Modelling the behaviour of soil-cooling tower-interaction

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Modelling the Behaviour of Soil-Cooling Tower-Interaction

By

Emad Hassan Aly

Department of Civil and Building Engineering
Loughborough University

Loughborough University

A dissertation thesis submitted in partial fulfilment of the requirements for the award of PhD, at Loughborough University

March 2007

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I dedicate this thesis to my wife and children for their patience and understanding during the time of this research.
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Modelling the Behaviour of Soil-Cooling Tower-Interaction

Abstract

Natural draught cooling towers belong to a category of exceptional civil engineering structures. These towers are an effective and economic choice among all technical solutions for the prevention of thermal pollution of natural water resources caused by heated cooling water in various industrial facilities. They are therefore widely used in most electric power generation units, chemical and petroleum industries and space conditioning processes. The cooling tower shell is the most important part of the cooling tower, both in technical and financial terms and also the most sensitive, since its collapse would put all or part of the cooling tower out of action for a considerable length of time.

In this thesis, the 2D and 3D behaviour of soil-cooling tower-interaction, via the idealisation of the structure and soil on the resulting parameters, have been investigated, taking into consideration the effect of temperature changes in the cooling tower on the simultaneous interaction of the cooling tower and underlying soil. The temperature effect has been considered because it plays an important role in the design of the cooling towers.

The capabilities of the two-dimensional Geotechnical Finite Element Analysis Program (GeoFEAP) have been updated in this project and the new version has been referred to as GeoFEAP2. New modelling capabilities and the ability to model 3D problems, with accompanying postprocessing features, were introduced, including 3D first order 8-noded hexahedrons. In addition, the Drucker-Prager yield criterion was programmed in GeoFEAP2 to model the elasto-plastic behaviour of the soil. A new 4-noded quadrilateral flat shell element, based on discrete the Kirchhoff’s quadrilateral plate bending element, was also added to the software to model the elastic behaviour of the cooling tower shell. Furthermore, this element was modified to accommodate a temperature profile. The new software (GeoFEAP2) was then validated for soil behaviour and using several standard widely-used benchmark problems and the results compared well with the analytical and/or numerical results obtained by other researchers. A 3D finite element model was created, comprising the cooling tower, columns support, foundation, and elasto-plastic soil behaviour.
The analyses of soil-cooling tower-interaction in this thesis have indicated the need to model the soil and structure as a combined problem, rather than by applying loads onto soil as geotechnical engineers' model, or by assuming the soil comprises springs and model the cooling tower, as structural engineers' model. The results have shown how unrealistic the latter two approaches are. In addition, the analysis necessitates the incorporation of thermal effects when modelling cooling tower problems. Moreover, from a design point of view, it has been recommended that circular footing with two cross-columns is better than pad footings and/or one column. Several other conclusions have been made that would improve the modelling of soil-cooling tower-interaction. Furthermore, the designer needs to ensure that enough modelling of soil conditions is done and an extensive site investigation is required to ensure that the variation in soil properties is represented correctly. Finally, the engineer needs also to ensure that the site tests performed to measure shear strength with depth via drilling and other methods needs to go deep enough into the ground to ensure that enough site information is available when designing the cooling tower.
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## Nomenclature

- \( a \) : Radius at throat level of cooling tower
- \{a\} : Nodal displacement vector
- \( A \) : Area of transfer surface per unit of volume tower
- \( A \) : Integer constant
- \( b \) : Characteristic dimension of the cooling tower shell, defined by equations (7.2) and (7.3)
- \( b_x, b_y, b_z \) : The body forces per unit volume in the \( x, y \) and \( z \) directions, respectively
- \([B]\) : Strain-displacement matrix
- \( c \) : Cohesion of the soil
- \{d\} : Displacement inside the element
- \( D \) : Flexural rigidity of the plate
- \[D]\) : Matrix describes the material moduli
- \( E \) : Young’s modulus
- \[E]\) : Matrix defined by equation (3.33)
- \( g \) : Magnitude of the acceleration due to gravity
- \( g_x, g_y, g_z \) : Components of the acceleration vector due to gravity in the \( x, y \) and \( z \) directions, respectively
- \( g_w \) : Water unit weight
- \( g \) : The gravitational force per unit mass
- \( G \) : Gas mass
- \( h_1, h_2 \) : Horizontal soil layers
- \( h_i \) : Arbitrary functions, chosen as displacement
- \( h_s, h_t \) : Vertical distances from the throat to the base and top of the cooling tower shell, respectively
- \( H \) : Enthalpy
- \( i, j, k \) : The unit vectors in the \( x, y \) and \( z \) directions, respectively
- \[J]\) : Jacobian matrix
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>Heat transfer coefficient</td>
</tr>
<tr>
<td>( k )</td>
<td>Permeability of the porous media</td>
</tr>
<tr>
<td>( k_l )</td>
<td>Parameter defined in equation (3.28)</td>
</tr>
<tr>
<td>( k_s )</td>
<td>Slope of ground depth of variation of ( c ).</td>
</tr>
<tr>
<td>( k_t )</td>
<td>Curvature parameter of the cooling tower shell defined by equation (7.5)</td>
</tr>
<tr>
<td>( K )</td>
<td>Specific permeability of the porous media</td>
</tr>
<tr>
<td>([K])</td>
<td>Stiffness matrix</td>
</tr>
<tr>
<td>([K])</td>
<td>Permeability matrix</td>
</tr>
<tr>
<td>( L )</td>
<td>Water rate</td>
</tr>
<tr>
<td>([L])</td>
<td>Matrix defined by equation (3.42)</td>
</tr>
<tr>
<td>( {m} )</td>
<td>Vector defined by equation (3.38)</td>
</tr>
<tr>
<td>( n )</td>
<td>Porosity of a porous medium</td>
</tr>
<tr>
<td>( n_j )</td>
<td>Directions cosines of the outward normal vector ( {n} )</td>
</tr>
<tr>
<td>( {n} )</td>
<td>Outward normal vector</td>
</tr>
<tr>
<td>([N])</td>
<td>Matrix of the shape functions</td>
</tr>
<tr>
<td>( p )</td>
<td>Load point (Chapter 6)</td>
</tr>
<tr>
<td>( p )</td>
<td>Mean pressure defined by equation (3.21)</td>
</tr>
<tr>
<td>( q )</td>
<td>Deviatoric stress defined by equation (3.22)</td>
</tr>
<tr>
<td>( Q )</td>
<td>Plastic potential function</td>
</tr>
<tr>
<td>( r_t, r_s )</td>
<td>Top and base radius of cooling tower respectively</td>
</tr>
<tr>
<td>( R_x, R_y, R_z )</td>
<td>Reaction at supports in the ( x, y ) and ( z ) directions, respectively</td>
</tr>
<tr>
<td>( S )</td>
<td>Closed curve surrounding the boundary area ( \Gamma ) of the domain</td>
</tr>
<tr>
<td>( t )</td>
<td>Time</td>
</tr>
<tr>
<td>( t )</td>
<td>Thickness of shell</td>
</tr>
<tr>
<td>( T ) and ( T )</td>
<td>Temperature</td>
</tr>
<tr>
<td>( T_1, T_2 )</td>
<td>Hot and water temperature, respectively</td>
</tr>
<tr>
<td>( u, v, w )</td>
<td>Meridional, circumferential and normal displacement, respectively</td>
</tr>
<tr>
<td>( u, v, w )</td>
<td>Components of the average seepage velocity in the ( x, y ) and ( z ) directions, respectively</td>
</tr>
<tr>
<td>( u_p, v_p, w_p )</td>
<td>Fluid velocity through the void patches in the ( x, y ) and ( z ) directions, respectively</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>( v_1, v_2 )</td>
<td>Vertical soil layers</td>
</tr>
<tr>
<td>( {v_n} )</td>
<td>Artificial seepage velocity</td>
</tr>
<tr>
<td>( V )</td>
<td>Effective tower volume,</td>
</tr>
<tr>
<td>( V )</td>
<td>Volume element which consists of both fluid and solid phases, ( = \Delta x \Delta y \Delta z )</td>
</tr>
<tr>
<td>( W_x, W_y, W_z )</td>
<td>Weight of the structure in the ( x, y ) and ( z ) directions, respectively</td>
</tr>
<tr>
<td>( x, y, z )</td>
<td>Cartesian co-ordinates</td>
</tr>
</tbody>
</table>

**Greek Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Coefficient of thermal expansion</td>
</tr>
<tr>
<td>( \alpha_t )</td>
<td>Parameter defined by equation (3.28)</td>
</tr>
<tr>
<td>( \beta_x, \beta_y )</td>
<td>Rotations of the normal to the undeformed middle surface in the ( xz ) and ( yz ) planes respectively</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Unit weight of the material</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>Boundary area of the domain ( \Omega )</td>
</tr>
<tr>
<td>( \Delta x, \Delta y, \Delta z )</td>
<td>Mesh sizes in ( x, y ) and ( z ) directions respectively</td>
</tr>
<tr>
<td>( \delta_{ij} )</td>
<td>Kronecker's function</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Strain</td>
</tr>
<tr>
<td>( \Theta )</td>
<td>Time integration parameter</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Circumferential angle from plane of the reference meridian</td>
</tr>
<tr>
<td>( \theta_L )</td>
<td>Lode's angle defined by equation (3.23)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Lamé's coefficient defined by equation (3.18)</td>
</tr>
<tr>
<td>( [A] )</td>
<td>Matrix for the transformation of coordinates from the global to the local axis</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Dynamic viscosity of the fluid</td>
</tr>
<tr>
<td>( \bar{\mu} )</td>
<td>Lamé's coefficient defined by equation (3.18)</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Poisson's ratio</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density</td>
</tr>
<tr>
<td>( \theta )</td>
<td>The angle between the outward normal to the body surface and downward vertical direction</td>
</tr>
</tbody>
</table>
\( \sigma \) Stress

\( \sigma_x, \sigma_y, \sigma_z \) Normal stress in \( x, y \) and \( z \) directions respectively

\( \tau \) Shear stress

\( \phi \) Angle of friction

\( [\Phi] \) Flux matrix defined by equation (3.43)

\( \varphi \) Angle between axis of revolution and normal to the shell's surface

\( \Omega \) Domain volume of the problem

### Subscripts

- \( 0 \) Initial
- \( as \) Air steam
- \( aw \) Saturated air at water temperature
- \( b \) Bending action
- \( f \) Fluid
- \( h_1, h_2 \) Horizontal soil layer
- \( m \) Membrane action
- \( pp \) Pore pressure
- \( v \) Void space or vertical soil layer
- \( v_1, v_2 \) Vertical soil layers
- \( w \) Water

### Superscripts

- \( (1) \) Value of the parameter at time \( t \)
- \( (2) \) Value of the parameter at time \( t + \Delta t \)
- \( - \) Value of the parameter at nodes
- \( * \) Virtual of the parameter
- \( e \) Elastic (Chapter 4), Element (Chapter 5)
- \( p \) Plastic
- \( o \) Degree
- \( T \) Transpose of a matrix/vector
CHAPTER 1

Introduction

1.1 Overview

The aim of this chapter is to introduce the structure of this thesis, the research conducted during this PhD and the background to it.

1.2 Aim and objectives

The aim of this project is to study the soil-structure interaction of cooling towers taking into consideration the effect of temperature changes in the cooling tower on the simultaneous interaction of the cooling tower and underlying soil. This will be investigated in 2D and 3D.

The objectives of this research are:

1. Implementing and examining the behaviour of a four noded quadrilateral flat shell element based on Kirchhoff’s assumptions and modifying it to accommodate a temperature profile.

2. Extending the capabilities of the two-dimensional Geotechnical Finite Element Analysis Program (GeoFEAP) (Espinoza et al., 1995) to model 3D soil-structure interaction problems, incorporating the modelling of thermal aspects of soil-structure interaction in both 2D and 3D.

3. Testing and validating this updated GeoFEAP2 software for two and three dimensions soil-structure interaction and shell element.
1.3 Background to the research

Natural draught cooling towers belong to a category of exceptional civil engineering structures. These towers are an effective and economic choice among all technical solutions for the prevention of thermal pollution of natural water resources caused by heated cooling water in various industrial facilities. They are therefore widely used in most electric power generation units, chemical and petroleum industries and space conditioning processes. The shell is the most important part of the cooling tower, both in technical and financial terms (30% to 45% of the total cost) and also the most sensitive, since its collapse would put all or part of the cooling tower out of action for a considerable length of time. Hence, the shell is one of the fundamental factors influencing the life of the cooling tower (Jullien et al., 1994).

In November 1965, three of the eight cooling towers suddenly collapsed under a storm at the Ferrybridge Power Station in Yorkshire (UK). Details of the collapse were reported extensively in the literature (e.g. CEGB, 1965; Pope, 1994). The collapse was attributed to insufficient vertical reinforcement to resist the uplift forces, and to wind-induced vibrations causing vertical cracking on shells (Bosman et al., 1998). In September 1973, a single 137 m tall cooling tower collapsed under moderate winds at Adeer Nylon Works power plant just off the Southwest coast of Scotland. Formal investigations into the collapse (ICI, 1974) concluded that meridional curvature imperfections in the shell were responsible for the failure. A third incident of a cooling-tower collapse in the UK occurred in Lancashire in January 1984, when one tower at Fiddlers Ferry Power Station collapsed in wind gusts exceeding 125 km/h, details of this collapse have been reported extensively in the literature (e.g. CEGB, 1984; Pope, 1994). As a result, much research (Bamu and Zingoni, 2005; Nasir et al., 2002) into the structural behaviour of hyperbolic axisymmetric shell structures has been carried out using the finite element method. Some of the areas of major interest include:

• Heat and mass transfer inside the cooling tower (e.g. Hawlader and Liu, 2002; Patel et al., 2004).
1.3 Background to the research

- Structural stability and foundation settlement (e.g. Min, 2004; Noh et al., 2003).
- Non-linear analysis of cooling tower shells (e.g. Lang et al., 2002; Waszczyszyn et al., 2000).
- Static and pseudo-static effects of wind and earthquake loading (e.g. Baillis et al., 2000; Orlando, 2001).
- Durability of cooling towers (e.g. Busch et al., 2002; Harte and Krätzig, 2002).

Other important aspects of investigating cooling tower behaviour is temperature and wind (Kloppers, 2003). However, the contribution of thermal stresses to the total stress distribution in cooling tower under combined loading (wind pressure, gravity load etc.) remains significant (Bosak and Flaga, 1996; Sharma and Boresi, 1980; Sudret et al., 2005). In addition, the total tower height is generally fixed by the thermal design (Busch et al., 2002). In this thesis, the attention is therefore given to the temperature effect, as this focus provides greater scope for original contributions. Effect of wind will be considered as future work, as stated in Section 9.2.

Heat transfer between the hot air inside the cooling tower and ambient air establishes a variable temperature field in the wall of the tower. The temperature difference between internal and external faces of the cooling tower shell causes bending moments in the shell (Ravinder et al., 1996). In particular, the temperature changes create stresses on the upper concrete part of the cooling tower, especially near the bottom of the tower. It should be noted that this thermal gradient causes the concrete to crack vertically along the meridians (Rao and Ramanjaneyulu, 1993). In addition, these stresses may then change the loading on the foundation and soil respectively. The temperature value at the internal face of the cooling tower is in general in the range of 28°C to 39°C (Kloppers, 2003; Kloppers and Kröger, 2003, 2005; Wittek and Krätzig, 1996). This is the standard operating temperature range which depends on the ratio of the water to gas mass flow \((L/G)\) of the cooling tower. It should be noted that temperature plays an important role in the design of cooling towers (Kim et al., 2001; Ravi et al., 1996). In particular, a change in the \((L/G)\) of the cooling tower will change the tower characteristics (Cheremisinoff and Cheremisinoff, 1989; Kloppers and Kröger, 2005). More design details are covered in parts 2, 3 and 4 of the British Standard BS4485 (1988a,b, 1996).

In general, two approaches are usually used for studying the soil-structure interaction. In the first, the reactions of the structure are modelled as loads applied directly to the soil (e.g.
1.3 Background to the research

Almeida and Paiva, 2004), neglecting the structure itself. In this case the soil behaviour is the main area of interest. When interest in the structural behaviour is more important, the soil is modelled using flexible springs (Jullien et al., 1994). These are however very approximate methods and do not account for the actual interaction between soil and structure. Less attention was given to the investigation of soil-structure interaction. If, however, the simultaneous interaction of soil and structure was modelled then more accurate results would be obtained.

The above indicates that the proper and more accurate way of modelling the soil and structure is to model them simultaneously using both geotechnical and structural approaches in order to fully understand the soil-structure interaction that occurs for cooling towers. To the best of the author's knowledge, no complete 3D model including both cooling tower and soil as one continuous medium has been modelled. Shu and Wenda (1990) modelled axisymmetric cooling tower-soil, excluding temperature, and concluded that structure-soil interaction must be considered in order to understand the truly dynamic behaviour of structure and soil. The effect of temperature, which has generally been ignored in early designs due to lack of adequate knowledge (Bamu and Zingoni, 2005), has to also be included as it is a major design criterion in cooling towers. The tower considered for this analysis is a column supported hyperbolic cooling tower with a height of 520 ft (158.5 m), which has dimensions in the range of the tallest modern cooling towers, earlier analysed by Gould (1985) and later by other researchers (e.g. Aksu, 1996; Iyer and Rao, 1990; Karisiddappa et al., 1998) for further studies.

The two-dimensional Geotechnical Finite Element Analysis Program (GeoFEAP) (Espinoza et al., 1995) is used in this project because it supports geomechanical analysis. The availability of the basic source code, in addition to being well-tested and used by various researchers (e.g. Bray, 2001; Pestana et al., 1996) facilitates the validation process. GeoFEAP is therefore updated in this project and the new version will be referred to as GeoFEAP2. This software will be used to create a 3D finite element model to comprise the shell element to represent the cooling tower, columns support, foundation, and elastic and elasto-plastic soil behaviour.
1.4 Structure of the thesis

This thesis is divided into the following remaining chapters:

- In Chapter 2, cooling towers' terminology and its two types, natural and mechanical draught, are presented. The main components of the natural draught tower, or simply cooling tower as considered during this research, are introduced. In addition, application of heat and mass transfer processes and Merkel theory are discussed to show the importance of temperature when designing these towers. Literature review of areas of interest for cooling towers is also introduced.

- The aim of Chapter 3 is to present the governing finite element equations for the soil-structure interaction analysis in two and three dimensions, which have been used in the present numerical modelling and software (GeoFEAP2). First, the theory of porous media, as a general case for soil, is introduced and then the governing equations (continuity and momentum) for the fluid flow through a porous medium are presented. In addition, soil parameters and elasto-plastic behaviour are discussed. Finally, an appropriate constitutive model, Drucker-Prager, is then chosen to be used in this study.

- In Chapter 4, validation of the new software (GeoFEAP2) when modelling the soil-structure interaction is investigated for elastic and elasto-plastic soil behaviour. This validation is studied for one- and two- horizontal and vertical soil layers in two and three dimensions with different types of elements and loads.

- The main aim of Chapter 5 is to introduce the four noded quadrilateral flat shell element which is used in this research to model the cooling tower shell. First, membrane and bending elements based on Kirchhoff’s assumptions are used to develop this element, then the stiffness matrix for it is presented. Thermal effects upon this flat shell element are presented.

- In Chapter 6, five standard widely-used 'obstacle course' benchmark shell problems, under self weight (Scordelis–Lo shell roof and hemicylindrical shell) and external load (pinched cylindrical shell with rigid end diaphragm, pullout of an open-ended cylindrical shell and hemispherical shell with an 18° hole), are presented to validate the 4-noded quadrilateral flat shell element which has been introduced in Chapter 5. These problems have been modelled using the GeoFEAP2 and the results are compared with those obtained by other researchers.
1.4 Structure of the thesis

• Comparisons between the 2D and 3D applied load, and between 3D applied load and combined cooling tower-foundation-soil are then investigated and examined in Chapter 7. In addition, the behaviour of the soil layer is investigated in the presence of temperature effect at the shell and a change in the effect of soil shear strength and soil angle of friction.

• In Chapter 8, the soil behaviour for different types of footings, circular and pad, affected by different types and configurations of columns are investigated in the presence of the temperature effect on the cooling tower shell. Vertical and horizontal soil layers are also considered.

• A summary and conclusions of this research with recommendations for future work are presented in Chapter 9.
CHAPTER 2

Significance of Temperature and Areas of Interest for Cooling Tower

2.1 Overview

The main objective of this chapter is to discuss the importance of temperature when designing cooling towers. It should be noted that thermal loading has generally been ignored in early designs of cooling towers due to lack of adequate knowledge (Bamu and Zingoni, 2005). Hence, cooling tower terminology, the application of heat transfer theory and literature review of areas of interest for cooling tower are discussed.

2.2 Introduction

Heat is discharged in power generation, refrigeration, petrochemical, steel processing and many other industrial plants. In many cases, this heat is discharged into the atmosphere with the aid of a cooling tower via a secondary cycle with water as the process fluid. In particular, when water changes state from liquid to vapour or steam, an input of heat energy must take place, known as the latent heat of evaporation. Cooling towers take advantage of this change of state by creating conditions in which hot water evaporates in the presence of moving air. Hence, cooling towers are a very important part of many chemical plants.

Cooling tower designs are generally based on the ambient air dry—bulb temperature, measured at, or near the ground with a corresponding dry adiabatic lapse rate. The average temperature of the air at the inlet of the tower may deviate significantly from the measured
air temperature near the ground due to temperature inversions. The effective inlet air temperature is higher than the measured air temperature at ground level because the tower draws in air from high above the ground (Kloppers and Kröger, 2005).

The reinforced concrete cooling tower is generally subjected to a dead-weight, temperature gradient load due to the temperature difference between the inside and outside of the shell and the effects of creep/shrinkage of concrete (Choi and Noh, 2000). Hence, a better understanding of evaporative cooling of water in cooling towers is necessary both for modernisation of the existing cooling towers, and for predicting the efficiency of newly designed ones because the development of new types of cooling towers requires the conduction of many thermal and hydraulic tests that are too costly (e.g. Dreyer and Erens, 1996; Fisenko et al., 2002).

2.3 Cooling tower terminology

The cooling towers built for industrial purposes are amongst the largest shell structures constructed in the form of hyperbolic shells of revolution supported by closely spaced inclined columns. Although the art of evaporative cooling is quite ancient, the first natural draught cooling tower was only constructed in 1916 at the Emma Pit in the Netherlands by the Dutch State Mines (Bowman and Benton, 1997). The world's tallest cooling tower is 200 m high and is situated at the Niederaussem power plant in Germany (Busch et al., 2002; Harte and Krätzig, 2002).

2.3.1 Types of cooling towers

Although there are many different types of cooling tower, they can be divided into two main categories; natural draught and mechanical draught towers, depending on the method by which air is moved through the tower. However, there are four major components which go into the make-up of a cooling tower (Hill et al., 1990):

1. the packing,
2. drift eliminators,
3. the water distribution system, and
4. the fans (except for natural draught towers).
The natural draught towers depend upon natural forces to move air through the pack and they are designed using very large concrete chimneys to introduce air through the media. These towers are an effective and economic choice among all technical solutions for the prevention of thermal pollution of natural water resources caused by heated cooling water in various industrial facilities (Min, 2004).

The mechanical draught towers utilise large fans to force air through circulated water. One of the major advantages of the mechanical draught tower, is that it can cool to a lower water temperature than a natural draught tower.

In this research, more attention is given to the natural draught cooling tower. However, more details about mechanical cooling towers can be found in Fisenko et al. (2004).

2.3.2 Main components of cooling towers

Figure 2.1 shows a typical natural draught cooling tower (Nuclear Power Plants Around the World, 2003), while figures 2.2 and 2.3 show the outside view and cross section, with the main components of it respectively. Three main parts can be seen in figures 2.2 and 2.3; shell, plant heat exchange process and soil. The main components inside the shell are discussed below:

1. Shell (or Casing): the structure enclosing the heat transfer process, necessary to carry the other main items as shown in figure 2.2. The temperature on the inner surface of this shell is in the range of 28°C to 39°C (Kloppers, 2003; Kloppers and Kröger, 2003, 2005; Wittek and Krätzig, 1996).

2. Air inlet: the position at which cool air enters and is normally protected by drip-proof louvers.

3. Air outlet: the position at which warm air leaves the tower as shown in figure 2.2. It is normally protected by a suitable grill.

4. Drift eliminators: they prevent water droplets from being carried away from the tower by the airstream. However, some water loss occurs and this process is called drift loss.

5. Warm water inlet: the point at which warm water enters the tower. The warm water comes from processes like air conditioning, manufacturing and electric power generation.
2.3 Cooling tower terminology

Figure 2.1: Typical natural draught cooling tower, Didcot, UK (Nuclear Power Plants

Figure 2.2: Diagram of a natural draught cooling tower.
2.3 Cooling tower terminology

Figure 2.3: Cross section diagram of a natural draught cooling tower.
6. Water distribution system: to spread the water as much as possible over the cross-section of the tower.

7. Packing (or Fill): it consists of a system of baffles which ensure maximum contact between water droplets and cooling air by maximising surface area and minimising water film thickness. In addition, they slow the progress of the warm water through the tower. The pack design is therefore very important.

8. Cold water basin (or Tank): to collect the cold water before returning to the plant heat exchange process. However, as cooling systems suffer many forms of corrosion and failure, some treatments are made before returning the cold water to the process (Herro and Port, 1993).

9. Cold water outlet: the point at which the cooled water leaves the tower and is sent back to the process.

10. Make-up water source: the point at which water added to the circulating water system to replace evaporation, leakage and drift loss.

Two zones exist; spray zone and rain zone. The spray zone is located between the hot water distribution system and the packing, while the rain zone is located between the packing and the basin. In addition, two interactions occur; soil-structure interaction and fluid-structure interaction.

Figures 2.2 and 2.3 show the cycle of water in the cooling tower. First, the plant heat exchange process produces warm water which enters the tower at the inlet hot water (5). Some of this water goes through drift eliminators (4) and carries away through the tower at air outlet (3). The remaining water is spread by a hot water distribution system (6) in the spray zone. The air enters the cooling tower at the air inlet (2) then the hot water from the spray zone meets the air at packing (7) and then droplet in the rain zone. Hence, the water is cold in this zone and collected at the cold water basin (8), which is fed by water from the make-up water source. Finally, this cold water goes through outside the cooling tower to the process at outlet cold water (9) and then a new cycle then starts.

2.3.3 Industrial applications

The cold water which goes through outside cooling towers is used for many applications, such as refrigeration plants, air compressors, engines, chemical and refinery plants and turbine
condenser cooling. In general, the most common applications of cooling towers are supplying cooled water for air conditioning, electric power generation and nuclear power stations.

2.4 Heat transfer

The objective of this section is to discuss the fundamental concepts of heat transfer and then its application to cooling towers.

2.4.1 Definitions

Heat is defined as energy transferred between a system and its surroundings as a result of temperature differences only (Resnick and Halliday, 1996). The science of heat transfer is devoted to the study of the processes of heat propagation in solid, liquid and gaseous bodies. Such processes, because of their highly diversified physico-mechanical nature, are very complex and usually proceed as a complete complex of heterogeneous phenomena.

There are three mechanisms by which heat transfer can occur; conduction, convection and radiation. Conduction is the process of heat transfer by molecular motion, supplemented in some cases by the flow of free electrons through a body (solid, liquid or gaseous) from a region of high temperature to a region of low temperature (Kakaç and Yener, 1995). Heat transfer by conduction also takes place across the interface between two bodies that are in contact when they are at different temperatures. The mechanism of heat conduction in liquids and gases has been postulated as the transfer of kinetic energy of the molecular movement. Transfer of thermal energy to a fluid increases its internal energy by increasing the kinetic energy of its vibrating molecules, and it is measured by the increase in its temperature. Thus heat conduction is the transfer of kinetic energy of the more energetic molecules in the high-temperature region by successive collisions to the molecules in the low-temperature region. Thermal radiation, or simply radiation, is heat transfer in the form of electromagnetic waves. All substances, solid bodies as well as liquids and gases emit radiation as a result of their temperature, and they are also capable of absorbing such energy.

The process of heat transfer develops in a different way depending on the physical properties of the investigated material. Heat transfer is especially affected by the following properties: thermal conductivity, specific heat, density, thermal diffusivity and viscosity. These
properties have definite magnitudes for each substance and, in general, are functions of the temperature, and some are also functions of the pressure.

2.4.2 Application to cooling towers

In the case of cooling towers two fluids are involved; air with moisture content up to saturation point and water which enters the tower at high temperature and leaves the tower cooled. The main source of the heat transferred in cooling towers is the latent heat or evaporative increment and this heat must be extracted from the water as it flows through the tower. In the next paragraph, the mechanisms by which the water is cooled are discussed.

Figure 2.4 shows the various ways in which a water droplet loses heat. The droplet is surrounded by a thin film of air which is saturated and remains almost undistributed by the passing air steam. The transfer of heat takes place in three ways, as shown in figure 2.4:

a. By radiation from the surface of the droplet; this is a very small proportion of the total amount of heat flow and is usually neglected.

b. By conduction and convection between water and air; the amount of heat transferred will depend on the temperatures of air and water. It is a significant proportion of the whole.

c. By evaporation, where $T_2 < T_1$; this accounts for the majority of heat transfer and so the whole process is termed evaporative cooling, about 90% of the heat energy is transferred via evaporation (Hawlader and Liu, 2002). Evaporation is the key to the successful operation of cooling tower so more discussion has been presented.

The evaporation that occurs when air and water are in contact is caused by the difference in pressure of water vapour at the surface of the water and in the air. In the cooling tower, the water and air streams are generally opposed so that cooled water leaving the bottom of the pack is in contact with the entering air. Similarly, hot water entering the pack will be in contact with warm air leaving the pack. Evaporation will take place throughout the pack. It should be noted that at the top of the pack the fact that the air is nearly saturated is compensated for by the high water temperature and consequently high vapour temperature.

A better understanding of evaporative cooling of water in cooling towers is necessary both for modernisation of the existing cooling towers and for predicting the efficiency of newly
designed ones (Fisenko et al., 2002) because the development of new types of cooling towers requires the performing several thermal and hydraulic costing tests (Dreyer and Erens, 1996). A detailed investigation of evaporative process in cooling towers can be found in Fisenko (1992), Petruchik and Fisenko (1999), Petruchik et al. (2001) and Fisenko et al. (2002) for natural draught cooling towers (open towers) and recently in Stabat and Marchio (2004) in indirect cooling towers (closed towers).

![Diagram of heat transfer](image)

Figure 2.4: Diagram of the various ways in which a water droplet loses heat; radiation, convection and evaporation.

