Variable speed control of a small wind turbine

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Variable Speed Control of a Small Wind Turbine

by

Simon H Lloyd

Doctoral Thesis
Submitted in partial fulfilment of the requirements
for the award of

Doctor of Philosophy of Loughborough University
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Abstract

An electronic controller has been developed for a wind turbine which uses a passive pitching mechanism and a permanent magnet generator. The turbine rotor is a 3 bladed, down wind horizontal axis design with a diameter of 3.4m. The machine, manufactured by Proven Engineering Ltd., produces 2.2kW at a wind speed of 12m/s and a rotor speed of 300rpm.

Passive regulation is achieved through a variation of blade pitch controlled by balancing the aerodynamic, centrifugal and spring forces acting on each blade.

A production machine has been instrumented and laboratory and field test data collected; from this data a mathematical model has been derived. A power electronic interface (DC-DC booster) was designed and built to transform the generator voltage to a fixed DC voltage. A controlled load is used together with feedback to the booster to set an appropriate load resistance according to operating conditions.

Current demand from the generator (used in the control) is derived either from the difference between the rotor speed and a reference speed, or directly as a function of the rotor speed (feed-forward control).

This thesis deals with the design and testing of the 3 compensators which govern the wind turbine control using both simulated and measured results. The overall objective of the controller is to maximise the energy yield from the wind turbine, subject to realistic constraints imposed by the power electronic design in the context of this particular design.
Acknowledgements

This project has had a strong experimental emphasis, and has involved a number of people to whom I am indebted.
I would like to acknowledge the help of the staff at the mechanical workshop at the Department of Electronic and Electrical Engineering, Loughborough University for their assistance during the early stages of the project.
I would also like to thank the staff and students at the Centre for Renewable Energy Systems and Technology (CREST) for providing the equipment and facilities and for both the practical engineering and theoretical help they have given. In particular I'd like to thank Dr Gordon Smith for his help with the power electronics, Dr Bill Forsythe for his help with aspects of the control analysis and also Dave Poole for the very practical engineering assistance he has given.
I would also like to thank Alan Derrick of the National Engineering Laboratory (NEL) for providing data on the Proven wind turbine.
Finally I would like to thank my supervisor, Dr David Infield, for his guidance and unceasing optimism, and the EPSRC for providing financial support through a studentship.
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<tr>
<td>CREST</td>
<td>Centre for Renewable Systems and Technology</td>
</tr>
<tr>
<td>DUWECS</td>
<td>Delft University Wind Energy Conversion System</td>
</tr>
<tr>
<td>EMF</td>
<td>Electro Motive Force</td>
</tr>
<tr>
<td>LQG</td>
<td>Linear Quadratic Gaussian</td>
</tr>
<tr>
<td>NEL</td>
<td>National Engineering Laboratory</td>
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<tr>
<td>PID</td>
<td>Proportional Integral Derivative</td>
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<tr>
<td>PWM</td>
<td>Pulse Width Modulator</td>
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<tr>
<td>SGA</td>
<td>Strain Guage Amplifier</td>
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<td>TSR</td>
<td>Tip Speed Ratio</td>
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<td>ZCD</td>
<td>Zero Crossing Detector</td>
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<tr>
<td>$a$</td>
<td>Axial Flow Induction Factor</td>
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<td>Current Regulator Low Pass Filter</td>
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<td>$G_{ph}$</td>
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<td>$G_{vlag}$</td>
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<td>$G_{WT}$</td>
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<td>$I$</td>
<td>Turbulence Intensity</td>
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---

X
\( I_c \) DC-DC Booster Capacitor Current
\( I_{ces} \) Snubber Capacitor Current
\( I_d \) Demanded Current
\( i_g \) Instantaneous Generator Current
\( I_{g_{\text{rms}}} \) Generator Phase Current
\( I_{gdc} \) Generator DC Current
\( I_L \) DC-DC Booster Inductor Current
\( I_{\text{out}} \) DC-DC Booster Output Current
\( J_g \) Generator Moment of Inertia
\( J_{\text{mot}} \) Motor Moment of Inertia
\( J_{\text{rot}} \) Rotor Moment of Inertia
\( J_T \) Wind Turbine Moment of Inertia
\( k_{\text{1-2}} \) State Space Transfer Function Coefficients
\( \text{\( K_{g1} \)} \) EMF versus Angular Velocity Gain
\( \text{\( K_{g2} \)} \) Generated Current versus Torque Gain
\( \text{\( K_{GLa_{1}} \)} \) Rate of Change of Load Conductance with respect to Duty Cycle \( \alpha_1 \)
\( \text{\( K_{GLa_{2}} \)} \) Rate of Change of Load Conductance with respect to Duty Cycle \( \alpha_2 \)
\( \text{\( K_{iQg} \)} \) Rate of Change of Generator DC Current with respect to Load Conductance
\( \text{\( K_{\text{irreg}} \)} \) Current Regulator Gain
\( \text{\( K_{qav} \)} \) Rate of Change of Aerodynamic Torque with respect to Wind Velocity
\( \text{\( K_{qaw} \)} \) Rate of Change of Aerodynamic Torque with respect to Angular Velocity
\( \text{\( K_{QgG} \)} \) Rate of Change of Generator Torque with respect to Load Conductance
\( \text{\( K_{gaw} \)} \) Rate of Change of Generator Torque with respect to Angular Velocity
\( \text{\( K_{f_{\text{fr}}} \)} \) Rate of Change of Frictional Torque with respect to Angular Velocity
\( \text{\( K_{\text{reg}} \)} \) Voltage Regulator Gain
\( \text{\( K_{\omega} \)} \) Rate of Change of Output Voltage with respect to Angular Velocity
\( \text{\( K_{WT} \)} \) Wind Turbine Steady State Gain
\( k_{xx} \) Partial Differentials
\( L \) Inductance
\( L_s \) Synchronous Reactance
\( m \) Mass
\( M \) DC-DC Booster Voltage Gain
\( n \) Number of Pole Pairs
\( N \) Number of Coil Turns
\( P_G \) Generator Power
\( P_{\text{loss}} \) Power Loss
\( Q_a \) Aerodynamic Torque
\( Q_{\text{diff}} \) Accelerating / Decelerating Torque
\( Q_f \) Frictional Torque
\( Q_s \) Shaft Torque
\( r \) Autocorrelation
\( r_{\text{ad}} \) Rotor Radius
\( r_b \) Internal Battery Resistance
\( r_1 \) DC-DC Booster Inductor Resistance
\( r_{\text{on}} \) MOSFET On State Resistance
\( r_s \) Sensing Resistor
\( r_{\text{st}} \) Generator Stator Resistance
\( r_t \) DC-DC Booster Total Input Resistance
\( R_L \) DC-DC Booster Load Resistance
\( R_{\text{ph}} \) Phase Load Resistance
\( s \) Laplace Transform Operator
\( s_{p1-pn} \) Pole Locations
\( s_{z1-zn} \) Zero Locations
\( S_{\text{vreg}} \) Voltage Regulator Sensitivity Function
\( t \) Time
\( t_d \) Time Delay
\( t_{\text{on}}, t_{\text{off}} \) DC-DC Booster on/off Time
\( T_{\text{vreg}} \) Voltage Regulator Complementary Sensitivity Function
\( v \) Wind Velocity
\( V_{\text{in}} \) DC-DC Booster Input Voltage
\( V_{\text{gdc}} \) Generator DC Voltage
\( V_{\text{grms}} \) Generator Voltage
\( V_L \) Synchronous Reactance Voltage Drop
\( V_{\text{out}} \) DC-DC Booster Output Voltage
\( V_{\text{ref}} \) Voltage Reference
\( V_{\text{st}} \) Stator Resistance Voltage Drop
\( X_{\text{op}} \) Operating Point Value
\( z_{\text{st}} \) Stator Impedance

XII
\( \alpha_1 \) Duty Cycle on Mosfet M_1
\( \alpha_2 \) Duty Cycle on Mosfet M_2
\( \gamma \) Exponential Filter Time Constant
\( \Delta t \) Sample Period
\( \theta X \) Small Signal Representation
\( \delta \) Torque Angle
\( \zeta \) Damping Ratio
\( \lambda \) Tip Speed Ratio
\( \rho \) Density
\( \sigma \) Variance
\( \Re \) Magnetic Reluctance
\( \tau \) Time Constant
\( \phi \) Magnetic Flux
\( \omega \) Angular Velocity
\( \Omega_m \) Measured Angular Velocity
\( \Omega_{\text{ref}} \) Reference Angular Velocity
Proven 2.2kW Wind Turbine
Chapter 1

Introduction

This chapter defines the objectives of this thesis and describes its overall organisation. A literature review summarises the relevant research concerning the control of variable speed wind turbines. An introduction is given to the Proven wind turbine dividing it into two sections: i) generator and ii) aerodynamics. This chapter ends with a brief description of basic wind turbine theory after which the main objectives are elaborated.

1.1 Thesis Organisation

This thesis details the work done during a three year project to design, build and test a power electronic interface. This uses a derived characterisation of the Proven wind turbine to run the machine at an optimal speed.

For the first two years of the project the Proven generator was in the CREST workshop coupled to a DC motor. During this time the machine was instrumented (Chapter 2) and a set of relationships for the generator were defined and used to derive a Matlab script model and a Simulink model (Chapter 3).

A set of performance characteristics were derived using data made available by the National Engineering Laboratory (NEL).

In addition to the central objective of optimising efficiency, a further objective is for the power electronic interface to provide a constant 400V DC link voltage. This is to provide a suitable power supply for a grid-linked inverter which will be
constructed as a follow on to the project. The design of this inverter is simplified by the provision of the constant 400V along with a signal, which determines the optimal power that should be being generated into the grid. Chapter 4 describes the design of a DC-DC booster capable of transforming the generator rectified variable DC voltage to a constant 400V DC and of providing the appropriate load.

Having defined the general model outline, Chapter 5 uses the technique of linearisation to further analyse the system. Chapter 6 uses the results to derive optimal parameters for use in the regulator electronics that control the power electronic interface.

With the controllers designed, Chapter 7 derives a Simulink model to test them, and to establish the effects of varying certain key parameters.

A major issue addressed by this project is the inevitability that the controllers have been designed using data recorded under non-ideal conditions and, due to variations between turbines may not be exactly matched to any particular machine. Ideally the machine in question would itself be used to derive a set of performance characteristics. This necessitates operating the machine at a fixed speed and using the resulting data to derive a performance characteristic which will be applicable at that speed. To operate the machine at a fixed speed requires a controller which itself will have been designed on an approximate basis; in this case using data supplied by NEL.

Chapter 8 uses the results from the fixed speed tests to derive a more accurate aerodynamic model. This improved model is then used in Chapter 9 to further improve the control strategy. Chapter 9 also presents alternative control schemes and compares their effectiveness.

The concluding Chapter 10 analyses the results using recorded data and further validates the model by comparing it with simulated data. It ends by suggesting further work that could improve the turbine controller design.
1.2 Literature Review

This thesis covers a broad range of engineering disciplines. These can be classified into four groups: i) permanent magnet generators, ii) aerodynamics, iii) control theory and iv) power electronics.

A general introduction to the Proven wind turbine is given by Proven[1.1]. Some of the key developments in the evolution of the machine are described together with examples of its applications. In this analysis the generator has been simplified and treated as a 'black box'.

Much more detailed descriptions of synchronous generators, in particular direct drive permanent magnet generators used for wind turbine applications are given by Spooner et al[1.2, 1.3]. The benefits of directly coupling permanent magnet (PM) generators to a rotor are clearly demonstrated and it is shown that the design of a very simple power train can be achieved with acceptable overall dimensions and with good efficiency. The emphasis is in general placed on their use for larger grid-linked wind turbines but the concepts are similar to those applied to the PM generator of the Proven turbine.

Feris[1.4] edited a collection of essays on wind energy conversion systems and the chapter by D Sharpe gives a very concise introduction to basic aerodynamic theory.

Control theory and its applications for wind turbines has for some time been studied by the Industrial Control Unit at Strathclyde University. One of their first papers by De La Salle[1.5] in 1990 is essentially a literature survey on the control of wind turbines. Leithead[1.6] highlighted the extent to which the advantages of variable speed operation are dependent on the systems dynamics and the turbulence of the wind. He also showed how the dynamics of a wind turbine are determined partly by the control system itself. He then went on to clarify a set of design goals for the control system.

Leithead[1.7, 1.8] continued by defining the roles and objectives for wind turbine control. He compared PI control design and classical Nyquist-Bode loop shaping and concluded that the classical methods are more suitable. Leithead also considered optimal control for the same problem. He concluded that conventional LQG has great difficulty in matching the classical technique by recognising the
difficulty in choosing the weighting matrices used in the cost function. In general the groups work is based on the variable pitch control of larger grid connected machines. He considers the control as being a tracking problem and in this he follows on from papers by Goodfellow[1.9] and Smith[1.10] which established the ideal torque speed characteristic based on the points at which the power coefficient is at a maximum. Goodfellow, using simulation studies, showed that by using variable speed operation a 3-4% improvement in energy capture is possible. He also states that such turbines must be capable of operation at 20% above rated speed.

The optimal control of wind turbines has been extensively dealt with by a group at Chalmers University. Novak[1.11] emphasised that for the design of a wind turbine control system a detailed knowledge of the system dynamics is essential. Although the report deals with an induction generator with frequency converter it also addresses more general configurations. Ekelund[1.12, 1.13] investigates the trade off between the objectives of generating maximum power and minimising the dynamic loads. He uses a linear quadratic controller and his results show it to be significantly more robust than a traditional PI controller at high wind speeds. Their work is mathematically intensive and their designs are yet to be implemented.

Optimal control strategies are described by Asher et al[1.14] who do implement their design. Their results show that the levels of wind turbulence have a large effect, as energy capture decreases when 'tracking' is lost due to turbulent wind conditions. Their conclusions can therefore be interpreted as suggesting that the optimal control of smaller machines is more problematic as they will be more susceptible to the effects of high turbulence.

A group at Delft University have also been studying control aspects of wind turbine design. Bongers[1.15] considered a wind turbine as a stochastic, non-linear multivariable system. He applied a linear quadratic optimal controller based on the entire wind turbine dynamical system. The group developed a computer code (DUWECs) to simulate flexible wind turbines. This was used to compare alternative control system designs.

Bossanyi[1.16] used adaptive control for control of a 200kW commercial turbine. He concluded that a model independent adaptive controller can perform satisfactorily
if an accurate measurement of the shaft speed is available. His control algorithm is computationally intensive and its implementation involves significant development costs.

Miller\textsuperscript{[1,17]} dealt with the optimisation of generated power by allowing a wind turbine with an induction generator to operate at its maximum power coefficient. His results show a considerable improvement in energy capture when compared with fixed speed controllers.

Conley\textsuperscript{[1,18]} implemented a control scheme for a vertical axis machine using rotor speed to set the load. His experimental results showed a general improvement over fixed load but that high levels of turbulence caused the control system to malfunction.

The comparison between variable speed and fixed speed operation taking into account the added cost of power electronics is dealt with in detail by McIver\textsuperscript{[1,19]}. His results, verified by simulation and by laboratory experiment show a 9-15\% improvement in net energy extraction. He also states that careful evaluation is required for each individual situation to confirm the economic benefits of moving to variable speed operation.

The optimisation of energy capture is dealt with by Holley\textsuperscript{[1,20]} using power electronics to adjust the speed of the generator independently of the grid frequency. This is accomplished with the added constraint of the design load limits of the turbine.

Based on the optimal shaft speed control concept for wind turbines, a paper by Perahia\textsuperscript{[1,21]} develops an algorithm and controller structure. A permanent magnet alternator battery charging system uses a half controlled bridge rectifier to implement the concept.

The optimisation of variable speed wind turbines is dealt with in a practical manner by Ernst\textsuperscript{[1,22]}. He compares tip speed ratio, feed-forward and hill climbing control methods. He concludes that feed-forward is the most suitable. He also states that it would benefit from being supplemented by a slow correcting loop which adapts the gain of the feed-forward controller.

Bleijs et al\textsuperscript{[1,23,1,24,1,25]} at Leicester University has studied the control of a variable speed wind turbine with a synchronous generator using a boost rectifier. A number
of configurations are considered though they all operate on the generator field winding. A dynamic model is developed and validated using measured time series. It is demonstrated that optimal performance can be achieved with a DSP based controller.

Nebel and Molley\cite{1,26} have considered the physical effects of the environment on wind turbines. They concluded that a 5% change in power is expected due to the variation in Reynolds Number, caused by changes in atmospheric pressure and temperature. They showed that the contamination effect on blades can lead to a 20% change in power especially for stall regulated machines. Power loss due to rainfall can also be as high as 20%. Power loss due to icing can reach 60%. They also noted large changes in generator power due to air density variations and varying turbulence intensities.

Simmons\cite{1,27} implemented a controller for a 1kW pitch controlled wind turbine. His experimental results indicated a 18.5% increase in mean energy capture for variable speed though his simulation results showed a 9.1% increase. He suggested that the poor quality rotor and the high levels of wind turbulence, which were not modelled, were the cause of the discrepancies.

The consensus amongst the published papers involved with modern control techniques is that their success is dependent upon an accurate system representation. They also all agree that because of uncertainties in the aerodynamic representation that optimal control is not a straightforward problem.

It is generally accepted that variable speed operation, if correctly implemented, will lead to increased energy extraction. As the cost of electronic power components decreases, the viability of variable speed operation increases.

To date most of the published work has concentrated on active pitch controlled turbines. Some researchers e.g. Ekelund have considered variable speed operation of stall regulated turbines. Very little attention has been paid to turbines with passive pitch control mechanisms (Bleijs is a notable exception). These can be expected to be the most demanding in that the aerodynamic response is continually changing. The Proven turbine provides a particular, and novel challenge, in that the rotor responds significantly to both rotor speed and wind load changes.
1.3 Generator Introduction

The generator is a multi phase, permanent magnet, synchronous machine. It has 8 pairs of magnets and 24 stator coils arranged as in figure 1.1.

The start and finish of each coil were bought out by the manufacturer so that the windings may be configured according to the application. Throughout this project the windings were arranged in star formation, as in figure 1.2. The neutral was used for the bench tests only. Once the machine became operational it was wired as a three-phase, three wire system.

Figure 1.1: Generator Magnet/Stator Coil Arrangement

Figure 1.2: Stator Coil Arrangement
1.4 Aerodynamic Introduction

The generator is specified as being capable of delivering 2.2kW at a wind speed of 12m/s. If the wind increases beyond this point, then there must be some form of regulation in order to avoid overloading the generator or allowing the machine to runaway.

The Proven wind turbine is unique in the manner in which it regulates the power extraction from the wind. The machine is down wind and so is able to cone its blades without any danger of the blades making contact with the tower. Each blade has two hinges (Appendix C). These are the flap hinge at 90° to the blade axis and the zebedee hinge at 45°. The flap hinge sets the cone angle. As the wind speed increases, the thrust on the blades increases and they are pushed downwind, which decreases the cone angle and also decreases the pitch angle, thereby reducing lift and aerodynamic torque. The zebedee hinge sets the pitch angle. The starting pitch is approximately 5°. This is adjustable and a precise determination of this angle is difficult. As the rotor speed increases the centrifugal forces pulling the blade away from the axis of rotation increase. This will flatten the zebedee hinge, altering the pitch towards stall, thereby reducing energy extraction.

This arrangement means that the machine can safely be left unloaded, and if full load is applied during high wind conditions, the regulation will act to limit the generator power to its rated value.

1.5 Basic Wind Turbine Theory

It can be calculated how much energy is contained in the wind within a given area at a given wind speed. The Betz criterion provides the accepted limit of 59% for the amount of this energy that can be extracted. This limit exists because the air must remain with sufficient energy to move away downwind of the turbine. The percentage of this limit that is actually extracted will decrease from an optimum if:

i) The blades are so close together or rotating so rapidly that a following blade moves into the turbulent air created by a previous blade.

ii) The blades are so far apart or rotating so slowly that much of the air passes...
through the cross section of the device without affecting a blade.

There is a definable relationship between the rotor speed and the wind speed known as the tip speed ratio (TSR). For a fixed pitch wind turbine there is a constant TSR at which energy extraction is optimised. In the case of the Proven, since the pitch angle varies with both rotational speed and wind speed, the optimum TSR is not fixed.

The objectives of this thesis are to define the relationship between this optimal TSR and the rotational speed, and then use this definition to set the generator load resistance to a value such that the wind turbine operates as closely as possible to its optimum rotor speed, for a given wind speed.

The relationship used for the characterisation of a wind turbine is known as the $G_p/A$ curve. $G_p$ is the power coefficient which is the ratio of how much power is extracted to how much power is available. The recognised symbol for TSR is $\lambda$. As $\lambda$ is increased from zero, the value of $G_p$ increases to a maximum at the optimal TSR, and then decreases.

For fixed pitch machines one $G_p/\lambda$ curve would be a sufficient characterisation, however with the Proven wind turbine there is some pitch angle change at below rated power, and so the $G_p/\lambda$ curve and therefore the optimal TSR will vary with rotational speed.

The objective of section 1.1 is therefore further clarified by stating it in five parts:

1) To derive a set of characteristics defining the wind turbine and power control system and use them to derive a simulation model.
2) To use the resulting model to design a fixed speed controller.
3) To implement the fixed speed controller and use the recorded data to derive a more accurate characterisation.
4) To use the improved model to derive an ideal control strategy.
5) To implement and test that ideal control strategy.

1.6 Control Philosophy

The controlled variable is the load resistance of the generator. This is set according to the effects of a stochastic input in a manner so that the tip speed ratio is kept
equal to the determined optimal value.

The value of the TSR depends upon both wind speed and rotor speed. It also depends upon the radius of the rotor which for the Proven wind turbine varies with the coning angle. The radius has been approximated as being constant throughout this thesis. The simplest form of energy maximisation control is to operate at a constant TSR. This necessitates knowledge of the wind speed. For a commercial system, the direct measurement of the wind speed is not a viable option. One method of control is to use the wind turbine as an anemometer, by measuring the rotor speed and its rate of change together with the load resistance, to infer a value for the wind speed. The disadvantage is that an accurate (noise free) measurement of the rotor speed is required. An alternative method, but which also requires measuring the rotor speed, is to allow the wind speed to affect the rotor speed and simply to set the load resistance as a direct (pre calculated) relationship to the current rotor speed. This method is known as feed-forward control. The advantages of this method are its ease of implementation and the fact that it allows the inherent dynamic characteristics of the machine to prevail. The lack of anticipatory control is the main disadvantage of this method. The effects of an increase in wind speed are not realised until there is a resultant increase in rotor speed. This means that the energy contained in turbulent winds can not be fully utilised.

This under utilisation of available energy is exacerbated by the effects of induction lag\(^{1.29}\). This phenomenon describes the dynamic, transient relationship between wind speed changes, and the resulting effect they have on the machines torque. If the wind speed increases then the delayed change to the rotational wake downstream of the turbine means that the induced flow is instantaneously less than the steady state value. This means that the torque will overshoot, followed by an exponential decay. If the wind speed decreases then the reverse happens. In the context of the already rather complex aerodynamics of the Proven turbine, the effect will be difficult to properly represent. In view of these considerations, induction lag has not been explicitly included in the models presented in this thesis.

In addition to the effects of induction lag, the machines inertia plays a role in
describing how the machine reacts to wind speed changes. Energy stored within the inertia is released or absorbed as the rotor speed decreases or increases. The rate of change of this stored energy depends on the machine's total inertia and upon the load resistance.

The effects of the upper and lower limits of the load resistance value have important implications. If the load resistance is at its maximum i.e. the machine is open circuit, then there is effectively no control being exerted. This means that if the wind speed decreases and the load is at its minimum value then the rotor speed will decrease at a rate determined by the machine's inertia and the frictional component. An increase in rotor speed can however be acted on by virtue of the fact that the machine is under control, until at the extreme, the load resistance is at its minimum value. At this point however the machine should be operating at rated power and the passive blade pitch mechanism will limit any further increase in generator power.

The load control of a wind turbine for variable speed operation is therefore complicated firstly by the highly non-linear nature of the aerodynamics and secondly by the saturation effects of the controlled variable.
Chapter 2

Instrumentation

2.1 Introduction

There are three areas for which instrumentation is required:

i) Electrical measurements. These are the current and voltage for the motor and the generator current and voltage.

ii) Torque. Transmitted through the shaft between the rotor and the generator.

iii) Angular velocity. The shaft is modelled as being non-elastic and so the rotor speed equals the generator speed.

Initially these transducers are for use with the test rig. For the operational machine an alternative method of measuring angular velocity was used.

The transducers and the DC motor to generator coupling are shown in figure 2.1.

![Figure 2.1: DC Motor to Generator Coupling](image)

This chapter describes the data acquisition system and then gives details on the design, construction, testing and calibration of the transducers required.
The transducers are then used to analyse the behaviour of the generator and the test rig.

2.2 Data Acquisition

The data acquisition system used is a PCL818HG supplied by Advantech Ltd. It gives 12 bit A/D conversion on 8 channels, 100kHz maximum sampling frequency and a programmable gain instrument amplifier for each channel. It also has a 1kB 'first in first out' (FIFO) memory on board. The system was calibrated according to the manufacturers recommendations\[2.1\] using its internal voltage reference and a supplied calibration program. The calibration was checked after three weeks use and found to not need any adjustment. There are three methods of data transfer:

i) Software Control.

ii) Interrupt Driven.

iii) Direct Memory Access (DMA).

The software method was first used. The speed of this method is dependent upon the computer speed. With the 286 computer originally used the minimum sampling period for six channels is 2 ms per channel. In addition there is a limit to how much data can be stored. The maximum array size for Pascal is 32250 words. This means that at the maximum sampling frequency, 6 channels can be sampled for 10.7 seconds. The time taken for the transfer of 1K words to the hard disc is 21ms. The hard disc has a higher priority over the acquisition cards interrupt and so no new data can be acquired during that 21ms.

Direct memory access was tried but a faster computer was purchased before the method could be perfected.

With interrupt drive software the 486 PC was capable of operating the PCL818 at 100kHz which for eight channels gives a minimum sampling time per channel of 0.08ms.

The interrupt software is written in Pascal with incorporated assembly language instructions for the interrupt service routine.

The Pascal listing of an interrupt driven data acquisition program is given in
Appendix Q. This program contains a number of procedures that can be included or excluded within the main program according to each individual test.

The *initialise* procedure sets the number of channels to be sampled and their respective gains. The sampling period is adjusted by setting the values of *cntr_1* and *cntr_2*. Note that each channel is sampled in sequence i.e. the system does not take instantaneous samples from all channels. For this reason the program written operates by taking a series of samples from each channel and computing the average. The number of samples taken is determined by the value of the variable *total*. The sampling period is therefore set by the user entering an integer value for *total* which when multiplied by the clock speed will be the sampling time. The results from each channel are used to compute the value of the variable being measured according to the calibration procedures. This is then stored and displayed on the screen.

### 2.3 Electrical Transducers

The DC current and voltage in the drive motor and the AC current and voltage output of the generator were originally measured using 'in house' built electronics. These must include some isolation in order to safeguard the PC, by preventing the possibility of any high voltages reaching it. The DC motor voltage *V_m* and the AC generator voltage *V_g* were measured using a potential divider followed by an opto-isolator. The DC motor current *I_m* and AC generator current *I_g* were measured by passing them through a 50A/50mV shunt and amplifying the resulting voltage signals before passing them through an opto-isolator.
The opto-isolator used for the DC signals is the 4N38 and for the AC signals is the H11A11. The specifications for each are the same, the only difference being that the H11A11 has two diodes arranged in parallel. The circuit diagrams for the motor current and voltage transducers are shown in figures 2.1 and 2.2 respectively. The resulting relationship between the measured parameter and the opto-isolator output is very non-linear and so a combination of offset voltages and gain adjustment had to be used to force the isolator to operate in its most linear region. This however gave problems with the signal to noise ratio and so a
compromise had to be reached. A non-linear relationship can however be tolerated as it can be represented as a polynomial.

The generator voltage and current are both alternating quantities. The resulting isolated waveform could either be full wave rectified with an active rectifier and sampled in the same way as the motor current and voltage, or software could be used to calculate the rms value. An active rectifier was built but the results were disappointing due to the non-linearities caused by the low frequency of the measured waveform. The software method was therefore chosen, even though it meant that a sample for the rms value would only be available once per period. Two separate methods for the software calculation of the rms value were tried.

The first involved summing all the samples over one period and calculating the rms according to the formula:

\[
V_{rms} = \sqrt{\frac{(v_1)^2 + (v_2)^2 + \ldots + (v_n)^2}{n}}
\]  

The second method involved calculating the rms value from maximum and minimum values of the sinusoidally varying waveforms.

It was found when the sets of results were plotted against a true rms reading digital voltmeter (DVM) that the second method was more accurate. This was because of the effect of the non linear opto isolator.

The current and voltage transducers were calibrated by recording the DVM reading alongside the PC output and deriving a polynomial relationship between them using the Excel database.

The electrical transducers described were used for the derivation of the machine parameters. When the wind turbine became operational more sophisticated hall effect transducers were used for measuring the generators rectified current and voltage. These proved far less problematic in that they had a linear relationship between their input and output. These were calibrated in a similar manner. The circuit diagrams and printed circuit board layouts are given in Appendix A.
2.4 Torque Transducer

There are a number of ways of measuring the torque being transmitted through a shaft\cite{2.2,2.3}. Many 'off the shelf' packages are available and altogether 12 companies were approached for quotes. Some methods involved cutting a section out of the shaft and replacing it with a purpose built torque transducer while others involved sending the shaft away to have strain gauges glued onto it. The quotes received ranged in price from £1100 to £6000.

Two methods of measuring torsional deformation were considered. The first involved the use of two slotted discs, one at each end of the shaft. The torque could theoretically be measured by calculating the time difference between the pulses received from each disc. It was realised that there would be a lot of work involved in the design of the electronic circuitry and coupling this to the data acquisition system.

The second method involved gluing strain gauges onto the shaft and measuring the torque by means of their change in resistance due to torsional deformation. This method has the advantage of the signal being analogue and so it can be sampled at any chosen frequency. This method was chosen, as it seemed the most straightforward even though there were concerns over the high gains necessary for amplifying such small changes in resistance. A mathematical treatment of the torque / torsional deformation / resistance change is given in section 2.4.1.

A strain gauge amplifier (SGA) was attached to the shaft and the power in and signal out connections were made via a set of three slip rings.

The strain gauges were purchased from TSM Ltd. (type TK9/1000/P/C). These are torque measuring gauges as they each have two gauges arranged at 90° on the same substrate (figure 2.3). The substrate is glued onto the shaft as shown in figure 2.3 so that each gauge is at 45° to the axis of rotation. This means that only the torsional component is measured.

Initially an SGA was bought from RS Components. The voltage output signal was converted to current loop using a purpose designed IC (XTR110). It was necessary to convert this signal to current loop since it had to be first transmitted through the slip rings and later with the wind turbine operational through 75m of twisted pair cable. This SGA turned out to be inadequate when it was tested,
since there was a large amount of drift at the gain required for this application. Consequently an SGA was bought from Datum Electronics Ltd. (type DPT02). This SGA had the advantage of providing a 4-20mA current loop output and will work at the high gain necessary with a reasonably low signal to noise ratio and low drift. The current output was converted to a voltage using a 250Ω resistor to give a 0 to 5V signal.

The slip ring assembly was bought from BGB Engineering Ltd. There was a major problem initially with this assembly as an irritating squeaking sound was produced, which may be amplified by the tower once the machine is operational. After much consultation with BGB the problem was solved to some extent by using a different material for the brush and using one brush only per slip ring.

2.4.1 Strain Gauge Application

Figure 2.3 shows the orientation of the strain gauges on the shaft.

![Figure 2.3: Strain Gauge Layout](image)

The area on the shaft was first thoroughly cleaned with alcohol. Each pair of strain gauges is glued at opposite sides of the shaft using standard superglue. The biggest problem was the soldering of the wires onto the gauges without damaging them.

Figure 2.4 shows the Wheatstone bridge arrangement.
The resistors are arranged so that for a steady state torque, $R_1$ and $R_4$ increase and $R_2$ and $R_3$ decrease in value. The four wires were passed to the SGA which provided a precision reference voltage $V_i$ and output a current representative of the difference $V_b - V_a$. If it is assumed that each resistor for the unstrained shaft has the same value $R$, and that each changes by the same amount $\Delta R$, for a change in torque then it can be shown that

$$\frac{V_b - V_a}{V_i} = \frac{\Delta R}{R}$$  \hspace{1cm} (2.2)

**Calculation of Strain Gauge Output Voltage**

$Q =$ Torque (N\text{m})

$d =$ shaft diameter $= 0.05\text{m}$

$I_p =$ polar second moment of area

$\tau =$ shear stress

$\varepsilon =$ resultant strain

$E =$ modulus of elasticity $= 210 \times 10^9 \text{ N/m.m}$ for steel

$\nu =$ Poissons ratio $= 0.3$ for steel

$L =$ length of strain gauge

$R =$ resistance of strain gauge $= 1000 \Omega$

$GF =$ gauge factor$= \frac{\Delta R}{R} = 2.1$ (for the gauges used)

$$\tau = \frac{dQ}{2I_p} \hspace{1cm} \text{and} \hspace{1cm} I_p = \frac{\pi d^4}{32}$$  \hspace{1cm} (2.3)
\[ \tau = \frac{16Q}{\pi d^3} \]  

(2.4)

\[ \varepsilon_{\text{max}} = \frac{\tau}{E} (1 + \nu) = \frac{16Q}{\pi d^3} \frac{1}{E} (1 + \nu) = \frac{\Delta L}{L} \]  

(2.5)

therefore

\[ \frac{\Delta R}{R} = GF \frac{\Delta L}{L} = GF \frac{16Q}{\pi d^3} \frac{1}{E} (1 + \nu) \]  

(2.6)

therefore

\[ \frac{\Delta R}{R} = 7.26 \times 10^{-7} Q = \frac{\Delta (V_b - V_a)}{V_i} \]  

(2.7)

So the torque is linearly related to the change in resistance.

The maximum torque transmitted through the shaft will be the rated power divided by the rated speed which gives \( Q_{\text{max}} = 63 \text{ Nm} \). This is the electrical power and so allowing for energy conversion losses, a full scale value of \( Q_{\text{max}} = 80 \text{ Nm} \) was adopted. Using equation 2.7 gives a full scale output \( \Delta (V_b - V_a) \) of the order of 0.7mV. This means an amplifier gain of 14000 must be used to produce a corresponding 0 to 10V signal output.

### 2.4.2 Torque Transducer Calibration

The rotor was locked and a static torque applied by attaching a bucket to the end of a radius arm attached to the generator end of the wind turbine. A program was written which sampled the SGA output every 40\( \mu \text{s} \). 50000 samples were taken over a two second period and the average calculated. The result was displayed on the screen along with the maximum and minimum values. The load was incrementally increased by filling the bucket with a precisely measured amount of water and for each step a DVM reading of the SGA output together with the computer display was recorded. The results showed that the average gap between the maximum and minimum samples compared with the total range gives a signal to noise ratio of approximately 1/60. The calibration was repeated at different times of the day (at different temperatures) and the results showed that the SGA does suffer from drift. The extent of the drift was such that the torque transducer in operational use will have to be self calibrating by recording a reference when there is no wind and using it to offset further readings. In addition to the DC drift
there was also an accompanying drift in gain. An attempt was made to quantify this by applying the maximum load at different times and recording the difference between unloaded output voltage and fully loaded output voltage. The results were inconclusive.

![Figure 2.1: Torque Meter Calibration](image)

Figure 2.5 shows the calibration results. The ordinate is the computer output average which is directly related to the SGA output. The abscissa is the mass, each division represents one cup of water which contains 310 ml and so weighs 0.31 kg. The distance from the axis of rotation to the point at which the bucket acts is 0.286 m. The bucket can hold 37 cups which therefore corresponds to a torque of 32.6 Nm. A 'best line fit' through the points of figure 2.5 was made using the Excel trendline tool. The gradient of this fit represents the relation between the shaft torque (as represented by the change in strain gauge resistance) and the SGA output.

### 2.5 Angular Velocity Transducer

Two separate methods for determining the angular velocity of the generator shaft were used. The first used a frequency to voltage converting IC (tachometer).
This was used during the bench testing stage as it has a fast response and high accuracy. This method would not ordinarily be used in a commercial application due to the manufacturing effort and materials involved and the fact that a power supply would have to be made available at the tower head. An alternative method was therefore developed which uses the time interval between zero crossings on the generator voltage waveform to calculate the angular velocity.

### 2.5.1 Tachometer Design and Calibration

The wind turbine was dismantled and a 3mm thick disc was bolted onto the brake hub collar. The disc has 180 holes, each 1.1 mm diameter, drilled around its circumference. A value of 1.1 mm was chosen to correspond to the diameter being a reasonable size for ease of mounting of the opto switch. A mounting plate was built with an opto-switch on it and arranged so that the infrared beam was directed through the holes. The circuit diagram of the opto-switch is shown in figure 2.6.

Originally the intention was to take the opto-switch output directly to the data acquisition unit and calculate the angular velocity by counting the number of pulses over a specified time period. Software was written to do this using the labcards Intel 8254 programmable counter/timer, but it was decided that the software complications that resulted were such that it made sense to have analogue voltage signals present at all channels.

![Image of circuit diagram](image)

**Figure 2.6: Opto Switch Circuit**

A 9400 frequency to voltage converter (F/V) IC is used to convert the opto-switch pulse output to a voltage proportional to its frequency. A circuit diagram for this
IC is shown in figure 2.7

Figure 2.7: 9400 Frequency to Voltage Converter

The data sheet states an accuracy of 0.05% over the range 1Hz to 100kHz. The basic equation relating voltage output to frequency input is

\[ V_{\text{out}} = V_{\text{ref}} C_{\text{ref}} R_{\text{int}} F_{\text{in}} \]  

(2.8)

A compromise between accuracy and speed of response has to be made when choosing component values. The ripple on the output voltage is inversely proportional to \( C_{\text{int}} \) and the input frequency. The time constant is equal to \( R_{\text{int}} C_{\text{int}} \). It is assumed that the speed of response of the system is sufficiently lower than the speed of response of the tachometer so that the emphasis can safely be placed on the accuracy. \( C_{\text{ref}} \) and \( R_{\text{int}} \) were chosen to give a 0 to 1V output that corresponded to the frequency range input 0 to 1800Hz corresponding to a generator speed of 0 to 600rpm. When the circuit was built there was seen to be a 100mV ripple. To reduce this, an operational amplifier arranged in differential mode was added to the 9400 output (figure 2.8).
The circuit has a unity DC gain and any AC components are amplified positively by the lower path and negatively by the upper path. When both paths have the same gain, the AC component will be cancelled. The circuit is effectively a low pass filter with the 3dB point set by the potentiometer. The trade-off with this circuit therefore involves sacrificing some speed of response in order to reduce the AC output ripple. This trade-off is made according to the application for which the tachometer is being used.

The tachometer was calibrated using a square wave generator, a frequency counter and a DVM. A program was written which took 5000 samples every 0.1s and calculated the average. The results were seen to be linear over the range 100Hz to 2000Hz. The lower frequency corresponds to an angular velocity of 4 rad/s which will be sufficient for deriving the machine parameters.

The time response was measured by supplying a square wave at 1800Hz, sampling the tachometer output and disconnecting the input. The time constant is found by timing the interval between the disconnection and the point where the response has reached 36.8% of its final steady state value. An average value of $\tau = 27\text{ms}$ was measured. This was done with the output filter set to remove 70% of the output voltage ripple at 900Hz. This is of such an order that when compared to the generators time response of several seconds it can safely be ignored.

2.5.2 Time Interval Measurement Circuit
The objective of the time interval measurement circuit is to measure the turbines angular velocity between $\omega_m = 15\, \text{rad/s}$ and $\omega_m = 35\, \text{rad/s}$. Fig 2.9 shows the block diagram representation.

The time interval circuit works by using a high frequency pulse signal to clock a counter between two reference pulses. The reference pulses are derived from a zero crossing detector (ZCD). Monostable '0' is set to hold its pulse output for 35ms. This is to prevent the speed transducer from registering false zero crossings. The maximum measurable rotor speed is therefore 45 $\text{rad/s}$. At each zero crossing the count value is transferred to a latch. The counter is reset and begins counting again until the next pulse. The 16 bit latch output therefore remains constant between zero crossings and is available to be sampled by the PC.

A 6.18Mhz crystal oscillator was chosen for its high stability. The output from the crystal is divided by 128 and passed through a comparator to give a 48 kHz
square wave. This signal is gated through an AND gate by the ZCD output. When this signal is at +5V the oscillator pulses are allowed through the gate to trigger a set of 4 counters arranged in series. When the output of counter '1' reaches its maximum, the terminal count signal ripples through to counter '2'. The 16 outputs of the four counters are taken to a latch which is triggered by the rising edge of the ZCD. The output of the latch is therefore representative of the turbine speed. The circuit has the additional feature of the latch triggering signal being available to the data acquisition board where it could be used as an interrupt signal.

**Time Interval Measurement Circuit Transfer Function**

The digital output of the time interval measurement circuit is latched and therefore is available to be sampled at any chosen frequency. The latch however is only updated at each positive going zero crossing. The resulting time delay is therefore dependent upon the angular velocity. At rated speed \( \omega = 35 \text{ rad/s} \) the frequency of the generated AC voltage is 22Hz. This gives a minimum time period between the latch output changes of 45ms.

A time delay is represented by a block with the transfer function \( e^{-t_s s} \). Frequency response analysis shows that a time delay has a destabilising effect. The transient response of a system with a time delay is not so easily determined because the number of roots is infinite. The approximate transient response is known as the Pade approximation[^2.4^]:

\[
e^{-t_s s} = \frac{1 - \frac{t_s}{2} s}{1 + \frac{t_s}{2} s}
\]  

(2.9)

This represents a unity gain, non minimum phase transfer function. It has a pole in the left hand plane and a corresponding zero at the same distance from the origin in the right hand plane.

### 2.6 Bench Test Rig Analysis

Having calibrated the torque transducer using a known static torque, its output voltage signal for the machine under various load conditions can be analysed.
When this signal is used in conjunction with the rotor speed signal then basic information on their relationship can be gained. Ideally under constant input conditions, the torque and rotor speed signals would be expected to remain constant. There are AC components of determinable frequency (using FFT analysis) in both the torque and the speed signals. The possible causes of these AC components are:

i) Generator imbalance.
ii) Elastic effects of shaft.
iii) AC ripple on motor input DC voltage.
iv) Strain gauges picking up interference from surrounding power systems.
v) Resonant frequency of test rig.

The frequency spectrum of the torque transducer output shows the main component to be at the rotor mechanical frequency. Using a dial meter and rotating the generator by hand showed some significant differences in the orientation of the magnets. This difference would cause a mechanical imbalance which is very hard to correct without appropriate equipment.

The velocity transducers output signal was analysed in a similar manner to the torque transducer. For steady state conditions, the levels of the velocity AC component depend upon the input power and the load conditions. This is because the generator imbalance was causing a resonance to be set up with the test rig. Obvious vibrations occurred at certain rotational speeds which died away as the speed was increased.
Chapter 3

Wind Turbine Model

3.1 Introduction

This chapter describes how a mathematical model for the complete wind turbine was derived.

The wind turbine model is split into two parts.

i) Generator Model

ii) Aerodynamic Model

The stator impedance and moment of inertia for the machine were measured. With the generator coupled to a DC motor a data acquisition system was used to collect measurements to determine relationships between key machine parameters. These relationships were then used to derive a model for the generator.

The aerodynamic model was derived at this stage of the work using data supplied by NEL. This was coupled to the generator model, and an integrated model of the wind turbine was derived.

