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The Interaction of Tyre and Anti-Lock Braking in Vehicle Transient Dynamics

By

Manish Jaiswal
B.Tech, M.Sc

A thesis submitted in partial fulfillment of the requirements for the award of the degree
Doctor of Philosophy of Loughborough University

Wolfson School of Mechanical and Manufacturing Engineering
Loughborough University

January 2009
This Thesis is dedicated
to my Grandmother
and
in the memory of
my late Grandfather
Abstract

The thesis presents an intermediate modelling approach to study transient behaviour of vehicle systems, with emphasis put on simplified yet accurate representation of important system elements. A representative non-linear vehicle model is developed in MATLAB/Simulink environment, where non-linear characteristics of tyre, suspension and braking system are included to capture the dynamic behaviour of a vehicle under transient conditions. The novel aspect of this work is the application of a representative full vehicle-tyre-ABS integrated set-up to study the complicated interaction between tyre and anti-lock braking, under a range of demanding operating conditions, including combined cornering and braking.

The modelling methodology involves development of low end vehicle models, based on the Newton-Euler formulation. Subsequently, an intermediate vehicle model is devised, where more details are incorporated such as additional DOF to capture the sprung mass motion in space, along with its non-linear interactions with the un-sprung masses, large angle effects, kinematics of steering/wheels and an appropriate tyre model suitable for transient manoeuvres. Particular attention is paid to the suspension system modelling, through inclusion of non-linear effects in springs, dampers, bump-stops, and anti-roll bars, along with the jacking and anti-dive effects using the virtual work method. The model also incorporates a hydraulic brake model, based on the reduced order brake system dynamics for realistic simulation of the braking manoeuvres.

A complex multi-body ADAMS/Chassis model, with much greater level of detail, has also been established to extensively compare and enhance the realistic behaviour of the intermediate vehicle model. During the simulation exercise, the intermediate vehicle model has shown good agreement with the complex ADAMS model, thus justifying the accurate representation of vehicle non-linear characteristics, particularly the suspension system. The realistic behaviour of the vehicle model is further ascertained with a reliable GPS enabled test vehicle, by performing number of manoeuvres on test tracks, including combined cornering and braking.

A representative 4-channel conventional ABS system is modelled and integrated in the intermediate vehicle model. The ABS adopts generic peak seeking approach, employing wheel deceleration and brake slip as control variables. External braking inputs, in form of stepped pressure pulses, are also separately used to represent the transient braking system dynamics.

In the current work, different transient tyre models based on the single point contact approach and using Magic Formula steady-state characteristics are applied, while studying the influence of their dynamic behaviour on the ABS system. By employing a representative ABS system in a multi-body vehicle model and considering the particularly demanding situation of combined braking/cornering, it is shown that the models which are adequate for pure braking might struggle when the complicated full vehicle dynamics are excited. It is shown that the first order relaxation length approach may not be sufficient to fully satisfy the requirements of an ABS braking, especially if the relaxation length is not modelled as a variable dependent on tyre slip. In comparison, the modelling approach, where the carcass compliances and contact patch properties are explicitly represented, can handle the oscillatory tyre behaviour associated with ABS braking, in a far more accurate manner. In comparison to the earlier studies, which were mostly conducted for straight-line braking, this thesis stresses the fact that the tyre behaviour can be influenced by the complex interaction of handling and braking, and hence the effect should be captured while investigating or evaluating the performance of a tyre model in relation with ABS simulation.

Keywords: vehicle handling, ABS braking, intermediate vehicle modelling, multi-body dynamics, transient tyre modelling, transient manoeuvres, vehicle testing
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Nomenclature

Chapter 2

\( \alpha_f \)  
Front tyre slip angle

\( \alpha_r \)  
Rear tyre slip angle

\( \delta_f \)  
Steer angle at the front wheel

\( \lambda \)  
Wheel slip

\( C_{af} \)  
Cornering stiffness for front tyre

\( C_{ar} \)  
Cornering stiffness for rear tyre

\( g \)  
Acceleration due to gravity

\( K_{us} \)  
Understeer coefficient

\( L_w \)  
Wheelbase of the vehicle

\( m_f \)  
Mass of the vehicle at the front axle

\( m_r \)  
Mass of the vehicle at the rear axle

\( r \)  
Yaw rate

\( R \)  
Radius of curvature of the turn

\( S_1, S_2 \)  
Threshold values of wheel slip

\( U \)  
Vehicle forward velocity

Chapter 3

\( \alpha \)  
Slip Angle

\( \alpha_f \)  
Front tyre slip angle

\( \alpha_r \)  
Rear tyre slip angle

\( \alpha_{1-4} \)  
Slip angle of the tyres

\( \bar{\alpha} \)  
Normalized slip angle

\( \beta \)  
Side slip angle of the vehicle

\( \bar{\beta} \)  
Normalized combined slip / camber angle

\( \gamma \)  
Camber angle

\( \gamma_{\text{total}} \)  
Equivalent camber angle

\( \Delta F_{xz} \)  
Vertical load transfer front

\( \delta_0 \)  
Steer angle at the central vehicle axis

\( \delta_{1-4} \)  
Steer angles at the wheels

\( \delta F_{x,y,z} \)  
Force components of the single point P

\( \delta_f \)  
Steer angle at the front wheels
Nomenclature

- \( \delta M_{x,y,z} \) Components of moments about the \( x,y,z \) of the local frame of reference
- \( \delta m \) Mass of the single point \( P \)
- \( \varepsilon_{x,y} \) Roll steer coefficient at front, rear
- \( \eta \) Unit normal along the kingpin axis
- \( \eta(k), \eta_l \) Coefficient for normalized force
- \( \theta \) First rotation about \( x \)-axis in Euler transformation
- \( \theta_{rd} \) Road inclination angle
- \( \lambda \) Lateral inclination
- \( \mu \) Coefficient of friction
- \( \mu_x \) Longitudinal friction coefficient
- \( \mu_y \) Lateral friction coefficient
- \( \nu \) Caster Angle
- \( \rho_{1-4} \) Contact patch deflections
- \( \rho_a \) Air mass density
- \( \Sigma F_{x,y,z} \) Sum of forces in the direction of \( x,y,z \) of the local frame of reference
- \( \Sigma M_{x,y,z} \) Sum of moments in the direction of \( x,y,z \) of the local frame of reference
- \( \sigma_{ax, ay, az} \) Sign function in relation to the pressure difference
- \( \nu_p \) Normal to the wheel plane
- \( \nu_p' \) Movement of \( \nu_p \) about the unit vector \( \eta \)
- \( \phi \) Second rotation about \( y \)-axis in Euler transformation
- \( \phi_{rd} \) Road camber angle
- \( \phi_{wheel,x} \) Equivalent roll-angles at the front, rear axle of the vehicle
- \( \psi \) Third rotation about \( z \)-axis in Euler transformation
- \( \Omega_B \) Rotational velocity of the local or moving frame of reference
- \( \omega \) Rotational velocity of the local frame of reference
- \( \omega_{1-4} \) Wheel rotational speed
- \( \omega_{x,y,z} \) The rotational velocity components of the local frame of reference
- \( \partial y/{\partial \phi} \) Roll camber coefficient
- \( \partial x_{1-4\text{contact}} \) Infinitesimal contact patch longitudinal displacements
- \( \partial y_{1-4\text{contact}} \) Infinitesimal contact patch lateral displacements
- \( \partial z_{1-4\text{contact}} \) Infinitesimal contact patch vertical displacements
- \( A \) Global or fixed frame of reference
- \( a \) Distance of front axle from origin of the local frame of reference
- \( a_{O_b/A} \) Acceleration of origin \( O_b \) related to \( A \)
- \( A_l \) Contact patch centre
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<td>Coefficient for calculating cornering stiffness</td>
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<td>$A_2$</td>
<td>Contact patch centre (deflected position)</td>
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<tr>
<td>$A_d$</td>
<td>Diaphragm area</td>
</tr>
<tr>
<td>$A_{mc}$</td>
<td>Cross-section area of the master cylinder bore</td>
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<tr>
<td>$A_p$</td>
<td>Projected frontal area of the vehicle</td>
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<td>$a_{p/A}$</td>
<td>Acceleration of a particle $P$ relative to frame $A$</td>
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<tr>
<td>$a_{p/B}$</td>
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<td>$A_t$</td>
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<td>$B$</td>
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<td>$b$</td>
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<td>$C_{rf/r}$</td>
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<td>$C_{gf,g}$</td>
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<td>$C_y$</td>
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Nomenclature

- \( d_{1,4} \): Horizontal shift of the normal load towards front of the contact patch
- \( D \): Distance of the centre of turn from the vehicle central axis
- \( e \): Longitudinal distance of un-sprung mass with origin of the local frame of reference
- \( F_{\text{app}} \): Apply force as function of output force
- \( F_{\text{app}, r} \): Forces required to fully open the orifices in the apply and release stages
- \( F_{b64} \): Bump stop forces
- \( F_{cfs} \): Seal friction forces for the primary and secondary master cylinders
- \( F_{cfs} \): Spring preload and seal friction force for reduced order brake model
- \( F_{cfs} \): Return spring forces for the primary and secondary master cylinders
- \( F_{cso}, F_{eso} \): Initial return spring forces for the primary and secondary master cylinders
- \( F_{a, y, z} \): Aerodynamic forces
- \( F_{d, d} \): Suspension damper forces with respect to the vehicle body
- \( F_{d} \): Diaphragm force
- \( F_{g, y, z} \): Gravitational forces
- \( F_{in} \): Input force from the brake pedal linkage to the pushrod
- \( F_{N} \): Normalized lateral force for the positive camber angles
- \( F_{\text{out}} \): Booster output force
- \( F_{p} \): Reaction force through the washer to the power piston
- \( F_{pr} \): Reaction force through the washer to the push rod
- \( F_{r14} \): Rolling resistance force at the wheels
- \( F_{rel} \): Release force as function of output force
- \( F_{\text{roll} 14} \): Anti-roll bar forces at the wheels
- \( F_{s} \): Return spring force
- \( F_{s0} \): Return spring preload
- \( F_{s44} \): Suspension spring forces with respect to the vehicle body
- \( F_{\text{ susp} 4} \): Suspension forces
- \( F_{s} \): Valve spring force
- \( F_{d, d} \): Longitudinal force at the tyres along the vehicle central axis
- \( F_{x\text{ sym}} \): Longitudinal force at the tyres in the wheel plane
- \( F_{x\text{ sym}} \): Normalized longitudinal force in the wheel plane
- \( F_{y\text{ sym}} \): Lateral force at the tyres in the wheel plane
- \( F_{y, f, r} \): Lateral force at front, rear tyre
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<td>$F_{\text{zo}}$</td>
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<td>$F_{\text{fr, r}}$</td>
<td>Vertical force at front, rear tyre</td>
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<td>$F_{\text{ps, cr}}$</td>
<td>Resulting vertical forces due to suspension secondary motion in long.</td>
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<td>$F_{\text{sy, cr}}$</td>
<td>Resulting vertical forces due to suspension secondary motion in lateral</td>
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<td>$g$</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>$G$</td>
<td>Camber stiffness</td>
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<td>$h$</td>
<td>Height of roll axis from sprung mass location in 3-DOF model</td>
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<tr>
<td>$h_1$</td>
<td>Height of roll axis from CG of vehicle in 3-DOF model</td>
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\( v_{x_{\text{wil}}} \)  Longitudinal velocity at the wheel centres with respect to the wheel plane
\( v_{y_{\text{i}}} \)  Lateral velocity at the wheel centres with respect to the vehicle body
\( v_{y_{\text{wil}}} \)  Lateral velocity at the wheel centres with respect to the wheel plane
\( W_{\text{body}_{\text{i}}} \)  Velocities of the vehicle sprung body at 4 corners
\( W_i \)  Total weight of the vehicle
\( W_{\text{f},\text{x}} \)  Front, rear vehicle load
\( X, Y, Z \)  Coordinates of fixed frame of reference A
\( x, y, z \)  Coordinates of the local frame of reference B
\( x_{mc} \)  Master cylinder piston displacement for reduced order brake model
\( x_{mcp}, x_{mes} \)  Primary and secondary piston displacements
\( x_{\text{pp}} \)  Power piston displacement from the rest state
\( [x_w, y_w, z_w] \)  Wheel centre position vector in the local or global frame of reference.
\( z_{\text{body}_{\text{i}}} \)  Vertical displacements of the vehicle sprung body (4 corners)
\( z_{\text{bs}_{\text{i}}} \)  Bump stop clearances
\( z_{\text{ss}_{\text{i}}} \)  Resultant suspension deflections
\( z_{\text{wheel}_{\text{i}}} \)  Vertical displacements of the wheels

Chapter 4

\( \alpha \)  Slip angle
\( \alpha_{r,\text{eq}} \)  Equivalent slip for residual torque at combined slip
\( \alpha_s \)  Slip angle with offset for combined slip
\( \alpha_{r,\text{eq}} \)  Equivalent slip for pneumatic trail at combined slip
\( \alpha_y \)  Slip angle with offset
\( \gamma \)  Camber angle
\( \theta \)  2nd rotation about X in Euler transformation
\( \kappa \)  Longitudinal slip
\( \kappa_s \)  Longitudinal slip with offset for combined slip
\( \kappa_x \)  Longitudinal slip with offset
\( \lambda_i \)  Lagrange multiplier
\( \lambda_{\text{fr}} \)  Scaling factor for the friction decaying with increasing slip
\( \lambda_{\text{ax,ay}} \)  Scaling factor for the peak friction coefficient in x and y direction
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\( P_{h3} \)  Variation of shift \( S_{hy} \) with inclination

\( P_{kx1} \)  Longitudinal slip stiffness \( K_{xx}/F_z \) at \( F_{z0} \)

\( P_{kx2} \)  Variation of slip stiffness \( K_{xx}/F_z \) with load

\( P_{kx3} \)  Exponent in slip stiffness \( K_{xx}/F_z \) with load

\( P_{ky1} \)  Maximum value of stiffness \( K_{yy}/F_z \)

\( P_{ky2} \)  Load at which \( K_{yy} \) reaches maximum value

\( P_{ky3} \)  Variation of \( K_{yy}/F_z \) with inclination

\( P_{vx1} \)  Vertical shift \( S_{vx}/F_z \) at \( F_{z0} \)

\( P_{vx2} \)  Variation of shift \( S_{vx}/F_z \) with load

\( P_{vy1} \)  Vertical shift in \( S_{vy}/F_z \) at \( F_{z0} \)

\( P_{vy2} \)  Variation of shift \( S_{vy}/F_z \) with load

\( P_{vy3} \)  Variation of \( S_{vy} \) with inclination

\( P_{vy4} \)  Variation of \( S_{vy} \) with inclination and load

\( q_{B1} \)  Trail slope factor for trail \( B_1 \) at \( F_{z0} \)

\( q_{B10} \)  Variation of slope \( B_1 \) with \( B_y \)

\( q_{B2} \)  Variation of slope \( B_1 \) with load

\( q_{B3} \)  Variation of slope \( B_1 \) with squared load

\( q_{B4} \)  Variation of slope \( B_1 \) with inclination

\( q_{B5} \)  Variation of slope \( B_1 \) with absolute inclination

\( q_{B6} \)  Residual moment slope factor at \( F_{z0} \)

\( q_{C1} \)  Shape factor \( C_1 \) for pneumatic trail

\( q_{D1} \)  Peak trail \( D_1 \)

\( q_{D2} \)  Variation of peak \( D_1 \) with load

\( q_{D3} \)  Variation of peak \( D_1 \) with inclination

\( q_{D4} \)  Variation of peak \( D_1 \) with squared inclination

\( q_{D6} \)  Peak residual moment \( D_r/(F_z \cdot r_0) \)

\( q_{D7} \)  Variation of peak factor \( D_r \) with load

\( q_{D8} \)  Variation of peak factor \( D_r \) with inclination
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<td>$S_{hy}$ Horizontal and vertical offset for the lateral force at pure slip</td>
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**Nomenclature**

- $y_x$: horizontal asymptote in Magic Formula
- $Y(X)$: Magic Formula output variables with offset, with $X$ as the primary input variables
- $y(x)$: Typical output variables of the Magic Formula, with $x$ as the primary input variables

### Chapter 6

- $\alpha$: Slip angle
- $\alpha'$: Transient slip angle
- $\alpha'_t$: Effective slip angle which account for the shift $\Delta \alpha$
- $\gamma$: Camber angle
- $\gamma_y$: Camber variation
- $\varepsilon_f$: Factor to avoid singularity while calculating transient slip
- $\varepsilon_{NL}$: Non-lagging fraction
- $\theta$: Angle between direction of wheel travel and tangent to the central contact line
- $\kappa$: Longitudinal slip
- $\kappa'$: Transient longitudinal slip
- $\lambda$: Longitudinal brake slip
- $\lambda_1, \lambda_{dres}$: Slip thresholds
- $\mu$: Friction coefficient
- $\mu_b$: Braking force coefficient
- $\mu_s$: Lateral force coefficient
- $\xi$: Distance between point P and S
- $\sigma_a$: Lateral relaxation length
- $\sigma_a^*$: Lateral relaxation length for the modified stretched-string tyre model
- $\sigma_{\alpha_0}$: Initial lateral relaxation length at $\alpha = 0$
- $\sigma_c$: Contact relaxation length
- $\sigma_\kappa$: Longitudinal relaxation length
- $\sigma_{\kappa_0}$: Initial longitudinal relaxation length at $\kappa' = 0$
- $\sigma_\kappa^*$: Longitudinal relaxation length for modified stretched-string tyre model
- $\phi$: Tumslip
- $\omega$: Wheel rotational speed
- $\omega_{th_{min}}$: Angular wheel deceleration minimum threshold value
- $\omega_{th_{max}}$: Angular wheel acceleration pronounced threshold value
- $\omega_{th_{max}}$: Angular wheel acceleration maximum threshold value
- $+A$: Pronounced wheel's peripheral acceleration threshold
Nomenclature

+\( a \)  \( \) wheel’s peripheral acceleration threshold
-\( a \)  \( \) Wheel peripheral deceleration threshold
-\( a_{\text{max}} \)  \( \) Maximum wheel peripheral deceleration
\( a \)  \( \) Half the contact length in the wheel plane direction
\( a, b \)  \( \) Stable and unstable region of the \( \mu \)-slip curve
\( B \)  \( \) Tangent point to the central contact line of the tyre
\( C \)  \( \) Intersection point of tangent to the central contact line and the wheel centre plane
\( c_{\text{ex}} \)  \( \) Longitudinal damping
\( c_{\text{ey}} \)  \( \) Lateral damping
\( C_{\text{\( \gamma \alpha \)}} \)  \( \) Cornering stiffness
\( C_{\text{\( \gamma f \)}} \)  \( \) Camber thrust stiffness
\( C_{\text{\( \gamma \text{Fx} \)}} \)  \( \) Longitudinal slip stiffness
\( C_{\text{\( \gamma \text{Fx} \)}} \)  \( \) Longitudinal tyre stiffness
\( C_{\text{\( \gamma f y \)}} \)  \( \) Lateral tyre stiffness at the road level
\( D \)  \( \) Intersection point of direction of wheel travel and tangent to the central contact line
\( d \)  \( \) Small positive number used in the calculation of reference velocity
\( d_{f z} \)  \( \) Non-dimensional vertical load increment
\( F_{\text{x}} \)  \( \) Longitudinal tyre force
\( F_{\text{xa}} \)  \( \) Longitudinal force acting on the wheel rim
\( F_{\text{y}} \)  \( \) Lateral tyre force
\( F_{\text{ya}} \)  \( \) Lateral force acting on the wheel rim
\( F_{\text{y,\text{NL}}} \)  \( \) The non-lagging part of the camber force
\( F_{\text{ym}} \)  \( \) Lateral tyre force at steady-state condition
\( F_{\text{z}} \)  \( \) Tyre vertical load
\( F_{\text{\text{\text{z}\}}0} \)  \( \) Reference normal load
\( G_{VK} \)  \( \) Weighing function
\( H_{\text{Ref}} \)  \( \) Gain value corresponding to the vehicle deceleration
\( I_{\text{\text{\text{wheel}}} \}} \)  \( \) Mass moment of inertia of the wheel assembly
\( k_{\text{\text{\text{ex}}} \}} \)  \( \) Longitudinal stiffness
\( k_{\text{\text{\text{ey}}} \}} \)  \( \) Lateral stiffness
\( M_{\text{b}} \)  \( \) Braking toque
\( m_{\text{c}} \)  \( \) Contact body mass
\( M_{r} \)  \( \) Road friction torque

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<td>Maximum road friction torque</td>
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<td>$M_z$</td>
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<td>$T$</td>
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<td>$u$</td>
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<td>$u_l$</td>
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<td>$V$</td>
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<td>Lateral deflection of the carcass</td>
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<td>$v_l$</td>
<td>Lateral displacement of the leading edge of the contact point</td>
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<tr>
<td>$V_{sx}$</td>
<td>Longitudinal slip velocity</td>
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</table>
Nomenclature

\[ V_{sx} \] Longitudinal slip velocity of the contact patch mass point
\[ V_{sy} \] Lateral slip velocity
\[ V_{sy}^{*} \] Lateral slip velocity of the contact patch mass point
\[ V_w \] Wheel lateral velocity perpendicular to the wheel plane
\[ X \] Longitudinal axis in the direction of wheel travel
\[ Y \] Lateral axis perpendicular to the direction of wheel travel
\[ Y_t \] Ordinate of point B
**List of Abbreviations**

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<td>ABS</td>
<td>Anti-Lock Braking System</td>
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<tr>
<td>ADAMS</td>
<td>Automatic Dynamic Analysis of Mechanical Systems</td>
</tr>
<tr>
<td>AMESim</td>
<td>Advanced Modeling Environment for performing Simulations of engineering systems</td>
</tr>
<tr>
<td>BS</td>
<td>British Standard</td>
</tr>
<tr>
<td>CAN</td>
<td>Controller Area Network</td>
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<tr>
<td>CG</td>
<td>Centre of Gravity</td>
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<tr>
<td>DADS</td>
<td>Dynamic Analysis and Design System</td>
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<tr>
<td>DAQ</td>
<td>Data Acquisition</td>
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<tr>
<td>DC</td>
<td>Direct Current</td>
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<tr>
<td>DOE</td>
<td>Design of Experiment</td>
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<tr>
<td>DOF</td>
<td>Degrees of Freedom</td>
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<tr>
<td>DSP</td>
<td>Digital Signal Processing</td>
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<tr>
<td>ECU</td>
<td>Electrical Control Unit</td>
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<tr>
<td>ESP</td>
<td>Electronic Stability Program</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>FRFs</td>
<td>Frequency Response Functions</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>HVE</td>
<td>Human-Vehicle-Environment</td>
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<tr>
<td>HVOSM</td>
<td>Highway Vehicle Object Simulation Model</td>
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<tr>
<td>IAVSD</td>
<td>International Association for Vehicle Systems Dynamics</td>
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<tr>
<td>LCA</td>
<td>Lower Control Arm</td>
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<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
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<tr>
<td>ISO</td>
<td>International Organization for Standardization</td>
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<td>LPM</td>
<td>Lumped Parameter Model</td>
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<td>LVDS</td>
<td>Light Vehicle Dynamics Simulation</td>
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<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>MADYMO</td>
<td>Mathematical Dynamic Modeling</td>
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<tr>
<td>MBF</td>
<td>Multi-Body Formation</td>
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<tr>
<td>MIRA</td>
<td>Motor Vehicle Industry Research Association</td>
</tr>
<tr>
<td>NAVDyn</td>
<td>Non-Linear Analysis of Vehicle Dynamics</td>
</tr>
<tr>
<td>NHTSA</td>
<td>National Highway Traffic Safety Administration</td>
</tr>
<tr>
<td>NVH</td>
<td>Noise Vibration Harshness</td>
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<tr>
<td>PC</td>
<td>Personal Computer</td>
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<tr>
<td>PID</td>
<td>Proportional–Integral–Derivative</td>
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<tr>
<td>PWM</td>
<td>Pulse Width Modulation method</td>
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<tr>
<td>RSM</td>
<td>Road Surfacing Material</td>
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<tr>
<td>SAE</td>
<td>Society of Automotive Engineers</td>
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<tr>
<td>SAM</td>
<td>Simulation and Analysis Model</td>
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<td>STI</td>
<td>Standard Tyre Interface</td>
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<tr>
<td>STI</td>
<td>Systems Technology Inc.</td>
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<td>SVC</td>
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<td>SWIFT</td>
<td>Short Wavelength Intermediate Frequency Tyre</td>
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<td>TCS</td>
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<td>TMPT</td>
<td>Tyre Model Performance Test</td>
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<td>UMTRI</td>
<td>University of Michigan Transportation Research Institute</td>
</tr>
<tr>
<td>UCA</td>
<td>Upper Control Arm</td>
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<td>VDANL</td>
<td>Vehicle Dynamics Analysis Non-Linear</td>
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<td>VDAS</td>
<td>Vehicle Dynamics Analysis Software</td>
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<tr>
<td>VDC</td>
<td>Vehicle Dynamics Control</td>
</tr>
<tr>
<td>VSA</td>
<td>Vehicle Stability Assist</td>
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<tr>
<td>XML</td>
<td>Extensible Markup Language</td>
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</table>
1 Introduction

1.1 Preamble

The study of the dynamic behaviour of vehicles using computer simulation has been one of the major areas of research for many years. Based on the application area, the vehicle systems used for performing these studies vary greatly in their capability, complexity and amount of data required. Looking at the vehicle modelling spectrum, on the one hand, there is multi-body formulation approach, where the commercially available packages have enabled the users to build complex vehicle models. In this approach the tendency is to represent most of the vehicle subsystems in great detail by employing rigid or flexible bodies with numerous joints and couplings. On the other hand, there is the classical modelling approach, where the equations of motion are mostly formulated by the users in the Newton-Euler form, resulting in simpler models with only a few degrees of freedom and a minimum number of parameters. While, the multi-body formulation approach is highly advantageous when it comes to iterative design optimization at both system and component levels, its requirement for large amount of data and the modelling effort often makes it a less preferable approach for people working outside the vehicle industry. The non-reliability of the large data sets (obtained from different sources) also acts as a constraint many times. In comparison, the simple models with far less parameters and faster computational times are mostly preferred for studying basic vehicle handling characteristics and vehicle control problems. However, their severe limitation is to adequately represent vehicle non-linear behaviour, especially during transient operating conditions.

Although, both the modelling approaches have their own set of advantages and drawbacks, its not always pragmatic to model a vehicle based on one or the other extremes. In the past, though, many authors such as Sayers and Han, (1996), Dickison and Yardley, (1993), Crolla et al, (1994), Allen and Rosenthal, (1994), Willumeit et al, (1992), and Lukowski and Medeksza, (1992) have advocated the need for reduced complexity of a vehicle model, it is not so straightforward to draw a line for the level of complexity a model or system in general should possess. At the end, whether it is customized or generic vehicle model, it is important that any decision to include the requisite amount of parameters or components details involve proper consideration of the problem area to be addressed.
Introduction

By combining the two aforementioned modelling extremes, a third modelling approach can be used, where using the Newton-Euler formulation for the derivation of equations of motion, it is possible to derive non-linear multi-body models for the simulation of handling dynamics (Ellis, 1994). This approach, also referred to as intermediate modelling approach (Zainul-Abidin, 2005), can provide sufficiently accurate results for use in prediction, assessment and optimisation studies. The models formulated, using Newton-Euler approach, although, require less computational effort than any method based on the constrained Lagrangian dynamics, but at the same time, they are susceptible to inaccuracies induced by various assumptions/simplifications made during the derivation process. However, an additional advantage of such intermediate models is that their relatively simple structure allows general trends to be observed in relation to vehicle parameters. This leads to useful conclusions which might be significantly more difficult to draw when working with more complex models which exhibit a large number of interactions. As a result, researchers often rely on simple models when it comes to parametric investigations, avoiding the use of high-end multi-body software packages (Renfroe et al, 2006, Hac, 2002, Allen et al, 1999).

Keeping in context the application area - for the studies involving critical handling and braking manoeuvres, it is imperative to include certain amount of detail in the vehicle model in a realistic manner, especially for representation of suspension, steering and tyre characteristics. However, when it comes to intermediate vehicle modelling, it is always a challenge to maintain an optimum balance between the two modelling extremes, as one has to ensure that the ease of parametric investigation along with computational efficiency is not lost, while keeping the correct amount of complexity in a vehicle model.

1.2 Problem Description

Vehicle braking and stability control has an important place in the study of handling performance. Over the years, a large effort has been expended in the development of various active systems to assist braking and directional control of the vehicle under various driving conditions, among which the prime ones such as anti-lock-braking system, traction control system, four wheel steering system, and vehicle stability control system have already made their way into production passenger vehicles. Different methods of applying control on these braking systems have been developed; from linear control to highly robust non-linear control. Whilst most of the control system studies are conducted using relatively simple
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vehicle models, the complexity of the model along with its various parameters and the severity of manoeuvres have significant effects on the performance of a particular control system. More studies are required to establish the influence of these parameters on the behaviour of the control systems, so that a wider knowledge-base is established for the simulation of real life driving situations.

Among the different active braking control systems, anti-lock braking system is the most commonly used, whose objective is to not only prevent the wheel from locking, but also retain an adequate cornering force for directional stability. To achieve its objective, the ABS system should be ideally capable of exploiting frictional force between the tyre and road surface to its maximum extent. However, the frictional force coefficient is a non-linear function of the wheel slip, where the longitudinal braking coefficient reaches its peak value at some intermediate value of brake slip, whereas the lateral force coefficient attains its maximum value at the lowest value of brake slip. In addition, the relation between the frictional force coefficient and brake slip (commonly represented as $\mu - \text{slip curve}$) also varies with the road surface and driving condition. This means, for efficient operation, the ABS system has to maintain the brake slip within an optimum range so that the tyre cannot only develop high braking force for stopping the vehicle, but also retain sufficiently large lateral force, while cornering. However, for performing slip control, the correct determination of the primary input parameter (i.e. wheel angular speed) holds significance as it directly affects the calculation or estimation of the wheel slip.

During an ABS operation, there is rapid fluctuation of the brake pressure at the wheels, as the ABS control performs brake pressure modulation by apply, release or hold cycles for the hydraulic pressure through the solenoid valves. The tyre-wheel assembly also experiences excitation from the road surface irregularities, which along with the brake pressure fluctuation translates into rapid fluctuation of the angular wheel speed as well as the brake slip. These fluctuations in the tyre-wheel assembly, when coupled with the load variations and the other transients present in the axle and suspension, have an adverse impact on the angular wheel speed signals. As the ABS system relies heavily on the information from the wheel speed sensors, any disturbance there can affect its overall functioning. These fluctuations or oscillations of the wheel speed are also a result of the tyre transient behaviour, which in the aftermath of the structural deformations (carcass, sidewall, belt and tread), due to various time bound external excitations causes lags in the longitudinal and lateral force response in relation to the tyre slippages. Hence, in an ABS simulation study, it
is quite crucial to have a dynamic tyre model, which apart from predicting accurate behaviour, can also handle fast transients and high slippages.

In the past, several authors (van der Jagt et al, 1989, Zegelaar and Pacejka, 1997, Jansen et al, 1999, Pauwelussen et al, 2003, Braghin et al, 2006) have applied transient tyre models for ABS studies, where these authors have either resorted to quarter vehicle models or preliminary runs for straight-line braking. In another study, van Zanten et al, (1989, 1990) have conducted detailed experimental investigation, where the authors studied transients in longitudinal and lateral tyre forces in relation to an ABS system, using experimental car fitted with rotating wheel dynamometer. The authors further extended their study to develop a transient tyre model, which was simulated in a modelled tyre test stand, so as to validate the experimental investigation. While, most of the aforementioned studies have tried to establish the effectiveness of the particular transient tyre model to study responses in relation to the high frequency input such as road undulations and brake torque variations. However, much scope still exists to study the interaction of tyre and anti-lock braking, particularly during combined cornering and braking manoeuvres, where due to interaction of handling and braking, the influence of tyre transience may be significantly different in relation to the straight-line braking.

Like in any simulation oriented research work, to conduct such a study, one has to confront an important issue i.e. the decision about the modelling approach and the overall complexity of the method. At the first sight, one can be tempted to go for a detailed approach where all the three components i.e. vehicle model, ABS braking system and tyre model are represented in significant detail, covering the various non-linear or the transient aspects of their behaviour. However, such an approach may not be entirely practical, as there is a good chance that all the above three components may need different modelling tools or environments, where issues like compatibility, computational time/effort, as well as effective integration can create undue difficulties. On the other hand, one may go with a fairly simplified modelling approach, such as those mostly followed in control oriented studies. However, this will be hard to justify as the key objective is the study of the influence of tyre transient behaviour, under the influence of other vehicle sub-systems, on ABS braking. Hence, keeping the overall direction of the research in a streamline manner necessitates a representative system modelling approach to be selected, where not only the vehicle model, but also the choice of other components such as transient tyre model and the ABS brake system is influenced by the idea that there needs to be an adequate balance between the
complexity and simplicity, while representing a dynamic vehicle system for transient analysis.

1.3 Overall Aims and Objectives

Keeping in perspective the problems stated in the preceding sections, the overall aim and objectives of the research project are listed below:

The aim of the present research is to conduct a simulation study to investigate the complicated interaction between tyre and anti-lock braking, under a range of demanding operating conditions, including combined cornering and braking. Conducting such a study in a representative full vehicle-tyre-ABS integrated set-up in a multi-body simulation environment may not only aid in better prediction of the tyre behaviour (in more demanding situations), but will also help in establishing an understanding of the transient tyre model requirements for performing vehicle braking and stability control studies.

Towards the achievement of aims, the following objectives have to be met.

Developing a representative vehicle model for transient analysis is the starting point in the research. The first objective is to develop a vehicle model, based on Newton-Euler formulation of equations of motion, with adequate representation of the various non-linearities. In order to critically access the vehicle model (also referred as intermediate vehicle model), a multi-body ADAMS model with much greater level of detail will be established, thus ascertaining the performance of the intermediate vehicle model for a range of transient handling manoeuvres. Later, an experimental test programme will be conducted to further evaluate the intermediate vehicle model against an instrumented vehicle on test track.

The second objective is to develop a representative ABS control algorithm, based on peak seeking approach, and integrate in with the intermediate vehicle model. Apart from capturing the important characteristics of a conventional ABS system in the control algorithm, large effort has to be expended to correctly establish the various ABS parameters and their values, through published work and by performing number of iterations.

The third objective is to explore the various approaches for transient tyre modelling based on the widely used semi-empirical Magic Formula characteristics, and to implement them in the intermediate vehicle model.
Finally, a series of simulation need to be conducted, in a full vehicle-tyre-ABS integrated modelling environment, to investigate the interaction between transient tyre response and anti-lock braking control, for a range of operating conditions, including combined cornering and braking. Additionally, to further investigate the tyre response under transient braking system dynamics, a stepped brake pressure input with short pulses will be applied, to compare the performance of the different tyre models.

To meet the above aims and objectives, the following work plan is devised:

1. Development of simple vehicle models, like those mostly used in control studies.

2. Development of a 10-DOF intermediate vehicle model in MATLAB-Simulink environment, incorporating sufficient DOFs to capture all the translational and rotational motions of the unsprung masses in space, along with adequate representation of the various sub-systems such as suspension, steering, wheel-tyre and driving/braking system.

3. Establishing a detailed multi-body dynamic model in ADAMS/Chassis environment, with non-linear characteristics of different assembled parts, including compliances in the body, suspension and steering sub-systems, and perform comparison with the intermediate vehicle model for a range of standard handling manoeuvres.

4. Ascertaining validity of the intermediate vehicle model against the actual instrumented vehicles by performing real life transient handling manoeuvres on test tracks.

5. Modelling a generic ABS braking system based on the conventional peak seeking approach, with the brake control cycle an adaptation of the Bosch work (Bosch, 1999).

6. Applying the various transient tyre models, based on Magic Formula characteristics, for investigating their interaction with anti-lock braking control, for straight-line and cornering manoeuvres on different surfaces, in a full vehicle simulation environment.

1.4 Structure of the thesis

The thesis is organised in seven chapters. A brief description of the work carried out in each chapter of the thesis is given below:
Chapter 1 – Introduction: The chapter presents some fundamental ideas in vehicle modelling, with an overview of problem description. The aims and objectives of the work are also defined.

Chapter 2 – Literature Review: This chapter presents a review of the earlier work published in the area of vehicle handling dynamics. The review of different vehicle modelling approaches is presented, along with its various sub-systems such as suspension, tyre, and braking system (including ABS brake modelling). The chapter also looks into the influence of transient tyre behaviour on ABS braking.

Chapter 3 – Theoretical Vehicle Modelling: A fundamental description of the theoretical vehicle model is discussed in this chapter, where the Newton-Euler approach is adopted for the formulation of equations of motion for a 6-DOF of sprung mass. The approach is extended to develop a 10-DOF intermediate vehicle model. The chapter also includes description of the hydraulic brake system model, as an integrated part of the vehicle model.

Chapter 4 – Multi-body Model and Simulation Results: This chapter presents a detailed multi-body vehicle model, which is established in ADAMS/Chassis environment. This chapter also covers the description of the PAC2002 tyre model, which is based on the latest version of Magic Formula tyre model. The chapter also presents the simulation results of various standard handling manoeuvres, performed on three different vehicle models i.e. 3-DOF non-linear model, 10-DOF intermediate model and multi-body vehicle model.

Chapter 5 – Experimental Vehicle Testing: This chapter presents the description of the instrumentation involved in the two different experimental test programs, along with the measurement results and their comparisons with the intermediate 10-DOF model.

Chapter 6 – Influence of Tyre Transients in Abs Braking: This chapter presents the different single point contact transient tyre models, based on semi-empirical Magic Formula characteristics. It also looks into the modelling and description of the ABS braking system, developed for the present work. Finally, the simulation work is presented, where the influence of tyre transients on ABS braking is investigated.

Chapter 7 – Overall Conclusion and Suggestion for Future Work: This chapter provides the overall summary of the present work, along with the critical assessment of the present approach as well as some suggestion for future research work.
2 Literature Review

2.1 Introduction

This chapter presents a review of the earlier work published in the area of vehicle handling dynamics. In particular, various vehicle modelling approaches and different aspects of sub-system modelling, which are suited for a range of handling and braking studies starting from steady-state to limit handling conditions are reviewed. Apart from investigating the influence of tyre and suspension non-linearities on vehicle handling behaviour, emphasis is put on the review of the influence of transient tyre behaviour on ABS braking. The list of related publications is provided at the end of the thesis. The literature review is presented in the following order:

- Vehicle Handling
- Vehicle Modelling
- Tyre Modelling
- Brake System
- ABS Braking

2.2 Vehicle Handling

As described in (Milliken and Milliken, 1995), when a vehicle negotiates a turn, it normally goes through three phases. The first phase is entry into the turn, where the yawing velocity ‘r’ and the lateral velocity ‘V’ build up from zero values associated with straight line motion to their values in a steady turn. This is called the ‘transient entry’ phase during which r and V change with time. The second phase is the ‘steady-state’ phase, where r and V, as well as the vehicle and tyre slip angles remain constant as the vehicle traverses along a path of nominally constant radius ‘R’. The final phase is the ‘transient exit’ phase, where r and V again change with time as they return to their eventual zero values for straight line motion.
2.2.1 Steady-State Handling

Steady-state handling is concerned with the directional behaviour of a vehicle under time invariant conditions, for example, a vehicle negotiating a curve of constant radius with a constant forward velocity. The vehicle, in this case, will have two simultaneous motions - rotation about its own CG and revolution around the centre of turn, both at a constant angular velocity ‘r’ (refer Figure 2-1). The steady-state handling is most commonly analyzed by following kinematic method of representation. In this analysis, although forces are considered, the response can be related in kinematic terms to the control inputs. Thus path curvature, yaw rate and lateral acceleration are expressed as functions of steer angle and forward speed. This type of approach helps in understanding the basic handling behaviour of a vehicle, which can be studied using two degrees of freedom model, also popularly known as bicycle model (shown in Figure 2-1). Many authors such as (Ellis, 1969, Milliken and Milliken, 1995, Dixon, 1996, Wong, 2001) have covered this approach in their text books.

![Figure 2-1: Steady-state handling representation using bicycle model](image_url)

The relationship between the front and rear slip angles ‘$\alpha_f$ ’ and ‘$\alpha_r$ ’, steer angle of the front tyres ‘$\delta_f$ ’, turning radius ‘R’, and wheel base ‘$L_w$ ’ can be expressed as (Pacejka, 1973a):

$$\delta_f - \alpha_f + \alpha_r = \frac{L_w}{R}$$ (2.1)
This indicates that the steer angle $\delta_t$ is not only a function of turning radius $R$, but also the front and rear slip angles $\alpha_f$ and $\alpha_r$. The front and rear slip angles can be expressed as functions of vehicle forward speed, cornering stiffness and vehicle mass distribution at the front and rear respectively. Thus:

$$\alpha_f = \left( \frac{m_f}{C_{af}} \right) \left( \frac{U^2}{R} \right)$$

$$\alpha_r = \left( \frac{m_r}{C_{ar}} \right) \left( \frac{U^2}{R} \right)$$

Substituting values of $\alpha_f$ and $\alpha_r$ in equation (2.1) yields:

$$\delta_t = \frac{L_u}{R} + \left( \frac{m_f}{C_{af}} - \frac{m_r}{C_{ar}} \right) \left( \frac{U^2}{R} \right)$$

The term $\left( \frac{m_f}{C_{af}} - \frac{m_r}{C_{ar}} \right)$ can be expressed as the under-steer coefficient $'K_{us}'$.

$$\delta_t = \frac{L_u}{R} + K_{us} \left( \frac{U^2}{R} \right)$$

Equation (2.4) is the fundamental equation governing the steady-state handling behaviour of a vehicle. It indicates that the steer angle required to negotiate a given turn depends on the wheel base, turning radius, forward speed, and the under-steer coefficient, which itself is a function of the weight distribution and tyre cornering stiffness.

The values of under-steer coefficient $K_{us}$ serve to classify the steady-state handling characteristics into three categories: neutral steer when $K_{us} = 0$, under-steer when $K_{us} > 0$ and over-steer when $K_{us} < 0$. For an under-steering vehicle, the steering angle needs to be increased, when it is accelerated in a constant radius turn. For an over-steering vehicle, the steering angle needs to be decreased, when it is accelerated in a constant radius turn, whereas for a neutral steering vehicle, the steering angle needs to be maintained at the same position, when it is accelerated in a constant radius turn.

The main factors controlling the steady-state handling characteristics of a vehicle are the weight distribution (including load transfer) and the cornering stiffness of the tyres. Various design and operational parameter affect the cornering stiffness of the tyres, such as tyre construction, inflation pressure, lateral load transfer during a turn, and application of driving or braking torque to the tyre during a turn. Also, the effects of roll steer, roll camber, and
compliance steer would be significant under certain circumstances and should be taken into account for a more comprehensive analysis of vehicle handling.

Looking at the three types of steady state handling behaviour, over-steer is not desired from a directional stability point of view. On the other hand, a small degree of under-steer is considered desirable for a certain level of lateral acceleration, such as 0.4 g (Wong, 2001), with increased values of under-steer at higher lateral accelerations.

Bundorf (1967) presented a method for measurement and prediction of the under-steering quality and the characteristic speed to determine some of the steady-state properties of passenger cars. In another paper, Bundorf and Leffert (1976) proposed a method to determine vehicle steady-state and transient response through cornering compliance, which includes the effects of roll, structural rigidity and suspension geometry. This method can be used to understand the influence of design specifications on handling behaviour of a vehicle. Milliken et al (1976) proposed steady-state force / moment diagrams to analyse stability and control of a car. The method portrays graphically vehicle handling performance, by employing yaw moment along with side force or lateral acceleration with side-slip and steering angles as variables. It is possible, for example, to reflect the entire manoeuvring envelope for a vehicle undergoing linear and angular acceleration, using normalized $C_{W_A Y}$ (yawing moment-lateral acceleration) diagram.

Pacejka (1973a) described a handling diagram (Figure 2-2), which is obtained by merging two diagrams with the same ordinates ($U^2/gR$) into one. The first diagram, obtained by plotting the difference in the slip angle of the front and rear tyres ($\alpha_f - \alpha_r$) against the non-dimensional lateral acceleration in g units ($U^2/gR$), produces the handling curve. The other diagram provides a number of radial straight lines, constituting the relationship between the lateral acceleration $U^2/gR$ and the path curvature $1/R$ for constant values of the forward speed $U$. Both diagrams, when combined together, can be used to study steady-state turning behaviour in relation to different values of steer angle, path radius and speed. The handling behaviour of a vehicle can be identified by the slope of the curve in the handling diagram. The negative slope leads to under-steering characteristics, whilst positive slope leads to over-steering characteristics and if the slope is zero, then this indicates a neutral steer characteristic. Pacejka (1973b) then applied a stability boundary in the handling diagram to access unstable motion of an over-steering vehicle, using slope of the handling curve at different values of lateral acceleration.
The handling diagrams can also be extended for more elaborate linear and non-linear models, where a possible shift in the vehicle behaviour from under-steer to over-steer can be studied. Pacejka (1973c) demonstrated the use of handling diagrams to study such behaviour, caused by non-linearities due to the tyres, steering compliances, unequal load transfer at front and rear, driving or braking forces, etc. In this case, he applied effective slip angles and effective cornering stiffness instead of real slip angles and cornering stiffness. Lukowski et al (1991) also used handling diagrams to study vehicle directional behaviour in both linear and non-linear ranges of tyre operation.

Another approach for steady-state analysis for directional control instabilities was adopted by Allen et al (1988), where they deployed bode plots for yaw rate-to-steering input transfer function. By considering the bandwidth of this transfer function as being proportional to the rear axle side force coefficient, they demonstrated that the bandwidth approaches zero, once the rear axle side forces reach a saturation mode under the influence of high lateral acceleration. Through a series of tests, involving straight line driving to pure / combined cornering and braking manoeuvres, they demonstrated that the transfer function approach can be used to illustrate the under-steer / over-steer behaviour of a vehicle.

Abe (1986) extended the analysis of vehicle motion from the basic steady-state analysis, where the longitudinal speed is kept constant, to the vehicle undergoing steady accelerations and decelerations. By using quasi-steady-state equilibrium equations, the vehicle turning behaviour under a fixed steering angle was predicted with an increase or decrease in vehicle speed; in acceleration or braking. The change in vehicle under-steer / over-steer behaviour
along with the characteristic line was then examined, along with the effects of drive arrangements and brake force distribution on the vehicle behaviour.

The International Organisation for Standards (ISO) specifies an open-loop test method for determining the steady-state circular driving behaviour of passenger cars, reproduced as the British Standard (BS ISO 4138, 2004). The three test methods specified under the standard are: constant-radius test method, constant steering-wheel angle test method, and the constant-speed test method.

The desired set of steady-state equilibrium conditions for speed, steering-wheel angle and turn radius can be obtained by holding one of these parameters constant, whilst varying another and measuring the third (the remaining parameter). Thus, in a constant-radius test method, the vehicle speed is varied and the steering-wheel angle is measured; in a constant steering-wheel angle test method, the vehicle speed is varied and the radius of turn is calculated from the vehicle motion. In a constant-speed test method, the radius of turn is varied and the steering-wheel angle is measured, otherwise the steering-wheel angle is varied and the radius is calculated. The measured parameters of interest include yaw velocity or lateral acceleration, which can be obtained by a rate gyro and an accelerometer. Yaw velocity gain and lateral acceleration gains can be obtained by the ratio of yaw velocity and lateral acceleration to the steer angle respectively. These parameters form measures of vehicle steady-state handling performance.

2.2.2 Transient Vehicle Handling

The steady-state responses deal with the final condition of a vehicle due to application of steering, which after a certain input is held constant. The steady-state does not depend upon the way in which the steering is applied. The transient response, on the other hand, is affected by the steering input pattern, which controls the time history of responses before the steady-state is reached. The transient state, as defined by (SAE, 1976), is attained when the vehicle responses, the external forces relative to the body, or the control positions are changing with time. It is important that any handling study undertaken for analysing the transient response should take into account the most important sources of nonlinearities, which generally comes into play at high levels of lateral acceleration, triggered by steep control positions. The nonlinearities associated with transient behaviour makes it imperative to use digital simulation to solve for the time responses.
The handling behaviour of a vehicle depends significantly on its transient response, which eventually manifests into drivers' subjective feel of overall handling. The transient response time may be defined as the time needed by the vehicle from the start of an input to reach a specific percentage of its final steady-state response. The percentage was assumed to be 50% by (Bickerstaff, 1976), whereas Whitcomb and Milliken (1956) assumed it to be 63%. In a turn, various parameters leading to transient response of a vehicle are suspension articulation, body roll, and generation of unequal tyre slip angles at the front and rear, and the inner and outer wheels.

Naude and Steyn (1993) conducted an investigation to evaluate transient handling characteristics of a vehicle in a double lane change manoeuvre. They employed a rigid-body model with detailed suspension representation. They also used a driver model to steer the vehicle along a prescribed path. The influence of vehicle, suspension and tyre characteristics on the handling performance of the vehicle was then investigated. Appel et al. (1992) studied the braking performance of a vehicle while cornering, using an in-house simulation package at the Mercedes-Benz. The yaw velocity response was found to be dependent on the longitudinal deceleration, with braking force distribution being the dominant factor influencing the directional behaviour. Antoun et al. (1987) applied full vehicle model, built in ADAMS platform at Ford Motor Company, to transient and limit handling responses using step-steer and lane change manoeuvres. The vehicle model was also correlated with experimental data, where some discrepancy in the response time was noted as compared to the test data, which was attributed to the absence of tyre relaxation length in their model. Hegazy et al. (2000) also conducted a transient handling analysis, using a multi-degree of freedom full vehicle model built in ADAMS. By subjecting the vehicle to a step-steering function, the vehicle responses were observed in terms of lateral acceleration, roll angles and load transfer.

A variety of transient response tests have been developed based on the different type of control inputs. The ISO International Standard specifies (BS ISO 7401, 2003) as open-loop lateral transient response test methods. The transient response behaviour of a vehicle is determined by generating characteristic values and functions in the time and/or frequency domains. The important characteristics, which can be determined are the time lags, response times, overshoot values, and relations of lateral acceleration and yaw velocity to the steering angle. As control inputs, these test methods specify step input and sinusoidal input (one period) in time domain and random input, pulse-steer input and/or continuous sinusoidal
input in the frequency domain. Primarily the following output variables are measured: steering-wheel angle, lateral acceleration, yaw velocity, and longitudinal velocity.

The other standard simulation tests performed for measuring transient handling responses are various lane change manoeuvres, J-Turn, Fishhook, braking in a turn (BS ISO 7975, 2006), double lane change (BS ISO 3888-1, 1999) and slalom testing.

2.3 Vehicle Modelling

Computer simulation techniques for vehicle handling / dynamics studies have continuously evolved over the years, more so from the time when Segel (1956-57) published his linear model for predicting handling responses. At the core of every computer simulation attempt lies the mathematical description of the vehicle, catering its dynamic behaviour. Such mathematical descriptions may vary in complexity from simple linear models with only few principal DOF to more elaborate non-linear multi-DOF models.

In 1956, Whitcomb and Milliken (1956), Segel (1956-57) of the Cornell Aeronautical Laboratory published the first major quantitative and theoretical analysis of vehicle handling in a series of papers. The final paper in the series by Whitcomb and Milliken (1956) studied the steady-state and transient motion of a vehicle using 2 DOF representation of yaw and side-slip degrees of freedom. The response characteristics of the vehicle were discussed along with the vehicle parameters that affect them. The study by Segel (1956-57) was one of the initial examples, when mathematical descriptions were used to closely predict the rigid body behaviour of a vehicle whilst cornering in response to the steering input. The equations of motion for his three degree of freedom model, derived in terms of stability derivatives, could predict the lateral dynamic behaviour of the vehicle up to a lateral acceleration of 0.3 g. The study, in overall perspective, was perceived by many as the pioneering work in vehicle handling dynamics. Aware of this perception, Segel (1993) made another publication, providing the historical review in the development of road-vehicle dynamics. Keeping in perspective his contribution, he highlighted the efforts made by others since the invention of motorcars, in the understanding of the road vehicle behaviour, ride and handling issues, and emphasis of directional response and stability in relation to the pneumatic tyre. He chose to divide the historical development of vehicle handling theory in three distinct periods: Period 1, which extends from the invention of the motorcar to the early 1930’s (basic behaviour of motor car in a curved path without an established understanding of tyre
behaviour); Period 2, which extends from the early 1930's to the end of 1952 (understanding of steady-state behaviour and directional stability of the pneumatic tyre, along with study of transient aspects in a turn); and Period 3, which extends from 1953 to the early 1990's (period which saw development of linear mathematical models for vehicle starting from his work at CAL, leading to tyre force and moment representations, and to the development of multi-body computer codes to conduct non-linear analysis for vehicle behaviour).

With the advances in computing, the final period, as identified by Segel (1993), saw rapid development and refinement of non-linear vehicle models – one of which was the ride and handling model of McHenry (1968) which formed the basis for HVOSM (Highway Vehicle Object Simulation Model) in the United States. The period also saw emergence of new modelling techniques and numerical methods. Crolla et al (1994) in his paper, broadly classified these modelling techniques into two categories. The first involves solution of differential equations of motion derived to represent vehicle behaviour, either directly by integration to produce simulation time histories, or via linearised versions to undertake stability analyses and frequency responses. The second approach uses multi-body system analysis to generate the equation itself. It relies on the general description of dynamical system as collection of rigid bodies connected by joint and internal forces, and acted upon by external forces. The author further classifies the important computational methods used for vehicle dynamic simulations in a further four categories, pointing their relative merits and shortcomings:

- Purpose-designed simulation codes, where equations of motion are developed for a specific vehicle, which is then embedded in a computer program. The parameters can be changed for simulation time histories, but the model is not alterable, and particular new features cannot easily be incorporated.

- Multi-Body simulation packages that are numerically based, such as ADAMS and DADS, where the equations generated in numerical form are solved directly using numerical integration routines embedded in the package. These equations cannot be inspected like those in the purpose-designed simulation codes.

- Multi-body simulation packages that are algebraic such as AUTOSIM, where the equations are formulated in the symbolic form and often uses an independent solver. The
equations are not regenerated, every time the parameters are changed, unlike those in the multi-body dynamic packages.

- Toolkits, which are collection of analysis routines that formulate and process the equations of motion and provide results. The VDAS package developed by the authors (Crolla et al, 1994) which uses semi-automatic equation generation approach, was classified under this category.

The paper by Crolla (1996) provides a review of vehicle dynamics theory and its contribution to practical vehicle design, with a focus on actively controlled components, such as active suspension and four wheel steering. Through the paper, the author brings forward the debate about the extent to which theoretical understanding in vehicle dynamics has translated into practice, vis-à-vis the application of sophisticated simulation tools and packages. Another interesting review of some of the main contributions that led to the development of models and tools in vehicle dynamics, was given by Blundell (1999a). Rahnejat (2000) also provided some interesting insight into the historical evolution and application of multi-body dynamics from fundamental physics of motion.

Suresh and Gilmore (1994) classified vehicle modelling methods in two approaches, denoting them as lumped parameter model approach (LPM) and multi-body formation approach (MBF). The paper explained the differences between the two approaches from various viewpoints, such as efficiency / computation time, flexibility for modification, required modelling effort, and potential uses.

The research carried out at Leeds University have led to the development of a design toolkit for use in the prediction of vehicle ride and handling behaviour (Crolla et al, 1994). The design toolkit; VDAS (Vehicle Dynamics Analysis Software) embodies all the commonly required calculations for vehicle dynamics studies. In addition to the standard range of calculation for ride / handling behaviour such as natural frequencies, mode shapes, frequency responses, steady-state handling diagrams etc, the package also incorporates control algorithms for active/semi-active suspension application. It includes a library of ride and handling models of varying degrees of complexity, and also has flexibility to add new models.

Sayers (1989) presented the symbolic multi-body analysis package AUTOSIM for the automatic generation of simulation codes. Developed at UMTRI (University of Michigan
Transportation Research Institute) using computer language Common Lisp, the package can perform coding optimisation and generate custom simulation programs for a specific set of problem An 18-DOF generic vehicle model was developed by Sayers and Han (1996) for handling and braking analysis, using the AUTOSIM code generator. The model was compared with a full multi-body vehicle model (IAVSD Ilitis multi-body benchmark) using AUTOSIM for simulation time, Eigen values and time history plots. Through the paper, the authors advocated the use of relatively simple models for specific vehicle handling / braking studies, for fast run-time performance without relying heavily on the component level details.

Hogg et al (1992) described the use of a modelling tool in the vehicle development phase, particularly in the design and prediction of vehicle handling and stability. The application of their functional vehicle model, SAM (Simulation and Analysis Model), along with a detailed ADAMS model, was explained in this regard. Dickison and Yardley (1993) explained the development of the SAM model and its experimental validation against real vehicle data. The 17 DOF SAM model incorporated non-linear suspension and steering characteristics, modelled as look-up tables. It also included first order tyre lag, using the earlier version of Magic Formula Tyre Model. The development work was carried at Lotus Engineering, to aid ride and handling improvement of its new vehicles, particularly from the point of active suspension systems.

