Digital simulation of a gas-turbine generating unit

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DIGITAL SIMULATION

OF

A GAS-TURBINE GENERATING UNIT

BY

William Wing-Cheung Hung, B.Sc., C.Eng., M.I.E.E.

A Doctoral Thesis
Submitted in Partial Fulfilment of the
Requirements for the Award of the Degree of
Doctor of Philosophy

of

Loughborough University of Technology

January, 1983

Supervisors: S. Williams, B.Sc., Ph.D., C.Eng., M.I.E.E.
Professor I.R. Smith, B.Sc., Ph.D., D.Sc., C.Eng., F.I.E.E.
Department of Electronic and Electrical Engineering.

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Most existing synchronous machine stability studies are based on the assumption that the machine is operating on an infinite busbar of constant voltage and frequency. The present study however, is concerned mainly with the dynamic stability of the type of gas-turbine generating unit commonly used in remote sites, such as off-shore oil rigs and desert areas, where the grid is far from stiff and is liable to become unstable in the event of a severe system disturbance. Because of this, there is a pressing need for an accurate representation of the system, to enable investigations to be made into problems concerned with the system response and to consider any improvements that may be effected in the associated control scheme.

The initial stage of the investigation involves the development of digital models for a synchronous machine, using both direct-phase co-ordinates and a stationary 2-axis representation. Symmetrical and unsymmetrical faults at the generator terminals are simulated and comparisons with test results are made. Despite being more restricted in its range of application than the direct-phase representation, the 2-axis model is used throughout the remainder of the investigations described in the thesis, mainly because of its computational convenience.

The increasing use of large induction motor drives makes it necessary to study their effect on system stability, especially in the case of small gas-turbine power systems. Various types of switching transients on both small and large squirrel-cage induction motors are therefore studied. The successful development of the synchronous and induction machine models is followed by a fault simulation of a composite system containing both types of machines.
The machine investigation is followed by the simulation of a conventional steam turbo-alternator unit operating on an infinite grid system. Various types of system disturbances are simulated and the effect of generator saturation on the system response is considered. Following this study, a mathematical model of a gas-turbine generating unit is developed, with emphasis given to operation in an isolated grid situation. To validate this model, correlations between predicted and test results on a typical installation are presented.
ACKNOWLEDGEMENTS

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The author is greatly indebted to the General Electric Company and his previous employer, GEC Gas Turbines Limited, for the GEC Fellowship award which enabled him to be seconded to Loughborough University of Technology to carry out his research work. Financial assistance from the Science Research Council is also acknowledged.

A particular debt of gratitude is due to those former colleagues at GEC Gas Turbines Limited for their advice and assistance in obtaining the system design information, and the preparation and execution of site tests. Thanks are also due to the staff of the Computer Department at Loughborough University of Technology for running his many computer programs.

The author would also like to express his gratitude to the staff of Merz and McLellan for typing the text and producing the diagrams in the thesis.

The greatest debt, however, is to the author's wife for her tolerance, persistent encouragement and support which helped him through the most crucial period of this project.
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LIST OF PRINCIPAL SYMBOLS

(Other symbols are defined as they occur)

**Electrical machines**

\( v, i \) instantaneous voltage and current

\( V, I \) r.m.s. voltage and current (except those in eqns. 2.1, 2.2, 2.5 and 2.6)

\( R, r \) resistance

\( L \) inductance

\( M \) mutual inductance

\( X, x \) inductive reactance

\( Z, z \) impedance

\( f \) frequency

\( \psi \) flux linkage

\( \delta \) load angle

\( \theta \) rotor angular position

\( \dot{\theta}, \omega \) rotor angular speed

\( \ddot{\theta} \) rotor angular acceleration

\( u \) mechanical speed

\( J \) moment of inertia

\( H \) inertia constant

\( T_e \) electrical torque

\( T_t \) transmitted torque

\( T_L \) loss torque

\( P \) power

\( n_{pp} \) number of pair of poles

\( t \) time

\( p \) Heaviside operator, \( \frac{d}{dt} \)
Electrical machines, continued

\( s \) \hspace{1cm} \text{Laplace transform operator}

\( x_d, x_q \) \hspace{1cm} \text{synchronous reactances}

\( x'_d \) \hspace{1cm} \text{transient reactance}

\( x''_d, x''_q \) \hspace{1cm} \text{subtransient reactances}

\( x_a \) \hspace{1cm} \text{armature leakage reactance}

\( T'_d \) \hspace{1cm} \text{short-circuit transient time constant}

\( T'_{do} \) \hspace{1cm} \text{open-circuit transient time constant}

\( T''_d, T''_q \) \hspace{1cm} \text{short-circuit sub-transient time constants}

\( T''_{do}, T''_{qo} \) \hspace{1cm} \text{open-circuit sub-transient time constants}

\( T_a \) \hspace{1cm} \text{armature time constant}

\textbf{Subscripts}

\( a, b, c \) \hspace{1cm} \text{armature phases}

\( f, ff \) \hspace{1cm} \text{field circuit}

\( d, q \) \hspace{1cm} \text{direct and quadrature axes, respectively}

\( D, DD, Q, QQ \) \hspace{1cm} \text{direct and quadrature dampers, respectively}

\( 1, 2 \) \hspace{1cm} \text{rotor and stator circuits, respectively}

\( m \) \hspace{1cm} \text{mutual}

\( o \) \hspace{1cm} \text{base or open circuit}

\( 0 \) \hspace{1cm} \text{initial or zero sequence component}

\( l \) \hspace{1cm} \text{leakage}

\( a \) \hspace{1cm} \text{armature}

\( t \) \hspace{1cm} \text{generator terminal or transmission system}
Excitation control systems

\[ v_t, v_T \] terminal voltage

\[ v_{ref}, v_{REF} \] reference voltage

\[ v_{r1}, v_{r2} \] output voltages from magnetic amplifiers 1 and 2, respectively

\[ v_f, E_{FD} \] exciter output voltage (after rectification)

\[ v_{rf}, v_{ef} \] stabilising signal voltages derived from magnetic amplifier 2 and exciter, respectively

\[ v_{r1 \ max}, v_{r1 \ min} \] saturation limits for magnetic amplifier 1

\[ v_{r2 \ max}, v_{r2 \ min} \] saturation limits for magnetic amplifier 2

\[ K_i \] gain constant of voltage transformer and rectifier circuit

\[ K_c \] gain constant of comparator

\[ K_{r1}, K_{r2} \] gain constants of magnetic amplifiers 1 and 2, respectively

\[ K_e \] gain constant of exciter and rotating rectifier

\[ K_{rf}, K_{ef} \] gain constants of feedback stabilising circuits

\[ T_{r1}, T_{r2} \] time constants of magnetic amplifiers 1 and 2, respectively

\[ T_e \] exciter time constant

\[ T_{rf1}, T_{rf2} \] time constants of amplifier stabilising circuit

\[ T_{ef1}, T_{ef2} \] time constants of exciter stabilising feedback circuit

\[ B_{r1}, B_{r2} \] bias constants of amplifiers 1 and 2, respectively

\[ B_e \] bias constant of exciter

\[ V_R \] regulator output voltage

\[ K_A \] regulator gain

\[ K_E \] exciter constant related to self-excited field
Excitation control systems, continued

- $K_F$: regulator stabilising circuit gain
- $T_R$: regulator input filter time constant
- $T_A$: regulator amplifier time constant
- $T_E$: exciter open-circuit time constant
- $T_{F1}, T_{F2}$: regulator stabilising feedback circuit time constants
- $S_E$: exciter saturation function
- $V_{RMAX}$: maximum value of $V_R$
- $V_{RMIN}$: minimum value of $V_R$
- $E_{FDMAX}$: maximum value of $E_{FD}$
- $E_{FDMIN}$: minimum value of $E_{FD}$

Steam turbine and governor system

- $u_{sg}$: sleeve movement
- $u_0$: speeder gear setting
- $u_{tv}$: throttle valve position
- $u_{vo}$: preset constant of throttle valve
- $P_s$: steam power admitted to turbine
- $P_t$: shaft power input to generator
- $K_{sg}$: gain constant of centrifugal governor
- $K_{hy}$: gain constant associated with servomechanisms between centrifugal governor and throttle valve
- $K_v$: transfer constant of throttle valve
- $T_{hy1}, T_{hy2}$: time constants related to servomechanisms
- $T_s$: steam turbine time delay
Gas turbine and governor system

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<td>$Q_F$</td>
<td>fuel flow</td>
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<td>power turbine speed</td>
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<td>$N_G$</td>
<td>gas generator speed</td>
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<td>$P_2$</td>
<td>compressor delivery static pressure</td>
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<td>$P_4$</td>
<td>gas generator exhaust gas static pressure</td>
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<td>$T_4$</td>
<td>exhaust cone temperature</td>
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<tr>
<td>$M_4$</td>
<td>exhaust gas mass flow</td>
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<td>EGP</td>
<td>exhaust gas power</td>
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<td>$\mu_{NG}$</td>
<td>gain or sensitivity of the corresponding gas-turbine variables on fuel flow</td>
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<td>$\mu_{P2}$</td>
<td>lead/lag ratio for $P_2$ and $P_4$</td>
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<td>$\mu_{T4}$</td>
<td>lead/lag ratio for $T_4$</td>
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<td>$\mu_{EGP}$</td>
<td>lead/lag ratio for EGP</td>
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<td>$a_{P2}$</td>
<td>gas generator rotor inertia time constant</td>
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<td>$a_{T4}$</td>
<td>combustion chamber time constant</td>
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<td>$a_{EGP}$</td>
<td>exhaust duct time constant</td>
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<td>$T_{TC}$</td>
<td>exhaust gas temperature thermocouple time constant</td>
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<tr>
<td>$C_P$</td>
<td>specific heat at constant pressure</td>
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<td>$\gamma$</td>
<td>ratio of specific heats</td>
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<td>ambient static pressure</td>
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<td>ambient temperature</td>
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<td>$\eta_T$</td>
<td>power turbine isentropic efficiency</td>
</tr>
<tr>
<td>$V_c$</td>
<td>least gate output voltage</td>
</tr>
</tbody>
</table>
**Gas turbine and governor system, continued**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_d$</td>
<td>demand voltage</td>
</tr>
<tr>
<td>$V_{P2}$</td>
<td>$P_2$ feedback voltage</td>
</tr>
<tr>
<td>$V_{CDP}$</td>
<td>compressor discharge pressure demand voltage</td>
</tr>
<tr>
<td>$V_{ACL}$</td>
<td>acceleration fuel schedule limit voltage</td>
</tr>
<tr>
<td>$V_{DCL}$</td>
<td>deceleration fuel schedule limit voltage</td>
</tr>
<tr>
<td>$V_{TD}$</td>
<td>throttle demand voltage</td>
</tr>
<tr>
<td>$\theta$</td>
<td>throttle angle</td>
</tr>
<tr>
<td>$V_\theta$</td>
<td>throttle angle feedback voltage</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION
CHAPTER 1
INTRODUCTION

The degree of sophistication of power system simulation has undergone considerable change over the past decade. With the advent of powerful and high-speed computer facilities, power-system analysts have been able to simulate system components as accurately as the available information will allow.

Synchronous machine stability studies have been a subject of interest for many years, although much of the work presented has been based on steam- or hydro-turbine generating sets. However, in the present study, emphasis is placed on dynamic stability studies of a gas-turbine generating unit.

Aero-type gas-turbine engines have been widely adopted as prime movers for electrical power generation units for over two decades. The fully automatic start-up capability and the fast run-up characteristics of such engines have made them particularly suitable for peak-load lopping and standby power supply purposes. In terms of size, weight and adaptability to a wide range of fuels (from natural gas to crude or residual oil), the gas turbine is far superior to other forms of prime mover. For this, and for many other reasons, the gas turbine has lent itself directly to the fast growing oil and gas production industries, not only as an ideal prime mover for electrical power generation, but also for gas compression and injection and crude oil pumping.

Unlike steam-turbine or hydro-electric generators, gas-turbine generators are commonly connected to a small grid system, or even used in isolated operation as in the off-shore platform. Such types of systems are liable to become unstable after a severe system disturbance and, indeed, the relatively small inertia constant of the gas turbines further aggravates this problem. In addition, electric motors are often used in oil fields to drive pumps and compressors for
processing, product exporting and water re-injection. A continual variation of the system load is inevitable and these variations are often on a step-change basis and could possibly be as high as 40 per cent of the standing load. Consequently, variations in the system voltage and frequency are unavoidable. An effective control system is therefore required to maintain system stability following a system disturbance, as failure to do so will cause an inevitable plant shut-down, from which a loss of production and considerable damage to the plant may result. Because of this, there is an increasing demand for an accurate model of such a system, to enable the system response to be investigated and improvements to the associated control system to be made. However, no such detailed model has yet been reported in the literature and one of the main objectives of this thesis is therefore to develop an accurate model for a complete gas-turbine generating unit, comprising a generator, a brushless excitation control system and an aero-type gas-turbine engine with its electronic and hydraulic governor control system.

The investigation begins with the digital simulation of a synchronous machine. The development of a circuit-theory approach to the solution of transient problems in electrical machines has evoked considerable interest in different techniques for the solution of the associated differential equations. Various transformations, such as α–β–0 or d–q–0 etc., were introduced to eliminate the time-varying coefficients of the basic equations, thereby allowing a solution to be obtained by the use of operational methods. However, the application of such methods is somewhat limited because of the assumptions necessary during the analysis.

Following the advent of modern computers, numerical methods can now be employed efficiently for solving non-linear differential equations. In such circumstances, a mathematical model, using either a stationary 2-axis or a 3-phase representation, can be used for simulation studies. The 2-axis model is widely adopted for investigations involving symmetrical operation of a synchronous machine 2-5, while the phase model is mainly used for unsymmetrical fault studies 6-9. The direct-phase model also enables the non-ideal properties of a real machine, such as the higher harmonic components of the airgap m.m.f., to be included in the machine studies 9,10.
Synchronous machine models based on both representations are developed in the following two chapters of the thesis, with the results of simulations of various types of faults at the generator terminals being presented and comparisons with test results being made.

Gas-turbine generators are commonly installed in plants (e.g. oil fields and petroleum refineries) where as much as 85 per cent of the connected loads are induction motors, with capacities ranging from fractional horse-power to megawatts. The dynamic behaviour of such machines can therefore have a marked effect on the system performance under transient conditions.

A mathematical model describing the dynamic behaviour of a 3-phase single-cage induction motor, based on the stationary 2-axis approach, is developed and its validity is checked by comparison with transients recorded on a 5.6 kW single-cage machine subsequent to various types of switching likely to be encountered in practice. For the simulation of large induction motors, the program is modified to include 'deep-bar' effects in the rotor. However, rather than using 2-coils on each rotor axis, the more direct approach described in Section 4.4 is used. The accuracy of such a model is again verified by comparing test and predicted transients for motor short-circuit and open-circuit faults.

The successful development of mathematical models for both synchronous generators and induction motors has led to a study of the dynamic behaviour of systems containing both types of machines. The system simulated in the thesis corresponds to a typical industrial plant, in which induction motors are partly fed by the plant generators, with the balanced supply being taken from the main grid via a transmission system. The response of the plant machines following a short-circuit fault occurring at the transmission line is investigated.

Before proceeding with the simulation of a gas-turbine/generator model, a mathematical model of a conventional steam-turbine/generator system comprising a generator, an excitation control system, a steam turbine and its governing system is developed. Based on this model, the effect of magnetic saturation on the generator
response following various types of system disturbances is investigated and comparisons with test results are made.

Various types of excitation system are used with gas-turbine generators, but the brushless scheme employing silicon diodes is by far the most widely adopted. The derivation of a mathematical model for such a system, based on the IEEE type 2 representation, is presented in Chapter 6.

This investigation is followed by the simulation of an aero-type gas turbine using a linearised-model approach. To extend the validity of the model to large disturbances, engine parameters are varied as functions of engine speed.

The demand for an ever-increasing gas-turbine performance has brought about the need for a greatly improved accuracy of speed control. The governor control system for a gas-turbine generator working under a widely and continuously varying load requirement is extremely complex. Based on design information, a detailed mathematical model for such a governor control system is developed in the thesis.

Having completed the gas-turbine/generator model, the final stage of the investigation involves the simulation of various types of switching transients on this model and its validity is thereby assessed using site-test results.

The mathematical models developed in this thesis are expressed in state-variable form. This approach provides a systematic means of assembling the system equations in a form which can conveniently be solved by a digital computer using numerical integration methods. In recent years, various numerical methods for the computer solution of differential equations have become available and an efficient predictor-corrector routine developed by Gear is chosen for the present investigation.

The computer programs developed throughout the investigation are written in Fortran IV and run on an ICL 1904A machine.
CHAPTER 2

MATHEMATICAL MODELS

FOR

A 3-PHASE SYNCHRONOUS MACHINE
CHAPTER 2
MATHEMATICAL MODELS FOR A 3-PHASE SYNCHRONOUS MACHINE

Based on modern machine theory, a phase model for an idealised 3-phase 2-pole synchronous machine is developed in this chapter. By the application of 2-axis theory, an alternative representation (i.e. a stationary 2-axis model) is derived therefrom.

In the past, p.u. systems have been discussed in relation to the circuit aspects of both machines and power systems. However, the approaches adopted have differed significantly, depending on the individual applications. In addition, there is often a too-ready assumption that p.u. systems are theoretically self-evident, requiring and receiving only a minimum explanation. Consequently, the whole subject appears diffuse and confusing. The p.u. system employed for the present study is therefore clearly specified.

2.1 - Representation of an idealised 2-pole synchronous machine

The essential features of an idealised 2-pole synchronous machine are shown in fig. 2.1. The rotor carries three identical and symmetrically-spaced windings a, b and c, one for each armature phase, while the stator contains a field winding f and two damper windings D and Q.

To expedite analysis, the machine is idealised by means of the following assumptions.

a. The airgap m.m.f. due to armature currents is sinusoidally distributed. This may be justified from the standpoint that the windings are distributed so as to minimise all harmonics as far as possible.
b. Slotting effects are ignored, by assuming the distributed windings to be formed from closely-spaced conductors of negligible diameter.

c. The effects of saturation and hysteresis in the magnetic circuits and eddy currents in the armature iron are ignored. (The effect of saturation on machine performance is considered later in Section 5.1.1).

d. Each circuit of the real machine can be represented by a single coil.

2.2 - Direct-phase representation

The synchronous machine to be modelled is assumed to be connected to an infinite busbar through a transmission system, each phase of which can be represented by a series lumped impedance. If the resistance and inductance of each phase are included in the corresponding armature terms, the machine can be regarded as connected directly to the infinite system.

2.2.1 - Derivation of voltage equations

The sign convention adopted for the electrical quantities is positive for motoring action and negative for generating action, which implies that externally applied voltages and currents flowing into the machine are taken as positive. The advantages of this convention are that it

a. agrees with conventional circuit theory

b. introduces the minimum number of negative signs into the machine equations

c. agrees with Kron's Tensor Theory which treats all machines on a common basis.
The absolute voltage equations relating the six circuits of an idealised 3-phase synchronous machine (i.e. f, D, Q, a, b and c) may be expressed by the matrix equation,

\[
[V] = [R] [I] + p([L][I])
\]

where

\[
[V] = [V_f, V_D, V_Q, V_a, V_b, V_c]^t
\]
\[
[I] = [I_f, I_D, I_Q, I_a, I_b, I_c]^t
\]
\[
[R] = \text{Diagonal} \begin{bmatrix} R_f, R_D, R_Q, R_a, R_b, R_c \end{bmatrix}
\]

\[
[L] = \begin{bmatrix}
L_{ff} & L_{fd} & L_{fq} & L_{fa} & L_{fb} & L_{fc} \\
L_{df} & L_{DD} & L_{DQ} & L_{Da} & L_{Db} & L_{Dc} \\
L_{Qf} & L_{QD} & L_{QQ} & L_{Qa} & L_{Qb} & L_{Qc} \\
L_{af} & L_{aD} & L_{aQ} & L_{aa} & L_{ab} & L_{ac} \\
L_{bf} & L_{bD} & L_{bQ} & L_{ba} & L_{bb} & L_{bc} \\
L_{cf} & L_{cD} & L_{cQ} & L_{ca} & L_{cb} & L_{cc}
\end{bmatrix}
\]

in which like subscripts denote a self-inductance and unlike subscripts represent a mutual inductance between the corresponding windings.

It follows from considerations of the structure of a salient-pole machine that the inductances of the machine comprise \(^{16}\) (a) the constant field circuit and damper circuit self and mutual inductances, (b) the armature self and mutual inductances containing even space harmonic terms (c) the stator and rotor mutual inductance containing odd space harmonic terms.
If the angular variations of the various machine inductances are derived using magnetic circuit theory\(^{16}\), the resulting machine inductance matrix has the form

\[
\begin{array}{|c|c|c|c|c|}
\hline
L_f & M_{FD} & M_{af} \cos \theta_a & M_{af} \cos \theta_b & M_{af} \cos \theta_c \\
\hline
M_{FD} & L_D & M_{aD} \cos \theta_a & M_{aD} \cos \theta_b & M_{aD} \cos \theta_c \\
\hline
\vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
M_{af} \cos \theta_a & M_{aD} \cos \theta_a & M_{aQ} \sin \theta_a & L_{ao} & M_{so} \\
& & & +L_{a2} \cos 2\theta_a & +L_{a2} \cos 2\theta_c \\
M_{af} \cos \theta_b & M_{aD} \cos \theta_b & M_{aQ} \sin \theta_b & M_{so} & L_{ao} \\
& & & +L_{a2} \cos 2\theta_c & +L_{a2} \cos 2\theta_b \\
M_{af} \cos \theta_c & M_{aD} \cos \theta_c & M_{aQ} \sin \theta_c & M_{so} & L_{ao} \\
& & & +L_{a2} \cos 2\theta_b & +L_{a2} \cos 2\theta_c \\
\hline
\end{array}
\]

where \( L_{ao}, M_{so} \) and \( L_{a2} \) are the inductance coefficients of the armature\(^3\), with the voltage equations including the parameters of the transmission system being as given in eqn. 2.5.
<table>
<thead>
<tr>
<th>Vf</th>
<th>Vd</th>
<th>Vq</th>
<th>Va</th>
<th>Vb</th>
<th>Vc</th>
<th>( I_f )</th>
<th>( I_d )</th>
<th>( I_q )</th>
<th>( I_a )</th>
<th>( I_b )</th>
<th>( I_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_f + pL_f )</td>
<td>( pM_{FD} )</td>
<td>( pM_{af} \cos\theta_a )</td>
<td>( pM_{af} \cos\theta_b )</td>
<td>( pM_{af} \cos\theta_c )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( pM_{FD} )</td>
<td>( R_d + pL_d )</td>
<td>( pM_{ad} \cos\theta_a )</td>
<td>( pM_{ad} \cos\theta_b )</td>
<td>( pM_{ad} \cos\theta_c )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( R_q + pL_q )</td>
<td>( pM_{aq} \sin\theta_a )</td>
<td>( pM_{aq} \sin\theta_b )</td>
<td>( pM_{aq} \sin\theta_c )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_a = \frac{pM_{af} \cos\theta_a}{pM_{aq} \sin\theta_a} )</td>
<td>( \frac{R_a + R_1 + pL_1}{p(M_{ao} + L_{a2} \cos 2\theta_a)} )</td>
<td>( \frac{p(M_{so} + L_{a2} \cos 2\theta_c)}{p(M_{so} + L_{a2} \cos 2\theta_b)} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_b = \frac{pM_{af} \cos\theta_b}{pM_{aq} \sin\theta_b} )</td>
<td>( \frac{R_a + R_2 + pL_2}{p(M_{so} + L_{a2} \cos 2\theta_a)} )</td>
<td>( \frac{p(M_{so} + L_{a2} \cos 2\theta_c)}{p(M_{so} + L_{a2} \cos 2\theta_b)} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_c = \frac{pM_{af} \cos\theta_c}{pM_{aq} \sin\theta_c} )</td>
<td>( \frac{R_a + R_3 + pL_3}{p(L_{ao} + L_{a2} \cos 2\theta_c)} )</td>
<td>( \frac{p(M_{so} + L_{a2} \cos 2\theta_b)}{p(M_{so} + L_{a2} \cos 2\theta_a)} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where, \( R_1, R_2, R_3 \) – resistances of transmission system phases a, b and c respectively

\( L_1, L_2, L_3 \) – inductances of transmission system phases a, b and c respectively
2.2.2 - Normalisation of voltage equations

In studies of electrical machines and power systems, it is advantageous to normalise the circuit quantities to a p.u. basis rather than to work in actual units.

The basis for the armature voltage and current are sometimes taken as the r.m.s. and sometimes as the peak of these quantities at rated conditions. Peak quantities are taken here and the armature is assumed star-connected, irrespective of the actual connection. Peak values are preferred because the present study is concerned with 2-axis theory and various transient problems in which the peak values are more significant. Additionally, the p.u. current required in the rotor to produce the same fundamental flux density wave as the rated 3-phase armature currents is unity, rather than \( \sqrt{2} \) as when an r.m.s. base is selected.

In order to refer the field and damper circuits to the armature, the corresponding turns ratio has to be defined. Theoretically, there is an infinite number of choices, but the one adopted here is the \( X_{ad} \) base, as recommended by Rankine \(^{20}\) Concordia \(^{21}\) and Harris, Lawrenson and Stephenson \(^{22}\). In this system, base field current \( I_{fo} \) is defined as that field current which induces in each stator phase a peak voltage of \( X_{ad} I_{ao} \) during balanced synchronous operation,

\[
\text{where } X_{ad} \text{ is the reactance (in ohms) corresponding to the armature reaction,}
\]

\[
\text{and } I_{ao} \text{ is the peak rated armature current (in amperes).}
\]

This voltage is in fact equal to that caused by the armature reaction produced by balanced 3-phase unit-peak armature currents (acting on the direct-axis), and clearly the principle of an equal mutual effect between the stator and the rotor is met by this definition. Since this airgap flux is central to all aspects of machine performance, the only meaningful way in which one winding (or a set of windings) can be evaluated in terms of another is through its effectiveness in developing the basic m.m.f. wave. In view of this, it is not surprising that the
preferred choice leads to a p.u. system which reflects most closely the physical features of the machine.

Similar definitions to those used for the field winding may be developed for the base damper currents. Hence, the base electrical quantities for the idealized machine are

\[ V_{ao} \] - peak rated phase voltage
\[ I_{ao} \] - peak rated phase current
\[ V_{A0} \] - total apparent power \( (= \frac{3}{2} V_{ao} I_{ao}) \)
\[ Z_{ao} \] - base impedance \( (= V_{ao}/I_{ao}) \)

\[ I_{f0}, I_{D0}, I_{Q0} \] \( \text{base quantities for the field,} \)
\[ V_{f0}, V_{D0}, V_{Q0} \] \( \text{D-damper and Q-damper circuits.} \)

With the base quantities specified above, normalisation of the phase-model equations can be performed. For the sake of simplicity, eqn. 2.5 is partitioned as,

\[
\begin{bmatrix}
V_r \\
V_{abc}
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
I_r \\
I_{abc}
\end{bmatrix}
\]

where \( r \) represents \( f, D \) and \( Q \) as required. After normalisation, the p.u. equations become,

\[
\begin{bmatrix}
V_r \\
V_{abc}
\end{bmatrix} =
\begin{bmatrix}
\frac{3}{2} z_{11} & z_{12} \\
\frac{3}{2} z_{21} & z_{22}
\end{bmatrix}
\begin{bmatrix}
I_r \\
I_{abc}
\end{bmatrix}
\]
where \( V_r = V_r / V_{ro} \)
\( V_{abc} = V_{abc} / V_{ao} \)
\( I_r = I_r / I_{ro} \)
\( I_{abc} = I_{abc} / I_{ao} \)

and, if \( N_r = I_{ro} / Z_{ao} \)

then,
\( z_{11} = N_r^2 Z_{11} / Z_{ao} \)
\( z_{12} = N_r Z_{12} / Z_{ao} \)
\( z_{21} = N_r Z_{21} / Z_{ao} \)
\( z_{22} = Z_{22} / Z_{ao} \)

The lack of reciprocity of the mutual inductance between the stator and the rotor circuits shown in eqn. 2.7 can easily be restored, by transferring the factor 3 from the impedance matrix to the current vector matrix \( i_r \).

With eqn. 2.5 being normalised to the specified base quantities, the p.u. mutual inductances corresponding to \( M_{af} \) and \( M_{aD} \) become numerically equal and the final form of the p.u. phase-model voltage equation is shown in eqn. 2.11. Comparing the symbols in eqns. 2.5 and 2.11, shows that the upper case letters of eqn. 2.5 are replaced by lower case letters in eqn. 2.11, when the p.u. form is used, except for \( L \) and \( M \). From eqn. 2.11 onwards, \( L \) and \( M \) will be taken as the p.u. self and mutual inductances respectively.

