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Sensitivity analysis for comparison, validation and physical-legitimacy of neural network-based hydrological models

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Abstract

This paper addresses the difficult question of how to perform meaningful comparisons between neural network-based hydrological models and alternative modelling approaches. Standard, goodness-of-fit metric approaches are limited since they only assess numerical performance and not physical legitimacy of the means by which output is achieved. Consequently, the potential for general application or catchment transfer of such models is seldom understood. This paper presents a partial derivative, relative sensitivity analysis method as a consistent means by which the physical legitimacy of models can be evaluated. It is used to compare the behaviour and physically rationality of a generalised linear model and two neural network models for predicting median flood magnitude in rural catchments. The different models perform similarly in terms of goodness-of-fit statistics, but behave quite distinctly when the relative sensitivities of their parameters are evaluated. The neural solutions are seen to offer an encouraging degree of physical legitimacy, in their behaviour, over that of their generalised linear modelling counterpart – particularly when overfitting is constrained. This indicates that neural solutions are preferable models for transferring into ungauged catchments. Thus, the importance of understanding both model performance and physical legitimacy when comparing neural models with alternative modelling approaches is demonstrated.

Keywords

Index flood, neural network, partial derivative, sensitivity analysis, ungauged catchment, generalised model.
1. INTRODUCTION

This paper presents an approach for delivering greater meaning from the comparison of artificial neural network (ANN) models with alternative modelling approaches in hydrological studies. ANN-based hydrological models are most commonly applied as black-box tools and the internal mechanisms by which the model output is generated are not normally explored in hydrological terms. Used in this way, an ANN’s primary purpose is the optimisation of complex, non-linear relations between a specific set of hydrological input and output data, and standard goodness-of-fit procedures may, therefore, be considered an adequate basis by which to compare its performance to that of other models (Klemes, 1986; Refsgaard and Knusden, 1996). Indeed, assessments of goodness-of-fit have been widely used in comparative hydrological modelling studies to argue that ANN models can perform as well as, or better than alternative modelling approaches (e.g. Shrestha and Nestmann, 2009; Mount and Abrahart, 2011). However, such arguments are informed solely by the degree of optimisation that is achieved by each model. They say nothing about the means by which different models achieve their performance and the relative merits of these alternative means. Indeed, when ANN models are applied solely as black-boxes, their potential relative to other modelling approaches can never be properly understood in a generalised or transferrable manner because the extent to which their modelling mechanisms conform to physically-based, hydrological domain knowledge remains untested (Howes and Anderson, 1988; Sargent, 2011). Consequently, critical questions about whether ANN modelling mechanisms are more or less reflective of real-world hydrological processes than alternative models are seldom addressed directly (Minns and Hall, 1996; Abrahart et al., 2011), and the relative extent to which they are able to deliver hydrological process insights (i.e. Caswell’s (1976) model duality) is not normally evaluated. The purpose of this paper is to present a method by which these questions may be addressed.
More informative approaches to model comparison are required that explicitly consider the internal behaviours of the different models and assess them according to their conformance with the logical, rational and physical expectations of the modeller (c.f. Robinson, 1997). This process is termed model legitimisation and is discussed in a philosophical context by Oreskes et al., (1994) and an applied, hydrological modelling context by Mount et al., (in press). Sensitivity analysis (Hamby, 1994) is an important and effective means by which the legitimacy of a hydrological model may be explored. It has been widely applied in conceptual and physically-based modelling over several decades (e.g. McCuen, 1973; Beven and Binley, 1992; Schulz and Huwe, 1999; Radwan et al., 2004; Pappenberger et al., 2008; Mishra, 2009; Zhang et al., 2012). A variety of approaches have been used including local (e.g. Turanayi and Rabitz, 2000; Spruill et al., 2000; Holvoet et al., 2005; Hill and Tiedeman, 2007), regional (e.g. Spear and Hornberger, 1980) and global-scale methods (Muleta and Nicklow, 2005; Salteli et al., 2008). By contrast, sensitivity analysis has not been widely adopted in ANN modelling studies beyond a few, isolated examples (Sudheer, 2005; Nourani and Fard, 2012). This is presumably because the equations that relate inputs and outputs in an ANN are considered complex, inaccessible and difficult to interpret (Aytek et al., 2008; Abrahart et al., 2009), making exploration of model sensitivity via direct analysis of the governing equations difficult. Nonetheless, recent progress has been made (Yeung et al., 2010) and relative sensitivity analysis techniques for ANNs have made it possible to assess the internal, mechanistic legitimacy of such models (Abrahart et al., 2012b; Mount et al., in press). However, the focus of these studies has so far been restricted to mechanical considerations. The application of sensitivity analysis to evaluate the physical legitimacy of ANN-based hydrological models, and thus the degree to which they can be generalised and transferred, remains an outstanding task.

In this paper we apply a sensitivity analysis method that can be used to compare the physical legitimacy of ANN-based hydrological models and alternative model counterparts in a direct manner. We exemplify the method by comparing the
performance and physical legitimacy of a pair of ANN-based models and an established
generalised linear model (GLM) for median flood magnitude prediction in ungauged
catchments in the UK. First order, partial derivatives of each model’s response function
are computed, interpreted and used as a consistent means by which the physical
legitimacy of each model can be evaluated and compared. This focus on response
function behaviour is distinctly different to past efforts to assess the physical legitimacy
of ANN models, which have traditionally explored internal structural components, such as
weights (Abrahart et al., 1999; Olden and Jackson, 2002; Anctil et al., 2004; Kingston et
al., 2003; 2005; 2006; 2008) and units (Wilby et al., 2003; Jain et al., 2004; Sudheer
and Jain, 2004; See et al., 2008; Fernando and Shamseldin, 2009; Jain and Kumar,
2009). However, the uniqueness of ANN structures means that the information derived
from them cannot easily be compared directly with that derived from alternative models
with different internal structures - thus limiting the comparative value of the information.
To overcome this problem, we here assess the physical legitimacy of an ANN’s overall
response function using a standard relative sensitivity-based method that can be
consistently and directly replicated across a range of alternative model types and that is
widely understood and accepted by hydrologists. Consequently, an evaluation of the
physical legitimacy of the means by which each model’s performance is obtained
accompanies the usual assessments of output validity; enabling the extent to which each
model delivers a transferable, general solution to be considered.