The research carried out at System Technologies Inc. (Allen et al, 1988) led to the development of VDANL (Vehicle Dynamics Analysis Non-Linear) program. Allen and Rosenthal (1994) described the requirements for a vehicle dynamic simulation model and discussed the importance of identifying complexity of the vehicle model based on the desired response behaviour. In the same vane, in another paper, the detail requirements for a tyre model for vehicle dynamics simulation was given by Allen et al (1995). The VDANL model, in general, is based on a 13 DOF lumped parameter model with no bounce or pitch of the sprung mass, but roll and bounce DOF for the front and rear un-sprung masses. The vehicle model was predominantly used to assess vehicle lateral/directional, as well as rollover stability (Allen et al, 1991, Allen et al, 1999). The recent validation of the vehicle model, with all its additional features, was given in Allen et al (2002), where it was compared against test results for different manoeuvres such as double-lane change, fishhook, and pulse steer. Tandy et al (1992) employed the VDANL model to conduct sensitivity analysis, where the model input parameter change is predicted to cause a specified change in
the model output. Using an inverse sensitivity analysis technique, changes in the vehicle parameters such as mass, cg height, various stiffness values, etc are predicted to attain a specified handling dynamics parameter such as yaw rate or roll angle.

Based on the work carried out at the Engineering Dynamics corporation, a detailed vehicle model was presented by Day (1995). The model known as HVE was part of an integrated simulation environment, called HVE (Human-Vehicle-Environment) (Day, 1994), and provided a pre-programmed object-oriented vehicle model, where the user can carry out vehicle handling studies by developing a vehicle and particular environment in a 3D graphical user interface. The HVE interface had an integrated set of editors, where databases exist for creating visual models of humans, vehicles and environments. As an extension to the above work, Day et al (2001) presented a new vehicle simulation model (SIMON) for vehicle handling, as well as collision studies. The vehicle model includes a 6 degrees of freedom sprung mass, along with multiple axles with up to five degrees of freedom per axle. The vehicle model integrated various features present in the HVE simulation environment, and could be used for various applications such as handling for single and articulated vehicles in both simple and limiting manoeuvres. It was used for vehicle design analysis, for collision simulation and rollover events.

Nalecz (1992) describes development and validation of a 3 dimensional vehicle dynamics model; LVDS (Light Vehicle Dynamics Simulation) at the university of Missouri. The 8 DOF LVDS model utilizes 2 coupled models: a handling model derived using Newton-Euler formulation and a rollover model derived using Lagrangian formulation. In addition to the steering, driving, and braking system, tyre and an aerodynamic model, the overall model utilizes multiple kinematic configurations of the front and rear suspension systems, and could be used for a range of vehicle dynamics analyses. Lukowski and Medeksza (1992), presented a 10 DOF model, based on Lagrange's equations. Since the equations were expressed in terms of moving coordinate system, rather than inertial coordinate system, the Lagrange's equations were derived, using quasi-coordinates. Demerly and Youcef-Toumi (2000) presented a 8 DOF model NAVDyn (Non-Linear Analysis of Vehicle Dynamics), built using the MATLAB / Simulink environment. With the primary focus on the handling and braking requirements, the reduced order model incorporated only 4 DOF for the sprung mass, neglecting pitch and bounce degrees of freedom. The effects of suspension / steering kinematics and compliances were included without resorting to the vertical dynamics of the suspension system. The model was validated against test data, where some differences were
noted on vertical load transfer during braking, which was attributed to its lack of freedom in pitch.

While most of the above programs or models utilize a limited set of equations, solved by symbolic or a numeric code, eventually resulting in a lesser degrees of freedom computationally efficient models; still in practice most modelling work is dominated by complex multi-body-dynamic vehicle models. These can account for the effects of flexible couplings between vehicle sub-systems, geometrical and other non-linearities, as well as various kinematic constraints (Blundell and Harty, 2004). The underlying theory required for modelling such elaborate dynamic systems is presented by Rahnejat (1998), Blundell and Harty (2004) and Shabana (2005), and is primarily based on the application of the constrained Lagrangian multi-body dynamics. In general, the modelling process involves the assignment of a frame of reference and six DOF to each rigid-body component of the vehicle. The influence of mechanical joints and couplings is modelled mathematically by linking the various DOFs of the connected bodies, using holonomic / non-holonomic constraints. The reaction forces within the constraints are calculated as the Lagrange multipliers (Rahnejat, 1998, Blundell and Harty, 2004 and Shabana, 2005). The solution is by simultaneous handling of differential-algebraic system of equations.

The research effort in the mid 60’s and 70’s at the University of Michigan led to the development of large rigid multi-body dynamics’ solvers (Orlandea et al., 1976a, 1976b), eventually resulting in the commercial package ADAMS (Automatic Dynamic Analysis of Mechanical Systems). A detailed account of various multi-body approaches in vehicle dynamics and associated software packages is provided in Kortum and Sharp (1993), where an attempt was made to conduct a comprehensive exercise to benchmark numerous commercially available multi-body computer codes, used for rail and road vehicle applications. The exercise organised by IAVSD (International Association for Vehicle System Dynamics), identified two benchmarks for road vehicles: modelling of Bombardier Iltis vehicle and a five-link suspension system. The exercise, at the end, could not produce very conclusive results as similar levels of modelling consistency could not be obtained with the alternative codes. Also, the results were affected by the fact that major multi-body simulation package vendors like ADAMS and DADS chose to have a limited participation in the benchmark exercise. A discussion about the different multi-body codes could also be found in (Kortum and Sharp (1991) and Sharp (1994).
The multi-body formation approach not only develops to model the system as close as possible to the real system, it also helps the designer to fine tune the design at component level by using highly iterative design optimisation. Hegazy et al (2000) demonstrated the use of such an approach in solving complex vehicle handling problems. A series of papers by Blundell (1999a, b, 2000a, b) provides a comprehensive description of the application of multi-body methodology. Through these papers, he demonstrated the comparative influence of different suspension and tyre modelling approaches on vehicle handling simulations.

2.3.1 Complexity of Vehicle Model

The subject of modelling refinement or the amount of detail required in a vehicle model for conducting full handling studies is a contentious issue, rather similar to the use of numerous computation codes or mathematical modelling approaches. While the multi-body formulation approach holds its own advantages in terms of accuracy and prediction of real behaviour, it also poses problems for the uninitiated outside the vehicle industry to access the huge sets of data required to build vehicle models. Willumeit et al (1992) presented two simpler vehicle models, arguing its computational efficiency and effectiveness in solving particular problem associated with dynamics of vehicle. The argument has found favour in vehicle studies related to control, however, for studies involving large roll and pitch angles, one has to be careful while discretizing the system, as over-simplification of the model can often affect the accuracy of the results. The overall approach of lumped parameter modelling or intermediate vehicle modelling is also error prone as it demands much effort from the analyst, in deriving the equations of motion, coding and validating them, although the advent of mathematical tools such as Mathematica and MATLAB has somewhat reduced the complexity of the task.

While many authors such as Willumeit et al (1992), Dickison and Yardley (1993), Allen and Rosenthal (1994), Crolla et al (1994), and Sayers and Han (1996) have advocated the need for reduced complexity; it is not so straight forward to draw a line for the level of complexity a model in general should possess. It rather varies with the aims and objectives of the investigation to be carried out. The vehicle handling / dynamics models, in general, can be classified in two categories: linear and non-linear models.
2.3.1.1 Linear Models

The most common example of linear model is the 2-DOF 'bicycle model', having only the lateral and yaw degrees of freedom. The linear models are mostly suitable for low level of lateral acceleration (0.3 - 0.4 g), and are generally employed for handling studies, which does not require the complications introduced by suspension or various other non-linear vehicle characteristics. The use of such models in control related studies have gained in ascendancy in last decade or so.

The most common assumptions of these models are:
- Linear tyre behaviour
- Small steering angle
- Lateral tyre forces not being sensitive to small changes in load
- Limited DOF of sprung mass; mostly neglecting pitch and vertical translation
- Constant forward speed
- Assumption of small angles

Many authors have covered the use of linear models in their text books such as Ellis (1994), Milliken and Milliken (1995) and Wong (2001). Also, Pacejka (1973a), Willumeit et al (1992) demonstrated that linear models can be effectively used to study problems concerning the dynamics of a vehicle.

2.3.1.2 Non-Linear Models

The inclusion of non-linearities in the vehicle model becomes imperative for studies involving higher lateral accelerations, where the non-linearities due to the following effects become significant:
- Tyre forces and moments
- Bump and rebound stop forces
- Effect of Suspension kinematics
- Non-linear spring and damper characteristics
- Steering and suspension compliances
Anti-roll bar effects

These non-linearities can be tackled either by an intermediate model, which includes representative non-linearities in a compact formulation, or the detailed multi-body approach, which includes various sources of non-linearities in a complex formulation.

2.3.2 Suspension System

Suspension system, being an important part of a road vehicle, has a major influence in its handling performance. Over the years, many studies have been conducted to improve understanding of suspension system kinematic and its influence on vehicle handling behaviour. In one of pioneering studies on suspension secondary motion, Hales (1964-65) conducted a theoretical analysis of the lateral properties of suspension systems. Using suspension derivatives, he studied the lateral motions of a wheel, resulting from prescribed motions of the vehicle body for a range of suspension systems such as double wishbone, Macpherson strut, swing axle, beam axle, and beam axle-panhard rod. Furthermore, the effects of suspension design on the steady-state roll angle, suspension jacking, and load transfer were investigated. The early use of suspension derivatives are also mentioned by Ellis (1969). Using a root locus analysis, Chen and Guenther (1991) investigated the influence of suspension roll stiffness on handling responses, where it was found to have more apparent impact on over-steer vehicles rather than the under-steering ones, particularly at high speeds.

For simulation studies, the suspension characteristics included in a vehicle model can have a significant bearing on the accuracy of the results. For vehicle handling analysis, the suspension system representations vary from the very basic (Pacejka (1973a, b, c) to the very detailed with complete information about the kinematics and compliances (Hegazy et al, 2000 and Mirza et al, 2005). Pacejka (1973a, b, c) in his study took into account the effect of weight transfer, roll angle and steer compliance, but did not include the non-linearities arising from suspension geometry and assembly constraints. Willumeit et al (1992) considered non-linear spring and damper characteristics, in addition to some effects of suspension kinematics in their non-linear two tracks model. Dickison and Yardley (1993) employed a generic suspension modelling approach, where an individual suspension is represented as swing arm attached to the body at the instantaneous hub pivot axis with an additional steering swivel axis for the front wheels. All the suspension characteristics were
modelled as look-up tables. Sayers and Han (1996) also used a generic suspension modelling approach, employing simplified suspension kinematics and compliances, along with non-linear spring dampers, which overall can represent any kind of independent suspension.

Ross-Martin *et al* (1992) applied the concept of a force roll centre (also see (Dixon, 1987)) to define suspension system characteristics, in their vehicle model developed at the University of Bath (hydraulic and dynamic systems simulation package BATH). In the method, the lateral forces were assumed to act on the vehicle body at the front and rear roll centres, but it did not constrain the body to roll about these points relative to the wheels. The roll centre, which is determined by the tyre contact patch paths were represented in terms of polynomial functions, relying on the measured vehicle data.

Ward *et al* (2001) discussed the development of a generic suspension model for mid-class passenger vehicles. Rather than relying on the suspension geometry, a parametric approach was considered, where characteristic curves were used to describe the change in position and velocity of the wheel carriers along with castor and camber angles, relative to the vehicle body variables such as angle of the lower suspension link. The equivalent spring and damper forces exerted on the vehicle body were then determined.

Various researchers, while using lumped parameter or intermediate modelling approaches, have often included effects of suspension kinematics through characteristics curves or non-linear function sets. There have been instances where detailed suspension kinematics has been incorporated in these vehicle modelling environments. Makita (1999) employed differential-algebraic equations to represent kinematic constraints for suspension linkage and obtained a Jacobian matrix for an independent suspension system. The equation formulation was achieved by symbolic manipulation tool ‘Mathematica’, which could then be applied with any ordinary differential equation-based vehicle model for handling simulations. However, the author did not consider elasto-kinematics of the suspension system.

Many researchers including Plochl and Lugner (2000), Schuller *et al* (2003), Zainul-Abidin (2005) incorporated suspension elasto-kinematics in full vehicle simulation studies. However, the inclusion of complete kinematic details can lead to a significant increase in computation times. Watanabe and Sayers (2004) compared the behaviour of two suspension modelling methods, employing CarSim and AutoSim. The fully non-linear model, where the 3D motion of wheel spindle in space was specified through tabulated data had 20 % longer
computation time when compared to the relatively simple model with linear kinematics of the wheel spindle. However, the non-linear model gave better predictions for roll and pitch at higher vertical movement of wheel spindles. Sharp (2000) studied the influence of suspension kinematics on pitching, bouncing, and tyre load transfer, to step inputs of braking and driving torques at the front and rear axles. The author applied different anti-dive designs and studied translational and rotational anti-dive influences, varying the % anti-dive.

Lee et al (1996) performed a DOE (Design of Experiment) study to rank severity of various suspension factors to selected handling output parameters, deploying a 10 DOF vehicle model, in addition to a detailed FEA (Finite Element Analysis) model for quarter car suspension. The author conducted number of linear cornering / braking simulations to qualitatively evaluate the effect of numerous suspension variables on vehicle handling response. Later, using DOE and FEA, the influence of body structure, tyre rates, suspension components, hub bearing compliances, bushing rates etc on the overall suspension force compliances were qualitatively determined.

Researchers have also used complex multi-body approach to study the influence of suspension characteristics on vehicle handling behaviour. Blundell (1999b) developed a range of vehicle models in ADAMS for handling simulation, employing different suspension representation. Apart from using the detailed suspension linkage model, incorporating bushing compliances, he employed three other simplified suspension representations, known as lumped mass model (suspension movement in vertical direction only), swing arm model (individual swing arm for each suspension to rotate about the instant centre), and roll stiffness model (body rotation about a single roll axis). The roll stiffness model was shown to agree well with the detailed linkage model, and also with the actual test results, in comparison to the other simpler suspension representation (Blundell, 2000b). Using full vehicle simulations in ADAMS, Mirza et al (2005) investigated the dynamic influence of suspension parameters on a vehicle experiencing steering drift during braking, where the front suspension’s lower control arm bushing was found to have a significant influence on lateral drift of a vehicle.

McGuire and Guenther (1993) studied the effect of including longitudinal compliance link in conventional suspension with respect to vehicle behaviour, using ADAMS. The inclusion of compliance resulted in an increase in the lateral stiffness of the vehicle without significantly affecting its vertical body displacement. However, it also increases the magnitude of
compliance steer angle, which can lead to instability if an obstacle is struck. Park et al (2003) developed a simplified suspension kinematics model in ADAMS using laboratory measurements, tuned by optimization methods, for full vehicle simulation.

With the design of active suspension system in mind, Kim et al (1999) investigated the relationship between suspension linkage structure (layout, strut mount point and inclination, swing arm length) on the equivalent values of suspension parameters, which were obtained using identification method for an ADAMS quarter car model. Kim and Ro (2000), in another study, developed a two-mass model of a quarter car suspension system using linearization and model reduction techniques. The authors demonstrated that a complex suspension linkage model could be reasonably represented by a reduced order suspension system model, which could provide an efficient way to handle control studies.

2.3.2.1 Modelling for Limit Handling Conditions

The suspension modelling requirements, considered for any vehicle simulation should address the concerns of the particular study. The pure lateral/directional dynamics concerns mainly revolve around yawing, rolling and lateral acceleration responses due to a steering input, whereas longitudinal dynamics is concerned mainly with longitudinal acceleration and the pitching response due to throttle and braking inputs. For these studies, the most important requirement for a suspension/wheel assembly is the tyre model force / moment response characteristics vis-à-vis normal load, slip ratio, camber and slip angle, and secondly the suspension system response characteristics, keeping account of the primary and secondary motions of the wheel hub. The above requirements, aided with the inertial dynamics of sprung and unsprung masses, gravity effects, aerodynamics, and some consideration of compliances and force lags should suffice for most steady-state or transient vehicle handling studies, unless the aim is to study any specific component or design parameter.

The requirement, however, becomes a little different for limit handling/stability manoeuvres, which include spin-out and roll-over. Since these manoeuvres involve considerable variations in roll and pitch angles, the equation formulation should accommodate large angles. The vertical dynamics concerning tyre normal loads, road surface input, and suspension heave between sprung and unsprung masses becomes important, and so does the tyre deflection as it directly affects the normal load. Also, the wheel position determination becomes important, especially when modelling roll-over. In the suspension model, the
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different force mechanisms such as auxiliary roll stiffness (anti-roll bar), bump stops, jacking/anti-dive/anti-squat, roll steer, compliance and tyre camber angle, affect the vehicle behaviour and should be part of the suspension modelling requirements. This section reviews some of the past studies, where different suspension representation were used for conducting extreme handling or stability simulations, including roll-over.

In the VDANL model, the suspension characteristics, including bump stop and damper jounce and rebound, squat/lift effects, wheel camber etc. were defined using body roll as the reference (Alien et al, 1999). The front and rear unsprung masses, with their own roll inertia, were connected to the body through the compliant pin joints. The model also incorporated lateral tyre compliance along with vertical tyre deflection, in addition to a full range of slip, camber and load included with the tyre model; STIREMOD (Allen et al, 1997), which is also popularly known as STI tyre model (Systems Technology Inc.) and is based on the composite slip formulation. Using VDANL model, Alien et al (1999) conducted directional/rollover stability analysis, where they employed phase plane representation and bifurcation points to show transition from stable to unstable behaviour.

For severe limit manoeuvres, Day et al, (2001) located the exact position and orientation of wheel centre by using terrain coordinates extracted through the HVE environment (Day, 1994) in order to determine tyre radial deflection. The model used a secondary tyre stiffness multiplier for large tyre deflections. Transformation matrices and Euler angles were employed to incorporate large angles encountered in such manoeuvres. The equations of motion for the sprung mass included contribution of inertial coupling of the unsprung masses in the vehicle fixed $x$ and $y$ directions, an approach which is also followed by other researchers such as Eger et al (1998). Sayers and Riley (1996) also presented a detailed account of the vehicle / suspension modelling, which can address severe yaw and roll behaviour of heavy trucks using the TruckSim program.

Hac (2002) adopted an analytical approach to study secondary effects resulting from suspension and tyre compliance on the propensity to roll-over. The different secondary effects considered were lateral movement of the vehicle CG during body roll, suspension jacking forces, changes in the track width due to suspension kinematics, tyre lateral compliance, gyroscopic forces and dynamic overshoot in the roll angle. These effects were then translated in terms of an expression for lateral acceleration at the roll-over threshold. The analytical results were supported by separate simulation tests conducted using a 16 DOF
vehicle model. The author suggested that the most important factor affecting lateral acceleration is the lateral displacement of the CG, followed by lateral compliance of tyre, suspension kinematic effects and changes in the CG height. In the study, the author derives jacking force effects using roll centre height, fixed in the symmetric vertical plane of a vehicle. However, in reality the roll centre does not stay symmetrical across the transverse plane. Gerrard (1999) analysed the mechanism of jacking for independent suspensions, using the concept of original roll centre, combined roll centre and roll centre migration. Through the study, the author proposed a set of tools in the form of simple equations for the concept suspension designers.

2.4 Tyre Modelling

Pneumatic tyre is commonly recognized as one of the most influential components of a vehicle, because it is the only component that contacts the road surface, and eventually transmits the forces and moments required to control the vehicle's motion. As the performance of a vehicle depends significantly on the forces and moments generated in the contact patch, tyre behaviour becomes crucial to most of the performance related measures in a modern day vehicle, such as ride, comfort, durability, fuel efficiency, NVH, and last but not the least vehicle handling and stability.

Pacejka (1979) identified different classifications based on the tyre functions and the way tyre is mathematically represented. Differentiating between the primary and secondary tasks the tyre performs, its functions were grouped based on the responses, which either reflect the symmetrical (in-plane of tyre) or anti-symmetrical behaviour (out-of-plane). The tyre functions were further regrouped based on the steady-state and transient nature of these responses. The classification of tyre models based on their mathematical descriptions can also be found in Pacejka and Sharp (1991), where the authors broadly classified the tyre models as physical tyre models and empirical tyre models. A physical tyre model, in its complex form is based on detailed representation of the tyre structure and the interaction of contact element with ground. These mostly require computationally intensive numerical methods for solution, whereas, simple physical models rely on good representation of the basic contact geometry, aided with elastic and friction forces, and can be solved analytically. On the other hand, empirical methods are based on experimental test results, which are used to fit certain mathematical functions for the tyre forces and moments. This approach is far
more computationally effective than the physical models and has gained in popularity over the years. Though the modelling approaches are termed physical and empirical, the classification itself is not strictly distinct, as physically based approaches often employ empirical observations and correction factors, whereas empirical methods often take into account physical principles. This has resulted in modelling approaches that are often called semi-physical, semi-analytical, or semi empirical, making the whole classification more a continuum rather than distinct. However, in this thesis use is made of broad physical and empirical terms, while referring to the different modelling approaches.

2.4.1.1 Physical Tyre Models

In a study of cornering properties of tyres, Fiala (1954) proposed a model based on theoretical principles. Using simplified geometrical representation, he derived expression for lateral force and aligning moment by assuming the carcass centreline to behave as an elastically supported beam subjected to a point load (lateral) at its midpoint, with its deflection approximated by a quadratic polynomial. This tyre model later became a standard part of ADAMS multi-body package (i.e. ADAMS/Tyre, 2005). Dugoff et al (1970) analysed the influence of longitudinal / lateral tyre stiffness and the coefficient of friction at the tyre-road interface on the vehicle steering and braking responses. Using the Fiala tyre model as the base, they applied many assumptions such as keeping the carcass centreline deformation as constant, and expressing the tyre road friction coefficient to be independent of the normal pressure, as well as a linear function of the relative sliding velocity. Sakai (1981a, b, c, 1982) also used the Fiala model as the base of his eventual model to extend study of cornering properties of tyre to braking and traction. He conducted experimental studies to investigate tyre deformation in relation to slip angle, slip ratio, camber, load, internal pressure etc. He also studied rubber friction in relation to vertical load, sliding velocity and temperature. He applied all the findings to calculate six components of tyre forces and moments using numerical integration. In another series of papers Gim and Nikravesh (1990, 1991a, b) presented an analytical tyre model for both pure and combined slips, where tyre dynamic properties were formulated as functions of slip ratio, slip angle, camber angle and other dynamic parameters. Bernard et al (1977) used an approximate method in their tyre model with a trapezoidal load distribution and an anisotropic friction coefficient, which varied linearly with sliding velocity. The approximation method concerns the transitions from adhesion to sliding (see Pacejka and Sharp (1991)).
The physical models based on analytical modelling make a number of simplifications, most notably the way the pressure distribution and friction coefficient are represented. This can often form a fundamental limitation to represent realistic tyre behaviour, especially during braking and traction related studies. Many of these issues were discussed by Sharp and El-Nashar (1986), who developed a steady-state tyre model mainly for use in computer simulations, applying physical principles. In the multi-spoke tyre model, the carcass and tread block deflections were represented through the radial spokes, located in a single plane with individual longitudinal, lateral and radial tip flexibilities. The tyre model required a few parameters to perform simulation and could cover full range of steady-state operations. Mavros et al (2004) also developed numerical steady-state tyre models using the brush modelling approach, where they considered different non-linear characteristics such as viscoelastic properties of rubber and velocity dependent stick-slip frictional behaviour. While the simple version of their steady-state tyre model assumed a parabolic pressure distribution, the advanced version considered the actual normal pressure distribution, and could predict a more realistic self-aligning moment.

2.4.1.2 Empirical Tyre Models

Although the physical models provide good insight into the theoretical behaviour of the tyres, they often rely on complex representation of the structural properties of the tyres. This often leads to a relatively large computational effort, which can be not feasible in vehicle handling/stability simulations. The use of experimental data to understand and represent tyre characteristics using force and moment curves for a range of slip under the influence of camber and load, was initially conceived in early 1950's, as demonstrated by (Fonda, 1956-57). However, representing large set of data in vehicle simulation studies were never easy, and researchers tried many mathematical functions to describe tyre cornering force characteristics, such as exponential, arctangent, parabolic, hyperbolic tangent, where most of these resulted in crude approximations (Pacejka and Sharp, 1991). The use of tables of data along with an interpolation algorithm was not considered feasible for simulation, and also it was found to be rather inadequate for efficient use and for optimization studies. The use of Fourier series and higher order polynomials yielded good results, but had a number of issues such as large number of coefficients, waviness in the fitted curves, and also deviations for extrapolations beyond the fitted range, as pointed out by Bakker et al (1987). Sitchin (1983) addressed some of these issues with a polynomial representation, where he divided the
curves into five regions such that the uniformity and continuity were maintained in each region. He then applied different equations for curve fit within each region.

The most popular model of the empirical modelling method is the Magic Formula tyre model, which was an outcome of the joint project between Volvo and Delft University. Initially presented by Bakker et al. (1987), the model has undergone continual development (Bakker et al., 1989, Pacejka and Bakker, 1993, Pacejka and Besselink, 1997). The model was based on the use of mathematical functions such as sine and arctan, to fit characteristics of longitudinal force in relation to slip ratio, and lateral force and aligning moment in relation to slip angle. The model employed lesser number of coefficients (obtained using steady-state testing) as compared to the use of polynomials. This produces very close curve fit. The effect of ply steer, conicity, camber and rolling resistance were incorporated in the model by using shift factors, which generates anti-symmetric curves with axes shifted in the horizontal and vertical directions from the origin of the characteristic curves. The characteristics represented for pure braking and cornering conditions were extended for combined cornering and braking using normalized composite slip quantities. This was further improved by Bayle et al. (1993), who suggested an empirical method employing cosine-arctangent weight functions instead of physically-based formulae to represent combined slip. Pacejka and Bakker (1993) introduced pneumatic trail to calculate the aligning moment using a hill-shaped cosine function, which further simplified the calculation of aligning moment under combined slip conditions. Schuring et al. (1993) developed BNPS model based on the Magic Formula concept, with the objective of automating the process of generating coefficients from the raw experimental data so that it can be easily implemented for vehicle handling studies. The model is also popularly referred as the ‘Smithers’ model.

Apart from producing close curve fits, the advantage which the Magic Formula enjoys over the previously developed empirical curve fit methods is the fact that it employs physically meaningful quantities such as slip stiffness values and peak values. It also allows calculation of forces and moments for conditions which deviate from the original imposed conditions of measurement. Over the years, the Magic Formula tyre model has become the popular choice for vehicle handling simulation in complex multi-body environments such as ADAMS. Blundell (2000a,b) conducted a study on tyre modelling using the Magic formula and Fiala Tyre models, along with the interpolation routine, which was used as a benchmark for comparing the two models. The tyre data set was generated by physical testing at Dunlop tyres and also at flat bed tyre test machine at Coventry University, and the tyre models were
then simulated by developing ADAMS virtual tyre test rig. Except Fiala Model, the remaining two tyre models were implemented in ADAMS using Fortran subroutines. Later, he performed full vehicle handling simulation in ADAMS, where the combinations of the aforementioned tyre models with his different vehicle models were evaluated, in a double-lane change manoeuvre. In relation to the Magic Formula model, Blundell (2000a,b) addressed some of the issues with the Fiala tyre model such as its reliance on less number of parameters which are easy to physically interpret, and its relative computational efficiency, along with its limitation such as representation of lateral force offsets, camber thrust, and aligning moment at high slip angle. The authors work in tyre / vehicle modelling has led to the development of an intermediate tyre model, primarily intended for use in computer simulation environment such as ADAMS, to evaluate flat road vehicle handling performance (Blundell and Harty (2007)). Though the tyre model deals with the pure slip condition only, it can address some of the limitation of the FIALA tyre model without resorting to large number of parameters. Hegazy et al (2000a) also performed a study of different tyre models, using an ADAMS single tyre test rig. The tyre models used in the exercise were: Magic Formula, Smithers tyre model, Arizona tyre model, and STI tyre model. The aforementioned tyre models were compared against the experimental tyre measurement, where the Magic Formula was shown to have the closest match.

2.4.1.3 Transient Tyre Modelling

The transient characteristics of tyre relate to the time varying responses of the tyre, as a result of the non-linear effects of tyre structure (such as carcass elasticity and viscoelastic relaxation). Researchers using physical modelling approach, often include finite width of the contact patch, carcass compliance, and belt inertia in their models so that force and moment responses to time varying longitudinal, lateral and yaw excitation can be studied. Zhou et al (1999) extended (Sharp and El-Nashar, 1986) model for transient conditions, where three planes of spoke were employed (equally spaced) across the width of the tyre, as opposed to a single plane used in its steady-state version. The transient model includes a rigid bristle base with freedoms to translate longitudinally and laterally with respect to the wheel hub. As the spokes or bristles located in each of the three planes represent tyre structural elements at different locations across the width, the transient tyre behaviour requires updating the states of all the spokes under consideration in time domain. The semi-analytical tyre model by Mastinu et al (1997) contained separate carcass and tread flexibilities, where the carcass
deformation was calculated using finite element methods, whereas analytical formulation was used in the contact region for the tread, which was represented using longitudinal and lateral elastic elements. Mavros et al (2005a) developed a transient brush tyre model, which consists of a discretized flexible belt with damping and inertia, connected to a separate discretized viscoelastic tread with inertia. The belt elements were connected to the wheel rim via the carcass, represented by viscoelastic bristles with circumferential and lateral degrees of freedom. They applied the viscoelastic interconnections between the adjacent bristles, which exhibited lesser oscillatory tendency of the bristles and also lower lateral force generation in comparison to the independent bristles model, when exposed to the transient conditions, as demonstrated in another study by Mavros et al (2005b).

Pacejka (1972) presented the string theory for studying transient behaviour of tyres. In the string theory, the tyre model consist of single or parallel strings, which are kept under a certain pretension by a uniform radial force distribution, representative of inflation pressure in tyres. The strings are elastically supported with respect to wheel centre plane in the axial direction, but restricted from moving in the circumferential direction, with a finite length established over its contact with the road surface. Also, finite contact width arises when parallel strings are used. The longitudinal deformation is addressed by the elastic tread elements, which are assumed to be flexible in the radial direction only. By taking assumption of small lateral deformation and considering the condition of continuity in slope at the leading edge, the lateral displacement of the string at the leading edge could be represented by first order differential equation, leading to the calculation of lateral forces and aligning moment. A tyre parameter significant in the transient analysis is the relaxation length, which also appears in the equation for lateral deflection in the stretched string model, and is expressed as the square root of the ratio of effective total tension acting on the string and the lateral carcass stiffness per unit length of the tyre circumference. Researchers have often employed relaxation length as a parameter in their study to account for the compliance of the carcass with respect to the rim, that is responsible for the lag in the response to lateral and longitudinal slip (Pacejka and Besselink, 1997). The study on relaxation length, with an aim to quantify it through experiment, was conducted in the past by Loeb et al (1990).

The semi-empirical tyre models such as the Magic Tyre Formula in its original form represent the steady-state characteristics of the tyre, which is mostly suited for low slip applications. Pacejka and Besselink (1997) extended the Magic Formula tyre model for transient behaviour, though limited to cover a frequency range up to 15 Hz and for
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wavelength of motion and road undulation, which are large enough in relation to the tyre contact length (>1.5 m). The authors adopted a single contact point model, which deflects horizontally with respect to the lower part of the rim. A point mass, attached to the moving contact patch, enables the calculation of forces and moments acting between the road and tyre as a response to the slip velocity of the point mass and to the camber angle (Figure 2-3). The carcass compliance together with the slip model of the contact patch enabled the model to address the wheel load dependent lag and also the reduction in lag with an increase in slip. The model also included the contribution of three important gyroscopic effects related with belt distortions. The latest Magic Formula model ‘PAC2002’ (Kuiper and Van Oosten, 2007) used two methods to account for tyre transient behaviour; the stretched string model and the contact mass model. The contact mass model also considered yaw rotation while generating differential equations for the contact mass. The model also included turn-slip effect to generate a new formula for tyre spin and parking, thus extending its application for low speed. Further description of this model can be seen in the chapter 6.

Figure 2-3: Single contact point tyre mass model (Pacejka and Besselink, 1997)

Zegelaar and Pacejka (1996, 1997) developed an in-plane rigid ring model to study in-plane tyre responses on uneven roads and subject to brake torque variations. The rigid ring, representing pneumatic tyre-wheel assembly is modelled as three components; the tyre tread band in the form of a circular ring, a rim and sidewalls. The tyre ring and rim are connected through horizontal, vertical and rotational springs and dampers, representing sidewalls and pressurized air, with the tyre road interface modelled as residual stiffnesses and a first order slip model using the relaxation length concept. The rigid ring has 4 degrees of freedom in the form of horizontal and vertical displacements of the ring, and rotation of the rim and the ring. The stationary slip characteristics are represented by the theoretical brush approach. The model was able to describe the deformation of the flexible tyre ring accurately, with
excitations provided as effective inputs. Maurice et al (1998) later extended the model to out-of-plane applications, by adding lateral displacement and rotation about the vertical axis in the rigid ring model.

Maurice et al (1999) presented a tyre model which takes into account the short wavelength sideslip variations, recognising the fact that with short wavelengths, unlike the lateral force, the self-aligning moment cannot be represented by the first order relaxation length concept. The author called his model “a pragmatic tyre model”, as it represents tyre behaviour by means of relatively simple differential equations. Using the brush model as the reference model, the analytical frequency response functions of the lateral force and the self-aligning moment (through pneumatic trail) are generated with respect to the variations in side slip. The pneumatic trail is obtained by a phase-leading system in series with the first-order system for the lateral force. The equations are finally solved in the time domain, where it can be combined with the Magic Formula steady-state characteristics for use in vehicle simulations.

More recently, an advanced model for short wavelength was developed, as an extension to the work of Zegelaar and Pacejka (1997) and Maurice et al (1999), using the rigid ring in-plane and out-of-plane concepts with higher flexible deformation modes. It can cover very short wavelengths (>10 cm) with frequency range of (60-80 Hz), and uses steady-state tyre slip characteristics of the Magic Formula. The model known as SWIFT (Short Wavelength Intermediate Frequency Tyre Model) was an outcome of joint work carried by Delft University and TNO automotive and has found particular application in ride, handling and comfort. Recently, it has been extended to a three-dimensional contact model that accounts for the highly non-linear tyre-enveloping properties (Jansen et al, 2005). Other commercial tyre models such as FTire and RMOD-K have also gained considerable attention for their application in ride and comfort studies. The above three models were comprehensively discussed at a CCG Seminar held in Vienna, September 2004, an outline of which is given in a study by (Lugner et al, 2005). The above advanced dynamic tyre models were tested under Tyre Model Performance Test (TMPT), which provided a comprehensive trial study for a possible comparative evaluation of tyre models for handling and high frequency behaviour (including response to short road unevenness, smaller than the tyre contact patch), under a virtual test rig implemented in three different multi-body software programs (ADAMS, DADS and SIMPACK), with a prerequisite of testing the tyre model with at-least one multi-body package.
2.4.1.4 Application of Transient Tyre Modelling in Braking Studies

This section reviews the studies, where tyre dynamic or transient behaviour has been studied under braking, involving brake torque variations such as in ABS.

Zegelaar and Pacejka (1997) used their rigid ring model to study tyre responses to brake torque variations in a stepwise fashion (increases in brake pressure). Wheel lock braking was also simulated on a single wheel. The authors compared their results with an experiment, which was performed on a rotating drum with a hydraulic servo system used to control the brake pressure. The results of both studies matched well, with deviations observed at large slip values, which were attributed to the slip characteristics of the brush model. It was felt that this could be improved by using Magic Formula for the slip characteristics. In the wheel-lock braking, the experiment showed stick-slip phenomenon – something which could not be simulated as the tyre model used a constant $\mu$ brush model. The authors also calculated the frequency response functions (FRFs), where the response of longitudinal force with brake torque and wheel slip variations was obtained, which were used to evaluate some of the tyre parameters.

van der Jagt et al (1989) studied the dynamic effects of longitudinal and vertical vibrations of the tyre and suspension, excited by road irregularities on the ABS performance. They modelled the tyre-wheel assembly suspended with springs and dampers in both the horizontal and vertical directions. Rolling tyre quasi-static deformation with mass and inertia of tyre belt under the influence of normal and tangential road excitation were also taken into account. A basic ABS algorithm was used to perform the braking simulation on a flat and wavy road, where large variations in brake slip on wavy road were observed. These were caused by horizontal and vertical wheel vibrations, which deteriorated the ABS performance and could pose serious problems for any control logic.

Jansen et al (1999) studied the influence of in-plane tyre dynamics on ABS braking, by employing three different tyre modes on a quarter vehicle model built in MADYMO. The three models used were; a steady-state model and a transient model, based on the Magic Formula characteristics, as well as a rigid ring model (Zegelaar and Pacejka, 1997). The authors compared the tyre models by applying some step excitation, where the influence of belt oscillation (rigid ring model) was notable in all the responses. The frequency response plots further established that the steady-state model could be used for brake torque
variations below 10 Hz, whereas the transient model could be used for brake torque variations up to 30 Hz. The rigid ring model with a high frequency range has no such restrictions. In the ABS simulation, it was observed that the rigid ring model was more reliable than the other two models, especially after observing the influence of tyre belt dynamics on ABS performance.

Pauwelussen et al (2003) performed a dynamic study using the SWIFT tyre model, where they investigated different in-plane oscillation problems through predefined road undulations such as steps, cleats, sinusoidal inputs, and also through brake torque variations. Furthermore, a sensitivity study was conducted to ascertain the influence of tyre parameters (such as sidewall stiffnesses and belt mass/inertias) on the Eigen frequencies and relative damping. Additionally, they compared the SWIFT model with the steady-state and transient Magic Formula tyre models by conducting similar parameter study using a quarter car ADAMS model, with a focus on the influence of belt dynamics. Finally, a full vehicle ABS simulation, using a professional ESP system, was conducted for different combination of road friction and vehicle speed, showcasing the significance of including belt dynamics for a proper ABS simulation.

Braghin et al (2006) developed a new relaxation length tyre model after a series of tests on a Flat Track machine. The model ‘MF-Relax’ was based on the Magic Formula characteristics and used a first order differential equation for determining the relaxation length as a function of both normal load and slippage. The author compared his model with two other advanced tyre models; the Magic Formula transient model which uses a first order relaxation length and the Enhanced Nonlinear Transient Tyre Model developed by Pacejka (2006). After determining the parameters of all the three models by elaborate experimental testing, an ABS braking manoeuvre was performed using a real test car as well as with a multi-Body vehicle model, where the three tyre models were simulated. The longitudinal acceleration and ABS brake pressure time history showed that both the MF-Relax model and the enhanced model compared well with the experiment, except at low speeds where slippage becomes critical. The ABS controller, however, could not produce the same performance when the Magic Formula transient model was used, and failed to stop the car in the expected time.

van Zanten et al (1989, 1990) conducted a detailed experimental study to investigate the transient effect of tyre during an ABS operation, and to simulate the effect using a transient tyre model integrated into a test stand. In the initial study, the authors carried out
measurement and simulation of longitudinal dynamics only. However, in the later paper, they extended the work to include lateral dynamics also. Overall, for measurement, an experimental test car was equipped with rotating wheel dynamometers for measuring wheel forces and moments, along with other sensors (including those of ABS) and associated electronics. The correlation between the oscillations in the wheel speed and the braking force caused by the pressure pulses, along with high frequency oscillations in the lateral tyre forces were observed. In addition, the phase shift between the slip angle and the lateral tyre force could be reproduced. Considering the transient effects of tyre (particularly sidewalls and tread), a tyre model with rigid belt, elastically connected to the rim via elastic sidewalls, was developed. The contact condition was represented by a tread model based on brush modelling approach. Through the simulation and experimental study, the author stressed the importance of including tyre transient effects for the ABS study.

2.5 Brake System

In vehicle handling / braking studies, the brake torque is often represented through simple expressions or through use of linear braking models with compact representation of intrinsic characteristics, as used in VDANL (Alien et al., 1988). However, the studies in which driver/vehicle interaction or braking control is involved, the transience involved in a brake system response from the pedal force to brake torque on the wheel, becomes important in a dynamic simulation. The study of complete brake system dynamics has been conducted in the past for number of reasons:

- Component / system design and performance improvement
- Driver /vehicle interaction
- Study of braking characteristics in a dynamic simulation
- Application in control system

In this thesis, the dynamic characteristics of a brake system are incorporated, keeping in view the last two reasons. This section provides a brief review of some of the studies regarding dynamics of a passive braking system.

The functioning of a conventional brake system of a passenger car is illustrated in Figure 2-4. When a driver presses the brake pedal, the force from the pedal after being multiplied by the pedal/lever ratio reaches the brake booster. The hydraulic or pneumatic brake booster
further amplifies it by a gain. The booster exerts a force against the piston in the master cylinder, which pressurises the brake fluid in the master cylinder. This pressurised fluid is forced out of the master cylinder and into the wheel cylinders, through the brake lines. The flow into the rear brake cylinders is passed through proportioning valves, which are normally attached integrally to the master cylinder. The hydraulic pressure at the wheel cylinder is translated into the friction pads, which eventually applies the brake torque at the disk / drum to resist the motion of the wheels.

![Diagram of the conventional brake system](image)

**Figure 2-4: The conventional brake system**

Fisher (1970) developed a comprehensive model of a complete brake system, including the dynamics of brake pedal, vacuum booster, master cylinder and the brake lines. The model has 18 states, including the transient characteristics of a brake system, with frequency and thermal response data obtained using an experimental setup. The author discussed its application in design of ABS systems, but the model in general is too complicated for a control study or even for an efficient full vehicle simulation. Khan *et al* (1994) used (Fisher, 1970) model as the base to develop an analytical model for an applied braking system, where they included proportioning valves and improvised the vacuum booster dynamics. The model has 10 states and was constructed using the bond graph technique. The model was validated against laboratory experiments, performed on a bench setup of the brake system.

Gerdes and Hedrick (1999) presented a reduced order model of brake system dynamics, incorporating simplified brake hydraulics and vacuum booster with transient effects. The model was originally developed for longitudinal control of automated highway vehicles for the Californian PATH program. It incorporated many critical aspects for control system
application such as hysteresis and disturbance modelling. The hydraulic system has originally four states in relation to fluid capacity of each wheel, which could even be reduced to a single state by assuming a lumped fluid capacity for the entire brake system. The model was verified against experiment, where very close response from pedal force to wheel brake pressure were established.

Fortina (2003) modelled a passive brake system, comprising a booster, a tandem master cylinder, callipers, and pipes using a commercial software AMESim®, which is especially used for hydraulic simulation with features to physically model a component. The model was experimentally verified through a braking test bench. The author extended the brake system model by attaching the hydraulic units of ABS and VDC (vehicle dynamic control), which was then integrated with a vehicle model and a control system, both built in MATLAB / Simulink to perform a vehicle simulation.

Delaigue and Eskandarian (2004) developed a vehicle braking system model to predict vehicle stopping distances. The brake characteristics were represented using a second order function for fluid pressure, which were translated into braking torque through a first order load dependent function and use of a correction factor for heat dissipation. The vehicle model took different factors into account for stopping distance prediction such as pavement properties, road slope, suspension characteristics, brake types (disk/drum), drag/wind influence, tyre properties (dimension, load and pressure), driver behaviour (reaction time and motion dynamics), and ABS characteristics. The model was validated for different braking configurations and a range of vehicles, where simulated distance was found to be reasonably close to the gathered track test results.

2.6 Anti-Lock Braking System (ABS):

The continuous development of passenger car brake systems have resulted in powerful and reliable systems, which are capable of providing optimum retardation and effective braking under normal operating conditions. However, when a vehicle undergoes critical conditions, such as those encountered with wet or slippery road surfaces or panic braking by driver, the vehicle is susceptible to wheel locking. As a result, the vehicle steering response is lost and it can lose traction and stability. An Anti-lock braking system becomes very useful in these situations as it would detect incipient locking action at one or more wheels in time, and react
by modulating the brake pressure on individual wheels. This action could prevent the wheel from locking and keep the tyre slip within a desired range.

An anti-lock braking system primarily consist of three major parts 1) electronic sensors for measuring the wheel velocities, brake pressure, vehicle velocity and/or vehicle acceleration 2) an electrical control unit (ECU), which is usually a microprocessor based system and 3) electrically controllable valves or pumps to control the pressure in brake cylinders or chambers. A general loop of an ABS control, which uses hydraulic modulator with solenoid valves, is shown in Figure 2-5.

![ABS control loop diagram](image)

In practice, tyre slip is difficult to determine accurately due to the lack of a practical and cost effective means to determine the linear speed of the tyre centre during braking. Hence, some easily measurable parameters such as the angular speed and angular deceleration of the wheel and linear deceleration of the vehicle are used to formulate the control strategy in an ABS system. The angular speed and angular deceleration of the wheel are derived from the wheel sensors with electromagnetic pulse signals. The signals are transmitted to the electronic control unit for processing. Once the signals are processed in the control unit, the measured parameter and/or those derived from them are compared with the corresponding reference values. When certain conditions that indicate the impending lock-up are met, a command signal is sent to the modulator to release the brake. During the braking process, the operating condition of the tyre and the vehicle are continuously monitored by sensors and the control unit. After the danger of wheel locking is predicted and the brake is released, another module in the control unit will determine the point at which the brake needs to be reapplied. In passenger car ABS mostly hydraulic regulator valves are used such that they can increase, decrease or hold the brake line pressure. Each wheel has its isolation and exhaust valves and
through opening and closing of these valves, the required hydraulic state can be attained. An accumulator is also incorporated which temporarily absorbs the flow surge which occurs during the pressure decrease phase.

Figure 2-6 shows a typical plot of wheel speed profile during ABS control. When the brakes are initially applied, the drop in the wheel speeds is more or less in accord with the vehicle speed (section 1 in the Figure 2-6). If the brakes are applied at a higher value, or the road surface is slippery, the speed of one or more wheels begins to drop rapidly (section 2), which indicates that the tyre has gone through the peak of its $\mu$-slip curve and is heading towards lockup condition. The ABS intervenes at this juncture and releases the brakes on those wheels before lockup can occur (section 3). Once the wheel speed picks up again, the brakes are reapplied. By working in this fashion, the ABS tries to keep each tyre near the peak point of the $\mu$-slip curve, or around a desired slip value according to the particular control strategy.

![Figure 2-6: Wheel speed cycling during ABS braking (Gillespie, 1992)](image)

2.6.1 ABS Application

The anti-lock braking systems can be traced back to 1930's, when Bosch first applied an ABS patent for the passenger car (SAE Publication, 1996), but the first application came in early 50's when Dunlop's Maxaret system was introduced in aircrafts (Rowell and Gritt, 1986). Though, in 1950s and 1960s, some development of ABS application was carried out, the real growth of ABS took place in the 1970s, primarily due to the developments in integrated electronics and microprocessor technologies. At that time, vehicle manufacturers like Ford and Chrysler introduced ABS systems in their vehicles, which were based on analogue principles. The first ABS system based on digital electronics was introduced by Bosch in 1978, which found its way to Mercedes Benz (Bosch, 1999).
Since the introduction of first commercial ABS in the 1970s, the ABS has gone through various generations of development. Over these years, various methods for ABS control were introduced, which can be broadly grouped under two categories: generic ABS methods and advanced slip control ABS methods. This section briefly reviews some of the control studied conducted in the past for ABS.

2.6.1.1 Generic ABS Method

The starting ABS control systems were mostly developed empirically through iterative design and test techniques. The use of theoretical approaches and systematic studies were later reported by authors such as Guntur and Ouwerkerk (1972), who investigated the conditions for prediction and reselection (brake release and reapplication) in an ABS cycle. In another paper, Guntur (1974) studied the characteristics of brake pressure modulator and signal processing unit, looking at various design aspects such as rise rate / decay rate of wheel cylinder pressure, time delays etc. In general, his approach can be termed as a conventional peak-seeking approach, where the algorithm is designed to contain the wheel slip within a range, close to the optimal slip value that corresponds to the peak point of the \( \mu \)-slip curve. The ABS control system by Bosch (1999) also uses the conventional peak-seeking approach, where the wheel angular acceleration and deceleration thresholds are used to determine the brake control cycle. By keeping the angular deceleration within certain thresholds, the brake slip could be checked and the wheel could be prevented from locking. The ABS system based on the peak-seeking approach is quite simple to implement in a full vehicle simulation environment, as demonstrated by Day and Roberts (2002).

The controller presented by Bowman and Law (1993) is based on a concept similar to wheel angular acceleration / deceleration threshold, but uses the wheel angular slip instead. Two threshold values of wheel slip \( S_1 \) and \( S_2 \) (both as a function of tyre slip angle) were used so as to keep the requirements of lateral manoeuvring. Once the braking is initiated, the brake pressure is allowed to increase, until the wheel slip (\( \lambda \)) exceeds the lower threshold \( S_1 \). The brake pressure is then maintained at a constant level, so that the rate of slip increase can be reduced. If the wheel slip continues to increase or reaches the upper threshold \( S_2 \), the brake pressure is released. By doing so, the wheel slip re-enters the region bounded by \( S_1 \) and \( S_2 \), where again the brake pressure is held constant at a level it was when it first passed the upper limit \( S_2 \). Figure 2-7 shows the slip range \( S_1 \) and \( S_2 \) of generic peak seeking ABS for a range of slip angle. The control law can be put as:
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\[ \lambda > S_2, \quad \text{Brake pressure is released} \]

\[ \lambda < S_1, \quad \text{Brake pressure is increased} \]

\[ S_1 < \lambda < S_2, \quad \text{Brake pressure is held constant} \]

Figure 2-7: Slip range of generic peak seeking ABS (Bowman and Law, 1993)

Zeller (1984) followed an analytical method to design an ABS system, where he used nonlinear feedback control techniques to examine response of dynamic components such as the modulator. By performing analysis of time and frequency responses, the wheel angular acceleration was found to be the preferred motion feedback variable for generating a stable antilock limit cycle.

Ozdalyan (1999) used ADAMS to simulate the performance of an anti-lock braking system. He initially developed a slip control ABS algorithm for a quarter vehicle model, built in ADAMS, where he investigated the influence of change in road condition from dry to wet on ABS braking, using the Fiala tyre model (Ozdalyan and Blundell, 1998). He then extended the work to half vehicle and full vehicle models, where he applied a realistic ABS model based on the work by (Guntur, 1975). Finally, he studied the influence of ABS and tyre parameters on the braking outcome on dry and wet road surfaces. The study was conducted using steady-state tyre characteristics, with the inclusion of transient tyre effects suggested as part of future work.

Watanabe and Noguchi (1990) presented an ABS algorithm based on wheel acceleration control, which compensates for the road disturbance and road friction coefficient. The basic feature of which was to allow for the continuous shifting of the wheel acceleration threshold.
by the disturbance amplitude of the wheel acceleration, and to determine the duration for brake pressure reduction by various road friction coefficient. Harned et al (1969) presented measurement of tyre brake force characteristics related to wheel slip, with an objective of enabling wheel slip control systems to adapt to a wide range of tyre-road operating conditions. The measurements covered a wide range of commercial tyre types on dry, wet and icy road surfaces, at different test speeds. The measurements were conducted on an instrumented tyre brake trailer equipped with an ABS controller, attached to a tow truck. The dependence of wet road characteristics on road construction, water cover depth, and tread wear were demonstrated.

The vehicle velocity, as discussed earlier, is difficult to measure because of the expensive sensors and additional wiring which reduces the reliability of the system. Kuo and Yeh (1994) used Harned et al (1969) tyre-road characteristics to develop a slip estimation scheme for an ABS control system, using a single wheel model. Jiang and Gao (2000) proposed an adaptive non-linear filter approach to directly estimate the vehicle velocity, based on the wheel velocities. The estimated vehicle velocity then provides an estimate of slip, which is compared with the desired slip value to control the wheel slip.

2.6.1.2 Advanced Slip Control ABS

The idea of advanced slip control is primarily based on clear structure of closed loop control, where the control algorithm calculates or estimates the actual wheel slip, compares it with the desired or optimal value, and then adjusts the amount of brake pressure based on the wheel slip error. Over the years, the study of advanced control strategies for ABS system has increasingly become more popular. The classification of these control strategies is not so straightforward as they can be differentiated in a number of ways, such as: model free or model based, parallel structure or variable structure, adaptive or feedback, conventional or hybrid. However, it is easy to list them in their popular form such as: sliding mode, fuzzy logic, neural network etc.

Sliding mode control (or variable structure control) has appeared more frequently in the ABS literature in the last decade or so. The main idea behind the sliding mode control is to restrict the system motion in a sliding plane, where a predetermined function of error is zero. It is known to be robust to parameter uncertainties and external disturbances, as shown by Lin et al (1993). Using an electric brake actuator, the author developed an ABS control
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system for wheel slip regulation and conducted a sensitivity study, where it was shown that
the non-linear sliding mode controller was more robust to parametric variations when
compared with a linear feedback controller. Will et al (1998) presented a non-linear control
system that combines sliding mode based optimizer and PID controller for an ABS system.
The sliding mode optimizer can perform an online search for the optimal wheel slip, and is
coupled with the PID controller to regulate the brake torque so as to control the wheel slip.
Wu and Shih (2003) built a non-linear mathematical model of the anti-lock braking system
and conducted an ABS study, using two sliding mode controllers, on a quarter-car model. Of
the two sliding mode controllers, the pulse width modulation method (PWM) reduced the
chattering effect, which is associated with the switching control method as an effect of
discontinuous control signals. Mills et al (2002) conducted an ABS performance study on
full vehicle model to investigate the effects of transient load shifting. An analytical approach
was used to develop the ABS modulator dynamics, using the bond graph technique. The
authors then adopted two individual control strategies on full vehicle model :- 1) Model free
proportional-integral controller design, with gains scheduled into a look-up table. 2) Model
based discrete non-linear controller, composed of sliding model control algorithm and
inverse brake modulator model. Both the controllers gains were fine tuned using simulation
and experiments, and then full vehicle ABS simulations were conducted, where the model
based controller performance was marginally better than the model free controller. However,
it was felt that the transient load shifting effect on the model based controller should be
further investigated.

Fuzzy logic controllers, contrary to the sliding mode controllers, do not require a
mathematical model of the plant. In addition, they have an inherently parallel structure,
which allows the controller to respond immediately, once a new situation arises (Mauer,
1995). Also, most of the complex systems used in cars involve multiple parameters and
optimization of these parameters is mostly based on the engineering expertise. All this makes
the fuzzy logic a popular candidate for control applications, which is already used in a
number of commercial vehicles for different systems such as ABS, engine, automatic
gearbox control etc. (von Altrock, 1994). Mauer et al (1994) presented a fuzzy logic
controller and a decision logic network, which identifies the current road conditions based on
current and past readings of the slip ratio and brake pressure. The controller detects
impending wheel lock immediately and avoids excessive slipping. Layne et al (1993)
introduced the idea of fuzzy model reference learning control to maintain an adequate
performance even under adverse road conditions. The controller utilizes a learning mechanism which observes the plant outputs and adjusts the rules in a direct fuzzy controller so that the overall system behaves like a reference model.

Other control strategies such as neural network have also been studied, but have shown limited prospect for commercial ABS application. The method is based on the premises that the physical system can be sufficiently instrumented during network training so the accurate evaluation of the effect of control actions is possible. However, in a system like ABS, it may be costly to obtain the detailed data that would be required to exploit the full capabilities of neural methods (Davis et al, 1992).

2.7 Chapter Remarks

A detailed discussion of the work published in the field of vehicle dynamics and simulation is presented in this chapter. Apart from exploring the various modelling approaches related to the vehicle and its sub-systems such as suspension, tyre and brake, the chapter also looks into the non-linear behaviour aspects of these systems in transient conditions. The level of detail required in vehicle model, be it a vehicle or its sub-systems, has always been a contentious issue. The discussion in the chapter further extends some of the arguments about complexity of the vehicle model, about suspension and tyre modelling, and suitability of their application in transient vehicle handling (including limit handling). The chapter further establishes the understanding that in relation to the commercial multi-body software packages which are widely used in the industry, the relatively simpler solutions might offer comparable accuracy and additional benefits such as fewer parameters and reduced computational cost along with the distinct advantage of extending them in control related studies. Keeping this in view, the following chapter looks into the fundamental description of the theoretical vehicle model, based on the Newton-Euler approach, which is subsequently applied to develop a 10-DOF intermediate vehicle model. Apart from presenting a brief overview of the work published in ABS control, the present chapter also discusses the work carried out in transient tyre modelling, with particular reference to its application in ABS braking studies. Through the limited literature published in the above area, it is clear that more study is still required to establish the influence of tyre transients in ABS braking in a full vehicle simulation environment, under a variety of operating conditions.
3 Theoretical Vehicle Modelling

3.1 Introduction

A fundamental description of the theoretical vehicle model is discussed in this chapter with consideration given to use of appropriate coordinate systems and generalized motion of a particle leading to rigid body motion of the vehicle body. Newton-Euler approach is adopted for the formulation of equations of motion for the 6-DOF of sprung mass. The approach is also used to initially formulate 2-DOF and 3-DOF vehicle models, and to finally develop a 10-DOF intermediate vehicle model. The 10-DOF model also hosts a reduced order hydraulic brake system model; the description of which is also included in the chapter.