3.2 Generator Equivalent Circuit

The simplified equivalent circuit for a single-phase of a permanent magnet, synchronous generator is shown in figure 3.1
where $L_s$ and $r_{st}$ are the synchronous reactance and resistance. The output load $R_{ph}$ is considered as being purely resistive. $L_s$ is approximated as remaining constant for all $\omega$ and $I_g$. Using Kirchoff’s voltage law\[^3.1\] gives:

\[
E_g = V_L + V_{rst} + V_g
\] (3.1)

Substituting $V_L = n\omega L_s \frac{di_{st}}{dt} = j\omega nL_s I_g$, $(V_{rst} = I_g r_{st})$ and $(V_g = I_g R_{ph})$ into equation 3.1 gives:

\[
\frac{I_g}{E_g} = \frac{1}{R_{ph} + r_{st} + j\omega nL_s}
\] (3.2)

where $\omega$ is the mechanical frequency in rad/s and the number of pole pairs is $n = 4$.

The effect of the synchronous reactance is to produce a phase difference between $e_g$ and $i_g$. More important in this instance is the voltage drop $V_L$. Figure 3.1 can be viewed as a potential divider. So:

\[
V_g = \frac{R_{ph}}{\sqrt{(R_{ph} + r_{st})^2 + (4\omega L)^2}} E_g
\] (3.3)

### 3.3 Machine Parameter Measurement

#### 3.3.1 Generator Stator Impedance

Chapter 1 described how the stator windings are wired in star arrangement. At this stage the machine is considered as having a three-phase, four wire arrangement. The stator impedance refers to the resistance and inductance (synchronous reactance) between one phase and neutral.
**Stator Resistance**

The resistance of each phase was measured by connecting it in series with a fully charged lead acid battery, a rheostat and an ammeter. The current was adjusted in steps and for each step the voltage across the phase and the current through it were measured. The result is a straight line, the gradient of which is equal to the phase resistance, $R = \frac{\Delta V}{\Delta I}$. Each phase was measured as $r_{st} = 1.6 \Omega$.

**Synchronous Reactance**

Four separate methods were used to measure the synchronous reactance.

i) Inductance meter.

ii) RLC tuned circuit

iii) LR circuit (measuring time constant)

iv) Wheatstone bridge circuit

The inductance meter uses a Wheatstone bridge arrangement. A dial is turned until a minimum meter reading is attained. The sensitivity is increased and the process repeated. The results gave $L_s = 7 \, mH$. No difference could be seen when varying the rotor position. The other two phases also gave $7 \, mH$.

Knowing an approximate value for the inductance means that theoretically a tuned circuit could be used. This could be supplied with a sinusoidal voltage from a frequency meter. As the frequency is varied the amplitude of the voltage across the coil could be seen to vary. The results were not precise and their validity depended on knowing an accurate value for the capacitor.

The stator coils are air cored and so their inductance should be constant. This is because for an air cored coil the flux is linearly related to the current:

$$\phi = \frac{Ni}{R} \quad (3.4)$$

where $\phi$ is the flux, $N$ is the number of turns and $R$ is the reluctance. From equation 3.4:

$$\frac{d\phi}{dt} = \frac{N}{R} \frac{di}{dt} \quad (3.5)$$

The voltage across an inductor is proportional to the rate of change of flux. (Faraday's Law).

$$V_L = N \frac{d\phi}{dt} \quad (3.6)$$
Using equations 3.5 and 3.6 gives:

\[ V_L = \frac{N^2}{s} \frac{di}{dt} = L_s \frac{di}{dt} \]  

(3.7)

Using equations 3.6 and 3.7 gives:

\[ L_s = N \frac{d\phi}{di} \]  

(3.8)

Knowing the stator resistance means that if the time constant of an LR circuit can be measured then the inductance can be calculated. The circuit used is shown in figure 3.2.

![Circuit Diagram](image)

**Figure 3.2: Circuit for Measuring Synchronous Reactance**

The switch is double pole. One pole is used to switch a battery in series with a resistance across the coil, the other pole is used to trigger the data acquisition board. The sampling period was set to 40 µs.

From figure 3.2, using equation 3.7

\[ E_b = i r_t + L_s \frac{di}{dt} \]  

(3.9)

where \( r_t = r_b + r + r_{st} \). Taking the Laplace Transform\(^{[3.2]}\) of equation 3.9 and rearranging gives:

\[ i(s) = \frac{E_b}{r_t + sL_s} = \frac{E_b}{r_t} \left( \frac{r_t L_s}{s + \frac{r_t L_s}{r_s}} \right) \]  

(3.10)

Taking the inverse transform of equation 3.10 with a step input gives:

\[ i(t) = \frac{E_b}{r_t} \left( 1 - e^{-\frac{r_s}{L_s}t} \right) \]  

(3.11)
Where \( i(t) \) is the instantaneous current. The time constant is \( \tau = \frac{L_s}{r} \) and is equal to the time taken for the voltage across the inductor and the current in the inductor to have completed 63.2% of its total change. As \( t \to \infty \), \( e^{-\frac{t}{\tau}} \to 0 \) and so \( i \to \frac{E_s}{r} \) which is the steady state current. When the circuit is first switched on with the coils de-energised, the voltage across the stator increases rapidly, reaches a peak and then decays exponentially with a time constant \( \tau \) as the coil becomes magnetised. At time \( t = \frac{L_s}{r} \) the voltage across the coil will be 36.8% of the initial peak voltage. The resistance \( r_s \) is the sum of the stator resistance (calculated as \( r_{st} = 1.6 \Omega \)) and an additional resistor \( r = 1 \Omega \). The internal resistance of a fully charged lead acid battery will be low in comparison to \( r \) and has been ignored.

A program was written which used the data acquisition system to measure the time constant which may then be used with a knowledge of \( r_s \) to calculate \( L_s \). The program was run ten times and the average calculated. The rotor was moved through 5° and the process repeated. After each measurement the coil was short-circuited to disperse the stored magnetic energy. The difference between the inductance for direct and quadrature axes was too small to be reliably measured using this technique.

At 0° \( L_s = 8.2 \) mH
At 22.5° \( L_s = 7.7 \) mH
Average \( L_s = 7.95 \) mH

A source of inaccuracy was the problems encountered with clamping the rotor at a precise orientation. The process was repeated for the three phases and no discernible difference in values was measured.

In an attempt to increase the accuracy, a Wheatstone bridge was used with the stator coil as one arm and a changeover switch used to switch the polarity of the battery. There was no noticeable increase in accuracy using this method.

The machines stator inductance is therefore taken as \( L_s = 8 \) mH and equal for all three phases and remaining constant for all rotor positions.

### 3.3.2 Torque Angle

The load torque produced by a synchronous generator is proportional to the sine of its torque angle (also known as load angle \( \delta \)) which is the angle by which the
magnetic axes of the stator and rotor are displaced\(^{3.3}\). The relation between this angle and the load torque is sinusoidal. There is therefore an inherent limit (or pullout power) which occurs at \(\delta = 90^\circ\).

An experiment to measure the torque angle was performed by blocking off all but one hole on the tachometer disc. There is therefore a pulse output for each revolution. This pulse was used to trigger the oscilloscope which displayed the voltage waveform. As the load was varied the pulse should be seen to be occurring at a different point in the voltage cycle. The problem is that the DC motor and the generator are not matched. The full load torque could not be supplied without overloading the motor by a factor of four.

The gradient between the torque angle and the torque at the lower end of the torque range will be small. There was no measurable variation in the torque angle with varying generator power seen using this method. In an effort to increase accuracy a data acquisition program was written which used the pulse to trigger a counter. The results were again inconclusive due to the difficulty involved with a precise determination of the voltage peak.

### 3.3.3 Moment of Inertia (J)

The moment of inertia is a measure of a rotating machine's resistance to changes in angular velocity. To derive a model it is necessary to calculate the moment of inertia for both the generator and the wind turbine rotor.

Two methods have been used for the motor and generator. The first used measurements of the physical dimensions of the major components and the distance of their centre of mass from the axis of rotation. The second method involved wrapping a rope around the shaft, taking it through a pulley attached to the ceiling and allowing a known mass to fall to the floor. The terminal velocity and the number of revolutions before and after the weight hits the floor can be used to calculate the inertia. These methods are referred to as the measurement and the calculation method. The wind turbine rotor inertia is calculated using the first method.
Moment of Inertia Measurement

**Generator**  The generator consists of four major parts.

i) Magnet backing disc  
ii) Magnets  
iii) Rotor hub plate  
iv) Transmission shaft  

Measurements were made for each of these and used in standard formulae to calculate their moments of inertia. The total generator inertia is the sum of its constituent parts.

**Magnet backing disc**  Each disc is 230mm in diameter ($d_{\text{disc}} = 0.23$) and 10mm thick ($t_{\text{disc}} = 0.01$). Each has a set of triangular slots cut near to its centre to allow air to flow over the stator coils. For the inertia estimation the disc is approximated as being solid. Using the density of steel as $\rho_{st} = 7800 \text{ kg/m}^3$.

$$J_{\text{disc}} = \frac{\pi}{2} \rho_{st} t_{\text{disc}} d_{\text{disc}}^4 = 0.69 \text{ kgm}^2 \tag{3.12}$$

**Magnet**  Each magnet measures $b \times h \times t_m = 100 \text{ mm} \times 100 \text{ mm} \times 25 \text{ mm}$. Each magnet's centre of mass is $d_m = 180 \text{ mm}$ from the axis of rotation. Approximating the magnets as having the density of standard steel means the mass of each magnet is $M_{\text{mag}} = b \times h \times t_m \times \rho_{st} = 1.95 \text{ kg}$.

$$J_{\text{mag}} = M_{\text{mag}} d_m^2 + t_m \rho_{st} \frac{bh}{12} \left(b^2 + h^2\right) = 0.066 \text{ kgm}^2 \tag{3.13}$$

**Rotor Hub Plate**  The plate to which the blades are attached is a 5mm thick triangular shaped disc with the vertices bent at an angle of $\approx 30^\circ$. It can be estimated as being a circular disc with diameter $d_{\text{rot}} = 0.2 \text{ m}$.

$$J_r = \frac{\pi}{2} \rho_{st} t_{\text{rot}} d_{\text{rot}}^4 = 0.098 \text{ kgm}^2 \tag{3.14}$$

**Transmission Shaft**  As for the disc.

$$J_{\text{tr}} = \frac{\pi}{2} \rho_{st} t_{\text{tr}} d_{\text{tr}}^4 = 0.076 \text{ kgm}^2 \tag{3.15}$$

The total estimated generator inertia is

$$J_g = 2J_{\text{disc}} + 16J_{\text{mag}} + J_r + J_{\text{tr}} = 2.61 \text{ kgm}^2 \tag{3.16}$$
**Motor** The estimation of the physical dimensions of the cylindrical rotor is: diameter $d_{\text{mot}} = 0.12\, m$ and length $t_{\text{mot}} = 0.18\, m$.

$$J_{\text{mot}} = 0.75\frac{\pi}{2}\rho_{\text{cu}} t_{\text{mot}} d_{\text{mot}}^4 = 0.39\, kgm^2$$  \hspace{1cm} (3.17)

where the density of copper is $\rho_{\text{cu}} = 8900\, kg/m^3$ and a 75% fill factor has been assumed.

**Moment of Inertia Calculation**

Figure 3.3 shows the experimental arrangement for calculating a rotating machines moment of inertia.

An expression for the energy just before and just after the weight hits the floor is derived. Substituting one into the other eliminates the frictional effects. The result is an expression containing measurable parameters which can be used to derive the moment of inertia.

The data acquisition system was used to record the pulses from the slotted opto-switch (180 pulses per revolution) for the generator and a hall effect device on the back of the motor which gave 10 pulses per revolution. A second channel was connected to the falling mass which becomes $+5\, V$ when it contacts with the silver foil on the floor. The recorded data can be used to calculate the number of turns before and after the weight hits the floor and the angular velocity at the moment of impact.
Summing the energy before the weight hits the floor.

\[ mgh = \frac{1}{2} J \omega_i^2 + Q_f \theta_A + \frac{1}{2} m v_t^2 \]  

(3.18)

where \( v_t \) is the terminal velocity which is equal to the product of the angular velocity and the radius, and \( \theta_A \) is the number of radians gone through before the weight hits the floor. Summing the energy after the weight hits the floor.

\[ Q_f \theta_B = \frac{1}{2} J \omega_i^2 \]  

(3.19)

from which

\[ Q_f = \frac{1}{2 \theta_B} J \omega_i^2 \]  

(3.20)

Substituting \( v_t = R \omega_t \) and equation 3.20 into equation 3.18 and rearranging gives

\[ J = \frac{mgh - \frac{1}{2} mR^2 \omega_t^2}{\frac{1}{2} \omega_t^2 \left( 1 + \frac{\theta_A}{\theta_B} \right)} \]  

(3.21)

A Pascal program was written which recorded the data from the two channels and then processed it to calculate \( J \). This allowed multiple estimates to be easily made and so gave an indication of the accuracy of the method. Three separate weights were used and the program ran ten times for each. An average and a standard deviation were calculated which gave

\[ J_g = 2.36 \pm 0.32 \, kgm^2 \]  

(3.22)

and

\[ J_{mot} = 0.16 \pm 0.05 \, kgm^2 \]  

(3.23)

**Conclusion of Moment of Inertia Assessment**

The measured results were consistently higher than those calculated by 10% for the generator and 140% for the motor. The discrepancy for the motor can be explained by inaccuracies in the measurements since small errors for the diameter measurement will have a large effect. In the case of the generator, increased access to its constituent parts enabled more accurate measurements to be made. Due to the increased number of estimates made for measurement, it can be concluded that the calculated results are the most accurate.
Calculation of Wind Turbine Rotor Inertia

Appendix C describes how the inertia of a Proven blade is calculated. Each blade is calculated as having an inertia of 2.11 kgm\(^2\) which gives a total inertia of

\[ J_{rot} = 6.33 \text{ kgm}^2 \] (3.24)

### 3.3.4 Bench Testing Procedure

The bench testing procedures can be divided into two parts. i) Steady state and ii) Dynamic tests. The steady state tests involved adjusting the DC motor voltage in steps and for each step adjusting the generator load. The dynamic tests involved applying a step input either by shutting the power off, or switching the generator load. The effects were similar and so the emphasis was placed on the motor power shut down since account could more easily be taken of the effects of the motor. The general data acquisition program is described in section 2.2. Adjustments were made to the sampling time, program run time and the number of channels used according to the specific objectives of each test. Additional procedures were included for data manipulation for example averaging, instantaneous to rms conversion and deglitching (filtering). In some cases the raw instantaneous data was recorded and post-processing programs used in Matlab and Excel.

A three-phase load was used initially. This consisted of nine 1 kW fire bar elements each of 50 Ohm resistance. Each element is able to be switched on or off. There were therefore four possible loads: no load, 50\(\Omega\), 25\(\Omega\), and 16.7\(\Omega\). The tests were repeated using a DC load. The three phases were rectified with a full wave bridge rectifier. This load is adjustable from 255\(\Omega\) to 8\(\Omega\) in 15 steps.

### 3.3.5 EMF Versus Angular Velocity

Faraday’s law states that a generated voltage is proportional to the rate of change of magnetic flux. The rate of change of flux for a permanent magnet synchronous generator is proportional to the speed at which the magnets travel past the coils which for a direct drive machine is equivalent to the rotor speed. The electromotive force (EMF), denoted as \(E_g\) is the open circuit output rms voltage per phase
and can be calculated when the machine is loaded by adding the voltage drop across the stator impedance to the output voltage (equation 3.1). The generators angular velocity was increased in steps and for each step the values of \( \omega, V_g \) and \( I_g \) were recorded. This was repeated for each of the AC loads. \( E_g \) was calculated by substituting the measured values for \( \omega, V_g \) and the load resistance \( R_{ph} \) into equation 3.3 and plotting against \( \omega \) in \( \text{rad/s} \).

![Graph of EMF Versus Angular Velocity for Four Loads](image)

Figure 3.4: EMF Versus Angular Velocity for Four Loads

Figure 3.4 shows that the four relationships are coincident and the gradient can be calculated as

\[
E_g = K_{g1} \omega \tag{3.25}
\]

with \( K_{g1} = 3 \).

### 3.3.6 Torque Versus Angular Velocity

Figure 3.5 shows a typical plot of the instantaneous angular velocity and torque with their DC components removed and their AC components amplified.
Figure 3.5: Torque and Angular Velocity Time Series

Ideally these traces would both be flat. The waveforms go through one cycle for every revolution and they can therefore be attributed to the generator imbalance (see section 2.6). It can be seen that there is a 90° phase shift between them with the speed lagging the torque. When the rate of change of speed is at its greatest the torque is at its maximum.

Newton's second law states that for a rigid body in rotation about a fixed axis the rate of change of velocity is directly proportional to the torque and is inversely proportional to the inertia.

\[ T_m = \frac{J_m}{\tau} \quad T_g = \frac{J_g}{\tau} \]

Figure 3.6: Transmission Shaft and Flywheels

This means that the difference between the input torque and the generator torque plus losses is equal to the moment of inertia multiplied by the angular acceleration. The system can be treated as consisting of two flywheels at opposite ends of a shaft supported by two bearings (figure 3.6).
The moment of inertia for the motor and generator are $J_m$ and $J_g$ respectively. The torque as measured by the strain gauges is $Q_s$. Equating torques gives:

$$Q_s = Q_m - J_m \frac{d\omega_m}{dt} - Q_f - Q_{ml}$$  \hspace{1cm} (3.26)$$

$$Q_s = Q_g + J_g \frac{d\omega_g}{dt} + Q_f + Q_{gl}$$  \hspace{1cm} (3.27)$$

where $Q_{ml}$ and $Q_{gl}$ are the motor and generator losses respectively and $Q_f$ is the frictional torque. The shaft can be considered as being non-elastic. Therefore $\omega_m = \omega_g = \omega$ and $\dot{\omega}_m = \dot{\omega}_g = \dot{\omega}$. From 3.26 and 3.27

$$Q_m - Q_g - Q_{loss} = J_T \frac{d\omega}{dt}$$  \hspace{1cm} (3.28)$$

Where $J_T = J_m + J_g$ is the combined motor and generator inertia. Figure 3.7 shows the relationship for the ideal case ($Q_{loss} = 0$) for the motor and generator torque versus speed.

![Motor and Generator Torque Versus Angular Velocity for the Ideal Case](image)

At standstill with a motor torque applied the generator will begin to accelerate and $Q_g < Q_m$. The difference $Q_m - Q_g$ is attributed to energy being stored in the inertia. The speed will increase until $Q_m = Q_g$ as a result of changes to $Q_m$, $Q_g$ or both, at which point the speed will become constant.

The relationship between the rate of change of speed and the difference between the motor torque and the generator torque will be the reciprocal of the combined inertia.
motor and generator inertia. The angular velocity can therefore be calculated by dividing this difference by $J_T$ and integrating the result over time.

$$\omega = \frac{1}{J} \int Q_{\text{diff}} dt$$

(3.29)

3.3.7 Generated Current Versus Torque

It can be shown that the current generated by a synchronous machine is approximately proportional to the input torque\(^3\). It is approximate because the voltage drop across the stator inductance is neglected. This is a reasonable approximation as the frequency of the generator current is low.

Figure 3.8 shows the torque as measured from the strain gauges plotted against the shaft speed for the four balanced AC loads.

![Figure 3.8: Torque Versus Angular Velocity for the Four AC Loads](image)

As the load is increased the gradient increases. For the no load case the input torque required to overcome the frictional losses can be measured as approximately 4 Nm. If it is assumed that the relationship between the generator current and torque can be calculated from

$$K_{p2} = \frac{Q_s - Q_f}{I_g}$$

(3.30)

then for each sample $K_{p2}$ can be calculated. Figure 3.9 shows the results.
Figure 3.9: Calculation of $K_g2$ for Three Loads

Ideally the three lines would be horizontal and coincident. The results for the 50 $\Omega$ load are higher because the difference between $V_g$ and $E_g$ will be smaller for lower currents and so the machine will resemble more that of the ideal machine. A further reason is the inaccuracies involved in measuring small values of current and torque. $K_g2$ can be seen to be increasing for the other two loads from 8.5 at low speeds to 9.5 at high speeds. There are two reasons for this increase. Firstly the loss torque has been taken to be constant at 4 $Nm$ whereas it will be proportional to speed. Secondly the effects of the stator synchronous reactance have been ignored. These will become more dominant at the higher speeds (higher generator frequencies). $K_g2$ can be set at the average value

$$K_g2 = 9 \text{ Nm/A}$$  \hspace{1cm} (3.31)

If the mechanical input power is equated with the electrical output power then for a resistive load $K_g2 = 3K_g1 = 9$. For the non-ideal case therefore, $K_g2$ should be less than nine.

### 3.3.8 Frictional Effects

Friction is a force that tends to oppose the direction of motion of a rigid body. There are a number of different forms of friction. Static friction (Coulomb effect)
is the force required to initiate movement or to overcome what is known as the 'cold weld' effect. Constant friction is a force that is independent of speed and must be equal to zero if the machine is stationary. Its effect can be mainly attributed to bearing and slip ring loss. Velocity friction is proportional to the angular velocity and is caused by windage loss, which is the propeller action of the generator. There is also some contribution from the 'stray load losses' which will be a frictional effect proportional to velocity squared and is caused by eddy currents being set up in the surrounding metal work which will tend to resist a change in velocity.

The torque that acts on the machines inertia is the difference between the input torque and the generator torque plus losses. These losses must therefore be expressed as a torque, which is a function of the angular velocity.

\[ Q_f = a_1 \omega^2 + a_2 \omega + a_3 \]  \hspace{1cm} (3.32)

Note that this expression is true only if the machine is rotating.

For a DC motor the theoretical developed torque is directly proportional to the motor current. For an unloaded generator a motor current of 6.5 A is required before the turbine began moving. The current increases slowly until \( \omega = 10 \text{ rad/s} \) after which the rate of change of motor current to angular velocity increases.

Figure 3.10 shows three graphs of torque versus angular velocity for the unloaded generator. These were recorded at different times of the day i.e. at different temperatures. Each has a polynomial fitted using the Excel spreadsheet. At such low levels of torque the signal to noise ratio of the strain gauge signal is very low. The plots also demonstrate the major problem with the strain gauge amplifier used, which is that of drift. An additional disadvantage of this method for determining \( Q_f \) is that the losses associated with the motor are inseparable from those associated with the generator. Note that the speed has been taken to twice its rated value. At speeds up to 30 rad/s the frictional torque can be seen to be proportional to speed.
Figure 3.10: No Load Torque versus Angular Velocity

A more sophisticated method of deriving $Q_f$ versus $\omega$ would be to use the deceleration curve. Recording the velocity and time between a motor power shut down and the machine coming to rest means that if the experiment is performed for the generator plus motor followed by the motor alone, then the effects of the motor
can be excluded. The generator was open circuit throughout. Once a model has been developed, the effect of the generator current on $Q_f$ can be investigated by recording the deceleration curve when the machine is loaded. A rough estimate of the time taken to come to rest was made and the data acquisition sampling period set so as 4000 samples are recorded during this period. The velocity of the decelerating motor and generator was recorded using the output of the frequency to voltage converter (section 2.5.2). The velocity of the decelerating motor alone was recorded from a hall effect transducer mounted on the back of the motor which gave 10 pulses per revolution.

The Matlab Polyfit.m function was used to derive a 2\textsuperscript{nd} order relationship between $\omega$ and time. This resulted in:

$$\omega_{mg} = 0.1278t^2 - 5.598t + 55.332 \quad (3.33)$$

for the motor / generator and

$$\omega_m = 0.101t^2 - 8.728t + 53.9178 \quad (3.34)$$

for the motor alone.

These two deceleration curves must now be manipulated so as they both have $\omega_t=0 = 50 \text{ rad/s}$. Subtracting 50 and solving equation 3.33 for $\omega_{mg} = 0$ gives $t = 0.9737$. Substituting $t = t' + 0.9737$ into equation 3.33 gives

$$\omega_{mg} = 0.1278t^2 - 5.4768t + 50 \quad (3.35)$$

so

$$\dot{\omega}_{mg} = 0.2556t - 5.598 \quad (3.36)$$

for the motor / generator.

Similarly subtracting 50 and solving equation 3.34 for $\omega_m = 0$ gives $t = 0.4513$. Substituting $t = t' + 0.4513$ into equation 3.34 gives

$$\omega_m = 0.101t^2 - 8.6368t + 50 \quad (3.37)$$

so

$$\dot{\omega}_m = 0.202t - 8.6368 \quad (3.38)$$
for the motor alone.

A Matlab program Accal.m (Appendix D) was written which uses equations 3.36 and 3.38 to derive a relationship between \( \omega_m \) and \( \dot{\omega}_m \) and between \( \omega_{mg} \) and \( \dot{\omega}_{mg} \). The results are

\[
\omega_{mg} = 0.7 \times 10^{-3} \omega_{mg}^2 - 99.8 \times 10^{-3} \omega_{mg} - 2.113
\]  
(3.39)

and

\[
\dot{\omega}_m = 0.1 \times 10^{-3} \omega_m^2 - 27.7 \times 10^{-3} \omega_m - 7.3778
\]  
(3.40)

Equations 3.39 and 3.40 must now be converted by multiplying by their respective inertia (section 3.3.3) so that they represent torque. This results in

\[
Q_{mg} = 1.7 \times 10^{-3} \omega_{mg}^2 - 0.252 \omega_{mg} - 5.326
\]  
(3.41)

and

\[
Q_m = 8.96 \times 10^{-6} \omega_m^2 - 4.43 \times 10^{-3} \omega_m - 1.18
\]  
(3.42)

The difference \( Q_{mg} - Q_m \) will be representative of the frictional torque attributed to the generator alone.

\[
Q_f = Q_{mg} - Q_m = 1.72 \times 10^{-3} \omega_g^2 - 0.247 \omega_g - 4.145
\]  
(3.43)

The standstill frictional torque \( Q_{mgss} = -5.326 \) is slightly higher than that approximated from the no load torque versus rotor speed curves of figure 3.10. This difference can be attributed to torque transducer inaccuracies.

### 3.4 Generator Model

To derive a model for the generator for use in the Simulink dynamic simulation software it is necessary to express the generator current in terms of the generator speed. From Figure 3.1, \( V_g = E_g - I_g z_t \) where \( z_t = \sqrt{(r_{st})^2 + (4\omega L)^2} \). Substituting \( E_g = 3\omega \) and \( V_g = I_g R_{ph} \) into 3.3 gives

\[
I_g = G_{ph} \left( 3\omega - I_g \sqrt{(r_{st})^2 + (4\omega L)^2} \right)
\]  
(3.44)

The input for the transfer function represented by equation 3.44 is the angular velocity which is the integral of the torque difference divided by the inertia. The
block diagram representation is shown in figure 3.11 where the generator block is represented by equation 3.44 and where \( G_{ph} \) is the load conductance, which is the reciprocal of the load resistance.

![Electro Mechanical Model](image)

Figure 3.11: Electro Mechanical Model

### 3.4.1 Matlab Script Model

Figure 3.11 can be represented by a sequence of Matlab expressions. The rate of change of speed is

\[
\frac{d\omega}{dt} = \frac{\omega_{n+1} - \omega_n}{\Delta t} = \frac{1}{J} Q_{diff} = \frac{1}{J} (Q - Q_f - Q_g) \tag{3.45}
\]

Appendix D gives the listing for the Matlab script file `sswdet.m`. This program finds the steady state solution for a given \( Q \) and \( R_{ph} \). The program begins by setting \( \omega_n \) to a start value and solves the quadratic equation 3.45. The difference between \( \omega_{n+1} \) and \( \omega_n \) is used at the end of each iteration to determine whether a steady state condition has been reached.

### 3.4.2 Simulink Model

Figure 3.12 is the Simulink representation of the block diagram figure 3.11 and equation 3.44.
Figure 3.12: Simulink Representation of Electro Mechanical Model

3.4.3 Generator Model Validation

The validation is divided into two sections. These are the static and the dynamic validation.

The static validation is also split into two parts. The first validates the electrical aspects and the second validates the whole generator model. The dynamic validation involved comparing a recorded deceleration curve resulting from the motor power being turned down under various loads, with a predicted response.

Static Validation

**Electrical Validation** The first part of the static validation did not involve using the data acquisition system. Measurements were made on the DC side only with digital voltmeters and the speed was measured with a hand held tachometer. The following procedure was used. A speed was chosen and the motor input voltage set so that the machines ran at that speed for no load. The load was
increased and for each step increase, the motor input voltage was increased so as the machines were brought back to the chosen speed. At each step, recordings were made of the generator current and the output voltage. The DC load was used in this test so that smaller load resistance steps could be used. The chosen speed was increased from 10 rad/s to 30 rad/s in 5 rad/s steps and the process repeated for each step. The results of $I_{gdc}$ versus $V_{gdc}$ for the chosen speeds are shown in figure 3.13.

![Figure 3.13: Generator V-I characteristic for fixed speeds of 10, 15, 20, 25 and 30 rad/s](image)

For an ideal generator these lines would be horizontal. The amount they droop by gives an indication of the losses. For $\omega = 30$ rad/s at $I_{gdc} = 8$ A there is a 30V drop. Converting $I_{gdc}$ and $V_{gdc}$ into their equivalent rms values per phase and dividing $V_{grms}$ by $I_{grms}$ gives 1.98Ω. The calculated stator impedance at this speed is $\sqrt{(4\omega L_s)^2 + (r_{st})^2} = 1.86$ Ω. The difference is attributable to the voltage drop across the diode bridge. The diode loss can be accounted for by subtracting 1.2 V from the DC voltage drop. This gives $z_{st} = 1.85$ Ω.
Electro-mechanical Validation  Three sets of static results have been used. Each set consists of four sections corresponding to the four AC loads. Each section of each set consists of 50 samples. Each sample was calculated by taking the average of 2000 samples. A Pascal procedure was written to calculate the rms current from the 2000 instantaneous readings. Another procedure was used to remove outliers from the recorded 2000 samples. The data acquisition sampling time was set so as approximately 20 periods were recorded at rated speed (corresponding to five revolutions).

A Matlab script file *statst.m* (Appendix D) was written to run the Simulink model until a steady state condition has been reached using the recorded torque and the corresponding $G_{ph}$ as the inputs. The resulting speed and generator current were used to calculate the percentage error. A similar addition was made to the Matlab script file *sswdet.m*. As before there was no difference between the two models.

Over the speed range 20 rad/s to 35 rad/s the percentage errors for $\omega_{diff} = \omega_{med} - \omega_{sim}$ and $I_{gdiff} = I_{gmed} - I_{gsim}$ averaged -30% for $G_{ph} = 0.02 S$, -20% for $G_{ph} = 0.04 S$ and -10% for $G_{ph} = 0.06 S$. The reasons for these inaccuracies can be solely attributed to the torque transducer. If the third order polynomials fitted by Excel to the no load torque versus angular velocity relationship from figure 3.10 are used for the frictional feedback torque, then the errors are greatly reduced. This does not imply that the derived $Q_f$ versus $\omega$ relationship in section 3.3.3 is incorrect nor that any of the no load fits are more representative of the real frictional effect. The seemingly more accurate representation is the one that uses as a torque frictional versus speed relationship the one derived using the same data used for the test. The fact that there is a widely spaced error range using the three no load relationships indicates that the calibration and in particular the offset of the torque transducer is causing large errors. This is substantiated by the fact that the errors for $\omega_{diff}$ and $I_{gdiff}$ both decrease as the angular velocity and generator current increase.

Dynamic Validation

The dynamic validation involved evaluating the error between a measured and a simulated angular velocity deceleration curve.
For $G_{ph} = 0$ both the Matlab script model and the Simulink model gave a 5% error at $w = 20\, \text{rad/s}$. Since there is no electrical load applied the contribution to the errors caused by $K_g1$ and $K_g2$ can be discounted. This therefore validates the mechanical half of the electro-mechanical model.

Further validation is possible by applying an electrical load at rated speed and step decreasing either the power to the motor or the load resistance. This method has the advantage that the torque transducer is not used. The disadvantage is that there are increased frictional effects caused by current flow in both the motor and generator. It is difficult to distinguish between the effects of the DC motor and those of the generator. The motor effects are minimised by a power shut down as opposed to a generator load change.

Note that the load conductance used in the models is the DC load conductance multiplied by 1.92 to transform it to the single-phase load conductance. The derivation of this constant will be given in Chapter 4.

The percentage errors between simulated and measured results for three selected loads plotted against the machines angular velocity are given in figure 3.14. The points marked '+' are for no load, $G_{ph} = 0\, S$, those marked 'o' are for $G_{ph} = 0.05768\, S$ ($R_{LDC} = 33.3\, \Omega$) and those marked 'x' are for $G_{ph} = 0.14\, S$ ($R_{LDC} = 14\, \Omega$). These caused the machines to come to rest in 12 seconds, 4 seconds and 2 seconds respectively. The sampling time was adjusted for each so
as 4000 samples were taken during the time it took for the machines to come to rest. To compute the errors a cubic spline fit was made so as both have the same number of elements. Ideally the starting speed for each would have been the same. Because of the mismatch between the motor and the generator it was not possible to take the loaded generator to the speed used for the no load case without seriously overloading the motor. Obviously there will be 0% error at the start. The maximum error of -7% is for the middle load. In general the simulated deceleration decays more slowly at the higher speeds than the measured and faster at the lower speeds. Both the torque feedback relationship and the inertia could be adjusted to give identical deceleration curves but this would be done on a trial and error basis and so the derived parameters must be adhered to.

Further validation and investigations into the models accuracy were made by adjusting some of the key parameters. Decreasing $K_{g2}$ by 10% resulted in a slight improvement at the higher speeds and higher loads but a deterioration at the lower end. Other parameters were varied in a similar manner and it was found that for all round accuracy the values calculated in section 3.3 give the lowest difference between measured and simulated results.

### 3.5 Aerodynamic Model

The aerodynamic block of the wind turbine model is the part which contains the most significant non-linearities. The basic aerodynamic theory explained in Chapter 1 showed that if the relationship between the torque coefficient $C_q$ and the tip speed ratio $\lambda$ is known ($C_q/\lambda$ curve), then the wind velocity $v$ and the rotor speed $\omega$ can be used to calculate the aerodynamic torque $Q_a$. The expression relating the aerodynamic torque to the torque coefficient $C_q$ and the wind velocity is\textsuperscript{[1,4]}:

$$Q_a = \frac{C_q}{\frac{1}{2} \pi \rho r^3_{ad} v^2}$$  \hspace{1cm} (3.46)

where $C_q = f(\omega)$ and $\lambda = \frac{\omega r_{ad}}{v}$ where $r_{ad}$ is the radius of the rotor and is considered as being constant for all $\omega$.

The passive pitching mechanism complicates the aerodynamic block. Ideally the aerodynamic performance would be defined by a series of $C_q/\lambda$ curves for a cor-
responding series of rotor speeds. The wind turbine at Loughborough was not operational at this stage of the work and to define the aerodynamic block a set of data for a similar wind turbine, tested at NEL, was used.

3.5.1 Performance Characteristic Derivation

The data provided by NEL consists of a 5Mb Excel file. It contains 5000, 10 minute average samples of wind speed, rotor speed, generated power etc. NEL used a standard Proven controller with a dump load consisting of three banks of halogen lamps. During the course of their measurement campaign one of these banks failed. The resulting data for the partially loaded machine has been ignored.

The data is first split into five groups based on turbine speed. Each group was transferred to Matlab. The resulting $G_p/\lambda$ curves showed a great deal of scattering and further data manipulation is obviously necessary. Two methods were used.

The first method removed all outlying data by calculating the equations of lines superimposed onto the $G_p/\lambda$ curve and then removed all those data points which lay further from the line than some maximum tolerance. The Matlab file cfilt.m used for this method is given in Appendix D. The Matlab least squares program was used to derive a polynomial fit to the remaining data points.

The second method used a binning and averaging technique. An average was calculated for the data in each $0.1 \lambda$ band. The results for the lowest speed band are ignored as there is no real power being produced at these speeds. From a visual comparison of the structure of the resulting performance curves from the two methods, the second was considered superior. The resulting $C_p/\lambda$ and $C_q/\lambda$ curves from this method for the four highest bands are shown in figure 3.15 and 3.16 together with the polynomial fits. Equation 3.47 gives the polynomial fits. As the speed increases the tip speed ratio at which $G_p_{\text{max}}$ occurs increases. For the lower three speed bands as the speed is increased the $C_p_{\text{max}}$ value decreases with the exception of the highest speed band though this may be misleading as there is far less data in this speed band.
Figure 3.15: Power Coefficient Characteristics for 4 Speed Bands

Figure 3.16: Torque Coefficient Characteristics for 4 Speed Bands

\[
\begin{align*}
\omega < 28.5 & \quad C_p = 0.0046\lambda^3 - 0.1008\lambda^2 + 0.6609\lambda - 1.1596 \\
28.5 \leq \omega < 31.5 & \quad C_p = 0.0037\lambda^3 - 0.085\lambda^2 + 0.5923\lambda - 1.1086 \\
31.5 \leq \omega < 34.5 & \quad C_p = 0.0023\lambda^3 - 0.057\lambda^2 + 0.4265\lambda - 0.8234 \\
34.5 \leq \omega & \quad C_p = 0.0017\lambda^3 - 0.048\lambda^2 + 0.409\lambda - 0.9078
\end{align*}
\] (3.47)

3.5.2 Simulink Aerodynamic Model

The Simulink representation of equation 3.46 is shown in figure 3.17.
At the centre of this block diagram is the look up table. This is a one-dimensional table of the overall $C_q/\lambda$ curve. A $C_q$ value is produced for every $\lambda$ input. This is done by fitting a cubic spline to a range of points centred around the input $\lambda$. The actual data points produced by the binning process can therefore be used as opposed to their polynomial fits.

### 3.5.3 Matlab Model

The Matlab model for the aerodynamic section has an advantage over the Simulink model in that it can use all four of the estimated $C_q/\lambda$ curves. The subroutine `cp1calc.m` listed in Appendix D contains the four derived polynomials. It is provided with the value for $\omega$ and $\lambda$. It uses two of the four fits to calculate two values of $C_p$ and then based on the position of $\omega$ between the centre of the two speed bands, derives $C_p$.

### 3.6 Wind Turbine Model Conclusions

This chapter has outlined the process of determining by measurement or calculation the values of the parameters and the relationships between them of the
individual blocks that make up the complete wind turbine. Where possible each
derivation has been validated using alternative derivation techniques.

The static validation of the mechanical section did not achieve its full purpose
but it nevertheless highlighted two areas. Firstly that the torque transducer at
low levels of torque cannot be relied upon and secondly that a small variation in
the frictional feedback torque has a large effect on the generator model accuracy.

Two separate models have been derived in this chapter. The Simulink model is
simply the combination of the electro-mechanical and the aerodynamic models.
A Matlab script file has been written wtmod.m (Appendix D). The program that
calls the wtmod subroutine sets the initial values and the sampling time. Wtmod.m
is called for each time step and updates the speed and the generator current.
The load resistance and wind speed can then be set in the calling program to test the
system with a variety of input signals.

For ease of use in terms of varying parameters, inputs etc, Simulink has a major
advantage as it provides a visual block diagram which can easily be manipulated.
It also gives real time outputs in the form of 'scope plots'. This model can be
used to gain a general understanding of the system. For increased accuracy the
Matlab model was used. The major drawback of the Simulink model is that
the passive pitching mechanism is not modelled because of the restriction of the
one-dimensional look up table. This means the rotor blades are approximated
as having a fixed pitch angle. For a fixed wind speed as the load is reduced the
rotor will speed up beyond its rated speed. The Simulink model is therefore only
suitable as an approximate model for small input changes below rated power.

Analysis of resulting data should take account of the inaccuracies bought about
by the data manipulations involved in the derivation of the $C_q/\lambda$ curve. Varying
this curve by fixed percentage offsets and comparing the results with the same
percentage variation of other key parameters revealed how significant these inac­
curacies can be when compared to the approximations made for the derivation of
the generator model.
3.6.1 Wind Turbine Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator Resistance</td>
<td>1.6Ω per phase</td>
</tr>
<tr>
<td>Stator Inductance</td>
<td>8mH per phase</td>
</tr>
<tr>
<td>Generator Moment of Inertia</td>
<td>2.36 kgm²</td>
</tr>
<tr>
<td>Rotor Moment of Inertia</td>
<td>6.33 kgm²</td>
</tr>
<tr>
<td>$K_{g1} = E_g / \omega$</td>
<td>3 V/rads⁻¹</td>
</tr>
<tr>
<td>$K_{g2} = (Q_s - Q_f) / I_g$</td>
<td>9 Nm/A</td>
</tr>
<tr>
<td>Frictional Torque</td>
<td>$Q_f = 1.72 \times 10^{-3} \omega_g^2 - 0.247 \omega_g - 4.145$</td>
</tr>
</tbody>
</table>
Chapter 4

Power Control

4.1 Introduction

Power drawn from the generator will be controlled by the circuit shown in figure 4.1. It is anticipated that in the future, grid connection using an 'H' bridge converting DC voltage to an AC current will be used. Such a system would require a constant input of around 400V DC. To achieve this a boost converter was chosen to hold a constant DC voltage as the machines speed varied. For the tests, output power was controlled and dissipated through a PWM switched resistor.

Over the normal operating speed range of the wind turbine the rectified output voltage will be varying from 140 to 240 V.

The objectives for the power control circuit are:

i) Maintain a constant 400 V DC voltage over the operating range.

ii) To control the PWM switched resistor and the DC-DC booster gain so as to provide an appropriate load conductance to suit the turbine characteristic. The conductance is to be adjustable via a 0 to 10 V analogue output signal from the data acquisition circuit.

This chapter describes how these objectives are met by the design, simulation, building and testing of a DC-DC booster. The basic theory is described in section 4.2 together with the derivation of a set of transfer functions using the state space averaging technique pioneered by Middlebrook\(^{[4.1]}\). Section 4.3 addresses
the central design issues.

A Matlab model is detailed in section 4.4. The results can be compared with those derived mathematically from the transfer functions. Having validated the transfer functions, further analysis can be carried out using standard classical control system techniques.

The electronic control regulator designs are described in section 4.5 together with the power supplies, the pulse width modulator circuit and the protection circuits. The chapter ends with the operational testing of the booster circuit.

The main aim of the research is to achieve the physical construction of a working system. The emphasis is therefore based on practical issues, in particular the availability of components. Although a brief outline is given on the design and construction of a DC choke for the boost converter the choke actually used was already available in the laboratory. The capacitors used were also available in the laboratory. Having fixed values for these components made the design process a lot easier but nevertheless an outline is given here of a standard design methodology.

4.2 Mathematical Theory

![Figure 4.1: DC-DC Booster. Basic Circuit Diagram.](image)

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The basic theory of operation of the circuit (figure 4.1) is as follows. The mosfet switch $M_1$ is in one of two states.

1) Switch on. The rectifier output is short-circuited by the inductor. The diode is reverse biased thus isolating the output stage. The current will ramp upwards, and energy is stored in the inductor at a rate dependent on $V_{in}$ and $L$.

2) Switch off. The inductor current cannot change instantaneously and so current will be forced through the diode to the capacitor and load resistance. The current will ramp downwards with its rate of change dependent upon $(V_{in} - V_{out})$ and $L$. The result is that energy is stored in the inductors magnetic field for $t_{on}$ and then released to be stored as an electric field in the capacitor during $t_{off}$. With the correct sizing of the inductor, capacitor and load resistor the result will be a voltage gain that is dependent upon the duty cycle of the applied switching waveform.

There are two possible modes of operation:

i) Continuous. The inductor current never becomes zero.

ii) Discontinuous. The inductor current during the switch on period ramps down far enough for it to become zero.

Throughout this chapter the design of the booster will be considered for the continuous mode.

Figure 4.2 is a timing diagram for the idealised circuit.

