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Buffer-aided Cooperative Networks

by

Zhao Tian

A Doctoral Thesis submitted in partial fulfilment of the requirements
for the award of the degree of Doctor of Philosophy (PhD)

August 2015

Advanced Signal Processing Group,
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I dedicate this thesis to my loving parents, J Zhao and X Tian, and my girlfriend, Q Pan.
Abstract

In this thesis new methods are presented to achieve performance enhancement in wireless cooperative networks. In particular, techniques to improve diversity gain, throughput and minimise the transmission delay are described.

A buffer-aided amplify-and-forward max-link relay selection scheme for both symmetric and asymmetric channels is introduced. This approach shows that the max-link scheme is most effective over the traditional max-SNR scheme when the source-to-relay and relay-to-destination links are symmetric. The closed form expressions for the outage probability and average packet delay of the proposed scheme under both symmetric and asymmetric channel configurations is derived. The diversity order and the coding gain of the AF max-link scheme is analytically provided. Then a novel relay selection scheme with significantly reduced packet delay is proposed. Both the outage performance and average packet delay of the proposed scheme are analysed. The analysis shows that, besides the diversity and coding gains, the proposed scheme has average packet delay similar to that of a non buffer-aided relay system when the channel SNR is sufficiently high thereby.

A novel buffer-aided link selection scheme based on network-coding in a multiple hop relay network is proposed. Compared with existing approaches, the proposed scheme significantly increases the system throughput. This is achieved by applying data buffers at the relays to decrease the outage probability and using network-coding to increase the data rate. The closed-form expressions of both the average throughput and packet delay are derived. The proposed scheme has not only significantly higher throughput than both the traditional and existing buffer-aided max-link scheme, but also smaller average packet delay than the max-link scheme.
A decode-and-forward buffer-aided relay selection for the underlay cognitive relay networks in the presence of both primary transmitter and receiver is presented. A novel buffer aided relay selection scheme for the cognitive relay network is proposed, where the best relay is selected with the highest signal-to-interference-ratio among all available source-to-relay and relay-to-destination links while keeping the interference to the primary destination within a certain level. A closed-form expression for the outage probability of the proposed relay selection scheme is obtained.

Finally, a novel security buffer-aided decode-and-forward cooperative wireless networks is considered. An eavesdropper which can intercept the data transmission from both the source and relay nodes is considered to threaten the security of transmission. Finite size data buffers are assumed to be available at every relay in order to avoid having to select concurrently the best source-to-relay and relay-to-destination links. A new max-ratio relay selection policy is proposed to optimise the secrecy transmission by considering all the possible source-to-relay and relay-to-destination links and selecting the relay having the link which maximises the signal to eavesdropper channel gain ratio. Two cases are considered in terms of knowledge of the eavesdropper channel strengths: exact and average gains, respectively. Closed-form expressions for the secrecy outage probability for both cases are obtained. The proposed max-ratio relay selection scheme is shown to outperform one based on max-min-ratio relay scheme.
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Statement of Originality

The contributions of this thesis are mainly on the improvement of cooperative network performances with applying buffer on relay nodes and the outage probability, throughput and average delay analysis in the context of multi-relay selection. The novelty of the contributions is supported by the following international journal papers:

In Chapter 3, both symmetric and asymmetric channel configuration for buffer-aided relay selection with AF protocol are presented. Outage probability, diversity gain, coding gain and average delay have been analysis. The results have been published in:


The contribution of Chapter 4 is a significant reduced packet delay transmission scheme applied on buffer-aided relay networks. Both symmetric and asymmetric channel configuration have been considered. The novelty of this work is supported by the following work:


In Chapter 5, a novel buffer-aided link selection scheme based on network-coding in a multi-hop relay network is presented. This scheme significantly increases the system throughput when compare with existing approaches. The novelty of this work is supported by the following work:

3. Z. Tian, Y. Gong, G. Chen, Z. Chen and J. A. Chambers, “Buffer-aided Link Selection with Network-coding in Multi-hop Networks”, 3rd revi-
In Chapter 6, outage probability analysis in the decode-and-forward buffer-aided relay selection in cognitive relay networks is presented. The novelty of this work is reinforced by the following work:


In Chapter 7, A new max-ratio relay selection policy to optimize the secrecy transmission is presented. The novelty of this work is published by the following work:

Acknowledgements

I AM DEEPLY INDEBTED to my supervisor Dr. Alex Gong for his kind interest, generous support and constant advice throughout the past three years. I have benefitted tremendously from his rare insight, his ample intuition and his exceptional knowledge. This thesis would never have been written without his tireless and patient mentoring. It is my very great privilege to have been one of his research students. I wish that I will have more opportunities to work with him in the future.

I am extremely thankful to Professor Jonathon A. Chambers and Professor Sangarapillai Lambotharan, Dr. Gaojie Chen, Dr. Yanfeng Liang, Dr. Lu Ge, Yusen Chen, Rushan Pun, Tom Southall, Yixin Deng, Yu Liang and Fu Xu for their support and encouragement.

I am also grateful to all my colleagues Olusegun Awe, Bokamoso Basutli, Gaia Rossetti, Tasos Deligiannis and Yu Wu in the Advanced Signal Processing Group for providing a stable and cooperative environment within the Advanced Signal Processing Group.

Most importantly, I really can not find appropriate words or suitable phrases to express my deepest and sincere heartfelt thanks, appreciations and gratefulness to my parents, my grandmothers, my brothers, and my girlfriend for their constant encouragement, attention, prayers and their support in innumerable ways throughout my PhD and before.

Finally, I would like to thank everyone else who helped me to successfully realise this thesis.

Zhao Tian
August, 2015
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<td>3G</td>
<td>Third Generation</td>
</tr>
<tr>
<td>3GPP</td>
<td>3rd Generation Partnership Project</td>
</tr>
<tr>
<td>4G</td>
<td>Fourth Generation</td>
</tr>
<tr>
<td>5G</td>
<td>Fifth Generation</td>
</tr>
<tr>
<td>ACK</td>
<td>Acknowledgment</td>
</tr>
<tr>
<td>AF</td>
<td>Amplify-and-Forward</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>BPCU</td>
<td>Bits Per Channel Use</td>
</tr>
<tr>
<td>BRS</td>
<td>Best Relay Selection</td>
</tr>
<tr>
<td>CC</td>
<td>Coded Cooperation</td>
</tr>
<tr>
<td>CCI</td>
<td>Co-Channel Interference</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CP</td>
<td>Cyclic Prefix</td>
</tr>
<tr>
<td>CRC</td>
<td>Cyclic Redundancy Check</td>
</tr>
<tr>
<td>Acronym</td>
<td>Definition</td>
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</tr>
<tr>
<td>CRN</td>
<td>Cognitive Relay Network</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>D2D</td>
<td>Device-to-Device</td>
</tr>
<tr>
<td>DF</td>
<td>Decoded-and-Forward</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
</tr>
<tr>
<td>FIC</td>
<td>Full Interference Cancellation</td>
</tr>
<tr>
<td>FIFO</td>
<td>First-In-First-Out</td>
</tr>
<tr>
<td>FSA</td>
<td>Fixed Spectrum Access</td>
</tr>
<tr>
<td>FSIC</td>
<td>Full Inter-Relay Self Interference Cancellation</td>
</tr>
<tr>
<td>HRS</td>
<td>Hybrid Relay Selection</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>Independent and Identically Distributed</td>
</tr>
<tr>
<td>INR</td>
<td>Interference-to-Noise Ratio</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-Symbol Interference</td>
</tr>
<tr>
<td>LTE</td>
<td>Long-Term Evolution</td>
</tr>
<tr>
<td>MGF</td>
<td>Moment Generating Function</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input-Multiple-Output</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
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<tr>
<td>MMRS</td>
<td>Max-Max Relay Selection</td>
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<td>Acronym</td>
<td>Definition</td>
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<tr>
<td>MRC</td>
<td>Maximum Ratio Combiner</td>
</tr>
<tr>
<td>NLOS</td>
<td>Non-Line-of-Sight</td>
</tr>
<tr>
<td>ODSTC</td>
<td>Orthogonal Distributed Space-Time Code</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>PEP</td>
<td>Pairwise Error Probability</td>
</tr>
<tr>
<td>PU</td>
<td>Primary User</td>
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<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>SIMO</td>
<td>Single-Input-Multiple-Output</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal-to-Noise plus Interference Ratio</td>
</tr>
<tr>
<td>SIR</td>
<td>Signal-to-Interference-Ratio</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-Input-Single-Output</td>
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<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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<tr>
<td>SU</td>
<td>Secondary User</td>
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<tr>
<td>TDBC</td>
<td>Time-Division Broadcast</td>
</tr>
<tr>
<td>WiFi</td>
<td>Wireless Fidelity</td>
</tr>
<tr>
<td>WiMax</td>
<td>Worldwide Interoperability for Microwave Access</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>⌊ ⌋</td>
<td>Floor operator</td>
</tr>
<tr>
<td>H</td>
<td>Channel convolution matrix</td>
</tr>
<tr>
<td>G</td>
<td>Block equalizer</td>
</tr>
<tr>
<td></td>
<td>Absolute value</td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>⊙</td>
<td>Schur-Hadamard (elementwise) product</td>
</tr>
<tr>
<td>(.)^H</td>
<td>Hermitian transpose operator</td>
</tr>
<tr>
<td>(.)^T</td>
<td>Transpose operator</td>
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<tr>
<td>(.)^†</td>
<td>Pseudo-inverse</td>
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<td>⌊ ⌋</td>
<td>Floor operator</td>
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<tr>
<td>Γ(.)</td>
<td>Gamma operator</td>
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<tr>
<td>Ψ(.)</td>
<td>Digamma function</td>
</tr>
<tr>
<td>Ψ(.).′</td>
<td>Trigamma function</td>
</tr>
<tr>
<td>Λ</td>
<td>Diagonal matrix</td>
</tr>
<tr>
<td>a_i</td>
<td>ith column vector of mixing matrix</td>
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<tr>
<td>A</td>
<td>Mixing matrix</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
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<td>------------</td>
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<tr>
<td>$diag(b)$</td>
<td>Diagonal matrix with vector $b$ on its main diagonal</td>
</tr>
<tr>
<td>$C_u$</td>
<td>Covariance of signals $u$</td>
</tr>
<tr>
<td>$E{\cdot}$</td>
<td>Expectation operator</td>
</tr>
<tr>
<td>$I$</td>
<td>Identity matrix</td>
</tr>
<tr>
<td>$I(y_i, y_j)$</td>
<td>Mutual information between variables $y_i$ and $y_j$</td>
</tr>
<tr>
<td>$\text{kurt}(\cdot)$</td>
<td>Kurtosis</td>
</tr>
<tr>
<td>$L$</td>
<td>Order of filter</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of mixtures</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of sources</td>
</tr>
<tr>
<td>$\text{Neg}(\cdot)$</td>
<td>Negentropy</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
</tr>
<tr>
<td>$R$</td>
<td>Covariance matrix</td>
</tr>
<tr>
<td>$\text{Re}{\cdot}$</td>
<td>Real part</td>
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<tr>
<td>$S$</td>
<td>Source matrix</td>
</tr>
<tr>
<td>$\text{sgn}(\cdot)$</td>
<td>Signum function</td>
</tr>
<tr>
<td>$T$</td>
<td>Number of time samples</td>
</tr>
<tr>
<td>$V$</td>
<td>Whitening matrix</td>
</tr>
<tr>
<td>$\text{Var}(\cdot)$</td>
<td>Variance</td>
</tr>
<tr>
<td>$w_i$</td>
<td>Estimated unmixing vector corresponding to the $i$th IC</td>
</tr>
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<td>$W$</td>
<td>Estimated unmixing matrix</td>
</tr>
<tr>
<td>$X$</td>
<td>Input data matrix</td>
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<td>Symbol</td>
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Chapter 1

INTRODUCTION

The wireless revolution is transforming the existing global telecommunications networks into an integrated system that will provide a broad class of ubiquitous communications services to customers anywhere, anytime, in motion or fixed. Unlike fiber or coaxial cables which is almost free of interference, signal propagation through a wireless channel has to combat more difficulties such as fading, co-channel interference, adjacent channel interference and greater additive noise [1]. The design of a reliable wireless system is difficult due to the random nature of the wireless channel, and the diversity of environment in which they are likely to be deployed [2]. Recently, with the fast development of Internet of things and mobile Internet, the demands for high-speed data applications, such as social networking, machine-to-machine communication, and high-quality wireless video streaming, have been growing exponentially [3]. The next generation wireless communication systems are anticipated to provide energy and spectral efficiency growth by a factor of at least 10, and the are throughput growth by a factor of at least 25 [4].
1.1 Multi-Input Multi-Output

Designing very high speed wireless links that offer good range capability and quality-of-service (QoS) in non-link-of-sight (NLOS) environment constitutes a significant research and engineering challenge. However, the performance of wireless communication systems at the link level is limited by multi-path fading, multi-path inter-symbol interference (ISI), interference created from co-channel users (CCI) and thermal noise [5]. These limitations offer a number of technical challenges for reliable wireless communication systems. One approach to overcome these challenges is to use multiple antenna wireless communication systems, which have received a great deal of attention due to the diversity gain and increased the reliability of wireless link [6]. Under suitable channel fading conditions, having both multiple transmit and multiple receive antennas (i.e., a MIMO channel), which is included in Fig.1.1, can provide an additional spatial dimension for communication to potentially combat multipath fading propagation effects and yields a capacity gain as compared with a conventional single-input single-output (SISO) system. MIMO has already become a major focus for long-term evolution (LTE) and fourth generation (4G) cellular system and it has been applied in 802.11x systems wireless fidelity (WiFi), 802.16x worldwide interoperability

![Figure 1.1. Schematic of a MIMO Communication System](image-url)
The core idea in MIMO systems is space-time signal processing in which time is complemented with the spatial dimension inherent in use of multiple spatially distributed antennas. MIMO effectively takes advantages of random fading and when available, multipath delay spread, for multiplying transfer rates [8]. Some significant advantages will be presented as follows, such as array gain, diversity gain and multiplexing gain.

First, array gain is a key parameter in MIMO communication systems, which a power gain of the transmitted signal can be achieved by applying multiple-antennas at the transmitter and/or receiver. Through correlative combination technique and an antenna array, in addition with the knowledge of channels, the average signal-to-noise (SNR) at the receiver can be improved significantly.

Second, diversity is an important method in wireless channels to combat fading. A MIMO system consists of $N_t \times N_s$ SISO links, if the signals transmitted over each of these links experience independent fading then the diversity order of the system is given by $N_t \times N_s$. Thus the diversity order in a MIMO systems scales linearly with the product of the number of receive and transmit antennas.

Finally, the key differentiating advantage of MIMO systems is practical throughput enhancement which is referred as multiplexing gain and it can be realised through a technique known as spatial multiplexing. Figure 1.2 shows the basic principle of spatial multiplexing for a $2 \times 2$ MIMO system. The symbol stream to be transmitted is split into two half-rate sub-streams and modulated to form the signal $x_1$ and $x_2$ that are transmitted simultaneously from separate antennas. Under favorable channel conditions, the spatial signatures of these signals ($[y_{11}, y_{12}]$ and $[y_{21}, y_{22}]$) induced at the receive antennas are well separated (ideally orthogonal). The receiver can then extract the two sub-streams, $x_1$ and $x_2$, which it combines to give the original symbol stream, $x$. 
Figure 1.2. Spatial Multiplexing

The channel capacity is a measure of the maximum amount of information that can be transmitted over a channel and received with a low probability of error at the receiver. The ergodic capacity of a SISO channel is the ensemble average of the information rate over the distribution of the channel $h_{sd}$ [2], which is given by

$$C_{SISO} = E(\log_2(1 + \rho|h_{sd}|^2)),$$  \hspace{1cm} (1.1.1)

where $h_{sd}$ is a complex Gaussian random channel coefficient, and $\rho$ is the average SNR ratio at the receiver branch and $E(\cdot)$ denotes the statistical expectation over all channel realisations. And the ergodic capacity of a MIMO system is given by [9]

$$C_{MIMO} = E(\log_2(\det(I_{Nd} + \frac{\rho}{Ns}HH^*))),$$  \hspace{1cm} (1.1.2)

where $I_{Nd}$ is the $Nd \times Nd$ identity matrix, $H$ is the $Nd \times Ns$ normalised channel response matrix and $\det(\cdot)$ denotes the matrix determinant. When the channel is square and orthogonal ($HH^* = I$), then with an independent and identically input distribution, (1.1.2) reduces to

$$C_{MIMO}^* = NdE(\log_2(1 + \frac{1}{Nd}\rho))$$  \hspace{1cm} (1.1.3)

Hence $N = Nd = Ns$ parallel channels are created within the same frequency bandwidth for no additional transmit power. Capacity scales linearly with
number of antennas for increasing SNR, i.e. capacity increases by $N \text{b/s/Hz}$ for every 3dB increase in SNR. In general, it can be shown that an orthogonal channel of the form described above maximises the Shannon capacity of a MIMO system. For the independent and identically distributed (i.i.d.) fading MIMO channel model described earlier, the channel realisations become approximately orthogonal when the number of antennas used is very large. When the number of transmit and receive antennas is not equal, $N_d \neq N_s$ the increase in capacity is limited by the minimum of $N_d$ and $N_s$. This increase in channel capacity is called multiplexing gain.

Fig. 1.3 illustrates the ergodic capacity of a MIMO flat fading wireless link with an equal number of $N_s$ and $N_d$ antennas at the source and destination node. It can be seen that the ergodic capacity increases with SNR and with the number of antennas at the transmitter and the receiver. For example at SNR = 20 dB the capacity increases approximately eight-fold when the SISO link is replaced by an $8 \times 8$ MIMO link. Therefore, when the channels between all antennas are uncorrelated, MIMO can offer major increase in capacity proportional to $\min(N_s, N_d)$.

![Ergodic capacity of a MIMO fading channel](image)

**Figure 1.3.** Ergodic capacity for different MIMO sizes
Therefore, a MIMO point-to-point system can effectively provide array gain, diversity gain, multiplexing gain and ergodic capacity gain. However, the requirements of multiple-antenna terminals increases the system complexity and the separation between the antennas increases the terminal size. Furthermore, MIMO systems suffer from the effect of path loss and shadowing, where path loss is the signal attenuation between the source and destination nodes due to propagation distance, while the shadowing is the signal fading due to objects obstructing the propagation path between the source and destination nodes [6, 10, 11]. Wireless cooperative networks can provide some solutions to deal with the aforementioned problems.

1.2 Wireless Cooperative Networks

“The increasing demand for wireless multimedia and interactive Internet services, along with rapid proliferation of a multitude of communications and computational gadgets, are fueling intensive research efforts on the design of novel wireless communication systems architectures for high speed, reliable and cost effective transmission solutions” [11]. However, the performance of wireless communication systems at the link level is limited by multi-path propagation effects, which lead to inter-symbol interference (ISI), path loss, and interference from other users in the form of co-channel interference (CCI) [12]. These limitations proved a number of technical challenges for reliable wireless communication systems. One approach to address these challenges, which had been presented in previous section, is MIMO.

Another approach to address this problem is a newly developed technique know as multi-user cooperative diversity that allows a single antenna user to achieve transmit diversity benefits by sharing their physical resource through a virtual transmit and receive antenna array [13]. Such cooperative relay networks have become a useful technique that can achieve the
same advantage as MIMO systems whilst avoiding some of their disadvantages. Therefore, they have recently been adopted for different new wireless systems such as 3GPP LTE-Advanced [14]. In addition, they have been considered in different wireless system standards such as WiMAX standards (IEEE 802.16j and IEEE 802.16m) [15] and WiFi standards (IEEE 802.11s and IEEE 802.11n) [16].

The origin of cooperative communication can be traced back to the work [17] on the relay channel. Their relay channel model includes a source node, a relay node and a destination node, as shown in Figure 1.4. This model can be decomposed into a broadcast channel (where the source node A transmits the signals, the relay node B and the destination node C receive the signals) and a multiple access channel (where the source node A transmits the signals, the relay node B retransmits the signals received from node A, and the destination node C receives signals from node A and node B).

![Figure 1.4. Relay channel model](image)

[17] demonstrates that the capacity of a discrete memoryless Additive White Gaussian Noise (AWGN) relay channel is better than that of the source-destination channel. Cooperative relay networks can potentially provide several gains, i.e., cooperative diversity gain, cooperative multiplexing gain and pathloss gain. Cooperative diversity gain can be obtained in proportion to the number of independent channels in the cooperative relay network, which depends on the number of relay nodes and the environment [18].
For example, in a frequency-flat channel, the maximum cooperative diversity gain $G_d = N_s \times N_r \times N_d$, where $N_r$ is the number of single-antenna relay nodes. The cooperative spatial multiplexing gain effectively equals the number of independent channels over which different information can be transmitted, which can improve capacity or transmission data rates. Finally, using intermediate relay nodes helps in avoiding the pathloss problem because dividing the propagation path between the source and destination nodes into at least two parts yields transmit power gains because the total resultant pathloss of part of the whole path is less than the pathloss of the whole path [19]. This advantage of the cooperative relay network can be referred to as pathloss gain.

### 1.3 Cooperative Scheme

Cooperative communication within homogeneous network can be realised with two cooperation schemes: fixed relaying and user cooperation. The fixed relaying scheme is very similar to the relay channel model shown in Figure 1.4. In this scheme, a fixed relay node is placed between the source node and the destination node in advance and it is wirelessly connected with the source node and the destination node. This relay node does not have any information to transmit, but forwards the information it receives. Unlike fixed relaying, user cooperation is more flexible. The source nodes can act as relay nodes to forward the information of cooperative partners in addition to sending their own information. As a result, these terminals should have such functions as signal forwarding and simple routing. By the ways the relay node processes the source nodes information, user cooperation can be further divided into the following schemes: Amplify-and-Forward (AF), Decode-and-Forward (DF), Coded Cooperation (CC). In the following sections, the basic principles of these schemes and a roughly comparison will be presented. To
simplify the description, only the case with single relay node will be discussed here. The cases with multiple relay nodes are similar.

1.3.1 Amplify and Forward

The Amplify and Forward method simply amplify the signal received by the relay before transmitting it to the destination. This method was first proposed by J. N. Laneman and G. W. Wornell [20], and is ideal when the relay station has minimal computing power. The concept of the Amplify and Forward technique is shown in Fig.1.5

![Amplify and Forward Technique](image)

**Figure 1.5.** Amplify and Forward Technique

In Phase 1, the source node transmits the signals by way of broadcasting, while the destination node and the relay node receive the signals. In Phase 2, the relay node amplifies the powers of the signals received from the source node and forwards them to the destination node. In Phase 3, the destination node combines and decodes the signals received from the source node in Phase 1 and the relay node in Phase 2 so as to restore the original information. AF is also called non-regenerative relaying scheme and it is basically a processing method for analog signals. Compared with other schemes, AF is the simplest. Besides, as the destination node can receive independent fading signals from the source and relay nodes, full diversity gain and good performance can be achieved with this scheme.

However, one major drawback of this method is that the noise in the
signal is also amplified at the relay station, and the destination receives two independently faded versions of the signal.

### 1.3.2 Decode and Forward

The Decode and Forward technique is the most preferred technique of processing data in the relay nodes, and is closet to the idea of a traditional relay. DF scheme was first presented by Sendonaris et al [21]. Similarly to AF, signal processing in DF scheme can also be simplified into three phrases, as shown in Figure 1.6

![Figure 1.6. Decode and Forward Technique](image)

In Phase 1 and Phase 3, DF scheme processes the signals the same way as AF. In Phase 2, the relay node decodes and detects the signals received from the source node before it forwards the signals to the destination node. Hence, DF is also called regenerative relaying scheme. Clearly, DF is essentially a digital signal processing scheme. Although noise propagation problem will not take place, the signal processing in DF largely depends on transmission performance of source-relay channel. If Cyclic Redundancy Check (CRC) is not implemented in coding, full diversity orders can not be obtained. Moreover, the errors brought by the relay node during signal demodulation and decoding will accumulate with the increase of hops, thus affecting diversity advantage and relay performance.
1.3.3 Coded Cooperation

In AF and DF, the relay node always repeatedly forwards the information received from the source node, which often leads to decreased usage of the degrees of freedom. To solve the problem, Hunter et al. [22] integrated channel coding into cooperative communication and proposed CC scheme. Signal processing in CC scheme is shown in Figure 1.7.

![Figure 1.7. Coded Cooperation Technique](image)

In CC scheme, different segments of each user’s code words can be sent via two different fading paths. Each user correctly decodes the information received from cooperative partners and then re-encodes them before forwarding them. With redundant information bits being repeatedly transmitted through different spaces, the system performance is improved. In CC scheme, each mobile terminal achieves diversity and coding gains by re-encoding and transmitting different redundant bits, thus the system performance is greatly enhanced. Moreover, this scheme does not require information feedback between cooperative nodes. When a relay node cannot correctly decode the information bits, it automatically reverts back to non-cooperative mode, ensuring the system efficiency.
1.4 Relay Selection in Cooperative Networks

Recently, relay selection has been proposed as an attractive solution to improve the performance of conventional cooperative networks. For example, in cooperative wireless networks, the relay nodes have different locations so each transmitted signal from the source node to the destination node must pass through different paths causing different attenuations within the signals received at the destination which results in reducing the overall system performance. Therefore, to minimise this effect and benefit from cooperative communication, high quality paths should be chosen by using relay selection techniques. Furthermore, some works [13,18,23] have shown that full cooperative diversity order can be achieved with the relay selection scheme.

![Figure 1.8. A cooperative network with a relay selection scheme, where only the second relay is employed in the second stage of the transmission.](image)

In Fig. 1.8, a transmitter broadcasts its signal toward all the relay nodes at the first stage; the best relay can then be selected, by using local measurements of the instantaneous channel conditions between the source-relay and the relay-destination, and then used to transmit its received signal to the destination node during the second stage. No direct link between the source and destination is assumed due to path loss and shadowing. A relay
Section 1.5. Challenges and Thesis Contributions

selection scheme can be exploited in both DF and AF relaying schemes. In the literature, the DF relay selection scheme has been investigated in [24,25] over Rayleigh fading channels. The performance of a DF relay network has been provided in [26] over a Nakagami-m fading channel.

Max-min selection policy for AF networks can generally be used to choose the best relay node to help the source to transmit its signal to the destination node, which as below [18]

$$ R_{best} = \arg \max_i \min( |h_{sr_i}|^2, |h_{rd_i}|^2) \quad \text{Policy I} $$

where $h_{sr_i}$ and $h_{rd_i}$ are channel links between the source-relay and relay-destination. On the basis of this policies, some works in [27–29] have been considered to select the best relay from a cooperative AF network. In this thesis, an exact selection policy will be provided to obtain an accurate outage probability in cooperative AF networks.

1.5 Challenges and Thesis Contributions

Cooperative transmission can enhance the reliability of data transmission, provide broader and low-cost coverage and combat shadowing and fading effects [13]. For cooperative networks with multiple relay nodes that can assists the source transmission through either multi-hop relaying or cooperative diversity, relay selection has been introduced as an efficient cooperative technique. The main advantages of relay selection is the implementation simplicity, as it does not require complex physical layer transmission or explicit synchronisation process. Although relay selection is introduced as a general radio design technique for several cooperative applications with different optimisation targets, in this thesis, its basic form is re-examined. More specifically, the simple AF and DF cooperative networks where a source communicate with a destination through a set of half-duplex relay nodes with
the objective of performance optimisation are focused. In order to overcome the limitation of traditional relay selection scheme, recently in works [30], data buffers are introduced at the relay nodes which allow the selection of a different relay for reception and transmission in order to extract further diversity gain. However, the most existing works [31–33] are mainly investing the relay node with DF protocol which is not easier to implement, as well as lower level of security, thus in this thesis, the AF protocol for buffer-aided relay cooperative network is considered. Moreover, the diversity gain achieved by applying buffer on relay also with the cost on the packet delay, thus, in this thesis, a novel selection scheme with significant reduced average packet delay is introduced for buffer-aided relay cooperative network.

In addition, buffer-aided relay can be applied to many communication scenarios where requires high diversity gains, thus in this thesis, three different communication scenarios, multi-hop, cognitive and security, are reconsidered to apply the buffer on relays in order to improve the performances on throughput, diversity gain, secrecy capacity gain, respectively. In summary, the contributions of this thesis can be summarised into five main parts:

1. The outage performance of an AF relay system which exploits buffer-aided max-link relay selection is fully investigated. Both asymmetric and symmetric source-to-relay and relay-to-destination channel configurations are considered. The closed-form expressions for the outage probability, and analyse the average packet delays are derived. The diversity order is proved between \( N \) and \( 2N \) (where \( N \) is the relay number), corresponding to a relay buffer size between 1 and \( \infty \) respectively. Furthermore, the coding gain is analytically presented.

2. A novel relay selection scheme for buffer-aided relay cooperative network with significantly reduced packet delay is proposed. Both the outage performance and average packet delay of the proposed scheme are analysed.
The analysis shows that, besides the diversity and coding gains, the proposed scheme has average packet delay similar to that of a non buffer-aided relay system when the channel SNR is sufficiently high thereby, making it an attractive scheme for practical systems.

3. A novel buffer-aided link selection scheme based on network-coding in a multiple hop relay network is proposed. Compared with existing approaches, the proposed scheme significantly increases the system throughput. This is achieved by applying data buffers at the relays to decrease the outage probability and using network-coding to increase the data rate. The closed-form expressions of both the average throughput and packet delay are derived. The proposed scheme has not only significantly higher throughput than both the traditional and existing buffer-aided max-link scheme, but also smaller average packet delay than the max-link scheme, making it an attractive scheme in practice.

4. A DF buffer-aided relay selection for underlay cognitive relay networks in the presence of both primary transmitter and receiver is investigated. A novel buffer aided relay selection scheme for the cognitive relay network is proposed, where the best relay is selected with the highest signal-to-interference-ratio (SIR) among all available source-to-relay and relay-to-destination links while keeping the interference to the primary destination within a certain level. A new closed-form expression for the outage probability of the proposed relay selection scheme is obtained. Both simulation and theoretical results are shown to confirm performance advantage over the conventional max-min relay selection scheme, making the proposed scheme attractive for cognitive relay networks.

5. The security of transmission in buffer-aided decode-and-forward cooperative wireless networks is considered. An eavesdropper which can intercept the data transmission from both the source and relay nodes is considered to threaten the security of transmission. Finite size data buffers are assumed to
be available at every relay in order to avoid having to select concurrently the best source-to-relay and relay-to-destination links. A new max-ratio relay selection policy is proposed to optimise the secrecy transmission by considering all the possible source-to-relay and relay-to-destination links and selecting the relay having the link which maximises the signal to eavesdropper channel gain ratio. Two cases are considered in terms of knowledge of the eavesdropper channel strengths: exact and average gains, respectively. Closed-form expressions for the secrecy outage probability for both cases are obtained, which are verified by simulations. The proposed max-ratio relay selection scheme is shown to outperform one based on max-min-ratio relay scheme.

1.6 Structure of Thesis

To simplify the understanding of this thesis and its contributions, its structure is summarised as follows:

In Chapter 1, a general introduction to cooperative communication systems was presented. Then two cooperative communication protocols and relay selection scheme were introduced. In addition, because a buffer-aided relay network has been used as an application for all the proposed schemes or selections followings, a brief introduction to relay network’s performance measurements was provided in order to better understanding the advantages of applying buffer on relays.

In Chapter 2, a literature review on current buffer-aided relay system is presented. A three-node buffer-aided relay system is first introduced. Two buffer-aided relay selection schemes are then described. Finally, a brief introduction to Markov Chain, Little’s Law, Trellis Diagram and queue, which are tools of analysis buffer-aided system, is presented.

In Chapter 3, An AF buffer-aided relay system which exploit max-link
selection is investigated. Both symmetric and asymmetric source-to-relay
and relay-to-destination channel configurations are considered. The outage
performance of the buffer-aided AF relay selection is analysed and the results
of numerical simulations shows that the outage performance gain of AF max-
link scheme over the traditional max-SNR scheme is more significant for
symmetric channels. The average packet delays for both asymmetric and
symmetric channels are analysed. Both the diversity order and coding gain
of AF max-link scheme are investigated.

In Chapter 4, A novel buffer-aided relay selection scheme which can
significantly reduce packet delay is proposed. Though applying data buffers
at the relay nodes significantly improves the data transmission of cooperative
networks, but the performance gain is often at the price of long packet delays.
The reduced packet delay is achieved by giving higher priority to select the
relay-to-destination than the source-to-relay links, so that the data queueing
lengths at the relay buffers are minimised. Both the closed-form expression
for outage probability and average packet delay are derived and verified with
numerical results. In addition, the asymptotic performance is analysed.

In Chapter 5, A novel multi-hop link selection scheme which seamlessly
integrates max-link selection and physical layer network coding is proposed.
By this approach, the average throughput is significantly improved over both
low and high SNR ranges. A new analysis tool to obtain the average through-
put of the proposed scheme is described. The closed-form expression of the
average packet delay of the proposed scheme is derived and verified with
simulations.