2. EXEMPLAR: COMPARING GLM AND ANN-BASED MODELS FOR UNGAUGED
CATCHMENT PREDICTION IN THE UK

The modelling of hydrological responses in ungauged catchments remains an important
focus of research for hydrologists; especially as the majority of the world’s river
catchments remain ungauged or poorly gauged. In such catchments, the application of
distributed physically-based models and statistical approaches is hampered by a lack of
input parameter knowledge and datasets. Consequently, lumped models, which relate
broad physiographic, hydrogeologic and climatologic catchment descriptors to flood frequency curves, have long been recognised as offering potential (Rodriguez-Iturbe and Valdes, 1979; Grover et al., 2002).

The standard UK method (Natural Environment Research Council, 1975; Vogel and Kroll, 1992; Schriber and Demuth, 1997) models the relationship between the median of the annual flood series ($Q_{MED}$) and a set of regionalised catchment descriptors for rivers in the national, gauged network. The modelled relationship is then applied to ungauged catchments and used to estimate $Q_{MED}$, which is subsequently multiplied by a standard, dimensionless growth curve to estimate flood frequency (Institute of Hydrology, 1999).

Four catchment descriptors are used in the standard UK methodology:

1. $AREA$ (catchment area in km$^2$);
2. $SAAR$ (standard-period average annual rainfall in mm);
3. $FARL$ (flood attenuation due to reservoirs and lakes);
4. $BFIHOST$ (baseflow index derived from HOST data, Boorman et al., 1995).

These catchment descriptors can be thought of as physical controls of $Q_{MED}$ potential. $SAAR$ controls the hydrological inputs to the catchment, $AREA$ controls the scaling of the catchment response, whilst $BFIHOST$ and $FARL$ control the degree of buffering of the input-output signal.

Of central importance to the above method is the model that is used to relate $Q_{MED}$ and the catchment descriptors. These relationships are non-linear and not well represented by standard multiple linear regression. Therefore, the most recent UK method described applies a range of non-linear transformations within a generalised linear modelling (GLM) framework (Kjeldsen et al., 2008; Kjeldsen and Jones, 2009; Kjeldsen and Jones, 2010). The end product is a non-linear regression equation (see Equation 1) from which $Q_{MED}$ can be estimated directly from the four catchments descriptors.
ANN models are also very effective at optimising complex, non-linear relations in hydrological data (American Society of Civil Engineers 2000a; 2000b; Maier and Dandy, 2000; Dawson and Wilby, 2001; Maier et al., 2010; Abrahart et al., 2010; 2012b) and a number of studies have highlighted their potential in ungauged catchment prediction (Liong et al., 1994; Mutiah et al., 1997; Hall and Minns, 1998; Hall et al., 2000; Dastorani and Wright, 2001; Dawson et al., 2006; Dastorani et al., 2010). Indeed, the UK relationship between QMED and catchment descriptors has also been modelled using ANNs and been shown to deliver comparable levels of fit when compared to GLMs (Dawson et al., 2006). However, it remains unclear whether the two modelling approaches are similarly comparable with respect to their physical legitimacy. Models with greater physical legitimacy should be more generally transferrable to new catchment settings. Therefore, determining the physical legitimacy of each model is an important element in delivering a physically-informed evaluation of how robustly it can be expected to transfer from the gauged catchments upon which it is developed, to the ungauged catchments in which it is intended to be applied.

In the following sections, the importance of evaluating both model performance and physical legitimacy in ANN model comparisons is exemplified by contrasting the performance and legitimacy of the standard GLM method for QMED prediction with two different ANN-based model counterparts. Its use as an example is particularly appropriate because:

1. The model inputs and outputs are all physical-based measurements, meaning that patterns observed in inputs and output relations can be interpreted directly in physical terms;
2. The number of model inputs is relatively small;
3. First order partial derivatives can be computed for the GLM and directly compared with those of the ANN-based models;
4. The results of the analysis have real-world relevance and application.
2.1 Data

A GLM model and two counterpart ANN models for $Q_{MED}$ estimation are developed for comparison, with the model inputs conforming to the four used in the standard UK methodology. These inputs were extracted from a pre-filtered set of HiFlows-UK rural catchment data, available at (http://www.environment-agency.gov.uk/hiflows/97503.aspx). AREA values are derived from the Centre for Ecology and Hydrology’s Integrated Hydrological Digital Terrain Model (based on a 50m grid) and represent surface catchment area projected onto a horizontal plane, draining to the gauging station (Marsh and Hannaford, 2008: 5). SAAR values are derived from UK precipitation records over the standard period 1961-1990. FARL provides a guide to the degree of flood attenuation attributable to reservoirs and lakes above the gauging station. The index ranges from zero (complete attenuation) to one (no attenuation) with values $<0.8$ representing a substantial influence on flood response. BFIHOST is derived from the HOST (Hydrology of Soil Types) soil data classification and ranges from zero (impermeable) to one (completely permeable). In undisturbed catchments, a strong association exists between Baseflow Index (derived from archived gauged daily mean flows) and BFIHOST. The relationships between $Q_{MED}$ and AREA, SAAR and FARL are positive, whilst that between $Q_{MED}$ and BFIHOST is negative.