### 2.4.3 Merkel theory

The most generally accepted theory of the cooling tower heat transfer process is that developed by Merkel (1925). This theory states that the total heat transfer taking place at any position in the tower is proportional to the difference between the total heat of the air at that point and the total heat of the air saturated at the same temperature as the water at the same point. The Merkel equation can be written as follows:

\[
\frac{kAV}{L} = \int_{T_2}^{T_1} \frac{dT}{H_{aw} - H_{as}}
\]

(2.1)
2.4 Heat transfer

where
\[
\frac{kAV}{L} = \text{tower characteristic (or Merkel number)},
\]
\[
k = \text{heat transfer coefficient},
\]
\[
A = \text{area of transfer surface per unit of volume tower},
\]
\[
V = \text{effective tower volume},
\]
\[
L = \text{water rate},
\]
\[
T_1 = \text{hot water temperature},
\]
\[
T_2 = \text{cold water temperature},
\]
\[
H_{aw} = \text{enthalpy of saturated air at water temperature}, \text{and}
\]
\[
H_{as} = \text{enthalpy of air steam}.
\]

The relation between saturated air enthalpy and temperature is not a simple linear function of temperature so numerical integration is usually required to solve this integral (Dreyer and Erens, 1996). Hence, the tower characteristic value can be calculated by solving equation (2.1) with the Chebyshev numerical method (Fox and Parker, 1968),

\[
\frac{kAV}{L} = \frac{T_1 - T_2}{4} \left( \frac{1}{\Delta h_1} + \frac{1}{\Delta h_2} + \ldots \right) \tag{2.2}
\]

where
\[
\Delta h_1 = \text{value of } (H_{aw} - H_{as}) \text{ at } T_2 + 0.1 (T_1 - T_2)
\]
\[
\Delta h_2 = \text{value of } (H_{aw} - H_{as}) \text{ at } T_2 + 0.4 (T_1 - T_2)
\]
\[
\Delta h_3 = \text{value of } (H_{aw} - H_{as}) \text{ at } T_1 - 0.4 (T_1 - T_2)
\]
\[
\Delta h_4 = \text{value of } (H_{aw} - H_{as}) \text{ at } T_1 - 0.1 (T_1 - T_2)
\]

Thermodynamics dictate also that the heat removed from the water must be equal to the heat absorbed by the surrounding air. This relation can be written in a simple form as

\[ (\text{Water Heat} + \text{Air Heat})_{\text{in}} = (\text{Water Heat} + \text{Air Heat})_{\text{out}} \]

This expression can be written in mathematical form as follows:

\[
\frac{L}{G} = \frac{H_2 - H_1}{T_1 - T_2} \tag{2.3}
\]

where
\[
L/G = \text{liquid to gas mass flow ratio},
\]
\[
H_2 = \text{enthalpy of air-water mixture at exhaust wet-bulb temperature}, \text{and}
\]
\[
H_1 = \text{enthalpy of air-water mixture at inlet wet-bulb temperature}.
\]
The tower characteristic \((kAV/L)\) varies with the \((L/G)\) ratio, which can be deduced from equations (2.2) and (2.3). Careful and accurate analysis of cooling towers are needed to ensure a precise determination of cooling water temperature. The importance of \((L/G)\) in the design of cooling tower is discussed below.

2.5  Design and manufacture of cooling towers

Cheremisinoff and Cheremisinoff (1989) and Burger (1989) described the principal criteria on which the design and manufacture of cooling towers is based as:

1. Achieving maximum contact between air and water in the tower by optimising the design of tower packing and water distribution system.

2. Minimising the loss caused by water spray escaping from the tower; control of spray loss is very important in eliminating the risk of infectious diseases being transmitted to people by the warm moist air.

3. Relating the design of the tower to the volume flow rate of the water to be cooled, ambient air wet bulb temperature, warm water input temperature and cooled water output temperature.

4. Understanding and controlling the problems arising from the quality of the water such as corrosion, fouling and growth of bacteria.

5. Taking due account of space limitations at the tower's location and controlling the noise from it.

It is important to notice the following three key points in cooling tower design:

1. A change in wet bulb temperature (due to atmospheric conditions) will not change the tower characteristic \((kAV/L)\).

2. A change in the cooling range will not change \((kAV/L)\).

3. Only a change in the \((L/G)\) will change \((kAV/L)\).

The best way to design the cooling tower is to find a proper \((L/G)\) satisfying such sizes of cooling tower. The \((L/G)\) is the most important factor in designing the cooling tower and related to the construction and operating cost of cooling tower (Kim et al., 2001).
2.6 Literature review of areas of interest for cooling towers

To avoid thermal pollution of lakes and rivers, the heat input to the cooling tower should be removed artificially using a cooling system. Among many solutions, hyperbolic natural draught cooling towers are considered to be very effective and most economical. Cooling towers, especially natural draught ones, are therefore widely used in most electric power generation units, chemical and petroleum industries and space conditioning processes. In large power stations only natural draught cooling towers are able to recover the large quantity of water required for the cooling system. Careful and accurate analysis of cooling towers are desirable to ensure a precise determination of cooling water temperature. However, cooling tower shell design practice in the late 1950s and early 1960s did not have access to finite element techniques and depended on less accurate methods (Bosman et al., 1998). The first approach to the finite element solution of axisymmetric shells was presented by Grafton and Strome (1963). However, in 1965 three of the eight cooling towers at the Ferrybridge ‘C’ power station in UK collapsed, as shown in figure 2.5. Much research into the behaviour of hyperbolic axisymmetric shell structures has since been carried out since the development of the finite element method. Areas of interest during the last few decades have included:

1. Thermal effects (eg. Fisenko (1992), Mertes and Wendisch (1997), Al-Nimr (1998), Su et al. (1999), Kim et al. (2001), Fisenko et al. (2002), Hawlader and Liu (2002), Vlasov et al. (2002), Stabat and Marchio (2004), Patel et al. (2004)). It is found that the droplets and jets in the rain zone of a cooling tower are formed on shedding of water from the sheets of the pack. As a rule, the radius of the droplets is quite large, and an appreciable fraction of water falls dawn in the form of jets. As this takes place, the mean radius of droplets in the rain zone may several times exceed the radius of droplets in the spray zone.

Sharma and Boresi (1980) studied the thermal stresses of cooling tower supported on a spring foundation and subjected to thermal and gravity loads. Furthermore, temperature field in the tower is assumed to be a symmetric with a linear variation through the thickness of the tower and an arbitrary variation along the height. In addition, the effects of the weight of the tower are included. The results show that the difference in temperature distributions on the inner and outer cooling tower surfaces give rise to significant thermal stresses in the tower.
2.6 Literature review of areas of interest for cooling towers

Figure 2.5: (a) A cooling tower comes crashing to the ground during high winds at Ferrybridge 'C' Power Station in 1965. The aftermath of the incident; (b) Three of the eight cooling towers were completely destroyed (University of Bristol, 2002).

2. Structural stability and foundation settlement (e.g. Gould (1985), Bosman (1985), Gupta and Maestrini (1986), Meschke et al. (1991), Krätzig and Zhuang (1992), Eckstein and Nunier (1998), Bosman et al. (1998), Wittek and Meiswinkel (1998), Choi and Noh (2000), Lackner and Mang (2002), Nasir et al. (2002), Noh et al. (2003), Min (2004)). It is found that the hyperbolic shape is the most economical solution of axisymmetric shells under various combinations of the following static loading (i) self weight and static wind loading, and (ii) self weight, static wind loading plus liquid pressure. Curvature, height and shell-wall thickness are important parameters for such structures. The effect of these parameters on free vibration response has been studied.

3. Non-linear analysis of cooling tower shells (e.g. Gopalkrishnan et al. (1993a), Gopalkrishnan et al. (1993b), Zahlten and Borri (1998), Wittek and Meiswinkel (1998), Lang et al. (2002), Waszcyszyn et al. (2000)). For the linear static and dynamic determination of the state of stress in these shells by the finite element method, ring elements are often employed. The basic idea of a ring element is to introduce Fourier series as trial functions for the unknown displacements in the circumferential direction of the shell. Thus, only the meridional direction has to be subdivided in separate elements,
whereas one single ring element is used in circumferential direction.

4. Static and pseudo-static effect of wind and earthquake loading (e.g. Lee and Gould (1967), Abu-Sitta and Davenport (1970), Sollenberger et al. (1980), Kato et al. (1986), Zahlten and Borri (1998), Niemann and Köpper (1998), Su et al. (1999), Baillis et al. (2000), Orlando (2001), Witasse et al. (2002)). The results give the main parameters of a concrete model which determine the behaviour of the towers. These parameters are the strength of the concrete under tension and the shape of the softening part after cracking. All authors agree with the fact that under this load, the failure occurs through collapse and not because of a buckling problem.

Horr and Safi (2002) investigated the dynamic analysis of cooling towers in the case of earthquake excitation. In this case, it can be assumed that the earthquake motions were applied at the structural support points. Hence, all displacements at the base level were assumed to depend only on the earthquake-generation waves. However, the structural response to any earthquake excitation not only depends on the dynamic characteristics of the structure itself but also on the relative mass and stiffness properties of the soil and the structure. Hence, the soil media has been modelled using conventional solid tetrahedral elements without any mass property. For the cooling tower structure, the conventional eight noded shell element has been used to model the curved shell. The conventional beam element is used to model the columns. It appeared from the results that a spherical zone of soil media with the triple size of the cooling tower radius has been affected substantially by the vibration of the superstructure.

5. Durability of cooling towers (e.g. Krätzig et al. (1998), Busch et al. (2002), Harte and Krätzig (2002)). It is found that natural draught cooling towers balance the technical requirements of an efficient energy supply with appropriate means for protection of the environment. What is the reason for this? Even in the most efficient fossil or nuclear power units, only about 45% of the generated heat is converted into electric energy. The remaining 55% is discharged into the environment, mainly through the smoke-stack and the cooling water. To avoid thermal pollution of rivers, lakes and seashores by using their water for cooling, natural draught cooling towers are most effective in minimising the need of water. Thus, they are able to balance environmental factors as well as investment and operating costs with the demands of a reliable energy supply. In addition, the cooling towers can be used for the discharge of the cleaned flue-gases. Due to environmental requirements, the flue-gas from fossil-fired boiler units has to be
cleaned of sulphur and nitrogen oxides by washing in an alkaline liquid. This process cools down the flue-gas, thus losing the thermodynamic energy needed to disperse it through the smokestack. Therefore, since 1983, the cleaned flue-gas stream has been injected into the natural draught of the cooling tower via large pipes, supported on steel or concrete pipe bridges. In 1996, a new idea was added an inflow in a high position with pipes supported in the wall of the shell. This optimises the efficiency of the flue-gas dispersion by avoiding bends in the pipes, thus achieving less flow resistance and higher energy output.

In general the quadrilateral shell element is the most common element in the modelling of cooling tower shells and bar elements for modelling the column supports, as reported by P. Gould (2004) and T. Hara (2004). However, triangular elements are suitable elements if the wind load has been taken into account (S. Fisenko, 2004). More details will be given in the next two chapters.

Reinforced concrete (RC) of cooling towers is thin shell structures composed of concrete and reinforcement and have a large concrete surface area. The configuration of a cooling tower is usually a hyperboloidal shell of revolution. Therefore, to evaluate the stresses and displacements of these structures, an axisymmetric analysis may be selected using a shell of revolution finite element. This scheme is a quite efficient way to solve a cooling tower shell which has no deviation from the perfect axisymmetric configuration from a geometry and load point of view (Hara and Gould, 2002). Otherwise, local-global analysis may be the suitable analysis to be used as introduced recently by Gould and Hara (2002) and Hara and Gould (2002).

2.7 Conclusion

Hyperbolic cooling towers are widely used in most industrial processes because of the following reasons;

1. the hyperbolic shape is more efficient in drafting of air compared to conical or cylindrical shapes, and

2. the hyperbolic shape has greater structural strength due to double curvature of the shell.
The importance of thermal effects in cooling tower problems, and the importance of temperature in the design of cooling towers has been shown in this chapter. The effect of temperature is a major design criterion in cooling towers and therefore has to be considered in the soil-structure interaction. The water flow and heat transfer processes in a natural draught cooling tower are strongly coupled, because of the following reasons (Majumdar et al., 1983):

1. The amount of air flow through the tower depends on the difference between the density of air inside the tower and that of ambient air.

2. The density of air inside the tower depends on the extent of heat and mass transfer from water to air in the fill and spray region.

3. Heat and mass transfer from water to air depends on the available air flow rate.

Areas of interest for cooling tower studies during the last few decades include:

1. Thermal effects.

2. Static and pseudo-static effect of wind and earthquake loading.


5. Durability of cooling towers.
CHAPTER 3

Numerical Modelling and Governing Equations of Soil

3.1 Overview

The main objective of this chapter is to present the governing finite element equations for the soil-structure interaction analysis in two and three dimensions which have been used in the present numerical modelling and software (GeoFEAP2). To achieve this objective, the theory of porous media, as a general case for soil, is introduced and then the governing equations (continuity and momentum) for the fluid flow through a porous medium are presented. In addition, soil parameters and elasto-plastic behaviour will be discussed. An appropriate constitutive model will then be chosen to be used in this research.

3.2 Finite element method

In solving engineering problems, it is necessary to obtain approximate numerical solutions rather than exact closed-form solutions which are very difficult to obtain, or in a lot of cases, unobtainable. The finite element (FE), boundary element (BE), finite difference (FD), finite volume (FV) and spectral methods are examples of numerical methods used to obtain approximate solutions. However, in modern engineering analysis it is rare to find a project that does not require some type of finite element analysis (FEA). In particular, since the evolution of the term “finite element” by Clough in 1951, there have been significant developments in the finite element method. The finite element method became even more popular with the advancement of computers and development of various efficient programming languages.
3.2 Finite element method

During the last two decades, the practical advantages of FEA in stress analysis, structural dynamics and thermal analysis have made it a “standard” solution tool. The importance and stages of FEM will be discussed in the next two sections.

3.2.1 The importance of FEM

The finite element method (FEM) has the following advantages over most other numerical analysis methods (Cook et al., 2002):

1. It is applicable to any field problem (e.g. stress analysis, heat transfer, etc).
2. No geometric restriction, i.e. FEA has the ability to handle truly arbitrary geometry.
3. Boundary conditions and loading are not restricted, i.e. FEA can deal with nonhomogeneous materials.
4. Material properties are not restricted to isotropy.
5. Components that have different behaviour and different mathematical descriptions can be combined.

The above features mean that systems of arbitrary shape that are made of numerous different material regions can be considered. Each material can have constant properties or the properties may vary with spatial location. Moreover, a large amount of freedom in prescribing the loading conditions and in the postprocessing of items such as the stresses and strains can be added.

3.2.2 Stages of FEM

FEM can be defined as dividing all systems into their individual components or elements, whose behaviour is easily understood, and then rebuilding the original system from such components to study its behaviour. Figure 3.1 shows the stages of the finite element analysis. In general, the following steps are applied to obtain the field quantity;

1. Discretising the continuum which means dividing the continuum or solution region to elements. A variety of element shapes may be used.
2. Selecting interpolation functions to assign nodes to each element and then represent the variation of the field variable over the element. The polynomial degree is chosen depending on the number of nodes assigned to the element and the nature and number of unknowns at each node.
3.3 Governing equations of a porous medium

3. Finding the element properties of the individual elements. One of three approaches can be used; the direct approach, the variational approach and the weighted residual approach.

4. Assembling the element properties to obtain the system equations. The matrix equations that express the behaviour of the individual elements are combined to form the matrix equations that express the behaviour of the entire system.

5. Imposing the boundary conditions. Before solving the global matrix equations they have to be modified to account for the boundary conditions of the problem.

6. Solving the system equations. Applying steps 4 and 5 gives a set of simultaneous equations that are solved to obtain the unknown nodal values of the problem.

7. The solution of the system equation is used to calculate other important parameters.

3.3 Governing equations of a porous medium

3.3.1 Porous media terminology

Porous media can be defined as materials which consist of a solid or semi-solid matrix with an interconnected void (Pop and Ingham, 2001). In nature, porous materials contain an interconnected three-dimensional network of capillary channels which are non-uniform in size and shape and are commonly referred to as pores. The interconnectedness of the pores allows the flow of fluid through the material. There are two types of fluid flow through the material; 'single-phase flow' and 'two-phase flow'. In 'single-phase flow', the void is saturated by a single fluid while in 'two-phase flow', a liquid and a gas, or two immiscible liquids, share the void. In general, the distribution of pores with respect to shape and size is irregular within a natural porous media. The flow quantities, such as the fluid velocity and pressure, are also irregular. However, in typical experiments the space-average (macroscopic) quantities of interest are measured over areas that cross many pores and change in a regular manner with respect to both space and time and these averaged quantities are now amenable to theoretical treatment (Nield and Bejan, 1999). In geotechnical engineering the porous media is soil and the fluid is water.
3.3 Governing Equations of a Porous Medium
3.3 Governing equations of a porous medium

3.3.2 Porosity

The porosity, $n$, is defined as the fraction of the total volume of the medium to that which is occupied by the void space and thus $(1 - n)$ is the fraction that is occupied by the solid region. The porosity is one of the most important geometrical properties for natural media, where $n$ does not normally exceed a value of about 0.6. For soils, it is between 0.43 to 0.54 (Scheidegger, 1974).

3.3.3 Methods of deriving equation of a porous medium

In this section, the governing equations for the fluid flow through a porous medium are described. The mass conservation equation is derived in a similar way to that used in Newtonian fluids, whereas the momentum equation is an experimental law first postulated by Darcy (1856). On the microscopic scale, the fluid velocity within a porous medium is seen to be highly random, fluctuating rapidly in both magnitude and direction. The full equations which govern the fluid flow through a porous medium are complicated and hence very difficult to solve and therefore some assumptions have to be made about the porous media in order to average out these complicated features.

To derive the governing equations, in terms of the averaged variables, one begins with the standard fluid equations which are obeyed by the fluid and then obtains the macroscopic equations by averaging over areas or volumes containing many pores. There are two ways to do the averaging: spatial and statistical. In the spatial approach, a macroscopic variable is defined as an appropriate mean over a sufficiently large representative elementary volume (r.e.v.) which yields the value of that variable at the centroid of the r.e.v. The length scale of the r.e.v. is taken to be much larger than the pore scale and much smaller than the length scale of the macroscopic flow domain. This result, obtained using r.e.v., is assumed to be independent of the size of the r.e.v. In the statistical approach, the variables are averaged over an ensemble of possible pore structures which are macroscopically equivalent. The statistical information has to be based on a single sample and to avoid this difficulty the statistical homogeneity is assumed. The results obtained using the previous two approaches are essentially the same if the deriving relations between the space-averaged variables are only concerned, i.e. their fluctuations are neglected. Thus it is appropriate to use the simpler approach, the spatial averaging of the variables.
3.3 Governing equations of a porous medium

The main idea is to consider the porous media on a large scale, such that the pores and the solid matrix become indistinguishable so the porous media can be regarded as a homogenous medium. Cartesian co-ordinates $x$, $y$ and $z$ and the volume element $V = \Delta x \Delta y \Delta z$ is now considered. It is assumed that the medium is isotropic, so that the porosity $n$ equals the fraction of void space $V_v$ to the total volume $V$, i.e.

$$n = \frac{V_v}{V}$$  \hspace{1cm} (3.1)

It must be noted that volume changes in a soil occur because the volume of voids changes.

![Diagram](image)

Figure 3.2: Diagram of (a) Cartesian co-ordinates system, (b) fluid velocity, $u_p$, through the void patches in the $x$ direction, and (c) the components of the averaged seepage velocity $u$, $v$ and $w$ in the $x$, $y$ and $z$ directions respectively.

3.3.4 Continuity equation

In an approach analogous to the treatment of mean velocities in turbulent flow, the seepage (called also volumetric flux) velocity is introduced, based on the r.e.v. concept. Thus a continuum model can be used, along with a Cartesian reference frame, whereby sufficiently large volume elements compared to the pore volume are considered such that reliable volume averaging can be achieved. There is a distinction between two average fluid velocities flow through the porous media. The seepage average velocity, $v$, is the average of the fluid velocity over the volume $V_v$, which is taken with respect to a volume element which consists of both fluid and solid phases, and the averaged seepage velocity components $u$, $v$ and $w$ in the $x$, $y$
3.3 Governing equations of a porous medium

and $z$ directions respectively, at the centroid of $V$ are defined by

$$u = \frac{1}{V} \int_{V} u_p \, dV_v$$

(3.2a)

$$v = \frac{1}{V} \int_{V} v_p \, dV_v$$

(3.2b)

$$w = \frac{1}{V} \int_{V} w_p \, dV_v$$

(3.2c)

where $u_p$, $v_p$ and $w_p$ are the $x$, $y$ and $z$ components of the velocity over the void patches in the $x$, $y$ and $z$ directions respectively. The intrinsic average velocity, $V$, is the average of the fluid velocity over the volume $V_f$, which is taken with respect to a volume element which consists of only the fluid. The Dupuit-Forchheimer relationship combines the seepage velocity and the intrinsic velocity as follows (Nield and Bejan, 1999)

$$\{v\} = n \{V\}$$

(3.3)

Once one has a continuum to deal with, the usual arguments can be applied and then the governing differential equations, expressing the conservation laws, can be derived.

The continuity equation, which expresses the conservation of mass, is derived by considering an elementary unit volume of the medium. Equating the rate of increase of mass within the volume $V$, $\frac{\partial (n \rho_f)}{\partial t}$, to the net mass flux into the volume, $-\nabla \cdot (\rho_f \{v\})$, the following equation is obtained

$$\frac{\partial (n \rho_f)}{\partial t} + \nabla \cdot (\rho_f \{v\}) = 0$$

(3.4)

where $\rho_f$ is the density of the fluid and $t$ is the time. On noting that $n$ is independent of $t$ and for an incompressible fluid, $\rho_f$ = constant, then equation (3.4) becomes:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

(3.5)

### 3.3.5 Momentum equation

Darcy (1856) carried out experiments on a steady state, unidirectional flow in a uniform porous medium and he discovered that the fluid velocity was directly proportional to the applied pressure gradient and inversely proportional to the dynamic viscosity of the fluid, $\mu$, when gravity is neglected. This may be expressed in one-dimension by

$$u = -\frac{K}{\mu} \frac{\partial p}{\partial x}$$

(3.6)
3.3 Governing equations of a porous medium

where $p$ is the pressure and $K$ is the specific permeability of the medium. Thus it is indepen­dent of the nature of the fluid and depends only on the geometry of the medium. $K$ is therefore the most important physical property of a porous medium. Hence, some authors classify porous medium depending on their ability to be permeated by a fluid.

The generalisation to three dimensions of the Darcy equation (3.6) is given by:

$$\{v\} = -\frac{1}{\mu} [K].\nabla p$$

where $[K]$ is a second-order tensor called the permeability matrix and takes the form

$$[K] = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

and $\nabla$ is the gradient operator and for Cartesian co-ordinates $\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}$.

3.3.5.1 Special case

In the isotropic medium (a medium whose structures and properties in the neighbourhood of any point are the same relative to all directions through the point) the permeability is given by $K_{ij} = K\delta_{ij}$, where $K$ is a scalar permeability and $\delta_{ij}$ is Kronecker’s function defined as,

$$\delta_{ij} = \begin{cases} 1 & \text{when } i = j \\ 0 & \text{when } i \neq j \end{cases}$$

Hence equation (3.7) becomes

$$\{v\} = -\frac{K}{\mu} \nabla p$$

One of the generalisations of equation (3.10) is (Pop and Ingham, 2001)

$$\{v\} = -\frac{K}{\mu} (\nabla p - \rho \sin (\phi) \ g)$$

where $\phi$ is the angle between the outward normal to the body surface and downward vertical direction and $g$ is the gravitational force per unit mass of the fluid, where $g_x$, $g_y$ and $g_z$ are the components of the acceleration vector due to gravity in the $x$, $y$ and $z$ directions respectively. Nield and Bejan (1999) discussed in detail extensions of Darcy’s equation (3.6) which include both the inertia and viscous terms.
3.3.5.2 Geotechnical engineering formulation

The porous medium is taken as soil, saturated by water, i.e. \( \rho_f = \text{constant} \). The geotechnical engineering symbols are used, i.e. \( \frac{k}{\mu} = \gamma_w \) and \( p = \sigma_{pp} \), where \( k \) is Darcy’s coefficient of permeability, \( \gamma_w \) is the unit weight of water and \( \sigma_{pp} \) is the excess pore pressure. Equation (3.7) can be re-written as

\[
\begin{align*}
\frac{\partial}{\partial x} \left( k_{xx} \frac{\partial \sigma_{pp}}{\partial x} + k_{xy} \frac{\partial \sigma_{pp}}{\partial y} + k_{xz} \frac{\partial \sigma_{pp}}{\partial z} \right) \\
+ \frac{\partial}{\partial y} \left( k_{yz} \frac{\partial \sigma_{pp}}{\partial x} + k_{yy} \frac{\partial \sigma_{pp}}{\partial y} + k_{yy} \frac{\partial \sigma_{pp}}{\partial z} \right) \\
+ \frac{\partial}{\partial z} \left( k_{zz} \frac{\partial \sigma_{pp}}{\partial x} + k_{zy} \frac{\partial \sigma_{pp}}{\partial y} + k_{zz} \frac{\partial \sigma_{pp}}{\partial z} \right) = 0
\end{align*}
\]

(3.13)

Substituting these relations into equation (3.5), for the steady-state case results in

In the isotropic case, this equation reduces to Laplace’s equation which governs a number of other physical phenomena.

If \( n \) is dependent on \( t \) (unsteady), \( n_i = \epsilon_{ii} \), where \( \epsilon_{ii} \) is the volumetric strain of a soil element, and the components of the permeability tensor are constants;

\[
\frac{k_{ij}}{\gamma_w} \frac{\partial^2 \sigma_{pp}}{\partial x_j^2} = \frac{\partial \epsilon_{ii}}{\partial t}
\]

(3.14)

3.4 Soil parameters and behaviour

One of the most important tasks in the application of soil mechanics to engineering problems is the study of soil behaviour under load. In the design of structures engineers rely upon the laws of applied mechanics and determine the stresses and strains in structural elements on the basis of a few physical characteristics of the construction materials.
3.4 Soil parameters and behaviour

3.4.1 Elasto-plastic theory

Elasto-plastic theory has gained widespread acceptance in numerical simulations of practical geotechnical engineering problems due to its extreme versatility and accuracy in modelling real engineering materials behaviour. This theory provides an excellent framework available in which to formulate constitutive models that can realistically simulate real soil behaviour (Potts and Zdravkovic, 1999). The theory of soil plasticity is concerned with the analysis of stress and strain in the plastic range of soil media. Applied to the design of foundations and retaining structures, it represents a necessary extension of the theory of elasticity in that it furnishes more realistic estimates of load-carrying capacities against failure, and in addition, it provides better estimates of settlements or displacements when subjected to its working load (Chen, 1975).

Elasto-plastic models are based on the assumption that the principal directions of accumulated stress and incremental plastic strain coincide. They require the following piece of information for their definition: a yield function which separates purely elastic from elasto-plastic behaviour; a plastic potential (or flow rule) which prescribes the direction of plastic straining, and a set of hardening/softening rules which describe how the state parameters (e.g. strength) vary with plastic strain (or plastic work). It assumes elastic behaviour prior to yield and can therefore utilise the benefits of both elastic and plastic behaviour.

The idealised constitutive relation, or stress-strain curve, is capable of reflecting the three most important characters of the real soil. Firstly, the elastic response is pronounced at lower loads. Secondly as the load is increased near ultimate, the actual curve has already bent over considerably so that the tangent modulus at this stage is merely a fraction of the initial elastic modulus. The perfectly plastic idealisation represents the condition at which the modulus ratio approaches zero. Finally, the plastic behaviour of soil is observed by having a residual strain when a complete unloading takes place beyond the elastic range. This is in contrast to the nonlinear elastic idealisation where unloading follows the initial path and the strain is fully recoverable. This last characteristic gives a distinction between a plastic and an elastic soil, as shown in figure 3.3.
3.4 Soil parameters and behaviour

3.4.2 Elasto-plastic governing equations

3.4.2.1 Elastic behaviour

With an elastic material, the state of stress is only a function of the current state of deformations, and is characterised by complete reversibility (i.e. the mechanical work done by an external load is regained when the load is statically removed). The relation between the stress $\sigma_{ij}$ and strain $\epsilon_{ij}$ tensors in the elasticity can be expressed by the Cauchy elastic expression as follows:

$$\sigma_{ij} = \sigma (\epsilon_{ij}^e)$$  \hspace{1cm} (3.15)

which written in the incremental form as follows:

$$d\sigma_{ij} = D_{ijkl} \, de_{kl}^e$$  \hspace{1cm} (3.16)

where $D_{ijkl}$ represents the components of a rank four stiffness tensor, $[D]$, which describes the material moduli, and the superscript $e$ refers the elastic behaviour. In general, $[D]$ has $3^4 = 81$ components. However, due to the symmetry of both stress and strain tensors, the 81 possibilities are reduced to 36 independent components in total for a general 3D case. In addition, it can be reduced even further for some special cases (Chen and Han, 1988). The most general form of the isotropic elastic stiffness tensor of $D_{ijkl}$ has the following representation

$$D_{ijkl} = \lambda \delta_{ij}\delta_{kl} + \mu (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$  \hspace{1cm} (3.17)
where $\delta_{ij}$ is Kronecker's function, defined by equation (3.9), and $\lambda$ and $\mu$ are Lamé's coefficients defined as

$$\lambda = \frac{\nu E}{(1 + \nu)(1 - 2\nu)}$$

and

$$\mu = \frac{E}{2(1 + \nu)}$$

(3.18)

where $E$ and $\nu$ are Young's modulus and Poisson's ratio respectively. It should be noted that the elastic isotropic behaviour of soils obeys Hooke's law with constant Poisson's ratio. In addition, a linear elastic law has been used, i.e. $E$ and $\nu$ are constants.

### 3.4.2.2 Elasto-plastic formulation

The elasto-plastic formulation can be expressed in general incremental form as follows:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p$$

(3.19)

where the superscript $p$ refers to the plastic behaviour and the plastic strain increment ($d\varepsilon_{ij}^p$) is expressed as

$$d\varepsilon_{ij}^p = d\lambda \frac{\partial Q}{\partial \sigma_{ij}}$$

(3.20)

where $Q$ is the plastic potential function which defines the relative magnitudes of the various components of plastic deformation. The direction of the incremental plastic strain is orthogonal ($d\varepsilon_{ij}^p$) to $Q$ as associated flow is assumed.

### 3.4.3 State variables

Material models are more conveniently described using the invariant of the stress tensor ($\sigma_{ij}$). In geotechnical material modelling, stress-strain relationships are usually represented using the following stress variables: mean pressure ($p$), deviatoric stress ($q$) and Lode angle ($\theta_L$) (Espinoza et al., 1995). For use in yield functions, these variables can be expressed in terms of principal variants as follows (Nayak and Zienkiewicz, 1972):

$$p = \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33})$$

(3.21)

$$q = \left(\frac{1}{2} ((\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2) + 3 \left(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2\right)\right)^{\frac{1}{2}}$$

(3.22)

$$\sin(3 \theta_L) = \frac{-3\sqrt{3}}{2} \left| \begin{array}{ccc}
\frac{2}{3}\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \frac{2}{3}\sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \frac{2}{3}\sigma_{33}
\end{array} \right|^{\frac{1}{2}}$$

(3.23)

where $-\frac{\pi}{6} \leq \theta_L \leq \frac{\pi}{6}$. 

