Low frequency noise generated by industrial gas turbines

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Low Frequency Noise Generated by Industrial Gas Turbines

by

Gia Kroeff

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of
Doctor of Philosophy of Loughborough University
On the 9th September 2004

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Abstract

Noise generated from industrial gas turbines is an increasing environmental concern. The silencing of the exhaust from industrial gas turbines is an important element of current designs as it can affect efficiency, space, noise and gas emissions. However, exhausts are very costly and the lower the frequency, the higher is the cost involved in trying to attenuate the noise due to the amount of material and space necessary to implement the exhaust system.

In this work, the sources of noise from an exhaust system of a particular gas turbine are investigated. Improvements in understanding the unsteady behaviour of the flow in the exhaust system could potentially lead to an increase in efficiency, a reduction in noise emissions, a decrease in the cost of exhaust mufflers and improved location for the plants.

This work presents the experimental approach used to identify the major sources of noise and how these results were then used to create a model that could represent the sources identified. As the frequency components generated by the flow are low, this work concentrates on understanding the mechanisms that generate the low frequency noise.

Results show that the major source of noise is the jet leaving the engine exhaust and that the main acoustic source is of dipole nature.
To:

- Björn Petersson, for all his guidance, patience and friendship. Without him this work would never have finished,
- Jim McGuirk, Jon Carrotte and Charith Jayatunga for all their help with the aerodynamic aspects of this research,
- Rolls-Royce and CNPq for funding this work,
- Friends and colleagues from Loughborough University and TU Berlin, in particular Andy, Bill, Kevin, Les, Matt, Nicola and Paul for their support and encouragement,
- My beloved husband, Phil, for his endless help, patience, love and continuous inspiration for new ideas,
- My parents and my family, that despite the distance kept encouraging me throughout this time in many different ways,
- My great friends Renata, Rita and Simone for their friendship and example of dedication and perseverance,
- Bauzer, Vitor and Ester for their support at the early stages of this research,
- To all of you, perhaps not listed here, that helped me in one way or another to finish this work,

My many many thanks.
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<td>cylinder's radius</td>
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<td>constant</td>
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<td>$a$</td>
<td>inferior limit of the integration</td>
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<td>$b$</td>
<td>constant</td>
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<tr>
<td>$B_r$</td>
<td>constant</td>
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<td>$B_n$</td>
<td>force distribution constant</td>
</tr>
<tr>
<td>$b$</td>
<td>superior limit of the integration</td>
</tr>
<tr>
<td>$CF$</td>
<td>abbreviation for Centre Shaft</td>
</tr>
<tr>
<td>$C_\phi$</td>
<td>constant</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of sound in the air</td>
</tr>
<tr>
<td>$CDC$</td>
<td>abbreviation for Continuous Ring</td>
</tr>
<tr>
<td>$DDR$</td>
<td>abbreviation for Discrete Dipole Ring</td>
</tr>
<tr>
<td>$d$, $D$</td>
<td>diameter of duct</td>
</tr>
<tr>
<td>$d$</td>
<td>infinitesimal dipole lever distance</td>
</tr>
<tr>
<td>$\hat{D}$</td>
<td>dipole strength</td>
</tr>
<tr>
<td>$e$</td>
<td>unitary vector pointing outward from the dipole centre to the receiver position</td>
</tr>
<tr>
<td>$\hat{F}_w$</td>
<td>complex, frequency dependent dipole force</td>
</tr>
<tr>
<td>$f(z)$</td>
<td>force profile in the $z$ direction</td>
</tr>
<tr>
<td>$\hat{f}(z)$</td>
<td>Fourier transform of $z$</td>
</tr>
<tr>
<td>$f(y)$</td>
<td>inverse Fourier transform</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Schroeder frequency</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency</td>
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<td>$G_k$</td>
<td>Green's function</td>
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<tr>
<td>$g$</td>
<td>generic profile in the $z$ direction</td>
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<tr>
<td>$H_n^{(1)}$</td>
<td>$n$th-order Hankel functions of the first kind</td>
</tr>
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<td>$H_n^{(2)}$</td>
<td>$n$th-order Hankel functions of the second kind</td>
</tr>
<tr>
<td>$IGV$</td>
<td>abbreviation for Inlet Guide Vanes</td>
</tr>
<tr>
<td>$i$</td>
<td>complex number $i = \sqrt{-1}$</td>
</tr>
<tr>
<td>$I$</td>
<td>acoustic intensity</td>
</tr>
<tr>
<td>$i$</td>
<td>element position in the $x$ direction</td>
</tr>
<tr>
<td>$j$</td>
<td>element position in the $\phi$ direction</td>
</tr>
<tr>
<td>$k$</td>
<td>wave number</td>
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<tr>
<td>$l, L$</td>
<td>length</td>
</tr>
<tr>
<td>$L_w$</td>
<td>Sound Power Level</td>
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<tr>
<td>$M$</td>
<td>Mach number</td>
</tr>
<tr>
<td>$m$</td>
<td>constant</td>
</tr>
<tr>
<td>$N$</td>
<td>number of intervals</td>
</tr>
<tr>
<td>$n$</td>
<td>constant</td>
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<tr>
<td>$n$</td>
<td>modes</td>
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<tr>
<td>$O$</td>
<td>order of magnitude</td>
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<td>$OGV$</td>
<td>Outlet Guide Vanes</td>
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<tr>
<td>$\hat{p}$</td>
<td>complex acoustic pressure</td>
</tr>
<tr>
<td>$p$</td>
<td>acoustic pressure</td>
</tr>
<tr>
<td>$p^*$</td>
<td>conjugate acoustic pressure</td>
</tr>
<tr>
<td>$\tilde{p}$</td>
<td>Fourier transform of pressure $p$ in the $z$ direction</td>
</tr>
<tr>
<td>$P$</td>
<td>position of a measurement point</td>
</tr>
<tr>
<td>$R$</td>
<td>gas constant</td>
</tr>
<tr>
<td>$R$</td>
<td>distance between the dipole centre and the measurement position</td>
</tr>
<tr>
<td>$R$</td>
<td>spherical coordinate</td>
</tr>
<tr>
<td>$r$</td>
<td>certain position in the radial direction</td>
</tr>
<tr>
<td>$RANS$</td>
<td>Reynolds Averaged Navier Stokes</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds Number</td>
</tr>
<tr>
<td>$Re$</td>
<td>real part of a number</td>
</tr>
<tr>
<td>$R_p$</td>
<td>radial position of point $P$</td>
</tr>
<tr>
<td>$\hat{S}$</td>
<td>monopole sources of strength</td>
</tr>
<tr>
<td>$St$</td>
<td>Strouhal number</td>
</tr>
<tr>
<td>$S$</td>
<td>area</td>
</tr>
<tr>
<td>$SPL$</td>
<td>sound pressure level</td>
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<tr>
<td>$T$</td>
<td>reverberation time</td>
</tr>
<tr>
<td>$T$</td>
<td>absolute temperature</td>
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<tr>
<td>$t$</td>
<td>time</td>
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<td>$V$</td>
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<tr>
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<tr>
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<td>reference acoustic power</td>
</tr>
<tr>
<td>$W$</td>
<td>radiated acoustic power</td>
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Normalized power

Power of a unitary dipole at a certain far-field position for unitary source strength.

Acceleration distribution

Amplitude for the position $z_o$

Position in $Z$

Large integer number

Wavelength

Thickness of the initial shear layer

Spherical and cylindrical coordinate

Function of $\phi$

Function of $\gamma$

Adiabatic constant

Angle between the dipole lever and the line going from the centre of the dipole to the observer position

Fourier transform in $z$

Spherical coordinate

Angle between dipole and observer

Density

Gaussian function variance

Radiation efficiency

Angular frequency

Auxiliary function in the $R$ direction

Auxiliary function in the $\phi$ direction

Auxiliary function in the $t$ direction

Sub-scripts:

Far field

Number of sources

Radial direction

Referent to measuring position P

In the $Z$ direction
\( \phi \) in the \( \phi \) direction

\( i,j,k \) \( x, \phi \) and \( z \) directions respectively

\( \text{top} \) radius values at a more external position in the radial direction

\( \text{bottom} \) radius values at a more internal position in the radial direction
Chapter 1 Introduction

The advent of the gas turbine was a landmark in engineering history. Since its introduction, it has been used in many applications and by a range of industries. The possibility of affordable air flights led to a lucrative market in gas turbines for civil aircraft. On the industrial side, many uses have been and continue to be developed including electricity generation and natural gas pumping.

Nevertheless, with new potentials for marketing of a product come the challenges for the manufacturers. In the civil aircraft market, increased competition between the airline companies has forced gas turbine manufacturers to strive for improvements in efficiency and durability. In the industrial market, competition has led to the same requirements but with the addition of improved reliability.

Increased awareness of the environment led to the introduction of noise regulations with respect to both civil and industrial gas turbines. These requirements will become more rigorous as time passes and technology improves. Although each industrial gas turbine application has its own challenges, the exhaust systems used are somewhat similar and have the same requirements of an increase in efficiency, a reduction in space, low noise emission and low gas emissions. Therefore, in this research in order address the noise generated by an industrial gas turbine, aspects such as noise characteristics, flow complexity and issues related to system build-up are considered.

A cross-section of an industrial gas turbine is shown in Figure 1, from Rolls-Royce ([1], [2]). The fumes from the turbine exit travel through vanes that act to straighten the flow, which goes into a diffuser prior to entering the exhaust collector. The air is then diffused as it enters the collector box area. Part of the flow reaches the collector box (volute) wall and is forced to turn vertically. The remainder of the flow turns downwards first, and due to the shape of the exhaust case, that restricts the flow from going further downwards and has a round shape that facilitates the flow movement, eventually also turns upwards. Once all the flow has turned 90°, the air passes through a long muffler in order to attenuate the noise.
The silencing of the exhaust from industrial gas turbines is an important but very costly element of current designs. Nowadays, it is necessary to introduce very long mufflers at the exit of the gas turbine to attenuate the noise such that it meets environmental regulations. The attenuation of sound with silencers ([3],[4]) uses both reflective and dissipative mufflers. Dissipative mufflers employ appropriate materials that dissipate some of the sound energy and are generally used to attenuate high frequencies (since for lower frequencies the absorber dimensions have to be very large). Reactive mufflers are based on the reflection of the sound waves, due to the change in impedance through the muffler. Although reactive mufflers work at low frequency, it is necessary to use extended compartment lengths in order to ensure attenuation. Therefore, the lower the frequency, the higher is the cost involved in trying to attenuate the sound due to the amount of material and space necessary to implement the exhaust system.

The most challenging characteristic in the spectrum obtained from the industrial gas turbine considered here is a pronounced peak of O(10Hz) (Figure 2). At such low frequencies the wavelength is very large. In order to try to attenuate the low frequencies components long exhaust mufflers of around 7 meters are attached to the industrial gas turbines. This means that a costly element is added to each turbine. This system is still not long enough to effectively reduce the low frequency component identified in the measurements, which would require even longer exhaust mufflers. Therefore, instead of adopting the traditional improvements in industrial gas turbines of attenuating the propagation of the sound generated in their exhaust system, in this work attention is given to reducing the noise at the source. With that aim, the source mechanisms and the physical description of sound generated aerodynamically in
exhaust systems of industrial gas turbines are investigated. The investigation of aerodynamic noise sources can be a joint process with efficiency considerations of the gas turbine as the flow characteristics reducing the efficiency may be the same as those of sound generation. Understanding the unsteady behaviour of the flow in the exhaust system could potentially lead to increases in efficiency, reduction of noise emissions, a decrease in the cost of exhaust mufflers and better locations of the plants. As the emitted noise is dominated by the low frequency range, this work concentrates on the mechanisms that generate the low frequency noise.

There are several causes of aerodynamic fluctuations and hence many possible noise sources in a combination of a typical annular exhaust diffuser dumping gas into a typical exhaust volute box. Experimental and computational modelling of acoustics (with the help of aerodynamic experimental and computational modelling [5]) is carried out, in order to identify the characteristics and behaviour of the main aero-acoustic sources present. Combustion effects have not been considered due to the fact that the combustion noise spectra has peaks happen at much higher frequencies [12]. It is also noticed that the low frequency component appears in the industrial gas turbine but not in the equivalent aero-engine. As combustion happens in both engines, this also emphasised that excluding the combustion phenomena should not compromise the results. However attention is taken during preliminary experimental phase of this research (Chapter 4) to verify that combustion is not one of the major sources at low frequencies. As the understanding of the major acoustic sources is the main concern,
and as the characteristics of the muffler system only "colours" the source spectral characteristics [3], in this work, the system is studied up to the volute and the muffler's influence is excluded.

In summary, some of the challenges in characterising the noise from an industrial gas turbine are related to:

- How to deal with the high temperatures in the system;
- What is the best compromise in considering noise attenuation at the source, path or receiver;
- How to create an adequate model to represent the real engine in order to capture the sources of noise;
- How to isolate the sources of noise;
- How to take into consideration the resources and space available.

The use of a scaled experimental model to simulate the real industrial gas turbine is motivated by practical and economical reasons (more details are given in Chapter 4). Therefore a model containing the major features of the real engine is developed (Figure 3 & 4). Despite possessing similar characteristics in terms of generated flow power, the model does not account for the combustion effect and does not include the exhaust muffler present in the real engine.

The experimental facility used in this work consists of a plenum (Figure 3) and the test section (rig itself – Figure 4). As shown in Figure 3, atmospheric air passes through an inlet duct into an acoustically lined centrifugal fan room. The air is subsequently drawn into the fan and pumped along an interconnecting duct to a plenum, which is also acoustically lined and contains a series of acoustic baffles. Within this plenum, the air has a velocity of approximately 0.3 m/s (M~0.001), which settles most flow instabilities generated by the flow. In addition, the acoustic cladding absorbs the majority of the noise generated by the fan and motor. Air enters the working section via an inlet flare. The flow then passes through inlet guide vanes (IGVs) that rotate the flow in such a way to represent the flow characteristics generated by the rotating parts of a real engine. Further downstream, the flow passes through the Outlet Guided Vanes (OGS) that straighten the flow back in the same way they are used in a real engine, prior to entering the diffuser, as shown in Figure 4. The inlet Mach number is kept constant at 0.08. Flow exiting the diffuser is dumped into an acoustic volute box that
turns the flow 90° before expelling it into the atmosphere. The IGV geometry can be altered in order to simulate the changes produced in the swirl angle distribution. The three sets presented to the IGVs correspond to 30%, 70% and 100% engine operating powers and vary from high levels of swirl to low levels of swirl respectively. The Reynolds number, based on diffuser inlet annulus height, is approximately $1.3 \times 10^5$. The OGVs are positioned immediately upstream of the diffuser inlet, and at 10 IGV blade chords downstream of the IGVs. It is possible to operate the facility with or without several of its components in order to acoustically evaluate the components in combination or in isolation.

It is important to emphasize that both the experimental and theoretical investigation in this work are based on the $1/7^{th}$ scale aerodynamic model of the real engine. Therefore, from here on all the results presented (experimental and theoretical) are related to the characteristics and dimensions encountered on the $1/7^{th}$ scale model. Although in terms of dimensions and aerodynamic characteristics the model is considered at $1/7^{th}$ scale, in terms of acoustics characteristics it is scaled as $1/4^{th}$. This is based on the fact that added to the dimension changes, the experimental model runs at ambient temperature and therefore does not present the temperature effects that occur in the real turbine (temperatures of the order $O(1000K)$) and therefore the frequencies are scaled based on $f = \sqrt{\frac{RT}{\lambda}}$ [6].

![Figure 3 Fan and plenum – schematic drawing of the test facility](image)
As all the experiments and theoretical modelling are related to the 1/4\textsuperscript{th} acoustic scale model, the low frequency range of interest is then about four times higher than the real engine. In this way, peaks like the $\Omega(10\text{ Hz})$ (Figure 2) and the frequency range of interest (one decade) would be expected to be around two octaves higher in the model.

Exploiting the fact that the experimental facility can be assembled in several different configurations, several tests were conducted in order to account for possible different flow phenomena caused by the presence or absence of each element. For more information about the experiments, please refer to Chapter 4 and Appendix I.

Figure 5 shows the sound power level against frequency for the configuration without the volute for two power conditions (more details on the sources are presented on Chapter 4). The spectra show a decay in sound power with frequency up to approximately 600 Hz followed by a sudden increase in level and another decay. The frequency where the sudden increase occurs corresponds to the cut-off-frequency at which the higher order modes start to propagate\cite{7}.
Irrespective of configuration analysed, the results show the same slope below the cut-off frequency (frequency above which a particular mode pattern propagates) and this fact is the key element used in the source investigations. This decay is present regardless of the complexities encountered in some of the configurations, even when every element is removed and the only remaining component is the duct, which produces a jet into the free space.

Another major feature of the results are the peaks present in the low frequency region below the cut-off. These are plane wave resonances.

Given the interest in attenuating the low frequency components (as they represent the critical operational restrictions in the real engine) an investigation into the major types of sources that could be generating the characteristic sloping spectrum and that could be exciting the duct resonances at low frequencies is conducted.

Generally, in analysing aerodynamic noise, three major types of sources are considered: monopoles, dipoles and quadrupoles. Monopoles are normally associated with expansions, dipoles associated with vortex shedding and quadrupoles associated with turbulence. As the flow in the exhaust system of the industrial gas turbine is complex, it is likely that all the sources mentioned are present. However, the acoustic power radiated from each of those sources is different. Figure 6 shows the normalised sound power against Mach number for different types of sources following Morse and
Ingard [8]. Considering that the speeds used in the test rig are low, the source that would radiate most efficiently is the monopole, followed by the dipole.