The duty cycle is defined as the ratio of the on time to the switching period.

$$\alpha_1 = \frac{t_{on}}{t_{on} + t_{off}}$$  \hspace{1cm} (4.1)

Initially consider the circuit as ideal. The average voltage across the inductor must be zero (otherwise the current would be infinite). This means that area A = area B. This gives

$$V_{in}t_{on} + (V_{in} - V_{out})t_{off} = 0$$  \hspace{1cm} (4.2)

which gives the voltage gain as

$$M = \frac{V_{out}}{V_{in}} = \frac{1}{1 - \alpha_1}$$  \hspace{1cm} (4.3)

For a lossless circuit the output power equals the input power which gives the current ratio as

$$\frac{I_{out}}{I_{Lav}} = 1 - \alpha_1$$  \hspace{1cm} (4.4)
To achieve $V_{out} = 400 \, V$ with $140 \, V < V_{in} < 240 \, V$, the duty cycle must be $0.65 > \alpha_1 > 0.4$. For a constant power output of 2.2 kW, $I_{out} = 5.42 \, A$ but the input current will range from 15.5 A to 9.05 A.

4.2.1 State Space Averaging

The steady state transfer function $\frac{V_{out}}{V_{in}}$ and the dynamic transfer functions $\frac{\delta V_{out}}{\delta V_{in}}$ and $\frac{\delta^2 V_{out}}{\delta V_{in}^2}$ are derived in terms of the state space variables $A$, $B$ and $C$ in Appendix E. This section derives the state space variables in terms of the component values. The diode and capacitor are taken as being ideal but the resistance of the inductor and the on state resistance of $M_1$ have been included. Also included is the sensing resistor $r_s$ which will be used to determine the input current for use in the control regulator described later in this chapter.

The circuit is considered as being in one of two states. A set of mathematical expressions is derived for each state using Kirchoff’s current and voltage laws. The chosen states are the input current and the output voltage, $\begin{bmatrix} I_{Lav} \\ V_{out} \end{bmatrix}$.

1) Period A: Switch on.
Let \( r_t = r_l + r_{on} + r_s \)

\[
L \frac{dI_{Lav}}{dt} + r_t I_{Lav} = V_{in} \tag{4.5}
\]

\[
C \frac{dV_{out}}{dt} + I_{out} = 0 \tag{4.6}
\]

\[
\begin{bmatrix}
I_{Lav} \\
\dot{V}_{out}
\end{bmatrix} =
\begin{bmatrix}
-\frac{1}{L} & 0 \\
0 & -\frac{1}{CRL}
\end{bmatrix}
\begin{bmatrix}
I_{Lav} \\
V_{out}
\end{bmatrix} +
\begin{bmatrix}
\frac{1}{L} \\
0
\end{bmatrix} V_{in} \tag{4.7}
\]

\[
\dot{X} = A_1 X + B_1 V_{in} \tag{4.8}
\]

\[
Y = C_1^T X \tag{4.9}
\]

The system output is \( V_{out} \) so \( C_1^T = \begin{bmatrix} 0 & 1 \end{bmatrix} \)

2) Period B: Switch off Neglecting the forward voltage drop of the diode.

\[
L \frac{dI_{Lav}}{dt} + I_{Lav} r_l + V_{out} = V_{in} \tag{4.10}
\]

\[
C \frac{dV_{out}}{dt} + I_{out} = I_{Lav} \tag{4.11}
\]

\[
\begin{bmatrix}
\dot{I}_{Lav} \\
\dot{V}_{out}
\end{bmatrix} =
\begin{bmatrix}
-\frac{1}{L} & -\frac{1}{L} \\
\frac{1}{C} & -\frac{1}{CRL}
\end{bmatrix}
\begin{bmatrix}
I_{Lav} \\
V_{out}
\end{bmatrix} +
\begin{bmatrix}
\frac{1}{L} \\
0
\end{bmatrix} V_{in} \tag{4.12}
\]

\[
\dot{X} = A_2 X + B_2 V_{in} \tag{4.13}
\]

\[
Y = C_2^T X \tag{4.14}
\]

where \( C_2^T = \begin{bmatrix} 0 & 1 \end{bmatrix} \)
Figure 4.4: Switch Off Circuit

The averaged equations are produced over a period using the switch duty cycle as a weighting factor. Let $\alpha_1' = 1 - \alpha_1$

$$A = A_1 \alpha_1 + A_2 \alpha_1' = \begin{bmatrix} -\frac{R}{L} \alpha_1 - \frac{V}{L} \alpha_1' - \frac{1}{L} \alpha_1' \\ \frac{1}{C} \alpha_1' \\ -\frac{1}{CR_L} \end{bmatrix}$$ (4.16)

$$B = B_1 \alpha_1 + B_2 \alpha_1' = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$ (4.17)

$$C^T = C_1^T \alpha_1 + C_2^T \alpha_1' = \begin{bmatrix} 0 & 1 \end{bmatrix}$$ (4.18)

$$X = AX + BV_{in}$$ (4.19)

$$Y = C^T X$$ (4.20)

From Appendix E the steady state transfer function is

$$\frac{V_{out}}{V_{in}} = -C^T A^{-1} B$$ (4.21)

$$= \frac{\alpha_1'}{\frac{1}{R_L} (r_t \alpha_1 + r_I \alpha_1') + (\alpha_1')^2}$$ (4.22)

As $r_t \to 0$, $\frac{V_{out}}{V_{in}} \to \frac{1}{1 - \alpha_1}$ as for the ideal equation 4.3.

From Appendix E the dynamic transfer function is

$$\frac{\dot{V}_{out}(s)}{\dot{V}_{in}(s)} = C^T (sI - A)^{-1} B$$ (4.23)
where

\[ k_1 = \frac{1}{LC} \alpha_1' \]  (4.25)

\[ a_1 = \frac{1}{CR_L} + \frac{r_t}{L} \alpha_1 + \frac{r_t}{L} \alpha_1' \]  (4.26)

\[ a_0 = \frac{1}{CR_L} \left( \frac{r_t}{L} \alpha_1 + \frac{r_t}{L} \alpha_1' \right) + \frac{1}{LC} \left( \alpha_1' \right)^2 \]  (4.27)

The input voltage will have a relatively low bandwidth corresponding to the machines inertia.

From Appendix E the dynamic transfer function for the duty cycle is

\[ \frac{\tilde{V}_{out}(s)}{\tilde{\alpha}_1(s)} = CT(sI - A)^{-1} \left[ (A_1 - A_2) \bar{X} + (B_1 - B_2) V_{in} \right] + (C_{1T} - C_{1T}) \bar{X} \]  (4.28)

\[ (B_1 - B_2) = 0, \text{ and } (C_{1T} - C_{1T}) = 0 \text{ so } \frac{\tilde{V}_{out}(s)}{\tilde{\alpha}_1(s)} = CT(sI - A)^{-1} \left[ (A_1 - A_2) \bar{X} \right] V_{in} \]

\[ = k_2 \frac{s + b_0}{s^2 + a_1 s + a_0} \]  (4.29)

where

\[ k_2 = \frac{1}{Cr_t \alpha_1 + Cr_t \alpha_1' + CR_L \left( \alpha_1' \right)^2} \]  (4.30)

\[ b_0 = \frac{r_t}{L} - \frac{\left( \alpha_1' \right)^2 R_L}{L} \]  (4.31)

The denominator is the same as for \[ \frac{\tilde{V}_{out}(s)}{\tilde{V}_{in}(s)} \]

This transfer function has a zero at

\[ s_z = -\frac{r_t}{L} + \frac{\left( \alpha_1' \right)^2 R_L}{L} \]  (4.32)

and a pair of complex poles at

\[ s_{p1,2} \approx \frac{1}{2CR_L} \pm \frac{1}{2} \sqrt{\left( \frac{1}{CR_L} \right)^2 - \frac{4 \left( \alpha_1' \right)^2}{LC}} \]  (4.33)

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4.3 DC-DC Booster Design

4.3.1 Inductor Design

A rule of thumb method is to set the switching frequency and specify the maximum acceptable inductor current ripple. Initially choose $f_s = 18 \ kHz$ so that it is outside the audio range and low enough to not cause excessive switching losses. Using equation 3.7:

$$\frac{V_L}{L} = \frac{i_{\text{max}} - i_{\text{min}}}{t_{\text{on}}}$$ (4.34)

For a 10% ripple at full load with $I_{\text{in}} = 15.5 \ A$: $i_{\text{max}} - i_{\text{min}} = 1.55 \ A$. So $L = 3.3 \ mH$.

Having found a value for $L$ the available core size can be used to calculate the number of turns necessary. The design of an inductor using standard core dimensions is described in detail in Mohan. There are many iterations and design trade off decisions that have to be made. The main factors that need to be taken into account are:

i) Compromise between resistive losses, skin depth considerations, wire type and wire dimensions.

ii) Compromise between switching frequency and switching losses.

iii) Thermal considerations.

As an example the largest available E type core from Phillips is an E65 which has a window area of $528 \ mm^2$. A flux density of $B = 0.2 \ T$ is assumed. If an industrially acceptable current density of $3 \ A/mm^2$ is taken then a maximum inductor current of $15.5 \ A$ will mean a necessary wire diameter of $2.56 \ mm$. The number of turns of this diameter wire that will fit into this window assuming a 100% fill factor is $N = 80$. The inductance achievable with this number of turns is $L = 0.56 \ mH$ which is less than 20% of the required value.

The options are:

i) Reduce the required inductance value by permitting a higher current ripple.

ii) Increase the switching frequency thereby incurring higher switching losses.

iii) Use a double wound inductor.

iv) Stack the cores.

v) Find an alternative supplier of inductor cores.
As has already been mentioned a more than suitable DC choke was available and so this obviated the need for any severe design trade off decisions. The inductance value of the available choke is \( L = 8.5 \) mH. Its parameters were measured using the same technique for measuring the stator impedance (see section 3.3.1).

### 4.3.2 Capacitor Design

For a times 2 safety margin the capacitors voltage rating must be at least 100% greater than the maximum output voltage. The capacitance is chosen for an acceptable voltage ripple under worst case conditions.

The output voltage ripple is equal to the change in charge over time \( t_{on} = \frac{Q}{f_s} \) divided by the capacitance.

\[
\Delta V_{out} = \frac{\Delta Q}{C} = \frac{I_{out}a_1}{Cf_s}
\]  
(4.35)

from which

\[
C = \frac{a I_{out}}{\Delta V_{out} f_s}
\]  
(4.36)

where \( \Delta V_{out} \) is the output voltage ripple. The result suggests that the higher the capacitance, the lower the output voltage ripple. The disadvantage of large capacitor sizes is that the inrush current may damage other components.

There was available in the CREST workshop a number of 40 \( \mu F \), 600 V capacitors. Using three of these in parallel results in 120 \( \mu F \) which will give a voltage output ripple of \(<0.5\%\). This is calculated using equation 4.36 with \( \alpha = 0.75 \), \( I_{out} = 5.5 \) A, and \( f_s = 18 \) kHz. The capacitance was measured using an RC circuit and the data acquisition board in a similar way to the stator inductance measurement (section 3.3.1).

### 4.3.3 Snubber Design

The design of snubbers is individual to a particular circuit and its layout. As such it is partly based on observations and measurements and partly based on circuit design.

The reasons for using snubber networks are:

i) Alternative path for current at turn on therefore current spikes are limited.
ii) Limits voltage spikes at turn off caused by stray inductance.

iii) Reduces switching loss.

iv) Prevents overheating by distributing switching loss between the snubber resistor and the mosfet.

The design is split into two parts, turn off and turn on snubbing.

The DC-DC booster and load controller contain two mosfets. The procedure used for the turn off snubber is the same for both. The turn off snubber is more important for the chopping mosfet $M_2$ as the stray inductance from the bar fire elements. For the switching mosfet the stray inductance is minimised by physically locating the diode and mosfet close to each other.

**Turn Off Snubber**

![Snubber Network](image)

Figure 4.5: Snubber Network

The mosfet has a peak voltage rating which must not be exceeded (= 600V). Including a 20% safety factor it is necessary to keep $V_{DS} < 480V$. Assume at turn off the current $I_d$ decreases at a constant rate. So assuming $I_t$ is constant (due to circuit inductance) over the switching period $I_{csb}$ will increase at the same rate. So $V_{DS}$ will increase parabolically with its rate of increase proportional to the size of the snubber capacitor. Figure 4.6 is the timing diagram for the switch off
transient. At turn off the load current is diverted into $C_{sb}$ via $D_{sb}$.

$$I_d = I_t - I_{c_{sb}} = I_t - \left(\frac{I_t}{t_t}\right)t$$  \hspace{1cm} (4.37)

At time $t_u$:

$$Q_{c_{sb}} = \int_0^{t_u} I_{c_{sb}}\,dt = C_{sb}V_{DS}$$  \hspace{1cm} (4.38)

so

$$C_{sb} = \frac{1}{V_{DS}} \int_0^{t_u} \frac{I_t}{t_t}\,dt = \frac{I_t t_u^2}{2V_{DS}t_t}$$  \hspace{1cm} (4.39)

substituting $t_u = t_t$ and $V_{DS} = V_{c_{sb}}$. From observations at low power $t_t = 1.1\mu s$. At full power $I_{tau} = 15.5\ A$ and $V_{out} = V_{DS} = 400\ V$. This gives $C_{sb} = 16\ nF$. The nearest preferred value for snubber capacitors with a 2000 $V$ rating is $C_{sb} = 22\ nF$. Note that the voltage rating must have sufficient safety factor as it decreases with increasing frequency.

The snubber resistance was chosen so that the sum of the snubber discharge current and the inductor current at turn on does not exceed the mosfet pulsed current rating. The initial resistor current is $\frac{V_{DS}}{r_{reb}}$. This is added to the load current at turn on. The pulsed current rating for the mosfet used is 100A. $I_{L_{max}} + \frac{V_{DS}}{r_{reb}} < 100$. So

$$r_{reb} > \frac{V_{DS}}{100 - I_{L_{max}}}$$ \hspace{1cm} (4.40)
So $r_{sb} > 5 \Omega$. In addition the resistor value must be sufficiently low for the snubber capacitor to discharge completely during the on time of the mosfet. The capacitor discharges with a time constant $\tau_{sb} = r_{sb}C_{sb}$. Multiplying by 5 for the safety factor gives $t_{on \min} \geq 5r_{sb}C_{sb}$ so

$$r_{sb} \leq \frac{t_{on \min}}{5C_{sb}} \Rightarrow r_{sb} \leq 202 \Omega \quad (4.41)$$

The power rating requirement for $r_{sb}$ depends upon the switching frequency.

$$P_r = \frac{1}{2} C_{sb} V_{DS}^2 f_s = 32 W \quad (4.42)$$

This is too high and it is therefore necessary to reduce the snubber capacitor size by 5 (reducing the rise time of $V_{DS}$) thereby reducing the necessary snubber resistor power handling capability to 6.8 W.

A non-inductive snubbing resistor of $r_{sb} = 15 \Omega$ was tried with a snubbing capacitor of $C_{sb} = 4.7 \text{nF}$. This necessitated a decrease in the switching time from 1.1$\mu$s to 0.7$\mu$s. There is a slight overshoot in $V_{DS}$ but this is the price that has to be paid for reducing the switching loss.

**Leakage Inductance Snubber**

The chopping mosfet is in series with a set of fire bar elements which include some stray inductance. This will reduce the rate of rise of current which will increase the switching loss. The solution is to put a series combination of a resistor and diode in parallel with the load. At turn off the current in the stray inductance will be diverted through the diode and its stored energy released as heat in the resistor. The resistor will, ideally, be as small as possible and can even be omitted if a sufficiently high current carrying diode is used.

**4.3.4 Heat Dissipation**

Switching devices can fail because of over voltage, over current or over temperature. Standard design techniques and snubber protection circuits limit the effects of the first two. The third is prevented by adequate heat sinking.

Heat is created within the mosfet during the 'on' state and during switching transients. Snubbing will minimise this loss by externally dissipating some of the
energy and by minimising turn on and turn off times. The losses will be proportional to switching frequency. For the selected mosfets used the 'on' resistance $r_{on}$ at $20^\circ C$ is 0.3 $\Omega$ and at the anticipated operating temperature is 0.5 $\Omega$.

Consider first the booster mosfet. The average current in the output capacitor is zero. The mosfet rms current is $I_{Drms} = I_L\sqrt{\alpha_1}$. If $I_{out} = 5.42$ A then using equation 4.4 gives a maximum ($\alpha_1 = 0.65$) mosfet current of $I_{Drms} = 12.5$ A. The on state loss will therefore be $(12.5)^2 r_{on} = 78 \text{ W}$. The switching loss is less easy to determine and strictly should include the diode recovery current. A simple approach is to integrate the power during the switching transient over the rise time.

$$P_{loss(tr)} = \int_0^t i_D v_{DS} dt = \frac{1}{2} V_{DS} I_L t_r$$ (4.43)

Assume that the rise and fall times are equal, $t_r = t_f$ so the average power loss $= V_{DS} I_L t_r f_s$. The rise and fall times were measured as 0.7 $\mu$s so $P_{loss(tr)} = 78 \text{ W}$. This is quite large and further effort should reduce the turn on and turn off times to approximately one tenth of these values. The total power loss (worst case) is therefore 156 $\text{ W}$. Take the junction temperature as $100^\circ C$ and the ambient air temperature as $20^\circ C$. The heat flow will be through a series connected pair of thermal resistors $\theta_{j-s}$ and $\theta_{s-a}$ which represent the junction to heat sink thermal resistance and the heat sink to ambient air thermal resistance. To allow a safety margin $\theta_{j-s}$ can be ignored which gives $\theta_{s-a} = 0.75^\circ C/\text{W}$. The heat sink used has $\theta = 0.48^\circ C/\text{W}$.

A similar design process should be performed for the output chopping mosfet. The heat dissipation requirements will be lower due to the lower switching frequency and current but in this instance the heat sink chosen is the same as that used for the boosting mosfet.

4.3.5 Determination of Switching Frequency

As the switching frequency is increased the size of the inductor and capacitor can be reduced whilst maintaining the same current and voltage ripple. This decreases their physical dimensions and therefore their cost. The disadvantage of high frequency switching is that the switching loss increases thereby decreasing the overall efficiency.
The choice of switching frequency is therefore a compromise between maintaining efficiency and having to use large passive components. In this instance the choice was somewhat arbitrary since available components were used. The emphasis is therefore on efficiency and so a reasonably low value of $f_s = 18 \text{kHz}$ was chosen so as to be outside the audible range.

4.4 Mathematical Modelling

Two methods are used for the modelling of the DC-DC booster. The first uses the discretised differential equations in a Matlab script file and switches between the ON state and the OFF state according to the required duty cycle. The second uses the transfer functions derived using the state space averaging technique.

4.4.1 Matlab Script Model

The discretised differential equations are:

i) Switch on

$$\frac{I_{L(n+1)} - I_{L(n)}}{\Delta t} = \frac{1}{L} \left( V_{\text{in}} - r_l I_{L(n+1)} \right) \tag{4.44}$$

$$\frac{V_{c(n+1)} - V_{c(n)}}{\Delta t} = -\frac{V_{c(n)}}{R_C C} \tag{4.45}$$

ii) Switch off

$$\frac{I_{L(n+1)} - I_{L(n)}}{\Delta t} = \frac{1}{L} \left( V_{\text{in}} - V_{c(n)} - r_l I_{L(n)} \right) \tag{4.46}$$

$$\frac{V_{c(n+1)} - V_{c(n)}}{\Delta t} = \frac{1}{C} \left( I_{L(n+1)} - \frac{V_{c(n)}}{R_L} \right) \tag{4.47}$$

The program $dcsim.m$ is given in Appendix F. The switching frequency has been set at $f_s = 18 \text{kHz}$. This means that the program should be in each state for $27.778 \mu s$ for a duty cycle of $\alpha_1 = 0.5$. Taking $\Delta t = 1 \mu s$ means the program is an approximation as it will be in each state for $28 \mu s$. At the end of each $\Delta t$ time period the input current and output voltage are updated using the appropriate pair of discrete equations.

The disadvantage of this method is the length of time necessary for the program to complete. The program was translated into Pascal to speed it up but the
problems are now with memory availability. This method is therefore not suitable for design purposes. Its objectives were however satisfied in that the outputs were successfully used for the validation of the state space model.

The simulated results of the booster being switched on with $V_{in} = 200 \, V$ and $R_{Leff} = 200 \, \Omega$ are shown in figures 4.7 and 4.8. The diode has been approximated as being ideal. An ideal voltage source is also assumed. The initial current surge reaches 45 A at approximately 3 ms after switch on. The voltage reaches 690 V at approximately 6.5 ms after switch on. The large turn on current is proportional to the rate of change of voltage multiplied by the capacitance. This inrush current mostly supplies charge to the capacitor. Reducing the size of the capacitor reduces the current peak but also increases the output voltage ripple. Note that in practise, such a severe step input from both $V_{in}$ and $\alpha_1$ would not occur. The rate of change of charge on the capacitor would not be so great because the output voltage is generally held constant or changes at a much lower rate than it would if a step were applied to the input voltage. If the control circuit is switched on with the wind turbine running a soft start sequence must be used. The voltage source has been assumed to be ideal and so in practise the voltage and current overshoots would be limited by the stator impedance which effectively increases the size of the inductance $L$. At a steady state operating point a step input will result in a typical second order lightly damped exponentially decaying sinusoid.

The resonant frequency can be measured from the interval between the peak and trough of figure 4.7 as $t_p = 12.5 \, ms$ which gives $\omega_n = 483.3 \, rad/s$. The damping ratio can be calculated from a measurement of the overshoot $M_p = 0.725$ using $M_p = e^{-\zeta \pi/\sqrt{1-\zeta^2}}$ as $\zeta = 0.1$.

### 4.4.2 State Space Transfer Function Model

The small signal transfer functions derived in 4.2.1 are:

\[
G_1 (s) = \frac{\hat{V}_{out}(s)}{\hat{V}_{in}(s)} = \frac{k_1 \frac{1}{s^2 + a_1 s + a_0}}
\]

and

\[
G_2 (s) = \frac{\hat{V}_{out}(s)}{\hat{a}_1(s)} = \frac{k_2 \hat{V}_{in}(s) \frac{s + b_0}{s^2 + a_1 s + a_0}}
\]
The coefficients are defined in section 4.2.1. Note that $G_1(s)$ and $G_2(s)$ have the same characteristic equation. Using the same parameters as used for the Matlab model the resonant frequency and damping ratio can be calculated as $\omega_n = \sqrt{a_0} = 497.3$ rad/s and $\zeta = \frac{a_1}{2a_0} = 0.097$. These are virtually identical to those measured from the time response figures 4.7 and 4.8. This proves that there has been no mathematical errors in the derivation of the transfer function equations.

The steady state gains can be checked as $\lim_{s \to 0} G_1(s) = \frac{k_1}{a_0} = 1.98$ and $\lim_{s \to 0} G_2(s) = \frac{k_2a_0}{a_0} V_{in} = -774$. Ideally these steady state gains would be 2 and -800 respectively. The losses are responsible for the discrepancy.

A Matlab program `stspav.m` (Appendix F) was written which uses the standard file `ss2tf.m` to transform the state space equations to transfer functions. The results were checked by evaluating the coefficients of equations 4.48 and 4.49. A step response was generated using `step.m`. The results were identical to those obtained from `dcsim.m` if the diode is treated as a short circuit. The transfer function can only be used for small signal analysis and assumes the system is operating in continuous mode.

$G_2(s)$ has a zero at $s_z = -\frac{a_1}{L} + \frac{(a_1)^2 R_L}{L} \approx \frac{(a_1)^2 R_L}{L}$ i.e. it has a zero in the right
half plane. This type of transfer is classified as being non-minimum phase. For a minimum phase system the total loss resistance \( r_t \) must be greater than \( (\alpha'_1)^2 R_L \). Using the minimum value of \( \alpha'_1 \) and \( R_L \) means that \( r_t \) must be greater than 9.2\( \Omega \) for the zero to move into the left half plane. It is obviously unrealistic for \( r_t \) to be this high since the efficiency would be low due to power losses. The presence of this zero does not mean the system is unstable. Its effect is to cause a phase shift of \( \approx 180^\circ \) between \( \alpha_1 \) and \( V_{out} \). It is approximately 180° because of its large relative distance from the origin. This phase shift is counteracted by the fact that the transfer function has a negative sign in it. An increase in \( \alpha_1 \) therefore gives an increase in the voltage ratio which concurs with equation 4.3. The dynamic effect of this zero is that a step increase in \( \alpha_1 \) will cause an initial decrease in \( V_{out} \) before it begins to increase. This effect can be simulated using both \textit{dcsim.m} \textit{and step.m} on the state space model. When the system is first switched on with \( V_{in} = 200 \text{V} \) and \( V_{out(t=0)} = 0 \text{V} \) then for approximately 1.2\( \text{ms} \) the output voltage dips down to -20\( \text{V} \) before beginning its rise to 690\( \text{V} \).

There is a conflict now between the control systems design and the power system design. To improve the response and to have an acceptable damping ratio would mean increasing \( r_t \) to greater than 9.2\( \Omega \) and decreasing \( L \) and \( C \). The effects
would be reduced efficiency and higher current and voltage ripple which from a power system design are undesirable. If $r_t$ were increased and is still less than $9.2 \Omega$ then for higher values of $R_L$ the system may become unstable since moving the zero towards the origin will reduce its phase shift from $180^\circ$. The effects of the initial opposite direction of tracking will become more pronounced until for some $r_t$ there may be a sustained oscillation resulting from a step input.

### 4.4.3 Root Locus Analysis

The location of the roots give an understanding of the transient behaviour. In this instance the characteristic equations are the same. To validate the root locus derivation, the location of the set of poles of $G_1(s)$ are evaluated as $s_{p1,p2} = -48.04 \pm j495.03$ using the same set of parameters as used in the Matlab model $dcsim.m$. These result in a natural frequency of oscillation of $\omega_n = \sqrt{(48.04)^2 + (495.03)^2} = 497.3 \text{ rad/s}$ and a damping ratio of $\zeta = \cos \left( \tan^{-1} \left( \frac{495.03}{48.04} \right) \right) = 0.096$ which are the same as those measured from the time response graphs resulting from $dcsim.m$.

The Matlab script file $stspav.m$ (Appendix F) is used for the plotting of the root locus. The effective load resistance is set at $75 \Omega$ and incremented in steps by $50 \Omega$ to $1000 \Omega$. For each step the roots of the characteristic equation are calculated and plotted (figure 4.9). This shows the root locus for $\alpha_1 = 0.4$ (marked with an $x$) and $\alpha_1 = 0.65$ (marked with an $o$) as $R_L$ is varied. As the load is decreased the roots move to the right indicating decreasing stability. They will never cross into the right half plane as they asymptotically approach a point in the complex plane. This point is defined as $\text{real} (s_{p1,p2}) = -\frac{495.03}{L} - \frac{48.04}{L}$ as $R_L \to \infty$. This means that if the loss resistances that makes up $r_t$ are reduced to zero then there would be a sustained oscillation as a result of a step change at the input for an unloaded system. The value of $r_t$ must therefore be greater than zero and less than a value for which the proximity of the zero to the origin starts to cause stability problems. Efficiency has a high priority and so as $r_t \to 0$ the clamp on the maximum load resistance must be decreased to avoid instability problems which may arise if for instance a fault occurs on the current regulator and $\alpha_1$ stays constant. At the extreme this may cause problems with too high a starting torque and a compromise will have to be reached. The imaginary part stays relatively
Figure 4.9: DC-DC Booster Root Locus Plot

constant over varying $R_L$ since it is dominated by the term $\frac{(\alpha_1)^2}{L_C}$. Due to the high imaginary part the angle between a line drawn from each of the extreme poles to
the origin remains relatively unchanged with varying $R_L$. This means that both
the damping ratio and the frequency of oscillation do not change a lot with the
varying load. Taking $\alpha_1 = 0.4$ as an example the root positions for $R_L = 75\,\Omega$
and $R_L = 1000\,\Omega$ are $-87.4 \pm j345.7$ and $-36.12 \pm j345.4$ respectively. These
correspond to $\zeta = 0.245$ and $\zeta = 0.104$ and $\omega_n = 356.5\,\text{rad/s}$ and $\omega_n = 347.28\,\text{rad/s}$.

4.4.4 Conclusions to Mathematical Model Analysis

The natural frequency of oscillation at an average $R_L = 200\,\Omega$ (800 W power
output) is $f_n = 80\,\text{Hz}$. There will need to be some noise filtering in the voltage
and current regulator circuits. This value for $f_n$ is relatively low and so it may
be envisaged that the compensators will go some way towards counteracting the
effects of the lightly damped circuit. In addition, the duty cycle $\alpha_1$ should not
have any high frequency components which could lead to unstable behaviour. The
voltage input also should be mostly low frequency because of the inertial effects.
of the wind turbine.

If the system were to be switched on with a voltage present at its input then the over voltage protection circuit would switch within approximately 3ms. It is essential therefore that a soft start sequence is used.

4.5 Power Control Hardware

This section describes the design and construction of the ancillary electronic hardware circuits that make up the complete DC-DC booster. Figure 4.10 shows the

![DC-DC Booster Diagram](image)

Figure 4.10: DC-DC Booster

DC-DC booster with the control circuitry. The general aim (described in more detail in the control system design chapter) is to control the generator current by means of the booster gain. The voltage is held at 400 V by adjusting the load resistance according to the output voltage. A software compensator is used for speed control and this is also described in a later chapter.
The three-phase rectifier is described and the relation between its input and output voltages and currents are derived. The design of the current and voltage regulators is next described. Section 4.5.3 describes the design of the pulse width modulator circuit and the mosfet driver. Section 4.5.4 describes the general hardware layout followed by the design of the DC power supplies. The final section is the design and implementation of the wind turbine protection circuits.

4.5.1 AC to DC Rectification

The system is three-phase, three wire i.e. there is no neutral connection. The rectifier used is shown in figure 4.11. This is known as a three-phase, six pulse full bridge diode rectifier. The circuit is fully analysed in Mohan[4,2]. The waveform time series is shown in figure 4.12. To obtain the average value of the output DC voltage it is sufficient to consider only one of the six segments and obtain its average over a \( \frac{\pi}{6} \) rad interval.

\[
V_d = v_{yn} = \sqrt{2}V_{LL} \cos \omega t \quad -\frac{\pi}{6} < \omega t < \frac{\pi}{6}
\]  

(4.50)

where \( V_{LL} \) is the rms value of the line to line voltage. By integrating \( v_{yn} \) the volt second area \( A \) is given by:

\[
A = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sqrt{2}V_{LL} \cos \omega t \, dt = \sqrt{2}V_{LL}
\]  

(4.51)

Figure 4.11: Three Phase Bridge Rectifier

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and therefore dividing $A$ by the $\frac{\pi}{3}$ interval gives

$$V_{dc} = \frac{3}{\pi} \sqrt{2} V_{LL} = \frac{3}{\pi} \sqrt{2} \sqrt{3} V_{LN} = 2.3 V_{LN} \quad (4.52)$$

The relation between the input and output current is derived under the assumption that the full current commutates from one diode to the next i.e. that the inductance $L_s$ is zero. This circuit was simulated using Pspice and the effects of including the stator inductance of the test generator were to reduce the output current by less than 1%. So

$$I_d = \sqrt{3} I_m = \frac{\sqrt{3}}{\sqrt{2}} I_{grms} = 1.2 I_{grms} \quad (4.53)$$

The generator is taken as being balanced and so its line conductance, $G_{ph} = \frac{I_{grms}}{V_{LN}}$ (used in Chapter 3) is related to the DC conductance, $G_{dc} = \frac{I_{dc}}{V_{dc}}$ using equations 4.52 and 4.53 as $G_{ph} = 1.92 G_{dc}$.

### 4.5.2 Regulators

The block $G_{ci}$ in figure 4.10 is a software compensator that sets a demanded current $I_d$ according to the difference between the measured rotor speed and a
desired rotor speed. A description and the implementation of $G_{e1}$ is given in section 8.2 once the control system design process has been carried out.

The current and voltage regulator blocks are represented as $G_{e2}$ and $G_{e3}$ in figure 4.10. Each consists of some signal conditioning circuits, a standard PID controller and an opto-isolator. The opto-isolator is essential to keep the high voltage power side separate from the instrumentation side. The gain blocks within the current and voltage regulators are denoted by the subscripts $I$ and $V$ respectively.

**PID Compensator**

Figure 4.13 is the circuit diagram for the PID compensator. The voltages $V_p$, $V_I$ and $V_D$ are represented by the equations

\[
V_p = -\frac{R_2}{R_1} K_p V_{err} = -K_p V_{err} \tag{4.54}
\]

\[
V_I = -\frac{K_p}{C_1 R_3} \int V_{err} = -K_I \frac{V_{err}}{s} \tag{4.55}
\]

\[
V_D = -C_2 R_4 K_p \frac{dV_{err}}{dt} = -K_D s V_{err} \tag{4.56}
\]
where $K_{p1}$, $K_{p2}$ and $K_{p3}$ are all $>0$ and $<1$ and $s$ is the Laplace operator.

The summing op amp combines and inverts these so if $R_5 = R_6 = R_7 = R_8$ then

$$V_{PID} = V_{err} \left( \frac{s^2 + sK_p + K_i}{s} \right)$$  \hspace{1cm} (4.57)

The implementation and component selection for the PID compensators is described in section 8.2 once the design process has been carried out.

**Current Regulator**

The objective of the current regulator is to adjust the duty cycle on the DC-DC booster such that the generated DC current is equal to the demanded current input signal. The circuit diagram and the printed circuit board layout are given in Appendices G and I.

The generator DC current is measured from the voltage drop across a 0.0251Ω resistor $r_s$ on the negative side of the rectifier. As the current increases the input voltage to the current regulator $V_{rs}$ decreases until at full rated power $V_{rs} = -0.25\,V$. To maintain a steady state therefore if $V_{rs}$ decreases then the amplification of the booster is required to decrease which is achieved by decreasing $a_1$. If the demanded current signal $V_{id}$ decreases then the amplification should also decrease. The first op amp $X_1$ inverts and amplifies $V_{rs}$. $G_{IX1} = \frac{R_3}{R_1} = -33$. $X_2$ is a low pass filter which removes noise generated by the switching action.

$$G_{IX2} = \frac{K_{IX2}}{1 + s\tau_{IX2}}$$  \hspace{1cm} (4.58)

where $K_{IX2} = \frac{R_3}{R_4} = -1$, $\tau_{IX2} = C_1R_4 = 1.10^{-3}$. The filter therefore has a 3dB point at $f = 160\,Hz$. A test input voltage was used to define the transfer function for the opto-isolator. So as the device operated in its most linear region a DC offset signal was added by the potential divider connected to $X_3$ which acted as a unity gain summing amplifier. The output of the opto-isolator is on the instrumentation side. The signal is buffered and inverted by $X_4$. The combination of $X_3$, the opto-isolator and $X_4$ can be represented by a single equation which was found by measurement to be $G_{Opt} = 0.93$.

The demanded current signal from the PC is inverted and filtered by $X_5$. The
transfer function can be calculated in the same way as for X2.

\[ G_{IX5} = \frac{-1}{1 + s \left( \frac{1}{6.6 \times 10^3} \right)} \] (4.59)

The filter has a 3dB point at 1060 Hz. This is necessary to reduce the high frequency component of the discrete signal from the PC. The filtered demanded and measured signals are passed through a difference op amp X6 whose output \( V_{err} \) is the input to the PID compensator.

\[ V_{err} = G_{IX5} V_{Id} - G_{IX1}.G_{IX2}.G_{Iop}.V_{rs} \]

\[ = - \frac{1}{1 + s \left( \frac{1}{6.6 \times 10^3} \right)} V_{Id} - \frac{31}{1 + s \left( \frac{1}{1.7 \times 10^3} \right)} V_{rs} \] (4.60)

which can be approximated as \( V_{err} = -V_{Id} - 31V_{rs} \) which is their difference because \( V_{rs} \) is below ground potential. The error signal is the input to the PID compensator. The output of the current compensator is proportional to \( \alpha_1 \) and this needs to increase as a result of a decrease in \( V_{err} \). The output is therefore inverted by X11 before being transmitted to the pulse width modulator. A 7.5 V zener diode is used at the output to ensure there is never any possibility of the full 10V being applied to the PWM since this would represent \( \alpha_1 = 1 \) and mean an infinite gain is demanded. The output is therefore in the range 0 to 7.5V which corresponds to \( 0 < \alpha_1 < 0.75 \).

**Voltage Regulator**

The objective for the voltage regulator is to set the duty cycle on the output chopping mosfet \( M_2 \) so as the output voltage \( V_{out} \) is held at 400 V. Eventually the output mosfet and load will be replaced with a grid-linked inverter and the voltage regulator output signal will be used to set the power that is injected into the grid. If \( V_{out} \) becomes greater than 400 V then the duty cycle should go up so as \( M_2 \) spends more time in its 'on' state. This will decrease the effective load resistance thereby increasing the average output current. The circuit diagram and printed circuit board layout for the voltage regulator are given in Appendices H and J.

The output voltage is attenuated by a potential divider \( R_1, R_2 \) and \( C_1 \) in figure
4.10. The input to the inverting op amp X1 is $\gamma V_{\text{out}}$ where

$$\gamma = \frac{R_2}{R_1 + R_2 + j\omega C_1 R_1 R_2} = \frac{9.9 \times 10^{-3}}{1 + s \left(\frac{1}{150s}\right)}$$

(4.61)

This attenuates the 400 V to 3.96 V and filters the signal with a 3dB point of $f = 7.3$ Hz. The necessity for having such a low 3dB point will be dealt with subsequently.

$X1$ filters and inverts the signal

$$G_{VX1} = \frac{-1.715}{1 + s \left(\frac{1}{150s}\right)} \approx -1.7$$

(4.62)

The reference signal $V_{\text{ref}}$ is a negative voltage which is compared with the output of X1.

$$V_{\text{err}} = -\gamma G_{VX1} V_{\text{out}} - V_{\text{ref}} = G_{\text{vlag}} V_{\text{out}} - V_{\text{ref}}$$

(4.63)

This error signal is the input to the PID compensator. The output has a DC component added by X7 so it is operating in the linear region of the opto-isolator. The opto-isolator output has a DC component added so as it is in the range 0 to 10 V before being buffered and inverted by X8. The combination of X7, the opto-isolator and X8 were found by measurement to be $G_{\text{vop}} = 1.8$.

### 4.5.3 Pulse Width Modulator

The PWM circuit generates a square wave with its duty cycle proportional to the input voltage. The circuit diagram and printed circuit board layout for the PWM and mosfet driver are given in Appendices K and L. A block diagram is shown in figure 4.14. The sawtooth waveform is generated using a standard 555 timer. The 555 is specified as having a frequency stability of 1%. The waveform is produced using a JFET current source to charge a 1500 pF timing capacitor. The output charges to $\frac{i}{2}$ of the supply voltage and then rapidly discharges through the npn discharge transistor of the 555. The current source is not ideal and so the 555 output is not a pure sawtooth since there is some exponential component. A LM311 comparator is used to compare the 555 output with the input reference signal. The output will be high when the sawtooth is less than the input signal. The PWM signal is next opto-isolated. This is to increase the adaptability of the controller. Having an isolator here means that the reference signal can be
generated straight from the PC if necessary. The opto-isolator output is passed through another comparator which amplifies and buffers the signal before applying it to the mosfet driver. The relationship between a steady state input voltage and the output duty cycle is virtually a straight line with approximately 1% deviation due to the non-linearity of the exponential sawtooth. $R_1$ and $R_2$ have been chosen so as an input voltage of 0 to 5 V gives an output duty cycle of 0 to 1. This results in an overall gain between the input voltage and output duty cycle of 0.2.

The mosfet driver must be placed as physically close as possible to the mosfet to reduce the effects of any stray inductance. The driver consists of an npn and a pnp transistor. These decrease the switch transition time which will have been affected by the gate to source capacitance of the mosfet. The emitters of these two transistors are connected to the mosfet gate via an inverse pair of diodes one of which has an 8.2 Ω resistor in series with it. These prevent any high frequency ringing component from reaching the mosfet. The disadvantage of using this diode arrangement in the gate drive circuit is that the rise and fall times of the gate signal are increased. A 12 V zener diode and a 1k resistor are connected between the mosfet gate and ground as close as possible to the mosfet to prevent any voltage spikes from destroying it.

### 4.5.4 DC Power Supplies

Opto-isolators have been used throughout to keep the power and instrumentation signals separate so as to prevent the possibility of any high voltages from reaching
the PC. This means that separate DC power supplies must be provided. This need would be obviated in a commercial application as a PC would not be necessary for control or for data acquisition. Four power supplies are used, PSU1 to PSU4. Their location and the circuits each supplies is given in the cabinet layout diagram in Appendix M.

Each power supply is virtually identical. The power supply for the mosfet drivers has a higher transformer VA rating and has additional smoothing capacitors since high currents are demanded for short periods of time during switching transients.

4.5.5 Protection Circuits

The power mosfets need protecting from temperature, voltage, and current. The mosfets used for $M_1$ and $M_2$ (STE40N60) are rated at $100^\circ C$, $600$ V and $25$ A.

The temperature is detected by a sensor bolted onto the mosfet heat sink close to the mosfet. If the heat sink temperature rises above $28^\circ C$ the sensor will switch on the appropriate fan mounted above the heat sink. It will switch off when the temperature drops below $22^\circ C$.

In the event of an over voltage on the output capacitor or over current through the inductor the protection circuit must shut down the wind turbine until a manual reset is initiated. There should be a latched indication as to whether a voltage or current overshoot caused the shut down. The circuit diagram for the protection circuit is shown in Appendix N. The input for the current protection circuit is from the sensing resistor $r_s$. The input for the voltage protection circuit is from a resistive potential divider across $V_{out}$. The signals are compared with preset voltages on the potentiometers $P_1$ and $P_2$. These are set so as the respective comparator output goes to the positive rail if $V_{out}$ becomes greater than $450$ V or $I_L$ becomes greater than $15$ A. Once a comparator output has gone to the positive rail it will remain there until the appropriate push to break button is pressed. The results of a positive signal from either of the comparators is to turn the transistor on. This provides current to the gate of the thyristor which acts as a short circuit on the three-phase rectifier output. The protection circuits must be capable of being manually reset as oscillations may be set up as a result of a fault condition. They also need to have indicators for diagnostic purposes.
Figure 4.15: Oscilloscope Plot of Turn On Transient. Channel 1 is $V_{\text{out}}$ and Channel 2 is $I_L$.

4.6 Operational Testing of DC-DC Booster

Three separate voltage sources were used. A 12 V lead acid battery, a 220 V lead acid battery bank and the three-phase mains via a variac and rectifier. The input voltage was connected through a fuse to the DC-DC booster. The testing was done in stages.

The circuit operation was checked with the 12 V battery. The duty cycles $\alpha_1$ and $\alpha_2$ were disconnected from their respective regulators and potentiometers used to set their values. Setting the load resistance and varying the booster gain confirmed the voltage and current ratio equations 4.3 and 4.4. Two methods were used to obtain the dynamic response. The first simply involved a switch on the power input. The second involved applying a step to the switching duty cycle $\alpha_1$ by means of a potentiometer, a resistor and a switch. The current was measured using a Tektronix current probe. The resulting current and voltage $I_L$ and $V_{\text{out}}$ are shown in figure 4.15. These are almost identical waveforms to those predicted with $dcsim.m$ (figures 4.7 and 4.8). The results of the two separate input steps were indistinguishable. The measured time period of oscillation is $t_p = 9 \text{ ms}$ as opposed to the predicted $t_p = 12.5 \text{ ms}$. The measured overshoot on the voltage waveform is 0.77 which is slightly greater than the predicted value of 0.725. The
measured damping is $\zeta = 0.083$ i.e. the system is slightly less damped than that predicted. The differences can be explained by the fact that the oscilloscope waveform resulted from the 12 V battery being switched on with the load set at a comparable value to that used for dosim.e. Other possibilities for the differences are that the on state resistance was estimated as being too large and the fact that a 12 V battery is not an ideal source.