In Chapter 6, A buffer-aided max-ratio relay selection in the underlay
cognitive relay network is proposed. The proposed scheme provides a more
efficient way to handle the correlation among relay selection candidates than
existing approaches. The closed-form expression of the outage probability
for the proposed relay selection scheme is derived. The analysis not only
provides a deep insight in understanding the proposed scheme but also shows a potential approach to analyse similar systems in the future.

In Chapter 7, A novel max-ratio relay selection scheme is proposed for secure transmission in decode-and-forward relay networks with an eavesdropper which can intercept signals from both the source and relay nodes. Two cases are considered, which either the exact, or the average gain, for the eavesdropping channel is available. The closed-form expressions for the secrecy outage probability of the max-ratio scheme for both cases are derived.

Finally, in the last chapter which is Chapter 8, this thesis is concluded by summarising its contributions and suggestions are given for some future possible research directions.
Chapter 2

BACKGROUND OF BUFFER-AIDED COOPERATIVE COMMUNICATIONS

Cooperative communication increases the throughput and/or extend the coverage of wireless network. Because of the fixed schedule of transmission and reception for the relay, conventional cooperative networks cannot exploit the best potential of channel resources. Recent research has on the other hand found that, by introducing data buffers at the relays can significantly improve the transmission performances. Compared to conventional relaying protocols, these buffer-aided protocols provide significant gains in terms of throughput, diversity and signal-to-noise ratio.
2.1 Buffer-Aided Single-Relay Networks

The first three-node buffer-aided relay network was proposed by B. Xia et al. [30]. They considered a three-node relay network (including a source, a HD relay equipped with a buffer and a destination) in a slow fading model as is shown in Fig 2.1. The source-relay and relay-destination channels are assumed as block fading model which the channel coefficients are constant over one block but are independently varying from block to block. The relay uses the decode-and-forward protocol and there is no direct link between the source-destination.

Two buffering models, namely Fixed Buffering Relay Model and Dynamic Buffering Relay Model, are proposed for this three-node relay network.

2.1.1 Fixed Buffering Relay Model

In this model, the relay receives packets from the source in $N$ packets slots. These packets are not immediately transmitted to the destination. Instead, they are stored in the buffer in the next $N$ packet slots. If the instantaneous channel capacity between the relay and the destination is large enough for these packet slots, all the transmitted data can be successfully decoded at the destination. Otherwise some data will have to be dropped at relay. First-In-First-Out (FIFO) mode is applied in this model so that the later packets
Section 2.1. Buffer-Aided Single-Relay Networks

will stay in the buffer until the earlier packets are sent successfully.

Figure 2.2. Fixed buffering relay model (buffer size=3 packets)

For better understanding, Fig. 2.2 show how this model works when the buffer size $N = 3$ packets. The numbers in the boxes of the first and the second lines denote the instantaneous capacity (bits per channel use) for the source-relay channel and the relay-destination channel, respectively. The values in parentheses denote the related packet number. The values in the third and fourth lines denote the information (bits per channel use) for retransmission and buffering, respectively. It can also be identified which packet the information belongs to. The mean end-to-end capacity of this model is computed by averaging over all possible channel realisations of the three terminal network.

2.1.2 Dynamic Buffering Relay Model

This model is aiming to decrease the delay. It has the capability to keep each packet sent from the source for $N$ relay to destination packets slots, whereupon it is dropped if it has not been transmitted to the destination. If the instantaneous channel capacity of source-relay for one packet slot is larger than that of relay-destination for the next packet slot, the remainder
of this packet will be stored in the buffer. It will be transmitted to the destination when the instantaneous channel capacity between the relay and the destination is large enough in later packet slot, provided that it has not been dropped due to the limited buffer size of $N$ packets. Again, FIFO mode is applied to this model so the later packets will be put into the buffer unless the earlier packets in the buffer are sent successfully.

For better understanding, Fig. 2.3 shows how this model works when the buffer size $N = 3$ packets. It can be seen that when the buffer size is $N$, every packet can be stored in the buffer for a maximum of $N$ relay-destination packet slots in the dynamic buffering relay model. However, for the fixed buffering relay model, only the first packet after the dropping packet slot can achieve this maximum storage time.

B. Xia et al’s work is the first one introducing two buffering relay techniques to improve the capacity of relay networks in slow fading environments. It has shown that for any fading statistics, by using dynamic buffering relay model, which has a smaller delay that fixed buffering relay model, the capacity of relay networks is improved towards the ergodic capacity of a fast
fading channel. Although this ergodic capacity is achieved when the buffer size tends to infinity the asymptotic performance can be approached with finite and reasonable buffer sizes. However, the flexibility offered by relays with buffers has not been fully exploit since the schedule of when the source transmits and when a relay transmit is prefixed. In addition, slicing the data packet on relay based on the instantaneous channel capacity make it hard to implement on relay. Abandon the data packets which beyond the waiting threshold may cause the difficult at destination node to decode the signal transmitted from relay node.

2.2 Buffer-Aided Multi-Relay Networks

Using multiple relays to establish the communication between a source and destination allows further improvement of performance in terms of throughput and/or reliability compared to using a single relay. Many different protocols, including beamforming, space-time coding and relay selection, have been proposed for multi-relay networks. Relay selection has attracted much attention due to its good performance and simplicity of implementation. A typical relay system model is shown in Fig. 2.4, where there is one source node \( S \), one destination node \( D \) and \( N \) relay nodes \( R_k, 1 \leq k \leq N \). \( h_{sr_k}(t) \) and \( h_{rk_d}(t) \) are denoted as the frequency flat channel coefficients for \( S \rightarrow R_k \) and \( R_k \rightarrow D \) at time slot \( t \) respectively. Let \( \gamma_{sr_k} \triangleq |h_{sr_k}|^2 \frac{E_s}{N_0} \) denote the instantaneous SNR between the source and \( R_k \) and \( \gamma_{rk_d} \triangleq |h_{rk_d}|^2 \frac{E_s}{N_0} \) the instantaneous SNR between \( R_k \) and the destination. Here \( E_s \) is the energy available at the transmitting nodes and \( N_0 \) is the variance of the zero mean AWGN at the receiving nodes. As in most existing relay selection approaches, the destination node is assumed has exact channel state information (CSI) for all channels so that it can choose the best relay node for transmission.
Figure 2.4. The system model of the buffer-aided relay selection system.

A well-known relay selection protocol is best relay selection (BRS) [18]. In this protocol, for each packet transmission, the relay with the best end-to-end channels chosen for transmission out of $N$ available relays. The packet is transmitted to the selected relay in the first time slot, and the relay forwards the packets to the packet to the destination in the second time slot. Assuming the channels remain constant during both time slots. The Best relay selection policy can be analytically expressed as follows:

$$R_{\text{best}} \triangleq \arg \max_{R_k} \min \{ \gamma_{sr_k}, \gamma_{rd_k} \}. \quad (2.2.1)$$

BRS achieves a diversity gain of $N$. However, the BRS protocol is in general not able to simultaneously exploit the best available source-relay and relay-destination channels as the selected relay does not generally experience the best source-relay and relay-destination channels at the same time. Thus the significant performance gains can be achieved by equipping relays with buffers and exploiting the resulting additional degrees of freedom for relay
2.2. Buffer-Aided Multi-Relay Networks

2.2.1 Max-Max Relay Selection (MMRS)

The max-max relay selection [34] are proposed by A. Ikhlef et al. and is the first relay selection scheme with comprehensive diversity analysis. Given that the relay nodes are equipped with data buffers and can store the data received from the source, the max-max policy splits the relay selection decision in two parts and selects the relay with the best source-relay link for reception and the relay with the best relay-destination link for transmission. The max-max selection policy follows the conventional two-slot cooperative transmission where the first slot is dedicated for the source transmission and the second slot for the relaying transmission, but the relay node may not be the same for both phases of the protocol. The max-max relay selection policy can be written as

\[ R_{\text{receive}}^{\text{best}} = \arg \max_{R_k} \{ \gamma_{sr_k} \} \]
\[ R_{\text{transmit}}^{\text{best}} = \arg \max_{R_k} \{ \gamma_{rd_k} \}, \]  

(2.2.2)

where \( R_{\text{receive}}^{\text{best}} \) and \( R_{\text{transmit}}^{\text{best}} \) denote the relay selected for the first phase and the second phase of the cooperative protocol, respectively.

It has been proven that the max-max relay selection policy achieve full diversity equal to the number of the relays and provides a significant coding gain in comparison to the conventional max-min selection scheme. However, it is worth noting that for the above selection strategy to work properly, the buffer of no relay can be empty or full at any time such that all relays always have the option of receiving and transmitting. Obviously, for buffers of finite size this may not be possible since a buffer may become empty (full) if, for example, a relay enjoys repeatedly the best relay-to-destination link (source-to-relay) link. To overcome this issue, the hybrid relay selection scheme is
introduced next.

### 2.2.2 Hybrid Relay Selection

Since max-max relay selection only take into account the status of the channel, it can not avoid buffer over- and underflows. Authors in [34] proposed Hybrid Relay Selection (HRS) scheme to handle the scenario that the buffer of the relay selected for reception is full or the buffer of the relay selected from transmission is empty. HRS is a combination of MMRS when the buffer of the relay selected is neither full nor empty and BRS otherwise. Assume that every relay is equipped with data buffer $Q_k (1 \leq k \leq N)$ of finite size $L$. Let $\Psi(Q_k)$ denote the number of data packets in buffer $Q_k$. It is clear that $0 \leq \Psi(Q_k) \leq L$. Thus for HRS, the best relay for reception, $R_{\text{receive}}^{\text{receive best}}$, is selected according to

$$
R_{\text{receive}}^{\text{receive best}} = \begin{cases} 
R_{\text{best}}; & \text{if } \Psi(Q_k) = L - 1 \text{ or } \Psi(Q_k) = 0, \\
R_{\text{receive}}^{\text{receive best}}; & \text{otherwise},
\end{cases}
$$

and the best relay for transmission, $R_{\text{transmit}}^{\text{transmit best}}$, is selected according to

$$
R_{\text{transmit}}^{\text{transmit best}} = \begin{cases} 
R_{\text{best}}; & \text{if } \Psi(Q_k) = L - 1 \text{ or } \Psi(Q_k) = 0, \\
R_{\text{transmit}}^{\text{transmit best}}; & \text{otherwise},
\end{cases}
$$

where $R_{\text{best}}, R_{\text{receive}}^{\text{receive best}}$ and $R_{\text{transmit}}^{\text{transmit best}}$ are defined in (2.2.1) and (2.2.2), respectively.

The diversity of the HRS scheme is equal to the relay number $N$, since in every slot, the available links for both the source-to-relay and relay-to-destination transmission is $N$.

A. Ikhlef et al. were the first to investigate buffer-aided relay selection schemes. They proposed two new relay selection schemes, MMRS and HRS. Since the assumption that buffers at the relays are neither full nor empty for
MMRS is not practical for finite buffers, they proposed HRS. They used as a Markov chain to analyse the outage probability performance of the system. It’s found that while two schemes have the same diversity gain as BRS, they achieve coding gain up to 3dB. This work indicates that relays with buffers add additional flexibility in cooperative diversity systems.

### 2.2.3 Max-Link Selection

A limitation in above schemes comes from the fact that they still follow the traditional schedule that the time slots for the source and relay transmission are fixed. In order to relax this limitation, Krikidis et al. [33] proposed the max-link scheme, which allows each slot to be allocated dynamically to the source-relay or relay-destination transmission, according to the instantaneous quality of the links and the status of the relays’ buffers. More specifically, the max-link relay selection scheme fully exploit the flexibility offered by the buffers at the relay nodes and at each time selects the strongest link for transmission among all available links. A source-relay link is considered to be available when the corresponding relay node is not full and therefore can receive data from the source, while a relay-destination link is considered to be available when the relay node is not empty and thus can transmit sources data towards the destination. The proposed scheme always selects the strongest link at any time. If a source-relay link is the strongest link, the source transmits and the corresponding relay is selected for reception; on the other hand, if a relay-destination link is the strongest link, the corresponding relay is selected to forward data to the destination.

The max-link relay selection scheme is given by:

$$R_{\text{best}} = \arg \max_{R_k} \left\{ \bigcup_{R_k: \Psi(Q_k) \neq 0} \{ \gamma_{sr_k} \}, \bigcup_{R_k: \Psi(Q_k) \neq 0} \{ \gamma_{rd} \} \right\}$$  \hspace{1cm} (2.2.5)
The diversity of the max-link scheme, due to the selection of the best link among both the source-to-relay and relay-to-destination links, is of order between $N$ and $2N$. The diversity order depends on both the number of relays $N$ and the buffer size $L$, with the limit values reached for buffer sizes $L = 1$ and $L \to \infty$ respectively.

Unlike the MRRS and HRS approaches, the max-link relay selection policy for cooperative networks with finite buffers fully exploits the buffering capability at the relays and schedules transmission only through the strongest available channel link. The max-link scheme outperforms non-buffer-aided schemes by providing not only coding gain but also diversity gain especially for large buffer sizes. However, the max-link relay selection scheme only works as an efficient cooperative technique for systems with no latency constraints. Particularly, under asymmetric channel configurations that the average channel gain of source-relays links are lower than that of relays-destination links, the latency for average packet will significant increase.

2.3 Challenges and Opportunities

Buffering is a promising solution for cooperative networks and motivates the investigation of new protocols and relay selection schemes. However, most existing approaches have been mainly for DF relay systems. Though it is usually harder to analyse the probability distribution of the end-to-end SNR at the destination for AF protocol than that of DF counterpart, AF protocol retains the advantage of no decoding at the relays which makes it particularly attractive in many application such as mobile relays that are not always allowed to decode the source message. In Chapter 3 of this thesis, the outage performance of an AF buffer-aided relay system is comprehensively studied.
In addition, delay is a key issue in buffer-aided relaying protocols. Most of the buffer-aided relaying protocols in the literature can only be applied in systems without delay constraint [31–33, 35]. In practical, application including video streaming and web browsing, the delay must be limited. In Chapter 4 of this thesis, a novel buffer-aided relay selection scheme with significantly reduced packet delay is proposed.

Buffer-aided relaying has been extended to many different protocols. The authors in [36] proposed a buffer-aided protocol for two-way relaying, where at each time slot one of the three transmission phases is optimally selected such that the sum rate at both destinations is maximised, which leads to significant performance gains compared to conventional time-division broadcast (TDBC). In [31], the authors proposed a buffer-aided adaptive link selection protocol where the source and relay adapt the data rates at each time slot to maximise the overall throughput of the three-node relay network. In [35], the authors extended the buffer-aided adaptive link selection protocol to a three-node network employing BICM-OFDM where the frequency-selective fading channels were considered.

In this thesis, buffer-aided relay schemes are applied to different applications including multi-hop networks, cognitive networks and security networks. To be specific, Chapter 5 studies buffer-aided multi-hop system and shows that applying buffer-aided relaying and network-coding can significantly increase the system throughput. Chapter 6 investigates buffer-aided relay selection for underlay cognitive relay networks and Chapter 7 studies physical layer secrecy in buffer-aided cooperative wireless networks.

2.4 Summary

This chapter presented an overview of various buffer-aided relay schemes. A brief introduction of the three-node buffer-aided network with fixed buffering
relay model and dynamic buffering relay model was given. Three popular buffer-aided relay selection schemes were discussed. Finally, several challenges and opportunities of applying buffer-aided approaches are presented.
This chapter investigates the outage performance of an AF relay system which exploits buffer-aided max-link relay selection. Both asymmetric and symmetric source-to-relay and relay-to-destination channel configurations are considered. The closed-form expressions for the outage probability is derived, and the average packet delays is analysed. The diversity order is proven in range $N$ and $2N$ (where $N$ is the relay number), corresponding to a relay buffer size between 1 and $\infty$ respectively. The coding gain has also been analytically shown. Numerical results are given to verify the theoretical analyse.

3.1 Introduction

Relay selection can be applied in either non-regenerative (i.e. AF) or regenerative (i.e. DF) relay systems [37]. The max-min relay selection is often considered as an optimum DF relay selection scheme, in which the
best relay is selected with the highest gain among all of the minima of the source-to-relay and relay-to-destination channel gain pairs [18]. Although max-min schemes achieve diversity order of $N$ (where $N$ is the number of available relays), their performance is practically limited by the constraint that the best source-to-relay and relay-to-destination links for a packet transmission must be determined concurrently. Recent research has on the other hand found that, by introducing data buffers at the relays, this constraint can be relaxed to yield significant performance advantage in practical systems [31–34, 36, 38, 39].

An early example of buffer-aided relay selection is the max-max scheme [38]. In max-max relay selection, at one time slot $t$, the best link among all source-to-relay channels is selected, and a data packet is sent to the selected relay and stored in the buffer. At the next time slot $t+1$, the best link among all relay-to-destination channels is selected, and the selected relay (which is often not the same relay selected at time $t$) forwards one data packet from its buffer to the destination. In this way, the strongest links from both source-to-relay and relay-to-destination group channels are always selected so that it has significant coding gain over the traditionary max-min scheme.

The max-max relay selection approach still follows the traditional transmission order in which the source-to-relay and relay-to-destination transmissions always carry on in an alternative manner, with a diversity order of $N$ which is the same as that for the max-min scheme. In the recent max-link approach [31,33], this constraint on the transmission order is further relaxed so that, at any time, a best link is selected among all available source-to-relay and relay-to-destination links. Depending on whether a source-to-relay or a relay-to-destination link is selected, either the source transmits a packet to the selected relay or the selected relay forwards a stored packet to the destination. It is shown in [33] that the max-link relay selection not only has coding gain over the max-min scheme, but also has higher diversity order.
than both the *max-min* and *max-max* schemes. In particular, the diversity order can approach $2N$ when the relay buffer size is large enough.

While the buffer-aided relay selection describes a promising way forward in cooperative networks, existing approaches have been mainly for DF relay systems (e.g. [31–34, 36, 38, 39]). This naturally prompts the following two questions:

- *Is it necessary or not to apply buffer-aided relay selection in an AF relay network?* In the AF system, the relay simply amplifies and forwards the received signal to the destination. Because the AF does not decode the received packets, it is not only easier to implement but also has higher level of security than a DF system [40]. When data buffers are applied at the relays, another difference between DF and AF is that after “decoded digital data” or “received real signals” are stored in the buffers respectively. This brings up two implementation issues: quantisation and data storage. It is interesting to point out that because the relay works in the half-duplex mode, that is it receives a data packet at one time slot and forwards it out at another slot, a data buffer (of size 1) actually exists even in the traditional AF or DF relay systems. In order to store the data in the buffer, quantisation is always necessary for both AF and DF systems, no matter whether the buffers are used or not. Compared to its DF counterpart, therefore, buffer-aided AF relay selection has the extra implementation cost of storing quantized “real signals”, but it retains the advantage of no decoding at the relays, making it particularly attractive in many applications such as mobile relays which are not always allowed to decode the source messages.

- *How is the buffer-aided relay selection applied in AF cooperative networks?* In traditional AF relay selection, the best relay is selected
with the highest end-to-end SNR at the destination [41], which is
termed as the AF \textit{max-SNR} scheme in this chapter. When the AF
relays are equipped with data buffers, however, the traditional max-
SNR or its variants (e.g. [42–44]) cannot be used. This is because the
source-to-relay and relay-to-destination links are selected separately
which implies that the end-to-end SNR at the destination cannot be
obtained instantaneously. In this chapter, building upon traditional
relay selection in DF relay selection schemes, such as the max-min
scheme which may also be applied in an AF system (e.g. [45]), the DF
max-link approach is proposed to be applied in the AF buffer-aided
relay selection.

Of particular importance is the outage probability of the buffer-aided
AF relay selection system. In a DF system, generally, the outage probability
for the source-to-relay and relay-to-destination transmission can be obtained
separately and then combined to give the overall outage probability. In
contrast, the outage performance of an AF relay system depends on the
probability distribution of the end-to-end SNR at the destination, making it
usually harder to analyse than that of its DF counterpart. Particularly, as
when a relay buffer is introduced in the AF relays, the best source-to-relay
and relay-to-destination links for a packet transmission must be determined
at different times, thus they are generally selected from different numbers
of available links. As a result, the distribution of the end-to-end SNR no
longer follows the form of the MacDonald distribution as in traditional AF
\textit{max-SNR} relay selection [41]. This makes the outage performance of the
buffer-aided AF relay selection much more difficult to analyse than both
the traditional max-SNR scheme and the buffer-aided DF max-link scheme.
This is perhaps the main reason that AF buffer-aided relay selection has not
been well studied.

In this chapter therefore, buffer-aided AF max-link relay selection is care-
fully investigated. Unlike existing buffer-aided relay selection approaches (e.g. [34, 36, 39]), this chapter considers both symmetric and asymmetric channel configurations allowing the average gains for the source-to-relay and relay-to-destination channels to be different. Although the asymmetric channel assumption makes the analysis even more difficult, it represents a more practical scenario so that the results provides an important basis for new system design. The main contributions of this chapter are summarized as follows:

- Analyzing the outage probability of the AF max-link scheme for both asymmetric and symmetric channel configurations. Based on literature review, this is the first time asymmetric channels have been considered in buffer-aided relay selection, and the outage probability was been derived in closed-form for an AF buffer-aided relay selection scheme. Numerical simulations are used to verify the analysis. The results show that the outage performance gain of the AF max-link scheme over the traditional max-SNR scheme is more significant for symmetric channels. This gives important insight for designing buffer-aided relay systems: for example, power control at the source and relay nodes may be used to achieve symmetric channel configuration for better outage performance.

- Analyzing the average packet delays for both asymmetric and symmetric channels. The results show that when the relay-to-destination channels are stronger than the source-to-relay channels, the AF buffer-aided relay system introduces less delay. Therefore, the “best” delay and outage performance requires different channel conditions. This actually arises an interesting design topic for future study: how the delay and outage performance can be jointly optimised.

- Proving that the diversity order of the AF max-link relay selection
scheme is between $N$ and $2N$ (where $N$ is the number of relays), and the lower and upper diversity limits are reached when the relay buffer size $L$ is $1$ and $\infty$ respectively.

- Analytically showing the coding gain of the AF max-link scheme compared to the traditional AF max-SNR schemes.

The remainder of this chapter is organized as follows: Section 3.2 describes buffer-aided AF max-link relay selection; Section 3.3 derives the closed-form expressions for outage probability; Section 3.4 analyses the average packet delay; Section 3.5 studies the diversity order; Section 3.6 shows the coding gain; Section 3.7 includes numerical simulations to verify the analyse; finally Section 3.8 summarizes and concludes the chapter.

### 3.2 AF max-link relay selection

The system model of buffer-aided AF relay selection is shown in Fig. 3.1, where there is one source node $(S)$, one destination node $(D)$ and $N$ relay nodes $(R_k, 1 \leq k \leq N)$. All nodes operate in the half-duplex mode, that is they do not transmit and receive simultaneously. Each relay is equipped with a data buffer $Q_k$ $(1 \leq k \leq N)$ of finite size $L$ (in the number of data packets). The data packets in the buffer obey the “first-in-first-out” rule.

In this chapter, assuming no direct transmission link between the source and destination nodes\(^1\). Let $h_{sr_k}(t)$ and $h_{rk_d}(t)$ denote the frequency flat channel coefficients for $S \rightarrow R_k$ and $R_k \rightarrow D$ at time slot $t$ respectively. All channel coefficients are assumed independent and quasi-static Rayleigh fading such that they remain unchanged during one packet duration but independently vary from one packet time to another. The average $S \rightarrow R_k$

\(^1\)Including the direct link has little effect on the relay selection which is the main issue in this chapter.
Figure 3.1. The system model of the buffer-aided AF relay selection.

and $R_k \to D$ channel gains are assumed as

$$E[|h_{sr_k}(t)|^2] = \sigma_{h_{sr}}^2, \quad E[|h_{rd}(t)|^2] = \sigma_{h_{rd}}^2, \quad \text{for all } k,$$

respectively. It worth noting that, while all channels for $S \to R_k$ and $R_k \to D$ are i.i.d. respectively, symmetric channel configuration that is $\sigma_{h_{sr}}^2 = \sigma_{h_{rd}}^2$ is beyond the assumption on this chapter. Without losing generality, the noise variances assumed at all receiving nodes ($R_k$ and $D$) as the same. As in most existing relay selection approaches, the destination node is assumed to has exact CSI for all channels so that it can choose the best relay node for transmission\(^2\).

In max-link relay selection, the best transmission link is chosen with the highest channel SNR among all available source-to-relay and relay-to-destination links. A source-to-relay link is considered available when the buffer of the corresponding relay node is not full, and a relay-to-destination link is available when the corresponding relay buffer is not empty. If a source-to-relay link is selected, the source node transmits one data packet to the corresponding relay node, and the relay receives and stores the data

\(^2\)While the CSI is normally estimated with pilot symbols or channels, this detail is beyond the scope of this chapter.
packet in its buffer\(^3\). The number of data packets in the buffer is then increased by one. On the other hand, if a relay-to-destination link is selected, the corresponding relay transmits the earliest stored packet in the buffer to the destination, and the number of packets in the buffer is decreased by one. In general, the best selected relay node \( R_{\text{best}} \) (for either reception or transmission) can be expressed as

\[
R_{\text{best}} = \arg \max \bigg\{ \bigcup_{R_k : \Psi(Q_k) \neq L} \{ |h_{sr_k}|^2 \}, \bigcup_{R_k : \Psi(Q_k) \neq 0} \{ |h_{rd_k}|^2 \} \bigg\}, \tag{3.2.2}
\]

where \( \Psi(Q_k) \) gives the number of data packets in the buffer \( Q_k \).

Without losing generality, at time slot \( t \), \( S \rightarrow R_k \) is assumed as the strongest link so that the source transmits data packet \( s(t) \) to the relay \( R_k \). The received signal at \( R_k \) is given by

\[
y_{sr_k}(t) = \sqrt{P_s} h_{sr_k}(t) s(t) + n_{r_k}(t), \tag{3.2.3}
\]

where \( P_s \) is the average transmission power at the source and \( n_{r_k}(t) \) is the AWGN at \( R_k \) with mean zero and variance \( \sigma^2 \).

Then \( y_{sr_k}(t) \) is stored into the buffer \( Q_k \) and waits for its turn to be transmitted. Assuming that at the next \( \tau \)-th time slot, \( y_{sr_k}(t) \) is forwarded from \( R_k \) to the destination node. It is clear that \( \Psi(Q_k(t)) \leq \tau < \infty \), where \( \Psi(Q_k(t)) \) gives the number of data packets in the buffer \( Q_k \) at time \( t \). Since the relays exploit AF, at the time slot \( (t + \tau) \), the received signal at destination is given by

\[
y_{rd_k}(t + \tau) = \sqrt{P_{R_k}(t + \tau)} h_{rd_k(t + \tau)} y_{sr_k}(t) + n_d(t + \tau), \tag{3.2.4}
\]

\(^3\)The received signal needs to be quantized before it is stored in the buffer. As was mentioned in the introduction, quantization exists in any half-duplex relaying, either AF or DF, with or without buffers. The quantization noise can either be ignored or absorbed in the channel noise.
where \( n_d(t + \tau) \) is the noise at the destination node with mean zero and variance \( \sigma^2 \), and \( P_{R_k}(t + \tau) \) is the relay gain at \( R_k \) which is given by

\[
P_{R_k}(t + \tau) = \frac{P_s}{P_s|h_{sr_k}(t)|^2 + \sigma^2}, \tag{3.2.5}
\]

where all relay nodes are assumed to have the same average transmission powers as the source node, namely \( P_s \).

Substituting (3.2.3) into (3.2.4) gives

\[
y_{rd}(t + \tau) = \sqrt{P_s} \sqrt{P_{R_k}(t + \tau)h_{rd}(t + \tau)h_{sr_k}(t)s(t)} + n_d(t + \tau) + n'_{R_k}(t), \tag{3.2.6}
\]

where \( n'_{R_k}(t) = \sqrt{P_{R_k}(t + \tau)h_{rd}(t + \tau)n_{rk}(t)} \).

The outage performance of the buffer aided AF relay system will be derived at the next section.

### 3.3 Outage performance

The outage probability for the AF relay system can be defined as the probability that the instantaneous end-to-end SNR at the destination, \( \gamma_d \), falls below a certain target SNR \( \gamma_{th} \) such that

\[
P_{out} = P(\gamma_d \leq \gamma_{th}), \tag{3.3.1}
\]

where \( P(\cdot) \) denotes the probability of an event. The Markov chain is used to model the transitions between the states of the buffers, where the states describe the number of data packets at every buffer [33]. There are \((L+1)^N\) states in total, and the \( l^{th} \) state is expressed as

\[
s_l = [\Psi_l(Q_1), \ldots, \Psi_l(Q_N)], \quad l = 1, \ldots, (L+1)^N, \tag{3.3.2}
\]
where $\Psi_l(Q_k)$ gives the number of data packets in buffer $Q_k$ at state $s_l$. It is clear that $0 \leq \Psi_l(Q_k) \leq L$.

Suppose at time $t$, the state is at $s_j$. At time $t + 1$, if a source-to-relay link is selected, a packet is transmitted to the selected relay and the number of packets in the corresponding data buffer is increased by 1. On the other hand, if a relay-to-destination link is selected, a packet in the selected relay is forwarded to the destination. Then at the destination, suppose that if the packet can be successfully decoded, it is stored at the destination, or otherwise is discarded. In either case, the number of packets in the selected relay’s buffer is decreased by 1. Thus depending on which relay receives or transmits data, at time $t + 1$, the buffers may move from state $s_j$ to several possible states. Let $A$ denote the $(L + 1)^N \times (L + 1)^N$ state transition matrix, where the entry $A_{i,j} = P(X_{t+1} = s_i|X_t = s_j)$ which is the transition probability to move from state $s_j$ at time $t$ to state $s_i$ at time $(t + 1)$.

Suppose that when the data packet $s(t)$ is transmitted from the source to the destination through the best selected relay $R_k$, the strongest source-to-relay and relay-to-destination links are selected when the buffer state is at $s_i$ and $s_j$ respectively. It then follows from (3.2.6) that, the instantaneous end-to-end SNR at the destination for receiving $s(t)$ is obtained as

$$\gamma_d^{(s_i,s_j)}(t + \tau) = \frac{\gamma_{sr}^{(s_i)}(t) \gamma_{rd}^{(s_j)}(t + \tau)}{\gamma_{sr}^{(s_i)}(t) + \gamma_{rd}^{(s_j)}(t + \tau) + 1}. \tag{3.3.3}$$

where $\gamma_{sr}^{(s_i)}(t)$ and $\gamma_{rd}^{(s_j)}(t + \tau)$ are the instantaneous SNRs for the $S \rightarrow R_k$ and $R_k \rightarrow D$ links at time $t$ and $t + \tau$ respectively, and the superscripts $(s_i)$ and $(s_j)$ denote that the corresponding best links are selected when the buffer state is at $s_i$ and $s_j$ respectively. Because all channels are assumed

---

4 The discarded packet may need to be retransmitted. For example, in the TCP/IP protocol, the re-transmission is handled in the transport layer. The detailed implementation issue is beyond the scope of this chapter.
at all times are independent fading, for clearer exposition, the time indices \( t \) and \( \tau \) are ignored unless otherwise necessary in the rest of the chapter.

By considering all possible states for \( s_i \) and \( s_j \), the outage probability of the max-link AF relay selection is given by

\[
P_{\text{out}} = \sum_{s_i} \sum_{s_j} P(s_i) P(s_j) P(\gamma_{d}^{(s_i,s_j)} < \gamma_{th}),
\]

where \( P(s_i) \) and \( P(s_j) \) are the probabilities that the buffer state is at \( s_i \) and \( s_j \) respectively.

Below the derivation of \( P(\gamma_{d}^{(s_i,s_j)} < \gamma_{th}) \) and \( P(s_i) \) are shown.

### 3.3.1 \( P(\gamma_{d}^{(s_i,s_j)} < \gamma_{th}) \)

Suppose at one time the strongest link is selected when the buffer state is at \( s \). The buffer state \( s \) uniquely corresponds to a pair of \( \{K_{sr}^{(s)}, K_{rd}^{(s)}\} \), where \( K_{sr}^{(s)} \) and \( K_{rd}^{(s)} \) are the numbers of the available source-to-relay and relay-to-destination links respectively. Recall that a source-to-relay or relay-to-destination link is considered as “unavailable” if the buffer of the corresponding relay node is full or empty respectively.