**2.3 - Stationary 2-axis representation**

For ease of computation, the time-varying coefficients of eqn. 2.11 can be eliminated by transforming the armature variables to a set of new variables, relating to a reference frame fixed to the field system. Fig. 2.2 shows an idealised synchronous machine in a stationary 2-axis representation.
\[
\begin{array}{|c|c|c|c|c|}
\hline
v_f & r_f + pL_f & pM_{fD} & pM_{af} \cos \theta_a & pM_{af} \cos \theta_b & pM_{af} \cos \theta_c & i_f' \\
\hline
v_D & pM_{fD} & r_D + pL_D & pM_{af} \cos \theta_a & pM_{af} \cos \theta_b & pM_{af} \cos \theta_c & i_D' \\
\hline
v_Q & & & r_Q + pL_Q & pM_{aQ} \sin \theta_a & pM_{aQ} \sin \theta_b & pM_{aQ} \sin \theta_c & i_Q' \\
\hline
v_a & pM_{af} \cos \theta_a & pM_{af} \cos \theta_a & pM_{aQ} \sin \theta_a & r_a + r_1 + p(L_{ao} + L_1) + pL_{a2} \cos 2\theta_a & pM_{so} + pL_{a2} \cos 2\theta_c & pM_{so} + pL_{a2} \cos 2\theta_b & i_a \\
\hline
v_b & pM_{af} \cos \theta_b & pM_{af} \cos \theta_b & pM_{aQ} \sin \theta_b & r_a + r_2 + p(L_{ao} + L_2) + pL_{a2} \cos 2\theta_b & r_a + r_3 + p(L_{ao} + L_3) + pL_{a2} \cos 2\theta_c & pM_{so} & i_b \\
\hline
v_c & pM_{af} \cos \theta_c & pM_{af} \cos \theta_c & pM_{aQ} \sin \theta_c & r_a + r_3 + p(L_{ao} + L_3) + pL_{a2} \cos 2\theta_c & r_a + r_3 + p(L_{ao} + L_3) + pL_{a2} \cos 2\theta_c & & i_c \\
\hline
\end{array}
\]

where, \( i_f' = \frac{3}{2} i_f \), \( i_D' = \frac{3}{2} i_D \), \( i_Q' = \frac{3}{2} i_Q \)

\( r_1, r_2, r_3 \) - resistances of transmission system phases a, b and c respectively

\( L_1, L_2, L_3 \) - inductances of transmission system phases a, b and c respectively
2.3.1 - Transformation matrices

Based on their effectiveness in developing m.m.f., the 3-phase and 2-axis systems of figs. 2.1 and 2.2 are equivalent when,

\[
F_d = F_a \cos \theta_a + F_b \cos \theta_b + F_c \cos \theta_c
\]

and

\[
F_q = F_a \sin \theta_a + F_b \sin \theta_b + F_c \sin \theta_c
\]

If the fictitious armature coils have the same number of turns as the actual armature coils, then, since the 3-phase system produces a resultant m.m.f. of magnitude 3 times that of a peak phase m.m.f., the \(Z\) axis current required to produce an equivalent m.m.f. is 3 times the peak phase current. It can be shown that by selecting the base axis current as 3 times the base phase current \(i_{22}\), the amplitude of the peak phase current in balanced synchronous operations is equal to \(\sqrt{i_d^2 + i_q^2}\) which, at rated condition, is unity. A more important outcome of this choice is the achievement of a power invariant transformation, which leads to reciprocal mutual inductance terms. The p.u. axis currents can thus be expressed as,

\[
i_d = \frac{2}{3} (i_a \cos \theta_a + i_b \cos \theta_b + i_c \cos \theta_c)
\]

and

\[
i_q = \frac{2}{3} (i_a \sin \theta_a + i_b \sin \theta_b + i_c \sin \theta_c)
\]

The above transformation is however singular and to overcome this, a zero sequence current \(i_0\) defined by

\[
i_0 = \frac{1}{3} (i_a + i_b + i_c)
\]

is introduced as the third constraint on the phase currents.
The current transformation matrix then becomes,

$$
\begin{bmatrix}
    i_d \\
    i_q \\
    i_0 \\
\end{bmatrix}
= \begin{bmatrix}
    \cos \theta_a & \cos \theta_b & \cos \theta_c \\
    \sin \theta_a & \sin \theta_b & \sin \theta_c \\
    \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\end{bmatrix}
\begin{bmatrix}
    i_a \\
    i_b \\
    i_c \\
\end{bmatrix}
$$

or

$$i_{dq0} = C_i_{abc}$$

with the inverse transformation giving the phase currents in terms of the axis currents being,

$$
\begin{bmatrix}
    i_a \\
    i_b \\
    i_c \\
\end{bmatrix}
= \begin{bmatrix}
    \cos \theta_a & \sin \theta_a & 1 \\
    \cos \theta_b & \sin \theta_b & 1 \\
    \cos \theta_c & \sin \theta_c & 1 \\
\end{bmatrix}
\begin{bmatrix}
    i_d \\
    i_q \\
    i_0 \\
\end{bmatrix}
$$

or

$$i_{abc} = C^{-1} i_{dq0}$$

Similar transformation matrices can be applied to the voltage transformations,

i.e.

$$v_{dq0} = C v_{abc}$$

$$v_{abc} = C^{-1} v_{dq0}$$

2.3.2 - 3-phase to stationary 2-axis transformation

The normalised phase-model voltage equation may be rewritten as,

$$
\begin{bmatrix}
    v_fDQ \\
    v_{abc} \\
\end{bmatrix}
= \begin{bmatrix}
    \frac{3}{2} z_{11} & z_{12} \\
    \frac{3}{2} z_{21} & z_{22} \\
\end{bmatrix}
\begin{bmatrix}
    i_fDQ \\
    i_{abc} \\
\end{bmatrix}
$$

Applying the transformation matrices of the previous section to eqn. 2.21 yields eqn. 2.22 for the stationary 2-axis model with the details of the manipulation required being given in Appendix A.
where,
\[ r_{ff} = \frac{3}{2} r_f \]
\[ r_{DD} = \frac{3}{2} r_D \]
\[ r_{QQ} = \frac{3}{2} r_Q \]
\[ r_d + r_q = r_a \]
\[ L_{ff} = \frac{3}{2} L_f \]
\[ L_{DD} = \frac{3}{2} L_D \]
\[ L_{QQ} = \frac{3}{2} L_Q \]
\[ L_{md} = \frac{3}{2} M_a f = \frac{3}{2} M_{FD} \text{ (assume } M_{FD} = M_a f \text{)} \]
\[ L_{mq} = \frac{3}{2} M_a Q \]
\[ L_d = L_{ao} - M_{so} + \frac{3}{2} L_{a2} \]
\[ L_q = L_{ao} - M_{so} - \frac{3}{2} L_{a2} \]
\[ L_{0} = L_{ao} + 2M_{so} \]
2.4 - Torque equations

Following the positive sign convention of Section 2.2.1, positive torque is defined as a torque applied to the shaft in the positive direction of rotation, to bring out the analogy between electrical and mechanical quantities. Hence, positive voltage and current implies motor operation and positive torque and speed implies generator action.

The instantaneous torque applied to the shaft can be expressed in absolute value as

\[
T_t = T_J + T_D + T_S + T_L + T_e
\]

where

- \(T_t\) - applied torque
- \(T_J\) - inertia torque
- \(T_D\) - damping torque
- \(T_S\) - stiffness torque
- \(T_L\) - constant loss torque
- \(T_e\) - electrical torque, defined as positive when the airgap torque is transmitted into the machine from outside at a positive speed.

2.4.1 - P.U. system

In addition to the base quantities described in Section 2.2.2, time also needs to be normalised. It has been argued that this is unnecessary in the 3-phase/2-axis p.u. system, although it is common in American literature\(^{18-21,24,25}\) and is supported by Harris, Lawrenson and Stephenson\(^{22}\). The present author prefers to normalise time, because the machine equations are then unaffected by the number of pole pairs and the supply frequency. This choice also leads to a simple p.u. electrical torque equation and to a numerical equality between the machine p.u. reactance and p.u. inductance (i.e. \(x = L\)).
Since $\omega t$ is dimensionless and the selection of the synchronous speed $\omega_0$ as the base speed is a sensible choice, base time becomes automatically $\frac{1}{\omega_0}$,

i.e. $t_0 = \frac{1}{\omega_0}$  \hspace{1cm} 2.26

Having normalised time, the base differential operator $p_0$ is defined by

$$p_0 = \frac{1}{t_0} = \frac{1}{\omega_0}$$  \hspace{1cm} 2.27

Since the number of pole pairs $n_{pp}$ is not fundamental to the machine performance, it is desirable to exclude it from the major p.u. equations. To achieve this, the base for the mechanical angular velocity of the rotor $\omega_0$ is defined as $\omega_0$. Hence the p.u. rotor angular velocity becomes $\frac{n_{pp}}{n}$

$$\omega = \frac{\omega}{\omega_0} = \frac{\omega/n_{pp}}{\omega_0/n_{pp}} = \omega$$  \hspace{1cm} 2.28

and under this cancellation, the p.u. mechanical angular velocity of the rotor and the p.u. electrical angular frequency become numerically equal.

Other base quantities, such as those for the flux linkage, power, torque and polar moment of inertia, are defined by

$$\Psi_0 = \frac{V_{ao}}{\omega_0}$$  \hspace{1cm} 2.29

$$VA_0 = \frac{3}{2} V_{ao} I_{ao} (= \text{total apparent power})$$  \hspace{1cm} 2.30

$$T_0 = \frac{VA_0}{\omega_0}$$  \hspace{1cm} 2.31

$$J_0 = \frac{T_0}{\omega_0 / \omega_0}$$  \hspace{1cm} 2.32
In practice the inertia constant $H$ is normally given and the p.u. moment of inertia $J$ is derived therefrom by the equation

$$J = 2\omega_0 H$$  \hspace{2cm} 2.33

2.4.2 - P.U. torque equations

After the base torque has been selected, eqn. 2.25 can be expressed in p.u. form as

$$T_t = T_J + T_D + T_S + T_L + T_e$$  \hspace{2cm} 2.34

To express this equation in terms of the load angle $\delta$, a reference axis in space has to be defined. When a synchronous machine operates in conjunction with an infinite supply system, the supply voltage may be taken as a fixed reference in both magnitude and phase. If the machine is isolated from the infinite busbar, a fictitious reference axis has to be assumed for analytical purpose. It is convenient to take the rotor axis as the reference when it is running at synchronous speed under no-load condition. This definition is valid both when the machine is operating on and off a grid system.

If zero time is taken as the instant the phase a axis is in line with the field winding axis (see fig. 2.1), the no-load angular position of the rotor, or the reference axis position can be defined as $t\omega(0)$. With respect to this reference axis, the rotor position, speed and acceleration are then respectively

$$\theta = t + \delta$$
$$\dot{\theta} = \omega = 1 + p\delta$$
$$\ddot{\theta} = p^2 \delta$$  \hspace{2cm} 2.35

and the torque equation is

$$T_t = Jp^2 \delta + K_D p \delta + K_S \delta + T_L + T_e$$  \hspace{2cm} 2.36
where $K_D$ - damping coefficient
$K_S$ - rotating shaft stiffness coefficient

The electrical torque equations for the d-q model and the phase model are then respectively

$$T_e = \psi_d i_q - \psi_q i_d \quad 2.37$$

and

$$T_e = \frac{2}{3\sqrt{3}} \left[ \psi_a (i_c - i_b) + \psi_b (i_a - i_c) + \psi_c (i_b - i_a) \right] \quad 2.38$$

This completes all the equations required to form a phase model and a d-q model of a 3-phase synchronous machine.
Fig. 2.1 Idealised synchronous machine phase model representation

Fig. 2.2 Stationary 2-axis model
CHAPTER 3

DIGITAL METHODS FOR THE SIMULATION

OF

A 3-PHASE SYNCHRONOUS MACHINE
CHAPTER 3
DIGITAL METHODS FOR THE SIMULATION
OF A 3-PHASE SYNCHRONOUS MACHINE

The synchronous machine equations developed in Chapter 2 are not suitable for use with modern control theory, since treating a machine as a generalised electrical circuit produces an equation of the form,

\[ v = (r + pL) i \]  \hspace{1cm} 3.1

The equation for the machine needs therefore to be re-arranged into a state-variable representation, in which the machine is treated as a dynamic system having the general equation,

\[ \dot{x} = f(x, u) \]  \hspace{1cm} 3.2

where \( f \) is a continuous function of both the state variable \( x \) and the forcing function \( u \) and the resulting 1st-order differential equations may be solved on a digital computer using numerical integration methods.

The substantial upsurge of work in adopting and developing methods of numerical integration to computer solutions in recent years has provided numerous alternative techniques for solving differential equations. Irrespective of the particular system in which these equations are formulated, the total numerical work required in the solution depends to an important extent on the choice of the integration method and on the form of its implementation. The numerical integration method used in the present study is discussed briefly and the technique by which discontinuities in the solution were handled is also presented.

The validity of the resulting machine models is assessed by comparisons between the predicted and test results.
3.1 - State-space approach

Significant developments in control system theory in the past two decades have extended the study of dynamic system behaviour from the analysis of differential equation properties as exhibited by a single \( n^{th} \)-order equation to a more general state-space method using \( n \) sets of 1st-order differential equations.

The solution of large sets of differential equations describing the dynamics of a physical system presents at least two difficulties. Firstly, there is the problem of assembling the equations into a form that can conveniently be solved by a digital or analogue computer. Secondly, insight into the physical system behaviour may be lost or at least considerably blurred by their combination into large sets. The systematic format of the state-space representation provides a means for overcoming both of these difficulties. In addition, the state-space equations are expressed in a form that enables the roots of the system characteristic equation to be obtained easily using readily available computer subroutines.

In synchronous machine stability studies, the equation of motion giving the rotor acceleration \( \delta \) is replaced by two state equations with the load angle \( \delta \) and the rotor angular velocity \( \dot{\delta} \) as the state variables. Even when there are many other state variables, a 2-dimensional state-space (or phase-plane) diagram relating \( \delta \) and \( \dot{\delta} \) provides very useful information for the assessment of the machine stability.

3.1.1 - Choice of state variables

The choice of states used may be made in many ways, depending on the type of system, and consideration should be given to the concepts of controllability and observability. These concepts concern basically the ability to affect (control) and detect (observe) the dynamics of a system from both the inputs and outputs.

The state variables chosen for synchronous machines are generally either currents or flux linkages, depending on the individual application of the studies. For problems concerning operation of a
single synchronous machine, flux linkages are a favoured choice\textsuperscript{16,26}, because the state equation can be obtained in a direct manner and the coefficients retain a physical meaning. It is also unnecessary to compute new initial values for the state variables after each change of the system conditions or parameters, as total flux linkages cannot be changed instantaneously. On the other hand, it is perhaps more natural to take currents\textsuperscript{27,28} as the state variables when machines are interconnected through a transmission network, where the performance may be visualized more readily in terms of voltages and currents than in terms of flux linkages. However, the changes in currents and flux linkages are related by a non-singular linear transformation, so the choice of state variables may ultimately be ascribed merely to personal preference.

In the present study, the dynamic response of multi-machine system is examined and the latter choice is adopted.

3.1.2 - State-space representation of the direct-phase model

The direct-phase model equations (eqns 2.11 and 2.36) for the 3-phase synchronous machine can be re-arranged as,

\[
\begin{bmatrix} [P] \end{bmatrix} \begin{bmatrix} \dot{x} \end{bmatrix} = \begin{bmatrix} [Q] \end{bmatrix} \begin{bmatrix} x \end{bmatrix} + \begin{bmatrix} [S] \end{bmatrix} + \begin{bmatrix} [T] \end{bmatrix} \begin{bmatrix} u \end{bmatrix} \tag{3.3}
\]

where,

\[
\begin{bmatrix} \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{\delta}, \dot{\delta}', i_f', i'_D, i'_Q, i'_a, i'_b, i'_c \end{bmatrix}^t \tag{3.4}
\]

\[
\begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} \delta, \delta', i_f', i'_D, i'_Q, i_a, i_b, i_c \end{bmatrix}^t \tag{3.5}
\]

\[
\begin{bmatrix} u \end{bmatrix} = \begin{bmatrix} T_t, v_f \end{bmatrix}^t \tag{3.6}
\]
<table>
<thead>
<tr>
<th>( P ) =</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J )</td>
<td></td>
</tr>
<tr>
<td>( L_f )</td>
<td>( M_{FD} )</td>
</tr>
<tr>
<td>( M_{FD} )</td>
<td>( L_D )</td>
</tr>
<tr>
<td>( L_Q )</td>
<td>( M_{aQ} \sin^\theta_a )</td>
</tr>
<tr>
<td>( M_{af} \cos^\theta_a )</td>
<td>( M_{af} \cos^\theta_a )</td>
</tr>
<tr>
<td>( M_{af} \cos^\theta_b )</td>
<td>( M_{af} \cos^\theta_b )</td>
</tr>
<tr>
<td>( M_{af} \cos^\theta_c )</td>
<td>( M_{af} \cos^\theta_c )</td>
</tr>
</tbody>
</table>
\[ [q] = \begin{array}{cccc}
-\mathbf{K}_S & -\mathbf{K}_D & & \\
\mathbf{r}_f & & \omega_{Ma} \sin \theta_a & \omega_{Ma} \sin \theta_b & \omega_{Ma} \sin \theta_c \\
\mathbf{r}_D & & \omega_{Ma} \sin \theta_a & \omega_{Ma} \sin \theta_b & \omega_{Ma} \sin \theta_c \\
\mathbf{r}_Q & & -\omega_{MaQ} \cos \theta_a & -\omega_{MaQ} \cos \theta_b & -\omega_{MaQ} \cos \theta_c \\
\omega_{Ma} \sin \theta_a & \omega_{Ma} \sin \theta_a & -\omega_{MaQ} \cos \theta_a & +2\omega_l a_2 \sin 2\theta_a & 2\omega_l a_2 \sin 2\theta_c & 2\omega_l a_2 \sin 2\theta_b \\
\omega_{Ma} \sin \theta_b & \omega_{Ma} \sin \theta_b & -\omega_{MaQ} \cos \theta_b & 2\omega_l a_2 \sin 2\theta_c & +2\omega_l a_2 \sin 2\theta_b & 2\omega_l a_2 \sin 2\theta_a \\
\omega_{Ma} \sin \theta_c & \omega_{Ma} \sin \theta_c & -\omega_{MaQ} \cos \theta_c & 2\omega_l a_2 \sin 2\theta_b & 2\omega_l a_2 \sin 2\theta_a & +2\omega_l a_2 \sin 2\theta_c \\
\end{array} \]
and this form of matrix equation arrangement facilitates a computer manipulation of the system equations into a state-space form.

Between each integration step, the following auxiliary equations are computed to update the coefficients of the matrices,

\[
\omega = 1 + p \delta \quad 3.11 \\
\theta = t + \delta \quad 3.12 \\
\theta_a = \theta \quad 3.13 \\
\theta_b = \theta - \frac{2}{3} \pi \\
\theta_c = \theta + \frac{2}{3} \pi \\
v_a = V_m \sin \theta_a \quad 3.14 \\
v_b = V_m \sin \theta_b \\
v_c = V_m \sin \theta_c \\
T_e = \frac{2}{3\sqrt{3}} \left[ \psi_a (i_c - i_b) + \psi_b (i_a - i_c) + \psi_c (i_b - i_a) \right] \quad 3.15 \\
\]

where \( V_m \) is the p.u. peak phase busbar voltage.
### 3.1.3 State-space representation of the stationary 2-axis model

The voltage and torque equations (eqns 2.22 and 2.36) can be combined and re-arranged into the following general form.

\[
[P] \dot{[x]} = [Q] [x] + [S] + [T] [u]
\] 3.16

where:

\[
[x] = [\delta, \omega, i_f, i_d, i_Q, i_d', i_q]^t
\] 3.17

\[
[x] = [\delta, \omega, i_f, i_d', i_Q, i_d', i_q]^t
\]

\[
[u] = [T_t, v_f]^t
\] 3.18

\[
[P] =
\begin{bmatrix}
1 & & & & \\
& J & & & \\
& & L_{ff} & L_{md} & L_{md} \\
& & L_{md} & L_{DD} & L_{md} \\
& & & & L_{QQ} & L_{mq} \\
& & & & L_{md} & L_{md} \\
& & & & & L_{d} \\
& & & & & L_{mq} \\
& & & & & L_q
\end{bmatrix}
\] 3.19
\[ [S] = \begin{bmatrix}
- (T_e + T_L) \\
v_D \\
v_Q \\
v_d \delta(L_{md} i_Q + L_q i_Q) \\
v_q \delta(L_{md} i_f + L_{md} i_D + L_d i_d)
\end{bmatrix} \]

\[ [T] = \begin{bmatrix}
1 \\
1 \\
1 \\
1 \\
1
\end{bmatrix} \]

\[ [Q] = \begin{bmatrix}
1 \\
-K_S & -K_D \\
-r_{ff} & -r_{DD} \\
-r_{QQ} & -L_{mq} -r_d -L_q \\
L_{md} & L_{md} & L_d & -r_q
\end{bmatrix} \]

3.20

3.21

3.22
The equation for $T_e$ is given in eqn. 2.37 and expressions for $v_d$ and $v_q$ are derived as below.

Referring to Section 2.4.2, the position of the reference axis with respect to the axis of phase a is defined as $t$ when the rotor angular position is,

$$\theta = t + \delta \quad 3.23$$

Under balanced conditions, the a phase busbar voltage may be expressed as,

$$v_a = V_m \sin t$$

$$= V_m \sin (\theta - \delta)$$

$$= (-V_m \sin \delta) \cos \theta + (V_m \cos \delta) \sin \theta \quad 3.24$$

where $V_m$ is the p.u. busbar peak phase voltage.

Also, from eqn. 2.20,

$$v_a = v_d \cos \theta + v_q \sin \theta + v_0 \quad 3.25$$

where $v_0 = 0$ for balanced system.

Comparing eqns. 3.24 and 3.25, the following equations are formed.

$$v_d = -V_m \sin \delta$$

$$v_q = V_m \cos \delta \quad 3.26$$
3.2 - Numerical integration techniques

All numerical integration methods are inevitably approximate and numerical errors are propagated through the solution as the integration proceeds. The reduction of the truncation error, which is the major component of the numerical error, requires the use of a small step length and the selection of the maximum usable step length is a compromise between the accuracy required for the solution and the computing time which can be allowed.

If the rates-of-change of the variables during a solution vary widely, as in a stiff system, an integration method using a fixed step length throughout the solution is undesirable.

The presence of discontinuities in the solution sequence is generally of particular importance. Incorrect handling of such conditions may result in incorrect solutions or even instability.

3.2.1 - Choices of integration methods

The first group of methods, single-step algorithms such as the Runge-Kutta routine, has been widely adopted. Since single-step methods start each integration afresh and use only 1st-derivative values formed wholly within each step, they are inherently self-starting. This characteristic allows them to integrate through discontinuities without taking special precautions, and the step length can readily be changed. The methods therefore lend themselves directly to incorporation in power system simulations.

In the second group of methods, those of the multi-step kind, the solution in any current integration step draws on the solution and derivative values formed in previous steps. Prediction-correction sequences are commonly applied in this group, in which a prediction is made at the beginning of each step and followed by a correction usually applied iteratively. For solving the same set of differential equations, these methods can invariably be used with larger step lengths than their single-step counterparts. However, they are generally not self-starting and additional means for starting the algorithm is therefore required. In a variable step-length control scheme, the convenience of
relating the local truncation errors to the step size in these methods puts them at a considerable advantage over single-step routines.

Several studies\textsuperscript{29-31} confirm that the predictor-corrector routine is more efficient than the Runge-Kutta type, because the former method, for the same accuracy, requires fewer functional evaluations per step and offers the possibilities of using a larger step length. This advantage is more significant when dealing with larger problems, and is therefore the primary reason for employing the predictor-corrector routine in the present study.

3.2.2 - The selected numerical integration algorithm

The predictor-corrector routine based on the Adam-Bashforth-Moulton formulae and developed by Gear\textsuperscript{13-15} has been proved to be an efficient method\textsuperscript{31} and was therefore chosen for the present investigation. The original version\textsuperscript{13,14} is not as efficient as that developed by Kroghs\textsuperscript{32}, but the modified version using high-order formulae is more competitive. His algorithm also includes a very important feature which provides an option for handling stiff systems and is found to be very efficient\textsuperscript{31}.

The Gear routine integrates up to 30 differential equations over one step length $h$ at each entry. The value of $h$ can be specified by the user, but it is controlled by the subroutine to maintain the local truncation error within a specified tolerance. The order of the methods are also adjustable (up to 12 for the Adams method and 5 for the Gear stiffly-stable method) by algorithm, as to maximise the step size.

Truncating the Adams-Bashforth formulae to give a 1st-order predictor equation, in conjunction with the 1st-order Adams-Moulton formulae as the corrector equation, leads to a mathematically simple and self-starting predictor-corrector routine. Therefore, in the first entry to the subroutine for the solution, the order of the method is set to one to allow for self-starting. The order control mechanism can then increase the order to a desirable level.
The mechanism and mathematical details involved in estimating the single-step error and the solution of the step size and order of the method are well-documented in Gear's publications\textsuperscript{13-15}.

3.2.3 - Integration through discontinuities

In power system simulations, there are basically two types of discontinuities. The first comprises major discontinuities, such as those caused by fault inception, fault clearance and subsequent circuit breaker reclosure, which occur at pre-specified points in the solution. The second type arises from the limits on system variables (e.g. amplifier saturation) and the time of these occurrences in the course of the saturation is normally unpredictable.

Since the integration algorithm is self-starting, any major discontinuities can easily be handled by simply resetting the order of the integration method to one at the instant a discontinuity occurs. However, in handling randomly occurring discontinuities, a method to combat the risk of mathematical instability was derived by the present author and is presented in Appendix B.

3.3 - Fault simulations on synchronous machines

With the synchronous machine models expressed in state-variable form, and using the numerical integration routine selected, digital computer programs for both the stationary 2-axis and the direct-phase models are developed.

3.3.1 - Fault simulation on the stationary 2-axis model

The initial part of the program consists of an algorithm (Appendix C) which converts the manufacturer's machine data (e.g. $x_d$, $x_d'$, $x_d''$, $x_q$, $x_q''$, $T_d'$, $T_d''$, $T_q'$, $T_q''$, $T_a'$, $x_a'$, $H$, etc.) to the data (e.g. $r_{ff}$, $r_{DD}$, $r_{QQ}$, $r_a$, $L_{ff}$, $L_{DD}$, $L_{QQ}$, $L_d$, $L_q$, $L_{md}$, $L_{mq}$, $J$, etc.) required by the model equations.

Using the calculated machine data and the information on the machine initial operating condition, the matrices $[P]$, $[Q]$, $[S]$, $[T]$, ...
[x] and [u] of eqn. 3.3 are defined. Matrix [P] is inverted and the model equation is manipulated to the state-variable form.

\[ \dot{x} = [A]x + [F] + [B]u \]  

where

\[ [A] = [P]^{-1}[Q] \]

\[ [F] = [P]^{-1}[S] \]

\[ [B] = [P]^{-1}[T] \]

The resulting equation is solved by the predictor-corrector routine and, at the end of each integration step, the algebraic equations (eqns. 3.23 and 3.26) are computed.

Provided the machine parameters remain constant, matrices [A] and [B] are unchanged, while matrix [F], comprising non-linear elements, requires updating at every integration step.

A 3-phase short-circuit fault was applied at the terminals of a 15 MW generator from open-circuit at rated speed and 50 per cent rated voltage. The resulting transients were predicted and comparison with test results are made in fig. 3.1.

The generator data used in the computer program are listed in Table 3.1 and the base quantities of the machine parameters are defined in Appendix D.

If the machine speed is assumed constant, the machine equations can be solved analytically. Expressions for the 3-phase short-circuit transients of \( i_f \), \( i_d \), \( i_q \), \( i_a \) and \( T_e \) are given in Appendix E. Fig. 3.2 shows a comparison between the computed transients using the expressions derived and those using the model equations. Since the effect of machine speed variation is neglected in the analytical solution, the error in the prediction increases as machine speed falls. This phenomenon is evident in the results given in fig. 3.2.
3.3.2 - Fault simulations on the direct-phase model

The equations detailed in Appendix F were included in the computer program to convert the d-q parameters into phase form. The matrices [P] and [Q] of eqn. 3.3 are no longer constants, as in the case of the d-q model. Consequently, inversion of [P] and computation of matrices \([P]^{-1}[Q]\), \([P]^{-1}[S]\) and \([P]^{-1}[T]\) are required at every integration step.

For various types of fault occurring on a transmission system, the constraints imposed on, and the corresponding modifications required for, the system matrices are described in the following sections, and comparisons are made with test results.

3.3.2.1 - 3-phase symmetrical fault

For a 3-phase symmetrical fault, the constraints and modifications, which apply at the instant of the fault are,

\[
\begin{align*}
    v_a &= v_b = v_c = 0 \\
    r_1 &= r_2 = r_3 = r_{\text{fault}} \\
    L_1 &= L_2 = L_3 = L_{\text{fault}}
\end{align*}
\]

where \(r_{\text{fault}}\) and \(L_{\text{fault}}\) are respectively the fault resistance and inductance per phase measured from the generator terminals to the point of the fault.

A comparison of the predicted short-circuit transients with test results (identical to those used for verifying the d-q model) is made in fig. 3.3.

3.3.2.2 - Line-to-earth fault

For a fault on, say, phase c, the following modifications are required,
\[ V_c = 0 \]
\[ r_3 = r_{\text{fault}} \]  \hspace{1cm} 3.30
\[ L_3 = L_{\text{fault}} \]

A phase c to earth fault was applied to a 50 MW generator\textsuperscript{7,33} from open-circuit at rated speed and 50 per cent rated voltage. A comparison of predictions and test results is presented in fig. 3.4 with the generator data being given in Table 3.1. The agreement between the computed and test results is satisfactory and the errors in the predictions are likely to be caused by inaccurate damper data used in the analysis.

3.3.2.3 - Double line-to-earth fault

The simulation of a double line-to-earth fault from phases b and c requires the following modifications,

\[ V_b = V_c = 0 \]

\[ r_2 = r_3 = r_{\text{fault}} \]  \hspace{1cm} 3.31
\[ L_2 = L_3 = L_{\text{fault}} \]

The computed transients following such a fault on the 50 MW generator are shown in fig. 3.5, with the prefault conditions being identical with those of the previous section.

3.3.2.4 - Line-to-line fault

Faults not involving the neutral (or earth) cannot be handled directly by a 3-phase 4-wire model, unless the modifications suggested by Subramaniam and Malik\textsuperscript{6} are made. Alternatively, the 3-phase 3-wire model, derived in Appendix G, can be used for this purpose.
For a fault, say between phases b and c,

\[ v_{bc} = v_b - v_c = 0 \]  \hspace{1cm} 3.32

Hence,

\[ v_b = v_c \]  \hspace{1cm} 3.33

Also

\[ v_a + v_b + v_c = 0 \]  \hspace{1cm} 3.34

Therefore, at the instant of fault \( v_b = v_c = -\frac{v_a}{2} \)  \hspace{1cm} 3.35

In addition, \( r_2, r_3 \) and \( L_2, L_3 \) are made equal to \( r_{\text{fault}} \) and \( L_{\text{fault}} \) respectively.

With prefault conditions identical with those of Section 3.3.2.2, a line-to-line short-circuit was applied to the 50 MW generator, with the test results and corresponding computer predictions being compared in fig. 3.6.

3.3.3 - Comparison of the two types of models

The transient behaviour of a synchronous machine has been studied using both d-q and phase models, and their predictions are generally in good agreement with test results.

It has been shown that the direct-phase model enables a unified approach to be adopted in the study of both symmetrical and unsymmetrical faults. This is because the constraints imposed on the model for the simulation of various types of faults are in a direct relationship with the actual fault condition and do not involve any transformation, as in the d-q model.\(^{33}\) Also, the non-ideal properties of real machines, such as any higher harmonics in the spatial variations of the airgap m.m.f. and permeance, can be included in the machine analysis\(^9,10\).

However a major drawback of the phase model is its excessive use of computing time, when compared with that required by the d-q model. This is because the periodic nature of the state variables necessitates a small step length. In addition, extra time is involved in
the inversion of the matrix \( [P] \) and the calculation of matrices \( [P]^{-1} [Q] \), \( [P]^{-1} [S] \) and \( [P]^{-1} [T] \) in eqn. 3.3 at every integration step.