3.2 Equations of Motion of the Vehicle Body

3.2.1 Kinematic Equations of a Particle

In describing the kinematic equations of a particle, one must distinguish between the coordinate frame in which the vector components are specified and the coordinate frame in which the derivatives are obtained. The classical approach is to describe the motion in a reference frame which moves relative to a fixed global frame of reference. Figure 3-1 shows these two frames, where \( O_A \) represents the origin of the global or fixed frame of reference \( A (XYZ) \) and \( O_B \) represents the origin of the local or moving frame of reference \( B (xyz) \). The fixed frame and moving frame have orthogonal set of basis vectors \( I, J, K \) and \( i,j,k \) respectively.

Assume a vector \( c_1 \), measured in the moving frame of reference. The time derivative of the same vector in the fixed frame can be expressed as: (Rahnejat,1998)

\[
\left( \frac{dc_1}{dt} \right)_A = \left( \frac{dc_1}{dt} \right)_B + \omega \times c_1
\]

(3.1)

where, \( \left( \frac{dc_1}{dt} \right)_A \) = Time derivative of the vector in the fixed frame
\[ \left( \frac{dc_1}{dt} \right)_B = \text{Time derivative of the vector in the moving frame} \]

\[ \omega \times c_1 = \text{Cross product of the rotational velocity } \omega \text{ of the moving frame of reference and vector } c_1 \text{ with respect to the fixed frame of reference.} \]

In order to distinguish between the vector representation in the fixed and the moving frames of reference, subscripts A and B are used throughout this chapter.

### 3.2.1.1 Relative Velocity

![Graph showing general motion of a particle in fixed and moving frames of reference]

*Figure 3-1: General motion of a particle in the fixed / moving frames of reference*

The general motion of a particle \( P \) (Figure 3-1) with respect to the fixed and the moving frames of reference is represented by the following vectors.

- \( R_A \) Position vector of particle \( P \) with respect to the fixed frame of reference A.
- \( R_B \) Position vector of particle \( P \) with respect to the moving frame of reference B
- \( R_0 \) Position vector of the origin \( O_B \) with respect to the fixed frame of reference A

In order to determine the relative velocity and acceleration of particle \( P \) in the two reference frames, one can state the following vector relationship:
Theoretical Vehicle Modelling

\( R_A = R_0 + R_B \) \hspace{1cm} (3.2)

Let: \( R_B = x \cdot i + y \cdot j + z \cdot k \)

which leads to: \( R_A = R_0 + (x \cdot i + y \cdot j + z \cdot k) \)

Differentiating the equation with respect to time yields:

\[
\dot{R}_A = \dot{R}_0 + (\dot{x} \cdot i + \dot{y} \cdot j + \dot{z} \cdot k) + (x \cdot \dot{i} + y \cdot \dot{j} + z \cdot \dot{k})
\]

where \( i, j, k \) of the moving frame are not constant vectors, since their direction may change with respect to the fixed frame \( A \). \( \dot{x}, \dot{y}, \dot{z} \) are the projections of velocity of point \( P \) on the moving frame \( B \), hence:

\[
V_{P/B} = \dot{x} \cdot i + \dot{y} \cdot j + \dot{z} \cdot k
\]

(3.4)

Also, \( i, j, k \) of the moving frame may be represented as

\[
\dot{x} = \omega \times i, \dot{j} = \omega \times j, \dot{k} = \omega \times k
\]

where \( \omega \) is the angular velocity of the moving frame, and hence equation (3.3) results in:

\[
\dot{R}_A = \dot{R}_0 + V_{P/B} + \omega \times (x \cdot i + y \cdot j + z \cdot k)
\]

(3.5)

where \( R_B = x \cdot i + y \cdot j + z \cdot k \) is the position vector of particle \( P \) with respect to the moving frame \( B \), \( \dot{R}_o \) is the velocity of the origin \( O_B \) with respect to the fixed frame \( A \) \( (V_{O_B/A}) \) and \( \dot{R}_A \) is the velocity of the particle \( P \) with respect to the fixed frame \( A \) \( (V_{P/A}) \). Thus:

\[
V_{P/A} = V_{P/B} + V_{O_B/A} + (\omega \times R_B)
\]

(3.6)

3.2.1.2 Relative Acceleration

Similarly, one can find the relative acceleration by differentiating equation (3.3), thus:

\[
\ddot{R}_A = \ddot{R}_0 + (\ddot{x} \cdot i + \ddot{y} \cdot j + \ddot{z} \cdot k) + 2(\dot{x} \cdot \dot{i} + \dot{y} \cdot \dot{j} + \dot{z} \cdot \dot{k}) + (x \cdot i + y \cdot j + z \cdot k)
\]

(3.7)
where $\ddot{R}_A$ is the acceleration of a particle with respect to the fixed frame $A$ ($a_{p/A}$), $\ddot{R}_0$ is the acceleration of origin $O_B$ related to the fixed frame $A$ ($a_{o/A}$), and $a_{P/B} = \ddot{x} \cdot i + \ddot{y} \cdot j + \ddot{z} \cdot k$ is the acceleration of particle $P$ with respect to the moving frame $B$.

In the above equation, the term $2(\ddot{x} \cdot i + \ddot{y} \cdot j + \ddot{z} \cdot k)$ can be expressed as:

$$2(\ddot{x} \cdot i + \ddot{y} \cdot j + \ddot{z} \cdot k) = 2[\omega \times (x \cdot i + y \cdot j + z \cdot k)] = 2(\omega \times V_{p/B})$$  \hspace{1cm} (3.8)

Also, the term $(x \cdot \ddot{i} + y \cdot \ddot{j} + z \cdot \ddot{k})$ can be simplified using the relation $\ddot{i} = \ddot{\omega} \times i + \omega \times \ddot{i}$

This gives $\ddot{i} = \ddot{\omega} \times i + \omega \times (\omega \times i)$  \hspace{1cm} (3.9)

where $\ddot{\omega}$ is the angular acceleration of frame $B$. Similar relations apply to the unit vectors $\ddot{j}$ and $\ddot{k}$

Therefore, $x \cdot \ddot{i} + y \cdot \ddot{j} + z \cdot \ddot{k} = \ddot{\omega} \times (x \cdot i + y \cdot j + z \cdot k) + \omega \times [\omega \times (x \cdot i + y \cdot j + z \cdot k)]$

$$= \ddot{\omega} \times r + \omega \times [\omega \times R_B]$$  \hspace{1cm} (3.10)

From the above equations and also using the relation $a_o \times (b_o \times c_o) = b_o \cdot (a_o \cdot c_o) - c_o \cdot (a_o \cdot b_o)$, the following equation can be obtained:

$$a_{P/A} = a_{p/B} + a_{o/A} + \ddot{\omega} \times R_B + \omega \times (\omega \cdot R_B) - R_B \ddot{\omega}^2 + 2(\omega \times V_{p/B})$$  \hspace{1cm} (3.11)

where $a_{p/B} + a_{o/A}$ is the linear acceleration

$\ddot{\omega} \times R_B$ is the tangential acceleration

$\omega \times (\omega \cdot R_B) - R_B \ddot{\omega}^2$ is the centripetal acceleration

$2(\omega \times V_{p/B})$ is the coriolis acceleration

The above theory of describing rigid body motion through description of elementary particle kinematics forms the theoretical basis for studying kinematics of vehicle body motion. The velocity and acceleration of any point of the rigid body can thus be obtained, knowing its
position vector which remains the same with respect to the moving reference frame (attached to the moving vehicle body). However, in rigid body kinematics, it is often necessary to project velocities on the moving frames of reference. For this purpose, in addition to the theory for rigid body kinematics (already presented), it is essential to apply relative transformation, as a convenient way to switch between the frames of reference.

3.2.2 Relative Transforms

Using Euler transformation method (Rahnejat, 1998), objects in the moving frame of reference can be transformed to the fixed frame of reference and vice-versa. In the present analysis, ‘1-2-3’ transformation is used; which means that the first rotation is about the x-axis, the second about the y-axis and the third about the z-axis (referred to as roll, pitch and yaw). Let $X, Y, Z$ be the coordinates of the point $P$ in the reference Frame $A$ with origin $O_A$, and let $x, y, z$ be the coordinates of the same point in the reference Frame $B$ with origin $O_B$.

![Image of Relative Transformation]

It is possible to move from frame $B$ to frame $A$ by performing three sequential rotations and a linear translation of $B$ (Figure 3-2), until $B$ coincides with $A$. It should be noted that in this specific case, each rotation takes place with respect to the previous frame of reference, not the initial frame of reference.

**Figure 3-2: Relative transformation between the moving and fixed frames of reference**
Theoretical Vehicle Modelling

Thus, it is a case of sequential relative transforms:

i) Linear translation resulting coincidence of \( O_B \) with \( O_A \)

ii) 1st rotation \((\phi)\) about \( O_B \) \( x \) axis, resulting in \( O_A \) \( x'y'z' \)

iii) 2nd rotation \((\theta)\) about \( O_A \) \( y' \) axis, resulting in \( O_A \) \( x'y'z'' \)

iv) 3rd rotation \((\psi)\) about \( O_A \) \( z'' \) axis, resulting in coincidence with Frame \( A = O_AXYZ \)

The three transformation matrices (roll, pitch and yaw) for three different rotations can be expressed as:

\[
L(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \quad (3.12)
\]

\[
L(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad (3.13)
\]

\[
L(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.14)
\]

\[
(x' \ y' \ z')^T = L(\phi) \cdot (x \ y \ z)^T \quad 1^{st} \text{ intermediate frame of reference}
\]

\[
(x'' \ y'' \ z'')^T = L(\theta) \cdot L(\phi) \cdot (x \ y \ z)^T \quad 2^{nd} \text{ intermediate frame of reference}
\]

\[
(X \ Y \ Z)^T = L(\psi) \cdot L(\theta) \cdot L(\phi) \cdot (x \ y \ z)^T \quad 3^{rd} \text{ (final) frame of reference}
\]

The resulting overall transformation matrix shown below is obtained by multiplying these three transformation matrices in the following order \( L(\phi,\theta,\psi) = L(\psi) \cdot L(\theta) \cdot L(\phi) \)

\[
L(\phi,\theta,\psi) = \begin{bmatrix} \cos\theta \cdot \cos\psi & \sin\phi \cdot \sin\theta \cdot \cos\psi - \cos\phi \cdot \sin\psi & \cos\phi \cdot \sin\theta \cdot \cos\psi + \sin\phi \cdot \sin\psi \\ \cos\theta \cdot \sin\psi & \sin\phi \cdot \sin\theta \cdot \sin\psi + \cos\phi \cdot \cos\psi & \cos\phi \cdot \sin\theta \cdot \sin\psi - \sin\phi \cdot \cos\psi \\ \sin\theta & \sin\phi \cdot \cos\theta & \cos\phi \cdot \cos\theta \end{bmatrix} \quad (3.15)
\]
Theoretical Vehicle Modelling

3.2.3 Application of Kinematics Equations for Vehicle Dynamics Study

The above theory can be extended to vehicle motions following a similar approach of two different sets of axes: ‘A’ represents the global frame of reference, fixed to the ground, whilst ‘B’ represents a frame of reference attached to the vehicle and moving with it, thus, its position and orientation changes continually with respect to the global frame of reference.

The velocity and acceleration of a point P on the vehicle can be arranged in a matrix form using the kinematics equations. The velocities and accelerations are calculated with respect to the global/ground frame of reference ‘A’, as velocities and accelerations with respect to the moving set of axes ‘B’ are all equal to zero. However, it is much more convenient to use the projection of these velocities to the moving frame of reference ‘B’. In this way, one may gain a better perception of the situation, as it is very important to know the velocity vectors in certain directions specified by the frame of reference attached to the vehicle.

Equation (3.6) includes the translational motion of the origin of the moving frame of reference and is used for in the current analysis. In order to write this equation for the ground frame of reference it is essential to multiply the second term by the transformation matrix L, so that the components of the rotational velocities are projected to the ground frame of reference:

\[ V_{p/A} = V_{\theta_{h/A}} + L \left( V_{p/B} + (\Omega_B \cdot R_B) \right) \]  

(3.16)

The term \( \Omega_B \cdot R_B \) represents the cross product \( \omega \times R_B \) in a matrix form. By carrying out the cross product of the vectors, the matrix \( \Omega_B \) can be obtained so that \( \Omega_B \cdot R_B = \omega \times R_B \):

\[
\Omega_B = \begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix}
\]  

(3.17)

where \( \omega_x, \omega_y, \omega_z \) are the components of the moving frame’s rotational velocity.

Equation (3.16) represents the velocity of a point P of the vehicle with respect to the global frame of reference. This velocity should now be projected to the moving frame of reference by pre-multiplying the velocity \( V_{p/A} \) by the inverse transformation matrix \( L^{-1} \):
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$$V_{p/b} = L^t \cdot V_{p/a} = L^t \left[ V_{o_b/a} + L \left( V_{p/b} + \Omega_B \cdot R_B \right) \right]$$

Thus: $$V_{p/b} = L^t V_{o_b/a} + L^t L \left( V_{p/b} + \Omega_B \cdot R_B \right)$$

Hence: $$V_{p/b} = L^t V_{o_b/a} + \left( V_{p/b} + \Omega_B \cdot R_B \right)$$ \hspace{1cm} (3.18)

Equation (3.18) is written in a condensed matrix form. It is easy to obtain the velocity components in all the three directions of the axes of the moving frame as follows:

Let $[U \ V \ W]^T$ be the projections of the translational velocity to the moving frame of reference, so that $[U \ V \ W]^T = L^t V_{o_b/a}$.

Also let $[p \ q \ r]^T = [\omega_x \ \omega_y \ \omega_z]^T = \omega$, then the components of the velocity in the directions of the moving axes can be written:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} U \\ V \\ W \end{bmatrix} + \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} z \cdot q - y \cdot r \\ x \cdot r - z \cdot p \\ y \cdot p - x \cdot q \end{bmatrix}$$ \hspace{1cm} (3.19)

Because point P belongs to the vehicle, which is considered as a rigid body, the middle term of the right-hand side of the above equation becomes zero. Thus, the velocity of a point P becomes:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} U \\ V \\ W \end{bmatrix} + \begin{bmatrix} z \cdot q - y \cdot r \\ x \cdot r - z \cdot p \\ y \cdot p - x \cdot q \end{bmatrix}$$ \hspace{1cm} (3.20)

The acceleration of point P can be calculated by differentiating equation (3.18):

$$\dot{V}_{p/b} = dL^t V_{o_b/a} / dt + d \left( V_{p/b} + \Omega_B \cdot R_B \right) / dt$$

$$\Rightarrow \dot{V}_{p/b} = dL^t V_{o_b/a} / dt + d \left( \dot{R}_B + \Omega_B \cdot R_B \right) / dt$$

$$\Rightarrow \dot{V}_{p/b} = dL^t V_{o_b/a} / dt + \ddot{R}_B + \Omega_B \cdot R_B + 2 \cdot \Omega_B \cdot \dot{R}_B + \Omega_B \cdot \Omega_B \cdot R_B$$
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$$\Rightarrow a_x = \ddot{V}_{p/B} = \frac{dL^1V_{O_y/A}}{dt} + \Omega_B \cdot L^1V_{O_y/A} + \ddot{R}_B + \dot{\Omega}_B \cdot R_B + 2 \cdot \dot{\Omega}_B \cdot \dot{R}_B + \Omega_B \cdot \Omega_B \cdot R_B \ (3.21)$$

Equation (3.21), after carrying out the matrix calculations, can be written as:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix} + \begin{bmatrix} -V \cdot r + W \cdot q \\ -W \cdot p + U \cdot r \\ -U \cdot q + V \cdot p \end{bmatrix} + \begin{bmatrix} -x \cdot (q^2 + r^2) + y \cdot (p \cdot q - r) + z \cdot (p \cdot r + q) \\ -y \cdot (r^2 + p^2) + z \cdot (q \cdot r - p) + x \cdot (p \cdot q + r) \\ -z \cdot (p^2 + q^2) + x \cdot (p \cdot r - q) + y \cdot (q \cdot r + p) \end{bmatrix} \ (3.22)$$

3.2.4 Dynamic Equations of Motion

The dynamic equations of motion can be derived from the kinematic equations by applying the Newton – Euler method.

Newton's law for a single point P of the vehicle is expressed as:

$$\delta F_x = \delta m \cdot \ddot{u} \quad (3.23)$$

$$\delta F_y = \delta m \cdot \ddot{v} \quad (3.24)$$

$$\delta F_z = \delta m \cdot \ddot{w} \quad (3.25)$$

The effect of the sum of particles, which form the vehicle, is realised through integration:

$$\int \delta F_x = \int \delta m \cdot \ddot{u} \quad (3.26)$$

$$\int \delta F_y = \int \delta m \cdot \ddot{v} \quad (3.27)$$

$$\int \delta F_z = \int \delta m \cdot \ddot{w} \quad (3.28)$$

where the integration of particle masses can be equated to total mass of the vehicle

$$\int \delta m = m \quad (3.29)$$

Also, $$\int \delta m \cdot (x, y, z) = m \cdot (x_G, y_G, z_G) \quad (3.30)$$
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where \( x_c, y_c, z_c \) is the vector of coordinates of the position of the centre of mass of the vehicle with respect to the moving frame of reference.

The sum of all the forces in the directions \( x, y, z \) can be denoted by \( \Sigma F_x, \Sigma F_y, \Sigma F_z \), so that:

\[
\Sigma F_x = \int \delta F_x \quad \text{is the sum of forces in the direction of } x \text{ axis of frame } B
\]

\[
\Sigma F_y = \int \delta F_y \quad \text{is the sum of forces in the direction of } y \text{ axis of frame } B
\]

\[
\Sigma F_z = \int \delta F_z \quad \text{is the sum of forces in the direction of } z \text{ axis of frame } B
\]

Consequently, the combination of equations (3.22) - (3.30) yields:

\[
\Sigma F_x = m \cdot \left( \dot{U} - V \cdot r + W \cdot q \right) + m \left[ -x_G \cdot \left( q^2 + r^2 \right) + y_G \cdot \left( p \cdot q - r \right) + z_G \cdot \left( p \cdot r + q \right) \right]
\] (3.31)

\[
\Sigma F_y = m \cdot \left( \dot{V} - W \cdot p + U \cdot r \right) + m \left[ -y_G \cdot \left( r^2 + p^2 \right) + z_G \cdot \left( q \cdot r - p \right) + x_G \cdot \left( p \cdot q + r \right) \right]
\] (3.32)

\[
\Sigma F_z = m \cdot \left( \dot{W} - U \cdot q + V \cdot p \right) + m \left[ -z_G \cdot \left( p^2 + q^2 \right) + x_G \cdot \left( p \cdot r - q \right) + y_G \cdot \left( q \cdot r + p \right) \right]
\] (3.33)

Using the Euler momenta equations, one can derive the moment of the point \( P \) with respect to the moving frame as:

\[
\delta M_x = \delta m \cdot \left[ y \cdot \dot{w} - z \cdot \dot{v} \right]
\] (3.34)

\[
\delta M_y = \delta m \cdot \left[ z \cdot \dot{u} - x \cdot \dot{w} \right]
\] (3.35)

\[
\delta M_z = \delta m \cdot \left[ x \cdot \dot{v} - y \cdot \dot{u} \right]
\] (3.36)

The sum of all moments in the direction of \( x, y, z \) can be denoted by \( L, M, N \), so that:

\[
L = \Sigma M_x = \int \delta M_x \quad \text{is the sum of moments in the direction of } x \text{ axis of frame } B
\]

\[
M = \Sigma M_y = \int \delta M_y \quad \text{is the sum of moments in the direction of } y \text{ axis of frame } B
\]

\[
N = \Sigma M_z = \int \delta M_z \quad \text{is the sum of moments in the direction of } z \text{ axis of frame } B
\]
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\[ \Sigma M_x = \int \delta m \left[ y \cdot \left( \dot{W} \cdot U \cdot q + V \cdot p - z \cdot (p^2 + q^2) \right) + x \cdot (p \cdot r - q) + \dot{y} \cdot (q \cdot r - \ddot{q}) \right] \]  
(3.37)

\[ \Sigma M_y = \int \delta m \left[ z \cdot \left( \dot{W} \cdot U \cdot q + V \cdot p - z \cdot (p^2 + q^2) \right) - x \cdot (p \cdot r - q) + \dot{z} \cdot (q \cdot r - \ddot{q}) \right] \]  
(3.38)

\[ \Sigma M_z = \int \delta m \left[ x \cdot \left( \dot{V} \cdot W \cdot p + U \cdot r - y \cdot (r^2 + p^2) \right) + \dot{x} \cdot (p \cdot q + \dot{r}) + \dot{z} \cdot (r \cdot q + \ddot{r}) \right] \]  
(3.39)

Furthermore, the following relations apply for the mass and product moments of inertia:

\[ I_{xx} = \int (y^2 + z^2) \cdot \delta m \]  
(3.40)

\[ I_{yy} = \int (x^2 + z^2) \cdot \delta m \]  
(3.41)

\[ I_{zz} = \int (x^2 + y^2) \cdot \delta m \]  
(3.42)

\[ I_{yz} = \int y \cdot z \cdot \delta m \]  
(3.43)

\[ I_{zx} = \int z \cdot x \cdot \delta m \]  
(3.44)

\[ I_{xy} = \int x \cdot y \cdot \delta m \]  
(3.45)

Introducing the definitions of mass and product moments of inertia into the Euler equations yields:

\[ \Sigma M_x = I_{xx} \cdot \ddot{p} - (I_{yy} - I_{zz}) \cdot q \cdot r + I_{yz} \cdot (r^2 - q^2) + I_{zx} \cdot (p \cdot q + \dot{r}) + I_{xy} \cdot (p \cdot r - q) \]  
(3.46)

\[ + m \cdot y \cdot (\ddot{W} \cdot U \cdot q + V \cdot p) - m \cdot z \cdot (\ddot{V} \cdot W \cdot p + U \cdot r) \]

\[ \Sigma M_y = I_{yy} \cdot \ddot{q} - (I_{zz} - I_{xx}) \cdot p \cdot r + I_{xz} \cdot (p^2 - r^2) + I_{yx} \cdot (q \cdot r + \dot{p}) + I_{zx} \cdot (q \cdot p - \dot{r}) \]  
(3.47)

\[ + m \cdot z \cdot (\ddot{U} \cdot V \cdot r + W \cdot q) - m \cdot x \cdot (\ddot{W} \cdot U \cdot q + V \cdot p) \]
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\[ \Sigma z = I_z \cdot \ddot{r} - (I_{xx} \cdot I_{yy}) \cdot p \cdot q + I_{yy} \cdot (q^2 \cdot p^2) - I_{yy} \cdot (r \cdot p + \dot{q}) + I_{xx} \cdot (r \cdot q - \dot{p}) + m \cdot x_g \cdot (\dot{V} \cdot W \cdot p + U \cdot r) + m \cdot y_g \cdot (\dot{U} \cdot V \cdot r + W \cdot q) \]  

(3.48)

Below, the six generic differential equations of motion obtained by the application of Newton-Euler method are grouped together:

\[ \Sigma f_x = m \cdot (U - V \cdot r + W \cdot q) + m \cdot \left[ -x_g \cdot (q^2 + r^2) + y_g \cdot (p \cdot q - \dot{r}) + z_g \cdot (p \cdot r + \dot{q}) \right] \]  

(3.49)

\[ \Sigma f_y = m \cdot (V - W \cdot p + U \cdot r) + m \cdot \left[ -y_g \cdot (r^2 + p^2) + z_g \cdot (q \cdot r - \dot{p}) + x_g \cdot (p \cdot q + \dot{r}) \right] \]  

(3.50)

\[ \Sigma f_z = m \cdot (W - U \cdot q + V \cdot p) + m \cdot \left[ -z_g \cdot (p^2 + q^2) + x_g \cdot (p \cdot r - \dot{q}) + y_g \cdot (q \cdot r + \dot{p}) \right] \]  

(3.51)

\[ \Sigma M_x = I_{xx} \cdot \dot{p} - (I_{xx} - I_{yy}) \cdot q \cdot r + I_{yy} \cdot (r^2 - q^2) - I_{xx} \cdot (p \cdot q + \dot{r}) + I_{yy} \cdot (p \cdot r - \dot{q}) + m \cdot y_g \cdot (\dot{W} \cdot U \cdot q + V \cdot p) - m \cdot z_g \cdot (\dot{V} \cdot W \cdot p + U \cdot r) \]  

(3.52)

\[ \Sigma M_y = I_{yy} \cdot \dot{q} - (I_{yy} - I_{xx}) \cdot p \cdot r + I_{xx} \cdot (p^2 - r^2) - I_{yy} \cdot (q \cdot r + \dot{p}) + I_{xx} \cdot (q \cdot p - \dot{r}) + m \cdot z_g \cdot (\dot{U} \cdot V \cdot r + W \cdot q) - m \cdot x_g \cdot (\dot{W} \cdot U \cdot q + V \cdot p) \]  

(3.53)

\[ \Sigma M_z = I_{zz} \cdot \dot{r} - (I_{xx} - I_{yy}) \cdot p \cdot q + I_{yy} \cdot (q^2 - p^2) - I_{yy} \cdot (r \cdot p + \dot{q}) + I_{xx} \cdot (r \cdot q - \dot{p}) + m \cdot x_g \cdot (\dot{V} \cdot W \cdot p + U \cdot r) - m \cdot y_g \cdot (\dot{U} \cdot V \cdot r + W \cdot q) \]  

(3.54)

### 3.3 Two Degrees of Freedom Bicycle Model

Two degrees of freedom model (Figure 3-3), also known as bicycle model, can be developed from the equations of motion by neglecting all degrees of freedom except the lateral translation and yaw about the z axis. The input to the system becomes steering wheel angle and the forward speed (U), which is considered to be constant.

The model derived is only suitable for steady-state low speed cornering (low lateral acceleration) due to the assumptions made, and the linearity of the model. For higher lateral acceleration other effects such as body roll, suspension effects, load transfer and tyre load sensitivity become more significant and, thus, the model is no longer suitable.
The following significant assumptions are made for the 2 DOF model:

- No rolling, pitching or bounce motion
- No lateral or longitudinal load transfer
- Linear tyre models
- Small angle assumption
- Constant forward velocity
- No gravity or aerodynamic effect

Figure 3-3: Two degrees of freedom model

3.3.1 Equations of Motion

The concept of fixed axes to the body of the vehicle is applied to the 2-DOF model. The equations of motion for the 2-DOF model become:
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\[ m(\dot{V} + Ur) = \Sigma F_y \]  
\[ I_{xz} \dot{r} = \Sigma M_z \]  

where \( \Sigma F_y \) and \( \Sigma M_z \) are equivalent force and yaw moment about the centre of gravity of the vehicle.

The lateral force and yaw moment can be expressed as:

\[ \Sigma F_y = C_{af} \alpha_f + C_{ar} \alpha_r \]  
\[ \Sigma M_z = aC_{af} \alpha_f - bC_{ar} \alpha_r \]  

where \( C_{af}, C_{ar} \) are the cornering stiffness coefficients of the front and rear tyres, and \( \alpha_f, \alpha_r \) are the average slip angles of the left and right side for the front and rear tyres.

Considering small angles, the front and rear slip angles become:

\[ \alpha_f = \delta_0 \left( \frac{V + ar}{U} \right) \]  
\[ \alpha_r = -\left( \frac{V - br}{U} \right) \]  

Putting the terms in equation (3.55) and (3.56), force and moment equations become:

\[ m(\dot{V} + Ur) = C_{af} \left( \delta_0 \left( \frac{V + ar}{U} \right) \right) + C_{ar} \left( - \left( \frac{V - br}{U} \right) \right) \]  
\[ = -(C_{ar} + C_{af}) \frac{V}{U} + (-aC_{af} + bC_{ar}) \dot{r} + (C_{af}) \delta_0 \]  

\[ I_{xz} \dot{r} = aC_{af} \left( \delta_0 \left( \frac{V + ar}{U} \right) \right) - bC_{ar} \left( - \left( \frac{V - br}{U} \right) \right) \]  
\[ = (-aC_{af} + bC_{ar}) \frac{V}{U} + (-a^2C_{af} - b^2C_{ar}) \dot{r} + (aC_{af}) \delta_0 \]  

The equations (3.61) and (3.62) can be arranged in a state space representation, where the lateral and yaw acceleration \( (\dot{V}, \dot{r}) \) are solved, using lateral velocity \( (V) \) and the yaw rate \( (r) \).
as state vectors. The input or control vector will be the steering wheel angle (δ). The total lateral acceleration in that case will be \( \dot{V} + U_r \).

### 3.4 Three Degrees of Freedom Model

![Figure 3-4: Three degrees of freedom model](image)

#### 3.4.1 Equations of Motion

The equations of motion used in the two degrees of freedom model can be extended to include the roll dynamics of the vehicle, by knowing its suspension characteristics (Ellis, 1969). Assume the vehicle is moving on a flat plane surface and at a steady forward speed and that there is no bounce and pitching movements. This means that the motions that occur are those due to yaw, side-slip and roll of the vehicle body.

The following significant assumptions are made for the 3-DOF model:

- No pitching or bounce motion
- No longitudinal load transfer
- Small angle assumption
- Constant forward velocity
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- No aerodynamic effect
- Linear suspension characteristics

In the 2-DOF model, a single mass was used to represent the total vehicle, with the local frame of reference fixed at the CG of the vehicle. For 3-DOF model, a slightly different approach is adopted, where the vehicle is represented by two masses \(m_s\) and \(m_u\), for the sprung and un sprung part of the vehicle respectively. The location of the two masses is defined with respect to the centre of gravity (CG) of the vehicle, with the sum of their moments about the CG being zero. Also, the local frame of reference does not coincide with the CG, but is located vertically in line with the CG of the vehicle, positioned at the roll axis. This axis is assumed parallel to the ground instead of the consideration of an inclined roll axis, which connects the front and rear roll centres (see Figure 3-4).

Lateral velocity at the sprung mass centre \(m_s\) is \(v_s = V + ph + rc\) \hspace{1cm} (3.63)

Therefore, the lateral acceleration, \(\dot{v}_s = \dot{V} + \dot{ph} + \dot{rc} + r\dot{c}\) \hspace{1cm} (3.64)

As the location of sprung mass from the CG is constant in the longitudinal direction, the term \(\dot{c}\) in equation (3.64) can be equated to longitudinal velocity \(U\).

Therefore, \(\dot{v}_s = \dot{V} + \dot{ph} + \dot{rc} + r\dot{c} + \dot{U}\) \hspace{1cm} (3.65)

Lateral velocity at the un sprung mass centre \(m_u\) is \(v_u = V - re\) \hspace{1cm} (3.66)

Therefore, the lateral acceleration, \(\dot{v}_u = \dot{V} - \dot{re} - r\dot{c}\) \hspace{1cm} (3.67)

Similarly, the term \(-\dot{c} = U \Rightarrow\)

Therefore, \(\dot{v}_u = \dot{V} - \dot{re} + \dot{U}\) \hspace{1cm} (3.68)

The net lateral force due to sprung and un sprung mass lateral acceleration can be expressed as:

\[
\sum F_y = m_s \ddot{v}_s + m_u \ddot{v}_u
\]

Substituting equations (3.65) and (3.68) into equation (3.69) gives:
\[ \sum F_y = (m_s + m_u)(\dot{V} + Ur) + m_h \dot{p} + (m_s c - m_u e) \dot{r} \] (3.70)

But, \( m_s c = m_u e \) \& \( m_s + m_u = m_r \)

Therefore, \( \sum F_y = m_r (\dot{V} + Ur) + m_h \dot{p} \) (3.71)

In the above equation for the lateral force, it is interesting to note that the first term takes into account the total mass of the vehicle \( m_r \) whereas the second term only uses the sprung mass \( m_s \). The first term is the lateral acceleration term, which is influenced by the total mass of the vehicle, whereas the second term signifies the contribution of the roll component of the sprung mass only in the lateral force generation. For the case, when the vehicle is represented as a single mass, which coincides with the local frame of reference, the term 'h' in equation (3.71) becomes zero and the equation reduces to the form used in the 2-DOF model. Overall, this provides an important aspect of defining equations of motion for the vehicle body, while using sprung and un-sprung mass representation and will be adopted further in the more elaborate 10-DOF vehicle model.

The yaw moment for 3-DOF model can be defined using equation (3.54). In this case, since the centre of mass is assumed to be just above the origin of the frame of reference \( O \) and the vehicle is symmetrical about the plane defined by the axes \( Ox \) and \( Oy \) (i.e. \( x_G = 0, y_G = 0 \) \& \( I_{xy} = I_{yz} = 0 \)), the yaw moment can be simplified to:

\[ \sum M_z = I_{zz} \dot{r} - I_{x z} \dot{p} \] (3.72)

Similarly, the roll moment can be taken from equation (3.52), with \( m z_G = -m z_h \)

Therefore, \( \sum M_x = I_{xx} \dot{p} - I_{xz} \dot{r} + m_s h (\dot{V} + Ur) \) (3.73)

In the above two equations, the term \( I_{zz} \) represents the moment of inertia of the whole vehicle about the \( z \) axis, whereas the term \( I_{xz} \) represents the moment of inertia of the sprung mass only about the \( x \) axis.

Therefore, \( I_{xx} = I_{cc(x)} + m_z h^2 \) \& \( I_{xz} = -m_z h c \)
where the product of inertia is negative since $h$ is negative with respect to the frame of reference.

### 3.4.2 External Forces and Moments

The calculation of lateral forces for this model can be carried out by including the roll camber and roll steer effects. Thus:

Lateral force at the front axle

$$ F_{yf} = C_{af} \alpha_f + C_{afR} \alpha_{fR} + \left( C_{yl} \frac{\partial \gamma}{\partial \phi_{fl}} + C_{yr} \frac{\partial \gamma}{\partial \phi_{fr}} \right) \varphi \tag{3.74} $$

Lateral force at the rear axle

$$ F_{yr} = C_{ar} \alpha_r + C_{arR} \alpha_{rR} + \left( C_{yl} \frac{\partial \gamma}{\partial \phi_{rl}} + C_{yr} \frac{\partial \gamma}{\partial \phi_{rr}} \right) \varphi \tag{3.75} $$

The left and right side forces can be linearly approximated as:

$$ F_{yf} = C_{af} \alpha_f + \left[ C_{yl} \frac{\partial \gamma}{\partial \phi_{fl}} \right] \varphi \tag{3.76} $$

$$ F_{yr} = C_{ar} \alpha_r + \left[ C_{yl} \frac{\partial \gamma}{\partial \phi_{rl}} \right] \varphi \tag{3.77} $$

Here, the front and rear slip angles can be modified by including the roll steer coefficient $\varepsilon_f = \frac{\partial \delta_f}{\partial \varphi}$ and $\varepsilon_r = \frac{\partial \delta_r}{\partial \varphi}$.

Now, the front steer angle becomes

$$ \delta_f = \delta_0 + \varepsilon_f \varphi $$

Hence, from equation (3.59)

$$ \alpha_f = \delta_0 + \varepsilon_f \varphi - \frac{V + ar}{U} \tag{3.78} $$

Substituting equation (3.78) into equation (3.76), the front tyre force becomes:

$$ F_{yf} = C_{af} \left( \delta_0 + \varepsilon_f \varphi - \frac{V + ar}{U} \right) + \left( C_{yl} \frac{\partial \gamma}{\partial \phi_{fl}} \right) \varphi \tag{3.79} $$

Similarly, the rear slip angle from equation (3.60)

$$ \alpha_r = \varepsilon_r \varphi - \frac{V - br}{U} \tag{3.80} $$
Substituting equation (3.80) into equation (3.77), the rear tyre force becomes:

\[ F_{y_r} = C_{ar} \left( \varepsilon_r \phi - \frac{(V - br)}{U} \right) + \left[ C_{nr} \frac{\partial \gamma}{\partial \phi_r} \right] \phi \]  

(3.81)

Thus, the net lateral force becomes:

\[ \Sigma F_y = C_{ar} \left( \delta_0 + \varepsilon_t \phi - \frac{(V + ar)}{U} \right) + \left( C_{nf} \frac{\partial \gamma}{\partial \phi_t} \right) \phi + C_{ar} \left( \varepsilon_r \phi - \frac{(V - br)}{U} \right) + \left( C_{nr} \frac{\partial \gamma}{\partial \phi_r} \right) \phi \]  

(3.82)

The final lateral force equation after rearranging the terms can be expressed as:

\[ \Sigma F_y = \frac{(C_{ar} + C_{ar})}{U} V + \left( C_{ar} \right) \delta_0 + \left( \frac{-aC_{st} + bC_{ar}}{U} \right) \]  

(3.83)

\[ + \left( C_{ar} \varepsilon_r + C_{nf} \frac{\partial \gamma}{\partial \phi_t} + C_{ar} \varepsilon_r + C_{nr} \frac{\partial \gamma}{\partial \phi_r} \right) \phi \]

The total yaw moment can be written as:

\[ \Sigma M_z = aF_{y_t} + bF_{y_r} \]  

(3.84)

Putting the terms \( F_{y_t} \) & \( F_{y_r} \) from equations (3.79) and (3.81) into equation (3.84) yields:

\[ \Sigma M_z = a \left( C_{ar} \left( \delta_0 + \varepsilon_r \phi - \frac{(V + ar)}{U} \right) + \left( C_{nf} \frac{\partial \gamma}{\partial \phi_t} \right) \phi \right) \]

\[ - b \left( C_{ar} \left( \varepsilon_r \phi - \frac{(V - br)}{U} \right) + \left( C_{nr} \frac{\partial \gamma}{\partial \phi_r} \right) \phi \right) \]  

(3.85)

The total yaw moment after rearranging the terms can be expressed as:

\[ \Sigma M_z = \frac{(-aC_{st} + bC_{ar})}{U} V + \frac{(-a^2C_{st} - b^2C_{ar})}{U} \]  

(3.86)

\[ + \left( aC_{st} \varepsilon_t + aC_{st} \frac{\partial \gamma}{\partial \phi_t} - bC_{st} \varepsilon_t - bC_{st} \frac{\partial \gamma}{\partial \phi_t} \right) \phi \]

The total roll moment \( L \) consists of moments due to suspension spring and damping coefficients (both front and rear) as well as the moment due to vehicle CG.
Total suspension moment due to spring and damper can be expressed as:

\[ L_{\text{susp}} = -\left( K_{\text{sf}} + K_{\text{sr}} \right) \phi - \left( C_{\text{sf}} + C_{\text{sr}} \right) p \]  

(3.87)

Where, the -ive sign is due to the moment acting against the roll deflection/roll angular velocity. The subscript 'f' and 'r' denotes the front and rear axle respectively. The terms \( K \) and \( C \) represent the roll stiffness and damping coefficients and can be expressed in terms of linear suspension spring and damping coefficients as:

\[ K_{\text{sf}} = 2K_t \tau_{\text{f}}^2 \; ; \; K_{\text{sr}} = 2K_t \tau_{\text{r}}^2 \]  

(3.88)

\[ C_{\text{sf}} = 2C_t \tau_{\text{f}}^2 \; ; \; C_{\text{sr}} = 2C_t \tau_{\text{r}}^2 \]

Moment due to vehicle CG can be expressed (noting that for small angles: \( \sin \phi = \phi \)):

\[ L_G = W_t h_f \phi \]  

(3.89)

Total roll moment about the roll axis is the sum of equations (3.87) and (3.89), thus:

\[ \Sigma M_x = -\left( K_{\text{sf}} + K_{\text{sr}} \right) \phi - \left( C_{\text{sf}} + C_{\text{sr}} \right) p + W_t h_f \phi \]  

(3.90)

The effect of anti-roll bar can also be incorporated by adding the anti-roll bar coefficient to the roll stiffness coefficient \( K_{\phi} \) in the equation (3.90), for both front and rear axle.
3.4.3 Load Transfer

Load transfer for the front and rear axles can be found by equating the suspension roll moment with moments generated by the vertical and lateral tyre forces (refer Figure 3-5).

Moments about front roll centre can be expressed as:

\[-(F_{xa} - F_{xa}) \cdot t_{rf} - (F_{ya} + F_{ya}) \cdot r_{tf} = -(K_{ref} \phi + C_{ref} P)\]  

(3.91)

Now: \[F_{xa} + F_{ya} = W_{tf}\] (front load)  

(3.92)

Putting equation (3.92) into equation (3.91) yields:

\[W_{tf} - 2F_{xa} = -(K_{ref} \phi + C_{ref} P) \cdot \frac{1}{t_{rf}} + (F_{ya} + F_{ya}) \cdot \frac{h_{tf}}{t_{rf}}\]  

(3.93)

\[F_{xa} = (K_{ref} \phi + C_{ref} P) \cdot \frac{1}{2t_{rf}} \cdot (F_{ya} + F_{ya}) \cdot \frac{h_{tf}}{2t_{rf}} + \frac{W_{tf}}{2}\]  

(3.94)

\[F_{xa} = -(K_{ref} \phi + C_{ref} P) \cdot \frac{1}{2t_{rf}} + (F_{ya} + F_{ya}) \cdot \frac{h_{tf}}{2t_{rf}} + \frac{W_{tf}}{2}\]  

(3.95)

Now, load transfer for the front axle can be expressed as:

\[\Delta F_{xa} = \frac{F_{xa} - F_{xa}}{2}\]  

(3.96)

Substituting equations (3.94) and (3.95) into equation (3.96) yields:

\[\Delta F_{xa} = (K_{ref} \phi + C_{ref} P) \cdot \frac{1}{2t_{rf}} \cdot (F_{ya} + F_{ya}) \cdot \frac{h_{tf}}{2t_{rf}}\]  

(3.97)

Hence, the front vertical forces can be expressed in terms of the static load and load transfer at the front axle as:

\[F_{xa} = \frac{W_{tf}}{2} + \Delta F_{xa}\]  

(3.98)

\[F_{xa} = \frac{W_{tf}}{2} - \Delta F_{xa}\]  

(3.99)
Similarly, the equations for rear axle load transfer can be expressed as:

\[
\Delta F_z = (K_{pr} \varphi + C_{pr} \rho) \frac{1}{2t} - (F_{za} + F_{za}) \frac{h}{2t} \tag{3.100}
\]

\[
F_{za} = \frac{W_t}{2} + \Delta F_z \tag{3.101}
\]

\[
F_{za} = \frac{W_t}{2} - \Delta F_z \tag{3.102}
\]

The equations for lateral force, yaw moment and roll moment are summarized below:

\[
m_t (\dot{V} + U_r) + m_s h \ddot{\rho} = -\left(\frac{C_{af} + C_{at}}{U}\right) V + \left(\frac{C_{af} \delta_0}{U}\right) + \left(\frac{-aC_{af} + bC_{at}}{U}\right) r
\]

\[
+ \left(\frac{C_{af} \varepsilon_f + C_{af} \varepsilon_r}{U}\right) m + \frac{C_{af} \varepsilon r}{U} + \frac{C_{at} \delta}{U} \varphi \right) \tag{3.103}
\]

\[
I_{xx} \ddot{\rho} - I_{xx} \ddot{\rho} = \left(\frac{-aC_{af} + bC_{at}}{U}\right) V + \left(\frac{-a^2C_{af} - b^2C_{at}}{U}\right) r + aC_{af} \delta_0
\]

\[
+ \left(\frac{aC_{af} \varepsilon_f + aC_{af} \varepsilon_r}{U}\right) - \frac{bC_{at} \varepsilon_r}{U} \frac{\partial \gamma}{\partial \phi_r} \frac{\partial \gamma}{\partial \phi_r} \tag{3.104}
\]

\[
I_{xx} \ddot{\rho} - I_{xx} \ddot{\rho} + m_s h (\dot{V} + U_r) = -\left(K_{pr} + K_{pr}\right) \varphi - \left(C_{pr} + C_{pr}\right) \rho + W_t h \varphi \tag{3.105}
\]

The 3-DOF model can be solved by using the state space representation or through ordinary differential equations, using MATLAB/Simulink. Although, the above mentioned equations of motion have employed linear tyre characteristics, it is easy to use Magic Formula characteristics for the lateral force, under pure slip conditions. In this study, the 3-DOF model is used for comparative purposes (as demonstrated in chapter 4) against the more elaborate 10-DOF model as well as the complex multi-body ADAMS model for different transient manoeuvres, performed at constant speed.

### 3.5 Intermediate Vehicle Model

In the previous sections, the low-end vehicle models with 2 and 3 DOF were derived, using Newton-Euler formulation. While, these models are appropriate for steady-state handling
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analysis, they generally lack accuracy, when it comes to transient handling manoeuvres (demonstrated later in chapter 4). This section will focus on the development of a more elaborate non-linear multi-body model, using the same Newton-Euler formulation. Apart from incorporating additional degrees of freedom to capture all the translational and rotational motion in space, special attention is paid to the inclusion of adequate non-linear characteristics of suspension, tyre and steering in a realistic manner. This approach of vehicle modelling is often termed as Intermediate vehicle modelling, as its complexities lies in between the two approaches: low end vehicle models (demonstrated in previous sections) and the complex multi-body vehicle models based on the constrained Lagrange equation (chapter 4). As opposed to large number of interactions involved in a complex multi-body model, this approach relies on simplified assumptions, yet could produce accurate results, as demonstrated later in chapter 4 and 5.

The complete vehicle model includes the sprung mass and four un-sprung masses, comprising the wheels, tyres and part of the suspension, as shown in Figure 3-6. While the sprung-mass is given complete six DOF motion in space, the un-sprung masses move only in the vertical direction, generating another four degrees of freedom. It should be noted that the additional DOF to capture the rotation of the wheels about their spin axes is not counted while expressing the total DOF of the vehicle model. Hence, it this thesis the model will be commonly referred to as 10-DOF model.

The equations of motion of the sprung mass are expressed with respect to the vehicle SAE frame of reference (SAE, 1976), which is attached to the sprung part of the vehicle body, comprising three translational and three rotational degrees of freedom. The choice of expressing it in the SAE frame of reference agrees with common practice (Ellis, 1994) and relates to the fact that most external forces such as tyre and aerodynamic forces are more readily expressed in the vehicle local frame of reference than in the global frame. Furthermore, the results are more informative when presented as vehicle-based velocities/displacements than when referred to the global frame of reference. The vehicle is assumed to be geometrically symmetrical about the X-Z plane of the SAE frame. However, the general case is considered, in which the vehicle is not inertially symmetrical about the same plane (i.e. the various products of inertia need not equal zero). The only restriction adopted is that the origin of the SAE frame lies at the same longitudinal position as the centre of gravity of the vehicle. This requirement has no mathematical significance and is considered only to achieve general comparability of the results with the implications of
analytical results obtained by simple bicycle models, where the origin of the SAE frame is usually taken at the position of centre of gravity (Pacejka, 2006). The global/fixed frame of reference is assumed to lie underneath the local frame of reference at the ground level. This frame of reference is used to observe vehicle path trajectory during various manoeuvres. As discussed in section (3.2.2), the transformation between the two frames of reference is carried out using Euler 1-2-3 transformation. The vehicle model and its coordinate system can be seen in Figure 3-6.

![Figure 3-6: Vehicle model coordinate system representation](image)

The forces and moments acting on the vehicle sprung mass are based on Euler’s equation, written with respect to the vehicle local SAE frame of reference. The following forces and moments are considered in this vehicle model:

- The tyre longitudinal forces arising from the driving and braking torque.
- The tyre lateral forces generated during cornering.
- Gravitational and aerodynamic forces acting on the sprung mass.
- Suspension component forces (spring-damper, bump stop, suspension reaction, and anti-roll bar)
- All moments generated from the above forces.
3.5.1 Sprung Mass Dynamics:

The equations of motion for the three translational and three rotational DOF, acting on sprung mass (refer section 3.2.4) are given below:

\[ \Sigma_x = m_r \left( \dot{U} \cdot V \cdot r + W \cdot q \right) - m_s \left[ x_G \cdot (q^2 + r^2) - y_G \cdot (q \cdot r - p) - z_G \cdot (r \cdot p + \dot{q}) \right] \]  
(3.106)

\[ \Sigma_y = m_r \left( \dot{V} \cdot W \cdot p + U \cdot r \right) - m_s \left[ y_G \cdot (r^2 + p^2) - z_G \cdot (q \cdot r - p) - x_G \cdot (p \cdot q + \dot{r}) \right] \]  
(3.107)

\[ \Sigma_z = m_r \left( \dot{W} \cdot U \cdot q + V \cdot p \right) - m_s \left[ z_G \cdot (p^2 + q^2) - x_G \cdot (p \cdot r - q) - y_G \cdot (q \cdot r + p) \right] \]  
(3.108)

\[ \Sigma_{M_x} = I_{xx} \cdot \dot{p} \left( I_{zz} - I_{xy} \right) \cdot q \cdot r + I_{zy} \left( \dot{r}^2 - q^2 \right) - I_{xy} \cdot (p \cdot q + \dot{r}) + I_{xy} \cdot (p \cdot r - q) \]  

\[ + m_s \cdot y_G \cdot \left( \dot{W} \cdot U \cdot q + V \cdot p \right) - m_s \cdot z_G \cdot \left( \dot{V} \cdot W \cdot p + U \cdot r \right) \]  
(3.109)

\[ \Sigma_{M_y} = I_{yy} \cdot \dot{q} \left( I_{zz} - I_{xx} \right) \cdot p \cdot r + I_{zx} \left( \dot{r}^2 - p^2 \right) - I_{zy} \cdot (q \cdot r + \dot{p}) + I_{zy} \cdot (q \cdot p - \dot{r}) \]  

\[ + m_s \cdot z_G \cdot \left( \dot{U} \cdot V \cdot r + W \cdot q \right) - m_s \cdot x_G \cdot \left( \dot{W} \cdot U \cdot q + V \cdot p \right) \]  
(3.110)

\[ \Sigma_{M_z} = I_{zz} \cdot \dot{r} \left( I_{xx} - I_{yy} \right) \cdot p \cdot q + I_{xy} \left( \dot{q}^2 - p^2 \right) - I_{xy} \cdot (r \cdot p + \dot{q}) + I_{xy} \cdot (r \cdot q - \dot{p}) \]  

\[ + m_r \cdot x_G \cdot \left( \dot{V} \cdot W \cdot p + U \cdot r \right) - m_r \cdot y_G \cdot \left( \dot{U} \cdot V \cdot r + W \cdot q \right) \]  
(3.111)

In the above equations, \( U \), \( V \), \( W \) denote the three translational velocities of the sprung mass along the X, Y and Z axes of the SAE frame respectively, while \( p \), \( q \), \( r \) are the rotational speeds (roll, pitch, yaw) about the same axes. The left-hand-side terms in equations (3.106) - (3.111) denote the net forces in the direction of the X, Y and Z axes, or the net moments about the same axes. In terms of inertial properties, \( m_r \) denotes the total mass of the vehicle, including the mass of the un-sprung components. As explained earlier in the 3-DOF model, the longitudinal and lateral force equations ((3.106)-(3.107)) employs the total mass \( (m_r) \) as well as the sprung mass \( (m_s) \) terms. On the other hand, when dealing with the vertical force (equation (3.108)), it is more appropriate to use only the sprung mass term \( (m_s) \), as the vertical dynamics of the un-sprung mass will be considered separately. Similarly, while writing equations for the net roll and pitch moments, only the sprung mass term \( (m_s) \) appears (refer equations (3.109)-(3.110)), where as in the case of yaw moment (equation (3.111)), the total vehicle mass \( (m_r) \) is considered.
An alternative and mathematically better approach would be to use the sprung mass $m_s$ throughout equations (3.106)-(3.111) and provide all un-sprung masses with additional lateral and longitudinal degrees-of-freedom. This would bring the model closer to its complex multi-body alternatives, increasing unnecessarily the computational cost of the simulation. It is important to emphasize that the treatment of mass presented herein is a simplification which aims to distribute the mass more appropriately between various degrees-of-freedom, without increasing their number and without introducing additional constraints. In the same spirit, parameters $x_G, y_G, z_G$ indicate the distance of the cg of the complete vehicle from the origin of the SAE frame of reference. According to the restriction described earlier regarding the longitudinal position of the cg, the term $x_G = 0$ lie on the origin of the frame of reference.

Finally, $I_{xx}, I_{yy}$ indicate the sprung mass moments of inertia about the X and Y axes, whereas $I_{zz}$ denotes the moment of inertia of the full vehicle about the Z axis. The products of inertia $I_{xy}, I_{zx}, I_{yz}$ are all calculated considering the sprung mass only. The forces on the left-hand-side of equations (3.106)-(3.108) include those developed at the tyres’ contact patches, the gravitational forces, suspension component forces, as well as the aerodynamic forces.

### 3.5.1.1 Transformation of Forces and Moments:

In many cases, the roll and pitch angles of the vehicle body (denoted $\phi$ and $\theta$, respectively) can be assumed small enough so that the X-Y plane of the SAE frame is considered always parallel to a flat road. Under these circumstances the calculation of forces $\Sigma F_x, \Sigma F_y$ and $\Sigma F_z$ is a rather straightforward procedure. However, under extreme cornering and/or braking manoeuvres, large roll and pitch angles require the determination of the exact position of the SAE frame with respect to the global frame of reference. In this manner, a tyre force which is parallel to the road is not assumed parallel to the X-Y plane, or, conversely, a vertical suspension force is not assumed parallel to the Z-axis of the SAE frame. To solve the problem, some of the forces on the left-hand-side of equations (3.106)-(3.108) are multiplied by a transformation matrix resulting from two successive rotations $\phi, \theta$ (roll and pitch) as shown below:

$$L(\phi, \theta) = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ \sin \phi \cdot \sin \theta & \cos \phi & \sin \phi \cdot \cos \theta \\ \cos \phi \cdot \sin \theta & -\sin \phi & \cos \phi \cdot \cos \theta \end{bmatrix}$$ (3.112)
The fact that the yaw angle of the vehicle body is not considered different from that of tyre or suspension means that only two successive rotations are considered for transformation of forces. In addition, the angles of rotation cannot be assumed small, which means the angles $\phi, \theta, \psi$ are not calculated by direct integration of rotational velocities $p, q, r$. Instead, an angular velocity transformation is required, similar to the transformations used frequently in aircraft dynamic analysis (Katz, 1997). Such transformations relate the instantaneous rotational velocities as expressed in the vehicle frame of reference to the angular rates as expressed in the global frame of reference. Provided that $p, q, r$ are calculated by solution of the differential equations (3.109) - (3.111), the following transformation provides the corresponding rates in the global frame of reference (Katz, 1997).

$$
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
\sin \phi \cdot \tan \theta & \cos \phi \cdot \tan \theta \\
0 & \cos \phi \\
\sin \phi \cdot \tan \theta & \cos \phi \cdot \tan \theta
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
$$

(3.113)

Finally, integration of equation (3.113) yields the corresponding angles for use in the transformation matrix described by relation (3.15) and (3.112). Accordingly, the net moments on the left-hand-side of equations (3.109) - (3.111) are calculated, based on the forces expressed in the SAE frame of reference (i.e. following transformation).

Equations (3.106) - (3.111) can be further simplified and re-arranged in the following generic form. This form is obtained by assuming the local frame of reference to coincide with the vehicle CG and the vehicle to be symmetrical about XY and YZ planes.

Longitudinal dynamics: $m_T \cdot \ddot{U} = \Sigma F_x - m_T \cdot (W \cdot q - V \cdot r)$

(3.114)

Lateral dynamics: $m_T \cdot \ddot{V} = \Sigma F_y - m_T \cdot (U \cdot r - W \cdot p)$

(3.115)

Vertical dynamics: $m_T \cdot \ddot{W} = \Sigma F_z - m_T \cdot (V \cdot p - U \cdot q)$

(3.116)

Roll moment: $I_{xx} \cdot \dot{p} - I_{xz} \cdot \dot{r} = \Sigma M_x + \left( I_{yy} - I_{zz} \right) \cdot q \cdot r + I_{xz} \cdot p \cdot q$

(3.117)

Pitch moment: $I_{yy} \cdot \dot{q} = \Sigma M_y + \left( I_{zz} - I_{xx} \right) \cdot p \cdot r - I_{xz} \cdot \left( p^2 - r^2 \right)$

(3.118)

Yaw moment: $I_{zz} \cdot \dot{r} - I_{xz} \cdot \dot{p} = \Sigma M_z + \left( I_{xx} - I_{yy} \right) \cdot p \cdot q - I_{xz} \cdot r \cdot q$

(3.119)
The state variables in these equations are the velocity components \([U, V, W, p, q, r]\). The right hand sides of the above equations have products of two state variables, referred to as gyroscopic terms and as a result these equations can not be solved using state-space representation. The combined approach of MATLAB – Simulink is adopted here to define the vehicle model, which will be explained later in section (3.7).

The net forces \(\Sigma F_x, \Sigma F_y\), and \(\Sigma F_z\) acting on the sprung mass (equation (3.114)-(3.116)), can be written in the following vector form:

\[
\begin{bmatrix}
\Sigma F_x \\
\Sigma F_y \\
\Sigma F_z
\end{bmatrix} =
\begin{bmatrix}
F_{x1} + F_{x2} + F_{x3} + F_{x4} \\
F_{y1} + F_{y2} + F_{y3} + F_{y4} \\
F_{z1} + F_{z2} + F_{z3} + F_{z4}
\end{bmatrix} + 
\begin{bmatrix}
F_{Gx} \\
F_{Gy} \\
F_{Gz}
\end{bmatrix} + 
\begin{bmatrix}
F_d \\
F_d \\
F_d
\end{bmatrix}
\]  

(3.120)

where \(F_{x1}, F_{x2}, F_{x3}, F_{x4}\) denotes the tyre longitudinal forces (refer equation (3.141))

\(F_{y1}, F_{y2}, F_{y3}, F_{y4}\) denotes the tyre lateral forces (refer equation (3.142))

\(F_{z1} + F_{z2} + F_{z3} + F_{z4}\) denotes the suspension forces (refer equations (3.139))

\(F_{Gx}, F_{Gy}, F_{Gz}\) denotes the gravitational forces (refer equation (3.163))

\(F_d\) denotes the aerodynamic forces (refer equation (3.209))

Since, the tyre longitudinal and lateral forces are generated in the road plane; they need to be transformed to vehicle SAE frame of reference using the transformation matrix expressed in equation (3.112). On the other hand, the gravitational and aerodynamic forces are defined with respect to the vehicle SAE frame of reference and hence, require no transformation.

