time-variation from the data-signal components in \{r'_i\} due to the frequency offset. The frequency offset corrector includes a phase-locked loop estimator (135-152) (not shown in Figure 6.2) which produces an estimate \(\hat{\phi}_i\) of \(\phi_i\) using the received signal samples \{p_i\} and the detected data-symbols \{s'_i\}. It is assumed here that \(\phi'_i=\phi_i\), so that the signal sample at the frequency offset corrector output is given by,
\( r_i = \sum_{h=0}^{g} s_{i-h} y_h + u_i \) \hspace{1cm} (6.15)

where

\( u_i = v_i e^{-j\phi_i} \) \hspace{1cm} (6.16)

and the \( \{u_i\} \) are the complex-valued noise components in the \( \{r_i\} \). The \( \{u_i\} \) are correlated by both the receiver filter and by the effect of the frequency offset, the overall correlation being expressed by eqn. 5.87. Here the correlation between the \( \{u_i\} \) due to the frequency offset is neglected since it is very small, and \( \phi_i \) in eqn. 6.16 is set to zero. Thus the only correlation effects considered are those due to the receiver filter. However, the variance of the \( \{u_i\} \) is independent of the frequency offset (eqn. 5.88), and when the adaptive linear filter introduces no gain or attenuation, this variance is equal to the variance of the \( \{w_i\} \) (eqn. 6.11).

Now, the signal samples \( \{r_i\} \) are fed to the detector which uses those together with an estimate \( Y' \) of \( Y \) to give at its output the detected data symbols \( \{s'_i\} \) after a delay of \( nT \) seconds. Of course, the detector has prior knowledge of the 64 possible values of the data-symbol \( s_i \). The channel estimator (Figure 6.2) which is considered in Section 3.9, uses the detected data symbols \( \{s'_i\} \) together with the \( \{r_i\} \) to form the estimate \( Y' \). Since channel estimation is beyond the scope of this work, perfect estimation is assumed, i.e. \( Y' = Y \).

Finally, the detected data-symbols \( \{s'_i\} \) are decoded into the \( \{a'_k\} \), the detected values of the transmitted binary digits \( \{a_k\} \). The decoding process is considered in Section 6.2.4.
6.2.3 Systems C and D

Systems C and D both transmit QAM signals, and the corresponding model of the data transmission system is shown in Figure 6.3. The theoretical analysis of the QAM system is considered in Section 4.2. The information to be transmitted is a sequence of binary digits \( \{a_k\} \), where

\[ a_k = 0 \text{ or } 1 \quad (6.17) \]

the \( \{a_k\} \) being statistically independent and equally likely to have either binary value. The \( \{a_k\} \) are fed, at a rate of 19200 bit/s, to the encoder which gives at its output the corresponding sequence of data symbols \( \{s_i\} \), where

\[ s_i = s_{1,i} + js_{2,i} \quad (6.18) \]

\[ s_{1,i}, s_{2,i} = \pm 1, \pm 3, \pm 5, \pm 7 \quad (6.19) \]

in system C and,

\[ s_{1,i}, s_{2,i} = \pm 1, \pm 3, \pm 5, \pm 7, \pm 9, \pm 11, \pm 13, \pm 15 \quad (6.20) \]

in system D, and \( j = \sqrt{-1} \). It is assumed here that \( s_i = 0 \) for \( i \leq 0 \), so that \( s_i \) is the \( i \)th transmitted data-symbol. From eqns. 6.18 to 6.20, the signal fed to the telephone circuit is a 64 or 256-level QAM signal, for systems C and D, respectively. The data-symbols \( \{s_i\} \) are obtained from the \( \{a_k\} \), by the encoder, after being differentially encoded (Section 6.2.4). The \( \{a_k\} \), and therefore the \( \{s_i\} \), are statistically independent and equally likely to have any of their possible values. The QAM signal generator (Figure 6.3) converts the input data-symbols \( \{s_i\} \) into a serial stream of signal elements, with a carrier frequency \( f_c = 1600 \) or 1800 Hz, depending on the transmitter filter A in use. This will be considered further in Section 6.3.5. In case of system C, the 64-point QAM signal-elements are transmitted at 3200 bauds, and in the case of
FIGURE 6.3: Model of Systems C and D
system D, the 256-point QAM signal-elements are transmitted at 2400 bauds. Each signal-element itself is real-valued and comprises the sum of two double sideband suppressed carrier amplitude modulated elements, with their carriers in phase quadrature, the symbol values carried by the "in phase" and "quadrature" elements being given, respectively, by $s_{1,i}$ and $s_{2,i}$, that is, by the real and imaginary parts of the corresponding data-symbol $s_i$. The resulting QAM signal is fed to the telephone circuit, which introduces linear amplitude and phase distortions into the transmitted signal. It is assumed here that stationary white Gaussian noise with zero mean and a two-sided power spectral density of $\frac{1}{2}N_0$ is added to the data signal at the output of the telephone circuit. The linear demodulator in Figure 6.3 includes at its input the bandpass filter $C$, the absolute value of whose transfer function matches the amplitude spectrum of the QAM signal at the input of the telephone circuit. The filter, removes the noise components outside the frequency band of the signal without excessively distorting it. The filtered signal is fed to two linear coherent demodulators whose reference carriers are in phase quadrature and have the same frequency as that of the received QAM signal. The demodulated signals at the outputs of the in-phase and quadrature coherent demodulators are taken to be real and imaginary valued, respectively, so that the resultant demodulated baseband signal $p(t)$ is complex-valued.

The white Gaussian noise at the output of the telephone circuit gives the complex-valued Gaussian noise waveform $w(t)$, at the output of the demodulator. $w(t)$ represents the noise component in $p(t)$.

It will be assumed here that the transmitter filters $A$ and the low-pass filters $B$ (Figure 6.3) have transfer functions $A(f)$ and $B(f)$,
respectively, which are such that

$$A(f) = B(f) = 0 \text{ for } |f| > f_c$$  \hspace{1cm} (6.21a)

and that $f_c$ is such that

$$f_c \geq \frac{1}{2T}$$  \hspace{1cm} (6.21b)

where $\frac{1}{T}$ is the signal-element rate. Although now both $A(f)$ and $B(f)$ are not strictly bandlimited as shown in Figs. 4.2 and 4.5, all the relevant theory developed in Section 4 is still applicable here, with the difference that the integration limits $\{- \frac{1}{2T}, \frac{1}{2T}\}$ in eqns. 4.29, 4.30, 4.34 and 4.48 are replaced by $\{-f_c, f_c\}$. The condition in eqn. 6.21a is necessary to remove the high frequency components in eqn. 4.14 in order to obtain eqn. 4.16, which is taken here to represent the baseband model of the QAM system. Eqns. 6.21a and 6.21b allow the theory to be extended to the case when the signal-element rate is below the Nyquist rate of filters $A$ (Figure 6.3).

The waveform $p(t)$ at the output of the linear baseband channel (Figure 6.3) is now sampled once per signal element at times $t=iT$, $i=1,2,\ldots$, to give the signal samples $\{p_i\}$, where

$$p_i = \sum_{h=0}^{\infty} s_i - h d_h + w_i$$  \hspace{1cm} (6.22)

and

$$d_h = d(hT)$$

$$w_i = w(iT)$$

and $d(t)$ is the complex-valued impulse response of the linear baseband channel (Figure 6.3) formed by the QAM signal generator, telephone circuit and linear demodulator. It is assumed here to be of finite duration, for practical purposes, so that $d_h = 0$ for $h<0$ and $h>p$, any delay in transmission being neglected. The $\{w_i\}$ are Gaussian random
variables with zero mean and a variance $\sigma_w^2$, and are slightly correlated by the receiver filter $C$, because of the deviation of the transfer function of this filter from the ideal rectangular form (eqns. 4.30-4.35).

The samples $\{p_i\}$ are fed to the adaptive linear filter which, when correctly adjusted, replaces the roots of the $z$-transform of the sampled impulse response given by the $\{d_h\}$, which lie outside the unit circle, by the complex-conjugates of their reciprocals. The properties and adaptive structure of this filter are considered in Sections 3.2.3, 3.5 and 3.9. The signal sample $r_i$ at the output of the adaptive linear filter is given by

$$r_i = \sum_{h=0}^{g} s_{i-h} y_h + u_i \quad (6.23)$$

where the $\{y_h\}$ are given by the $(g+1)$-component complex-valued vector

$$Y = y_0, y_1, \ldots, y_g \quad (6.24)$$

which represents the sampled impulse response of the linear baseband channel, sampler and adaptive linear filter in cascade (Figure 6.3). Again, the delay in transmission over the linear baseband channel and adaptive filter is neglected here so that $y_h=0$ for $h<0$ and $h>g$. Clearly, the $z$-transform of the vector $Y$ has no roots outside the unit circle.

The $\{u_i\}$ (the noise components in the $\{r_i\}$) are Gaussian random variables with the same statistics as those of the $\{w_i\}$, since the adaptive linear filter performs a pure phase (orthogonal) transformation. Of course, any gain or attenuation introduced by this filter affects the data-signal as well as the noise so that the overall signal-to-noise ratio is not changed. When the adaptive linear filter introduces no gain or attenuation, the variance of the $\{u_i\}$ is given by the variance of the $\{w_i\}$.

The detector (Figure 6.3) uses the received signal samples $\{r_i\}$
together with an estimate $Y'$ of the sampled impulse response $Y$ (eqn. 6.24) to give, after a delay of $nT$ seconds, the detected data symbols $\{s_i\}$. The detector, of course, has prior knowledge of the possible values of the data symbol $s_i$. The estimate $Y'$ of $Y$ is produced by the channel estimator which is considered in Section 3.9. Since channel estimation is beyond the scope of this work, perfect estimation is assumed, i.e. $Y' = Y$.

Finally, the detected data symbols $\{s_i\}$ are decoded into the sequence of the detected binary digits $\{a_k\}$, which are the detected values of the binary digits $\{a_k\}$ fed to the transmitter.

6.2.4 Encoder and Decoder

In the data transmission systems A, B, C and D, shown in Figs. 6.1, 6.2 and 6.3, respectively, the stream of binary digits $\{a_k\}$, where
\[ a_k = 0 \text{ or } 1, \quad (6.25) \]
is fed, at a rate of 19200 bit/s, to the encoder. This converts the $\{a_k\}$ into a corresponding stream of data symbols $\{s_i\}$, where $s_i$ is given by eqns. 6.3, 6.10 and 6.18 to 6.20 for systems A, B, C and D respectively. The encoder gives the $\{s_i\}$ at its output at rates of 6400, 3200, 3200 and 2400 symbols per second for the four systems, respectively. At the receiver, the decoder converts the detected data symbols $\{s_i\}$ back into the corresponding detected binary digits $\{a'_i\}$.

In system A, the encoding process of the $\{a_k\}$ into the $\{s_i\}$ (eqn. 6.3) is as follows. The sequence of binary digits $\{a_k\}$ is divided into separate groups of 3 adjacent digits $a_{3(i-1)+1}, a_{3(i-1)+2}, a_{3(i-1)+3}$. The first binary digit $a_{3(i-1)+1}$ in the $i$th group is differentially recoded into the binary digit $\overline{a}_{3(i-1)+1}$ (with possible values 0 or 1), using the
previously differentially coded digit $\overline{a}_{3(i-2)+1}$ according to Table 6.1, and the binary coded word $\overline{a}_{3(i-1)+1}a_{3(i-1)+2}a_{3(i-1)+3}$ is then recoded into the data-symbol $s_i$ according to Table 6.2. As can be seen from Table 6.2, the first binary digit $\overline{a}_{3(i-1)+1}$ determines the sign of the corresponding data symbol $s_i$ whereas the other two bits give its absolute value.

At the receiver in system A, and following the detection of the data-symbol $s_i$, the corresponding detected values of $\overline{a}_{3(i-1)+1}a_{3(i-1)+2}a_{3(i-1)+3}$ are determined from Table 6.2 using the detected value of $s_i$ in place of its actual value. Next, the detected binary digit $\overline{a}_{3(i-1)+1}$ is differentially decoded into $a_{3(i-1)+1}$, according to Table 6.1, using $\overline{a}_{3(i-1)+1}$ and $\overline{a}_{3(i-2)+1}$.

As can be seen from Tables 6.1 and 6.2, a change of a multiple of $180^\circ$ in the phase of the reference carrier in any carrier link included in the telephone circuit, following a large signal-carrier phase change, does not change the values of $a_{3(i-1)+2}$ and $a_{3(i-1)+3}$ nor can it lead to a prolonged burst of errors in the $\{a_{3(i-1)+1}\}$. To further reduce the error rate in the $\{a_{k}\}$, for a given error rate in the $\{s_i\}$, Gray coding is used in Table 6.2.

The encoding and decoding process used in systems B, C and D is a development of a technique outlined in Ref. (131) and it is described in Ref. (118). Both the encoder and decoder in systems B and C are exactly the same, since in both systems, the data-symbol $s_i$ may have 64 complex values defined by eqn. 6.10 (or equivalently by eqns. 6.18-6.19). Therefore any reference to system C in this section applies equally well to system B.
### TABLE 6.1: Differential Coding in System A

<table>
<thead>
<tr>
<th>$\alpha_3(i-1)+1$</th>
<th>$\overline{\alpha_3(i-2)+1}$</th>
<th>$\overline{\alpha_3(i-1)+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

### TABLE 6.2: Coding of the $\{s_i\}$ in System A

<table>
<thead>
<tr>
<th>$\overline{\alpha_3(i-1)+1}$</th>
<th>$\alpha_3(i-1)+2$</th>
<th>$\alpha_3(i-1)+3$</th>
<th>$s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>-7</td>
</tr>
</tbody>
</table>
The stream of binary digits \( \{a_k\} \) fed to the encoder is divided into separate groups of \( n \) adjacent digits, where

\[ n = 6 \text{ or } 8 \]

for systems C and D, respectively. To simplify the terminology, the 

\( h \)th binary digit in the \( i \)th group, i.e. \( a_{n(i-1)+h} \) is known here as \( a_{i,h} \), 

so that the \( i \)th group is known as \( a_{i,1}, a_{i,2}, \ldots, a_{i,n} \). The first two binary digits, \( a_{i,1} \) and \( a_{i,2} \), in the \( i \)th group are differentially coded in the encoder to give the corresponding two binary digits \( \bar{a}_{i,1} \) and \( \bar{a}_{i,2} \) (with possible values of 0 or 1), according to Table 6.3. In the case of system C, the resulting group of 6 binary digits \( \bar{a}_{i,1}, a_{i,2}, a_{i,3}, a_{i,4}, a_{i,5}, a_{i,6} \), considered here as the corresponding binary-coded number, is recorded into the appropriate data-symbol \( s_i \), according to Fig. 6.4. In the case of system D, the resulting group of 8 binary digits \( \bar{a}_{i,1}, \bar{a}_{i,2}, a_{i,3}, \ldots, a_{i,8} \) is recoded into \( s_i \) (eqns. 6.18 and 6.20) according to Fig. 6.5. 

The latter shows only the all positive quadrant, the other three quadrants being related to this in exactly the same way as in Fig. 6.4. 

Thus, the first two binary digits in any binary-coded number determine the quadrant containing \( s_i \), and the remaining digits determine the position of \( s_i \) in the quadrant. The latter digits in any quadrant are the same as those in all positive quadrant, if this is rotated to coincide with the given quadrant.

Following the detection of \( s_i \), at the receiver, the corresponding sequence of \( n \) detected binary digits \( \bar{a}_{i,1}, \bar{a}_{i,2}, a_{i,3}, \ldots, a_{i,n} \) is determined from Fig. 6.4 or Fig. 6.5, for system C or D, respectively, using, of course, the detected values of \( s_{1,i} \) and \( s_{2,i} \) (eqns. 6.18-6.20) in place of their actual values. The detected values of \( a_{i,1} \) and \( a_{i,2} \) are then determined from Table 6.3 using the detected values of \( \bar{a}_{i-1,1} \).
TABLE 6.3: Differential Coding of Binary Digits in Encoder in Systems B, C and D

<table>
<thead>
<tr>
<th>$\alpha_i, \overline{\alpha_i, 2}$</th>
<th>$\overline{\alpha_{i-1}, \alpha_{i-1, 2}}$</th>
<th>$\overline{\alpha_i, \alpha_{i, 2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>00</td>
<td>00</td>
</tr>
<tr>
<td>01</td>
<td>00</td>
<td>01</td>
</tr>
<tr>
<td>11</td>
<td>00</td>
<td>11</td>
</tr>
<tr>
<td>10</td>
<td>00</td>
<td>10</td>
</tr>
<tr>
<td>00</td>
<td>01</td>
<td>01</td>
</tr>
<tr>
<td>01</td>
<td>01</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>01</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>01</td>
<td>00</td>
</tr>
<tr>
<td>00</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>01</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>00</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>01</td>
</tr>
<tr>
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<td>10</td>
</tr>
<tr>
<td>01</td>
<td>10</td>
<td>00</td>
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<tr>
<td>11</td>
<td>10</td>
<td>01</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>$s_{2,i}$</td>
<td>101111</td>
<td>101101</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>•</td>
<td>•</td>
</tr>
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<td>$s_{1,i}$</td>
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**FIGURE 6.4:** Encoding of $\{s_i\}$ in Systems B and C
FIGURE 6.5: Encoding of \( \{ s_i \} \) in System D
It can be seen from Figs. 6.4 and 6.5 and Table 6.3, that a shift of a multiple of $90^\circ$ in the phase of the reference carrier in any carrier link included in the telephone circuit, in the case of system B, or in the phase of the reference carriers of the coherent demodulators in systems C and D, following a large signal-carrier change, does not change the values of $a_{i,1}^{'}, a_{i,3}^{'}, a_{i,4}^{'}, \ldots, a_{i,n}^{'}$ corresponding to any given value of $s_i^{'}$, nor can it lead to a prolonged burst of errors in the $\{a_i^{'}\}$ for $h=1$ or 2. To reduce further the error rate in the $\{a_i^{'}\}$, for a given error rate in the $\{s_i^{'}\}$, the coding in Fig. 6.4 and 6.5 is as near as possible to Gray coding, the exact realisation of this being unfortunately not attainable with the given signal, when differential coding is also used. Gray coding is nevertheless achieved over each quadrant in Figs. 6.4 and 6.5.

6.3 SAMPLED IMPULSE RESPONSES

In systems A, B, C and D, the transmitted data are detected at the receiver from the received signal samples given by eqns. 6.7, 6.15 and 6.23, in the presence of the intersymbol interference represented by the sampled impulse responses which are given by eqns. 6.8, 6.14 and 6.24, for the four systems, respectively. Of course, $Y$ is real in system A whereas it is, in general, complex-valued in systems B, C and D, but in every case, it is assumed here to be time-invariant.

With the accurate removal of the frequency offset, the real-part of the impulse response of the linear baseband channel in system A (Fig. 6.1) (which is of interest here) is given by eqn. 5.41 as,
\[
y_1(t) = \frac{1}{\sqrt{2}} g(t) * h(t) * c(t) \tag{6.26}
\]

where \( \frac{1}{\sqrt{2}} g(t), h(t) \) and \( c(t) \) are the impulse responses of the transmitter filter \( G \), telephone circuit and receiver filter \( C \), respectively, (Fig. 6.1). Clearly, the bandwidth of the transmitter and receiver filters is approximately equal to the bandwidth of the ideal voice frequency channel. As far as the transmitted data signal is concerned, \( y_1(t) \) in eqn. 6.26 may be viewed as a cascade of a single linear filter and the telephone circuit. The impulse response of this filter is given by
\[
x(t) = g(t) * c(t) \tag{6.27}
\]
and \( \frac{1}{\sqrt{2}} x(t) \) represents the overall filtering carried out on the data signal at both the transmitter and receiver (i.e. \( \frac{1}{\sqrt{2}} x(t) \) is the impulse response of the equipment filters in system A). Thus \( y_1(t) \) (eqn. 6.26) becomes
\[
y_1(t) = \frac{1}{\sqrt{2}} x(t) * h(t) \tag{6.28}
\]

The impulse response of the linear baseband channel in system B is given by eqn. 5.69, which is
\[
y_2(t) = \frac{1}{2} [g(t) + j\hat{g}(t)] * h(t) * c(t) \tag{6.29}
\]
where accurate removal of the frequency offset is assumed. \( \hat{g}(t) \) is the Hilbert transform of \( g(t) \). Using eqn. 6.27 also here gives
\[
y_2(t) = \frac{1}{2} [x(t) + j\hat{x}(t)] * h(t) \tag{6.30}
\]
so that the linear filter whose impulse response is \( \frac{1}{2} (x(t) + j\hat{x}(t)) \) represents the overall filtering carried out, both, at the transmitter and the receiver in system B. \( \hat{x}(t) \) is the Hilbert transform of \( x(t) \).

Also, when the reference frequency has the same phase as the carrier frequency, in the demodulation process in systems C and D, the
impulse response of the linear baseband channel given by eqn. 4.16

\[ y_3(t) = a(t) * \{[h(t) * c(t)]e^{-j2\pi f_c t}\} * b(t) \]  

(6.31)

which, according to eqn. 4.13 may be written as,

\[ y_3(t) = \{[a(t)e^{j2\pi f_c t}] * h(t) * c(t) * [b(t)e^{j2\pi f_c t}]\}e^{-j2\pi f_c t} \]  

(6.32)

\(a(t)\) is the impulse response of the transmitter filter in the QAM

signal generator (Fig. 6.3), \(h(t)\) and \(c(t)\) are the impulse responses of

the telephone circuit and receiver filter \(C\), respectively, and \(b(t)\) is

the impulse response of the lowpass filter in the coherent demodulator.

Let

\[ x'(t) = a(t)e^{j2\pi f_c t} * c(t) * b(t)e^{j2\pi f_c t} \]  

(6.34)

then \(y_3(t)\) becomes,

\[ y_3(t) = [x'(t) * h(t)]e^{-j2\pi f_c t} \]  

(6.35)

\(x'(t)\) represents the overall filtering carried out in the passband of

the QAM signal at both the transmitter and the receiver in systems \(C\) and \(D\).

The Fourier transforms of \(y_1(t)\), \(y_2(t)\) and \(y_3(t)\) are, according to eqns.

6.28, 6.30 and 6.35, given by

\[ Y_1(f) = \frac{1}{\sqrt{2}} X(f)H(f) \]  

(6.36)

\[ Y_2(f) = \begin{cases} X(f)H(f), & f>0 \\ 0 & f<0 \end{cases} \]  

(6.37)

and

\[ Y_3(f) = X'(f+f_c)H(f+f_c) \]  

(6.38)

respectively, where \(H(f)\), \(X(f)\) and \(X'(f)\) are the Fourier transforms

of \(h(t)\), \(x(t)\) and \(x'(t)\), respectively. Clearly, \(\frac{1}{\sqrt{2}} X(f)\) represents

the transfer function of the equipment filters in system \(A\). In system

\(B\) (eqn. 6.37), the transfer function of the equipment filters is given

by \(X(f)\) for \(f>0\), and it is zero for \(f<0\). In systems \(C\) and \(D\), which are
both QAM systems, the transfer function of the equipment filters, in the baseband, is represented by $X'(f+f_c)$ (eqn. 6.38). But from eqn. 6.34,

$$X'(f) = A(f-f_c) C(f) B(f-f_c)$$  \hspace{1cm} (6.39)

where $A(f)$, $C(f)$ and $B(f)$ are the Fourier transforms of $a(t)$, $c(t)$ and $b(t)$, respectively. Clearly, from eqn. 6.39,

$$X'(f+f_c) = A(f) C(f+f_c) B(f)$$  \hspace{1cm} (6.40)

Since both $A(f)$ and $B(f)$ are zero for $|f|>f_c$ (eqn. 6.21a), $X'(f)$ (eqn. 6.39) is zero for $f<0$ and $f>2f_c$. It should be noted here that we are dealing with the model of the QAM system on the basis of the complex representation of its input and output signals, and eqns. 6.39 and 6.40 are valid only when the two signals at the outputs of the coherent demodulators (Figs. 4.1 and 6.3) are taken as the real and imaginary parts of the received complex-valued signal. When these two signals are handled individually, eqns. 6.39 and 6.40 (and indeed eqn. 4.16) become invalid in the general case. From eqn. 6.38, $X'(f)$ represents, as far as the input and output complex-valued signals are concerned, the transfer function of the equipment filters in the passband of the telephone circuit whose transfer function is $H(f)$. This should not be confused with the equivalent passband representation of the equipment filters as seen by the transmitted QAM signal itself, where the transfer function is complex-conjugate symmetric about $f=0$, since the QAM signal is purely real.

For the four systems to be tested under the same conditions of linear distortion (as in Section 5.5), and with the same telephone circuit, the overall filtering (in the passband of the telephone circuit) carried out by the equipment filters must be the same.
Therefore, $X'(f)$ is chosen, according to eqns. 6.36-6.39, such that

$$X'(f) = X(f) \quad 0 \leq f \leq f_c$$
$$= 0 \quad \text{elsewhere}$$  \hspace{1cm} (6.41)

From eqn. 5.52, the average transmitted energy per bit at the input of the telephone circuit in system A is given by

$$E_{01} = 7 \int_{-\infty}^{\infty} \frac{1}{2} |G(f)|^2 \, df$$
$$= 7 \int_{0}^{\infty} |G(f)|^2 \, df$$  \hspace{1cm} (6.42)

where the average energy per bit in the data-symbol $s_i$ in eqn. 6.3 is 7. $|G(f)|$ is the absolute value of the Fourier transform of $g(t)$ and it is clearly an even function of $f$. Similarly, from eqn. 5.85, the average transmitted energy per bit at the input of the telephone circuit in system B is given by

$$E_{02} = 7 \int_{0}^{\infty} |G(f)|^2 \, df$$  \hspace{1cm} (6.43)

where, again, the average energy per bit in a data-symbol $s_i$ in eqn. 6.10 is 7.

In systems C and D, and according to eqn. 4.48, the average transmitted energy per bit at the input of the telephone circuit is given by

$$E_{03} = e_0 \int_{-f_c}^{f_c} |A(f)|^2 \, df$$  \hspace{1cm} (6.44)

where $e_0$ is the average energy per bit in a data symbol $s_i$ in eqns. 6.18-6.19 in system C and eqns. 6.18 and 6.20 in system D. Notice that the integration limits in eqn. 6.44 are for $A(f)$ as defined by eqn. 6.21a. In eqn. 4.48, the integration limits are given specifically for the case when the transmitter filter is used at the Nyquist rate. Of course, eqn. 6.44 is more general since it covers cases when the
element rate is lower than the Nyquist rate of the transmitter filter.

When the lowpass filter at the output of the coherent demodulator (Fig. 6.3) has a very sharp transition from the passband to the stopband, then (eqn. 6.21a)

\[ a(t) * b(t) = a(t) \quad (6.45) \]

and eqn. 6.34 becomes,

\[ x'(t) = a(t)e^{i2\pi f_c t} * c(t) \quad (6.46) \]

Now, let

\[ |G(f)| = 0 \quad \text{if} \quad |f| > 2f_c, \quad (6.47a) \]

then, to satisfy eqn. 6.41, comparing eqn. 6.46 with eqn. 6.27 gives,

\[ A(f-f_c) = G(f), \quad 0 \leq f \leq 2f_c \]

\[ = 0 \quad \text{elsewhere} \quad (6.47b) \]

and consequently, eqn. 6.44 becomes,

\[ \varepsilon_{03} = \int_{0}^{2f_c} |G(f)|^2 df \quad (6.48) \]

According to eqns. 6.18-6.20, \( \varepsilon_0 = 7 \), in system C and \( \varepsilon_0 = 21.25 \) in system D. \( \varepsilon_0 \) is of course the average energy per bit in a data symbol \( s_i \).

Consequently, the choice in eqn. 6.41 implies equal average transmitted energy per bit in systems A, B and C. The average transmitted energy per bit in system D is higher by \( 10\log \left( \frac{21.25}{7} \right) = 4.82 \text{ db} \).

The sampled impulse responses of the linear baseband channels in the four systems are obtained in accordance with the above assumptions.

6.3.1 Frequency Characteristics of the Telephone Circuits

The tests carried out on systems A, B, C and D have used four different models of the telephone circuit known as circuits 1-4. The attenuation and group-delay characteristics of telephone circuits 1-4 are shown in Figs. 6.6 to 6.9, respectively. The telephone circuit 1 and 2 introduce typical levels of attenuation and group-delay
distortion, and telephone circuits 3 and 4 are close to the typical worst
circuits (on the public switched telephone network in the U.K.) normally
considered for the transmission of data at 9600 and 1200 bit/s,
respectively. The attenuation and group-delay characteristics of the four channels, sampled at frequency intervals of 50 Hz, are given
in Table 6.4. Let $A_t(f)$ and $\theta_t(f)$ be the attenuation and group delay,
respectively, of a telephone circuit at frequency $f$, then (eqns. 2.2-2.4)

$$A_t(f) = -20 \log_{10} |H(f)| \quad \text{[in db]}$$

(6.49)

and

$$\theta_t(f) = \frac{1}{2\pi} \frac{d\phi_t(f)}{df} \quad \text{[in sec.]}$$

(6.50)

where

$$H(f) = |H(f)| e^{j\phi_t(f)}$$

(6.51)

$|H(f)|$ being the absolute value of $H(f)$ and

$$\phi_t(f) = \tan^{-1}\left[\frac{\text{Imaginary}[H(f)]}{\text{Real}[H(f)]}\right]$$

(6.52)

6.3.2 Frequency Characteristics of Equipment Filters

The term "equipment filters" here means the resultant of all filters,
used at the transmitter and the receiver. Three different sets of
equipment filters have been used in the tests. The attenuation and group-
delay characteristics of equipment filters 1, 2 and 3 are shown in Figs.
6.10-6.12, respectively, where they are considered as operating on the
transmitted bandpass signal. Filter 1 has, over the passband of the
telephone circuit, a transfer function shaped as sine-squared, (23) which
is given by

$$X(f) = \sin^2\left(\frac{2\pi f}{6400}\right) \quad -3200<f<3200 \text{ Hz}$$

(6.53)

so that its peak is at $f=1600$ Hz. Clearly, zero phase-characteristic
is assumed. Filter 2 has a flatter amplitude response over the passband,
(a) Attenuation Characteristic

(b) Group-Delay Characteristic

FIGURE 6.6: Telephone Circuit 1
(a) Attenuation Characteristic

(b) Group-Delay Characteristic

FIGURE 6.7: Telephone Circuit 2
(a) Attenuation Characteristic

(b) Group-Delay Characteristic

FIGURE 6.8: Telephone Circuit 3
Figure 6.9: Telephone Circuit 4

(a) Attenuation Characteristic

(b) Group-Delay Characteristic
Attenuation Characteristic

FIGURE 6.10: Transmitter and Receiver Filters 1
(a) Attenuation Characteristic

(b) Group-Delay Characteristic

FIGURE 6.11: Transmitter and Receiver Filters 2
FIGURE 6.12: Transmitter and Receiver Filters 3

(a) Attenuation Characteristic

(b) Group-Delay Characteristic
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<th>TELEPHONE CIRCUIT 2 ATT. G.D. (db)</th>
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\[
\begin{align*}
H(4): & = H(41) \cdot \left(\frac{p}{q}\right) \\
& = \frac{20^2}{49} \\
& = \frac{400}{49}
\end{align*}
\]
so that it is effectively a little bit wider than filter 1. Filter 3 has even wider bandwidth than filter 2. The attenuation and group-delay characteristics of filters 2 and 3, sampled at frequency intervals of 50 Hz are given in Table 6.5. The characteristics of filter 1 are obtained by sampling eqn. 6.53 directly.

Let $A_e(f)$ and $\theta_e(f)$ be the attenuation and group-delay, respectively, of a filter, at the frequency $f$, then,

$$A_e(f) = -20 \log_{10} |X(f)|$$  \hspace{1cm} (6.54) \\
and

$$\theta_e(f) = \frac{1}{2\pi} \frac{d\phi_e(f)}{df}$$  \hspace{1cm} (6.55) \\
where

$$X(f) = |X(f)| e^{j\phi_e(f)}$$  \hspace{1cm} (6.56) \\

$|X(f)|$ being the absolute value of $X(f)$,

$$\phi_e(f) = \tan^{-1} \frac{\text{Imaginary} [X(f)]}{\text{Real} [X(f)]}$$  \hspace{1cm} (6.57) \\
and $X(f)$ is as defined for eqns. 6.36, 6.37 and 6.41.

6.3.3 Computation of the Sampled Impulse Responses

From eqns. 6.49-6.57,

$$X(f)H(f) = 10^{-A_0(f)/20} e^{j\phi_0(f)} , f > 0$$

$$= X(-f)H(-f) , f < 0$$  \hspace{1cm} (6.58) \\
where $X(f)$ and $H(f)$ are the complex conjugates of $X(f)$ and $H(f)$, respectively,

$$A_0(f) = A_t(f) + A_e(f)$$  \hspace{1cm} (6.59) \\
and

$$\phi_0(f) = \phi_t(f) + \phi_e(f)$$

$$= 2\pi \int [\theta_t(f) + \theta_e(f)] df + c_0$$  \hspace{1cm} (6.60) \\
c_0 being the integration constant. $A_t(f)$ and $A_e(f)$ are the attenuation characteristics, and $\theta_t(f)$ and $\theta_e(f)$ are the group-delay characteristics.
of the telephone circuit and equipment filters, respectively. Substituting eqn. 6.58 in eqns. 6.36-6.37 results in

\[ Y_1(f) = \frac{1}{\sqrt{2}} 10^{-A(f)/20} e^{j\phi(f)} , f > 0 \]

\[ = Y_1(-f) , f < 0 \]  

(6.61)

and

\[ Y_2(f) = 10^{-A(f)/20} e^{j\phi(f)} , f > 0 \]

\[ = 0 , f < 0 \]  

(6.62)

From eqns. 6.38, 6.41 and 6.58,

\[ Y_3(f) = 10^{-A(f+c)/20} e^{j\phi(f+c)} , |f| < f_c \]

\[ = 0 , \text{ elsewhere.} \]  

(6.63)

To find the sampled impulse response of the linear baseband channel in any of the four systems, the corresponding \( Y(f) \) (eqns. 6.61-6.63) is sampled and fed to an inverse discrete Fourier transform (IDFT). Let \( \Delta f \) be the sampling period in the frequency domain, \( T \) the sampling period in the time domain and \( N \) the number of samples in the IDFT, then

\[ \Delta f = \frac{1}{NT} \]  

(6.64)

\( NT \) represents, of course, the length, in seconds, of the resultant sampled impulse response. Clearly, to avoid aliasing in the time domain, \( \Delta f \) must be small enough. In the actual computations it has been found that setting \( \Delta f = 25 \) Hz gives very good accuracy (such that the changes in the values of the sampled impulse response at its edges become negligible when \( \Delta f \) is further decreased).

The sampling of the \( Y_1(f) \), \( Y_2(f) \) and \( Y_3(f) \), at frequency intervals of 25 Hz, requires the sampling of \( A_t(f) \), \( \theta_f(f) \), \( A_e(f) \) and \( \theta_e(f) \) at the same intervals. But since the latter are given in Tables 6.4 and 6.5
sampled at intervals of 50 Hz, they have been over-sampled by linear-interpolation.

The integration process in eqn. 6.60 has been carried out numerically according to the formula,

\[ \phi_k = \phi_{k-1} + 2\pi [\theta_{t,k} + \theta_{e,k}] \Delta f, \quad k=0,1,2,\ldots \]  

(6.65)

where

\[ \phi_k = \phi_0(k\Delta f) \]  

(6.66)

\[ \theta_{t,k} = \theta_{t}(k\Delta f) \]  

(6.67)

and

\[ \theta_{e,k} = \theta_{e}(k\Delta f) \]  

(6.68)

The constant \( c_0 \) in eqn. 6.60, represents a constant phase shift, but since this phase shift is effectively eliminated in the resultant sampled impulse response at the output of the adaptive linear filter (which performs a pure orthogonal transformation on the signal), the constant \( c_0 \) has no effect on the resultant response, and therefore it has been set to zero.

As mentioned before, the overall filtering carried out on the signal in the passband is represented by \( \frac{1}{\sqrt{2}} x(t) \) and \( \frac{1}{2} [x(t)+j\hat{x}(t)] \) in systems A and B, respectively, and by \( x'(t) \) in systems C and D, where \( x(t) \) and \( x'(t) \) are defined by eqns. 6.27 and 6.34, and their Fourier transforms are related to each other by eqn. 6.41. In the tests carried out here it is assumed that this filtering, apart from a scaling factor, is equally shared between the transmitter and the receiver. In systems A and B, this assumption gives (eqns. 6.27, 6.36 and 6.37)

\[ G(f) = C(f) = \sqrt{X(f)} \]  

(6.69)

In systems C and D, the transfer function of the equipment filters, in the baseband, is given by \( X'(f+f_c) \) (eqn. 6.40). When the lowpass filters at the output of the coherent demodulators (Figs. 4.1 and 6.3)
have a very sharp transition from the passband to the stopband, then
from eqns. 6.21a and 6.40,
\[ X'(f+f_c) = A(f) C(f+f_c) \] (6.70)
and when the filtering is equally shared between the transmitter and the
receiver, then,
\[ A(f) = C(f+f_c) = \sqrt{X'(f+f_c)} \] (6.71)
which with eqn. 6.41 gives,
\[ A(f) = C(f+f_c) = \sqrt{X(f+f_c)} \]
\[ = 0 \quad \text{elsewhere} \] (6.72)
It should be noted in eqn. 6.72 that \( C(f+f_c) \) for \(-f_c < f < f_c\) is, as far
as the complex-valued signal at the output of the linear baseband channel
in the QAM system (Figs. 4.1 and 6.3) is concerned, the transfer function
of the receiver filter in the baseband. Of course, the transfer function
of the receiver filter, when considered as operating on the QAM signal
itself, is \( C(f) \).

The sampled impulse responses of the linear baseband channels in
the different systems obtained by the IDFT have been scaled such that
\[ \int_{-\infty}^{\infty} |G(f)|^2 df = \int_{-\infty}^{\infty} |C(f)|^2 df = 2 \] (6.74)
in systems A and B, and
\[ \int_{-f_c}^{f_c} |A(f)|^2 df = \int_{-f_c}^{f_c} |C(f+f_c)|^2 df = 1 \] (6.75)
in systems C and D, where eqns. 6.69 and 6.72 have been used. Eqns.
6.74-6.75, together with eqns. 6.42, 6.43 and 6.44 imply that the
average transmitted energy per bit \( \mathbb{E}_0 \) at the input of the telephone
circuit is 7.7, 7 and 21.25 for systems A, B, C and D, respectively.

To achieve the different sampling rates, eqn. 6.64 has been used
with $\Delta f = 25$ Hz. Thus, for every sampling rate $f_s = \frac{1}{T}$, the number of points $N$ in the IDFT is given by

$$N = \frac{f_s}{25}$$

(6.76)

The sampled impulse responses of the linear baseband channels in the different systems must originally be obtained by sampling the continuous waveforms resulting from applying the inverse Fourier transform on the transfer functions given by eqns. 6.61-6.63. In Appendix E, it is shown that when $Y(f)$ is sampled and fed to an IDFT, the resultant samples must be scaled by the sampling rate $f_s$ to give the required samples of the continuous waveform.

It should be noted here that the IDFT mentioned above is the standard transform. When a computer subroutine is used, correction must be applied to any non-standard scaling introduced by the subroutine.

6.3.4 The Minimum Phase Sampled Impulse Responses

As mentioned before, the adaptive linear filter, when used ahead of the detector and at high signal-to-noise ratios, replaces the roots of the $z$-transform of the sampled impulse response of the linear baseband channel, which lie outside the unit circle, by the complex conjugates of their reciprocals. This process is a pure phase transformation and should not introduce any gain or attenuation in the signal energy. However, some of the detectors to be described later may be greatly simplified if the first non-zero component of the resultant sampled impulse response is real with value 1, and this, of course requires all components of the sampled impulse response to be divided by the value of its first component. Now, this scaling must also be introduced into
the noise components in the received signal samples (eqns. 6.7, 6.15 and 6.23), in order to preserve the signal-to-noise ratio, which is not affected by the linear phase transformation. In the computer simulation tests, the signal samples are formed as suggested by eqns. 6.7, 6.15 and 6.23 where the noise components are generated separately. In practice, the whole signal, of course, passes through the adaptive linear filter and any scaling introduced there applies to both the signal and noise components in the signal samples \( \{r_i\} \).

6.3.5 Tables of the Minimum Phase Sampled Impulse Responses

The resultant sampled impulse response so far represents the intersymbol interference in the received signal samples \( \{r_i\} \) (Figs. 6.1, 6.2 and 6.3, and eqns. 6.8, 6.14 and 6.24). Tables 6.6 to 6.12 give the sampled impulse responses corresponding to different systems and different combinations of telephone circuits and equipment filters. The contents of these tables are as follows:

Table 6.6: System A. Telephone circuits 1-4 with equipment filters 1. The sampling rate is 6400 samples/second.

Table 6.7: System A. Telephone circuits 1-3 with equipment filters 3. The sampling rate is 6400 samples/second.

Table 6.8: System B. Telephone circuits 1-4 with equipment filters 2. The sampling rate is 3200 samples/second.

Table 6.9: System B. Telephone circuits 1-3 with equipment filters 3. The sampling rate is 3200 samples/second.
TABLE 6.6: Minimum Phase Sampled Impulse Responses in System A with Equipment Filters 1 and Telephone Circuits 1-4. The Sampling Rate is 6400 sample/second. (actual value = γ × given value)

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TABLE 6.7: Minimum Phase Sampled Impulse Responses
in System A with Equipment Filters 3 and Telephone Circuits
1-3. The Sampling Rate is 6400 sample/second.
(actual value = γ × given value)

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γ | 0.7368 | 0.5762 | 0.3220
TABLE 6.8: Minimum Phase Sampled Impulse Responses in System B with Equipment Filters 2 and Telephone Circuits 1-4. The Sampling Rate is 3200 sample/second.

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\[ \gamma \]

\[ \text{actual value} = \gamma \times \text{given value} \]
### TABLE 6.9: Minimum Phase Sampled Impulse Responses in System B with Equipment Filters 3 and Telephone Circuits 1-3. The Sampling Rate is 3200 sample/second.

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\[ \text{actual value} = \gamma \times \text{given value} \]

\[ \gamma = 0.5923 \quad 0.5458 \quad 0.2366 \]
TABLE 6.10: Minimum Phase Sampled Impulse Responses in System C with Equipment Filters and Telephone Circuits 1-4. The Sampling Rate is 3200 sample/second. $f_c = 1600$ Hz.

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$\gamma = 0.3723$ \hspace{1cm} 0.3545 \hspace{1cm} 0.1791 \hspace{1cm} 0.0905

actual value $= \gamma \times$ given value
TABLE 6.11: Minimum Phase Sampled Impulse Responses in System C with Equipment
Filters 2 and Telephone Circuits 1-4. The Sampling Rate is 3200 sample/second. 
\[ f_c = 1800 \text{ Hz.} \]

<table>
<thead>
<tr>
<th>TELEPHONE CIRCUIT 1</th>
<th>TELEPHONE CIRCUIT 2</th>
<th>TELEPHONE CIRCUIT 3</th>
<th>TELEPHONE CIRCUIT 4</th>
</tr>
</thead>
<tbody>
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<td>REAL PART</td>
<td>IMAGINARY PART</td>
<td>REAL PART</td>
<td>IMAGINARY PART</td>
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<td>1.0000</td>
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\[ \gamma = \begin{bmatrix} 0.4362 \\ 0.3699 \\ 0.1749 \\ 0.0658 \end{bmatrix} \]

actual value = \( \gamma \times \) given value
TABLE 6.12: Minimum Phase Sampled Impulse Responses in System D with Equipment Filters 2 and Telephone Circuits 1-4. The Sampling Rate is 2400 sample/second. $f_c = 1800$ Hz.

<table>
<thead>
<tr>
<th>TELEPHONE CIRCUIT 1</th>
<th>TELEPHONE CIRCUIT 2</th>
<th>TELEPHONE CIRCUIT 3</th>
<th>TELEPHONE CIRCUIT 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>REAL PART</td>
<td>IMAGINARY PART</td>
<td>REAL PART</td>
<td>IMAGINARY PART</td>
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$\gamma = 0.8296$, $0.7182$, $0.3630$, $0.1623$

actual value = $\gamma \times$ given value
Table 6.10: System C. Telephone circuits 1-4 with equipment filters 1. The sampling rate is 3200 samples/second. \( f_c = 1600 \) Hz.

Table 6.11: System C. Telephone circuits 1-4 with equipment filters 2. The sampling rate is 3200 samples/second. \( f_c = 1800 \) Hz.

Table 6.12: System D. Telephone circuits 1-4 with equipment filters 2. The sampling rate is 2400 samples/second. \( f_c = 1800 \) Hz.

The sampled impulse responses in these tables are scaled such that the first component is set to 1. Correct values may be obtained by multiplying all components of the sampled impulse response by the corresponding scalar \( \gamma \) which is given in the tables.

6.3.6 Noise Generation and the Signal-to-Noise Ratio

As mentioned before, the linear adaptive filter in Figs. 6.1 to 6.3 does not change the statistics of the noise components in the received signal samples. Therefore, and by comparing Fig. 6.1 with Fig. 5.2, Fig. 6.2 with Fig. 5.5 and Fig. 6.3 with Fig. 4.1, the noise components \( \{u_i\} \) in eqns. 6.7, 6.15 and 6.23 may be considered as the noise samples of \( u(t) \) in eqns. 5.45, 5.70 and 4.17 respectively. From eqn. 5.45, and when the effect of the frequency offset on the noise process is neglected, the \( \{u_i\} \) in eqn. 6.7, in system A, are obtained by sampling the waveform

\[
u_i(t) = n(t) * c(t)
\]

where the sampling rate is 6400 samples per second. The \( \{u_i\} \) in system B, when the effect of the frequency offset is neglected (eqn. 5.70), are
the samples of
\[ u_2(t) = \frac{1}{\sqrt{2}} n(t) \ast (c(t) + j\hat{c}(t)) \]
(6.78)
where the sampling rate now is 3200 samples/second. \( \hat{c}(t) \) is the Hilbert transform of \( c(t) \). It should be noted here that the frequency offset has no effect on the variances of \( u_1(t) \) and \( u_2(t) \), as shown in eqns. 5.55 and 5.88. In systems C and D, the \( \{u_i\} \) are obtained by sampling \( u(t) \) in eqn. 4.17, which is
\[ u_3(t) = \sqrt{2}[(n(t) \ast c(\omega))e^{-j(2\pi f_c t+\phi)}] \ast b(t) \]
(6.79)
at rates of 3200 and 2400 samples/second, respectively.

In eqns. 6.77 to 6.79, \( n(t) \) is a real-valued white Gaussian noise waveform, with zero mean and a two-sided power spectral density \( \frac{1}{2}N_0 \).

From eqns. 5.55, 5.88 and 4.34, the variances of \( u_1(t) \), \( u_2(t) \) and \( u_3(t) \) are,
\[ \sigma_{u_1}^2 = \frac{1}{2}N_0 \int_{-\infty}^{\infty} |C(f)|^2 df \]
(6.80)
\[ \sigma_{u_2}^2 = N_0 \int_{0}^{\infty} |C(f)|^2 df \]
(6.81)
and
\[ \sigma_{u_3}^2 = N_0 \int_{-f_c}^{f_c} |C(f) + f_c|^2 df \]
(6.82)
respectively. The integration limits in eqn. 6.82 are for the transmitter and receiver filters, as given in eqn. 6.72. From eqns. 6.74 and 6.75, eqns. 6.80-6.82 become,
\[ \sigma_{u_1}^2 = \frac{1}{2}N_0 \]
(6.83)
\[ \sigma_{u_2}^2 = N_0 \]
(6.84)
and
\[ \sigma_{u_3}^2 = N_0 \]
(6.85)

Now, defining the signal-to-noise ratio \( \psi \) to be
\[ \psi = 10 \log_{10} \left( \frac{N_0}{1N_0} \right) \]
(6.86)
where, of course, \( \mathcal{E}_0 \) is the average transmitted energy per bit at the input of the telephone circuit, and noting that \( \mathcal{E}_0 \) is given as \( \mathcal{E}_{01} \) and \( \mathcal{E}_{02} \) (eqns. 6.42 and 6.43) for systems A and B, and as \( \mathcal{E}_{03} \) (eqn. 6.44) for systems C and D, then from eqns. 6.83 to 6.86,

\[
\sigma_{u_1}^2 = 10^{-\psi/10} \cdot 2 \cdot \mathcal{E}_{01}
\]

(6.87)

\[
\sigma_{u_2}^2 = 10^{-\psi/10} \cdot 2 \cdot \mathcal{E}_{02}
\]

(6.88)

and

\[
\sigma_{u_3}^2 = 10^{-\psi/10} \cdot 2 \cdot \mathcal{E}_{03}
\]

(6.89)

From eqns. 6.42-6.44 and 6.74-6.75, \( \mathcal{E}_{01} = 7 \) (system A), \( \mathcal{E}_{02} = 7 \) (system B), and \( \mathcal{E}_{03} = 7 \) and 21.25, for systems C and D, respectively. Notice that the average transmitted energy per bit and the noise variance in systems A, B, and C are the same.

To generate the noise sequence \( \{u_i\} \) in any of the four systems, the noise discrete-time model discussed in Appendix F is used. In system A, the \( \{u_i\} \) are originally the samples of \( u_1(t) \) (eqn. 6.77), where the sampling rate is 6400 samples/second. For this case, it is shown in Appendix F (Section F.1) that these \( \{u_i\} \) may be obtained by passing a sequence of statistically independent Gaussian random variables with zero mean and variance \( \sigma_{u_1}^2 \) through a linear transversal filter whose tap gains \( \{c_i\} \) are obtained by sampling \( c(t) \) at a rate not less than the Nyquist rate of \( c(t) \) itself and scaled such that \( \sum c_i^2 = 1 \). System A has been tested with equipment filters 1 and 3. When filter 1 is used, no noise correlation is introduced into the noise components, therefore the \( \{c_i\} \) have been taken to be: \( c_0 = 1 \) and \( c_i = 0 \) for \( i \neq 0 \). When filter 3 is used, and as may be seen from Fig. 6.12, the sampling rate 6400 samples/second is a little below the Nyquist rate of the receiver filter, whose
characteristics are obtained by dividing those in Fig. 6.12 by 2 (or equivalently by taking the square root of $X(f)$, eqns. 6.54 and 6.69). Therefore, the $\{c_i\}$ are obtained from these characteristics at a sampling rate of 12800 samples/second and scaled so that $\sum_i c_i^2 = 1$. The $\{u_i\}$ are now obtained by taking every second sample of the output of the transversal filter formed by the $\{c_i\}$. It should be noted here that the variance of the output is still given by $\sigma^2_{u_i}$.

In system B, and as can be seen from eqn. 6.78, the $\{u_i\}$ are complex-valued, and may be obtained by passing a sequence of statistically independent complex-valued Gaussian random variables with zero mean and variance $\sigma^2_{u_i}$ (eqn. 6.88) through a linear transversal filter whose tap gains $\{c_i\}$ are now complex-valued (Appendix F, Section F.3). The $\{u_i\}$ are required here at a rate of 3200 samples/second. When system B has been tested with equipment filters 2, the sampling rate is a little above half the Nyquist rate for this filter (eqns. 6.54 and 6.69 and Fig. 6.11). Thus, the $\{c_i\}$, where (eqn. F.78)

$$c_h = k \int_0^{3200} C(f) e^{j2\pi fh/3200} df,$$

are spaced at delays (time intervals) of $1/3200$ seconds and scaled (by $k$, eqn. 6.90) such that $\sum_i |c_i|^2 = 1$. The input signal is now fed to the filter whose tap gains are the $\{c_i\}$ at a rate of 3200 sample/second and the required $\{u_i\}$ are those at the output of the filter. When system B is tested with equipment filters 3 (Fig. 3.12), the sampling rate (3200 samples/second) is a little below half the Nyquist rate for this filter. Therefore the $\{u_i\}$ have been generated at a rate of 6400 samples/second ($M=2$, eqn. F.88). Thus, the $\{c_i\}$, where

$$c_h = k \int_0^{3400} C(f)e^{j2\pi fh/6400} df,$$

(6.91)
are spaced at delays of 1/6400 seconds and scaled (by $k$, eqn. 6.91) such that $\sum |c_i|^2 = 1$. The input signal is now fed to the linear transversal filter with the tap gains $\{c_i\}$ at a rate of 6400 samples/second and the required $\{u_i\}$ are obtained by taking every second sample at the output of the filter.

In systems C and D, the $\{u_i\}$ which are the samples of $u_J(t)$ (eqn. 6.79), are obtained by passing a sequence of statistically independent complex-valued Gaussian random variables with zero mean and variance $\sigma_u^2$ (eqn. 6.89) through a linear transversal filter whose tap gains are given by the $\{c_i\}$ which are the samples of the waveform $\{[c(t)e^{-j(2\pi ft+a)}]b(t)\}$, as described in Appendix F (Section F.2). In fact, $u(t)$ (eqn. 6.79) is band-limited to $-f_c$ to $f_c$ Hz by $B(f)$ (eqn. 6.21a). In other words, when $B(f)$ is rectangular over the frequency band $-f_c$ to $f_c$, then the spectral shaping of $u(t)$ is determined, over this band, by the Fourier transform of $[c(t)e^{-j2\pi ft+a}]b(t)$ which is $C(f+f_c)$ for $-f_c < f < f_c$ and zero elsewhere.

In system C, with equipment filters 2, the required sampling rate is 3200 samples/second. As can be seen from Fig. 6.11 and eqns. 6.54 and 6.72, and since $f_c = 1800$ Hz, $C(f+f_c)$ has a very negligible amplitude outside the frequency band $\{-1400, 1400\}$ Hz. Thus, the sampling rate 3200 sample/second is a little above the Nyquist rate. Consequently, the tap gains $\{c_i\}$, where (eqn. F.56)

$$c_h = k \int_{-1400}^{1400} C(f+1800)e^{j2\pi fh/3200} df$$

are spaced at delays (time intervals) of 1/3200 seconds, and scaled (by $k$, eqn. 6.92) such that

$$\sum |c_i|^2 = 1$$

The input signal to the linear transversal filter with tap gains $\{c_i\}$
is fed at a rate of 3200 sample/second, and the signal samples at this filter output are the required \( \{u_i\} \).

When system C is used with equipment filters 1, noise correlation is neglected. Here the \( \{c_i\} \) are such that \( c_0 = 1 \) and \( c_i = 0 \) for \( i \neq 0 \).