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3.4 Soil parameters and behaviour

3.4.4 Plasticity models

The behaviour of an elasto-plastic material is governed by a yield criterion and an associated flow rule (equations 3.19 and 3.20). The yield function definitions are usually based on a set of existing shapes. A few of the most prominent examples of yield functions are: Von Mises, Drucker-Prager (Drucker, 1953, 1956, 1964; Drucker and Prager, 1952), Mohr-Coulomb, Cam-Clay (Roscoe et al., 1958, 1963; Schofield and Wroth, 1968), Parabolic model (Mentrey and Willam, 1995) and Lade’s yield functions (Lade, 1977; Lade and Duncan, 1975).

On using equations (3.21) to (3.23) the surface for several classical yield conditions can be given as (Zienkiewicz and Taylor, 2000):

1. Tresca:

\[ F = \frac{2}{\sqrt{3}} q \cos(\theta) - Y = 0 \]  

where \( Y \) is a function which depends on an isotropic strain hardening parameter.

2. von Mises:

\[ F = \sqrt{\frac{2}{3}} (q - Y) = 0 \]  

3. Mohr-Coulomb:

\[ F = p \sin(\phi) + \frac{q}{3} \left( \cos(\theta_L) - \frac{1}{\sqrt{3}} \sin(\phi) \sin(\theta_L) \right) - c \cos(\phi) = 0 \]  

where \( c \) and \( \phi \) are the cohesion and the angle of friction for the soil respectively.

4. Drucker-Prager:

\[ F = 3 \alpha_1 p + \frac{q}{3} - k_l = 0 \]  

where

\[ \alpha_1 = \frac{2 \sin(\phi)}{\sqrt{3}(3 - \sin(\phi))} \quad \text{and} \quad k_l = \frac{6 c \cos(\phi)}{\sqrt{3}(3 - \sin(\phi))} \]  

Both criteria 1 and 2 are well verified in metal plasticity. Mohr-Coulomb or Drucker-Prager surfaces are frequently used to model soils, concrete and other frictional materials (Drucker and Prager, 1952).
3.4 Soil parameters and behaviour

3.4.5 Choice of the constitutive model

The elasto-plastic Drucker-Prager yield criterion is chosen for modelling the soil in this research because it can represent soil dilatancy and its parameters can be related to the physical soil properties (cohesion and friction angle) in a rather straightforward way. In addition, the implementation of its constitutive law is similar to that required for more complex constitutive laws. Furthermore, despite its relative simplicity, it can lead to reasonable agreement between the results of simulations and observations (Xu et al., 2003).

Drucker and Prager (1952) discussed an extension of the well known von Mises yield condition which included the hydrostatic component of the stress tensor. A comprehensive review of the subject is given by Chen (1975). The extended von Mises yield function, as viewed in three-dimensional principal stress space, is a cone with the space diagonal as its axis. As shown in figure 3.4, the space diagonal is a line defined by \( \sigma_1 = \sigma_2 = \sigma_3 \), where \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the principal stresses.

\[
\sqrt{3} c \cot(\phi)
\]

Figure 3.4: Von Mises and Drucker-Prager yield surfaces (Zienkiewicz and Taylor, 2000).

Many researchers have used this constitutive model to investigate the elasto-plastic behaviour of the soil (e.g. Bardet, 1990; Bousshine et al., 2001; Davidson and Chen, 1978; Loret and Prevost, 1986; Monazen and Neményi, 1999; Noorzaei et al., 1995; Oettl et al., 1998; Veen et al., 1999; Xu et al., 2003). Equations (3.27) and (3.28) indicate that Drucker-Prager’s
3.5 Governing finite element equations

model is defined by two constants: one defines the cohesion (shear strength); and the second defines the dependence on the confining pressure, related to the angle of friction in soil mechanics. In addition, this model represents one of the idealisations of real soil which entails appropriate elastic constants, a yield function and a flow rule. Furthermore, from a mathematical point of view, the Drucker–Prager’s criterion is the most convenient choice because of its simplicity and its straightforward numerical implementation (Oettl et al., 1998) and resembles the behaviour of real engineering materials more closely (Loret and Prevost, 1986).

3.5 Governing finite element equations

3.5.1 Continuity equation

The principle of virtual work is applied to the equation (3.14)

\[ \int_{\Omega} \sigma_{pp}^{*T} \left( \frac{\partial^2 \sigma_{pp}}{\partial x_j^2} + \frac{\partial \epsilon_{ii}}{\partial t} \right) d\Omega = 0 \]  

where \( \Omega \) is the domain volume of the problem, shown in figure 3.5, and \( \sigma_{pp}^{*T} \) is an arbitrary function which varies with \( x_i \), representing a virtual excess pore pressure. The divergence theorem of equation (3.29) is given by

\[ - \int_{\Omega} \left( \frac{\partial \sigma_{pp}^{*}}{\partial x_j} \right)^T k_{ij} \frac{\partial \sigma_{pp}}{\partial x_j} d\Omega + \int_{\Gamma} \sigma_{pp}^{*T} k_{ij} \frac{\partial \sigma_{pp}}{\partial x_j} n_j d\Gamma + \int_{\Omega} \sigma_{pp}^{*T} \frac{\partial \epsilon_{ii}}{\partial t} d\Omega = 0 \]  

where \( n_j \) are the direction cosines of the outward normal vector \( \{\mathbf{n}\} \) to the closed curve surrounding the boundary area \( \Gamma \) of the domain, shown in figure 3.5. Equation (3.30) represents ‘principle of virtual power’ and forms the starting point for obtaining the FE equations, in much the same way as the principle of virtual work can be used to obtain the FE equations for the stress analysis.

The nodal displacement and excess pore pressures are the primary unknowns. The displacement is assumed to vary over a FE mesh according to

\[ \{\mathbf{u}\} = [\mathbf{N}_u] \{\mathbf{a}\} \]  

where \( \{\mathbf{u}\} \) is the displacement inside the element, \( [\mathbf{N}_u] \) is a matrix containing the shape functions for the element and \( \{\mathbf{a}\} \) is the vector listing all the nodal displacement associated with an element. The excess pore pressures are assumed to vary over the same mesh.
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![Problem domain and boundary](image)

Figure 3.5: Problem domain $\Gamma$ and boundary $\Omega$.

According to

$$\{\sigma_{pp}\} = [\bar{N}_{pp}] \{\bar{\sigma}_{pp}\} \quad (3.32)$$

On differentiating the last equation results in

$$\frac{\partial \{\sigma_{pp}\}}{\partial x_j} = [E] \{\bar{\sigma}_{pp}\} \quad \text{where} \quad [E] = \frac{\partial [\bar{N}_{pp}]}{\partial x_j} \quad (3.33)$$

It should be noted that different shape functions are used for displacement $[\bar{N}_{a}]$, which may vary in a quadratic function, and pore pressures $[\bar{N}_{pp}]$, may vary in a linear function, over an element. The virtual excess pore pressures $\{\sigma_{pp}^*\}$ are assumed to vary according to the same shape function as the excess pore pressures

$$\{\sigma_{pp}^*\} = [\bar{N}_{pp}] \{\bar{\sigma}_{pp}^*\} \quad (3.34)$$

On differentiating this equation results in

$$\frac{\partial \{\sigma_{pp}^*\}}{\partial x_j} = [E] \{\bar{\sigma}_{pp}^*\} \quad (3.35)$$

The strains $\{\epsilon\}$ are given by

$$\{\epsilon\} = [B] \{a\} \quad (3.36)$$

where $[B]$ is the strain matrix. Finally, the volumetric strain is defined by

$$\epsilon_v = \{m\}^T \{\epsilon\} \quad (3.37)$$
3.5 Governing finite element equations

where the vector \( \{m\} \) is defined as

\[
\{m\} = \{1 \ 1 \ 1 \ 0 \ 0 \ 0\}^T \tag{3.38}
\]

Substituting from (3.36) into (3.37) equation

\[
\varepsilon_{ii} = \{m\}^T [B] \{a\} \tag{3.39}
\]

Substituting from equations (3.33), (3.35), (3.36) and (3.39) into equation (3.30) results

\[
\{\bar{\sigma}^*_{pp}\}^T \left( \int_{\Omega} [\bar{N}_{pp}]^T \{m\}^T [B] d\Omega \right) \frac{d\{a\}}{dt} - \{\bar{\sigma}^*_{pp}\}^T \left( \int_{\Omega} [E]^T \gamma_w [E] d\Omega \right) \{\bar{\sigma}_{pp}\}
\]

\[
= \{\bar{\sigma}^*_{pp}\}^T \left( \int_{\Gamma} [\bar{N}_{pp}]^T \{v_n\} d\Gamma \right) \tag{3.40}
\]

where \( \{v_n\} \) is called the artificial seepage velocity normal to the boundary, with components \( k_i \gamma_w \frac{\partial \sigma_{pp}}{\partial x_j} n_j \). \( \{\bar{\sigma}^*_{pp}\}^T \) can be cancelled from the last equation to give

\[
[L]^T \frac{d\{a\}}{dt} - [\Phi] \{\bar{\sigma}_{pp}\} = \int_{\Gamma} [\bar{N}_{pp}]^T \{v_n\} d\Gamma \tag{3.41}
\]

where \([L]\) is the coupling matrix and defined as

\[
[L] = \int_{\Omega} [B]^T \{m\} [\bar{N}_{pp}] d\Omega \tag{3.42}
\]

and \([\Phi]\) is the flux matrix and defined as

\[
[\Phi] = \int_{\Omega} [E]^T \gamma_w [E] d\Omega \tag{3.43}
\]

Equation (3.41) is a first-order differential equation which may be integrated with respect to time from \( t \rightarrow t + \Delta t \)

\[
\int_t^{t+\Delta t} \left( [L]^T \frac{d\{a\}}{dt} \right) dt - [\Phi] \int_t^{t+\Delta t} \{\bar{\sigma}_{pp}\} dt = \int_{\Gamma} \left( \int_t^{t+\Delta t} [\bar{N}_{pp}]^T \{v_n\} dt \right) \int_{\Gamma} \tag{3.44}
\]

The following approximations are considered

\[
\int_t^{t+\Delta t} \{\bar{\sigma}_{pp}\} dt = \left( 1 - \Theta \right) \{\bar{\sigma}_{pp}\}^{(1)} + \Theta \{\bar{\sigma}_{pp}\}^{(2)} \Delta t \tag{3.45}
\]

and

\[
\int_t^{t+\Delta t} \{v_n\} dt = \left( 1 - \Theta \right) \{v_n\}^{(1)} + \Theta \{v_n\}^{(2)} \Delta t \tag{3.46}
\]

where \( \{\bar{\sigma}_{pp}\}^{(1)} = \{\bar{\sigma}_{pp}(t)\}, \{\bar{\sigma}_{pp}\}^{(2)} = \{\bar{\sigma}_{pp}(t + \Delta t)\}, \{v_n\}^{(1)} = \{v_n(t)\}, \{v_n\}^{(2)} = \{v_n(t + \Delta t)\} \)

and \( \Theta \) is the time integration parameter. \( \Theta \) is usually in the range 0 to 1 (Britto and Gunn, 1987) and its value defines the way that the variable varies during the time interval. Moreover, the above approximation is called an Euler explicit algorithm if \( \Theta \) equals zero and an
Euler implicit algorithm if $\Theta$ equals one.

Substituting from equations (3.45) and (3.46) into equation (3.44) results in the following equation;

$$\begin{align}\nonumber
\{L\}^T \{a\}_{t+\Delta t} - \{\Phi\} \left( (1 - \Theta) \{\bar{\sigma}_{pp}\}^{(1)} + \Theta \{\bar{\sigma}_{pp}\}^{(2)} \right) \Delta t \\
\nonumber
= \int_{T} \{\bar{N}\}^T \left( (1 - \Theta) \{\bar{v}_n\}^{(1)} + \Theta \{\bar{v}_n\}^{(2)} \right) \Delta t \ dT
\end{align}\tag{3.47}$$

Booker and Small (1975) shows that the stability of integration occurs when $\Theta \geq \frac{1}{2}$. The last equation can be written as

$$\begin{align}
\{L\}^T \{\Delta a\} - \{\Phi\} \Delta t \Theta \{\Delta \bar{\sigma}_{pp}\} = \{\Phi\} \Delta t \{\bar{\sigma}_{pp}\}^{(1)} \\
+ \int_{T} \{\bar{N}\}^T \left( \Theta \{\Delta \bar{v}_n\} + \{\bar{v}_n\}^{(1)} \right) \Delta t \ dT
\end{align}\tag{3.48}$$

where $\{\Delta a\} = \{a(t + \Delta t)\} - \{a(t)\}$, $\{\Delta \bar{\sigma}_{pp}\} = \{\bar{\sigma}_{pp}\}^{(2)} - \{\bar{\sigma}_{pp}\}^{(1)}$ and $\{\Delta \bar{v}_n\} = \{\bar{v}_n\}^{(2)} - \{\bar{v}_n\}^{(1)}$.

### 3.5.2 Equilibrium equations

The equilibrium equations are

$$\begin{align}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = b_x \\
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = b_y \\
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = b_z
\end{align}\tag{3.49a-b-c}$$

where $\sigma_x$, $\sigma_y$ and $\sigma_z$ are the normal stress, $b_x$, $b_y$ and $b_z$ are the body forces per unit volume in the directions of $x$, $y$ and $z$ axes respectively, and $\tau_{xy}$, $\tau_{xz}$, $\tau_{yz}$, $\tau_{zx}$ and $\tau_{yz}$ are the shear stress components. These equations can be written in tensor form as

$$\begin{align}
\frac{\partial \sigma_i}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} - b_i = 0 \quad i \neq j
\end{align}\tag{3.50}$$

where $(1) \equiv (x)$, $(2) \equiv (y)$, $(3) \equiv (z)$, $x_1 \equiv x$, $x_2 \equiv y$ and $x_3 \equiv z$. The principle of virtual work of the last equation is

$$\begin{align}
\int_{\Omega} h_i^T \left( \frac{\partial \sigma_i}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} - b_i \right) \ d\Omega = 0
\end{align}\tag{3.51}$$
3.5 Governing finite element equations

where \( h_i \) are arbitrary functions and called weighting functions. On applying the divergence theorem to the last equation results

\[
- \int_\Omega \left( \frac{\partial h_i}{\partial x_i} \right)^T \sigma_i \, d\Omega + \int_\Gamma h_i^T \sigma_i \, n_i \, d\Gamma \\
- \int_\Omega \left( \frac{\partial h_i}{\partial x_j} \right)^T \tau_{ij} \, d\Omega + \int_\Gamma h_i^T \tau_{ij} \, n_j \, d\Gamma \\
- \int_\Omega h_i^T b_i \, d\Omega = 0
\]

(3.52)

When \( h_i \) is identified as the displacement (i.e. \( h_i = u_i \)) then

\[
\frac{\partial h_i}{\partial x_i} = \epsilon_i
\]

(3.53)

therefore equation (3.52) becomes

\[
- \int_\Omega \epsilon_i^T \sigma_i \, d\Omega + \int_\Gamma u_i^T (\sigma_i \, n_i + \tau_{ij} \, n_j) \, d\Gamma - \int_\Omega u_i^T b_i \, d\Omega = 0
\]

(3.54)

The last equation can be written in matrix form as

\[
\int_\Omega \{\epsilon\}^T \{\sigma\} \, d\Omega = \int_\Gamma \{u\}^T \{\tau\} \, d\Gamma - \int_\Omega \{u\}^T \{b\} \, d\Omega
\]

(3.55)

where \( \{\tau\} \) is a vector with components \( \tau_i = \sigma_i n_i + \tau_{ij} n_j \) which are called 'tractions' and the term \( \int_\Gamma \{u\}^T \{\tau\} \, d\Gamma \) represents the work done by these tractions on the boundary of the continuum. In order to emphasise in the virtual work principle that the strains are not necessarily caused by stresses, but can be arbitrary as long as they are compatible, it is common to denote the virtual strains and displacements by superscript \( \ast \), i.e. \( \{\epsilon\} \) and \( \{u\} \). Hence, like the relations (3.31) and (3.36) the following relations are considered

\[
\{u\} = [N_u] \{a\} \quad \text{and} \quad \{\epsilon\} = [B] \{a\}
\]

(3.56)

and equation (3.54) is written in the incremental form as

\[
\int_\Omega \{\epsilon\}^T \{\Delta\sigma\} \, d\Omega = \int_\Gamma \{u\}^T \{\Delta\tau\} \, d\Gamma - \int_\Omega \{u\}^T \{\Delta b\} \, d\Omega
\]

(3.57)

The linear elastic stress-strain relation for the material is written as

\[
\{\sigma\} = [D] \{\epsilon\}
\]

(3.58)

where \( \{\sigma\} \) is the effective stress and the components of tensor \([D]\) are defined by equation (3.17). On using relations (3.32), (3.36) and (3.58) the following equation

\[
\{\Delta\sigma\} = \{\Delta\sigma\} + \{m\} \{\Delta\sigma_{pp}\}
\]

(3.59)
can be written as

$$\{\Delta \sigma\} = [D] [B] \{\Delta a\} + \{m\} \[\tilde{N}_{pp}\] \{\Delta \tilde{\sigma}_{pp}\} \tag{3.60}$$

Substituting equations (3.56) and (3.60) into equation (3.55) results in

$$\{a^*\}^T \left( \int_{\Omega} [B]^T [D] [B] d\Omega \right) \{\Delta a\} + \{a^*\}^T \left( \int_{\Gamma} [B]^T \{m\} \tilde{\sigma}_{pp} \right) d\Gamma = \int_{\Gamma} [N] [D] \{a(\Gamma)\} d\Omega \tag{3.61}$$

$$\{a^*\}^T$$ can be cancelled to give

$$[K] \{\Delta a\} + [L] \{\Delta \tilde{\sigma}_{pp}\} = \int_{\Gamma} [N] [D] \{\Delta \tau\} d\Gamma + \int_{\Omega} [N] [D] \{\Delta b\} d\Omega \tag{3.62}$$

where $[K]$ is the stiffness matrix, defined as

$$[K] = \int_{\Omega} [B]^T [D] [B] d\Omega \tag{3.63}$$

Equations (3.49a) and (3.62) are used to establish a solution at time $t + \Delta t$ from the solution at time $t$. Thus the solution can be marched forward in time from $t = 0$. In summarising, these equations can be written as

$$\begin{bmatrix} K & L \\ L^T & -\Phi \Delta t \Theta \end{bmatrix} \begin{bmatrix} \Delta a \\ \Delta \tilde{\sigma}_{pp} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{3.64}$$

where

$$\{F_1\} = \int_{\Gamma} [N_u]^T \{\Delta \tau\} d\Gamma + \int_{\Omega} [N_u]^T \{\Delta b\} d\Omega \tag{3.65}$$

and

$$\{F_2\} = [\Phi] \Delta t \{\tilde{\sigma}_{pp}\} + \int_{\Gamma} [\tilde{N}_u]^T \left( \Theta \{\Delta v_n\} - \{v_n\}^{(1)} \right) \Delta t d\Gamma \tag{3.66}$$

where $\{F_1\}$ and $\{F_2\}$ are vectors that can be evaluated from loads and solution values at time $t$.

### 3.6 GeoFEAP and GeoFEAP2

#### 3.6.1 Capabilities of GeoFEAP

GeoFEAP is a general purpose geotechnical finite element program for static analysis of two-dimensional nonlinear axisymmetric soil-structure interaction problems developed at the University of California at Berkeley by Espinoza et al. (1995). This program is based on the well-validated general purpose finite element analysis program (FEAP) and the earlier
version of FEAP is described in Zienkiewicz and Taylor (1989, 1991).

GeoFEAP was modified to provide additional capabilities for solving problems of interest to geotechnical engineers. A FE analysis from pre-processing to post-processing is prescribed by a variety of macro commands, which allow the user to perform an analysis in a batch mode, an interactive mode or a combination of these modes. The program also includes a graphical processor to allow the analyst to readily visualise the results even during the analysis. This processor includes the capabilities of displaying meshes, stress/strain contours and displacement vectors. Figure 3.6 shows the general GeoFEAP layout. This flexibility allows the inclusion of a wide variety of solution schemes, which can be essential when solving problems using different non-linear stress-dependent soil constitutive models.

![GeofEAPlayout.png](attachment:GeofEAPlayout.png)

**Figure 3.6: General GeoFEAP layout.**

Four types of elements are implemented within the GeoFEAP element library:

1. 2-noded bar element,
3.6 GeoFEAP and GeoFEAP2

(2) 2-noded beam element,

(3) 4-noded interface element, and

(4) 3, 4, 5, 6, 7, 8 and 9-noded soil elements.

A combination of one or more element types allows for realistic analysis of earth embankments, retaining walls and reinforced soil slopes.

The program can model a variety of geotechnical problems with different soil models. Presently, the structural bar and beam elements are modelled as a linear isotropic elastic material. Interface elements may be used to model the non-linear behaviour of soil-structure interfaces or shear planes within a soil mass (Clough and Duncan, 1969; Goodman et al., 1968). The soil response can be represented by:

1. non-linear stress-dependent incrementally elastic hyperbolic model (Duncan et al., 1980),
2. Drucker-Prager's model assuming an elastic-perfectly plastic material,
3. non-linear stress-dependent elasto-plastic model (Lade and Duncan, 1975), or
4. modified cam-clay model (Roscoe and Burland, 1968).

To solve non-linear governing equations, the Newton-Raphson technique is used to find a solution which satisfies the global equilibrium of the analysed system. Both drained and undrained analyses can be performed.

3.6.2 Limitations of GeoFEAP

To get more accurate results, the simultaneous interaction of soil and structure have to be modelled, as explained in Section 1.3. GeoFEAP has the following limitations:

1. It is only a 2D axisymmetric program.
2. Shell elements are not implemented. However, a shell element is needed to model the cooling tower shell.
3. The program does not model 3D soil-structure interaction problems.
4. It deals with the static analysis soil-structure interaction only.
3.7 Conclusion

3.6.3 New features programmed into GeoFEAP

Regarding to the previous limitations of GeoFEAP on modelling the 3D soil-cooling tower-interaction, the following new features are programmed into GeoFEAP:

1. Extending the capabilities of GeoFEAP to model footings and soil, using first order 8–noded hexahedron element. The 3D simultaneous interaction of soil and structure problems can be therefore investigated.

2. Extending the Drucker–Prager yield criterion to 3D and implementing it.

3. Computing the deviatoric stress and mean pressure, as the most parameters of importance to geotechnical engineers and modellers.

4. Implementing 4–noded quadrilateral flat shell element based on discrete Kirchhoff’s quadrilateral plate bending element. This element is used to model the elastic behaviour of curved structural which has one small dimension (a thickness normal to the remaining surface coordinates) compared to the other dimensions of the surface. The model is also formulated in terms of force resultants which are computed by integration of stress components over the cross-sectional thickness of the shell.

5. Modifying this element to accommodate a temperature profile.

6. Incorporating the modelling of thermal aspects of soil-structure interaction in both 2D and 3D.

After programming these new features, the software was modified and re-programmed, and renamed “GeoFEAP”. The soil-cooling tower-interaction using GeoFEAP2 are then investigated taking into consideration the temperature changes in the cooling tower. In addition, the effect of the modelling method of soil-cooling tower-interaction are also studied, via the idealisation of the structure and soil on the resulting parameters.

3.7 Conclusion

In this chapter, the 3D governing finite element equations for the soil have been derived. The soil parameters, elasto-plastic behaviour and reasons for choosing the appropriate constitutive Drucker–Prager’s model in this analysis have been described. In addition, the governing finite element equations of soil-structure interaction have been presented. These governing finite element equations and the Drucker–Prager’s model have been modelled in
GeoFEAP2. The Drucker–Prager’s model requires two parameters; one defines the cohesion (shear strength), and the second defines the dependence on the confining pressure, related to the angle of friction in soil mechanics.
CHAPTER 4

Validation of the Software for Soil Model

4.1 Overview

The aim of this chapter is to validate the new software (GeoFEAP2) when modelling the soil-structure interaction, as discussed in Chapter 3, for elastic and elasto-plastic soil behaviour. This validation will be performed for one- and two- horizontal and vertical soil layers in two and three dimensions with different types of elements and loads. The investigated materials in this chapter are assumed to be homogenous, isotropic and have constant physical properties.

4.2 Elastic behaviour of soil

In this section, the elastic behaviour of soil has been investigated to validate the software. The soil is assumed to comprise one- and two- horizontal and vertical layers. It has been investigated in 2D and 3D and modelled with 3-noded triangle and 4-noded quadrilateral elements. Although the elastic type of model may not provide an accurate representation of the actual behaviour of the soil, a number of geotechnical problems can be analysed successfully with this soil model. The elastic soil model also allows the analyst to compare the results with closed-form solutions available for some linear elastic problems (e.g. Poulos, 1967; Poulos and Davis, 1974).
4.2 Elastic behaviour of soil

4.2.1 2D circular flexible footing: 4-noded quadrilateral elements

The following example simulates the response of a soil undergoing an external surface load to compare with the original example by Poulos and Davis (1974). Figure 4.1 shows the geometric and stratigraphic characteristics of this problem. The corresponding finite element mesh of 4-noded quadrilateral elements, comprising 140 elements and 165 nodes, has been defined taking advantage of symmetry, as shown in figure 4.2. The mesh on the left side of the symmetric line is considered for this example. The boundary conditions for the mesh are as follows: the outer vertical boundary is restrained in the \( x \)-direction and is free to move in the \( y \)-direction. The base of the layer is restrained in both \( x \) and \( y \) directions. The soil’s elastic properties are assumed to be Young’s modulus of 180,000 lb/ft\(^2\) and Poisson’s ratio of 0.4 with an applied force of 1500 lb/ft\(^2\). Imperial units have been used in order to be able to compare with the original example by Poulos and Davis (1974).

Figure 4.3 shows the resultant displacement vector plot. Each vector represents the magnitude (by its length) and the direction (by its orientation) of the displacement. The relative magnitude of each of the vectors and their orientation clearly indicate the mechanism of failure as shown in figure 4.4 which shows the settlement contour in the \( y \)-direction. It can be seen from these figures that the displacements and contours decrease with depth and away from the load as expected, which agrees with Poulos and Davis (1974). It is found that the vertical displacement of a point just beneath the edge of the loading strip, i.e. at the solid circled node, is 0.072 ft. The percentage error of this result is therefore zero compared with the closed-form elastic solution, which is 0.072 ft Poulos and Davis (1974).

\[ E = 180,000 \text{ lb/ft}^2 \]
\[ v = 0.4 \]
\[ f = 1500 \text{ lb/ft}^2 \]

Figure 4.1: Geometric and stratigraphic characteristics for problems 4.2.1, 4.2.3 and 4.2.4.
4.2 Elastic behaviour of soil

Figure 4.2: Finite element mesh for problem 4.2.1.

Figure 4.3: Displacement resultant vector for problem 4.2.1.
4.2 Elastic behaviour of soil

Total vertical displacement (ft)

<table>
<thead>
<tr>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.06E-01</td>
</tr>
<tr>
<td>-8.68E-02</td>
</tr>
<tr>
<td>-6.72E-02</td>
</tr>
<tr>
<td>-4.76E-02</td>
</tr>
<tr>
<td>-2.79E-02</td>
</tr>
<tr>
<td>-8.27E-03</td>
</tr>
</tbody>
</table>

Current View
Min = -1.25E-01 ft
X = 5.00E+01 ft
Y = 3.00E+01 ft
Max = 1.14E-02 ft
X = 0.00E+00 ft
Y = 3.00E+01 ft

Figure 4.4: Settlement contours in the y-direction for problem 4.2.1.

4.2.2 2D circular flexible footing: 3-noded triangle elements

The problem considered in this section is a linear elastic drained soil layer of finite depth subject to a uniform circular flexible footing with a radius of 4 m to compare with the original example by Poulos (1967) and Harr (1966). The corresponding finite element mesh of this problem has been defined taking advantage of symmetry, as shown in figure 4.5. As shown in this figure, the mesh consists of 80 elements and 54 nodes with 3-noded triangle elements where the outer vertical boundary is restrained in the x-direction and is free to move in the y-direction. The base of the layer is restrained in both x and y directions. Young’s modulus for the elastic soil layer is equal to $3 \times 10^3$ kPa, with Poisson's ratio of 0.25 and an applied pressure of 30 kPa.

Figures 4.6 and 4.7 show the displacement resultant vector and settlement contour in the y-direction respectively. It can be seen from these figures that the displacements and contours decrease with depth and away from the load as expected. The calculated vertical displacement at the center, i.e. at node 6, and edge of the applied pressure, i.e. at node 18, are 54.2 mm and 32.0 mm, respectively. Poulos (1967) presented a theoretical solution which gives a central displacement of 55 mm for a layer of the same thickness. In addition, Harr (1966) presented an approximate solution, which gives a foundation edge settlement of
31 mm. The percentage errors are therefore 1.4% and 3.0% compared with Poulos (1967) and Harr (1966) respectively.

Figure 4.5: Finite element mesh for problem 4.2.2.

Figure 4.6: Displacement resultant vector for problem 4.2.2.

Figure 4.7: Settlement contour in the y-direction for problem 4.2.2.
4.2 Elastic behaviour of soil

4.2.3 Effect of Young’s modulus on the settlement of 2D axisymmetric circular footing on two horizontal soil layers

The response of two horizontal soil layers subject to an external circular footing has been investigated in this section. Figures 4.1, 4.2 and 4.8 show the problem in question. In figure 4.2, the corresponding finite element mesh of 140 4-noded quadrilateral elements and 165 nodes has been presented, taking advantage of symmetry. It should be noted, as shown in figure 4.8, that \( h = a = 10 \) ft, \( E_{h_1} \) and \( E_{h_2} \) are Young’s modulus of soil layers \( h_1 \) and \( h_2 \) respectively and \( \nu_{h_1} \) and \( \nu_{h_2} \) are Poisson’s ratios of soil layers \( h_1 \) and \( h_2 \) respectively, where \( E_{h_1} = 180,000 \times A_h_1 \) lb/ft\(^2\), \( E_{h_2} = A_h_2 \times 180,000 \) lb/ft\(^2\) and \( \nu_{h_1} = \nu_{h_2} = 0.4 \), where \( A_h_1 \) and \( A_h_2 \) are integer constants assumed as follows:

\[ A_{h_j} = 1 \quad \text{and} \quad 1 \leq A_{h_i} \leq 50 \quad \text{such that} \quad \frac{E_{h_i}}{E_{h_j}} = A_{h_i} \quad \text{where} \quad i, j = 1, 2 \quad \text{and} \quad i \neq j \quad (4.1) \]

The boundary conditions for the mesh are the same as explained in Section 4.2.1 with an applied force of 1500 lb/ft\(^2\).

