Unsteady inlet condition generation for Large Eddy Simulation CFD using particle image

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Unsteady Inlet Condition Generation for Large Eddy Simulation CFD Using Particle Image Velocimetry

Mark Robinson

Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University
Department of Aeronautical and Automotive Engineering
February 2009

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Abstract

In many areas of aerodynamics the technique of Large Eddy Simulation (LES) has proved a practical way of modelling the unsteady phenomena in numerical simulations. Few applications are as dependent on such an approach as the prediction of flow within a gas turbine combustor. Like any form of Computational Fluid Dynamics (CFD), LES requires specification of the velocity field at the inflow boundary, with much evidence suggesting the specification of inlet turbulence can be critical to the resultant accuracy of the prediction. While a database of time-resolved velocity data may be obtained from a precursor LES calculation, this technique is prohibitively expensive for complex geometries. An alternative is to use synthetic inlet conditions obtained from experimental data.

High-speed Particle Image Velocimetry (PIV) is used here to provide planar velocity data at up to 1kHz temporal resolution in two test cases representative of gas turbine combustor flows (a vortex generator in a duct and an idealised combustor). As the data sampling rate is approaching a typical LES time-step it introduces the possibility of applying instantaneous experimental data directly as an inlet condition. However, as typical solution domain inlet regions for gas turbine combustor geometries cannot be adequately captured in a single field of PIV data, it is necessary to consider a method by which a synchronous velocity field may be obtained from multiple PIV fields that were not captured concurrently.

A method is proposed that attempts to achieve this by a combined process of Linear Stochastic Estimation and high-pass filtering. The method developed can be generally applied without a priori assumptions of the flow and is demonstrated to produce a velocity field that matches very closely that of the original PIV, with no discontinuities in the velocity correlations. The fidelity and computational cost of the method compares favourably to several existing inlet condition generation methods.

Finally, the proposed and existing methods for synthetic inlet condition generation are applied to LES predictions of the two test cases. There is shown to be significant differences in the resulting flow, with the proposed method showing a marked
reduction in the adjustment period that is required to establish turbulent equilibrium downstream of the inlet. However, it is noted the presence of downstream turbulence generating features can mask any differences in the inlet condition, to the extent that the flow in the core of the combustor test case is found to be insensitive to the inlet condition applied at the entry to the feed annulus for the test conditions applied here.

Keywords: Particle Image Velocimetry, Large Eddy Simulation, Gas Turbine Combustors, Inlet Conditions, Linear Stochastic Estimation, Proper Orthogonal Decomposition.
Acknowledgements

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Special thanks go to Dr Adrian Spencer and Prof. Jim McGuirk, whose advice, support and experience have been of unquantifiable benefit. They both have my utmost respect and admiration, and it has been a pleasure to work with them.

Further thanks go to three groups of people who have made the experience of producing this work far more enjoyable: The UTC tea-school, the department 6-a-side football team and “Team PIV”.

Thank you to my friends and family, especially my parents, for their continued support. And finally to Heather, who is my inspiration.

Mark Robinson, 2009
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<td>ACARE</td>
<td>Advisory Council for Aeronautic Research in Europe</td>
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<tr>
<td>ACF</td>
<td>Autocorrelation Function</td>
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<tr>
<td>CAEP</td>
<td>ICAO Committee on Aviation Environmental Protection</td>
</tr>
<tr>
<td>CCF</td>
<td>Cross Correlation Function</td>
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<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
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<td>CLF</td>
<td>Courant-Friedrich-Lewy</td>
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<tr>
<td>CO₂</td>
<td>Carbon Dioxide</td>
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<td>CRVP</td>
<td>Counter Rotating Vortex Pair</td>
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<td>DFG</td>
<td>Digital Filter-based Generation</td>
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<tr>
<td>DGV</td>
<td>Doppler Global Velocimetry</td>
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<td>DNS</td>
<td>Direct Numerical Simulation</td>
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<td>Eddy Viscosity Hypothesis</td>
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<td>FFT</td>
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<td>FoV</td>
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<td>HMN</td>
<td>Hoest-Madsen and Nielsen (sub-cell filtering correction)</td>
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<td>ICAO</td>
<td>International Civil Aviation Organization</td>
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<tr>
<td>LDA</td>
<td>Laser Doppler Anemometry</td>
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<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
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<td>RAM</td>
<td>Random Access Memory</td>
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<td>RANS</td>
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<td>REP</td>
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SGS – Subgrid-scale
sLSB – Supplemented Linear Stochastic Estimation
SVC – Spatial Velocity Correlation
SVD – Singular Value Decomposition
TKE – Turbulent Kinetic Energy
## Index of Nomenclature

**Lower case letters:**
- \(a\) – discrete velocity range or ‘bin’
- \(a\) – weighting coefficient
- \(b\) – filter coefficients
- \(d_p\) – primary bleed port diameter
- \(f\) – focal length
- \(k\) – wavenumber
- \(k\) – scaling factor
- \(n\) – integer
- \(p\) – static pressure
- \(p_\text{e}\) – point reference signal
- \(r\) – spatial separation
- \(s\) – spatial separation in a coordinate axis
- \(t\) – time
- \(u, v, w\) – Cartesian fluctuating velocity
- \(u'\) – residual velocity (after filtering); incoherent velocity field
- \(w\) – Hann window function
- \(x, y, z\) – Cartesian directions
- \(y^+\) – non-dimensionalised distance from the wall (equation 1.18)
- \(z\) – confidence band

**Upper case letters:**
- \(C\) – Confidence factor
- \(C_s\) – Smagorinsky constant
- \(D\) – Interrogation cell area
- \(E\) – energy
- \(E_K\) – kinetic energy
- \(G\) – filter function
- \(H\) – reference length
- \(K\) – Turbulent Kinetic Energy (TKE)
- \(L\) – integral lengthscale
- \(N_c\) – number of velocity components
- \(N_{\text{on}}\) – cut-on POD mode
- \(N_{\text{samp}}\) – gross number of data set samples
- \(N_{\text{fl-samp}}\) – effective number of data set samples
- \(N_{\text{I-samp}}\) – number of independent data set samples
- \(N_{\text{terms}}\) – Number of terms defining a filter function
- \(N_x\) – number of samples in range ‘x’
- \(P_1\) – 1<sup>st</sup> correlation peak intensity
- \(P(x)\) – Probability of event ‘x’
- \(Q\) – Q-factor; signal to noise ratio (equation 1.21)
- \(R\) – Rotation
- \(R_{ij}\) – Cross Correlation Function (equation 3.8)
- \(T\) – Translation
- \(S_{ij}\) – strain rate tensor (equation 1.3)
- \(U, V, W\) – Cartesian instantaneous velocities
- \(U_c\) – friction velocity (equation 1.19)
Greek letters:
\( \delta_{ij} \) – Kronecker delta
\( \varepsilon \) – turbulent energy dissipation rate
\( \varepsilon_x \) – error in property \( x \)
\( \eta \) – Kolmogorov length microscale
\( \mu \) – molecular viscosity
\( \rho \) – density
\( \tau \) – temporal separation
\( \tau_{ij} \) – shear stress
\( \tau_\eta \) – Kolmogorov time microscale
\( v \) – kinematic viscosity
\( \nu_e \) – effective viscosity
\( \nu_T \) – SGS eddy viscosity
\( \phi_k \) – \( k \)th spatial POD mode
\( \omega \) – frequency
\( \Delta \) – filter width
\( \Lambda_x \) – Rotational momentum about the \( x \)-axis

Sub-script:
1,2,3 – horizontal, vertical, through-plane component of a given FoV (unless otherwise stated)
\( c \) – camera coordinate system
\( cai+ \) – value conditionally averaged to positive tail of \( i \)-component PDF
\( cai- \) – value conditionally averaged to negative tail of \( i \)-component PDF
\( r \) – value at \( r \)th reference point

down – downstream value
est – estimated value
i,j,k – tensor dimensions
max – maximum value
meas – measured value
min – minimum value
(\( o \) – value at an arbitrary point
\( r \) – reference value
\( u_p \) – upstream value
\( \text{wall} \) – value at the wall
\( w \) – world coordinate system

Highlight:
bold – matrix property
italic – axis dimension

Punctuation:
\( \cdot \) – filtered property
\( \sim \) – coherent (correlated) fluctuation
\( \wedge \) – Fourier coefficient
\( \| \| \) – magnitude
\{ \} – time average
\( ' \) – RMS (standard deviation)
\( M \) – property in master frame
\( S \) – property in slave frame
\( \rightarrow \) – vector
Chapter 1

Introduction
The aerodynamics of almost every flow of practical consequence is to some extent governed by turbulence - be it external or internal flows around or within aeronautical or automotive systems, hydrodynamics, oceanography, meteorology, aero-acoustics, external or internal building flows or biomedical flows, among others. Since turbulence is fundamentally an unsteady flow phenomenon, the ability to understand, predict and utilise the properties of unsteady flow is of paramount importance to fluids engineering.

Observation of the nature of unsteady flow by experiment has been conducted since the time of Leonardo Da Vinci (1452-1519) (Libby, 1471), with theories and measurement techniques developing accordingly since. However, with the 20th century advent of digital computers came the ability to solve the discretised equations of a fluid continuum, opening the potential to gain unprecedented access to numerical solutions and predictions of a variety of increasingly complex flow scenarios.

The development of Computational Fluid Dynamics (CFD) has thus continued apace for the last 60 years (Libby, 1471), with most methods based around solving various simplifications of the governing equations of continuum fluid mechanics: the Navier-Stokes equations. The most widely used approach is to attempt to predict only the time-averaged statistical flow properties via solution of Reynolds-Averaged Navier-Stokes (RANS) equations, which introduce the need for a turbulence model (Pope, 1731). A solution to the unsteady Navier-Stokes equations may be obtained without modelling through the use of Direct Numerical Simulation (DNS). However, the computational cost of DNS is extremely high: despite the first application of this concept being in 1972 (Orszag & Patterson, 1641), it still remains unfeasible for all but low to moderate Reynolds number cases (Pope, 1731). Turbulent flows at high Reynolds numbers relevant to engineering problems contain a very large range of length and time scales in the flow. Since over 99% of the effort in DNS is devoted to solving the smallest of these scales, despite these containing only a small proportion of the turbulent energy, it is logical to consider an unsteady approach which resolves the larger scales only. This approach is known as Large Eddy Simulation (LES).

LES has expanded from its initial origins in meteorology to become applicable to a wide range of engineering geometries. However, with new technologies come new
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challenges. As with all numerical simulations, LES requires the specification of boundary conditions, in particular for the velocity field anywhere where there is flow into the calculation domain (i.e. inlet conditions). Whereas it has become common practice to make use of experimentally-derived knowledge (if available) to provide inlet conditions for RANS CFD, such predictions do not require intrinsic time-resolved properties of the velocity field. The use of experiments to obtain synthetic LES inlet conditions (i.e. synthesised without numerical simulation using a priori experimentally-derived or empirical knowledge) is therefore relatively new and suitable methods must be developed and validated.

Particle Image Velocimetry (PIV) is, in many ways, an experimental technique closely analogous to LES. By illuminating a seeded flow and capturing and correlating two images separated by a short period of time, spatially discretised instantaneous velocity information over a planar field of view may be extracted. Repeating this process at many time instances results in a time series over the illuminated field. Thus, like LES, the flow is resolved within the limitations of the spatial grid resolution and temporal sampling rate.

Potentially, this technique could be utilised to specify time-resolved accurate inlet conditions for LES CFD, just as the more traditional measurement techniques of pressure probes, hot-wires and Laser Doppler Anemometry (LDA) form the basis of many experimentally derived RANS CFD inlet conditions. This potential has not yet been studied or realised. Hence, this thesis addresses the challenges and details the development, testing and validation of the various new and existing methods available for utilising PIV data to provide unsteady LES inlet conditions.

It will be useful in the discussions and descriptions that follow to have a specific application example in mind when developing a PIV driven LES inlet condition technique. Of all the engineering contexts to which LES can be, and is being, applied, a gas turbine combustor is a quintessential example of a system that necessitates study with an unsteady approach (Schlüter183). The prevalence of large-scale structures and the dominance of turbulent mixing make this an ideal, industry-relevant test case for application of both LES and PIV. The fluid mechanics of a gas-turbine combustor have thus been chosen to justify and influence the research performed herein. The
subsequent section provides a brief introduction to the geometry and flow within a gas turbine combustor, and the context within which LES CFD becomes a powerful development tool to help meet future challenges. However, although the investigations subsequently described will be set in the context of gas-turbine combustors, it is stressed they are intended to be generally applicable to the wide range of scenarios to which LES can be applied. Given the critical role played by both LES and PIV in the research reported here, it is first necessary to gain an appreciation of the state-of-the-art of both LES and PIV, and these are hence the subjects of the sections 1.2 and 1.3 respectively, before the current project objectives are set out.

### 1.1 An Introduction to Gas Turbine Combustors

Gas turbine combustor flows are used as the test case for proving this technology throughout this investigation. Although there are variations in the layout of the combustor, the majority of designs follow the basic cross-section shown in Fig 1.1.

As the air exits the compressor the velocity is too high for combustion and must be diffused, first in a pre-diffuser and then a dump diffuser. Across a typical engine cycle the overall air/fuel ratio in the combustor may vary from 45:1 to 130:1.
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Therefore, in order to burn close to the stoichiometric ratio (which for kerosene is \(~15:1\) ), it is essential the air is introduced in stages (Cohen et al.\[^{[11]}\]). Typically, for current rich burn systems, around 20\% of the air is directed through the fuel spray nozzle into the primary combustion zone; the nozzle imparts a swirl on the flow which then mixes with atomised fuel; the remainder of the compressor discharge are goes into the feed annuli. A further 20\%-30\% of the air enters the chamber through dilution ports downstream of the fuel injector. The interaction of the primary port jets and the swirled air anchors the recirculation and increases the turbulent mixing, thereby ensuring fuel is evenly burned. Some air enters through further holes in the dilution zone, which ensures the exit temperature profile is as designed, and hence does not produced ‘hot streaks’ which could damage the turbine blades (Cohen et al.\[^{[11]}\]). The remaining air (approximately 40\%) is introduced progressively along the flame tube in order to provide cooling air, often by a combined process of film and effusion cooling (Chua\[^{[9]}\]). This insulates the chamber walls from the hot combustion air.

Increasingly, the design of the combustor is motivated by the need to reduce emissions. In the near future the aero-engine industry faces significant challenges to meet the environmental objectives set out by the Advisory Council for Aeronautic Research in Europe (ACARE) by 2020. These include a 50\% reduction in CO\(_2\) and 80\% reduction in NO\(_x\) emissions relative to the CAEP2 of 1996 (ACARE\[^{[11]}\]). As seen in Fig 1.2, this represents a significant advance over existing engines.

![Fig 1.2 – Future NO\(_x\) emissions targets\[^{[103]}\)](image-url)
Whereas CO$_2$ is reduced via improved fuel consumption (from an improvement in the overall system efficiency), NO$_x$ emissions originate from high temperature (i.e. near-stoichiometric) combustion. The highly unsteady flows emanating from the fuel-injector and dilution ports govern the creation of so-called 'hot-spots' where NO$_x$ is created. Thus combustion engineers are embracing LES as a viable way of resolving these unsteady phenomena and better understanding the combustion process. An overview of the challenges of combustor design and some potential future solutions has been presented at an earlier stage in this project and can be found in Robinson (2005)\cite{811}.

1.2 Large Eddy Simulation

Turbulence at high Reynolds numbers is characterised by unsteady three-dimensional rotational fluctuations over a large range of length and time scales. It is the way in which these motions are represented, either by being resolved or modelled, that dictates the fidelity and ultimately the computational cost of a given simulation.

If a flow is considered in an Eulerian context, the mass and momentum conservation characteristics of that flow are described by the Navier-Stokes equations. The energy equation is not considered here, since the focus of the work in this thesis is on non-reacting incompressible, constant fluid property flow, i.e. $\rho$, $\mu$ (and hence $\nu$) are constant. For these conditions, the Navier-Stokes equations may be written in tensor notation as:

$$\frac{\partial}{\partial x_i} (U_i) = 0$$  \hspace{1cm} (1.1)

$$\frac{\partial U_i}{\partial t} + \frac{\partial}{\partial x_j} (U_i U_j) = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} (2\nu S_{ij})$$  \hspace{1cm} (1.2)

Where the strain rate tensor, $S_{ij}$, is defined as,

$$S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$$  \hspace{1cm} (1.3)
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and the velocity and pressure are instantaneous properties at any point in the flow.

The technique of Direct Numerical Simulation (DNS) solves these Navier-Stokes equations for all length and time scales present in the flow, and hence produces a complete realisation of the flow within the limits of the boundary conditions used. However, the computational cost is excessive and increases approximately with the cube of the overall Reynolds number, restricting the current use of DNS to flows with Reynolds number of the order $10^4$ (Piomelli et al.\cite{71}).

As mentioned previously, the most developed and widely used approach in CFD is to use a RANS method. Here a Reynolds decomposed form of the Navier-Stokes equations is considered, splitting instantaneous velocities, $U_i$, into time-averaged, $<U_i>$, and fluctuating, $u_i$, components, resulting in only the time-averaged flow field being solved. Turbulence effects are accounted for through the appearance of the Reynolds stress terms, $\left\langle u_i u_j \right\rangle$. Closure of these must be obtained through a turbulence model. This process obviously leads to a loss of information, with the instantaneous fluctuations not predicted.

Large Eddy Simulations go some way to bridging the gap between DNS and RANS. The velocity field is filtered into a 'resolved' part and a 'residual' part. The resolved field contains the larger eddies and is explicitly calculated, the residual field is modelled in a similar way to a RANS calculation. This method is based on the knowledge that the large eddies tend to contain the majority of the turbulence energy and are flow-dependent, while the small eddies are more adequately modelled by empirical assumptions, being generally more universal and isotropic in character (Pope\cite{73}). Despite the use of some modelling LES is still a computationally expensive technique. Large scale motions must be spatially and temporally resolved by the numerical solver, which leads to fine grids and small time steps (although not to the extent of DNS), resulting in a computational cost between that of RANS and DNS (Pope\cite{73}). The three dimensional nature of turbulence means LES must be conducted on a three dimensional grid, even if the (average) flow is intrinsically one...
or two dimensional. Also, gathering the required time-averaged statistics requires long run times.

1.2.1 Governing Equations

The separation of LES from DNS occurs through the use of a low-pass spatial filter applied to the velocity field. The general form of the filter, as presented by Pope\textsuperscript{[73]}, can be written as:

\[
\overline{U}(x,t) = \int G(r,x)U(x-r,t)dr
\]  

(1.4)

Where \(G\) is the filter function and \(\overline{U}\) is the filtered (resolved) velocity field. The filter function is usually defined as possessing a filter width, \(\Delta\), which determines the size of the eddies removed by the filtering. As defined in the 1D case for simplicity, the three most commonly used filter functions are the top hat filter:

\[
G(x) = \begin{cases} 
\frac{1}{\Delta} & \text{if } |x| \leq \Delta/2 \\
0 & \text{otherwise}
\end{cases}
\]

(1.5)

the Gaussian filter:

\[
G(x) = \sqrt{\frac{6}{\pi \Delta^2}} \exp \left( -\frac{6x^2}{\Delta^2} \right)
\]

(1.6)

and, as described best in wavenumber space, the sharp Fourier cut-off filter:

\[
\hat{G}(k) = \begin{cases} 
1 & \text{if } k \leq \pi/\Delta \\
0 & \text{otherwise}
\end{cases}
\]

(1.7)

where the caret \(^\wedge\) denotes the complex Fourier coefficient (Piomelli \textit{et al.}\textsuperscript{[71]}). Whereas the top-hat and Gaussian filters will have an attenuation effect on a range of wavenumbers, the sharp Fourier cut-off will only remove frequencies above the cut-off wavenumber. In practice a Gaussian filter is always used in conjunction with a
sharp Fourier cut-off. If not applied explicitly by the application of equation 1.4, a filter may be implied by the spacing of the computational grid. In the latter case the filter function can be considered a top-hat filter. More details on the application of these filters to velocity field analysis is given in section 3.1.3.1.

As a consequence of the filtering, the velocity field is now defined by the decomposition:

$$U(x, t) = \underline{U}(x, t) + u'(x, t)$$

(1.8)

where $u'$ is the residual velocity field. Equation 1.8 may appear analogous to a Reynolds decomposition, however the important difference is that $\underline{U}(x, t)$ is still a time-dependent (unsteady) field. By applying equation 1.8 to the Navier-Stokes equations given above, the following filtered forms, may be obtained.

$$\frac{\partial \underline{U}_i}{\partial x_i} = 0$$

(1.9)

$$\frac{\partial \underline{U}_i}{\partial t} + \frac{\partial}{\partial x_j} (\underline{U}_i \underline{U}_j) = -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j} (2\nu \overline{S}_{ij} - \tau_{ij})$$

(1.10)

These equations are solved in LES to give the motion of the large (resolved) scales. The effect of the small scales is introduced through the subgrid-scale (SGS) stress term, $\tau_{ij}$, which arises from the difference between the filtered product of the velocities, $\underline{U}_i \underline{U}_j$, and the product of the filtered velocities, $\underline{U}_i \underline{U}_j$, hence:

$$\tau_{ij} = \underline{U}_i \underline{U}_j - \underline{U}_i \underline{U}_j$$

(1.11)

An SGS model must be used to account for this term.
1.2.2 Sub-Grid Scale Models

The function of an SGS model is to transfer the correct amount of energy between the resolved and residual scales. Almost all models make the assumption of a forward energy cascade, i.e. turbulent energy generated in the large scales is transferred through progressively smaller scales until the level where it is dissipated through viscosity at the Kolmogorov microscales. This is illustrated schematically in Fig 1.3. Methods are available (Leith\textsuperscript{46}, Mason & Thompson\textsuperscript{53}) to allow for backward energy transfer, i.e. from small to large scales, but these will not be considered here.

Most current SGS models trace their origins to the Smagorinsky model. Smagorinsky\textsuperscript{90} originally proposed a method based on an Eddy Viscosity Hypothesis (EVH) for use in weather prediction, although it has since been adapted to a wide range of applications. Such models are based on the introduction of a scalar eddy viscosity, $\nu_\tau$, defined by a linear relationship between the SGS stresses and the resolved strain-rate tensor, $\bar{S}_{ij}$, such that:

$$\tau_{ij} - \frac{\delta_{ij}}{3} \tau_{kk} = -2\nu_\tau \bar{S}_{ij} \quad (1.12)$$

Fig 1.3 – Schematic kinetic energy transfer in a forward energy cascade
For practical coding reasons the $\tau_{kk}$ term in equation 1.12 is generally accounted for in the solving of the pressure term, thus equation 1.10 is implemented as,

$$\frac{\partial U_i}{\partial t} + \frac{\partial}{\partial x_j} (U_i U_j) = -\frac{1}{\rho} \frac{\partial p^*}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu_e \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right)$$

(1.13)

where,

$$p^* = p + \frac{1}{3} \tau_{kk}$$

(1.14)

and $\nu_e$ is the effective viscosity given by the sum of the molecular kinematic and SGS eddy viscosities, such that:

$$\nu_e = \nu + \nu_T$$

(1.15)

Generally, the eddy viscosity is obtained algebraically from the resolved flow scales, avoiding the expense of solving further transport equations. This is commonly justified through a production/dissipation equilibrium assumption, implying an energy spectrum shape invariant with time (Popel [73]). Such an assumption is valid as small scales exist over much shorter time scales and hence react to change effectively instantaneously.

Smagorinsky [90] used the equilibrium assumption, along with the magnitude of the strain-rate tensor, $|\overline{S}| = (2\overline{S} - \overline{S})^{1/2}$, and the filter width, $\Delta$ (or local grid size), yielding the eddy viscosity as,

$$\nu_T = (C_s \Delta)^2 |\overline{S}|$$

(1.16)

Various studies have attempted to evaluate the Smagorinsky constant, $C_s$ (Lilly [48], Deardorff [12], McMillan et al. [55]) and found it can be flow dependent. Typically a value of $C_s \approx 0.18 - 0.23$ is used, although in the presence of shear, near solid
boundaries or in transitional flows, values as low as 0.065 may be required (Piomelli\(^{70}\)). A dynamic calculation of the local \( C_s \) is available through the implementation of a method developed by Germano et al.\(^{21}\). This removes the need for pre-determined constants and facilitates the backscatter of energy by allowing \( C_s^2 \) to become negative, though this can cause flow instability (Veloudis\(^{101}\)) and is usually suppressed by some form of spatial averaging. Dynamic implementation will not be considered in this project.

A commonly used modification to correct for the over prediction of eddy viscosity in resolved near-wall regions is to use Van Driest damping. This uses the non-dimensional distance from the wall, \( y^+ \), to modify equation 1.16 to be of the form,

\[
\nu_t = [C_s A (1 - e^{-y^+ / \kappa})] |S| \tag{1.17}
\]

where,

\[
y^+ = \frac{U_* |y - y_{wall}|}{\nu} \tag{1.18}
\]

\[
U_* = \sqrt{\frac{\tau_{wall}}{\rho}} \tag{1.19}
\]

This form of the SGS model will be used throughout this investigation (see Chapter 6). For a more extensive review of SGS models, the reader is referred to Veloudis\(^{101}\).

### 1.2.3 Inlet Conditions

In any numerical calculation, the distribution of all flow variables must be specified at any inlet boundaries (Versteeg & Malalasekera\(^{106}\)). In a RANS context these can be obtained from time-averaged experimental data or through a method of 'educated guessing' to give a mean profile and some simple turbulence statistics (such as an intensity and a length scale). For example, measurement of an inlet mass flow rate, possibly with the addition of an empirical profile (e.g. from knowledge of boundary
layers, shear layers, channel flows or other commonly-encountered flow features), allows a local mean velocity to be estimated; assumption of a turbulence intensity allows the inlet turbulence energy to be scaled and a guess at the largest eddy size from geometry information allows a length scale estimate. However, for LES the unsteady nature of the flow must also be specified and this proves to be much more problematic in practice.

For many flows of engineering application, inlet conditions have been shown to be critical to the resultant accuracy of LES predictions (Glaze & Frankel, McMullan et al., Schönsfeld et al.), especially with regard to the higher-order statistics and turbulent structures (George & Davidson). However, in some situations it is possible to reproduce the time-dependency of the inlet conditions adequately and affordably.

If a sufficient amount of geometry upstream of the region of interest can be simulated as part of the LES prediction, and the inlet conditions there are far-removed from the region of interest and the solution close to the region of interest develops naturally within the simulation, the flow may be considered independent of the inlet condition applied. The feasibility of this approach is heavily dependent on the complexity of the flow and geometry, with the computational cost of resolving large regions of flow rendering it impractical for many applications.

In simulations where the geometry is repeated both upstream and downstream of the solution domain, periodic boundary conditions may be applied in the streamwise direction with flow at the exit plane being recycled to the inlet. Provided the domain is of sufficient length to contain the largest turbulent structures, self-sustained turbulence is achieved. Upon reaching a statistically stationary state the flow will be determined by the repeating geometry and will be independent of the ‘inlet condition’ applied initially. A similar technique may also be applied for developing flows, such as boundary layers, though this requires appropriate rescaling of the velocity profile between the exit and inlet planes.

If the solution geometry does not allow any of these methods to be used, the alternative is to generate a database of unsteady velocity data to use as an inlet
condition. The database may be read and re-read as required as the velocity data is applied to the inlet of the main simulation. Two strategies are in common use to obtain this: precursor simulation and synthetic generation.

For some turbulent flows the use of a precursor simulation allows multiple instantaneous samples to be extracted from an auxiliary LES calculation, which is usually a simplified or cyclic representation of the geometry upstream of the main LES calculation. This precursor technique is widely accepted to be a good approach provided the auxiliary calculation provides a good representation of the real flow, such as in Apte et al.\cite{4} for a coaxial jet, Veloudis et al.\cite{102} for a constricted channel flow, Lund et al.\cite{50} for a developing boundary layer (with appropriate re-scaling in the precursor simulation) and Schlüter\cite{83} for a gas turbine engine. The latter is of special interest as the precursor calculation was based on RANS CFD, and so is an example of an interface between a RANS simulation of a compressor and an LES of a combustor. The unsteady fluctuations were provided from a periodic pipe flow and scaled to match the mean and turbulence profiles of the RANS (although why fully-developed pipe flow turbulence should be appropriate for a compressor exit flow was not justified). However, the computational cost of any precursor simulation rises rapidly with the complexity of the upstream geometry, to the point where modelling the upstream region may be more expensive than the region of interest and thus the method becomes impractical. Equally, in some cases, adopting a precursor approach may simply move the question of where to specify inlet conditions for LES further upstream. An alternative to the precursor method is clearly necessary.

Synthetic inlet condition generation offers the potential of providing computational simulations with high fidelity experimentally-derived data for complex geometries, without the need for excessive computation. However, the extent to which the experimental data is used has a direct effect on the quality, complexity and cost of the generated data. The analysis and development of synthetic velocity field inlet conditions forms a major part of this thesis and is discussed at length in chapter 3.
1.2.4 Previous Work on LES Applied to Combustor Flows

The entire field of development of LES methods and their application to engineering situations is too broad a subject to tackle here. It has already been stated that the case of a gas turbine combustor is a promising area of application for LES as it has become clear that its prediction benefits substantially from unsteady simulation Schütter[83]. Testing and development of the work in this project is therefore to be undertaken using combustor-relevant flow examples, and hence discussion here is limited to analogous previous studies. A review of some general applications and recent advances is provided by Moin[61].

Simulating an entire combustor geometry in LES is obviously computationally expensive, and hence the trend has been to look at components in isolation. Where LES has so far been applied to realistic fully annular combustor geometries (containing approximately 20 fuel injector sectors, see Fig 1.4) it requires an excessively large mesh (up to 100 million control volumes!) and consequently massive processing capabilities (Fatica & Alonso[17]). Although this allows the resolution of circumferential instabilities, such studies are more relevant in the context of code development and computer science due to their demands for a multi-processor code architecture and efficient memory allocation.

Fig 1.4 – Geometry and instantaneous velocity contours within a full P&W 6000 combustor simulation[29]
More often combustor studies consider a one-burner sector, applying periodic boundary conditions in the circumferential direction. Reynolds et al. describe one of a number of studies aimed at predicting the reactive flow in a Pratt and Whitney 6000 combustor using LES. Cold predictions were first conducted during the CFD model development process on a mesh of 1.4 million control volumes, with the simulation taking 13,500 CPU hours for an explicit solver using 32 processors on an IBM SP3 machine. The prescription of inlet conditions is described in more detail in Schliüt and Schlier et al. Mean velocity and turbulence levels were established at pre-diffuser entry from an upstream RANS compressor simulation, with synthetic turbulence fluctuations superimposed from a database obtained from an auxiliary LES of a fully-developed turbulent pipe flow. LES was chosen over RANS for combustor prediction because of its superior ability to capture flows where mixing is dominated by large scale structures. Ham et al. continued this LES study on a single injector sector introducing complex models of spray break-up, evaporation and turbulent combustion. Again, a cold flow case was initially investigated and results for various pressure drops and mass flow splits were found to be within 10% of experimentally obtained validation data, which could be considered a reasonable error band. Cold flow simulations were made on a 35 million control volume mesh, taking 8 CPU hours per time step. No reference is made to the inlet conditions used.

Several studies have neglected the pre-diffuser and dump cavity entirely, modelling only the combustor internal geometry. Di Mare et al. is one such example where only a single combustor can has been modelled, based on the geometry of the Rolls-Royce Tay engine. Primary and secondary port jets were resolved, but porous walls mimicked by imposing a fixed mass flow boundary instead of solid walls in the dilution zone. Inlet conditions at the fuel injector were, by admission, crude, with a random noise corresponding to a turbulence intensity of around 20% imposed on a flat mean velocity profile at the swirler exit. Some attempt was made to account for correlations within the flow by ‘grouping’ the imposed fluctuations in space and time in blocks corresponding to an assumed integral length and time scale. Turbulence fluctuations were neglected in the dilution jets. The flow features compared very favourably with experimental observations, although it was noted that the results were sensitive to the turbulence intensity and precise definition of the geometry at the inlet. Selle et al. also noted the difficulty in defining inlet profiles for injectors when the
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geometry of the swirler vanes is not resolved, although, by contrast, this work asserted that inlet velocity fluctuations could be neglected owing to the dominance of the turbulent activity caused by high velocity gradients inside the combustor. Apte et al.\textsuperscript{[4]} conducted LES on a geometry similar to those above, again with no resolution of the swirler vanes: the injector was modelled as a coaxial jet with prescribed swirl. Here the inlet conditions were obtained from a pre-cursor simulation. They found significant improvement over RANS methods in the prediction of particle-dispersion characteristics.

It should be noted that the above studies only dealt with the combustor internal geometry. McGuirk and Spencer\textsuperscript{[54]} have shown, albeit in a RANS context, that coupling exists between the internal and external flow. Thus, if a simulation is to be conducted of the internal flow alone, ideally the boundary conditions specified to represent the external flow should be based on measurements at the fuel injector exit both the upper and lower supply annulus ducts. Spencer et al.\textsuperscript{[95]} have attempted to follow this approach in their predictions of a realistic combustor geometry containing 4.3 million grid cells using both LES and RANS methods. Synthetic inlet conditions were applied variously at the fuel injector and annulus inlets. They found the mixing process was significantly affected by the details of the inlet conditions used, especially in the fuel injector, with LES offering a much closer prediction of experimental profiles than RANS.

Another example, which emphasises one important flow component of a gas-turbine combustor internal flowfield is the case of multiple impinging jets in cross flow. This simulates the primary zone of the combustor in the absence of the swirling injector flow. Spencer and Adumitrosie\textsuperscript{[94]} conducted an LES study of this test case and compared it to both experimental and RANS methods. The results show that, in addition to improved prediction of turbulence levels, LES can provide extra data such as the scalar-velocity correlations and the bi-modal PDF of the axial velocity at the free stagnation point of the impinging jets. Inlet conditions were prescribed from an experimental mean profile, onto which was superimposed a random Gaussian distribution to the measured RMS levels. Geyer et al.\textsuperscript{[22]} used a similar test case to provide a general evaluation of the capabilities of LES, as well as providing velocity profiles, PDF’s and correlations that correspond fairly well with experimental
measurements. LES sources of error are discussed, which, in the case of spatial filtering and sampling errors, are equally applicable to PIV and will be discussed in section 5.1. A further discussion on sources of numerical error in LES is offered by Ghosal\textsuperscript{[23]}.

These few examples serve to illustrate the increasing interest in using LES CFD for combustor flows, but also the importance, and as yet insufficient understanding, of producing an optimum strategy for specifying the inlet conditions for such simulations, which maximises the available knowledge of inherent flow features identified in external measurements. Further discussion on studies relating to the application of LES in the context of inlet condition usage will be given in chapter 3.

1.3 Particle Image Velocimetry

The primary method of obtaining experimental measurements in this investigation will be through the use of Particle Image Velocimetry (PIV). PIV was selected since, as stated above, it is a technique that can provide the spatial and temporal resolution required for inlet condition generation methods for LES, as mentioned in previous sections and as will be introduced in chapter 3.

PIV has really only become feasible during the last 30 years and can still be considered as ‘under development’. A history of the development of PIV is not provided here, and interested readers are referred to Hollis\textsuperscript{[32]} and Grant\textsuperscript{[26]}. PIV is essentially a flow visualisation and quantification technique. In planar PIV, such as used here, the flow to be measured is seeded with an appropriate particulate material (i.e. flow-following at relevant timescales, and light reflective) and is illuminated by a pulsed laser light sheet in a flow plane where measurements are to be obtained. This plane is then either viewed orthogonally by a single camera, in the case of monoscopic PIV (which only allows the resolution of the two in-plane velocity components), or obliquely using two cameras, for stereoscopic PIV (which additionally allows the through-plane component to be resolved) (Prasad\textsuperscript{[74]}).
Two images of the flow are then taken in a repeated time-sequenced manner, usually with a form of digital photography, timed to coincide with the laser pulses. The double image capture is repeated a number of times, dependent on the storage capabilities of the camera, to produce a data set containing a given number of double images. Each double image is processed to obtain an instantaneous vector field and, subsequently, a data set (time-series) of instantaneous vector field samples. Vectors are obtained by sub-dividing the image field of view (FoV) into a series of interrogation cells (normally of size 32×32 pixels) and performing a cross-correlation of each double image pair. The resultant vectors represent the modal displacement of the imaged particles within each interrogation cell, and knowledge of the time between laser pulses allows the calculation of the instantaneous two or three component velocity vector field for mono- or stereoscopic PIV respectively.

The entire PIV process can be thought of as a series of stages: acquisition, processing, validation, and analysis. The following 4 sub-sections describe briefly the theory and standard practices of each stage independently. Further details on elements specific and of importance to the current project are presented in chapter 2.

1.3.1 Acquisition

The first stage of the PIV process is to obtain the raw images. Whereas subsequent stages follow a fairly consistent procedure, the set-up parameters for performing the measurements may change substantially depending on the fluid used and the geometry being tested. Flexibility and patience are thus required in optimising the set-up.