In system D with equipment filters 2, where the sampling rate is 2400 samples/second, this rate is a little below the Nyquist rate for \( C(f+1800) \) over the frequency band \( \{-1400, 1400\} \) Hz. \( f_c = 1800 \) Hz. Therefore, the \( \{c_i\} \) are now given by

\[
\begin{align*}
    c_k &= k \int_{-1400}^{1400} C(f+1800) e^{j2\pi fh/4800} df \\
    &= k C(1800) e^{j2\pi f/4800} \\
    \text{(6.93)}
\end{align*}
\]

so that the spacing (delay interval) is now \( 1/4800 \) second. Of course, the \( \{c_i\} \) are again scaled (by \( k \), eqn. 6.93) such that

\[\sum |c_i|^2 = 1\]

The input signal to the linear filter with tap gains \( \{c_i\} \) is fed now at a rate of 4800 samples/second, and the required \( \{u_i\} \) are obtained by taking every second sample of the output of the linear transversal filter.

6.4 DETECTION PROCESSES

The detection processes used here for the detection of data signals transmitted at 19200 bit/s over the given telephone circuits, are developments of near-maximum likelihood detection processes of the type considered in Section 3.8. In order to clarify the description of the detectors, the basic principles of near-maximum likelihood detection will be first outlined.
6.4.1 Near-Maximum Likelihood Detection

Let $S_k$, $R_k$ and $U_k$ be the $k$-component row vectors (sequences) whose $i$th components are $s_i$, $r_i$ and $u_i$, respectively, for $i=1,2,\ldots,k$, where,

$$
 r_i = \sum_{h=0}^{\infty} s_{i-h} y_h + u_i \tag{6.94}
$$

is the received signal sample at the detector input (eqns. 6.7, 6.15 and 6.23). Also, let $X_k$, $Z_k$ and $W_k$ be the $k$-component row vectors whose $i$th components are $x_i$, $z_i$ and $w_i$, respectively, for $i=1,2,\ldots,k$, where $x_i$ is a possible value of $s_i$,

$$
 z_i = \sum_{h=0}^{\infty} x_{i-h} y_h \tag{6.95}
$$

and $w_i$ is the possible value of $u_i$ satisfying,

$$
 r_i = z_i + w_i \tag{6.96}
$$

In the $k$-dimensional unitary vector space containing the vectors $R_k$, $Z_k$ and $W_k$, the square of the (unitary) distance between the vectors $R_k$ and $Z_k$ is

$$
 |W_k|^2 = \sum_{i=1}^{k} |w_i|^2 \tag{6.97}
$$

where $|w_i|$ is the absolute value of $w_i$. When all real and imaginary parts of the $\{u_i\}$ are statistically independent Gaussian random variables with zero mean and a fixed variance (which is not in fact the case here, since the $\{u_i\}$ are slightly correlated), the maximum-likelihood vector $X_k$, which gives an estimate of the vector $S_k$ is its possible value such that $|W_k|^2$ is minimized. If the correlation between neighbouring $\{u_i\}$ is not too high (widely separated $\{u_i\}$ being, of course, uncorrelated), it can be assumed for practical purposes, that the smaller the value of $|W_k|^2$ associated with any vector $X_k$, the better is the estimate of $S_k$ that is given by this $X_k$. $|W_k|^2$ is said to be the "cost" of $X_k$. 
Just prior to the receipt of the sample $r_k$, the detector holds in store $u$ different $n$-component vectors $\{Q_{k-1}\}$, where

$$Q_{k-1} = x_{k-n} x_{k-n+1} \ldots x_{k-1}$$

and $x_i$ can take on any of the possible values of $s_i$. Normally, $n \geq g$.

Each vector $Q_{k-1}$ is formed by the last $n$ components of the corresponding vector $X_{k-1}$, which represents a possible received sequence of data-symbols $\{s_i\}$. Associated with each vector $X_{k-1}$ is its cost $|W_{k-1}|^2$, which is therefore also the cost of the corresponding $Q_{k-1}$. On the receipt of $r_k$, the detector operates on the stored vectors $\{Q_{k-1}\}$ to give the detected value of the data-symbol $s_{k-n}$, and it then determines $u$ vectors $\{Q_k\}$ together with their costs $\{|W_k|^2\}$ which are stored in place of the previous vectors and costs. (113-117) The detector here attempts to include, among the $u$ stored vectors, the vector $Q_k$ given by the last $n$ components of the particular vector $X_k$ with the smallest cost over all possible vectors $\{X_k\}$.

All that has been said above about the unitary vector-space applies also to the corresponding Euclidean vector space, which is in fact a special case of the former such that all components are real-valued. Therefore, the principles of the detection process just outlined apply to system A, which handles purely real signals, as well as systems B, C and D, which handle complex-valued signals.

The detectors to be described are basically modifications of detectors used in a 9600 bit/s modem, (116) which are originally all modifications of system 1 described in Section 3.8.

6.4.2 Detector 1

The detector here handles real signals, so that it is to be used with
system A. Just prior to the receipt of the signal sample \( r_k \), the detector holds in store \( \mu \) \( n \)-component vectors \( \{Q_{k-1}\} \), where

\[
Q_{k-1} = x_{k-n} x_{k-n+1} \ldots x_{k-1}
\]

(6.99)

and \( x_i \) may have any of the eight possible values of \( s_i \) in eqn. 6.3, each vector \( Q_{k-1} \) being associated with its cost \( |w_{k-1}|^2 \). On the receipt of the signal sample \( r_k \), each of the \( \mu \) stored vectors \( \{Q_{k-1}\} \) is expanded into eight \( (n+1) \)-component vectors \( \{P_k\} \), where,

\[
P_k = x_{k-n} x_{k-n+1} \ldots x_k
\]

(6.100)

The first \( n \) components of the eight \( \{P_k\} \) derived from any one \( Q_{k-1} \) are as in the original \( Q_{k-1} \) and the last component \( x_k \) takes on the eight different values of \( s_i \) (eqn. 6.3). The cost of each vector \( P_k \) is evaluated as

\[
|w_k|^2 = |w_{k-1}|^2 + |r_k - \sum_{h=0}^{8} x_{k-h} y_h|^2
\]

(6.101)

The detector next selects the vector \( P_k \) with the smallest cost \( |w_k|^2 \) and takes the value of the first component \( x_{k-n} \) of this vector as the detected value \( s_{k-n} \) of the data-symbol \( s_{k-n} \). All vectors \( \{P_k\} \) for which \( x_{k-n} \neq s_{k-n} \) are now discarded and the first component of each of the remaining vectors \( \{P_k\} \) (including that with the smallest cost) is omitted, to give the corresponding \( n \)-component vectors \( \{Q_k\} \), from which, \( \mu \) vectors \( \{Q_k\} \) are then selected as follows. The first of the selected vectors \( \{Q_k\} \) is taken as that with the smallest cost. If the remaining vectors contain any originating from the vector \( Q_{k-1} \) with the smallest cost \( |w_{k-1}|^2 \), the detector selects from this set the vector \( Q_k \) with the smallest cost \( |w_k|^2 \) and then from all \( \{Q_k\} \) (excluding the two just selected) the \( \mu-2 \) vectors with the smallest costs. If the remaining vectors do not include any originating from the \( Q_{k-1} \) with the smallest cost \( |w_{k-1}|^2 \),
the detector simply selects the $\mu - 1$ vectors $\{Q_k\}$ with the smallest costs. The detector now has a total of $\mu$ vectors $\{Q_k\}$ which are stored together with their costs $|W_k|^2$ waiting for the receipt of the next signal sample.

Although it has been mentioned that detector 1 is for use with system A (real signals), it may equally be used with systems B, C and D (complex signals), where now each vector $Q_{k-1}$ is expanded into 64, 64 or 256 vectors $\{P_k\}$, for systems B, C or D, respectively, instead of just 8 vectors as for system A, and the components $\{x_i\}$ of the vectors $\{Q_k\}$ and $\{P_k\}$ can have any of the possible values of $s_i$ in the corresponding system. Clearly, the number of expanded vectors $\{P_k\}$, from which the $\mu$ vectors $\{Q_k\}$ to be selected, is now 64$\mu$ or 256$\mu$ which require an excessive amount of storage, computations and search operations, compared with the case when system A is used.

Detector 1 is basically a simplification of the Viterbi-algorithm detector considered in Section 3.4, where the storage and computation requirements are dramatically reduced so that the detector becomes implementable in practice. Of course, substituting the very large number of survivors, required by the VA detector, by a small number ($\mu$) of vectors $\{Q_k\}$, will, no doubt, incur some loss in tolerance to additive white Gaussian noise, the loss being insignificant when binary or quaternary signals are used. From the method of operation of detector 1 it is evident that it falls within the family of reduced state Viterbi detectors, considered in Section 3.7, whose tolerance to additive white Gaussian noise is shown in Ref.(112) to approach that of the VA detector as the signal-to-noise ratio increases without limit. Therefore, the detector is expected
to show no significant loss in performance relative to the VA detector, at high signal-to-noise ratios.

Since the number of stored vectors in this detector is very small compared with the number of survivors of the VA detector, the straightforward selection of $\mu$ vectors $\{Q_k\}$ with the smallest costs $\{|W_k|^2\}$ may sometimes result in losing the vector which represents the best estimate of the corresponding data sequence. When the channel introduces severe amplitude distortion and when the number of signal levels is relatively high, the detector may not recover that vector again (this has been actually observed by computer simulation), causing consequently an endless error burst. The selection, where possible, of at least one vector $Q_k$ originating from the vector $Q_{k-1}$ with the smallest cost, reduces the possibility of losing that best estimate, and has actually been found to reduce significantly the average length of error bursts. Also, the straightforward selection of the $\mu$ vectors $\{Q_k\}$ with the smallest costs has the danger of all vectors becoming the same at some time, which reduces the effective number of the stored vectors. This is prevented by discarding all vectors $\{P_k\}$ for which $x_{k-n} \neq s_{k-n}$, which ensures that, if these vectors are all different at the start of transmission, no two or more of them can subsequently become the same (see Section 3.8, system 2).

6.4.3 Detector 2

Detector 2 is a modification of detector 1, where the storage and computation requirements are further reduced, so that it may be used with systems B, C and D. The following description is intended for systems B and C only, where the number of possible values of $s_1$ is 64. System D,
where the number of possible values of \( s_i \) is 256, is a straightforward modification of this.

Just prior to the receipt of the signal sample \( r_k \), the detector holds in store \( \mu \) \( n \)-component vectors \( \{Q_{k-1}\} \) where

\[
Q_{k-1} = x_{k-n}, x_{k-n+1}, \ldots, x_{k-1}
\]

and \( x_i \) may have any of the 64 possible values of \( s_i \) (eqn. 6.10), each vector \( Q_{k-1} \) being associated with its cost \( |W_{k-1}|^2 \). On the receipt of \( r_k \), each of the \( \mu \) vectors \( \{Q_{k-1}\} \) is expanded into four \( (n+1) \)-component vectors \( \{P_k\} \), where

\[
P_k = x_{k-n}, x_{k-n+1}, \ldots, x_k
\]

The first \( n \) components of the four \( \{P_k\} \) derived from any \( Q_{k-1} \) are as in the original \( Q_{k-1} \) and the last component \( x_k \) takes on the four different values \( \pm 4 \pm 4j \), where \( j = \sqrt{-1} \). The cost of each vector \( P_k \) is evaluated as

\[
C_k = |W_{k-1}|^2 + |r_k - \sum_{h=0}^{g} x_{k-h}y_h|^2
\]

From the four \( \{P_k\} \), originating from the vector \( Q_{k-1} \) with the smallest cost \( |W_{k-1}|^2 \), is now selected the vector \( P_k \) with the smallest cost \( C_k \), and then, from the remaining \( 4\mu-1 \) vectors \( \{P_k\} \) are selected the \( \mu-1 \) vectors with the smallest costs \( \{C_k\} \), to give a total of \( \mu \) selected vectors \( \{P_k\} \). No vector is, of course, selected more than once. Each of the selected vectors is next expanded into four vectors \( \{P_k\} \) whose first \( n \) components are again as in the original vector \( Q_{k-1} \), and to the given value of the last component \( x_k \) (now one of the four values \( \pm 4 \pm 4j \)) are added the four different values \( \pm 2 \pm 2j \). The cost \( C_k \) (eqn. 6.104) of each expanded vector \( P_k \) is then re-evaluated. From the \( \{P_k\} \) originating from the vector \( Q_{k-1} \) with the smallest cost \( |W_{k-1}|^2 \) is now selected the vector
$P_k$ with the smallest cost $C_k$, and then, from the remaining $4u-1$ vectors \{$P_k$\} are selected the $u-1$ vectors with the smallest costs, to give a total of $u$ selected vectors \{$P_k$\}. Each of the selected vectors is next expanded into four vectors \{$P_k$\}, whose first $n$ components remain unchanged, and to the given value of the last component $x_k$ are added the four different values $\pm 1 \pm j$. The cost of each expanded vector is then evaluated as,

$$|W_k|^2 = |W_{k-1}|^2 + |r_k - \sum_{h=0}^{3} x_{k-h}y_h|^2$$

(6.105)

where $x_k$ here may have any of the 64 different possible values of $s_i$. Now the detector has $4u$ expanded vectors \{$P_k$\} each being associated with its cost $|W_k|^2$. From these vectors, the detector then proceeds to detection of the data-symbol $s_k-n$ and the selection of the $u$ vectors \{$Q_k$\}, together with their costs \{|$W_k|^2$\}, exactly as in detector 1.

The technique just described is a process of treble expansion which is suitable for use with 64-point complex-valued data-symbols, as in systems B and C. For this detector to be used with system D, where a data-symbol may take on 256 possible complex values (eqns. 6.18 and 6.20), the treble expansion becomes a quadruple expansion. The latter proceeds exactly as does the treble expansion technique with the difference that four expansion processes are now involved instead of three. Thus, $x_k$ takes on one of the four values $\pm 8 \pm j 8$ in the first expansion, and to $x_k$ are then added the values $\pm 4 \pm j 4, \pm 2 \pm j 2$ and $\pm 1 \pm j$ in the second, third and fourth expansion, respectively. In fact, the treble expansion technique may be generalized to the multiple expansion technique, where the number of possible complex values of a data symbol is $m^2$, and $m=2^l$, $l=1,2,3,\ldots$. Clearly, the multiple expansion involves $l$ expansions, in the first of which $x_k$ takes on the four values $\pm 2^{l-1} \pm j 2^{l-1}$, and in the $i$th expansion
(i=2,3,\ldots,n) are added to the given value of x_k the four values \(\pm 2^{i-1}
\pm j2^{i-1}\).

The main advantage of the multiple expansion technique over the arrangement of detector 1 is that it requires the calculation of \(4\mu\) costs and the selection of \(\mu\) vectors out of \(4\mu\) expanded vectors, the two processes being repeated \(\ell\) times per received signal sample, whereas detector 1 requires the calculation of \(\mu m^2 = \mu 2^2\ell\) costs and the selection of \(\mu\) vectors out of \(\mu m^2\) expanded vectors per received signal sample. Clearly, the multiple expansion reduces the number of cost calculations by the ratio \(\mu 2^{2\ell}/4\mu k\) whose significance increases rapidly with \(\ell\). As far as the selection of the \(\mu\) vectors is concerned, the advantage of the multiple expansion over detector 1 is obvious, but the amount of the gain here depends on the selection technique actually employed.

In detector 1, all possible values of the data-symbol \(s_k\) are involved in the expansion of any vector \(Q_{k-1}\), so that in the selection process all these values are available to be in the selected vectors. In the multiple expansion technique and after the first expansion and selection process, the component \(x_k\) of any selected vector \(P_k\) has one of the four values \(\pm 2^{i-1}\pm j2^{i-1}\) which represents the centre points of the four quadrants in the signal constellation (represented by x on Fig. 6.13, in the 64 point system). When the selected vector \(P_k\) is expanded into four vectors \(\{P_k\}\) by adding, to the given value of \(x_k\), the corresponding four values, the detector assumes here, for a given initial vector \(P_k\) and a given initial value of \(x_k\) (in the centre of a quadrant), that the only values of \(s_k\) which will be further considered are those which lie in that quadrant, i.e., for one vector \(P_k\) selected in the first expansion, the detector eliminates the possibility of further taking into account the values of \(s_k\) in the
FIGURE 6.13: The Selection Process in Detector 2
other three quadrants. When, as a result of the first expansion and selection process, not all the four vectors \( \{ P_k \} \), originating from the same vector \( Q_{k-1} \), have survived, the possible values of \( s_k \) in the quadrants corresponding to the non-accepted \( \{ P_k \} \) are effectively discarded. Further elimination and discarding processes are performed in the following expansions and selections, so that the final selection process does not take into account all possible values of \( s_k \) for every vector \( Q_{k-1} \), as in detector 1. This, of course, should degrade the performance of the multiple expansion technique compared with detector 1. At high signal-to-noise ratios, however, the possibility of eliminating (over the successive expansions and selections) the quadrant or the part of the quadrant, in which \( s_k \) lies (as far as a vector \( Q_{k-1} \) is concerned), becomes small, and the degradation is expected to be negligible.

6.4.4 Detector 3

This is another approach to reduce the storage and computation requirements of detector 1. Just prior to the receipt of the signal sample \( r_k \), the detector holds in store \( \mu \) vectors \( \{ Q_{k-1} \} \), each being associated with its cost \( |W_{k-1}|^2 \). On the receipt of \( r_k \), each of the stored vectors \( \{ Q_{k-1} \} \) is expanded into 8 or 16 vectors \( \{ P_k \} \) for systems B and C or D, respectively, the first \( n \) components of these \( \{ P_k \} \) being as in the original \( Q_{k-1} \) and the last component \( x_k \) taking on the 8 values \( \pm 1, \pm 3, \pm 5 \) and \( \pm 7 \), for systems B and C, and the 16 values \( \pm 1, \pm 3, \pm 5, \pm 7, \pm 9, \pm 11, \pm 13 \) and \( \pm 15 \) for system D. From the value of

\[
d_k = r_k - \sum_{h=0}^{\infty} x_{k-h} y_h
\]

where

\[
e_k = r_k - \sum_{h=1}^{\infty} x_{k-h} y_h
\]

(6.106)
and

\[ y_0 = 1.0 + j0.0 \]  \hspace{1cm} (6.108)

as given in Tables 6.8-6.12, the detector then selects the \( \rho \) vectors \( \{P_k\} \) for which the absolute values of the real part of \( d_k \) are minimum. Thus, the detector selects the \( \rho \) vectors \( \{P_k\} \) for which the given possible values of \( x_k \) are closest to the real part of \( e_k \) (eqn. 6.107), this being achieved by simple threshold comparison techniques, and not involving the evaluation of any costs. In addition to the above operation, each of the stored vectors \( \{Q_{k-1}\} \) is also expanded into the corresponding 8 or 16 vectors \( \{P_k\} \), for systems B and C or D, respectively, where as before the first \( n \) components are as in the original \( Q_{k-1} \), but now the last component \( x_k \) takes on the 8 values \( \pm j, \pm 3j, \pm 5j \) and \( \pm 7j \), in systems B and C, and the 16 values \( \pm j, \pm 3j, \pm 5j, \pm 7j, \pm 9j, \pm 11j, \pm 13j \) and \( \pm 15j \) in system D. The detector then forms \( d_k \) (eqn. 6.106) for each expanded vector \( P_k \) and selects the \( \rho \) vectors \( \{P_k\} \) for which the absolute values of the imaginary part of \( d_k \) are minimum, i.e. for which the given values of \( x_k \) are closest to the imaginary part of \( e_k \), this again being achieved by a threshold comparison technique. The detector has now selected \( 2\rho \) vectors \( \{P_k\} \) originating from each stored vector \( Q_{k-1} \). From each of these \( \mu \) sets of \( 2\rho \) vectors \( \{P_k\} \) the detector then forms the \( \rho^2 \) vectors \( \{P_k\} \) whose first \( n \) components are as in the original \( Q_{k-1} \) and the \( \{x_k\} \) (now complex-valued) are given by all combinations of the real and imaginary values of \( x_k \) in the vectors \( \{P_k\} \) just selected. Let

\[ \lambda = \rho^2 \]  \hspace{1cm} (6.109)

The detector now evaluates the cost

\[ |w_k|^2 = |w_{k-1}|^2 + |d_k|^2 \]  \hspace{1cm} (6.110)

for each of these \( \lambda \) vectors \( \{P_k\} \) in each set, using the real and imaginary parts of \( d_k \) already evaluated.
As a result of the expansion process just described, the detector holds $\mu \lambda$ vectors $\{P_k\}$. It then proceeds to the detection of the data-symbol $s_{k-n}$ and the selection of $\mu$ vectors $\{Q_k\}$, together with their costs $\{|W_k|^2\}$, exactly as in detector 1.

In the process of forming the $\lambda$ vectors $\{P_k\}$, originating from one vector $Q_{k-1}$, $x_k$ effectively takes on the $\lambda$ complex values in the square of $\rho^2 \times \rho^2$ points whose centre $\theta$ is closest to $e_k$, in the signal constellation, as shown in Fig. 6.14 (for the 64 points systems and where $\rho=4$). Let the coordinates of $\theta$ be $(\theta_1, \theta_2)$, then the $\lambda$ points (or the $\lambda$ values of $x_k$) in the corresponding square, when $\rho$ is even, are given by

$$x_k = (\delta_1 + j\delta_2) + (\theta_1 + j\theta_2)$$

(6.111)

where

$$\delta_1, \delta_2 = \pm1, \pm3, \ldots, \pm(\rho-1).$$

(6.112)

The $\rho^2$ points $(\delta_1 + j\delta_2)$ form, in fact, the square of $\rho^2 \times \rho^2$ points centred at the origin. Thus, an alternative and a simpler approach to determine the required $\lambda$ values of $x_k$, when $\lambda$ is even, is to determine the value of $(\theta_1 + j\theta_2)$ and add it to the $\lambda$ values given by eqn. 6.112. This, in fact, saves the detector performing the threshold comparisons whose number increases with $\rho$. From Fig. 6.14, $\theta_1$ and $\theta_2$ may have even-integer values only, since the real and imaginary parts of a signal point take on odd-integer values only. Consequently, $\theta_1$ and $\theta_2$ may be obtained by truncating the real and imaginary parts of $e_k$, respectively, each to the nearest even-integer. For values of $e_k$ nearer to the outer points in the signal constellation, the described procedure may result in values of $x_k$ which are outside the signal point space. For example, if $e_k$, in Fig. 6.14, has the value $(2.5+j5.1)$, $\theta_1$ and $\theta_2$ become 2 and 6, respectively, causing the square to be shifted upwards to include values.
FIGURE 6.14: The Selection Process in Detector 3
of $x_k$ whose imaginary parts are greater than 7. Clearly, this may be prevented if $\theta_2$, in this example, is limited to have a value of 4. In general, when the number of points in the signal space is $m^2$ ($m$ even), then $\theta_1$ and $\theta_2$ may be determined from

$$
\begin{align*}
\theta_1 &= \left[ \frac{\epsilon_{1,k}^{m-1}}{2} \right] \times 2^{-m+2} \\
\theta_2 &= \left[ \frac{\epsilon_{2,k}^{m-1}}{2} \right] \times 2^{-m+2}
\end{align*}
$$

(6.113)

where $\epsilon_{1,k}$ and $\epsilon_{2,k}$ are the real and imaginary parts of $c_k$, and $[.]$ means the largest integer smaller than (.). In binary arithmetic, the operation $[a/2] \times 2$ is equivalent to zeroing the least significant bit of the integer part and the whole fractional part of the binary-coded number representing (a). The resultant values of $\theta_1$ and $\theta_2$ are then limited to be such that

$$|\theta_1|, |\theta_2| \leq (m-p)$$

(6.114)

Thus, the $\lambda$ values of $x_k$ may be obtained simply by forming $c_k$ (eqn. 6.107), determining $\theta_1$ and $\theta_2$ according to eqns. 6.113 and 6.114 and adding $(\theta_1 + j\theta_2)$ to the $\lambda$ values of $(\delta_1 + j\delta_2)$ in eqn. 6.112.

When $\rho$ has an odd value, the $\lambda$ values of $x_k$ may by similar argument be shown to be given by eqn. 6.111, but now $\delta_1, \delta_2, \theta_1$ and $\theta_2$ are given by

$$\delta_1, \delta_2 = 0, \pm 2, \pm 4, \ldots, \pm (p-1)$$

(6.115)

and

$$
\begin{align*}
\theta_1 &= \left[ \frac{\epsilon_{1,k}^{m}}{2} \right] \times 2^{-m+1} \\
\theta_2 &= \left[ \frac{\epsilon_{2,k}^{m}}{2} \right] \times 2^{-m+1}
\end{align*}
$$

(6.116)

and both $\theta_1$ and $\theta_2$ must be limited to satisfy eqn. 6.114.

The cost of each vector $P_k$ is now evaluated as

$$|W_k|^2 = |W_{k-1}|^2 + |\epsilon_k - x_k|^2$$

(6.117)
Detector 3 requires the calculation of $\mu\lambda$ cost values and the selection of $\mu$ vectors out of $\mu\lambda$ expanded vectors. Since detector 1 has to calculate $\mu m^2$ cost values and to select $\mu$ vectors out of $\mu m^2$ expanded vectors, the relative advantage of detector 3 becomes higher when $\lambda$ becomes smaller.

Again, like detector 2, detector 3 is expected to show some degradation in performance compared with detector 1, since each vector $Q_{k-1}$ is expanded into $\rho^2$ vectors $\{P_k\}$, which is small compared with $m^2$ vectors $\{P_k\}$ in case of detector 1. At high signal-to-noise ratios, and as far as a given vector $Q_{k-1}$ is concerned, the most likely value of $x_k$ is that closest to $e_k$, and consequently, it is most likely to be within the selected square, which suggests a negligible degradation. In detector 2, when only one vector $P_k$, originating from a given vector $Q_{k-1}$, has survived after the first expansion, then the quadrant in which $e_k$ lies (given $Q_{k-1}$), is accepted for further considerations, and when $e_k$ is closer to the real and imaginary axis in the signal space, then the points in this quadrant are not necessarily the closer to $e_k$. This, effectively gives some advantage to detector 3, which always considers signal points closer to $e_k$, than does detector 2.

6.4.5 Detector 4

This is a development of detector 3, in which a delay of one sampling period is introduced and two signal samples are involved in the expansion and selection process.

Just prior to the receipt of the signal sample $r_{k+1}$ (not $r_k$) the detector holds in store $\mu$ vectors $\{Q_{k-1}\}$ (eqn. 6.102) each being
associated with its cost $|v_{k-1}|^2$. On the receipt of $r_{k+1}$, the detector proceeds exactly as does detector 3, where it forms $\mu\lambda$ vectors $\{P_k\}$ and evaluates, for each, the cost $|w_k|^2$ (eqn. 6.110 or 6.117). The signal sample $r_{k+1}$ is, as yet, not involved in any process. Now, the detector modifies the costs $\{|w_k|^2\}$ in such a way which assumes that it knows the value of $s_{k+1}$, the first component of its transmitted data-element has just arrived, as a part of $r_{k+1}$. Therefore, the detector forms, for each of the $\mu\lambda$ vectors $\{P_k\}$, the value

$$d_k' = r_{k+1} - \sum_{h=0}^{g} x_{k+1-h}y_h$$

$$= e_k' - x_{k+1}$$

where

$$e_k' = r_{k+1} - \sum_{h=1}^{g} x_{k+1-h}y_h$$

and $y_0=1.0$, as given by eqn. 6.108. The detector now determines for each vector $P_k$, the value of $x_{k+1}$ for which $|d_k'|$ is minimum. Of course, the resultant value of $x_{k+1}$ is one of the possible values of $s_{k+1}$. Let this value of $x_{k+1}$ be given by

$$x_{k+1} = \beta_1 + j\beta_2$$

(6.120)

The cost $|w_k|^2$ of each vector $P_k$ is now modified by the corresponding value of $(\beta_1 + j\beta_2)$ to give

$$c_k = |w_k|^2 + |e_k - \beta_1 + j\beta_2|^2$$

(6.121)

and then the detector proceeds to the detection of the data-symbol $s_{k-n}$ and the selection of the $\mu$ vectors $\{Q_k\}$, exactly as detector 1, but using now the modified costs $\{c_k\}$ instead of the $\{|w_k|^2\}$. The selected vectors $\{Q_k\}$ are then stored together with their corresponding costs $\{|w_k|^2\}$ waiting for the receipt of the next signal sample. After
the selection process, the \( \{C_k\} \) are not required further and are therefore discarded.

The value of \( x_{k+1} \) (eqn. 6.120) that minimises \( |d_k'| \) is, according to eqn. 6.118, the possible value of \( s_1 \) closest to \( e_k' \) in the signal-point space, whose real and imaginary parts may be shown to be given by

\[
\beta_1 = \left[ \frac{e_1' + m}{2} \right] \times 2^{-m+1}
\]

\[
\beta_2 = \left[ \frac{e_2' + m}{2} \right] \times 2^{-m+1}
\]

(6.122)

where \( e_{1,k} \) and \( e_{2,k} \) are the real and imaginary parts of \( e_k' \), \( m \) is the number of points in the signal constellation and \([\cdot]\) has the same meaning as defined for eqn. 6.113. When \( e_k' \) has a large amplitude so that

\( |\beta_1|, |\beta_2| > (m-1) \), then \( \beta_1 \) and \( \beta_2 \) are limited as required, to be such that

\( |\beta_1|, |\beta_2| = m-1 \). \( (m-1) \), in fact, represents the largest possible value of the real and imaginary parts of a data-symbol.

Detector 4, by modifying the costs \( |W_k|^2 \) to \( \{C_k\} \) effectively removes the intersymbol interference of \( s_{k+1} \) in \( r_{k+1} \) (on the assumption that the given vector \( P_k \) represents a correct estimate of the corresponding data sequence \( \{s_{k-n+1}\} \), \( i=0,1,\ldots,n \) and at the same time involves \( r_{k+1} \), which carries further information about \( s_k' \), and so improves the detection process. The improvement is expected to be higher when \( y_0 \), the first component of the sampled impulse response, has a significantly smaller amplitude than the second component.

The penalty paid here in improving the performance of detector 3 is in fact the re-calculation of the \( \mu \lambda \) costs in eqn. 6.121. Determining the values of \( \beta_1 \) and \( \beta_2 \) in eqn. 6.122 is however a simple process and does not represent significant additional complexity.
ASSESSMENT OF SYSTEMS AND COMPUTER SIMULATION RESULTS

Computer simulation tests over the models of the four telephone circuits, whose characteristics are shown in Figs. 6.6 to 6.9, have been carried out on systems A, B, C and D, using various combinations of equipment filters (Figs. 6.10-6.12) and detectors (Section 6.4) to determine in each case the tolerance to additive white Gaussian noise. The various systems and detectors are summarised in Table 6.13. Examples of computer programs, which have been written in FORTRAN to simulate the different systems, are presented in Appendices J, K, L and M. The minimum phase sampled impulse responses \( \{ Y \} \) (eqns. 6.8, 6.14 and 6.24) are obtained as described in Section 6.3 and are given, for the different arrangements, in Tables 6.6 to 6.12. As mentioned before, the telephone circuits 1 and 2 introduce typical levels of attenuation and group-delay distortion, and the telephone circuits 3 and 4 are close to the typical worst circuits (on the public switched telephone network in the U.K.) normally considered for the transmission of data at 9600 and 1200 bit/s, respectively.\(^{116,118}\)

The performances of the different systems, with the various combinations of equipment filters and detectors, over the four telephone circuits are shown in Figs. 6.15 to 6.24, where every given curve is labelled with the relevant information. The label "system A1" means "system A" with "detector 1", and the label "system B3" means "system B" with "detector 3". \( \mu \) is the number of stored vectors held by the detector and \( n \) is the delay in the detection of a data-symbol. \( \lambda \) is defined by eqn. 6.109. The signal-to-noise ratio (\( \psi \) \( \text{db} \)) is given by eqn. 6.86. The 95% confidence limits of the results are better than \( \pm 0.5 \) \( \text{db} \) over telephone circuits 1-3. Over telephone circuit 4, the 95%
TABLE 6.13: Various Systems with the Different Arrangements Used in the Computer Simulation Tests

<table>
<thead>
<tr>
<th>Data transmission System/Detector*</th>
<th>Equipment Filters</th>
<th>Telephone Circuits</th>
<th>Element-rate ($s^{-1}$)</th>
<th>$f_c$ (Hz)</th>
<th>Performance shown in Figs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
<td>1-3</td>
<td>6400</td>
<td>-</td>
<td>6.15-6.17</td>
</tr>
<tr>
<td>A1</td>
<td>3</td>
<td>1-3</td>
<td>6400</td>
<td>-</td>
<td>6.22-6.24</td>
</tr>
<tr>
<td>B2,B3,B4 Equalizer B</td>
<td>2</td>
<td>1-4</td>
<td>3200</td>
<td>-</td>
<td>6.18-6.21</td>
</tr>
<tr>
<td>B3, Equalizer B</td>
<td>3</td>
<td>1-3</td>
<td>3200</td>
<td>-</td>
<td>6.22-6.24</td>
</tr>
<tr>
<td>C2</td>
<td>1</td>
<td>1-3</td>
<td>3200 1600</td>
<td>6.15-6.17</td>
<td></td>
</tr>
<tr>
<td>C2,C3,C4 Equalizer C</td>
<td>2</td>
<td>1-4</td>
<td>3200 1800</td>
<td>6.18-6.21</td>
<td></td>
</tr>
<tr>
<td>D3, Equalizer D</td>
<td>2</td>
<td>1-4</td>
<td>2400 1800</td>
<td>6.18-6.21</td>
<td></td>
</tr>
</tbody>
</table>

*A1: system A with detector 1

Equalizer B: system B with the conventional non-linear equalizer
confidence limits of the results are slightly worse than ±0.5 db for curves more on the left hand side of Fig. 6.21 and about ±1 db for curves on the right hand side.

Although the given results do not cover all possible combinations of the different systems, equipment filters, telephone circuits and detectors, they are arranged to give a clear idea of the relative performances of the various combinations. To allow a direct comparison of the performances of detectors 1-4 with the performance of the optimum non-linear equalizer (Section 3.2.3), the latter has been tested with systems B and C over telephone circuits 1-4, and the corresponding results are shown in Figs. 6.18-6.24. The label "Non-linear equalizer A or B" means system A or B with the optimum non-linear equalizer. The performance of system D with the non-linear equalizer is also shown on Figs. 6.18-6.21, for telephone circuits 1-4, respectively, where these results have been obtained by Mr. M.J. Fairfield (non-linear equalizer B in reference 118).

The correct adjustment of the adaptive linear filter (Figs. 6.1-6.3) has been assumed throughout, so that the results obtained represent upper bounds to the performances obtainable with an adaptively adjusted filter. In general, the more severe the signal distortion, the less accurate is the adjustment of the adaptive filter and hence the greater the degradation in tolerance to noise relative to the corresponding system having a correctly adjusted filter. (118) Also, perfect estimation of the sampled impulse response of the linear baseband channel, sampler and adaptive linear filter has been assumed, so that, again, the obtained results represent upper bounds to the systems performances in that respect. With a near time-invariant channel, such as a telephone circuit
(after having accurately removed the effects of the frequency offset in systems A and B and where no significant phase jitter is present in the received signal carrier in systems C and D), this can be approximated to quite accurately, so long as the signal distortion introduced by the channel is not too severe.\footnote{118}

Computer-simulation tests over the given channels have shown that the phase of the sampling instants at the receiver has a negligible effect on the performance of any system, other than on that of the optimum non-linear equalizer when system B is used with equipment filters 3 (Figs. 6.22-6.24) and when system D is used with equipment filters 2 (Figs. 6.18-6.21). The sampling phase used here, for each channel has therefore been selected to give a good performance of the non-linear equalizer, with a tolerance to noise within about 0.5 db of optimum, which is probably as good as would be achieved in practice.\footnote{118} The lack of influence of the sampling phase on the performance of the non-linear equalizer when systems B and C are used with equipment filters 2 (Figs. 6.18-6.21) is due to the sampling rate which is now a little above the Nyquist rate. In Appendix G, it is shown that, when the sampling rate is not less than the Nyquist rate and when the sampled impulse response of the channel is linearly processed by an allpass network to be a minimum phase (as is the case here), the resultant sampled impulse response is independent of the sampling phase. This has been confirmed by tests.

The relative performances of detectors 2, 3 and 4, with systems B, C and D and equipment filters 2, are shown in Figs. 6.18-6.21. The performance of detector 1 with systems B, C and D has not been actually determined because of the very long computation time required for the
simulation. But, as has been shown in Sections 6.4.2 to 6.4.5, detectors 2, 3 and 4 are sub-systems of detector 1, and therefore they can not have a better performance than does detector 1. However, detector 1 is far too complicated to be used, in practice, with systems B, C and D.

As can be seen from Figs. 6.18-6.21, and for $\mu=32$ and $\lambda=16$, detectors 3 and 4 have the same performance over the milder channels (telephone circuits 1 and 2), but over the worst channel (telephone circuit 4) and at a bit error rate of $4 \times 10^{-3}$, detector 4 has an advantage of about 0.75 db. For the same number of stored vectors ($\mu=32$), detectors 3 and 4 have a slight advantage over detector 2, for telephone circuits 1 and 2, but this advantage becomes more significant (about 4 db) over telephone circuit 4. Over telephone circuit 3, the three detectors have almost the same performance, in particular, at lower error rates. This, in fact, gives an advantage to detector 3 over the other detectors, since it is the simplest and its performance, over the more realistic channels for this application, is not significantly worse. As shown in Section 6.4.4, the performance of detector 3 is expected in general to be better than that of detector 2, which is, more or less, confirmed by the given results. The almost similar performances of detectors 2, 3 and 4 over the milder channels suggest that these detectors are actually achieving the best which can be done with the given signal distortion, and detector 1 is not expected to have a significant advantage under the same conditions.

Detectors 3 and 4 show a slightly better performance over telephone circuit 1 (Fig. 6.18) than they do over telephone circuit 2 (Fig. 6.19).
This is essentially because telephone circuit 2 introduces more amplitude distortion at the higher frequencies than does telephone circuit 1, as can be seen from Figs. 6.6 and 6.7. Over telephone circuits 3 and 4, detector 4 loses about 7 and 14 db (Figs. 6.20-6.21), respectively, relative to its performance over telephone circuit 1 (Fig. 6.18). This is in fact due to the relatively narrow bandwidth of telephone circuits 3 and 4 (Figs. 6.8-6.9) which, in addition to representing more severe signal distortion, causes high loss in the transmitted signal energy because of the considerable mismatch between the attenuation characteristics of these circuits and the attenuation characteristics of the transmitter filter. The latter are given by dividing those of filters 2 (Fig. 6.11) by 2.

Over the mildest of the four telephone circuits, i.e. over telephone circuit 1, and when equipment filters 2 are used with systems B and C, detectors 3 and 4 (Fig. 6.18) lose about 5.5 db in tolerance to additive Gaussian noise relative to their performances over a perfect channel (the sampled impulse response of which has only one non-zero component with value 1.0) which has been determined experimentally (by simulation) and is given in Fig. 6.25. The given loss of 5.5 db is probably due to the use of the equipment filters 2, which introduce the most severe band-limiting that could reasonably be considered as appropriate for a transmission rate of 19200 bit/s. (118) When these filters are replaced by equipment filters 3 (Fig. 6.12) whose bandwidth is wider than that of filters 2 by about 400 Hz, the performance of detector 3 is improved by about 2 dB over telephone circuits 1,2 and 3, as can be seen from Figs. 6.22-6.24. Now, detector 3 loses over telephone circuit 1 no more.
than 3.5 db relative to the performance of the corresponding system over the perfect channel (Fig. 6.25).

The performances of detector 2, when systems B and C are used with equipment filters 1 (Fig. 6.10), are shown in Figs. 6.15 to 6.17 for telephone circuits 1, 2 and 3, respectively. Comparing these with the corresponding curves in Figs. 6.18-6.20 show that filters 1 do not cause a significant degradation, over telephone circuits 1-3, relative to the case when filters 2 are used. Although filters 1 have a narrower bandwidth, (Figs. 6.10 and 6.11), and consequently should cause more signal distortion, the expected degradation is probably offset by the fact that uncorrelated noise is used with these filters, unlike all other tests, in which the noise has been appropriately correlated. However, filters 1 are not appropriate to be used in practice because they do not introduce sufficient filtering at the lower frequencies which is essential to remove the interference of the Main's hum.

From curves labelled "systems B, and C." (where i represents the detector i) in Figs. 6.15-6.21, it can be seen that systems B and C have the same performance when the same detector is used and under the same conditions of signal distortion. This confirms the theoretical results obtained in Section 5.5, in which it was shown that the two-dimensional (Hilbert transform pair) baseband system exhibits an exact equivalence to the QAM system, when both systems operate at the same signalling rate and use the same signal constellation (Fig. 6.4), as is the case of systems B and C here. On the other hand, the performance of system A with detector 1 is slightly worse than the performance of systems B and C with detectors 2 or 3, when the number of stored vectors is the same (μ=32) in
both cases (Figs. 6.15-6.17 and 6.22-6.24), the relative difference being less than 1 db. When \( \mu = 8 \), the performance of system A with detector I is inferior by at least 1.5 db to the performance of system B with detector 3 over telephone circuits 1-3 (Figs. 6.22-6.24). This suggests that the performance of system A becomes closer to the performance of systems B and C when the number of stored vectors \( \mu \) increases, and the performance of the three systems is consequently expected to become the same when a true maximum-likelihood detector is used, as has been shown in Section 5.5. Of course, system A transmits an 8-level one-dimensional baseband signal at a rate of 6400 bauds, and systems B and C transmit a 64-point two-dimensional signal (signals with two components in phase quadrature) at a rate of 3200 bauds, and the performances of the three systems over the perfect channel are similar, as shown theoretically in Section 5.5.1 and as confirmed by simulation results given in Fig. 6.25.

System D has a similar structure to that of system C except that it transmits a 256-point signal at a rate of 2400 baud. According to eqn. 6.48, where \( e_0 = 7 \) for system C and 21.25 for system D, the average transmitted energy per bit in system D is greater by 4.8 db than that in system C, when the same transmitter filter is used in both cases. This means that system D should lose 4.8 db in tolerance to additive white Gaussian noise over the perfect channel compared with system C. But as has been shown earlier, systems B and C with filters 3 and over telephone circuit 1 (which by no means forms a perfect channel, as can be seen from Table 6.9) lose no more than 3.5 db relative to their performance over the perfect channel (Figs. 6.22 and 6.25), which means that systems B and C over this typical telephone circuit have a better performance than can
system D have over a perfect channel. Therefore, system D can not compete with systems B and C on the grounds of improved performance, whatever detection technique it uses. However, when system D is used with filters 2 and detector 3, but with only 6 stored vectors, its performance is not significantly worse than the performances of systems B and C with the same equipment filters 2 and detector 3, but with 32 stored vectors (Figs. 6.18-6.21). Of course, system D is more efficient than systems B and C, if the system performance and the detector complexity are to be taken together into account in the evaluation of the systems efficiency. The superiority of system D is, in fact, due to the reduced level of intersymbol interference in the received signal, because the sampling rate is here at 2400 samples/second, which is below the Nyquist rate for filters 2 (when considered in the baseband, of course). When filters 2 are replaced by filters 3, system B (and therefore system C) regains its advantage over system D as can be seen from Figs. 6.18-6.24, and this advantage becomes bigger over the milder channels (circuits 1 and 2). This is because filters 3 have now a wider bandwidth and therefore introduce a lower level of intersymbol interference compared with the case when filters 2 are used. The use of filters 3 with system D does not reduce the intersymbol interference significantly here, whereas it allows more noise power into the received signal at the receiver, which may lead to a poorer performance of this system. Even when the level of intersymbol interference is significantly reduced, the performance of system D can not improve over its performance over the perfect channel. The latter is worse, by at least 1 db, than the performance of system B with filters 3 over telephone circuit 1, as has been shown earlier.
As far as the optimum non-linear equalizer is concerned, it is useless with systems B and C and filters 2, as can be seen from Figs. 6.18-6.21. That is not only because of the big degradation in its tolerance to additive Gaussian noise, but also because this equalizer can not recover from an error burst once this started. The non-linear equalizer with system D and equipment filters 2 gives a much better performance than the previous equalizer, but the performance here is worse than can be achieved by the better of the considered systems by at least 4, 4.4, 4.5 and 6 db over telephone circuits 1, 2, 3 and 4, respectively, when filters 2 are also used (Figs. 6.18-6.21). The best of what can be achieved with the non-linear equalizer is when systems B and C are used with filters 3. The corresponding results with system B are shown in Figs. 6.22-6.24, for telephone circuits 1, 2 and 3, respectively. Here again, system B with detector 3 and filters 3 gains about 3.5, 3.5 and 7 db over this equalizer, for telephone circuits 1, 2 and 3, respectively.

A near-maximum likelihood detector is clearly preferable to the non-linear equalizer for several reasons. First is the better tolerance of this detector to additive white Gaussian noise which is not less than 3.5 db, as shown above. Secondly, error extension effects are much more serious with the non-linear equalizer than with the near-maximum likelihood detectors. In fact, it has been observed from the computer simulation tests that the near-maximum likelihood detector recovers from an error burst very rapidly whereas the non-linear equalizer sometimes may or may not recover from the error burst. This advantage of the near-maximum likelihood detector over the non-linear equalizer becomes greater when the channel introduces into the transmitted signal more amplitude distortion. Thirdly, no particular sampling phase is required by the near-maximum
likelihood detector (under the conditions considered here) to give its optimum performance, whereas it is essential to have the received signal sampled at certain time instants in order to obtain the best performance of the non-linear equalizer. Actually, when the sampling rate is not far below the Nyquist rate, the sampling instant has a negligible effect on the total signal energy as seen by a maximum or a near-maximum likelihood detector, since both operate on all received signal-element components to detect the data symbol, but the sampling instant has a great effect now on the signal energy as seen by the non-linear equalizer which detects the data symbol from the first arriving component of the corresponding received data element. The non-linear equalizer has no inherent advantage in tolerance to phase jitter, on account of the reduced delay in detection, since "early" detection can always be used for the channel estimator and signal-phase control loops, in a near-maximum likelihood detector.\(^{118}\)

In fact, no noticeable change in the error rate has been observed when \(n\) was reduced to \(n=6\), as shown in Figs. 6.18-6.20.

Figs. 6.26 and 6.27 show the effects on the bit error rate of inaccuracies in the estimates made by the receiver of the level and carrier phase (system C) or frequency offset phase (system B), respectively, for the given systems when operating over telephone circuits 1 and 3 with equipment filters 2 and detector 3. Although the corresponding tests assume a fixed deviation in the signal level and phase, these results give a good idea of the effects of small and random changes in the signal level and phase actually produced by the channel estimator and the phase-locked loop. Of course, a better evaluation of these effects on the performances of the given systems may be obtained by considering the mean
square error, caused by the estimator and phase-locked loop which are to be used in an actual implementation, as an additive noise whose effects are already given in Figs. 6.15-6.24. However, it can be seen from Figs. 6.26 and 6.27 that, for the correct operation of the detector, very accurate estimates are required of the received signal level and phase. It is in fact evident from these results that the factor which limits the highest achievable transmission rate over telephone circuits is most likely to be the accuracy to which the receiver can adjust the adaptive linear filter and the accuracy to which it can estimate the sampled impulse response $Y$ (including, of course, the signal level and phase).

As mentioned earlier, an overall improvement of about 2 db has been achieved when equipment filters 2 are replaced by equipment filters 3, over telephone circuits 1-3. Over telephone circuit 4 a very small or no improvement is expected with a similar arrangement, because of the excessively narrow bandwidth of this circuit (Fig. 6.9). Here, equipment filters with wider bandwidth can not reduce the intersymbol interference significantly whereas more noise power will be passed by the receiver filter. Indeed, even if the improvement of 2 db could be obtained, and even with the correct adjustment of the adaptive filter, there is here a low tolerance to noise and the errors furthermore occur in long bursts. When due allowance is made for the further degradation in performance introduced by the adaptive adjustment of the linear filter, it seems unlikely that a satisfactory performance will be obtained with any of the given systems over telephone circuit 4. However, when operating over telephone circuits 1-3, the performances of systems B and C with equipment filters 3 and detector 3 are quite promising and suggest that
satisfactory operation should be obtainable in practice, provided a suitable arrangement is used for the adjustment of the adaptive filter and so long as the phase jitter in the received signal is not too severe.\textsuperscript{[118]}
FIGURE 6.15: Performance of Different Systems Over Telephone Circuit 1 with Equipment Filters 1
FIGURE 6.16: Performance of Different Systems Over Telephone Circuit 2 with Equipment Filters 1
FIGURE 6.17: Performance of Different Systems Over Telephone Circuit 3 with Equipment Filters 1
FIGURE 6.18: Performance of Different Systems Over Telephone Circuit 1 with Equipment Filters 2
FIGURE 6.19: Performance of Different Systems Over Telephone Circuit 2 with Equipment Filters 2
FIGURE 6.20: Performance of Different Systems Over Telephone Circuit 3 with Equipment Filters 2
FIGURE 6.21: Performance of Different Systems Over Telephone Circuit 4 with Equipment Filters 2
FIGURE 6.22: Performance of Different Systems Over Telephone Circuit 1 with Equipment Filters 3
FIGURE 6.23: Performance of Different Systems Over Telephone Circuit 2 with Equipment Filters 3
FIGURE 6.24: Performance of Different Systems Over Telephone Circuit 3 with Equipment Filters 3
FIGURE 6.25: Performance of Systems A, B and C Over the Channel that Introduces No Linear Distortion
DISCREPANCY IN THE ESTIMATE OF RECEIVED SIGNAL LEVEL (db)

FIGURE 6.26: Variation of Error Rate With An Inaccuracy in the Estimate of the Level of the Received Signal
FIGURE 6.27: Variation of Error Rate With An Inaccuracy in the Estimate of the Carrier Phase of the Received Signal
7. THE DETECTION OF DIGITAL DATA SIGNALS TRANSMITTED AT 9600 BIT/S OVER HF RADIO LINKS

7.1 INTRODUCTION

A high frequency (HF) radio voice-channel has a bandwidth of 3 kHz in the frequency band 3-30 MHz. The voice-frequency signal, whose spectrum extends from 300 to 3000 Hz is transmitted over the HF channel by a process of linear modulation at the transmitter to shift its spectrum into the HF band, and linear demodulation at the receiver to bring it back to its original frequency band. As mentioned in Section 2.4, typical HF channels are characterized by multipath propagation and fading, and signals transmitted over these channels arrive at the receiver suffering from severe time-varying amplitude distortion. This distortion introduces severe degradations into the quality of analogue communications and it has an even worse effect on digital data communications.

When the transmitted data signal has been subjected to severe amplitude distortion such as that encountered over HF radio channels, which may vary considerably during any one transmission or from one transmission to another, a useful overall improvement in tolerance to additive white Gaussian noise can be achieved by using a maximum-likelihood
detector in place of a non-linear equalizer.\textsuperscript{(115)} Unfortunately, the maximum-likelihood detector, implemented as a Viterbi-algorithm detector (Section 3.4), faces two major problems when used in such an application. First, when the sampled impulse response of the channel contains more than one or two non-zero components, this detector becomes too complex because of the excessive amount of storage and computation required. The second problem is the implementation of the adaptive whitened matched filter (Section 3.5) which is required ahead of the Viterbi-detector for this to achieve its best performance, and which must be capable of tracking rapid changes in the HF channel characteristics. The first problem may be overcome by forcing the sampled impulse response of the channel to have a small number of components by means of an adaptive linear filter ahead of the detector\textsuperscript{(102-107)} (Section 3.6). This technique may lead to an acceptably simple Viterbi detector, when the channel characteristics vary very slowly with time, but at the expense of possible noise enhancement and correlation, where the adaptive filter performs some amplitude equalization in order to give the desired impulse response and so ceases to be a whitened matched filter.\textsuperscript{(108-109,115-116)} When the channel characteristics vary considerably with time, such as is the case here, there may be great difficulties in holding the adaptive linear filter correctly adjusted at all times to give the desired impulse response.\textsuperscript{(115)} The second problem of implementing the whitened matched filter adaptively may be overcome by fixing the receiver filter, oversampling
the received signal (that is, sampling at, say, twice the data element rate) and appropriately modifying the Viterbi detector. Of course, the penalty paid here is in the further increase in the detector complexity where now not only the sampled impulse response of the channel may contain more than few non-zero components, but also the detector is operating on say two received signal samples per data symbol, which at least doubles the detector complexity.

In these applications where the channel is used at the Nyquist rate, the whitened matched filter may be replaced by a lowpass filter which has a rectangular transfer function and a cutoff frequency (in Hz) at half the signal element rate (in bauds), this being followed by a sampler which samples the received signal once per data element (Section 3.5). Here, no adaptive adjustment of a whitened matched filter is required, and the Viterbi detector should achieve its optimum performance even when the channel characteristics vary considerably with time, as long as these characteristics are perfectly known to the receiver. Again, the complexity of the Viterbi detector here becomes unacceptable when the sampled impulse response of the channel includes more than a very few components, but this may be overcome if the detector is preceded by the linear adaptive filter which forms the first part of the optimum non-linear equalizer (Sections 3.2.3 and 3.5). When this filter is correctly adjusted, it has the effect of concentrating the energy of the sampled impulse response in its first few components, thus reducing its overall length and leading to a simpler Viterbi detector. Unfortunately there may be difficulties in
holding this linear filter adaptively adjusted when the channel undergoes large and rapid changes, as can happen over HF radio links.(115)

The alternative approach to the Viterbi detector is to use a reduced-state Viterbi detector (Section 3.7), and in particular its more practical and realistic version, the near-maximum likelihood detector (Section 3.8). This technique has been used successfully in the detection of data signals transmitted at 9600 bit/s(116) and 19200 bit/s (Section 6) over models of telephone lines. In both cases, the near-maximum likelihood detectors used are much simpler than the Viterbi detector while their performance is maintained close to optimum, mainly through the inclusion (ahead of the detectors) of the adaptive linear filter, which forms the first part of the corresponding optimum non-linear equalizer. Clearly, if the near-maximum-likelihood detector is to be used in the present application it must be capable of handling the severe time-varying distortion introduced by the HF channel into the transmitted signal without relying on the adaptive linear filter, which, as before, may be very difficult to hold correctly adjusted. Basic changes in the method of operation of the detector are now required to compensate for the absence of the adaptive filter; (114) and the appropriate development of the resulting system has led to a basic detection process suitable for use over HF radio links at a transmission rate of 2400 bit/s. (51,115) For correct detection to be achieved here, the receiver only has to estimate the sampled impulse response of the channel, and this can be achieved more rapidly and more accurately than can the adjustment of the adaptive linear filter. (92,121-122) The arrangement is therefore potentially better suited, than is that with the adaptive filter, to applications of
digital data transmission over HF radio links.

The detection process used successfully at a transmission rate of 2400 bit/s with a 4-level QAM signal over the given HF radio link\(^{(51,115)}\) has been tested at the much higher transmission rate of 9600 bit/s, with a 16-level QAM signal, but satisfactory operation is no longer obtained regardless of the adjustment of the system parameters. Tests with the same basic detection process, but now using a 4-level QAM signal at 9600 bit/s, have again failed, the performance here being in fact significantly poorer than that of the previous system. It has therefore been found necessary to carry out some fundamental changes in the method of operation of the detector, leading to a more sophisticated detection process.