Having confirmed correct predictable operation of the circuit with the 12 V battery, the 220 V battery was connected and the circuit gain turned up. At each step increase in power the switching waveforms were monitored and the temperature of the switching devices checked with an infra red thermometer. Once the circuit reached 2.5 kW it was left to soak test for an hour. The switching mosfet case temperature was seen to stabilise at 40°C. Therefore the junction temperature is well within its safe operating region. The chopping mosfet settled at 30°C.

The disadvantage of using a battery is that the input voltage is fixed. The mains provides a variable source but it has a frequency component which is higher than that which would be generated by the wind turbine. As these frequencies are comparable to the natural frequency of the booster then care must be taken in the analysis of the results. The general operation of the control regulators can be checked using this variable source. The two PWM signals were reconnected and the PID compensators set to proportional only. At steady state operation the errors at the input to the PID compensators were seen to decrease as their respective gains were turned up. The steady state errors were measured over a range of input voltages and the results used to confirm theoretical results. The effects of adding some integral action were looked at but it was felt at this stage that the operational testing of the complete DC-DC booster plus control electronics must be left until a design strategy has been implemented. Once this is complete some further testing on the mains will be carried out before the circuit is connected to the wind turbine.
Chapter 5

Linearised Model

5.1 Introduction

In this section a linearisation technique is presented. It has been shown in Chapters 3 and 4 that the wind turbine / DC-DC booster system is non-linear and that there is a set of operating conditions at which the machine should 'ideally' be operated. Linearisation involves taking a discrete set of operating points, and for each one considering a small signal perturbation. Keeping the perturbation small enough means that the system may be considered as being linear and standard analysis techniques may be applied.

The linearisation procedure presented here is based on the expansion of the non-linear function into a Taylor series about the operating point\(^{[5,1]}\). The higher order terms in the Taylor series may be neglected if the variables deviate by small enough amounts from their operating points.

The system is split into 3 major sections:

i) Aerodynamic subsystem

ii) Electro-mechanical subsystem

iii) DC-DC booster subsystem

An equation is derived for the linearised coefficients of each subsystem. These equations are listed in the Matlab script file coeff.m (Appendix O). The user supplies an operating point wind speed and tip speed ratio and the program calculates the other variables. The calculations are based on the \(C_q/\lambda\) curve supplied
by NEL. (Section 3.5.1). Once the linearised coefficients are calculated and a controller designed (Chapter 6) the wind turbine is run at constant speed and measurements taken to derive another $C_q/\lambda$ curve which will be more representative of the actual turbine for which the controller is to be designed.

### 5.2 Aerodynamic Model

The aerodynamics of the wind turbine are represented by a single block in figure 5.1 with the turbine angular velocity ($\omega$) and the wind velocity ($v$) as its inputs and the aerodynamic torque ($Q_a$) as its output.

![Aerodynamic Block](image)

Figure 5.1: Aerodynamic Subsystem

The small signal representation for the aerodynamic block will be:

$$\delta Q_a = \frac{\partial Q_a}{\partial v} \delta v + \frac{\partial Q_a}{\partial \omega} \delta \omega$$  \hspace{1cm} (5.1)

for a small variation in torque $\delta Q_a$, in terms of small variations in $\omega$ and $v$, $\delta \omega$ and $\delta v$. Having chosen an operating point for the wind velocity $v_{op}$ which fixes the turbine speed $\omega_{op}$ these partial differentials will be constants.

Let $K_{Qaw} = \frac{\partial Q_a}{\partial v}$ and $K_{Qaw} = \frac{\partial Q_a}{\partial \omega}$

The aerodynamic block can now be represented as in figure 5.2.

### 5.3 Electro-Mechanical Model

The resultant torque that is applied to the turbine inertia to accelerate or decelerate the machine is the difference between the applied aerodynamic torque $Q_a$ and
the electrical generator torque $Q_g$ plus the loss torque $Q_f$ (section 3.4). This resultant, when divided by the combined rotor and generator inertia and integrated gives the turbine angular velocity as shown in figure 5.3. It has been assumed that the transmission shaft linking the rotor to the generator is infinitely stiff.

$K_{Q\omega\omega}$ is the frictional coefficient which can be assumed constant at $\omega_{op}$ (see section 3.3.8).

A change in the load conductance ($\delta G_{ph}$) or a change in the turbine velocity ($\delta \omega$). A step increase in load conductance will cause an instantaneously corresponding increase in the generator current which in turn will cause an increase in the generator torque. This has assumed that the generator torque is directly related to the generator
current (section 3.3.7). Similarly an increase in the turbine velocity will lead to an increase in the generator voltage which leads to an increase in torque. The linearised electro-mechanical model can be represented by equation 5.2 and figure 5.4.

\[ \delta Q_g = K_{QgG} \delta G_{ph} + K_{Qg\omega} \delta \omega \]  

(5.2)

where \( K_{QgG} = \frac{\partial Q_g}{\partial G_{ph}} \) and \( K_{Qg\omega} = \frac{\partial Q_g}{\partial \omega} \).

Combining figures 5.2, 5.3 and 5.4 provides a single block diagram representation (figure 5.5) for the complete linearised wind turbine. The inputs are the changes in load conductance and wind speed and the output is the change in angular velocity.

This block diagram can be manipulated so as it conforms to the standard control system multiple input, single output format with a single forward element and a single feedback element (figure 5.6). From figure 5.5.

\[ \delta \omega = \frac{1}{J_s} (K_{Qav} \delta v + K_{Qaw} \delta \omega - K_{QgG} \delta G_{ph} - K_{Qg\omega} \delta \omega - K_{Qf\omega} \delta \omega) \]

\[ \Rightarrow J_s \delta \omega = (K_{Qaw} - K_{Qg\omega} - K_{Qf\omega}).\delta \omega + K_{Qav} \delta v - K_{QgG} \delta G_{ph} \]

\[ \Rightarrow (J_s - K_{Qaw} + K_{Qg\omega} + K_{Qf\omega}).\delta \omega = K_{Qav} \delta v - K_{QgG} \delta G_{ph} \]

\[ \Rightarrow \delta \omega = \frac{K_{Qav}}{J_s - K_{Qaw} + K_{Qf\omega}} \delta v - \frac{K_{QgG}}{J_s - K_{Qg\omega} - K_{Qaw} + K_{Qf\omega}} \delta G_{ph} \]

\[ \Rightarrow \delta \omega = \frac{K_{Qav}}{1 + \frac{1}{J_s} (K_{Qg\omega} - K_{Qaw} + K_{Qf\omega})} \delta v - \frac{K_{QgG}}{1 + \frac{1}{J_s} (K_{Qg\omega} - K_{Qaw} + K_{Qf\omega})} \delta G_{ph} \]

(5.3)
This can further be broken down so as it can be expressed as a first order lag system as in figure 5.7.

\[ G_{WT} = \frac{G}{1 + GH} = \frac{K_{WT}}{1 + \delta \tau_{WT}} \]  

(5.4)

where the wind turbine gain \( K_{WT} = \frac{1}{K_{Q_{g\omega}} - K_{Q_{a\omega}} + K_{Q_{f\omega}}} \)

and the wind turbine time constant \( \tau_{WT} = \frac{J}{K_{Q_{g\omega}} - K_{Q_{a\omega}} + K_{Q_{f\omega}}} \)

### 5.4 Combining Wind Turbine and DC-DC Booster Models

The conductance at the input and the output of the DC-DC booster are represented by:

\[ G_{dc} = \frac{I_{pdc}}{V_{gdc}} \text{ and } G_{out} = \frac{I_{out}}{V_{out}} \]

\( G_{dc} \) is the DC load conductance 'as seen' by the wind turbine generator. Its small signal representation is a result of changes in the duty cycles \( \alpha_1 \) on the boosting
mosfet $M_1$ and $\alpha_2$ on the chopping mosfet $M_2$. Figure 5.8 is the DC-DC booster block diagram reproduced from Chapter 4.

The duty cycle on the mosfet $M_1$, ($\alpha_1$) is set according to the compensator $G_{c3}$ acting on the difference between the desired current ($I_d$) and the generator current ($I_{gdc}$).

The duty cycle on the mosfet $M_2$, ($\alpha_2$) is set according to the compensator $G_{c3}$ acting on the difference between the desired voltage ($V_{ref}$) and the output voltage ($G_{vlag}V_{out}$) where $G_{vlag}$ is the attenuation of the potential divider $R_1$ and $R_2/C_1$ and the low pass filter in the voltage regulator circuit as described in section 4.5.2.

The change in load conductance $\delta G_{dc}$ is the summation of the outputs from the 2 compensators.

The generator voltage $V_{gdc}$ (labelled as $V_{in}$ in Chapter 4) can be approximated for
small signal analysis as being equal to a constant multiplied by the turbine speed minus the voltage dropped across the generators impedance. If $\alpha_1$ is held constant, then allowing $\alpha_2$ to vary will cause $G_{dc}$ to vary. Similarly if the output voltage $V_{out}$ varies it will lead to the input to $G_{c3}$ varying, the output of which is $\alpha_2$. A similar argument can be applied for $\alpha_1$, if $\alpha_2$ is held constant. Changes in the load conductance ($\delta G_{dc}$) are therefore attributable to changes in the outputs of both compensators $G_{c2}$ and $G_{c3}$. Figure 5.9 is the block diagram for the derivation of the change in load conductance.

The signal representing the desired current is derived by the PC software and is applied to the summing amplifier in the current regulator. There are 2 methods used for its derivation:

i) via a compensator $G_{c1}$ acting on the difference between the desired speed and the measured speed.

ii) as a function of the measured speed.

Both these alternatives will be investigated but the linearised controller will be developed for option i) since to implement option ii) will then merely involve
adapting the outermost loop by replacing the compensator $G_{c1}$ with a gain equal to the gradient of the derived relationship between speed and current at the chosen operating point.

The measured speed is the digital signal from the time interval measurement board (section 2.5.2).

The block diagram describing the wind turbine controller is shown in figure 5.10 with the input as the desired change in speed $\delta \omega$. The wind velocity $\delta v$ acts as a disturbance input since there is no control over it.

$G_{vlag}$ and $G_{ilag}$ (section 4.5.2) are the low pass filters used to reduce noise on the signals representing the output voltage and generator DC current respectively.

5.5 Derivation of Steady State Operating Points

To illustrate the linearised model a typical operating point is selected: $v_{op} = 8 \ m/s$ and $\lambda_{op} = 6.5$. The rotor speed can be calculated as:

$$\omega_{op} = \frac{\lambda_{op} \cdot v_{op}}{r_{ad}} = 30.4 \ rad/s$$

where $r_{ad}$ is the rotor radius = 1.71 m.

From section 3.5 the torque parameter $C_q$ was assumed to be directly related to $\lambda$ and at these operating points will be $C_q = 0.043$.

The torque coefficient $C_q = \frac{Q_a}{\frac{1}{2} \pi r_{ad}^3 v^3}$ is used to calculate a value for the aerodynamic torque as $Q_a = 25.7 \ Nm$. 

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Figure 5.10: Linearised Block Diagram
The frictional torque $Q_f$ at this speed can be calculated using equation 3.43. 

$$Q_f = f(\omega) = -10 \text{ Nm}$$

$Q_f$ is negative because the direction in which it is considered to act for the derivation of equation 3.43 is opposite to the direction in which it acts for this chapter. It therefore must be multiplied by -1.

Under steady state conditions there is no acceleration or deceleration and so the generator torque is equal to the difference between the aerodynamic torque and the frictional torque:

$$Q_g = Q_a - (-Q_f) = 15.6 \text{ Nm}$$

The generator rms current in one phase (assuming a balanced load) has been assumed (section 3.3.7) to be directly related to the generator torque by:

$$I_{grms} = \frac{Q_g}{g} = 1.73 \text{ A}$$

The stator impedance can be found from the values measured for the stator resistance and inductance from section 3.3.1 and $\omega_{op}$ as:

$$z_{st} = \sqrt{(r_{st})^2 + (4\omega_{op}l)^2} = 1.87 \text{ \Omega}$$

The generator phase voltage has been shown (section 3.3.5) to be proportional to the rotational speed:

$$e_{grms} = 3.\omega_{op} = 91.2 \text{ V}$$

The rms voltage per phase can be calculated as:

$$V_{grms} = e_{grms} - I_{grms}z = 88 \text{ V}$$

The generator DC voltage at the output of the 3 phase rectifier (neglecting the voltage drop across the diodes) is directly related to $V_{grms}$ (section 4.5.1, equation 4.52) as:

$$V_{gdc} = 2.3V_{grms} = 205 \text{ V}$$

The generator DC current has been shown (section 4.5.1, equation 4.53) to have a direct relation to the rms line current as:

$$I_{gdc} = 1.2I_{grms} = 2.1 \text{ A}$$

For a voltage output from the DC-DC booster of 400V the duty cycle $\alpha_1$ on the boosting mosfet $M_1$ has been found from section 4.2 to be

$$\alpha_1 = 1 - \frac{V_{gdc}}{V_{out}} = 0.49$$

The relation between the input and output currents of the DC-DC booster has been found to be:
\[ I_{\text{out}} = (1 - \alpha_1) \cdot I_{\text{plc}} = 1.1 \ A \]

The effective output resistance is directly related to the duty cycle \( \alpha_2 \) on the chopping mosfet as:

\[ R_{\text{out}} = \frac{R_b}{\alpha_2} = \frac{V_{\text{out}}}{I_{\text{out}}} \]

where \( R_b = 75\Omega \) is the resistance of the fire bar dump load.

Finally, \( \alpha_2 = \frac{R_b \cdot I_{\text{out}}}{V_{\text{out}}} = 0.2 \)

### 5.6 Linearised Coefficient Derivation and Evaluation

This section gives details on the derivation of each of the blocks of the linearised block diagram of figure 5.10. Having derived an equation for each block the steady state parameters calculated in section 5.4 for the operating points \( \nu_{op} = 8 \ m/s \) and \( \lambda_{op} = 6.5 \) are used to derive the transfer function that describes the linearised small signal relationship between its input and its output.

#### 5.6.1 Rate of change of aerodynamic torque with respect to wind velocity \( K_{Qav} \)

A change in wind speed will cause a change in the aerodynamic torque. If it is assumed that the angular velocity stays constant then the change in aerodynamic torque is equal to the product of the wind speed change and the constant \( K_{Qav} \).

\( K_{Qav} \) and \( K_{Qav} \) are determined by the shape of the \( C_q/\lambda \) curve and where on that curve (in particular which side of the peak) the machine is being operated.

\( K_{Qav} \) is calculated using the expression for the torque coefficient and the calculated gradient of the \( C_q/\lambda \) curve at the chosen operating point.

\[ K_{Qav} = \frac{\partial Q_a}{\partial \nu_{op}} \quad (5.5) \]

The wind turbine aerodynamics are characterised by two coefficients:-

i) torque coefficient \( C_q \)

ii) power coefficient \( C_p \)

These are plotted against the tip speed ratio \( \lambda = \frac{\omega R_{ad}}{\nu} \).
The torque coefficient is related to the square of the wind speed by:

$$C_q = \frac{Q_a}{\frac{1}{2} \pi \rho r_{ad}^3 v^2} = \frac{Q_a}{K_a v^2} \quad (5.6)$$

where $K_a = \frac{1}{2} \pi \rho r_{ad}^3 = 9.43$ for an air density $\rho = 1.2$. 

From equation 5.6

$$Q_a = K_a v^2 C_q$$

The gradient of the $C_q/\lambda$ curve is represented locally by the linear relationship

$$C_q = K_{1a} \lambda + c_1$$

so that

$$Q_a = K_a v^2 (K_{1a} \lambda + c_1) = K_a K_{1a} \lambda v^2 + K_a c_1 v^2 \quad (5.7)$$

Substitution of $\lambda = \frac{\omega r_{ad}}{v}$ into equation 5.7 gives

$$Q_a = K_a K_{1a} \omega r_{ad} v + K_a c_1 v^2 \quad (5.8)$$

Differentiation of equation 5.8 results in

$$\frac{\partial Q_a}{\partial v} = K_a K_{1a} \omega r_{ad} + 2 K_a c_1 v = K_{Qav} \quad (5.9)$$

The Matlab data file cqtsrj.mat contains 2 columns each with 66 rows. These represent the co-ordinates of points on the smoothed $C_q/\lambda$ curve as derived using the filtering and binning techniques described in section 3.5.1 using the NEL data. These are the co-ordinates used in the Simulink model unimod.m. They are also used in the Matlab script files coeff.m (listed in Appendix 0). Coeff.m has 2 input arguments $\omega_{op}$ and $v_{op}$. From these $\lambda_{op}$ (the variable $tsr_{op}$ in coeff.m) is calculated. (Alternatively $\lambda_{op}$ is given and $\omega_{op}$ calculated). A control loop is used and the variable $tsr$ is incremented until $tsr(n) > tsr_{op}$. The values of $tsr$ either side of $tsr_{op}$, $tsr(n+1)$ and $tsr(n-1)$ are then used as inputs to the Matlab script file polyfit.m which uses the least squares fit method to find the best fit through the 3 points.

Since $C_q = K_{1a} \lambda + c_1$. 

$\lambda_{op} = 6.5 \Rightarrow C_{qop} = 0.043$. Using coeff.m, this gives $K_{1a} = -0.0051$ and $c_1 = C_{qop} - K_{1a} \lambda_{op} = 0.0765$

Using equation 5.9 and the values found for $K_1, K_{1a}$ and $c_1$ gives

$$K_{Qav} = 9 \quad (5.10)$$
5.6.2 Rate of change of aerodynamic torque with respect to angular velocity $K_{Qaw}$

From figure 5.2 it is clear that if the wind velocity is constant, then a change in aerodynamic torque is attributable to the product of a change in angular velocity and the constant $K_{Qaw}$.

$$K_{Qaw} = \frac{\partial Q_a}{\partial \omega}$$

and so from equation 5.8

$$K_{Qaw} = K_a K_{1a} r_{ad} \psi_{op}$$

Using the values of $K_a$ and $K_{1a}$ derived in section 5.6.1

$$K_{Qaw} = -0.66$$

(5.11)

5.6.3 Rate of change of generator torque with respect to angular velocity $K_{Qgw}$

The linearised electro-mechanical model (figure 5.4) has 2 inputs, $\delta G_{ph}$ and $\delta \omega$. If $\delta G_{ph}$ is held constant then a change in angular velocity will result in a change in generator torque $\delta Q_g$ equal to $K_{Qgw} \delta \omega$. The constant $K_{Qgw}$ contributes to the time constant for the wind turbine block.

The electrical torque has been approximated as being proportional to the generator current. The relationship has been found by measurement (section 3.3.7) to
be: \( Q_g = 9 I_{\text{grms}} \) Nm

The electrical frequency is related to the angular velocity (mechanical frequency) by the number of pole pairs \( (n = 4) \). From the generator characterisation (section 3.3) the following values were determined:

stator impedance = \( z_{st} = \sqrt{(r_{st})^2 + (4\omega l)^2} \)

total impedance = \( z_t = \sqrt{(r_{st} + R_{ph})^2 + (4\omega l)^2} \)

where \( r_{st} = \) stator resistance = 1.6 \( \Omega \)

and \( l = \) stator inductance = 8 mH.

The rms current \( I_{\text{grms}} = \frac{e_{\text{rms}}}{z_t} \), where \( e_{\text{rms}} \) is the rms emf of the generator. Thus

\[
Q_g = 9 I_{\text{grms}} = \frac{27\omega}{\sqrt{(r_{st} + R_{ph})^2 + (4\omega l)^2}} \quad (5.12)
\]

Let \( Q_g = \frac{u}{v} \)

where \( u = 27\omega \) and \( v = \sqrt{(r_{st} + R_{ph})^2 + (4\omega l)^2} = \sqrt{k} \).

Thus \( \frac{\partial Q_g}{\partial \omega} = \frac{v \frac{\partial u}{\partial \omega} - u \frac{\partial v}{\partial \omega}}{u^2} \)

and \( \frac{\partial u}{\partial \omega} = 27 \)

hence \( \frac{\partial v}{\partial \omega} = \frac{\partial k}{\partial \omega} \frac{\partial v}{\partial k} = 32l^2\omega \cdot \frac{1}{\sqrt{(r_{st} + R_{ph})^2 + (4\omega l)^2}} = \frac{16l^2\omega}{\sqrt{(r_{st} + R_{ph})^2 + (4\omega l)^2}} \)

finally \( \frac{\partial Q_g}{\partial \omega} = \frac{\sqrt{(r_{st} + R_{ph})^2 + (4\omega l)^2} \cdot 27 - 27\omega \cdot \frac{16l^2\omega}{\sqrt{(r_{st} + R_{ph})^2 + (4\omega l)^2}}}{(r_{st} + R_{ph})^2 + (4\omega l)^2} \)

\[
= \frac{27(r_{st} + R_{ph})^2}{((r_{st} + R_{ph})^2 + (4\omega l)^2)^{\frac{3}{2}}} \approx \frac{27}{r_{st} + R_{ph}} = K_{Qgw}
\]

Knowing \( V_{\text{gdc}} \) and \( I_{\text{gdc}} \) means that the resistance as 'seen' at the input to the DC-DC booster can be calculated.

\( R_L = \frac{V_{\text{gdc}}}{I_{\text{gdc}}} = 97.6 \ \Omega \)

From section 4.5.1, \( G_{ph} = 1.92G_{dc} \) which gives \( R_{ph} = \frac{1}{1.92} R_{dc} = 50.5 \Omega \)

The constant \( K_{Qgw} \) can now be calculated;

\( K_{Qgw} = 0.52 \quad (5.13) \)

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5.6.4 Rate of change of frictional torque with respect to angular velocity $K_{Q_f \omega}$

$$K_{Q_f \omega} = \frac{\partial Q_f}{\partial \omega}$$ (5.14)

From section 3.3 the frictional torque was found to be related to angular velocity by

$$Q_f = 1.72 \times 10^{-3} \omega^2 - 0.247 \omega - 4.145$$

so $$\frac{\partial Q_f}{\partial \omega} = 3.44 \times 10^{-3} \omega - 0.247$$, and

$$K_{Q_f \omega} = -0.14$$ (5.15)

5.6.5 Wind Turbine Transfer Function $G_{WT}$

The results from the last 3 sections can be used to derive the transfer function for the combined aerodynamic and electro-mechanical models. The wind turbine steady state gain $K_{WT}$ is given in section 5.3 as equation 5.4:

$$K_{WT} = \frac{1}{K_{Q_g \omega} - K_{Q_a \omega} + K_{Q_f \omega}} = 0.97$$ (5.16)

and the wind turbine time constant, $\tau_{WT}$ is

$$\tau_{WT} = \frac{J_r}{K_{Q_g \omega} - K_{Q_a \omega} + K_{Q_f \omega}}$$ (5.17)

where $J_r = 8.7 \text{kgm}^2$ is the total moment of inertia. (Section 3.3.3). So at the chosen operating point $\tau_{WT} = 8.4 = \frac{1}{0.12}$.

These terms are combined as in equation 5.4 to give the wind turbine transfer function:

$$G_{WT} = \frac{K_{WT}}{1 + s(\tau_{WT})} = \frac{0.97}{1 + s(\frac{1}{0.12})}$$ (5.18)

5.6.6 Rate of change of output voltage with respect to angular velocity $K_{v \omega}$

$$K_{v \omega} = \frac{\partial V_{out}}{\partial \omega}$$ (5.19)

The output voltage will ideally be held constant at 400V through a combination of the DC-DC booster amplifying the generator voltage and the chopping mosfet...
applying an appropriate load. From figure 3.1, the output rms voltage is equal to the generator emf stepped down by the potential divider of $R_{ph}$ and $z$. Hence

\[
V_{\text{rms}} = e_g \left( \frac{R_{ph}}{R_{ph} + z} \right)
\]  
(5.20)

The generator emf has been shown to be directly related to the angular velocity (section 3.3.5).

For the linearised model, the gradient of the $V_{\text{gdc}}$ versus $\omega$ relationship at $\omega = \omega_{op}$ is required. This gradient will be affected by the amount of boost due to the duty cycle of mosfet $M_1$ (figure 5.8). The change in load conductance can be split into two parts: one attributed to the gain controlled by $G_{c2}$ and the other related to the load controlled by $G_{c3}$. $K_{vw}$ is calculated by taking the inner loop separately, that is, by assuming the duty cycle $\alpha_1$ is held constant at that value required to amplify the input voltage $V_{\text{gdc}}$ to 400V. Then using the ideal steady state relationship between $V_{\text{out}}$ and $V_{\text{gdc}}$ from equation 4.3:

\[
V_{\text{out}} = \frac{V_{\text{gdc}}}{1 - \alpha_1} = \frac{2.3V_{\text{rms}}}{1 - \alpha_1} = \frac{2.3 e_g (\frac{R_{ph}}{R_{ph} + z})}{1 - \alpha_1}
\]

Since $e_g = 3\omega$ at the operating point as described in section 5.5,

\[
V_{\text{out}} = \frac{6.9}{1 - \alpha_1} \left( \frac{R_{ph}}{R_{ph} + z} \right) \omega
\]

The gradient required is thus

\[
\frac{\partial V_{\text{out}}}{\partial \omega} = \frac{6.9}{1 - \alpha_1} \left( \frac{R_{ph}}{R_{ph} + z} \right) = \frac{6.9}{1 - \alpha_1} \left( \frac{1}{1 + zG_{ph}} \right)
\]

Where $G_{ph} = \frac{1}{R_{ph}} = \frac{1}{50.5} S$

For the operating points $\lambda_{op} = 6.5$ and $\nu_{op} = 8$ m/s the duty cycle $\alpha_1$ has been found (section 5.5) to be $\alpha_1 = 0.49$ and $z_{st} = \sqrt{(r_{st})^2 + (4\omega L)^2} = 1.87$. This gives the gain at the chosen operating point as:

\[
K_{vw} = 13
\]  
(5.21)

### 5.6.7 Low pass filter on voltage regulator $G_{vlag}$

As already described, smooth operation requires filtering of the measured voltage signal. The predominant frequency in $V_{\text{out}}$ is the AC component of the rectified input voltage. This frequency is proportional to the rotor speed as described in
section 4.5.1. $G_{uag}$ represents the gain between the output voltage and the input to the summing junction where $G_{uag} V_{out}$ is compared with $V_{ref}$. $\gamma$ is the action of the potential divider $R_1/(R_2/C_1)$ as described in section 4.5.2 (equation 4.61). From equation 4.63, $G_{uag} = -\gamma G_{VX1} = 16.8 \times 10^{-2}$. This block is considered as part of the compensator $G_{c3}$ which is dealt with in section 6.2.

5.6.8 Voltage Regulator Gain $K_{vreg}$

$K_{vreg}$ is the gain between the voltage regulator compensator output and the output of the pulse width modulator circuit. The gain between the PID output and the PWM input has been shown in section 4.5.2 to be $G_{vop} = 1.8$. The PWM (section 4.5.3) has been shown to have a gain of 0.1 (section 4.5.2). The duty cycle for $M_2$ is given by $a_2 = 0.18 G_{c3} V_{diff}$, and hence

$$K_{vreg} = 0.18$$ (5.22)

5.6.9 Rate of change of Load Conductance with Respect to the Duty Cycles $a_2$ and $a_1$. $K_{GLa2}$ and $K_{GLa1}$.

It has been shown how the load conductance can be seen as being attributable to the two duty cycles $a_1$ and $a_2$. The relative proportions accounted for by each one are determined from the respective partial derivatives of $G_{dc}$.

$K_{GLa2}$ is found by holding $a_1$ constant and finding the effect changing $a_2$ has on $G_{ph}$.

$$K_G_{La2} = \frac{\partial G_{dc}}{\partial a_2}$$ (5.23)

The load conductance $G_{dc}$ is that conductance as 'seen' at the output of the rectifier. Its value will be determined by the duty cycles present on the boost and load control mosfets, i.e.

$G_{dc} = \frac{I_{ph}}{V_{gdc}} = f (a_1, a_2)$

The output load conductance is that conductance 'seen' at the output of the DC-DC booster. This is dependent upon the duty cycle of the chopping mosfet. If this mosfet is permanently on, $(a_2 = 1)$, then $G_{out} = 1/R_6$ where $R_6$ is the resistance of the dump load. If the mosfet is permanently off, $(a_2 = 0)$, then $G_{out} = 0$ i.e. the
output is open circuit. In general:

\[ G_{out} = \frac{I_{out}}{V_{out}} = \frac{\alpha_2}{R_b} \]  

(5.24)

From section 4.2 (equation 4.3) the relation between the input and output voltages of the DC-DC booster was found to be:

\[ \frac{V_{out}}{V_{gdc}} = \frac{1}{1 - \alpha_1} \]  

(5.25)

and

\[ \frac{I_{out}}{I_{gdc}} = 1 - \alpha_1 \]  

(5.26)

\[ G_{dc} = \frac{I_{gdc}}{V_{gdc}} = \frac{I_{gdc}}{V_{out}(1 - \alpha_1)} = \frac{I_{out}}{(1 - \alpha_1) V_{out}(1 - \alpha_1)} \text{ which gives} \]

\[ G_{dc} = \frac{\alpha_2}{R_b} \frac{1}{(1 - \alpha_1)^2} \]  

(5.27)

and so

\[ K_{G_{L\alpha_2}} = \frac{1}{R_b (1 - \alpha_1)^2} \]  

(5.28)

For the steady state operating points chosen:

\[ K_{G_{L\alpha_2}} = 0.051 \]  

(5.29)

The gradient of the relationship between \( G_{dc} \) and \( \alpha_2 \) will therefore be dependent upon the duty cycle \( \alpha_1 \) as shown in figure 5.12.

\( K_{G_{L\alpha_2}} \) is calculated by holding \( \alpha_2 \) constant and finding the effect changing \( \alpha_1 \) has on \( G_{dc} \).

\[ K_{G_{L\alpha_1}} = \frac{\partial G_{dc}}{\partial \alpha_1} \]  

(5.30)

\[ G_{dc} = \frac{\alpha_2}{R_b} \frac{1}{(1 - \alpha_1)^2} \]  

(5.31)

Let \( u = \alpha_2 \) and \( v = R_b (1 - \alpha_1)^2 = R_b (1 - 2\alpha_1 - \alpha_1^2) \) so

\[ G_{dc} = \frac{u}{v} \]

\[ \frac{\partial G_{dc}}{\partial \alpha_1} = \frac{v \frac{\partial u}{\partial \alpha_1} - u \frac{\partial v}{\partial \alpha_1}}{v^2} \]

where \( \frac{\partial u}{\partial \alpha_1} = 0 \) and \( \frac{\partial v}{\partial \alpha_1} = R_b (-2 - 2\alpha_1) = -2R_b (1 + \alpha_1) \) and so

\[ K_{G_{L\alpha_1}} = \frac{2R_b \alpha_2 (1 + \alpha_1)}{R_b^2 (1 - \alpha_1)^4} = \frac{2\alpha_2 (1 + \alpha_1)}{R_b (1 - \alpha_1)^4} \]  

(5.32)
Figure 5.12: Load Conductance as a function of Duty Cycle $\alpha_2$ for $\alpha_1 = 0.2$ to 0.7 in 0.1 steps. Drawn using the Matlab script file glal2.m listed in Appendix 5.2.

For the operating points chosen:

$$K_{GL\alpha_1} = 0.12$$  

(5.33)

The gradient of the relationship between $G_{dc}$ and $\alpha_1$ will therefore be dependent upon both the duty cycles $\alpha_1$ and $\alpha_2$ as shown in figure 5.13.

5.6.10 Current Regulator Gain $K_{ireg}$

$K_{ireg}$ is the gain from the current PID compensator output to the duty cycle $\alpha_1$ at the boosting mosfet $M_1$. It includes the output op amp and the pulse width modulation circuit. These can be regarded as a single gain. From section 4.5.2 $G_{IX11} = -1$. The magnitude of $K_{ireg}$ is therefore equal to the PWM gain.

$$K_{ireg} = -0.1$$
Figure 5.13: Load Conductance as a function of Duty Cycle $\alpha_1$ for $\alpha_2 = 0.1$ to 1 in 0.1 steps. Drawn using the Matlab script file glal1.m listed in Appendix 5.3.

5.6.11 Low pass filter on current regulator board $G_{ilag}$

As with the voltage regulator, the current control also requires that its input be filtered. $G_{ilag}$ represents the transfer function between $K_i Q_g \delta Q_g$ and $\delta I_g$ (figure 5.10). On the current regulator board it comprises the filters and amplifiers X1 to X4 (including the opto-isolator) that operate on the signal derived from the voltage drop across the sensing resistor $r_s$ (Appendix G). Its output is representative of the generator DC current at the input to the difference op amp X6.

The gain for the current regulator circuit comprising X1, X2, X3, the opto isolator and X4 is equal to

$$G_{ilag} = \frac{31}{1 + s\tau_{IX2}} \quad (5.34)$$

From section 4.5.2 a preliminary value for $\tau_{IX2} = \frac{1}{1 \times 10^5}$ is chosen but this may be modified at a later stage in the design process.
5.6.12 Rate of change of generator DC current with respect to generator torque $K_{iQg}$

This response relates to the feedback from $\delta Q_g$ to $\delta I_{gdc}$ shown in figure 5.10

$$K_{iQg} = \frac{\partial I_{gdc}}{\partial Q_g}$$

(5.35)

The generator current per phase $I_{g rms}$ has been found from section 3.3.7 to be:

$I_{g rms} = \frac{Q_g}{g} \text{ A}$

and from section 4.5.1

$I_{gdc} = 1.2I_{g rms}$, hence

$$K_{iQg} = \frac{\partial \left( \frac{1.2Q_g}{g} \right)}{\partial Q_g} = 0.13$$

(5.36)

5.6.13 Rate of change of generator torque with respect to load conductance $K_{QgG}$

$$K_{QgG} = \frac{\partial Q_g}{\partial G_{dc}} = \frac{1}{1.92} \frac{\partial Q_g}{\partial G_{ph}}$$

(5.37)

From equation 5.12

$$Q_g = \frac{27\omega}{\sqrt{(r_{st} + R_{ph})^2 + (4\omega l)^2}}$$

hence

$$\frac{\partial Q_g}{\partial G_{ph}} = \frac{v\frac{\partial G_{ph}}{\partial G_{ph}} - u\frac{\partial v}{\partial G_{ph}}}{v^2}$$

(5.38)

where $u = 27\omega \Rightarrow \frac{\partial u}{\partial G_{ph}} = 0$

and $v = \sqrt{(r_{st} + R_{ph})^2 + (4\omega l)^2}$

Substituting into equation 5.38 and rearranging gives

$$\frac{\partial Q_g}{\partial G_{ph}} = \frac{27\omega \left( r_{st} + \frac{1}{G_{ph}} \right)}{G_{ph}^2 \left[ \left( r_{st} + \frac{1}{G_{ph}} \right)^2 + (4\omega l)^2 \right]}$$

(5.39)

At the chosen operating point this results in

$$K_{QgG} = 401$$

(5.40)
A table of parameter values and gains, for \( v = 6 \), 8 and 10 m/s and for tip speed ratios \( \lambda = 5 \) and 6.5 is given below.

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Chapter 6

Controller Design

6.1 Introduction

This chapter deals with the selection of sets of parameters for the 3 compensators $G_{c1}$, $G_{c2}$ and $G_{c3}$ using the linearisation method described in Chapter 5 and the resulting linearised block diagram of figure 5.10.

The Matlab script files used in the design process are listed in Coeff.m (Appendix O). These determine the steady state characteristics at a chosen operating point. $Gc1desa.m$, $gc2desa.m$ and $gc3desa.m$ (Appendix O) are the script files for each compensator design. Parameters shown in italics are those used in these programs. Each compensator produces a signal that is derived as a result of the difference between a reference and a measured variable.

$G_{c1}$ is a software compensator implemented on the control PC. It acts on the difference between the measured speed, which is the digital output from the time interval measurement circuit, and a reference speed which can either be set as a constant or calculated as a result of the measurement of other variables. The output of $G_{c1}$ is provided by an analogue output from the PC and represents the demanded current ($I_d$). In general, if the measured speed drops below the reference speed then the demanded current should decrease thereby decreasing the load on the machine and allowing it to accelerate.

$G_{c2}$ is a hardware compensator. Its construction is explained in detail in section 4.5.2. $G_{c2}$ acts on the difference between the demanded current as derived from
$G_{cl}$ and a filtered representation of the generator current as measured from the voltage drop across the sensing resistor $r_s$. The output from $G_{c2}$ is used to set the duty cycle ($\alpha_1$) on the booster mosfet $M_1$. In general, an increase in wind speed will result in an increase in rotor speed. The demanded current should increase and if it becomes greater than the generator current then the DC-DC booster will increase its amplification and the output voltage will increase. This will in turn cause the output load conductance to increase.

$G_{c3}$ is a hardware compensator built on similar lines to $G_{c2}$ and described fully in section 4.5.2. It acts on the difference between an attenuated / filtered version of the output voltage and a reference voltage which is set by a potentiometer and therefore stays constant. The output of $G_{c3}$ is used to set the duty cycle ($\alpha_2$) on the chopper mosfet $M_2$ and is therefore representative of the output load conductance.

If the output voltage increases then the duty cycle $\alpha_2$ of the chopper mosfet will increase thereby increasing the effective conductance. This will lead to the output voltage decreasing since the turbine speed decreases due to the additional load.

The model used is a linearised one and so it should be kept in mind that the resulting set of compensator parameters will only be applicable at the operating point around which the model was linearised. Account should be taken of this in the design of $G_{cl}$ since, as a software compensator, it can easily be designed to adapt its parameters in some prescribed manner according to the operating point. The hardware compensator parameters are fixed and so their determination must take into account the broad range of operating conditions under which these compensators are expected to function.

Dealing with a system with 3 separate compensators presents some difficulty in that adequate steps must be taken to ensure that they will work together effectively. This can be achieved, even though the first compensator to be dealt with must initially be designed in isolation to the other two. The design process must therefore be an iterative one. Once the parameters for all 3 compensators are determined the overall system must be considered and if necessary the steps repeated until a satisfactory solution is attained. In addition a further iterative procedure must be adopted to ensure satisfactory operation under all feasible op-
erating conditions, both above and below the turbine rated power and both on and off the ideal operating points defined by $\lambda = \lambda_{opt}$.

If $G_{e3}$ is dealt with first, the voltage regulator loop can be represented as a single block. Moving the takeoff point at $\delta Q_g$ to the other side of $G_{WT}$ in figure 5.10 would then mean that the system could be represented as a unity feedback system with the demanded current as input and the generator current as output. The compensator $G_{e2}$ can then be designed and the system represented again as a standard feedback system with the velocity transducer in the feedback path and demanded speed as the input and actual speed as the output. The compensator $G_{e1}$ can then be designed with the discrete nature of the sampling process being taken into account.

In general the design process represents a tuning of the compensatory action to the inherent dynamic characteristics of the wind turbine and DC-DC booster. High frequency components are not generally encountered and so attempting to control the machine with rapid changes in load conductance may be counter productive in terms of high frequency torque components and resulting fatigue problems. In addition if the machine is to be eventually grid-linked, problems may be experienced by the inverter with the quality of the generated waveform.

6.2 Voltage Regulator Compensator Design

The voltage regulator compensator refers to the chain of circuitry from the output of the summing op amp to the input signal to the pulse width modulator. It includes some gain and non-linearities primarily due to the opto-isolator. Details on these can be found in section 4.5.2. These are ignored throughout most of the design process as their effects will be shown to be insignificant when compared to the inaccuracies brought about by the errors involved in the estimation of system parameters.

Figure 6.1 shows the voltage regulator loop rearranged from figure 5.10 with the voltage reference as input and the generator voltage as output. The wind velocity acts as a disturbance and the resulting aerodynamic torque is summed with the generator torque before acting as an input to the wind turbine block $G_{WT}$. 
The input $\delta V_{ref}$ is negative (equation 4.63) so the change in duty cycle $\delta \alpha_2$ on the chopping mosfet is

$$\delta \alpha_2 = K_{vreg} G_{c3} (G_{vlag} \delta V_{out} - \delta V_{ref})$$

where $G_{vlag} = -\gamma G_{VX1} = \frac{16.8 \cdot 10^{-3}}{1 + \frac{1}{43}}$

The duty cycle on the chopper mosfet should decrease as a result of a decrease in $\delta V_{out}$ since this will decrease the load conductance. Similarly the duty cycle should increase as a result of a decrease in $\delta V_{ref}$.

Making the input positive and then changing the sign of the $\delta Q_g$ input means that the block diagram may be rearranged and simplified as in figure 6.2.

If $\delta V_{out} \neq 0$, the compensators $G_{c3}$ and $G_{c2}$ must react in such a way so as to reduce $\delta V_{out}$ towards zero so as to avoid stressing the power electronic components and the wind turbine. In isolating $G_{c3}$ it must be assumed that the contribution to the change in load conductance of the booster mosfet is zero, i.e. $\delta G_{L1} = 0$.

$K_F = K_{vreg} K_{GL1} K_{QG} = 3.7$ at the operating point used in Chapter 5.

The objective for the voltage regulator compensator is to keep the DC-DC booster output voltage constant at a required value as set by $V_{ref}$. The loop can therefore be considered as a tracking or regulator control system. Under normal conditions $\delta V_{ref} = 0$ since it is set by a potentiometer.

Classical root locus and bode plot design procedures are used. In addition to tracking $V_{ref}$ the system must reject disturbances from $\delta v$ and prevent them from...
Figure 6.2: Simplified Linearised Block Diagram of Voltage Regulator Loop

being transmitted to the output. To judge how effectively these two objectives are achieved by a particular compensator, the closed loop frequency response plots of the two transfer functions are plotted. In addition, the root locus is used to give an indication of the time response.

Although ideally a fast response is required, this is usually associated with a large overshoot unless some derivative action is used. The key problem associated with derivative action is one of noise amplification. There will be a component of the generator DC voltage (ripple) with a frequency that is representative of the rotor speed. The controller is expected to regulate down to a turbine speed of 20 rad/s which corresponds to an electrical frequency of 80 rad/s. The bandwidth generally associated with wind speed variation is of the order of 10 rad/s. To achieve a balance between retaining control up to 10 rad/s and rejecting frequency components greater than 80 rad/s means the low pass filter $G_{vlag}$ is chosen to have a 3dB point at 50 rad/s. The component values chosen in section 4.5.2 gave a 3dB point at 46 rad/s.

$$G_{vlag} = \frac{16.8 \times 10^{-3}}{s \left(\frac{1}{46}\right) + 1} = \frac{g_{vlagnum}}{g_{vlagden}}$$

(6.1)

It has been explained in Chapter 4 how the devices used in the DC-DC booster are selected as a compromise between excessive cost and power rating. Each device

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used is capable of withstanding a 50% over voltage or over current surge when the machine is operating at rated power. This means that it is critical that the damping is kept at such a value that there is never any possibility of more than a 20% overshoot. This implies, for example, that $V_{\text{out}}$ must never exceed 440V. The compensator must be designed with this in mind and if necessary some sacrifice in transient performance (rise time) may be made to ensure low values of overshoot.

Figure 6.2 is a multiple input, single output system. The input $\delta V_{\text{ref}}$ is the preset voltage that the output is required to track. The input $\delta v$ is the change in wind velocity. The output voltage is required to stay constant in spite of variations on this input and so it is considered as a disturbance.

Deriving a closed loop transfer function for each input and plotting its magnitude and phase versus frequency will give an indication of the tracking and disturbance rejection properties of the selected compensator.