In the present study, attention is focused on transient studies of systems consisting of a synchronous generator, an excitation control system, a prime mover and its governing system. The computing time required to simulate such a system using a phase model is excessive. In addition, the investigations covered in the remaining part of the thesis are based on balanced operation of the generator, for which a d-q model is preferable. If the analysis of unsymmetrical faults is required, the generator model can easily be converted to the well-proven phase model.
TABLE 3.1
GENERATOR DATA

<table>
<thead>
<tr>
<th>Generator Ratings</th>
<th>18.875 MVA</th>
<th>62.5 MVA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15.1 MW</td>
<td>50 MW</td>
</tr>
<tr>
<td></td>
<td>13.8 kV</td>
<td>11 kV</td>
</tr>
<tr>
<td></td>
<td>60 Hz</td>
<td>50 Hz</td>
</tr>
<tr>
<td></td>
<td>1800 r.p.m.</td>
<td>3000 r.p.m.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>18.875 MVA</th>
<th>62.5 MVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchronous reactance</td>
<td>1.78</td>
<td>1.7</td>
</tr>
<tr>
<td>Transient reactance</td>
<td>0.272</td>
<td>0.19</td>
</tr>
<tr>
<td>Sub-transient reactance</td>
<td>0.194</td>
<td>0.135</td>
</tr>
<tr>
<td>Transient time constant (short-circuit)</td>
<td>1.12 s</td>
<td>1.0 s</td>
</tr>
<tr>
<td>Sub-transient time constant (short-circuit)</td>
<td>0.045 s</td>
<td>0.025 s</td>
</tr>
<tr>
<td>Armature time constant</td>
<td>0.192 s</td>
<td>0.182 s</td>
</tr>
<tr>
<td>Armature leakage reactance</td>
<td>0.12</td>
<td>0.122</td>
</tr>
<tr>
<td>Armature resistance</td>
<td>0.0023</td>
<td>0.0024</td>
</tr>
<tr>
<td>Field circuit resistance</td>
<td>0.00064</td>
<td>0.00059</td>
</tr>
<tr>
<td>Inertia constant</td>
<td>2.5 s</td>
<td>5.0 s</td>
</tr>
</tbody>
</table>

(All parameters are expressed in p.u. unless otherwise specified)
Fig. 3.1a Computed and test results for a 3-phase short-circuit test (15 MW generator)
Fig. 3.1b Computed results for a 3-phase short-circuit test
(15 MW generator)
Fig. 3.2 Comparison of model predictions and analytical solutions on 3-phase short-circuit transients (15 MW generator)
Fig. 3.3 3-phase short-circuit transients
(15 MW generator)
Fig. 3.4 Line-to-earth fault transients
(50 MW generator)
Fig. 3.5  Predicted transients on double-line-to-earth fault
(50 MW generator)
Fig 3.6 Line-to-line fault transients (50 MW generator)
CHAPTER 4

DIGITAL SIMULATION OF INDUCTION MOTORS

AND

THEIR INTERCONNECTION WITH SYNCHRONOUS MACHINES
CHAPTER 4
DIGITAL SIMULATION OF INDUCTION MOTORS AND THEIR INTERCONNECTION WITH SYNCHRONOUS MACHINES

The use of digital computers has facilitated many refinements in system component representation and a good accuracy of prediction has generally been made possible. Through more refined system studies, it has become evident that as a consequence of improved alternator representation, it is often unsatisfactory to assume all electrical loads to be of constant impedance.

Induction motors often form a major part of a system load and their transient characteristics have been explored in depth during the past two decades. Most investigations\textsuperscript{34-39} have been concerned with the transient behaviour of a single machine supplied from an infinite source. As the use of large induction motors increases, the coupling effect of the common transformer and line becomes appreciable, especially during transient conditions. A notable contribution to this problem was made by Lewis and Marsh\textsuperscript{40} who gave a qualitative discussion and an approximate analysis of power station auxiliary motors subjected to an interruption of the motor busbar supply, as during busbar transfer. Humpage, Durrani and Carvalho\textsuperscript{41} studied the transient stability of groups of interconnected synchronous and induction machines, following short-circuit faults on the system. Kalsi and Adkins\textsuperscript{42} analysed a short-circuit on a system consisting of an induction motor and a synchronous generator connected to an infinite busbar through an impedance, and compared the transients measured on a micro-machine system with those computed by methods employing different approximations. They observed that the approximate method of eliminating the terms in the stator-voltage equations corresponding to the rate-of-change of stator flux linkage, although reasonably accurate for a system containing only synchronous machines, may introduce appreciable errors in the case of a system in which induction motors form a substantial part of the loading. This was explained on the basis
that during a transient the speed deviation of such machine from the initial value is very much larger than that of a synchronous machine of comparable rating.

Induction motors often form a major part of a gas-turbine generator load and their effect on the system during transient condition can be significant. Accurate induction motor models, including the rate-of-change of stator flux linkage and any speed variation, are therefore required for the present investigation.

4.1 - Effects of induction motors on power system studies

The increasing use of large motor devices makes it necessary that their effect on the system is carefully studied. The accurate model developed in this Chapter provides a basis for the study of various problems associated with such motors.

4.1.1 - Increase of system fault level

The contribution of induction motors to the system fault level has been of interest to system analysts for many years. In 1920, Doherty and Williamson concluded, by testing a 150 hp motor, that the induction motor does contribute to the system fault level, but this contribution was considered to be unimportant because the motor load was insignificant compared with the system capacities and also because of the slow operating circuit breakers employed. However, the increasing size of induction motors, together with the development of fast acting protective gear and circuit breakers have made such motor fault current contribution appreciable, and its effect is often included in studies of present-day systems.

The contribution to fault current of an induction motor can be calculated using a simple empirical method or by means of the motor transient and sub-transient reactances and time constants. Nevertheless, the accurate induction motor model developed in this chapter provides an accurate estimate of the motor fault current contribution to the system and hence assists in determining the circuit breaker ratings. Thus the margins which have had to be allowed in
switchgear ratings to cover unknown effects due to machine behaviour can be reduced.

4.1.2 - Motor recovery problem

Transient voltage reductions in power systems, such as those arising during short-circuit faults, can force induction motors on the system into operating regions from which, depending on the motor load characteristics, they may be unable to recover. The increasing size of induction motors and the concentration of induction motor loads, particularly where these are supplied over substantial transmission distances, emphasize this form of possible load instability.

During the initial part of the motor recovery period, the motor draws an excessive current from the system resulting in a reduction in its terminal voltage. An excessive voltage reduction will extend the recovery period and may lead to motor overheating.

To shorten the recovery time, the system impedance between the power supply point and the motor terminal should be reduced. However, this increases the system fault level. The system impedance is therefore a compromise between the motor recovery time and the system fault level.

It is a widely adopted practice that, in the event of a voltage depression, all non-vital machines are allowed to trip on undervoltage protection, while vital machines remain connected to the supply for as long as possible. Should the supply be restored, these machines will accelerate and restore the major operation of the plant. Failure to do so will cause an inevitable plant shut down, loss of production and considerable damage to the plant often results.

4.1.3 - Motor starting problem

Failures of direct-on-line started cage motors over 300 kW are often the result of designs which do not take fully into account the thermal and mechanical stresses imposed by the initial surge current in the windings. The symmetrical peak of this inrush current can be some 7 to 10 times the rated r.m.s. motor current and its unsymmetrical
component may result in the peak current rising to as high as 10 to 16 times the rated current\textsuperscript{45}.

An accurate induction motor model provides a means of exploring the motor starting problem in the following areas.

a. The excessive voltage drop which delays the start-up period and thereby causes motor overheating.

b. The high transient airgap torque which may cause damage to both the motor and the driven system components.

4.2 - Stationary 2-axis representation of a 3-phase induction motor

Due to the uniformity of its airgap, the axes of the induction machine reference frame can be attached to either its primary or its secondary windings. To analyse it as a special case of a synchronous machine, the reference frame of an idealised induction machine model is assumed to be attached to its secondary member. To simplify the analysis, the d-axis is chosen to coincide with the axis of phase a of the secondary winding, as shown in fig. 4.1(a).

Since an induction machine is symmetrical along both axes, the resistances, leakage and mutual inductances of the direct and quadrature axes are identical.

Hence,

\[
\begin{align*}
    r_{d_1} &= r_{q_1} = r_1 \\
    r_{d_2} &= r_{q_2} = r_2 \\
    L_{d_1} &= L_{q_1} = L_1 \\
    L_{d_2} &= L_{q_2} = L_2 \\
    L_{ad_1} &= L_{aq_1} = L_{m_1} \\
    L_{ad_2} &= L_{aq_2} = L_{m_2}
\end{align*}
\]
If the base secondary current is selected on the $X_{ad}$ base principle described in Section 2.2.2, $L_{m1}$ and $L_{m2}$ are numerically equal,

\[ i.e. \quad L_{m1} = L_{m2} = L_m \]  

4.2

With reference to the 2-axis representation of the synchronous machine in fig. 2.2, elimination of the field winding leads to a model which is identical with that of the induction machine shown in fig. 4.1(b). Hence the voltage equation for the induction machine becomes,

\[
\begin{align*}
\begin{bmatrix}
V_{d2} \\
V_{q2} \\
V_{d1} \\
V_{q1}
\end{bmatrix} &=
\begin{bmatrix}
L_m^p & 0 & L_m^p & 0 \\
0 & L_m^p & 0 & L_m^p \\
L_m^p & -\omega' L_m & r_1 + L_1 p & -\omega' L_1 \\
-\omega' L_m & L_m^p & -\omega' L_1 & r_1 + L_1 p
\end{bmatrix}
\begin{bmatrix}
i_{d2} \\
i_{q2} \\
i_{d1} \\
i_{q1}
\end{bmatrix}
\end{align*}
\]  

4.3

where $\omega'$ is similarly defined as $\omega$ with the exception that $\omega'$ is for the induction motor while $\omega$ is for the synchronous machine. Similar notation applies to other variables and parameters of the induction motor model.

For computational purpose, the complete induction machine matrix equation is presented in the form,

\[
[P] [\dot{x}] = [Q] [x] + [S] + [T] [u]
\]  

4.4

where

\[
[\dot{x}] = [\dot{\delta}', p\dot{\delta}', \dot{i}_{d2}, \dot{i}_{q2}, \dot{i}_{d1}, \dot{i}_{q1}]^t
\]  

4.5

\[
[x] = [\delta', p\delta', i_{d2}, i_{q2}, i_{d1}, i_{q1}]^t
\]
\[ [u] = [T_t^t, v_{d_1}, v_{q_1}]^t \]

\[ [P] = \]

<p>| | | | | |</p>
<table>
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<tr>
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<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>J'</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L_2</td>
<td>L_m</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L_2</td>
<td>L_m</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L_m</td>
<td>L_1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>L_m</td>
<td>L_1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ [Q] = \]

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
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<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-K_{S'}</td>
<td>-K_{D'}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-r_2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-r_2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-L_m</td>
<td>-r_1</td>
<td>-L_1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>L_m</td>
<td>L_1</td>
<td>-r_1</td>
<td></td>
</tr>
</tbody>
</table>
The non-differential equations are,

\[
\begin{align*}
\nu_{d1} &= -V_m \sin \delta' \\
\nu_{q1} &= V_m \cos \delta' \\
T'_e &= L_m (i_{d2} i_{q1} - i_{d1} i_{q2})
\end{align*}
\]

where \( V_m \) is the p.u. busbar peak phase voltage.
4.3 - Transient behaviour of a small single-cage induction motor

Experimentally obtained switching transients from a 5.6 kW induction motor\(^{46}\), having the data and base values given in Tables 4.1 and 4.2 respectively, are used to validate the model developed in the previous section.

4.3.1 - Direct-on-line starting

A comparison of test and predicted motor transient current and electromagnetic torque following a no-load direct-on-line start of the motor is presented in fig. 4.2 and the agreement between the two sets of results is generally good. However, the peaks of the predicted current in the first few cycles are somewhat lower than the measured peaks, owing mainly to the starting currents being sufficiently high to saturate the leakage flux paths and thereby to reduce the leakage reactances.

The magnitude of the oscillatory torque peak during starting is measured as about 4.4 times the full-load torque and the oscillatory component dies away in about two to three cycles of the supply frequency. The pulsating component in the starting torque can be interpreted, in d-q-0 theory, as a result of the reaction between the asymmetrical flux on one axis and the alternating current on the other, and also the reaction between the asymmetrical current on one axis and the alternating flux on the other. With constant full-load torque applied to the shaft, the predicted starting transients are shown in fig. 4.3. The oscillatory electromagnetic torque persists for a longer period than during the no-load start but the magnitude of the first peak remains unaltered.

The oscillatory torque, particularly in large induction motors, can persist for a relatively longer time owing to the higher inertia and lower p.u. winding resistances than in small motors. One consequence of this is the slow rise in speed and a severe disturbance to the supply busbar voltages. In addition, the prolonged starting time may cause overheating of the motor.
With transient torque depending on the asymmetrical currents, the torque oscillations which follow a supply switching are a function of the time at which the switching occurs, with respect to the voltage waveform. Investigations\(^{35,47}\) show that the oscillatory torque can be significantly reduced by the use of non-simultaneous switching of the supply, such that each phase is connected to its respective voltage when this voltage has its peak value.

### 4.3.2 - Disconnection and reconnection of power supply

In the simulation of power supply interruption to an induction motor, the contactor is idealised by assuming zero impedance before interruption and infinite impedance thereafter.

Figs. 4.4 and 4.5 show reconnection transients following two different lengths of interruption of the supply to the unloaded motor.

Following a supply interruption, the induction motor effectively functions as a synchronous generator without a prime-mover, but with an exponentially decaying field excitation. The trapped rotor flux induces in the open-circuited armature windings a voltage which decays at a rate dependant upon the rotor time constant and the motor/load inertia. If the motor is re-switched to the supply after a short interruption, the decaying rotor flux causes the re-switching transient to be more severe than when the rotor is electrically inert. The principle factors governing the magnitude of the re-switching transients are the mode of connection (simultaneous or non-simultaneous), the motor load, the length of supply interruption and the difference between the instantaneous stator and supply voltages at the instant of supply reclosure. Maximum re-switching transients are to be expected when simultaneous closure occurs with the stator and supply voltages 180° out of phase. A re-switching transient current peak in excess of 13 times the full-load peak current has been recorded\(^{48}\).

The accuracy of the computed induced stator e.m.f. affects the predictions of the re-switching transients. Errors in such calculations are cumulative and re-switching transients are likely to be
more accurately predicted when the time of disconnection is short. This fact is clearly illustrated in figs. 4.4 and 4.5.

At no-load, the motor re-switching transients are normally less severe than when on load, because the initial speed is close to synchronous and the decay in speed during disconnection is relatively slow. Hence, the induced stator e.m.f. remains nearly co-phasal with the supply voltage and the reconnection transient is relatively small. The above phenomenon is clearly demonstrated in figs. 4.5 and 4.6.

4.3.3 - Plugging

To retard a motor by reversing the phase sequence of the supply voltage causes very severe re-switching transients. Fig. 4.7 shows the transients of an unloaded motor subsequent to a supply phase reversal. The transient torque peak is about three times the starting torque peak. The discrepancies between predicted and measured torques shown in fig. 4.7 may be attributed both to mechanical and to electrical factors. Plugging is a very drastic operation and the effects of eddy currents and saturation are more pronounced than in the tests described previously. At a slip greater than unity, discrepancies between measured and calculated steady-state torques are well-known and the additional loss in both the iron and copper has to be taken into account if it is required to improve the predictions.

4.4 - Transient behaviour of a deep-bar-cage induction motor

Large induction motors are in common use, and many of these have deep-bar-cage windings to increase the starting torque and to limit the starting current. Fig. 4.8 shows the measured variations in the rotor resistance and reactance of a micro induction motor as a function of slip. This model motor was designed to simulate the starting and running characteristics of an 1800 hp deep-bar-cage induction motor for establishing the theory of the transient performance of a large machine. If the dynamic performance study of the motor is carried out by analytical means, based on the concept of operational impedance, it is necessary to represent the deep-bar cage by an equivalent double-cage rotor. This is normally achieved by first
determining the actual admittance locus of the machine, then fitting the nearest double-cage type locus to it. Finally, the transient and sub-transient parameters so obtained are used to calculate the dynamic performance of the machine.

Rather than using a double-cage approach\textsuperscript{11,42}, a deep-bar rotor is represented in the present study by a single coil on each rotor axis and the values of the rotor resistance $r_2$ and reactance $x_2$ are varied as functions of the rotor slip. This approach is more direct since the variations of $r_2$ and $x_2$ with slip are read directly into the computer program instead of using this information to derive parameters for an equivalent double-cage rotor.

\textbf{4.4.1 - 3-phase short-circuit fault}

The model motor was connected to an infinite busbar system through an impedance of $0.0142 + j0.0496$ p.u.\textsuperscript{42} and a symmetrical short-circuit was applied at the motor terminals for 0.142 s. The prefault busbar voltage, current and slip were respectively 1.05, 1.32 and 0.01 p.u. and the good correlation obtained between measured and computed transients is shown in fig. 4.9.

The principal machine data is given in table 4.3 and the curves of load torque/speed and stray load loss/primary current, used in the computer program, are shown in fig. 4.10.

\textbf{4.4.2 - Effect of stray load loss}

Stray load loss in a motor arises from non-uniform current distribution in the copper and the additional core losses produced in the iron by distortion of the magnetic flux by the load current. To allow for stray loss in efficiency calculations, BS 269\textsuperscript{49}, recommends a value of 0.5 per cent at full load, on a basis of continuous maximum rating. For a number of machines ranging from 415 hp to 1190 hp, Schwarz\textsuperscript{50} found the full load stray loss to be between 0.68 per cent and 1.9 per cent. Barton\textsuperscript{51} measured the stray loss of a 7.5 hp motor at full load to be 5 per cent. This shows that the effect of stray load loss depends to a large extend on the size of the motor.
If the stray loss is neglected in the motor short-circuit fault simulation, the motor speed recovery is very much faster (see fig. 4.9). The full load stray loss of the model motor used in the present investigation is 3 per cent which represents 0.6 per cent for the full-scale motor. Neglecting the stray loss in a large motor is unlikely therefore to have such an adverse effect on the machine recovery as that shown in fig. 4.9. Nevertheless, if the loss is not included, the calculated recovery time will be under-estimated.

4.4.3 - Open-circuit fault

Fig. 4.11 shows computed and measured motor transients after a momentary open-circuit fault. The line impedance between the busbar supply and the motor was $0.012 + j 0.01$ p.u. and the prefault busbar voltage, current and slip were respectively 1.05, 1.29 and 0.009 p.u. Although the fault duration was the same as that for the short-circuit test, the motor speed dip and hence the recovery time can be seen to be now considerably less. The computed electrical torque oscillates for a few cycles following reconnection of the supply and the peak torque reaches 4.5 times the full-load torque.

4.4.4 - Effect of rotor trapped flux

Under normal operating conditions, the flux in the rotor of an induction motor cannot fall instantaneously to zero should the machine become isolated from the supply through a period of short-circuit or by complete isolation from the power source. During both conditions, the motor possesses mainly kinetic energy as there is no input power supply. In addition to the loss associated with friction and windage, the kinetic energy is further dissipated as internal heating. The electrical torque generated by the interaction of the currents in the stator circuits and the rotor trapped flux retards the machine.

It is evident from figs. 4.9 and 4.11 that the speed dip in the short-circuit fault is considerably greater than that of the open-circuit fault. This is attributed to the large armature short-current current, which dissipates more heat and hence develops a greater retarding torque.
4.5 - Dynamic simulation of a composite system containing both synchronous and induction machines

The composite system shown in fig. 4.12 forms a typical power supply system, in which large induction motors form an important part of the load and the supply is taken from the grid through a transmission line, with one or more generators installed for emergency purposes. For simulation considerations, the induction motors and the synchronous generators are represented respectively by a single motor and a single generator.

4.5.1 - Axis transformation

The 2-axis equations of each machine express the voltages and currents in a reference frame fixed to its direct and quadrature axes. To relate these quantities between the two machines, an axis-transformation is required to refer them to a common reference frame (D, Q) rotating at synchronous speed. The phasor relation between the common reference frame and the synchronous machine reference frame d, q are shown in fig. 4.13. The transformation of \( i_d' \), \( i_q' \) to \( i_{gD} \), \( i_{gQ} \) and of \( v_{gD} \), \( v_{gQ} \) to \( v_d \), \( v_q \) may be stated as,

\[
\begin{align*}
\begin{bmatrix}
i_{gD} \\
i_{gQ}
\end{bmatrix} &=
\begin{bmatrix}
\cos \delta & -\sin \delta \\
\sin \delta & \cos \delta
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} \\
\begin{bmatrix}
v_d \\
v_q
\end{bmatrix} &=
\begin{bmatrix}
\cos \delta & \sin \delta \\
-\sin \delta & \cos \delta
\end{bmatrix}
\begin{bmatrix}
v_{gD} \\
v_{gQ}
\end{bmatrix}
\end{align*}
\]

4.13

4.14
Similarly, the axis transformation equations for the induction motor can be written as,

$$
\begin{bmatrix}
 i_{mD} \\
i_{mQ}
\end{bmatrix}
= \begin{bmatrix}
 \cos \delta' & -\sin \delta' \\
 \sin \delta' & \cos \delta'
\end{bmatrix}
\begin{bmatrix}
 i_d_1 \\
i_q_1
\end{bmatrix}
$$

\[ 4.15 \]

$$
\begin{bmatrix}
 v_{d_1} \\
v_{q_1}
\end{bmatrix}
= \begin{bmatrix}
 \cos \delta' & \sin \delta' \\
 -\sin \delta' & \cos \delta'
\end{bmatrix}
\begin{bmatrix}
 v_{mD} \\
v_{mQ}
\end{bmatrix}
$$

\[ 4.16 \]

### 4.5.2 - Computation procedures

Simulation of the composite system requires solution of the following three sets of equations at each integration step.

a. Differential equations of the machines.

b. Frame transformation equations.

c. Algebraic equations of the network.
With the machine state variables obtained from the previous integration step, the computation for the current step may be summarized as follows:

a. The axis currents $i_d$, $i_q$ for the synchronous machine and $i_{d1}$, $i_{q1}$ for the induction motor are transformed to $I_{gD}$, $I_{gQ}$ and $I_{mD}$, $I_{mQ}$ in the common reference frame using eqns. 4.13 and 4.15 respectively.

b. The derivatives of the axis currents are also transformed as in a.

Hence, \[
\begin{align*}
\dot{i}_d' + \dot{i}_q' &= I_{gD} + I_{gQ} \\
\dot{i}_{d1}' + \dot{i}_{q1}' &= I_{mD} + I_{mQ}
\end{align*}
\]

Hence, \[
\begin{align*}
\dot{i}_d' + \dot{i}_q' &= I_{gD} + I_{gQ} \\
\dot{i}_{d1}' + \dot{i}_{q1}' &= I_{mD} + I_{mQ}
\end{align*}
\]

\[
\begin{align*}
\dot{i}_bD &= \dot{i}_gD + \dot{i}_mD \\
\dot{i}_bQ &= \dot{i}_gQ + \dot{i}_mQ
\end{align*}
\]

\[
\begin{align*}
\dot{i}_bD &= \dot{i}_gD + \dot{i}_mD \\
\dot{i}_bQ &= \dot{i}_gQ + \dot{i}_mQ
\end{align*}
\]

while the axis voltages across the transmission line are,

\[
\begin{align*}
\Delta V_{bD} &= r_t i_{bD} + L_t (i_{bD} + i_{bQ}) \\
\Delta V_{bQ} &= r_t i_{bQ} + L_t (i_{bQ} - i_{bD})
\end{align*}
\]

where $r_t$ and $L_t$ are respectively the resistance and the inductance of the transmission line.
d. The axis voltages at the junction of the two machines \( v_{JD} \) and \( v_{JQ} \) are calculated from:

\[
\begin{align*}
v_{JD} &= v_{BD} - \Delta v_{BD} \\
v_{JQ} &= v_{BQ} - \Delta v_{BQ}
\end{align*}
\]

Where \( v_{BD} \) and \( v_{BQ} \) are the axis components of the busbar voltage.

Since the busbar voltage is chosen as the reference vector, its axis components are,

\[
\begin{align*}
v_{BD} &= 0 \\
v_{BQ} &= \text{peak phase voltage}
\end{align*}
\]

e. The voltages \( v_{JD} \) and \( v_{JQ} \) obtained from eqn. 4.22 are transformed to axis voltages \( v_d, v_q \) for the generator and \( v_{d1}, v_{q1} \) for the motor, using eqns. 4.14 and 4.16 respectively. \( (v_{gD} = v_{mD} = v_{JD}; v_{gQ} = v_{mQ} = v_{JQ}) \).

f. With the updated axis voltages, the machine dynamic equations are solved using a numerical integration routine.

g. With the updated values of the state variables repeat the above procedures until the simulation time elapses.

4.5.3 - Fault simulation on a composite system

The validity of the system model was checked by simulating a momentary 3-phase short-circuit fault at an intermediate point \( F \) of the transmission line (refer to fig. 4.12). Comparisons of the predicted and test results are presented in fig. 4.14. The principal generator parameters used in the computer program are listed in table 4.4 whilst those for the induction motor are detailed in Section 4.4.
The agreement between the test and computed results is generally good. The calculated load angle $\delta$ shows a temporary dip, referred to as a 'back swing', before an eventual rise. This phenomenon can arise because, during the short-circuit, the machine is effectively isolated from the external busbar supply and the 'flux wave' of the armature is stationary with respect to the armature. The consequent induced currents in the rotor circuits cause power losses. These losses, together with the armature short-circuit power losses, produce a unidirectional retarding torque. This torque decays rapidly, but its initial value may be considerably larger than the pre-fault torque and it may thus be able to produce an initial back swing as in the present investigation. The initial variation of the computed generator electrical torque, slip and rotor angle, shown in fig. 4.15, gives a better insight into the above phenomenon. The first peak of the pulsating electrical torque $T_e$ rises to about 2.3 p.u. while the input torque is about 0.15 p.u. In the first cycle, it can be seen from fig. 4.15 that the average value of $T_e$ exceeds the shaft torque and retards the generator. As the unidirectional torque decays, the rotor starts to accelerate. The negative swing of $T_e$ at the instant of fault clearance is caused by the synchronising power from the external supply.
### TABLE 4.1
**INDUCTION MOTOR DATA**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>5.6 kW</td>
</tr>
<tr>
<td>Rated line voltage</td>
<td>400 V</td>
</tr>
<tr>
<td>Rated current</td>
<td>11.9 A</td>
</tr>
<tr>
<td>Supply frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Rated speed</td>
<td>955 r.p.m.</td>
</tr>
<tr>
<td>Stator connection</td>
<td>Δ</td>
</tr>
<tr>
<td>Rated torque</td>
<td>56 N m</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>2.45 Ω</td>
</tr>
<tr>
<td>Rotor resistance (refer to stator)</td>
<td>2.45 Ω</td>
</tr>
<tr>
<td>Stator leakage inductance</td>
<td>0.017 H</td>
</tr>
<tr>
<td>Rotor leakage inductance (refer to stator)</td>
<td>0.017 H</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>0.42 H</td>
</tr>
<tr>
<td>Rotor inertia</td>
<td>0.102 kg m²</td>
</tr>
<tr>
<td>Mechanical loss torque</td>
<td>$0.06 + 1.2 u^2 \text{ N m}$</td>
</tr>
</tbody>
</table>
TABLE 4.2
INDUCTION MOTOR BASE AND P.U. DATA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base stator voltage $V_{ao}$</td>
<td>326.56 V</td>
</tr>
<tr>
<td>Base stator current $I_{ao}$</td>
<td>16.83 A</td>
</tr>
<tr>
<td>Base power $VA_o$</td>
<td>8.244 kVA</td>
</tr>
<tr>
<td>Base stator impedance $Z_{ao}$</td>
<td>19.4 Ω</td>
</tr>
<tr>
<td>Base frequency $\omega_o$</td>
<td>314.159 rad/s</td>
</tr>
<tr>
<td>Base inductance $L_o$</td>
<td>0.0618 H</td>
</tr>
<tr>
<td>Base speed $\nu_o$</td>
<td>104.72 rev/s</td>
</tr>
<tr>
<td>Base torque $T_o$</td>
<td>78.7 Nm</td>
</tr>
<tr>
<td>Base inertia $J_o$</td>
<td>0.000266 kg m²</td>
</tr>
<tr>
<td>Stator resistance $\sigma_1$</td>
<td>0.042 p.u.</td>
</tr>
<tr>
<td>Rotor resistance $\sigma_2$</td>
<td>0.042 p.u.</td>
</tr>
<tr>
<td>Stator self-inductance $L_1$</td>
<td>2.36 p.u.</td>
</tr>
<tr>
<td>Rotor self-inductance $L_2$</td>
<td>2.36 p.u.</td>
</tr>
<tr>
<td>Mutual inductance $L_m$</td>
<td>2.265 p.u.</td>
</tr>
<tr>
<td>Rotor inertia $J'$</td>
<td>42.5 p.u.</td>
</tr>
<tr>
<td>Mechanical loss torque $T'_L$</td>
<td>$0.00076 + 0.0152 v^2$ p.u.</td>
</tr>
</tbody>
</table>
### TABLE 4.3
MICRO INDUCTION MOTOR DATA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>3.0 kVA</td>
</tr>
<tr>
<td>Rated line voltage</td>
<td>220.0 V</td>
</tr>
<tr>
<td>Rated current</td>
<td>7.87 A</td>
</tr>
<tr>
<td>Armature resistance $r_1$</td>
<td>0.0094</td>
</tr>
<tr>
<td>Armature leakage reactance $x_1$</td>
<td>0.143</td>
</tr>
<tr>
<td>Magnetising reactance $x_m$</td>
<td>3.41</td>
</tr>
<tr>
<td>Rotor resistance (starting)</td>
<td>0.033</td>
</tr>
<tr>
<td>Rotor resistance (running)</td>
<td>0.0072</td>
</tr>
<tr>
<td>Rotor reactance (starting)</td>
<td>0.058</td>
</tr>
<tr>
<td>Rotor reactance (running)</td>
<td>0.128</td>
</tr>
<tr>
<td>Transient reactance $x'_d$</td>
<td>0.292</td>
</tr>
<tr>
<td>Subtransient reactance $x''_d$</td>
<td>0.184</td>
</tr>
<tr>
<td>Time constants</td>
<td></td>
</tr>
<tr>
<td>$T'_d$</td>
<td>0.1 s</td>
</tr>
<tr>
<td>$T''_d$</td>
<td>0.014 s</td>
</tr>
<tr>
<td>Inertia constant</td>
<td>2.0 s</td>
</tr>
</tbody>
</table>

(All parameters are in p.u. unless otherwise specified)
### TABLE 4.4
MICRO ALTERNATOR DATA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>3.0 kVA</td>
</tr>
<tr>
<td>Rated line voltage</td>
<td>220 V</td>
</tr>
<tr>
<td>Rated current</td>
<td>7.87 A</td>
</tr>
<tr>
<td>Magnetizing reactances $x_{md}$</td>
<td>2.66</td>
</tr>
<tr>
<td>$x_{mq}$</td>
<td>2.45</td>
</tr>
<tr>
<td>Armature leakage reactance $x_a$</td>
<td>0.19</td>
</tr>
<tr>
<td>Armature resistance $r_a$</td>
<td>0.0197</td>
</tr>
<tr>
<td>Field leakage reactance $x_{ff}$</td>
<td>0.1489</td>
</tr>
<tr>
<td>Field resistance $r_{ff}$</td>
<td>0.0015</td>
</tr>
<tr>
<td>Transient reactance $x'_d$</td>
<td>0.336</td>
</tr>
<tr>
<td>Subtransient reactances $x''_d$</td>
<td>0.232</td>
</tr>
<tr>
<td>$x''_q$</td>
<td>0.264</td>
</tr>
<tr>
<td>Time constants $T'_{do}$</td>
<td>6.06 s</td>
</tr>
<tr>
<td>$T'_d$</td>
<td>0.63 s</td>
</tr>
<tr>
<td>$T''_d$</td>
<td>0.008 s</td>
</tr>
<tr>
<td>$T''_q$</td>
<td>0.01 s</td>
</tr>
<tr>
<td>Inertia constant $H$</td>
<td>3.5 s</td>
</tr>
</tbody>
</table>

(All parameters are in p.u. unless otherwise specified)
Fig. 4.1 Representations of an induction motor
Fig. 4.2 No-load direct-on-line motor starting transients (5.6 kW motor)
Fig. 4.3 Predicted full-load direct-on-line motor starting transients (5-6 kW motor)
Fig. 4.4 Motor transients following a supply interruption
(5.6 kW motor)
Fig. 4.5 Motor transients following a supply interruption (5-6 kW motor)
Fig. 4.6 Predicted transients on a fully-loaded motor following a supply interruption (5.6kW motor)
Fig. 4.7 Motor transients following a plugging operation (5.6 kW motor)
Fig. 4.8 Variation of rotor resistance and reactance with slip
(micro induction motor)
Fig. 4.9 Motor transients following a short-circuit fault
(micro induction motor)
Fig. 4.10 Micro induction motor characteristics

torque/speed curve

stray load loss, p.u.

speed, p.u.

primary current, p.u.

stray load loss
Fig. 4.11 Motor transients following an open-circuit fault
(micro induction motor)
Fig. 4.12 Schematic diagram of the composite system

![Schematic diagram of the composite system](image)

\[ r_t = 0.0062 \text{ p.u.} \quad L_t = 0.2 \text{ p.u.} \]
\[ r_{b1} = 0.0045 \text{ p.u.} \quad L_{b1} = 0.031 \text{ p.u.} \]
\[ r_{b2} = 0.0065 \text{ p.u.} \quad L_{b2} = 0.093 \text{ p.u.} \]

Fig. 4.13 Axis transformation phasor diagram

![Axis transformation phasor diagram](image)
Fig. 4.14 System transient response to a temporary short circuit
Fig. 4.15 Phenomenon of generator rotor back-swing
CHAPTER 5

DYNAMIC RESPONSE ANALYSIS

OF

A STEAM TURBO-ALTERNATOR UNIT
CHAPTER 5
DYNAMIC RESPONSE ANALYSIS OF A STEAM TURBO-ALTERNATOR UNIT

With the continual growth in power systems and their interconnection, there is a corresponding continual economic pressure to install larger generating units. Recent advances in steam turbine design, generator cooling methods and electrical insulation techniques have meant that the increase in machine rating has however not been accompanied by a corresponding increase in physical size, and indeed, the inertia constant of typical unit has been significantly reduced\textsuperscript{52,53}. The changes in generator design have also caused certain unit parameters, such as the generator short-circuit ratio and the transient reactance, to move in a direction less favourable to the maintenance of unit (system) stability during a system disturbance. The large scale use of high-voltage transmission and the complex interconnection of machines further aggrevate this problem.