In this section, a model of an HF channel is presented and the new near-maximum likelihood detector is described. Results of computer simulation tests on this detector, when operating on a 9600 bit/s data signal that has been transmitted over the given HF channel model, are given in both cases, when the detector assumes perfect knowledge about the channel sampled impulse response and when this is estimated using a recently developed estimator\(^{(183)}\) which is designed to operate on the given channel model. A 16-level QAM signal is used throughout the tests at an element rate of 2400 bauds.

7.2. MODEL OF THE HF RADIO CHANNEL

Many HF links show "Rayleigh fading" characteristics\(^{(41)}\) in that an unmodulated carrier is received with a Rayleigh distribution in amplitude and a Gaussian distribution in power spectra. A non-fading (specular)
component may also be received, giving a Rician amplitude distribution.\(^{41}\)

The rapidity of fading is usually expressed in terms of the fading rate, which is defined, for a single carrier, as the average number of downward crossings per second of the envelope through its median value.\(^{180}\) Fading rates of up to 10 fades per second may be encountered,\(^{42}\) but most often they are in the range 4-15 fades per minute.\(^{23}\)

A signal transmitted over an HF radio link may be received through two or more paths, each with Rayleigh fading properties and all of roughly equal attenuation but differing appreciably in their arrival times at the receiver. The differential time delays may exceed 6ms, but most often are less than 3ms.\(^{23,34,36,180}\)

The variations in the effective height of the ionospheric layers introduce Doppler shifts in each of the multipath components. The combination of these with the small frequency offset introduced by the radio equipment results in a frequency offset in the received voice-frequency signal, which varies slowly with time and may have values up to 30 Hz,\(^{40}\) but usually has the value in the range 0 to ±5Hz.\(^{23}\)

To compare the performance of two or more systems over an actual HF channel, they must be tested simultaneously, because propagation and channel conditions vary randomly and cannot be accurately repeated at other times.\(^{179}\) Therefore, a channel simulator,\(^{163-182}\) when it is valid, has the advantages of accuracy, regularity of performance, repeatability, availability of a large number of channel conditions and lower cost;\(^{179}\) when used in place of the HF channel in comparing different systems. Channel simulators are usually designed to provide channel models with up to five independent propagation paths,\(^{179,181}\) each having its own parameters such as the differential time delay, attenuation, frequency shift and frequency spread.
In a single propagation path (i.e. one sky-wave), the depth of a fade may sometimes be so large as to constitute the effective total loss of signal, but the probability of such a deep fade is lower in the presence of two sky-waves (propagation paths), because it requires the simultaneous occurrence of deep fades in the two sky-waves. Thus, as the number of the independent sky-waves increases, so the probability of the signal as a whole being in a deep fade decreases. When a detector is designed to detect digital data signals transmitted over HF radio channels, it must be capable of handling those fading conditions under which the instantaneous signal-to-noise ratio becomes very low. Clearly, when the detector operates satisfactorily over those channels having more frequent deep fades, regardless of the fading rate, it should give a satisfactory performance on those channels having less frequent deep fades, as long as the signal spread in time is within given limits. From this standpoint, a detector operating satisfactorily over a two sky-waves model is expected to be capable of handling more than two sky-waves, and therefore, an HF channel simulator (122,185) with two independent sky-waves has been used here in developing the detector.

Fig. 7.1 shows a model of a Rayleigh fading sky-wave. For an input signal x(t), the output signal z(t) is given by

\[ z(t) = x(t)q_1(t) + \hat{x}(t)q_2(t) \]  

(7.1)

where \( \hat{x}(t) \) is the Hilbert transform of \( x(t) \) and \( q_1(t) \) and \( q_2(t) \) are two random processes which meet the following requirements:

1. Each must possess a zero-mean Gaussian probability density function with the same variance.
2. Each must possess a power spectrum that is Gaussian in shape and with the same r.m.s. frequency (Fig. 7.2), i.e.,
\[ x(t) \]

**INPUT SIGNAL**

\[ q_1(t) \]

**HILBERT TRANSFORM**

\[ q_2(t) \]

**OUTPUT SIGNAL**

\[ z(t) \]

**FIGURE 7.1:** Model of Rayleigh-fading Sky-wave

\[ |Q_1(f)|^2 = |Q_2(f)|^2 \]

**FIGURE 7.2:** Power Spectrum of \( q_1(t) \) and \( q_2(t) \)
\[ |Q_1(f)|^2 = |Q_2(f)|^2 = \exp(- \frac{f^2}{2f_{\text{r.m.s.}}^2}) \] (7.2)

where \( Q_1(f) \) and \( Q_2(f) \) are the spectra of \( q_1(t) \) and \( q_2(t) \), respectively, and \( f_{\text{r.m.s.}} \) is the r.m.s. frequency and is given by

\[ f_{\text{r.m.s.}} = \frac{\text{fading rate}}{1.475} \] (7.3)

3. \( q_1(t) \) and \( q_2(t) \) are uncorrelated.

The frequency spread, \( f_{\text{sp}} \), introduced by each of \( q_1(t) \) and \( q_2(t) \) into a single-carrier waveform is given by

\[ f_{\text{sp}} = 2f_{\text{r.m.s.}} = 1.356 \times \text{fading rate} \] (7.4)

which means that 1 Hz frequency spread is equivalent to 44.2 fades per minute.

To generate \( q_1(t) \) and \( q_2(t) \), two independent real-valued white Gaussian noise processes \( n_1(t) \) and \( n_2(t) \), both with zero mean and unity two-sided power spectral density, may be fed to two linear filters, where the square of the absolute value of the transfer function of each filter is given by eqn. 7.2. Let the impulse response of the filter be \( \gamma(t) \) with the Fourier transform \( \Gamma(f) \), then,

\[ \Gamma(f) = \exp(- \frac{f^2}{4f_{\text{r.m.s.}}^2}) \] (7.5)

whose inverse Fourier transform \( \gamma(t) \) is given by

\[ \gamma(t) = \frac{1}{\sqrt{2\pi t_1}} \exp(- \frac{t^2}{2t_1^2}) \] (7.6)

where

\[ t_1 = \frac{1}{2\sqrt{2\pi f_{\text{r.m.s.}}}} \] (7.7)

The output signals of the two filters represent \( q_1(t) \) and \( q_2(t) \) which
clearly satisfy the required properties. From eqn. 7.5, the cut-off
(-3 db) frequency of the filter is,

\[ f_3 = 1.17741 \text{ f}_{\text{r.m.s.}} \]
or, from eqn. 7.4,

\[ f_3 = 0.588705 \text{ f}_{\text{sp}} \quad (7.8) \]

The filter characterized by eqns. 7.5 and 7.6 may be approximated
by a Bessel filter, the transfer function of which in the s-plane is

\[ H(s) = \frac{d_0}{L \sum_{k=0}^{\infty} d_k s^k} \tag{7.9} \]

where \( L \) is the filter order, \( s \) is the Laplacian variable and

\[ d_k = \frac{(2L-k)!}{2^{n-L} k!(L-k)!} \tag{7.10} \]

Both the frequency and impulse responses of the Bessel filter tend
towards Gaussian as the filter order is increased. \( (194) \)

As a practical choice, the filter has been chosen to be of the fifth
order, i.e. \( L=5 \). Hence, eqn. 7.9 becomes

\[ H(s) = \frac{945}{s^5 + 15s^4 + 105s^3 + 420s^2 + 945s + 945} \tag{7.11} \]

where \( d_0 \) in eqn. 7.9 is chosen such that \( H(0)=1 \). The poles of \( H(s) \) in
eqn. 7.11 are \( (189) \)

\[
\begin{align*}
p_1 &= -3.64674 \pm j 0.0 \\
p_{2,3} &= -3.35196 \pm j 1.74266 \\
p_{4,5} &= -2.32467 \pm j 3.57102
\end{align*}
\]

so that eqn. 7.11 may be written as,

\[ H(s) = \frac{945}{\prod_{i=1}^{5} (s-p_i)} \quad (7.13) \]
Substituting $s$ by $j\Omega$, where $\Omega$ is the angular frequency and $j = \sqrt{-1}$, eqn. 7.13 gives the frequency response of the filter as,

$$H(\Omega) = \frac{945}{5 \prod_{i=1}^{5} (j\Omega - p_i^i)}$$ \hspace{1cm} (7.14)

whose cut-off (-3 db) angular frequency is,

$$\Omega_3 = 2.4274 \text{ rad/s}$$ \hspace{1cm} (7.15)

In order to be able to change the cut-off frequency of the filter, let $\omega$ be the new angular frequency variable such that

$$\omega = c\Omega$$ \hspace{1cm} (7.16)

where

$$c = \frac{\omega_3}{\Omega_3} = \frac{2\pi f_3}{\Omega_3}$$ \hspace{1cm} (7.17)

and $f_3$ is the required cut-off frequency. Substituting for $\Omega_3$ from eqn. 7.15, eqn. 7.17, becomes,

$$c = 2.58844 f_3$$ \hspace{1cm} (7.18)

Now, from eqns. 7.14 and 7.16, the frequency response of the wanted filter is,

$$H(\omega) = \frac{945c^5}{5 \prod_{i=1}^{5} (j\omega - p_i^i)}$$ \hspace{1cm} (7.19)

where the $\{p_i^i\}$ are the poles of the wanted filter and are given by,

$$p_i^i = cp_i = 2.58844 f_3 p_i$$ \hspace{1cm} (7.20)

Redefining the $s$-plane such that $j\omega = s$, and substituting the value of $c$ in eqn. 7.19 give

$$H(s) = \frac{d_0'}{5 \prod_{i=1}^{5} (s - p_i^i)}$$ \hspace{1cm} (7.21)

where

$$d_0' = 109805.05 f_3^5$$ \hspace{1cm} (7.22)
In the HF channel model used later to study the detector, the frequency spread has been assumed to have one of the three possible values 0.5, 1.0 and 2.0 Hz. From eqn. 7.8, the corresponding cut-off frequencies are 0.2943, 0.5887 and 1.1774 Hz, respectively. The values of \( d'_0 \) and the poles \( \{ p'_i \} \) in eqn. 7.21 are given in Table 7.1 for the given three values of \( f_3 \), where eqns. 7.12, 7.20 and 7.22 have been used.

The actual tests have been carried out by computer simulation which required the filter to be implemented digitally. Using the impulse-invariance technique, the transfer function \( H(s) \) in eqn. 7.21 becomes in the \( z \)-plane

\[
H(z) = \frac{G_0}{\prod_{i=1}^{5} (1-A_i z^{-1})}
\]

(7.23)

where \( G_0 \) is a constant and the \( \{ A_i \} \) are the poles of \( H(z) \) in the \( z \)-plane corresponding to the \( \{ p'_i \} \). The values of \( G_0 \) and the \( \{ A_i \} \) are given in Table 7.2 for the three given values of the cut-off frequency. These values assume a sampling rate of 50 samples/second, i.e. the sequence at the filter output is equivalent to the sequence obtained by sampling the waveform at the output of the original filter at 50 samples/second. In general, there is no direct one-to-one mapping of the \( \{ p'_i \} \) into the \( \{ A_i \} \), but since \( \Gamma(f) \) (eqn. 7.5) has a value less than \(-5000\) db at 50 Hz, for the given values of \( f_{sp} \), the used transformation technique degenerated into the simple mapping \( A_i = e^{p'_i T'} \), where \( T' = 1/50 \) second.

The actual required sampling rate is 4800 samples/second, but at this rate, the \( \{ A_i \} \) need to be expressed by very high accuracy or otherwise the filter characteristics may be seriously in error. To obtain the required sampling rate, the discrete-time signal at the output of the filter is oversampled by interpolation.
TABLE 7.1: Poles of the 5th Order Analogue Bessel Filter for the Different Values of the Frequency Spread

<table>
<thead>
<tr>
<th>$f_{sp}$ [Hz]</th>
<th>$f_3$ [Hz]</th>
<th>$d_0$</th>
<th>$p_1^1$</th>
<th>$p_2^1p_3^1$</th>
<th>$p_4^1p_5^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.2943</td>
<td>242.42</td>
<td>-2.7785</td>
<td>-2.554±j1.3277</td>
<td>-1.7712±j2.7208</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5887</td>
<td>7764.124</td>
<td>-5.557</td>
<td>-5.1078±j2.6555</td>
<td>-3.5424±j5.44</td>
</tr>
<tr>
<td>2.0</td>
<td>1.1774</td>
<td>248451.98</td>
<td>-11.114</td>
<td>-10.2156±j5.311</td>
<td>-7.0848±j10.883</td>
</tr>
</tbody>
</table>

TABLE 7.2: Poles of the 5th Order Digital Bessel Filter for the Different Values of the Frequency Spread

<table>
<thead>
<tr>
<th>$f_{sp}$ [Hz]</th>
<th>$c_0^{-1}$</th>
<th>$A_1$</th>
<th>$A_2,A_3$</th>
<th>$A_4,A_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>325623.4</td>
<td>0.9460</td>
<td>0.9500±j0.0252</td>
<td>0.9638±j0.0524</td>
</tr>
<tr>
<td>1.0</td>
<td>16121.6</td>
<td>0.8950</td>
<td>0.9018±j0.0479</td>
<td>0.9262±j0.1010</td>
</tr>
<tr>
<td>2.0</td>
<td>893.06</td>
<td>0.8010</td>
<td>0.8109±j0.0863</td>
<td>0.8477±j0.1872</td>
</tr>
</tbody>
</table>

TABLE 7.3: The Coefficients of the Digital Filter in Fig. 7.3

<table>
<thead>
<tr>
<th>$f_{sp}$ [Hz]</th>
<th>$c_0^{-1}$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$c_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>325623.4</td>
<td>0.9460</td>
<td>1.9000</td>
<td>0.903135</td>
<td>1.9276</td>
<td>0.9316562</td>
</tr>
<tr>
<td>1.0</td>
<td>16121.6</td>
<td>0.8950</td>
<td>1.8036</td>
<td>0.8155376</td>
<td>1.8524</td>
<td>0.8680474</td>
</tr>
<tr>
<td>2.0</td>
<td>893.06</td>
<td>0.8010</td>
<td>1.6218</td>
<td>0.6650064</td>
<td>1.6954</td>
<td>0.7536391</td>
</tr>
</tbody>
</table>
Now, the discrete-time random process \( \{q_1\} \) at the output of the
digital filter (and after having been oversampled to a rate of 4800
sample/second) satisfies the required properties of each of \( q_1(t) \) and
\( q_2(t) \) (eqn. 7.2) over the frequency band \(-2400\) to \(2400\) Hz. The filter is
here fed at its input by a sequence of statistically independent Gaussian
random variables with zero mean and a fixed variance, and in Appendix F,
it is shown that such a sequence represents a bandlimited white Gaussian
noise waveform which has the required power density over the passband and
which is sampled at the Nyquist rate. For reasons to be given later, \( \sigma_0 \)
in eqn. 7.23 and Table 7.2, is arranged such that the variance of the \( \{q_1\} \)
is given by
\[
\sigma_q^2 = 0.25 \quad (7.24)
\]
when the variance of the input sequence is 1.

Fig. 7.3 shows the implementation of the filter as three sections in
cascade. The coefficients \( \{c_i\} \) are given by,
\[
\begin{align*}
c_1 &= A_1 \\
c_2 &= A_2 + A_3 \\
c_3 &= A_2 A_3 \\
c_4 &= A_4 + A_5 \\
c_5 &= A_4 A_5
\end{align*}
\]
(7.25)
and their values are given in Table 7.3. The sequence \( \{n_1\} \) in Fig. 7.3 is
obtained from a random number generator which gives a sequence of
statistically independent Gaussian variables with zero mean and variance 1.0.

As shown in Fig. 7.1, a Rayleigh fading sky-wave model needs the
two signals \( q_1(t) \) and \( q_2(t) \). The corresponding sequences \( \{q_{1,1}\} \) and \( \{q_{2,1}\} \)
are generated exactly in the same manner as the \( \{q_1\} \) in Fig. 7.3, where
a different sequence \( \{n_1\} \) is used at each filter input.

As mentioned earlier, the HF radio link is assumed here to be
formed of two independently Rayleigh fading sky-waves, the difference
INPUT SEQUENCE of statistically independent Gaussian random variables with zero mean and variance 1.0

\[ T_f = \frac{1}{50} \text{ second} \]

\[ \{n_1\} \]

\[ G_0 \text{ and } c_1 \text{ to } c_5 \text{ are given in Table 7.3} \]

**FIGURE 7.3:** 5th Order Digital Bessel Filter for Generating the \{q_1\}
between the arrival times at the receiver being \( \tau \) seconds. The corresponding model of the HF radio link is shown in Fig. 7.4. The two signals \( q_1(t) \) and \( q_2(t) \) give the first sky-wave and the two signals, \( q_3(t) \) and \( q_4(t) \) give the second sky-waves. \( q_3(t) \) and \( q_4(t) \) are statistically independent of \( q_1(t) \) and \( q_2(t) \) but possess the same properties. When the signal at the input of the HF link is \( x(t) \), then from Fig. 7.4, the output signal \( z(t) \) is

\[
z(t) = x(t)q_1(t) + \hat{x}(t)q_2(t) + x(t-\tau)q_3(t) + \hat{x}(t-\tau)q_4(t)
\]

where \( \hat{x}(t) \) is the Hilbert transform of \( x(t) \).

7.3 THE TRANSMISSION OF QAM DATA SIGNALS OVER THE HF LINK MODEL

Fig. 7.5 shows a model of a serial synchronous data transmission system in which a QAM signal is transmitted over a model of an HF channel with two independently Rayleigh fading sky-waves. The differential time delay of the two sky-waves is \( \tau \) seconds (Fig. 7.4). The input signal to the system is the stream of data elements \( \sum_i s_i \delta(t-i\tau) \), where \( s_i \) is a scalar and may have one of finite number of complex values, and \( \frac{1}{T} \) is the signal-element rate in bauds. The input filter \( A' \) has the real-valued impulse response \( a'(t) \) and its function is to shape the transmitted signal spectrum so that it appropriately fits the voice-frequency band of the HF channel, after having quadrature amplitude modulated a carrier whose frequency is \( f_c \). The transfer function \( A'(f) \) of filter \( A' \), is assumed here such that

\[
A'(f) = 0 \text{ for } f < -f_c + kf_{sp} \text{ and } f > f_c - kf_{sp}
\]

where \( f_{sp} \) is the largest value of the frequency spread expected to be introduced by the HF channel into the transmitted QAM signal, and \( k \) is
**FIGURE 7.4:** Model of HF Radio Link with Two Rayleigh-fading Sky-waves
FIGURE 7.5: Model of the QAM Data Transmission System Over an HF Radio Link
an integer the value of which will be discussed later. The QAM signal 
\( x_2(t) \) is the real part of the complex signal \( x_1(t) \) at the output of the
linear modulator and is fed to the transmitter filter \( G \) of the radio
equipment. The impulse response of this filter is \( g(t) \) and has a transfer
function \( G(f) \). The radio transmitter uses single sideband suppressed
carrier amplitude modulation (SSS-AM) to shift the voice-band spectrum to
the HF band, whereas the radio receiver linearly demodulates the received
signal to return its spectrum back to the voice-band. The two processes
of linear modulation and demodulation ideally do not introduce into the
signal any distortion\(^{(23)}\) apart from that introduced by the radio
equipment filters and the HF transmission path itself. Therefore these
two processes are not shown in Fig. 7.5. Instead, the baseband model of
the HF channel in the voice-band is represented by the inner dashed box.
The radio-receiver filter \( D \) has the impulse response \( d(t) \) and the voice-
band transfer function \( D(f) \). The different types of additive noise
likely to be introduced by the HF link into the signal are neglected and
the only additive noise assumed here is a white Gaussian noise \( n(t) \) with
zero mean and two-sided power spectral density of \( \frac{1}{4}N_0 \), which is added to
the signal at the output of the HF radio link.\(^{(51,115)}\) The two-Rayleigh-
fading-sky-wave model is exactly as that given in Fig. 7.4. The receiver
filter of the data modem (Filter C) has bandpass characteristics and
removes the noise frequencies outside the data signal-band without
excessively distorting that signal. The impulse response of this filter
is \( c(t) \). The noisy, Rayleigh-fading and distorted QAM signal at the
output of filter \( C \) is coherently demodulated through multiplications by
the complex-valued reference signal \( \sqrt{2} e^{-j2\pi f_c t} \), where \( f_c \) is equal to the
average instantaneous carrier frequency of the received signal, thus
eliminating any constant frequency offset may be present in the received QAM signal, but not tracking the small variations in the signal carrier frequency introduced by the HF radio link.\(^{(51,115)}\) Clearly, this is equivalent to the assumption that there is no constant frequency offset in the received signal and therefore
\[
f' = f_c
\]
the case which will be assumed here. The complex-valued demodulated signal is now lowpass filtered by filter B' where the high frequency components of the signal are removed. The impulse response of filter B' is \(b'(t)\) whose Fourier transform \(B'(f)\) is bandlimited such that
\[
B'(f) = 0 \quad \text{for} \quad |f| > f_c
\]
From Fig. 7.5, the real-valued QAM signal at the output of the radio-transmitter filter \(G\) is,
\[
x(t) = \text{Re}\left[\sqrt{2} \sum_i s_i a'(t-iT)e^{j2\pi f_c t} \right] \ast g(t)
\]
or equivalently,
\[
x(t) = \frac{1}{\sqrt{2}} \left[ \sum_i s_i a(t-iT)e^{j2\pi f_c t} + s_i^* a^*(t-iT)e^{-j2\pi f_c t} \right]
\]
where \(s_i^*\) and \(a^*(t)\) are the complex conjugates of \(s_i\) and \(a(t)\), respectively, and (see eqn. 4.13),
\[
a(t-iT) = a'(t-iT) \ast (g(t)e^{-j2\pi f_c t})
\]
which represents the overall filtering carried out at the transmitting side on the baseband signal. The Hilbert transform of \(x(t)\) is given by
\[
\tilde{x}(t) = x(t) \ast f(t)
\]
where \(f(t)\) is the impulse response of the Hilbert transform whose Fourier transform is\(^{(186)}\)
\[ F(f) = \begin{cases} j & f < 0 \\ 0 & f = 0 \\ -j & f > 0 \end{cases} \quad (7.34) \]

From eqn. 7.31, eqn. 7.33 becomes,

\[ \hat{x}(t) = \frac{1}{\sqrt{2}} \left[ \sum_s a(t-iT)e^{j2\pi fc_t} \right] \ast f(t) + \sum_s a^*(t-iT)e^{-j2\pi fc_t} \ast f(t) \]  

which may be rewritten as (eqn. 4.13),

\[ \hat{x}(t) = \frac{1}{\sqrt{2}} \left[ \sum_s a(t-iT) \ast f(t)e^{-j2\pi fc_t}e^{j2\pi fc_t} + \sum_s a^*(t-iT) \ast f(t)e^{j2\pi fc_t}e^{-j2\pi fc_t} \right] \quad (7.36) \]

Now, according to eqn. 7.32, \( A(f) \), the Fourier transform of \( a(t) \), is band-limited to the frequency band of \( A'(f) \) (eqn. 7.27). But, the Fourier transform of \( f(t)e^{-j2\pi fc_t} \), i.e. \( F(f+c) \), has the value \(-j\) over the frequency band \(-c\) to \( c\), whereas the Fourier transform of \( f(t)e^{j2\pi fc_t} \), i.e. \( F(f-c) \), has the value \( j\) over the same frequency band, as shown in Fig. 7.6. Consequently, by taking the Fourier transform of \( \hat{x}(t) \), replacing \( F(f+c) \) and \( F(f-c) \) by their corresponding values over the frequency band of \( A(f), (-c, c) \), and taking then the inverse Fourier transform, eqn. 7.36 becomes,

\[ \hat{x}(t) = \frac{1}{\sqrt{2}} \left[ \sum_s (-j)a(t-iT)e^{j2\pi fc_t} + j a^*(t-iT)e^{-j2\pi fc_t} \right] \quad (7.37) \]

From Fig. 7.4 and 7.5, the output signal of the two-Rayleigh-fading-sky-wave model is given by

\[ z(t) = x(t)q_1(t) + \hat{x}(t)q_2(t) + x(t-\tau)q_3(t) + \hat{x}(t-\tau)q_4(t) \quad (7.38) \]

where \( x(t) \) and \( \hat{x}(t) \) are given by eqns. 7.31 and 7.37 respectively.

Thus, the signal at the input to the radio-receiver filter (Fig. 7.5) is
FIGURE 7.6: Frequency Responses Involved in Eqn. 7.36
\[ z(t) = \frac{1}{\sqrt{2}} \left[ \sum s_i a(t-iT)(q_1(t)-jq_2(t))e^{j2\pi f_c t} + s_i^* a^*(t-iT)(q_1(t)+jq_2(t))e^{-j2\pi f_c t} + s_i a(t-iT)(q_3(t)-jq_4(t))e^{j2\pi f_c (t-t)} + s_i^* a^*(t-iT)(q_3(t)+jq_4(t))e^{-j2\pi f_c (t-t)} \right] \quad (7.39) \]

or equivalently,

\[ z(t) = \frac{1}{\sqrt{2}} \left[ \sum s_i h_i(t-iT)e^{j2\pi f_c t} + s_i^* h_i^*(t-iT)e^{-j2\pi f_c t} \right] \quad (7.40) \]

where,

\[ h_i(t-iT) = a(t-iT)(q_1(t)-jq_2(t)) + a(t-iT)(q_3(t)-jq_4(t))e^{-j2\pi f_c t} \]

When \( T \) is constant, as will be assumed here, the factor \( e^{-j2\pi f_c t} \) is a complex-valued scalar with absolute value 1.0, and since \( q_3(t) \) and \( q_4(t) \) are statistically independent with zero mean, it has no effect on the statistical properties of \( [(q_3(t)-jq_4(t))e^{-j2\pi f_c t}] \) nor does it affect the power spectrum of this signal. Consequently, \( h_i(t-iT) \) may be written as

\[ h_i(t-iT) = a(t-iT)(q_1(t)-jq_2(t)) + a(t-iT)(q_3(t)-jq_4(t)) \quad (7.41) \]

The signal at the output of the linear demodulator in Fig. 7.5 is

\[ r(t) = \sqrt{2} \left\{ z(t) * d(t) * c(t) \right\} e^{-j2\pi f_c t} * b'(t) + \sqrt{2} \left\{ n(t) * c(t) \right\} e^{-j2\pi f_c t} * b'(t) \quad (7.42) \]

which may be written as (eqn. 4.13)

\[ r(t) = \sqrt{2} \left[ z(t)e^{-j2\pi f_c t} \right] * b(t) + u(t) \quad (7.43) \]

where

\[ b(t) = \left\{ [d(t) * c(t)]e^{-j2\pi f_c t} \right\} * b'(t) \quad (7.44) \]

which represents the overall filtering carried out on the signal at the receiving end, and

\[ u(t) = \sqrt{2} \left\{ n(t) * c(t) \right\} e^{-j2\pi f_c t} * b'(t) \quad (7.45) \]
which represents the additive Gaussian noise component in $r(t)$. Taking into account that $f'_c = f_c$ (eqn. 7.28), eqns. 7.40 and 7.43 give

$$r(t) = \sum_i [s_i h_i(t-iT) + s_i^* h_i^*(t-iT)e^{-j4\pi f_c t}] * b(t) + u(t)$$

(7.46)

Now, $h_i(t-iT)$ (eqn. 7.41) consists of the time-invariant impulse response $a(t)$ and the random components $q_1(t)$ to $q_4(t)$. As mentioned before, each of these random components has a Gaussianly-shaped power spectral density the r.m.s. frequency of which is of the order of a few Hz, and which extends, according to eqn. 7.2, over all frequencies. But since this power density decreases very sharply when $f$ increases (according to eqn. 7.5, the r.m.s. frequency is attenuated by 2.2 db relative to the d.c. component, and when $f=10 f_{r.m.s.}$, the attenuation becomes 217 db), the frequency response of $h_i(t-iT)$ may be considered to be strictly bandlimited and given by the Fourier transform of $a(t)$ dispersed both upwards and downwards by less than $10 f_{r.m.s.}$. Accordingly to eqn. 7.32, the Fourier transform of $a(t)$ is bandlimited to the frequency band of $A'(f)$, the transfer function of $a'(t)$, which is defined by eqn. 7.27. Thus, if $k$ in eqn. 7.27 is equal $5 (f_{sp} = 2f_{r.m.s.}$, eqn. 7.4), then the frequency response of $h_i(t-iT)$ will be strictly band-limited such that,

$$|H(f)| = 0 \quad |f| > f_c$$

Hence, the Fourier transform of $h_i^*(t-iT)e^{-j4\pi f_c t}$ in eqn. 7.46 will be outside the passband of the lowpass filter whose impulse response is $b(t)$ (the Fourier transform of $b(t)$ is zero for $|f| > f_c$ according to eqns. 7.29 and 7.44) and therefore it is blocked by this lowpass filter. Thus, eqn. 7.46 becomes,

$$r(t) = \sum s_i y_i(t-iT) + u(t)$$

(7.47)

where
\[ y_i(t-iT) = h_i(t-iT) \ast b(t) \]

or, from eqn. 7.41,

\[ y_i(t-iT) = a(t-iT)[q_1(t)-j\,q_2(t)] + a(t-r-iT)[q_3(t)-j\,q_4(t)] \ast b(t) \]

which is the time-varying impulse response of the linear baseband channel in Fig. 7.5.

Eqn. 7.47 represents the baseband model of the QAM system over the HF radio link. This model is shown in Fig. 7.7. The impulse response \( a(t) \) of the overall equivalent transmitter filter is given by eqn. 7.32 and the impulse response \( b(t) \) of the overall equivalent receiver filter is given by eqn. 7.44. The noise component in \( r(t), u(t) \) is given by eqn. 7.45, and it is a complex-valued Gaussian random process whose autocorrelation function \( R_u(\tau) \) is shown in Appendix A to be given by,

\[ R_u(\tau) = N_0 \int_{-f_c}^{f_c} |C(f+f_c)|^2 \left| B'(f) \right|^2 e^{j2\pi f \tau} df \]

(7.49)

where the integration limits are given for \( B'(f) \) (eqn. 7.29). \( C(f) \) is the transfer function of the bandpass filter \( C \) (Fig. 7.5). Of course, \( C(f) \) has the same bandwidth as the spectrum of the transmitted QAM signal which approximately extends over the frequency band \(-2f_c\) to \(2f_c\), as can be seen from eqns. 7.27, 7.31 and 7.32.

It is shown in Appendix A that the autocorrelation functions of each of the real and imaginary parts of \( u(t) \) are given by \( \text{Re} \left[ R_u(\tau) \right] \), whereas the imaginary part of \( R_u(\tau) \) expresses the cross-correlation between the two parts of \( u(t) \), and when \( R_u(\tau) \) is purely real, then the real and imaginary parts of \( u(t) \) become uncorrelated. This can be achieved if \( C(f) \) (over positive frequencies) is symmetric about \( f_c \). The
FIGURE 7.7: Baseband Model of the QAM System Over the HF Radio Link
variance of \( u(t) \) is equal to \( R_u(0) \), and is given by

\[
\sigma_u^2 = N_0 \int_{-f_c}^{f_c} |C(f+f_c)|^2 |B'(f)|^2 df
\]  
(7.50)

The average transmitted energy per signal element at the output of the transmitter filter whose impulse response is \( a(t) \) (Fig. 7.7), is

\[
E_1 = E\left[ \int_{-\infty}^{\infty} |s_i a(t-it)|^2 dt \right]
= \frac{2}{s_i} \int_{-\infty}^{\infty} |A(f)|^2 df
\]  
(7.51)

where Parseval's theorem is used, \( E[.] \) denotes expected value and

\[
\frac{2}{s_i^2} = E[|s_i|^2]
\]  
(7.52)

The average energy per signal element at the input of the receiver filter whose impulse response is \( b(t) \) (Fig. 7.7), is

\[
E_2 = E\left[ \int_{-\infty}^{\infty} |s_i [a(t-it)(q_1(t)-jq_2(t))a(t-\tau-it)(q_3(t)-jq_4(t))]|^2 dt \right]
= \frac{2}{s_i^2(q_1(t)+q_2(t)+q_3(t)+q_4(t))} \int_{-\infty}^{\infty} |A(f)|^2 df
\]  
(7.53)

where \( q_1(t), q_2(t), q_3(t) \) and \( q_4(t) \) are the variances of \( q_1(t), q_2(t), q_3(t) \) and \( q_4(t) \) respectively. In the model of the HF channel assumed here, the four variances are equal and each is given by eqn. 7.24, i.e. the value of each is 0.25. This choice enables \( E_2 \) (eqn. 7.53) to have the same value as \( E_1 \) (eqn. 7.51). In other words, the two sky-waves do not introduce, on average, any gain or attenuation into the transmitted signal. (This assumption simplifies the calculation of the signal-to-noise ratio in the computer simulation tests).
7.4 MODEL OF DATA TRANSMISSION SYSTEM USED IN THE TESTS

7.4.1 Basic Assumptions

The data transmission system is a synchronous serial system, with a 16-level QAM signal and an adaptive detection process, as shown in Fig. 7.8, where the baseband model of the QAM system in Fig. 7.7 is used here. The information to be transmitted is a sequence of binary digits \( \{a_k\} \), where,

\[
a_k = 0 \text{ or } 1
\]

(7.54)

the \( \{a_k\} \) being statistically independent and equally likely to have either binary value. The \( \{a_k\} \) are fed, at a rate of 9600 bit/s, to the encoder which gives at its output the corresponding sequence of data-symbols \( \{s_i\} \) at a rate of 2400 symbol/s, where,

\[
s_i = s_{1,i} + js_{2,i}
\]

(7.55)

and \( s_{1,i} = \pm 1 \) or \( \pm 3 \) and \( s_{2,i} = \pm 1 \) or \( \pm 3 \). Thus, the \( s_{1,i} \) and \( s_{2,i} \), which are the real and imaginary parts of \( s_i \), are 4-level data symbols whereas \( s_i \) is a 16-level data symbol. It is assumed that \( s_i = 0 \) for \( i \leq 0 \), so that \( s_i \) is the \( i^{th} \) transmitted data symbol. The \( \{s_i\} \), for \( i > 0 \), are statistically independent and equally likely to have any of their 16 possible values.

The details of the encoder which converts the \( \{a_k\} \) into the \( \{s_i\} \) will be given later on.

The data symbols \( \{s_i\} \) are fed to a model of the QAM system over the HF radio link. The impulse response of the linear baseband channel is time-varying, and at time \( t = iT \) is given by

\[
y_i(t-iT) = \{a(t-iT)[q_1(t)-jq_2(t)] + a(t-T-iT)[q_3(t)-jq_4(t)]\}*b(t)
\]

(7.56)

where

\[
a(t) = a'(t) * (g(t)e^{-j2\pi f_c t})
\]

(7.57)

and

\[
b(t) = \{[d(t) * c(t)]e^{-j2\pi f_c t}\}*b'(t)
\]

(7.58)
FIGURE 7.8: Model of Data Transmission System Used in the Tests
a(t) is the impulse response of the transmitter filter A and b(t) is the impulse response of the receiver filter B (Fig. 7.8). \( a'(t), g(t), d(t), c(t) \) and \( b'(t) \) are the impulse responses of filters \( A', G, D, C \) and \( B' \), respectively (shown in the original model of the QAM system, Fig. 7.5). Eqns. 7.56, 7.57 and 7.58 have been previously given as eqns. 7.48, 7.32, and 7.44, respectively (see eqn. 7.28). The carrier frequency of the QAM signal has the value

\[
f_c = 1800 \text{ Hz} \tag{7.59}
\]

The demodulated baseband signal \( r(t) \) at the output of the QAM system model (Fig. 7.8) comprises the stream of data elements \( \{s_i y_i(t-iT)\} \) to which is added the stationary zero-mean complex-valued baseband Gaussian noise waveform \( u(t) \) originating from the real-valued white Gaussian noise waveform \( n(t) \), as given by eqn. 7.45. The waveform \( r(t) \) is sampled, once per data-symbol \( s_i \), at the time instants \( iT \). The sampling rate is assumed to be correct so that there is no gain or loss in the sampling phase over one transmission, but the sampling phase itself is selected at random. The delay in transmission is, for convenience, taken to be such that the first potentially non-zero sample of a received signal-element arrives with no delay.

The complex-valued sample of \( r(t) \) at time \( t=iT \) is now

\[
r_i = \sum_{h=0}^{L} s_{i-h} y_i,h + u_i \tag{7.60}
\]

where

\[
y_{i,h} = y_{i-h}(hT) \tag{7.61}
\]

\[
u_i = u(iT) \tag{7.62}
\]

and \( y_{i,h} = 0 \) for \( h<0 \) and \( h>g \), for practical purposes. The sequence of complex values given by the vector

\[
Y_i = y_{i,0}, y_{i,1}, \ldots, y_{i,g} \tag{7.63}
\]
is taken to be the sampled impulse response of the linear baseband channel in Fig. 7.8 at time $t=iT$. In the computer simulation tests, the vector $Y_i$ is obtained by sampling the \{y_i(t-iT)\} at a rate of 2400 samples/second. Since the process is performed in the discrete-time domain, and to avoid any aliasing likely to occur when any of the $q_1(t)$, $q_2(t)$, $q_3(t)$ or $q_4(t)$ is changing rapidly, the convolution in eqn. 7.56 is carried out with a sampling rate of 4800 samples/second, which is well above the Nyquist rate for filters A and B, whose characteristics are given in Section 7.4.3. The functions $a(t)$, $a(t-\tau)$ and $b(t)$ have been jointly sampled at three different random phases so that the tests do not assume any particular sampling phase. The functions $q_1(t)$, $q_2(t)$, $q_3(t)$ and $q_4(t)$ are generated as described in Section 7.2 (Fig. 7.3). Let the three sequences $A_1$, $A_2$ and $B$ represent the three impulse responses $a(t)$, $a(t-\tau)$ and $b(t)$, respectively, sampled at 4800 samples/second, i.e.,

$$A_1 = a_{1,0}, a_{1,1}, \ldots, a_{1,p}$$  \hfill (7.64)  

$$A_2 = a_{2,0}, a_{2,1}, \ldots, a_{2,p}$$  \hfill (7.65)  

and  

$$B = b_0, b_1, \ldots, b_p$$  \hfill (7.66)  

where  

$$a_{1,k} = a(k \frac{T}{2})$$  

$$a_{2,k} = a(k \frac{T}{2} - \tau)$$  

$$b_k = b(k \frac{T}{2})$$  \hfill (7.67)  

and $\frac{1}{T}$ is the data-symbol rate of 2400 symbols/second. For practical purposes, $a(t)=b(t)=0$ for $t<0$ and $t>(p+1-\rho')\frac{T}{2}$, $\rho'$ being an integer such that $\tau=\rho'\frac{T}{2} + \tau'$ and $\tau' < \frac{T}{2}$. This implies that $a_{1,k}=b_k=0$ for $k<0$ and $k>(\rho-\rho')$ and $a_{2,k}$ are for $k<\rho'$ and $k>\rho$. Clearly, the multipath propagation delay $\tau$ is expressed as a whole number of sampling periods $\rho'$ plus a sampling phase $\tau'$. Let also the four components $q_1(t)$, $q_2(t)$, $q_3(t)$ and $q_4(t)$ be
sampled at 4800 samples/second, and the corresponding resultant samples up to time instant \( t = iT \), be represented by the four sequences

\[
Q_{1,i} = q_{1,1} q_{1,2} \cdots q_{1,2i} \\
Q_{2,i} = q_{2,1} q_{2,2} \cdots q_{2,2i} \\
Q_{3,i} = q_{3,1} q_{3,2} \cdots q_{3,2i} \\
Q_{4,i} = q_{4,1} q_{4,2} \cdots q_{4,2i}
\]

(7.68)

and

\[
Q_{k,i} = q_{k,1} q_{k,2} \cdots q_{k,2i}
\]

where \( q_{k,h} = q_k \left( h \frac{T}{2} \right) \), \( k = 1, 2, 3 \) and 4.

Now, according to eqns. 7.56 and 7.61, the components of the vector \( Y_i \) (eqn. 7.63), at time \( t = iT \), are given, from eqns. 7.64 to 6.68, by the following expression

\[
y_{i,h} = \sum_{k=0}^{2h} a_{1,k} (q_{1,2(i-h)} + k) - jq_{2,2(i-h)+k} + a_{2,k} (q_{3,2(i-h)} + k) - jq_{4,2(i-h)+k} b_{2h-k}
\]

(7.69)

for \( h = 0, 1, 2, \ldots, p \), so that \( g \) in eqn. 7.63 is equal to \( p \) and therefore it is a function of \( T \). Notice that the \( \{y_{i,h}\} \) are given at a sampling rate of 2400 samples/second. In Appendix H it is shown that for \( y_{i,h} \) in eqn. 7.69 to be an exact representation of that in eqn. 7.61, it must be multiplied by the inverse of the sampling rate of the \( \{a_k\} \) and \( \{b_k\} \)

i.e. \( y_{i,h} \) in eqn. 7.69 must be multiplied by \( \frac{T}{2} \) to be equal to \( y_{i,h} \) in eqn. 7.61. However, when the value in eqn. 7.69 is used in the simulation, then this implies the multiplication of the signal by \( \frac{2}{T} \), and therefore the signal energy will be scaled by \( \frac{4}{T^2} \). In this case, and in order to have a correct definition of the signal-to-noise ratio, similar scaling must be introduced in the noise power. Therefore, eqn. 7.69 will be used here as it is and the scaling factor will be taken account of in the calculation of the signal-to-noise ratio (Section 7.4.4).
The detector in Fig. 7.8 operates on the received signal samples \( \{r_i\} \) to give at its output the sequence of data symbols \( \{s'_i\} \) which are the detected values of the transmitted data symbols \( \{s_i\} \). Here, the detector introduces a delay of \( n \) sampling periods (\( nT \) seconds) in detecting a data symbol, i.e. the data symbol \( s_i \) is detected on the receipt of the signal sample \( r_{i+n} \). Of course, the detector in practice has no prior knowledge of the sampled impulse responses \( \{Y_i\} \) of the channel (eqn. 7.63), and normally uses instead the predictions of these responses \( \{Y_{i,i-\ell-1}\} \), the latter being produced by the channel estimator (Fig. 7.8), which in turn uses the "early" detected data symbols \( \{s''_i\} \) and the corresponding received signal samples \( \{r_i\} \) to form the predictions. The delay in the early detection of the \( \{s_i\} \) is \( \lambda \), where \( \lambda \ll n \). Clearly, the early detected data symbols \( \{s''_i\} \) may be less reliable decisions than the \( \{s'_i\} \), because \( \lambda \ll n \) (see Sections 3.4 and 3.8 for the delay in detection).

But since the prediction error in the \( \{Y_{i,i-\ell-1}\} \) is dependent on the prediction interval (\( \lambda T \) seconds, Section 7.6) and since it becomes larger when the value of \( \lambda \) increases, the predictions \( \{Y_{i,i-\ell-1}\} \) are usually more accurate than the predictions \( \{Y_{i,i-n-1}\} \) that are produced using the detected data symbols \( \{s'_i\} \), where the prediction interval is \( n \) sampling intervals. The estimation and prediction techniques employed here are those given in Ref. (183) and are briefly considered in Section 7.6. The estimator aims normally to minimize the mean-square error

\[
\bar{e}^2_i = E[|Y_{i,i-\ell-1} - Y_i|^2]
\]  
(7.70)

over all \( \{i\} \).

The first series of tests on the detector (by computer simulation, of course) assume a perfect knowledge of the \( \{Y_i\} \) i.e. it assumes that
\[ Y_{i, i-k-1} = Y_i \]  \hspace{1cm} (7.71)

which implies in this case that \( e_i^2 = 0 \) for all \( \{i\} \).

The detected data symbols \( \{s_i'^{i-n}\} \) are fed into the decoder (Fig. 7.8) which gives at its output the corresponding binary digits \( \{a_k'\} \), where,

\[ a_k' = 0 \text{ or } 1 \]  \hspace{1cm} (7.72)

the \( \{a_k'\} \) being the detected values of the transmitted binary digits \( \{a_k\} \).

### 7.4.2 Differential Encoding and Decoding

In converting the \( \{a_k\} \) into the \( \{s_i\} \) by the encoder at the transmitter (Fig. 7.8), and the \( \{s_i'\} \) into the \( \{a_k'\} \) by the decoder at the receiver, differential coding is employed to prevent a serious error-extension effect (that is, a prolonged burst of errors in the \( \{a_k'\} \)) resulting from a large and rapid carrier phase change during a deep fade which results in a subsequent phase shift of a multiple of 90° (in the complex number plane) of all components of \( Y_i' \) relative to \( Y_i \). To reduce the error rate in the \( \{a_k'\} \), for a given error rate in the \( \{s_i'\} \), the coding must, in addition, be as near as possible to Gray coding, the exact realisation of this being unfortunately not attainable with the given signal. The coding and decoding processes assumed here are an application of a technique outlined in Reference (131) and described in Reference (119).

The stream of binary digits \( \{a_k\} \) to be transmitted is divided into adjacent groups of four digits, such that the \( i \)th group \( a_{4(i-1)+1}, a_{4(i-1)+2}, a_{4(i-1)+3}, a_{4(i-1)+4} \) is converted by the encoder into the corresponding data symbol \( s_i \). The first two binary digits, \( a_{4(i-1)+1} \) and \( a_{4(i-1)+2} \), in the \( i \)th group, are recorded in the encoder to give the corresponding two binary digits \( \overline{a}_{4(i-1)+1} \) and \( \overline{a}_{4(i-1)+2} \) (with possible values 0 or 1).
according to Table 7.4. The resulting group of four binary digits $\overline{a}_4(i-l)+1, \overline{a}_4(i-l)+2, \overline{a}_4(i-l)+3, \overline{a}_4(i-l)+4$, considered here as the corresponding binary-coded number is encoded into the appropriate data-symbol $s_i$ (eqn. 7.55), according to Fig. 7.9. Thus the first two binary digits in the binary-coded number determine the quadrant containing $s_i$, and the remaining two digits determine the position of $s_i$ in the quadrant. The latter two digits in any quadrant are the same as those in the all-positive quadrant, if this is rotated to coincide with the given quadrant.

Following the detection of $s_i$, at the receiver, the corresponding sequence of four detected binary digits $\overline{a}'_4(i-l)+1, \overline{a}'_4(i-l)+2, \overline{a}'_4(i-l)+3, \overline{a}'_4(i-l)+4$ is determined from Fig. 7.9, using, of course, the detected values of $s_{1,i}$ and $s_{2,i}$ (eqn. 7.55) in place of their actual values. The detected values of $a'_4(i-l)+1$ and $a'_4(i-l)+2$ are then determined from Table 7.4, using the detected values of $\overline{a}'_4(i-2)+1, \overline{a}'_4(i-2)+2, \overline{a}'_4(i-1)+1$ and $\overline{a}'_4(i-1)+2$.

It can be seen from Fig. 7.9 and Table 7.4 that a shift of a multiple of 90° in the phase relationship between the reference carrier in the coherent demodulator (Fig. 7.5) and the received signal carrier, such as can occur following a deep fade, does not change the values of $a'_4(i-l)+3$ and $a'_4(i-l)+4$ corresponding to any given value of $s'_i$, nor can it lead to a prolonged burst of errors in the $\{a'_4(i-l)+1\}$ and $\{a'_4(i-l)+2\}$. Gray coding is achieved over each quadrant in Fig. 7.9.

7.4.3 Frequency Characteristics and Sampled Impulse Responses of the Equipment Filters Used in the Tests

In Fig. 7.8, the overall filtering carried out on the baseband signal at the transmitting side is represented by filter A whose impulse
<table>
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<th>$a_{4(i-1)+1}a_{4(i-1)+2}$</th>
<th>$\overline{a}<em>{4(i-2)+1}\overline{a}</em>{4(i-2)+2}$</th>
<th>$\overline{a}<em>{4(i-1)+1}\overline{a}</em>{4(i-1)+2}$</th>
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</table>
FIGURE 7.9: Encoding of the \( s_i \)
response is $a(t)$, whereas at the receiving end is represented by filter $B$ whose impulse response is $b(t)$, where $a(t)$ and $b(t)$ are given by eqns. 7.57 and 7.58 as

$$a(t) = a'(t) * (g(t)e^{-j2\pi f_c t})$$
and

$$b(t) = [(d(t) * c(t)e^{-j2\pi f_c t})] * b'(t)$$

The details of the linear baseband channel in Fig. 7.8 are given in Fig. 7.5, in which filter $A'$ is the baseband signal shaping filter, filter $C$ is the bandpass filter at the input of the linear demodulator, and filter $B'$ is the lowpass filter at the output of the linear demodulator. $A'$, $C$ and $B'$ are the modem filters, where the transfer functions of filters $A'$ and $B'$ should satisfy eqns. 7.27 and 7.29, respectively, and the transfer function of filter $C$ should ideally be flat over the bandwidth of the transmitted QAM signal. Filters $G$ and $D$ are the transmitter and receiver filters, respectively, in the radio set, and both, in cascade, form a voice-frequency channel.

When the HF radio link introduces no fading, attenuation or multipath propagation, then the impulse response of the linear baseband channel in Fig. 7.8 is, according to eqn. 7.56, given by $(a(t)*b(t))$, which ideally, and for the optimum performance of the detection process, should be a minimum phase$^{(119)}$ and also such that $|A(f)| = |B(f)|^{(23)}$. In the detection of data signals transmitted at 19200 bit/s (Section 6), an adaptive linear filter is used ahead of the detector (Figs. 6.1-6.3) to convert the sampled impulse response of the linear baseband channel to be a minimum phase. But, as mentioned in Section 7.1, this filter is difficult to hold adaptively adjusted in the present application, because of the relatively rapid changes of the channel characteristics, and therefore it
has been discarded here. However, since the equipment filters, in both the data and ratio sets, are under the control of the equipment designer, they may be designed and fixed to satisfy the minimum phase condition.

In developing the detection process to be described later, and in order to test it under considerably less favourable conditions, the minimum phase condition has been relaxed and a set of equipment filters with quite severe phase distortion has been used. The attenuation and group delay characteristics, corresponding to the transfer function of the radio filters $G$ and $D$ in cascade over the positive frequencies, are shown in Fig. 7.10 and given in Table 7.5 sampled at frequency intervals of 50 Hz. Since both, $g(t)$ and $d(t)$ are real-valued, the characteristics over the negative frequencies are such that $(G(f)D(f))$ is complex-conjugate symmetric about $f=0$. The attenuation and group delay characteristics of the modem filters $A', C$ and $B'$ in cascade, in the passband of the radio filters over positive frequencies, i.e. the characteristics corresponding to the impulse response

$$
{a'(t) * [c(t)e^{-j2\pi f_c t}] * b'(t)]e^{j2\pi f_c t}
$$

are shown in Fig. 7.11, and given (sampled at frequency intervals of 50 Hz) as filter 2 in Table 6.5 (Section 6). The corresponding characteristics over the negative frequencies are zero, according to eqns. 7.27 and 7.29. The resultant attenuation and group delay characteristics of the linear baseband channel (Fig. 7.8 and eqn. 7.56) are shown in Fig. 7.12, where again they are considered in the passband of the radio filters (or equivalently of the QAM signal, Fig. 7.5) over positive frequencies, and where the HF link introduces no fading, attenuation and multipath
FIGURE 7.10: Frequency Characteristics of the Clansman VRC 321 Radio Filters
**TABLE 7.5: Attenuation and Group Delay Characteristics of Radio Filters in Cascade**

<table>
<thead>
<tr>
<th>FREQUENCY (Hz)</th>
<th>ATT. (db)</th>
<th>G.D. (m.s.)</th>
<th>FREQUENCY (Hz)</th>
<th>ATT. (db)</th>
<th>G.D. (m.s.)</th>
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</tr>
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</tr>
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<td>2.00</td>
</tr>
<tr>
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<td>3450</td>
<td>8.25</td>
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</tr>
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</table>
FIGURE 7.11: Frequency Characteristics of the Modem Transmitter and Receiver Filters in Cascade (in the passband of the QAM Signal)
FIGURE 7.12: Transmitter and Receiver Filters of the Modem and Radio Equipment in Cascade (in the Passband of the QAM Signal When there is No Fading or Multipath Propagation)
propagation. In other words, the characteristics in Fig. 7.12 are those corresponding to the impulse response

\[ (a(t) \ast b(t)) e^{j2\pi fc t} \]

Clearly, the Fourier transform of this impulse response is zero for \( f < 0 \) (eqns. 7.27, 7.29, 7.57 and 7.58). The attenuation and group delay characteristics corresponding to each of \( a(t) \) and \( b(t) \) (eqn. 7.56) are obtained by shifting those in Fig. 7.12 by \( f_c = 1800 \) Hz to the left and dividing them by 2. Clearly, \( a(t) \) and \( b(t) \) have thus the same characteristics.

The impulse responses \( a(t) \) and \( b(t) \) are then obtained, sampled at 4800 samples/second, from their characteristics using the inverse discrete Fourier transform (IDFT). The resultant sampled impulse responses \( \{a_1, k\} \) and \( \{b_k\} \) (eqns. 7.64 and 7.66) are given in Table 7.6, each with a different sampling phase (the \( b_k \) being advanced relative to the \( a_{1,k} \) by \( 0.6/4800 \) seconds). The sequence \( \{a_{2,k}\} \) (eqn. 7.65) is obtained by sampling \( a(t) \) (at a rate of 4800 sample/second), but after introducing a delay of \( \tau \) seconds. As mentioned in Section 7.4.1 (the comments about eqns. 7.64-7.67), \( \tau \) may be expressed as a whole number (\( \rho' \)) of sampling periods, plus a sampling phase \( \tau' < \frac{T}{2} \) (\( T = 1/2400 \) second). Thus, the \( \{a_{2,k}\} \) are obtained by inserting \( \rho' \) zero-valued samples ahead of a set of samples which are obtained by sampling \( a(t) \) at 4800 samples/second and which are delayed, relative to the \( \{a_{1,k}\} \) by \( \tau' \) seconds. An example of the \( \{a_{2,k}\} \) is given also in Table 7.6 with \( \tau = 1 \) m.s. Here, \( \rho' = 4 \) and \( \tau' = 0.8/4800 \) second.

In the second stage of the work and after having developed the detection process to operate satisfactorily with perfect channel estimation (eqn. 7.71), actual estimation and prediction processes have been involved. With the given filter characteristics, however, satisfactory
TABLE 7.6: Examples of the Sampled Impulse Responses of the Two Sky-waves (with no fading) and the Receiver Filter, Obtained by Sampling the Corresponding Non-minimum Phase Waveforms at 4800 Samples/second.