![Figure 4.8: Finite element mesh for problem 4.2.3.](image)

Figure 4.9 shows the vertical displacement of a point just beneath the edge of the loading strip, i.e. at node 161, with

\[ 1 \leq \frac{E_{h_i}}{E_{h_j}} \leq 50 \quad \text{where} \quad i, j = 1, 2 \quad \text{and} \quad i \neq j \quad \text{(4.2)} \]
4.2 Elastic behaviour of soil

It is noted that the vertical displacement at node 161, where $\frac{E_{h1}}{E_{h2}}$ increases, is greater than the vertical displacement at node 161, where $\frac{E_{h2}}{E_{h1}}$ increases, and it decreases as $\frac{E_{h1}}{E_{h2}}$ or $\frac{E_{h2}}{E_{h1}}$ increases (Poulos and Davis, 1974). It should be also noted that the vertical displacement at node 161, when $E_{h1} = E_{h2}$, is equal to, as expected, 0.072 ft as discussed in Section 4.2.1.

![Figure 4.9: Vertical displacement (ft) at node 161 as a function of $E_{h1}/E_{h2}$ and $E_{h2}/E_{h1}$](image)

Figures 4.10 and 4.11 show the settlement at the interface level, i.e. at $y = 20$ ft where the nodes are numbered 91 to 105 as seen in figure 4.8, between the two soil layers of the elastic soil when $\frac{E_{h2}}{E_{h1}} = 1, 10, 20, 30, 40$ and 50 where $i, j = 1, 2$ and $i \neq j$. It is noted that the maximum modulus of the vertical settlement occurs at the central line and as expected it decreases as $\frac{E_{h1}}{E_{h2}}$ or $\frac{E_{h2}}{E_{h1}}$ increases. In addition,

$$\left| \frac{E_{h1}}{E_{h2}} \right|_k - \left| \frac{E_{h2}}{E_{h1}} \right|_{k+i+1} > \left| \frac{E_{h1}}{E_{h2}} \right|_{k+i+1} - \left| \frac{E_{h2}}{E_{h1}} \right|_{k+i+19}$$

(4.3)

where $i, j = 1, 2, \ i \neq j, \ k = 1, 10, 20, 30, \ l = \{0, 1\}$ as $k = \{1\}$ otherwise.

However, the vertical settlement at the central line decreases very rapidly from $\frac{E_{h2}}{E_{h1}} = 1$ to 10, i.e. when the strength of soil layer $h_2$ is greater than the strength of soil layer $h_1$ in this range of $\frac{E_{h2}}{E_{h1}}$, and then decreases slowly from $\frac{E_{h2}}{E_{h1}} = 10$ to 30 and becomes a constant in the range 30 to 50. This means that the major affected range for changing the ratio $\frac{E_{h2}}{E_{h1}}$ is 1 to 10 and there is no effect when this ratio is greater than 30.
4.2 Elastic behaviour of soil

Figure 4.10: Vertical displacement (ft) at the interface level, \( y = 20 \) ft, when \( \frac{E_{\text{D}_1}}{E_{\text{D}_2}} = 10, 20, 30, 40 \) and 50.

Figure 4.11: Vertical displacement (ft) at the interface level, \( y = 20 \) ft, when \( \frac{E_{\text{S}_2}}{E_{\text{S}_1}} = 10, 20, 30, 40 \) and 50.
4.2 Elastic behaviour of soil

Figures 4.12 and 4.13 show the settlement at $y = 20$ ft for $\frac{E_{h1}}{E_{h2}} = \frac{E_{h2}}{E_{h1}} = 10$ and 50 respectively. As shown in these figures, the modulus of the vertical settlement, as expected, decreases rapidly when $\frac{E_{h1}}{E_{h2}} = \frac{E_{h2}}{E_{h1}}$ because the volume of soil layer $h_1$ is two times of soil layer $h_2$.

![Figure 4.12: Vertical displacement (ft) at the interface level, $y = 20$ ft, when $\frac{E_{h1}}{E_{h2}} = \frac{E_{h2}}{E_{h1}} = 10$.](image)

![Figure 4.13: Vertical displacement (ft) at the interface level, $y = 20$ ft, when $\frac{E_{h1}}{E_{h2}} = \frac{E_{h2}}{E_{h1}} = 50$.](image)
4.2.4 Effect of Young’s modulus on the settlement of 2D axisymmetric circular footing on two vertical soil layers

In this section, the response of two vertical soil layers undergoing an external flexible footing has been investigated with the effect of Young’s modulus. Figures 4.1, 4.2 and 4.14 show the geometric and stratigraphic characteristics of this problem. The corresponding finite element mesh of 280 4-noded quadrilateral elements and 330 nodes has been defined, as shown in figure 4.2. In figure 4.14, the whole mesh of the studied problem is included, not half of it as in examples 4.2.1 and 4.2.3. This is due to Young’s modulus not being the same either side of the central line of symmetry which is taken as a separate level between soil layers \( V_1 \) and \( V_2 \). Furthermore, \( E_{v_1} \) and \( E_{v_2} \) are Young’s modulus of soil layers \( v_1 \) and \( v_2 \) respectively and \( \nu_{v_1} \) and \( \nu_{v_2} \) are Poisson’s ratios of soil layers \( v_1 \) and \( v_2 \) respectively, where \( E_{v_1} = 180,000 \) lb/ft\(^2\), \( E_{v_2} = 180,000 \times A_{y_2} \) lb/ft\(^2\) and \( \nu_{v_1} = \nu_{v_2} = 0.4 \), where \( A_{y_2} \) is an integer constant. The boundary conditions for the mesh are the same for the problem studied in Section 4.2.1 with an applied force of 1500 lb/ft\(^2\).

![Finite element mesh for problem 4.2.4.](image)

Figure 4.14: Finite element mesh for problem 4.2.4.

Figure 4.15 shows the vertical displacement of the points just beneath the edge of the loading strip, i.e. at node 161 in soil layer \( v_1 \) and node 320 in soil layer \( v_2 \), with

\[
1 \leq \frac{E_{v_2}}{E_{v_1}} = A_{y_2} \leq 50
\]

(4.4)

It is noticed that the absolute value of the vertical displacement at node 320 is equal to or less than the absolute value of the vertical displacement at node 161. In addition, the vertical
displacement at node 320 increases more rapidly than the vertical displacement at node 161 in the range \( 1 \leq A_{v2} \leq 5 \) and then both curves increase slowly in the range \( 5 < A_{v2} \leq 50 \) such that the difference between these curves is big. The vertical displacement at node 161, in soil layer \( v_1 \), is 25 times of the vertical displacement at node 320, in soil layer \( v_1 \). However, it is constant in this range, i.e. \( 5 < A_{v2} \leq 50 \). It should be also noted that, when \( E_{v1} = E_{v2} \), the vertical displacement at node 161, as expected, is equal to the vertical displacement at node 320 = 0.072 ft as discussed in Section 4.2.1.

![Vertical displacement at nodes 161 and 320 as a function of \( E_{v1} / E_{v2} \)](image)

Figure 4.15: Vertical displacement at nodes 161 and 320 [ft] as a function of \( E_{v2} / E_{v1} \).

Figures 4.16 to 4.20 show the settlement contours in the \( y \)-direction when \( E_{v2} / E_{v1} = 1, 20, 50, 70 \) and 100 respectively. The symmetric results (contours) of the vertical displacement have been noticed in figure 4.16 when \( E_{v2} / E_{v1} = 1 \) and from figures 4.17 to 4.20, as expected, the contours of the vertical displacement are non-symmetric although the two soil layers have been affected by equal loads. In addition, these contours decrease as \( A_{v2} \) increases, i.e. when soil layer \( v_2 \) becomes stronger than soil layer \( v_1 \). This means that as \( A_{v2} \) increases as soil layer \( v_2 \) acts as a solid material which affects the behaviour of soil layer \( v_1 \) as well. This results that the vertical displacement contours being concentrated in the weaker soil layer \( v_1 \).
4.2 Elastic behaviour of soil

Figure 4.16: Settlement contour in the $y$-direction for problem 4.2.4 when $\frac{E_{soil}}{E_{v}} = 1$.

Figure 4.17: Settlement contour in the $y$-direction for problem 4.2.4 when $\frac{E_{soil}}{E_{v}} = 20$.

Figure 4.18: Settlement contour in the $y$-direction for problem 4.2.4 when $\frac{E_{soil}}{E_{v}} = 50$. 


4.2 Elastic behaviour of soil

Figure 4.19: Settlement contour in the y—direction for problem 4.2.4 when \( \frac{E_{cr}}{E_{v1}} = 70 \).

Figure 4.20: Settlement contour in the y—direction for problem 4.2.4 when \( \frac{E_{cr}}{E_{v1}} = 100 \).
4.2 Elastic behaviour of soil

4.2.5 3D soil-structure interaction with a column

The aim of this section and Section 4.2.6 is to validate the software and model for the 3D soil-structure interaction undergoing an external load as a base for studying the 3D soil-cooling tower-interaction problem which will be investigated in Chapters 7 and 8.

Figure 4.21 shows the geometric and stratigraphic characteristics of a 3D soil-structure interaction with a column having its concrete footing sitting on the soil. The corresponding finite element mesh, shown in figures 4.22 and 4.23, consists of 332 first order 8-noded hexahedron elements with 879 nodes where the column is modelled by a 2-noded bar element. The boundary conditions for the mesh are as follows: the outer vertical boundary is restrained in the \( x \) and \( y \) directions and is free to move in the \( z \)-direction. The base of the soil layer is restrained in the \( x \), \( y \) and \( z \) directions. The soil's and structure's elastic properties are assumed to be Young's modulus of \( 10^4 \) kPa and \( 30 \times 10^6 \) kPa, Poisson's ratio of 0.3 and 0.2, and a unit weight of 20 kN/m\(^3\) and 24 kN/m\(^3\), respectively (Das, 2002; Hunt, 1984). These parameters would be representative of clay and concrete. In addition, the column's properties are assumed to be Young's modulus of \( 20 \times 10^6 \) kPa and a cross section of 0.1 m\(^2\) with an applied force of 1000 kN in the opposite of the \( z \)-direction.

The displacement and stress results in the \( x \), \( y \) and \( z \) directions have been found to be symmetric around the central lines, as shown in figure 4.22. These results are the same at the solid circled nodes. In addition, it is found, as expected, that \( F_z = R_z \) and \( R_x = R_y = 0 \), where \( F_z \) is the applied force in the \( z \)-direction and \( R_x \), \( R_y \) and \( R_z \) are the total reaction in \( x \), \( y \) and \( z \) directions respectively.

Figures 4.24 and 4.25 show the top view for the displacement contours at \( z = 5 \) m and \( z = 4 \) m respectively while figure 4.26 presents a side view for the settlement contours in the \( z \)-direction. It can be seen from these figures that the displacement contours decrease with depth and away from the load as expected. In addition, the major settlement occurs at the soil-structure interaction surface which is shown in figure 4.22. Figure 4.27 shows the deviatoric stress values, which are defined by equation (3.22) and measured in kPa, as a function of \( x \) and \( y \), measured in m, at the soil-structure interaction interface, which is shown in figure 4.22, before and after applying the load. The maximum deviatoric stress value occurs, as expected, at the middle of this interface, as shown in figure 4.27, and is equal to 494.8 kPa.
Figure 4.21: Geometric and stratigraphic characteristics for problem 4.2.5.
4.2 Elastic behaviour of soil

Figure 4.22: Finite element mesh, top view, for problem 4.2.5.

Figure 4.23: Finite element mesh, side view, for problem 4.2.5.
4.2 Elastic behaviour of soil

Figure 4.24: Settlement contours, top view, at \( z = 5 \) m in the \( z \)-direction for problem 4.2.5.

Figure 4.25: Settlement contours, top view in the \( z \)-direction, layer \( h_1 \) – layer \( h_2 \) interface, i.e. \( z = 4 \) m, for problem 4.2.5.
4.2 Elastic behaviour of soil

![Figure 4.26: Settlement contours, side view in the z-direction, for problem 4.2.5.](image)

![Figure 4.27: Deviatoric stress values at the soil-structure interaction surface, shown in figure 4.22, before and after applying the load.](image)
4.2 Elastic behaviour of soil

4.2.5.1 Two horizontal soil layers

The response of two horizontal soil layers subject to an external point load has been investigated in this section. Figures 4.21, 4.22 and 4.28 show the problem in question, where \( E_{h_1} \) and \( E_{h_2} \) are Young's modulus of soil layers \( h_1 \) and \( h_2 \) respectively and \( \nu_{h_1} \) and \( \nu_{h_2} \) are Poisson's ratios of soil layers \( h_1 \) and \( h_2 \) respectively, where \( E_{h_1} = 10^4 \times A_{h_1} \text{ kPa} \), \( E_{h_2} = 10^4 \times A_{h_2} \text{ kPa} \) and \( \nu_{h_1} = \nu_{h_2} = 0.3 \), where \( A_{h_1} \) and \( A_{h_2} \) are integer constants. In the present analysis \( A_{h_1} \) and \( A_{h_2} \) are assumed as in equation (4.1). The properties of the other materials for this problem and boundary conditions for the mesh are the same as for the problem studied in Section 4.2.5.

![Figure 4.28: Finite element mesh, side view, for problem 4.2.5.1.](image)

Figures 4.29 to 4.33 show the displacement contour in the \( z \)–direction when \( \frac{E_{h_1}}{E_{h_2}} = 10, 20, 30, 40 \) and 50. It can be seen from these figures that the displacement in the \( z \)–direction decreases as \( \frac{E_{h_1}}{E_{h_2}} \) increases, i.e. when soil layer \( h_1 \) is stronger than soil layer \( h_2 \). On the other hand, figures 4.34 to 4.38 show the displacement contour in the \( z \)–direction when \( \frac{E_{h_2}}{E_{h_1}} = 10, 20, 30, 40 \) and 50. It can be seen from these figures that the displacement in the \( z \)–direction increases as \( \frac{E_{h_2}}{E_{h_1}} \) increases, i.e. when soil layer \( h_2 \) is more stronger than soil layer \( h_1 \).
4.2 Elastic behaviour of soil

Figure 4.29: Settlement contours in the $z$-direction when $\frac{E_{hl}}{E_{h2}} = 10$ for problem 4.2.5.1.

Figure 4.30: Settlement contours in the $z$-direction when $\frac{E_{hl}}{E_{h2}} = 20$ for problem 4.2.5.1.
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Figure 4.31: Settlement contours in the z-direction when $\frac{E_h}{E_h} = 30$ for problem 4.2.5.1.

Figure 4.32: Settlement contours in the z-direction when $\frac{E_h}{E_h} = 40$ for problem 4.2.5.1.
4.2 Elastic behaviour of soil

Figure 4.33: Settlement contours in the z-direction when $\frac{P_{h2}}{P_{h1}} = 50$ for problem 4.2.5.1.

Figure 4.34: Settlement contours in the z-direction when $\frac{P_{h2}}{P_{h1}} = 10$ for problem 4.2.5.1.
4.2 Elastic behaviour of soil

Figure 4.35: Settlement contours in the z-direction when $\frac{E_{h_2}}{E_{h_1}} = 20$ for problem 4.2.5.1.

Figure 4.36: Settlement contours in the $z$-direction when $\frac{E_{h_2}}{E_{h_1}} = 30$ for problem 4.2.5.1.
4.2 Elastic behaviour of soil

Figure 4.37: Settlement contours in the $z$-direction when $\frac{E_{h2}}{E_{h1}} = 40$ for problem 4.2.5.1.

Figure 4.38: Settlement contours in the $z$-direction when $\frac{E_{h2}}{E_{h1}} = 50$ for problem 4.2.5.1.
4.2 Elastic behaviour of soil

Figure 4.39 shows the vertical displacement contours at the interface level, i.e. at $z = 4$ m and $x = 5$ m, when $\frac{E_{h_1}}{E_{h_2}}$ and $\frac{E_{h_2}}{E_{h_1}} = 10, 20, 30, 40$ and 50, respectively. It is shown in figure 4.39 that the vertical settlement decreases very rapidly from $\frac{E_{h_1}}{E_{h_2}} = 1$ to 10, i.e. when the strength of soil layer $h_1$ is greater than the strength of soil layer $h_2$ in this range, and then decreases slowly from $\frac{E_{h_1}}{E_{h_2}} = 10$ to 50. However, figure 4.40 shows that the vertical settlement decreases rapidly from $\frac{E_{h_2}}{E_{h_1}} = 1$ to 10, i.e. when the strength of soil layer $h_2$ is greater than the strength of soil layer $h_1$ in this range, and then decreases slowly from $\frac{E_{h_2}}{E_{h_1}} = 10$ to 50.

![Figure 4.39: Vertical displacement [m] at level $z = 4$ m when $\frac{E_{h_1}}{E_{h_2}} = 10, 20, 30, 40$ and 50.](image)

![Figure 4.40: Vertical displacement [m] at level $z = 4$ m when $\frac{E_{h_2}}{E_{h_1}} = 10, 20, 30, 40$ and 50.](image)
4.2 Elastic behaviour of soil

This means that the affected range for changing the ratios of \( \frac{E_{h_2}}{E_{h_1}} \) and \( \frac{h_2}{h_1} \) is between 1 to 10. It should be noted that the settlement in the case of \( \frac{E_{h_2}}{E_{h_1}} \) is bigger than the settlement of \( \frac{E_{h_2}}{E_{h_1}} \) because the soil volume in the first case is four times of the second case, as shown in figure 4.28. This is the same conclusion obtained on investigating the 2D two horizontal soil layers in Section 4.2.3.

Figure 4.41 shows the displacement contour in the z-direction of the same soil with the same properties except that \( \nu_{h_2} = 0.4999 \), i.e. soil layer \( h_2 \) is assumed to be undrained being loaded in the short term, where pore water is not allowed to drain from the soil sample during loading, the increase in stress is carried by the pore water, called the excess pore water pressure. It is well-known that the standard displacement finite element formulation of elastic problems fails when the material becomes incompressible, i.e. volume remains constant, or Poisson's ratio \( \nu \) reaches 0.5, see equation (3.18). A simple method to side-step this difficulty is to use Poisson's ratio close to 0.5 but not equal to it. This strategy is widely adopted in a number of commercial geotechnical finite element softwares (Phoon et al., 2003).

It can be seen from figure 4.41 that the displacement in the z-direction decreases at soil layer \( h_2 \), compared with figure 4.26, i.e. when \( \nu_{h_2} = \nu_{h_1} \) and \( E_{h_2} = E_{h_1} \), because \( \nu_{h_2} \) is bigger than \( \nu_{h_1} \).

![Figure 4.41: Settlement contours in the z-direction when \( \nu_{h_2} = 0.4999 \) for problem 4.2.5.1.](image-url)
4.2 Elastic behaviour of soil

4.2.5.2 Two vertical soil layers

In this section, the response of two vertical soil layers undergoing an external point load has been investigated. Figures 4.21, 4.22 and 4.42 show the geometric and stratigraphic characteristics, finite element mesh in top and side views of this problem respectively. In figure 4.42, $E_{V1}$ and $E_{V2}$ are Young’s modulus of soil layers $V_1$ and $V_2$ respectively and $\nu_{V1}$ and $\nu_{V2}$ are Poisson’s ratios of soil layers $V_1$ and $V_2$ respectively, where $E_{V1} = 10^4$ kPa, $E_{V2} = 10^4 \times A_{V2}$ kPa and $\nu_{V1} = \nu_{V2} = 0.3$, where $A_{V2}$ is an integer constant. The properties of the other materials for this problem and boundary conditions for the mesh are the same for the problem studied in Section 4.2.5.

![Finite element mesh, side view, for problem 4.2.5.2.](image)

Figure 4.42: Finite element mesh, side view, for problem 4.2.5.2.

Figures 4.43 to 4.47 show the displacement contour in the $z-$direction when $\frac{E_{V2}}{E_{V1}} = A_{V2} = 10, 20, 30, 40$ and $50$. It can be seen from these figures that the displacement in the $z-$direction decreases at soil layer $V_2$ as $A_{V2}$ increases, i.e. when soil layer $V_2$ is more stronger than soil layer $V_1$. In addition, the contours of the vertical displacement are non-symmetric although the two soil layers have been affected by equal loads. Furthermore, these contours decrease as $A_{V2}$ increases, i.e. when soil layer $V_2$ becomes stronger than soil layer $V_1$. This means that as $A_{V2}$ increases as soil layer $V_2$ acts as a solid material which affects the behaviour of soil
4.2 Elastic behaviour of soil

layer \( v_1 \) as well. This means that the vertical displacement contours being concentrated in the weak soil layer \( v_1 \).

Figure 4.43: Settlement contours in the \( z \)-direction for problem 4.2.5.2 when \( A_{v_2} = 10 \).

Figure 4.44: Settlement contours in the \( z \)-direction for problem 4.2.5.2 when \( A_{v_2} = 20 \).
4.2 Elastic behaviour of soil

Figure 4.45: Settlement contours in the z-direction for problem 4.2.5.2 when $A_{v_2} = 30$.

Figure 4.46: Settlement contours in the z-direction for problem 4.2.5.2 when $A_{v_2} = 40$. 
4.2 Elastic behaviour of soil

4.2.5.2

Total displacement in z-direction (m)

<table>
<thead>
<tr>
<th>Level</th>
<th>Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.58E-02</td>
<td></td>
</tr>
<tr>
<td>-1.32E-02</td>
<td></td>
</tr>
<tr>
<td>-1.06E-02</td>
<td></td>
</tr>
<tr>
<td>-7.91E-03</td>
<td></td>
</tr>
<tr>
<td>-5.28E-03</td>
<td></td>
</tr>
<tr>
<td>-2.64E-03</td>
<td></td>
</tr>
</tbody>
</table>

Current View
Min = -1.85E-02 m
X = 0.00E+00 m
Y = 5.00E+00 m
Z = 5.00E+00 m
Max = 0.00E+00 m
X = 0.00E+00 m
Y = 0.00E+00 m
Z = 0.00E+00 m

Figure 4.47: Settlement contours in the z-direction for problem 4.2.5.2 when $A_{v2} = 50$.

Figure 4.48 shows the displacement contours in the z-direction of the same soil with the same properties except that $v_2 = 0.4999$, i.e. soil layer $v_2$ is assumed to be undrained, as explained in Section 4.2.5.1. It can be seen from this figure that the displacement in the z-direction decreases at soil layer $v_2$, compared with the figure 4.26, i.e when $v_{v2} = v_{v1}$ and $E_{v2} = E_{v1}$, because $v_{v2}$ is bigger than $v_{v1}$.

4.2.6 3D soil-structure interaction for one-storey building

The following example extends the problem which has been investigated in Section 4.2.5 and the results will be compared with those obtained by Kocak and Mengi (2000). Figure 4.49 shows the geometric and stratigraphic characteristics of a one-storey building having an elastic floor, which stands, at its corners on columns supported by rigid square footings. The corresponding finite element mesh is shown in figures 4.50 (a),(b),(c) and (d) for top view of soil, side view of soil, footings and floor respectively.

The boundary conditions for the mesh are as follows: the outer vertical boundary is restrained in the $x$ and $y$ directions and is free to move in the $z$-direction. The base of the soil layer is restrained in the $x$, $y$ and $z$ directions.

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4.2 Elastic behaviour of soil

Young's modulus is assumed to be $10^4$ kPa for the elastic soil and $20 \times 10^5$ kPa for the rigid footings, columns and roof. Poisson's ratio is assumed as 0.2 for the elastic soil, rigid footings, columns and roof (Kocak and Mengi, 2000). In addition, the column's cross section and moment of inertia are assumed to be $0.16 \text{ m}^2$ and $0.00213 \text{ m}^4$ respectively with an applied force of 100 kN at point A, as shown in figure 4.49 (a).

Figure 4.50 (e) shows the vertical displacement contours at the roof. It is shown that, as expected, the maximum value is at the load point, i.e. at point A. In addition, the displacement value decreases as the total distance between any point in the roof and point A increases. It is also found that the force at the point B is $-99.2$ kN, an error of 1.8% compared with the value $-97.4$ kN obtained by Kocak and Mengi (2000) who proved that the interaction between the footings may be ignored, with respect to the element forces, due to symmetry of the structure.
Figure 4.49: (a) Geometric and stratigraphic characteristics for the soil-structure interaction problem considered in Section 4.2.6 and (b) top view for this problem.
4.2 Elastic behaviour of soil

Figure 4.50: Finite element mesh for the soil-structure interaction problem considered in Section 4.2.6 for (a) top view of the soil, (b) side view of the soil (c) top view of the rigid footings, (d) top view of the roof, and (e) vertical displacement contours at the roof.
4.3 Elasto-plastic behaviour of soil

4.3.1 Introduction to the problem

One of the most critical parameters required by an engineer when designing shallow foundations is the bearing capacity of the underlying soil. For simple cases, either exact solutions, or upper and lower bounds can be found when the soil behaves in an elasto-plastic manner. However, for the majority of actual geotechnical situations, the assumptions made when deriving plasticity solutions provide a very crude idealisation of reality. (El-Hamalawi, 1997). The prediction of collapse loads under plastic flow conditions can be a significant numerical challenge to simulate accurately. The limiting load-bearing capacity of soil structures forms one of the classic problems of geomechanics. There are essentially three types of solution methods: limit equilibrium methods (e.g. Meyerhof, 1951; Terzaghi, 1943), slip-line field methods (e.g. Prandtl, 1921; Sokolovskii, 1960; Wu, 1976) and limit analysis methods (e.g. Boulbibane and Ponter, 2005; Griffiths and Fenton, 2001).

In this section, the problem under consideration will be determining the bearing capacity of a single soil layer and upon which a flexible square smooth footing of width b and weight q acts, as shown in figure 4.51. The flexibility property means that any loading is uniform and can be represented by a surface surcharge pressure applied to the surface of the soil immediately below the position of the footing, i.e. the force is applied in the y—direction. If the footing is smooth, then the other boundary condition that must be applied to the nodes under the footing is that the horizontal nodal forces are zero.

![Figure 4.51: Geometric and stratigraphic characteristics for problem 4.3.](image-url)
4.3 Elasto-plastic behaviour of soil

The soil is assumed to be an elasto-plastic material which obeys the Drucker–Prager model, as introduced in Section 3.4.5, with Young’s modulus \( E \) of \( 10^5 \) kPa, Poisson’s ratio \( \nu \) of 0.4999, cohesion \( c \) of 100 kPa and friction angle of \( \phi \) 0° (Potts and Zdravković, 2001). The corresponding finite element mesh of 3–noded triangles and 4–noded quadrilateral elements, comprising 130 elements and 177 nodes, has been defined taking advantage of symmetry, as shown in figure 4.52. The two vertical sides of the mesh have been restrained in the horizontal direction, while the base of the mesh was not allowed to move in either the vertical or horizontal directions.

![Finite element mesh for problem 4.3](image)

Figure 4.52: Finite element mesh for problem 4.3.

### 4.3.2 Bearing capacity of a strip footing

The classic solution for the collapse load derived by Prandtl (1921) is a worthy problem for comparison purposes, where the failure mechanism is shown in figure 4.53. In this figure, the foundation is assumed to be a strip foundation, represented by a 2D plane strain finite element model. Terzaghi and Peck (1967) derived the closed-form solution for this problem as follows:

\[
\bar{q}_{\text{max}} = (2 + \pi) \times c = 5.14 \times c
\]  

where \( \bar{q}_{\text{max}} \) is the ultimate bearing capacity of the footing.

Figure 4.54 shows the displacement resultant vector and contours for the horizontal displacement for problem 4.3. It can be seen from this figure that the displacements and contours
4.3 Elasto-plastic behaviour of soil

decrease with depth and away from the load as expected, which agrees with the failure mechanism, as shown in figure 4.53.

\[ q_{\text{max}} = (2 + \pi)c \]

Figure 4.53: Failure mechanism of a strip footing on a frictionless soil (Prandtl’s wedge problem).

Figure 4.54: Displacement resultant vectors and contours of the horizontal displacement for problem 4.3.2.

Figure 4.55 shows the bearing capacity of a strip footing versus vertical displacement of the footing. The final value of the bearing capacity is 516 kPa, an error of 0.01% compared to the analytical solution 514 kPa, calculated using equation (4.5), which is negligible.

Figure 4.55: Bearing capacity of a strip footing for problem 4.3.2.
4.3.3 Bearing capacity of a circular footing

If the foundation is circular, the solution to the problem considered in Section 4.3.2 becomes as follows (Shield, 1955)

\[ \bar{q}_{\text{max}} = 5.69 \times c \] (4.6)

Figure 4.56 shows the displacement resultant vector and contours for the horizontal displacement of problem 4.3 under the axisymmetric assumption where the loading is also symmetric about the center line. It can be seen from this figure that the displacements and contours decrease with depth and away from the load as expected, which agrees with the failure mechanism, as shown in figure 4.53. Figure 4.57 shows the bearing capacity of a circular footing versus vertical displacement of the footing. The final value of the bearing capacity is 571 kPa, an error of 0.01% compared with the analytical solution 569 kPa, see equation (4.6), which is negligible.

![Figure 4.56](image1)

![Figure 4.57](image2)
4.4 Conclusion

In this chapter, the new software (GeoFEAP2) has been validated when modelling the soil-
structure interaction. 2D circular flexible footings have been analysed using quadrilateral
and triangular elements and the results compared well with analytical solutions.

The effect of Young’s modulus on the settlement of 2D axisymmetric circular footing has
been investigated when the soil is divided into two horizontal layers \( h_1 \) and \( h_2 \) correspond-
ing to the upper and lower layers of soil beneath the foundation respectively. It was found
that the vertical displacement decreases, as expected, with depth and away from the load.
In addition, the vertical displacement, where \( \frac{E_{h_1}}{E_{h_2}} \) increases, is higher than the vertical dis-
placement at the same point, where \( \frac{E_{h_1}}{E_{h_2}} \) increases, and it decreases as \( \frac{E_{h_1}}{E_{h_2}} \) or \( \frac{E_{h_2}}{E_{h_1}} \) increases.
Moreover, the maximum modulus of the vertical settlement occurs at the central line and,
as expected, it decreases as \( \frac{E_{h_1}}{E_{h_2}} \) or \( \frac{E_{h_2}}{E_{h_1}} \) increases. Furthermore, the vertical settlement at the
central line decreases very rapidly from \( \frac{h_2}{h_1} = 1 \) to \( 10 \), i.e. when the strength of the lower
soil layer \( h_2 \) is greater than the strength of the upper soil layer \( h_1 \) in this range of \( \frac{E_{h_1}}{E_{h_2}} \), and
then decreases slowly from \( \frac{h_2}{h_1} = 10 \) to \( 30 \) and becomes a constant in the range \( 30 \) to \( 50 \).
This means that the major affected range for changing the ratio \( \frac{E_{h_1}}{E_{h_2}} \) is 1 to 10 and there is
no effect when this ratio is greater than 30. Finally, the modulus of the vertical settlement
at the interface level, as expected, decreases rapidly when \( \frac{h_2}{h_1} = \frac{E_{h_2}}{E_{h_1}} \) because the volume of
soil layer \( h_1 \) is two times of the volume of soil layer \( h_2 \).