Perhaps the most crucial aspect of the physical set-up is the correct choice and delivery of the seeding. A particle must both accurately follow the flow and scatter sufficient light to be recorded. Previous investigations by Raffel et al.[75] have shown suitable seeding can be obtained by the use of 20μm Polyamid particles for use in water, or atomised Shell Ondina oil, with a maximum diameter of 2μm, for use in air. Raffel et al.[75] state that for 95% detection during processing there should be a minimum of 5 particles per interrogation cell. As most water-based systems are recirculating the issues of seeding distribution are negated, however in airflow this is often not the case and care must be taken that seeding density is both sufficient and
homogeneously distributed. Particle size and density should be adjusted until a particle image diameter of around the optimum 2 pixels for the selected FoV is achieved (Westerweel\textsuperscript{[108]}).

Care should be taken when arranging and calibrating the camera to the desired FoV size. If the test facility has not been designed specifically with PIV in mind optical access may be difficult and practical problems such as glare may need to be overcome. Often this is just a matter of painting any reflective surfaces or using a polarising filter on the camera. When designing for any given FoV size the user should generally select a lens and camera position to give the minimum required area. Large fields may result in insufficient particle size and poor spatial resolution. In the experience of the author, a field of approximately 150mm in water and 80mm in air are the advisable upper limits when using the above seeding: if a greater area is required the user should consider splitting the FoV and taking separate measurements.

Calibration of the camera image is absolutely necessary in order to map the real-world dimensions to the camera pixel units (LaVision\textsuperscript{[44]}). This process must be repeated every time the distance or viewing conditions between the camera and the object plane is changed. When a camera views normal to the object plane and there is no distortion (almost always the case in monoscopic PIV), this is simply a process of linear scaling. It is only necessary to know the separation and location of two points within the object plane to calibrate the entire image. However, if this is not the case and the camera views the object plane obliquely (as in the case of stereoscopic PIV), or there is image distortion, a calibration plate with known, equally spaced marks on it must be used, such as in Fig 1.5. When oblique viewing is required a device called a Scheimpflug adaptor may be employed on the camera lens to maintain focus over an inclined plane (for details on the Scheimpflug principle see Merklinger\textsuperscript{[58]}).
In order to resolve the out-of-plane motion for stereoscopic PIV, calibration at two or more locations in the through-plane direction is required. This can either be achieved by using a two-level calibration plate (Fig 1.5b) with one calibration image per camera, or by the relative translation of a flat-calibration plate (Fig 1.5a)) between two calibration images for each camera. In the experience of the author, the two-level calibration plate, if available, is generally the most reliable method of providing calibration as it removes the ambiguity of translating the plate and/or camera system. Further discussion on this will be given in section 2.2.3.

Two common mapping functions used to define the transformation between the world coordinate system and the camera coordinate system are the camera pinhole model (Willert[111]) and the 3rd order polynomial fit method (Solof et al.[192]). The former, as illustrated in Fig 1.6, uses the calibration image to calculate six parameters that define the rotation, \( R \), and translation, \( T \), between the world coordinates, \( X_w \), and camera coordinates, \( X_c \), for a given focal length to a 'pinhole' point, \( f \), such that,

\[
X_c = R X_w + T
\]  

(1.20)

Note the focal length of the pinhole model will be close to, but not the same as, the focal length of the lens. First and second order radial distortions may also be accounted for about the principle point on the camera CCD chip. The use of a Scheimpflug adaptor will move this point away from the centre of the chip.
Fig 1.6 – Camera pinhole model (Willert\textsuperscript{111})

Alternatively, the image distortion may be described by a polynomial of up to 3\textsuperscript{rd} order for both directions in the plane of the image. The coefficients of the polynomial are then determined by a non-linear least-squares fit to the calibration image.

The polynomial fit approach may be beneficial if the image contains strong distortions, which could stop the pinhole model converging. However, extrapolation of the polynomial fit is poor and requires calibration marks covering the whole image. The pinhole model has the added advantages that it can take into account the thickness of the calibration plate when using a set-up that requires the cameras to view from opposite sides and can take advantage of the self-calibration technique proposed by Wienke\textsuperscript{110}, further described in section 1.3.2.

When capturing flow areas that extend beyond a single FoV, multiple fields (i.e. PIV data sets at differing locations) may be captured to avoid having to increase the FoV and hence reduce the vector resolution. Regardless of the calibration method used, in the experience of the author, a positional error of order 1 pixel should be expected in either the location or the scale factor between the camera and world coordinate systems for each field. Thus, for a typical FoV, one should be prepared to accept
discontinuities between intersecting fields of up to 1 mm. A correction for this phenomenon will be discussed in section 2.2.3.

Two key timings must be considered when taking images: the sampling time (the time between the capture of the first frames of successive double image pairs) and the inter-frame time (the time between the capture of each frame of a double image pair). The former gives the temporal resolution of the data and is generally the limiting factor in controlling the range of frequencies in the turbulence spectrum that can be resolved. With the latest PIV equipment, sample rates of 1kHz are available at 1 mega-pixel resolution. This is the reason why PIV may now be considered to have sufficient resolution to be applicable to LES inlet boundary conditions. The inter-frame time is dependent on the flow being captured and the optical set-up. Keane and Adrian\cite{36} suggest a 'one quarter rule' is employed when determining inter-frame time: namely that the resulting mean displacement magnitude of the particles in each interrogation cell should be less than one quarter of the cell’s dimension, in order to minimise particle pair loss due to out of plane motions. Thus, for a standard 32x32 pixel interrogation cell the displacement should be less than 8 pixels. However, as the calculated displacement error increases with a decrease in the displacement, a maximum displacement of 8 pixels should be targeted.

Laser light sources are used to give high intensity light that can be easily formed into a light-sheet. Care must be taken to ensure the sheet adequately illuminates the entire FoV. Generally the light sheet should be as thin as possible to eliminate any out of plane motion (Raffel\textit{ et al.}\cite{75}), however in circumstances where the out of plane motion is large relative to the in-plane motion, or when taking stereoscopic data, the thickness may need to be increased to allow sufficient in-plane displacement to be captured within the chosen inter-frame time.

1.3.2 Processing

In a PIV context the term processing refers to the conversion of raw images into vector fields. As mentioned previously, the image is spatially discretised into interrogation cells in which the modal displacement (and the known inter-frame time) allows inference of a single velocity vector to be associated with that cell. Within
each cell the spatial correlation between the double image frames is calculated via Fast Fourier Transform, resulting in a correlation map with the peak intensity at the average displacement. A Gaussian fitting scheme is generally employed to determine peak location (Keane & Adrian\textsuperscript{[37]}), typically resulting in a correlation peak location accurate to within 0.1 pixels. A typical correlation map is shown in Fig 1.7.

![Correlation Peak](image)

**Fig 1.7 – Typical correlation map (LaVision\textsuperscript{[44]})**

As outlined by Lecerf \textit{et al.}\textsuperscript{[42]}, correlations performed on the image pairs of each camera infer the projected particle displacement for that camera. Knowledge of the relative calibrated positions of the cameras can then be used to extract the in-plane components (and through-plane component in stereoscopic PIV).

The above calculation is optimised when all particles move homogeneously, however this is not necessarily the case. In regions of strong velocity gradients or strong local rotation Keane and Adrian\textsuperscript{[36]} show that the resulting effect is a broadening and diminishing of the correlation peak, with a resultant bias towards slower velocities. This may lead to increased uncertainty or even non-detection of the 'correct' peak. These effects can be remedied by careful selection of the interrogation cell size and inter-frame time, to ensure minimum velocity variation with each cell and minimise the loss of particle image pairs between frames.

An instant measure of the quality of the peak obtained is the Q-factor, defined as the ratio of the highest peak (i.e. the assumed correct displacement) to the next highest (deemed to be noise). Thus, if $P$ indicates intensity, LaVision\textsuperscript{[44]} define:
This quantifies the signal to noise ratio: if the Q-factor is greater than 2 the peak is probably correct, whereas values closer to one indicate it is probably false.

The choice of size of interrogation cell to use is governed by two criteria: the cell should be small enough that the homogeneity assumption is a good assumption (i.e. for turbulent flow it should be of the order of the Kolmogorov length scale) and it should contain sufficient particles to obtain a valid correlation. The former criteria is almost always relaxed since this would result in unacceptably small FoV sizes for all high Reynolds number fluids, and would also make it difficult to achieve the 5 particles per cell mentioned previously. For almost all practical situations a cell size of 32×32 pixels is sufficient, though it is common practice to perform multiple passes, usually with an initially higher cell size. Multi-pass processing allows a shift to be applied to the cell position of subsequent passes based on the local velocity vector inferred by previous passes, ensuring the right particles are correlated and improving the signal-to-noise ratio\(^{[44]}\). Using a larger cell size (say 64×64 pixels) to conduct the initial pass ensures even large velocity vectors are calculated with high confidence. Vector grid density may be increased by overlapping the interrogation cells, so that parts of the image are used more than once. The standard overlap is 50% of cell size, thereby increasing the number of vectors by a factor of 4.

The above concessions on interrogation cell size lead to the resultant velocity field being effectively low-pass filtered, with eddies smaller than the cell size smoothed out. This leads to the phenomenon of sub-cell filtering, which can have a significant effect on the statistics derived from a velocity field measured by PIV. While dimensional analysis may be used to gain an \textit{a priori} estimate of the Kolmogorov scales relative to the experimental parameters (Pope\(^{[73]}\)), a quantification and correction for this effect is best performed \textit{a posteriori}, using the captured and processed velocity vector field (Sheng \textit{et al.}\(^{[86]}\)). This is further detailed in section 3.1.2.3, since this aspect of PIV data is of crucial significance to its use for accurate unsteady velocity field representation.
Similarly, in the temporal sense, dynamic averaging occurs when the real particle motion is averaged over the time between the two recorded images due to the discrete time sampling of the image pair. Ideally, the inter-frame time should be set to less than or equal to the local Kolmogorov time scale; although, once again this must be balanced with the requirement above to optimise the pixel shift. Dynamic averaging affects each vector in isolation and is independent of the time series resolution, which is wholly governed by the available sampling rate.

If the camera pinhole model was used in the calibration of the camera, self-calibration may be performed before vector calculation takes place. Self-calibration corrects for any misalignment between the calibrated image plane and the laser plane. Firstly, a disparity vector is calculated by performing a cross-correlation on the de-warped (i.e. camera to world coordinate) images from each camera, akin to that used for planar velocity vector calculation. Although ideally these images should look the same, as the particles are distributed throughout the light sheet an ensemble average from around 50 images is typically required to gain an accurate correlation peak and thus an accurate disparity vector. The map of disparity vectors is then used to triangulate the true position of the light sheet relative to the calibrated viewing plane and thus correct the camera calibration models. Multiple passes may be used to gain better fits, though often the disparity will converge after two or three passes (Wieneke[110]).

1.3.3 Validation

Once the derived vector field has been established a validation process must be conducted to ensure sufficient confidence that the results may be considered genuine. The amount of validation required is thus a function of the quality of the data obtained. The identification of spurious vectors is conducted by outlining a set of criteria that a valid vector should meet.

The most reliable method is to compare any particular vector to its neighbours. Westerweel et al.[101] describe the now widely-used method by which a vector is evaluated against the average magnitude and standard deviation of the surrounding 8 vectors. If the vector lies more than a defined factor of the standard deviation outside
the magnitude it is considered spurious. This is based on the assumption that if a vector is substantially different to those surrounding it then continuity is not satisfied and the vector must be false. The factor used is dependent on the turbulence, but typically 2.0 is acceptable for most situations.

The vector may also be evaluated via the Q-factor given in equation 1.21. A threshold value may be set, below which the vector is rejected. This is useful for good quality data, but for data sets with high noise there is the possibility of removing genuine vectors. The threshold should therefore be set fairly low; typically 1.3 is used.

An alternative validation method is to use some pre-determined knowledge. This may be by imposing set limits to the velocity, outside of which a vector is considered spurious, or by masking out certain regions where it is known no valid vectors exist, such as walls. Whereas masking can be confidently used (and has the added benefit of reducing the computational time), in the experience of the author, pre-defined limits should only be set after other validation methods have been exhausted.

In order to maintain a continuous field, any vector that is rejected by the above methods must be replaced. Replacement in the PIV hardware/software supplied by LaVision\(^{[44]}\) is implemented by considering the next highest peak in the correlation map. This peak is then checked against the validation criteria and, if found also to be invalid, the process is repeated for the 3\(^{rd}\) and 4\(^{th}\) peaks. Subsequent peaks are considered to be noise and are never evaluated. If no alternate vector can be found the standard practice is to perform linear interpolation based on the surrounding vectors. This will obviously cause some smoothing and is no substitute for good data: typically, if more than 5% of the data is interpolated the user should consider revising the experimental setup and re-taking the data.

1.3.4 Analysis

The final stage of PIV measurements is to extract useful information from the calculated velocity field. This can be both in the context of providing a final ‘sanity check’ on the data, now that derived statistics can be obtained, but also in order to present the data in a compact and descriptive way.
Since providing every instantaneous vector field (potentially over 3000 samples) is excessive, methods of quantifying the spatially coherent turbulence data, which separates PIV from point based measuring techniques, become of prime importance. As well as point-based first and second moment statistics (time-averaged mean and RMS velocities; Reynolds stresses), PIV may be used to obtain two-point statistics (velocity correlations; length/time scales; power spectra) and provide data for turbulent structure identification techniques (Proper Orthogonal Decomposition; Linear Stochastic Estimation). The optimisation and implementation of such methods is a major focus of this investigation and is dealt with in chapter 3.

1.3.5 Previous Work on PIV Applied to Complex 3D Flows

As with section 1.2.4 above, a full review of the use of PIV in engineering situations is too broad a subject to cover here. It is worthwhile, however, to review a selection of previous work on the use of PIV for complex turbulent flow field measurement in applications similar to that used here.

Hollis gives an excellent account of PIV set-up and processing in combustor-relevant geometries in both air and water flows, using a monoscopic low frame-rate system. Of particular relevance to this project are the analysis methods used, described therein to extract information on the scales of turbulent flow captured. Similarly, Midgley et al. show how even low-speed, monoscopic data can be used to extract unsteady flow data on coherent vortex structures in the swirling region downstream of a fuel injector. To this end the concepts of Reynolds decomposition, Probability Density Functions (PDFs) and conditional averaging, supplemented in Midgley by two-point correlations along with Proper Orthogonal Decomposition (POD), proved to be very successful and will be used extensively in the current work. These ideas are explained more fully in chapter 3.

The work of Wernet is a good example of an attempt to capture time-resolved PIV in challenging circumstances. Images are captured at 30Hz downstream of both a single stage transonic axial compressor and a centrifugal compressor, requiring the use of dedicated viewing windows and a periscope probe for light sheet delivery. Of
particular importance is the emphasis placed on achieving the correct seeding distribution, with a combined global and local delivery system required. This highlights the potential difficulties of achieving an even distribution of seeding whilst using a heavier-than-air particulate in a non-recirculating airflow experiment. Careful attention must be paid to highly swirled and wake regions, where seeding density may be reduced to unworkable levels. The techniques used to ensure an even seeding distribution for the measurements made in this project are outlined in chapter 2.

Several attempts have been made to extract a through-plane component from monoscopic data. Often, as in Fujisawa and Staoh, this involves estimation from the continuity equation, although this often requires assumptions about the flow or measurements in other locations. Willert et al. introduce the additional technique of Doppler Global Velocimetry (DGV) to obtain the through-plane velocities, whilst using PIV to capture the in-plane velocities. As such a set-up only requires viewing perpendicular to the image plane it has the potential to alleviate some of the difficulties in viewing obliquely for stereoscopic PIV. However, currently DGV is unproven for unsteady data and only has a measurement resolution of order 1m/s (Willert et al.). This is clearly of significance for the use of experimental data for LES inlet conditions since it is a pre-requisite for such use that the data should be of all three velocity components at the same time instant. For PIV, this can also be achieved via the stereoscopic, two-camera approach.

Willert et al. give a useful discussion on the set-up of stereoscopic PIV in air flow. They note how the relative positions of the cameras and laser must be optimised to provide even light scattering and how optical access and glare increasingly become an issue as seeding is deposited on the viewing surface. Lang et al. and Parker et al. both successfully use stereoscopic PIV in water flows, with both employing water filled prisms to alleviate the optical distortions associated with oblique viewing, as illustrated in Fig 1.8.

High-speed PIV has been shown to be possible up to 20kHz in Williams et al., albeit with a wet-film camera. Again, the main problem encountered was achieving the correct seeding, which had to be carefully monitored to attain the correct level. Also highlighted is the spatial filtering analogy with LES and the need for high
sample rates when extracting statistics for LES comparison. Spatial filtering results from the use of an interrogation cell size larger than the Kolmogorov length scale, thereby introducing low-pass filtering into the measured velocity field in an analogous way to the filtering applied in LES. The identification and correction for this phenomenon is significant in the context of inlet condition generation and will be discussed further in section 3.1.2.3. Tanahashi et al.\textsuperscript{[98]} demonstrate 1kHz stereoscopic PIV (combined with simultaneous Planar Laser Induced Fluorescence (PLIF) of two different species) in a combusting flame in airflow and note the critical extra insight that can be gained from simultaneously capturing three components.

This selection of previous studies show the suitability of the PIV technique for providing detailed unsteady flow analysis, using complex geometries, provided careful consideration is given to the optimisation of the data acquisition stage. The major challenges are to provide sufficient optical access to the flow being imaged, particularly when using stereoscopic PIV, and to ensure the seeding particulate is evenly distributed at sufficient concentration across the FoV. A description of the techniques used to achieve these criteria for the experiments conducted in this project is given in chapter 2.
1.4 Project Objectives

Section 1.2 has detailed fundamental ideas behind LES CFD and its potential for resolving unsteady flow aspects of engineering scenarios. However, the cost of generating a time-dependent velocity field to apply at an LES inlet plane via precursor LES calculation has indicated the importance of developing experimentally-derived, synthetic generation techniques. Section 1.3 has outlined how the temporally and spatially resolved velocity field obtained via the PIV technique has, in principle, the potential to form the basis of such a synthetic method, although little work to investigate this has so far been reported.

The primary aim of this thesis is thus to develop, quantify and validate a generalised and widely applicable methodology for providing unsteady, spatially and temporally resolved velocity fields utilising the full potential of the PIV technique, for direct application as an LES inlet condition generation method.

A list of project objectives may thus be stated as:

- To select one or more suitable test cases, relevant to gas turbine combustor fluid mechanics, and to apply the above procedures to the capture of PIV data, using both air and water as the working fluid. The captured data should be suitable for use as inlet conditions for LES predictions of the test cases.
- To define the optimal method by which the available PIV instrumentation may be used to provide the three velocity components necessary for inlet condition generation.
- To develop methods for analysis of measured data sets to provide any derived statistics or velocity field processing that may be necessary for inlet condition generation.
- To evaluate existing methods of unsteady synthetic inlet condition generation in terms of their benefits, drawbacks, requirements and useful applications.
- To develop and verify a new method of inlet generation for use with high sample rate PIV and evaluate this against the existing methods.
- To apply, and where necessary modify, an existing LES code to allow simulation of the selected test cases with a variety of unsteady inlet conditions.
• To perform and analyse LES predictions of the test cases and evaluate the implications of the inlet conditions used.

1.5 Thesis Structure

The remainder of this thesis is dedicated to describing how the objectives outlined in the previous section have been addressed and completed. Chapter 2 will describe the experimental facilities used and how the PIV technique has been applied in them. Chapter 3 details various methods of unsteady velocity field analysis and the techniques for using these methods for generation of synthetic inlet conditions in existence at the start of the current research. A new inlet condition generation method is then introduced in chapter 4. This is validated and compared to existing synthetic methods in chapter 5. Chapter 6 details the computational techniques required for simulation using these synthetic methods. Some results of these observed in a combustor relevant test case are presented in chapter 7. The thesis concludes with a summary of the major achievements of the present research and suggestions for future work in chapter 8.
Chapter 2

Experimental Facilities and Instrumentation
The basics of the process behind obtaining PIV data has already been outlined in chapter 1, along with an appreciation of previous PIV studies applicable to the work in this thesis. This chapter details the choice and design of experimental facilities and the capturing and analysis of the resultant data for this project. All velocity field measurements were undertaken using PIV.

Two test geometries were considered as part of this project: an idealised combustor flow using water as the fluid medium and a vortex generator using airflow. Both geometries were commissioned as part of this project and will be described in some detail. A few measurements were also performed in a real (engine representative) combustor geometry using airflow. Data from this experiment will not be discussed for the remainder of this thesis, but details of the geometry and an overview of the captured data are provided in Appendix A for future reference.

The following section details the high-speed PIV instrumentation utilised throughout. Subsequent sections discuss the test geometries used.

### 2.1 High-Speed PIV System

All experimental testing within the present investigation took advantage of a newly commissioned high-speed PIV system made available at the start of this project. The system is commercially available and is supplied by LaVision GmbH. The general arrangement is shown in Fig 2.1, and the technical specifications in Table 2.1.

![PIV system arrangement](image)
### Experimental Facilities and Instrumentation

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Table 2.1 – PIV system specifications

All PIV data capture uses LaVision’s software packages, DaVis 7.2 (LaVision[^44]). This software controls the acquisition, processing and validation stages, outputting a data file for each vector field in the time series. It may also be used to calculate the mean and RMS vector fields from the data set. A facility is available to export all data files into Matlab for further analysis and processing.

Following the recommendations of Hollis[^32] and LaVision[^44], and initial investigations using the current test geometries, the following parameters were employed for all PIV processing:

- Calibration via the pinhole model, with stereoscopic self-calibration, for all stereoscopic data.
Multi-pass processing, consisting of three passes (the initial one with an interrogation cell size of 64x64 pixels, the remaining two at 32x32 pixels).

- 50% cell overlap at each pass.
- ‘Standard’ (FFT) second-order correlation.
- Vector post-processing after each pass, with vectors removed if found to have Q-factor less than 1.3 or exceed the RMS of the neighbouring vectors by a factor of 2. Vectors removed after the final processing pass are estimated by interpolation.

2.2 Idealised Combustor

The major test case examined in this project is that of single sector of an annular combustor geometry simplified from a typical modern engine configuration (e.g. similar to a Rolls-Royce Phase 5 combustor, as currently used in Rolls-Royce’s Trent 500 series of engines). It was designed specifically as an idealised test case that could be easily replicated as a computational geometry, but which reproduced the defining features of a real combustor flow.

2.2.1 Test Rig

All testing was conducted in the Loughborough University Horizontal Water Flow Tunnel. For isothermal and incompressible flow, using water as a working fluid is feasible and convenient. By matching the Reynolds number to an equivalent size airflow case, this has the benefit of maintaining the dynamic characteristics of the flow whilst significantly reducing velocities (by a factor of approximately 15). This allows a greater portion of the turbulence spectrum to be captured with a given temporal resolution of PIV. Additionally, flow seeding is more easily achieved in water since recirculation around the rig ensures an even distribution of particles, with minimum deposits on the test geometry to affect the optical quality of the FoV. As suggested in section 1.3.1, 20μm Polyamid particles are used for seeding the flow throughout.
A schematic of the facility is shown in Fig 2.2. This was originally designed by Behrouzi and McGuirk\textsuperscript{[5]}, with Perspex test section walls allowing excellent optical access from any angle. The test section is of a rectangular cross-section with dimensions $380\text{mm}$ (horizontal) $\times$ $303.4\text{mm}$ (vertical). Variable return valves downstream of the flow regulating pump allow variation of the flow velocity within the test section. Mounted next to the test section, a Dantec Dynamics three-axis traverse system is available for supporting the camera and lasers. This allows automated positioning of the PIV system to a resolution of $6.25\mu\text{m}$ in all three directions.

![Fig 2.2 - Horizontal Water Flow Tunnel Schematic](image)

The test geometry itself represents one full sector and two half sectors of an annular combustor, although converted to a planar Cartesian geometry for simplicity, i.e. no curvature is included. The geometry is simplified to include only the flame tube wall, supply annuli and a burner arm. The burner arm is not replicated in detail, consisting of a $30\text{mm}$ diameter rod, joined at right angles to the combustor head, to provide both support and to simulate the generation of disturbances which pass into the annulus and are presented to the ports. The combustor head itself is sealed: there is no representation of the injector flow. One row of ports is used to simulate the primary jets in a combustor, with two evenly spaced holes per burner 'sector', one of which is centred behind the burner arm (note; for convenience and optical access the two half burner arms that should be present at the test rig side walls are omitted). Although circular chuted ports are often present in real combustors, these would be unnecessarily complicated to generate as a computational grid, especially for the in-house LES code selected for the purposes of this thesis. Spencer\textsuperscript{[93]} showed that the port shape, in particular the use of chutes, can have a significant effect on the flow in the core region, and it was decided to use square, non-chuted ports, which may be
easily generated computationally. Fig 2.3 shows the test geometry, with dimensions measured from the manufactured rig. This geometry is designed to contain sufficient features of the external and internal (primary zone) fluid mechanics of a real combustor to present a representative and sufficiently complex test case.

Fig 2.3 - Idealised combustor test geometry (all dimensions in mm)

The burner arm can be rotated about the combustor head to allow the coordinate system to be inverted so that all data capture in the annuli can be performed adjacent to the bottom of the tunnel. This avoids the inconvenience of having to reposition the camera and laser for different annuli. Circular shims can be placed between the burner arm and tunnel wall to allow the annuli heights to be set as equal. All data will be presented in the orientation shown in Fig 2.3, with the annulus on the same side as the burner arm termed the ‘upper’ annulus.

Based on the dimensions of the test section, the port size, \( d_p \), was fixed (for the basis of comparison to the engine geometry, an equivalent-area circular port diameter of
24.0mm was used) and all subsequent dimensions scaled to give proportions analogous to a typical engine combustor. The Reynolds number based on the primary port conditions was matched between test rig and engine geometries at ~30,000, giving a bulk velocity in the tunnel of 0.1662 m/s and a Reynolds number based on the tunnel height of 56,534. The mass flow rate through each annulus downstream of the row of port holes was fixed at 13% of the total flow by metering plates in the annulus exits, 345.7mm downstream of the port holes. Each metering plate contained 40x10.5mm holes, giving a total outlet area of 3,463mm² per annulus. Table 2.2 details the comparative design dimensions of the current and Rolls-Royce Phase 5 geometries. All dimensions shown are in millimetres.

<table>
<thead>
<tr>
<th>Item</th>
<th>Current geometry</th>
<th>Relative to (d_p)</th>
<th>Phase 5 geometry(^{[27]})</th>
<th>Relative to (d_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Primary) port size ((d_p))</td>
<td>(\sqrt{4(21.3\times21.3)/\pi}) (= 24.0^a)</td>
<td>(d_p)</td>
<td>13.8(^b)</td>
<td>(d_p)</td>
</tr>
<tr>
<td>Flame tube height</td>
<td>201</td>
<td>8.23(d_p)</td>
<td>120.0</td>
<td>8.70(d_p)</td>
</tr>
<tr>
<td>Flame tube exit height</td>
<td>76.6(^c)</td>
<td>3.19(d_p)</td>
<td>78.2</td>
<td>5.67(d_p)</td>
</tr>
<tr>
<td>Core length</td>
<td>535.5(^c)</td>
<td>22.22(d_p)</td>
<td>250.0</td>
<td>18.12(d_p)</td>
</tr>
<tr>
<td>Annulus height</td>
<td>45.2</td>
<td>1.88(d_p)</td>
<td>26.2</td>
<td>1.90(d_p)</td>
</tr>
<tr>
<td>Flame tube wall thickness</td>
<td>6.0(^d)</td>
<td>0.25(d_p)</td>
<td>2.0(^e)</td>
<td>0.14(d_p)</td>
</tr>
<tr>
<td>(Primary) port spacing</td>
<td>95.0</td>
<td>3.96(d_p)</td>
<td>55.8(^f)</td>
<td>4.04(d_p)</td>
</tr>
</tbody>
</table>

\(^a\) Current square holes of width 21.3 are equivalent area to a circular diameter of 24; \(^b\) average value; \(^c\) combustor exit area reduced to avoid core recirculation; \(^d\) minimum thickness possible due to manufacturing limitations; \(^e\) estimate; \(^f\) 7.5\(^e\) equivalent spacing

Table 2.2 – Test rig comparative dimensions

During commissioning of the geometry, several initial PIV data sets were captured to confirm the gross features of the flow. Fig 2.4 shows the wall boundary layer profiles at the spanwise (z-direction) midpoint of the test section, 250mm upstream of the burner arm.
Profiles are normalised by the tunnel bulk velocity. Although there are small differences in the profiles between the 4 walls, with a maximum difference of the order 5% near the walls, the velocity is near-uniform (<2% difference) in the central (80%) section of the tunnel, from which data will be taken.

During initial commissioning, it was discovered that extra bracing was required in the core region to stop the rig flexing out of square, resulting in non-symmetrical impingement of the port jets. This bracing was fitted against the spanwise extremities of the test section, against the tunnel walls. The bracing was rearward of $x = 150\text{mm}$ and thus should not affect the flow in the core region or the optical access to it. Additionally, the exit taper was extended, to the extent shown in Fig 2.3, after reverse flow was noticed at the exit. PIV data captured subsequently showed the jet impingement to be symmetrical (Fig 2.5) and no reverse flow at combustor exit (Fig 2.6).
2.2.2 Choice and Arrangement of Measurement Locations

It is intended to carry out LES calculations of the flow in this idealised combustor geometry (reported below in chapter 7). Hence, the extent of the computational domain and the associated selection of locations for data capture must be made with consideration to both resulting computational demands and the validity of the selected domain as a test case. Clayton and Jones\textsuperscript{[10]} and Spencer et al.\textsuperscript{[95]} computationally, and Spencer and McGuirk\textsuperscript{[96]} experimentally have shown that the internal flow within the primary zone of a combustor is dominated by the behaviour of the primary zone impinging jets, which are in turn affected by the profile of the flow and nature of the turbulence in the supply annuli. An LES prediction whose inlet plane lies within the combustor annuli upstream of the ports should thus adequately model the coupling...
between the annulus/core flows whilst negating the need to carry out CFD predictions of the whole combustor external aerodynamics. Choosing inlet planes in the annuli for a potential LES also allows good optical access for PIV and has the added advantage of resolving the burner arm wake. Although some flow coupling will exist between the flows in the upper and lower annuli this will invariably be very weak and thus does not necessitate the capture of data in both annuli simultaneously.

The proposed layout of the intended LES computational domain is thus of the form shown in Fig 2.7. The simulated region will cover the central ‘sector’ of the combustor (covering up to the centreline of the ports on either side of the combustor sector centreline), allowing periodic boundary conditions to be employed in the z-direction. CFD inlet planes will cover both the upper and lower annuli at the upstream end of this region, with CFD outlet planes in the downstream annuli and also at the core exit. This implies for inlet condition purposes the capture of experimental data for all three velocity components over the yz-plane of the annulus inlet plane (x = 0 in Fig 2.3) and, for validation purposes, additional data over one or more xy-planes within the core combustor flow.

For the remainder of this thesis, the annulus inlet region will be normalised by the nominal annulus height, H = 45.2mm. Also, for clarity, the transverse co-ordinate value used to present velocity data in the annuli will be assumed to have its origin on the respective annulus inner wall (i.e. the flame tube wall for both inner and outer annuli). Using the tunnel bulk velocity and assuming a 50% mass flow split through each annulus, an annulus bulk velocity, $U_{ref} = 0.558 \text{ m/s}$, is obtained and will be used for all velocity normalisation. Fig 2.8, taken from PIV data sets in both annuli, indicates this is a valid assumption with the measured values within 5% of this figure.
The layout and dimensions of the measured PIV fields over each annulus inlet plane will largely dictate the quality and ease of generation of the resultant CFD inlet condition. As well as containing sufficient spatial resolution to depict adequately the velocity field, the measured data must be time-synchronised for all three velocity
components across the whole CFD inlet plane. If the latter condition is not met, any discontinuity in the flow field would result in a loss of velocity correlation across that region. It will be seen in chapter 3 that this has been shown to cause premature decay of the local turbulence in LES.

However, only two PIV instrumentation arrangements exist which could satisfy this three component synchronisation condition implicitly. Either the entire inlet plane must be captured in one field of PIV data, or multiple cameras must be used to capture portions of the inlet plane simultaneously.

![PIV Instruments](image)

**Fig 2.9** – Possible stereoscopic PIV arrangements for synchronised data capture in upper annulus; **PIV field(s), LES inlet plane region,**

* a) Whole inlet in one field b) Multi-camera, multi-field

As illustrated generically for a stereoscopic PIV setup in Fig 2.9, the former of these approaches would result in significant regions of useless data with the inlet region itself poorly resolved (approximately $16 \times 62$ vectors). While the latter arrangement achieves much higher inlet resolution (approximately $62 \times 236$ vectors), to maintain synchronicity all PIV fields would have to be captured at the same time, necessitating
the use of at least 4 cameras (8 for stereoscopic PIV). This was certainly beyond the availability of the current project and probably of most research institutions!

Given current computational resources, it is common practice to resolve a combustor annulus height (y-direction) using between 30-60 grid cells in LES (Clayton & Jones\textsuperscript{[10]}, Spencer \textit{et al.}\textsuperscript{[95]}). Since, for this project specifically, there is a desire to resolve and monitor the turbulent structures within the annulus flow, it is expected that the annulus resolution for CFD predictions will be towards the upper end of that range. Therefore, it is prudent to capture PIV that also has at least similar spatial resolution, thus fixing the field size at approximately the annulus height. Clearly, capturing the whole inlet in one field would be inadequate. Data must be captured using fields of a similar size and orientation to the multi-camera setup (Fig 2.9b), but one at a time since only two cameras are available. Thus an appropriate method is required to generate synchronised LES inlet condition data from the initially unsynchronised raw PIV data. A methodology to this end will be discussed in chapter 4.

At the outset of the project only two-component (monoscopic) PIV could be obtained due to only one high-speed camera being available. Data in the inlet plane was captured across 5 fields spread spanwise across the LES inlet plane. Fixing the field dimensions on the annulus height gives a FoV of approximately 50mm. Four of these fields are arranged as in Fig 2.9b, centred at \( z = -75, -25, 25 \) and 75mm. Additionally, in order to give more flexibility in the reconstruction of the burner arm wake, a fifth field centred at \( z = 0 \) was captured. For each location a 50Hz and a 1kHz data set (containing 1024 and 3072 samples respectively) was taken. The former is in order to obtain better statistical analysis (due to its longer overall sampling time) and the latter for maximal temporal resolution. As viewing normal to the \( yz \)-plane is not possible in this test rig, a mirror was installed in the annulus exit, allowing viewing through the lower wall, as shown in Fig 2.10. The mirror is \( 50 \times 50 \) mm square, mounted 200mm downstream \( x = 0 \) and accounted for a blockage of approximately 5%. PIV tests taken of the axial velocity in the annulus just upstream of the mirror show it to have negligible effect on the flow (see Fig 2.11).
In order to obtain velocities in the through-plane \((x)\) direction, additional two component (2c) data were taken over \(xy\) and \(xz\) planes intersecting the inlet plane. Measurements were taken over 10 \(xy\)-fields and 12 \(xz\)-fields, evenly spaced across each annulus. All fields represent a FoV of 50mm and were captured at 50Hz and 1kHz.

Subsequent to the capture of the above 2c-data, a second high-speed camera became available, allowing the capture of stereoscopic PIV to be used and, most importantly, for all three velocity components to be captured simultaneously. Comparison of the
2c- and 3c-data also served as a useful experimental accuracy validation exercise. Five fields of stereoscopic data have been captured in each annulus at 50Hz and 1kHz, centred at the same locations as the 2c yz-fields noted above. The experimental setup is akin to that in Fig 2.9. Although the FoV is limited in the y-direction to around 50mm, elongation due to perspective effects gives a field width of around 65mm. Calibration is conducted using a two sided calibration plate (Fig 2.12) of known thickness placed in the annulus inlet. Perspective in the out-of-plane direction is obtained by traversing the cameras by 1mm in the x-direction (thereby avoiding the need to translate the plate accurately whilst in-situ), with a calibration image taken at each location by both cameras. The pinhole model, as detailed in section 1.3.1, and subsequently self-calibration as detailed in section 1.3.2, determine the effective image correction. Scheimpflug mounts are employed for both cameras, with uniform focus achieved at an angle of approximately 19°.

Modification was required to the standard water tunnel setup to allow imaging in the yz-plane when using stereoscopic PIV. Due to the cameras now viewing obliquely at an air-Perspex-water interface, distortion of the image was observed in the direction normal to the interface. The solution was to mount two removable, triangular, water filled prisms onto the side of the rig, with sides angled at 45° (see Fig 2.13), allowing the cameras to view normal to the air/Perspex/water interface, as used previously by Lang et al.\(^{43}\) and Parker et al.\(^{46}\).
2.2.3 Sample PIV Results and Validation

All experimental data has been captured using the 'best practice' approaches detailed in section 1.3. Camera calibration is performed before data capture at each test location. Since clear reference points are visible in the PIV images (namely the upper and lower walls of the annulus), linear calibration (from two known reference points) has been conducted for all monoscopic data sets, with no calibration plate needed. The inter-frame time is optimised to give a maximum particle shift of 8 pixels for each data set, which normally necessitates an inter-frame time of approximately 500µs. However, when taking data at 1kHz the maximum inter-frame time available is 460µs, which typically results in a maximum particle shift of approximately 7.5 pixels. The experimental parameters are adjusted until a data set is obtained with over 95% first choice vectors and an average Q-factor of over 2.2. However, for fields in the yz-plane, this had to be relaxed to 90% and 2.0 respectively.

