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How do symbolic and non-symbolic numerical magnitude processing relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior

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ABSTRACT

Many studies have tested the association between number magnitude processing and mathematics achievement. However, results appear to be quite different depending on the number format used. When using symbolic numbers (digits), data consistent and robust across studies and populations have been found, with weaker performance associated with weak math achievement and dyscalculia. However, when using non-symbolic format (dots), many conflicting findings are reported. These inconsistencies might be explained by methodological issues. Alternatively, it might be that the processes measured by non-symbolic tasks are not particularly critical for school-relevant mathematics. A few neuroimaging studies have also shown the brain signature of these effects. During numerical magnitude processing, the degree of brain activation (mostly in parietal areas) varies with the children's degree of math achievement, but the consistency of such relationships for symbolic and non-symbolic processing is unclear. These neurocognitive data provide ground for educational interventions, which seem to have positive effects on children's numerical development in typical and atypical populations.

Keywords: mathematical difficulties; magnitude representation; comparison; dyscalculia, approximate number system.

INTRODUCTION

One important way in which cognitive neuroscience has made successful connections to educational research is by drawing attention to the importance of numerical magnitude processing as a foundation for higher-level numerical and mathematical skills (e.g., Butterworth et al., 2011; De Smedt et al., 2010). Over the last decade, this has fuelled research aimed at investigating the relationship between individual differences in numerical magnitude processing skills and arithmetic achievement in typically developing children as well as studies probing whether children with atypical mathematical development or developmental dyscalculia (DD) are impaired in their abilities to process numerical magnitudes. Such research is beginning to lay the foundations for the design and evaluation of educational interventions that foster numerical magnitude processing.

One of the outstanding questions in this emerging body of research is whether processing magnitudes in either symbolic (digits) or non-symbolic (dots) formats or both is crucial for successful mathematics achievement. Such research can pinpoint more precisely the mathematical content that should be included in specific interventions.

Beyond educational applications, establishing whether symbolic or non-symbolic numerical magnitude processing skills, or both, are predictive of children's mathematics achievement is of theoretical importance too. While non-symbolic representations of numerical magnitudes are thought to be shared across species and can already be measured in early infancy (Cantlon, 2012), symbolic representations are uniquely human and relatively recent cultural inventions to provide abstract representations of numerical magnitude. Thus, by investigating the relationship between, on the one hand symbolic and non-symbolic numerical magnitude processing and, on the other, children's mathematical achievement, larger questions concerning the role of evolutionary ancient skills for the acquisition of uniquely human number skills and representations can also be constrained. In this

contribution, we provide an integrative review of the existing body of data that has dealt with this question.

DEVELOPMENT OF NON-SYMBOLIC NUMBER PROCESSING

The nature and role of typically developing children's magnitude representations have been commonly explored with magnitude comparison tasks (Box 1). Nonsymbolic (dot) comparison tasks are frequently thought to index the precision or acuity of representations within the approximate number system (ANS), a system which allows individuals to represent and process numerical magnitude information. Representations within the ANS are noisy and become increasingly imprecise with increasing magnitude. Individuals with more precise ANS representations perform more accurately and faster on magnitude comparison tasks and they show smaller effects of ratio or distance. Typically developing children also show an increase in the precision of ANS representations over developmental time (e.g., Halberda & Feigenson, 2008).

It has been hypothesized that performance on non-symbolic magnitude comparison tasks is related to mathematics achievement, but the evidence to support this proposal is mixed (Table 1). A number of studies have found that dot comparison performance is related to prior, concurrent and future mathematics achievement. However, many studies have failed to find such a significant relationship (see Table 1 for a summary). One possible explanation for these contrasting findings is that there is no standardized version of the dot comparison task. Studies vary in the size of the dot arrays, the way in which visual characteristics of the dots are controlled, the length of time the displays are presented and the performance measures used. This final point is particularly important as the range of possible measures includes mean accuracy, (median) RT, Weber Fraction (w) estimates, and distance or ratio effects, which may be calculated in a number of ways on the basis of accuracy or RT. These measures capture different aspects of participants' performance, they are not interchangeable

