Novel forms of inverse analysis to characterise properties of fibre-matrix composites

This item was submitted to Loughborough University’s Institutional Repository by the/an author.

Additional Information:

• A Doctoral Thesis. Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University.

Metadata Record: https://dspace.lboro.ac.uk/2134/14198

Publisher: © Paul J. Sherratt

Please cite the published version.
This item was submitted to Loughborough University as a PhD thesis by the author and is made available in the Institutional Repository (https://dspace.lboro.ac.uk/) under the following Creative Commons Licence conditions.

**Attribution-NonCommercial-NoDerivs 2.5**

You are free:

- to copy, distribute, display, and perform the work

Under the following conditions:

**Attribution.** You must attribute the work in the manner specified by the author or licensor.

**Noncommercial.** You may not use this work for commercial purposes.

**No Derivative Works.** You may not alter, transform, or build upon this work.

- For any reuse or distribution, you must make clear to others the license terms of this work.
- Any of these conditions can be waived if you get permission from the copyright holder.

Your fair use and other rights are in no way affected by the above.

This is a human-readable summary of the Legal Code (the full license).

Disclaimer:

For the full text of this licence, please go to:
http://creativecommons.org/licenses/by-nc-nd/2.5/
Please note that fines are charged on ALL overdue items.

For Reference Only

0402697243
NOVEL FORMS OF INVERSE ANALYSIS TO CHARACTERISE PROPERTIES OF FIBRE-MATRIX COMPOSITES

By

Paul Jonathan Sherratt. B.Eng.(Hons)

A Doctoral Thesis Submitted in Partial Fulfilment of the Requirements for the Award of Doctor of Philosophy of Loughborough University.

March, 2002.

ABSTRACT

Novel approaches to the determination of material properties and damage parameters of fibre-matrix composites using inverse analysis are presented. In inverse analysis system identification techniques are used to update some form of mathematical model (normally a FE model) using data from an over-determined number of tests.

Initially, pultruded GFRP box-section beams are subjected to a quasi-static impact or bending crush. The results of the impact tests are presented to corroborate those in the literature that have been obtained using simpler geometries such as flat plates.

In the first form of inverse analysis, a model-updating approach is applied to progressive tearing damage in pultruded composite box-section beams. The difference between empirical data (from a programme of three-point bend tests) and a FE model is minimised by a genetic algorithm to produce an optimal solution. The solution is in the form of a FE model that can be subsequently analysed to determine the structural integrity of the damaged specimen.

Secondly, a unidirectional composite disc from the same GFRP pultruded section is analysed in diametral compression to both verify and improve the validity of the diametral compression test in determining the material properties. Coupons are cut from damaged specimens and test results are presented. The strain distribution within the disc is compared to known laminate theory in order to process data obtained by speckle-shearing interferometry.

Finally, speckle-shearing interferometry is used to characterise the response of the pultruded box-section exhibiting progressive tearing damage. Out-of-plane displacement-gradient data is used to determine and characterise damaged regions or flaws.

The differences between the need to perform a programme of unequivocal static tests and the collection of full-field optical data are highlighted. It is shown that the shearing interferometry approach is the superior method.

KEYWORDS: Composite, Inverse Analysis, Speckle Shearing Interferometry, Material Characterisation.
ACKNOWLEDGEMENTS

I wish to thank my supervisor Dr. Andrew Nurse, for his direction and enthusiasm without which this thesis would not have been possible.

Thanks to my colleague David Panni for his collaboration on the Lightweight Booms Project and for the exchange of ideas throughout the course of our research.

My thanks also to the members of the Structural Integrity Research Group within the Wolfson School of Mechanical and Manufacturing Engineering at Loughborough University. Most notably Mr. Dave Britton for his technical assistance and unique sense of humour. Thanks to undergraduate student Mohamad Rizal Awang for his assistance with the diametral compression tests.

I am also grateful to Prof. Jon Huntley, Dr. Jon Petzing and Mr. Jamal Sheikh-Ibrahim for sharing their considerable knowledge and expertise of optical engineering and image processing.

The research was funded by J.C. Bamford Excavators Ltd. and the Engineering and Physical Sciences Research Council (EPSRC).

Special thanks to my Mother, for her constant encouragement to further my education and the sacrifices she made to support me when I did.

Thanks to Donna for her patience and encouragement, especially whilst writing-up.

Dedicated to the memory of my grandfather, Alfred John Chambers-Booth.

"The length of this document defends it well against the risk of its being read."

- Sir Winston Churchill
NOTATION

\( \beta \)  
Angle between major fibre axis and loading direction

\( P \)  
Applied Load

\( I_0 \)  
Background Intensity

\( A \)  
Boom Angle

\{P\}  
Causes (direct or inverse problem)

\( S \)  
Compliance tensor (or compliance matrix)

\( v \)  
Deflection

\( a \)  
Direction cosine

\( T \)  
Direction cosine matrix

\( \gamma \)  
Engineering shear strain

\( I_o \)  
Intensity of object wavefront

\( I \)  
Intensity or Second Moment of Area

\( \eta \)  
Interaction ratio (also parameter used in rule of mixtures)

\( P_{i,j} \)  
Matrix used in phase unwrapping

\( I_M \)  
Modulation Intensity

\( \xi \)  
Parameter used in rule of mixtures

\( \phi \)  
Phase

\( \Delta \phi \)  
Phase change

\( \Delta i,j \)  
Phase Difference in the x-direction at location (i,j)

\( \alpha \)  
Phase Step

\( \nu \)  
Poisson’s ratio

\{U\}  
Response (direct or inverse problem)

\( \omega \)  
Rotation

\( \delta x, \delta y \)  
Shear in Michelson interferometer (x, horizontal and y vertical)

\( G \)  
Shear modulus

\( \tau \)  
Shear stress

\( L \)  
Span or Length

\( k \)  
Stiffness (load/displacement).

\( C \)  
Stiffness tensor (or stiffness matrix)

\( \varepsilon \)  
Strain
\( \sigma \) Stress

\([Z]\) System (direct or inverse problem)

\( \theta \) Tilt angle of mirror in Michelson interferometer

\( \tilde{S} \) Transformed compliance tensor (or matrix)

\( \tilde{C} \) Transformed stiffness tensor (or matrix)

\( f \) Volume Fraction of Fibres

\( \lambda \) Wavelength

\( \phi_w \) or \( \psi \) Wrapped Phase

\( E_f \) Young's Modulus of Fibres

\( E_m \) Young's Modulus of Matrix

\( E \) Young's modulus or elastic modulus
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSI</td>
<td>British Standards Institute</td>
</tr>
<tr>
<td>BVID</td>
<td>Barely Visible Impact Damage</td>
</tr>
<tr>
<td>CAI</td>
<td>Compression After Impact</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge-Coupled Device</td>
</tr>
<tr>
<td>CFRP</td>
<td>Carbon Fibre Reinforced Plastic</td>
</tr>
<tr>
<td>DCB</td>
<td>Double Cantilever Beam</td>
</tr>
<tr>
<td>DCT</td>
<td>Discrete Cosine Transform</td>
</tr>
<tr>
<td>ENF</td>
<td>End-Notch Flexure</td>
</tr>
<tr>
<td>ESPI</td>
<td>Electronic Speckle Pattern Interferometry</td>
</tr>
<tr>
<td>FCT</td>
<td>Fourier Cosine Transform</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FMC</td>
<td>Fibre-Matrix Composite</td>
</tr>
<tr>
<td>FRP</td>
<td>Fibre-Reinforced Plastic</td>
</tr>
<tr>
<td>GFRP</td>
<td>Glass Fibre Reinforced Plastic</td>
</tr>
<tr>
<td>IKE</td>
<td>Incident Kinetic Energy</td>
</tr>
<tr>
<td>JCB</td>
<td>J. C. Bamford Excavators Ltd.</td>
</tr>
<tr>
<td>MRI</td>
<td>Magnetic Resonance Imaging</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>NDE</td>
<td>Non-Destructive Evaluation</td>
</tr>
<tr>
<td>PC</td>
<td>Personal Computer</td>
</tr>
<tr>
<td>PEEK</td>
<td>Poly Ether Ether Ketone</td>
</tr>
<tr>
<td>PZT</td>
<td>Piezo-Electric Transducer</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning Electron Microscopy</td>
</tr>
<tr>
<td>SHPB</td>
<td>Split Hopkinson Pressure Bar</td>
</tr>
<tr>
<td>SSI</td>
<td>Speckle Shearing Interferometry</td>
</tr>
<tr>
<td>TPB or 3PB</td>
<td>Three-Point Bend</td>
</tr>
<tr>
<td>TPBI</td>
<td>Three-Point Bend Instrument</td>
</tr>
<tr>
<td>UD</td>
<td>Uni-Directional</td>
</tr>
</tbody>
</table>
# CONTENTS

Abstract I  
Certificate of Originality II  
Acknowledgements III  
Notation and Abbreviations IV  

1 INTRODUCTION ........................................................................................................ 5  
1.1 PUBLICATIONS........................................................................................................ 8  

2 LITERATURE REVIEW .......................................................................................... 10  
2.1 INTRODUCTION ...................................................................................................... 10  
2.2 LAMINATE AND PLATE THEORY........................................................................... 12  
2.3 IMPACT DAMAGE OF FIBRE-MATRIX COMPOSITES ........................................ 16  
  2.3.1 Characterisation of Impact .............................................................................. 18  
  2.3.2 Impact Test Techniques .................................................................................... 19  
  2.3.3 Failure Modes ................................................................................................... 20  
  2.3.4 Failure Mechanism Due to Low or Quasi-Static Impact ............................... 21  
  2.3.5 Effect of Matrix and Fibre Properties on Impact Performance .................... 22  
  2.3.6 Effect of Fibres on Impact Performance ....................................................... 23  
  2.3.7 Effect of the Matrix on Impact Performance ................................................ 24  
  2.3.8 Effect of Fibre-Matrix Bonding on Impact Performance .............................. 24  
2.4 LOW VELOCITY IMPACT RESPONSE OF FIBRE-MATRIX LAMINATES ............. 25  
2.5 DETERMINATION OF MATERIAL PROPERTIES OF FIBRE-MATRIX COMPOSITES ............................................................................................................ 31  
  2.5.1 Manufactured Coupon Tests ............................................................................ 31  
  2.5.2 Removed Coupon Tests .................................................................................... 36  
2.6 NON-DESTRUCTIVE EVALUATION OF COMPOSITE COMPONENTS ............. 39  
  2.6.1 Speckle Interferometry Methods .................................................................... 41  
2.7 BEHAVIOUR OF PULTRUDED SECTIONS ............................................................ 45  

3 BACKGROUND THEORY ....................................................................................... 48  
3.1 INTRODUCTION ................................................................................................... 48
3.2 ELASTIC PROPERTIES OF FIBRE-MATRIX COMPOSITES ........................................ 49
  3.2.1 Elastic Behaviour of Unidirectional Laminae (Plies) ........................................ 49
3.3 INVERSE ANALYSIS ......................................................................................... 59
3.4 SPECKLE SHEARING INTERFEROMETRY ..................................................... 63
  3.4.1 Phase Stepping .......................................................................................... 67
  3.4.2 Determination of the Phase from Phase-stepped Intensity Maps .................. 70
  3.4.3 Two-Dimensional Phase Unwrapping ...................................................... 72

4 DESIGN OF A LIGHTWEIGHT TELEHANDLER BOOM ................................ 77
  4.1 INTRODUCTION ......................................................................................... 77
  4.2 DESIGN CONSIDERATIONS ......................................................................... 80
  4.3 LIGHTWEIGHT MATERIALS .......................................................................... 82
  4.4 IMPACT BEHAVIOUR OF COMPOSITE BEAMS ....................................... 85
    4.4.1 Experimental Procedure ....................................................................... 86
    4.4.2 Results of Investigation ....................................................................... 87
  4.5 LOAD CHART PREDICTIVE TOOL ................................................................ 94
    4.5.1 Defining Component Positions using Vector Algebra ............................ 95
  4.6 PROTOTYPE DESIGN .................................................................................... 102
    4.6.1 Prototype Aims and Objectives ............................................................. 102
    4.6.2 Modelling and Analysis ....................................................................... 102
    4.6.3 Prototype Manufacture ......................................................................... 104
    4.6.4 Design Details ...................................................................................... 105
  4.7 CONCLUSION ............................................................................................... 110

5 INVERSE ANALYSIS OF A GFRP DISC IN DIAMETRAL COMPRESSION .......... 111
  5.1 INTRODUCTION .......................................................................................... 111
  5.2 DIAMETRAL COMPRESSION TEST TO CHARACTERISE DAMAGE IN GFRP
      BOX-SECTION .............................................................................................. 112
    5.2.1 Removal of Coupons ............................................................................ 113
    5.2.2 Diametral Compression Test Methodology .......................................... 115
    5.2.3 Finite Element Analysis ...................................................................... 117
    5.2.4 Results .................................................................................................. 118
    5.2.5 Conclusions .......................................................................................... 122
5.3 DISC ANALYSIS BY SPECKLE SHEARING INTERFEROMETRY ............... 124
5.3.1 Experimental Equipment and Procedure .......................... 124
5.3.2 Results and Discussion ........................................... 127
5.3.3 Analysis of Phase Maps ........................................... 129

5.4 DETERMINATION OF ORTHOTROPIC PROPERTIES .................... 131
5.4.1 Introduction .................................................. 131
5.4.2 Methodology .................................................. 134
5.4.3 Results ...................................................... 137
5.4.4 Conclusions ................................................... 139

6 INVERSE ANALYSIS OF DAMAGE IN BOX-SECTION COMPOSITES USING MULTIPLE LOADS ................................. 140
6.1 INTRODUCTION .................................................. 140
6.2 A GENETIC ALGORITHM TO RESOLVE PROGRESSIVE TEARING DAMAGE .......................... 141
6.3 DAMAGE IN PULTRUDED BOX-SECTIONED COMPOSITES ............. 143
6.4 THREE-POINT BEND INVESTIGATION .................................. 144
6.4.1 Theory ...................................................... 144
6.4.2 Three-Point Bend Test Arrangement .............................. 146
6.4.3 Test Programme ................................................ 147
6.5 THREE-POINT BEND DATA AND ANALYSIS ........................... 151
6.5.1 Three-Point Bend Data .......................................... 151
6.5.2 Results from the Genetic Algorithm ................................ 157
6.5.3 Secondary Results from the FE Model ............................. 160
6.5.4 Validation of the FE Model (GA Output) ........................... 160
6.6 DISCUSSION ...................................................... 162

7 INVERSE ANALYSIS OF BOX-SECTION COMPOSITES USING SPECKLE SHEARING INTERFEROMETRY: A FEASIBILITY STUDY 164
7.1 INTRODUCTION .................................................. 164
7.2 EXPERIMENTAL PROCEDURE .................................... 165
7.3 RESULTS FROM INITIAL SSI TESTS ................................. 166
7.3.1 Improvements to Experimental Procedure ......................... 168
7.4 THE FEASIBILITY OF USING SPECKLE SHEARING INTERFEROMETRY TO INSPECT STRUCTURAL COMPONENTS ............................................................... 169
  7.4.1 Test Programme .......................................................................... 170
  7.4.2 Proposed Algorithm .................................................................... 171
  7.4.3 Numerical Trials ......................................................................... 173
  7.4.4 Initial Results .............................................................................. 175
7.5 CONCLUSIONS ...................................................................................... 176

8 CONCLUSIONS ............................................................................................ 177
  8.1 THE DESIGN OF A LIGHTWEIGHT TELEHANDLER BOOM ...................... 177
  8.2 DETERMINATION OF ORTHOTROPIC PROPERTIES ................................. 178
  8.3 DAMAGE DETECTION .......................................................................... 179

9 RECOMMENDATIONS FOR FURTHER WORK ............................................. 180

REFERENCES .................................................................................................. 180

APPENDICES ................................................................................................... 192
Chapter 1: Introduction

1 INTRODUCTION

The word composite means "made up of different parts". When applied to materials this means on a macroscopic scale. Composite materials are thought of as relatively new yet have been around for centuries: wattle and daub was a popular construction material in 14th century Britain. The principle is simple; two materials are combined such that the properties of the whole are superior to those of the constituent parts. This work concerns one category of composite materials, fibre-matrix composites (also known as fibre-reinforced plastics). As the name suggests these materials consist of fibres bound in a matrix or resin and are the most common form of composite materials. For decades steel and other metals have been the materials of choice for the structural engineer. The expanding use of composites is indicative of the benefits offered by this genre of materials.

It is almost impossible to read anything concerning fibre-matrix composites without encountering some version of the phrases "high strength-to-weight" and "high stiffness-to-weight". These are the desirable properties of the material that cause such demand from industry. Composites differ from metals in that they are anisotropic, that is the orientation of the fibres can be designed to optimise the material properties of a component in a particular direction or location. This allows more efficient structures to be designed particularly in applications where weight is important.

Much of the research in the field is devoted to impact damage and with good reason. Fibre-matrix composites are susceptible to impact events that can produce significant damage. This damage can reduce the strength of the material by over 50% whilst still going undetected by the human eye. This phenomenon is termed 'Barely Visible Impact Damage' (BVID) and has been the subject of intensive research.

Composite materials were initially developed for aerospace applications. Early components were evaluated on weight, even at the expense of cost. As the technology becomes more accessible and the cost becomes more competitive,
composites are being utilised in more ‘lower technology’ applications and are increasingly found in primary structural applications. Manufacturing techniques such as pultrusion are allowing composites to become more economically viable compared with more traditional materials.

A potential market for structural composites is in the production of ‘yellow goods’, construction machines. The reduction of overall vehicle weight offers similar benefits to those reaped by the automotive industry, such as down rating of suspension and brakes. Additionally, a growth market for many construction equipment manufacturers is the telehandler sector. This is a high value market where the benefits of composite materials could deliver a competitive advantage.

J.C. Bamford Excavators Ltd. (JCB) produce a range of telehandlers for agricultural and construction markets. The heart of the machine is a telescopic boom that can be extended to place palletised loads at height. Currently the sections of the boom are made of steel and JCB were quick to identify the performance improvements that lightweight materials could offer. Thus they sponsored this research into developing alternatives to the steel boom using advanced materials. With this mandate the use of fibre-matrix composites was quickly identified as offering maximum benefits in terms of weight reduction.

This thesis contains the results of the JCB project plus the academic research undertaken to arrive at a suitable solution. Whenever new technology is introduced into industry there are many questions regarding the design, manufacture and maintenance of new components. In this specific case, that of a fibre-matrix boom, the major area of concern was damage to the component and the accompanying loss of strength. This would have an effect on the product life, the machine performance and could cause catastrophic failure. Since there was no previous knowledge regarding the operational performance of a composite boom the prototypes would require extensive testing and close monitoring. Thus the Lightweight Booms spawned research into related areas where there were gaps in existing knowledge.
Chapter 1: Introduction

An investigation into the impact behaviour of fibre-matrix box-section was conducted. Samples were subjected to transverse, quasi-static, three-point bend impact or bending crush to determine failure modes. Furthermore, some means of determining the elastic properties was required since this gave an indication of stiffness and a loss of stiffness indicated damage was present. The resulting research into material characterisation is presented. The technique can be applied to the JCB project or used in a more general context.

Chapter 2 presents a review of the existing knowledge appertaining to composite materials. Particular attention is given to impact performance, impact damage and common means of determining material properties. Chapter 3 expands on this by presenting lamina theory, an outline of inverse analysis and details of speckle-shearing interferometry.

The fourth chapter contains the relevant detail of the Lightweight Booms project commissioned by J.C. Bamford Excavators Ltd. The Lightweight Booms project was a major exercise in its own right but is limited to a single chapter for brevity. Chapter 5 shows how speckle-shearing interferometry can be used to obtain the empirical data for inverse analysis. In this case the technique is applied to the analysis of a circular disc coupon in diametral compression.

Chapter 6 presents a means of determining progressive tearing damage in composite box-section by inverse analysis. The difference between empirical data (gathered by a programme of three-point bend tests) and numerical data is minimised by a genetic algorithm.

Chapter 7 combines elements from chapters 5 and 6 to examine the feasibility of analysing damage in box-section composites using speckle-shearing interferometry.

Finally, conclusions are drawn and recommendations for further work are made in chapters 8 and 9 respectively.
1.1 Publications

The research undertaken during the course of this project has given rise to many publications, listed below.


2 LITERATURE REVIEW

2.1 Introduction

Composite materials were initially developed for use in the aerospace industry where strength-to-weight ratios were the primary concern. The majority of early research was funded by N.A.S.A. and the United States military. Composite structures were soon in use on a variety of safety critical applications, mainly on military aircraft. The technology filtered down into civil aircraft but the primary consideration remained the reduction of the specific weight of the structure. Associated research focussed on the use of laminated composites in thin-walled structural components. Material and manufacturing costs were a secondary consideration. Today the use of composite materials is expanding into more and more markets where the cost of the components is becoming increasingly important. Composites are now widely used in marine, automotive and civil engineering (Fibreforce 1999; Maunsell 1999). In these markets composite materials must often compete on a cost basis with traditional materials. An excellent example of this is an article from the Automotive Engineer (Mann 1998) that compares the merits of various materials aimed at reducing the weights of automobiles. These new markets have resulted in new avenues of research as the use of composite materials becomes more widespread and diverse.

Research into composites materials has been extensive and (over forty years) the volume of knowledge accumulated is immense. Much of the derived theories have been published in numerous books (Hull and Clyne 1996) and there have been numerous attempts to categorise this work (Bogdanovich and Sierakowski 1999).

One of the earliest identified weaknesses of composite materials was their post-impact residual strength. This was a major concern as a loss in strength resulting from impact could have a catastrophic effect on any load bearing structure. The occurrence of impacts in aircraft is highly probable and can occur relatively frequently. Bird-strikes, bullets, other projectiles and even dropped tools can occur and result in a reduction in strength. Impact damage in fibre-matrix composites can be barely visible to the human eye yet the reduction in strength is commonly around 50%. This problem was a major concern and a great deal of
research has been conducted solely on the effect of impact damage on composite materials.
2.2 **Laminate and Plate Theory**

The fundamental theory behind composite materials is that the whole is stronger than the constituent parts. Perhaps the simplest explanation of this principle is given by Daniel in his 1998 William Murray Lecture (Daniel 1998). Single fibres are stiff and strong but their strength is highly variable as it is dominated by flaws. A bundle of fibres shows greatly reduced scatter, reduced further still when the bundle is set in a matrix or resin. The matrix supports the fibres so that a flaw in a fibre does not disable the entire fibre, only a short length around the failure. This is a very simplistic example when compared with many accepted theories and mathematical models.

A good summary of accepted theory is provided by Hull and Clyne (Hull and Clyne 1996). The most basic theory assumes that a lamina can be considered as two discrete layers of the constituents bonded together. The widely used 'Rule of Mixtures' is derived from this basic assumption and provides a good approximation when determining axial properties.

\[
E_1 = (1-f)E_m + fE_f
\]

- Young's Modulus of Composite, \(E_1\)
- Volume fraction of fibres, \(f\)
- Young's Modulus of Matrix, \(E_m\)
- Young's Modulus of Fibres, \(E_f\)

The determination of transverse properties is more complex. Young's modulus in the transverse direction (\(E_2\)) can be determined using a 'Reuss model'. This is similar in principle to the rule of mixtures, although it gives a poor approximation of \(E_2\).

\[
E_2 = \left( \frac{f}{E_f} + (1-f)/E_m \right)^{-1}
\]

In practice the rule of mixtures gives a good approximation for \(E_1\) because the strain distribution within the material is homogenous. The Reuss model for \(E_2\) gives a poor approximation because the fibres constrain the matrix and the
resultant strain field is inhomogeneous. The non-uniform strain field, coupled with stress concentrations adjacent to the fibres make a significant contribution to the behaviour of the material but are not accounted for in the Reuss model.

A more accurate representation is obviously needed, the most commonly used is the Halpin-Tsai equation (Halpin and Tsai 1967). This introduces additional parameters and gives a good approximation of transverse modulus when compared with empirical data.

\[
E_2 = \frac{E_m (1 + \xi \eta f)}{(1 - \eta f)}
\]

Where:

\[
\eta = \left( \frac{E_f}{E_m} - 1 \right) \left( \frac{E_f}{E_m + \xi} \right)
\]

\(\xi\) is a parameter but its usual value is close to unity.

Hull goes on to compare the Halpin-Tsai prediction with experimental values. More complex models are available such as the Ashelby model but the majority of modelling is now performed using finite element analysis.

The complexity of composites is already a problem even when considering a single lamina. These problems are compounded when considering multiple fibre layers at differing orientations. The Halpin-Tsai prediction also provides a good approximation of the shear modulus. Hull points out that even at high fibre content both the transverse and shear modulus of the composite are very close to the modulus of the matrix.

For an aligned fibre composite there are three different Poisson's ratios. The large number of elastic constants requires a standard notation presented by Nye (Nye 1985). This is discussed further by both Halpin (Halpin 1992) and Harris (Harris 1999). Halpin categorises and defines some common terms that are widely used.

- HOMOGENOUS - uniform material, properties constant throughout.
- ISOTROPIC - Material properties are not a function of orientation, i.e. constant in any plane.
• **ORTHOTROPIC** - Only three mutually perpendicular planes of symmetry pass through a point.

• **ANISOTROPIC** - There are no planes of material property symmetry that pass through a point.

For isotropic materials Hooke's Law can be expressed by four common equations.

\[
\sigma_1 = (\epsilon_1 + \nu \epsilon_1) \cdot \frac{E}{1 - \nu^2}
\]

\[
\sigma_2 = (\epsilon_2 + \nu \epsilon_1) \cdot \frac{E}{1 - \nu^2}
\]

\[
\tau_{12} = \gamma_{12} \cdot G
\]

\[
G = \frac{E}{2(1 + \nu)}
\]

For anisotropic materials there are eighty-one elastic constants although these are reduced to 21 due to symmetry. Nye defined Hooke's Law using tensors and matrices, for isotropic, anisotropic and orthotropic materials. The properties of individual plies or lamina are often highly anisotropic, this can be reduced by stacking a number of plies with differing fibre orientations to form a laminate. The properties of the laminate can be predicted from the component plies provided that certain assumptions are made. The assembly is assumed to be flat and thin with no through-thickness stresses and all edge effects are neglected. These assumptions are widely accepted and called 'Kirchoff assumptions'.

Other excellent sources on lamina theory are Introduction to Composite Materials (Tsai and Hahn 1980) and Introduction to Composite Materials Design (Barbero 1998). A comprehensive review of plate theory literature is also provided by Altenbach (Altenbach 1998).

Tuttle et al. studied the biaxial loading of thin, 8-ply panels and recorded out-of-plane displacement data using shadow Moire and dial gauges (Tuttle, Singhatanadgid et al. 1999). Tuttle et al. identified that most buckling analyses use numerical methods (FEA) but opted for a different approach. The authors gathered a great deal of empirical data and performed their buckling analysis.
using an algorithm based on the Galerkin method. Their buckling load prediction showed a standard deviation of +/- 15%. This was attributed to material variation within the test samples and imperfections within the loading frame. The authors concluded that the buckling load of a 'practical' composite laminate might be 15% or more below the predicted value.
2.3 Impact Damage of Fibre-Matrix Composites

Much of the early research into impact damage has been summarised in later work. Of the many reviews of previous literature there are two most widely recognised and referred to. Cantwell & Morton reviewed over one hundred papers and identified key parameters that govern impact response (Cantwell and Morton 1991). This paper is referenced in most of the work produced since 1991. Cantwell & Morton highlight the main differences between metals and composites with regard to impact. Metals absorb energy through elastic and plastic deformation. Although this can result in permanent damage it's effect on the load-bearing capability of the component is usually small. In addition the reduction in the load-bearing capability of the structure can be predicted easily and accurately using fracture mechanics principles. Composites provide the engineer with two problems. Firstly there is little scope for plastic deformation, impact energy is generally absorbed by creating large areas of fracture. Secondly the effect on the load-bearing capability of the structure is often unknown because the damaged area is difficult to predict and complex in nature. Cantwell goes on to describe the most common test procedures and also the effect of the fibre and matrix on the impact resistance of the composite, discussed later. Several conclusions are drawn most notably:-

- The ability of the fibres to absorb strain energy is perhaps the most important factor in determining impact resistance of the material.
- The inter-lamina fracture toughness (Mode II) should be high to maximise the post-impact compressive strength.
- The stacking sequence of the plies determines the ability of the composite to absorb elastic energy and the failure mode.
- Data taken from laboratory tests is not necessarily representative of larger structures. The geometry of the sample is a factor when determining its impact resistance.
- Varying the strain rate affects the material properties of the target and therefore it's response. Higher velocities tend to induce a more localised
response. Thus static test results are not necessarily applicable to dynamic behaviour.

A large body of work has been undertaken by Abrate who summarises previous research both in his book (Abrate 1998) and in a comprehensive literature review (Abrate 1994). Both of these items summarise impact theory from contact laws to beam and plate theories. Modelling of impact events and damage prediction are also presented. Abrate focuses on the modelling of impact events based on accepted theories. He goes on to discuss the effect of many variables on the impact event and the numerous models used for damage prediction. Abrate concludes that the damage resistance of the material is governed by the matrix properties while the fibres and their orientation determine the stiffness.

A similar book has been authored by Sierakowski and Newaz looking at damage tolerance of composites and providing several examples of analytical models (Sierakowski and Newaz 1995).

The majority of recent research has concentrated on low-velocity impact. Richardson and Wisheart reviewed much of this work (Richardson and Wisheart 1994) and made several important observations.

- Low-velocity impact is defined as an event that can be treated as quasi-static. Thus the upper limit of low-velocity is dependant on the properties and stiffness of the target and the mass and stiffness of the impactor.
- High-velocity impact is dominated by stress waves and leads to localised damage as the larger structure has insufficient time to respond.
- Post-impact performance is dependant on the damage mode.
- Relatively little research has been undertaken into the response of pultruded sections.
2.3.1 Characterisation of Impact

For reasons discussed above the majority of research was undertaken using composite samples in coupon form. These coupons were basic flat plates that were clamped in various configurations and then subjected to an impact. The force of the impact was determined by the speed of the impacter and its mass. Most testing is categorised simply by the magnitude of the impact velocity, although there is no clear definition. The test categories are typically, in order of increasing velocity, Quasi-static, low-velocity, high-velocity and hyper-velocity. The resulting damage incurred by the sample depends on a more complex set of criteria. It is often difficult to compare research and there have been attempts to characterise the impact (Christoforou and Yigit 1998) but these have been restricted to flat plates and coupons. Christoforou identifies key parameters that can be used to predict the type of response of the coupon.

Hyper-velocity and high-velocity impacts are usually concerned with bullets or other projectiles and tests have particular relevance to military applications. Low velocity impacts have been defined as between 1 to 10 metres per second (Wisheart 1996). The boundary between high and hyper-velocity is less clearly defined but impact velocities at these levels are not relevant to this project.

The mass or inertia of the impacter is the other main variable. This is usually determined by the impact event that is being simulated. Impacters have been used in a variety of shapes with a simple sphere being the most common.

Research relevant to the project is concerned with how the fibre and matrix properties, manufacturing method and lay-up affect the resistance of the composite to impact. The failure modes can provide valuable insight into predicting how a composite boom would behave post-impact. There have been many attempts to develop various computer-simulations of impact behaviour but to date there is no single theory or software item that can accurately predict the failure mode (Herakovich 1998). This is due to the complexity and the variations within the fibre and matrix. An interesting study is underway (Hinton, Kaddour et al. 1999) that compares the latest theories against experimental results. This study highlights the difficulty in accurately predicting the failure of composite materials. A key point made by Hinton et al. is the actual definition of failure and how it may be application specific.
2.3.2 Impact Test Techniques

The majority of impact problems can be divided into two categories: low velocity impact by a large mass and high velocity impact by a smaller mass. Ideally the impact test should simulate the impact conditions to which the materials will be subjected to in the field. Several 'standard' tests are used, described by numerous authors (Hollaway 1978; Cantwell and Morton 1991). Since the thrust of current research is restricted to low-velocity impact the tests concerning high-velocity impacts are ignored. Interested readers can examine additional work by Cantwell & Morton (Cantwell and Morton 1989) that compares the response of laminates to both high and low velocity impact.

The Charpy test was developed for testing metals and plastics but was used for most of the early impact tests on composites. The test involves a pendulum that can be instrumented. The test is simple but can provide a great deal of information. Most samples are short, thick beams that incorporate a notch opposite the impact site of the pendulum. This test is only suitable for continuous fibre composites and there are some doubts over the effect of specimen geometry on the test results. The Izod test is similar to the Charpy test but the specimen is clamped vertically as a cantilever beam. The pendulum impacts the free end of the beam but the test suffers from similar problems as the Charpy. Information on these tests applied to GFRP was published by Johnson (Johnson 1986).

Drop-weight impact tests are common and involve a mass falling from a known height to strike a horizontal specimen. The specimens are usually flat coupons or plates although a greater range of geometry's can be tested. The velocity of the impactor can be determined from equations of motion or is often measured optically. The impactor can be instrumented to give force-time data.

Other authors have utilised standard tests originally developed for plastics, notably the falling dart method (B.S.I. 1985).

Hydraulic test machines are often used with a double-cantilever beam (three-point bend) arrangement. The great advantage of these machines is that the strain rate of the impact can be varied. Further test data can be obtained from bonded strain gauges.

- 19 -
2.3.3 *Failure Modes*

Herakovich defines four main modes of failure resulting from impact (Herakovich 1998). These are matrix mode, delamination mode, fibre mode or penetration. Each of these modes results from one or more micro-level failures. Micro-level failure mechanisms include:

- Fibre fracture – the tensile load causes the maximum stress or strain of the fibre to be exceeded.
- Fibre buckling – where the axial compressive stress causes the fibres to buckle.
- Fibre splitting – the maximum hoop stress of the fibres is exceeded.
- Fibre pull-out – resulting from fibre fracture.
- Fibre/matrix de-bonding – resulting from fibre fracture.
- Matrix cracking – strength of the matrix is exceeded.
- Radial cracking – the maximum hoop stress of the matrix is exceeded.

Damage manifests itself as local failures stemming from one or more of these micro-level damage mechanisms. The local failure may lead to further failures in the immediate vicinity, this is called damage accumulation and can lead to catastrophic failure of the composite structure.

Matrix mode is the cracking of the matrix parallel to the fibres as a result of tensile, compressive or shear forces. This mode leads to delamination of the plies. Delamination mode is probably the most common and is the separation of the plies resulting from high inter-laminar stresses. Delaminations are formed as a result of matrix cracks.

Fibre mode is generally the most catastrophic failure mode as the fibres are the primary load-bearing component. Fibre failure is the breaking of the fibres due to tensile load or fibre buckling caused by axial compressive loads.

Penetration is self-explanatory: the complete perforation of the impacted surface. Penetration generally occurs as a result of high strain rate, when fibre failure reaches a critical level. This type of failure is most commonly encountered in high or hyper-velocity impacts resulting from small projectiles.
2.3.4 Failure Mechanism Due to Low or Quasi-Static Impact

Although the orientation of fibres and other variables will effect the failure mode, most composite laminates seem to display matrix cracks and delaminations in a similar way. This is the well-known 'pine-tree' damage pattern discussed by many authors and summarised by Abrate (Abrate 1998).