Because all channels are assumed to be independently Rayleigh fading, the instantaneous SNR for every channel, \( \gamma_w \ (w \in \{sr_k, rd_k\}) \), is independently exponentially distributed. Then based on the theory of order statistics [46], the cumulative distribution function (CDF) of the selected channel gain, \( \gamma_w^{(s)} \), is given by

\[
F_{\gamma_w^{(s)}}(x) = (1 - e^{-\frac{x}{\bar{\gamma}_{sr}}})^{K_{sr}^{(s)}} (1 - e^{-\frac{x}{\bar{\gamma}_{rd}}})^{K_{rd}^{(s)}}, \quad w \in \{sr_k, rd_k\},
\]

where \( \bar{\gamma}_{sr} = \frac{P_s \sigma^2_{hr}}{\sigma^2_{hr}} \) and \( \bar{\gamma}_{rd} = \frac{P_s \sigma^2_{hr}}{\sigma^2_{hr}} \) which are the average SNR-s for the source-to-relay and relay-to-destination channels respectively. Differentiating (3.3.5) with respect to \( x \) gives the probability density function (PDF) of
\[ f_{\gamma_w}(x) = \frac{K_{sr}}{\gamma_w} e^{-\frac{x}{\gamma_w}} (1 - e^{-\frac{x}{\gamma_w}})^{K_{sr} - 1}(1 - e^{-\frac{x}{\gamma_rd}})^{K_{rd}} \]
\[ + \frac{K_{rd}}{\gamma_rd} e^{-\frac{x}{\gamma_rd}} (1 - e^{-\frac{x}{\gamma_rd}})^{K_{rd} - 1}(1 - e^{-\frac{x}{\gamma_sr}})^{K_{sr}}, \quad w \in \{sr_k, rd_k\}. \]

(3.3.6)

Supposing the strongest source-to-relay and relay-to-destination links are selected when the buffer state is at \( s_i \) and \( s_j \) respectively, and because all channels are assumed to be mutually independent, thus

\[ f_{\gamma(s_i),s_j}(x, y) = f_{\gamma(s_i)}(x) f_{\gamma(s_j)}(y), \quad \text{ (3.3.7)} \]

and

\[ P(\gamma_d(s_i, s_j) \leq \gamma_{th}) = \int \int_{x+y+\gamma_d < \gamma_{th}} f_{\gamma(s_i),s_j}(x, y) \, dx \, dy, \quad \text{ (3.3.8)} \]

which becomes

\[ P(\gamma_d(s_i, s_j) < \gamma_{th}) = 1 + \sum_{m,n} \sum_{(m,n) \neq (0,0)} C^m_{K_{sr}} C^n_{K_{rd}} (-1)^{m+n}2^n e^{-M_2 \gamma_{th}} \sqrt{M_4 M_{\gamma_{th}}} \]
\[ \cdot \left[ K_{sr}^{-1} K_{rd}^{-1} \sum_{a_1=0}^{K_{sr}} \sum_{a_2=0}^{K_{rd}} (-1)^{a_1+a_2} C^{a_1}_{K_{sr}} C^{a_2}_{K_{rd}} \frac{e^{-M_1 \gamma_{th}}}{\sqrt{M_1}} B(1, 2 \sqrt{M_1 M_4 M_{\gamma_{th}}}) \right] \]
\[ + K_{sr}^{-1} K_{rd}^{-1} \sum_{a_3=0}^{K_{sr}} \sum_{a_4=0}^{K_{rd}} (-1)^{a_3+a_4} C^{a_3}_{K_{sr}} C^{a_4}_{K_{rd}} \frac{e^{-M_2 \gamma_{th}}}{\sqrt{M_2}} B(1, 2 \sqrt{M_2 M_4 M_{\gamma_{th}}}) \],

(3.3.9)
where

\[
M_1 = \frac{1}{\gamma_{sr}} + \frac{a_1}{\gamma_{sr}} + \frac{a_2}{\gamma_{rd}}, \quad M_2 = \frac{1}{\gamma_{rd}} + \frac{a_3}{\gamma_{rd}} + \frac{a_4}{\gamma_{sr}}, \quad M_4 = \frac{m}{\gamma_{sr}} + \frac{n}{\gamma_{rd}}, \quad M_{\gamma_{th}} = \gamma_{th}(\gamma_{th} + 1),
\]

\((3.3.10)\)

and \(B\) denotes the modified Bessel function of the second kind \([47]\) and \(C_N^M = M!/\left[N!(M - N)!\right]\) denotes the binomial coefficient.

Since the integration area of \((3.3.8)\) is closed by the curve \(\frac{\gamma_{th}(x+1)}{x-\gamma_{th}}, \ x \geq 0\) axis and \(y \geq 0\) axis, the integration can be split into three parts as

\[
P(\gamma_d(x, y) \leq \gamma_{th}) = \int_{\gamma_{th}}^{\gamma_{th}} \int_{0}^{\gamma_{th}} f_{\gamma_{s}(x), \gamma_{d}(y)}(x, y) \, dx \, dy \bigg| \begin{array}{c}
A \\
B \\
C
\end{array} \]

\((3.3.11)\)

Parts \(A\) and \(B\) can be obtained as

\[
A = \int_{\gamma_{th}}^{\gamma_{th}} \int_{0}^{\gamma_{th}} f_{\gamma_{s}(x), \gamma_{d}(y)}(x, y) \, dx \, dy \\
= \left[1 - F_{\gamma_{s}(x)}(\gamma_{th})\right] F_{\gamma_{d}(y)}(\gamma_{th})
\]

\((3.3.12)\)

\[
B = \int_{0}^{\gamma_{th}} \int_{\gamma_{th}}^{\infty} f_{\gamma_{s}(x), \gamma_{d}(y)}(x, y) \, dx \, dy \\
= F_{\gamma_{s}(x)}(\gamma_{th})
\]
respectively. Part $C$ can be further divided into parts $C_1$ and $C_2$ as

$$
C = \int_{\gamma_{th}}^{\infty} f_{\gamma_{th}}(y) \int_{\gamma_{th}}^{\gamma_{th}(y+1)} y^{-\gamma_{th}} f_{\gamma_{th}}(x) \, dx \, dy
$$

$$
= \int_{\gamma_{th}}^{\infty} f_{\gamma_{th}}(y) F_{\gamma_{th}}(\gamma_{th}(y+1)) \, dy - \left[ 1 - F_{\gamma_{th}}(\gamma_{th}) \right] F_{\gamma_{th}}(\gamma_{th}).
$$

(3.3.13)

Noticing $C_2$ is equal to part $A$, and part $C_1$ can be calculated as follows.

First applying a binomial expansion for $F_{\gamma_{th}}(\gamma_{th}(y+1))$ yields

$$
F_{\gamma_{th}}(\gamma_{th}(y+1)) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} C_{K_{s,r}}^{m} C_{K_{r,d}}^{n} (-1)^{m+n} e^{-m \gamma_{th} y^{-\gamma_{th}}} e^{-n \gamma_{th} y^{-\gamma_{th}}} e^{-M_{4} \gamma_{th}(y+1)}
$$

(3.3.14)

Let $M_{4} = \frac{m}{\gamma_{sr}} + \frac{n}{\gamma_{rd}}$ and substitute (3.3.14) into part $C_1$ gives

$$
C_1 = \left[ 1 - F_{\gamma_{th}}(\gamma_{th}) \right] F_{\gamma_{th}}(\gamma_{th}).
$$

(3.3.15)
which is further split into another two parts as $C_{11}$ (for $(m, n) = (0, 0)$) and $C_{12}$ (for $(m, n) \neq (0, 0)$) respectively. Noticing $C_{11}$ is actually equal to $1 - B$ as is shown in (3.3.12).

Applying a binomial expansion for $f_{\gamma_{rkd}}(y)$ gives

$$
f_{\gamma_{rkd}}(y) = \sum_{a_1=0}^{K_{sr}} \sum_{a_2=0}^{K_{rd}} C_{a_1}^{s_{r}} C_{a_2}^{d_{k}} (-1)^{a_1+a_2} \frac{K_{r}^{(s_{j})}}{\bar{r}_{sr}} e^{-M_{1}y} \tag{3.3.16}
$$

$$
+ \sum_{a_3=0}^{K_{rd}} \sum_{a_4=0}^{K_{sr}} C_{a_3}^{d_{k}} C_{a_4}^{s_{r}} (-1)^{a_3+a_4} \frac{K_{r}^{(d_{k})}}{\bar{r}_{rd}} e^{-M_{2}y}
$$
where \( M_1 = \frac{1}{\gamma_{sr}} + \frac{a_1}{\gamma_{sr}} + \frac{a_2}{\gamma_{rd}} \) and \( M_2 = \frac{1}{\gamma_{rd}} + \frac{a_3}{\gamma_{rd}} + \frac{a_4}{\gamma_{sr}} \). Thus for \( C_{12} \),

\[
C_{12} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} C_{K_{sr}^{(s)}}^{m} C_{K_{rd}^{(s)}}^{n} (-1)^{m+n}.
\]

\[
= \sum_{(m,n) \neq (0,0)} \sum_{a_1=0}^{\infty} \sum_{a_2=0}^{\infty} \left( \frac{K_{sr}^{(s)} - K_{rd}^{(s)}}{\gamma_{sr}} \right) (-1)^{a_1+a_2} \frac{K_{sr}^{(s)}}{\gamma_{sr}} \cdot B(1, 2\sqrt{M_1 M_1 M_{\gamma_{th}}})
\]

\[
+ \sum_{a_3=0}^{\infty} \sum_{a_4=0}^{\infty} \left( \frac{K_{rd}^{(s)} - K_{sr}^{(s)}}{\gamma_{rd}} \right) (-1)^{a_3+a_4} \frac{K_{rd}^{(s)}}{\gamma_{rd}} \cdot B(1, 2\sqrt{M_2 M_2 M_{\gamma_{th}}})
\]

\[
= \sum_{(m,n) \neq (0,0)} \sum_{a_1=0}^{\infty} \sum_{a_2=0}^{\infty} \left( \frac{K_{sr}^{(s)} - K_{rd}^{(s)}}{\gamma_{sr}} \right) (-1)^{a_1+a_2} \frac{K_{sr}^{(s)}}{\gamma_{sr}} \cdot B(1, 2\sqrt{M_1 M_1 M_{\gamma_{th}}})
\]

\[
+ \sum_{a_3=0}^{\infty} \sum_{a_4=0}^{\infty} \left( \frac{K_{rd}^{(s)} - K_{sr}^{(s)}}{\gamma_{rd}} \right) (-1)^{a_3+a_4} \frac{K_{rd}^{(s)}}{\gamma_{rd}} \cdot B(1, 2\sqrt{M_2 M_2 M_{\gamma_{th}}})
\]

where \( M_{\gamma_{th}} = \gamma_{th}(\gamma_{th} + 1) \) and \( B \) denotes the modified Bessel function of the second kind [47].
3.3.2 \( P(s_l) \)

Because the average channel gains for the \( S \rightarrow R_k \) and \( R_k \rightarrow D \) links are not the same, at any time the probabilities to select the source-to-relay and relay-to-destination transmission are also not the same. This is very different from existing buffer-aided relay selection schemes (e.g. the max-link approach in [33]) where the selection of any available link is equally likely. With this observation, divide all states which can be moved from \( s_l \) into two sets as

\[
U_{s_l}^{S \rightarrow R} = \left\{ \bigcup_{1 \leq n \leq (L+1)^N} s_n : s_n - s_l \in Q^{S \rightarrow R} \right\}
\]

\[
U_{s_l}^{R \rightarrow D} = \left\{ \bigcup_{1 \leq n \leq (L+1)^N} s_n : s_n - s_l \in Q^{R \rightarrow D} \right\},
\]

(3.3.19)

where \( Q^{S \rightarrow R} \triangleq \{ \bigcup_{1 \leq k \leq K} + e_k \} \), \( Q^{R \rightarrow D} \triangleq \{ \bigcup_{1 \leq k \leq K} - e_k \} \), and \( e_k \) is the vector where its \( k \)-th element is one and all other elements are zero. To be specific \( U_{s_l}^{S \rightarrow R} \) contains all states to which \( s_l \) can move when a source-to-relay link...
is selected and $U_{s_l}^{R\rightarrow D}$ contains all states to which $s_l$ can move when a relay-to-destination link is selected. Let $p_{S\rightarrow R}^{(s_l)}$ and $p_{R\rightarrow D}^{(s_l)}$ be the probabilities that the source-to-relay and relay-to-destination transmissions are selected at state $s_l$, respectively. It is clear that $p_{S\rightarrow R}^{(s_l)} + p_{R\rightarrow D}^{(s_l)} = 1$.

On the other hand, because all source-to-relay channels are assumed i.i.d. fading and all relay-to-destination channels are also i.i.d. fading, the selection of one particular link within either $U_{s_l}^{S\rightarrow R}$ or $U_{s_l}^{R\rightarrow D}$ is equally likely. Therefore, the probabilities to select a source-to-relay or relay-to-destination link at state $s_l$ are given by

$$p_{S\rightarrow R}^{(s_l)} = \frac{1}{K_{sr}^{(s_l)}} p_{S\rightarrow R}^{(s_l)} (1 - p_{R\rightarrow D}^{(s_l)}),$$

respectively.

With these observations, the $(i, j)$-th entry of the state transition matrix $A$ is expressed as

$$A_{i,j} = \begin{cases} 
   p_{S\rightarrow R}^{(s_j)} = \frac{1}{K_{sr}^{(s_j)}} (1 - p_{R\rightarrow D}^{(s_j)}), & \text{if } s_i \in U_{s_j}^{S\rightarrow R}, \\
   p_{R\rightarrow D}^{(s_j)} = \frac{1}{K_{rd}^{(s_j)}} p_{R\rightarrow D}^{(s_j)}, & \text{if } s_i \in U_{s_j}^{R\rightarrow D}, \\
   0, & \text{elsewhere},
\end{cases} $$

(3.3.21)

Because the transition matrix $A$ in (3.3.21) is column stochastic and irreducible\(^5\), the stationary state probability vector is obtained as (see [49, 50])

$$\pi = (A - I + B)^{-1} b,$$

(3.3.22)

where $\pi = [\pi_1, \cdots, \pi_{(L+1)K}]^T \in \mathbb{R}^{1 \times (L+1)K}$, $b = (1, 1, \ldots, 1)^T \in \mathbb{R}^{1 \times (L+1)K}$, $I$ is the $(L+1)K \times (L+1)K$ identity matrix and $B_{n,l}$ is an $n \times l$ all unity element matrix. Or in the stationary state, $\pi_l = \lim_{t \to \infty} P(s_l)$ for $l = 1, \cdots, (L+1)K$.

---

\(^5\)Column stochastic means all entries in any column sum up to one, irreducible means that it is possible to move from any state to any state [48, 49].
Below $p_{R \rightarrow D}^{(s_l)}$ in (3.3.21) is derived.

### 3.3.3 $p_{R \rightarrow D}^{(s_l)}$: probability of selecting the relay-to-destination transmission at state $s_l$

If there are no relay-to-destination links available (or $K_{rd}^{(s_l)} = 0$), $p_{R \rightarrow D}^{(s_l)} = 0$. On the other hand, if there are no source-to-relay links available (or $K_{sr}^{(s_l)} = 0$), $p_{R \rightarrow D}^{(s_l)} = 1$. For other cases, $p_{R \rightarrow D}^{(s_l)}$ is given by

$$p_{R \rightarrow D}^{(s_l)} = P(x < y) = \int_{x<y} f_{XY}(x,y) dxdy$$

(3.3.23)

where $x$ and $y$ are the maximum SNR-s from the $K_{sr}^{(s_l)}$ number of source-to-relay and $K_{rd}^{(s_l)}$ number of relay-to-destination links respectively, and $f_{XY}(x,y)$ is the joint PDF of $x$ and $y$. Because $x$ and $y$ are mutually independent, thus

$$f_{XY}(x,y) = f_X(x)f_Y(y) = \frac{K_{sr}^{(s_l)} K_{rd}^{(s_l)}}{\gamma_{sr} \gamma_{rd}} e^{-\left(\frac{x}{\gamma_{sr}} + \frac{y}{\gamma_{rd}}\right)} (1 - e^{-\frac{x}{\gamma_{sr}}})^{K_{sr}^{(s_l)}-1} (1 - e^{-\frac{y}{\gamma_{rd}}})^{K_{rd}^{(s_l)}-1}. $$

(3.3.24)

where $f_X(x)$ and $f_Y(y)$ are the PDF-s of $x$ and $y$ respectively. Substituting (3.3.24) into (3.3.23) yields

$$p_{R \rightarrow D}^{(s_l)} = \int_{0}^{\infty} \int_{0}^{y} \frac{K_{sr}^{(s_l)} K_{rd}^{(s_l)}}{\gamma_{sr} \gamma_{rd}} e^{-\left(\frac{x}{\gamma_{sr}} + \frac{y}{\gamma_{rd}}\right)} (1 - e^{-\frac{x}{\gamma_{sr}}})^{K_{sr}^{(s_l)}-1} (1 - e^{-\frac{y}{\gamma_{rd}}})^{K_{rd}^{(s_l)}-1} dxdy$$

(3.3.25)

$$= \frac{K_{rd}^{(s_l)}}{\gamma_{rd}} \int_{0}^{\infty} e^{-\frac{y}{\gamma_{rd}} (1 - e^{-\frac{y}{\gamma_{rd}}})^{K_{rd}^{(s_l)}-1} (1 - e^{-\frac{y}{\gamma_{sr}}})^{K_{sr}^{(s_l)}}} dy.$$
Applying a binomial expansion to \((1 - e^{-\frac{y}{\bar{r}_{rd}}})K_{rd}^{(s_l)} - 1\) and \((1 - e^{-\frac{y}{\bar{r}_{sr}}})K_{sr}^{(s_l)}\) gives

\[
\begin{align*}
(1 - e^{-\frac{y}{\bar{r}_{rd}}})K_{rd}^{(s_l)} - 1 &= \sum_{m=0}^{K_{rd}^{(s_l)}-1} C_m^{K_{rd}^{(s_l)}-1} (-1)^m e^{-\frac{ym}{\bar{r}_{rd}}}, \\
(1 - e^{-\frac{y}{\bar{r}_{sr}}})K_{sr}^{(s_l)} &= \sum_{n=0}^{K_{sr}^{(s_l)}} C_n^{K_{sr}^{(s_l)}} (-1)^n e^{-\frac{yn}{\bar{r}_{sr}}}.
\end{align*}
\]

(3.3.26)

Then \(p_{R \rightarrow D}^{(s_l)}\) is obtained

\[
\begin{align*}
p_{R \rightarrow D}^{(s_l)} &= \frac{K_{rd}^{(s_l)} K_{sr}^{(s_l)} - 1}{\bar{r}_{rd}} \sum_{m=0}^{K_{rd}^{(s_l)}-1} \sum_{n=0}^{K_{sr}^{(s_l)}} C_m^{K_{rd}^{(s_l)}-1} C_n^{K_{sr}^{(s_l)}} (-1)^{m+n} \int_0^{\infty} e^{-\frac{ym}{\bar{r}_{rd}} - \frac{yn}{\bar{r}_{sr}}} dy \\
&= \sum_{m=0}^{K_{rd}^{(s_l)}-1} \sum_{n=0}^{K_{sr}^{(s_l)}} C_m^{K_{rd}^{(s_l)}-1} C_n^{K_{sr}^{(s_l)}} (-1)^{m+n} \frac{K_{rd}^{(s_l)} \bar{\gamma}_{sr}}{\bar{\gamma}_{sr} + \bar{\gamma}_{sr} m + \bar{\gamma}_{rd} n}.
\end{align*}
\]

(3.3.27)

### 3.3.4 A special case: symmetric \(S \rightarrow R\) and \(R \rightarrow D\) channels

with \(\sigma^2_{h_{sr}} = \sigma^2_{h_{rd}}\)

In this section, a special case is considered that the average channel gains for the source-to-relay and relay-to-destination links are the same, or \(\sigma^2_{h_{sr}} = \sigma^2_{h_{rd}}\). Under this symmetric channel scenario, the probabilities to select any available source-to-relay and relay-to-destination link at state \(s_l\) at any time are the same. Thus (3.3.20) can be simplified as

\[
p_{s_l}^+ = p_{s_l}^- = \frac{1}{K^{(s_l)}}, \quad l = 1, \cdots, (L + 1)^N, \tag{3.3.28}
\]

where \(K^{(s_l)} = K_{sr}^{(s_l)} + K_{rd}^{(s_l)}\) which is the total number of available links (including both source-to-relay and relay-to-destination links) at state \(s_l\). Then the state transition matrix is given by

\[
A_{i,j} = \begin{cases} 
\frac{1}{K^{(s_l)}}, & \text{if } s_l \in U_j, \\
0, & \text{elsewhere},
\end{cases} \quad j = 1, \cdots, (L + 1)^N, \tag{3.3.29}
\]
where $U_j$ is the set of all possible states to which can be moved from $s_j$ at the next time slot.

The stationary state probability vector is then obtained by substituting (3.3.29) into (3.3.22). Alternatively, because at any time the probability to select one available link is uniform and every link corresponds to one transition of states, the stationary probability for a state is proportional to its corresponding number of available links so that

$$\pi_j = \lim_{t \to \infty} P(s_j) = \frac{K(s_j)}{\sum_{i=1}^{(L+1)^N} K(s_i)}. \quad (3.3.30)$$

For the proof of (3.3.30) please refer to examples 1.9.6 and 1.9.7 in [49].

Next, the outage probability for the “symmetric” channel, $P_{\text{symmetric}}(\gamma_d^{(s_i,s_j)}, \gamma) < \gamma_{th}$ is calculated, when the strongest source-to-relay and relay-to-destination links are selected at state $s_i$ and $s_j$ respectively. By letting $\bar{\gamma} = \bar{\gamma}_{sr} = \bar{\gamma}_{rd}$, and following the similar procedure in Section 3.3.1, it can be obtained

$$P_{\text{symmetric}}(\gamma_d^{(s_i,s_j)}, \gamma) < \gamma_{th} = 1 + \frac{K(s_j)}{\bar{\gamma}} \cdot \sum_{n=0}^{K(s_j)-1} \sum_{m=1}^{K(s_i)} C_n^{K(s_j)-1} C_m^{K(s_i)} (-1)^{m+n} 2e^{-\frac{\gamma_{th}}{\bar{\gamma}}(1+m+n)} \cdot \sqrt{\frac{m\gamma_{th}(\gamma_{th} + 1)}{(n+1)}} B \left(1, \frac{2}{\bar{\gamma}} \sqrt{m\gamma_{th}(\gamma_{th} + 1)(n+1)} \right). \quad (3.3.31)$$

Finally, substituting (3.3.30) and (3.3.31) into (3.3.4) gives the overall
outage probability for the symmetric channel configuration as

\[
P_{\text{symmetric}}^{\text{out}} = \sum_{s_i} \sum_{s_j} \frac{K(s_i)}{\sum_{l=1}^{(L+1)^N} K(s_l) \sum_{l=1}^{(L+1)^N} K(s_l)} \frac{K(s_j)}{

\left(1 + \frac{K(s_j)}{\eta} \sum_{n=0}^{K(s_j) - 1} \sum_{m=1}^{C_{K(s_j) - 1}^n} C_{K(s_i) - 1}^m \left(-1\right)^{m+n} 2^{\frac{\gamma th}{\eta}} \left(1 + m + n\right) \right) \sqrt{\frac{m \gamma th (\gamma th + 1)}{(n + 1)}} B \left(1, \frac{2}{\eta} \sqrt{m \gamma th (\gamma th + 1)(n + 1)} \right) \right). \tag{3.3.32}
\]

Next, the average delay introduced in such networks

### 3.4 Average Packet Delay

In the AF max-link scheme, at a transmission node (either the source or a relay), a data packet can only be transmitted out if the corresponding link is selected. This brings up two issues: first, the packets may not arrive at the destination in order; second, each packet may suffer from different delay within the systems. While the first issue can be easily handled by for instance numbering every packet, the delay becomes a main issue in buffer-aided relay selection systems [32].

In general, a packet delay includes delays at both the source and selected relay nodes, which are denoted as \(D_s\) and \(D_r\) respectively. A simple example is illustrated in Fig. 3.2, where there are three packets \((s(1), s(2)\) and \(s(3))\) transmitted out consecutively from the source. The transmission time-span for every packet is represented by a horizontal bar in Fig. 3.2, where \(D_s\) and \(D_r\) indicate the delay time slots at the source and relay nodes respectively, and \(S - R\) and \(R - D\) indicate the transmission time slots for source-to-relay and relay-to-destination respectively. For example, packet \(s_1\) is transmitted from the source to a relay node at time slot 2. After that, packet \(s_2\) waits for three time slots (slots 3, 4 and 5) and is then transmitted to a relay. After
Section 3.4. Average Packet Delay

$s_2$ arrives at the relay at slot 6, it waits for another four time slots (slots 7-10) before it is eventually transmitted to the destination at slot 11. Thus the delays for $s_2$ at the source and relay nodes are 3 and 4 respectively in this example. Fig. 3.2 also shows that the packets arrive at the destination in the order of $[s_1, s_3, s_2]$, which is clearly not the same as the transmission order.

**Figure 3.2.** An example of packet delay in the AF max-link scheme.

It particularly worth highlight that, while different packets may suffer from different delays, the system throughput (or the average data rate) of the AF max-link scheme is not scarified. This is because that, at any time slot, there is always one link selected for transmission. Therefore, when a packet is “waiting” for transmission at a node, another packet must be transmitted at another node. Assuming there are $M$ packets in total, as each packet takes two time slots for transmission (excluding the waiting time), if $M$ is large enough, the overall transmission time to deliver all packets is approximately $2M$. Therefore, the system average throughput is $\eta = \frac{M}{2M} = 0.5$, which is the same as that for the classic three-node “$S \rightarrow R \rightarrow D$” relay system [51].

According to Little’s law [52], the average packet delay at the node $i$ can be obtained as

$$E[D_i] = \frac{E[Q_i]}{\eta_i},$$

(3.4.1)

where $E[Q_i]$ and $\eta_i$ are the average queuing length and throughput at the
node.

In the following two subsections, the average packet delay is derived at the source and relay nodes respectively.

### 3.4.1 Average packet delay at the source

Because all data are transmitted from the same source node, the average throughput at the source node is the same as that for the overall system which is given by

$$ s = \sum_{l=1}^{(L+1)^N} \pi_l $$

On the other hand, if suppose the source always has data to transmit, the queuing length at the source depends on how fast the data leave the source, which again depends on the probability that a source-to-relay link is selected. Considering all buffer states at the relay, the probability that a source-to-relay link is selected can be obtained as

$$ p_{S \rightarrow R} = \sum_{s_l} P(s_l) \cdot (1 - p_{R \rightarrow D}^{(s_l)}) $$

where $p_{S \rightarrow R}$ is the stationary probability for state $s_l$ which is obtained in (3.3.22), and $p_{R \rightarrow D}^{(s_l)}$ is the probability to select a relay-to-destination link at state $s_l$ which is given by (3.3.27). Alternatively, for any fixed sized buffers, the number of data packets arriving at all the relays must be equal to that leaving these relays, because no data packet can stay in a relay node forever and fail to reach the destination. Thus

$$ p_{S \rightarrow R} = p_{R \rightarrow D} = 1/2 $$

This implies that the average queuing length at the source node is

$$ \mathbb{E}[Q_s] = 1/2 $$

Substituting (3.4.2) and (3.4.4) into (3.4.1) gives the average packet delay

$$ \eta_s = \eta = 1/2 $$
at the source node as

\[ E[D_s] = \frac{E[Q_s]}{\eta_s} = 1. \quad (3.4.5) \]

It is worth highlighting that (3.4.5) holds for both symmetric and asymmetric channel scenarios.

### 3.4.2 Average packet delay at the relay

Because the probabilities to select any of the relays are the same, the average packet delays at any of the relay are also the same, so the average throughput at any relay is given by

\[ \eta_r = \frac{\eta}{N} = \frac{1}{2N} \quad (3.4.6) \]

Let \( Q_r^{(s_l)} \) be the queuing length (or the average number of packets) for the selected relay at the buffer state \( s_l \). Considering all buffer states \( s_l \), the average queuing length at the selected relay is obtained as

\[ E[Q_r] = \frac{(L+1)^N}{2N} \sum_{l=1}^{N} \pi_l Q_r^{(s_l)}, \quad (3.4.7) \]

Substituting (3.4.6) and (3.4.7) into (3.4.1) gives the average packet delay at the relay as

\[ E[D_r] = \frac{1}{2N} \sum_{l=1}^{N} \pi_l Q_r^{(s_l)} \quad (3.4.8) \]

Finally, combining the delay at the source and relay nodes gives the overall average delay in the AF max-link system as

\[ E[D] = E[D_s] + E[D_r] = 1 + \frac{1}{2N} \sum_{l=1}^{N} \pi_l Q_r^{(s_l)}. \quad (3.4.9) \]

On the other hand, if the source-to-relay and relay-to-destination channels are symmetric (i.e. \( \sigma_{p_{sr}}^2 = \sigma_{p_{rd}}^2 \)), the average packet delay at the relay in (3.4.8) can be obtained as \( E[D_r] = L/2 \), and the overall average delay becomes \( E[D] = 1 + NL \).
3.4.3 Numerical examples

The extensive numerical simulation have been performed which all well match the above delay analysis. Some of the results are shown in Tables I and II, where for fair comparison, let $\bar{\gamma}_{sr}(\text{dB}) + \bar{\gamma}_{rd}(\text{dB}) = 40\text{dB}$ in all cases. It is clearly shown that, with increased relay number $N$ and larger buffer size $L$, the delays increased. Moreover, if the relay-to-destination link SNR is stronger than the source-to-relay SNB, the delay decreased. This is not surprising because higher relay-to-destination SNR implies that the relay-to-destination link is more likely to be selected and the data are more quickly forwarded to the destination.

<table>
<thead>
<tr>
<th>$(N, L) = (2, 2)$</th>
<th>$D_{\text{ave}}$</th>
<th>$(\bar{\gamma}<em>{sr}, \bar{\gamma}</em>{rd})$ [dB]</th>
<th>Simulation</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10 30</td>
<td>2.03</td>
<td>2.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15 25</td>
<td>2.29</td>
<td>2.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 20</td>
<td>4.99</td>
<td>5.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25 15</td>
<td>7.69</td>
<td>7.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30 10</td>
<td>7.97</td>
<td>7.97</td>
</tr>
</tbody>
</table>

Table 3.2. Average packet delays

<table>
<thead>
<tr>
<th>$(N, L) = (4, 4)$</th>
<th>$D_{\text{ave}}$</th>
<th>$(\bar{\gamma}<em>{sr}, \bar{\gamma}</em>{rd})$ [dB]</th>
<th>Simulation</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10 30</td>
<td>2.04</td>
<td>2.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15 25</td>
<td>2.42</td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 20</td>
<td>17.04</td>
<td>17.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25 15</td>
<td>31.56</td>
<td>31.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30 10</td>
<td>31.95</td>
<td>31.95</td>
</tr>
</tbody>
</table>

The diversity order of the scheme will be considered next.

3.5 Diversity Order

In order to show the diversity order of the AF max-link scheme, suppose all channels are i.i.d. such that $\sigma_{h_{sr}}^2 = \sigma_{h_{rd}}^2 = \sigma_h^2$, and then the outage
probability is given in (3.3.32). The diversity order can be defined as

\[ d = - \lim_{\bar{\gamma}_h \to \infty} \frac{\log_{10} P_{\text{out}}}{\log_{10} \bar{\gamma}_h}, \quad (3.5.1) \]

where \( \bar{\gamma}_h = \frac{(P_s \sigma^2_k)}{\sigma^2} \) which is the average SNR for every channel. However substituting (3.3.32) into (3.5.1) does not explicitly show the diversity order. Instead, the upper and lower bounds of the outage probability will be derived first, from which the diversity order is obtained; then the minimum and maximum diversity orders will be obtained when the relay buffer sizes are 1 and \( \infty \) respectively.

### 3.5.1 Outage probability bounds

Noting \( (s_i)_{sr}^k = \gamma_{sr} h_{sr}^k |^2 \), \( (s_j)_{rd}^k = \gamma_{rd} h_{rd}^k |^2 \), and from (3.3.3), thus

\[ \lim_{\bar{\gamma}_h \to \infty} \gamma_{d}^{(s_i,s_j)} = \frac{\gamma_{sr}^k \gamma_{rd}^k}{\gamma_{sr}^k + \gamma_{rd}^k}. \quad (3.5.2) \]

Since \( \gamma_{sr}^k > 0 \) and \( \gamma_{rd}^k > 0 \), thus

\[ \frac{1}{2} \min(\gamma_{sr}^k, \gamma_{rd}^k) \leq \frac{\gamma_{sr}^k \gamma_{rd}^k}{\gamma_{sr}^k + \gamma_{rd}^k} \leq \min(\gamma_{sr}^k, \gamma_{rd}^k). \quad (3.5.3) \]

From (3.5.2) and (3.5.3), thus

\[ P_e^L \leq \lim_{\bar{\gamma}_h \to \infty} P(\gamma_d^{(s_i,s_j)} < \gamma_{th}) \leq P_e^U, \quad (3.5.4) \]

where \( P_e^L = P(\min(\gamma_{sr}^k, \gamma_{rd}^k) < \gamma_{th}) \) and \( P_e^U = P(1/2 \min(\gamma_{sr}^k, \gamma_{rd}^k)) \) which are the lower and upper bounds for \( \lim_{\bar{\gamma}_h \to \infty} P(\gamma_d^{(s_i,s_j)} < \gamma_{th}) \) respectively.