and may show different relationships with mathematics achievement (Mundy & Gilmore, 2009; Price et al. 2012). However, as shown in Table 1, studies that have or have not found a significant relationship cannot be easily differentiated by factors such as the dot comparison measure employed or the range of numbers used in a straightforward manner, since both positive and negative evidence has been found for the various performance measures and there is no clear pattern to suggest that a particular performance measure of non-symbolic numerical magnitude processing is particularly sensitive in the way it relates to individual differences in mathematics achievement.

SYMBOLIC PROCESSING DEVELOPMENT

The development of symbolic number processing has been typically investigated by means of magnitude comparison tasks that involve Arabic digits (Box 1). Performance on this task improves with age (Holloway & Ansari, 2009; Sekuler & Mierkiewicz, 1977) and is also characterized by an effect of distance or ratio. Scores on this task are not straightforward to interpret, as they might reflect the nature of underlying ANS representations, or the mapping between symbols and the ANS representations, or alternatively the nature of symbolic representations themselves, which may or may not be linked to non-symbolic ones. Nevertheless, children's performance on these symbolic comparison tasks has been found to be robustly and significantly correlated with concurrent and future mathematics achievement 1 or 2 years later (Table 2). This relationship appears to be very consistent for overall RT on the symbolic comparison task. On the other hand, similar associations with performance measures such as accuracy and distance/ratio effects have been observed in most, but not all studies (Table 2).

ATYPICAL NUMERICAL DEVELOPMENT: DEVELOPMENTAL DYSCALCULIA

Developmental dyscalculia (DD) is a persistent and specific disorder of numerical development and mathematical learning despite normal intelligence and scholastic

opportunities. Several authors have proposed that DD arises from a fundamental impairment in the representation of numerical magnitudes (e.g., Butterworth, 1999, 2005; Wilson & Dehaene, 2007). This hypothesis has been tested with numerical magnitude comparison tasks (see Table 3). Data on symbolic comparison tasks has led to very consistent results showing weaker performance in DD than in controls. This difference is the most robust in terms of global RTs (e.g., De Smedt & Gilmore, 2011; Landerl et al., 2004; Landerl & Kölle, 2009; Rousselle & Noël, 2007) yet group differences in the size of the distance effect (Rousselle & Noël, 2007) and error rate (Rousselle & Noël, 2007) have also been observed. In other words, an impairment in symbolic number processing among children with DD has been demonstrated using a variety of different dependent measures.

However, when nonsymbolic stimuli have been employed to measure numerical magnitude processing in DD, the results have been far from conclusive. On the one hand, some studies support the hypothesis of a deficient ANS in individuals with DD and showed reduced ANS acuity (Mazzocco et al., 2011a; Piazza et al., 2010), slower and less accurate performance (Mussolin et al., 2010) or less precise estimates of dot collections (Mazzocco, et al. 2011a; Mejias et al. 2012) in DD compared to typically achieving children. On the other hand, others failed to observe any significant difference between DD and controls in nonsymbolic comparison (De Smedt & Gilmore, 2011; Iuculano et al., 2008; Landerl & Kölle, 2009; Rousselle & Noël, 2007), although, in those studies, DD children displayed significant and systematic impairments in symbolic magnitude comparison. This contradictory pattern of results could partly be due to methodological differences. For instance, only Piazza et al. (2010) and Mazzocco et al. (2011a) measured the acuity of the ANS by calculating the index w and found a difference between DDs and controls. The other studies used the distance or the ratio effect as an indicator of ANS precision: Some of these studies found significant differences between DD and controls (e.g., Mussolin et al., 2010;

Price et al., 2007) whereas others did not (e.g., De Smedt & Gilmore, 2011; Landerl & Kölle, 2009).