The data from which our models are derived are almost identical to those from which the GLM that is published in the revitalised UK Flood Estimation Handbook (Kjeldsen et al., 2008) has been developed, and full particulars of the Hi-Flows UK data set can be found in this handbook. A statistical summary of our dataset is provided in Table 1. Some minor discrepancies exist between the data used in this study and that used by Kjeldsen et al., (2008) due to our use of the public release version of HiFlows-UK 3.02 rather than the pre-release version originally used. Specifically, our dataset comprises 597 rural
catchment records rather than the 602 used previously, and we use an unadjusted flood attenuation variable.

2.2 Model development procedures

Three models were developed for comparison:

1. QMEDGLM - a GLM developed on all 597 catchment records, using the methodology outlined in Kjeldsen et al. (2008);

2. \( \text{ANN}_A \) – an optimised ANN, selected from 180 candidate solutions of varying complexity and training iterations according to both its goodness-of-fit performance and avoidance of evident overfitting.

3. \( \text{ANN}_B \) – a purposely over-trained version of \( \text{ANN}_A \) in which the number of training iterations was artificially extended to deliver an overfitted solution. It is included as a means of exemplifying the impact of ANN overfitting on the physical legitimacy of a network response function.

\( \text{QMED}_{\text{GLM}} \) was developed in accordance with the method of Kjeldsen et al. (2008). Despite the minor differences in the dataset noted above, the resultant regression equation (Equation 1) remains almost identical to Kjeldsen et al.’s original:

\[
QMED_{\text{GLM}} = 8.6704 \text{AREA}^{0.8568} \text{1.5509}^{\text{STAE}} \text{FARL}^{3.3662} \text{0.0380}^{\text{BFIHOST}}
\]  

(1)

\( \text{ANN}_A \) and \( \text{ANN}_B \) comprise a Multi-Layer Perceptron (MLP), with one hidden layer, trained using error back propagation (Rumelhart et al., 1986). The basic structure of these networks is shown schematically in Figure 1. The ANN consists of a number of units or neurons arranged in three layers (although additional hidden layers can be incorporated). The units in the input layer distribute the inputs to the units in the hidden layer, which in turn pass their outputs to the output layer (usually consisting of a single output neuron). Each neuron consists of a weighted set of inputs and an activation
function – typically the logistic sigmoid function (Equation 2). The output from a single unit is calculated by applying this sigmoid function to the weighted sum of its inputs.

\[ f(x) = \frac{1}{1 + e^{-x}} \]  \hspace{1cm} (2)

Training such networks using back propagation involves presenting the ANN with training data; calculating the error of the network’s output with respect to the observed values; propagating this error backwards through the network; and adjusting the input weights to the neurons accordingly (to reduce this error). This process must be repeated many times, making minor adjustments to the weights each cycle (or epoch), until the ANN begins to map input values to the correct output response. The amount by which the weights are adjusted each time can be manipulated by using a learning rate multiplier. Readers that are unfamiliar with ANN concepts, structures and training methods are referred to Kattan et al., (2011) or Nelson, (2011).

The simplicity of this ANN has enabled the development of computational methods for delivering first-order partial derivatives of its response function (Hashem, 1992), which we subsequently use as the basis for our comparative assessment of model legitimacy (see Section 3). This standard ANN has been successfully used in many hydrological studies in the past (Abrahart et al., 2012a) and provides an established non-linear modelling benchmark for ANN studies and a starting point against which more novel approaches can subsequently be compared (Mount et al., 2012). Whilst it is recognised that more advanced ANN structures might arguably deliver some additional optimisation advantages, the computational methods required to quantify their response function partial derivatives, and hence deliver directly comparable assessments of their physical legitimacy, are not readily available. Their use is thus avoided in this study.

\( \text{ANN}_A \) was developed using the approach described in Dawson et al. (2006) in which a large number of candidate ANNs are trained on a random subset of the data, partitioned
according to a 60% calibration to 40% cross-validation ratio. Although there is no agreed standard for splitting the data, this ratio is widely accepted in hydrological modelling (Mount and Abrahart, 2011; See and Openshaw, 2000). 180 candidate models containing 2, 3, 4, 5, 6, 7, 8, 9, 10 hidden units were developed with each candidate being trained for up to 20,000 epochs in steps of 1,000, using a learning rate of 0.1 and a momentum value of 0.9. Each candidate model was cross-validated using the remaining 40% as a means of preventing overfitting (Giustolisi and Laucelli, 2005; Piotrowski and Napierkowski, 2013). Overfitting of each candidate solution was evaluated according to its cross-validation scores, and the candidate solution displaying the best optimisation performance, whilst avoiding apparent overfitting, was selected as the final model.

ANN_A has nine hidden units, and is trained for 4000 epochs. ANN_B, which we adopt as an example of an overfitted ANN, is structurally identical to ANN_A. However its training epochs have been artificially extended to ten times that of ANN_A (i.e. 40,000 epochs) to promote overfitting. The network unit weights and biases are provided in Table 2 and are used as the inputs to Equation 8, from which relative sensitivity can be computed.