Supposing the total numbers of available links for buffer state \( s_i \) and \( s_j \) are given by \( K^{(s_i)} \) and \( K^{(s_j)} \) respectively, the lower bound \( P_e^L \) can be
obtained as

\[ P_{e}^{L} = P(\min(\gamma_{s_{i}r_{k}}, \gamma_{s_{j}d}) < \gamma_{th}) \]

\[ = 1 - (1 - F_{X}(\gamma_{th}))(1 - F_{Y}(\gamma_{th})) \]

\[ = (1 - e^{-\frac{\gamma_{th}}{\gamma_{h}}} K^{(s_{i})}) + (1 - e^{-\frac{\gamma_{th}}{\gamma_{h}}} K^{(s_{j})}) - (1 - e^{-\frac{\gamma_{th}}{\gamma_{h}}} K^{(s_{i})}) (1 - e^{-\frac{\gamma_{th}}{\gamma_{h}}} K^{(s_{j})}). \]  

(3.5.5)

Further noting that \( e^x \approx 1 + x \) for very small \( x \), and ignoring the high order terms, thus

\[ \lim_{\tilde{\gamma}_{h} \to \infty} P_{e}^{L} = \left( \frac{\tilde{\gamma}_{th}}{\tilde{\gamma}_{h}} \right)^{\min\{K^{(s_{i})}, K^{(s_{j})}\}}. \]  

(3.5.6)

Then it can be obtained that

\[ \lim_{\tilde{\gamma}_{h} \to \infty} \frac{\log_{10} P_{e}^{L}}{\log_{10} \tilde{\gamma}_{h}} = \min\{K^{(s_{i})}, K^{(s_{j})}\}. \]  

Further noting that \( N \leq K^{(s_{i})} \leq 2N \) and \( N \leq K^{(s_{j})} \leq 2N \), thus

\[ N \leq - \lim_{\tilde{\gamma}_{h} \to \infty} \frac{\log_{10} P_{e}^{L}}{\log_{10} \tilde{\gamma}_{h}} \leq 2N. \]  

(3.5.7)

On the other hand, the upper bound \( P_{e}^{U} \) can be obtained as

\[ P_{e}^{U} = P(1/2 \min(\gamma_{s_{i}r_{k}}, \gamma_{s_{j}d}) < \gamma_{th}) \]

\[ = 1 - (1 - F_{X}(2\gamma_{th}))(1 - F_{Y}(2\gamma_{th})) \]

\[ = (1 - e^{-\frac{2\gamma_{th}}{\gamma_{h}}} K^{(s_{i})}) + (1 - e^{-\frac{2\gamma_{th}}{\gamma_{h}}} K^{(s_{j})}) - (1 - e^{-\frac{2\gamma_{th}}{\gamma_{h}}} K^{(s_{i})}) (1 - e^{-\frac{2\gamma_{th}}{\gamma_{h}}} K^{(s_{j})}). \]  

(3.5.8)

Then following the similar procedure as that for \( P_{e}^{L} \), thus

\[ \lim_{\tilde{\gamma}_{h} \to \infty} P_{e}^{U} = \left( \frac{\tilde{\gamma}_{th}}{\tilde{\gamma}_{h}} \right)^{\min\{K^{(s_{i})}, K^{(s_{j})}\}}. \]  

(3.5.9)

and

\[ N \leq - \lim_{\tilde{\gamma}_{h} \to \infty} \frac{\log_{10} P_{e}^{U}}{\log_{10} \tilde{\gamma}_{h}} \leq 2N. \]  

(3.5.10)
It is clear from (3.5.6) and (3.5.9) that, when \( \bar{\gamma}_h \to \infty \), \( \log_{10} P_e^L \) and \( \log_{10} P_e^U \) have the same gradients against \( \log_{10} \bar{\gamma}_h \). Then using (3.5.7) and (3.5.10) in (3.5.4), it is derived as

\[
N \leq - \lim_{\bar{\gamma}_h \to \infty} \log_{10} P\left( \gamma_{d}^{(s_i,s_j)} < \gamma_{th} \right) \log_{10} \bar{\gamma}_h \leq 2N. \tag{3.5.11}
\]

Particularly, if \( K^{(s_i)} = K^{(s_j)} = K \), thus

\[
- \lim_{\bar{\gamma}_h \to \infty} \log_{10} P\left( \gamma_{d}^{(K,K)} < \gamma_{th} \right) = K. \tag{3.5.12}
\]

Finally, because (3.5.11) holds for every \( s_i \) and \( s_j \), from (3.3.4), the diversity order of the max-link AF relay selection can be obtained as

\[
N \leq d \leq 2N. \tag{3.5.13}
\]

It is clear that the diversity order \( d \) is a function of both the relay number \( N \) and buffer size \( L \). Below the upper and lower limits of the diversity order are performed to be reached when \( L = 1 \) and \( L \to \infty \) respectively.

### 3.5.2 Buffer size \( L = 1 \)

If the buffer size \( L = 1 \), the available number of links at any state is \( N \), or \( P(K^{(s_i)} = N) = 1 \) for all \( s_i \). Then from (3.3.4), the outage probability is given by

\[
P_{out}^{(L=1)} = P(\gamma_{d}^{(N,N)} < \gamma_{th}). \tag{3.5.14}
\]

Furthermore from (3.5.12), the diversity order for \( L = 1 \) can be obtained as

\[
d^{L=1} = - \lim_{\bar{\gamma}_h \to \infty} \frac{P_{out}^{(L=1)}}{\log_{10} \bar{\gamma}_h} = N. \tag{3.5.15}
\]
3.5.3 Buffer size $L \to \infty$

If the buffer size is $L$, there are $(L - 1)^N$ states which are neither full nor empty so that their corresponding number of available links is $2N$. Since the total number of buffer states is $(L + 1)^N$, the number of states whose corresponding links is not $2N$ is $(L + 1)^N - (L - 1)^N$. Thus the probability that the available link is not $2N$ is given by

$$P(K \neq 2N) = \sum_{K^{(s_j)} \neq 2N} \pi_{s_j}, \quad (3.5.16)$$

where $K^{(s_j)}$ and $\pi_{s_j}$ are the total number of available links and stationary probability for the state $s_j$ respectively. Substituting (3.3.30) into (3.5.16), and recalling that $N \leq K^{(s_j)} \leq 2N$ for all $j$, thus

$$P(K \neq 2N) = \sum_{K^{(s_j)} \neq 2N} \frac{K^{(s_j)}}{\sum_{i=1}^{(L+1)^N} K^{(s_i)}} \leq \sum_{D_j \neq 2N} \frac{2N}{\sum_{i=1}^{(L+1)^N} N} \leq 2 \sum_{K^{(s_j)} \neq 2N} \frac{N}{N[(L+1)^N - 1]} = 2 \frac{(L + 1)^N - (L - 1)^N}{(L + 1)^N - 1} \quad (3.5.17)$$

It is clear from (3.5.17) that $\lim_{L \to \infty} P(K \neq 2N) = 0$.

Therefore, if $L \to \infty$, the outage probability in (3.3.4) can be simplified as

$$P^{(L \to \infty)}_{out} = P(\gamma^{(2N,2N)}_d < \gamma_{th}). \quad (3.5.18)$$

Then from (3.5.12), the diversity order for $L \to \infty$ is obtained as

$$d^{L \to \infty} = - \lim_{\gamma_h \to \infty} \frac{P^{(L \to \infty)}_{out}}{\log_{10} \gamma_h} = 2N. \quad (3.5.19)$$

Lastly, the coding gain of the approach is performed.
3.6 Coding gain

Compared with the traditional max-SNR relay selection scheme, the AF max-link scheme has not only diversity but also coding gain. In order to highlight the coding gain, suppose that the relay buffer size of the max-link scheme is $L = 1$. Then the diversity orders for both the max-link and max-SNR schemes are $N$, and the outage performance advantage of the AF max-link over the max-SNR scheme comes from the coding gain.

From (3.5.14), when $L = 1$, the outage probability of the AF max-link scheme is given by $P_{\text{out}}^{(L=1)} = P(\gamma_d^{(N,N)} < \gamma_{th})$ whose lower and upper bounds ($P_e^L$ and $P_e^U$ respectively) can be obtained using (3.5.4). As is shown in Section 3.5.1, when the channel SNR $\bar{\gamma}_h \to \infty$, $\log_{10} P_e^L$ and $\log_{10} P_e^U$ have the same gradients against $\log_{10} \bar{\gamma}_h$. This implies that, for $\bar{\gamma}_h \to \infty$, it must have

$$10 \log_{10} P_{\text{out}}^{(L=1)} = \alpha + 10 \log_{10} P_e^L; \quad (3.6.1)$$

where $0 \leq \alpha \leq \log_{10}(P_e^U/P_e^L)$ which is a small constant.

Substituting (3.5.5) into (3.6.1), and ignoring the high order terms in the context of high SNR, it has

$$\lim_{\bar{\gamma}_h \to \infty} 10 \log_{10} P_{\text{out}}^{(L=1)} = \alpha + 10 \log_{10} \left( \frac{2 \bar{\gamma}_h}{\gamma_{th}} \right)^N \quad (3.6.2)$$

On the other hand, in the traditional max-SNR scheme, the best relay is selected that maximises the SNR at the destination. To be specific, if the relay $R_k$ is selected, the end-to-end SNR at the destination can be obtained as

$$\gamma_d^{(R_k)} = \frac{\gamma_{sr_k} \gamma_{rk_d}}{\gamma_{sr_k} + \gamma_{rk_d} + 1}, \quad (3.6.3)$$

where $\gamma_{sr_k}$ and $\gamma_{rk_d}$ are the instantaneous channel SNRs for $S \to R_k$ and $R_k \to D$ links respectively. Similar to (3.5.4), assuming over high SNR, the
lower and upper bounds for \( P(\gamma_d^{(R_k)} < \gamma_{th}) \) can be obtained as

\[
P(\min(\gamma_{sr_k}, \gamma_{rd}) < \gamma_{th}) \leq P(\gamma_d^{(R_k)} < \gamma_{th}) \leq P(1/2 \min(\gamma_{sr_k}, \gamma_{rd}) < \gamma_{th}).
\]

(3.6.4)

Because the best relay in the max-SNR scheme is selected among \( N \) pairs of source-to-relay and relay-to-destination links that maximises (3.6.3), the outage probability can be obtained as

\[
P_{out}^{(\text{max-SNR})} = [P(\gamma_d^{(R_k)} < \gamma_{th})]^N
\]

(3.6.5)

Substituting (3.6.4) into (3.6.5) gives

\[
[P(\min(\gamma_{sr_k}, \gamma_{rd}) < \gamma_{th})]^N \leq P_{out}^{(\text{max-SNR})} \leq [P(1/2 \min(\gamma_{sr_k}, \gamma_{rd}) < \gamma_{th})]^N.
\]

(3.6.6)

For the similar reasons in obtaining (3.6.1), at high SNR, it must have

\[
10 \log_{10} P_{out}^{(\text{max-SNR})} = \beta + 10 \log_{10}[P(\min(\gamma_{sr_k}, \gamma_{rd}) < \gamma_{th})]^N
\]

(3.6.7)

where \( \beta \) is a small positive constant. Because the channel SNRs are exponentially distributed, it has

\[
[P(\min(\gamma_{sr_k}, \gamma_{rd}) < \gamma_{th})]^N = \left((1 - e^{-\frac{\gamma_{th}}{\bar{h}}}) + (1 - e^{-\frac{\gamma_{th}}{\bar{h}}}) - (1 - e^{-\frac{\gamma_{th}}{\bar{h}}})(1 - e^{-\frac{\gamma_{th}}{\bar{h}}})\right)^N.
\]

(3.6.8)

Substituting (3.6.8) into (3.6.7), and ignoring the high orders assuming high SNR, it has

\[
\lim_{\gamma_{th} \to \infty} 10 \log_{10} P_{out}^{(\text{max-SNR})} = \beta + 10 \log_{10} \left(\frac{2 \gamma_{th}}{\gamma_{th}}\right)^N
\]

(3.6.9)

Finally, from (4.5.16) and (3.6.9), when the buffer size \( L = 1 \), the coding gain of the AF max-link scheme over the traditional AF max-SNR scheme
is given by

\[ C^{(L=1)}(\text{dB}) = \lim_{\gamma_n \to \infty} \left( 10 \log_{10} P_{\text{out}}^{(\text{max-SNR})} - 10 \log_{10} P_{\text{out}}^{(L=1)} \right) \]

\[ = 10(N - 1) \log_{10} 2 + (\beta - \alpha) \]

\[ \approx 10(N - 1) \log_{10} 2, \quad (3.6.10) \]

where the approximation in (3.6.10) comes from the fact that both \( \alpha \) and \( \beta \) are small positive constants.

Recall that data buffers (with size 1) also exist at the relays in a traditional relay selection scheme, as the data packets need to be stored in the relay at one time and forwarded to the destination at the next time. It is clear from (3.6.10) that, even with \( L = 1 \), the AF max-link scheme still has better outage performance than the traditional AF max-SNR scheme because of the coding gain. It is also shown in (3.6.10) that more relays lead to higher coding gain. Only when \( N = 1 \), does the coding gain disappear because then both the max-link and max-AF schemes reduce to the standard three nodes relay system.

While the coding gain analysis above is for buffer size \( L = 1 \), it is also useful in understanding the more general case with other buffer sizes, where the coding gain also exists. In general, the coding gain depends on the number of available links for selection, which again depends on both the relay number \( N \) and buffer size \( L \). With larger \( L \) and \( N \), it has larger coding gain. This will be verified in the simulations in the next section.

3.7 Simulations and discussions

In this section, numerical results are shown to verify the analyse in this paper. In the simulations below, the average transmission powers for all transmission nodes is set as \( P_s = 1 \), the noise variances for all receiving nodes are set as \( \sigma^2 = 1 \). All simulation results are obtained with 1,000,000
Monte Carlo runs.

### 3.7.1 Outage performance of the AF max-link scheme

Fig. 3.3 verifies the outage probability expression in (3.3.32) with simulation results under various scenarios. It is clearly shown that in all cases the theoretical analysis well matches the simulation results. Both Fig. 3.3 (a) and (b) show that the best outage performance is obtained when the source-to-relay and relay-to-destination channels are symmetric.

Fig. 3.4 (a) and (b) show the outage performance against different buffer lengths $L$ for symmetric and asymmetric channel configurations respectively, where the relay number is fixed at $N = 3$. It is clearly shown that the outage performance improves with larger buffer size $L$, but the improvement is less significant when $L$ becomes larger. It is shown in Fig. 3.4 (a) and (b) that, when $L = 50$ and $L = 20$, the outage performance is almost the same as that for $L \to \infty$ for the symmetric and asymmetric channel configurations respectively. Therefore, in practice, the full outage order $2N$ can be achieved with finite buffer sizes. It is also shown that, with larger buffer size $L$, the outage performance improvement in the symmetric channel (Fig. 3.4 (a)) is much more significant than that in the asymmetric channel (Fig. 3.4 (b)).

Fig. 3.5 shows how the outage performance changes with different relay numbers $N$ for a fixed buffer size $L = 8$, where the asymmetric channel configuration with $\bar{\gamma}_{sr} = 30$dB and $\bar{\gamma}_{rd} = 25$dB is considered. It is clearly shown that the outage performance improves with more relays. The results for other channel configurations are similar so they are not presented.
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3.7.2 Outage performance comparison between the AF max-link and max-SNR schemes

Fig. 3.6 compares the proposed AF max-link and traditional max-SNR schemes in symmetric and asymmetric channels. For fair comparison, let
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Figure 3.4. Outage probability performance of the max-link scheme for different buffer lengths \( L \).

\( \gamma_{sr}(\text{dB}) + \gamma_{rd}(\text{dB}) = 40\text{dB} \) in all cases. It is clearly shown that, for both the AF max-link and max-SNR schemes, the best outage performance is achieved in the symmetric channel. Moreover, the outage performance advantage of the AF max-link scheme over the traditional max-SNR scheme is also more significant in the symmetric than in the asymmetric channels. For example,
when the target SNR=10dB, the outage probability differences between the max-link and max-SNR are approximately as large as 28dB for symmetric channels, and only about 2dB for asymmetric channels\textsuperscript{6}.

This can be explained as follows: In the AF max-link scheme, as is shown in (3.3.4), the outage performance depends on both the outage probability for every buffer state and the distributions of the buffer states, because different buffer states may correspond to different available links for the relay selection. On the one hand, the outage probability for a given buffer state is always minimised in the symmetric channel. This is because, as is shown in the outage bound in Section 3.5.1, the outage probability for any buffer state depends on the minimum SNR of the source-to-relay and relay-to-destination channels, which is clearly minimised in the symmetric channels. On the other hand, if the channels become more asymmetric, the relay buffers are more likely to be full or empty, corresponding to fewer available links, which also deteriorates the outage performance.

\textsuperscript{6}Outage probability in dB = 10 log(outage probability)
In comparison, the traditional AF max-SNR scheme does not have buffer states and the available links for selection are always equal to the relay numbers. Thus the outage performance solely depends on the minimum SNR of the source-to-relay and relay-to-destination channels, and is optimum in symmetric channels. Therefore, when the channels become more asymmetric, there are two and one deteriorating factors in the outage performance for the max-link and max-SNR respectively, so that the outage performance of the max-link deteriorates faster than that of the max-SNR scheme. Therefore, compared with the traditional relay selection scheme, the buffer-aided max-link scheme is most effective in the symmetric channel configuration.

![Figure 3.6](image_url)

**Figure 3.6.** Outage performance comparison between the AF max-link and max-SNR schemes with different channel configurations.

### 3.7.3 Diversity order and coding gain

In order to show the diversity gain, Fig. 3.7 considers a symmetric channel configuration for which $\bar{\gamma}_{sr} = \bar{\gamma}_{rd} = 25$dB. As is proved in Section 3.5, the diversity orders of the AF max-link scheme are $N$ and $2N$, when the buffer sizes are $L = 1$ and $L \to \infty$ respectively. On the other hand, the
diversity order of the max-SNR is $N$. Therefore, the max-link schemes with $(N, L = 1)$ and $(N, L \to \infty)$ have the same diversity orders as those for the max-SNR with $N$ and $2N$ respectively, which is clearly verified in Fig. 3.7.

It is interesting to observe that, because of the coding gain, the max-link scheme with $(N = 5, L = 1)$ has significant better outage performance than the max-SNR scheme with $N = 5$, though they have the same diversity orders. Fig. 3.7 shows that, when SNR = 14dB, the outage probability difference between max-SNR with $N = 5$ and max-link with $(N = 5, L = 1)$ is approximately 11dB, which well matches the approximate coding gain obtained from (3.6.10) that is $10(N - 1) \log_{10} 2 = 12$dB when $N = 5$.

On the other hand, for the max-link scheme $N = 5, L \to \infty$, the available link for every buffer state is $2N = 10$. Then following the similar procedure in Section 3.5, the coding gain of the max-link with $N = 5, L \to \infty$ over the max-SNR with $2N = 10$ can be obtained as approximately $10(2N - 1) \log_{10} 2 = 27$dB. But Fig. 3.7 shows that, when SNR = 14dB, the outage probability difference between the max-SNR with $N = 10$ and max-link with $(N = 5, L \to \infty)$ is approximately 31dB, which well matches the analytical result.

![Figure 3.7](image-url)  

**Figure 3.7.** Diversity order and coding gain of the AF max-link scheme.
3.8 CONCLUSION

In this chapter, the performance of the buffer-aided AF max-link relay selection scheme for both symmetric and asymmetric channels have been studied in detail. The closed form expressions for the outage probability of the proposed scheme has been derived. The results showed that the max-link scheme is most effective over the traditional max-SNR scheme when the source-to-relay and relay-to-destination links are symmetric. The average packet delay of the max-link scheme under both symmetric and asymmetric channel configurations have also been derived. The diversity order of the AF max-link scheme has been proved to be between $N$ and $2N$, where the lower and upper limits were obtained when the buffer size is 1 and $\infty$ respectively. The coding gain of the max-link scheme over the traditional max-SNR scheme has also been analytically showed. Finally, extensive numerical simulations were given to verify the analyse in this chapter.
Applying data buffers at the relay nodes significantly improves the data transmission of cooperative networks, but the performance gain is often at the price of long packet delays. In this chapter, a novel relay selection scheme with significantly reduced packet delay is proposed. Both the outage performance and average packet delay of the proposed scheme are analysed. The analysis shows that, besides the diversity and coding gains, the proposed scheme has average packet delay similar to that of a non buffer-aided relay system when the channel SNR is sufficiently high thereby, making it an attractive scheme for practical systems.

4.1 Introduction

Relay selection provides an attractive way to harvest the diversity gain in multiple relay cooperative networks [51] and [53]. A typical relay selection system is shown in Fig. 4.1, which includes one source node (S), one des-
tion node ($D$) and $N$ relay nodes ($R_k$, $1 \leq k \leq N$). The relays may apply either DF or AF protocols [37]. Performance analysis of AF relay selection schemes shows that full diversity order can be achieved with the best selected relay ([54] and [29]).

![Figure 4.1. The system model of the relay selection system.](image)

The purpose of relay selection is to choose the ‘best’ relay among all available relay nodes to forward the data from the source to the destination. In the classic max-min relay selection scheme, the best relay is selected with the highest gain among all of the minima of the source-to-relay and relay-to-destination channel gain pairs [18]. While the max-min schemes achieve diversity order of $N$, their performance is practically limited by the constraint that the best source-to-relay and relay-to-destination links for a packet transmission must be determined concurrently. Recent research has on the other hand found that introducing data buffers at the relays yield significant performance advantage in practical systems [33,38]. Buffer-aided relays have also been used in other applications including adaptive link selection [31, 32], cognitive radio networks [55] and physical layer network security [40].

Typical buffer-aided relay selection schemes include the max-max [38] and max-link [33] schemes. In max-max relay selection, at one time slot $t$, the best link among all source-to-relay channels is selected, and a data packet
is sent to the selected relay and stored in the buffer. At the next time slot \((t + 1)\), the best link among all relay-to-destination channels is selected, and the selected relay (which is often not the same relay selected at time \(t\)) forwards one data packet from its buffer to the destination. The max-max scheme has significant coding gain over the traditional max-min scheme.

In the max-link scheme [33], at any time, the best link is selected among all available source-to-relay and relay-to-destination links. Depending on whether a source-to-relay or a relay-to-destination link is selected, either the source transmits a packet to the selected relay or the selected relay forwards a stored packet to the destination. As a result, the max-link relay selection not only has coding gain over the max-min scheme, but also higher diversity order than both the max-min and max-max schemes.

The performance gain of the buffer-aided relay network is however at the price of much higher average packet delay. In the non buffer-aided relay selection scheme (e.g. the max-min scheme), the delay for every packet through the network is always two time slots, corresponding to the source-to-relay and relay-to-destination transmission respectively. In the buffer aided approach, on the contrary, when a packet is transmitted to a relay node, it is stored in the buffer and will not be forwarded to the destination until the corresponding relay-to-destination link is selected. As a result, different packets in the buffer-aided relay network may endure different delays, depending on instantaneous channel conditions. In either the buffer-aided max-max or max-link scheme, the average packet delay increases linearly with both the relay number and buffer size. On the other hand, in order to achieve high performance gain, relay numbers and buffer size in the max-max or max-min scheme needs to be set as high as possible. This makes the existing buffer-aided relay selection schemes unsuitable in most applications where packet delays must be limited. For example, in the TCP/IP network, a packet need to be re-transmitted or even discarded if the acknowledge-
ment (ACK) from the receiver does not arrive at the transmitter within a certain time duration. Because this may be due to either the channel outage or the long packet delay, the long packet delay has the same effect on the data transmission as the channel outage. Therefore, the packet delay is an important issue in buffer-aided approaches [35]. The packet delay can be controlled by maximising the system throughput, subject to the constraints on the arrival and leaving data rate of the relay nodes (e.g. [31]). In order to make the optimisation problem feasible, the buffer sizes in these approaches are often assumed to be infinite large. Investigate reducing the packet delay in buffer-aided relay selection with limited buffer sizes becomes very attractive.

In this chapter, therefore a novel buffer-aided relay selection scheme with significantly reduced packet delay is proposed. This is achieved by giving higher priority to select the relay-to-destination than the source-to-relay links, so that the data queuing lengths at the relay buffers are minimised. The main contributions of this chapter are listed as follows:

- **Proposing a novel relay selection scheme.** Based on the literature review, this is the first buffer-aided relay scheme with reduced packet delay for limited buffer sizes at the relays.

- **Deriving the closed-form expression for outage probability.** In the proposed scheme, because the probabilities to select different links are different, the outage analysis is significantly different from existing buffer-aided approaches. Successful derivation of the outage probability also provides the basis to analyse the packet delay and other performance indices of the proposed scheme.

- **Determining a closed-form expression for the average packet delay.** Using Little’s law, the average packet delay of the proposed scheme is analytically obtained.
• **Analysing the asymptotic performance that the channel SNR goes to infinity.** The asymptotic performances including diversity order, coding gain and average packet delay for infinite channel SNR, are analysed and compared with the existing max-max and max-min schemes. These are important to understand the system. The analysis shows that the proposed scheme has slightly higher diversity order than the max-max scheme, and lower diversity order than the max-link scheme. But unlike the max-max or max-link scheme whose average packet delay increases linearly with the buffer size and relay number, the proposed scheme always has packet delay of two time slots when the channel SNR is sufficiently high.

The remainder of this chapter is organised as follows: Section 4.2 proposes the novel buffer-aided relay selection scheme with reduced packet delay; Section 4.3 analyses the outage probability; Section 4.4 analyses the average packet delay; Section 4.5 studies the asymptotic performance; Section 4.6 shows simulation results to verify the performance of the proposed scheme; and Section 4.7 concludes the chapter.

### 4.2 Buffer-aided relay selection with reduced delay

The system model of buffer-aided relay selection is the same as that shown in Fig. 4.1, except that every relay is equipped with a data buffer $Q_k$ ($1 \leq k \leq N$) of finite size $L$. Suppose the relays exploit DF transmission in this chapter. The channel coefficients for $S \rightarrow R_k$ and $R_k \rightarrow D$ at time slot $t$ are denoted as $h_{sr_k}(t)$ and $h_{rd_k}(t)$ respectively. Suppose that all channels are i.i.d. quasi-static Rayleigh fading\(^1\), so that the channel gains $\gamma_{ij}(t)$ are exponentially distributed with the same average gain as $\bar{\gamma} = \mathbb{E}[|h_{sr_k}(t)|^2] = \mathbb{E}[|h_{rd_k}(t)|^2]$ for all $k$. Supposing that channel coefficients remain unchanged

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\(^1\)While the analysis in this chapter is based on the i.i.d. channel assumption, it can be generalised to the case that every link has different average channel gain.
during one packet duration but independently vary from one packet time to another. Without losing generality, the noise variances at all receiving nodes ($R_k$ and $D$) are normalised to unity.

In the existing buffer-aided relay selection schemes including the max-max and max-link schemes, the average packet delay increases linearly with both the relay numbers and buffer size. The large delay is due to the packets queuing at the buffers. This can be seen, for example, in the max-link scheme where the first packet $s_1$ is sent to relay $R_k$ at time $t = 1$. Suppose that there is no data in all relay buffers initially and there are $K$ available source-to-relay and relay-to-destination links at time $t = 2$. Then the probability that $s_1$ is forwarded to the destination at $t = 2$ is $1/K$. This is because the max-link scheme always selects the link with the strongest channel SNR among all available source-to-relay and relay-to-destination links. In other words, it is most likely (with probability of $(K - 1)/K$) that $s_1$ remains stored in the buffer of $R_k$ at $t = 2$, leading to one extra time slot in packet delay. It is clear that this extra delay may be avoided by forwarding $s_1$ to the destination immediately at $t = 2$, once the corresponding $R_k \rightarrow D$ link is not in outage (though the link may not be the strongest link).

This leads to a new principle of buffer-aided relay selection: that is to transmit the packets already in the buffers as fast as possible. This translates into giving higher priority to select the relay-to-destination links; only when no relay-to-destination link can be selected, are the source-to-relay links considered. As a result, the packet queuing lengths at the relay buffers are minimised, and so is the average packet delay.

To be specific, at time slot $t$, the link selection rule is as follows:

1. Choose the link with highest channel SNR among all available relay-to-destination links ($|h_{rkD}(t)|^2$). If the chosen link is not in outage, the corresponding relay forwards on a packet from its buffer to the destination.
Section 4.3. Outage Probability

2. Otherwise, if the selected link in step 1) is in outage or there are no available relay-to-destination links at time \( t \), choose the link with the highest channel SNR among all available source-to-relay links \(|h_{sr_k}(t)|^2\). If the selected link is not in outage, the source transmits one packet to the corresponding relay and the packet is stored in the buffer. Otherwise outage occurs.

4.3 Outage Probability

At any time, the numbers of data packets in the buffers of both relays forms a “state”. Because there are \( N \) relays and every relay has a buffer of size \( L \), there are \((L + 1)^N\) states in total. The \( l \)-th state vector is defined as

\[
\mathbf{s}_l = [\Psi_l(Q_1), \cdots, \Psi_l(Q_N)], \quad l = 1, \cdots, (L+1)^N, \tag{4.3.1}
\]

where \( \Psi_l(Q_k) \) gives the number of data packets in buffer \( Q_k \) at state \( s_l \). It is clear that \( 0 \leq \Psi_l(Q_k) \leq L \).

Every state corresponds to one pair of \((K_{sr}^{(s_l)}, K_{rd}^{(s_l)})\), where \( K_{sr}^{(s_l)} \) and \( K_{rd}^{(s_l)} \) are the numbers of available links for source-to-relay and relay-to-destination transmission, respectively. A source-to-relay link is considered available when the buffer of the corresponding relay node is not full. At state \( s_l \), the number of available source-to-relay links \( K_{sr}^{(s_l)} \) can be obtained as

\[
K_{sr}^{(s_l)} = \sum_{k=1}^{N} \Phi^+_l(Q_k), \tag{4.3.2}
\]

where \( \Phi^+_l(Q_k) = 0 \) if the buffer \( Q_k \) is full and \( \Phi^+_l(Q_k) = 1 \) otherwise. On the other hand, a relay-to-destination link is available when the corresponding relay buffer is not empty, and the number of available relay-to-destination links \( K_{rd}^{(s_l)} \) is given by

\[
K_{rd}^{(s_l)} = \sum_{k=1}^{N} \Phi^-_l(Q_k), \tag{4.3.3}
\]
where \( \Phi^{-l}(Q_k) = 0 \) if the buffer \( Q_k \) is empty and \( \Phi^{-l}(Q_k) = 1 \) otherwise. It is clear from (4.3.2) and (4.3.3) that \( 0 \leq K_{sr}^{(s_l)} \leq N \) and \( 0 \leq K_{rd}^{(s_l)} \leq N \). Specifically, if none of the buffers is full or empty, all links are available such that \( K_{sr}^{(s_l)} = K_{rd}^{(s_l)} = N \).

Considering all possible states, the outage probability of the proposed buffer-aided scheme can be obtained as

\[
P_{out} = \sum_{l=1}^{(L+1)^N} \pi_l p_{\text{out}}^{s_l},
\]

(4.3.4)

where \( \pi_l \) is the stationary probability for state \( s_l \), and \( p_{\text{out}}^{s_l} \) is the outage probability at state \( s_l \). In the following two subsections, \( p_{\text{out}}^{s_l} \) and \( \pi_l \) are derived respectively.

### 4.3.1 \( p_{\text{out}}^{s_l} \): outage probability at state \( s_l \)

According to the relay selection rule of the proposed scheme, at state \( s_l \), outage occurs if and only if all available links are in outage. Further noting the i.i.d. assumption of all links, the outage probability for state \( s_l \) is given by

\[
p_{\text{out}}^{s_l} = (P(C_i < r_t))^{K_{sl}},
\]

(4.3.5)

where \( P(C_i < r_t) \) is the probability that a single link becomes outage, \( r_t \) is the target data rate and \( K_{sl} = K_{sr}^{(s_l)} + K_{rd}^{(s_l)} \) which is the total number of available links at state \( s_l \).