Another factor that can explain this incoherent profile is the age of the children tested (see Noël & Rousselle, 2011; Table 3). Indeed, a dissociation appears between the studies that tested younger (6 to 9-year-olds) *versus* older children (10-year-olds and above) with only the latter group showing significant differences for tasks using non-symbolic numbers. According to Noël and Rousselle (2011), this developmental profile suggests that the first deficit seen in DD children is specific to the magnitude processing of symbolic numbers and not to the ANS. DD children would indeed be impaired in their development of an exact representation of natural numbers (Box 2) and this would explain their difficulties in manipulating exact numbers and doing exact calculation. This, in turn, would prevent them from refining their ANS in the same way as typically developing children do and would explain why difference in number acuity between DD and control children only appears later in development. Schooling, and more specifically mathematics classes have been shown to increase the acuity of the ANS (Dehaene et al., 2008; Piazza al. in press; however see Zebian & Ansari, 2012 for data indicating that literacy and schooling affect symbolic but not non-symbolic numerical magnitude processing). As children with DD would be slow and error prone in these mathematical activities, they would possibly benefit less from these mathematics activities on their ANS acuity, relative to their control peers. This could explain why the difference in number acuity between DD and control children only appears later in development.

BRAIN IMAGING DATA

There have been a growing number of efforts to uncover which brain regions might underlie the associations between numerical magnitude processing and mathematics achievement. In studies with both children and adults, the left and the right intraparietal sulci (IPS) have been found to be important neural correlates of numerical magnitude processing

(see Ansari, 2008; Cohen-Kadosh et al., 2008; Dehaene et al., 2003; see Kaufmann et al., 2011 for a meta-analysis in children), with evidence suggesting that there is increasing specialization of the parietal cortex for numerical magnitude processing over developmental time (Ansari et al., 2005; Ansari & Dhital, 2006; Cantlon et al., 2009).

Moving beyond localization, a very small set of recent studies have started to indicate that the degree to which the parietal cortex is activated during numerical magnitude processing in children is related to individual differences in their mathematics achievement. Specifically, in a functional Magnetic Resonance Imaging (fMRI) study, Bugden et al. (2012) demonstrated that the degree to which the left IPS is modulated by numerical ratio, during a symbolic number comparison task, is related to standardized measures of arithmetic fluency (over and above reading fluency) in 8-10 year old children. In other words, those children who exhibited a larger symbolic ratio effect on activity in the left IPS also displayed relatively stronger performance on the standardized tests of speeded arithmetic. In another set of recent studies (Cantlon & Li, 2013; Emerson & Cantlon, 2012), children viewed educational videos (Sesame Street) that had mathematical content, while their brain activity was recorded using fMRI. The degree of activity coupling (functional connectivity) between frontal and parietal brain regions during the viewing of these clips as well as how similar the brain activation of children was to that of a group of adults was found to be related to standardized measures of children's mathematics achievement. However, these studies cannot specifically constrain our understanding of the brain regions that underlie the association between symbolic and non-symbolic numerical magnitude processing and children's mathematics achievement, since they did not explicitly address such relationships. To the best of our knowledge there does not exist a study that reveals an association between brain activation during non-symbolic number processing and individual differences in mathematical achievement in typically developing children.

fMRI research with children with DD has revealed a largely inconclusive picture with respect to brain regions that might mediate the association between magnitude processing and DD. While some studies have shown atypical activation patterns of the parietal cortex (such as reduced distance effects on brain activation) in children with DD relative to their typically developing peers for non-symbolic (Price et al., 2007) and symbolic (Mussolin et al., 2010) numerical magnitude processing (see also Kaufmann et al., 2011 for a meta-analysis), other studies have not revealed any differences in the parietal cortex during non-symbolic number processing between children with and without DD, instead showing differences in regions related to task difficulty (Kucian et al., 2011).