Although it is likely that all three types of sources exist in the flow, the source strength of the quadrupoles required to radiate the same sound power as a monopole and a dipole would be respectively of the order \( O(M^2) \) and \( O(M) \) bigger for Mach numbers below one. Therefore, in this work, monopoles and dipoles are considered to be the likely sources generating noise at low frequency.

![Figure 6 Sound Power against Mach number for different types of sources](image)

A review of the literature on airborne noise in turbines is presented in Section 1.1. Although there are many types of sources in industrial gas turbines, emphasis is given to the aspects that are relevant for this work, such as: 1) aerodynamically generated monopole and dipole sources (as they are the most likely candidates for noise at low frequency) and 2) jet noise which is present in exhaust systems such as the one analyzed. This literature review is not intended to be all encompassing, but aims to present aspects relevant to the subject in question.

No temperature effects are included in this work as the experimental facility runs at ambient temperature. Also, no consideration is given to aspects relevant to high-speed flow in air (convection/refraction), as the Mach number involved is very low. Although combustion is not present in the scale model experiments, some of its effects are considered, as combustion is normally an important monopole source.
1.1 Airborne noise in gas turbines

The major sources of airborne noise in gas turbines consist of turbo-machinery noise [9], combustion noise [10], duct noise [11] and exhaust jet noise ([10], [12], [13]).

1.1.1 Turbo-machinery noise

In exhaust systems of gas turbine engines the ‘turbo-machinery’ present is the rotating and stationary blades used in the turbine for which the aero-acoustic flow aspects are similar to other turbo-machinery components. Turbo-machinery flows normally generate significant tonal components superimposed on broadband noise. The spectra they generate depend on the tip speed. The blade passing frequency and its harmonics are superimposed on a broadband component. The noise generation process is a combination of many mechanisms. Internal disturbances (blade wakes, vortices, turbulence) together with inflow disturbance generate a blade response that causes unsteady surface pressure on the blade. These responses are coupled to the duct, which has a modal behaviour and is responsible for the transmission. The origins of fluctuating pressures on the blades are incident vortical disturbances. The velocity component normal to the blade chord is responsible for the blade pressure fluctuations. There are models considering the response for periodic [14] and random vortical disturbances [15]. Although there exist both experimental and analytical techniques describing the lift theory, the transmission and the coupling with the duct, the dominant generating mechanism of the broadband noise remains to be properly understood. It seems to be independent of the inflow characteristics but dependent on the blade loading [15].

1.1.2 Combustion noise

The combustion noise is characterized by its broadband radiated power, its spectrum and directional distribution. However, due to the resonance of the combustor, the radiated noise has sharp peaks superimposed on the broadband noise. Combustion
noise results when a volume of mixture expands at constant pressure as it is rapidly heated by combustion. Such sources behave as acoustic monopoles ([16] and [17]).

### 1.1.3 Duct noise

Another important aspect is that the engine duct system is excited by the aerodynamic noise sources generated at the exit of the system. The duct system might only introduce peaks in the spectra corresponding to flow-excited resonances [18] and/or change the inflow/outflow boundary conditions for the aero-acoustic problem. The propagation of acoustic disturbance in ducts varies with the dimensions of the duct [19], annular parts ([20], [21], [22], [23]), speed changes [24], level of swirl ([25], [10], [26], [27], [28], [29]), and variation of the impedance at the end [30]. Other turbulent sources are caused by effect of surfaces ([15], [31], [32], [16], [33] [34]), flow over objects ([35], [32]), diffusers ([36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50]), splitters ([51], [52], [32], [12], [53]), nozzles ([17], [9], [54], [55], [56], [57]) and boundary layer conditions ([51], [58], [59], [60], [61], [62], [63], [64], [65]). These may be significant depending on the frequency of interest.

### 1.1.4 Jet noise

**Noise concern**

Concern about jet noise started in the late 1940’s when the advantages of the jet engine led to an increase in its use. The studies of Lighthill ([66], [67], [68]) are of major relevance in this field. He describes the sound generated aerodynamically using a static distribution of monopole, dipole and quadrupole sources noise sources. He also showed that the acoustic power radiated by the monopole, dipole and quadrupole are respectively proportional to the flow velocity to the power $V^4$, $V^6$ and $V^8$ respectively. His studies were complemented by several experimental investigations that verified the eighth-power law for quadrupole type sources and confirmed other broadband features of the jet theory related to convective amplification with the Mach number. Other authors, ([69], [70]) presented the effects of reflection on these sources. The modification of Lighthill theory to take into account solid surfaces was introduced by Curle [31] and approached afterwards by other researchers ([13], [71]) showing that the presence of solid boundaries gives rise to a dipole field due to the force with which the
boundary acts upon the fluid. Nowadays, although the sound power levels have come down, the number of gas turbines in operation has increased and the regulations are more strict, which promotes continuing research ([72],[73]).

**Jet Characteristics**

When fluid emerges into a stagnant or more slowly moving background fluid, mechanical stresses results that cause the interface to break up in violent turbulent fluctuations, forming a jet. A jet is constituted of three regions as presented in Figure 7 initial shear layer, potential core and mixing layer.

![Jet structure](image)

*Figure 7 Jet structure*

The potential core is the region where the characteristics of the flow upstream of the jet formation are preserved. Outside the shear layer is the mixing layer that spreads from the jet exit and at its final state gives rise to a fully developed jet. For a plane mixing layer (from 1 to 4 diameters downstream) and for a Reynolds number greater than $10^5$, the flow structure is self-preserving meaning that the average properties of the turbulence and of the mean flow are similar except for a change in scale [32]. At low jet Reynolds numbers, the disturbances associated with instability waves can be seen in the shear layer rolling up into toroidal vortices in the transitional flow region. To accommodate the growth, neighbouring structures undergo continuous change. Sometimes they group together to form ‘vortex pairing’ [74]. At other times a large structure can abruptly disintegrate. The preferred lengths scale for maximum spatial growth is $7\delta$ to $8\delta$ [51], where $\delta$ is the thickness of the initial shear layer. In the final
stages of transition, some authors argue that the vortex structures persist in the region where the flow is fully turbulent. Others suggest that large vortex structures arise naturally in the turbulent flow. At the end of the mixing layer the regular vortices disappear breaking up into turbulent structures, resulting in large localized pressure fluctuations [75]. The length of the transitional region where the toroidal vortices are found depends on the Reynolds number. The transition region is shortened as Reynolds number increases, although the structure at higher Reynolds numbers is not well known.

**What generates sound?**

A jet noise spectrum is characterized by its broadband characteristics and by certain tonal components.

The broadband content of jet noise mechanisms normally fall into two types. Closer to the jet exit, the flow velocity is constant and proportional to the exit velocity, whereas downstream in the mixing region the convection velocity decreases with increasing distance from the jet exit. Therefore, each jet regions determines a different region in the sound spectrum. The contribution from the mixing region generally will be high frequency, while that from the fully developed region normally control the low frequency sound. The midrange frequencies are determined by the flow at the end of the potential core region [76]. Although the characterization of jet noise spectra based on the jet regions is in general accepted and valid, the fact that the mixing layer is mainly composed of a mean velocity field, a large eddy motion and a main turbulent motion introduces some instabilities to the flow. These instabilities can mask the noise generated by each of the jet zones giving rise to different noise characteristics.

The large-scale turbulence (normally occurring at the end of the jet core) is directly responsible for the generation of the dominant sound at high velocities. However, it is still not clear whether large-scale turbulence generates sound at low Mach numbers [51], as the crucial factor is the effective phase velocity of the wave, which is small for subsonic flows. Crighton [52] says that large-scale ordered structures are responsible for jet noise production around the spectral peak and that vortex paring creates sound. As Reynolds number increases, the length of the transition region where the vortex pairing occurs decreases. Therefore, large structures are perhaps not the main source at low frequencies. On the other hand, some authors argue that in the final stages of
transition, the vortex structures persist in the region where the flow is fully turbulent. In addition, large vortex structures arise naturally in the turbulent flow. The question of vortex rings and vortex paring and the resulting localized pressure fluctuations as a source of intense noise generation has been considered by many researchers, but remains unanswered for a jet at high Re, where the turbulent diffusion process acts to smear out such peaks in the pressure fluctuations. There seems to be a consensus that the other sources of turbulent flow noise at subsonic speeds are small compared with the noise generated by turbulent mixing, although not many authors have done experiments at very low Mach numbers and little is known for high Reynolds numbers (it is known that the region in which the paring can occur is smaller but it is not clear when paring disappears completely). On the other hand, [14] shows that the paired smaller scale vortices dominate the noise generated at low speeds and high Reynolds number. The large structure of the turbulence in the mixing region of a jet possesses similar structures that are coherent and extend in the direction of their convection. The amplitude of the unstable disturbances and their sub-harmonics grow initially exponentially within both space and time and are convected downstream with phase speed of about 0.6 of the mean speed.

In relation to generation of tonal components in the jet spectra, jets generate vortex noise normally at low Re numbers [32]. Ring tones are one class of jet tones that involve axis-symmetric modes of the jet. The feedback structures are generated by the vortex shedding from the ring. The feedback disturbances reinforce axis-symmetric modes of the jet.

The acoustic power spectrum of a jet generally is proportional to $V^8$ [66] and for subsonic speeds the predicted spectrum follows $1/f^2$ at high frequencies and $f^2$ at low frequencies [12].

Sometimes, the dependence on $V^8$ does not happen ([51], [15], [77]). Some experimental studies, including [51], have shown dependence of noise intensity on $V^6$ at low mach numbers, although they assumed that this dependence is due to:

- A non-uniform flow at the jet exit (jet efflux that is either turbulent or bubbly) also called lip noise, which is sound generated by turbulence near the exit plane of an open tube [15], [77];
- Edge dipole sound;
- Interference of surfaces in the lab.
- That it takes about 10 length scales downstream from duct lip to reach $V^8$ law, and therefore, for low Re the law of $V^4$ and $V^6$ appear near the outlet of the jet because of the long acoustic wavelength.

In most of those papers however the main interest was on high-speed jets. Although they have highlighted the appearance of $V^6$, not much emphasis, unfortunately, has been given to the identification of the proper source as most of the references found relate to higher speeds.

**How is jet noise modelled?**

One of the most important modelling approaches is Lighthill's theorem. Lighthill replaced the fine-scale motion of turbulent flows (he called them small eddies) by a volume distribution of equivalent acoustic sources throughout the entire flow field. In this analogy, the sources are embedded in a uniform medium at rest, in which the sources may move but not the fluid. His equation for the sound power in the far field is proportional to the mean jet velocity to the eighth power ($V^8$) and proportional to the square of the jet's diameter ($d^2$). His experimental studies not only verified the eighth power law (for Mach numbers between 0.3 - 0.9), but also confirmed the other broad features of the theory relating to convective amplification with Mach number and consequent changes in the directivity spectra. The necessary modifications in Lighthill's theory to deal with flow-acoustic interactions are considered in [15] and [51]. Other modifications to include solid surfaces were first introduced by Curle [31] and subsequently by other researchers. Although not considered in this work, the effects of convection in high-speed jets are studied by Ffowcs-Williams [78] and later by [79]. Lilley ([80],[81]) suggested a modification to Lighthill's approach using a convected-wave treatment in Lighthill's equations to account for the distortion in the fields that occurs due to strong interaction between the flow and disturbance wave motions caused by the flow [15]. Changes in Lilley's equation to emphasize the effects of non-uniform density are given by [82]. Other models that do not use Lighthill's analogy are also used ([51] and [15]).

Another important theory was introduced by Powell [83]. He investigated what characteristics of the vortex (or eddy) generate sound and discovered that vortex formation is the main noise mechanism. He isolated a ring and studied the changes in
fluid motion caused by the ring, showing that changes in vorticity and changes in fluid momentum can be seen as a dipole and a quadrupole respectively. He showed that the rate of change of the force applied to the fluid by the vortex motion determines the radiated pressure, which is the fundamental relationship for the dipole motion. Comparisons of Powell's results with Lighthill and Curle's results are shown by [32]. After Powell, vortex sound became almost a subject of its own and many authors approached it ([84], [85], [86], [87], [88], [89], [90], [91], [92], [93], [94], [95], [96], [97], [98], [99], [100] and others)

The methods above are mainly used to account for "small eddies", which are the small turbulent structures in the flow. During the seventies and eighties it was discovered that a jet is made up of large turbulence structures and fine-scale turbulence. With those discoveries, different models to approach the large turbulence structures, that had not been considered before, were introduced ([101]), [102], [103], [104]).

To model the large turbulent structures several approaches are used [51], such as:

**Discrete wave model:** they model large turbulence structures using instability waves. Most of them use only a single frequency wave. They are normally used to identify the dominant frequency of the large structures ([105]; [101]; [106]);

**Computational aeroacoustics:** methods like Large Eddy Simulations (LES) and Direct numerical simulation (DNS) would be very useful as they would give the temporal variation for pressure and velocity, but at the moment they takes a long computational time and/or are still limited to low Reynolds number;

**Discrete vortex model:** the large structures are modelled as vortices. These models are normally the simplest and more efficient form of representing the structure, although sometimes they are limited to a particular geometry and to conservation of vorticity for the cases that they are modelling ([108], [109]).

In computational aeroacoustics, recent advances are offering other approaches [110] for calculating jet noise, although this still remains a difficult task due to the wide range of spatial and temporal frequencies. They include techniques such as: DNS ([36], [111], [112], [113]), unsteady Reynolds-average Navier-Stokes simulations (RANS) [114] and k-epsilon [115] and LES ([116], [117], [118], [119], [120], [121], [122], [123], [124]). Although all those methods offer reasonable solutions, there are several difficulties involved in each of them. DNS, is very complete but still does not offer the
possibility to account for high Reynolds number and RANS can be used for high Reynolds numbers but can only account for coherent structures. LES is an intermediary solution that computes Navies-Stokes equations for the large scales and uses RANS for the smaller scales. Due to these characteristics LES has been chosen to be used in this work as a way of inputting the flow characteristics (as a source) to the acoustics model. Literature results using LES so far have been used together with Lighthill’s equations or as input for acoustics models based on the Kirchhoff theorem ([125], [126], [118], [127], [128], [129]).

Studies of discrete vortex models were originally based on experimental work that showed the formation of vortex rings for certain flow conditions. Goldstein [15] shows that for $160 < \text{Re} < 1200$ the flow from a jet becomes unstable generating vortex rings although other researchers say that vortex rings can persist at higher Reynolds numbers. From these observations, some models were created to describe large scales as vortex rings or a combination of vortex rings: Paul Pao [130] considers the jet radiation from vortex rings as beam patterns. In one approach, theoretical and experimental work was done to construct a model for the shedding of vortex rings [131] in order to simulate the real-time pressure variation in the acoustic region near a jet. The results show promise with respect to the near field, but show a complicated analysis in the far field. A similar approach is taken in [132] where the large-scale eddy structure of a circular jet is calculated from a potential flow model that includes the vorticity distribution. The model exhibits all of the observed general features of high Re numbers jet flows. The acoustic far field is calculated from the predicted unsteady vorticity distribution measured experimentally. Tang [133] shows a model for the noise production due to the activities of the vortex paring and due to the interaction and decay of the coherent structures. He says that the sound generated by a vortex is correlated with acceleration and deceleration of the vortical elements of both the thin vortex rings in the pairing stage and the relatively thicker vortex rings during the breakdown process. This suggested that the noise produced through the pairing of vortices and through the ring breakdown comes from the same mechanism of rate of change of the propagation velocity of the vortical elements. In observing these large structures formation, another finding is that when the flow is excited, these structures could remain present in the flow. Vaslov [134] shows the toroidal vortex ring that occurs when the flow is excited. With this finding, some researchers analysing large-scale structures started focussing on what could be achieved by exciting the flows (e.g. reduction of the broadband noise [135]) and therefore few researchers considered
closely what happens with the large structures at higher Reynolds numbers, especially if the Mach number is low, which is the flow characteristics of interest in this thesis. An adapted discrete vortex model is used as the acoustic model for flows at low mach numbers and high Reynolds numbers. Laufer [131] was one of the few that treated low Mach numbers and relatively high Reynolds numbers. His results show that for 0.05 – 0.2 Mach numbers most of the radiation occurs along the first diameter of the jet and gives some physical insights into the sources of sound.

1.2 Hypothesis Formulation

Throughout this research, the overall question of how to determine the noise sources responsible for the majority of the low frequency noise generation is addressed and is the underlining hypothesis driving this research. It has led to some answers (presented in Section 1.1 and the next chapters) and to other questions that are here stated as two other hypotheses.

The question of whether monopoles or dipoles are the major source of low frequency noise in this work is still open as an unknown until two particular experiments helped to clarify this question (more details are given in Chapter 4). Figure 5 shows that the system resonance peaks are present in most measurements. A particular case where only the duct and the diffuser are present (Figure 8) showed that the presence of the diffuser made the resonances more evident. In geometrical terms, the only difference between the two cases is the presence of the diffuser, which means that the main aerodynamic effects that could be generating sound are similar in both cases.

The fact that both cases excite the system, together with the fact that the diffuser makes the resonances more evident, show that the aerodynamic source exciting the systems are present in both cases. Given that the flow phenomena that constitutes the source happens in both cases and the only difference between the two is in the exit characteristics, the source mechanism exciting the flow is therefore located at the exit. For a source at the exit of an open pipe system to be able to excite a system, it must be a pressure source as it is the only source type that can excite a modal position of zero pressure (i.e. the pressure node at the exit of a open duct). The fact that the source is a pressure source excludes the possibility of monopoles being the main source (as they are volume velocity sources).
With the possibility of a monopole source excluded, the second most likely source to be radiating at low frequencies would be the dipole source [136].