It should be noted that the GLM and ANN models utilise the available data records differently during model development. Whilst the GLM uses all 597 records to define the model, each candidate ANN uses only the first 400 records to refine the model, and the remaining 197 records to constrain it via cross-validation. Indeed, the apparent inconsistency with which the GLM and ANN models use the available data could be cited as an argument to negate the fairness of a direct comparison between them. However, this stance fails to credit that both models do use all of the data in the model development process; they just use it in a characteristically different manner that reflects the fundamental differences between each method. In this sense, the models are comparable; not because they use the same data in the same way, but rather because each one’s use of the data is equally appropriate and justifiable in the context of its own model development method.
3. MODEL PERFORMANCE AND PHYSICAL LEGITIMACY ASSESSMENT

3.1 Model performance evaluation

Each model’s performance was evaluated using standard goodness-of-fit metrics to deliver output validation. To ensure a consistent approach the metrics were generated using HydroTest (http://www.hydrotest.org.uk): a standardised, open access web site that performs the required numerical calculations (Dawson et al. 2007; 2010). Each model’s performance is evaluated using \( RMSE \) (Root Mean Squared Error) and \( R^2 \) (R-squared – the Coefficient of Determination) – providing an overall measure of model performance; \( MSRE \) (Mean Squared Relative Error) and \( MSLE \) (Mean Squared Logarithmic Error) – providing two additional measures of performance which place greater emphasis on errors occurring in lower magnitude predictions. These comparative performance statistics are defined as:

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (Q_i - \hat{Q}_i)^2} \tag{3}
\]

\[
R^2 = \left( \frac{\sum_{i=1}^{n} (Q_i - \bar{Q})(\hat{Q} - \bar{Q})}{\sqrt{\sum_{i=1}^{n} (Q_i - \bar{Q})^2 \sum_{i=1}^{n} (\hat{Q}_i - \bar{\hat{Q}})^2}} \right)^2 \tag{4}
\]

\[
MSE = \frac{1}{n} \sum_{i=1}^{n} (Q_i - \hat{Q}_i)^2 \tag{5}
\]

\[
MSLE = \frac{1}{n} \sum_{i=1}^{n} (\ln Q_i - \ln \hat{Q}_i)^2 \tag{6}
\]

where \( Q_i \) is observed index flood value \( i \) (of \( n \) values), \( \hat{Q}_i \) is the modelled value \( i \), \( \bar{Q} \) is the mean of the observed data, and \( \bar{\hat{Q}} \) is the mean of the modelled data.
3.2 Physical legitimacy

Following the recent studies of Abrahart et al., (2012b) and Mount et al., (in press), the physical legitimacy of each model was assessed by means of relative, first-order partial derivative sensitivity analysis (see Hamby, 1994 for an overview of sensitivity analysis approaches). Partial derivative sensitivity analysis elucidates the patterns of influence that each model input has on the output (and vice versa) across the output range – thus revealing the internal behaviour of the model response function. First order derivatives reveal the separate behaviours associated with each model input. When using partial derivatives in model comparison studies, it is necessary to standardise derivative values to rates to avoid the difficulties associated with comparing absolute values derived from different inputs with different ranges (Nourani and Fard, 2012). Patterns of relative sensitivity can then be used to directly compare the internal response function behaviour of different models, and legitimacy of these behaviours can then be evaluated according to how well the relative sensitivity patterns conform to the logical, rational and physical expectations of the modeller. The relative sensitivity ($RS_i$) of the output from a model (O) with respect to input ($I_i$) can be calculated as:

$$RS_i = \frac{\partial O}{\partial I_i} \frac{I_i}{O}$$  \hspace{1cm} (7)

Partial derivatives can be computed for ANNs via the application of a backward chaining partial differentiation rule as outlined in Hashem (1992). Adapted from Hashem’s more general rule, for an ANN with sigmoid activation functions (i.e. of standard type, as used in our case study), one hidden layer, i input units, j hidden units and one output unit (O), the partial derivative of a network’s output can be calculated with respect to each of its inputs as:
\[
\frac{\partial O}{\partial I_i} = \sum_{j=1}^{n} w_{ij}w_{jo}h_j(1-h_j)O(1-O) \tag{8}
\]

In which \(w_{ij}\) is the weight from input unit \(i\) to hidden unit \(j\); \(w_{jo}\) is the weight from hidden unit \(j\) to the output unit \(O\); \(h_j\) is the output of hidden unit \(j\); and \(O\) is the output from the network.

One important difference between calculating partial derivatives for multiple input, single output GLMs and ANN models should, however, be noted. When computing partial derivatives of a GLM, there is no need to vary the values of the other inputs to investigate the range of sensitivity responses under different input conditions. This is because GLMs deliver a simple additive response function, such that the relative sensitivity for any one variable will involve only that variable, given that all other parts of the expression will cancel out, during the process of scaling the other variables. Hence relative sensitivity values for each input to the QMED\textsubscript{GLM} model (Equation 1) can be computed according to Equations 9-12. The final relative sensitivities of the QMED\textsubscript{GLM} model are provided in Equations 13-16.

\[
\frac{\partial QMED}{\partial AREA} = \frac{0.8568QMED}{AREA} \tag{9}
\]

\[
\frac{\partial QMED}{\partial SAAR} = \frac{1864.05QMED}{SAAR^2} \tag{10}
\]

\[
\frac{\partial QMED}{\partial FARL} = \frac{3.3662QMED}{FARL} \tag{11}
\]

\[
\frac{\partial QMED}{\partial BFIHOST} = -6.5385QMED.BFIHOST \tag{12}
\]

\[
RS_{AREA} = 0.8568 \tag{13}
\]

\[
RS_{SAAR} = 1864.05 / SAAR \tag{14}
\]

\[
RS_{FARL} = 3.3662 \tag{15}
\]

\[
RS_{BFIHOST} = -6.5385 \cdot BFIHOST^2 \tag{16}
\]
The same is not true for ANNs, which are not constrained to produce simple, additive response functions. When computing partial derivatives for an ANN it is therefore necessary to isolate the pattern of relative sensitivity of each input variable in turn by holding the other inputs at fixed values so that the patterns of sensitivity associated with each variable can be interpreted within the context of the other variable states. To this end we adopt a simple three-step methodology:

Step 1: Compute 25th percentile, median and 75th percentile values for each input variable in the data set;
Step 2: Holding all other variables at either 25th percentile, median or 75th percentile, vary each input variable in turn from across the range of observed values;
Step 3: Plot results and interpret the resultant graphs.