Similarly the total moments \(\Sigma M_x, \Sigma M_y\), and \(\Sigma M_z\) acting on the sprung mass (equation (3.117)-(3.119)), can be represented in the following vector form:

\[
\begin{bmatrix}
\Sigma M_x \\
\Sigma M_y \\
\Sigma M_z
\end{bmatrix} =
\begin{bmatrix}
M_{x1} + M_{x2} + M_{x3} + M_{x4} \\
M_{y1} + M_{y2} + M_{y3} + M_{y4} \\
M_{z1} + M_{z2} + M_{z3} + M_{z4}
\end{bmatrix} + 
\begin{bmatrix}
M_d \\
M_d \\
M_d
\end{bmatrix}
\]  

(3.121)

where, \(M_{x1}, M_{x2}, M_{x3}, M_{x4}\) denotes the roll moment (refer equations (3.144))

\(M_{y1}, M_{y2}, M_{y3}, M_{y4}\) denotes the pitch moment (refer equations (3.145))
$M_{z_1}, M_{z_2}, M_{z_3}, M_{z_4}$ denotes the yaw moment (refer equations (3.146))

$M_d$ denotes the aerodynamic moments (refer equation (3.210))

The transformation of forces and moments described in this section provides a relatively simple method to take care of the large roll and pitch angles occurring during extreme cornering and/or braking manoeuvres. However, a much better approach would be to include another moving coordinate system, which can be attached to the unsprung mass of the vehicle. Such a coordinate system is often referred to as undercarriage coordinate system (Kiencke and Nielsen, 2000), whose orientation can be very similar to the vehicle SAE coordinate system, except that it does not roll and pitch with the vehicle body. However, it can experience roll and pitch angles due to road camber and any inclination. The equation of motion for the unsprung mass could then be written for the translational and rotational dynamics with its own mass and inertial terms. This approach would be more appropriate to handle complex interactions between sprung and unsprung masses, during transient manoeuvres on non-flat road surfaces. However, this approach would increase the complexity of the model and involve transformation of forces and motion variables between the two moving frame of references.

### 3.5.2 Unsprung Mass Dynamics

The four unsprung masses, which include wheels, tyres and suspensions, are modelled as mass-spring-damper systems. The wheels and part of the suspension mass are modelled as single DOF mass-spring-damper systems. The tyres are considered as linear spring-damper systems which connect the un-sprung masses to the road. At their top ends, the un-sprung masses are connected to the vehicle body using springs and dampers representing linear wheel-rates and non-linear damping functions. The non-linear effects due to suspension geometry are also accommodated in the model using simplified functions. The total suspension force, at each unsprung mass, is modelled by incorporating spring/damper, anti-roll bar, bump stop, and suspension secondary motions. A schematic showing the vertical dynamics of an un-sprung mass is shown in Figure 3-7.
3.5.2.1 Spring-Damper Forces

The spring-damper forces can be expressed as the wheel rate and damping functions of the relative motion of the un-sprung masses with respect to the body, in the vertical direction. The spring forces, thus, become functions of relative vertical displacement and the damper forces become functions of relative velocity. The spring forces with respect to the vehicle body can be expressed as:

\[
F_{s_1} = -K_f \cdot \left[ z + \frac{1}{2} \cdot \frac{\varphi - a \cdot \theta - z_{\text{wheel},1}}{z_{\text{body},1}} \right] = -K_f \cdot \left( z_{\text{susp},1} \right) \tag{3.122}
\]

\[
F_{s_2} = -K_f \cdot \left[ z + \frac{1}{2} \cdot \frac{\varphi - a \cdot \theta - z_{\text{wheel},2}}{z_{\text{body},2}} \right] = -K_f \cdot \left( z_{\text{susp},2} \right) \tag{3.123}
\]

\[
F_{s_3} = -K_f \cdot \left[ z + \frac{1}{2} \cdot \frac{\varphi + b \cdot \theta - z_{\text{wheel},3}}{z_{\text{body},3}} \right] = -K_f \cdot \left( z_{\text{susp},3} \right) \tag{3.124}
\]

\[
F_{s_4} = -K_f \cdot \left[ z + \frac{1}{2} \cdot \frac{\varphi - b \cdot \theta - z_{\text{wheel},4}}{z_{\text{body},4}} \right] = -K_f \cdot \left( z_{\text{susp},4} \right) \tag{3.125}
\]

Similarly, the damper forces with respect to the vehicle body can be expressed as:
In equation (3.126) - (3.129), the damping coefficients for compression and extension are taken to be the same. The non-linear damping behaviour of the suspension system can be captured such that when a wheel moves upwards, it generates a smaller damping force than when it moves downwards. This non-linear effect allows an upward bump from the road profile to have a small impact on the vehicle body, while the vertical wheel oscillations are still effectively damped during the downward movement of the wheel (Kieneke and Nielsen, 2000). The modified expression with different bump/rebound damper settings (refer Figure 3-8) can thus be written as:

\[
F_{d_i} = -0.5 \cdot \left[1 - \text{sgn}\left(\hat{z}_{\text{susp},i}\right)\right] \cdot C_{nl, i} \cdot \left(\hat{z}_{\text{susp},i}\right) - 0.5 \cdot \left[1 + \text{sgn}\left(\hat{z}_{\text{susp},i}\right)\right] \cdot C_{nl, i} \cdot \left(\hat{z}_{\text{susp},i}\right)
\]  

(3.130)

Where, the subscript ‘i’ denotes the wheel number. Overall, in the equations (3.122)-(3.130), p and q are the roll and pitch rates of the vehicle body, z and W are the vertical displacement and velocity of the vehicle body, \(\phi\) and \(\theta\) are the roll and pitch angles as calculated by integration of equation (3.113). \(z_{\text{body}}\) and \(W_{\text{body}}\) denote the vertical displacement and velocity of the four corners of the vehicle sprung body and \(z_{\text{wheel}}\) denotes the vertical displacements of the four wheels. \(z_{\text{susp}}\) is the resultant suspension deflection. \(K_f\) / \(K_r\) and \(C_f\) / \(C_r\) indicate linear spring stiffness and damping respectively. \(C_{nl}\) indicate non-linear damping function with subscripts ‘c’ and ‘e’ standing for compression and extension respectively. Finally, a and b denote the distance of the origin of the SAE frame from the front and rear axles respectively and \(t_f\) and \(t_r\) denote the front and rear half-track lengths.
3.5.2.2 Bump Stop Forces

The bump stop force could be expressed as the function of bump stop clearance \((z_{bs})\). The following equation provides the value of bump stop force, when suspension deflection exceeds the bump stop clearance. \(K_{bs}\) in this case denotes bump stop stiffness. The bump stop force at the \(i^{th}\) wheel can be expressed as:

\[
F_{bs_i} = K_{bs_i} [-z_{susp_i} + \text{sgn}(z_{susp_i}) \cdot z_{bs_i}] \quad \text{for} \quad |z_{susp}| > z_{bs_i}
\]

\[
F_{bs_i} = 0 \quad \text{else}
\]

(3.131)

3.5.2.3 Anti-roll Bar Forces and Moments:

The anti-roll bar forces can be calculated by considering the moment generated due to the roll angle of the body. However, by inclusion of four additional degrees of freedom in the form of vertical wheel displacements, an additional moment will be generated by uneven vertical displacements of the wheels on the opposite sides of the vehicle (Figure 3-9). These uneven displacements can occur, when one of the wheels runs over a road bump or a kerb or even hit a low profile pot-hole, thus generating an additional moment in the anti-roll bar. This moment due to vertical displacement of the wheel can be calculated by including additional equivalent roll-angles \(\varphi_{wheel}\) at the front and rear axle of the vehicle.

Front wheel angle: \(\tan\varphi_{wheel_f} = \frac{z_{wheel_i} - z_{wheel_i}}{t_{rf}} \Rightarrow\)
A simplified expression for anti-roll bar moment can be obtained as a product of the roll stiffness and the roll angle. However, with the additional moment due to uneven wheel displacement, the equivalent roll-angle $\varphi_{\text{wheel}}$ is added to the actual roll angle $\varphi$, to give the total anti-roll bar moment as:

$$M_{\text{roll}} = - K_{\text{roll}} \cdot (\varphi + \varphi_{\text{wheel}})$$  \hspace{1cm} (3.134) $$

$$M_{\text{roll}} = - K_{\text{roll}} \cdot (\varphi + \varphi_{\text{wheel}})$$  \hspace{1cm} (3.135) $$

The anti-roll bar forces at all the four wheels can be expressed as:
The sign of the wheel angle can change, depending upon displacement direction of the two opposite wheels.

### 3.5.2.4 Suspension Rigid Reactions

The vertical motion of the centre of the tyre contact patch is associated with secondary lateral and longitudinal motions, as a result of the complex suspension geometries used in practice. Hence, an infinitesimal vertical displacement $\partial Z_{\text{contact}}$ is almost always coupled with displacements $\partial X_{\text{contact}}$ and $\partial Y_{\text{contact}}$ (Figure 3-10). This observation allows treatment of the suspension as a kinematic mechanism, where the kinematic input point is chosen as the centre of the tyre contact patch with two possible input-motion directions, i.e. lateral and longitudinal. The output is taken as the vertical motion of the tyre contact centre with respect to the vehicle body. If the lateral, $F_Y$, and longitudinal, $F_X$, forces at the centre of the contact patch are known, application of the virtual work method (Merian and Kraige, 1993b,a) yields the resulting vertical forces applied on the sprung mass, as described in relations (3.137) and (3.138).

\[
F_{z_{yi}} = - (F_{y_{yi}}) \left( \frac{\partial y_{\text{contact}}}{\partial Z_{\text{contact}}} \right)_{i=1-4}
\]

\[
F_{z_{xi}} = - (F_{x_{yi}}) \left( \frac{\partial x_{\text{contact}}}{\partial Z_{\text{contact}}} \right)_{i=1-4}
\]

where subscript ‘i’ denotes the wheel number.
It should be emphasized that equations (3.137) and (3.138) hold true for all four corners of the vehicle as long as the vehicle-based SAE frame of reference is used, both for the expression of the displacements and forces. The suspension geometry in this case is characterized by the relation of the infinitesimal displacement, which could be obtained numerically by either simulation or physical experiment using test rig. In this case, the data is obtained using Multi-Body ADAMS code for a Macpherson strut suspension. Figure 3-11 shows the plot of $\partial_{X_{\text{contact}}}/\partial_{Z_{\text{contact}}}$ and $\partial_{Y_{\text{contact}}}/\partial_{Z_{\text{contact}}}$ against wheel vertical displacement.

Figure 3-10: Suspension geometry effect on tyre contact patch displacement

The vertical forces caused by a rigid body suspension reactions, primarily induce roll and pitch moments which oppose the normal roll and pitch moments caused by lateral and longitudinal forces. Hence, equations (3.137) and (3.138) account for all jacking, anti-dive, anti-roll and related phenomena, offering an alternative to the frequently used roll-centre concept (Gerrard, 1999) and other similar treatments. The application of the virtual work method requires only the establishment of the lateral and longitudinal displacements as functions of the vertical displacement of the contact centre. Subsequently, it allows the nonlinear treatment of rigid suspension reactions, avoiding complications related to phenomena such as roll-centre migration (Gerrard, 1999).
The net suspension force with respect to the vehicle body at each corner of the vehicle thus becomes:

\[ F_{\text{susp}} = F_s + F_d + F_{bs} + F_{es} + F_{zy} \cdot F_{\text{roll}} \]  

(3.139)

The term \( F_s, F_d, F_{bs}, F_{es}, F_{zy}, \) and \( F_{\text{roll}} \) stands for component forces from spring, damper, bump stop, suspension rigid reactions, and ant-roll bar respectively. The anti-roll bar force is having a negative sign as it was originally expressed with respect to the wheel, as shown in equation (3.136).

### 3.5.2.5 Equations of Motion for the Unsprung Mass

The equations of motion for the four unsprung masses can be solved using Newton’s second law of motion, balancing various forces acting on the wheel due to suspension components, tyre vertical load, and wheel inertia. The equation for the vertical motion of the unsprung mass for the \( i^{th} \) wheel can be expressed as:

\[-F_{\text{susp}} \cdot K_{\text{tyre}} \left( z_{\text{wheel},i} \right) \cdot C_{\text{tyre}} \left( \dot{z}_{\text{wheel},i} \right) + M_{\text{wheel},i} \cdot \dot{z}_{\text{wheel},i} - M_{\text{wheel},i} \cdot \ddot{z}_{\text{wheel},i} = 0 \quad i=1:4 \]  

(3.140)
In the above equation, the vertical force due to tyre deflection $F_z$ is expressed as a function of wheel displacement only, with road disturbance taken as zero in this case.

### 3.5.2.6 Tyre Forces and Moments

The tyre model in this thesis is based on the Magic Formula approach (Pacejka and Bakker, 1993), where slip ratio and vertical load are primarily used to calculate tyre forces at each contact patch. Tyre modelling is covered later. The traction and cornering forces from the tyre model are generated in the wheel plane, and are then projected along the vehicle central axis using the wheel steer angle $\delta$. Here, the forces and velocities in the wheel plane are represented using the definition of the tyre SAE frame of reference (SAE, 1976).

\[
F_{x_i} = (F_{x_i})_{\text{tyre}} \cdot \cos \delta_i - (F_{y_i})_{\text{tyre}} \cdot \sin \delta_i \quad \text{for } i=1,4 \quad (3.141)
\]
\[
F_{y_i} = (F_{x_i})_{\text{tyre}} \cdot \sin \delta_i + (F_{y_i})_{\text{tyre}} \cdot \cos \delta_i \quad \text{for } i=1,4 \quad (3.142)
\]

![Diagram showing tyre moments and position vectors for all wheels](image)

**Figure 3-12: Tyre moments and position vectors for all wheels**

The above forces after transformation, along with the suspension forces generate moments about the vehicle SAE frame of reference. The generated moments which act in the roll, pitch and yaw directions require the information about the position of the local frame of reference with respect to the tyre contact patch. These position vectors for all the four wheels can be expressed as (refer Figure 3-12).
Theoretical Vehicle Modelling

\[
\begin{align*}
\mathbf{r}_1 &= \begin{bmatrix} a - t_{rf} / 2 & h_0 - z + z_{wheel_1} \end{bmatrix}^T \\
\mathbf{r}_2 &= \begin{bmatrix} a & t_{rf} / 2 & h_0 - z + z_{wheel_2} \end{bmatrix}^T \\
\mathbf{r}_3 &= \begin{bmatrix} -b & - t_{rf} / 2 & h_0 - z + z_{wheel_3} \end{bmatrix}^T \\
\mathbf{r}_4 &= \begin{bmatrix} -b & t_{rf} / 2 & h_0 - z + z_{wheel_4} \end{bmatrix}^T
\end{align*}
\]

(3.143)

The roll, pitch and yaw moments are, therefore, calculated as:

**Roll moments:**

\[
\begin{align*}
M_{x_1} &= -(h_0 - z + z_{wheel_1}) \cdot F_{y_1} - (t_{rf} / 2) \cdot F_{\text{susp}_1} \\
M_{x_2} &= -(h_0 - z + z_{wheel_2}) \cdot F_{y_2} + (t_{rf} / 2) \cdot F_{\text{susp}_2} \\
M_{x_3} &= -(h_0 - z + z_{wheel_3}) \cdot F_{y_3} - (t_{rf} / 2) \cdot F_{\text{susp}_3} \\
M_{x_4} &= -(h_0 - z + z_{wheel_4}) \cdot F_{y_4} + (t_{rf} / 2) \cdot F_{\text{susp}_4}
\end{align*}
\]

(3.144)

**Pitch moments:**

\[
\begin{align*}
M_{y_1} &= (h_0 - z + z_{wheel_1}) \cdot F_{x_1} - (a) \cdot F_{\text{susp}_1} \\
M_{y_2} &= (h_0 - z + z_{wheel_2}) \cdot F_{x_2} - (a) \cdot F_{\text{susp}_2} \\
M_{y_3} &= (h_0 - z + z_{wheel_3}) \cdot F_{x_3} + (b) \cdot F_{\text{susp}_3} \\
M_{y_4} &= (h_0 - z + z_{wheel_4}) \cdot F_{x_4} + (b) \cdot F_{\text{susp}_4}
\end{align*}
\]

(3.145)

**Yaw moments:**

\[
\begin{align*}
M_{z_1} &= (t_{rf} / 2) \cdot F_{x_1} + (a) \cdot F_{y_1} \\
M_{z_2} &= -(t_{rf} / 2) \cdot F_{x_2} + (a) \cdot F_{y_2} \\
M_{z_3} &= (t_{rf} / 2) \cdot F_{x_3} - (b) \cdot F_{y_3} \\
M_{z_4} &= -(t_{rf} / 2) \cdot F_{x_4} - (b) \cdot F_{y_4}
\end{align*}
\]

(3.146)

3.5.3 Wheel Dynamics

3.5.3.1 Tyre Slip Angle:

Tyre slip angles for the four wheels can be calculated, based on the wheel steer angle and the longitudinal and lateral velocity components at the wheel centre.
The longitudinal velocities at the wheel centre with respect to the vehicle body can be expressed as (see Figure 3-13):

\[ v_{x1} = U + \left( \frac{t_{r}^2}{2} \right) \cdot r \]
\[ v_{x2} = U - \left( \frac{t_{r}^2}{2} \right) \cdot r \]
\[ v_{x3} = U + \left( \frac{t_{r}^2}{2} \right) \cdot r \]
\[ v_{x4} = U - \left( \frac{t_{r}^2}{2} \right) \cdot r \]  

Similarly, the lateral velocities can be expressed as:

\[ v_{y1} = V + a \cdot r \]
\[ v_{y2} = V + a \cdot r \]
\[ v_{y3} = V - b \cdot r \]
\[ v_{y4} = V - b \cdot r \]  

Also, the longitudinal and lateral velocity with respect to the wheel plane at the \( i^{th} \) wheel can be expressed as:

\[ v_{xi} = v_{xi} \cdot \cos(\delta_i) + v_{yi} \cdot \sin(\delta_i) \]  
\[ v_{yi} = -v_{xi} \cdot \sin(\delta_i) + v_{yi} \cdot \cos(\delta_i) \]
The theoretical vehicle modelling states:

The tyre slip angle at the $i^{th}$ wheel can be expressed as:

$$\alpha_i = \delta_i - \arctan \left( \frac{v_{x_i}}{v_{y_i}} \right)_{i=1...4} \quad (3.151)$$

### 3.5.3.2 Wheel Slip Ratio:

The longitudinal slip ratio can be calculated based on the longitudinal wheel velocity (with respect to the wheel plane) and rotational speed $\omega$ of the wheel.

The longitudinal slip ratio at the $i^{th}$ wheel can be expressed as:

$$s_{x_i} = \frac{\omega_i \cdot r_e - v_{x_{ai}}}{v_{x_{ai}}} \quad (3.152)$$

where $r_e$ represents the effective rolling radius of the tyre (refer section 4.6.5.3).

The lateral slip can be calculated as the ratio of lateral velocity to the longitudinal wheel velocity, both defined with respect to the wheel plane.

The lateral slip at the $i^{th}$ wheel takes the form:

$$s_{y_i} = \frac{-v_{y_{ai}}}{v_{x_{ai}}} \quad (3.153)$$

### 3.5.3.3 Rolling Resistance Force

The rolling resistance force is the force generated at the tyre contact patch, which opposes the wheel motion and is a major retardation force at high vehicle speed. Much of the rolling resistance takes place because of the hysteresis losses in cyclic deformation of the rubber, which is inherently viscoelastic in nature. This leads to heat losses and as a result the reaction to the normal load occurs with a lag, which means the normal load acts at a distance 'd' towards front of the contact patch from the centre of the wheel. The net effect is a rolling resistance moment about the spin axis of the wheel, which opposes forward rolling of the wheel. Under steady-state condition, the rolling resistance force $F_R$ is generated in the contact patch, which causes a moment about the spin axis, thus balancing the rolling resistance moment. The rolling resistance force can be expressed as:
Theoretical Vehicle Modelling

\[ F_{R_i} = \frac{F_{zc} \cdot d}{r_{ni}} \quad i=1:4 \quad (3.154) \]

The above relationship is also shown in Figure 3-15. The magnitude of the rolling resistance force depends mainly on the load and the surface characteristics and is also influenced by vehicle speed, tyre temperature, inflation pressure and tyre material and design. The following approximation can be used to represent rolling resistance.

\[ F_{R_i} = F_{zc} \cdot \left( A_r + B_v \cdot v_{x_{in}} \right) \cdot \text{sgn}(v_{x_{in}}) \quad i=1:4 \quad (3.155) \]

In the above equation, \( A_r \) is a constant, which accounts for the road surface characteristics, whereas \( B_v \) accounts for the effect of forward speed. For low and medium speeds \( B_v \) can be neglected. The value of \( A_r \) for different road surfaces is given in Table 3-1.

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Concrete</th>
<th>Medium Hard</th>
<th>Soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger Car</td>
<td>0.015</td>
<td>0.08</td>
<td>0.30</td>
</tr>
<tr>
<td>Heavy Truck</td>
<td>0.012</td>
<td>0.06</td>
<td>0.25</td>
</tr>
<tr>
<td>Tractors</td>
<td>0.02</td>
<td>0.04</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 3-1: Rolling resistance coefficient \( A_r \), (Gillespie, 1992)

### 3.5.3.4 Steering Kinematics

![Steering geometry](image)

Figure 3-14: Steering geometry
Theoretical Vehicle Modelling

The effect of steering kinematics on the tyre forces is modelled here by determining equivalent camber angle. Figure 3-14 shows the steering geometry, where $\nu_p$ represents the normal to the wheel plane, $\eta$ represents the unit normal along the kingpin axis, $\nu$ represents the caster angle, $\gamma$ represents the camber angle, and $\lambda$ represents the lateral inclination angle.

The rotation of the wheel plane normal can be expressed in relation to the camber angle, which itself is modelled as a function of wheel vertical displacement ($z_{\text{wheel}}$).

\[ \nu_p = \begin{pmatrix} 0 \\ \cos \gamma \\ \sin \gamma \end{pmatrix} \]  
\[ \text{(3.156)} \]

Also, the unit vector along the kingpin axis can be expressed in terms of lateral inclination angle and caster angle

\[ \eta = \begin{pmatrix} \cos \lambda \cdot \sin \nu \\ \sin \lambda \\ \cos \lambda \cdot \cos \nu \end{pmatrix} \]  
\[ \text{(3.157)} \]

The movement of $\nu_p$ about the unit vector $\eta$ can be expressed as: (Huston and Liu, 2001)

\[ \nu_p' = (\nu_p \cdot \eta) \eta (1 - \cos \delta) + \nu \cos \delta + \sin \delta (\eta \times \nu_p) \]  
\[ \text{(3.158)} \]

The above vector can be used to determine the equivalent camber angle, which can be expressed as:

\[ \gamma_{\text{total}} = \arcsin (\nu_p' \cdot n_s) \]  
\[ \text{(3.159)} \]

where $n_s$ denotes the road surface normal with reference to the vehicle coordinate system.

3.5.4 Vehicle Traction

As the current vehicle model was not intended for the study of vehicle traction characteristics or traction control, the details of power and drive train were not considered in the model development. However, vehicle traction is represented using a simple PID controller, which enables the vehicle model to maintain constant forward speed under
various manoeuvres and ultimately achieve comparability with constant speed simulation and experimental results, obtained through complex multi-body ADAMS model and on-road vehicle tests (see chapters 4 & 5). The controller regulates the driving torque at the driven wheels, by deploying the difference between target and actual forward speed as the feedback error. Alternatively, the model can generate torque at the wheels, using engine characteristics curve in form of look-up tables (engine torque v speed and throttle position) and the information of gear train and differential ratios. This feature was used, while validating the model with the experimental test results, where the information of engine speed, throttle and gear position were extracted from the test car CAN network and then fed into the model for generating drive torque at the wheels (refer chapter 5 for results).

![Figure 3-15: Torque and forces acting on wheel](image)

The rotational speed ($\omega$) of the wheel can be calculated by applying Newton’s second law for rotational dynamics. By balancing the wheel inertia, brake torque ($M_b$), driving torque ($T_d$), tyre longitudinal force ($F_{x,tyre}$), and rolling resistance force ($F_R$), the differential equation takes the following form (refer to Figure 3-15):

$$\dot{\omega} = \frac{T_d - (F_{x,tyre} - F_R) \cdot r_w - M_b}{I_{wheel}} \quad (3.160)$$

The term ($r_w$) in the equation (3.160) refers to the static tyre radius, which relates the static load ($F_{zo}$) to the tyre spring stiffness ($K_{tyre}$), as shown in Figure 3-15:

$$r_w = r_0 - \frac{F_{zo}}{K_{tyre}} \quad (3.161)$$
The rotational dynamics of all wheels are included in the study, to facilitate simulation of the variation of longitudinal slip angle under braking manoeuvres.

### 3.5.5 Gravity Forces:

The gravitational force applies to the centre of mass of the vehicle. The gravitational force can be transformed into vehicle SAE co-ordinate system, using the transformation matrix defined in section (3.2.2), thus incorporating the instantaneous body roll and pitch angle. In addition, the effect of road inclination ($\theta_{rd}$) and road camber ($\varphi_{rd}$) could also be incorporated by using resultant angles ($\theta - \theta_{rd}$) and ($\varphi - \varphi_{rd}$), in the transformation matrix of equation (3.112). A positive inclination in road means an upward inclined road, whereas a positive camber means a road which raises the left hand side of the vehicle. As the yaw angle in this case is zero, the transformation matrix takes the form:

$$L(\varphi, \theta)_{\varphi, \theta, \theta_{rd}, \varphi_{rd}} = \begin{bmatrix} \cos(\theta - \theta_{rd}) & 0 & -\sin(\theta - \theta_{rd}) \\ \sin(\varphi - \varphi_{rd}) \cdot \sin(\theta - \theta_{rd}) & \cos(\varphi - \varphi_{rd}) & \sin(\varphi - \varphi_{rd}) \cdot \cos(\theta - \theta_{rd}) \\ \cos(\varphi - \varphi_{rd}) \cdot \sin(\theta - \theta_{rd}) & -\sin(\varphi - \varphi_{rd}) & \cos(\varphi - \varphi_{rd}) \cdot \cos(\theta - \theta_{rd}) \end{bmatrix}$$

(3.162)

The forces generated by the gravitational effect, thus becomes:

$$\begin{bmatrix} F_{G_i} \\ F_{G_i} \\ F_{G_i} \end{bmatrix} = L(\varphi, \theta)_{\varphi, \theta, \theta_{rd}, \varphi_{rd}} \cdot \begin{bmatrix} 0 \\ 0 \\ M_s \cdot g \end{bmatrix} = \begin{bmatrix} 0 \\ -\sin(\theta - \theta_{rd}) \\ \sin(\varphi - \varphi_{rd}) \cdot \cos(\theta - \theta_{rd}) \\ \cos(\varphi - \varphi_{rd}) \cdot \cos(\theta - \theta_{rd}) \end{bmatrix} \cdot M_s \cdot g$$

(3.163)

### 3.5.6 Wheel Lift-off

![Figure 3-16: Vehicle experiencing large roll angle](image)
To simulate wheel lift-off case, it is important that the position of wheel contact is determined with respect to the global frame of reference. The vehicle velocity at origin ‘O_B’ in the local frame of reference can be obtained after solving the equations of motion (3.106) - (3.111). To establish the velocity with respect to the global frame of reference ‘O_A’ (refer Figure 3-16), the following transformation matrix (from local to global frame) is used:

\[
L(\varphi, \theta, \psi) = \begin{bmatrix}
\cos \theta \cdot \cos \psi & \sin \varphi \cdot \sin \theta \cdot \cos \psi - \cos \varphi \cdot \sin \psi & \cos \varphi \cdot \sin \theta \cdot \cos \psi + \sin \varphi \cdot \sin \psi \\
\cos \theta \cdot \sin \psi & \sin \varphi \cdot \sin \theta \cdot \sin \psi + \cos \varphi \cdot \cos \psi & \cos \varphi \cdot \sin \theta \cdot \sin \psi - \sin \varphi \cdot \cos \psi \\
-\sin \theta & \sin \varphi \cdot \cos \theta & \cos \varphi \cdot \cos \theta
\end{bmatrix}
\] (3.164)

The velocity in the global frame of reference thus becomes:

\[
[U, V, W]_G = L(\varphi, \theta, \psi) \cdot [U, V, W]_L
\] (3.165)

Position of the origin of the local frame ‘O_B’ with respect to the global frame is obtained by integrating equation (3.165) with initial conditions as [0, 0, -h_0]

\[
[x, y, z]_G = \int [U, V, W]_G \, dt
\] (3.166)

Position of the wheel centre ‘C_1’ with respect to the origin of the local frame of reference ‘O_B’ (refer Figure 3-16):

\[
[x_w, y_w, z_w]_L = \begin{bmatrix}
a & a & -b & -b \\
-t_{df}/2 & t_{df}/2 & -t_{tf}/2 & t_{tf}/2 \\
h_0 - r_{w_1} - z + z_{wheel_1} & h_0 - r_{w_2} - z + z_{wheel_2} & h_0 - r_{w_3} - z + z_{wheel_3} & h_0 - r_{w_4} - z + z_{wheel_4}
\end{bmatrix}
\] (3.167)

where \(z\) is the vertical displacement of the origin of the local frame of reference, obtained after integrating the velocity in the local frame, and \(z_{\text{wheel}}\) is the wheel centre displacement after solving the differential equation (3.140).

Now, position of the wheel centre ‘C_1’ with respect to the global frame of reference can be expressed as:

\[
[x_w, y_w, z_w]_G = [x, y, z]_G + L(\varphi, \theta, \psi) \cdot [x_w, y_w, z_w]_L
\] (3.168)

Similarly, the position of contact patch centre can be determined with respect to the global frame of reference in order to work out the contact patch deflection \(p\).
The position of the contact patch centre ‘A₁’ with respect to the origin of the local frame of reference ‘O₀’ can be written as:

$$\begin{bmatrix} a & -b \\ -t_\alpha/2 & -t_\alpha/2 \end{bmatrix}$$

And the position of the contact patch centre ‘A₁’ with respect to the global frame of reference can be expressed as:

$$[x_c, y_c, z_c]_G = [x, y, z]_G + L(\varphi, \theta, \psi) \cdot [x_c, y_c, z_c]_L$$

Once the global positions of the contact patch and the wheel centre are known, a 3d line can be drawn between the two points (refer Figure 3-17). The intersection of this line and a plane, which represents the road surface, can provide another point ‘A₂’. The distance between A₁ and A₂ provides the contact patch deflection ρ.

3.5.7 Tyre Model:

The tyre model used here is based on the non-dimensional or normalised tyre characteristics (Milliken and Milliken, 1995). This technique based on Magic Formula characteristics is an extension to the similarity concept, as provided in (Radt and Milliken, 1983). The semi-empirical Magic Formula model relies heavily on the test measurement data to derive its parameters by regression process (van Oosten and Bakker, 1993). The determination of
Magic Formula parameters is a tedious task as it involves gathering of large test data through different runs of a single tyre for a range of loads, longitudinal slips, side slip and camber angles. The test of a single tyre may often cause rapid tyre wear, meaning multiple tyres have to be used resulting in variations in results. Furthermore, similar sets of tests would have to be repeated if the front and rear tyres are different and also if the effects of inflation pressure is critical.

The non-dimensional or normalised data technique provides data compression on the measured tyre data, which results in data from various test conditions (such as several different loads) to fall on a single curve. It makes use of the ratios of the raw data in such a way that the dimensions of the data are eliminated. For example, normalised lateral force could be achieved by dividing the lateral force by the product of friction coefficient and load. This non-dimensional way of tyre data treatment leads to dimensionless variables such as normalized lateral force, normalized longitudinal force and normalized aligning torque as a function of normalized slip angle, inclination angle and slip ratio.

In the ‘Magic Formula’ equations, the longitudinal tyre forces are primarily function of slip ratio between wheel speed and vehicle forward speed and lateral forces are function of the sideslip angle. The forces are also function of wheel camber angle and load.

Using a pure slip characteristic calculation of slip angle and vertical load, the normalized lateral force $\bar{F}_{y_{nm}}$ is defined as:

$$\bar{F}_{y_{nm}} = \frac{F_{y_{nm}}}{\mu_y \cdot F_z}$$

(3.171)

The peak force is represented as a function of load and its is assumed that $\mu_x \cdot F_z$ is equal to $\mu_y \cdot F_z$

$$\text{Peak Force} = \frac{\mu \cdot F_z}{1 + \left(1.5 \cdot \frac{F_z}{m_r \cdot g}\right)^3}$$

(3.172)

The normalized slip angle is defined as:

$$\bar{\alpha} = \frac{C_x \cdot \tan \alpha}{\mu_y \cdot F_z}$$

(3.173)
where \( C_a \) is the cornering stiffness at zero sideslip angle

Cornering stiffness is also represented as a function of load and here the cornering stiffness is assumed to be equal to the longitudinal stiffness \( K_x \)

Cornering stiffness = \( a_1 \cdot (1 - \exp(-a_2 \cdot F_z)) \)  

(3.174)

Using the ‘Magic Formula’, the normalized lateral force \( \bar{F}_{\gamma_{ns}} \) is represented by:

\[
\bar{F}_{\gamma_{ns}} = D' \cdot \sin\left(C' \cdot \arctan\left(B' \cdot \left(1 - E'\right) \cdot \bar{\alpha} + (E'/B') \cdot \arctan(B' \cdot \bar{\alpha})\right)\right)
\]

(3.175)

where \( B', C', D', E' \) represents the parameters for solid curve for “Magic Formula”

Also, the camber angle can be represented as normalized camber angle

\[
\bar{\gamma} = \frac{G \cdot \sin \gamma}{\mu \cdot F_z}
\]

(3.176)

where \( G \) is the camber stiffness or the ratio of camber thrust to camber angle.

When the slip and camber angle occur simultaneously, a normalized combined slip / camber angle \( \bar{\beta} \) can be defined for the positive camber angles:

\[
\bar{\beta} = \frac{\bar{\alpha}}{1 - \bar{\gamma} \cdot \text{sgn} \alpha}
\]

(3.177)

For combined slip / camber angles, the normalized lateral force \( \bar{F}_N \) can be defined for the positive camber angles (\( \bar{\gamma} > 0 \)),

\[
\bar{F}_N = \frac{\bar{F}_{\gamma_{ns}} - \bar{\gamma}}{1 - \bar{\gamma} \cdot \text{sgn} \alpha}
\]

(3.178)

\( \bar{F}_N \) reduces to \( \bar{F}_{\gamma_{ns}} \) of equation (3.171) for zero camber angles (\( \bar{\gamma} = 0 \)). Also \( \bar{F}_N \) is a function of \( \bar{\beta} \), and when the slip angle is zero, \( \bar{\beta} \) is zero (equation (3.177)), and so \( \bar{F}_N \) becomes zero as well. This means normalized lateral force \( \bar{F}_{\gamma_{ns}} \) becomes equal to normalized camber angle \( \bar{\gamma} \) (equation (3.178)) for zero slip angle.
Similar to the normalized lateral force, the normalized longitudinal force $F_{\text{x,\scriptscriptstyle{n}}} / \mu_s \cdot F_Z$ can be also be described, for the case of pure traction or braking with zero slip and camber angles.

$$F_{\text{x,\scriptscriptstyle{n}}} / \mu_s \cdot F_Z$$  \hspace{1cm} (3.179)

and normalized slip ratio $\overline{s}$ as:

$$\overline{s} = \frac{K_s \cdot s_s}{\mu_s \cdot F_Z}$$  \hspace{1cm} (3.180)

where $s_s$ is the slip ratio given by

$$s_s = \frac{\omega \cdot R_s - v_s}{v_s}$$

Using the combined slip characteristics calculation for slip ratio and slip angle for cases, when traction or braking occurs in a turn, a combined normalized slip variable is needed:

$$\kappa = \sqrt{\overline{s}^2 + (\overline{\alpha})^2}$$  \hspace{1cm} (3.181)

and also a normalized resultant force is given as:

$$R_{\text{tyre}} = \sqrt{\left(\overline{F_{\text{y,\scriptscriptstyle{n}}}}\right)^2 + \left(\overline{F_{\text{x,\scriptscriptstyle{n}}}}\right)^2}$$  \hspace{1cm} (3.182)

Since normalized slip angle is replaced with the combined normalized slip variable $\kappa$, the normalized resultant force $R_{\text{tyre}}$ can be taken as:

$$R_{\text{tyre}} = D' \cdot \sin \left( C' \cdot \arctan (B' \cdot ((1 - E') \cdot \kappa + (E' / B') \cdot \arctan (B' \cdot \kappa))) \right)$$  \hspace{1cm} (3.183)

Another necessary equation is obtained from measured data as:

$$\overline{F_{\text{y,\scriptscriptstyle{n}}}} / \overline{F_{\text{x,\scriptscriptstyle{n}}}} = \frac{\eta(\kappa) \cdot \tan \alpha}{s_s}$$  \hspace{1cm} (3.184)

where $\eta(\kappa)$ is the multiplier necessary to make equation (3.184) hold for both small and large slip angles and slip ratios.
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The \( \eta(\kappa) \) function is of the form:

\[
\begin{cases}
0.5 \cdot [1 + \eta_0] - 0.5 \cdot [1 - \eta_0] \cdot \cos(0.5 \cdot \kappa) & \text{for } |\kappa| \leq 2\pi \\
1 & \text{for } |\kappa| > 2\pi
\end{cases}
\]

For small value of \( \kappa \), \( \eta \) is equal to \( \eta_0 \), whereas for large value of \( \kappa \), \( \eta \) is equal to 1.

\( \eta_0 \) is determined from the cornering stiffness \( C_s \), longitudinal stiffness \( K_x \), lateral friction coefficient \( \mu_y \), and longitudinal friction coefficient \( \mu_x \):

\[
\eta_0 = \frac{C_s \cdot \mu_x}{K_x \cdot \mu_y}
\]  

(3.185)

By substituting equation (3.185) into equation (3.184), for values of \( \kappa \) and using equation (3.182), the non-dimensional lateral and longitudinal forces can be separated as follows:

\[
\overline{F}_{\text{tyre}} = \eta(\kappa) \cdot R_{\text{tyre}}(\kappa) \cdot \frac{\tan \alpha}{\sqrt{s_x^2 + \eta^2 \cdot \tan^2 \alpha}}
\]  

(3.186)

\[
\overline{F}_{\text{swr}} = R_{\text{tyre}}(\kappa) \cdot \frac{s_x}{\sqrt{s_x^2 + \eta^2 \cdot \tan^2 \alpha}}
\]  

(3.187)

Also by substituting both equation (3.186) and equation (3.187) into equation (3.171) and equation (3.179), the final lateral force and longitudinal force generated at the tyre contact patch can be obtained.

### 3.5.8 Steering Geometry

The effect of Ackerman steer geometry is incorporated in the vehicle model. Referring to Figure 3-18, the wheel steer angles at the front wheels are \( \delta_1 \) and \( \delta_2 \), whereas at the rear wheels, they are assumed to be zero, considering that no camber or steer affect the rear suspension geometry.
The steer angle at the front wheels and also at the central vehicle axis can be expressed in relation to its distance from the centre of turn, thus:

\[
\tan \delta_0 = \frac{L_w}{D} \quad (3.188)
\]

\[
\tan \delta_1 = \frac{L_w}{(D + \text{trf}/2)} \quad (3.189)
\]

\[
\tan \delta_2 = \frac{L_w}{(D - \text{trf}/2)} \quad (3.190)
\]

From eq (3.188) and (3.189) \( D = \frac{L_w}{\tan \delta_0} \) and \( D + \text{trf}/2 = \frac{L_w}{\tan \delta_1} \)

\[
\Rightarrow \quad \frac{L_w}{\tan \delta_0} + \text{trf}/2 = \frac{L_w}{\tan \delta_1} \quad (3.191)
\]

\[
\Rightarrow \quad \frac{L_w + (\text{trf}/2 \cdot \tan \delta_0)}{\tan \delta_0} = \frac{L_w}{\tan \delta_1} \quad (3.192)
\]

\[
\Rightarrow \quad \tan \delta_1 = \frac{\tan \delta_0}{1 + \frac{\text{trf}/2}{L_w} \cdot \tan \delta_0} \quad (3.193)
\]
Now, using small angle approach for $\delta_0$ and $\delta_1$, and applying the binomial series:

$$(1 + x)^n = 1 + n \cdot x + \frac{n \cdot (n - 1) \cdot x^2}{2!} + \frac{n \cdot (n - 1) \cdot (n - 2) \cdot x^3}{3!} + \ldots$$

$$\Rightarrow (1 + x)^{-1} = 1 - x$$

Equation (3.193) takes the form:

$$\delta_1 = \frac{\delta_0}{1 + \frac{\text{trf}/2}{L_w} \cdot \delta_0} = \delta_0 \cdot (1 + \frac{\text{trf}/2}{L_w} \cdot \delta_0)^{-1}$$

$$\Rightarrow \delta_1 \approx \delta_0 \cdot (1 - \frac{\text{trf}/2}{L_w} \cdot \delta_0) \text{ and } \delta_2 \approx \delta_0 \cdot (1 + \frac{\text{trf}/2}{L_w} \cdot \delta_0)$$

If $P_a$ is the proportion of Ackerman, then $\delta_1$ and $\delta_2$ become:

$$\delta_1 \approx \delta_0 \cdot (1 - \frac{P_a \cdot \text{trf}/2}{L_w} \cdot \delta_0)$$

$$\delta_2 \approx \delta_0 \cdot (1 + \frac{P_a \cdot \text{trf}/2}{L_w} \cdot \delta_0)$$

Thus, in matrix form the correction for steering input becomes:

$$\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3 \\
\delta_4
\end{bmatrix} = \begin{bmatrix}
1 & -1 \\
1 & 1 \\
0 & 0 \\
0 & 0
\end{bmatrix} \cdot \begin{bmatrix}
\delta_0 \\
\text{trf}/2 \\
\text{trf}/2 \cdot \delta_0
\end{bmatrix}$$

(3.200)

where the matrix $\begin{bmatrix}
1 & -1 \\
1 & 1 \\
0 & 0 \\
0 & 0
\end{bmatrix}$ is termed as steer coefficient $K_{steer}$.

The steering system also incorporates variable steering gear ratio, where the steering road wheel angle is expressed as a function of steering hand wheel angle, in form of look-up table. Figure 3-19 shows the variable gear ratio plot. The variable gear ratio adjusts for some
of the effects of steering compliances. The effects of lateral and longitudinal force compliances are also included, while calculating the steering wheel angle at the road wheel.

3.5.9 Aerodynamic Forces and Moments

Aerodynamic forces and moments are generated by considering presence of an air stream around the vehicle. The Aerodynamic forces and moments affect the vehicle behaviour in a significant way, as the air stream around the vehicle influences its translational and rotational motion. These induce forces in the form of drag, lift and side direction, and generate corresponding rolling, yawing, and pitching moments. The aerodynamic forces and moments are expressed by the following equations:

Drag Force: $F_d = \frac{1}{2} \rho_a \cdot V_{wr}^2 \cdot A_p \cdot C_D$  \hspace{1cm} (3.201)

Side Force: $F_y = \frac{1}{2} \rho_a \cdot V_{wr}^2 \cdot A_p \cdot C_y$  \hspace{1cm} (3.202)

Lift Force: $F_z = \frac{1}{2} \rho_a \cdot V_{wr}^2 \cdot A_p \cdot C_L$  \hspace{1cm} (3.203)

Rolling Moment: $M_{d_z} = \frac{1}{2} \rho_a \cdot V_{wr}^2 \cdot A_p \cdot La \cdot C_{Mx}$  \hspace{1cm} (3.204)

Pitching Moment: $M_{d_y} = \frac{1}{2} \rho_a \cdot V_{wr}^2 \cdot A_p \cdot La \cdot C_{My}$  \hspace{1cm} (3.205)
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Yawing Moment: \( M_{y} = \frac{1}{2} \cdot \rho_{a} \cdot V_{\text{rel}}^{2} \cdot A_{p} \cdot L \cdot C_{M_{y}} \)  

(3.206)

where \( \rho_{a} \) denotes the air mass density, \( V_{\text{rel}} \) is the relative wind velocity, \( A_{p} \) is the projected frontal area, \( C_{D} \), \( C_{Y} \) and \( C_{L} \) represents the coefficient for drag force, side force and lift force respectively, and \( C_{M_{x}} \), \( C_{M_{y}} \) and \( C_{M_{z}} \) represents the coefficient for rolling moment, pitching moment and yawing moment respectively.

In this study, the air mass density \( \rho_{a} \) is assumed to be constant, where as the relative wind velocity is approximated by assuming stationary air movement so that the total relative velocity is only induced by the vehicle velocity, given as:

\[
V_{\text{rel}} = \sqrt{U^2 + V^2}
\]

(3.207)

The aerodynamic force and moment coefficients in equations (3.201) - (3.206) are function of relative wind velocity angle, which can be approximated as vehicle slip angle, under stationary air condition:

\[
\beta = \tan^{-1}\left( \frac{V}{U} \right)
\]

(3.208)

The aerodynamic forces (equations (3.201) - (3.203)) can be expressed in vehicle SAE coordinate system in the following manner:

\[
F_{d} = \begin{bmatrix}
-F_{d_{x}} \cdot \text{sgn}(U) \\
-F_{d_{y}} \cdot \text{sgn}(\beta) \\
-F_{d_{z}}
\end{bmatrix}
\]

(3.209)

Similarly, the aerodynamic moments (equations (3.204)-(3.206)) can be expressed as:

\[
M_{d} = \begin{bmatrix}
-M_{d_{x}} \cdot \text{sgn}(\beta) \\
M_{d_{y}} \cdot \text{sgn}(U) \\
-M_{d_{z}} \cdot \text{sgn}(\beta)
\end{bmatrix}
\]

(3.210)

The aerodynamic force and moment coefficients, used in this study, in relation to a range of vehicle slip angle are shown in Table 3-2.
Table 3-2: Aerodynamic force coefficients (Yip et al,1992)

<table>
<thead>
<tr>
<th>$\beta$ (deg)</th>
<th>$C_D$</th>
<th>$C_Y$</th>
<th>$C_L$</th>
<th>$C_{Mx}$</th>
<th>$C_{My}$</th>
<th>$C_{Mz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.41</td>
<td>0.00</td>
<td>0.35</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>0.43</td>
<td>0.10</td>
<td>0.44</td>
<td>0.05</td>
<td>0.11</td>
<td>0.05</td>
</tr>
<tr>
<td>10</td>
<td>0.49</td>
<td>0.24</td>
<td>0.56</td>
<td>0.10</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>15</td>
<td>0.52</td>
<td>0.42</td>
<td>0.71</td>
<td>0.20</td>
<td>0.11</td>
<td>0.18</td>
</tr>
<tr>
<td>20</td>
<td>0.59</td>
<td>0.60</td>
<td>0.95</td>
<td>0.29</td>
<td>0.12</td>
<td>0.22</td>
</tr>
<tr>
<td>25</td>
<td>0.68</td>
<td>0.72</td>
<td>1.05</td>
<td>0.39</td>
<td>0.12</td>
<td>0.24</td>
</tr>
</tbody>
</table>

3.6 Brake System Model

The brake system incorporated in the vehicle model is adapted from a previous study carried out by (Gerdes and Hedrick, 1999). Instead of a detailed representation of the complete brake system as outlined in (Fisher, 1970) and (Khan et al, 1994), a reduced order model of the brake system dynamics is used. This model considers simplified brake hydraulics and a vacuum booster. The proportioning valves are not included in the hydraulic circuit in the current study, as the brake system model is intended to serve as a platform for future extension to an ABS hydraulic model. When there is no ABS in a vehicle, the proportioning valve plays a significant role in limiting the pressure rise at the rear wheels, during high level of deceleration, thus preventing the rear wheels to lock

3.6.1 Vacuum Booster Model

The vacuum booster, located on the vehicle’s firewall, acts like a force amplifier, enhancing the force applied by the driver pedal, by exploiting the pressure differential between the atmosphere and the engine manifold vacuum. The vacuum chamber as shown in Figure 3-20 consists of two air chambers - vacuum chamber and apply chamber - separated by a diaphragm. The vacuum chamber is connected to the engine manifold, whereas the apply chamber is alternately connected to the atmospheric pressure, when stays sealed, or opens up to the vacuum chamber, depending upon the control valve settings i.e. if control valve is in the apply stage, hold stage or release stage respectively. The control valve modulates the airflow in the vacuum booster, based on the force response from the brake pedal, and thus provides a mechanical feedback mechanism to set the three stages of booster operation. The operation of these stages is modelled by taking into consideration the static force balance and
air flow dynamics, including effects such as vacuum booster hysteresis, which occurs due to reaction washer deformation and master cylinder seal friction.

During normal driving, when no brakes are applied, the control valve remains in the release stage as shown in Figure 3-20, connecting the apply chamber to the vacuum chamber and thus both chambers stay at engine manifold pressure. The pushrod and power piston in this case are pushed against limit stops by their respective return springs. During braking, the force from the brake pedal is transmitted to the pushrod through the pedal linkage. The pushrod, which is held back by the valve spring, is pushed ahead, once the input force overcomes the valve spring preload, thus sealing the apply chamber. Further movement of pushrod opens the control valve and as a result connect the apply chamber to the atmospheric pressure. This initiates the apply stage (Figure 3-21), where the atmospheric air flows into the apply chamber, resulting in a pressure rise across the diaphragm, eventually pushing the power piston to the left, once the pressure rise overcomes the return spring preload. As a result, the brake system is pressurised, forcing the fluid out of the master cylinder and into the wheel cylinders. The rise in master cylinder pressure causes a portion of the resulting force, to be fed back to the pushrod, through the rubber reaction washer. This reaction force acts to close the control valve, providing the feedback mechanism for the boosters internal control loop. The closing of the valve brings the booster to the hold stage (Figure 3-21), where no air is admitted and the chambers remain sealed. If the pedal force is
reduced, the booster returns to the release stage, thus allowing the air to drain from the apply chamber to the vacuum chamber and finally to the intake manifold through a check valve.

![Diagram of apply and hold stage of control valve](image)

**Figure 3-21: Apply and hold stage of control valve**

### 3.6.1.1 Control Valve Model

The static behaviour of the vacuum booster can be modelled by balancing the forces acting on the pushrod and power piston individually. Here, the inertial effects of the pushrod and power piston are neglected as they are quite small compared to the forces in the booster.

\[
F_{in} - F_{vs} - F_{pr} = 0 \quad (3.211)
\]

\[
F_{d} - F_{rs} + F_{vs} + F_{pp} = 0 \quad (3.212)
\]

where \(F_{vs}\) and \(F_{rs}\) denote the forces in the valve springs and return spring respectively. \(F_{in}\) denotes the input force to the pushrod from the pedal linkage. \(F_{d}\) denotes the diaphragm force, which could be represented as:

\[
F_{d} = A_{d} \cdot (P_{a} - P_{v}) \quad (3.213)
\]

where \(P_{a}\) and \(P_{v}\) denotes the pressures in apply and vacuum chamber respectively. \(A_{d}\) denotes the area of diaphragm, which is assumed to be equal for both chambers. Further, the terms \(F_{pr}\) and \(F_{pp}\) in equations (3.211) and (3.212) represent the force fed back through the reaction washer to the pushrod and power piston respectively, which can be equated to the output force of the booster \(F_{out}\).

\[
F_{pr} + F_{pp} = F_{out} \quad (3.214)
\]
Rearranging equations (3.211) - (3.214), to obtain an overall force balance as:

\[
F_{\text{out}} = \begin{cases} 
F_d + F_{\text{in}} - F_{\text{rs}} & \text{for } F_d + F_{\text{in}} > F_{\text{rs}} \\
0 & \text{else}
\end{cases}
\] (3.215)

The condition in equation (3.215) accounts for the case when the return spring presses the power piston against the limit stop. Equation (3.215) is sufficient to determine the output booster force \(F_{\text{out}}\), as \(P_a\), \(P_v\) and \(x_{\text{pp}}\) can all be determined from the state variables of the booster and master cylinder models (defined subsequently). The return spring force \(F_{\text{rs}}\) can be represented in terms of return spring preload \(F_{\text{rs0}}\) and spring constant \(K_{\text{rs}}\).

\[
F_{\text{rs}} = F_{\text{rs0}} + K_{\text{rs}} \cdot x_{\text{pp}}
\] (3.216)

To determine the different stages of booster operation, Gerdes and Hedrick (1999) described an approach, which included effect of hysteresis, occurring largely due to reaction washer deformation, as well as also due to seal friction. In this approach, the stages of booster operation are determined by representing the reaction washer deformation and valve spring forces into a form of force value, denoted as \(F_{\text{app}}\) and \(F_{\text{rel}}\), as shown in Figure 3-22 – the boundary values of which as a function of output force \(F_{\text{out}}\), are obtained by setting an idealised reaction washer model experiment (Gerdes and Hedrick, 1999).

Figure 3-22: Stages of booster operation (Gerdes and Hedrick, 1999)
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The relationship defining the various stages is given as:

\[ F_{in} < F_{ref} \Rightarrow \text{release} \]
\[ F_{ref} \leq F_{in} \leq F_{app} \Rightarrow \text{hold} \]
\[ F_{app} < F_{in} \Rightarrow \text{apply} \]  \hspace{1cm} (3.217)

3.6.1.2 Vacuum Booster Dynamics

While the control valve model represents more of a static behaviour, the air flow associated with the apply and vacuum chambers determines the dynamic behaviour of the booster. During the apply stage, the air flows into the apply chamber, resulting in a pressure rise and thus forces the diaphragm forward. This causes compression of air in the vacuum chamber, which results in pressure rise that decays once the air flows through the check-valve into the manifold. During the hold stage, slight leakage takes place between the two chambers, through the control valve seal, which is attached to the pushrod by a light spring (refer Figure 3-20). As a result of this leakage, the pressure in the apply chamber decays, thus maintaining a constant pressure difference across the diaphragm. During the release stage, the two chambers are connected again, allowing air flow from the apply chamber to the vacuum chamber.

The air flow dynamics in the vacuum booster is modelled assuming ideal gas behaviour and isothermal expansion. The pressure in the apply and vacuum chambers can be written as:

\[ P_a = \frac{m_a \cdot R \cdot T}{V_{ao} + A_d \cdot x_{pp}} \]  \hspace{1cm} (3.218)
\[ P_v = \frac{m_v \cdot R \cdot T}{V_{vo} - A_d \cdot x_{pp}} \]  \hspace{1cm} (3.219)

where \( V_{ao} \) and \( V_{vo} \) denote the initial chamber volumes, \( R \) denotes the gas constant, and \( T \) is the temperature.

The state equations for \( m_a \) and \( m_v \), which represent the air masses in the apply and vacuum chamber respectively, can be stated as:
where $C_{aa}$, $C_{av}$, and $C_{leak}$ stand for linearized flow coefficients for flow from the atmosphere to the apply chamber, between the apply and vacuum chambers during release and between the apply and vacuum chambers during hold, respectively. The term $\dot{m}_{vm}$ denotes the flow rate through the check valve. As this valve allows air to flow only from the vacuum chamber to the manifold, the flow equation for the check valve is:

$$\dot{m}_{vm} = \begin{cases} -C_{vm} \cdot (P_v - P_{man} - P_0) & \text{for } P_v > P_{man} + P_0 \\ 0 & \text{or else} \end{cases} \quad (3.222)$$

where $C_{vm}$ is the linearised flow coefficient, $P_0$ denotes the pressure offset required to open the valve, and $P_{man}$ is the manifold pressure, which is taken as constant in the current model.

The vacuum booster model, in its final form, has one input ($F_{in}$) and two states ($m_a, m_v$), and is coupled to the brake hydraulics (through $X_{pp}$).

The flow coefficients used in equations (3.220) - (3.222) are based on the assumption of an incompressible flow. The above coefficients can be modelled for a variable orifice size, where the effective orifice area, based on the given valve model, increases linearly with the relative displacement of the pushrod and power piston, until the orifice is fully opened. The flow coefficients $C_{aa}$ and $C_{av}$ can be expressed as:

$$C_{aa} = \begin{cases} C_{aa} & F_{in} > F_{app} + \bar{F}_{app} \\ \frac{C_{aa}}{C_{aa} \cdot \left( \frac{F_{in} - F_{app}}{\bar{F}_{app}} \right)} & \text{Else} \end{cases} \quad (3.223)$$

$$C_{av} = \begin{cases} \bar{C}_{av} + \bar{C}_{leak} & F_{in} < F_{rel} \cdot \bar{F}_{rel} \\ \frac{C_{av}}{C_{av} \cdot \left( \frac{F_{rel} - F_{in}}{\bar{F}_{rel}} \right)} + \bar{C}_{leak} & \text{Else} \end{cases} \quad (3.224)$$
where $F_{ap}$ and $F_{rel}$ denote the forces required to fully open the orifices in the apply and release stages, which are shown graphically in Figure 3-22 by dotted lines, whereas $C_{as}$ and $\bar{C}_{as}$ denotes the corresponding flow coefficients.