The effect of Young’s modulus on the settlement of 2D axisymmetric circular footing has
been modelled when the soil is divided into two vertical layers \( v_1 \) and \( v_2 \), where \( E_{v_2} \geq E_{v_1} \).
It was found that the vertical displacement at a node in the soil layer \( v_2 \) is equal to the
vertical displacement at a node corresponding to the same position symmetrically in the soil
layer \( v_1 \) when \( E_{v_1} = E_{v_2} \). However, the vertical displacement at a node in the soil layer
\( v_2 \) is less than the vertical displacement at a node corresponding to the same position sym-
metrically in the soil layer \( v_1 \) when \( E_{v_1} < E_{v_2} \). In addition, the vertical displacement at a
node corresponding to the same position symmetrically in the soil layer \( v_1 \) increases more
rapidly than the vertical displacement a node in the soil layer \( v_2 \) in the range \( 1 \leq \frac{E_{v_2}}{E_{v_1}} \leq 5 \)
and then increases slowly in the range \( 5 < \frac{E_{v_2}}{E_{v_1}} \leq 50 \). Moreover, the vertical displacement
at a node in soil layer \( v_1 \), is 25 times of the vertical displacement at an equivalent node in
soil layer \( v_1 \) and is constant in this range, i.e. \( 5 < \frac{E_{v_2}}{E_{v_1}} \leq 50 \). The settlement contours in
the vertical direction when \( \frac{E_{v_2}}{E_{v_1}} = 1, 20, 50, 70 \) and 100 have been also discussed. It was
found that these vertical settlement contours decrease, as expected, with depth and away from the load. In addition, these contours are symmetric when \( \frac{E_{v2}}{E_{v1}} = 1 \) and, as expected, non-symmetric when \( \frac{E_{v2}}{E_{v1}} > 1 \), although the two soil layers have experienced equal loads. Furthermore, these contours decrease as \( \frac{E_{v2}}{E_{v1}} \) increases, i.e. when the soil layer \( v_2 \) becomes stronger than the soil layer \( v_1 \). This means that as \( \frac{E_{v2}}{E_{v1}} \) increases as the soil layer \( v_2 \) acts as a solid material which affects the behaviour of the soil layer \( v_1 \) as well. This results in the vertical displacement contours being concentrated in the weak soil layer \( v_1 \).

The 3D soil-structure interaction undergoing an external load has been investigated as a base for studying the 3D soil-cooling tower-interaction problem. It was found that the displacement contours decrease with depth and away from the load as expected. In addition, the major settlement occurs at the soil-structure interaction surface. Moreover, the displacement and stress results in the \( x, y \) and \( z \) directions have been found to be symmetric around the central line. Furthermore, the maximum deviatoric stress value occurs, as expected, at the middle of soil-structure interaction interface.

The response of the 3D two horizontal soil layers, \( h_2 \) and \( h_1 \) corresponding to the upper and lower soil layers beneath the foundation respectively, structure interaction undergoing an external load has been investigated. It was found that the vertical displacement decreases, as expected, with depth and away from the load. In addition, the vertical displacement in the vertical direction decreases as \( \frac{E_{h1}}{E_{h2}} \) increases, i.e. when the lower soil layer \( h_1 \) is stronger than the upper soil layer \( h_2 \), and increases as \( \frac{E_{h2}}{E_{h1}} \) increases, i.e. when the upper soil layer \( h_2 \) is stronger than the lower soil layer \( h_1 \). Furthermore, the vertical displacement contours at the interface level decrease very rapidly from \( \frac{E_{h1}}{E_{h2}} = 1 \) to 10, i.e. when the strength of the lower soil layer \( h_1 \) is greater than the strength of the upper soil layer \( h_2 \) in this range, and then decrease slowly from \( \frac{E_{h1}}{E_{h2}} = 10 \) to 50. However, the vertical settlement contours at the interface level decrease rapidly from \( \frac{E_{h2}}{E_{h1}} = 1 \) to 10, i.e. when the strength of the upper soil layer \( h_2 \) is higher than the strength of the lower soil layer \( h_1 \) in this range, and then decrease slowly from \( \frac{E_{h2}}{E_{h1}} = 10 \) to 50. This means that the affected range for changing the ratios of \( \frac{E_{h1}}{E_{h2}} \) and \( \frac{E_{h2}}{E_{h1}} \) is between 1 to 10. It was also noted that the settlement when \( \frac{E_{h1}}{E_{h2}} \) is bigger than the settlement when \( \frac{E_{h2}}{E_{h1}} \) because the soil volume in the first case is taken as four times of the soil volume in the second case. This is the same conclusion obtained on investigating 2D axisymmetric circular footing problem when the soil is divided into two horizontal layers. If the upper soil layer \( h_2 \) is assumed as undrained, the vertical displacement decreases at the
upper soil layer \( h_2 \) because \( \nu_{h_2} \) is bigger than \( \nu_{h_1} \).

The response of the 3D two vertical soil layers, \( v_1 \) and \( v_2 \), structure interaction undergoing an external load has been studied. It was found that the vertical displacement contours decrease, as expected, with depth and away from the load. In addition, these contours in the vertical direction decrease at the soil layer \( v_2 \) as \( \frac{E_{v_2}}{E_{v_1}} \) increases, i.e. when the soil layer \( v_2 \) becomes stronger than the soil layer \( v_1 \). Furthermore, these contours are symmetric when \( \frac{E_{v_2}}{E_{v_1}} = 1 \) and, as expected, non-symmetric when \( \frac{E_{v_2}}{E_{v_1}} > 1 \), although the two soil layers have experienced equal loads. This means that as \( \frac{E_{v_2}}{E_{v_1}} \) increases as the soil layer \( v_2 \) acts as a solid material which affects the behaviour of the soil layer \( v_1 \) as well. This means also that the vertical displacement contours will being concentrated in the weak soil layer \( v_1 \). This is the same conclusion obtained on investigating 2D axisymmetric circular footing problem when the soil is divided into two vertical layers. If the soil layer \( v_2 \) is assumed as undrained, the vertical displacement contours decrease at the soil layer \( v_2 \) because \( \nu_{v_2} \) is bigger than \( \nu_{v_1} \).

The 3D soil-structure-interaction for one-storey building undergoing an external load has been modelled. It was found that the maximum value occurs, as expected, at the load point. In addition, the displacement value decreases as the total distance between any point in the roof and the load point increases, and the results compared well with the numerical solution obtained by Kocak and Mengi (2000).

The bearing capacity of strip and circular footings have been studied. It was found that the displacement resultant vector and contours for the horizontal displacement decrease with depth and away from the load as expected, which agrees with the failure mechanism, and the bearing capacities correspond well with the closed-form solution.
CHAPTER 5

Implemented Shell Element

5.1 Overview

The four noded quadrilateral flat shell element is introduced in this chapter. Membrane and bending elements based on Kirchhoff’s assumptions are used for development of this element, the stiffness matrix for it is then presented. In addition, thermal effects upon this flat shell element are presented.

5.2 Introduction

A shell is a continuum which is bounded by two curved surfaces separated by the thickness which may be constant or vary either gradually or abruptly therefore shells are amenable to treatment by the continuum theory, in this case, the theory of elasticity. Two types are to be considered; thin and thick shell. If the thickness is considerably smaller than the principal radii of curvatures of the bounding surfaces then the shell is defined as a thin shell; if not, it is termed thick. In addition, if each bounding surface is generated by the rotation of a plane curve about a common axis, a shell of revolution is produced. Shell structures are one of the most important structural elements, as they are widely used in a variety of engineering applications. In this research, it has been assumed that the material from which the shell is constructed is homogenous, isotropic and perfectly elastic so that Hooke’s law can be used.
5.3 Types of shell elements

Shell elements are widely used to model the curved geometry of a structure. Shell elements based on classical shell theory are very difficult to develop. Many simplifying approximations are involved in the development. These types of elements are very efficient in modelling the curved geometry of the structure. However, because of the complexities involved, the alternative approach of modelling the structure with series of flat elements, which is simpler and easier to implement, became more popular for the analysis of shell structures.

The main elements of shell structures include triangular and quadrilateral shells; low-order and high-order shell elements; shallow and deep shells; flat and curved shell element; thick and thin element; axisymmetric and sandwich shells. The axisymmetric shell structures, generated by rotating a plane curve generator around an axis of rotation to form a circumferentially closed surface, have many applications such as hyperbolic cooling towers. These structures are thin and the first attempt to apply the finite element analysis to them treated the shell surface as being made up of flat plate elements (either triangle or quadrilateral in shape) (Djoudi and Bahai, 2004). The two most widely adopted approaches in the finite element analysis of general shells are (1) use of curved elements based on a suitable shell theory and (2) approximation of the curved structure by an assemblage of flat shell elements.

Although the first plate bending element was introduced in 1961, elements which are adequate for general shell analysis only became available in the 1970s. The published literature on modelling of plates and shells in the linear and non-linear regimes and their application to dynamic or vibration analysis of structures has grown extensively. The long history of gradual improvements in the design of finite shell elements has been reviewed by MacNeal (1998) for the period 1956 to 1998, Yang et al. (2000) for the period 1985 to 2000 and Mackerle (2002) for the period 1999 to 2001. Table 5.1 outlines the summary of plate and shell elements development (Mackerle, 2002).

Needless to say that research on the design of plate and shell elements continues to this day. However, numerous shell finite elements have been proposed and yet there is a consensus that there are still difficulties in analysing general shell structures. In addition, it is difficult to identify which shell elements are the most effective elements currently available, and how to arrive at more efficient shell analysis procedures. It may be therefore fair to state that no single theory has proven to be general and comprehensive enough for the entire range.
of applications (Chapelle and Bathe, 1998). Elements used in cooling tower shell will be discussed in Section 2.6.

Table 5.1: Summery of plate and shell element development (Mackerle, 2002).

<table>
<thead>
<tr>
<th>Element development</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>First membrane element</td>
<td>1956</td>
</tr>
<tr>
<td>First plate bending element</td>
<td>1961</td>
</tr>
<tr>
<td>Kirchhoff plate elements</td>
<td>1961 to 1970</td>
</tr>
<tr>
<td>Discrete Kirchhoff plate and shell elements</td>
<td>1969</td>
</tr>
<tr>
<td>First Mindlin shell element (8-noded quadrilateral)</td>
<td>1969</td>
</tr>
<tr>
<td>Reduced integration</td>
<td>1971</td>
</tr>
<tr>
<td>Local-global shell elements</td>
<td>1971</td>
</tr>
<tr>
<td>4-noded quadrilateral and 3-noded triangle</td>
<td>1976 to 1982</td>
</tr>
<tr>
<td>9-noded quadrilateral</td>
<td>1985 to 1990</td>
</tr>
<tr>
<td>High-order shell elements</td>
<td>1988 to recent</td>
</tr>
</tbody>
</table>

5.4 Elastic shell theory

5.4.1 Membrane theory

The membrane theory is based on a momentless plane stress system. It is applicable only if the following physical conditions are met:

1. The properties of the shell's material are constant across the thickness of the shell.

2. The shell's bending rigidity is negligible compared to its membrane rigidity. In other words, the shell is exceedingly thin, i.e. \[
\frac{\text{thickness}}{\text{radii of curvature}} \ll 1.
\]

3. The changes in radii of curvature are negligible.

5.4.2 Bending theory

Love (1888) established the bending theory of thin elastic shells for linear problems. In this theory, the behaviour of thin shells are approximated by the following assumptions:

1. The material of the shell is isotropic and homogenous.
2. The displacements are small.

3. The thickness of the shell is small compared with the radii of curvature of its middle surface.

4. The stress components normal to the middle surface (shear and axial) are small compared to stresses tangent to the shell’s surface and may be neglected in the stress-strain relationship.

5. Normals to the middle surface remain normal to it and undergo no strain during deformation. Hence, the transverse shear deformations are not taken into account.

The assumption that deflections are small, allows the assumption that the equations of equilibrium with respect to both the original (undeformed) and deformed geometry are the same. This, with Hook’s law, ensures linear elastic behaviour. The physical interpretation of assumption 5 is that the transverse shear deformations are not taken into account. This is called Kirchhoff’s assumption because the same assumption was proposed by Kirchhoff (1876) to introduce the theory of plates.

Many researchers have tried to generalise Love’s theory and this has led to the introduction of additional theories of thin shell. One of these is the first order approximation theory in which all of Love’s original assumptions are preserved. The well-known work in this category was done by Reissner (1941). Reissner was the first to use the lines of curvatures to derive the shell’s theory equations based on Love’s theory. Figure 5.1 shows the membrane, bending and coupled stress of the shell.

Natural draught cooling towers are an effective and economic choice among all technical solutions for the prevention of thermal pollution of natural water resources caused by heated cooling water in various industrial facilities. Cooling towers are shells of double curvature that resist applied forces primarily through in-plane membrane action. These shells can be more than 150 m in height and 60 m in base diameter. These towers have at most a wall thickness of about 20 to 25 cm. In general, both membrane and bending actions are present in the shell. However, it should be noted membrane stresses are the most important stresses in the designing of a cooling tower shell (ACI-ASCE, 1977). Hence, the shell behaviour is seen as a superposition of membrane and plate bending actions. Finite elements are constructed by simply combining plate bending and plane stress stiffness matrices. This approach will be discussed in Sections 5.5, 5.6 and 5.7.
5.5 Membrane elements

5.5.1 Introduction

Membrane elements are used for analysing structures subjected to in-plane forces. These elements are used to model the behaviour of shear wall, stiffened sheet construction, and membrane action in shells. Assuming that the structure is in the $xy$ plane, the displacements at any point of the structure are $u$ and $v$, the translation in the $x$ and $y$ directions, respectively. The stresses of interest are the normal stresses $\sigma_x$ and $\sigma_y$ and the shearing stress $\tau_{xy}$. The normal stress in the direction perpendicular to the plane of structure is considered to be zero.

Isoparametric elements are useful for modelling structures with irregular boundaries; since these elements can have curved sides. They are formulated in the natural coordinate system that maps the element geometry in terms of natural coordinates regardless of the orientation of an element in the global coordinate system; however, the relationship between the two systems must be used in the element formulation (Cook, 1995).

Irons (1966) introduced the concept of isoparametric elements in stiffness methods. Ergatoudis et al. (1968) developed shape functions to formulate the element stiffness matrix for four noded isoparametric quadrilateral element. The four noded isoparametric quadrilateral plane element is used to develop the quadrilateral flat shell element in this study.
5.5 Membrane elements

5.5.2 Elasticity constitutive model

Two dimensional elasticity problems typically involve structures that are very thin and the loads are applied in the direction in the plane of the structure. Consider a structure in the $xy$ plane with thickness $t$ along the $z$-direction. When in-plane forces are applied to the structure, the displacements at any discrete point of the structure located by the coordinates $(x, y)$, are

$$\{U_m\} = \{u \ v\}^T \quad (5.1)$$

where, $u$ and $v$ are the $x$ and $y$ components of the displacement and the subscript $m$ refers to the membrane element. The stresses and strains are given respectively by,

$$\{\sigma\} = \{\sigma_x \ \sigma_y \ \tau_{xy}\}^T \quad \text{and} \quad \{\epsilon\} = \{\epsilon_x \ \epsilon_y \ \gamma_{xy}\}^T \quad (5.2)$$

Two classes of plane elasticity problems are considered; plane stress and plane strain. The conditions of them are described in the following sections.

5.5.2.1 Plane stress condition

When the structure is subjected to forces in its own plane, the state of deformations and stresses is called the plane stress condition. If a plate is very thin and is only subjected to inplane forces, then the displacements and stresses normal to the plane of the plate are negligible. Assuming that the thin plate is in the $xy$ plane, the stresses $\sigma_z = 0$, $\tau_{yz} = 0$, $\tau_{zx} = 0$ and $\epsilon_z \neq 0$. For isotropic material properties the elastic stress-strain relationship for the plane stress condition is,

$$\{\epsilon\} = [E] \{\sigma\}$$

where $[E]$ is the material matrix expressed as,

$$[E] = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{bmatrix} \quad (5.4)$$

where $E$ is the modulus of elasticity for the material and $\nu$ is Poisson’s ratio.

5.5.2.2 Plane strain condition

When a prismatic solid is subjected to a uniform load normal to its axis and the solid is divided into thin plates then each plate will have inplane forces, i.e. the forces will be in the direction of the plane of the plate. This condition is called the plane strain condition. For
5.5 Membrane elements

the plane strain condition $\epsilon_z = 0$, $\epsilon_{yz} = 0$, $\epsilon_{xz}$ and $\sigma_z \neq 0$. The material matrix $[E]$ for the plane strain condition and an isotropic material is given by,

$$[E] = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} 1 - \nu & 0 & 0 \\ \nu & 1 - \nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$ (5.5)

5.5.3 Quadrilateral plane stress element

The quadrilateral plane stress element used in this study is the four noded isoparametric quadrilateral element, shown in figure 5.2. This element has two degrees of freedom per node for a total of eight degrees of freedom per element. The formulation of the element stiffness matrix is described below.

To develop the isoparametric quadrilateral plane stress element, the master element must be defined in the natural coordinate system $(\xi, \eta)$, as shown in figure 5.3. The relationship between the natural coordinate system and the global coordinate system can be defined using Lagrange interpolating functions as follows:

$$x(\xi, \eta) = \sum_{i=1}^{4} N_i x_i \quad \text{and} \quad y(\xi, \eta) = \sum_{i=1}^{4} N_i y_i$$ (5.6)

where $N_i$, $i = 1, 2, 3, 4$, are the shape functions for the four noded quadrilateral element in the natural coordinate system. The shape functions are,

$$\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} (1 - \xi)(1 - \eta) \\ (1 + \xi)(1 - \eta) \\ (1 + \xi)(1 + \eta) \\ (1 - \xi)(1 + \eta) \end{bmatrix}$$ (5.7)

Similarly, the relationship between displacements in the natural coordinate system and the nodal displacements can be written in the following manner,

$$u(\xi, \eta) = \sum_{i=1}^{4} N_i u_i \quad \text{and} \quad v(\xi, \eta) = \sum_{i=1}^{4} N_i v_i$$ (5.8)

To obtain the element stiffness matrix, the strain-displacement matrix must be determined. When using isoparametric elements, the element geometry is defined in the natural coordinate system and hence the strain-displacement matrix must be transformed to natural coordinates. The transformation matrix used to convert the strain-displacement matrix from the element local coordinate system to the natural coordinate system is called Jacobian's matrix, which can be defined as,

$$[J_m] = \begin{bmatrix} J_{m11}^{11} & J_{m11}^{12} \\ J_{m21}^{11} & J_{m22}^{12} \end{bmatrix}$$ (5.9)
\[ J_{m}^{11} = \frac{\partial x}{\partial \xi} = \sum_{i=1}^{4} \frac{\partial N_i}{\partial \xi} x_i \]
\[ J_{m}^{12} = \frac{\partial y}{\partial \xi} = \sum_{i=1}^{4} \frac{\partial N_i}{\partial \xi} y_i \]
\[ J_{m}^{21} = \frac{\partial x}{\partial \eta} = \sum_{i=1}^{4} \frac{\partial N_i}{\partial \eta} x_i \]
\[ J_{m}^{22} = \frac{\partial y}{\partial \eta} = \sum_{i=1}^{4} \frac{\partial N_i}{\partial \eta} y_i \]  

Figure 5.2: Four noded quadrilateral plane stress element.

Figure 5.3: Four noded quadrilateral element in natural coordinate system \((\xi, \eta)\).

Thus, the strain-displacement relationships for the four noded isoparamatric quadrilateral element are,

\[ \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix} \]  

The derivatives of the horizontal and vertical displacements with respect to \(x\) and \(y\) in terms of Jacobian's matrix are,

\[ \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{bmatrix} = [J_m]^{-1} \begin{bmatrix} \frac{\partial x}{\partial \xi} \\ \frac{\partial y}{\partial \eta} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{bmatrix} = [J_m]^{-1} \begin{bmatrix} \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \xi} \end{bmatrix} \]  

where \([J_m]^{-1}\) is the inverse of Jacobian's matrix defined as

\[ [J_m]^{-1} = \frac{1}{J_m} \begin{bmatrix} J_{m}^{22} & -J_{m}^{12} \\ -J_{m}^{21} & J_{m}^{11} \end{bmatrix} \]
where \( J_m \) is the determinant of Jacobian's matrix defined as

\[
J_m = \det[J_m] = J_m^{11} J_m^{22} - J_m^{12} J_m^{21}
\]  

(5.14)

From equations (5.11) and (5.12), the strain-displacement relations can be obtained as function of \( \xi \) and \( \eta \) as follows:

\[
\begin{bmatrix}
\epsilon_x \\
\epsilon_y \\
\gamma_{xy}
\end{bmatrix} = \frac{1}{J_m} \begin{bmatrix}
J_m^{22} & -J_m^{12} & 0 & 0 \\
0 & 0 & -J_m^{21} & J_m^{11} \\
-J_m^{21} & -J_m^{11} & J_m^{22} & -J_m^{12}
\end{bmatrix} \begin{bmatrix}
\frac{\partial u}{\partial \xi} \\
\frac{\partial u}{\partial \eta} \\
\frac{\partial v}{\partial \xi} \\
\frac{\partial v}{\partial \eta}
\end{bmatrix}
\]

(5.15)

On differentiation equation (5.8), results in

\[
\begin{bmatrix}
\frac{\partial u}{\partial \xi} \\
\frac{\partial u}{\partial \eta} \\
\frac{\partial v}{\partial \xi} \\
\frac{\partial v}{\partial \eta}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} & 0 \\
\frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} & 0 \\
0 & \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} \\
0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta}
\end{bmatrix} \begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
u_4
\end{bmatrix}
\]

(5.16)

On substituting from equation (5.16) into (5.15), the following relationship is obtained,

\[
\{\epsilon\} = [A] [G] \{U_m^*\} = [B_m] \{\epsilon\}
\]

(5.17)

where \( \{\epsilon\} \) is defined in equation (5.2) and

\[
[A] = \frac{1}{J_m} \begin{bmatrix}
J_m^{22} & -J_m^{12} & 0 & 0 \\
0 & 0 & -J_m^{21} & J_m^{11} \\
-J_m^{21} & -J_m^{11} & J_m^{22} & -J_m^{12}
\end{bmatrix}
\]

(5.18)

\[
[G] = \begin{bmatrix}
\frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} & 0 \\
\frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} & 0 \\
0 & \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} \\
0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta}
\end{bmatrix}
\]

(5.19)

\[
\{U_m^*\} = \{u_1 \ v_1 \ u_2 \ v_2 \ u_3 \ v_3 \ u_4 \ v_4\}^T
\]

(5.20)
5.6 Bending elements

and

\[ [B_m] = [A] [G] \]  \hspace{1cm} (5.21)

where the superscript \( e \) refers to the quantity for element. Equation (5.21) is called the strain-displacement relation. Hence, the element stiffness matrix for the plane stress condition is given by,

\[ [k_m] = t \int_A \int [B_m]^T [E] [B_m] \, dx \, dy \]  \hspace{1cm} (5.22)

where \( t \) is thickness of the element, considered as a constant, \( A \) is area of the element and \([E]\) is defined by equation (5.4). Equation (5.22) must be integrated with respect to the natural coordinates \((\xi, \eta)\), therefore on using the following relationship

\[ dx \, dy = J \, d\xi \, d\eta \]  \hspace{1cm} (5.23)

Equation (5.22) can be rewritten as,

\[ [k_m]_{8 \times 8} = t \int_{-1}^{+1} \int_{-1}^{+1} [B_m(\xi, \eta)]_{8 \times 3}^T [E]_{3 \times 3} [B_m(\xi, \eta)]_{3 \times 8} \, J_m(\xi, \eta) \, d\xi \, d\eta \]  \hspace{1cm} (5.24)

5.6 Bending elements

5.6.1 Introduction

There has been considerable interest in the development of plate bending elements ever since their use became popular for representing the bending behaviour of shell elements. Many plate bending elements have been developed. Hrabok and Hrudey (1984) presented a review of all plate bending elements as a part of the study on the effectiveness of plate bending elements.

Quadrilateral plate bending elements are used in formulating shell elements for the analysis of regular shaped shell structures. However, shear locking is a phenomenon attributed to the predominance of the transverse shear energy over the bending energy. Due to the presence of a parameter in the transverse shear strain energy that is inversely proportional to the thickness of the element, the transverse shear energy becomes very large compared to the bending energy as the thickness of the element becomes very small and hence results in an overly stiff solution. In other words the bending energy becomes very small or negligible compared to the shear energy, resulting in spurious modes and incorrect results.
5.6 Bending elements

To avoid the shear locking problem, the shear strain energy is completely neglected and the discrete Kirchhoff's theory is applied. This has the effect of relating the shell normal rotations in terms of the transverse displacement and its derivatives (or slopes) at discrete locations (usually at the corner and mid-side nodes) and thus eliminating the shell normal rotations at these locations. Batoz and Tahar (1982) developed a four noded quadrilateral element based on the discrete Kirchhoff's theory in which the transverse shear strain is neglected. They considered transverse shear strain to be presented in the element in the initial development and then removed the transverse shear strain terms by applying the discrete Kirchhoff constraints.

5.6.2 Bending of flat plates

Bending of flat plates is similar to bending of beams; the former is more complicated because plate bending is two dimensional while the bending of beams is one dimensional. The behaviour of plates mainly depends on the plate thickness. In this study, thin plates with small deformations are considered.

The bending properties of this plate can be used in the development of flat shell elements. There are three basic assumptions in the theory of bending for thin plates (Timoshenko and Woinowsky-Krieger, 1959)

1. The mid-surface of the plate remains unstretched during deformations.

2. Points straight and normal to the mid-surface of the plate before bending remain straight and normal to the mid surface after bending.

3. Transverse shear stresses are small compared to normal stresses and hence can be neglected.

These assumptions are known as Kirchhoff's hypothesis and are applicable to the bending of the thin plates with small deflections. Consider an isotropic plate of uniform thickness $t$ with the $xy$ plane as the principal plane. According to the theory of bending for a thin plate, the plate is in the plane stress condition and hence all stresses vary linearly over the thickness of the plate.

The moments per length can be represented as follows:

$$
M_x = \int_{-t/2}^{t/2} \sigma_x z \, dz \\
M_y = \int_{-t/2}^{t/2} \sigma_y z \, dz \\
M_{xy} = \int_{-t/2}^{t/2} \tau_{xy} z \, dz
$$

(5.25)
where $M_x$ and $M_y$ are the moments in the $x$ and $y$ directions respectively, and $M_{xy}$ is the twisting moment. If $w$ is the transverse displacement of the plate, the displacement-curvature relationships for the thin plate can be written as follows:

$$k_x = -\frac{\partial^2 w}{\partial x^2} \quad k_y = -\frac{\partial^2 w}{\partial y^2} \quad k_{xy} = -2 \frac{\partial^2 w}{\partial x \partial y} \quad (5.26)$$

Consider a small section of the plate of length $dx$ in the $x$-direction. When a load is applied in the $z$-direction, the point $O$ on the mid-surface of the plate moves in the $z$-direction as the plate deforms due to bending, as shown in the figure 5.4.

![Figure 5.4: Bending of a thin plate (Cook et al., 2002).](image)

According to Kirchhoff’s assumption, a line that is straight and normal to the mid-surface before bending remains straight and normal to the mid-surface after bending (Cook et al., 2002). The displacements can be written as,

$$u = -z \frac{\partial w}{\partial x} \quad \text{and} \quad v = -z \frac{\partial w}{\partial y} \quad (5.27)$$

Thus, the strain-displacement relationship is given by

$$\begin{align*}
\{ \varepsilon_x \} &= \begin{bmatrix} -z \frac{\partial^2 w}{\partial x^2} \\
-2z \frac{\partial^2 w}{\partial x \partial y} \\
-2z \frac{\partial^2 w}{\partial y \partial x} \end{bmatrix} \\
\{ \varepsilon_y \} &= \begin{bmatrix} -z \frac{\partial^2 w}{\partial y^2} \\
-2z \frac{\partial^2 w}{\partial x \partial y} \\
-2z \frac{\partial^2 w}{\partial y \partial x} \end{bmatrix} \\
\{ \gamma_{xy} \} &= \begin{bmatrix} -z \frac{\partial^2 w}{\partial x \partial y} \end{bmatrix}
\end{align*} \quad (5.28)$$
Hence, the stress-strain relationship becomes
\[
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix} = \frac{E}{(1-\nu^2)} \begin{pmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2}
\end{pmatrix} \begin{pmatrix}
-z \frac{\partial^2 w}{\partial x^2} \\
-z \frac{\partial^2 w}{\partial y^2} \\
-2z \frac{\partial^2 w}{\partial x \partial y}
\end{pmatrix}
\] (5.29)

On substituting from equation (5.29) into equation (5.25) and integrating over the thickness of the plate, the following relation is obtained
\[
M_x = \frac{-Et^3}{12(1-\nu^2)} \left( \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right)
\]
\[
M_y = \frac{-Et^3}{12(1-\nu^2)} \left( \nu \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)
\]
\[
M_{xy} = \frac{-Et^3}{12(1-\nu^2)} (1-\nu) \left( \frac{\partial^2 w}{\partial x \partial y} \right)
\] (5.30)

From equations (5.30) and (5.26), the moment-curvature relationship is written as follows:
\[
\begin{pmatrix}
M_x \\
M_y \\
M_{xy}
\end{pmatrix} = \begin{bmatrix}
D & D\nu & 0 \\
D\nu & D & 0 \\
0 & 0 & D \left( 1 - \frac{\nu}{2} \right)
\end{bmatrix} \begin{pmatrix}
k_x \\
k_y \\
k_{xy}
\end{pmatrix}
\] (5.31)

where \(D = \frac{Et^3}{12(1-\nu^2)}\) is the flexural rigidity of the plate and the moment-curvature matrix, say \([D_b]\), is given by
\[
[D_b] = D \begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2}
\end{bmatrix}
\] (5.32)

### 5.6.3 Discrete Kirchhoff quadrilateral element (DKQ)

The quadrilateral thin plate bending element is efficient and useful for representing the bending part of flat shell elements. Batoz and Tahar (1982) developed a quadrilateral plate bending element based on the discrete Kirchhoff’s assumptions for thin plates. This discrete Kirchhoff quadrilateral element (DKQ) has 12 degrees of freedom, as shown in figure 5.5. The degrees of freedom at each node of the element are the transverse displacement \(w = w(x, y)\) in the direction normal to the \(xy\) plane, and the in-plane rotations \(\theta_x\) and \(\theta_y\) in the \(x\) and \(y\) directions respectively where
\[
\theta_x = \frac{\partial w}{\partial y} \quad \text{and} \quad \theta_y = -\frac{\partial w}{\partial x}
\] (5.33)
According to Kirchhoff's assumptions for thin plates, the strain energy of the element is

\[ U = \sum_e U_b^e \quad \text{with} \quad U_b^e = \frac{1}{2} \int \int \{ \chi \}^T \{ D_b \} \{ \chi \} \, dx \, dy \]  

(5.34)

where \( U_b^e \) is the element strain energy due to bending, and \( \{ D_b \} \) is defined in equation (5.32) and \( \{ \chi \} \) is given by

\[ \{ \chi \} = \begin{bmatrix} \frac{\partial \beta_x}{\partial x} \\ \frac{\partial \beta_x}{\partial y} \\ \frac{\partial \beta_y}{\partial y} + \frac{\partial \beta_x}{\partial x} \end{bmatrix} \]  

(5.35)

where \( \beta_x \) and \( \beta_y \) are the rotations of the normal to the undeformed middle surface in the \( xz \) and \( yz \) planes respectively. \( \beta_x \) and \( \beta_y \) should relate to the transverse displacement \( w \) in such a way that the final element has the characteristics of the Kirchhoff type element, i.e. the nodal variables must be \( w, \theta_x \) and \( \theta_y \) with respect to \( x \) and \( y \) at the four corner nodes and Kirchhoff's assumptions must be verified along the boundaries. To achieve that, \( \beta_x \) and \( \beta_y \) are defined by incomplete cubic polynomial expressions as follows:

\[ \beta_x = \sum_{i=1}^{8} N_i \beta_{x_i} \quad \text{and} \quad \beta_y = \sum_{i=1}^{8} N_i \beta_{y_i} \]  

(5.36)

where \( \beta_{x_i} \) and \( \beta_{y_i} \) are transitory nodal variables affected at the corner and mid-nodes of the quadrilateral element with straight sides, as shown in figure 5.6, and \( N_i(\xi, \eta), \ i = 1, \ldots, 8, \) where \( \xi \) and \( \eta \) for the parametric co-ordinates are the 8-noded serendipity element defined.
5.6 Bending elements

as

\[
\begin{bmatrix}
N_1 \\
N_2 \\
N_3 \\
N_4 \\
N_5 \\
N_6 \\
N_7 \\
N_8 \\
\end{bmatrix} = \frac{1}{4} \begin{bmatrix}
-(1 - \xi)(1 - \eta)(1 + \xi + \eta) \\
-(1 + \xi)(1 - \eta)(1 - \xi + \eta) \\
-(1 + \xi)(1 + \eta)(1 - \xi - \eta) \\
-(1 - \xi)(1 + \eta)(1 + \xi - \eta) \\
2(1 - \xi^2)(1 - \eta) \\
2(1 + \xi)(1 - \eta^2) \\
2(1 - \xi^2)(1 + \eta) \\
2(1 - \xi)(1 - \eta^2) \\
\end{bmatrix}
\]  

\[(5.37)\]

Figure 5.6: Geometry of the DKQ element (Batoz and Tahar, 1982).