For the uncompensated loop at the chosen example operating points there are 2 poles at $s = -0.12$ (from the wind turbine, section 5.6.5, equation 5.18) and at $s = -46$ (from the low pass filter, section 4.5.2, equation 4.6.1). The root locus will therefore consist of the loci leaving the poles and moving along the real axis towards each other until at $K_{p3} = 160$ they break away and move vertically towards $\pm\infty$. This indicates that for the proportional term there is no value of gain for which the system will become unstable and that for $K_{p3} = 160$ the loop is critically damped. Such a high value of gain would result in the system
behaving as a 'bang bang' control system i.e. the chopper mosfet duty cycle will tend to saturate at either of its extreme values. The root locus technique does not take this effect into account but for small signal analysis it nevertheless gives an indication of system performance. The characteristic equations for figures 6.2 and 6.3 are identical and so their root loci will be identical.

Ideally, the signal representing the output voltage should be equivalent to the reference voltage and so for a zero steady state error some integral action is required. The generalised transfer function for a P+I compensator is:

$$G_{c3} = \frac{K_{i3} \left(1 + s \left(\frac{K_{p3}}{K_{i3}}\right)\right)}{s}$$

(This is a modified version of equation 4.57).

Its inclusion will place a pole at the origin ($s = 0$) and a zero at $s = -\frac{K_{i3}}{K_{p3}}$.

The P+I compensator will move the breakaway point to $-0.12 < s_{brk} < 0$. The result will be a faster response than a purely proportional controller with some overshoot. This is because the added integral term is positioned in the feedback loop (figure 6.3).

The objective is now reduced to deciding on values for $K_{i3}$ and $K_{p3}$, i.e. on where to place the compensator zero and the value of gain to apply. To improve the transient response, the zero must be placed to the left of the wind turbine pole at $s = -0.12$. The effects of a P+I compensator can best be seen using a root locus diagram. The advantage of using root locus analysis is that for a chosen operating point the effects of gain variation is easily seen. The overall effect of 3 separate zero positions for a range of $K_{p3}$ can therefore be viewed on a single diagram. Figure 6.4 is the locus of the roots of the characteristic equation for 3 compensator zeros at $s = -0.5$, $s = -1$ and $s = -2$ as $K_{p3}$ is varied between $1.10^{-4}$ and 1 over 20 logarithmically spaced intervals. The breakaway point for the 3 example compensator positions all occur at approximately $K_{p3} = 0.01$. The loci then swing to the left with the angle of departure being determined by the position of the compensator zero. The damping ratio is the cosine of the angle between the real axis and a point on the root locus. As the zero moves towards the wind turbine pole the angle of departure increases and the point at which the root locus breaks away from the real axis moves further from the origin. This is because
Figure 6.4: Voltage Regulator Root Locus for 3 Zero Positions

the zero is tending towards cancelling the wind turbine pole. Ideally the gain will be set so as any movement along the loci due to system uncertainties will result in the damping ratio and the overshoot remaining constant. The damping ratio is dependent upon both the zero location and the gain. For a selected zero location as the gain is increased the damping ratio decreases to a minimum (determined by the zero location) and then increases as the gain is further increased. The gain will ideally be set so as the dominant roots lie at a point on the root loci that is touched by a straight line drawn from the origin. The overshoot $M_p$ and the damping are related by the standard expression:

$$M_p = e^{\frac{-\zeta \pi}{2}}$$

where $\zeta$ is the damping ratio.

To keep the overshoot to less than 20% therefore the damping ratio must be greater than 0.45. A rule of thumb method is to use this limit to specify how close to the wind turbine pole the compensator zero can be placed. If the compensator zero is placed to the right of the wind turbine pole then the root locus will move in the opposite direction moving into the right hand plane as the gain is increased resulting in an unstable system. The compensator zero position is a compromise between system robustness and fast response. Using the Matlab
script file gc3des.m, the compensator zero can be moved towards the wind turbine pole until the minimum damping ratio is equal to 0.45. Once the position of the zero is found, the gain for the point on the root locus which is tangential to a line drawn from the origin can be found. Choosing a zero position of $s = -0.6$ results in a minimum damping ratio of $\zeta = 0.46$. The gain at which this minimum damping ratio occurs is $K_{\alpha3} = 0.23$. The result is a P+I compensator with a transfer function:
$$G_{\alpha3} = \frac{0.23(1 + s(\frac{1}{s}))}{s}.$$
This results (for the operating points chosen) in a pair of dominant poles at $s = -0.08 \pm j0.12$ which gives a damping ratio of $\zeta = 0.46$.

The open loop transfer function bode plot is equivalent to that of a low pass filter with the 3dB point at $\omega = 0.17 \text{ rad/s}$. Adding a compensator with the parameters set using the root locus method results in a system with an infinite open loop DC gain which will give a zero steady state error and an increased bandwidth which gives a faster response.

Figure 6.5: Bode Plot of Sensitivity Function for Voltage Regulator Loop for uncompensated (0) and compensated (x).

The 2 closed loop transfer functions for this loop are known conventionally as the sensitivity function $S_{\text{vreg}}$ and the complementary sensitivity function $T_{\text{vreg}}$. The
open loop transfer function is labelled $L_{v_{\text{reg}}}$.

The model can be written as:

$$\delta V_{\text{out}} = G_{31}(s) \delta V_{\text{ref}} + G_{32}(s) \delta v$$

where $S_{\text{v_{reg}}} = G_{32}(s)$ is the transfer function between the output $\delta V_{\text{out}}$ and the wind disturbance $\delta v$ and $T_{\text{v_{reg}}} = G_{31}(s)$ is the transfer function between the output $\delta V_{\text{out}}$ and the voltage reference input $\delta V_{\text{ref}}$.

$$T_{\text{v_{reg}}} = G_{31}(s) = \frac{\delta V_{\text{out}}}{\delta V_{\text{ref}}} = \frac{G_{c3} K_F K_{vw} G_{WT}}{1 + G_{c3} K_F K_{vw} G_{WT} G_{v_{lag}}}$$

$$S_{\text{v_{reg}}} = G_{32}(s) = \frac{\delta V_{\text{out}}}{\delta v} = K_{Q_{av}} \frac{K_{vw} G_{WT}}{1 + G_{c3} K_F K_{vw} G_{WT} G_{v_{lag}}}$$

As noted earlier, $G_{31}(s)$ and $G_{32}(s)$ have the same denominator, that is they have the same characteristic equation. The numerators differ in that the zeros of the transfer function $S_{\text{v_{reg}}}$ come from the denominator of the compensator, $gc3den$, whilst the zeros of the transfer function $T_{\text{v_{reg}}}$ come from the numerator, $gc3num$. This is because the compensator is in the forward path for $T_{\text{v_{reg}}}$ and in the feedback path for $S_{\text{v_{reg}}}$.

The sensitivity functions before compensation (marked with an 'o') and after compensation (marked with an 'x') are shown in figure 6.5. At low frequencies the gain is low indicating that disturbance rejection has benefited from compensatory action in that the effects of the wind disturbance on the output voltage have been diminished. A further result of compensatory action is a $90^\circ$ phase lead. At frequencies above 1 rad/s the 2 plots become the same which means that at these higher frequencies the intrinsic inertial characteristics of the wind turbine are dominant.

The complementary sensitivity function (figure 6.6) also shows an improvement as a result of compensation. The low frequency gain has increased as has the bandwidth, both of which indicate that an improvement in the systems performance has been achieved.

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6.2.1 Representing the Voltage Regulator Loop as a Single Block

The voltage regulator loop can be represented by the transfer function:

\[ G_{\text{vreg}} = \frac{\delta \omega}{\delta G_{L1}} \]

Referring to figure 5.10.

\[ G_{\text{vreg}} = \frac{K_{O}G \cdot G_{WT}}{1 + K_{O}G \cdot G_{WT} \cdot K_{GLa2} \cdot G_{vreg} \cdot G_{c3} \cdot G_{vlag} \cdot K_{vw}} \]

Let \( G_{WT} = \frac{gwt\text{num}}{gwt\text{den}}, \ G_{c3} = \frac{gc3\text{num}}{gc3\text{den}}, \ G_{vlag} = \frac{gvlag\text{num}}{gvlag\text{den}} \)

and let \( G_{vreg} = \frac{gvreg\text{num}}{gvreg\text{den}1 + gvreg\text{den}2} \)

so

\( gvreg\text{num} = K_{O}G \cdot gwt\text{num} \cdot gc3\text{den} \cdot gvlag\text{den} \)

\( gvreg\text{den}1 = gwt\text{den} \cdot gc3\text{den} \cdot gvlag\text{den} \)

\( gvreg\text{den}2 = K_{O}G \cdot gwt\text{num} \cdot K_{GLa2} \cdot G_{vreg} \cdot gc3\text{num} \cdot gvlag\text{num} \cdot K_{vw} \)

Using the evaluated variables for the operating points \( v_{op} = 8 \text{ m/s} \) and \( \lambda_{op} = 6.5 \)

and the compensator \( G_{c3} \) as designed in section 6.2 results in
The linearised system block diagram of figure 5.10 can be manipulated as follows. Move the takeoff point at \( \delta Q_g \) to the other side of \( G_{WT} \) and therefore include within the feedback path a block with the transfer function \( \frac{1}{G_{WT}} \). Replace the inner loop transfer function \( G_{\text{reg}} \) with the single block \( G_{vreg} \). Combine \( K_{\text{ireg}} \) with \( G_{c2} \). Figure 6.7 is the resulting revised linearised system block diagram.

Taking the inner loop of figure 6.7 and rearranging so that the generated change in current is at the output results in figure 6.8.

\[
G_{vreg} = \frac{16s^2 + 728.6s}{s^3 (0.177) + s^2 (8.05) + s (1.44) + 0.74}
\]
6.3 Current Regulator Compensator Design

The current regulator compensator refers to the chain of circuitry between the output of the summing op amp and the input signal to the pulse width modulator. It sets the duty cycle on the booster mosfet according to the difference between the demanded current and the generated DC current.

Figure 6.8 is the linearised block diagram of figure 5.10 with the voltage regulator replaced with $G_{vreg}$ as derived in section 6.2.1.

The fundamental objective for the current regulator loop is to keep the generated DC current equal to the demanded current. This objective can only be satisfied if the current that is demanded is a realistic one for the conditions pertaining. Obviously the system can not achieve any significant current if there is no wind available.

For the linearised system, setting the wind speed and the rotor speed at specified operating points means that for a step change in demanded current there must be a steady state change in turbine speed (assuming the wind speed stays constant). Under normal working conditions however there would not be a change in demanded current unless the wind speed changed, causing the rotor speed to begin to change.

Design of the current regulator loop is therefore made under the premise that the demanded current is a valid one i.e. that the machine is capable of achieving the demanded current and delivering the prescribed amount of power.

To satisfy the fundamental objective ($\delta I_{gdc} = \delta I_d$) the linearised block diagram of figure 6.8 can be rearranged as figure 6.9 by expressing the forward path blocks as a single block. This represents a unity feedback system with the compensator acting on the error signal $\delta I_{diff} = \delta I_d - \delta I_{gdc}$.

With the compensator $G_{c2} = 1$; a step increase in $\delta I_d$ can be thought of as being instantaneously translated as a step increase in load conductance. Then it is evident that the demanded current will be achieved instantaneously followed by an exponential decay as the machine slows down with a time constant representing the inertia of the wind turbine. For a constant wind speed a step decrease in $\delta I_d$ resulting in a step decrease in load conductance will cause the generator current to exponentially decay. For the uncompensated current loop therefore there will be
Figure 6.9: Single Block Current Regulator

a steady state error. Intuitively this can be explained by the fact that a positive change in $\delta I_d$ allows the release of energy stored in the machine inertia whilst a negative change means that energy is being absorbed by the inertia. The rate at which energy is released is determined by the demanded current change whilst the rate at which energy is absorbed is determined by the wind speed.

The control objective is therefore the reverse of the classical compromise of improving response time whilst avoiding large overshoots. The objective for this compensator is more one of avoiding inflicting excessively high frequencies onto $\alpha_1$ as a result of demanded current change which if satisfied would stress the DC-DC booster mosfet.

At issue is whether to avoid responding to high demands in current with the design of $G_{e2}$ or to avoid demanding those high currents in the first place with the design of $G_{e1}$. In practice both these issues will be addressed.

Assuming that a step in $\delta I_d$ leads directly to a step in $\delta G_{dc}$ implies that the dynamics of the DC-DC booster are ignored. Although they will be at a much higher frequency when compared with the wind turbine dynamics, their effects are still important in so far as the life-span of the switching devices are concerned. For example, consider a step increase in $\delta I_d$ resulting in the duty cycle on the boosting mosfet becoming saturated at $\alpha_1 = 0.75$. The voltage gain is $\frac{V_{out}}{V_{dc}} = 4$ and the output voltage will increase according to the dynamics of the DC-DC booster. If these were such that the system was lightly damped then the output voltage would ordinarily overshoot if the chopping mosfet duty cycle were held constant. If the voltage regulator is operating correctly then the dynamics of the DC-DC booster would be reflected onto the generator current. Chapter 4 dealt with the
DC-DC booster dynamics and it was found that the system is lightly damped for all values of load conductance. This emphasises the necessity of avoiding high frequency components in the demanded current.

From figures 6.8 and 6.9

\[ G_{ifor} = K_{GLO1}.K_{iQf}. \frac{gvregnum.gwtden.gilagnum}{gvregden.gwtnum.gilagden} \]

\[ = \frac{gifornum}{giforden} \]

Inversion of the wind turbine block means that the pole at \( s = -0.12 \) in the voltage regulator loop becomes a zero in the current regulator loop. In addition to this zero, there is a zero at the origin from the \( G_{vreg} \) block since the compensator \( G_{c3} \) has a pole at the origin and is in the feedback path from \( \delta \omega \) to \( \delta G_{L1} \) and therefore acts as a zero in the closed loop transfer function. There is also a pair of complex poles at \( s = -0.09 \pm j0.29 \) (from \( gvregrden \)) and a pole on the negative real axis from \( gilagden \).

At the chosen operating point, using the compensator parameters for \( G_{c3} \) (section 6.2), the frequency domain representation of \( G_{ifor} \) is

\[ G_{ifor} = \frac{s (2.458 s^2 + 111.7 s + 13.9)}{0.0017 s^4 + 0.244 s^3 + 7.82 s^2 + 19.5 s + 30.35} \]

Expressing figure 6.9 as a closed loop transfer function:

\[ G_{ireg} = \frac{\delta I_{pred}}{\delta I_d} \]

\[ = \frac{G_{c2}.G_{ifor}}{1 + G_{c2}.G_{ifor}} \]

\[ = \frac{gc2num.giforum}{gc2den.giforden + gc2num.giforum} \]

\[ = \frac{igidnum}{igidden1 +igidden2} = \frac{igidnum}{igidden} \]

The characteristic equation is the denominator of the closed loop transfer function \( G_{ireg} \):

\[ K_1 gc2den.gvregrden.gilagden + K_2 gc2num.gvregrnum.gwtden = 0 \]

where \( K_1 = gwtnum \), and \( K_2 = K_{GLO1}.K_{iQf}.gilagnum \)

At low values of gain the roots are located at the poles \( gc2den \), \( gvregrden \), and \( gilagden \). As the gain is increased the root loci follow paths dictated by the
zeros gc2num, guregnum and gwtden. Figure 6.10 is the root locus for the uncompensated system as the gain $K_{p2}$ (the steady state gain of $G_{c2}$) is varied at logarithmically spaced intervals in 10 steps between 0.01 and 100.

The roots start at the pair of complex poles at $s = -0.13 \pm j0.25$. The departure angles are $\pm 30^\circ$. They meet the real axis for $K_{p2} = 1$ and then move in opposite directions towards the zeros at $s = 0$ and $s = -0.13$. This indicates firstly that the system is stable and secondly that, as the gain is increased, the damping increases, becoming critical at $K_{p2} = 1$.

Figure 6.11 is the corresponding open loop frequency response.

The shape is that of a high pass filter with a 3dB point of 0.7 rad/s. At low frequency the gain is low which corroborates the intuitive explanation given earlier in this section. For $G_{c2} = 1$ (uncompensated) there will be a steady state error in that the generator current can never be equal to the demanded current. At high frequency the gain is constant at 25dB with a $0^\circ$ phase difference.

Placing a lag compensator in series with $G_{ij0r}$ will level the open loop gain at low frequencies and leave the higher frequencies unaffected. There will still be some steady state error and possibly some problems with noise. Elimination of the steady state error will involve placing some integral action in series with a lag
filter. This will achieve high gain at low frequency with the gain rolling off at the higher frequencies. The integrator will place a pole at the origin cancelling the zero and the phase lag will place another pole on the real axis causing the angle of departure from the complex poles to change by approximately $180^\circ$ depending on where the phase lag pole is placed. The integrator should have a response which falls of at $-20 \cdot K_i$ dB per decade with a crossover frequency of $\omega = 1 \text{ rad/s}$.

Using the bode diagram of figure 6.11 and designing a compensator to achieve a high open loop gain up to a crossover at $\omega = 1 \text{ rad/s}$ and then rolling off at $20 \text{ dB/decade}$ will result in a compensator:

$$G_{c2} = \frac{1}{s} \frac{1}{s \left( \frac{1}{0.1} \right) + 1}$$

The resulting root locus is shown in figure 6.12 with the gain $K_{p2}$ being varied as before.

The roots now start at the lag pole at $s = -0.1$ and the complex pair of system poles. The roots move along the real axis towards the system zero at $s = -0.13$ and away from the complex poles towards $\pm \infty$. The loci are not asymptotic; that is they will cross into the right hand plane resulting in an unstable system for $K_{p2} > 25$. This situation is unacceptable. Although zero steady state errors
are very desirable they are not worth risking the possibility of instability for. Although theoretically, stability can be achieved by keeping the gain low, there is nevertheless some risk remaining due to parameter uncertainty, and the possibility of the machine operating away from its ideal operating point. Consequently a straightforward lag compensator is preferable. Figure 6.13 is the resulting root locus plot for a lag compensator with its pole at \( s = -0.1 \).

\[
G_{c2} = \frac{K_{p2}}{s \left( \frac{1}{0.1} \right) + 1}
\]  

(6.2)

One root locus starts at the pole at \( s = -0.1 \) and moves towards the zero at the origin. The second and third loci start at the pair of complex poles and move away with a departure angle of \( \pm 120^\circ \) and meet the real axis at \( s = -0.35 \) for \( K_{p2} = 0.3 \) before moving in opposite directions towards the zeros at \( s = -0.13 \) and \( s = -46 \). The forth locus starts at the pole at \( s = -100 \) and moves away towards infinity. The system is now stable for all values of gain.

The transient effects of varying \( K_{p2} \) will be negligible due to the dominant pole between the origin and \( s = -0.1 \). For a reasonably low steady state error, a high value of gain is desirable. For a reasonably fast response a low value of gain is desirable. There will never be zero steady state error because \( G_{c2} \) is a simple
low pass filter. Choosing $K_g2 = 10$ reflects a compromise between excessively sluggish response and a low steady state error. The choice is made on the basis that a slow response is desirable to avoid high frequency components in the load conductance as is a low steady state error, bearing in mind that the wind speed is never constant.

6.3.1 Representing the Current Regulator Loop as a Single Block

The current regulator loop can be represented by the transfer function:

$$G_{wid} = \frac{\delta \omega}{\delta I_d}$$

Figure 6.7 can be represented by figure 6.14 with $G_{wid}$ representing the current regulator loop. Using figure 6.8:

$$G_{wid} = \frac{G_{c2} \cdot K_{GLa1} \cdot G_{vreg}}{1 + G_{c2} \cdot K_{GLa1} \cdot G_{vreg} \cdot K_w \cdot G_{diag} \left( \frac{1}{G_{WT}} \right)}$$

$$= \frac{K_{GLa1} \cdot \frac{gc2num \cdot gvregrunm}{gc2den \cdot gvregruden}}{1 + K_{GLa1} \cdot K_{wt} \cdot \frac{gc2num \cdot gvregrunm \cdot gilagrunm \cdot gwtden}{gc2den \cdot gvregruden \cdot gilagden \cdot gwtum}}$$

Figure 6.13: Compensated Root Locus. Lag Only.
Figure 6.14: Simplified Linearised System Block Diagram

\[ \frac{gwidnum}{gwidden1 + gwidden2} \]

\[ \frac{gwidnum}{gwidden} \]

where:
\[ gwidenum = K_{GLa1}.gc2num.gvregnum.gilagden.gwtnum \]
\[ gwidden1 = gc2den.gvregden.gilagden.gwtnum \]
\[ gwidden2 = K_{GLa1}.K_{i,Qr}.gc2num.gvregnum.gilagnum.gwtden \]

For the compensators chosen for \( G_{c3} \) and \( G_{c2} \) at \( v_{op} = 8 \text{ m/s} \) and \( \lambda_{op} = 6.5 \),
\[ G_{wid} = \frac{s}{(s + 1.7)(s + 0.14)(s + 0.04)} \]

### 6.4 Speed Regulator Compensator Design

The compensator \( G_{c1} \), as has already been discussed, is the most flexible and adaptable of the three compensators since it consists of a sequence of Pascal commands contained within the procedure ‘idset’. The variable used by this procedure to compute the desired current can either be the difference between the required and the measured speed ‘wdiff’, or it can be the measured speed alone. As discussed in Chapter 1, variable speed operation is generally more efficient than fixed speed when the optimal rotor speed is tracked. The first control objective is therefore that of deciding what that rotor speed should be, upon what
measurements it should be determined, and how to attain this desired speed in
the most efficient manner at any moment in time.
In practice variable speed operation can be achieved by computing in real time a
desired rotor speed based on system variable measurements and comparing this
with the measured speed and setting the demanded current accordingly.
A straightforward, alternative approach is to use feed-forward control and set
the desired current according to the measured speed alone using the relationship
relating the 'ideal' generator torque to turbine speed.
The open loop transfer function for figure 6.14 is:
\[ G_{ols} = \frac{G_{cl} \cdot G_{wi} \cdot D}{golsnum} = \frac{golsnum}{golsden} \]

**Fixed Speed Operation**

It has been stated that to operate the turbine at optimal efficiency a detailed
characterisation of the wind turbine aerodynamics needs to be made. To best
achieve this, the machine should be operated at fixed speed. Gathering data
under fixed speed operation will allow a \( C_p/\lambda \) curve to be derived for that speed.
The tip speed ratio at which the machine should be operated at that speed is
then given by the peak of the \( C_p/\lambda \) curve. Changing the fixed speed will lead to
a series of optimal tip speed ratios being derived. The reason for the variation in
the \( C_p/\lambda \) curve at different speeds is that the blade pitch angle varies according to
the turbine speed thereby altering the lift and drag characteristics. The process
of transforming the resulting data into an ideal generator current / speed curve
will be described in Chapter 8.
At fixed speed the wind turbine system is operating as a regulator. For a step
increase in wind speed the turbine speed will tend to increase. The demanded
current should therefore increase leading to an increased load on the turbine.
This should slow the machine down until the measured speed becomes equal to
the desired speed whereupon the demanded current should stay constant.
A P+I compensator will slow the rate of increase of turbine speed resulting from
a step increase in demanded speed and if a higher proportional gain is used then
the response time will be improved. A result of integral action is the possibility of
integral windup which is caused by an error at the compensator input persisting
long enough for the integrator output to become saturated. If the error changes sign then this control term will not respond until the integrator has ramped up or down far enough to change sign itself. The rate at which the integrator ramps must therefore be selected according to the dynamics of the wind turbine. For a system like this which is not always controllable, such as when the wind speed drops too low to provide useful torque, integral windup can be a particular problem.

The open loop transfer function containing $G_{\text{wid}}$ represents a type 1 system. The zero at the origin (from the $G_c$ pole) means there is a non-zero steady state error.

**Deriving a Closed Loop Transfer Function for the Speed Regulator**

From figure 6.14,

$$G_{\omega \text{ref}} = \frac{\delta \omega}{\delta \omega_{\text{ref}}} = \frac{G_{c1}.G_{\text{wid}}}{1 + G_{c1}.G_{\text{wid}}.D}$$

$$= \frac{g_{c1}n_{um}.g_{\text{wid}}n_{um}.d_{den}}{g_{c1}n_{um}.g_{\text{wid}}n_{um}.d_{den} + g_{c1}n_{um}.g_{\text{wid}}n_{um}.d_{num}}$$

$$= \frac{\omega_{\text{ref}}n_{um}}{\omega_{\text{ref}}n_{den1} + \omega_{\text{ref}}n_{den2}} = \frac{\omega_{\text{ref}}n_{num}}{\omega_{\text{ref}}n_{den}}$$

The characteristic equation is the denominator of $G_{\omega \text{ref}}$.

The program $gc1des.m$ calculates the open and closed loop transfer functions for figure 6.14 and uses them to derive the root locus and the frequency response plots.

**Frequency Response Analysis**

The open loop frequency response plot for the uncompensated system is shown in figure 6.15. At frequencies $< 0.1 \text{ rad/s}$ the gain is decreasing which indicates that there is a non-zero steady state error. It is apparent that a P+I compensator would improve the low frequency response. The integral action gives an increasing gain as the frequency is decreased but it reduces the phase by $90^\circ$. The proportional action can be set to provide a phase margin that leads to an acceptable degree of stability. The high frequency response could be improved with a lead network. This would increase the phase at selected frequencies thereby increasing the phase margin and the systems relative stability.
Figure 6.15: Open Loop Frequency Response for Uncompensated Speed Regulator

Root Locus Analysis

The root locus technique can be used to confirm the beneficial effects of a lag lead network and graphically display the effects of varying the gain. The uncompensated root locus is shown in figure 6.16.

The dominant poles are at $s = -1.7$, $s = -0.14$ and $s = -0.04$. There is a zero at the origin due to the voltage compensator integrator $gc3den$. The system is non-minimum phase due to the zero in the right hand plane from the speed transducer (section 2.5.2). There is a matching pole in the left hand plane which means that the speed transducer will affect the phase but not the magnitude. The root locus shows the position of the dominant roots as the gain $K_{p1}$ is varied from $1.10^{-4}$ to 100 in 10 logarithmically spaced increments. The poles at $s = -1.7$ and $s = -0.14$ move towards each other and break away from the real axis for $K_{p1} = 0.4$. The pole at $s = -0.04$ moves towards the zero at the origin. The roots will never move into the right plane due to the presence of the pole from the speed transducer at $s = -\frac{2}{t_d} = -17$, where $t_d$ is the sampling time, (in this instance it is the time between consecutive updates in the demanded current, $t_d = 0.12s$). For $K_{p1} > 15$, any difference in speed greater than 1 rad/s will result in the demanded current being saturated at either $I_d = 0$ or $I_d = 15$. Under these conditions the system
control becomes 'bang bang' and, as has been mentioned already, this is generally considered as being undesirable.

A P+I compensator can be considered as a lag network with a pole at the origin. This pole cancels the system zero thus changing the system from a type 1 to a type 0. This reduces the ramp error coefficient to zero and improves the steady state error. This may be good enough for this particular application since the wind speed is never a constant and so the speed error is always changing.

If integral action only were used, then the breakaway point would move towards the origin between the poles at -0.04 and -0.14. This means instability would result at much lower values of gain. Using a P+I compensator and placing the zero between the pole at -0.04 and the pole at -0.14 means the breakaway point remains where it was for the uncompensated system. This is because the integrator pole cancels the zero at the origin and one branch of the root locus moves from the pole at -0.04 to the P+I zero. Adding a lead network with a zero between the system poles at -0.14 and -1.7 and a pole the other side of the system pole at -1.7, results in a compensator with a transfer function of the form:

\[
G_{c1} = K_{p1} \frac{s \left( \frac{1}{0.02} \right) + 1}{s \frac{1}{0.02}} \frac{s \left( \frac{1}{0.02} \right) + 1}{s \left( \frac{1}{2} \right) + 1} \quad (6.3)
\]
The resulting root locus and open loop frequency response plot for \( K_{pl} = 1 \) are shown in figures 6.17 and 6.18.

![Root Locus Plot](image)

**Figure 6.17: Root Locus of Compensated Speed Regulator**

The inclusion of the lag term has improved the steady state error. Theoretically this is very beneficial. It should be borne in mind that the signal input is discrete as it is derived from the speed transducer. The effects of the lag zero on the high frequency component of this signal will be large spikes appearing in the demanded current signal. These will be attenuated by the low pass filter \( G_{c2} \) but may still lead to problems. The philosophy of preventing spikes in the first place rather than attempting to repress them is preferable.

Using a standard PI controller without the lead lag terms may not be considered such a problem since the wind is never steady state and from figure 6.15 it is seen that there is 0dB gain at 0.4 rad/s for \( K_{pl} = 1 \). Such low frequencies can be regarded as steady state in this instance.

\[
1 + s \left( \frac{1}{0.09} \right)
\]

The selected compensator will be: \( G_{c1} = K_{pl} \frac{1}{s} \)

The root locus can be used to determine the value of gain \( K_{pl} \) to give a damping ratio of \( \xi = 0.7 \). This resulted in \( K_{pl} = 0.2 \).

The response to a step input on \( \omega_{ref} \) for the uncompensated and the compensated
systems is shown in figure 6.19 for a gain $K_{p1} = 0.2$. It can be seen that the compensator has dramatically improved the rise time. As $K_{p1}$ is increased the overshoot increases and the damping ratio decreases as predicted by the root locus (figure 6.17).

6.5 Robustness of Control System Design

The goal of a robust system design is to retain assurance of system performance in spite of model inaccuracies and operating point changes. Ideally the wind turbine will always operate at the peak of the appropriate $C_p/\lambda$ curve. Optimal control of the wind turbine will involve the machine generating a required current that is calculated from the machine speed according to a derived relationship. Emphasis should therefore be placed on the machines operating points along that curve. In the event that the tracking of the 'ideal' curve is lost then sub-optimal control can be tolerated as long as the system remains stable. Intuitively it is evident that if the wind turbine were operating to the left of the $C_q/\lambda$ peak then it would be unstable since an increase in wind speed would give a
Figure 6.19: Response to a 5 rad/s Step Input on $\omega_{ref}$ for Uncompensated and Compensated Speed Regulators

decrease in aerodynamic torque. In control terms this can be seen as the movement of a pole for $G_{WT}$ (equation 5.18) into the right hand plane. The location of this pole is determined by the parameters $K_{Qgw}$, $K_{Qaw}$ and $K_{Qfw}$.

Holding the wind speed constant at $v = 8 \text{ m/s}$ and increasing the tip speed ratio from $\lambda = 4$ to $\lambda = 7$ has the following effects:

i) $K_{Qgw}$ increases initially reaching its maximum value of 0.85 at $\lambda = 4.6$ and then decreases linearly.

ii) $K_{Qaw}$ decreases linearly from 2.7 becoming negative at peak $C_q$ then continues to decrease though at a slower rate due to the asymmetry of the $C_q/\lambda$ curve.

iii) $K_{Qfw}$ increases linearly (proportional to $\omega$) from -0.182 to -0.134.

Changing the tip speed ratio will affect the wind turbine time constant but in a way which depends upon the wind speed. Figure 6.20 shows the wind turbine time constant plotted against tip speed ratio for two wind speeds, $v = 8 \text{ m/s}$ (marked with an 'x') and $v = 12 \text{ m/s}$ (marked with an 'o'). There is a discontinuity at $\lambda = 4.8$. The peak of the $C_q/\lambda$ curve occurs at $\lambda = 5.6$. Intuitively it would be expected that the discontinuity would occur at the peak, however this assumes that the wind turbine is ideal. The effect of the frictional losses is to move the tip
Figure 6.20: Wind Turbine Time Constant Versus Tip Speed Ratio for $v = 8\text{m/s}$ (x) and $v = 12\text{m/s}$ (o)

speed ratio value at which the onset of instability occurs to the left. This value of TSR remains more or less constant over all wind speeds. As the wind speed increases the time constant decreases and remains constant over a broader range of TSR's. This suggests that the system is more responsive and more stable at the higher wind speeds.

The effects of both $K_{Qgw}$ and $K_{Qca}$ decreasing for $\lambda > 5.5$ is that their difference remains substantially constant. The result is that $\tau_{WT}$ becomes constant at $\tau_{WT} = 7.5$ for $\lambda > 6$.

The procedure adopted here to check the robustness of the controller is to set the tip speed ratio and vary the wind speed in steps from 6 to 12 m/s. For each step, the roots of the appropriate characteristic equation are plotted. Their movement as the wind speed is varied gives an indication of the system time response characteristics.

6.5.1 Voltage Regulator Robustness

From section 6.2 the dominant roots selected are at $s = -0.08 \pm j1.2$ for $v_{op} = 8$ m/s. As the wind speed is increased the roots move vertically towards the real
axis with the imaginary part becoming zero for $v_{op} = 10.5\ m/s$. This means the voltage regulator does not overshoot at the higher wind speeds, but the response time decreases. For lower values of TSR the system is unstable i.e. the roots are in the right hand plane.

6.5.2 Current Regulator Robustness

From section 6.3 the dominant roots selected are at $s = -0.12$ and $s = -0.005$. At $v_{op} = 6\ m/s$ the roots are at $s = -0.06$ and $s = -0.002$. As the wind speed is increased the roots move horizontally away from each other, one towards $-\infty$ and the other towards the origin becoming $s = -0.23$ and $s = -0.0009$ at $v_{op} = 12\ m/s$. For the lower tip speed ratio $\lambda = 5$ at $v_{op} = 6\ m/s$ the roots are complex at $s = -0.01 \pm j0.05$. As $v_{op}$ is increased they move towards the origin with the imaginary part becoming zero at $v_{op} = 10\ m/s$ and then moving in opposite directions as for the $\lambda = 6.5$ case. The conclusions are that for the lower tip speed ratios the damping is below critical so there will be some overshoot in the generator current resulting from a step in demanded current. If the controller were adjusted so as there is no overshoot at the lower tip speed ratios then at the higher, more ideal, tip speed ratios the system would be overdamped. The controller selected is a good compromise therefore between the avoidance of overshoots at low TSR's and a fast response at the higher more ideal TSR's.

6.5.3 Speed Regulator Robustness

Figure 6.21 is the root locus for the speed regulator as the wind speed is varied between 6 and 14 $m/s$ for $\lambda = 6.5$ (marked with an 'o') and $\lambda = 4.5$ (marked with an 'x').

At $\lambda = 6.5$ and $v_{op} = 6\ m/s$, with the chosen speed compensator, the roots are at $-1.2 \pm j2.07$, $-0.82$, $-0.1$ and at the origin. As the wind speed is increased the complex roots move diagonally away from the origin. The root at $-0.82$ moves initially towards the origin reaching its closest at $v_{op} = 10\ m/s$ and then changes direction ending up at $-0.8$ for $v_{op} = 14\ m/s$. The root at $-0.1$ moves towards the origin with its step size decreasing for the higher wind speeds.
Figure 6.21: Root Locus of Compensated Speed Regulator for $\lambda = 5$ (x) and $\lambda = 6.5$ (o) as the wind speed is Varied Between 6 and 12 m/s

For $\lambda = 5$ the complex roots move in a similar direction as for $\lambda = 6.5$ but their overall movement is less than half. The real roots both move towards the origin. It can be concluded that the system is robust in the sense that it cannot become unstable.
Chapter 7

Simulink Analysis

7.1 Introduction

A Simulink model has been developed to simulate the wind turbine and DC-DC booster system. The model has been used extensively for providing time response data and for checking the controller design and the effects of parameter variation.

Simulink™ is a program for simulating dynamic systems. As an extension to Matlab™ it adds many features specific to dynamic systems while retaining all of Matlab's general purpose functionality[7.1].

A major advantage of using the Simulink model is that non-linear characteristics, such as the saturation of variables, can be accounted for. The root locus technique assumes that there is infinite energy available to the input signal. In practice, as already discussed the load conductance is bounded to lie between 0.0 S (unloaded) and 0.21 S (fully loaded).

Initial values and variables are set using the Matlab script file coef2a.m. The progress of a simulation can be viewed while the simulation is running and the final results are made available in the Matlab workspace when a simulation is complete.
7.2 Block Diagram Description

Figure 7.1 is the Simulink block diagram representation of the wind turbine and DC-DC booster.

A number of assumptions have been made which will be explained and validated under each block heading.

The wind turbine is split into two parts: the aerodynamic block and the generator block. The wind velocity is derived from the addition of three inputs. These are each switched on or off as required. A wind time series input can also be added to determine the effects of different levels of turbulence and mean wind speed.

The aerodynamic torque $Q_a$ is produced by the aerodynamic block and input to the generator block along with the load conductance $G_{ph}$. The outputs from the generator block are the turbine speed, the generator current and the generator voltage. These are input to the speed compensator, current compensator and voltage compensator blocks respectively. These three compensator blocks provide the load conductance signal for the generator block.
7.2.1 Aerodynamic Block

Figure 7.2 is the contents of the aerodynamic block of figure 7.1.

![Aerodynamic Block Diagram](image)

The block is a representation of the torque coefficient equation

\[ C_q = \frac{Q_a}{\frac{1}{2} \pi \rho r^3 u^2} = \frac{Q_a}{9.43 \cdot u^2} \]

The aerodynamic section of the Proven wind turbine has been treated as a 'black box' in that the measured data is used to derive a \( C_q/\lambda \) curve which is used as a look up table within the aerodynamic block. Ideally a two-dimensional look up table would be used. The turbine velocity would be the second input. The added complexity involved in characterising the aerodynamic block as a two-dimensional array would not necessarily improve the results obtained from the simulation. This is justified by the fact that the simulation is to be used to verify the compensator design of Chapter 6. This involves small signal analysis, that is the study of small perturbations about an operating point. For each operating point the \( C_q/\lambda \) curve can be adjusted to resemble the \( C_q/\lambda \) characteristic that would be operating at that turbine speed.
7.2.2 Generator Block

Figure 7.3 shows the contents of the generator block of figure 7.1. This section of the wind turbine is based around the central fact that any change in the difference between the aerodynamic torque and the generator torque plus a frictional term will lead to the wind turbine accelerating or decelerating depending upon whether the difference is positive or negative. If the difference were a constant and remained constant then the angular velocity would change linearly with time, i.e. the acceleration would be constant. The angular velocity is the result of integral action on the torque difference divided by the total inertia. The shaft compliance has been neglected (i.e. it is assumed to be infinitely stiff).
The frictional effects are included as a "Simulink" function block which describes a non-linear relation between the frictional torque $Q_f$ and $\omega$ as derived in section 3.3.8.

The generator block consists of the implementation of the equations describing the electro-mechanical model derived in section 3.4. The generator emf has been shown to be linearly related to speed and combining this with the total load reactance results in a value for the generator current. The resulting generator torque has been shown to be approximately linearly related to the generator current and this is fed back to the summing junction with the shaft torque.

7.2.3 Voltage Compensator

Figure 7.4 shows the contents of the voltage compensator block of figure 7.1. The generator voltage is converted to a DC voltage and multiplied by the DC-DC booster gain to give the output voltage. This is filtered by the $g_{vlag}$ block and compared with a reference voltage. Note that the reference used is 400V and so the steady state gain of the $g_{vlag}$ block is equal to one. The difference is passed to the voltage compensator $G_{c3}$, the output of which is put through a saturation block to give $\alpha_2$ in the range 0 to 1. $\alpha_2$ is multiplied by the gain squared and
divided by the dump load resistance \( R_b = 75 \Omega \) to give the load conductance at the output of the DC-DC booster. This is multiplied by the square of the gain (equation 5.27) to give the conductance at the input to the DC-DC booster. This must be converted to the per phase load conductance by multiplying by 1.92 (section 4.5.1).

### 7.2.4 Current Compensator

Figure 7.5 shows the contents of the current compensator block of figure 7.1.

Figure 7.5: Current Compensator Block

The generator current is converted to DC, filtered and compared with the demanded current. The result \( I_{diff} \) is passed to the current compensator. The output is passed through a saturation block so as it is in the range 0 to 0.75. This represents the duty cycle \( \alpha_1 \) which is the signal that is applied to the boosting mosfet and is therefore representative of the gain. The gain is the reciprocal of \( (1 - \alpha_1) \), (see section 4.2).

### 7.2.5 Speed Compensator

Figure 7.6 shows the contents of the speed compensator block of figure 7.1.

The speed compensator block calculates the demanded current from the turbine speed. The speed transducer (section 2.5.2) is a time interval measurement circuit with a latched digital output. The sampling time varies with the rotor speed. The
data acquisition system is capable of sampling 6 channels at 0.24 ms. To reduce the effects of noise, 500 samples are taken and their average calculated. The control software can therefore be represented as a zero order hold block with a delay time of 0.12 s. Its input is the rotor speed and its output is the measured rotor speed.

### 7.3 Simulink Analysis of Compensator Design

The same procedure used in Chapter 6 to develop the compensator design is used here. Voltage, current and speed compensator designs are checked in that order. A major issue is the question of the type of input signal that is used. Step inputs represent a severe test but they give an insight into the behaviour of a compensator over a wide range of frequencies including steady state response. Under normal working conditions instantaneous changes in the system inputs are not encountered with the exception of the demanded current.

Initially the wind turbine is assumed to be operating to the right of the peak of the $C_q/\lambda$ curve. A step increase in wind velocity will lead to a step increase in
aerodynamic torque because the tip speed ratio will decrease and so the torque coefficient increases.

Once the correct functioning of the voltage, current and speed compensators is established the complete system will be analysed over a range of operating points to check stability.

7.3.1 Voltage Compensator

The voltage compensator is isolated by setting the value for the DC-DC booster duty cycle to $\alpha_1 = 0.49$ as derived in section 5.5.

Ordinarily the current compensator would assist in the response of the voltage compensator by altering the DC-DC booster gain in response to a change in the generator current caused by the load conductance changing. With the gain fixed, a change in $V_{out}$ or $V_{ref}$ is transmitted to the load conductance according to the expression:

$$G_{de} = 1.92 \cdot \left(\frac{(\text{gain})^2}{78}\cdot \text{sat}_{0-1} [G_{c3} (G_{vlag} V_{out} - V_{ref})]\right)$$

$sat_{0-1}$ represents the saturation of the duty cycle $\alpha_2$ which is constrained to lie between 0 and 1. At the operating point chosen for the controller design with $\alpha_1$ fixed, $G_L$ is constrained by $\alpha_2$ to lie between 0 and 0.1 $s$.

From section 6.2, a P+I controller was selected for $G_{c3}$.

$$G_{c3} = \frac{0.23 (1.67s + 1)}{s}$$

At the chosen operating point, $\alpha_2 = 0.2$. A step decrease/increase in $V_{diff}$ of $<- (0.2/0.23)$ or $>(0.8/0.23)$ would be sufficient to saturate $G_{dc}$ at its minimum/maximum value. $V_{ref}$ is set by a potentiometer and remains constant. The output voltage is filtered by $G_{vlag}$ and so instantaneous changes in $V_{diff}$ are not generally encountered.

To test the action of the voltage compensator by demanding a step change in $V_{out}$ would therefore be an unrealistic exercise. It does however demonstrate the shortcomings of the PI design process of section 6.2 by showing the effects of saturation. In addition, a further drawback of the linearisation technique is highlighted. This is the effect that a change in current can have on the generator voltage by the consequent change in stator loss voltage.

The response sequence for a step decrease in $V_{ref}$ is as follows. The duty cycle on
the chopping mosfet changes instantaneously followed by an exponential decay to a new steady state value. The output conductance is directly related to $\alpha_2$ and therefore has the same shape waveform. The output current will therefore respond with an instantaneous change. Since a decrease in output voltage is demanded then the inertial effects mean that the generator can supply the current pulse. The result is that the voltage dropped across the stator increases rapidly and so the generator voltage is decreased. Since for the isolated voltage regulator the DC-DC booster gain is fixed then the demanded decrease in output voltage is achieved even though the rotor speed has not yet decreased.