The leakage flow coefficient is modelled by assuming a linear variation over the length of reaction washer's hysteresis, taking into account that the spring provides more resistance to leakage as the force increases.

$$C_{\text{leak}} = \bar{C}_{\text{leak}} \cdot \left( \frac{F_{\text{app}} - F_{\text{in}}}{F_{\text{app}} - F_{\text{rel}}} \right)$$  \hspace{1cm} (3.225)

### 3.6.2 Hydraulics of Brake System

The master cylinder, brake lines, and wheel cylinders are included in this model, as components of brake system hydraulics (Figure 3-23), considering only those features that directly affect the dynamic behaviour of the brake system.

#### 3.6.2.1 Master Cylinder

The master cylinder, which converts the output force from the vacuum booster into hydraulic pressure by displacing the fluid into the rear and front brakes, is modelled as two pistons (primary / secondary), arranged concentrically in a single bore. The brake hydraulics is then
spilt into two circuits, each circuit containing a front brake and a diagonally opposite rear brake. During the application of brake, the power piston of the vacuum booster presses the primary piston on the tandem master cylinder, causing the pressure to rise, and as a result the brake fluid is forced into the primary brake line. The pressure rise in the primary cylinder also results in a force on the secondary piston, eventually forcing the fluid into the secondary brake line. Neglecting the small inertias of the pistons and assuming the hydraulics to consist of two sealed circuits, the pressure in each circuit is given by:

\[ P_{mcp} = \frac{(F_{out} - F_{esp} - F_{efp})}{A_{mc}} \]  
\[ P_{mes} = P_{mcp} - \frac{(F_{css} + F_{efs})}{A_{mc}} \]

where \( F_{esp} \) and \( F_{css} \) are the return spring forces and \( F_{efp} \) and \( F_{efs} \) are the seal friction forces for the primary and secondary cylinders, respectively, and \( A_{mc} \) denotes the cross-section area of the master cylinder bore. The seal friction is assumed here to follow a Coulomb friction model (Gerdes and Hedrick, 1999). For finite displacements, the spring forces are given by:

\[ F_{esp} = F_{espo} + K_{esp} \cdot (x_{mcp} - x_{mes}) \]  
\[ F_{css} = F_{esso} + K_{css} \cdot x_{mes} \]

where \( x_{mcp} \) and \( x_{mes} \) are primary and secondary piston displacements. This leads to the variable \( x_{pp} \) used in equations (3.218) and (3.219) to be expressed in the following expression, which reflects the fact that the booster cannot pull on the master cylinder:

\[ x_{pp} = \begin{cases} x_{mcp} & F_{out} > 0 \\ 0 & \text{Else} \end{cases} \]

3.6.2.2 Brake Lines and Wheel Cylinders

Since the hydraulics are modelled assuming an incompressible flow, the state variables are represented in terms of the volume of fluid displaced in each wheel cylinder. The choice of the volumes as state variables reflects from the fact that pressure changes at the master cylinder are not immediately translated into pressure rises at the wheels. Rather, it takes place once the fluid flows to the wheels. The primary and secondary displacements of the
piston can be expressed in terms of the volume of fluid displaced into each wheel’s cylinder ($V_{lf}$, $V_{rf}$, $V_{lr}$, and $V_{rr}$) in the following manner:

$$x_{mcs} = \frac{(V_{lf} + V_{rr})}{A_{mc}}$$  \hspace{1cm} (3.231)  

$$x_{mcp} = \frac{(V_{rf} + V_{lr})}{A_{mc} + x_{mcs}}$$  \hspace{1cm} (3.232)  

The displaced fluid volume can be related to the wheel pressure, by assuming brake lines and wheel cylinders to possess some fluid capacity, where a certain pressure in the brake line results in a certain displacement of the piston in the wheel cylinder. The pressure at each wheel can then be expressed as:

$$P_{wa} = P_{wa}(V_{\alpha}) \hspace{1cm} \alpha \in \{lf, rf, lr, rr\}$$  \hspace{1cm} (3.233)  

A general shape of brake system capacity can be obtained experimentally as shown in Figure 3-24. It can be seen that an initial flow takes place without an increase in pressure, caused by expansion in the lines and wheel cylinder seals, as well as knock-back of the calliper, after which the capacity may be approximated by a smooth function.

To generate the four hydraulic state equations for individual wheels, the flow to each wheel is modelled using Bernoulli’s equation in terms of pressures in the master cylinder and the wheel cylinders:
\[ V_a = \sigma_a \cdot C_{qa} \cdot \sqrt{P_{\text{mcs}} - P_{\text{wa}}} \]  
(3.234)

where \( \sigma_a = \text{sgn} (P_{\text{mcs}} - P_{\text{wa}}) \) \( \alpha \in \{\text{lf, rf, lr, rr}\} \)  
(3.235)

and: \( C_{qa} (\alpha \in \{\text{lf, rf, lr, rr}\} \) denote flow coefficients for the individual brake lines.

### 3.6.3 Brake Friction

The final output of the brake model is the friction force, which is generated at the brake pads, after translation from the wheel cylinder pressure through a calliper brake or a drum brake. The friction force between the pad and the rotor / drum generates braking torque. Neglecting the inertia of the callipers or shoe (small compared to the forces involved), the static model of the brake friction can be expressed as:

\[ M_{ba} = \begin{cases} K_{bu} \cdot (P_{\text{wa}} - P_{\text{poa}}) & P_{\text{wa}} \geq P_{\text{poa}} \\ 0 & \text{Else} \end{cases} \quad \alpha \in \{\text{lf, rr, rf, lr}\} \]  
(3.236)

where \( M_{ba} \) denotes the brake torque, \( K_{bu} \) is the brake effectiveness (speed dependent), and \( P_{\text{poa}} \) denotes the push-out pressure, below which the pads do not contact, resulting in no braking. This pressure, in physical terms, corresponds to the force required to overcome the return springs in a drum brake or calliper seal rollback in a disk brake. (Radlinski, 1987) in his study presented values of brake effectiveness, which are presented in Table 3-3.

<table>
<thead>
<tr>
<th>Brake Effectiveness</th>
<th>Speed (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>lb –ft / psi</td>
<td>30</td>
</tr>
<tr>
<td>Front Brakes</td>
<td>0.69-0.87</td>
</tr>
<tr>
<td>Rear Brakes</td>
<td>0.36-0.52</td>
</tr>
</tbody>
</table>

Table 3-3: Brake effectiveness (Radlinski, 1987)

### 3.6.4 Reduced State Hydraulic Model

The brake system model, presented so far, includes a four-state model of the brake hydraulics, in addition to a two-state vacuum booster model. The model can be further simplified by including only one hydraulic state (Gerdes and Hedrick, 1999). The idea for
having four individual hydraulic states was to provide a platform for ABS control, where individual wheel pressures can be modulated. However, a single state non-linear model can be used for normal braking applications, without ABS, where variations across wheels are not important.

In a single state hydraulic model, the brake force is represented as a whole, rather than distribution of this force among individual wheels, and the master cylinder is modelled as an equivalent single hydraulic circuit, as shown in Figure 3-25. The pressure in the master cylinder can be expressed as:

\[ P_{mc} = \frac{F_{out} - F_{cs} - F_{cf}}{A_{mc}} \]  

(3.237)

The equation is similar to that given in (3.226), with the terms \( F_{cs} \) representing the spring preload and \( F_{cf} \) representing the seal friction. The piston displacement in the master cylinder \( x_{mc} \) can be expressed as:

\[ x_{mc} = V_p / A_{mc} \]  

(3.238)

where \( V_p \) represents the equivalent displaced volume, which means the state equation for the flow can be expressed as:

\[ \dot{V}_p = \sigma \cdot C_q \cdot \sqrt{|P_{mc} - P_w|} \]  

(3.239)

where \( C_q \) represents the effective flow coefficient. The pressure in the equivalent wheel cylinder \( P_w \), can be modelled as lumped fluid capacity of the entire brake system:

\[ P_w = P_w (V_p) \]  

(3.240)
Finally, the brake torque $M_b$ takes the form:

$$M_b = \begin{cases} K_b \cdot (P_w - P_{po}) & P_w \geq P_{po} \\ 0 & \text{Else} \end{cases}$$

The brake effectiveness $K_b$, in the above equation reflects the entire brake system. The torque / pressure relationship can also be experimentally determined, as shown in Figure 3-26, which represents the overall brake effectiveness (in form of slope), suitable for the one state hydraulic model.

![Figure 3-26: Brake torque / pressure plot (Gerdes and Hedrick, 1999).](image)

The reduced order hydraulic brake model was integrated in the intermediate vehicle model, for normal braking application. A flow chart of the brake model is provided in Figure 3-27. It has total three number of states, where the first two states are for the flow rates in apply and vacuum chamber of the vacuum booster, and the third state is for the lumped fluid capacity of the master cylinder. The brake system essentially provides an open loop braking control, where the brake pedal force acts as an input to the system.
Figure 3-27: Flow chart of reduced order hydraulic brake model
3.7 Numerical Solution of the Intermediate Vehicle Model

The 10-DOF intermediate vehicle model was solved using combined MATLAB / Simulink approach, which eases the tedious task of solving the hand-written codes. The MATLAB was used to define the basic vehicle characteristics and initial conditions through use of functions and matrixes. The execution of Simulink model and post-processing of the results were also carried out in MATLAB. The Simulink was used to solve the differential equations in time domain and generate time histories for MATLAB workspace. The different vehicle subsystems (refer Figure 3-28) were defined in the Simulink environment, where the vehicle characteristics data was called using functions or look-up table. The combined MATLAB / Simulink environment also enabled an easy interface with the experimental testing data (chapter 5) through the use of data structures, and with the ABS code (written in C language - refer chapter 6), through S-Functions. The numerical simulation was carried out using 4th order Runge-Kutta solver, where the fixed integration size of either 0.01s or 0.001s was used.

Figure 3-28: Schematic of intermediate vehicle model in MATLAB / Simulink
3.8 Chapter Remarks

This chapter looks into the fundamental description of the vehicle modelling using Newton-Euler principles. The same principle is then applied to develop vehicle models, starting from 2-DOF to fully functional intermediate vehicle model. Also, the various sub-sections or components of the intermediate vehicle model is discussed in this chapter, along with its implementation in the MATLAB-Simulink environment. In the following chapter, the intermediate vehicle model’s comparison is drawn with the simplified 3-DOF vehicle model, along with the complex multi-body vehicle model (built in ADAMS), for standard handling manoeuvres. In addition, the intermediate vehicle model is also validated against the instrumented test vehicle, which will be covered later in chapter 5.
4 Multi-Body Model and Simulation Results

4.1 Introduction

One of the important objectives of this work was to develop a multi-body model and perform comparative study with the Intermediate vehicle model (reported in chapter 3). A detailed multi-body vehicle model was established in the ADAMS/Chassis environment for this purpose. This chapter, after a brief introduction to the theory of multi-body dynamics describes the developed multi-body vehicle model. The modelling of realistic vehicle behaviour is accomplished by incorporating the non-linear characteristics of different assembled parts or sub-systems, which includes the effects of compliances in the body, suspension and steering sub-systems. This chapter also covers the description of the PAC2002 tyre model, which is based on the latest version of Magic Formula (Pacejka, 2006). Apart from the steady-state characteristics, the tyre model developed here also incorporates transient effects through first order differential equations, using relaxation length as a parameter (covered later in chapter 6). Finally, the chapter presents the simulation results of various standard handling manoeuvres, undertaken to perform comparison and validation of the 10-DOF intermediate model (described in the previous chapter) against the detailed multi-body vehicle model.

4.1.1 Theory of Multi-Body Dynamics

The theory of multi-body includes the determination of equations of motion for rigid inertial elements, applied forces and moments, holonomic and non-holonomic constraint functions and the setting up of a Jacobian matrix. The equations of motion, formulated using Lagrangian dynamics for constrained systems can be represented as (for each part):

$$\frac{d}{dt} \left( \frac{\partial K}{\partial \dot{q}_i} \right) + \frac{\partial K}{\partial q_i} \frac{\partial V}{\partial \dot{q}_i} + \sum_{i=1}^{m} \lambda_i \frac{\partial C_i}{\partial q_i} = 0$$

(4.1)

where $K$ is the kinetic energy

$V$ is the potential energy

$m$ is the number of constraints for a given joint
Multi-Body Model and Simulation Results

C_i is the holonomic/non-holonomic constraint function

\lambda_i is the Lagrange multiplier

F_{q_j} = -\frac{\partial V}{\partial q_j} is the generalised forces according to Euler.

\{q_j\} = \{X, Y, Z, \psi, \theta, \phi\}_T^T is the generalised Eulerian co-ordinate set. In ADAMS this is based on 3-1-3 Euler frame of reference, where \psi is the 1st rotation about Z, \theta is the 2nd rotation about X, and \phi is the final rotation about Z.

These differential equations of motion are solved together with the imposed constraints by joints and joint primitives, which are represented as scalar algebraic functions:

C_i(q_j) = 0 \hfill (4.2)

The differential equation set, the applied forces and reactions due to the various sources of compliance, and the scalar constraint functions are solved simultaneously in small time steps. The vector of unknowns includes the system state variables; position, velocity and acceleration of all parts, and the Lagrange multipliers, representing the joint reactions. Thus, in matrix form the set of equations are represented (Rahnejat, 1998) by:

\[ [J]\{q, \lambda\}_T^T = \{F_a\} \hfill (4.3) \]

where \[ J \] is the Jacobian matrix

\{q, \lambda\}_T^T is the required solution vector in small time steps

\[ F_a \] is the applied force

The solution method is fully described in chapter 5 of (Rahnejat, 1998).

4.2 ADAMS Multi-Body Vehicle Model

The multi-body vehicle model used in this study is built using ADAMS/Chassis environment, which is one of the modules offered in the ADAMS software, dedicated for vehicle dynamics' analysis. ADAMS/Chassis module offers a comprehensive library of vehicle components and sub-systems. One of the advantages in using ADAMS/Chassis is
Multi-Body Model and Simulation Results

that it enables simulation of full-vehicle dynamic events such as steady-state drift, double lane change, constant radius turn, etc, as well as half-vehicle events such as dynamic load case etc. These events are readily available within an extensive list of ride, handling and durability events. Figure 4-1 shows a graphical representation of the vehicle model built in the ADAMS/Chassis environment.

![Multi-Body model of passenger car in ADAMS chassis](image)

**Figure 4-1: Multi-Body model of passenger car in ADAMS chassis**

4.3 ADAMS/Chassis Work Modes

ADAMS/Chassis is divided into four work modes: Build, Test, Review and Improve. The Build mode is for editing model data and changing system configuration. The Test mode is for building and running a model. The Review mode allows visualizing analysis results using ADAMS/Post Processor, which has two formats: reports and plots. A majority of standard ADAMS/Chassis events have either a report or a plot, or both. The improve mode is for refining models with ADAMS/Insight, where it is possible to create sophisticated experiments for measuring the performance of the model, and also to analyze the results of the experiments, through a collection of statistical tools.

4.4 ADAMS/Chassis Data Structure

The ADAMS/Chassis data structure comprises vehicle databases, systems, sub-systems, property files and XML data format, the relationship of which is illustrated in Figure 4-2.
Multi-Body Model and Simulation Results

A Database is a file folder or directory (with the extension .vdb), where each model's data is stored. The database contains sub-directories, also called tables (.tbl), for the different file types. There are three major classes of files in the database: system, subsystem and property files. These classes of files are stored using the XML (Extensible Markup Language) data format, and therefore have an .xml extension.

The System files are also known as vehicle configuration files. They are stored in systems.tbl directory of the vehicle database. System files are the first step in visual data editing. There are three types of systems in ADAMS/Chassis: front, rear, and full. The system files contain sub-system references and system parameters.

The Sub-system files are stored in subsystems.tbl directory of the vehicle database. The primary contents of sub-system files include: hardpoints, parts, connectors (bushings), property file references, construction options, and sub-system parameters (if applicable). The
different sub-system types are: body, front suspension, steering gear, steering column, rear suspension, front wheel and tyres, rear wheel and tyres. Also, there are some optional sub-systems such as loading, instrumentation, traction, brakes, controls etc.

The Property files represent data for a particular component, and it contains: object attributes, SPLINE, parts and connectors. Property files can also reference other property files (for example, stabilizer bar property file references a bushing property file for the mounts). The different types of property files and their locations are listed in Table 4-1.

<table>
<thead>
<tr>
<th>Property Files</th>
<th>Location</th>
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<tbody>
<tr>
<td>Aerodynamic forces</td>
<td>aero_forces.tbl</td>
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<td>Dampers</td>
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<td>Steering assistants</td>
<td>steering_assists.tbl</td>
</tr>
<tr>
<td>Tyre models</td>
<td>tires.tbl</td>
</tr>
</tbody>
</table>

Table 4-1: List of property files

4.5 ADAMS/Chassis Coordinate System

The ADAMS/Chassis coordinate system is represented such that the positive X goes from the front of the vehicle to its rear, the positive Y goes from centerline of the vehicle to the right, and the positive Z points upwards. The origin of the coordinate system is located some place in front of the vehicle with the Y=0 as the centerline and the Z=0 is some place below the vehicle. Due to the varying wheel and tyre sizes Z=0 does not correspond to the ground patch for a particular tyre. The result of choosing Z=0 to be below the vehicle is that one would not have negative Z values, when describing the vehicle geometry. Figure 4-3 illustrates the representation of the coordinate system. When comparing it with the intermediate vehicle model developed in MATLAB / Simulink (which uses the SAE coordinate representation), the direction of the coordinate system in ADAMS differs only in the Z direction. This means the positive Z for the ADAMS model will be negative Z for the intermediate model and vice-versa.
4.6 Vehicle Sub-systems

The vehicle model developed in the ADAMS/Chassis includes the following vehicle sub-systems: body, front and rear suspension, steering gear, steering column, traction, brakes, front wheels, rear wheels, and loading. The following sections describe the important sub-systems contained in the vehicle model. For simplification, the steering gear and the steering column, and also both the front and rear wheels/tyres are described in the same section.

4.6.1 Vehicle Body

The body sub-system contains information about the body parts, aerodynamic forces, body compliance, bushing-related-to-chassis etc. The body sub-system file is stored in the subsystems.tbl sub-directory of the vehicle database, which contains data related to the joints, location of the part mounts, mass and inertia of different parts etc. Table 4-2 shows the body compliance in the form of bushing stiffness at different mounts.

<table>
<thead>
<tr>
<th>Name</th>
<th>Left Type</th>
<th>Left Requests</th>
<th>Left K-X</th>
<th>Left K-Y</th>
<th>Left K-Z</th>
<th>Left K-RX</th>
<th>Left K-RY</th>
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<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 4-2: List of joints and bushings connecting body mounts
The vehicle body in this model is taken to be a rigid body, the general characteristics of which are listed in Table 4-3. The characteristics computed by SVC (static vehicle characteristics) are generally based upon the compliance matrix for a vehicle suspension. Loosely, this matrix is defined as the wheel centre deflections relative to the body due to unit forces and moments applied at the wheel centres. The compliance matrix is computed by inverting the Jacobian matrix formed by ADAMS and then manipulating the resultant matrix to remove the body's 6 degrees of freedom, including the effect of the tyres.

### General Characteristics

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Total</th>
</tr>
</thead>
<tbody>
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<td>Total weight</td>
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<td>17.86E+03</td>
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<tr>
<td>Front ground reaction</td>
<td>N</td>
<td>9407.26</td>
</tr>
<tr>
<td>Rear ground reaction</td>
<td>N</td>
<td>8450.73</td>
</tr>
<tr>
<td>Total roll inertia</td>
<td>kg·mm²</td>
<td>581.5E+06</td>
</tr>
<tr>
<td>Total pitch inertia</td>
<td>kg·mm²</td>
<td>3.246E+09</td>
</tr>
<tr>
<td>Total yaw inertia</td>
<td>kg·mm²</td>
<td>3.605E+09</td>
</tr>
<tr>
<td>Total product lxy</td>
<td>kg·mm²</td>
<td>2.250E+06</td>
</tr>
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<tr>
<td>Sprung product lyz</td>
<td>kg·mm²</td>
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<td>517.03</td>
</tr>
<tr>
<td>Sprung c.g. height</td>
<td>mm</td>
<td>545.27</td>
</tr>
<tr>
<td>Wheelbase</td>
<td>mm</td>
<td>3028.73</td>
</tr>
</tbody>
</table>

**Table 4-3: Vehicle characteristics**

### 4.6.1.1 Aerodynamic Modelling

Aerodynamic property file is stored in the aero_forces.tbl subdirectory of a vehicle database. In vehicle body, aerodynamics forces and moments are modelled through this routine.
Aerodynamics' modelling provides the sensitivities of a particular vehicle to wind gusts. The vehicle front section area is fixed to 2.30 m² and air density is taken as 1.220 kg/m³. This routine requires wind tunnel aerodynamic coefficient data for the vehicle being modelled and they must be taken according to SAE J1594 conventions for Vehicle Aerodynamics Terminology. In ADAMS/Chassis, the aerodynamic modelling routine can be carried out by either one or two wind force points of application. This option depends on the source and type of wind tunnel data. Europe tends to require two points, whereas data in the U.S. is typically resolved for a single point. To apply aerodynamic forces at two positions on the body, the ADAMS' GFORCE statement must be duplicated at both points. In this vehicle model the two force points method was used.

A SPLINE function for wind velocity and angle of flow allows simulating wind fans, chaotic wind forces, etc. The wind properties can be dependent on distance (useful for wind fan modelling) or on time (useful to investigate vehicle sensitivity versus speed). The wind velocity and angle are given with respect to the ground frame of reference. The routine calculates relative wind speed and direction based on the vehicle velocity and yaw angle. The different aerodynamic coefficients used in the vehicle model are shown in Figure 4-4 (represented in terms of SPLINE).

![Aerodynamic coefficients in terms of wind velocity angle](image)

*Figure 4-4: Aerodynamic coefficients in terms of wind velocity angle*
4.6.2 Front Suspension

The front suspension in this vehicle model is an SLA suspension with coil spring system. In this section the main set of data blocks, parts and characteristics of the suspension system are described. In ADAMS/Chassis, the suspension and steering systems are demarcated by including the inner tie rod ball as part of the front suspension, while the rack or centre link points are considered as steering data. The front suspension sub-system files is stored in the subsystems.tbl directory of the vehicle database, containing information about hardpoints, parts’ cg, mass and inertial data, alignment information etc.

4.6.2.1 Hardpoints:

<table>
<thead>
<tr>
<th></th>
<th>UCA front</th>
<th>2</th>
<th>UCA rear</th>
<th>3</th>
<th>LCA front</th>
<th>4</th>
<th>LCA rear</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Lower ball joint</td>
<td>7</td>
<td>Upper ball joint</td>
<td>9</td>
<td>Wheel centre</td>
<td>10</td>
<td>Contact patch</td>
</tr>
<tr>
<td>11</td>
<td>Spindle align</td>
<td>12</td>
<td>Tierod outer</td>
<td>14</td>
<td>Tierod inner</td>
<td>58</td>
<td>Spring seat upper</td>
</tr>
<tr>
<td>59</td>
<td>Spring seat lower</td>
<td>61</td>
<td>ARB bushing (left)</td>
<td>62</td>
<td>ARB bushing (right)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>Droplink bar (left)</td>
<td>64</td>
<td>Droplink bar (right)</td>
<td>65</td>
<td>Droplink external (left)</td>
<td>66</td>
<td>Droplink external (right)</td>
</tr>
<tr>
<td>75</td>
<td>Bumper upper (left)</td>
<td>76</td>
<td>Bumper upper (right)</td>
<td>71</td>
<td>Bumper lower (left)</td>
<td>72</td>
<td>Bumper lower (right)</td>
</tr>
<tr>
<td>77</td>
<td>Rebound upper (left)</td>
<td>78</td>
<td>Rebound upper (right)</td>
<td>73</td>
<td>Rebound lower (left)</td>
<td>74</td>
<td>Rebound lower (right)</td>
</tr>
<tr>
<td>55</td>
<td>Damper upper</td>
<td>56</td>
<td>Damper lower</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-4: Front suspension geometrical hardpoints (ADAMS/Chassis, 2005)
In defining the suspension system, the first major block of data is the geometry point data or geometric hardpoints. The geometrical hardpoints of all the joints in this suspension system are highlighted in Figure 4-5, the names of which are further listed in Table 4-4. These hardpoints are defined in ADAMS/Chassis as the standard set of required points for both the left and right front quarter suspensions.

4.6.2.2 Mass and Inertial Data

The mass and rotational inertias of the modeled suspension components can be specified in the Part tab in each sub-system file. The rotational inertias of all parts are assumed to be in design coordinates, except for the tyre/wheel/rotor part. This part is referenced with the Z axis as the spin axis to simplify model building. The product of inertia terms are given for only the left side components. The mass and inertia table is followed by the C.G. location data block. Furthermore, if tyre/wheel/rotor part coordinate is specified as 0, 0, 0, the C.G. of the part can be directly placed at the wheel centre.

4.6.2.3 Suspension Bushings

![Bushing Slope (spline data)](image)

Figure 4-6: Bushing slope for UCA (radial direction)

An important part of any suspension is the bushing rates and their damping characteristics. ADAMS/Chassis stores individual bushing property files in the bushings.tbl sub-directory of a vehicle database. The bushing rates (3 translational and 3 rotational) should be specified as
the slope of the static load deflection curve, with the units entered as a positive value in N/mm. ADAMS/Chassis will interpret a negative rate as the SPLINE or ARRAY statement required by the nonlinear bushing routines. Figure 4-6 shows the translational bushing data of the upper control arm in the radial (x) direction, expressed as a SPLINE statement.

4.6.2.4 Spring Data

Front suspensions typically have a spring coil, a torsion bar, or leaf springs. The coil spring (used in this model) is modelled with a linear rate specification. The coil’s spring data can also possess a non-linear specification, where the independent axis data can either be specified in terms of spring length or deflection/displacement. The information such as rates, capacities and free lengths are given in the coil spring chart. However, the only data which ADAMS/Chassis actually reads and uses is the spring rate, free length L, and the coil diameter. ADAMS/Chassis stores all these information in the spring property files in the springs.tbl sub-directory of a vehicle database.

4.6.2.5 Shock Absorber Data

![Front Damper (spline data)](image)

Figure 4-7: Front damper characteristics

The shock absorbers are modelled as a force versus velocity SPLINE in ADAMS/Chassis, and no parts or bushings are included (except in the case of struts). The shock absorbers’ property files are stored in the dampers.tbl sub-directory of a vehicle database, containing
the lower and upper shock part information (the mounts), as well as optional bushing specification. The sign convention for shock absorber SPLINE is such that an extension returns a positive velocity and force, and compression is regarded as having a negative velocity and force. The velocity unit is mm/sec, whereas force is in Newton. The non-linear characteristic plot of the front damper is shown in Figure 4-7.

4.6.2.6 Suspension Bumpers

The suspension bumpers are defined by BumpStopPair (jounce) and ReboundStopPair (rebound) objects in the sub-system data file. These property files are stored in the bumpstops.tbl sub-directory of a vehicle database. The sub-system file contains the metal-to-metal rate, suspension attachment, and property file reference, and additionally for rebound bumpers the free length (clearance) is also included. The property format is the same for jounce and rebound bumpers. Here, it's allowed to have the metal-to-metal bump stop located at a separate point than the jounce bumper. By default the jounce bumper metal-to-metal stop is at the same position as the jounce bumper, and when the deflection of the bumper goes beyond the bumper height, the metal-to-metal force is applied. Figure 4-8 is a pictorial representation of the jounce bumper, where the hardpoint pairs 'bumper_mtl_upper' and 'bumper_mtl_lower' define the metal-to-metal stop at a different position. The point 'bumper_mtl_upper' is the metal-to-metal engagement point on the body, whereas point 'bumper_mtl_lower' is the engagement point on the suspension. The default orientation for the jounce bumper is parallel to the global Z direction (pointing upwards).

![Figure 4-8: Jounce bumper (ADAMS/Chassis, 2005)](image-url)
The equation for the jounce bumper force is as follows:

\[
\text{Force}_z = a \cdot (\text{Disp}_z) + b \cdot (\text{Disp}_z)^2 + c \cdot (\text{Disp}_z)^3
\]  
(4.4)

where the variable 'a' is the linear rate, 'b' is the quadratic rate, 'c' is the cubic rate defined in the bumper table, and \( \text{Disp}_z \) is equal to the displacement of the rubber. Instead of a polynomial fit, using the above equation, the jounce bumper force can also be represented by a SPLINE as shown by a non-linear plot in Figure 4-9.

The equation for the metal-to-metal force is as follows:

\[
\text{Force}_z = \text{STEP}(\text{Disp}_z, \text{BumpHeight}, 0, \text{BumpHeight} + 1 \cdot 10^6)
\]  
(4.5)

Where, the \text{STEP} function is a function of the 'z' displacement of the jounce bumper points, and is activated when the jounce bumper deflection is greater than the height of the bumper, and \( \text{BumpHeight} \) represents the bumper height value (stored in the bumper property file).

The rebound bumper is shown in Figure 4-10. The equation for the rebound bumper is similar to the jounce bumper, expressed as:

\[
\text{Force}_z = a \cdot (\text{Disp}_z) + b \cdot (\text{Disp}_z)^2 + c \cdot (\text{Disp}_z)^3
\]  
(4.6)

where \( \text{Disp}_z \) is equal to the displacement of the rubber.
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\[
\text{Disp}_z = \frac{(\text{DM}(\text{mar77, mar73}) - \text{free\_length} + \text{abs}(\text{DM}(\text{mar77, mar73})) - \text{free\_length})}{2} \tag{4.7}
\]

\(\text{Disp}_z\) vanishes when the distance between the markers at points 73 and 77 is less than the free length, and it becomes equal to \((\text{DM}(\text{mar77, mar73}) - \text{free\_length})\) when the distance is greater than the free length.

![Figure 4-10: Rebound bumper (ADAMS/Chassis, 2005)](image)

The equation for the metal-to-metal force is:

\[
\text{Force}_{m} = \text{STEP}(\text{Disp}_z, \text{BumpHeight}, 0, \text{BumpHeight} + 1, \text{MetalRate} \cdot (\text{Disp}_z - \text{BumpHeight})) \tag{4.8}
\]

where the \(\text{STEP}\) function relates the distance between the markers at points 73 and 77, and is activated when the jounce bumper deflection is greater than the height of the bumper.

4.6.2.7 Stabilizer Bar

The front suspension model contains a beam element type of stabilizer bar, and is represented in ADAMS/Chassis using the property file, stored in the sub-directory stabilizer\_bars.tbl. As the beam element model is a more physical representation of the system, the model does not have to be adjusted to get the correct roll rate. The beam-element stabilizer bar model needs an array of points for various parts of the bar profile (20 in the current case), which is shown in Figure 4-11. The mass and inertia of the parts along with the material properties of the beam element such as its elastic modulus, Poisson’s ratio,
density etc. are defined in the property file. A pictorial representation of the stabilizer bar is shown in Figure 4-12 with different hardpoints, where the location of the ‘ARB_bushing’ is specified via the connection point 61-62. The endpoint becomes the 'droplink_bar' point (63-64), connecting the stabilizer bar to the link, and the 'droplink_external' point (65-66) connecting link to the LCA. All these hardpoints are represented by joints or bushings, with their stiffness and damping rate defined in the property files.

![Figure 4-11: Bar profile of stabilizer bar](image)

![Figure 4-12: Beam element stabilizer bar (ADAMS/Chassis, 2005)](image)

### 4.6.2.8 Front Suspension Characteristics

The suspension characteristics in ADAMS/Chassis are based on the suspension geometry, the suspension compliance matrix, or both. Suspension geometry refers to the position and orientation of the suspension parts relative to the ground frame of reference as the suspension is articulated through its ride, role and steer motions. For example, the orientation of the wheel spindle axle is used to compute the toe and camber angles. The suspension compliance matrix refers to the incremental movements of the suspension due to the
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application of incremental forces at the wheel centres. ADAMS/Chassis computes the suspension compliance matrix at each solution position as the suspension is articulated through its motion. Characteristics such as suspension ride rate and aligning torque camber compliance are computed based on the compliance matrix. The compliance matrix for a system, $[C]$, is defined as the partial derivatives of displacements with respect to the applied forces:

$$[C] = \left[ \frac{\partial X}{\partial F} \right]$$

(4.9)

If a system is assumed to be linear, the compliance matrix can be used to predict the system movement due to force inputs as:

$$\{\Delta X\} = [C] \cdot \{\Delta F\}$$

(4.10)

From this perspective, matrix element $c_{ij}$ is the displacement of system degree of freedom $i$ due to a unit force at degree of freedom $j$. ADAMS/Chassis uses a $12 \times 12$ matrix relating the motion of the left and right wheel centres to units forces and torques applied to the wheel centres. This matrix has the form shown below:

$$
\begin{bmatrix}
X \text{ left wheel} & C(1, 1), C(1, 2) \ldots C(1, 12) \\
Y * & C(2, 1), C(2, 2) \ldots C(2, 12) \\
Z * & C(3, 1), C(3, 2) \ldots C(3, 12) \\
A X * & \ldots \\
A Y * & \ldots \\
A Z * & \ldots \\
X \text{ right wheel} & C(7, 1) \\
Y * & C(8, 1) \\
Z * & C(9, 1), C(9, 2) \\
A X * & \ldots \\
A Y * & \ldots \\
A Z * & C(12, 1), C(12, 2) \ldots C(12, 12) \\
\end{bmatrix}
\begin{bmatrix}
F X \text{ left wheel} \\
F Y \\
F Z \\
T X \\
T Y \\
T Z \\
F X \text{ right wheel} \\
F Y \\
F Z \\
T X \\
T Y \\
T Z \\
\end{bmatrix}
$$

For example, element $C(3,3)$ is the vertical motion of the left wheel centre due to a unit vertical force applied at the same wheel centre. Element $C(3,9)$ is the vertical motion of the left wheel centre due to a unit vertical force applied at the right wheel centre. For an independent suspension without a stabilizer bar, $C(3,9)$ is zero since a vertical force on the right wheel does not cause motion of the left wheel. The other elements of the compliance matrix are defined in the same manner.
The front suspension characteristics used in this model are listed in Table 4-5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsprung mass (total)</td>
<td>Kg</td>
<td>108.00</td>
</tr>
<tr>
<td>Unsprung cg height</td>
<td>mm</td>
<td>297.93</td>
</tr>
<tr>
<td>Roll centre height</td>
<td>mm</td>
<td>92.42</td>
</tr>
<tr>
<td>Wheel centre rise</td>
<td>mm</td>
<td>117.3E-03</td>
</tr>
<tr>
<td>Static loaded tyre radius</td>
<td>mm</td>
<td>299.14</td>
</tr>
<tr>
<td>Track width</td>
<td>mm</td>
<td>1557.73</td>
</tr>
<tr>
<td>Axle distance from vehicle cg</td>
<td>mm</td>
<td>1433.25</td>
</tr>
<tr>
<td>Toe angle</td>
<td>deg</td>
<td>6.51E-02</td>
</tr>
<tr>
<td>Caster angle</td>
<td>deg</td>
<td>7.65</td>
</tr>
<tr>
<td>Camber angle</td>
<td>deg</td>
<td>-5.55E-01</td>
</tr>
<tr>
<td>Kingpin angle</td>
<td>deg</td>
<td>7.76</td>
</tr>
<tr>
<td>Scrub radius</td>
<td>mm</td>
<td>12.28</td>
</tr>
<tr>
<td>Caster trail</td>
<td>mm</td>
<td>36.66</td>
</tr>
<tr>
<td>Toe change</td>
<td>deg/mm</td>
<td>-6.33E-03</td>
</tr>
<tr>
<td>Caster change</td>
<td>deg/mm</td>
<td>1.68E-02</td>
</tr>
<tr>
<td>Camber change</td>
<td>deg/mm</td>
<td>-2.18E-02</td>
</tr>
<tr>
<td>Roll camber coefficient</td>
<td>deg/deg</td>
<td>6.96E-01</td>
</tr>
<tr>
<td>Percentage roll steer</td>
<td>%</td>
<td>7.22</td>
</tr>
<tr>
<td>Track change</td>
<td>mm/mm</td>
<td>112.7E-03</td>
</tr>
<tr>
<td>Wheelbase change</td>
<td>mm/mm</td>
<td>-39.93E-03</td>
</tr>
<tr>
<td>Wheel rate</td>
<td>N/mm</td>
<td>22.63</td>
</tr>
<tr>
<td>Single bump wheel rate</td>
<td>N/mm</td>
<td>33.16</td>
</tr>
<tr>
<td>Ride rate</td>
<td>N/mm</td>
<td>20.85</td>
</tr>
<tr>
<td>Tyre rate</td>
<td>N/mm</td>
<td>264.81</td>
</tr>
<tr>
<td>Percent anti-dive/braking</td>
<td>%</td>
<td>19.60</td>
</tr>
<tr>
<td>Percent anti-lift/accel</td>
<td>%</td>
<td>0.00</td>
</tr>
<tr>
<td>Wheel hop natural freq.</td>
<td>Hz</td>
<td>11.61</td>
</tr>
</tbody>
</table>

Table 4-5: Front suspension characteristics

### 4.6.3 Rear Suspension

As the rear suspension in the Multi-body model is also based on SLA with coil spring, the modelling approach is similar to the one described earlier for the front suspension. Figure 4-13 shows the rear suspension system, with the geometrical hardpoints listed in Table 4-6. The rear suspension characteristics are listed in Table 4-7.
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Figure 4-13: Rear suspension (ADAMS/Chassis, 2005)

<table>
<thead>
<tr>
<th></th>
<th>UCA front</th>
<th>UCA rear</th>
<th>LCA front</th>
<th>LCA rear</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lower ball joint</td>
<td>Upper ball joint</td>
<td>Wheel centre</td>
<td>Contact patch</td>
</tr>
<tr>
<td>2</td>
<td>Spindle align</td>
<td>Damper upper</td>
<td>Spring seat lower</td>
<td>ARB bushing (left)</td>
</tr>
<tr>
<td>3</td>
<td>Damper lower</td>
<td>Droplink bar (right)</td>
<td>ARB bushing (right)</td>
<td>Droplink external (right)</td>
</tr>
<tr>
<td>4</td>
<td>Subframe front (left)</td>
<td>Subframe front (right)</td>
<td>Subframe rear (right)</td>
<td>Subframe rear (right)</td>
</tr>
<tr>
<td>5</td>
<td>Bumper upper (left)</td>
<td>Bumper upper (right)</td>
<td>Bumper lower (left)</td>
<td>Bumper lower (right)</td>
</tr>
<tr>
<td>6</td>
<td>Rebound lower (left)</td>
<td>Rebound lower (right)</td>
<td>Rebound upper (left)</td>
<td>Rebound upper (right)</td>
</tr>
</tbody>
</table>

Table 4-6: Rear suspension geometrical hardpoints (ADAMS/Chassis, 2005)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsprung mass (total)</td>
<td>Kg</td>
<td>100.26</td>
</tr>
<tr>
<td>Unsprung cg height</td>
<td>mm</td>
<td>298.73</td>
</tr>
<tr>
<td>Roll centre height</td>
<td>mm</td>
<td>142.74</td>
</tr>
<tr>
<td>Wheel centre rise</td>
<td>mm</td>
<td>21.18E-03</td>
</tr>
<tr>
<td>Static loaded tyre radius</td>
<td>mm</td>
<td>300.94</td>
</tr>
<tr>
<td>Track width</td>
<td>mm</td>
<td>1559.17</td>
</tr>
<tr>
<td>Axle distance from vehicle cg</td>
<td>mm</td>
<td>1595.48</td>
</tr>
<tr>
<td>Camber Angle</td>
<td>deg</td>
<td>-1.02</td>
</tr>
<tr>
<td>Camber change</td>
<td>deg/mm</td>
<td>-23.69E-03</td>
</tr>
<tr>
<td>Roll camber coefficient</td>
<td>deg/deg</td>
<td>663.4E-03</td>
</tr>
<tr>
<td>Percentage roll steer</td>
<td>%</td>
<td>-1.22</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Track change</th>
<th>mm/mm</th>
<th>175.2E-03</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheelbase change</td>
<td>mm/mm</td>
<td>-94.19E-03</td>
</tr>
<tr>
<td>Wheel rate</td>
<td>N/mm</td>
<td>23.76</td>
</tr>
<tr>
<td>Single bump wheel rate</td>
<td>N/mm</td>
<td>27.40</td>
</tr>
<tr>
<td>Ride rate</td>
<td>N/mm</td>
<td>21.80</td>
</tr>
<tr>
<td>Tyre rate</td>
<td>N/mm</td>
<td>264.82</td>
</tr>
<tr>
<td>Percent anti-dive/braking</td>
<td>%</td>
<td>95.54</td>
</tr>
<tr>
<td>Percent anti-lift/accel</td>
<td>%</td>
<td>55.21</td>
</tr>
</tbody>
</table>

Table 4-7: Rear suspension characteristics

4.6.4 Steering System

The vehicle model built in ADAMS/Chassis has a rack and pinion type steering system, (shown in Figure 4-14), with a variable gear ratio and power boost. The geometrical hardpoints of the steering system are listed in Table 4-6. The steering system contains various data blocks and part information, such as hardpoints, mass and inertial data, bushings, friction data, steering gear data, steering column data, steering damper data, power boost, torsion bar etc. All these data are stored in various property files in the sub-directories of the vehicle database. The steering gear and the steering column, which represent different sub-systems, have their data stored in the subsystems.tbl directory of the vehicle database, whereas the data for the power boost and torsion bar are contained in different property files, stored in steering_assists.tbl directory of the vehicle database. Some of the important data files required for modelling steering system are discussed below:

![Rack and Pinion Steering System](image)

Figure 4-14: Steering system (ADAMS/Chassis, 2005)
Multi-Body Model and Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Upper ujoint</th>
<th>Lower ujoint</th>
<th>Steering wheel centre</th>
<th>Tilt ujoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td></td>
<td>34</td>
<td>30</td>
<td>31</td>
</tr>
<tr>
<td>15</td>
<td>Pinion pivot</td>
<td>16</td>
<td>Centre of rack</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>42</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>44</td>
</tr>
</tbody>
</table>

Table 4-8: Steering system geometrical hardpoints (ADAMS/Chassis, 2005)

4.6.4.1 Steering Gear Data

The gear ratio is specified in mm of rack travel per revolution (360 degrees) of the steering wheel for the rack and pinion steering gears. The overall ratio refers to the total steering ratio and is used in the understeer calculations. As the steering system in this vehicle model employs a non-linear rack-and-pinion gear, a SPLINE function is used to define the pinion angle-to-rack displacement relationship. The non-linear gear SPLINE contains pinion rotation (deg) for the X values, and rack displacement (mm) for the Y values, where positive X values (for right turn) correspond to negative Y values, and vice versa. Figure 4-15 shows the plot of non-linear rack-and-pinion gear using a SPLINE.

![Nonlinear Rack and Pinion Gear](image)

Figure 4-15: Nonlinear gear SPLINE

The steering gear data chart for the advanced rack and pinion gear model contains information on the piston, torsion bar, and power assisted system specifics, apart from the basic information such as the gear ratio, overall steering system ratio, and the damping values. The torsion bar data, required by ADAMS/Chassis for the advanced gear, is the diameter, length and the stop angle. ADAMS/Chassis then builds a BEAM to model the torsion bar. The data for the advanced rack and pinion gear model is given below:
4.6.4.2 Steering Column Data

In ADAMS/Chassis, the steering column information is entered in a simple table, having options to specify compliance, damping and lash characteristics. This table is similar for all models, whether they are complex or simple. Lash in intermediate shaft slip joint can be specified in degrees. A spring, shown as the 'rate in slip joint' (N-mm/deg) is required in order to reach equilibrium. The steering column compliance (N-mm/deg) is the combined compliance of the entire steering system for the simple gear model. In this model, a linear column compliance method is used, but there is an option for specifying a non-linear column compliance, which can be a SPLINE, with X units of deg and Y units of N-mm.

4.6.5 Tyre / Wheel System

ADAMS/Chassis includes data of wheels and tyres in the front and rear wheel sub-system files, which contain information about mass and inertia of the wheel assembly, tyre/wheel axis orientation, force and moment scale factor, conicity and ply steer values, etc. The tyre property file is stored in the tires.tbl sub-directory of the vehicle database, which essentially contains various parameters required for tyre modelling. In ADAMS, tyre modelling is carried out using ADAMS/Tire Module, which is a set of shared object libraries that ADAMS/Solver calls through the DIFSUB, GFOSUB, GSESUB subroutines. These subroutines calculate the forces and moments that tyres exert on a vehicle as a result of the interaction between the tyres and the road surface.
4.6.5.1 Tyre Model

ADAMS/Tire has a range of tyre models for handling, ride, comfort, and durability analysis. Since the tyre model in this study has to suit the various handling requirements, the PAC2002 tyre model is used, which is based on the Pacejka 2002 version of the Magic Formula. The PAC2002 tyre model describes the tyre behaviour for rather smooth roads (road obstacle wavelengths longer than the tyre radius) up to the frequency of 8 Hz, which makes the model applicable for all generic vehicle handling and stability simulations.

4.6.5.2 STI Interface and Axis Systems

The PAC2002 model is linked to the ADAMS/Solver using the TYDEX STI (Standard Tyre Interface) conventions (ADAMS/Tyre, 2005). The STI interface passes the following information to the tyre model: position and velocities of the wheel centre, orientation of the wheel, and tyre and road parameters. The tyre model routine calculates the vertical load and slip quantities based on the position and speed of the wheel with respect to the road. The input for the Magic Formula consists of the wheel load \(F_z\), the longitudinal and lateral slip \((x, \alpha)\), and inclination angle \((\gamma)\) with the road. The output from the Magic Formula are the forces \((F_x, F_y)\) and moments \((M_x, M_y, M_z)\) in the contact point between the tyre and the road, which are transferred to the wheel centre and returned to the ADAMS/Solver through STI.

![Figure 4-16: ISO-C coordinate system (ADAMS/Tyre, 2005)](image)

For the transformation of variables between the wheel centre and tyre road contact point, TYDEX STI specifies the use of two different axes system, each having an ISO orientation, but with different origins. The ISO-C axis system is used for calculating translational and
rotational velocities, and for outputting the force and moments at the wheel hub. In the ISO-C axis system (Figure 4-16), the origin lies at the wheel centre, the longitudinal axis ‘x_c’ is parallel to the road and lies in the wheel plane (x_c-z_c plane), and the lateral axis ‘y_c’ is normal to the wheel plane and, therefore, parallel to the wheel’s spin axis.

![Figure 4-17: ISO-W coordinate system (ADAMS/Tyre, 2005)](image)

Forces and moments calculated by PAC2002 are generated in the ISO-W axis system. The origin of the ISO-W axis system (Figure 4-17) is the road contact-point, defined by the intersection of the wheel plane, the plane through the wheel carrier, and the road tangent plane. In this case, the longitudinal axis ‘x_w’ lies in the road tangent plane along the intersection of the wheel plane and the road tangent plane, the vertical axis ‘z_w’ is perpendicular to the road tangent plane and points upwards, whereas the lateral axis ‘y_w’ lies in the road tangent plane and is perpendicular to the x_w and z_w axes.

### 4.6.5.3 Tyre Model Inputs

As tyre forces and moments are calculated at the tyre road contact point, the position and force and motion variables are referred to the ISO-W axis system. The overall wheel kinematics is shown in Figure 4-18.

The tyre deflection \( \rho \) can be calculated using the free rolling tyre radius \( r_0 \), loaded tyre radius \( r_l \) (distance of the wheel centre to the contact point), and a correction for the tyre radius growth due to the rotational tyre speed:

\[
\rho = r_0 - r_l + q_{vl} \cdot r_0 \cdot \left( \frac{\omega \cdot r_b}{V_0} \right)^2
\]  

(4.11)
The effective rolling radius $r_c$, used to calculate the rotational speed of the tyre, can be represented in relation to the tyre deflection as:

$$r_c = r_0 + q_{v1} \cdot r_0 \cdot \left( \frac{\omega \cdot r_0}{V_0} \right)^2 - \rho_{t,0} \cdot \left[ D_{R_{eff}} \cdot \arctan \left( B_{R_{eff}} \cdot \rho^d \right) + F_{R_{eff}} \cdot \rho^d \right]$$  \hspace{1cm} (4.12)

where, $\rho_{t,0}$ is the nominal tyre deflection, expressed as:

$$\rho_{t,0} = \frac{F_{t,0}}{C_z}$$  \hspace{1cm} (4.13)

And, $\rho^d$ is the dimensionless radial tyre deflection, expressed as:

$$\rho^d = \frac{\rho}{\rho_{t,0}}$$  \hspace{1cm} (4.14)

In the absence of different coefficients, tyre deflection $\rho$ can be simply defined as the difference between the free tyre radius $r_0$ and the loaded tyre radius $r_l$ as:

$$\rho = r_0 - r_l$$  \hspace{1cm} (4.15)

The longitudinal slip velocity $V_{sx}$ in the contact point can be defined using the longitudinal speed $V_x$, the wheel rotational velocity $\omega$, and the effective rolling radius $r_c$ as:

$$V_{sx} = V_x - \omega \cdot r_c$$  \hspace{1cm} (4.16)
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The lateral slip velocity $V_{sy}$ is equal to the lateral speed in the contact point with respect to the road tangent plane, thus:

$$V_{sy} = V_y$$

(4.17)

The practical slip quantities such as longitudinal slip $\kappa$ and slip angle $\alpha$ are calculated with the above slip velocities in the contact point as:

$$\kappa = \frac{V_{sy}}{|V_x|}$$

(4.18)

$$\tan \alpha = \frac{V_{sy}}{|V_x|}$$

(4.19)

In the vertical direction, the MF tyre is modelled as a linear spring and damper, having one point of contact (C) with the road, which is valid for road obstacles with a wavelength larger than the tyre radius (refer Figure 4-18). The normal load $F_z$ of the tyre is calculated using the linear vertical tyre stiffness $C_z$ and tyre damping $K_z$, according to:

$$F_z = C_z \cdot \rho + K_z \cdot \dot{\rho}$$

(4.20)

The vertical stiffness can also be represented as tyre load-deflection characteristics, instead of a linear vertical tyre stiffness $C_z (= q_{rel} \cdot F_{\rho0} / r_o)$.

4.6.5.4 Magic Formula Characteristics

In the Magic Formula, mathematical functions are used to depict steady-state tyre characteristic under a range of operating conditions, which primarily includes:

- Pure cornering slip conditions: cornering with a free rolling tyre
- Pure longitudinal slip conditions: braking or driving the tyre without cornering
- Combined slip conditions: simultaneous cornering and longitudinal slip

For pure slip conditions, the lateral force $F_y$ as a function of the slip angle $\alpha$, and the longitudinal force $F_x$ as a function of longitudinal slip $\kappa$, have a similar shape (sine - arctangent). The self-aligning moment $M_z$ is calculated as a product of the lateral force $F_y$ and the offset of lateral force from the contact point, called pneumatic trail $t$, added with the
residual moment $M_{zr}$. The pneumatic trail as a function of the slip angle has a cosine shape. The general shape of these curves under pure slip conditions is shown in Figure 4-19.

Figure 4-19: Characteristic curves under pure slip conditions (Bakker et al, 1989)

The Magic Formula used to describe the above characteristic curves, for a given load and camber, can be expressed in the sine and cosine form as:

$$y(x) = D \cdot \sin\left[ C \cdot \arctan \left\{ B \cdot x - E \cdot \left( B \cdot x - \arctan(B \cdot x) \right) \right\} \right]$$  \hspace{1cm} (4.21)

$$y(x) = D \cdot \cos\left[ C \cdot \arctan \left\{ B \cdot x - E \cdot \left( B \cdot x - \arctan(B \cdot x) \right) \right\} \right]$$  \hspace{1cm} (4.22)

The sine version of the Magic Formula (4.21) typically represents an anti-symmetric curve, passing through the origin ($x = y = 0$), and which rises with an increase in slip, reaching a maximum value and subsequently tend to a horizontal asymptote. To include the effects of ply-steer and conicity, and possibly rolling resistance, an offset $S_H$ and $S_V$ are added in the equation, which causes $F_x$ and $F_y$ not to pass through the origin. As a result of this offset, a new set of coordinates $Y(X)$ arise, as shown in Figure 4-20. Hence:

$$Y(X) = y(x) + S_V$$  \hspace{1cm} (4.23)

$$x = X + S_H$$  \hspace{1cm} (4.24)

where $X$ represents the primary input variables, which can be longitudinal slip or slip angle, and $Y$ represents the output variables, which can be longitudinal force or lateral force.
The difference in the shape of the characteristic curves are determined by several factors, such as B, C, D and E. D-factor is the peak factor and determines the peak of the characteristic curve, with respect to the central x-axis and for $C \geq 1$.

At high slip, force in equation (4.21) converges to a value, given in the form of horizontal asymptote $y_a$. Thus:

$$y_a = D \cdot \sin \left( \pm C \frac{\pi}{2} \right) \quad (4.25)$$

The shape factor $C$ controls the limiting value of the curve and thereby determines the shape of the resulting curve. The shape factor may be computed from the peak factor D and the height of the horizontal asymptote $y_a$ as:

$$C = 1 \pm \left( 1 - \frac{2}{\pi} \cdot \arctan \frac{y_a}{D} \right) \quad (4.26)$$

B is called the stiffness factor and controls the slope of the curve at the origin. The product $BCD$ is the slope at the origin, which can be referred to as the longitudinal slip stiffness or cornering stiffness.

E is the curvature factor, which can be expressed in terms of B and C and the slip location $x_m$ at the curve peak

$$E = \frac{B \cdot x_m - \tan \left\{ \pi / (2 \cdot C) \right\}}{B \cdot x_m - \arctan (B \cdot x_m)} \quad (\text{If } C > 1) \quad (4.27)$$
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The asymmetry caused by the effect of camber on the lateral force curve can be accommodated by making the curvature factor dependent on the sign of the slip value:

\[ E = E_0 + \Delta E \cdot \text{sgn}(x) \]  \hspace{1cm} (4.28)

This can also help in addressing the asymmetry in the \( F_x \) versus \( \kappa \) variation, when comparing tractive and braking forces.

![Figure 4-21: Cosine version of the magic formula (Pacejka, 2006)](image)

The cosine version of the Magic Formula, as expressed in equation (4.22), represents a hill-shaped curve, where the peak decays on both sides with an increasing slip angle. The cosine version (Figure 4-21) is used to generate pneumatic trail and also residual torque, both of which decays with the slip angle. The pneumatic trail is expressed as:

\[ t(\alpha_t) = D_t \cdot \cos \left[ C_t \cdot \arctan \left( B_t \cdot \alpha_t - E_t \cdot \left( B_t \cdot \alpha_t - \arctan(B_t \cdot \alpha_t) \right) \right) \right] \]  \hspace{1cm} (4.29)

\[ \alpha_t = \tan \alpha + S_{\text{lit}} \]  \hspace{1cm} (4.30)

where, \( S_{\text{lit}} \) represents the horizontal shift of the pneumatic trail.

And, residual torque \( M_{\omega} \) is expressed as:

\[ M_{\omega}(\alpha_t) = D_t \cdot \cos \left[ \arctan(B_t \cdot \alpha_t) \right] \]  \hspace{1cm} (4.31)

\[ \alpha_t = \tan \alpha + S_{\text{lit}} \]  \hspace{1cm} (4.32)

where, \( S_{\text{lit}} \) represents the horizontal shift of the residual torque, which corresponds to the slip angle, where the side force diminishes.
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![Graph of aligning moment along with residual torque](image)

Figure 4-22: Plot of aligning moment along with residual torque (Pacejka, 2006)

The aligning moment $M_z$ can then be obtained by multiplying the side force $F_y$ with the pneumatic trail $t$, whilst adding the usually small residual torque $M_{\sigma r}$. The graph of the aligning moment along with its relation with pneumatic trail and residual torque is shown in Figure 4-22.

$$M_z = -t \cdot F_y + M_{\sigma r} \tag{4.33}$$

In the cosine version, the factor $D$ represents the peak value. $C$ is a shape factor, which determines the level of the horizontal asymptote $y_a$ (the limiting value). The factor $C$ can be expressed as:

$$C = \frac{2}{\pi} \arccos \frac{y_a}{D} \tag{4.34}$$

The factor $B$ here influences the curvature of the peak and the factor $E$ influences the shape at large value of slip, and governs the point $x_0$, where the curve becomes zero (intersects with the x axis). The factor $E$ can be expressed as:

$$E = \frac{B \cdot x_0 - \tan \left\{ \pi / (2 \cdot C) \right\}}{B \cdot x_0 - \arctan (B \cdot x_0)} \quad \text{(If } C > 1) \tag{4.35}$$
In the earlier version of Magic Formula (Pacejka, 1993), the tyre characteristics for combined slip was modeled using physically based formula. The latest version of Magic Formula deploys a more empirical method using weighting function, which when multiplied with the original pure slip functions produce the interaction effects of $\kappa$ on $F_y$ and of $\alpha$ on $F_x$. The weighting functions have a hill shape, which is represented by the cosine version of the Magic Formula.

$$G = D \cdot \cos \left[ \arctan (B \cdot x) \right]$$

(4.36)

where $G$ is the resulting weighing factor and $x$ is either $\kappa$ or $\tan \alpha$. $D$ represents the peak value, $C$ determines the horizontal asymptote of the hill’s base, and $B$ determines the sharpness of the hill and is mainly responsible for the shape of the weighting functions. In addition, factor $E$ can be added in order to improve the approximation, in particular at large values of slip, so that the weighing functions remain positive for all slip conditions.