Thus, physically speaking, if variable states in our study are held at the 25th percentile (or the 75th percentile in the case of the inverse BFIHOST measure), the resultant scenario under test is representative of relatively small, dry catchments with high permeability and high flood attenuation: i.e. low catchment QMED potential. Conversely, when variables states are held at the 75th percentile (with BFIHOST at the 25th percentile), the resultant scenario under test will be representative of relatively large, wet catchments with low permeability and low attenuation: i.e. high catchment QMED potential.

4. RESULTS

4.1 Independence

Figure 2 and Table 3 present an overview of the data showing the relationships that exist between each of the five variables. AREA is not correlated with any of the other three parameters (correlation coefficient ranging from -0.07 to -0.02). There is a negative
correlation between \textit{SAAR} and \textit{BFIHOST} (correlation coefficient of -0.42) and a similar strength negative relationship between \textit{SAAR} and \textit{FARL} (correlation coefficient of -0.39). The only positive correlation is that between \textit{BFIHOST} and \textit{FARL} (correlation coefficient of 0.11). These weak relationships indicate a reasonable degree of linear independence between the four variables. The strength of the linear relationship between each of the parameters and \textit{QMED} ranges from a correlation coefficient score of 0.76 for \textit{AREA} to -0.07 for \textit{FARL}. The strong linear relationship between \textit{QMED} and \textit{AREA}, contrasts with the relative sensitivity scores presented later in this paper for the multiple linear regression model, and in so doing emphasises the additional insights provided by sensitivity analysis over basic statistical measures.

\textbf{4.2 Model skill}

Figures 3, 4 and 5 present scatter diagrams of observed versus modelled index flood values for the GLM, ANN\textsubscript{A} and ANN\textsubscript{B} models. The full dataset is depicted in each scatter plot. Figures 3 and 4 reveal comparable amounts of predictive skill for the GLM and ANN\textsubscript{A} model. Both plots, indeed, appear to show a reasonable degree of model performance at lower levels, but typically under-estimate the higher magnitude flood events. In contrast the ANN\textsubscript{B} model appears to perform well across the range of flood event magnitudes and seems very close to correctly modelling the two largest flood events.

Although Figures 3, 4, and 5 provide an interpretive view of the accuracy of the three models, Table 4 provides a more objective, numerical contrast by providing comparative performance statistics for each of the models. It shows that while the ANN\textsubscript{B} model is undoubtedly the most accurate overall according to the RMSE and R\textsuperscript{2} measures, the GLM is more accurate at modelling low flood indices. Although there appears to be a significant difference between the MSRE statistics of the GLM and the ANN\textsubscript{A} model (0.19 and 16.12 respectively) these results need to be treated with caution. A very basic model, that simply predicts the index flood for every catchment as 1 m\textsuperscript{3}/s, results in a
MSRE statistic of 0.93 – better than both the ANN models and not too dissimilar from the GLM. One would not seriously contemplate using such a simple model as a prediction of the index flood in an ungauged catchment so it brings into question the suitability of the MSRE as an appropriate measure of performance. It indicates that a model needs to make only a handful of errors at lower levels (which may not be too far from the observed values) to result in a poor MSRE result. This emphasises the importance of using multiple evaluation criteria and understanding the limitations of individual error measures.

Although the scatter diagrams show reasonably similar performance at lower levels, one or two over/under predictions have skewed the results. A more appropriate measure of performance at lower levels is perhaps the MSLE used by Pokhrel et al. (2012) – the results of which are also presented in Table 4. In this case, although the GLM outperforms the ANN<sub>A</sub> and ANN<sub>B</sub> models, the results are not too dissimilar. For the simple model (producing 1 m<sup>3</sup>/s for each case) the MSLE is calculated as 15.36 – significantly higher than the more complex models. Given that the ANN<sub>B</sub> performs reasonably well for low QMED values, and better than the GLM at large QMED values, where prediction is normally more problematic, the goodness-of-fit statistics suggest that ANN<sub>B</sub> could be considered a reasonable alternative to GLM.

5. SENSITIVITY ANALYSIS AND PHYSICAL INTERPRETATION OF MODELS

5.1 GLM

Relative sensitivity plots for the GLM are provided in Figure 6 which are calculated using Equations 13-16. AREA and FARL are both used as simple scaling variables in the model such that the index flood magnitude increases proportionally for larger catchments with lower flood attenuation. The model behaves in a manner that larger catchments produce consistently larger floods, but the overall significance of this behaviour is relatively small.
In a simplistic, conceptual sense, this is physically legitimate behaviour and one would expect the catchment area to act as a proportionally-consistent driver of flood magnitude with a ratio close to unity, as a larger catchment will have proportionally greater hydrological inputs. Importantly, \textit{FARL}, as a driver, is shown to be around four times more important than \textit{AREA}; a pattern that perhaps highlights the overriding importance of in-channel buffering of flood peaks by lakes and reservoirs in the model.