On applying Kirchhoff's assumptions the following relations are obtained at the corner

\[
\begin{bmatrix}
\beta_{x_i} + \frac{\partial w}{\partial x_i} \\
\beta_{y_i} + \frac{\partial w}{\partial y_i} \\
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
\end{bmatrix} \quad i = 1, 2, 3, 4
\]

\[(5.38)\]

and at the mid-nodes as follows

\[
\beta_{s_k} + \frac{\partial w}{\partial s_k} = 0 \quad k = 5, 6, 7, 8
\]

\[(5.39)\]

where \(s\) represents the co-ordinate along the element boundary and \(\frac{\partial w}{\partial s_k}\) is the derivative of the transverse displacement with respect to \(s\) at the mid-node \(k\), defined by

\[
\frac{\partial w}{\partial s_k} = -\frac{3}{2I_{ij}} (w_i - w_j) - \frac{1}{4} \left( \frac{\partial w}{\partial s_i} + \frac{\partial w}{\partial s_j} \right)
\]

\[(5.40)\]
5.6 Bending elements

where \( i, j = 1, 2, 3, 4, k = 5, 6, 7, 8 \) is the mid-node of the sides \( ij = 12, 23, 34, 41 \) respectively and \( l_{ij} \) is the length of the side \( ij \). The rotation normal to the sides at the mid-nodes varies linearly as

\[
\beta_{n_k} = \frac{1}{2} (\beta_{n_i} + \beta_{n_j}) = -\frac{1}{2} (w_{n_i} + w_{n_j}) \quad (5.41)
\]

It should be noted that the nodal variable \( w \) is not defined in the interior of the element and it appears at the four corner nodes through equation (5.40). In addition, Kirchhoff’s assumptions are satisfied along the entire boundary of the element since \( \frac{\partial w}{\partial s} \) and \( \beta_n \) are both quadratic expressions along the element sides.

The explicit expression of the rotations \( \beta_x \) and \( \beta_y \) of a general quadrilateral, in terms of the final DKQ nodal variables, is

\[
\{U_b\} = \{w_1 \ \theta_{x_1} \ \theta_{y_1} \ w_2 \ \theta_{x_2} \ \theta_{y_2} \ w_3 \ \theta_{x_3} \ \theta_{y_3} \ w_4 \ \theta_{x_4} \ \theta_{y_4}\}^T \quad (5.42)
\]

where \( \theta_{x_i} = \frac{\partial w}{\partial s_i} \) and \( \theta_{y_i} = -\frac{\partial w}{\partial s_i}, \ i = 1, 2, 3, 4 \). On using the component vectors of the shape functions, the quantities \( \beta_x \) and \( \beta_y \) are expressed in terms of the nodal displacements as

\[
\beta_x = [H^x(\xi, \eta)] \{U_b\} \quad \text{and} \quad \beta_y = [H^y(\xi, \eta)] \{U_b\} \quad (5.43)
\]

\( H^x(\xi, \eta) \) and \( H^y(\xi, \eta) \) are the component vectors of the shape functions given by

\[
[H^x(\xi, \eta)] = \begin{bmatrix}
\frac{3}{2} (a_5 N_5 - a_8 N_8) \\
b_5 N_5 + b_8 N_8 \\
N_2 - c_5 N_5 - c_8 N_8 \\
\frac{3}{2} (a_6 N_6 - a_5 N_5) \\
b_6 N_6 + b_5 N_5 \\
N_3 - c_6 N_6 - c_5 N_5 \\
\frac{3}{2} (a_7 N_7 - a_6 N_6) \\
b_7 N_7 + b_6 N_6 \\
N_4 - c_7 N_7 - c_6 N_6 \\
\frac{3}{2} (a_8 N_8 - a_7 N_7) \\
b_8 N_8 + b_7 N_7 \\
N_5 - c_8 N_8 - c_7 N_7
\end{bmatrix}
\]

\[
[H^y(\xi, \eta)] = \begin{bmatrix}
\frac{3}{2} (d_5 N_5 - d_8 N_8) \\
-N_2 - e_5 N_5 - e_8 N_8 \\
\frac{3}{2} (d_6 N_6 - d_5 N_5) \\
-N_3 - e_6 N_6 - e_5 N_5 \\
\frac{3}{2} (d_7 N_7 - d_6 N_6) \\
-N_4 - e_7 N_7 - e_6 N_6 \\
\frac{3}{2} (d_8 N_8 - d_7 N_7) \\
-N_5 - e_8 N_8 - e_7 N_7 \\
\frac{3}{2} (d_9 N_9 - d_8 N_8) \\
-N_6 - e_9 N_9 - e_8 N_8 \\
\frac{3}{2} (d_10 N_{10} - d_9 N_9) \\
-N_7 - e_{10} N_{10} - e_9 N_9
\end{bmatrix}
\]

where \( a_k = -\frac{x_{ij}}{l_{ij}}, b_k = \frac{3x_{ij}y_{ij}}{4l_{ij}^3}, c_k = \frac{1}{4} \left( \frac{x_{ij}^2 - 2y_{ij}^2}{l_{ij}^2} \right), d_k = -\frac{y_{ij}}{l_{ij}^2}, e_k = \frac{1}{4} \left( \frac{y_{ij}^2 - 2x_{ij}^2}{l_{ij}^2} \right) \), where \( x_{ij} = x_i - x_j, y_{ij} = y_i - y_j \) (where \( i = 1, 2, 3, 4 \) et \( j = 2, 3, 4, 1 \), \( l_{ij}^2 = x_{ij}^2 + y_{ij}^2 \) (for \( ij = 12, 23, 34, 41 \)), where \( k = 5, 6, 7, 8 \) for \( ij = 12, 23, 34, 41 \), respectively.

The Jacobian’s matrix \([J_b]\) of the transformation between the parent and the actual element is defined by

\[
[J_b] = \frac{1}{4} \begin{bmatrix}
x_{21} + x_{34} + \eta(x_{12} + x_{34}) & y_{21} + y_{34} + \eta(y_{12} + y_{34}) \\
x_{32} + x_{41} + \xi(x_{12} + x_{34}) & y_{32} + y_{41} + \xi(y_{12} + y_{34})
\end{bmatrix}
= \begin{bmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{bmatrix}
\quad (5.45)
\]
The strain-displacement relation is given by substitution from equation (5.43) into equation (5.35)

\[ \{\chi\} = [B_b] \{U_b\} \]  

(5.46)

where \([B_b]\) is the strain-displacement matrix defined as

\[
[B_b] = \begin{bmatrix} \frac{\partial H^x}{\partial x} \\ \frac{\partial H^y}{\partial y} \\ \frac{\partial H^z}{\partial x} + \frac{\partial H^x}{\partial y} \end{bmatrix} = \begin{bmatrix} j_{11} \frac{\partial H^x}{\partial x} + j_{12} \frac{\partial H^y}{\partial y} \\ j_{21} \frac{\partial H^x}{\partial x} + j_{22} \frac{\partial H^y}{\partial y} \end{bmatrix}
\]

(5.47)

where the components of \(\frac{\partial H^x}{\partial x}\), \(\frac{\partial H^y}{\partial y}\), \(\frac{\partial H^z}{\partial x}\) and \(\frac{\partial H^z}{\partial y}\) are expressed in terms of the derivatives of the shape function defined in equation (5.37) and \(j_{11}, j_{12}, j_{21}\) and \(j_{22}\) are the components of the inverse of Jacobian's matrix \([J_b]\) defined as

\[
[J_b]^{-1} = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix} = \frac{1}{J_b} \begin{bmatrix} J_{22} & -J_{12} \\ -J_{21} & J_{11} \end{bmatrix}
\]

(5.48)

where \(J_b\) is the determinant of Jacobian's matrix given by

\[
J_b = \frac{1}{8} (y_{42} x_{31} - y_{31} x_{42} + \xi (y_{34} x_{21} - y_{21} x_{34}) + \eta (y_{41} x_{32} - y_{32} x_{41}))
\]

(5.49)

Hence, the stiffness matrix for the DKQ element is given in the standard manner by

\[
[k_b] = \int_A [B_b]^T [D_b] [B_b] \, dx \, dy
\]

(5.50)

and in the natural coordinates \((\xi, \eta)\) by

\[
[k_b(\xi, \eta)]_{12 \times 12} = \int_{-1}^{+1} \int_{-1}^{+1} [B_b(\xi, \eta)]_{12 \times 3}^T [D_b]_{3 \times 3} [B_b(\xi, \eta)]_{3 \times 12} J_b(\xi, \eta) \, d\xi \, d\eta
\]

(5.51)

5.7 Flat shell elements

5.7.1 Introduction

Shells with cylindrical shapes or regular curved surfaces can be modelled using rectangular or quadrilateral flat shell elements. Zienkiewicz and Taylor (2000) recommended modelling curved surfaces by a series of flat shell elements, rather than using the more complex curved shell elements. They suggested developing a built up element by combining membrane and plate bending elements to develop a flat shell elements.
MacNeal and Harder (1988) suggested that the higher order elements take three times more solution effort than the lower order elements. Another drawback of higher order elements is the use of high order numerical integration schemes to avoid spurious zero energy modes. Lower order elements require a large number of elements to model the structure but they require less computational effort and hence are still cheaper as compared to the higher order elements. However, the effectiveness of the element and accuracy of results of the lower order elements largely depends on the type of the element selected for the formulation of the shell element.

Flat shell elements are developed by combining membrane elements containing two in-plane translational degrees of freedom and plate bending elements containing two rotational degrees of freedom and one out-of-plane translational degree of freedom, as shown in figure 5.7. Since the in-plane rotational degrees of freedom are not included, that leaves null or zero values in the stiffness matrix. The null values for the in-plane rotational degrees of freedom, generally called drilling degrees of freedom, gives singularity in structure stiffness matrix if all the elements are co-planar.

The simplest method adopted to remove the rotational singularity is to add a fictitious rotational stiffness. However, Yang et al. (2000) suggested that, although this method solves the problem of singularity it creates a convergence problem that sometimes leads to poor results. However, the majority of the flat shell elements are developed by inducing the fictitious rotational stiffness to remove the singularity. Recent developments include using membrane elements with rotational degrees of freedom to develop an efficient flat shell element.
5.7 Flat shell elements

5.7.2 Development of stiffness matrix

The general formulation of the stiffness matrix for flat shell elements is discussed in this section. In general, shell elements have six degrees of freedom at each node. The nodal displacements of the shell element are,

\[ \{U_{(mb)}_i\} = \{u_i \ v_i \ w_i \ \theta_{x_i} \ \theta_{y_i} \ \theta_{z_i}\} \quad i = 1, 2, 3, 4 \quad (5.52) \]

The membrane and bending stiffness at each node is of size 2x2 and 3x3, respectively, and is represented as \([k_m]_{2x2}\) by equation (5.24), and \([k_b]_{3x3}\) by equation (5.51), respectively. Hence, the stiffness matrix at each node of the shell element is of size 6x6 and is represented as, \([k_s]_{6x6}\).

The assembly of the stiffness matrices of membrane and bending components at each node will result in a zero value on the diagonal corresponding to the rotational degree of freedom \(\theta_z\) since this displacement is not considered in the membrane or bending element. This zero stiffness for the drilling degree of freedom creates a singularity in the structure stiffness matrix when all the elements are coplanar and there is no coupling between the membrane and bending stiffness of the element. There are two ways to deal with this singularity.

The first approach for removing the singularity in the structure stiffness matrix is to substitute an approximate value for the diagonal value of the stiffness of drilling degree of freedom. Although, this solves the problem of singularity from the structure stiffness matrix, it sometimes does not represent actual behaviour of the element because of the fact that a fictitious stiffness has been added. The second approach is to develop a higher order membrane element that includes the drilling degree of freedom. This approach is less efficient since higher order displacement functions are needed for the membrane stiffness matrix and hence a higher order numerical integration scheme is required.

In this study the first approach is used since it is easier to implement and is more efficient. This approach of including the fictitious stiffness for the drilling degree of freedom closely approximates the behaviour of the shell. The element stiffness matrix for the shell element is first assembled by superimposing the membrane stiffness and bending stiffness at each node. The null values of the stiffness corresponding to the drilling degree of freedom are then replaced by approximate values. This approximate value is taken to be equal to \(10^{-3}\).
5.7 Flat shell elements

times the maximum diagonal value in the element stiffness matrix. The stiffness matrix at each node of the shell element \([k_s]_i\) can thus be represented as,

\[
[k_s]_i = \begin{bmatrix}
[k_m]_{2i \times 2i} & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \vdots & \vdots & \cdots & 0 \\
0 & 0 & \vdots & \vdots & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & \frac{\max((k_s)_{4i,4i})}{1000}
\end{bmatrix}
\]  (5.53)

where \(k_s\) is element stiffness for the shell element.

5.7.3 Coordinate transformation

A shell is a three dimensional structure and it is often convenient to define the geometry of shell structure in the global coordinate system. However, to generate the element stiffness matrix for the membrane and plate bending elements, the elements have to be defined in the element local plane, and element local coordinates are required to calculate the stiffness of these elements. Since the flat shell elements considered in this study are based on a combination of membrane and plate bending elements it is thus necessary to use local coordinates for computing the element stiffness matrix of the flat shell elements. The transformation between global coordinates and local coordinates is required to generate the element local stiffness matrix in the local coordinate system. The stiffness matrix must be then transformed to the global coordinate system.

Figure 5.8 shows the global and local coordinate axis for the quadrilateral element. The element plane is defined by creating two vectors intersecting each other and passing through the mid points of the sides 23 and 34 of the quadrilateral.

Assuming that the local x-axis of the quadrilateral is parallel to the vector, say \(\{V_z\}\), passing through nodes \(l\) and \(j\), then \(\{V_z\}\) is given by

\[
\{V_z\} = \begin{bmatrix} x_j - x_l \\ y_j - y_l \\ z_j - z_l \end{bmatrix}^T = \begin{bmatrix} x_{ji} \\ y_{ji} \\ z_{ji} \end{bmatrix}^T
\]  (5.54)
where \( x_j, y_j \) and \( z_j \) represent the global coordinates at node \( j, j = 1, 2, 3, 4 \). The direction cosine \( \{\lambda_x\} \) for the local \( x \)-axis is given by

\[
\{\lambda_x\} = \frac{1}{l_{ji}} \{V_x\}
\]  

(5.55)

where \( l_{ji}^2 = x_{ji}^2 + y_{ji}^2 + z_{ji}^2 \) is the length of the vector. A reference vector \( V_R \), defining the plane of the element is obtained by creating a vector through nodes \( i \) and \( k \) of the element, as shown in figure 5.8.

\[
\{V_R\} = \{V_{ik}\}
\]  

(5.56)

![Figure 5.8: Coordinate transformation for quadrilateral element.](image)

The normal to plane is obtained by the vector cross product of \( \{V_z\} \) and \( \{V_R\} \)

\[
\{V_z\} = \{V_z\} \times \{V_R\}
\]  

(5.57)

Hence, the direction cosine for the local \( z \)-direction \( \{\lambda_z\} \) is obtained by normalising vector \( \{V_z\} \) with respect to its length, as given in equation (5.55). The local \( y \)-axis is obtained by the vector cross product of the vector in the local \( x \)-direction and vector in the local \( z \)-direction as follows

\[
\{V_y\} = \{V_x\} \times \{V_z\}
\]  

(5.58)

The direction cosine for the local \( y \)-direction \( \{\lambda_y\} \) is obtained by normalising vector \( \{V_y\} \) with respect to its length, as given in equation (5.55).
5.8 Thermal effects

The matrix for the transformation of coordinates from the global to the local axis is given by

\[ [A]_{3x3} = \begin{bmatrix} \{\lambda_x\}_{3x1} & \{\lambda_y\}_{3x1} & \{\lambda_z\}_{3x1} \end{bmatrix} \] (5.59)

The relation between the local coordinate system, \(xyz\), and the global coordinate system, \(XYZ\), can be written as

\[ \begin{bmatrix} x \\ y \\ z \end{bmatrix}^T = [A] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}^T \] (5.60)

and the transformation of the stiffness matrix from the local to the global system is given by

\[ [K_s] = [T]^T [k_s] [T] \] (5.61)

where \([K_s]\) is the stiffness matrix in the global coordinate system and \([T]\) is the transformation matrix given by

\[ [T] = \begin{bmatrix} \vdots & 0 & 0 & 0 \\ \vdots & 0 & 0 & 0 \\ \vdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \vdots \\ 0 & 0 & 0 & \vdots & [A] \\ 0 & 0 & 0 & \vdots \end{bmatrix} \] (5.62)

5.8 Thermal effects

When the cooling tower is in service, it is subjected to the combination of the following loads (Sudret et al., 2005):

- self-weight, corresponding to a mass density,
- wind pressure,
- internal depression due to air circulation in service, which is supposed to be constant all over the tower, and
- thermal gradient \(\Delta T\) within the shell thickness. This is computed from the difference between the inside- and outside-air temperatures and from the air/concrete heat transfer coefficient.

Heat transfer between the hot air inside the cooling tower and ambient air establishes a variable temperature field in the wall of the tower. This temperature field is, in general,
5.9 Conclusion

a function of the meridional, circumferential and normal coordinates associated with the tower's middle surface. The variation in the temperature field gives rise to appreciable thermal stresses, especially near the bottom of the tower.

The initial strains may be due to many causes and temperature change is the most frequent case. For an isotropic material in an element

$$\{\epsilon_0\} = \Delta T \begin{bmatrix} \alpha_x & \alpha_y & \alpha_z & 0 & 0 & 0 \end{bmatrix}^T$$ (5.63)

where $\alpha_x$, $\alpha_y$ and $\alpha_z$ are the coefficient of thermal expansion in the $x$, $y$ and $z$ directions, respectively, and $\Delta T$ is the temperature change. For the isotropic materials, i.e. $\alpha_x = \alpha_y = \alpha_z = \alpha$, equation (5.63) becomes

$$\{\epsilon_0\} = \alpha \Delta T \{m\}$$ (5.64)

where $\{m\} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix}^T$. Hence the total strain $\{\epsilon^*\}$ and total stress $\{\sigma^*\}$ can be written as

$$\{\epsilon^*\} = \{\epsilon\} - \{\epsilon_0\}$$ (5.65)

where $\{\epsilon_0\}$ is the initial strain which produced by temperature change. It should be noted that on replacing $\{\epsilon\}$ and $\{\sigma\}$ by $\{\epsilon^*\}$ and $\{\sigma^*\}$ respectively, the finite element analysis remain the same as in Sections 5.5 and 5.6. Hence, the equilibrium equation becomes

$$[k_s] \{U_m\} = \{F\} + \{F_T\}$$ (5.66)

where $\{F\}$ is the concentrated loads applied at nodes and $\{F_T\}$ is the global thermal load.

5.9 Conclusion

In this chapter, the four noded quadrilateral flat shell element, with six DOF at each node, were described based on Kirchhoff's assumptions. This element was used to model the elastic behaviour of curved structural which have one dimension small (a thickness normal to the remaining surface coordinates) compared to the other dimensions of the surface. The thermal effects upon this flat shell element were then presented, and incorporated into the element, and into GeoFEAP2. In addition, the elements used to model the cooling tower shell and its columns support have been discussed.

The bending elements are popular for representing the bending behaviour of shell elements, where the quadrilateral plate bending elements are used in formulating shell elements for
the analysis of regular shaped shell structures. However, shear locking is a phenomenon attributed to the predominance of the transverse shear energy over the bending energy. Due to the presence of a parameter in the transverse shear strain energy that is inversely proportional to the thickness of the element, the transverse shear energy becomes very large compared to the bending energy as the thickness of the element becomes very small and hence results in an overly stiff solution. To avoid this shear locking problem, the shear strain energy is completely neglected and discrete Kirchhoff’s theory is applied, in which the transverse shear strain is neglected. This has the effect of relating the shell normal rotations in terms of the transverse displacement and its derivatives (or slopes) at discrete locations (usually at the corner and mid-side nodes) and thus eliminating the shell normal rotations at these locations. In summary, the transverse shear strain is presented in the four noded quadrilateral element in the initial development and is then removed by applying discrete Kirchhoff’s constraints.
CHAPTER 6

Validation of the Implemented Shell Element

6.1 Overview

Five standard widely-used 'obstacle course' benchmark shell problems are presented in this chapter to validate the 4-noded quadrilateral flat shell element, with six DOF at each node, which has been introduced in Chapter 5. These examples include a shell under self-weight (Scordelis–Lo's roof and hemicylindrical shell) and a shell under external load (pinched cylindrical shell with rigid end diaphragm, pullout of an open-ended cylindrical shell and hemispherical shell with an 18° hole). These problems have been modelled using the GeoFEAP2, where the materials are assumed to be homogenous and isotropic. In addition, the behaviour is assumed to be elastic and the force resultant computed by integration of stress components over the cross-sectional thickness. The results are to be compared with those obtained by other researchers. To achieve that, it should be noted that the used units, $u$, $v$ and $w$ are replaced by $u_x$, $u_y$ and $u_z$ respectively, and $x$, $y$ and $z$ directions are chosen as in the original examples.

6.2 Shell under self-weight

6.2.1 Scordelis–Lo shell roof

Scordelis–Lo's problem is extremely useful for determining the ability of an element to accurately solve complex states of membrane strain (Carpenter and Ong, 1985), the result is
6.2 Shell under self-weight

compared with the result given by MacNeal and Harder (1985). The typical shell, as shown in figure 6.1, is used in many civil engineering applications and was developed by Scordelis and Lo (1964). Since then, this classical shell has being used as a benchmark problem for shell structure by many authors and to test the validation of any new shell element (eg. MacNeal and Harder (1985), Briassoulis (1997), Chapelle and Bathe (1998), MacNeal (1998), Zienkiewicz and Taylor (2000), Rao and Shrinivasa (2001), Lee and Bathe (2002), Chapelle et al. (2003), Dau et al. (2004), Reddy (2004) and Charmpis and Papadrakakis (2005)).

![Figure 6.1: Geometry and boundary conditions for Scordelis–Lo’s roof.](image)

This shell is loaded vertically by its uniformly distributed dead weight of intensity 90 psi. It is supported by rigid diaphragms at its two ends and it is free along the longitudinal sides. The pertinent data of the roof are: radius $R = 25$ in, length $L = 50$ in, thickness $t = 0.25$ in, arc AB $\theta = 40^\circ$, Young’s modulus of elasticity $E = 4.32 \times 10^8$ psi and Poisson’s ratio $\nu = 0$. The boundary condition for this problem is as follows: $u_z = 0$, $\theta_x = \theta_y = 0$ along arc OD, $u_x = 0$, $\theta_y = \theta_z = 0$ along side OB and $u_x = u_z = 0$, $\theta_y$ along arc BC, as shown in figure 6.1.

By making use of geometrical and loading symmetry, only one quarter of the roof is represented by the shell element with $20 \times 20$ elements. As expected, deflection occurs due to the
self-weight of the shell. The value of vertical displacement at point D is $-0.311$ in, by error of 0.2\% compared with the exact value given by MacNeal and Harder (1985) as $-0.309$ in. This deflection is the maximum displacement value, as expected and shown at point D in figures 6.2 to 6.4 which present the resultant displacement, displacement contours in the $x$ and $y$ directions respectively.

Figure 6.2: Resultant displacement for Scordelis–Lo's problem.

Figure 6.3: Displacement contours in the $x$–direction for Scordelis–Lo's problem.
6.2 Shell under self-weight

Figure 6.4: Displacement contours in the y—direction for Scordelis—Lo’s problem.

6.2.2 Hemicylindrical shell

The aim of this section is to compare the $N_{xx}$, $N_{yy}$ and $N_{xy}$ stresses by those obtained by Pfüger (1957). Figures 6.5 (a) and (b) show the geometric, stratigraphic characteristics, boundary conditions and finite element mesh of the whole domain for the considered problem, where the radius $R = 4$ m, length $L = 15$ m, thickness $t = 0.05$ m, Young’s modulus of elasticity $E = 3 \times 10^7$ kPa and Poisson’s ratio $\nu = 0$. This shell is loaded vertically by its uniformly self-weight of 3.75 kN/m².

Under the self-weight of this shell, it moves downwards, as expected. The stresses $N_{xx}$, $N_{yy}$ and $N_{xy}$ at element 314, as shown in figure 6.5 (b), are $-30.10$ kN/m², $-9.86$ kN/m² and $-14.57$ kN/m², respectively, with a small percentage error of 0.13%, 1.2% and 0.27%, respectively, compared with the analytical results $-30.06$ kN/m², $-9.74$ kN/m² and $-14.61$ kN/m², respectively given by Pfüger (1957).
6.3 Shell under external load

6.3.1 Pinched cylindrical shell with rigid end diaphragm

The pinched cylinder has rigid end diaphragms and is subjected to a point load, as shown in figure 6.6 (a). This is discussed in this section and the results are compared with the results obtained by Sze et al. (2004). Due to the ability of shell elements to represent both the inextensional bending actions and very complex membrane stress states (Liu and To, 1998), this example is another well-known benchmark problem (eg. Flügge (1973), MacNeal (1989), Kreja et al. (1997), Cho and Roh (2003) and Charmpis and Papadrakakis (2005)). The geometrical and material properties are: \( R = 100 \) in, \( L = 200 \) in, \( t = 1 \) in, \( E = 30 \times 10^3 \) psi, \( \nu = 0.3 \) and the point load is \( p = 12 \times 10^3 \) lb (Sze et al., 2004). The boundary conditions are, \( u_x = u_y = \theta_z = 0 \) for arc AD, \( u_x = \theta_y = \theta_z = 0 \) for side DC, \( u_y = \theta_x = \theta_z = 0 \) for side AB and \( u_z = \theta_x = \theta_y = 0 \) for arc BC, as shown in figure 6.6 (a).

Only one-eighth of the cylinder is considered due to symmetry, as shown in figure 6.6 (b), with \( 20 \times 20 \) elements. It is found that the deflection at the load point, i.e. at point C, is
6.3 Shell under external load

-86.82 in, by error of 4.28% compared with the numerical deflection -83.10 in obtained by Sze et al. (2004).

Figure 6.6: (a) Geometry and boundary conditions for the pinched cylindrical shell, and (b) mesh for one-eighth of the cylinder.

6.3.2 Pullout of an open-ended cylindrical shell

The pullout of an open-ended cylindrical shell being pulled by a pair of radial forces, as shown in figure 6.7, is modelled in this section and the result is compared with the result obtained by Sze et al. (2004). The geometrical and material properties are: \( R = 4.953 \) in,
6.3 Shell under external load

L = 10.35 in, t = 0.094 in, \( E = 10.5 \times 10^6 \) psi, \( \nu = 0.3125 \) and the point load is \( p = 40 \times 10^3 \) lb.

Owing to symmetry, one-eighth of the shell is modelled, as in Section 6.3.1. The displacement in the \( x \)-direction at point B is -4.613 in, by error of 1.34% compared with the numerical result -4.551 in obtained by Sze et al. (2004).

![Figure 6.7: Geometry and boundary conditions for the pullout of an open-ended cylindrical shell.](image)

6.3.3 Hemispherical shell with an 18° hole

The hemispherical shell subjected to alternating loads at the equator is a popular widely-used benchmark (e.g. MacNeal and Harder (1985), Stander et al. (1989), Simo et al. (1990), Chroscielewsk et al. (1992), Sansour and Bufler (1992), Schieck et al. (1999), Zienkiewicz and Taylor (2000) and Gruttmann and Wagner (2005)). This example is modelled and the result is compared with this given by MacNeal and Harder (1985).

This problem is a very challenging example for several reasons. It is an excellent test of the ability of an element to handle truly three-dimensional finite rotations. It is also useful in checking the invariance of the elements to rigid body rotations and the sensitivity to membrane locking. Another feature which makes the problem challenging is that both membrane and bending strains contribute significantly to the radial displacement at the loading point (Zhu and Zacharia, 1996).
Figures 6.8 (a) and (b) show the geometry of this hemispherical shell with a central hole of 18° in the top and side views respectively. The properties of this hemispherical shell are as follows: radius $R = 10$ in, thickness $t = 0.04$ in, Young's modulus $E = 6.825 \times 10^7$ psi and Poisson's ratio $\nu = 0.3$ under the action of two inward and two outward forces 90° apart. The boundary conditions for this example are: free for nodes on the $xy$ plane, $u_y = \theta_x = \theta_z = 0$ for nodes on the $xz$ plane and $u_x = \theta_y = \theta_z = 0$ for nodes on the $yz$ plane.