The above factors, together with the complexity of a modern turbo-generator, have led to an increasing demand for accurate modelling of the system, such that a suitable and sophisticated control system can be designed to ensure safe and reliable operation under all conditions.

A mathematical model for a steam turbo-alternator unit is developed in this chapter and the simulation of various types of system disturbance is presented.

5.1 - Mathematical model for a steam turbo-alternator unit

The system analysed is shown in schematic form in fig.5.1. It consists of a single synchronous generator, an excitation system, a prime mover with its governor system and a transmission line connecting the generator through a step-up transformer to an infinite system.
5.1.1 - Generator representation including the effect of saturation

The system of equations expressed in the d-q-0 reference frame, as given in Section 3.1.3, is used to represent the turbo-generator. The inductance coefficients contained in these equations are dependent on the precise flux condition in the machine. The effect of saturation gives rise to a non-linear relationship between the m.m.f. and the corresponding flux. An international survey\textsuperscript{54} has revealed that generator models allow for saturation in a variety of different ways, and the method adopted in this study is to modify the machine mutual inductances by a saturation factor.

In order to obtain an expression for the variation in the effective reactance of the generator\textsuperscript{2}, the following assumptions are made in relation to a flux path comprising laminated iron in the stator, solid iron in the rotor and the airgap between them.

a. the leakage reactance of each winding is assumed to retain a constant value, independent of flux conditions

b. the leakage fluxes do not contribute to the iron saturation, which is therefore determined by the total mutual flux as calculated from its direct and quadrature components

c. all the mutual reactances on the same axis have numerically equal p.u. values

d. both axes are assumed to be equally affected by saturation.

Following the assumptions made in (b) and (c), the magnitude of the airgap flux can be expressed as

\[ \Psi_{ag} = \sqrt{L_{md}^2 (i_f + i_D + i_d)^2 + L_{mq}^2 (i_Q + i_q)^2} \]  

5.1
when saturation is neglected this becomes,

\[ \psi_{agu} = \sqrt{L_{mdu}^2 (i_F + i_D + i_d)^2 + L_{mqu}^2 (i_Q + i_q)^2} \] 5.2

where \( L_{mdu} \) and \( L_{mqu} \) are the unsaturated values of \( L_{md} \) and \( L_{mq} \) respectively.

If, \( k = \frac{\psi_{ag}}{\psi_{agu}} \) is a non-linear function accounting for saturation, then

\[ L_{md} = k L_{mdu} \] 5.3
\[ L_{mq} = k L_{mqu} \]

Hence,

\[ L_{mq} = \lambda L_{md} \] 5.4

where \[ \lambda = \frac{L_{mqu}}{L_{mdu}} \] 5.5

This simple relationship between \( L_{md} \) and \( L_{mq} \) allows eqn. 5.1 to be simplified to

\[ \psi_{ag} = L_{md} \sqrt{(i_F + i_D + i_d)^2 + \lambda^2 (i_Q + i_q)^2} \]
\[ = L_{md} \sqrt{H_{agd}^2 + \lambda^2 H_{agq}^2} \]
\[ = L_{md} \cdot H_{ag} \] 5.6

where \( H_{ag} \) is proportional to the total airgap m.m.f.

When a generator is on open circuit, the airgap flux is,

\[ \psi_{ag} = L_{md} i_F \] 5.7

and on multiplying both sides by \( \omega_0 \),

\[ v_{qo} = x_{md} i_f \] 5.8
which is the familiar expression for the generator open-circuit characteristic. The non-linear relationship between $v_{ag}$ and $H_{ag}$ can therefore be derived from the generator open-circuit characteristic. Their non-linear relationship can then be expressed by a polynomial of degree $n$ having $n+1$ coefficients.

\[ v_{ag} = a_0 + a_1 H_{ag} + a_2 H_{ag}^2 + \ldots + a_n H_{ag}^n \]  

5.9

In evaluating the coefficients $a_0, a_1, a_2, \ldots a_n$, a well-established curve fitting sub-routine based on Chebyshev's least-square technique is used. The generator open-circuit characteristic used in the computer program is shown in fig. 5.2.

To include saturation in the generator model, the total airgap m.m.f. and the equivalent airgap flux are evaluated at the beginning of each integration step. Using eqns. 5.6 and 5.4, the mutual inductances, and hence the inductance matrix of the generator are up-dated for the next integration step.

The generator and transmission system data and the base quantities are shown in Appendix H.1. Since there is no direct information on the quadrature-axis circuits, these are assumed to be numerically equal to the corresponding direct-axis terms, with the exception that $x_q$ is assumed equal to 0.9 $x_d$. As suggested by Shackshaft\(^2\), the computed damper winding resistances are reduced by a factor of four to account for the inability of the model to represent the skin effect of the solid rotor surface.

### 5.1.2 Excitation control system

The transfer function block diagram for the excitation control system is shown in fig. 5.3 and the assumptions made in the derivation of the equations which follow are:

a. The transfer characteristics of the magnetic amplifiers and exciter may be represented by the curve shown in fig. 5.4 and their time response may be represented by a simple time lag.
b. The parameters given in Appendix H.2 are assumed to be constant under any operating conditions.

c. The characteristics of the rectifiers are assumed bi-linear.

d. The var-limiter is inoperative.

- Voltage feedback circuit
  \[ v_{er} = K_c (v_{ref} - K_i v_t) \]  

- Magnetic amplifier 1
  \[ v_{r1} = B_{r1} + \frac{K_{r1}}{1 + T_{r1p}} (v_{er} - v_{ef} - v_{rf}) \]
  \[ v_{r1\text{min}} < v_{r1} < v_{r1\text{max}} \]  

- Magnetic amplifier 2
  \[ v_{r2} = B_{r2} + \frac{K_{r2}}{1 + T_{r2p}} v_{r1} \]
  \[ v_{r2\text{min}} < v_{r2} < v_{r2\text{max}} \]  

- Exciter
  \[ v_f = B_e + \frac{K_e}{1 + T_{ep}} v_{r2} \]  

- Amplifier stabiliser
  \[ v_{rf} = \frac{K_{rf} T_{rf1} v_{r2}}{1 + T_{rf2p}} \]  

- Exciter stabiliser
  \[ v_{ef} = \frac{K_{ef} T_{ef1} v_f}{1 + T_{ef2p}} \]  

The above system equations are transformed into state-space form, as shown in eqn. 5.16, with time normalised to \( 1/\omega_0 \) s, and the
<table>
<thead>
<tr>
<th>$v_{r1}$</th>
<th>$\frac{-1}{T_{r1}}$</th>
<th>$\frac{K_{r1}}{T_{r1}}$</th>
<th>$\frac{K_{r1}}{T_{r1}}$</th>
<th>$\frac{B_{r1}}{T_{r1}} + VCF_{VR1}$</th>
<th>$\frac{K_{r1}}{T_{r1}}$</th>
<th>$\frac{K_{r1}}{T_{r1}}$</th>
<th>$\frac{K_{r1}}{T_{r1}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{r2}$</td>
<td>$\frac{K_{r2}}{T_{r2}}$</td>
<td>$\frac{-1}{T_{r2}}$</td>
<td>$\frac{B_{r2}}{T_{r2}} + VCF_{VR2}$</td>
<td>$v_{r2}$</td>
<td>$\frac{B_{r2}}{T_{r2}} + VCF_{VR2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_{f}$</td>
<td>$\frac{K_{e}}{T_{e}}$</td>
<td>$\frac{-1}{T_{e}}$</td>
<td>$\frac{K_{r}}{T_{r}}$</td>
<td>$v_{f}$</td>
<td>$\frac{K_{e}}{T_{e}} + VCF_{VF}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_{rf}$</td>
<td>$\frac{K_{r}}{T_{r}} \frac{T_{rf1}}{T_{r} T_{rf2}}$</td>
<td>$\frac{K_{r}}{T_{r}} \frac{T_{rf1}}{T_{r} T_{rf2}}$</td>
<td>$\frac{-1}{T_{rf2}}$</td>
<td>$v_{rf}$</td>
<td>$\frac{K_{e}}{T_{e}} \frac{T_{rf1}}{T_{r} T_{rf2}} \frac{B_{e}}{T_{r} T_{rf2}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_{ef}$</td>
<td>$\frac{K_{e}}{T_{e}} \frac{T_{ef1}}{T_{e} T_{ef2}}$</td>
<td>$\frac{K_{ef}}{T_{e}} \frac{T_{ef1}}{T_{e} T_{ef2}}$</td>
<td>$\frac{-1}{T_{ef2}}$</td>
<td>$v_{ef}$</td>
<td>$\frac{K_{ef}}{T_{e}} \frac{T_{ef1}}{T_{e} T_{ef2}} \frac{B_{e}}{T_{e} T_{ef2}}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
system data are listed in Appendix H.2. $VCF_{VR1}$, $VCF_{VR2}$ and $VCF_{VF}$ shown in eqn. 5.16 are the clamping functions for the 1st and 2nd stage amplifiers and the exciter output voltages. The values of these clamping functions are adjusted at each integration step, to ensure their corresponding output voltages operate within their specified limits. The method of introducing clamping functions in the differential equations for handling randomly occurring discontinuities is presented in Appendix B.

5.1.3 - Steam turbine and its speed governor

A simplified representation of the steam turbine and its speed control system is shown in block diagram form in fig. 5.5, in which the steam power admitted to the turbine through the throttle valve is controlled by the governing system in accordance with the speed of the shaft.

The equations incorporated in the present model, preceded by the main assumptions on which they are based, are:

a. The boiler is assumed to sustain a constant input power to the throttle valve during any transient period.

b. There is no time lag between the shaft speed and the governor sleeve movement.

c. The delays in the servomechanisms between the centrifugal governor and the steam throttle valve may be represented by a 2nd-order time lag.

d. The delay caused by the entrained steam in the turbine system may be represented by a single time lag.
- Centrifugal governor

\[ u_{\text{sg}} = u_0 - K_{\text{sg}} \rho_\delta \]

\[ 0 < u_{\text{sg}} < 1 \]

\( u_{\text{sg}} \) corresponds to the centrifugal governor sleeve movement and \( u_0 \) represents the speeder gear setting.

\( u_{\text{sg}} = 0 \) - flyballs are fully out

\( u_{\text{sg}} = 1 \) - flyballs are fully in

- Servo control system

\[ u_{\text{tv}} = u_{\text{vo}} + \frac{K_{\text{hy}}}{(1 + T_{\text{hy1}})(1 + T_{\text{hy2}})} u_{\text{sg}} \]

\[ 0 < u_{\text{tv}} < 1 \]

\( u_{\text{tv}} \) is proportional to the governor valve position.

\( u_{\text{tv}} = 0 \) - valve is fully close

\( u_{\text{tv}} = 1 \) - valve is fully open

- Throttle valve

\[ P_s = K_v u_{\text{tv}} \]

where \( P_s \) is the steam power admitted to the steam turbine.

- Turbine

\[ P_t = \frac{1}{1 + T_s \rho} P_s \]

\[ T_t = P_t / u \]

where \( P_t \) is the shaft power applied to the generator rotor and \( T_t \) is the shaft torque corresponding to \( P_t \).
The above equations are re-arranged in a state-variable form, as shown in eqn. 5.22 with time normalised to $1/\omega_0$ s, and the values of the associated parameters are shown in Appendix H.3.

\[
\begin{align*}
\dot{u}_{sg} &= -\frac{1}{T_{hy1}} \\
\dot{u}_{tv} &= \frac{1}{T_{hy2}} - \frac{1}{T_{hy2}} \\
\dot{p}_t &= \frac{K_v}{T_s} - \frac{1}{T_s}
\end{align*}
\]

\[
\begin{align*}
\dot{u}_{sg} &= \frac{K_{hy} K_{sg}}{T_{hy1}} p_0 \\
\dot{u}_{tv} &= \frac{u_{vo}}{T_{hy2}} + VCF_{UTV} \\
\dot{p}_t &= \frac{K_{hy}}{T_{hy1}} u_o
\end{align*}
\]

where $VCF_{UTV}$ is the clamping function for $u_{tv}$

5.2 - Comparison of site-test and computed results

The validity of the steam turbo-alternator model which has been developed is examined by comparing the model predictions with results obtained from a full scale site test on a 30 MW unit.

5.2.1 Step change in voltage regulator reference level

The response of the generator and its excitation system following a step change of a.v.r. reference setting, corresponding to a change of generator terminal voltage from 12 to 10 kV, was examined. Comparisons between computed and test results are made in fig. 5.6 and the initial and final steady-state readings of the generator quantities are recorded in Table 5.1.

Both Table 5.1 and fig. 5.6 show good agreement between test and computed results, except for a slight over estimate of the field voltage. This discrepancy is caused by the failure of the simulation to take into account variations in the field resistance with temperature.
Using the variable inductance model, the computed value for \( L_{md} \) varies between 1.682 p.u. at the terminal voltage of 12 kV to 1.796 p.u. at 10 kV, compared with an unsaturated value of 1.86 p.u. used in the constant inductance model. The effect of this error is clearly demonstrated in the results shown in fig. 5.6 and Table 5.1.

5.2.2 - Load rejection test

This test was initiated by operating the 132 kV circuit breaker with the generator operating at full-load and rated power factor. Comparisons of test and computed results are shown in figs. 5.7 and 5.8 and Table 5.2.

Despite being better correlated with the measured excitation current for both steady-state and transient conditions, the model including generator saturation effect yields less accurate predictions of transient stator voltage compared with the simple model in which saturation is neglected.

Comparing the predictions made by the two models, fig. 5.7 shows that the one including saturation gives a slower rise and a subsequent slower decay of the terminal voltage. This is because in the variable inductance model, the increase of generator flux level following the full-load rejection reduces the mutual inductances and hence decreases the rate of rise of the generator terminal voltage. The subsequent decay of generator flux level resulting from the a.v.r. action increases the mutual inductances towards their final steady-state values. Consequently, the rate of decay of the generator terminal voltage is lower than that of the constant inductance model.

5.2.3 - Symmetrical 3-phase fault

A 3-phase fault was applied at the high voltage terminals of the step-up transformer, with the generator at rated load and power factor, for a period of 0.38 s. Comparison of test and computed results are made in Table 5.3 and fig. 5.9. In general, the discrepancy between the test and computed results using the variable inductance model is larger than in studies using the constant inductance model. However, the former model gives very good agreement during
the initial part of the rotor angle swing, and this is to be compared with the poor agreement in the latter model. Furthermore, the predicted steady-state conditions using the variable inductance model correlate better with the test results.

When saturation is not included, the computed load angle swing is comparable with that presented by Harley and Adkins. When studying the merits of using generator models of different complexity, Hammons and Winning reported that the discrepancy between the first swing of the computed and test rotor angles is larger in a model with an allowance made for saturation than in a simple representation. This phenomenon is similar to that experienced by the present author. By introducing the effects of generator saturation and rotor eddy current, Humpage and Saha obtained a drastic improvement in their predictions. These may lead to a wrong conclusion that the introduction of rotor eddy current effects is the prime reason for such an improvement. In fact, however, they neglected the transmission system resistance in their computation. Judging from the value of this resistance, it appears to be negligible, although in a short-circuit test it plays any important role in determining the rotor angle swing. Neglect of this parameter during a short-circuit transient causes a reduction in the retarding torque, corresponding to its power loss, and this extra torque is available for further acceleration of the rotor. Fig. 5.10 illustrates the effect of neglecting the transmission system resistance on the rotor angle swing and a comparison with predictions from reference 3 is made in fig. 5.11. This indicates that it is not justifiable for Humpage and Saha to neglect this resistance in their computation. If they had not made this assumption, the accuracy of their predictions of the rotor angle swing would be similar to that of the present author.

It is evident from fig. 5.9(a) that the predicted rotor angle negative swing is considerably less than that measured. This discrepancy is mainly caused by the failure of the simulation to take into account the steam valve opening rate limit. If this limit had been included in the model, the rise of the applied torque to the rotor would be slower than that shown in fig. 5.9(b) and the prediction of the rotor angle swing would hence be improved.
### TABLE 5.1

**COMPARISONS OF SITE-TEST AND COMPUTED RESULTS**

**FOR STEP CHANGE OF REFERENCE VOLTAGE**

<table>
<thead>
<tr>
<th>System quantities</th>
<th>Initial readings</th>
<th>Final readings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Computed</td>
<td>Computed</td>
</tr>
<tr>
<td></td>
<td>Test</td>
<td>Test</td>
</tr>
<tr>
<td></td>
<td>Variable</td>
<td>Variable</td>
</tr>
<tr>
<td></td>
<td>inductance</td>
<td>inductance</td>
</tr>
<tr>
<td></td>
<td>Constant</td>
<td>Constant</td>
</tr>
<tr>
<td></td>
<td>inductance</td>
<td>inductance</td>
</tr>
<tr>
<td>Stator voltage (kV)</td>
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<td>9.95</td>
</tr>
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<td></td>
<td>12.0</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>12.0</td>
<td>10</td>
</tr>
<tr>
<td>Field voltage (V)</td>
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</tr>
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<td>98.8</td>
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<td>89.36</td>
<td>74.45</td>
</tr>
<tr>
<td>Field current (A)</td>
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<td>108</td>
</tr>
<tr>
<td></td>
<td>147</td>
<td>114.4</td>
</tr>
<tr>
<td></td>
<td>132.76</td>
<td>110.65</td>
</tr>
</tbody>
</table>
## TABLE 5.2

**COMPARISONS OF SITE-TEST AND COMPUTED RESULTS**

**FOR FULL-LOAD REJECTION TEST.**

<table>
<thead>
<tr>
<th>System quantities</th>
<th>Initial readings</th>
<th>Final readings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test</td>
<td>Computed</td>
</tr>
<tr>
<td>Load (MW)</td>
<td>29.8</td>
<td>30</td>
</tr>
<tr>
<td>Load (MVar)</td>
<td>25.3</td>
<td>25</td>
</tr>
<tr>
<td>Stator voltage (kV)</td>
<td>12.1</td>
<td>12.1</td>
</tr>
<tr>
<td>Stator current (kA)</td>
<td>-</td>
<td>1.86</td>
</tr>
<tr>
<td>Field voltage (V)</td>
<td>236</td>
<td>266</td>
</tr>
<tr>
<td>Field current (A)</td>
<td>386</td>
<td>395</td>
</tr>
<tr>
<td>Load angle (Elec. deg)</td>
<td>32.5</td>
<td>38.36</td>
</tr>
</tbody>
</table>
### Table 5.3

**Comparisons of Site-Test and Computed Results for Symmetrical Short-Circuit Test.**

<table>
<thead>
<tr>
<th>System quantities</th>
<th>Initial readings</th>
<th>Final readings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test</td>
<td>Computed</td>
</tr>
<tr>
<td></td>
<td>Variable inductance</td>
<td>Constant inductance</td>
</tr>
<tr>
<td>Load (MW)</td>
<td>29.76</td>
<td>30.0</td>
</tr>
<tr>
<td>Load(MVAR)</td>
<td>22.6</td>
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</tr>
<tr>
<td>Stator voltage (kV)</td>
<td>13.05</td>
<td>13.04</td>
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<tr>
<td>Stator current (kA)</td>
<td>-</td>
<td>1.65</td>
</tr>
<tr>
<td>Field voltage (V)</td>
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</tr>
<tr>
<td>Field current (A)</td>
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</tr>
<tr>
<td>Load angle (Elec.deg)</td>
<td>36.0</td>
<td>35.5</td>
</tr>
</tbody>
</table>
Fig. 5.1 Schematic diagram of a steam turbo-alternator system

Fig. 5.2 Goldington generator open-circuit characteristic
Fig 5.3 Block diagram of the excitation control system
Fig. 5.4  Straight-line representation of operating characteristics of magnetic amplifiers and exciter

Fig. 5.5  Block diagram of the prime mover and its speed governor system
Fig. 5.6 Generator response to step demand in generator line voltage from 12 to 10 kV
Fig. 5.7 Generator response to 30MW load rejection test
Fig. 5.8 Generator speed variations following the 30MW load rejection.
Fig. 5.9a Generator transients following 3-phase short-circuit fault at full load
Fig. 5.9b Generator transients following 3-phase short-circuit fault at full load
Fig. 5.10 Effect of transmission system resistance on the rotor angle swing

Fig. 5.11 Comparison of computer predictions with those of reference 3
CHAPTER 6

COMPUTER REPRESENTATION

OF

A GAS-TURBINE-GENERATOR

EXCITATION SYSTEM
CHAPTER 6
COMPUTER REPRESENTATION OF A GAS-
TURBINE-GENERATOR EXCITATION SYSTEM

With the advent of high power semi-conductor devices, generator excitation systems have developed from schemes based on d.c. commutator machines and electro-mechanical automatic voltage regulators (a.v.r's) to installations employing static thyristor exciters of several megawatts capacity.

In parallel with the development of components for the excitation system, the whole area of turbine-generator control technology has received considerable attention. Thus, the problem of the dynamic stability of the overall unit when the excitation is controlled has been extensively explored \(^56-60\). The use of supplementary signals in the excitation controller to improve system stability has been demonstrated in a number of theoretical and practical applications \(^61-66\). However, all studies so far undertaken are related to excitation systems for either steam-turbine or hydro-electric systems.

In the present study, emphasis is placed on excitation systems for gas-turbine generators, with the specific requirements for this type of system being discussed. A mathematical model for a typical brushless excitation system for a 15 MW unit is developed, with the validity of the model being assessed by comparison with site-test results.

6.1 - General requirements for gas-turbine-generator excitation system

The excitation system for a gas-turbine-driven generator is basically the same as that for a conventional turbo-generator. Nevertheless, since gas-turbine generators commonly operate on a weak supply system, the effectiveness of their excitation system in maintaining system stability following a system disturbance is more
significant than that of generators operating on an infinite grid. Some
gas-turbine generators, although installed normally for peak-load
generation, require also to be capable of isolated operation in case of
emergency or, on occasions, as cranking units.

Direct-on-line starting of large induction motors on gas-
turbine generators is often required, and it is common practice for a 3
to 5 MW induction motor to be started directly from a 10 to 15 MW
generator. Fast-excitation response and high field-forcing levels are
therefore required to restrain the voltage depression and recovery time
to within their specified limits. In some cases, generators with lower
than normal reactances are required to reduce the maximum voltage dip
(i.e. by minimising the initial voltage drop across the generator
transient or sub-transient reactance).

6.1.1 - Brushless excitation scheme

Various types of excitation system are used with gas-turbine
generators, but a brushless scheme employing silicon diodes is by far
the most widely adopted method. The elimination of commutators, slip
rings and any associated brushgear leads to an excitation system which
is reliable, compact, requires minimum attendance and, most important
of all, is free from electrical sparking. Those characteristics are
especially favourable in gas-turbine applications, since such units are
commonly operated on sites which are remote, unattended and contami-
nated with inflammable gas.

6.1.2 - Specific requirements for the excitation controller

In certain circumstances, additional excitation controller
features, not essential in a conventional system, may be incorporated in
a gas-turbine/generator system. These features are discussed below.

6.1.2.1 - Quadrature current compensation

Quadrature current compensation is commonly embodied in an
a.v.r. to ensure even reactive load sharing between
parallel-connected generators. Gas-turbine generators are
commonly paralleled, with virtually no reactance between corresponding terminals. Without any form of reactive power sharing control, even a relatively small mismatch of individual a.v.r's may result in overloading of some of the units. However, the compensation is not essential for steam- or hydro-turbine/generator systems, since paralleling is normally on the high-voltage side of the generator transformers.

6.1.2.2 - Overfluxing control

Gas-turbine plants, especially those installed for peak-load generation or standby power supply, are normally specified to be capable of synchronizing and operating at a reduced system frequency. To avoid overfluxing of the main generators and their associated transformers, in the event of sustained emergency operation at reduced frequency, maximum volts-per-hertz excitation limits are incorporated in their excitation systems. This limiting circuit is designed to give a precise rate of fall of generator voltage with frequency, as the latter falls below the operating limit (normally set at 95% of the rated frequency).

In a limited capacity power system, a suddenly applied load may cause the system frequency to fall to an unacceptable level. However, the effect of the overfluxing control may ease the situation by suppressing the generator terminal voltage in accordance with the generator frequency, which in turn reduces the effective load on the generator.

A volts-per-hertz relationship is not essential for generator run-up, when overfluxing can be avoided by switching on the excitation at near synchronous speed. However, its incorporation into the excitation controller allows the excitation system to be energised at a lower generator speed, and a smooth and fast build-up of generator voltage can hence be achieved. The resulting reduction in overall run-up time becomes significant when the generators are used for peak-load lopping and emergency standby purposes.
The brushless excitation system involved in the present investigation is shown schematically in fig. 6.1. As shown in fig. 6.2, the pilot exciter, main exciter and diode wheel form a self-contained unit which overhangs at the non-drive end of the generator. The permanent-magnet pilot exciter is a 20-pole rotating-field machine, while the main exciter is a 6-pole rotating-armature machine. The diodes are connected in a 3-phase bridge configuration with each bridge arm having two series-connected diodes.

The associated a.v.r. responds to any deviation of generator voltage from a zener-stabilised reference setting. The resulting error signal is amplified to control the firing pulses for the thyristor bridge at the power output stage, which in turn controls the excitation to the main exciter. A schematic diagram of the a.v.r. is shown in fig. 6.3.

A mathematical representation of the a.v.r. is initially derived using the component model approach. However, in order to adopt a universal approach to the representation of excitation system, the model so described is modified and coupled with an exciter model to confine with the IEEE format 12, 67. Such a computer representation of excitation systems has been widely adopted and has formed a consistent frame of reference whereby manufacturers can respond to a user's request for excitation system data.

6.2.1 - Component model of a.v.r.

Under the assumptions that the overfluxing control is inoperative and that the quadrature current compensation is set to zero, the mathematical representation of the a.v.r. shown in fig. 6.3 is derived as below.
6.2.1.1 - Input filter

The input smoothing circuit is assumed to have unity gain and a single time constant $T_1$. 

\[ \frac{E_{RR}}{V_t - V_{REF}} = \frac{1}{1+sT_1} \]  

6.2.1.2 - Amplifier circuit

The transfer function for the amplifier circuit shown in fig. 6.4 is

\[ \frac{V_A}{E_{RR}} = K_1 \frac{(1+sT_2)(1+sT_3)}{(1+sT_4)(1+sT_5)} \]  

the derivation of which is detailed in Appendix I.