\textit{SAAR} and \textit{BFIHOST} function as more complex drivers of \textit{QMED} and their relative sensitivities vary considerably. Indeed, in certain data ranges each has the potential to become the most influential driver of index flood magnitude. However, their specific patterns of relative sensitivity prove difficult to legitimise in simplified, physical terms. The proportionally greater sensitivity of index flood magnitude to increases in wetness in low rainfall catchments, as opposed to ones possessing high rainfall, does not correspond well with broad hydrological notions. The expectation would be to find low antecedent moisture in low rainfall catchments to result in enhanced infiltration, reduced propensity for Hortonian overland flow and correspondingly lower index flood sensitivity compared to higher rainfall catchments. This suggests that there is a substantive runoff buffering mechanism in wet catchments that is not present in dry ones. Whilst one may postulate that factors such as different vegetation types in dry and wet catchments may buffer flood responses differently, it is difficult to envisage their impact being sufficient to produce the magnitude of difference observed in the relative sensitivity plot. Moreover, the pattern appears counter to notions of antecedent moisture which would be expected to be lower in dry catchments and, therefore, would act to proportionally reduce catchment runoff and index flood magnitude.

Similarly, the sensitivity of the index flood to catchment permeability is counter to basic physical principles with index floods seen to be an order of magnitude more sensitive to a unit change in permeability in a highly permeable catchment when compared with the same proportional change in an impermeable one. Whilst the overall negative relative
sensitivity of $QMED$ to $BFIHOST$ is conceptually legitimate, the specific pattern is difficult to legitimise physically as is the magnitude of the relative sensitivity observed relative to that of the other variables.

The sensitivity analysis thus indicates only partial physical legitimacy of the GLM, with the pattern of sensitivity of $QMED$ to $SAAR$ and $BFIHOST$ being particularly difficult to rationalise.

5.2 ANN_A

Relative sensitivity plots for the ANN_A model are provided in Figure 7. Importantly, none of the plots exhibit the extreme, localised sensitivity variability that one would expect from an over-fitted model (see ANN_B below), which in the context of the model skill statistics reported above, suggests ANN_A offers a reasonable solution. ANN_A is characterised by generally lower relative sensitivity values in comparison to those observed for the GLM, coupled with enhanced complexity in the sensitivity responses across each variable’s data range, the form of which is strongly influenced by the values of the other variables.

The relatively high sensitivity of $QMED$ to $AREA$ highlights the central importance of catchment size as a determinant of index flood magnitude in this model. This pattern of behaviour is an approximate counterpart of the GLM plot. Relative sensitivity remains roughly consistent at a value close to 1 and $AREA$ is seen to act as a scaling variable in a physically-legitimate manner. However, the same degree of legitimacy is not observed in either the low or high $QMED$ potential plots. Here opposing trends in the relative sensitivity are observed. When all other inputs are set to high $QMED$ potential, proportional changes in catchment area of small catchments is seen to have almost 10 times the impact on $QMED$ than the same proportional change in large catchments. The
pattern reverses when inputs are set to low $QMED$ potential. This model behaviour is very difficult to legitimise in physical terms.

Low values associated with $BFIHOST$ highlight the general insensitivity of $QMED$ to catchment permeability in this model. As expected, $BFIHOST$ has a generally negative influence on $QMED$ such that as permeability increases, $QMED$ reduces. A general increase in $QMED$’s sensitivity to $BFIHOST$ is observed as the other inputs are set to increasing levels of $QMED$ potential. This indicates an increased importance of permeability as a constraint on index flood magnitude in catchments with high potential for generating large index floods. However, the very low magnitude of the sensitivities observed makes it difficult to draw any clear conclusions about the physical legitimacy of the patterns observed beyond the fact that $BFIHOST$ is clearly not a particularly important driver of $QMED$.

In contrast to the GLM, $FARL$ acts as a relatively modest driver of $QMED$, indicating that the $ANN_A$ model is less heavily influenced by in-channel controls of peak discharge magnitude than the GLM. In simplistic physical terms, one would expect a reduction in flood attenuation to drive a proportional increase in $QMED$, and the positive relative sensitivity plots confirm this basic assumption. However, the precise form of the sensitivity relationship between $QMED$ and $FARL$ is more difficult to legitimise. The GLM represents the relationship as one of simple scaling and this same basic pattern exists for low and median $QMED$ potential plots across medium to high $FARL$ data ranges (i.e. medium to low levels of attenuation) where relative sensitivity is consistently about 0.5. However, at lower $FARL$ data ranges the proportional response of $QMED$ to change in $FARL$ reduces substantially to 0.1. When other inputs are set to high $QMED$ potential, the decreasing trend is consistent across all $FARL$ ranges. This is less easily rationalised and is most likely attributable to the scarcity of catchments with low $FARL$ values in the data resulting in a lack of data constraint on the form of the ANN model covering this data range, irrespective of the values of the other inputs.
The pattern of sensitivities observed for SAAR can only be partially legitimised in generalised physical terms. At a very simplistic level, the scaling behaviour of SAAR observed in the low QMED potential plot is perhaps reasonable given that proportionally wetter catchments should indeed result in proportionally greater floods. However, the patterns observed in the median and high QMED potential plots possess elements that are both physically rational and irrational. The increasing sensitivity to SAAR at low and mid data ranges could feasibly be explained in terms of antecedent moisture. Indeed, the on-average lower antecedent moisture in dry catchments could be expected to result in a smaller proportion of the rainfall contributing to runoff; leading to reduced hydrograph flashiness and proportionally lower QMED sensitivity to SAAR in dryer catchments. Similarly, the decline in sensitivity in the upper data ranges could be argued to be due to the fact that the catchment is already so wet that any additional rainfall makes relatively little difference to the index flood. However, this explanation ignores the role of overland, Hortonian flow in saturated, wet catchments which one would expect to drive an increase in the relative sensitivity in the upper data ranges. Finally, the negative relative sensitivity observed in the extreme upper ranges of the high QMED potential plot is physically-irrational as it suggests that proportionally increasing the catchment wetness will reduce the proportional response in QMED; in extreme cases even resulting in a reduction in QMED.