![Figure 6.8: The hemispherical shell (with 18° central hole) subjected to alternating loads, (a) top view and (b) side view.](image)

By making use of the symmetry of the geometry, only one quadrant of the hemisphere is analysed and each of the two alternating point loads on the quadrant is $p = 1.0$ lb. The deflection at either one of the alternating load points is 0.097 in, by error of 0.03% compared with the theoretical value 0.094 in given by MacNeal and Harder (1985).

### 6.4 Conclusion

To validate the implemented 4–noded quadrilateral shell element, the following five standard widely-used benchmark shell problems have been modelled in this chapter using GeoFEAP2:

- under self weight: Scordelis–Lo's roof and hemicylindrical shell,
- under external load: pinched cylindrical shell with rigid end diaphragm subjected to a point load, pullout of an open-ended cylindrical shell being pulled by a pair of radial forces.
forces, and hemispherical shell with an $18^\circ$ hole subjected to alternating loads at the equator.

It was found that the deflection in the vertical direction, displacement in the horizontal plane perpendicular to the vertical direction, and $N_{xx}$, $N_{yy}$ and $N_{xy}$ stresses compared well with the analytical and/or numerical solutions given by MacNeal and Harder (1985), Sze et al. (2004) and Pfüger (1957).
CHAPTER 7

2D and 3D Study of Soil-Cooling Tower-Interaction

7.1 Overview

Comparisons between the 2D and 3D applied load, and between 3D applied load and combined cooling tower-foundation-soil are then investigated and examined in this chapter. In addition, the behaviour of the soil layer is investigated in the presence of temperature effect at the shell and a change in the effect of soil shear strength, $c$, and soil angle of friction, $\phi$.

7.2 Introduction

As shown in Chapter 2, the natural draught cooling towers are structures that are used to reduce the temperature of the cooling-cycle water in thermal power plants. These towers are large-scale civil engineering structures. The shell is the most important part of the cooling tower, both in technical and financial terms (30% to 45% of the total cost) and also the most sensitive, since its collapse would put all or part of the cooling tower out of action for a considerable length of time. Hence, the shell is one of the fundamental factors influencing the life of the cooling tower (Jullien et al., 1994).

A brief geometry of the shell of revolution, shown in figure 7.1, is now introduced as a base for studying the geometry of the cooling tower shell. A plane curve, also called a meridian and defined by angle $\theta$, rotates about an axis lying in the plane of the curve to produce a surface of revolution. If this plane curve is a straight line, the shell is conical or cylindrical.
7.2 Introduction

The surface of revolution is also called the middle surface of the shell. Any point at this surface can be described by first specifying the meridian on which it is located and secondly by specifying a quantity, called a parallel circle and defined by angle $\varphi$, that varies along the meridian and is constant on a circle around the axis of the shell (Jawad, 1994).

![Diagram of a surface of revolution](image)

Figure 7.1: Coordinates, basic definitions and geometry of a surface of revolution.

Self-weight remains the most basic loading for cooling towers (Zingoni, 1999), where it is controlled by two factors, the chosen material properties and high level structural system (Liao and Lu, 1996). Hence, the tower considered for the analysis is a column-supported hyperbolic cooling tower under dead load, 520 ft tall (158.5 m) earlier analysed by Gould (1985) and later by other researchers (Aksu, 1996; Iyer and Rao, 1990; Karisiddappa et al., 1998) for further studies. It should be noted that the height of this tower is in the range of the tallest modern cooling towers which vary from around 60 m to in excess of 200 m (Busch et al., 2002; Zingoni, 1999).

The geometry of the whole structure and soil, i.e. combined cooling tower-soil case, is shown in figure 7.2. The variable thickness shell is supported by 36 pairs of columns where these columns are circular in cross-section with a radius of 2.0 ft. The geometrical details of the shell, columns, footing and soil used in the analysis are presented in Table 7.1 and the thickness of the cooling tower shell is given in Table 7.2, where the acceleration due to gravity, $g$, is taken as 32 ft/s$^2$. It should be noted that the width and height of the soil are
assumed to be $2 \times (5 \times r_f)$ and $4 \times r_f$, respectively (Potts and Zdravkovic, 2001).

Figure 7.2: Geometric and stratigraphic characteristics for the combined cooling tower-soil case.
Table 7.1: Geometrical details for the hyperbolic cooling tower (Gould, 1985) and soil used in the analysis.

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Parametric value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell</td>
<td></td>
</tr>
<tr>
<td>Height above throat level, $h_t$</td>
<td>150 ft</td>
</tr>
<tr>
<td>Height below throat level, $h_s$</td>
<td>370 ft</td>
</tr>
<tr>
<td>Radius at top, $r_t$</td>
<td>125 ft</td>
</tr>
<tr>
<td>Radius at throat level, $a$</td>
<td>115 ft</td>
</tr>
<tr>
<td>Radius at bottom, $r_s$</td>
<td>200 ft</td>
</tr>
<tr>
<td>Thickness at top</td>
<td>1.33 ft</td>
</tr>
<tr>
<td>Thickness at throat level</td>
<td>0.75 ft</td>
</tr>
<tr>
<td>Thickness at bottom</td>
<td>4 ft</td>
</tr>
<tr>
<td>Columns</td>
<td></td>
</tr>
<tr>
<td>Number of column pairs</td>
<td>36</td>
</tr>
<tr>
<td>Height of column</td>
<td>30 ft</td>
</tr>
<tr>
<td>Radius of column</td>
<td>2 ft</td>
</tr>
<tr>
<td>Circular spacing of column pairs</td>
<td>10°</td>
</tr>
<tr>
<td>Footing</td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td>10 ft</td>
</tr>
<tr>
<td>Width</td>
<td>10 ft</td>
</tr>
<tr>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td>825 ft</td>
</tr>
<tr>
<td>Width</td>
<td>2050 ft</td>
</tr>
</tbody>
</table>

Table 7.2: Material properties used in the analysis (Gould (1985), Bowles (1988)).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Shell</th>
<th>Column</th>
<th>Footing</th>
<th>Soil</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus</td>
<td>0.576 × 10^6</td>
<td>0.72 × 10^6</td>
<td>0.72 × 10^6</td>
<td>0.3 × 10^5</td>
<td>kips/ft²</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Weight density</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.004</td>
<td>kips/ft³</td>
</tr>
</tbody>
</table>
7.2 Introduction

Generally, as shown in figure 7.2, the meridional shape of the hyperboloid cooling tower, as a closed shell, consists of a lower and an upper hyperbola branches, which both meet at the throat. Hence, it is efficient to choose the origin of the axes at this throat. The equation of the generating curve in Cartesian coordinates becomes

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

where \(a\) is the radius at throat, \(z\) is the vertical coordinate along the axis of revolution and \(b\) is a characteristic dimension of the shell that may evaluated as follows (Gould, 1985)

\[
b = \frac{ah_t}{\sqrt{(r_t^2 - a^2)}}
\]

for the upper hyperboloid, i.e. \(z > 0\), and

\[
b = \frac{ah_s}{\sqrt{(r_s^2 - a^2)}}
\]

for the lower hyperboloid, i.e. \(z < 0\). In equations (7.2) and (7.3), \(r_t\) and \(r_s\) are the top and base radius, respectively, and \(h_t\) and \(h_s\) are the vertical distances from the throat to the top and to the base of the shell, respectively.

If \(R\) is the horizontal radius of the parallel circle then \(R^2 = x^2 + y^2\). Substituting \(R\) into equation (7.1) results in

\[
\frac{R^2 - z^2}{a^2 - b^2} = 1
\]

By defining the curvature parameter, \(k_t\), as follows (Gould, 1977)

\[
k_t = \sqrt{1 + \frac{a^2}{b^2}}
\]

the meridian curve may be rewritten as follows:

\[
R^2 - (k_t^2 - 1) z^2 = a^2
\]

The steps in this analysis are as follows:

1. Establishment of meridional geometry.

2. Selection of discretisation pattern, where the parts of this soil-cooling tower-interaction project are modelled as follows:

(a) shell using the implemented 4-noded shell element, as introduced in Chapter 5,
7.3 2D and 3D cooling tower cases

(b) columns support using 3D 2-noded bar element (P. Gould, 2004; T. Hara, 2004),
(c) footing using first order 8-noded hexahedron element, and
(d) drained soil using 4-noded quadrilateral element in 2D, and first order 8-noded
hexahedron element in 3D, as introduced in Chapter 4.

3. Determination of the loading and its type. Self-weight and temperature loads are
considered in this analysis.

The following three cases are then investigated:

1. (a) Model the 3D shell and columns structure without the soil and foundation, but
with fixities representing the effect of the underlying soil and foundation.
   (b) then using the z-reaction of (a) in the 3D applied load case.

2. 2D axisymmetric applied load.

3. 3D combined soil-structure interaction.

7.3 2D and 3D cooling tower cases

7.3.1 3D structure case

Figures 7.3 (a) and (b) show the 3D discretisation of the hyperboloidal shell and columns
support, for isometric and elevation views, respectively, where its geometric and stratigraphic
characteristics are shown in figure 7.2, comprising 998 nodes and 540 elements. The top view
of this mesh is shown in figure 7.4. In addition, Table 7.3 presents the thickness of the cooling
tower shell. The horizontal radius at every elevation is computed on using equations (7.1)
to (7.3). The base of every column is restrained in the x, y and z directions. The material
properties of the shell and columns are given in Table 7.2.

Every supporting column is a device for attaching the tower structure to the foundation.
Hence, these columns are very critical components of the cooling tower and their behaviour
is dominated by axial forces and motion (Yang and Kapania, 1983). It is therefore desirable
to use a sophisticated column element that contains axial displacements as degrees of free-
dom.
The axial force resultant is computed by integration of the axial stress component over the cross-sectional area of the member (Espinoza et al., 1995).

Figure 7.3: 3D discretisation for the hyperboloidal shell and columns support, (a) isometric view and (b) elevation view.

After conducting the finite element analysis, it was found that this 3D shell-columns case, as expected, is in static equilibrium under the self-weight, i.e.

\[
\begin{align*}
\sum W_x &= \sum R_x = 0 \\
\sum W_y &= \sum R_y = 0 \\
\sum W_z &= -\sum R_z = 4.0631 \times 10^5 \text{ kips}
\end{align*}
\]

(7.7 a) (7.7 b) (7.7 c)

where \( W_x, W_y \) and \( W_z \) are the weight of the structure in the \( x, y \) and \( z \) directions, respectively, and \( R_x, R_y \) and \( R_z \) are the reaction at supports in the \( x, y \) and \( z \) directions, respectively. It should be noted that the \( \sum W_z \) value will be considered as the applied load force in the next section.
Table 7.3: Thickness of the cooling tower shell (Gould, 1985).

<table>
<thead>
<tr>
<th>Elevation from throat (ft)</th>
<th>Shell thickness value (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>From To</td>
<td></td>
</tr>
<tr>
<td>+145 +150</td>
<td>1.33</td>
</tr>
<tr>
<td>+130 +145</td>
<td>1.25</td>
</tr>
<tr>
<td>+100 +130</td>
<td>1.00</td>
</tr>
<tr>
<td>+50 +100</td>
<td>0.75</td>
</tr>
<tr>
<td>0 +50</td>
<td>0.75</td>
</tr>
<tr>
<td>-50 0</td>
<td>0.75</td>
</tr>
<tr>
<td>-100 -50</td>
<td>0.80</td>
</tr>
<tr>
<td>-150 -100</td>
<td>0.85</td>
</tr>
<tr>
<td>-200 -150</td>
<td>0.90</td>
</tr>
<tr>
<td>-250 -200</td>
<td>0.95</td>
</tr>
<tr>
<td>-300 -250</td>
<td>1.00</td>
</tr>
<tr>
<td>-350 -300</td>
<td>2.50</td>
</tr>
<tr>
<td>-370 -350</td>
<td>4.00</td>
</tr>
</tbody>
</table>
Due to the self-weight of the cooling tower shell, stresses are produced on the wall. Figures 7.5 and 7.6 show the meridional and circumferential stresses resultant versus height, respectively, at the 0° meridian, i.e. at the midpoint between the two adjacent supports. These stresses, as expected, decrease as the height decreases from the bottom base. However, the meridional stress decreases rapidly when $-370 \text{ ft} < z < -300 \text{ ft}$ because the thickness of the shell increases rapidly at this range from 1 ft to 4 ft. Figures 7.5 and 7.6 show good agreement with the results obtained by Gould (1985) and Iyer and Rao (1990).

Figure 7.5: Meridional stress resultant versus height.

Figure 7.6: Circumferential stress resultant versus height.
7.3 2D and 3D cooling tower cases

7.3.2 3D applied load excluding temperature effect

Figures 7.7 and 7.8 show the side and details of top views for the 3D finite element mesh of the applied load case, comprising 9504 nodes and 2304 elements, and an applied force of $4.0631 \times 10^5$ kips, as shown in figure 7.8(c). It should be noted that this force is the same as $z-$reaction from equation (7.7 c) in the 3D structure case, i.e. shell-columns case. The drained soil is assumed to be an elasto-plastic material which obeys Drucker–Prager’s model, as introduced in Section 3.4.5, where $c = 2$ kips/ft$^2$ and $\phi = 0^\circ$ (Kérdi, 1974).

The boundary conditions for the mesh are as follows: the outer vertical boundary is restrained in the $x$ and $y$ directions and is free to move in the $z-$direction. The base of the layer is restrained in the $x$, $y$ and $z$ directions. For comparison purposes, it should be noted that the axes have been taken, in this problem and the remaining 3D analysis, at the same position as in figure 7.3.

![Figure 7.7: Side view of the 3D discretisation for the applied load case.](image)

Figures 7.9 and 7.10 show the deviatoric stress and mean pressure, respectively, for the 3D applied load at the AA-axis, as shown in figure 7.8(c), and $z = -405$ ft, $-410$ ft, $-615$ ft,
7.3 2D and 3D cooling tower cases

-820 ft and -1025 ft, respectively. In these figures, the vertical arrows refer to the position of the applied load. Both deviatoric stress and mean pressure decrease as the absolute value of $z$ increases where the values are much greater at $z = -405$ ft and $z = -410$ ft, respectively. In addition, the highest value of the deviatoric stress and mean pressure occur, as expected, at node A, shown in figure 7.7, at level $z = -405$ ft. Moreover, the deviatoric stress and mean pressure decrease, as expected, with the depth and away from the structure and they tend to zero as $x$ tends to the boundaries, i.e. away from the effect of the shell weight. More attention is therefore given to the values at $z = -405$ ft and $z = -410$ ft in the remaining study.

7.3.3 2D axisymmetric applied load excluding temperature effect

To compare the vertical settlement with the 3D applied load investigated in the last section, a 2D axisymmetric applied load is investigated, where mesh is shown in the right hand side of the central line in figure 7.7 is considered. The boundary conditions, shown in figure 7.11, for the mesh are as follows: the outer vertical boundary is restrained in the $x-$direction and is free to move in the $y-$direction. The base of the layer is restrained in the $x$ and $y$ directions.

Figure 7.11 shows the settlement contours in the $y-$direction for this problem. These contours decrease, as expected, with the depth and away from the load. However, these contours are not symmetric around the load axis, as for the examples studied in Chapter 4, because the central line has not been taken at the load axis.

Figures 7.12 (a) to (e) show the comparison of the vertical settlement [ft] for the 3D and 2D axisymmetric applied load at $z = -405$ ft, -410 ft, -615 ft, -820 ft and -1025 ft, respectively. This settlement decreases, as expected, with the depth and away from the structure, which tends to zero as $x$ tends to the boundaries, i.e. away from the effect of the shell weight. In addition, the maximum settlement at $z = -405$ ft and $z = -410$ ft occurs, as expected, under the point load and then decreases as $z$ tends to the central line. It is also shown in these figures that there is a very big difference between the 2D and 3D analysis, with maximum errors of about 78% and 76% at $z = -405$ ft and $z = -410$ ft, respectively, although they undergo the same failure mechanism. This means that the analysis of the 3D applied load case has to be considered with more attention to the settlement at $z = -405$ ft and $z = -410$ ft levels.
7.3 2D and 3D cooling tower cases

Figure 7.8: Top view for the 3D discretisation of the applied load case (a) finite element mesh, (b) title of the material types, and (c) geometry and load applied onto the footing, represented by the vertical arrows.
7.3 2D and 3D cooling tower cases

Figure 7.9: Deviatoric stress for the 3D applied load at the AA-axis, as shown in figure 7.8(c), and $z = -405$ ft, $-410$ ft, $-615$ ft, $-820$ ft and $-1025$ ft, respectively.

Figure 7.10: Mean pressure for the 3D applied load at the AA-axis, as shown in figure 7.8(c), and $z = -405$ ft, $-410$ ft, $-615$ ft, $-820$ ft and $-1025$ ft, respectively.
7.4 3D combined soil-structure interaction

In this section, the full 3D cooling tower-foundation-soil model, as shown in figure 7.2, is considered excluding the temperature effect, to compare with the 3D applied load case, and including the temperature effect. The mesh of this model is a combination of the shell-columns mesh, as shown in figure 7.3, and foundation-soil, as shown in figure 7.7. The boundary conditions is considered the same as explained in Section 7.3.2.

7.4.1 Excluding temperature effect

The aim of this section is to compare the deviatoric stress, mean pressure and settlement at the AA-axis and $z = -405$ ft and/or $-410$ ft for the 3D applied load and soil-structure interaction (SSI), excluding the temperature effect.

Figures 7.13 and 7.14 show the comparison of the deviatoric stress value for the 3D applied load and combined SSI cases at the AA-axis and $z = -405$ ft and $z = -410$ ft, respectively, excluding the temperature effect. Again, this value decreases with the depth and away from the structure and it tends to zero as $x$ tends to the boundaries and central line. In addition, on comparing the maximum deviatoric stress value, it is found that there are errors of 36% and 22% at $z = -405$ ft and $z = -410$ ft, respectively.
Figure 7.12: Comparison of the vertical settlement (ft) for the 2D and 3D axisymmetric applied load cases at $z = (a) \, -405 \, ft$, (b) $-410 \, ft$, (c) $-615 \, ft$, (d) $-820 \, ft$ and (e) $-1025 \, ft$. 
7.4 3D combined soil-structure interaction

Figure 7.13: Comparison of the deviatoric stress for the 3D applied load and combined SSI cases at the AA-axis and $z = -405$ ft, excluding temperature effect.

Figure 7.15 shows the comparison of the mean pressure value for the 3D applied load and combined SSI cases at the AA-axis and $z = -405$ ft, excluding the temperature effect. Again, this value decreases with the depth and away from the structure and it tends to zero as $x$ tends to the boundaries and central line. Moreover, on comparing the maximum mean pressure value, it is found that there is an error of 27.6%.

Figures 7.16 and 7.17 show the comparison of the vertical settlement value for the 3D applied load and combined SSI cases at the AA-axis and $z = -405$ ft and $-410$ ft, respectively, excluding the temperature effect. Again, this value decreases with the depth and away from the structure and it tends to zero as $x$ tends to the boundaries. Furthermore, on comparing the maximum settlement value, it is found that there are errors of 97.0% and 95.6% at $z = -405$ ft and $-410$ ft, respectively, although they undergo a similar failure mechanism.

The results of this section show that there is a large error when comparing the deviatoric stress, mean pressure and vertical settlement values between the 3D applied load and combined SSI cases. Hence, a 3D full model of the soil-cooling tower-interaction has to be therefore considered.
Figure 7.14: Comparison of the deviatoric stress for the 3D applied load and combined SSI cases at the AA-axis and \( z = -410 \) ft, excluding temperature effect.

Figure 7.15: Comparison of the mean pressure for the 3D applied load and combined SSI cases at the AA-axis and \( z = -405 \) ft, excluding temperature effect.
7.4 3D combined soil-structure interaction

Figure 7.16: Comparison of the settlement for the 3D applied load and combined SSI cases at the AA-axis and $z = -405$ ft, excluding temperature effect.

Figure 7.17: Comparison of the settlement for the 3D applied load and combined SSI cases at the AA-axis and $z = -410$ ft, excluding temperature effect.
7.4 3D combined soil-structure interaction

7.4.2 Including temperature effect

In this section, deviatoric stresses and settlement are investigated with the effect of temperature on the cooling tower’s shell, as introduced in Section 5.8. $\Delta T$ is assumed as follows:

$$\Delta T = \frac{1}{t} (T_{in} - T_{ex})$$

(7.7)

where $t$ is the thickness of the shell, where its value at every elevation is defined in Table 7.3, $T_{in}$ and $T_{ex}$ are the interior and exterior wall temperature of the cooling tower, respectively. $(T_{in} - T_{ex})$, or simply $T$ considered in the remaining analysis, is equal to 45°C in winter and 70°C in summer resulting from adding 25°C for sunshine (Meschke et al., 1991). The coefficient of thermal expansion, $\alpha$, is assumed as $10^{-5}/°C$ in this analysis (Smith et al., 1996).

Figures 7.18 and 7.19 show the comparison of the deviatoric stress for the 3D combined SSI case when $T = 0°C$, 45°C and 70°C at $z = -405$ ft and $z = -410$ ft, respectively. These figures show that the maximum value of the deviatoric stress increases as $T$ increases. This is because the increasing of the stresses in the shell due to the temperature effect, especially near the base of the tower as explained in Section 7.3.1. However, as expected, the increasing of this maximum value at $z = -405$ ft is more clear compared to the increasing value at $z = -410$ ft. In addition, from these figures, it is also noticed that the deviatoric stress value decreases, as expected, with the depth and away from the structure and it tends to zero as $x$ tends to the boundaries and to the central line.

Figures 7.20 and 7.21 show the comparison of the settlement for the 3D combined SSI case when $T = 0°C$, 45°C and 70°C at $z = -405$ ft and $z = -410$ ft, respectively. These figures show that the settlement increases as the temperature value increases. This is because the increasing of the deviatoric stress value due to the increasing of the temperature value, as explained in the previous paragraph. The behaviour of the soil layer is similar for different temperatures at the same vertical level, experiencing the failure mechanism explained in Section 4.3.2. However, temperature does not have a major effect on the deviatoric stress of the soil, but does affect the cooling tower shell.
7.4 3D combined soil-structure interaction

Figure 7.18: Comparison of the deviatoric stress for the 3D combined SSI case when \( T = 0^\circ\text{C}, 45^\circ\text{C} \) and \( 70^\circ\text{C} \) at \( z = -405 \) ft.

Figure 7.19: Comparison of the deviatoric stress for the 3D combined SSI case when \( T = 0^\circ\text{C}, 45^\circ\text{C} \) and \( 70^\circ\text{C} \) at \( z = -410 \) ft.
7.4 3D combined soil-structure interaction

Figure 7.20: Comparison of the settlement for the 3D combined SSI case when $T = 0^\circ C$, $45^\circ C$ and $70^\circ C$ at $z = -405$ ft.

Figure 7.21: Comparison of the settlement for the 3D combined SSI case when $T = 0^\circ C$, $45^\circ C$ and $70^\circ C$ at $z = -410$ ft.
7.5 Parametric study of the 3D cooling tower

7.5.1 Effect of soil shear strength \((c)\)

Figures 7.22, 7.23 and 7.24 show the comparison of the deviatoric stress for the 3D SSI and \(T = 0^\circ C, 45^\circ C\) and \(70^\circ C\) at \(z = -410\) ft for \(c = 0.2\) kips/ft\(^2\), i.e. for very soft clay, \(c = 2\) kips/ft\(^2\), i.e. for hard clay, and \(c = 13\) kips/ft\(^2\), i.e. for very hard clay, respectively, where \(\phi = 0^\circ\) (Kérdi, 1974). In these figures, it is shown that the maximum value of the deviatoric stress increases slightly as the temperature increases at the same value of \(c\), and also increases as \(c\) increases at the same value of temperature. It is also noticed that the deviatoric stress decreases away from the structure and it tends to zero when the absolute value of \(x\) increases away from the effect of the shell weight. In addition, the deviatoric stress decreases as the absolute value of \(x\) decreases from the point load to the central line. Furthermore, this stress increases at the central line as \(c\) increases. This is because the increasing of \(c\) makes the soil more resisting of transferring the stress from the point load.

![Figure 7.22](image)

Figure 7.22: Comparison of the deviatoric stress for the 3D combined SSI case when \(T = 0^\circ C, 45^\circ C\) and \(70^\circ C\) at \(z = -410\) ft, where \(c = 0.2\) kips/ft\(^2\).
Figure 7.23: Comparison of the deviatoric stress for the 3D combined SSI case when $T = 0°C$, $45°C$ and $70°C$ at $z = -410$ ft, where $c = 2$ kips/ft$^2$.

Figure 7.24: Comparison of the deviatoric stress for the 3D combined SSI case when $T = 0°C$, $45°C$ and $70°C$ at $z = -410$ ft, where $c = 13$ kips/ft$^2$. 

7.5 Parametric study of the 3D cooling tower
7.5 Parametric study of the 3D cooling tower

7.5.2 Effect of soil angle of friction (\(\phi\))

Figures 7.25, 7.26 and 7.27 show the comparison of the deviatoric stress for the 3D SSI and \(T = 0^\circ\text{C}, 45^\circ\text{C} \text{ and } 70^\circ\text{C}\) at \(z = -410\) ft for \(\phi = 0^\circ, 10^\circ \text{ and } 20^\circ\), respectively, where \(c = 2\) kips/ft\(^2\). In these figures, it is shown that the maximum value of the deviatoric stress increases slightly as the temperature increases for the same value of \(\phi\), and also increases slightly as \(\phi\) increases for the same value of temperature. It is also noticed that the deviatoric stress decreases as the absolute value of \(x\) increases away from the effect of the shell weight, and also decreases as the absolute value of \(x\) decreases from the point load to the central line.

![Comparison of the deviatoric stress for the 3D combined SSI case when \(T = 0^\circ\text{C}, 45^\circ\text{C} \text{ and } 70^\circ\text{C}\) at \(z = -410\) ft, where \(\phi = 0^\circ\).]
7.5 Parametric study of the 3D cooling tower

Figure 7.26: Comparison of the deviatoric stress for the 3D combined SSI case when $T = 0^\circ\text{C}, 45^\circ\text{C}$ and $70^\circ\text{C}$ at $z = -410$ ft, where $\phi = 10^\circ$.

Figure 7.27: Comparison of the deviatoric stress for the 3D combined SSI case when $T = 0^\circ\text{C}, 45^\circ\text{C}$ and $70^\circ\text{C}$ at $z = -410$ ft, where $\phi = 20^\circ$. 
7.6 Conclusion

Self-weight and temperature remain the most important basic loading for cooling towers. Hence, columns-supported hyperbolic cooling tower under dead load, which has dimensions in the range of the tallest modern cooling towers, has been modelled. It was found that this 3D shell-columns case, as expected, is in static equilibrium under the self-weight. Due to this load, stresses are produced at the shell wall, where the meridional and circumferential stresses decrease, as expected, as the height of the cooling tower decreases from the bottom base. However, the meridional stress decreases rapidly at the bottom \( \frac{1}{3} \) of the cooling tower height because the thickness of the shell increases rapidly at this range until it becomes the same radius of the column for achieving the stability of the structure. Furthermore, these stresses were in good agreement and compared well with the numerical solutions obtained by other authors. Comparisons have been made between the 2D and 3D applied load, 3D applied load and combined soil-cooling tower-interaction, and 3D combined soil-cooling tower-interaction excluding and including the temperature effect upon the cooling tower shell.

In the 3D applied load analysis, it was found that the deviatoric stress and mean pressure values decrease as the soil depth increases where these values are much greater at the ground and SSI levels. In addition, the highest value of the deviatoric stress and mean pressure occur, as expected, at the middle of the ground level. Moreover, the deviatoric stress and mean pressure values decrease, as expected, with the depth and away from the structure and they tend to zero as the horizontal distance tends to the boundaries, i.e. away from the structure effect. More attention was therefore given to the results at ground and final SSI levels in the remaining study.

In the 2D axisymmetric applied load analysis, the settlement contours decrease, as expected, with the depth and away from the load and they tend to zero as the horizontal distance tends to the boundaries, i.e. away from the structure effect. However, these contours are not symmetric around the load axis, because the central line has not been taken at the load axis. In addition, the maximum settlement at ground and final SSI levels occurs, as expected, under the point load and then decreases as the horizontal distance tends to the central line. Furthermore, it was also found that there is a very big difference between the 2D and 3D applied load analysis, with maximum errors at ground and final SSI levels, although they undergo the same failure mechanism. This means that the analysis of the 3D applied load
7.6 Conclusion

case has to be considered when compared with the 2D axisymmetric applied load case.

The 3D soil-cooling tower-interaction excluding and including the temperature effect have been investigated and compared with the 3D applied load, where more attention has been given to ground and final SSI levels. On comparing the deviatoric stress, mean pressure and vertical settlement values for 3D applied load and SSI excluding the temperature effect, it was found that these values decrease with the depth and away from the structure and they tend to zero as the horizontal distance tends to the boundaries and central line. In addition, comparing between these two cases for maximum deviatoric stress, mean pressure and vertical settlement have indicated that there are large errors at ground and final SSI levels. This means that, the 3D full model of the soil-cooling tower-interaction has to be therefore considered when compared with the 3D applied load case.

When including the temperature effect, it was found that the deviatoric stress value decreases, as expected, with depth and away from the structure and it tends to zero as the horizontal distance tends to the boundaries and to the central line. Due to increasing of the stresses at the cooling tower shell in presence of the temperature effect, the maximum value of the deviatoric stress increases as the temperature increases, and therefore the vertical settlement increases as the temperature value increases as well. It should be noted that the increasing of the maximum value of the deviatoric stress and vertical settlement at the ground level is more clear compared to their increasing at the final SSI level. However, temperature does not have a major effect on the deviatoric stress of the soil, but does affect the cooling tower shell as explained in Chapter 2.

The behaviour of the soil layer for the 3D soil-cooling tower-interaction has been investigated with the effect of the soil shear strength, $c$, and soil angle of friction $\phi$. The effect of $c$ indicated that the deviatoric stress decreases, as expected, away from the structure and it tends to zero when the absolute of the horizontal distance increases away from the structure effect. In addition, the maximum value of the deviatoric stress increases slightly as the temperature increases at the same value of $c$, and also increases as $c$ increases at the same value of temperature. Moreover, the deviatoric stress decreases as the absolute value of the horizontal distance decreases from the point load to the central line. Furthermore, this stress increases at the central line as $c$ increases. This is because the increasing of $c$ indicates an increase in the shear strength of soil.
7.6 Conclusion

The effect of $\phi$ indicated that the maximum value of the deviatoric stress increases slightly as the temperature increases for the same value of $\phi$, and also increases slightly as $\phi$ increases for the same value of temperature. It was also noticed that the deviatoric stress decreases as the absolute value of the horizontal distance increases away from the structure effect, and also decreases as the absolute value of the horizontal distance decreases from the point load to the central line. The designer needs therefore to ensure that enough modelling of soil conditions is done and an extensive site investigation is required to ensure that the variation in soil properties is represented correctly.
CHAPTER 8

Effect of Different Designs for Cooling Tower

8.1 Overview

The soil behaviour for the same cooling tower shell studied in Chapter 7 for different types of footings, circular and pads, affected by different types of columns is investigated in this chapter. Vertical and horizontal soil layers are also considered.