Independently, speed variation tests were conducted for several setups of the test rig (Figure 9 shows one of them) and they showed that the variation in power when the speed is increased is proportional to $V^6$, which indicates, once more, according to speed ratios to acoustic power ([16],[32]) that the main source present in the system is a dipole.

![Figure 8 $L_W$: Duct only](image)

![Figure 9 $L_W$: Duct Only – Speed Variation](image)
Therefore, another hypothesis of this work is that the main low frequency source contributor is of dipole type. A secondary hypothesis originated from the fact that the fluctuations occurring at the exit of the duct or of the diffuser occur all around their perimeter. As the sources are dipoles present all around the duct perimeter and possibly in the external region, the secondary hypothesis is that: the source can be represented as a ring of dipoles or as a truncated cylinder of dipoles.

To approach these hypotheses, Chapter 2 presents the analytical development of the two competing models for the sources. Numerical results for the models are presented on Chapter 3 and comparison with experimental results is presented on Chapter 4. An example using LES as the source input is presented on Chapter 5 and concluding remarks are given on Chapter 6.
Chapter 2 Analytical Modelling

As presented in Chapter 1, the principal hypothesis in this work is that the main source of noise at low frequency is generated by a dipole. There are several flow phenomena that could generate dipoles sources in a jet leaving a duct, such as duct lip characteristics or low speed internal separation. However, vortex pairing or unsteady pressure field near the duct walls and aerodynamic jet instabilities at low speeds are assumed as the major aero-acoustics sources in this work. The exact behaviour of those aerodynamic movements is random and not completely known. This could mean that several different “dipole type” sources could exist or could be present at the same time.

The “dipole type” sources are modelled here to calculate the far field radiation from a low speed jet. The main objective is to get an indication of how the source mechanisms of the low speed jet can be modelled for the purpose of understanding the resulting noise radiation. A qualitatively correct model gives a possibility to affect the sound radiation by manipulating the very source of the sound.

This chapter, therefore, argues the case for two competing models that could represent the low frequency noise source, depending on which is the most relevant character of the source. They are simplified models as the interest is only in the far-field radiation.

The first model, the Continuous Ring (CR) is physically developed to represent the first stages of a jet. The large flow structures start at the duct edge and mark the beginning of the mixing layer. The turbulence in the shear layer around the potential core forms the shedding of vortex rings that grow and are convected downstream. Depending on the flow conditions this vortex may form pairs creating structures as presented in Figure 10.

Due to forward fluid movement, vortices move downstream the flow. This movement has been represented as a moving ring by Maestrello [137] and Davies [9]. Here it is chosen to create a source covering the jet core to account for vortex generation that occurs on its surroundings and the possibility of rings rolling or paring.
Therefore the Continuous Ring (CR) is created to represent continuous sources that could be happening over an extended length of the jet.

![Diagram of Continuous Ring (CR)](image)

**Figure 10** Example of (a) vortex shedding, (b) vortex pairing

A second competing model is created to represent a more discrete and localized concentration of dipole sources: a Discrete Dipole Ring (DDR) model.

![Diagram of Discrete Dipole Ring (DDR)](image)

**Figure 11** Vorticity generated when jet is being formed

The discrete dipole ring model might represent surface pressure instabilities, exit discontinuities and local vorticity. The ring accounts for the fact that the dipole formation can occur all around the jet (**Figure 11**).

A single dipole ring located at the diffuser exit is postulated here to represent adequately the primary acoustic source at low frequencies.
It is assumed that the pressure inside the jet core is greater than outside, so that the inner region of the jet can be neglected. Therefore the impedance of the air inside the jet core is not considered. It also assumes the sources are not compact, so that they can not interact with each other in a near field to form more complex higher order sources. This chapter presents an introduction to dipole sources (as they are the base for all the modelling developed in this work) and the analytical development of the two competing models.

### 2.1 Dipole

This section covers the concept and equations for a dipole. This has already been presented in several standard bookwork, but is included here for completeness.

A dipole consists of two equal monopole sources of strength $S$ that are separated by an infinitesimal distance $d$ and are pulsating $180^\circ$ out of phase with each other as shown in Figure 12. Figure 13 shows that whilst one source expands the other source contracts. The result is that the fluid near the two sources moves back and forth to produce the sound. A dipole source does not radiate sound equally in all directions. The directivity pattern of that there are two regions where sound is radiated well, and two regions where sound cancels.

![Figure 12 Dipole source, consisting of two monopoles $S$, creating a dipole $\hat{D}$](image-url)
Figure 13 Pressure field created by a dipole source [138]

When the distance \( \bar{d} \) is small enough that \( kd \ll 1 \), the dipole is considered a point source. In this case the superimposed pressure is given by:

\[
\hat{p} = \hat{D} \cdot \nabla_s G_k(x/x_s)
\]

Where \( \hat{D} \) is the dipole strength given by \( \hat{S} \cdot \hat{d} \) and \( G_k(x/x_s) \) is the Green's function, associated with the boundary conditions. If the dipole is considered to be in free space, the acoustic pressure becomes:

\[
\hat{p}(R) = -\nabla_s \left( \frac{\hat{D} e^{ikR}}{R} \right)
\]

\[
\hat{p}(R) = -\hat{D} \cdot \hat{e}_R [ikR - 1] \frac{e^{ikR}}{R^2}
\]

Where \( \hat{e}_R \) is the unit vector pointing outward from the dipole centre to the receiver position \( \hat{e}_R = \frac{x - x_s}{R} \). When a concentrated point force \( \hat{F} \) is applied to a fluid the compression and rarefaction activity create a dipole field. This can be represented by an oscillating thin disk ([6]) as shown in Figure 14 provided its radius is small compared to the wavelength. As the disk has an associated frontal area and exerts a net force into the fluid the acoustic source created in this manner can be described as a pressure source.
The relation between the dipole source strength and the force is given by:

\[ \hat{D} = \frac{\hat{F}}{4\pi} \]

So that the acoustic pressure becomes:

\[ \hat{p}(R) = -\frac{\hat{F} \cdot \hat{e}_R}{4\pi} \left[ ikR - 1 \right] \frac{e^{ikR}}{R^2} \]

\[ \hat{p}(R,\theta,\phi) = -\frac{1}{4\pi} F_\omega \cos(\theta)(ikR - 1) \frac{e^{ikR}}{R^2} \]  \hspace{1cm} (1)

Where \( k \) (cycles/m) is the wave number, \( \rho \) (Kg/m\(^3\)) is the density and \( R \) (m) is the vector distance between the dipole centre and the measurement position. \( \hat{F}_\omega \) is the complex, frequency dependent dipole force. The angle \( \theta \) is the angle between the dipole axis and the radius vector \( R \) to the measurement point, as presented in Figure 12.

The radial velocity is given by:

\[ \hat{v}_R(R,\theta,\phi) = \left( \frac{1}{i\omega\rho} \right) \frac{\partial}{\partial R} p(R,\theta,\phi) \]

\[ \hat{v}_R(R,\theta,\phi) = \frac{i(-2 + 2ikR + k^2R^2)\cos(\theta)F_\omega e^{ikR}}{4\pi \omega k \rho R^2} \]
The time-averaged intensity is given by:

\[ I = \frac{1}{2} (p v^* + p^* v) \]

\[ I = \frac{\cos^2(\theta) \hat{F} \cdot k^3}{16\pi^2 \rho \omega R^3} \]  

(3)

Hence, the radiated power is given by (e.g. [139]):

\[ W = \int_0^{2\pi} \int_0^\infty r^2 \sin\theta d\theta d\phi \]  

(4)

\[ W = \frac{F_a \cdot k^3}{12\pi \rho c} \]  

(5)

The intensity equation shows, as expected, that the dipole has a dependence on the angle \( \theta \), creating an eight-shaped dipole directional field. The pressure amplitude has its maximum at the dipole axis and for \( kR \geq 1 \), it is inversely proportional to the distance from the source. The radiated power expression shows the dependence on the frequency squared \( (k^2) \) and on the source strength.

Aero-acoustics examples of dipoles are: forces such as lift and drag (propellers at low frequencies; rotor, fan and compressor noise), airframe noise (undercarriage gear noise, cavity resonances in the undercarriage, surface roughness), flap noise (vortex passing close to surface) and general vorticity.

### 2.2 Continuous Ring (CR)

This section presents the development of the Continuous Ring model. This includes the development of the equations for the sound power emitted by the Continuous Ring and some insight on energy distribution aspect in a jet.
2.2.1 Acoustic power formulation

An expression for the acoustic pressure in the far field due to the presence of a Continuous Cylinder Source is developed as a function of the source’s acceleration \( \dot{w} \), and accounts for source variations in the \( \phi \) and in the \( z \) direction. The sound propagation is solved using cylindrical coordinates \( (r, \phi, z) \) as show in Figure 15.

![Figure 15 Continuous Ring – cylindrical coordinates](image)

The wave equation in cylindrical coordinates satisfying the equations of motion (e.g. [140]) is:

\[
\frac{\partial^2 \hat{p}}{\partial t^2} + \frac{1}{r} \frac{\partial \hat{p}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \hat{p}}{\partial \phi^2} + \frac{\partial^2 \hat{p}}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 \hat{p}}{\partial t^2}
\]

The dependence is considered to be harmonic \( (e^{-i\omega t}) \) and will be omitted from here on:

\[
\frac{\partial^2 \hat{p}}{\partial r^2} + \frac{1}{r} \frac{\partial \hat{p}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \hat{p}}{\partial \phi^2} + \frac{\partial^2 \hat{p}}{\partial z^2} + k^2 \hat{p} = 0
\]

To account for non-periodic motion in the \( z \)-direction, which is more suitable for representing the jet core fluctuations, a Fourier transform in \( z \) is applied to the Helmholtz equation. The partial differential equation governing the pressure transform \( \hat{p}(r,\phi,z) \) is then:

\[
\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \left[ k^2 - k_z^2 \right] + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) \hat{p}(r,\phi,k_z) = 0
\]

(6)
Considering the solution as a group of standing and progressive harmonic waves, the solutions, using the method of separation of variables (e.g. [141]), can be of the form:

\[ \ddot{p} = \Pi(r)\Phi(\phi)\Gamma(\gamma)T(t) \]  

(7)

Substituting (7) in (6):

\[ \frac{r^2}{\Pi(r)} \left[ \frac{\partial^2 \Pi(r)}{\partial r^2} + \frac{1}{r} \frac{\partial \Pi(r)}{\partial r} \right] + \left[ k^2 - \gamma^2 \right] = -\frac{1}{\Phi(\phi)} \frac{\partial^2 \Phi(\phi)}{\partial \phi^2} \]

The left-hand side depends only on \( r \) and the right side depend only on \( \phi \). Assuming \( (-n^2) \) to be the constant that both sides equal to and solving for \( \phi \) gives:

\[ \frac{1}{\Phi(\phi)} \frac{\partial^2 \Phi(\phi)}{\partial \phi^2} = -n^2 \]

\[ \Phi(r, \phi, \gamma) = C_\phi \cos(n\phi) \]  

(8)

Solving for \( r \) gives:

\[ \frac{1}{\Pi(r)} \left[ \frac{\partial^2 \Pi(r)}{\partial r^2} + \frac{1}{r} \frac{\partial \Pi(r)}{\partial r} \right] + \left[ k^2 - \gamma^2 \right] \frac{m^2}{r^2} = 0 \]

Transforming now gives Bessel’s differential equation [142], which is known and has exact solutions.

\[ \Pi(r) = g(\zeta) = g(\sqrt{k^2 - \gamma^2} r) \quad \zeta = \sqrt{k^2 - \gamma^2} r \]

\[ \frac{d^2}{d\zeta^2} g(\zeta) + \frac{1}{\zeta} \frac{d}{d\zeta} g(\zeta) + \left[ 1 - \frac{n^2}{\zeta^2} \right] g(\zeta) = 0 \]

The exact solution (solving for \( R \)) using Bessel’s function, assuming that there are only progressive waves, is given by:
\[ \Pi(r) = A_n H_n^{(0)}(\sqrt{k^2 - \gamma^2} r) + B_n H_n^{(2)}(\sqrt{k^2 - \gamma^2} r) \]  

(9)

Where \( H_n^{(0)}(\xi) \) and \( H_n^{(2)}(\xi) \) are the \( n \)th-order Hankel functions of the first and second kind respectively. \( H_n^{(0)} \) decays from the axis \( r = 0 \) and \( H_n^{(2)} \) increases from it. Substituting (8) and (9) in (7) gives:

\[ \bar{p}(r,\phi,\gamma) = \left[ A_n H_n^{(0)}(\sqrt{k^2 - \gamma^2} r) + B_n H_n^{(2)}(\sqrt{k^2 - \gamma^2} r) \right] \cos(n\phi) \]  

(10)

\[ k_r = \sqrt{k^2 - \gamma^2} \]

\[ \bar{p}(r,\phi,\gamma) = \left[ A_n H_n^{(0)}(k_r r) + B_n H_n^{(2)}(k_r r) \right] \cos(n\phi) \]

In order to ensure that the pressure is the same at \( \phi \) and \( \phi + 2\pi \) the values of \( n \) must be integers.

Since \( e^{-i\omega} \) has been chosen as our time-harmonic dependence \( H_n^{(0)}(k_r r) \) represents an outgoing wave travelling in the direction of increasing \( r \) (as it decays in amplitude when outward travelling from the source) and \( H_n^{(2)}(k_r r) \) represents an incoming wave travelling in the direction of decreasing \( r \) (as it increases in amplitude when inward travelling to the source). The radiated acoustic field from a cylinder is assumed to propagate in free space; therefore there will be no reflected waves. The radiation from the end of the cylinder is not considered in this analysis (Figure 16).

The radius of the cylinder is chosen to be \( a > 0 \), as shown in Figure 15, avoiding the use of \( r = 0 \) in the equation (9). As the cylinder is radiating into free space, there are no reflected waves, so that \( B_r = 0 \) in equation (9).
\[ \tilde{p}(r,\phi,y) = A_n H_n^{(0)}(\sqrt{k^2 - \gamma^2}r)\cos(n\phi) \]

Since the sum of all possible solutions of the Helmholtz equation is also a solution [143]:

\[ \tilde{p}(r,\phi,y) = \sum_{n=0}^{\infty} A_n H_n^{(0)}(\sqrt{k^2 - \gamma^2}r)\cos(n\phi) \]  \hspace{1cm} (11)

This equation has been used by several authors and shows that the pressure is given by a summation over the \( n \) circumferential modes. Physically, \( H_n^{(0)} \) represents the azimuthal modal numbers as presented in Figure 17 for the first three circumferential modes. In this sense, \( n = 0 \) can be thought of as a breathing mode, which corresponds to a monopole, \( n = 1 \) corresponds to a dipole, \( n = 2 \) corresponds to a quadrupole and so forth (e.g. [144]).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{modes.png}
\caption{Figure 17 Modes of vibration of a ring}
\end{figure}

Consider first that dipoles sources are created on the surface of the cylinder with no particular orientation (Figure 18).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{cylinder_dipoles.png}
\caption{Figure 18 Cylinder of Dipoles}
\end{figure}
Under steady-state conditions the acceleration is spatially periodic for a surface surrounding the whole volume. For infinite cylinders, the spatial periodicity can be restricted to the coordinate $\phi$ and the sound field can be expressed by a Fourier series in $\phi$. Therefore the force distribution at the boundary is given by:

$$F(z, \phi) = -\rho \sum_{n=0}^{\infty} B_n f(z) \cos(n\phi)$$  \hspace{1cm} (12)

Where $f(z)$ accounts for a single decay profile in $z$. At the boundary $r = a$, the boundary condition is [136]:

$$\nabla p + \rho \frac{\partial u}{\partial t} = F$$

It’s Fourier expansion then becomes:

$$\frac{\partial \tilde{p}(r, \phi, \gamma)}{\partial r} = -\rho \sum_{n} \tilde{W}_n \tilde{g}(\gamma) \cos(n\phi) + \sum_{n} B_n \tilde{f}(\gamma) \cos(n\phi)$$  \hspace{1cm} (13)

Where $\tilde{W}_n$ is the acceleration distribution constant, $B_n$ is the force distribution constant, $\gamma$ is the wave number in the $z$ direction and $g$ and $f$ are the profiles in the direction.

This equation has been used in standard bookwork with the volume flux term. Here the force term has been added to represent the dipoles influence.