<table>
<thead>
<tr>
<th>{a_{1,k}}</th>
<th>{a_{2,k}}</th>
<th>{b_{k}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Part</td>
<td>Imaginary Part</td>
<td>Real Part</td>
</tr>
<tr>
<td>-0.000552</td>
<td>0.007237</td>
<td>0.0000</td>
</tr>
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<td>0.033576</td>
<td>0.0000</td>
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<tr>
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<td>0.010714</td>
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<tr>
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<tr>
<td>-0.006351</td>
<td>0.005422</td>
<td>0.078945</td>
</tr>
</tbody>
</table>

The \{a_{1,k}\} give the first sky-wave (eqn. 7.64)

The \{a_{2,k}\} give the second sky-wave, with delay \(\tau=1\) ms (eqn. 7.65)

The \{b_{k}\} give the sampled impulse response of the receiver filter (eqn. 7.66)
operation has not always been obtainable, in particular at the higher fading rates. That is because the sampled impulse response of the linear baseband channel, in the absence of fading and multipath propagation is not minimum phase (slow rise towards the peak as shown in Table 7.6), which not only reduces the tolerance of the detection process to additive noise, but also enhances the prediction error in the channel estimator by increasing the minimum value of \( \varepsilon \), \( \varepsilon \) being the early detection delay (Fig. 7.8). These effects will become clearer after presenting the detection and estimation processes. Thus, to achieve the best operation, it becomes necessary to use equipment filters with minimum phase.

As can be seen from Fig. 7.10, the attenuation characteristic of the radio filters has a value close to zero over the frequency band 600 to 2800 Hz, which implies that the attenuation characteristic in Fig. 7.12 is almost entirely determined by the modem filter characteristic (Fig. 7.11 and Table 6.5). Therefore if the radio filters (Fig. 7.10) are adjusted to have a linear phase, and the modem filters are adjusted to be minimum phase, then the resultant characteristics obtained by combining both together will be approximately minimum phase, because the radio filters form now an almost ideal voice channel which introduces no distortion into the transmitted signal. Of course, if the radio filters introduce significant amplitude distortion, then they should also have a minimum phase for the overall impulse response to be minimum phase. In fact, the characteristics in Fig. 7.10 are those of an actual radio set (Clansman VRC 321), and on the assumption that these may be manually adjusted to have a linear phase, the characteristics of the linear baseband channel in the absence of fading and multipath propagation may
quite accurately be approximated by those of the modem filters. Therefore, in testing the detection process with actual estimation, the attenuation characteristics of filters A and B (Fig. 7.8) have been obtained by dividing those of filter 2 in Table 6.5 (Fig. 7.11) by 2 and shifting them to the left by $f_c = 1800$ Hz.

When both $a(t)$ and $b(t)$ are minimum phase, then $y_i(t - iT)$ (eqn. 7.56) is also a minimum phase, so long as there is no multipath propagation. Now, and depending on the sampling phase, the $\{y_{i,h}\}$ (eqn. 7.69) may or may not be a true minimum phase sequence, but nevertheless, the time interval from the start to the peak of the $\{y_{i,h}\}$ will be much shorter than it is when $a(t)$ and $b(t)$ are not minimum phase. In fact, the optimum performance of the detection process requires the sequence $\{y_{i,h}\}$ to be a minimum phase, but this is unattainable without a linear adaptive filter ahead of the detector and this filter is not employed here for reasons mentioned before (Section 7.1). Consequently, forcing both $a(t)$ and $b(t)$ to be minimum phase is the best which can be done here.

To obtain the $\{y_{i,h}\}$ (eqn. 7.69), the $\{a_{1,k}\}, \{a_{2,k}\}$ and $\{b_k\}$ (eqns. 7.64-7.66), corresponding to the minimum phase impulse responses $a(t)$ and $b(t)$ have been obtained as follows. First, the characteristics of filter 2 in Table 6.5 (Fig. 7.11) have been divided by 2 and shifted to the left by 1800 Hz. By means of the inverse DFT of the transfer function corresponding to these characteristics, the sequence $\{a'_k\}$ has been obtained with a sampling rate of 4800 sample/second. The $\{a'_k\}$ are then converted into a minimum phase sequence $\{a''_k\}$ by replacing the roots of the z-transform of the $\{a'_k\}$ which lie outside the unit circle in the z-plane by the complex-conjugates of their reciprocals (81,195) (Section 3.2.3). The $\{a''_k\}$ represent now the minimum phase impulse
response a(t) sampled at a certain phase such that the \( \{ a_n \} \) give a minimum phase sequence. In order to obtain any other sampling phase, the \( \{ a_n \} \) have been oversampled at 64 times the original sampling rate. Here, the DFT of the \( \{ a_n \} \) has been obtained with a frequency sampling interval of 25 Hz. Since the sampling rate is 4800 sample/second, the number of components in the DFT is 4800/25=192. Now, this DFT has been injected by 12096 zero-valued components so that the total number of components become 12288. The injection process is such that the 97th to the 192nd components of the original DFT become the 12193rd to 12288th components and the 97th to the 12192nd components are set to zero. This process is equivalent to increasing the sampling rate (in the time domain) from 4800 to 4800×64=307200 samples/second. The inverse DFT gives now the minimum phase impulse response a(t) sampled at this rate. Let the resultant sequence be denoted \( \{ \hat{a}_k \} \). The \( \{ a_{1,k} \} \) (eqn. 7.64) are then obtained by taking every 64th sample of the \( \{ \hat{a}_k \} \). The \( \{ a_{2,k} \} \) (eqn. 7.66) are obtained from the \( \{ \hat{a}_k \} \) also by taking every 64th sample, but now with a delay of \( \tau \) seconds, relative to the \( \{ a_{1,k} \} \). \( \tau \) is expressed as a whole number \( \rho' \) of sampling periods \( \frac{T}{2} \) plus a sampling phase \( \tau'.\frac{T}{2} \) (\( T=1/2400 \) second). Thus \( \{ a_{2,k} \} \) are obtained by inserting \( \rho' \) zero-valued samples ahead of a set of samples which are obtained by taking every 64th sample of the \( \{ \hat{a}_k \} \), these samples being delayed by \( \tau' \) relative to the \( \{ a_{1,k} \} \) (\( \tau' \) is obtainable here with a maximum error of half the sampling period of the \( \{ \hat{a}_k \} \), i.e., with an error not greater than 1.63 \( \mu \)s). An example of the \( \{ a_{1,k} \} \) and \( \{ a_{2,k} \} \) is shown in Table 7.7, for \( \tau=1 \) ms (\( \rho'=4 \) and \( \tau'=0.8/4800 \) second). Since it is assumed here that a(t) and b(t) have the same characteristics, the \( \{ b_k \} \) (eqn. 7.65) are also obtained
TABLE 7.7: Examples of the Sampled Impulse Responses of the Two Sky-waves (with no fading) and the Receiver Filter, Obtained by Sampling the Corresponding Minimum Phase Waveforms at 4800 Samples/Second.

<table>
<thead>
<tr>
<th>{a_{1,k}}</th>
<th>{a_{2,k}}</th>
<th>{b_k}</th>
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The \{a_{1,k}\} give the first sky-wave (eqn. 7.64)
The \{a_{2,k}\} give the second sky-wave with delay $\tau =1$ms (eqn. 7.65)
The \{b_k\} give the sampled impulse response of the receiver filter (eqn. 7.66)
by taking every 64th sample of the \( \{a_k\} \). An example of the \( \{b_k\} \) is given in Table 7.7, where the sampling phase is chosen such that these \( \{b_k\} \) represent a minimum phase sequence. In other words, the \( \{b_k\} \) in Table 7.7 are exactly as the \( \{a''\} \).

In both cases, when the minimum phase condition has been relaxed (Table 7.6) and when the equipment filters are forced to have a minimum phase characteristic, the effective bandwidth of the data signal is determined by the modem filters (Fig. 7.11) which is in fact narrower than the actually available bandwidth in the radio set (Fig. 7.10). This implies that an unnecessary severe bandlimiting of the data signal is assumed which represents more amplitude distortion and which may be reduced by widening the bandwidth of the modem filters. However, if the detection and estimation processes give satisfactory operation with the given distortion, this should ensure satisfactory operation when wider bandwidth modem filters are used or when the radio filters cause more severe bandlimiting of the data signal.

7.4.4 Noise Generation and the Signal-To-Noise Ratio

The complex-valued Gaussian random process \( u(t) \) which is added to the data signal (Fig. 7.8) originates from the real-valued white Gaussian noise process \( n(t) \) which is added to the signal at the output of the HF radio link (Fig. 7.5). The \( \{u_i\} \), which represent \( u(t) \) sampled at 2400 samples/second, are the noise components in the received signal samples \( \{r_i\} \) (eqn. 7.60). \( u(t) \) is given by eqn. 7.45 as

\[
u(t) = \sqrt{2} \{n(t) * c(t)\} e^{-j2\pi f_c t} * b'(t) \quad (7.73)
\]

where eqn. 7.28 is used. The auto-correlation function of \( u(t) \) is given
by eqn. 7.49, as
\[ R_u(\tau) = N_0 \int_{-F}^{F} |C(f + f_c)|^2 |B'(f)|^2 e^{j2\pi f \tau} df \] (7.74)
where \( N_0 \) is the power spectral density of \( n(t) \), \( C(f) \) and \( B'(f) \) are the transfer functions of filters \( C \) and \( B' \), respectively (Fig. 7.5).

Now, from eqn. 7.58, the transfer function of filter \( B \) (Fig. 7.8) is given by
\[ B(f) = D(f + f_c)C(f + f_c)B'(f) \] (7.75)
where \( D(f) \) is the transfer function of the radio receiver filter \( D \), whose characteristics are obtained by dividing those in Fig. 7.10 by 2.

But since these characteristics represent an almost ideal voice-channel over the passband of the modem filters, as mentioned in Section 7.4.3, then, from eqn. 7.75,
\[ |C(f + f_c)| |B'(f)| \approx |B(f)| \]
with a negligible error. Thus, eqn. 7.74 becomes,
\[ R_u(\tau) = N_0 \int_{-F}^{F} |B(f)|^2 e^{j2\pi f \tau} df \] (7.76)

Clearly, the replacement of eqn. 7.74 by eqn. 7.76 is equivalent to the assumption that \( n(t) \) is added to the data signal at the input of the radio receiver filter \( D \) (Fig. 7.5). Eqn. 7.76 agrees with the model in Fig. 7.5, only when filter \( D \) has a wider bandwidth than does filter \( C \), which is the case here. Eqn. 7.76 is used here for convenience.

The variance of \( u(t) \) is given by \( R_u(0) \), which is
\[ \sigma_u^2 = N_0 \int_{-F}^{F} |B(f)|^2 df 
= N_0 \int_{-\infty}^{\infty} |b(t)|^2 dt \] (7.77)
where the Parseval's theorem is used. \( b(t) \) is given in Tables 7.6 and 7.7 sampled at 4800 samples/second (the \( \{b_k\} \)), which is clearly above the Nyquist rate for filter \( B \). In Appendix E (eqn. E.5) it is shown that
\[
\int_{-\infty}^{\infty} |b(t)|^2 dt = \frac{T}{2} \sum_{k} |b_k|^2
\]

where \( \frac{2}{T} \) is the sampling rate of the \( \{b_k\} \) (T = 1/2400 second). Thus, eqn. 7.77 becomes

\[
\sigma_u^2 = \frac{N_0 T}{2} \sum_{k} |b_k|^2
\]  

(7.78)

Let the signal-to-noise ratio be defined as

\[
\psi = 10 \log_{10} \left( \frac{\varepsilon_0}{N_0} \right) \text{ [db]}
\]  

(7.79)

where \( \varepsilon_0 \) is the average transmitted energy per bit at the input of the HF link (at the input of filter G, Fig. 7.5). Again, because filter G has a negligible effect on the power spectrum of the transmitted data signal, \( \varepsilon_0 \), at the input of the HF radio link, has the same value as \( \varepsilon_0 \) at the output of filter G (at the output of filter A, Fig. 7.8). The average transmitted energy per signal element at the output of filter G is given by eqn. 7.51. But since a data symbol \( S_i \) carries 4 bits of information (eqn. 7.55), then from eqn. 7.51,

\[
\varepsilon_0 = \frac{\varepsilon_0}{4} \int_{-\infty}^{\infty} |A(f)|^2 df
\]

\[
= \frac{\varepsilon_0}{4} \int_{-\infty}^{\infty} |a(t)|^2 dt
\]  

(7.80)

where the Parseval's theorem is used. \( a(t) \) is given in Tables 7.6 and 7.7 (the \( \{a_{1,k}\} \)) sampled at 4800 samples/second, which is also above the Nyquist rate for filter A. Therefore, eqn. E.5 (Appendix E) may again be used so that \( \varepsilon_0 \) becomes

\[
\varepsilon_0 = \frac{\varepsilon_0}{4} \frac{T}{2} \sum_{k} |a_{1,k}|^2
\]  

(7.81)

Now, taking into account that eqn. 7.69 introduces a scaling factor into the signal equal to \( \frac{2}{T} \) (Appendix H), and that therefore the signal energy is multiplied by \( \frac{4}{T^2} \), \( \varepsilon_0 \) becomes,
\[ e_0 = \frac{s_1^2}{4} \frac{2}{T} \sum |a_{1,k}|^2 \]  

(7.82)

which with eqns. 7.78 and 7.79 give,

\[ \psi = 10 \log_{10} \left( \frac{1}{2\sigma_u^2} \sum |a_{1,k}|^2 \sum |b_k|^2 \right) \]  

(7.83)

In Tables 7.6 and 7.7, the \( \{a_{1,k}\}, \{a_{2,k}\} \) and the \( \{b_k\} \) are scaled such that \( \sum |a_{1,k}|^2 = \sum |a_{2,k}|^2 = \sum |b_k|^2 = 1 \), these being the values assumed in the tests. Thus, eqn. 7.83 becomes,

\[ \psi = 10 \log_{10} \frac{5}{\sigma_u^2} \]  

(7.84)

where \( s_1 = 10 \) (eqn. 7.55). From eqn. 7.84,

\[ \sigma_u^2 = 5 \times 10^{-\frac{\psi}{10}} \]  

(7.85)

To generate the \( \{u_i\} \) (eqns. 7.60 and 7.62), the noise discrete-time model described in Appendix F (Section F.2) is used. Here a sequence of complex-valued statistically independent Gaussian random variables with zero mean and variance \( \sigma_u^2 \) is fed, at a rate of 4800 samples/second, to a linear transversal filter whose tap gains are given by the \( \{b_k\} \) (Tables 7.6 and 7.7) which clearly are spaced at delays (time intervals) of \( 1/4800 \) second. The variance of the output sequence is given by \( \sigma_u^2 \) because \( \sum |b_k|^2 = 1 \). The \( \{b_k\} \), as the tap gains of the linear transversal filter, introduce the required correlation into the noise sequence, as can be seen from eqns. 7.76 and Appendix F (Section F.2). The resultant noise samples at the output of the linear transversal filter are now given at a rate of 4800 samples/second. The \( \{u_i\} \), which are required at a rate of 2400 samples/second are then formed by every second sample of the output of the linear transversal filter.
7.5 THE DETECTOR

7.5.1 Outline

As mentioned in Section 7.1, the near-maximum likelihood detection technique considered in Section 3.8 has been employed here. The detector to be described is a development of a detection process which was originally designed for the detection of digital data signals transmitted over HF channels at a rate of 2400 bit/second. Before describing the detection process itself, the basic principles of the maximum and near-maximum likelihood detectors will be briefly described.

Let \( S_k \), \( R_k \) and \( U_k \) be the k-component row vectors (sequences) whose ith components are \( s_i \), \( r_i \) and \( u_i \), respectively, for \( i = 1, 2, \ldots, k \), where

\[
\begin{align*}
r_i &= \sum_{h=0}^{\infty} s_{i-h} y_h + u_i \quad (7.86)
\end{align*}
\]

Also let \( X_k \), \( Z_k \) and \( W_k \) be the k-component row vectors whose ith components are \( x_i \), \( z_i \) and \( w_i \), respectively, for \( i = 1, 2, \ldots, k \), where \( x_i \) has one of the 16 possible values of \( s_i \) (eqn. 7.55),

\[
\begin{align*}
z_i &= \sum_{h=0}^{\infty} x_{i-h} y_h \quad (7.87)
\end{align*}
\]

and \( w_i \) is the possible value of \( u_i \) satisfying

\[
\begin{align*}
r_i &= z_i + w_i \quad (7.88)
\end{align*}
\]

It is assumed here and throughout the rest of this section (Section 7) that \( x_i = 0 \) for \( i < 0 \) as has been previously assumed about the \( \{s_i\} \).

In the k-dimensional "unitary" vector space containing the vectors \( R_k \), \( Z_k \) and \( W_k \), the square of the unitary distance between the vectors \( R_k \) and \( Z_k \) is,

\[
|W_k|^2 = \sum_{i=1}^{k} |w_i|^2 = |W_{k-1}|^2 + |r_k - z_k|^2 \quad (7.89)
\]
where \(|w_i|\) is the absolute value (modulus) of \(w_i\). When all real and imaginary parts of the \(\{u_i\}\) are statistically independent Gaussian random variables with zero mean and the same variance (which is not in fact quite the case here), the maximum likelihood vector \(X_k\) is its possible value such that \(|w_k|^2\) is minimized. When the data-symbols \(\{s_i\}\) are statistically independent and equally likely to have any of their 16 possible values (as is the case here), \(S_k\) is equally likely to have any of its 16 different possible values, and the maximum likelihood vector \(X_k\) is now the possible value of \(S_k\) most likely to be correct. \(|w_k|^2\) is said to be the "cost" of \(X_k\), since the smaller its value the more likely it becomes (under the assumed conditions) that \(X_k\) is correct.

The basic principle behind a near-maximum likelihood detector (which is a form of reduced-state Viterbi-algorithm detector considered in Section 3.7), operating with an adaptive linear filter (as is the case in Section 6), is that just prior to the receipt of \(r_k\), the detector holds \(u\) different vectors \(\{X_{k-1}\}\), each of which is stored together with the associated cost \(|w_{k-1}|^2\). On the receipt of \(r_k\) each stored vector \(X_{k-1}\) is expanded into 16 vectors \(\{X_k\}\), where the first \(k-1\) components \(\{x_i\}\) are as in the original vector \(X_k\) and the last component \(x_k\) takes on its 16 different possible values. The cost \(|w_k|^2\) of each vector \(X_k\) is next evaluated (according to eqns. 7.86-7.89), to give 16\(u\) vectors \(\{X_k\}\) together with their costs \(|w_k|^2\). From the 16\(u\) vectors are now selected \(u\) vectors according to some criterion (Sections 6.4.2 to 6.4.5). The selected vectors are then stored together with their costs. At the end of the received signal message, the vector \(X_k\) with the smallest cost is taken as the detected value of \(S_k\). In practice, this would involve
excessive storage, so that each vector $X_k$ is replaced by the corresponding vector

$$Q_k' = x_{k-n+1} x_{k-n+2} \cdots x_k$$

(7.90)
given by the last $n$ components of the particular $X_k$, where $n > g$ ($g+1$ is the number of components in $Y_i$, eqn. 7.63), and the cost of $Q_k'$ is the cost of $X_k$, i.e. $|W_k|^2$. The data-symbol $s_{k-n+1}$ is now detected as the value of $x_{k-n+1}$ in the vector $Q_k'$ having the smallest cost.

Where no adaptive linear filter is used ahead of the detector, as is the case here and for reasons given in Section 7.1, it may often happen that,

$$|y_{i,0}| < |y_{i,h}|$$

(7.91)

for some $h$ in the range 1 to $g$ (eqn. 7.63) (for example, when the $\{y_i,h\}$ are taken to be those $\{a_{i,k}\}$ in Table 7.6). Under these conditions, the arrangement just described does not usually operate efficiently (113). This is essentially because $x_k$, the last component of $Q_k'$, no longer significantly affects the corresponding cost $|W_k|^2$, which means that $x_k$ is selected almost arbitrarily and suggests that it would be better not to select $x_k$ at all, at this stage. The detection process must now be modified as follows. (114)

Suppose that $y_{i,f}$ is the first component of $Y_i$ (eqn. 7.63) that has a "significant" magnitude. $f$ can be taken, for example, (115) to be the smallest integer such that

$$||y_{i,f}|| > 0.7 ||y_{i,h}||$$

(7.92)

where $||y||$ is the sum of the magnitudes of the real and imaginary parts of $y_i$, (120) and

$$||y_{i,h}|| > ||y_{i,h}||$$

(7.93)

for all $\{h\}$ other than $h=f$. The detector now temporarily ignores the first $f$ components $s_i y_{i,0} s_i y_{i+1} s_i y_{i+2} \cdots s_i y_{i+f-1}, f-1$ of the $i$th received
signal-element $s_i y_i(t-iT)$ (notice the relationship between the samples \(\{y_i,h\}\) and the impulse responses \(\{y_i(t-iT)\}\) defined by eqn.7.61) until the (f+1)th component $s_i y_{i+f,f}$ is received, when all received components of the signal element are taken account of in the detection process. This means that, in the different sequences of the \(\{x_i\}\) held in the store during the time interval between the receipt of $r_{k+f}$ and $r_{k+f+1}$, the receiver sets to zero all \(\{x_i\}\) for which $i>k$, which is the equivalent to setting to zero the first $h$ components of the vector $Y_{k+h}$ (eqn. 7.63), for $h=1,2,\ldots,f$.

Just prior to the receipt of $r_{k+f}$, the detector holds in store $\mu$ different $(n-f)$-component vectors \(\{Q_{k-1}\}\), where

\[ Q_{k-1} = x_{q} x_{q+1} \ldots x_{k-1} \]  
\[ q = k+f-n \] (7.94) (7.95)

each vector being associated with the corresponding cost

\[ d_{k-1} = |W_{k-1}|^2 + \sum_{j=0}^{f-1} |r_{k+j} - \sum_{h=1}^g x_{k-h} y_{k+j,h}|^2 \] (7.96)

which is also stored. In the evaluation of \(d_{k-1}\), the detector here considers all \(\{x_i\}\) for $i\leq k-1$ and ignores all \(\{x_i\}\) for $i>k-1$. Thus, $d_{k-1}$ is obtained from $|W_{k+f-1}|^2$ by setting to zero all \(\{x_i\}\) for $i>k-1$, where $|W_{k+f-1}|^2$, according to eqn. 7.89, is given by,

\[ |W_{k+f-1}|^2 = |W_{k-1}|^2 + \sum_{j=0}^{f-1} |r_{k+j} - \sum_{h=0}^g x_{k+j-h} y_{k+j,h}|^2 \]
\[ = |W_{k-1}|^2 + \sum_{j=0}^{f-1} |r_{k+j} - \sum_{h=-j}^g x_{k-h} y_{k+j,h}|^2 \] (7.97)

On the receipt of $r_{k+f}$, each of the $\mu$ stored vectors \(\{Q_{k-1}\}\) is expanded into 16 $(n-f+1)$-component vectors \(\{P_k\}\), where

\[ P_k = x_q x_{q+1} \ldots x_k \] (7.98)

in which the first $n-f$ components are as in the original vector $Q_{k-1}$ and the last component $x_k$ takes on its 16 different possible values. Each of
the resulting $16\mu$ vectors $\{P_k\}$ has the cost

$$d_k = |W_{k-1}|^2 + \sum_{j=0}^{f} |r_{k+j} - \sum_{h=0}^{g-j} x_{k-h} y_{k+j+h}|^2$$

(7.99)

which is determined using the appropriate stored value of $|W_{k-1}|^2$, assuming here that the value of $f$ has not changed. Clearly, $d_k$ is obtained from $|W_{k+f}|^2$ by setting to zero all $\{x_i\}$ for $i>k$. The detector now selects from the vectors $\{P_k\}$ a set of $\mu$ vectors $\{Q_k\}$ according to some appropriate criterion and then stores the selected vectors together with their costs.$^{(114,115)}$ A vector $Q_k$ is, of course, here given by the last $n$ components of the corresponding vector $P_k$ and it has the same cost $d_k$. The detected value of $s_q$ (eqn. 7.95) is taken as the value of $x_q$ in the vector $P_k$ having the smallest cost. Clearly, the delay in detection is $n$ sampling intervals.

Over an HF radio link, the value of $f$ may vary considerably from one vector $Y_i$ to the next, leading to undue complexity in the arrangement just described.$^{(115)}$ The method of computing the costs $\{d_k\}$ has therefore been substantially modified to tolerate this effect, leading to the detection process in the 2400 bit/s modem.$^{(51,115)}$ The latter effectively performs the operations just described but in a recursive manner. Unfortunately, the arrangement does not operate satisfactorily at 9600 bit/s. In the 2400 bit/s modem, a 4-level QAM signal at an element rate of 1200 bauds has been used,$^{(51,115)}$ whereas in the 9600 bit/s modem a 16-level QAM signal at an element rate of 2400 bauds is usually preferred.$^{(116,131)}$ In fact, increasing the signal-element rate, but within the Nyquist rate of the channel, allows the use of signals with a smaller number of levels, which are more tolerant to additive noise than those with more levels.$^{(23)}$

Of course, the use of the 16-level QAM signal here is imposed by the
required bit rate, i.e. 9600 bit/s. With the considerable increase in signal distortion, due to doubling the signal-element rate, together with the replacement of a 4-level QAM signal by a 16-level QAM signal, it becomes necessary to introduce some basic modifications into the detection process just outlined. These modifications may be clarified by the following comments.

1. Since the detector ignores all \( \{x_i\} \) for \( i > k \), in the evaluation of the costs \( d_k \) in eqn. 7.99, it is effectively considering the corresponding residual intersymbol interference (\( \sum_{h=-j}^{j} s_{k-h} y_{k+j, j+h} \) for \( j = 1, 2, \ldots, f \)) as "additive noise," and the level of this intersymbol interference increases for some combinations of \( s_{k+1}, s_{k+2}, \ldots, s_{k+f} \) as \( f \) does. In the complete absence of the "external" additive noise, the residual intersymbol interference must be such that error-free signal detection is always assured. Bearing this in mind, \( f \) must be determined to maximize the instantaneous overall signal-to-noise ratio involved in the selection of \( x_k \) in a given vector \( P_k \) (eqn. 7.98).

2. The effect of the residual intersymbol interference may also be reduced by eliminating its biggest component for example. Let the component \( y_{k+f, e} \) of the vector \( Y_{k+f} \) be such that

\[
|y_{k+f, e}| > |y_{k+f, h}|
\]  
(7.100)

for \( h = 0, 1, 2, \ldots, f-1 \), \( e \neq f \) and \( e \neq h \). Let also eqn. 7.99 be modified to be

\[
d_k = |W_{k-1}|^2 + \sum_{j=0}^{f-1} |r_{k+j} - \sum_{h=0}^{g-j} x_{k-h} y_{k+j, j+h}|^2 +
+ |r_{k+f} - \sum_{h=0}^{g-f} x_{k-h} y_{k+f, h} - x_{e+f} y_{k+f, e}|^2
\]  
(7.101)

where \( x_{k-e+f} \) has one of the 16 possible values of \( s_{k-e+f} \) for which \( d_k \).
is minimum. In effect, that value of \( x_{k-e+f} \) is most likely to be the correct value of \( s_{k-e+f} \), and the process in eqn. 7.101 is equivalent to the removal of the corresponding intersymbol interference in \( r_{k+f} \), which is the largest amongst all components in the residual intersymbol interference. \( d_k \) in eqn. 7.101 is expected therefore to be more accurate than that given by eqn. 7.99. Of course, eqn. 7.101 becomes the same as eqn. 7.99 when \( x_{k-e+f} \) is put to zero.

In accordance with the previous two comments, it is shown in Section 7.5.2 that the optimum value of \( f \) is that which maximizes the quantity,

\[
\Delta_i = |y_{k+i,i}| - \sum_{h=0}^{i-1} |y_{k+i,h}|
\]

over all values of \( i \) from 1 to \( g \), where \( \phi \) is the integer smaller than \( i \) such that

\[
|y_{k+i,\phi}| > |y_{k+i,h}|
\]

for \( h=0,1,\ldots,i-1 \) and \( h\neq\phi \). Eqn. 7.102 has been derived for the worst case when the residual intersymbol interference, given by \( \sum_{h=-j}^{j-1} s_{k-h} y_{k+j,h} \) for \( j=1,2,\ldots,f \), takes its biggest possible value, i.e. for a specific combination of the sequence \( \{s_{k+1},s_{k+2},\ldots,s_{k+f}\} \).

3. The magnitudes \( |y_{i,h}| \) of the component values \( y_{i,h} \) of \( Y_i \), as \( h \) increases from 0 to \( g \), normally form two peaks, which are introduced by the two sky-waves. It has been observed experimentally that \( f \), as given by eqn. 7.102, most often takes on one or other of two particular values, each of which closely precedes the corresponding peak in the \( |y_{i,h}| \) and persists over a large number of consecutive received samples. When \( f \) takes its larger value, although it is optimum, the magnitude of the residual intersymbol interference also
takes a bigger value so that for a given total overall instantaneous signal-to-noise ratio, the margin allowed for the "external" additive noise becomes smaller. On the other hand, a sub-optimum value of \( f \), which does not satisfy eqn. 7.102 but precedes the first peak, does not necessarily mean the presence of error in determining \( x_k \) (for a given \( P_k \)) for all possible combinations of \( s_{k+1}, s_{k+2}, \ldots, s_{k+f} \) and for most of these combinations, the margin allowed for the additive noise may be greater because the magnitude of the residual intersymbol interference is now smaller. Based on this argument, it seems that \( f \) must be always chosen in such a way as to be biased towards its smaller value. But this then does not always satisfy comment 1, and errors may occur for some combinations of \( s_{k+1}, s_{k+2}, \ldots, s_{k+f} \), even where there is no additive noise. Thus the detector is modified in the following manner. When \( f \), determined according to eqn. 7.102, has its smaller value, then the detector proceeds as described before, and the costs of the vectors \( \{P_k\} \) (eqn. 7.98) are computed exactly as in eqn. 7.101. When \( f \), determined according to eqn. 7.102, has its larger value, then the detector uses in addition to this another value designated \( f' \) which is smaller than \( f \) and precedes the first peak, and now, for both \( f \) and \( f' \), the detector computes two sets of costs \( \{d_k\} \) and \( \{d'_k\} \), respectively, and selects some of the vectors \( \{Q_k\} \) according to the \( \{d_k\} \) and the remaining according to the \( \{d'_k\} \). Details of the process are considered in Section 7.5.3.

4. Since \( x_k \) has different values in all vectors \( \{P_k\} \) originating from the same vector \( Q_{k-1} \), the \( f+1 \) terms under summation in eqn. 7.99 must be evaluated for every vector \( P_k \) separately. Now, when eqn. 7.99 is replaced by (115)
\[
d_k = |w_{k-1}|^2 + \sum_{j=0}^{f-1} |r_{k+j} - \sum_{h=1}^{g-j} x_{k-h} y_{k,j+h}|^2 + \\
+ |r_{k+f} - \sum_{h=0}^{g-f} x_{k-h} y_{k,f+h}|^2 ,
\]  

(7.104)

then the \( f \) terms under summation do not involve \( x_k \) (\( h \) starts with 1 instead of 0) and therefore they are the same for all vectors \( \{\mathbf{p}_k\} \) originating from the same vector \( \mathbf{q}_{k-1} \). The last term only in eqn. 7.104 involves \( x_k \) and therefore must be evaluated separately for every \( \mathbf{p}_k \). Clearly, eqn. 7.104 represents a simplified, but less accurate version of eqn. 7.99, where now the \( f \) terms under summation have to be evaluated only once for all vectors \( \{\mathbf{p}_k\} \) originating from the same vector \( \mathbf{q}_{k-1} \), and the effect of \( x_k \) on \( d_k \) appears only in the last term (involving \( r_{k+f} \)). With the increased signal distortion in the present application, this simplification, together with all the other modifications mentioned before, gave unsatisfactory results. But when \( x_k \) has been involved in the last two terms under summation in eqn. 7.99, satisfactory operation has been achieved. This, together with eqn. 7.101 give,

\[
d_k = |w_{k-1}|^2 + \sum_{j=0}^{f-2} |r_{k+j} - \sum_{h=1}^{g-j} x_{k-h} y_{k,j+h}|^2 + \\
+ |r_{k+f-1} - \sum_{h=0}^{g-f+1} x_{k-h} y_{k,f-1,h}|^2 + \\
+ |r_{k+f} - \sum_{h=0}^{g-f} x_{k-h} y_{k,f+h} - x_{k-e} y_{k,f+e}|^2
\]

(7.105)

Eqn. 7.105 is a simplified but less accurate version of eqn. 7.101. However, tests have shown no noticeable difference in performance when eqn. 7.105 was used instead of eqn. 7.101.
7.5.2 Evaluation of $f$

In the selection of the required $\mu$ stored vectors $\{Q_k\}$ from the $16\mu$ expanded vectors $\{P_k\}$ (eqn. 7.98), the selected vectors are those with the smallest values of $d_k$. Since $x_k$ is the only component, in any one of the selected vectors, which is likely to have any of its 16 possible values due to the selection process (the other $\{x_{k-i}\}_{i=1,2,\ldots}$ have previously been determined), the minimum value of $d_k$ over all possible values of $x_k$, given a vector $Q_{k-1}$ (eqn. 7.94), must be such that the selected value of $x_k$ is most likely to be the correct value of $s_k$. As can be seen from eqns. 7.99, 7.101, 7.104 and 7.105, $d_k$ is a function of $f$, and therefore the value of $x_k$ corresponding to the minimum value of $d_k$ varies as $f$ takes different values. Thus, by allowing $f$ to have one value in the range 1 to $p \leq g$, where $p$ is the largest permitted value of $f$, the value of $f$ must be chosen such that the selected value of $x_k$, corresponding to the minimum value of $d_k$, is most likely to be the correct value of $s_k$. With the correct application of this criterion in determining the value of $f$ before every process of expanding and selecting the $\mu$ stored vectors, every component in any vector $Q_k$ is a candidate to have the correct value of the corresponding data-symbol. This allows therefore the assumption that one of the vectors $\{Q_k\}$ is likely to be the correct value of the corresponding transmitted data sequence. Although this assumption is not true all the time (otherwise it implies that the detector makes no errors), it is at high signal-to-noise ratio true for most of the time (otherwise the entire objective of having the detector becomes meaningless). This assumption will be used later on.

In the following derivation, eqn. 7.99 will be used, but the result will be appropriately modified to be consistent with eqn. 7.105 which has
been actually used in the detection process. In a vector $P_k$, originating from a given vector $Q_{k-1}$, and for a specific value of $f$, the value of $x_k$ that minimizes $d_k$ in eqn. 7.99, effectively minimizes the quantity,

$$\beta_k = \sum_{i=0}^{f} |r_{k+i} - \sum_{h=0}^{g-i} x_{k-h} y_{k+i,h+i}|^2$$  

(7.106)

With the assumption that the given vector $Q_{k-1}$ is the correct value of the data sequence $\{s_{k-i}\}$, $i=1,2,\ldots,q$ (eqns. 7.94-7.95), $\beta_k$ becomes,

$$\beta_k = |(s_k - x_k)y_{k,0} + u_k|^2 + \sum_{i=1}^{f} |(s_k - x_k)y_{k+i,i} +$$

$$+ \sum_{h=0}^{i-1} s_{k+i-h} y_{k+i,h} + u_{k+i}|^2,$$

(7.107)

where the $\{r_{k+i}\}$ are given by eqn. 7.60. Defining,

$$x_i = \sum_{h=0}^{i-1} s_{k+i-h} y_{k+i,h}, \text{ for } i=1,2,\ldots,f,$$

$$= 0 \text{ for } i=0$$

(7.108)

eqn. 7.107 becomes,

$$\beta_k = \sum_{i=0}^{f} |(s_k - x_k)y_{k+i,i} + r_{i}|^2$$

(7.109)

The $\{r_i\}$ in eqn. 7.109 represent the resultant intersymbol interference in $\beta_k$ due to setting to zero all $\{x_i\}$ for which $i>k$, on the receipt of the signal sample $r_{k+f}$, as mentioned in Section 7.5.1. Now, when $x_k$ may have any value whatsoever, the absolute minimum value of $\beta_k$ is associated with the value of $x_k$ for which the derivative of $\beta_k$ with respect to $x_k$ is zero. Since $x_k$ is complex-valued, where

$$x_k = x_{1,k} + jx_{2,k}$$

(7.110)

and $x_{1,k}$ and $x_{2,k}$ are statistically independent (the same as the two parts of $s_k$), the minimum value of $\beta_k$ requires the partial derivatives of $\beta_k$ with respect to $x_{1,k}$ and $x_{2,k}$ both to be zero. From eqn. 7.109,
putting the two partial derivatives of $\beta_k$ with respect to $x_{1,k}$ and $x_{2,k}$ to zero gives the values of $x_{1,k}$ and $x_{2,k}$ as

$$\gamma_{1,k} = \frac{1}{2} \left( \sum_{i=0}^{f} y_{k+i,i}(\nu_i^* + u_{k+i}) \right)$$

and

$$\gamma_{2,k} = \frac{1}{2} \left( \sum_{i=0}^{f} |y_{k+i,i}|^2 \right)$$

respectively, where * denotes complex conjugate and $s_{1,k}$ and $s_{2,k}$ are the real and imaginary parts of $s_k$. The values of $x_{1,k}$ and $x_{2,k}$ in eqn. 7.111 give the absolute minimum value of $\beta_k$ (the second order partial derivatives of $\beta_k$ with respect to $x_{1,k}$ and $x_{2,k}$ can not be zero), but since $x_k$ may have one of 16 possible discrete values (the values of $s_k$, eqn. 7.55), the minimum value of $\beta_k$ achievable in practice corresponds to the possible values of $x_{1,k}$ and $x_{2,k}$ nearest to $\gamma_{1,k}$ and $\gamma_{2,k}$, respectively. Thus, selecting $x_k$ to minimize $\beta_k$ (eqn. 7.109) is equivalent to the simple threshold "detection" of $s_{1,k}$ and $s_{2,k}$ from the noisy samples $\gamma_{1,k}$ and $\gamma_{2,k}$ (eqn. 7.111), respectively (the word "detection" used here is just to clarify the principle of determining the values of $x_{1,k}$ and $x_{2,k}$ from $\gamma_{1,k}$ and $\gamma_{2,k}$, and has nothing to do with the actual detection process of the data symbols $s_k$)). Let

$$s = \frac{\sum_{i=0}^{f} y_{k+i,i} \nu_i^*}{\sum_{i=0}^{f} |y_{k+i,i}|^2}$$

and

$$v_k = \frac{\sum_{i=0}^{f} y_{k+i,i} u_{k+i}}{\sum_{i=0}^{f} |y_{k+i,i}|^2}$$

(7.112)

(7.113)
then, \( \gamma_{1,k} \) (eqn. 7.111) becomes

\[
\gamma_{1,k} = s_{1,k} + \delta + v_k \tag{7.114}
\]

\( \delta \) is the resultant intersymbol interference in \( \gamma_{1,k} \) due to setting to zero all \( \{x_i\} \) for which \( i > k \), on the receipt of the signal sample \( r_{k+f} \).

\( v_k \) is the real-valued "noise" component in \( \gamma_{1,k} \). For a given value of \( f \), the \( \{v_k\} \) are Gaussian random variables (eqn. 7.113) with zero mean (as the \( \{u_i\} \)) and a variance given by

\[
\sigma_v^2 = \mathbb{E}[v^2] = \frac{\sigma_u^2}{\sum_{i=0}^{f} |y_{k+i}|^2} \tag{7.115}
\]

where \( \sigma_u^2 \) is the variance of the complex-valued noise sequence \( \{u_i\} \) and \( \mathbb{E}[.] \) denotes the expected value of [.] . It is assumed here that the \( \{u_i\} \) are statistically independent.

Now, from eqns. 7.108 and 7.112, and for a given value of \( f \), \( \delta \) may have \( m^{2f} \) different equally likely values corresponding to the \( m^{2f} \) possible combinations of the data-sequence \( s_{k+1}, s_{k+2}, \ldots, s_{k+f} \), where \( m^2 \) is the number of possible values a data-symbol may have (here \( m^2 = 16 \), eqn. 7.55). In the complete absence of additive noise (where the \( \{u_i\} \) and therefore the \( \{v_i\} \) in eqn. 7.113 are all zero), and to ensure that the selected value of \( x_{1,k} \) (as its nearest possible value to \( \gamma_{1,k} \)) is equal to \( s_{1,k} \), the maximum possible value of \( |\delta| \) must be such that

\[
|\delta|_{\text{max}} < 1 \tag{7.116}
\]

where the difference between two adjacent values of \( s_{1,k} \) is 2, as can be seen from eqn. 7.55. Clearly, \( \delta \), and therefore \( |\delta|_{\text{max}} \), are functions of \( f \) as can be seen from eqns. 7.108 and 7.112. Thus, all values of \( f \), for which the inequality 7.116 is not satisfied, are unacceptable here. In the presence of the additive noise, the selected value of \( x_{1,k} \) (as its
nearest possible value to $y_{1,k}$ (eqn. 7.114) becomes more likely equal to the value of $s_{1,k}$ when the quantity $(1-|\delta|_{\text{max}})$ has a greater value. Thus, the required value of $f$ is that for which this quantity is maximum. But, the selected value of $f$ here, affects the variance of the "noise" component $v_k$ in eqn. 7.114, as can be seen from eqn. 7.115, and this, in turn, affects the selected value of $x_{1,k}$. Clearly, the effect of $v_k$ on the selection of $x_{1,k}$ becomes smaller as the quantity $(\sum_{i=0}^{f} |y_{k+i,i}|^2)^{\frac{1}{2}}$ is increased, since this leads to a smaller value of $\sigma_v^2$. Hence, the value of $f$, for which the selected value of $x_{1,k}$ is most likely to be the correct value of $s_{1,k}$' is that for which $\Delta'(f)$ is maximum, where

$$\Delta'(f) = \left( \sum_{i=0}^{f} |y_{k+i,i}|^2 \right)^{\frac{1}{2}} (1-|\delta|_{\text{max}})$$ (7.117)

From eqns. 7.108 and 7.112, and for $f>0$, we have

$$\delta = \frac{1}{\left( \sum_{i=0}^{f} |y_{k+i,i}|^2 \right)} \left\{ \sum_{i=1}^{f} \text{Re}[y_{k+i,i}] \sum_{h=0}^{i-1} s_{k+i-h} y_{k+i+h}^{*} \right\}$$

$$= \frac{1}{\left( \sum_{i=0}^{f} |y_{k+i,i}|^2 \right)} \left[ \sum_{i=1}^{f} \sum_{h=0}^{i-1} s_{1,k+i-h} (y_{1,k+i,i} y_{1,k+i,h}^{*} + y_{2,k+i,i} y_{2,k+i,h}^{*} + y_{2,s_{k+i-h},k+i-h}^{*}) - y_{1,k+i,i} y_{2,k+i,h}^{*} \right]$$ (7.118)

where $y_{1,k+i,h}$ and $y_{2,k+i,h}$ are the real and imaginary parts of $y_{k+i,h}$, respectively. Eqn. 7.118 may be re-arranged in the following form,
From eqn. 7.119, the maximum value of $|\delta|$ over all possible sequences $s_{k+1}, s_{k+2}, \ldots, s_{k+f}$ is given by

$$|\delta|_{\text{max}} = \frac{f}{(\sum_{i=0}^{f} |y_{k+i, i}^*|^2)} \left\{ \sum_{h=1}^{f} \left[ \text{Re} \left[ \sum_{i=h}^{f} y_{k+i, i} y_{k+i, i-h}^* \right] + \text{Im} \left[ \sum_{i=h}^{f} y_{k+i, i} y_{k+i, i-h}^* \right] \right] \right\}$$

(7.120)

where (eqn. 7.55)

$$|s_{1,k}|_{\text{max}} = |s_{2,k}|_{\text{max}} = (m-1)$$

and $m=4$ in the present application. Now, for any complex scalar $(a+jb)$, it may be shown that

$$(a^2+b^2)^{1/2} \leq |a| + |b| \leq \sqrt{2}(a^2+b^2)^{1/2}$$

so that eqn. 7.120 may be written as

$$|\delta|_{\text{max}} \leq \frac{\sqrt{2}(m-1)}{(\sum_{i=0}^{f} |y_{k+i, i}^*|^2)} \left\{ \sum_{h=1}^{f} \left[ \sum_{i=h}^{f} y_{k+i, i} y_{k+i, i-h}^* \right] \right\}$$

(7.121)

Replacing $|\delta|_{\text{max}}$ in eqn. 7.117 by the largest value it may have in eqn. 7.121 gives,

$$\Delta'(f) = \left( \sum_{i=0}^{f} |y_{k+i, i}^*|^2 \right)^{1/2} - \frac{\sqrt{2}(m-1)}{(\sum_{i=0}^{f} |y_{k+i, i}^*|^2)} \left\{ \sum_{h=1}^{f} \left[ \sum_{i=h}^{f} y_{k+i, i} y_{k+i, i-h}^* \right] \right\}$$

(7.122)

for $f=1, 2, \ldots, p$. When $f=0$, then from eqns. 7.108 and 7.112, $\delta=0$.

Hence, eqn. 7.117 gives

$$\Delta'(0) = |y_{k,0}|$$

(7.123)

The value of $f$, $f=0, 1, \ldots, p$, for which $\Delta'(f)$ in eqns. 7.122 and 7.123 is maximum, is the wanted value for $x_{1,k}$ to be most likely the correct value of $s_{1,k}$ when $\delta_k$ and therefore $d_k$ is minimized. Applying the
same procedures, carried out on the selection of $x_{1,k}$ from $\gamma_{1,k}$ (eqn. 7.111), also on the selection of $x_{2,k}$ from $\gamma_{2,k}$, leads again to the same result, so that the wanted value of $f$ is that for which $\Lambda'(f)$ in eqns. 7.122 and 7.123 is maximum.

Eqns. 7.122 and 7.123 have been derived assuming that eqn. 7.99 is used in computing the value of $d_k$. When eqn. 7.105 is used, which is the case here, eqn. 7.122 becomes,

$$\Lambda''(f) = \left( \frac{1}{f-1} \sum_{i=0}^{f-1} |y_{k+i,i}|^2 \right)^{1/2} - \frac{\sqrt{2(m-1)}}{\left( \sum_{i=0}^{f-1} |y_{k+i,i}|^2 \right)^{1/2}} \times \left( \frac{1}{f-1} \sum_{h=1}^{f} \sum_{i=0}^{f-1} y_{k+i,i} y_{k+i,i+h}^* \right)$$

(7.124)

for $f=1,2,\ldots,p$. In the evaluation of $\Lambda''(f)$, $y_{k+f,e}$ must be set to zero, since according to eqn. 7.105, the intersymbol interference $(s_{k-e+f} y_{k+f,e})$ is effectively removed, for $x_{k-e+f}$ as defined. For $f=0$, $\Lambda''(f)$ is equal to $\Lambda'(f)$ in eqn. 7.123.

The evaluation of $\Lambda''(f)$ in eqn. 7.124 involves an excessive amount of operations. A less accurate but simpler formula may be obtained by assuming that $d_k$ is determined by,

$$d_k = |w_{k-1}|^2 + \sum_{i=0}^{f-1} |r_{k+i} - \sum_{h=1}^{g-f} x_{k-h} v_{k+i,i+h}|^2 + |r_{k+f} - \sum_{h=0}^{g-f} x_{k-h} v_{k+f,f+h} - x_{k-e+f} v_{k+f,e}|^2$$

(7.125)

where $x_{k-e+f}$ has one of its 16 possible values for which $d_k$ is minimum, and $y_{k+f,e}$ has the largest amplitude amongst all $\{y_{k+f,h}\}$ for $h=0,1,\ldots,f-1$. In Eqn. 7.125, the value of $f$ for which $x_k$ is most likely to have the correct value of $s_k$ when $d_k$ is minimized (of course over all possible values of $x_k$), results from minimizing,
\[ \Delta(f) = |y_{k+f,f} - \sqrt{2(m-1)} \sum_{h=0}^{f-1} |y_{k+f,h}| \] 

for \( f=1,2,\ldots,p \). Eqn. 7.126 results from eqns. 7.122 or 7.124 simply by letting \( i=f \) only. Again, \( \Delta(f) \), for \( f=0 \), is equal to \( \Delta'(f) \) in eqn. 7.123.

In the tests carried out on the detector (to be described later), a better performance has been obtained when eqn. 7.126 was replaced by

\[ \Delta(f) = |y_{k+f,f} - \sum_{h=0}^{f-1} |y_{k+f,h}| \] 

and that may be because eqn. 7.105 has been actually used by the detector to evaluate \( d_k \), not eqn. 7.125. Of course, and for the optimum result in this case, eqn. 7.124 should have been used. However, spot check tests showed no noticeable difference when eqn. 7.124 was replaced by eqn. 7.127. Clearly, the latter is much simpler to be used in practice.

7.5.3 Detection Process for the 9600 bit/s modem

The detection process developed here, for the detection of digital data transmitted over HF radio channels at 9600 bit/s, and outlined in Section 7.5.1 proceeds as follows.

Let the largest permitted value of \( f \) be \( p \), where \( p \leq g \). Just prior to the receipt of the signal sample \( r_{k+p} \), the \( p+1 \) vectors \( \{Y'_{k+i,k+i-2-1}\} \) for \( i=0,1,\ldots,p \), are available to detector (from the estimator), where

\[ Y'_{k+i,k+i-2-1} \] is the prediction of \( Y_{k+i} \) formed by the estimator, and \( Y_{k+i} \) is the sampled impulse response at \( t=(k+i)T \) of the linear baseband channel in Fig. 7.8. To simplify the notation, the vector \( Y'_{k+i,k+i-2-1} \) will be denoted \( Y_{k+i} \) in this section, and whenever the vector

\[ Y_i = y_i,0',y_{i,1},\ldots,y_{i,g} \] is encountered here, it must be taken as representing the prediction \( Y'_{i,i-2-1} \).
Also, just prior to the receipt of the signal sample \( r_{k+p} \), the detector holds in store \( u \) \((n-p)\)-component vectors \( \{Q_{k-1}\} \), where
\[
Q_{k-1} = x_q x_{q+1} \cdots x_{k-1},
\]
\[
q = k+p-n
\]  
and \( x_i \) has a possible value of the transmitted data symbol \( s_i \), each vector being associated with its cost \( |W_{k-1}|^2 \) and the \( p \) quantities \( \{z_{k-1,i+1}\} \) for \( i=0,1,\ldots,p-1 \), where
\[
z_{k-1,i+1} = \sum_{h=1}^{g-i} x_{k-h} y_{k+i,i+h}
\]  
and \( z_{k-1,i+1} \) is an estimated value of the data signal \( \sum_{h=0}^{g} s_{k+i-h} y_{k+i,h} \) in \( r_{k+i} \) on the assumption that the first \( i+1 \) components \( y_{k+i,0}, y_{k+i,1}, \ldots, y_{k+i,i} \) of \( Y_{k+i} \) have negligible amplitudes, so that they are set to zero.

On the receipt of the signal sample \( r_{k+p} \), the detector sets \( f \) to the value of \( i \) that maximizes the quantity
\[
\Delta_i = |y_{k+i,i} - \sum_{h=0}^{i-1} |y_{k+i,h}| \text{ for } i \neq 0 \]
\[
= |y_{k,0}| \text{ for } i = 0
\]  
over all values of \( i \) from 0 to \( p \) where \( \phi \) is the integer less than \( i \) such that
\[
|y_{k+i,\phi}| > |y_{k+i,h}|
\]  
for \( h=0,1,\ldots,i-1 \) and \( h \neq \phi \). Eqn. 7.132 is, in fact, eqn. 7.127, and the exclusion of \( |y_{k+i,\phi}| \) in eqn. 7.132 takes account of the fact that the intersymbol interference \( x_{k+i-\phi} y_{k+i,\phi} \) when \( f \) is set to the corresponding value of \( i \), is removed (eqn. 7.105).

In association with \( f \), the detector holds also the quantity \( e<\phi \)
which is such that
\[
|y_{k+f,e}| > |y_{k+f,h}|
\]  
\[
(7.134)
\]
for \( h=0,1,2,\ldots,f-1 \) and \( h \neq e \). Clearly, \( y_{k+f,e} \) has the largest magnitude amongst the first \( f \) components of \( y_{k+f} \). Since \( f \) and \( e \) may vary with \( k \), their values determined immediately after the receipt of \( r_{k+p} \) will be referred to as \( f(k) \) and \( e(k) \), respectively.

In addition to the quantities \( f(k) \) and \( e(k) \), the detector employs three further quantities \( f'(k), e'(k) \) and \( \lambda(k) \), where \( f'(k) \) and \( e'(k) \) can be considered as alternative values of \( f(k) \) and \( e(k) \), respectively, and \( \lambda(k) \) controls the mode of operation of the detector (outlined in comment 3 in Section 7.5.1). \( f'(k), e'(k) \) and \( \lambda(k) \) are determined as follows, using the values of \( f(k) \) and \( e(k) \) given by eqns. 7.132 and 7.134.

1. If \( f(k)>f(k-1) \), \( \lambda(k)=1 \).
2. If \( f(k)=f(k-1) \), \( \lambda(k)=\lambda(k-1) \).
3. If \( f(k)<f(k-1) \) and \( |y_{k+f(k),f(k)}|>|y_{k-1+f(k-1),f(k-1)}| \), \( \lambda(k)=0 \).
4. If \( f(k)<f(k-1) \) and \( |y_{k+f(k),f(k)}|\leq|y_{k-1+f(k-1),f(k-1)}| \),

then \( \lambda(k)=\lambda(k-1) \), \( e(k) \) is set to the value of \( e(k-1) \) or \( f(k) \) depending on \( |y_{k-1+f(k-1),e(k-1)}| \) being larger than \( |y_{k+f(k),f(k)}| \) or not, and \( f(k) \) is set to its previous value \( f(k-1) \), so that the finally accepted values of \( f(k) \) and \( e(k) \) in this case are not those determined most recently from eqns. 7.132 and 7.134. Since a change in the value of \( f(k) \), determined by eqn. 7.132, is normally followed by a relatively long period with no change, the value of \( f(k-1) \) is effectively remembered until condition 4 is no longer satisfied.

Now, whenever \( \lambda(k) \neq \lambda(k-1) \), then \( e'(k)=e(k-1) \) and \( f'(k)=f(k-1) \), and whenever \( \lambda(k)=\lambda(k-1) \), then \( e'(k)=e'(k-1) \) and \( f'(k)=f'(k-1) \). Clearly, \( e'(k) \) and \( f'(k) \) change their values when \( f(k) \) does.