The weighing function assumes the value of unity in the case of pure slip ($\kappa$ or $\alpha$ equal to 0). For combined slip condition, for example, when a brake slip $\kappa$ is introduced to a tyre cornering at a slip angle $\alpha$, the relevant weighting function for $F_y$ (expressed as function of $\kappa$ in this case) may first show a slight increase in magnitude (exceeds unity at low slip), but soon reaches its peak, after which a continuous descent occurs. The same case can be seen for longitudinal force, where the weighting function initially becomes more than unity at low values of slip angle, but eventually decreases with an increase in the slip angle, resulting in a decrease in the longitudinal force with increasing in slip.

### 4.6.5.5 Magic Formula Steady State Equations

The steady-state equations for PAC2002 the Magic Formula tyre model are listed here for longitudinal and lateral forces and the aligning moment, each at pure and combined slip conditions. Further details of the equation set can be found in chapter 4 of (Pacejka, 2006).

#### 4.6.5.5.1 Longitudinal Force at Pure Slip

$$F_x = F_{x0} (\kappa, F_z, \gamma)$$

(4.37)

$$F_{x0} = D_x \cdot \sin \left[ C_x \cdot \arctan \left\{ B_x \cdot \kappa_x - E_x \cdot (B_x \cdot \kappa_x - \arctan (B_x \cdot \kappa_x)) \right\} \right] + S_{vx}$$

(4.38)
Multi-Body Model and Simulation Results

\[ \kappa_z = \kappa + S_{hz} \]  

(4.39)

where, the different factors can be expressed in terms of the coefficients as:

\[ C_z = p_{cz1} \]  

(>0)  

(4.40)

\[ D_z = \mu_z \cdot F_z \]  

(>0)  

(4.41)

\[ B_z = K_{xz} / (C_z \cdot D_z) \]  

(4.42)

\[ \mu_z = \left( p_{dz1} + p_{dz2} \cdot df_z \right) \cdot \left( 1 - p_{dz3} \cdot \gamma_z^2 \right) \cdot \lambda_{xz}^* \]  

(>0)  

(4.43)

\[ E_z = \left( p_{ez1} + p_{ez2} \cdot df_z + p_{ez3} \cdot df_z^2 \right) \cdot \left\{ 1 - p_{ez4} \cdot \text{sgn}(\kappa_z) \right\} \]  

(\leq 1)  

(4.44)

where \( df_z \) is the non-dimensional vertical load increment, expressed as ratio of nominal load:

\[ df_z = \frac{F_z - F_{z0}}{F_{z0}} \]  

(4.45)

The longitudinal slip stiffness \( K_{xz} \) is given by:

\[ K_{xz} = F_z \cdot \left( p_{xz1} + p_{xz2} \cdot df_z \right) \cdot \exp \left( p_{xz3} \cdot df_z \right) \]  

(4.46)

\[ (K_{xz} = B_z \cdot C_z \cdot D_z = \partial F_{z0} / \partial \kappa_z \text{ at } \kappa_z = 0) \]

\[ S_{hz} = (p_{hz1} + p_{hz2} \cdot df_z) \]  

(4.47)

\[ S_{yz} = F_z \cdot \left( p_{yz1} + p_{yz2} \cdot df_z \right) \]  

(4.48)

4.6.5.5.2 Lateral Force at Pure Side Slip

\[ F_y = F_{y0} (\alpha, \gamma, F_z) \]  

(4.49)

\[ F_{y0} = D_y \cdot \sin \left[ C_y \cdot \arctan \left\{ B_y \cdot \alpha_y - E_y \cdot \left\{ B_y \cdot \alpha_y - \arctan \left( B_y \cdot \alpha_y \right) \right\} \right\} \right] + S_{vy} \]  

(4.50)

\[ \alpha_y = \alpha + S_{ly} \]  

(4.51)
Multi-Body Model and Simulation Results

where, the different factors can be expressed in terms of the coefficients as:

\[ C_y = p_{Cy1} \quad (>0) \]  \hspace{1cm} (4.52)

\[ D_y = \mu_y \cdot F_z \]  \hspace{1cm} (4.53)

\[ \mu_y = \left( p_{Dy1} + p_{Dy2} \cdot df_z \right) \cdot \left( 1 - p_{Dy3} \cdot \gamma^2 \right) \cdot \lambda_{py} \quad (>0) \]  \hspace{1cm} (4.54)

\[ E_y = \left( p_{Ey1} + p_{Ey2} \cdot df_z \right) \cdot \left\{ 1 - \left( p_{Ey3} + p_{Ey4} \cdot \gamma \right) \cdot \text{sgn}(\alpha_y) \right\} \quad (\leq 1) \]  \hspace{1cm} (4.55)

The cornering stiffness \( K_{y\alpha} \) is given by:

\[ K_{y0} = p_{ky1} \cdot F_{z0} \cdot \sin \left[ 2 \cdot \arctan \left( F_z / \left( p_{ky2} \cdot F_{z0} \right) \right) \right] \]  \hspace{1cm} (4.56)

\[ K_{y\alpha} = K_{y0} \cdot \left( 1 - p_{ky3} \cdot \gamma \right) \]  \hspace{1cm} (4.57)

\[ \left( K_{y\alpha} = B_y \cdot C_y \cdot D_y = \partial F_{z0} / \partial \alpha_y \right. \text{ at } \alpha_y = 0 \]  \hspace{1cm} (4.58)

\[ B_y = K_{y\alpha} / \left( C_y \cdot D_y \right) \]  \hspace{1cm} (4.59)

\[ S_{by} = \left( p_{by1} + p_{by2} \cdot df_z \right) + p_{by3} \cdot \gamma \]  \hspace{1cm} (4.60)

\[ S_{vy} = F_z \cdot \left\{ \left( p_{vy1} + p_{vy2} \cdot df_z \right) + \left( p_{vy3} + p_{vy4} \cdot df_z \right) \cdot \gamma \right\} \]  \hspace{1cm} (4.61)

Also, the camber stiffness \( K_{y\gamma0} \) is given by:

\[ K_{y\gamma0} = p_{by3} \cdot K_{y0} \cdot F_z \cdot \left( p_{vy3} + p_{vy4} \cdot df_z \right) \]  \hspace{1cm} (4.62)

4.6.5.5.3 Aligning Moment at Pure Slip

\[ M_z = M_{z0} (\alpha, \gamma, F_z) \]  \hspace{1cm} (4.63)

\[ M_{z0} = -t \cdot F_{z0} + M_{\pi} \]  \hspace{1cm} (4.64)

where, the equation for the pneumatic trail \( t \) is:
Multi-Body Model and Simulation Results

\[ t(\alpha_t) = D_t \cdot \cos \left[ C_t \cdot \arctan \left\{ B_t \cdot \alpha_t - E_t \cdot \left( B_t \cdot \alpha_t - \arctan \left( B_t \cdot \alpha_t \right) \right) \right\} \right] \]  \hspace{1cm} (4.64)

\[ \alpha_t = \alpha + S_{llt} \]  \hspace{1cm} (4.65)

And, the equation for residual torque \( M_{\alpha} \) is:

\[ M_{\alpha}(\alpha_t) = D_t \cdot \cos \left[ \arctan \left( B_t \cdot \alpha_t \right) \right] \]  \hspace{1cm} (4.66)

\[ \alpha_t = \alpha + S_{llt} \]  \hspace{1cm} (4.67)

where, the different factors can be expressed in terms of coefficients as:

\[ B_t = \left( q_{bl1} + q_{bl2} \cdot df_z + q_{bl3} \cdot df_z^2 \right) \cdot \left( 1 + q_{bl4} \cdot \gamma + q_{bl5} \cdot |\gamma| \right) \]  \hspace{1cm} (4.68)

\[ C_t = q_{cl1} \]  \hspace{1cm} (4.69)

\[ D_t = F_z \cdot \left( q_{dl1} + q_{dl2} \cdot df_z \right) \cdot \left( 1 + q_{dl3} \cdot \gamma + q_{dl4} \cdot \gamma^2 \right) \cdot \frac{r_0}{F_{l0}} \]  \hspace{1cm} (4.70)

\[ E_t = \left( q_{el1} + q_{el2} \cdot df_z + q_{el3} \cdot df_z^2 \right) \cdot \left( 1 + q_{el4} + q_{el5} \cdot \gamma \right) \cdot \left( \frac{2}{\pi} \right) \cdot \arctan \left( B_t \cdot C_t \cdot \alpha_t \right) \]  \hspace{1cm} (4.71)

\[ S_{llt} = q_{llt1} + q_{llt2} \cdot df_z + \left( q_{llt3} + q_{llt4} \cdot df_z \right) \cdot \gamma \]  \hspace{1cm} (4.72)

\[ B_t = q_{bt9} + q_{bt10} \cdot B_t \cdot C_t \]  \hspace{1cm} (4.73)

\[ D_t = F_z \cdot \left[ \left( q_{dt9} + q_{dt7} \cdot df_z \right) + \left( q_{dt8} + q_{dt9} \cdot df_z \right) \cdot \gamma \right] \cdot r_0 \]  \hspace{1cm} (4.74)

The aligning moment stiffness can be approximated as:

\[ K_{z0} = -t \cdot K_{yz} \left( \approx -\frac{\partial M_{z}}{\partial \alpha} \text{ at } \alpha = 0 \right) \]  \hspace{1cm} (4.75)

4.6.5.5.4 Longitudinal Force at Combined Slip

\[ F_x = F_{x0} \cdot G_{xz}(\alpha, \kappa, F_z) \]  \hspace{1cm} (4.76)

where, \( G_{xz} \) is the weighting function of the longitudinal force at pure slip, expressed as:
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$$G_{x\alpha} = \cos \left[ C_{x\alpha} \cdot \arctan \left\{ B_{x\alpha} \cdot \alpha_s - E_{x\alpha} \cdot \left( B_{x\alpha} \cdot \alpha_s - \arctan \left( B_{x\alpha} \cdot \alpha_s \right) \right) \right\} / G_{x\alpha 0} \right] (\geq 0) \quad (4.77)$$

$$G_{x\alpha 0} = \cos \left[ C_{x\alpha 0} \cdot \arctan \left\{ B_{x\alpha 0} \cdot S_{l\alpha x} - E_{x\alpha} \cdot \left( B_{x\alpha 0} \cdot S_{l\alpha x} - \arctan \left( B_{x\alpha 0} \cdot S_{l\alpha x} \right) \right) \right\} \right] \quad (4.78)$$

$$\alpha_s = \alpha + S_{l\alpha x} \quad (4.79)$$

$$B_{x\alpha} = r_{b\alpha} \cdot \cos \left[ \arctan \left( r_{b\alpha} \cdot \kappa \right) \right] \quad (\geq 0) \quad (4.80)$$

$$C_{x\alpha} = r_{c\alpha} \quad (4.81)$$

$$E_{x\alpha} = r_{e\alpha} + r_{e\alpha 2} \cdot df_z \quad (\leq 1) \quad (4.82)$$

$$S_{l\alpha x} = r_{l\alpha x} \quad (4.83)$$

### 4.6.5.5 Lateral Force at Combined Slip

$$F_y = F_{y0} \cdot G_{yx} \left( \alpha, \kappa, \gamma, F_z \right) + S_{vyx} \quad (4.84)$$

where, $S_{vyx}$ is the 'k' induced side force, and $G_{yx}$ is the weighting function of the lateral force at pure slip, which can be written as:

$$G_{yx} = \cos \left[ C_{yx} \cdot \arctan \left\{ B_{yx} \cdot \kappa_s - E_{yx} \cdot \left( B_{yx} \cdot \kappa_s - \arctan \left( B_{yx} \cdot \kappa_s \right) \right) \right\} / G_{yx 0} \right] (\geq 0) \quad (4.85)$$

$$G_{yx 0} = \cos \left[ C_{yx 0} \cdot \arctan \left\{ B_{yx 0} \cdot S_{l\yx} - E_{yx} \cdot \left( B_{yx 0} \cdot S_{l\yx} - \arctan \left( B_{yx 0} \cdot S_{l\yx} \right) \right) \right\} \right] \quad (4.86)$$

$$\kappa_s = \kappa + S_{l\yx} \quad (4.87)$$

$$B_{yx} = r_{b\yx} \cdot \cos \left[ \arctan \left( r_{b\yx} \cdot \left( \alpha - r_{b\yx 3} \right) \right) \right] \quad (\geq 0) \quad (4.88)$$

$$C_{yx} = r_{c\yx} \quad (4.89)$$

$$E_{yx} = r_{e\yx} + r_{e\yx 2} \cdot df_z \quad (\leq 1) \quad (4.90)$$

$$S_{l\yx} = r_{l\yx} + r_{l\yx 2} \cdot df_z \quad (4.91)$$
\[ S_{vyx} = D_{vyx} \cdot \sin \left( r_{vy5} \cdot \arctan \left( r_{vy6} \cdot \kappa \right) \right) \] (4.92)

\[ D_{vyx} = \mu_y \cdot F_z \cdot \left( r_{vy1} + r_{vy2} \cdot df_z + r_{vy3} \cdot \gamma \right) \cdot \cos \left( \arctan \left( r_{vy4} \cdot \alpha \right) \right) \] (4.93)

### 4.6.5.5.6 Aligning Moment at Combined Slip

\[ M_z = -t \cdot F_y + M_{nz} + s \cdot F_z \] (4.94)

\[ t \left( \alpha_{t,eq} \right) = D_z \cdot \cos \left[ C_1 \cdot \arctan \left( B_1 \cdot \alpha_{t,eq} - E_1 \cdot \left( B_1 \cdot \alpha_{t,eq} - \arctan \left( B_1 \cdot \alpha_{t,eq} \right) \right) \right] \] (4.95)

\[ F_y = F_y \cdot S_{vyx} \] (4.96)

\[ M_{nz} = M_{nz} \left( \alpha_{t,eq} \right) = D_z \cdot \cos \left[ \arctan \left( B_1 \cdot \alpha_{t,eq} \right) \right] \] (4.97)

\[ s = r_0 \cdot \left( s_{s2} + s_{s2} \cdot \left( F_y / F_{s0} \right) + \left( s_{s2} + s_{s2} \cdot df_z \right) \cdot \gamma \right) \] (4.98)

\[ \alpha_{t,eq} = \arctan \sqrt{\tan^2 \alpha_t + \left( \frac{K_{sx}}{K_{sy}} \right)^2 \cdot \kappa^2 \cdot \text{sgn}(\alpha_t)} \] (4.99)

\[ \alpha_{t,eq} = \arctan \sqrt{\tan^2 \alpha_t + \left( \frac{K_{sx}}{K_{sy}} \right)^2 \cdot \kappa^2 \cdot \text{sgn}(\alpha_t)} \] (4.100)

### 4.7 Vehicle Simulation

In line with the project aims and objectives, the performance of the theoretical vehicle model is compared and verified against the detailed multi-body dynamics model. A number of transient handling manoeuvres are first conducted in ADAMS Chassis, which has a large library of standard events, covering a wide range of ride, handling and durability tests. The same transient manoeuvres are then carried out using the intermediate vehicle model (described in chapter 3), so as to compare the handling behaviour of the two different modelling approaches. In this study, the selected handling manoeuvres were restricted to the open loop test environment, with input variables as vehicle speed and steering hand wheel angle. This allows repeating the same manoeuvres in the intermediate vehicle model, using...
the same steering input, and without resorting to any controller for the driver’s steering or braking input. However, some of the tests are performed in the cruise control mode — a condition which is replicated in the theoretical vehicle model with the use of a PID controller for maintaining constant vehicle speed. Out of the total number of tests conducted, the simulations involving double lane change, single lane change and sinusoidal steering are carried out under the cruise control mode, where as J-turn, Fishhook manoeuvre, and step-steer are performed with traction switched off.

The theoretical vehicle modelling, as mentioned in chapter 3, initially involved development of basic vehicle models, such as a 2-DOF bicycle model and a 3-DOF non-linear model, before gradually shifting to a relatively complex intermediate vehicle model with 10-DOF. The non-linear 3-DOF model (section 3.4) was also used here along with the 10-DOF intermediate model (section 3.5) and the detailed multi-body model (section 4.2) for comparative simulation runs. The inclusion of the 3-DOF model was primarily to establish a basic comparison with the 10-DOF model, which eventually helps studying the effects of adding more complexity into a simple vehicle model. The model was built in MATLAB / Simulink environment, and included some non-linearities in terms of inclined roll axis, front and rear stabilizer bars and a normalised Magic Formula tyre model (Milliken and Milliken, 1995). The basic three degrees of freedom of sprung mass comprises translation in the lateral direction, and rotations in yaw and roll directions.

The 3-DOF model does not possess any degree of freedom in the longitudinal direction. Instead it has a pre-specified longitudinal speed as a constant parameter throughout the course of a simulation run. Thus, it can easily be used for handling tests which are carried out under the cruise control mode, such as double lane change, single lane change and a sinusoidal steering manoeuvre. However, some other manoeuvres are not deemed suitable with a 3-DOF model, such as a J-Turn, step-steer and Fishhook manoeuvres, which are carried out with switched off traction, leaving the vehicle in a deceleration mode throughout the manoeuvre.

The static vehicle characteristics of both 10-DOF and 3-DOF models are matched by obtaining a SVC plot from the ADAMS/Chassis. Additionally, in the ADAMS model, the mass and inertia of both cargo and occupants is included in the loading sub-system file. This effect is accounted in the theoretical vehicle modelling by incorporating additional sprung mass weight distribution at the front and the rear.
4.7.1 Vehicle Handling Tests

4.7.1.1 Single-Lane Change

Lane change is one of the most important handling manoeuvres, which is performed mostly in a simulation environment in an early phase of vehicle development, so that the behaviour of a vehicle can be studied in the same way as in the practical tests on road. The lane change simulation is used to evaluate ease of control and stability of a vehicle in a single lane change manoeuvre. The physical test is conducted by setting up a pylon course with 10 foot wide lanes. Two cones are placed at the entrance and two cones are placed at the exit. The vehicle drives in a straight line through the entrance cones, changes into the adjacent lane, and then drives straight through the exit cones. The throttle is controlled by the driver to maintain constant speed during the event. This test can be run at any speed and any lateral acceleration level, but the lateral acceleration should be kept below a level, where the vehicle commences to slide.

In the present simulation, the lane change was carried out by keeping the vehicle at a constant speed of 100 km/h and the simulation was performed for a duration of 10 sec. The width of the lane change course was set as 3.66 m (12 feet) and the length of the lane change course was set as 10.67 m (35 feet). The prescribed steering input used in the manoeuvre is shown in Figure 4-23.

The vehicle response plots for lateral acceleration, yaw rate, roll angle, side-slip angle, and vehicle path location for all the three models are shown in (Figure 4-23). It can be seen that the responses in all the cases are more or less closely matched. The lateral acceleration and yaw rate of the 10-DOF model is slightly larger than that of the ADAMS model, where as the 3-DOF model responses exhibit large overshoots. In the vehicle path location plot (last graph of Figure 4-23), the 10-DOF model is shown to have more under-steer behaviour in relation with the ADAMS model. The 3-DOF model, on the other hand, exhibits largely under-steer behaviour. The lack of proper representation of suspension and other non-linearities is quite apparent in these plots.
Multi-Body Model and Simulation Results

Figure 4-23: Vehicle handling responses for single-lane change

Figure 4-24: Lateral force plots for single-lane change
Multi-Body Model and Simulation Results

The lateral force plot is shown in Figure 4-24, where a reasonably close agreement can be observed between the various models. The 3-DOF model, because of its large roll, seems to exhibit higher lateral load transfer, resulting in higher peaks in both wheel jounce and rebound. The 10-DOF model, although exhibits higher peaks compared to multi-body model in wheel rebound, the lateral load transfer is less as compared to multi-body model. This could be partly attributed to the difference in the suspension characteristics of the two models. Also, the lateral tyre forces in both 3-DOF and 10-DOF model, unlike the ADAMS model, commences from zero, which could be attributed to the fact that horizontal shift was not considered in the Magic Formula lateral force equations, in both the models. The difference in the lateral force curves in the steady-state part of the manoeuvre, clearly explains this effect.

The vertical force plots in Figure 4-25 show close overall behaviour between the ADAMS model and the 10-DOF models, with some differences between the two during the front wheel rebound. The 3-DOF model again shows higher peaks in comparison to other the two models, and its inadequate representation in the lateral and roll stiffness can be easily observed. Overall, the results of the three models are closely matched, with the plots of the 10-DOF model showing close proximity with the general behaviour of the ADAMS model.

![Figure 4-25: Vertical force plots for single-lane change](image-url)
4.7.1.2 Double-Lane Change

The double-lane-change manoeuvre is an inherently subjective test, standardised by the International Organization for Standardization (ISO), with recommended standards under ISO 3888 (BS ISO 3888-1, 1999). The primary objective of the test is to determine the transient response behaviour of the vehicle subjected to one period of quasi-harmonic variation of the vehicle steering angle. It essentially represents a method to determine transient road-holding ability, with which closed loop control can be tested in a situation similar to that occurring in real traffic.

To perform Double Lane Change testing the vehicle has to be driven at a pre-specified speed through the course shown in Figure 4-26. The dimensions of the track are given in Table 4-3. The track resembles that of a vehicle performing a rather tight overtaking manoeuvre, while the driver is free to correct the steer-angle at will. In the present study the test is carried out in an open-loop fashion, with a prescribed steering input (refer first graph of Figure 4-23). The vehicle longitudinal speed is maintained at 100 Km/h, using the traction controller in the ADAMS model, and the same is achieved using a PID controller in the 10-DOF model.

![Diagram](image)

**Figure 4-26: Double lane-change track and designation of sections**

<table>
<thead>
<tr>
<th>Section</th>
<th>Length</th>
<th>Lane offset</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>-</td>
<td>1.1 x vehicle width + 0.25</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>3.5</td>
<td>1.2 x vehicle width + 0.25</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>-</td>
<td>1.3 x vehicle width + 0.25</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>-</td>
<td>1.3 x vehicle width + 0.25</td>
</tr>
</tbody>
</table>

**Table 4-9: Dimensions of the double lane-change track (BS ISO 3888-1, 1999)**
Multi-Body Model and Simulation Results

The response plots for vehicle lateral acceleration, yaw rate, roll angle, side slip angle, and vehicle path location are shown in Figure 4-23, where good agreement between the various models can clearly be seen. The lateral acceleration and yaw rate peaks of the 3-DOF model are somewhat higher than the ADAMS and 10-DOF models, but with its limited degrees of freedom and lack of proper suspension and steering representation, the results are quite reasonable. The results of the 10-DOF model match those of the ADAMS model quite closely, though exhibiting an under-steer behaviour similar to the single lane change plot. Furthermore, a minor response lag appears in the 10-DOF model with respect to the results of the ADAMS model, as seen in the plots of the lateral acceleration and yaw rate. The steering system in the 10-DOF model does not have the detail representation of the ADAMS model for non-linear bushing, viscous damping and friction. These characteristics can lead to transient response lags in the vehicle behaviour – an effect which can be observed later in the sinusoidal steering test.

![Figure 4-27: Vehicle handling responses for double-lane change](image)

The lateral force plots (Figure 4-28) also show reasonable agreement between the three models. Similar to single lane change manoeuvre, the 10-DOF model shows slightly higher peaks in both front and rear wheels rebound as compared to multi-body model. Also, the
steady-state shift in the lateral force can be again observed in the ADAMS plot from the start of the manoeuvre. Though easy to incorporate, the horizontal and vertical shifts were not considered, while using the Magic Formula Coefficients in 10-DOF model, which explains the reason for this shift. The 3-DOF model shows high peaks at the turns, which again can be related to the higher responses in the lateral and roll direction.

![Lateral force plots for double-lane change](image)

**Figure 4-28: Lateral force plots for double-lane change**

In the vertical force plot, shown in Figure 4-25, the 10-DOF model again matches very well with the ADAMS model. Considering the different jounce and rebound characteristics as well as roll stiffness of the two models, the results are reasonably close. The 3-DOF model again shows higher peaks, as a result of a higher lateral load transfer. In general, the 10-DOF model follows the much detailed ADAMS model very closely in this manoeuvre, which again highlights the significance of adding details in the basic model like the 3-DOF one, by incorporating unsprung mass dynamics, additional degrees of freedom, and adequate non-linearities, particularly while defining suspension characteristics.
Multi-Body Model and Simulation Results

4.7.1.3 Sinusoidal Steering

The sinusoidal steering test determines the vehicle response behaviour with harmonic steering motion, while driving the vehicle at a constant speed. In this study, simulations were conducted by driving the vehicle at a constant speed of 70 Km/h, with a sinusoidal steering input of magnitude 90 degrees at the steering hand wheel.

The response plots for vehicle lateral acceleration, yaw rate, roll angle, side slip angle, and vehicle path location are shown in Figure 4-30. Once again the 10-DOF model shows a close match with the ADAMS model as shown in the yaw rate, lateral acceleration and the roll angle peaks. In Figure 4-23 some under-steer behaviour of the 10-DOF model is again visible. However, the point to observe here is the phase lag between the 10-DOF and the ADAMS model in some of the plots; particularly the yaw rate and side slip. The ADAMS model, with its steering compliances, damping, and roll steer characteristics, seems to add some transient lag in the vehicle response behaviour. This behaviour, previously observed in the double lane change, now becomes quite clear with sinusoidal steering input. The 10-DOF model counters for the first order tyre lag, using relaxation length, but does not have a
provision to account for the steering non-linearities except for the Ackerman steer and the variable steer ratio. The influence of the steering related lag can be studied in isolation in the future by conducting a more severe steering input such as a frequency sweep test. However, the overall behaviour of the 10-DOF model is quite satisfactory. The 3-DOF model, on the other hand, shows large differences in the lateral and roll responses, which eventually results in the vehicle deviating from the original path location (last graph of Figure 4-30).

![Graphs showing vehicle handling responses for sinusoidal steering](image)

**Figure 4-30: Vehicle handling responses for sinusoidal steering**

In the lateral force plots, shown in Figure 4-31, the 10-DOF and ADAMS models show similar response patterns. Some differences in the peak responses can be seen, with the ADAMS model exhibiting higher lateral load transfer as compared to the 10-DOF model. The compliances included in the ADAMS model can further lead to additional lateral load transfer. The same observation can be made for the vertical force plots (Figure 4-32), where again the load peaks for the ADAMS model can be seen to be slightly higher. The 3-DOF model, on the other hand, exhibits much higher peaks, which again highlights its lack of stiffness in the lateral and roll direction. However, both lateral and vertical force plots of the 3-DOF model show phase differences in the response when compared to the ADAMS model. This again highlights the importance of tyre lags and other response lags, which were not incorporated in a relatively simple 3-DOF model.
Figure 4-31: Lateral force plots for sinusoidal steering

Figure 4-32: Vertical force plots for sinusoidal steering
Multi-Body Model and Simulation Results

4.7.1.4 J Turn Analysis

The J-turn test is a single steer manoeuvre test used to study the transient handling properties of a vehicle at limiting cornering conditions. The test is conducted by driving at a constant speed and applying a pre-set steering hand wheel angle of 90 degrees. The steering input approximates a steep ramp increase in steering-wheel angle at a rate of 512 deg/sec. This manoeuvre assesses the vehicle's open loop response to an almost step input and is often used to evaluate vehicle properties, particularly in relation to roll stability. For more severe situations, such as vehicle rollover, NHTSA has suggested their J-Turn and J-Turn with pulse braking. These tests are representative of rollover resistance of on-road vehicles, during un-tripped manoeuvres, and are performed with two directional steer, to the left and to the right (NHTSA, 2001). In the current J-Turn simulation, the vehicle velocity was set to 70 Km/h and steering wheel angle of 90 degree was applied for a left hand turn. As the traction control is switched off in this manoeuvre, the vehicle speed tails-off through the course of the manoeuvre.

![Figure 4-33: Vehicle handling responses for J-turn](image)

The response plots for vehicle lateral acceleration, yaw rate, roll angle, side slip angle, and vehicle path location are shown in Figure 4-33, where it can be seen that both the ADAMS and the 10-DOF model's responses correlate well. The ADAMS model exhibits a slight
overshoot in its yaw rate response, in relation to the 10-DOF model. This can be attributed to the additional suspension and steer compliances incorporated in the more detailed ADAMS model. The weight distribution of the actual vehicle is such that an inherently under-steering behaviour is expected. This behaviour, however, is emphasized by the additional compliance built into the ADAMS model. Another observation concerns the roll behaviour of the two models. The 10-DOF model exhibits a sudden spike at the transient phase of the roll response, whereas the ADAMS model exhibits a smooth behaviour at the same point. This again can be attributed to the compliances built into the ADAMS model apart from its non-linear roll stiffness and damping, which overall helps in its roll response.

![Figure 4-34: Lateral force plots for J-turn](image)

In the lateral force plot (Figure 4-34), differences in the results can be seen under steady-state conditions between the ADAMS and the 10-DOF models. This can be partially due to the fact that horizontal and vertical shifts were not considered in the 10-DOF model, and hence the lateral forces originate from zero. The other reason can be difference in the jounce and rebound characteristics of the two models. The ADAMS model exhibits slightly large roll angle, which can lead to an increase in the lateral load transfer. This effect is reflected in the vertical load plot (Figure 4-35), where the difference between the ADAMS and the 10-DOF model can be observed in the front wheel loads. The spikes in the vertical load plots of
the 10-DOF during the transient phase again demonstrates the lack of compliances and non-linear roll damping in the 10-DOF model. However, irrespective of the little quantitative differences in the results, the 10-DOF model follows the ADAMS model reasonably well, which is significant for comparative purposes.

Figure 4-35: Vertical force plots for J-Turn

4.7.1.5 Fishhook Manoeuvre

The Fish Hook manoeuvre assesses the transient handling properties of a vehicle at limiting handling conditions. The test is conducted by driving at a constant speed, before bringing the vehicle in neutral and applying a pre-set steering wheel angle, holding, and then applying another pre-set steering wheel angle in the reverse direction. The period of time for which the driver holds the first steering wheel angle before applying the counter steering, is known as the steering dwell time. The steering input rate approximates a step input at 512 deg/sec of rotational velocity to the steering hand wheel, during first and second steer.

To determine more severe limiting handling condition such as vehicle roll stability, a few other similar sort of test manoeuvres are prescribed, such as Fixed Timing Fishhook and Roll Rate Feedback Fishhook, where the steering hand wheel angle is selected once a lateral
acceleration of 0.3g is achieved on either side. The two manoeuvres differ in how the steering dwell time is defined (NHTSA, 2001). In the current Fishhook manoeuvre, the initial vehicle speed is set as 50 Km/h, with a steering hand wheel angle of 90 degrees to the left and steering reversal angle of 180 degrees to the right (first graph of Figure 4-36). The steering dwell time is set as 0.65 seconds.

Figure 4-36: Vehicle handling responses for fishhook

The response plots for vehicle lateral acceleration, longitudinal speed, yaw rate, roll angle, and vehicle path location are shown in Figure 4-36, where the results for both the models agree very well. Due to step steering input, the 10-DOF model exhibits a sudden spike in the transient phase of the roll plot, which as explained earlier, can be attributed to the lack of compliances and non-linear roll damping as compared to the ADAMS model. The roll behaviour of the 10-DOF model, however, matches that of the ADAMS model under the steady-state condition. The yaw and lateral acceleration of the 10-DOF model are marginally higher, which is visible in the steady-state phase of the plots. However, this does not affect the path location of the 10-DOF model with respect to that of the ADAMS model (last graph of Figure 4-36). The reason can be that the vehicle speed in the 10-DOF model does not trail-off as much as that in the ADAMS model (second graph of Figure 4-36), and hence compensates for the extra yaw rate.
Multi-Body Model and Simulation Results

Figure 4-37: Lateral force plots for fishhook

Figure 4-38: Vertical force plots for fishhook
The lateral force plot, as shown in Figure 4-37, once again shows the lateral shift in the ADAMS plot under the steady-state condition. The reason explained for the earlier tests holds true in this case as well. The vertical force plot, as shown in Figure 4-38, shows a close match between the two models. Although in transient phase, there are minor variations in the peaks of the two models, considering the difference in their suspension and anti-roll bar characteristics, the overall conformance is acceptable.

4.7.1.6 Step Steer

This step steer test is used to evaluate the transient handling and steady-state directional control response characteristics of a vehicle when a step input is given to the steering wheel. The test starts by driving the vehicle in a straight line at a constant speed, then abruptly applying a pre-set steering wheel angle. The steering wheel input is such that it gives a desired steady-state lateral acceleration. In this study, the step steer was performed by driving the vehicle at 100 km/h, before applying a step steer of 45 degrees (first graph of Figure 4-39) and switching the throttle off. The same test can also be performed under cruise control or by maintaining a constant throttle.

Figure 4-39: Vehicle handling responses for step steer
The response plots for vehicle lateral acceleration, yaw rate, roll angle, sideslip and vehicle path location are shown in Figure 4-39, where the results for both the models show very close agreement. The yaw rate and lateral acceleration of the two models matches well in the transient phase, before moving to the steady-state, where the 10-DOF model generates slightly higher yaw rate. The roll behaviour of the 10-DOF model, as observed in the previous manoeuvres, differs from the ADAMS model in the transient phase. This behaviour could be addressed in future by including suspension and roll compliances in the 10-DOF model.

![Lateral force plots](image)

**Figure 4-40: Lateral force plots for step steer**

The lateral force plots of the two models are shown in Figure 4-40, where similar comparative behaviour can be observed as that of J-turn or Fishhook manoeuvre, with ADAMS model again showing lateral shift at the start of the manoeuvre. The same could be said for the vertical plots (Figure 4-41), where some differences in the lateral load transfer occurs, owing to the individual suspension and anti-roll bar characteristics of the two models. However, the overall response of 10-DOF model shows an acceptable behaviour, particularly in terms of directional control, which is obvious in its comparison of path location with the much detailed ADAMS model (last graph of Figure 4-39).
4.7.1.7 Transient Manoeuvre on a Surface with Uneven Friction

Thus far the models have been tested under conditions which promote operation in the nonlinear area. In all cases, good agreement is observed between the results generated by the intermediate model and those obtained by the more elaborate multi-body model. The success of the intermediate model can be attributed mainly to the careful consideration of the influence of suspension mechanics in vehicle handling dynamics. A realistic representation of the suspension and a modelling approach appropriate for the treatment of large roll angles is critical for the simulation of severe manoeuvres which are likely to lead to vehicle rollover. Although the vehicle under consideration is inherently stable with respect to roll (due to large front and rear tracks), a couple of simulation studies are conducted to explore its handling behaviour at limiting conditions.

The first test involves a step-steer input of more than $100^\circ$ at an initial forward speed of 120 km/h. Road-tyre friction is assumed even and the peak factor ‘$D$’ of the Magic Formula is modified to generate a peak force corresponding to a coefficient of friction $\mu = 1$ under a static cornering load. It is important to note that during this test the forward speed is allowed to reduce as a result of the steering input, i.e. the forward speed PID controller is deactivated.
Multi-Body Model and Simulation Results

The yaw-rate response depicted in Figure 4-42 indicates under-steering behaviour, while the tyre vertical loads show clearly that all the four tyres remain in contact with the ground.

![Graphs showing yaw-rate and other vehicle responses](image)

**Figure 4-42: Responses to a step-steer manoeuvre on a surface with even friction**

The second test case involves an identical steer input performed at the same initial speed of 120 km/h. The coefficient of road-tyre friction starts at a value of \( \mu = 0.5 \). However, during the manoeuvre, the two outer tyres suddenly enter a part of the road where friction is significantly higher, attaining a value of \( \mu = 2.5 \). Although this value of friction appears rather unrealistic for a passenger vehicle running on a normal road, it can be used as a means of simulating the situation where tyre side-forces increase abruptly as a result of the wheels meeting with an obstacle, such as the edge of a low pavement.

A composite friction scaling factor \( \lambda_{psy}^* \) (Pacejka, 2006) is used in \( x \) and \( y \) direction to modify the peak factor ‘D’ of the Magic Formula (refer equations (4.43) and (4.54)).

\[
\lambda_{psy}^* = \frac{\lambda_{psy}}{\left(1 + \lambda_{\mu} \cdot \frac{V_s}{V_0}\right)}
\]

(4.101)

where \( \lambda_{\mu} \) represents the scaling factor for the friction decaying with increasing slip, \( \lambda_{psy} \) represents the scaling factor for the peak friction coefficient, \( V_s \) represents the slip velocity
Multi-Body Model and Simulation Results

(V_x and V_y for the longitudinal and lateral direction respectively) and V_0 represents the reference velocity.

From the results shown in Figure 4-43, it can be observed that a discontinuity appears more distinctly in the lateral acceleration, yaw rate and roll rate responses. This discontinuity coincides with the moment in time when the two outer tyres enter the part of the road with increased friction. Although the manoeuvre starts with a lower lateral acceleration due to reduced road friction, the subsequent increase in friction is sufficient to cause lift-off for both inner wheels after approximately 1.5 seconds. It is interesting to note that in terms of its yaw rate response the vehicle remains stable, persisting on its under-steering qualities. Finally, the inherently stable character of the vehicle in terms of its roll motion is highlighted by the modest roll angle, even at high lateral accelerations (above 1.5g).

![Graphs showing responses to a step-steer manoeuvre on a surface with varying friction](image)

Figure 4-43: Responses to a step-steer manoeuvre on a surface with varying friction

4.8 Chapter Remarks

The following observations can be made, while comparing the intermediate vehicle model with the complex multi-body model for different transient handling manoeuvres:
- The intermediate vehicle model, despite using relatively simplified representation, is able to produce close agreement with the complex multi-body model during various transient handling simulations.

- The inclusion of adequate DOF, as well as steering, suspension and tyre characteristics in an intermediate modelling approach has a significant bearing on the simulation outcome of such critical handling manoeuvres. This can be clearly observed in the comparative simulation runs involving the 3-DOF vehicle model, where the 3-DOF model, due to its overly simplified vehicle characteristics, fell short to satisfy the performance requirements for those transient handling manoeuvres.

- In addition to the transient handling manoeuvres, the intermediate vehicle model can also be employed to study the pre-roll over event or vehicle's rollover propensity, as demonstrated in the final simulation run with uneven friction, in the current chapter. The inclusion of adequate features in the intermediate vehicle model (refer chapter 3) such as the ability to handle large roll and pitch angles as well as wheel lift-off cases becomes very significant for such extreme manoeuvres, in addition to the correct representation of vehicle suspension, anti-roll bars and wheel geometry effects.

- The intermediate vehicle model, though matched well with the complex ADAMS model, but certain differences in the handling responses during sharp steering transient manoeuvres such as J-Turn and Fishhook were observed. The lack of steering and suspension elasto-kinematics and other compliances as well as non-linearities which may influence the secondary motion of the unsprung mass components can be the prime reason for such behaviour.

- The commercial multi-body package like ADAMS/Chassis possesses a wide range of capabilities, which can not easily be bettered by an intermediate vehicle modelling approach. However, as demonstrated in this work, it may still offer comparable accuracy, along with other benefits such as fewer parameters and reduced computational cost. The flexibility of extending such models for control related studies is an additional advantage, which will be explored further in chapter 6.

- In the following chapter, the intermediate vehicle model validation against the instrumented test vehicles is presented.
5 Experimental Vehicle Testing

5.1 Test Introduction

In the previous chapter, the 10-DOF model was compared with a detailed multi-body ADAMS model. The results were in good agreement. However, the validation of the model against measurements from a real vehicle is essential. For this reason, the theoretical vehicle model was validated against experimental on-road vehicle tests, on two different occasions.

On the first occasion, the test car was already equipped with a data logging system and several existing sensors. Calibration was carried out for the already installed sensors before testing. Two different manoeuvres were undertaken at the Donington Park circuit. These were a Lane change and a J-turn. Unfortunately, due to poor weather condition and constraints imposed on test track times, the test run for the J-turn could not be carried out on a proper track, but on a paddock. Hence, the results obtained were not adequate for a proper comparison. The lane change results are presented in this chapter, comparing them with the 10-DOF model predictions.

To further test the accuracy of the intermediate model, a second series of experimental measurements were later carried out on a different vehicle, at the MIRA (Motor Vehicle Industry Research Association) Proving Grounds. The vehicle was equipped with an RT3200 GPS/Inertial measurement system which provided measures of all the six vehicle motions, while the steer-angle and other information such as rotational wheel speeds were obtained from the vehicle’s CAN network. Testing was carried for a range of test manoeuvres, from pure cornering to one which involved both cornering and braking. The test vehicle, most of the time, was operated in its non-linear region. This was done purposefully to validate the 10-DOF model’s responses, as the accuracy of simplified models usually deteriorates, when exposed to such conditions.

This chapter presents the description of some of the instrumentation, involved in both the test programs, along with the measurement results and their comparisons with the intermediate 10-DOF model predictions.
5.2 Description of the First Experimental Test

The test vehicle I (Figure 5-1) has a 2.0 litre, V6 4-stroke gasoline engine. It employs an independent front suspension, with a Macpherson Strut, and Quad-link type independent rear suspension system, and has a front wheel drive, using a rack and pinion steering system.

![Figure 5-1: The test vehicle](image)

5.2.1 Test Instrumentation

The following sensors were primarily employed in the tests:

- 3 accelerometers for measuring longitudinal, lateral and vertical accelerations
- 3 rate gyros for measuring roll, pitch and yaw rates
- A potentiometer for measuring the steering angle
- 2 magnetic pick-up sensors for measuring wheel speed at the rear wheels

5.2.1.1 Vehicle Translational Sensor

![Figure 5-2: Linear accelerometer](image)

To measure the translational motions of the vehicle along the three axes (longitudinal, lateral and vertical), three similar accelerometers were used (Figure 5-2), all of which were placed as closely as possible to the CG location of the vehicle (Figure 5-3). The Schaevitz DC-Operated accelerometers were capable of measuring accelerations up to ± 2g.
5.2.1.2 Vehicle Rotational Sensor

To measure the angular motions of the vehicle about the three axes (roll, pitch and yaw rates), three highly sensitive British Aerospace gyroscopes were used. These vibrating structure gyroscopes are solid-state devices (see Figure 5-4), which provide an output voltage proportional to the rate of turn applied to the sensitive axis. These were placed as close as possible to the accelerometers (see Figure 5-3).

5.2.1.3 Vehicle Steering Angle Sensor

Figure 5-3: The positions of accelerometers and gyroscopes

Figure 5-4: Bipolar type single axis vibrating structure gyroscope

Figure 5-5: Potentiometer for measuring the steering angle
A linear rotary potentiometer was used to measure the steering wheel angle (Figure 5-5). The sensor is positioned parallel to the steering column and connection between the two components is achieved by using a belt. Since this is a linear potentiometer, calibration is quite straightforward. This is performed by increasing the steering wheel angle by a certain constant degree, and the changes in the voltage can be plotted to obtain an estimate of the gain between the steering angle and the sensor voltage output.

5.2.1.4 Vehicle Wheel Speed Sensor

For measuring the wheel speeds, magnetic “pick-up” sensors were deployed on the rear left and right wheels (see Figure 5-6). The sensors were attached very close to the brake disc caps, where 40 teeth are available to provide pulses per rotation, which can then be used to calculate the actual wheel speed. The pulses were fed into a frequency-to-voltage converter system, which then generated a steady voltage output. By knowing the angular velocity of the wheel and its radius the vehicle forward speed can be determined.

5.2.2 Calibration

Calibration is an important step in any vehicle testing activity since proper calibration can help in ensuring reliable outcome from the test data. For each sensor, a minimum of three samples of data sets were taken and an average value was obtained. For all translational sensors, the calibration was conducted by pointing the sensors both downward, towards the ground, and upward away from it. By doing so, one can measure the output voltage for ±1g. The measurement non-linearity was found to be less than ± 0.05% for 1g. The sensor output can, therefore, be considered as almost linear for the entire test range.
Experimental Vehicle Testing

The original calibration of the rotational sensors was conducted by the British Aerospace Systems & Equipment. Since the cost to send these sensors for calibration was prohibitive and also dismantling the sensors was seen as unadvisable, an alternative way to calibrate the sensors was devised. The simplest approach is by driving the vehicle around a constant radius with a constant speed for a fixed number of circles. By knowing this information, one can then calculate the speed (deg/sec) of the vehicle traveling around a circle, which relates to its voltage output for yaw gain. The same procedure needs to be repeated for the roll and pitch sensors as well.

Calibration of the steering wheel sensor is straight forward. By turning the steering wheel to the left or to the right by 90 degree per step, the change in the output voltage can be recorded. Since the sensor is a rotational linear transducer, the gain of the sensor can be obtained. The wheel speed sensors were calibrated by driving the car at a specific road speed and recording the voltage once the car reaches steady-state.

5.2.3 Data Acquisition system

Figure 5-7: Schematic diagram of data logging system

The mobile data acquisition (DAQ) system, fitted to the vehicle, was based around a 16 channel, 100 kHz DAQ board that is controlled by a laptop PC through its parallel port. Figure 5-7 shows the schematic of the complete data logging system, physically installed in the vehicle boot (Figure 5-8). The LabView V-5.1 software was used as user-interface for logging the acquired signals using a standard template file with 200 Hz sampling rate.
5.2.4 On-Road Test Result

The on-road test should ideally have been carried out on a smooth flat surface. However, because of certain constraints, the tests were conducted in the paddock area of the Donnington Park circuit. Since the J-turn results were not adequate for comparison, only the lane change results are presented in this section.

The experimental results for single lane change are presented along with the simulation output from the 10-DOF model in Figure 5-9. The test was carried out at a forward speed of 100 km/h. The steering input (Figure 5-9 (a)) in this case was not exactly similar to the one used in section 4.7.1.2, as the test was conducted on a road without any markings. For this purpose, the 10-DOF model was run by taking the steering input from the test results to make a fair comparison. In the lateral acceleration plot (Figure 5-9 (b)), it can be seen that the two results conform closely as far as the general trend is concerned. The simulation was performed by keeping the velocity constant (PID controller), whereas in the on-road test, there would be slight fluctuations in the longitudinal velocity of the vehicle, which is very difficult to maintain at a constant value. The lateral acceleration is proportional to square of the velocity, which means even a slight variation in the speed could lead to a quite visible difference when compared to the simulation results. Considering the overall adverse test conditions, the lateral acceleration plot is a fairly reasonable match. The same can also be said of the longitudinal acceleration plot (Figure 5-9 (c)) and roll angle plot (Figure 5-9 (d)).
Though the graphs are made smooth for plotting, the test results clearly show a significant amount of noise, even after reaching the steady-state condition. Considering, the suspension non-linearities, bushing compliances, and other non-linearities in the vehicle sub-systems, not forgetting the road irregularities, the overall results show fairly good agreement. This can be further improved upon by carrying out testing on a proper test track.

![Steering Angle - Experiment (deg)](image1)

![Lat Acceleration (g)](image2)

![Longitudinal Acc (g)](image3)

![Roll Angle (deg)](image4)

Figure 5-9: On-road test results for single-lane change

5.3 Description of the Second Series of Experimental Test

The test vehicle 2 used for the second experiment has a 3.5 litre V8 Diesel engine, with a rear wheel drive system. It employed Macpherson Strut suspensions at both front and rear, and a rack and pinion steering system. The general description of the vehicle can be found in the description of the Multi-body model, described in Chapter 4, which is based on the same test vehicle.
The vehicle was equipped with an RT3200 differential GPS/Inertial measurement system, from Oxford Technical Solution Ltd. The RT3200 contains an instrumentation set installed in the vehicle, comprising antennas, inertial sensors, GPS receiver, data storage and CPU. The schematic of the RT3200 system is shown in Figure 5-12. The instrumentation set, in this case has a single antenna installed on the roof of the vehicle, parallel to the longitudinal axis of the vehicle (refer Figure 5-10). The antenna acquires signal from the GPS satellite and passes it to the receiver unit, providing the absolute positions, velocities and Euler angles (heading, pitch and roll) of the vehicle. These measurements from the GPS receivers are used in the navigation computer to update the position and velocity measured by the inertial sensors.

![Figure 5-10: Single antenna on the roof of the vehicle](image)

The inertial measurement unit (IMU), positioned approximately at the centre of the vehicle’s axis system (refer Figure 5-11) with three accelerometers and three gyros (angular rate sensors), computes the accelerations and the angular velocities of the vehicle. The accelerometers are all mounted at 90 degrees to each other so that they can measure each direction independently. The three angular rate sensors are mounted in the same three directions as the accelerometers. The signal is then fed into the navigation computer through DSP (digital signal processing). The DSP unit processes the data through filters and compensation algorithms, and also provides calibration of the accelerometers and angular rate sensors, ensuring the direction of the acceleration and angular rate measurement accuracy to be better than 0.01 degrees.
The Information from the DSP and the two GPS receivers is fed into the navigation computer, which runs on a real-time operating system so that the outputs are made in determined finite time durations. The sampling process in the IMU is synchronised to GPS time so that the 100Hz measurements from the RT3200 are synchronised to GPS. If a base station is involved, the differential corrections from the base station (via radio modems) can be supplied directly to the GPS receiver to improve the positioning accuracy.
The outputs of the system are derived directly from the Strapdown navigator, which uses a WGS-84 model of the Earth, the same as GPS uses. This is an elliptical model of the Earth rather than a spherical one. The role of the Strapdown navigator is to convert the measurements from the accelerometers and angular rate sensors to position, velocity and orientation, and while doing so, the navigator algorithm compensates for the Earth's curvature, rotation and Coriolis accelerations. Figure 5-13 shows a basic overview of the Strapdown navigator.

![Diagram of Strapdown Navigator](image)

**Figure 5-13: Overview of the strapdown navigator (RT3000, 2005)**

The Kalman filter used in the RT3200 is used to apply corrections to several places in the Strapdown navigator, including position, velocity, heading, pitch, roll, angular rate bias and the scaling factor and acceleration bias. After all the correction, the angular rates are finally integrated to give heading, pitch and roll angles, whereas the accelerations are integrated to give velocity, which is then integrated to give position.

Apart from the GPS/Inertial measurement system, the test vehicle also employs the RT-CAN unit, through which the various other parameters from vehicle's CAN network are obtained such as the steering wheel angle, rotational wheel speeds, engine torque and speed, gear and throttle positions, master cylinder pressure etc. The RT-CAN unit is an interface converter that accepts the RT3200 NCOM output and converts it for transmission over a CAN interface. All this data is finally post-processed to obtain the necessary information.

While post-processing the user has to be clear about the angles adopted in the GPS/Inertial measurement system, so as to derive any position, velocity or acceleration parameters at a different position or with respect to another coordinate frame. In the GPS/Inertial

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measurement system, the Euler angle outputs are three consecutive rotations (first heading, then pitch and finally roll) that transform a vector measured in the navigation co-ordinate frame to the body co-ordinate frame. The navigation co-ordinate frame is the orientation on the Earth at the current location with axes of North, East and Down. For example, the forward and lateral velocities can be found by rotating the velocities in the navigation co-ordinate frame to be in the direction of the vehicle (using the heading angle). The heading angle stands for the popular convention of the yaw angle, as used in the intermediate vehicle model.

5.3.1 Test Manoeuvres

The experimental test runs were carried out at the MIRA Proving Ground, using primarily the Dunlop handling circuit and also the Steering Pad for different handling manoeuvres. The location of the circuit in the MIRA Proving Ground is shown by the markers in Figure 5-14. The handling circuit with its high grip Delugrip RSM surface and different constant radii bends, was used to conduct different limit handling manoeuvres, whereas the Steering Pad with its flat surface and marked constant radii circles was used for the constant steering test. The results of different handling manoeuvres are presented in the following sub-section. In all the handling plots, the steering hand wheel angle and the corresponding initial forward velocity used in the 10-DOF model is extracted from the test runs to facilitate comparisons.

![Figure 5-14: MIRA proving ground (MIRA-Brochure, 2008)](image-url)
5.3.1.1 Arbitrary Steer Input

This test run was performed at the Dunlop circuit of the MIRA Proving Ground, where the vehicle was subjected to an arbitrary steering input, as shown in the first graph of Figure 5-15. The forward speed was maintained at approximately 50 km/h, which, given the magnitude of the steering, proved sufficient for the achievement of medium to high values of lateral acceleration. The results obtained by the 10-DOF model show good agreement with experimental results in terms of yaw rate, lateral acceleration and roll angle, as demonstrated in the third graph of Figure 5-15, and both graphs of Figure 5-16 respectively. The vehicle speed in the 10-DOF model was kept constant using the PID controller, as shown in the forward velocity plot of Figure 5-15. The vehicle path plot (last graph of Figure 5-15) also shows close agreement between the 10-DOF model and the test vehicle. Though, the 10-DOF model exhibits slight under-steer behaviour, but considering the transient nature of the manoeuvre, the overall results are very closely matched.

![Graphs showing vehicle handling responses](image)

Figure 5-15: Vehicle handling responses to an arbitrary steer-input
5.3.1.2 Step-Steer Manoeuvre

This test was again carried out at the Dunlop circuit of the Mira Proving Ground, which involved a severe step-steer manoeuvre at a forward speed of approximately 50 km/h. The steering-input and the forward velocity plots are shown in Figure 5-17. The experimental test was initiated in cruise control mode, but the control was lost in the middle of the manoeuvre. The exact reason was not known, however it was felt that it could have happened because of the presence of some wet patches and also some side wind gusts, during the measurements. As the test did not involve constant speed, the 10-DOF model simulation was performed by switching off the PID controller, which means the vehicle speed trails-off during the course of the manoeuvre. Though not an ideal way to compare the results, reasonably good correlation between experimental and simulation results, was evident, as seen in the handling response plots in Figure 5-18. The experimental yaw rate and lateral acceleration responses show slight oscillatory behaviour compared to the corresponding simulation results. These oscillations can be seen even when the steering remains constant. Since both the experimental and simulation yaw-rate responses appear to have almost identical rise rates, it is possible that the observed oscillation is due to arbitrarily changing road tyre friction conditions. Provided the accuracy and consistency of the measuring system is assumed, this might be attributed to relatively wet patches or side-wind gusts, during the measurement.
5.3.1.3 Double Lane Change

The double lane change manoeuvre was conducted on the Dunlop track, though without any markings, which means that the steering input in this case was in accord with the ISO standard, explained in section 4.7.1.2. The vehicle in this case was driven with a constant speed of 65 km/h (keeping the cruise control on), with steering input as shown in Figure 5-19. The forward velocity in the 10-DOF model was maintained constant using the PID controller.
Figure 5-19: Steering wheel angle and forward velocity in double lane change

The response plots for vehicle lateral acceleration, yaw rate, roll angle, and vehicle path location are shown in Figure 5-20, which shows a reasonably close match between the 10-DOF model and the experimental run. The interesting observation in this plot is the 10-DOF model response in the 3rd and 4th turns, which is rather smooth when compared to sharp response of the test vehicle as an outcome of the sharp steering in the 3rd and 4th turns. It seems the 10-DOF model rigid behaviour generates lags in its overall response during very sharp steering inputs. The 10-DOF model exhibits a small under-steering behaviour as compared to the test vehicle, which can be seen in the vehicle track plot (last graph of Figure 5-20).

Figure 5-20: Vehicle handling responses in double lane change

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5.3.1.4 Constant Steering Manoeuvre

This test run was carried out on the Steering Pad at the MIRA Proving Ground, where the vehicle was driven around a circle of 26 m radius, at a constant speed of approximately 50 km/h. The steering wheel angle and the forward velocity plots are shown in Figure 5-21. Although the vehicle was driven continuously for number of circles, only that portion of the result is used here, where the vehicle forward velocity remained constant, so as to allow for a fair comparison with the 10-DOF model.

![Figure 5-21: Steering wheel angle and forward velocity in constant steering test](image1)

![Figure 5-22: Vehicle handling responses in constant steering test](image2)
The response plots for the vehicle lateral acceleration, yaw rate, roll angle, and vehicle path location are shown in Figure 5-22, which show reasonably close agreement. In this case the 10-DOF model was simulated using set velocities and orientation in the sprung mass equation, as part of the initial condition for the simulation. Although, this allows the 10-DOF model to be compared to the experimental outputs, extracted from a middle of a test run, the other dynamics such as weight transfer and accelerations were hard to match. This led to small discrepancies in the roll angle plots. Considering, the initial conditions were very different for this steady-state manoeuvre, the overall comparison is quite acceptable.

5.3.1.5 Combined Cornering and Braking Manoeuvre

The previously described test manoeuvres were mostly carried out at almost constant speed and hence could be easily compared with the 10-DOF model results, simulated through the use of the PID speed controller. The comparisons allowed the behaviour of the theoretical vehicle model to be ascertained vis-à-vis test vehicle performance under a number of real life transient manoeuvres. Furthermore, in the process to experimentally validate the 10-DOF vehicle model, an attempt is made in this section to establish its behaviour in manoeuvres involving both cornering and braking.