A spatially non-periodical Fourier transform is used for $z$ (e.g. [145]). The transform is presented below:

$$\tilde{f}(\gamma) = \int_{-\infty}^{\infty} f(z)e^{-i\gamma z} \, dz \hspace{1cm} f(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\gamma)e^{i\gamma z} \, d\gamma$$  \hspace{1cm} (14)

Substituting equation (11) into (13) at $r = a$, gives:
\[ A_n = \frac{-\rho \tilde{W}_n \tilde{g}(\gamma) + B_n \tilde{f}(\gamma)}{(k^2 - \gamma^2)^{\frac{5}{2}} H_n^{(0)}[(k^2 - \gamma^2)^{\frac{3}{2}} a]} \]  

(15)

Substituting (15) into (11) results in:

\[ \tilde{p}(r, \phi, \gamma) = \sum_n \frac{-\rho \tilde{W}_n \tilde{g}(\gamma) + B_n \tilde{f}(\gamma)}{(k^2 - \gamma^2)^{\frac{5}{2}} H_n^{(0)}[(k^2 - \gamma^2)^{\frac{3}{2}} a]} H_n^{(0)}[(k^2 - \gamma^2)^{\frac{3}{2}} r] \cos(n\phi) \]

not considering the volume flux term (that represents the monopole influence as the monopole is a velocity source) gives:

\[ \tilde{p}(r, \phi, \gamma) = \sum_n \frac{B_n \tilde{f}(\gamma)}{(k^2 - \gamma^2)^{\frac{5}{2}} H_n^{(0)}[(k^2 - \gamma^2)^{\frac{3}{2}} a]} H_n^{(0)}[(k^2 - \gamma^2)^{\frac{3}{2}} r] \cos(n\phi) \]

So that the inverse Fourier transform finally gives the acoustic pressure field:

\[ p(r, \phi, z) = \frac{1}{2\pi} \sum_n B_n \cos(n\phi) \int_{-\infty}^{\infty} \frac{\tilde{f}(\gamma) H_n^{(0)}[(k^2 - \gamma^2)^{\frac{3}{2}} r]}{(k^2 - \gamma^2)^{\frac{5}{2}} H_n^{(0)}[(k^2 - \gamma^2)^{\frac{3}{2}} a]} d\gamma \]  

(16)

In the far field, the Hankel function can be approximated by [146]:

\[ H_n^{(0)}(x) = \left( \frac{2}{\pi x} \right)^{\frac{3}{2}} (-i)^n \exp \left( ix - i\frac{\pi}{4} \right) \]

Therefore the pressure becomes:

\[ p(r, \phi, z) = \frac{1}{2\pi} \frac{2^{\frac{3}{2}} e^{-\frac{i\pi}{4}}}{(ar)^{\frac{3}{2}}} \sum_n (-i)^n B_n \cos(n\phi) \int_{-\infty}^{\infty} \frac{\tilde{f}(\gamma)e^{\sqrt{k^2 - \gamma^2}} e^{ir\gamma}}{(k^2 - \gamma^2)^{\frac{3}{2}} H_n^{(0)}[\sqrt{k^2 - \gamma^2 a]} d\gamma} \]

The acoustics pressure in spherical coordinates (Figure 19) is given by:
The integrand is of the form $\Phi \exp(i\Psi)$, which is suitable for stationary phase integration [151]. The modulus and phase of the integrand are respectively:

$$\Phi(\gamma) = \frac{f_n(\gamma)}{(k^2 - \gamma^2)^{k/2} H_n^{(0)}(\sqrt{k^2 - \gamma^2}a)}$$

$$\Psi(\gamma) = R(\sqrt{k^2 - \gamma^2} \sin(\theta) + \gamma \cos(\theta))$$

The stationary phase integration can be constructed by using:

$$I = \frac{(2\pi)^{k/2} \phi(\gamma) \exp(\pm i(\pi/4) + i\Psi(\gamma))}{|\partial^2\Psi/\partial \gamma^2|^{k/2}}$$

Therefore the pressure is given by:

$$p(r,\phi,z) = \frac{e^{i\mathbf{k}\cdot\mathbf{z}}}{kR\pi\sin(\theta)} \sum_n (-i)^n B_n \cos(n\phi) \frac{f(k\cos(\theta))}{H_n^{(0)}(k\sin(\theta)a)}$$

Hence the power in the far field in the direction per unit of solid angle is:

$$W = \frac{R^2}{2\rho c} \int_0^\pi \left[ \int_0^{2\pi} |p(R,\theta,\phi)|^2 \sin(\theta) d\theta \right] d\phi$$
This equation gives the sound power for the Continuous Ring source. This shows the dependence of the power in relation to the number of modes chosen in \( \phi \) direction. This equation is also proportional to the source force squared. The sound power is then normalized dividing by the cylinder surface area:

\[
\overline{W} = \frac{W}{(2\pi a_{\text{cylinder}})} \tag{18}
\]

This normalisation is used in Chapter 3 for the comparison of the models, as the Discrete Ring source has a much smaller length.

### 2.2.2 Profile in the z-direction

This section is intended to give an overview of how the energy contained in the Continuous Ring is spread spatially. This is done by analysing the \( f \) profile. The physical shape of the \( f \) profile is assumed to be known and to be invariant with the angle \( \phi \), as presented in Figure 20.
The profile for \( f \) (Figure 21) is chosen taking into consideration the following characteristics for a typical jet energy profile as observed in real jets: it starts small at the duct exit, grows reaching a peak and decays [147]. In order to satisfy those characteristics a Gaussian function is chosen to be used with a small standard deviation, as it approximately represents the peak/decay behaviour with the advantage of having a simple Fourier transform. Although this function is not strictly correct for describing the behaviour as it tails on both sides are infinite, it is used here for comparison with the finite profile. Although this function is not strictly correct for describing the behaviour, as it tails on both sides are infinite, it is used here for comparison with the finite profile. The Gaussian function is given by:

\[
f(z) = e^{-\frac{1}{\sigma^2}(z-\Delta z)^2}
\] (19)

where \( \sigma \) is the variance and \( \Delta z \) gives the peak position. The Fourier transform of (19) using (14) is:

\[
\tilde{f}(\gamma) = e^{-\gamma \Delta z} \sqrt{\pi \sigma^2} e^{-\pi^2 \gamma^2 \sigma^2}
\]

Figure 21 Amplitude of a Gaussian function representing the energy profile in the z direction

Now considering aspects observed in this particular research, from the experimental results (presented in Chapter 4) it is known that the flow phenomena happening at more than one diameter away from the duct exit has little influence on the low frequency noise under consideration. Results from hot wire measurements taken near the jet exit
also indicated that the fluctuation level near the exit is stronger in other areas of the jet [148]. For this reason and due to the fact that finite profiles are more realistic, they are introduced here for comparison against the infinite profile in order to verify the relevance of each type of profile and the length of the cylinder, using the Continuous Ring Model (Figure 20).

To represent a finite profile, a triangular function is used. The triangular function is chosen such that it contains the same internal area as a normalised Gaussian function. The function is presented below:

\[
\begin{align*}
  f(z) &= 0, \quad -\infty < z < -\frac{1}{2} \\
  f(z) &= \frac{2y_o}{2z_o + L} z + \frac{y_o L}{2z_o + L}, \quad \frac{1}{2} < z < z_o \\
  f(z) &= \frac{2y_o}{2z_o - L} z - \frac{y_o L}{2z_o - L}, \quad z_o < z < \frac{1}{2} \\
  f(z) &= 0, \quad \frac{1}{2} < z < \infty
\end{align*}
\]

In order to be able to compare the finite and infinite profiles in Chapter 3, the area underneath the curve is chosen to be equal to an elementary area. For that, the value of \( y_o \) has to be \( y_o = \frac{\gamma}{2} \). The Fourier transform of the triangular function is:

\[
\tilde{f}(k) = \frac{e^{-2\gamma(z + z_o)}}{2\gamma^2 L^2 \pi^2} y_o \left( 2e^{2\gamma i z_o} + e^{2\gamma i z_o} (-1 + 2\gamma i \pi z_o) + e^{2\gamma i z_o} (-1 + 2\gamma i \pi z_o) \right)
\]

To verify the behaviour of extremely localised sources, leading to a ring instead of a cylinder using the Continuous Ring model, a Delta Dirac Function is used. The Delta Dirac function and its Fourier transform are given below:

\[
\begin{align*}
  f(z - z_o) &= \delta(z - z_o) \\
  \tilde{f}(\gamma) &= \int_{-\infty}^{\infty} \delta(z - z_o) e^{-2\gamma i \pi z} dz = e^{-2\gamma i \pi z_o}
\end{align*}
\]

This will be used in the comparisons presented in Chapter 3.
2.3 Ring model

The Discrete Dipole Ring model is developed as a set of $N$ discrete dipoles on a circle centred on the geometric centre of the duct/diffuser exit, and equally spaced from each other around the ring (as presented in Figure 22). The dipole sources are at a distance $R_n$ from the observation point. Here, as the sources are discrete, a pure operator in the radial direction is used instead of the cylindrical wave equation.

\[ \hat{p}(R, \theta, \phi) = -\frac{\hat{F} \cdot \hat{e}_R}{4\pi} [\frac{ikR - 1}{R^2}] e^{ikR} \]

Figure 22 Schematic representation of the discrete dipole ring model
The acoustic pressure at some point in the far field ($\hat{p}_f$) due to the presence of the discrete dipole ring is given by the sum of all the $n$ dipoles present:

$$\hat{p}_f = \sum_n \hat{p}_n$$

And the radial acoustic velocity ($\hat{v}_f$) at the same point is given by:

$$\hat{v}_f = \frac{1}{i \omega \rho} \frac{\partial}{\partial R} \hat{p}_f \quad \text{at } R = R_p.$$ 

The time-averaged intensity at each measurement position is calculated by:

$$I = \frac{1}{2} \left[ \text{Re}(\hat{p}_f) \text{Re}(\hat{v}_f^*) + \text{Im}(\hat{p}_f) \text{Im}(\hat{v}_f^*) \right]$$

The radiated power is obtained by integrating the intensity in the far field with respect to the angles $\theta$ and $\phi$ (Figure 23) over a spherical surface.

$$W = \int_0^{2\pi} \int_0^\pi IR^2 \sin \theta d\theta d\phi$$
Both the sound intensity and sound power are dependent on the angle that each dipole has in relation to the measured point and the strength of each dipole. Euler angles [152] are used for the determination of these angles. The DDR intensity and power are calculated numerically and the results are presented in Chapter 3.

For comparison involving different number of sources in DDR, a normalisation for the sound power is used by dividing the sound power by the number of dipole sources.
Chapter 3  Numerical Results

In this chapter the numerical results for the models shown in Chapter 2 are presented. These results are then compared with experimental findings in Chapter 4 to validate the models.

Section 3.1 shows the numerical issues associated with the present calculations. Sections 3.2 and 3.3 look into radiation efficiency aspects of the Continuous Ring model and the Discrete Dipole Ring respectively. Section 3.5 and 3.6 will focus on source characterisation discussing jet noise spectra characteristics and aerodynamic spectra respectively.

The results presented in this chapter include plots of sound power level ($L_w$) against Helmholtz number ($kl$), where $l$ is the length of the duct chosen to be the same as the length of the experimental duct ($l = 1.5m$). The $kl$ range of interest is up $kl = 6$. A reference power of $W_o = 1 \times 10^{-12}W$ is used.

Intensity plots are generated against the angles $\theta$ or $\phi$. The intensity plots were taken at $R = 36m$ away from the centre of the ring (this distance is chosen to make sure that the contours are not in the near field for the lower frequencies) and the results are obtained at every degree. All the intensity plots are in $W/m^2$ through a plane looking into the duct. The atmospheric conditions are considered as standard ISO conditions.

3.1 Numerical Issues

For the sound power calculations on the Continuous Ring (Equation 17), numerical integration is used by applying the Expanded Trapezoidal Rule [150] with 64 interior point elements. The error implicit in this method is of the order:

$$O\left(\frac{(b-a)^3 f^*}{N^2}\right)$$
where \( a \) and \( b \) are the lower and upper limits of the integration, \( N \) is the number of intervals used and \( f \) is the function being integrated.

Other sources of numerical error such as the elementary area used in the power calculations for the Discrete Dipole Ring (Equation 21) were relatively small when the elementary areas are formed by varying one degree in \( \theta \) by one degree in \( \phi \).

### 3.2 Case Studies - Continuous Ring

In order to address radiation efficiency aspects related to the Continuous Ring model, case studies are set up. They are used to verify the influence of the force profile \( f \) in the \( z \) direction and the force profile in the \( \phi \) direction on the radiated noise. These calculations use the model defined in Section 2.2, in particular using Equation 22 for the sound power. The integral in this equation is calculated numerically using the Expanded Trapezoidal Rule [150]. The sums for the first three circumferential modes \( (n = 0, 1, 2) \) are included in this analysis as the interest relies on low frequencies and the first three modes are the most efficient radiators.

The following values apply for the variables, unless they are being varied for a particular example.

\[
\begin{align*}
L &= 1.5m \quad \text{length of the cylinder (chosen to be the same as the experimental investigation in Chapter 4)} \\
D &= 0.15m \quad \text{diameter of the cylinder (chosen to be the same as the experimental investigation in Chapter 4)} \\
z_o &= D \quad \text{position in } z \text{ of the peak of the } z \text{ (chosen to be at one diameter away from the duct exit)} \\
\gamma_o &= \quad \text{amplitude of the peak position in the } z \\
\sigma &= L \quad \text{variance of the Gaussian function}
\end{align*}
\]
Influence of z-profile

The z-profile is physically representing here the influence of the length of the jet core in the z direction.

In order to check the influence of the force z-profile in the radiation efficiency of the Continuous Ring model, the function $f$ is chosen to be represented by 3 functions: an infinite Gaussian function, a finite triangular function and a Dirac Delta function (representing respectively a infinite, finite and a localised source in the jet core). A sketch of these functions is given in Figure 20. Although an infinite jet core does not exist, the Gaussian function is chosen as one of the functions to be used in this comparison, due to the similarities that it presents to the physical jet characteristics.

To undertake these comparisons, these functions are normalised such that the area underneath the function is equal to one. This normalisation was chosen so that all the force profiles have the same radiating area. This normalisation could also be done by normalising the energy (by looking into the force squared) but this approach has not been considered here.

Figure 25 presents the sound power level for different variances in the Gaussian function. The Gaussian function is normalised such that the area underneath the function is equal to one. In this way, the results can be compared with other functions with the same unitary area. The results show that the $L_w$ is higher at low frequencies. This figure also shows that when the variance is reduced, the $L_w$ increases. This happens because at low frequencies the shape of a source is not so relevant, which means that only the amplitude peaks become relevant. In the Gaussian function, by reducing the variance, peaks of higher amplitudes are being created, which represent higher force amplitudes.
To check the influence of the peak position on the $L_w$, two peak positions in the Gaussian function are chosen ($z_o = D/4$, $z_o = 3D$). Figure 26 shows that varying the peak position does not influence the spectrum.

Figure 27 shows the influence of the diameter of the cylinder when a Gaussian function is used as the force distribution in the z-profile. The results show that at low frequencies when the cylinder's diameter is increased $L_w$ increases, as bigger objects have a bigger radiating surface and therefore the radiation efficiency is higher.
For the cases when a finite cylinder is considered, a triangular function is used. The effect of reducing the cylinder length (Figure 28) increases the noise level and makes the spectrum flatter at low frequencies. The level increase is due to the fact that by having kept the radiating area constant (whilst reducing the size of the cylinder) the force amplitudes become higher, as presented in the Gaussian function case. However, in this case, the increase in amplitude is also followed by a decrease in the source dimension, which reduces the radiation efficiency. The influence of these two phenomena together explains the "convergence" to a certain level at the low frequency end.

At higher frequencies, the cylinder spectrum shape is not affected, but the level increases (Figure 28). A finite flat finite profile in z is also shown in this figure to explore the behaviour of the finite profiles further. The results show that for the same length the presence of a peak increases the $L_w$ slightly. Reducing the length of the cylinder tends to flatten the spectrum at low frequencies and increase the $L_w$ at higher frequencies.
Varying the peak position for the triangular function also influences the shape and thus has a very small effect in the $L_w$ (Figure 29). Although the difference is small, it is bigger than the difference presented for the Gaussian function. This highlights that for smaller size sources the position of the source could have a small effect in $L_w$.

The influence of the cylinder diameter is also verified with the triangular function (Figure 30) that shows that the increase in diameter increases the sound power level as bigger objects have a bigger radiation area.
The results when a Dirac Delta function is used as the profile in z, for different peak positions is presented in Figure 31 and confirms that the ring position is not relevant. Figure 32 shows that by increasing the diameter of the cylinder when a Dirac Delta is used as the z-profile, increases the $L_w$, in the same way as for the other z-profiles.
The following two plots (Figure 33 and Figure 34) present a comparison between the three z-profiles presented for two different cylinder lengths (variance for the Gaussian function). It shows that the smaller the lengths of the cylinder, the more the results approach the Delta Dirac function, as expected.
In order to address radiation efficiency aspects related to the Discrete Dipole Ring model several case studies have been set up. The aim of this analysis is to pin down ways in which a dipole (or groups of dipoles) could be placed in a discrete ring in order to be able to generate low frequency noise. For that, a dipole is chosen to be the first test case as it is the simpler case and its results are used for comparison and normalization of the Discrete Dipole Ring results. Then, other dipoles are added to the ring, so that there combined effect in the radiation can be analysed.

These combinations of sources are important when trying to understand the phenomena happening in the flow. For instance, there could be a series of localised fluctuations in the gas turbine exhaust flow happening due to flow characteristics when passes by the diffuser scarf angle and diffuser. These fluctuations could also be originated from the jet flow characteristics. Overall, these force fluctuations could be forming dipoles in phase or out of phase from each other situated around a ring and the interaction between these sources may or may not form dipole-type sources.

The model defined in Section 2.3 is used in these calculations. The diameter of the ring (chosen to be the same as the as the experimental investigation in Chapter 4) is \( D = 0.15m \).
A single dipole is considered here for reference purposes for the comparison with other combinations of sources that is presented later. (b)

Figure 35(a) shows radiated power results for a single dipole when its force profile in $k \lambda$ is flat (constant amplitude against wavenumber domain). The result shows that the sound power level follows $k^2$ as expected. Once again, for reference, when compared with other cases and to test that the code developed for the calculation of Euler angles, the dipole's intensity is plotted. The result shows as expected the characteristic directional dipole lobes (Figure 35).