6.2.1.3 - Firing control circuit

The steady-state characteristic of this circuit is given in fig. 6.5 and the firing angle $\alpha$ can be expressed as

\[ \alpha = - \frac{K_{FA}}{(1+sT_6)} V_A + B_{FA} \]  

The time constant $T_6$ represents the combined inherent time delay in both the firing control and the thyristor bridge circuit. It may be calculated approximately from

\[ T_6 = \frac{1}{2T_f} \]  

where $f$ is the input frequency to the thyristor bridge.
6.2.1.4 - Thyristor bridge

The output characteristic of a half-controlled single-phase thyristor bridge is well-known and the output voltage \( V_R \) expressed in terms of the firing angle \( \alpha \) is

\[
V_R = \frac{V_{\text{MAX}}}{\pi} (1+\cos \alpha)
\]

The peak value of the input voltage \( V_{\text{MAX}} \) is determined by the field forcing requirement (i.e. \( V_{R\text{MAX}} \)). On rearranging eqn. 6.5

\[
V_{\text{MAX}} = V_{R\text{MAX}} \frac{\pi}{1 + \cos \alpha_{\text{MIN}}}
\]

so that, if

\[
\alpha_{\text{MIN}} = 15^\circ
\]

then

\[
V_{\text{MAX}} = 1.598 V_{R\text{MAX}}
\]

6.2.2 - Exciter and diode bridge representation

The exciter and rotating diode bridge are assumed to have unity gain and a single time constant. The non-linear function \( S_E \) shown in fig. 6.6, multiplied by the generator field voltage \( E_{FD} \), represents the increased excitation requirement caused by exciter saturation and armature reaction. Fig. 6.7 illustrates how \( S_E \) is determined from the exciter open-circuit and load curves. It is suggested that \( S_E \) should be specified at two different exciter voltages, normally field forcing level \( E_{FD\text{MAX}} \) and 0.75 \( E_{FD\text{MAX}} \).

6.2.3 - IEEE representation

Fig. 6.6 shows the IEEE type 2 computer representation for an excitation system containing rotating rectifiers. To obtain such a representation, the a.v.r. model developed in Section 6.2.1 is modified as below.
The firing control circuit and the thyristor bridge shown in fig. 6.3 are assumed to be linear, with a gain of $K_2$ and a single time constant $T_6$. The resulting transfer function of these circuits is combined with that of the a.v.r. amplifier to form

$$\frac{V_R}{V_{ERR}} = \frac{K_1 K_2}{(1+sT_4)(1+sT_5)(1+sT_6)}$$

An equivalent transfer function, derived from the IEEE representation, is given by

$$\frac{V_R}{V_{ERR}} = \frac{K_A (1+sT_{F1})(1+sT_{F2})}{(1+sT_A)(1+sT_{F1})(1+sT_{F2})+sK_A K_F}$$

By comparing eqns. 6.9 and 6.10, expressions relating the fictitious constants of the IEEE representation and the available system parameters may be derived as

$$K_A = K_1 K_2$$
$$T_{F1} = T_2$$
$$T_{F2} = T_3$$
$$T_A + T_{F1} + T_{F2} + K_A K_F = T_4 + T_5 + T_6$$
$$T_A(T_{F1} + T_{F2}) + T_{F1}T_{F2} = T_6(T_4+T_5)+T_4 T_5$$
$$T_A T_{F1} T_{F2} = T_4 T_5 T_6$$

Although no unique solution exists for $T_A$ and $K_F$, the value of $T_A$ is known to be negligible compared with $K_F$, so that to a close approximation

$$T_A = T_6$$
$$K_F = \frac{1}{K_A} \left( T_4 + T_5 - T_{F1} - T_{F2} \right)$$

The resulting state-space equation for the excitation system, in the IEEE format, is given in eqn. 6.19.
\[ \begin{array}{c|c|c|c|c|c|c} 
\cdot x_1 & - \frac{1}{T_R} & \cdot & \cdot \cdot \cdot & \cdot & \cdot \cdot \cdot & \cdot \cdot \cdot \cdot \cdot \\
\cdot x_2 & - \frac{K_A}{T_A} & - \frac{1}{T_A} & \cdot & \cdot \cdot \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \cdot \cdot \cdot \cdot \\
\cdot x_3 & \cdot & \cdot \frac{1}{T_E} & - \frac{S_E + K_E}{T_E} & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \cdot \cdot \cdot \cdot \\
\cdot x_4 & \cdot \cdot \cdot \cdot \cdot & \cdot \cdot \frac{1}{\omega_o} & - \frac{1}{T_{F1}} & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \cdot \cdot \cdot \cdot \\
\cdot x_5 & \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot & \cdot \cdot \frac{\omega_o K_F}{T_{F1} T_{F2}} & \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot & \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot & \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\
\end{array} \]

where, \( VCF_{VR} \) is a clamping function for \( V_R \)
6.3 - Excitation system step-response tests

Step changes of excitation were carried out on a typical 15 MW gas-turbine generator, running on open-circuit and at rated speed, by applying step changes to the a.v.r. reference voltage.

Using the computer model developed above, changes of +10% and -10% in the generator terminal voltage were simulated, and comparisons with test results are made in figs. 6.8 and 6.9 respectively. The corresponding generator and excitation system data are given in tables 3.1 and 6.1. Derivation of the excitation system data is detailed in Appendix J.

Fig. 6.8 shows that both the measured and predicted a.v.r. outputs increase to ceiling level almost instantaneously after the application of the +10% step change. Following a field forcing duration of 0.12 s, both voltages reduce sharply and the measured output is in fact suppressed to zero for a period of 0.15 s, while the predicted output does not reach zero level at all. The predicted generator voltage overshoot is consequently higher than that obtained by test. However, the general agreement between test and predicted result is good.

Following the 10% step-down change, the measured generator voltage, shown in fig. 6.9, decays faster than the model predicts and this results in the delayed recovery of the computed a.v.r. output. The discrepancy is thought to be caused mainly by a lack of precision of the exciter time constant used in the simulation.

It is understood that the effective time constant of the exciter varies, depending on its loading condition. It is a maximum, approximately equal to the exciter field time constant, when the exciter is on no-load, and a minimum when the exciter is on short-circuit. For any other load condition, its value lies somewhere between these two extremes.

For the present study, the exciter time constant $T_E$ is taken as the open-circuit time constant. However, due to the non-linear feedback function $S_E$, the effective exciter time constant becomes
With the exciter being loaded by the corresponding generator field circuit, the measured exciter load characteristic is given in fig. 6.10. Since this load curve is approximately linear throughout the operating range, $S_E$ is assumed constant. At the calculated value of 2.21 (see Appendix I) and with $K_E$ at unity, the effective exciter time constant becomes $T_E/3.21$. Fig. 6.11 illustrates the effect of $T_E$ on the system response.

During the commissioning stage of a gas-turbine generator, excitation step-response tests are used to optimise the values of the a.v.r. amplifier lead-lag compensation components (fig. 6.4). This optimisation is normally carried out on a trial and error basis, which is clearly both time and fuel consuming. However, using the present model, a set of optimised components could be determined and used as the starting values for the test. The effect of the a.v.r. lead-lag compensation circuit capacitors $C_1$ and $C_2$ on the system response is clearly demonstrated in figs. 6.12 and 6.13.
TABLE 6.1

EXCITATION SYSTEM DATA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
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</tr>
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<td>$K_F$</td>
<td>0.018</td>
</tr>
<tr>
<td>$T_R$</td>
<td>0.02 s</td>
</tr>
<tr>
<td>$T_A$</td>
<td>0.0017 s</td>
</tr>
<tr>
<td>$T_E$</td>
<td>1.02 s</td>
</tr>
<tr>
<td>$T_{F1}$</td>
<td>1.632 s</td>
</tr>
<tr>
<td>$T_{F2}$</td>
<td>0.232 s</td>
</tr>
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<td>$V_{RMAX}$</td>
<td>20.0 p.u.</td>
</tr>
<tr>
<td>$V_{RMIN}$</td>
<td>0.0 p.u.</td>
</tr>
<tr>
<td>$S_{EMAX}$</td>
<td>2.21</td>
</tr>
<tr>
<td>$S_{EMIN}$</td>
<td>2.21</td>
</tr>
</tbody>
</table>
永久磁铁

A.V.R.

驱动信号

原动机

Fig. 6.1 Schematic diagram of a brushless excitation scheme
Fig. 6.2 Overhung brushless rotor
Fig. 6.3 Schematic diagram of an automatic voltage regulator
Fig. 6.4 A.V.R. amplifier circuit

Fig. 6.5 Firing control circuit characteristic
Fig 6.6 IEEE type 2 excitation system representation
Fig. 6.7 Determination of $S_E$
Fig. 6.8 Generator response to 10% step-up demand in stator voltage
Fig. 6.9 Generator response to 10% step-down demand in stator voltage
Fig. 6.10 Exciter characteristic
Fig.6.11 Effect of $T_E$ on generator response
Fig 6.12 Effect of $C_1$ on generator response
Fig. 6.13 Effect of $C_2$ on generator response
CHAPTER 7

SIMULATION

OF

GAS-TURBINE DYNAMIC PERFORMANCE
CHAPTER 7

SIMULATION OF GAS-TURBINE DYNAMIC PERFORMANCE

During the last thirty or so years, the gas turbine has developed from a rather primitive device of low thermal efficiency to a highly sophisticated system of greatly improved performance. The major developments in gas-turbine technology invariably originated in the aero-engine field, although much of this work has subsequently been utilised in applications such as electrical power generation, oil and natural gas pumping, and automotive, railway and marine propulsion.

The improvements obtained in thermodynamic performance have resulted from continuous advances in both aerodynamics and metallurgy, but have usually arisen at the expense of an increased engine complexity and a considerable development cost. The provision of suitable control systems for complex gas-turbine installations is of prime importance, yet designing such control systems with modern control design techniques, which are entirely mathematical, requires an extensive knowledge of the engine transient behaviour. However, this cannot be obtained experimentally until the development program is well advanced and the mechanical integrity of the engine has been established. A satisfactory mathematical model of the engine dynamic would undoubtedly provide insight into the engine response problem and allow the control system to be developed simultaneously with the engine. The judicious use of such a model could permit control schedules to be investigated with minimum test-bed running and without endangering the engine. Since it is common for customers to demand guarantees on engine response rate at the proposal stage, an accurate model is of considerable importance.
7.1 - Review of previous work

Considerable work, both theoretical and experimental, has been carried out on gas-turbine dynamics. However, for security reasons, it is difficult to obtain access to much of this unless it has been declassified.

The steady-state performance of gas-turbine has been well understood for some years. Gas-turbine dynamic studies began in the early 1950s, when the twin-spool engine was introduced. Much of this work was pioneered by NACA and their theoretical approach was to assume the gas-turbine to be a linear system and to devise equations for small perturbations about an equilibrium point.

The importance of the ability to predict engine dynamics from steady-state data was soon realised and work was concentrated on methods of calculating the rotor inertia time constant. Among these methods, the one developed by the Lucas company has been widely adopted.

In the mid-fifties, consideration was given to the simulation of the gas-turbine as a non-linear system, using the flow and work compatibility approach. With such an approach, Larrowe and Spencer developed an analogue computer simulation of gas-turbine engines using the characteristics of the engine components (e.g. compressor, combustion chamber and turbine). The development of the component model was a great improvement in gas-turbine simulation, because the prediction of engine dynamics at the design stage, for both large and small perturbations, became possible. Problems such as that of achieving a surge-free acceleration could also be studied before any engine test run.

A major difficulty with early analogue work was the representation of the compressor characteristic. The earliest approach to this used an electro-mechanical map reader, which was unreliable, inaccurate and slow to respond. Saravanamutto overcame this by using single-function generators to represent families of similar curves; a function of two variables being represented by three functions of a single variable.
With the advent of the digital computer, more emphasis was placed on digital simulation. Using both analogue and digital approaches, methods for improving the dynamic response of a twin-spool engine were proposed and were verified experimentally with good accuracy.

7.2 - Gas-turbine simulation techniques

Gas-turbine models can be classified according to the different possible engine types, for example, single-spool, twin-spool or 3-spool, etc. Nevertheless, from the standpoint of mathematical modelling, the breakdown is not very important, since the same modelling principles are in general applicable regardless of the type of engine.

For transient studies, any gas-turbine model must include, as a minimum, the dynamic equation of the rotor. In addition to this, the model may also include aerothermodynamics, combustion phenomena, heat-soak characteristics and many other transient considerations. A model including all the dynamic processes existing in a given system is, of course, the most exact description possible, although it is naturally extremely complex. As usual, a decision to include or neglect the non-sophisticated dynamics depends to a large extent on the accuracy required. It is therefore a task for research workers to optimise the models, such that it is sufficiently simple to be practical but accurate enough to be useful.

7.2.1 - Component model

A gas turbine comprises a number of components; with the behaviour of each of these being individually well-understood, the relationship between the components is fixed by the physical layout of the engine and by the thermodynamic behaviour of each engine component. Thus, when all the component characteristics and engine layout are known, a gas turbine is precisely defined and its dynamic behaviour can be expressed mathematically.

In the digital simulation of a gas turbine, the first requirement is the selection of a set of engine parameters which permit
the operation of each engine component to be determined, provided that the externally controlled variables are specified. This set of parameters forms the engine state vector and the number of parameters required depends on the complexity of the engine.

If the state vector and the given control variables correspond to a steady-state operating point, the conditions of flow and work compatibility are obtained. However, if not, there will be a net torque on the compressor/turbine assembly and an acceleration of the mass at some point in the engine. The difference between the turbine torque and the compressor torque can be used to find the rotor acceleration, while the mass flow discrepancy between two adjacent components can be used to compute the rate of pressure rise at the intercomponent volume (the mass storage capacity of the engine is assumed to be concentrated between the engine components). The latter is achieved by assuming a value for the intercomponent volume and applying the gas laws.

The component approach to gas-turbine simulation allows a clear insight into its dynamic behaviour, and the simulation is valid over the whole engine operating range. Nevertheless the resulting model is complex and the computer storage required for the gas-turbine characteristics, including double-variable functions (e.g. the compressor characteristic), is considerable. This situation becomes even more important when the technique is applied to multi-spool engines.

7.2.2 - Linear model

A gas turbine can be represented by a linear model with constant coefficients. This concept is based on the fact that an engine can be represented as a collection of multivariable functions, which can be linearised by writing them in total differential form. The major limitation to a linearised model is, of course, the limited range of excursion from the base point which is permissible without the introduction of excessive error. However, the method can be extended to cover larger engine speed variations by scheduling the engine parameters against speed, and such an approach has been extensively used in gas-turbine control studies.
7.3 - Gas-turbine arrangement

The gas turbine simulated in the present study is a Rolls-Royce Avon gas generator driving a free power turbine. This type of gas generator, developed from the Avon single-shaft turbo-jet aero-engine, has a 17-stage axial compressor, a 3-stage turbine and a combustion chamber of the turbo-annular type containing eight straight-flow flame tubes.

In the twin-shaft gas-turbine arrangement of fig. 7.1, the high-pressure turbine drives the compressor, with the combination acting as a gas generator for the low-pressure free power turbine. This arrangement gives the advantage of a large power output with a relatively simple mechanical arrangement and low capital cost. Due to its relatively low rotating inertia, this type of gas generator gives the overall unit a rapid transient power response and, with a suitably designed free power turbine, the unit is capable of a quick start to full power from cold (typically 2 mins for 15 MW). A disadvantage, however, is that a shedding of electrical load can lead to excessive overspeeding of the power turbine, and the control system must be designed to minimise this adverse effect.

7.4 - Gas-turbine mathematical model

Since the gas-turbine model forms only a small part of a complex gas-turbine/generator model, and to confine the computer program to an amenable size, a piecewise linear representation for such an engine is employed for the present investigation. Details of its derivation are given in the following section.

7.4.1 - Basic dynamics of gas-turbine engines

The development of linearised equations describing the dynamic response of a gas turbine is based on the hypothesis that transient thermodynamic and flow processes may be considered as quasi-static, that is these processes continuously progress from one equilibrium state to another along an equilibrium curve. This assumption permits functional relationships to be written between the different input and output variables.
In a gas-turbine engine, the fuel flow \( Q_F \) is a true independent variable with respect to the engine, whilst the engine speed \( N_G \) is an independent variable with respect to the thermodynamic cycle but a dependent variable in the inertia/speed relationship. The dependent variables chosen for the present investigation are the compressor discharge pressure \( P_2 \), the exhaust gas pressure \( P_4 \), the exhaust gas temperature \( T_4 \) and the exhaust gas power \( EGP \). For equilibrium running, with specified engine geometry and inlet conditions, each engine performance parameter can be defined as a function of the fuel flow \( Q_F \). Under transient conditions this is no longer true, but the quasi-static principle enables each of these parameters to be related to the fuel flow \( Q_F \) and the other independent variable which is the engine speed \( N_G \). With the extra variable, the departure of the engine parameters from their steady-state line can be defined.

If the engine output torque \( T_Q \) is expressed as

\[
T_Q = f(Q_F, N_G)
\]  

Then, to a linear approximation

\[
\Delta T_Q = \frac{\partial T_Q}{\partial Q_F} \Delta Q_F + \frac{\partial T_Q}{\partial N_G} \Delta N_G
\]

where \( T_{Q0} \) is the initial steady-state torque.

Also,

\[
\Delta T_Q = J \Delta N_G
\]

so that,

\[
\Delta N_G = \mu_{NG} \left( \frac{1}{1 + sT_G} \right) \Delta Q_F
\]

where

\[
T_G = \frac{J}{\frac{\partial T_Q}{\partial N_G} T_{Q0}}, \text{ the engine time constant}
\]

and

\[
\mu_{NG} = \frac{\partial T_Q}{\partial Q_F} T_{Q0}, \text{ the gain or sensitivity of the speed to fuel flow}
\]
For small deviations, the expression for \( \mu_{NG} \) becomes
\[
\mu_{NG} = \left. \frac{\partial N_G}{\partial Q_F} \right|_{T_{Q0}} \quad 7.8
\]

Thus, for small perturbations, the engine speed response to a change of fuel flow is completely defined by the engine inertia time constant \( T_G \) and the relative speed gain \( \mu_{NG} \). The evaluation of \( \mu_{NG} \), which is \( \frac{\partial N_G}{\partial Q_F} \) at a steady operating point, does not present any difficulty, as it can be found directly from the local slope of the \( N_G \) versus \( Q_F \) steady-state running line (fig. 7.2b). It only then remains to derive the engine inertia time constant. Although several different methods\(^{72, 75-77} \) are available for obtaining this, it was in fact obtained, in the present study, from a test curve relating the variation of \( T_G \) to \( N_G \) (fig. 7.3a).

The transfer functions for the other output variables, such as \( P_2, P_4, T_4 \) and EGP are derived as follows:
\[
\Delta P_2 = \frac{\partial P_2}{\partial Q_F} \bigg|_{P_{20}} \cdot \Delta Q_F + \frac{\partial P_2}{\partial N_G} \bigg|_{P_{20}} \cdot \Delta N_G \quad 7.9
\]

Substituting \( \Delta N_G \) from eqn. 7.5,
\[
\Delta P_2 = \left[ \frac{\partial P_2}{\partial Q_F} \bigg|_{P_{20}} + \frac{\partial P_2}{\partial N_G} \bigg|_{P_{20}} \mu_{NG} \frac{1}{(1 + s T_G)} \right] \Delta Q_F \quad 7.10
\]
or,
\[
\Delta P_2 = \mu_{P2} \frac{1 + s \frac{aP_2 T_G}{\mu_{P2}}}{(1 + s T_G)} \cdot \Delta Q_F \quad 7.11
\]

where,
\[
\mu_{P2} = \mu_{NG} \frac{\partial P_2}{\partial N_G} \bigg|_{P_{20}} + \frac{\partial P_2}{\partial Q_F} \bigg|_{P_{20}} \quad 7.12
\]

\[
aP_2 = \frac{\partial P_2}{\partial Q_F} \bigg|_{P_{20}} / \mu_{P2} \quad 7.13
\]
Similarly, the transfer characteristics of $P_4$, $T_4$ and EGP are respectively

$$\Delta P_4 = \mu_{P_4} \left( \frac{1 + s a_{P_4} T_G}{1 + s T_G} \right) \Delta Q_F \quad 7.14$$

$$\Delta T_4 = \mu_{T_4} \left( \frac{1 + s a_{T_4} T_G}{1 + s T_G} \right) \Delta Q_F \quad 7.15$$

$$\Delta EGP = \mu_{EGP} \left( \frac{1 + s a_{EGP} T_G}{1 + s T_G} \right) \Delta Q_F \quad 7.16$$

Expressions for $\mu_{P_4}$, $a_{P_4}$, $\mu_{T_4}$, $a_{T_4}$, $\mu_{EGP}$ and $a_{EGP}$ are all similar to those for $P_2$.

The above equations show that the transfer characteristics of all the dependent variables can be expressed in the following general form.

$$\Delta x = \mu_x \left( \frac{1 + s a_x T_G}{1 + s T_G} \right) \Delta Q_F \quad 7.17$$

If the engine is subjected to a step increase of fuel flow $q_F$, the response of the dependent variables, expressed in the time domain, is therefore

$$x = \mu_x q_F \left[ 1 - (1 - a_x) e^{-\frac{t}{T_G}} \right] \quad 7.18$$

which shows that the response is an initial step change of $\mu_x a_x q_F$ followed by an exponential rise with a time constant $T_G$. The initial change is attributable to the instantaneous heat effect of the abrupt fuel change, as distinct from the later change due to the ensuing speed variation.

To allow for the packing lags of the combustion chamber and the exhaust duct, the gas-turbine dynamic equations derived above can be modified to include the time constants $T_C$ (combustion chamber) and
\[ \Delta N_G = \mu_{NG} \frac{1}{(1 + s^{T_G})(1 + s^{T_C})} \Delta Q_F \]  
7.19

\[ \Delta P_2 = \mu_{P2} \frac{(1 + s a_{P2} T_G)}{(1 + s^{T_G})(1 + s^{T_C})} \Delta Q_F \]  
7.20

\[ \Delta P_4 = \mu_{P4} \frac{(1 + s a_{P4} T_G)}{(1 + s^{T_G})(1 + s^{T_C})(1 + s^{T_D})} \Delta Q_F \]  
7.21

\[ \Delta T_4 = \mu_{T4} \frac{(1 + s a_{T4} T_G)}{(1 + s^{T_G})(1 + s^{T_C})(1 + s^{T_D})} \Delta Q_F \]  
7.22

\[ \Delta EGP = \mu_{EGP} \frac{(1 + s a_{EGP} T_G)}{(1 + s^{T_G})(1 + s^{T_C})(1 + s^{T_D})} \Delta Q_F \]  
7.23

7.4.2 - Digital simulation program

The linearised gas-turbine-model equations (i.e. eqns. 7.19-7.23) can be represented by the block diagram shown in fig. 7.4, the equation for which can be expressed in state-variable form as,

\[
\begin{align*}
\dot{x}_1 &= -\frac{1}{T_G} x_1 + \frac{1}{T_C} x_2 + \frac{1}{T_D} x_3 + \frac{1}{T_G} \Delta Q_F \\
\dot{x}_2 &= \frac{1}{T_C} x_1 - \frac{1}{T_C} x_2 \\
\dot{x}_3 &= \frac{1}{T_D} x_2 - \frac{1}{T_D} x_3
\end{align*}
\]

7.24
\[ \Delta N_G = \mu_{NG} \cdot x_2 \]
\[ \Delta P_2 = \mu_{P2} \left( x_2 + a_{P2} T_G \cdot x_2 \right) \]
\[ \Delta P_4 = \mu_{P4} \left( x_3 + a_{P4} T_G \cdot x_3 \right) \]
\[ \Delta T_4 = \mu_{T4} \left( x_3 + a_{T4} T_G \cdot x_3 \right) \]
\[ \Delta EGP = \mu_{EGP} \left( x_3 + a_{EGP} T_G \cdot x_3 \right) \]

\[ \Delta Q_F = Q_F - Q_{F0} \]
\[ N_G = N_{G0} + \Delta N_G \]
\[ P_2 = P_{20} + \Delta P_2 \]
\[ P_4 = P_{40} + \Delta P_4 \]
\[ T_4 = T_{40} + \Delta T_4 \]
\[ EGP = EGP_0 + \Delta EGP \]

where, \( P_{20}, P_{40}, T_{40}, EGP_0 \) and \( Q_{F0} \) are the initial steady-state values.

Since the variation of the engine variables in the above equations are based on deviations from their corresponding initial steady-state values (i.e. \( P_{20}, P_{40}, Q_{F0} \) etc.), the simulation study is restricted to small disturbances. To improve the accuracy of the model for larger disturbances, the equations for the dependent variables are modified to the following forms.
\[ P_2 = P_{2\text{NG}} + a_{P2} \mu_{P2} (Q_F - Q_{F\text{NG}}) \]

\[ P_4 = P_{4\text{NG}} + a_{P4} \mu_{P4} (Q_F - Q_{F\text{NG}}) \]

\[ T_4 = T_{4\text{NG}} + a_{T4} \mu_{T4} (Q_F - Q_{F\text{NG}}) \]

\[ EGP = EGP_{\text{NG}} + a_{EGP} \mu_{EGP} (Q_F - Q_{F\text{NG}}) \] 7.27

where \( P_{2\text{NG}}, P_{4\text{NG}}, T_{4\text{NG}}, EGP_{\text{NG}} \) and \( Q_{F\text{NG}} \) are functions of engine speed.

The output shaft power, developed by the expansion of the gas generator exhaust gas through the power turbine can be expressed as

\[ P_T = n_T. EGP \] 7.28

The term \( n_T \), defined as the isentropic efficiency of the power turbine, can be expressed as a function of \( N_T/C \), where \( N_T \) is the power turbine speed and \( C \) the gas velocity resulting from the expansion of gas through the power turbine, and calculated from

\[ C = \sqrt{2 \cdot 1000 \cdot C_p \cdot T_{45}} \] 7.29

where \( T_{45} \) is the temperature drop across the turbine resulting from an isentropic expansion from \( P_4 \) to \( P_5 \)

or, \( T_{45} = T_4 [1 - (P_5/P_4) (\gamma - 1)/\gamma] \) 7.30

where \( \gamma \) is the ratio of specific heats.

Since the time constants, lead/lag ratios and steady-state gas turbine parameters are engine-speed dependent, while \( n_T \) is a function of \( N_T/C \), the computer program is written to store these characteristics and to enable the parameters to be updated throughout the simulation period.
A least-square cubic-spline curve-fitting technique is used in the computer program to fit curves to the input data which are obtained from the given engine characteristics. The resulting fitted engine characteristic curves used for the present study are shown in figs. 7.2, 7.3, 7.5 and 7.6.

The cubic-spline consists of a number of cubic polynomial segments joined end to end with continuity in the first and second derivatives at the joints. The third derivative is in general discontinuous. The capability of handling discontinuous curves is especially useful in the present application, because some gas-turbine characteristics have fast changing slopes or even discontinuities. The routine not only evaluates the function of the cubic-spline, but also the first three derivatives at any prescribed point on the x-axis. The gains, which are defined as the slope of the gas-turbine steady-state operating curves, can hence be evaluated and are given in fig. 7.7.

The computer program is arranged such that, at the end of each integration step, the time constants and lead/lag ratios corresponding to the computed engine speed \( N_G \) are obtained from the fitted curves shown in fig. 7.3. However, the corresponding values of the gas-turbine dependent variables and gains are not readily available, since as shown in figs. 7.2 and 7.7, these parameters are plotted against fuel flow rather than engine speed. Consequently an additional steady-state operating characteristic relating the steady-state fuel flow \( Q_F \) to the computed engine speed \( N_G \) and shown in fig. 7.5 is required. The fuel flow \( Q_{FNG} \), corresponding to the computed engine speed \( N_G \), can then be applied to figs. 7.2 and 7.7 to obtain the required parameters.

Finally, with the updated values of \( T_4', P_4 \) and \( N_T \), the ratio \( N_T/C \) is computed and the corresponding power turbine isentropic efficiency \( \eta_T \) is obtained from the fitted curve of fig. 7.6. Hence the output shaft power of the turbine can be evaluated using eqn. 7.28.
Fig. 7.1 Gas-turbine engine layout and station numbering
Fig. 7.2 Gas-turbine steady-state operation curves
Fig. 7.3 Gas-turbine time constants and lead-lag ratios
Fig. 7.4 Linear gas-turbine model
Fig 7.5 Fuel flow versus engine speed characteristic

Fig 7.6 Power turbine isentropic efficiency characteristic
Fig. 7.7 Computed curves for gains versus fuel flow
CHAPTER 8

GAS-TURBINE CONTROL SYSTEM
CHAPTER 8  
GAS-TURBINE CONTROL SYSTEM

Gas-turbine control can be achieved by hydraulic, pneumatic, mechanical, electrical or electronic means, or by a suitable combination of any of these. The fuel control system of an aero-type industrial gas turbine can be a standard aircraft engine hydromechanical control unit if liquid fuel is used. Alternatively, a fuel control system specially designed for industrial application can be employed.

With the increasing availability of electronic integrated circuits and the corresponding substantial improvement in reliability, the development of gas-turbine control has been in the direction of replacing mechanical parts by electrical and electronic components. In parallel with this has been the development of pumps and control valves to work with fuels of low lubricity and of a corrosive nature.

The general aspects of gas-turbine control are discussed in this chapter and are followed by a general description of the governor control system for the engine to be investigated.

8.1 - Basic control system requirements

Land and sea based gas-turbine engines are often operated from remote stations, or, in the limit, are completely automated. An integrated control strategy is therefore required, to embrace the pre-start checks, automatic engine start-up and shut-down sequences, load regulating, together with a fully-automated supervisory and surveillance system. For electrical power generation, additional facilities such as automatic synchronisation, load sharing and load shedding are required. These requirements have to be met whether the engine is started or run on liquid or gaseous fuel and they may call for automatic reversion from one fuel to another.
Several aspects of gas-turbine fuel control, with particular reference to electrical power generation, are discussed in this section. Although the control philosophy is generally directed towards land or sea based engines, it can usually be applied with minor modification to aircraft gas turbines.

8.1.1 - Engine starting

Starting a gas turbine involves a sequence of steps to bring the engine from rest to its idling speed. The engine is first cranked by a starter (e.g. an electric motor) to a preset speed, before admitting a prescribed level of fuel flow to the engine for ignition. The successful ignition of fuel is followed by an increase in fuel flow, in accordance with the starting fuel schedule. The choice of this schedule depends upon the engine requirements, but the flexibility of an electronic controller enables a choice of, for instance, scheduling fuel flow versus engine speed or a ramp of fuel flow with time to be used. As the gas generator reaches a speed at which the compressor discharge pressure (C.D.P.) is sufficiently high to become a reliable control parameter, the starting fuel schedule becomes redundant.

8.1.2 - Acceleration fuel schedule

The life expectancy of a gas turbine is influenced as much by conditions during starting and acceleration as during continuous operation. Sharp temperature gradients within the engine, even at low temperature, can be more harmful than continuous operation at higher temperature.