Flow within the upper annulus inlet region is dominated by a transient and intermittent counter-rotating vortex pair (CRVP) emanating from separation off the burner arm. A similar, although considerably less powerful, CRVP is observed in the lower annulus. This appears to originate from separation off the corner of the burner arm that protrudes into the capture streamtube heading into the lower annulus. Fig 2.14 shows instantaneous vectors in the upper annulus inlet plane data from a yz-field centred on the rig centreline (z = 0). The samples are captured at an interval of 0.1
seconds. The differences in the two vector fields and the lack of a CRVP in Fig 2.14b illustrate the unsteady and intermittent nature of the vortices.

![Fig 2.14 - Instantaneous upper annulus vectors at a) t = 0, b) t = 0.1 s](image)

![Fig 2.15 - Mono- and stereoscopic viewing and plane intersections in upper annulus; a) xy- (blue) and yz- (green) planes, b) xz- (pink) and yz- (green) planes; yz stereo (and mono as in Fig 2.10), xy mono, xz mono, intersection lines](image)

The mean and RMS profiles of the 2c velocity data taken in the orthogonal xy- and xz-planes were found to be closely matched to that taken in the yz-plane on the lines at which they intersect (see Fig 2.15). If only monoscopic data were to be used when developing a methodology for obtaining a synchronous three-component velocity field over the LES inlet plane, data from these orthogonal planes must provide information for obtaining the through yz-plane velocity component. It is thus necessary to check whether data extracted over only the intersecting lines of the orthogonal planes could be used to interpolate data over the whole yz-plane to provide
all three velocity components needed to construct the LES inlet conditions without excessive loss of accuracy.

Using the stereoscopic data as a datum, several interpolation schemes were trialled for interpolating the RMS velocity fluctuation of the through-plane component (measured directly across lines intersecting the orthogonal planes, see Fig 2.15) onto a sample region of the $yz$ inlet plane. The RMS axial velocity fluctuation was used in this test since it was expected that interpolation errors would probably be larger for second moment quantities. The results may be seen in Fig 2.16 for linear, bi-harmonic spline and Kriging (Chao-yi$^{[8]}$) interpolation schemes. The three vertical and two horizontal lines of directly measured source data that intersect this $yz$ region are shown in red. The quality of each interpolation is evaluated based on the mean difference from the stereoscopic datum RMS field across this sample region.

Fig 2.16 - Sample interpolations for $u'$ from orthogonal PIV fields;

- Source data lines; brackets: mean difference from stereoscopic PIV
Although reconstruction of the \(yz\)-plane data was reasonably successful, no scheme was able to achieve a mean difference from the stereoscopic PIV datum of less than 10\% for this or other data sets sampled. Additionally, although time average statistics can be interpolated, the non-synchronous capture of the orthogonal planes means that instantaneous data cannot (as there is no knowledge of phase information between data sets captured at different times). Use of monoscopic data would therefore imply the need for a method for generating synchronous time-series data from knowledge of just first and second moments, and perhaps spatial correlations deduced from these statistics – by no means an easy task. It was therefore decided that only stereoscopic data sets are viable as the basis for inlet condition generation.

However, the two-component and three-component data may still be compared to assess repeatability and accuracy of the data. Fig 2.17 shows a comparison between mono- and stereoscopic camera data sets across the upper annulus on the test-section centreline \((z = 0)\), following to the arrangements illustrated in Fig 2.15a. The plots show good agreement for both the time-averaged mean and RMS data, especially given the proportionally strong through-plane component.
Despite best efforts being taken to optimise the experimental setup, errors such as camera or laser sheet misalignment, variations in the test rig operation (e.g. pump fluctuations, cleanliness, tolerances during re-fitting) or ambient conditions (e.g. temperature) and the inherent errors within the PIV process (see section 5.1) will inevitably affect the results of the tests. Several consequences were observed and are discussed below, along with the empirical corrections developed to minimise their effect on the subsequent use of multi-plane PIV for inlet condition generation. These
Consequences refer essentially to the impact of measurement errors on the continuity between separate (but spatially neighbouring) data sets and should not be confused with errors associated with the vector calculation evaluation detailed in sections 1.3.2 and 1.3.3.

Noticeable error in the time-averaged statistics may be caused by the occurrence of spurious samples within a data set. Such 'rogue' datasets can occur when the processing algorithm rejects too many vectors within a single sample and subsequently replaces them with interpolated data. This often results in non-physical triangular patterns in the statistical contours. The optimal method for removal of such samples is to examine the vector choice matrix produced during the validation stage of the vector calculation (see section 1.3.3). In addition to the velocity field, DaVis allows export of a 'choice' matrix, with first choice vectors rated 1, through to fourth choice vectors rated 4 and interpolated data rated 5. Spurious samples produce an uncommonly high choice rating (typically with >50% of vectors rated 5), and may be identified as those samples having an average choice rating of more than 4 or an average rating that is a large number of standard deviations from the data set mean (a factor of 6 is typically used here). Spurious samples in data sets to be used for statistical comparison may be removed, however this leads to discontinuities if instantaneous data is required and thus such samples are best replaced by a vector field linearly interpolated from the previous and following time samples.

Another consequence of measurement errors of prime importance to inlet condition generation is any slight discontinuity in statistics observed between two spatially adjoining fields of PIV that were not taken concurrently. A correction has been developed to account for this effect based on the assumption that such discontinuity errors can be decomposed as in equation 2.1 below, i.e. location errors have constant offsets across each data set, $\Delta x$ and $\Delta y$, in each in-plane dimension and are scaled by a constant factor, $k_i$; velocity errors have a constant offset, $\Delta U_i$.

$$U_i(x, y, t) = k_i[U_i(x_{meas} + \Delta x, y_{meas} + \Delta y, t)]_{meas} + \Delta U_i$$ (2.1)
The final correction is determined by evaluation of these parameters through comparison with neighbouring fields. One field must be chosen as a datum from which its neighbours may be sequentially corrected (starting from the datum and working outwards). The location correction offsets, $\Delta x$ and $\Delta y$, are determined by maximising the correlation between the time-averaged mean velocities of the two fields in the region where they overlap. This is performed through a process of trial and error, moving the corrected field relative to the datum. Typically, these offsets are rarely more than 1mm and never more than 3mm for typical FoVs. The factor and offset, $k_i$ and $\Delta U_i$, for a given velocity field is then determined by performing a least squares fit for time-averaged velocities in the overlap region. As an error in the calibration procedure for stereoscopic PIV could result in a discrepancy in any, some, or all components of the calculated particle displacement, this correction must be performed separately for each velocity component. For in-plane components the correction factor is of order 10%, although often considerably less. Such factors have no effect on the overall mass flow as data may still be scaled to the bulk velocity.

Table 2.3 shows the correction parameters calculated for five 50Hz stereoscopic data sets in the upper annulus, using the field centred at $z = 0$ as a datum. Fig 2.18 shows the effect of this correction, and the intuitively more continuous velocity field that results.

<table>
<thead>
<tr>
<th>Field</th>
<th>$\Delta x$</th>
<th>$\Delta y$</th>
<th>$k_U$</th>
<th>$\Delta U/\text{U}_{\text{ref}}$</th>
<th>$k_V$</th>
<th>$\Delta V/\text{U}_{\text{ref}}$</th>
<th>$k_W$</th>
<th>$\Delta W/\text{U}_{\text{ref}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z = 25$</td>
<td>0.954</td>
<td>0.412</td>
<td>1.027</td>
<td>-0.016</td>
<td>1.031</td>
<td>0.007</td>
<td>1.065</td>
<td>-0.002</td>
</tr>
<tr>
<td>$z = -25$</td>
<td>0.493</td>
<td>0.750</td>
<td>1.001</td>
<td>-0.014</td>
<td>0.843</td>
<td>0.004</td>
<td>1.006</td>
<td>0.075</td>
</tr>
<tr>
<td>$z = 75$</td>
<td>-2.911</td>
<td>0.288</td>
<td>0.501</td>
<td>0.014</td>
<td>0.661</td>
<td>-0.005</td>
<td>1.065</td>
<td>-0.005</td>
</tr>
<tr>
<td>$z = -75$</td>
<td>2.186</td>
<td>1.409</td>
<td>0.905</td>
<td>-0.025</td>
<td>0.997</td>
<td>-0.002</td>
<td>1.131</td>
<td>-0.063</td>
</tr>
</tbody>
</table>

Table 2.3 – Typical correction parameters
After the above corrections were applied a z-direction skew in the through yz-plane velocity component of the stereoscopic data sets became apparent. To illustrate this, it is convenient to define a 'local' mean velocity, \( \langle U \rangle_{\text{local}} \), averaged across the annulus height at each z-wise location in the annulus, such that

\[
\langle U \rangle_{\text{local}}(z) = \frac{1}{H} \int_0^H \langle U \rangle(y, z) dy
\]

\( \langle U \rangle_{\text{local}} \) was found to vary linearly from 1.07\( U_{\text{ref}} \) at \( z/H = -2 \) to 0.92\( U_{\text{ref}} \) at \( z/H = 2 \), as shown Fig 2.19. This is outside the 5% limits that could be tolerated as acceptable experimental error, and in any case the clear spanwise linear bias is suspicious. Analysis of monoscopic data in orthogonal planes taken in the annuli and upstream of the burner arm shows variations in bulk velocity across the annulus of approximately 5%, but there is no overall skewness trend (see Fig 2.19).
Experimental Facilities and Instrumentation

![Graph](image)

Fig 2.19 – \( \langle U \rangle_{\text{local}} \) variation in the upper annulus

- stereoscopic, + monoscopic, — annulus \( U_{\text{bulk}} \)

The source of this skewness was found to lie in the calibration procedure for the stereoscopic configuration. Stereoscopic calibration is notoriously difficult, as is evident by the amount of recent literature devoted to it (Lacerf et al.\cite{42}, Prasad\cite{74}, Soloff et al.\cite{92}, Wieneke\cite{110}, Willert\cite{111}). Testing here showed the skewness error was due to the use of a flat calibration plate (Fig 2.12). Stereoscopic data was captured in the \( yz-\)plane 90mm upstream of the burner arm using both flat and two-level (Fig 1.5b) calibration plates. This data, seen in Fig 2.20 (with the integral from equation 2.2 evaluated over the tunnel height, 303.4mm, in this case), showed the skewing phenomenon is not present when a two-level calibration plate is used.

![Graph](image)

Fig 2.20 – \( \langle U \rangle_{\text{local}} \) variation measured 90mm upstream of the burner arm;

- flat plate, — two-level plate, blue tunnel \( U_{\text{bulk}} \)
Due to time limitations and the observed systematic nature of the calibration error it was argued that, rather than retake all the stereoscopic data using a two-level plate, a correction scheme could be applied to the existing stereoscopic data. A linear least-squares fit to the local time-averaged bulk velocity per unit width was used to determine the z-wise gradient of the required correction to remove the skew. The correction was applied to the measured through-plane (U) velocity in the stereoscopic data such that the gradient became zero across the annulus width. No correction was applied to the other components. The corrected profiles then showed good agreement with the monoscopic data sets and, as with the above corrections, were normalised by \( U_{\text{ref}} \) to remove any mass-flow implications.

Fig 2.21 – z-wise bulk velocity per unit width in the upper annulus, showing skew correction; – original stereoscopic data, — skew gradient, + monoscopic data, corrected stereoscopic data, — annulus \( U_{\text{bulk}} \) (corrected gradient)

Figs 2.23 to 2.24 below show time-averaged contours for the three velocity components, as well as their RMS velocity fluctuation magnitude, over the five fields taken in each annulus, as calculated from DaVis, with both of the above corrections applied. Note the RMS velocity fluctuation magnitude is calculated as:

\[
|u| = \sqrt{u'^2 + v'^2 + w'^2} \tag{2.3}
\]
For future reference, the five fields will be referred to by the letters A through E, as in Fig 2.22. Note the implied hierarchical structure of the fields within this figure; since field C contains the majority of the unsteady motions and is used as the datum field for statistical correction, data in this field is given preference over data in fields B and D, which in turn are given preference over fields A and E.

![Fig 2.22 - Annulus field arrangement](image)

Although the first order time-averaged statistics presented here only give a brief overview of the flowfield characteristics, it is still possible to gain an appreciation of the challenges for inlet condition generation. The mean flowfield is generally symmetrical about the centreline, with a small bias in the axial velocity towards the flame tube walls. The CRVP described above is primarily seen in the in-plane mean components, and is stronger in the upper annulus. Outside of this region the velocity profile is approximately $z$-wise constant, with low turbulence intensity. However, although the CRVP in the upper annulus is largely confined to fields B, C and D (within one annulus height of the centreline), Fig 2.24a shows that no single field could adequately capture it entirely. Further statistical analysis methods which will be applied to these data sets will be presented in chapter 3. The inherent errors within the PIV process that may have a subsequent effect on any inlet conditions generated using this data are discussed in section 5.1.

As mentioned previously, monoscopic PIV data was also captured in the internal region (primary zone) of the combustor to allow validation of the LES results. Time-averaged vectors and contours of RMS velocity fluctuation magnitude are presented in Fig 2.25. The flow in this region is dominated by jets from the port holes, which impinge symmetrically. Two notable periodic flow features are present in the turbulence field. The first is a bi-modal ‘flapping’ of the jets, resulting in the inclination of the stagnation line of their impingement upon each other. This effect is
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evident in the two frames of instantaneous vectors shown in Fig 2.26 and is similar to the phenomenon captured by Hollis[32] for a more realistic geometry. The frequency of this flapping is approximately 7Hz. There is also a periodic shedding of vortices within the jet shear layers, which appear to originate from the separation at the port. Such a feature is evident as the 'rippling' of the instantaneous shear layers and is also the source of the high RMS within these regions. This flow feature was also measured and discussed in detail by Hollis[32]. With use of high speed PIV it has been possible to measure the frequency of the vortex shedding at approximately 90Hz. It was found the power spectra of the velocity fluctuations (calculated as described below in section 3.1.2.5) in the primary zone of the combustor were generally too noisy to definitively identify the structures described above, nor were they adequately reproduced by Proper Orthogonal Decomposition (POD) of the data (described in section 3.1.3.2). The frequencies quoted above are thus estimated from observation of the PIV vector time series.

Fig 2.23 – Time averaged U velocity, a) upper annulus, b) lower annulus
Fig 2.24 – RMS fluctuation magnitude, a) upper annulus, b) lower annulus

Fig 2.25 – Core velocity field, a) Time averaged vectors, b) RMS fluctuation magnitude
2.3 Vortex Generator

Although the test case described in section 2.2, using water as the flow medium, provides the best basis for an inlet condition technique development, it was also desired to examine the performance of this technique in an airflow experiment, since the range of frequencies to be resolved in air flows will typically be larger than in a waterflow experiment, including energy containing vortices further outside the frequency range resolvable by the PIV instrumentation. Since this is a more complex problem it was decided to design a simplified, but still combustor-relevant airflow test case. A vortex generator in a rectangular channel represents a suitable test case, with a CRVP generated of a similar nature to that observed in the wake of the combustor burner arm, as seen in section 2.2.3. It also allows a simple computational domain to be generated.

2.3.1 Test Rig Design and Setup

Previous studies of vortex flows, such as by Pauley & Eaton\textsuperscript{[68]} experimentally and Biswas & Chattopadhyay\textsuperscript{[61]} and Deb \textit{et al.}\textsuperscript{[13]} computationally, have shown a CRVP to be maintained for over 5 channel heights downstream of a vortex generator within a channel over a range of Reynolds numbers. A rig was designed based on these...
studies, with a cross section such that it could adequately be captured in two PIV FoVs. A diagram of the test section is shown in Fig 2.27. The vortex generator itself is a delta wing type, as used by Biswas and Chattopadhyay\textsuperscript{[6]}. It has an aspect ratio of 1 and is set at an incidence of $26^\circ$, as detailed in Fig 2.28.

![Diagram of test section](image1)

**Fig 2.27 – Test section showing computational domain**
- **LES domain**
- **inlet plane**
- **validation data**

![Diagram of vortex generator](image2)

**Fig 2.28 – Vortex generator detail**

LES calculations will be performed in a computational domain which covers the region downstream of the vortex generator, as indicated in Fig 2.27. This necessitates PIV data for inlet condition generation to be taken at $x = 130\text{mm}$ (measured from the inlet datum) and a set of validation data will be taken at a station further downstream, at $x = 250\text{mm}$.
The rectangular channel test section is mounted into an existing airflow facility, previously commissioned for capture of PIV data (Midgley\textsuperscript{[59]}). A schematic of this facility is shown in Fig 2.29. The test channel exhausts into a 140mm diameter circular pipe, in which an orifice plate of diameter 100mm is placed for mass flow measurement. The rig is driven by a centrifugal fan, which exhausts to atmosphere. A water manometer connected to static pressure tappings upstream and downstream of the orifice plate was used to set the mass flow, using equation 2.4.

\[
\dot{m} = C_d \rho A_1 \left[ \frac{2 (h_1 - h_2) \rho_{H, \rho g}}{\rho} \left( \frac{A_1}{A_2} \right)^2 - 1 \right]^{1/2} \tag{2.4}
\]

Where the $1$ and $2$ subscripts denote values at the upstream and downstream pressure tapping locations respectively, and ‘h’ denotes the height of the water level above a datum on the manometer. The orifice plate discharge coefficient, $C_d$, was assumed to be 0.63 (Reader-Harris \textit{et al.}\textsuperscript{[76]}).

\begin{figure}
\centering
\includegraphics[width=0.9\textwidth]{fig2.29}
\caption{Vortex generator rig schematic}
\end{figure}

To ensure a representative turbulence spectrum, testing was conducted at a Reynolds number of 87,500, based on the channel height ($H=43.5\text{mm}$), resulting in a test section bulk velocity of $29.2\text{m/s}$ and a pressure drop across the orifice plate of $30.1\text{mmH}_2\text{O}$. For future normalisation of data taken in this geometry the bulk velocity and channel height are used. Using this notation, the trailing edge of the
vortex generator is at $x/H = 2.34$ and the inlet and outlet planes of the LES domain are situated at $x/H = 3$ and $x/H = 5.75$ respectively.

To ensure the vortex structure would be maintained along the length of the measurement/computational domain and that there were negligible upstream effects caused by exhausting into the larger 140mm diameter pipe, a RANS CFD simulation was conducted prior to commencing PIV testing, using the commercial code Fluent 6.2. The geometry encompassed the test section channel, vortex generator and larger downstream pipe and was discretised using approximately 900,000 tetrahedral cells. The flow conditions were as described above and a $k$-$\varepsilon$ model was used for turbulence.

Fig 2.30 shows contours of velocity magnitude on the centreline of the test section and on the inlet and outlet planes of the LES computational domain. It can be seen from Fig 2.32 that the CRVP shed from the vortex generator is predicted to be maintained throughout the computation domain. As would be expected, the legs of the vortex pair move apart due to the influence of the lower wall, dissipating the strong centreline downwash seen in the wake of the vortex generator in Fig 2.31. The simulation suggests the vortex pair interacts strongly with the side walls by the time it reaches the outlet plane (Fig 2.32). Although no side wall would be present in an analogous combustor annulus, the simulation of the downstream evolution of this strong vortex pair will be heavily dependent on the fidelity of the inlet condition reconstruction, and thus this geometry provides an excellent test case. The exhausting effect of the jet into the downstream pipe has negligible effect on the upstream flow, although a slight bias can be seen in the velocity profile due to residual downwash from the CRVP.
Fig 2.30 – Contours of velocity magnitude (m/s) around vortex generator

Fig 2.31 – V velocity (m/s) over xy-plane at over duct centreline.
All PIV data capture and subsequent inlet condition generation efforts for this test case used the stereoscopic PIV technique, viewing through the side walls of the section with the light sheet entering from below. Calibration was performed using a two-level calibration plate of the type outlined in Fig 1.5b, hence the data does not suffer from the same skewness problems discussed in the previous section and no corrections for this were necessary. Due to the difficulty of locating the plate within the geometry, the cameras were calibrated outside of the test section and traversed vertically to the required test location. Scheimpflug mounts, pinhole-model calibration and self-calibration were used as described in section 2.2.2.

As is often the case in airflow PIV, the delivery of seeding is one of the most challenging aspects of the setup. Atomised Ondina oil was used as a seed medium. Initially, the settling pipe noted in Fig 2.29 was not present, with the seeding injected directly into the test section inlet. This was found to produce a poor distribution of seeding across the measurement planes. An acceptable solution was to add a 200mm
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diameter, 700mm long settling pipe upstream of the test section, with the seeding injected radially into the inlet of the pipe via multiple jets. This allowed the seeding particles opportunity to diffuse across the whole duct cross-section and produced an even seeding distribution within the test section.

2.3.2 PIV Results and Validation

All data were captured and processed using the ‘best practice’ approaches detailed in section 1.3. An inter-frame time of $11\mu s$ was required to give the desired maximum particle shift of 8 pixels. The statistical discontinuities between fields observed in section 2.2.3 were, as expected, far less pronounced in the airflow data taken here, and hence no correction was required as noted above. This is due to a combination of better calibration techniques developed as a result of the water flow investigations and the shorter timescales for the larger energy containing motions involved in the airflow experiment, resulting in a higher number of independent samples per data set.

With the PIV FoV constrained by the channel height (for reasons discussed in section 2.2.2), the channel cross section can be captured in two adjoining fields centred at $z/H = \pm 0.6$. However, to ensure optimal statistical matching and allow greater flexibility of inlet generation, a third field has been taken centred at $z/H = 0$. The image resolution in this frame was halved in order to capture double the number of samples. Only a 1kHz data set was required at each measurement location as the lower airflow timescales mean this is a statistically large enough time-history to be used for both statistic gathering and instantaneous samples. Further discussion on the quantification of convergence errors is given in section 5.1.4.

Figs 2.33 to 2.34 show the time-averaged mean and RMS statistics for data taken at the LES inlet and outlet planes. The CRVP is found to exist stably and coherently at a location of $z/H = \pm 0.6$ on the inlet plane. By the outlet plane, the vortices exist about a third of the way up the side walls. It can be seen that the CRVP is represented largely in the mean flow field, with the peaks in RMS coming from the impingement of the vortices with the lower wall.
It may also be seen, especially in the U-velocity component, that there are regions of very low velocity and high RMS near the channel walls and at the extremities of the PIV fields. This is a consequence of insufficient seeding in these regions, despite the efforts described in the previous section. Often, camera focus must be sacrificed at the limits of the PIV field in order to optimise the focus over the entire field. This is especially true for the small tracer particles used in airflow testing. The intermittent vector computation in such regions can cause an overestimation of the local RMS velocity fluctuation. While it was possible to trim the data from fields that overlap near the centreline of the channel to remove areas with high vector choice values (and therefore give optimal statistic matching between fields), no such overlap of data was available in the channel corners.

When LES was subsequently performed on this test geometry (using the range of inlet condition types described later in this thesis), it was found that these regions of low velocity, particularly in the left hand corner of the domain, heavily affected the computational solution. The simulations showed the underestimation of the axial velocity and overestimation of the RMS seen in the inlet plane data continued into the domain. This created an effective 'wake' in the flow, which appears to be responsible for the left-hand vortex undergoing a premature impingement with the side wall. This is shown in the TKE contours in Fig 2.35, extracted at a plane just downstream of the inlet plane and on the exit plane from one of the simulations conducted. The resulting generation of TKE largely masked any differences in the inlet condition. The right-hand vortex was less affected by wall interactions, though some initial adjustment in the vortex trajectory and generation of TKE were observed that may result from lower wall interaction. Some differences were noted between results for the different inlet conditions simulated, but the findings of this are adequately and more reliably described by the idealised combustor test case.

The computational results of this test case are thus not presented in this thesis, though they can be found in Robinson\textsuperscript{[79]}. The challenges of generating unsteady inlet conditions using PIV data obtained in airflow (with reduced resolution of the turbulence spectrum) are still relevant, and will be discussed in Chapter 5. This test case serves to represent how LES simulations using synthetic PIV inlet conditions may be adversely affected by regions of low fidelity data in the source PIV velocity.
As perfect data quality is realistically unachievable, especially when conducting PIV in airflow, any inlet condition generation method using experimentally derived data must either be robust enough to cope with such deficiencies or there must be sufficient correction, and, where necessary, recapture of the raw PIV field to ensure regions of bad or mismatched data are not fed into the CFD.

Fig 2.33 – Time-averaged velocity (contours of through-plane velocity),
   a) $x/H = 3$, b) $x/H = 5.75$
Fig 2.34 - RMS fluctuation magnitude, a) $x/H = 3$, b) $x/H = 5.75$

Fig 2.35 - Typical RMS fluctuation magnitude predicted by LES, a) $x/H = 3.5$, b) $x/H = 5.75$
2.4 Chapter Summary

This chapter has provided details on the high-speed PIV equipment that has been used for velocity field data capture during this work. It has also detailed the geometries, test procedures and velocity fields captured within two gas turbine combustor relevant test cases: an idealised combustor in water flow and a vortex generator in airflow. Deficiencies have been noted in the PIV data in the latter test case that make it unsuitable for comparing different inlet condition generation methods.

The only velocity field analysis provided thus far has been the time-averaged mean and RMS velocities produced by the DaVis PIV software. No attempt has been made to describe the nature of the turbulence within these test cases or how these data may be used to provide synthetic inlet conditions.

The following chapter details some of the currently available analysis methods that may be used to extract turbulence information from PIV velocity fields and follows on to discuss existing methodologies that use such analyses to derive synthetic unsteady inlet conditions for LES simulations.
Chapter 3

Unsteady Velocity Field Analysis and Review of Existing Inlet Condition Generation Methods
It was indicated in section 1.2.3 that the generation of synthetic inlet conditions for LES simulations via the use of experimental data would allow the possibility of obtaining high-fidelity unsteady velocity field data for complex geometries, at a cost far less than a precursor simulation. The previous chapter has detailed how the PIV technique can capture unsteady velocity data that is both temporally and spatially well resolved. However, as noted in section 1.3.4, PIV produces a large amount of data that must be reduced and analysed in order to maximise its usability. This chapter presents currently existing methodologies for converting raw PIV data sets into computational inlet conditions. It is first necessary to address the various forms of data analysis relevant to the inlet condition generation methods being considered, and this is the topic of section 3.1. Section 3.2 then discusses the application of the synthetic inlet generation methods themselves.

3.1 Velocity Field Analysis Methods

During the course of this project, various analysis tools have been packaged together in Matlab format. The workings of the resulting computer programme, Xact, are discussed in Robinson\cite{Robinson}. The following sections give an overview of these analysis techniques. Illustrative examples are given of each method applied to 50Hz PIV data sets taken from fields B and D as shown in Fig 2.22, near the centreline in the upper annulus inlet of the idealised combustor. It should be noted that these analyses are all applied to the resolved PIV velocity field. However, the over-bar notation used in chapter 1 will be neglected here for simplicity where not required.

3.1.1 One-point Statistics

Analysis of one-point statistics gives an appreciation of the variation in space of time-averaged properties. As such, they represent a ‘first port of call’ when evaluating a flow field.

3.1.1.1 First and Second Moments

Although one-point first moment statistics are the simplest form of data reduction, they provide a useful starting point in PIV analysis. The concept of producing a mean
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and RMS is based on the principle of a Reynolds decomposition of the instantaneous velocity field, \( U \), into a time-independent mean, \( \langle U \rangle \), and a time-dependent fluctuation, \( u \), such that:

\[
U_i(x, t) = \langle U_i \rangle(x) + u_i(x, t) \tag{3.1}
\]

for each velocity component, \( i \). The spatial range, \( x \), in this case, is the PIV FoV. As each data set is made up of a finite number of samples, \( N_{\text{samp}} \), at discrete time intervals, the mean velocity may be calculated from,

\[
\langle U_i \rangle(x) = \frac{1}{N_{\text{samp}}} \sum_{n=1}^{N_{\text{samp}}} U_i(x, t_n) \tag{3.2}
\]

Likewise, a root mean squared (RMS) velocity fluctuation, \( u' \), can be defined such that:

\[
u_{i}'(x) = \sqrt{\frac{1}{N_{\text{samp}}} \sum_{n=1}^{N_{\text{samp}}} u_{i}^2(x, t_n)} \tag{3.3}
\]

Mean and RMS data calculated in this way by PIV DaVis software has already been presented in Figs 2.23-2.24 and 2.33-2.34. The Turbulent Kinetic Energy (TKE), \( K \), is defined as half of the sum of the squares of the RMS, as in equation 3.4:

\[
K_i(x) = \frac{1}{2} \langle u_{i}'^2(x) \rangle \tag{3.4}
\]

Whereas the RMS quantifies the magnitude of an individual component fluctuation at a given point, the TKE gives the total amount of energy contained in the fluctuating field and is often a better way of identifying significant turbulent structures. However, both the RMS and TKE as measured by PIV are subject to the possible filtering effect from the unresolved sub-cell turbulence scales as noted in chapter 1. If uncorrected, this may lead to an under-estimation of the RMS and TKE. Sub-cell filtering and its effect on the RMS and thus TKE will be discussed in section 3.1.2.3.
Reynolds stresses are obtained by taking the covariance of any two components, thus:

$$\langle u_i u_j \rangle(x) = \frac{1}{N_{\text{samp}}} \sum_{n=1}^{N_{\text{samp}}} u_i(x, t_n) u_j(x, t_n)$$ (3.5)

It will be discussed in section 3.2 how the nature of turbulence is governed by its inherent correlations. Thus, as shear stresses \((i \neq j)\) indicate an interdependency between velocity components, they are an important characteristic in the evaluation of a velocity field. One advantage of stereoscopic PIV is the ability to capture all three shear stresses, as opposed to the single in-plane stress available from monoscopic PIV. Discrete regions of high shear stress (with a magnitude approaching that of the TKE) accompanied by the checkerboard pattern apparent in the sample shown in Fig 3.1 are often indicative of large scale transient structures within the data set. However, they do not indicate the form of these structures or specifically their interdependence with other points in space.

![Fig 3.1 – \(\langle vw \rangle\) Reynolds (shear) stress contours](image)

### 3.1.1.2 Time Signals and PDFs

It is often useful to view the time history of one or more components at a given point in the flow domain as a visualisation of the level and nature of turbulence. Although
little quantitative information may be gained from such a time signal in itself, various periodic structures or scales may be identified from it to guide future analysis.

The time history signal has a more useful role in identifying the grouping of the fluctuations via the subsequent calculation of a Probability Density Function (PDF). The range of velocities over which the time signal fluctuates is divided into a series of "bins", $\bar{a}$, such that the PDF is defined as the probability a fluctuation is greater than a lower limit, $\bar{a}_1$, and less than an upper limit, $\bar{a}_2$:

$$P(\bar{a}) = P(\{U_i > \bar{a}_1\} \cap \{U_i < \bar{a}_2\})$$

(3.6)

Thus for a given discrete signal, the PDF is calculated from Libby[47].

$$P(\bar{a}) = \lim_{(\bar{a}_2-\bar{a}_1) \to 0} \frac{1}{(\bar{a}_2-\bar{a}_1)} \left[ \lim_{N_{\text{samp}} \to \infty} \frac{N_a}{N_{\text{samp}}} \right]$$

(3.7)

Where $N_a$ denotes the number of samples in bin $\bar{a}$ and $N_{\text{samp}}$ the total sample size. For the finite number of samples in a PIV data set the PDF can be adequately represented by setting the bin width to one twentieth of the velocity range (i.e. $(\bar{a}_2 - \bar{a}_1) = 0.05 \times (U_{\text{max}} - U_{\text{min}})$). A sample time signal and its respective PDF are shown in Fig 3.2. This particular example shows a bi-modal PDF, where the velocity signal commonly exists in one of two states. Such behaviour is evident from the large amplitude, low frequency oscillation in the time signal. Results of this form are often seen at jet impingement points or where large scale transient structures exist, such as indicated by Fig 3.1 above.

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3.1.2 Two-point Statistics

Whereas shear stresses give the one-point correlation between two different velocity components, various two-point statistics may be employed to given the relation between one (or more) velocity components at different points in space and/or time.

Turbulence is characterised by motions over a wide range of length and time scales. The concept of the energy cascade, introduced by Richardson\textsuperscript{[78]} and described in some detail by Pope\textsuperscript{[73]}, explains how turbulent energy is generated in the larger scales and transferred through progressively smaller scales by vortex stretching. At the Kolmogorov microscale ($\eta$) the viscosity acts to dissipate energy and $\eta$ is thus the smallest length scale of turbulent motion present. It follows therefore that there is always some finite distance and time over which flow coherence is observed. A successful LES inlet condition generation scheme must aim to contain not just the correct magnitude of turbulent energy, but its distribution over space and time, and to reflect its correct spatial and temporal coherence.

The following sections detail the developed ‘best practice’ approach to two-point PIV analysis; for a more comprehensive review the reader is referred to Hollis\textsuperscript{[32]}, Pope\textsuperscript{[73]} and Robinson\textsuperscript{[80]}. 

Fig 3.2 – Point history at $z/H = 0$, $y/H = 0.52$ (idealised combustor),
a) time signal, b) PDF
3.1.2.1 Correlations and Conditional Averaging

Velocity correlations, and subsequently length and time scales, provide some quantification of the size and mutual relationship of turbulent regions within a FoV and, along with the shear stresses, will be shown to characterise the most important features of the turbulence. The Cross Correlation Function (CCF), \( R \), between any two points, separated by the vector \( \mathbf{r} \) in space and by \( \tau \) in time, can be defined as:

\[
R_{ij}(x, r, \tau) = \frac{\langle u_i(x, t)u_j(x + r, t + \tau) \rangle}{\sqrt{\langle u_i^2(x, t) \rangle} \sqrt{\langle u_j^2(x + r, t) \rangle}} \tag{3.8}
\]

Equation 3.8 has two simplifications from which integral scales are defined: the Spatial Velocity Correlation (SVC), where correlations are performed at the same point in time \( (\tau = 0) \), and the Autocorrelation Function (ACF), performed at the same point in space \( (\mathbf{r} = 0) \).

The technique of conditional averaging, although more generally a method of identifying coherent structures, is included here (because it has been used in this way throughout this project) as an aide to visualising the contribution to a PIV velocity correlation map of specific turbulence structures. Conditional averaging is used here to calculate the ensemble average of a velocity component at a point, \((x_0, y_0)\), in the data field subject to the constraint that the instantaneous fluctuating velocity at that point is a factor, \(k\), times greater than the RMS at \((x_0, y_0)\). This highlights the fluctuations that lie in the ‘tails’ of the PDF distribution at that point, as shown in Fig 3.3.

Mathematically, the conditionally averaged fluctuations for a given component from the positive and negative regions of the PDF, \(\langle u_{+} \rangle\) and \(\langle u_{-} \rangle\), may be described as:
\[ \langle u_{i,aj+}(x) \rangle = \frac{1}{N_{ca+}} \sum_{k=1}^{N_{cmp}} a_{j+}(t_k) u_i(x,t_k) \quad a_{j+} = \begin{cases} 0 & u_j(x_o,y_o,t_k) < k u_j'(x_o,y_o) \\ 1 & u_j(x_o,y_o,t_k) \geq k u_j'(x_o,y_o) \end{cases} \quad N_{ca+} = \sum_{k=1}^{N_{cmp}} a_{j+} \]

\[ \langle u_{i,aj-}(x) \rangle = \frac{1}{N_{ca-}} \sum_{k=1}^{N_{cmp}} a_{j-}(t_k) u_i(x,t_k) \quad a_{j-} = \begin{cases} 0 & u_j(x_o,y_o,t_k) > k u_j'(x_o,y_o) \\ 1 & u_j(x_o,y_o,t_k) \leq k u_j'(x_o,y_o) \end{cases} \quad N_{ca-} = \sum_{k=1}^{N_{cmp}} a_{j-} \]  

(3.9)

It has been shown in Hollis \cite{32} and is seen in Fig 3.4 that, for a particular correlation, plotting in-plane velocity vectors conditionally averaged by the velocity component and location of the particular correlation in question (i.e. \(i=j\) in equation 3.9), gives a good visualisation of the large scale structures contributing to that correlation. A value of \(k=1.5\) is usually sufficient to identify the dominant structures.

Fig 3.4 shows a sample \(ww\)-component CCF distribution overlaid with vectors conditionally averaged by the \(w\)-velocity. Note that the use of the positive, \(\langle u_{ca+} \rangle\), or negative, \(\langle u_{ca-} \rangle\), tails of the PDF will almost always produce identical results, but with the vector direction reversed. The CCF in this plot shows the strength and extent of the linkage between the turbulent fluctuations at the point \((z/H, y/H) = (0, 0.52)\) and every other point in the two fields shown. The large areas of positive and negative correlation are indicative of large scale flow coupling with the reference point due to
turbulent structures within the flow field. As a distribution such as this may be obtained for every point in a given field the CCF may be used to describe the spatial distribution of the various flow structures within a FoV. Since the horizontal component of the conditionally averaged vectors is closely matched to the CCF it follows that the CCF has been influenced by structures of a similar form to those shown. Note that a continuous correlation across the two fields is obtained only because the correlation point was chosen in the overlapping region. The non-synchronous capture of the two data sets would otherwise render this impossible.