Taken together, while neuroimaging methods are being used to constrain our understanding of the association between numerical magnitude processing and mathematics skills in both children with and without DD, there are currently too few studies, often with relatively small sample sizes, to allow for clear-cut conclusions to be drawn.

EDUCATIONAL INTERVENTIONS

Various attempts have been made to design educational interventions to foster the development of numerical magnitude processing. These types of interventions have been embedded in larger-scale kindergarten programs for children from low-income communities (Dyson et al., 2013; Griffin, 2004) and children at-risk for DD (Toll et al., 2013). These programs comprised a wide variety of numerical activities, including number recognition, counting, comparing sets, playing board games, etc., and have been shown to have significant effects on children's understanding of numbers and tests of early numeracy when they enter formal schooling. From these interventions, it is, however, not possible to determine the precise effects of stimulating numerical magnitude processing.

More relevant are therefore intervention studies that only focused on very specific aspects of numerical magnitude processing, as reviewed in Table 4 and Box 3. These

interventions have been presented in game-like formats using both symbolic and non-symbolic stimuli, and have been shown to have positive effects on children's numerical magnitude processing. It is important to note that these intervention effects have been mainly observed on symbolic but not non-symbolic measures of numerical magnitude processing (Table 3). The effects of these interventions seem to generalize to other mathematical skills, such as arithmetic (e.g., Ramani & Siegler, 2011; Siegler & Ramani, 2009) and standardized measures of mathematics achievement (Obersteiner et al., 2013; but see Rasanen et al., 2009 and Wilson et al., 2009), which suggests that numerical magnitude processing might be causally related to children's mathematics achievement.

Most of the existing interventions have been applied to kindergarteners or children from low-income backgrounds, yet surprisingly few studies have focused on older children or children with DD. Wilson et al. (2006) and Kucian et al. (2011) showed that computerized interventions significantly improved children with DD's numerical magnitude processing skills. Both studies did not include a control group who did not receive the intervention, which makes it difficult to evaluate whether these improvements were related to the intervention or to other factors, such as maturation or repeated testing. Interestingly, data by Vilette et al. (2010) indicate that in children with DD a short computerized game that focuses on the numerical meaning of symbolic numbers leads to larger improvements in (symbolic) number processing and calculation than a game that only taps into exact calculation, without specific attention to the numerical meaning of symbolic numbers. In all, it will be crucial for future research to investigate whether the interventions reviewed above also improve the numerical skills of children with DD.

A next step will be to investigate how brain activity changes in response to the educational interventions reviewed above, an approach that has been successfully applied in the field of reading (McCandliss, 2010). Only one study has examined the effect of a

computerized numerical training program “*Rescue Calcularis*” on brain activity in children with and without DD (Kucian et al., 2011) and revealed significant neuroplastic changes of the intervention in both groups. Future research is, however, needed to pinpoint more carefully how these interventions affect brain activity in children.

SUMMARY AND CONCLUSIONS

One of the most robust findings in the literature that sought to uncover the association between numerical magnitude processing and mathematics achievement is that children who are better in determining which of two symbolic numbers is the largest have higher achievement in mathematics. Relatedly, children with DD show significant deficits in their ability to compare symbolic numbers. These data may suggest that children with low mathematics achievement or DD have difficulties in mapping symbols to their ANS representation or, alternatively, that they fail to adequately construct a system for the representation of symbolic number that is fundamentally different from the ANS (see Box 2). Data on non-symbolic comparison tasks, however, have been inconclusive so far, in both typically developing and DD populations. These inconsistencies might be explained by differences in the age of the participants, the stimuli used as well as indices that were calculated to tap into nonsymbolic processing and mathematics achievement. While there may well be important methodological issues that obscure the relationship between non-symbolic magnitude processing and mathematics achievement in many of the studies we have reviewed, it is nevertheless important to note that the correlations between symbolic numerical magnitude processing and mathematics achievement do not appear to be subject to such constraints. In view of this, it can be argued that such relationships are more robust and that the difficulty in finding relationships between non-symbolic numerical magnitude processing and mathematics achievement may indicate that the kinds of representations and

processes measured by these tasks are not particularly critical for children's development of school-relevant mathematical competencies.