Since \( C_i = \log_2(1 + \gamma_i) \) and the SNR \( \gamma_i \) is exponentially distributed, it has

\[
P(C_i < r_t) = F_{\gamma}(\Delta) = \left(1 - e^{-\frac{\Delta}{\gamma}}\right),
\]

(4.3.6)

where \( F_{\gamma}(.) \) is the CDF of \( \gamma_i \) and \( \Delta = 2^{r_t} - 1 \).
Substituting (4.3.6) into (4.3.5) gives

\[ p_{\text{out}}^{s_l} = F_{\gamma}(\Delta)^{K_{st}} = \left(1 - e^{-\frac{\Delta}{\bar{\Delta}}}ight)^{K_{st}}. \] (4.3.7)

### 4.3.2 \( \pi_l \): stationary probability of the state \( s_l \)

Let \( A \) denote as the \((L + 1)^N \times (L + 1)^N\) state transition matrix, where the entry \( A_{n,l} = P(X_{t+1} = s_n|X_t = s_l) \) is the transition probability that the state moves from \( s_l \) at time \( t \) to \( s_n \) at time \( (t + 1) \). Suppose without losing generality that at time slot \( t \), the state is at \( s_l \).

At the next time \((t + 1)\), if the selected link becomes outage, the state \( s_l \) remains unchanged. Or the probability that \( s_l \) remains unchanged at time \((t + 1)\) is \( p_{\text{out}}^{s_l} \).

Otherwise if the selected link is not in outage, depending on which link is selected for transmission, \( s_l \) may move to several possible states at time \((t + 1)\). In the proposed relay selection scheme, in order to minimise the packet delay, the relay-to-destination links have higher priority to be selected than the source-to-relay links. As a result, the probabilities to select the source-to-relay and relay-to-destination links are not the same, which is very different from existing buffer-aided relay schemes such as the max-link scheme in [33].

With this observation, all states which can be moved from \( s_l \) can be divided into two sets as:

\[
U_{s_l}^{S\rightarrow R} = \left\{ \bigcup_{1 \leq n \leq (L+1)^N} s_n : s_n - s_l \in Q^{S\rightarrow R} \right\} \\
U_{s_l}^{R\rightarrow D} = \left\{ \bigcup_{1 \leq n \leq (L+1)^N} s_n : s_n - s_l \in Q^{R\rightarrow D} \right\},
\] (4.3.8)

where \( Q^{S\rightarrow R} \triangleq \left\{ \bigcup_{1 \leq k \leq K} + e_k \right\} \), \( Q^{R\rightarrow D} \triangleq \left\{ \bigcup_{1 \leq k \leq K} - e_k \right\} \), and \( e_k \) is the vector where its \( k \)-th element is one and all other elements are zero. To be specific, if a source-to-relay link is selected, the number of packets in the selected
buffer is increased by one, and then state $s_l$ moves to $s_n$ with $s_n - s_l \in Q^{S\rightarrow R}$. Therefore, the set $U^{S\rightarrow R}_{s_l}$ contains all states to which $s_l$ can move when a source-to-relay link is selected. Similarly, $U^{R\rightarrow D}_{s_l}$ contains all states to which $s_l$ can move when a relay-to-destination link is selected.

The probabilities that all available source-to-relay links and source-to-relay links are in outage are given by

$$p_{S\rightarrow R}^{out} = \left(1 - e^{-\frac{\Delta}{\gamma}}\right)^{K_{sr}^{(s_l)}}.$$ (4.3.9)

$$p_{R\rightarrow D}^{out} = \left(1 - e^{-\frac{\Delta}{\gamma}}\right)^{K_{rd}^{(s_l)}}.$$ (4.3.10)

respectively. In the proposed scheme, a relay-to-destination link is selected if not all of the $K_{rd}^{(s_l)}$ available relay-to-destination links are in outage. Thus the probability to select one relay-to-destination link at state $s_l$ is given by

$$p_{s_l}^{rd} = \frac{1}{K_{rd}^{(s_l)}} (1 - p_{R\rightarrow D}^{out}) = \frac{1}{K_{rd}^{(s_l)}} \left(1 - \left(1 - e^{-\frac{\Delta}{\gamma}}\right)^{K_{rd}^{(s_l)}}\right).$$ (4.3.11)

On the other hand, because a source-to-relay link is selected only when all relay-to-destination links are in outage and not all source-to-relay links are in outage, the probability to select one source-to-relay link at state $s_l$ is given by

$$p_{s_l}^{sr} = \frac{1}{K_{sr}^{(s_l)}} p_{R\rightarrow D}^{out} \left(1 - p_{S\rightarrow R}^{out}\right) = \frac{1}{K_{sr}^{(s_l)}} \left(1 - \left(1 - e^{-\frac{\Delta}{\gamma}}\right)^{K_{sr}^{(s_l)}}\right) \left(1 - \left(1 - e^{-\frac{\Delta}{\gamma}}\right)^{K_{rd}^{(s_l)}}\right).$$ (4.3.12)

With these observations, the $(n,l)$-th entry of the state transition matrix $A$ is expressed as

$$A_{n,l} = \begin{cases} p_{s_l}^{sr}, & \text{if } s_n = s_l, \\ p_{s_l}^{rd}, & \text{if } s_n \in U^{R\rightarrow D}_{s_l}, \\ p_{s_l}^{sr}, & \text{if } s_n \in U^{S\rightarrow R}_{s_l}, \\ 0, & \text{elsewhere}, \end{cases}$$
where $p_{sl}^n$, $p_{sl}^-$ and $p_{sl}^+$ are given by (4.3.7), (4.3.10) and (4.3.11) respectively.

Because the transition matrix $A$ in (4.3.12) is column stochastic, irreducible and aperiodic\(^2\), the stationary state probability vector is obtained as (see [49,50])

$$\pi = (A - I + B)^{-1}b,$$

where $\pi = [\pi_1, \cdots, \pi_{(L+1)^N}]^T$, $b = (1, 1,\ldots, 1)^T$, $I$ is the identity matrix and $B_{n,l}$ is an $n \times l$ all one matrix.

Finally, substituting (4.3.12) and (4.3.13) into (4.3.4) gives the outage probability as

$$P_{out} = \sum_{l=1}^{(L+1)^N} \pi_l p_{sl}^n = \text{diag}(A)\pi$$

$$= \text{diag}(A)(A - I + B)^{-1}b,$$

where $\text{diag}(A)$ is a vector consisting of all diagonal elements of $A$.

### 4.4 Average Packet Delay

The delay of a packet in the system is the duration between the time when the packet leaves the source node and the time when it arrives the destination. Because it takes one time slot to transmit a packet from the source to a relay node, the average packet delay in the system is given by

$$\bar{D} = 1 + \bar{D}_r,$$

where $\bar{D}_r$ is the average delay at the relay nodes.

Because suppose all channels are i.i.d., the average delay through every relay node is the same. Without losing generality, the average delay through relay $R_k$ is analysed below. Based on Little’s Law [52], the average packet delay $\bar{D}_r$ is given by

\[D = 1 + \bar{D}_r\]
delay at $R_k$ is given by

$$D_r = D_k = \frac{\bar{L}_k}{\bar{\eta}_k},$$  \hspace{1cm} (4.4.2)

where $\bar{L}_k$ and $\bar{\eta}_k$ are the average queuing length and average throughput at $R_k$ respectively.

The average queuing length at $R_k$ is obtained by averaging the queueing lengths of the buffer $Q_k$ over all states, or it has

$$\bar{L}_k = \sum_{l=1}^{(L+1)N} \pi_l \Psi_l(Q_k)$$ \hspace{1cm} (4.4.3)

where recall that $\Psi_l(Q_k)$ gives the number of packets (or the buffer length) of buffer $Q_k$ at state $s_l$, and $\pi_l$ is given by (4.3.13).

On the other hand, because the probabilities to select any of the relays are the same, the average throughput at $R_k$ is given by

$$\bar{\eta}_k = \frac{\bar{\eta}}{N}$$ \hspace{1cm} (4.4.4)

where $\bar{\eta}$ is the average throughput of the overall system network which is given by

$$\bar{\eta} = R(1 - P_{out}),$$ \hspace{1cm} (4.4.5)

where $R$ is the average data rate (without considering the outage probability) of the system and $P_{out}$ is given by (4.3.14).

In the proposed scheme, suppose there are $M$ data packets for transmission in total. Because each packet transmission takes two time slots (not necessarily consecutively) if there is no outage, the overall transmission time to deliver all packets is $2M$. Therefore, the average data rate is $R = (M/2M) = 1/2$. This states that, although different packets in the proposed scheme may have different delays in the system, the average data rate is the same as that of the classic three-node “$S \rightarrow R \rightarrow D$” relay system [51].
Then substituting \((R = 1/2)\) and (4.4.5) into (4.4.4) gives

\[
\bar{\eta}_k = \frac{1 - P_{\text{out}}}{2N}.
\] (4.4.6)

Finally, substituting (4.4.3) and (4.4.6) into (4.4.2), and further into (4.4.1), gives the average packet delay in the proposed scheme as

\[
\bar{D} = 1 + \frac{2N \sum_{l=1}^{(L+1)N} \pi_l \Psi_l(Q_k)}{1 - P_{\text{out}}}.
\] (4.4.7)

### 4.5 Asymptotic Performance

This section analyse the asymptotic performance of the proposed scheme when the average channel SNR goes to infinity. While it assumes asymmetric channel scenarios, the average channel SNR for source-to-relay and relay-to-destination link can be respectively expressed as

\[
\bar{\gamma}_{sr} = \bar{\gamma} \quad \text{and} \quad \bar{\gamma}_{rd} = \beta \bar{\gamma},
\] (4.5.1)

where \(\alpha\) and \(\beta\) are positive real constants, and \(\bar{\gamma}\) is normalised average channel SNR. Particularly for symmetric channel assumption, it has \(\alpha = \beta\).

Below it is first to derive the asymptotic outage probability for \(\bar{\gamma} \to \infty\), from which the diversity order, coding gain and average packet delay of the proposed scheme are obtained.

#### 4.5.1 Asymptotic outage probability

When the average channel SNR \(\bar{\gamma} \to \infty\), it is clear from (??) that the probability to select one relay-to-destination link at any state \(s_l\) is given by

\[
\lim_{\bar{\gamma} \to \infty} P_{s_l}^{R \to D} = 1, \quad \text{if} \ K^{(s_l)}_{rd} \neq 0.
\] (4.5.2)
This implies that, when $\bar{\gamma} \to \infty$, any packets in the relay buffers will be forwarded to the destination until all buffers are empty. Only when all buffers are empty, is a new packet transmitted to one of the relays.

Therefore, when $\bar{\gamma} \to \infty$, the buffers can only be in two possible states: $S^{(0)}$ and $S^{(1)}$, where $S^{(0)}$ is the state that all buffers are empty, and $S^{(1)}$ contains a group of $N$ possible states and every state in $S^{(1)}$ corresponds to the case that only one of the buffers has one packet and all other buffers are empty. Then from (4.5.2), the outage probability of the system is given by

$$\lim_{\bar{\gamma} \to \infty} P_{\text{out}} = P(S^{(0)}) \cdot p_{\text{out}}^{S^{(0)}} + P(S^{(1)}) \cdot p_{\text{out}}^{S^{(1)}},$$

where $P(S^{(0)})$ and $P(S^{(1)})$ are the probabilities that buffers are in states $S^{(0)}$ and $S^{(1)}$ respectively, and $p_{\text{out}}^{S^{(0)}}$ and $p_{\text{out}}^{S^{(1)}}$ are the outage probabilities for the states in $S^{(0)}$ and $S^{(1)}$ respectively.

It assumes that, at time $t$, all buffers are empty so that the state is in $S^{(0)}$. Then one packet will be transmitted to one of the relays at the next time, and the state is in $S^{(1)}$ at $(t+1)$. From (4.5.2), with probability 1, the packet in the buffer is forwarded to the destination at $(t+2)$ and the state returns to $S^{(0)}$. This process continues until all packets are transmitted. With this observation, therefore, it has

$$P(S^{(0)}) = P(S^{(1)}) = \frac{1}{2} \quad (4.5.4)$$

When the buffers are in state $S^{(0)}$, there are $N$ available source-to-relay links and no available relay-to-destination links. Then it has

$$p_{\text{out}}^{S^{(0)}} = \left(1 - e^{-\frac{\Delta}{\bar{\gamma} \sigma}}\right)^N. \quad (4.5.5)$$

On the other hand, when the buffers are in state $S^{(1)}$, if the buffer size $L = 1$, there are $(N - 1)$ available source-to-relay links and one available
relay-to-destination links. If $L \geq 2$, there are $N$ available source-to-relay links and one available relay-to-destination link. Or the total number of available source-to-relay links for state $S^{(1)}$ is given by

$$K_\infty = \begin{cases} 
N - 1, & L = 1 \\
N, & L \geq 2
\end{cases} \quad (4.5.6)$$

Then the outage probability for the states in $S^{(1)}$ is given by

$$p_{\text{out}}^{S^{(1)}} = \left(1 - e^{-\frac{\Delta}{7sr}}\right)^{K_\infty} \cdot \left(1 - e^{-\frac{\Delta}{7rd}}\right). \quad (4.5.7)$$

Substituting (4.5.4), (4.5.5) and (4.5.7) into (4.5.3) gives

$$\lim_{\gamma \to \infty} P_{\text{out}} = \frac{1}{2} \cdot \left(1 - e^{-\frac{\Delta}{7sr}}\right)^N + \frac{1}{2} \cdot \left(1 - e^{-\frac{\Delta}{7sr}}\right) K_\infty \cdot \left(1 - e^{-\frac{\Delta}{7rd}}\right)$$

$$= \frac{1}{2} \cdot \left(1 - e^{-\frac{\Delta}{7sr}}\right)^N + \frac{1}{2} \cdot \left(1 - e^{-\frac{\Delta}{7sr}}\right) K_\infty \cdot \left(1 - e^{-\frac{\Delta}{7rd}}\right) \quad (4.5.8)$$

### 4.5.2 Diversity order

The diversity order can be defined as

$$d = - \lim_{\gamma \to \infty} \frac{\log P_{\text{out}}}{\log \gamma}. \quad (4.5.9)$$

If the buffer size $L = 1$, substituting (4.5.8) (by letting $K_\infty = N - 1$) into (4.5.9), and further noting that $e^x \approx 1 + x$ for very small $x$, it has the
diversity order for $L = 1$ as

$$d^{(L=1)} = -\lim_{\hat{\gamma} \to \infty} \frac{\log \left[ \frac{1}{2} \cdot \left( \frac{\Delta}{\hat{\gamma}} \right)^{N-1} \cdot \left( \frac{\Delta}{\hat{\gamma} + \Delta} \right) \right]}{\log \hat{\gamma}}$$

$$= -\lim_{\hat{\gamma} \to \infty} \frac{\log \left[ \frac{1}{2} \cdot \left( \frac{1}{\hat{\gamma}} \right)^{N-1} \left( \frac{1}{\hat{\gamma}} + \frac{1}{\Delta} \right) \left( \frac{\Delta}{\hat{\gamma}} \right) \right]}{\log \hat{\gamma}}$$

$$= -\lim_{\hat{\gamma} \to \infty} \frac{\log \left[ \frac{1}{2} \cdot \left( \frac{1}{\hat{\gamma}} \right)^{N-1} \left( \frac{1}{\hat{\gamma}} + \frac{1}{\Delta} \right) \left( \frac{\Delta}{\hat{\gamma}} \right)^N \right]}{\log \hat{\gamma}} - N \log \hat{\gamma}$$

$$= N$$

On the other hand, if the buffer size $L \geq 2$, the asymptotic outage probability is obtained by letting $K_\infty = N$ in (4.5.8). Then it has

$$\lim_{\hat{\gamma} \to \infty} P_{out}^{(L \geq 2)} = \lim_{\hat{\gamma} \to \infty} \left[ \frac{1}{2} \cdot \left( \frac{\Delta}{\hat{\gamma}} \right)^N \cdot \left( \frac{\beta \hat{\gamma} + \Delta}{\beta \hat{\gamma}} \right) \right].$$

(4.5.11)

Because

$$\lim_{\hat{\gamma} \to \infty} (\beta \hat{\gamma}) < \lim_{\hat{\gamma} \to \infty} (\beta \hat{\gamma} + \Delta) < \lim_{\hat{\gamma} \to \infty} (2 \cdot \beta \hat{\gamma}),$$

(4.5.12)

Thus with same approach in (4.5.13), the diversity order for $L \geq 2$ can be obtained

$$N < d^{(L \geq 2)} < N + 1$$

(4.5.13)

### 4.5.3 Coding gain

It shows below that, compared with the traditional non-buffer-aided max-min relay selection scheme, the proposed relay selection scheme also has coding gain. The coding gain is defined as the SNR difference (in dB) between the max-min and proposed schemes to achieve the same outage probability. In order to highlight the coding gain, the buffer size of the proposed scheme is set as $L = 1$ so that the diversity order for the max-min and proposed schemes are both $N$. From the above definition, the coding gain can be
obtained as

\[ C (\text{dB}) = - \lim_{\bar{\gamma} \to \infty} \frac{\Delta P (\bar{\gamma})}{d} \quad (4.5.14) \]

where \( d = N \) which is the diversity order for both scheme \((L = 1)\) and

\[ \Delta P (\bar{\gamma}) = 10 \log P_{\text{out}}^{(\text{max-min})} (\bar{\gamma}) - 10 \log P_{\text{out}}^{(L=1)} (\bar{\gamma}), \quad (4.5.15) \]

where \( P_{\text{out}}^{(\text{max-min})} (\bar{\gamma}) \) and \( P_{\text{out}}^{(L=1)} (\bar{\gamma}) \) are the outage probabilities at the SNR \( \bar{\gamma} \) for the max-min and proposed schemes (with \( L = 1 \)) respectively.

From (4.5.8) and (4.5.13), it has

\[ \lim_{\bar{\gamma} \to \infty} 10 \log P_{\text{out}}^{(L=1)} = 10 \cdot \log \left[ \frac{1}{2} \cdot \left( \frac{1}{\alpha} \right)^{N-1} \left( \frac{1}{\alpha + \frac{1}{\beta}} \right) \right] + \lim_{\bar{\gamma} \to \infty} 10 \cdot \log \left( \frac{\Delta}{\bar{\gamma}} \right)^N \quad (4.5.16) \]

While for the tradition max-min scheme, it has

\[ \lim_{\bar{\gamma} \to \infty} 10 \log P_{\text{out}}^{(\text{max-min})} = \lim_{\bar{\gamma} \to \infty} 10 \cdot \log \left( \frac{\Delta}{\alpha \bar{\gamma}} + \frac{\Delta}{\beta \bar{\gamma}} \right)^N = 10 \cdot \log \left( \frac{1}{\alpha} + \frac{1}{\beta} \right)^N + \lim_{\bar{\gamma} \to \infty} 10 \cdot \log \left( \frac{\Delta}{\bar{\gamma}} \right)^N \quad (4.5.17) \]

Substituting (4.5.16) and (4.5.17) into (4.5.15) gives

\[ \lim_{\bar{\gamma} \to \infty} \Delta P (\bar{\gamma}) = -10 \cdot \log \left[ \frac{1}{2} \left( \frac{\beta}{\alpha + \beta} \right)^{N-1} \right]. \quad (4.5.18) \]

Finally, substituting (4.5.18) into (4.5.14) gives the coding gain of the proposed scheme as

\[ C (\text{dB}) = \frac{-10 \cdot \log \left[ \frac{1}{2} \left( \frac{\beta}{\alpha + \beta} \right)^{N-1} \right]}{N} \quad (4.5.19) \]

It can be easily obtained that, for symmetric channel configuration with \( \alpha = \beta \), the coding gain is 3dB.
Section 4.6. Simulations and Discussions

4.5.4 Average packet delay

It has been shown that, when $\bar{\gamma} \to \infty$, the buffer states can only be in either $S^{(0)}$ or $S^{(1)}$. Thus for any relay buffer $Q_k$, it either is empty or contains one packet. When all buffers are empty or the state is in $S^{(0)}$, a new packet is transmitted to any of the relays with equal probability of $1/N$. Further from (4.5.4) that $P(S^{(1)}) = 1/2$, the probability that $Q_k$ contains one packet is given by

$$P(Q_k = 1) = P(S^{(1)}) \cdot \frac{1}{N} = \frac{1}{2N}. \quad (4.5.20)$$

Thus, when $\bar{\gamma} \to \infty$, the average buffer length at relay $R_k$ is given by

$$\lim_{\bar{\gamma} \to \infty} \bar{L}_k = 1 \cdot P(Q_k = 1) = P(S^{(1)}) \cdot \frac{1}{N} = \frac{1}{2N}. \quad (4.5.21)$$

From (4.4.6), and noticing that $\lim_{\bar{\gamma} \to \infty} P_{out} = 0$, the average throughput at relay $Q_k$ is given by

$$\lim_{\bar{\gamma} \to \infty} \eta_k = \frac{\lim_{\bar{\gamma} \to \infty}(1 - P_{out})}{2N} = \frac{1}{2N} \quad (4.5.22)$$

Finally, substituting (4.5.21) and (4.5.22) into (4.4.2), and further into (5.6.1), gives the average packet delay for $\bar{\gamma} \to \infty$ as

$$\lim_{\bar{\gamma} \to \infty} \bar{D} = 1 + \frac{1/(2N)}{1/(2N)} = 2. \quad (4.5.23)$$

It is clearly shown in (4.5.23) that, when SNR is high enough, the average packet delay of the proposed scheme is the same as that for the non-buffer-aided schemes.

4.6 Simulations and Discussions

This section verifies the proposed scheme with numerical simulations, where the results for max-link and max-min scheme are also shown for comparison.
In the simulation below, the average transmission powers for all transmission nodes and the noise variances for all receiving nodes are all normalised to unity. The transmission rates in all schemes are set as $r_t = 2 \text{ bps/Hz}$. All simulation results are obtained with 1,000,000 Monte Carlo runs. Particularly for the proposed scheme, both the theoretical analysis and simulation results are shown. In all simulations for outage probabilities and average delays, the simulation results perfectly match the theoretical analysis in the proposed scheme, which very well verifies the analysis in the paper.

In order to fully reveal the proposed scheme, both the symmetric and asymmetric channel configurations are considered below.

### 4.6.1 Symmetric channel configuration: $\bar{\gamma}_{sr} = \bar{\gamma}_{rd}$

In the first simulation, symmetric channel scenario that the source-to-relay and relay-to-destination links have same channel SNR-s is considered.

Fig. 4.2 (a) and (b) compare the outage probabilities and average packet delays for the non-buffered max-min, traditional max-link and proposed schemes respectively, where the relay number is fixed at $N = 3$, and $\alpha = \beta = 1.5$ and $\bar{\gamma} = 10 \text{ dB}$ in (4.5.1) so that $\bar{\gamma}_{sr} = \bar{\gamma}_{rd} = 15 \text{ dB}$. Fig. 4.2 (a) shows that, when the buffer size $L = 1$, the proposed and max-link have the same outage probabilities, where both have significantly better outage performance than the traditional non-buffer-aided max-min scheme. This is because that, when $L = 1$, both max-link and proposed schemes have diversity order of $N = 3$, and coding gains of 3dB over the max-min scheme. When the buffer size increases to $L = 5$, the proposed scheme has slightly better outage performance than that for $L = 1$. This well matches the asymptotic analysis that, when $L \geq 2$, the diversity order is larger than $N$ but smaller than $(N + 1)$ for the proposed scheme. On the other hand, for the max-link scheme, the outage performance improves more significantly with larger buffer size. This is because that diversity order of the max-link
scheme goes up with the buffer size, until it reaches $2N$ when $L \to \infty$.

Fig. 4.2 (b) shows that, even for $L = 1$, the average delay of the max-link scheme is at least twice as much that for the proposed scheme. When the buffer size increases to $L = 5$, the average packet delay of the proposed scheme still maintains at 2 in high SNR range, which is the same as that for $L = 1$. On the other hand, when $L = 5$, the average packet delay of the max-link scheme increases to 18 at high SNR-s, which is 9 times larger than that of the proposed scheme.

To further compares the delay performance of the max-link and proposed schemes in symmetric channels, Fig. 4.3 (a) and (b) show the average packet delay vs the buffer size and relay number respectively, where the channel SNR-s in both schemes are set as 10 dB. In Fig. 4.3 (a), the relay number is fixed at $N = 2$, and the buffer size varies from 1 to 20. In Fig. 4.3 (b), the buffer size is fixed at $L = 10$, but the relay number varies from 1 to 10. It is clearly shown in both Fig. 4.3 (a) and (b) that, the average packet delay for the proposed scheme remains at a constant value of 2. On the other hand, the packet delay in the max-link scheme goes up linearly with either $N$ or $L$, which is effectively ‘out of control’.

In order to reveal the diversity order and coding gain of the proposed scheme, Fig. 4.4 compares the outage probabilities of the proposed and non-buffer-aided max-min scheme at very high SNR-s, where the relay number is set as $N = 4$ and all results are from theoretical analysis. First the coding gain is clearly 3 dB by comparing the max-min and proposed scheme with $L = 1$. For example, to achieve the outage probability of $10^{-34} = -340$ dB, the SNR-s for the max-min and proposed scheme with $L = 1$ are about 85 and 88dB respectively, so that the coding gain is clearly $88 - 85 = 3$ dB. The diversity order of the proposed scheme is also clearly shown to be $(N, N + 1)$ for $L \geq 2$. For example, as is illustrated in the figure, for the proposes scheme with $L = 8$, the SNRs to achieve the outage probabilities
Figure 4.2. Outage probabilities and average delay among different schemes, where $\bar{\gamma}_{sr} = \bar{\gamma}_{rd} = 10$ dB.
Figure 4.3. Average packet delay comparison between the max-link and proposed schemes.
of $-315$ and $-340$ dB are about 78 and 84 dB, respectively. Then according to the diversity order definition in (4.5.9), the diversity order is obtained as $(340 - 315)/(84 - 78) = 4.17$, which is clearly between $N = 4$ and $N + 1 = 5$. Thus the results in Fig. 4.4 very well verify the coding gain and diversity order analysis in Section ??.

![Figure 4.4. Diversity order and Coding gain of the proposed scheme.](image)

In summary, Table I compares the asymptotic performance (including diversity order, coding gain and average delay) that $\gamma \to \infty$ for the different schemes, where for comparison another buffer-aided relay selection scheme (max-max) is also included in the table.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>max-min</th>
<th>max-max</th>
<th>max-link</th>
<th>proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diversity order</td>
<td>$N$</td>
<td>$N$</td>
<td>$[N, 2N)$</td>
<td>$(N, N + 1)$</td>
</tr>
<tr>
<td>Coding gain</td>
<td>0 dB</td>
<td>3 dB</td>
<td>3 dB</td>
<td>3 dB</td>
</tr>
<tr>
<td>Average delay</td>
<td>$2$</td>
<td>$(NL/2) + 1$</td>
<td>$NL + 1$</td>
<td>2</td>
</tr>
</tbody>
</table>

In the proposed scheme, the diversity order is $N$ for buffer size $L = 1$, and between $N$ and $(N + 1)$ for any buffer size $L \geq 2$. In the max-max scheme, the diversity order is $N$, but the buffers need to be carefully initialised to achieve the diversity [38]. In the max-link scheme, the diversity order increases with
buffer size $L$. Specifically, the diversity order of the max-link scheme is $N$ when $L = 1$, and $2N$ when $L \to \infty$. The coding gains of all of the buffer-aided schemes (including the max-max, max-link and proposed) are 3 dB. On the other hand, the average packet delay of the max-link scheme is about twice of that of the max-max scheme. In either the max-max or max-link scheme, the average packet delay increases linearly with both the relay number $N$ and buffer size $L$. In the proposed scheme, when $\bar{\gamma} \to \infty$, the average delay is fixed at 2 which is the same as that for the non buffer-aided max-min scheme. Thus for symmetric channel configuration, while the proposed scheme has lower diversity gain than the max-link scheme, its average delay is significantly reduced. The proposed scheme is also superior to the max-max scheme in both outage and delay performance.

It worth to highlight that Table I is only for the symmetric channel configuration. For asymmetric channels, the schemes performs significantly differently as will be shown below.

4.6.2 Asymmetric channel configuration: $\bar{\gamma}_{sr} > \bar{\gamma}_{rd}$

In Fig. 4.5, asymmetric channels that source-to-relay links are stronger than relay-to-destination links in average is considered, where $\alpha = 2$, $\beta = 1$ and $\bar{\gamma} = 10$ dB in (4.5.1) so that $\bar{\gamma}_{sr} = 20$ dB and $\bar{\gamma}_{rd} = 10$ dB, and relay number is fixed at $N = 3$.

It is very interesting to observe in Fig. 4.5 (a) that, for both $L = 1$ and $L = 5$, the outage performance of the proposed scheme is significantly better than the max-link scheme! This is because that, when the source-to-relay links are stronger than relay-to-destination links, the max-link scheme is more likely to select the source-to-relay links so that the buffers are more likely full. This effectively decreases the number of the available source-to-relay links (as a full buffer cannot receive more data), leading to fewer diversity order. On the other hand, in the proposed scheme, while the chan-
nel condition gives higher priority to the source-to-relay selection, the selection rule gives higher priority to the relay-to-destination link selection. This leads to a more ‘balanced’ buffers at the relays, or fewer full or empty buffers, which again increases the diversity order.

Fig. 4.5 (b) shows that, the average delay of the max-link even worse than that in symmetric channels. This is because the buffers are more likely to be full, or higher queuing length at buffers. On the contrary, the average delay for the proposed scheme is still as low as about 2 at high SNR range.

Therefore, when $\bar{\gamma}_{sr} > \bar{\gamma}_{rd}$, the proposed scheme has better performance in both outage probability and average delay than the max-link scheme.

### 4.6.3 Asymmetric channel configuration: $\bar{\gamma}_{sr} < \bar{\gamma}_{rd}$

Fig. 4.6 assumes that the source-to-relay link is weaker than the relay-to-destination link in average, where $\alpha = 1$, $\beta = 2$ and $\bar{\gamma} = 10$ dB in (4.5.1) so that $\bar{\gamma}_{sr} = 10$ dB and $\bar{\gamma}_{rd} = 20$ dB, and relay number is set as $N = 3$.

It is interesting to observe in Fig. 4.6 that, the max-link and proposed schemes have similarly performance both in outage and average delay. This is because that, stronger relay-to-destination links ‘naturally’ give higher priority to select the relay-to-destination links. But even under this channel assumption, the average packet delay is still better constrained in the proposed scheme than in the max-link scheme, particularly in low SNR ranges.

### 4.6.4 Summary

It has shown the outage and average delay performance under different channel configurations. To be specific, for symmetric $S \rightarrow R$ and $R \rightarrow D$ channels, the max-link scheme has better outage performance than the proposed. But when $S \rightarrow R$ links are stronger, the proposed scheme performs better in outage than the max-link. On the other hand, when $R \rightarrow D$ links are stronger, the max-link and proposed scheme have similar outage per-
Figure 4.5. Outage probabilities and average delay among different schemes, where $\bar{\gamma}_{sr} = 20$ dB and $\bar{\gamma}_{rd} = 10$ dB.
Section 4.6. Simulations and Discussions

Figure 4.6. Outage probabilities and average delay among different schemes, where $\bar{\gamma}_{sr} = 10$ dB and $\bar{\gamma}_{rd} = 20$ dB.
formance. Therefore, if the relay nodes are evenly spread within an area as in many practical systems, it is reasonable to expect that the outage performance of the proposed and max-link schemes are similar. This will be left for future study. It also worth highlight that, in all cases, the proposed scheme has significantly better outage performance than the non-buffer-aided schemes.

On the other hand, in all channel configuration, the average delay of the proposed scheme is well constrained close to the classic 3-node relay network (which is 2). This is supported by both theoretical analysis and numerical simulations. However, in the max-link scheme, the delay is simply ‘out of control’, increasing linearly with both the relay number and buffer size. Further considering that the delay considered in this paper is only for the 2-hop relay network and will be much more serious in a multihop network, the traditional max-link scheme has very limited application. The proposed scheme is certainly a more attractive alternative in applying buffers in relay network.

Finally the observation that the proposed scheme performs differently under different channel conditions also suggests new design approaches in practical systems: i.e. relay nodes can be selected to achieve both outage and delay improvement. This interesting issue will be left for future study.

4.7 Conclusion

This chapter proposed a novel buffer-aided relay selection scheme. Both the outage performance and average packet delay of the proposed scheme were analysed. Unlike the traditional buffer-aided schemes, the proposed scheme achieves outage performance gains without increasing the average packet delay, making it an attractive scheme in practical applications. Numerical simulation results were also shown to verify the proposed scheme.
Chapter 5

BUFFER-AIDED LINK SELECTION WITH NETWORK-CODING IN MULTI-HOP NETWORKS

This chapter proposes a novel buffer-aided link selection scheme based on network-coding in a multiple hop relay network. Compared with existing approaches, the proposed scheme significantly increases the system throughput. This is achieved by applying data buffers at the relays to decrease the outage probability and using network-coding to increase the data rate. The closed-form expressions of both the average throughput and packet delay are derived. The proposed scheme has not only significantly higher throughput than both the traditional and existing buffer-aided max-link scheme, but also smaller average packet delay than the max-link scheme, making it an attractive scheme in practice.