![Fig 3.4 - $R_{11}$ and $\langle u_{i,iso} \rangle$ at $z/H = 0$, $y/H = 0.52$ (idealised combustor), $\tau = 0$](image)

3.1.2.2 Integral Lengthscales

A lengthscale, by definition, gives quantitative information on the size of turbulent structures present in the flow. As the largest eddies carry the bulk of the energy containing turbulent motions in a flow, lengthscales tend to characterise the geometry of these energetic turbulent motions. A quantification of the smallest scale motions is given by an estimate of the Kolmogorov lengthscales, and will be presented in section 3.1.2.3. Integral lengthscales are directly calculable from the SVC (defined above), and are defined as the integral with respect to distance in the direction $x_k$ of the SVC:

$$kL_q(x) = \int_0^1 R_{ij}(x,r)dr$$

(3.10)
Where $r$ now represents the separation vector displacement $x_k$. In calculations of integral length scale only correlations where $i=j$ are usually considered. Given the planar nature of PIV, only integration in the in-plane directions (i.e. $k=1,2$) is possible. Thus, four lengthscales may be defined from monoscopic data and six from stereoscopic data. For convenience, lengthscales integrated along their component direction (i.e. $i=j=k$) are termed 'longitudinal', and those integrated perpendicular (i.e. $i=j\neq k$) are termed 'lateral'. Analysing the longitudinal and lateral lengthscales gives an appreciation of the homogeneity of the turbulence.

Fig 3.5 shows the SVC distribution over separations, $\Delta s$, in both the longitudinal and lateral directions at an arbitrary point. The area under the plots, from the origin to infinity in either direction, is the realisation of the integral in equation 3.10. For discrete PIV data this is calculated from the sum of the areas of the trapezoids formed between points in the distribution. In practical calculations, however, it is not possible to integrate to infinity and hence it is standard practice to integrate up to the first crossing of the $\Delta s$-axis, which is generally a very close approximation to the true definition for most practical flows with relatively localised turbulence (Hollis\textsuperscript{[32]}).

![Fig 3.5 - SVC distributions for $R_{11}$ at $z/H = 0$, $y/H = 0.52$ (idealised combustor), taken across field D.](image)

Fig 3.5 - SVC distributions for $R_{11}$ at $z/H = 0$, $y/H = 0.52$ (idealised combustor), taken across field D,  
- Longitudinal  
- Lateral  
- Exponential model
Consideration is required as to what should be done in calculating the integral lengthscale if the available data is curtailed, such as occurs at the edge of the data field, and hence never reaches the zero crossing point. Holli’s\textsuperscript{32} showed this ‘missing’ data can be adequately approximated by an exponential form:

\[
R_{ij} = \left( \frac{R_{ij,\text{curtailed}}}{e^{-\Delta S_{\text{curtailed}}}} \right) e^{-\Delta S} \tag{3.11}
\]

Where the ‘curtailed’ subscript denotes the value at curtailment, i.e. the last value in the SVC distribution. The area under this curve can then be easily calculated to give an estimation of the missing integral contribution. The contribution of the exponential model is seen in Fig 3.5.

Whereas an ACF will always produce a symmetrical distribution about the zero-separation point, it is clearly seen in Fig 3.5 how inhomogeneity or obstacles in the flow may cause the SVC to be asymmetrical. Thus, calculation of the resultant lengthscale from equation 3.10 will differ depending on the direction the integral is evaluated in. Robinson\textsuperscript{80} showed the optimum method is to use a confidence weighting approach (as developed by Holli’s\textsuperscript{32}) based on the relative proportion of data curtailment in the distributions. A confidence factor, $C$, is introduced such that the lengthscale is defined as:

\[
L_{ij} = \frac{C_{ij,\text{up}} L_{ij,\text{up}} + C_{ij,\text{down}} L_{ij,\text{down}}}{C_{ij,\text{up}} + C_{ij,\text{down}}} \tag{3.12}
\]

Where the ‘up’ and ‘down’ subscripts refer to the upstream (negative separation) and downstream (positive separation) sides of the SVC distribution, and,

\[
C_{ij} = 1.125 - \left( 1.25 R_{ij,\text{curtailed}} \right) \begin{cases} 0 & R_{ij,\text{curtailed}} > 0.9 \\ 0.1 & 0.1 \leq R_{ij,\text{curtailed}} \leq 0.9 \\ 1 & R_{ij,\text{curtailed}} < 0.1 \end{cases} \tag{3.13}
\]
The use of confidence weighting has been shown to give a more consistent lengthscale distribution close to data boundaries whilst not affecting the calculation away from boundaries.

Due to the spatially discretised nature of PIV this methodology alone is often insufficient for extracting the true integral lengthscale. The effect of sub-cell filtering of the unresolved PIV scales on the measured integral lengthscale must be considered. The phenomenon of sub-cell filtering and its correction is presented in the next section.

### 3.1.2.3 Sub-cell scales and correction

When considering any derived statistic it is important to recall there is a portion of the *true* experimental flow field that is not captured by PIV: those scales which fall within the dimensions of an interrogation cell and hence are subject to sub-cell filtering. Saarenrinne *et al.*[82] highlight how this is analogous to unresolved LES scales, in that the true velocity field, \((U(\mathbf{x},t))\), is filtered according to a filter function associated with the cell size, \(G(\mathbf{r})\), integrated over the interrogation cell area, \(D\), such that:

\[
\overline{U}(\mathbf{x},t) = \int_{D} G(\mathbf{r},\mathbf{x}) U(\mathbf{x} - \mathbf{r},t) d\mathbf{r}
\]  

(3.14)

Where \(\overline{U}\) is the velocity vector resolved for a given PIV interrogation cell area located at \(\mathbf{x}\). Sheng *et al.*[88] use this analogy to develop a method of estimating the turbulent energy dissipation rate, \(\varepsilon\), from the filtered (PIV) velocity field. From equation 3.15 below this can then be used to estimate the Kolmogorov length scale, and hence gain an appreciation of the level of sub-cell filtering, which will almost certainly be spatially variant across the FoV.

\[
\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}
\]  

(3.15)
Given that the resolved PIV field may be viewed as being governed by a filtered version of the Navier-Stokes equations (with the instantaneous velocity field filtered as in equation 3.14), a resolved PIV kinetic energy ($\bar{E}_K$) transport equation can be obtained as described by Sheng et al.\cite{Sheng1981}:

$$\frac{\partial \bar{E}_K}{\partial t} + \bar{U}_j \frac{\partial \bar{E}_K}{\partial x_j} = \frac{\partial}{\partial x_j} \left( -\delta_j \bar{U}_j \frac{\bar{P}}{\rho} - \tau_{ij} \bar{U}_i + \nu \frac{\partial \bar{E}_K}{\partial x_j} \right) - \nu \frac{\partial \bar{U}_i}{\partial x_j} \frac{\partial \bar{U}_j}{\partial x_j} + \tau_{ij} \bar{S}_{ij}$$

(3.16)

The final term in equation 3.16 represents the energy transferred to the non-resolved scales from the resolved scales, assuming zero backscatter of energy, as previously discussed in the context of SGS modelling in section 1.2.2. Sheng et al.\cite{Sheng1981} show that if the assumption of dynamic equilibrium between energy production and dissipation holds, this energy transfer to non-resolved vortices is then equal to the dissipation rate at the Kolmogorov scales. Thus, via evaluation of the SGS stress using equation 1.12, an estimate of the dissipation rate may be calculated from the final term in equation 3.16. An SGS model, such as the Smagorinsky model (equation 1.16), may be used to calculate the eddy viscosity and hence $\tau_{ij}$, although, according to Sheng et al.\cite{Sheng1981}, the evaluated dissipation rate is relatively insensitive to the SGS model selected.

The calculated dissipation rate may then also be used to estimate the Kolmogorov lengthscale (equation 3.15) and timescale:

$$\tau_\eta = \left( \frac{\nu}{\varepsilon} \right)^{1/2}$$

(3.17)

This provides the user with the ideal upper limit for the inter-frame time for avoidance of dynamic averaging, although as mentioned in section 1.3.1, this must in practice be balanced with experimental constraints. It is in this context that the calculation of the Kolmogorov scales is used throughout this project and results using this approach are presented for the current test cases during the discussion of PIV error analysis in section 5.1.

\footnote{Sheng et al.\cite{Sheng1981} present the final term as $2\tau_{ij} \bar{S}_{ij}$, although this can be shown to be erroneous}
The thesis of Hollis\textsuperscript{[32]} provides a detailed description of the phenomenon of PIV sub-cell filtering and the techniques developed to account for its effect on the derived turbulence statistics. The major approach presented and validated there followed the proposals of Hoest-Madsen & Nielsen\textsuperscript{[31]} (here referred to as the HMN method). It is clear that the magnitude of the error in the RMS produced in a single interrogation cell due to sub-cell filtering (the level of 'lost' fluctuating energy) is related to the cell size and the local lengthscale. The assumption is made that the flow is homogeneous, isotropic and two-dimensional within each interrogation cell, although, as shown by Keane & Adrian\textsuperscript{[36]} and discussed in section 1.3.2, this is not necessarily the case. It then becomes possible (see Hoest-Madsen & Nielsen\textsuperscript{[31]}) to approximate the ratio of the measured RMS, \(u'_{\text{meas}}\), to the true RMS, \(u'_{\text{true}}\), by the following exponential relationship:

\[
\frac{u'_{\text{meas}}}{u'_{\text{true}}} = \begin{cases} 
\frac{\ln(\Delta X/L_{\text{true}})}{\frac{0.3235(\Delta X/L_{\text{true}})}{0.2181 \ln(\Delta X/L_{\text{true}}) + 0.7501}} & \text{for } (\Delta X/L_{\text{true}}) < 1 \\
\frac{0.2181 \ln(\Delta X/L_{\text{true}}) + 0.7501}{-0.2181 \ln(\Delta X/L_{\text{true}}) + 0.7501} & \text{for } (\Delta X/L_{\text{true}}) \geq 1
\end{cases}
\]  

(3.18)

Where \(\Delta X\) is the PIV cell size and in turn, Hollis\textsuperscript{[32]} showed that the true lengthscale, \(L_{\text{true}}\), is given by the following equation:

\[
\frac{L_{\text{true}}}{L_{\text{meas}}} = \begin{cases} 
\frac{\ln(\Delta X/L_{\text{true}})}{\frac{0.5141(\Delta X/L_{\text{true}})}{0.2300 \ln(\Delta X/L_{\text{true}}) + 0.6230}} & \text{for } (\Delta X/L_{\text{true}}) < 0.65 \\
\frac{0.2300 \ln(\Delta X/L_{\text{true}}) + 0.6230}{-0.2300 \ln(\Delta X/L_{\text{true}}) + 0.6230} & \text{for } (\Delta X/L_{\text{true}}) \geq 0.65
\end{cases}
\]  

(3.19)

Where \(L_{\text{meas}}\) represents the lengthscale evaluated from applying equation 3.10 to the measured turbulent velocity field. Equations 3.18 and 3.19 represent two equations for two unknowns (\(u'_{\text{true}}\) and \(L_{\text{true}}\)); these are solved iteratively to extract at each PIV vector location a correction to the raw measured RMS (and thus TKE) and lengthscale calculated from the raw (resolved) PIV data. Corrected and uncorrected lengthscale and TKE contours are shown in Figs 3.6 and 3.7 respectively.
Fig 3.6 – $L_{11}$ integral lengthscale, a) uncorrected, b) sub-cell corrected
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Fig 3.7 – Turbulent Kinetic Energy contours a) uncorrected b) sub-cell corrected

Fig 3.8 – $\Delta X/L_{true}$ contours
As the uncorrected RMS and lengthscales calculated from the measured PIV data contain only contributions from the larger (resolved) flow scales (those not filtered by the discrete cell size), this results in an underestimation of the true RMS/TKE and an overestimation of the true lengthscale. The distribution of \( \Delta X/L_{\text{true}} \) calculated in correcting the above data is shown in Fig 3.8. Note that \( \Delta X/L_{\text{true}} < 0.2 \) for much of the domain and hence by equations 3.18 and 3.19, \( L_{\text{true}} > 0.9 \, L_{\text{meas}} \) and \( u'_{\text{true}} < 1.07 \, u'_{\text{meas}} \), suggesting the effects of sub-cell filtering on the PIV data in these data sets is be modest.

Whereas a correction may be applied safely to resolved statistics by applying a correction factor, there is no current reliable method of recovering the lost information in the instantaneous velocity field. For PIV data captured with low spatial resolution (i.e. where \( \Delta X/L_{\text{true}} \) approaches or exceeds unity), the resultant broadening of the correlation peak may alter the nature of the turbulence, making the instantaneous velocity field unsuitable for use as an LES inlet condition. The acceptable level of sub-cell filtering in this context is thus closely related to similar concerns and estimates made when setting the LES grid size, which as stated previously, governs the flow scales that can be resolved by LES. This will be discussed in more detail in sections 4.3 and 5.1.2.

3.1.2.4 Autocorrelation and Integral Timescales

The local ACF distribution is obtained by consideration of equation 3.8 when \( r = 0 \). For PIV data the temporal offset, \( \tau \), must be an integer multiple of the sample time, thus \( \tau = N_{\text{at}} \Delta t \). A sample ACF is shown in Fig 3.9 for both 50Hz and 1kHz data sets taken at the same point. Analysis of the ACF can be of particular use in identifying periodicity within the flow. Care must be taken when comparing ACFs taken at different sample rates: a high sample rate will generally resolve the low \( \Delta t \) correlations well, but may not have a sufficiently long time sample to capture long-term periodicity (as in Fig 3.9b). Likewise, data taken at a lower sample rate may have poor resolution of the correlation peak, which can lead to over-prediction of the resultant timescale.
Fig 3.9 – Autocorrelation function at z/H = 0, y/H = 0.63 (idealised combustor),
a) 50Hz, b) 1kHz

By analogy with equation 3.10 an integral timescale can be defined:

$$T_{ij}(x) = \int_0^\infty R_{ij}(x,\tau)d\tau$$

(3.20)

As previously, only correlations where i=j are considered here for timescale calculations. Since all integrations are taken along the time axis there is no ‘third dimension’ as appears in lengthscales, hence two timescales may be defined for monoscopic PIV and three for stereoscopic PIV. The integrals are evaluated as for the lengthscales, however, as mentioned previously, the correlation peak of an ACF distribution is always symmetrical and thus no weighting is required.

Fig 3.10 shows the integral timescale as calculated from the 1kHz sample data sets. The regions of high timescale generally correspond to the regions of high energy, widely-correlated turbulence seen previously.
It will be shown in chapter 5 that the integral timescale of a data set can have a significant effect on its statistical convergence, since a large timescale, relative to the PIV sample rate, reduces the amount of independent samples available.

### 3.1.2.5 Power Spectra

Power spectra are used to determine how turbulent kinetic energy is distributed among different eddy sizes (Pope\(^{[73]}\)). Lynn\(^{[51]}\) shows that the power spectrum, \(E_{ij}\), may be obtained from a Fourier transform of the ACF such that:

\[
E_{ij}(x, \omega) = 2 \int_{0}^{\infty} R_{ij}(x, \tau) \cos(\omega \tau) d\tau
\]  

(3.21)

(Strictly, for a finite ACF the result should really be termed an 'energy spectrum', though the term 'power spectrum' serves to be used interchangeably (Lynn\(^{[51]}\)). If \(i=j\) then \(E_{ij}\) represents the Power Spectral Density (PSD) of the \(i^{th}\) velocity component; if \(i \neq j\) then the result is a coherence function. If the CCF is used (i.e. the correlation is offset in space as well as time) the result is a cross-spectral density.

Spectra obtained from PIV may be very noisy or poorly resolved, especially for data sets at a low sample rate, which can make interpretation and filtering of the signal
problematical. To counteract this, the spectra at a given point are calculated in this project as the mean of the spectra of the 9 vector points surrounding it. Spectra may also be calculated as the mean of multiple spectra obtained by down-sampling of a point signal (i.e. reducing the sample rate by considering every $N$ samples). This however exaggerates the poor resolution problem and is better suited to higher resolution measurement techniques (such as hot-wire anemometry) than PIV.

Whereas the PSD quantifies the distribution of energy in the time domain when deduced from the ACF at a point in space, it follows that a similar distribution could be obtained in the spatial domain from the SVC for a given point in time. Thus a time-averaged wavenumber spectrum is defined for the separation in space, $s$, across an in-plane coordinate direction:

$$P_n(x_i, k) = \left( \int_{-\infty}^{\infty} R_{n}(x_i, s) \cos(ks) ds \right)$$

(3.22)

Although a typical PIV data set gives far less resolution across a spatial axis than a time axis, and thus the wavenumber spectrum is less well resolved, time-averaging across the data set was found to produce smooth spectra that give a good indication of the sizes of structures within a flowfield.

Fig 3.11 shows an example of both a PSD and a wavenumber spectrum. To optimise the resolution these are taken from a 1kHz data set. The PSD gradient shows excellent agreement with Kolmogorov's $f^{-5/3}$ approximation. These spectra show the good resolution possible from 1kHz PIV in a water flow experiment, with resolution of a large proportion of the energy spectrum (the Kolmogorov time scale in this region is of the order $10^{-3}$ seconds). The spectra show that, even in the region of the CRVP, a spectrum shape typical of developed turbulence is evident. Henning & Ehrenfried\[30\] show PIV spectra will converge to a minimum noise value at high frequencies, above which the energy remains constant. The beginning of this phenomenon is just visible in Fig 3.11a above 200Hz. This measurement noise was shown in that study to be independent of interrogation cell size for a cell width of greater than 48 pixels (albeit for grid turbulence).
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3.1.3 Identifying Coherent Structures

The concept of a turbulent 'eddy' is something that, according to Pope\textsuperscript{[73]} "eludes precise definition". Yet, observation of turbulent flows shows a range of vortical structures that display at least a semblance of local coherence. Since the larger of these structures may have a relatively long lifetime, they can have a significant effect on any simulated flow far downstream of the inlet. For the study of unsteady mixing flows, of which the core region of a combustor is a prime example, it is imperative that these structures are recognised and replicated in any simulation and in the specified inlet conditions.

Two point statistics can give quantitative trends of the size, range and life-time of structures within the data set as a whole. However, many flows are irregular and aperiodic, and thus they cannot describe the shape, evolution or influence of the various different structures over their life-cycle.

In this thesis, three methods of extracting coherent structures from a planar turbulence field are presented: filtering, Proper Orthogonal Decomposition (POD) and Linear Stochastic Estimation (LSE). This is by no means an exhaustive list, but alternative analyses based on approaches such as swirl strength (Zhou et al.\textsuperscript{[116]}), vorticity-based conditional sampling (Jeong et al.\textsuperscript{[34]}) and wavelets (Meneveau\textsuperscript{[57]}), while deserving of mention, have found no particular use in this thesis and are thus not discussed in detail.

Fig 3.11 – a) PSD at $z/H = 0, \ y/H = 0.52$  b) Wavenumber spectrum across $y/H = 0.52$
Additionally, for the purposes of LES analysis it is also necessary to consider vortex identification within a volumetric velocity field. For this reason, the calculation of the vorticity field, and tracking of vortex centres are also discussed in this section.

3.1.3.1 Filtering

A filter may be defined as any system that transmits a certain range (or ranges) of frequency and rejects others (Lynn[51]). Filters may be broadly categorised into four types, (illustrated in Fig 3.12): Low-pass, high-pass, band-pass and band-stop. The first two only transmit frequencies below or above a certain cut-off or cut-on frequency respectively. The latter two only transmit or reject frequencies within a defined range.

![Figure 3.12 - Idealised filter response characteristics;](image)

In the context of turbulence measurement, filters allow the separation of the frequency or wavenumber spectrum (depending on whether the filter is applied in space or time) to isolate eddies of a particular size. This concept has already been introduced in the context of LES decomposition and sub-grid filtering, where a low pass spatial filter is applied to the velocity field in equation 1.4. For a discrete velocity signal, the filtering process may be written (Lynn[51]):

\[
\bar{U}(x,t) = \sum_{n=0}^{N_{max}} G(n\Delta x)U(x-n\Delta x,t)
\]  

(3.23)
Where $N_{\text{terms}}$ is the number of terms defining the filter. This filter could alternatively be applied in time, such that:

$$\bar{U}(x, t) = \sum_{n=0}^{N_{\text{time}}} G(n\Delta t)U(x, t-n\Delta t) \quad (3.24)$$

A filter function, such as the top hat, Gaussian or sharp Fourier cut-off (equations 1.5, 1.6 and 1.7 respectively) may be used to define the coefficient, $G$. However, as the identification of coherent structures often requires the isolation of a small range of frequencies, the sharp Fourier cut-off is of most relevance here. Improved filter performance is possible by using a window function, which essentially ‘trims’ the filter function outside of a given range. The window function utilised throughout this thesis is the commonly-used Hann window, which factors the filter function by a window function, $w$, as in equation 3.25. More details on window functions are given in Lynn\[^{51}\].

$$w(n) = 0.5 \left[ 1 - \cos \left( \pi \left( \frac{n}{N_{\text{terms}}} + 1 \right) \right) \right] \quad (3.25)$$

Fig 3.13a) shows the distribution of several filter functions, each defined by 201 terms. Fig 3.13b) shows the resultant spectrum of a data set of white noise low-pass filtered by these functions. The filter width, $\Delta$, corresponds to a cut-off wavenumber, $k_c$. 

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Although the wide range of possible filtering techniques allows flexibility, the resultant wide range of parameters may make the filtered data difficult to interpret and optimise. As such, a certain amount of *a priori* knowledge is required.

Fig 3.14 compares an instantaneous PIV data sample with the same sample low-pass spatially filtered using a sharp Fourier filter with a Hann window of filter width $\Delta = 6\Delta x$. Notice the attenuation of the high frequency 'noisy' fluctuations and resultant increased clarity of the large scale structures.
3.1.3.2 Proper Orthogonal Decomposition

Proper Orthogonal Decomposition (also known as Karhunen-Loéve Decomposition) is a method of extracting ordered, energetic mode shapes present within a data field. In the context of velocity data this approach gives an optimal time-averaged representation of the most energetic turbulent motions in a particular data set.

The standard calculation method follows that described in Midgley\[59\]. A fluctuation field for a given velocity component represented via a set of \(N_{\text{samp}}\) discrete time samples may be represented by a set of spatial functions, \(\varphi_k(x)\), and uncorrelated temporal coefficients, \(a_k(t)\), such that,

\[
\mathbf{u}(x, t_i) = \sum_{k=1}^{N_{\text{samp}}} a_k(t_i) \varphi_k(x) \quad \text{for } i = 1, \ldots, N_{\text{samp}} \tag{3.26}
\]

Where \(k\) is the mode number, and a maximum number of POD modes equal to the number of discrete time samples in the data set may be defined. The spatial functions are obtained by firstly constructing the matrix \(\mathbf{A}\):

\[
\mathbf{A} = \begin{bmatrix}
\mathbf{u}(x_1, t_1) & \cdots & \mathbf{u}(x_1, t_{N_{\text{samp}}}) \\
\vdots & \ddots & \vdots \\
\mathbf{u}(x_m, t_1) & \cdots & \mathbf{u}(x_m, t_{N_{\text{samp}}})
\end{bmatrix} \tag{3.27}
\]

where \(m\) is the number of spatially variant vectors in the 2D PIV field per time sample and \(x_i\) is the two-dimensional location \((x, y)_i\). The spatial correlation matrix, \(\mathbf{R}\), between all points in the field may be obtained from equation 3.28.

\[
\mathbf{R} = \frac{1}{N_{\text{samp}}} (\mathbf{A} \mathbf{A}^T) \tag{3.28}
\]

Sirovich\[89\] has shown that the spatial modes result from the eigenvectors of \(\mathbf{R}\). It was shown in Chatterjee\[71\] that these may be obtained from the Matlab function for
Singular Value Decomposition (SVD) (a step-by-step guide to the process is given in Midgley\textsuperscript{[59]}), giving:

\[ \mathbf{R} = \mathbf{U\Sigma\Phi}^T \]  

(3.29)

Where \( \mathbf{U} \) is an \( N_{\text{samp}} \times N_{\text{samp}} \) matrix, \( \mathbf{\Phi} \) is an \( m \times m \) matrix, and \( \mathbf{\Sigma} \) is an \( N_{\text{samp}} \times m \) matrix with all zero elements except on the leading diagonal, where the elements are the singular values arranged in descending order. The columns of \( \mathbf{\Phi} \) are the eigenvectors; hence, by taking the \( k^{th} \) column of \( \mathbf{\Phi} \) across the field the \( k^{th} \) spatial POD mode can be extracted.

Unfortunately, using this method involves performing an SVD on the very large \( (m \times m) \) correlation matrix. This is computationally very expensive and thus a modification known as the "snap-shot" method, as proposed by Sirovich\textsuperscript{[89]}, is generally used to reduce the problem to an \( N_{\text{samp}} \times N_{\text{samp}} \) matrix (a factor of 35 smaller for a typical data set). Rather than considering the correlations, \( \mathbf{R} \), the matrix \( \mathbf{C} \) is generated such that:

\[ \mathbf{C} = \frac{1}{N_{\text{samp}}} (\mathbf{A}^T \mathbf{A}) \]  

(3.30)

Performing an SVD on \( \mathbf{C} \) now produces the coefficients \( \kappa \) as eigenvectors. Linear combination of the \( k^{th} \) column of eigenvectors with the fluctuation matrix then yields the \( k^{th} \) spatial POD mode:

\[ \varphi_k(x) = \sum_{i=1}^{N_{\text{samp}}} \kappa_k(t_i)A(x, t_i) \]  

(3.31)

Thus \( N_{\text{samp}} \), rather than \( m \), POD modes are obtained. Although \( N_{\text{samp}} \) modes may be calculated, it has been found that around 500 interdependent samples are sufficient to give convergence of lower modes. What constitutes an independent sample for a given data set is discussed in section 5.1.4.
In equation 3.29 the singular values on the diagonal of the $\Sigma$ matrix are the square roots of the eigenvalues, $\lambda$, of the A matrix. They are representative of the energy contained within each mode. Hence the cumulative proportion of energy contained within each mode, $E_k$, can be defined as,

$$
E_k = \frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{N_{eig}} \lambda_i}
$$

(3.32)

The cumulative distribution of the eigenvalues gives a representation of the turbulent kinetic energy contained across the POD modes. As the first POD mode should be the best representation of the turbulent energy in the flow it is typical that around 20\% of the total energy is contained within that mode. This figure tends to increase with Reynolds number (Patte-Rouland et al.\cite{Patte-Rouland}).

Fig 3.15 show the vectors of the first spatial POD mode, $\varphi_1$, over the two example PIV fields from the idealised combustor upper annulus. Note that as the decomposition for each is optimal there is not necessarily a continuous transition between the two, as can be seen from the dissimilar profiles across $z/H = 0$ in Fig 3.16, even though the distribution of energies across the modes (Fig 3.17) is very similar. Also note the similarity of the decomposed structures with those extracted via conditional averaging in Fig 3.4.
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Fig 3.15 – Vectors of 1st spatial POD mode

Fig 3.16 – Profile of 1st spatial POD mode across z/H = 0
- $v_{k=1}$, field B; $w_{k=1}$, field B; $v_{k=1}$, field D; $w_{k=1}$, field D
Whereas a full reconstruction using equation 3.26 returns the instantaneous velocity field, a reconstruction using less than $N_{\text{snap}}$ modes gives a new velocity field containing only contributions from the higher-energy modes, and thus isolating the energy-containing structures. In order to obtain a reconstruction from equation 3.26, rearrangement is required to give the temporal coefficients, $a_k$. However, since the “snap-shot” method is used to calculate $\phi_k$, the matrix is no longer square, and thus cannot be directly inverted. Thus $a_k$ is obtained from:

$$a = u \phi^T \left( \phi \phi^T \right)^{-1}$$

and may subsequently be applied to equation 3.34 where $N_{\text{modes}}$ is the number of modes used in reconstruction.

$$u_k(x,t) = \sum_{k=1}^{N_{\text{modes}}} a_k(t) \phi_k(x) \quad \text{for } i = 1, ..., N_{\text{modes}}$$

Fig 3.18 shows the temporal coefficients of the $1^{\text{st}}$ and $2^{\text{nd}}$ POD modes of the left-hand field decomposition shown above. This can be used in conjunction with time series to evaluate the periodicity and frequency of structures within a flowfield. As with the time series, Fourier decomposition can be used to obtain spectra from this data. In the case below, the first POD mode has clearly extracted the dominant periodic motion present in the flow.
The wide ranging and optimal applicability of POD is reflected in numerous and varied studies which have recently utilised it, such as vortex identification in swirling flows (Graftieaux et al.\textsuperscript{[25]} and Midgley\textsuperscript{[59]}), flow prediction for sensing and control (Taylor & Galuser\textsuperscript{[110]}), flame analysis (Kodal et al.\textsuperscript{[40]}) and (of most relevance to the present research project) velocity field extrapolation (Druault et al.\textsuperscript{[116]}), amongst others. Section 3.2.4 will introduce a study by Perret et al.\textsuperscript{[69]} that uses a conditioned random time series to artificially generate the temporal coefficients, $a_k$, thereby increasing the sample rate of the data set.

### 3.1.3.3 Linear Stochastic Estimation

Linear Stochastic Estimation (LSE), recently introduced in the context of inlet generation by Druault et al.\textsuperscript{[116]} after initial proposition by Adrian (1977)\textsuperscript{[2]}, takes advantage of the statistical information contained within the two-point correlation tensor to isolate coherent structures within a flowfield. The assumption is made that the instantaneous velocity signal at a given point in space may be estimated through knowledge of the time history at one or more known reference locations and the spatial velocity correlation between the signal point and the reference locations. The technique enables a relatively coarse spatial grid of time-resolved (i.e. high sample rate) reference locations to allow reconstruction of (synchronous) instantaneous...
fluctuations over a spatially well resolved grid of points at which only time-averaged information is known.

LSE can of course only ever reconstruct those structures that display coherence with the locations at which the reference signals are measured. By extension of the Reynolds decomposition, the fluctuating velocity field may be considered as made up of coherent, \( \bar{u} \), and incoherent, \( u' \), motions such that,

\[
U_i(x,t) = \langle U_i \rangle(x) + \bar{u}_i(x,t) + u'_i(x,t)
\]  

(3.35)

In LSE the coherent fluctuation field, \( \bar{u}(x,t) \), refers to those motions within the instantaneous filed that are correlated with the \( N_r \) reference signals, \( p_r(t) \). These are used to provide an estimation of the coherent field, such that,

\[
\bar{u}_{\text{est}}(x,t) = \sum_{r=1}^{N_r} a_r(x)p_r(t)
\]  

(3.36)

Where \( a_r(x) \) is the solution of the equations,

\[
\langle u(x,t)p_q(t) \rangle = \sum_{r=1}^{N_r} a_r(x)p_r(t)p_q(t) \quad q = 1, ..., N_r
\]  

(3.37)

The reference signal, \( p_r \), can be chosen as any flow variable (e.g. pressure, Druault et al.\textsuperscript{16}, Taylor & Glauser\textsuperscript{100}, or velocity, Adrian & Moin\textsuperscript{35}), as long as its two-point correlation with the fluctuation, \( u(x,t) \), can be obtained.

Although LSE is essentially a form of conditional sampling (given the weighting supplied by the velocity correlation), it has the advantage over more conventional techniques that the only parameter to be optimised is the location of the reference point(s). As the estimation in equation 3.36 essentially derives from the linear term of a Taylor expansion (Adrian & Moin\textsuperscript{35}), improvement of the estimation may be obtained by using higher-order terms. Previous literature (Debiasi et al.\textsuperscript{14}, Naguib et
al.\textsuperscript{[63]} has shown the use of quadratic stochastic estimation to be beneficial over LSE, although this method is not used in this project.

Fig 3.19 shows a comparison between a typical sample of PIV fluctuations and the same field reconstructed by LSE using signals from the 6 evenly distributed reference points (analogous to the arrangement of a hot-wire rake) shown in Fig 3.19 as red circles at $z/H = 0.2, 0.55, 0.7; y/H = 0.3, 0.75$. Clearly, although the large scale structures are replicated, the medium and small scale structures are not. It will shown in chapter 4 how LSE is heavily dependent on the number and arrangement of the reference points used. The issue of how the incoherent motions can be added to the coherent fluctuations to recreate a complete fluctuating field has not been previously addressed and is a specific consideration of the present work, to be addressed in more detail in chapter 4.

![Typical fluctuation field, a) PIV b) LSE from 6 reference points](image)

### 3.1.3.4 Vorticity and Vortex Centre Tracking

Although the methods outlined above are of use in the inlet condition generation process, they are either too computationally expensive (POD) or difficult to generalise (LSE and filtering) to be of regular use in the analysis of the three-dimensional velocity fields calculated by LES. A more aggressive data reduction technique is required which can identify the flow structures within a volumetric space.
Vorticity is the local circulation of the fluid about a particular axis, as defined for the mean flow about the $x$-axis in equation 3.38. Vorticity has been found to be useful in the identification of the location and development of large scale flow structures.

\[ \omega_x = \frac{\partial \langle v \rangle}{\partial z} - \frac{\partial \langle w \rangle}{\partial y} \]  
(3.38)

As a gradient-based technique, the vorticity field does not always lend itself to unique interpretation, especially in boundary layers (where the background shear is high everywhere (Jeong et al.\(^{[34]}\))). Its use as a flow visualisation technique is therefore only recommended in time-averaged flows. Nevertheless, in these circumstances the streamwise vorticity can be a useful measure of the diffusion of a vortex.

Fig 3.20 shows an example of the through-plane ($x$) vorticity of a time-averaged flow field. Note this is the only component of vorticity available from planar monoscopic PIV data. This highlights the CRVP present in the mean flow. The skewed nature of the structures is probably a result of the annulus-wide skew noted in the previous chapter, which has not been corrected for in this data. Also evident are the high levels of shear present in the boundary layers.

Fig 3.20 – $\omega_x$ vorticity magnitude in the time-averaged flow field
Although the centre of an ideal vortex is located at the point of maximum vorticity, for real flows this is not a reliable method. In flows containing dominant vortices, the method of Grosjean et al.\textsuperscript{[28]} (or the similar method used by Graftieaux et al.\textsuperscript{[25]}) has been used to identify the vortex centre location. The former method is based on the maximisation of the normalised angular momentum about a point, $x_p$, at a given axial location such that $x_p$ is selected to maximise $f(x_p)$:

$$f(x_p) = \frac{1}{(2N_L + 1)^2} \sum_{i=1}^{m} \frac{\vec{r} \times \langle \vec{U}(x_i) \rangle}{|\vec{r}||\langle \vec{U}(x_i) \rangle|}$$  \hspace{1cm} (3.39)

Where $r$ is the radius of the point $x_i$ from the point $x_p$, $N_L$ is the number of layers used in the calculation and $m$ is the number of vectors lying within $N_L$ layers of $x_p$ on a given $yz$-plane. The number of layers, illustrated schematically in Fig 3.21, should be selected to encompass the majority of the vortex, although this has a dramatic effect on the computation time. It was found that, for a typical velocity field, 8 layers were suitable for obtaining the vortex centre. By applying cubic splines to interpolate the maximum of the functional surface, $f(x_p)$, the vortex centre point may be identified at higher resolution than the measurement grid.

In Grosjean et al.\textsuperscript{[28]} the vortex centres calculated in each frame of a PIV data set are used to obtain a two-dimensional PDF of the vortex centre location. However, when applied to a velocity field within a volume (such as the output of LES data) the path of a vortex may be determined from the piecewise connection of the centre points. Although this is computationally expensive on an instantaneous basis, it is feasible when a vortex exists stably in the mean flow field for the length of the LES domain.

Fig 3.22 shows the calculated vortex centre within an example PIV field has identified the ‘intuitive’ vortex centre. Note that fields may be sub-divided for the tracking of more than one vortex within a given field provided they each occupy a discrete area of space.
Fig 3.23 shows a PDF of the calculated vortex centre location within the instantaneous flow field of the same data set. This is a useful way of visualising the bi-polar location resulting from the highly periodic nature of the CRVP in this data.

Fig 3.21 – Method for swirl centre location nomenclature schematic,
- 1st layer
- 2nd layer
- 3rd layer
- 4th layer

Fig 3.22 – Time averaged vectors showing the calculated vortex centre (+)
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Fig 3.23 – PDF of vortex centre location in the instantaneous flow field

3.2 Synthetic Inlet Generation Methods

3.2.1 Overview

The attraction of using an LES CFD inlet condition velocity field generated by a synthetic method using experimentally-derived data has been outlined in section 1.2.3. The aim of any synthetic inlet condition generation method must be to maximise the fidelity of the synthetic data with respect to the experimental flowfield from which it is derived. This is obviously maximised by the extent to which experimental data is actually used. Broadly, synthetic inlet conditions may be categorised by a “Hierarchy of Fidelity”, as shown in Fig 3.24 (it is assumed that available measured mean velocity data is nearly always used and emphasis here is placed on the effect of specification of turbulent fluctuations in the inlet condition).
Inlet conditions at each level and their application are discussed in the following sections. It is worth noting that Fig 3.24 could include a “Level 0” for LES that has been conducted with turbulent fluctuations neglected entirely in the inlet condition specification, but this approach and Level 1 are almost synonymous.