As previously discussed matrix cracking is usually the first type of failure encountered. Matrix cracking typically occurs in planes parallel to the fibre direction in unidirectional layers. The matrix cracks start under the edges of the impactor in the upper and middle layers. These cracks are formed by the very high shear stresses in the transverse direction and are often termed shear cracks. They are characteristically aligned at approximately 45 degrees. The matrix crack at the bottom of Figure 2-1 on the non-impacted surface is also typical. This crack is caused by the high tensile bending stresses at that point and as such is termed a bending crack. The bending crack is vertical and sits beneath a delamination between the middle and lower plies. Delaminations are oblong or ‘peanut-shaped’ (Cantwell and Morton 1991) with their major axis in the direction of the fibres of the lower layer. The delamination is caused by a bending mismatch and its size is relative to the difference in orientation between the two adjacent lamina.

Therefore a lay-up of 0/90 degrees produces the greatest area of delamination.

---

![Diagram](image.png)

Figure 2-1 Typical Low-Velocity Impact Damage in a 0/90/0 Laminate
Davies performed a number of tests in an attempt to verify a FE model that would predict the threshold for delaminations in CFRP and GFRP plates (Davies, Zhang et al. 1994). The samples were subjected to a low-velocity impact and the author used damage/force maps to predict the damage and its initiation. The FE model gave a good indication of threshold for CFRP plates but it was less accurate for thick GFRP plates, which the author attributed to the strain-rate sensitivity of the stiffness and fracture toughness of GFRP.

2.3.5 Effect of Matrix and Fibre Properties on Impact Performance

Impact damage in composites is different from metals in two main ways. Firstly, as previously mentioned, damage detection in metals is relatively easy as the damage occurs at the impacted surface. In composite materials the damage is usually visible on the non-impacted surface, if visible at all, making detection difficult.

Secondly metals are more ductile and thus absorb large amounts of energy during an impact. The metal may flow at its yield stress for strains up to 20% (Richardson and Wisheart 1994). Composites on the other hand are generally brittle and thus can only absorb energy in elastic deformation and via damage mechanisms. Damage mechanisms are discussed above and even a small amount of damage can result in significant reductions in strength. In order to maintain its material properties the composite must absorb all of the energy by elastic deformation. Due to the nature of the material, both fibres and resins, it cannot suffer a great deal of deformation. This has led to design standards for composite materials adopting a design failure strain of only 0.5%, some sources quote 0.4% (Matthews and Rawlings 1994).
2.3.6 **Effect of Fibres on Impact Performance**

It is generally accepted that the ability of the fibres to store elastic energy is the main factor in determining a composite's impact toughness (Cantwell and Morton 1991). The fibres themselves are relatively brittle, their strain-to-failure is relatively poor compared to metals:

- Carbon fibre: Strain-to-failure of 0.5 – 2.4%
- E-Glass fibre: Strain-to-failure around 3.2%

Typically, E-glass fibres can absorb three times the elastic energy of carbon fibres. This is reflected in the impact performance of GFRP that is considerably better than CFRP. The impact properties of Carbon are notoriously poor and it is often improved by adding E-glass or aramid (Kevlar) fibres to it. The resulting hybrid has improved impact properties but the difference in modulus of the fibres leads to design problems. There is also a considerable body of work investigating the effect of strain rate on the impact performance of the fibres. Carbon is generally thought of as not being strain rate dependent, i.e. its properties do not vary with strain rate. E-glass fibres are known to be affected by strain rate. It has been shown that the modulus and stiffness both increase with increasing strain rate. There is however, very little information to accurately assess the effect of strain rate on the impact resistance of composite systems.

Fibre length is known to affect the impact performance of the composite. Generally, the longer the fibre, the better the performance. Short fibre composites are susceptible to puncturing. Long fibre composites tend to behave with greater ductility and absorb more energy. It has also been shown that composites with longer fibres have increased fatigue performance.
2.3.7 Effect of the Matrix on Impact Performance

The most common matrix material is epoxy, a thermosetting polymer. Epoxy is brittle and has poor resistance to crack growth. Various methods have been tried and are under development to improve the fracture toughness of thermosetting resins. A variety of additives have been used including rubber, thermoplastic particles and synthetic plasticizing agents. Increasing the fracture toughness can be achieved but is commonly accompanied by a reduction in mechanical properties of the resin. The improvement in fracture toughness of the resin tends to have a lesser effect on the overall composite due to the brittle nature of the fibres.

The use of thermoplastic resins such as Poly (Ether Ether Ketone) or PEEK can give a considerable increase in fracture toughness. The use of thermoplastic resins was for some time held back because of the many problems associated with them. Thermal instability, poor chemical resistance and creep have restricted their use. Problems with the bond strength between the matrix and the fibres were also encountered. Gradually these problems are being overcome and thermoplastic resins are becoming more competitive.

2.3.8 Effect of Fibre-Matrix Bonding on Impact Performance

The adhesion between the matrix and the fibres is obviously of critical importance. Generally fibres are treated to improve the bonding with the matrix. Poor matrix-fibre bonding can result in catastrophic failures at relatively low transverse stresses due to fibre pull-out.

A further consideration on impact resistance is the geometry of the component itself. Complex components have been tested and it has been shown that where a component is allowed to buckle the impact damage is less. Components impacted near stiffeners or ribs showed a reduction in damage tolerance as the damage propagated along the stiffener.
2.4 **Low Velocity Impact Response of Fibre-matrix Laminates**

There have been many studies of impact response and the effect of the various parameters discussed above.

The effect of ply orientation has most recently been discussed by Sun & Tao (Sun and Tao 1998) who compared the inter-lamina toughness of various orientations. Sun used an end-notch-flexure (ENF) arrangement with multiple lamina layers of varying orientations. Teflon film is used to insert pre-cracks in one end of the specimen. Sun found that the delamination always grew in the same interface and deduced that the matrix crack jumped from one interface to another until its progress was stopped by a zero degree ply.

Lagace and Wolf (Lagace and Wolf 1995) compared the impact damage resistance of differing composite lay-ups. They make two important definitions: damage resistance is the ability of the structure to withstand impact, damage tolerance is the ability of the structure to perform post-impact. Four different materials in a variety of configurations were tested using a specially designed rig. Damage was evaluated visually, using x-rays and by thermal de-ply. Lagace and Wolf found that the shape of the force-time response remained constant while the contact force and duration increased with impacter velocity. Delamination size increased with bending mismatch between plies as discussed in section 2.3.4. They deduced that contact force is a key parameter in assessing impact damage resistance. Force-time histories did not vary much for different laminates. An important conclusion drawn is that damage resistance is both a material and a structural response.

The majority of research considers thin laminates, usually less than 8mm thick and made of carbon fibre based composites. An interesting comparison is provided by Zhou and Davies who test samples of glass-fibre reinforced plastics (GFRP) with thicknesses of 10mm and 25mm (Zhou and Davies 1995). They used a drop weight set-up and compared the impact response with finite element modelling. Post-impact damage was assessed both visually and ultrasonically. Further static tests were conducted to determine the strain-rate effect and indentation law for the coupons. The results show good correlation between static and low velocity force-displacement data. The results from the two different
specimen thicknesses show similar characteristics when normalised for thickness. Zhou and Davies conclude that:

- Impact force and incident kinetic energy are sufficient to characterise impact behaviour.
- Maximum achievable impact load reaches a threshold value when a large delamination is formed. This highlights the value of impact force in any predictive algorithm.
- Strain-rate has a negligible effect on the size of the damage zone but does affect the peak impact force.

Another interesting analysis involved low-velocity impact in woven fibre composites (Shim and Siow 1998). A series of drop-weight tests were performed with two different impacter masses and two different contact tip radii. Shim confirmed earlier research that delamination area varied linearly with impact energy. The tests also found that a smaller impacter produced a greater delamination than a larger impacter with identical impact energy. Woven fibre composites have fewer matrix rich areas and Shim deduced that delaminations in woven fibre composites might result more from fibre breakage than matrix failure. The research also found that variability in the impact response increased with impact energy. The residual strength of the samples was also tested and if was found that the compressive strength of the samples degraded more than the tensile strength. This is now accepted as fact and is due to the compressive failure being dominated by local fibre buckling, accentuated by delaminations. The tensile strength is controlled by fibre fracture where delaminations have far less effect.

Robeson (Robeson 1998) used a five point bending arrangement to test flat coupons produced from carbon fibres reinforced with epoxy and also PEEK. Robeson observed transverse shear cracks in the matrix at 45-degree angles and also delaminations. These forms of damage frequently occurred together but were observed independently. The characteristic bending or tensile crack was also observed opposite the impact point on the non-impacted face. Robeson varied the ply orientation and stacking sequence, observing the different failure modes and their locations in the various plies. It was found that zero degree plies were best at resisting damage from transverse loading, they also tended to stop the
propagation of transverse shear/matrix cracks. Robeson concluded that the 5-point bend arrangement gave a good correlation to low-velocity impact tests.

Kelkar et al. (Kelkar, Sankar et al. 1998) considered low-velocity impact on pre-loaded composites of three different thicknesses. They compared their experimental results with a finite element model. Kelkar concluded that the application of a tensile pre-load had several significant effects:

- Application of a pre-load lowered the impact energy threshold needed to initiate damage.
- The amount of damage on the non-impacted face was higher for pre-loaded samples.
- The damage mechanisms in thick and thin laminates are different.

Zhou (Zhou 1996) tested thick circular coupons of glass-fibre reinforced polyester (GFRP) using a drop-weight impact rig. He determined damage by C-scan and determined residual compressive strength using a 'compression after impact' (CAI) test. The CAI test involved applying a compressive axial load to a sample laterally supported to prevent buckling. To minimise the damage to the contact area of the specimen the edges were 'potted' in epoxy and machined parallel. Zhou deduced that the incident kinetic energy (IKE) gives only an approximate measure of damage unless all of the IKE is absorbed by delamination. As a result Zhou also recommends the use of impact force as a measure of impact. Using the two measures allows the examination of the strain rate effect and a better indication of damage initiation. Zhou identifies that there is a distinct threshold force above which damage increases in an unstable manner. The use of damage area to assess residual strength should be used with caution (if at all). Different orientations of damage shape with the same area give very different results. The given example compares an elliptical damage area with its major axis in the loading direction with an ellipse with its minor axis in the loading direction. The residual compressive strength of the former is higher than the latter although the areas are identical. Several relevant conclusions were drawn:

- Delamination and fibre breakage are the dominant damage mechanisms.
• The use of IKE to assess damage tolerance should be treated with caution, particularly for thick laminates. Impact force provides a useful additional measure.
• Residual compressive strength of impacted panels shows reductions of 50%.
• Laminate thickness has a negligible effect on residual compressive strength.
• Completely delaminated panels can still retain 20-30% of their original compressive strength.

The use of force as a measure of impact resistance has been discussed in further detail by several authors (Lagace, Williamson et al. 1992; Jackson and Poe-Jr 1993). Lagace co-authored further research with Wardle on using impact force to link quasi-static and low-velocity impact tests (Wardle and Lagace 1998a; Wardle and Lagace 1998b). Quasi-static tests are often easier to conduct and more readily accessible, they can also provide better results than impact tests. Wardle and Lagace promote the use of quasi-static testing in producing impact damage as the impact event is better controlled. The results obtained show very similar and in some cases identical plots from the two test types. However they highlight factors that may affect this relationship and are yet to be fully researched.

Previously, Swanson investigated transverse impact of laminates in an attempt to determine limits for which the quasi-static approximation is suitable (Swanson 1992). Swanson reported that it was usual practice to compare the natural frequency of the quasi-static response with the lowest natural frequency of the targeted sample. Since the natural frequency of the sample can be hard to determine Swanson proposed that the ratio of the impact mass to the equivalent mass of the structure (sample) could be used in a similar manner. This observation was based purely on empirical data and not written theory.

Hsiao and co-workers investigated the strain-rate effect of transverse impact on thick laminates (Hsiao, Daniel et al. 1999). They used a drop-weight test machine to impact a thick laminate (72 plies). To compliment this they used a split Hopkinson pressure bar (SHPB) to conduct tests at higher velocities. They found that there was a significant strain-rate effect on the transverse compressive behaviour of the UD laminate.

Zhou attempted to predict the impact damage thresholds of glass-fibre composites using circular discs of differing size and thickness (Zhou 1995). Experimental
data for static and low-velocity impact tests was compared to predicted values based on interlaminar shear strengths of the material. Assuming that the post-impact performance is governed by the largest delamination gave good comparison between experimental and calculated results. Zhou therefore concluded that damage initiation dominated by a delamination could be predicted using a relatively simple theoretical model.

Jelf and Fleck stated that there was no consensus in the literature on the compressive failure mechanism of UD laminates (Jelf and Fleck 1992). The authors summarised the current theories at the time. Data was gathered from Celanese compression testing of UD carbon fibre-PEEK (Poly-ether-ether-ketone) laminates. The authors concluded that the predominant failure mechanism in fibre-matrix UD composites is plastic micro-buckling.

The compressive failure of a damaged laminate is also detailed by Xiong and co-workers (Xiong, Poon et al. 1995). By obtaining C-scan images of damaged laminates and applying numerical techniques they were able to predict the compressive failure. The analytical methods used were verified via experimentation using a compression-after-impact test. C-scans of damaged laminates showed the common pine-tree damaged pattern but the simulation used an elliptical area of degraded material properties. The level of degradation was determined from the buckling stress of the largest sub-laminate formed by the delamination. Predictions compared favourably with experimental results.

Gottesman et al. (Gottesman, Girshovich et al. 1994) tested rectangular coupons using a drop-weight rig. Post-impact assessment was carried out by ultrasonic and X-ray evaluation, CAI tests and Moire interferometry. The interferometry was used to evaluate buckling of sub-laminates by video capture of changing fringe patterns. The authors used a damage model that considered only the major delaminations with the testing used to confirm the accuracy of the model. A good correlation was found between analytical and experimental results.

Ireman et al. (Ireman, Thesken et al. 1996) performed coupon tests to examine the growth of delaminations. A variety of tests were performed and the coupon behaviour was also monitored using Moire interferometry. Ireman determined that changes in specimen geometry produced different results. Cyclic loading was used to examine delamination growth but it was found that matrix cracks and the location of the initial defect have a strong influence on damage growth. The
research was complimented with a paper by Wiggenraad et al. (Wiggenraad, Aoki et al. 1996) who as part of the same project tested structural elements. The structural elements tested were common aircraft components; honeycomb (sandwich) panels and various skin-stiffened panels. The residual compressive strength is predicted as a function of damage size and compared to test data. Of particular interest is the growth of delaminations around skin stiffeners that causes them to de-bond leading to failure of the panel.

Abrate includes a section in his book devoted to impact behaviour of beams (Abrate 1998). He defines indentation as the difference between the displacement of the impacter and the back face of the impacted laminate. Abrate also states that statically determined contact laws can be used where strain-rate effects are negligible. Hertz contact law does apply to composite materials even though they are orthotropic. Abrate identifies that for the flexural loading of beams the local deformations are superimposed onto the overall deflection of the beam.

Abu-Farsakh studied the effect of material and geometric non-linearity on the response of laminated beams (Abu-Farsakh, Barakat et al. 2000). The author used FEA and a new material model to investigate the effect on boron/epoxy and graphite/epoxy UD composites. The empirical data for different configurations is presented in great detail.
2.5 Determination of Material Properties of Fibre-Matrix Composites

The determination of material properties of fibre-matrix composites can be broken down into two main areas of research. Firstly, the manufacture of coupons to determine the material properties of a fibre-matrix composite. This data is then used in the design of components. Secondly, the removal of coupons from composite components either as a quality control type of test or alternatively to determine damage of an impacted component. These tests are of course destructive as they involve removal of material.

2.5.1 Manufactured Coupon Tests

A large volume of research has been conducted in this area. Of particular interest is the Iosipescu test originally designed for single shear testing of metals. In his original paper Iosipescu (Iosipescu 1967) used photoelasticity to prove that none of the existing methods of the time caused metals samples to develop pure shear failure conditions. He derived a new test procedure using a rectangular, notched coupon that induced static single-shear loading (Figure 2-2). The sample and loading structure gave a uniform stress distribution in the failure section of the sample.

![Figure 2-2 Iosipescu Specimen (Iosipescu 1967)]
Adams and Walrath (Adams and Walrath 1987) reviewed the status of the Iosipescu test in 1987. Their paper documented recent research into the Iosipescu test including its use with orthotropic materials and the experiences of other researchers. The paper discusses the application of the test to orthotropic materials and compares theory with finite element analysis (FEA) and experimental data. Other methods are also mentioned including the Arcan specimen test. Adams considers the Iosipescu test with respect to a unidirectional carbon-fibre reinforced plastic (CFRP). Load-point crushing is identified as a problem area when using fibre-matrix composites. They concluded that the test is reliable and shear properties of the material can be determined.

Adams continues his investigation in a paper with Lewis (Adams and Lewis 1995). The authors perform a strain analysis of a unidirectional (UD) CFRP Iosipescu specimen using strain gauge rosettes. Other researchers had observed the premature lateral cracking of the specimen from the notch roots, when applied to composite materials. It had been shown by finite element analysis that the occurrence of these cracks did not influence the subsequent shear-strength measurement. The authors deduced that it therefore should be possible to machine away the material from this area and this formed part of their current study by using specimens with different notch configurations. The phenomenon of load-point crushing was observed and found to be greatest in materials with low transverse stiffness (highly orthotropic) such as fibre-matrix composites.

Pierron has conducted a large body of research on the Iosipescu test. A paper co-authored with Vautrin and Harris (Pierron, Vautrin et al. 1995) compares the shear modulus of an isotropic material obtained from the Iosipescu test with the shear modulus obtained from a tensile test. The results showed excellent agreement between the two methods. A finite element model was presented to account for non-uniform stress distribution in the Iosipescu coupon. The authors presented their experimental data that showed a difference between Iosipescu derived shear modulus and a tensile derived shear modulus of between two and five percent. This experimental data was used to validate a finite element model. The results from the test and the FE model allowed the method to be applied to orthotropic materials with confidence.

Little work has been undertaken on the Arcan test (relative to the Iosipescu test) and its application to fibre-matrix composites. Hung and Liechti (Hung and
Liechti 1997) evaluated the specimen for determining the shear modulus of a unidirectional 19mm thick plate from which Arcan specimens were cut in different orientations. The test was originally used for the shear testing of polymers and uses a more compact geometry than the Iosipescu test, thus reducing bending effects. The shear properties of the material can be more readily determined in all orientations. Strains in the Arcan sample were measured using both Moire interferometry (Cloud 1995) and resistance strain gauges. The authors concluded that the Arcan specimen could be used but attention to proper specimen alignment was needed to minimise errors.

Grediac and Pierron (Grediac and Pierron 1998) defined a new test and coupon geometry for determining the four in-plane stiffness components of an orthotropic material. The new specimen was T-shaped, the authors suggested that a larger specimen would average out local discrepancies in mechanical properties. They
used a ‘model-updating’ technique whereby an algorithm minimised an objective function. The objective function being the difference in experimental and numerical data. The T-shaped geometry was proposed because each unknown parameter was dominant in a particular area of the specimen. The authors used the virtual fields approach to derive four linear equations with unknowns as the four stiffness components. This approach was validated by numerical simulations. The advantage of this approach is that only one specimen and one test is required to determine the four elastic constants. The specimen is mounted on standard tensile test machine fixtures. It does require full-field strain measurement. The method used by the authors was subsequently criticised by Prabhakaran (Prabhakaran 2000) with the original authors exercising their right to reply (Grediac, Pierron et al. 2000).

The research was followed up in a further paper by Grediac, Pierron and Surrel (Grediac, Pierron et al. 1999). The authors applied a similar theory to a fibre-matrix composite plate. The method relies on the inverse analysis of heterogeneous stress fields over large areas of the specimen. A detailed experimental procedure is presented. The full-field data is obtained using moiré interferometry (phase-shifted grid method).

Pierron and Grediac (Pierron and Grediac 2000a) then proceed to apply the virtual fields technique to an Iosipescu-type specimen. Once more the method is

---

Figure 2-4 T-Shaped Specimen (Grediac and Pierron 1998)
reliant on whole-field measurement to map shear strain in the gauge section of the specimen. The authors highlighted the major drawback of the Iosipescu specimen, the shear stress in the gauge section is not exactly uniform therefore correction factors are needed. This had been regarded as a problem but the authors used this to their advantage. All three components of stress are present in the Iosipescu specimen. Obviously there is shear in the gauge section but there is also bending stress in the notch root and compressive transverse stress near the inner loading points (where other authors have reported local crushing). Pierron and Grediac studied the effect of errors on the system and deemed that their method was feasible. Identified stress components were accurate to less than one percent. An additional advantage was that no FEA was needed.

Pierron, Zhavoronok and Grediac then adapted the virtual fields method to thick laminated tubes (Pierron, Zhavoronok et al. 2000). The authors identified that components produced from glass-fibre reinforced plastic (GFRP) usually required thicker wall sections and therefore the plane strain and plane stress assumption of classical laminate theory is not valid. Full three-dimensional stress and strain states must be accounted for in design. Torsion and Iosipescu tests are frequently used to determine behaviour and properties. Alternative methods are the model-updating approach but the authors point out that their method requires no model or iteration. Again the virtual field method is used and requires whole-field data obtainable from optical methods. Using this technique the authors were able to determine the through-thickness stiffness of thick laminated composite tubes.

Pierron and Grediac then produced a further paper on determining the through thickness properties of thick composites using the Iosipescu test (Pierron and Grediac 2000b). Again the principle of virtual work was used in conjunction with a modified Iosipescu-type specimen. The specimen was modified so as not to produce a pure shear stress field but to produce a complex stress field. By removing the notch all plane stress components became present. The authors adjusted the fixture until all the components were equal in magnitude and then adopted the virtual fields approach.
2.5.2 Removed Coupon Tests

This group of tests involve the removal of small coupons of material from key locations of much larger composite components. Research in this area has generally focussed on the removal of circular disks from composite skins, shells or plates. These discs are subsequently analysed by the diametral compression test, also called the 'Brazilian disc test'. The diametral compression test was originally used to determine the splitting tensile strength of rock specimens (A.S.T.M. 1995). The disc is mounted in a tensile test machine between two platens and in the original test was loaded to failure. Okubo first presented the solution for the stress distribution in an orthotropic circular disc in diametral compression, many authors reference his work (Okubo 1939; Okubo 1952).

Chisholm et al. (Chisholm, Hahn et al. 1989) applied the diametral compression test to pultruded composite rods as a quality control test. Several factors affect the properties of the rods including viscosity of the resin, pulling force, pulling speed and rate of cure. The rods were used in the manufacture of ropes for tethering offshore installations, so quality control was critical. The test was accomplished by cutting a short specimen between two and thirty millimetres long and loading the specimen to failure. Shorter rods yielded at higher strengths and Chisholm found that the length effect could be described by Weibull theory.

Sanford applied the least-squares method to the photoelastic analysis of a disc in diametral compression (Sanford 1980). The analysis uses whole-field data and requires a PC-based algorithm to determine the stress-optic coefficient. Data could be taken from locations where confidence is greatest, from the fringe pattern directly (no interpolation).

Okada applied the diametral compression test to unidirectional silicon carbide fibre-reinforced glass (Okada 1990). The author found that the strength showed no dependence on fibre orientation at angles greater than thirty degrees (from the loading axis). At these orientations the tensile stress in the thickness direction governs failure. The crack occurs in the plane of the disc at a consistent load. At angles less than thirty degrees the stress on the inclined plane of the fibres governs fracture with the crack initiating at the loading point. Okada observed crushing of the sample at the loading points. These crushed zones accounted for around 10% of the disc diameter and correction factors were needed.
Chapter 2: Literature Review

Fahad used plaster of paris discs to investigate the effect of a ground flat (Fahad 1996), the purpose of which was to alleviate the stress concentration at the loading point. Fahad reminds us that the maximum tensile stresses increase perpendicular to the loading axis and are proportional to the applied load. For the test to yield useful results the fracture must be initiated by tensile stresses. Since failure occurs along the diametral plane of loading it is assumed to be caused by the nominal tensile stress. Some believe that failure is initiated under the loading points and there is disagreement on the failure mechanism. Fahad provides justification for this by highlighting that strength values obtained from diametral compression tests are always much lower than for other types of test. The author examines the effect of a flat ground on the loading point by using a quarter-disk finite element model. Fahad concludes that the point load causes failure due to shear and compressive stresses at the loading point. The results indicate that a flattened area of 0.2D (D is diameter of the disc) can be used to obtain an accurate tensile strength and that this finding is consistent with brittle fracture theories.

Lemmon and Blackketter conducted a stress analysis study on an orthotropic material under diametral compression (Lemmon and Blackketter 1996). They used both orthotropic and isotropic disks and recorded strain using photoelasticity and strain gauges bonded to the non-illuminated surface. The investigation was completed by analysing a quarter-disc and whole-disc FE model. The authors highlight that stress symmetry cannot be assumed when the principal material directions were not aligned parallel or perpendicular to the loading axis since there is no coupling between shear and normal stresses. Therefore a whole disk model must be used. The application of the load to the model was modelled for three different loading arcs. The study used 50mm diameter, 5mm thick GFRP discs. A photoelastic coating was bonded to the surface of the disk and strain gauges bonded to the opposite face. The set-up was calibrated first with an isotropic disc. Adjustments were made for the pre-stress and stiffening effects of the coating. The authors concluded that the stress field is significantly affected by the orthotropy of the specimen therefore isotropic stress calculations are invalid. A difference of up to 45% for normal stress was quoted. The principle material axes should be aligned to the loading axes to avoid shear stresses. The load should be applied over at least 8% of the disc circumference to obtain more uniform tensile stresses and avoid crushing.
Stanley and Garroch produced two papers that used the diametral compression test as its basis (Garroch and Stanley 1999; Stanley and Garroch 1999). However the authors measured the thermal response of a GFRP disc using a sensor in the centre. This thermoclastic signal was measured for several orientations. The authors found that the signal adopted a cosine function variation. They concluded that this method could be used to determine the principle material axes of the disc (if unknown) by locating the maximum and minimum thermal signal. This procedure could also be used to measure the fibre content.

Prabhakaran and Xu presented a method for determination of the elastic constants using a circular disc with resistance strain gauges bonded to the surface (Prabhakaran and Xu 2000). The disc used was cut from a pultruded GFRP panel, the gauges were bonded along two lines, parallel and perpendicular to the pultrusion direction. The disc diameter was 152.4mm although the author confirms a smaller disc could be used. The disc was loaded along pultrusion direction and then perpendicular to it and the gauges continually recorded the strain up to the maximum load. The author uses the solution for the stress distribution presented by Okubo (Okubo 1952). The results obtained compare well with values obtained from tensile specimens.
2.6 **Non-Destructive Evaluation of Composite Components**

Zalameda, Farley and Smith (Zalameda, Farley et al. 1994) presented a technique for in-service, non-destructive evaluation (NDE) of primary aircraft structures. The technique involved using thermal inspection to identify flaws and then applying ultrasonic detection to quantify the damage. Using this technique they were able to detect flaws in the composite and also de-bonds between the composite skin and the stiffeners. To confirm their results they also cut their samples into thin wafers and used dye penetrant techniques to photograph each wafer. The composite panels under investigation contained through-thickness reinforcement and Zalameda concluded that this made damage detection more difficult although good correlation was found between the results from the three methods.

Cawley (Cawley 1994) compared various methods for the commercial NDE of large composite structures. He also makes the point that laminates are damage tolerant and therefore only relatively large defects (10-20mm delaminations) need to be found. This is an important point although the geometry and wall thickness of the structure will determine the critical size of the defect. Cawley considers thermography, laser ultrasound and shearography as the three main methods. He also quotes a system cost of over £100,000 for each of these methods. In the case of shearography this figure is greatly exaggerated. Cawley proposes the use of ultrasonic lamb waves as a cheaper alternative. This is a similar technique to ultrasound but measures a line at each transducer rather than a point. Cawley also points out that transient thermography and shearography will not readily detect small flaws at depth.

Laermann (Laermann 1998) describes the application of inverse analysis to the NDE of primary composite structures. He describes the two types of inverse problems; over-determined (1st kind) and under-determined (2nd kind). Laermann then shows how the technique of inverse analysis can be applied to an engineering problem, using a truss structure as an example.

Cunha and Piranda (Cunha and Piranda 2000) present a method of determining the stiffness properties of composite tubes from dynamic tests. They use the sensitivity method, which is a form of model updating whereby an algorithm minimises the difference between empirical data and a modelled response. The
use of tubes allows multiaxial tests to be performed and the identification of elastic properties is possible from a single test. The algorithm compares the eigensolutions from a finite element model with the solutions from experimental data. Cunha goes on to examine the effect of experimental errors on the system and presents his results.

Prabhakaran and Chermahini used the non-linear least-squares method and strain gauges to determine the elastic constants of a fibre-reinforced composite (Prabhakaran and Chermahini 1984). The authors highlight problems experienced using other methods in determining the elastic constants particularly in-plane shear modulus. A proposal is made to use a single (photoelastic) specimen to determine all the in-plane elastic constants. The specimen configuration is an orthotropic half-plane subjected to a concentrated edge load. Unidirectional CFRP plate was used to verify the proposed method. The plate was loaded perpendicular to the reinforcement direction. Strain gauges were positioned at various angles to the reinforcement direction. To determine photoelastic parameters the authors used unsaturated polyester and styrene monomer resin as this had the same refractive index as the E-glass fibres. It was concluded that the least-squares method was ideally suited to obtaining material properties from multiple data points and can derive all in-plane properties from a single photoelastic specimen.
2.6.1 Speckle Interferometry Methods

Information on speckle interferometry can be found in several books, notably Cloud (Cloud 1995) and Rastogi (Rastogi 2001). Cloud explains the huge expectation that science had for the laser and the subsequent disappointment when images of laser-illuminated surfaces appeared 'grainy', limiting effective resolution. This was the first appearance of laser speckle, once thought to be a major obstacle and now the staple of numerous measurement systems. Speckle occurred due to the minute roughness of the illuminated surface and is an interference phenomenon. It has given rise to a whole series of measurement techniques based on laser speckle.

Shearography, or Speckle Shear Interferometry (SSI) offers numerous advantages over other optical methods.

- It is relatively fast.
- It does not require a highly stable environment.
- It can be used in brightly lit conditions.
- It uses simple optics.

Toh and co-workers and Shang and co-workers used time-average shearography to detect debonds in GFRP beams by vibrating the beam at various frequencies. They found that the technique could be used to detect debonds and delaminations giving a good indication of size. Higher frequencies gave better results and this was attributed to the small flaws resonating at the higher frequencies. The technique was applied to beams (Toh, Shang et al. 1991) and vacuum-stressed plates (Shang, Toh et al. 1991). A subsequent paper by Shang, Tham and Chau applied the same technique to a metal-foam-metal adhesively bonded structure and GFRP plates (Shang, Tham et al. 1995). The authors used a model-updating algorithm to minimise the difference between experimental and calculated results. The model was only tested on circular-shaped flaws.

Maji, Satpathi and Zawaydeh made an assessment of shearography for the assessment of large-scale structures such as bridges (Maji, Satpathi et al. 1997). They encountered problems with the CCD camera in resolving speckles. Also determined that the maximum inspection area at any one time was 100mm square. Relative movement was a factor as the in-plane deflection of a sample bridge structure could be up to 20mm between frames, thus the system would have to be
attached to the structure. The authors deduced that for an acceptable ratio between object distance and image distance the viewing distance would need to be 200 to 300 metres, which they deemed impractical.

Steinchen, Yang and Kupfer noted that shearography measured displacement derivatives directly (i.e. strain) and it was therefore easier to detect flaws (Steinchen, Yang et al. 1997). Their paper gives a good description of a Michelson-type interferometer. They identified that measurements can be recorded in real-time and also discussed the potential for this type of system in NDE.

Richardson et al. used phase-stepped ESPI to detect flaws in GFRP pultruded panels (Richardson, Zhang et al. 1998). They compared their results with data from ultrasonic C-scans. They identified that although ESPI could give real-time measurements it was not practical to perform phase-stepping in real-time. Different levels of damage were inflicted on the GFRP panels using an instrumented drop-weight impact-testing machine. Coupons were restrained by a circular clamp and subjected to four different impact velocities. The damaged coupons were then excited using a force-transducer hammer and phase-stepping ESPI was performed. Noise reduction and fringe enhancement were achieved using fast Fourier transforms before extraction of the optical phase data. The intensity patterns were then used to identify damaged areas. The same samples were then tested by an ultrasonic C-scan flaw detector. Finally the samples were sectioned with a diamond saw, polished and dye-enhanced to highlight damage. The authors concluded that damage was not visible on the impacted surface but the ESPI system was able to highlight internal damage that appeared in intensity patterns and phase maps. The results were confirmed by C-scan and sectioning data. It was found that the damaged area reported by sectioning was larger than the other two and this was attributed to additional mechanical loading during the sectioning process. The authors also noted that the damaged area had a linear relationship with absorbed impact energy. The authors proposed that the intensity fringe approach can be carried out in real-time but with the disadvantage of speckle noise. The phase map approach gave better results but could not be performed in real-time.
Steinchen et al investigated the potential and limitations of shearography for strain analysis (Steinchen, Yang et al. 1998). The authors identify that shearography offers the following benefits over other optical techniques.

- Requires no surface preparation
- Immune to ambient vibration
- Less sensitive to environmental factors
- Determine strains directly

A good explanation of shearography is given and a good explanation of phase shifting using a piezoelectric transducer (PZT) to move the mirror a small distance. They also highlight that large rigid body motion can lead to speckle decorrelation.

![Schematic Set-up of Digital Shearography (Steinchen, Yang et al. 1998)](image)

Chen produced a comprehensive literature review of over 170 papers related to digital shearography (Chen 2001). Perhaps the most important point made is that shearography is based on common path interference.

Picart, Lolive and Berthelot used a dual beam laser speckle interferometer to study the indentation of foam core material for use in sandwich structures (Picart, Lolive et al. 2001). A pockels cell was used as a phase-shifter. They used the test data to verify a FE model of the indentation test and concluded that ESPI is a good tool for validation purposes.

Gregory presented a basic paper on the application of shearography for the inspection of primary aircraft structures (Gregory 2001). Shearography can
provide rapid NDE capability and can also be used in process control. The technique requires the target to be under some load but the applied stress can be a fraction of the service load.

Pezzoni and Krupka presented a similar paper that gave a good description of a shearography set-up and identified its use in defect detection, structural analysis and fatigue inspection (Pezzoni and Krupka 2001). They applied the technique to a composite panel and also to a complete tail unit from a helicopter. They concluded that the system was able to detect defects in large-scale structures and could reduce inspection times.
2.7 Behaviour of Pultruded Sections

The behaviour of composite sections can be very different from the behaviour of plates or laminates. Increasingly composite sections are offered as replacements for steel sections especially in weight critical applications (Fibreforce 1999). This is especially true of pultruded composite sections due to the ability of the pultrusion process to produce high volume at low cost. The advantages of using sections are obvious. GFRP has low stiffness so deflection can cause problems. The use of sections (I, box, C etc.) increase the second moment of area of the structure thus maximising the stiffness-to-weight ratio.