5.1 Introduction

Relay networks have been well investigated as attractive transmission schemes in wireless communications [44]. Current research mainly focuses on 2-hop
relay networks in which every data packet passes over two hops in the process of being transmitted from the source to destination through a relay node [56]. Relatively less has been put in studying relay networks with more than two hops. A multi-hop relay network can be seen in many scenarios. A typical example is device-to-device (D2D) communications in a cellular system, where some mobile users may directly communicate with each other (D2D communications) rather than through the base station (cellular communications) [57, 58]. Because the transmission powers for the D2D mobile users are usually strictly limited to avoid interfering with the base station, multi-hop transmission can be required for D2D communications [59].

Conventionally the multi-hop links are consecutively selected for data transmission. Recent research shows that applying data buffers at the relays significantly improves the transmission performance. Due to the presence of data buffers at the relays, when a data packet arrives at a node, it may not be immediately forwarded to the next node. Instead, other links with better SNR may be selected for data transmission. This so called adaptive link selection ([31,32]) is particularly useful in a D2D cellular system, because the base station often has knowledge of the CSI of the D2D links to coordinate the interference between the cellular and D2D communications. As a result, the base station can always select the best link for data transmission, rather than following the conventional hopping sequence.

Buffer-aided relay transmission has attracted much attention recently. Besides the aforementioned adaptive link selection, buffer-aided relays have also been used in applications including relay selection [30,33,34,60], cognitive radio networks [55] and physical layer network security [40]. Of particular interest is the max-link relay selection scheme due to its excellent outage performance [33]. In max-link relay selection, at any time slot, the link with the strongest channel SNR among all possible source-to-relay and relay-to-destination links is always selected for data transmission, leading
to diversity order of $2N$ if the buffer size is large enough (where $N$ is the number of relays). The max-link scheme can be straightforwardly used in multi-hop link selection, simply by selecting the link with the highest SNR among all possible multi-hop links at any time slot.

Of particularly interest in performance analysis of such multi-hop relay networks is the average throughput which is given by

$$\bar{\eta} = \bar{R}(1 - P_{out}), \quad (5.1.1)$$

where $P_{out}$ is the outage probability of the system and $\bar{R}$ is the average data rate (without considering the outage). It is known that the max-link scheme significantly reduces the outage probability. However, the max-link scheme still has the same data rate as the conventional scheme, because in both schemes only one link is selected for data transmission at any time slot. On the other hand, since the outage probability tends to zero when the SNR goes to infinity, it is clear from (5.1.1) that the average throughput mainly depends on the average data rate over the high SNR range. This implies that the max-link selection scheme mainly improves the system throughput over the low SNR range.

On the other hand, it is well known that physical layer network coding can be used to increase the data rate of a two-way relay network, where two source nodes exchange data packets through a single relay node [61]. To be specific, in physical layer network coding scheme, the two sources can transmit packets to, or receive packets from, the relay node simultaneously. Thus the data rate can reach 1 packet per time slot, rather than $1/2$ as in the conventional approach. This encourages author to apply physical layer network coding in multi-hop relay selection to increase the data rate. This can be achieved by simultaneously selecting two or more links for data transmission. As a result, the throughput of the multi-hop network over the
high SNR range can be improved.

In this chapter, a novel multi-hop link selection scheme which seamlessly integrates max-link selection and physical layer network coding is proposed so that the average throughput is significantly improved over both low and high SNR ranges. The main contributions of this chapter are listed as follows:

- **Proposing a novel buffer-aided network-coding link selection scheme for the multi-hop relay network.** The proposed scheme has significantly higher throughput than existing buffer-aided max-link schemes.

- **Describing a new analysis tool to obtain the average throughput of the proposed scheme.** Both the outage probability and average data rate are successfully derived to obtain the average throughput of the proposed scheme.

  - First, the outage probability analysis is based on a Markov chain representation of the buffer states, which is much more involved than those for existing approaches (e.g. [33]) due to the complicated link selection rules. Particularly, a trellis diagram is described to derive the transition probabilities between buffer states, based on which the outage probability is obtained.

  - Secondly, in the proposed multi-hop scheme, due to the simultaneous link transmissions, the calculation of the average data rate is far from straightforward. In this chapter, a trellis diagram is described to successfully obtain the average data rate. The analysis not only provides deep insight in understanding a multi-hop relay network, but also provides guidance in analysing similar systems.

- **Deriving the closed-form expression of the average packet delay of the proposed scheme.** The average packet delay is an important issue in
buffer-aided schemes. The analysis shows that the proposed scheme not only has larger throughput, but also shorter packet delay, than the max-link scheme, making it an attractive scheme in practice.

The remainder of this chapter is organised as follows: Section 5.2 shows the system model of the $N$-hop relay network; Section 5.3 proposes the buffer-aided network-coding link selection scheme; Section 5.4 and 5.5 analyse the outage probability and average data rate of the proposed scheme respectively; Section 5.6 analyses the average packet delay; Section 5.7 includes simulation results to verify the proposed scheme; finally Section 5.8 concludes the chapter.

5.2 $N$-hop relay network

The system model of the $N$-hop relay network is shown in Fig. 5.1, which includes are one source node ($S$), one destination node ($D$) and $(N-1)$ relay nodes ($R_1, \cdots, R_{N-1}$). Suppose that there are no direct links between two nodes separated by two hops or more, and all relays apply the decode-and-forward (DF) protocol and operate in the half-duplex mode.

Figure 5.1. The system model of the $N$-hop relay network.

For later use, the hopping links are consecutively named as $link_1, link_2, \cdots, link_N$ respectively, as is shown in Fig. 5.1. The channel coefficient and gain for $link_i$ at time slot $t$ is denoted as $h_i(t)$ and $\gamma_i(t) = |h_i(t)|^2$ respectively. Suppose that all channel links are i.i.d. quasi-static Rayleigh fading\(^1\), so that the channel gains $\gamma_i(t)$ are exponentially distributed with

\(^1\)While the analysis in this chapter is based on the i.i.d. channel assumption, it can be generalised to the case that every link has different average channel gain.
the same average gain as $\bar{\gamma} = E|h_i(t)|^2$ for all $i = 1, \cdots, N$. Without losing generality, suppose that transmission powers and all noise variances are normalized to unity.

In the traditional $N$-hop scheme, $link_1, link_2, \cdots, link_N$ are consecutively used for data transmission. If the transmission rates at all nodes are the same as $r_t$, the average data rate of the traditional scheme is given by

$$\bar{R}^{\text{(traditional)}} = \frac{1}{N} r_t.$$  \hfill (5.2.1)

In this chapter, the channels are assumed as quasi-static so that the coefficients remain unchanged during one hop interval but independently vary from one hop to another. Because every link accounts for $1/N$ of the total transmission time, and further considering that all channels are i.i.d. Rayleigh fading, the outage probability of the traditional $N$-hop relay network is obtained as

$$P_{\text{out}}^{\text{(traditional)}} = \frac{1}{N} P(C_1 < r_t) + \frac{1}{N} P(C_2 < r_t) + \cdots + \frac{1}{N} P(C_N < r_t)$$

$$= 1 - e^{-\frac{\Delta}{r_t}},$$  \hfill (5.2.2)

where $\Delta = 2^{r_t} - 1$, $C_i = \log_2(1 + \gamma_i)$ which is the instantaneous capacity for $link_i$.

Substituting (5.2.1) and (5.2.2) into (5.1.1) gives the average throughput of the traditional $N$-hop scheme as

$$\eta^{\text{(traditional)}} = \frac{1}{N} e^{-\frac{\Delta}{r_t}} r_t.$$  \hfill (5.2.3)

\textsuperscript{2}Eq. (5.2.2) holds when channels vary at every hop (for fair comparison among different schemes). The outage probability may be different for other assumptions. For example, if suppose the channels remain unchanged for $N$ consecutive time slots, the outage probability is then given by $P(\min\{C_i\} < r_t)$. 
5.3 Buffer-aided link selection based on network-coding

In this section, apply the buffers at the relays to reduce the outage probability and use physical layer network coding to increase the data rate will be performed first. Then a novel link selection scheme for the multi-hop relay network by integrating the buffer-aided and network coding approaches is proposed.

5.3.1 Decrease the outage probability with buffers at the relays

The max-link relay selection scheme described in [33] can be straightforwardly applied in multi-hop link selection. To be specific, in buffer-aided link selection, every relay is equipped with a data buffer of size $L$. Suppose that the relay $R_i$ has buffer $Q_i$, where $i = 1, \cdots, N - 1$. At any time slot, when a data packet arrives at a relay node, it is stored in the buffer. At the next time slot, unlike the traditional scheme, the stored data packet is not necessarily forwarded to the next node. Instead the link with the highest SNR among all of the “available” links is selected for data transmission. A link is considered available if the buffers of the corresponding transmitting and receiving nodes are not empty and full respectively. Thus in the max-link scheme, the link for data transmission is selected as

$$link = \arg \max_{\text{link} \in \mathcal{A}} \{\gamma_i\},$$  \hspace{1cm} (5.3.1)

where $\mathcal{A}$ is the set containing all available links, and recall that $\gamma_i$ is the instantaneous channel SNR for $\text{link}_i$.

In the max-link scheme, because only one link is selected for data transmission at any time, the average data rate is still the same as that in the traditional scheme which is given by

$$\bar{R}_{(\text{max-link})} = \frac{1}{N} r_t,$$  \hspace{1cm} (5.3.2)
Then the average throughput of the max-link scheme is given by

\[
\bar{\eta}^{\text{(max-link)}} = \frac{1}{N} \left( 1 - P_{\text{out}}^{\text{(max-link)}} \right) r_t, \tag{5.3.3}
\]

where \( P_{\text{out}}^{\text{(max-link)}} \) is the outage probability of the max-link scheme which can be obtained by following similar analysis as in [33].

Because \( P_{\text{out}}^{\text{(max-link)}} < P_{\text{out}}^{\text{(traditional)}} \), the throughput of the buffer-aided max-link scheme is higher than that of the traditional scheme. On the other hand, because the max-link and traditional schemes have the same data rate, and further noting that \( P_{\text{out}}^{\text{(max-link)}} \to 0 \) when \( \text{SNR} \to \infty \), the two schemes have similar throughput when the SNR is high enough. This indicates that the buffer-aided link selection mainly improves the throughput over the low SNR range.

### 5.3.2 Increase the data rate with network coding

Suppose at one time slot, all odd numbered links transmit data at the same time, and at the next time slot all even numbered links transmit simultaneously. Thus a relay node may receive data from both the previous and next nodes. Without losing generality, at time \( t \), suppose that node \( R_i \) receives data from its previous node \( a \) and next node \( b \) simultaneously. Then the received signal at relay \( R_i \) at time slot \( t \) is given by

\[
y_i(t) = h_i(t)x_a(t) + h_{b,i}(t)x_b(t) + n_i(t), \tag{5.3.4}
\]

where \( (x_a(t)) \) and \( x_b(t) \) are the data packets transmitted from nodes \( a \) and \( b \) respectively, \( h_{b,i}(t) \) is the channel coefficient for the \( b \to R_i \) link, and \( n_i(t) \) is the noise at node \( R_i \).

It is clear from (5.3.4) that \( h_{b,i}(t)x_b(t) \) forms the inter-relay interference. Because \( x_b(t) \) is transmitted from \( R_i \) to node \( b \) previously, it can be stored at \( R_i \). With the principle of physical layer network coding [61], the inter-relay
interference can be completely removed from (5.3.4), so that the received
signal at $R_i$ becomes

$$y_i(t) = h_i(t)(x_a(t)) + n_i(t).$$  \hspace{1cm} (5.3.5)

Therefore, with physical layer network coding, all odd (or even) num-
bered links can be used for data transmission simultaneously without caus-
ing any inter-relay interference. As an example, the network coding based
transmission scheme for the 4-hop relay network is shown in Fig. 5.2.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig5.2}
\caption{Network-coding based 4-hop relay transmission.}
\end{figure}

As is illustrated in Fig. 5.2, on average it only takes two hops to transmit
one data packet from $S$ to $D$, no matter how many hops there are in the
relay network. Thus the data rate for the network-coding based scheme is
given by

$$\bar{R}^{(\text{net-coding})} = \frac{1}{2} r_t.$$ \hspace{1cm} (5.3.6)

On the other hand, when the odd numbered links are used for data trans-
mission, the outage occurs when $\min_{i \in \text{odd}} \{C_i\} < r_t$. When the even numbered
links are used for transmission, the outage occurs when $\min_{j \in \text{even}} \{C_j\} < r_t$. Because each of the odd and even numbered links takes half of the total
transmission time, the outage probability of the the network-coding scheme

is obtained as
\[
P_{\text{out}}^{(\text{net-coding})} = \frac{1}{2} P\left( \min_{i \in \text{odd}} \{C_i\} < r_t \right) + \frac{1}{2} P\left( \min_{j \in \text{even}} \{C_j\} < r_t \right)
\]
\[
= \frac{1}{2} \left( 1 - e^{-\frac{N_o \Delta}{\gamma}} \right) + \frac{1}{2} \left( 1 - e^{-\frac{N_e \Delta}{\gamma}} \right)
\]
\[
= 1 - \frac{1}{2} \left( e^{-\frac{N_o \Delta}{\gamma}} + e^{-\frac{N_e \Delta}{\gamma}} \right),
\]
where \(N_o = \lceil N/2 \rceil\) and \(N_e = \lfloor N/2 \rfloor\) which are the numbers of odd and even numbered links respectively, \(\lceil \cdot \rceil\) and \(\lfloor \cdot \rfloor\) round up and down the embraced value to the nearest integer respectively. Comparing (5.2.2) and (5.3.7) clearly shows that \(P_{\text{out}}^{(\text{net-coding})}\) is larger than that for the traditional scheme.

Substituting (5.3.6) and (5.3.7) into (5.1.1) gives the average throughput of the network-coding based scheme for the 2-hop relay network as
\[
\bar{\eta}^{(\text{net-coding,2-hop})} = \frac{1}{4} \left( e^{-\frac{N_o \Delta}{\gamma}} + e^{-\frac{N_e \Delta}{\gamma}} \right) r_t.
\]

Because the outage probability \(P_{\text{out}} \to 0\) when the SNR \(\to \infty\), the average throughput is mainly determined by the data rate when the SNR is large enough. Thus over the high SNR range, the average throughput of the \(N\)-hop network with the network-coding scheme is always approximately \(r_t/2\). On the other hand, when the SNR \(\to -\infty\), the outage probability \(P_{\text{out}} \to 1\) so that the throughput is more determined by the outage probability than by the data rate. This implies that, when the SNR is very small, the network-coding based scheme has lower throughput than the traditional scheme. Therefore, the network-coding scheme improves the throughput over the high SNR range.

### 5.3.3 Buffer-aided network-coding link selection

In order to increase the average throughput over all SNR ranges, a novel link selection scheme by integrating the buffer-aided max-link and network-
coding approaches is proposed. This is achieved by adding simultaneous link transmission in the buffer-aided link selection rules.

Generalising from the network-coding scheme, any links separated by two hops or more can be simultaneously selected for data transmission. Let \( N_s \) denote as the number of simultaneously transmitting links at one time slot. For the \( N \)-hop relay network, it has

\[
1 \leq N_s \leq \lceil N/2 \rceil \tag{5.3.9}
\]

For any \( N_s \), there exist \( D(N_s) \) possible link selections, which is represented by the selection vector as

\[
\text{link}^{(N_s)} = [\text{link}^{(N_s)}(1), \ldots, \text{link}^{(N_s)}(D(N_s))], \tag{5.3.10}
\]

where \( \text{link}^{(N_s)}(i) \) is the \( i \)th link selection for \( N_s \) simultaneous link transmission. For later use, let denote \( \text{link}_{i_1+\cdots+i_n} \) as the simultaneous transmission of \( \text{link}_{i_1}, \ldots, \text{link}_{i_n} \).

For example, in the 4-hop relay network, it has \( 1 \leq N_s \leq 2 \), and

\[
\text{link}^{(N_s=1)} = [\text{link}_1, \text{link}_2, \text{link}_3, \text{link}_4] \\
\text{link}^{(N_s=2)} = [\text{link}_{1+3}, \text{link}_{1+4}, \text{link}_{2+4}] \tag{5.3.11}
\]

The principle of the proposed scheme is to provide as many links for simultaneous transmission as possible. To be specific, at time slot \( t \), the link(s) for transmission is/are selected following the rules below:

**Step 1:** First, let \( N_s = \lceil N/2 \rceil \), and find the selection vector \( \text{link}^{(N_s)} \), or list all possible link selections for \( N_s \) simultaneous link transmissions.

- If none of the link selections in \( \text{link}^{(N_s)} \) are available, then go to **Step 2**.
- Otherwise, use the max-min scheme to choose the best \( N_s \) simul-
taneous link transmission among all available links in $\text{link}^{(N_s)}$, as

$$link_b^{(N_s)} = \arg \max_{\text{link}^{(N_s)(i)} \in A} \left\{ \min_{\text{link}_i \in \text{link}^{(N_s)(i)}} \{\gamma_i\} \right\}, \quad (5.3.12)$$

where $A$ is the set containing all available links.

- Check whether the link selection $link_b^{(N_s)}$ is in outage or not.
  - If $link_b^{(N_s)}$ is in outage, then no $N_s$ simultaneous link transmission is possible at time $t$ and go to Step 2.
  - Otherwise select $link_b^{(N_s)}$ for data transmission at time $t$.

**Step 2:** Let $N_s \leftarrow (N_s - 1)$ and repeat Step 1 until $N_s = 1$.

In order to better understand the proposed link selection rule, consider a 4-hop relay network as an example. Suppose at time slot $t$, all links are available except $\text{link}_3$. Then the selection vectors for available links are obtained by removing all selections containing $\text{link}_3$ in (5.3.11), so that it has

$$\text{link}^{(N_s=1)} = [\text{link}_1, \text{link}_2, \text{link}_4]$$

$$\text{link}^{(N_s=2)} = [\text{link}_{1+4}, \text{link}_{2+4}] \quad (5.3.13)$$

Then the links are selected as follows.

**Step 1:** Let $N_s = 2$, and find the best selection of 2 simultaneous link transmission as

$$link_b^{(N_s=2)} = \arg \max \{\min\{\gamma_1, \gamma_4\}, \min\{\gamma_2, \gamma_4\}\} \quad (5.3.14)$$

Suppose that the solution from (5.3.14) is $link_b^{(N_s=2)} = \text{link}_{1+4}$. Then check whether $\min\{C_1, C_4\} < r_t$ or not.
If ‘no’, $link_{1+4}$ is not in outage and is selected for data transmission at time slot $t$.

Otherwise, go to Step 2.

**Step 2:** Let $N_s = 1$, and find the best selection of single link transmission as

$$link_b^{(N_s=1)} = \arg\max \{\gamma_1, \gamma_2, \gamma_4\} \quad (5.3.15)$$

Suppose that the solution from (5.3.15) is $link_b^{(N_s=1)} = link_2$. Then check whether $\min\{C_2\} < r_t$ or not

- If ‘no’, then choose $link_2$ for data transmission.
- Otherwise, outage occurs.

The proposed buffer-aided network-coding scheme takes advantages of both the network-coding and max-link schemes. On the one hand, because higher link selection priority is given to simultaneous transmission, the average data rate is higher than that of the traditional scheme. Particularly, when $\text{SNR} \to \infty$, it has $P_{out} \to 0$ so that the average throughput of the proposed scheme is $r_t/2$, which is the same as that for the network-coding scheme. On the other hand, in the proposed scheme, the outage occurs only if all available links are in outage. This is similar to the max-link scheme. Therefore, the outage performance of the proposed and max-link scheme are similar.

From (5.1.1), the average throughput of the proposed scheme is given by

$$\bar{\eta}^{(\text{buffer-code})} = \bar{R}^{(\text{buffer-code})} \left(1 - P_{out}^{(\text{buffer-code})}\right), \quad (5.3.16)$$

where $P_{out}^{(\text{buffer-code})}$ and $\bar{R}^{(\text{buffer-code})}$ are the outage probability and average data rate of the proposed scheme, which are given by (5.4.14) and (5.5.9) obtained in the following two sections respectively.
5.4 Outage probability

At any time, the numbers of data packets in the relay buffers form a “state”. Because each buffer has size L, there are \((L + 1)^{N-1}\) states in total, where the \(i\)-th state vector is defined as

\[
s_i = [\Psi_i(Q_1), \Psi_i(Q_2), ..., \Psi_i(Q_{N-1})], \quad i = 1, ..., (L + 1)^{N-1},
\]

where \(0 \leq \Psi_i(Q_k) \leq L\) for all \(k = 1, ..., N - 1\) which is the buffer length (or the number of data packets in the buffer) of \(Q_k\) at state \(s_i\). At every time, depending on which link(s) is/are selected for transmission, the state may move to several possible states at the next time, forming a Markov chain.

Considering all possible states, the outage probability of the buffer-aided network-coding scheme can be obtained as

\[
P_{\text{out}}^{(\text{buffer-code})} = \sum_{i=1}^{(L+1)^{N-1}} \pi_i p_{out}^{s_i}, \quad (5.4.2)
\]

where \(\pi_i\) and \(p_{out}^{s_i}\) are the stationary probability and outage probability for state \(s_i\) respectively.

In the following two subsections, \(p_{out}^{s_i}\) and \(\pi_i\) are derived respectively.

5.4.1 \(p_{out}^{s_i}\): outage probability for state \(s_i\)

According to the link selection rules of the proposed buffer-aided network-coding scheme, at state \(s_i\), outage occurs only if all available links are in outage. Recalling that a link is available when the buffers of the corresponding transmission and receiving nodes are not empty and full respectively, define the available-link vector for the state \(s_i\) in the \(N\)-hop network as

\[
a_i = [a_i(1), a_i(2), ..., a_i(N)]
\]

(5.4.3)
where \( a_i(n) \) can only be ‘1’ or ‘0’, indicating that the corresponding link \( n \) is available or not available at state \( s_i \) respectively. For instance, in the 4-hop example in Section 5.3.3 where the buffers are at the state that all links except link3 are available, it has \( a_i = [1 \ 1 \ 0 \ 1] \).

Because all channels are i.i.d., the outage probability for state \( s_i \) is given by

\[
p_{out}^{s_i} = (P(C_i < r_t))^{|a_i|^+}, \tag{5.4.4}
\]

where \( P(C_i < r_t) \) is the probability that a single link suffers outage, and \(|a_i|^+\) is the total number of available links at state \( s_i \) which is the number of ‘1’-s in \( a_i \).

Because \( C_i = \log_2(1 + \gamma_i) \) and the SNR \( \gamma_i \) is exponentially distributed, it has

\[
P(C_i < r_t) = F_{\gamma}(\Delta) = \left(1 - e^{-\frac{\Delta}{\gamma}}\right), \tag{5.4.5}
\]

where \( F_{\gamma}(\cdot) \) is the CDF of \( \gamma_i \) and \( \Delta = 2^{r_t} - 1 \). Substituting (5.4.5) into (5.4.4) gives

\[
p_{out}^{s_i} = F_{\gamma}^{|a_i|^+} = \left(1 - e^{-\frac{\Delta}{\gamma}}\right)^{|a_i|^+}, \tag{5.4.6}
\]

where \( \Delta \) is ignored in \( F_{\gamma}(\Delta) \) without causing any confusion.

### 5.4.2 \( \pi_i \): the stationary probability of state \( s_i \)

In order to obtain the stationary probability \( \pi_i \) for every state, it first need to calculate the state transition matrix \( A \) which is an \((L+1)^{N-1}\) by \((L+1)^{N-1}\) matrix, where the entry \( A_{j,i} = P(X_{t+1} = s_j | X_t = s_i) \) is the transition probability that the state moves from \( s_i \) at time \( t \) to \( s_j \) at time \( (t+1) \).

Suppose that the buffer state is \( s_i \) at time slot \( t \). If outage occurs, the buffer state remains at \( s_i \) at the time slot \( (t+1) \). Otherwise, \( s_i \) may move to several possible states at \( (t+1) \), which are denoted as \( s_{j_1}, \ldots, s_{j_{Q_i}} \) respectively. The state transition from \( s_i \) to \( s_{j_q} (j_q \in \{j_1, \ldots, j_{Q_i}\}) \) is the
result of one particular link selection which is represented by the selection vector defined as

\[ \text{sel}_i^{(j_q)} = [\text{sel}_i^{(j_q)}(1), \ldots, \text{sel}_i^{(j_q)}(N)], \tag{5.4.7} \]

where \( \text{sel}_i^{(j_q)}(n) \) can only take values of 1 or 0, indicating the corresponding \( \text{link}_n \) is selected or not respectively. For example, in the 4-hop network, \( \text{sel}_i = [1 \ 0 \ 0 \ 1] \) represents the link selection of \( \text{link}_{1,4} \). With these observations, it has

\[ A_{j,i} = \begin{cases} p_{\text{out},i}^{s_i}, & j = i \\ P\left(\text{sel}_i^{(j)}\right), & j \in \{j_1, \ldots, j_{Q_i}\} \\ 0, & \text{otherwise} \end{cases} \tag{5.4.8} \]

where \( P\left(\text{sel}_i^{(j)}\right) \) is the probability to choose the link selection \( \text{sel}_i^{(j)} \) at state \( s_i \). While \( p_{\text{out},i}^{s_i} \) is given by (5.4.6), and \( P\left(\text{sel}_i^{(j)}\right) \) is calculated below.

According to the proposed link selection rules, the link selection at state \( s_i \) depends on the outage events at every available link, where the priority is given to as many simultaneous link transmissions as possible. Only when a link is both available and not in outage, may it be used for data transmission. Define the \textit{good}-link vector to indicate whether the links are ‘good’ or not for data transmission at state \( s_i \) as

\[ \text{g}_i = [g_i(1), g_i(2), \ldots, g_i(N)] \]

where \( g_i(n) \) indicates three states and can be take values of ‘1’, ‘−1’ or ‘0’, \( g_i(n) = 1 \) indicates that the corresponding \( \text{link}_n \) is not only available but also not in outage, \( g_i(n) = −1 \) indicates that \( \text{link}_n \) is available but in outage, and \( g_i(n) = 0 \) indicates that \( \text{link}_n \) is not available.

Comparing (5.4.3) and (5.4.9) shows that, for every state \( s_i \), it corresponds to one available-link vector \( a_i \), which again corresponds to a set of
good-link vectors including all possible link outages of the available links. Because the state \( s_i \) has \( |a_i|^+ \) available links, there are \( G_i = (|a_i|^+) + \cdots + (|a_i|^+ - 1) \) good-link vectors for \( s_i \), denoted as \( g_i^{(1)}, \ldots, g_i^{(G_i)} \) respectively, where \( (|a_i|^+) \) is the (combination) probability that \( n \) links become outage among all \( |a_i|^+ \) available links. The probability of the \( k \)-th good-link vector is obtained as

\[
P\left(g_i^{(k)}\right) = \bar{F}_{|g_i^{(k)}|^+} F_{|g_i^{(k)}|^-}, \quad k = 1, \ldots, G_i
\]

where \(|g_i^{(k)}|^+\) and \(|g_i^{(k)}|^-\) give the number of ‘1’-s and ‘−1’-s in \( g_i^{(k)} \) respectively, and \( \bar{F}_\gamma = 1 - F_\gamma \) which is the probability that a single link is not in outage.

From the proposed link selection rules, for every good-link vector, it may lead to several possible link selections, depending on the channel gains at the current time slot. On the other hand, one link selection may also correspond to several good-link vectors. As a result, a 2 stage trellis-like diagram for the state \( s_i \) can be formed, as is illustrated in Fig. 5.4 for the 4-hop network. At the first stage, there are \( G_i \) nodes, where each node corresponds to one good-link vector \( g_i \). At the second stage, there are \( Q_i \) nodes, each corresponding to one link selection vector \( \text{sel}_i \).

Suppose that the \( k \)-th node at stage 1, \( g_i^{(k)} \), leads to \( N_k \) nodes at stage 2, denoted as \( \text{sel}_i^{(n_1)}, \ldots, \text{sel}_i^{(n_{N_k})} \) respectively. Because the channels are i.i.d., the probabilities for the paths from node \( g_i^{(n)} \) to any of these \( N_k \) nodes at stage 2 are the same, or it has

\[
P\left(g_i^{(k)} \rightarrow \text{sel}_i^{(j)}\right) = \begin{cases} P\left(g_i^{(k)}\right) \frac{1}{N_k}, & j \in \{n_1, \ldots, n_{N_k}\} \\ 0, & \text{otherwise} \end{cases}
\]

Then further from (5.4.8), the transition probability from \( s_i \) to \( s_j \) is the summation of the probabilities of all paths that end at the node \( \text{sel}_i^{(j)} \), which
is given by

\[ A_{j,i} = P(\text{sel}_i^{(j)}) = \sum_{k=1}^{G_i} P(\text{g}_i^{(k)} \rightarrow \text{sel}_i^{(j)}), \quad j \in \{j_1, \cdots, j_{Q_i}\} \]  \hspace{1cm} (5.4.12)

Substituting (5.4.12) into (5.4.8), and applying it on all states, the state transition matrix \( A \) can be obtained.

Because the transition matrix \( A \) is column stochastic, irreducible and aperiodic\(^3\), the stationary state probability vector is obtained as (see [49] and [50])

\[ \pi = (A - I + B)^{-1} b, \]  \hspace{1cm} (5.4.13)

where \( \pi = [\pi_1, \cdots, \pi_{(L+1)^N-1}]^T \in \mathbb{R}^{1 \times (L+1)^N-1}, b = [1, \cdots, 1]^T \in \mathbb{R}^{1 \times (L+1)^N-1} \), \( I \) is \((L + 1)^N-1 \times (L + 1)^N-1\) identity matrix and \(B_{n,l}\) is an \(n \times l\) all one matrix.

Finally, substituting (5.4.6) and (5.4.13) into (5.4.2) gives the outage probability of the overall system as

\[ P_{\text{out}}^{(\text{buffer-code})} = \sum_{i=1}^{(L+1)^N-1} \pi_i P_{\text{out}}^{s_i} = \text{diag}(A) \pi \]  \hspace{1cm} (5.4.14)

where \( \text{diag}(A) \) is the vector consisting of all diagonal elements of \( A \).

**Illustration - the 4-hop relay network**

In order to better understand the above analysis, an example of the 4-hop relay network with buffer size of \(L = 4\) is given. As an illustration, the state transition for the state \(s_i = [2 \ 0 \ 2]\) is considered, or the buffer lengths at nodes \(R_1, R_2\) and \(R_3\) are 2, 0 and 2 respectively. This is actually the same example in Section 5.3.3 where the selection rules are explained. As is shown

\(^3\)Column stochastic means all entries in any column sum up to one, irreducible means that is is possible to move from any state to any state, and aperiodic means that it is possible to return to the same state at any steps [48], [49]
in Fig. 5.3, there are 5 possible states that \( s_i \) can move to at time \( (t + 1) \), denoting as \( s_{j_1}, \ldots, s_{j_5} \) respectively, and each \( s_{j_q} \) corresponds to one link selection.

![State transition diagram](image)

**Figure 5.3.** State transition diagram for the state \( s_i = [2 0 2] \) in the 4-hop relay network with buffer size of \( L = 4 \).

At state \( s_i = [2 0 2] \), all links except \( \text{link}_3 \) are available, so that the available-link vector is given by

\[
a_i = [1 1 0 1]
\]

The trellis diagram for the transition probability of state \( s_i \) is shown in Fig. 5.4, where there are 7 nodes (good-link vectors) at stage 1, and 5 nodes (link selection vectors) at stage 2. The probabilities for every link selection can be obtained from Fig. 5.4. For example, for \( \text{link}_{2+4} \) which is highlighted in red, it has

\[
A_{j_1, i} = P(s_i \rightarrow s_{j_1}) = P(\text{sel}_1^{(j_1)})
= P\left(g_i^{(1)} = [1 1 0 1]\right) \frac{1}{2} + P\left(g_i^{(4)} = [-1 1 0 1]\right) 1
= \frac{1}{2} F^3_\gamma + F_\gamma F^2_\gamma
\]
Section 5.5. Average data rate

In this section, the concept of “effective” hops is first introduced and then a trellis diagram is used to obtain the average data rate.

5.5.1 Effective hops

In the proposed $N$-hop link selection scheme, although every data packet needs to go through the $N$ hops consecutively to reach the destination, at some time slots, several packets may be simultaneously transmitted at different links. Thus on average, it takes fewer than $N$ time slots to deliver one packet to the destination, or the number of ‘effective’ hops to transmit one packet is fewer than $N$. To be specific, at one time slot, if several packets are transmitted simultaneously, this time slot is only counted as one effective hop for one of the packets.

In order to better understand the influence of the simultaneous transmission on the effective hop number, an example of the 4-hop network is given as is shown in Fig. 5.2. Specifically, for data packet $x(2)$, it has the following observations:

**Figure 5.4.** Trellis diagram for the transition probability for the state $s_i = [2 \ 0 \ 2]$ in the 4-hop relay network.
At time slot $t = 1$, data packets $x(2)$ and $x(1)$ are simultaneously transmitted over $link_1$ and $link_3$ respectively. Suppose that the time slot is counted as one effective hop only for the packet at the link with the lowest number. At $t = 1$, the lowest numbered link is $link_1$. Thus $t = 1$ contributes one effective hop only for $x(2)$ transmission, but not for $x(1)$ transmission.