This comparative study was conducted by making use of parameters obtained through the test vehicle CAN network, which was interfaced with the RT-CAN unit of the GPS/Inertial measurement system. The parameters obtained were engine speed, throttle position, gear position, engine/friction torque, and brake master cylinder pressure. By deploying the engine characteristic curve in the form of a look-up table (speed, torque and throttle) and using information of the gear position, the torque at the driven wheels can be nominally calculated. The engine characteristic curve, and 5 speed gear train and differential gear ratios are known apriori. The brake torque was modelled by employing the hydraulic brake unit of the 10-DOF model, described earlier in chapter 3. In this case, as the brake pedal force was not available, the master cylinder pressure was used directly as an input to the system, cutting-off the vacuum booster /master cylinder assembly, but using the brake hydraulics/ wheel cylinder assembly to workout the brake torque at the wheels.

The results of two experimental runs, involving combined cornering and braking, are shown in this section, along with the simulation results. The first run was carried out at the Dunlop circuit of the MIRA Proving Ground, where the vehicle was initially subjected to a lane
change manoeuvre at a constant speed, following which a sharp steering input was made, forcing the vehicle to perform a U-turn. Braking was initiated just prior to the sharp steering input, forcing the vehicle to trail-off during the turn. The vehicle was again put in acceleration mode, once it came out of the sharp turn, allowing the vehicle to regain its speed, before being finally subjected to braking towards the end. The parameters obtained from test vehicle CAN network are shown in Figure 5-23, where plots of steering input, throttle and gear position, engine friction torque, engine speed and brake master cylinder pressure are shown respectively.

![Figure 5-23: Parameters obtained from vehicle CAN network](image)

The above mentioned parameters were used in the 10-DOF model to simulate the exact manoeuvre. The plots of vehicle forward velocity and the brake torque at the wheels are shown in the Figure 5-24. Considering the fact that traction at the wheels were provided, without a detailed representation of the drive train model, the vehicle forward velocity plot for the 10-DOF models shows a reasonably good agreement with the test car. Moreover, it demonstrates the successful integration of hydraulic brake system model with the 10-DOF model.
Figure 5-24: Forward velocity and brake torque plots of 10-DOF model

The response plots for vehicle lateral acceleration, yaw rate, roll angle, and vehicle path location are shown in Figure 5-25, which again show reasonably close match between the 10-DOF model predictions with measured responses. Slight differences occur in the roll angle plot, which considering the nature of the manoeuvre (sharp U-Turn), is of acceptable nature. The vehicle track plot also shows a close agreement with the 10-DOF model, with a slight drift towards the outside of the turn. This could partly be attributed to the relatively under-steering characteristics of the 10-DOF model and also to the differences in the traction braking torques generated at the front and rear wheels, in comparison with the test vehicle.

Figure 5-25: Handling response plots in lane change manoeuvre involving braking
The second run was performed at the Steering Pad circuit of the MIRA Proving Ground, where the vehicle was initially subjected to a fixed steering angle at a constant speed for a number of circles, following which the vehicle was made to follow a little triangular path, by initially steering to its left and then to its right. Braking was initiated just prior to the first turn, forcing the vehicle to trail-off during the turn. After the first turn, the vehicle was again provided with some acceleration, while it was driven in a straight line leading up to the second turn, with a final brake application towards the very end. The parameters obtained from the test vehicle CAN network are shown in Figure 5-26, where plots of the steering input, throttle and gear position, engine friction torque, engine speed and brake master cylinder pressure are shown respectively. The plots are self-explanatory, with engine speed and throttle position appearing at constant values, prior to the braking input. The brake master cylinder pressure peak (last graph of Figure 5-26) can be seen at the onset of the first turn, just after the constant steering position. The plots of vehicle forward velocity and the brake torque at the wheels are shown in the Figure 5-27, where it can be seen that the forward velocity of the 10-DOF models again follows that of the test car quite closely.

![Figure 5-26: Parameters obtained from vehicle CAN network](image-url)
The response plots for vehicle lateral acceleration, yaw rate, roll angle, and vehicle path location are shown in Figure 5-28, which again show reasonable agreement with the predictions of the 10-DOF model. The roll angle plot for the test vehicle shows some fluctuations about a constant steering position, the exact reason for which is not known, but there could be some issues with that particular circular strip of the Steering Pad, as in the case of an earlier run (reported above) (refer to 5.3.1.4). The vehicle track plot for the 10-DOF model shows some difference in the path location, as compared to the test vehicle, while it moves through the triangular strip. Considering the open loop nature of the test and the relative under-steering characteristics of the 10-DOF model, the small drift incurred
during the circular runs can yield this final deviation. Overall, the response of the 10-DOF model in both the manoeuvres was of acceptable nature, and it helped in establishing the behaviour of the 10-DOF model in combined cornering and braking manoeuvres.

5.4 Chapter Remarks

The comparison of the intermediate vehicle model against the experimental on-road vehicle tests presented in this chapter further ascertains the realistic and reasonably accurate behaviour of the vehicle model. The second series of experimental tests, conducted on a proper track with a more reliable GPS/Inertial measurement system, helped in experimentally evaluating the intermediate vehicle model for different real life handling manoeuvres, performed at constant speed in an open loop fashion. Additionally, by using the information from the CAN network of the actual vehicle, the performance of the intermediate vehicle model was further validated for combined cornering and braking manoeuvres. In the process, it also allowed to successfully test the mathematical model of the hydraulic brake unit, integrated to the intermediate vehicle model (refer chapter 3). Since most of the earlier test manoeuvres were conducted in cruise control mode, the PID controller used in the intermediate vehicle model, for constant speed control, was sufficient to satisfy its traction requirements. However, as demonstrated in the last manoeuvre, through the use of basic engine characteristics data, the intermediate vehicle model can be extended for conducting any traction related studies, in future. The modular structure adopted while building intermediate vehicle model can come very handy in this situation, as it can facilitate the integration of a drive-train model with relative ease.

This chapter concludes the assessment and validation of the intermediate vehicle model, where it was overall deemed suited to study the range of critical transient handling manoeuvres, from pure cornering to cornering plus braking, and also the much extreme manoeuvres on varying friction surfaces that could even lead to vehicle rollover. As an extension to the work carried so far, the following chapter will look into the application of the intermediate vehicle model to study the influence of tyre / vehicle transients on ABS braking.
6 Influence of Tyre Transience in ABS Braking

6.1 Introduction

Steady-state tyre models such as the semi-empirical Magic Formula are ideally suited for steady-state handling characteristics. Their application can be extended to transient handling, provided that the vehicle is restricted to very low frequency manoeuvres. However, in situations where the wheel motion is subjected to time dependent variations in driving and braking commands, the steady-state tyre models are found inadequate for representing the transient response. The lag caused in the response of the tyre stems from that the fact that the longitudinal and lateral tyre deformations have a phase lag with the slip variations (a viscoelastic characteristic). In semi-empirical Magic Formula tyre model, this response behaviour is often represented using the relaxation length concept by deploying first order relaxation length, which accounts for the carcass compliance with respect to the rim in both the longitudinal and lateral directions. Kuiper and Van Oosten (2007) applied a stretched string model to represent tyre transient behaviour, where the tyre belt was modelled as stretched-string, attached to the wheel rim by longitudinal and lateral springs. The tyre force and moments were than calculated using the standard Magic Formula coefficients with inclusion of transient slip as an input to the model. Another approach adopted by Pacejka and Besselink (1997) was the contact mass model, where the tyre carcass compliance and slip properties were represented separately. In this case, the carcass spring was used in the model explicitly. Furthermore, the contact patch was given some small mass and inertial properties.

The inclusion of dynamic characteristics such as inertia, damping and stiffness in the tyre model influences its transient response behaviour. The tyre transient response characteristics can have a significant influence in different vehicle handling applications, particularly so in the ABS braking simulation, which involves wheel speed oscillations caused by rapid changes in the wheel brake pressure. In the past, researchers have used transient tyre models for ABS applications (van der Jagt et al, 1989, Zegelaar and Pacejka, 1997, Jansen et al, 1999, Pauwelussen et al, 2003, Braghin et al, 2006). However, the influence of tyre transient behaviour in the actual functioning of ABS braking cycle has not been investigated in an extensive manner.
This chapter describes the three different single point contact transient tyre models, based on the semi-empirical Magic Formula characteristics. The chapter also presents the modelling and description of the ABS braking system, developed for the current research. Finally, the simulation work is presented, where the influence of tyre transients on ABS braking is investigated. The performance of different transient tyre models is analysed, both for ABS operation as well as for external step variation in brake pressure, under a range of operating conditions, including combined cornering and braking.

6.2 Relaxation Length

The relaxation length is an important parameter to study tyre transient effects due to time lagged responses of a tyre to various external inputs such as slip and camber. The tyre longitudinal and lateral flexibility, being the source of such time lagged responses, can be incorporated as a first order dynamic equation, as shown in the past by various researchers. In an early work, Owen and Bernard (1982) studied the effect of lateral force build up on vehicle directional response, where the effect of tyre lateral flexibility was modelled as a first order delay response to the changes in slip angle, using the relaxation length concept. Though the approach was straight forward, the model was not applicable at low speed as the vehicle speed appeared in the denominator of the equation, which caused numerical instability.

Ellis (1994) developed a lateral elastic tyre model for transient response where the time varying lateral deflection, in relation to the wheel rim, was modelled using a first order differential equation. The author employed a time lag constant, known as relaxation time constant in the equation, with the solution of the differential equation i.e. lateral deflection expressed as exponential time lag. The lateral force, which is a product of lateral deflection and cornering stiffness, as a result builds up to a steady-state value through an exponential function.

Loeb et al (1990) conducted experiments to quantify the relaxation length of a fully rolling steered tyre. The lateral force transient response to the steering input was measured, relating a step steer to a tyre rolling against a rotating drum. The corresponding lateral tyre force showed exponential response, typical of a first order dynamic system. The system time constant was then estimated by determining the time required to reach 63% of the final steady-state response. Loeb et al (1990) also performed an analytical prediction of tyre
transient response, using a single point contact tyre model, employing a first order differential equation, similar to the one used by Ellis (1994).

Heydinger et al (1991) also used a first order dynamic equation, where they employed tyre side force as a variable instead of the tread deflection, with the solution of the differential equation, yielding a lagged side force response. Heydinger et al (1991) further used a second order tyre slip angle dynamic equation, with the idea of adequately representing the under-damped characteristics of tyre at high speeds. The lagging slip angle, used as a variable in the equation, generates lag in the lateral force and the aligning moment. This second order dynamic response of side force in relation to the slip angle was based on the experimental results, with the equation employing terms such as tyre path natural frequency and speed dependent tyre damping ratio. Although the model with second order dynamic equation was shown to generate better results, the slip model did not directly take into account the damping and stiffness associated with the carcass flexibility, nor did it include the mass and inertial effects in the contact patch, all of which lead to a second order response function, as visible in the contact mass model.

Bernard and Clover (1995) extended this approach to both longitudinal and lateral dynamics, using both longitudinal slip and lateral slip angle as time lagged variables for the respective first order differential equations. Also by choosing slip as a variable, the authors addressed the numerical problem occurring at low vehicle speeds.

Park and Nikravesh (1997) used a multi-body approach to model tyre longitudinal and lateral flexibility, instead of a time lagged function. The tyre model, in this case, comprised an additional ring body, which was kinematically restrained to the main wheel body through spring and damper elements, providing translational and rotational degrees of freedom to the ring body. Using this approach generates forces and moments that determine the motion of the ring body similar to a conventional tyre model, which are then transferred to the wheel hub through the flexible joints. In the process the response lag caused by the longitudinal and lateral flexibility of the tyre is taken into account.

The following section deals with the kinematic relations for the tyre contact with the road surface, which leads to the first order differential equation for the time lagged tyre transient response, using longitudinal and lateral relaxation lengths.
6.2.1 Lateral Relaxation Length

The first order differential equation for lateral flexibility can be determined, when considering a stretched string model with massless structure. Figure 6-1 shows the contact condition of a pneumatic tyre undergoing a small lateral displacement. The central contact line of a tyre, with its displaced position, under the influence of lateral force follows the path of the wheel. Assuming small lateral displacement so that the point on the central contact line ‘B’ stays in touch with the ground (i.e. no sliding occurs), then a kinematic constraint function can be established for the central line. Here, the lateral displacement of the leading edge of the contact point B with the wheel centre plane is expressed as \( v_1 \). Provided the contact condition is satisfied, the tangent to the central contact line at point B intersects the wheel plane at point C, forming an angle \( \alpha' \). The same tangent intersects with the line defining the direction of wheel travel (i.e. the X axis at point D), with the intersection angle expressed as \( \theta \).

The following relationship can then be established:

\[
\tan \theta = \frac{dY}{dX} \tag{6.1}
\]

\[
\alpha = \alpha' + \theta \tag{6.2}
\]

This leads to \( \tan \alpha = \tan(\alpha' + \theta) \) \( \tag{6.3} \)
after expansion, \[ \tan \alpha = \frac{\tan \alpha' + \tan \theta}{1 - \tan \alpha' \cdot \tan \theta} \] (6.4)

This gives: \[ \tan \alpha - \tan \alpha' \cdot \tan \theta = \tan \alpha' + \tan \theta \] (6.5)

As the above angles are considered to remain small, the second term on the left side of equation (6.5) can be omitted. So equation (6.5) can be written as:

\[ \tan \alpha = \tan \alpha' + \tan \theta \] (6.6)

The equation for the central contact line of the tyre contact patch outside of the contact point can be expressed as:

\[ \tan \alpha' = \frac{v_1}{\sigma_\alpha} \] (6.7)

Where, \( \sigma_\alpha \) denotes the lateral relaxation length and \( \alpha' \) stands for the transient slip.

Substituting equation (6.7) into equation (6.6) yields:

\[ \tan \alpha = \frac{v_1}{\sigma_\alpha} + \tan \theta \] (6.8)

The ordinate \( Y_1 \) of point B can be expressed as:

\[ Y_1 = \frac{v_1}{\cos \alpha} \left( a - v_1 \cdot \tan \alpha \right) \cdot \sin \alpha \] (6.9)

Equation (6.9) can be rewritten as:

\[ Y_1 = v_1 \cdot \left( 1 - \sin^2 \alpha \right) \cdot \cos \alpha + a \cdot \sin \alpha \] (6.10)

Thus: \[ Y_1 = v_1 \cdot \cos \alpha + a \cdot \sin \alpha \] (6.11)

Differentiating equation (6.11) with respect to time yields:

\[ \frac{dY_1}{dt} = \frac{dv_1}{dt} \cdot \cos \alpha + v_1 \cdot \frac{d\cos \alpha}{dt} + a \cdot \frac{d\sin \alpha}{dt} \] (6.12)
Equation (6.1) can be expressed in terms of time derivative as:

\[
\tan \theta = \frac{\frac{dY}{dt}}{\frac{dX}{dt}}
\]

(6.13)

Where, \( \frac{dX}{dt} \) can be expressed as:

\[
\frac{dX}{dt} = U_w \cdot \cos \alpha
\]

(6.14)

Substituting equations (6.12) and (6.14) into equation (6.13) yields:

\[
\tan \theta = \left( \frac{dv_1}{dt} \cdot \cos \alpha + v_1 \cdot \frac{dcos \alpha}{dt} + a \cdot \frac{dsin \alpha}{dt} \right) \cdot \frac{1}{U_w \cdot \cos \alpha}
\]

(6.15)

Assuming small angle condition, equation (6.15) can be expressed as:

\[
\tan \theta = \left( \frac{dv_1}{dt} + a \cdot \frac{d\alpha}{dt} \right) \cdot \frac{1}{U_w}
\]

(6.16)

Substituting equation (6.16) into equation (6.8) gives:

\[
\alpha = \frac{v_1}{\sigma_\alpha} + \left( \frac{dv_1}{dt} + a \cdot \frac{d\alpha}{dt} \right) \cdot \frac{1}{U_w}
\]

(6.17)

Equation (6.17) can be expressed in the following form:

\[
\frac{dv_1}{dt} + \frac{v_1}{\sigma_\alpha} \cdot |U_w| = \alpha \cdot |U_w| \cdot a \cdot \frac{d\alpha}{dt}
\]

(6.18)

where, \( \frac{d\alpha}{dt} \) is the term associated with the turn slip, and in case turn slip = 0, equation (6.18) reduces to the following form:

\[
\frac{dv_1}{dt} + \frac{v_1}{\sigma_\alpha} \cdot |U_w| = \alpha \cdot |U_w|
\]

(6.19)

The above first order differential equation provides the lateral displacement for the leading edge of the contact point, under the influence of the slip angle.
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Equation (6.19) can also be expressed directly in terms of lateral force $F_y$, where the lateral force is expressed as product of transient slip $\alpha'$ and cornering stiffness $C_{fa}$ as:

$$F_y = C_{fa} \cdot \alpha' \tag{6.20}$$

Also, the internal reaction force that balances the lateral force can be expressed as the product of lateral tyre stiffness at road level $C_{ty}$ and lateral deflection $v_i$ as:

$$F_y = C_{ty} \cdot v_i \tag{6.21}$$

Through equations (6.20), (6.21) and (6.7), the following relation applies for the lateral relaxation length:

$$\sigma_{\alpha} = \frac{C_{fa}}{C_{ty}} \tag{6.22}$$

Substituting equation (6.20) into equation (6.7) yields:

$$v_i = \sigma_{\alpha} \cdot \frac{F_y}{C_{fa}} \tag{6.23}$$

Hence, equation (6.19) can be written as:

$$\frac{\sigma_{\alpha}}{C_{fa}} \cdot \frac{dF_y}{dt} + \frac{1}{C_{fa}} \cdot F_y = \alpha \cdot |U_w| \tag{6.24}$$

The steady-state tyre force $F_{yss}$ is:

$$F_{yss} = C_{fa} \cdot \alpha \tag{6.25}$$

Substituting equation (6.25) into equation (6.24) yields:

$$\sigma_{\alpha} \cdot \frac{dF_y}{dt} + F_y = F_{yss} \cdot |U_w| \tag{6.26}$$

The above equation shows the first order time lagged tyre force response to the slip angle variations. The lateral force response at constant forward velocity and step input of slip angle can be expressed as:
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\[ F_y = F_{ys} \cdot \left(1 - e^{-\frac{u_{nt}}{\sigma_x}}\right) \]  \hspace{1cm} (6.27)

at time \( t = \frac{\sigma_x}{U_w} \), \( F_y = 0.63 \cdot F_{ys} \) \hspace{1cm} (6.28)

The above equation explains the relaxation length as the travelled distance, where the lateral force reaches 63% of its steady-state value, under the application of a step rise in the slip angle of the tyre.

6.2.2 Longitudinal Relaxation Length

The first order differential equation for longitudinal flexibility can be determined in a similar manner to that of Bernard and Clover (1995). Figure 6-2 shows the contact condition of a pneumatic tyre undergoing small longitudinal displacement. The original wheel slip point S is located in the wheel centre plane and after displacement attains the instant position \( S' \), which is at a longitudinal distance \( u \) from the spin axis. Considering a small longitudinal displacement, the point \( S' \) follows the road surface and hence satisfies the non-sliding condition. A hypothetical point P is assumed here at the wheel centre plane, which is fixed to the vehicle SAE frame of reference and travels with the velocity \( U_w \). The undeformed point S located at a distance \( \xi \) from the point P has a velocity \( r_e \cdot \omega \), and the distance of the displaced point \( S' \) from the point P is taken as \( \sigma_x \).

![Figure 6-2: Longitudinal relaxation length](image)
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Now, the transient longitudinal slip at point $S'$ can be defined as:

$$\kappa' = \frac{\sigma_x - \xi}{\sigma_x} \quad (6.29)$$

Differentiating equation (6.29) with respect to time yields:

$$\frac{d}{dt} \kappa' = \frac{\sigma_x \cdot (\sigma_x - \xi) - (\sigma_x - \xi) \cdot \sigma_x}{\sigma_x^2} \quad (6.30)$$

where, $\sigma_x = U_w$ and $\xi = \tau \cdot \omega$

This leads to

$$\frac{d}{dt} \kappa' = \frac{\sigma_x \cdot (U_w - \tau \cdot \omega) - (\sigma_x - \xi) \cdot U_w}{\sigma_x^2} \quad (6.31)$$

Rearranging equation (6.31) gives,

$$\sigma_x \frac{d}{dt} \kappa' + \kappa' \cdot U_w = (U_w - \tau \cdot \omega) \quad (6.32)$$

where the right hand term can be equated to the longitudinal slip velocity. Furthermore, the equation can be represented for the either direction of the wheel forward motion as:

$$\sigma_x \frac{d}{dt} \kappa' + \kappa' \cdot |U_w| = -V_{ss} \quad (6.33)$$

where, the longitudinal relaxation length $\sigma_x$ can be defined in a similar manner to the lateral relaxation length, using the longitudinal tyre stiffness $C_{Fx}$ at road level and the longitudinal slip stiffness $C_{fx}$:

$$\sigma_x = \frac{C_{fx}}{C_{Fx}} \quad (6.34)$$

6.3 Transient Tyre Models

The transient tyre models used in this thesis primarily employ semi-empirical Magic Formula characteristics for force and moment generation. For the force and moment characteristics, the latest version of the Magic Formula model ‘PAC 2002’ is used, description of which is provided in chapter 4. Pacejka (2006) has described single contact
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point approach to represent transient characteristics in Magic Formula tyre model, along with its applications. Because of its simplicity and ease of numerical simulation owing to the Magic Formula characteristics, the tyre models based on single contact point theory are used here.

Although the first order differential equations described in the section-6.2 were based on a linear theory, they can be extended for the non-linear range of slip operation. Here, equation (6.19) is used to obtain the transient slip, which is then used as an input to the Magic Formula tyre model. An alternative approach may be to deploy equation (6.26), in which case the steady-state force from the Magic Formula model can be used as an input to the differential equation. However, it may lead to incorrect results because of side force response lag the normal load may not correspond to the side force. This problem may be exacerbated under limiting conditions, where the tyre may be predicted to be still in an adhesion regime, whilst in reality full sliding may be prevalent (Pacejka, 2006). Hence in this study, the transient lateral and longitudinal slips are used as inputs to the Magic Formula Model.

Another issue concerning the transient tyre models is the use of relaxation length in the model. The relaxation length depends on the amount of slip, as well as vertical load. With an increase in the side-slip value, the tyre shows a quicker response, which means the relaxation length decreases with increasing slip (Higuchi and Pacejka, 1997). On the other hand, with an increase in the vertical load, the relaxation length also increases. The following tyre models differ in the way the relaxation length is considered, apart from the tyre carcass compliances and contact patch slip properties.

6.3.1 Stretched-String Tyre Model

The stretched-string tyre model is based on the relaxation length concept, as described in section 6.2, to incorporate the transient tyre behaviour. Adopted from the work by (Pacejka, 2006), the model follows a simplified form of the string method, originally developed to study the tyre shimmy motion (Pacejka, 1972). This approach is also used in the PAC 2002 version of the Magic Formula (Kuiper and Van Oosten, 2007). Here the tyre belt is modelled as a stretched string, which is suspended to the rim by longitudinal and lateral springs. Figure 6-3 shows a top-view of the stretched string model for the transient tyre behaviour. The first order relaxation length model is used, which accounts for the carcass compliance.
with respect to the rim in both longitudinal and lateral directions. When the tyre is rolling, the first point that comes into contact with the road adheres to the road with no sliding. Therefore, a lateral deflection of the string arises which depends on the slip angle and the lateral deflection of the previous points that precedes it into contact.

![Diagram of a stretched-string tyre model](image)

**Figure 6-3: Top view of the stretched-string tyre model (Kuiper and Van Oosten, 2007)**

The lateral deflection $v_l$ of the string at the initial point of contact with the road can be calculated, using the following differential equation:

$$\frac{1}{U_w} \frac{dv_l}{dt} + \frac{v_l}{\sigma_x} = \tan \alpha + a \cdot \phi$$

$$(6.35)$$

where, $\sigma_x$ represents the relaxation length in the lateral direction. The turnslip $\phi$ can be neglected at radii larger than 10 m. After multiplying with $U_w$, the equation can be transformed to:

$$\sigma_x \frac{dv_l}{dt} + |U_w| \cdot v_l = \sigma_x \cdot V_{sy}$$

$$(6.36)$$

When the tyre is rolling, the lateral spring deflection depends on the lateral slip velocity, but at standstill the deflection depends solely on the relaxation length $\sigma_x$, which is a measure for the lateral stiffness of the tyre. This allows the tyre to respond to a slip speed, when rolling and behave like a spring at standstill.

Similarly, the deflection of the string in the longitudinal direction can be expressed as:
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\[
\sigma_x \frac{du_t}{dt} + |U_w| \cdot u_t = -\sigma_x \cdot V_{ss} \quad (6.37)
\]

Here, both longitudinal and lateral relaxation lengths are defined in terms of vertical load as (ADAMS/Tyre, 2005):

\[
\sigma_x = F_x \cdot \left( p_{\tau x1} + p_{\tau x2} \cdot df_x \right) \cdot \exp \left( p_{\tau x3} \cdot df_x \right) \cdot \left( r_0 / F_{\sigma} \right) \quad (6.38)
\]

\[
\sigma_u = p_{\tau y1} \cdot F_{\sigma} \cdot \sin \left[ 2 \cdot \arctan \left( \frac{F_x}{(p_{\tau y2} \cdot F_{\sigma})} \right) \right] \cdot \left( 1 - p_{\gamma y3} \cdot |\gamma| \right) \cdot r_0 \quad (6.39)
\]

where, the parameters \((p_{\tau x1}, p_{\tau x2}, p_{\tau x3})\) and \((p_{\tau y1}, p_{\tau y2}, p_{\gamma y3})\) are PAC 2002 tyre parameters, which are determined experimentally. \(F_{\sigma}\) is the reference normal load, \(df_x\) is the relative variation of the normal load \(F_x\) with respect to the reference load \(F_{\sigma}\), \(r_0\) stands for unloaded tyre radius, and \(\gamma\) denotes the camber variation.

The longitudinal slip and slip angle can be calculated, based on the string deformation as:

\[
\kappa' = \frac{u}{\sigma_x} \cdot \text{sign} \left( U_w \right) \quad (6.40)
\]

\[
\alpha' = \tan \left( \frac{v}{\sigma_u} \right) \quad (6.41)
\]

The Magic Formula equations based on the transient slip quantities \(\kappa'\) and \(\alpha'\) can be evaluated to calculate the transient forces and moments as:

\[
F_x = F_x \left( \alpha', \kappa', F_x \right) \quad (6.42)
\]

\[
F_y = F_y \left( \alpha', \kappa', \gamma, F_x \right) \quad (6.43)
\]

\[
M_z = M_z \left( \alpha', \kappa', \gamma, F_x \right) \quad (6.44)
\]
6.3.2 Modified Stretched-String Tyre Model

In the previous stretched string tyre model, the relaxation length is expressed as a variable of vertical load. In that case, the tyre slippage is not taken into account, while defining the relaxation length, which means the relaxation length does not change with an increasing slip angle, unlike what happens in practice. To overcome the limitations of the bare (simple) stretched string model, (Pacejka, 2006) suggested a stretched string model with an elastic tread element. Figure 6-4 shows the deflected string model with tread elements, which are shown at various levels of steady-state side slip.

![Figure 6-4: Stretched-string model with tread elements (Pacejka, 2006)](image)

The relaxation length can also be defined as the distance between the leading edge of the contact and the point of intersection of the wheel plane and elongation of the straight contact line (as shown in Figure 6-4). The relaxation length or the 'intersection length' \( \sigma^* \) decreases with an increase in the slip \( \alpha \) in this model.

The lateral relaxation length \( \sigma^*_a \) can be expressed as the ratio of lateral deflection \( v_1 \) and the transient lateral slip \( \tan \alpha' \) as:

\[
\sigma^*_a = \frac{v_1}{\tan \alpha'} \quad (6.45)
\]

The first order relaxation length equation in this model is similar to the stretched string model (equation (6.36)), with the relaxation length \( \sigma_a \) replaced by \( \sigma^*_a \).
The relaxation length in this model is determined by considering the transient effects of variation in vertical load as well as slip. The $\sigma_a^*$ can be represented as:

\[
\sigma_a^* = \frac{1}{C_{Fy}} \frac{F_y}{\tan \alpha'} = \frac{\sigma_{ao}}{C_{F_{ao}}} \frac{F_y}{\tan \alpha'} \approx \frac{\sigma_{ao}}{C_{F_{ao}}} \frac{F_y'}{\tan \alpha'_F} + C_{F_{ao}} \cdot \epsilon_{F_i}.
\]  

(6.47)

where, $\sigma_{ao}$ denotes the initial relaxation length at $\alpha = 0$, which is given as:

\[
\sigma_{ao} = \frac{C_{F_{ao}}}{C_{Fy}}.
\]  

(6.48)

In equation (6.47), the last term contains the term $\epsilon_F$ so as to avoid singularity in the equation (Pacejka, 2006). For the same reason, a shift $\Delta \alpha$ is added to the transient slip $\alpha'$ to arrive at $\alpha'_F$. Another approach may also be adopted, where the transient slip $\tan \alpha'$ is used directly in equation (6.46) and thus the term $\sigma_a^*$ is avoided. However, it means that $\tan \alpha'$ will need to be determined from the lateral deflection $v_i$. Higuchi and Pacejka (1997) followed this approach where the transient slip is determined from the lateral deflection $v_i$, using the inverse steady-state tyre characteristics $F'_y\left(\alpha'\right)$. This requires information about the lateral force – transient slip slope *apriori*, and also the equations need to be linearised by considering small deviations from steady-state situation. Alternatively, Higuchi and Pacejka (1997) also adopted a more practical approach, where they used an estimation algorithm to determine $\alpha'$ from the deflection value $v_i$ and then use the estimated slip in equation (6.46).

However, in the present work, the relaxation length $\sigma_a^*$ (from equation (6.47)) is directly used in equation (6.46) to avoid any complexity in the calculations. In order to avoid an algebraic loop while carrying out the numerical simulation in Simulink, the value of $\sigma_a^*$ is used from the previous time step in the model. The final force value from the tyre model is then obtained in a conventional way (equation (6.43)), after computing the transient slip $\alpha'$ from equation (6.45).
Similar to the lateral relaxation model, the first order differential equation for the longitudinal deflection can be represented as:

\[
\frac{du_i}{dt} + \frac{1}{\sigma^*_x} \left| U_{w} \right| \cdot u_i = \left| U_{w} \right| \cdot \kappa = -V_{xx}
\] (6.49)

where, the longitudinal relaxation length \( \sigma^*_x \) can be obtained using the following expression:

\[
\sigma^*_x = \frac{1}{C_{F_x}} \cdot \frac{F_{x}}{\kappa'} = \frac{\sigma_{so}}{C_{F_k}} \cdot \frac{F_{y}}{\kappa'} \cdot \frac{\sigma_{so}}{C_{F_k}} \cdot \frac{\left| F_{x}' \right| + C_{F_x} \cdot e_F}{\left| \kappa' \right| + e_F}
\] (6.50)

The term \( \sigma_{so} \) represents the initial relaxation length at \( \kappa' = 0 \), which is given as:

\[
\sigma_{so} = \frac{C_{F_k}}{C_{F_k}}
\] (6.51)

The transient slip can then be computed from the longitudinal deflection \( u_i \) through:

\[
\kappa' = \frac{u_i}{\sigma^*_x}
\] (6.52)

The final force value from the tyre model is then calculated in a conventional way (refer to equation (6.42)).

### 6.3.3 Contact Mass Transient Tyre Model

The stretched-string models described in the previous sections are based on the relaxation length concept, which takes into account the combined effect of carcass compliance and contact patch slip properties, and in the process accommodate the lag in the response to lateral and longitudinal slips. Since the response lag is dependent on the vertical load and the wheel slip variations, the relaxation length in the modified stretched string model reduces at a higher level of slip. However, this approach is numerically not very stable and also the situation at combined slip becomes cumbersome (Pacejka and Besselink, 1997). To overcome this problem, a slightly different approach can be followed, based on the separation of the carcass compliance and contact slip properties by explicitly incorporating carcass springs in the model. This approach was initially employed by Pacejka and Besselink...
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(1997), based on an earlier work by van der Jagt et al (1989), in their 1997 version of the Magic Formula transient tyre model, which was later incorporated in the PAC 2002 tyre model (Kuiper and Van Oosten, 2007). Figure 2-3 shows the structure of the contact mass transient tyre model. In the model, the contact patch with inertia is defined such that it can deflect in the circumferential and lateral directions with respect to the lower part of the rim. The mass point coincides with point \( S^* \), which is the displaced contact point from the original wheel plane. The velocity of point \( S^* \) constitutes the slip speed of the contact point. Here, it should be noted that the carcass compliance together with the slip model of the contact patch automatically takes care of the vertical load dependent lag, in addition to a decrease in lag with the increasing (combined) slip (Pacejka, 2006).

![Figure 6-5: Top and side view of the contact mass model (Pacejka, 2006).](image)

By introducing contact patch mass \( m_c \), carcass stiffness \( k_{cxy} \) and damping ratios \( c_{cxy} \), the equations of motion for the contact can be represented in the following form, employing one degree of freedom model for the single point mass:

\[
\begin{align*}
    m_c \cdot \ddot{V}_x^* + c_{cx} \cdot \dot{u} + k_{cx} \cdot u &= F_x (\kappa, \alpha', F_x) \\
    m_c \cdot \ddot{V}_y^* + c_{cy} \cdot \dot{v} + k_{cy} \cdot v &= F_y (\alpha', \kappa', \gamma, F_x) \cdot F_{y, NL}
\end{align*}
\]

(6.53)  
(6.54)

where, \( u \) and \( v \) represents the longitudinal and lateral deflection of the carcass, \( F_x, F_y \) represents the longitudinal and lateral force acting from the ground to contact patch. \( F_{y, NL} \) represents the non-lagging part of the camber force, which is assumed to act directly on the wheel rim. The force response to camber variation is instantaneous and not lagged, as shown by experimental evidence (Pacejka, 2006). This non-lagging camber force acts in the direction opposite to the steady-state side force, and is caused by the non-symmetric
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distortion of the cross-section of the lower part of the tyre (Pacejka, 2006). The non-lagging camber force part can be approximated by a linear relation:

\[ F_{y,NL} = G_{VK} \cdot e_{NL} \cdot C_{Fy} \cdot \gamma \]

where \( \gamma \) denotes the camber, \( G_{VK} \) denotes the weighing function, \( C_{Fy} \) denotes the camber thrust stiffness, and \( e_{NL} \) denotes the non-lagging fraction.

In addition, the contact mass model also includes a simple relaxation length model to enable computations near zero speed.

\[ \sigma_c \frac{dk'}{dt} + |U_w| \cdot k' = -V_{ss}' \quad (6.55) \]

\[ \sigma_c \frac{d\alpha'}{dt} + |V_w| \cdot \alpha' = -V_{sy}' \quad (6.56) \]

From these equations the transient slip quantities \( k', \alpha' \) are obtained, which act as an input in the steady-state slip force model (right hand side of equations (6.53) and (6.54)). The contact relaxation length \( \sigma_c \) can be given a small value such as half the contact length. The equations (6.55) and (6.56) do not respond to load changes.

The longitudinal and lateral deflection rates required in equations (6.53) and (6.54) can be computed by the difference in the slip velocities:

\[ \dot{u} = V_{ss}' - V_{ss} \quad (6.57) \]

\[ \dot{v} = V_{sy}' - V_{sy} \quad (6.58) \]

where, \( V_{ss}, V_{sy} \) denote the wheel slip velocity of the slip point S, which is attached to the wheel rim at distance \( r_e \) below the wheel centre in the wheel centre plane. \( r_e \) denotes the effective rolling radius (refer (4.12)). The \( V_{ss}, V_{sy} \) can be represented as:

\[ V_{ss} = U_w - r_e \cdot \omega \quad (6.59) \]

\[ V_{sy} = V_w - r_e \cdot \dot{\gamma} \quad (6.60) \]
Once the longitudinal and lateral deflections are obtained by solving equations (6.53) and (6.54), the longitudinal and lateral forces acting on the wheel rim, finally, become:

\[ F_{xa} = c_{ex} \cdot \dot{u} + k_{ex} \cdot u \]  
\[ F_{ya} = c_{ey} \cdot \dot{v} + k_{ey} \cdot v \] 

In the aforementioned equations of contact mass model, the yaw rate terms are not included in the equations. The yaw terms can be easily incorporated in the model, which will enable the contact body deflection in the yaw direction to be evaluated (Kuiper and Van Oosten, 2007). Also, the gyroscopic effects related with belt distortions can be taken into account.

### 6.4 Anti-Lock Braking System

The Anti-lock Braking system used in this thesis is based on a conventional peak seeking approach, where the wheel angular deceleration and slip thresholds are used to determine the brake control cycle, which is an adaptation of the Bosch work (Bosch, 1999). The ABS brake system is modelled here using an S-function, where the characteristics of solenoid valves are represented in the algorithm itself. The detailed dynamics of the ABS modulator is not considered here, primarily because of lack of data and also it would have required much detailed work on ABS control itself, thus taking the focus away from the current research. In the past, many researchers such as Mills et al. (2002) have included modulator dynamics in their non-linear ABS control. However, they have often relied on linearised vehicle equations to arrive at a solution. The approach used in this work is based on the representation of ABS control algorithm using a simplified model free closed loop control, which is then integrated into a detailed rigid body vehicle model. A similar approach has been used in the past by researchers such as Ozdalyan and Blundell (1998) and Day and Roberts (2002), where they applied generic ABS control in a detailed full vehicle model.

In this thesis, the control of brake pressure takes place in the ABS System. The individual brake pressure at the wheel is then translated into brake torque using the relationship expressed in the brake system model, described in chapter 3. The variables used in the basic control cycle are adjusted, depending upon the operating conditions such as high or low friction surface, representing surface such as dry concrete, wet concrete and dry packed snow. The road disturbance in this model is neglected and also any disturbance initiated by
engine is not considered, as the braking is assumed to take place in the drive disengaged condition.

6.4.1 ABS Theory

As the primary function of an anti-lock braking system is to prevent wheel locking while maintaining the directional stability of a vehicle at all times, it should be capable of exploiting the frictional force between tyre and road surface to the maximum. In case of braking of a single wheel model, with an assumption that the drive torque from the engine is in a disengaged condition, there will be two major torques that would act on the wheel. One is the brake torque $M_b$, which acts to reduce the wheel velocity. The other is the moment generated by the road friction force $F_x$, which acts in the opposite direction to the applied brake torque. The friction force $F_x$ on the one hand accelerates the wheel velocity, whilst on the other slows down the vehicle velocity. The difference between the two torques leads to an angular acceleration $\dot{\omega}$ of the wheel as:

$$\dot{\omega} = \frac{F_x \cdot r_e - M_b}{I_{\text{wheel}}}$$  (6.63)

where, $I_{\text{wheel}}$ is the mass moment of inertia of the wheel assembly about its centre and $r_e$ is the effective rolling radius of the tyre. When the difference between $F_x \cdot r$ and $M_b$ is positive, the wheel accelerates, and when it is negative, the wheel decelerates. The brake torque $M_b$ in the latter case decelerates the wheel, but the friction force $F_x$ from the tyre eventually stops the vehicle.

When accelerating or braking forces are acting on a wheel, the tyre contact patch is subjected to the slip phenomena, in the longitudinal and lateral direction. The longitudinal brake slip can be given as

$$\lambda = \frac{\omega \cdot r_e - U_w}{U_w}$$  (6.64)

where $U_w$ denotes the vehicle velocity transformed at the wheel, and $\omega \cdot r_e$ denotes wheel peripheral velocity. When the wheel peripheral velocity is equal to the vehicle velocity, no wheel slip exists. This corresponds to the free rolling condition. On the other hand, when the
wheel peripheral velocity is equal to zero, the brake slip is equal to -1, which means the wheel is locked.

The brake force coefficient $\mu_b$ acting between tyre and road surface is a non linear function of the braking slip $\lambda$. Figure 6-6 shows the relationship of brake force coefficient with brake slip, also known as $\mu$-slip curve. The braking force coefficient $\mu_b$ is zero at $\lambda = 0$. It increases rapidly and reaches a peak value at some intermediate value of the brake slip $\lambda$, generally between 0.1 and 0.3, depending on the tyre and road conditions. Once attaining the peak, it decreases with an increase in the brake slip. The sector of the curve ‘a’ where the brake force coefficient increases is called the stable range, whereas the sector ‘b’ where the brake force coefficient falls is called the unstable range.

Figure 6-6 also shows the plot of lateral force coefficient $\mu_s$ with the brake slip $\lambda$. The lateral forces generated at the tyres during cornering provide the lateral stability, counteracting the centrifugal forces, which try to throw the vehicle outwards from its path. The coefficient of lateral force $\mu_s$ rises to its maximum value with no brake slip. As the brake slip increases, the lateral force coefficient drops gradually at the start, and more rapidly with the increase in the brake slip. At the bottom of the curve, the locking of the wheel occurs. A locked wheel has no lateral stability and less longitudinal braking friction forces. Therefore braking with wheels in locked situation causes longer stopping distance and higher lateral instability.

![Figure 6-6: $\mu$-slip curve (Austin and Morrey, 2000)](image-url)
During the start of braking, the wheel slip begins to rise. An increase in the brake torque causes a reduction in the wheel peripheral velocity and thus leads to an increase in the slip velocity (i.e. $U_w - \omega \cdot r_c$). The frictional force from the tyre $F_x$ causes the wheel peripheral velocity to increase and thus leads to a decrease in the slip velocity. In the stable part of the $\mu$-slip curve (Figure 6-6), an increase in the brake slip $\lambda$ leads to a higher frictional force $F_x$ as a result of an increase in the braking force coefficient $\mu_b$. The higher frictional force $F_x$ reverses the wheel slip to a small value. However, in the unstable part of the $\mu$-slip curve, an increase in the brake slip $\lambda$ leads to a lower frictional force $F_x$ as a result of a reduction in the braking force coefficient $\mu_b$, which eventually causes the wheel slip to increase continuously. During normal braking, if the brake torque is too slow, the wheel slip may stop increasing and starts to decrease before the brake force coefficient reaches its peak point. However, if excessive brake torque is applied, the brake slip may jump straight to a large value. The brake force coefficient in this case will pass its peak point and reach somewhere in the unstable part of the $\mu$-slip curve. If the brake torque is not reduced quickly at this point, the increase in the braking effort will lead to a rapid increase in wheel slip, and eventually wheel locking occurs. By detecting the time when the peak point could be reached, an ABS system reduces the brake torque accordingly so as to avert the wheel lock up.

Figure 6-7: Relationship between braking and road frictional torque (after Bosch, 1999).
The relationship between the braking torque $M_b$ and the road frictional torque $M_r \left( = F_x \cdot r_c \right)$ is shown in Figure 6-7, along with the plot of the peripheral wheel deceleration with time. The braking torque increases linearly over time, whereas the road friction torque follows the braking torque with a slight lag $T_s$ for as long as the braking process remains in the stable region of the $\mu$-slip curve. Once the road friction torque reaches its maximum value $M_{r,\text{max}}$, it enters the unstable region, where the braking torque $M_b$ continues to increase, whereas the road friction torque $M_r$ cannot rise and drops gradually. During this period, the differential $M_b - M_r$, which remains minimal in the stable region, quickly attains larger proportions. Also, the peripheral deceleration, which is restricted to low values in the stable region increases rapidly, through transition to the unstable region. The threshold value of wheel peripheral deceleration, beyond which the wheel is susceptible to locking, is often employed by the ABS system to determine the slip rate corresponding to optimal braking.

Ideally, an ABS system should be keeping the wheel slip at the peak point of the $\mu$-slip curve, but the characteristics of the braking force coefficient changes with the condition of tyre and road surface, as shown in Figure 6-8. For example, the peak value and slip ratio is different for a dry road surface as compared to an icy road surface. This makes it difficult to hold the braking force coefficient at the peak, just based on the slip ratio. Also, staying at the peak point can affect the lateral stability of the vehicle. Thus, a balance needs to be made between the lateral stability, which is best at $\lambda = 0$, and frictional force $F_x$, which usually is good for $\lambda$ between 0.1 and 0.3. Many control strategies work by keeping the slip ratio $\lambda$ near to a value of 0.2.

![Figure 6-8: $\mu$-slip curve for different surfaces (Bosch, 1999).](image-url)
The brake force coefficient $\mu_b$ and the lateral force coefficient $\mu_s$ change their peak values with the tyre slip angle. The typical tyre lateral force and brake force coefficients as a function of brake slip for different slip angle are shown in Figure 6-9. It can be seen that for the brake force coefficient, the optimum slip value at which the peak value occurs, increases with the higher values of tyre slip angle. In case of the lateral force, with an increase in the slip angle, the lateral force coefficient shifts towards a higher value. This happens as lateral acceleration rate increases with an increase in the slip angle, leading to a higher value of lateral force coefficient. The target for an ABS system, in general, is to maintain these friction coefficients within a maximum range under different operating conditions, so that the maximum braking effort could be obtained from the road surface, without compromising on lateral stability. This ensures a minimum stopping distance and good directional stability; an optimal compromise between the different performance requirements would be needed.

![Figure 6-9: $\mu$-slip curve as a function of slip angle (Austin and Morrey, 2000)](image)

### 6.4.2 ABS System Description

In a passenger car, different layouts of ABS system can be used, where the number of channels and sensors can vary in some particular combinations, the most common of which are four channels and four sensors, three channels and three sensors, and three channels and four sensors. The chosen layout eventually affects the directional control, stability and braking distance of a vehicle during its straight line motion, while cornering, or even on an asymmetric surface with different adhesion coefficients for the left and right side tyres (Leiber and Czinezel, 1983). In a four channels and four sensors' layout (used in this thesis), the front two tyres are controlled individually, based on the information obtained by the
respective sensors. The rear tyres, however, are jointly controlled, based on a select-low operating mode, which means the two tyres are controlled jointly with the same brake pressure, using the information from the slowest of the two tyres. The other option is the select-high operating mode, where the brake pressure in both the tyres is controlled jointly by using the information from the faster tyre. The select-low operating mode ensures vehicle directional stability when braking on an asymmetrical road or in a turn, in contrast to the select-high operating mode. In the latter case, the inside tyre or the lower friction side tyre would lock-up fast, while the opposite side tyre would develop a higher braking force, resulting in an adverse effect on the vehicle directional stability.

In a four channels and four sensors’ layout, the control system monitors the wheel speed at each of the four sensors, during the vehicle motion. This layout comprises two brake circuits with either front/rear or diagonally opposite splits. Figure 6-10 shows a four channels ABS layout with a diagonal split, where each wheel has two 2/2 solenoid valve configuration. When an impending wheel lock is detected, the control system responds by actuating the solenoid valves for the affected wheels in the hydraulic modulator. At the front wheels, the individual solenoid valves control their respective wheels’ responses to ensure the wheel’s maximum potential for effective braking regardless of the condition of the other wheel. At the rear axle, due to the select-low operating mode, two different solenoid valves are used to control the rear wheels’ brake pressure together.

![Figure 6-10: A four-channels’ ABS layout with diagonal split (Bosch, 1999)](image-url)
6.4.2.1 Solenoid Valve Configuration

In the Bosch ABS control cycle, the solenoid valve configuration is based on the five pressure gradient solution (Leiber and Czinczel, 1979), which essentially determines the way the brake pressure is varied in a control cycle. The different pressure gradients are: quick pressure increase, quick pressure decrease, pressure retention, quick pressure increase, and slow stepwise pressure increase. These pressure variations are shown in the brake pressure plot of Figure 6-12. The pressure gradients can be achieved through a 3/3-solenoid valve (Figure 6-11), which is a solenoid-actuated directional-control valve with three different ports and three operating positions (Bosch, 1999). In a four channels and four sensors' layout, each wheel has its own 3/3 solenoid valve and the pressure modulation in each wheel brake cylinder is carried out by the control unit so as to regulate the pressure build-up, pressure hold, and pressure reduction operations.

In the pressure build-up phase, the solenoid is at its non-energised position, which allows unrestricted braking pressure build-up during normal braking or in the pressure increase phase when ABS is active. In this stage, the two springs in the armature i.e. main spring and auxiliary spring exerts mutually opposite forces. As the main spring is under higher tension than the auxiliary unit, the resulting force opens the input valve, thus connecting the master cylinder port and the wheel cylinder port. In the pressure holding phase, the valve is energised by half of the maximum current and as a result the armature shifts until the input valve is closed. This leads to interruption of the passage between the master cylinder port and the wheel cylinder port, thus preventing any further increase in the brake pressure. In this position the armature remains in the intermediate position, where both the input and output valves are closed and, thus holds the brake pressure at a constant level.

In the pressure reduction phase, the maximum current is applied to the winding, which allows the armature to overcome the force exerted by both the springs, and thus opens the outlet valve. In this position, as the input valve remains closed and the output valve is opened, the brake pressure is released through the passage between the wheel cylinder port and the return line or the accumulator. Once sufficient pressure is discharged from the wheel cylinder, the solenoid valve normally returns to the pressure holding phase. The above configuration, not only allows continuous operation, but also provides stepped increase or reduction in the brake pressure. The stepped build-up in the brake pressure is achieved by applying a pulse train to the solenoid valve between the first and second valve positions.
The 3/3 solenoid valve configuration was originally used in the Bosch ABS 2S version (Bosch, 1999). Later, in Bosch ABS 5.0 version, it was replaced with two 2/2 solenoid valves (see Figure 6-10), which employs two regulator valves in form of isolation and exhaust valves. During normal braking and pressure increase situation, the isolation valve remains open and the exhaust valve remains closed. This allows pressure from the master cylinder to act directly upon the wheel cylinder. During the pressure hold situation, both isolation and exhaust valves are closed off, which keeps the pressure to the wheel cylinder constant regardless of the driver input. For a pressure reduction situation, the isolation valve is closed and the exhaust valve is open, which means the brake fluid flows back into the accumulator and the pressure in the wheel cylinder decreases accordingly. In general the isolation valve is of normally open type and the exhaust valve is normally a close type, which ensures that during normal braking, the ABS system has no effect.

The later version of Bosch ABS (version 5.0) with 2/2 valve configuration improved the safety and also was designed for integration with the traction control system (TCS). However, the basic control cycle remains the same, irrespective of the different valve configuration. It should be noted that the structure of the control algorithm in this thesis is adopted from the basic ABS control cycle and is not based on any particular ABS version of Bosch.
6.4.2.2 ABS Control Variables

As control variables play a key role in the functioning of an ABS system, its selection is an important part in any particular control philosophy, which eventually determines the efficiency of an ABS system. The basis for the control variables is provided by the signals from the wheel speed sensors, which are utilized by the control unit to calculate wheel peripheral deceleration, brake slip, reference vehicle speed and vehicle deceleration. The variables such as wheel peripheral deceleration and brake slip, on their own, are not suitable for use as a controlled variable as the reaction that a driven wheel displays is vastly different from that of a non-driven wheel to braking (Bosch, 1999). These variables, however, can be used in certain logical combination to obtain satisfactory results. As the brake slip cannot be measured directly due to lack of a practical and cost effective means to determine longitudinal speed of the tyre centre, the control unit therefore calculates a reference vehicle velocity for optimal brake slip conditions. The reference vehicle velocity may be determined, using information from wheel speed sensors (Jiang and Gao, 2000) or through combination of sensors and an estimation algorithm (Bowman and Law, 1993).

The control variables in this thesis are selected based on the information of brake slip and wheels’ peripheral acceleration / deceleration, which are then used in certain combination in the brake control cycle. As the work involves pure simulation, the vehicle velocity is directly available through equations of motion. For any future extension of the current research, it is not unduly difficult to incorporate the calculation of reference velocity by using wheel speed information in the vehicle model, as shown by Jiang and Gao (2000).

6.4.2.3 ABS Control Cycle

A typical Bosch ABS control cycle is shown in Figure 6-12, for road surfaces with high friction coefficient. In the cycle, the brake pressure application is divided into eight phases, based on different brake pressure gradients. Phase 1 shows the start of the control cycle, where the output brake pressure is kept equal to the input pressure (from the master cylinder), until the wheel’s peripheral deceleration moves beyond the defined threshold (-\(a\)) at the end of phase 1. In phase 2, the brake pressure is held constant as the solenoid valves move to the pressure hold position. The brake pressure is not reduced in this phase as the deceleration threshold (-\(a\)) could have been exceeded with the friction force coefficient still in the stable range of the \(\mu\)-slip curve. This phase continues until the wheel’s peripheral
velocity $V_r$ exceeds the slip threshold velocity $\lambda_1$. The slip threshold velocity is based on the reference velocity $V_{ref}$ which is determined by a preset deceleration and hence reduces at the start of phase 2. In the current work, the reference velocity $V_{ref}$ is determined using a nonlinear filter method, which is explained later in section (6.4.3.1).

![Figure 6-12: Typical plot of a bosch abs braking cycle (Bosch, 1999)](image)

In phase 3, the brake pressure is reduced as the solenoid valve moves to the pressure release position. This phase continues until the wheel’s peripheral deceleration exceeds the threshold ($-a$) again. In phase 4, the brake pressure is held constant, where the wheel’s peripheral acceleration exceeds the threshold ($+a$). This phase continues until the wheel’s peripheral acceleration exceeds the pronounced threshold ($+A$). In phase 5, the brake pressure continues to increase for as long as the wheel’s peripheral acceleration remains above the threshold ($+A$). In phase 6, brake pressure is maintained at a constant value for the time the peripheral acceleration stays above the threshold ($+a$). This phase is indicative of the fact that the wheel has entered the stable range in the $\mu$-slip curve but still has not reached its maximum braking potential. In phase 7, the brake pressure is built-up in stages as the solenoid valve applies a stepwise pressure increase. In this stage greater braking performance is achieved close to peak friction, while minimising the wheel lock-up potential. This phase continues until the wheel’s peripheral deceleration exceeds the threshold ($-a$) again. In phase 8, the brake cycle returns to phase 3, without waiting for the threshold $\lambda_1$, and finally a new control cycle starts.
6.4.3 ABS Implementation in the Vehicle Model

The ABS algorithm used in this work is based on the Bosch control cycle (Figure 6-12), as stated earlier. However, there are some changes made to the Bosch control cycle, while adopting it for the current vehicle model. These changes are primarily concerned with the threshold conditions used for different phases (section 6.4.2.3), as well as system variables in the control cycle. Figure 6-13 shows the plot of the ABS control cycle used in this thesis.

![ABS control cycle](image)

Figure 6-13: ABS control cycle

The actual algorithm used in a commercial ABS system is much more detailed, as it incorporates a number of practical considerations. Also, due to its proprietary nature, the data for a commercial ABS algorithm is not easily accessible. The algorithm used in this work relies more on the concept rather than the exact nature of the data, as the focus is to apply a realistic ABS system for a tyre transient study, as opposed to using a detailed ABS system for design or performance related studies. Also, instead of modelling the dynamics of the ABS hydraulic modulator, the algorithm takes into account the intrinsic characteristics of the modulator in a simplified way. These characteristics include delays associated with the opening and closing of solenoid valves, brake pressure apply and release rates, apply delay while the solenoid is in the pressure holding phase, filters used for deceleration measurement and also the characteristics of a pulse train where the solenoid valve holds and builds up the
pressure in a stepwise manner. The other characteristics of a control cycle such as deceleration and slip thresholds are also represented through the algorithm.