Mahmood looked at the crush strength characteristics of GFRP box-section beams (Mahmood, Jeryan et al. 1990) from a crash worthiness perspective. Both axial crush tests and bending crush tests were conducted using samples manufactured from unidirectional prepeg material with varying lay-ups. The axial crush tests were conducted using 300mm (12") long specimens mounted between to flat platens. Mahmood reported local buckling as the failure mechanism for the thin-walled samples while the thick-walled samples showed progressive crushing. The bending crush tests were conducted using three-point bend configuration with square and rectangular box-section samples, the load was applied mid-span. Failure occurred under the impacter in the compression flange, Mahmood observed shearing at the top corners of the section. A four-point bend test was also performed and again cracks on the top corners were observed.

Barbero and Turk investigated the beam-column behaviour of pultruded I-section beams (Barbero and Turk 2000). They applied simultaneous axial and bending
loads to investigate the buckling mode. The authors used shadow Moire to measure the out-of-plane displacement in real-time. The shadow Moire provided flange deformation data that could be used in design or to validate a numerical model. Similar work was conducted by Kabir and Sherbourne who performed an investigation into the effect of non-uniform stress gradients on the local buckling of I-section pultruded beams (Kabir and Sherbourne 1999).

Wisheart conducted a thorough investigation into the impact behaviour of pultruded ‘planks’ for use in freight container construction (Wisheart 1996). The planks were already in commercial use for decking and bridge structures. Wisheart used in instrumened falling weight impact test machine to determine the response to a range of impact velocities and masses. The author concluded that the strain-rate effect was negligible. It was found that delaminations occur due to poor through-thickness impact resistance of the laminates and low inter-lamina strengths. A comprehensive review of associated literature is also presented.

Wisheart and Richardson (Wisheart and Richardson 1999) performed drop-weight tests on pultruded cross-sections. Damage was assessed by optical microscopy, ultrasonic C-scanning and thermal de-ply analysis. To obtain a three-dimensional picture of the damage pattern the impacted samples were cut into transverse strips. The force-deflection curves reported by the authors show a 'knee', also noted in earlier research (Zhou and Davies 1995). This 'knee' is a change in stiffness of the sample prior to reaching the peak force. The pultruded samples contained a high content of unidirectional (UD) fibres sandwiched between two layers of chopped strand or continuous filament mat. Wisheart and Richardson concluded that in bending the crack at the non-impacted face in the continuous filament mat was critical as it leads directly to delamination in the lower interface. Shear cracks in the unidirectional layer were also reported, also leading to delamination in the lower interface.

Tabiei et al. presented an investigation into the optimum profile of box-sectioned beams for roadside vehicle restraint barriers (Tabiei, Svenson et al. 1998). The authors used various pultruded GFRP sections and configuration of sections. They performed drop-weight and quasi-static tests using a 3-point bend arrangement and transverse loading. The key factor for this type of application is the amount of energy absorbed by the damage mechanisms. The authors tested a
variety of samples including 50 x 50mm box-section with wall thickness of 3mm. They reported a similar failure mode to Mahmood et al. (Mahmood, Jeryan et al. 1990) with the top corners of the sample exhibiting shear cracks. Tabiei also tested an identical sample (50 x 50 x 3mm) but wrapped in two layers of 0-90 fabric. The sample absorbed more energy than the unwrapped sample and there were no shear cracks. The author also found that peak force did not vary with drop height.

Gan reported that under bending box section beams develop local buckling in the compressed flange, causing premature failure (Gan, Ye et al. 1999). The author presented several pultruded shapes that featured internal stiffeners to increase resistance to local buckling. A FEA of the different proposed shapes was conducted to determine the optimum profile to minimise buckling.

Palmer used a quasi-static three-point bend simulation to investigate the failure mechanism of pultruded GFRP box-section (Palmer, Bank et al. 1998). The samples used were 76.2 x 76.2mm square box-section with 6.25mm wall thickness. Palmer describes the initial linear load-deflection (elastic) response of the sample and then the sudden and drastic loss of stiffness. The loss of stiffness is coincident with rupture at the top corners of the specimen originating under the impactor and the growth of shear cracks. Palmer describes this type of failure as 'progressive tearing' and reports that it appears to be a function of wall thickness. It is suggested that beyond a certain ratio of cross-section to wall thickness progressive tearing failure will not occur. The authors present some preliminary FE results, the output of which (they state) is not fully understood.

Chotard investigated the effect of impact on pultruded C-section open channels (Chotard and Benzeggagh 1998). An instrumented falling weight test machine was used to impact a sample at differing impact energies using two different impactor shapes. The authors investigated the damage by several methods; ultrasonic C-scan, X-ray tomography, magnetic resonance imaging (MRI), scanning electron microscopy (SEM) and sectioning. It is important to note that MRI requires the sample to be immersed in water and needs an open crack in order to be effective. The results are presented in detail and the authors identify that a large impactor produces less delamination than a smaller one. The location of the delamination is also different for the two types of impactor.
3 BACKGROUND THEORY

3.1 Introduction

This chapter contains three sections that expand on key areas of knowledge presented in the literature review. This should afford the reader greater understanding of these important topics and their application in the following chapters.

Firstly, an explanation of lamina theory is given including tensor notation and how the compliance and stiffness tensors relate to more recognised elastic constants.

Next the principle of inverse analysis is discussed along with how it can be used to determine damage in fibre-matrix composites.

Finally the optical technique of speckle shearing interferometry is explained, including the use of phase stepping to determine the 'wrapped' phase. The phase must be unwrapped and a method of two-dimensional phase unwrapping is provided using the discrete Fourier cosine transform.
3.2 Elastic Properties of Fibre-Matrix Composites

3.2.1 Elastic Behaviour of Unidirectional Laminae (Plies)

Fibre-matrix composites typically comprise of layers or laminae stacked in a predetermined arrangement. It is generally assumed in theoretical analyses that each lamina is homogenous, that is the fibre arrangement and volume fraction is uniform within the lamina (Hull and Clyne 1996). Fibres within a lamina may be continuous or short and can be aligned in one or more directions. Commonly laminae consist of continuous unidirectional fibres and this configuration of lamina is often called a ‘ply’. Composite materials consist of laminae stacked in various configurations to form laminates. The mechanical properties of the laminate depend on the number, thickness and fibre-orientation of the plies (or laminae).

The properties of laminae have been discussed in the literature review. The determination of the axial, transverse and shear stiffness by the ‘rule of mixtures’ and the more robust Halpin-Tsai model is outlined. This is covered in more detail by Hull (Hull and Clyne 1996) and other texts (Halpin 1992; Barbero 1998; Berthelot 1998).

The stress state at any point in a body is described by the stress tensor $\sigma_{ij}$, which has nine components. The use of tensors to identify stress and strain fields is widely used and readers are referred to Nye for further detail on notation (Nye 1985). The tensor $\sigma_{ij}$ is the stress acting in the ‘$i$’ direction on a plane normal to the ‘$j$’ direction. If ‘$i = j$’ then the stress is a normal stress. If ‘$i \neq j$’ then the tensor represents a shear stress. Shear stresses are often written ‘$\tau_{ij}$’ and unless the body is rotating there are only six independent shear stress tensors (because ‘$\tau_{ij} = \tau_{ji}$’).

Application of a shear stress produces angular rotations defined by the relative displacement tensor ‘$e$’ representing both rotation of the body and angular strain. These components are identified by the following expression.

$$e_y = \frac{1}{2}(e_{yy} + e_{\mu\mu}) + \frac{1}{2}(e_{yy} - e_{\mu\mu}) = e_y + \omega_y$$

Equation 3-a
Where $\varepsilon_y$ is the strain tensor and $\omega_y$ is the rotation tensor. It is important to note that the engineering shear strain ($\gamma_y$) differs from the tensorial shear strain ($\varepsilon_y$) by a factor of two. This is due to the engineering shear strain being determined using the shear modulus and a single shear stress. In most practical cases the value of the engineering shear strain recorded is due to the action of a pair of shear stresses. This relationship can be seen from Figure 3-1 and is expressed below in Equation 3-b.

$$\gamma_y = 2\varepsilon_y = \varepsilon_y + e_{ij}$$

Equation 3-b

Figure 3-1 shows three combinations of shear and rigid body rotation. Considering these conditions and Equation 3-b the relationship between the variables is described by the following equations. (In Figure 3-1, ‘i’ is the abscissa and ‘j’ is the ordinate)

Pure Shear (left-hand image) ($\omega_y = 0$) $e_y = e_{ij} = \frac{\gamma_y}{2} = \varepsilon_y$

Pure Rotation (centre image) ($\varepsilon_y = \gamma_y = 0$) $e_y = -e_{ij}$

Simple Shear (right-hand image) ($e_{ij} = 0$) $e_y = \gamma_y = 2\varepsilon_y$
The relationship between the tensorial shear strain and the stress tensor is defined by the stiffness tensor ‘C’, such that,

\[ \sigma_{ij} = C_{ijkl} \varepsilon_{kl} \]

Equation 3-c

The stiffness tensor has 81 independent constants and Equation 3-c represents nine equations. The effect of symmetry will simplify these equations in the majority of cases. A body in static equilibrium will have symmetrical stress and strain tensors and this will reduce the number of independent constants in C to 36. It has become common practice for the relationship between the stiffness, stress and strain tensors to be expressed by matrices. The relationship can be expressed as:

\[ \sigma_{p} = C_{pq} \varepsilon_{q} \]

Equation 3-d

Where ‘p’ and ‘q’ are integers between 1 and 6 with ‘p’ equivalent to ‘ij’ and ‘q’ equivalent to ‘kl’ thus translating the tensor notation into matrix notation.

<table>
<thead>
<tr>
<th>Tensor Notation</th>
<th>11</th>
<th>22</th>
<th>33</th>
<th>23, 32</th>
<th>31, 13</th>
<th>12, 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Notation</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Using this notation Equation 3-d can be written in matrix form as shown below in Equation 3-e.

\[
\begin{bmatrix}
\sigma_{1} \\
\sigma_{2} \\
\sigma_{3} \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix} = 
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\
C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\
C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{1} \\
\varepsilon_{2} \\
\varepsilon_{3} \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix}
\]

Equation 3-e
For practical applications it is often more helpful to express the observed strains in terms of applied stresses. Equation 3-d can be re-written in the form (matrix notation),

\[ \varepsilon_p = S_{pq} \sigma_q \]

Equation 3-f

Where ‘S’ is the compliance tensor. Additional expressions are needed to compensate for the relationship between engineering and tensorial strains as mentioned above.

Elastic strain energy equations allow the number of independent constants to be further reduced in number to 21. These 21 constants are for a completely anisotropic material. Most fibre-matrix composite laminae are at the very least orthotropic. Orthotropic materials have three mutually perpendicular planes of symmetry, reducing the number of independent constants to 9. The stiffness matrix presented in Equation 3-e can therefore be reduced to Equation 3-g.

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 & \varepsilon_1 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 & \varepsilon_2 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 & \varepsilon_3 \\
0 & 0 & 0 & C_{44} & 0 & 0 & \gamma_{23} \\
0 & 0 & 0 & 0 & C_{55} & 0 & \gamma_{31} \\
0 & 0 & 0 & 0 & 0 & C_{66} & \gamma_{12}
\end{bmatrix}
\]

Equation 3-g

It can be assumed that a lamina is in a plane stress state (\(\sigma_3, \tau_{23}, \tau_{31}\) are zero). This assumption is valid for thin lamina although increasing thickness will lead to a poor approximation due to the effect of through-thickness constraints. If the above assumption is made for an orthotropic lamina then Equation 3-f can be further reduced to (in matrix form) Equation 3-h.
Also Equation 3-d can be reduced to Equation 3-i.

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & 0 \\
S_{12} & S_{22} & 0 \\
0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}
\]

Equation 3-h

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & 0 \\
C_{12} & C_{22} & 0 \\
0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}
\]

Equation 3-i

Equation 3-h and Equation 3-i can be expanded to give the individual equations that can subsequently be re-written as shown below.

\[
S_{11} = \frac{1}{E_1}
\]

\[
S_{12} = \frac{-\nu_{12}}{E_1} = \frac{-\nu_{21}}{E_2}
\]

\[
S_{22} = \frac{1}{E_2}
\]

\[
S_{66} = \frac{1}{G_{12}}
\]

\[
C_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}
\]

\[
C_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}}
\]

\[
C_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}
\]

\[
C_{66} = G_{12}
\]

Where \( E_1 \) is the elastic or Young’s modulus in the fibre direction, often called the axial or longitudinal Young’s modulus. \( E_2 \) is the Young’s modulus perpendicular to the fibre direction (but within the plane of the lamina), commonly called the transverse modulus. \( G_{12} \) is the shear modulus defined as the ratio of the shear stress to the shear strain, i.e.

\[
G_y = \frac{\tau_y}{\gamma_y}
\]

It is clear from the above expressions that there is no interaction between normal and shear stresses and strains. Normal stresses result in normal strains and shear
strains are produced solely by shear stresses. This assumption is invalid in instances where the loading axes do not coincide with the principal axes of the lamina or ply. For a lamina loaded at an angle to the fibre direction but within the plane of the lamina the stress state is defined relative to the fibre axis. External stresses \((\sigma_x, \sigma_y, \tau_{xy})\) are referenced to the fibre axis \((\sigma_1, \sigma_2, \tau_{12})\). It should be noted that these are not the principal stresses conventionally represented by this notation.

![Figure 3-2 Schematic of Angle Beta between Loading Axis (x) & Major Fibre Axis (1)](image)

Figure 3-2 Schematic of Angle Beta between Loading Axis (x) & Major Fibre Axis (1)

The referencing to the fibre axes is most commonly performed using Equation 3-j that expresses a second rank tensor with respect to a new co-ordinate system.

\[
\sigma'_{ij} = a_{ik}a_{lj}\sigma_{kl}
\]

Equation 3-j

Direction cosines are represented by ‘\(a\)’. For example, \(a_{jk}\) is the direction cosine of the new ‘\(j\)’ axis referred to the old ‘\(l\)’ axis. For a loading angle ‘\(\beta\)’ between the stress axis ‘\(x\)’ and the major fibre axis ‘\(l\)’ the direction cosines are,

\[
\begin{align*}
a_{11} &= \cos \beta \\
a_{12} &= \cos(90 - \beta) = \sin \beta \\
a_{21} &= \cos(90 + \beta) = -\sin \beta \\
a_{22} &= \cos \beta
\end{align*}
\]

Equation 3-k
The direction cosines for all three applied stresses must be evaluated in a similar manner. This gives the following matrix expression.

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
\cos^2 \beta & \sin^2 \beta & 2 \cos \beta \sin \beta \\
\sin^2 \beta & \cos^2 \beta & -2 \cos \beta \sin \beta \\
-\cos \beta \sin \beta & \cos \beta \sin \beta & \cos^2 \beta - \sin^2 \beta
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
\]

Equation 3-1

The direction cosine matrix can be assigned the letter ‘T’, such that,

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
\cos^2 \beta & \sin^2 \beta & 2 \cos \beta \sin \beta \\
\sin^2 \beta & \cos^2 \beta & -2 \cos \beta \sin \beta \\
-\cos \beta \sin \beta & \cos \beta \sin \beta & \cos^2 \beta - \sin^2 \beta
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
\]

Equation 3-m

The matrix [T] can also be used to transform strains although once more allowances need to be made for the factor of two involved in engineering strain. The direction cosine matrix for transforming engineering strain becomes [T'],

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix} =
\begin{bmatrix}
\cos^2 \beta & \sin^2 \beta & \cos \beta \sin \beta \\
\sin^2 \beta & \cos^2 \beta & -\cos \beta \sin \beta \\
-2 \cos \beta \sin \beta & 2 \cos \beta \sin \beta & \cos^2 \beta - \sin^2 \beta
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

Equation 3-n
To find the strains relative to the loading direction we simply re-arrange Equation 3-n and use the inverse of \([T']\).

\[
[T']^{-1} = \begin{bmatrix}
\cos^2 \beta & \sin^2 \beta & -\cos \beta \sin \beta \\
\sin^2 \beta & \cos^2 \beta & \cos \beta \sin \beta \\
2\cos \beta \sin \beta & -2\cos \beta \sin \beta & \cos^2 \beta - \sin^2 \beta
\end{bmatrix}
\]

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = [T']^{-1} \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix} = [T']^{-1} [S] \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}
\]

We can now substitute for the stresses in the fibre direction from Equation 3-m to give,

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = [T']^{-1} [S] [T] \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
\]

Equation 3-o

We replace the matrix expression in Equation 3-o with the matrix \([S]\) called the transformed compliance matrix. The elements of \([S]\) are derived by concatenation of the three matrices; \([T']^{-1}, [S], [T]\) Thus Equation 3-o now becomes,

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = [S] \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
\]

Equation 3-p

The elements of Equation 3-p (transformed compliance tensors) can all be expressed in terms of the four elastic constants \((S_{11}, S_{12}, S_{22}, S_{66})\) and the angle \(\beta\). As \(\beta\) tends to zero then the transformed compliance matrix will tend to the original compliance matrix \((\mathbf{S} \rightarrow [S])\). The expressions for the transformed
compliance tensors are presented below along with the expressions for the transformed stiffness tensors $[\bar{C}]$, derived in a similar manner.

\[
\begin{align*}
\bar{S}_{11} &= S_{11} \cos^4 \beta + S_{22} \sin^4 \beta + (2S_{12} + S_{66}) \cos^2 \beta \sin^2 \beta \\
\bar{S}_{12} &= S_{12} (\cos^4 \beta + \sin^4 \beta) + (S_{11} + S_{22} - S_{66}) \cos^2 \beta \sin^2 \beta \\
\bar{S}_{22} &= S_{11} \sin^4 \beta + S_{22} \cos^4 \beta + (2S_{12} + S_{66}) \cos^2 \beta \sin^2 \beta \\
\bar{S}_{16} &= (2S_{11} - 2S_{12} - S_{66}) \cos^3 \beta \sin \beta - (2S_{22} - 2S_{12} - S_{66}) \cos \beta \sin^3 \beta \\
\bar{S}_{26} &= (2S_{11} - 2S_{12} - S_{66}) \cos \beta \sin^3 \beta - (2S_{22} - 2S_{12} - S_{66}) \cos^3 \beta \sin \beta \\
\bar{S}_{66} &= (4S_{11} + 4S_{22} - 8S_{12} - 2S_{66}) \cos^2 \beta \sin^2 \beta + S_{66} (\cos^4 \beta \sin^4 \beta)
\end{align*}
\]

\[
\begin{align*}
\bar{C}_{11} &= C_{11} \cos^4 \beta + C_{22} \sin^4 \beta + (2C_{12} + 4C_{66}) \cos^2 \beta \sin^2 \beta \\
\bar{C}_{12} &= C_{12} (\cos^4 \beta + \sin^4 \beta) + (C_{11} + C_{22} - 4C_{66}) \cos^2 \beta \sin^2 \beta \\
\bar{C}_{22} &= C_{11} \sin^4 \beta + C_{22} \cos^4 \beta + (2C_{12} + 4C_{66}) \cos^2 \beta \sin^2 \beta \\
\bar{C}_{16} &= (C_{11} - C_{12} - 2C_{66}) \cos^3 \beta \sin \beta - (C_{22} - C_{12} - 2C_{66}) \cos \beta \sin^3 \beta \\
\bar{C}_{26} &= (C_{11} - C_{12} - 2C_{66}) \cos \beta \sin^3 \beta - (C_{22} - C_{12} - 2C_{66}) \cos^3 \beta \sin \beta \\
\bar{C}_{66} &= (C_{11} + C_{22} - 2C_{12} - 2C_{66}) \cos^2 \beta \sin^2 \beta + C_{66} (\cos^4 \beta \sin^4 \beta)
\end{align*}
\]

The elastic response of the material is fully defined by either one of the transformed compliance matrix or the transformed stiffness matrix. Due to the widespread understanding of engineering constants it is often simpler to define the response of the material in these terms. This is relatively straightforward using Equation 3-h or Equation 3-i though using compliance tensors simplifies the expressions.

\[
\begin{align*}
E_x &= \frac{1}{\bar{S}_{11}} \\
E_y &= \frac{1}{\bar{S}_{22}} \\
G_{xy} &= \frac{1}{\bar{S}_{66}} \\
\nu_{xy} &= -\frac{E_y}{E_x} \bar{S}_{12} \\
\nu_{yx} &= -\frac{E_x}{E_y} \bar{S}_{12}
\end{align*}
\]

It was discussed previously that the interactions between normal stresses and shear strains, and also shear stresses and normal strains become significant as
‘β’ increases. These interaction terms are $S_{16}$ and $S_{26}$. These terms are used to define two further engineering constants (interaction ratios) that represent this interaction effect.

$$\eta_{xyc} = E_x S_{16}$$
$$\eta_{yyc} = E_y S_{26}$$

The interaction ratio $\eta_{xyc}$ describes the ratio of the shear strain $\gamma_{xy}$ to the normal strain $\varepsilon_x$, produced by the normal stress $\sigma_x$. The magnitude of the interaction ratio is proportional to the shear strain induced by an applied normal stress and is also dependent on the tensile stiffness in the loading (x) direction. The interaction terms are zero when the major fibre axis (1) or the transverse fibre axis (2) coincide with the loading direction (x), i.e. $\beta = 0^\circ, \beta = 90^\circ$. 
3.3 **Inverse Analysis**

Section 3.2 presented a means of characterising fibre-matrix composites using stiffness and compliance tensors to describe a set of elastic constants. When designing with composite materials these constants are generally known. In practice there are many situations where it is necessary to determine the elastic properties of the material. Inverse analysis allows the material properties to be determined in a non-destructive manner.

The structural integrity of fibre-matrix composites can be limited by defects such as delamination flaws, porosity and other forms of ‘damage’, either introduced during the manufacture of the component or resulting from an in-service event. Structural inverse analysis can be used to detect these flaws in laminated composite structures. Normally delamination and other sub-surface flaws can only be detected using laboratory-based techniques. This proves costly in terms of both equipment and the resulting downtime of the structure. Inverse analysis offers the opportunity to perform on-site structural inspections at relatively little cost by combining empirical data from static tests with data from finite element analysis. The finite element model can also be used to determine the residual strength of the structure and calculate the remaining lifetime under known service loads.

The majority of problems are ‘direct’ problems; the response is determined from known external inputs. Finite element analysis is normally performed to solve a direct problem, the response of a structure; discretised into elements of defined geometry and material properties is determined from external inputs (e.g. loads). This statement can be represented in matrix from by:

\[
\{P\} = [Z]\{U\}
\]

**Equation 3-q**

Where \(\{P\}\) represents the ‘causes’, \(\{U\}\) the ‘response’ and \([Z]\) the ‘system’ matrix (defining the geometry and materials). Generally, the solution of Equation 3-q is well defined and the vast majority of structural problems solved by the finite element method take this form. In conventional FE analysis the responses,
\{U\}, represent the calculated displacements for a given stiffness matrix, \([Z]\), from a prescribed set of boundary conditions \(\{P\}\).

Consider a problem of similar form where either the causes \(\{P\}\) or the system \([Z]\) are unknown and must be determined from the response \(\{U\}\). This is termed an inverse problem (Hensel 1991). Inverse problems take two general forms (Laermann 1998). If \(\{P\}\) contains unknowns, the problem is an inverse problem of the first kind and boundary conditions can be formulated as unknowns. In cases where the system \([Z]\) contains unknowns, this belongs to inverse problems of the second kind. Problems that involve the determination of material properties take this form where \([Z]\) is the stiffness matrix containing unknowns. The standard tensile test method for determining the Young’s Modulus of steel and other metals (dog-bone specimen) is a basic inverse problem of the second kind. These types of inverse problems can be under-deterministic due to the response \(\{U\}\) containing fewer elements than the system \([Z]\). Additional information is often needed and can be obtained from further unequivocal testing or by adding constraints to the solution based on scientific experience.

In a finite element (FE) model the structure is divided into elements with each element having characteristics defined in terms of the stiffness matrix. The stiffness matrix typically depends on the geometric properties of the elements and on the extensional modulus, \(E\) and the shear modulus, \(G\).

Cracks and delaminations for example, can be represented by de-equivalencing the mid-side node between two, eight-sided quadrilateral elements (Figure 3-3a). A longer crack can be considered as a series of adjacent de-equivalenced noded cracks in a line, although in practice the crack would open as suggested by the dashed line in Figure 3-1b.

(a) Single de-equivalenced noded crack. (b) Longer de-equivalenced crack.

Figure 3-3 Crack representation in FE models by de-equivalencing of mid-side nodes.
A solution is required for the linear system defined as:

\[ g(x, y) = \zeta \cdot f(x, y) \]

Where \( g(x,y) \) is the damaged FE model and \( \zeta \) is the mathematical operator acting on the input \( f(x,y) \) that represents the solution to each individual pair of de-equivalenced mid-side nodes in the FE model. The solution for each pair may be determined and then used to find \( \zeta \) by selecting combinations of \( f(x,y) \) that yield the minimum difference between the response of the FE model and the damaged structure.

The inverse analysis procedure begins with the gathering of empirical data, for example via a series of unequivocal static tests. Usually, inverse problems are poorly defined and a significant amount of data is required to solve for the unknowns in an over-deterministic sense to obtain reliable solutions. For each test, the responses (displacements, strains or stresses) due to a series of known inputs (loads or deflections) are measured. It is important to understand that the input load or deflection does not have to be similar in magnitude to the service load of the structure. In many cases, the applied load or deflection can be orders of magnitude lower than the loads or deflections usually experienced. The responses of the structure can be measured by a variety of means; transducers, strain gauges, whole-field optical techniques to name a few. System identification techniques are then used to determine the damage in the FE model by degrading the orthotropic constants of elasticity for certain elements or by the de-equivalencing of nodes to create cracks. Stiffness values in the mesh are adjusted so that the results of the FE model match the measured response in an appropriate manner. It is worth noting that the FE model is the form taken by the optimal solution produced by the inverse analysis. The FE model can now be used for further analysis of the structure or component, for example by determining the residual strength or predicting the remaining life under actual service loads.

Inverse analysis in this form was first applied to large civil engineering structures that could be considered as a framework model. Response data was obtained from strain gauges attached to the individual members of the framework and the stiffness of each was calculated to determine any damage. Damaged composite
Structures differ in many ways from the civil-engineering frameworks for which inverse analysis was first developed. Fibre-matrix composites made up of several individual laminae are inherently three-dimensional with orthotropic properties whereas the early framework models were often considered one-dimensional. Line/bar elements are typically used to construct FE models of frameworks with only a single value of elastic modulus to characterise the element stiffness. Conversely, the three-dimensional nature of a composite structure allows the FE model to have more degrees of freedom and to work with two or three-dimensional elements. The elements in the composite FE model may be sized to resolve defects of a certain scale at pertinent locations. The disadvantage is that there must be sufficient empirical data collected to be able to solve the prescribed set of unknowns and this may be a function of the number of elements in the FE mesh.
3.4 Speckle Shearing Interferometry

In order to perform inverse analysis, particularly when using the model-updating approach, a relatively large quantity of empirical data is needed to quantify the response of the material. This data can be obtained from a variety of methods however optical techniques are 'whole-field' allowing a vast amount of data to be collected for a single test. Additionally, optical methods are non-destructive and non-contacting and the chosen method is already used in damage detection applications.

Speckle Shearing Interferometry (SSI) is also called shearing interferometry and shearography. The technique uses the phenomenon of 'laser speckle' by combining two speckle fields to give interference fringes. It is closely related to other speckle methods such as Electronic Speckle Pattern Interferometry (ESPI) although tends to be more robust. Interested readers are referred to Wykes (Wykes 1982) for information on using ESPI to measure displacements and a review paper by Petzing and Tyrer (Petzing and Tyrer 1998). SSI allows the determination of displacement derivatives (strains) to be determined directly. Its use has increased as PC technology and image-processing software has become more powerful. SSI was originally developed at Loughborough University by Leendertz and Butters (Leendertz and Butters 1973) to overcome errors experienced with ESPI measurements due to environmental vibrations.

Figure 3-4 shows an optically rough target illuminated by a collimated beam of laser light, the incident light is scattered by the surface of the target. Each point
on the CCD (charge-coupled device) array receives an optical wave made up of many waves received from different points on the target. The distance travelled by the component waves can vary from zero to many wavelengths depending on the roughness of the surface and the geometry of the target. Interference of the waves arriving at an arbitrary point on the CCD array cause the intensity of light at that point to be anything from totally dark to very bright. An adjacent point on the CCD array is likely to have a different brightness and this variation is the cause of laser speckle. This type of speckle pattern is termed ‘objective speckle’. It is important to understand that ‘laser speckle’ is an interference phenomenon in its own right.

Speckle techniques rely on the speckle moving with the surface of the target for two known inputs (e.g. loads). Two speckle patterns, one for each input are analysed to derive a result. The two speckle patterns must stay marginally superimposed in order to stay correlated. Speckle decorrelation occurs when the original speckle moves so far that it cannot be related back to its original position. This places limits on the magnitude of the range of deformation or movement that can be recorded.

The technique used in this investigation utilised a Michelson interferometer that intercepted the object wave front before it passed to a CCD camera. The Michelson interferometer is a well-known device in optical engineering and is shown in Figure 3-5. It consists of a beam-splitter cube and two adjustable mirrors.
The wave front from the specimen or target passes into the interferometer and is split into two. One beam is incident on mirror 1 and is reflected back through the cube to the CCD array. The other half of the wave front is incident on mirror 2 that is positioned at a slight angle ($\theta$) to either the horizontal or vertical axis to induce a small vertical or horizontal shear. The reflected beam passes back through the beam splitter and is incident on the CCD camera. The beam that travels through to the 'fixed' mirror (1) acts as a reference beam for the beam that is incident on mirror 2.

It should be noted that the difference in angle between the reference beam and the beam reflected by the tilted mirror is $2\theta$. The two beams recombine at the CCD camera to produce a further speckle pattern. The use of the Michelson interferometer makes SSI a common path type of interferometer, which explains its robustness and relative immunity to environmental conditions.

Each pixel on the CCD camera effectively receives light from two different points on the object wave front. In a similar manner to ESPI, intensity maps for the initial and deformed states are recorded. The intensity map of the deformed state is subtracted from the intensity map of the initial state to give a fringe map. In SSI the images relative to the CCD plane are sheared by amounts $\Delta x$ or $\Delta y$ depending on the tilt angle of mirror 2 ($\theta$). The resultant sheared fringe patterns are sometimes referred to as 'shearograms'.

When the object is deformed an arbitrary point (A) on the object surface is displaced to a new position (A') and an adjacent point (B) is displaced to (B') where:-

\[
A = (x, y) \\
B = (x + \Delta x, y) \\
A' = (x + \Delta x, y + \Delta y, w) \\
B' = (x + \Delta x + u + \Delta u, y + \Delta v, w + \Delta w)
\]
Deformation of the object introduces a phase change \( \phi \) between the two points; this can be expressed in relative terms by Equation 3-r for a small horizontal shear \( \partial x \).

\[
\Delta \phi(x, y) = \frac{2\pi}{\lambda} \partial x \left[ \sin \theta \frac{\partial u}{\partial x}(x, y) + (1 + \cos \theta) \frac{\partial w}{\partial x}(x, y) \right]
\]

Equation 3-r

Where \( \partial w/\partial x \) is the first-order partial derivative of the out-of-plane displacement and \( \partial u/\partial x \) is the first-order partial derivative of the in-plane displacement of the object. A similar expression is used for vertical shear (Equation 3-s).

\[
\Delta \phi(x, y) = \frac{2\pi}{\lambda} \partial y \left[ \sin \theta \frac{\partial u}{\partial y}(x, y) + (1 + \cos \theta) \frac{\partial w}{\partial y}(x, y) \right]
\]

Equation 3-s

The in-plane strain term \( (\partial u/\partial x) \) can be eliminated by illuminating the target normal to its surface. For normal illumination the following expression is used (for horizontal shear).

\[
\frac{\partial w}{\partial x}(x, y) = \frac{\lambda}{2\partial x}
\]

Equation 3-t
3.4.1 Phase Stepping

Consider Figure 3-5 where mirror 1 functions as the reference arm of the Michelson interferometer and mirror 2 provides the shear, \( \delta x \) or \( \delta y \) as described above. The objective speckle is displayed on the CCD camera; each pixel will have a measurable intensity. However, the intensity of the pixels can only be related to a phase ranging from \( 0 \) to \( 2\pi \), it is modulo \( 2\pi \). The fringe maps can be interpreted by counting the fringes to determine the fringe order. Then multiplying this by a factor related to the set-up geometry, wavelength and in some cases material properties. This process tends to be slow and laborious and sometimes there are insufficient fringes to allow interpolation. The main reason that this approach has worked so well for so long is the ability of the human eye to identify the centre of fringes. With the improvement in PC and image processing technology an obvious ‘improvement’ was to replicate the human eyes means of detecting fringe centres using digital processing. However, this method has been proved ineffective in both consistently mapping fringe patterns and determining the fringe order. The technique still requires human input to function reliably and is still unable to process fringe patterns containing noise, including speckle noise.

Instead of trying to mimic the human eye, a more robust approach is to utilise the PC-based system in a manner for which it was designed. A CCD camera allows a good intensity map to be captured and stored on the computer accurately in a very short time period. The PC can perform complex calculations on the images and deliver the results in pictorial form. The issue is how to derive results from the original captured intensity maps. This is where phase stepping has been utilised although the technique has been widely used for many years in other optical applications.
Chapter 3: Background Theory

Figure 3-6  Schematic of Interferometer with Phase Step/Shift

Consider the 'ideal' interferometer shown in Figure 3-6 where the objective is to determine the out-of-plane displacement ($d_z$) of a single point P on the target object. The object is illuminated by coherent light ($I_i$) striking the target at normal to its surface. A scattered object wave ($I_o$) travels from P to a point detector, Q. The interferometer is completed by a reference beam ($I_r$) incident on Q and close to the optical axis. This creates a series of interference fringes at Q indicated by the black and grey lines in Figure 3-6. Assume that a dark fringe (black line) passes through the detector, Q that outputs a minimum signal indicating an intensity ($I_1$) close to zero. The target object is then deformed so that point P is displaced a distance $d_z$ along the optical axis to a new position P'. The interference fringe pattern moves relative to the detector, Q because the object wave ($I_o$) has undergone a phase change ($\phi_2$) given by:

$$\phi_2 = \frac{2\pi}{\lambda} 2d_z = \frac{4\pi d_z}{\lambda}$$

The detector, Q will output a new signal, $I_2$, which can be expressed in terms of $I_1$, $\phi_2$ and $I_{max}$ (the maximum signal output from Q).

$$I_2 = \frac{I_{max}}{2} + I_1 + \frac{I_{max} - I_1}{2} \cos \phi_2$$

Equation 3-u
However Equation 3-u contains two unknowns ($\phi_2$ and $I_{\text{max}}$) and thus further information is needed to solve it. This information is provided by introducing a known phase-shift ($\phi_{\text{shift}}$) into the reference beam ($I_r$) of the interferometer. The output signal from detector ($I_3$) is recorded for this known phase-step.

\[
I_3 = \frac{I_{\text{max}} + I_1}{2} + \frac{I_{\text{max}} - I_1}{2} \cos(\phi_2 + \phi_{\text{shift}})
\]

Equation 3-v

Equation 3-u and Equation 3-v are two independent equations that can be solved to obtain our two unknowns ($\phi_2$ and $I_{\text{max}}$) and $\phi_2$ can be evaluated to give $d_z$.