It is important to point out that the existing body of studies have typically employed standardized or curriculum measures of mathematics achievement, which encompass a range of mathematical skills (e.g. number fact knowledge, conceptual understanding, strategy use and proficiency). It is likely that any meaningful relationship between numerical magnitude representations and mathematics will vary across different mathematical skills. In other words, numerical magnitude processing will be more important for some aspects of mathematical competencies than others. Thus, more specific measures of mathematical performance will be needed to explore such specific associations (e.g., Vanbinst et al., 2013).

Relatively few longitudinal studies have been conducted to investigate how the associations between numerical magnitude processing and mathematics achievement change with age. Such research is, however, necessary to unravel the developmental trajectory of these associations. These data will also help to reveal developmental changes in impairments in numerical magnitude processing, i.e. the precise time course as to when deficits in symbolic and/or non-symbolic processing emerge.

Intervention research indicates that board games and computer games can be used to effectively foster children's symbolic representations of magnitude. These games seem to have effects on more general measures of mathematics achievement, although not all studies have observed such generalization effects and not all studies have used matched control groups to evaluate the specificity of any training effects observed. It should be noted that the studies reviewed above have typically included both symbolic and non-symbolic interventions. Future studies should contrast interventions that focus on non-symbolic processing, symbolic processing or both, to evaluate which of these interventions has the largest impact on children's numerical magnitude processing and more generally on their

mathematical development. This type of research is also necessary to determine whether children's symbolic and/or nonsymbolic processing skills are causally related to their mathematics achievement. In view of the above- reviewed studies, it would even be more compelling to run these studies across different ages, to verify which type of intervention is appropriate at which age.

From a practical point of view, the existence of computer games to foster children's understanding of numerical magnitudes is extremely relevant for the early intervention of at-risk children. Such games allow teachers and parents not only to stimulate this knowledge but also provide a motivating environment. This is particularly interesting in view of the increasing availability of tablets and smartphones, which offer opportunities to practice these skills at home.

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TABLES

Table 1

The nature of the relationship between nonsymbolic (dot) comparison task performance and mathematics achievement in typically developing participants. The dot comparison measure(s) used and number range of the task are given in brackets.

Relationship between dot comparison performance and mathematics	
Significant	Nonsignificant
Children	Children
Halberda et al. (2008) [<i>w</i> ; 5-16] ^a	Holloway & Ansari (2009) [NDE; 1-9]
Mundy & Gilmore (2009) [acc; 1-9]	Mundy & Gilmore (2009) [NDE; 1-9]
Inglis et al. (2011) [<i>w</i> ; 5-22]	Soltesz et al. (2010) [acc, RT, NRE; 4-20]
Libertus et al. (2011) [acc, <i>w</i> , RT; 4-15]	Lonneman et al. (2011) [NDE; 4-6]
Mazzocco et al. (2011b) [acc, <i>w</i> ; 1-14] ^a	Ferreira et al. (2012) [acc; 20-44]
Bonny & Lourenco (2013) [<i>w</i> , acc; 4-12]	Sasanguie et al. (2012a) [RT/error, NDE; 1-9]
Libertus et al. (2013) [acc, <i>w</i> , RT ; 4-15]	Sasanguie et al. (2012b) [RT/error, NDE; 1-9] ^a
	Vanbinst et al. (2012) [NDE; 1-9]
	Fuhs & McNeil (2013) [acc; 1-30]
	Kolkman et al. (2013) [acc; 1-100]
	Sasanguie et al. (2013) [<i>w</i> , acc; 6-26] ^a
Adults	Adults
Lyons & Beilock (2011) [<i>w</i> ; 1-9]	Inglis et al. (2011) [<i>w</i> ; 9-70]
Halberda et al. (2012) [<i>w</i> , RT; 5-20]	Castronovo & Göbel (2012) [<i>w</i> ; 12-40]
Libertus et al. (2012) [<i>w</i> ; 5-20]	Price et al. (2012) [<i>w</i> , NDE; 6-40]
Lourenco et al. (2012) [acc; 5-14]	

Acc = accuracy; NDE = numerical distance effect; NRE = numerical ratio effect; RT = response time; w = estimates of Weber fraction; ^aLongitudinal data.