It is assumed that $\alpha_2$ does not become saturated. The effects of saturation are more pronounced at the chosen operating point for a step increase in $V_{ref}$. Figure 7.7 shows the time response results for the load conductance and the output voltage for a 10% step increase in $V_{ref}$ for two zero locations $s = -0.6$ and $s = -2$.

![Graph 1](image1.png)

![Graph 2](image2.png)

Figure 7.7: Load conductance and output voltage response to a step input on $V_{ref}$ for the zeros $s = -0.5$ and $s = -2$.

Figure 7.7 demonstrates the effects of both saturation and integral wind up. The step increase in $V_{ref}$ causes a step decrease in the output load conductance due to the proportional action of the PI compensator. The load conductance saturates
At $G_{dc} = 0$. With the gain fixed and the generator current at its minimum, the output voltage is constrained to increase with the rotor speed increase. Once the rotor speed has increased so as the generator voltage when multiplied by the gain equals the required output voltage then the load conductance will increase to its new steady state value. The rate at which it increases and its overshoot are determined by the position of the $G_{c3}$ zero. The different times at which the load conductance begins its increase are caused by the different amounts of integral windup from the two zeros. For the zero furthest from the origin the integrator output has ramped down further meaning it has further to go for its output to reach its new steady state value. Moving the zero towards the origin therefore means that the load conductance will turn on earlier and there will be less overshoot in $V_{out}$ because the integrator is less saturated since it is closer to being cancelled by the zero.

The procedure used in section 6.2 specified a maximum overshoot of 20% for a step input in $V_{ref}$. With the zero location $s = -0.6$, the simulation shows that the load conductance and output voltage do confirm the design by giving a 20% overshoot in response to a 10% step increase in $V_{ref}$. Moving the zero towards the origin decreases the overshoot and also slows the rise time. Moving away from the origin has the opposite effect.

The simulation shows that the chosen position for the zero represents a good compromise between achieving a fast response whilst avoiding the effects of excessive integral wind up.

A step increase in the wind velocity of 0.8 m/s (10%) demonstrate the beneficial effects of compensation. With P compensation only the result is a 0.4% increase in $V_{out}$. The effects of the integral action are to reduce the steady state error to zero. The output voltage overshoots by 0.04% and then exponentially decays. The tip speed ratio and aerodynamic torque change instantaneously with the step input resulting in a torque imbalance at the input to the generator integrator which leads to the speed increasing exponentially. With $V_{ref}$ constant a 20% overshoot is only possible if the gain is reduced to $K_{r3} = 0.05$. This means that with $K_{r3} = 0.23$ there will be very little chance of any substantial overshoots occurring as a result of wind speed changes.
7.3.2 Current Compensator

Reconnecting $\alpha_1$ to the current compensator and disconnecting the demanded current output of the speed compensator isolates the current loop. The effect of step changes in demanded current can now be assessed.

Section 6.3 resulted in:

$$G_{c2} = K_p^2 \frac{1}{s(\frac{1}{10})+1} = \frac{10}{10s + 1}$$

The output from the speed transducer is a digital signal. This signal remains constant for a length of time dependent on the turbine speed. The computer processes this signal and provides a value for the demanded current that remains constant for 0.12s. The demanded current will therefore be a stepped signal. Large high frequency steps in load conductance must be avoided as they will lead to excessive current spikes which may damage the switching components.

The current compensator is a low pass filter. There are 2 parameters which can be varied: the gain $K_p^2$ and the pole location. Figure 7.8 shows the effects of varying $K_p^2$. A step in demanded current from $I_d = 2.2$ to $I_d = 2.42$ at $t = 10s$ is input. The response for 2 values of gain, $K_p^2 = 5$ and $K_p^2 = 20$ are shown.

![Figure 7.8: Demanded Current Step Input for $K_p^2 = 5$ and $K_p^2 = 20$.](image)

As the gain is increased the steady state error decreases and the overshoot and resonant frequency both increase. The gain $K_p^2 = 10$ is therefore a reasonable
compromise between low steady state error with a reasonably fast response and without an excessive overshoot.

Setting the gain as $K_p^2 = 10$, figure 7.9 shows the step response for 2 pole locations $s = -0.05$ and $s = -0.2$ which are either side of the chosen pole location.

![Figure 7.9: Demanded Current Step Input for Two Pole Locations: $s = -0.05$ and $s = -0.2$.](image)

As the pole is moved away from the origin the overshoot and resonant frequency increase though the steady state error is unaffected.

The selected pole position is therefore a good compromise between achieving a fast response and minimising the effects of overshoot.

### 7.3.3 Speed Compensator

**Fixed Speed**

The demanded current signal is reconnected to the speed compensator output. For fixed speed operation the design of $G_{c1}$ can be tested by applying a step input on $\omega_{\text{ref}}$ under steady state conditions. Note that steady state conditions never actually exist. The step input is applied when the speed is changing at less than $0.1\text{rad/s/s}$. Section 6.4.1 derived a compensator for $G_{c1}$ of the form

$$G_{c1} = K_p^1 \frac{s^2 \omega_{\text{ref}} + 1}{s} = 0.2 \frac{11.1s + 1}{s}$$
Figure 7.10 shows the results of applying a step decrease in $\omega_{ref}$ from 30 rad/s to 27 rad/s at time $t=10$ seconds for the uncompensated and compensated systems.

![Graph showing time response]

Figur 7.10: Time Response for a Step Decrease in Referance Speed for Compensated and Uncompensated Speed Regulators.

The compensated speed regulator shows a dramatic improvement. The difference between the reference and the actual speed decreases as the gain is increased.

Simulation shows that the gain could be increased by a factor of 4 before any speed oscillations occur. The demanded current now exhibits excessive spikes. Figure 7.11 and figure 7.12 show the demanded current and the generator current respectively for 2 values of gain, $K_{p1} = 0.2$ and $K_{p1} = 0.8$ for the $3 \text{ rad/s}$ demanded decrease in rotor speed. The discrete nature of the demanded current signal is associated with the time delay of the speed transducer. For a negative speed change the demanded current saturates at 15 A for the higher gain. The generator current overshoots and the demanded current saturates at 0 A. If the gain were further increased the oscillations would be sustained resulting in instability. The time delay of the speed transducer accentuates this effect.
The selection of $K_{p1}$ is therefore a compromise between low steady state error on the speed difference and excessively high frequency components on the demanded current which will result in reduced lifetime for the electronic components, and increased fatigue of the wind turbine drive train. The gain selected in section 6.4.1 is a reasonably good compromise though for an added safety margin the system would benefit from a slightly lower value.
Variable Speed

The simplest variable speed strategy (feed-forward control) sets the demanded current according to a fixed function, such as $I_d = 0.006 \omega^2 + 0.033 \omega + 0.55$. This is not necessarily the 'ideal' relationship. Derivation of the 'ideal' relationship is dealt with in chapter 9. The purpose of this chapter is to ensure the machine will track a given relationship.

Under steady state conditions, for every wind speed, there is a corresponding rotor speed for maximum energy transfer. For a small amplitude, sinusoidally varying wind speed, the rotor speed and therefore the demanded current and load conductance will be sinusoidal (assuming a linear system response). The tip speed ratio is also periodic. Ideally it will be a constant (for constant TSR operation) at the optimal tip speed ratio, but because of the phase difference between the wind speed input and the rotor speed it will oscillate. The peak to peak value of this oscillation is directly related to this phase difference. If the phase difference were reduced by increasing the time constant of the low pass filter $G_{c2}$, then the result would be high frequency spikes on the demanded current resulting from wind gusts. The peak to peak value will affect how non-linear the resulting $C_q$ is. This in turn will determine the efficiency of the extraction of energy from the wind.

Even with a high gain on $G_{c2}$ there will still be some phase difference. This is because the effects of a change in wind speed can not be instantaneously transmitted to a change in rotor speed. To change the speed of a rotating machine necessitates the providing of an extra amount of energy, equivalent to the product of the machine inertia and its rate of speed change. The best that can be expected is a balance between excessive overshoots and a sufficiently fast rate of change of speed.

Figure 7.13 shows the tracking capabilities of the feed-forward controller. A sinusoidal wind speed oscillating at 2 rad/s between 6 and 10 m/s is applied. Superimposed on the 'ideal' $I_d$ versus $\omega$ characteristic is the actual generator current $I_{gdc}$. If the difference between $I_d$ and $I_{gdc}$ is plotted then a mean error (after the machine has settled) of 0.057 Amps is seen. This represents an error of 2.8%. Further improvements are possible as has been pointed out but they would be
detrimental in terms of overshoots during high wind gusts.

Figure 7.13: Feedforward Controller Tracking for 2 rad/s to 8 m/s sinusoid on $v$.

Figure 7.14: $I_d$ and $I_{gdc}$ versus Time for the Feedforward Controller with $v = 6$ to 10 m/s at 2 rad/s

Figure 7.14 shows the instantaneous waveforms for $I_d$ and $I_{gdc}$ as the wind speed varies between 6 and 10 m/s at a frequency of 2 rad/s. Clearly shown is the effect of the discrete nature of $I_d$ and the resulting oscillatory patterns on $I_{gdc}$. The general conclusion is that some sacrifice in performance is made in an effort to protect the DC-DC booster components and provide a smooth output power.
waveform. How much of a sacrifice this amounts to will become apparent with the full system simulation studies. A difficulty is involved in specifying the required overall performance. This may well differ according to the type of inverter that is to be used. Once the main priority of allowing a safety margin between the maximum current and voltage handling capabilities of the switching devices is catered for the gain may need to be further reduced depending on external circumstances. A continuation of this section and a comparison with other control strategies, is contained in Chapter 9 following the data analysis in Chapter 8.
Chapter 8

Fixed Speed Measurements and Current Demand Specification

8.1 Introduction

The wind turbine control system has been designed provisionally using a linearised approach based on data supplied by NEL. In this chapter the fixed speed controller is implemented and operational data recorded. This data is then used to improve the aerodynamic model. The Simulink turbine model can then be improved and, once validated can be used as the basis for further control system design improvements.

This chapter gives details on how data is recorded and on the procedures used for processing the recorded data. As has been outlined in Chapter 3, the aerodynamic characteristics have been treated as a 'black box'. Chapters 5, 6 and 7 were based on the premise that the demanded current, selected under a given set of conditions, is optimal. An objective of this chapter is therefore to define what the optimal demanded current is, and how it is related to the rotor speed. Achieving this involves deriving a set of performance characteristics as was done with the NEL data in Chapter 3. The implementations of the 3 controllers, using the transfer functions derived in Chapter 6 for fixed speed operation, are dealt with first. The methods used for data gathering, storage and analysis are then covered, followed by a description of the issues involved in deriving a set of performance
characteristics leading to the derivation of an ideal relationship between demanded current and rotor speed.

8.2 Fixed Speed Controller Implementation

This section describes how the 3 controllers, designed for fixed speed operation in Chapter 6, are implemented. Chapter 5 gave the generalised description for the design of the PID controllers. This section is a continuation of that chapter, and derives the component values.

Voltage Compensator

Appendix H is the voltage regulator circuit diagram.
From section 6.2: \( G_{c3} = \frac{0.23(1.67s+1)}{s} = 0.38 + \frac{0.23}{s} \)
The proportional gain is attained with a feedback resistor 0.38 times the value of the series resistor in figure 4.13.
The integral part is attained with \( \frac{1}{RC} = 0.23 \). Choosing \( C = 47\mu F \) gives \( R = 92k \).

Current Compensator

Appendix G is the current regulator circuit diagram.
From section 6.3: \( G_{c2} = \frac{10}{20s+1} \) which is a low pass filter. This is implemented by placing a capacitor in parallel with the feedback resistor on the proportional part of the current compensator. The capacitance is calculated from \( R_fC = 20 \).
The steady state gain is attained by selecting a resistor for the input to the proportional element such that \( \frac{R}{R_{ina}} = 10 \).

Speed Compensator

From section 6.4: \( G_{c1} = 0.2 \cdot \frac{11.1s+1}{s} = 2.22 + \frac{0.2}{s} \)
This is implemented in the idset procedure by splitting the \( L_d \) equation into 2 parts. The first is the proportional part equal to \( k_{p1} \cdot wdiff \) and the second is the integral part equal to \( k_{i1} \cdot \text{diffint} \) where \( \text{diffint} \) is updated for each pass by:
\[ \text{wdiff}\text{int} = (0.2 \times t_d \times \text{wdiff}) + \text{wdiff}\text{int}, \] where \( t_d = 0.12s \) is the sampling time of the data acquisition and control program.

### 8.3 Experimental Design

For flexibility a PC is used both to implement the speed compensator aspect of the control and to collect data. The experimental arrangement comprises of the PC plus an 8 channel A/D converter plus all the associated transducers as detailed in Chapter 2.

The torque transducer output became corrupted when the DC-DC booster is operating. This was caused by the high levels of electro magnetic interference transmitted from the generator when its output is being chopped. Shielding the strain gauges made no difference and the use of the torque transducer on the operational wind turbine had to be abandoned.

#### 8.3.1 Control and Monitoring Software

Appendix P is the flowchart and listing of the Pascal program \textit{wset.pas} used for fixed speed operation. The program sets the reference speed according to the generator current. Every 10 minutes data is transferred to hard disc and the reference speed adjusted by 2 rad/s if necessary. Originally this reference speed was set manually but there is the potential problem of the machine operating above rated power if the wind speed increases significantly when the turbine is operating at a low reference speed. This is because the passive pitching mechanism relies mainly on centrifugal forces for its operation which are proportional to the square of the rotor speed. Thus if the rotor speed is kept low by applying a large load at the higher wind speeds there is the possibility of the generator becoming overloaded. This however is diminished to a certain extent as the blade pitch will change with the degree of coning which is related to the wind speed (thrust on the rotor).

In addition with the rotor speed fixed the tip speed ratio will decrease with increasing wind leading to the machine operating to the left of the \( C_{\lambda}/\lambda \) peak and the resulting instability problems as discussed in Chapter 6.
A Pascal program was written so as the wind turbine can be left unattended yet still working at fixed speed with the assurance that it won't become overloaded. The reference speed is increased by 2 rad/s if the average generator current is greater than twice that of the previous 10 minute period. It is decremented if it is less than half that value.

8.4 Data Description

The data recorded is the uncalibrated, raw data from the A/D converter on the data acquisition board. Each converted value will be an integer between 0 and 4095 that is representative of the quantity being measured. There are 9 columns representing wind speed, demanded current, DC current, reference speed, DC voltage, output current, output voltage, yaw misalignment and turbine speed in that order. The demanded current, reference speed and turbine speed are stored as real numbers. The sampling time was set at 2.4 seconds. The sampling frequency for the data acquisition program was set so as 500 samples are taken in 0.12s. Each row of the resulting data is therefore the average of 22 sets, each of which is the average of 500 samples. This chapter is based on 50000 stored samples taken over a 30 hour period during early December when the wind was reasonably constant from the south east. This is the wind direction at this particular turbine site from which there is the least turbulence.

8.5 Data Analysis

The recorded uncalibrated data is converted within Matlab using the calibration equations derived in Chapter 2.

8.5.1 Data Integrity

The integrity of the data can be checked by plotting the ratio of the emf voltage (calculated from $V_{gdc}$ and $I_{gdc}$) and the rotor speed. This ratio has been shown
(section 3.3.5) to be a constant. The result showed a deviation from this constant with a scattering of ±3% indicating that the 3 channels are reasonably consistent representations of the actual variables.

The wind speed data integrity is not so easily checked. One method is to derive power versus wind speed and compare with the manufacturers claims but the reasons for any discrepancies cannot be solely attributed to corrupt data. The integrity of this data is especially important in that one method used to derive power coefficients uses the cube of the wind speed. The wind speed measurement is the weak link in the chain of data analysis and the effects of varying its mean value will be investigated. The voltage and current transducers have been directly checked using the 3 phase mains supply through a variac.

8.5.2 Wind Speed Correction

A fundamental problem with wind turbine performance analysis is how to measure the wind speed 'experienced' by the wind turbine. Placing the anemometer at an equivalent height on an adjacent building is possible but the large amount of turbulence caused by surrounding trees and buildings means there would be virtually no correlation between this anemometer reading and the wind speed at the rotor. Moreover, flow distortion due to the major obstacles, are expected to bias the flow values in an unpredictable manner.

The downwind design of the machine enables upwind measurement to be made close to the plane of the rotor using an anemometer attached to a boom on the nacelle (figure 8.1). The anemometer is placed at a distance of 1.2 times the rotor radius. The IEA recommendations is for 2 times the rotor radius but it was realised that there would be excessive vibration as well as poor structural integrity leading to potential fatigue problems at this distance. Figure 8.1 shows the position of the anemometer with respect to the wind turbine.

The wind speed measured by the anemometer is not therefore the same as the wind speed 'experienced' by the rotor. The difference is due to the blockage presented by the rotor disc and this blockage is related to the tip speed ratio or the effective solidity of the rotor disk. Ideally, for power performance purposes, the wind speed that would have existed at the position of the rotor, had the wind turbine not
been there, is required. The anemometer reading therefore needs correcting to compensate for the presence of the wind turbine. Two methods have been used to calculate this correction.

**Correction Method 1**

This method uses the uncorrected wind velocity to calculate the power coefficient. The axial flow induction factor \( a \) is calculated based on actuator disk momentum theory from\(^{11,4} \):

\[
C_p = 4a (1 - a)^2 = 4a^3 - 8a^2 + 4a
\]

The velocity deficit upstream from the rotor can then be calculated from\(^{8,1} \):

\[
\alpha' = a \left( 1 - \frac{a}{\sqrt{(1 + \gamma^2)}} \right) \text{ where } \gamma = \frac{D_{wp}}{r_{ad}} = 1.2
\]

Where \( D_{wp} \) is the distance from the anemometer to the rotor.

From this \( \alpha' = 0.232a \)

and

\[
v_{cor} = \frac{v}{1 - a}
\]

The Matlab script file \( wvcor.m \) (Appendix R) is used to correct the wind velocity.
data based on method 1. The result is a 3% increase in the mean wind speed for the data set used.

**Correction Method 2**

The disadvantage of method 1 is that no account is taken of the drag force on the blades and so the correction will be underestimated. The second method uses blade element momentum theory. Software written by David Sharpe has been used to calculate the axial induction factor at given tip speed ratios for a rotor of similar profile and dimensions to the Proven WT2200. Figure 8.2 shows the theoretically calculated axial induction factor versus tip speed ratio. The Matlab script file `interp1.m` was used to interpolate this fit and thereby derive a value for the axial induction factor at each tip speed ratio for the data set used. Equation 8.1 is then used with $\alpha' = 0.232a$ to calculate the corrected wind velocity. The result is a 9% increase in the mean wind speed for the data set used. This larger correction is considered more reliable.
8.5.3 Performance Characteristics

Two methods can be used to derive a set of performance characteristics. Each method has advantages and disadvantages in terms of the procedures used for data analysis and the effects on them of inherent wind speed measurement errors.

Method 1: Deriving $C_p/\lambda$ curves Using a Binning Technique

This method splits the data into 3 separate speed bands, $<20$ rad/s, 20-25 rad/s and $>25$ rad/s. For the centre of each speed band there is an optimal tip speed ratio for which energy transfer is maximised as indicated by the location of $C_{p_{\text{max}}}$. Interpolating between these sets of curves will result in the derivation of a relationship between the 'ideal' demanded current and the rotor speed.

The Matlab script file datv.m (Appendix R) is used to transform the recorded data to the actual values as outlined in the calibration section. Datv.m also filters the data by removing samples with rotor speed readings of $<12$ or $>32$ rad/s. For the data set used this involved removing ~ 2%. These are data glitches caused by false triggering of the speed transducer.

The purpose of running the machine at constant speed is that the effects of the machines inertia can be ignored. Figure 8.3 is the unbinned overall $C_p/\lambda$ data

![Figure 8.3: Unbinned Overall $C_p/\lambda$ Points Excluding Machines Inertia](image)

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set. This is obviously distorted with a blank region beneath $C_p/\lambda$ points at the higher tip speed ratios. This suggests that the errors involved in approximating the resulting torque are more pronounced at the lower wind speeds and higher rotor speeds. Including a term in the resultant torque equal to the product of the total inertia and the rate of change of speed to compensate for energy moving in and out of the machine inertia makes a considerable improvement. Figure 8.4 is

![Graph](image)

**Figure 8.4: Unbinned Overall $C_p/\lambda$ Points Including Machines Inertia.**

the same data set used for figure 8.3 but with a $J \frac{d\omega}{dt}$ term included in the torque calculation. The result is that the anomalous feature is no longer apparent.

For each sample `datu.m` calculates the resultant aerodynamic torque as the addition of the generator torque (calculated from the generator current), the frictional torque and the inertial torque. Multiplying this torque with the rotor speed gives the mechanical power. The torque and power coefficients can then be calculated and plotted against the tip speed ratio to give the performance characteristics.

The Matlab file `bin.m` was used to find the average $C_p$ and $C_q$ values for each TSR band of 0.2.

The performance characteristics for the uncorrected wind velocity measurements could immediately be seen to have errors by comparing the power that would be produced at a particular wind speed with the manufacturers claims. As discussed
in section 8.5.2, method 2 gave 9% and an additional 1% can be attributed to the blockage presented by the generator. The resulting corrected $C_p/\lambda$ curves for the 3 speed bands are shown in figure 8.5.

![Figure 8.5: Corrected $C_p/\lambda$ Curves for the 3 Speed Bands.](image)

As the speed is increased the power coefficient and the ideal tip speed ratio increase. Note that for the higher speed band the data to the right of $C_{p_{\text{max}}}$ is more suspect. This is to be expected since at the higher rotor speed, to operate in this region there must simultaneously be low wind speed, a situation not generally encountered except when there is a sudden lull in the wind when the rotor is spinning fast. The discontinuity is therefore due to the scarcity of data in this region.

### Method 2: Deriving $C_p/\lambda$ curves from Power versus Wind Speed

This method is based on transforming the power wind speed curve for a fixed rotor speed. This has the advantage of fitting functions to data based on the wind speed rather than the wind speed cubed as used for the binning method. In theory this can reduce statistical errors which arise from the fitting procedure and from the wind speed error.
The results from datv.m are used by the Matlab file da.m to derive mechanical power versus wind speed for a set of rotor speeds. The data is filtered to remove samples which are outside a range centred on $\omega \pm 1\%$. Theoretically a cube law relationship exists between power and wind speed. The problem with finding this relationship is the limited wind speed band over which data is available. A cubic fit extrapolated to outside the narrow region of wind speed data will be unreliable. A linear fit was therefore made. Figure 8.6 shows the results for the 4 speed bands.

Figure 8.6: Mechanical Power Versus Wind Speed for Rotor Speeds 22, 24, 26 and 28 rad/s.

Each linear fit can be expressed by:

$$P_m = k.v + c$$  \hspace{1cm} (8.2)

Using the power coefficient expression and substituting for $P_m$ gives:

$$C_p = \frac{P_m}{\frac{1}{2}\pi \rho r_{ad}^2 v^3} = \frac{k.v + c}{\frac{1}{2}\pi \rho r_{ad}^2 v^3}$$  \hspace{1cm} (8.3)

substituting $v = \frac{\omega r_{ad}}{\lambda}$ gives

$$C_p = \frac{k.\omega r_{ad}.\lambda^2 + c\lambda^3}{k_v (\omega r_{ad})^3}$$  \hspace{1cm} (8.4)
where \( k_t = \frac{1}{2} \pi \rho r_{ad}^2 \).

The maximum \( C_p \) can be found by setting the derivative of \( C_p \) with respect to \( \lambda \) to 0.

\[
\frac{dC_p}{d\lambda} = \frac{1}{k_t (\omega r_{ad})^2} (2k \omega r_{ad} \lambda + 3c \lambda^3)
\]

which gives

\[
\lambda_{pk} = \frac{-2k \omega r_{ad}}{3c}
\]

Figure 8.7 shows the resulting set of \( C_p/\lambda \) curves, which are obviously inadequate. The table shown below gives the 4 rotor speeds, their gradients, offsets and \( \lambda_{pk} \)'s.

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>( k )</th>
<th>( c )</th>
<th>( \lambda_{pk} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>36.4</td>
<td>-20</td>
<td>45.6</td>
</tr>
<tr>
<td>24</td>
<td>74.8</td>
<td>-100</td>
<td>20.5</td>
</tr>
<tr>
<td>26</td>
<td>152.2</td>
<td>-500</td>
<td>9.02</td>
</tr>
<tr>
<td>28</td>
<td>300</td>
<td>-700</td>
<td>13.7</td>
</tr>
</tbody>
</table>

Figure 8.7: \( C_p/\lambda \) Curves Using Power Versus Wind Speed Method

It is evident from this table that the \( C_p/\lambda \) curves do not have their peaks anywhere near where they would be expected with the exception of \( \omega = 26 \text{ rad/s} \) which has
its peak at $\lambda = 9$ which although too high, is nevertheless more believable than the other 3.

The disadvantage of this method compared to the binning method is that the fitting of a polynomial generates more errors than the binning. Consequently the results of the binning approach (method 1) are considered to give the best estimate of the rotor aerodynamic characteristics for this application.

8.5.4 Derivation of 2D Simulink Look Up Table

The initial controllers were designed using a single dimensional look up table for a single overall $C_q/\lambda$ curve derived from the NEL data. This was considered acceptable considering the degree of data manipulation that had to be undertaken to derive this single $C_q/\lambda$ curve. In contrast, the data recorded for the turbine tested on the Loughborough University campus, because of the higher sampling rates, gave a much smoother set of performance curves at different rotor speeds. These are consequently much more amenable to the calculation of a 2 dimensional look up table. Figure 8.8 shows the $C_q/\lambda$ curves from method 1 together with polynomial fits. Comparing figure 8.5 with figure 8.8 shows that the $C_p$ peak and

![Graph showing corrected $C_q/\lambda$ curves using Method 1 and their associated polynomial fits.](image)

**Figure 8.8:** Corrected $C_q/\lambda$ curves Using Method 1 and Their Associated Polynomial Fits
the \( C_q \) peak positions are similar. Generally the peak of the \( C_p/\lambda \) curve would be located further to the right of the peak of the \( C_q/\lambda \) curve. The reduced distance between these peaks can be explained by the varying frictional losses which are usually not properly included in most performance characteristic representations. Note that there are 2 fits for the \( \omega > 25 \text{ rad/s} \) curve. The first is for the data only. If this fit were used for the derivation of a look up table to be used in Simulink, then large errors would result if at \( \omega > 25 \text{ rad/s} \) the tip speed ratio becomes greater than 7.5. A solution is to estimate where the curve will meet the \( x \) axis using a data set for the unloaded wind turbine and force the fit to curve round so as it can be used over the entire tip speed ratio range. Comparing the 2 fits, the limitations of this compromise approach are not too severe.

The polynomial fits for the \( C_q/\lambda \) curves for the 3 speed bands are:

\[
C_{qa} = -0.0015\lambda^2 + 0.0185\lambda - 0.037
\]
\[
C_{qb} = -0.0013\lambda^2 + 0.019\lambda - 0.036
\]
\[
C_{qc} = -0.0003\lambda^2 + 0.007\lambda + 0.0135
\]

An array of tip speed ratio values from 3.6 to 12 in steps of 0.2 is used to derive \( C_q \) values. The result is a table with 3 columns and 42 rows which is accessed by the Simulink 2D look up table. This replaces the original 1D look up table developed in Chapter 3, with the additional input from the rotor speed. Figure 8.9 is a 2 dimensional representation of this look up table, and as required the surface is well behaved.

If the rotor speed is within the band 16 to 28.5 \( \text{rad/s} \) then Simulink will interpolate a value for \( C_q \) using the 3 polynomial fits. If the rotor speed is outside this band then Simulink will extrapolate a value. A potential source of error is that as the speed becomes greater than 28.5 \( \text{rad/s} \) the \( C_q \) value will continue to increase. This is unrealistic since the actual \( C_q \) should decrease due to the passive pitching mechanism. This issue can be resolved by inserting a 4th row of zeros for a rotor speed of 38 \( \text{rad/s} \). This value was chosen as simulation results gave maximum rotational speed under no load conditions at steady state high winds of 35 \( \text{rad/s} \) which is the same maximum steady state speed that the unloaded machine under test was seen to reach. This is because the aerodynamic torque is limited because the blade pitch angle is such as to cause the machine to be operating in its stall
region under these extreme conditions.

8.5.5 Validation of Simulink Model

The 2D Simulink look up table replaces the 1D look up table derived from the NEL data in the final Simulink model of Chapter 7. The extra input is the modelled (actual) rotor speed (as opposed to the measured rotor speed). The recorded wind data from a particular data set must be pre-processed so as it represents the actual wind velocity according to the corrections given in section 8.5.2. The resulting wind velocity time series may then be used as the input to the Simulink model with the initial rotor speed set in the generator integrator block. Having run the simulation there are 2 sets of data: recorded and simulated. The Matlab script file spline.m was used to add data to the recorded set using interpolation, and to ensure the two sets contain the same number of elements. In an ideal world these two sets would be identical, but in view of the number of approximations made, this is unrealistic. The result of plotting simulated versus measured rotor speeds is shown in figure 8.10. The results show a 12.9% deviation between them at $\omega = 30 \text{ rad/s}$. The majority of the non-linearities lie within the aerodynamic section of the model and in this context the differences can be considered acceptable. The
model can therefore be considered as sufficiently accurate.

Figure 8.10: Simulink Model Validation for Rotor Speed

8.6 Derivation of Ideal Operating Points

For feed-forward control it is necessary to derive a relationship between the rotor speed and the ideal generator DC current. Three methods are used here. The first derives a square law relationship between demanded current and rotor speed ignoring second order effects. The second includes frictional effects and the third includes both friction and the effects of the passive pitching mechanism.

8.6.1 Basic Square Law Relationship

Using equation 3.46: $C_q = \frac{Q_a}{\frac{1}{2} \pi \rho r_{ad} u^2}$ and the tip speed ratio $\lambda = \frac{\omega r_{ad}}{v}$ means that the aerodynamic torque can be expressed as:

$$Q_a = KC_q \frac{1}{\lambda^2} \omega^2$$  \hspace{1cm} (8.8)

where $K = \frac{1}{2} \pi \rho r_{ad}^5 = 27.6$
A first approximation is to take the generator current as being proportional to the aerodynamic torque. The aim is therefore to operate the machine at a fixed tip speed ratio. Taking a single $C_p/\lambda$ curve to represent the turbine, the optimal tip speed ratio occurs at $\lambda_{\text{opt}} = 6.5$ at which $C_p = 0.28$ and $C_q = 0.043$. The demanded current can therefore be expressed as being equal to a constant multiplied by the square of the rotor speed, i.e.

$$I_d \approx \frac{1.82}{9} KC_q \frac{1}{\lambda_{\text{opt}}} \omega^2 = 0.0058 \omega^2.$$  \hspace{1cm} (8.9)

### 8.6.2 Square Law Relationship Including Frictional Effects

To account for losses, the frictional torque can be subtracted to give the generator torque. The revised operating curve is:

$$I_d = 0.0058 \omega^2 - \frac{1.2}{9} Q_f$$ \hspace{1cm} (8.10)

The frictional component $Q_f = f(\omega)$ was determined in section 3.3.8.

The demanded current can now be expressed as a polynomial in $\omega$:

$$I_d = 5.57 \times 10^{-3} \omega^2 + 0.033 \omega + 0.55$$ \hspace{1cm} (8.11)

Implementation is straightforward. Equation 8.11 is substituted into the $idset$ procedure in $wset.pas$. However there are 2 major problems with this approach:

i) The efficiency is maximised only over the specific speed range for which the $C_p/\lambda$ curve is appropriate. Comparing the overall $C_p/\lambda$ relation with the 3 speed dependent $C_p/\lambda$ curves shows that the overall optimal tip speed ratio is approximately equivalent to the tip speed ratio for the mid speed range.

ii) There is a high demanded current at the low rotor speeds resulting in a high starting torque, this gives a high cut-in wind speed which limits energy capture. Using $\lambda_{\text{opt}}$ for the higher speed range will alleviate i) but accentuate ii). A possible answer is to have 3 separate $I_d$ versus $\omega$ relationships. The problems with this are the discontinuities at the boundaries. These could lead to the wind turbine hunting at one of the boundaries until the wind speed changes.
8.6.3 Including Effects of Passive Pitching

An alternative approach is to express $C_{q\text{opt}}$ and $\lambda_{\text{opt}}$ as functions of rotor speed. The performance characteristic curve figures 8.5 and 8.8 can be used to define the relationships between the optimal tip speed ratio and rotor speed and between the torque coefficient and rotor speed. Table 8.1 lists these values.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$\lambda_{\text{opt}}$</th>
<th>$C_q \circ \lambda_{\text{opt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>6.4</td>
<td>0.02</td>
</tr>
<tr>
<td>22.5</td>
<td>7.2</td>
<td>0.031</td>
</tr>
<tr>
<td>28.5</td>
<td>7.5</td>
<td>0.051</td>
</tr>
</tbody>
</table>

(8.12)

A problem with this approach, and a potential source of error, is the fact that there are only 3 values for each parameter derived from the 3 $C_p/\lambda$ curves. In deriving the relationship care must be taken to ensure that $\lambda_{\text{opt}}$ and $C_{q\text{opt}}$ do not behave uncharacteristically outside of the limits set by the 3 speed bands chosen. Figures 8.11 and 8.12 show $\lambda_{\text{opt}}$ versus $\omega$ and $C_{q\text{opt}}$ versus $\omega$ respectively. A $2^{nd}$ order polynomial trendline was fitted and the resulting equation was used to extrapolate to $\omega = 12$ and $\omega = 35$ rad/s. This procedure was repeated for the $C_{q\text{opt}}$ versus $\omega$ relationship (figure 8.12). The resulting polynomial fits are:

![Figure 8.11: Optimal Tip Speed Ratio as a Function of Rotor Speed.](image-url)
Equations 8.13 and 8.14 can then be used to calculate the desired aerodynamic torque for each rotor speed by substituting them into equation 8.7. The desired DC current can be found by substituting the desired torque into:

\[ I_{gdc} = \frac{1.2}{9} (Q_a - Q_f) \]  

(8.15)

Having found \( I_d \) as a function of \( \omega \), a polynomial function can be fitted and used in the control program to calculate \( I_d \). The problems are the errors involved in the polynomial fit. A less error prone method would be to use a look up table. This has the additional advantage which is that it can be manipulated at the lower rotor speeds to give a smoother start up torque characteristic. This is the approach actually implemented.

A Pascal program was written \( \text{calcld.pas} \) (Appendix R) which incremented \( \omega \) by 0.1 between 17 and 35 \( \text{rad/s} \). For each \( \omega \), \( C_{qopt} \) and \( \lambda_{opt} \) were calculated. These can then be used to calculate a value for \( I_d \) in terms of \( \omega \). Pascal is used so that the control method can be incorporated into the control and data acquisition program.

Figure 8.12: Optimal Torque Coefficient as a Function of Rotor Speed.

\[ \lambda_{opt} = -0.0058\omega^2 + 0.3482\omega + 2.3262 \]  

(8.13)

and

\[ C_{qopt} = 0.0001\omega^2 - 0.0034\omega + 0.0402 \]  

(8.14)
as a look up table. A Matlab version was also written to be used in the simulation model. The resulting $I_d$ versus $\omega$ relationship is shown in figure 8.13.

![Graph showing demanded current as a function of rotor speed.](image)

Figure 8.13: Demanded Current as a Function of Rotor Speed. Calculated from the Polynomial Relationship of Figures 8.11 and 8.12.

### 8.6.4 Derivation of the Pascal Look Up Table

Equations 8.11, 8.12 and 8.7 are used in a repeat until loop to build a look up table by incrementing $\omega$ by 0.1 rad/s and for each step calculating $I_d$. This Pascal procedure is placed in the *initialise* procedure in the data monitoring and control program. The procedure was also written in an Excel spreadsheet. Using Excel meant that data can be easily added and the results seen instantaneously. This was done at the lower end in order to smooth off the point where the $I_d$ versus $\omega$ characteristic meets the x axis. This is necessary to avoid a step in demanded current which could lead to the machine hunting at this speed at low wind speeds. The look up table approach has the disadvantage of some quantisation error because the table will always be of limited size. Using steps of 0.1 rad/s means the table will have 180 rows. There is a finite amount of memory available and a table of this size represents a reasonable compromise between low quantisation error and the use of available memory space. The look up table is accessed in the
idset procedure which rounds off the rotor speed reading to the nearest 0.1 rad/s. This is viewed as being consistent with the expected resolution of the rotor speed measurement transducer.
Chapter 9

Control Strategy Description

9.1 Introduction

Chapters 5, 6 and 7 used a linearised model to design a set of controllers. Chapter 8 then used these controllers to produce a more accurate model which was validated using a recorded wind time series by comparing simulated and measured results. This improved model can now be used as a tool with which alternative control strategies can be tried and tested, before implementation.

To compare control strategies it is necessary to provide the system model (including the controller in question) with an identical set of wind time series. These wind time series can be synthesised to have prescribed statistical properties. Section 9.2 describes how sets of wind speed data, each with different mean, variance and temporal structure are derived. This is preferable to real wind data for system evaluation purposes.

For each group of control strategies there are many parameters that can be varied. Decisions have to be made therefore as to establishing the most effective ways of applying the derived wind time series to each control group and then varying the parameters within that group in an attempt to optimise the machine under all operating conditions. The results of each simulation then need to be compared. The methods used for this comparison are just as important as the methods used for parameter variation. To further complicate this procedure (especially if the hardware compensators are adjusted), the wind conditions (i.e. the mean wind
speed, turbulence level and structure) will affect the end result. A solution to this may be to analyse the measured wind time series and design a controller for typical wind conditions at a given site (in this instance, Loughborough University). This may be of limited commercial use however unless a method for on site tuning based on wind speed measurements can be derived or an adaptive controller designed. For quantitative analysis, a performance index is assigned. This is a number reflecting how well the control strategy achieves the objectives assigned to it under a given set of operating conditions i.e. for a particular wind time series. A methodology for performance comparison is developed in section 9.3.

Sections 9.4, 9.5 and 9.6 deal with the 3 major groups of control system investigated. The final section of this chapter compares these 3 groups and discusses their ease of implementation.

9.2 Synthesis of Wind Time Series

A range of methods exist, many of which are computationally demanding such as spectral inversion. One of the least numerically intensive methods is the random walk. The resulting time series will have a gaussian probability density function and an exponential autocorrelation function, equivalent to a simple Dryden spectrum. It is well suited to modelling wind turbulence.

The first order random walk method is based around equation 9.1.

\[
(U_{t+1} - \bar{U}) = (U_t - \bar{U}) r + \varepsilon_t
\]

(9.1)

where \(U_{t+1}\) is the instantaneous value of the wind speed at time \(t+1\). \(U_t\) is the value at time \(t\), \(\bar{U}\) is the required mean value, \(r\) is the autocorrelation at unit lag and \(\varepsilon_t\) a gaussian noise term with variance given by \(\sqrt{(1-r^2)}\). Equation 9.1 can be rewritten as 9.2.

\[
U_{t+1} = rU_t + \bar{U} (1 - r) + \varepsilon_t
\]

(9.2)

The turbulence intensity \(I\) is defined by

\[
I = \frac{\sigma}{\bar{U}}
\]

(9.3)

where \(\sigma\) is the variance.
Three mean wind speeds 6, 9 and 12 m/s are used. For each mean wind speed two values of \( L \) are chosen: 50m and 120m, where \( L \) is the longitudinal length scale and is a measure of the turbulent eddies related to the scale of topography. The wind spectrum \( F(\omega) \) is assumed to follow the Kaimal model:

\[
F(\omega) = \frac{0.064 \sigma \left( \frac{f_0}{f_0} \right)}{1 + 0.064 \left( \frac{f}{f_0} \right)^{1.57}}
\]

(9.4)

where

\[
f = \frac{\omega z}{U}
\]

(9.5)

and

\[
f_0 = \frac{0.041 z}{L}
\]

(9.6)

where \( z \) is the height of the wind turbine. The variance is chosen according to the desired mean wind speed and turbulence intensity. From the spectrum, the autocorrelation \( r \) is calculated using

\[
r(t) = \int F(\omega) \cos(2\pi \omega t) dt
\]

(9.7)

The resulting value of \( r \) is then used with equation 9.2 to derive a wind time series. Appendix S lists the Matlab programs used in deriving the wind time series. \textit{Sts.m} derives a frequency spectrum from which \( r \) is calculated using a chosen mean and variance. \textit{Wser.m} uses this value of \( r \) to derive the wind time series based on the first order random walk method. \textit{Corer.m} is the autocorrelation program which uses the derived time series to calculate \( r \) to compare with the desired \( r \).

### 9.3 Performance Index Evaluation

To compare and contrast control systems with identical inputs an appropriate measure of performance must be developed. This measure is derived to reflect the objectives for which the control system was developed. For a fixed speed controller the measure, or index, should reflect the extent to which the rotor speed deviates from the desired speed.

For optimal control, the objective is to keep the power coefficient at a maximum value. For any tip speed ratio the simulated power coefficient can be compared...
with the power coefficient that could be achieved if the rotor speed is set correctly. The deviation of this ratio from unity can be used as a measure of the systems performance. This method is beneficial for the analysis of recorded data. For simulation studies where different control schemes (for output maximisation) are provided with identical wind time series it is sufficient to simply look at the mean generator power.

There are a range of possible performance indices, based on the difference between a desired value and the obtained value. The simplest is to take their ratio and use the standard deviation as a measure of performance. More sophisticated methods involve taking the integral of the error, the integral of the error squared and finally the integral of the error squared multiplied by the time.

In addition to maximising power output, a further objective is to maximise power quality i.e. to minimise the frequency of torque variations. These objectives conflict to some extent. In general, a major reason for reducing torque fluctuations is to reduce the fatigue damage to the machine. The Proven wind turbine has been over engineered to some extent and so emphasis can be placed on power maximisation. The power quality itself however is also an issue and so the level of compromise attained will depend ultimately on how the generator is to be used. If the machine is to be grid linked then the response of the inverter to power fluctuations needs to be accounted for.

Chapter 7 dealt with the power quality issue to some extent by ensuring the two hardware compensators do not behave in a 'bang bang' mode. Ekelund[1,13] has dealt with this compromise by deriving a performance index that encompasses both the level of torque fluctuations and the deviations of generator power from an ideal value. A supposition may be made here that if the switching components in the DC-DC booster are capable of surviving the fluctuations in demanded current then it can be assumed that there are no undue stresses being put onto the wind turbine. The final test will be whether the over current/voltage trip is activated during high winds. If this occurs frequently then some control parameter needs adjusting. Without gain scheduling, a further compromise is necessary. Inevitably the one in a hundred year wind gust will cause the protection circuit to switch. Relaxing the controller gain will mean less optimal energy capture at the lower,
more common wind speeds. The trade off is a difficult one. Most wind turbines do shut themselves down in high winds and so this may be considered as a reasonable price to pay for the increased optimality at the lower wind speeds. The decision to be made therefore is under what conditions it is acceptable for the wind turbine to shut itself down. Making this decision on a theoretical basis is not straightforward. The procedure used here is to analyse the recorded data at the time of any over voltage/current and decide on the basis of that analysis what action, if any, needs to be taken.

A further indicator of performance is the machines' operation at above rated power. The Proven wind turbine used at Loughborough did not achieve its manufacturer's claims. The maximum power output is of the order of 1.8kW. During high winds a gust will cause it to peak to 2.2kW. The machines self governing system is evidently in need of further adjustment. The DC-DC booster was statically tested to 3kW and the protection circuit was set to operate if \( I_{\text{gdc}} > 15A \) or \( V_{\text{out}} > 500V \). The performance of the machine at above rated power is therefore not an issue which affects energy yield significantly.