The problem is represented graphically in Figure 3-7.

![Graph of Detector Output against Phase](image)

The problem presented here is an oversimplification of the phase-stepping technique. It would be unlikely that the initial output of the detector would be a totally dark fringe and therefore this output would not represent the minimum intensity. In most cases the output of the original state of the object is being subtracted from the deformed state so everything is in the form of a phase difference.
Additionally, the example presented is for a single narrow beam interferometer. For intensity maps that can be many thousands of pixels in size the principle remains the same but obviously greater computational effort is required. Further complexity is introduced if the displacement of the object is greater than half a wavelength. Phase changes greater than $\lambda/2$ but less than $\lambda$ require careful use of positive and negative signs in the equations. It can be seen from Figure 3-7 that for the intensity $I_i$ recorded at Q there are multiple possible positions for the displaced point P represented by $P'$, $P''$ and $P'''$.

Phase stepping for the SSI set-up described previously is accomplished by introducing the phase-step in the reference arm of the Michelson interferometer. In practice this is achieved by mounting mirror 1 of the Michelson interferometer (Figure 3-5) on a piezo-electric transducer (PZT).

3.4.2 Determination of the Phase from Phase-stepped Intensity Maps

Intensity maps of most interferometric measurements can be represented by Equation 3-w.

$$I(x, y) = I_0(x, y) + I_M(x, y) \cos \phi(x, y)$$

Equation 3-w

Where $I_0$ is the background intensity, $I_M$ is the modulation intensity and $\phi$ is the phase. $I(x,y)$ is the intensity at point $(x,y)$ of the image, or pixel $(x,y)$ of the CCD array. Equation 3-w contains three unknowns. The introduction of a known phase-step is a common technique in solving for the three unknowns, interested readers are referred to Cloud and Huntley for further examples (Cloud 1995; Huntley 2001).

In this instance the popular Four-Frame Algorithm was used to solve the simultaneous equations for $I_0$, $I_M$ and $\phi$. As the name suggest the technique involves the capture of four intensity maps (in this case shearograms) denoted by $I_{x,y}(N)$ where N=4. For each image a phase step is introduced ($\alpha$). Considering an arbitrary pixel for clarity Equation 3-w can be re-written in the form,
\[ I(N) = I_0 + I_M \cos(\phi + N\alpha) \]

The value of \( \alpha \) is generally chosen to be \( \pi/2 \) although in some texts this is referred to as \( \lambda/4 \). The above expression can be expanded into four simultaneous equations.

\[
\begin{align*}
I(0) &= I_0 + I_M \cos(\phi) \\
I(1) &= I_0 + I_M \cos(\phi + \frac{\pi}{2}) = I_0 - I_M \sin \phi \\
I(2) &= I_0 + I_M \cos(\phi + \pi) = I_0 - I_M \cos \phi \\
I(3) &= I_0 + I_M \cos(\phi + \frac{3\pi}{2}) = I_0 + I_M \sin \phi
\end{align*}
\]

Re-arranging the above leads to expressions for the phase and the modulation.

\[
\begin{align*}
\phi_w(0) &= \tan^{-1} \left[ \frac{I(3) - I(1)}{I(0) - I(2)} \right] \\
I_M(0) &= \sqrt{\frac{[I(3) - I(1)]^2 + [I(0) - I(2)]^2}{2}}
\end{align*}
\]

Equation 3-x

The above expressions are really estimations of the true values. Systematic and random measurement errors, particularly in the calibration of the phase-shift device (PZT) introduce errors. The phase is assigned the subscript 'w' to indicate that it is wrapped; this is explained in the following section.
3.4.3 *Two-Dimensional Phase Unwrapping*

The solution for the phase presented in the preceding chapter does not necessarily indicate the absolute phase for the grid of points or pixels. This is due to the periodic nature of the functions used in its calculation. It is the ‘wrapped’ phase with modulo $2\pi$ and is more correctly identified by the symbol $\phi_w$ or $\gamma$. The method of two-dimensional phase unwrapping presented here is based on the work by Ghiglia and Romero (Ghiglia and Romero 1994) and thus their nomenclature is used, $\gamma$, where,

$$\gamma_{i,j} = \phi_{i,j} + 2\pi k$$

$$-\pi < \gamma_{i,j} \leq \pi$$

$$i = 0,1,2,......,M-1$$

$$j = 0,1,2,......,N-1$$

Where $k$ is an integer and $(i,j)$ defines the grid location. Given the wrapped phase values we need to determine the unwrapped phase values ($\phi_{i,j}$) that correlate in the least squares sense. The first step of the process is to determine the phase difference between adjacent pixels or grid locations. A wrapping operator ‘$W$’ is defined that wraps all values of its argument to the range $-\pi \rightarrow +\pi$ by addition or subtraction of an integral number of $2\pi$ radians from the argument, i.e.

$$W\{\phi_{i,j}\} = \gamma_{i,j}$$
This allows the computation of phase differences with respect to both indices to be expressed as,

\[
\Delta_{i,j}^x = W \{ \psi_{i+1,j} - \psi_{i,j} \} \\
i = 0 \ldots M - 2 \quad \text{otherwise, } \Delta_{i,j}^x = 0 \\
j = 0 \ldots N - 1
\]

\[
\Delta_{i,j}^y = W \{ \psi_{i,j+1} - \psi_{i,j} \} \\
i = 0 \ldots M - 1 \quad \text{otherwise, } \Delta_{i,j}^y = 0 \\
j = 0 \ldots N - 2
\]

Where the superscripts 'x' and 'y' refer to the phase differences in the ‘i’ and ‘j’ indices respectively.

The solution \( \phi_{i,j} \) that minimises the following expression is the least-squares solution.

\[
\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (\phi_{i+1,j} - \phi_{i,j} - \Delta_{i,j}^x)^2 + \sum_{i=0}^{M-1} \sum_{j=0}^{N-2} (\phi_{i,j+1} - \phi_{i,j} - \Delta_{i,j}^y)^2
\]

This can be represented by a simplified equation,

\[
\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1} - 4 \phi_{i,j} = \Delta_{i,j}^x - \Delta_{i,j-1}^x + \Delta_{i,j}^y - \Delta_{i,j-1}^y
\]

Equation 3-y

And also expressed as,

\[
p_{i,j} = (\phi_{i+1,j} - 2 \phi_{i,j} + \phi_{i-1,j}) + (\phi_{i,j+1} - 2 \phi_{i,j} + \phi_{i,j-1})
\]

Equation 3-z

where

\[
p_{i,j} = (\Delta_{i,j}^x - \Delta_{i-1,j}^x) + (\Delta_{i,j}^y - \Delta_{i,j-1}^y)
\]

Equation 3-aa
Chapter 3: Background Theory

The expression of \( p_{i,j} \) in terms of \( \phi \) is a discretised form of Poisson's equation for an M x N grid, with Neumann boundary conditions,

\[
p(x, y) = \frac{\partial^2}{\partial x^2} \phi(x, y) + \frac{\partial^2}{\partial y^2} \phi(x, y)
\]

The Neumann boundary conditions are directly imposed on the discretised version by the least-squares expression and the limits imposed on the phase difference, \( \Delta \).

Ghiglia and Romero proposed the following algorithm for two-dimensional phase unwrapping.

a. Operate a two-dimensional forward discrete cosine transform (DCT) on the array of values, \( p_{i,j} \) to produce the transformed values, \( \hat{p}_{i,j} \).

b. Modify the values \( \hat{p}_{i,j} \) in the transform plane to obtain \( \hat{\phi}_{i,j} \).

c. Perform the inverse DCT on \( \hat{\phi}_{i,j} \) to obtain the least squares unwrapped phase values, \( \phi_{i,j} \).

Fourier transforms, although complex in appearance find many uses in practical physics and engineering. Generally only a background understanding of the transforms is required in order to apply them effectively. An excellent introduction to Fourier transforms is provided by James (James 1995) and a more in-depth source of reference is authored by Walker (Walker 1991).

Generally the Fourier Cosine Transform (FCT) takes the following form (Yip 1996) for a function \( f(t) \),

\[
FCT[f(t)] = F_c(\omega) = \int_{-\infty}^{\infty} f(t) \cos(\omega t) dt
\]

and the inverse FCT takes the form (for \( t \geq 0 \)),

\[
FCT^{-1}[F_c(\omega)] = \frac{2}{\pi} \int_{0}^{\infty} F_c(\omega) \cos(\omega t) d\omega
\]
The functions $f(t)$ and $F_\omega(\omega)$ are said to form a FCT pair. The discrete cosine transform (DCT) is a discretised approximation of a FCT. The theory and application of DCT’s is covered in great detail by Rao and Yip (Rao and Yip 1990), including their use on two-dimensional arrays. DCT’s take various forms but are identifiable because they replace the integral sign of the FCT with a summation.

Ghiglia and Romero employed a two-dimensional DCT of the form,

$$
C_{m,n} = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} 4x_{i,j} \cos \left( \frac{\pi}{2M} m(2i+1) \right) \cos \left( \frac{\pi}{2N} n(2j+1) \right)
$$

$$
0 \leq m \leq M - 1
$$

$$
0 \leq n \leq N - 1
$$

$$
= 0, \text{ otherwise}
$$

Equation 3-bb

The inverse DCT of Equation 3-bb takes the form,

$$
x_{i,j} = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \omega_1(m) \omega_2(n) C_{m,n} \cos \left( \frac{\pi}{2M} m(2i+1) \right) \cos \left( \frac{\pi}{2N} n(2j+1) \right)
$$

$$
0 \leq i \leq M - 1
$$

$$
0 \leq j \leq N - 1
$$

$$
= 0, \text{ otherwise}
$$

Equation 3-cc

Where,

$$
\omega_1(m) = \begin{cases} 1/2 & m = 0 \\ 1 & 1 \leq m \leq M - 1 \end{cases}
$$

$$
\omega_2(n) = \begin{cases} 1/2 & n = 0 \\ 1 & 1 \leq n \leq N - 1 \end{cases}
$$
For the desired solution, \( \phi_{i,j} \), Equation 3-cc is written in the form,

\[
\phi_{i,j} = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \omega_1(m) \omega_2(n) \hat{\phi}_{m,n} \cos \left( \frac{\pi}{2M} m(2i+1) \right) \cos \left( \frac{\pi}{2N} n(2j+1) \right)
\]

Equation 3-dd

Equation 3-dd can then be substituted into Equation 3-z. A similar procedure of expansion and substitution can be carried out for \( p_{i,j} \) (Equation 3-aa) to give an exact solution for the phase in the DCT domain, Equation 3-ee.

\[
\hat{\phi}_{i,j} = \frac{\hat{p}_{i,j}}{2 \left( \cos \frac{\pi i}{M} + \cos \frac{\pi j}{N} - 2 \right)}
\]

Equation 3-ee

The unwrapped phase can now be determined by the inverse DCT of Equation 3-ee. Ghiglia and Romero proved that this method automatically imposes the equivalent of Neumann boundary conditions on the solution.

Huntley identifies potential problems with this approach (Huntley 2001). Localised areas of noise can cause underestimates in the phase values of nearby noise-free areas. Huntley provides an example of this by performing the algorithm on a noisy phase map and then re-wrapping the output. The rewrapping did not generate a phase map identical to the initial one; rather it underestimated the initial state by around 20%. This problem can be improved by the introduction of weighting factors to account for regions of data. Ghiglia and Romero presented a method for this. The major drawback is that the unwrapping algorithm becomes iterative and increased computational effort is required, typically an order of magnitude increase.
4 DESIGN OF A LIGHTWEIGHT TELEHANDLER BOOM

4.1 Introduction

The telescopic handler or telehandler first appeared in the late 1970's. Although it was a new concept, it borrowed much of its technology from earlier vehicles, such as the rough-terrain forklift. Combined with clever marketing and product engineering the design of the first telehandler was already very advanced. Compelling evidence for this is the machine itself; the overall design has changed little in the intervening years (Figure 4-1).

Telehandlers were an instant success, their forward reach allowed loads to be positioned over obstacles, even at height. The construction industry recognised the value of the telehandler for on-site materials handling. The focal point of the machine is a telescopic arm, usually with two or three hollow sections or booms. Each boom is of rectangular cross-section up to five metres in length. At the time of writing telehandler booms are exclusively made of steel.

Telehandlers are produced for two major markets; agriculture and construction. Agricultural machines tend to be smaller and typically have only two boom sections or stages. Construction machines are often called ‘pick and place’
machines and typically have three or four stages and a greater payload. The operating envelope for these machines is massive with lift heights of over 13 metres and forward reach of almost 10 metres. It was these ‘pick and place’ construction machines on which the lightweight boom would deliver the maximum benefit.

Telehandler performance is indicated by a load-chart provided by the manufacturer. The load-chart indicates the limits within which a certain payload can be lifted safely. It also indicates the payload at the extremities of the operating envelope. The safe operating limits are determined by the point at which the load causes the machine to become unstable, with an applied factor of safety. The rated value on the load chart is commonly around eighty percent of the actual tipping load.

Figure 4-2 Telehandler Load-Chart (JCB-Sales 1999)

Most telehandlers have excellent performance when the boom is retracted due to the relatively low moment that the payload exerts about the front axle of the machine. As the boom extends especially when reaching forward the moment of the payload increases greatly. Additionally, the weight of the boom sections and the various components (dead-weight) now acts in front of the axle increasing the tipping moment about the front axle still further. This tipping moment is
countered by the mass of the components within the main chassis of the vehicle plus the addition of a counter-weight at the rear. To give an indication of the significance of this dead-weight it is estimated that for every brick lifted at full outreach the machine lifts the equivalent of four bricks in dead-weight.

It is obvious that reducing the mass of the boom components will deliver performance benefits. This was the driving force behind the research into the utilisation of lightweight materials for the boom components. Again performance for these types of machine are measured using a load-chart, improvements could be made in one of three areas:-

1) Increasing the payload for the same load-chart reach envelope.
2) Increasing the reach envelope for the same load-chart payloads.
3) Reducing the counter-weight giving a cost saving for the same performance.
4.2 Design Considerations

The popularity of telescopic handlers is largely due to their versatility, it is easy to change between attachments and the machine can perform a wide variety of tasks. This presents great challenges to companies trying to design or develop the machines. The seemingly endless number of applications generate many and varied forces on the machine structure and particularly the boom. Producing a machine that can withstand all of the varied applications for an acceptable working life (usually ten years) is a difficult task that requires a finely balanced compromise between the desirable characteristics.

The boom needs to withstand the bending stresses and dynamic loads incurred when the boom angle is adjusted sharply or 'judder' occurs. Not only must the boom provide the strength required to lift the load but it must also be rigid enough to prevent an unacceptable amount of deflection. What is a reasonable amount of deflection? This is a subjective judgement that is difficult to quantify, often a perception of the end-user. To design a boom to do pure lifting work is relatively simple. In the majority of cases, the limiting factor for lift at the extremes of reach and height is the stability of the machine. The machine is likely to overbalance long before the stresses in the boom rise to critical levels. The design of the boom can be based around the design of cranes, which is covered by numerous international standards. This does not account for the additional loads experienced by telehandlers but provides a suitable starting point. It is possible to put a significant torsional load on the boom when using forks or large buckets. The transportation of off-centre or extremely wide loads over rough terrain is an example. This exerts combined dynamic bending and torsional loads on the boom (also encountered during re-handling work).

It is when a bucket is fitted and the machine is used for re-handling, back grading or digging that the most complex loads are encountered. The boom is exposed to torsional, lateral bending and compressive loads in numerous combinations and these loads are cyclic and dynamic. There are no design standards that apply and the boom designs are largely based on design experience and strain gauge data.

Other applications that induce unusual loads include using the crab steer mode while cleaning up material against a wall or other barrier. This induces lateral bending into the boom. Some applications require the boom to be fitted with a
rock-hammer. Whilst this is only rare, the load and vibration place large stresses on the boom and accelerate fatigue.

As mentioned previously the variety of applications that telehandlers are used for result in the machine being exposed to a vast array of environmental conditions. These range from extremes of temperature to abrasive particles such as sand and cork dust. This is a major factor in the choice and success of any new material that must maintain its mechanical properties in extreme environments.

The complexities of the various loading configurations have largely been bypassed by developing a test for the worse case scenario. Named the 'tear-out' test, it involves driving the machine into a steel post set deep into the ground. The machine is fitted with a bucket and driven in first gear against the post with the boom fully extended. At the same time as applying full tractive effort, the driver attempts to lift the boom and crowd back the bucket. The bucket is positioned so that the post is offset from the centre of the boom axis. This places maximum strain on the boom, chassis and hydraulics and is a good indication of machine strength. The loads are significant, as the tractive effort in first gear is approximately 1.5 tonnes at the wheels (assuming a friction factor, $\mu$ of 0.8 with the ground). The combination of boom lift, tractive force and torsion due to the offset load is the worst example of machine abuse and the most strenuous test of boom strength.
4.3 Lightweight Materials

At the time of writing, all telehandler booms are exclusively made from steel. A thorough investigation was performed to determine the suitability and possible benefits of utilising advanced materials. It is possible to reduce the weight using higher grades of steel; quite simply by using stronger steel you require less material and thus less mass. The limiting factor in this case is often buckling and thus the maximum weight reduction for this method is around 18%.

Other metallic alloys were considered, most notably aluminium and its alloys. Aluminium alloys are utilised in the manufacture of telescopic booms for access platforms. They do deliver a weight saving but this is compromised by an accompanying drop in rigidity. Other metals were considered including Magnesium, Titanium, Beryllium and Lithium alloys. Titanium offers little benefit, as although it has a higher yield strength than steel the buckling of the boom section is the limiting factor on wall thickness giving no opportunity for weight reduction. Magnesium alloys most often contain aluminium (typically up to 9%) and can be manufactured using techniques similar to the alloys of aluminium. Magnesium alloys have a relatively high cost and this is usually offset by their ease of machining. Since telehandler booms are fabricated from plate there is little machining required and thus the benefits of Magnesium alloys cannot be fully exploited. The alloys of Beryllium and Lithium are as yet used very little outside the aerospace industry. The knowledge base is still growing for these metals and commercial availability is still some way off.

The other major group of materials under consideration was composite materials. A simple definition of the term ‘composite material’ is a material consisting of two or more constituent materials combined so that the material properties of the whole are better than those of the individual constituents. This encompasses a wide range of diverse materials including metal-matrix composites, sandwich structures and fibre-matrix composites.

The use of metal-matrix composites (MMC’s) is almost exclusively limited to the aerospace sector. MMC’s consist of a base metal and a filler material. The base metal is usually a lightweight metal alloy, commonly aluminium. The filler material is either particles or short fibres but can be continuous fibres. Currently particular fillers are the most common (e.g. Alumina or Silicon Carbide with an
aluminium alloy base). MMC's can approach the mechanical properties of steel yet still deliver a weight saving of up to 40%. However, they are still a relatively new technology, availability and cost eliminates them from this application for the foreseeable future.

Sandwich Structures or stressed-skin composites consist of two outer skins bonded to an inner core. Core materials fall into two categories, firstly solid core materials such as lightweight foams or ceramics. These are used when the properties of the core material creates a sandwich structure for a specific function, for example fire resistant panels or insulating panels. The second category is of more relevance to the project, the core material is a honeycomb produced from steel, aluminium or aramid. This type of sandwich structure has very high strength-to-weight and stiffness-to-weight ratios due to their increased second moment of area. Sandwich structures are now widely used in the construction industry as structural panels and flooring. The use of this type of composite for a telehandler boom would raise several issues. Due to the nature of the material, the fabrication of the boom would have to rely on adhesive bonding. The ability of the sandwich structure to withstand in-plane loading was also a major concern. Finally, the impact resistance and performance of the honeycomb structures was relatively poor and internal flaws would be difficult to detect. These factors counted against this type of composite for this application.

The final group of composite materials is fibre-matrix composites or fibre-reinforced plastics (FRP). These are by far the most common and have filtered down from aerospace into 'lower technology' industries. The fibres are arranged within a matrix and carry the majority of the load. They can be aligned to optimise the mass of the structure. The matrix or resin serves to hold the fibres in place, protect them and help transmit the load to the fibres. The scope for design with this type of material is massive, different fibres can be used (e-glass, s-glass, carbon, aramid) in a variety of arrangements and in combination with different matrix materials, commonly epoxies, polyesters and vinyl esters. It was this flexibility and the availability of the technology that proved attractive for this application.

The material of choice for the lightweight telehandler boom was fibre-matrix composites. This was due to the availability of the materials, the scope for design
and the potential for weight reduction of up to 60% (versus the current steel boom).
4.4 Impact Behaviour of Composite Beams

It was identified at an early stage in the lightweight booms project that there was little data available on the behaviour of composite box-section beams under transverse, quasi-static impact. The research that had been undertaken tended to concentrate on the energy absorption of the composite section and not its post-impact performance. It was recognised that there were two likely sources of impact damage, tool drops and transverse impact.

Tool drops are self-explanatory and there was no further data gathered in this area for several reasons. Firstly, because there was significant research available as the problem is significant in the aerospace sector where thin-walled composite shell structures are widely used. Secondly, the telehandler boom would have a much greater wall thickness and be more resistant to this type of event. Thirdly, the likelihood of an item such as a tool falling onto the structure is unlikely during production.

Transverse impact was the area of greatest concern. Consider the telehandler in operation lowering a payload onto a platform at height. The operator could conceivably lower the boom onto the platform edge. The edge of the platform would strike the lower surface of the boom normal to its longitudinal axis. Since this face of the boom is in compression, the effect of any damage could be catastrophic. In addition, there was little research relative to this type of impact. What little there was tended to focus on the energy absorption of the structure rather than its post-impact residual strength.

A test programme was devised to investigate the damage and failure mechanism of composite box-section beams under this type of impact. The samples used were commercially available pultruded box-section beams. This gave a good approximation to the anticipated final lay-up of the boom with a high percentage of unidirectional fibres parallel to the longitudinal axis of the section.
4.4.1 Experimental Procedure

The samples selected were pultruded box-section beams with external dimensions of 51mm x 51mm and a nominal wall thickness of 3.2mm. The sections consisted of unidirectional glass fibres sandwiched between outer layers of chopped strand mat (randomly orientated short fibres). The fibres were bound in an epoxy resin and the outer surface covered by a thin veil resistant to ultra-violet light (Figure 4-3).

![Figure 4-3 Schematic of Fibre Lay-Up of Test Sample](image)

The sections were produced in six metre lengths although the selected length for the sample was 600mm. This length was chosen so the width-to-length ratio of the sample was representative of the width-to-length ratio of the current telehandler boom. The anticipated impact event involved the lowering of the boom onto an immovable object such as a wall at relatively low velocity. It was decided that the three-point bend test was a reasonable compromise between simulating this event and simplicity. The boom would be simply supported at either end and struck mid-span by a knife-edge type impactor. The impact speed would be quasi-static; some authors have referred to this type of test as ‘crushing’ (Mahmood, Jeryan et al. 1990).

For each test the sample would be placed in a purpose-built test rig mounted on a hydraulic tensile test machine (Figure 4-4). The sample would then be struck by...
the impact, the severity of the impact governed by limiting the displacement of the impactor. The resulting event is a combination of quasi-static impact and crushing of the section. This corresponds closely with the anticipated event in the field.

Figure 4-4  Hydraulic Test Machine and Test Rig

Information was gathered both visually and by outputting the test data to an X-Y plotter. The plotter recorded the load-displacement curve of the impactor. Initial runs were used to identify the peak force of the impact and the displacement at which it occurred.

4.4.2 Results of Investigation

Visual observation of the tests revealed that the sample failed in a manner unique to this type of composite materials. Referred to as ‘progressive tearing’ (Palmer, Bank et al. 1998) the failure involves the separation of the impacted surface from the shear webs (side walls) of the section. This tearing occurs due to the high content of unidirectional fibres aligned parallel to the longitudinal axis of the sample. When load is applied to the upper surface the material attempts to transfer the load into the shear webs. This creates high shear stresses at the upper corner of the sample that exceed the shear strength of the composite, provided mainly by the matrix.

The initial behaviour of the sample was linear until cracks began to propagate. Cracking could be heard before any visible signs could be observed. The peak
force was consistently around 6KN corresponding to a displacement of between 9 and 10mm. At this peak force catastrophic tearing failure occurred as matrix cracks grew along the boundary of the top flange and the shear webs. As impacter displacement increased the cracks grew and the impacter began to perforate the shear webs (in-plane crushing). This crushing of the shear webs, due to their poor in-plane strength, resulted in V-shaped notches forming and fibre-breakage around the impact site.

Figure 4-5 shows how the damage occurs. The impacted surface, X (shown in red) is deflected more than the remaining faces of the sample. X is separated from the two shear webs, Y by the matrix cracks described above. As the displacement of the impacter (shown in blue) increases it crushes the shear webs, Y creating V-shaped notches. Further increases in displacement cause the shear webs to buckle outwards. At very high displacements (45mm) there are matrix cracks visible at the interface of the Y and Z surfaces.

![Figure 4-5 Schematic of Progressive Tearing Failure](image)
Figure 4-6 shows the load-displacement plots of two random samples at the maximum displacement possible with the experimental set-up. At this displacement the damage was severe and was easily observed by the naked eye. It was the area around the peak force that was of interest as this was the region where any barely visible impact damage (BVID) would occur. Further investigations were conducted around this region. It was found that the load-displacement plots contained hysteresis (Figure 4-7), since these tests were at displacements where no shear web perforation was visible it was assumed that either the energy was being absorbed by damage mechanisms within the composite or the energy was being absorbed by the propagation of the matrix cracks (progressive tearing). It should be noted that at a displacement of 2.5mm there was no hysteresis.
The initial investigation into the damaged sample involved the removal of rectangular coupons from the centre of each of the four faces. These coupons were relatively large (280mm x 35mm) and were removed with a diamond saw. The coupons were then subjected to a simple laboratory based three-point bend test to determine any degradation in material properties. Fundamental beam theory gives us the relationship shown in Equation 4-a for a beam in 3 point bending with a load applied mid-span.

\[ v = \frac{PL^3}{48EI} \]

**Equation 4-a**

Where ‘v’ is the deflection of the beam, P is the applied load, L is the span, E is the Young’s modulus and I is the second moment of area. By measuring values of deflection (v) at incrementally applied loads (P) and plotting them on a graph the gradient (v/P) can be determined. This gradient can be used in Equation 4-a to determine a value for Young’s modulus (E) for a known span (L) and second moment of area (I). Young’s modulus is often used as a damage parameter in assessing composite structures as damage results in a loss of stiffness. The degradation of Young’s modulus is a common method of representing damage in FE models.
The above technique is a simple inverse problem; we are measuring the response or effects and applying our knowledge of the causes to determine the system (i.e. Young’s Modulus).

During this investigation it was found that one of the four sides of the box-section was consistently thinner than the remaining three and showed increased deflection per unit load. A normalised plot of load versus deflection/thickness revealed that the increased deflection per unit load was not solely related to the thickness of the coupons. Some reduction in the material properties must have resulted during the manufacturing process. A sample load-deflection plot for one set of four coupons is shown below.

![Figure 4-8 Coupon Inverse Analysis Plot (10mm Deflection)](image)

The results of the inverse analysis showed that at the deflections considered there was no reduction in residual properties of the coupons. Therefore it can be assumed that the energy absorbed by the sample was the work done in producing the matrix crack at the corners of the box section.

<table>
<thead>
<tr>
<th>Coupon</th>
<th>2.5mm Defln</th>
<th>5mm Defln</th>
<th>10mm Defln</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>16.35</td>
<td>17.88</td>
<td>16.97</td>
</tr>
<tr>
<td>Ya</td>
<td>17.09</td>
<td>16.80</td>
<td>17.32</td>
</tr>
<tr>
<td>Yb</td>
<td>17.06</td>
<td>16.55</td>
<td>16.24</td>
</tr>
<tr>
<td>Z</td>
<td>16.83</td>
<td>13.30</td>
<td>17.81</td>
</tr>
</tbody>
</table>

![Figure 4-9 Table of Young’s Modulus for Three Sets of Coupons](image)

It can be seen that all of the values shown in Figure 4-9 are approximately equal with the exception of the lower surface of the sample deflected 5mm. This value
The results obtained indicated that the effect of the impact damage on the faces of the box-section was insignificant. The assumption could be made that the damage was concentrated at the corners. The cracks on the X-Y corners grew as the displacement increased. The separation between the two faces was clearly visible at large deflections. Smaller cracks at the Y-Z corners became visible as the deflection increased further. The Y surfaces were penetrated by the impactor due to their poor in-plane strength. V-shaped notches were visible and fibre breakage was present around the impact site. At higher deflections matrix cracks (shear cracks) could be seen on the Y surfaces, characteristically at 45 degrees. This behaviour can be explained by the properties of the samples tested. The layers of unidirectional fibres parallel to the longitudinal axis give the section good longitudinal properties. The transverse properties are provided by layers of chopped strand mat. The material lay-up is designed to maximise the longitudinal properties of the section, little of the transverse strength is provided by the fibres. Hence cracks form at the corners where the high shear stresses exceed the material properties supplied largely by the resin (Figure 4-10).

![Figure 4-10](image)

It was clear from the observed damage and the initial tests that the damage mechanism was progressive tearing as described in the literature (Palmer, Bank et al. 1998). It was also concluded that there was no significant damage within the
walls of the specimen at the lower displacements. To quantify the damage would be difficult by standard material testing procedures. It would be impossible to obtain a suitable sample due to the damage being local to the corners of the box-section. It was clear that some form of damage assessment would be needed that could analyse the entire section in a non-destructive manner. Additionally further testing was needed to reinforce the theory that there was no or very little damage in the walls of the section. Furthermore it was discovered that there was tremendous variability in both the geometry and the mechanical properties of the sample. Variability is not uncommon in composite materials but the magnitude of the variability experienced in the initial study was larger than expected. This is likely to be a result of poor process control during the manufacture of the pultruded sample and/or sagging of the composite whilst curing.
Chapter 4: Design of a Lightweight Telehandler Boom

4.5 Load Chart Predictive Tool

As described in the introduction the performance of a telehandler is measured in terms of a load-chart (Figure 4-2). This defines the extremes of the operating envelope for the machine and also the limits to which loads can be positioned. Currently the load-charts are produced using empirical data gathered from a series of simple experiments. The boom is extended at pre-determined increments and weights are attached until the tipping load is achieved. This weight is recorded and after applying a factor of safety is plotted as a point on the load chart. This process is repeated for different boom angles to arrive at the finished load-chart.

At an early stage of the project it was clear that a predictive model was needed to determine the load-chart mathematically. It was obvious that the use of lightweight components would enhance the performance but there was no indication of the magnitude of this improvement. Thus the need for an analytical, predictive tool was recognised to quantify the performance benefits of proposed lightweight components.

Consider the side elevation of a telehandler, in simple terms the problem can be likened to a balance with the front axle of the machine as the fulcrum. For equilibrium the moment due to the payload and the components in front of the fulcrum must equal that due to the components behind the front axle. This problem becomes more complex when the extension and angle of the boom are considered, but a simple two-dimensional equilibrium approach will still provide a good approximation. Thus this approach became the basis for the first iteration of what was termed the ‘load-chart generator’.

The initial approach was linear and although the results were good there were problems in re-positioning of components. The changing of certain parameters required whole sections of program code to be re-written. It was clear that a more modular and simplistic approach was needed. An investigation into a vector approach revealed that the scalar (dot) product could be used to solve the calculation. Each component within the machine was positioned using a vector relative to a datum. The boom pivot was selected as the datum. The centres of gravity of all components were positioned relative to it for zero boom extension and zero boom angle. The vector approach is described in detail below.
4.5.1 Defining Component Positions using Vector Algebra

Given two vectors ‘A’ and ‘B’ and the angle between them (θ) the scalar or dot product is defined as (Bajpai, Mustoe et al. 1989),

\[ A.B = |A||B|\cos \theta = B.A \]

The scalar product is commutative and can also be written as,

\[ A.B = |A|(|B|\cos \theta) = (|A|\cos \theta)|B| \]

If we replace the vector ‘B’ with the vector ‘n’, a unit vector then the above expression can be rewritten to express the component of vector ‘A’ in the direction of the unit vector ‘n’.

\[ n.A = |n||A|\cos \theta = |A|\cos \theta \]

Equation 4-b

This is the form of the scalar product that the load chart generator uses to determine the horizontal distance of each mass from the front axle (fulcrum) of the machine.

Consider the diagram shown in Figure 4-11 defining point ‘H’, the centre of mass of an arbitrary component on the telehandler boom. At a boom angle ‘A’ of zero
and a ram extension ‘X’ of zero the point ‘H’ is defined by the co-ordinates (I, - J). The origin (0,0) is the boom pivot of the telehandler (shown as a red dot).

Figure 4-12 shows that the position of ‘H’ is dependant on the boom extension. If the component is on the outer boom (first stage) of the machine it will be unaffected by the extension. If the component is on the second stage of the boom it will be a function of the extension. If it is on the third stage it will be a function of twice the extension ‘2X’ and so on for multi-stage telehandlers. For simplicity the point ‘H’ is assumed to be on the second stage and thus a function of ‘H’.

In addition the position of ‘H’ is also dependant on the boom angle ‘A’. For any position of ‘H’ defined by ‘X’ and ‘A’ the horizontal distance of ‘H’ from the front axle is required to calculate the tipping moment. In our simplistic two-dimensional balance approach the horizontal distance can be defined as normal to gravity. For simplicity it is easier to reference the co-ordinate system (x,y) to the boom. The ‘x’ axis is parallel to the longitudinal axis of the boom, Figure 4-13.
To determine the tipping moment the distance ‘L’ must be calculated. If the machine stands on horizontal ground then ‘L’ is the horizontal distance between the boom pivot (the origin) and point ‘H’. If we define ‘H’ by a vector the advantage of using the scalar product becomes clear.

In Figure 4-14 we define the vector ‘m’ that defines the point ‘H’. We can now use the scalar product in the form of Equation 4-b to determine the component of the vector ‘m’ in the direction of the unit vector ‘n’, this is the horizontal distance ‘L’.

\[ n \cdot m = |n||m| \cos \theta \]

Note that this expression is in terms of ‘\( \theta \)’, the angle between ‘n’ and ‘m’. To obtain an expression in terms of boom angle (A) the scalar product is expressed in terms of its mutually orthogonal vector components (i, j) in the co-ordinate axes.
(x, y). Since the scalar product of perpendicular vectors is zero only the like components are multiplied together giving,

\[ n.m = (n_1i + n_2j) \cdot (m_1i + m_2j) = n_1m_1 + n_2m_2 \]

In the defined co-ordinate system (referenced to the boom) we know the vector ‘m’. We need to determine the unit vector ‘n’ at an angle of ‘A’ from the ‘x’ axis. This is given simply by Pythagoras,

\[ \cos A \]
\[ \sin A \]

Thus the expression for the distance ‘L’ can be written paying attention to positive and negative signs as,

\[ L = n.m = \begin{pmatrix} \cos A \\ -\sin A \end{pmatrix} \begin{pmatrix} l + X \\ -J \end{pmatrix} \]

Where ‘l’ and ‘J’ define the location of the centre of mass of the component and ‘X’ is the ram extension.