For each of the model inputs the behaviour of the ANN model is seen to be particularly influenced by the states of the input variables. When these are set to their median values (i.e. indicative of median QMED potential), the majority of the relative sensitivity plots indicate that the response function produces a model behaviour that can be physically-legitimised. However, this legitimacy is less certain when other variables are set at their 25th percentile values (i.e. indicative of low QMED potential) and completely breaks down when set at their 75th percentile value (i.e. indicative of high QMED potential). Indeed, under the latter condition, AREA, FARL and SAAR drive QMED in a
manner that is particularly difficult to explain in hydrological terms. Crucially then, a link can be made between the lack of physical legitimacy in the model’s behaviour in the upper and lower quartiles of the solution space and a lack of coincident data points which exist there to constrain the form of the ANN model.

5.3 ANN$_B$

Relative sensitivity plots for the ANN$_B$ model are provided in Figure 8. This ANN model is intentionally over-fitted and the impact of this over-fitting is clearly seen in the relative sensitivity plots. The degree of local variability in relative sensitivity is very exaggerated when compared to ANN$_A$ with variables switching between both negative and positive responses in QMED at different data ranges. QMED responds to AREA and SAAR (the most influential drivers in the model) in an irrational manner with high magnitude, localised variation in relative sensitivity being particularly characteristic of the patterns observed. The relative sensitivity plots of QMED to AREA and SAAR are characterised by complex polynomial forms with no consistent trends in the relationship. The patterns observed are indicative of data over-fitting and lack any physical legitimacy.

Relative sensitivity of QMED to FARL behaves in a more constrained manner than AREA or SAAR, ranging from +0.8 to -0.3 indicating the relative lack of sensitivity to this variable in ANN$_B$. However, the sensitivity plots for low and median QMED potential show both positive and negative responses at different data ranges. Indeed, these plots suggest that in certain data ranges, a proportional decrease in flood attenuation will see a proportional reduction in flood magnitude: a result that lacks physical legitimacy. The high QMED potential plot is very similar to that of ANN$_A$.

Relative sensitivity of BFIHOST to QMED is very muted with this variable being an almost irrelevant driver of index flood magnitude when other variables are set to low and median QMED potential. Localised complexity in the relative sensitivity is observed,
particularly across low BFIHOST values where low and median QMED potential plots switch between positive and negative relative sensitivity values in a physically-irrational manner. The high QMED potential plot is perhaps more rational as it displays a flatter, negative response which indicates a negative scaling behaviour.

In contrast with ANN\textsubscript{A}, local variation in relative sensitivity for AREA and SAAR becomes highly exaggerated when other variables are held at their low QMED potential values. This again highlights difficulties of fitting a ‘bottom heavy’ physically-legitimate ANN model, through upper regions of a solution space that lack sufficient coincident higher magnitude data points to constrain the form of the model.

5.4 Physical legitimacy

The broad physical legitimacy of the different model sensitivity plots are compared in Table 5. It is clear that none of the models behave in a manner that can be physically rationalised for all input variables. The GLM displays a basic level of physical legitimacy in the behaviour of AREA and FARL but this is lacking for SAAR and BFIHOST drivers. ANN\textsubscript{A} displays varying degrees of physical legitimacy in the sensitivity between QMED and each of the input variables, with the least rational responses occurring when other variables are set to the high QMED potential values. However, in all cases, when other variables are set to their median values, the relative sensitivities of the ANN are physically legitimate at least in part. Indeed, in this sense ANN\textsubscript{A} arguably performs better than its GLM counterpart albeit delivering slightly less favourable goodness-of-fit. ANN\textsubscript{B} is over-fitted and the patterns observed in its relative sensitivity plots cannot be legitimised in a physical sense. However, this lack of model legitimacy is in contrast to the goodness-of-fit statistics which indicate ANN\textsubscript{B} to be the best model. Thus, developing techniques that can deliver a clear physical or mechanistic interpretation of input relative sensitivity analysis patterns in ANN modelling scenarios represents an important consideration for future research. Indeed, the presented results serve as a clear
demonstration of the dangers associated with evaluating models on the basis of statistical performance validation approaches alone.

6. SUMMARY AND CONCLUSIONS

This paper has addressed the difficult question of how to make meaningful comparisons between artificial neural network-based hydrological models and alternative modelling approaches. Comparisons which are based solely on goodness-of-fit metrics (i.e. the standard black-box approach presented in much of the literature) are very limited because they only consider model performance and not the means by which the performance is obtained. The commonly encountered limitation of metric equifinality, in which metric scores for the models being compared are insufficiently different to enable conclusive differentiation of the best or preferred model, is evident in our results. Our example of median flood modelling provides a clear demonstration of this with the fit scores obtained by the ANN and GLM models delivering inconclusive evidence about relative overall model performance.