Similarly, $t = 2$ contributes one effective hop for $x(2)$ transmission, but not for $x(1)$.

At $t = 3$, $x(3)$ and $x(2)$ are simultaneously transmitted over $link_1$ and $link_3$ respectively. Because the lowest numbered link is $link_1$, $t = 3$ contributes one effective hop only for $x(3)$ transmission, but not for $x(2)$.

Similarly $t = 4$ is counted as one effective hopping time for $x(3)$, but not for $x(2)$.

Therefore, although $x(2)$ goes through all 4 hops to reach the destination, only $t = 1$ and $t = 2$ are counted as its effective hopping times, or the number of effective hops for $x(2)$ transmission is 2. This leads to the following rule as:

*Effective hopping rule: at any time slot ‘$t$’, if multiple data packets are transmitted simultaneously, the time slot is only counted as one effective hopping time for the packet transmitted at the lowest numbered link.*

In the proposed buffer-aided network coding scheme, because different simultaneous link transmissions may be selected at different time slots, different data packets have different numbers of effective hops and the average data rate is obtained as

$$R_{(buffer-code)}^{(buffer-code)} = \frac{1}{n} r_t, \quad (5.5.1)$$
where \( \bar{n} \) is the average number of effective hops to transmit one data packet.

### 5.5.2 Trellis diagram to obtain \( \bar{n} \)

Below the trellis diagram is used to analyse average number of effective hops \( \bar{n} \). In the proposed \( N \)-hop relay scheme, for any data packet, it must go through all links \( (link_1, \ldots, link_N) \) consecutively to reach the destination. Suppose that at time slot \( t \), a packet needs to go through \( link_n \). There exist several possible link selections to make this happen: either only \( link_n \) is selected, or \( link_n \) is selected simultaneously with other links. On the other hand, the link selections only depend on the buffer states at time slot \( t \), but not on other packet transmissions. With this observation, an \( N \)-stage trellis diagram is described to represent all possible link selections for one packet transmission, as is illustrated in Fig. 5.5 for the 4-hop network. Every stage contains a set of nodes, where every node corresponds to one possible link selection to pass through the corresponding link.

Trellis nodes at adjacent stages are inter-connected, forming ‘paths’ from stage 1 to \( N \). The total number of paths is given by

\[
N_p = N_t^{(1)} \times \cdots \times N_t^{(N)}
\]

where \( N_t^{(n)} \) is the number of trellis nodes at stage \( n \). Every path corresponds to one combination of link selections for a packet passing through the network.

Supposing that the \( k \)-th path consists of the \( n_k \)-th trellis node at the \( n \)-th stage, the \( k \)-th path is represented as

\[
\text{path}_k = \{ \text{sel}^{(1_k)}, \ldots, \text{sel}^{(N_k)} \}, \quad k = 1, \ldots, N_p.
\]

where \( \text{sel}^{(n_k)} \) is defined in (5.4.7) which is the \( n_k \)-th link selection for a packet passing through \( link_n \).
In order to obtain the number of effective hops for the \( k \)-th path, define a binary function \( \mathcal{H}(\text{sel}^{(n_k)}) \). If \( \mathcal{H}(\text{sel}^{(n_k)}) = 1 \), then the corresponding transmission at stage \( i \) contributes one effective hop; otherwise if \( \mathcal{H}(\text{sel}^{(n_k)}) = 0 \), no effective hop is contributed at this stage. From the *effective hopping rule*, a time slot is counted as one effective hopping time for a packet, only if there are no other packets transmitting simultaneously at lower numbered links. Thus it has

\[
\mathcal{H}(\text{sel}^{(n_k)}) = \begin{cases} 
1, & L(\text{sel}^{(n_k)}) < i \\
0, & L(\text{sel}^{(n_k)}) = i
\end{cases}
\]  

(5.5.4)

where \( L(\text{sel}^{(n_k)}) \) gives the index of the first ‘1’ in the selection vector \( \text{sel}^{(n_k)} \).

Then the number of effective hops for path \( k \) is then given by

\[
N_e(path_k) = \sum_{n=1}^{N} \mathcal{H}(\text{sel}^{(n_k)})
\]  

(5.5.5)

On the other hand, the probability to choose path \( k \) is given by

\[
P(path_k) = \prod_{n=1}^{N} P(\text{sel}^{(n_k)}),
\]  

(5.5.6)

where \( P(\text{sel}^{(n_k)}) \) is the probability to select \( \text{sel}^{(n_k)} \) which is given by

\[
P(\text{sel}^{(n_k)}) = \sum_{i=1}^{(L+1)^{N-1}} \pi_i P_i \left( \text{sel}^{(n_k)} \right)
\]  

(5.5.7)

for which \( P(\text{sel}^{(n_k)}) \) is the probability to select \( \text{sel}^{(n_k)} \) at state \( s_i \) which is given by (5.4.12).

Then from (5.5.5) and (5.5.6), the average number of effective hops is obtained by averaging over all paths in the trellis as

\[
\bar{n} = \sum_{k=1}^{N_p} N_e(path_k) P(path_k)
\]  

(5.5.8)
Substituting (5.5.8) into (5.5.1) gives the average data rate as

\[ R^{(\text{buffer-code})} = \frac{1}{\sum_{k=1}^{N_p} N_e(\text{path}_k) P(\text{path}_k)} r_t, \]  

(5.5.9)

### 5.5.3 An illustration of the hopping trellis diagram for the 4-hop relay network

Fig. 5.5 shows the hopping trellis diagram for the 4-hop relay network, where there are 4 stages (or columns) corresponding to a packet passing through \( \text{link}_1 \) to \( \text{link}_4 \) respectively. At stage 1, there are 3 nodes corresponding to 3 selection vectors, namely \([1 \ 0 \ 0 \ 0]\), \([1 \ 0 \ 1 \ 0]\) and \([1 \ 0 \ 0 \ 1]\) respectively. It worth note that the first element of all of the three vectors at stage 1 is 1. Therefore, for a packet to go through \( \text{link}_1 \), it must correspond to one of these selection vectors. Similarly, there are 2, 2 and 3 trellis nodes at stage 2, 3 and 4 respectively.

![Hopping trellis diagram for the 4-hop relay network.](image)

It is clear in Fig. 5.5 that there are \( 3 \times 2 \times 2 \times 3 = 36 \) paths for the 4-hop relay network, where each path corresponds to one combination of link selections for a packet passing through the network. For example, the \( k \)-th path, which is highlighted with red, is represented as

\[ \{ \text{sel}^{(1_k)}, \text{sel}^{(2_k)}, \text{sel}^{(3_k)}, \text{sel}^{(4_k)} \} = \{ [1 \ 0 \ 0 \ 1], [0 \ 1 \ 0 \ 0], [1 \ 0 \ 1 \ 0], [1 \ 0 \ 0 \ 1] \}, \]  

(5.5.10)
which corresponds to the combination of selections as $\text{link}_{1+4}$, $\text{link}_2$, $\text{link}_{1+3}$ and $\text{link}_{1+4}$ consecutively.

Table 5.1 lists the effective hops at every stage for the path in (5.5.10). Particularly, at stage 1, $\text{sel}^{(1_k)} = [1 \ 0 \ 0 \ 1]$ which is the link selection for the first hop for this path. It is clear that the index of the first ‘1’ is $\mathcal{L} \left( \text{sel}^{(1_k)} \right) = 1$ which is equal to the hop index (or the first hop). Thus $\text{sel}^{(1_k)}$ contributes one effective hop for this path, or it has $\mathcal{H} \left( \text{sel}^{(1_k)} \right) = 1$. On the other hand, at stage 4, although $\text{sel}^{(4_k)} = \text{sel}^{(1_k)} = [1 \ 0 \ 0 \ 1]$, $\text{sel}^{(4_k)}$ does not contribute one effective hop for this path. This is because that, for $\text{sel}^{(4_k)}$, the hop index is now 4 which is not equal to the index of the first ‘1’ (which is still 1).

<table>
<thead>
<tr>
<th>$\text{sel}^{(n_k)}$</th>
<th>$[1 \ 0 \ 0 \ 1]$</th>
<th>$[0 \ 1 \ 0 \ 0]$</th>
<th>$[1 \ 0 \ 1 \ 0]$</th>
<th>$[1 \ 0 \ 0 \ 1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$\mathcal{L} \left( \text{sel}^{(n_k)} \right)$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\mathcal{H} \left( \text{sel}^{(n_k)} \right)$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.1. Effective hops for the path in (5.5.10)

Then from (5.5.5), the number of effective hops for the path in (5.5.10) is given by $\sum_{n=1}^{N} \mathcal{H} \left( \text{sel}^{(n_k)} \right) = 1 + 1 + 0 + 0 = 2$. The probability to choose this packet is given by

$$P(path_k) = P([1 \ 0 \ 0 \ 1])P([0 \ 1 \ 0 \ 0])P([1 \ 0 \ 1 \ 0])P([1 \ 0 \ 0 \ 1]) (5.5.11)$$

### 5.6 Average Packet Delay

The delay of a packet in the $N$-hop network is defined as the duration between the time when the packet is ready to transmit at the source and the time when it arrives the destination. In the non-buffer-aided schemes (e.g. the traditional or network-coding based scheme), when a packet reaches one node, it will be immediately forwarded to the next node at the following
time slot, so that the delay for every packet is $N$ time slots. On the other hand, in the buffer-aided scheme, because the data packets may queue at the relay nodes, the packet delay also includes the queuing time. It particularly worth noting that the packet delay is different from the number of effective hops, where the latter does not take into account of the queueing times at the relays.

In the proposed $N$-hop scheme, for every data packet, the queuing times at relays are different, and so are the packet delays. In general, the average packet delay in the network is the combination of the average delays at all transmission nodes as

$$
\bar{D}^{(\text{buffer-code})} = \bar{D}_s + \sum_{k=1}^{N-1} \bar{D}_{r_k}, \quad (5.6.1)
$$

where $\bar{D}_s$ and $\bar{D}_{r_k}$ are the average delays at source $S$ and relay $R_k$ respectively.

Using Little’s law [52], the average delay at node $i$ can be obtained as

$$
\bar{D}_i = \frac{\bar{L}_i}{\bar{\eta}_i}, \quad i \in \{s, r_k\} \quad (5.6.2)
$$

where $\bar{L}_i$ and $\bar{\eta}_i$ are the average queuing length and average throughput at node $i$ respectively.

Because all nodes are connected in series, the average throughput at every node is the same, which is equal to the system average throughput as

$$
\bar{\eta}_i = \bar{\eta}^{(\text{buffer-code})}, \quad i \in \{s, r_k\}, \quad (5.6.3)
$$

where $\bar{\eta}^{(\text{buffer-code})}$ is given by (5.3.16).

The average queuing lengths are derived as below. First, for the source node, because the “starting” time for a packet delay duration is when the packet is ready to transmit at the source. The queuing length at the source
is always 1, or it has

\[ \bar{L} = 1. \] (5.6.4)

On the other hand, the average length at a relay node is obtained by averaging the buffer lengths over all buffer states as

\[ \bar{L}_i = \frac{(L+1)^{N-1}}{N-1} \sum_{l=1}^{(L+1)^{N-1}} \pi_l \Psi_l(Q_i), \quad i \in \{s, r_k\} \] (5.6.5)

where \( \Psi_l(Q_i) \) gives the number of packets (or the buffer length) of buffer \( Q_i \) at state \( s_l \).

Substituting (5.6.3), (5.6.4) and (5.6.5) into (5.6.2), and further into (5.6.1), gives the proposed average packet delay in the buffer-aided network-coding scheme as

\[ \bar{D}_{\text{buffer-code}} = 1 + \frac{\sum_{i=1}^{L+1} \sum_{k=1}^{N-1} \pi_l \Psi_l(Q_k)}{\bar{D}_{\text{buffer-code}}}. \] (5.6.6)

It is interesting to compare the average packet delays of the two buffer-aided schemes: the max-link and proposed schemes respectively. On the one hand, the proposed scheme has higher throughput than the max-link scheme, or \( \bar{D}_{\text{buffer-code}} > \bar{D}_{\text{max-link}} \). On the other hand, because of the simultaneous data transmission in the proposed scheme, the data packets move more quickly through the system, resulting in shorter queuing lengthes at the relays, than the max-link scheme. From Little’s law (as is shown in (5.6.2)), the average packet delay of the proposed scheme is significantly smaller than that of the max-link scheme.

5.7 Simulation

In this section, numerical results are shown to verify the proposed scheme in this chapter. In all simulations, the transmission powers and the noise powers are normalised to unity, the transmission rates are set as \( r_t = 1 \), all channels...
are i.i.d. Rayleigh fading, and the channel coefficients remains unchanged
during one hopping time slot but vary independently from one time slot
to another. Both the simulation and theoretical results are shown, where
the simulation results are obtained by averaging over 100,000 independent
runs. Other parameters including the buffer size and number of hops are set
individually for every simulation.

5.7.1 Average system throughput

In this simulation, the average system throughput among different schemes
are compared.

Fig. 5.6 (a) and (b) compare the average throughput for the traditional,
network-coding, max-link and proposed buffer-aided and network-coding-
based schemes for 3-hop ($N = 3$) and 5-hop ($N = 5$) relay networks respec-
tively. In both the max-link and proposed schemes, the buffer sizes for the
3-hop and 5-hop network are set as $L = 3$ and $L = 4$ respectively. First,
it is clearly shown that, in both 3-hop and 5-hop networks, the theoretical
throughputs of the proposed scheme well match the simulation results, which
verifies the throughput analysis in this chapter. For all other schemes, only
the simulation results are shown for better exposition. As is expected, in
both 3-/5- hop networks, the network-coding scheme can achieve the maxi-
mum throughput of $1/2$ at high SNRs (e.g. SNR $>20$ dB), but it has lower
throughput than the traditional scheme for small SNRs. This is because,
compared with the traditional scheme which has data rate of $R = 1/N$,
though the network-coding scheme increases the data rate to $R = 1/2$, it
also increases the outage probability. For the max-link scheme, it signifi-
cantly increases the throughput at low SNRs, but has the same throughput
of $1/N$ as the traditional scheme at high SNRs. Therefore, both Fig. 5.6 (a)
and (b) verify the analysis that the network-coding and max-link schemes
improve the throughput at high SNRs and low SNRs respectively.
On the other hand, it is clearly shown in Fig. 5.6 that the proposed buffer-aided network-coding scheme takes advantage of both network-coding and max-link schemes, leading to significantly improvement in throughput at all SNR ranges. Particularly, when the SNR is large enough, the proposed scheme has the same maximum throughput of $1/2$ as the network-coding scheme.

Fig. 5.7 compares the average throughput vs the buffer size $L$ between the max-link and proposed schemes for the 3-hop relay network. It is clearly shown that, for every buffer size $L$, the proposed scheme always has higher throughput than the max-link scheme, where the former can reach the date rate of $1/2$ and the latter can only reach $1/3$ when the SNR is very large. In both schemes, the average throughput becomes higher with larger buffer size, but the improvement becomes less significant when the buffer size is larger. For example, the throughput difference between those for $L = 10$ and $L = 5$ is trivial in both schemes.

### 5.7.2 Average packet delay

This simulation investigates the average packet delay. Table 5.2 compares the theoretical analysis (based on (5.6.6)) and simulation results of the proposed buffer-aided network-coding scheme for both 3-hop and 5-hop network, where the channel SNR is set as 15 dB. It is clearly shown that, in both networks, the theoretical analysis very well matches the simulation results. Together with the results in Fig. 5.7, it shows that it is not necessary to have a very large buffer size $L$ as otherwise it not only has little improvement in throughput but also unnecessarily increases the average packet delay.

Table 5.3 compares average packet delays between the max-link and proposed schemes for the 3-hop relay network. It is clearly shown that, while both schemes have larger average packet delays with larger buffer size $L$, the max-link scheme has approximately 50% larger average packet delay.
Figure 5.6. Throughput comparison among traditional, network-coding, max-link and buffer-aided network-coding schemes.

than the proposed scheme. This well matches our expectation in Section 5.6.

5.8 Conclusion

In this chapter, a novel buffer-aided network-coding link selection scheme for the $N$-hop relay network is proposed. The proposed scheme applied buffers
Section 5.8. Conclusion

Figure 5.7. Throughput vs buffer length $L$, for the max-link and buffer-aided network-coding schemes in the 3-hop relay network.

<table>
<thead>
<tr>
<th>Buffer size</th>
<th>3-hop Average Delay</th>
<th>5-hop Average Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=1</td>
<td>3.09</td>
<td>3.09</td>
</tr>
<tr>
<td>L=3</td>
<td>7.15</td>
<td>7.18</td>
</tr>
<tr>
<td>L=5</td>
<td>11.28</td>
<td>11.30</td>
</tr>
</tbody>
</table>

Table 5.2. Average packet delays of the buffer-aided network-coding scheme

<table>
<thead>
<tr>
<th>Channel SNR</th>
<th>3-hop Schemes</th>
<th>L=5</th>
<th>L=10</th>
<th>L=20</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 dB</td>
<td>Proposed</td>
<td>13.10</td>
<td>24.07</td>
<td>45.86</td>
</tr>
<tr>
<td></td>
<td>Max-link</td>
<td>18.37</td>
<td>33.53</td>
<td>62.51</td>
</tr>
<tr>
<td>20 dB</td>
<td>Proposed</td>
<td>12.12</td>
<td>22.22</td>
<td>42.42</td>
</tr>
<tr>
<td></td>
<td>Max-link</td>
<td>17.49</td>
<td>32.63</td>
<td>61.28</td>
</tr>
</tbody>
</table>

Table 5.3. Average packet delays comparison between the max-link and proposed scheme in the 3-hop network.

at the relays to decrease the outage, and used network-coding to increase the data rate. As a result, the throughput at all SNR ranges is increased. New analysis tools are described to analyse the outage probability and average data rate, based on which the average throughput of the proposed scheme was successfully obtained. The analysis of the average packet delay is also performed. The analysis shows that, the proposed scheme not only has higher throughput, but also lower average packet delay, than the existing
buffer-aided max-link scheme, making it an attractive approach in the multi-hop network.
This chapter investigates DF buffer-aided relay selection for the underlay cognitive relay networks in the presence of both primary transmitter and receiver. A novel buffer aided relay selection scheme for the cognitive relay network is proposed, where the best relay is selected with the highest SIR among all available source-to-relay and relay-to-destination links while keeping the interference to the primary destination within a certain level. A closed-form expression for the outage probability of the proposed relay selection scheme is obtained. Both simulation and theoretical results are shown to confirm performance advantage over the conventional max-min relay selection scheme, making the proposed scheme attractive for the cognitive relay networks.
6.1 Problem Statement

Cognitive radio (CR) provides a promising way to improve spectrum efficiency by allowing primary and secondary users to share frequency bands. While spectrum sharing in a CR network can be realised through various approaches including spectrum underlay, overlay and interweave [62], due to its straightforwardly practical implementation, the underlay approach is of particular interest in which the interference from the secondary users to the primary users is strictly limited. On the other hand, relay selection provides an efficient way to harvest the diversity gain in the cognitive relay network (CRN) [63]. With only the best relay is selected for transmission, not only the system complexity but also the interference to the primary user is significantly lower than that when all relays participate transmission. Therefore, it is attractive to apply the cooperative network in the cognitive radio system, which forms the cognitive relay network with advantages from both cognitive radio and cooperative relay technologies. A typical relay selection system in CRN is shown in Fig. 6.1, where there are one primary source (PS), one primary destination (PD), one secondary source node (SS), one secondary destination node (SD) and a number of relays $SR_k, k \in (1, 2, ..., N)$. The channel coefficients and gains for the channel “$a \rightarrow b$” are labelled as $h_{ab}$ and $\gamma_{ab} = |h_{ab}|^2$ respectively.

As is shown in Fig. 6.1 (a), if the relay $SR_k$ is selected to receive data from the secondary source SS, due to the interference from the primary source PS, the received signal at $SR_k$ is given by

$$y_{sr_k} = \sqrt{P_{ss}h_{sr_k}s} + h_{pr_k}\sqrt{P_{ps}s'} + n_{rk}, \quad (6.1.1)$$

where $s$ and $s'$ are transmission signal vectors from SS and PS respectively, $P_a$ represents the transmission power at the node $a$ ($a \in \{ss, sr_k, ps\}$) and
Section 6.1. Problem Statement

(a) Source to relay transmission

(b) Relay to destination transmission

Figure 6.1. Throughput comparison among traditional, network-coding, max-link and buffer-aided network-coding schemes.

\( \mathbf{n}_{rk} \) is the noise vector at \( SR_k \). On the other hand, as is shown in Fig. 6.1 (b), if the relay \( SR_k \) is selected to forward the data to the secondary destination \( SD \), the received signal at \( SD \) is given by

\[
\mathbf{y}_{rd} = \sqrt{P_{srk} h_{rk} \mathbf{s}} + h_{pd} \sqrt{P_{ps} \mathbf{s}'} + \mathbf{n}_d;
\]

where \( \mathbf{n}_d \) is the noise vector at \( SD \).

In the underlay cognitive system, the secondary transmission nodes including \( SS \) and \( SR_k \) are only allowed to share the spectrum with the primary user \( PD \) if the corresponding interfering power to \( PD \) is below a pre-defined level \( I_{th} \) such that \( P_{ss} \gamma_{sp} \leq I_{th} \) and \( P_{srk} \gamma_{rk,p} \leq I_{th} \). With these power...
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constraints, the received SIR at the selected relay $SR_k$ and the destination $D$ become

\[
\text{SIR}_{sr_k} = \frac{P_{ss}|h_{sr_k}|^2}{P_{pr_k}|h_{pr_k}|^2} = \frac{I_{th}\gamma_{sr_k}}{\gamma_{sp}\gamma_{pr_k}}, \quad \text{SIR}_{rd} = \frac{P_{sr_k}|h_{rd_k}|^2}{P_{pd}|h_{pd}|^2} = \frac{I_{th}\gamma_{rd_k}}{\gamma_{rd_p}\gamma_{pd}}
\]

(6.1.3)

respectively, where the transmission power at the primary source $P_{ps}$ is normalised to unit without losing generality.

The objective of the relay selection in the CRN is to choose the “best” relay node such that the corresponding end-to-end capacity from $SS$ to $SD$ is maximised, subject to the constraint that the interferences at the primary destination $PD$ are below a certain level. Of particular interest in the CRN is the interference-limited scenario that the interference power from the primary source is dominant relative to the noise so that the noise effects can be ignored [64], and then the capacity is mainly dependent on the SIR. It is known that, if the relay node applies DF, the corresponding end-to-end capacity is the minimum of those for the source-to-relay and relay-to-destination links. Therefore, the traditional max-min relay selection can be generalised for the relay selection in the CRN, where the best relay node is selected with the maximum SIR of \( \min\{\text{SIR}_{sr_k}, \text{SIR}_{rd}\} \) from all available relay nodes. Or from (6.1.3) the max-min scheme chooses the best relay node for the CRN as

\[
R_{\text{best-min}}^{\text{max}} = \arg\max_{R_k} \left\{ \min \left\{ \frac{I_{th}\gamma_{sr_k}}{\gamma_{sp}\gamma_{pr_k}}, \frac{I_{th}\gamma_{rd_k}}{\gamma_{rd_p}\gamma_{pd}} \right\} \right\}.
\]

(6.1.4)

As is shown in (6.1.3), because the transmission power from the source to every relay $SR_k$ is limited according to the same interference constraint at the primary destination $PD$, there exists a common term “\( \gamma_{sp} \)” in every SIR$_{sr_k}$. Similarly, because the every $SR_k \rightarrow SD$ transmission suffers from the same interference from the primary source $PS$, there also exists a common term “\( \gamma_{pd} \)” in every SIR$_{rd}$. These two common terms make all of the “min” terms within the “max” operation in (6.1.4) become correlated.

This translates into the fact that the best relay is selected among dependent “candidates”, or the full diversity cannot be achieved even when all relevant channel coefficients are i.i.d.. This is very different from the conventional relay selection where the best relay is usually selected among independent candidates. The correlation among selection candidates is thus the key issue in the CRN relay selection scheme.

Relay selection in the CRN has attracted much attention recently. In [65], a max-min based relay selection similar to (6.1.4) was proposed in the CRN, though only the primary destination $PD$ is available (no primary source $PS$) in the system. Similar to (6.1.4), the candidates for the relay selection in [65] are also correlated. However, the outage analysis in [65] assumed that there exist multiple independent links between secondary source and primary destination so that the candidates for relay selection become independent. This is not correct because there is only one secondary sources and primary destination in the system respectively, or there are no multiple secondary source to primary destination links. Some earlier works (e.g. [66]) in the cognitive relay network also failed to consider the correlation in the relay selection. The correlation in the cognitive relay selection was identified in [67], and a “half” DF relay selection scheme was proposed where in the first phase the source broadcasts data to all relays and only in the second phase applies the relay selection that only the selected relay is used for data transmission. Similar relay selection approach was also considered in [68] so that the outage performance can be analysed. While the “half” relay selection successfully avoids the correlation issue in the CRN relay selection, it is at the price of losing efficiency because all relays (rather than only the selected relay) are involved in transmission in the first phase. Alternatively, the correlation in the CRN relay selection may also be avoided by assuming the link between the secondary and primary sources as constant, but this only applies to some particular systems such as when the secondary source
and primary user have little mobility [69].

Most current CRN relay selection approaches (including the aforementioned) assume there is no primary source for simplicity. In practice, both primary transmitter and receiver may present, in which the interference from the primary transmitter to the secondary users cannot be ignored [64, 70]. This motivates us to investigate relay selection in the more general CR network as is shown in Fig. 6.1. On the other hand, it is recently recognised that the performance of conventional relay selection can be further improved by relaxing the constraint that the best source-to-relay and relay-to-destination links for a packet transmission must be determined altogether. This is achieved by introducing data buffer at the relay nodes (e.g. [33, 38]). Of particular interest is the buffer-aided max-link relay selection where the best link is always selected among all available the source-to-relay and relay-to-destination links [33]. In this chapter, a max-ratio relay selection for the cognitive relay network is proposed, where the selected relay achieves the highest SIR at the secondary destination while satisfying the interference constraint at the primary receivers. The main contributions of this chapter are list as following:

- Proposing the buffer-aided max-ratio relay selection in the underlay cognitive relay network. Because the best source-to-relay and relay-to-destination links are selected separately, the proposed scheme provides a more efficient way to handle the correlation among relay selection candidates than existing approaches, a key issue in the CRN relay selection. As far as the authors’ aware, this is also the first relay selection in the CRN with both primary transmitter and receiver available.

- Deriving the closed-form expression of the outage probability for the proposed relay selection scheme. Because both the primary transmitter and receiver are present, the analysis is much more involved.
than those for both the conventional and the existing cognitive relay selection schemes. The analysis not only provides a deep insight in understanding the proposed scheme but also shows a potential approach to analyse similar systems in the future.

6.2 Max-ratio Relay Selection

In the buffer aided relay selection, each relay is equipped with a data buffer $Q_k$ ($1 \leq k \leq N$) of finite size $L$ (in the number of data packets), and the data packets in the buffer follow the “first-in-first-out” rule. The secondary transmission related channels can be divided into three groups: secondary transmission channels $h_{sr_k}$ and $h_{rd_k}$ for $SS \rightarrow SR_k$ and $SR_k \rightarrow SD$ respectively, secondary interfering channels $h_{pr_k}$ and $h_{pd_k}$ for $PS \rightarrow SR_k$ and $PS \rightarrow SD$ respectively, and primary interfering channels $h_{sp}$ and $h_{rp_k}$ for $SS \rightarrow PD$ and $SR_k \rightarrow PD$ respectively. Suppose all channels are quasi-static Rayleigh fading so that the channel coefficients keep unchanged during one packet duration but independently vary from one packet time to another. Channels within every group are i.i.d. fading, but channels for different groups may have different average gains, or it has $\gamma_{sr_k} = \gamma_{rd_k}$, $\gamma_{pr_k} = \gamma_{pd_k}$ and $\gamma_{sp} = \gamma_{rp_k}$ for all $k$. This is a more practical assumption than those in many existing approaches where all channels are assumed to be i.i.d. fading (e.g. [33, 68, 71]).

Suppose the secondary source $SS$ and relays $SR_k$ have the CSI knowledge from themselves to the primary destination respectively, so that the transmission powers at $SS$ and $SR_k$ can be determined. Moreover, suppose that the secondary destination node has global CSI\(^1\) and buffer state information for all relays, and selects a relay for transmission through an error-free feedback channel [74].

---

\(^1\)The CSI is usually estimated through pilots and feedback (e.g. [72]), and the CSI estimation without feedback may also be applied (e.g. [73]). Further detail of the CSI estimation is beyond the scope of this chapter.
In the max-ratio relay selection, at any time, the best transmission link with the highest SIR is selected among all available source-to-relay and relay-to-destination links. A source-to-relay or a relay-to-destination link is considered available when the buffer of the corresponding relay node is not full or empty respectively. To the specific, if a source-to-relay link is selected, the source node transmits one data packet to the corresponding relay node. If the selected relay can successfully decode the data, the decoded packet is stored in the buffer and the number of data packets in the buffer is increased by one. On the other hand, if a relay-to-source link is selected, the corresponding relay transmits the earliest stored packet in the buffer to the destination. If the destination can successfully decode the packet, the number of packets in the buffer is decreased by one.

The best selected relay (either for transmission or reception) in the max-ratio scheme can be obtained as

$$R_{\text{best}}^\text{max-ratio} = \arg \max_{SR_k} \{ (\text{SIR}_{sr_k}, \text{SIR}_{r_kd}) \mid \text{for all available links} \}. $$

Because the source-to-relay and relay-to-destination links are determined separately, from (6.1.3), the max-ratio relay selection rule can be expressed as

$$R_{\text{best}}^\text{max-ratio} = \arg \max_{SR_k} \left\{ \max_{SR_k: \Psi(Q_k) \neq L} \left\{ \frac{I_{th} \gamma_{sr_k}}{\gamma_{sp} \gamma_{pr_k}} \right\}, \max_{SR_k: \Psi(Q_k) \neq 0} \left\{ \frac{I_{th} \gamma_{r_kd}}{\gamma_{pd} \gamma_{rd_k}} \right\} \right\}, $$

(6.2.1)

where $$\Psi(Q_k)$$ gives the number of data packets in the buffer $$Q_k$$.

The outage probability can be defined as the probability that the selected link is in outage as

$$P_{out} \triangleq \begin{cases} \mathbb{P}\{(1/2) \log_2 (1 + \text{SIR}_{sr_k}) < r_t \} & \text{for relay reception}, \\ \mathbb{P}\{(1/2) \log_2 (1 + \text{SIR}_{r_kd}) < r_t \} & \text{for destination reception}, \end{cases} $$

(6.2.2)

where $$r_t$$ is the target rate, and the factor 1/2 captures the fact that it takes two time slots to transmit any packet from the source to the destination.
6.3 Outage Probability Analysis

This section analyses the outage probability of the max-ratio relay selection in the CRN. At any time, the numbers of data packets in every buffer form a “state”. Because there are $N$ available relays and every relay is equipped with a buffer of size $L$, there are $(L + 1)^N$ states in total. The $l$-th state vector is defined as

$$s_l = [\Psi_l(Q_1), \cdots, \Psi_l(Q_K)], \quad l = 1, \cdots, (L + 1)^N$$  

(6.3.1)

where $\Psi_l(Q_k)$ gives the number of data packets in buffer $Q_k$ at state $s_l$. It is clear that $0 \leq \Psi_l(Q_k) \leq L$.