8.2 Introduction

The same hyperbolic cooling tower shell, that has been investigated in Chapter 7, is considered for further study in this chapter. The same geometrical details for the shell and soil, and material properties, introduced in Tables 7.1 and 7.2 respectively, are considered as well. The 3D combined soil-structure, with different types of footings and columns, are investigated in Sections 8.3 and 8.4. However, different geometric, stratigraphic characteristics and mesh, are used and described in the next paragraphs.

Figures 7.2 and 8.1 show the geometric and stratigraphic characteristics for the cooling tower shell, two columns attached to circular and pad footings respectively, and soil. In addition, figures 8.2 and 8.3 (a), (b) and (c) show the top, isometric and elevation views, respectively, for the cooling tower shell, circular footing with two columns and one column, respectively.
8.2 Introduction

Figures 8.4 (a), (b) and (c) show the top, isometric and elevation views, respectively, for the cooling tower shell, and two columns attached to pad footings. Moreover, figure 8.5 shows the design and construction of two typical natural draught cooling towers with one column attached to pad footings. Furthermore, figures 8.6 (a), (b) and (c) show the top, isometric and elevation views, respectively, for the cooling tower shell, pad footings and two columns, one column, respectively.

It should be noted that the thickness of the cooling tower shell being studied here is shown in figure 7.3 and further listed in Table 7.3. In addition, the origin of the axes are assumed at the throat as explained in Chapter 7.

The same steps used in Chapter 7 will be applied here, i.e.

1. (a) Model the 3D shell and columns structure without the soil and foundation, but with fixities representing the effect of the underlying soil and foundation.
   (b) then using the $z$-reaction of (a) in the 3D applied load case.

2. 2D axisymmetric applied load.

3. 3D combined soil-structure interaction.

These steps are repeated for the most commonly used arrangements in practice (Gould, 1985; Wittek and Krätzig, 1996)

1. one column attached to circular footing.

2. two columns attached to pad footings.

3. one column attached to pad footings.

It is found that the 3D full model of the cooling tower, with different types of columns and footings, and soil has to be considered and compared with the 3D applied load case which has to be assumed and compared with the 2D axisymmetric applied load. It should be noted that results in the next few sections are taken at the central line in the case of one column and with an angle of $5^\circ$, i.e. along the AA axis as shown in figure 7.8, for the case of two columns.
8.2 Introduction

Figure 8.1: Geometric and stratigraphic characteristics for the cooling tower shell, two columns, pad footings and soil case.
Figure 8.2: 3D discretisation for the cooling tower shell, two columns and circular footing where (a) top view, (b) isometric view and (c) elevation view.
Figure 8.3: 3D discretisation for the cooling tower shell, one column and circular footing where (a) top view, (b) isometric view and (c) elevation view.
Figure 8.4: 3D discretisation for the cooling tower shell, two columns and pad footings where (a) top view, (b) isometric view and (c) elevation view.
Figure 8.5: Typical natural draught cooling tower in Trino Vercellese, Italy (a) design and construction of two towers, and (b) one column attached to pad footings (Bierrum, 2005).
8.2 Introduction

Figure 8.6: 3D discretisation for the cooling tower shell, one column and pad footings where (a) top view, (b) isometric view and (c) elevation view.
8.3 Type of footings

8.3.1 Circular footing

In this section, the behaviour of soil is investigated for the case of circular footing attached to two columns and one column, as shown in figures 8.2 and 8.3, respectively.

Figures 8.7 (a), (b) and (c) and 8.8 (a), (b) and (c) show the comparison of the deviatoric stress at $z = -405$ ft and $z = -410$ ft, respectively, for the case of circular footing with affected to one column and two columns for (a) $T = 0^\circ C$, (b) $T = 45^\circ C$ and (c) $T = 70^\circ C$. As shown in these figures, the deviatoric stress decreases with the depth and away from the structure and it tends to zero when the absolute value of $x$ increases away from the effect of the shell weight. In addition, the deviatoric stress decreases as the absolute value of $x$ decreases from the point load to the central line. Moreover, it is also noticed that the deviatoric stress in the case of one column is slightly higher than the deviatoric stress in the case of two columns for different temperature values. This is because in the two columns case, the sum of this stress in the $x$ and $y$ directions is equal to zero, as in equations (7.7 a, b) so the stress in the $z$-direction is only distributed to the foundation and then to the soil. However, total deviatoric stress is transferred to the soil via the foundation. In addition, this difference increases as the temperature value and the absolute value of $x$ increases.

Figures 8.9 (a), (b) and (c) and 8.10 (a), (b) and (c) show the comparison of the vertical settlement at $z = -405$ ft and $z = -410$ ft, respectively, for the case of circular footing with one column and two columns for (a) $T = 0^\circ C$, (b) $T = 45^\circ C$ and (c) $T = 70^\circ C$. These figures show that the maximum settlement, which occurs at the point load, in the one column case is higher than the maximum settlement in the two columns case at $z = -405$ ft and $z = -410$ ft. This is expected because the deviatoric stress in the one column case is higher than the maximum settlement in the two columns case. In addition, the settlement value decreases away from the structure to the boundaries and to the central line. Furthermore, this value tends to zero as $x$ tends to the boundaries.

Discussion in this section indicates that the deviatoric stress and settlement in the two columns case are smaller than those in the one column case, where the foundation is designed as a circular footing. Hence, from a design point of view, it is recommended that the two columns are better than the one column.
8.3 Type of footings

(a) circular footing
$T=0^\circ C$
- one column
- two columns

(b) circular footing
$T=45^\circ C$
- one column
- two columns
8.3 Type of footings

Figure 8.7: Comparison of the deviatoric stress at $z = -405$ ft in the case of circular footing with one column and two columns for (a) $T = 0^\circ C$, (b) $T = 45^\circ C$ and (c) $T = 70^\circ C$. 
8.3 Type of footings

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(a) Circular footing $T=0^\circ C$

- One column
- Two columns

(b) Circular footing $T=45^\circ C$

- One column
- Two columns
8.3 Type of footings

Figure 8.8: Comparison of the deviatoric stress at $z = -410$ ft in the case of circular footing with one column and two columns for (a) $T = 0^\circ$C, (b) $T = 45^\circ$C and (c) $T = 70^\circ$C.
8.3 Type of footings

![Graph of settlement in the vertical direction for circular footings at T=0°C and T=45°C with one column and two columns](image)

(a)

(b)
Figure 8.9: Comparison of the vertical settlement [ft] at \( z = -405 \) ft in the case of circular footing with one column and two columns for (a) \( T = 0^\circ C \), (b) \( T = 45^\circ C \) and (c) \( T = 70^\circ C \).
8.3 Type of footings

(a)

(b)
8.3 Type of footings

Figure 8.10: Comparison of the vertical settlement [ft] at \( z = -410 \) ft in the case of circular footing with one column and two columns for (a) \( T = 0^\circ C \), (b) \( T = 45^\circ C \) and (c) \( T = 70^\circ C \).
8.3 Type of footings

8.3.2 Pad footings

In the current section, the behaviour of soil is investigated in the case of pad footings attached to two columns and one column, as shown in figures 8.4 and 8.6, respectively.

Figures 8.11 (a), (b) and (c) and 8.12 (a), (b) and (c) show the comparison of the deviatoric stress at $z = -405$ ft and $z = -410$ ft, respectively, for the case of pad footings attached to one column and two columns for (a) $T = 0\degree C$, (b) $T = 45\degree C$ and (c) $T = 70\degree C$. As shown in these figures, the deviatoric stress decreases with the depth and away from the structure and it tends to zero when the absolute value of $z$ increases away from the effect of the shell weight. In addition, the deviatoric stress decreases as the absolute value of $z$ decreases from the point load to the central line. Moreover, the deviatoric stress at $z = -405$ ft and $z = -410$ ft in the case of one column is slightly higher than the deviatoric stress in the case of two columns for different temperature values. This is because in the two columns case, the sum of this stress in the $x$ and $y$ directions is equal to zero, as in equations 7.7 a,b,c so the stress in the $z$-direction is only distributed to the foundation and then to the soil. However, the total deviatoric stress is transferred to the soil via the foundation, i.e. for the same reason explained in the previous section. Furthermore, this difference increases at $z = -405$ ft and decreases at $z = -410$ ft as the temperature value increases.

Figures 8.13 (a), (b) and (c) and 8.14 (a), (b) and (c) show the comparison of the vertical settlement at $z = -405$ ft and $z = -410$ ft, respectively, for the case of pad footings attached to one column and two columns for (a) $T = 0\degree C$, (b) $T = 45\degree C$ and (c) $T = 70\degree C$. These figures show that the maximum settlement, which occurs at the point load, in the one column case is higher than the maximum settlement in the two columns case at $z = -405$ ft and $z = -410$ ft. This is expected because the deviatoric stress in the one column case is higher than the maximum settlement in the two columns case. In addition, the settlement value decreases away from the structure to the boundaries and to the central line.

Discussion in this section indicates that the deviatoric stress and settlement in the two columns case are smaller than those in the one column case, where the foundation is designed as pad footings. Hence, from a design point of view, it is again recommended that the two columns is better than the one column.
8.3 Type of footings

(a) pad footings
T=9°C
- one column
- two columns

(b) pad footings
T=45°C
- one column
- two columns
Figure 8.11: Comparison of the deviatoric stress at $z = -405$ ft in the case of pad footings with one column and two columns for (a) $T = 0^\circ$C, (b) $T = 45^\circ$C and (c) $T = 70^\circ$C.
8.3 Type of footings

(a) Pad footings
T=5°C
-○ one column
-× two columns

(b) Pad footings
T=45°C
-○ one column
-× two columns
Figure 8.12: Comparison of the deviatoric stress at $z = -410$ ft in the case of pad footings with one column and two columns for (a) $T = 0^\circ C$, (b) $T = 45^\circ C$ and (c) $T = 70^\circ C$. 
8.3 Type of footings

(a) 

(b)
8.3 Type of footings

Figure 8.13: Comparison of the vertical settlement [ft] at \( z = -405 \) ft in the case of pad footings with one column and two columns for (a) \( T = 0^\circ C \), (b) \( T = 45^\circ C \) and (c) \( T = 70^\circ C \).
8.3 Type of footings

(a) pad footings
T=0°C
- one column
- two columns

(b) pad footings
T=45°C
- one column
- two columns
Figure 8.14: Comparison of the vertical settlement [ft] at $z = -410$ ft in the case of pad footings with one column and two columns for (a) $T = 0^\circ C$, (b) $T = 45^\circ C$ and (c) $T = 70^\circ C$. 
8.4 Type of columns

8.4.1 One column

The behaviour of soil for the case of one column sitting on circular and pad footings, as shown in figures 8.3 and 8.6, respectively, is studied.

Figures 8.15 (a), (b) and (c) and 8.16 (a), (b) and (c) show the comparison of the deviatoric stress at $z = -405$ ft and $z = -410$ ft, respectively, for the case of one column sitting on circular and pad footings for (a) $T = 0^\circ$C, (b) $T = 45^\circ$C and (c) $T = 70^\circ$C. As noticed in these figures, the deviatoric stress decreases with the depth and away from the structure and it tends to zero when the absolute value of $x$ increases away from the effect of the shell weight. In addition, figures 8.15 and 8.16 show that the maximum deviatoric stress in the case of pad footings is much higher than the deviatoric stress in the case of circular footing for different temperature values at $z = -405$ ft. Moreover, this difference increases as the temperature value increases. The big difference between the circular and pad footings cases is because the circular footing spreads the stress over a big area under the columns rather than the pad footings. However, the deviatoric stress at $z = -410$ ft in the case of pad footings is slightly smaller than the deviatoric stress in the case of circular footing for different temperature values.

In the one column case, it is noticed that the deviatoric stress and settlement in the circular footing case are smaller than those in the pad footings case. Hence, from a design point of view, it is recommended that the circular footing is better than the pad footings.
8.4 Type of columns

(a) Deviatoric stress [kip/ft^2]

(b) Deviatoric stress [kip/ft^2]

One column
T=45°C
- Circular footing
- Pad footings
8.4 Type of columns

Figure 8.15: Comparison of the deviatoric stress at z = -405 ft in the case of one column attached to circular and pad footings for (a) T = 0°C, (b) T = 45°C and (c) T = 70°C.
8.4 Type of columns

(a) and (b) show the deviatoric stress [kip/ft²] as a function of x [ft] for one column T=0°C and T=45°C. The graphs illustrate the distribution of deviatoric stress for circular footings and pad footings.
Figure 8.16: Comparison of the deviatoric stress at $z = -410$ ft in the case of one column attached to circular and pad footings for (a) $T = 0^\circ$C, (b) $T = 45^\circ$C and (c) $T = 70^\circ$C.
8.4 Type of columns

8.4.2 Two columns

The behaviour of soil for the case of two columns sitting on circular and pad footings, as shown in figures 8.2 and 8.4, respectively, is studied.

Figures 8.17 (a), (b) and (c) and 8.18 (a), (b) and (c) show the comparison of the deviatoric stress at \( z = -405 \) ft and \( z = -410 \) ft, respectively, in the case of two columns sitting on circular and pad footings for (a) \( T = 0^\circ C \), (b) \( T = 45^\circ C \) and \( T = 70^\circ C \). As shown in these figures, the deviatoric stress decreases with the depth and away from the structure and it tends to zero when the absolute value of \( x \) increases away from the effect of the shell weight. In addition, these figures show that the deviatoric stress in the case of pad footings is much greater than the deviatoric stress in the case of circular footing for different temperature values. Moreover, this difference increases as the temperature value increases. As explained in Section 8.4.1, the big difference between the circular and pad footings cases is because the circular footing spreads the stress over a big area under the columns rather than the pad footings. However, the deviatoric stress at \( z = -410 \) ft in the case of pad footings is slightly smaller than the deviatoric stress in the case of circular footing for different temperature values. Furthermore, this difference increases as the temperature value increases.

Comparison of the vertical settlement for one column and two columns sitting on circular and pad footings has been investigated at \( z = -405 \) ft and \( z = -410 \) ft for \( T = 0^\circ C \), \( 45^\circ C \) and \( 70^\circ C \). As explained in Section 8.3, in the one column and two columns cases, the settlement in the pad footings case is higher than the settlement in the circular footing case at \( z = -405 \) ft and \( z = -410 \) ft. This is again expected because the stress in the pad footings case is higher than the stress in the circular case. It is also found that the settlement value decreases away from the structure to the boundaries and to the central line as discussed throughout this chapter.

In the two columns case, it is noticed that the deviatoric stress and settlement in the circular footing case are smaller than those in the pad footings case. Hence, from a design point of view, it is again recommended that the circular footing is better than the pad footings.
8.4 Type of columns

![Diagram showing the distribution of deviatoric stress for two columns at different temperatures.]

- **two columns**
  - $T=0^\circ\text{C}$
  - $T=45^\circ\text{C}$

- Symbols:
  - • circular footing
  - • pad footings
8.4 Type of columns

Figure 8.17: Comparison of the deviatoric stress at $z = -405$ ft in the case of two columns attached to circular and pad footings for (a) $T = 0^\circ C$, (b) $T = 45^\circ C$ and (c) $T = 70^\circ C$. 
8.4 Type of columns

(a) Two columns, $T=0^\circ C$
- Circular footing
- Pad footings

(b) Two columns, $T=45^\circ C$
- Circular footing
- Pad footings

Dependent stress (kPa)

x [m]
8.4 Type of columns

Figure 8.18: Comparison of the deviatoric stress at $z = -410$ ft in the case of two columns attached to circular and pad footings for (a) $T = 0^\circ$C, (b) $T = 45^\circ$C and (c) $T = 70^\circ$C.
The soil layers are divided vertically at the central line. It is assumed that the soil layers in the right and left hand sides of the central line have a shear strength of \( c = 2 \text{ kips/ft}^2 \) and \( c = 3 \text{ kips/ft}^2 \), respectively. Figures 8.19 (a) and (b) show the comparison of the deviatoric stress in the case of two columns sitting on circular footing and vertical soil layers at (a) \( z = -405 \text{ ft} \) and (b) \( z = -410 \text{ ft} \), respectively, for \( T = 0^\circ \text{C} \). As noticed in these figures, the deviatoric stress decreases, as expected, with the depth and away from the structure and it tends to zero when the absolute value of \( x \) increases away from the effect of the shell weight. These figures show also that the deviatoric stress for \( c = 3 \text{ kips/ft}^2 \) is, as expected, higher than the deviatoric stress for \( c = 2 \text{ kips/ft}^2 \).

Figures 8.20 (a) and (b) show a comparison of the vertical settlement in the case of two columns sitting on circular footing and vertical soil layers at (a) \( z = -405 \text{ ft} \) and (b) \( z = -410 \text{ ft} \), respectively, for \( T = 0^\circ \text{C} \). From these figures, it can be seen that the modulus of the vertical settlement for \( c = 3 \text{ kips/ft}^2 \) is higher than the modulus of the vertical settlement for \( c = 2 \text{ kips/ft}^2 \). This is because the stress in the case of \( c = 3 \text{ kips/ft}^2 \) is higher than the stress in the case of \( c = 2 \text{ kips/ft}^2 \), as explained in the last paragraph. Again, it is also seen that the settlement value decreases away from the structure to the boundaries and to the central line as discussed through this chapter.

The designer needs therefore to ensure that enough modelling of soil conditions is done and an extensive site investigation is required to ensure that the variation in soil properties is represented correctly.
Figure 8.19: Comparison of the deviatoric stress in the case of two columns attached to circular footing and vertical soil layers at (a) $z = -405$ ft and (b) $z = -410$ ft.
8.5 Vertical soil layers

Figure 8.20: Comparison the vertical settlement [ft] in the case of two columns attached to circular footing and vertical soil layers at (a) \( z = -405 \) ft and (b) \( z = -410 \) ft.
8.6 Horizontal soil layers

In this section, the soil layers are divided horizontally where $c$ is modelled linearly with the depth $z$ by using the following equation (Wood, 2001)

$$c = c_0 (1 + k_s z)$$

where $c_0$ is the value of $c$ at the surface, assumed as 2 kips/ft$^2$, and $k_s$ is the slope of ground depth of variation of $c$.

Figures 8.21 (a), (b) and (c) show a comparison of the deviatoric stress for one soil layer and horizontal soil layers at $z = -410$ ft in the case of two columns sitting on circular footing for (a) $T = 0^\circ C$, (b) $T = 45^\circ C$ and $T = 70^\circ C$. These figures show that the deviatoric stress in the case of the horizontal soil layers is higher than the deviatoric stress in the case of one soil layer for $T = 0^\circ C$. However, they are identical for $T = 45^\circ C$ and $T = 70^\circ C$, except near the boundaries, where again the deviatoric stress in the case of the horizontal soil layers is higher than the deviatoric stress in the case of one soil layer.

The designer needs therefore to ensure that enough modelling of soil conditions is done and an extensive site investigation is required to ensure that the variation in soil properties is represented correctly. Most soils have an increasing shear strength with depth rather than a constant value. The engineer needs to ensure that the site tests are performed to measure shear strength with depth via drilling and other methods. He also needs to go deep enough into the ground to ensure that enough site information is available when designing the cooling tower.
8.6 Horizontal soil layers

(a) T=0°C
- one soil layer
- horizontal soil layers

(b) T=45°C
- one soil layer
- horizontal soil layers
Figure 8.21: Comparison of the deviatoric stress for one soil layer and horizontal soil layers at $z = -410$ ft in the case of two columns attached to circular footing for (a) $T = 0^\circ$C, (b) $T = 45^\circ$C and (c) $T = 70^\circ$C.
8.7 Conclusion

To investigate which type of columns is better for cooling tower design practice, soil behaviour has been investigated in the presence of one column and two columns when they are attached to circular and pad footings. It was found that the deviatoric stress decreases with the depth and away from the structure and it tends to zero when the absolute value of the horizontal distance increases away from the effect of the structure weight. In addition, the deviatoric stress decreases as the absolute value of the horizontal distance decreases from the point load to the central line. Moreover, it is also noticed that the deviatoric stress in the case of one column is slightly higher than the deviatoric stress in the case of two columns for different temperature values. This is because the sum of the stresses in the horizontal plane in the case of two columns are equal to zero so the stress in the vertical direction is only distributed to the foundation and then to the soil. However, in the case of one column the total deviatoric stress is transferred to the soil via the foundation. In addition, this difference increases as the temperature value and the absolute value of the horizontal distance increases. The maximum settlement, which occurs at the point load as expected, in the one column case is higher than the maximum settlement in the two columns case. This is expected because the deviatoric stress in the one column case is higher than the maximum settlement in the two columns case. This means that the deviatoric stress and settlement in the two columns case are smaller than those in the one column case, where the foundation is designed as a circular footing. Hence, from a design point of view, it is recommended that two columns are better than one column.

To investigate which type of footings is better for cooling tower design practice, soil behaviour has been investigated in the presence of circular and pad footings when they are affected by one column and two columns. It was found that the deviatoric stress decreases with the depth and away from the structure and it tends to zero when the absolute value of the horizontal distance increases away from the effect of the structure weight. In addition, the maximum deviatoric stress in the case of pad footings is very higher than the deviatoric stress in the case of circular footing for different temperature values. Moreover, this difference increases as the temperature value increases. The big difference between the circular footing and pad footings cases is because the circular footing spreads the stress over a big area under the columns rather than the pad footings. This means that the deviatoric stress and therefore settlement in the circular footing case are smaller than those in the pad footings case. Hence, from a design point of view, it is recommended that circular footings
are better than pad footings.

Regarding the conclusions stated in the last two paragraphs, more attention has been given to the case of two columns sitting on a circular footing when the soil layers are divided vertically and horizontally with a change in the soil shear strength, \( c \). It was found that the deviatoric stress decreases, as expected, with the depth and away from the structure and it tends to zero when the absolute value of the horizontal distance increases away from the effect of the structure weight. In addition, the deviatoric stress increases as \( c \) increases. Furthermore, the vertical settlement therefore increases as \( c \) increases. Most soils have therefore an increasing shear strength with depth rather than a constant value. Hence, the engineer needs to ensure that the site tests performed to measure shear strength with depth via drilling and other methods needs to go deep enough into the ground to ensure that enough site information is available when designing the cooling tower.
CHAPTER 9

Conclusions and Future Work

9.1 Conclusions

In this thesis, the 2D and 3D behaviour of soil-cooling tower-interaction, via the idealisation of the structure and soil on the resulting parameters, have been investigated.

The analyses in the past few chapters have indicated the need to model the soil and structure as a combined problem, rather than by applying loads onto soil as geotechnical engineers model, or by assuming the soil comprises springs and model the cooling tower, as structural engineers model. The results have shown how unrealistic the latter two approaches are. In addition, the background to the research indicated that the shell is one of the fundamental factors influencing the life performance and behaviour of the cooling tower.

The availability of the basic source code of the 2D GeoFEAP, in addition to supporting geomechanical analysis and being well-tested and used by various researchers, encouraged using and updating this software in this project. The new software, called GeoFEAP2, is used to create a 3D finite element model to comprise the shell element to represent the cooling tower shell, columns support, foundation, and elastic and elasto-plastic soil behaviour.

The software GeoFEAP was improved by adding new modelling analysis and postprocessing features such as first order 8–noded hexahedron elements to model the footings and soil in 3D. In addition, Drucker–Prager criterion was programmed in GeoFEAP2 to model the elasto-plastic behaviour of the soil in 3D. A new 4–noded quadrilateral flat shell element, based on discrete Kirchhoff's quadrilateral plate bending element, was also added to the software to model the elastic behaviour of the cooling tower shell. Furthermore, this element
was modified to account for temperature.

It is found that the $\frac{d}{dt}$ is the most important factor in designing the cooling tower and related to the construction and operating cost of cooling tower. Careful and accurate analysis of cooling towers are then needed to ensure a precise determination of cooling water temperature. This necessitates the incorporation of thermal effects when modelling cooling tower problems. This has been emphasised in the analysis chapters of this thesis, where the effect of heat has indicated the extra stresses developing in the structure.

In this research, Drucker–Prager's yield criterion was chosen to model the elasto-plastic soil behaviour for the following reasons:

1. It can represent soil dilatancy and its parameters can be related to the physical soil properties in a rather straightforward way.

2. Despite its relative simplicity, it can lead to reasonable agreement between the results of simulations and observations (Xu et al., 2003).

3. This model represents one of the idealisations of real soil which entails appropriate elastic constants, a yield function and a flow rule. The parameters required by this model are readily available and can be measured in the lab or on site based on standard site investigation techniques.

4. From a mathematical point of view, Drucker–Prager's criterion is the most convenient choice because of its simplicity and its straightforward numerical implementation and resembles the behaviour of real engineering materials more closely.

The new software (GeoFEAP2) has been validated when modelling the soil-structure interaction for;

1. elastic and elasto-plastic soil behaviour,

2. one soil layer and two horizontal and vertical soil layers,

3. two and three dimensions,

4. different types of elements and loads,

5. effect of Young's modulus and Poisson's ratio, and

6. bearing capacity of strip and circular footings.
The results compared well with analytical and/or numerical solutions obtained by other researchers.

Based on Kirchhoff’s assumptions, the four noded quadrilateral flat shell element, with six DOF at each node, were introduced. This element was used to model the elastic behaviour of curved structural which have one dimension small (a thickness normal to the remaining surface coordinates) compared to the other dimensions of the surface. The thermal effects upon this flat shell element were then presented, and incorporated into the element, and into GeoFEAP2. To validate this element, five standard widely-used benchmark shell problems have been modelled using GeoFEAP2. It was found that the results compared well with the analytical and/or numerical solutions given by other authors.

The geometry of the shell of revolution, and therefore cooling tower shell, columns support, foundation and soil, have been introduced. The following steps have then been applied:

1. selection of discretisation pattern, where the parts of this soil-cooling tower interaction project are modelled as follows:
   
   (a) shell using the implemented 4–noded shell element.
   (b) columns support using 3D 2–noded bar element.
   (c) footings using first order 8–noded hexahedron element.
   (d) drained soil using 4–noded quadrilateral element in 2D, and first order 8–noded hexahedron element in 3D.

2. determination of the loading and its type.

These elements have been modelled for the most commonly used arrangements in practice; one column or two columns attached to circular footing or pad footings. The following steps have been applied to the following cases:

1. (a) Model the 3D shell and columns structure without the soil and foundation, but with fixities representing the effect of the underlying soil and foundation.
   (b) then using the vertical reaction of (a) in the 3D applied load case.

2. 2D axisymmetric applied load.

3. 3D combined soil-structure interaction case.
In every case, it was found that the 3D full model of the soil-cooling tower-interaction has to be considered compared with the 3D applied load case which has to be assumed compared with the 2D axisymmetric applied load case.

It was found that the deviatoric stress and settlement in the two columns case are smaller than those in the one column case, where the foundation is designed as a circular footing. It was also shown that the deviatoric stress and therefore settlement in the circular footing case are smaller than those in the pad footings case. Hence, from a design point of view, it is recommended that two columns are better than one column and circular footings are better than pad footings.

Regarding this conclusion, more attention was given to the case of two columns sitting on a circular footing when the soil layers are divided vertically and horizontally with a change in the soil shear strength. It was found most soils have an increasing shear strength with depth rather than a constant value. Hence, the engineer needs to ensure that the site tests performed to measure shear strength with depth via drilling and other methods need to go deep enough into the ground to ensure that enough site information is available when designing the cooling tower.

### 9.2 Recommendations for future work

Several conclusions have been inferred from this thesis. In the next sections, several recommendations for future work will be made.

#### 9.2.1 Local-global model

The local-global model combines in a single analysis, axisymmetric shell elements, transitional shell elements, general shell elements and column elements as shown in figure 9.1 (a). Moreover, figure 9.1 (b) shows the element discretisation with various elements.

Rotational shell elements ($L_1$) are employed in the axisymmetric portion of the shell. The element has five degrees of freedom (DOF) per node, with the rotations regarded as independent along with the displacements, thus accounting for admitting transverse shearing strains. General shell elements ($L_3$) are employed in the local zone, where deviations from axisymmetric behaviour are contained. The element has five DOF per node, which are three
9.2 Recommendations for future work

Figure 9.1: Local-global FE model of (a) a cooling tower, where $L_1 = \text{rotational shell elements}$, $L_2 = \text{transitional shell elements}$, $L_3 = \text{general shell elements}$ and $L_4 = \text{discrete column elements}$; (b) element discretisation with various elements (Hara and Gould, 2002).
9.2 Recommendations for future work

translations, referred to the global Cartesian directions, and two rotations about axis in the meridional and circumferential directions of the shell of revolution. In order to achieve the continuity of displacements between the rotational and general shell elements, a layer of transitional shell elements are inserted ($L_2$). It has point nodes as well as a nodal circle to ensure smooth continuity with the adjacent rotational shell element. If the shell is supported on a discrete system of column, a standard beam element ($L_4$) with six DOF per node is deployed. The nodal DOF for this element are six, three in the global Cartesian directions and three rotations about the same axes. When this element is attached to a node of a general shell element, which has only five DOF per node, an appropriate transformation must be used to provide robust compatibility.

9.2.2 Effect of wind

The cooling efficiency is seriously dependent on the environmental conditions, such as the temperature and speed of the cross winds (Su et al., 1999). The SSI will then be investigated (e.g. Noorzaei et al., 2006; Viladkar et al., 2006) taking into account:

1. the reinforced concrete (RC) comprising of the shell (e.g. Hara et al., 1994; Min, 2004; Noh et al., 2003; Viladkar et al., 2006; Wittek and Meiswinkel, 1998)
2. non-uniform temperature at the top of the tower (Su et al., 1999),
3. effect upon the adjacent buildings, as shown in figure 9.2 (a), or group of towers, as shown in figures 9.2 (b) and (c) (e.g. Niemann and Köpper, 1998; Orlando, 2001). However, more attention has to be given to the analysis of groups of towers because they are often in close proximity to each other or to other buildings so that wind-induced pressures can be significantly different from those on an isolated tower.

Wind speeds should be derived in accordance with BS4485 (1996) while details of its loading pressure upon the shell is explained in BS4485 (1996).

9.2.3 Effect of earthquake

Horr and Safl (2002) investigated the dynamic analysis of cooling towers in the case of earthquake excitation. In this case, it can be assumed that the earthquake motions were applied at the structural support points. Hence, all displacements at the base level were assumed to depend only on the earthquake-generation waves. However, the structural response to any earthquake excitation not only depends on the dynamic characteristics of the structure itself.
Figure 9.2: Adjacent buildings to (a) one cooling tower, (b) group of five cooling towers, and (c) group of eight cooling towers (Nuclear Power Plants Around the World, 2003).
but it depends also on the relative mass and stiffness properties of the soil and the structure. It should be noted that the footing-footing interaction is important in this dynamic analysis (Kocak and Mengi, 2000) and has to be investigated.
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