For every stored vector \( Q_{k-1} \), the detector next evaluates the quantity,
so that the detector now holds \( p+1 \) quantities \( \{ z_{k-i,i+1} \} \) for \( i = 0, 1, \ldots, p \), (eqns. 7.131 and 7.135). The detector then expands each of the \( u \) stored vectors \( \{ Q_{k-1} \} \) into 16 \((u-p+1)\)-component vectors \( \{ P_k \} \), where

\[
P_k = x_q, x_{q+1}, \ldots, x_k
\]

where \( q \) is given by eqn. 7.130 and \( x_k \) takes on the 16 possible values of \( s_k \). Now, when \( \lambda(k) = 1 \), it evaluates for each of the expanded vectors the four quantities \( z_{k, f(k)-1} \), \( z_{k, f(k)} \), \( z_{k, f'(k)-1} \) and \( z_{k, f'(k)} \), whereas, when \( \lambda(k) = 0 \), it evaluates just the two quantities \( z_{k, f(k)-1} \) and \( z_{k, f(k)} \). The symbol \( z_{k, i} \) is here defined to be,

\[
z_{k, i} = \sum_{h=0}^{g-i} x_{k-h} y_{k+i, i+h}
\]

where \( z_{k-1, i+1} \) is given by eqn. 7.131. The first of the two subscripts of \( z \) (\( k-1 \) or \( k \)) determines the particular function (eqn. 7.131 or 7.137), and the second subscript, when added to the first, gives the time location of the corresponding received signal sample. \( z_{k-1, i+1} \) and \( z_{k, i} \) are both estimated values of the data signal \( \sum_{h=0}^{g} s_{k+i-h} y_{k+i, h} \) in \( r_{k+i} \), where the first \((i+1)\) and the first \(i\) components of \( y_{k+i} \) are set to zero in \( z_{k-1, i+1} \) and \( z_{k, i} \) respectively.

Now, for each of the 16u expanded vectors \( \{ P_k \} \), the detector evaluates the quantities,

\[
c_i = |r_{k+i} - z_{k-1, i+1}|^2, \quad i = 0, 1, \ldots, f(k)-2
\]

\[
c_{f(k)-1} = |r_{k+f(k)-1} - z_{k, f(k)-1}|^2
\]

and \( c_{f(k)} \), which is the minimum value of
\[ \theta_f(k) = |r_{k+f(k)} - z_{k,f(k)} - x_{k+f(k)-e(k)}y_{k+f(k),e(k)}|^2 \]  

(7.140)

over the 16 possible values of \( x_{k+f(k)-e(k)} \). Since \( y_{k+f(k),e(k)} \) is normally the component with the largest magnitude of the first \( f(k) \) components of \( Y_{k+f(k)} \), the symbol \( x_{k+f(k)-e(k)} \) can be taken to be the one of the symbols \( x_{k+1}, x_{k+2}, \ldots, x_{k+f(k)} \) with the largest component in \( r_{k+f(k)}' \) and its possible values are considered in the evaluation of \( c_{f(k)} \). This operation, effectively removes the intersymbol interference introduced by \( z_{k+f(k)-e(k)}y_{k+f(k),e(k)} \) in \( r_{k+f(k)} \), as explained in comment 2 in Section 7.5.1. The cost of each vector \( P_k \) is finally determined by the corresponding value of

\[ d_k = |W_{k-1} f(k) + \sum_{i=0}^f c_i | \]  

(7.141)

which is computed using the appropriate stored value of \( |W_{k-1}|^2 \). Eqn. 7.141 is, in fact, just another but an exactly equivalent form of eqn. 7.105, where it is here recursively evaluated through eqn. 7.137.

When \( \lambda(k) = 1 \), the detector evaluates, in addition to the \( \{c_i\} \) (eqns. 7.138-7.140), the corresponding quantities,

\[ c'_i = c_i \quad \text{for } i = 0, 1, \ldots, f'(k)-2 \]  

(7.142)

\[ c'_{f'(k)-1} = |r_{k+f'(k)-1} - z_{k,f'(k)-1}|^2 \]  

(7.143)

and \( c'_{f'(k)} \), which is the minimum value of

\[ \theta'_{f'(k)} = |r_{k+f'(k)} - z_{k,f'(k)} - x_{k+f'(k)-e'(k)}y_{k+f'(k),e'(k)}|^2 \]  

(7.144)

over the 16 possible values of \( x_{k+f'(k)-e'(k)} \). The alternative cost \( d'_k \) of each vector \( P'_k \) is finally determined by the corresponding value of

\[ d'_k = |W_{k-1}|^2 + \sum_{i=0}^{f'(k)} c'_i \]  

(7.145)

which is computed using the appropriate stored value of \( |W_{k-1}|^2 \). When
\[ \lambda(k) = 0, \] the \( \{c_i^l\} \) and \( d_k' \) are not required. Notice here, that the \( \{d_k'\} \) are evaluated when \( f(k) \) takes on its larger values. The significance of the \( \{d_k'\} \) is explained in comment 3 in Section 7.5.1.

Having determined the costs of the vectors \( \{P_k\} \), the detector selects the vector \( P_k \) with the smallest cost \( d_k \) (not \( d_k' \)) and the value of the first component \( x_q \) (i.e. \( x_{k+p-n} \)) of this vector is now taken as the detected value \( s'_q \) (i.e. \( s'_{k+p-n} \)) of the data-symbol \( s_q \) (i.e. \( s_{k+p-n} \)). The delay in detection is, of course, \( n \) sampling periods. Also, the component \( x_{k+p-z} \) of the given selected vector \( P_k \) is taken as the "early" detected value \( s''_{k+p-z} \) of the data-symbol \( s_{k+p-z} \). The \( \{s_{i-z}''\} \) are used by the estimator to form the predictions \( \{Y_{i-1}^{i-2}\} \) of the \( \{Y_i\} \) (Fig. 7.8). Of course, \( z \leq n \). The detector next discards all vectors \( \{P_k\} \) for which \( x_q \neq s'_q \) and removes the first component \( x_q \) from each of the remaining vectors (including that with the smallest cost), to give the corresponding vectors \( \{Q_k\} \). The first of the selected vectors \( \{Q_k\} \) is then taken as that with the smallest cost \( d_k \). If the vectors \( \{Q_k\} \) include the group of 16 that originated from the selected vector \( Q_{k-1} \) with the smallest cost, the second selected vector \( Q_k \) is the one of this group with the smallest cost \( d_k \) (excluding from the group, of course, the already selected vector, if this also originated from the vector \( Q_{k-1} \) with the smallest cost).

Depending on whether or not the second vector \( Q_k \) was available for selection, the detector selects from the remaining vectors \( \{Q_k\} \) the \( \{i\mu-2\} \) or \( \{i\mu-1\} \) vectors with the smallest costs \( \{d_k\} \). \( \mu \) is assumed to be even. The remaining \( \mu \) vectors \( \{Q_k\} \) are finally selected as those with the smallest costs \( \{d_k\} \) or \( \{d_k'\} \), respectively, depending upon whether \( \lambda(k) = 0 \) or 1. Once the \( \mu \) vectors \( \{Q_k\} \) have been selected, the costs \( \{d_k\} \) and \( \{d_k'\} \) are discarded, since they are not required in the following detection.
processes. For each of the \( m \) selected vectors \( \{Q_k\} \), the detector now evaluates the quantities

\[
|w_k|^2 = |w_{k-1}|^2 + |r_k - z_{k,0}|^2
\]  
(7.146)

and

\[
z_{k,i} = z_{k-1,i+1} + x_k y_{k+i,i}
\]  
(7.147)

for \( i=1,2,\ldots,p \), which are stored together with the \( m \) vectors, ready for the next detection process. Notice that eqn. 7.147 represents eqn. 7.131 just prior to the receipt of \( r_{k+p+1} \). Clearly, the evaluation of the \( \{z_{k,i}\} \) is achieved recursively, on the reception of the successive signal samples, and this technique, including the evaluation of the \( \{c_i\} \) in eqn. 7.138-7.140, greatly simplifies the evaluation of \( d_k \) given by eqn. 7.105, without changing its value. This technique of evaluating the \( \{z_{k,i}\} \) has been adopted from Reference (115).

The basic operation of the detection process just described and the reasons for its choice are outlined in comments 1-4 in Section 7.5.1 which may be summarised as follows:

1. The choice of \( f \) according to eqn. 7.132 is close to the optimum, as has been shown in Section 7.5.2.

2. The intersymbol interference represented by \( s_{k+f(k)} - e(k) y_{k+f(k)}, e(k) \) in \( r_{k+f(k)} \) represents the component of the largest magnitude in the residual intersymbol interference \( \sum_{h=0}^{f(k)-1} s_{k+f(k)} - h y_{k+f(k)}, h \) which is ignored in the evaluation of \( z_{k,f(k)} \), and this component has effectively been removed through the evaluation of \( \theta_{f(k)} \) (eqn. 7.140).

3. When \( f(k) \) takes its smaller value, then \( \lambda(k)=0 \) and the selection of the \( m \) vectors \( \{Q_k\} \) is based on the values of the costs \( \{d_k\} \), but when \( f(k) \) takes its larger value, then \( \lambda(k)=1 \) and both sets of costs \( \{d_k\} \) and \( \{d_k'\} \) are used in the selection of the \( \{Q_k\} \).
4. When \( f(k) \), determined by eqn. 7.132, changes its value downwards to its smaller value, the values just before and after the change are considered equally acceptable. But since the smaller value is now still remembered in \( f'(k) \) so that both values may be involved in the selection of the vectors \( \{Q_k\} \), \( f(k) \) is not allowed to have the smaller value (according to eqn. 7.132) until there is no doubt that the larger value of \( f \) is not an appropriate one. This is controlled according to conditions 3 and 4 previously listed.

7.6 THE ESTIMATOR (183,184)

The estimator employed in the tests has recently been developed to be used in estimating the sampled impulse response of time-varying channels of the type considered here, and it has been chosen for this application because of its potential advantage in tolerance to noise over other estimators. The details of this estimator are given in Reference (183) and its basic function (184) will be presented here.

The estimation process that determines the estimate (prediction) \( Y_{i,i-2-1} \) of the sampled impulse-response \( Y_i \) uses the conventional linear feedforward transversal-filter estimator. (92,122,183) Thus, following the receipt of the signal sample \( r_{i+\delta+1} \) (by the detector, Fig. 7.8) at time \( t=(i+\delta+1)T \) and before the early detection of \( s_{i+1} \), the channel estimator uses the received sample \( r_i \), the early detected data-symbols \( s''_1,s''_1-1,...,s''_i-\delta \) and the vector,

\[
Y_{i,i-1} = Y_{i,i-1},0,Y_{i,i-1,1},...Y_{i,i-1,\delta}
\]  

(7.148)

which is the prediction of \( Y_i \) from the estimates of \( Y_{i-1},Y_{i-2},... \), to give the error signal
The estimator now generates the vector,

\[ Y'_i = y'_{i,0}, y'_{i,1}, \ldots, y'_{i,g} \]  

which is the updated estimate of \( Y_i \), and is given by

\[ y'_{i,h} = y'_{i,i-1,h} + c e_i (s''_{i-h})^* \]  

for \( h = 0, 1, \ldots, g \). \((s''_{i-h})^*\) is the complex conjugate of \( s''_{i-h} \) and \( c \) is a small positive real constant. Based on \( Y'_i \), the detector now, unlike other arrangements, \((92,122)\) forms an improved estimate \( F_i \) of \( Y_i \) which is then used to obtain predictions of \( Y_{i+1} \) and \( Y_{i+t+1} \).

In forming the improved estimate \( F_i \), the estimator makes use of the available prior knowledge of the HF radio link. It assumes that this has two Rayleigh fading sky-waves, with a constant relative delay in their arrival at the receiver and introducing a frequency spread of no more than 2 or 3 Hz into the data signal. It assumes furthermore that the correct sampling rate is used at the receiver. Under these conditions, the sampled impulse response of the linear baseband channel in Fig. 7.8, at time \( t = iT \), can be taken to be

\[ Y_i = \lambda_i L + \mu_i M \]  

where \( L \) and \( M \) are fixed \((g+1)\)-component vectors, with complex-valued components, and the complex-valued scalar parameters \( \lambda_i \) and \( \mu_i \) vary with \( i \). \((183)\) The vectors \( \lambda_i L \) and \( \mu_i M \) are, in fact, the sampled impulse responses corresponding to the first and second sky-waves, at time \( t = iT \), such that, in the absence of the first or second sky-wave, \( Y_i \) becomes \( \mu_i M \) or \( \lambda_i L \) respectively (this can be seen from eqn. 7.69). It is clear from eqn. 7.152 that \( Y_i \) must at all times lie in the 2-dimensional subspace spanned by \( L \) and \( M \) in the \((g+1)\)-dimensional unitary vector space containing...
all (g+1)-component vectors over the complex field. Since \( L \) and \( M \) are fixed, so also is the subspace.

During the synchronisation process at the start of transmission, the estimator obtains two estimates of \( Y_i \) at well spaced time instants, and these estimates must be quite accurate. The Gram-Schmidt orthonormalization process is then applied to the corresponding two estimates to give the orthonormal vectors \( A_i \) and \( B_i \), that lie, at least approximately in the 2-dimensional subspace containing \( Y_i \). \( A_i \) and \( B_i \) can now be considered as estimates of the orthonormal (g+1)-component vectors \( A \) and \( B \) which are such that

\[
Y_i = a_iA + b_iB
\]

The estimate \( Y'_i \) (eqns. 7.150 and 7.151) formed by the feedforward transversal-filter estimator is now projected orthogonally onto the 2-dimensional subspace spanned by \( A_i \) and \( B_i \), to give an improved estimate

\[
F_i = a_iA_i + b_iB_i
\]

where

\[
a_i = Y_i^*A_i^* \tag{7.155}
\]

and

\[
b_i = Y_i^*B_i^* \tag{7.155}
\]

\( A_i^* \) and \( B_i^* \) being the conjugate transposes of \( A_i \) and \( B_i \), respectively. If \( A_i \) and \( B_i \) lie exactly in the 2-dimensional subspace spanned by \( A \) and \( B \), \( F_i \) is a better estimate of \( Y_i \) than \( Y'_i \). In practice, \( A_i \) and \( B_i \) are unlikely to lie exactly in the subspace, and furthermore, since \( L \) and \( M \) may in fact vary slowly with time, so also may the subspace. This means that, for satisfactory operation, the subspace spanned by \( A_i \) and \( B_i \) should be adjusted adaptively to track the received signal. \( Y'_i \) is now used to adjust the subspace spanned by \( A_i \) and \( B_i \), and hence to determine the vectors \( A_{i+1} \) and \( B_{i+1} \) that span a subspace slightly closer to \( Y'_i \). It has been shown (183) that, for the minimum change in the subspace, given
that $|Y'_i - F'_i|$ is not large and $F'_i$ is to be moved towards $Y'_i$, the estimator forms the vectors

$$A'_{i+1} = A'_i + \eta \alpha'_i (Y'_i - F'_i) \quad (7.157)$$

and

$$B'_{i+1} = B'_i + \eta \beta'_i (Y'_i - F'_i) \quad (7.158)$$

where $\eta$ is a small positive real constant. The movement of $F'_i$ here is orthogonal to both $A'_i$ and $B'_i$. The estimator then applies the Gram-Schmidt orthonormalization process to the vectors $A'_{i+1}$ and $B'_{i+1}$ to give the required orthonormal vectors $A_{i+1}$ and $B_{i+1}$ as follows,

$$A_{i+1} = \frac{A'_{i+1}}{A'_{i+1} \cdot A_{i+1}} A'_{i+1} \quad (7.159)$$

$$B_{i+1} = \frac{B''_{i+1}}{B''_{i+1} \cdot B_{i+1}} B''_{i+1} \quad (7.160)$$

where

$$B''_{i+1} = B'_i \cdot B''_{i+1} - B'_i \cdot A_i A_i A_{i+1} \quad (7.161)$$

The estimator next evaluates the predictions $\alpha'_{i+1,i}$ and $\beta'_{i+1,i}$ of $\alpha_{i+1}$ and $\beta_{i+1}$, as follows, using a degree-1 least-square fading memory prediction (191)

$$e_{\alpha,i} = \alpha_i - \alpha_{i,i-1} \quad (7.162)$$

$$e_{\beta,i} = \beta_i - \beta_{i,i-1} \quad (7.163)$$

$$\alpha'_{i+1,i} = \alpha'_{i,i-1} + (1-\theta^2) e_{\alpha,i} \quad (7.164)$$

$$\alpha'_{i+1,i} = \alpha'_{i,i-1} + (1-\theta^2) e_{\alpha,i} \quad (7.165)$$

$$\beta'_{i+1,i} = \beta'_{i,i-1} + (1-\theta^2) e_{\beta,i} \quad (7.166)$$

$$\beta'_{i+1,i} = \beta'_{i,i-1} + (1-\theta^2) e_{\beta,i} \quad (7.167)$$

where $\theta$ is a positive real constant in the range 0 to 1 and generally close to 1. $\alpha'_{i+1,i}$ and $\beta'_{i+1,i}$ are dummy parameters introduced in the recursive process and give the rates of change with $i$ of $\alpha_i$ and $\beta_i$, respectively. The estimator now forms, the vector,

$$Y'_{i+1,i} = \alpha'_{i+1,i} A_{i+1} + \beta'_{i+1,i} B_{i+1} \quad (7.168)$$
which is the one-step prediction of $Y_{i+1}$ that is used in the feedforward transversal-filter estimator, on the receipt of the signal sample $r_{i+\ell+2}$ (at the input to the detector) to generate the error-signal $e_{i+1}$ (eqn. 7.149) and hence the updated $Y_{i+1}'$ (eqns. 7.150 and 7.151) and repeat the recursion given by eqns. 7.154-7.168. After having formed $Y_{i+1}'$, the estimator also forms the $(\ell+1)$-step prediction of $y_{i+1}$ as

$$Y_{i+\ell+1,i} = a_{i+\ell+1,i}A_{i+1} + \beta_{i+\ell+1,i}B_{i+1}$$  \hspace{1cm} (7.169)

where

$$a_{i+\ell+1,i} = a_{i+1,i} + \ell a_{i+1,i}$$  \hspace{1cm} (7.170)

and

$$\beta_{i+\ell+1,i} = \beta_{i+1,i} + \ell \beta_{i+1,i}$$  \hspace{1cm} (7.171)

The prediction $Y_{i+\ell+1,i}$ is required by the detector together with the received signal sample $r_{i+\ell+1}$ for the early detection of $s_{i+1}$ and the final detection of $s_{i-n+\ell+1}$.

7.7 COMPUTER SIMULATION TESTS

Computer simulation tests have been carried out on the detection process described in Section 7.5, when operating in the 9600 bit/s serial data transmission system considered in Section 7.4 and shown in Fig. 7.8. A complete computer simulator of the system is given in Appendix N. The model of the HF radio link used in the tests has two independent Rayleigh fading sky-waves, each introducing the same average attenuation and the same frequency spread into the data signal, such that the average attenuation over the radio link is 0 db and the frequency spread is $1/2$, 1 or 2 Hz, with a differential multipath propagation delay between the two sky-waves of 0.5, 1 or 3 ms. The parameters of the various channels used
in the tests are given in Table 7.8, and examples of the sampled impulse responses of the equipment filters are given in Tables 7.6 and 7.7. Three different sampling phases have in fact been used in the tests and show no noticeable variation in performance with the sampling phase. The signal-to-noise ratio is taken to be $\psi$ db, where $\psi$ is defined by eqn. 7.79.

The results of the tests are given in Figs. 7.13, 7.14 and 7.15. In Fig. 7.13 it is assumed that the detector has a perfect knowledge of the $\{Y_i\}$ (eqn. 7.71), but no particular conditions have been imposed on the received signal phase characteristics, so that the waveform of a received signal element at the output of the linear baseband channel in Fig. 7.8, in the absence of multipath propagation, has a non-minimum phase characteristic (the corresponding sampled impulse response $Y_i$ is obtained according to eqn. 7.69 using the $\{a_{1,k}\}, \{a_{2,k}\}$ and $\{b_k\}$ in Table 7.6).

The results given in Fig. 7.14 are for the same sampled impulse responses, but now the detector uses the prediction $Y_{i,i-2-1}^!'$ of the sampled impulse response $Y_i$, this prediction being produced by the channel estimator which uses the received signal samples $\{r_i\}$ and the early detected data symbols $\{s_{i-2}\}$ (Fig. 7.8), as described in Section 7.6. The results given in Fig. 7.15 have been obtained using the prediction $Y_{i,i-2-1}^!'$ as above, but now the waveform of a received signal element at the output of the linear baseband channel (Fig. 7.8) in the absence of multipath propagation, is minimum-phase (the corresponding sampled impulse response is obtained according to eqn. 7.69 using the $\{a_{1,k}\}, \{a_{2,k}\}$ and $\{b_k\}$ in Table 7.7).

Each of the curves given in Figs. 7.13-7.15 (apart from those taken from Reference (51)), has involved not less than 12 individual measurements of the error rate in the $\{a_k\}$ and each of these has involved the transmission of 100,000 data symbols $\{s_i\}$. In any two measurements, different
### TABLE 7.8: Channels used in the Tests

<table>
<thead>
<tr>
<th>Channel</th>
<th>Frequency spread introduced into the data signal [Hz]</th>
<th>Differential multipath propagation delay between the two sky waves [m.s.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>0.5</td>
</tr>
</tbody>
</table>
random number sequences are used in the generation of the \( \{s_i\}, \{u_i\} \) and
the Rayleigh fading of the two sky-waves, thus minimizing the influence
of the choice of the random-number sequences on the results. In all
tests, the detector uses 16 stored vectors \( \{Q_k\} (\mu=16) \) with a delay in
detection of 41 sampling periods \( (n=41) \). The delay in the detection of
the early detected data symbols \( \{s_i^{\prime}\}, \{u_i^{\prime}\} \), is given individually for
each curve in Figs. 7.14 and 7.15. The values of the parameters \( c \) and \( \theta \)
used by the estimator (eqns. 7.151 and 7.164-7.167) are taken to be 0.01
and 0.9, respectively, as these values are considered as optimum.\(^{183}\)
Since the model of the HF radio link used here assumes a constant
differential multipath propagation delay between the two sky-waves over
any one transmission, the value of \( \eta \) in eqns. 7.157 and 7.158 should be
close to zero, when the very first two estimates \( A_0 \) and \( B_0 \) are obtained
by the estimator assuming the usual synchronization process at the start
of transmission. The given results in Figs. 7.14 and 7.15 assume, in
fact, that \( A_0 \) and \( B_0 \) are obtained by applying the Gram-Schmidt ortho-
normalization process on two perfectly accurate vectors \( Y_0 \) and \( Y_{-k} \) (which
represent the sampled impulse responses of the linear baseband channel
in Fig. 7.8 at times \( t=0 \) and \( t=-kT \)), where \( k \) is such that
\[
|Y_0|^{-1} |Y_{-k}|^{-1} Y_0^* Y_{-k}^* \leq 0.5
\]
\( (7.172) \)
Therefore, and under the given conditions, \( \eta \) in eqns. 7.157 and 7.158 has
been assumed to be zero. A spot check has been made, however, to see the
effect of the value of \( \eta \) on the performance of the system, and it has been
found that values of \( \eta \approx 0.001 \) do not cause any noticeable change in the
system's performance.

The curve labelled "2400 bit/s modem" in Fig. 7.13 shows the
performance of a 2400 bit/s serial modem, with a near-maximum likelihood
detector having perfect knowledge of the channel (system 6, Reference (51)),
and has been included for the purpose of calibration, since this modem
performs much better than the corresponding parallel modem (51) whose
performance is shown also in Fig. 7.14. The inclusion of these two curves
in the results gives some feel for the magnitudes of the signal-to-noise
ratio.

The inferior performance of the 9600 bit/s system over channels 3 and
4, in the case of perfect channel estimation (Fig. 7.13), is essentially
due to the increase in signal distortion that results when the differential
multipath propagation delay \( \tau \) is increased from 1 (channels 1 and 2) to
3 ms. (Table 7.8). The frequency spread introduced by the sky-waves,
however, begins to affect the performance at the lower error rates. This
is essentially because the detector here has exact knowledge of the \( \{ Y_i \} \),
and, for a given value of \( \tau \), two channels with different values of the
frequency spread only differ significantly during the deepest fades, when
the rate of change of \( Y_i \) with \( i \), measured relative to \( Y_i \) itself, is the
most rapid. Since the lower the error rate, the more of the errors
occur only during the deepest fades, the performance should become
increasingly sensitive to the frequency spread, as the error rate reduces,
and this is what has been found to occur (Fig. 7.13). Of course, and
based on the same argument, the performance of the system over channel
5 is expected to be half way between its performances over channels 1
and 2, and the performance over channel 6 is expected to be half-way
between its performances over channels 3 and 4 (Table 7.8 and Fig. 7.13).
It can be seen from Fig. 7.13 that at an error rate of 1 in 10^4, over
channel 1, the 9600 bit/s modem loses about 10 db in tolerance to noise
compared with the 2400 bit/s modem. In fact, the 9600 bit/s modem, when operating over a perfect channel (the sampled impulse response contains only one non-zero component with value 1.0), should lose 4 dB, essentially because the 9600 bit/s modem uses a 16-point QAM signal and the 2400 bit/s modem uses a 4-point QAM signal. The further 6 dB difference in tolerance to noise is due to the increased signal distortion in the 9600 bit/s modem, especially during deep fades. In fact, the signal-element rates are 1200 and 2400 bauds in the 2400 and 9600 bit/s modems, respectively.

When actual estimation of the sampled impulse response of the linear baseband channel has been involved, correct operation is obtained over channels 1 and 3 but at a reduction of about 6 and 4 dB, respectively, in tolerance to noise at error rates around 1 in $10^4$ (Figs. 7.13 and 7.14), whereas over channels 2 and 4-6 (Table 7.8) error-free operation, even in the absence of additive noise, is no longer always obtained, when a non-minimum phase equipment filter is used, implying incorrect operation of the system under the assumed conditions. The inaccuracies in the estimates of the $\{y_i\}$ are now themselves introducing errors into the detected data-symbols. But as shown in Fig. 7.15, when the impulse response of the linear baseband channel (Fig. 7.8) has been adjusted to be minimum phase in the absence of multipath propagation, correct operation is achieved over all channels listed in Table 7.8, excluding the channels 4 and 6. The use of minimum phase equipment filters, not only enables the value of $\zeta$, the delay in the early detection of the $\{s_{1-2}\}$, to be reduced, but also improves the detector performance because it reduces the number of "non-significant" components of the sampled impulse response which are temporarily ignored by the detector (thus reducing the value of $p$, eqn.
Unfortunately, the advantage of using the minimum phase equipment filters has been marginal over those channels with a differential multi-path propagation delay of 3 ms, so that over channel 3 only a small improvement has been achieved, as can be seen from Figs. 7.14 and 7.15, and no improvement has been obtainable over channels 4 and 6 (not shown in Figs. 7.14 and 7.15). Here, the relatively large value of the delay between the two sky-waves dominates the signal distortion introduced by the HF channel. Over channel 3, the value of \( \lambda \) could not, in fact, be reduced below 30 sampling periods without a large degradation in the system performance. Actually, 3 ms delay is equivalent to at least 7 sampling periods, which when added to 18 (the number of samples resulting from convolving the \( \{a_{1,k}\} \) with the \( \{b_k\} \) in Table 7.7, according to eqn. 7.69), gives a sampled impulse response of at least 25 components and a value of \( p \) (eqn. 7.132) not less than 10. With such a sampled impulse response of the linear baseband channel, reliable values of the early detected data-symbols \( \{s''_{1-2}\} \) need an appreciable delay in detection, since otherwise the decision directed process of channel estimation produces a larger error in the channel estimate, which, in turn, causes an increased error rate in the \( \{s''_{1-2}\} \). In the evaluation of this effect, it is not sufficient, in fact, to consider the long term average of the estimation noise as defined by eqn. 7.70, since under the given conditions the estimation error has a non-stationary nature. This has been actually observed in the computer simulation tests. When the value of the differential multipath propagation delay is reduced to 1 ms, which is equivalent to 2 sampling periods, the number of components in the sampled impulse response decreases from 25 to 20 and the value of \( p \) from 10 to 5, so that \( \lambda \) could be reduced to 18, for channels 1 and 5. Under these
conditions and when operating over channel 1, the system achieves a remarkable improvement of 4 db in tolerance to noise (at an error rate of 1 in $10^4$) relative to its performance when a non-minimum phase equipment filter is used (Fig. 7.14 and 7.15). Furthermore, the system performs quite satisfactorily now over channel 5 (Fig. 7.15) after a complete failure over this channel with non-minimum phase filters. Over channel 2, where the frequency spread is 2 Hz, a smaller value of $\tau$ is required in order to obtain satisfactory predictions $\{Y_i^t,i-\tau-1\}$, but as before, this value of $\tau$ must not be too small to avoid unreliable values of the $\{S_i^t\}$. In Fig. 7.15, the performance of the system over channel 2 with $\tau=16$ is shown, where it is clear that the system now requires a very high signal-to-noise ratio in order to give a satisfactory operation. Although the system does not perform as well over channel 2 as over channels 1 and 5, which also have a 1 m.s. differential multipath propagation delay, it is nevertheless basically operating correctly under the very severe fading conditions here. As can be seen from the curves for channels 1,5 and 2 in Fig. 7.15 (which have the same value of the differential multipath propagation delay), increasing the frequency spread from 0.5 (channel 1) to 1 Hz (channel 5) degrades the performance by about 4 db, at an error rate of 1 in $10^4$, whereas increasing the frequency spread to 2 Hz (channel 2) causes a substantial degradation in performance. This is to be compared with the results given in Fig. 7.13, where the value of the frequency spread starts to affect the performance of the system only at the lower error rates, when perfect channel estimation is assumed. Clearly, the degradation is due to the prediction process, where the sampled impulse response of the channel at time $t=(i+\tau+1)T$ is to be predicted at time $t=iT$ (eqn. 7.169).
The performance of the system over channel 7 (Table 7.8) is shown in Fig. 7.15. The fading rate here is the dominant factor in determining the performance of the system. At the lower signal-to-noise ratios, the curve for channel 7 extends almost horizontally over the range 20-30 db (not shown on Fig. 7.15), where the error rate has been found to be less than 1 in 10 in all measurements carried out. After the sharp decrease in the error rate with the signal-to-noise ratio, at $\psi=38$ db, the curve tends again to be almost horizontal, as can be seen from Fig. 7.15. In both cases, at the lower and the higher error rates, error bursts seem to be starting when the signal is in a deep fade, but at the lower signal-to-noise ratios, the error burst is long and the system recovers from the error burst only after a long delay, whereas at the higher signal-to-noise ratios the system recovers from the error burst more rapidly.

An important feature of the system, which has been observed in the computer simulation tests is that it can recover from an error burst under almost all considered conditions, even at the higher error rates, implying that the system does not need repetitive re-synchronization processes.
FIGURE 7.13: Performance Over Channels 1-4 With Perfect Channel Estimation and Equipment Filters with Non-Minimum Phase Impulse Responses (Table 7.6)
FIGURE 7.14: Performance Over Channels 1 and 3 With Actual Channel Estimation and Equipment Filters with a Non-Minimum Phase Impulse Response (Table 7.6)
![Figure 7.15: Performance on Different Channels with Actual Channel Estimation and Equipment Filters with a Minimum Phase Impulse Response (Table 7.7)](image-url)
In the computer simulation tests carried out on the 19200 bit/s digital data signal transmitted over telephone lines (Section 6), it has been assumed that the receiver has a perfect knowledge of the impulse response of the channel, the carrier frequency-and-phase (in systems C and D) and the frequency offset value (in systems A and B). Also, the adaptive linear filter (Figs. 6.1-6.3) is assumed to be correctly adjusted so that the sampled impulse response of the channel and this filter together is minimum phase. The results obtained represent therefore the upper bounds to the performance of an actual system. A spot check has been made, however, to test the effect of inaccuracies in the estimates of the amplitude and phase of the received signal, on the performance of some of the systems tested. It has been found here that the transmission of digital data at a speed as high as 19200 bit/s requires in fact quite a good accuracy in the estimates of the amplitude and phase of the received signal (Figs. 6.26 and 6.27). A final decision as to the suitability of the suggested systems requires therefore more tests which involve actual adaptive adjustment of the linear adaptive filter, actual channel estimation and actual carrier phase or frequency offset estimation. A detailed study of the hardware requirements should also be carried out to determine the cost effectiveness of each of the tested systems.

In the tests carried out on the 9600 bit/s digital data signal transmitted over an HF radio link, actual channel estimation has been involved
so that the obtained results are quite informative in that respect. An important assumption has been made, however, about the reference frequency of the coherent demodulator of the QAM signal at the receiver (Fig. 7.5). It has been assumed in Section 7.3 that no constant frequency offset is present in the demodulated signal. In practice, any inaccuracy in the estimate of the carrier frequency and phase at the receiver will appear as a time-variation in the sampled impulse response of the linear baseband channel which already varies with time due to fading and multipath propagation. Therefore, tests involving a practical carrier frequency- and-phase estimator are required to determine the performance of the system under the more practical conditions.

A two Rayleigh fading sky-wave model with a constant differential multipath propagation delay has been assumed in testing the 9600 bit/s modem. As mentioned in Section 7.2, the received signal may contain more than two sky-waves in practice. But a system which shows satisfactory operation with two sky-waves only is expected also to operate satisfactorily when there are more than two sky-waves (for reasons considered in Section 7.2) so long as the spread of the received signal in time is within some limits (i.e. the time delay between the arrival of the first sky-wave and the last sky-wave should be within the values considered in the two sky-wave model). This needs to be confirmed experimentally using a model of the HF radio channel with more than two sky-waves.

As mentioned before, the results obtained over the faster fading channel are quite encouraging and suggest the possibility of considering the employed detection and estimation techniques for use with mobile radio communications. This point is well worth further investigation.
As can be seen from Figs. 7.10 and 7.11, the radio equipment filters (which belong to a practical radio system) have a much wider frequency band than do the modem filters, which implies that the available bandwidth is not fully exploited in the tests of the 9600 bit/s over the HF radio link. Therefore, there is a scope of improving the performance of the system over the faster fading channels by widening the bandwidth of the modem filters. The spread of the received signal in time is in fact dominated by the effect of the differential multipath propagation delay, and widening the filter bandwidth will have a negligible effect on the total signal spread in time, especially when a minimum phase signal is used, but it has a considerable effect on the spread in time of each received sky-wave individually, where this time spread becomes less and may lead to a better channel estimation, which is of greater importance over the faster fading channels.
9. CONCLUSIONS

The project has been concerned with the transmission of digital data signals over voice-frequency channels such as telephone lines and HF radio links, where the main impairment is additive noise and linear distortion. The characteristics of these channels have been reviewed together with the most important detection techniques.

The detailed study of QAM and baseband signals has shown the equivalence between these signals and has demonstrated the suitability of baseband signals for transmission over telephone lines, even in the presence of frequency offset. The investigation has however shown that baseband signals have no fundamental advantage over QAM signals and are no less complex, so that QAM data transmission systems are probably to be preferred in practice.

The baseband channel models of the different QAM data transmission systems considered here have been derived as a result of a systematic study, which can be used to obtain models for other linear modulation methods.

In Section 6, the transmission of digital data at 19200 bit/s over telephone lines is investigated. Since the large majority of private-line telephone circuits in the U.K. are considerably better than those of the telephone circuit 3 (Fig. 3.8), it seems that the transmission of digital data at 19200 bit/s may well be feasible over private lines, provided only that the effects of phase jitter are not too severe. The
preferred arrangement of a 19200 bit/s modem uses system B or C (Sections 6.2.2 and 6.2.3) together with detector 3 (Section 6.4.4) and equipment filters 3 (Fig. 6.12).

The detection and estimation processes described in Sections 7.5 and 7.6, respectively, for use over HF radio links, are well worth further study as potential techniques for the serial transmission of digital data at 9600 bit/s over HF radio links. The real problem here is the design of a satisfactory channel estimator (whose details are beyond the scope of this investigation) and it has been shown that, with the correct operation of such an estimator, satisfactory operation is obtained over all channels tested.
APPENDICES
The noise component in the received signal $r(t)$ (eqn. 4.15) in the QAM system is given by eqn. 4.17 as

$$u(t) = \sqrt{2} \left[ (n(t) * c(t)) e^{-j(2\pi f_c t + \theta)} \right] * b(t) \quad (A.1)$$

where $n(t)$ is a stationary white Gaussian noise with zero mean and two-sided power spectral density $\frac{1}{2}N_0$, $c(t)$ is the impulse response of a bandpass filter whose transfer function is shown in Fig. 4.4, and $b(t)$ is the impulse response of a lowpass filter with the transfer function shown in Fig. 4.5. $f_c$ is the carrier frequency, whose value is at the centre of $|C(f)|$ over positive frequencies (Fig. 4.4), and it is such that $f_c > \frac{1}{2T}$.

Under the assumed conditions, the noise signal $n(t) * c(t)$, in eqn. A.1, may be replaced by the noise signal $n_1(t) * c(t)$, where $n_1(t)$ is a stationary Gaussian noise signal with zero mean and two-sided power spectral density $\frac{1}{2}N_0$ over the frequency band of $C(f)$ only. The power spectral density of $n_1(t)$ outside the frequency band of $C(f)$ is zero. Thus, $n_1(t)$ is a bandpass stationary white Gaussian process, centred at $f_c$ and has a bandwidth of $\frac{1}{T}$ Hz (over the positive frequencies, Fig. 4.4). It may be shown that $n_1(t)$ may be expressed as a sum of two processes

$$n_1(t) = n_c(t) \cos 2\pi f_c t + n_s(t) \sin 2\pi f_c t \quad (A.2)$$

where $n_c(t)$ and $n_s(t)$ are lowpass stationary white Gaussian processes each with zero mean and two-sided power spectral density $N_0$ over the frequency band $-\frac{1}{2T}$ to $\frac{1}{2T}$ Hz and zero elsewhere. Furthermore, for the
given $|C(f)|$, the autocorrelation functions of $n_c(t)$ and $n_s(t)$ are equal\(^{(198)}\)

\[
R_{n_c}(\tau) = R_{n_s}(\tau) \tag{A.3}
\]

whereas their cross-correlation function is given by

\[
R_{n_c n_s}(\tau) = 0 \tag{A.4}
\]

i.e. they are uncorrelated. Clearly, eqn. A.2 may be re-written as

\[
n_1(t) = \frac{1}{2} \left[ (n_c(t)+jn_s(t))e^{-j2\pi f_c t} + (n_c(t)-jn_s(t))e^{j2\pi f_c t} \right] \tag{A.5}
\]

Now, replacing $n(t)$ by $n_1(t)$ in eqn. A.1 and using eqn. 4.13, gives

\[
u(t) = \sqrt{2} \left[ (n_1(t)e^{-j2\pi f_c t}) \ast (c(t)e^{-j2\pi f_c t}) \right] e^{-j\theta} \ast b(t) \tag{A.6}
\]

and then using eqn. A.5, $u(t)$ becomes

\[
u(t) = \frac{1}{\sqrt{2}} \left[ (n_c(t)-jn_s(t)) \ast c(t)e^{-j(2\pi f_c t+\theta)} \ast b(t) \right] \tag{A.7}
\]

bearing in mind that $B(f)$ (Fig. 4.5), the Fourier transform of $b(t)$, is bandlimited to $-\frac{1}{2T}$ to $\frac{1}{2T}$ Hz, so that the terms involving the factor $e^{-j4\pi f_c t}$ are eliminated. In eqn. A.7 it is assumed that $\theta$ is constant.

Let

\[
u(t) = u_1(t) + ju_2(t) \tag{A.8}
\]

where $u_1(t)$ and $u_2(t)$ are the real and imaginary parts of $u(t)$, respectively, then, from eqn. A.7 results

\[
u_1(t) = \frac{1}{\sqrt{2}} \left[ n_c(t)\ast c(t)\cos(2\pi f_c t+\theta)-n_s(t)\ast c(t)\sin(2\pi f_c t+\theta) \right] \tag{A.9}
\]

and

\[
u_2(t) = \frac{1}{\sqrt{2}} \left[ -n_s(t)\ast c(t)\cos(2\pi f_c t+\theta)-n_c(t)\ast c(t)\sin(2\pi f_c t+\theta) \right] \tag{A.10}
\]

where $b(t)$ has been dropped because its transfer function (Fig. 4.5) has the same spectral shaping as $n_c(t)$ and $n_s(t)$. Also let,

\[
\begin{align*}
v_1(t) &= n_c(t)\ast c(t)\cos(2\pi f_c t+\theta) \\
v_2(t) &= n_s(t)\ast c(t)\sin(2\pi f_c t+\theta) \\
v_3(t) &= n_s(t)\ast c(t)\cos(2\pi f_c t+\theta) \\
v_4(t) &= n_c(t)\ast c(t)\sin(2\pi f_c t+\theta)
\end{align*}
\tag{A.11}
\]
then \[ u_1(t) = \frac{1}{\sqrt{2}}[v_1(t)-v_2(t)] \] (A.12)
and \[ u_2(t) = \frac{1}{\sqrt{2}}[-v_3(t)-v_4(t)] \] (A.13)

From eqns. A.12 and A.13, the autocorrelation functions of \( u_1(t) \) and \( u_2(t) \)
are, respectively, given by

\[
R_{u_1}(\tau) = \frac{1}{2}[R_{v_1}(\tau)+R_{v_2}(\tau)-R_{v_1^*}v_2+R_{v_2^*}v_1] \quad (A.14)
\]

and

\[
R_{u_2}(\tau) = \frac{1}{2}[R_{v_3}(\tau)+R_{v_4}(\tau)+R_{v_3^*}v_4+R_{v_4^*}v_3] \quad (A.15)
\]

Now, \( v_1(t), v_2(t), v_3(t) \) and \( v_4(t) \) all are obtained by passing random
processes through time-invariant systems, therefore, from the Weiner-
Khintchine theorem, eqn. A.11 and noting that the Fourier transforms of
\( c(t)\cos(2\pi f_c t+\theta) \) and \( c(t)\sin(2\pi f_c t+\theta) \) are, respectively, given by

\[
\frac{1}{2}[C(f-f_c)e^{j\theta} + C(f+f_c)e^{-j\theta}]
\]
and

\[
\frac{1}{2}[-C(f-f_c)e^{j\theta} + C(f+f_c)e^{-j\theta}],
\]

we get,

\[
R_{v_1}(\tau) = N_0 \int_{-1/2T}^{1/2T} \frac{1}{4}|C(f-f_c)e^{j\theta} + C(f+f_c)e^{-j\theta}|^2 e^{j2\pi\tau df} dt
\]

\[
R_{v_2}(\tau) = N_0 \int_{-1/2T}^{1/2T} \frac{1}{4}[-C(f-f_c)e^{j\theta} + C(f+f_c)e^{-j\theta}|^2 e^{j2\pi\tau df} dt
\]

where, of course, \( n_c(t) \) and \( n_s(t) \) are both bandlimited to \(-\frac{1}{2T}\) to \(\frac{1}{2T}\) Hz
and each has a spectral density of \( N_0 \). Using the fact that \( |C(f)|^2 = C(f)C^*(f) \)
where \( C^*(f) \) is the complex conjugate of \( C(f) \), \( R_{v_1}(\tau) \) and \( R_{v_2}(\tau) \) become

\[
R_{v_1}(\tau) = N_0 \int_{-1/2T}^{1/2T} \frac{1}{4}|C(f-f_c)|^2 + |C(f+f_c)|^2 +
\]

\[
+C(f-f_c)C^*(f+f_c)e^{j2\theta} + C(f+f_c)C^*(f-f_c)e^{-j2\theta} e^{j2\pi\tau df}
\]

and

\[
R_{v_2}(\tau) = N_0 \int_{-1/2T}^{1/2T} \frac{1}{4}|C(f-f_c)|^2 + |C(f+f_c)|^2 -
\]

\[
-C(f-f_c)C^*(f+f_c)e^{j2\theta} - C(f+f_c)C^*(f-f_c)e^{-j2\theta} e^{j2\pi\tau df}
\]

(A.16)
Similarly, it may be shown that,
\[ R_{v_3}(\tau) = R_{v_1}(\tau) \] (A.18)
and
\[ R_{v_4}(\tau) = R_{v_2}(\tau) \] (A.19)
since \( n_s(t) \) and \( n_c(t) \) in eqns. A.11 have the same spectral densities as mentioned before.

Let \( W_{v_1,v_2}(f) \) be the cross-power density of \( v_1(t) \) and \( v_2(t) \), then the cross-correlation function of \( v_1(t) \) and \( v_2(t) \) is given by
\[ R_{v_1,v_2}(\tau) = \int_{-\infty}^{\infty} W_{v_1,v_2}(f)e^{j2\pi \tau f} df \] (A.20)

Now, from eqns. A.11\(^{193}\)
\[ W_{v_1,v_2}(f) = W_{n_c,n_s}(f)H(f) \] (A.21)
where \( W_{n_c,n_s}(f) \) is the cross-power density of \( n_c(t) \) and \( n_s(t) \) and
\[ H(f) = \frac{1}{4}[C(f-f_c)e^{j\theta} + C(f+f_c)e^{-j\theta}] [-jC(f-f_c)e^{j\theta} + jC(f+f_c)e^{-j\theta}] \] (A.22)
But\(^{193}\)
\[ W_{n_c,n_s}(f) = \int_{-\infty}^{\infty} R_{n_c,n_s}(\tau)e^{-j2\pi \tau f} d\tau = 0 \] (A.23)
because of eqn. A.4. Thus, \( W_{v_1,v_2}(f) = 0 \) (eqn. A.21) and therefore eqn. A.20 gives
\[ R_{v_1,v_2}(\tau) = 0 \] (A.24)
In the same way it may be shown that
\[ R_{v_2,v_1}(\tau) = R_{v_3,v_4}(\tau) = R_{v_4,v_3}(\tau) = 0 \] (A.25)
\[ R_{u_1}(\tau) = \frac{1}{4} N_0 \int \left[ |C(f-f_c)|^2 + |C(f+f_c)|^2 \right] e^{j2\pi \tau f} df \] (A.26)
and
\[ R_{u_2}(\tau) = R_{u_1}(\tau) \] (A.27)
Notice that $R_{u_1}(\tau)$ and $R_{u_2}(\tau)$ are real-valued and even, because

$$[|C(f-f_c)|^2 + |C(f+f_c)|^2]$$

is an even function of $f$.

From eqns. A.12 and A.13, the cross-correlation function of $u_1(t)$ and $u_2(t)$ is given by

$$R_{u_1,u_2}(\tau) = \frac{1}{4} [-R_{v_1,v_3}(\tau)+R_{v_1,v_4}(\tau)+R_{v_2,v_3}(\tau)+R_{v_2,v_4}(\tau)]$$  \hspace{1cm} (A.28)

From eqns. A.11, and following the same procedures as in eqns. A.20 to A.23, it may be shown that

$$R_{v_1,v_3}(\tau) = R_{v_2,v_4}(\tau) = 0$$  \hspace{1cm} (A.29)

whereas

$$R_{v_1,v_4}(\tau) = \int_{-\infty}^{\infty} W_{v_1,v_4}(\xi) e^{j2\pi f_{c}\tau} d\xi$$  \hspace{1cm} (A.30)

and

$$R_{v_2,v_3}(\tau) = \int_{-\infty}^{\infty} W_{v_2,v_3}(\xi) e^{j2\pi f_{c}\tau} d\xi$$  \hspace{1cm} (A.31)

where $W_{v_1,v_4}(\xi)$ and $W_{v_2,v_3}(\xi)$ are the cross-power densities of $(v_1,v_4)$ and $(v_2,v_3)$, respectively, and given by

$$W_{v_1,v_4}(\xi) = W_{n_c,n_c}(\xi)H(\xi)$$  \hspace{1cm} (A.32)

$$W_{v_2,v_3}(\xi) = W_{n_s,n_s}(\xi)H^*(\xi)$$  \hspace{1cm} (A.33)

where

$$W_{n_c,n_c}(\xi) = W_{n_s,n_s}(\xi) = 0, \quad -\frac{1}{2T} < f < \frac{1}{2T}$$  \hspace{1cm} (A.34)

and $H(\xi)$ is given by eqn. A.22.

From eqns. A.28-A.34 results

$$R_{u_1,u_2}(\tau) = \frac{1}{4} N_0 \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{1}{2T} [-H(\xi)+H^*(\xi)] e^{j2\pi f_{c}\tau} d\xi$$  \hspace{1cm} (A.35)

which, with eqn. A.22 gives

$$R_{u_1,u_2}(\tau) = \frac{1}{4} N_0 \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \left[ |C(f-f_c)|^2 - |C(f+f_c)|^2 \right] e^{j2\pi f_{c}\tau} d\xi$$  \hspace{1cm} (A.36)
Notice here that \( R_{u_1, u_2}(\tau) \) is an odd real-valued function of \( \tau \), because \( |C(f-f_c)|^2 - |C(f+f_c)|^2 \) is an odd function of \( f \). This means that,

\[
R_{u_1, u_2}(-\tau) = -R_{u_1, u_2}(\tau) \tag{A.37}
\]

but (193)

\[
R_{u_2, u_1}(\tau) = R_{u_1, u_2}(-\tau) \tag{A.38}
\]

hence

\[
R_{u_2, u_1}(\tau) = -R_{u_1, u_2}(\tau) \tag{A.39}
\]

Now, let the autocorrelation function of the complex-valued \( u(t) \) (eqn. A.8) be defined as

\[
R_u(\tau) = E[u^*(t)u(t+\tau)] \tag{A.40}
\]

where \( u^*(t) \) is the complex conjugate of \( u(t) \), then

\[
R_u(\tau) = R_{u_1, u_2}(\tau) + j[R_{u_1, u_2}(\tau) - R_{u_2, u_1}(\tau)] \tag{A.41}
\]

which, according to eqns. A.27 and A.39, reduces to

\[
R_u(\tau) = 2R_{u_1, u_2}(\tau) + j2R_{u_1, u_2}(\tau) \tag{A.42}
\]

From eqn. A.42, it is clear that the autocorrelation function of each of the real and imaginary parts of \( u(t) \) is given by half the real part of \( R_u(\tau) \), whereas the cross-correlation function of these parts is given by half the imaginary part of \( R_u(\tau) \). The important result, which may be derived from eqn. A.42, is that, for this given case, the real and imaginary parts of \( u(t) \) are uncorrelated when \( R_u(\tau) \) is purely real-valued.

Now, eqns. A.26, A.36 and A.42 give,

\[
R_u(\tau) = N_0 \int_{-1/2T}^{1/2T} \left| C(f-f_c) \right|^2 e^{j2\pi f\tau} df \tag{A.43}
\]

which is to be proved.
APPENDIX B

NOISE CORRELATION IN THE ONE-DIMENSIONAL BASEBAND SYSTEM

The noise component in the received signal \( r(t) \) (eqn. 5.44) in the one-dimensional baseband data transmission system (Fig. 5.2) is given by eqn. 5.45 which is

\[
u(t) = \text{Real}\{n(t)*[c(t)+j\hat{c}(t)]e^{-j2\pi pt}\}
\]

where \( n(t) \) is a stationary white Gaussian noise process with zero mean and two-sided power spectral density of \( \frac{1}{2N_0} \). \( \hat{c}(t) \) is the Hilbert transform of \( c(t) \), and \( p \) is the frequency offset value which is constant.

Let

\[
\begin{align*}
n_1(t) &= n(t) * c(t) \\
n_2(t) &= n(t) * \hat{c}(t)
\end{align*}
\]

then eqn. B.1 becomes,

\[
u(t) = \text{Real}\{[n_1(t)+jn_2(t)]e^{-j2\pi pt}\}
\]

which gives

\[
u(t) = n_1(t)\cos(2\pi pt) + n_2(t)\sin(2\pi pt)
\]

The autocorrelation function of \( u(t) \) is defined as

\[
R_u(\tau) = E[u(t)u(t+\tau)]
\]

where \( E[.\] denotes an ensemble average. Since the two trigonometric functions in eqn. B.5 are deterministic functions of time, they are not involved in the ensemble average in eqn. B.6. Thus, from eqns. B.5 and B.6 results,
\[ R_1(\tau) = R_{n_1}(\tau) \cos(2\pi pt) \cos(2\pi p(t+\tau)) + \]
\[ + R_{n_2}(\tau) \sin(2\pi pt) \sin(2\pi p(t+\tau)) + \]
\[ + R_{n_1,n_2}(\tau) \cos(2\pi pt) \sin(2\pi p(t+\tau)) + \]
\[ + R_{n_2,n_1}(\tau) \sin(2\pi pt) \cos(2\pi p(t+\tau)) \]  
(B.7)

Let the power spectral densities of \( n_1(t) \) and \( n_2(t) \) (eqns. B.2 and B.3) be denoted \( W_{n_1}(f) \) and \( W_{n_2}(f) \), respectively, then

\[ W_{n_1}(f) = \frac{1}{N_0} |C(f)|^2 \]  
(B.8)

and

\[ W_{n_2}(f) = \frac{1}{N_0} |\hat{C}(f)|^2 \]  
(B.9)

where \( C(f) \) and \( \hat{C}(f) \) are the Fourier transforms of \( c(t) \) and \( \hat{c}(t) \), respectively. \( \hat{C}(f) \) is, in fact, given by

\[ \hat{C}(f) = C(f)F(f) \]  
(B.10)

where

\[ F(f) = \begin{cases} j & f < 0 \\ 0 & f = 0 \\ -j & f > 0 \end{cases} \]  
(B.11)

From eqns. B.10 and B.11, it is clear that \( |\hat{C}(f)|^2 = |C(f)|^2 \), so that eqns. B.8 and B.9 give,

\[ W_{n_1}(f) = W_{n_2}(f) = \frac{1}{N_0} |C(f)|^2 \]  
(B.12)

Now, by the Wiener-Khintchine theorem,

\[ R_{n_1}(\tau) = \int_{-\infty}^{\infty} W_{n_1}(f)e^{j2\pi ft}\,df \]  
(B.13)

and

\[ R_{n_2}(\tau) = \int_{-\infty}^{\infty} W_{n_2}(f)e^{j2\pi ft}\,df \]  
(B.14)

Thus, from eqns. B.12-B.14,

\[ R_{n_1}(\tau) = R_{n_2}(\tau) = \frac{1}{N_0} \int_{-\infty}^{\infty} |C(f)|^2 e^{j2\pi ft}\,df \]  
(B.15)

On the other hand,
where \( W_{n_1 n_2}(f) \) is the cross-power spectra of \( n_1(t) \) and \( n_2(t) \) (eqns. B.2-B.3) and is given by (193)

\[
W_{n_1 n_2}(f) = C(-f)C(f)W_n(f)
\]  

\[
= C(-f)C(f)(\tilde{N}_0)
\]  

where \( W_n(f) \) is the power spectral density of \( n(t) \). Eqns. B.10, B.16 and B.17 give

\[
R_{n_1 n_2}(\tau) = \mathcal{N}_0 \int_{-\infty}^{\infty} C(-f)\hat{C}(f)e^{j2\pi f\tau}df
\]

\[
= \mathcal{N}_0 \int_{-\infty}^{\infty} |C(f)|^2P(f)e^{j2\pi f\tau}df
\]  

(B.18)

Similarly, it may be shown that

\[
R_{n_2 n_1}(\tau) = \mathcal{N}_0 \int_{-\infty}^{\infty} C(f)\hat{C}(-f)e^{j2\pi f\tau}df
\]

\[
= \mathcal{N}_0 \int_{-\infty}^{\infty} |C(f)|^2P(-f)e^{j2\pi f\tau}df
\]  

(B.19)

According to eqn. B.11,

\[
F(-f) = -F(f)
\]  

(B.20)

so that eqns. B.18 and B.19 give

\[
R_{n_2 n_1}(\tau) = -R_{n_1 n_2}(\tau)
\]  

(B.21)

Now, since \( R_{n_1}(\tau) = R_{n_2}(\tau) \) (eqn. B.15) and using eqn. B.21, eqn. B.7 becomes,

\[
R_u(\tau) = R_{n_1}(\tau)\cos(2\pi \tau) + R_{n_1 n_2}(\tau)\sin(2\pi \tau)
\]

\[
= \text{Real}\{[R_{n_1}(\tau) + jR_{n_1 n_2}(\tau)]e^{-j2\pi \tau}\}
\]  

(B.22)

Replacing \( R_{n_1}(\tau) \) and \( R_{n_1 n_2}(\tau) \) from eqns. B.15 and B.18 in eqn. B.22 gives

\[
 R_u(\tau) = \text{Real}\{\mathcal{N}_0 e^{-j2\pi \tau} \int_{-\infty}^{\infty} |C(f)|^2[1+jF(f)]e^{j2\pi f\tau}df\}
\]  

(B.23)
or equivalently,

\[ R_u(\tau) = N_0 \text{Real}\left\{ \int_0^\infty |C(f)|^2 e^{j2\pi(f-p)\tau} df \right\} \]  \hspace{1cm} (B.24)

where, according to eqn. B.11,

\[ 1 + jF(f) = \begin{cases} 2 & f > 0 \\ 0 & f < 0 \end{cases} \]  \hspace{1cm} (B.25)
APPENDIX C

NOISE CORRELATION IN THE TWO-DIMENSIONAL BASEBAND SYSTEM

The noise component in the received signal r(t) (eqn. 5.68 and Fig. 5.5) in the two-dimensional baseband data transmission system is given by eqn. 5.70 which is

\[ u(t) = \frac{1}{\sqrt{2}} [n(t) \ast (c(t) + j\hat{c}(t))]e^{-j2\pi pt} \]  

(C.1)

where \( n(t) \) is a white Gaussian noise with zero mean and two-sided power spectral density \( \frac{1}{2}N_0 \). \( \hat{c}(t) \) is the Hilbert transform of \( c(t) \), and \( p \) is the frequency offset value which is constant.