Before discussing the application of the various ‘Levels’, it is prudent to assess the current state of usage of these various types of inlet condition in published LES predictions. A survey was conducted of all studies of unsteady, turbulent flows within the last 5 years (2003-2007) in the International Journal of Computational Fluid Dynamics. The results are shown in Fig 3.25. It is perhaps surprising to note the proportion of studies that either neglect the inflow fluctuations entirely (Level 0) or completely disregard the use of measured turbulence experimental data (Levels 3 and 4). While the use of steady inlet conditions may be acceptable in flows with significant downstream turbulence-generating features, such as free shear layers, wakes or obstructions, the lack of consideration given to using high-quality experimental data shows there is much scope for improvement in this area.
3.2.2 Level 1

The simplest method of obtaining an unsteady inlet flow, and the method used in much early work on LES, is to take no account of most structural details of the turbulence at all, merely the magnitude of it. The Level 1 approach essentially consists of the addition of noise (usually Gaussian in nature), scaled to match an available measurement (or guess) of the local TKE, superimposed upon a mean profile. This type requires only very rudimentary experimental measurements, e.g. measurement of the mean velocity profile and a simple measurement (or a guess) at the level of turbulence intensity. The profiles may even be estimated from similarity to related flow profiles, or be spatially invariant (e.g. if only a mass flow rate is measured). However, a random field lacks spatial and temporal correlations, and hence has an energy spectrum that is approximately uniform over the whole frequency range. This is not consistent with realistic turbulent spectra, which show much higher energies in the low frequency region. Simulations with inlet conditions of this type often require an adaptation zone downstream of the inlet for realistic turbulence to develop (Perret et al. [69]), especially because the inlet (white noise) turbulence decays very rapidly in the streamwise direction and ‘true’ turbulence has to develop in the numerical prediction from the remnants of the white noise fluctuations. Essentially, a
transition to turbulence process has to take place and, particularly if the flow is developing near a solid surface, this can take a substantial streamwise distance.

Clayton and Jones (2006)\textsuperscript{[10]} performed LES predictions of impinging jets in cross-flow with and without an annulus feeding the jets. White noise fluctuations were applied at inflow boundaries in the upstream annulus and jet ports respectively. The prediction that allowed annulus turbulence to develop matched the experimental measurement better (although admittedly with still too low turbulence levels). The prediction with just jet port turbulence showed the white noise to be insufficient in representing the upstream geometry. Spencer \textit{et al.}\textsuperscript{[95]} used a more realistic combustor geometry and compared steady (Level 0) and white noise inlet conditions in the annuli and injector. Although the unsteady LES inlet case showed an improvement in the resultant core flow, there were still significant differences compared with PIV measurements showing use of the Level 1 approach is still too crude. Fig 3.26 below shows two snapshots of instantaneous velocity vectors within the core of the combustor considered in Spencer \textit{et al.}\textsuperscript{[95]}. While the back and forth 'flapping' of the jets was well predicted, the behaviour appeared insensitive to the annulus inlet conditions. The core region as a whole was found to be far more sensitive to the turbulence conditions prescribed at the fuel injector. Schlüter\textsuperscript{[83]} trialled white noise to facilitate unsteady behaviour across a RANS/LES interface in a coupled compressor/combustor simulation. It was specifically noted that the lack of correlation caused the input turbulence to dissipate very quickly.

However, there is some evidence to suggest that this simple treatment of unsteady inlet conditions is sufficient in some cases, provided the flow development downstream of the inlet is such that it allows the turbulence to develop rapidly (and hence 'forget' the errors in the turbulence condition at the inlet). For example, García-Villalba \textit{et al.}\textsuperscript{[19]} found that an LES with an inlet boundary placed just upstream of a radial swirler was better for the prediction of the resultant coherent structures than one placed too close to the swirler exit. Similarly, Schönfeld & Poinset\textsuperscript{[85]} found the applied inlet turbulence condition at the inlet of a similar swirler had little effect.
3.2.3 Level 2

Level 2 may be defined as the approach in which a turbulent velocity field is generated and superimposed on a mean profile by way of a filtering process applied to a random noise signal such that the filtered unsteady signal matches measured second-order single-point statistics. However, due to computational and/or experimental restrictions assumptions must be made about the nature of the two-point velocity correlations to make this generation process feasible. There are two methods of flow conditioning in this manner in common use: the method of Klein et al.\textsuperscript{39}, which is referred to here as Digital Filter-based Generation (DFG), and the method of conditioning by way of Fourier harmonics, referred to here as spectral forcing. The two methods may be considered to be analogous operations in the time and frequency domains respectively and are described separately in the following sections.

3.2.3.1 Digital Filter-based Generation

DFG is a two-stage process for conditioning (filtering) a random white-noise field of zero mean and unit variance. Firstly, a random velocity field, $u(x,t)$, is convoluted with a correlation (filter) function to give a correlated velocity field, $\tilde{u}_i(x,t)$ using a modified form of equation 3.23. Thus in the 1D spatial sense:
\[ \bar{u}_i(x_j, t) = \sum_{n=-N}^{N} b_j(n)u_i(x_j + n, t) \] (3.40)

Where \( N \) is the support width of the filter. The filter coefficients, \( b_j \), are determined such as to achieve a specified two-point spatial velocity correlation in the filtered field. Klein et al.\(^{391}\) show that the filter-function may be related to the two-point spatial correlation,

\[ R_{ij}(k\Delta x) = \frac{\sum_{n=-N+k}^{N} b_j(n)b_j(n-k)}{\sum_{n=-N}^{N} b_j^2(n)} \] (3.41)

If \( R_{ij} \) is known (see below) the filter coefficients \( b_j \) can be determined from the solution of equation 3.41. Note, a three-dimensional version of the filter is simply formed by the convolution of three one-dimensional filters (equation 3.42) so that the method may be understood by focusing on just a 1D filter.

\[ b_{ijk} = b_i \cdot b_j \cdot b_k \] (3.42)

The spatial correlation, \( R_{ii} \), is taken from an assumed distribution shape, usually parameterised in terms of a required lengthscale.

Equation 3.41 assumes spatially invariant filter coefficients, hence modification may be required to produce fields with continuously varying length scales and/or correlation distributions, such as has been demonstrated by Veloudis et al.\(^{102}\). Klein et al.\(^{391}\) recommend the use of a constant length scale with a Gaussian correlation distribution, found in homogeneous turbulence,

\[ R_{ii}(x, r) = \exp\left(-\frac{\pi r^2}{4 L_i(x)^2}\right) \] (3.43)
Hence, for a constant lengthscale, \( L_{ii} = n_{Ax} \Delta x \), an expression is obtained such that, the 2-point correlation in the \( i \)-direction at any point in the inlet plane is given by,

\[
R_{ii}(k\Delta x) = \exp \left( -\frac{\pi}{4} \frac{k^2}{N_{Ax}^2} \right)
\]  

\[
(3.44)
\]

Klein et al.\textsuperscript{[39]} recommend the filter support for all directions takes a value of \( N \geq 2n_{Ax} \), i.e. twice the integral length scale. Thus the length scale and grid spacing (or time step in the through-plane direction) have a strong influence over the number of operations required in equation 3.40 and hence the overall computational time and memory requirements of the convolution. Since for LES the grid spacing is ideally of the order one sixth of the local energy containing lengthscale (Pope\textsuperscript{[73]}), the convolution stage is clearly computationally expensive if solved directly. However, the convolution may be calculated in the frequency domain, using a Fast Fourier Transform (FFT). Details of this process are given in Veloudis et al.\textsuperscript{[102]}, quoting an improvement in computation time of around 70%. Quantification of the computational expense of DFG, especially in regard to other methods, will be discussed further in chapter 5.

The convoluted field, \( \bar{u}_i \), must be scaled to ensure that after the convolution \( \langle \bar{u}_i \rangle = 0 \) and \( \langle \bar{u}_i \bar{u}_j \rangle = \delta_{ij} \) (although this already should be the case). After the convoluted field is generated (as described above) a further step must be taken since up to now the convoluted velocity field has zero mean, unity variance and no shear stresses. The following transformation is made:

\[
U_i = \langle U_i \rangle + a_{ij} \bar{u}_j
\]

\[
(3.45)
\]

Where \( a_{ij} \) is a matrix of coefficients such that,
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\[
a = \begin{pmatrix}
\langle u_1^2 \rangle^{1/2} & 0 & 0 \\
\langle u_1 u_2 \rangle & \left( \langle u_2^2 \rangle - a_{21}^2 \right)^{1/2} & 0 \\
\langle u_1 u_3 \rangle & \left( \langle u_3 u_2 \rangle - a_{31}^2 \right) & \left( \langle u_3^2 \rangle - a_{32}^2 \right)^{1/2}
da_{11} & a_{12} & a_{13}
da_{21} & a_{22} & a_{23}
da_{31} & a_{32} & a_{33}
\end{pmatrix}
\] (3.46)

In these equations, \( \langle U_i \rangle \) and \( \langle u_i u_j \rangle \) are the first and second order statistics (usually taken from measurement) at each point in the inlet field. The matrix in equation 3.46 was obtained by Lund et al.\(^{[50]} \), the derivation of which has been included for completeness in Appendix B. The resultant time dependent velocity field thus now displays the measured mean velocity vector and Reynolds stress tensor with specified length and time scales and has a two point correlation distribution following equation 3.44.

Klein et al.\(^{[39]} \) have applied the DFG method to a plane jet at moderate Reynolds number. They found the downstream mean and turbulence profiles were a much closer match to measured behaviour than the use of random noise. They also claim (although do not explicitly show) that the method is much more adaptable to generally available experimental data than spectral methods.

Veloudis et al.\(^{[102]} \) have noted the prohibitive nature (in a computational sense) of applying DFG with spatially varying length scales and propose a method allowing a variation in the filter coefficients between a small number of discrete zones. They also show how the time-step used to generate the DFG data (\( \Delta t_{\text{DFG}} \)) may be larger than the LES time step (\( \Delta t_{\text{LES}} \)), with data linearly interpolated after the synthetic database is created. They recommend setting the DFG time-step such that \( n_{t} \geq 10 \). Although this method produces a small (albeit non-proportional) increase in computational cost and memory and will not account for rapid variation between zones, results show an improvement in the prediction of a constricted channel flow over 'conventional' DFG, and a large improvement over scaled noise.

DFG thus provides a promising method of producing synthetic inlet conditions within the boundaries of its assumptions. In addition to the above studies, it is beginning to
gain popularity for the generation of inflows that do not contain large-scale coherent structures (e.g. turbulent opposed nozzle flow (Stein et al.\cite{stein1971}), urban meteorology (Xie & Castro\cite{xie2011}). Although it is primarily a method requiring first and second order one-point velocity statistics, it requires experimental data of sufficient quality so as to provide also the turbulence length and time scales. Given PIV is more conducive to providing the length and time scales and Reynolds stresses needed for DFG than the alternative method of spectral forcing (see below), DFG will be used to demonstrate the viability of level 2 inlet conditions later in this thesis.

3.2.3.2 Spectral Forcing

Although not considered further in this project, the technique of spectral forcing is widely used in the literature and thus merits mention here as an alternative to DFG. Although many variations in the technique exist, most current methods are derived from the approach of Kraichnan\cite{kraichnan1967} and later Lee et al.\cite{lee1988}. The fluctuation field at an inlet plane is formed by superimposing $N_{\text{samp}}$ Fourier harmonics, as in the generic form (Maruyama et al.\cite{maruyama2005}, Smirnov et al.\cite{smirnov2005}),

$$\bar{u}_j(x,t) = \sum_{n=1}^{N_{\text{samp}}} \left[ A_{in} \cos \left( k_{jn} x_j + \omega_n t \right) + B_{in} \sin \left( k_{jn} x_j + \omega_n t \right) \right]$$  \hspace{1cm} (3.47)

Where $A_{in}$ and $B_{in}$ are the Fourier coefficients corresponding to the wavenumbers and frequencies, $\tilde{k}$ and $\tilde{\omega}$, appropriately scaled by assumed length and time scales. An assumption must generally also be made as to the form of the turbulence spectrum, or this could be a further measured input if a sufficiently resolved experimental spectrum is available. Since, via equation 3.21, the spectra are directly related to the two-point velocity correlations, this approach is a direct analogy of the DFG convolution stage and hence it is the assumed spectral shape that is used to determine $A_{in}$ and $B_{in}$, resulting in similar computational issues surrounding the spatially constant or varying nature of these assumptions. However, three-dimensional spectra are difficult to obtain experimentally, and must in general be reconstructed on a regular Cartesian grid (Kempf et al.\cite{kempf2005}).

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Of particular interest to this project are those studies comparing spectral forcing methods to other inlet generation techniques. Glaze & Frankel\cite{24} have performed LES of a turbulent jet using both scaled white noise and a form of spectral forcing. They note that, unlike the scaled noise, the spectral LES showed no dissipation of the inlet turbulence downstream of the jet nozzle. However, they also note a significant difference in the flow field between these studies and ones which resolve the upstream nozzle. These differences appear to originate from an 'adjustment period' downstream of the inlet plane whilst sustainable turbulence is established, characterised by an over-prediction of the turbulence intensity and under-prediction of the mean velocity profiles. Jones & Wille\cite{35} have performed LES for a plane jet in cross flow using both spectral and precursor inlet conditions. They notice practically identical first and second moments (although make no reference to the presumed difference in computational time).

Smirnov et al.\cite{91} take the analogy with DFG a step further by conditioning the generated velocity fields by the Reynolds stress tensor in a similar way to that described in the previous section. This results in an improved prediction of anisotropic turbulence, as well as the obvious advantages over scaled noise.

### 3.2.4 Level 3

The classification of the Level 3 approach encompasses methods that do not require any a priori assumptions about the nature of the two-point correlations (or spectra), but stop short of fully resolving the instantaneous fluctuations across the entire inlet (this is classified as Level 4). Crucially, this implies that only methods which take as inputs the measured correlation field between all points in the inlet plane (i.e. without any need for assumed spatial invariance or assumed spectra/correlation shapes) may be categorised as Level 3. Although the Level 3 approach does not explicitly use measured instantaneous data directly at each point in the inlet plane, the use of all or part of the measured instantaneous velocity field across all or part of the inlet is the most reliable way of ensuring the experimental velocity correlations are maintained.

In theory, DFG could be used, taking as input spatially varying measured two-point correlation distributions to produce a Level 3 method. However, as stated above, the
requirement for spatially varying filter coefficients is not met explicitly by equation 3.41 and would thus require modification. Also the computational cost is likely to far exceed that of a precursor simulation.

As seen from Fig 3.25, there is relatively little evidence in the literature for the use of Level 3 inlet conditions. This is perhaps largely due to the historically insufficient temporal or spatial resolution of available experimental data. However, a few studies do exist.

Maruyama et al.\footnote{1521} used three-dimensional hot-wire probes to obtain time signals at 12 locations in a boundary layer growing on a surface with roughness elements. Non-synchronous, two-point correlations were also obtained from hotwires over a much finer measurement grid. Fluctuations on the fine grid were determined from conditional interpolation of the Fourier coefficients of the reference signals, weighted by the cross-spectral densities. It was found the resultant inlet represented the low frequency experimental data well, but energy in the high frequency range and Reynolds shear stresses were smaller than expected. Although hotwires were used in this case, an analogy with PIV may be drawn from the use of a large FoV (as in Fig 2.9a).

In a comparable study, Druault et al.\footnote{1161} used the decomposition presented in equation 3.35 to reconstruct the flow at an experimental/computational interface in a turbulent mixing layer. 33 two-component hotwires were used to capture simultaneous instantaneous velocities at the interface location in the streamwise and either transverse or spanwise directions. LSE was used to reconstruct the coherent part of the instantaneous velocities at an arbitrary spanwise plane, given the structures within the mixing layer are spanwise-homogenous, though details are not provided on which velocity components were used to condition the reconstruction. Interpolation and extrapolation of the measured velocity correlation tensor was required to obtain the correlation over the computational grid. POD was used to ‘fill in the gaps’ by using two important properties of the decomposition: the repetition within each POD mode of asymptotic behaviour within the flow properties and the degeneration of POD modes into Fourier harmonics for homogeneous flows. The former allows the measured correlations to be extrapolated easily and reliably outside of the
measurement region through extrapolation of the individual POD modes and subsequent reconstruction of the correlation by, via:

\[ R_{ij}(x,r) = \sum_{k=1}^{N_{\text{mp}}} \lambda_k \phi_{i,k}(x) p_{j,k}(x+r) \]  

(3.48)

The latter assumption allows the spanwise spatial modes to be modelled beyond the measurement region, whilst maintaining the periodic requirement of the numerical simulation in this direction. White noise was used to simulate the incoherent part of the fluctuation. A comparison of the structures within the resultant simulation is given against a pre-cursor simulation, an inlet condition of uncorrelated random noise (Level 1), and inlet condition adjusted to have the correct temporal or spatial correlations (Level 2). The results are shown in Fig 3.27, illustrating the superior performance of the proposed hybrid LSE+POD+white noise method. An adjustment period was observed in the Reynolds stresses immediately downstream of the interface, before they reached an asymptotic state. This is attributed to an underestimation of the shear stress by approximately 50% within the generated inlet, which could be a result of the admitted lack of optimisation of the supplementary white noise fluctuations. However, although this method produced well-defined coherent structures, it relies on certain \textit{a priori} knowledge about the flow in question, such as the suitability of POD as an extrapolation tool and the optimal location of the hot-wire rakes. This method therefore requires development to be applicable in the general sense, and this is a prime objective of the work reported in the remainder of this thesis.
Using the same geometry as in the previous study, Perret et al.\textsuperscript{[69]} captured temporally under-resolved stereoscopic PIV to generate inflow data. This takes advantage of the superior spatial resolution of PIV, but lacks the temporal resolution of the hotwires used previously (data was captured at 1Hz). One field of PIV was taken and POD used as above to extrapolate the data in the spanwise and transverse directions. To reconstruct the temporal behaviour of the flow with an appropriate time-step for LES, a randomly generated series, conditioned to an idealised spectrum (as in spectral forcing), was modified to provide the temporal POD coefficients ($a_k(t)$ from equation 3.25). The delimiting frequencies of the conditioning spectrum were from measured hot-wire data. Although this technically reduces the method to a Level 2 inlet condition, a simultaneous hotwire measurement or high-speed PIV system could alternatively have been used. The one point statistics of the mixing layer were well represented, with a notable improvement on the 50% deficit in shear stress noted in
the previous study. The power spectra, although representative of the experimental data, do not contain the peaks typically found in mixing layer data. Although the spatial correlations are very well reproduced, the random origins of the temporal coefficients lead to some loss of coherent structures within the generated velocity field. Rapid development of turbulence within the resultant LES simulation is again observed, though the TKE development is perhaps less favourable than the previous study.

The method by Perret et al.\cite{69} was once again dependent on known and favourable flow characteristics. For a general case where more than one PIV field is required to capture the inlet physics, there is no guarantee the POD modes would be continuous between fields and thus the method would not be applicable. To the knowledge of the author, this is the only previous study where the spatial data resolution of PIV has been utilised for inlet condition generation.

The research presented in the remainder of thesis is intended to overcome the shortfalls of these two pieces of work

3.2.5 Level 4

For an approach to be considered “Level 4” the experimental data must be of sufficient temporal and spatial resolution that no instantaneous generation of any synthetic data to “fill-in” either spatial or temporal gaps in the experimental data is required and the measured fluctuations may be interpolated directly onto the LES grid with no loss of information. In this sense the inlet flow would then behave exactly as a precursor simulation, matching all statistics over the entire inlet domain.

With the current state-of-the-art of PIV described in chapter 2, such a method would only be possible for low Reynolds number, spatially confined flows. The extension to realistic geometries would require multiple high-speed cameras in a setup akin to Fig 2.9b, resulting in instrumentation costs far beyond the reaches of most institutions. To the knowledge of the author, no such studies have taken place and are unlikely to be feasible for the foreseeable future.
Chapter 4

Development of a New Inlet Condition Generation Technique
Previous work on various inlet condition generation techniques has been discussed in chapter 3. However, none of these methods are able to generate the 'Level 3' type of inlet condition explained in Fig 3.24 for realistic geometries, using multiple frames of PIV. This chapter describes the process of developing a methodology whereby the analysis techniques described in the previous chapter may be utilised to produce such a Level 3 method for application to LES inlet conditions.

When considering the application of PIV data as input for a potential LES inlet condition, it is quickly apparent that the required data region rarely falls within the near-square fields of view obtained from PIV. Taking the example of geometries applicable to a gas turbine combustor, none of the potential inlet planes for LES – such as the OGV exit, pre-diffuser exit, or annulus inlet – could be adequately described by a single field of PIV. It has already been outlined in section 2.2.2 how multiple fields of PIV are thus required to capture data of sufficient resolution to be used in LES.

For statistical-based methods, such as using scaled white-noise or the first and second-moments as in a digital filter method, the use of multiple fields does not represent a problem. However, for a method based on using the instantaneous velocity field as input, the loss of spatial correlation between points in neighbouring fields when these are separately and independently obtained would very quickly destroy any coherent structures that may have been captured. The challenge in generating this class of inlet condition from multiple fields of PIV is thus producing a velocity field that has both the required statistics and maintains measured spatial correlations.

Taking these requirements into consideration, it is prudent to re-evaluate the instantaneous velocity field decomposition used by Druault et al.\textsuperscript{[16]}, Kodal et al.\textsuperscript{[40]} among others, and presented in equation 3.35. Rather than separating the turbulence field into coherent and incoherent components, it is more here useful to consider the decomposition in terms of the correlated and uncorrelated velocities. Thus,

$$U_i(x, t) = \langle U_i \rangle(x) + \bar{u}_i(x, t) + u'_i(x, t)$$  (4.1)
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where $\bar{u}$ is the portion of the unsteady velocity field in a given PIV field which is correlated with that in a neighbouring PIV field and $u'$ is the portion of the field that is uncorrelated. This more precise and relevant definition will be used for the remainder of this thesis. It also serves to highlight that the method of producing the correlated scales in the final inlet condition field ("sewn together" from several PIV fields) must involve reconstruction of the time-varying velocity field from measured neighbouring fields, whilst the uncorrelated portion may simply be obtained from a decomposition of single measured field data. The two unsteady portions of the velocity field are then superimposed upon the time-averaged velocity field for individual measured PIV fields at each location on the desired LES grid.

The technique of Linear Stochastic Estimation was selected for correlated field reconstruction.

4.1 Correlated Field Reconstruction

It was detailed in section 3.1.3.3, how through application of the LSE technique, two non-synchronous fields of velocity signals may be 'conditioned' into a single, synchronised field using the known signal conditions at one or more reference points. Thus, a velocity signal (for the 'correlated' unsteady component) at any given point in the 'unified' field, created from the separately measured PIV data sets, may be estimated using LSE provided the two-point correlation is known between that point and a set of reference signals. Applying this situation to PIV data (taken in two separate fields or data sets) it is possible to form a conditioned velocity field over one single unified measurement field, as long as spatial correlations can be obtained across each of the PIV measurement fields from reference signals taken in the region where the two fields spatially overlap.

It is convenient to designate the PIV field from which reference velocities are taken as the 'master' field and the PIV field over which the conditioned velocity field is to be reconstructed as the 'slave' field, as shown in Fig 4.1. The master PIV field itself may be directly interpolated onto the desired output grid and thus, within the limits of
sub-grid filtering effects (see section 4.3), will replicate exactly the PIV statistics. Careful selection of the master field can thus go a long way to optimising the fidelity of the resultant velocity field. The following reconstruction method need only be applied to the slave field(s).

The \( S \) and \( M \) superscripts denote data within the slave and master planes respectively. There are \( N_r \) reference points in the overlap region, i.e. \( x_r^M \) or \( x_r^S \) where \( r = 1 \ldots N_r \) and \( x_r^M \) and \( x_r^S \) are coincident for any given \( r \). With reference to equation 3.36 the LSE reconstruction is now obtained as,

\[
\vec{u}^{S}_{est}(x, t^M) = \sum_{r=1}^{N_r} a_r(x)U^M(x_r^M, t^M) 
\]

(4.2)

Where \( x \) represents the point(s) in the slave frame being reconstructed and the coefficients \( a_r(x) \) are the solution of:

\[
\langle U^S(x, t^S) U^S(x_q^S, t_q^S) \rangle = \sum_{r=1}^{N_r} a_r(x) \langle U^S(x_r^S, t) U^S(x_q^S, t) \rangle \quad q = 1, \ldots, N_r
\]

(4.3)

The resultant slave velocity field can of course only contain those structures which give rise to finite correlations between points in the slave field and points in the master field (precisely, in the overlap zone with the master field). A perfect estimation (i.e. realisation of the \( \vec{u} \) term in equation 4.1) would therefore only be possible if the master and slave fields were overlapped completely. Crucially, the created \( \vec{u}^{S}_{est} \) velocity field is now synchronous with fluctuations in the master field, but
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has the correct level of spatial correlations, as far as it can, where there exists genuine correlations between points in the slave field and overlap region.

There is, of course, no perfect estimation, and much focus has been directed to the optimisation of the LSE procedure given that the overlap region will, in practice, only cover a small proportion of the slave field. This can be problematic as it is largely dependent on the flowfield in question and the quality of the raw PIV data. One key measure of the quality of the outcome of the procedure is the fraction of turbulent kinetic energy contained within the parent PIV field that is returned in the reconstructed field. A Reconstructed Energy Percentage (REP) can be defined as:

\[
\text{REP} = \frac{\sum_{x} \left( u_{i,\text{est}}^{S}(x, t^{M}) u_{i}^{S}(x, t^{M}) \right)_{\text{LSE}}}{\sum_{x} \left( u_{i}^{S}(x, t^{M}) u_{i}^{S}(x, t^{M}) \right)_{\text{PIV}}} \times 100 \quad i = 1 \ldots N_{e}
\]

(4.4)

where \( N_{e} \) is the number of velocity components reconstructed. This was found to be a useful differentiating factor for comparing reconstructed datasets which gave otherwise closely matched statistical profiles.

To eliminate any interpolation effects, initial testing was based on co-located master and slave measurement point locations, i.e. using exactly the same data set for the master and slave fields and reconstructing the velocities at the same locations as the original vector grid. The test data were chosen so as to require reconstruction of the large-scale vortical structures detailed in section 2.2.3. With reference to Fig 2.22 for example, field B was chosen to be the ‘slave’ field and reconstructed using data from its overlapping region with field D, but with the ‘master’ signals taken in this instance from field B itself, rather than field D. Given the overlap region is restricted, it is not possible to reconstruct the slave velocity field exactly, since some fluctuations in the frame will not be correlated with the overlapped region. The example ‘LES inlet plane’ onto which the velocity field was reconstructed was chosen to have the same dimensions as field B, with the grid lines intersecting at the PIV vector locations. Indeed, in later CFD predictions, the LES inlet plane grid was also chosen to have similar resolution to the PIV fields used here.
In these tests it was found that the number and arrangement of reference signals used was one of the major influencing factors in both maximising the REP value achieved and the computation time of the LSE reconstruction. The reference points in an overlap region can be arranged in an $N \times M$ array, where, with reference to Fig 4.1, $N$ is the number of columns (i.e. number of points in the direction normal to the field interface) and $M$ the number of rows (i.e. number of points tangential to the interface). Fig 4.2 shows the REP and the computation time for various arrangements of points in this initial (collocated master and slaves) test.

As expected, the relationship between number of reference points and calculation time is approximately linear. While an increase in the overall number of points produces an increase in reconstructed energy for any given arrangement, for $N \geq 3$ the REP is independent of $N$. Because 95\% of the possible reconstructed energy can be obtained using only 60\% of the available points (as seen from the solid and dashed lines in Fig 4.2a), around 100 points are generally used to reconstruct a given slave field. It is also prudent to maximise the number of points lying on the boundary edge (i.e. minimising $N$), and so optimal reconstruction is achieved with points arranged using values of $N=3$ and $M=M_{\text{max}}$. Such a trend can only be claimed for square or near-square PIV fields with highly-rectangular overlap regions (such as across fields in Fig 2.22), i.e. $M \gg N$. It is likely that deviations from this arrangement would significantly alter the optimum reference point choice.
Provided the respective cross-correlations are known, any signal may be used to reconstruct a velocity field. For example, tests were conducted using temporally offset signals and correlations, although no subsequent benefit was found. A small improvement was found by varying the master velocity component the field is conditioned by. Although the REP could be improved by around 2% by conditioning each slave component through its respective master signal, significant differences may be observed subsequently in the statistical profiles. The most reliable method is to condition each slave velocity component by the master velocity component containing the maximum TKE. However, caution should be exercised in weakly sheared flows, when there is little correlation between components as this method can have a detrimental effect on the SVC reconstruction.

It is also necessary to consider the practicalities of applying this technique to real data sets, i.e. when the two PIV fields and prescribed output grid are not collocated. This implies some sort of interpolation to obtain the desired reference signals and correlations, since the reference points from the master and slave planes are no longer guaranteed to be at exactly the same locations. Three methods were considered: ‘nearest point’ interpolation on the respective PIV grids; interpolation of the master reference point velocities onto the slave PIV grid locations and interpolation of the slave correlations to the reference points on the master grid. These are illustrated schematically in Fig 4.3.

![Fig 4.3 - Schematic of possible slave/master interpolations](image-url)

---

Slave  Master
For testing of non-collocated data, field B was again reconstructed as the slave plane, but now using field D as the master. Generally, energy fractions reconstructed on such grids were lower by approximately 10% in relation to the collocated case, regardless of the interpolation method used. Tests on collocated, but unsynchronised data sets showed this discrepancy was due to the use of separate fields rather than data from separate instances in time.

It was found the optimum method involved interpolation of slave correlations, with 49.4% of the original TKE reconstructed. However, it should be noted that interpolation to the nearest point is simpler in terms of coding and computation (calculation time may be cut by as much as 75%), and the resultant REP within one percent of the optimum value for this initial test case (though the performance was less comparable when lower PIV grid resolutions were used).

When the output grid is not collocated with either of the PIV grids, a further interpolation step is required, regardless of the method used above. This may be performed before reconstruction (by interpolation of the slave field correlations) or after reconstruction (by interpolation of the resultant velocity field). The latter showed an improvement in REP of the order of 5% in the LSE field, although this was not translated into closer statistical matching after the addition of the uncorrelated noise (see section 4.2). Interpolation after reconstruction is, however, more computationally expensive as it requires interpolation of $N_x \times N_y \times N_{samp}$ vectors, as opposed to $N_x \times N_y \times N_t$ correlations, which is generally at least a factor of 3 less. As such, interpolation of the correlations prior to reconstruction was used exclusively for non-collocated grids.

Care must be taken when interpolating the correlation fields, especially on PIV grids that are relatively coarse in comparison to the local lengthscale. The use of a bi-linear interpolation can result in large under or over estimation of the turbulent energy in certain locations. The use of a bi-cubic scheme, such as that described in Vetterling et al. (1988)[104] is recommended. Caution is also advised with regard to the quality of the initial PIV data sets. LSE is very sensitive to statistical differences between the master and slave fields in the overlap region, which can result in dramatic over- or under-estimations of the velocity in the overlapping region. This is especially
important since the reference points are taken from the edge of a field, where vector quality is often at its poorest. It is often wise to crop the raw PIV fields to remove the influence of bad vectors, in addition to the correction between neighbouring fields discussed in section 2.2.3.

A comparison of the LSE reconstruction with an original PIV field can be seen in Figs 4.4 to 4.11. The reconstruction is from the collocated and synchronised master/slave arrangement described above. The analysis techniques used here follow the procedures detailed in chapter 3.

A clear pattern emerges from the analysis of the reconstruction that is very encouraging near the reference point locations, but becomes progressively weaker as the correlations reduce further away from the overlap, where the reconstruction captures less of the overall turbulence. (Note the original PIV contains contributions to turbulence from both correlated and uncorrelated sources, the reconstructed field only addresses the former). This can be clearly seen in the TKE contours, where the two fields are well matched around \( z/H = 0 \), but diverge elsewhere. With the matching of the spatial correlation fields between the synthetic and real data being a major attraction of this technique (compared to Level 2, for instance), it is encouraging that a similar pattern is seen in the cross-correlation maps, with the reconstruction again matching very closely near to the centreline. Obviously, further away from the centreline, only progressively larger scale structures will attract a correlation hence the broadening of the peaks. Encouragingly, the vectors conditionally averaged at a point in the overlap show near-identical structures to that of the original PIV (see Fig 4.5). The peak broadening is clearly more pronounced in Fig 4.6, taken at a point that is weakly correlated with the overlap region, although the overall profile trends are still analogous. Given that a broadening of the correlation peaks will produce an increase in the integral lengthscale, it is unsurprising that both lengthscales in the LSE field are generally larger. It is perhaps surprising that the \( \text{L}_{11} \) profile is not closer in the region of the overlap, given the relative similarities of the correlation maps, but this could just be due to exaggeration from boundary curtailment effects as described in section 3.1.2.2.
Analysis of the spatial POD modes extracted from the unsteady data confirms the existence of much of the LSE reconstructed energy within the larger, coherent motions. It is clearly seen from Figs 4.9 and 4.10 that the first POD mode is almost identical, whereas the second, although similar, is starting to identify different structures. The plot of cumulative kinetic energy across the POD modes as a fraction of the PIV TKE, shown in Fig 4.11, is indicative of this. The vast majority of the LSE reconstructed energy is present in the first POD mode, which contains a similar amount to the PIV field. However, whereas the PIV energy is more evenly distributed across the higher modes in a manner typical of real flows, there is little energy stored in the higher modes of the LSE reconstruction.

Fig 4.4 – TKE contours, a) PIV, b) LSE; ‘overlap’ region

Fig 4.5 – \( w w \) cross-correlation field about \( z/H = -0.1, y/H = 0.63 \), a) PIV, b) LSE

Vectors conditional averaged from \( w > 1.5w' \); ‘overlap’ region
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Fig 4.6 – $ww$ cross-correlation field about $z/H = -0.84$, $y/H = 0.63$, a) PIV, b) LSE

Vectors conditional averaged from $w > 1.5 w'$; 'overlap' region

Fig 4.7 – $L_{11}$ integral lengthscale, a) PIV, b) LSE'; 'overlap' region

Fig 4.8 – $L_{22}$ integral lengthscale a) PIV b) LSE'; 'overlap' region
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Fig 4.9 – 1st POD mode of w-velocity, a) PIV, b) LSE; □ ‘overlap’ region

Fig 4.10 – 2nd POD mode of w-velocity a) PIV b) LSE; □ ‘overlap’ region

Fig 4.11 – Cumulative energy across first 20 POD modes
Finally, we examine the case of a typical fluctuation field as reconstructed by LSE (Fig 4.12). The medium scale vortex near the overlap region is well reconstructed (even though it is only intermittent in the field history). However, the larger scale structures in the centre of the field obviously do not display correlation with the reference signal region and are thus not reconstructed. It is the non-presence of these types of structure that contribute to the over-estimation of the lengthscales in Fig 4.7.

It is thus clear the deficiencies in the LSE technique lie in the reconstruction of those fluctuations that do not correlate with the overlapping region. In order to gain an improved synthetic realisation of the original PIV flow field it is necessary to find a supplementary method that can add these scales back into the reconstructed synchronous field.

![Fig 4.12 - Typical fluctuation field, a) PIV b) LSE from 96 reference points](image)

**4.2 Uncorrelated Structures Reconstruction**

Several methods of supplementing the LSE reconstructed data through addition of a secondary velocity field were evaluated. The purpose of this is to improve the match to the statistics of the parent PIV field without destroying the synchronicity derived from the LSE. Evaluation is thus considered based on the superimposed correlated and uncorrelated fields, rather than in isolation.

As observed in section 4.1, the correlations contributing to the LSE reconstruction are heavily influenced by the large-scale, energy-containing flow structures present.
Therefore the PIV energy spectrum is adequately reconstructed in the low frequency region. Ideally, a supplementary method is required to match the missing energy proportion by introduction of fluctuations in the higher frequency region.

Several methods of adding uncorrelated energy were evaluated, with the results discussed below. The simplest method was the addition of white noise. This was introduced by scaling a normally-distributed random field to a variance such that at each location it accounted for the unresolved energy fraction.

A second method was based on a POD reconstruction. It was detailed in section 3.1.3.2 how POD can be used to analyse a velocity field identifying a hierarchy of structures ordered in terms of decreasing contribution to the overall energy. From this, a velocity field can be reconstructed containing only the most energetic structures present by using, for example, only the first N modes. A supplementation method was therefore investigated whereby a reconstruction of the last N modes may be first generated from an overall POD analysis, containing the lesser energetic structures. This is to be added to the LSE reconstructed (correlated) unsteady field. Data from the original (unsynchronised) slave PIV field is used as the basis of the reconstruction. Since only modes containing structures uncorrelated with the master plane are to be extricated, the correlations from the LSE should not be adversely affected when these are added back in to the LSE reconstructed fields described in the previous section. Thus, equation 3.34 becomes,

$$u'(x,t) = \sum_{k=N_{\text{on}}(x)}^{N_{\text{amp}}} a_k(t)p_k(x)$$

(4.5)

The choice of the cut-on mode, \(N_{\text{on}}\), may vary from point to point across the reconstructed field. The value is chosen such that the sum of the energies in the POD modes \(N_{\text{on}}\) to \(N_{\text{amp}}\) is equal to the unresolved energy fraction in the LSE field at that point. However, as \(N_{\text{on}}\) must be an integer, only a discrete range of energy fractions can be realised. In reality the energy that is required to be supplemented could take any value. For this reason, a weighted average has been taken of velocity fields resulting from two reconstructions in which \(N_{\text{on}}\) has been rounded up and down to the

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integers nearest to the required value, i.e. adjacent modes which, when used as the cut-on mode, result in energies above and below the required energy fraction.