Figure 35 (a) $L_w$ Dipole (b) Dipole intensity
When one dipole is placed in the ring the intensity pattern shows a distortion towards the side where the dipoles are placed (Figure 36) as expected.

![Figure 36 Intensity for a dipole in the ring](image-url)

When two dipoles are placed in the ring separated 180 degrees from each other, the results vary according to whether the dipoles are in or out of phase in relation to each other. When the dipoles are in phase they both move outwards in relation to the ring, but in opposite directions in relation to each other. For wavelengths big compared to the size of the ring, this creates a cancellation effect, reducing the power radiated by the dipoles. If the dipoles are out of phase, they move in the same direction in relation to each other and given the high wavelength this creates an increase in radiated power (Figure 37 (a)). It is important to highlight that this effect disappears with increase in frequency. Figure 37 (b) and (c) show the resultant intensity patterns for dipoles in phase and out of phase respectively. The dipoles with the same phase present slightly narrower lobes than a single dipole whilst two dipoles with opposite phase present slightly larger lobes than a single dipole.
To verify the effect of introducing more dipoles to the ring, the number of dipoles is increased. The results show that the radiated power still increases with $k_l$ but does not follow $k^2$ in the same way as a single dipole does (Figure 38(a)). This behaviour is observed to Helmholtz number up to around 20. The intensity results show that with increasing number of dipole sources in phase, there comes a point (for each radius where the intensity plot is produced) where the sources are well superimposed and the pattern of each individual dipole can no longer be seen any longer (Figure 38(b)). However the same is not observed when random phase forces are introduced into the dipoles (Figure 39). The fact that some dipoles have higher individual forces prevents the development of an even intensity pattern. Physically, this could mean for instance...
that at the beginning of a jet, if all the vortex formed are in phase they probably won’t present a dipole-type source when acting together. On the other hand, if these dipoles are out of phase, they could be generating dipole-type spectra despite the fact that the intensity patterns don’t necessarily show dipole structures.

Figure 38 Ring with different number of dipoles in the radial direction at constant (force with constant amplitude and phase) (a) $L_w$ (b) Intensity
Figure 39 Ring with different number of dipoles in the radial direction (force with random phase)

(a) $L_N^r$ (b) Intensity
Another aspect considered is the direction of the dipoles in relation to the ring. Figure 40 shows the results for when the dipoles face the axial direction. From the results it can be seen that when dipoles of constant forces are all radially oriented they interact more with each other cancelling their contributions and therefore the radiated power is lower. On the other hand, when the dipole forces are random forces (Figure 41), this effect almost disappears and the $k^2$ increase of power with frequency continues to happen.

When the dipoles are introduced at 45°, the behaviour is similar to when the dipoles are facing the $z$-direction but with lower amplitude. This is due to the fact that the components in the $x$-direction tend to cancel each other and the main contribution is then in the axial ($z$) direction.

---

**Figure 40** $L_w$ - Ring with 8 dipoles at constant force

**Figure 41** $L_w$ - Ring with 8 dipoles at random phase force
### 3.4 Comparison Cases

One way of comparing the Continuous Ring and the Discrete Dipole Ring models is to consider a situation where both show essentially the same behaviour. Therefore, it is chosen to make them both oscillate as a dipole source. To make the CR oscillate as a single dipole source (Figure 42(a)) only its \( n=1 \) mode is considered. On the other hand, to have the DDR oscillating as a dipole several dipoles were distributed symmetrically around the ring, with phase angles as in Figure 42(b). All the results are normalised on the cylinder surface area for the CR and on the ring perimeter for the DDR.

Comparing the acoustic power results (Figure 43) one can see that the slope in the frequency range of interest is the same for the dipole, DDR and Continuous Ring, but the levels for CR are lower. At higher frequencies, the cylinder effects start to influence the \( L_w \) and levels drop.
As a more general case, the models are compared using some of the results presented previously. The CR configuration used here is using the first 3 modes in $\phi$ and the different $z$-profiles. The DDR model uses 8 dipole sources radially orientated and equally distributed around the ring. Figure 44 shows the results for constant forces. Although both models show similar sound power levels, their trend characteristics in frequency are different. DDR shows the same increase with frequency as a dipole would, whereas the CR model shows a decay. When forces with random amplitude and phase are considered, the results (Figure 45) show that the DDR preserves the dipole trend in frequency (as it is only dipole based), whereas the CR does not, probably because multiple modes in $\phi$ are being considered.
None of these results shows the effect of variation of the force term in frequency. This is approached in the next sections.

### 3.5 General jet noise spectra characteristics

In the previous sections, noise radiation efficiency aspects were considered. In order to focus on some of the source characterisation aspects that are be presented in Section 3.6, some major characteristics of jet noise spectra are given here to help in the later analysis.

The jet noise spectrum is discussed in Chapter 1 and has the shape given in Figure 46. The sound power level increases parabolically along with increase in frequency until it reaches a peak and then decays with increasing frequency. This spectrum is presented using the Strouhal number to non-dimensionalise it. The Strouhal number is given by:

\[
St = \frac{fD}{V}
\]

where \( f \) is a characteristic frequency, \( D \) is a characteristic diameter and \( V \) is the mean flow speed.
Figure 46 shows some values for the characteristics of jet noise peak sound power levels found in the literature. The Strouhal number at the peak generally varies from 0.1 to 0.4 for $M \leq 0.9$. For low Mach number flows not many data is found, but for the lowest speeds found, the Strouhal number at the peaks lies between 0.1 and 0.2.

![Figure 46 Jet noise spectra general shape [154]](image)

<table>
<thead>
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<th>Strouhal Number</th>
<th>0.11</th>
<th>0.1</th>
<th>0.4</th>
<th>0.2</th>
<th>0.2</th>
<th>0.33</th>
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<td>Mach Number</td>
<td>0.4</td>
<td>0.435</td>
<td>0.36</td>
<td>&lt;0.7</td>
<td>0.36</td>
<td>0.9</td>
</tr>
<tr>
<td>Jet diameter (m)</td>
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<td>0.12</td>
<td>0.05</td>
<td>N/A</td>
<td>0.025</td>
<td>0.051</td>
</tr>
<tr>
<td>Peak frequency (Hz)</td>
<td>300</td>
<td>N/A</td>
<td>1000</td>
<td>N/A</td>
<td>1000</td>
<td>2000</td>
</tr>
<tr>
<td>Reference</td>
<td>[12]</td>
<td>-</td>
<td>[153]</td>
<td>[154]</td>
<td>[12]</td>
<td>[12]</td>
</tr>
</tbody>
</table>

Table 1 Strouhal number obtained from jet noise literature

Assuming $St = 0.2$, as suggested by [154], considered as appropriate for the case considered in this thesis ($M = 0.08, D = 0.3$), the corresponding frequency and Helmholtz number are given by:

$$St = \frac{fD}{V}$$

$$f = 0.2 \frac{V}{D} = 0.2 \frac{30}{0.3} = 20Hz$$

$$kl = \frac{\omega l}{c} = \frac{2\pi f}{c}l = \frac{2\pi 20}{340} \cdot 0.3 \approx 0.1$$
This $kl$ should then correspond to the jet noise peak and will be used for comparison with experimental results.

3.6 Aerodynamic Source Spectra

The results presented so far for both models are all with a source term constant in frequency, looking into radiation efficiency aspects. This section, on the other hand, covers some source characterisation aspects by introducing a "generic" profile. Other aspects will be cover in Chapter 5, where an example of LES data as the source input will be presented.

In order to get to a generic result that is somehow realistic, an appropriate spectrum profile for the pressure sources has to be considered. This generic spectrum is then introduced into the models as an example of a source spectrum.

3.6.1 Aerodynamic jet spectrum

To introduce an appropriate generic profile for the sources, the classical spectrum of turbulence that was described by Kolmogorov and others is considered. This spectrum represents the turbulent fluctuations and associated cascade process that occurs in turbulent flow. The Kolmogorov energy spectrum is divided in three parts: an energy containing range, an inertial range and a dissipation range (Figure 47).
The first region in the spectrum has anisotropic large eddies and is the one containing most of the energy. This region normally shows a parabolic \((k^2)\) increase in the energy levels with increase in \(k\). In reality, most measurements are taken only in one direction and therefore only a one-dimensional spectrum is measured. In the one-dimensional spectrum this region becomes flat. The large energy-containing eddies are then stretched and broken down into smaller eddies that form the inertial sub-range.

The inertial-sub range is the range where the motions are dominated by inertial effects. It contains isotropic small eddies and the energy is transferred successively to smaller eddies. In this range the energy-spectrum function \([155]\) presents the Kolmogorov's \(-\frac{5}{3}\) decay spectrum. The turbulence is finally broken down and converted into internal energy in the dissipation range. The energy is dissipated by viscosity. In this range the energy decays exponentially with the wave number \((k)\).

To represent the energy spectra characteristics, here a model of the spectrum \([155]\) is used. This model can be scaled in two different ways. One that makes the low frequency part of the spectra converge (large scale-normalization) and another that makes the high frequency range of it converge (small scale normalization). Given that the major interest in this work is in the low frequency range, the large-scale normalization is used here.

If one considers a jet issuing from a pipe then turbulence is generated by the shear layer around the periphery of the jet. These shear layers grow, due to mixing, until eventually they meet on the centreline – this represents the end of the so-called 'jet potential core'. The scale of the turbulence is reflected in the size of the potential core. Noise is produced by fluctuations in static pressure – and these are created by turbulence. Hence in general it is logical to assume that the scales of the turbulent velocity fluctuations also reflect the scales associated with the pressure fluctuations. For isotropic turbulence various calculations have been done and have yield formulae such as \([156]\):

\[
p' = \rho \cdot \nu \cdot v^2
\]

which supports the argument of using the energy profile as the force profile. Although a jet does not show only isotropic fluctuations, especially at low frequencies, the generic energy spectra, is here used to exemplify a type of input source data. For that, a model energy spectra proposed by Pope \([155]\) is used.
3.6.2 Results of introducing aerodynamic jet spectrum into CR & DDR

This section presents the results using the aerodynamic profile as the source force decay in frequency in both models.

![Graph showing decay in frequency](image)

**Figure 48** $L_w$ CR model using triangular function for $z$-profile

Introducing the aerodynamic $kl$ profile into the Continuous Ring model source term, gives a very accentuated decay in $kl$ as shown in Figure 48. This decay happens both for constant force or not.

The same examples presented in section 3 are used here with the aerodynamic profile applied to DDR so that the results can be compared. Figure 49 shows that for a single dipole the introduction of the frequency dependent source term makes the spectrum to increase with frequency, reach a peak and then decrease with increase in frequency, with a decay slope is 12 dB/decade. The peak using these particular profile happens at $kl=0.5$, which is much higher than what is expected for a typical low speed jet spectra.
When 8 dipoles are placed around the ring and the dipoles are all in phase (Figure 50), the results show that there is a significant reduction in level when the dipoles are oriented in the radial direction. When sources with random phase are considered (Figure 51), the results both in the longitudinal direction and in the radial direction present similar results for the peak amplitude ($k_l=0.3$) and decay rate of 12 dB/decade at lower frequencies and 25 dB/decade at high frequencies.
Overall, the introduction of a generic aerodynamic profile is useful as a preliminary visualisation of what the real spectra is likely to be. For instance, in the case of a simple dipoles, once the aerodynamic profile has been included as the source input $L_w$ presents more realistic values than the ones showed in Figure 35, where the level of energy would just keep rising with frequency. However, the assumptions made to use this model, mean that it is primarily not so valid at low frequencies. Therefore, more appropriate source values will be introduced from LES calculations and discussed on Chapter 5.

3.7 Summary of findings

This chapter shows the analysis of the results for the two analytical models presented in Chapter 2.

Radiation efficiency aspects are considered for both models. In the Discrete Dipole Ring, the major finding is the fact that dipoles displaced in a ring with forces out of phase, can create dipole-type noise spectrum.

For the Continuous Ring using a constant $k_l$ profile, the results show that:

- The position of the force peak is not very relevant.
- The sound power level decays with increase in frequency.
- For a given force profile, the bigger the radiating area the higher the sound power level is at low frequencies, for a given radiation surface, the higher the sound radiated will be.

When addressing the source characterisation aspects a "generic" aerodynamic profile has been used as an example of input source force in both models. The results shows sound power levels with accentuated decays in frequency. This leads to the fact already known that the jet the "generic" aerodynamic model is not completely valid at the first stages of the jet, as the flow at that stage is formed of other structures (such as vortex) than only isotropic turbulence. Nonetheless, these results are used in Chapter 4 for comparison with the experimental results.
Chapter 4 Experimental Investigation

In Chapter 1 the hypothesis of dipoles as the main source of noise in industrial gas turbines at low frequency is introduced. In this chapter, the experiments that verified this hypothesis are analysed in more detail. This chapter also shows how some of the other possible noise mechanisms generating noise at low frequency were determined, excluded and/or considered to be of little relevance.

To find out about the possible sources, an experimental model of the gas turbine is used and several components varied (or removed) to analyse each different flow aspect. Details regarding the preliminary work that lead to the design and characteristics of the experimental facility are presented in Section 4.1.

A set of initial measurements performed to qualify the room is presented in Section 4.2. Acoustic measurements are undertaken in order to search for the possible source mechanisms of noise generation at low frequency as well as to quantify the swirl noise, the noise contribution of the volute, noise contribution of the diffuser and jet noise. A semi-reverberant room is used to determine the sound power spectra from the jet. Section 4.3 shows the different test conditions and test procedures considered to identify the low frequency noise sources in the industrial gas turbine considered. The results are then analysed (Section 4.4) and compared with the theoretical modelling (Section 4.5).

4.1 Experimental Facility Design

The high temperatures and gases present in industrial gas turbines present several difficulties and risks when measurements are to be undertaken. To resist high temperatures the equipment has to be much more robust and there is a need for greater safety precautions. Added to that, the costs associated in building and operating a full size engine are extremely high. These reasons suggest, that an experimental model of the engine should be build.
In designing the experimental model, several factors are considered:

- Costs and physical resources to build the facility;
- Adaptable configurations in order to explore the possible isolated conditions that could be generating noise;
- Ability to capture the major aerodynamic features associated with noise in an industrial gas turbine exhaust system;
- Practical to manufacture and operate.

A summary of the necessary decisions to accommodate these considerations from the acoustics perspective is presented below. More details related to the aerodynamic aspects associated with the build up of this rig are given by [5].

Space and costs restrict the scale model to be somewhere between 1/4\textsuperscript{th} and 1/10\textsuperscript{th} of the real engine. The two major factors that can increase significantly the cost of the rig are related to the high temperatures (combustion aspects) and moving parts of the engine such as turbine.

In order to be able to run the experimental model at ambient temperature, one has to consider the relevance of combustion noise in the low frequency range. As the literature suggests, combustion noise happens at much higher frequencies [12], and therefore excluding it does not affect the low frequency range. It is also clear that the industrial gas turbine has the same combustion chamber as the equivalent aero-engine, which does not present high low frequency noises. This re-emphasized that combustion is not one of the major sources of low frequency noise and that it could be excluded from the experimental facility. From the aerodynamics point of view, the characteristics of the flow after passing the combustion chamber are not considered to be very relevant to subsequent flow characteristics due to the fact that the turbine effects are strong enough to mask other flow characteristics. In order to take into consideration the temperature differences between the real engine and the experimental model, the noise spectrum is expected to present to have a shift in frequency as presented in Chapter 1.

With combustion excluded from the experimental model, the question of what could drive the experimental model arises. This could be done using electrical moving systems to rotate the turbine blades. However due to the complexity of components and cost
associated with these types of rigs, it is decided to build an experimental model without moving parts. For that, a fan is used to drive the rig and fixed blades (IGVs and OGVs) are used to simulate the aerodynamic effects.

Considering the high sound power level generated by the fan (93dB(A) measured at 1m distance from the open fan inlet), a plenum is added to the experimental facility (Figure 3). As shown in Figure 3, atmospheric air passes through an inlet duct into an acoustically lined centrifugal fan room. The fan is located within an enclosure, lined with noise absorbing material of 0.5m thicknesses.

The air is drawn into the fan and pumped along an interconnecting duct (made of flexible rubber to prevent vibration transmission from the fan to the duct) to a plenum, which is also acoustically lined and contains a series of acoustic baffles to attenuate the fan noise. From the aerodynamic point of view the plenum provides a uniform entry flow into the test section inlet. For that, the air has a velocity of approximately 0.3 m/s (M~0.001), which settles most flow instabilities generated by the flow.

Once the air leaves the plenum it enters the working section (rig itself – Figure 4) via an inlet flare. The inlet flare is placed at the entrance of the rig to attenuate flow instabilities in the flow entering the rig.

The flow then passes through inlet guide vanes (IGVs) that rotate the flow in such a way to represent the flow characteristics generated by certain power conditions of a real engine. This is done by varying amounts of swirl in the annular passage downstream of the power turbine at different power settings. The swirl angle variations simulating different power settings are achieved by using different sets of IGVs (100%, 70% and 30% power respectively). For the 100% power condition, the levels of swirl are small whilst for the 30% power condition these levels are very accentuated.

Further downstream, the flow passes through the Outlet Guided Vanes (OGVs) that straighten the flow back (to essentially the axial direction) in the same way they are used in a real engine, prior to entering the diffuser, as shown in Figure 4. “For the generic exhaust system geometry 16 OGVs upstream of the diffuser inlet were chosen as a typical design feature” (ref1). The OGVs are designed for the standard operation condition of 100% power, which originates small levels of swirl in other conditions.
The "duct" is a structure designed to hold the IGVs and OGVs. Its length is chosen with the objective of having the volute at 1.5m away from every laboratory walls to avoid acoustics interference.