This approach allows the position of the centre of mass of each component to be calculated very simply as the boom extends and the boom angle varies. The load-chart generator is written in Mathcad software, each component is given an initial location when the ram extension and boom angle are both zero. This initial location is a vector and is identified in the program code by the prefix ‘COG’ (centre of gravity). Thus the initial location of the outer boom is termed ‘COGOUT’, the intermediate boom ‘COGINT’ and the inner boom ‘COGINN’. Every component is defined in this manner. The second step in the program is to determine the location of the component at varying extensions, X and boom.
angles, A. This is very straightforward due to the vector approach, all that is required is to operate the scalar product on the position vector and the unit vector ‘n’. For example the outer boom does not extend therefore is unaffected by the extension, X. The inner boom (third) will be extended by a value of 2X. Both will still be affected by the boom angle ‘A’. Thus the program calculates the position of each component as a function of X and A, given the prefix ‘POS’.

For example:-

\[ \text{POSOUT}(X, A) = \text{COGOUT} \left( \frac{\cos A}{-\sin A} \right) \]
\[ \text{POSINN}(X, A) = \left[ \text{COGINN} + \frac{2X}{0} \right] \left( \frac{\cos A}{-\sin A} \right) \]

Components on the chassis of the vehicle are independent of either ‘X’ or ‘A’. Each component has its position defined by this simple function.

The next phase of the program is to determine the position of the fulcrum relative to the datum (boom pivot). This is relatively straightforward, the generator allows the angle of the ground to be entered to determine the stability of the machine on an incline or decline. The program now has the positions of all components defined, as a function of X and A, relative to the front axle of the telehandler. The masses of the components are entered at this point, given the prefix ‘M’, thus ‘MOUT’ is the mass of the outer boom.

The program can now calculate the moment that each component exerts about the front axle as a function of X and A (e.g. MOMOUT). In our simplistic two-dimensional model the program can now simply sum these moments to determine whether the net moment causes the telehandler to tip over. At this stage it also calculates the moment for eight different payloads. These payloads are predetermined lines on the load-chart and are usually increments of either 500kg or 1000kg up to the maximum payload.

The values of X and A are now entered by defining a maximum and minimum value and a number of increments for both variables. Increasing the number of increments will increase the resolution of the program but will also increase the processing time. The program now reads the values of X and A into the moment function and produces an array of moments for each component. The size of the array is dependant on the resolution defined by the user. These arrays can be
summed to give the total moment due to all components at each inputted X and A. The point at which tipping occurs can be seen in these arrays as the values will change from negative to positive.

To further refine the program, a sub-routine was added that for each payload would interpolate the values of the final array and determine the tipping load for each angle. This routine also defines the operating envelope of the boom and finally plots the tipping load on a chart (Figure 4-16).

The vertical lines in Figure 4-16 correspond to the payload lines whose values are listed on the right-hand side as 'MPL'. The line on the far left represents a payload of 330kg and the line on the far right 3700kg. The generator correlated well with the charts produced by JCB. This particular example does not include the four-degree decline that the machine is tested on.

The major discrepancies between the calculated chart and the actual chart occur as the lift height increases. This is due to the simplistic approach taken. The load-chart generator only considers longitudinal stability and at increasing heights the limiting criteria is usually the lateral stability. This is why manufacturers load-charts show a decrease in maximum payload at higher lift heights. Although the
current programme deals in only two dimensions the vector approach allows the programme to be expanded into three dimensions. Although highly desirable by the manufacturer the development of the programme was outside the scope of the project. Recently the programme has been developed to calculate lateral stability and is now used by JCB to predict the stability of new designs or the effect of proposed changes.

The programme in its two dimensional form was adequate for the needs of the project. It allowed the value of lightweight materials to be determined. It also allowed various concept changes to be evaluated because it was straightforward to move various components and determine the effect on performance.
4.6 Prototype Design

4.6.1 Prototype Aims and Objectives

As previously mentioned it was determined that fibre-matrix composites offered the greatest benefits in terms of weight reduction. Once this was established it was clear that a prototype would be needed to evaluate fully the proposed design. This prototype boom would be mounted on an existing telehandler with four major objectives:

a) To demonstrate that composite materials could withstand the in-service lifting loads.

b) To prove the manufacturing feasibility of large-scale fibre-matrix box-section.

c) To provide a component that could be used to evaluate the long-term performance (fatigue, impact and wear)

d) To evaluate operator and customer reaction to the boom and evaluate the deflection.

It is important to note that this would be the first time a telehandler had been fitted with a boom made from anything but steel.

4.6.2 Modelling and Analysis

The design process used two main tools to arrive at the final prototype design. The first was a piece of software called CoDA (Composites Design and Analysis). CoDA is a tool produced by the National Physical Laboratory that allows different fibre lay-ups plus different fibres and resins to be evaluated. The proposed lay-up is inputted including thickness and orientation of laminates and the overall mechanical properties of the composite material are evaluated from its constituent part. CoDA also provides a simple analysis package that determines the bending stresses and deflections of simple sections under a variety of loading conditions. Although limited in terms of modelling the CoDA package allowed
different configurations of composites box-section beams to be evaluated in three-point and cantilever bending.

Although CoDA was useful as a starting point it did not account for local stress concentrations or analyse stresses in individual laminates. To obtain a more detailed model Lusas FE modeller was used. Lusas FE is a finite element modeller that specifically caters for composite materials by allowing the lay-up and orientation data to be entered. It also allows the stresses in individual laminates to be analysed and will also indicate areas of high local stress.

Although these two packages provided invaluable data there was concern over the variability of material properties of composite materials. This had been highlighted by the variability experienced during the three-point bend analysis (section 4.4). It was a concern that the material properties could vary greatly depending on the constituent materials used and the manufacturing technique employed. Therefore the results of the models were used to obtain an order of magnitude and not an absolute value.

Further complications were experienced in simulating the loading conditions of the telehandler in service. Enough is known about the loading conditions to be able to design a steel boom within prescribed factors of safety. However for a composite boom, the complex nature of the microstructure interactions involved in fracture of the material means more conservative factors of safety are required. In aircraft components factors of safety of 400% are typical.

This uncertainty augmented the need for a prototype and prototype-testing programme to validate the numerical modelling. A comparison could be made between the prototype under test (strain gauge data) and the finite element solution.
4.6.3 Prototype Manufacture

There are numerous processes that are suited to producing this type of component although the size of the box-section excludes some processes from consideration. The size of the prototype causes problems in terms of handling and production capacity but perhaps more importantly the thick-walled section generates large quantities of exothermic heat. The processes short-listed as feasible for prototype production were resin-transfer moulding (RTM), pultrusion, filament winding and prepreg lay-up.

The intermediate boom of a three-stage telehandler was chosen as the prototype component. The intermediate boom was relatively simple in terms of geometry but was exposed to complex and demanding loads. In addition the prototype boom could be mounted on an existing machine with only slight modifications thus allowing the prototype to be put through a full range of tests. It should be noted that the use of a composite intermediate boom with a steel inner boom would not offer the maximum weight reduction benefits. The telehandler model chosen was a three-stage pick and place machine with chain-driven boom extension and retraction.

It was decided to use carbon fibres in the production of the prototype. Whilst glass fibres are significantly less expensive their modulus of elasticity was lower and therefore they suffered from excessive deflection. A glass-fibre composite boom section would require a large wall thickness to achieve an acceptable level of stiffness. This increased wall thickness would offset directly against both the mass of the boom and the production time. A carbon-fibre composite boom would require a much thinner wall section, comparable with that of the current steel boom and would deliver a far greater weight saving. At time of writing the relative cost of carbon fibres was also falling increasing their economic viability. For prototype production the use of prepreg material provided the best means of producing prototype components effectively.

The existing intermediate boom is made from 6mm high strength steel formed into a U-shaped channel. This is welded to a closing plate of the same material with an 8mm wall thickness. The carbon-fibre prototype has a uniform wall thickness of 10mm on all faces yet still delivers a weight saving of 66%.
4.6.4 Design Details

One of the major drawbacks of fibre-matrix composites compared to steel is the ability of the material to withstand surface wear. The current steel design uses nylon-based wear pads on which the sections slide in and out. These wear pads are mounted in pairs on each of the four faces of the section at either end. The pads provide the only means of transmitting the load between the three sections and thus the wear is high. The pads are sacrificial and designed to run on steel. If the pads were to run directly on a fibre-matrix composite they would destroy the surface (mainly resin) of the composite extremely quickly. This would be accelerated in-service by the ingress of grit and other small particles. To eliminate this concern it was decided to bond thin steel strips to the surface of the composite upon which the wear pads would run. These thin strips were termed 'running strips' and for the prototype would be ex-stock stainless steel angle sections and would be bonded along the corners of the prototype internally and externally (Figure 4-17). These strips would offer a secondary benefit of helping to transmit the load from the upper flanges to the shear webs. This would help prevent progressive tearing failure of the prototype as experienced during quasi-static impact testing (section 4.4). The strips would also provide a limited amount of impact protection and would also help to distribute point loads from chain and ram mounts through the section. It should be clear that the running strips are bonded to the box-section after it has been fully cured.
The lay-up of the fibres is the most critical factor in the design of any composite component. The lay-up determines the mechanical properties of the component and also determines its resistance to impact. For the box-section prototype boom, the longitudinal axis is designated as zero degrees and the transverse direction is designated as ninety degrees. As previously reported, the boom will be subjected to complex static and dynamic loading combining bending and torsional loads. The bending load will be the primary load and will result in high stresses in the zero direction in the top and bottom flanges. A large proportion of longitudinal fibres will account for these stresses. The side walls (shear webs) serve to transfer the bending load between the top and bottom flanges by in-plane shear. Thus the shear webs of the section have a high content of fibres aligned at \( \pm 45 \) degrees to resist the shear forces (that act at 45 degrees).

Torsional loads will be equally distributed between the four faces of the box-section because the cross-sectional shape is square. Torsional loads are transmitted by in-plane shear and thus the use of \( \pm 45 \)-degree fibres is again desirable.

Figure 4-17 Exploded View of Prototype Boom
In addition to bending and torsion the effect of localised loads must also be given due consideration. For the telehandler section under consideration these localised loads are produced from the contact of the wear pads on the structure. The concentrated out of plane bending loads result in high through-thickness stresses and local bending stresses. These stresses are accommodated by continuous 90-degree fibres running around the outer circumference of the section.

The final consideration for the fibre lay-up is that it is good practice to ensure that it is balanced and symmetrical. This will minimise the level of residual stresses within the section. In addition non-symmetrical lay-ups commonly lead to warping or deformation during manufacture and/or curing.

Lusas FE software was used to evaluate the stresses and strains within the material based on the framework described above. This is important, as fibre-matrix composites are susceptible to internal flaws; FE analysis allows the stresses and strains in individual plies to be determined. A conservative approach was used in the FE analysis. In operation the component will be exposed to combined bending and torsion. For simplicity the lay-up to withstand the worst case bending load was determined and then the lay-up to withstand the worst-case torsional load was determined. These two fibre lay-ups were then combined to give the final solution. The worst-case bending load applied to the model was 2700 Kg at 7-metre extension. In reality the machine would become unstable before it reached an extension of 3 metres with this payload. The worst-case torsional load is modelled by applying an offset load at a distance of 500 millimetres from the longitudinal axis of the beam. Subsequent finite element analysis indicated that this procedure ensured a factor of safety of at least 500%, though not accounting for interfaces and localised loads.

The nature of composite materials is such that there are many permutations of lay-up for even the simplest component. A genetic algorithm was therefore used to optimise the lay-up, giving an effective and efficient structure. This approach has been used successfully to optimize the strength and weight of composite laminates (Le-Riche and Haftka 1997).
Chapter 4: Design of a Lightweight Telehandler Boom

Figure 4-18  Generic Configuration of Fibre Lay-Up

Figure 4-18 shows the general lay-up of the fibres for the shear walls (SW), the top flange (TF) and the bottom flange (BF). The genetic algorithm was used to optimise the thickness of each ply within the overall wall thickness. The results from the algorithm gave ply ratios of:

**Top and Bottom Flange**
- **Fibre Orientation:**
  - 90°
  - ±45°
  - 0°
  - ±45°

- **Ply Thickness Ratio:**
  - 4 1 1 4

**Shear Walls**
- **Fibre Orientation:**
  - 90°
  - ±45°

- **Ply Thickness Ratio:**
  - 8 11 11

The fibres were high modulus carbon fibres and bonding them directly to the stainless steel running strips presented a problem due to the possibility of galvanic corrosion and also problems due to thermal mismatch. To prevent this a
glass-fibre ply was added to separate the carbon fibre plies from the stainless steel strips.

Detailed FE analysis was performed on the boom model. The Lusas FE software allowed individual fibre layers to be examined for failure stresses. A sample plot of Tsai-Hill contours is shown below in Figure 4-19. Panni provides further information on the FE model and stress analysis (Panni 2002).

Figure 4-19  Stress Contours of the Outermost Layer (Tsai-Hill) under Combined Bending and Torsional Loads
4.7 Conclusion

The telehandler is a versatile machine produced for a growing market. The use of steel booms limits the operating envelope of the machine by contributing dead weight that affects the stability of the machine at reach and height. The reduction in weight of the boom sections using advanced lightweight materials would yield performance benefits to the manufacturer.

An investigation into lightweight materials revealed that fibre-matrix composites would offer the maximum benefits in terms of weight reduction and also manufacturing flexibility. An in-depth study developed a prototype design and fibre lay-up to enable a field test to be undertaken. The production of a prototype that can be fitted to an existing platform will allow a rigorous testing programme to be completed. This will provide data to corroborate the finite element and analytical data already gathered. The prototype will be the first non-steel boom to be fitted to a telehandler.

A fibre-matrix composite boom will deliver massive weight savings that will drastically increase the scope for performance improvements. Telehandlers will be able to lift payloads higher and reach further making them more attractive to the market. This will give JCB a competitive advantage in the telehandler market.
5 INVERSE ANALYSIS OF A GFRP DISC IN DIAMETRAL COMPRESSION

5.1 Introduction

The diametral compression test has been widely used for the testing of isotropic materials, particularly brittle materials. It has also been termed the ‘Brazilian Disc’ test or the indirect tensile test. Like most test methods, although initially intended for isotropic materials, it has been subsequently used in the testing of orthotropic materials such as fibre-matrix composites.

The test was designed to determine the splitting tensile strength of rock specimens. It originated due to the difficulty of configuring a suitable pull-test for rock-type materials (A.S.T.M. 1995). The test utilises the fact that tensile stresses are formed when two diametrically opposed forces compress a disc. The maximum tensile stresses grow perpendicular to the loading direction and are proportional to the applied load. Isotropic materials fail along the plane of the applied load although there has been much debate as to whether failure is caused by tensile stresses or whether it initiates at the loading points (Fahad 1996).

The stress distribution for an orthotropic disc was first presented mathematically by Okubo (Okubo 1939) who later developed his earlier work in a further paper (Okubo 1952). Okubo’s work was intended for wooden discs but the development of composite materials spawned new research into the use of the test for material characterisation. Until recently, the effect of material orthotropy and fibre orientation on the stress state had not been fully determined (Lemmon and Blackketter 1996). Lemmon used strain gauges and photoelasticity to determine the effect of fibre orientation and loading arc on the test results. It was determined that applying the load over an area equivalent to eight percent (2 x 4%) of the circumference of the disc reduced the likelihood of localised crushing and increased the uniformity of the tensile stresses in the vertical axis of the disc.
5.2 Diametral Compression Test to Characterise Damage in GFRP Box-Section

The purpose of the current investigation was to determine whether inverse analysis could be used to characterise the material properties of a glass-fibre reinforced plastic (GFRP) disc. By using inverse analysis the material properties of the coupon could be determined. The use of a disc coupon as opposed to other methods allows the fibre direction to be varied relative to the axis of loading. Part of this investigation was to determine whether a single disc could be tested in various orientations to provide the empirical data needed to perform inverse analysis.

The failure mechanism of pultruded composite box-section has already been identified. The progressive tearing damage observed is concentrated in the upper corners of the section. The surfaces of the pultruded section appeared free from damage. The removal of disc coupons from each of the four surfaces of a pultruded specimen would allow the material properties of the wall to be characterised along the length of the damaged sample (Figure 5-1).

![Figure 5-1 Pultruded Box-Section with 40mm Diameter Coupons Removed](image-url)
5.2.1 Removal of Coupons

In order to perform the test, circular coupons had to be removed from the faces of the damaged box section. In previous work the coupons had been relatively large (one coupon per face) and rectangular in shape. These were easily removed by diamond saw and cut to the correct dimensions. The removal of circular coupons presented two main problems. Firstly, the diamond saw was ill suited to profiling small circular shapes. Secondly, the coupons needed to be removed in close proximity and thus there was little ‘room to manoeuvre’, it would be difficult removing one coupon without damaging the next. These two factors made the use of the diamond saw impracticable for removing numerous small coupons from a single face.

The requirement was to remove the coupons without affecting the material properties. In reality this would be impossible, but the damage inflicted by the removal process should be small enough to be considered insignificant.

Avoiding this collateral damage is a major challenge when considering machine tools for cutting fibre-matrix composites. An interesting paper by Bhatnagar et al. (Bhatnagar, Ramakrishnan et al. 1995) highlights the common problems. Usual damage mechanisms experienced are fibre pull-out, delaminations, burning and ‘fuzzing’. Additional problems are encountered due to the effect of fibre orientation on the cutting operation. Chip formation and cutting force varies with fibre orientation and cutting direction. Interestingly, Bhatnagar relates the cutting mechanism to the shear strength of the material in different orientations, determined by the Iosipescu shear test. The findings presented indicated that there were numerous problems associated with machining fibre-matrix composites and therefore it was likely that a significant amount of additional damage would be incurred during the cutting process.

Another option available for removal of the disc coupons was laser cutting. The laser profiling systems available within the department were CNC controlled and this would allow accurate and rapid removal of multiple disc coupons. Investigations by Caprino et al. (Caprino, Tagliaferri et al. 1995a; Caprino, Tagliaferri et al. 1995b) reported that better cut quality was reported with GFRP than with CFRP (Carbon Fibre Reinforced Plastic). The biggest problem associated with laser cutting of GFRP was matrix combustion (the majority of
matrix materials were thermosetting). To minimise this Caprino used a high volume output of inert gas (168 litres/min argon). Various configurations of GFRP laminates were tested and it was found that higher volume fractions required increased laser power. Caprino also found that high cutting speeds, between 150 and 500 millimetres per minute, gave superior cut quality.

It was decided to remove the coupons from the box section using a laser profiling system. The box section was marked up with lines parallel to the fibre direction and each coupon was labelled so that its position and fibre orientation could be determined, this can just be seen in Figure 5-1. Two sizes of discs were removed, 15mm and 40mm diameter. All the discs removed from a single box section were of the same diameter. An industrial $CO_2$ cutting laser was used with the following settings.

<table>
<thead>
<tr>
<th>Power</th>
<th>Pulsed 8mA (140W), 4KHz, 30ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nozzle</td>
<td>1mm with 1.5mm stand-off</td>
</tr>
<tr>
<td>Gas</td>
<td>1 bar inert</td>
</tr>
<tr>
<td>Cut Speed</td>
<td>200 mm/min</td>
</tr>
</tbody>
</table>

The box section was positioned on the working table of the machine. Protective steel strips were placed inside the section to prevent damage to the back face of the section. The discs were profiled and then removed before proceeding to the next face. The profiled discs had charred edges although visually this appeared to be localised to the circumference of the disc. The coupons were then catalogued to ensure that they could be traced to their original positions.
5.2.2 **Diametral Compression Test Methodology**

The investigation into the effectiveness of the diametral compression test was conducted in three parts for undamaged and damaged samples. Firstly a single disc specimen was tested under a small load ($P$) for angles of $\beta$ between 0 and 360 degrees in 10-degree increments. Where $\beta$ is the angle between the loading axis and the primary fibre axis. This can be seen pictorially in Figure 5-2.

![Figure 5-2 Schematic of Diametral Compression Test](image)

Next, a single disc specimen was loaded in a similar manner but only for $0 \leq \beta \leq 90$ in 10-degree increments. Finally multiple disc specimens were loaded for $0 \leq \beta \leq 90$ with a single disc being compressed at one value of $\beta$ only. Tests were conducted with both 15mm and 40mm diameter discs to determine if there were any size effects on the observed stiffness. The load ‘$P$’ was applied by an Instron 4411 tensile testing machine. The sample was compressed between two flat platens. Data was collected via a PC and imported into a Microsoft (MS) Excel spreadsheet where a load-deflection plot was produced. Sample load-deflection curves are presented in Figure 5-3 that shows a sample loaded at $\beta = 0, 30, 45, 60$ and $90$ degrees. The tests were conducted in order to determine the gradient of the linear portion of the load-deflection curve; this was termed the stiffness, $k$. The load-deflection curve was initially non-linear and this was attributed to both the taking up of slack within the loading system and also the platens crushing the charred zones around the circumference of the disc. Beyond the linear section (increasing load) the gradient began to drop off in a manner typical of many failure curves. This indicated that the stiffness of the specimen
had been degraded, a sign that additional damage had been inflicted on the coupon.

Each disc was positioned between the platens such that there was just sufficient friction between the surface of the platen and the disc coupon to maintain the disc in its vertical position. The position of the crosshead was set to zero for this position. The compressive load was applied to the disc by displacing the crosshead a small distance. This distance was a function of both the size of the disc and the fibre orientation. The distance was varied between 0.3 and 1mm to produce a suitable linear load-displacement curve for further analysis. It was anticipated that this would be sufficient to produce the required gradient whilst minimising the secondary damage inflicted on the sample.

![Disc Compression Tests](image)

**Figure 5-3** Load-Deflection Curves for Disc Samples at Different Angles
5.2.3 Finite Element Analysis

A simplistic FE model of the composite disc was developed using Lusas Modeller. The material properties applied were assumed to be homogenous and are presented in Figure 5-4.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Young’s Modulus, $E_1$</td>
<td>17.2 GPa</td>
</tr>
<tr>
<td>Transverse Young’s Modulus, $E_2$</td>
<td>5.5 GPa</td>
</tr>
<tr>
<td>Shear Modulus, $G_{12}$</td>
<td>2.9 GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio, $\nu_{12}$</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Figure 5-4 Material Properties for Disc FE Model

Results were obtained in the form of the nodal displacement of the model in the y-direction. The stiffness, $k$, could be calculated from the nodal displacement and the combined loading. The model was analysed for multiple values of $\beta$ as described for the empirical test samples. The FE model appeared to give acceptable results although it was clear that a more complex model could represent the loading conditions more effectively. The FE mesh and the deformed model are shown in Figure 5-5. The objective of the FE model was to provide data against which the empirical results could be compared.
5.2.4 Results

The ultimate output from each test was in the form of a stiffness value, $k$, representing the gradient of the load-displacement curve. These stiffness values could then be plotted against the angle (relative to the loading axis) of the fibre orientation, $\beta$. The data was plotted graphically with stiffness, $k$ as the ordinate and angle, $\beta$ as the abscissa. Sample plots are presented in this section with further data available in the appendices.

Figure 5-6 shows a typical plot obtained when the empirical data is plotted against the FE results. The least squares fit was used to plot the best-fit curve to a sixth order polynomial trend line. Initial investigations utilised MATLAB programming language to perform the curve-fitting function. However, comparison between MATLAB and the Microsoft Excel trend line function revealed minimal discrepancies. Thus Microsoft Excel was used to perform the curve fitting as this required less data manipulation and reduced processing time.

Figure 5-6 shows 15mm and 40mm undamaged coupons plotted against FE results. Multiple discs were used so each disc was compressed just once. Discs were tested at angles of 0 to 90 degrees in 10-degree increments. The value of ‘$k$’ was normalised to compensate for varying disc thickness.
Figure 5-7 shows plots of damaged coupons against undamaged coupons. The undamaged coupons in this case were removed from a section that was subjected to 'minimum damage'. The plots indicate that the damaged discs had higher stiffness than the undamaged discs. This can be attributed to either the variability in material properties of the material or inaccuracies with the test method. In this case it is likely to be the former, it is assumed that no damage was suffered by the sample in the area of the coupons and the 'increased stiffness' reported in Figure 5-7 is due to increased material properties of the original box-section sample. This variability in material properties is a common problem with fibre-matrix composites.

Figure 5-6 and Figure 5-7 use a new disc coupon for each increment of $\beta$. Thus 10 disc coupons are required to fully characterise a disc, assuming 10-degree increments. The figures repeat the values from $0 \leq \beta \leq 90$ to give a full plot. Ideally a single disc coupon should be used for all angles of $\beta$. This would require that the diametral compression test does not degrade the properties of the coupon in any way. This was investigated and sample results are presented below, Figure 5-8.
If we compare Figure 5-8 with Figure 5-6 it can be seen that a single, rotated coupon does experience degradation in material properties. This would be reduced by limiting the number of angles for the test. The above data indicates that the diametral compression test cannot be used with confidence on a single coupon at multiple angles.

Another phenomenon that was encountered was the attenuation of the sinusoidal curve. An example of this is shown in Figure 5-9. The diametral compression test returns a similar value of stiffness for all angles apart from those close to 90 and 270 degrees. This is when the major fibre axis of the disc is parallel to the loading axis ($\beta$ is equal to zero or $2\pi$). This would again indicate that the diametral compression test is not suited to providing the empirical data needed for inverse analysis.
Figure 5-9 Stiffness Plot Showing Attenuation
5.2.5 Conclusions

The results of the investigation were not as expected. Three factors were investigated:

1. The use of a single disc coupon tested at multiple values of $\beta$.
2. The use of multiple disc coupons, each one tested at a single value of $\beta$.
3. The effect of different size discs (15mm and 40mm diameter)

It was found that the use of a single disc specimen at multiple values was impractical as the coupons stiffness degraded with each test. This not only ruled out this method of testing but also questioned the validity of the whole programme as damage was being done by the test on the samples. Even an accurate value of stiffness would be called into question. If the test was repeated would the coupon get progressively less stiff with each test?

The use of multiple disc coupons yielded improved results albeit far from the expected (FE) values. Since each disc was subjected to a single test there was no risk of subsequent discs having degraded stiffness. However, this test method is impractical since multiple discs are required to characterise the disc behaviour with respect to $\beta$. Thus numerous coupons from either multiple samples or multiple areas of a single sample are needed. It must either be assumed that material properties are identical between box-section samples (or even different faces) or it must be assumed that discs taken from different areas within a section have the same stiffness. Both of these assumptions are dubious due to the nature of fibre-matrix properties and the high variability of their elastic properties. Additionally a sample containing, for example delaminations, would not have uniform stiffness within a single face and thus multiple disc coupons would not have identical stiffness.

Finally the use of different sized disc coupons was addressed. Larger (40mm) coupons appeared to give better results than smaller ones (15mm) in terms of approaching the theoretical (FE) curve. However, there remains almost a fifty percent discrepancy and also a poor approximation to a sinusoidal curve.

Although further empirical data and perhaps an improved finite element model is needed it can be deduced from the investigation that the diametral compression test is not a useful means of obtaining empirical data for inverse analysis.
Furthermore the test set-up and results are subject to great variation, which can be attributed to the high degree of variability within the material. The test is difficult to define due to varied response of each disc as mentioned above plus the variation in stiffness in with respect to $\beta$. 
5.3 Disc Analysis by Speckle Shearing Interferometry

The use of shearing interferometry to analyse a disc in compression offers numerous advantages over the method presented in section 5.2. Primarily, that the technique allows whole-field strain data to be determined for the disc. This means that damaged regions can not only be detected but also characterized (by inverse analysis). Additionally, due to the sensitivity of the interferometer, a much smaller load can be used. This will help to prevent additional damage being inflicted on the disc coupons. For this investigation only 40mm diameter discs were analysed.

5.3.1 Experimental Equipment and Procedure

The discs were loaded in an identical manner to section 5.2.2 except that the two flat platens were replaced by concave jaws as shown below, Figure 5-10. The concave jaws were used to alleviate some of the point load problems discussed at the outset of this chapter. The discs were sprayed with matt white paint or white powder to eliminate problems associated with reflectance, particularly at normal illumination angles.

![Figure 5-10 GFRP Disc in Concave Jaws](image)

The principle of Speckle Shearing Interferometry (SSI) is described in detail in the Theory chapter and therefore only its application to the diametral compression test is discussed here. The SSI set-up used is not a state-of-the-art set-up but is more than capable of the tasks demanded from it. The set-up consists of a vertically polarised Elforlight G4 laser system (Figure 5-11A) with a power of 100mW and a wavelength of 532nm. The beam is expanded by an objective lens
(Figure 5-11B) and subsequently collimated by a plano-convex lens (Figure 5-11C). As with most lasers the intensity is represented by a Gaussian distribution and thus the relationship between the lenses is optimised to provide uniform collimated illumination of the disc (Figure 5-11D). The object wavefront then passes through a Michelson interferometer (Figure 5-11E) to a CCD camera (Figure 5-11F). The reference arm of the Michelson interferometer contains a mirror mounted on a PZT (piezo-electric transducer). The voltage is applied to the PZT by a manually adjusted DC voltage calibrator. The CCD camera is a Hitachi Denshi KP-M1 black and white camera that is linked to a frame store. The frame store is capable of outputting a shearogram by storing a reference image and subtracting the real-time signal from the stored image. The frame store was made especially for this purpose. The output from the frame store is connected to a PC via a PCI capture card.

To analyse a disc it is first mounted between the jaws of the tensile test machine. A pair of small locating pegs protrude from the upper and lower jaws to ensure the disc is located in the vertical plane. The jaws are closed until there is just enough friction to hold the disc in place. This is termed the zero load state but in fact a slight compressive load is present. Next a shear is introduced into the interferometer. The image of the disc recorded by the CCD camera is first adjusted to give no shear. This is done by adjusting the horizontal and vertical angles of the mirrors in the Michelson interferometer. When there is no shear the two images will coincide and Michelson fringes will be visible. The PZT-driven
mirror in the reference arm is not adjusted for the remainder of the experiment. The other 'static' mirror is angled so a shear is introduced to image received by the CCD camera. The shear is horizontal or vertical, not both. It should also be made clear that the angle between the illuminating beam and the object wavefront must be in the same plane as the shear.

![Figure 5-12 Schematic of Horizontal and Vertical Shear](image)

Once this step has been completed the reference image is captured by the frame store for the zero load state. The image displayed on the PC monitor is now a real-time shearogram. The sensitivity of the system can be seen by placing your hand under the target object, thermal currents can be seen on the shearogram. If the hand is placed close enough to the target (in this case the disc) fringes appear on the shearogram due to thermal stresses.

A compressive load is now applied to the disc. Care must be taken to ensure that the load induces sufficient deformation to form interference fringes but not so great that decorrelation takes place. Once the desired load has been applied four images are taken, \( I_0, I_1, I_2, I_3 \) where the phase difference \( \phi_n \) for a captured image \( I_n \) is,

\[
\phi_n = n\Delta
\]

In most cases, including this procedure, the phase-step (\( \Delta \)) is assigned the value of \( \pi/2 \). Thus the phase-difference for the four captured images are,

\[
\begin{align*}
\phi_0 &= 0 \\
\phi_1 &= \frac{\pi}{2} \\
\phi_2 &= \pi \\
\phi_3 &= \frac{2\pi}{3}
\end{align*}
\]
The captured shearograms are then used to derive a wrapped phase map. This is done using the commonly used four-frame algorithm described in the theory chapter. The resulting phase map is then smoothed by a median filter before being unwrapped.

5.3.2 Results and Discussion

In practice the algorithm and filtering are performed using MathCAD software. Figure 5-13 shows the four phase-stepped images inputted to the algorithm and the resulting wrapped phase output. It was clear from the results obtained that there was considerable speckle noise and a more advanced filter would be needed to derive a reasonable result.

![Figure 5-13 Sample Inputs and Output of Four-Frame Algorithm](image)

Six disc samples were tested each in three orientations ($\beta = 0, 45, 90^\circ$). Wrapped phase maps are included in the appendices. The phase data was unwrapped using the DCT-based algorithm described in Chapter 3. Phase maps of wrapped and unwrapped phase are shown in Figure 5-14. Note the high level of speckle noise...
in Figure 5-14a derived from a PC-based MathCAD algorithm with a simple median filter. Figure 5-14c is a similar test configuration but with better image processing performed on a workstation using a high level programming language.
5.3.3 Analysis of Phase Maps

The associated phase changes for the horizontal and vertical directions are given by:

\[ \Delta \phi_x = \frac{2\pi}{\lambda} \left( \sin \theta \frac{\partial u}{\partial x} + (1 + \cos \theta) \frac{\partial w}{\partial x} \right) \delta_x \]

\[ \Delta \phi_y = \frac{2\pi}{\lambda} \left( \sin \theta \frac{\partial u}{\partial y} + (1 + \cos \theta) \frac{\partial w}{\partial y} \right) \delta_y \]

Equation 5-a
Where $u$, $v$, and $w$ are the in-plane and out-of-plane components of displacement. Equation 5-a shows that shearograms are whole-field representations of displacement derivatives, however, quantitative strain information is not yet available except in a convolved form comprising both in-plane and out-of-plane components. To remedy this, the illumination angle may be directed normal to the surface of the object to measure the out-of-plane derivatives of displacement only ($\partial w/\partial x$ or $\partial w/\partial y$) which may simply be subtracted from either expression where appropriate.
5.4 Determination of Orthotropic Properties

5.4.1 Introduction

Several basic approaches are available for the characterisation of orthotropic properties in composite materials. The obvious procedure, utilising tensile specimens oriented at different angles to the principal directions, is quite feasible but requires many specimens and is tedious. This is exacerbated when considering that properties of composites may vary from one location to the next in the same component. As an alternative, simple test specimens may be produced in simple shapes and under various loading states the properties may be determined in an over-deterministic sense. This requires the collection of whole-field experimental data or the attachment of a large number of electrical-resistance strain gauges at unambiguous locations. The most well-known approach to determining orthotropic properties is through use of the losipescu specimen. Whilst this approach has come in for some criticism recently the main reason this approach has not been adopted here is the machining required to produce the relatively complex geometries. This would be quite time-consuming for large batches.

Prabhakaran and Chermahini (Prabhakaran and Chermahini 1984) used a half-plane specimen bonded with a photoelastic coating and strain gauges, and obtained properties using a least-squares approach. More recently, Grediac et al (Grediac and Pierron 1998) proposed a T-shaped specimen, though the shape of the specimen is complex and would be time-consuming to manufacture. Arguably the most convenient geometry to use is the circular disc loaded in compression, as it is simple to manufacture and analyse. Stanley and Garroch (Stanley and Garroch 1999) have shown that this specimen may be used to obtain orthotropic properties using thermoelasticity and, hence, avoided the more cumbersome approach of attaching a photoelastic coating or large numbers of strain gauges. Prabhakaran (Prabhakaran and Xu 2000) also used a circular disc specimen but in conjunction with strain gauges to yield material properties. Both methods of Stanley and Garroch, and of Prabhakaran rely on the closed-form solution for the orthotropic disc in compression. This suffers from the need for pin-point location of data points to be entered into the relevant equations and since both strain
gauges and thermoelastic data is collected from regions of finite size this approach will always be approximate. In the quest for a routine enabling data to be collected rapidly one could also question the use of strain gauges (and also reflection photoelasticity) for the time-consuming preparation needed before results can be obtained and the use of thermoelasticity that needs cyclic loads.