An ideal acceleration control is one that allows the engine to accelerate at a reasonably fast rate, without the engine being driven into the surge region or overheating the engine components. The inability to detect impending stall has led to the almost universal adoption of acceleration by a fuel scheduling method. By judicious selection of the acceleration fuel schedule, considerable improvements in engine transient response can be made.

Acceleration control, based upon the gas generator exhaust gas temperature versus engine speed schedule, is one typical method
used for starting. The transient performance is unaffected by changing fuels or by differences in the heating value of the fuel when this type of control is used. This may become an important consideration when dealing with gas or dual-fuel burning engines, where the fuel heating value may vary. However control developments have been limited by practical difficulties in achieving suitable accuracy, response and life of the thermocouples and transducers required.

Other methods, such as control of the rate-of-change of exhaust gas temperature, fuel-flow versus engine speed or fuel-flow versus compressor discharge pressure (C.D.P.) schedules are also used. In practice, the latter scheme is by far the most widely adopted and is used in the control system employed in the present investigation. Fig. 8.1 shows a typical characteristic for an acceleration fuel schedule based upon the C.D.P. and it is evident that at high output operating conditions, the fuel flow is restricted by the exhaust gas temperature rather than by the engine surge boundary.

8.1.3 - Deceleration fuel schedule

The availability of rapid deceleration is generally a basic requirement of an industrial gas turbine used for electrical power generation. Following load rejection, a rapid reduction of fuel flow is required to limit the maximum speed rise. However, there is a minimum level to which the fuel flow can be reduced before flame out occurs, resulting from a sudden increase in the air-fuel ratio. A deceleration fuel schedule is therefore required to minimise the turbine speed rise in the event of load rejection, but not the flaming-out of the engine.

A fuel scheduling method based on the C.D.P. is commonly used for deceleration control and a typical characteristic is shown in fig. 8.1.

8.1.4 - Output power control

The control parameter for steady engine running depends on the application of the engine. For electrical power generation, it is normally the power turbine speed, and hence the generator output
frequency. Accurate governor control is clearly required to maintain the system frequency to within specific limits.

Load sharing between parallel-connected machines is conventionally achieved by the governor-droop matching method. In small isolated grid systems, isochronous governing (i.e. zero error speed control) can eliminate or reduce the frequency-droop effect. The simplest approach is to run one engine isochronously and the other(s) in a droop mode. However, the isochronous engine is the only one which will sense the load change, with the other(s) contributing a fixed output. An alternative method is to compare the system frequency with a reference integrating device, and to use the resulting error signal to adjust the speed reference settings of all the paralleled machines until zero error is achieved.

8.1.5 - Topping governor

For engine protection purposes, C.D.P., gas generator speed and exhaust gas temperature are monitored and compared with their prescribed limits in the topping governor. Should any of these exceed a safe value, the topping governor overrides the main governor and reduces the fuel flow until the engine is brought back to within a safe operating level.

8.2 - General description of the governor control system

The gas-turbine governor control system described in this section is used to control a split-shaft, liquid-fuel gas-turbine/generator installation. However, it can easily be converted to control a range of industrial gas turbines for various applications and is also suitable for either liquid, gas or dual-fuel burning engines.

The major electrical and electronic components for the control system are housed in the governor control cubicle (fig. 8.2) which forms part of the unit control panel with the major components for the hydraulic fuel system being housed in the fuel valve package (fig. 8.3).
The electronic governor system monitors and senses the vital performance and operating data of the gas turbine and controls the fuel scheduling system, to ensure the engine is operating within its safety limits during steady-state or transient conditions. Fig. 8.4 shows a schematic diagram of the gas-turbine governor control system.

8.2.1 - Electronic governor

The electronic governor, shown in fig. 8.5, monitors the power turbine speed, the gas generator speed and the exhaust gas temperature. After comparison with their corresponding datum settings, these quantities are represented by electrical signals. In order to produce a progressive and responsive control system, the signals are fed to a diode function least gate, or a lowest win logic gate, where the least negative signal (i.e. least power demand signal) is chosen as the final control function. In addition to these signals, there are three other signals entering the least gate.

a. compressor discharge pressure datum setting $V_{P2L}$ - to limit the maximum C.D.P. demand.

b. acceleration limit $V_{ACC}$ - to minimise the gas generator speed overshoot during engine starting.

c. zero voltage clamp - to suppress the least gate output to zero until the power turbine speed reaches its minimum controlled level.

The final control voltage $V_c$ is directed to the increased jump and rate limit circuit, where $V_c$ is modified, if required, to ensure the rate of increase of power demand to the gas generator is not excessive. The output voltage $V_d$ is compared with the C.D.P. feedback voltage $V_{P2}$, with the error signal being amplified and fed to the air fuel controller A.F.C. The output voltage from the A.F.C. $V_{TD}$ is compared with the throttle angle feedback voltage $V_\theta$ at the throttle amplifier and the output of which controls the fuel valve actuator.
8.2.2 - Air fuel controller

The air fuel controller, shown in fig. 8.6, acts as an interface between the electronic governor and the fuel valve package. The C.D.P. demand voltage $V_d$ from the governor is fed to the A.F.C., where limits on the throttle valve position relative to the C.D.P. during acceleration and deceleration are imposed, before returning to the rack governor as a throttle demand voltage $V_{TD}$.

During the gas generator starting sequence, the A.F.C. schedules a progressive opening of the throttle valve until the C.D.P. has risen sufficiently to be an accurate control parameter.

8.2.3 - Hydraulic fuel control system

The major components of the liquid fuel hydraulic control system shown in fig. 8.7 are housed in the fuel valve package (fig. 8.3). High pressure fuel from the gas generator driven pump is delivered to the fuel valve package inlet via a strainer, before passing to the throttle valve which controls the fuel flow to the burners. The throttle valve is mounted with an adapter to an actuator, which receives an electrical positioning signal from the electronic governor system. Excessive fuel delivered to the metering valve is directed from an overspill line to the H.P. fuel pump inlet via a flow indicator. A fast acting electrically operated shut-off valve isolates the fuel supply to the burner. To avoid any excessive pressure build-up in the fuel system in the event of closure of the shut-off valve a solenoid-operated bypass valve is incorporated.

The Woodward 1907 type liquid fuel valve shown in fig. 8.8 controls fuel flow as a function of its metering sleeve angular position. This is achieved by maintaining a constant pressure drop across the fuel metering ports.

The actuator used in conjunction with the throttle valve is a Woodward type TM55p electrohydraulic device which consists of three parts: a torque motor servo valve, a spring centred four-land spool valve and a double-sided equal-area servo-piston linking a rotary output shaft.
The torque motor servo valve shown in fig. 8.9 utilises two oil jet nozzles and a flapper to generate differential pressures to move the spool valve. This in turn controls the servo-piston which is capable of exerting a torque of 70 lb-ft (for 1000 p.s.i. hydraulic oil pressure) over an angular movement of 45°. Through the assembly, the 45° actuator movement is converted to 60° valve movement.

The response of the valve is faster than 100 ms for end to end travel. However, to limit the rate of increase of fuel supply to the gas generator, a restrictor is fitted in the actuator to increase the valve opening time to about 6-8 s for the full travel.

The throttle position feedback voltage to the governor is derived from a potentiometer which is mounted on the output shaft of the actuator.
Fig. 8.1 Acceleration and deceleration fuel schedules
Fig. 8-2 Governor control cubicle
Fig. 8.3 Fuel valve package (liquid)
Fig. 8.4 Schematic diagram of gas-turbine control system
Fig. 8.5 Electronic governor

Fig. 8.6 Air fuel controller
Fig. 8.7 Liquid fuel control system schematic
Fig. 8.8 Fuel valve schematic (liquid)

Fig. 8.9 Actuator schematic (liquid)
CHAPTER 9

DIGITAL SIMULATION

OF

A GAS TURBINE AND ITS CONTROL SYSTEM
In this chapter, a mathematical model of the gas-turbine control system, described in section 8.2, is derived and presented in a state-variable form. A brief description of the computer program used for the dynamic simulation of the gas-turbine and its governor control system is also included.

9.1 - Derivation of control system model

Based on design information for the governor control system, equations describing the dynamic behaviour of the system are derived below.

9.1.1 - Power turbine speed feedback circuit

Fig. 9.1a shows the power turbine speed amplifier circuit, in which the voltage \( V'_{NT} \) derived from the power turbine speed signal \( (V'_{NT} = 5.03 \text{ V at rated speed}) \) is compared with the speed reference voltage \( V_{NTref} \). The resulting error signal is amplified to form the power turbine speed demand voltage \( V_{NT} \).

The potentiometer RV11 is set to give the required power turbine speed-droop characteristic and the setting for the present investigation is 4 per cent of the rated speed.

To eliminate any possibility of the generator motoring following closure of its circuit breaker to the grid system, the contact of the block-load circuit shown in fig 9.1(a) closes at the same instant that the circuit breaker closes. This gives a step change to the amplifier output voltage and hence an increase of the gas-turbine output power.
A transfer function block diagram for the power turbine speed channel is presented in fig. 9.1(b), from which the associated state equations follow as

\[ x_{13} = -\frac{1}{T_{NT2}} x_{13} + B_{NT} - C_{NT} \cdot N_T \]  \hspace{1cm} 9.1

\[ V_{NT} = x_{13} + T_{NT1} x_{13} \]  \hspace{1cm} 9.2

where \[ B_{NT} = C_{NTSG} \cdot (B_{NTS} - B_{NTT}) \]  \hspace{1cm} 9.3

\[ C_{NT} = C_{NTSG} \cdot C_{NTT} \]  \hspace{1cm} 9.4

The effect of the block-load circuit is simulated by an appropriate adjustment of the value of \( B_{NTS} \) at the time the generator circuit breaker operates.

9.1.2 Exhaust gas temperature feedback circuit

The average gas generator exhaust gas temperature is monitored and the resulting signal is amplified to provide a feedback signal \( V_{T4} \) to the temperature controller output amplifier circuit shown in fig. 9.2(a).

The controller has both proportional and integral terms, with any error at the input driving the controller output to saturation. If the gas turbine is operating at an exhaust gas temperature which is below its datum setting, the gas turbine will be controlled by other parameters (e.g. power turbine speed, gas generator speed, etc.). As a result, the controller amplifier output voltage \( V_{T4} \) (fig 9.2(a)) increases until the amplifier is saturated (say at 12 V). In the event of a sharp temperature rise towards a level beyond its datum setting, the controller output \( V_{T4} \) has to be integrated from 12 V to 7 V (assume 7 V is the control voltage \( V_c \) required to maintain the temperature at its datum level) to limit the excessive temperature rise. Depending on the integration rate and the amount of proportionate action, the decay of \( V_{T4} \) may not be sufficiently fast to avoid any overshoot in the temperature. To minimise this adverse effect, an overshoot inhibit circuit is added to restrain \( V_{T4} \) to a maximum of 0.5 V above the control voltage (i.e. \( V_c \)). This implies that when the temperature
tends to exceed its datum setting, the controller output has only to decrease by 0.5 V to take control of the gas turbine.

A transfer function block diagram for the temperature controller, incorporated with the overshoot inhibit feature, is shown in fig. 9.2(b) and the relationship between the exhaust gas thermocouple time constant $T_{TC}$ and the engine speed is given in fig. 9.3.

If the maximum controlled exhaust gas temperature is limited to $645^\circ C$, the voltage for the corresponding datum setting is

$$B_{TS} = B_{TC} + C_{TC} (645 + 273)$$  \hspace{1cm} (9.5)

The state equations associated with the exhaust gas temperature feedback circuit are

$$x_{14} = - \frac{1}{T_{TC}} x_{14} + \frac{C_{TC} T_4}{T_{TC}}$$  \hspace{1cm} (9.6)

$$x_{15} = - \frac{C_{TCR}}{\omega_o} x_{14} + \frac{C_{TCR} B_{T4}}{\omega_o} + V_{CF T4}$$  \hspace{1cm} (9.7)

$$V_{T4} = x_{15} + T_{TCR} x_{15}$$  \hspace{1cm} (9.8)

where

$$B_{T4} = B_{TS} - B_{TC}$$  \hspace{1cm} (9.9)

$$V_{CF T4} = \text{voltage clamping function for } V_{T4}$$

(refer to Appendix B)

### 9.1.3 - Gas generator speed feedback circuit

Fig. 9.4(a) shows the gas generator speed channel output amplifier circuit, in which the voltage $V_{NG}$ derived from the gas generator speed signal ($V_{NG} = 5.03$ V at 100 per cent speed) is compared with the datum setting, with the resulting error signal being amplified to form the gas generator speed demand voltage $V_{NG}$. An overshoot inhibit circuit, similar to that described in Section 9.1.2 for the exhaust gas temperature controller, is installed to limit $V_{NG}$ to a maximum of 0.5 V above the control voltage $V_c$ when the gas generator speed is not in control.
A signal flow diagram for the gas generator speed feedback circuit is shown in fig. 9.4(b), from which the following equations are derived.

\[ B_{NGS} = B_{NGT} + C_{NGT} \cdot 7700 \]  

where 7700 r.p.m. is the gas generator speed datum setting

\[ x'_{16} = \frac{1}{T_{NGT}} x_{16} + \frac{C_{NGT}}{T_{NGT}} N_G \]  
\[ x'_{17} = \frac{C_{NGSG}}{\omega_0} x_{16} + \frac{B_{NG}}{\omega_0} + V_{CF_{NG}} \]  
\[ V_{NG} = x_{17} + \frac{T_{NG}}{\omega_0} x'_{17} \]

where \( B_{NG} = C_{NGSG} (B_{NGS} - B_{NGT}) \)
\( V_{CF_{NG}} \) = voltage clamping function for \( V_{NG} \)

9.1.4 - Increased jump and rate limit circuit

A simplified increased jump and rate limit circuit is shown in fig. 9.5(a), with the input voltage being derived from the diode function least gate and the output voltage \( V'_d \) being fed to the C.D.P control amplifier as the C.D.P. demand voltage.

Based on the equivalent circuit shown in fig. 9.5(b), with the diode D41 open-circuit, the output voltage \( V'_d \) is

\[ V'_d = \frac{C_{41}}{C_{42}} \left\{ \frac{1 + s C_{42} R_{44}}{1 + s C_{41} R_{43}} \right\} \frac{V_{41}}{R_{44}} \left( \frac{1}{R_{48}} + \frac{1}{s C_{42} R_{48}} \right) \frac{V_{42}}{R_{48}} \]  

or, to a close approximation

\[ V'_d = -\frac{C_{41}}{C_{42}} V_{41} - \frac{1}{s C_{42} R_{48}} V_{42} \]

Any decrease in \( V_c \) (i.e. an increase in power demand) causes the output of the amplifier \( A_{41} \) to saturate at about -12V. The 2nd-stage amplifier \( A_{42} \) then operates as an integrator, with the integration rate being a function of \( C_{42} \) and \( R_{48} \) and the voltage \( V_{42} \).
The integration action is terminated when the virtual earth at the amplifier $A_{41}$ is re-established.

If $V_c$ is increased (i.e. a decrease in power demand), the output of amplifier $A_{41}$ saturates at approximately 12V, diode $D_{41}$ becomes conducting and the effective summing resistance of the integrator is greatly reduced. Consequently, $V_d$ decreases rapidly to re-establish a new equilibrium condition.

The circuit of fig. 9.5(a) provides thereby a means of limiting the rate of increase of power demand on the gas generator, without restricting any governor action in the reduction of output power.

Two integration rates are available in the circuit to enable a faster rate of increase of $V_d$ at high power levels. This is achieved by switching off transistor $T_{41}$ and allowing $V_{42}$ to rise to 6.2V (i.e. the maximum voltage across the zener $Z_{41}$). The transistor $T_{41}$ is normally set to be switched off when $V_d$ reaches a level corresponding to the power demand required to maintain the generator running at rated speed on no load.

For response reasons, a 'backlash' circuit is included to allow for a small step increase in $V_d$ from the steady condition before the imposed rate limit is reached and this effect is represented by the term $C_{41}V_{41}$ in eqn. 9.16.

The power demand voltage $V_d$ is limited to a minimum set by the potentiometer $RV_{42}$.

The increased jump and rate limit circuit shown in fig. 9.5(a) may be represented by the signal flow diagram shown in fig. 9.5(c). When $V_c$ is within the maximum limit, the transfer relationship between $V_d$ and $V_c$ is

$$V_d = \frac{C_{JR} (1 + T_{CJR}/s)}{1 + C_{JR} (1 + T_{CJR}/s)}$$  \hspace{1cm} 9.17

$$= 1.0 \text{ (provided that } C_{JR} \gg 1)$$
so that \( V_d\) is equal to \( V_c\). The differential equation describing the dynamic behaviour of the circuit is

\[
x_4 = -\frac{T_{CJR}}{\omega_0} x_4 + \frac{T_{CJR}}{\omega_0} V_c + V_{CFJRL}
\]

9.1.5 - C.D.P. feedback circuit

The gas generator compressor delivery pressure (C.D.P. or \( P_2\)) is monitored by a pressure transducer, which converts a 100-1100 kPa (or 1-11 bar absolute) pressure signal to a 4-20 mA current signal. This in turn is fed to the governor, where it is converted into a corresponding voltage signal \( V_{P2}\). The associated circuit diagram and transfer function block diagram are shown in fig. 9.6.

9.1.6 - C.D.P. output amplifier and air fuel controller

A schematic diagram of the C.D.P. output amplifier and the air fuel controller A.F.C. is shown in fig. 9.7(a) with the corresponding signal flow diagram being given in fig. 9.7(b). The C.D.P. demand voltage \( V_d\) from the increased jump and rate limit circuit is compared with the C.D.P. feedback voltage \( V_{P2}\) with the resulting error signal being amplified and passed to the A.F.C. The voltages corresponding to the acceleration (\( V_{ACL}\)) and deceleration (\( V_{DCL}\)) fuel limits are derived from \( V_{P2}\) through the shaper circuits and the logic circuits. The throttle demand voltage \( V_{TD}\) at the output of the A.F.C. is limited such that the corresponding throttle position is within the prescribed limits for the working C.D.P. Under steady-state conditions, \( V_{TD}\) is equal to \( V_{CDP}\) and the A.F.C. circuit is effectively bypassed.

When the throttle demand voltage is controlled by either of the fuel schedule limits, the C.D.P. amplifier output is prevented from being driven into saturation by a voltage clamping circuit. This clamping circuit and the amplifier circuit itself are shown in fig. 9.8. When diodes D61 and D62 are not conducting, the amplifier output voltage is
\[ V_{CDP} = \frac{1}{C_{61} R_{62}} \left( \frac{1 + s C_{61} R_{64}}{s} \right) \left( \frac{R_{62}}{R_{61}} V_d' - \frac{R_{62}}{R_{61}} V_{P2} + \frac{R_{62}}{R_{63}} V_{idle} \right) \]

and with the inputs normalised to \( V_{P2} \)

\[ V_{CDP} = \frac{1}{C_{61} R_{62}} \left( \frac{1 + s C_{61} R_{64}}{s} \right) \left( \frac{R_{62}}{R_{61}} V_d' - \frac{R_{62}}{R_{61}} V_{P2} + \frac{R_{62}}{R_{63}} V_{idle} \right) \]

or,

\[ V_{CDP} = C_{ADL} \left( \frac{1 + s T_{ADL}}{s} \right) (V_d' - V_{P2} + V_{idle}) \]

where \( C_{ADL} = \frac{1}{C_{61} R_{62}} = 3.036 \text{ s}^{-1} \)

\[ T_{ADL} = C_{61} R_{64} = 1.62 \text{ s} \]

\[ V_d = \frac{R_{62}}{R_{61}} V_d' = 0.27 V_d' \]

\[ V_{idle} = \frac{R_{62}}{R_{63}} V_{idle} = 0.1 V_{idle} \]

Other equations related to the C.D.P. amplifier and the A.F.C. are listed as follows

\[ x_5 = \frac{C_{ADL}}{\omega_0} (V_d + V_{idle} + V_{P2}) + VCF_{CDP} \]

\[ V_{CDP} = x_5 + T_{ADL} x_5 \]

\[ V_{ACL} = B_{P2} + C_{P2} \cdot V_{P2} \]

\[ V_{DCL} = C_{DL} \cdot V_{ACL} \]

whenever \( V_{CDP} \) exceeds either of the fuel schedule limits (i.e. above \( V_{ACL} \) or below \( V_{DCL} \)), the clamping function \( VCF_{CDP} \) shown in eqn.9.23 will be applied to restrain \( V_{CDP} \) to \( \pm 0.1 \text{ V} \) of the prevailing throttle demand voltage \( V_{TD} \) (i.e. \( V_{TD} \pm 0.1 \text{ V} \)).
9.1.7 - Throttle control loop

A schematic diagram for the throttle control loop and a circuit diagram for the throttle amplifier are shown respectively in figs. 9.9(a) and 9.10. The throttle demand voltage from the A.F.C. is compared with the fuel valve actuator angle feedback voltage \( V_\theta \) at the throttle amplifier, the output of which controls the actuator output shaft position.

The inherent 'stiction' problem on the spool valve in the fuel valve actuator is overcome by superimposing an a.c. signal (\( V_{\text{dither}} \) of fig. 9.10) onto the throttle position signal. The amplitude and frequency of this dither signal is adjusted to set the spool valve in motion but not causing the servo piston to response.

The transfer function block diagram of the throttle control loop is presented in fig. 9.9(b). To accommodate the unidirectional rate limit characteristic of the actuator, the block diagram is re-arranged to take the form of fig. 9.9(c) and the associated state equations are as follows:

\[
\begin{align*}
\dot{x}_6 &= \frac{x_7}{\omega_0} \\
\dot{x}_7 &= \frac{x_8}{\omega_0} \\
\dot{x}_8 &= \frac{V_{\text{TD}}}{\omega_0} - \frac{x_7}{T_{A3}} - \frac{x_8}{T_{A3}} - \frac{x_{10}}{T_{A3}} \\
\dot{x}_9 &= \frac{x_6}{T_{A3}} \cdot \frac{\omega_0}{T_{A4}} + \frac{x_7}{T_{A4}} \cdot \frac{C_{TA}}{T_{A4}} - \frac{x_9}{T_{A4}} \\
&\quad - \frac{x_{10}}{T_{A4}} + V_{\text{CFTRL}} \\
\dot{x}_{10} &= \frac{x_9}{\omega_0}
\end{align*}
\]
9.1.8 – Liquid fuel throttle valve

The actuator, through the linkage assembly, controls the metering sleeve angular position of the throttle valve; this in turn regulates the fuel flow to the gas turbine.

An approximate mathematical model for the valve is given by the second order system shown in fig. 9.11, while fig. 9.12 shows the valve calibration for the engine tested. The state equations for the model are:

\[
\begin{align*}
    \dot{x}_{11} &= \frac{x_{12}}{\omega_0} \\
    \dot{x}_{12} &= \frac{x_{10}}{\omega_0} - \frac{x_{11}}{T_{H2}} - \frac{x_{12}}{\omega_0} \frac{T_{H1}}{T_{H2}}
\end{align*}
\]

9.2 – Gas turbine and its control system

When the control system model derived in the preceding sections is combined with the gas-turbine model developed in Section 7.3, the combined system may be represented by the signal flow diagram of fig. 9.13. Since the generator has not been coupled to the gas-turbine model at this stage, the power turbine speed loop is not closed.

9.2.1 – State-space representation

The differential equations of the combined system are presented in the following state-space form,

\[
\dot{x}_G = A_G \cdot x_G + C_G + B_G \cdot u_G
\]

where matrices \(A_G\), \(C_G\), \(B_G\) and \(u_G\) are respectively defined in eqns. 9.35 - 9.38.

For ease of handling and convenience of carrying out modifications to the system, the system matrix is divided into four subsets corresponding to (a) gas generator, (b) increased jump and rate limit circuit and C.D.P. amplifier, (c) throttle control loop and liquid fuel valve and (d) least gate input signals.
9.2.2 - Computer program

The digital computer program written to simulate the gas turbine and its governing system can be summarised as

a. Read in system data

b. Fit curves to the gas-turbine characteristics using a least-square curve-fitting technique (figs. 7.2, 7.3, 7.5, 7.6 and 9.3)

c. Set the following potentiometers to give the corresponding voltages

i. pressure idle setting, \( V_{\text{idle}} \)
ii. gas generator speed datum setting, \( B_{\text{NGS}} \)
iii. exhaust gas temperature datum setting, \( B_{T4S} \)
iv. maximum C.D.P. datum setting, \( V_{\text{P2L}} \)

d. At the given gas generator speed \( N_G \), determine the corresponding gas-turbine parameters (e.g. \( \text{EGP}_{NG} \), \( P_{2NG} \), \( P_{4NG} \), \( T_{4NG} \) and \( Q_{FNG} \)) from the fitted performance curves (figs. 7.5 and 7.2)

e. With the given power turbine speed \( N_{TO} \) and the predicted values of \( P_{4NG} \) and \( T_{4NG} \), the efficiency of the power turbine \( \eta_T \) is determined from the fitted characteristic (fig. 7.6)

f. Compute the power turbine speed demand voltage setting \( B_{NTS} \) from the predicted compressor discharge pressure \( P_{2NG} \)

g. Calculate the throttle demand voltage \( V_{TD} \) and the actuator angle \( \theta \) from \( Q_{FNG} \)

h. Compute the initial values of the state variables

i. Apply logic to simulate the function of the least gate in choosing the minimum demand signal as the final control voltage \( V_C \)
j. When $V_{T4}$ and $V_{NG}$ are not in control of the gas turbine, limit their maximum values to $V_c + 0.5$ V

k. Apply the jump and rate limit, if required

l. Compute the acceleration and deceleration fuel schedule limits, $V_{ACL}$ and $V_{DCL}$

m. Apply these voltage limits to $V_{CDP}$, to ensure the throttle demand voltage $V_{TD}$ is within the fuel schedule limits

n. When $V_{CDP}$ is outside either of its limits, apply the voltage clamping function to ensure $V_{CDP}$ lies between $V_{TD} - 0.1$ V and $V_{TD} + 0.1$ V

o. Apply throttle rate limit, if required

p. From the fitted curves (figs. 7.2, 7.3, 7.5 and 9.3), determine the following gas turbine parameters $T_G$, $T_C$, $T_D$, $T_{TC}$, $a_{P2}$, $a_{T4}$, $a_{EGP}$, $a_{NG}$, $a_{P2}$, $a_{P4}$, and $a_{T4}$

q. Calculate the state equation matrices

r. Solve the differential equations using a predictor-corrector routine

s. Calculate $E_{GP}$, $N_{G}$, $P_{2}$, $P_{4}$, $T_{4}$ and $Q_{F}$ from the updated values of the state variables

t. With the current gas generator speed, redefine $E_{GP_{NG}}$, $P_{2_{NG}}$, $P_{4_{NG}}$, $T_{4_{NG}}$ and $Q_{FG_{NG}}$ from the fitted curves given in figs. 7.5 and 7.2

u. Repeat from i. with the updated values of integration time, state variables and forcing functions until the pre-set simulation period elapses.
Fig. 9.1 Power turbine speed feedback

(a) Output power amplifier circuit

(b) Transfer function block diagram
Fig. 9.2 Exhaust gas temperature feedback
Fig. 9.3 Variation of exhaust gas temperature thermocouple time constant with engine speed
Fig. 9.4 Gas generator speed feedback
Fig. 9.5 Increased jump and rate limit circuit
Fig. 9.6 CDP feedback
Fig. 9.7 C.D.P. amplifier and air fuel controller
Fig. 9.8  C.D.P. output amplifier
Fig. 9.9 Throttle control loop
Fig. 9.10 Throttle amplifier circuit

Fig. 9.11 Liquid throttle valve transfer function block diagram
Fig. 9.12 throttle valve calibration
Fig. 9.13 Signal flow diagram for the gas turbine and its governor control system.
CHAPTER 10

DYNAMIC SIMULATION OF

A COMPLETE GAS-TURBINE GENERATING UNIT
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DYNAMIC SIMULATION OF A COMPLETE GAS-TURBINE GENERATING UNIT

In this chapter, the mathematical models developed for the generator (chapter 2 and 3), the excitation control system (chapter 6), and the gas turbine and its governor control system (chapter 7 and 9) are coupled to form a complete gas-turbine/generator model. Governor and a.v.r. step-response tests, together with load acceptance and rejection tests, were carried out on a 15 MW gas-turbine generating unit and typical test results are compared with corresponding computer predictions, in order that the validity of the model can thereby be assessed.

The gas turbine and governor data used in the computer model are identical to those described in chapter 7 and 9, while the generator and the excitation system data employed in the program are listed in Tables 10.1 and 10.2 respectively.

10.1 - Governor step-response tests

When the governor is in the automatic control mode, as shown in fig. 8.4, the governor output is determined by the power turbine speed reference setting. On the other hand, if the control is in the manual mode, the governor output is controlled by the C.D.P. demand reference setting.

Ideally, a governor step-response test should be carried out by applying a step change to the power turbine speed reference setting, with the governor being in the automatic control mode. However, such test results are unavailable, and the transients obtained by applying step changes to the C.D.P. demand setting, with the governor on manual control, are therefore used for the present investigation.
10.1.1 - Step up response

A step change in the C.D.P. demand voltage setting $V_c$ from 1.82V to 2.50V was simulated on the gas-turbine/generator model and a comparison of the computed and test results is made in fig. 10.1, with the initial and final steady-state values being tabulated in Table 10.3.

The transients shown in fig. 10.1 are dominated by the increased jump and rate limit circuit, which allows $V_d$ to increase initially by a small step before the application of the ramp rate limit. The good correlation between computed and test results confirms that the characteristic of the increased jump and rate limit circuit has been correctly implemented. Although typical gas generator steady-state operating curves (fig. 7.2) are used in the computer program, the predicted steady-state values for the engine variables $P_2$, $T_4$, $Q_F$, etc. at a given engine speed, shown in Table 10.3, are in good agreement with the test results.

10.1.2 - Step down response

Fig. 10.2 shows the predicted and test gas-turbine response following a step change of $V_c$ from 1.97V to 1.30V. A comparison of the measured and computed initial and final steady-state values is made in Table 10.4.

A sudden decrease of $V_d$ causes the C.D.P. amplifier output to fall below the deceleration limit set by the air fuel controller, and the throttle demand voltage shown in fig. 10.2 is therefore controlled by the deceleration fuel limit until the C.D.P. amplifier output recovers to a level higher than the prevailing deceleration limit. Since the deceleration fuel schedule is a function of $V_{P2}$, it is evident from fig. 10.2 that the faster the decay of the measured $V_{P2}$, compared with the predicted value, the lower the actual deceleration fuel schedule limit and hence the quicker reduction of the actual fuel flow to the gas turbine. This results in a further decay of the measured $V_{P2}$. Any discrepancies between the measured and the computed response are likely to be caused by the inaccurate value of the engine time constant used in the computer program. With this parameter reduced by 20 per
cent from the typical value given in fig. 7.3, the accuracy of the computer predictions shown in fig. 10.2 is significantly improved.