Finally, a method was trialled in which the original slave PIV field was high-pass filtered at a cut-on frequency defined such that the resultant energy matched the local unresolved energy fraction in the LSE field. As with the previous method this should remove the structures that were correlated with the master field. It is not the intention of such a method to replicate the difference between the PIV and LSE spectra for each frequency, however. Doing this was found to introduce large-scale, uncorrelated fluctuations that act to destroy the inherent correlations of the LSE. As such a 251 term high-pass sharp Fourier filter with a Hann window is required (i.e. using equation 3.24 and 3.25, where \( N_{\text{terms}} \) is 251) to allow optimal replication of the low frequency region. Fig 4.13 shows a plot of the generic power spectra as observed at a typical point in a reconstructed field after supplementation of the filtered PIV. Note that the filtered PIV does not match the unresolved energy at a given frequency, but uses the maximal amount of high frequency fluctuations to match the integral of the unresolved scales.

![Figure 4.13 - Generic power spectra](image)

A comparison of these methods, along with the original PIV and LSE data, is shown in Figs 4.15 to 4.20. The plots show profiles of various quantities across the lines \( y/H = 0.41, 0.63, \) and 0.85 shown in Fig 4.14, i.e. evenly distributed below, across and
above the CRVP observed in the experiment. Data is reconstructed across PIV field B, with field B as the slave and field D as the master.

Fig 4.14 - Field B analysis slice locations (shown with time-averaged vectors of the original PIV data)

From Fig 4.15 it is seen that all supplementary methods allow TKE profiles of the PIV data to be replicated successfully. Similarly, the \( \langle wv \rangle \) Reynolds stress statistic is well reproduced. However, other statistics more dependent on spatial correlations show significant differences. The addition of scaled noise appears to over-reduce the lengthscales consistently. This is not surprising since the noise added lacks spatial correlation and therefore has no inherent lengthscale. The much-flattened line in Fig 4.20 indicates this is due to the partial destruction of the large-scale structures reconstructed from by the LSE.

Generally, the addition of lower-energy POD modes and filtered PIV both produce a very good replication of PIV statistics. This is especially true of the spectra and POD energy distributions, which give confidence that the spatial structures within the flow are well replicated. The filtered PIV in particular shows very consistent replication.
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Fig 4.15 – Turbulent Kinetic Energy

Fig 4.16 – \langle wv \rangle Reynolds stress
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Fig 4.17 – $L_{11}$ integral lengths

Fig 4.18 – $L_{22}$ integral lengths
Although statistically there is little to choose between the POD and filtered PIV methods, concern was raised about the flexibility of the POD method. It was found that, since the first two POD modes may contain up to 60% of the local TKE, there were many regions in the flow where the cut-on mode, N_{on}, is less than 2 (i.e. first
mode data is contributing to the uncorrelated field). Clearly, unless the LSE reconstruction is exceptionally good, this implies fluctuations correlated with the master plane are being introduced on top of the LSE. There is no such problem when using the filtered PIV method since the cut-on frequency may be chosen to take any value between zero (if LSE has reconstructed no energy, hence the slave field will be identical to the original PIV) and infinity (if the LSE has reconstructed all the energy, hence the slave plane is identical to the LSE field). The filtered PIV approach will thus be used as the supplementary method to reconstruct the uncorrelated field for the remainder of this thesis. For convenience, the supplemented LSE technique will henceforth be referred to as sLSE.

It is worth at this point reconsidering the example fluctuation field presented in Fig 4.12. Fig 4.21 shows the original fluctuation field (from PIV), a synthetic field (generated from LSE alone) and a combined synthetic field from correlated and uncorrelated portions (via sLSE). The uncorrelated field provides additional structures that do not significantly alter the correlated field but do not exactly match those missing from the PIV field either. Thus, sLSE can never reconstruct an identical flowfield to that of the original PIV since the correlations that caused those fluctuations are lost. It can, however, reconstruct an equivalent flowfield with the same range of structures distributed across the velocity field. The relative quality of this reconstruction with regard to existing methods will be discussed in the following chapter.
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Fig 4.21 – Typical fluctuation field, a) $u$ from PIV, b) $\bar{u}$ from LSE, c) $\bar{u} + u'$ from sLSE

4.3 Unresolved Sub-grid Energy

The previous two sections have detailed an effective technique for reconstructing the flow field captured in one or more PIV fields and adapting them for use as an LES inlet condition. It is however important to recall that, notwithstanding the inherent experimental errors of the PIV technique, detailed in section 1.3, the discretisation of the PIV vector field will inevitably lead to a degree of turbulent energy loss. While the uncertainties arising from this will be quantified in section 5.1, it is important to consider here how this ‘missing’ part of the turbulence field will affect the overall reconstruction.

The effect of sub-cell filtering is tangible and can be quantified using the HMN method through equation 3.18. By simple manipulation of this equation, the amount of TKE that is unresolved for any given location in a PIV flow field is obtained.
Assuming that an inlet generation technique completely recreates the PIV energy \( E_{\text{PIV}} = E_{\text{IG}} \) (which from Fig 4.15 is a valid assumption), it is thus possible to state that the total energy, \( E_{\text{tot}} \), is given by:

\[
E_{\text{tot}} = E_{\text{IG}} \times f\left(\frac{\Delta X_{\text{PIV}}}{L_{\text{true}}}ight)
\]  

(4.6)

The strong analogy with LES of the sub-cell filtering has already been noted. Hence it is reasonable to assume a near identical relationship to equation 4.6 for an LES geometry, such that:

\[
E_{\text{tot}} = E_{\text{LES}} \times f\left(\frac{\tilde{\Lambda}_{\text{LES}}}{L_{\text{true}}}ight)
\]  

(4.7)

where the \( E_{\text{LES}} \) is the TKE resolved by LES, and \( \tilde{\Lambda}_{\text{LES}} \) is the LES filter width (exclusively set to the implicit filter width of \( \sqrt[3]{\Delta x \Delta y \Delta z} \) during this project).

Assuming that a given reconstruction can replicate the correct lengthscale at a given location, the following link between the energies required and reconstructed is obtained:

\[
E_{\text{LES}} = E_{\text{IG}} \times \frac{f\left(\frac{\Delta X_{\text{PIV}}}{L_{\text{true}}}ight)}{f\left(\frac{\tilde{\Lambda}_{\text{LES}}}{L_{\text{true}}}ight)}
\]  

(4.8)

It is clear that as \( \Delta X_{\text{PIV}} \to \tilde{\Lambda}_{\text{LES}} \) the level of the PIV sub-cell filtering approaches that which is accounted for by the LES sub-grid scale model. Thus the case where a PIV velocity field contains insufficient energy to replicate the full range of resolved LES scales is when the PIV interrogation cell size is greater than that of the LES filter width, i.e. \( E_{\text{LES}}/E_{\text{IG}} > 1 \). Re-arranging equation 4.8, \( E_{\text{LES}}/E_{\text{IG}} \) may be defined as a ‘correction factor’, which is the amount the local reconstructed TKE must be factored by to contain sufficient energy to match that resolved by LES. It will be seen in section 5.1.2 that this correction factor is close to or below one throughout the annuli inlets of the idealised combustor considered in this thesis. This gives confidence that the effects of sub-grid filtering will be negligible, and thus no further correction methods are applied. However, this is not the case for the vortex generator and will
not necessarily be the case for other geometries. No method is used within this thesis to account for this unresolved energy fraction and, as such, the development of a correction methodology, either by \textit{a priori} correction of the PIV data or the addition of a further supplementary velocity field, could be the subject of further research.
Chapter 5

Validation and Assessment of New and Existing Inlet Condition Generation Methods
Chapter 3 presented several existing methods by which PIV data can be used to generate unsteady inlet conditions for LES. Chapter 4 has proposed a new method by which instantaneous PIV data can be manipulated in order to generate a higher fidelity, spatially correlated inlet velocity field.

This chapter aims to validate this process of generating PIV-derived inlet fields by comparison of existing and developed methods. In order to gain context for this comparison it is first necessary to gain an appreciation of the inherent errors in the PIV process that may subsequently have been passed through to influence inlet condition generation.

5.1 PIV Error Analysis

It has been discussed in section 1.3 how the final accuracy of the PIV measurement process is sensitive to the experimental setup, flow field and vector computation parameters. Chapter 2 has outlined how these aspects have been accounted for (and where possible the errors minimised) for the PIV data taken during the present project. However, even for an ideal optimisation of measurement set-up parameters, any given PIV system will still give rise to some errors that largely arise due to the discretisation from a continuum field to samples in time and space. The following potential sources of error have been identified for PIV data:

- Sub-pixel particle displacement accuracy
- Sub-cell filtering
- Dynamic averaging
- Statistical convergence

Each of these error sources will be considered and assessed for the current PIV data before its use as the input to inlet condition generation methods.

5.1.1 Sub-pixel Particle Displacement Accuracy

It has been mentioned in section 1.3.2 how there is an unavoidable error of up to 0.1 pixels in the location of the correlation peak defining the particle displacement. This leads to an uncertainty in the velocity measurement. Since the particle displacement
is limited in the data taken here to 8 pixels, this implies an uncertainty of at least ±1.25% is present in each vector computation.

For a stereoscopic set-up, there is the slight complication that the physical particle displacement is not that observed by the camera. This is due to the oblique viewing angle (see Fig 5.1):

![Diagram of stereoscopic particle displacement viewing uncertainty](image)

Fig 5.1 – Stereoscopic particle displacement viewing uncertainty

- Particle position at \( t = t_0 \)
- Particle position at \( t = t_0 + \Delta t \)
- Displacement uncertainty

where the \( c \) subscript denotes the plane of the camera and \( \varepsilon_{Ax} \) etc. are the uncertainties in the measurement of the respective particle shift. Thus, as the stereoscopic vector computation is obtained from the following simultaneous equations:

\[
\begin{pmatrix}
\cos\alpha_1 & \sin\alpha_1 \\
\cos\alpha_2 & \sin\alpha_2
\end{pmatrix}
\begin{pmatrix}
U_z \\
U_x
\end{pmatrix}
=
\begin{pmatrix}
U_{x,1} \\
U_{x,2}
\end{pmatrix}
\] (5.1)

it follows that the in-plane uncertainty in the correlation peak may be obtained by the same transformation. For an idealised stereoscopic experimental set-up with both cameras coplanar with the FoV and viewing at \( \alpha_1 = \alpha_2 = 45^\circ \) (which matches closely to the experimental set-up used in this project, as detailed in chapter 2), a correlation error of 0.1 pixels translates to a displacement error equivalent to 0.0707 pixels. This reduction obviously does not occur in the vertical (\( y \)) component.
Since the idealised combustor data to be presented below used a constant inter-frame time of 460μs and a near-constant pixel size of approximately 0.06mm, it follows there will be constant uncertainty of ±0.0092m/s in the x- and z-velocities and ±0.013m/s in the y-velocity across each annulus, equivalent to ±1.65% and ±2.34% of $U_{\text{ref}}(0.558m/s)$ respectively.

For the vortex generator test case ($\Delta t = 11\mu s$, $d_{\text{pix}} = 0.065$) the uncertainties are ±0.418m/s in the x- and z-velocities and ±0.591m/s in the y-velocity, i.e. ±1.43% and ±2.03% of $U_{\text{ref}}(29.2m/s)$.

This uncertainty has zero mean and thus time-averaged values will not be affected beyond the convergence limits detailed below in section 5.1.4, however, it is present in all instantaneous data and will have the effect of increasing the RMS by an amount equivalent to a displacement of 0.1 pixels. As non-uniform movement of particles within an interrogation cell causes the uncertainty, it is closely related to sub-cell filtering but will, to some extent, counteract its effects. A flow region with little sub-cell variation will produce a near-Gaussian correlation curve with very accurate sub-pixel accuracy. Thus, the figures quoted here represent a maximum error associated with this phenomenon: it may be expected that these uncertainties will only be approached in regions of high-intensity, small scale fluctuations.

### 5.1.2 Sub-cell Filtering

Section 3.1.2.3 has discussed how the HMN method can be used to provide a correction for the low-pass filtering effects of sub-cell filtering/averaging. As with the sub-pixel location uncertainty, these errors will not appear in the mean flow, but may have a significant effect on the turbulence statistics.

Section 4.3 presented a correction factor that may be used to gain an appreciation of the relative levels of sub-cell/sub-grid filtering, present in PIV and LES flow fields, defined as the ratio of the TKE resolved on the LES and PIV grids ($E_{\text{LES}}/E_{\text{PIV}}$). Fig 5.2 shows this correction factor across fields A-E of the idealised combustor upper and lower annuli and the vortex generator, calculated using equation 4.8. The LES grids used are described in chapter 6.
Fig 5.2 – Sub-grid correction factor, $E_{LES}/E_{IG}$, a) Idealised combustor, upper annulus, b) idealised combustor, lower annulus, c) vortex generator

Clearly, the correction factor across the idealised combustor inlet planes is close to or below 1 over the majority of the inlet region. There are some regions of energetically under-resolved flow far from the upper annulus burner arm wake (where the local TKE is particularly low) and in the lower annulus, though the energy deficit is rarely more than 10% of the measured TKE. This gives confidence that the effects of sub-cell filtering for this geometry will be negligible.

For the vortex generator test case, there appears to be a consistent energy deficit of between 10 and 20% within the measured turbulence field, though this may rise to 50% in regions of high turbulence. Note this infers larger errors in the airflow (vortex generator) experiment compared to the water flow (idealised combustor) experiment.
This evidence provides strong support for the choice of a water flow experiment as the primary test case in this investigation. As mentioned in section 4.3, no method has been used to account for this unresolved energy fraction and thus it is important to bear in mind that the resolved velocity field at the inlet plane of a resultant LES will not contain an energy spectrum in equilibrium.

Were the measured velocity fields to be used for DNS calculations, it would be necessary to resolve all of the flow scales (i.e. have no sub-cell filtering present). A method of quantifying the absolute levels of sub-cell filtering is therefore included here for the purposes of future comparisons and validation.

As stated previously, the Kolmogorov microscales can be used to gain an appreciation of the level of spatial and temporal filtering present in the captured PIV data. Fig 5.3 shows the distribution of the Kolmogorov lengthscale over the LES inlet condition planes measured in the two annuli of the idealised combustor, calculated using equation 3.15. The values shown match closely with those suggested by Hollis. Given the frame dimensions and interrogation cell size used, the smallest resolved length scale in the PIV data will be of order 2mm. This is at least an order of magnitude greater than the microscale for much of the region, and suggests a moderate amount of sub-cell filtering will be present.

Applying the HMN corrections to these fields, the under-prediction of the measured RMS magnitude in comparison to the true RMS, $\Delta u'$, is obtained. This is shown as a fraction of the reference velocity in Fig 5.4. Generally, the trend is as indicated by the Kolmogorov scales, with a low-level of filtering (generally <2%) for much of the domain, rising in the regions of higher turbulence (around 3% in the region of the CRVP). As the HMN is dependent on the derived integral lengthscale it is unsurprising that inconsistent values are obtained around the boundaries of some fields, where lengthscale computation can be problematical.
Fig 5.3 – Kolmogorov length scale (m), a) upper annulus, b) lower annulus

Fig 5.4 – RMS under-prediction due to sub-cell filtering, a) upper annulus, b) lower annulus

Fig 5.5 shows the Kolmogorov lengthscale across the vortex generator inlet plane. Given an interrogation cell size of the order of 2mm in the measurements made across this plane, the Kolmogorov lengthscale is around two orders of magnitude smaller than the resolved scales, suggesting sub-grid filtering may have a significant effect on these data sets. This is shown to be the case in Fig 5.6, where large values and variations in the levels of RMS under-prediction are observed. As expected for an
airflow case the values are generally higher than for water flow, with an under-prediction of around 3% for much of the domain, but this rises to as much as 30% in the vicinity of the vortex impingement.

Fig 5.5 – Kolmogorov length scale, vortex generator channel at \( x/H = 3 \)

Fig 5.6 – RMS under-prediction due to sub-cell filtering, vortex generator channel at \( x/H = 3 \)

5.1.3 Dynamic Averaging

It was explained in section 1.3.2 how dynamic averaging may affect the final RMS of a data set if the inter-frame time used for data capture is greater than the Kolmogorov timescale.

The Kolmogorov timescale, as calculated from equation 3.17, is shown in Fig 5.7 for the idealised combustor. Given that an inter-frame time of 460\( \mu \)s was used in the capture of all 1kHz data sets, this suggests the effects of dynamic averaging within the data will be minimal.
Similarly, Fig 5.8 shows the Kolmogorov timescale across the vortex generator inlet. Given an inter-frame time of 11μs was used for capture of these data sets, some dynamic averaging may occur in the centre of the vortices. For this experiment, the Shell Ondina oil used as a seeding particulate could potentially introduce a further averaging effect given its density is larger than that of the fluid medium (air) and thus will have a slower response time. However, Hollis\textsuperscript{[32]} shows that the response time for such a particle is approximately 10μs, which is less than the inter-frame time, and thus should not influence results.

![Fig 5.7 - Kolmogorov time scale, a) upper annulus inlet, b) lower annulus inlet](image)

![Fig 5.8 - Kolmogorov time scale, vortex generator channel at \(x/H = 3\)](image)
5.1.4 Statistical Convergence

As with any statistical parameter, the accuracy of flow information extracted from PIV is dependent on the number of independent samples from which the statistic is calculated. Inevitably, the use of more samples gives a better representation of the population as a whole and thus increases the fidelity to which the captured PIV data represents the source flow field.

It has already been shown that, in order to capture as much of the turbulence field as possible, data should be generated using the highest sample rate available. However, if we use the common definition of a statistically-independent sample (as used by Westerweel et al.\textsuperscript{[10]}, and Weisgraber & Liepmann\textsuperscript{[106]}, among others), it is the number of samples separated by at least one integral timescale that defines the number of statistical samples within a data set. Thus a high sample rate may be detrimental to the resultant statistical accuracy.

The convergence of time-averaged first and second order statistics has been shown (Hollis\textsuperscript{[32]}) to be well represented by normalised standard error estimates, such as presented by Montgomery & Runger\textsuperscript{[62]},

\begin{equation}
\varepsilon_{(u)} = \frac{z u'}{U_{ref}} \sqrt{\frac{1}{N_{1-samp}}} \tag{5.2}
\end{equation}

\begin{equation}
\varepsilon_{u'} = \frac{z u'}{U_{ref}} \sqrt{\frac{1}{2N_{1-samp}}} \tag{5.3}
\end{equation}

\begin{equation}
\varepsilon_{(uu)} = \frac{z u'^2}{U_{ref}^2} \sqrt{\frac{2}{N_{1-samp}}} \tag{5.4}
\end{equation}

where \( z \) relates to the confidence band. If the error is assumed to be normally distributed then \( z = 2.576 \) for a 99\% confidence band. \( N_{1-samp} \) is the number of statistically independent samples used. \( u' \) represents the true RMS value, although this is adequately represented by using a value calculated from all available samples.
However, for use with high-speed PIV, the definition of confidence limits is clouded by the uncertainty in the true number of independent samples, not only due to the spatial variation of timescales across the domain but because each vector is itself an average of a number of particle displacements. The latter is the reason why the standard error limits reported for low sample rate (i.e. statistically independent) PIV data sets are often pessimistic when compared with the sampled statistics (such as in Hollis et al.\textsuperscript{[33]}).

To quantify the convergence error in this project we have adapted the standard estimate definitions to be based on the \textit{effective} number of independent samples, \(N_{E\text{-samp}}\), i.e. the number of samples that result in an appropriate portion of statistic blocks falling within the limits of a given confidence band. An example of this procedure is shown for mean and RMS velocities in Fig 5.9 using a range of block sizes for a 99\% confidence band. The sample is taken from a 50Hz data set within the CRVP of the idealised combustor.

![Fig 5.9 - Convergence of PIV sampling error, a) Mean b) RMS](image)

\[
N_{\text{amp}} = \triangle 5, \square 10, \blacktriangle 20, \blacktriangle 50, \times 100, + 200, \diamond 500, \bigcirc 1000,
\]

- standard error,  
- effective error

The error analysis in Fig 5.9 suggests that the 1024 sample data set effectively contains 243 independent samples. This value appears consistent with observation, with a timescale of 0.22s in this region indicating approximately 93 independent samples and sub-cell averaging of the (approximately) 5 particles per interrogation cell accounting for the additional factor of 2.6. Thus, re-applying equations 5.2 and
5.3, we may assume for a 99% confidence band that the mean results are within ±2.64% and the RMS within ±1.86% of their true values.

Table 5.1 shows the convergence errors as calculated by this procedure averaged across the inlet regions of both test geometries for each velocity component. Each data set contains 3072 samples.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>x-component</th>
<th>y-component</th>
<th>z-component</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N_{E,samp}</td>
<td>\varepsilon_{t,x}</td>
<td>\varepsilon_{a,x}</td>
</tr>
<tr>
<td>Idealised</td>
<td>1250</td>
<td>±0.45</td>
<td>±0.32</td>
</tr>
<tr>
<td>Combus tor, Upper annulus</td>
<td>1926</td>
<td>±0.15</td>
<td>±0.11</td>
</tr>
<tr>
<td>Idealised</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Combus tor, Lower annulus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vortex</td>
<td>891</td>
<td>±0.65</td>
<td>±0.46</td>
</tr>
</tbody>
</table>

Table 5.1 – Inlet geometry convergence errors (all errors shown as % of U_{ref})

It is apparent that generally the averaged convergence errors are below 1%. The errors are seen to be greater in regions where high turbulence (such as the CRVP in each geometry) is observed. Thus, caution is required when considering the error at a given point: in regions of high TKE the errors may be significantly greater than that quoted. For example, in the CRVP of the idealised combustor the maximum error is ±8.95%.

### 5.1.4 Error Summary

The previous sections show that the only significant errors inherent in the PIV data are those arising from sub-pixel particle location accuracy and statistical convergence. The level of sub-cell filtering errors, although shown to be significant in the vortex generator test case, are a function of the LES grid used and are thus not included here.

Hollis[^32] provides a Pythagorean method by which these errors may be combined:
\[
\frac{\varepsilon_i}{U_{\text{ref}}} = \sqrt{\left(\frac{\varepsilon_{Ax_y}}{\Delta x_{\text{ref}}}\right)^2 + \left(\frac{\varepsilon_{u_y}}{U_{\text{ref}}}\right)^2}
\]  

(5.5)

Extracting the maximum of this error across the inlet flow fields yields the results shown in Figs 5.10 and 5.11. The regions of large error are seen to coincide very closely with the regions of high RMS (and thus TKE) in Figs 2.24 and 2.34. When considering the fidelity of the inlet conditions generated by the various methodologies it is important to consider that all PIV data is subject to an inherent uncertainty of at least 2\%, rising to as much as 6\% in regions of high turbulence.

![Fig 5.10 - Maximum inherent PIV error, a) upper annulus, b) lower annulus inlet](image)

![Fig 5.11 - Maximum inherent PIV error, vortex generator channel at x/H = 3](image)
5.2 Inlet Generation Method Comparison

A number of strategies for inlet generation have been presented in the previous chapters. Fig 3.24 showed these can be categorised into a "Hierarchy of Fidelity" according to the extent to which the experimental data is used and the resultant assumptions that have to be made. In order to compare current and developed techniques, we return to this hierarchy by selecting one methodology from each category to be used for comparison. In all cases the generated fluctuations are superimposed upon an experimentally obtained time-averaged profile. For the purposes of these comparisons, inlet conditions are generated to match the time step of the source PIV data.

Level 1 is represented by superimposing Gaussian white noise with a variance scaled such that it matches the experimental TKE. Since there is little extra computational cost, and the experimental data is available, the TKE level will be spatially variant across the inlet region. Note that this is an improvement on many of the studies discussed in section 3.2, which use only constant or one-dimensionally varying TKE levels.

DFG represents inlet conditions of Level 2 type. It has previously been stated that PIV is more conducive to producing the required statistics for DFG than the alternative, spectral forcing. Modification is made to allow for two-dimensional spatial variation of the Reynolds stress tensor (i.e. the Reynolds stresses, and thus the coefficient $a$, are spatially variant in equation 3.46). The computational time-saving methods of Veloudis et al.\textsuperscript{102} are incorporated, i.e. an optimisation of the DFG time-step and the use of FFT for filter convolution. However, the time step used was generally found to be adequate to resolve the time scales within the given tolerance.

The supplemented LSE approach (sLSE), as detailed in the previous chapter, is used to represent Level 3 inlet conditions.

As stated in section 3.2, Level 4 inlet conditions are not yet feasible given current experimental technologies.
All methodologies will be evaluated using the PIV data as standard. The stated aim of any synthetic inlet generation procedure must be to replicate as exactly as possible the flow field at the inlet to the computational geometry, and to do so as efficiently as possible with the minimum of computational and experimental time and effort.

Here we consider the two criteria separately: section 5.2.1 will consider the velocity field generated by each methodology; section 5.2.2 will quantify the cost of generating each method.

5.2.1 Velocity Field Comparison

For the evaluation of the velocity field all inlet conditions are generated over a plane taken as an example LES inlet, covering the region \(-1 \leq z/H \leq 1\) and \(0 \leq y/H \leq 1\) of the upper annulus of the idealised combustor. Source PIV data is taken from fields B and D (Fig 2.22), thus encompassing the CRVP in the burner arm wake. The LES grid is taken to be regular and collocated, comprising of \(125 \times 65\) points. This is approximately the same density as the source PIV grid, largely decoupling issues of grid interpolation from the inlet comparisons. 50Hz data is used to ensure the optimum statistical matching between the fields.

For the purpose of the Level 3 reconstruction, field D is chosen as the master field. This is also reflected in the field hierarchy of the other methods as data in the overlapping region is taken from this field.

Where appropriate for multi-method comparisons, profiles are shown along the lines \(y/H = 0.41, 0.63,\) and \(0.85,\) i.e. evenly distributed below, across and above the CRVP. The PIV fields, LES inlet and profile slice arrangement are shown in Fig 5.12.
Fig 5.12 – Fields B and D slice arrangement, example LES inlet (shown with time-averaged vectors of the original PIV data)

Fig 5.13 shows the distribution of TKE for each method. As all methods are ultimately scaled to match the PIV TKE it is unsurprising that they each replicate the profile well. The step change difference between the two fields is of the order 10% of $U_{ref}$. This originates from experimental errors discussed in chapters 1 and 2 and the inherent PIV errors discussed in section 5.1. This difference is exacerbated due to the poor statistical match between frames B and D mentioned in chapter 2.2.3. When inlet conditions are generated for the final LES calculation, this difference will be reduced given the improved match between frames B, C and D.

As observed previously, the Level 3 inlet replicates well the Reynolds shear stress shown in Fig 5.14. Also as expected, the Reynolds stress transformation of the Level 2 DFG process implicitly ensures the profile matches very closely to the PIV. However, as no mechanism exists to recreate a correlation in the Level 1 inlet, no shear stress variation is observed, underlining the major limitations of this method.

The spatial velocity correlations are shown in Fig 5.15. As these are, to some extent, used to define the different levels of fidelity, they are indicative of the methods as a whole. As with the shear stress, the lack of specified correlation in a Level 1 inlet condition leads to no SVC between any two points as is evident in Fig 5.15a. The Gaussian correlation assumption of the Level 2 inlet is unmistakeable, but is clearly a poor representation of the experimental distribution (Fig 5.15d). The Level 3 inlet gives a good representation of the measured distribution over much of the domain,
with some noise observed in the lower correlation regions where the effect of the LSE is less marked. Crucially, the correlation is continuous across the interface.

For completeness, Fig 5.16 shows the effect of interpolating non-synchronous PIV fields directly on to the LES grid, as a ‘pseudo-Level 4’ approach. The sharp discontinuity in the correlation is evidence of why this is not a feasible inlet condition generation approach. For a true Level 4 inlet this would not be the case though because, as stated previously, this would require synchronous capture of the entire inlet region in either one or more PIV fields. Some weak correlation is observed with the neighbouring field, although this is likely to be a consequence of the periodicity of the flow in this case.

From Fig 5.17, the constant lengthscale assumption of the Level 2 inlet appears reasonable for the $L_{11}$ lengthscale (although not in the region of the CRVP where the measured lengthscale varies by ±50%). However, for the rapidly varying $L_{22}$ lengthscale (Fig 5.18) the assumption is less acceptable. The spatially-variant velocity correlation used in the generation of the Level 3 inlet results in a close replication of the lengthscale profiles. It should be noted that the $L_{11}$ lengthscale calculated from the PIV data is affected by the curtailment (and subsequent discontinuity) of the velocity correlations at the field edges. As such the profile of the Level 3 inlet (which has continuous correlation across the overlap) may be a closer representation of the experimental field. The Level 1 velocity field is uncorrelated and thus shows no appreciable lengthscale.

Analysis of the spatial POD modes gives some indication of the turbulent structures contributing to the derived statistics. As mentioned previously, due to the optimum nature of the decomposition, caution must be exercised when comparing modes of adjacent fields as the spatial modes are not necessarily continuous even for statistically similar data sets. A structure that is isolated to one field will be of proportionally higher energy in that field than the inlet as a whole and thus the distribution of energies of the PIV fields and generated inlet conditions should not be expected to be identical. However, as in this case the lower modes will be approximately symmetrical about $z/H = 0$, they may be used as a basis for evaluation.
The Level 3 inlet gives a very close representation of the first POD mode in Fig 5.19. However, even by the second POD mode (Fig 5.20) the discontinuity between PIV fields is evident, though the Level 3 profile appears to follow the structure of frame D. The fact the Level 2 inlet gives a reasonably good approximation to the POD modes (and the second mode in particular) appears surprising given the strength of approximations used. However, analysis of the vectors of the spatial modes as a whole (Fig 5.21, for the 1st mode) shows the Level 2 inlet does not contain the vortical structures frequently observed in the lower modes of the experimental data. Contrastingly, the Level 3 inlet contains a very close representation of the PIV mode shape, previously presented in Fig 3.15. As the Level 1 inlet does not contain any local flow coherence, it is inevitable the resultant POD modes show no discernable structures.

The POD mode energy distribution, wavenumber and power spectra (Figs 5.22-5.24) should perhaps be considered together in order to appreciate how these structures are distributed throughout the flow. It should be noted the power spectra in Fig 5.24 were generated using 1kHz data in order to capture a wider bandwidth of frequencies.

The Level 2 inlet shows markedly, though consistently, different distributions to the PIV fields. It becomes apparent from all analyses that much of the turbulent energy is contained in the medium to large scales of flow (roughly corresponding to the ‘energy containing’ scales of Pope (2000)[73]), thus leading to non-physical attenuation of the signal in the high and very low frequency regions.

While the Level 3 inlet shows a similar distribution of energies across the flow scales to the PIV, especially in wavenumber space, analysis of the power spectra in Fig 5.24 demonstrates the limitations of the assumptions used to apply the decomposition of equation 4.1. While the spectrum in Fig 5.24a is taken in the overlap region and thus the Level 3 spectrum matches closely that of the PIV, Fig 5.22b show an attenuation of the low frequencies and amplification in the high frequencies. This is caused by the presence of large-scale uncorrelated structures in the PIV slave frames (such as those noted in Fig 4.21), which are not replicated by the high-pass filtering.
It should be noted that, although the results discussed above have all been taken from a waterflow test case, very similar trends were observed for data captured in the vortex generator test case in airflow. Capturing data in airflow inevitably leads to curtailment of the power spectra for a given data sampling rate. Contrasting Fig 5.25 with the well resolved spectra in Fig 5.24, this is obviously the case. Despite this, the trends discussed above are repeated, with the Level 3 methods offering a close representation of the experimental profile for both PSD and wavenumber spectra.

In summary, the Level 2 and 3 methods replicate the first and second order statistics well, though the Level 2 approach does not replicate physically representative turbulent structures. Its use should therefore be limited to flows corresponding closely to its assumptions (i.e. constant length scale; no dominant structures). In contrast, the Level 3 approach can reproduce these structures very well provided they are correlated with the master field. Thus the allocation of master and slave fields should only be taken after careful consideration of the flow field. The Level 1 method should only be used as a last resort.

On the basis of the evidence presented here, it is considered that the Level 3 approach proposed in the present thesis, whilst not perfect, is an improvement on evaluated inlet condition generation methods and has been adequately validated in the tests described above to warrant future investigations and applications as a synthetic inlet condition generation methodology.
Fig 5.13 – Reconstructed profiles of Turbulent Kinetic Energy across the inlet plane

Fig 5.14 – Reconstructed profiles of \( \langle vw \rangle \) Reynolds stress across the inlet plane
Fig 5.15 – Reconstructed spatial velocity correlations, $R_{11}$, about $z/H = 0, y/H = 0.63$ across the inlet plane, a) Level 1, b) Level 2 c) Level 3 d) Original PIV

Fig 5.16 – Reconstructed spatial velocity correlations, $R_{11}$, about $z/H = 0, y/H = 0.63$ across the inlet plane by direct (non-synchronous) interpolation
Fig 5.17 – Reconstructed profiles of $^1L_{11}$ integral lengthscales across the inlet plane

Fig 5.18 – Reconstructed profiles of $^2L_{22}$ integral lengthscales across the inlet plane
Fig 5.19 – Reconstructed profiles of 1st spatial POD modes, $\varphi_1$, of w-velocity across the inlet plane

Fig 5.20 – Reconstructed profiles of 2nd spatial POD mode, $\varphi_2$, of w-velocity across the inlet plane
Fig 5.21 – Reconstructed 1\textsuperscript{st} spatial POD modes, $\varphi_1$, across the inlet plane

a) Level 2, b) Level 3

Fig 5.22 – Cumulative reconstructed POD energy across the inlet plane
Fig 5.23 – Reconstructed wavenumber spectra of w-velocity across sections of the inlet plane

Fig 5.24 – Reconstructed power spectra from 1kHz data set at points on the inlet plane, a) z/H = 0, y/H = 0.63, b) z/H = -0.44, y/H = 0.63
5.2.2 Comparative Costs

In order to make any synthetic inlet generation methodology viable it must offer a benefit over performing a precursor calculation. As a sufficiently detailed simulation can provide high enough fidelity data to replicate almost any flow scenario, experimentally derived methods must offer either reduced equipment costs and/or time savings. However, since a full pre-cursor calculation may be impossible or impractical for many engineering simulations, synthetic inlet condition generation may be the only viable option. The choice of method is then largely dependent upon the complexity of the test case, the resources available and the quality of inlet data required. Initially, we evaluate synthetic methods only, assuming PIV data of sufficient quality is available.

5.2.2.1 Cost of Generating Synthetic Data

Appendix C details a dimensional analysis of each stage of the four candidate synthetic inlet generation methodologies in terms of the overall number of required operations and maximum memory requirement. Operations common to all methods (such as loading the PIV data and interpolating the mean profiles) are neglected. Each operation may be expressed in terms of any or all of 6 parameters:

- \( N_i \) – Number of grid points in the inlet plane horizontal direction
- \( N_j \) – Number of grid points in the inlet plane vertical direction
- \( N_c \) – Number of velocity components reconstructed
- \( N_{M} \) – Number of time steps required for reconstruction
- \( n_{\Lambda} \) – Ratio of the local turbulent lengthscale to local LES grid spacing (Level 2 only)
- \( \frac{A_{m}}{A_{\text{tot}}} \) – Ratio of the area of the inlet plane covered by the master PIV field to the total inlet plane area (Level 3 only)

Using the example geometry of the previous section it is reasonable to assume the following approximate values for these parameters:

\[
N_i \approx 100, \; N_j = 50, \; N_c = 3, \; N_{M} \approx 3000, \; n_{\Lambda} \approx 10, \; \frac{A_{m}}{A_{\text{tot}}} = 0.5 \quad (5.6)
\]

Applying these values to the processes shown in Appendix C and neglecting lower order terms, the results shown in Table 5.2 are obtained.

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of operations Dimension</th>
<th>Order</th>
<th>Maximum memory storage Dimension</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>( N_i \times N_j \times N_c \times N_{M} )</td>
<td>( O[10^7] )</td>
<td>( N_i \times N_j \times N_c \times N_{M} )</td>
<td>( O[10^7] )</td>
</tr>
<tr>
<td>Level 2</td>
<td>( 77 \times n_{\Lambda}^4 \times (N_j+4n_{\Lambda}) \times (N_j+4n_{\Lambda}) \times N_c \times N_{M} )</td>
<td>( O[10^{14}] )</td>
<td>( N_i \times N_j \times N_c \times N_{M} )</td>
<td>( O[10^7] )</td>
</tr>
<tr>
<td>Level 3</td>
<td>( 6 \times N_i \times N_j^2 \times N_c \times N_{M} \times \left( 1 - \frac{A_{m}}{A_{\text{tot}}} \right) )</td>
<td>( O[10^{10}] )</td>
<td>( N_i \times N_j \times N_c \times N_{M} )</td>
<td>( O[10^7] )</td>
</tr>
<tr>
<td>Level 4</td>
<td>( N_i \times N_j \times N_c \times N_{M} )</td>
<td>( O[10^7] )</td>
<td>( N_i \times N_j \times N_c \times N_{M} )</td>
<td>( O[10^7] )</td>
</tr>
</tbody>
</table>

Table 5.2 – Inlet generation operation dimensional analysis

The processing time of each method is dominated by a particular stage of the generation process. For the DFG (Level 2) method, this occurs during the convolution of the random field with the assumed correlation filter. The influence on this process of the relative size of the local lengthscale is apparent. Similarly, the sLSE (Level 3) method is dependent on the number of reference points used (assumed here to be \( 3N_j \)). Notably, the memory requirement is unchanged regardless of method
used. However, even for this relatively simple scenario, the memory requirement is already of the order 100MB.