The experimental facility design should incorporate a generic model of an industrial gas turbine exhaust system containing the main components associated with the exhaust system such as a diffuser, volute and rectangular exit duct.

Diffusers are normally used in industrial gas turbines to increase the pressure in the system. The aerodynamics details associated with the design of the diffuser and its scarf angle are presented in [5]. From the acoustics perspective, as the diffuser is considered to be one of the possible low frequency noise sources, therefore it is made with adjustable configurations in order to explore different potential relevant noise sources.

Struts are included inside the diffuser to hold the centre shaft in place. Although this involves detailed aerodynamic investigation, its effect is not considered to be of relevance in the acoustics noise generation at low frequency as the struts are very thin, which under relative high speed generated much higher frequencies than the ones being considered as the issue in the engine.

Flow exiting the diffuser is dumped into an acoustic volute box that turns the flow 90° before expelling it into the atmosphere. A rubber seal is placed around the contact area between the diffuser and the volute to prevent vibration transmission to the volute, so that its effects could be measured in isolation.

Once the major experimental facility arrangements have been thought, details regarding its configuration are considered. The scale factor between the model and typical engine sizes is then chosen based on aerodynamic and acoustic measurement requirements of running at the same Mach and Reynolds number as the real engine. Due to cost/space restrictions related to the choice of fans it is decided to run the rig at similar a Reynolds number to the real engine to guarantee turbulent conditions in the flow. However this means that the rig runs at lower Mach number then the real engine, which could potentially introduce differences in the noise sources. Jayatunga [5] shows how the scale value of the rig came out to be 1/7\textsuperscript{th} scale based mainly on the options of fans operating conditions.
In order to simulate the changes produced in the swirl angle distribution, the IGV geometry can be altered. The three sets used for the IGVs correspond to 30%, 70% and 100% engine operating powers and vary from high levels of swirl to low levels of swirl respectively. The Reynolds number, based on diffuser inlet annulus height, is approximately $1.3 \times 10^5$. It is possible to operate the facility with or without several of its components in order to acoustically evaluate the components in combination or in isolation.

Details regarding the aerodynamic commissioning of the rig and its operating point are presented in [5]. The non-dimensional mass flow rate is used to guarantee the same flow conditions during the measurements. This operating condition is chosen to be a non-dimensional parameter involving pressure and temperature and is directly proportional to the Mach number. For the all the acoustics measurements presented in this chapter, the inlet Mach number is kept constant at 0.08.

### 4.2 Room qualification

**Reverberation Time**

The purpose of this measurement is to check the acoustic behaviour of the laboratory that is used for noise measurements. The acoustical properties of the lab are investigated by measuring its reverberation time and calculating the Schroeder frequency. Reverberation time is calculated using the decay rates in each third octave band.

Figure 52 shows one of the reverberation time curves. The results show that the variation obtained in the reverberation time measurements is small such that a quasi-diffuse sound field can be expected. Using the average reverberation time obtained from the measurements, the Schroeder frequency is calculated ($f_s = 140\, Hz$). The Schroeder frequency is obtained from [156]. In the equation below, $T$ is the reverberation time (in s) and $V$ is the room volume (in $m^3$).
\[ f_s = 2000 \sqrt{\frac{T}{V}} \]

Part of the frequency range of interest is below this limit, so that in order not to compromise the measurements, a greater number of measurement points and time averages must be used to guarantee more reliable results.

![Graph showing reverberation time plot](image)

**Figure 52**: Example of the reverberation time plot obtained

**Background Sound**

Background noise measurements (Figure 53) are taken in the lab in order to make sure that no other components in and around the room would influence the measurements using the rig. The results show that the background sound power levels are around 10 dB below any measurements taken (apart from a sharp peak at 400 Hz) with the model running, which is an encouraging result that background levels do not interfere in the source identification.
Structure-borne sound

The properties of the rig are investigated to check for the influence of any structural radiation. The measurements are taken on both the supports of the rig and on one of the plenum walls. The sound power [157] is calculated using the equation below for the supports:

\[ W = \sigma \rho c S v^2 \]

where \( \sigma \) is the radiation efficiency (conservatively set equal to 1), \( \rho \) is the air density, \( c \) is the speed of sound in the air, \( S \) is the radiating area of the object and \( v \) is the spatially averaged velocity of vibration obtained from the measurement. The radiated sound power level is calculated from:

\[ L_p = 10 \log \left( \frac{W}{10^{-12}} \right) \]

The results (Figure 54) show that the structural sound intensity is very small even having calculated the sound power level conservatively.
Fan Noise

A silencer system is designed and built according to Figure 3 to isolate the fan noise from the noise generated in the rig as presented in Section 4.1. To check whether the fan noise is isolated, sound pressure level measurements of the fan are taken and compared with measurements taken in the lab with the fan running but without the rig working. This is possible by closing the rig aperture and opening the plenum door. The results show that when the fan is running the background $L_w$ is higher at high frequencies (Figure 55), but it is still low enough not to interact with the source.
4.3 Noise source identification

To identify the noise sources present in the flow, each part of the exhaust system is isolated and the sound power for each configuration measured. In this way, the contribution of each component and its interaction with others is analysed.

4.3.1 Data acquisition

The sound pressure measurements are taken with a 1/2" (Bruel & Kjaer 4133) microphone connected via pre-amplifier to an FFT-analyser (Hewlett Packard). The readings of sound pressure levels were averaged over a time of 200 seconds. A wind protection is placed around the microphone to prevent wind-generated noise from interfering. The calibration of the microphone is done using a 250 Hz piston phone.

4.3.2 Case studies

The experimental programme to obtain the low frequency noise characteristics of the industrial gas turbine is described in this section. To evaluate the sound power levels for different configurations, measurements are conducted in the semi-reverberant room. Each case is tested at three different inlet flares Mach numbers (0.07, 0.08 and 0.09 whenever possible) and for three different IGV configurations. Given that the experimental rig does not have any rotating component, the IGVs are then vanes that turn the flow in such a way that it represents the same characteristics that occur in the real gas turbine when it is rotating. The IGV configurations tested were representative of 100%, 70% and 30% power conditions of the gas turbine. In this section only the experiments for \( M = 0.08 \) and 100% power conditions are presented. The cases considered in this thesis are explained below.

Case 1: IGVs + OGVs + Centre shaft + Scarf diffuser

This case (Figure 56) considers the whole model characteristics excluding the volute so that the contribution of the rest of the rig can be evaluated. Different IGVs are used to consider the effect of swirl.
Case 2: IGVs + OGVs + Centre shaft + Scarf Diffuser + Wall

In order to gradually introduce the effects of the volute, a wall is placed in front of the flow (Figure 57). The wall’s position is varied relative to the diffuser exit: positions 1 and 2 are 10 cm and 40 cm away from the diffuser respectively.

Case 3: IGVs + OGVs + Centre shaft + Scarf diffuser + Volute

This case considers the complete test rig as designed to simulate the industrial gas turbine. This is an important case to compare with the real engine and to account for the volute contribution to noise. This contribution could be the flow impinging on the
back part of the volute, or the re-circulation in its bottom part could generate some of the noise sources.

Figure 58 Lateral view Case 3

Case 4: IGVs + OGVs + Centre shaft + Plane diffuser

This is the same as Case 1, but with a plane diffuser at the exit instead of a scarf diffuser. The introduction of a plane diffuser came to clarify whether or not the diffuser's cant angle (cut that the diffuser has) is influencing the noise characteristics (low frequency slope) by altering the swirl effect on the tangential velocity or by releasing vortices out of phase for the jet formation.

Figure 59 Lateral view Case 4
Case 5: IGVs + no OGVs + Centre shaft + Plain diffuser

In this case the OGVs are removed and the plain diffuser is used. The objectives of this measurement are to verify whether there is interaction between the two sets of vanes and check the swirl effect.

![Figure 60 Lateral view Case 5](image)

Case 6: Centre shaft + Plain diffuser

No set of blades is considered in this case and therefore the influence of the centre shaft can be verified. The centre shaft is held in position by three 6 mm rods placed at the IGV and at the OGV positions.

![Figure 61 Lateral view Case 6](image)
Case 7: Centre shaft + Scarf diffuser

Case 7 is the same as Case 6, but with the scarf diffuser at the exit instead of the plane diffuser. Having the two sets of vanes removed (therefore no swirl) is a way of analysing the influence of the scarf angle of the diffuser without having to consider any swirl or flow separation caused by the vanes.

Figure 62 Lateral view Case 7

Case 8: Plain diffuser

In this case the centre shaft and both sets of blades are removed, leaving only the external duct and the plain diffuser.

Figure 63 Lateral view Case 8
Case 9: Duct only

In this case (Figure 64), only the external duct of the scaled model is used (without IGVs, OGVs, centre shaft or diffuser). The objective of this measurement is to account for the duct characteristics and for the contribution of a simple jet formation leaving the system.

4.3.3 Sound power measurement

Sound power measurements are taken to determine the overall characteristics of the spectra of the system for each configuration.

The sound power for each testing configuration is obtained from sound pressure measurements in fixed positions of the lab and calculated from ([158], [159]):

$$L_w = SPL - 10 \log_{10} \frac{T}{T_o} + 10 \log_{10} \frac{V}{V_o} + 10 \log_{10} \left( 1 + \frac{S \lambda}{8V} \right) - 10 \log_{10} \left( \frac{B}{1000} \right) - 14$$

where $SPL$ is the mean pressure level, $T$ is the reverberation time, $V$ is the volume of the room, $S$ is the total surface area, $\lambda$ is the wave length at the centre frequency of the frequency band and $B$ is the barometric pressure. The uncertainty inherent in these techniques according to ISO 3745 for measurements in a semi-reverberant room is 1.5 dB. The $L_w$ is measured in 15 positions around the room.
4.4 Results and analysis

Figure 65 shows the sound power level against frequency, for the 100% and 70% IGVs power conditions. The spectrum shows a decay of 13 dB/decade for up to approximately 600Hz followed by a sudden increase in magnitude and another decay. The sharp increase in level at 600Hz corresponds to the cut-off frequency that is the frequency at which the higher order modes start to propagate.

Above the cut-off frequency the spectrum falls off with increasing frequency. At ~600Hz the (1,0) mode dominates the (0,0) mode and the same happens at ~1000Hz for the (2,0) mode. The characteristics of the spectrum above the cut-off frequency are similar for all cases including the centre shaft at all the power conditions. For this reason and because the main interest is not in the higher frequency content, the region above the cut-off will not be commented on henceforth.

The peaks below the first cut-off frequency (~600Hz) indicate plane wave resonances. Some are related to the system resonances and others to unstable frequencies of the flow.

The first phenomenon that could be contributing to the original noise levels and general noise characteristics is swirl. Comparing the results for 100% power and 70% power one can see that the increase in level of swirl (from 100% power to 70% power) increases the sound power level. As this happens in all the cases and speeds considered, it is concluded that at high swirl incidence (more than 14° at the tip) the higher the level of swirl the higher the noise it generates. The physical mechanism behind it is not completely understood, but some indication is presented in the discussion.
The results of the sound power level against frequency are presented in Figure 66 for a wall perpendicular to the flow, for the 100% power condition. The figure shows that the sound power level for this case when compared to Case 1, follows the same basic characteristics (no change in magnitude is observed in the decaying slope), but presents a pronounced peak around 45 Hz. The peak around 45 Hz peak coincides with one of the duct resonance frequencies, but the possibility of the wall dimensions generating flow separation at its end cannot be excluded. Figure 67 shows the variation of the sound power level against frequency when the position of the wall is varied. Moving the wall forward does not change the spectrum characteristics apart from increasing the peak at 45 Hz. The fact that altering the wall position doesn't affect the overall level shows that the interaction fluid/wall is not so relevant and is not one of the main noise sources. The effect of moving the wall closer to the diffuser only makes the resonance peaks higher than the peaks created by wall in position 2. This can be explained as the presence of the surface creates another dipole source that excites the system even more (if the source was the same it would also affect the broad band content of the spectra).

Certain measurements present a very evident peak at 100 Hz. This is not due to flow noise but due to electrical noise due to mains interference.
Measurements including the volute are taken and the power spectrum against frequency (Figure 69) shows the same increase in level for certain low frequencies when compared with the case where the wall is placed in front of the flow. The frequencies are shifted down as the length of the system has increased (duct + volute length). This shows that the flow impinging on the volute is exciting the resonant frequencies of the system “duct+volute”. The shift in frequency presented in Figure 69 is then due to the different systems considered.
Comparing the sound power measurements against frequency for the case with and without volute (Figure 70), shows that the flow characteristics with the presence of the volute - apart from exciting some of the system resonances - tends not to change the overall sound power level at low frequencies. This is an encouraging result as the recirculation on the bottom of the volute (Figure 68) was originally thought to be a significant source of noise at low frequency given its size and the possibility of this recirculation being a monopole (due to its continuous volumetric expansion and contraction), which is an efficient radiator at low frequencies. This measurement, therefore excludes aerodynamic noise sources in the volute from being one of the major sources of noise at low frequency. The fact that the system resonances are amplified by the introduction of the volute/walls indicates that the source is speed sensitive, but more investigation is still necessary.

![Figure 68 Axial velocity contours at diffuser exit](image)

Although not many data from the real engine were available, it is known that the typical sound power level acquired from it has a very similar decaying slope (14 dB/decade, Figure 1.2) to the experimental model, regardless of not being operated at the same temperatures and at the same Mach numbers. This is an encouraging result as it means that most sources of interest are being represented in the experimental model. It also gives confidence that the low frequency sources are present in the experimental model without the volute, as the slope is the same with even stronger amplitudes.
Given that the sound power measurements including the aerodynamic noise sources present in the volute show that the aerodynamic effects happening in the volute (such as the recirculation the bottom and the flow impinging against the back wall) are not one of the major sources of noise, further experiments were taken without it in order to try to identify the other possible source mechanisms, such as: swirl, jet fluctuations, blade flow interactions, aerodynamic noise sources happening due to the presence of the diffuser and the diffuser scarf angle.

Comparing the sound power level measurement versus frequency for the cases with plane diffuser and with the scarf diffuser, it can be seen that the effect of the sources related to the cant angle (Figure 71) is to make the system resonate more (higher levels
at the peaks). However, adding a plane diffuser does not affect the overall decaying characteristics. Therefore, the alteration of the swirl effect on the tangential velocity (and therefore the cant angle) is not the main contributor to the characteristic decaying slope present in the sound power level at low frequencies. Checking if the aerodynamic noise sources in the cant angle are responsible for the possible pressure variation at the exit of the diffuser (thought to be the release of vortex out of phase for the jet), the cant angle increases the level at the resonances but the level of the decay slope is kept the same. This shows that the aerodynamic effects generated by the presence of the cant angle are not the main cause of pressure variation at the exit of the diffuser, although the source is still there with the cant angle present.

![Frequency vs. Sound Power Level](image)

**Figure 71** $L_p$ - IGVs + OGVs + CS

Figure 72 shows the comparison of sound power level against frequency between the case with and without OGVs. It shows that the level without the OGVs is higher than with OGVs at low frequencies. This suggests that the flow interaction between the two sets of vanes helps to stabilize the diffuser and is not a significant source of noise.
When both sets of vanes are removed, the plot of sound power level against frequency (Figure 73) shows that the levels without the blades are higher than with blades. This reinforces the fact that the presence of the blades helps to stabilise the flow. Above the cut-off frequency, the presence of the blades makes the higher order modes stronger.

Comparing Case 7 with Case 6 (Figure 74), shows that changes in the flow due to the presence of the scarf angle only makes the system resonate even more, showing therefore that there is a source at the end of the diffuser (regardless of the diffuser angle) present in both cases.
When the centre shaft is removed from the system, the sound power level against frequency presents a significant reduction in amplitude (Figure 75) due mainly to the reduction in the flow speed. The slope of the curve remains the same at low frequencies, highlighting that the basic source mechanism is still there regardless of the presence of the centre-shaft and blades. At higher frequencies it is clear that the flow characteristics change due to presence of the centre shaft makes the higher order modes appear.

It is important to emphasize that in this experiment the diffuser is not operating on the stable side of the design conditions as its Length/Diameter dimension has been altered.
when the centre shaft is removed. Because of that, phenomena like separation and instabilities could be occurring inside the diffuser which could perhaps explain why the decay in the sound power level not being as high as expected for the difference in area ratio in between the duct and the diffuser.

When the diffuser is removed from the duct, the sound power level shows (Figure 76) an increase in level compared with the free duct setup, which is expected since the flow velocity increases. The slope in the $L_w$ remains the same (decay of 13 dB/decade up to 200Hz). Although the general noise signature is the same for both cases the flow characteristics is due to the presence of the diffuser makes the duct resonate more. The difference between the $L_w$ of these two cases is shown in Figure 76.

![Figure 76 $L_w$ - Duct only](image)

4.5 General comments

With these experiments it is possible to know that the phenomena responsible for the decaying slope present at low frequency is present even in the simplest case (Duct only – simple jet). The fact that an externally placed source located near the exit of a duct (as there is no other significant sources inside the duct) makes the system resonate indicates that this source is a pressure source. This is due to the fact that at the exit of a pipe, the acoustic pressure in each mode is zero. To excite a zero pressure point one needs a pressure source. This is analogous to a beam or a plate with velocity nodal
lines. A force (pressure source) cannot drive the structure at a velocity nodal line but a velocity source can. A velocity source cannot drive the structure at an anti-node (force nodal line) whereas a force source can.