However, the limitations of goodness-of-fit metrics are arguably more fundamental if there is a requirement to compare the transferability of each model from one hydrological context to another. In such cases, the physical legitimacy of each model must also be evaluated and compared in a direct manner. Models used in ungauged catchment prediction are a good example of those that must ultimately be transferred, and that therefore require evaluation of their physical legitimacy. This study has presented a consistent means by which the physical legitimacy of ANN models can be evaluated and compared with alternative modelling approaches. The application of relative sensitivity analysis in our median flood modelling example has enabled the physical legitimacy of two ANN-based models to be compared directly with the GLM counterpart used as standard in the UK. Tables 4 and 5 provide clear evidence that a general ANN modelling approach can deliver models as good as the GLM approach.
currently used in the UK Flood Estimation Handbook, both in terms of their performance and their legitimacy. Whilst the paper does not purport to be a competition between ANNs and GLMs, in this isolated case the evidence does lend some support to the view that ANN-based models may have some advantages over their GLM counterparts. However, one can only build good physically-legitimate ANN models if ample data of sufficient quality exist, and if the model development process is sound. It is also evident from this evaluation that ANN solutions can only deliver physical legitimacy if issues such as overfitting are avoided.

To conclude it is clear that comparing ANN models to alternative approaches on the basis of goodness-of-fit is insufficient, and that sensitivity analysis offers an important means by which the physical legitimacy of ANN models can be compared with that of counterpart models. Indeed, hydrological modellers using ANNs can and should be striving to evaluate the physical legitimacy of their models as well as their performance. By applying sensitivity analysis to ANN models a sense of trust is introduced that goes part of the way to addressing one of the key issues in the international ANN river forecasting research agenda of Abrahart et al. (2012a) – specifically the need for advanced diagnostic techniques that can help counter criticisms of the black-box nature of such models (e.g. Babovic, 2005). It is, therefore, surprising that it remains almost entirely absent from ANN studies and highlights the importance of a broader research agenda to develop robust, computational sensitivity analysis methods across the range of data-driven techniques currently being used in hydrological modelling. Such an agenda should include additional investigations that more fully explore the impact of different architectural structures in ANN models especially the potential bearing that internal complexity might have on the relative sensitivity of solutions to particular types of hydrological modelling problem.
REFERENCES


Table 1 Statistical summary of catchment descriptors

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
<th>Minimum</th>
<th>Maximum</th>
<th>25&lt;sup&gt;th&lt;/sup&gt; Percentile</th>
<th>75&lt;sup&gt;th&lt;/sup&gt; Percentile</th>
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<tbody>
<tr>
<td>AREA (km²)</td>
<td>148.70</td>
<td>1.63</td>
<td>4586.97</td>
<td>68.00</td>
<td>327.81</td>
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<td>BFIHOST</td>
<td>0.47</td>
<td>0.20</td>
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<td>FARL</td>
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<td>0.65</td>
<td>1.00</td>
<td>0.96</td>
<td>1.00</td>
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<tr>
<td>SAAR (mm)</td>
<td>1096</td>
<td>558</td>
<td>2848</td>
<td>830</td>
<td>1375</td>
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<tr>
<td>QMED</td>
<td>43.54</td>
<td>0.14</td>
<td>992.85</td>
<td>12.92</td>
<td>117.71</td>
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Table 2 Network weights and biases. Input neurons I1 - I4 (AREA, BFIHOST, FARL, SAAR respectively); Hidden neurons H1 – H9; Output neuron O (QMED)

<table>
<thead>
<tr>
<th>Neuron</th>
<th>Weight ANNa</th>
<th>Weight ANNb</th>
<th>Neuron</th>
<th>Bias ANNa</th>
<th>Bias ANNb</th>
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<tbody>
<tr>
<td>H1</td>
<td>-0.596</td>
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<td>H3</td>
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<tr>
<td>H7</td>
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<td>H8</td>
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<td>H9</td>
<td>-0.720</td>
<td>0.278</td>
<td>O</td>
<td>0.282</td>
<td>1.707</td>
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<table>
<thead>
<tr>
<th>Neuron</th>
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<th>Weight</th>
<th>Neuron</th>
<th>Weight</th>
<th>Weight</th>
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<tr>
<td>I1</td>
<td>2.112</td>
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### Table 3 Correlation matrix for model variables

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<tr>
<th></th>
<th>AREA</th>
<th>BFIHOST</th>
<th>FARL</th>
<th>SAAR</th>
<th>QMED</th>
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<td>AREA</td>
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<td>-0.07</td>
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<tr>
<td>FARL</td>
<td>1.00</td>
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<td>1.00</td>
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### Table 4 Numerical accuracy of different models under test

<table>
<thead>
<tr>
<th></th>
<th>GLM</th>
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<th>ANN&lt;sub&gt;B&lt;/sub&gt;</th>
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<tr>
<td>RMSE (m³/s)</td>
<td>43.09</td>
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<td>R²</td>
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<td>MSLE</td>
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Table 5  Physical legitimacy of GLM and ANN models.

<table>
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<tr>
<th>Input Variable</th>
<th>QMED potential of other catchment variables</th>
<th>Does the pattern of sensitivity response conform to conceptual notions of physically-rationality?</th>
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<td><strong>AREA</strong></td>
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<td><strong>SAAR</strong></td>
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<td><strong>FARL</strong></td>
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<td></td>
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<tr>
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<td></td>
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Figure 1 Typical feed forward ANN structure
Figure 2 Scatter plot matrix of model variable with linear regression lines fitted
**Figure 3** GLM versus QMED
Figure 4 ANN$_A$ model versus QMED
Figure 5 $ANN_b$ model versus $QMED$
Figure 6 Relative sensitivity of $QMED$ to model inputs: GLM
Figure 7 Relative sensitivity of QMED to model inputs: $\text{ANN}_A$
Figure 8 Relative sensitivity of $QMED$ to model inputs: $AN_{B}$