It is important to bear in mind that these values are based on the dimensions of the necessary equations and, as such, may not appear in their optimum form. Increased processing time may often be used to reduce memory storage and vice versa, although this does not necessarily result in a proportional transfer. Also, this analysis does not take account of the time taken to conduct each operation. For example, the interpolations for Level 4 require an order of magnitude more processing time than the random number generations of Level 1. Table 5.3 shows the sampled processing time required to construct the 1kHz velocity fields containing 3,000 samples as presented in section 5.2.1 on the computer detailed in Table 2.1. Although the relative order of magnitude difference between the methods is less than suggested by Table 5.3, a computation time of the order days for the Level 2 method begins to make the method prohibitively expensive, especially given the relatively simple test scenario used. Such data also illustrates the obvious attraction of the Level 1 method. It is also of note that, were data of sufficient resolution available to allow the use of a Level 4 inlet condition, it would require many more data points than the amount used in the current example, and therefore the calculation time would be increased accordingly.

<table>
<thead>
<tr>
<th>Method</th>
<th>Measured generation time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>23.2</td>
</tr>
<tr>
<td>Level 2</td>
<td>79,700</td>
</tr>
<tr>
<td>Level 3</td>
<td>2,840</td>
</tr>
<tr>
<td>Pseudo-</td>
<td></td>
</tr>
<tr>
<td>Level 4</td>
<td>147</td>
</tr>
</tbody>
</table>

Table 5.3 – Sample inlet generation computation times

Similarly, it was noted how the memory requirements may also differ from those detailed in Table 5.2. For some flows the redundant terms outlined in Appendix C may become significant and make inlet generation unfeasible with the available equipment. This was especially noted for the Level 2 inlet.
5.2.2.2 Cost in Comparison to Precursor Simulation

In order to evaluate the relative cost of obtaining experimentally-derived inlet conditions in comparison with conducting a precursor simulation we must first define what constitutes the inlet generation process.

To remove the development process from consideration (and thus consider only the application of each method) it is necessary to neglect the creation of the geometry, both in terms of the rig manufacture and mesh generation. Similarly, it must be assumed the necessary equipment is available, such as a high-speed PIV system, computational hardware and a suitable LES code. It could be argued that the capture of the required PIV data could be neglected also, although it will be included here for completeness.

The computation time of a precursor simulation will vary excessively depending on the complexity of the upstream geometry used and the fidelity to which it is modelled. It is assumed here that the computation time is of the same order as that of the main simulation. Likewise, this encompasses a period of flow convergence before velocity data is sampled. It should be noted that for geometries where the upstream fluctuations may be modelled as a channel flow, Schlüter\[83\] reports the precursor calculation time may be two orders of magnitude less.

Based on the experience of the author during the course of this project, the time taken to obtain PIV velocity data for an established rig and PIV system is limited by the vector processing, rather than the data capture. An estimate of one day per field of PIV plus one day set-up time is appropriate. It should be noted that Level 1 and 2 inlet conditions require only low sample rate (e.g. 50Hz) data sets for statistics, rather than the time resolved (1kHz) data required by Level 3. While this may have an effect on the data storage requirements, it will have negligible effect on the overall computational time.

Thus, based on the two test cases considered in this project, the following computation times may be estimated.
Table 5.4 – Inlet generation times [s]