Therefore, these experiments show that the sources responsible for the decaying slope and for the resonance peaks are present in the free jet. Also, the source exciting the duct is a pressure source. Monopole sources were therefore not considered to be relevant as these velocity sources would not make the system resonate. Putting aside the monopoles that initially were expected to be the most likely radiators at low frequencies as show in Figure 1.6, the second most likely type of source is the dipole. The theory of having the dipole as the source agrees with the findings of the sources being a pressure source. The fact that the main source is a pressure source does not exclude the possibility of existence of other higher order pressure sources. The only reason to keep the dipoles as the most probable source is due to the fact that dipoles are more efficient radiators at low frequencies.

It is still not known whether the source responsible for the decaying slope is the same one responsible for the resonance peaks. To investigate those questions, velocity variation measurements are undertaken. The results are shown in Figure 7.7, which presents the $L_w$ against frequency for the free jet at some different Mach numbers. The result shows that the $L_w$ increases with the increase in speed and it is possible to check that the increase in the $L_w$, follows $V^6$. This verifies as an independent experiment that the pressure source is of dipole nature (as dipoles follow the $V^6$ power law). The fact that the decaying slope as a whole followed the same increase in magnitude when the speed increases, shows that the sources under investigation remained the same and are dipoles.

Although this result is expected from the fact that the frequencies being analysed are low, they contradict most of what has been presented in jet noise theory, which says that jet noise is represented by quadrupoles and follows the $V^8$ relation. Another mismatching characteristic between what is observed in these experiments and what was seen in the jet theory, is that many papers present that the low frequency noise is originated by aerodynamic sources at the end of the jet core, more then 5 diameters away from the jet exit. This does not happen in the experiments in this work, which is clear from the cases where the aerodynamic flow characteristics due to the volute and
wall were present. Both the volute and wall when placed no further than one diameter away from the jet exit, present a $L_w$ spectra, which is not much different from those flows without any interference, which show that the sources must be closer then one diameter away.

![Figure 77 $L_w$ - Duct only](image)

Doing a more careful search it is clear that most of the $V^8$ results presented in the literature were for Mach numbers above 0.3 and that many experiments that approached low speed jets ended up finding $V^6$ results, as presented in Chapter I. Most of them concluded that there was possibly some error in the lower speed experiments and that further analysis was not being taken as the lower speeds were not of great interest.

Some clarification came with paper by Laufer [131], where he shows that for a jet with a Mach number between 0.005 and 0.2, the jet flow presents different characteristics that are not necessarily present in higher Mach number jet flows. He shows that these characteristics are the dominant sources at low frequency and that most of its radiation happens along the first diameter, which is also observed in this research. He describes the sources as "generated by convective instability waves developed in the shear layer". He did not suggest however that they were dipoles.

Trying to understand physically, which flow phenomena could create the dipole(s), the most probable phenomenon is the low frequency magnitude associated with periodic vortex formation in the jet. The slope is possibly due to the potential core
characteristics of the jet leaving the duct and its instabilities. Their rapid variation in amplitude produces a periodic thickening and thinning of the shear layer generating a pressure field that develops into acoustic waves (as presented by Laufer (ref!)).

To summarise, this section presents the analysis of the experimental investigation showing mainly that regardless of the experimental setup the noise spectra always shows a decaying slope at low frequencies. Taking the investigations a step forward it is observed that the main sources responsible for this slope are dipole-type sources probably created by vortex formation in the jet. These vortex formations take place all around the duct (forming a ring) and can happen in more then one downstream location (forming a cylinder). The ring and cylinder configurations for the dipole sources are presented in the theoretical modelling (Chapter 3) and the comparison between the experimental results and the theoretical results are given in Section 4.6.

4.6 Comparison with the model

This section presents the comparison between experimental and theoretical results in order to validate the model.

The experimental result used for comparison is Case 9, where only the duct is left and the flow structure presented is a jet. This experimental result shows that the major feature of the spectrum presented in the low frequency range, is a constant decaying slope that happens in all configurations analyzed.

As the pure jet case is the simplest case that presents the decaying slope and the one with most similarities with the models proposed, it is used here for comparison with the models results. The DDR model is set up for this comparison with 8 dipoles around the ring facing the axial direction and using a generic Kolmogorov spectrum as the source input for each dipole.

Figure 78 presents the $L_w$ results against Helmholtz number for the jet measurements and the DDR model. The plot shows that the results obtained from the DDR model are much lower than the ones measured in the experiments and also don’t seem to present the same decay. These differences could be due to the use of a generic profile, but if they were the real profile they could invalidate the DDR model. However, there could
still be a possibility of these results being different when a proper source input data is used for each particular location. This is discussed on Chapter 5 when LES sources are introduced into the model.

![Figure 78 $L_{sw}$ - Comparison between jet experimental results and DDR model](image)

The DDR model also shows that if there is a single dominant dipole in the x direction, it could be generating the same decaying characteristics as in the experiments. Although the possibility of having one single dipole in the x direction is very unlikely (as this giant dipole would have to be present in all the experiments undertaken), the possibility of it happening could not be disproved.

![Figure 79 $L_{sw}$ - Comparison between jet experimental results and CR model](image)
Comparing the CR results with the same experimental result (Figure 79) shows that when the aerodynamic source profile in kl is included, the $L_w$ decay is much more accentuated than the experimental results. The results for CR with forces with a predetermined profile in frequency are also presented for reference on how much the source frequency content can change the results. This could be due to the fact that generic profiles used do not necessary represent all the flow characteristics at first stages of the jet. A more appropriate profile will be introduced with the LES calculations.
Chapter 5  Example: source input using Large Eddy Simulations

In Chapter 3, the results from the theoretical investigation are presented using a generic source. When these results are compared with the experimental investigation it is shown that most theoretical results present a similar slope in the noise spectra. However, it is clear that there is a huge difference in the levels found.

Therefore, the objective of this chapter is to shows the use of Computational Fluid Dynamics (CFD) in the investigation of the acoustic power. More specifically this chapters will show the use of LES technique as an example of source input data for the CR and DDR models.

To achieve that an LES example of a jet flow leaving a duct will be used. This example will have as many as characteristics as possible from the experimental facility, so that once the LES data is input into the CR and DDR model the sound power level output form this models can be compared with the experimental data.

An overview of the LES method and setup used for the calculations is presented in Section 5.1. Some of the parameters used in the LES calculations and LES results are presented Section 5.2. The LES results are then analyzed and placed into the acoustic model (section 5.3) and the results are discussed.

5.1 Large Eddy Simulations

Large Eddy Simulation is a method for computation of three-dimensional turbulent time-dependent turbulent flow at moderate to high Reynolds number. Turbulent motion is a random, unsteady process. Its behaviour is of difficult prediction through computer simulations.

Current methods are normally based on Reynolds Averaged Navier Stokes (RANS) [5] analyses that rely on models to represent the effect of turbulence within a flow field. The quality of the results depends on the quality of the model and its applicability to
the type of flow field being studied. RANS is also more focused on average flow distributions and therefore not highlighting the transients aspects observed in the flow.

LES as being time-dependent is capable of dealing with the transient aspects and can be more accurate because it reduces the amount of modelling necessary by directly computing part of the flow characteristics.

Turbulent flow is composed of coherent patches of swirling fluid called eddies. These range in size from large to little swirls of air. In LES, the large-scale dominant turbulent motion is computed directly and uncoupled from the smaller and less important turbulent eddies, which are modelled.

A filter is used to determine which eddies are to be modelled and which are to be resolved. The governing flow equations employed for LES are obtained by filtering the time-dependent Navier-Stokes equations in space. The filtering process eliminates eddies whose scales are smaller than the filter and the resulting equations are governed by the dynamics of large eddies. The sub-grid-scale stresses resulting from the filtering operation require modelling.

The computer program LU-LES developed at Loughborough University for Large-Eddy Simulation is used for predictions in the current investigation [149].

A brief description of the governing equations of fluid flow and modelling approach used in this programme is given below. Some insight is also given on the procedure involved in the mesh generation, discretisation and convergence monitoring used in the code. The boundary conditions applicable to the example given in this work are also presented.

LES simulations are used in this work only as an example of a way of inputting the source pressure fluctuations into the theoretical models. This work does not intend to focus in LES details and techniques.
5.1.1 Governing equations and Modelling Approach

The fluid flow can be represented by mathematical statements that govern the equations for the conservation laws of mass, momentum and energy [160], which here is done using by using time-dependent, three-dimensional incompressible Navier-Stokes equations.

In this code, the large eddies are resolved directly solving this equations, whilst small eddies are modelled using a turbulence model.

A turbulence model is a computational procedure that models the characteristics of turbulence. However the complexity of turbulence makes it unlikely that any single model will be able to represent all turbulent flows. For this particular application, it is unnecessary to resolve the details of the turbulent fluctuations and therefore the Smagorinsky Eddy Viscosity turbulence model [161].

5.1.2 Mesh Generation

In order to solve the governing equations numerically it is necessary to divide area under investigation into discrete points. Grid is the definition for the arrangement of these discrete points throughout the flow field. The task of grid generation is a significant operation in CFD as it is linked to the methods used in the CFD solver. This grid can also be divided into blocks to speed up the computational effort required to achieve a solution.

The main components in these calculations, consist of flow leaving a duct, which then is transformed into a jet. Predictions carried out on this geometry will give an indication of the impact of the pressure fluctuations that happen in the jet under the specified conditions for this case. For this particular case, the grid topology and the generated grids for this configuration is discussed below.

The calculations are set up using the dimensions and blocks as shown in Figure 80. This representation has the intension to represent as much as possible the conditions existent in the experimental measurements. Blocks One and Five consists of the lateral areas around the duct. These areas exist in the experimental facility and are also included in
the calculation as the jet tends to "use" the air under these areas in part of its development. This configuration also avoids the possibility of including boundary conditions near the duct exit, which could affect the flow. Blocks Two and Four consist of the lateral area around the jet. These areas are included as part of the jet flow will grow into those directions. Block 3 represents the area where the jet core will be located and will be flowing forward. The number of elements in each block is given below in the axial, radial and angular direction respectively.

Block One: \((32 \times 16 \times 32)\)
Block Two: \((64 \times 16 \times 32)\)
Block Three: \((64 \times 8 \times 32)\)
Block Four: \((64 \times 32 \times 32)\)
Block Five: \((18 \times 32 \times 32)\)

The detailed view of the grid is given on Figure 81. The grid is more refined near the centreline and near the rigid wall boundary conditions to avoid numerical errors and issues related to eddy sizes respectively.

This is grid a coarse grid, that might not represent every detail in the flow, but which is hopefully fine enough to pick up the large-scale pressure fluctuations occurring in the flow.
Figure 8.1 View of LES grid
Discretisation of the equations and Convergence

The program uses the finite volume method [162] to discretize the Navier-Stokes equations [149], which requires appropriate differencing of the convective and diffusive fluxes at each face of the control volume.

Time discretization uses the explicit scheme of Adams-Bashforth [163] and spatial discretization of the filtered Navier-Stokes equations is done using central differencing scheme [163].

In terms of convergence, the flow field is allowed to develop in time until it reaches a steady level of energy, where statistical quantities such as mean velocities and Reynolds stresses are accumulated.

For the turbulent model convergence an iterative process is required until the maximum normalised residual (over all the nodes) for the velocity and turbulence variables to be less than 1.0x10⁻⁶.

Boundary Conditions

The four types of boundary conditions used to determine all the necessary conditions to solve the algebraic equations are presented below. Accurate prediction of any flow depends on the quality of the boundary conditions specified by the user.

Inflow: In the jet flow predictions it is important to specify accurate boundary conditions at the inlet of the computational domain (i.e. end of the duct), where the boundary conditions must be as close as possible to the flow conditions that actually exist in the experiments that are being simulated. At the inflow plane, the values of the three velocity components are prescribed. In this example, this is achieved by specifying a mean flow and some perturbation at the beginning of the duct. The calculations for the duct are then kept running until the flow reaches the desired turbulent state at the end of the duct. The data acquires at the end of the duct is then used as the input for the jet calculations.
Outflow: It is important to note that the location of the outflow plane must be selected far away from geometrical disturbances, which sometimes cause recirculation in the vicinity of the boundary, such that part of the boundary has incoming flow. In this calculation, convective boundary conditions are used as the outflow boundary conditions with convection velocity based on the mean value.

Wall: Rigid wall conditions are applied for the duct inlet plane and duct walls. This boundary condition requires that all velocity components at the wall are zero both in the direction normal and parallel to the wall surface. This condition requires greater number of grid to be considered as the mean wall eddies decay forming smaller sub layers.

Centre-line: In the centre-line of the grid, a symmetry boundary is used here to reduce the size of the computational domain. This implies that the velocity vector components normal to the central symmetry plane must be set to zero and the gradient in the direction normal to it of the other components is also zero.

Constant velocity: A fixed velocity value of $v_e = 0.075 m/s$ is used in the external lateral surface of the grid. This value is determined from experimental entrainment rates for a free pipe.

In this section an overview of the LES technique together with specific aspects of flow predictions applicable to the current investigation (in terms of grid generation and inflow boundary condition specification) are discussed. The LES results and introductions of these results into the models are given on Sections 5.2 and 5.3.

5.2 LES calculation Parameters and results

To run the LES calculations for a free jet, several parameters have to be set up in the LU-LES computer program. The grid details are presented in Section 5.1 and the other relevant initial parameters are given in Table 2:
The choice for duct inlet flow speed and Reynolds number are made to keep the same conditions as the experimental calculations. The time step ($\Delta t$) is chosen so that the required precision needed for the aerodynamic variables is reached. The choice of time step and the number of steps saved take into consideration the fact that the aerodynamic pressure fluctuation output spectra has to have enough points to guarantee good results in the frequency range of relevance for the acoustics analysis/comparisons (avoiding therefore aliasing). The total time used was a compromise between leaving the calculation running for a long enough period (so that the flow has achieved its convergence) and minimisation of computational time involved in the calculation. In general, data obtained from the calculations are saved once the energy levels go through a few cycles after reaching its top level to prove that they are stable. Here, due to time constraints and computational availability, the data has been collected as soon as the energy levels are in its first cycle at the top level of energy.

Figure 84 shows the longitudinal snapshot of axial velocity. From that, one can see in red the formation of the jet core (with constant speed profile), the shear layer (where localised vorticity is being formed) and the mixing layer (where the jet expands laterally and the speed is reduced). This shows that a clear jet formation is being generated from the calculations. Figure 85 gives an example of a snapshot for the pressure around the jet. This figure clearly shows large regions of pressure variation at the first stages of the jet, that if they were to be presented as a movie would show the pressure fluctuations from one time step to the other. As this happens all around the cylinder and in more then one downstream location, this figure also gives an indication that the overall concept chosen for the theoretical modelling is in line with the findings.
Figure 84 Snap shot of LES calculations - axial velocity (m/s)

Figure 85 Snap shot of LES calculations - pressure
Figure 86 Spectra calculated from the LES calculations [164] at 3 positions aligned with the duct radius \( r = 0.15 \text{m} \) at three positions downstream from the duct exit. (red: at the duct exit, blue: 5 mm downstream, green: 50 mm downstream)

Figure 86 shows the sound pressure levels at a position near the duct exit. This results are a courtesy from Manners [164], using a data sample from the LES calculations presented in this work, and Lighthill analogy [68] for the calculation of jet noise. This is presented here to show the roll off at high frequencies follows the \(-5/3\) power law which is reasonable decay for turbulence as presented in Section 3.6.1. At higher frequencies above 2kHz there is a lot of aliasing noise, which is to be expected given the use of low order finite differencing in the calculations (as the interest is in the lower frequencies). This figure also shows that the sound pressure levels are very low. Although these levels could increase by up to 12-15 dB if higher levels of turbulence are used in the inlet this level is still lower than expected. The overall shape at low frequencies shows low Mach number jet noise characteristics. The peak level is at a higher frequency then expected (should be 30Hz considering \( \text{St} = 0.2 \) as the average for this speed of jet flow). However, the time history is not long enough to give best accuracy at the low frequency end. Despite that, a decay of around 12dB per decade is also present. At frequencies below the peak the levels drop, which is what would be observed in the experiments if there wasn’t room interference at such low frequencies.
5.3 Calculation of discrete forces using LES input

The interface between the LES results and the acoustic models (Chapter 2 & 3) is presented here. The LES results using the LU-LES code [149] outputs the pressure time history for each point in the 3D grid (radial, circumferential and longitudinal directions). A snapshot of the contours generated by these pressures is presented in Section 5.2. The interest in this section is to calculate forces based on the pressure difference between two regions. This force would then form the basic dipole force. The explanation on how this force is then applied into the theoretical models is presented in Sections 5.4 and 5.5.

To facilitate the explanation, the pressure difference regions are presented here as a unitary cell, with a pressure difference ($\Delta p$) between two opposite faces of elementary area $\Delta S$, as presented in Figure 87.

![Figure 87 Schematic visualization of a dipole force F, generated from LES pressure difference.](image)
The elementary force is therefore given by:

\[ p_{\text{top}} = \frac{p(n_{\phi}, n_z, r_{\text{top}}) + p(n_{\phi+1}, n_z, r_{\text{top}}) + p(n_{\phi}, n_{z+1}, r_{\text{top}}) + p(n_{\phi+1}, n_{z+1}, r_{\text{top}})}{4} \]

\[ p_{\text{bottom}} = \frac{p(n_{\phi}, n_z, r_{\text{bottom}}) + p(n_{\phi+1}, n_z, r_{\text{bottom}}) + p(n_{\phi}, n_{z+1}, r_{\text{bottom}}) + p(n_{\phi+1}, n_{z+1}, r_{\text{bottom}})}{4} \]

\[ dS_{\text{top}} = (z(n_z + 1) - z(n_z)) * r_{\text{top}} * (\phi(n_{\phi+1}) - \phi(n_{\phi})) \]

\[ dS_{\text{bottom}} = (z(n_z + 1) - z(n_z)) * r_{\text{bottom}} * (\phi(n_{\phi+1}) - \phi(n_{\phi})) \]

and \( n \) is the \( n \)-th element in the specified coordinate direction.