10.2 - A.V.R. step-response test

With the generator on open-circuit and running at rated speed, a step change in the generator demand voltage from 90 per cent to 100 per cent of the rated value was applied. The computed and measured transients are shown in fig. 10.3 and their agreement can be seen to be generally good.

10.3 - Load rejection and acceptance tests

The terminals of the gas-turbine generator were connected to a water load tank via a generator circuit breaker. Load acceptance and rejection tests were initiated by operation of the generator circuit breaker.

The main objective of these tests was to ensure that the excitation and governor control systems were capable of maintaining system stability following a load switching operation. In response to the speed rise subsequent to a load rejection, the governor system is required to be able to reduce the fuel flow to a minimum level (limited by the deceleration fuel schedule) in the shortest possible time, to avoid any excessive and possibly catastrophic overspeed.

At the time of conducting these tests, due to technical difficulties, the excitation system was in the manual control mode and hence the dynamic response on the excitation control system could not be examined. However, test results obtained from other gas-turbine generators are used to support the excitation system response predictions. The transients of the gas turbine and its governor control system following various types of load switching were recorded and have provided valuable information for verifying the gas-turbine/generator model.
10.3.1. - Load rejection tests

With the generator running at rated speed and the excitation supply switched off, the generator terminals were connected to the load tank by closure of the circuit breaker. The excitation system was then energised and the exciter field current was increased gradually until a 3 MW unity power factor load was achieved. The unit was kept at this load for a short period to allow the engine to become thermally stabilised, before the circuit breaker was opened to reject the 3 MW load. Since the excitation was on manual control, the excitation supply was switched off simultaneously to avoid any overvoltage occurring on the stator windings.

Fig. 10.4 shows the measured and computed transients following the 3 MW load rejection, with the initial and final steady-state values being given in Table 10.5. The power turbine is shown to reach a maximum speed of 6576 r.p.m. (i.e. 101.2 per cent of rated speed) after 1.2 s and to settle finally at about 6550 r.p.m. (i.e. 100.75 per cent of rated speed). The initial drop in $V_C$ from 3.49V to 2.47V is caused by closure of the power turbine speed governor amplifier block-load switch (fig. 9.1(a)) at the instant the circuit breaker opens. The subsequent decay of $V_C$ after about 0.9 s is the result of the droop characteristic of the power turbine speed channel. The effect of the increased jump and rate limit circuit is exhibited in the transient response of $V_d$ between about 2.0 s and 5.5 s. Despite the complexity of the model, the general agreement between the computed and test results is good.

If the a.v.r. had been in operation during the test, the excitation system transients would be similar to the predictions given in fig. 10.5. It is interesting to note that the generator terminal voltage, following opening of the circuit breaker, does not initially increase, but rather decreases before a subsequent rise. A similar phenomenon can be observed in the test results, given in fig. 10.6, obtained from a 22 MW gas-turbine generator subsequent to rejection of an 8 MW unity power factor load. The theory of such phenomenon is explained in reference 21. The final control element of the a.v.r. for the 22 MW gas-turbine generator is of the fully-controlled thyristor bridge type and an inverting output, as evident in fig. 10.6, is therefore possible.
Following successful simulation of the 3 MW load rejection, test results for 5 MW and 7.5 MW rejections were examined and comparisons between predicted and test results are made in figs. 10.7 and 10.8. The results shown are again generally in good agreement.

10.3.2 - Load acceptance test

Before the generator circuit breaker was closed, the excitation field current was increased to a pre-determined level. Load acceptance was initiated by closure of the circuit breaker and the system transients were recorded. Comparisons with the computer predictions are made in fig. 10.9.

Following closure of the circuit breaker, the power turbine speed starts to fall and the speed governor output voltage $V_{NT}$ increases accordingly. The least gate output voltage $V_c$ shown in fig. 10.9, follows $V_{NT}$, until a limit imposed by the maximum C.D.P. datum is reached. From then on, $V_c$ is limited to a maximum of 6.69V. Although the least gate output voltage is at its maximum value, the demand voltage $V_d$ is restrained by the increased jump and rate limit circuit and which in turn controls the rate of increase of output power to the turbine. Consequently no acceleration power is available until 3.5 s after the load was accepted. The maximum speed dip and recovery time are recorded as about 9 per cent and 8 s respectively. As the power turbine speed recovers, the speed governor regains control over the least gate output (i.e. $V_c$ follows $V_{NT}$) and starts to reduce the fuel flow to the gas generator in order to limit the power turbine speed overshoot subsequent to the speed recovery.

The effective resistance of the load tank varies, depending on its temperature, the purity of the water and any turbulence in the tank. However in the computer program, this load is represented as a constant resistance. This may be one reason for the discrepancy, shown in fig. 10.9, between the measured and computed transient power turbine speed.

If the a.v.r. had been in operation, the transients of the excitation system and generator voltages would be similar to those shown in fig. 10.10. For comparison, transients obtained from a 22 MW
gas-turbine generating unit following an 8 MW unity power factor load acceptance test are presented in fig. 10.11.

10.4 - Symmetrical short-circuit test

The objective of a symmetrical short-circuit test on a generator is to ascertain the effect of the excitation and governor control systems on the generator performance under such a severe disturbance. The capability of the rotating rectifying bridge in handling the resulting high field current can also be examined. However, the chance of carrying out such test on a power system is remote and it is therefore essential to have an accurate model to study this type of severe system fault.

The gas-turbine generator is assumed to be connected to an infinite system via a step-up transformer and a transmission line, with impedances equal to those given in Appendix H. A symmetrical 3-phase fault is then applied to the high-voltage terminals of the generator transformer, with the generator at rated load and power factor, for a period of 0.37 s.

The prediction of the system transients are shown in fig. 10.12. Comparison with the short-circuit transients of the steam turbo-generator given in fig. 5.9(a) shows that the frequency of the gas-turbine generator rotor oscillation is higher, because its inertia constant is only about one-half that of the steam turbo-generator.

Since the increased rate limit circuit of the gas-turbine governor system provides a means of restraining the rate of increase of power demand on the gas generator, without restricting any governor action in the reduction of output power, the resulting gas-turbine power immediately after the transient is below the prefault level. This will eventually be restored, with the time required depending mainly on the increased rate limit setting of the governor. Fig. 10.13 shows a steady increase in the demand voltage $V_d$ and the time required to reach its prefault value is estimated as about 11 s after the time of fault inception.
If the gas-turbine generator is connected to a large system, its temporary power reduction immediately after the short-circuit fault may not be a significant factor in the system recovery. But, if the system is small, this temporary power deficiency may cause a temporary depression of the system frequency.

In a steam-turbine governor system, there is an increased rate limit imposed by the rate of opening of the steam valve. However, this characteristic has not been included in the Goldington steam turbo-generator model of Chapter 5 because such information is unavailable. In fact, most research workers have not included any considerations of such a constraint. As a result, the problem of temporary power deficiency does not appear in the predictions given in fig. 5.8(b). If the valve opening rate limit had been included in the steam turbo-generator model, due to the high inertia constant and slow governor response, the power deficiency following the fault would be less significant compared with that of the gas turbine.

To study the effect of fault duration on the generator rotor angle swing, fault periods of 0.35, 0.37 and 0.39 s were simulated. The corresponding load angle transients are shown in fig. 10.14. It was found that the fault has to be cleared in less than 0.39 s in order to maintain the machine stability.

Without the governor control action, the increase of load angle swing, shown in fig. 10.15, is significant. This indicates that the gas-turbine governor control has a significant effect on unit/system stability and an accurate gas turbine and governor model, compatible to that of the excitation control system, is therefore required.

10.5 - Estimation of generator voltage and frequency variations on application of induction motor load

When induction motors are to be started from systems of limited capacity, as in most gas-turbine generating plant, the question of voltage regulation is of prime importance from several points of view. Among these are the possibility of the motor being unable to start due to insufficient torque, the flickering lights, the dropping out of
undervoltage relays and the possible stalling of other induction or synchronous motors already connected.

In a limited capacity plant, frequency depression resulting from motor starting transients is another important consideration.

Gas-turbine generator manufacturers are therefore often required to guarantee the maximum voltage and frequency dips and their recovery time subsequent to the application of a large induction motor load. A mathematical model which is capable of predicting such transients at the tender stage would be of considerable value.

Since the information available on the motor to be started, at the tender stage, is often limited to its rating, starting MVA and power factor, it is an accepted approximation to assume the motor load to remain constant at a given starting current during the initial run-up period. This implies that the motor remains at standstill and is the worst condition. In such a case, the estimations of voltage and frequency dips and their recovery time following the connection of an induction motor to the gas-turbine generator can be obtained by applying an impedance load to the gas-turbine/generator model. The value of the impedance is determined by the fact that it will draw a current corresponding to the motor starting current when rated voltage is applied. Based on this assumption, the generator voltage and frequency transients following the starting of various size induction motor from an unloaded gas-turbine generator are shown in fig. 10.16, and the maximum frequency and voltage dips and the voltage recovery time are summarised in Table 10.6.

In addition to the approximation of constant load impedance, further assumptions made during the estimation of the maximum frequency dips are

a. the generator voltage recovers to its nominal value after the initial dip (i.e. zero quadrature current compensation in a.v.r. is assumed).

b. the motor run-up time is longer than the time required to reach the predicted maximum system frequency dip. If the
motor reaches its nominal speed when the frequency shown in fig. 10.16 has fallen to $f_1$ and is still decaying. To a good approximation, the maximum frequency dip is about $f_1$.

At a given motor starting current, the gas-turbine/generator model developed provides a means of predicting the voltage and frequency transients and, if required, the system components and control equipments can be modified or re-designed to fulfil these specifications.

For a detailed study of an induction motor starting from a gas-turbine generator, the induction motor models developed in Chapter 4 can be incorporated into the gas-turbine/generator model. Further discussion on the simulation of such a system is presented in Section 11.2.
### TABLE 10.1

**GENERATOR DATA**

<table>
<thead>
<tr>
<th>Parameters</th>
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<tbody>
<tr>
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<tr>
<td>$x'_d$</td>
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<td>$x''_d$</td>
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</tr>
<tr>
<td>$x_q$</td>
<td>1.6</td>
</tr>
<tr>
<td>$x''_q$</td>
<td>0.2</td>
</tr>
<tr>
<td>$T'_d$</td>
<td>1.1 s</td>
</tr>
<tr>
<td>$T''_d$</td>
<td>0.035 s</td>
</tr>
<tr>
<td>$T''_q$</td>
<td>0.035 s</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>$H$</td>
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(All parameters are expressed in p.u. unless otherwise specified)
### TABLE 10.2
EXCITATION SYSTEM DATA

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<td>$K_F$</td>
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</tr>
<tr>
<td>$T_A$</td>
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<tr>
<td>$T_E$</td>
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<tr>
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<td>$S_{EMAX}$</td>
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<td>$S_{E75MAX}$</td>
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(All parameters are expressed in p.u. unless otherwise specified)
## TABLE 10.3
GOVERNOR STEP-UP RESPONSE TEST RESULTS

<table>
<thead>
<tr>
<th>System variables</th>
<th>Initial readings</th>
<th>Final readings</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Test</td>
<td>Computed</td>
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<tr>
<td>$V_c$(V)</td>
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<td>1.82</td>
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<td>$V_{p2}$(V)</td>
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<td>1.32</td>
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<tr>
<td>$V_{TD}$(V)</td>
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<td>$V_9$(V)</td>
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<td>5550</td>
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<td>632</td>
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<td>$Q_F$ (MW)</td>
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### TABLE 10.4
**GOVERNOR STEP-DOWN RESPONSE TEST RESULTS**

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</thead>
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<td>Computed</td>
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<td>( V_c(V) )</td>
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<tr>
<td>( V_{TD}(V) )</td>
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<td>0.92</td>
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<td>( V_{\theta}(V))</td>
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<td>( N_G ) (r.p.m.)</td>
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<tr>
<td>( T_q ) (°K)</td>
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<td>633</td>
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<tr>
<td>( Q_F ) (MW)</td>
<td>15.1</td>
<td>14.8</td>
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### TABLE 10.5

**3 MW LOAD REJECTION TEST RESULTS**

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<th>System variables</th>
<th>Initial readings</th>
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<tr>
<td></td>
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<td>Load (MW)</td>
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<tr>
<td>$V_t$ (kV)</td>
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<tr>
<td>$I_a$ (A)</td>
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<td>$N_T$ (r.p.m.)</td>
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<td>6500</td>
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<tr>
<td>$N_G$ (r.p.m.)</td>
<td>6750</td>
<td>6700</td>
</tr>
<tr>
<td>$P_2$ (kPa)</td>
<td>579</td>
<td>563</td>
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<tr>
<td>$T_4$ ($^\circ$K)</td>
<td>679</td>
<td>725</td>
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<td>$Q_F$ (MW)</td>
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<td>$V_d$ (V)</td>
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<tr>
<td>$V_{P2}$ (V)</td>
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<tr>
<td>$V_{TD}$ (V)</td>
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<td>1.69</td>
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<tr>
<td>$V_{\Theta}$ (V)</td>
<td>1.73</td>
<td>1.69</td>
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TABLE 10.6
ESTIMATIONS OF VOLTAGE AND FREQUENCY VARIATIONS
DURING MOTOR STARTING

<table>
<thead>
<tr>
<th>Motor rating (MW)</th>
<th>Motor starting MVA</th>
<th>Maximum voltage dip (%)</th>
<th>Voltage recovery time (s) (reach 97% of rated voltage)</th>
<th>Maximum frequency dip (%)</th>
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<tr>
<td>1</td>
<td>6.25</td>
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<td>1.5</td>
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<tr>
<td>2.5</td>
<td>15.6</td>
<td>20</td>
<td>1.33</td>
<td>12</td>
</tr>
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</table>
Fig. 10.1 Gas-turbine governor step-up response test
Fig. 10.2 Gas-turbine governor step-down response test
Fig. 10.3 AVR. 10% step-response test
Fig. 10.4 3 MW load rejection test
Fig. 10.5 Prediction of excitation system and generator voltage transients following a rejection of 3 MW load.
Fig. 10.6 Generator and a.v.r. response on 8 MW load rejection
Fig. 10.7 5 MW load rejection test
Fig. 10.8 7.5 MW load rejection test
Fig. 10.9 2 MW load acceptance test
Fig. 10.10 Predictions of excitation system and generator voltage transients following an acceptance of 2 MW load.
Fig. 10.11 Generator and a.v.r. response on 8 MW load acceptance
Fig 10.12 Generator and excitation system transients following a symmetrical 3-phase short-circuit
Fig. 10.13 Gas turbine and governor transients following a 3-ph symmetrical short-circuit fault.
Fig. 10.14 Rotor angle swing following various fault durations

Fig. 10.15 Effect of governor action on the rotor angle swing
Fig. 10.16 Motor starting transients
CHAPTER 11

RECOMMENDATIONS

FOR

FURTHER STUDIES
CHAPTER 11
RECOMMENDATIONS FOR FURTHER STUDIES

Throughout the investigation described in this thesis, the theoretical work has been substantiated by extensive test results. The high level of agreement evident between experimental and predicted results gives considerable confidence in both the formulation of the theory and the adaptation of the modelling technique, and more importantly encourages the extension of the present work to those areas described below.

11.1 - Unsymmetrical fault studies on gas-turbine generators

The application of the gas-turbine/generator model presented in Chapter 10 is restricted to symmetrical operation or fault studies. However, the area of study can be extended to the investigation of unsymmetrical faults, by replacing the existing 2-axis generator model with the well-proven phase model of Chapter 3.

11.2 - Interaction of gas-turbine generators and induction motors

A mathematical model describing the dynamic behaviour of a composite system, consisting of a regulated gas-turbine generator and an induction motor, can be developed from the gas-turbine/generator model of Chapter 10 and the synchronous generator/induction motor model of Chapter 4.

The model can be arranged either to simulate the gas-turbine generator/motor combination being connected to the infinite grid via a transmission system (as an industrial plant gas-turbine power supply system) or in isolation (as an offshore platform gas-turbine power supply system). Valuable information such as the maximum generator voltage and frequency dips and their recovery time following a direct-on-line motor start can be obtained at the design stage. If
required, the generator can be re-designed to limit the maximum voltage dip, by lowering the generator transient reactance, or the gas-turbine acceleration limit may need to be increased to limit the frequency dip.

For detailed studies of the starting of large induction motors, especially those with a long run-up time, the voltage droop resulting from the operation of the a.v.r. quadrature current compensation and overfluxing control may need to be considered. Neglect of this may lead to an underestimate of the motor run-up time. An accurate model is therefore required to determine the ratings of the excitation system components to ensure they can withstand the long period when field forcing is applied during motor starting.

The model can also be used to study the system transients following various types of motor switching likely to be encountered in practice.

11.3 - Interaction of multi-gas-turbine-generator system

With strong electrical coupling, generators with similar characteristic on a common power system can be considered as lumped together for the purpose of studying the overall frequency variation. The gas-turbine model described in this thesis can be used for such studies.

In practice, it is common for various types of gas-turbine generating units from different manufacturers to be operating on the same isolated grid system. In such circumstances, the units are likely to have different ratings, inertia constants and fuel scheduling schemes. As a consequence of this, oscillations of power between the machines may be expected. Speed governors do not normally assist in the damping of these oscillations, and in fact because of their inherent time lags, they usually tend to aggravate the situation by producing negative damping and possibly leading to an unstable system. A multi-gas-turbine-generator system model could be developed to study such dynamic interacting problem.
For most multi-machine studies of steam-turbine or hydroelectric generators on an infinite system, the representation of the prime movers and the governor control systems are confined to very simple models. This simple representation is acceptable in large system studies, because the frequency variation following a system disturbance is often small, and the contribution of the prime mover to the short-term transient is unlikely to be significant. However, in most gas-turbine power systems, the capacity of each individual unit may be 20 per cent or more of the total system capacity and the loss of any one generator leads to a relatively large frequency drop and to a long recovery time. In addition, the inertia constants of aero-type gas turbines are generally smaller than those of steam or hydro turbo-generators. The effect of prime-mover control on the transient response of a gas-turbine power system is therefore significant and a detailed representation of such a system is required, even for multi-machine studies. Nevertheless, the degree of complexity of each gas-turbine representation must be a compromise between the cost of computing and the accuracy required.

11.4 - Integrated control system for a gas-turbine generator

For short-term transient studies, due to the slow-acting mechanical governors of steam and hydro turbines, excitation control has been considered as a major means of maintaining generator/system stability. However, the introduction of an electro-hydraulic governor system, offering a fast response to speed variation and the possibility of operating with additional control signals (e.g. rotor acceleration), has made the prime-mover control an effective means of improving system transient stability. The response of the gas-turbine speed governor system is even faster and it is therefore important to include its effect, in addition to the excitation control, in system stability studies. The advantages of integrating both excitation and prime-mover controls appear to be considerable 98-100, but because of the interactions between the control loops, the design of suitable control schemes is complicated, and multi-variable control techniques are required.

Modern control theory provides a suitable approach to the design of multi-variable control systems and such design methods can
be classified under two broad categories: those in which all the loops are designed together, and those in which the loops are designed sequentially using classical control techniques, taking into account the interactions which occur.

In the former category, the state-space approach is probably the best known method and its application to turbo-generator control has received considerable attention in the past two decades. The controller is designed by optimising a quadratic performance index in the time domain, using an nth-order linear approximation to the system to be controlled. Suboptimal techniques for incomplete state feedback have also been applied to turbo-generator system.

The second category of multi-variable control system design methods are those in which individual loops are designed sequentially. One such technique is Rosenbrock's Inverse Nyquist Array method where the emphasis is on the separation of the system into virtually non-interacting loops. This is achieved through the use of compensator matrices to give a diagonally dominant system. The individual loops are then designed using classical frequency response methods which are well suited to interactive graphic computing.

Although most of the work on application of modern control theory to power system studies are concerned with the design of multi-variable control system for steam turbo-generator, similar techniques could be applied to gas-turbine-generator control systems.
CHAPTER 12
CONCLUSION

This thesis has presented a detailed mathematical model of a gas-turbine/generator system comprising a generator, an excitation control system, a gas turbine and its governor control system. Despite the large number of non-linearities inherent in the system, especially in the gas turbine, the model has been designed to enable both large disturbances and small perturbations to be studied over the entire operating range.

The derivation of synchronous machine models based on direct-phase co-ordinates and on the stationary 2-axis reference frame forms the initial part of the investigation. The per-unit system, in relation to machine analysis, is complicated and has been discussed and used by many authors 16,19-22, but the approaches adopted often differ significantly. To avoid any possible confusion over this topic, the base quantities used in the present study are clearly defined. Since machine parameters supplied by manufacturers are not directly applicable to phase and to d-q models, equations relating the three types of per-unit machine parameters are derived. These equations are incorporated into computer programs to facilitate the computer manipulation of the set of generator data obtained from the manufacturer to those required for the machine models.

The accuracy of both machine models has been verified by test results. Because of the computational simplicity of the 2-axis representation, a machine model in this form was used during the major part of the present investigation. However, for unsymmetrical fault studies, the phase model needs to be used.

If the behaviour of synchronous machines is to be accurately simulated in power-system studies, it is essential that their excitation control systems are modelled in sufficient detail. Based on the widely adopted IEEE representation, a mathematical model describing the
dynamic behaviour of a typical gas-turbine brushless excitation control system was formulated. It was demonstrated that the generator and excitation system model can be used to alleviate the need for extensive trial and error optimisation of the a.v.r. lead-lag compensation component settings by field test.

It was intended to study the gas-turbine engine behaviour using the component model approach, but since the individual component characteristics of the engine (e.g. compressor and turbine characteristics) were not readily available, a piecewise-linear model, with the engine parameters being biased by engine speed, had to be adopted. Throughout the investigation, emphasis was placed on the control aspect of a gas-turbine/generator set, so that a piecewise-linear representation is perhaps a better choice than a component model. In addition, the model of the gas-turbine engine forms only a small part of a complex gas-turbine/generator model and it is therefore desirable to use a linearised representation so as to confine the computer program to an acceptable size.

Following the successful derivation of the gas-turbine governor control system model, a complete gas-turbine/generator model was formulated. With the finalised model, various types of switching transients were simulated and comparisons with test results were made. Despite the fact that the engine characteristics used in the investigation are mostly typical engine curves, the predictions are generally in good agreement with test results.

As the developed gas-turbine/generator model undoubtedly provides insight into the system response problem and allows improvements to the control system to be considered. The judicious use of such a model could reduce the amount of test-bed running required and permit control schedules, or even totally new concepts of control, to be investigated without endangering the system and the personnel. This could result in a significant reduction in both the overall development time and cost. Furthermore, since it is common for customers to demand guarantees on maximum voltage and frequency variations and their recovery time following the application or rejection of a specific load, such a model is of considerable importance.
Although the work described in this thesis generally relates to a typical 15 MW gas-turbine/generator installation, the method of analysis and the model developed can readily be applied to similar types of system.

The dynamic behaviour of induction motors, especially those in the MW range, has significant effects on the stability of small gas-turbine power systems and predictions of the corresponding transient behaviour are necessary at an early design stage. Mathematical models of a small single-cage induction motor and a large deep-bar-cage motor have therefore been developed to simulate various types of transients, such as direct-on-line starting, disconnection and reconnection of the power supply to the motor, plugging and 3-phase short-circuit fault. The predictions are generally in good agreement with test results.

Due to the types of test results available, most simulations have been based on the assumption that the induction motors are operating on an infinite supply system. However, a synchronous generator/induction motor system model has been developed to enable the dynamic behaviour of an induction motor operating on a weak supply system to be studied. Since the equations for each machine are written in their own reference frame, the development of a combined system requires axis transformation to bring these axes to a common reference frame. Based on this model, various switching transients and fault conditions can be predicted and the interaction of the synchronous and induction machines can be studied.

In addition to the gas-turbine/generator model, a mathematical model describing the dynamic behaviour of a complete steam turbo-generator system has also been developed. With this model, various system disturbances such as a step change of the a.v.r. reference setting, a full-load rejection and a symmetrical short-circuit fault were simulated and good correlations with test results were obtained. It was found that the introduction of magnetic saturation into the generator model, improves the steady-state predictions but does not always improve the transient predictions. Similar phenomenon has been reported elsewhere \(^4\) and perhaps the method for the implementation of saturation requires further detailed appraisal, with special reference to saturation along the \(q\)-axis \(^{114, 115}\).
To a good approximation, appropriately selected and constant magnetising impedance can be used to study transients in which machine flux is falling. During most fault conditions, the flux falls and the current rises to a high value. Saturation in the leakage-flux paths is then more important than in the main-flux path and moreover, its effect is so complicated that it is difficult to make allowance for such an effect. However, it is essential to allow for saturation if the flux rises as in the case of load rejection tests.

Throughout the present investigation, system dynamic equations are presented in a state-space form and the predictor-corrector routine has been used for the computer solution of these equations. Since this is a multi-step method, the simulation of randomly occurring discontinuities such as amplifier saturation, increased rate limits in the electronic governor and the hydraulic actuator, etc., presents a problem. However, this problem has been overcome by the present author using the method described in Appendix B which has been proved extremely successful.

Areas of further research relating to the present studies are discussed in Chapter 11.
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APPENDICES
APPENDIX A

3-PHASE TO 2-AXIS TRANSFORMATION

Eqn. 2.11 for the phase model of the generator can be re-written as

\[
\begin{bmatrix}
  \mathbf{V}_{fDQ} \\
  \mathbf{V}_{abc}
\end{bmatrix}
= \begin{bmatrix}
  \frac{3}{2} z_{11} & z_{12} \\
  \frac{3}{2} z_{21} & z_{22}
\end{bmatrix}
\begin{bmatrix}
  \mathbf{i}_{fDQ} \\
  \mathbf{i}_{abc}
\end{bmatrix}
\]

and this is to be transformed to the following equation for a d-q machine model

\[
\begin{bmatrix}
  \mathbf{V}_{fDQ} \\
  \mathbf{V}_{dq0}
\end{bmatrix}
= \begin{bmatrix}
  z'_{11} & z'_{12} \\
  z'_{21} & z'_{22}
\end{bmatrix}
\begin{bmatrix}
  \mathbf{i}_{fDQ} \\
  \mathbf{i}_{dq0}
\end{bmatrix}
\]

The relationship between the impedance matrices of the two models are derived in the following sections.

A.1 - Field and damper circuits

The voltage equations for the field and damper circuits for the two models can be written as

\[
\text{dq model, } [\mathbf{V}_{fDQ}] = [z'_{11}] [\mathbf{i}_{fDQ}] + [z'_{12}] [\mathbf{i}_{dq0}] \quad A.3
\]

\[
\text{phase model, } [\mathbf{V}_{fDQ}] = \left[ \frac{3}{2} z_{11} \right] [\mathbf{i}_{fDQ}] + [z_{12}] [\mathbf{i}_{abc}] \quad A.4
\]
Applying eqn. 2.18 to eqn. A.4, the phase model equation becomes

\[
[v_{fDQ}] = \left[\frac{3}{2} z_{11}\right] [i_{fDQ}] + [z_{12}] [C]^{-1} [i_{dq0}] \quad \text{A.5}
\]

By comparing eqns. A.3, A.5 and 2.11, the following result can be obtained.

\[
[z'_{11}] = \frac{3}{2}
\]

\[
\begin{array}{ccc}
M_{fDP} & r_{fD} + L_{fP} & M_{fDP} \\
M_{fDP} & r_{D} + L_{Dp} & r_{Q} + L_{Qp} \\
M_{fDP} & & \\
\end{array}
\quad \text{A.6}
\]

\[
[z'_{12}] = \frac{3}{2}
\]

\[
\begin{array}{ccc}
M_{afP} & & \\
M_{afP} & & \\
M_{afP} & M_{aQP} & \\
\end{array}
\quad \text{A.7}
\]

A.2 - Armature circuit

The voltage equations for the armature circuits for both models can be written as

dq model, \quad [v_{dq0}] = [z'_{21}] [i_{fDQ}] + [z'_{22}] [i_{dq0}] \quad \text{A.8}

phase model, \quad [v_{abc}] = \left[\frac{3}{2} z_{21}\right] [i_{fDQ}] + [z_{22}] [i_{abc}] \quad \text{A.9}

Since, \quad [v_{abc}] = [C]^{-1} [v_{dq0}]

\[
[i_{abc}] = [C]^{-1} [i_{dq0}] \quad \text{A.10}
\]
Eqn. A.9 becomes

\[ [\nu_{dq0}] = [C] \left[ \frac{3}{2} z_{21} \right] [i_{fDQ}] + [C] [z_{22}] [C]^{-1} [i_{dq0}] \quad A.11 \]

Comparing eqns. A.8, A.11 and 2.11.

\[
z_{21}' = \frac{3}{2}
\]

\[
\begin{array}{ccc}
M_{afP} & M_{afP} & \omega M_{aQ} \\
-\omega M_{af} & -\omega M_{af} & M_{aQ^p}
\end{array}
\]

\[
z_{22}' =
\]

\[
\begin{array}{ccc}
r_d+L_dP & \omega L_q \\
-\omega L_d & r_q+L_qP \\
 & r_0+L_0P
\end{array}
\]

Where

\[
L_d = L_{ao} - M_{so} + \frac{3}{2} L_{a2} \quad A.14
\]

\[
L_q = L_{ao} - M_{so} - \frac{3}{2} L_{a2}
\]

\[
L_0 = L_{ao} + 2M_{so}
\]

\[
r_d = r_q = r_0 = r_a \quad A.15
\]
Thus, the impedance matrix for the d-q-0 model is

$$
[z'] = \begin{pmatrix}
\frac{3}{2}(r_f + L_f p) & \frac{3}{2}M_D p & \frac{3}{2}M_a f p \\
\frac{3}{2}M_D p & \frac{3}{2}(r_D + L_D p) & \frac{3}{2}M_a f p \\
\frac{3}{2}M_a f p & \frac{3}{2}M_a f p & \frac{3}{2}(r_Q + L_Q p) & \frac{3}{2}M_a Q p \\
\frac{3}{2}M_a f p & \frac{3}{2}M_a f p & \frac{3}{2}M_a Q p & r_d + L_d p & \omega L_q \\
\frac{3}{2}M_a f & \frac{3}{2}M_a f & \frac{3}{2}M_a Q p & -\omega L_d & r_q + L_q p \\
\frac{3}{2}M_a f & \frac{3}{2}M_a f & \frac{3}{2}M_a Q p & -\omega L_d & r_q + L_q p \\
& & & & r_0 + L_0 p
\end{pmatrix}
$$
APPENDIX B

SIMULATION OF SYSTEM LIMITS

In digital simulation studies, randomly occurring discontinuities such as those caused by amplifier saturation, if not correctly handled, will result in an incorrect solution or even numerical instability. The method proposed here involves the addition of a clamping function VCF to the state equation of the system wherever limits are imposed.