In the ABS control cycle adopted here, the angular wheel deceleration is used as a control variable rather than wheel peripheral deceleration. This has no conceptual basis and is just used for convenience as the wheel rotational speed can be directly differentiated without multiplying it with the effective rolling radius. However, the differentiating circuit in a control system often employs filters for signal processing. This means a cut-off frequency is needed, which can filter out the noise at a lower frequency such that any faulty deceleration threshold can be avoided. In this research work, a low-pass filter is chosen with a cut-off frequency of 15 Hz. In phase 3 and 8 of the ABS control cycle (Figure 6-13), the brake pressure is reduced until the angular wheel deceleration becomes positive, which is slightly different from the condition used in the Bosch ABS system. In phase 4, the brake pressure is held constant for a specified time duration (also referred to as the apply delay) or until the angular wheel acceleration exceeds the pronounced threshold value ($\dot{\omega}_{th,\text{max}}$). A similar condition is used in phase 6, where the brake pressure is held constant for a specified time duration or until the angular wheel acceleration drops below the acceleration threshold ($\dot{\omega}_{th,\text{max}}$). In phase 7, the step pressure increase is attained by using an adjustable duty cycle, where the time duration for the pressure build up and pressure hold phase is specified. This represents the pulse train for modulating the brake torque so that the maximum braking potential is achieved. In the brake control cycle, the brake pressure apply and release rates are important parameters, which depend on the characteristics of the modulator. However, here these rates are kept adjustable, based on the ABS operating conditions. The list of parameters used in the ABS control cycle along with their description is provided in Table 6-1. The flow chart of the ABS braking control cycle is shown in Figure 6-14.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel minimum angular deceleration threshold $\dot{\omega}_{th,\text{min}}$</td>
<td>This parameter sets the value for the wheel angular deceleration, and decides when to release the brake pressure. It signifies the value beyond which the wheel is susceptible to locking, and is based on the optimal slip condition for ABS braking.</td>
</tr>
<tr>
<td>Wheel maximum angular acceleration threshold $\dot{\omega}_{th,\text{max}}$</td>
<td>This parameter is used in the control cycle to decide when to stop the brake release phase and also where to start the step brake pressure build-up phase once a wheel reaches a lower slip value.</td>
</tr>
</tbody>
</table>
Influence of Tyre Transience in ABS Braking

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel pronounced angular acceleration threshold $\omega_{th, pmax}$</td>
<td>This parameter is used in the control cycle to decide when to start the quick brake pressure increase phase. Its value may be 8-10 times of wheel maximum angular acceleration threshold $\omega_{th, max}$.</td>
</tr>
<tr>
<td>Brake pressure apply rate</td>
<td>This parameter stores the rate of output pressure rise. Two different values may be used in the control cycle, where the high value is used for the initial brake pressure apply rate and the lower value for the subsequent brake pressure apply rate. This is done to avoid suspension and drivetrain resonances, which may get triggered by high pressure fluctuation in ABS cycle (Bosch, 1999).</td>
</tr>
<tr>
<td>Brake pressure release rate</td>
<td>This parameter stores the solenoid valves pressure decay rate. Unlike the apply rate, single value is used in this case.</td>
</tr>
<tr>
<td>Pressure build up time</td>
<td>This parameter stores the time duration for which the input valve stays open during the step brake pressure build-up phase.</td>
</tr>
<tr>
<td>Pressure hold time</td>
<td>This parameter stores the time duration for which the input and output valves are closed during the step brake pressure build-up phase, so that the output brake pressure is held constant.</td>
</tr>
<tr>
<td>Apply delay</td>
<td>This parameter stores the value of the delay period preceding the brake pressure build-up phase.</td>
</tr>
<tr>
<td>Release delay</td>
<td>This parameter stores the value of the delay period preceding the brake pressure release phase.</td>
</tr>
<tr>
<td>Vehicle velocity threshold</td>
<td>This parameter stores the minimum vehicle velocity, below which the ABS is not actuated. In the current work, its value is set as 2 m/s.</td>
</tr>
<tr>
<td>Minimum threshold pressure</td>
<td>This parameter stores the minimum system threshold pressure, below which the ABS is not actuated.</td>
</tr>
<tr>
<td>Maximum threshold pressure</td>
<td>This parameter stores the value of the maximum brake pressure that can be generated by ABS system for individual wheel cylinders.</td>
</tr>
<tr>
<td>Reference velocity $V_{Ref}$</td>
<td>The reference velocity is used to calculate the slip threshold velocity in the ABS brake cycle. It is determined based on the information obtained from the individual wheel velocities.</td>
</tr>
<tr>
<td>Slip threshold $\lambda_1$</td>
<td>The slip threshold is used in the first ABS cycle, where it decides when to initiate the brake release valve.</td>
</tr>
</tbody>
</table>

Table 6-1: Parameters used in the implemented ABS control cycle
Figure 6-14: Flow chart of the implemented ABS control cycle
6.4.3.1 Reference Velocity in ABS Cycle

The reference velocity $V_{\text{ref}}$ used in the control cycle is determined using the information from the wheel velocities, using an adaptive non-linear filter method. Here, the gain $H_{\text{ref}}$, which reflects the vehicle deceleration, is first established using the peak values of the wheel velocities in an ABS cycle. Figure 6-15 shows the plot of the wheel velocity peaks and the reference velocity during an ABS cycle.

![Figure 6-15: Reference velocity plot](image)

At the start of the braking cycle, the reference velocity is assumed to be equal to the wheel velocity, marked as point '1' on the Figure 6-15. The reference velocity value stays equal to the wheel velocity, until the point where certain threshold value is crossed and phase 2 in the ABS brake cycle commences. The reference velocity at this point decreases based on an initial value of $H_{\text{ref}}$, which is chosen as a constant deceleration value for a particular road surface. The wheel velocity also drops, but exceeds the reference velocity at point '2' during the accelerating phase. This indicates that the initial value of the gain $H_{\text{ref}}$ has been large and so the reference velocity is allowed to follow the wheel velocity until point '3'. At point '3' the wheel velocity reaches its first peak value and the slope between point '1' and '3' is used to determine the new value of $H_{\text{ref}}$. As $H_{\text{ref}}$ acts as a gain in the non-linear filter, the reference velocity $V_{\text{ref}}$ assumes the peak value of the wheel velocity. In the subsequent wheel oscillations, whenever the wheel velocity reaches its peaks, such as point '4', '5', and '6',...
the gain $H_{\text{Ref}}$ is adjusted so that it reflects the current vehicle deceleration. If the wheel velocity exceeds the reference velocity $V_{\text{Ref}}$, it shows that the previous value of gain $H_{\text{Ref}}$ was set a little higher and thus leads to a lower value of the reference velocity. The reference velocity is then assigned the same value as that of the wheel velocity, as a wheel during braking cannot exceed the reference velocity value. In this process, the gain value of $H_{\text{Ref}}$ is always adjusted based on the slope between the current peak and the starting point of the ABS operation i.e. at point ‘1’, rather than the slope between the two adjacent peaks. This is achieved in order to filter out the effect of any noisy peaks.

The equation for the nonlinear filter can be written as:

$$V_{\text{Ref}} = -H_{\text{Ref}} \cdot \text{Sat}(V_{\text{Ref}} - V_w, d)$$  \hspace{1cm} (6.65)

where, $V_w$ is the input value representing the wheel velocity, $V_{\text{Ref}}$ is the output value representing the reference velocity, and $H_{\text{Ref}}$ is the gain which reflects the vehicle deceleration and its initial value is chosen as a constant deceleration value. Sat(*) is a saturation function, which is defined as:

$$f(x) = \text{Sat}(x,d) = \begin{cases} 1, & \text{when } x > d \\ -1, & \text{when } x < -d \\ x/d, & \text{else} \end{cases}$$  \hspace{1cm} (6.66)

where d is a small positive number such as 0.01. The saturation function is chosen to avoid numeric oscillations, for the case when the value ($x = V_{\text{Ref}} - V_w$) becomes zero.

The non-linear filter described above has a similar functioning to that of a bang-bang controller, where the output $V_{\text{Ref}}$ converges to the input $V_w$ in steady-state. The change of $V_{\text{Ref}}$ is limited only by the gain $H_{\text{Ref}}$ in this filter, which is continuously adjusted to reflect the vehicle deceleration based on the wheel velocity peaks. This methodology was initially presented by Jiang and Gao, (2000) to estimate the vehicle longitudinal velocity in their ABS control. Though, in this simulation work, the vehicle velocity is directly available through equations of motion, but the methodology can be adopted to establish the reference velocity at each wheel, as explained above. The reference velocity at each wheel can then be used to obtain the slip threshold velocity $\lambda_1$, which acts as a variable in the ABS brake cycle. The slip threshold velocity (shown in Figure 6-15) can be represented as:
\[ \lambda_t = V_{\text{Ref}} \cdot (1 - s_{\text{Ref}}) \]  \hspace{1cm} (6.67)

where, \( s_{\text{Ref}} \) represents a threshold slip value used in the wheel brake cycle.

### 6.5 ABS Simulation

During a critical manoeuvre, the response behaviour of a vehicle, while being intervened by an active control system such as ABS, is affected by the tyre oscillations that occur due to the brake torque variations and road unevenness. For performing a simulation study of such a critical manoeuvre, the tyre model should be able to respond to the dynamic changes which it encounters during an ABS operation, as the transient response of a tyre can have a significant bearing on the reliable prediction of the vehicle behaviour.

The influence of tyre transient behaviour on the anti-lock braking response is studied in the current research by applying the earlier described transient tyre models and the ABS system. The ABS control subroutine (written in C-code) was integrated into the vehicle model using the S-Function, which provides an external interface to MATLAB-Simulink environment. This combined approach of MATLAB-Simulink linked with S-function, provided a robust yet computationally efficient way of arriving at a solution. Also, the modelling of transient tyre in MATLAB-Simulink was achieved in a straightforward way, as the single contact point transient tyre models (described in section-6.4) are built on the semi-empirical Magic Formula tyre characteristics. The only care needed while solving the transient tyre models was for the occurrence of algebraic loops, which were avoided by using certain information (such as transient slip) from the previous time step. This whole approach of modelling detailed vehicle model with a realistic ABS and transient tyre characteristics can be employed to perform comparative studies where the influence of different transient tyre models on the response of anti-lock braking system is investigated.

In this study, The ABS simulation is carried out for straight line braking as well as braking during cornering on various surfaces. In addition, the responses of different tyre models to step variations in brake pressure is also analysed. The road-tyre friction for different surfaces is modelled by using the scale factor for the peak friction coefficients, such that the peak factor '\( D \)' of the Magic Formula is modified to generate a peak force corresponding to the following coefficient of friction under static corner load.

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The brake force coefficient for a road surface varies with the longitudinal slip, and has a strong dependence on the road condition (texture, construction, temp, contamination) along with the vehicle speed and the tyre condition, as shown in a study by Harned et al (1969). However, a constant value is selected here (as shown in Table 6-2) for a particular type of road surface, which is in relation with the vehicle speed used in the manoeuvres. The different analysis results are presented in the following sub-sections:

### 6.5.1 Straight-Line Braking on Dry Surface

The ABS simulation results for straight-line braking on a dry surface are compared for the three different tyre models. Figure 6-16 shows the result for stretched-string tyre model, where only the front wheel plots are displayed. The initial vehicle speed was kept as 70 km/h (19.4 m/sec) and the brake was applied at 1 sec. The peripheral wheel speed plot is shown in Figure 6-16 (a) along with the vehicle longitudinal speed projected at each wheel. At lower speeds, the wheel speed fluctuates past the vehicle speed, during the brake release phase. This behaviour is also visible from the brake slip plot (Figure 6-16 b), where the slip becomes momentarily positive. The fact that the relaxation length in the stretched-string tyre model does not vary with slip means that the effective damping at the contact is reduced and the oscillatory behaviour is promoted.

The brake output pressure profile is shown in Figure 6-16 (d), which is similar to the pressure characteristics described earlier in Figure 6-13. The number of cycles per second for the front wheel is around 6, which is within an acceptable range encountered in a commercial ABS system. Here, it should be noted that the parameters used for ABS braking in these results were selected by number of trial runs, such that an optimised braking performance can be simulated for a particular tyre model-road surface combination. Also, same ABS parameters and control cycle were used for all the three transient tyre models, so that a fair judgement can be made on their performance.
Influence of Tyre Transience in ABS Braking

Figure 6-16: Straight-line braking with stretched-string model (V = 70 Km/h, dry)

Figure 6-17: Straight-line braking with modified stretched-string model (V = 70 Km/h, dry)
Influence of Tyre Transience in ABS Braking

Figure 6-18: Straight-line braking with contact mass model (V = 70 Km/h, dry)

Figure 6-17 shows the result for the modified stretched-string tyre model, which shows a response profile similar to that of stretched-string tyre model, with the longitudinal slip staying in the negative range. The same pattern can be seen for the contact mass transient tyre model, shown in Figure 6-18. During an ABS operation, the correct representation of the transient slip becomes crucial, particularly at lower wheel speed. As the response lag in both these tyre models take into account the variations in the wheel slip along with the load, it results in a more acceptable performance. In Table 6-3, a comparison of the individual braking distance of the vehicle is shown for the three tyre models. As evident, the braking distance is more or less similar for the three tyre models, with stretched-string model exhibiting marginally higher value.

<table>
<thead>
<tr>
<th>Tyre Models</th>
<th>Braking Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretched-String transient tyre model</td>
<td>25.82</td>
</tr>
<tr>
<td>Modified stretched-string tyre Model</td>
<td>25.21</td>
</tr>
<tr>
<td>Contact Mass transient tyre model</td>
<td>25.29</td>
</tr>
</tbody>
</table>

Table 6-3: Braking distance and average brake force comparison (V = 70 Km/h, dry, st-line)
In the previous ABS run, the control cycle employed deceleration threshold for the brake release phase. This is evident in the wheel angular acceleration plots, such as Figure 6-18 (c), where the brake release phase starts only after a certain angular acceleration threshold is reached, which eventually causes the wheel angular acceleration to fluctuate within a certain range. However, if the wheel deceleration alone is used as a control variable for the brake release phase, the brake slip fluctuation in the control cycle may not be restrained within a narrow slip range ((Figure 6-18 b), which is often desired for optimum frictional force based on the peak value of the \( \mu \)-slip curve. A slightly different approach can be adopted in the control cycle described in section (6.4.2.3), where a slip threshold can be combined along with the deceleration threshold to determine the brake release phase. In this case, if the wheel longitudinal slip exceeds a predetermined slip threshold, the brake pressure is released and the control cycle is resumed from phase 3. This approach of slip and deceleration threshold as control variables can be used to compare the ABS braking performance of the three transient tyre models, as conducted previously.

Figure 6-19 shows the results for straight line ABS braking on a dry surface, using the stretched-string tyre model, while employing brake slip as an additional control variable. The wheel speed plot in Figure 6-19 (a) shows that the wheel speed fluctuates past the vehicle speed, once the vehicle starts slowing down. This behaviour is also visible in the brake slip plot (Figure 6-19 b), where rapid oscillations in brake slip can be seen, once the vehicle speed slows down. In comparison to the stretched-string tyre model, the modified stretched-string tyre model and the contact mass model results display the smooth operation of the ABS control, as shown in Figure 6-20 and Figure 6-21 respectively. For both the tyre model, the wheel speed fluctuates below the vehicle speed for every brake cycle, with the brake slip staying within an optimal slip range.

This set of simulation is attempted to see how the stretched-string tyre model behaves, when the brake slip is additionally used as a control variable. The results very clearly demonstrate that with the stretched string tyre model, the ABS control fails to respond in a desirable manner. As the relaxation length for the stretched-string tyre model is a function of the vertical load only, without being affected by the wheel slip, the tyre model lacks the accuracy to provide enough damping at the contact. Overall, the use of brake slip as a control variable again highlights the limitation of the stretched-string tyre model for ABS operations.
Influence of Tyre Transience in ABS Braking

Figure 6-19: Stretched-string model plots ($V = 70$ Km/h, dry, slip as control variable)

Figure 6-20: Modified stretched-string model plots ($V = 70$ Km/h, dry, slip as control variable)
Influence of Tyre Transience in ABS Braking

Figure 6-21: Contact mass model plots ($V = 70$ Km/h, dry, slip as control variable)

6.5.2 Straight-Line Braking on Wet Surface

Similar to the straight-line braking on a dry surface, the ABS simulation results for a wet surface are compared for the three different tyre models, employing deceleration as a control variable. Figure 6-22 shows the result for stretched-string tyre model, where only the front wheel plots are displayed. The result again highlights the undesirable behaviour of the stretched-string tyre model at lower vehicle speed. As evident from the plots of wheel speed (Figure 6-22 a) and the longitudinal slip (Figure 6-22 b), the wheel speed keep fluctuating past the vehicle speed. As the friction force saturates earlier on the wet surface, a slight release of brake pressure accelerates the wheel very rapidly. In the stretched-string tyre model, the relaxation length which is representative of the tyre lag does not vary with changes in steady-state slip. As the relaxation length is still higher, the tyre response lag is still greater and this produces more oscillatory behaviour.
Figure 6-22: Straight-line braking with stretched-string model ($V = 70$ Km/h, wet)

Figure 6-23: Straight-line braking with modified stretched-string model ($v = 70$ km/h, wet)
Influence of Tyre Transience in ABS Braking

In comparison to the stretched-string tyre model, both the modified stretched-string and the contact mass tyre model demonstrates a well-behaved ABS cycle, as shown in Figure 6-23 and Figure 6-24 respectively. The brake pressure profile in both the cases is quite repetitive and smooth, which can be attributed to the fact that the external disturbances such as road irregularities were not considered in the ABS model. It should be noted that the ABS control parameters were kept the same for all the tyre models, and thus similar brake pressure profiles can be observed for all the three models, until 4.2 sec, where the stretched-string tyre model start exhibiting undesirable responses. Also, the stretched-string tyre model takes larger braking distance in comparison with the other two tyre models, as shown in Table 6-4.

<table>
<thead>
<tr>
<th>Tyre Models</th>
<th>Braking Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretched-String transient tyre model</td>
<td>41.33</td>
</tr>
<tr>
<td>Modified stretched-string tyre Model</td>
<td>40.16</td>
</tr>
<tr>
<td>Contact Mass transient tyre model</td>
<td>40.26</td>
</tr>
</tbody>
</table>

Table 6-4: Braking distance for the three transient tyre models (V = 70 Km/h, wet, st-line)
6.5.3 Straight-Line Braking on Snow Surface

The results of straight-line braking for a dry packed snow surface are also compared for the three tyre models. Figure 6-25 shows the results for the stretched-string transient tyre model, where only the front wheel plots are displayed. As observed earlier, the profile of wheel speed, longitudinal slip, wheel angular acceleration and brake wheel pressure display repetitive cycles at higher speeds. However, at lower speeds, it starts showing undesirable behaviour, which again highlights the inability of the stretched-string tyre model to provide realistic contact damping, and thus failing to contribute towards the fast attenuation of brake force oscillations. When comparing the behaviour of stretched-string tyre model on three different surfaces, it can be observed that the performance of the stretched-string tyre model deteriorates from high friction to low friction surfaces. On snow, the friction force saturates earlier and with it the relaxation length approaches zero and the compliant behaviour of the carcass deteriorates. This is not captured by the stretched-string tyre model, which continues to exhibit a significant oscillatory behaviour related to the unrealistically high relaxation length value.

![Graphs showing straight-line braking with stretched-string model](image)

*Figure 6-25: Straight-line braking with stretched-string model (V = 70 Km/h, snow)*
Figure 6-26: Straight-line braking with modified stretched-string model (V = 70 Km/h, snow)

Figure 6-27: Straight-line braking with contact mass model (V = 70 Km/h, snow)
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The ABS results for the modified stretched-string and the contact mass tyre model are shown in Figure 6-26 and Figure 6-27 respectively. Similar to the dry and wet surface results, those for the snow surface are again well behaved for the two tyre models.

A comparison of the vehicle braking distance during straight-line manoeuvre on the snow surface is shown in Table 6-5. The braking distance for the stretched-string tyre model is close to the other two tyre models, though marginally on a higher side again. However, looking at the results of all the three surfaces, it can be observed that for straight-line manoeuvre, the braking distance in itself does not provide a very useful insight to judge the performance of the three tyre models.

<table>
<thead>
<tr>
<th>Tyre Models</th>
<th>Braking Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretched-String transient tyre model</td>
<td>82.83</td>
</tr>
<tr>
<td>Modified stretched-string tyre Model</td>
<td>82.46</td>
</tr>
<tr>
<td>Contact Mass transient tyre model</td>
<td>82.27</td>
</tr>
</tbody>
</table>

Table 6-5: Braking distance for the three transient tyre models (V = 70 Km/h, snow, st-line)

6.5.4 ABS Braking while Cornering on Dry Surface

Similar to the straight-line braking, the ABS simulation for braking while cornering is also conducted for the three transient tyre models. The initial vehicle speed was again kept at 70 km/h (19.4 m/sec), with a steering hand wheel input of 90° (counter-clockwise) applied at 0.2 sec, and once the vehicle moves into a left side corner, the braking was initiated at 1 sec. Figure 6-28 shows the results for the stretched-string tyre model, where the wheel speed and longitudinal slip for the individual wheels are shown with an offset. This is for sake of clarity in the presentation. As shown in Figure 6-28, the wheel speed and the longitudinal slip plots exhibits large and undesirable oscillations at lower speed. As the wheel speed fluctuates past the vehicle speed, the longitudinal slip (Figure 6-28 b) and the braking force (Figure 6-28 d) attains positive value momentarily. This undesirable fluctuation of longitudinal or brake slip at lower speed also leads to a sudden drop in brake output pressure, as evident from the outer front wheel plot (Figure 6-28 c). Here, only three curves appear in the brake pressure plot (Figure 6-28 c), as both the rear wheels are operated on select-low mode, where same brake output pressure is used. In comparison to the straight-line braking on a dry surface, the oscillatory behaviour of the stretched-string tyre model is more visible during a cornering manoeuvre, which further highlights the overall limitation of the model.
Figure 6-28: Braking while cornering with stretched-string model (V = 70 Km/h, dry)

Figure 6-29: Lateral slip / force plots with stretched-string model (V = 70 Km/h, dry)

Figure 6-29 shows the lateral slip and the lateral force plots for the stretched-string tyre model. In comparison with the longitudinal slip, the lateral slip is relatively well behaved for most part of the brake cycle. However, towards end of the simulation, the lateral slip plot exhibits undesirable response pattern, which can be attributed to the fact that the friction forces at the front wheels saturates during the same time. In the current tyre models, the coupling between longitudinal and lateral transient dynamics is not modelled, and hence a further study may be required to investigate the correlation between the longitudinal and the lateral response of a tyre, in more detail.
The results for the modified stretched-string tyre model and the contact mass model are shown in Figure 6-30 to Figure 6-33 respectively. In comparison with the stretched-string tyre model, the ABS results for the modified stretched-string and the contact mass tyre model are again well behaved. As the vehicle is steered towards left, the inner front tyre (front left) attains its maximum friction potential much earlier than the outer front tyre, and hence the brake pressure build-up is less for the inner front tyres, resulting in a reduced longitudinal braking force. Consequently, the lateral or cornering force also drops for the inner front tyres. The plots of wheel speed, longitudinal and lateral slip, brake output
pressure, and longitudinal and lateral force for both the tyre models show a similar pattern. In relation to the stretched-string tyre model, both the modified stretched-string and the contact mass tyre model can provide realistic contact damping, thus contributing towards the fast attenuation of brake force oscillations.

![Graphs showing wheel speed, brake output pressure, and lateral force plots with contact mass model](image)

**Figure 6-32: Braking while cornering with contact mass model (V = 70 Km/h, dry)**

![Graphs showing lateral slip and force plots with contact mass model](image)

**Figure 6-33: Lateral slip / force plots with contact mass model (V = 70 Km/h, dry)**

During straight-line manoeuvre, the performance of the three transient tyre models was not very clearly distinguishable in terms of braking distance comparison. However, during cornering manoeuvre, where more complicated interaction takes place, the performance of the tyre models is more reflective in terms of the braking effectiveness. Figure 6-34 shows
the vehicle path curve for the three transient tyre models, where the vehicle with stretched-string model travels a longer distance and also drifts slightly towards the inside of the turn. This behaviour can be further understood from the average braking force value at the individual wheels during the ABS cycle. As shown in Table 6-6, the average brake force at the front wheels is much lower for the stretched-string tyre model, in comparison with the other two tyre models. The difference is more prominent for the outer front wheel, where the brake force attains positive value towards the end of simulation run (Figure 6-28 d). The fact that the braking force saturated much earlier for the stretched-string tyre model, leads to a reduction in its braking effectiveness, resulting in higher braking distance (Table 6-6). Overall, the stretched-string tyre model, with its limited representation of the relaxation length, is once again found inaccurate in a more dynamic situation of combined ABS braking with cornering.

<table>
<thead>
<tr>
<th>Tyre Models</th>
<th>Braking Distance (m)</th>
<th>Average Brake Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FL Wheel</td>
<td>FR Wheel</td>
</tr>
<tr>
<td>Stretched-String transient tyre model</td>
<td>29.44</td>
<td>2910</td>
</tr>
<tr>
<td>Modified stretched-string tyre Model</td>
<td>25.5</td>
<td>3255</td>
</tr>
<tr>
<td>Contact Mass transient tyre model</td>
<td>25.55</td>
<td>3281</td>
</tr>
</tbody>
</table>

Table 6-6: Braking distance and average force (V = 70 Km/h, dry, braking with cornering)

Figure 6-34: Vehicle path curve for ABS braking while cornering (V = 70 Km/h, dry)
6.5.5 ABS Braking while Cornering on Wet Surface

The ABS braking comparison for the three transient tyre models, while the vehicle is cornering on a wet surface, again shows similar trends as observed earlier. Figure 6-35 shows the ABS results for the stretched-string tyre model on a wet surface. In comparison to the cornering manoeuvre on a dry surface, the performance of the stretched-string tyre model on a wet surface deteriorates further. Here, as the speed drops, all the four wheels can be seen exhibiting large oscillations (Figure 6-35 a), with the longitudinal slip (Figure 6-35 b) and the braking force (Figure 6-35 d) attaining positive values. The inaccurate representation of the relaxation length eventually effects the operation of the ABS cycle, which is evident from the various response plots of Figure 6-35. Under this situation, the tyre response lags do not correctly correspond to the delays in the valve opening and closing for the brake apply, release and hold cycles. As an after-effect, during the brake apply phase, the wheel speed instead of dropping down oscillates close to the vehicle speed. This means even a slight release of brake pressure can accelerate the wheel very rapidly, allowing it to move past the vehicle speed, resulting in positive slip and force values, and eventually reducing the brake effectiveness.

![Figure 6-35: Braking while cornering with stretched-string model (V = 70 Km/h, wet)](image-url)
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Figure 6-36: Braking while cornering with modified stretched-string model ($V = 70$ Km/h, wet)

Figure 6-37: Braking while cornering with contact mass model ($V = 70$ Km/h, wet)
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In comparison with the stretched-string tyre model, the ABS plots for both the modified stretched-string model and the contact mass model are rather well behaved, as shown in Figure 6-36 and Figure 6-37 respectively. In all the figures, the wheel speed and longitudinal slip profile for individual wheels are plotted with an offset, for clarity of presentation. The correct representation of the relaxation length or the response lags in these two tyre models means that the ABS cycle corresponds in a desirable manner to the tyre transient response.

While, comparing the lateral slip and force plots for the three transient tyre models, it can be observed that the lateral slip / force plots for the stretched-string tyre model (Figure 6-38) show some abruptness after 3 sec of simulation, around the time when the longitudinal slip starts fluctuating inconsistently. On the other hand, the lateral slip / force plots for the modified stretched-string model and the contact mass model are well behaved (Figure 6-39 and Figure 6-40 respectively). The overall braking distance taken by the vehicle with the stretched-string tyre model is also much longer in comparison with the other two tyre models (see Figure 6-41).

Figure 6-38: Lateral slip / force plots with stretched-string model (V = 70 Km/h, wet)

Figure 6-39: Lateral slip / force plots with modified stretched-string model (V = 70 Km/h, wet)
Figure 6-40: Lateral slip / force plots with contact mass model (V = 70 Km/h, wet)

Figure 6-41: Vehicle path curve for ABS braking while cornering (V = 70 Km/h, wet)

6.5.6 ABS Braking while Cornering on Snow Surface

The ABS braking performance for the three transient tyre models, while the vehicle is cornering, is concluded by comparing their performance on the snow surface. Figure 6-42 shows the ABS results for the stretched-string tyre model on the snow surface. As evident from the results, the stretched-string tyre model is not able to provide enough damping at the contact. The longitudinal force plots (Figure 6-42 d) shows large fluctuations, with the peaks reaching positive values during the brake release phase. The slip and wheel speed plots also indicate similar behaviour from the very start of the simulation. The results are consistent with the previous simulation runs and it once again highlights the fact that the braking performance of the stretched-string tyre model deteriorates from high to low friction surfaces. In comparison, the ABS plots for both the modified stretched-string tyre model
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(Figure 6-43) and the contact mass model (Figure 6-44) show fairly uniform and consistent behaviour. Although, the uniformity or the repetitive nature of the ABS pressure cycles are reflective of the fact that the road disturbances were not considered in the ABS simulation, the results, nevertheless, emphasise the ability of the two transient tyre models to simulate ABS braking on low friction surfaces.

Figure 6-42: Braking while cornering with stretched-string model (V = 70 Km/h, snow)

Also, for all the three tyre model cases, the rear right wheel (outer) exhibits maximum longitudinal force fluctuations. Since the ABS brake system employs select-low configuration for the rear wheels, the rear right wheel, being on the outside of the turn, does not reach its maximum force potential. Hence, during the brake release phase, the wheel speed accelerates and almost reaches the vehicle speed. As the vehicle slows down, the above effects become more predominant, which is evident from the slip and longitudinal force plots of all the three transient tyre models. For the modified stretched-string tyre model (Figure 6-43), the relaxation length reduces with an increase in slip, which helps in attenuating the longitudinal force oscillations, due to the presence of contact damping. The same holds true for the contact mass model (Figure 6-44), where the carcass deformation reduces with an increase in slip, thus keeping the longitudinal force oscillations under check. The relaxation length of the stretched-string tyre model (Figure 6-42), on the other hand,
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does not vary with slip and, hence, the longitudinal force response has the same lag at low speeds as that at high speeds. The unrealistically high relaxation length and lack of contact damping, eventually leads to higher force fluctuations for the stretched-string tyre model.

Figure 6-43: Braking while cornering with modified stretched-string model (V = 70 Km/h, snow)

<table>
<thead>
<tr>
<th>Tyre Models</th>
<th>Braking Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dry</td>
</tr>
<tr>
<td>Stretched-String transient tyre model</td>
<td>29.44</td>
</tr>
<tr>
<td>Modified stretched-string tyre Model</td>
<td>25.5</td>
</tr>
<tr>
<td>Contact Mass transient tyre model</td>
<td>25.55</td>
</tr>
</tbody>
</table>

Table 6-7: Braking distance for the three transient tyre models while cornering (V = 70 Km/h)

The vehicle path curves for the three transient tyre models are shown in Figure 6-45. Both the modified stretched-string tyre model and the contact mass model show similar vehicle path curves, whereas the stretched-string tyre model not only takes much longer in braking distance, but also the vehicle path deviates towards the inside of the turn. Table 6-7 shows the braking distance comparison for all the three tyre models, where the braking distance for the stretched-string tyre model is consistently on a higher side, in comparison to the other
two tyre models, on various surfaces. The results once again reiterates the fact that limitations of the stretched-string tyre, in terms of braking effectiveness, is exposed during cornering manoeuvre, in comparison to the earlier conducted straight-line braking.

Figure 6-44: braking while cornering with contact mass model (V = 70 Km/h, snow)

Figure 6-45: Vehicle path curve for ABS braking while cornering (V = 70 Km/h, snow)
6.6 Braking Simulation with Step Variation in the Brake Pressure

The simulation results presented so far involved closed loop ABS braking, where the influence of tyre transient behaviour was investigated for a range of operating conditions. The analysis results clearly highlight the limitations of the stretched-string model in correctly predicting the tyre behaviour in ABS applications. In comparison, both the modified stretched-string model and the contact mass model, not only represented the tyre dynamic behaviour in a more realistic manner, but also produced similar results. The similarity in both these tyre models' results can be attributed to the fact that the current ABS brake model, while simulating the rapid brake pressure build-up, hold and reduction cycles (mostly repetitive in nature), tested only certain characteristics of a tyre model, namely the primary one being its ability to handle fast changing slip and wheel oscillations. In comparison, an ABS system in a production vehicle has to deal with a number of other transients such as those originating from the brake modulator and the brake lines as an outcome of the pressure transients due to the brake fluid hydraulics and opening and closing of the valves, as well as irregularities of the road surface etc. Consequently, the tyre in a production vehicle is exposed to more dynamic behaviour, than what is simulated in the current ABS model. Keeping in mind the above facts, a new set of simulations are conducted, where a step brake pressure pulse is employed to excite the transients in the tyre model. As opposed to the ABS simulation, the brake pressure in this case follows an open-loop cycle, where a pre-determined step brake pressure is applied to the wheels without any feedback to modulate the pressure cycle. Additionally, a short pressure pulse is added at the beginning of every pressure step so that the dynamic behaviour of the transient tyre models can be studied further. In-line with the previous simulation runs, comparisons between the three transient tyre models is again made, with the analysis results presented below.

Figure 6-46 shows the plots of the stretched-string tyre model with a step brake pressure variation. The vehicle in this case is driven in a straight-line with an initial speed kept as 70 km/h (19.4 m/sec). A brake pressure with ramp input is initially applied to all the four wheels (front and rear distribution) at 0.5 seconds, before the step brake input is initiated at 1.5 seconds. The stepped brake pressure input with short pressure pulses is shown in Figure 6-46 (c), where the overall brake pressure gradually increases through the course of the simulation, but the small step rise in brake pressure ensures that the vehicle decelerates gradually. As a result, the brake slip stays on the low side during the simulation (Figure 6-46...
b) with no danger of wheel locking. The interesting part of the simulation, however, is the tyre response to the excitation provided by the short pressure pulses. As evident from Figure 6-46 (b), the longitudinal slip oscillates with every pressure pulse and the oscillations are carried until the next pressure step. It clearly appears that the oscillations are not damped quickly, which is also evident from the longitudinal force response (Figure 6-46 d). The oscillation increases as the speed drops, again highlighting the limitation of the relaxation length approach in the stretched-string model. As the relaxation length in this case does not vary as function of slip, its value stays higher at low speeds. A higher value of relaxation length not only causes more lag in the force response, but also leads to a larger percentage overshoot. This is in line with the findings of Pauwelussen et al (2003).

![Figure 6-46: Step braking with stretched-string model (V = 70 Km/h, dry, st-line)](image)

The modified stretched-string tyre model plots are shown in Figure 6-47. For the same brake pressure input, the modified stretched-string tyre model shows better response, as evident from the longitudinal slip (Figure 6-47 b) and longitudinal force (Figure 6-47 d) profiles. The oscillations in this case are damped much quicker and also as the relaxation length reduces with an increase in slip, the force response is less oscillatory when compared with the stretched-string tyre model. In both the stretched-string tyre models, the only source of
damping is the friction at the contact. In the simple stretched-string tyre model, this damping mechanism is combined with the relaxation length, which stays constant in relation with the slip. This promotes oscillations, as the higher value of relaxation length delays the force response. In the modified stretched-string tyre model, the relaxation length reduces and with it the tendency of the tyre to oscillate.

The contact mass tyre model plots are shown in (Figure 6-48). As opposed to the ABS simulation performed earlier, where the modified stretched-string tyre model and the contact mass model showed similar behaviours, the response of the contact mass model in this case shows improvement in relation to the modified stretched-string tyre model. This can be attributed to the fact that the contact mass model includes additional damping in the carcass, which leads to further suppression of the oscillations in comparison to the modified stretched-string tyre model. As a consequence, the longitudinal slip and force oscillations are damped much quicker (Figure 6-48 b,d) and the response overshoot is also reduced, particularly at low speeds.

Figure 6-47: Step braking with modified stretched-string model (V = 70 Km/h, dry, st-line)
To further look into the transient behaviour of the three tyre models, the previous simulations are repeated, this time at a low speed. The vehicle initial speed this time is kept at 34 km/h, with similar brake input initiated at 0.5 seconds. Figure 6-49 shows the plots of the stretched-string tyre model. As evident from the results, the slip oscillations, following the excitations from the pressure pulses, are of much higher amplitude. At reduced speed, the damping also reduces and the oscillations with large overshoots are carried until the next pressure step. In comparison, the results from the modified stretched-string tyre model show some improvement (Figure 6-50), with longitudinal slip staying negative (Figure 6-50 b), as opposed to the stretched-string tyre model, where the slip fluctuations reach positive values (Figure 6-49 b). However, the oscillations in the modified stretched-string tyre model are not damped quickly and persist until the next pressure step, as evident from the longitudinal force plot (Figure 6-50 d). The contact mass model (Figure 6-51), on the other hand, demonstrates much improved performance, where the oscillations not only dampen quickly, but the overall amplitude is also reduced. This is also evident from the longitudinal force plot (Figure 6-51 d), which after initial oscillations due to the excitation provided by the short pressure pulse, settles down before the next pressure step.
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Figure 6-49: Step braking with stretched-string model \((V = 34 \text{ Km/h, dry, st-line})\)

Figure 6-50: Step braking with modified stretched-string model \((V = 34 \text{ Km/h, dry, st-line})\)
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In the previous simulations, the transient response of the three tyre models, upon excitation by the stepped pressure pulses were discussed. The stepped braking simulation was conducted at reduced speed. From the results, it can be concluded that at lower speeds the dynamic behaviour of the tyre becomes more critical, particularly for the situations which involves rapid brake pressure or brake torque fluctuations like the ones encountered in an ABS system. As observed in the previous simulations, except for the contact mass transient model, the other two tyre models exhibit large fluctuations, which take much longer time to settle. The high amplitude longitudinal force fluctuations leads to wheel speed disturbances, which eventually affect the performance of the ABS system, as the controller may have to deal with corrupt signals.

As a final simulation run, stepped braking with short pressure pulses is applied in a cornering manoeuvre. The vehicle initial speed was kept as 70 km/h (19.4 m/sec), with a steering hand wheel input of 90° provided at 0.2 seconds from the start of the simulation. Similar to the previous simulation runs, the brake pressure with ramp input was initiated at 0.5 seconds, while the vehicle enters its steady-state phase. Figure 6-52 shows the plots for the stretched-

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**Figure 6-51: Step braking with contact mass model (V = 34 Km/h, dry, st-line)**

In the previous simulations, the transient response of the three tyre models, upon excitation by the stepped pressure pulses were discussed. The stepped braking simulation was conducted at reduced speed. From the results, it can be concluded that at lower speeds the dynamic behaviour of the tyre becomes more critical, particularly for the situations which involves rapid brake pressure or brake torque fluctuations like the ones encountered in an ABS system. As observed in the previous simulations, except for the contact mass transient model, the other two tyre models exhibit large fluctuations, which take much longer time to settle. The high amplitude longitudinal force fluctuations leads to wheel speed disturbances, which eventually affect the performance of the ABS system, as the controller may have to deal with corrupt signals.

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string tyre model. The brake pressure (Figure 6-52 c) follows an open loop cycle, where the predetermined stepped input with short pressure pulses was set such that it provides enough excitation to the wheels, but also guards against wheel locking. As the brake pressure or brake torque is equally distributed on the left and right side wheels, the inner wheels which have lower velocities compared to the outer wheels, achieve their friction potential and attain higher values of longitudinal slip (refer Figure 6-52 b). The outer wheels, being at higher speed, do not achieve their friction potential and stays at lower values of slip. Thus the outer wheels are more susceptible to rapid oscillations due to the applied excitations of the short pressure pulses. This behaviour is evident from both the longitudinal slip as well as longitudinal force plots, which exhibits larger overshoots and longer settling times.

The limitations of the stretched-string tyre model can be further expressed by the dynamic force-slip plots, shown in Figure 6-53. After, every step increase in the brake pressure or brake torque, the longitudinal slip increases as well as the longitudinal force after a certain lag due to the relaxation length of the tyre. This means that while slip increases or decreases, the longitudinal force responds after certain lag. The circular loops in the dynamic force-slip plot (Figure 6-53) reflect this particular oscillatory behaviour, where every single step pressure change leads to a new set of loops with slip moving to a new steady-state value. The transient response of the tyre model can also be identified from the nature of the circular loops. The dynamic force-slip plot of the outer front tyre (Figure 6-53 b) exhibits large loops, which are a direct reflection of its oscillatory behaviour, as evident from the longitudinal force and slip plots of Figure 6-52. The inner tyre dynamic force-slip plots, on the other hand, exhibits comparatively short loops which agree with the less oscillatory nature of the inner tyre response. Overall, these plots once again confirm that the stretched-string tyre model, with its limited representation of the relaxation length, cannot accurately address the requirements of an ABS braking, where undesirable oscillations due to the external excitations can provide misleading information to the controller.
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Figure 6-52: Step braking with stretched-string model (V = 70 Km/h, dry, cornering)

Figure 6-53: Slip-force plot with stretched-string model (V = 70 Km/h, dry, cornering)
The modified stretched-string tyre model plots are shown in Figure 6-54. The outer wheel slip plot (Figure 6-54 b), in this case, exhibits less oscillatory response, where the slip settles soon after some initial oscillations. As the oscillations are quickly damped, the dynamic force-slip plots (Figure 6-55) show relatively smaller loops; the size of which increases once the vehicle speed drops to low values. The contact mass transient tyre model, in comparison with the other two tyre models, shows far better response (refer Figure 6-56). As observed in the previous simulation runs, the inclusion of the dynamic characteristics such as inertia, damping and stiffness in the tyre model influences its overall damping behaviour, which can be verified by the dynamic force-slip plots in Figure 6-57. The force-slip plots, in this case, exhibit relatively short loops, which signifies the fact that the oscillations were not only damped much quickly, but also the longitudinal force response lag in relation to the slip is also reduced with speed. In addition, the loops in the force-slip plots can be distinguished for every brake pressure step cycle, as the slip settles down long before the next pressure pulse excitation. 

Figure 6-54: Step braking with modified stretched-string model (V = 70 Km/h, dry, cornering)
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Figure 6-55: Slip-force plot with modified stretched-string model ($V = 70$ Km/h, dry, cornering)

Figure 6-56: Step braking with contact mass model ($V = 70$ Km/h, dry, cornering)
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Figure 6-57: Slip-force plot with contact mass model (V = 70 Km/h, dry, cornering)

6.7 Chapter Remarks

This chapter has shown the influence of tyre transient behaviour on the performance of an ABS system. The dynamic behaviour of three transient tyre models have been analysed by employing a realistic ABS system in a full vehicle model. Although all the three transient tyre models are based on the widely followed semi-empirical Magic Formula tyre characteristics, they significantly differ in the way the tyre carcass compliances and contact patch slip properties are represented. The stretched-string tyre model, where the transient behaviour is represented through the first order relaxation length approach, has its limitations in providing realistic contact damping, and as a result promotes higher oscillations. Unlike the stretched-string tyre model, the relaxation length in the modified stretched-string tyre model varies with tyre slippages, such that with an increase in slip the relaxation length decreases. This means that the tyre response lag in both longitudinal as well as lateral directions reduce at higher slips, which eventually leads to better performance of the modified stretched-string tyre model in ABS braking. The contact mass model, in comparison, follows a slightly different approach, where the carcass compliance and contact slip properties are modelled separately. Instead of the normal relaxation length approach, the inclusion of single point
contact mass along with the carcass springs, represents the dynamic characteristics of a tyre in a more realistic manner. During ABS simulation, both the modified stretched-string tyre model and the contact mass model showed similar performance for straight-line as well as cornering manoeuvre. This similarity is observed since the employed ABS brake model does not take into account certain external disturbances, as well as transient pressures of the brake lines and modulator dynamics; all of which can potentially affect the tyre responses. However, in the second set of simulations, when the stepped brake pressure with short pulses were used as input, the contact mass model exhibited reduced oscillatory behaviour and quicker response time to the brake pressure excitations. The additional damping in the carcass for the contact mass model leads to further suppression of the oscillations, in comparison to the modified stretched-string tyre model. Based on the results, it can be safely concluded that the contact mass transient tyre model is much better suited for an ABS simulation study. For the tyre models based on the first order relaxation length approach, the high amplitude longitudinal force fluctuations may lead to wheel speed disturbances, eventually affecting the performance of the ABS system, as the controller may have to deal with corrupt signals.

The overall observations of this research and the need for any future work are outlined in chapter 7.
7 Conclusions and Suggestions for Future Work

7.1 Overall Conclusions and Contribution to Knowledge

The thesis presents an intermediate vehicle system modelling approach to study vehicle transient handling behaviour, by stressing on simplified yet accurate modelling of the important elements. The approach is further extended and applied to ABS control study, where the influence of tyre transience on ABS braking is investigated.

One of the starting tasks in the research work was to develop low-end vehicle models (few DOFs), employing Newton-Euler formulation of equations of motion. The 3-DOF model, developed using this approach, includes some non-linear characteristics, but also has a number of simplifications. Though largely appropriate for steady-state handling analysis, it lacks in accuracy when it comes to transient handling manoeuvres. This was demonstrated during its comparative study with the larger intermediate model and the ADAMS multi-body dynamics model.

The intermediate vehicle model, built in MATLAB/Simulink environment, incorporates additional DOF to capture all the translational and rotational motions of the unsprung masses in space, along with a realistic representation of the various sub-systems such as vehicle body (sprung mass), suspension, steering, wheel-tyre and driving/braking system. Here, particular attention is paid to the suspension system modelling, where the various non-linear effects are adequately represented through springs, dampers, bump-stops, anti-roll bars, as well as rigid suspension constraints (using the virtual work method). During handling analysis, it is shown that accurate modelling of suspension is of paramount importance, especially for critical handling conditions. Additionally, the inclusion of large angular motions, as well as the kinematic effects at suspension and steering/wheels to simulate the wheel lift-off, has allowed the intermediate model to perform limiting handling manoeuvres. This is shown by conducting a few critical handling manoeuvres, involving cornering on surfaces with uneven friction. The vehicle's propensity to roll-over is then assessed, as the tyres suddenly enter areas of significantly higher friction.

In the thesis, a detailed multi-body vehicle model is also developed in the ADAMS/Chassis environment, which incorporates the effects of compliances in the body, suspension and
Conclusions and Suggestions for Future Work

steering sub-systems, in addition to the detailed representation of each rigid-body component of the vehicle. During the transient vehicle handling comparison task, the intermediate vehicle model has agreed well with the complex ADAMS model, which justifies the use of simplified yet reasonably accurate vehicle modelling approach for critical handling manoeuvres. Some minor differences in the results can also be attributed to lack of compliance effects in the intermediate vehicle model, along with a few secondary effects such as roll and bump steer, non-linear damping related to the steering and anti-roll bar etc.

The intermediate vehicle model is further validated through a number of vehicle tests, conducted on a proper track. The test vehicle fitted with a reliable GPS/Inertial measurement system helped in experimentally evaluating the intermediate vehicle model for different handling manoeuvres, where the intermediate vehicle model responses closely matched with the test vehicle one's. During the actual tests, combined cornering and braking manoeuvres were performed, which were also simulated by the intermediate vehicle model, using a set of information from the test vehicle CAN network, such as master cylinder brake pressure, engine speed/torque, gear position etc., so that both driving and braking command at the wheels can be replicated. This approach has allowed to successfully test the mathematical model of the hydraulic brake unit, based on the work by Gerdes and Hedrick (1999), and integrated to the intermediate vehicle model for realistic simulation of braking manoeuvres. The adopted model uses reduced order brake system dynamics, employing simplified brake hydraulics and vacuum booster and intends to serve as a platform for future extension to a complete ABS hydraulic model.

The ABS system, developed in the current work, follows a conventional peak-seeking approach, where control variables such as wheel angular deceleration and slip thresholds are used to determine the various stages of the ABS control cycle. Instead of modelling the detailed dynamics of an ABS system, the algorithm takes into account the intrinsic characteristics of the modulator in a simplified way, which includes apply and release rates, valve timings and delays, filters characteristics etc. The ABS system also employs an adaptive non-linear filter method to determine the reference velocity in the brake control cycle. Though, the various parameters used in the present ABS control were based on numerous optimization trial runs, as well as some published data, the existing vehicle-ABS system can be used to perform a more thorough optimization or parameter based study. Similarly, a more detailed study based on different control variables can also be conducted, as shown in chapter 6. The work, overall, demonstrates that an ABS control, based on
Conclusions and Suggestions for Future Work

relatively simplified modelling approach can be effectively applied for conducting crucial performance study in a full vehicle simulation environment.

One of the primary aims of the present work was to explore different transient tyre modelling approaches, based on semi-empirical Magic Formula characteristics and to study their influence on ABS braking, under a range of demanding operating conditions. The three different transient tyre models (i.e. the stretched-string model, modified stretched-string model and the contact mass model), applied in the current work, are based on the single point contact approach, which uses simplified and approximate characteristics as compared to the original stretched-string modelling approach. Nevertheless, for relatively low frequency transient and oscillatory vehicle motions, the single point contact approach can provide reasonably accurate prediction of the transient responses (Pacejka, 2006). Also, the simplicity and ease of numerical simulation, owing to the Magic Formula characteristics, further adds to its advantage.

To investigate the influence of tyre transients on ABS braking response, a number of braking runs were conducted using the transient tyre models, for straight line as well as cornering manoeuvres, on different road surfaces. The following are some of the key findings of the research in this area:

1) For the stretched-string transient tyre model, rapid oscillations of wheel speed and brake slip can be seen during ABS braking, especially at lower vehicle speeds. The behaviour of the model becomes more undesirable on low friction surfaces, while cornering and also during low speed manoeuvres. The oscillatory tyre behaviour may eventually prove detrimental to the braking performance, as disturbances of this type in wheel speed signals can often corrupt the working of an ABS controller. The braking distance is also on a slightly higher side as compared with the other two transient tyre models. In the stretched-string tyre model, the relaxation length does not vary with slip, which means at higher values of slip towards the end of simulation, the relaxation length is still higher than would be expected. Overall, the stretched-string tyre model is not found to be accurate enough to provide realistic contact damping, and as a result promotes higher oscillations.

2) In comparison, the modified stretched-string tyre model shows rather smooth and well behaved responses, during the various ABS runs. Apart from the relaxation length being expressed as a variable of both tyre slippage and vertical load, it also considers elastic tread
elements for longitudinal and lateral deflection, which is neglected in the stretched-string tyre model. Though the modified stretched-string model is relatively cumbersome to solve, the initial simulation results demonstrated that it can adequately handle rapid and oscillatory responses associated with ABS braking. However, considering the fact that a commercial ABS system has to encounter much more severe conditions, a second set of braking simulation was conducted, where a stepped brake pressure input with short pressure pulses was directly applied to the wheels so as to simulate a more realistic transient condition associated with the internal system dynamics. Here, the modified stretched-string tyre model’s response, though much better than the stretched-string tyre model, still exhibited large overshoots, with the oscillations taking longer time to dissipate. Through the later simulations, the limitations of the modified stretched-string tyre model became quite apparent, especially for more severe transient conditions.

3) The contact mass transient tyre model, on the other hand, displays overall smooth behaviour during the entire exercise. As opposed to the above two transient tyre models, the contact mass model is based on the separate representation of the carcass compliance and contact slip properties. The inclusion of the contact patch (with inertia), such that it can deflect in the circumferential and lateral direction with respect to the lower part of the rim, and the carcass springs (with damping and stiffness) represents the dynamic characteristics of a tyre in a more realistic manner. This approach automatically takes care of the vertical load dependent lag, along with accounting for reduction in lag with the increasing (combined) slip. The additional damping in the carcass for the contact mass model leads to further suppression of the oscillations, in comparison to the modified stretched-string tyre model. Based on the analysis results, it can be safely said that the contact mass transient tyre model, with reasonably accurate transient response behaviour, is much better suited for an ABS simulation environment.

4) Through the number of ABS simulations performed under a range of operating conditions, including combined cornering and braking, it is shown that the tyre models which are adequate for pure braking might struggle when the complicated full vehicle dynamics are excited. It is also shown that the tyre behaviour can be influenced by the complex interaction of handling and braking, during a cornering manoeuvre. It is, therefore, advisable to incorporate combined cornering and braking as important criteria, while investigating or evaluating the performance of a tyre model in relation with ABS simulation.
7.2 Achievements of Aims

The overall aims of the research project, as underlined at the start of the thesis, were accomplished successfully. A brief description of the level of task achieved is listed below:

1) Development of simpler vehicle models for non-linear vehicle behaviour, and extending it further to develop intermediate vehicle model by incorporating sufficient degrees of freedom and adequate non-linear characteristics of the different parts.

2) Inclusion of various non-linear effects of the vehicle subsystem, particularly suspension, helped in generating realistic response behaviour. A number of other features such as steering/wheel kinematics and large angle effects further facilitated the vehicle simulation study under limit handling conditions, such as a pre-roll-over event.

3) Establishing a complex multi-body model in ADAMS/Chassis, with much greater level of details. Its library of standard test events eased the task of conducting critical evaluation of the intermediate vehicle model under various transient handling manoeuvres.

4) The experimental tests, conducted with a reliable GPS enabled vehicle, further ascertained the realistic behaviour of the intermediate vehicle model under transient handling manoeuvres. It also involved combined cornering and braking manoeuvres, which were simulated with the intermediate vehicle model, through its hydraulic brake unit, by making use of certain information from the test vehicle CAN network.

5) The development and integration of a conventional ABS system in the intermediate vehicle model provided a platform to perform tyre transients' influence study in a full vehicle simulation environment. The modular structure of the intermediate vehicle model in MATLAB/Simulink environment means that the existing modelling set-up can be used for implementation of different control strategies in future. Feasibility also exists to externally interface or integrate other modules, such as a detailed hydraulic ABS model.

6) The study of the transient tyre models in a full vehicle simulation environment in conjunction with ABS braking yielded good results. The comparative influence of the three transient tyre models on the working of an ABS system under different operating conditions was critically assessed, along with the investigation of the modelling requirements and the relative importance of the dynamic characteristics of a tyre.
Conclusions and Suggestions for Future Work

7.3 Critical Assessment of the Approach

In the intermediate vehicle model, the non-linearities associated with the vertical motion of a complex suspension system is adequately represented through the non-linear spring damper, along with the secondary motion effects through the virtual work principle. However, this approach cannot accommodate the longitudinal and lateral displacements of the contact patch due to bushings and other compliance effects, which eventually affect the wheel kinematics and also the effective track width. These effects become more prominent during limit handling conditions, while a vehicle experiences large roll or pitch angles. The inclusion of elasto-kinematics and compliance effects of suspension can certainly aid the vehicle behaviour under such conditions.

The limited DOF (only vertical) for the tyre-wheel assembly meant that the effect of longitudinal deflection of the wheel can not be considered, while calculating the wheel slip. For accurate representation of the wheel slip, especially during braking on uneven roads, the above effect needs to be incorporated in the tyre-wheel assembly. The excitations originating from the road surface were also not taken into account in the present research. While, this can be a safe assumption for a flat road surface, it is inadequate when it comes to uneven roads; the influence of road undulations cannot be neglected. This effect can also be easily incorporated so that the same study can be extended for uneven roads.

The generic ABS model developed for the present research did not consider detailed dynamics / hydraulics of an ABS modulator, and hence the transient behaviour originating from the underlying system dynamics cannot be simulated with the present ABS model. While, the above effect may be worth attempting in future for a more detailed investigation of tyre transient effects in relation with the ABS system dynamics, the level of complexity of the vehicle model may need to be scaled down, as the complex interaction of various sub-systems in a vehicle may cause serious problems in the workings of an ABS controller. For similar reasons, conducting such studies in a commercial multi-body tool such as ADAMS is less desirable, especially due to the issues involved with the integration of externally built dynamic ABS braking system with the ADAMS model.

The contact mass model can represent the transient and oscillatory tyre behaviour in a reasonable accurate manner by including dynamic characteristics such as inertia, damping and stiffness in the model. However, it has its limitations in representing the deformation
distribution in the contact area, along with the high frequency oscillatory tyre behaviour. While, these effects were comprehensively captured in the past by researchers such as (Mavros et al, 2005a,b), who developed a transient brush model, incorporating discretised tread and belt elements with viscoelastic characteristics, the complexities involved in integrating such detailed tyre models in a generic multi-body vehicle environment often acts as a stumbling block. The transient tyre modelling based on semi-empirical Magic Formula characteristics offers ease of numerical simulation, as shown in the current research, but at the same time are restrictive in their overall representation of the tyre transient characteristics.

7.4 Suggestions for Future Work

Taking into account the aforementioned limitations of the highlighted approach, certain additional details can be incorporated in the intermediate vehicle model to improve its behaviour under limiting handling conditions. As a starting step, it will be interesting to take into account the influence of suspension and steering compliances, along with the detailed wheel-tyre kinematics so that the vertical forces at the tyre can be accurately modelled, while a wheel encounters large angular motions. For performing a detailed pre-rollover simulation, a detailed representation of the contact interaction between the tyres and potential obstacles, along with a 3-dimensional representation of surface terrain can be investigated. It will also be interesting to model the relaxation length with respect to large camber and slip variations, as demonstrated in a study by (Higuchi and Pacejka, 1997), so that the influence of tyre response lag can be studied for limiting handling conditions.

The transient tyre models used in the present study fitted well with the scope of the present research. However, considering the limitations of these transient tyre models for high frequency application, it would be more pragmatic to apply a more advanced tyre model for future studies. FTire is one such tyre model, which can satisfy the requirements for any future extension of the current work. Though, not as computationally efficient as the transient tyre models applied in the present research, the FTire model can be easily integrated into a multi-body simulation environment and retains an advantage over others in handling road irregularities and description of sudden obstacles. Its operation in high frequency domain (up to 120 Hz) means that the tyre model can be used to perform
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parameter study in relation to frequency response function, as an extension to the present study.

Apart from incorporating small effects such as road undulations and longitudinal DOF in the tyre-wheel assembly, the present modelling-set up can be further extended to integrate a drive-train model. This would provide a natural extension to the current research, where the study of drive torque variations in relation to the dynamic behaviour of a tyre can be conducted in a full vehicle simulation environment. The inclusion of a drive-train model can also open up more avenues for future research, where traction control or limited slip control studies can be implemented, using a similar line of approach.

Finally, the extension of the present hydraulic brake system to a fully non-linear ABS hydraulic model can also be conducted in the future. However, this is not a straightforward task, as the discrete nature of the actuator/modulator dynamics would require a more robust control system. In the past, studies have been conducted where approximate mathematical model of the ABS hydraulic system with a variable structure control was used. Hitherto, such studies have involved simplistic quarter vehicle models (Wu and Shih, 2003) or relatively simple vehicle models with partial representation of ABS hydraulics (Mills et al., 2002). There has been no reported study, which comprises different non-linear elements such as a fully non-linear vehicle model with complete ABS hydraulics and an integrated transient tyre model, together with a robust controller.
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Bibliography


**Appendix**

**A) Vehicle Parameters**

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## Appendix

### B) Tyre Parameters

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