Table 2

The nature of the relationship between symbolic (digit) comparison task performance and mathematics achievement in typically developing participants. The digit comparison measure(s) used and number range of the task are given in brackets.

Relationship between digit comparison performance and mathematics	
Significant	Nonsignificant
Children	Children
Durand et al. (2005) [acc; 3-9]	Lonneman et al. (2011) [NDE; 1-3]
De Smedt et al. (2009) [acc, RT, NDE, 1-9] ^a	Ferreira et al. (2012) [acc ; 1-9]
Holloway & Ansari (2009) [RT, NDE; 1-9]	Sasanguie et al. (2012b) [NDE; 1-9] ^a
Mundy & Gilmore (2009) [acc, NDE; 1-9]	Sasanguie et al. (2013) [NDE; 1-9] ^a
Bugden & Ansari (2011) [RT, NDE; 1-9]	
Lonneman et al. (2011) [RT, NDE; 4-6]	
Sasanguie et al. (2012a) [RT/error, NDE; 1-9]	
Sasanguie et al. (2012b) [RT/error; 1-9] ^a	
Vanbinst et al. (2012) [NDE; 1-9]	
Kolkman et al. (2013) [acc; 1-100]	
Sasanguie et al. (2013) [RT; 1-9] ^a	
Adults	Adults
Lyons & Beilock (2011) [acc; 1-9]	
Castronovo & Göbel (2012) [RT, NDE; 31-99]	

Acc = overall accuracy; RT = overall response time; NDE = numerical distance/ratio effect.

^aLongitudinal data.

Table 3

Comparison of the performance of children with developmental dyscalculia (DD) and matched controls (C) in symbolic or non-symbolic number comparison tasks.

Study	Age (years)	Symbolic significant	Symbolic ns	Non-symbolic significant	Non-symbolic ns
De Smedt & Gilmore (2011)	6	RT: [1-9]	NDE(RT)		RT, NDE(RT): [1-9]
Rousselle & Noël (2007)	7	RT, Size(RT), NDE(RT), Acc: [1-9]			RT, NDE(RT), Acc: [5-28]
Landerl & al. (2004)	8-9	RT : [1-9]	NDE(RT)		
Iuculano & al. (2008)	8-9	RT: [1-9]			RT, Acc: [1-9] RT: [10-58]
Landerl & Kölle (2009)	8-10	RT: [1-9] RT, Acc: [21-98]	NDE(RT)		RT, NDE(RT): [20-72]

Landerl, Fussenegger & al. (2009)	8-10	RT: [1-9], RT: [21-98]	NDE(RT)	RT: [20-72]	
Piazza & al. (2010)	10	-		<i>W</i> : [12-40]	RTs
Mussolin & al. (2010)	10-11	NDE(Acc): [1-9]	Acc, RT	NDE(Acc): [1-9]	Acc, RT
Price & al. (2007)	12	-		NDE(RT): [1-9]	Acc, NDE(acc), RT
Anderson & al., (2012)	11-13	RT, NDE(RT): [1-9] RT:[2-digit]			RT: [2-8]
Mazzocco & al. (2011a)	14	-		<i>w</i> : [5-16]	

Acc = overall accuracy; RT = overall response time; NDE = numerical distance/ratio effect