If the state equation relating an amplifier output voltage \( x \) is

\[
x = ax + bu,
\]

then to impose saturation limits \( x_{\text{max}} \) and \( x_{\text{min}} \) on the amplifier, it is necessary to modify eqn. B.1 to the form

\[
x = ax + bu + VCF
\]

The value of VCF is adjusted by the following equations at every integration step

\[
VCF = -ERR_x \left( \frac{x - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} \right)^n \quad \text{if } ERR_x > 0 \tag{B.3}
\]

\[
VCF = -ERR_x \left( \frac{x - x_{\text{max}}}{x_{\text{min}} - x_{\text{max}}} \right)^n \quad \text{if } ERR_x < 0 \tag{B.4}
\]

where \( ERR_x = ax + bu \)

\( n \) = an even integer and the effect of its value on the limiting characteristic is shown in fig. B.1.
Eqns. B.3 and B.4 show that as \( x \) approaches its limits, VCF approaches \(-\text{ERR}_x\) and \( x \) of eqn. B.2 approaches zero. This implies that \( x \) cannot exceed its limiting values.

Fig. B.1  Effect of \( n \) on the limiting characteristic

Clamping function removed

\( x_{\text{max}} \)

the value of \( n \) increases
APPENDIX C

DERIVATION OF d-q MODEL PARAMETERS FROM THE MANUFACTURER SUPPLIED DATA

The machine parameters suitable for the d-q model (e.g. \( r_{ff} \), \( r_{DD} \), \( r_{QQ} \), \( r_{d} \), \( r_{q} \), \( L_{ff} \), \( L_{DD} \), \( L_{QQ} \), \( L_{d} \), \( L_{q} \), \( L_{md} \), \( L_{mq} \)) are required to be derived from data supplied by the manufacturers (e.g. \( x_{d} \), \( x_{d}' \), \( x_{q} \), \( x_{q}' \), \( x_{a} \), \( T_{d} \), \( T_{d}' \), \( T_{q} \), \( T_{q}' \), \( T_{a} \) and \( r_{a} \)).

Based on the equivalent circuits for a synchronous machine shown in fig. C.1, an approximate physical interpretation of the machine reactances and time constants can be made and the required information can hence be deduced.

Thus, under steady state condition

\[
x_{d} = x_{a} + x_{md}
\]

\[\quad + \quad x_{md} = x_{d} - x_{a} \quad \text{C.1}\]

\[
x_{q} = x_{a} + x_{mq}
\]

\[\quad + \quad x_{mq} = x_{q} - x_{a} \quad \text{C.2}\]

and if no damping is present

\[
x_{d}' = x_{a} + \frac{x_{md} x_{ff}}{x_{md} + x_{ff}}
\]

\[\quad + \quad x_{ff} = x_{md} + \frac{(x_{d}' - x_{a}) x_{md}}{x_{d} - x_{d}'} \quad \text{C.3}\]
\[ x'_q = x_q \]

\[ T'_d = \frac{1}{\omega_q^{\text{off}}} \left[ x_{\text{ffl}} + \frac{x_{\text{md}} x_a}{x_{\text{md}} + x_a} \right] \]

\[ + \quad r_{\text{ff}} = \frac{1}{\omega_q^{\text{off}}} \left[ x_{\text{ffl}} + \frac{x_{\text{md}} x_a}{x_d} \right] \]

\[ C.4 \]

If damping is present

\[ x''_d = x_a + \frac{x_{\text{md}} x_{\text{ffl}} x_{\text{DDI}}}{x_{\text{md}} x_{\text{ffl}} + x_{\text{md}} x_{\text{DDI}} + x_{\text{ffl}} x_{\text{DDI}}} \]

\[ + \quad x_{\text{DD}} = x_{\text{md}} + \frac{x_{\text{md}} x_{\text{ffl}} (x''_d - x_a)}{x_{\text{md}} x_{\text{ffl}} - (x_{\text{md}} + x_{\text{ffl}})(x''_d - x_a)} \]

\[ C.5 \]

\[ x''_q = x_a + \frac{x_{\text{mq}} x_{\text{QQ}}}{x_{\text{mq}} + x_{\text{QQ}}} \]

\[ + \quad x_{\text{QQ}} = x_{\text{mq}} + \frac{(x''_q - x_a)x_{\text{mq}}}{x_q - x'_q} \]

\[ C.6 \]

\[ T''_d = \frac{1}{\omega_q^{\text{DD}}} \left( x_{\text{DDI}} + \frac{x_{\text{md}} x_{\text{ffl}} x_a}{x_{\text{md}} x_{\text{ffl}} + x_{\text{md}} x_a + x_{\text{ffl}} x_a} \right) \]

\[ + \quad r_{\text{DD}} = \omega_q^{\text{DD}} \left( x_{\text{DDI}} + \frac{x_{\text{md}} x_{\text{ffl}} x_a}{x_{\text{md}} x_{\text{ffl}} + x_{\text{md}} x_a + x_{\text{ffl}} x_a} \right) \]

\[ C.7 \]

\[ T''_q = \frac{1}{\omega_q^{\text{QQ}}} \left( x_{\text{QQI}} + \frac{x_{\text{mq}} x_a}{x_q} \right) \]

\[ + \quad r_{\text{QQ}} = \omega_q^{\text{QQ}} \left( x_{\text{QQI}} + \frac{x_{\text{mq}} x_a}{x_q} \right) \]

\[ C.8 \]
If the armature resistance $r_a$ is unavailable, it can be derived from $T_a$. Since $T_a$ applies to the decay of the $d,c$ component of the current in the armature circuit, and the leakage flux path is considered to vary between $x''_d$ and $x''_q$, $T_a$ is commonly taken as

$$T_a = \frac{1}{\omega_0 r_a} \frac{x''_d + x''_q}{2} \quad \text{C.9}$$

Hence

$$r_d = r_q = r_a = \frac{1}{\omega_0 T_a} \frac{x''_d + x''_q}{2} \quad \text{C.10}$$

---

Fig. C.1  Approximate equivalent circuits for a synchronous machine
(a)  $d$-axis equivalent circuit
(b)  $q$-axis equivalent circuit
APPENDIX D

BASE QUANTITIES FOR THE 15 MW GENERATOR

D.1 - Generator ratings

- 18.875 MVA
- 15.1 MW
- 13.8 kV
- 60 Hz
- 1800 r.p.m.

D.2 - Stator base quantities

\[ V_{ao} = 18.875 \text{ MVA} \]
\[ V_{ao} = 11.266 \text{ kV} \]
\[ I_{ao} = 1.117 \text{ kA} \]
\[ Z_{ao} = 10.086 \Omega \]
\[ \omega, f = 377 \text{ rad/s or } 60 \text{ Hz} \]
\[ L_o = \frac{Z_{ao}}{\omega} = 0.027 \text{ H} \]
\[ \psi_{ao} = I_{ao} L_o = 30.0 \]

D.3 - Rotor base quantities

From the open-circuit characteristic of the generator, the field current required to produce rated open-circuit phase voltage \( V_{ao} \) on the air gap line is 224 A \( I_f \).

Since, \[ V_{ao} = \omega L_{afd} I_f \]

Therefore, \[ L_{afd} = 0.133 \text{ H} \]
Also, \[ x_{ad} = x_d - x_a \]  
\[ = 1.78 - 0.12 \]  
\[ = 1.66 \text{ p.u.} \]  

\[ L_{ad} = 1.66 \times L_0 = 0.045 \text{ H} \]  

Let base field current \( (I_{fo}) \) be defined as the current which induces in each stator phase a peak open-circuit voltage \( \omega_0 L_{ad} I_{ao} \).

Thus,
\[ L_{ad} I_{ao} = L_{afd} I_{fo} \]  
\[ I_{fo} = I_{ao} \frac{L_{ad}}{L_{afd}} \]  
\[ = 1.117 \times \frac{0.045}{0.133} \]  
\[ = 378 \text{ A} \]  

For power invariance
\[ V_{fo} I_{fo} = V A_o \]  
\[ V_{fo} = \frac{18.875}{378} \]  
\[ = 50 \text{ kV} \]  
\[ Z_{fo} = \frac{V_{fo}}{I_{fo}} = 132.2 \Omega \]
APPENDIX E

EXPRESSIONS FOR THE 3-PHASE SHORT-CIRCUIT TRANSIENTS

The equations for the analytical solution of the generator 3-phase short-circuit transient currents and electrical torque are given as follows: 16

\[ i_d = \nu q_0 \left[ \frac{1}{x_d} + \left( \frac{1}{x'_d} - \frac{1}{x_d} \right) e^{\frac{-t}{T_d}} + \left( \frac{1}{x''_d} - \frac{1}{x_d} \right) e^{\frac{-t}{T''_d}} \right] - \frac{\nu q_0}{x''_d} e^{\frac{-t}{T_a}} \cos \omega_0 t \]

\[ i_q = \frac{\nu q_0}{x''_q} e^{\frac{-t}{T_a}} \sin \omega_0 t \]

\[ i_a = i_d \cos (\omega_0 t + \lambda) + i_q \sin (\omega_0 t + \lambda) \]

\[ i_f = i_{f0} + i_{f0} \left( \frac{x_d - x'_d}{x'_d} \right) e^{\frac{-t}{T_d}} - \frac{T_D}{T_d} e^{\frac{-t}{T''_d}} - \frac{T_D}{T_d} e^{\frac{-t}{T_a \cos \omega_0 t}} \]

\[ T_e = \nu q_0^2 e^{\frac{-t}{T_a}} \left[ \frac{1}{x_d} + \left( \frac{1}{x'_d} - \frac{1}{x_d} \right) e^{\frac{-t}{T_d}} + \left( \frac{1}{x''_d} - \frac{1}{x_d} \right) e^{\frac{-t}{T''_d}} \right] \sin \omega_0 t + \frac{\nu q_0^2}{2} e^{\frac{-2t}{T_a}} \left( \frac{1}{x''_q} - \frac{1}{x''_d} \right) \sin 2\omega_0 t \]

where \( T_D = x_{DD} / (\omega_0 r_{DD}) \)

\( \lambda \) = angle between the axis of phase a and the direct axis at the instance of short circuit applied.
APPENDIX F
CONVERSION OF d-q MODEL PARAMETERS TO PHASE MODEL PARAMETERS

The machine parameters required for the phase model are:

\[ r_f', \quad r_D', \quad r_Q', \quad r_a \]
\[ L_f', \quad L_D', \quad L_Q', \quad L_{ao}, \quad L_{a2}, \quad M_{so} \]
\[ M_{af}, \quad M_{aQ}, \quad M_{fD} \]

Using the relationship given in eqns. 2.23 and 2.24, the above parameters can be expressed in terms of the d-q model parameters as follows:

\[ r_f = \frac{2}{3} r_{ff} \]
\[ r_D = \frac{2}{3} r_{DD} \]
\[ r_Q = \frac{2}{3} r_{QQ} \]
\[ r_a = r_d = r_q \quad \text{F.1} \]

\[ L_f = \frac{2}{3} L_{ff} \]
\[ L_D = \frac{2}{3} L_{DD} \]
\[ L_Q = \frac{2}{3} L_{QQ} \]
\[ M_{af} = M_{fD} = \frac{2}{3} L_{md} \quad \text{F.2} \]
\[ M_{aQ} = \frac{2}{3} L_{mq} \]
\[ L_{ao} = \frac{x_a}{\omega_o} + \frac{M_{af} + M_{aQ}}{2} \]

\[ L_{a2} = \frac{M_{af} + M_{aQ}}{2} \]

\[ M_{so} = -\frac{M_{af} + M_{aQ}}{4} \]
APPENDIX G
DERIVATION OF A 3-PHASE 3-WIRE MODEL

With an isolated neutral the sum of the phase currents is always zero, i.e.

\[ i_a + i_b + i_c = 0 \]

or

\[ i_c = -i_a - i_b \]  \hspace{1cm} (G.1)\]

This constraint allows the rank of the impedance matrix of eqn. 2.11 to be reduced by one and the voltage equation for the 3-phase 3-wire model is derived as follows.

Let \( C_{43} \) be the connection matrix defined as

\[ i = C_{43} i_m \]  \hspace{1cm} (G.2)\]

or

\[
\begin{bmatrix}
  i_f' \\
  i_D'
  i_Q'
  i_a
  i_b
  i_c
\end{bmatrix}
=\begin{bmatrix}
  1 & & & & & \\
  & 1 & & & & \\
  & & 1 & & & \\
  & i_a & i_b & i_c & & \\
  & & & & -1 & -1
\end{bmatrix}
\begin{bmatrix}
  i_f' \\
  i_D'
  i_Q'
  i_a
  i_b
\end{bmatrix}
\]  \hspace{1cm} (G.3)\]
For power invariance

\[ v_m = C_{43}^t \cdot v \]  

\[ \begin{array}{c|c|c|c}
\hline
v_f & v_D & v_Q & v_{a-c} \\
\hline
1 & 1 & 1 & 1 \\
\hline
\end{array} \]

\[ \begin{array}{c|c|c|c|c|c}
\hline
v_f & v_D & v_Q & v_a & v_b & v_c \\
\hline
\hline
\end{array} \]

The equation for the impedance \( Z_m \) becomes

\[ Z_m = C_{43}^t \cdot Z \cdot C_{43} \]

\[ = \begin{array}{cccc}
Z_f & Z_{fD} & Z_{fQ} & Z_{fa-fc} \\
Z_{fD} & Z_D & Z_{DQ} & Z_{Da-Dc} \\
Z_{fQ} & Z_{DQ} & Z_Q & Z_{Qa-Qc} \\
Z_{fa-fc} & Z_{Da-Dc} & Z_{Qa-Qc} & Z_{c+Z_{ab}} \\
Z_{fb-fc} & Z_{Db-Dc} & Z_{Qb-Qc} & Z_{b+Z_c} \\
\end{array} \]

and a complete 3-phase 3-wire model voltage equation is shown in eqn. G.8.
<table>
<thead>
<tr>
<th>$v_f$</th>
<th>$v_f + pL_f$</th>
<th>$pM_{FD}$</th>
<th>$pM_{af}(\cos\theta_a - \cos\theta_c)$</th>
<th>$\sqrt{3}pM_{af}\sin\theta_a$</th>
<th>$i_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_D$</td>
<td>$pM_{FD}$</td>
<td>$v_D + pL_D$</td>
<td>$pM_{af}(\cos\theta_a - \cos\theta_c)$</td>
<td>$\sqrt{3}pM_{af}\sin\theta_a$</td>
<td>$i_D$</td>
</tr>
<tr>
<td>$v_Q$</td>
<td>$r_Q + pL_Q$</td>
<td>$pM_{aQ}(\sin\theta_a - \sin\theta_c)$</td>
<td>$-\sqrt{3}pM_{aQ}\cos\theta_a$</td>
<td></td>
<td>$i_Q$</td>
</tr>
<tr>
<td>$v_a - v_c$</td>
<td>$pM_{af}(\cos\theta_a - \cos\theta_c)$</td>
<td>$pM_{af}(\cos\theta_a - \cos\theta_c)$</td>
<td>$pM_{aQ}(\sin\theta_a - \sin\theta_c)$</td>
<td>$2r_a + r_1 + r_3 + (2L_{ao} - 2M_{so} + L_1 + L_3)p$</td>
<td>$i_a$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$r_a + r_3 + (L_{ao} - M_{so} + L_3)p + 3pL_{a2}\cos 2\theta_b$</td>
<td></td>
</tr>
<tr>
<td>$v_b - v_c$</td>
<td>$\sqrt{3}pM_{af}\sin\theta_a$</td>
<td>$\sqrt{3}pM_{af}\sin\theta_a$</td>
<td>$-\sqrt{3}pM_{aQ}\cos\theta_a$</td>
<td>$r_a + r_3$</td>
<td>$i_b$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$2r_a + r_2 + r_3 + (2L_{ao} - 2M_{so} + L_2 + L_3)p + 3pL_{a2}\cos 2\theta_a$</td>
<td></td>
</tr>
</tbody>
</table>

G.8
APPENDIX H

PRINCIPAL DATA FOR GOLDFINGTON STEAM TURBO-GENERATOR UNIT

H.1 - Generator and transmission system

Supplied data

<table>
<thead>
<tr>
<th>Generator</th>
<th>ratings</th>
<th>37.5 MVA 0.8 pf 11.8kV 3000 r.p.m</th>
</tr>
</thead>
<tbody>
<tr>
<td>x (_d)</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>x' (_d)</td>
<td>0.271</td>
<td></td>
</tr>
<tr>
<td>x'' (_d)</td>
<td>0.171</td>
<td></td>
</tr>
<tr>
<td>T' (_d)</td>
<td>0.93 s</td>
<td></td>
</tr>
<tr>
<td>T'' (_d)</td>
<td>0.025 s</td>
<td></td>
</tr>
<tr>
<td>x (_a)</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>r (_a)</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>5.3 s</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transformer</th>
<th>r (_{tx})</th>
<th>1.35% on 100 MVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>x (_{tx})</td>
<td>35.4% on 100 MVA</td>
<td></td>
</tr>
</tbody>
</table>

| Transmission | r \(_{tl}\) | 4.6% on 100 MVA |
| line        | x \(_{tl}\) | 12.65% on 100 MVA |

(All parameters are in p.u. unless otherwise specified)
Calculated data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base stator voltage</td>
<td>9.68 kV peak per phase</td>
</tr>
<tr>
<td>Base stator current</td>
<td>2.59 kA peak per phase</td>
</tr>
<tr>
<td>Base power</td>
<td>37.5 MVA</td>
</tr>
<tr>
<td>Base stator impedance</td>
<td>3.73 Ω</td>
</tr>
<tr>
<td>Base frequency</td>
<td>100π rad/s</td>
</tr>
<tr>
<td>Base field voltage</td>
<td>153.5 kV</td>
</tr>
<tr>
<td>Base field current</td>
<td>244A</td>
</tr>
<tr>
<td>Base field impedance</td>
<td>630 Ω</td>
</tr>
<tr>
<td>Xmd</td>
<td>1.86</td>
</tr>
<tr>
<td>Xmq</td>
<td>1.675</td>
</tr>
<tr>
<td>Xf1</td>
<td>0.14</td>
</tr>
<tr>
<td>XddI</td>
<td>0.04</td>
</tr>
<tr>
<td>XqqI</td>
<td>0.04</td>
</tr>
<tr>
<td>Xa</td>
<td>0.14</td>
</tr>
<tr>
<td>Xt</td>
<td>0.1802</td>
</tr>
<tr>
<td>Rff</td>
<td>0.00107</td>
</tr>
<tr>
<td>Rdd</td>
<td>0.00318</td>
</tr>
<tr>
<td>Rqq</td>
<td>0.00318</td>
</tr>
<tr>
<td>Ra</td>
<td>0.002</td>
</tr>
<tr>
<td>Rt</td>
<td>0.0223</td>
</tr>
</tbody>
</table>

(All parameters are in p.u. unless otherwise specified)
## H.2 - Excitation system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_i$</td>
<td>1</td>
</tr>
<tr>
<td>$K_c$</td>
<td>0.00159</td>
</tr>
<tr>
<td>$K_{r1}$</td>
<td>52.0</td>
</tr>
<tr>
<td>$K_{r2}$</td>
<td>12.2</td>
</tr>
<tr>
<td>$K_e$</td>
<td>3.06</td>
</tr>
<tr>
<td>$K_{rf}$</td>
<td>0.0139</td>
</tr>
<tr>
<td>$K_{ef}$</td>
<td>0.00525</td>
</tr>
<tr>
<td>$B_{r1}$</td>
<td>65.4 V</td>
</tr>
<tr>
<td>$B_{r2}$</td>
<td>-146.7 V</td>
</tr>
<tr>
<td>$B_e$</td>
<td>-15.0 V</td>
</tr>
<tr>
<td>$T_{r1}$</td>
<td>0.044 s</td>
</tr>
<tr>
<td>$T_{r2}$</td>
<td>0.1 s</td>
</tr>
<tr>
<td>$T_e$</td>
<td>0.2 s</td>
</tr>
<tr>
<td>$T_{rf1}$</td>
<td>0.1 s</td>
</tr>
<tr>
<td>$T_{rf2}$</td>
<td>0.1 s</td>
</tr>
<tr>
<td>$T_{ef1}$</td>
<td>2.0 s</td>
</tr>
<tr>
<td>$T_{ef2}$</td>
<td>2.0 s</td>
</tr>
<tr>
<td>$V_{r1\text{max}}$</td>
<td>51.6 V</td>
</tr>
<tr>
<td>$V_{r1\text{min}}$</td>
<td>1.8 V</td>
</tr>
<tr>
<td>$V_{r2\text{max}}$</td>
<td>227 V</td>
</tr>
<tr>
<td>$V_{r2\text{min}}$</td>
<td>0 V</td>
</tr>
</tbody>
</table>

## H.3 - Turbine and governor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{hy}$</td>
<td>1.33</td>
</tr>
<tr>
<td>$K_v$</td>
<td>1.42</td>
</tr>
<tr>
<td>$K_{sg}$</td>
<td>0.001088</td>
</tr>
<tr>
<td>$u_{vo}$</td>
<td>-0.267</td>
</tr>
<tr>
<td>$T_{hy1}$</td>
<td>0.1 s</td>
</tr>
<tr>
<td>$T_{hy2}$</td>
<td>0.188 s</td>
</tr>
<tr>
<td>$T_s$</td>
<td>0.49 s</td>
</tr>
</tbody>
</table>
APPENDIX I

DERIVATION OF TRANSFER FUNCTION FOR THE A.V.R AMPLIFIER

The a.v.r. amplifier circuit shown in fig. 6.4 can be represented by the equivalent circuit shown in fig. 1.1.

By applying Thevenin's theorem at A-A the circuit of fig. 1.1, can be simplified to that given in fig. 1.2.

Fig. 1.1 Equivalent circuit for the a.v.r. amplifier

Fig 1.2 Simplified equivalent circuit
where \[ V_{AA} = \frac{1 + sC_2 R_2}{1 + sC_2 (R_2 + R_4)} \] 
\[ z_{AA} = \frac{R_4 (1 + sC_2 R_2)}{1 + sC_2 (R_2 + R_4)} \] 
\[ z = \frac{R_3 (1 + sC_1 R_1)}{1 + sC_1 (R_1 + R_3)} \]

Apply Kirchhoff's current law at point \( x \)

\[ \frac{V_i}{R_5} + \frac{V_{AA}}{z + z_{AA}} = 0 \]

and on substituting for 1.1, 1.2 and 1.3 into 1.4, the following transfer function can be obtained.

\[ \frac{V_o}{V_i} = -\frac{1}{R_i} \frac{R_3 (1 + sC_1 R_1) [1 + sC_2 (R_2 + R_4)] + R_4 (1 + sC_2 R_2) [1 + sC_1 (R_1 + R_3)]}{(1 + sC_2 R_2) [1 + sC_1 (R_1 + R_3)]} \]

Since, \( R_3 \gg R_4 \)

Therefore, \[ \frac{V_o}{V_i} = -\frac{R_3}{R_5} \frac{(1 + sC_1 R_1) [1 + sC_2 (R_2 + R_4)]}{[1 + sC_1 (R_1 + R_3)] (1 + sC_2 R_2)} \]

or,

\[ \frac{V_o}{V_i} = -K_1 \frac{(1 + sT_2) (1 + sT_3)}{(1 + sT_4) (1 + sT_5)} \]

where \( K_1 = \frac{R_3}{R_5} \)

\( T_2 = C_1 R_1 \)
\( T_4 = C_1 (R_1 + R_3) \)
\( T_3 = C_2 (R_2 + R_4) \)
\( T_5 = C_2 R_2 \)
APPENDIX J

CALCULATION OF EXCITATION SYSTEM DATA

The excitation system data required for the IEEE type 2 representation are derived as follows:

J.1 - Input filter time constant $T_R$

\[ T_R = 0.02 \text{ s} \]

J.2 - A.V.R. time constant $T_A$

From eqns. 6.4 and 6.17, $T_A$ can be expressed as

\[ T_A = \frac{1}{2f} \]

\[ = \frac{1}{2 \times 300} \quad \text{(given } f = 300 \text{ Hz)} \]

\[ T_A = 0.0017 \text{ s} \]

J.3 - A.V.R. stabilising circuit time constants $T_{F1}$ and $T_{F2}$

The a.v.r. amplifier compensation circuit components shown in fig. 6.4 are given as

\[ R_1 = 120 \text{ k}\Omega \]
\[ C_1 = 13.6 \mu\text{F} \]
\[ R_2 = 220 \Omega \]
\[ C_2 = 22.67 \mu\text{F} \]
\[ R_3 = 2.2 \text{ M}\Omega \]
\[ R_4 = 10 \text{ k}\Omega \]
\[ R_5 = 22.9 \text{ k}\Omega \]

J.4
From eqns. 6.12 and 1.9
\[ T_{F1} = T_2 = C_1 R_1 \]

\[ T_{F1} = 1.632 \text{ s} \]  \hspace{1cm} J.5

From eqns. 6.13 and 1.11
\[ T_{F2} = T_3 = C_2 (R_2 + R_4) \]

\[ T_{F2} = 0.232 \text{ s} \]  \hspace{1cm} J.6

J.4 - Exciter time constant \( T_E \)

The exciter open-circuit time constant is given to be 1.02 s, therefore

\[ T_E = 1.02 \text{ s} \]  \hspace{1cm} J.7

J.5 Exciter constant related to exciter field \( K_E \)

This constant is normally taken as unity \(^{12} \),

i.e. \( K_E = 1.0 \)  \hspace{1cm} J.8

J.6 - Base field voltage \( E_{FD0} \)

The exciter base voltage \( E_{FD0} \) is defined as that voltage required to produce rated generator voltage on the airgap line. In the present investigation this value is found to be 26.8 V

i.e. \( E_{FD0} = 26.8 \text{ V} \)  \hspace{1cm} J.9
Fig. J.1 shows that $E_{FDMAX}$ is the p.u. exciter output voltage at the field forcing condition, while $V_{RMAX}$ is a fictitious maximum a.v.r. output voltage defined as the corresponding p.u. voltage on the exciter air gap line.
With the exciter field forcing current of 11.8A, it follows from fig. 6.10 that

\[ E_{FDMAX} = 167.0 \, V/26.8 \, V = 6.23 \, \text{p.u.} \quad \text{J.10} \]

\[ V_{RMAX} = 536.0 \, V/26.8 \, V = 20.0 \, \text{p.u.} \quad \text{J.11} \]

also,

\[ E_{FDMin} = 0.0 \quad \text{J.12} \]

\[ V_{RMin} = 0.0 \quad \text{J.13} \]

**J.8 - Exciter saturation factors \( S_{E_{MAX}} \) AND \( S_{E_{0.75MAX}} \)**

The relationship between \( E_{FD} \) AND \( V_R \) shown in fig. 6.6 can be written as

\[ E_{FD} = \frac{1}{K_E} (V_R - S_E E_{FD}) \quad \text{J.14} \]

which on re-arranging gives

\[ S_E = \frac{V_R - K_E}{E_{FD}} \quad \text{J.15} \]

therefore,

\[ S_{E_{MAX}} = \frac{V_{RMAX} - K_E}{E_{FDMAX}} \quad \text{J.16} \]

\[ = \frac{20.0}{6.23} - 1 \]

\[ S_{E_{MAX}} = 2.21 \quad \text{J.17} \]
similarly,

\[ S_{E0.75\text{MAX}} = \frac{V_{R0.75\text{MAX}} - K_E}{0.75 E_{\text{FDMAX}}} \]

\[ = \frac{402}{125.25} - 1 \]

\[ S_{E0.75\text{MAX}} = 2.21 \]

J.9 - Voltage regulator forward gain \( K_A \)

The absolute gain of the a.v.r. amplifier \( K_1 \), given in eqn. 1.8 is

\[ K_1 = \frac{R_3}{R_5} \]

\[ = \frac{2.2 \, M\Omega}{22.9k\Omega} \]

\[ = 96 \]

and the absolute gain across the firing control circuit and the thyristor bridge was measured to be about 5.33.

Therefore the overall absolute gain of the a.v.r. is

\[ K_{A\text{abs}} = 96.0 \times 5.33 \]

\[ = 512 \]
When the generator was operating on no load but at rated voltage and speed, the measured a.v.r. output $V_{Rabs}$ was about 15.1 V. To achieve this voltage, the required absolute error voltage at the a.v.r. amplifier input (corresponding to ERR of fig. 6.6) is

$$ERR_{abs} = \frac{15.1}{512} = 0.0295 \text{ V}$$

Since the a.v.r. is set such that 1 p.u. generator terminal voltage corresponds to about 13 V at the a.v.r. amplifier input, $ERR_{abs}$ is hence normalised to 13 V,

$$ERR = \frac{ERR_{abs}}{13} = 0.00227 \text{ p.u.}$$

With an exciter field current of 2.2 A at the no load condition, the corresponding $V_R$ can be obtained from the exciter airgap line shown in fig. 6.10.

$$i.e. \quad V_R = \frac{101.0}{26.8} = 3.77 \text{ p.u.}$$

Therefore, the p.u. a.v.r. forward gain is

$$K_A = \frac{3.77}{0.00227} = 1660.0$$
With reference to eqn. 6.18, the expression for the absolute value of \( K_F \) can be written as:

\[
K_{F\text{abs}} = \frac{1}{K_{A\text{abs}}} \left( T_4 + T_5 - T_{F_1} - T_{F_2} \right)
\]

where \( T_4 = C_1 (R_1 + R_3) = (13.6 \ \mu F)[(0.12+2.2)M\Omega] = 31.55 \ s \)

\[ T_5 = C_2 R_2 = (22.67 \ \mu F)(0.22K\Omega) = 0.0045 \ s \]

Substituting from eqns. J.5, J.6, J.21 and J.27 enables \( K_{F\text{abs}} \) to be calculated as

\[
K_{F\text{abs}} = \frac{1}{512} (31.55+0.0045-1.632-0.232) = 0.058
\]

When the generator is on no load at rated voltage, the required a.v.r. output voltage is 15.1 V.

i.e. \( V_{R\text{abs}} = 15.1 \ V \)

With an absolute gain of 0.058, the corresponding stabilising circuit feedback voltage is

\[
V_{RF\text{Babs}} = 15.1 \times 0.058 = 0.876 \ V
\]

If this is normalised to 13 V, then

\[
\frac{V_{RF\text{B}}}{13} = 0.067 \text{ p.u.}
\]

Therefore,

\[
K_F = \frac{V_{RF\text{B}}}{V_R} = \frac{0.0674}{3.77} = 0.018 \text{ p.u.}
\]