For the application in question the section is pultruded and it may be assumed that the orthotropic axes of the material are known a priori, i.e. they are aligned with the axis of pultrusion. Consequently, a simplified laminate expression may be used assuming the xy-axes are the orthotropic axes. The expression of Hooke’s law presented in Chapter 3 can be reduced to the following, in terms of the stiffness tensors, \(C\):

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & 0 \\
C_{12} & C_{22} & 0 \\
0 & 0 & C_{33}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

Equation 5-b

Consequently, to determine the four independent material constants \(E_1\), \(E_2\), \(v_{12}\), and \(v_{21}\) it is necessary to relate stresses with strains to determine, initially, the elements of the stiffness matrix, \(C_{11}\), \(C_{12}\), \(C_{22}\), and \(C_{33}\), using the results of unequivocal testing as in the inverse problem, and then, finally, to determine \(E_1\), \(E_2\), \(v_{12}\), and \(v_{21}\) from four independent material constants \(C_{11}\), \(C_{12}\), \(C_{22}\), and \(C_{33}\).

The proposed approach, however, is generic and may be applied to situations where the principal axes of the material are unknown. Figure 5-15 shows the geometry of the specimen, the square region of data collection within the circular disc and the boundary conditions used.
Figure 5-15  Geometry of Orthotropic Disc of Diameter (d) Compressed by a Force (P)
5.4.2 Methodology

The mechanical properties of the material are determined from the circular discs using data collected by SSI. In comparison to Stanley and Garroch (Stanley and Garroch 1999) the disc is also subjected to two diametrically opposed compressive loads and two orthogonal loading axes are used. In contrast, however, the number of data points used numbers many thousands whereas in the thermoelastic approach data is only collected from the disc centre. Consequently, the amount of information available is many more orders of magnitude overdetermined and even allowing for noise should yield accurate answers. In obtaining this large amount of data the proposed use of shearing interferometry does not suffer from time-consuming preparation, once the optical elements are in place, and does not need cyclic loads.

According to the principles of inverse analysis it is necessary initially to build a mathematical model of the disc. In other works this has been performed using the closed form elastic solution of Okubo (Okubo 1952). However, this is vulnerable to non-linear contact effects, such as finite deformations, and local crushing damage at the points of load application. Stanley and Garroch, and Prabhakaran successfully used this solution though the methods used relied on having accurate positions for the data points. These methods also preclude use of the remainder of the disc from which useful data may be collected. Instead, a more analytic approach is proposed based on simple summation of the boundary conditions around the largest square that may fit within the disc, Figure 5-15. It is reasonable to assume that data collected from within this square is well removed from the loading points thereby avoiding problems associated with the contact problem.

The quality and quantity of the available data may be further quantified. Consider the square consists of pixel data forming an array typically of 256 by 256 data points. Huntley (Huntley 1998) showed that for speckle measurements of strain, the average value of the strain recorded over a region of 16 by 16 pixels has the accuracy of a single electrical resistance strain gauge. Consequently, if the data region in Figure 5-15 consists of 256 by 256 it may equivocally be considered to be that of an array of 16 by 16 strain gauges. This is a significant advance over that of Prabhakaran who used only three strain gauges. The proposed approach is
also less dependent on accurate positions of the data points since errors are more likely to be ‘averaged’ out.

Consider that the boundary conditions for the array \((0 \leq m \leq M - 1, 0 \leq n \leq N - 1)\) may be stated in the following form:

\[
\begin{align*}
\sum_{m=0}^{M-1} (\sigma_x)_{m,0} \Delta_x t &= 0, \\
\sum_{m=0}^{M-1} (\sigma_x)_{m,N-1} \Delta_x t &= 0, \\
\sum_{m=0}^{M-1} (\tau_{xy})_{m,0} \Delta_y t &= 0, \\
\sum_{m=0}^{M-1} (\tau_{xy})_{m,N-1} \Delta_y t &= 0, \\
\sum_{n=0}^{N-1} (\sigma_y)_{0,n} \Delta_x t &= P, \\
\sum_{n=0}^{N-1} (\sigma_y)_{M-1,n} \Delta_x t &= P, \\
\sum_{n=0}^{N-1} (\tau_{xy})_{0,n} \Delta_y t &= 0, \\
\sum_{n=0}^{N-1} (\tau_{xy})_{M-1,n} \Delta_y t &= 0.
\end{align*}
\]

Equation 5-c

Where \(P\) is the applied load, \(t\) is the disk thickness, and \(\Delta_x, \Delta_y\) are the pixel dimensions. The pixel dimensions may be assumed to be constant and of value \(d / 256 \sqrt{2}\) if the pixel array is 256 by 256. From Equation 5-b the boundary conditions may be expressed using strains in the form:

\[
\begin{align*}
\sum_{m=0}^{M-1} C_{11} (\varepsilon_x)_{m,0} \Delta_x t + \sum_{m=0}^{M-1} C_{12} (\varepsilon_y)_{m,0} \Delta_y t &= 0, \\
\sum_{m=0}^{M-1} C_{11} (\varepsilon_x)_{m,N-1} \Delta_x t + \sum_{m=0}^{M-1} C_{12} (\varepsilon_y)_{m,N-1} \Delta_y t &= 0, \\
\sum_{n=0}^{N-1} C_{22} (\varepsilon_x)_{0,n} \Delta_x t + \sum_{n=0}^{N-1} C_{22} (\varepsilon_y)_{0,n} \Delta_y t &= P, \\
\sum_{n=0}^{N-1} C_{22} (\varepsilon_x)_{M-1,n} \Delta_x t + \sum_{n=0}^{N-1} C_{22} (\varepsilon_y)_{M-1,n} \Delta_y t &= P, \\
\sum_{m=0}^{M-1} C_{33} (\gamma_{xy})_{m,0} \Delta_y t &= 0, \\
\sum_{m=0}^{M-1} C_{33} (\gamma_{xy})_{m,N-1} \Delta_y t &= 0, \\
\sum_{n=0}^{N-1} C_{33} (\gamma_{xy})_{0,n} \Delta_x t &= 0, \\
\sum_{n=0}^{N-1} C_{33} (\gamma_{xy})_{M-1,n} \Delta_x t &= 0.
\end{align*}
\]

Equation 5-d

Thus, there are eight relationships to determine the four unknown coefficients \(C_{11}, C_{12}, C_{22},\) and \(C_{33}\) where \(\varepsilon_x, \varepsilon_y,\) and \(\gamma_{xy}\) are strains measured at each pixel in a whole-field sense.
To formulate equations of the form shown in Equation 5-d one effectively needs to sum measured strains along the four edges of the square of the data collection region for each orientation of the disc examined. The results are likely to be robust in spite of the probability of noise that affects speckle measurements since noise will be averaged out over the summation of the strains over a length of 256 pixels. It should be noted that the strains along the edge of the square are wrapped.
5.4.3 *Results*

Results were obtained for each disc extracted from a metre length of pultruded box-section GFRP. Each disc was oriented at either $0^\circ$ or $90^\circ$ and loaded in diametral compression to enable whole-field maps for the horizontal and vertical strains to be obtained Figure 5-16.

![Figure 5-16 Shearing Interferometry Wrapped Phase Maps for Composite Disc-in-Compression Test: (a) $0^\circ$ orientation; (b) $90^\circ$ orientation.](image)

These (phase) maps were unwrapped and the stresses were calculated about the square as shown in Figure 5-15. The unwrapped phase maps (Figure 5-17) need to be calibrated with a constant and this is obtained by enforcing at each corner of the square that the shear stress at the $45^\circ$ incline is zero, in other words from the Mohr circle, $\sigma_x = \sigma_y$.

![Figure 5-17 Shearing Interferometry Unwrapped Phase Maps for Composite Disc-in-Compression Test: (a) $0^\circ$ orientation; (b) $90^\circ$ orientation.](image)
A deterministic approach was used to obtain $E_1$ and $E_2$ for each orientation of the disc using the MathCAD programme included in the Appendices. The values obtained from all the discs orientations and subsequently used in future chapters to characterise the material were:

$$E_1 = 16.641 \text{ GPa}, \quad E_2 = 3.924 \text{ GPa}$$

For the GFRP material used in the production of the pultruded box-section beam, the elastic moduli in the principal fibre axes of the material is $E_1 = 17.2 \text{ GPa}, \quad E_2 = 5.5 \text{ GPa}$ (figures supplied by the manufacturer). Considering this layer of uniaxial fibres is sandwiched between thin layers of randomly oriented fibres (or chop-strand matting) to make up the laminate, these results seem reasonable.
5.4.4 Conclusions

The enclosed work has yielded an accurate and robust approach to the determination of orthotropic material properties that may be applied in an industrial environment where rapid turnover of results is essential. The method developed has been applied in this case to the problem of extracting the average material properties from all four sides of a pultruded GFRP box-section beam sample. The following conclusions are drawn:

- A method based on whole-field SSI experimentation has been developed to determine orthotropic material properties using an approach in which the stresses are summed about the data collection zone and matched with the applied boundary conditions of the disc.

- The developed approach is expected to produce a faster turnover of results compared to strain gauges since no gauges need to be attached and avoids the need for cyclic loading required in thermoelasticity.

- There is no need for accurate position measurements except for the largest square that may fit inside each disc.

- It is possible to use as many as 65,000 different data points in the algorithm thereby improving accuracy and robustness. An unwrapping approach is used using all the internal data to ensure the unwrapped strain results required along the edges of the data collection zone are reliable.

- In the problem considered a total number of six discs were used to characterise the GFRP material used to manufacture the box-section beam to give average values of $E_1 = 16.624$ GPa, $E_2 = 3.924$ GPa. These results compare well with known solutions.
6 INVERSE ANALYSIS OF DAMAGE IN BOX-SECTION COMPOSITES USING MULTIPLE LOADS

6.1 Introduction

The analysis of a pultruded box-sectioned composite beam is one potential application of inverse techniques. Pultruded box-sections suffer what has been termed 'progressive tearing' damage (Palmer, Bank et al. 1998) when subjected to transverse loading. This tearing is the separation of the sidewalls from the loaded surface of the section (Figure 6-1). It is visible at the top corners of the section and is a matrix crack formed when the shear stresses in this region exceed the threshold for the matrix material.

Figure 6-1  Progressive Tearing Damage in Pultruded Sample

This chapter explains how a series of unequivocal static tests are performed on damaged sections to determine the state of any damage using inverse analysis. The sections are tested in three-point bending at different spans and locations. The information from these tests provides stiffness values that are used to update a finite element model that represents the tearing damage by de-equivalencing mid-side nodes. A genetic algorithm is used to obtain the solutions, avoiding many of the problems associated with more established methods. The results are produced in the form of a finite element model that can conveniently be used to assess the structural integrity of the damaged component.
6.2 *A Genetic Algorithm to Resolve Progressive Tearing Damage*

Inverse or system identification problems are essentially minimisation problems. The difference between the measured response of an experimental specimen and that of an analytical (e.g. FE) model for identical inputs is minimised (Louis, Zhao et al. 1997). The difference between the two responses under identical inputs is represented by an objective function. For simple cases the objective function can be minimised by adopting an exhaustive search of all damage scenarios. For more complex scenarios, traditional gradient-based techniques can be used although these may converge to local minima and not global minima. Additionally, gradient-based techniques are ill suited to solving optimisation problems containing discrete parameters.

The use of a genetic algorithm (GA) negates the need to make exhaustive searches of the solution space that can be very large. Unlike gradient-based methods the GA does not tend to converge to local optima, quickly identifying optimum areas of the solution space. In other words they save the analyst from having to assess every conceivable solution, which in most cases would be extremely laborious. The nature of the GA makes it highly suited to handling discrete or discretised data.

**Figure 6-2 Outline of Genetic Algorithm**
The GA was used as an optimisation tool and as such, the detailed discussion of its development is outside the scope of thesis, readers are referred to an excellent book by Goldberg (Goldberg 1989). An overview of evolutionary algorithms is provided by Dasgupta and Michalewicz (Dasgupta and Michalewicz 1997). Conceptually GA’s are based on the survival of the fittest and each numerical solution (phenotype) is represented by a string-like code (genotype). These string-like representations (often binary) are manipulated using simple genetic operators. Each string represents the genetic code for a member of the population. The algorithm evaluates each member of the population and assigns them a fitness value based on how well the solution they represent minimises the objective function. The algorithm then selects the fittest members of the population to become parents and combines them to produce offspring. The algorithm uses probability to execute the above procedure with a bias towards the fittest members of the population. GA’s tend to converge quickly as each new generation gets fitter, retaining the characteristics that lead to high fitness values. The use of a genetic algorithm to minimise an objective function between a measured and computed response has been termed a ‘model-updating’ approach. Notably, it has been adopted by Cunha et al. to determine the elastic constants of composite laminates (Cunha, Cogan et al. 1999) and tubes (Cunha and Piranda 2000), also by Chou and Ghaboussi to detect structural damage in truss bridges (Chou and Ghaboussi 2001).
6.3 Damage in Pultruded Box-sectioned Composites

The test samples for this illustrative application were GFRP pultruded box-sections consisting of alternate layers of unidirectional fibres and discontinuous chopped strand mat bound in an epoxy resin. They were subjected to quasi-static impact (bending crush, (Mahmood, Jeryan et al. 1990)) equidistant between two simple supports. The impacted samples contained progressive tearing damage. The test procedure and the damage mechanism are described in the preceding chapter.

The damage inflicted on the box-sectioned composites consists of two longitudinal cracks of approximately equal length between the upper surface and the two vertical walls. The length of these cracks appears to be a function of the impactor displacement that causes them. The damaged test samples were inspected by an inverse procedure to reveal the length of the cracks in a FE model. The damage for each of the different test pieces was categorised as being 'Low', 'Medium' or 'High'. Characteristics of these three damage levels are outlined in Figure 6-3.

<table>
<thead>
<tr>
<th>Damage</th>
<th>Impacter Displacement</th>
<th>Crack Length (2a)</th>
<th>Mean Crack Length (2a)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOW</td>
<td>8mm</td>
<td>5 to 60mm</td>
<td>37.2mm</td>
<td>No other damage</td>
</tr>
<tr>
<td>MEDIUM</td>
<td>10mm</td>
<td>60 to 100mm</td>
<td>73.1mm</td>
<td>Slight indentation around impact site</td>
</tr>
<tr>
<td>HIGH</td>
<td>30mm</td>
<td>100 – 220mm</td>
<td>173.1mm</td>
<td>Fibre breakage around impact site, 'notching' of shear walls.</td>
</tr>
</tbody>
</table>

Figure 6-3 Characteristics of Impact Damaged Pultruded Composite Box-Section

However, due to the chaotic nature of fibre-matrix failures etc. it is difficult to establish exact crack length sizes. Therefore, for each level of damage the impact test was repeated for ten samples to obtain average results.
6.4 Three-Point Bend Investigation

6.4.1 Theory

It should be clear from the outset that the use of three-point bend to both inflict damage and quantify it by inverse analysis is merely coincidental. The three-point bend test lends itself to this configuration of sample and is the simplest, most effective means of accomplishing the required task. Rudimentary beam theory tells us that for a simply supported beam with a concentrated mid-span load:

\[ v = \frac{P.L^3}{48.E.I} \]

Equation 6-a

Where, ‘v’ is the deflection, ‘P’ is the applied load, ‘L’ is the span, ‘E’ is the Elastic (Young’s) Modulus and ‘I’ is the second moment of area (Benham and Crawford 1987). Equation 6-a can be re-written in the form:

\[ E = \frac{P.L^3}{v.48.I} \]

Equation 6-b

The elastic modulus is a measure of the stiffness of the material. Since damage suffered by a composite material results in a loss of stiffness then the Elastic modulus can be used to give a measure of the damage. This is particularly useful when modelling damaged composites in finite element software as individual elements can be assigned a degraded elastic modulus to simulate damaged regions or cracks (Davies, Zhang et al. 1994).

Equation 6-a and Equation 6-b assume that the second moment of area, ‘I’ is constant. The dimensional variability between samples has already been reported. Although the variation in cross-section was relatively high, due to dimensional inaccuracies during the manufacturing process, (wall thickness’ between samples could vary by as much as 0.3mm), its effect on the second moment of area was small enough to be considered negligible. Data from a random selection of
samples was analysed and the effect of the cross-section on the calculated second moment of area gave a maximum deviation of 0.3% from the mean value.

It can be seen from Equation 6-b that the determination of the elastic modulus is dependant on four variables. If the span, ‘L’ and the second moment of area, ‘I’ are known then the magnitude of the elastic modulus is dependant on ‘P/ν’.

This is the gradient of the load-deflection curve. Thus, the gradient of the curve can be used as a measure of damage between identical samples in identical three-point bend configurations (span). This gradient is termed the stiffness and assigned the letter ‘k’.

The above assumptions were made based on the experimental data acquired during initial testing. The assumptions are not valid for situations where the crack length is very long and the span is small. In this case, the bending mode of the beam is observed to change. The upper surface acts as an independent beam in 3-point bend with the supports aligned coincident with the crack tip. Thus the system appears to function in a compound manner. This requires confirmation by further experimentation that is considered outside the scope of this research.
6.4.2 *Three-Point Bend Test Arrangement*

The three-point bend test used to quantify the damage uses a relatively small load. A value of 1 KN was chosen, compared with a peak force of 6 KN observed during the impact test. The smaller load is used to minimise the chance of any additional damage being inflicted on the sample during the three-point bend (TPB) analysis. The damaged sample was mounted in a purpose-built test rig and an Instron Model 4411 was used to apply the load. The rig consisted of a steel box-section that had locating holes drilled along its length, these were used to attach two pairs of side plates, the position of which could be adjusted to vary the span. The side plates were used to locate two steel cylindrical mounts upon which the sample rested. The load was applied mid-span by a further cylinder, perpendicular to the longitudinal axis of the sample. This configuration is shown in Figure 6-4 and further clarified by Figure 6-5 that shows a side view of the lower section of the rig.

![Figure 6-4 Instron 4411 Tensile Test Machine with TPB Rig](image)

The Instron test machine was PC-controlled and thus data capture of the load-displacement curve was straightforward. The PC controlled the displacement of the impacter and logged the output from the load cell. This data was exported via
a comma-separated-variable file to a spreadsheet where the data was plotted and
the gradient determined.

![End View of TPB Rig Showing Cylindrical Mounts and Locating Holes for Span Variation](image)

Figure 6-5

6.4.3 Test Programme

Ten samples with 'low' damage, ten samples with 'med' damage and ten samples
with 'high' damage (defined in Figure 6-3) were tested with three undamaged
samples, see Figure 6-7. The undamaged samples provided a datum against which
the degradation in stiffness could be measured. The samples were subject to TPB
tests at three separate spans; maximum-span, half-span, quarter-span. These were
terms used to describe the different configurations, in reality the dimensions of
the three spans were as follows:

\[
\begin{align*}
\text{max span} & = 527\text{mm} \\
\text{half span} & = 276\text{mm} \\
\text{quarter-span} & = 124\text{mm}
\end{align*}
\]

The samples were tested in two configurations at half-span and four
configurations at quarter-span. The various configurations are shown in Figure
6-6, the left-hand side of the figure showing the max, half and quarter span
configurations and the right-hand side showing the four positions for the quarter-
span tests. The half-span tests were also performed at the centre of the sample to
complete the programme.
The samples were subjected to the TPB tests four times, each time the sample was rotated axially by ninety degrees so that in each orientation the load was applied to a different face. The permutations of each test, coupled with the number of samples resulted in an extensive testing programme (Figure 6-7).

To keep the number of tests manageable the number of damaged samples tested at quarter-span was reduced from ten to three. This reduced the time to complete the programme considerably as each of the samples at quarter-span was tested in four different orientations at four different positions. The three samples were chosen at random from the batch of ten.
### Chapter 6: Inverse Analysis of Damage in Box-Section Composites Using Multiple Loads

#### Figure 6-7 Test Matrix for Three-Point Bend Investigation

<table>
<thead>
<tr>
<th>DAMAGE</th>
<th>SPAN</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
<th>None</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum (527mm)</td>
<td>10 x 4</td>
<td>10 x 4</td>
<td>10 x 4</td>
<td>3 x 4</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>'Half' Span (276mm)</td>
<td>10 x 4</td>
<td>10 x 4</td>
<td>10 x 4</td>
<td>3 x 4</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>- End Position</td>
<td>10 x 4</td>
<td>10 x 4</td>
<td>10 x 4</td>
<td>3 x 4</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>- Centre Position</td>
<td>10 x 4</td>
<td>10 x 4</td>
<td>10 x 4</td>
<td>3 x 4</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>'Quarter' Span (124mm)</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>- Position 1</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>- Position 2</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>- Position 3</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>- Position 4</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>3 x 4</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>128</td>
<td>128</td>
<td>128</td>
<td>72</td>
<td>588</td>
<td></td>
</tr>
</tbody>
</table>

It can be seen from Figure 6-7 that the volume of data collected is large. For each completed test, a load-displacement curve was plotted and the gradient of the linear region determined to give a measure of stiffness. The load-displacement curves varied in form considerably between the minimum and maximum damage levels. For a minimum damage sample the curve was a straight line whereas the maximum damage sample often had two points of inflection before it approached a linear region (Figure 6-8). It was observed that the initial non-linear behaviour was due to the loading bar negotiating damaged material outside the original dimensions of the sample to make full contact with the loading surface. The maximum damaged tests were often terminated prematurely, particularly when the contact point was coincident with the impact site. The termination was intended to prevent additional damage being inflicted on the sample.
Figure 6-8 Load-Displacement Curve for Maximum Damaged Sample at Full Span

As previously discussed the gradients of the load-displacement curve gave a measure of stiffness. Thus by plotting the stiffness results for each sample and each face a comparison can be drawn to the undamaged sample. The most effective method of visualising the results was by plotting the stiffness of the four faces on a radar chart. This allows the four axes to correspond to the four faces of the sample while plotting a considerable amount of data on a single chart.
Chapter 6: Inverse Analysis of Damage in Box-Section Composites Using Multiple Loads

6.5 **Three-Point Bend Data and Analysis**

6.5.1 **Three-Point Bend Data**

As described in section 6.4.3 the generated data was plotted in the form of radar charts. These charts plotted the gradient of the load-displacement curve (stiffness, \( k \)) from each test. The value plotted on the axes of the radar chart had units of Newtons per millimetre.

The mean of the undamaged results was then plotted on the charts displaying the damaged data, for reference. It can be clearly seen that the damaged samples suffered the greatest reduction in stiffness when loaded at the impacted face (X). By comparison, the stiffness reduction when loaded in the opposite face (Z) was relatively small. This considerable difference has great potential for damage detection, as even for the minimum damaged sample where damage was barely visible the stiffness at the impacted surface was almost 40% less than the stiffness at the Z-surface. This difference is summarised in Figure 6-9.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Mean Stiffness ( k_x ) (N/mm)</th>
<th>Mean Stiffness ( k_z ) (N/mm)</th>
<th>Residual Stiffness ( (k_x/k_z) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Damage</td>
<td>683.9</td>
<td>1083.8</td>
<td>0.63</td>
</tr>
<tr>
<td>Medium Damage</td>
<td>445.7</td>
<td>1085.3</td>
<td>0.41</td>
</tr>
<tr>
<td>Maximum Damage</td>
<td>98.4</td>
<td>848.0</td>
<td>0.12</td>
</tr>
</tbody>
</table>

**Figure 6-9**  Mean Stiffness Comparison (Maximum Span)

It should be noted that when the loading bar of the TPB instrument is coincident with the damaged region at maximum damage levels, the stiffness obtained is very low. This also applies for other tests where the damaged region may be aligned over the supports. The tests are usually terminated early to prevent additional damage being inflicted on the sample. This results in the load-displacement curve not reaching its ultimate loading path. As previously mentioned, the curves of heavily damaged samples commonly exhibited two points of inflection. The tests were often terminated when the deflection reached a maximum value, which was positioned within the two points of inflection, thus giving a lower gradient and value of stiffness.
The plots also appear to display a slight skew, i.e. $k_{Y_1} > k_{Y_2}$. This was a concern as the surfaces were tested in the order X, Y1, Z, Y2 and a possible explanation for this was that the TPB testing was inflicting secondary damage on the sample. However the samples when re-tested gave repeatable results and so a need for this explanation was effectively eliminated. Several explanations have been put forward for this phenomena but further testing is needed to fully validate this behaviour.

The quarter-span TPB tests were conducted at four positions as defined in Figure 6-6, position one being the centre of the sample and each successive position being indexed along half a span (half of a quarter span). Fewer samples were used for these tests due to the increased number of permutations.
Figure 6-10  Mean Stiffness of All Samples at Maximum Span

Figure 6-11  Mean Stiffness of All Samples at Half Span (End)
Chapter 6: Inverse Analysis of Damage in Box-Section Composites Using Multiple Loads

Figure 6-12  Mean Stiffness of All Samples at Half Span (Middle)

Figure 6-13  Mean Stiffness of All Samples at Half Span (End)
Chapter 6: Inverse Analysis of Damage in Box-Section Composites Using Multiple Loads

Figure 6-14  Mean Stiffness of All Samples at ¼ Span (Pos’n 2)

Figure 6-15  Mean Stiffness of All Samples at ¼ Span (Pos’n 3)
### Figure 6-16 Mean Stiffness of All Samples at ¼ Span (Pos'n 4)

<table>
<thead>
<tr>
<th>MAX SPAN</th>
<th>X</th>
<th>Y1</th>
<th>Z</th>
<th>Y2</th>
</tr>
</thead>
<tbody>
<tr>
<td>UD</td>
<td>1079</td>
<td>1056</td>
<td>1038</td>
<td>1085</td>
</tr>
<tr>
<td>MIN</td>
<td>684</td>
<td>977</td>
<td>1064</td>
<td>853</td>
</tr>
<tr>
<td>MED</td>
<td>446</td>
<td>867</td>
<td>1085</td>
<td>676</td>
</tr>
<tr>
<td>MAX</td>
<td>98</td>
<td>553</td>
<td>848</td>
<td>494</td>
</tr>
<tr>
<td>HALF-END</td>
<td>X</td>
<td>Y1</td>
<td>Z</td>
<td>Y2</td>
</tr>
<tr>
<td>UD</td>
<td>2592</td>
<td>2562</td>
<td>2680</td>
<td>2519</td>
</tr>
<tr>
<td>MIN</td>
<td>2560</td>
<td>2094</td>
<td>2015</td>
<td>2133</td>
</tr>
<tr>
<td>MED</td>
<td>2554</td>
<td>1942</td>
<td>1464</td>
<td>1758</td>
</tr>
<tr>
<td>MAX</td>
<td>2323</td>
<td>1356</td>
<td>490</td>
<td>1379</td>
</tr>
<tr>
<td>HALF-MID</td>
<td>X</td>
<td>Y1</td>
<td>Z</td>
<td>Y2</td>
</tr>
<tr>
<td>UD</td>
<td>2714</td>
<td>2534</td>
<td>2702</td>
<td>2510</td>
</tr>
<tr>
<td>MIN</td>
<td>1275</td>
<td>2268</td>
<td>2820</td>
<td>1667</td>
</tr>
<tr>
<td>MED</td>
<td>626</td>
<td>2054</td>
<td>2812</td>
<td>1459</td>
</tr>
<tr>
<td>MAX</td>
<td>153</td>
<td>1222</td>
<td>2102</td>
<td>1013</td>
</tr>
</tbody>
</table>

### Figure 6-17 Summary of Mean Stiffness Values for All Tests (N/mm)

<table>
<thead>
<tr>
<th>QTR-1</th>
<th>X</th>
<th>Y1</th>
<th>Z</th>
<th>Y2</th>
</tr>
</thead>
<tbody>
<tr>
<td>UD</td>
<td>3653</td>
<td>3815</td>
<td>4143</td>
<td>3725</td>
</tr>
<tr>
<td>MIN</td>
<td>1267</td>
<td>2389</td>
<td>4426</td>
<td>3138</td>
</tr>
<tr>
<td>MED</td>
<td>981</td>
<td>1970</td>
<td>4023</td>
<td>2685</td>
</tr>
<tr>
<td>MAX</td>
<td>159</td>
<td>1270</td>
<td>3289</td>
<td>1401</td>
</tr>
<tr>
<td>QTR-2</td>
<td>X</td>
<td>Y1</td>
<td>Z</td>
<td>Y2</td>
</tr>
<tr>
<td>UD</td>
<td>3933</td>
<td>3639</td>
<td>4111</td>
<td>3867</td>
</tr>
<tr>
<td>MIN</td>
<td>4659</td>
<td>2771</td>
<td>2645</td>
<td>3219</td>
</tr>
<tr>
<td>MED</td>
<td>4135</td>
<td>2344</td>
<td>1750</td>
<td>2928</td>
</tr>
<tr>
<td>MAX</td>
<td>3850</td>
<td>1666</td>
<td>1203</td>
<td>1546</td>
</tr>
<tr>
<td>QTR-3</td>
<td>X</td>
<td>Y1</td>
<td>Z</td>
<td>Y2</td>
</tr>
<tr>
<td>UD</td>
<td>4244</td>
<td>4076</td>
<td>4474</td>
<td>3884</td>
</tr>
<tr>
<td>MIN</td>
<td>4613</td>
<td>3548</td>
<td>4648</td>
<td>3810</td>
</tr>
<tr>
<td>MED</td>
<td>4591</td>
<td>3695</td>
<td>4271</td>
<td>3756</td>
</tr>
<tr>
<td>MAX</td>
<td>4542</td>
<td>2662</td>
<td>4399</td>
<td>2654</td>
</tr>
<tr>
<td>QTR-4</td>
<td>X</td>
<td>Y1</td>
<td>Z</td>
<td>Y2</td>
</tr>
<tr>
<td>UD</td>
<td>4163</td>
<td>4007</td>
<td>4472</td>
<td>3896</td>
</tr>
<tr>
<td>MIN</td>
<td>4760</td>
<td>3649</td>
<td>4605</td>
<td>3919</td>
</tr>
<tr>
<td>MED</td>
<td>4586</td>
<td>3744</td>
<td>4554</td>
<td>3583</td>
</tr>
<tr>
<td>MAX</td>
<td>4531</td>
<td>3567</td>
<td>4485</td>
<td>3713</td>
</tr>
</tbody>
</table>

**Table Notes:**
- UD: Unidirectional
- MIN: Minimum
- MED: Median
- MAX: Maximum
6.5.2 Results from the Genetic Algorithm

Damage due to a quasi-static impact on one of the faces of a composite box section beam is assumed to exist as longitudinal cracks running along the corners of the upper contact face. At the heart of the GA is the fitness value and this can only be determined by solving the forward deterministic problem for each solution in the population. In this instance the analytical model is a finite element model and the cracks are represented by de-equivalencing given nodes along the corners of the model. It is important to define the solution space as carefully as possible to avoid problems and ambiguous results. Experience is needed to understand the nature of the damage likely to occur in service given a prescribed environment. It was clear that the damage takes the form of longitudinal ‘tearing’ cracks at both edges of the upper surface that increase in severity as the applied load/deflection increases. Consequently, the solution space should only consist of FE models with these longitudinal cracks covering a series of elements to enable short and long cracks to be incorporated. There is no point, for example, enabling the solution space to consist of FE models that have transverse cracks if they are not encountered in the damage. This would only serve to make the optimisation more complex. However, the number of possible combinations involving de-equivalenced nodes may still be large, particularly if small elements are used to accurately resolve the size of defects.

The first step in implementing an inverse analysis of this kind is to construct a suitable analytical (FE) model to represent the geometry and loading, Figure 6-19(a). A representative mesh that could be used to model the damage witnessed in the experiments is shown Figure 6-19(b). The size of the elements used gives the resolution to which the damage could possibly be detected. Therefore, the mesh shown has elements sized longitudinally to enable crack increments that correspond to the three categories of damage as previously defined in Figure 6-3. Of course, this mesh has been constructed with the benefit of experimental experience. This should be seen as another benefit of inverse analyses that one may refine the techniques by learning from experience.

The genetic algorithm considered here has been created in visual basic script. This routine allows all the genetic operations and the solutions to be run within the Lusas FE package. The script automates the creation of a basic finite element
model containing the crack configuration described by each solution. This is then solved for a number of different loading cases. The extracted stiffness is then used to determine the fitness of solutions for the GA. As well as controlling the GA, the script generates a log file that records the history of each run, including a list of all individuals and their fitness values. This log file can be exported to Microsoft Excel for post-processing of the data. When the GA is applied and the results obtained the FE meshes in each case are updated to produce a good fit to the experimental measurements of stiffness. The current performance of the GA is indicated by Figure 6-18 and Figure 6-20.

It is important to note the rapid convergence of the GA and also the high success rate for determining the exact solution. An element size of 20mm was used in the FE model incorporated into the GA. Solutions that were not an exact match to experimental data were mainly those with very small crack lengths, indicating that a smaller element size is needed to arrive at an acceptable solution.

It is necessary to note that to resolve more accurate information regarding the damage then the mesh needs to be more refined. However, this then requires further information to process thereby necessitating more unequivocal testing. The spans used in the TPBI are as small as a practically possible though one could incrementally move the instrument along the beam and stop at more places to take measurements.

As mentioned earlier the same FE mesh may then have actual service loads applied to determine its residual strength Figure 6-19(c) and (d).

<table>
<thead>
<tr>
<th>Clean Input Data</th>
<th>Do we find Damage?</th>
<th>Do we find Correct Surface?</th>
<th>Do we define crack length?</th>
<th>Do we locate correct position?</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>93%</td>
<td>100%</td>
<td>86%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Noisy Input Data (+/- 5%)</th>
<th>Do we find Damage?</th>
<th>Do we find Correct Surface?</th>
<th>Do we define crack length?</th>
<th>Do we locate correct position?</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>68%</td>
<td>72%</td>
<td>54%</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6-18  GA Performance – Finding an Exact Solution
Chapter 6: Inverse Analysis of Damage in Box-Section Composites Using Multiple Loads

(a) Geometry of experimental model under three-point bend test

(b) FE mesh

(c) Stresses in damaged model

(d) Local stresses in damaged region

Figure 6-19  FE Model with Progressive Tearing Damage

Figure 6-20  GA Convergence: Objective Function (Ordinate) against Iterations (abscissa) (Darker line is fittest member, Light grey line is average fitness of the population)
6.5.3 **Secondary Results from the FE Model**

As previously mentioned the solution provided by the GA is in the form of a FE model. The major advantage of this is that it allows the subsequent analysis of the model to determine residual strength and service life of the structure under loading.

The literature describes many theories on composite failure the main ones being Hill, Tsai-Hill and Tsai-Wu (Barbero 1998). Tsai-Wu was selected as the failure criterion for this analysis as it provided the best results when compared to experimental data. Additionally the Tsai-Wu theory contains additional stress terms that give greater flexibility when considering certain types of geometry.