Table 4

Cognitive interventions that focused on numerical magnitude processing

Study	Sample size	Age (years)	Participant group(s)	Intervention	Control	Duration	Outcome(s)	Main Effect(s)
Whyte & Bull (2008)	54	3.8	Middle-class and working class	Linear number BG Nonlinear number BG	Linear color BG	6 × 25 min	Symbolic comparison	Number BG > Color BG
Ramani & Siegler (2008)	124	4.9	Low-income	Number BG	Color BG	4 × 15-20 min	Symbolic comparison	Number BG > Color BG Effect remained after 9 weeks
Siegler & Ramani (2009)	88	4.8	Low-income	Linear number BG	Circular number BG Numerical activities control	5 × 15-20 min	Symbolic comparison Arithmetic	Linear BG > Circular + numerical control
Ramani & Siegler (2011)	88	4.0	Middle-upper class	Linear number BG	Circular number BG Numerical activities control	5 × 15-20 min	Symbolic comparison Arithmetic	Linear BG > Circular + numerical control Linear BG showed transfer to arithmetic
Wilson et al. 2006	9	8.1	DD	NRG	None	16 × 20-30 min	Symbolic comparison Nonsymbolic comparison Addition Subtraction	Performance increased only in symbolic comparison and subtraction
Wilson et al. 2009	53	5.6	Low SES	NRG	Reading control game	6 × 20 min	Symbolic comparison Nonsymbolic comparison	NRG > Reading on symbolic comparison

							Addition	No improvements in nonsymbolic comparison.
Räsänen et al. 2009	59	6.6	SEN-children, TD	NRG, Graphogame Math	No game	15 × 10-15min	Symbolic comparison	Interventions improved symbolic comparison
							Addition	No other effects
Obersteiner et al. 2013	147	6.9	TD	NRG - Exact NRG – Approximate NRG Exact + Approximate	Language game	10 × 30 min	Symbolic comparison Nonsymbolic comparison Approximate calculation Arithmetic	NRG > Language game Approximate NRG showed largest effects on comparison tasks
Vilette et al. 2010	20	11.0	DD, TD	The Estimator	Numerical games without estimation	5 × 20 min	Standardized number battery (ZAREKI) Addition, subtraction	The Estimator > numerical games
Kucian et al. 2011	32	9.5	DD, TD	Rescue calcularis		25 × 15 min	Standardized number battery (ZAREKI)	Both groups improved

Note. Only studies that included numerical magnitude comparison tasks or mathematics achievement tests as outcome measures were included and only these outcome measures were reported in the table. BG = Board Game. NRG = Number Race Game. DD = Developmental Dyscalculia. TD = Typically Developing children. SEN = Special Educational Needs.

BOXES

BOX 1: Magnitude Comparison Tasks

The nature of numerical representations is typically explored using magnitude comparison tasks. In a standard **nonsymbolic** magnitude comparison task, participants are shown two dot arrays – or sequences of sounds – and asked to select the more numerous. The difficulty of making this decision is manipulated by varying the ratio or the numerical distance between the two arrays. For example, it is more difficult to distinguish 12 and 9 dots (ratio 0.75; numerical distance 3) than it is to distinguish 12 and 6 dots (ratio 0.5; numerical distance 6). Typical measures of performance include overall accuracy, response time (RT), ratio or distance effects or the w index. This Weber fraction (w) can be calculated on the basis of the participants' performance across different ratios, and provides a measure of the acuity of ANS representations. Individuals with a smaller w have more precise ANS representations than those with a larger w .

To increase the possibility that participants use the number of dots rather than visual characteristics of the displays (e.g., dot size, density, total area), the dot arrays are typically constructed in such a manner so that these characteristics do not correlate with numerosity across the task, i.e. dot size, density and area vary across the experiment. However, recent data by Gebuis & Reynvoet (2012) indicate that it is impossible to perfectly control for these non-numerical parameters and that the number of items in a set cannot be extracted independently of visual cues. While the existing studies all controlled for non-numerical parameters in their experimental design, the degree to which some visual properties of the stimuli are controlled for varies between them and this might also account for the differences in the results they obtained. In other words, it is unclear how participants use the various non-numerical visual characteristics of the stimuli to guide their decision as to which array of dots

is larger and how this process might differ between children who have various levels of mathematical competence. On the other hand, the data by Gebuis & Reynvoet (2012) also call into question the degree to which non-symbolic number processing can truly be measured.