Although the procedure for force calculation presented here shows the development for a single cell, the same can be applied for multiple cells (representing concentrated pressure regions) in the fluid domain. This then account for the possibility of dipoles of bigger size to be considered.

### 5.3.1 Input of discrete forces into the Continuous Ring Model

The procedure chosen for introducing discrete sources into the Continuous Ring is presented here. This assumes that dipole forces (as calculated in the previous section) are facing the radial direction and occur between two concentric layers of the cylinder.

To introduce these forces into the CR model, the acoustic pressure given by Equation 16 is used, since the LES results are in cylindrical coordinates:

\[ p(r, \phi, z) = \frac{1}{2\pi} \sum_{n} B_n \cos(n\phi) \int_{-\infty}^{\infty} \frac{\tilde{f}(\gamma)H_n^0\left((k^2 - \gamma^2)^{\frac{1}{2}}\right)r e^{i\gamma}}{(k^2 - \gamma^2)^{\frac{1}{2}}H_n^0\left((k^2 - \gamma^2)^{\frac{1}{2}}\right)d\gamma} \]

However here to preserve the spatial phase in \( \phi \) and to consider the fact that \( \phi \) and \( z \) are inter-related, the acoustic pressure becomes:
In this equation the term $B_{n,y}(\gamma)$ represents the continuous force distribution in the $z$ direction for a given $\phi$. Considering that in the LES calculations the space is discrete and therefore is not represented by a continuous defined function, the force distribution here will be considered as:

$$F(\omega, y, k_{\phi}) = B_{n,y}(\gamma)$$

The discretized pressure is given by:

$$p(\omega, \phi, z) = \sum_{k_y=0}^{N_y-1} \sum_{r=0}^{N_z-1} F(\omega, k_{y}, \gamma) e^{-2i\pi \gamma_{y}} e^{-2i\pi \phi_{z}}$$

for $\gamma_{y} = 0, 1, 2, \ldots, N_{y} - 1$

$$\phi_{z} = 0, 1, 2, \ldots, N_{z} - 1$$

The force data originated from the LES calculations is in a non-transformed form ($F(t, \phi, z)$). In order to transform this force to fit the required format for the acoustic pressure equation ($F(\omega, k_{\phi}, y)$), the following steps are applied:

**Step One:** Choose the region to be analysed. The regions considered here are:

- all points in the $\phi$ direction
- all points in the $z$ direction from the duct exit to 0.6m downstream (as from the experimental investigation it was found that the flow characteristics happening further downstream are not so relevant for the low frequency noise generation)
- the concentric $r$ layers chosen are $n_r = -1$ and $n_r = 1$ (just below and above the duct lip respectively – this is chosen as the majority of the instabilities originated form the shear layer happen in that region)

**Step Two:** Calculate the Fourier discrete transform in the $\phi$ direction (Fourier transform is chosen to be used here, instead of the single coefficient Fourier Series
presented with the previous results in order to preserve the phase information at each grid location):

\[ F(t, z, k_\phi) = \frac{1}{N_{\phi}} \sum_{n_z = 0}^{N_z - 1} F(t, z, n_\phi) e^{\frac{-2i\pi n_z k_\phi}{N_z}}, \text{ for } k_\phi = 1, 2, ..., N_{k_\phi} - 1 \]

**Step Three:** Calculate the Fourier transform in the z direction:

\[ F(t, \gamma, k_\phi) = \frac{1}{N_{\gamma}} \sum_{n_z = 0}^{N_z - 1} F(t, z, n_\phi) e^{\frac{-2i\pi n_z \gamma}{N_z}}, \text{ for } \gamma = 1, 2, ..., N_{\gamma} - 1 \]  \hfill (24)

**Step Four:** Transform the data from the time domain to the frequency domain:

\[ F(\omega, \gamma, k_\phi) = \sum_{n_\gamma = 0}^{N_\gamma - 1} F(t, \gamma, k_\phi) e^{\frac{-2i\pi n_\gamma \omega}{N_\gamma}}, \text{ for } f = 1, 2, ..., N_{\omega} - 1 \]  \hfill (25)

Step Five: input \( F(\omega, \gamma, k_\phi) \) into the Continuous Ring Model.

5.3.1 LES input into Discrete Dipole Ring

The procedure of interfacing the LES results as input into the Discrete Dipole Ring is presented here. This assumes that dipole forces are calculated as presented at the beginning of Section 5.3. The force data originated from the LES calculations is in a non-transformed form (\( F(t, \phi, z) \)). In order to transform this force to fit the required format for the acoustic pressure equation (\( F(\omega, \phi, z) \)), the following steps are applied:

**Step One:** Chose the region to be analysed. The regions considered here is:

- all points in the \( \phi \) direction (a dipole is placed at each \( \phi \))
- the position \( z = 0.15m \) downstream from the duct exit is chosen to place the ring, as it is one of the regions presenting high amount of fluctuation when hot-wire measurements were taken in the experimental facility.
- the concentric \( r \) layers chosen are \( n_r = -1 \) and \( n_r = 1 \) (just below and above the duct lip respectively – this is chosen as the majority of the instabilities originated form the shear layer happen in that region)

**Step Two:** The force data for each dipole at each time step is discrete Fourier transformed into the frequency domain.

**Step Three:** The Transformed data for each dipole is input into the Discrete Dipole Ring model, by introducing each elementary force on equation (18)

### 5.4 Results of LES input into the models

#### 5.4.1 Results with Continuous Ring

Figure 88 shows the sound power levels for the case when LES force data is introduced as the source term into the Continuous Ring model. This result shows that a similar decay in frequency to the experimental results is presented. This result emphasised once more that dipole forces present in a jet could generate the decay shown in the spectrum.

However, a significant difference in the level is observed. Although this difference are not covered in this work, they could be due to several reasons (together or in isolation), such as:

- the possibility of a missing factor in the CR model: to accommodate the LES calculations, the Continuous Ring model consists of a cylinder “covered” with dipoles at the same radius and \( z \) positions. In reality these source will not be necessarily happening in this manner. As presented on Chapter 3 (and recreated in Figure 88), if for instance the profiles in the \( z \) direction differ, the radiation efficient will also be affected. Despite that, the difference observed for the profiles chosen were not as significant as the ones presented in the CR/LES case.
- number of cells used in LES grid: the grid used in the LES calculations is a coarse grid, that although might not represent every detail in the flow, is created with the intention of looking at the large scale pressure fluctuations occurring in the flow.
However, this does not exclude the possibility of large areas of fluctuations being able to be created from smaller fluctuation areas; and if these smaller areas are missing, the pressure fluctuation effect will be compromised.

- the way that the cells are grouped together: for the LES example using CR, a group of cells in between two radius positions ($r = 0.14$ and $r = 0.16$ m) from the LES calculation was chosen. However it is clear from Figure 85 that this region does not cover all the fluctuation presented in that snapshot. This could be causing a reduction in levels as the dipole characteristics are only being partially represented.

- total time of LES calculations: as mentioned earlier, in order to reach stable energy levels, a long simulation time is required. In this simulations, due to time constraints and computational availability this calculations are run until the level of energy decays for the first time, suggesting that it has reached its stable levels. The length of the time history also affects directly the low frequency content of the spectrum.

level of turbulence at the duct entrance: higher levels of turbulence at the entrance could increase the levels up to 15 dB. This would still not be enough to reach the levels obtained in the measurements.

![Figure 88](image)

**Figure 88** $\ell_{eq}$ – LES input into CR

### 5.4.2 Results with Discrete Dipole Ring

When the LES results are introduced into the DDR model, the results show, that the decay rate is the same (12dB/decade) for both cases, on the other hand the levels, however, are much lower than the experimental results.
This difference in level could be happening due to several causes, like:

- the possibility of a missing factor in the DDR model: to accommodate the LES calculations, the Continuous Ring model consists of a ring with a certain number of dipoles oriented in the same manner, at the same radius for one z position. This configuration might not be adequate in the reality. For instance, if dipoles are slightly angled in the LES calculations, part of the contribution from each dipole would not be accounted for. The same could happen if the concentric layer chosen are not capturing the overall pressure fluctuations.
- LES issues as presented in the previous section.

5.5 Summary

This Chapter presented an example of using LES calculations as the source term for the two theoretical models presented in Chapter 2. For that the pressure fluctuations obtained from the LES calculations are used to calculate dipole force terms that are then input into the analytical acoustic models.

The results reinforce the conclusion that at low speeds dipoles are the major source contributors for the $L_w$ spectra, as the introduction of LES forces (acting as dipoles) into the models generates a similar decay in the spectrum.
These results show that the models presented do not represent in totality the physical behaviour encountered in the experiments as the $L_w$ generated by the models are much lower than the experimental levels. This could be due to constraints regarding the ways the models have been designed and to issues regarding the LES data.

Overall, the use of LES calculations into these models demonstrates that to cover all the possible combinations of LES levels of data input and variations in the models in trying to achieve the sound power levels comparable with the experimental results would be an immense task (if at all possible). This led to the conclusion that the source present at the experimental investigation, cannot be fully represented by a ring of dipoles or by a truncated cylinder of dipoles.
Chapter 6  Concluding Remarks

The work presented in this thesis is concerned on the low frequency noise generated aerodynamically in industrial gas turbines.

For that, the overall question of: How to determine the noise sources responsible for the majority of the low frequency noise? is addressed throughout this thesis and is one of its hypothesis.

To find out the possible sources of low frequency noise, an experimental model of the industrial gas turbine is built. To investigate this hypothesis, a series of experiments is carried out. These experiments explored the major aerodynamic effect in isolation. From that, several aerodynamic effects are excluded from being the generator of noise and others are analysed more closely given their potential significant contribution in the low frequency noise range. Therefore, one of the conclusions from this thesis is that: the major low frequency noise sources in an industrial gas turbine are isolated. The major contributor for the low frequency noise being the jet flow up to one diameter away from the duct exit.

The isolation of sources rises other questions, that became another hypothesis of this work. This hypothesis is that the main low frequency source contributor is a dipole-type source. By critically examining the above idea in this thesis, it is found that the acoustic sources more likely to be generating low frequency noise are monopoles or dipoles given the fact that they radiate more energy at low frequency in comparison with higher order sources.

The question of what is the major noise source in the industrial gas turbine exhaust system: monopoles or dipoles is then confirmed by two independent processes. One showed that the major aerodynamic source generating noise happens at the exit of an open pipe system. This therefore confirmed that this source is an acoustic pressure source, as only pressure source would excite an open pipe when situated at the exit of a pipe. As monopoles are volume sources, they were then excluded. However, this experiment had not excluded the possibility of the existence of higher order pressure sources to such as quadrupoles. The second experiment conducted speed variation tests
for several setups of the test rig. They showed that the variation in power when the
speed is increased is proportional to $V^6$, which leads to the conclusion that the main
source present in the system is a dipole-type source.

From the acoustic experiments taken and some aerodynamic insight it is also possible
to narrow down the regions where the aerodynamic sources generating noise could be
based: all around the duct from the end of the duct to one diameter downstream from it.
This originated the secondary hypothesis, which states that: the source can be
represented as a ring of dipoles or as a truncated cylinder of dipoles.

To verify this hypothesis, two models were created and intermediate results showed
that both models present similar the trend behaviour at low Helmholtz numbers when
LES force sources are introduced into the models.

This thesis also showed a case where the transient pressures from LES calculations
were used as input for the Continuous Ring and the Discrete Dipole Ring model. The
results show that the characteristic decay is still present, reinforcing the fact that dipole
forces can generate low frequency noise in jets. On the other hand, the sound power
levels obtained are much lower than the experimental result, indicating that there is a
factor/element missing in the models. This could be due to restrictions in the model
representation of the physical phenomena or limitations on number of ways for
introducing the LES data into the models. Overall, the use of LES calculations into
these models demonstrates that to cover all the possible combinations of LES data
input and variations in the models in trying to achieve sound power levels comparable
with the experimental results would be an immense task, which would defeat the
intension of having a simplified model.

From this results, the conclusion that the source can be represented as a ring of
dipoles or as a truncated cylinder of dipoles can be drawn.
References


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Appendix I  Aero Acoustic Volute Test Facility

This facility has been especially designed and constructed to examine and identify the causes of aerodynamic instability and hence possible noise sources. Tests can be carried out on combinations of typical annular exhaust diffusers dumping into a typical exhaust volute box in an attempt to improve both loss and noise performance of these systems.

Test Facility Schematic

As shown in Figure A. 1, atmospheric air passes through an inlet duct into an acoustically lined centrifugal fan room. The air is subsequently drawn into the fan and pumped along an interconnecting duct to a plenum, which is also acoustically lined and contains a series of acoustic baffles. Within this plenum the air has a velocity of approximately 0.3 m/s (Mach number ~ 0.001) ensuring aerodynamic noise generation is virtually eliminated. In addition the acoustic cladding absorbs the majority of the noise generated by the fan and motor, resulting in sound power measurements at inlet to the working section which are essentially at background levels. Air enters the 1/7th scale working section via an inlet flare prior to passing through inlet and outlet guide vanes (IGV’s & OGV’s) and a diffuser as shown in Figure A. 2. Flow exiting the diffuser is dumped into an acoustic volute box before being expelled to atmosphere via an
exhaust duct, which is also acoustically lined. Figure A.3 and Figure A.4 present photos with more details of the test section.

Figure A.2 Experimental facility test section (scale model)

Figure A.3 Photo experimental facility test section (scale model)
Figure A. 4 Zoom inside the volute

Control system

The fan is driven by a 37 kW motor and is capable of delivering 4 m$^3$/s of air at 600 mm H$_2$O above ambient pressure. Control of the air supply is governed by a Fenner Speedranger variable AC drive unit. Closed loop feedback is provided by a shaft encoder which enables the speed of the fan to be maintained to within 0.1 r.p.m. A PC is used in conjunction with a 16-bit data acquisition system to monitor the pressure and temperature at inlet to the facility, allowing accurate maintenance of the inlet Mach number.

Operating Conditions

The IGV geometry can be altered in order to simulate the changes produced in the radial swirl angle distribution presented to the OGV’s, at 30%, 70% and 100% engine operating powers. The Reynolds number, based on diffuser inlet annulus height, is approximately 1.3 x 10$^5$. The OGV’s are positioned immediately upstream of diffuser inlet, and to 10 OGV blade chords downstream of the IGV’s. It is possible to operate
the facility with or without the volute in order to evaluate the acoustics of the IGV, OGV and diffuser combination in isolation.

**OGV's**

![OGV's](image)

**Figure A. 5 IGV's and OGV's in a setup without the volute**

**Instrumentation**

The need to obtain complete aerodynamic area surveys of the flow at various planes within the diffuser requires instrumentation (pitot tubes, 5-hole pressure probes and hot wires) to be traversed in both radial and circumferential directions. Radial movement is achieved by utilising a tailor-made miniature traverse mechanism housed within a rotating section of the centre spool. The axial position can be altered by positioning blank spool casings between the traverse section and OGV exit. The traverse itself consists of a precision linear guide attached to a pulley wheel on a fixed lead screw. The driving force is provided by a P.C. controlled stepper motor which is connected to the pulley by a miniature toothed belt. The positional accuracy of the assembly is ± 0.025mm. To facilitate circumferential traversing the centre spool is divided into two sections. The upstream section is mounted rigidly to the outer casing with loads being carried through the guide vanes (fig. 5.3). The rotating centre spool, located downstream of the OGV's, is supported by two bearings positioned within the upstream section. The circumferential drive is provided by a P.C. controlled stepper motor also located within the upstream section, connected to the rotating spool through a gearbox. The positional accuracy of the assembly is ± 0.01°. Static tappings are located throughout the diffuser at 90° intervals from TDC. Hot wire measurements can be made at numerous locations within the exhaust volute. Acoustic measurements can
be made at numerous locations within the volute for information in the near field, or probes can be placed around the laboratory in order to obtain data in the far field. Sound pressure data are taken using Brüel & Kjær microphones and are subsequently processed using a Hewlett-Packard analyser. 1/8" microphones (protected with a cone) and 1/2" microphones (protected with a foam windscreen) are used for measurements in the near and far field respectively.
Appendix II Continuous Ring Model: Separated contribution of \( n=1 \) (dipole) mode

Figure A 6 Gaussian function with different variances

Figure A 7 Gaussian functions with different peak positions
Figure A 8 SWL for different cylinder diameters using a Gaussian function

Figure A 9 Triangular function with different cylinder lengths
Figure A 10 Triangular function with different peak positions

Figure A 11 SWL for different cylinder diameters using a triangular function
Figure A 12: Dirac Delta function with different positions of $z_0$

Figure A 13: SWL for different cylinder diameters using a Dirac Delta function
Appendix III Experimental Results for 70% and 30% blades

Chapter 4 presented the measurements taken with 100% IGV blades. This Appendix presents the equivalent measurements for the 70% and 30% IGV blades.

Equivalent comparisons for the 70% IGV blades:

Figure A 14

Figure A 15
Figure A 16

Figure A 17
Figure A 18

Figure A 19
Equivalent comparisons for the 30% IGV blade:

Figure A 20

Figure A 21
Figure A 22

Figure A 23
Figure A 24

Figure A 25
Figure A 26

Figure A 27