6.5.4 **Validation of the FE Model (GA Output)**

The FE model produced by the GA (Figure 6-19) was vertically restrained at the lower edge of each end. Restraints were used to ensure that the modelled supports and impactor remained co-linear during the analysis. The validity of the FE model was tested by introducing a small vertical crack in the shear web of the box-section model directly under the impactor (Figure 6-21). This crack had been observed in the damaged samples. The displacement of the impactor was applied to the model compensating for the length of the vertical crack, i.e. the base of the vertical crack was coincident with the displaced co-linear mid-span nodes. The reaction loads at the supports were then determined and compared with the original load-displacement curve of the impact event. The cracks were then iteratively adjusted until the loads produced by the FE model matched those from the original experiment. For example, if the loads from the FE model were higher than the empirical result then this would indicate that the FE model was too stiff, thus the length of the vertical crack needed to be increased.

The validated models were then analysed using Tsai-Wu failure criterion. Figure 6-22 shows the Tsai-Hill plot where the dark blue areas represent a value of zero and the red areas indicate a value of 1.1 (where 1 predicts failure). It can be seen
that the model fails at the tips of the longitudinal matrix cracks and this concurs with the observed damage mode of progressive tearing.

Figure 6-21  Plot of Deformed Mesh for Crack Length 2a=73.1mm

Figure 6-22  Plot of Tsai-Hill Failure Contours for Crack Length 2a=73.1mm
6.6 Discussion

It can be seen from the proceeding sections that there is a large amount of effort required to accumulate sufficient empirical data. There is also some doubt as to what qualifies as sufficient data, do we need more tests? Have we done too many? The results obtained from the TPB show several interesting results.

Firstly the variation in the TPB data gives an indication of the variability of the material properties of this type of composite. This variation is a combination of the random nature of composite materials and inconsistencies due to the manufacturing process. Pultrusion technology is relatively new compared to more established production techniques. Dimensional controls are currently inferior to other processes largely due to the lack of market demand for tight tolerances. An ever-increasing market for pultruded sections will ultimately require manufacturers to improve their quality control. Even sections with similar geometric size and shape can exhibit different mechanical properties. The use of randomly oriented fibres and inconsistencies within the laminate, particularly around the corners in the case of box-section, lead to material variability within a production batch or even within a single length. It is due to this variability that the number of tests must be relatively high so that a mean value can be obtained to minimise the effect of material property fluctuation.

It can be seen from the radar plots and data presented in section 6.5.1 that simply by observing the plots we can detect damage. Furthermore, the difference in recorded stiffness when loading at the impacted face and the opposite face provides a simple means of detecting damage. Determining the location of the damage is a more complex problem, particularly where the damaged region is coincident with one of the supports. This alignment can be identified in the radar plots as they appear inverted, rotated through pi radians. This gives a much lower value of stiffness especially in maximum damaged samples due to localised deflections and penetration of the impactor. It is possible that this arrangement will lead to further propagation of the matrix cracks.

Results from the GA algorithm have been good and as mentioned previously this is in part due to the ability to develop the algorithm based on experience. The major weakness of the GA in its current form is the inability to quantify very small cracks. This is attributed to the current level of resolution (large element
size) of the FE model. It is anticipated that reducing the element size will deliver an improved result; further empirical data may or may not be needed. It should be noted that increasing the resolution of the FE mesh would increase the computational effort required and thus the run-time of the GA.

The FE model produced by the GA has been validated against the load-displacement data and progressive tearing damage of actual samples by introducing a small vertical crack into the FE mesh. The length of this vertical crack is estimated from the observed crack on the damaged sample and iteratively adjusted to produce the required stiffness that mimics the behaviour of the sample. A Tsai-Hill failure plot reveals that the box-section will fail at the tip of the longitudinal matrix (shear) cracks, which is consistent with the progressive tearing failure observed in this investigation and by other authors.
7 INVERSE ANALYSIS OF BOX-SECTION COMPOSITES USING SPECKLE SHEARING INTERFEROMETRY: A FEASIBILITY STUDY

7.1 Introduction

Chapter 6 presented a procedure whereby multiple three-point bend tests were performed on pultruded sections to provide empirical response data. The difference between this response data and a numerical model was minimised by a genetic algorithm to produce a solution representing the damaged section. This form of inverse analysis has been termed 'model-updating'. It should be clear that the empirical data collected was immense and required considerable time performing laboratory-based tests.

By contrast, Chapter 5 showed how speckle-shearing interferometry could be used to obtain the empirical data. SSI is an optical whole-field technique and thus a large quantity of data is obtained for a relatively short test. The analysis of removed disc coupons loaded in diametral compression was used to provide response data for inverse analysis.

It is a relatively straightforward and obvious step to perform SSI on the box-section directly. This offers numerous benefits over the preceding methods. Primarily that the test can be performed in-situ, without removing the disc coupons thereby alleviating the problems associated with the removal procedure. The accuracy, repeatability and implementation of the diametral compression test is also no longer a concern. The effect of the loading method on the behaviour of the coupons is no longer an issue as the loading points can be some distance away from the area under analysis.

This chapter investigates the feasibility of using SSI to characterise material properties of pultruded GFRP box-section beams. Initial results are presented and recommendations for improving the method are discussed.
7.2 Experimental Procedure

The SSI system used in this investigation was identical to the system detailed in chapter 5. The box-section was mounted in a purpose-built fixture. The fixture clamped the section at one end, positioning the opposite end of the section under a loading bar fitted to an Instron 4411 tensile test machine. The fixture was fixed to the base of the tensile testing machine. This set-up represented a cantilever beam arrangement, shown in Figure 7-1.

![Figure 7-1 Configuration of Sample for SSI](image)

The samples used were identical to the damaged box-section samples used in chapter 6, displaying varying stages of progressive tearing damage. The SSI set-up was aligned so that the damaged mid-span region was illuminated. The collection of SSI data is identical to the phase-stepping method described in chapter 5.
7.3 Results from Initial SSI Tests

It was clear from an early stage in the experimental programme that the applied load needed to be relatively small. Due to the low elastic modulus of the GFRP material a small load applied to the sample caused sufficient deflection for speckle decorrelation to occur. In order to achieve a result the speckle size was increased relative to that used in the analysis of the discs in diametral compression.

The initial results were poor and need refinement. Due to the decorrelation problem only a very small load (10N) was used and thus the stress and strain difference between the unloaded and loaded states was very small. Fringe patterns could only be seen on heavily damaged samples and were still difficult to interpret, Figure 7-2. Further wrapped phase maps are presented in the appendices. Wrapped phase maps were determined for both x-surface and y-surface illumination. It was clear that improvements to the experimental set-up were needed to obtain acceptable data. Figure 7-3 shows the unwrapped phase map from the DCT-based algorithm, it is clear that no meaningful data can be determined from this image.
Figure 7-2  Wrapped Phase Map of Maximum Damaged Sample (X-Surface)

Figure 7-3  Unwrapped Phase Map of Maximum Damaged Sample (X-Surface)
7.3.1 Improvements to Experimental Procedure

There were several areas where improvements to the collection of SSI phase data could be made.

The major problem was decorrelation occurring before the sample was sufficiently strained to obtain fringe patterns. This was due to the deflection of the sample under loading. It is anticipated from initial tests that this problem can be alleviated by changing the set-up such that the laser illumination is in the same plane as the loading direction. The current set-up has the loading direction perpendicular to the plane of illumination. This change will mean that the speckles will move parallel to the viewing axis and thus decorrelation is less likely, particularly for normal illumination.

Additionally, the damaged region is currently equidistant between the support and the applied load. The deflection experienced by the sample can be reduced by decreasing the distance between the illuminated region and the support, or by decreasing the span. This requires re-configuration of the test rig for the samples under analysis. For a continuous or long pultruded length the illuminated region can readily be positioned close to a support therefore minimising the deflection (of the illuminated region).

There is considerable speckle noise present in the wrapped phase data. The current median filter used in the four-frame phase-stepping algorithm is very crude. A more refined filter would reduce speckle noise giving a clearer phase map. The advantages of an improved filter have been highlighted in Chapter 5.
7.4 The Feasibility of Using Speckle Shearing Interferometry to Inspect Structural Components

Defect detection in composite materials typically involves lengthy laboratory-based inspection techniques. For many industries this equates to increased maintenance costs and prohibits many companies from benefiting from the structural advantages of composites. The new approach has evolved from the Lightweight Booms project; it provides a low-cost inspection solution using data from shearing interferometry fused with a numerical strain function. The pathology of impact or fatigue damage is determined at each service interval by assessment of the spatial frequencies in the function. The main application will be composite structures; however, the technique is generic and applicable to other forms of structure and whole-field experimental data.

It is hypothesised that by archiving the results as a strain function in discrete cosine transform (DCT) form, damage may be detected by direct comparison of the data with that of a previous inspection. Series coefficients of the DCT depend in number on the size of the CCD array used to collect data and are found in an over-deterministic sense by fusing raw shearing interferometry data with a finite-difference equivalent of the compatibility equation for orthotropic materials. Damage is detected and quantified by degradation of the series terms in the strain function when the results are compared between one service interval and the next. The damage may be determined as either long-term degradation (fatigue) or localized defects (impact) through quantitative analysis of the inspection results.

Tests to verify the suitability of the approach have been performed on pultruded GFRP box-section beams subjected to different levels of impact damage. By comparing the strain data with an undamaged sample it is shown that damage is revealed using the coefficients of the strain functions. Initial results are presented using simulated (numerical) data produced using the finite element (FE) method. These are followed up by some early observations regarding actual experimental data.
Chapter 7: Inverse Analysis of Box-section Composites using Speckle Shearing Interferometry

Shearing interferometry systems enable the displacement gradients to be measured in a whole-field sense based upon the relationships:

\[
\Delta \phi_x = \frac{2\pi}{\lambda} \left( \sin \theta \frac{\partial u}{\partial x} + (1 + \cos \theta) \frac{\partial w}{\partial x} \right) \delta_y
\]

\[
\Delta \phi_y = \frac{2\pi}{\lambda} \left( \sin \theta \frac{\partial v}{\partial y} + (1 + \cos \theta) \frac{\partial w}{\partial y} \right) \delta_x
\]

**Equation 7-a**

Where \( u, v, \) and \( w \) are the in-plane and out-of-plane components of displacement, \( \theta \) is the angle of inclination of the imaging system with the object surface, \( \delta_x \) and \( \delta_y \) are the shearing distances, and \( \lambda \) is the wavelength. In simple terms, using orientations enables the displacement gradients \( \partial u/\partial x, \partial u/\partial y, \partial v/\partial x, \) and \( \partial v/\partial y \) to be obtained and hence:

\[
\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
\]

**Equation 7-b**

7.4.1 Test Programme

To validate the proposed approach, undamaged and damaged test samples were produced from pultruded box-section beams made from GFRP as described in previous chapters. Damaged samples exhibit local ‘crushing’ with axial progressive tearing cracks of length \((2a)\), depending on the load. The proposed inspection technique will be verified by applying the service loads as if the beam was a cantilever and comparing the resulting coefficients of the strain functions for data obtained both before and after the impact event.
7.4.2 Proposed Algorithm

The strains in a component, under representative service loads, may be determined from displacement gradients measured using the optical technique of shearing interferometry. If damage is sustained by a composite material due to impact delamination, void growth, propagation of fatigue cracks etc. there is an associated decrease in local values of the elastic stiffness that will be revealed through measurements of strain. Therefore, the hypothesis is that the measurement of strains during a routine service interval should enable damage to be detected since the measured strains will differ from those of the previous inspection.

One potential problem with this approach, however, is that it is impossible to maintain the same experimental protocol between one inspection and the next. The optical detector cannot be repeatedly pointed at exactly the same location without introducing an alignment error. This would be extremely problematic likely to lead to ‘decorrelation’ problems if one attempted to compare strain data for each pixel. The proposed strain function approach, however, provides a solution through its co-ordinate system \((m,n)\). The anticipated error will be characterised by a simple geometrical misalignment of the object within the detector’s domain and the co-ordinate system can be corrected accordingly by scaling, rotation, and translation.

In the proposed method the strains from a given service inspection for the same loading case are ‘archived’ as a (mathematical) strain function in the following discrete cosine transform form:

\[
\phi_{m,n} = \frac{4}{MN} \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} \phi_{p,q} \cos \left[ \frac{\pi p}{P} \left( m + \frac{1}{2} \right) \right] \cos \left[ \frac{\pi q}{Q} \left( n + \frac{1}{2} \right) \right]
\]

Equation 7-c

Where \(M\) and \(N\) are the number of rows and columns in the pixel data, and \(0 \leq m \leq M\) and \(0 \leq n \leq N\).

The coefficients of the series terms depend on the geometry and applied forces on the strain field within a given ‘window’ of inspection. Briefly, these coefficients are found in an over-deterministic sense from the available shearing data for \(\varepsilon_x\).
and ε, that is ‘smoothed’ to satisfy the compatibility condition through a finite differencing process. The quantities P and Q depend on the number of pixels in the measurement array (typically 256 by 256). This approach is similar to that first used by Nurse (Nurse 2000) for the comparison of experimental and numerical (finite element) data. As a consequence of this approach the ‘true’ strains are determined as a function of the coordinates (m, n) rather than as pixel data.

Between service intervals one would expect that if damage occurred, whether by a sudden impact event or through more long-term, degradation such as fatigue, the coefficients of these strain functions would be adjusted accordingly. By way of an illustrative example consider the two strain distributions shown in Figure 7-4. The first (Figure 7-4a and b) contains the usual (undamaged) horizontal strain map for a beam in bending. The second (Figure 7-4c and d) assumes there is some loss of stiffness (due to damage) and the neutral axis has shifted. The DCT strain function approach produces a ‘fingerprint’ of the strain revealing a difference in the coefficients. In Figure 7-4 only the terms for n=0 are plotted (as the only change is in the vertical sense) but it can be seen that the changes indicate the occurrence of damage.

In Figure 7-4 the DCT coefficients are of a ‘zigzag’ appearance as only the cosine terms have any meaning; the other sine terms of expression Equation 7-c are zeros in this particular problem. It is expected here, although not shown, that by subtracting the DCT results of the undamaged problem from those of the damaged problem that the nature of the damage may be determined.
7.4.3 Numerical Trials

Before being tested using experimental data the proposed algorithm was first applied to simulated (numerical) data. This is to verify the theoretical basis of the method so that countermeasures may then be implemented if the proposed method is found to be affected by noise, whether it is systematic or experimental noise.

A thorough series of trials are necessary to fully evaluate the process against the claims made of its practicability. It is not possible here to mention all the results obtained but a few are produced using numerical data based on Finite Element Analysis (FEA). In the FEA simulations a pultruded GFRP box-section beam is loaded as a cantilever as shown in Figure 7-5. Various degrees of ‘damage’ are incorporated as different lengths of horizontal progressive tearing cracks. These
are modelled in the FEA simulations as de-equivalenced nodes between elements. Small vertical cracks that characterise this form of impact damage are also incorporated. Further details of this and the actual experimental tests performed to develop the damage in the beams have been presented in earlier chapters.

![FE Model of Cantilever Beam](image)

The proposed approach to determine the strain function was performed using data from the region around the damaged area. Several crack lengths were examined in the ranges $60\text{mm} \leq 2a \leq 120\text{mm}$; the total length of each specimen is $300\text{m}$ with $30\text{mm}$ height. For each crack length, no attempt was made to collect data from exactly the same region except for the same test case, i.e. strains in the $x$ and $y$ directions were collected simultaneously for each crack length. Furthermore, no attempt was made to collect data from within the free boundaries of the specimen. The method is able to cope with the presence of free boundaries through the use of masks. Another test case with no crack, i.e. the undamaged case, was needed for comparison.
7.4.4 Initial Results

Figure 7-6 shows the 2-D ‘fingerprints’ of the DCT coefficients of the sum of the strains, $\varepsilon_x + \varepsilon_y$, for the undamaged and the three crack lengths considered. These maps are 2-D equivalents of the 1-D plots shown in Figure 7-4. The resulting coefficients are plotted according to $(m,n)$ and scaled to be between 0...255 in order that they can be shown in greyscale. In the damaged cases the results of DCT have been determined having subtracted the DCT results of the undamaged case; i.e. those shown in Figure 7-6a. It is clear that there is damage present from the three fingerprints of the cracked strain maps in that appearance is substantially different from the undamaged case. It is less clear however at present how the increase in crack length may be detected and the work continues in this direction.

![Figure 7-6](image_url)

**Figure 7-6** DCT Coefficient Plots (from FE Simulated Data) versus $(m,n)$ for Different Crack Lengths
Conclusions

This research proposes a low-cost solution based on optical techniques to inspect components quickly and cheaply during a routine service interval. A benefit of the approach to be exploited here is that the results are archived using spatial frequencies in the above function and not as pixel data. Initial experimental results are encouraging in that similar spatial distributions for the sum of the principal strains have been determined. The appearance of noise is a little confusing since the appearance of damage itself is a phenomenological source of noise. The existence of cracks, for example, leads to discontinuities in the strain maps where the compatibility equation is not satisfied and the same phenomena occurs when experimental noise is present. Overall, however, the early results are highly encouraging and the research continues.

In the proposed method, shearing interferometry experiments are performed to provide data for the $x$- and $y$-direction strains. The series coefficients of the stress function are determined from the strain data at each service interval assuming the same test loading conditions and damage will be detected by comparing the coefficients for each spatial frequency. In other words, if the coefficients disagree between one service interval and the next then damage has occurred.

Damage due to localised impact is revealed through a difference at higher spatial frequencies. Cyclic loading can cause widespread propagation of fatigue micro-cracks that will be revealed by differences at lower spatial frequencies. Therefore, the proposed technique offers a low-cost means of in-situ inspection where the pathology of damage is revealed using a quantitative approach normally used in signal processing. There is no need (in general) for individual component disassembly and, therefore, this more rapid means of inspection promises to minimise loss of income due to maintenance.
8 CONCLUSIONS

8.1 The Design of a Lightweight Telehandler Boom

- The current literature contains gaps in knowledge pertaining to the behaviour of pultruded sections (of complex cross-section) under quasi-static transverse impact (bending crush), particularly regarding the post-impact performance of the section.

- A CFRP telehandler boom has been designed and a prototype is currently being manufactured. The composite boom will deliver a weight saving in excess of 60% over the current steel design. The performance increases from utilising lightweight booms will give J.C. Bamford Excavators Ltd. a competitive advantage helping to increase its market share.

- A method of predicting the benefits of lightweight components on the performance of telehandlers has been developed using mechanics and vector algebra.

- The lay-up of the prototype boom has been optimised by a genetic algorithm to give an efficient structure.

- The prototype boom is the first non-metallic boom to be fitted to a telehandler.
8.2 *Determination of Orthotropic Properties*

- Circular disc coupons can be removed from GFRP sections using CNC-controlled lasers.

- The diametral compression test cannot be used repeatedly on a single disc as the test causes degradation in the material properties. Empirical stiffness measurements were approximately 50% that of theoretical values provided by finite element analysis. Larger disc coupons (40mm) gave a better approximation than smaller coupons (15mm).

- The bending tests (presented in Chapter 4) and the diametral compression tests (Chapter 5) highlighted the problems of material variability within pultruded sections, especially when using relatively small coupons.

- The diametral compression test as employed in this work, is not suited to providing the empirical data for inverse analysis.

- The use of speckle-shearing interferometry allows whole-field data to be collected from a disc loaded in diametral compression. A smaller load can be used to minimise the degradation of the sample. Fewer tests are needed to characterise the material.

- The filtering of speckle noise is critical to deriving a worthwhile set of whole-field strain maps.

- The results show that this technique approximates closely to the elastic properties of the GFRP section quoted by the manufacturer.
8.3 **Damage Detection**

- The use of multiple loads to provide empirical data for inverse analysis has been shown to be an effective technique in characterising progressive tearing damage in GFRP box sections. Although the data required is relatively large and its collection is time consuming.

- The analysis of GFRP pultruded box-sections by speckle-shearing interferometry provides the opportunity for fast, low cost, in-situ inspection of structural components.

- The magnitude and position of the load must be given careful consideration to minimise the risk of speckle decorrelation due to large deflections.

- The initial results presented indicate that the proposed technique of comparing the series coefficients of the stress function is feasible.

- Damage is revealed as a difference in the higher spatial frequencies between routine service inspections.
9 RECOMMENDATIONS FOR FURTHER WORK

• The load chart generator presented in chapter 4 should be further developed to include the third dimension. This will allow a more accurate prediction of stability at height by accounting for lateral stability. This will prove a highly useful tool with which to evaluate design changes or fresh designs.

• The prototype will be tested within the coming months. The data from the testing programme needs to be analysed and iterative improvements to the composite boom design made.

• Further disc coupons need to be analysed using speckle shearing interferometry to investigate the material variability found in this investigation.

• Further analysis is required to determine the optimum number of unequivocal tests in order to characterise progressive tearing damage in GFRP sections. This will indicate whether the technique presented is feasible in terms of time and effort required to conduct the unequivocal static testing.

• Refinement of the speckle shearing interferometry equipment is needed to reduce the occurrence of speckle decorrelation. Improvements to the test rig will allow the damaged region to be moved closer to the supports thus reducing the deflection in the illuminated area.

• The technique proposed for detecting damage by comparison of the coefficients of the strain function requires further development. Additionally, the performance of the method when using experimental data needs to be determined. Furthermore, a clearer explanation of the results presented here is needed in order to fully comprehend the effect of damage on the series coefficients of the stress function.
REFERENCES


References


JCB-Sales (1999). JCB 532/537 Loadalls. www.jcb.co.uk


References


APPENDICES

a. MathCAD Programme: Load-Chart Generator
b. Additional Diametral Compression Test Results
c. MathCAD Programme: Four-Frame Algorithm
d. Diametral Compression: Wrapped Phase Data
e. MathCAD Programme: DCT-based Phase Unwrapping Algorithm
f. MatchCAD Programme: Determination of Orthotropic Properties
g. Additional Radar Plots from Three-Point Bend Tests
Load Chart Calculation
Simplified Example

Vectors are all from boom pivot and are the assembled positions. For example RINT will change with extension.

**Masses of Components**

\[
\begin{align*}
\text{ROUT} & := (217, 39) \text{ mm} & \text{MOUT} & := 442.4 \text{ kg} & \text{MRAM1} & := 162 \text{ kg} \\
\text{COGOUT} & := (1362, -199) \text{ mm} & \text{MINT} & := 242.2 \text{ kg} & \text{MRAM2} & := 108.4 \text{ kg} \\
\text{RINTO} & := (3159, 39) \text{ mm} & \text{MINN} & := 341.4 \text{ kg} & \text{MCAR} & := 216.2 \text{ kg} \\
\text{RINTI} & := (93, -243) \text{ mm} & \text{MTH} & := 5800 \text{ kg} \\
\text{RINN} & := (2773, -243) \text{ mm} & \text{g} & := 9.807 \frac{m}{s^2} \\
\text{RNOSE} & := (3815, -1020) \text{ mm} & \\
\text{COGINN} & := \text{RNOSE} + (1360.5, 673.5) \text{ mm} & \\
\text{COGINT} & := \text{RINTI} + (1612, 26) \text{ mm} & 
\end{align*}
\]

Note above mass of TH does NOT include fuel or hydraulic oil or displacement rams.
Quoted figure is 5800 kg so MTH may have been adjusted to allow for these items.

\[
\begin{array}{c|c}
\text{MPL} & \\
330 & kg \\
500 & \\
1000 & \\
1500 & \\
2000 & \\
2500 & \\
3000 & \\
3700 & 
\end{array}
\]

Assumption: Inner extension ram is in a horizontal plane when boom angle is zero.

CofG of forks, payload and carriage are all related to RNOSE and are considered to be independant of the boom angle (wrt RNOSE).
POSOUT(X, A) := COGOUT: \[
\begin{pmatrix}
\cos(A) \\
-\sin(A)
\end{pmatrix}
\]

POSINT(X, A) := \[
\begin{pmatrix}
X \\
0
\end{pmatrix}
\begin{pmatrix}
\cos(A) \\
-\sin(A)
\end{pmatrix}
\]

POSINN(X, A) := \[
\begin{pmatrix}
2-X \\
0
\end{pmatrix}
\begin{pmatrix}
\cos(A) \\
-\sin(A)
\end{pmatrix}
\]

POSNOSE(X, A) := RNose + \[
\begin{pmatrix}
2-X \\
0
\end{pmatrix}
\begin{pmatrix}
\cos(A) \\
-\sin(A)
\end{pmatrix}
\]

POSCAR(X, A) := POSNOSE(X, A) + 147.5mm

POSPL(X, A) := POSNOSE(X, A) + 725mm
Calculation of Ram Boss Positions and Ram C of G

\[
\text{ROUTX} := \text{ROUT} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \text{ROUTY} := \text{ROUT} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \text{RINTIX} := \text{RINT} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \text{RINTIY} := \text{RINT} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

\[
\text{RINTOX} := \text{RINTO} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \text{RINTOY} := \text{RINTO} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \text{RINNX} := \text{RINN} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \text{RINNY} := \text{RINN} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

\[
\text{POSER11}(X, A) := \begin{pmatrix} \sqrt{\text{ROUTX}^2 + \text{ROUTY}^2} \cos(A) + \text{ROUTY} \sin(A) \sqrt{\text{ROUTX}^2 + \text{ROUTY}^2} \\ \sqrt{\text{ROUTX}^2 + \text{ROUTY}^2} \sin(A) + \text{ROUTY} \cos(A) \sqrt{\text{ROUTX}^2 + \text{ROUTY}^2} \end{pmatrix}
\]

\[
\text{POSER12}(X, A) := \begin{pmatrix} \sqrt{(\text{RINTOX} + X)^2 + \text{RINTOY}^2} \cos(A) + \text{RINTOY} \sin(A) \sqrt{(\text{RINTOX} + X)^2 + \text{RINTOY}^2} \\ \sqrt{(\text{RINTOX} + X)^2 + \text{RINTOY}^2} \sin(A) + \text{RINTOY} \cos(A) \sqrt{(\text{RINTOX} + X)^2 + \text{RINTOY}^2} \end{pmatrix}
\]

\[
\text{POSER21}(X, A) := \begin{pmatrix} \sqrt{(\text{RINTIX} + X)^2 + \text{RINTIY}^2} \cos(A) + \text{RINTIY} \sin(A) \sqrt{(\text{RINTIX} + X)^2 + \text{RINTIY}^2} \\ \sqrt{(\text{RINTIX} + X)^2 + \text{RINTIY}^2} \sin(A) + \text{RINTIY} \cos(A) \sqrt{(\text{RINTIX} + X)^2 + \text{RINTIY}^2} \end{pmatrix}
\]

\[
\text{POSER22}(X, A) := \begin{pmatrix} \sqrt{(\text{RINNX} + 2X)^2 + \text{RINNY}^2} \cos(A) + \text{RINNY} \sin(A) \sqrt{(\text{RINNX} + 2X)^2 + \text{RINNY}^2} \\ \sqrt{(\text{RINNX} + 2X)^2 + \text{RINNY}^2} \sin(A) + \text{RINNY} \cos(A) \sqrt{(\text{RINNX} + 2X)^2 + \text{RINNY}^2} \end{pmatrix}
\]

\[
\text{COGRAM1}(X, A) := \text{POSER11}(X, A) + \frac{1343}{2942} (\text{POSER12}(X, A) - \text{POSER11}(X, A))
\]

\[
\text{COGRAM2}(X, A) := \text{POSER21}(X, A) + \frac{1286}{2680} (\text{POSER22}(X, A) - \text{POSER21}(X, A))
\]

\[
\text{POSRAM1}(X, A) := \text{COGRAM1}(X, A) \begin{pmatrix} \cos(A) \\ -\sin(A) \end{pmatrix}, \quad \text{POSRAM2}(X, A) := \text{COGRAM2}(X, A) \begin{pmatrix} \cos(A) \\ -\sin(A) \end{pmatrix}
\]
**Centre of Gravity of Tele-handler & Position of Front Axle.**

**Beta** := 0 deg  
Beta is angle of TH with ground.  
+ive Beta means TH is travelling uphill.  
-ive means TH is travelling downhill.  
AXLE is posn of f/axle c/f when beta is zero.  
COGTH is posn of CofG of TH when beta is zero.

**AXLE** := \begin{pmatrix} 2460 \\ -1300 \end{pmatrix} mm  
AXLE is posn of f/axle c/f when beta is zero.

**COGTH** := AXLE + \begin{pmatrix} -2000 \\ 300 \end{pmatrix} mm  
COGTH = \begin{pmatrix} 0.46 \\ -1 \end{pmatrix} m

**AXLEX** := AXLE \begin{pmatrix} 1 \\ 0 \end{pmatrix}  
**AXLEY** := AXLE \begin{pmatrix} 0 \\ 1 \end{pmatrix}  
**COGTHX** := COGTH \begin{pmatrix} 1 \\ 0 \end{pmatrix}  
**COGTHY** := COGTH \begin{pmatrix} 0 \\ 1 \end{pmatrix}  

**FAXLE** is the position of the front axle relative to the boom pivot at angle beta.  
**POSTH** is the pos'n of the CofG of the TH at angle beta.

**FALPHA** := \sin \left( \frac{AXLEY}{\sqrt{AXLEX^2 + AXLEY^2}} \right)  
\text{FALPHA} = \pm 27.854 \text{ deg}

**FAXLE** := \begin{pmatrix} \sqrt{AXLEX^2 + AXLEY^2 \cos(BETA + FALPHA)} \\ \sqrt{AXLEX^2 + AXLEY^2 \sin(BETA + FALPHA)} \end{pmatrix}  
\text{FAXLE} = \begin{pmatrix} 2.46 \\ -1.3 \end{pmatrix} m

**CALPHA** := \sin \left( \frac{COGTHY}{\sqrt{COGTHX^2 + COGTHY^2}} \right)  
\text{CALPHA} = \pm 65.298 \text{ deg}

**PSNTH** := \begin{pmatrix} \sqrt{COGTHX^2 + COGTHY^2 \cos(BETA + CALPHA)} \\ \sqrt{COGTHX^2 + COGTHY^2 \sin(BETA + CALPHA)} \end{pmatrix}  
\text{PSNTH} = \begin{pmatrix} 0.46 \\ -1 \end{pmatrix} m

**POSTH(X, A)** := PSNTH \begin{pmatrix} \cos(A) \\ -\sin(A) \end{pmatrix}

**POSTH(X, A)** := PSNTH \begin{pmatrix} \cos(A) \\ -\sin(A) \end{pmatrix}

**POSAXLE(X, A)** := FAXLE \begin{pmatrix} \cos(A) \\ -\sin(A) \end{pmatrix}
MOMOUT(X, A) := (POSOOUT(X, A) – POSAXLE(X, A))·MOUT·g
MOMINT(X, A) := (POSOINT(X, A) – POSAXLE(X, A))·MINT·g
MOMINN(X, A) := (POSOINN(X, A) – POSAXLE(X, A))·MINN·g
MOMCAR(X, A) := (POSCAR(X, A) – POSAXLE(X, A))·MCAR·g
MOMRAM1(X, A) := (POSRAM1(X, A) – POSAXLE(X, A))·MRAM1·g
MOMRAM2(X, A) := (POSRAM2(X, A) – POSAXLE(X, A))·MRAM2·g
MOMTH(X, A) := (POSTH(X, A) – POSAXLE(X, A))·MTH·g
MOMPL0(X, A) := (POSPL(X, A) – POSAXLE(X, A))·MPL0·g
MOMPL1(X, A) := (POSPL(X, A) – POSAXLE(X, A))·MPL1·g
MOMPL2(X, A) := (POSPL(X, A) – POSAXLE(X, A))·MPL2·g
MOMPL3(X, A) := (POSPL(X, A) – POSAXLE(X, A))·MPL3·g
MOMPL4(X, A) := (POSPL(X, A) – POSAXLE(X, A))·MPL4·g
MOMPL5(X, A) := (POSPL(X, A) – POSAXLE(X, A))·MPL5·g
MOMPL6(X, A) := (POSPL(X, A) – POSAXLE(X, A))·MPL6·g
MOMPL7(X, A) := (POSPL(X, A) – POSAXLE(X, A))·MPL7·g

RANGE VARIABLES (X & A)

XLO := 0m  XHI := 3.4m  XN := 10
ALO := 0deg  AH := 70deg  AN := 10

\[ x_{i,n} := \frac{XLO + i \cdot \frac{XHI - XLO}{XN - 1}}{\frac{AH - ALO}{AN - 1}} \]

\[ a_{j,n} := ALO + j \cdot \frac{AH - ALO}{AN - 1} \]

\( X \) is extension of rams and \( A \) is the boom angle

OUT\(_{i,j}\) := MOMOUT\( (x_{i,n}, a_{j,n}) \)
RAM1\(_{i,j}\) := MOMRAM1\( (x_{i,n}, a_{j,n}) \)
INT\(_{i,j}\) := MOMINT\( (x_{i,n}, a_{j,n}) \)
RAM2\(_{i,j}\) := MOMRAM2\( (x_{i,n}, a_{j,n}) \)
INN\(_{i,j}\) := MOMINN\( (x_{i,n}, a_{j,n}) \)
CAR\(_{i,j}\) := MOMCAR\( (x_{i,n}, a_{j,n}) \)
TH\(_{i,j}\) := MOMTH\( (x_{i,n}, a_{j,n}) \)

\( P_{i,j} \) := MOMPL\( (x_{i,n}, a_{j,n}) \)
\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline 
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0.378 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0.756 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 1.133 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline 
4 & 1.511 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 1.889 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 2.267 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 2.644 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 3.022 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 3.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[x_{\text{ind}} = 1.511 \text{ m} \quad \theta_{\text{ind}} = 31.111 \text{ deg}\]

\[
\text{SUBTOTAL} := \text{OUT} + \text{RAM1} + \text{INT} + \text{RAM2} + \text{INN} + \text{CAR} + \text{TH} \\
\text{TOTAL0} := \text{SUBTOTAL} + \text{PL0} \\
\text{TOTAL1} := \text{SUBTOTAL} + \text{PL1} \\
\text{TOTAL2} := \text{SUBTOTAL} + \text{PL2} \\
\text{TOTAL3} := \text{SUBTOTAL} + \text{PL3} \\
\text{TOTAL4} := \text{SUBTOTAL} + \text{PL4} \\
\text{TOTAL5} := \text{SUBTOTAL} + \text{PL5} \\
\text{TOTAL6} := \text{SUBTOTAL} + \text{PL6} \\
\text{TOTAL7} := \text{SUBTOTAL} + \text{PL7}
\]
CALCULATE RAM TIPPING EXTENSIONS

NB the number at the end of a variable name, 0-7 refers to the load case applied.

ie 0=330kg, 1=500kg, 2=1000kg, 3=1500kg, 4=2000kg, 5=2500kg, 6=3000kg, 7=3700kg

The following routine calculates the ram extension at which the machine becomes unstable for each angle between 0 and XN-1.