Symbolic comparison tasks typically have the same format, except that the quantities are represented as Arabic digits, or in some studies, number words. Similar effects of distance or ratio on performance are observed when people perform this task.

BOX 2: The nature of symbolic representations

For several authors (see Carey, 2004; 2009; Noël et al., 2008; Wiese, 2003, 2007), symbolic numbers do not acquire their meaning from a mapping with the ANS. Indeed recent research with adults suggests that the association between numerical symbols and the ANS may be much weaker than has been traditionally assumed (Lyons et al., 2012). Rather, learning the meaning of number words leads to the emergence of a new numerical representation that would be exact and with a semantic content based on the ordinal information enclosed in the symbol sequence. This exact representation would then connect with the ANS and this mapping would contribute to the increase of the precision of the ANS (Piazza et al. 2010; Piazza et al., in press).

BOX 3: Interventions that foster numerical magnitude processing

Number board games

These games are similar to existing board games, such as snakes and ladders or chutes and ladders, and consist of consecutively numbered spaces, on which children have to move their token, depending on the number they produced via a spinner or dice. The games can differ in the spatial organisation of the numbered spaces (linear or circular). It is argued that these games provide multimodal cues to connections between symbols and their respective quantities: the larger the number indicating how many squares their token needs to be moved, the larger the distance the child needs to move the counter, the larger the number of moves to be made and the number of number words to be spoken (Siegler, 2009).

Number Race Game

The Number Race Game (Wilson et al., 2006) was specifically designed as a remediation program for children with DD. The game provides training in comparing numbers and tries to establish links between numbers and space by asking children to position their counter on a board, depending on the number of coins they earned during the comparison of two presented numbers. Numbers are presented in nonsymbolic and symbolic formats. At higher levels, small addition and subtraction problems are also included. The game is adaptive and presents stimuli depending on the level of the learner.

The Estimator

Estimator is a computer game for children with DD that aims to develop the connection between exact and approximate number representations in addition and subtractions (Vilette et al., 2010). A calculation problem ($12+23=$) appears on the screen and children have to indicate the approximate position of the answer on a 0-100 number line. If this approximation is correct, the result of the calculation is presented and the next trial starts. If the estimate is

incorrect, the number corresponding to the position of the cursor on the number line appears and the child is invited to estimate again.

Rescue Calcularis

In this game, children control a spaceship to rescue the planet “Calcularis” (Kucian et al., 2011), by travelling through 10 planets through which the child can refuel the spaceship to arrive at Calcularis. On each planet, children have to solve a series of problems of increasing difficulty: they have to position their spaceship on a number line, depending on the Arabic digit, collection of dots or arithmetic problem displayed on the spaceship.

BOX 4: Outstanding questions

- How does the association between numerical magnitude processing and mathematics achievement change over developmental time? Is there a sensitive period for this association which should be the target of education?
- How do numerical symbols acquire developmentally their semantic meaning and how might this process be disrupted in children with DD?
- What is the role of non-numerical parameters (such as density of the arrays, the areas of individual dots, their luminance etc.) that covary with non-symbolic numerical magnitude stimuli in the typical and atypical development of the ANS?
- Are there cultural differences in the development of numerical magnitude representations?
- What are the effects of numerical magnitude interventions on children (with DD)'s numerical magnitude skills? Do these effects generalize to other mathematical skills (e.g., arithmetic, complex calculation, etc.) ? Do these effects sustain over longer time periods?
- How do brain structure and function change in response to numerical magnitude interventions?