Let

\[ n_1(t) = n(t) \ast c(t) \]  

(C.2)

\[ n_2(t) = n(t) \ast \hat{c}(t) \]  

(C.3)

and

\[ \nu(t) = u_1(t) + ju_2(t) \]  

(C.4)

then, the real and imaginary parts of \( \nu(t) \) are given by

\[ u_1(t) = \frac{1}{\sqrt{2}} [n_1(t)\cos(2\pi pt) + n_2(t)\sin(2\pi pt)] \]  

(C.5)

and

\[ u_2(t) = \frac{1}{\sqrt{2}} [n_2(t)\cos(2\pi pt) - n_1(t)\sin(2\pi pt)] \]  

(C.6)

respectively.

The autocorrelation functions of each of \( u_1(t) \) and \( u_2(t) \) are given by (200-201)
\[ R_{u_1} (\tau) = E[u_1(t)u_1(t+\tau)] \]  
\[ R_{u_2} (\tau) = E[u_2(t)u_2(t+\tau)] \]  
respectively, where \( E[.] \) denotes an ensemble average. The cross-correlation between \( u_1(t) \) and \( u_2(t) \) is given by (200-201) 
\[ R_{u_1,u_2} (\tau) = E[u_1(t)u_2(t+\tau)] \]  
where (193) 
\[ R_{u_2,u_1} (\tau) = R_{u_1,u_2} (-\tau) \] 

Now, from eqns. C.5 and C.6, and since the trigonometric functions are deterministic functions of time (\( p \) is constant),

\[ R_{u_1} (\tau) = \frac{1}{2} \left[ R (\tau) \cos(2\pi pt) \cos(2\pi p(t+\tau)) + \right. 
\left. + R (\tau) \sin(2\pi pt) \sin(2\pi p(t+\tau)) - 
\left. \right] n_1 \right) \] 
\[ + R (\tau) \cos(2\pi pt) \sin(2\pi p(t+\tau)) + 
\left. \right] n_2 \right) \] 
\[ + R (\tau) \sin(2\pi pt) \cos(2\pi p(t+\tau)) \] 
\[ + R (\tau) \sin(2\pi pt) \cos(2\pi p(t+\tau)) \] 
(C.11)

\[ R_{u_2} (\tau) = \frac{1}{2} \left[ R (\tau) \cos(2\pi pt) \cos(2\pi p(t+\tau)) + 
\left. \right] n_2 \right) \] 
\[ + R (\tau) \sin(2\pi pt) \sin(2\pi p(t+\tau)) + 
\left. \right] n_1 \right) \] 
\[ - R (\tau) \cos(2\pi pt) \sin(2\pi p(t+\tau)) - 
\left. \right] n_2 \right) \] 
\[ - R (\tau) \sin(2\pi pt) \cos(2\pi p(t+\tau)) \] 
\[ - R (\tau) \sin(2\pi pt) \cos(2\pi p(t+\tau)) \] 
(C.12)

and

\[ R_{u_1,u_2} (\tau) = \frac{1}{2} \left[ R (\tau) \cos(2\pi pt) \cos(2\pi p(t+\tau)) - 
\right. 
\left. \right] n_1 \right) \] 
\[ - R (\tau) \sin(2\pi pt) \sin(2\pi p(t+\tau)) - 
\left. \right] n_2 \right) \] 
\[ - R (\tau) \cos(2\pi pt) \sin(2\pi p(t+\tau)) + 
\left. \right] n_1 \right) \] 
\[ + R (\tau) \sin(2\pi pt) \cos(2\pi p(t+\tau)) \] 
\[ + R (\tau) \sin(2\pi pt) \cos(2\pi p(t+\tau)) \] 
(C.13)
where $R_{n_1}(\tau)$ and $R_{n_2}(\tau)$ are the autocorrelation functions of $n_1(t)$ and $n_2(t)$, respectively, whereas $R_{n_1,n_2}(\tau)$ is the cross-correlation function of $n_1(t)$ and $n_2(t)$ and

$$R_{n_2,n_1}(\tau) = R_{n_1,n_2}(-\tau) \quad (C.14)$$

Now, $n_1(t)$ and $n_2(t)$ in eqns. C.2 and C.3 are the same as those given by eqns. B.2 and B.3 in Appendix B. Thus, from eqns. B.15, B.18 and B.21,

$$R_{n_1}(\tau) = R_{n_2}(\tau) \quad (C.15)$$
$$R_{n_2,n_1}(\tau) = -R_{n_1,n_2}(\tau) \quad (C.16)$$

where

$$R_{n_1}(\tau) = \frac{1}{N_0} \int_{-\infty}^{\infty} |C(f)|^2 e^{j2\pi f \tau} df \quad (C.17)$$
$$R_{n_1,n_2}(\tau) = \frac{1}{N_0} \int_{-\infty}^{\infty} |C(f)|^2 F(f) e^{j2\pi f \tau} df \quad (C.18)$$

and $F(f)$ is given by eqn. B.11. Now, and according to eqns. C.15-C.16, eqns. C.11-C.13 become

$$R_{u_1}(\tau) = \frac{1}{4}[R_{n_1}(\tau)\cos(2\pi \tau) + R_{n_1,n_2}(\tau)\sin(2\pi \tau)] \quad (C.19)$$
$$R_{u_2}(\tau) = R_{u_1}(\tau) \quad (C.20)$$

and

$$R_{u_1,u_2}(\tau) = \frac{1}{4}[R_{n_1,n_2}(\tau)\cos2\pi \tau - R_{n_1}(\tau)\sin2\pi \tau] \quad (C.21)$$

According to eqns. C.14 and C.16

$$R_{n_1,n_2}(\tau) = -R_{n_1,n_2}(-\tau) \quad (C.22)$$

which means that $R_{n_1,n_2}(\tau)$ is an odd function of $\tau$. But since $R_{n_1}(\tau)$ is an even function of $\tau$ (property of autocorrelation functions), then $R_{u_1}(\tau)$ and $R_{u_2}(\tau)$ are even functions of $\tau$ (as they should be) and $R_{u_1,u_2}(\tau)$ is an odd function of $\tau$, which gives,

$$R_{u_1,u_2}(\tau) = -R_{u_1,u_2}(-\tau) \quad (C.23)$$
Therefore
\[ R_{u_2,u_1}(\tau) = -R_{u_1,u_2}(\tau) \]  \hspace{1cm} (C.24)

because
\[ R_{u_1,u_2}(\tau) = R_{u_2,u_1}(\tau) \]  \hspace{1cm} (C.25)

Defining now the autocorrelation function of the complex process \( u(t) \) (eqn. C.4) as
\[ R_u(\tau) = E[u^*(t)u(t+\tau)] \]
\[ = E[(u_1(t) - ju_2(t))(u_1(t+\tau) + ju_2(t+\tau))] \]  \hspace{1cm} (C.26)

where \( u^*(t) \) is the complex conjugate of \( u(t) \), then
\[ R_u(\tau) = R_{u_1}(\tau) + R_{u_2}(\tau) + j[R_{u_1,u_2}(\tau) - R_{u_2,u_1}(\tau)] \]  \hspace{1cm} (C.27)

which, with eqns. C.20 and C.24 give
\[ R_u(\tau) = 2R_{u_1}(\tau) + 2jR_{u_1,u_2}(\tau) \]  \hspace{1cm} (C.28)

From eqns. C.19, C.21 and C.28,
\[ R_u(\tau) = R_{n_1,n_2}(\tau)\cos(2\pi\tau) + R_{n_1,n_2}(\tau)\sin(2\pi\tau) + j[R_{n_2,n_1}(\tau)\cos(2\pi\tau) - R_{n_1,n_2}(\tau)\sin(2\pi\tau)] \]
\[ = [R_{n_1,n_2}(\tau) + jR_{n_2,n_1}(\tau)]e^{-j2\pi\tau} \]  \hspace{1cm} (C.29)

But, from eqns. C.17 and C.18,
\[ R_{n_1,n_2}(\tau) = \frac{1}{N_0} \int_{-\infty}^{\infty} |C(f)|^2 [1 + jF(f)]e^{j2\pi f\tau} df \]  \hspace{1cm} (C.30)

so that eqn. C.29 becomes,
\[ R_u(\tau) = N_0 \int_{0}^{\infty} |C(f)|^2 e^{j2\pi(f-p)\tau} df \]  \hspace{1cm} (C.31)

where \( F(f) \) has been replaced from eqn. B.11. Eqn. C.31 is which had to be derived.
From eqn. C.28, it is clear that the autocorrelation functions of each of \( u_1(t) \) and \( u_2(t) \) are given by \( \Re[R_u(\tau)] \) whereas the imaginary part of \( R_u(\tau) \) expresses the presence of the correlation between the real and imaginary parts of \( u(t) \), and when \( R_u(\tau) \) is purely real, then these parts are uncorrelated.
APPENDIX D

The effect of changing the sign of every second sample of a sampled impulse response on the minimum distance between noise-free signal-vectors in the vector space may be viewed as follows.

Let the sampled impulse response of the channel under consideration be given by the \((g+1)\)-component vector \(Y\), where,

\[
Y = y_0, y_1, \ldots, y_g \tag{D.1}
\]

and let \(S_k\) and \(Z_k\) be the \(k\)-component vectors whose \(i\)th components are \(s_i\) and \(z_i\), respectively. \(s_i\) is a data symbol and may have one of a finite set of discrete values, and

\[
z_i = \sum_{h=0}^{g} s_{i-h} y_h \tag{D.2}
\]

In the signal vector space, containing all possible vectors \(\{Z_k\}\) corresponding to all possible data sequences \(\{S_k\}\), the distance between any two vectors \(Z_{1,k}\) and \(Z_{2,k}\) is given by

\[
d = |Z_{1,k} - Z_{2,k}| \tag{D.3}
\]

where \(|X|\) represents the length of the vector \(X\). The minimum value of \(d\), denoted \(d_{\text{min}}\), determines the performance of the maximum-likelihood detector which detects the \(\{s_i\}\) from the \(\{z_i\}\), when to the latter are added statistically independent Gaussian random variables having relatively small values (giving a high signal-to-noise ratio).

Let \(S(z)\), \(Z(z)\) and \(Y(z)\) be the \(z\)-transforms of the sequences \(S_k\), \(Z_k\) and \(Y\), respectively, then eqn. D.2 may be written in terms of the \(z\)-transforms as

\[
Z(z) = S(z)Y(z) \tag{D.4}
\]
Now, let $z$ be replaced by $(-z)$ in eqn. D.4, which gives

$$Z(-z) = S(-z)Y(-z) \quad (D.5)$$

But changing the sign of $z$ in $Z(z), S(z)$ and $Y(z)$ corresponds to changing the sign of every second component in the sequences $Z_k, S_k$ and $Y$. Thus, changing the sign of every second component of $Y$, if accompanied by changing the sign of every second component of the data sequence $S_k$, results in a change in the sign of every second component of the corresponding signal sequence $Z_k$. In other words, changing the sign of every second component of $Y$ corresponds to the joint transformation of all possible sequences $\{S_k\}$ and $\{Z_k\}$ into the sequences $\{S_k'\}$ and $\{Z_k'\}$, respectively, where a sequence $S_k'$ or $Z_k'$ is obtained by changing the sign of every second component of the corresponding sequence $S_k$ or $Z_k$, respectively. But since any sequence $S_k$ or $S_k'$ belong to the same set of all possible sequences $\{S_k\}$, i.e. since

$$\{S_k\} \equiv \{S_k'\}, \quad (D.6)$$

changing the sign of every second component of $Y$ corresponds to transforming the set $\{Z_k\}$ into the set $\{Z_k'\}$, where for any sequence $Z_k$ in the first set, there is a sequence $Z_k'$ in the other set which is obtained by changing the sign of every second component of $Z_k$. It should be noted here that eqn. D.6 does not represent a point-to-point mapping, but expresses the fact that both sets $\{S_k\}$ and $\{S_k'\}$ have the same content.

Now, in the signal vector space containing all possible vectors $\{Z_k'\}$, the distance between any two vectors $Z'_{1,k}$ and $Z'_{2,k}$ is given by

$$d' = |Z'_{1,k} - Z'_{2,k}| \quad (D.7)$$

But, the vector $Z'_{1,k} - Z'_{2,k}$ is obtained by changing the sign of every second component in the vector $Z_{1,k} - Z_{2,k}$, and this change has no effect on the vector's length. Hence, eqn. D.7 may be written as
\[ d' = |z_{1,k} - z_{2,k} | \]  

which if compared with eqn. D.3 gives,

\[ d' = d \]  

and the transformation of the signal vector space containing the \( \{z_k\} \) into the signal vector space containing the \( \{z'_k\} \) by changing the sign of every second sample of the sampled impulse response, preserves relative distances between signal vectors, including, of course, the minimum distance \( d_{\min} \). As a result, the assumed transformation does not affect the performance of the maximum likelihood detector.
APPENDIX E

THE SCALING RELATIONSHIP BETWEEN THE FOURIER
TRANSFORM AND THE DISCRETE FOURIER TRANSFORM

Let $Y(f)$ be a bandlimited spectrum whose inverse Fourier transform is the continuous waveform $y(t)$. From Parseval's theorem we have,

$$\int_{-\infty}^{\infty} |Y(f)|^2 df = \int_{-\infty}^{\infty} |y(t)|^2 dt \quad (E.1)$$

Let $y(t)$ now be sampled at not less than the Nyquist rate to give the samples $\{y_h\}$, where

$$y_h = y(hT) \quad (E.2)$$

and $T$ is the sampling period. Now $y(t)$ may be expressed as (195)

$$y(t) = \sum_{h=-\infty}^{\infty} y_h \frac{\sin\pi \frac{t}{T} - h}{\pi \left(\frac{t}{T} - h\right)} \quad (E.3)$$

where the $\left\{\frac{\sin\pi \left(\frac{t}{T} - h\right)}{\pi \left(\frac{t}{T} - h\right)}\right\}$ are a set of orthonormal functions. Thus,

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \sum_{h=-\infty}^{\infty} |y_h|^2 \int_{-\infty}^{\infty} \left[\frac{\sin\pi \left(\frac{t}{T} - h\right)}{\pi \left(\frac{t}{T} - h\right)}\right]^2 dt \quad (E.4)$$

The integral on the right-hand side is equal to $T$ (193). Therefore, eqn. E.4 becomes,

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = T \sum_{h=-\infty}^{\infty} |y_h|^2 \quad (E.5)$$

Eqns. E.1 and E.5 give,

$$\int_{-\infty}^{\infty} |Y(f)|^2 df = T \sum_{h=-\infty}^{\infty} |y_h|^2 \quad (E.6)$$
On the other hand, if \( Y(f) \) is sampled at frequency intervals of \( \Delta f \) Hz to give the spectrum samples \( \{Y_k\} \), where

\[
Y_k = Y(k\Delta f)
\] (E.7)

and when \( \Delta f \) is small enough so that \( Y(f) \) can be recovered from the \( \{Y_k\} \) with negligible error, then

\[
\int_{-\infty}^{\infty} |Y(f)|^2 df = \Delta f \sum_k |Y_k|^2
\] (E.8)

which with eqn. E.6 give,

\[
\Delta f \sum_k |Y_k|^2 = T \sum_{h=-\infty}^{\infty} |y^*_h|^2
\] (E.9)

Now, when the \( \{Y_k\} \) are fed to an inverse discrete Fourier transform (IDFT), the resultant samples \( \{y'_h\} \) satisfy (81)

\[
\sum_k |Y_k|^2 = N \sum_h |y^*_h|^2
\] (E.10)

where \( N \) is the number of samples used in the IDFT. But \( \Delta f \), \( N \) and \( T \) are related by (81)

\[
\Delta f = \frac{1}{NT}
\] (E.11)

so that eqns. E.10 and E.11 give,

\[
\Delta f \sum_k |Y_k|^2 = \frac{1}{T} \sum_h |y^*_h|^2
\] (E.12)

Comparing now eqn. E.12 with eqn. E.9 gives

\[
\sum_h |y^*_h|^2 = \frac{1}{T^2} \sum_h |y^*_h|^2
\] (E.13)

so that, when the shape of the envelope of the \( \{y^*_h\} \) and \( \{y^*_h\} \) is the same, then,

\[
y^*_h = \frac{1}{T} y^*_h
\] (E.14)

which means that the samples obtained by means of the IDFT from a given sampled spectrum must be scaled by the sampling rate to give the \( \{y^*_h\} \).

The latter are obtained by sampling (at the Nyquist rate) the continuous waveform which is the inverse Fourier transform of the given spectrum.
APPENDIX F

THE DISCRETE-TIME MODEL OF ADDITIVE NOISE

IN DATA TRANSMISSION SYSTEMS

F.1 BASIC MODEL

In the analysis and simulation of data transmission systems it is conventional to assume that the only additive noise in the received signal is a white Gaussian noise which is added at the output of the transmission path. (52-162) Fig. F.1 shows the part of the data transmission system concerned with the additive noise (see also Figs. 3.2, 3.9, 4.1, 5.2 and 5.5). The data signal m(t), at the output of the transmission path, is corrupted by additive white Gaussian noise n(t) with zero mean and a two-sided power spectral density $\frac{1}{2}N_0$. The noisy data signal z(t) is fed to a receiver filter, which ideally should be matched to the signal m(t), where this filter removes the noise frequencies outside the signal bandwidth without excessively distorting the data signal itself. In the analysis to come, the general case is considered, where the receiver filter may or may not be a matched filter. This filter has a real-valued impulse response c(t) and a transfer function C(f) such that,

$$C(f) = 0, \quad |f| > W \text{ Hz} \quad (F.1)$$

The signal r(t), at the output of the receiver filter is now sampled at a rate of $\frac{1}{T_1}$ to give the signal sequence $\{r_n\}$, from which the transmitted data are detected. No particular relationship is assumed here between the sampling rate $\frac{1}{T_1}$ and the transmitted signal element rate $\frac{1}{T}$, but it will be assumed that
WHITE GAUSSIAN NOISE \( n(t) \)

DATA SIGNAL \( m(t) \) → \( z(t) \) → \( c(t) \) → \( r(t) = p(t) + u(t) \) → \( \{ r_i \} \)

FIGURE F.1: Model of Additive Noise in Data Transmission Systems

WHITE GAUSSIAN NOISE \( n(t) \)

DATA SIGNAL \( m(t) \) → \( z'(t) \) → \( c(t) \) → \( r(t) = p(t) + u(t) \) → \( \{ r_i \} \)

FIGURE F.2: Additive Noise Model Equivalent to the Model in Fig. F.1.
where $W$ is defined by eqn. F.1. From Fig. F.1,

$$r(t) = p(t) + u(t)$$  \hspace{1cm} (F.3)

where

$$p(t) = m(t) * c(t)$$  \hspace{1cm} (F.4)

and

$$u(t) = n(t) * c(t)$$  \hspace{1cm} (F.5)

$p(t)$ and $u(t)$ are the signal and noise components, respectively in $r(t)$. The signal at the sampler output is now given by

$$r_i = p_i + u_i$$ \hspace{1cm} (F.6)

where

$$p_i = p(it_1)$$ \hspace{1cm} (F.7)

and

$$u_i = u(it_1)$$ \hspace{1cm} (F.8)

The $\{u_i\}$ are the noise components in the received signal samples $\{r_i\}$.

Now, in the computer simulation of the digital data transmission system, the different signals in the system are processed in the discrete-time domain in a digital form. In fact, in Fig. F.1, the $\{r_i\}$ are the only signals which have a discrete-time nature and which may be processed directly by computer simulation, but the continuous signals, and in particular, the noise $n(t)$, need to be modelled correctly by the corresponding discrete-time signals in order to obtain valid simulation results.

Digital processing of analogue signals requires first the quantization of these signals both in amplitude and in time. Quantization in amplitude depends in fact on the computer in use, and usually has an entirely negligible effect on the results of simulation, especially if a powerful computer is used. Therefore, it will be assumed here that there is no quantization in amplitude (e.g. eqns. F.7 and F.8). In what follows, the interest is in generating the $\{u_i\}$ (eqn. F.6) by computer
so that their statistics correspond exactly to those of the \( \{ u_i \} \) in Fig. F.1.

First let the statistics of the \( \{ u_i \} \) in Fig. F.1 be determined.

The autocorrelation function of \( u(t) \) (eqn. F.5) is, according to the Wiener-Khintchine theorem, given by

\[
R_u(\tau) = \int_{-\infty}^{\infty} |U(f)|^2 e^{i2\pi f \tau} df
\]  

where \( |U(f)|^2 \) is the spectral density of \( u(t) \). From eqn. F.5,

\[
|U(f)|^2 = \frac{1}{2N} |C(f)|^2
\]  

where \( \frac{1}{2N} \) is, of course, the two-sided power spectral density of \( n(t) \), and \( C(f) \) is given by eqn. F.1. Thus, eqn. F.9, becomes,

\[
R_u(\tau) = \frac{1}{2N} \int_{-W}^{W} |C(f)|^2 e^{i2\pi f \tau} df
\]  

The value of \( R_u(\tau) \), for a given value of \( \tau \), gives the correlation between \( u(t) \) and \( u(t+\tau) \). At \( \tau=kT \), \( R_u(kT) \) gives the correlation between \( u(t) \) and \( u(t+kT) \), or equivalently between \( u(iT) \) and \( u((i+k)T) \). In other words, the correlation between two samples \( u_i \) and \( u_{i+k} \) (eqn. F.8) is given by

\[
R_u(kT) = \frac{1}{2N} \int_{-W}^{W} |C(f)|^2 e^{i2\pi f kT} df
\]  

The variance of the \( \{ u_i \} \) is given by \( R_u(0) \) which is clearly independent of \( \tau \) and therefore of the sampling period \( T \). Thus, the variance \( \sigma^2 \) is given by

\[
\sigma^2 = \frac{1}{2N} \int_{-W}^{W} |C(f)|^2 df
\]  

\( R_u(kT) \) and \( \sigma^2 \) are the two values of interest here. Therefore, the \( \{ u_i \} \) generated by computer simulation must have the same statistical properties.

Now, let a lowpass filter with an impulse response \( d(t) \) and a transfer function
be used to pre-filter the noise process \( n(t) \) in Fig. F.1 before adding it to the signal. The arrangement looks then as shown in Fig. F.2. The received signal \( r(t) \) is again given by eqn. F.3, where now \( u(t) \) is given by

\[
u(t) = n(t) \ast d(t) \ast c(t)
\]

instead of eqn. F.5. But, according to eqns. F.1, F.2 and F.14,

\[
D(f)C(f) = C(f)
\]

which gives

\[
d(t) \ast c(t) = c(t)
\]

and consequently \( u(t) \) in eqn. F.15 is the same as \( u(t) \) in eqn. F.5.

Hence, Figs. F.1 and F.2 are, as far as the received signal \( r(t) \) is concerned, equivalent and therefore the model in Fig. F.1 can be replaced by that in Fig. F.2. Again, Fig. F.2 may, according to eqn. F.3, be rearranged as shown in Fig. F.3. The two samplers here are synchronized, and \( r_i, p_i \) and \( u_i \) are defined by eqns. F.6-F.8. Clearly, Fig. F.3 is, as far as the signal samples \( \{r_i\} \) are concerned, equivalent to Fig. F.1, and the statistics of the \( \{u_i\} \) in Fig. F.3 are still given by eqns. F.12 and F.13.

Consider now the noise samples \( \{u'_i\} \) in Fig. F.4, where \( n(t), d(t) \) and \( T_1 \) are as defined before, whereas the \( \{c_i\} \) are given by the vector \( C \),

\[
C = c_{-\rho}, c_{-\rho+1}, \ldots, c_0, c_1, \ldots, c_n
\]

and

\[
c_h = c(hT_1)
\]

\( c(hT_1) \) is the value of \( c(t) \) at \( t=hT_1 \), where it is assumed, for practical purposes, that \( c(t) \approx 0 \) for \( t<\rho T_1 \) and \( t>T_1 \). The vector \( C \) represents the sampled impulse response of the receiver filter. Clearly, the \( \{c_h\} \) are obtained by sampling \( c(t) \) at a rate which is not less than the Nyquist rate, according to eqns. F.1 and F.2. The Fourier transform of the \( \{c_h\} \) is given by (195)
FIGURE F.3: An Alternative Arrangement of the Model in F.2.

FIGURE F.4: Noise Correlation in the Discrete-Time Domain

FIGURE F.5: Discrete-Time Model for Noise Computer-Simulation
\[ \tilde{C}(f) = \sum_{h=-p}^{p} c_h e^{-j2\pi fhT_1} \]

\[ = \frac{1}{T_1} \sum_{i=-\infty}^{\infty} C(f + \frac{i}{T_1}) \]  

\[ \tilde{C}(f) \] is periodic with period \( \frac{1}{T_1} \). From eqn. F.20 and for \( i=0 \),

\[ \tilde{C}(f) = \frac{1}{T_1} C(f) , \quad -\frac{1}{2T_1} \leq f \leq \frac{1}{2T_1} \]  

because the sampling rate is not less than the Nyquist rate (the adjacent periods of \( \tilde{C}(f) \) do not overlap).

From Fig. F.4, the noise samples \( \{u_i\} \) are given by

\[ u_i = \sum_{h=-p}^{p} w_i c_h \]  

where

\[ w_i = w(iT_1) \]

and

\[ w(t) = n(t) * d(t) \]

Since the \( \{c_i\} \) represent a discrete-time time-invariant linear system, the autocorrelation function of the \( \{u_i\} \) is given by

\[ R_u(k) = \sum_{h=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} c_h c_{h+j} R_w(k-j) \]  

where \( R_w(k) \) is the autocorrelation function of the noise sequence \( \{w_i\} \).

Since \( c_h = 0 \) for \( h<0 \) and \( h>2 \) (eqn. F.18), eqn. F.25 becomes,

\[ R_u(k) = \sum_{h=-p}^{p} \sum_{j=-p}^{j} c_h c_{h+j} R_w(k-j) \]  

Now, the autocorrelation function of the \( \{w_i\} \) is given by

\[ R_w(k) = R_w(kT_1) \]  

where, from Fig. F.4 and according to the Wiener-Khintchine theorem,

\[ R_w(\tau) = \int_{-\infty}^{\infty} |W(f)|^2 e^{j2\pi f\tau} df \]

\[ = \frac{1}{4N_0} \int_{-\infty}^{\infty} |D(f)|^2 e^{j2\pi f\tau} df \]  

\[ (F.28) \]
since \( |W(f)|^2 \), the power spectral density of \( w(t) \) is given by
\[
|W(f)|^2 = \frac{1}{2} N_0 |D(f)|^2.
\] (F.29)

By replacing for \( D(f) \) from eqn. F.14, eqn. F.28 gives,
\[
R_w(\tau) = \frac{N_0}{2T_1} \frac{\sin \frac{\pi \tau}{T_1}}{\frac{\pi \tau}{T_1}}
\] (F.30)

Thus, \( R_w(k) \) in eqn. F.27 becomes,
\[
R_w(k) = \frac{N_0}{2T_1} k = 0
\]
\[
= 0 \quad k \neq 0
\] (F.31)

or equivalently,
\[
R_w(k) = \frac{N_0}{2T_1} \delta(k)
\] (F.32)

where \( \delta(k)=1 \) for \( k=0 \) and 0 for \( k\neq 0 \). Replacing \( R_w(k) \) in eqn. F.26 gives,
\[
R_u(k) = \frac{N_0}{2T_1} \sum_{h=-\rho}^{\rho} c_h c_{h+j} \delta(k-j)
\]
\[
= \frac{N_0}{2T_1} \sum_{h=-\rho}^{\rho} c_h \frac{\sin \frac{\pi (k-h)}{T_1}}{\frac{\pi (k-h)}{T_1}}
\]
\[
= \frac{N_0}{2T_1} \sum_{h=-\rho}^{\rho} c_h c_{h+k}
\] (F.33)

From eqn. F.19, and using the inverse Fourier transform results
\[
c_{h+k} = c((h+k)T_1)
\]
\[
= \int_{-W}^{W} C(f) e^{j2\pi f(k+h)T_1} df
\] (F.34)
\( C(f) \) being defined by eqn. F.1. Using eqn. F.34 in eqn. F.33 gives,
\[
R_u(k) = \frac{N_0}{2T_1} \sum_{h=-\rho}^{\rho} c_h \int_{-W}^{W} C(f) e^{j2\pi f(k+h)T_1} df
\]
\[
= \frac{N_0}{2T_1} \int_{-W}^{W} C(f) \left[ \sum_{h=-\rho}^{\rho} c_h e^{j2\pi fhT_1} \right] e^{j2\pi fkT_1} df
\] (F.35)
But the term between brackets in eqn. F.35 is the complex conjugate of \( \tilde{C}(f) \) in eqn. F.20. Thus,

\[
R_u(k) = \frac{N_0}{2T_1} \int_{-W}^{W} |C(f)|^2 e^{j2\pi ft_1} df
\]  

(F.36)

Over the integration range, \( \tilde{C}(f) \) is given by eqn. F.21 (see Eqn. F.2). Therefore, eqn. F.36 becomes,

\[
R_u(k) = \frac{N_0}{2T_1} \int_{-W}^{W} |C(f)|^2 e^{j2\pi ft_1} df
\]  

(F.37)

which when compared with eqn. F.12, gives,

\[
R_u(k) = \frac{1}{T_1^2} R(kT_1)
\]  

(F.38)

with the result that the noise sequence \( \{u_i\} \) generated according to Fig. F.4 have the same auto-correlation function as that of the \( \{u_i\} \) (Fig. F.3) but scaled by \( \frac{1}{T_1^2} \). The variance of the \( \{u_i\} \) is given by \( R_u(0) \),

\[
\sigma_u^2 = \frac{N_0}{2T_1} \int_{-W}^{W} |C(f)|^2 df
\]

(F.39)

where \( \sigma_u^2 \) is given by eqn. F.13.

In the actual computer simulation process, the \( \{w_i\} \) need not be produced as suggested by Fig. F.4. According to eqn. F.24, \( w(t) \) is a Gaussian random process with zero mean because \( n(t) \) is a Gaussian process with zero mean and \( d(t) \) is a linear system, and therefore, the \( \{w_i\} \) (eqn. F.23) are Gaussian random variables with zero mean. But according to eqn. F.32, the \( \{w_i\} \) are uncorrelated and therefore statistically independent. Hence, the \( \{w_i\} \) may be obtained in the computer simulation from a random number generator which produces a sequence of statistically independent Gaussian random variables with zero mean and variance \( \sigma_w^2 \).
Thus, the discrete-time model which produces the \( \{u_i\} \) looks as shown in Fig. F.5. The \( \{w_i\} \) have a zero mean and a variance \( \sigma_w^2 \) which is related to the variance of the \( \{u_i\} \) by (195)

\[
\sigma_u^2 = \sigma_w^2 \sum_{h=-p}^{p} c_h^2
\]  

(F.40)

In the actual simulation process, the \( \{c_i\} \) may be scaled such that

\[
\sum_{h=-p}^{p} c_h^2 = 1
\]

(F.41)

It is then sufficient to put \( \sigma_w^2 = \sigma_u^2 \), where \( \sigma_u^2 \) is given by eqn. F.13, to obtain the \( \{u_i\} \) correctly correlated and scaled.

The model established so far requires the condition in eqn. F.2 to be satisfied. Let \( \frac{1}{T} \) be the rate at which the \( \{r_i\} \) are obtained from \( r(t) \) (Fig. F.1). When the receiver filter is such that

\[
\frac{1}{T} > 2W \text{ Hz}
\]  

(F.42)

then

\[
\frac{1}{T_1} = \frac{1}{T}
\]  

(F.43)

and the model is used as described. But when

\[
\frac{1}{T} < 2W \text{ Hz}
\]  

(F.44)

then one would choose \( M \) as the smallest integer such that

\[
\frac{M}{T} > 2W \text{ Hz}
\]  

(F.45)

and put

\[
\frac{1}{T_1} = \frac{M}{T}
\]  

(F.46)

Now, the \( \{u_i\} \) in Fig. F.5 are produced at a rate of \( \frac{M}{T_1} \); the required noise sequence is then obtained by sub-sampling these \( \{u_i\} \) to the required rate (by taking every \( M \)th sample of the \( \{u_i\} \)).
F.2 THE DISCRETE-TIME MODEL OF NOISE IN THE QAM SYSTEM

The noise component in the received signal at the output of the linear baseband channel in the QAM system (Fig. 4.1) is given by eqn. 4.17, which is

\[ u(t) = \sqrt{2} \left[ n(t) \ast c(t) \right] e^{-j(2\pi fc t + \theta)} \ast b(t) \]  

\[ (F.47) \]

\( n(t) \) is a white Gaussian noise waveform with zero mean and a two-sided power spectral density \( N_0 \). \( c(t) \) is the impulse response of the bandpass receiver filter, whose transfer function is \( C(f) \). No particular restrictions are assumed here on the bandwidth of \( C(f) \). \( b(t) \) is the impulse response of a lowpass filter with a rectangular transfer function \( B(f) \) which, in the general case, is bandlimited such that

\[ B(f) = 1 \text{ for } |f| < W_1 \text{ Hz} \]

\[ = 0 \text{ elsewhere} \]  

\[ (F.48) \]

\( f_c \) is the carrier frequency of the QAM signal and \( \theta \) is constant.

In Appendix A, the autocorrelation function of \( u(t) \), \( R_u(\tau) \) (eqn. A.40), has been defined as,

\[ R_u(\tau) = E[u(t)^*u(t+\tau)] \]  

\[ (F.49) \]

where \( E[.] \) denotes an ensemble average and \( u(t)^* \) is the complex conjugate of \( u(t) \). For \( u(t) \) as given by eqn. F.47, \( R_u(\tau) \) is given by (eqn. A.43)

\[ R_u(\tau) = N \int_{-W_1}^{W_1} |C(f+c)|^2 e^{j2\pi f \tau} df \]  

\[ (F.50) \]

where, of course, the spectrum of \( u(t) \) (eqn. F.47) is bandlimited as \( B(f) \) (eqn. F.48). Now, let \( u(t) \) be sampled at a rate of \( \frac{1}{T_1} \), which does not necessarily have any particular relationship with the transmitted signal-element rate \( \frac{1}{T} \), but it satisfies

\[ \frac{1}{T_1} \geq 2W_1 \text{ Hz} \]  

\[ (F.51) \]

From eqn. F.50, the autocorrelation function of the \( \{u_i\} \), the samples of
\( u(t) \) at \( t = iT_i \), \( i = 1, 2, \ldots \), is given by

\[
R_u(k) = N_0 \int_{-W_1}^{W_1} |C(f + f_c)|^2 e^{i2\pi fkT_1} df \tag{F.52}
\]

\( R_u(k) \) is the complex-valued autocorrelation function between any two samples \( u_i \) and \( u_{i+k} \). The variance of the \( \{u_i\} \) is given by \( R_u(0) \),

\[
\sigma_u^2 = N_0 \int_{-W_1}^{W_1} |C(f + f_c)|^2 df \tag{F.53}
\]

It should be noted here that the \( \{u_i\} \) are in general complex-valued (eqn. F.47). The autocorrelation functions of both, the real and imaginary parts of the \( \{u_i\} \) are equal and each is given by \( \text{Real}[R_u(k)] \). The cross-correlation function of the real and imaginary parts of the \( \{u_i\} \) is given by half the imaginary part of \( R_u(k) \) which is skew-symmetric about \( k = 0 \) (see Appendix A).

The \( \{u_i\} \) may be obtained by computer simulation in the same manner as described in Section F.1. The discrete-time model is still given by Fig. F.5, where now, the \( \{w_i\} \) and the \( \{c_i\} \) are complex-valued. Both, the real and imaginary parts of the \( \{w_i\} \) are generated in the same way as the \( \{w_i\} \) in the previous section. Thus, all the real and imaginary parts of the \( \{w_i\} \) are statistically independent Gaussian random variables with zero mean. The variance of each of the real and imaginary parts of the \( \{w_i\} \) is given by \( \frac{1}{2} \sigma_w^2 \), so that the autocorrelation function of the \( \{w_i\} \) is now given by

\[
R_w(k) = \sigma_w^2 \delta(k) \tag{F.54}
\]

Eqn. F.54 expresses the fact that the \( \{w_i\} \) are uncorrelated and that the overall variance is equal to the sum of the variances of the real and imaginary parts of the \( \{w_i\} \). In Fig. F.5, the \( \{c_i\} \) are now given by the vector,
\[ C = c_{-p}, c_{-p+1}, \ldots, c_0, c_1, \ldots, c_n \]  
\text{(F.55)}

where,
\[ c_h = \int_{-W_1}^{W_1} C(f+h) e^{j2\pi fhT_1} df \]  
\text{(F.56)}

Notice here that the \( \{c_i\} \) are not the same as those given by eqn. F.34. \( c_h = 0 \) for \( h < -\rho \) and \( h > \lambda \), for practical purposes.

Now, it will be shown that the \( \{u_i\} \) in Fig. F.5 have the same statistics as those given by eqns. F.52 and F.53. First, the Fourier transform of the \( \{c_i\} \) (eqn. F.55) is given by (195)
\[ \tilde{C}(f) = \sum_{h=-\rho}^{\lambda} c_h e^{j2\pi fhT_1} \]  
\text{(F.57)}

where
\[ C'(f) = C(f+h) \quad -W_1 < f < W_1 \]  
\[ = 0 \quad \text{elsewhere} \]  
\text{(F.58)}

\( \tilde{C}(f) \) is periodic with period \( \frac{1}{T_1} \). According to eqns. F.51 and F.58, the adjacent periods of \( \tilde{C}(f) \) (eqn. F.57) do not overlap, so that
\[ \tilde{C}(f) = \frac{1}{T_1} C'(f) = \frac{1}{T_1} C(f+h) \quad -W_1 < f < W_1 \]  
\text{(F.59)}

Now, let the auto-correlation function of the \( \{u_i\} \) at the output of the model in Fig. F.5 be defined as
\[ R^*_u(k) = E[u^*_i u_{i+k}] \]  
\text{(F.60)}

where \( u^*_i \) is the complex conjugate of \( u_i \), and \( u_i \) is given by (Fig. F.5),
\[ u_i = \sum_{h=-\rho}^{\lambda} w_{i-h} c_h \]  
\text{(F.61)}

From eqns. F.60 and F.61,
\[
R'_u(k) = \sum_{h=-\rho}^{\rho} \sum_{n=-h-\rho}^{h-\rho} c_h^* c_{h+n} R_w(k-n)
\]
\[
= \sum_{h=-\rho}^{\rho} \sum_{n=-h-\rho}^{h-\rho} c_h^* c_{h+n} R_w \delta(k-n)
\]
\[
= \sigma_w^2 \sum_{h=-\rho}^{\rho} c_h^* c_{h+k}
\]

where eqn. F.54 is used, and \(c_h^*\) is the complex conjugate of \(c_h\). The definition of \(R'_u(k)\) in eqn. F.60 corresponds to the definition in eqn. F.49. It may be shown, from eqns. F.60-F.62, that \(R'_u(k)\) is in general complex-valued and that the autocorrelation functions of both the real and imaginary parts of the \(\{u_i\}\) are equal and each is given by \(\text{Real}[R'_u(k)]\). Also, the cross-correlation function of the real and imaginary parts of the \(\{u_i\}\) is given by half the imaginary part of \(R'_u(k)\) and it is skew-symmetric about \(k=0\). Therefore, and in order to relate the statistics of the \(\{u_i\}\) in Fig. F.5 to those of the required samples of \(u(t)\) (eqn. F.47), \(R_u(k)\) and \(R'_u(k)\) (eqns. F.52 and F.62) must be related to each other.

From eqns. F.56 and F.62,
\[
R'_u(k) = \sigma_w^2 \sum_{h=-\rho}^{\rho} c_h^* \int_{-W_1}^{W_1} C(f+f_c) e^{j2\pi f(h+k)T_1} df
\]
\[
= \sigma_w^2 \int_{-W_1}^{W_1} C(f+f_c) \sum_{h=-\rho}^{\rho} c_h^* e^{j2\pi f_h T_1} e^{j2\pi f k T_1} df
\]
\[
= \sigma_w^2 \int_{-W_1}^{W_1} C(f+f_c) C^*(f) e^{j2\pi f k T_1} df
\]

where eqn. F.57 is used. But over the integration range, \(C(f)\) is given by eqn. F.59. Hence,
\[
R'_u(k) = \frac{\sigma_w^2}{T_1} \int_{-W_1}^{W_1} |C(f+f_c)|^2 e^{j2\pi f k T_1} df
\]
Eqns. F.52 and F.64 are, apart from a scaling factor, exactly the same, and therefore the model given by Fig. F.5 and eqns. F.54-F.56 ensures the correct correlation of the noise samples in the simulation process.

When the \( \{c_h\} \) are scaled such that
\[
\sum_{h=-\rho}^{\rho} |c_h|^2 = 1 \tag{F.65}
\]
then (Fig. F.5)
\[
\sigma_u^2 = \sigma_w^2 \tag{F.66}
\]

Therefore, the correct value of the variance of the simulated \( \{u_1\} \) may be obtained by setting the variance of the \( \{w_1\} \) (Fig. F.5) to the value of \( \sigma_u^2 \) (eqn. F.53) together with scaling the \( \{c_h\} \) to satisfy eqn. F.65.

Clearly, the \( \{u_1\} \) are now obtained at not less than the Nyquist rate of \( u(t) \) (eqn. F.47), as can be seen from eqns. F.47, F.48 and F.51. When the required sampling rate \( \frac{1}{T} \) is such that \( \frac{1}{T} \geq 2W_1 \), then \( \frac{1}{T} \) is set to
\[
\frac{1}{T_1} = \frac{1}{T} \tag{F.67}
\]
But when
\[
\frac{1}{T} < 2W_1 \tag{F.68}
\]
then one would choose \( M \) as the smallest integer such that
\[
\frac{M}{T} \geq 2W_1 \tag{F.69}
\]
and set
\[
\frac{1}{T_1} = \frac{M}{T} \tag{F.70}
\]

Now, the \( \{u_1\} \) in Fig. F.5 are produced at a rate of \( \frac{M}{T} \). The required noise sequence is obtained by subsampling these \( \{u_1\} \) to the required rate.

---

F.3 THE DISCRETE-TIME MODEL OF NOISE IN THE TWO-DIMENSIONAL BASEBAND SYSTEM

The noise component \( u(t) \) in the received signal \( r(t) \) (eqn. 5.68) in the two-dimensional baseband system is given by eqn. 5.70, which, in
the absence of the frequency offset is

\[ u(t) = \frac{1}{\sqrt{2}} \{n(t) \ast [c(t) + j\hat{c}(t)]\} \]  

(F.71)

where, as before, \( n(t) \) is a real-valued white Gaussian noise waveform with zero mean and a two-sided power spectral density \( \frac{1}{2}N_0 \), and \( c(t) \) is the real-valued impulse response of the receiver filter whose transfer function \( C(f) \) is such that

\[ C(f) = 0 \quad |f| > W \]  

(F.72)

\( \hat{c}(t) \) is the Hilbert transform of \( c(t) \). The received signal \( r(t) \) is sampled at the receiver at a rate of \( \frac{1}{T'} \) samples per second. The sampling rate is assumed here such that

\[ \frac{1}{T'} \geq W \]  

(F.73)

and it is not necessarily related to the transmission element rate \( \frac{1}{T} \) in any way. The noise components \( \{u_i\} \) in the received signal samples \( \{r_i\} \) are obtained by sampling \( u(t) \) (eqn. F.71) at times \( t=iT_1, i=1,2, \ldots \).

In Appendix C, the auto-correlation function of \( u(t) \) is defined by eqn. C.26 as

\[ R_u(\tau) = E[u(t)u(t+\tau)] \]  

(F.74)

and is shown to be given by (eqn. C.31),

\[ R_u(\tau) = N_0 \int_{-W}^{W} |C(f)|^2 e^{i2\pi f\tau} df \]

\[ = N_0 \int_{0}^{W} |C(f)|^2 e^{i2\pi f\tau} df \]  

(F.75)

where, as assumed, the frequency offset \( \nu = 0 \) and eqn. F.72 has been used. The autocorrelation function of the \( \{u_i\} \) is given by \( R_u(kT_1) \), which is

\[ R_u(k) = N_0 \int_{-W}^{W} |C(f)|^2 e^{i2\pi fkT_1} df \]  

(F.76)
$R_u(k)$ is in general complex-valued and gives the correlation between any two noise samples $u_i$ and $u_{i+k}$. The $\{u_i\}$ are, in general, complex-valued (eqn. F.71). The autocorrelation functions of both the real and imaginary parts of the $\{u_i\}$ are equal and each is given by $\text{Real}[R_u(k)]$. The cross-correlation function of the real and imaginary parts of the $\{u_i\}$ is given by half the imaginary part of $R_u(k)$, which is skew-symmetric about $k=0$ (Appendix C).

As in the QAM system (Section F.2), the $\{u_i\}$ may be obtained here as shown in Fig. F.5, where the $\{w_i\}$ are complex-valued Gaussian random variables with zero mean and variance $\sigma_w^2$, and the autocorrelation function of the $\{w_i\}$ is given by eqn. F.54. The $\{c_i\}$ in Fig. F.5 are in general complex-valued and are given by

$$C = c_{-p}, c_{-p+1}, \ldots, c_0, \ldots, c_{p} \quad \text{(F.77)}$$

where

$$c_h = \int_0^W C(f)e^{j2\pi fhT_1} df \quad \text{(F.78)}$$

$c_h = 0$ for $h < -p$ and $h > p$, for practical purposes.

The autocorrelation function of the $\{u_i\}$ in Fig. F.5 is given by eqn. F.62, where $u_i$ is given by eqn. F.61. From eqns. F.62 and F.78,

$$R'_u(k) = \sigma_w^2 \sum_{h=-p}^p c_h^* \int_0^W C(f)e^{j2\pi fh}e^{j2\pi hT_1} df$$

$$= \sigma_w^2 \int_0^W C(f) \left[ \sum_{h=-p}^p c_h^* e^{j2\pi fhT_1} \right] e^{j2\pi hT_1} df \quad \text{(F.79)}$$

Now, the Fourier transform of the $\{c_i\}$ may be shown to be given by (195)

$$\tilde{C}(f) = \sum_{h=-p}^p c_h e^{-j2\pi fhT_1}$$

$$= \frac{1}{T_1} \sum_{i=-\infty}^{\infty} C'(f+\frac{i}{T_1}) \quad \text{(F.80)}$$
where

\[ C'(f) = C(f) \quad 0 < f < W \]
\[ = 0 \quad \text{elsewhere} \]  \hspace{1cm} (F.81)

\( \tilde{C}(f) \) is periodic with period \( \frac{1}{T_1} \). According to eqn. F.73, the adjacent periods of \( \tilde{C}(f) \) do not overlap (no aliasing occurs) so that \( \tilde{C}(f) \) over the frequency band 0 to \( W \) is given by (eqn. F.80)

\[ \tilde{C}(f) = \frac{1}{T_1} C'(f) \]
\[ = \frac{1}{T_1} C(f) \quad 0 < f < W \]  \hspace{1cm} (F.82)

In eqn. F.79, the term between brackets represents the complex conjugate of \( \tilde{C}(f) \) (eqn. F.80), which, over the integration range in eqn. F.79, is given by eqn. F.82. Thus, eqn. F.79 becomes

\[ R_u(k) = \frac{\sigma_w^2}{T_1} \int_0^{N} |C(f)|^2 e^{j2\pi fkT_1} df \]  \hspace{1cm} (F.83)

Thus, by setting the variance of the \( \{w_i\} \) (Fig. F.5) to

\[ \sigma_w^2 = N_0 T_1 \]  \hspace{1cm} (F.84)

then eqn. F.83 becomes the same as eqn. F.76, and the \( \{u_i\} \) in Fig. F.5 may be taken to represent the samples of \( u(t) \) (eqn. F.71) at times \( t=iT_1 \), \( i=1,2, \ldots \).

Clearly, the \( \{u_i\} \) are now obtained at a rate of \( \frac{1}{T_1} \), which is half the Nyquist rate of the receiver filter (eqn. F.73). When the required sampling rate \( \frac{1}{T} \) is such that \( \frac{1}{T} > W \), then

\[ \frac{1}{T_1} = \frac{1}{T} \]  \hspace{1cm} (F.85)

But when

\[ \frac{1}{T} < W \]  \hspace{1cm} (F.86)

then one would choose \( M \) as the smallest integer such that

\[ \frac{M}{T} > W \]  \hspace{1cm} (F.87)
and set \[ \frac{1}{T_1} = \frac{M}{T} \] (F.88)

Now, the \( \{ u_i \} \) in Fig. F.5 are produced at a rate of \( \frac{M}{T} \). The required noise sequence is obtained by subsampling these \( \{ u_i \} \) to the required rate.
APPENDIX G

THE EFFECT OF THE SAMPLING PHASE ON SEQUENCES THAT ARE LINEARLY FILTERED TO BE MINIMUM PHASE

Let $y(t)$ be a continuous waveform whose spectrum $Y(f)$ is bandlimited such that

$$Y(f) = 0 \quad |f| > W$$  \hspace{1cm} (G.1)

and let the sequence $\{y_h\}$ be obtained by sampling $y(t)$ at a rate of $\frac{1}{T}$, where $\frac{1}{T} \geq 2W$, so that the sampling rate is not less than the Nyquist rate.

Let

$$y_h = y(hT)$$  \hspace{1cm} (G.2)

then $y(t)$ may be written in terms of the $\{y_h\}$ as

$$y(t) = \sum_{h=-\infty}^{\infty} y(hT) \frac{\sin \pi(t-hT)}{\pi(t-hT)}$$  \hspace{1cm} (G.3)

Now, when the sampling phase is changed such that

$$y_h = y(hT+\tau) \quad , \quad |\tau| < \frac{T}{2}$$  \hspace{1cm} (G.4)

then, the representation in eqn. G.3 becomes,

$$y(t) = \sum_{h=-\infty}^{\infty} y(hT+\tau) \frac{\sin \pi(t-hT-\tau)}{\pi(t-hT-\tau)}$$  \hspace{1cm} (G.5)

The Fourier transform of $y(t)$ in eqn. G.5 is given by

$$Y(f) = \sum_{h=-\infty}^{\infty} y(hT+\tau) \int_{-\infty}^{\infty} \frac{\sin \pi(t-hT-\tau)}{\pi(t-hT-\tau)} e^{-j2\pi ft} dt$$
But the Fourier transform of the \( \{y_h\} \) is given by (195)

\[
Y_1(f) = \sum_{h=-\infty}^{\infty} y_h e^{-j2\pi fhT} \tag{G.7}
\]

and when \( y_h \) is given by eqn. G.4, then eqns. G.6 and G.7 give

\[
Y(f) = T e^{-j2\pi f\tau} Y_1(f) , \quad -\frac{1}{2T} < f < \frac{1}{2T} \tag{G.8}
\]

Eqn. G.8 implies that the sampling phase (expressed here as \( \tau \), eqn. G.4) appears in the spectrum of the sampled signal as a pure linear phase shift, or equivalently as a pure time-delay. Eqn. G.8 gives,

\[
|Y(f)| = T|Y_1(f)| , \quad -\frac{1}{2T} < f < \frac{1}{2T} \tag{G.9}
\]

where \( |Y(f)| \) and \( |Y_1(f)| \) are the amplitudes of \( Y(f) \) and \( Y_1(f) \) respectively. Clearly, from eqn. G.9, the amplitude spectrums of both the continuous waveform and the sequence of its samples, when the sampling rate is not less than the Nyquist rate, are related to each other by the scalar \( T \) but independently of the sampling phase. In other words, the amplitude spectrum of the sequence of the samples \( \{y_h\} \) is independent of the sampling phase with which these \( \{y_h\} \) are obtained from \( y(t) \).