\[
\text{Alpha}_0 := \text{submatrix}(\text{TOTAL}_0, XN - 1, XN - 1, 0, XN - 1) \\
\text{Beta}_0 := \text{submatrix}(\text{TOTAL}_0, 0, 0, 0, XN - 1), \quad \text{Alpha}_1 := \text{submatrix}(\text{TOTAL}_1, XN - 1, XN - 1, 0, XN - 1) \\
\text{Gradient}_0 := \frac{\text{Alpha}_0 - \text{Beta}_0}{(\text{Xind}_{XN-1} - \text{Xind}_0) \frac{1}{1m}} \\
\text{RamExtension}_0 := -\left( \left( \frac{\text{Beta}_0}{\text{Gradient}_0} \right)^T \right) \\
\text{Beta}_1 := \text{submatrix}(\text{TOTAL}_1, 0, 0, 0, XN - 1) \\
\text{Gradient}_1 := \frac{\text{Alpha}_1 - \text{Beta}_1}{(\text{Xind}_{XN-1} - \text{Xind}_0) \frac{1}{1m}} \\
\text{RamExtension}_1 := -\left( \left( \frac{\text{Beta}_1}{\text{Gradient}_1} \right)^T \right) \\
\text{Alpha}_2 := \text{submatrix}(\text{TOTAL}_2, XN - 1, XN - 1, 0, XN - 1) \\
\text{Beta}_2 := \text{submatrix}(\text{TOTAL}_2, 0, 0, 0, XN - 1), \text{Alpha}_3 := \text{submatrix}(\text{TOTAL}_3, XN - 1, XN - 1, 0, XN - 1) \\
\text{Gradient}_2 := \frac{\text{Alpha}_2 - \text{Beta}_2}{(\text{Xind}_{XN-1} - \text{Xind}_0) \frac{1}{1m}} \\
\text{RamExtension}_2 := -\left( \left( \frac{\text{Beta}_2}{\text{Gradient}_2} \right)^T \right) \\
\text{Beta}_3 := \text{submatrix}(\text{TOTAL}_3, 0, 0, 0, XN - 1) \\
\text{Gradient}_3 := \frac{\text{Alpha}_3 - \text{Beta}_3}{(\text{Xind}_{XN-1} - \text{Xind}_0) \frac{1}{1m}} \\
\text{RamExtension}_3 := -\left( \left( \frac{\text{Beta}_3}{\text{Gradient}_3} \right)^T \right) \\
\text{Alpha}_4 := \text{submatrix}(\text{TOTAL}_4, XN - 1, XN - 1, 0, XN - 1) \\
\text{Beta}_4 := \text{submatrix}(\text{TOTAL}_4, 0, 0, 0, XN - 1) \\
\text{Gradient}_4 := \frac{\text{Alpha}_4 - \text{Beta}_4}{(\text{Xind}_{XN-1} - \text{Xind}_0) \frac{1}{1m}} \\
\text{RamExtension}_4 := -\left( \left( \frac{\text{Beta}_4}{\text{Gradient}_4} \right)^T \right) \\
\text{Beta}_5 := \text{submatrix}(\text{TOTAL}_5, 0, 0, 0, XN - 1) \\
\text{Gradient}_5 := \frac{\text{Alpha}_5 - \text{Beta}_5}{(\text{Xind}_{XN-1} - \text{Xind}_0) \frac{1}{1m}} \\
\text{RamExtension}_5 := -\left( \left( \frac{\text{Beta}_5}{\text{Gradient}_5} \right)^T \right)
\[
\begin{align*}
\text{Alpha}_6 & := \text{submatrix}(\text{TOTAL}_6, XN - 1, XN - 1, 0, XN - 1) \\
\text{Beta}_6 & := \text{submatrix}(\text{TOTAL}_6, 0, 0, 0, XN - 1) \\
\text{Gradient}_6 & := \frac{\text{Alpha}_6 - \text{Beta}_6}{(x_{\text{ind}}_{XN-1} - x_{\text{ind}}_0)\frac{1}{1m}} \\
\text{RamExtension}_6 & := -\left(\frac{\text{Beta}_6}{\text{Gradient}_6}\right)^T \\
\text{Alpha}_7 & := \text{submatrix}(\text{TOTAL}_7, XN - 1, XN - 1, 0, XN - 1) \\
\text{Beta}_7 & := \text{submatrix}(\text{TOTAL}_7, 0, 0, 0, XN - 1) \\
\text{Gradient}_7 & := \frac{\text{Alpha}_7 - \text{Beta}_7}{(x_{\text{ind}}_{XN-1} - x_{\text{ind}}_0)\frac{1}{1m}} \\
\text{RamExtension}_7 & := -\left(\frac{\text{Beta}_7}{\text{Gradient}_7}\right)^T \\
\text{BLCC} & := \frac{\text{POSNOSE}(0m, 0\text{deg})}{m} \quad \text{BLCC} = 3.815 \\
\text{BLOC} & := \frac{\text{POSNOSE}(3.4m, 0\text{deg})}{m} \quad \text{BLOC} = 10.615 \\
\text{These are conversion factors to ensure correct units} \\
\text{L}_4 & := \frac{\text{POSPL}(0m, 0)}{m} - \frac{\text{POSNOSE}(0m, 0)}{m} \quad \text{L}_4 = 0.725 \\
\text{L}_3 & := 0 \begin{bmatrix} \text{RNOSE} & 0 \end{bmatrix}^{\frac{-1}{m}} \quad \text{L}_3 = 1.02 \\
\text{L}_2 & := \begin{bmatrix} \text{AXLE} & 0 \end{bmatrix} \quad \text{L}_2 = 2.46 \\
\text{RADTYRE} & := 0.65 \\
\text{L}_{\text{PIV\_VER}} & := \begin{bmatrix} \text{AXLE} & 0 \end{bmatrix} + \text{RADTYRE} \quad \text{L}_{\text{PIV\_VER}} = 1.95
\end{align*}
\]
BLT is the Boom Length at Tipping, where 0, 1, 2, 3... refers to the applied loading

\[
\begin{align*}
BLT_0 & := \text{RamExtension}_0 \cdot 2 + BLCC \\
BLT_1 & := \text{RamExtension}_1 \cdot 2 + BLCC \\
BLT_2 & := \text{RamExtension}_2 \cdot 2 + BLCC \\
BLT_3 & := \text{RamExtension}_3 \cdot 2 + BLCC \\
BLT_4 & := \text{RamExtension}_4 \cdot 2 + BLCC \\
BLT_5 & := \text{RamExtension}_5 \cdot 2 + BLCC \\
BLT_6 & := \text{RamExtension}_6 \cdot 2 + BLCC \\
BLT_7 & := \text{RamExtension}_7 \cdot 2 + BLCC \\
\end{align*}
\]

**CONVERT BOOM TIPPING LENGTHS FROM POLAR TO CARTESIAN COORDINATES**

\[
\begin{align*}
Y_0 & := L_{PIV\_VER} + \text{BLT}_0 \cdot \sin \left( \text{aind}_0 - \arctan \left( \frac{L_2}{\text{BLT}_0} \right) \right) - L_3 \\
X_0 & := \text{BLT}_0 \cdot \cos \left( \text{aind}_0 - \arctan \left( \frac{L_2}{\text{BLT}_0} \right) \right) + L_4 - \text{RAD\_TYRE} \cdot L_{AXL} \\
Y_1 & := L_{PIV\_VER} + \text{BLT}_1 \cdot \sin \left( \text{aind}_1 - \arctan \left( \frac{L_2}{\text{BLT}_1} \right) \right) - L_3 \\
X_1 & := \text{BLT}_1 \cdot \cos \left( \text{aind}_1 - \arctan \left( \frac{L_2}{\text{BLT}_1} \right) \right) + L_4 - \text{RAD\_TYRE} \cdot L_{AXL} \\
Y_2 & := L_{PIV\_VER} + \text{BLT}_2 \cdot \sin \left( \text{aind}_2 - \arctan \left( \frac{L_2}{\text{BLT}_2} \right) \right) - L_3 \\
X_2 & := \text{BLT}_2 \cdot \cos \left( \text{aind}_2 - \arctan \left( \frac{L_2}{\text{BLT}_2} \right) \right) + L_4 - \text{RAD\_TYRE} \cdot L_{AXL} \\
Y_3 & := L_{PIV\_VER} + \text{BLT}_3 \cdot \sin \left( \text{aind}_3 - \arctan \left( \frac{L_2}{\text{BLT}_3} \right) \right) - L_3 \\
X_3 & := \text{BLT}_3 \cdot \cos \left( \text{aind}_3 - \arctan \left( \frac{L_2}{\text{BLT}_3} \right) \right) + L_4 - \text{RAD\_TYRE} \cdot L_{AXL} \\
Y_4 & := L_{PIV\_VER} + \text{BLT}_4 \cdot \sin \left( \text{aind}_4 - \arctan \left( \frac{L_2}{\text{BLT}_4} \right) \right) - L_3 \\
X_4 & := \text{BLT}_4 \cdot \cos \left( \text{aind}_4 - \arctan \left( \frac{L_2}{\text{BLT}_4} \right) \right) + L_4 - \text{RAD\_TYRE} \cdot L_{AXL} \\
\end{align*}
\]
\[ Y_{51} := L_{PIV\_VER} + BLT_{51} \cdot \sin\left( \alpha_{51} - \text{atan}\left( \frac{L_2}{BLT_{51}} \right) \right) - L_3 \]

\[ X_{51} := BLT_{51} \cdot \cos\left( \alpha_{51} - \text{atan}\left( \frac{L_2}{BLT_{51}} \right) \right) + L_4 - \text{RAD\_TYRE} - L_\text{AXL} \]

\[ Y_{61} := L_{PIV\_VER} + BLT_{61} \cdot \sin\left( \alpha_{61} - \text{atan}\left( \frac{L_2}{BLT_{61}} \right) \right) - L_3 \]

\[ X_{61} := BLT_{61} \cdot \cos\left( \alpha_{61} - \text{atan}\left( \frac{L_2}{BLT_{61}} \right) \right) + L_4 - \text{RAD\_TYRE} - L_\text{AXL} \]

\[ Y_{71} := L_{PIV\_VER} + BLT_{71} \cdot \sin\left( \alpha_{71} - \text{atan}\left( \frac{L_2}{BLT_{71}} \right) \right) - L_3 \]

\[ X_{71} := BLT_{71} \cdot \cos\left( \alpha_{71} - \text{atan}\left( \frac{L_2}{BLT_{71}} \right) \right) + L_4 - \text{RAD\_TYRE} - L_\text{AXL} \]

**DEFINE MINIMUM BOOM LENGTH ENVELOPE**

Range of Angles for plotting of Boom Envelope (θ1)

\[ \theta_1 := \begin{bmatrix} 0 \\ 10 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \\ 90 \end{bmatrix} \text{- deg} \]

\[ Y_{\text{ARM\_MIN}} := L_{PIV\_VER} + \text{BLCC} \cdot \sin\left( \theta_1 - \text{atan}\left( \frac{L_2}{\text{BLCC}} \right) \right) - L_3 \]

\[ X_{\text{ARM\_MIN}} := \text{BLCC} \cdot \cos\left( \theta_1 - \text{atan}\left( \frac{L_2}{\text{BLCC}} \right) \right) + L_4 - \text{RAD\_TYRE} - L_\text{AXL} \]

**DEFINE MAXIMUM BOOM LENGTH ENVELOPE**

\[ Y_{\text{ARM\_MAX}} := L_{PIV\_VER} + \text{BLOC} \cdot \sin\left( \theta_1 - \text{atan}\left( \frac{L_2}{\text{BLOC}} \right) \right) - L_3 \]

\[ X_{\text{ARM\_MAX}} := \text{BLOC} \cdot \cos\left( \theta_1 - \text{atan}\left( \frac{L_2}{\text{BLOC}} \right) \right) + L_4 - \text{RAD\_TYRE} - L_\text{AXL} \]
PLOT LOAD CHART

Theoretical Load Chart

\[
MPL = \begin{pmatrix}
330 \\
500 \\
1 \times 10^3 \\
1.5 \times 10^3 \\
2 \times 10^3 \\
2.5 \times 10^3 \\
3 \times 10^3 \\
3.7 \times 10^3
\end{pmatrix} \text{ kg}
\]
APPENDIX B: DIAMETRAL COMPRESSION TEST RESULTS
DIAMETRAL COMPRESSION TEST RESULTS

Independent 15mm Discs from Undamaged Shear Web

Single Rotated 15mm Disc from Undamaged Shear Web
Independent 15mm Discs from Undamaged Shear Web

Independent 15mm Discs from Damaged Top Flange
Independent 15mm Discs from Undamaged Top Flange

Independent 15mm Discs from Undamaged Shear Web
Comparison between Independent 15mm and 40mm Undamaged Discs

Independent 15mm Discs from Undamaged Shear Web
Independent 15mm Discs from Damaged Shear Web

Independent 40mm Discs from Undamaged Shear Web
\begin{align*}
I_0 &:= \text{READBMP}("i0.tif") \\
I_1 &:= \text{READBMP}("i1.tif") \\
I_2 &:= \text{READBMP}("i2.tif") \\
I_3 &:= \text{READBMP}("i3.tif")
\end{align*}

\begin{align*}
\text{SIZE} &:= 350 \\
\text{XMIN} &:= 130 \\
\text{YMIN} &:= 60
\end{align*}

\begin{align*}
\text{SUBI0} &:= \text{submatrix}[I_0, \text{YMIN}, (\text{YMIN} + \text{SIZE} - 1), \text{XMIN}, (\text{XMIN} + \text{SIZE} - 1)] \\
\text{SUBI1} &:= \text{submatrix}[I_1, \text{YMIN}, (\text{YMIN} + \text{SIZE} - 1), \text{XMIN}, (\text{XMIN} + \text{SIZE} - 1)] \\
\text{SUBI2} &:= \text{submatrix}[I_2, \text{YMIN}, (\text{YMIN} + \text{SIZE} - 1), \text{XMIN}, (\text{XMIN} + \text{SIZE} - 1)] \\
\text{SUBI3} &:= \text{submatrix}[I_3, \text{YMIN}, (\text{YMIN} + \text{SIZE} - 1), \text{XMIN}, (\text{XMIN} + \text{SIZE} - 1)]
\end{align*}

\begin{align*}
\text{M} &:= \text{rows}(\text{SUBI0}) \\
\text{N} &:= \text{cols}(\text{SUBI0}) \\
\text{m} &:= 1..\text{M} - 1 \\
\text{n} &:= 1..\text{N} - 1
\end{align*}

\begin{align*}
\text{TOP} &:= \text{SUBI3} - \text{SUBI1} \\
\text{BOTTOM} &:= \text{SUBI0} - \text{SUBI2}
\end{align*}

\begin{align*}
\text{PHI}_{m,n} &:= \text{if}( \text{BOTTOM}_{m,n} = 0,\pi, \text{atan2}(\text{TOP}_{m,n}, \text{BOTTOM}_{m,n}) ) \\
\text{IMAGE}_{\text{PHI}} &:= (\text{PHI} + \pi) \frac{255}{2\pi}
\end{align*}
SMOOTHED PHI

\[
P := \text{rows(IMAGE\_PHI)} \quad Q := \text{cols(IMAGE\_PHI)}
\]
\[
p := 1..P - 2 \quad q := 1..Q - 2
\]
\[
\text{PHI}_{p,q} := \text{median(submatrix(IMAGE\_PHI, p - 1, p + 1, q - 1, q + 1))}
\]

\[
\text{WRITEBMP(“phi\_w.bmp”) := PHI}
\]
### WRAPPED PHASE DATA

**HORIZONTAL 3mm SHEAR, ANGLED ILLUMINATION**

<table>
<thead>
<tr>
<th>Disc</th>
<th>( \alpha = 0^\circ )</th>
<th>( \alpha = 45^\circ )</th>
<th>( \alpha = 90^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disc 1</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
</tr>
<tr>
<td>Disc 2</td>
<td><img src="image4" alt="Image" /></td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
</tr>
<tr>
<td>Disc 3</td>
<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /></td>
<td><img src="image9" alt="Image" /></td>
</tr>
<tr>
<td>Disc 4</td>
<td><img src="image10" alt="Image" /></td>
<td><img src="image11" alt="Image" /></td>
<td><img src="image12" alt="Image" /></td>
</tr>
<tr>
<td>Disc 5</td>
<td><img src="image13" alt="Image" /></td>
<td><img src="image14" alt="Image" /></td>
<td><img src="image15" alt="Image" /></td>
</tr>
<tr>
<td>Disc 6</td>
<td><img src="image16" alt="Image" /></td>
<td><img src="image17" alt="Image" /></td>
<td><img src="image18" alt="Image" /></td>
</tr>
</tbody>
</table>
HORIZONTAL 3mm SHEAR, NORMAL ILLUMINATION

<table>
<thead>
<tr>
<th>Disc</th>
<th>$\alpha = 0^\circ$</th>
<th>$\alpha = 45^\circ$</th>
<th>$\alpha = 90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disc 1</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>Disc 2</td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td>Disc 3</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
<tr>
<td>Disc 4</td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
<tr>
<td>Disc 5</td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td><img src="image15.png" alt="Image" /></td>
</tr>
<tr>
<td>Disc 6</td>
<td><img src="image16.png" alt="Image" /></td>
<td><img src="image17.png" alt="Image" /></td>
<td><img src="image18.png" alt="Image" /></td>
</tr>
</tbody>
</table>
### VERTICAL 3mm SHEAR, ANGLED ILLUMINATION

<table>
<thead>
<tr>
<th>Disc</th>
<th>$\alpha = 0^\circ$</th>
<th>$\alpha = 45^\circ$</th>
<th>$\alpha = 90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disc 1</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>Disc 2</td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td>Disc 3</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
<tr>
<td>Disc 4</td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
<tr>
<td>Disc 5</td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td><img src="image15.png" alt="Image" /></td>
</tr>
<tr>
<td>Disc 6</td>
<td><img src="image16.png" alt="Image" /></td>
<td><img src="image17.png" alt="Image" /></td>
<td><img src="image18.png" alt="Image" /></td>
</tr>
</tbody>
</table>
# VERTICAL 3mm SHEAR, NORMAL ILLUMINATION

<table>
<thead>
<tr>
<th>Disc</th>
<th>$\alpha = 0^\circ$</th>
<th>$\alpha = 45^\circ$</th>
<th>$\alpha = 90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disc 1</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
</tr>
<tr>
<td>Disc 2</td>
<td><img src="image4" alt="Image" /></td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
</tr>
<tr>
<td>Disc 3</td>
<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /></td>
<td><img src="image9" alt="Image" /></td>
</tr>
<tr>
<td>Disc 4</td>
<td><img src="image10" alt="Image" /></td>
<td><img src="image11" alt="Image" /></td>
<td><img src="image12" alt="Image" /></td>
</tr>
<tr>
<td>Disc 5</td>
<td><img src="image13" alt="Image" /></td>
<td><img src="image14" alt="Image" /></td>
<td><img src="image15" alt="Image" /></td>
</tr>
<tr>
<td>Disc 6</td>
<td><img src="image16" alt="Image" /></td>
<td><img src="image17" alt="Image" /></td>
<td><img src="image18" alt="Image" /></td>
</tr>
</tbody>
</table>
APPENDIX E: DCT PHASE UNWRAPPING ALGORITHM
DISCRETE COSINE TRANSFORM METHOD FOR PHASE UNWRAPPING

REF: D.C. GHIGLIA and L.A. ROMERO.

\[ \alpha_{\text{wrapped}} := \text{READBMP("ALPHA1.BMP")} \]
size := rows(\alpha_{\text{wrapped}}) \quad M := \text{size} \quad N := \text{size}

\[
\text{wrap}(A, B) := \left[ \arg \left[ \exp \left[ i \pi \left( \frac{A - B}{32} \right) \right] \right] \right] \frac{32}{\pi} \Delta x_{\text{size}+1, \text{size}+1} := 0 \quad \Delta y_{\text{size}+1, \text{size}+1} := 0
\]

\[
i := 0 \ldots M - 2 \quad j := 0 \ldots N - 1 \quad \Delta x_{i+1, j+1} := \text{wrap}(\alpha_{\text{wrapped}}_{i+1, j}, \alpha_{\text{wrapped}}_{i, j})
\]
\[
i := 0 \ldots M - 1 \quad j := 0 \ldots N - 2 \quad \Delta y_{i+1, j+1} := \text{wrap}(\alpha_{\text{wrapped}}_{i, j+1}, \alpha_{\text{wrapped}}_{i, j})
\]
\[
i := 0 \ldots M - 1 \quad j := 0 \ldots N - 1 \quad \rho_{i, j} := \frac{\Delta x_{i+1, j+1} + \Delta x_{i, j+1}}{2} + \frac{\Delta y_{i+1, j+1} - \Delta y_{i, j}}{2}
\]

\[
\rho_{\text{DCT}, m, n} = \sum_{i=0}^{\text{size}-1} \sum_{j=0}^{\text{size}-1} 4 \rho_{i, j} \cos \left( \frac{\pi}{2 \cdot \text{size}} \cdot (2 \cdot i + 1) \right) \cos \left( \frac{\pi}{2 \cdot \text{size}} \cdot (2 \cdot j + 1) \right)
\]

\[
\rho_{\text{F}2, \text{size}-1, \text{size}-1} := 0 \quad \alpha_{\text{F}} := \text{ICFFT}(\rho)
\]

\[
m := 0 \ldots \text{size} - 1 \quad n := 1 \ldots \text{size} - 1 \quad \text{CCT}_{1, m, n} := 2 \cdot \text{Re}(\alpha_{\text{F}1, m, n} + \alpha_{\text{F}2, \text{size}-n, m})
\]

\[
m := 0 \ldots \text{size} - 1 \quad n := 0 \ldots \text{size} - 1 \quad \text{CCT}_{2, m, n} := 2 \cdot \text{Re}(\alpha_{\text{F}1, \text{size}-n, m} + \alpha_{\text{F}2, m, n})
\]

\[
\text{CCT}_{i, j} := \text{if}(i > j, \text{CCT}_{2, i, j}, \text{CCT}_{1, i, j})
\]

\[
\rho_{\text{DCT}_{i, j}} := \text{CCT}_{i, j} \cos \left( \frac{\pi \cdot i}{2 \cdot \text{size}} \right) \cos \left( \frac{\pi \cdot j}{2 \cdot \text{size}} \right)
\]

\[
m := 0 \ldots \text{size} - 1 \quad n := 1 \ldots \text{size} - 1 \quad \text{CCT}_{1, m, n} := -2 \cdot \text{Im}(\alpha_{\text{F}1, m, n} - \alpha_{\text{F}2, \text{size}-n, m})
\]
\[
m := 1 \ldots \text{size} - 1 \quad n := 0 \ldots \text{size} - 1 \quad \text{CCT}_{2, m, n} := -2 \cdot \text{Im}(\alpha_{\text{F}1, \text{size}-n, m} + \alpha_{\text{F}2, m, n})
\]

\[
\text{CCT}_{i, j} := \text{if}(i > j, \text{CCT}_{2, i, j}, \text{CCT}_{1, i, j})
\]

\[
\rho_{\text{DCT}_{i, j}} := \rho_{\text{DCT}_{i, j}} \sin \left( \frac{\pi \cdot i}{2 \cdot \text{size}} \right) \sin \left( \frac{\pi \cdot j}{2 \cdot \text{size}} \right)
\]

\[
m := 0 \ldots \text{size} - 1 \quad n := 1 \ldots \text{size} - 1 \quad \text{CCT}_{1, m, n} := -2 \cdot \text{Im}(\alpha_{\text{F}1, m, n} + \alpha_{\text{F}2, \text{size}-n, m})
\]
\[
m := 1 \ldots \text{size} - 1 \quad n := 0 \ldots \text{size} - 1 \quad \text{CCT}_{2, m, n} := -2 \cdot \text{Im}(\alpha_{\text{F}1, \text{size}-n, m} - \alpha_{\text{F}2, m, n})
\]

\[
\text{CCT}_{i, j} := \text{if}(i > j, \text{CCT}_{2, i, j}, \text{CCT}_{1, i, j})
\]

\[
\rho_{\text{DCT}_{i, j}} := \rho_{\text{DCT}_{i, j}} \sin \left( \frac{\pi \cdot i}{2 \cdot \text{size}} \right) \cos \left( \frac{\pi \cdot j}{2 \cdot \text{size}} \right)
\]
\[
\alpha_{C_{t_i,j}} := 2 \cos \left( \frac{\pi \cdot i}{\text{size}} \right) + 2 \cos \left( \frac{\pi \cdot j}{\text{size}} \right) - 4 \quad \alpha_{DCT_{i,j}} := \left\{ \begin{array}{ll}
\alpha_{C_{t_i,j}} & = 0, \quad \alpha_{DCT_{t_i,j}}, \ \rho_{DCT_{t_i,j}} \\
\alpha_{C_{t_i,j}} & > 0, \quad \alpha_{DCT_{t_i,j}}, \ \rho_{DCT_{t_i,j}} \\
\end{array} \right.
\]

\[
m := 0 \ldots \text{size} - 1 \quad n := 0 \ldots \text{size} - 1 \quad w_{m,n} := 1 \quad w_{m,0} := 0.5 \quad w_{0,n} := 0.5 \quad w_{0,0} := 0.25
\]

\[
\alpha_{DCT_{i,j}} = \frac{1}{\text{size}} \sum_{m=0}^{\text{size}-1} \sum_{n=0}^{\text{size}-1} \alpha_{DCT_{m,n}} w_{m,n} \cos \left[ \frac{\pi}{2 \cdot \text{size}} \cdot m \cdot (2 \cdot i + 1) \right] \cos \left[ \frac{\pi}{2 \cdot \text{size}} \cdot n \cdot (2 \cdot j + 1) \right]
\]

\[
\alpha_{2_{i,j}} = \alpha_{DCT_{i,j}} w_{i,j} \cos \left( \frac{\pi \cdot i}{2 \cdot \text{size}} \right) \cos \left( \frac{\pi \cdot j}{2 \cdot \text{size}} \right) \quad \alpha_{2_{\text{size}-1, \text{size}-1}} = 0 \quad \alpha_F := \text{CFFT}(\alpha_2)
\]

\[
m := 0 \ldots \text{size} - 1 \quad n := 1 \ldots \text{size} - 1 \quad \text{CCT}_{1_{m,n}} := 2 \cdot \text{Re}(\alpha_{F_{m,n}} + \alpha_{F_{m,2 \cdot \text{size} - n}})
\]

\[
m := 1 \ldots \text{size} - 1 \quad n := 0 \ldots \text{size} - 1 \quad \text{CCT}_{2_{m,n}} := 2 \cdot \text{Re}(\alpha_{F_{m,n}} + \alpha_{F_{2 \cdot \text{size} - m,n}})
\]

\[
\text{CCT}_{i,j} := \text{if}(i > j, \text{CCT}_{2_{i,j}}, \text{CCT}_{1_{i,j}}) \quad \alpha_{\text{unwrap}_{i,j}} := \alpha_{\text{unwrap}_{i,j}} - \text{CCT}_{i,j}
\]

\[
\alpha_{2_{i,j}} = \alpha_{DCT_{i,j}} w_{i,j} \sin \left( \frac{\pi \cdot i}{2 \cdot \text{size}} \right) \sin \left( \frac{\pi \cdot j}{2 \cdot \text{size}} \right) \quad \alpha_F := \text{CFFT}(\alpha_2)
\]

\[
m := 0 \ldots \text{size} - 1 \quad n := 1 \ldots \text{size} - 1 \quad \text{CCT}_{1_{m,n}} := -2 \cdot \text{Im}(\alpha_{F_{m,n}} - \alpha_{F_{m,2 \cdot \text{size} - n}})
\]

\[
m := 1 \ldots \text{size} - 1 \quad n := 0 \ldots \text{size} - 1 \quad \text{CCT}_{2_{m,n}} := -2 \cdot \text{Im}(\alpha_{F_{m,n}} + \alpha_{F_{2 \cdot \text{size} - m,n}})
\]

\[
\text{CCT}_{i,j} := \text{if}(i > j, \text{CCT}_{2_{i,j}}, \text{CCT}_{1_{i,j}}) \quad \alpha_{\text{unwrap}_{i,j}} := \alpha_{\text{unwrap}_{i,j}} - \text{CCT}_{i,j}
\]

\[
\alpha_{2_{i,j}} = \alpha_{DCT_{i,j}} w_{i,j} \sin \left( \frac{\pi \cdot i}{2 \cdot \text{size}} \right) \cos \left( \frac{\pi \cdot j}{2 \cdot \text{size}} \right) \quad \alpha_F := \text{CFFT}(\alpha_2)
\]

\[
m := 0 \ldots \text{size} - 1 \quad n := 1 \ldots \text{size} - 1 \quad \text{CCT}_{1_{m,n}} := -2 \cdot \text{Im}(\alpha_{F_{m,n}} + \alpha_{F_{m,2 \cdot \text{size} - n}})
\]

\[
m := 1 \ldots \text{size} - 1 \quad n := 0 \ldots \text{size} - 1 \quad \text{CCT}_{2_{m,n}} := -2 \cdot \text{Im}(\alpha_{F_{m,n}} - \alpha_{F_{2 \cdot \text{size} - m,n}})
\]

\[
\text{CCT}_{i,j} := \text{if}(i > j, \text{CCT}_{2_{i,j}}, \text{CCT}_{1_{i,j}}) \quad \alpha_{\text{unwrap}_{i,j}} := \alpha_{\text{unwrap}_{i,j}} + \text{CCT}_{i,j}
\]

\[
\text{max}(\alpha_{\text{unwrap}}) = 411.749 \quad \text{min}(\alpha_{\text{unwrap}}) = -485.25 \quad \text{range} := \text{max}(\alpha_{\text{unwrap}}) - \text{min}(\alpha_{\text{unwrap}}) \quad \alpha_{\text{unwrap}} := \alpha_{\text{unwrap}} - \text{min}(\alpha_{\text{unwrap}})
\]

\[
\text{range} = 897 \quad \text{WRITEPRN("unalpha.prn") := \alpha_{\text{unwrap}} \quad \text{WRITEBMP("unalpha.bmp") := \alpha_{\text{unwrap}} \cdot 255 / \text{range}}
\]
APPENDIX F: ORTHOTROPIC PROPERTIES ALGORITHM

Loughborough University
data _ EX := READBMP("h0degthick")
data _ Ey := READBMP("v0degthick")
size := \frac{\text{rows}(\text{data}\_\text{EX})}{\sqrt{2}}
i := 0..\text{size} - 1 \quad j := 0..\text{size} - 1
\text{wrap\_EX}_{i,j} := \text{data}\_\text{EX}_{i+18,j+18} \quad \text{wrap\_Ey}_{i,j} := \text{data}\_\text{Ey}_{i+18,j+18}

\text{data}\_\text{EX} \quad \text{data}\_\text{Ey} \quad \text{wrap}\_\text{EX} \quad \text{wrap}\_\text{Ey}

\text{WRITEBMP("wrap\_EX.bmp")} := \text{wrap}\_\text{EX}
\text{\varepsilon}_X := \text{READPRN("un\_EX.pm")}
\text{WRITEBMP("wrap\_Ey.bmp")} := \text{wrap}\_\text{Ey}
\text{\varepsilon}_Y := \text{READPRN("un\_Ey.pm")}

\text{\varepsilon}_X \quad \text{\varepsilon}_Y

\text{load\_x} := -100 \quad \text{load\_y} := -150 \quad \Delta x := 5 \quad \Delta y := 5 \quad \lambda := 620 \times 10^{-3} \quad \theta := 50 \times \frac{\pi}{180} \quad \nu := 0.33 \quad t := 3 \quad d := 40

\varepsilon_X := \varepsilon_X + 0 \quad \varepsilon_Y := \varepsilon_Y - 256

\varepsilon_{X_{i,j}} := \frac{\lambda}{\Delta x \sin(\theta)} \left( 1 - \frac{1}{256} \right)
\varepsilon_{Y_{i,j}} := \frac{\lambda}{\Delta y \sin(\theta)} \frac{\text{load\_x}}{256} \frac{1}{\text{load\_y} - 256}

size := \text{rows}(\varepsilon_X) \quad size = 92
i := 0..\text{size} - 1 \quad j := 0..\text{size} - 1

\varepsilon_x := \varepsilon_x + 0 \quad \varepsilon_y := \varepsilon_y - 256

\varepsilon_{X_{i,j}} := \frac{\lambda}{\Delta x \sin(\theta)} \left( 1 - \frac{1}{256} \right)
\varepsilon_{Y_{i,j}} := \frac{\lambda}{\Delta y \sin(\theta)} \frac{\text{load\_x}}{256} \frac{1}{\text{load\_y} - 256}

size := \text{rows}(\varepsilon_X) \quad size = 92
i := 0..\text{size} - 1 \quad j := 0..\text{size} - 1

\begin{align*}
\text{Fy}_1 & := \frac{t}{2} \frac{d}{\text{size} \sqrt{2}} \frac{\nu}{1 - \nu} \left( \sum_{j=0}^{\text{size}-1} \varepsilon_{X_{0,j}} + \sum_{j=0}^{\text{size}-1} \varepsilon_{X_{\text{size}-1,j}} \right) \\
\text{Fy}_2 & := \frac{t}{2} \frac{d}{\text{size} \sqrt{2}} \frac{1}{1 - \nu} \left( \sum_{j=0}^{\text{size}-1} \varepsilon_{Y_{0,j}} + \sum_{j=0}^{\text{size}-1} \varepsilon_{Y_{\text{size}-1,j}} \right) \\
\text{Fx}_1 & := \frac{t}{2} \frac{d}{\text{size} \sqrt{2}} \frac{\nu}{1 - \nu} \left( \sum_{j=0}^{\text{size}-1} \varepsilon_{X_{0,j}} + \sum_{j=0}^{\text{size}-1} \varepsilon_{X_{\text{size}-1,j}} \right) \\
\text{Fx}_2 & := \frac{t}{2} \frac{d}{\text{size} \sqrt{2}} \frac{1}{1 - \nu} \left( \sum_{j=0}^{\text{size}-1} \varepsilon_{Y_{0,j}} + \sum_{j=0}^{\text{size}-1} \varepsilon_{Y_{\text{size}-1,j}} \right)
\end{align*}

\begin{bmatrix}
\text{Fx}_1 \\
\text{Fx}_2
\end{bmatrix}
\begin{bmatrix}
\text{Fy}_1 \\
\text{Fy}_2
\end{bmatrix}
= \begin{bmatrix}
\text{Fy} \\
\text{Fx}
\end{bmatrix}
\text{E} := \begin{bmatrix}
\text{Fx}_1 & \text{Fx}_2
\end{bmatrix}
\begin{bmatrix}
\text{Fy}_1 \\
\text{Fy}_2
\end{bmatrix}
^{-1}
\begin{bmatrix}
0 \\
-200
\end{bmatrix}
\begin{bmatrix}
3.924 \\
16.641
\end{bmatrix}
APPENDIX G: RADAR PLOTS FROM TPB TESTS

Loughborough University
RADAR PLOTS FROM THREE-POINT BEND TESTS

Stiffness Plot: Undamaged Samples at All Quarter-Span Positions

Stiffness Plot: Minimum Damage at Quarter Span, Position 1
Stiffness Plot: Minimum Damage at Quarter Span, Position 2

Stiffness Plot: Minimum Damage at Quarter Span, Position 3
Appendices

Stiffness Plot: Minimum Damage at Quarter Span, Position 4

Stiffness Plot: Medium Damage at Quarter Span, Position 1
Appendices

Stiffness Plot: Medium Damage at Quarter Span, Position 2

Stiffness Plot: Medium Damage at Quarter Span, Position 3
Stiffness Plot: Medium Damage at Quarter Span, Position 4

Stiffness Plot: Maximum Damage at Quarter Span, Position 1
Stiffness Plot: Maximum Damage at Quarter Span, Position 2

Stiffness Plot: Maximum Damage at Quarter Span, Position 3
Stiffness Plot: Maximum Damage at Quarter Span, Position 4