### 9.4 Exponential Filtering

The monostable '0' in the time interval measurement circuit (figure 2.8, section 2.5.2) is designed to ensure that any noise on the voltage waveform will not cause multiple zero crossings leading to spikes on the rotor speed reading. The time period chosen for this monostable is such as to allow for a maximum measured speed of 45 \( \text{rad/s} \). This limits the effects of voltage spikes for 35ms after the zero crossing but there is still the possibility of spikes occurring after the 35ms time period has elapsed.

All three controllers from Chapter 8 use the measured rotor speed as the basis of their control action. It has been shown (section 6.3) that large positive rates of change in the demanded current can have disastrous effects by causing large voltage spikes which can destroy the switching mosfet. For this reason snubbers and an over voltage/current trip were installed. In addition to these safeguards, there is some low pass filtering of the DC-DC booster gain signal on the current
compensator circuit. A further safeguard would be to filter the rotor speed signal at its source. This may be done using a digital exponential filter which is represented by the equation:

$$\hat{u}_{t+1} = \gamma \hat{u}_t + (1 - \gamma) u_{t+1}$$  \hspace{1cm} (9.8)$$

where $u_{t+1}$ is the speed transducer output at time $t+1$ and $\hat{u}_{t+1}$ is the speed that is used for control. This is a low pass filter with the time constant set according to $\gamma$ which is a number between 0 and 1. Setting $\gamma = 0$ gives an infinite time constant and a flat response whilst $\gamma = 1$ means a zero time constant and the output remaining constant. The choice of $\gamma$ will therefore represent a compromise between achieving fast response and limiting the effects of possible spurious rotor speed readings. There is an additional trade-off which is that the lower the time constant the greater the phase shift will be leading to less robust system control. Figure 9.1 is the Simulink representation of an exponential filter. Choosing a value for $\gamma$ using the Simulink model is problematic because the precise nature of the noise on the rotor speed signal is unknown. Choosing a value for $\gamma$ using the actual hardware is also difficult because its effects will be indistinguishable from the effects of the low pass filter used on the current compensator. If the

Figure 9.1: Speed Transducer Exponential Filter

value for $\gamma$ using the Simulink model is problematic because the precise nature of the noise on the rotor speed signal is unknown. Choosing a value for $\gamma$ using the actual hardware is also difficult because its effects will be indistinguishable from the effects of the low pass filter used on the current compensator. If the
time constant of this filter were reduced, then there is the possibility of damage being caused to the switching mosfet. The procedure used should therefore use the Simulink model to investigate the effects of varying $\gamma$ and then implement the filter on the actual system and investigate the effects of varying $\gamma$ over a smaller range. The Simulink subsystem shown in figure 9.1 can be added to the Simulink model of figure 7.1.

Simulations show that adding band limited white noise and incrementing $\gamma$ shows improvements both in terms of power quality and mean power. With a sample period of $\Delta t = 0.05$ and a power spectral density height of 0.2, as $\gamma$ becomes $> 0.92$ the mean power begins to drop off.

The effectiveness of this filter depends upon how the speed transducer signal is to be used. For feedforward control the demanded current is set according to the rotor speed. The current regulator is a low pass filter which should prevent any high frequency noise on the speed transducer output from reaching the switching mosfet. The exponential filter is virtually a necessity for the wind speed estimator model as the rate of change of speed is used for control.

9.5 Software Control

Section 4.5.2 described the construction of the DC-DC converter hardware compensators. Provision has been made in the form of opto isolators on the mosfet driver boards for direct application of an analogue signal to set the two duty cycles. The data acquisition board used has only one analogue output. Using multiplexing and sample hold electronics this output could be used to directly set the two duty cycles. The generator current and the output voltage could be measured and isolated before being used as reference signals to the PC. The advantage would be that gain scheduling or adaptive type control systems could then be used. In view of the success of the hardware compensators the advantage gained may not be worth the time and energy involved in the development of this system and this has not been implemented.
9.6 Fixed Speed Controller

The objective is obviously to keep the rotor speed constant at a required value. The primary purpose of this controller is to use the measured results to derive a set of performance characteristics. In view of the small size of the machine and high levels of turbulence the objective was satisfied to an extent which was considered reasonable. Simulation showed that the speed stayed in a band ±3% either side of the reference speed for $\omega_{ref} = 30 \text{ rad/s}$. Measured results confirmed this. At lower reference speeds the performance deteriorated but this is expected as the compensators were designed for $\omega_{ref} = 30 \text{ rad/s}$, and of course this can only be maintained if sufficient wind exists.

Although the fixed speed controller is generally thought of as being designed to keep the rotor turning at a constant speed it can still form the basis of more sophisticated control strategies. This can be done simply by allowing the reference speed to be selected according to operating conditions. The controller described in section 8.3.1 is a simple example of this. The reference speed is set every 10 minutes according to the average current generated over that 10 minute period. This can be viewed as being a form of adaptive control with the reference speed set according to the wind speed. If the 10 minute time period were reduced then the design becomes less of a fixed speed controller and more of a tracking controller.

The disadvantage of this system is that the generator current needs to be measured. For the derivation of performance characteristics this was done using the hall effect current transducer. This is feasible but increases the cost to a level which, for a commercial application, may be considered unacceptable. The current is also of course measured for the DC-DC booster with a sensing resistor. The value of this resistor has to be made low so as not to compromise the efficiency of the DC-DC conversion. The resulting signal to noise ratio is rather low, so that extensive filtering is required which inevitably leads to inaccuracies.

The sequence of control is as follows. The measured current is used to set the reference speed which is compared with the measured speed and their difference used to set a demanded current. This is again compared with the same measured current to set the DC-DC converter gain. Although the current measurement is used twice, the first is an average current taken over a relatively long time period.
and the second is common to all control strategies. The errors associated with the current measurement are accounted for in the simulation model by adding random noise to the current transducer output. The level and bandwidth of the noise were set to values commensurate with those typically resembling the noise generated by the sensing resistor as measured directly with a current probe.

A lot of time and effort could be spent at this stage of the project on applying various wind time series to the model, deriving a set of performance indices and varying each parameter in turn. The linearised model must be used for each reference speed to derive the set point for the compensator parameters about which each is varied in turn. Plotting the parameter values against a performance index will in some cases show a parameter value for which the performance index is minimised. The problem is that it is impossible to test for all combinations of parameter values. In addition, the procedure resulting in an improved set of parameters, once implemented, will result in an improved set of performance characteristics which must then be used to update the simulation model, at which point the entire process could be repeated.

Initially the medium mean wind speed and medium turbulence wind time series was used as the model input. The model was run for various reference speeds. Plotting the generator power against wind speed showed clearly how as the reference speed was increased the generator power increases to a maximum and then decreases. It is clear therefore that unless the reference speed can be adjusted automatically the fixed speed controller will only ever be optimal at a particular wind speed. This is entirely as expected from consideration of the $C_p/\lambda$ curve.

Figure 9.2 is the resulting rotor speed time series for 2 reference speeds ($\omega_{ref} = 32$ and $\omega_{ref} = 26$ rad/s) using the same wind time series. It can be seen that the variance is higher for the lower rotor speed. This exemplifies the fact that for fixed speed operation the controller needs to be tuned for that particular reference speed. This was done in Chapters 5 and 6 for $\omega_{ref} = 30$ rad/s. In this particular case the generated power is highest for the higher rotor speed but for lower mean wind speeds it could be the other way round.
Figure 9.2: Rotor Speed Time Series for 2 Reference Speeds.

9.7 Feedforward Controller

This controller is a type of tracking regulator. The control system is reduced to two controllers. The first sets the gain of the DC-DC booster according to the difference between the demanded current and the measured current. The second sets the output load conductance according to the difference between the desired output voltage and the measured output voltage. The speed compensator is replaced with a specific control law as derived in section 8.6. This relationship is used to calculate a desired current solely on the basis of measured speed. It can be classified as feed-forward since the system output is not compared with any other state variable before being used as the basis of the system's input.

The standard Proven controller is an example of this type of controller. It uses the generator voltage to set the load. From a telephone conversation with the manufacturer, it was identified that the load cut in at 90V and saturated at 110V, with a linear relationship between these two levels.

Both measured and simulated results show that the tracking achieved (i.e. the performance of the two hardware controllers) is near ideal and difficult to improve upon. Plotting the rotor speed against the generator DC current and superimpos-
ing the chosen \( I_d \) versus \( \omega \) relationship shows them to be virtually coincident. The emphasis for improving energy transfer must therefore be placed upon the selection of the relationship between the desired current and the rotor speed. Section 8.6 discussed 3 types of feedforward controllers.

### 9.7.1 Basic Square Law Relationship

Section 8.6.1 showed that for the simplest implementation \( I_d = 0.0058 \omega^2 \). Using this relationship will give a demanded current at rated speed of \( \approx 6A \). At this speed and current the machine is operating at 60\% of its rated power. For the wind time series used, the mean generated power is 515 W.

To generate rated power at rated speed the square law relationship would be \( I_d = 0.011 \omega^2 \). The results of this control law are much lower rotor speeds (\(< 20 \text{ rad/s}\) as opposed to \(> 30 \text{ rad/s}\) for the first relationship). The consequence of this lower rotor speed is a drop in the mean generated power to 298 W. Neither of these equations can therefore be considered as being acceptable and they are far from optimal.

### 9.7.2 Including Frictional Effects

Using equation 8.11, which includes the frictional effects, has a similar result to the second square law relationship. Too much load is being applied at the lower rotor speeds which means the machine is not capable of reaching its ideal operating speed. The mean generated power is however improved marginally, as might be expected, for the standard input time series to 306 W.

### 9.7.3 Including Effects of Passive Pitching

Implementing equations 8.13 and 8.14 as a Simulink look up table results in a considerable improvement in controller performance. The mean generated power is now 1029 W for the same wind time series. The reason for the doubling in mean generated power is that the machine is allowed to operate at higher rotor speeds where the efficiency in terms of wind energy extraction and mechanical to
electrical energy conversion is much greater. This increase more than justifies the
work required to identify these functions of rotor speed.

![Graph showing derived Id vs n relationships](image)

Figure 9.3: Derived $I_d$ vs $\omega$ relationships

Figure 9.3 shows the 3 derived $I_d$ versus $\omega$ relationships. At $\omega = 18$ rad/s the top
graph is the square law relationship. The next down is the square law relation­
ship including frictional effects. The bottom graph includes the effects of passive
pitching. Neither of the first two can be considered as suitable for this machine
because of the higher demanded current at the low rotor speeds which effectively
prevents the machine from reaching the higher rotor speeds where the efficiency is
greater. It is possible that a lower cut-in wind speed could be achieved by further
reducing the gradient of the function for low rotor speeds. A steeper function at
high rotor speeds could also be beneficial.

9.8 Wind Speed Estimator

If the wind speed at the rotor were known in real time, it could be used together
with the measured rotor speed and a knowledge of the turbine performance char­
acteristics to set the load conductance to achieve a rotor speed giving the ideal
tip speed ratio.
Optimal control strategies are often implemented by running a software representation of the system in parallel with the plant that is being controlled. The basic outline in this case is shown in figure 9.4. It has been shown theoretically, that if the instantaneous values for the rotor speed, the rate of change of rotor speed and the generator current are known, then the wind speed can be calculated. This is effectively using the wind turbine itself as an anemometer. The wind speed so estimated, can then be used to set the 'ideal' load conductance. There is the opportunity of further enhancing this system by using a state estimator to predict the wind speed in the short term. How far ahead this prediction can be considered as valid will depend upon the wind turbulence, the noise levels and the sampling period.

Measurable parameters are input to this model and unmeasurable parameters are estimated. Outputs are compared and any differences between them are used to adjust the estimators parameters.

This method was not physically implemented but a Simulink model for the wind speed estimator was derived. Implementation of this control strategy would in-
volve writing in Pascal the representation for the model. The program would be run on the data acquisition and control computer. This may seem counterproductive as the cost of a PC is obviously greater than the cost of an anemometer. Once implemented however the entire program could be put onto an eprom thus bringing the cost down though the initial development costs would be high.

Figure 9.5: Wind Speed Estimator Block Diagram

Figure 9.5 shows a potential control system outline for the wind speed estimator. The wind turbine and the DC-DC booster are represented as a single block with the wind speed and demanded current as its inputs and the rotor speed and generated current as its outputs. The transducers are represented as low pass filters, each with an additional input representing noise. The outputs from the transducers are used to estimate the aerodynamic torque which together with the rotor speed estimate is input to a look up table. The output from this table (its derivation to be described) will be the wind speed estimate \( V_{est} \). This can then be used to calculate an estimate for the tip speed ratio. The ideal tip speed ratio can be represented as being a function of rotor speed (equation 8.13). This can be compared with the estimated tip speed ratio and their difference used as the input to a PID controller which will provide a value for the demanded current.
The optimality of this process will depend upon the accuracy of the wind speed estimate which in turn depends on the accuracy with which the turbines aerodynamics have been characterised. It will also depend upon the selection of the parameters for the PID controller.

9.8.1 Derivation of Wind Speed Estimator Look Up Table

As already discussed, look up tables can speed up implementation. In this case the table consists of a grid with each square containing a value for the wind speed. The x axis is the turbine speed in 0.5 rad/s steps from 16 to 34 rad/s and the y axis is the aerodynamic torque in 0.5 Nm steps from 0 to 50 Nm.

The program sets the wind speed and the load conductance and runs the model until a steady state condition (rate of change of speed squared less than 1) is reached. The resulting speed and torque are then used as the co-ordinates which define where that wind speed is placed. A two dimensional plot of the results is shown in figure 9.6. The gaps in the table have been filled by fitting a polynomial to the non-zero elements in each row and using it to interpolate and extrapolate torque values. Each polynomial will be the relation between the wind velocity

Figure 9.6: Wind Speed Estimator Look Up Table
and the aerodynamic torque at a particular speed.

### 9.8.2 Simulation of Wind Speed Estimator Model

Using the derivative Simulink block, in the absence of noise, the estimated and input wind speeds are virtually identical. To represent the speed transducer dynamics, the derivative block must be replaced with a delay block, a summing junction and a gain representing the equation: \( \frac{d\omega}{dt} = \frac{\omega_n - \omega_{n-1}}{\Delta t} \). The effects of the speed transducer were approximated by using a constant \( \Delta t = 0.05\,\text{s} \).

The effects of varying \( \Delta t \) only become apparent for \( \Delta t > 0.1 \). This indicates that for a successful wind speed estimate, in the absence of any noise, the sampling frequency must be greater than 10 Hz. The speed transducer output is latched and so, although it can be sampled at any chosen frequency, its output will only change at a rate determined by the rotor speed. The time interval between zero crossings therefore effectively determines the system's minimum sampling time. At 30 rad/s, \( \Delta t = 0.05 \) and at 16 rad/s \( \Delta t = 0.098 \). The latter is very close to the maximum sampling period. Simulation showed that at low rotor speeds the wind speed estimate is less accurate. At rotor speeds less than 16 rad/s there is a large occurrence of spurious noise on the wind speed estimate which could result in inappropriate loads being applied. There would therefore need to be a section added to the control software which prevents any demanded current signal from reaching the current compensator if \( \omega < 20 \,\text{rad/s} \). This is not viewed as a particular problem as very little power is available at these lower rotor speeds (see figure 9.3).

The result of adding noise to the transducer outputs clearly demonstrate the fact that to rely solely on an estimate of the tip speed ratio to set the demanded current would create a very unrobust system.

When this method is combined with a feed-forward controller the detrimental effects of noise and long sampling periods are diminished to an extent. Whether this system is more optimal than the feed-forward controller alone and whether it justifies the high increase in complexity will depend upon the levels of noise and the setting of the PID parameters.
9.8.3 Tuning the Wind Speed Estimator PID Controller

It must first be assumed that the wind speed estimate and consequent tip speed ratio estimate are accurate. The validity of this assumption depends, as discussed above, to a large extent on the expected noise levels. Using an exponential filter on the rotor speed measurement alleviates the effects of noise to some extent as discussed in section 9.4, but it also reduces the response time.

The demanded current can be calculated as a combination of the look up table output and the PID controller output:

$$I_d = f(\omega) + PID(\lambda_{est} - \lambda_{opt})$$  \hspace{1cm} (9.9)

A standard Zeigler Nicholls approach to gain setting may be used by putting a step input on the wind speed and for proportional control only, increasing the gain until the onset of instability. Standard formulae\(^{[9.2]}\) can be used to calculate parameters for the PID controller based on the values of \(k_{crit}\) and \(P_{crit}\) where \(k_{crit}\) is the limiting value of gain for stability and \(P_{crit}\), is the time period of oscillation. Implementation of this approach gives:

$$k_p = 0.5k_{crit}$$
$$k_p = 0.45k_{crit} \quad T_i = 0.83P_{crit}$$ \hspace{1cm} for P control
$$k_p = 0.6k_{crit} \quad T_i = 0.5P_{crit} \quad T_d = 0.125P_{crit}$$ \hspace{1cm} for P+I control

$$k_p = 0.6k_{crit}$$
$$T_i = 0.5P_{crit} \quad T_d = 0.125P_{crit}$$ \hspace{1cm} for P+I+D control

(9.10)

The problem with this method is that the mean wind speed has a major effect on \(k_{crit}\). For a mean wind speed change from 8 to 6 m/s, \(k_{crit} = 0.45, P_{crit} = 5.4\) s. For a wind speed change from 10 to 8 m/s, \(k_{crit} = 1.24, P_{crit} = 5.2\) s. Note that the step change must be negative because \(k_{crit}\) increases with wind speed. This confirms the conclusions of section 6.5.3 where it was shown that the systems robustness increases with wind speed. With fixed PID parameters this control system can only ever be optimal at one particular wind speed. The system is therefore a prime candidate for gain scheduling.

In conclusion it can be stated that extensive field trials would be necessary to perfect a control strategy based solely on an estimate of the wind speed. If this method were combined with feed-forward control and a self tuning controller de-
signed, then there may be some benefits. Its major disadvantage is its susceptibility to disturbances both from noise and from turbulent wind conditions. The Loughborough University wind turbine is situated in a highly turbulent area and the DC-DC booster used generates high levels of noise. Design of a wind speed estimator in this instance would therefore be problematic.

9.9 Hill Climbing Control

With the exception of the basic feed-forward controller the foregoing controllers have relied on the characterisation of the wind turbine and in particular on the derivation of a set of aerodynamic performance characteristics. Hill climbing attempts to remedy this drawback by using a very simple adaptive type control scheme. The generator current and voltage are measured and the power calculated. A step change in load resistance will either increase or decrease this generated power. If the step change results in an increase then a further step is taken in the same direction. If power decreases then the direction is reversed. Tanaka[9.3] used this method for the control of a small (100W) wind turbine. His conclusions are that it showed benefits over the constant load case but that it was very intolerant of wind turbulence. For larger machines this problem would be exacerbated as their larger inertia means that the time step would have to be increased to account for energy stored in the inertia. As the time step is increased the system becomes even more prone to the detrimental effects of wind turbulence. Use of this method as an exclusive form of control can therefore be discounted.

9.10 Control Strategy Comparison

Each of the control systems considered have advantages and disadvantages in terms of energy capture, power quality and ease of implementation. The fixed speed controller achieved its purpose which was to acquire data for the derivation of a set of performance characteristics. Numerous studies (section 1.1) have shown that in terms of energy capture, variable speed operation is
superior and the data recorded has substantiated this. The reason for using a DC link is to avoid having to run the machine at fixed speed, in other words it was assumed at the outset that it is less efficient in terms of energy capture. Adjusting the reference speed according to operating conditions showed some improvement but essentially this can not be considered as fixed speed operation. The minor disadvantage of this system is that a reasonably accurate measurement of generator current is required.

The feed-forward controller investigated constitutes a very simple approach to system control. Its ease of implementation is its main advantage. The other advantage is that it is less dependant upon the current measurement. The key challenge is the derivation of the demanded current versus rotor speed relationship. It has been demonstrated that the approach which included frictional effects, and the effects of the passive pitching mechanism, gave the maximum energy capture and it was realised that further improvements could be made by manipulating the curve of figure 9.3 so that it had still lower gradient at low rotor speeds and a higher gradient at the high rotor speeds.

If the wind speed as measured by the boom anemometer were corrected and used as the basis of control, then the load conductance could be set before the machine has had a chance to react to any changes in wind speed. Theoretically this would represent an ideal control system. Regrettably this approach is regarded as being too expensive and unreliable for a small stand alone wind turbine such as the Proven.

If a reliable wind speed estimate could be provided then the wind speed estimator model would be attractive. It has been shown that estimating the wind speed is possible using rotor speed, the rate of change of rotor speed and the generator current measurement. Simulations show that as noise is added the effectiveness of this approach quickly degrades. Using an exponential filter on the rotor speed signal and a low pass filter on the generator current signal helps noise removal. Inclusion of these filters however degrades the performance of the control system compensators and so the selection of their time constants is a compromise between sacrificing some performance whilst maintaining a sufficiently accurate representation of the wind speed. The design of this control system using Simulink is difficult
because the precise nature of the noise is unknown. A further disadvantage of the wind speed estimator model is its reliance on accurate performance characteristics. The difference between the $C_p/\lambda$ curves from NEL and those derived for the CREST machine demonstrate how much variability there can be between different examples of the same commercially produced turbine. These differences were assumed to be caused by the starting pitch angle for the blades on the two machines being different. Successful implementation of this strategy would involve analysis of long periods of recorded data, and parameter adjustment.

Combining the wind speed estimator with the feed-forward controller shows promise but again the question is whether the ends justify the means. The tuning of the PID controller for the wind speed estimator is such that it is only ever going to be optimal at one particular wind speed and so the use of gain scheduling would be virtually essential in this application.

To conclude, the feed-forward controller using the look up table is the most effective control system in terms of energy capture and it is straightforward to implement. Its performance could be surpassed by the wind speed estimator if time and effort were spent on the following:

i) Accurate characterisation. Ideally done in real time to cater for manufacturers tolerances and different blade setup.

ii) Noise minimisation.

iii) Optimal gain scheduling.

iv) Wind state prediction.

If an accurate wind speed estimator could be designed, it could most effectively be used in combination with the feed-forward controller. The advantage of this combined approach is that lower gains would be required for the feed-back term which increases the stability margin.

As always in engineering, it can be expected that there will be differences between theory and practice. Actual implementation and experimental evaluation of the variable speed controller has been left for the final chapter.
Chapter 10

Final Results, Conclusions and Suggestions for Further Work

10.1 Summary of Results

The work presented so far has resulted in a functional power electronic interface and a partially validated simulation model. It has also shown how an optimal control strategy is defined and implemented. Whether this control strategy uses the most ideal demanded current versus rotor speed relationship, and whether the controllers have been optimally designed, is best determined by a prolonged measurement campaign.

At the end of this chapter are a set of time series sampled at four second intervals, recorded over a 13.3 minute period. The controller implemented uses the look-up table of demanded current versus rotor speed derived in Chapter 9.

The measured wind speed, corrected using the method outlined in Chapter 8 is shown in figure 10.1. From this it can be seen that there are high levels of turbulence typical for this site at Loughborough. As already mentioned the wind turbine is surrounded by buildings and trees at a comparable height.

The recorded (corrected) wind time series is used as the input to the unified model so that the resulting simulated time series can be compared with the measured results. The Simulink model uses cubic spline interpolation to derive values for the wind speed between the 4 second samples. The model is therefore not expected
to follow high frequency variations.

Figure 10.2 shows the simulated and measured rotor speed. The lower time series is their difference (simulated minus measured). Below rotor speeds of $20 \text{ rad/s}$ the model has some difficulty in correctly predicting the rotor speed. This is to be expected since at low wind speeds any model errors become increasingly significant. This is not viewed as being a significant problem since limited power is available under these conditions, reassuringly, the accuracy of the model increases with increasing wind speed. In spite of this, the error averages $4 \text{ rad/s}$ over the sample period which corresponds to approximately $17\%$. The measured rotor speed has a greater high frequency component than the model is able to reproduce. This reflects the difficulty with which the model deals with the effects of turbulence.

Figures 10.3 and 10.4 show the simulated and measured demanded current and generator current respectively. On first inspection it may appear that the demanded current is unrelated to the generator current. For instance at time $t = 500\text{s}$ a full load current is demanded. This is due to the measured rotor speed increasing rapidly to $35 \text{ rad/s}$. The simulated rotor speed does not react so fast and only reaches $25 \text{ rad/s}$ and so the simulated demanded current is less than that measured. The measured generator current increases but before it has had a chance to become equal to the demanded current, the wind has dropped and the rotor begins to decelerate rapidly because of the persistence of the high demanded current due to the low pass filtering of the current compensator. As a result of the rapid deceleration, the demanded current drops to zero allowing the machine to begin speeding up again.

Figure 10.5 shows the simulated and the measured output voltage. The output voltage is equal to the DC generator voltage and varies with the rotor speed at below cut-in wind speed since there is no gain being applied to the DC-DC booster. It remains fixed at $400 \text{ V}$ for lengths of time corresponding to the wind speed being above cut-in. The simulated output voltage shows better performance in terms of the frequency of deviations from $400 \text{ V}$. There could be a number of reasons for this. Most likely is the cumulative effect of small errors from each stage of the model. The model shows some overshoot which is equal to that permitted in the design process of Chapter 6. The fact that the measured output voltage does not
show this overshoot is most likely due to the 4 second sampling period acting as a low pass filter.

At wind speeds below cut-in, the simulated output voltage is not exactly coincident with that measured. This does not necessarily mean the model is deficient since, as already mentioned, below cut-in wind speed accuracy is not such a great issue. The simulated and measured generator power time series (figure 10.6) are very similar to the output current time series since the only time any power is being generated is when the output voltage is held constant at 400V. The measured output power appears to have a greater high frequency component. One reason for this is that the sampling period is different for each. A more accurate method of establishing the similarity between the two is to integrate over time. Figure 10.7 shows the simulated and measured energy yield time series. The two plots are very similar which re-emphasises the point made earlier concerning the insignificance of the low accuracy of the model at below cut-in wind speeds. The difference between output power and generated power give an efficiency for the DC-DC booster of approximately 85%. This is consistent with that predicted in Chapter 4.

Figures 10.8 to 10.13 present selected relationships of interest between the key parameters.

Figure 10.8 is the demanded current versus rotor speed. The demanded current is the value read from the look up table corresponding to the measured rotor speed. Ideally this graph would be an exact copy of the look up table used (figure 8.13). For each measured rotor speed sample a corresponding value for the demanded current is derived from the look up table. A spurious rotor speed reading will therefore result in a correspondingly spurious recorded demanded current. Deviations can only be caused by errors introduced by the sampling process, by quantisation or by errors in the data acquisition process. False readings are known to be caused by spurious zero crossings due to voltage spikes. Their removal with further filtering would degrade the dynamic performance of the controller and a compromise was reached in Chapter 6. A commercial implementation would do well to concentrate on an intrinsically low noise system. However this is an EMC problem and outside the scope of this thesis.

Figure 10.9 shows the generator current versus rotor speed. This should ideally
be the same as the demanded current. There are a number of data points that are not on the specified relationship between rotor speed and generator current. These spurious data points represent periods where the generator current is less than its optimal value. The nature by which the rotor speed is measured means that erroneous measurements are always greater than the true value. In addition the percentage error is greater at the lower speeds because monostable '0' (figure 2.9) holds its output for 35 ms which means the maximum measurable rotor speed is 45 rad/s (section 2.5.2). The effect of these errors are more easily seen using figure 10.10 which shows the demanded current versus the generator current. The errors are above the trendline which maps $I_d$ to $I_{gdc}$ i.e. there are a number of samples for which the demanded current is greater than the generator current. If this graph is plotted as a movie sequence with the most recent points joined by a line of a different colour then it can clearly be seen that the majority of the errors occur during times of low angular velocity. A positive speed error gives a positive demanded current error which cannot be met by the wind turbine as there is insufficient energy available in the wind at that time. Further proof of this effect can be seen by plotting the demanded current against time over a small time period. There are spikes which can only be attributed to the presence of noise.

Ideally all the points on figure 10.10 would lie on a line at 45° to the horizontal. The fact that the gradient is less than one is indicative of steady state error (i.e. the lack of any integral action in the current regulator circuit, for reasons discussed in Chapter 6). This means that the non-linearity of the opto-isolator used in the current regulator will have a strong influence on the generator current / demanded current relationship. At higher rotor speeds (higher demanded currents), the gain of the current regulator is higher. The result is that the actual generator current is greater than that demanded at rotor speeds greater than 24 rad/s. Section 9.6 showed that the theoretical derivation of the ideal current versus rotor speed did not have a sufficiently high gradient at the higher rotor speeds. The non-linearity can therefore be said to enhance the performance by providing that higher gradient. A precise analysis of this effect is necessary so as it may be accounted for once the method for theoretically deriving the ideal current / speed
relationship is perfected, but this would be future work.

Figure 10.11 shows the output power versus wind velocity. The points marked 'o' are the results of binning the data in this graph. The points marked 'EI' are the results of simple extrapolation of the binned results. This gives an output power of 2.2kW at a wind velocity of 11.4 m/s. Taking the losses of the DC-DC booster into account means that this is in excess of that predicted by the manufacturer. In practice however the passive pitching mechanism reduces the generator power at the higher wind speeds and so a simple extrapolation is not really justifiable.

Figures 10.12 and 10.13 show the tip speed ratio error versus rotor speed and wind speed respectively. The TSR error is the difference between the actual tip speed ratio as calculated from measured wind velocity and rotor speed, and the ideal tip speed ratio which is a function of the measured rotor speed. It can be seen that the errors are only of any significant size outside the working range i.e. for less than a 4m/s wind velocity and for a rotor speed of less than 20 rad/s. The lack of integral action combined with increased errors due to the sampling process can be held responsible for increased errors at lower wind / rotor speeds. At rotor speeds above 22 rad/s and wind speeds above 5.5 m/s the TSR error becomes negative i.e. the actual TSR is to the right of the ideal TSR. This confirms the robustness analysis of section 6.5 where it was shown that the system becomes more robust as the wind speed and rotor speed increase. In a sense this feature is beneficial. Chapter 8 showed that the peaks of the $C_p/\lambda$ and $C_q/\lambda$ curves occur at similar TSR's. This means that to operate the wind turbine at the peak of the $C_p/\lambda$ curve means that deviations of TSR may cause the system to move to the left of the $C_q/\lambda$ peak thereby inducing instability. This would be especially problematic at the higher wind speeds where the system is operating at peak power. The fact that the measured TSR increases with wind speed ensures that the system remains stable. Inevitably there is a compromise here between energy yield and the potential onset of instability. There is perhaps some room for improvement and this could form the basis of useful future work. 10 minute averaged data recorded over long periods of time would be required to analyse the effects of key parameter variations.

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10.2 General Conclusions and Further Work

10.2.1 Statement of Contribution by the Author
The Proven generator was coupled to a DC motor, instrumentation installed and a mathematical model derived. The aerodynamics were initially modelled using a set of data from the National Engineering Laboratory. A DC-DC booster was designed and constructed. Voltage and current control regulators were designed using linearisation techniques together with classical control system methods. A speed transducer was designed and constructed which used the zero crossings from the generator voltage waveform to derive a digital representation of the rotor speed. Once the machine became operational, the system was tested, and recorded data was used to further validate the model. The model was then used to assess the design of the DC-DC booster control electronics. The machine was run at fixed speed and a more accurate (and appropriate) set of performance characteristics was defined. These were used to construct a more accurate model. A set of synthesised wind time series were used with this model to test and enhance several variable speed control system designs. An improved controller was implemented and evaluated by experiment. The resulting recorded data was used to further validate the model.

10.2.2 Conclusions
The conclusions of both simulations and observations is that the feed-forward controller is the most appropriate form of control both in ease of implementation and its reaction to turbulent wind conditions. Feed-forward control simply involves calculating the load conductance as a function of the rotor speed. This method worked by allowing the dominant characteristics of the passive changes in blade geometry along with the machines inertia to determine the rotor speed. This rotor speed was in turn used to set the load conductance to an appropriate value for those wind conditions.
This method of control effectively uses the wind turbine as an energy store. Energy is released when the wind speed drops and absorbed when it rises. The disadvantage is that the controlled variable reacts to the effect the wind speed has
on the turbine. In theory therefore, control based on the wind speed estimator should outperform the feed-forward controller. This has been shown not to be the case due to the increased reliance on accurate rotor speed readings. In view of this, the most ideal method may be a combination of wind speed estimation and feed-forward control. The control algorithm for the implementation of this method would be computationally intensive (section 9.7) and it is left as a follow on project. Whether the benefits would be justified by the costs would have to be carefully considered.

A fundamental feature of the control system is that there is no integral action on the demanded current / measured current error. It was found that its inclusion would lead to a less robust system. This means the non-linearity of the opto-isolator used in the current regulator has a profound effect in that the gain of the current compensator reduces proportionally with the error. The effects of this were minimised using offset and gain adjustments but a limit is reached due to the signal to noise ratio becoming unacceptably low.

This project has involved the control of a wind turbine situated in what would ordinarily be classified as an unacceptable location. Figure 10.14 shows a plan view with the wind turbine (marked WT) situated midway between two buildings 70m apart. The wind turbine is on a 12.5m tower and the buildings are approximately 10m high. In addition (not shown in figure 10.14) there is a group of trees also 10m high placed just to the north of the wind turbine. The resulting highly turbulent wind conditions make the control an especially difficult task, and for more favourable placed wind turbines some of the constraint imposed by the turbulence on the design process may be relaxed.

The successful operation of the system has been achieved. A proven capability for tracking a demanded current means that the emphasis for an optimised control system can be placed on the derivation of the current versus speed relationship. The results are weakened by the errors involved in assessing the true wind speed since this is used for the derivation of the performance characteristics.

A system model of proven accuracy (figure 10.7) has been developed, using Matlab/Simulink, which provides an important tool for controller design.
10.2.3 Further Work

Final tuning of the current speed relationship should be made using the results from a sustained measurement campaign. This will involve deriving a set of performance characteristics which can be used to derive a current versus speed relationship. This should then be implemented and the process repeated. Each improvement can be assessed using power performance analysis techniques which can simply involve plotting the generator power versus the wind speed. If each data set is taken over a long enough period then the effects of air pressure, temperature, rainfall etc. will be averaged out.

The next stage of this project will be the design and construction of a grid-linked inverter. This will use the 400V DC link voltage as its power input. The demanded current signal can then be used to determine the power that is generated into the grid.
Figure 10.1: Corrected Wind Speed Time Series
Figure 10.2: Rotor Speed Time Series. Simulated (x) and Measured. (Lower Trace is their Difference)
Figure 10.3: Demanded Current Time Series. Simulated (x) and Measured.
Figure 10.4: Generator Current Time Series. Simulated (x) and Measured.
Figure 10.5: Output Voltage Time Series. Simulated (x) and Measured.
Figure 10.6: Simulated (x) and Measured Output Power
Figure 10.7: Simulated (dashed) and Measured Energy Yield Time Series.
Figure 10.8: Demanded Current Versus Rotor Speed
Figure 10.9: Generated Current Versus Rotor Speed
Figure 10.10: Demanded Current Versus Generator Current
Figure 10.11: Generator Power Versus Wind Velocity
Figure 10.12: Tip Speed Ratio Error Versus Rotor Speed
Figure 10.13: Tip Speed Ratio Error Versus Wind Speed

Figure 10.14: Plan View of Wind Turbine Location
References

1.2 Spooner E.; Direct Drive, Grid Connected, Modular Permanent Magnet Generators. Proc BWEA conf 1994.
1.3 Spooner E.; Modular Permanent Magnet Wind Turbine Generators. Industry Applications 31st conf 1996.
1.4 Freris L.; Wind Energy Conversion Systems. Prentice Hall.
1.7 Leithead W.; Role and Objectives of Control for Wind Turbines. IEE Proc Vol 138 pt C, No 2.
1.20 Holley W,: Optimal Quasistatic Control of Variable Speed Wind Turbines. AWEA proc 1989.
2.1 Adventech Users Manual.
Appendix A

Hall Effect Transducers

Voltage Transducer

Current Transducer

Power Supply
Appendix B

Time Interval Measurement
Circuit PCB Layout
Appendix C

Calculation of the Moment of Inertia of the Proven Rotor

\[ \rho = \text{Material Density} \]
\[ P = \text{Perimeter of section} \]
\[ C_h = \text{Chord at root} \]
\[ C_t = \text{Chord at tip} \]
\[ C_r = \text{Chord at distance } r \text{ from root} \]
\[ t = \text{Skin thickness} \]
\[ dM_r = \text{Mass of shaded portion at distance } r \text{ from root} \]
\[ dr = \text{width of shaded portion} \]
\[ M = \text{Blade mass} \]
\[ \mu = \text{Mass per unit length} \]
\[ r_m = \text{Distance of centre of mass from root} \]
$I_0 = \text{Moment of inertia about own centre of mass}$

$$C_r = C_h \left( 1 + \left( \frac{C_o - 1}{L} \right) r \right) = C_h (1 + C_r r)$$

where $C_o = \frac{C_h}{C_h}$

$$dM_r = \rho PtC_r dr$$

$$M = \int_0^L dM_r = \frac{1}{2} \mu L (1 + C_o)$$

$1^{st}$ moment about root $= \int_0^L r dm = \frac{1}{6} \mu L^2 (1 + 2C_o)$

Centre of mass is at a distance $r_m$ from root.

$$r_m = \frac{1^{st} \text{ moment}}{\text{Mass}} = \frac{1}{3} L \left( \frac{1 + 2C_o}{1 + C_o} \right)$$

$r_m$ was found experimentally by balancing the blade and measuring from the root.

measured $r_m = 0.64m$

calculated $r_m = 0.69m$

The difference is due to the aerofoil section not being extended to the root.

$2^{nd}$ moment about root $= \int_0^L r^2 dm = \frac{1}{12} \mu L^3 (1 + 3C_o)$

Mass moment of inertia $= 2^{nd} \text{ moment} - \left( \frac{(1^{st} \text{ moment})^2}{\text{Mass}} \right)$

$$= \frac{1}{36} \mu L^3 \left( \frac{1 + 4C_o + C_o^2}{1 + C_o} \right) = I_{yy}$$

$$\frac{I_{yy}}{M} = \frac{1}{18} L^2 \left( \frac{1 + 4C_o + C_o^2}{1 + 2C_o + C_o^2} \right)$$

Moment of inertia about own centre of mass $= \frac{1}{18} ML^2 \left( \frac{1 + 4C_o + C_o^2}{1 + 2C_o + C_o^2} \right)$

Moment of inertia about axis not passing through centre of mass

$I = I_0 + MR^2$

Blade

$I_{\text{blade}} = 1.828 \text{Kg} \text{m}^2$

Plate A

$I_{\text{plate A}} = MR^2 = 0.031 \text{Kg} \text{m}^2$

Plate B

$I_{\text{plate B}} = 0.218 \text{Kg} \text{m}^2$

Spring

$I_{\text{spring}} = 0.032$

The total inertia is the sum of all constituent parts.
\[ J_{\text{rot}} = 6.11Kgm^2 \]
%accal Derives a relationship between angular velocity and the rate of change of angular velocity.
count = 1;
for t=0:0.1:15,
    wm(count)=(0.101*t*t)-(8.6368*t)+50;
    if wm(count)<0
        radsm(count)=0;
    end
    wmdot(count)=(0.202*t)-8.6368;
    wmt(count)=(0.1278*t*t)-(5.4768*t)+50;
    wmtdot(count)=(0.2556*t)-5.4768;
    time(count)=t; count=count+1;
end
coeffdm=polyfit(wm,wmdot,1);
fitm=polyval(coeffdm,wm);
coeffdmt=polyfit(wmt,wmtdot,1);
fitmt=polyval(coeffdmt,wmt);
coeffdtn=coeffdmt*2.52;
coeffdnn=coeffdm*0.16;
coeffdt=coeffdtn/2.36;
fitt=polyval(coeffdt,wmt);
wtcal3;
plot(time,wt)
end

%wtcal3 Subroutine of Accal.m.
wt(1)=50;
time(1)=0;
count=2;
for t=0:0.01:15,
    wtdot=(coeffdt(1)*wt(count-1))-coeffdt(2);
    wt(count)=wt(count-1)-(wtdot*0.01);
end
time(count)=t;
count=count+1;
end
end

%sswsdet Calculates the steady state solution for the generator model with a given torque and load resistance.
vw=[0,100,-40,10];
vi=[0,100,-10,40];
kg1=3;kg2=9;rl=50;rg=1.6;
a1=-1.72e-3;a2=0.247;a3=4.145;
z=rg;L=8e-3;
for count = 1:93,
  diff=1;
  wtemp=0;
  if count> 27,
    rl=25;
  end
  if count> 58,
    rl=16.67;
  end
  numb(count)=0;
  while diff>0.0001
    kt = (kg1*kg2)/(rl + z);
    A = -a1;
    B = -a2 - (kt);
    C = (stres2c(count,3))-a3;
    qua = (B^2) - (4*A*C);
    w(count) = (-B - sqrt(qua))/(2*A);
    z = sqrt((4*w(count)*L)^2 + 2.56);
    diff = w(count) - wtemp;
    wtemp = w(count);
    numb(count)=numb(count)+1;
  end;
  Ig(count) = (kg1*w(count))/(rl+z);
  wdiff(count) = w(count) - stres2c(count,1);
  Idiff(count) = Ig(count) - stres2c(count,2);
  wper(count) = (wdiff(count)/stres2c(count,1))*100;
  iper(count) = (Idiff(count)/stres2c(count,2))*100;
end
subplot(2,1,1);
plot(wper,'.');
ylabel('rad/s % error');
grid axis(vw) subplot(2,1,2);
plot(iper,'.');
ylabel('amps % error');
grid axis(vi)
end
%statst.m  Uses the recorded steady state data as the input to
the Simulink model.
Ierr=0;werr=0;
set_param('genmd1/cond','Value','0.02');
for count = 1:93,
    if count > 27,
        set_param('genmd1/cond','Value','0.04');
    end
    if count > 58,
        set_param('genmd1/cond','Value','0.06');
    end
set_param('genmd1/Torque','Value',stres2c(count,3));
linsim('genmd1', [0,30])  %Simulates for 30s.
Idiff(count) = Igrms(length(Igrms)) - stres2c(count,2);
Iper(count) = (Idiff(count)*100)/stres2c(count,2);
wdiff(count) = w(length(w)) - stres2c(count,1);
wper(count) = (wdiff(count)*100)/stres2c(count,1);
end
end

%cflit.m  Derives Cp lambda curves from NEL data by drawing
lines on the cp lambda graph and removing all data that lies on one
side of the line.
[m,n] = size(cp);
kc = (0.3 - 0.025)/(3.5 - 11.8);
cc = 0.025 - (kc*11.8);
kd = 0.3/(6.4 - 10);
cd = 0.025 - (kd*10);
kh = 0.25/(6 - 3.5);
ch = 0.025 - (kh*3.5);
kg = 0.25/(2 - 11);
cg = 0.025 - (kg*11);
kf = 0.25/(5 - 9);
cf = 0.025 - (kf*9);
ke = 0.3/(8.4 - 0);
ce = 0.025 - (ke*0);
for count = 1:m,
    if cp(count) < 0.025;
        cp(count) = 0;
        tsr(count) = 0;
    end;
    if tsr(count) < 2;
        cp(count) = 0;
        tsr(count) = 0;
    end;
    if cp(count) > 0.3;
        cp(count) = 0;
        tsr(count) = 0;
end
end;
if tsr(count) > 11;
    cp(count) = 0;
    tsr(count) = 0;
end;
deltx = tsr(count)-((cp(count)-cc)/kc);
delty = cp(count)-((tsr(count)*kc)+cc);
if (deltx > 0) & (delty > 0);
    cp(count) = 0;
    tsr(count) = 0;
end;
deltx = tsr(count)-((cp(count)-cd)/kd);
delty = cp(count)-((tsr(count)*kd)+cd);
if (deltx > 0) & (delty > 0);
    cp(count) = 0;
    tsr(count) = 0;
end;
deltxh = tsr(count)-((cp(count)-ch)/kh);
deltyh = cp(count)-((tsr(count)*kh)+ch);
deltxg = tsr(count)-((cp(count)-cg)/kg);
deltyg = cp(count)-((tsr(count)*kg)+cg);
deltxf = tsr(count)-((cp(count)-cf)/kf);
deltyf = cp(count)-((tsr(count)*kf)+cf);
if (deltxh > 0) & (deltyh < 0) & ((deltxg < 0) & (deltyg < 0)) & ((deltxf < 0) & (deltyf < 0));
    cp(count) = 0;
    tsr(count) = 0;
end;
deltx = tsr(count)-((cp(count)-ce)/ke);
delty = cp(count)-((tsr(count)*ke)+ce);
if (deltx < 0) & (delty > 0);
    cp(count) = 0;
    tsr(count) = 0;
end;
end

%wtmod Subroutine. Calculates model parameters for each time step.
%Matlab simplified dynamic model of wind turbine generator.
tr = 1.71*w/v(n);
cp1calc;
qw = (cp*11*(v(n)^2))/tsr;
qf = (1.72e-3*(w)^2) - (0.247*w) - 4.135;
eg = 3*w;
ig = eg/(sqrt((4*8e-3*(w)^2) + 2.56) + rl);
qg = 9*ig;
wdot = (qw - qf - qg)/J;

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\[ w = w + (w \cdot d) \]

end

\%cp1calc  Subroutine of wtmod. Calculates Cp using polynomial fits.

if \( w < 28.5 \),
\[ \begin{align*}
x_1 &= 0.0046; \quad x_2 = -0.1008; \quad x_3 = 0.6609; \quad x_4 = -1.1596; \\
y_1 &= x_1; \quad y_2 = x_2; \quad y_3 = x_3; \quad y_4 = x_4; \\
range &= 0; 
\end{align*} \]
end

if \( w \geq 28.5 \) & \( w < 31.5 \),
\[ \begin{align*}
x_1 &= 0.0046; \quad x_2 = -0.1008; \quad x_3 = 0.6609; \quad x_4 = -1.1596; \\
y_1 &= 0.0037; \quad y_2 = -0.085; \quad y_3 = 0.5923; \quad y_4 = -1.1086; \\
w_{\text{min}} &= 28.5; \\
range &= 1; 
\end{align*} \]
end

if \( w \geq 31.5 \) & \( w < 34.5 \),
\[ \begin{align*}
x_1 &= 0.0037; \quad x_2 = -0.085; \quad x_3 = 0.5923; \quad x_4 = -1.1086; \\
y_1 &= 0.0023; \quad y_2 = -0.057; \quad y_3 = 0.4265; \quad y_4 = -0.8234; \\
w_{\text{min}} &= 31.5; \\
range &= 1; 
\end{align*} \]
end

if \( w \geq 34.5 \),
\[ \begin{align*}
x_1 &= 0.0017; \quad x_2 = -0.048; \quad x_3 = 0.409; \quad x_4 = -0.9078; \\
y_1 &= x_1; \quad y_2 = x_2; \quad y_3 = x_3; \quad y_4 = x_4; \\
range &= 0; 
\end{align*} \]
end

\[ \begin{align*}
\text{cpx} &= x_1^*(\text{tsr(n)})^3 + x_2^*(\text{tsr(n)})^2 + x_3^*\text{tsr(n)} + x_4; \\
\text{cpy} &= y_1^*(\text{tsr(n)})^3 + y_2^*(\text{tsr(n)})^2 + y_3^*\text{tsr(n)} + y_4; \\
\text{if range} &= 1, \\
\quad k &= (w - w_{\text{min}})/3; \\
\text{else} \\
\quad k &= 1; \\
\text{end} \\
\text{cp}(n) &= (k*\text{cpy}) + ((1-k)*\text{cpx}); \\
\text{if cp}(n) < 0, \\
\quad \text{cp}(n) &= 0; \\
\text{end} \\
\text{end} \]

end
Appendix E

State Space Transfer Function Derivation

Proof of Equation 4.21, 4.23 and 4.28.
During the ON interval $\dot{X} = A_1X + B_1V_{in}$ and $Y = C_1^TX$
During the OFF interval $\dot{X} = A_2X + B_2V_{in}$ and $Y = C_2^TX$
where $X = \begin{bmatrix} I_L \\ V_{out} \end{bmatrix}$, $\dot{X} = \frac{dX}{dt}$, $Y = V_{out}$

Averaged equations are produced over a period using the switch duty cycle as a weighting factor

$$\dot{X} = [A_1\alpha_1 + A_2\alpha'_1]X + [B_1\alpha_1 + B_2\alpha'_1]V_{in} \quad (E.1)$$

$$Y = [C_1^T\alpha_1 + C_2^T\alpha'_1]X \quad (E.2)$$

Small AC perturbations are introduced to produce the static and dynamic components. Using $\sim$ and - to represent the AC and DC signals respectively.