Now, when the sequence \( \{y_h\} \) is filtered by means of an all-pass network to be a minimum phase sequence, then the filtering process is a pure phase transformation (81, 195) and therefore, the amplitude spectrum of the resultant sequence is also given by \( |Y_1(f)| \). On the other hand, the phase characteristic of a minimum phase sequence is entirely determined by its amplitude spectrum, (195) and therefore the resultant minimum phase
sequence is entirely determined by $|Y_1(f)|$. But since the amplitude spectrum $|Y_1(f)|$ is independent of the sampling phase when the sampling rate is not less than the Nyquist rate, then the resultant minimum phase sequence will be the same whatever is the sampling phase at which the original sequence is obtained.
THE EQUVALENCE BETWEEN THE CONTINUOUS-TIME AND THE DISCRETE-TIME CONVOLUTIONS

Let \( a(t) \), \( b(t) \) and \( y(t) \) be related by

\[
y(t) = a(t) * b(t)
\]

Let also the \( \{a_k\} \) and \( \{b_k\} \) be obtained by sampling \( a(t) \) and \( b(t) \), respectively, at not less than the Nyquist rate, where,

\[
a_k = a(kT) \quad \text{(H.2)}
\]
\[
b_k = b(kT) \quad \text{(H.3)}
\]

and \( \frac{1}{T} \) is the sampling rate. Using the set of \( \frac{\sin x}{x} \) orthogonal functions, \( a(t) \) and \( b(t) \) may be given in terms of the \( \{a_k\} \) and \( \{b_k\} \), respectively, as

\[
a(t) = \sum_k a_k \frac{\sin \pi(t-kT)}{T} \quad \text{(H.4)}
\]
\[
b(t) = \sum_h b_h \frac{\sin \pi(t-hT)}{T} \quad \text{(H.5)}
\]

Substituting \( a(t) \) and \( b(t) \) from eqns. H.4 and H.5 in eqn. H.1 gives

\[
y(t) = \sum_k \sum_h a_k b_h \int_{-\infty}^{\infty} \frac{\sin \pi(t-kT)}{T} \frac{\sin \pi(t-hT)}{T} \, dr \quad \text{(H.6)}
\]

and the sample of \( y(t) \), \( y_i \) at \( t=iT \) is

\[
y_i = y(iT) = \sum_k \sum_h a_k b_h \int_{-\infty}^{\infty} \frac{\sin \pi((i-k)T-\tau)}{T} \frac{\sin \pi(\tau-hT)}{T} \, d\tau \quad \text{(H.7)}
\]
Since the function $\frac{\sin x}{x}$ is an even function of $x$, eqn. H.7 may be written as,

$$y_i = \sum_k a_k \sum_h b_h \int_{-\infty}^{\infty} \frac{\sin \frac{\pi (\tau-(i-k)T)}{T}}{\sin \frac{\pi (\tau-hT)}{T}} dt$$

(H.8)

Now, the integral in eqn. H.8 is zero for all $(i-k)\neq h$ (orthonormal functions). Thus,

$$y_i = \sum_k a_k b_{i-k} \int_{-\infty}^{\infty} \left( \frac{\sin \frac{\pi (\tau-(i-k)T)}{T}}{\sin \frac{\pi (\tau-(i-k)T)}{T}} \right)^2 dt$$

(H.9)

The integral in eqn. H.9 is equal to $T(193)$. Hence,

$$y_i = T \sum_k a_k b_{i-k}$$

(H.10)

Thus, the $\{y_i\}$ may be obtained by multiplying by $T$ the sequence resulting from convolving the two sequences $\{a_k\}$ and $\{b_k\}$ with each other.
SIMULATOR FOR SYSTEM A WITH DETECTOR 1, CHAPTER 6.

DIMENSION Y(200),IS(200),IQ(65,100),IOQ(257,100),U(65)
DIMENSION N(65),R(100),D(513),UUU(513),YS(200),YH(200)
DIMENSION IOQQ(513),M(65),IQ1(513),MM(65)
DIMENSION Z(65,100),IZ(65,100),ZP(513),C(100)
DIMENSION CH1(200),CH2(200),DF1(200),DF2(200),CH3(200)
DIMENSION UU(257),ISD(200),DF3(200)
DIMENSION X(200)
DIMENSION YR(200),SN(200)
INTEGER A,SYSTEM
INTEGER CHH1,CHH11,CHH2,CHH22,CHH3,CHH33,DIF1,DIF2,DIF3
REAL MEAN

NY : NO. OF COMPONENTS IN THE SAMPLED IMPULSE RESPONSE
LS : NO. OF COMPONENTS IN A STORED VECTOR
NO : NO. OF STORED VECTORS
IN : INITIAL VALUE FOR THE RANDOM NUMBER GENERATOR
SNR : SIGNAL-TO-NOISE RATIO IN DB
IEL : NO. OF TRANSMITTED DATA SYMBOLS

LOAD NY,LS,NO,IN,SNR,IEL.
LOAD SAMPLED IMPULSE RESPONSE OF LINEAR BASEBAND
CHANNEL (TABLES 6.6-6.7) IN LOCATIONS 2 TO NY+1
IN ARRAY Y. Y(1)=0.0

LOAD SAMPLED IMPULSE RESPONSE OF THE RECEIVER FILTER
IN ARRAY YR. (FOR NOISE CORRELATION)

MEAN=0.0
UU(257)=1E70
UUU(257)=1E70
U(33)=1E70
IP=2
IF1=2
IF=1
LSY=200-NY
LS11=LS+1
LS12=LS+IP
LS13=LS12+1
IM=1
IP1=IP-1
LSF=200-LS-IP
C
LSF1=LSF-1
NYP=NY-IP-1
UUU(513)=1E70
EL=FLOAT(IEL)
IPP=IP+1
NOB=8*NO
NN=(NO/8)-1
LSN=LS-NN
LS1=LS-1
IEL4=IEL/4
NO2=2*NO
NO1=NO-1

INITIALIZATION OF RANDOM NUMBER GENERATOR

CALL G05CBF(IN)
MISSVLS=0
MISSV=0
DO 1000 I=1,NO
   DO 1001 K=1,LS
      IQ(I,K)=1
   1001 CONTINUE
   U(I)=1E70
1000 CONTINUE
   U(I)=0.0
   DO 1005 I=1,200
      IS(I)=1
      ISD(I)=1
1005 CONTINUE
   DO 1002 I=1,IP
      IK=I+1
      R(IK)=0.0
      ZA=0.0
      DO 1003 K=IK,LS
         ZA=ZA+Y(K)
1003 CONTINUE
   DO 1004 K=1,NO
      Z(K,I)=ZA
1004 CONTINUE
1002 CONTINUE

STANDARD DEVIATION OF NOISE SAMPLES
SEE EQU. 6.74 & 6.87.

STDEV=(10**(-SNR/20.0))*SQRT(14.0)
IERS=0
IER1=0
IERD1=0
IER2=0
IERD2=0

LOOP 2 IS THE MAIN LOOP.
EVERY TIME I IS INCREMENTED, A DATA SYMBOL IS GENERATED,
A SIGNAL SAMPLE IS RECEIVED AND A DATA SYMBOL IS DETECTED
WITH DELAY IN DETECTION OF LS SAMPLING PERIODS

ARRAY IS CONTAINS THE TRANSMITTED DATA SYMBOLS
C ARRAY R CONTAINS THE RECEIVED SIGNAL SAMPLES
C
C ARRAY IQ CONTAINS THE STORED VECTORS.
C
IER3=0
IERD3=0
DO 2 I=1,IEL
DO 37 K=LSF,199
K1=K+1
IS(K)=IS(K1)
CH1(K)=CH1(K1)
CH2(K)=CH2(K1)
CH3(K)=CH3(K1)
DF1(K)=DF1(K1)
DF2(K)=DF2(K1)
DF3(K)=DF3(K1)
SN(K)=SN(K1)
37 CONTINUE
DO 38 K=1,IP
R(K)=R(K+1)
38 CONTINUE
XX=G05CAF(XX)
IF(XX-0.125)60,60,61
61 IS(200)=7
CH1(200)=-1
CH2(200)=1
CH3(200)=-1
GOTO4
62 IS(200)=5
CH1(200)=-1
CH2(200)=1
CH3(200)=1
GOTO4
64 IS(200)=3
CH1(200)=-1
CH2(200)=-1
CH3(200)=1
GOTO4
66 IS(200)=1
CH1(200)=-1
CH2(200)=-1
CH3(200)=-1
GOTO4
68 IS(200)=-1
CH1(200)=1
CH2(200)=-1
CH3(200)=-1
GOTO4
70 IS(200)=-3
CH1(200)=1
CH2(200)=-1
CH3(200)=1
GOTO4
72 IS(200)=-5
CH1(200)=1
CH2(200)=1
CH3(200)=1
GOTO4
73 IS(200)=-7
CH1(200)=1
CH2(200)=1
CH3(200)=-1
4 IF(CH1(200)+CH1(199))601,602,601
601 DF1(200)=1
GOTO 609
602 DF1(200)=-1
609 SUMM=0.0
SN(200)=G05DDP(MEAN,STDEV)
8 DO 5 IJ=1,NY
IL=201-IJ
SUMM=SUMM+IS(IL)*Y(IJ)
SUMM=SUMM+SN(IL)*YR(IJ)
5 CONTINUE
R(IPP)=SUMM
DO 200 IK=1,NO
Z(IK,IPP)=0.0
DO 201 K=1,NYP
Z(IK,IPP)=Z(IK,IPP)+IQ(IK,LS11-K)*Y(K+IPP)
201 CONTINUE
200 CONTINUE
DO 203 K=1,NO
C(K)=0.0
DO 204 IK=1,IF
CC=R(IK)-Z(K,IK)
C(K)=C(K)+CC*CC
204 CONTINUE
203 CONTINUE
711 DO 202 K=1,NO
KB=8*K
ZF(K8-7)=Z(K,IF1)+Y(IF1)*7
ZF(K8-6)=Z(K,IF1)+Y(IF1)*5
ZF(K8-5)=Z(K,IF1)+Y(IF1)*3
ZF(K8-4)=Z(K,IF1)+Y(IF1)
ZF(K8-3)=Z(K,IF1)-Y(IF1)
ZF(K8-2)=Z(K,IF1)-Y(IF1)*3
ZF(K8-1)=Z(K,IF1)-Y(IF1)*5
ZF(K8)=Z(K,IF1)-Y(IF1)*7
202 CONTINUE
DO 205 K=1,NO
UC=U(K)+C(K)
KB=8*K
RZ1=R(IF1)-ZF(K8-7)
RZ2=R(IF1)-ZF(K8-6)
RZ3=R(IF1)-ZF(K8-5)
RZ4=R(IF1)-ZF(K8-4)
RZ5=R(IF1)-ZF(K8-3)
RZ6=R(IF1)-ZF(K8-2)
RZ7=R(IF1)-ZF(K8-1)
RZ8=R(IF1)-ZF(K8)
RZ1=RZ1*RZ1
RZ2=RZ2*RZ2
RZ3=RZ3*RZ3
RZ4=RZ4*RZ4
RZ5=RZ5*RZ5
RZ6=RZ6*RZ6
RZ7 = RZ7 * RZ7
RZ8 = RZ8 * RZ8
D(K8-7) = UC + RZ1
D(K8-6) = UC + RZ2
D(K8-5) = UC + RZ3
D(K8-4) = UC + RZ4
D(K8-3) = UC + RZ5
D(K8-2) = UC + RZ6
D(K8-1) = UC + RZ7
D(K8) = UC + RZ8

205 CONTINUE
DO 410 IK = 1, NO
IK8 = 8 * IK
DO 420 IKK = 1, LS1
A = IQ(IK, IKK + 1)
IQQ(IK8-7, IKK) = A
IQQ(IK8-6, IKK) = A
IQQ(IK8-5, IKK) = A
IQQ(IK8-4, IKK) = A
IQQ(IK8-3, IKK) = A
IQQ(IK8-2, IKK) = A
IQQ(IK8-1, IKK) = A
IQQ(IK8, IKK) = A

CONTINUE
IQQ(IK8-7, LS) = 7
IQQ(IK8-6, LS) = 5
IQQ(IK8-5, LS) = 3
IQQ(IK8-4, LS) = 1
IQQ(IK8-3, LS) = -1
IQQ(IK8-2, LS) = -3
IQQ(IK8-1, LS) = -5
IQQ(IK8, LS) = -7

410 CONTINUE
DO 411 IK = 1, NO
UU(IK) = D(IK)

411 CONTINUE
DO 750 IK = 1, NO
IK8 = 8 * IK
A = IQ(IK, 1)
IQQQ(IK8-7) = A
IQQQ(IK8-6) = A
IQQQ(IK8-5) = A
IQQQ(IK8-4) = A
IQQQ(IK8-3) = A
IQQQ(IK8-2) = A
IQQQ(IK8-1) = A
IQQQ(IK8) = A

750 CONTINUE
DNOISE = 1E60
DO 701 IK = 1, NO
IF(D(IK) - DNOISE) 702, 701, 701
702 IADD = IK
DNOISE = D(IK)

701 CONTINUE
D(IADD) = 1E70
N(1) = IADD
IQUIADD = IQQQ(IADD)
IM = 1
DO 700 K = 1, NO1
K1 = K + 1
DNOISE=1E60
IADD=0
DO 703 IK=1,N08
  IF(IQQQ(IK)-IQIADD)703,704,703
704 IF(D(IK)-DNOISE)705,703,703
  IADD=IK
  DNOISE=D(IK)
  CONTINUE
703 DO 709 K=K1,NO
  N(K)=IADD
  D(IADD)=1E70
700 CONTINUE
GOTO708
707 DO 709 K=K1,NO
  N(K)=257
709 CONTINUE
IF(I.GT.LS)MISSV=MISSV+NO-K1
708 ISD(LSF)=IQIADD
  CHH11=CHH1
  CHH22=CHH2
  CHH33=CHH3
  IF(IQIADD+5)610,611,612
610 CHH1=1
  CHH2=1
  CHH3=1
  GOTO621
611 CHH1=1
  CHH2=1
  CHH3=1
  GOTO621
612 IF(IQIADD+1)613,614,615
613 CHH1=1
  CHH2=1
  CHH3=1
  GOTO621
614 CHH1=1
  CHH2=1
  CHH3=1
  GOTO621
615 IF(IQIADD-3)616,617,618
616 CHH1=-1
  CHH2=-1
  CHH3=-1
  GOTO621
617 CHH1=-1
  CHH2=-1
  CHH3=1
  GOTO621
618 IF(IQIADD-5)619,619,620
619 CHH1=-1
  CHH2=1
  CHH3=1
  GOTO621
620 CHH1=-1
  CHH2=1
  CHH3=-1
621 IF(CHH1+CHH11)622,623,622
622 DIF1=1
  GOTO 586
623 DIF1=-1
DO 570 IKK=1,LS
IQ(IK,IKK)=IQQ(N(IK),IKK)
CONTINUE

DO 560 IK=1,NO
NK=(N(IK)+7)/8
RRR=R(1)-Z(NK,1)-Y(1)*IQ(IK,LS)
UUU(IK)=U(NK)+RRR*RRR
DO 572 IKK=1,IP
ZZ(IK,IKK)=Z(NK,IKK+1)+IQ(IK,LS)*Y(IKK+1)
CONTINUE

DO 571 IK=1,NO
NK=(N(IK)+7)/8
RRR=R(1)-Z(NK,1)-Y(1)*IQ(IK,LS)
UUU(IK)=U(NK)+RRR*RRR
DO 572 IKK=1,IP
ZZ(IK,IKK)=Z(NK,IKK+1)+IQ(IK,LS)*Y(IKK+1)
CONTINUE

571 CONTINUE

DO 573 IK=1,NO
U(IK)=UUU(IK)
DO 574 IKK=1,IP
Z(IK,IKK)=ZZ(IK,IKK)
CONTINUE

573 CONTINUE

IF(I-LS-IP)2,576,576
576 IF(IQIADD.NE.IS(LSF))IERS=IERS+1
IF(CHH1.NE.CH1(LSF))IER1=IER1+1
IF(DIF1.NE.DF1(LSF))IERD1=IERD1+1
IF(CHH2.NE.CH2(LSF))IER2=IER2+1
IF(CHH3.NE.CH3(LSF))IER3=IER3+1
IF(I.NE.IEL4)GOTO2
DO 20 K=1,NO
KB=8*K
K1=KB-7
K2=KB-6
K3=KB-5
K4=KB-4
K5=KB-3
K6=KB-2
K7=KB-1
IADD=N(K)
WRITE(2,51)(IQ(K,IK),IK=1,32),UU(IADD)
20 CONTINUE

55 FORMAT(1H ,75X,4(F9.3,1X))
51 FORMAT(1H ,32I2,1X,5(F9.3,1X))
643 FORMAT(1H ,32I2,10X,F9.3)
2 CONTINUE
ERSY=IERS/EL
ER1=IER1/EL
ERD1=IERD1/EL
ER2=IER2/EL
ER3=IER3/EL
ERAV=(ER1+ER2+ER3)/3
ERDAV=(ERD1+ER2+ER3)/3
WRITE(2,57)SNR,IN,MISSV
57 FORMAT(1H ,', SNR=',F5.2,' IN=',I5,' MISSV=',I5)
WRITE(2,631)ERSY
WRITE(2,8444)ERAV,ERDAV
8444 FORMAT(1H ,', AV.BIT ERROR RATE=',F10.8,
1', DIF. BIT ERROR RATE =',F10.8)
631 FORMAT(1H ,15X,'SYMBOL ERROR RATE = ',F10.8)
300 CONTINUE
301 CONTINUE
STOP
END
SIMULATOR FOR SYSTEMS B & C WITH DETECTOR 2.

DIMENSION Y(200), IS(200), IQ(33, 100), IQQ(33, 100), U(65)
DIMENSION N(65), R(100), D(2500), UU(513), ISD(200)
DIMENSION IQQ(2500), M(65), MM(65), ZF(65), MEM(4), IQ1(65)
DIMENSION Z(33, 50), ZZ(33, 50), C(100)
INTEGER RH1(200), RH2(200), RH3(200), DFR1(200), DFR2(200)
INTEGER DFI1(200), DFI2(200), DFI3(200)
DIMENSION IH1(200), IH2(200), IH3(200)
INTEGER X(400)
COMPLEX YR(100), SH(200)
DIMENSION UU(257),
INTEGER DFR3(200)
INTEGER BIN(8, 8, 6), DCOD(16, 2)
COMPLEX SN, G05DD, DF3, CHH3, RR
INTEGER SYSTEM
COMPLEX Y, IS, IQ, IQQ, ISD, Z, ZZ, R, IQQQ, CH1, CH2, CH3, DF1, DF2
COMPLEX SUMM, CC, RRR, A, IQIADD, CHH11, CHH22, RR//3, CHH1, CHH2
INTEGER A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4, F1, F2, F3, G1, G2, G3
COMPLEX ZF, MEM, IQ1
REAL MEAN

NY : NO. OF COMPONENTS IN THE SAMPLED IMPULSE RESPONSE.
LS : NO. OF COMPONENTS IN A STORED VECTOR.
NO : NO. OF STORED VECTORS.
IN : INITIAL VALUE FOR THE RANDOM NUMBER GENERATOR.
SNR : SIGNAL-TO-NOISE RATIO IN DB.
IEL : NO. OF TRANSMITTED DATA SYMBOLS.

LOAD NY, LS, NO, IN, SNR, IEL.

LOAD SAMPLED IMPULSE RESPONSE OF LINEAR BASEBAND CHANNEL (TABLES 6.8-6.11) IN LOCATIONS 2 TO NY+1 IN ARRAY Y. Y(1)=0.0

LOAD SAMPLED IMPULSE RESPONSE OF RECEIVER FILTER IN ARRAY YR.
LOAD THE FOLLOWING TABLE IN ARRAY BIN FOR DIFFERENTIAL CODING. SEE FIG. 6.4.

```
1011111011011001011101111101111010111110111111111011111011110111111101101100111101111101111010101011010111101111010101101011111110111101011110101111010111110111110111101011111011111111101111110111111
```

IP = 2
IF1 = 2
IF = 1
MEAN = 0.0
IPR = 1
UU(257) = 1E70
UUU(257) = 1E70
U(33) = 1E70
LSY = 200 - NY
LS11 = LS + 1
LS12 = LS + IP
LS13 = LS12 + 1
IM = 1
IP1 = IP - 1
LSF = 200 - LS - IP
LSF1 = LSF - 1
NYP = NY - IP - 1
MEM(1) = CMPLX(1.0, 1.0)
MEM(2) = CMPLX(-1.0, 1.0)
MEM(3) = CMPLX(-1.0, -1.0)
MEM(4) = (1.0, -1.0)
N04 = 4 * NO
UUU(513) = 1E70
EL = FLOAT(IEL)
IPP = IP + 1
NO8 = 8 * NO
NO64 = NO * 64
NN = (NO / 8) - 1
LSN = LS - NN
LS1 = LS - 1
IEL4 = IEL / 4
NO2 = 2 * NO
NO1 = NO - 1
DO 9902 I = 1, 8
WRITE(2, 9903)((BIN(I, K, KK), KK = 1, 6), K = 1, 8)
9902 CONTINUE
9903 FORMAT(1H0, 8(6I1, 3X))

ALL LOCATIONS OF ARRAY DCOD WHICH DO NOT APPAER IN WHAT FOLLOWS CONTAIN 0.

DCOD(2, 2) = 1
DCOD(3, 1) = 1
DCOD(4, 1) = 1
DCOD(4, 2) = 1
DCOD(5, 1) = 1
DCOD(7,1)=1
DCOD(7,2)=1
DCOD(8,2)=1
DCOD(9,2)=1
DCOD(10,1)=1
DCOD(10,2)=1
DCOD(12,1)=1
DCOD(13,1)=1
DCOD(13,2)=1
DCOD(14,1)=1
DCOD(15,2)=1

INITIALIZATION OF RANDOM NUMBER GENERATOR.

CALL G05CBF(IN)
IERS=0
IERS1=0
IERS2=0
IER11=0
IER21=0
IER31=0
IER12=0
IER22=0
IER32=0
IERD11=0
IERD21=0
IERD31=0
IERD12=0
IERD22=0
IERD32=0
MISSVLS=0
MISSV=0
DO 1000 I=1,NO
    DO 1001 K=1,LS
        IQ(I,K)=CMPLX(1.0,1.0)
1001 CONTINUE
U(I)=1E70
1000 CONTINUE
U(1)=0.0
DO 1005 I=1,200
    IS(I)=CMPLX(1.0,1.0)
    ISD(I)=CMPLX(1.0,1.0)
1005 CONTINUE
DO 1002 I=1,IP
    R(I+1)=CMPLX(0.0,0.0)
    IK=I+1
    ZA=0.0
    DO 1003 K=IK,LS
        ZA=ZA+Y(K)
1003 CONTINUE
DO 1004 K=1,NO
    Z(K,I)=ZA
1004 CONTINUE
1002 CONTINUE

STANDARD DEVIATION OF NOISE SAMPLES.
SEE EQNS 6.74&6.88 FOR SYSTEM B AND EQNS.6.75&6.89 FOR SYSTEM C.

STDEV1=(10**(-SNR/20.0))*SQRT(14.0)
STANDARD DEVIATION OF EACH OF THE REAL AND IMAGINARY PARTS OF THE NOISE SAMPLES IS

\[ \text{STDEV} = \text{STDEV1} / \sqrt{2.0} \]

LOOP 2 IS THE MAIN LOOP.
EVERY TIME I IS INCREMENTED, A DATA SYMBOL IS TRANSMITTED, A SIGNAL SAMPLE IS RECEIVED AND A DATA SYMBOL IS DETECTED WITH DELAY IN DETECTION OF LS SAMPLING PERIODS.

ARRAY IS CONTAINS THE TRANSMITTED DATA SYMBOLS.
ARRAY R CONTAINS THE RECEIVED SIGNAL SAMPLES.
ARRAY IQ CONTAINS THE STORED VECTORS.
DO 2 I=1,IEL
DO 37 K=LSF,199
  K1=K+1
  IS(K)=IS(K1)
  RH1(K)=RH1(K1)
  RH2(K)=RH2(K1)
  RH3(K)=RH3(K1)
  IH1(K)=IH1(K1)
  IH2(K)=IH2(K1)
  IH3(K)=IH3(K1)
  DFR1(K)=DFR1(K1)
  DFR2(K)=DFR2(K1)
  DFR3(K)=DFR3(K1)
  DFI1(K)=DFI1(K1)
  DFI2(K)=DFI2(K1)
  DFI3(K)=DFI3(K1)
  SH(K)=SH(K1)
CONTINUE
DO 38 K=1,IP
  R(K)=R(K+1)
CONTINUE
XX=G05CAF(XX)
IF(XX-0.125)60,60,61
IF(XX-0.25)62,62,63
IF(XX-0.375)64,64,65
IF(XX-0.5)66,66,67
IF(XX-0.625)68,68,69
IF(XX-0.75)70,70,71
D1=7.0
  GOT04
D1=5.0
  GOT04
D1=3.0
  GOT04
D1=1.0
  GOT04
D1=-1.0
  GOT04
D1=-3.0
  GOT04
D1=-5.0
  GOT04
73 D1=-7.0
4 XX=G05CAF(XX)
   IF(XX-0.125)765,765,766
766 IF(XX-0.25)767,767,768
768 IF(XX-0.375)769,769,770
770 IF(XX-0.5)771,771,772
772 IF(XX-0.625)773,773,774
774 IF(XX-0.75)775,775,776
776 IF(XX-0.875)777,777,778
778 D2=-7.0
609 IS(200)=CMPLX(D1,D2)
  K1=(D1+9)/2
  K2=(D2+9)/2
  K2=9-K2
  RH1(200)=BIN(K2,K1,1)
  IH1(200)=BIN(K2,K1,2)
  RH2(200)=BIN(K2,K1,3)
  IH2(200)=BIN(K2,K1,4)
  RH3(200)=BIN(K2,K1,5)
  IH3(200)=BIN(K2,K1,6)
  IADD1=RH1(199)*8+IH1(199)*4+RH1(200)*2+IH1(200)+1
  DFR1(200)=DCOD(IADD1,1)
  DFR2(200)=DCOD(IADD1,2)
788 SUMM=CMPLX(0.0,0.0)
8  DO 5 IJ=1,NY
   I1=2@1-IJ
   SUMM=SUMM+IS(I1)*Y(IJ)
5 CONTINUE
G05DD=CMPLX(G05DDF(0.0,STDEV),G05DDF(0.0,STDEV))
SN=CMPLX(0.0,0.0)
SH(200)=G05DD
DO 6 IJ=1,NY
   I1=2@1-IJ
   SN=SN+SH(I1)*YR(IJ)
6 CONTINUE
R(IPP)=SUMM+SN
SIGMAS=SIGMAS+REAL(SUMM*CONJG(SUMM))
SIGMAN=SIGMAN+REAL(G05DD*CONJG(G05DD))
SIGMANR=SIGMANR+REAL(SN*CONJG(SN))
DO 200 IK=1,NO
200 CONTINUE
Z(IK,IPP)=CMPLX(0.0,0.0)
DO 201 K=1,NYP
   Z(IK,IPP)=Z(IK,IPP)+IQ(IK,LS11-K)*Y(K+IPP)
201 CONTINUE
200 CONTINUE
   DO 203 K=1,NO
C(K)=0.0
DO 204 IK=1,IF
CC=R(IK)-Z(K,IK)
C(K)=C(K)+REAL(CC*CONJG(CC))
204 CONTINUE
C(K)=C(K)+U(K)
ZF(K)=R(IF1)-Z(K,IF1)
DO 420 IK=1,LS1
IQQ(K,IK)=IQ(K,IK+1)
420 CONTINUE
203 CONTINUE
DO 211 K=1,NO
K4=4*(K-1)
DO 212 IK=1,4
K4=K4+1
IQQQ(K4)=MEM(IK)*4
RR=ZF(K)-IQQQ(K4)*Y(IF1)
RZ=REAL(RR*CONJG(RR))
D(K4)=C(K)+RZ
212 CONTINUE
211 CONTINUE
DO 213 K=1,NO1
DNOISE=1E60
DO 214 IK=1,NO4
IF(D(IK)-DNOISE)215,214,214
215 IADD=IK
DNOISE=D(IK)
214 CONTINUE
N(K)=IADD
D(IADD)=1E70
213 CONTINUE
DNOISE=1E60
DO 232 IK=1,4
IF(D(IK)-DNOISE)233,232,232
233 IADD=IK
DNOISE=D(IK)
232 CONTINUE
N(NO)=IADD
D(IADD)=1E70
DO 216 K=1,NO
IQ1(K)=IQQQ(N(K))
M(K)=(N(K)+3)/4
216 CONTINUE
DO 217 K=1,NO
K4=(K-1)*4
DO 218 IK=1,4
K4=K4+1
IQQQ(K4)=MEM(IK)*2+IQ1(K)
RR=ZF(M(K))-IQQQ(K4)*Y(IF1)
RZ=REAL(RR*CONJG(RR))
D(K4)=C(M(K))+RZ
218 CONTINUE
217 CONTINUE
DO 219 K=1,NO1
DNOISE=1E60
DO 220 IK=1,NO4
IF(D(IK)-DNOISE)221,220,220
221 IADD=IK
DNOISE=D(IK)
220 CONTINUE
N(K)=IADD
D(IADD)=1E70
219 CONTINUE
NO43=NO4-3
DNOISE=1E60
DO 235 IK=NO43,NO4
IF(D(IK)-DNOISE)236,235,235
236 IADD=IK
DNOISE=D(IK)
235 CONTINUE
N(NO)=IADD
D(IADD)=1E70
DO 222 K=1,NO
IQ1(K)=IQQQ(N(K))
MM(K)=M((N(K)+3)/4)
222 CONTINUE
DO 223 K=1,NO
K4=(K-1)*4
DO 224 IK=1,4
K4=K4+1
IQQQ(K4)=MEM(IK)+IQ1(K)
RR=ZF(MM(K))-IQQQ(K4)*Y(IF1)
RZ=REAL(RR*CONJG(RR))
D(K4)=C(MM(K))+RZ
224 CONTINUE
223 CONTINUE
DNOISE=1E63
DO 225 IK=1,NO4
IF(D(IK)-DNOISE)226,225,225
226 IADD=IK
DNOISE=D(IK)
225 CONTINUE
UU(1)=D(IADD)
D(IADD)=1E70
N(1)=MM((IADD+3)/4)
IQIADD=IQ(N(1),1)
IQ(1,LS)=IQQQ(IADD)
DNOISE=1E60
DO 238 IK=NO43,NO4
NK=(IK+3)/4
IF(IQ(MM(NK),1)-IQIADD)238,240,238
241 IF(D(IK)-DNOISE)239,238,238
239 IADD=IK
DNOISE=D(IK)
238 CONTINUE
UU(2)=D(IADD)
D(IADD)=1E70
N(2)=MM((IADD+3)/4)
IQ(2,LS)=IQQQ(IADD)
IF(UU(2).GT.1E60)N(2)=33
DO 227 K=2,NO1
K1=K+1
DNOISE=1E60
IADD=0
DO 228 IK=1,NO4
NK=(IK+3)/4
A=IQ(MM(NK),1)
IF(A-IQIADD)228,229,228
229 IF(D(IK)-DNOISE)230,228,228
230 IADD=IK
DNOISE = D(IK)
NKK = NK
228 CONTINUE
IF (IADD) 707, 707, 231
231 UU(K1) = D(IADD)
D(IADD) = 1E70
N(K1) = MM(NKK)
IQ(K1, LS) = IQQ(IADD)
227 CONTINUE
GOTO 708
707 DO 709 K = K1, NO
N(K) = 33
709 CONTINUE
IF (I.GT. LS) MISSV = MISSV + NO - K1
708 ISD(LSF) = IQIADD
DR = REAL(IQIADD)
DI = AIMAG(IQIADD)
K1 = (DR + 9) / 2
K2 = (DI + 9) / 2
K2 = 9 - K2
A1 = BIN(K2, K1, 1)
A2 = BIN(K2, K1, 2)
B1 = BIN(K2, K1, 3)
B2 = BIN(K2, K1, 4)
C1 = BIN(K2, K1, 5)
C2 = BIN(K2, K1, 6)
IADD1 = F3*8 + G3*4 + A1*2 + A2 + 1
A3 = DCOD(IADD1, 1)
B3 = DCOD(IADD1, 2)
811 DO 560 IK = 1, NO
DO 570
IKK = 1, LS1
IQ(IK, IKK) = IQQ(N(IK), IKK)
570 CONTINUE
560 CONTINUE
DO 571 IK = 1, NO
NK = N(IK)
RRR = R(1) - Z(NK, 1) - Y(1) * IQ(IK, LS)
RZ = REAL(RRR*CONJG(RRR))
UUU(IK) = U(NK) + RZ
DO 572 IKK = 1, IP
ZZ(IK, IKK) = Z(NK, IKK + 1) + IQ(IK, LS) * Y(IKK + 1)
572 CONTINUE
571 CONTINUE
761 CONTINUE
DO 573 IK = 1, NO
U(IK) = UUU(IK)
DO 574 IKK = 1, IP
Z(IK, IKK) = ZZ(IK, IKK)
574 CONTINUE
573 CONTINUE
F3 = A1
G3 = A2
IF (I - LS) 2, 576, 576
576 IF (IQIADD .NE. IS(LSF)) IERS = IERS + 1
IF (DR .NE. REAL(IS(LSF))) IERS1 = IERS1 + 1
IF (DI .NE. AIMAG(IS(LSF))) IERS2 = IERS2 + 1
IF (A1 .NE. RH1(LSF)) IER11 = IER11 + 1
IF (B1 .NE. RH2(LSF)) IER21 = IER21 + 1
IF (C1 .NE. RH3(LSF)) IER31 = IER31 + 1
IF (A2 .NE. RH1(LSF)) IER12 = IER12 + 1
IF(B2.NE.IH2(LSF))IER22=IER22+1
IF(C2.NE.IH3(LSF))IER32=IER32+1
IF(A3.NE.DFR1(LSF))IERD11=IERD11+1
IF(B3.NE.DFR2(LSF))IERD21=IERD21+1
IF(I.NE.IEL4)GOTO2

LL31=LS-31
DO 20 K=1,NO
   WRITE(2,51)(IFIX(REAL(IQ(K,IK))),IK=1,32),UU(K)
   WRITE(2,55)(IFIX(AIMAG(IQ(K,IK))),IK=1,32)
20 CONTINUE
55 FORMAT(1H,32I2)
51 FORMAT(1H0,32I2,SX,F10.4)
2 CONTINUE
ERSY=IERS/EL
ERSY1=IERS1/EL
ERSY2=IERS2/EL
ERSY1=(ERSY1+ERSY2)/2.0
ER11=IER11/EL
ER12=IER12/EL
ER21=IER21/EL
ER22=IER22/EL
ER31=IER31/EL
ER32=IER32/EL
ERD11=IERD11/EL
ERD21=IERD21/EL
ERAV=(ER11+ER12+ER21+ER22+ER31+ER32)/6.0
ERDAV=(ERD11+ERD21+ER21+ER22+ER31+ER32)/6.0
WRITE(2,57)SNR,IN,MISSV
57 FORMAT(1H0,20X,'SNR = ',F7.3,3X,'IN=',I5,' MISSV=',I5)
WRITE(2,812)ERSY
WRITE(2,815)ERAV,ERDAV
W
816 FORMAT(1H0,20X,ASNR = ',F7.3,3X,ASNR1 = ',F7.3)
631 FORMAT(1H,15X,'SYMBOL ERROR RATE = ','F10.8)
812 FORMAT(1H0,35X,'COMPLX SYMBOL ERROR RATE = ','F10.8)
815 FORMAT(1H0,15X,'OVERALL AV. BIT ERROR RATE = ','F10.8,20X,'OVERALL AV. DIF. BIT ERROR RATE = ','F10.8)
300 CONTINUE
301 CONTINUE
STOP
END
SIMULATOR FOR SYSTEMS B & C WITH DETECTOR 3 OR 4.

A STORED VECTOR IS EXPANDED INTO 16 VECTORES.

DIMENSION Y(200), IS(200), IQ(33, 100), IQQ(33, 100), U(65)  
DIMENSION N(65), R(100), D(2500), UUU(513), ISD(200)  
DIMENSION IQQQ(2500), M(65), MM(65), ZF(65), MEM(4), IQ1(65)  
DIMENSION Z(33, 50), ZZ(33, 50), C(100)  
INTEGER RH1(200), RH2(200), RH3(200), DFR1(200), DFR2(200)  
INTEGER DFI1(200), DFI2(200), DFI3(200)  
DIMENSION IH1(200), IH2(200), IH3(200)  
DIMENSION UU(257)  
INTEGER DFR3(200)  
INTEGER X(400)  
COMPLEX YR(100), SH(200)  
COMPLEX TAB1(4, 4), TAB2(5, 5)  
INTEGER BIN(8, 8, 6), DCOD(16, 2)  
COMPLEX SN, G05DD  
INTEGER SYSTEM  
COMPLEX Y, IS, IQ, IQQ, ISD, Z, ZZ, R, IQQQ, CH1, CH2, CH3, DF1, DF2  
COMPLEX SUMM, CC, RRR, A, IQIADD, CHH11, CHH22, CHH33, CHH1, CHH2  
COMPLEX DF3, CHH3, RR  
INTEGER A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4, F1, F2, F3, G1, G2, G3  
COMPLEX ZF, MEM, IQ1  
REAL MEAN

NY : NO. OF COMPONENTS IN THE SAMPLED IMPULSE RESPONSE.  
LS : NO. OF COMPONENTS IN A STORED VECTOR.  
NO : NO. OF STORED VECTORS.  
IN : INITIAL VALUE FOR THE RANDOM NUMBER GENERATOR.  
SNR : SIGNAL-TO-NOISE RATIO IN DB.  
IEL : NO. OF TRANSMITTED DATA SYMBOLS.  
IDELAY : DELAY IN DETECTION IN SAMPLING PERIODS.  
DETECTR=3 : DETECTOR 3.  
DETECTR=4 : DETECTOR 4.  
LOAD NY, LS, NO, IN, SNR, IEL, IDELAY, DETECTR.  
LOAD SAMPLED IMPULSE RESPONSE OF LINEAR BASEBAND  
CHANNEL ( TABLES 6.8-6.11) IN LOCATIONS 2 TO NY+1  
IN ARRAY Y. Y(1)=0.0
LOAD SAMPLED IMPULSE RESPONSE OF RECEIVER FILTER IN ARRAY YR.

LOAD THE FOLLOWING TABLE IN ARRAY BIN FOR DIFFERENTIAL CODING. SEE FIG. 6.4.

10111110110110010110011110111101011111011111
10111011011010011011110011110111100111101
101101010110110111100111111110101110101101
101101101101111001111100111101111010111111
001110111000011000111101101110100011010110111
00010100010000000000010101001000001100101101
001101101101100101110110101110110111101110
001111001111001110011101101101111111

MEAN=0.0
IPR=1
IP=2
IF1=2
IF=1
UU(257)=1E70
UUU(257)=1E70
U(33)=1E70
LSY=200-NY
LS11=LS+1
LS12=LS+IP
LS13=LS12+1
IM=1
IP1=IP-1
LSFF=200-LS-IP
LSF=LSFF+LS-IDELAY
LSF1=LSFF-1
LSDEL=1+LS-IDELAY
NYP=NY-IP-1
MEM(1)=CMPLX(1.0,1.0)
MEM(2)=CMPLX(-1.0,1.0)
MEM(3)=CMPLX(-1.0,-1.0)
MEM(4)=(1.0,-1.0)
NO4=4*NO
UUU(513)=1E70
EL=FLOAT(IEL)
IPP=IP+1
NO8=8*NO
NO64=NO*64
NN=(NO/8)-1
LSN=LS-NN
LS1=LS-1
IEL4=IEL/4
NO2=2*NO
NO1=NO1-1
NO11=NO1-1
NO44=NO4-4
NO47=NO4-7
NPR=NO-IPR
IPR1=IPR+1
DO 9902 I=1,8
WRITE(2,9903)((BIN(I,K,KK),KK=1,6),K=1,8)
9902 CONTINUE
9903 FORMAT(1H0,8(6I1,3X))

All locations of array DCOD which do not appear in what follows contain 0.

DCOD(2,2)=1
DCOD(3,1)=1
DCOD(4,1)=1
DCOD(4,2)=1
DCOD(5,1)=1
DCOD(7,1)=1
DCOD(7,2)=1
DCOD(8,2)=1
DCOD(9,2)=1
DCOD(10,1)=1
DCOD(10,2)=1
DCOD(12,1)=1
DCOD(13,1)=1
DCOD(13,2)=1
DCOD(14,1)=1
DCOD(15,2)=1
DO 250 I=1,4
DO 250 K=1,4
AI=2*I-1.0
BK=2*K-1.0
TAB1(I,K)=CMPLX(AI,BK)
250 CONTINUE
DO 249 I=1,5
DO 249 K=1,5
AI=(I-1)*2.0
BK=8.0-(K-1)*2.0
TAB2(I,K)=CMPLX(BK, AI)
249 CONTINUE

Initialization of random number generator.

CALL G05CBF(IN)
IERS=0
IERS1=0
IERS2=0
IER11=0
IER21=0
IER31=0
IER12=0
IER22=0
IER32=0
IERD11=0
IERD21=0
IERD31=0
IERD12=0
IERD22=0
IERD32=0
MISSVLS=0
MISSV=0
DO 1000 I=1,NO
DO 1001 K=1,LS
IQ(I,K)=CMPLX(1.0,1.0)
1001 CONTINUE
U(I)=1E70
1000 CONTINUE
U(1)=0.0
DO 1005 I=1,200
IS(I)=CMPLX(1.0,1.0)
ISD(I)=CMPLX(1.0,1.0)
1005 CONTINUE
DO 1002 I=1,IP
R(I+1)=CMPLX(I2I.I2I,I2I.I2I)
IK=I+1
ZA=0.0
DO 1003 K=IK,LS
ZA=ZA+Y(K)
1003 CONTINUE
DO 1004 K=1,NO
Z(K,I)=ZA
1004 CONTINUE
1002 CONTINUE
C
C STANDERD DEVIATION OF NOISE SAMPLES.
C SEE EQNS 6.74&6.88 FOR SYSTEM B AND EQNS.6.75&6.89 FOR SYSTEM C
C
STDEV1=(10**(-SNR/20.0))*SQRT(14.0)
C
C STANDERD DEVIATION OF EACH OF THE REAL AND IMAGINARY
C PARTS OF THE NOISE SAMPLES IS
C
STDEV=STDEV1/SQRT(2.0)
C
LOOP 2 IS THE MAIN LOOP.
EVERY TIME I IS INCREMENTED ,A DATA SYMBOL IS TRANSMITTED,
A SIGNAL SAMPLE IS RECEIVED AND A DATA SYMBOL IS DETECTED
WITH DELAY IN DETECTION OF IDelay SAMPLING PERIODS.
C
ARRAY IS CONTAINS THE TRANSMITTED DATA SYMBOLS.
ARRAY R CONTAINS THE RECEIVED SIGNAL SAMPLES.
ARRAY IQ CONTAINS THE STORED VECTORS.
DO 2 I=1,IEL
DO 37 K=LSFF,199
K1=K+1
IS(K)=IS(K1)
RH1(K)=RH1(K1)
RH2(K)=RH2(K1)
RH3(K)=RH3(K1)
IH1(K)=IH1(K1)
IH2(K)=IH2(K1)
IH3(K)=IH3(K1)
DFR1(K)=DFR1(K1)
DFR2(K)=DFR2(K1)
DFR3(K)=DFR3(K1)
DFII(K)=DIII(K1)
DFI2(K)=DII2(K1)
DFI3(K)=DFI3(K1)
SH(K)=SH(K1)

37 CONTINUE
DO 38 K=1,IP
R(K)=R(K+1)
38 CONTINUE
XX=G05CAF(XX)
IF(XX-0.125)60,60,61
61 IF(XX-0.25)62,62,63
63 IF(XX-0.375)64,64,65
65 IF(XX-0.5)66,66,67
67 IF(XX-0.625)68,68,69
69 IF(XX-0.75)70,70,71
71 IF(XX-0.875)72,72,73
60 D1=7.0
GOTO4
62 D1=5.0
GOTO4
64 D1=3.0
GOTO4
66 D1=1.0
GOTO4
68 D1=-1.0
GOTO4
70 D1=-3.0
GOTO4
72 D1=-5.0
GOTO4
73 D1=-7.0
4 XX=G05CAF(XX)
IF(XX-0.125)765,765,766
766 IF(XX-0.25)767,767,768
768 IF(XX-0.375)769,769,770
770 IF(XX-0.5)771,771,772
772 IF(XX-0.625)773,773,774
774 IF(XX-0.75)775,775,776
776 IF(XX-0.875)777,777,778
765 D2=7.0
GOTO609
767 D2=5.0
GOTO609
769 D2=3.0
GOTO609
771 D2=1.0
GOTO609
773 D2=-1.0
GOTO609
775 D2=-3.0
GOTO609
777 D2=-5.0
GOTO609
778 D2=-7.0
609 IS(200)=CMPLX(D1,D2)
K1=(D1+9)/2
K2=(D2+9)/2
K2=9-K2
RH1(200)=BIN(K2,K1,1)
IH1(200)=BIN(K2,K1,2)
RH2(200)=BIN(K2,K1,3)
IH2(200)=BIN(K2,K1,4)
RH3(200) = BIN(K2, K1, 5)
IH3(200) = BIN(K2, K1, 6)
IADD1 = RH1(199) * 8 + IH1(199) * 4 + RH1(200) * 2 + IH1(200) + 1
DFR1(200) = DCOD(IADD1, 1)
DFR2(200) = DCOD(IADD1, 2)

788 SUMM = CMPLX(0.0, 0.0)
8 DO 5 IJ = 1, NY
   1l = 201 - IJ
   SUMM = SUMM + IS(I) * Y(IJ)
5 CONTINUE

G05DD = CMPLX(G05DDF(0.0, STDEV), G05DDF(0.0, STDEV))
SN = CMPLX(0.0, 0.0)
SH(200) = G05DD
DO 6 IJ = 1, NY
   1l = 201 - IJ
   SN = SN + SH(I) * YR(IJ)
6 CONTINUE

R(IPP) = SUMM + SN
SIGMAS = SIGMAS + REAL(SUMM*CONJG(SUMM))
SIGMAN = SIGMAN + REAL(G05DD*CONJG(G05DD))
SIGMANR = SIGMANR + REAL(SN*CONJG(SN))
DO 200 IK = 1, NO
   Z(IK, IPP) = CMPLX(0.0, 0.0)
DO 201 K = 1, NYP
   Z(K, IK) = Z(I) * Y(K + IPP)
201 CONTINUE

200 CONTINUE
DO 203 K = 1, NO
   C(K) = 0.0
DO 204 IK = 1, IF
   CC = R(IK) - Z(K, IK)
   C(K) = C(K) + REAL(CC*CONJG(CC))
204 CONTINUE

C(K) = C(K) + U(K)
ZF(K) = R(IF1) - Z(K, IF1)
DO 420 IK = 1, LS1
   IQQ(K, IK) = IQ(K, IK + 1)
420 CONTINUE

203 CONTINUE
711 DO 202 K = 1, NO
   A = ZF(K) / Y(IF1)
   REA = REAL(A)
   AIA = AIMAG(A)
   IF(REA + 1.0) 251, 251, 252
251 IF(REA + 3.0) 253, 254, 254
252 IF(REA - 3.0) 255, 256, 256
255 IF(REA - 1.0) 257, 258, 258
253 IK = 1
   GOTO 259
254 IK = 2
   GOTO 259
256 IK = 5
   GOTO 259
257 IK = 3
   GOTO 259
258 IK = 4
259 IF(AIA + 1.0) 260, 260, 261
260 IF(AIA + 3.0) 262, 263, 263
261 IF(AIA - 3.0) 264, 265, 265
264 IF(AIA - 1.0) 266, 267, 267
262  IKK=5
   GOTO268
263  IKK=4
   GOTO268
265  IKK=1
   GOTO268
266  IKK=3
   GOTO268
267  IKK=2
268  KK=(K-1)*16
   DO 205  IJ=1,4
   DO 410  JI=1,4
   KK=KK+1
   IQQQ(KK)=TAB1(IJ,JI)-TAB2(IKK,IK)
   RR=ZF(K)-Y(IF1)*IQQQ(KK)
   RZ=REAL(RR*CONJG(RR))
   D(KK)=C(K)+RZ
   IF(DETECTR.EQ.3.0)GOTO8435
   RR=R(IF1+1)-Z(K,IF1+1)-Y(IF1+1)*IQQQ(KK)
   RR=RR/Y(IF1)
   REA=REAL(RR)+8
   AIA=AIMAG(RR)+8
   K1=REA/2
   D1=K1*2-7
   K2=AIA/2
   D2=K2*2-7
   IF(ABS(D1).GT.7.0)D1=D1*7.0/ABS(D1)
   IF(ABS(D2).GT.7.0)D2=D2*7.0/ABS(D2)
   RR=(RR-CMPLX(D1,D2))*Y(IF1)
   D(KK)=D(KK)+REAL(RR*CONJG(RR))
8435  CONTINUE
410  CONTINUE
205  CONTINUE
202  CONTINUE
   DNOISE=1E60
   NO16=NO*16
   DO 701  IK=1,NO16
   IF(D(IK)-DNOISE)702,701,701
702  IADD=IK
   DNOISE=D(IK)
701  CONTINUE
   UU(1)=D(IADD)
   D(IADD)=1E70
   N(1)=(IADD+15)/16
   IQIADD=IQ(N(1),LSDEL)
   IQ(1,LS)=IQQQ(IADD)
   DO 269  K=1,NO
   KK=(K-1)*16
   DNOISE=1E60
   UUU(K)=1E70
   A=IQ(K,LSDEL)
   IF(A-IQIADD)269,271,269
271  DO 270  IK=1,16
   KK=KK+1
   IF(D(KK)-DNOISE)272,270,270
272  IADD=KK
   DNOISE=D(KK)
270  CONTINUE
   IQ1(K)=IQQQ(IADD)
   UUU(K)=D(IADD)
D(IADD)=1E70
269 CONTINUE
N(2)=1
UU(2)=UUU(1)
IQ(2,LS)=IQ1(1)
IF(UUU(1).GT.1E60)N(2)=33
K1=1
DO 273 K=3,NO
KK=(K1-1)*16
DNOISE=1E60
DO 274 IK=1,16
KK=KK+1
IF(D(KK)-DNOISE)275,274,274
275 IADD=KK
DNOISE=D(KK)
274 CONTINUE
IQ1(K1)=IQQQ(IADD)
UUU(K1)=D(IADD)
D(IADD)=1E70
DNOISE=1E60
DO 276 IK=1,NO
IF(UUU(IK)-DNOISE)277,276,276
277 IADD=IK
DNOISE=UUU(IK)
276 CONTINUE
N(K)=IADD
IQ(K,LS)=IQ1(IADD)
UUU(K)=UUU(IADD)
UUU(IADD)=1E70
K1=IADD
273 CONTINUE
708 ISD(LSF)=IQIADD
DR=REAL(IQIADD)
DI=AIMAG(IQIADD)
K1=(DR+9)/2
K2=(DI+9)/2
K2=9-K2
A1=BIN(K2,K1,1)
A2=BIN(K2,K1,2)
B1=BIN(K2,K1,3)
B2=BIN(K2,K1,4)
C1=BIN(K2,K1,5)
C2=BIN(K2,K1,6)
IADD1=F3*8+G3*4+A1*2+A2+1
A3=DCOD(IADD1,1)
B3=DCOD(IADD1,2)
811 DO 560 IK=1,NO
DO 570 IKK=1,LS1
ZZ(IK,IKK)=Z(NK,IKK)+IQ(IK,LS)*Y(IKK+1)
570 CONTINUE
560 CONTINUE
DO 571 IK=1,NO
NK=N(IK)
RRR=R(1)-Z(NK,1)-Y(1)*IQ(IK,LS)
RZ=REAL(RRR*CONJG(RRR))
UUU(IK)=U(NK)+RZ
DO 572 IKK=1,IP
ZZ(IK,IKK)=Z(NK,IKK+1)+IQ(IK,LS)*Y(IKK+1)
572 CONTINUE
571 CONTINUE
CONTINUE
DO 573 IK=1,NO
U(IK)=UUU(IK)
DO 574 IKK=1,IP
Z(IK,IKK)=ZZ(IK,IKK)
574 CONTINUE
573 CONTINUE
F3=A1
G3=A2
IF(I-LS)2,576,576
576 IF(IQIADD.NE.IS(LSF))IERS=IERS+1
IF(DR.NE.REAL(IS(LSF)))IERS1=IERS1+1
IF(DI.NE.AIMAG(IS(LSF)))IERS2=IERS2+1
IF(A1.NE.RH1(LSF))IER11=IER11+1
IF(B1.NE.RH2(LSF))IER21=IER21+1
IF(C1.NE.RH3(LSF))IER31=IER31+1
IF(A2.NE.IH1(LSF))IER12=IER12+1
IF(B2.NE.IH2(LSF))IER22=IER22+1
IF(C2.NE.IH3(LSF))IER32=IER32+1
IF(A3.NE.DFR1(LSF))IERD11=IERD11+1
IF(B3.NE.DFR2(LSF))IERD21=IERD21+1
3 IF(I.NE.IEL4)GOTO2
LL31=LS-31
DO 20 K=1,NO
WRITE(2,51)(IFIX(REAL(IQ(K,IK))),IK=1,32),UU(K)
WRITE(2,55)(IFIX(AIMAG(IQ(K,IK))),IK=1,32)
20 CONTINUE
55 FORMAT(1H ,32I2)
51 FORMAT(1H0,32I2,5X,F10.4)
2 CONTINUE
ERSY=IERS/EL
ERSY1=IERS1/EL
ERSY2=IERS2/EL
ERSY1=(ERSY1+ERSY2)/2.Ø
ER11=IER11/EL
ER12=IER12/EL
ER21=IER21/EL
ER22=IER22/EL
ER31=IER31/EL
ER32=IER32/EL
ERD11=IERD11/EL
ERD21=IERD21/EL
ERAV=(ER11+ER12+ER21+ER22+ER31+ER32)/6.Ø
ERDAV=(ERD11+ERD21+ER21+ER22+ER31+ER32)/6.Ø
WRITE(2,57)SNR,IN,MISSV,SYSTEM,IDELAY,IPR
57 FORMAT(1H0,20X,' ASNR = ',F7.3,3X,' ASNR1 = ',F7.3)
WRITE(2,812)ASNR,ASNR1
812 FORMAT(1H0,35X,'COMPLX SYM. ERR. RATE = ',F10.8)
815 FORMAT(1H0,35X,'OVERALL AV. BIT ERR. RATE = ',F10.8,20X,
1 'OVERALL AV. DIF. BIT ERR. RATE = ',F10.8)
APPENDIX M

SIMULATOR FOR SYSTEM D WITH DETECTOR 3, CHAPTER 6.
A STORED VECTOR IS EXPANDED INTO 16 VECTORS.