<table>
<thead>
<tr>
<th>Method</th>
<th>Idealised Combustor</th>
<th>Vortex Generator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Generation</td>
</tr>
</tbody>
</table>

```

From Table 5.4 it is clear that, even if the cost of obtaining the PIV data is taken into account, all synthetic methods represent an order of magnitude saving over a precursor simulation. In most cases, if the PIV cost is disregarded, the saving is two orders of magnitude. As intimated previously, for simpler upstream geometry assumptions, this deficit could be reduced although the fidelity of such an inlet would be affected accordingly.

It should also be noted that, when the operations common to all synthetic methods (such as the import and export of data) are taken into account, the minimum generation time is of the order hours, regardless of the method used.

### 5.3 Chapter Summary

This chapter aimed to evaluate process of generating synthetic inlet conditions from experimental PIV data. Initially, an error analysis of the PIV data capture procedure identified four sources of error that all subsequent generation methods will be subject to, given the discretised nature of the data used for inlet condition generation. They are sub-pixel particle displacement accuracy, sub-cell filtering, dynamic averaging and statistical convergence. Of these, the errors arising from sub-pixel particle location accuracy and statistical convergence were shown to be of most consequence, in this case giving rise to unavoidable errors of at least 2% in the PIV data, rising to 6% in regions of high turbulence. It was also shown how the measured turbulence field for vortex generator test case contains a TKE deficit of up to 50% in some
regions compared to the level resolvable by the LES grid. No correction is available for this and it must be considered when evaluating results with this test case.

Subsequent methods of generating inflow velocity fields from this data were evaluated using the fidelity classification outlined in Fig 3.24. Levels 1 and 2 were represented by the commonly used methods of superposition of scaled Gaussian white noise and DFG respectively. Both methods were applied with two-dimensionally spatially-varying statistical properties, which is an improvement on many of the studies outlined in section 3.2, though the commonly-used assumption of a constant turbulence lengthscale was applied to the DFG method. Level 3 inlet conditions were represented by the newly-proposed supplemented LSE approach, as detailed in chapter 4. Although it was stated previously that the use of a Level 4 inlet condition is unfeasible given current data-capture equipment, the use of directly interpolated experimental data was considered where appropriate.

Using an example LES inlet plane taken from some of the experimental data detailed in chapter 2, the velocity fields produced by the various methods were compared. Through the application of the various velocity field analysis techniques discussed in section 3.1 it was concluded that, while the Level 2 and 3 methods produce a good representation of the first and second order statistics, the turbulence structures present in the Level 2 velocity field are not physically representative. The use of Level 2 inlets is therefore only advisable in flows that closely match the assumptions used in the generation process (i.e. constant lengthscale; no dominant structures). The level 3 approach offers an improvement to this technology, generating physically-representative turbulent structures correlated across the LES inlet plane. However, such performance is dependent on the arrangement and velocity correlations between the source PIV fields and thus careful consideration must be given to the flow field before the PIV field arrangement is allocated. The Level 1 method performed poorly, giving only a representation of the turbulent magnitude, with second order statistics and velocity correlation levels falling within statistical noise.

Comparison was also made of the relative cost of generating inlet conditions via each method. There was shown to be a progressive increase in the calculation time for each method of approximately one order of magnitude between the Level 1, 4, 3 and 2
methods respectively. A similar progression is evident in the memory storage requirements. While the calculation time, required by the Level 2 method, of a few days for the geometries and computing equipment used in this project was not excessive, its use with more complicated geometries may become restrictive, if not prohibitive. Although it is difficult to provide a generalised comparison against the use of a precursor calculation, it was estimated that each synthetic generation method would represent an order of magnitude time saving, even if the time taken to capture the experimental data was included.
Chapter 6

Computational Implementation
This chapter details the process by which a computational simulation was implemented, using the synthetic inlet condition methodologies evaluated in chapter 5. The contents encompass the selection and application of a suitable LES solver, the definition of a computational grid and the necessary choices and modifications to facilitate the use of the selected inlet condition generation methods. As discussed in chapter 2, only the idealised combustor test case is reported here, though a similar process was conducted for the vortex generator test case and can be found in Robinson[79].

6.1 Description of LES solver

All numerical simulations during the present research were performed using the Loughborough University in-house solver LU-LES. This is a multi-block structured, incompressible LES solver, which has a number of features making it attractive for application to the present project. An overview of its function and application will be provided here, however a detailed outline has previously been provided by Tang et al.[99], with a more detailed discussion in Wang[105].

The LES-filtered Navier-Stokes equations are discretised in space using second-order central differencing on a curvilinear, orthogonal mesh. Thus, the instantaneous versions of the filtered equations for the resolved momentum field, shown in equation 1.13, are first transformed from Cartesian into curvilinear coordinates, \( \xi \), becoming (from Pope[72])

\[
\frac{\partial \bar{U}_{\xi i}}{\partial t} + \frac{h_j}{h_i} \frac{\partial}{\partial \xi_j} \left[ h_i \left( \bar{U}_{\xi i} \bar{U}_{\xi j} - \nu_s \bar{S}_{\xi j} \right) \right] = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \xi_i} + \bar{H} \left( \bar{U}_{\xi i} \bar{U}_{\xi j} + 2 \nu_s \bar{S}_{\xi j} \right) - \bar{H} \left( \bar{U}_{\xi i} \bar{U}_{\xi j} + \nu_s \bar{S}_{\xi j} \right)
\]

where,

\[
\bar{S}_{\xi j} = \frac{\partial \bar{U}_{\xi i}}{\partial \xi_j} + \frac{\partial \bar{U}_{\xi j}}{\partial \xi_i} \bar{H} \left( \bar{U}_{\xi i} \bar{U}_{\xi j} - \bar{H} \bar{U}_{\xi j} \right) + 2 \delta_{ij} \bar{H} \bar{U}_{\xi k}
\]

(6.2)
\[ H_j^i = \frac{1}{h_j} \frac{\partial h_i}{\partial \xi_i} \] (6.3)

\( \bar{U}_{xi} \) is the filtered (i.e. resolved scale) curvilinear contravariant velocity component. \( h_i \) is the transformation scale factor for the Jacobian between the computational, \( J_i \), and curvilinear (physical) grids, such that:

\[ \xi_i = h_i J_i \] (6.4)

\(|h|\) is the volume ratio between the two coordinate frames and is the product of the scale factors. The finite volume method is applied using a staggered pressure-velocity grid system, thus avoiding the occurrence of pressure-velocity decoupling and the need for interpolation to resolve the mass-flux carrying velocity appearing in the cell face fluxes.

Temporal discretisation is applied explicitly via the second-order accurate Adams-Bashforth scheme, involving a two step (predictor/corrector) advancement of the velocity field. The first step produces a prediction for an intermediate velocity field, \( \hat{U} \):

\[ \frac{\hat{U}_i - U_i^n}{\Delta t} = \frac{3}{2} \gamma_i^n - \frac{1}{2} \gamma_i^{n-1} + \frac{1}{2} \frac{\partial p^n}{\partial x_i} \] (6.5)

where \( \gamma_i \) represents the contribution of the discretised convective and diffusive terms of equation 6.1. After this first predictor equation, a multi-grid 3-dimensional solution of a Poisson equation for pressure (derived from the continuity equation) is applied. Equation 6.6 is solved iteratively by imposing a divergence free condition for the new pressure field at the \( n+1 \) time step.

\[ \frac{\partial^2 p^{n+1}}{\partial \xi_i^2} = \frac{2}{3\Delta t} \frac{\partial \hat{U}_i}{\partial x_i} \] (6.6)
The multi-grid implementation speeds up convergence by iterating the solution on a succession of coarser grid, thereby removing low-frequency errors more quickly. This has been shown to reduce the solution time by up to 50% (Tang et al.\textsuperscript{[99]}). To maximise the number of multi-grid levels available, meshes should be designed to be divisible by $2^N$, for the largest $N$ possible. Finally, the intermediate velocity field is updated in a corrector step to provide the $n+1$ time step solution.

\[
\frac{U^{n+1} - \hat{U}_i}{\Delta t} = \frac{3}{2} \frac{\partial p^{n+1}}{\partial x_i} \tag{6.7}
\]

Filtering is applied implicitly through the computational grid, resulting in a top-hat filtered velocity field. This is analogous to the filtering inherent in the PIV (and thus inlet) velocity data, thereby giving credence to the sub-cell filtering assumptions made in section 4.3. The Smagorinsky method is used to model the unresolved sub-grid scale stresses, with van-Dreist damping terms as described in equation 1.17. Due to its implicit nature, the filter-width $\Delta$ is obtained from the cell volume, such that:

\[
\Delta = \sqrt[3]{\Delta x \Delta y \Delta z} \tag{6.8}
\]

Multi-block meshes, partitioned in three-dimensions, may be defined allowing the efficient and flexible use of multi-faceted, complex geometries. Each three-dimensional block is created by translating or rotating a 2D orthogonal curvilinear grid. All $xy$-planes for a given block are thus identical, greatly reducing the number of geometric quantities that require calculation and storage. Only the translational form is used in this investigation. All blocks are 6 sided, with only one boundary condition per side. Hence any two sides forming block-to-block interface must have the same grid dimensions.

The boundary condition treatment for block-to-block interfaces or periodic boundaries is very similar. A row of halo cells is defined at each appropriate block boundary, as shown in Fig 6.1. $U(1)$ and $U(N_{i+1})$ are not calculated directly; solution variable information is transferred from the corresponding cell at the opposite end of the block, thus:
Computational Implementation

\[ U(1) = U(N_i), \quad U(N_i + 1) = U(2) \] \hspace{1cm} (6.9)

Care must be taken when generating the computational grid to ensure the respective cell volumes are identical.

At outlet boundaries the velocity is first updated using a convective outflow condition as proposed by Pauley et al.\(^{[67]}\),

\[ \frac{\partial U_1}{\partial t} + U_{\text{bulk}} \frac{\partial U_1}{\partial x_i} = 0 \] \hspace{1cm} (6.10)

The result is then scaled to ensure mass conservation. The normal velocity over the outlet plane is scaled such that:

\[ U = U_{\text{calc}} \frac{\dot{m}_{\text{in}}}{\dot{m}_{\text{calc}}} \] \hspace{1cm} (6.11)

where the \( \text{in} \) and \( \text{calc} \) subscripts refer to the measured input and calculated values respectively.

A wall function is incorporated to solve for an instantaneous wall shear stress based on a logarithmic law assumption. The approach adopted in LU-LES can be found in
Schumann\textsuperscript{[86]} and is only activated if the first mesh point away from the wall is at $y^+ > 11.3$. Otherwise the wall is modelled by a laminar no-slip condition.

A modification was required to the code to allow the reading and implementation of an unsteady inlet boundary condition database and is described in section 6.3. A second modification allows the exportation of velocity data over a chosen plane of the domain at a given time-step interval. The data is extracted in a format analogous to PIV, and may subsequently be read into the Xact analysis program and analysed using the techniques outlined in chapter 3.

### 6.2 Mesh Generation

A three-dimensional computational mesh is required for the idealised combustor test case. Mesh generation was performed using the commercially available \textit{IcemCFD} package.

The geometry for the idealised combustor must adequately represent the smallest repeatable ‘sector’ of the experimental geometry outlined in chapter 2. It has already been discussed in that chapter that the LES domain is arranged with its inflow inlet planes within the inner and outer annuli extending downstream to encompass the annuli, port holes and core region. This allows the impact of the various inlet conditions on the flow solution in these regions to be assessed with minimal computational expense. One periodic sector is formed by limiting the spanwise dimensions to the mid-point between the first holes either side of the combustor centreline. The layout is thus broadly as described in Fig 2.7, although the annulus exit channel is curtailed at $x = 213.5\text{mm}$. The core taper extends to $x = 425\text{mm}$, with the thickness of the flame tube wall neglected. Due to the codes’ requirement of a single boundary condition per block face, this necessitates a 45 block geometry to be defined with 15 blocks resolving each of the upper annulus, lower annulus and core.
Fig 6.2 – Idealised combustor computational grid, a) zy-plane, b) xy-plane

Owing to the restrictions on time and computational power, it was decided to limit the grid size for this geometry to around 1.5 million cells. The grid density is manipulated to provide maximum resolution of the annuli and port jets, with a cell width of $\Delta \approx 1$ mm in these regions. The remaining regions use a graded mesh in one or more dimensions to minimise the grid size whilst maintaining block-to-block similarity. The successive size ratio is never more than 1.1. No attempt is made to resolve the near wall region, with the near-wall cells typically at $y^+ \approx 100$ for much of the domain. The resulting grid contains 1,695,744 cells and can be seen in Fig 6.2.

Unsteady inlet conditions, as described in section 6.3, are applied across the annulus inlet planes. Outlet boundary conditions are applied to the core and annuli exit planes. A modification to the code was necessary to maintain the correct outlet mass-flow split of 13% of the total inflow in each annulus. Thus equation 6.11 was implemented separately over each outlet zone: in the total annulus exit (i.e. upper and lower),

$$U_{ann} = U_{calc,ann} \frac{0.26 \dot{m}_{in}}{\dot{m}_{calc,ann}} \quad (6.12)$$

and in the core exit,
Computational Implementation

\[ U_{\text{core}} = U_{\text{calc,core}} \frac{(1 - 0.26)m_m}{m_{\text{calc,core}}} \]  \hspace{1cm} (6.13)

where the \( \text{ann} \) and \( \text{core} \) subscripts denote values from that outlet only. Periodic conditions were applied to all block faces on the spanwise extremities of the domain. No attempt is made here to adapt the inlet conditions to be explicitly periodic in this direction, given that the TKE in this region is low and far away from the region of interest. Block-to-block interfaces were applied to all internal faces in the annuli and core, as well as the port faces between annuli and core. All other faces were treated as walls.

### 6.3 Implementation of Unsteady Inlet Conditions

In order to provide a consistent, fair and efficient process for generating LES inlet conditions from PIV data using the methods outlined in chapter 5, all inlet condition data generation was conducted using a purpose-made Matlab package, \textit{InGen}. A flowchart detailing the program architecture is given in Appendix D. This program provides a common interface, statistics calculation, interpolation/extrapolation and export platform.

Interpolation, such as from the PIV to LES grids, is required for all generation methods for derived statistics and/or instantaneous velocities. The interpolation method must be optimised depending on required properties: instantaneous velocities were found to require a bi-linear (Vetterling \textit{et al.} \cite{104}) scheme to reduce computation time and smoothing effects; derived statistics require a bi-cubic scheme (Vetterling \textit{et al.} \cite{104}) to resolve local maxima and minima accurately. Extrapolation of both instantaneous velocities and statistics is often required to account for areas of the LES inlet field that are not covered by PIV fields. Linear extrapolation was found to be the most reliable method for resolving these regions.

Data is exported from \textit{InGen} in a binary file, which is read in a dedicated modification to the LU-LES inflow subroutine. Such files are typically \( 10 \times N_p \times N_p \times N_{\text{samp}} \) bytes,
resulting in files of around 200MB for both the idealised combustor and vortex generator. To avoid storing this amount of data in RAM during each simulation, only the samples corresponding to immediately before and after the current simulation time are read and stored as required. The velocity field at any intermediate time between these samples is linearly interpolated, as in Veloudis\textsuperscript{[101]}. Although this produces a discontinuity in the first time derivative of the inlet signal, using a more advanced method to produce a continuous signal (such as that of Lourenco \textit{et al.}\textsuperscript{[49]}) would require excessively large memory storage for this project. If the end of the inlet data set is reached during a simulation, the data set was re-read from the beginning. Although this produces a discontinuity in the turbulence field, no simulation run-time in this project exceeded twice the inlet data set length, and thus this was a rare event.

As the PIV data fields taken in the annuli of the idealised combustor have large overlapping areas of the annulus PIV fields (as illustrated in Fig 2.22), some flexibility was accorded with regard to which fields were used for reconstruction. Each annulus could be resolved using either 4 (A, B, D and E) or 5 (A to E) PIV fields. However, large statistical differences (up to 40% of the TKE) were noted in the region of the CRVP between the 1kHz data sets of fields B and D. This broadly coincides with the regions of high error outlined in Fig 5.10 and suggests the statistical convergence error is particularly large. As a discontinuity in this region could cause the premature dissipation of the structure, it was decided to resolve as much of the CRVP in one field as possible. This necessitates the use of field C and requires each annulus to be resolved by 5 PIV fields. Although this increases the memory requirement of the generation software, it results in much smaller statistical discontinuities across the inlet region, as may be seen from Fig 6.3, and a much closer reproduction of the profile observed in the 50Hz data.
Computational Implementation

![TKE in CRVP region](image)

Fig 6.3 – TKE in CRVP region, with annulus resolved using a) 4 fields, b) 5 fields

Inlet conditions of Levels 1, 2 and 3 were chosen in an attempt to replicate the results of Spencer et al.\(^{[95]}\) using a similar geometry and further demonstrate the influence of more advanced methods. It is therefore necessary to validate the application of these methods to a geometry more complex than previously demonstrated. Additionally, data was sampled from a steady state (Level 0) case, which was used to establish the initial flowfield (see section 6.4 for full details of the solution procedure). In order to be consistent with studies such as Spencer et al.\(^{[95]}\) and Clayton and Jones\(^{[10]}\), a uniform flow field was applied at the inlet plane for this case, with no representation of the CRVP.

The Level 1 and 2 methods were implemented as described in chapter 5, with the PIV statistics interpolated as necessary. The implied hierarchy of Fig 2.22 is retained, with statistics from field C taking preference over fields B and D, which take preference over fields A and E. Although Level 1 and 2 inlet conditions do not
require the use of time-resolved data sets, 1kHz data were used to ensure comparable statistics to the Level 3 inlet condition.

The Level 3 technique was optimised by selecting field C as the master field, resulting in the same hierarchical structure as for Levels 1 and 2. As implied by the flowchart in Appendix D, the reconstructed fields B and D subsequently act as master fields for the reconstruction of fields A and E respectively. Fig 6.4 shows the realistic and continuous reconstruction of the velocity correlations between fields A and B, despite the lack of high-energy turbulent structures in this region. This result is indicative of the fidelity of reconstruction across both annuli, which is comparable to that demonstrated in section 5.2.1.

![Fig 6.4 - Spatial velocity correlations, R_{11}, about z/H = -1.1, y/H = 0.81,](image)

a) Level 3, b) PIV

Although reconstruction using the sLSE technique is not critically limited by the computational or experimental equipment in this project, the choice of data-sets could be optimised should this be the case. The master field does not need to be captured at
the same image resolution as the slaves. Reducing the resolution allows either higher sample rates or longer sample times to be captured and thus reconstructed, although this reduces the proportion of directly interpolated fluctuations. Likewise, the slave correlations could be obtained from only 50Hz data, thereby reducing the required data storage and computational time and potentially improving accuracy. However, 1kHz data would still be required for the supplementary high-pass filtered field and the addition of this field could cause a loss of fidelity if the 1kHz and 50Hz are not statistically similar. For these reasons only full-resolution, 1kHz data sets were used for reconstruction here.

6.4 Solution and Data Capture Details

For explicit schemes the time-step must be carefully monitored to maintain numerical stability. Initial testing using the LU-LES code showed the stability of each run was primarily limited by the CFL (Courant-Friedrich-Lewy) condition, defined as:

\[
CFL = \Delta t \cdot \max \left( \frac{|U|}{\Delta x}, \frac{|V|}{\Delta y}, \frac{|W|}{\Delta z} \right)
\] (6.14)

To avoid solution divergence the CFL number should never be allowed to exceed 1. However, for accuracy of resolving the transient dynamics of the smallest structures resolved, the time step is typically set to produce smaller CFL numbers. For the idealised combustor, the CFL number was 0.2, resulting in a time step of 50\(\mu\)s. The vortex generator solution allowed a CFL number of up to 0.65 to be used, resulting in a time-step of 5\(\mu\)s.

To allow direct comparison with the PIV validation data, a plane of velocity data was exported from each test case at an equivalent sample rate of 1kHz. For the idealised combustor the plane extracted was the central symmetry plane. Given that statistical convergence was relatively slow for this test case, a comparable time series of 3,000 samples was captured, requiring 60,000 time steps in the data sampling stage. In the case of the vortex generator, 1,000 samples were sufficient to give statistical convergence across the exit plane requiring 200,000 time-steps.
Before data sampling can begin, each simulation was run for a ‘start-up’ period, which allows the flow to become established and statistically stationary. During this period the total amount of kinetic energy present in the solution domain was monitored. The start-up period was judged to be over when the kinetic energy reached a time-independent Level, as shown for a typical case in Fig 6.5.

![Fig 6.5 - Kinetic energy history in start-up phase](image)

To reduce run-time, simulations were started using an appropriate initial condition obtained from a previous simulation. Initially, this was obtained from preliminary testing using a steady, uniform velocity (Level 0) inlet condition. The established flow field was then applied as a start-up solution for the Level 1 simulation, which was, in turn, applied to the more complex methods. The initial Level 0 simulation took around 60,000 time steps to become statistically stationary. In the case of the idealised combustor, subsequent simulations required a start-up phase of only 30,000 time-steps. For the vortex generator, little reduction of the start-up time is observed, with each simulation requiring a start-up phase of 60,000 time-steps. Use of an initial Level 0 simulation has the additional advantage that, with little extra computational effort, a steady inlet condition case is obtained for comparison.
Four processors operating at around 3GHz each were available for simulation, allowing a maximum of four cases to be conducted at any time. Each time step required approximately 30s and 9s of computation time for the idealised combustor and vortex generator geometries respectively. According to the requirements of each geometry, a time plan was devised, as presented in Fig 6.6, which details the completion of all computational testing within 100 days.
Fig 6.6 - LES time plan
Chapter 7

Comparison of Computational Results for the Idealised Combustor Test Case
This chapter details the results of an LES-based computational study of the flow within the idealised combustor geometry presented in chapter 2, using the various inlet condition methods compared in chapter 5. The computational set-up is as described in the previous chapter.

This study is not aimed at being a full review of the implications of using the various inlet condition methods in LES, nor on how they directly influence the resultant predictions, such as combustion in a gas turbine engine. Rather, it aims to show that different approaches to inlet condition modelling can significantly influence the predicted flow field and this area is thus worthy of further study.

The figures presented in this chapter have been chosen to give the best representation of the flow features described in the text. Unless otherwise stated or shown, derived properties in other directions or from other components to those shown may be assumed to give broadly similar trends. All derived properties are calculated as described in the review of unsteady velocity field analysis presented in section 3.1.

As described in chapter 2, the simulation of the vortex generator test case is adversely affected by regions of intermittent vector calculation in the source PIV data at the corners of the computational domain, which make it difficult to compare the influence of the inlet conditions applied. As such the results of that test case do not merit inclusion here, but can be found in Robinson[79].

The following sections discuss the LES results from the idealised combustor geometry with the use of Level 0, 1, 2 and 3 inlet conditions. Section 7.1 details the flowfield in the upper annulus immediately downstream of the inlet plane and the influence of the inlet conditions applied. Section 7.2 considers the core of the combustor, after the flow exhausts through the primary ports, in the context of the PIV validation data taken in this region.
7.1 Upper Annulus Flow Field

The previous chapters of this thesis have shown the inflow into the combustor feed annuli, especially the upper annulus, is dominated by a CRVP shed from the burner arm. The vortex pair is transient and intermittent and produces large variations in the lengthscale and shear-stress fields. Chapter 5 showed the representation of this flow feature is the major difference between the inlet condition methods trialled. Note that the Level 0 simulation is a uniform axial flow and contains no representation of the CRVP.

The iso-surfaces of time-averaged W- (transverse) velocity and axial vorticity in Figs 7.1 and 7.2 show the CRVP is maintained in the mean flowfield by those simulations in which it existed in the inlet flow (i.e. Level 1, 2 and 3). In all cases it appears the vortices are maintained up to and, to a certain extent, beyond the primary ports. The Level 3 simulation maintains both the transverse velocity and high vorticity cores for longer than the other methods, though notably these structures do not appear to be entrained by the port jets. As the Level 0 inlet contained only a uniform flow field, the structures shown for this simulation exist purely as a result of the flow acceleration into the port holes. For all methods, a high wavenumber oscillation is present in the velocity and vorticity fields upstream of the port holes. This has the appearance of pressure-velocity decoupling, though the use of a staggered grid (as described in chapter 5) should negate this. It may be a consequence of the low turbulence level in these accelerating regions, though the exact cause remains unknown.

The TKE and $\langle vw\rangle$ shear stress iso-surfaces (Figs 7.3 and 7.4) give an indication of the extent to which the CRVP exists in the unsteady velocity field. In the Level 1 simulation both of these properties decay quickly downstream of the inlet plane, which is consistent with the observations of previous studies (Perret et al.\cite{69}, Schlüter\cite{83}). The Level 2 simulation shows an increase in TKE downstream of the inlet plane, though the shear stress loses its form quickly. This perhaps suggests the transient nature of the structures observed in the inlet flow is not maintained in this simulation. The Level 3 simulation maintains a relatively constant TKE and shear
stress profile up until the port holes (where the flow acceleration will presumably suppress the turbulence field).

Integrating the TKE over $yz$-planes along the domain gives an indication of the developed nature of the turbulence (through the extent to which a turbulence production/dissipation equilibrium exists), and thus the amount of ‘settling’ that is required downstream of the inlet plane. Fig 7.5 shows the TKE development in the upper annulus for the region $-H < z < H$ for each simulation. The results are presented as a proportion of the measured inlet TKE. The particular area of interest in this investigation is between the inlet plane and port hole. As suggested by the iso-surfaces and seen previously, the Level 1 simulation decays sharply downstream of the inlet, owing to its lack of spatiotemporal correlations. The TKE in the Level 2 simulations increases before reaching a steady state. This is likely to be a result of the upper-prediction of the high frequency fluctuations (shown for this geometry in Fig 5.24), which results in an initial under-prediction in the turbulence dissipation. The Level 3 simulation, by contrast, maintains an approximately constant level up to the port, requiring no adjustment period. This is strong evidence for the fidelity of the sLSE method.

Figs 7.6 to 7.8 show data one annulus height downstream of the inlet plane, and describe the flow as it is presented to the port for each simulation. The spatial velocity correlations of the $w$- (through plane) component in Fig 7.6 bear close resemblance to the inlet correlations shown in Fig 5.15, emphasising the dependency of this region on the inlet velocity field. The centreline mean profiles (Fig 7.7) support the conclusion from the iso-surface plots that the time-averaged velocity field is largely insensitive to the unsteady inlet condition applied, though the upwash from the vortex pair is maintained better with the higher fidelity methods (and, obviously, non-existent for the uniform steady case). The profiles of two turbulence properties, shown in Fig 7.8, show greater differences, with confirmation the Level 3 inlet gives less dissipation of the shear stress profile. Note the uniform lengthscale assumption used in the Level 2 inlet condition generation is still evident downstream, as is the spatially variant nature of the Level 3 method.
The following section will evaluate the effects these differing flow-fields induce after the flow has exhausted into the core of the combustor. PIV data is available in this region for comparison.

Fig 7.1 – Upper annulus iso-surfaces of time averaged $W$ velocity at $\langle W \rangle = \pm 0.075U_{ref}$. a) Level 0, b) Level 1, c) Level 2, d) Level 3
Fig 7.2 – Upper annulus iso-surface time averaged $x$-vorticity at $\omega_x = \pm U_{ref}/H$, (boundary layer vorticity neglected), a) Level 0, b) Level 1, c) Level 2, d) Level 3
Fig 7.3 – Upper annulus iso-surface Turbulent Kinetic Energy at $e_k = 0.03U_{ref}^2$

a) Level 0, b) Level 1, c) Level 2, d) Level 3
Comparison of Computational Results

Fig 7.4 – Upper annulus iso-surface $vw$-shear stress at $\langle vw \rangle = \pm 0.005U_{ref}^2$

a) Level 0, b) Level 1, c) Level 2, d) Level 3
Fig 7.5 - TKE integral over x-planes in upper annulus, for \(-H < z < H\)

Fig 7.6 - Spatial \(ww\)-velocity correlations (\(R_{33}\)) about \(x = H, y/H = 0.31, z/H = 0\)

a) Level 0, b) Level 1, c) Level 2, d) Level 3
Comparison of Computational Results

Fig 7.7 – Profiles in upper annulus across $x = H$, $z/H = 0$

a) Time averaged $U$ velocity, b) time averaged $V$ velocity

Fig 7.8 – Profiles in upper annulus across $x = H$, $z/H = 0$

a) $(vw)$ shear stress, b) $u'' L_{\text{nu}}$ Lengthscale
7.2 Comparison with PIV Validation Data in the Core

Data presented in this section compares the results of the LES simulations with the PIV validation data taken in the core of the combustor.

The streamlines in Fig 7.9 and time-averaged vectors in Fig 7.10 give an overview of the flow in this region. The flow is dominated by the port jets that impinge approximately symmetrically in all cases and cause recirculation both upstream and downstream of this impingement. Table 7.1 shows the mean impingement stagnation point location on the combustor centreline (i.e. the intersection point of lines of \( \langle U \rangle = 0 \) and \( \langle V \rangle = 0 \)), which is relatively constant for all cases. This is perhaps surprising considering the differing annuli streamlines between the simulations, particularly in the Level 0 case, seen in the previous section. Note the slight difference in y-location in the Level 0 case is probably due to the lack of a CRVP causing centreline upwash in the upper annulus.

<table>
<thead>
<tr>
<th></th>
<th>Stagnation point (x,y) location</th>
<th>Core recirculation, ( \frac{\langle \dot{m}<em>{(t)} \rangle</em>{s}}{\dot{m}_{\text{inlet}}} ) at ( x/H = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0</td>
<td>(87.5, 0.0)</td>
<td>8.72%</td>
</tr>
<tr>
<td>Level 1</td>
<td>(88.0, 4.0)</td>
<td>7.54%</td>
</tr>
<tr>
<td>Level 2</td>
<td>(87.0, 3.5)</td>
<td>9.57%</td>
</tr>
<tr>
<td>Level 3</td>
<td>(87.0, 2.5)</td>
<td>9.69%</td>
</tr>
<tr>
<td>PIV</td>
<td>(87.0, 4.0)</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 7.1 – Time-averaged stagnation point and recirculating mass flow in the central sector of the combustor core

Observation of the core vectors and streamlines, and the contours of time-averaged U (axial) velocity in Fig 7.11, show the recirculation is much stronger in the PIV than any of the LES cases. The streamlines suggest the recirculation is increased in the Level 3 case. Table 7.1 also shows the recirculation through a plane at \( x/H = 50\)mm (forward of the impingement point) for each case as a percentage of the inlet mass flow. This is obtained by integrating the recirculating velocity (i.e. where \( \langle U \rangle < 0 \)) within the central sector of the combustor core (-47.5mm < z < 47.5mm). Although
no PIV data is available in this plane, it is clear from the core streamlines and vectors that an increase in recirculation is a better representation of the experimental velocity field. As suggested by the streamlines, the Level 3 case predicts the greatest amount of recirculation, though both the Level 2 and 3 cases predict over 25% more recirculation than the Level 1 case. It is notable that the Level 0 case gives increased recirculation over the Level 1, though this likely to be due to the differing port inlet velocity profiles resulting from the lack of a CRVP in the annulus in the Level 0 case. The prediction of this recirculation is critical to the prediction of mixing within the primary zone and the turbulent interaction of the port jet flow with the fuel injector.

The differences in the turbulence field at the inlets to the ports, noted in the previous section, again result in only very modest differences in the core between the simulations. The TKE (Fig 7.12) appears insensitive to the annulus turbulence. All simulations show an overprediction in the TKE contained within the downstream shear layer. This results in a much shorter low turbulence 'potential core' within the LES jet, which, given this is present in the Level 0 case (with no annulus CRVP), appears to be due to the computational setup. Note the potential core of the jets in the PIV data is much longer than may be expected in a realistic combustor, such as measured by Griffiths[27] and Hollis[32]). This is due to the relative steadiness of the initial jet angle in this study, probably caused by the idealised upstream annulus and port geometry. The high TKE region observed near the lower annulus port is not physical and is a result of intermittent vector calculation from the large particle shifts in this region.

Analysis of the jet profiles has been conducted on the combustor centreline, half a port diameter, $d_p$, inside the core. Fig 7.14 suggests a reasonably good match in the location and profile of the jets within the mean flowfield between the LES and PIV, despite the flame tube wall thickness not being modelled in the LES, though the peak velocity in the y-direction is lower by around 10% in all LES cases. The jet flow angle is also significantly steeper in the PIV than in any of the LES cases (Fig 7.16a), and is insensitive to the inlet condition applied, though, as noted above, the resulting stagnation location is relatively consistent.
The double peak in the normal stresses in Fig 7.15 is indicative of shear layer vortex shedding as previously observed in the PIV data. The frequency of this oscillation appears close to that observed in the PIV data (approximately 80Hz), though the velocity field in this region is noisy, making it difficult to obtain a more precise figure. A bias exists in all LES simulations towards the downstream shear layer (visible in instantaneous data), causing an over-estimation in the \( \langle uu \rangle \) stress relative to the PIV. Fig 7.18 shows the shear stress in a cross section (xz-plane) of the upper annulus port jet at two locations downstream of the port, though no PIV validation data is available in this plane. The structures identified are consistent with those observed by Hollis [32] as being the result of a CRVP present on the leeward side of a jet cross-section in cross-flow. The profiles in this plane are again insensitive to the annulus inlet condition for this and other derived properties, and may be the source of the over-estimation of turbulence seen in the downstream shear-layer relative to the PIV. Further investigation and PIV validation data in the xz- and yz- planes in the core are required to confirm the differences seen, but the differences in jet angle and peak velocity, shear layer turbulence and post-impingement recirculation suggest there may be an error in the annulus bleed ratio, between the LES and PIV, which may have resulted from flexing of the test rig between initial commissioning and data capture. This may be exacerbated by insufficient resolution from the computational grid in the near port region causing excessive spatial filtering, giving large differences in the higher frequency region of PSDs at points within the core (Fig 7.16b and Fig 7.17b). The important result in the context of the varying inlet conditions is that the shedding is not correlated with the annulus flow, and thus may mask any differences in the port inlet flow field further downstream.

The bi-modal PDF in the shear layer well modelled in LES (Fig 7.17a). This PDF results from a periodic flapping of the two jets resulting in alternating asymmetric impingement, as observed in the PIV data presented in chapter 2. This is not the same phenomenon as the recirculatory surge observed to produce a similar bi-modal PDF by Hollis [32] and Spencer [93]. The peak in the power spectra at this location (Fig 7.17b) from the LES data sets correspond to the frequency of this flapping. It occurs at a slightly higher frequency than the PIV (approximately 10Hz), which is insensitive to the annulus inlet condition.
Figs 7.19 and 7.20 show the resultant effect on the flowfield upstream and downstream of the impingement point locations (normalised by the core height). Generally, as noted above, the LES under-predicts the upstream recirculation and overall TKE in this region. Although the high fidelity Level 2 and 3 inlet conditions show a slightly bigger recirculation peak (as suggested by Table 7.1), the differences between the simulations are small in comparison to the differences to the PIV. This is not consistent with the finding of Spencer et al.\textsuperscript{[95]}, which confirmed jet impingement dominates the large scale mixing in this region, but suggested the jet penetration improves with higher fidelity annulus inlet conditions. It is likely the current study suffers from the lack of unsteady interaction with swirling flow through the omission of a fuel injector, and from the crude representation of the port geometry. As found downstream of the (premature) wall impingement in the vortex generator test case, the presence of a dominant turbulence-generating flow feature appears to mask any differences in the upstream flow, and thus the inlet conditions also.
Comparison of Computational Results

Fig 7.9 – Streamlines released from annulus inlets over the plane $z/H = 0$

a) Level 0, b) Level 1, c) Level 2, d) Level 3, e) PIV
Fig 7.10 – Time averaged velocity vectors across $z/H = 0$,

a) Level 0, b) Level 1, c) Level 2, d) Level 3, e) PIV
Comparison of Computational Results

Fig 7.11 - Time averaged U velocity across $z/H = 0$,
a) Level 0, b) Level 1, c) Level 2, d) Level 3, e) PIV
Comparison of Computational Results

Fig 7.12 – Turbulent Kinetic Energy across $z/H = 0$,
a) Level 0, b) Level 1, c) Level 2, d) Level 3, e) PIV

Note: Given only monoscopic PIV data is available, only contributions from the in-plane ($u$ and $v$) velocity components are plotted
Comparison of Computational Results

Fig 7.13 – $uv$-shear stress across $z/H = 0$,

a) Level 0, b) Level 1, c) Level 2, d) Level 3, e) PIV
Comparison of Computational Results

Fig 7.14 – Profiles along a line 0.5d_p below upper port hole, z/H = 0,

a) Time averaged U velocity, b) time averaged V velocity

Fig 7.15 – Profiles along a line 0.5d_p below upper port hole, z/H = 0,

a) \( \langle uu \rangle \) Reynolds stress, b) \( \langle vv \rangle \) Reynolds stress
Comparison of Computational Results

Fig 7.16 – Point in the jet, 0.5d$_p$ below upper port hole, z/H = 0,
a) PDF of $xy$-plane flow angle, b) PSD of $u$-velocity

Fig 7.17 – Jet impingement stagnation point (at locations in Table 7.1)
a) PDF of $xy$-plane flow angle, b) PSD of $u$-velocity
Fig 7.18 – $uw$-shear stress over a plane i) $0.5d_p$, ii) $d_p$ below the port,
a) Level 0, b) Level 1, c) Level 2, d) Level 3
Comparison of Computational Results

Fig 7.19 – Time averaged U velocity within the core, $z/H = 0$

a) $x/H_{core} = 0.5$ (forward of stagnation), b) $x/H_{core} = 1.5$ (rearward of stagnation)

Fig 7.20 – Turbulent Kinetic Energy within the core, $z/H = 0$

a) $x/H_{core} = 0.5$ (forward of stagnation), b) $x/H_{core} = 1.5$ (rearward of stagnation)
7.2.1 General Discussion

The results from a LES simulation have been described, as obtained from the idealised combustor geometry described in section 2.2 and implemented as described in chapter 6. The effect of the use of several inlet conditions applied at the annulus inlet has been discussed, with particular focus rendered on the upper annulus and core regions of the combustor. There has been shown to be regions that display significant differences in the predicted flow fields as a result of the differing inlet conditions, though there are also regions where the influence is less keenly felt.

The CRVP shed from the burner arm and present in the inlet velocity field of the Level 1, 2 and 3 inlet conditions is maintained in the time-averaged flow field in all these cases along the upper annulus, both up to and beyond the port holes. Although the strength of the vortices in each case appears reduced by the effect of the port holes, they are not entrained into them. There are however significant differences in the turbulence field. The decay of TKE downstream of the inlet plane is indicative of the performance of the different inlet conditions. The Level 1 TKE decays very quickly, resulting in a turbulence field at the port inlet that is almost indistinguishable from the steady Level 0 case. The Level 2 case displays some readjustment in the initial flow field resulting in an increase in the overall TKE levels and the expense of mean-flow vorticity. However, the shear stresses are seen to decay quickly downstream of the inlet. By contrast the Level 3 inlet conditions show little adjustment of TKE or shear stress downstream of the inlet. In all cases, the velocity correlation and thus lengthscale distributions seen previously at the inlet plane are broadly maintained along the length of the annulus.

Overall, the LES simulates the major flow features within the combustor core well, with the jet profiles and shear layer vortices, impingement locations and oscillation, and the subsequent recirculation well represented. The LES also predicts the flow features such as shear layer vortex shedding and bi-modal impingement PDFs seen in previous work[32][95]. However, differences in the core flow predicted by each case are much less pronounced than those observed in the annuli. There is evidence that improved fidelity of annulus inlet condition gives increased core recirculation, with the Level 3 case predicting 28% more recirculation than the Level 1 case. However,
with many properties of the port jet shear layers appearing insensitive to the annulus condition, and without PIV validation data in orthogonal planes in the combustor, the mechanism behind the increased recirculation is unknown.

While every effort was made to match the computational and experimental test conditions, the results suggest there may be difference in the annulus bleed ratio between the PIV and LES. A lower port jet mass-flow rate in the LES cases would account for the shallower jet angle, increased turbulence generation in the downstream shear layer and reduced post-impingement recirculation.

The results do provide evidence that the presence of downstream turbulence generating features diminish the dependence on the inlet conditions used, though the general insensitivity of the core results is likely to be exacerbated by the lack of entrainment of the majority of the annulus CRVP into the ports. Further testing would be required at a lower bleed ratio (i.e. one where the CRVP is entrained) to provide a definitive answer.

It also seems likely the quality of the prediction in the core region was compromised by the simplifications made to the combustor geometry, which may have been too conservative. Although entailing additional computational expense, the use of circular and chuted port holes may have provided more representative flow physics within the jet and impingment region. Likewise, the omission of the fuel injector and pre-diffuser may have removed sources of unsteadiness both internal and external to the core region that would otherwise heighten the sensitivity of the turbulent mixing in the core to the onset turbulence conditions. The relatively coarse computational grid used in the impingement region may also have been insufficient in resolving the impingment velocity gradients, though the overall conclusion may have been unaffected. A computational geometry such as that used by Spencer et al.\textsuperscript{[95]} may be more suitable, though the difficulties of obtaining sufficient quality inlet condition data in such a geometry are highlighted in Appendix A.
Chapter 8

Conclusion
8.1 Summary

This thesis has developed, quantified and validated a generalised and widely applicable methodology for providing unsteady, spatially and temporally resolved velocity fields, maximising the current potential of the PIV technique, for direct application as an LES inlet condition generation method.

PIV input and validation data has been taken on two test cases representative of the fluid mechanics found within the combustors of modern gas-turbine aero engines. The developed inlet condition generation method has been compared with existing methods, both in the context of the cost and fidelity of the velocity field they produce and the resultant influence this exerts on LES calculations of the chosen test cases. Guidance has been given as to the optimal parameters for the test cases used in this thesis.

8.2 Conclusions

- Current experimental techniques (LDA, PIV, DGV) do not provide sufficient temporal and spatial resolution of the velocity field for it to be feasible to obtain unsteady inlet conditions for practical flows solely by interpolation of measured data. This is unlikely to change in the foreseeable future.

- A new method has been developed to provide high fidelity synthetic inlet conditions from multiple fields of non-synchronous PIV using a combined process of LSE and high-pass filtered instantaneous PIV data. The resultant velocity field is synchronised (having no discontinuities in the velocity correlation field) and the method requires no \textit{a priori} assumptions or restrictions regarding the flow field. The method is also seen to be applicable to air or water flows.

- The velocity field produced by the proposed method compares favourably to existing synthetic methods, matching closely the TKE, Reynolds stress tensor, and spatial velocity correlation field (and by extension the integral lengthscale) of the source PIV data. The method is also shown to produce physically representative turbulent structures.
Conclusion

- Inlet condition generation using the proposed method can lead to an order of magnitude time saving over DFG. All synthetic methods considered are conservatively estimated to save at least an order of magnitude in time over using a precursor simulation.

- LES of the idealised combustor shows significant differences in the flow, particularly in the turbulence field, between new and existing inlet condition generation methods. In the absence of downstream turbulence generating features (such as a large pressure drop or wall impingement), the inlet turbulence properties have influence far downstream of the inlet.

- The inlet condition methodology developed here undergoes much less adjustment of the turbulence field immediately downstream of the inlet plane. This is in marked contrast to the use of Gaussian noise (which decays quickly in both cases) and DFG (which, given a consistent bias towards large scale structures, over-predicts the turbulent production).

- LES predicts the major flow features in the core of the combustor well, but was largely insensitive to the annuli inlet conditions applied in this case. This may be a consequence of the bleed ratio used or the simplifications made in the choice of the geometry.

- Poor experimental data quality in only a small region of the inlet plane can have a significant effect on the resultant LES, as found here for a vortex generator test case. Care must be taken when obtaining stereoscopic data to account for the effects of oblique viewing, camera calibration (notably the calibration plate used) and insufficient seeding, which can all lead to a reduction in the output data quality.

- Sub-pixel particle location accuracy and statistical convergence of the data sample were the major sources of intrinsic error in the PIV process in this project (varying from 2% to 6% of $U_{ref}$). Sub-cell filtering errors may also be significant if the PIV vector resolution is lower than the LES sub-grid filter width, which should be a consideration when choosing the size of the PIV FoV.

- It has been found to be unsatisfactory to obtain the through-plane velocity from multiple monoscopic PIV fields. A three-component stereoscopic arrangement is therefore recommended.
8.3 Future Work

This study has shown the potential for implementing an experimentally derived inlet condition using PIV. Although the newly proposed method has performed well in the evaluation undertaken here, there are several potential areas of improvement worthy of further investigation.

The prediction of the reconstructed correlated motions in the slave PIV fields may be improved by the use of quadratic stochastic estimation, which was shown by Naguib et al.\textsuperscript{[63]} to offer improvement for LSE, though it is more complicated to implement. A larger body of evidence is also required to determine if the arrangement of the reference points and use of a single conditioning velocity component is optimal in a more general sense. A method of allowing for periodic boundary conditions to be employed whilst maintaining the correlation continuity is also required.

There may be potential to reconstruct the uncorrelated motions by a method other than the high-pass filtering of PIV data used here. Given that in widely correlated flows the motions extracted tend to be isotropic and devoid of large scale structures, it may be possible to obtain these from a method such as spectral forcing of randomly seeded data or from a hot-wire rake. While such methods reduce the amount of high speed PIV data required and may allow for temporal resolutions beyond what is achievable with current PIV equipment (potentially allowing for the energy deficit introduced through sub-cell filtering to be re-incorporated), such methods may not be applicable to weakly correlated data where the flexibility of the cut-on frequency used in the current method allows the fidelity of the PIV to be maintained. If PIV data is to be continually used, that is under-resolved relative to the LES time step, it may be judged feasible to use the method of Lourenco et al.\textsuperscript{[49]}, which should offer improvement in removing the first order discontinuities introduced by linear interpolation in time (as used here).

The test cases here ultimately did not show a large degree of sensitivity to the inlet condition used. While further studies with more varied geometries would be desirable, it may be possible to expand the field of study around the current test cases provided that the lessons learnt in this thesis are heeded. The vortex generator
channel should be widened to prevent wall impingement from contaminating the results. It may also be prudent to re-evaluate the use of a delta wing-style generator, given the vortices produced showed little correlation with each other. Inclined flow over a cylinder or half-cylinder may be more representative. Steps may also be taken to aide removal of regions of bad data which appeared to affect the results in this study. This may include the use of POD reconstruction to remove spurious fluctuations.

Given the large sensitivity in the upper annulus flowfield to the applied inlet condition, PIV validation data taken in this region and in the vicinity of the ports would be beneficial. This case could be extended to include additional annulus bleed ratios (specifically one in which the annulus CRVP is entrained into the ports) and more representative features, such a fuel injector or round and chuted ports. The former is of particular importance since Spencer et al.\textsuperscript{195} noted particular sensitivity to the turbulent conditions applied here.
References


[81] Robinson, M.D. – “PIV Measurements For Unsteady Inlet Condition Generation In LES-Based CFD Predictions”, MPhil to PhD Transfer Report, Department of Aeronautical and Automotive Engineering, Loughborough University, UK, 2005


References


Appendix A – Geometry and Test Rig for a Representative Combustor in Air Flow

Although the measurements in water flow described in the thesis were used for technique development, it was desired that such techniques should also be applicable to data sets captured in airflow facilities. The increased velocities in an airflow test result in proportionally much less of the turbulence spectra being resolved using the current PIV instrumentation.

A VULCAN Phase 5 isothermal airflow combustor rig, as commissioned by Griffiths\textsuperscript{[27]} for use with LDA measurements, was adapted and used as a preliminary evaluation of high speed PIV in realistic airflow geometries.

A.1 Test Rig Arrangement and Modification

The VULCAN rig comprises of a 45° sector of an annular Rolls-Royce Phase 5 combustor, a cross-section of which is shown in Fig A.1. The flame-tube itself is an actual (metal) production model, including three complete injectors. The outer casing is made from Perspex and designed to closely replicate the engine aerodynamics, thus maintaining the correct aerodynamic conditions in and around the flame tube. For details of the development procedure and rig dimensions the reader is referred to Griffiths\textsuperscript{[27]}.

The rig itself is mounted vertically, with the air drawn from an upper plenum into a negative pressure lower plenum. The inlet to the rig consists of a 1800mm long circular pipe of diameter 200mm. A bell-mouth, boundary layer trip and wire mesh in the inlet ensures a low turbulence intensity into the rig.

The rig is globally seeded from the upper plenum using atomised Shell Ondina oil. This seeding arrangement has been used previously and has been shown to produce particles of diameter 1-2\textmu m (Griffiths\textsuperscript{[27]}). As this seeding particulate is more dense...
than the test medium (air), a potential source of error is introduced due to the response lag of the particle. This will be discussed further in section 5.1.

The coordinate system is arranged such that the x-axis is orientated along the flame tube axis, its origin on the combustor heat shield. The rθ-plane forms a right-handed coordinate system with its origin in the centre of the head of the central injector.

Due to the availability of only one high-speed camera for the duration of this test, only monoscopic PIV was possible. As for the idealised combustor discussed in chapter 2, the choice of measurement locations are governed by the location of inlet planes in potential LES simulations. Previous studies on similar geometries (Reynolds et al.\cite{77}, Tang et al.\cite{99}) have modelled the pre- and dump diffusers in addition to the annuli and flame tube. Thus three-component velocity data is ideally required in both annuli inlet conditions and in the pre-diffuser of the central (repeating) burner sector. As the thickness of the pre-diffuser walls makes distortion free viewing difficult, the exit plane has been chosen for data capture. Ultimately, due to time limitations, only data at pre-diffuser exit has been captured, and the modification and set-up work detailed in this appendix is tailored towards that region.
The rig was originally commissioned for use with Laser Doppler Anemometry (LDA) and later modified by Hollis\textsuperscript{[32]} to allow optical access for PIV within the flame tube. Further modification was required as part of this project to facilitate data capture in the pre-diffuser and annuli. The side wall cooling ducts were removed (highlighted in Fig A.2) and the pressure tappings around the dump cavity and annuli, detailed in Fig A.1, were replaced with plane windows. Despite this, the planar nature of PIV, when applied to a sector rig made viewing orthogonal to the required planes difficult. As detailed in Fig A.3, viewing in the $x_r$- and $x_\theta$-planes is relatively unobstructed (although a true circumferential plane is obviously not possible, it is sufficient to gather statistics over multiple tangential planes, i.e. a pseudo-circumferential $xz$-plane). The flame tube prevented viewing the axial direction ($r_\theta$-plane) normally, and thus oblique viewing was required. An image correction for this situation is described by LaVision\textsuperscript{[44]} and is employed. This required the manufacture and use of custom made flat calibration plates (as shown in Fig A.4), which were used for all planes and placed in-situ in the appropriate plane of the per-diffuser. As only monoscopic data is to be taken, only one calibration image at each test location is required. Once again, due to the Cartesian nature of the PIV vector computation, the vector grid will be over an $rz$-plane, with the $z$-direction tangential to the local circumferential $\theta$-direction at the centre of the FoV.

![Fig A.2 - Required VULCAN Phase 5 rig modifications, a) sidewall duct removal, b) window replacement](image-url)
Appendix A

Fig A.3 – PIV viewing arrangements, a) \(xr\)-plane, b) \(x\theta\)-plane, c) \(r\theta\)-plane

Fig A.4 – Pre-diffuser calibration plates, a) \(xr\)-plane, b) \(x\theta\)-plane, c) \(r\theta\)-plane

The rig flow conditions were designed to match the Trent 700 engine take-off conditions used in previous projects (Griffiths\textsuperscript{[27]}, Hollis\textsuperscript{[32]}). This dictates a mass flow split down the inner and outer annuli exits of 12.14% and 3.94% of the inlet mass flow respectively, and is fixed by metering plates in the annuli exits. To ensure the rig is running at the expected condition, calibration data were obtained by way of a button hook probe traverse at pre-diffuser inlet (115mm upstream of the datum line, Fig A.1) and circumferential pressure tappings in the inner and outer annuli exits (198.5 and 184.5mm downstream of the datum line respectively, Fig A.1). Since no measurements were to be conducted during the present test within the flame-tube
core, the effect of the above modifications do not need to be considered here. The rig was set to condition at an inlet Mach number of 0.1205 on the pre-diffuser centreline. Given a typical atmospheric temperature of 21°C, this results in a bulk velocity of 27.51m/s at the pre-diffuser exit. This value will be used as the reference velocity for normalisation of data in this region. Lengths will be normalised by the exit height of 40.76mm. As with the idealised combustor, normalised radial dimensions are assumed to have their origin at the inner wall of the pre-diffuser channel.

Fig A.5 shows a comparison of the velocity profile across the inlet traverse with previous projects. Clearly it is in excellent agreement, with perhaps a slightly thinner boundary layer observed on the outer wall. Similarly, analysis of the pressure tappings (Fig A.6) shows good agreement, especially with the data of Hollis\textsuperscript{32}. Slight discrepancies are noted between the profiles of each data set, although within the central measurement region (±7.5°) these are well within acceptable experimental error.

![Graph](image-url)

**Fig A.5** – Pre-diffuser inlet traverse comparison, 
- Current; • Hollis\textsuperscript{32}; △ Griffiths\textsuperscript{27}
A.2 PIV testing and results

All data captured for this test case used the ‘best practice’ approaches detailed in section 1.3. The pre-diffuser flow field required an inter-frame time of 12\mu s to give a maximum particle shift of 8 pixels.

In all cases viewing, seeding and illumination were optimised to produce maximum data quality over the pre-diffuser exit plane, often to the detriment of other regions of the FoV. However, as seen in Fig A.7 for a sample data set in the \(x_r\)-plane, the raw PIV image in some regions is affected by excessive glare and/or shadows from the pre-diffuser walls. This has a significant negative effect on the quality of the resultant velocity data measured in these regions, generally causing a reduction in the mean velocities and large variations in the RMS. Although the data is of good quality in the centre and downstream of the pre-diffuser exit, data for approximately 20\% of the diffuser exit height, particularly near the inner wall, is not of sufficient quality for inlet condition generation.
Appendix A

Figs A.8 to A.10 illustrate the mean velocities measured across five $xr$-planes, three $x\theta$-planes and one $r\theta$-plane covering the central sector of the pre-diffuser exit. It is clear the data quality problems caused by glare and/or shadows, as described above, are present in all cases to some extent. The RMS velocity is approximately $\theta$-wise constant and, as seen in Fig A.7d, is below $15\%$ of $U_{ref}$ across the pre-diffuser exit plane. This indicates much lower turbulence intensity than measured for either the idealised combustor or vortex generator test cases.

It is also apparent from Fig A.10 that the radial velocity measured in the $r\theta$-plane is far in excess of that shown in Fig A.7d (i.e. measured in the $xr$-plane). This suggests the oblique viewing angle to this plane is causing the in-plane motion to be contaminated by the (much larger) axial velocity component.

![Fig A.7 - Pre-diffuser exit, $\theta = 0^\circ$. Shadow region, Glare region, Raw image, Mean axial velocity, Mean radial velocity, RMS velocity](image-url)
Fig A.8 – Pre-diffuser exit mean axial velocity over $xr$-planes at $\theta = 0^\circ$, $\pm 3.75^\circ$, $\pm 7.5^\circ$

Fig A.9 – Pre-diffuser exit mean axial velocity over $x\theta$-planes at $r/H_{\theta=0} = 0.2, 0.5, 0.8$

(Note: Radial dimension stretched for clarity)
Although the data taken using this geometry has shown high-speed PIV can be obtained for complex airflow geometries, the optical access to the planes tested did not allow monoscopic data of sufficient quality to be considered for use in inlet condition generation to be captured. Stereoscopic PIV could alleviate some of these issues by necessitating oblique viewing, although the time available in this project did not allow further data to be acquired. However, it could be argued that the low turbulence intensity and lack of large scale structures at pre-diffuser exit make it an unsuitable test case for the development of unsteady inlet condition generation methodologies. Future tests using this geometry could be better served by capturing data in the annuli inlets, which should present easier optical access and reduce the complexity of the subsequent LES simulation.
Appendix B – Reynolds Stress Transform

The requirement is to transform a series of velocities with no correlations between components to a set of velocities with defined stresses.

Let the velocities be arranged in the matrix, R, such that:

\[
R = \begin{bmatrix}
U_1 & V_1 & W_1 \\
U_2 & V_2 & W_2 \\
\vdots & \vdots & \vdots \\
U_N & V_N & W_N
\end{bmatrix}
\]

where

\[
\begin{align*}
\langle U \rangle & \equiv 0 \equiv \langle V \rangle \equiv \langle W \rangle \\
\langle UU \rangle & \equiv \langle VV \rangle \equiv \langle WW \rangle \\
\langle UV \rangle & \equiv 0 \equiv \langle UW \rangle \equiv \langle VW \rangle \\
\end{align*}
\]

\(
\implies \rightarrow \quad \text{as } N \rightarrow \infty
\)

Then

\[
R^T \cdot R \equiv N \cdot I \equiv \begin{bmatrix} N & 0 & 0 \\ 0 & N & 0 \\ 0 & 0 & N \end{bmatrix}
\]

However, we require Q, where

\[
Q = \begin{bmatrix}
\langle uu \rangle & \langle uv \rangle & \langle uw \rangle \\
\langle vu \rangle & \langle vv \rangle & \langle vw \rangle \\
\langle wu \rangle & \langle vw \rangle & \langle ww \rangle
\end{bmatrix}
\]

Note: \(\langle u \rangle = \langle v \rangle = \langle w \rangle = 0\)

\[
Q^T \cdot Q = S = \begin{bmatrix}
\langle uu \rangle & \langle uv \rangle & \langle uw \rangle \\
\langle vu \rangle & \langle vv \rangle & \langle vw \rangle \\
\langle wu \rangle & \langle vw \rangle & \langle ww \rangle
\end{bmatrix}
\]

(diagonally symmetric)

Is there a transformation such that: \(AR^T = Q^T\)?

This implies:

\[
(AR^T) \cdot (AR^T)^T = S
\]
Hence multiplying for one element of $S_{ij}$, (note: $S_{ij} = S_{ji}$)

$$(A R^T) (A R^T)^T => S_{ij} = \frac{1}{N} \sum_{k=1}^{N} [(a_{i1} U_k + a_{i2} V_k + a_{i3} W_k)(a_{j1} U_k + a_{j2} V_k + a_{j3} W_k)]$$

Cross multiply and remove $U_k V_k$ etc terms as $\to 0$ as $N \to \infty$

$$S_{ij} = \frac{1}{N} \sum_{k=1}^{N} \left( a_{i1} a_{j1} U_k^2 + a_{i2} a_{j2} V_k^2 + a_{i3} a_{j3} W_k^2 \right)$$

$$S_{ij} = a_{i1} a_{j1} \frac{\sum_{k=1}^{N} U_k^2}{N} + a_{i2} a_{j2} \frac{\sum_{k=1}^{N} V_k^2}{N} + a_{i3} a_{j3} \frac{\sum_{k=1}^{N} W_k^2}{N}$$

And since $U_k$, $V_k$ and $W_k$ have been generated such that $\langle U U \rangle \equiv 1 \equiv \langle V V \rangle \equiv \langle W W \rangle$

$$S_{ij} = a_{i1} a_{j1} + a_{i2} a_{j2} + a_{i3} a_{j3}$$

As $S$ is diagonally symmetric there is only 6 equations, therefore let

$$a = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

such that the transformation is not over-prescribed, then:

$$S_{11} = \langle uu \rangle \therefore a_{11} = \langle uu \rangle^{1/2}$$

$a_{12}$, $a_{13}$ and $a_{23} = 0$ by choice
Appendix C – Operation Analysis

The following is an operation count-based analysis of the computational processes and memory storage required for each stage of the four synthetic inlet generation methodologies evaluated in section 5.2. Each operation is expressed in terms of the parameters outlined in section 5.2.2.

For the purpose of this analysis it is assumed the LES grid is of approximately the same size and density as the source PIV measurement grids. Where appropriate, for the purposes of this analysis it is assumed all 3 components are required. At each stage the processing requirements and the total memory storage are denoted by \( P \) and \( M \) respectively.

Level 1: Scaled White Noise

1) Interpolate RMS profiles from PIV to LES grid
\[
\begin{align*}
P & : N_i \times N_j \times N_c \\
M & : N_i \times N_j \times N_c
\end{align*}
\]

2) Generate and scale random number
\[
\begin{align*}
P & : N_i \times N_j \times N_c \times N_{\Delta l} \\
M & : (1+N_{\Delta l}) \times N_i \times N_j \times N_c
\end{align*}
\]

Result:

Total processing operations: \((1+N_{\Delta l})\times N_i \times N_j \times N_c\)
Maximum memory storage: \((2+N_{\Delta l})\times N_i \times N_j \times N_c\)

Level 2: DFG

1) Calculate CCFs for lengthscales along component directions (equation 3.8)
\[
\begin{align*}
P & : N_i \times N_j \times N_{\Delta l} \times (N_l+N_j+N_{\Delta l}) \\
M & : N_i \times N_j \times (N_l+N_j+N_{\Delta l})
\end{align*}
\]

2) Integrate to find lengthscales (equation 3.10)
\[
\begin{align*}
P & : N_i \times N_j \times (N_l+N_j+N_{\Delta l}) \\
M & : N_i \times N_j \times N_c
\end{align*}
\]

3) Calculate Reynolds stresses (equation 3.5)
\[
\begin{align*}
P & : 2 \times N_i \times N_j \times N_c \times N_{\Delta l} \\
M & : 3 \times N_i \times N_j \times N_c
\end{align*}
\]

4) Interpolate Reynolds stresses and lengthscales from PIV and LES grid
\[
\begin{align*}
P & : 3 \times N_i \times N_j \times N_c \\
M & : 3 \times N_i \times N_j \times N_c
\end{align*}
\]

5) Determine characteristic (constant) lengthscales
P: \[ 3 \times N \times N \times N_c \]

6) Generate random number field

P: \[ (N_i + 4n_\lambda) \times (N_j + 4n_\lambda) \times 4n_\lambda \times N_c \]

M: \[ (N_i + 4n_\lambda) \times (N_j + 4n_\lambda) \times 4n_\lambda \times N_c + 2 \times N_i \times N_j \times N_c \]

7) Generate 1D filter equation (equation 3.44)

P: \[ N_c \times 4n_\lambda \]

M: \[ (N_i + 4n_\lambda) \times (N_j + 4n_\lambda) \times 4n_\lambda \times N_c + 2 \times N_i \times N_j \times N_c + N_c \times 4n_\lambda \]

8) Generate 3D filter equation

P: \[ (4n_\lambda)^3 \]

M: \[ (N_i + 4n_\lambda) \times (N_j + 4n_\lambda) \times 4n_\lambda \times N_c + 2 \times N_i \times N_j \times N_c + (4n_\lambda)^3 \]

9) Perform 3D convolution of random field and filter (equation 3.40). Note, as quoted by Veloudis et al. (2007)[102], convolution by FFT reduces the effective number of operations by 70%.

P: \[ 0.3 \times (4n_\lambda)^3 \times (N_i + 4n_\lambda) \times (N_j + 4n_\lambda) \times N_c \times 4n_\lambda \times N \times N_M \]

M: \[ (N_i + 4n_\lambda) \times (N_j + 4n_\lambda) \times 4n_\lambda \times N_c + 2 \times N_i \times N_j \times N_c + (4n_\lambda)^3 \]

+ \[ N_i \times N_j \times N_c \times N_M \]

10) Generate new slice of random data for each new time step

P: \[ N_i \times N_j \times N_c \times N_M \]

M: \[ (N_i + 4n_\lambda) \times (N_j + 4n_\lambda) \times 4n_\lambda \times N_c + 2 \times N_i \times N_j \times N_c + (4n_\lambda)^3 \]

+ \[ N_i \times N_j \times N_c \times N_M \]

11) Normalise filtered field to zero mean and unit variance

P: \[ 2 \times N_i \times N_j \times N_c \times N_M \]

M: \[ N_i \times N_j \times N_c \times (2+N_M) \]

12) Perform Reynolds stress transformation (equation 3.46)

P: \[ N_i \times N_j \times (6+N_c \times N_M) \]

M: \[ N_i \times N_j \times N_c \times (2+N_M) \]

Result:

Total processing operations:
\[ N_i \times N_j \times N_M \times (N_i + N_j + N_M) + N_i \times N_j \times (N_i + N_j + N_M) \]
\[ + 6 \times N_i \times N_j \times N_c \times N_M + 4 \times N_i \times N_j \times N_c + 6 \times N_i \times N_j \]
\[ + 4 \times (N_i + 4n_\lambda) \times (N_j + 4n_\lambda) \times n_\lambda \times N_c + 4 \times N_c \times n_\lambda + 64 \times n_\lambda^3 \]
\[ + 77 \times n_\lambda^4 \times (N_i + 4n_\lambda) \times (N_j + 4n_\lambda) \times N_c \times N_M \]

Maximum memory storage:
\[ (N_i + 4n_\lambda) \times (N_j + 4n_\lambda) \times 4n_\lambda \times N_c + 2 \times N_i \times N_j \times N_c + (4n_\lambda)^3 \]
\[ + N_i \times N_j \times N_c \times N_M \]
Level 3: sLSE

1) Calculate and interpolate slave field correlations from PIV to LES grid. Assume the number of reference points is equal to $3N_j$ (see section 4.1)

$$P: \quad 3 \times N_j \times N_j^2 \times N_c \times N_A \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

$$M: \quad 3 \times N_j \times N_j^2 \times N_c \times N_A \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

2) Calculate correlations between reference points

$$P: \quad 9 \times N_j^2$$

$$M: \quad 3 \times N_j \times N_j^2 \times N_c \times N_A \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

$$+ 9 \times N_j^2$$

3) Invert reference point correlation matrix

$$P: \quad 9 \times N_j^2$$

$$M: \quad 3 \times N_j \times N_j^2 \times N_c \times N_A \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

$$+ 9 \times N_j^2$$

4) Calculate $a$ coefficients (equation 3.37)

$$P: \quad (3 \times N_j \times N_j^2) \times (3 \times N_j) \times N_c \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

$$M: \quad N_j \times N_j^2 \times N_c \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right) \times (1 + 3 \times N_A)$$

$$+ 9 \times N_j^2$$

5) Reconstruct correlated velocity field from LSE (equation 3.36)

$$P: \quad 3 \times N_j \times N_j^2 \times N_c \times N_A \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

$$M: \quad N_j \times N_j \times N_c \times N_A \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

6) Interpolate velocity field for high-pass filtering

$$P: \quad N_j \times N_j \times N_c \times N_A \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

$$M: \quad 2 \times N_j \times N_j \times N_c \times N_A \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

7) Calculate required uncorrelated energy fraction

$$P: \quad 2 \times N_j \times N_j \times N_c \times N_A \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

$$M: \quad 2 \times N_j \times N_j \times N_c \times N_A \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

$$+ 2 \times N_j \times N_j \times N_c \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

8) Calculate filter cut-on frequency

$$P: \quad N_j \times N_j \times N_c \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$

$$M: \quad 2 \times N_j \times N_j \times N_c \times N_A \times \left(1 - \frac{A_m}{A_{\text{tot}}}\right)$$
Appendix C

\[ + N_p N_j N_c N_{N_M} \times \left( 1 - \frac{A_m}{A_{tot}} \right) \]

9) High pass filter slave velocity field

\[ \text{P: } N_i N_j N_c N_{N_M} \times \left( 1 - \frac{A_m}{A_{tot}} \right) \quad \text{M: } 2N_i N_j N_c N_{N_M} \times \left( 1 - \frac{A_m}{A_{tot}} \right) \]

10) Combine correlated and uncorrelated velocity fields (equation 3.35)

\[ \text{P: } N_i N_j N_c N_{N_M} \times \left( 1 - \frac{A_m}{A_{tot}} \right) \quad \text{M: } N_i N_j N_c N_{N_M} \times \left( 1 - \frac{A_m}{A_{tot}} \right) \]

11) Interpolate master field fluctuations from PIV to LES grid

\[ \text{P: } N_i N_j N_c N_{N_M} \times \frac{A_m}{A_{tot}} \quad \text{M: } N_i N_j N_c N_{N_M} \]

Result:

Total processing operations: 
\[ N_i N_j N_c N_{N_M} \times \frac{A_m}{A_{tot}} + 6N_i N_j N_c N_{N_M} \times \left( 1 - \frac{A_m}{A_{tot}} \right) \]
\[ + 18N_j^2 + 9N_i N_j N_c \times \left( 1 - \frac{A_m}{A_{tot}} \right) \]
\[ + N_i N_j N_c N_{N_M} \times \left( 1 - \frac{A_m}{A_{tot}} \right) + N_i N_j N_c \times \left( 1 - \frac{A_m}{A_{tot}} \right) \]

Maximum memory storage: 
\[ N_i N_j N_c N_{N_M} \]

Level 4: Interpolated PIV data

1) Interpolate fluctuations from PIV to LES grids

\[ \text{P: } N_i N_j N_c N_{N_M} \quad \text{M: } N_i N_j N_c N_{N_M} \]

Result:

Total processing operations: 
\[ N_i N_j N_c N_{N_M} \]

Maximum memory storage: 
\[ N_i N_j N_c N_{N_M} \]
Appendix D - InGen Program Architecture

Start InGen

Display error & direct to re-enter

Enter location of LU-LES

Do files exist?

Load and check x.bin, y.bin, z.bin block data

Select generation options and number of PIV files

Enter each location of PIV data files

Display invalid files by name

Re-enter each location of invalid PIV data files

Generation method

Calculate Mean, RMS & TKE, L'scales as required

Are all files valid?

Load and check PIV data

Enter each location of PIV data files

Interpolate Mean & RMS to LES grid over inlets

Generate time series based on scaled Gaussian noise

Interpolate Rstresses over LES inlet grid & calculate representative L'scale/lengthscale

Convoluted random field with a constant L'scale assumption

Transform data to match Rstresses

Level 1: Scaled Noise

Level 2: DFG

Interpolate Rstresses over LES inlet grid & calculate representative L'scale/lengthscale

Convoluted random field with a constant L'scale assumption

Transform data to match Rstresses

Level 3: sLSE

Specify master field(s) in each inlet region

Interpolate and calculate correlations over slave fields to LES inlet grid

LSE reconstruct slave field(s) from overlapping the master(s) on LES grid

Any remaining slave fields?

Yes

Reconstructed field(s) become master(s) to any adjoining slave(s)

High pass filter interpolated PIV to missing TKE fraction and superimpose on slave

Interpolate original slave PIV to LES inlet grid

No

Interpolate (original) master field from PIV to LES grids

Level 4: Direct Interpolation

Interpolate all fields from PIV to LES grids

Interpolate any missing data from surrounding cells

End InGen

Export velocity data in LU-LES readable binary file

Scale inlets to prescribed mass flow (if requested)

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