$X = \bar{X} + \xi$
$V_{out} = \bar{V}_{out} + \bar{V}$
$V_{in} = \bar{V}_{in} + \bar{V}_{in}$

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\[ \alpha_1 = \bar{\alpha}_1 + \tilde{\alpha}_1 \]
\[ \alpha'_1 = 1 - \alpha_1 = \bar{\alpha}'_1 - \tilde{\alpha}_1 \]

Introducing these perturbations into 0.1 and 0.2 gives

\[ \overline{X} + \tilde{X} = [A_1 (\bar{\alpha}_1 + \tilde{\alpha}_1) + A_2 (\bar{\alpha}'_1 - \tilde{\alpha}_1)] (\overline{X} + \tilde{X}) \]
\[ + [B_1 (\bar{\alpha}_1 + \tilde{\alpha}_1) + B_2 (\bar{\alpha}'_1 - \tilde{\alpha}_1)] (\overline{V}_{in} + \tilde{V}_{in}) \]

let \( A = A_1 \bar{\alpha}_1 + A_2 \bar{\alpha}'_1 \)
\( B = B_1 \bar{\alpha}_1 + B_2 \bar{\alpha}'_1 \)
\( C^T = C_1^T \alpha_1 + C_2^T \alpha'_1 \)

\[ \overline{X} + \tilde{X} = (A \overline{X} + B \overline{V}_{in}) + (A \tilde{X} + B \tilde{V}_{in}) + [(A_1 - A_2) \overline{X} + (B_1 - B_2) \tilde{V}_{in}] \bar{\alpha}_1 \]
\[ + [(A_1 - A_2) \tilde{X} + (B_1 - B_2) \overline{V}_{in}] \bar{\alpha}'_1 \]

and
\[ V_{out} = [C_1^T (\bar{\alpha}_1 + \tilde{\alpha}_1) + C_2^T (\bar{\alpha}'_1 - \tilde{\alpha}_1)] (\overline{X} + \tilde{X}) \]

then
\[ \tilde{V}_{out} + \tilde{V}_{out} = C^T \tilde{X} + C^T \overline{X} + (C_1^T - C_2^T) \tilde{X} \bar{\alpha}_1 + (C_1^T - C_2^T) \overline{X} \bar{\alpha}'_1 \]

The perturbed value is assumed to be much less than the steady state value and to linearise the small signal model only the first order linear perturbation terms are considered. In the steady state \( \overline{X} = 0 \) so that

\[ \overline{X} = (A \overline{X} + B \overline{V}_{in}) + (A \tilde{X} + B \tilde{V}_{in}) + [(A_1 - A_2) \overline{X} + (B_1 - B_2) \tilde{V}_{in}] \bar{\alpha}_1 \quad (E.3) \]

and
\[ \tilde{V}_{out} + \tilde{V}_{out} = C^T \overline{X} + C^T \tilde{X} + (C_1^T - C_2^T) \tilde{X} \bar{\alpha}_1 \]

steady state conditions are obtained from equation 0.3 by setting all the time derivatives and the perturbation terms to zero giving \( A \overline{X} + B \overline{V}_{in} = 0 \) so

\[ \overline{X} = -A^{-1}B \overline{V}_{in} \quad (E.4) \]

The small signal variation is therefore

\[ \tilde{X} = A \overline{X} + B \overline{V}_{in} + [(A_1 - A_2) \tilde{X} + (B_1 - B_2) \tilde{V}_{in}] \bar{\alpha}_1 \quad (E.5) \]

Similary the steady state term of the output voltage is

\[ V_{out} = C^T \overline{X} \quad (E.6) \]

and therefore the small signal variation for the output voltage is

\[ \tilde{V}_{out} = C^T \tilde{X} + [(C_1^T - C_2^T) \tilde{X}] \bar{\alpha}_1 \quad (E.7) \]
Under steady state conditions the duty cycle \( \alpha_1 \) attains a steady state value \( \bar{\alpha}_1 \) and equations 0.4 and 0.6 lead to a steady state DC transfer function.

\[
\frac{\bar{V}_{out}}{\bar{V}_{in}} = -C^T A^{-1} B \quad \text{(E.8)}
\]

The input to output transfer for a small AC signal is obtained by putting \( \bar{\alpha}_1 = 0 \) into equations 0.4 and 0.7 giving

\[
\bar{X} = A\bar{X} + B\bar{V}_{in} \quad \text{and} \quad \bar{V}_{out} = C^T \bar{X}
\]

Taking Laplace Transforms of these equations with initial conditions of zero.

\[
s\bar{X}(s) = A\bar{X}(s) + B\bar{V}_{in}(s)
\]

\[
(sI - A) \bar{X}(s) = B\bar{V}_{in}(s)
\]

\[
\frac{\bar{X}(s)}{\bar{V}_{in}(s)} = (sI - A)^{-1} B
\]

since \( \bar{V}_{out}(s) = C^T \bar{X}(s) = C^T (sI - A)^{-1} B \bar{V}_{in}(s) \)

\[
\frac{\bar{V}_{out}(s)}{\bar{V}_{in}(s)} = C^T (sI - A)^{-1} B \quad \text{(E.9)}
\]

The duty cycle to output transfer function for a small AC signal is obtained by putting \( \bar{V}_{in} = 0 \) into equations 0.5 and 0.6 giving

\[
\bar{X} = A\bar{X} + [(A_1 - A_2) \bar{X} + (B_1 - B_2) \bar{V}_{in}] \bar{\alpha}_1
\]

\[
\bar{V}_{out} = C^T \bar{X} + (C^T_1 - C^T_2) \bar{X} \bar{\alpha}_1
\]

taking Laplace Transforms as before

\[
s\bar{X}(s) = A\bar{X}(s) + [(A_1 - A_2) \bar{X} + (B_1 - B_2) \bar{V}_{in}] \bar{\alpha}_1(s)
\]

\[
\frac{\bar{X}(s)}{\bar{\alpha}_1(s)} = (sI - A)^{-1} [(A_1 - A_2) \bar{X} + (B_1 - B_2) \bar{V}_{in}] \bar{\alpha}_1(s)
\]

since \( \bar{V}_{out}(s) = C^T \bar{X}(s) + (C^T_1 - C^T_2) \bar{X} \bar{\alpha}_1(s) \)

\[
\frac{\bar{V}_{out}(s)}{\bar{\alpha}_1(s)} = C^T (sI - A)^{-1} [(A_1 - A_2) \bar{X} + (B_1 - B_2) \bar{V}_{in}] + (C^T_1 - C^T_2) \bar{X} \quad \text{(E.10)}
\]
Appendix F

Chapter 4 Software Listings

```matlab
%dcsim Simulates DC-DC booster
delt = 1e-6;
l = 8e-3; ron = 0.3; rs = 0.0251;
il(1) = 0;
vin = 200;
vl(1) = vin; vc(1) = 0;
c = 120e-6; rl = 200;
numb = 1;
while numb < 20000,
    temp = numb;
    while numb < temp+28,
        ton;
        numb = numb+1;
    end,
    temp = numb;
    while numb < temp+28,
        toff;
        numb = numb+1;
    end,
end,
end;

%stspav Transforms state space equations to transfer functions.
%Calculates root locus. Step response.
wrang= logspace(0,3,100);
l = 8e-3;
ron = 0.3;
rs = 0.0251;
vin = 200; vout=400;
c = 120e-6;
rl = 0.2; r=200;
rt=ron+rs+rl;
```

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D=[0];
ts=0:0.1e-3:15e-3;
al1=1-(vin/vout);
al1p=1-all1;
clf
hold
k1=all1p/(1*c);
a1=(1/(c*r))+(rt*all1/l)+(rl*all1p/l);
a2=(1/(c*r))*(rt*all1/l)+(rl*all1p/l)+((all1p-2)/(1*c));
k2=1/((c*rt*all1)+(c*rl*all1p)+(c*r*(all1p-2)));
b0=(rt/l)-(all1p-2*r/l);
n=0;
r=200;
for r=75:50:1000,
n=n+1;
A1=[-(rt/l) 0;0 -(1/(c*r))];
B1=[(1/l);0];
C1=[0 1];
A2=[-(rl/l) -(1/l);(1/c) -(1/(c*r))];
B2=[(1/l);0];
C2=[0 1];
A=(A1*all)+(A2*allp);
B=(B1*all)+(B2*allp);
C=(C1*all)+(C2*allp);
[num1,den1]=ss2tf(A,B,C,D,1);
Atf2=A;
Xbar=inv(A)*B*vin;
Btf2=(A1-A2)*Xbar;
Ctf2=C;
Dtf2=D;
[num2,den2]=ss2tf(Atf2,Btf2,Ctf2,Dtf2,1);
Y1=step(A,B,C,D,1,ts);
Y2=step(Atf2,Btf2,Ctf2,Dtf2,1,ts);
prl(:,n)=roots(den1);
%plot(ts,Y2)
plot(real(prl),imag(prl),'x')
pause
end;
Appendix G

Current Regulator Circuit Diagram
Appendix H

Voltage Regulator Circuit

Diagram
Appendix I

Current Regulator PCB Layout
Appendix J

Voltage Regulator PCB Layout
Appendix K

Pulse Width Modulator and Mosfet Driver
Appendix L

Pulse Width Modulator PCB Layout

![Pulse Width Modulator PCB Layout Diagram]
Appendix M

Hardware Cabinet Layout

PSU1
Mosfet
Drivers

PSU2
Regulators
Power Side

PSU3
Regulators
Instrumentation
Side, PWM

Fan
Heat Sink
Temp
Sensor
Mosfet
Driver

Fan
Heat Sink
Temp
Sensor
Mosfet
Driver

Contactor

Bridge
Heat Sink

Bridge
Heat Sink

Thyristor

Protection
Circuit

Voltage / Current
Transducers

Voltage
Regulator

Current
Regulator

PSU4
Speed
Transducer,
V/A
Transducers

Speed
Transducer

Data Acquisition and Control Computer

Terminal
Board

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Appendix N

Protection Circuit
Appendix O

Chapter 6 Software

%coef Calculates Operating Points
wop=30; %setup operating points for
vop=8; %angular velocity and wind velocity
rad=1.71; %rotor radius
rs=1.6; %stator resistance
l=8e-3; %stator inductance
vout=400; %required output voltage
load cqtsr %aerodynamic data
tsrop=wop*1.75/vop;
%calculate tip speed ratio
n=1; %calculate Cq gradient at operating point.
while n<20,
    if lambda(n)>tsrop,
        xtsr=[lambda(n-1),lambda(n),lambda(n+1)];
        ycq=[Cq(n-1), Cq(n), Cq(n+1)];
        tsrql=polyfit(xtsr,ycq,1);
        kla=tsrql(1);
        cl=tsrql(2);
        p=n;
        n=21;
    end
end
ka=0.5*pi*1.2*(rad)^3;
qa=Cq(p)*ka*(vop^2);
qf=(7.3e-4*(wop^2)-0.1*wop)+1.75;
qg=qa-qf;
igrms=qg/9;
igdc=1.18*igrms;
z=sqrt((1.6^2)+((4*wop*8e-3)^2)); %stator impedance
eg=3*wop;
\[ \text{vgrms} = eg \cdot (igrms \cdot z) \]
\[ \text{vdgc} = 2.54 \cdot \text{vgrms} \]
\[ \text{gl} = \text{igdc} / \text{vdgc} \]
\[ \text{alph1} = 1 - (\text{vdgc} / 400) \]
\[ \text{iout} = (1 - \text{alph1}) \cdot \text{igdc} \]
\[ \text{rop} = \text{vout} / \text{iout} \]
\[ \text{alph2} = 75 / \text{rop} \]
\[ j = 8 \]
\[ kqav = (k_\alpha \cdot k_\lambda \cdot wop \cdot 1.75) + (2 \cdot k_\alpha \cdot c_1 \cdot vop) \]
\[ kqaw = k_\alpha \cdot k_\lambda \cdot 1.75 \cdot vop \]
\[ kqgw = (((1 + (z \cdot gl)) \cdot 27 \cdot gl) - (27 \cdot (4 \cdot wop \cdot gl \cdot 8e-3)^2)) / (z \cdot (1 + (z \cdot gl))^2) \]
\[ kqgg = (27 \cdot wop) / ((1 + (z \cdot gl))^2) \]
\[ kqfw = (14e-3 \cdot wop) - 0.1 \]
\[ kvw = 12.64 \]
\[ gwtnur = 1 / (kqgw - kqaw + kqfw) \]
\[ gwtden = ([j / (kqgw - kqaw + kqfw)] - 1) \]
\[ gvlagnur = 4.8e-3 \]
\[ gvlagden = [0.02] \]

%%%glal1 %Calculates load conductance as a function of duty cycle alpha 1.
clf
hold on
for al2 = 0:0.1:1,
    n = 1;
    for all = 0:0.01:0.75,
        gl(n) = al2 / (75 * (1-all)^2);
        alp1(n) = all;
        n = n + 1;
    end;
    plot(alp1, gl)
end;

%%%glal2 %Calculates load conductance as a function of duty cycle alpha 2.
clf
hold on
for al1 = 0:0.1:0.75,
    n = 1;
    for al2 = 0:0.1:1,
        gl(n) = al2 / (75 * (1-all)^2);
        alp2(n) = al2;
        n = n + 1;
    end;
    plot(alp2, gl)
end;

%%%gc3dsc Calculates coefficients of voltage regulator loop.
%Plots root locus or frequency response diagrams
%clear prl
kf=kreg*kgl2*kqgg;
kp3=0.23;
wrange=logspace(-1,3,10);
for n=1:length(kp3),
g3num=[(1j*1/0.6) 1];
g3den=[1 0];
g31dena1=gwtden;
g31dena1c=conv(gc3den,gwtden);
g31denac=conv(g31dena1,gwtden);
g31denc=conv(g31denac,gwtden);
g32num=kqav*kvw*gwtum*gvlagden;
g32numc=kqav*kvw*gwtum*conv(gc3den,gvlagden);
g32denc=g31denc;
g31denb=kf*kvw*gwtum*gvlagnum;
g31denbc=kf*kvw*gwtum*conv(gc3num,gvlagnum);
g31denb=[0 0 g31denb];
g31denbc=[0 0 g31denbc];
g31den=g31dena+g31denb;
g31denc=g31denac+g31denbc;
g32num=kqav*kvw*gwtum*gvlagden;
g32numc=kqav*kvw*gwtum*conv(gc3den,gvlagden);
g32denc=g32denc;
g31denb=g31denc;
g31num=g31denb;
g31den=g31dena;
g31denc=g31denac;
[mgl,phl]=bode(g31num,g31den,wrange);
[mgt,pht]=bode(g31numc,g31denc,wrange);
[mgs,phs]=bode(g32num,g32den,wrange);
[mgsc,phsc]=bode(g32numc,g32denc,wrange);
[re,im]=nyquist(g31num,g31den,wrange);

%gvregnum=kqgg*gwtum*conv(gc3den,gvlagden);
gvregden1a=conv(gwtden,gc3den);
gvregden1=conv(gvregden1a,gvlagden);
gvregden2=kggg*kgl2*kreg*kvw*gwtum*conv(gc3num,gvlagnum);
gvregden=gvregden1+[0 0 gvregden2];
subplot(2,1,1);
semilogx(wrange,(20*log10(mgt)),'o');grid
hold on;
semilogx(wrange,(20*log10(mgtc)),'x');grid
ylabel('closed loop gain (dB)');

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%hold off
%subplot(2,1,2);semilogx(wrange,pht,'o');grid
%hold on; %xlabel('Frequency (rad/s)');
ylabel('closed loop phase (deg)');
%hold on
%subplot(2,1,2);semilogx(wrange,phtc,'x');grid
%hold on
%plot(re,im)
%hold on
%plot(phs,(20*log10(mg)),'g')
%hold on
%plot(phs,(20*log10(mg)),'rx')
%plot(phs(9),(20*log10(mg(9))),'rx')
%ngrid
%ghnum3=[0 ghnum3];
.cegc3=g31denac+(kp3(n)*g31denbc);
%cegc3=g32denc+(kp3(n)*g32denc);
%roots(cegc3)
prl(:,n)=roots(cegc3);
plot(real(prl),imag(prl),'x')
%v=[-2 0 -1 1];
%axis(v)
pause
end
end

%gc2des.m  Calculates coefficients of current regulator loop.
%Plots root locus or frequency response diagrams

clear prl
v=[-100 1 -10 10];
%axis(v)
hold on
wrange=logspace(-2,1,20);
kgl1=0.14;
kvreg=0.36;
gc2num=1;
gc2den=[10 1];
gilagnum=1;
gilagden=[0.01 1];
kiqg=0.136;
kiieg=0.15;
kp2=logspace(-1,3,20);
for n=1:length(kp2),
gifornum=kgl1*kiqg*gilagnum*conv(gvregnum,gwtden);
giforden=gwtnum*conv(gvregden,gilagden);
giregnum=gifornum;
giregden=giforden+[0 gifornum];
ceregden=giforden+(kp2(n)*[0 gifornum]);

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igidnum = conv(gc2num, gifornum);
igiddden1 = conv(gc2den, giforden);
igiddden2 = conv(gc2num, gifornum);
igiddden = igidden1 + [0 0 0 igidden2];
ceigid = igidden1 + (kp2(n) *[0 0 0 igidden2]);
[mguc, phsuc] = bode((kp(n) * gifornum), giforden, wrange);
[mgc, phsc] = bode((kp(n) *igidnum), igidden, wrange);
[re, im] = nyquist(ghnum, ghdend, wrange);
subplot(2,1,1); semilogx(wrange, (20*log10(mguc))); grid
hold on
subplot(2,1,2); semilogx(wrange, phsuc); grid
hold on
subplot(2,1,1); semilogx(wrange, (20*log10(mgc))); 'b'; grid
hold on
subplot(2,1,2); semilogx(wrange, phsc, 'b'); grid
% plot(re, im)
% plot(phs, (20*log10(mg)), 'g')
% plot(phs, (20*log10(mg)), 'ro')
% plot(phs(19), (20*log10(mg(19))), 'gx')
roots(ce)
pr1(:, n) = roots(ce);
% plot(real(prl), imag(prl), 'bx')
t = 0:0.1:20;
igstuc = step(giregnum, ceregden, t);
igstc = step(igidnum, ceigid, t);
% plot(t, igstuc)
% hold on
% plot(t, igstc, 'b')
pause
end
end

% gc1des Calculates coefficients of speed regulator loop.
% Plots root locus or bode diagrams.
wrange = logspace(-3, 1, 100);
clear prl
v = [-1.5 0.5 -0.5 0.5];
dnum = [-td 1];
dden = [td 1];
gc1num = [(1/0.6) 1];
gc1den = [1 0];
gc1ldnum = [(1/0.34) 1];
gc1ldden = [1/2 1];
gc1ldnum = [(1/0.5) 1];
gc1ldden = [1/2 1];
gc1num = conv(gc1ldnum, gc1ldnum);
%%gclnum=gclldnum;
%%%gclden=gclldden;
%%%gclden=[1];
%%%gclden=[1];
kpl=1:10;
kpl=logspace(-2,1,10);
kpl=0.2;
for n =1:length(kpl),
    widnum1=kglal*gwtnum*conv(gc2num,gvregnum);
    widnum=conv(widnum1,gilagden);
    widden1a=conv(gc2den,gvregden);
    widden1b=conv(gilagden,gwtnum);
    widden1=conv(widden1a,widden1b);
    widden2a=kglal*kiog*conv(gc2num,gvregnum);
    widden2b=conv(gilagnum,gwtden);
    widden2=conv(widden2a,widden2b);
    widden2=[0 0 widden2];
    widden=widden1+widden2;
    wwrrefnum1=conv(gc1num,widnum);
    wwrrefnum=conv(wwrefnum1,dden);
    wwrrefden1a=conv(gc1den,widden);
    wwrrefden1=conv(wwrefden1a,dden);
    wwrrefden2a=conv(gc1num,widnum);
    wwrrefden2=conv(wwrefden2a,dnum);
    wwrrefden2=[0 0 wwrrefden2];
    cewwref=wwrefden1+(kpl(n)*wwrefden2);
    wwrrefucnum=conv(widnum,dnum);
    wwrrefucden=conv(widden,dden);
    wwrrefolnum1=conv(gc1num,widnum);
    wwrrefolnum=conv(wwrefolnum1,dnum);
    wwrrefolden1=conv(gc1den,widden);
    wwrrefolden=conv(wwrefolden1,dden);
pri(:,n)=roots(cewwref);
   roots(cewwref)
end

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Appendix P

Flow Chart for Data Monitoring and Control

Start

Number = 1

SAMPLE

Number = Number + 1

Number = Total?

Ave1

Chk Con

Id_Set

Calc

Disp

Tstore = Tstore + 1

Tstore = S_per

Ave2

Q key Pressed?

Yes

Igdc_tot > 2*Igdc_tot_old?

Wref = Wref + 2

Stop

No

Igdc_tot < 0.5*Igdc_tot_old?

Wref = Wref - 2

No

Wref = Wref - 2

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No

Wref = Wref + 2

Stop
Appendix Q

Data Acquisition and Control Software

PROGRAM wset6;  {Constant speed data acquisition and control program}
USES crt,dos;
{$L 818hgtpn $F-, $E+}
PROCEDURE pcl818hg(func:integer; var param:word);external;
CONST
base = $0220;
_a_d_lo = base + 0;
_a_d_hi = base + 1;
mux_sca = base +2;
d_ip_ lo = base + 3;
dlo_ lo = base + 4;
dlo_hi = base +5;
pcl_sta = base +8;
pcl_con = base +9;
ctr_ena = base +10;
d_ip_hi = base +11;
cntr_1 = base +13;
cntr_2 = base +14;
ctr_con = base +15;
fifo_lo = base +23;
irq_con = $21;
irq_sta = $20;
irq_res = $20;
total = 500;
average = 50;
VAR
irq_mask,channel,aip_word,a_ip,a_in, a_op, dig_ip_lo, dig_ip_hi :WORD;
a_sto : ARRAY[0..8,0..500] OF WORD;
b_sto : ARRAY[0..8,0..100] OF REAL;
c_sto : ARRAY[0..8,0..60] OF REAL;
id_sto : ARRAY[0..500] OF REAL;
filename : text;
ascname : STRING[10];
ki,kp,kd,total7, idold,id,tmp,rl,dutcy,total2,total1,freq,total0,total3,total4,total5,
total6 : REAL;
wref, wdiff, wdiffold, Vgrms, speedref, vgref, igref, vmref, imref, didg_ip, wv, ywm
:REAL;
oldspeed, temp, speed1, speed,Vgac, Igac, Vgdc, Igdc, Vout, Iout, wm, caltorq, fric,
torq, torq, torq, torq, torq, torq, torq, torq, torq, torq, torq, torq, torq, torq,

Formats of REAL:
cou : LONGINT;
c2, tmpl, to_cou, nttrial, maxcha, det : INTEGER;
u, count, x1, x2, p, z, j, counter, number, nsta, nt : INTEGER j
Ch, up_dwn : CHAR;
Int0bSave : POINTER;
min, min1, min2, max, max1, max2 : INTEGER;
pos, u_d, ch_no, stop, k, counterx, countery, n, cutoff, diff, cutoff2 : INTEGER ;
igd, cuttot, cutoff1, comptor, compsp : REAL;
glitch : BOOLEAN;

PROCEDURE Data_Int; {data aquire interrupt}
INTERRUPT;
BEGIN
INLINE ($Fa);
REPEAT
UNTIL (port[pcl_sta] and $10) = 16;
a_ip := portw[a_d_lo] shr 4;
ch_no := ((port[a_d_lo] and $0f)) ;
a_sto[ch_no,number] := a_ip ;
port[pcl_sta] := $ff; {clears the int bit}
port[irq_sta] := irq_res; {reset interrupt controller}
if ch_no = 7 then number := number + 1;
dig_ip_lo := port[d_ip_lo] ;
dig_ip_hi := port[d_ip_hi];
a_sto[8,number] := ((dig_ip_hi*256)+dig_ip_lo);
id_sto[number] := id;
INLINE ($FB);
END; {procedure Data_Int}
if a_op > 50900 then
up_dwn := chr(100);
end;
if up_dwn = chr(100) then
begin
a_op := a_op - 200;
if a_op < 100 then
up_dwn := chr(117);
end;
portw[d10_lo] := a_op;
if up_dwn = chr(115) then
begin
{ wsto[count] := wdiff; }
count := count + 1;
end;
END; {procedure rset}

PROCEDURE save;
BEGIN
for n := 1 to 60 do
begin
total0 := total0 + c_sto[0,n];
total1 := total1 + c_sto[1,n];
total2 := total2 + c_sto[2,n];
total3 := total3 + c_sto[3,n];
total4 := total4 + c_sto[4,n];
total5 := total5 + c_sto[5,n];
total6 := total6 + c_sto[6,n];
total7 := total7 + c_sto[7,n];
total8 := total8 + c_sto[8,n];
end;
Append(filename);
write(filename,(total0 / 60):4:3);write(filename,',');
write(filename,(total1 / 60):4:3);write(filename,',');
write(filename,(total2 / 60):4:3);write(filename,',');
write(filename,(total3 / 60):4:3);write(filename,',');
write(filename,(total4 / 60):4:3);write(filename,',');
write(filename,(total5 / 60):4:3);write(filename,',');
write(filename,(total6 / 60):4:3);write(filename,',');
write(filename,(total7 / 60):4:3);write(filename,',');
write(filename,(total8 / 60):4:3);writeln(filename);
close(filename);
end; {save}

PROCEDURE idset;
BEGIN
if keypressed then
up_dwn := readkey;
oldspeed := speed;
speed := speed + 0.0000001;
wdiff := speed - wref;
id := (kp * wdiff) + (ki*idold) + (kd*(wdiff-wdiffold));
if id < 0 then id := 0;
if id > 15 then id := 15;
idold := id;
tmp := (6550*id);
tmp1 := round(tmp);
a_op := tmp1;
port[a_d_lo] := a_op;
wdiffold := wdiff;
END; {procedure idset}

PROCEDURE Initialise;
BEGIN
kp := 0.1;
ki := 1;
kd := 0.9;
a_op := 0;
idold := 0;
wdiffold := 0;
port[mux_sca] := $0; {set pointer to channel 0 (wind speed)}
port[a_d_hi] := $4; {range code = 0 to 10V}
port[mux_sca] := $1; {set pointer to channel 1 (3 phase current C1)}
port[a_d_hi] := $0; {range code = -5 to 5V}
port[mux_sca] := $2; {set pointer to channel 2 (3 phase volts V1)}
port[a_d_hi] := $0; {range code = -5 to 5V}
port[mux_sca] := $3; {set pointer to channel 3 (DC Amps C2)}
port[a_d_hi] := $0; {range code = -5 to 5V}
port[mux_sca] := $4; {set pointer to channel 4 (DC Volts V2)}
port[a_d_hi] := $0; {range code = -5 to 5V}
port[mux_sca] := $5; {set pointer to channel 5 (Output Amps C3)}
port[a_d_hi] := $0; {range code = -5 to 5V}
port[mux_sca] := $6; {set pointer to channel 6 (Output Volts V3)}
port[a_d_hi] := $0; {range code = -5 to 5V}
port[mux_sca] := $7; {set pointer to channel 7 (Yaw Misalignment)}
port[a_d_hi] := $0; {range code = -5 to 5V}
port[pcl_sta] := $ff; {clear the int bit}
port[mux_sca] := $70; {20 = start ch = 0, stop ch = 2}
port[pcl_con] := $b3; {set int - irq3}
port[ctr_con] := $74; {set ctrl _ counter 1 mode 2 r/w}
port[cntr_1] := $05; {set counter 1 low byte}
port[cntr_1] := $00; {set counter 1 high byte}
port[ctr_con] := $b4; {set ctr counter 2 mode 2 r/w}
port[cntr_2] := $05; {set counter 2 low byte}
port[cntr_2] := $00; {set counter 2 high byte}

256
port[ctr_ena] := $02; {set timer counter enable}
Assign(filename,c:\data\conv.dat);
END; {procedure initialise}

procedure setup;
begin
  total2 := 0; total1 := 0;
  total0 := 0; total3 := 0; total4 := 0;
  total5 := 0; total6 := 0; total7 := 0; speed := 0; speed1 := 0; didg_ip := 0;
  Vgac := 0; Igac := 0; Vgdc := 0; Igdc := 0; freq := 0;
  Vout := 0; lout := 0; n := 0;
end; {procedure setup}

procedure avel;
begin
  for n := 1 to total do
    begin
      total0 := total0 + a_sto[0,n];
      total1 := total1 + id_sto[n];
      total2 := total2 + a_sto[2,n];
      total3 := total3 + a_sto[3,n];
      total4 := total4 + a_sto[4,n];
      total5 := total5 + a_sto[5,n];
      total6 := total6 + a_sto[6,n];
      total7 := total7 + a_sto[7,n];
      didg_ip := didg_ip + a_sto[8,n];
    end;
  p:=1;
  wv := total0 / total;
  lgd := total1 / total;
  Vgac := total2 / total;
  Igdc := total3 / total;
  Vgdc := total4 / total;
  lout := total5 / total;
  Vout := total6 / total;
  ywm := total7 / total;
  speed := didg_ip / total;
end; {procedure avel}

procedure stol;
BEGIN
  b_sto[0,c2] := wv;
  b_sto[1,c2] := lgd;
  b_sto[2,c2] := Vgac;
  b_sto[3,c2] := Igdc;
END;
\[
b_{\text{sto}[4,c2]} := V_{\text{gdc}};
\]
\[
b_{\text{sto}[5,c2]} := I_{\text{out}};
\]
\[
b_{\text{sto}[6,c2]} := V_{\text{out}};
\]
\[
b_{\text{sto}[7,c2]} := y_{\text{wrn}};
\]
\[
b_{\text{sto}[8,c2]} := \text{speed};
\]
END; \{ procedure sto1 \}

\{ ----------------------------- \}

\{ \}

procedure ave2;
begin
for \( n := 1 \) to 100 do
begin
\[
total0 := total0 + b_{\text{sto}[0,n]};
\]
\[
total1 := total1 + b_{\text{sto}[1,n]};
\]
\[
total2 := total2 + b_{\text{sto}[2,n]};
\]
\[
total3 := total3 + b_{\text{sto}[3,n]};
\]
\[
total4 := total4 + b_{\text{sto}[4,n]};
\]
\[
total5 := total5 + b_{\text{sto}[5,n]};
\]
\[
total6 := total6 + b_{\text{sto}[6,n]};
\]
\[
total7 := total7 + b_{\text{sto}[7,n]};
\]
\[
total8 := total8 + b_{\text{sto}[8,n]};
\]
end;
\[
c_{\text{sto}[0,cl]} := total0 / 100;
\]
\[
c_{\text{sto}[1,cl]} := total1 / 100;
\]
\[
c_{\text{sto}[2,cl]} := total2 / 100;
\]
\[
c_{\text{sto}[3,cl]} := total3 / 100;
\]
\[
c_{\text{sto}[4,cl]} := total4 / 100;
\]
\[
c_{\text{sto}[5,cl]} := total5 / 100;
\]
\[
c_{\text{sto}[6,cl]} := total6 / 100;
\]
\[
c_{\text{sto}[7,cl]} := total7 / 100;
\]
\[
c_{\text{sto}[8,cl]} := total8 / 100;
\]
end; \{ procedure ave2 \}

\{ ----------------------------- \}

procedure ave3;
begin
for \( n := 1 \) to 60 do
begin
\[
total0 := total0 + b_{\text{sto}[0,n]};
\]
\[
total1 := total1 + b_{\text{sto}[1,n]};
\]
\[
total2 := total2 + b_{\text{sto}[2,n]};
\]
\[
total3 := total3 + b_{\text{sto}[3,n]};
\]
\[
total4 := total4 + b_{\text{sto}[4,n]};
\]
\[
total5 := total5 + b_{\text{sto}[5,n]};
\]
\[
total6 := total6 + b_{\text{sto}[6,n]};
\]
\[
total7 := total7 + b_{\text{sto}[7,n]};
\]
\[
total8 := total8 + b_{\text{sto}[8,n]};
\]
end;
end; {procedure ave3}

begin
  ChrScr;
  wv := wv*7.33e-3;
  Igdc := ((Igdc - 2047)*7.4e-3)+0.11;
  Iout := ((Iout - 2047)*5.57e-3)-0.078;
  Vgdc := (Vgdc - 2047)*0.265;
  Vout := ((Vout - 2047)*0.272)-2.83;
  ywm := (ywm-2047)*0.085;
  speed := (113807/(didg_ip+0.001));
  Vgac := 0.7*Vgac;
  wm := (1/((1/3)+(0.0237*Igdc)))*((Vgdc)/4.9)-(1.186*Igdc);
  fric := ((-7.2ge-4)*speed*speed) + (0.104*speed) + 1.75;
  gotoxy(5,6);write('lgdc=',(Igdc):4:2);write('Amps');
  gotoxy(5,8);write('Vgdc=',(Vgdc):4:2);write('Volts');
  gotoxy(5,10);write('Iout=',(Iout):4:2);write('Amps');
  gotoxy(5,12);write('Vout=',(Vout):4:2);write('Volts');
  gotoxy(5,14);write('Wind Velocity =',(wv):4:2);write('m/s');
  gotoxy(5,16);write('Yaw Misalignment =',(ywm):4:2);write('degrees');
  gotoxy(5,18);write('Generated Power =',(Igdc*Vgdc):4:2);write('Watts');
  gotoxy(40,3);write('speed=',(speed):4:2);write('rad/s');
  gotoxy(40,9);write('wdiff=',(wdiff):4:2);
end; {procedure disp}

begin
  wref := 26;
  irq_mask := port[irq_con]; {fetch the current irq mask}
  port[pcl_sta] := $ff; {clear interrupt request}
  GetIntVec($0b,Int0bSave); {fetch interrupt vector 0bH - irq3}
  SetIntVec($0b,@Data_Int); {set interrupt vector for data Int}
  port[mux_sca] := $70; {20 = start ch = 0, stop ch = 2}
  port[pcl_con] := $b3; {set int - irq3}
  - set up programmable interrupt controller-
  port[irq_con] := port[irq_con] AND $f7; {set PIC mask to enable irq5}
  INLINE ($FB); {enable interrupts}
end; {procedure init}

begin
  Initialise;
  REPEAT
c1 := 1;
REPEAT
  c2 := 1;
  REPEAT
    init;
    setup;
    REPEAT
      UNTIL number > total;
      port[irq_con] := irq_mask; \{reset PIC mask\}
      avel;
      idset;
      stol;
      disp;
      c2 := c2 + 1;
    UNTIL c2 > 100;
    setup;
    ave2;
    c1 := c1 + 1;
  UNTIL c1 > 60;
  setup;
  save
UNTIL stop = chr(113); \{keypressed; up_dwn := readkey; ch := #0;\}
END. \{main program\}
Appendix R

Chapter 8 Software Listings

% wvcor Wind speed correction.
k=0.5*pi*1.2*(1.71^2);
for p=1:length(wv),
    cp(p)=pm(p)/(k*(wv(p).^3));
    vrts=[4;8;4;cp(p)];
    atmp=roots(vrts);
    for n=1:3,
        if real(atmp(n))>0 & real(atmp(n))<0.5,
            a(p)=atmp(n);
        end;
    end;
end;
aprime=0.233*real(a);
wvcor=wv./(1-aprime);
end

% datv
% takes raw data, filters it and creates Cp vs tsr curves.
[r,c]=size(conwh);
p=1;
for n=1:r,
    uplim=35;
    lolim=10;
    if conwh(n,9)<uplim & conwh(n,9)>lolim, 
        % & conv(n,8)>1988 & conv(n,8)<2105,
        wv=conwh(:,1)*0.00733;
        igdc=((conwh(:,3)-2047.5)*0.0074)+0.11;
        vgdc=((conwh(:,5)-2047.5)*0.265);
        iout=((conwh(:,6))*0.00557)-0.078;
        vout=((conwh(:,7)-2047.5)*0.272)-2.83;
        ywm=((conwh(:,8)-2047.5)*0.085;
    end;
end;
id = conwh(:,2); wref = conwh(:,4); w = conwh(:,9); p = p + 1;

end;
end;
dwdt(1) = 0;
for n = 1:length(w)-1,
dwdt(n+1) = (w(n+1)-w(n))/2.2;
end;
dwdt = dwdt';
pg = iout.*vout;
qf = (1.72e-3.*(w.^2)) - (0.247.*w) - 4.135;
igrms = igdc * 0.82;
qg = 9 * igrms;
qt = qg - qf + (8 * dwdt);
vgrms = vgdc / 2.33;
eg = vgrms + (1.7 * igrms);
pt = w.*(qt);

% calid2 Calculates the demanded current versus speed relationship.
n = 1;
for wc = 17:0.1:35,
    wca(n) = wc;
cqr = (0.0005*wc*wc) - (0.0156*wc) + 0.14;
tsrr = (-0.0273*wc*wc) + (1.53*wc) - 12.7;
iddes3(n) = (7.203*cqr*wc*wc) / (tsrr*tsrr);
    n = n + 1;
end;
Appendix S

Chapter 9 Software Listings

%sts.m Derives a Frequency Spectrum for a Wind Time Series % from which the Autocorrelation is calculated.

t=100;
l=120;
ubar=12;
sig=1;
n=1;int_r=0;
r(1)=0;
fo=(0.041*z)/l;
for om = 0.0001:0.0001:0.1,
    omeg(n)=om;
    lomeg(n)=log10(om);
    f=(om*z)/ubar;
    fom(n)=(sig*0.164*(f/fo))*(1+(0.164*((f/fo)^1.67)));
    r(n+1)=(fom(n)*0.0001*cos(2*pi*om*n))+r(n);
    int_r = int_r + r(n+1);
    n=n+1;
end
int_r
end

%wser.m
%Uses a Derived Value for the autocorrelation to derive %a wind time series using the Random Walk Method.
n=10000;
t=0.1;k=2;
r=exp(k*(t));
seed=1;
rand('seed',seed);
rnd;
c=sqrt(1-(r^2))*randn(1,n);
w(1)=0;w(2)=0;w(3)=0;w(4)=0;
for ind=4:n,
    w(ind+1)=((0.993499*w(ind))+(0.017448*w(ind-1))-(0.218639*w(ind-2))
            +(0.13665*w(ind-3))) + e(ind);
end
y=fft(w);
pyy=y.*conj(y);
f=(1/(1*10000))*(0:10000);
lf=log10(f);
std(w)
end

%corer
%Calculates the Autocorrelation of a Derived Wind Time Series
%to Compare with the Desired Value.
n=3000;
for k = 0:400,
    r(k+1)=0;
    for ind = 1:n-k,
        r(k+1)=r(k+1)+(w(ind)*w(ind+k));
    end
    r(k+1)=r(k+1)/(n-k);
end