DIMENSION Y(200), IS(200), IQ(33, 100), IQQ(33, 100), U(65)
DIMENSION UU(257)
DIMENSION N(65), R(100), D(2500), UU(513), ISD(200)
DIMENSION IQQQ(2500), M(65), MM(65), ZF(65), MEM(4), IQ1(65)
DIMENSION Z(33, 50), ZZ(33, 50), C(100)
INTEGER RH1(200), RH2(200), RH3(200), DFR1(200), DFR2(200)
INTEGER DFR3(200), DFR4(200)
INTEGER DFI1(200), DFI2(200), DFI3(200)
INTEGER IH1(200), IH2(200), IH3(200), IH4(200), RH4(200)
INTEGER DFI4(200), BIN(16, 16, 8), DCOD(16, 2)
INTEGER X(200)
COMPLEX YR(100), SH(200)
COMPLEX TAB1(8, 8), TAB2(13, 13)
INTEGER A5, A6, B5, B6, C5, C6, F5, F6, G5, G6
COMPLEX SN, G05DD, DF3, CHH3, RR
INTEGER SYSTEM
COMPLEX Y, IS, IQ, IQQ, ISD, Z, ZZ, R, IQQQ, CH1, CH2, CH3, DF1, DF2
COMPLEX SUMM, CC, RRR, A, IQIADD, CHH11, CHH22, CHH33, CHH1, CHH2
INTEGER A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4, F1, F2, F3, G1, G2, G3
COMPLEX ZF, MEM, IQ1
REAL MEAN

NY : NO. OF COMPONENTS IN THE SAMPLED IMPULSE RESPONSE.
LS : NO. OF COMPONENTS IN A STORED VECTOR.
NO : NO. OF STORED VECTORS.
IN : INITIAL VALUE FOR THE RANDOM NUMBER GENERATOR.
SNR : SIGNAL-TO-NOISE RATIO IN DB.
IEL : NO. OF TRANSMITTED DATA SYMBOLS.
IDELAY : DELAY IN DETECTION.

LOAD NY, LS, NO, IN, SNR, IEL, IDELAY.
LOAD SAMPLED IMPULSE RESPONSE OF LINEAR BASEBAND CHANNEL IN LOCATIONS 2 TO NY+1 IN ARRAY Y. Y(1)=0.0
LOAD SAMPLED IMPULSE RESPONSE OF RECEIVER FILTER (FOR NOISE CORRELATION) IN ARRAY YR.

LOAD THE FOLLOWING TABLE IN ARRAY BIN (ACCORDING TO THE NEXT READ STATEMENT) FOR DIFFIRENTIAL CODING. SEE FIG. 6.5.
READ(1,901)(((BIN(I,K,IK),IK=1,8),K=9,16),I=1,8)
901 FORMAT(64I1)
DO 8334 I=1,8
DO 8334 K=1,8
DO 8334 IK=3,8
BIN(I,K,1)=1
BIN(I,K,2)=0
BIN(I+8,K,1)=0
BIN(I+8,K,2)=0
BIN(I+8,K+8,1)=0
BIN(I+8,K+8,2)=1
BIN(I,K,IK)=BIN(K,17-I,IK)
BIN(I+8,K,IK)=BIN(9-I,17-K,IK)
BIN(17-I,17-K,IK)=BIN(I,K,IK)
8334 CONTINUE
MEAN=0.0
IP=2
IF1=2
IF=1
UU(257)=1E70
UUU(257)=1E70
U(33)=1E70
LSY=200-NY
LS11=LS+1
LS12=LS+IP
LS13=LS12+1
IM=1
IP1=IP-1
LSFF=200-LS-IP
LSF=LSFF+LS-IDELAY
LSF1=LSFF-1
LSDEL=1+LS-IDELAY
NYP=NY-IP-1
MEM(1)=CMPLX(1.0,1.0)
MEM(2)=CMPLX(-1.0,1.0)
MEM(3)=CMPLX(-1.0,-1.0)
MEM(4)=(1.0,-1.0)
NO4=4*NO
UUU(513)=1E70
EL=FLOAT(IEL)
IPP=IP+1
NO8=8*NO
NO64=NO*64
NN=(NO/8)-1
LSN=LS-NN
LS1=LS-1
IEL4=IEL/4
NO2=2*NO
NO1=NO-1
NO11=NO1-1
NO44=NO4-4
NO47=NO4-7
NPR=NO-IPR
IPRI=IPR+1
WRITE(2,9903)((BIN(I,K,IK),IK=1,8),K=9,16),I=1,8)

9903 FORMAT(1H ,8(8I1,3X))

C
C
C ALL LOCATIONS OF ARRAY DCOD WHICH DO NOT APPEAR
C IN WHAT FOLLOWS ARE Ø .
DCOD(2,2)=1
DCOD(3,1)=1
DCOD(4,1)=1
DCOD(4,2)=1
DCOD(5,1)=1
DCOD(7,1)=1
DCOD(7,2)=1
DCOD(8,2)=1
DCOD(9,2)=1
DCOD(10,1)=1
DCOD(10,2)=1
DCOD(12,1)=1
DCOD(13,1)=1
DCOD(13,2)=1
DCOD(14,1)=1
DCOD(15,2)=1
DO 250 I=1,8
DO 250 K=1,8
AI=2*I-1.0
BK=2*K-1.0
TAB1(I,K)=CMPLX(AI,BK)
CONTINUE

DO 249 I=1,13
DO 249 K=1,13
AI=(I-1)*2.0-4.0
BK=20.0-(K-1)*2.0
TAB2(I,K)=CMPLX(BK,AI)
CONTINUE

C

INITIALIZATION OF RANDOM NUMBER GENERATOR.
CALL G05CBF(IN)
IERS=Ø
IERSI=Ø
IERS2=Ø
IER11=Ø
IER21=Ø
IER31=Ø
IER12=Ø
IER22=Ø
IER32=Ø
IERD11=Ø
IERD21=Ø
IERD31=Ø
IERD12=Ø
IERD22=Ø
IERD32=Ø
IER41=Ø
IER42=Ø
IERD41=Ø
IERD42=0
MISSVLS=0
MISSV=0
DO 1000 I=1,NO
   DO 1001 K=1,LS
      IQ(I,K)=CMPLX(1.0,1.0)
   1001 CONTINUE
   U(I)=1E70
1000 CONTINUE
   U(I)=0.0
   DO 1005 I=1,200
      IS(I)=CMPLX(1.0,1.0)
      ISD(I)=CMPLX(1.0,1.0)
   1005 CONTINUE
   DO 1002 I=1,IP
      R(I+1)=CMPLX(0.0,0.0)
      IK=I+1
      ZA=0.0
      DO 1003 K=IK,LS
         ZA=ZA+Y(K)
      1003 CONTINUE
      DO 1004 K=1,NO
         Z(K,I)=ZA
      1004 CONTINUE
   1002 CONTINUE

C STANDARD DEVIATION OF NOISE SAMPLES.
C SEE EQNS. 6.75&6.89.
C
STDEV1=(10**(-SNR/20.0))*SQRT(42.5)
C STANDARD DEVIATION OF EACH OF THE REAL AND IMAGINARY PARTS
C OF NOISE SAMPLES
C
STDEV=STDEV1/SQRT(2.0)
   DO 2 I=1,IEL
   DO 37 K=LSFF,199
      K1=K+1
      IS(K)=IS(K1)
      RH1(K)=RH1(K1)
      RH2(K)=RH2(K1)
      RH3(K)=RH3(K1)
      IH1(K)=IH1(K1)
      IH2(K)=IH2(K1)
      IH3(K)=IH3(K1)
      DFR1(K)=DFR1(K1)
      DFR2(K)=DFR2(K1)
      DFR3(K)=DFR3(K1)
      DFI1(K)=DFI1(K1)
      DFI2(K)=DFI2(K1)
      DFI3(K)=DFI3(K1)
      RH4(K)=RH4(K1)
      IH4(K)=IH4(K1)
      DFR4(K)=DFR4(K1)
      DFI4(K)=DFI4(K1)
      SH(K)=SH(K1)
37 CONTINUE
   DO 38 K=1,IP
      R(K)=R(K+1)
38 CONTINUE
\[
XX=G\text{CAF}(XX) \times 15 \\
XX=XX \times 2 + 1 \\
K1=XX/2 \\
K1=K1-2-15 \\
XX=G\text{CAF}(XX) \times 15 \\
XX=XX \times 2 + 1 \\
K2=XX/2 \\
K2=K2-2-15 \\
D1=K1 \\
D2=K2 \\
IS(200)=CMPLX(D1, D2) \\
K1=(K1+17)/2 \\
K2=(K2+17)/2 \\
K2=17-K2 \\
\]

\[
RH1(200)\text{=BIN}(K2,K1,1) \\
IH1(200)\text{=BIN}(K2,K1,2) \\
RH2(200)\text{=BIN}(K2,K1,3) \\
IH2(200)\text{=BIN}(K2,K1,4) \\
RH3(200)\text{=BIN}(K2,K1,5) \\
IH3(200)\text{=BIN}(K2,K1,6) \\
RH4(200)\text{=BIN}(K2,K1,7) \\
IH4(200)\text{=BIN}(K2,K1,8) \\
\]

\[
D1=RH1(199) \times 8 + IH1(199) \times 4 + RH1(200) \times 2 + IH1(200) + 1 \\
DFR1(200)\text{=DCOD}(IADD1,1) \\
DFR2(200)\text{=DCOD}(IADD1,2) \\
\]

\[
788 \text{SUMM=}CMPLX(0.0,0.0) \\
8 \text{DO} \ 5 \ IJ=1, NY \\
\quad I1=201-IJ \\
\quad SUMM=SUMM+IS(I1) \times Y(IJ) \\
5 \text{CONTINUE} \\
\text{G05DD=}CMPLX(G05DDF(0.0,STDEV),G05DDF(0.0,STDEV)) \\
\text{SN=}CMPLX(0.0,0.0) \\
\text{SH}(200)\text{=}G05DD \\
\text{DO} \ 6 \ IJ=1, NY \\
\quad I1=201-IJ \\
\quad SN=SN+SH(I1) \times YR(IJ) \\
6 \text{CONTINUE} \\
\text{R}(IPP)=SUMM+SN \\
\text{SIGMAS=}SIGMAS+REAL(SUMM*CONJG(SUMM)) \\
\text{SIGMAN=}SIGMAN+REAL(G05DD*CONJG(G05DD)) \\
\text{SIGMANR=}SIGMANR+REAL(SN*CONJG(SN)) \\
\text{DO} \ 200 \ IK=1, NO \\
\quad Z(IK,IPP)=CMPLX(0.0,0.0) \\
\quad DO \ 201 \ K=1, NYP \\
\quad Z(IK,IPP)=Z(IK,IPP)+IQ(IK,LS11-K) \times Y(K+IPP) \\
201 \text{CONTINUE} \\
200 \text{CONTINUE} \\
\text{DO} \ 203 \ K=1, NO \\
\quad C(K)=0.0 \\
\text{DO} \ 204 \ IK=1, IF \\
\quad CC=R(IK)-Z(K,IK) \\
\quad C(K)=C(K)+REAL(CC*CONJG(CC)) \\
204 \text{CONTINUE} \\
\quad C(K)=C(K)+U(K) \\
\quad ZF(K)=R(IF1)-Z(K,IF1) \\
\text{DO} \ 420 \ IK=1, LS1 \\
\quad IQQ(K,IK)=IQ(K,IK+1) \\
420 \text{CONTINUE} \\
203 \text{CONTINUE} \\
711 \text{DO} \ 202 \ K=1, NO
A = ZF(K) / Y(IF1)
REA = REAL(A)
AIA = AIMAG(A)
IF (REA .LT. -11.1) REA = -11.1
IF (REA .GT. 11.1) REA = 11.1
IF (AIA .LT. -11.1) AIA = -11.1
IF (AIA .GT. 11.1) AIA = 11.1
IK = (REA + 17) / 2 - 1
IKK = (17 - AIA) / 2 - 1
268 KK = (K - 1) * 16
DO 205 IJ = 3, 6
DO 410 JI = 3, 6
KK = KK + 1
IQQQ(KK) = TAB1(IJ, JI) - TAB2(IKK, IK)
RR = ZF(K) - Y(IF1) * IQQQ(KK)
RZ = REAL(RR * CONJG(RR))
D(KK) = C(K) + RZ

410 CONTINUE
205 CONTINUE
202 CONTINUE
DNOISE = 1E60
NO16 = NO * 16
DO 701 IK = 1, NO16
IF (D(IK) - DNOISE) 702, 701, 701
702 IADD = IK
DNOISE = D(IK)
701 CONTINUE
UU(1) = D(IADD)
D(IADD) = 1E70
N(1) = (IADD + 15) / 16
IQIADD = IQ(N(1), LSDEL)
IQ(1, LS) = IQQQ(IADD)
DO 269 K = 1, NO
KK = (K - 1) * 16
DNOISE = 1E60
UUU(K) = 1E70
A = IQ(K, LSDEL)
IF (A - IQIADD) 269, 271, 269
271 DO 270 IK = 1, 16
KK = KK + 1
IF (D(KK) - DNOISE) 272, 270, 270
272 IADD = KK
DNOISE = D(KK)
270 CONTINUE
IQ1(K) = IQQQ(IADD)
UUU(K) = D(IADD)
D(IADD) = 1E70
269 CONTINUE
N(2) = 1
UU(2) = UUU(1)
IQ(2, LS) = IQ1(1)
IF (UUU(1) .GT. 1E60) N(2) = 33
K1 = 1
DO 273 K = 3, NO
KK = (K1 - 1) * 16
DNOISE = 1E60
DO 274 IK = 1, 16
KK = KK + 1
IF (D(KK) - DNOISE) 275, 274, 274
275 IADD = KK
DNOISE = D(KK)

274 CONTINUE
IQ1(K1) = IQQQ(IADD)
UUU(K1) = D(IADD)
D(IADD) = 1E70
DNOISE = 1E60
DO 276 IK = 1, NO
IF (UUU(IK) - DNOISE) GT 277, 276, 276

277 IADD = IK
DNOISE = UUU(IK)

276 CONTINUE
N(K) = IADD
IQ(K, LS) = IQ1(IADD)
UU(K) = UUU(IADD)
UUU(IADD) = 1E70
K1 = IADD

273 CONTINUE

708 ISD(LSF) = IQIADD
DR = REAL(IQIADD)
DI = AIMAG(IQIADD)
K1 = (DR + 17) / 2
K2 = (DI + 17) / 2
K2 = 17 - K2
A1 = BIN(K2, K1, 1)
A2 = BIN(K2, K1, 2)
B1 = BIN(K2, K1, 3)
B2 = BIN(K2, K1, 4)
C1 = BIN(K2, K1, 5)
C2 = BIN(K2, K1, 6)
A5 = BIN(K2, K1, 7)
A6 = BIN(K2, K1, 8)
IADD1 = F5 * 8 + F6 * 4 + A1 * 2 + A2 + 1
A3 = DCOD(IADD1, 1)
B3 = DCOD(IADD1, 2)
F5 = A1
F6 = A2

811 DO 560 IK = 1, NO
DO 570 IKK = 1, LS1
IQ(IK, IKK) = IQQ(N(IK), IKK)

570 CONTINUE

560 CONTINUE
DO 571 IK = 1, NO
NK = N(IK)
RRR = R(1) - Z(NK, 1) - Y(1) * IQ(IK, LS)
RZ = REAL(RRR * CONJG(RRR))
UUU(IK) = U(NK) + RZ
DO 572 IKK = 1, IP
ZZ(IK, IKK) = Z(NK, IKK + 1) + IQ(IK, LS) * Y(IKK + 1)

572 CONTINUE

571 CONTINUE

761 CONTINUE
DO 573 IK = 1, NO
U(IK) = UUU(IK)
DO 574 IKK = 1, IP
Z(IK, IKK) = ZZ(IK, IKK)

574 CONTINUE

573 CONTINUE
IF (I - LS) GT 2, 576, 576

576 IF (IQIADD .NE. ILS(LSF)) IERS = IERS + 1
IF (DR .NE. REAL(IQIADD)) IERS1 = IERS1 + 1

579 CONTINUE
IF(DI.NE.AIMAG(IS(LSF)))IERS2=IERS2+1
IF(A1.NE.RH1(LSF))IER11=IER11+1
IF(B1.NE.RH2(LSF))IER21=IER21+1
IF(C1.NE.RH3(LSF))IER31=IER31+1
IF(A5.NE.RH4(LSF))IER41=IER41+1
IF(A2.NE.IH1(LSF))IER12=IER12+1
IF(B2.NE.IH2(LSF))IER22=IER22+1
IF(C2.NE.IH3(LSF))IER32=IER32+1
IF(A6.NE.IH4(LSF))IER42=IER42+1
IF(A3.NE.DFR1(LSF))IERD11=IERD11+1
IF(B3.NE.DFR2(LSF))IERD21=IERD21+1
IF(I.NE.IEL4)GOTO2
DO 20 K=1,NO
WRITE(2,51)(IFIX(REAL(IQ(K,IK))),IK=1,32),UU(K)
WRITE(2,55)(IFIX(AIMAG(IQ(K,IK))),IK=1,32)
20 CONTINUE
55 FORMAT(1H ,32I3)
51 FORMAT(1H0,32I3,5X,F10.4)
2 CONTINUE
ERSY=IERS/EL
ERSY1=IERS1/EL
ERSY2=IERS2/EL
ER11=IER11/EL
ER12=IER12/EL
ER21=IER21/EL
ER22=IER22/EL
ER31=IER31/EL
ER32=IER32/EL
ER41=IER41/EL
ER42=IER42/EL
ERD11=IERD11/EL
ERD21=IERD21/EL
ERAV=(ER11+ER12+ER21+ER22+ER31+ER32+ER41+ER42)/8.0
ERDAV=(ERD11+ERD21+ER21+ER22+ER31+ER32+ER41+ER42)/8.0
WRITE(2,57)SNR,IN,MISSV,SYSTEM,DELAY,IPR
57 FORMAT(1H ," SNR = ",F5.2," IN='",I5," MISSV='",
I15," SYSTEM '",I3," DELAY = '",I2," IPR = '",I2)
WRITE(2,812)ERSY
WRITE(2,631)ERSY1
WRITE(2,815)ERAV,ERDAV
ASNMR=10*ALOG10(SIGMAS/SIGNAN)
ASNMR1=10*ALOG10(SIGMAS/SIGNANR)
WRITE(2,816)ASNMR,ASNMR1
816 FORMAT(1H0,20X," ASNR = '",F7.3,3X,
ASNMR1 = '",F7.3)
631 FORMAT(1H ,15X,"SYMBOL ERROR RATE ='",F10.8)
812 FORMAT(1H0,35X,"COMPLEX SYMBOL ERROR RATE ='",F10.8)
815 FORMAT(1H0,15X,"OVERALL AV. BIT ERROR RATE ='",F10.8,20X,
1'OVERALL AV. DIF. BIT ERROR RATE ='",F10.8)
300 CONTINUE
301 CONTINUE
STOP
END
SIMULATOR FOR THE 9600 BIT/S DATA TRANSMISSION SYSTEM
OVER HF CHANNELS.

******************IMPORTANT**********************
ALL ARRAYS AND VARIABLES ARE ASSUMED TO HAVE ZERO INITIAL VALUES.

DIMENSION Y(50,50), IS(200), IQ(33,50), IQQ(33,50), U(65)
DIMENSION N(65), R(100), D(600), UUU(513), UU(257)
DIMENSION IQQQ(600), M(65), ZF(65), MEM(4), IQ1(65)
DIMENSION Z(33,50), ZZ(33,50), C(100)
INTEGER RH1(200), RH2(200), DFR1(200), DFR2(200), DFI1(200)
INTEGER DPI2(200)
DIMENSION IH1(200), IH2(200), XR(50), YI(50), YRD(50), YID(50)
DIMENSION YRI(50), SKY(4, 3000), YRR(50)
COMPLEX SH(200)
INTEGER X(200)
COMPLEX YQ1(7,7), YQ2(7,7), YQ3(7,7), YQ4(7,7), YQ5(7,7)
COMPLEX IQQQQ(600), YQ6(7,7)
DIMENSION DD(600), UU2(50)
DIMENSION YPEAK(50)
COMPLEX YT(51,50)
INTEGER BIN(4,4,4), DCOD(16,2)
COMPLEX RR1, RR2, RR3, CHH3, RR
INTEGER SYSTEM
COMPLEX Y, IS, IQ, IQQ, ISD, Z, ZZ, R, IQQQ, CH1, CH2, CH3, DF1, DF2, DF3
COMPLEX SUMM, CC, RR, A, IQIADD, CHH11, CHH22, CHH33, CHH1, CHH2
INTEGER A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4, F1, F2, F3, G1, G2, G3
COMPLEX ZF, MEM, IQ1
COMPLEX G05DD, SN
COMPLEX SUMM1
COMPLEX ALF, ALF1, ALF2, BET, BET1, BET2, EALF, EBET, ALFN, BETN
REAL MEAN
DATA IH1/0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0,1,
10,0,1,0,0,0,1,1,0,1,1,0,0,1,1,1,1,0,0,0,0,1,1,0,1,1,0,0,0,0,1,1,1,1,0,1,1,1,1/
FSP: FREQUENCY SPREAD IN HZ.
DEL: DIFFERENTIAL MULTIPATH PROPAGATION DELAY IN M.S.
NO: NO. OF STORED VECTORS.
IN: INITIAL VALE FOR RANDOM NUMBER GENERATOR FOR FADING SEQ.
IN1: INITIAL VALUE FOR RANDOM NUMBER GENERATOR FOR DATA AND
NOISE.

THETA : REAL POSITIVE CONSTANT (EQNS. 7.164). THETA = 0.9
CCC : REAL POSITIVE CONSTANT (EQN. 7.151). CCC = 0.01
EPS : REAL POSITIVE CONSTANT (EQNS. 7.157).
NPD : THE EARLY DETECTION DELAY IN SAMPLING PERIODS (1/2400 S)
LS : NUMBER OF COMPONENTS IN A STORED VECTOR. LS IS THE DELAY IN DETECTION.
SNR : SIGNAL-TO-NOISE RATIO IN DB.
IEL : NO. OF TRANSMITTED DATA SYMBOLS.
IF1 : LOCATION OF FIRST SIGNIFICANT COMPONENT (EQN7.132).
IP : MAX. PERMITTED VALUE OF IF1.
IPC : MAX. PERMITTED VALUE OF IF1 OVER THE FIRST SKYWAVE.
SR : SAMPLING RATE IN KS/S (SR = 2.4)

LOAD FSP, DEL, SR, NY, NO, LS, NPD, IPC, THETA, CCC, EPS, IN, IN1, IEL, SNR

LOAD THE REAL AND IMAGINARY PARTS OF THE SAMPLED IMPULSE RESPONSE OF THE FIRST SKYWAVE (TABLES 7.6 & 7.7) IN ARRAYS YR AND YI, RESPECTIVELY.

LOAD THE REAL AND IMAGINARY PARTS OF THE SAMPLED IMPULSE RESPONSE OF THE SECOND SKYWAVE (TABLES 7.6 & 7.7) IN ARRAYS YRD AND YID, RESPECTIVELY. THE FIRST NON-ZERO COMPONENTS ARE LOCATED IN YRD(1) & YID(1).

LOAD THE REAL AND IMAGINARY PARTS OF THE SAMPLED IMPULSE RESPONSE OF RECEIVER FILTER (TABLES 7.6 & 7.7) IN ARRAYS YRR AND YRI, RESPECTIVELY.

UU(257) = 1E70
UUU(257) = 1E70
U(33) = 1E70
MEAN = 0.0
DO 9900 I = 1, 4
   IL = (I - 1) * 16
   DO 9900 K = 1, 4
      K1 = (K - 1) * 4 + IL
      DO 9900 KK = 1, 4
         BIN(I, K, KK) = IH1(K1 + KK)
9900 CONTINUE
9901 FORMAT(4(4I1))
DO 9902 I = 1, 4
   WRITE(2, 9903)((BIN(I, K, KK), KK = 1, 4), K = 1, 4)
9902 CONTINUE
9903 FORMAT(1H0, 4(4I1, 5X))
DCOD(2,2)=1
DCOD(3,1)=1
DCOD(4,1)=1
DCOD(4,2)=1
DCOD(5,1)=1
DCOD(7,1)=1
DCOD(7,2)=1
DCOD(8,2)=1
DCOD(9,2)=1
DCOD(10,1)=1
DCOD(10,2)=1
DCOD(12,1)=1
DCOD(13,1)=1
DCOD(13,2)=1
DCOD(14,1)=1
DCOD(15,2)=1
IDEL=DEL*SR*2
INYY=48
INY=NY+IDEL
INYY1=INYY-1
INYYR=NY+IDEL/2
LSY=200-NY
LS11=LS+1
LS12=LS+IP
LS13=LS12+1
IM=1
IP1=IP-1
LSF=200-LS-IP
LSF1=LSF-1
NYP=INYYR-IP-1
NO16=16*NO
MEM(1)=CMPLX(1.0,1.0)
MEM(2)=CMPLX(-1.0,1.0)
MEM(3)=CMPLX(-1.0,-1.0)
MEM(4)=(1.0,-1.0)
NO4=4*NO
NO43=NO4-3
INY1=INY-1
UUU(513)=1E70
EL=FLOAT(I16)
IPP=IP+1
NO8=8*NO
NO64=NO*64
NN=(NO/8)-1
LSN=LS-NN
IPR1=IPR+1
IF=IP1
LS1=LS-1
IEL4=IEL/4
NO2=2*NO
NO1=NO-1
NO11=NO-1-NSEC

GENERATING THE FOUR FADING SEQUENCES.
DC0,CO1,CO2,CO3,CO4,AND CO5 ARE GIVEN IN TABLE 7.3 AS G0,C1,C2,
C3,C4 AND C5, RESPECTIVELY
NOTICE THAT THE SAMPLING RATE HERE IS 50 S/S

CALL G05CBF(IN)
DO 923 IK=1,4
DO 923 I=1,3000
FA12=FA11
FA11=FA1
FA1=DCG*G05DDF(0.0,1.0)-FA11*CO1-FA12*CO2
FA22=FA21
FA21=FA2
FA2=FA1-FA21*CO3-FA22*CO4
FA31=FA3
FA3=FA2-FA31*CO5
SKY(IK,I)=FA3
923 CONTINUE
DO 1000 I=1,NO
DO 1001 K=1,LS
IQ(I,K)=CMPLX(1.0,1.0)
1001 CONTINUE
U(I)=1E70
1000 CONTINUE
U(I)=0.0
DO 1005 I=1,200
IS(I)=CMPLX(1.0,1.0)
1005 CONTINUE
DO 1002 I=1,IP
R(I+1)=CMPLX(0.0,0.0)
IK=I+1
ZA=0.0
DO 1003 K=IK,LS
ZA=ZA+Y(K)
1003 CONTINUE
DO 1004 K=1,NO
Z(K,I)=ZA
1004 CONTINUE
1002 CONTINUE
C INITIALIZATION OF RANDOM NUMBER GENERATOR FOR DATA AND NOISE.
C CALL G05CBF(IN1)
C STANDERD DEVIATION OF EACH OF REAL AND IMAGINARY PARTS OF NOISE.
C STDEV=(10**(-SNR/20.0))*SQRT(2.5)
SIGMAS=0.0
SIGMAN=0.0
SIGMANR=0.0
ISTEP=20*SR
STIP=1.0/ISTEP
ILOP=IEL/ISTEP
C
C *************** MAIN LOOP ***********************
C
I=0
DO 926 LOP=1,ILOP
SB1=0.0
DO 897 K=1,INY
SB1=SB1+REAL(Y(IC,K)*CONJG(Y(IC,K)))
897 CONTINUE
SB2=0.0
DO 898 K=LSF,200
SB2=SB2+REAL(SH(K)*CONJG(SH(K)))
898 CONTINUE
SB2=SB2/(200-LSF)
IERT=(IERD11+IERD12+IERD21+IERD22)/4
IERT1=IERT
IERT1=IERT-IERT2
IERT2=IERT
WRITE(2,52)I,IERT,IERT1,IF,IIJ,SB1,SB2,Y(IC,IF1),Y(IC,IIJ)
      52 FORMAT(1H ,5(I5,2X),2X,6(F6.4,2X))
      IF(IERT1-IERTT1)270,271,272
      270 IF(IERT1)273,273,272
      271 IF(IERT1)275,275,272
      272 LBR=LBR+1
      GOTO275
      273 MB=MB+1
      LBR1=LBR1+LBR
      LBR=0
      275 LT=LT+1
      IF(MB.EQ.0)GOTO276
      BMD=(LBR1*ISTEP)/(SR*1000.0*MB)
      BMTB=(LT*ISTEP)/(SR*1000.0*MB)-BMD
      276 CONTINUE
      KUK=0
      FA2=FA1
      FB2=FB1
      FC2=FC1
      FD2=FD1
      FA1=SKY(1,LOP)
      FB1=SKY(2,LOP)
      FC1=SKY(3,LOP)
      FD1=SKY(4,LOP)
      CON1=(FA1-FA2)*STIP
      CON2=(FB1-FB2)*STIP
      CON3=(FC1-FC2)*STIP
      CON4=(FD1-FD2)*STIP
      DO 2 LLOOP=1,ISTEP
      I=I+1
      KUK=KUK+1
      FA=FA2-FB2+KUK*(CON1-CON2)
      FB=FA2+FB2+KUK*(CON1+CON2)
      FC=FC2-FD2+KUK*(CON3-CON4)
      FD=FC2+FD2+KUK*(CON3+CON4)
      FAA=FA-KUK*(CON1-CON2)/2
      FBB=FB-KUK*(CON1+CON2)/2
      FCC=FC-KUK*(CON3-CON4)/2
      FDD=FD-KUK*(CON3+CON4)/2
      DO 924 IK=1,INYY1
      I1=INYY-IK+1
      I2=INYY-IK+2
      DO 924 IKK=1,INYY
      YT(I2,IKK)=YT(I2-2,IKK)
      Y(I1,IKK)=Y(I1-1,IKK)
      924 CONTINUE
      DO 925 IK=1,INY
      YT(1,IK)=CMPLX((YR(IK)*FA+YI(IK)*FB),(YI(IK)*FA-YR(IK)*FB))
      YT(2,IK)=CMPLX((YR(IK)*FAA+YI(IK)*FBB),(YI(IK)*FAA-YR(IK)*FBB))
      925 CONTINUE
      DO 927 IK=1,INY
      YT(1,IK+IDEL)=YT(1,IK+IDEL)+CMPLX((YRD(IK)*FC+YID(IK)*FD),
      1(YID(IK)*FC-YRD(IK)*FD))
      YT(2,IK+IDEL)=YT(2,IK+IDEL)+CMPLX((YRD(IK)*FCC+YID(IK)*FDD),
      1(YID(IK)*FCC-YRD(IK)*FDD))
      927 CONTINUE
DETERMINING THE VERY FIRST TWO ESTIMATES OF THE SAMPLED
IMPULSE RESPONSE FOR THE ESTIMATOR.

IF(ORTHO.GT.0.0)GOTO5112
IF(I.NE.100)GOTO5123
ENG1=0.0
DO 5124 K=1,INYR
YT(51,K)=Y(1,K)
ENG1=ENG1+REAL(YT(51,K)*CONJG(YT(51,K)))
5124 CONTINUE
ENG1=SQRT(ENG1)
DO 5125 K=1,INYR
YT(51,K)=YT(51,K)/ENG1
5125 CONTINUE
IF(I.LT.200)GOTO5113
SUMM=CMPLX(0.0,0.0)
ENG=I1l.0
DO 5114 K=1,INYR
ENG=ENG+REAL(Y(IPP+LS+1,K)*CONJG(Y(IPP+LS+1,K)))
5114 CONTINUE
ENG=SQRT(ENG)
DO 5116 K=1,INYR
Y(49,K)=Y(IPP+LS+1,K)/ENG
SUMM=SUMM+YT(51,K)*CONJG(Y(49,K))
5116 CONTINUE
ORTHO=0.0
IF(CABS(SUMM).GT.0.5)GOTO5113
ORTHO=1.0
ORTHO=I
ENG=0.0
DO 5117 K=1,INYR
Y(50,K)=YT(51,K)-SUMM*Y(49,K)
ENG=ENG+REAL(Y(50,K)*CONJG(Y(50,K)))
5117 CONTINUE
ENG=SQRT(ENG)
ALF1=CMPLX(0.0,0.0)
BET1=CMPLX(0.0,0.0)
DO 5118 K=1,INYR
Y(50,K)=Y(50,K)/ENG
YT(55,K)=Y(IPP+NPD+1,K)
YT(51,K)=YT(51,K)
ALF1=ALF1+YT(51,K)*CONJG(Y(49,K))
BET1=BET1+YT(51,K)*CONJG(Y(50,K))
5118 CONTINUE
THET1=(1-THETA)**2
THET2=1-THETA**2
GOTO5112
DO 928 K=1,INY
Y(1,K)=CMPLX(0.0,0.0)
DO 928 KK=1,K
KK1=K-KK+1
Y(1,K)=Y(1,K)+YT(KK1,KK)*CMPLX(YRR(KK1),YRI(KK1))
928 CONTINUE
DO 929 K=1,INY
K1=K+INY
K3=0
Y(1,K1)=CMPLX(0.0,0.0)
K2=K+1
DO 929 KK=K2,INY
K3 = K3 + 1
KK1 = INYY - K3 + 1
Y(1, K1) = Y(1, K1) + YT(KK1, KK) * CMPLX(YRR(KK1), YRI(KK1))

929 CONTINUE
DO 302 K = 1, INYY, 2
K1 = (K + 1) / 2
Y(1, K1) = Y(1, K)
302 CONTINUE
K1 = INYY + 1
DO 303 K = K1, INYY
Y(1, K) = CMPLX(0.0, 0.0)
303 CONTINUE
GOTO 5115

THE ESTIMATOR (SECTION 7.6).

5112 ALF = CMPLX(0.0, 0.0)
BET = CMPLX(0.0, 0.0)
SUMM = CMPLX(0.0, 0.0)
DO 5121 K = 1, INYY
SUMM = SUMM + YT(50, K) * IS(101 - K)
5121 CONTINUE
SUMM = R(101 - NPD) - SUMM
IF(BUCO .EQ. 0.0) GOTO 963
IF(CABS(SUMM) .LT. SQRT(SB1)) GOTO 963
ENG = 0.0
DO 964 K = 1, INYY
SUMM1 = YT(50, K) - Y(IPP + LS + 1, K)
ENG = ENG + REAL(SUMM1 * CONJG(SUMM1))
964 CONTINUE
IF(ENG .LT. 0.05 * SB1) GOTO 960
963 DO 5122 K = 1, INYY
YT(51, K) = YT(50, K) + SUMM * CCC * CONJG(IS(101 - K))
5122 CONTINUE
GOTO 961
960 DO 962 K = 1, INYY
YT(51, K) = Y(IPP + LS + 1, K)
962 CONTINUE
961 DO 5119 K = 1, INYY
ALF = ALF + YT(51, K) * CONJG(Y(49, K))
BET = BET + YT(51, K) * CONJG(Y(50, K))
5119 CONTINUE
SUMM1 = 0.0
ENG = 0.0
ENG1 = 0.0
DO 8550 K = 1, INYY
SUMM = ALF * Y(49, K) + BET * Y(50, K)
SUMM = YT(51, K) - SUMM
Y(49, K) = Y(49, K) + EPS * CONJG(ALF) * SUMM
Y(50, K) = Y(50, K) + EPS * CONJG(BET) * SUMM
ENG = ENG + REAL(Y(49, K) * CONJG(Y(49, K))
SUMM1 = SUMM1 + Y(50, K) * CONJG(Y(49, K))
8550 CONTINUE
ENG = SQRT(ENG)
DO 8551 K = 1, INYY
Y(49, K) = Y(49, K) / ENG
Y(50, K) = Y(50, K) - SUMM1 * Y(49, K) / ENG
ENG1 = ENG1 + REAL(Y(50, K) * CONJG(Y(50, K)))
CONTINUE
ENGL=SQRT(ENG1)
DO 8552 K=1,INYR
   Y(50,K)=Y(50,K)/ENG1
8552  CONTINUE
EALF=ALF-ALFI
EBET=BET-BET1
ALF2=ALF2+EALF*THET1
ALF1=ALF1+ALF2+EALF*THET2
BET2=BET2+EBET*THET1
BET1=BET1+BET2+EBET*THET2
ALFN=ALF1+(NPD+IP-1)*ALF2
BETN=BET1+(NPD+IP-1)*BET2
DO 5120 K=1,INYR
   YT (50,K)=ALF1*Y (49,K)+BET1*Y (50,K)
   Y(1,K)=ALFN*Y(49,K)+BETN*Y(50,K)
5120  CONTINUE

THE DETECTOR (SECTION 7.5)

DO 37 K=101,199
   K2=K-100
   IS(K2)=IS(K2+1)
   K1=K+1
   IS(K)=IS(K1)
   RH1(K)=RH1(K1)
   RH2(K)=RH2(K1)
   IH1(K)=IH1(K1)
   IH2(K)=IH2(K1)
   DFR1(K)=DFR1(K1)
   DFR2(K)=DFR2(K1)
   DFI1(K)=DFI1(K1)
   DFI2(K)=DFI2(K1)
   SH(K-1)=SH(K1)
37  CONTINUE
DO 5111 K=1,LS
   K1=100-LS-1+K
   R(K1)=R(K1+1)
5111  CONTINUE
R(100)=R(1)
DO 38 K=1,IP
   I1=IP+2-K
   YPEAK(I1)=YPEAK(I1-1)-SQRT(REAL(Y(I1,K)*CONJG(Y(I1,K))))
   R(K)=R(K+1)
38  CONTINUE
XX=G05CAF(XX)
IF(XX-0.25)60,60,61
61  IF(XX-0.5)62,62,63
63  IF(XX-0.75)64,64,65
60  DL=3.0
   A1=-1.0
   B1=-1.0
   GOTO4
62  DL=1.0
   A1=1.0
   B1=-1.0
   GOTO4
64  DL=-1.0
A1=1.0
B1=1.0
GOTO4

D1=-3.0
A1=-1.0
B1=1.0

XX=G05CAF(XX)

IF(XX-0.25)767,767,768

IF(XX-0.5)771,771,772

IIF(XX-0.75)775,775,776

D2=3.0
A2=-1.0
B2=-1.0
GOTO609

D2=1.0
A2=1.0
B2=-1.0
GOTO609

D2=-1.0
A2=1.0
B2=1.0
GOTO609

D2=-3.0
A2=-1.0
B2=1.0

IS(200)=CMPLX(D1,D2)

K1=(D1+5)/2
K2=(D2+5)/2

RH1(200)=BIN(K2,K1,1)
IH1(200)=BIN(K2,K1,2)
RH2(200)=BIN(K2,K1,3)
IH2(200)=BIN(K2,K1,4)

IADD1=RH2(199)*8+IH2(199)*4+RH1(200)*2+IH1(200)+1
IADD2=RH1(200)*8+IH1(200)*4+RH2(200)*2+IH2(200)+1

DFR1(200)=DCOD(IADD1,1)
DFR2(200)=DCOD(IADD1,2)
DF11(200)=DCOD(IADD2,1)
DF12(200)=DCOD(IADD2,2)

SUMM=CMPLX(0.0,0.0)
SUMM1=CMPLX(0.0,0.0)

DO 5 I,J=1,INY,2
I1=201-(IJ+1)/2
SUMM1=SUMM1+IS(I1-1)*YT(2,IJ+1)
SUMM=SUMM+IS(I1)*YT(1,IJ)

CONTINUE

SIGMAS=SIGMAS+REAL(SUMM*CONJG(SUMM))
G05DD=CMPLX(G05DDF(0.0,STDEV),G05DDF(0.0,STDEV))
SIGMAN=SIGMAN+REAL(G05DD*CONJG(G05DD))
SN=CMPLX(0.0,0.0)

SH(200)=G05DD+SUMM
G05DD=CMPLX(G05DDF(0.0,STDEV),G05DDF(0.0,STDEV))
SH(199)=G05DD+SUMM1
SIGMAS=SIGMAS+REAL(SUMM1*CONJG(SUMM1))
SIGMAN=SIGMAN+REAL(G05DD*CONJG(G05DD))

DO 934 I,J=1,INY
I1=201-IJ
SN=SN+SH(I1)*CMPLX(YRR(IJ),YRI(IJ))

CONTINUE

SIGMANR=SIGMANR+REAL(SN*CONJG(SN))
R(IPP)=SN
DO 200 IK=1,NO
Z(IK,IPP)=CMPLX(0.0,0.0)
DO 201 K=1,NYP
Z(IK,IPP)=Z(IK,IPP)+IQ(IK,LS11-K)*Y(1,K+IPP)
201 CONTINUE
200 CONTINUE
AA=0.0
DO 310 IK=1,IP
AA=AA+SQRT(REAL(Y(1,IK)*CONJG(Y(1,IK))))
310 CONTINUE
YPEAK(1)=AA
DELTA2=1E-30
AA1=1E-30
DO 311 IK=2,IP
IA=IP+2-IK
AA=SQRT(REAL(Y(IK,IA)*CONJG(Y(IK,IA))))
IF(AA)311,311,320
311 IJJ1=IKK
DELTA=0.0
LK=IPP-IK
DO 312 IKK=1,LK
ABS1=SQRT(REAL(Y(IK,IKK)*CONJG(Y(IK,IKK))))
IF(ABS1-DELTA)312,313,313
312 CONTINUE
DELTA1=AA-YPEAK(IK)+DELTA
IF(DELTA1-DELTA2)311,311,308
308 IF=IP+1-IK
DELTA2=DELTA1
AA1=AA
IJJ=IJJ1
311 CONTINUE
IF(IF-IFF11)156,151,155
156 AA=CABS(Y(IC,IF1))
IF(AA-AA1)160,161,161
161 IJJ=IF+1
IF=IFF11
GOTO151
160 NSEC=0
IPR=1
GOTO150
155 NSEC=8
IPR=1
150 KF1=IFF1
KF=IFF1-1
KF11=IFF11
KB=IB
KC=IC
KJJ=IJJP
NO1=NO-1-NSEC
IPR1=IPR+1
151 IFF11=IF
IJJP=IJJ
IFF=IFF1
IFF1=IFF+1
IFF11=IFF-1
IB=IP+2-IP
IC=IP+2-IPF
IF(I.GT.IP)GOTO711
KF1=IFF1
KF = IF
KF11 = IF11
KB = IB
KC = IC
KJJ = IF
GOTO 2

711 DO 414 K = 1, 7, 2
AJ = K - 4
DO 414 IK = 1, 7, 2
BJ = IK - 4
 RR1 = CMPLX(AJ, BJ)
 YQ1(K, IK) = RR1*Y(IB, IF)
 YQ2(K, IK) = RR1*Y(IC, IF1)/Y(IC, IJJ)
 YQ3(K, IK) = RR1*Y(IC, IJJ)
 YQ4(K, IK) = RR1*Y(KB, KF)
 YQ5(K, IK) = RR1*Y(KC, KF1)/Y(KC, KJJ)
 YQ6(K, IK) = RR1*Y(KC, KJJ)

414 CONTINUE
DO 202 K = 1, NO
RR3 = (R(IF1) - Z(K, IF1))/Y(IC, IJJ)
DO 640 IIK = 1, LS1
IQQ(K, IIK) = IQ(K, IIK + 1)

640 CONTINUE
UUC = U(K)
DO 641 IIK = 1, IF11
CC = R(IIK) - Z(K, IIK)
UUC = UUC + REAL(CC*CONJG(CC))

641 CONTINUE
RR1 = R(IF) - Z(K, IF)
JI2 = (K - 1)*16
DO 205 IJ = 1, 7, 2
AJ = FLOAT(IJ - 4)
DO 410 JI = 1, 7, 2
BJ = FLOAT(JI - 4)
JI2 = JI2 + 1
IQQQ(JI2) = CMPLX(AJ, BJ)
RR2 = RR1 - YQ1(IJ, JI)
UUUC = UUC + REAL(RR2*CONJG(RR2))
RR = RR3 - YQ2(IJ, JI)
RERR = REAL(RR)
AIMRR = AIMAG(RR)
IF(RERR) 411, 412, 412

411 RERR = RERR + 2
GOTO 417
412 RERR = RERR - 2
417 IF(AIMRR) 413, 419, 419
413 AIMRR = AIMRR + 2
GOTO 415
419 AIMRR = AIMRR - 2
415 IF(RERR) 420, 421, 421
420 RERR = RERR + 1
GOTO 422
421 RERR = RERR - 1
422 IF(AIMRR) 423, 424, 424
423 AIMRR = AIMRR + 1
GOTO 425
424 AIMRR = AIMRR - 1
425 RR = CMPLX(RERR, AIMRR)*Y(IC, IJJ)
D(JI2) = UUUC + REAL(RR*CONJG(RR))

410 CONTINUE
205 CONTINUE
202 CONTINUE
   IF(NSEC.EQ.0)GOTO418
   DO 140 K=1,NO
      RR3=(R(KF1)-Z(K,KF1))/Y(KC,KJJ)
      UUC=U(K)
      DO 141 IIK=1,KF11
         CC=R(IIK)-Z(K,IIK)
         UUC=UUC+REAL(CC*CONJG(CC))
      CONTINUE
      RR1=R(KF)-Z(K,KF)
      JI2=(K-1)*16
      DO 142 IJ=1,7,2
         AJ=IJ-4
         DO 143 JI=1,7,2
            BJ=JI-4
            JI2=JI2+1
            IQQQQ(JI2)=CMPLX(AJ,BJ)
            RR2=RR1-YQ4(IJ,JI)
            RR=RR3-YQ5(IJ,JI)
            RRERR=REAL(RR)
            AIRMRR=AIMAG(RR)
            IF(RRERR)330,331,331
               330 RRERR=RRERR+2
               GOTO332
            331 RRERR=RRERR-2
            332 IF(AIRMRR)333,334,334
               333 AIRMRR=AIRMRR+2
               GOTO335
            334 AIRMRR=AIRMRR-2
            335 IF(RRERR)359,360,360
            359 RRERR=RRERR+1
               GOTO361
            360 RRERR=RRERR-1
            361 IF(AIRMRR)362,363,363
            362 AIRMRR=AIRMRR+1
               GOTO364
            363 AIRMRR=AIRMRR-1
            364 RR=CMPLX(RRERR,AIRMRR)*Y(KC,KJJ)
               DD(JI2)=UUUC+REAL(RR*CONJG(RR))
      CONTINUE
      142 CONTINUE
      140 CONTINUE
      418 DNOISE=1E60
      DO 701 IK=1,NO16
         IF(D(IK)-DNOISE)702,701,701
            702 IADD=IK
            DNOISE=D(IK)
      CONTINUE
      701 CONTINUE
      UU(1)=D(IADD)
      D(IADD)=1E70
      UU2(1)=DD(IADD)
      DD(IADD)=1E70
      N(I)=(IADD+15)/16
      IQIADD=IQ(N(I),1)
      IQ(1,LS)=IQQQ(IADD)
      IF(I.GT.100)GOTO356
      DO 357 K=1,LS1
      IQQ(N(I),K)=IS(LSF+K)
CONTINUE
IQ(1,LS)=IS(LSF+LS)

IF(NSEC.EQ.0)GOTO358
DO 344 K=1,NO
  KK=(K-1)*16
  DNOISE=1E60
  UUU(K)=1E70
  A=IQ(K,1)
  IF(A.NE.IQIADD)GOTO344
  DO 345 IK=1,16
    KK=KK+1
    IF(DD(KK)-DNOISE)346,345,345
  346  IADD=KK
    DNOISE=DD(KK)
  345  CONTINUE
  IQ1(K)=IQQQQ(IADD)
  UUU(K)=DD(IADD)
  M(K)=IADD
  DD(IADD)=1E70
  344  CONTINUE
K1=0
DO 347 IK=1,NSEC
  K=IK+NOll+1
  IF(K1.EQ.0)GOTO348
  IF(K1.EQ.33)GOTO353
  KK=(K1-1)*16
  DNOISE=1E60
  IADD=0
  DO 349 IKK=1,16
    KK=KK+1
    IF(DD(KK)-DNOISE)350,349,349
  349  IADD=KK
    DNOISE=DD(KK)
  348  CONTINUE
IF(IADD.EQ.0)GOTO348
  IQ1(K1)=IQQQQ(IADD)
  UUU(K1)=DD(IADD)
  M(K1)=IADD
  DD(IADD)=1E70
  347  CONTINUE
IADD=33
DNOISE=1E60
DO 351 IKK=1,NO
  IF(UUU(IKK)-DNOISE)352,351,351
  352  IADD=IKK
    DNOISE=UUU(IKK)
  351  CONTINUE
N(K)=IADD
  IQ(K,LS)=IQ1(IADD)
  UU(K)=UUU(IADD)
  UUU(IADD)=1E70
  D(M(IADD))=1E70
  K1=IADD
  347  CONTINUE
DO 336 K=1,NO
  KK=(K-1)*16
  DNOISE=1E60
  UUU(K)=1E70
  A=IQ(K,1)
  IF(A.NE.IQIADD)GOTO336
  DO 337 IK=1,16
KK=KK+1
IF(D(KK)-DNOISE)338,337,337
338 IADD=KK
DNOISE=D(KK)
337 CONTINUE
IQI(K)=IQQQ(IADD)
UUU(K)=D(IADD)
D(IADD)=1E70
DD(IADD)=1E70
336 CONTINUE
N(2)=1
UU(2)=UUU(1)
IQ(2,LS)=IQ1(1)
IF(UUU(1).GT.1E60)N(2)=33
UUU(1)=1E70
K1=1
DO 339 IK=2,NO11
K=IK+1
IF(K1.EQ.33)GOTO354
KK=(K1-1)*16
DNOISE=1E60
IADD=0
DO 340 IKK=1,16
KK=KK+1
IF(D(KK)-DNOISE)341,340,340
341 IADD=KK
DNOISE=D(KK)
340 CONTINUE
IF(IADD.EQ.0)GOTO355
IQI(K1)=IQQQ(IADD)
UUU(K1)=D(IADD)
D(IADD)=1E70
DD(IADD)=1E70
355 IADD=33
DNOISE=1E60
DO 342 IKK=1,NO
IF(UUU(IKK)-DNOISE)343,342,342
343 IADD=IKK
DNOISE=UUU(IKK)
342 CONTINUE
354 N(K)=IADD
IQ(K,LS)=IQ1(IADD)
UU(K)=UUU(IADD)
UUU(IADD)=1E70
K1=IADD
339 CONTINUE
708 DR=REAL(IQIADD)
DI=AIMAG(IQIADD)
NVEC=N(1)
IF(U(N(9)).LT.U(N(1)))NVEC=N(9)
IS(100)=IQ(NVEC,1+LS-NPD)
K1=(DR+5)/2
K2=(DI+5)/2
A1=BIN(K2,K1,1)
A2=BIN(K2,K1,2)
B1=BIN(K2,K1,3)
B2=BIN(K2,K1,4)
IADD1=F2*8+G2*4+A1*2+A2+1
A3=DCOD(IADD1,1)
B3=DCOD(IADD1,2)
A4=DCOD(IADD2,1)
B4=DCOD(IADD2,2)

808 DO 560 IK=1,NO
DO 570 IKK=1,LS1
IQQ(IK,IKK)=IQ(N(IK),IKK)
570 CONTINUE
560 CONTINUE
DO 571 IK=1,NO
NK=N(IK)
RRR=R(1)-Z(NK,1)-Y(IPP,1)*IQ(IK,LS)
RZ=REAL(RRR*CONJG(RRR))
UUU(IK)=U(NK)+RZ
DO 572 IKK=1,IP
ZZ(IK,IKK)=Z(NK,IKK+1)+IQ(IK,LS)*Y(IPP-IKK,IKK+1)
572 CONTINUE
571 CONTINUE
761 CONTINUE
DO 573 IK=1,NO
U(IK)=UUU(IK)
DO 574 IKK=1,IP
Z(IK,IKK)=ZZ(IK,IKK)
574 CONTINUE
573 CONTINUE
F1=A1
F2=B1
G1=A2
G2=B2
IF(ORTHO)2,2,576
576 IF(IQ IADD.NE.IS(LSF))IERS=IERS+1
IF(DR.NE.REAL(IS(LSF)))IERS1=IERS1+1
IF(DI.NE.AIMAG(IS(LSF)))IERS2=IERS2+1
IF(A1.NE.RH1(LSF))IER11=IER11+1
IF(B1.NE.RH2(LSF))IER21=IER21+1
IF(A2.NE.IH1(LSF))IER12=IER12+1
IF(B2.NE.IH2(LSF))IER22=IER22+1
IF(A3.NE.DFR1(LSF))IERD11=IERD11+1
IF(B3.NE.DFR2(LSF))IERD21=IERD21+1
IF(A4.NE.DFI1(LSF))IERD12=IERD12+1
IF(B4.NE.DFI2(LSF))IERD22=IERD22+1
IF(I.NE.IEL4)GOTO2
LL31=LS-31
DO 20 K=1,NO
WRITE(2,51)(IFIX(REAL(IQ(K,IK))),(IK=LL31,LS),UU(K)
WRITE(2,55)(IFIX(AIMAG(IQ(K,IK))),(IK=LL31,LS)
20 CONTINUE
55 FORMAT(1H ,32I2)
51 FORMAT(1H0,32I2,5X,F10.4)
2 CONTINUE
926 CONTINUE
EL=EL-IOPTH
ERSY=IERS/EL
ERSY1=IERS1/EL
ERSY2=IERS2/EL
IER11=IER11/EL
IER12=IER12/EL
IER21=IER21/EL
IER22=IER22/EL
IERD11=IERD11/EL
IERD12=IERD12/EL
ERD21=IERD21/EL
ERD22=IERD22/EL
ERAV=(ER11+ER12+ER21+ER22)/4.0
ERDAV=(ERD11+ERD12+ERD21+ERD22)/4.0
WRITE(2,57)SNR,IN,DEL,FSP,SYSTEM
   57  FORMAT(1H0,'SNR=',F5.2,' IN=',I5,' DEL=',F4.2,' FSP=',
            1F4.2,' SYSTEM=',I3)
WRITE(2,812)ERSY
WRITE(2,631)ERSY1
WRITE(2,631)ERSY2
WRITE(2,815)ERAV,ERDAV
   812  FORMAT(1H0,35X,'COMPLX SYMBOL ERROR RATE =',F10.8)
   815  FORMAT(1H0,15X,'OVERALL AV. BIT ERROR RATE =','F10.8,20X,
                         'OVERALL AV. DIF. BIT ERROR RATE =','F10.8)
   631  FORMAT(1H0,15X,'SYMBOL ERROR RATE =',F10.8)
   818  FORMAT(1H0, 'BURST MEAN DURATION =',F10.6, ' SEC.')
   819  FORMAT(1H0, 'MEAN TIME BETWEEN BURSTS=',F10.5, ' SEC.')
   820  FORMAT(1H0, ' IORTHO=',I5)
   816  FORMAT(1H0, ' SNR AT THE RECEIVER FILTER INPUT = ',F10.5)
   817  FORMAT(1H0, ' SNR AT THE RECEIVER FILTER OUTPUT= ',F10.5)
300 CONTINUE
301 CONTINUE
STOP
END
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