Suppose that state $s_l$ corresponds to the pair of $(K^{(s_l)}_{sr}, K^{(s_l)}_{rd})$, where $K^{(s_l)}_{sr}$ and $K^{(s_l)}_{rd}$ are the numbers of available links for source-to-relay and relay-to-destination transmission at state $s_l$ respectively. By considering all possible available links for $K^{(s_l)}_{sr}$ and $K^{(s_l)}_{rd}$, the outage probability of the overall system can be obtained as

$$P_{\text{out}} = \sum_{l=1}^{(L+1)^N} \pi_l p_{s_l}^{\text{out}}.$$  

(6.3.2)

where $p_{s_l}^{\text{out}}$ is the outage probability when the state is at $s_l$, and $\pi_l$ is stationary probability for the state $s_l$. The following two sub-sections show the calculation of $p_{s_l}^{\text{out}}$ and $\pi_l$ respectively.
6.3.1 $p_{out}^{s_l}$: outage probability for state $s_l$

Separating the common terms $\gamma_{sp}$ and $\gamma_{pd}$ in the max-ratio relay selection rule in (6.2.1) gives:

$$R_{best}^{\text{max-ratio}} = \arg \max_{SR_k} \left\{ \max_{\Psi(Q_k) \neq L} \left\{ \frac{I_{th} \gamma_{sr_k}}{\gamma_{pr_k}} \right\} \gamma_{sp}, \max_{\Psi(Q_k) \neq 0} \left\{ \frac{I_{th} \gamma_{rd_k}}{\gamma_{rp_k}} \right\} \gamma_{pd} \right\}. \quad (6.3.3)$$

For the state $s_l$, there are $K^{(s_l)}_{sr}$ and $K^{(s_l)}_{rd}$ terms in the first and second part maximisation within the “outer” max operation in (6.3.3) respectively. For clear expression, define $w_k = \frac{I_{th} \gamma_{sr_k}}{\gamma_{pr_k}}$, $w = \max \{w_k\}$ and $x = \frac{w}{\gamma_{sp}}$, corresponding to the relay selection from the source-to-relay links. Similarly, define $v_k = \frac{I_{th} \gamma_{rd_k}}{\gamma_{rp_k}}$, $v = \max \{v_k\}$ and $y = \frac{v}{\gamma_{pd}}$ for the relay-to-destination selection. And finally define $z = \max \{x, y\}$ to complete the max-ratio relay selection for the overall system. It is clear that $z$ gives the instantaneous SIR of the selected link. Thus the outage probability corresponding to state $s_l$ for the target rate $r_t$ is given by:

$$p_{out}^{s_l} = P(z < r_t) = F_Z(z = \Delta), \quad (6.3.4)$$

where $F_Z(z)$ is the CDF of $z$ and $\Delta = 2^{2r_t} - 1$ which is the target SNR. The CDF of $z$ is derived as below.

First, for exponentially distributed channel gains, the CDF of $w_k = \frac{I_{th} \gamma_{sr_k}}{\gamma_{pr_k}}$ can be obtained as $F_{W_k}(w_k) = \frac{w_k}{L_1 + w_k}$, where $L_1 = \frac{I_{th} \gamma_{sr_k}}{\gamma_{pr_k}}$ and $\gamma_{ab} = E[\gamma_{ab}]^2$ representing the average channel gain for channel $h_{ab}$. Because the common term $\gamma_{sp}$ is taken out of all $w_k$ and all channels are assumed to be i.i.d, all of $w_k$ are also i.i.d.. Further recalling that there are $K^{(s_l)}_{sr}$ source-to-relay links available, the CDF of $w = \max \{w_k\}$ is given by:

$$F_W(w) = (F_{W_k}(w))^{K^{(s_l)}_{sr}} = \left( \frac{x}{L_1 + x} \right)^{K^{(s_l)}_{sr}}. \quad (6.3.5)$$
Because \( w \) and \( \gamma_{sp} \) are independent, the CDF of \( x = w/\gamma_{sp} \) is obtained as

\[
F_X(x) = \int_0^\infty \left( \frac{x \gamma_{sp}}{L_1 + x \gamma_{sp}} \right) \frac{1}{\gamma_{sp}} e^{-\frac{\gamma_{sp}}{\gamma_{sp}}} d\gamma_{sp}.
\] (6.3.6)

The diversity gain from the source-to-relay selection is clearly reflected in (6.3.5). In fact, because the common factor \( \gamma_{sp} \) can be separated out, \( \max\{w_k\} \) and \( \max\{\frac{w_k}{\gamma_{sp}}\} \) lead to the same selected link, or they correspond to similar diversity gain. This is very different from the traditional max-min relay selection as is shown in (6.1.4), where the common factors \( \gamma_{sp} \) and \( \gamma_{pd} \) cannot be separated.

From (6.3.6), the closed-form expression of the CDF of \( x \) can be obtained as

\[
F_X(x) = \begin{cases} 
1, & \text{if } K_{sr}^{(s)} = 0, \\
1 - \frac{L_1}{\gamma_{sp}^2} e^{-\frac{\gamma_{sp}}{\gamma_{sp}}} \text{Ei}(1, \frac{L_1}{\gamma_{sp}^2}), & \text{if } K_{sr}^{(s)} = 1, \\
\left( \frac{\gamma_{sp}}{L_1} \right) K_{sr}^{(s)} - 1 \frac{\text{M}_G(\{0\},\{\},[[K_{sr}^{(s)}-1,K_{sr}^{(s)}],],\{\},\{\}]}{\Gamma(K_{sr}^{(s)})}, & \text{elsewhere},
\end{cases}
\] (6.3.7)

where \( \text{Ei}(1,a) = \int_1^\infty \frac{\exp(-ta)}{a} dt, a > 0 \), \( \Gamma(\bullet) \) is the gamma function, and \( \text{M}_G(\{\},\{\},[[\bullet],],\{\},\{\}) \) is the Meijer G function [75].

Now define the diversity order from the source-to-relay selection as

\[
d_{sr}^{(K_{sr}^{(s)})} = - \lim_{\gamma \to \infty} \frac{\log_{10}(F_X(x))}{\gamma},
\] (6.3.8)

where \( \gamma \) is the average channel gain. Numerical verification based on (6.3.7) reveals that the diversity order from the source-to-relay relay selection is close to \( K_{sr}^{(s)} \). This will be verified in the simulation section.

On the other hand, the CDF of \( y \) for the relay-to-destination selection can be obtained similarly to (6.3.7), and the diversity order for the relay-to-destination selection \( d_{rd}^{(K_{rd}^{(s)})} \) is also shown close to the number of the available relay-to-destination links \( K_{rd}^{(s)} \).
Finally, because the source-to-relay selection \( x \) and relay-to-destination selection \( y \) are independent, the CDF of \( z = \max(x, y) \) is obtained as \( F_Z(z) = F_X(z)F_Y(z) \). And the overall diversity order when the buffer state is at \( s_l \) (with the pair \((K_{sr}^{(s_l)}, K_{rd}^{(s_l)})\)) is given by

\[
d^t(K_{sr}^{(s_l)}, K_{rd}^{(s_l)}) = d^t_{sr}(K_{sr}^{(s_l)}) + d^t_{rd}(K_{rd}^{(s_l)})
\] (6.3.9)

### 6.3.2 The stationary distribution probability \( \pi_l \)

The Markov chain can be used to model the transitions between the buffers states. Suppose at time \( t \), the state is at \( s_n \). At time \( t + 1 \), if the received data can be successfully decoded, there must be one relay either receiving or transmitting a data packet, so that the number of packets in the corresponding buffer is increased or decreased by one respectively. Depending on which relay receives or transmits data, at time \( t + 1 \), the buffers may move from state \( s_l \) to several possible states. Let denote \( U_{s_l} \) as the set containing all states which can be moved from \( s_l \).

Because the channels within secondary transmission, secondary interfering and primary interfering channels are i.i.d. fading, it is clear from (6.1.3) that the SIR-s for all channels are i.i.d. so that the probability to select any link is equable as \( \frac{1}{(K_{sr}^{(s_l)} + K_{rd}^{(s_l)})} \). Further noting that the state remains unchanged if outage occurs (or the decoding is not successful), the probabilities that the state \( s_l \) moves to a state in \( U_{s_l} \) is given by

\[
p_{s_l} = \frac{1 - p_{out}^{s_l}}{K_{sr}^{(s_l)} + K_{rd}^{(s_l)}}.
\] (6.3.10)

Let \( A \) denote as the \((L + 1)^N \times (L + 1)^N\) state transition matrix, where the entry \( A_{n,l} = P(X_{t+1} = s_n|X_t = s_l) \) which is the transition probability to move from state \( s_l \) at time \( t \) to state \( s_n \) at time \( t + 1 \). With the above
analysis, it has

\[
A_{n,l} = \begin{cases} 
p_{sn}^s, & \text{if } s_n = s_n, \\
p_{sl}, & \text{if } s_n \in U_l, \\
0, & \text{elsewhere}, 
\end{cases} \tag{6.3.11}
\]

Because the transition matrix \(A\) is column stochastic, irreducible and aperiodic\(^2\), the stationary state probability vector is obtained as (see [33, 49, 50])

\[
\pi = (A - I + B)^{-1}b, \tag{6.3.12}
\]

where \(\pi = [\pi_1, \ldots, \pi_{(L+1)^N}]^T \in \mathbb{R}^{1 \times (L+1)^N}\), \(b = (1, 1, \ldots, 1)^T \in \mathbb{R}^{1 \times (L+1)^N}\), \(I\) is \((L + 1)^N \times (L + 1)^N\) identity matrix and \(B_{n,l} = 1, \forall n, l\).

### 6.3.3 Discussion

Substituting (6.3.4) and (6.3.12) into (6.3.2) gives the outage probability of the max-ratio scheme. Or the outage probability can be expressed in the matrix/vector form as

\[
P_{\text{out}} = \text{diag}(A)\pi, \tag{6.3.13}
\]

where \(\text{diag}(A)\) is the vector consisting of the diagonal elements of \(A\).

The overall outage probability is the “average” of the outage probability \(p_{\text{out}}^{s_l}\) over all possible \((K_{sr}^{(s_l)}, K_{rd}^{(s_l)})\). The distribution of \((K_{sr}^{(s_l)}, K_{rd}^{(s_l)})\) depends on both the number of relays \(K\) and the relay buffer size \(L\). Particularly, when the buffer size \(L = 1\), it always has \(K_{sr}^{(s_l)} + K_{rd}^{(s_l)} = N\). As a result, the diversity order from the overall scheme is close to \(N\).

In another extreme, if the relay buffer size \(L \to \infty\), similar to that in [33], it can be shown that probabilities for \(K_{sr}^{(s_l)} = N\) and \(K_{rd}^{(s_l)} = N\) are one.

\(^2\text{Column stochastic means all entries in any column sum up to one, irreducible means that it is possible to move from any state to any state, and aperiodic means that it is possible to return to the same state at any steps [48, 49].}\)
and the correspondingly the diversity order is close to $2N$. In general, the
diversity order of the max-ratio scheme is between $N$ and $2N$.

In the max-ratio relay selection, different packets may have different
delays because a packet can only be transmitted if the corresponding link is
selected. Of particularly interest is the average packet delay which includes
the delays at both the source and relay nodes. At the source node, because
every link are assumed as i.i.d., the probability that the source is selected for
transmission is $1/2$. In comparison, in the traditional relay selection scheme
without any relay buffers, because the source node always transmits at the
odd time slots and waits at the even time slots, the probability that the
source transmits at any time is also $1/2$. Therefore, the average delay at the
source node is same for the max-ratio and traditional schemes. Since the
delay at the source in the traditional relay selection is 1, the source delay in
the max-link scheme is also $D_{\text{ave}}^{(\text{Source})} = 1$.

On the other hand, the average packet delay at the relay node in the
max-ratio scheme can be obtained using the Little’s law [52]: the average
delay multiplying the throughput gives the average queuing length. In the
max-ratio scheme, because the it takes 2 time slots to delivered a packet
(though the 2 time slots may not be consecutive), the overall throughput
of the whole system is $1/2$. Because all channels are i.i.d., the probability
for a packet transmission via any of the relay is the same. Therefore, the
throughput at any relay is $1/2N$. Furthermore, from the Markov model of the
relay buffer, the average number of packets (queuing length) in a relay buffer
can be obtained as $\sum_{l=1}^{(L+1)N} \pi_l \Theta_l(k)$. Then from the Little’s law, the average
delay at the relay is given by:

$$D_{\text{ave}}^{(\text{Relay})} = \frac{1}{(1/2)/N} \sum_{l=1}^{(L+1)N} \pi_l \Theta_l(k) = NL \quad (6.3.14)$$

Combining the delay at the source and the relay then gives the overall average
delay in the max-ratio system as

\[ D_{\text{ave}} = D_{\text{ave}}^{\text{(Source)}} + D_{\text{ave}}^{\text{(Relay)}} = 1 + NL. \] (6.3.15)

### 6.3.4 Numerical examples

The extensive numerical simulation have been performed which all well match the above delay analysis. Some of the results are shown in Tables. 6.1, where \( \bar{\gamma}_{sr}(\text{dB}) = \bar{\gamma}_{rd}(\text{dB}) = 40\text{dB} \) in all cases. It is clearly shown that, with increased relay number \( N \) and larger buffer size \( L \), the delays increased.

<table>
<thead>
<tr>
<th>Buffer size</th>
<th>N=3</th>
<th>N=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=1</td>
<td>Simulation: 4.09</td>
<td>Theory: 4</td>
</tr>
<tr>
<td></td>
<td>Simulation: 6.28</td>
<td>Theory: 6</td>
</tr>
<tr>
<td>L=3</td>
<td>Simulation: 10.15</td>
<td>Theory: 10</td>
</tr>
<tr>
<td></td>
<td>Simulation: 16.82</td>
<td>Theory: 16</td>
</tr>
<tr>
<td>L=5</td>
<td>Simulation: 16.28</td>
<td>Theory: 16</td>
</tr>
<tr>
<td></td>
<td>Simulation: 26.78</td>
<td>Theory: 26</td>
</tr>
</tbody>
</table>

Table 6.1. Average packet delays of proposed DF buffer-aided max-SIR-link relay selection for an underlay CRN.

### 6.4 Numerical Simulations

In the simulations below, the pre-defined level \( I_{th} = 1 \), and the average channel gains are set as \( \bar{\tau}_{sp} = \bar{\tau}_{rp} = 10 \text{ dB} \) and \( \bar{\tau}_{prk} = \bar{\tau}_{pd} = 10 \text{ dB} \). The transmission power of primary transmitter and channel noise are normalised to unit.

Fig. 6.2 verifies the theoretical analysis for the proposed max-SIR-link scheme with simulations. it has performed extensive simulations with different number of relays and buffer sizes. While all simulation results match the theoretical analysis, only a few are shown in Fig. 6.2 for better illustration. It is clearly shown that the outage probability decreases as the number of relays and buffer size increases. For example, for target rate \( r_t = 0.5 \text{ bits per channel use (BPCU)} \), when the number of relays and buffers \((N, L)\) increase
**Section 6.4. Numerical Simulations**

Figure 6.2. Theoretical and simulation outage probability vs target rate for the proposed max-SIR-link relay selection, where $\gamma_{sr_k} = \gamma_{rd} = 30$ dB, from (2, 2) to (5, 5), the outage probability drops about 40 dB. This is because that higher diversity is obtained with more relays and higher coding gain is obtained with larger buffer size.

Fig. 6.3 compares the outage probabilities of the proposed max-SIR-link, conventional max-min and no relay selection schemes, where the number of relays is set as $N = 3$, the relay buffer sizes for the proposed approach are set as $L = 1, 50, \infty$ respectively, and for better illustration only theoretical results for the proposed scheme are shown. It is clearly shown that, even with buffer size $L = 1$, the proposed relay selection still has better outage performance than the conventional max-min scheme. This is due to the coding gain from the max-ratio approach. Particularly for the proposed max-ratio selection, it is clearly shown that the diversity order is close to $N = 3$ for $L = 1$ and close to $2N = 6$ for $L \to \infty$. This well matches the analysis in Section III. Fig. 6.3 also shows that, for the proposed approach, the outage performance improves with larger buffer size, but the improvement becomes less significant when the buffer size is large enough. Particularly

<table>
<thead>
<tr>
<th>$K$ and $L$</th>
<th>Theory</th>
<th>Simulation results</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 and 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 and 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 and 5</td>
<td></td>
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</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Target Rate</th>
<th>Outage Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>10^−7</td>
</tr>
<tr>
<td>1.5</td>
<td>10^−6</td>
</tr>
<tr>
<td>2</td>
<td>10^−5</td>
</tr>
<tr>
<td>2.5</td>
<td>10^−4</td>
</tr>
<tr>
<td>3</td>
<td>10^−3</td>
</tr>
<tr>
<td>3.5</td>
<td>10^−2</td>
</tr>
<tr>
<td>4</td>
<td>10^0</td>
</tr>
<tr>
<td>4.5</td>
<td>10^1</td>
</tr>
<tr>
<td>5</td>
<td>10^2</td>
</tr>
<tr>
<td>5.5</td>
<td>10^3</td>
</tr>
<tr>
<td>6</td>
<td>10^4</td>
</tr>
</tbody>
</table>

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with \( L = 50 \), the outage performance is almost the same as that for \( L \to \infty \).

\[ \text{Outage Probability} \]
\[ \begin{array}{c}
\text{SNR (} \lambda_{sr} = \lambda_{rd} \text{) dB} \\
\end{array} \]

\[ \begin{array}{c}
\text{Outage Probability} \\
\text{Non relay selection} \\
\text{Max-Min relay selection (} K = 3 \text{)} \\
\text{Proposed relay selection (} K = 3, L = 1 \text{)} \\
\text{Proposed relay selection (} K = 3, L = 50 \text{)} \\
\text{Proposed relay selection (} K = 3, L \to \infty \text{)}
\end{array} \]

**Figure 6.3.** The comparison of outage probability for different relay selection policies with \( N = 3 \) relays, \( L = 1, 50 \) and \( \infty \) versus the different channel SNR, where \( r_t = 1 \) BPCU.

### 6.5 Conclusions

This chapter proposed the DF buffer-aided max-SIR-link relay selection for an underlay CRN, in the presence of both primary source and destination. In the proposed scheme, the best relay corresponds to the highest SIR among all available source-to-relay and relay-to-destination links while keeping the interference at the primary user within a pre-defined level. The closed-form expression of the outage probability of the proposed scheme was obtained, which matches exactly the simulation results. Both theoretical and simulation results showed that the proposed scheme has significantly better outage performance than the conventional max-min scheme, making it an attractive scheme in the CRN.
This chapter considers the security of transmission in DF cooperative wireless networks. An eavesdropper which can intercept the data transmission from both the source and relay nodes is considered to threaten the security of transmission. Finite size data buffers are assumed to be available at every relay in order to avoid having to select concurrently the best source-to-relay and relay-to-destination links. A new max-ratio relay selection policy is proposed to optimise the secrecy transmission by considering all the possible source-to-relay and relay-to-destination links and selecting the relay having the link which maximises the signal to eavesdropper channel gain ratio. Two cases are considered in terms of knowledge of the eavesdropper channel strengths: exact and average gains, respectively. Closed-form expressions for the secrecy outage probability for both cases are obtained, which are verified
by simulations. The proposed max-ratio relay selection scheme is shown to outperform one based on max-min-ratio relay scheme.

7.1 Introduction

Traditionally, security in wireless networks has been focused on higher layers using cryptographic methods [76]. As the secrecy at higher layers becomes the subject of increasing potential attacks, there has been growing interest in implementing security at the physical layer. Described as early as in the 1970’s [77–80], physical layer security in wireless communications has regained much attention recently [81–85]. The purpose of physical layer security is to prevent eavesdroppers from intercepting the data transmitted between the source and intended destination. The secrecy is quantified by the secrecy capacity, or the maximum rate of reliable information sent from the source to the intended destination in the presence of eavesdroppers [80].

Recent research shows that cooperative communication not only significantly improves the transmission capacity for wireless networks (e.g. [56,86]), but also provides an effective way to improve the secrecy capacity. This is achieved by carefully designing the relays to maximise the information rate at the intended destination and minimise that at the eavesdroppers [87]. In general, there are three ways to improve the secrecy capacity in a cooperative network:

- **Distributed beamforming** - Distributed beamforming at the source and relay nodes can form a transmission null space at the eavesdroppers so that only the intended destination can receive the data from the source. For example, in [88,89], the secrecy capacity is maximised by optimising the transmission weights (or the beamforming weights) at the relay and source nodes. Distributed beamforming however requires high coordination (such as synchronisation and central optimisation).
among source and relay nodes, which usually requires high overhead in implementation, i.e. a large amount of information needs to be exchanged between the relay nodes.

- **Jamming** - A jammer is used to inject artificial interference into the system. When the injected interfering power is higher at the eavesdroppers than that at the intended destination, the secrecy capacity can be improved (e.g. [90–92]). This often requires the jammer to be located closer to the eavesdroppers than to the destination node, which is not always possible in practice.

- **Relay selection** - Relay selection can increase the secrecy capacity by choosing an appropriate relay node with “strong” transmission link to the intended destination node and “weak” link to the eavesdropper [93–96]. The relay selection provides an attractive way to improve the secrecy capacity with little overhead in implementation. The performance of the relay selection well depends on number of available links for selection, which again depends on not only the relay numbers but also the relay selection schemes. When the number of available links is limited, so is the secrecy performance. Thus it is important to investigate relay selection schemes to have the best secrecy performance for a given relay number, which is the main objective of this chapter.

In traditional relay selection to improve wireless communications, the best relay is selected with the strongest link connecting the source and destination (e.g. [18, 53]). It is shown in [93] that by using relay selection the secrecy capacity can also be improved, when the best relay is selected to maximise the signal to eavesdropper channel gain ratio, or the “gain ratio”, in short, in this chapter. The relay selection scheme in [93] is based on the scenario that the eavesdropper can only intercept the signals from the relays
but not the source node. This obviously limits its practical applicability. On the other hand, if the eavesdropper intercepts signals from both the relay and source nodes, similar to the standard max-min scheme [18], the relay selection in [93] can be easily generalised to the so-called max-min-ratio scheme where the best relay is selected having the maximum of all minimum gain-ratios for every pair of source-to-relay and relay-to-destination links. The detail of the max-min-ratio relay selection will be described in Section III later. As for the max-min relay selection, the max-min-ratio scheme selects the best source-to-relay and relay-to-destination links, concurrently. It is unfortunately unlikely that both links correspond to the highest gain ratio simultaneously, leading to significant performance degradation. One possible solution is to select a jammer from the available relays (e.g [94]), but this is at the price of injecting more inference not only to the intended receivers but also to other users in the system. As was shown in [94], including a jammer does not necessarily lead to improvement in secrecy capacity, and thus a switching scheme was described to activate/deactivate the jammer.

Recent research shows that, by introducing data buffers at the relays, it is possible to relax the constraint that the best source-to-relay and relay-to-destination links for a packet transmission must be determined concurrently, and achieve significant performance advantage [30,31,33,36,38,39]. A typical buffer-aided relay selection is the max-max scheme described in [38], where the strongest source-to-relay and relay-to-destination links are selected alternatively so that it has significant coding gain over the traditional max-min scheme. Because the max-max relay selection still follows the traditional transmission order, that is the source-to-relay and relay-to-destination transmissions always carry on in an alternative manner, it can only attain a diversity order of $N$ which is the same as that for the max-min scheme, where $N$ is the number of available relay nodes. In the recent max-link approach [33], this constraint on the transmission order is further relaxed so
that, at any time, a best link is selected among all available source-to-relay and relay-to-destination links. Depending on whether a source-to-relay or a relay-to-destination link is selected, either the source transmits a packet to the selected relay or the selected relay forwards a stored packet to the destination. It is shown in [33] that the max-link relay selection not only has coding gain over the max-min scheme, but also has higher diversity order than both the max-min and max-max schemes. In particular, the diversity order can approach $2N$ when the relay buffer size is large enough.

Inspired by the max-link scheme, in this chapter, a novel max-ratio relay selection scheme for secure transmission in the DF relay networks with an eavesdropper which can intercept signals from both the source and relay nodes is proposed. In the proposed max-ratio scheme, every relay is equipped with a data buffer, and the best link is always selected with the highest gain ratio among all available source-to-relay and relay-to-destination links. Two cases that either the exact, or the average gain, for the eavesdropping channel is available; the latter case is of particular interest in practice since it is not always possible to obtain the exact channel information describing the eavesdropping channels are considered. Both theoretical and simulation results show that the proposed max-ratio relay selection has significantly better performance than the conventional max-min-ratio scheme, describing an attractive way in secure wireless transmission. The main contributions of this chapter are summarised as follows:

- Propose the buffer-aided max-ratio relay selection policy for secure communications in a DF cooperative network. This is the first approach in using buffer-aided relay selection in secure transmission under the scenario that the eavesdropper can intercepts signals from both the source and relay nodes. Existing relay selection schemes for secure transmission mainly assume no source-to-eavesdropper link for simplicity (e.g. [93]). While in [96] both the source and relay to eaves-
dropper links are considered, the proposed scheme is for the two-way relay network and furthermore no closed form expression is obtained for the secrecy outage probability.

- Derive closed-form expressions for the secrecy outage probability of the max-ratio scheme for both cases that either the exact or average gains of the eavesdropping channels is available. Because the gain-ratios for the source-to-relay and relay-to-destination transmissions have different statistical distribution, the secrecy outage performance of the proposed max-ratio scheme is much more involved to analyse than existing approaches such as the relay selection scheme for secure transmission in [93] and the buffer-aided max-link scheme for wireless communications in [33]. In this chapter, the problem arising from such “unbalanced” distribution has been successfully solved, and the analysis also provides a useful way in analysing similar systems such as a typical relay selection scheme but without the usual assumption that the source-to-relay and relay-to-destination channels are i.i.d..

The remainder of this chapter is organised as follows: Section 7.2 describes the system model; Section 7.3 proposes the max-ratio relay selection scheme; Section 7.4 gives the general secrecy outage probability expression, followed by Section 7.6 and 7.5 which finalise the derivations of the secrecy outage probability for the cases with exact and average knowledge of the eavesdropping channels, respectively; Section 7.7 discusses the performance of the max-ratio scheme when the relay buffer sizes go to infinity; Section 7.8 gives numerical simulations to verify the proposed max-ratio scheme; Finally, Section 7.9 summarises the chapter.
7.2 System Model

The system model of the relay network with an eavesdropper is shown in Fig. 7.1, where there is one source node ($S$), one destination node ($D$), a set of $N$ relays \{R_1, R_2, ..., R_k, ..., R_N\}, and one eavesdropper ($E$) which can intercept signals from both the source and relay nodes. The relay nodes apply the DF protocol and perform the half-duplex mode so that they do not transmit and receive simultaneously.

Figure 7.1. Relay selection system model in secure transmission, where the eavesdropper intercepts signals from both the source and relay nodes.

In this model, there is no direct link between the source and the destination due to path loss or shadowing effects\(^1\). The channel coefficients for $S \rightarrow R_k$, $S \rightarrow E$, $R_k \rightarrow D$ and $R_k \rightarrow E$ at time $t$ are denoted as $h_{sr_k}(t)$, $h_{se}(t)$, $h_{rk,d}(t)$ and $h_{rk,e}(t)$ respectively, where similar subscripts are also used for other parameters to indicate different channels in this chapter. Suppose the channels are quasi-static Rayleigh fading so that the channel coefficients keep unchanged during one packet duration but independently vary from one packet time to another. Let all source-to-relay links are independent

\(^1\)Including the direct link has little effect on the relay selection which is the main issue in this chapter.
and identically distributed (i.i.d.) fading and that $\mathbb{E}|h_{sr_k}(t)|^2 = \tau_{sr}$ for all $k$; all relay-to-destination channel gains are also i.i.d. and $\mathbb{E}|h_{rd}(t)|^2 = \tau_{rd}$, as are all relay-to-eavesdropper channel gains for which $\mathbb{E}|h_{re}(t)|^2 = \tau_{re}$. The source-to-eavesdropper channel gain is denoted as $\mathbb{E}|h_{se}(t)|^2 = \tau_{se}$. It is noted that its not require any two of $\tau_{sr}$, $\tau_{rd}$, $\tau_{se}$ and $\tau_{re}$ to be the same, thereby representing a “practical” scenario. All noises are AWGN, and without losing generality the noise variances are all normalised to unity.

The transmission powers for source and relay nodes are all assumed to be $E_s$.

At time $t$, the instantaneous secrecy capacities for $S \rightarrow R_k$ and $R_k \rightarrow D$ are obtained as (e.g. see [81])

$$C_{sr_k}(t) = \max \left\{ \frac{1}{2} \log_2 \left( \frac{1 + E_s|h_{sr_k}(t)|^2}{1 + E_s|h_{se}(t)|^2} \right), 0 \right\}$$

$$\approx \max \left\{ \frac{1}{2} \log_2 \left( \frac{|h_{sr_k}(t)|^2}{|h_{se}(t)|^2} \right), 0 \right\}$$

$$C_{rd}(t) = \max \left\{ \frac{1}{2} \log_2 \left( \frac{1 + E_s|h_{rd}(t)|^2}{1 + E_s|h_{re}(t)|^2} \right), 0 \right\}$$

$$\approx \max \left\{ \frac{1}{2} \log_2 \left( \frac{|h_{rd}(t)|^2}{|h_{re}(t)|^2} \right), 0 \right\}$$

respectively, where the approximations hold at high SNR. With the DF applied at the relays, if the relay $R_k$ is selected for the data transmission, the instantaneous secrecy capacity for the overall system is obtained as

$$C_k(t) = \min\{C_{sr_k}(t), C_{rd}(t)\}.$$  \hspace{1cm} (7.2.2)

this is a baseline [97]

$$C_k(t) = \max \left\{ \frac{1}{2} \log_2 \frac{\min\{1 + E_s|h_{sr_k}(t)|^2, 1 + E_s|h_{rd}(t)|^2\}}{1 + E_s|h_{se}(t)|^2 + E_s|h_{re}(t)|^2} \right\}$$  \hspace{1cm} (7.2.3)

For convenience in development, the time index $t$ is ignored in the rest of the chapter unless necessary.
7.3 Max-ratio relay selection

Two relay selection schemes have been described in [93] to maximise the secrecy capacity, corresponding to the cases that knowledge of exact or the average gains of the eavesdropping channels is known. While the relay selection schemes in [93] are for the scenario that the eavesdropper can intercept the data from the relays but not the source, they can be generalised to the more general system described in Section 7.2 where both source-to-eavesdropper and relay-to-eavesdropper links are present. To be specific, if the exact knowledge of the eavesdropper channels is available, the max-min-ratio selection scheme can be obtained from that for Case 1 in [93] as

\[
R_{\text{case 1}}^{(\text{max-min-ratio})} = \arg \max_{R_k} \left\{ \min \left\{ \frac{|h_{sr_k}|^2}{|h_{se}|^2}, \frac{|h_{rk_d}|^2}{|h_{rk_e}|^2} \right\} \right\}, \tag{7.3.1}
\]

where \(|h_{sr_k}|^2/|h_{se}|^2\) and \(|h_{rk_d}|^2/|h_{rk_e}|^2\) are the gain-ratios for the source-to-relay and relay-to-destination transmission for relay \(R_k\), respectively. On the other hand, if only the knowledge of the average eavesdropper channels gains is known, the max-min-ratio selection scheme becomes

\[
R_{\text{case 2}}^{(\text{max-min-ratio})} = \arg \max_{R_k} \left\{ \min \left\{ \frac{|h_{sr_k}|^2}{\gamma_{se}}, \frac{|h_{rk_d}|^2}{\gamma_{rk_e}} \right\} \right\}. \tag{7.3.2}
\]

The max-min-ratio schemes in (7.3.1) and (7.3.2) are used as a benchmark for comparison in this chapter.

The performance of the max-min-ratio scheme is limited by the constraint that the best source-to-relay and relay-to-destination links must be determined concurrently. In this chapter, a new max-ratio selection scheme by making use of data buffers at the relays is proposed. To be specific, suppose that every relay is equipped with a data buffer \(Q_k\) \((1 \leq k \leq N)\) of finite size \(L\) (in the number of data packets), and the data packets in the buffer follow the “first-in-first-out” rule. At any time, the best transmission
link with the highest gain-ratio is selected among all available source-to-relay and relay-to-destination links. If a source-to-relay link is selected, the source node transmits one data packet to the corresponding relay node, and the relay decodes the packet. If the decoding is successful, the decoded packet is stored in the buffer and the number of data packets in the buffer is increased by one. On the other hand, if a relay-to-destination link is selected, the corresponding relay transmits the earliest stored packet in the buffer to the destination. If the destination can successfully decode the packet, the number of packets in the buffer is decreased by one. Depending on the knowledge of the eavesdropper channel, it has two selection criteria:

Case 1 - If the exact knowledge of all channels\(^2\), including the eavesdropping channels \(h_{se}\) and \(h_{re}\), are available, the max-ratio selects the best relay as

\[
R_{\text{case } 1}^{(\text{max–ratio})} = \arg \max_{R_k} \left\{ \max_{R_k: \Psi(Q_k) \neq L} \frac{|h_{sr_k}|^2}{|h_{se}|^2}, \max_{R_k: \Psi(Q_k) \neq 0} \frac{|h_{rk_d}|^2}{|h_{re}|^2} \right\},
\]

(7.3.3)

where \(\Psi(Q_k)\) gives the number of data packets in buffer \(Q_k\).

Case 2 - If only the average channel gains for the eavesdropping channels, i.e. \(\tau_{se}\) and \(\tau_{re}\), are available (but the exact knowledge for all other channels are known), the best relay for the max-ratio scheme is selected as:

\[
R_{\text{case } 2}^{(\text{max–ratio})} = \arg \max_{R_k} \left\{ \max_{R_k: \Psi(Q_k) \neq L} \frac{|h_{sr_k}|^2}{\tau_{se}}, \max_{R_k: \Psi(Q_k) \neq 0} \frac{|h_{rk_d}|^2}{\tau_{re}} \right\},
\]

(7.3.4)

Of particular interest is the secrecy outage probability of the max-ratio relay selection scheme, which is investigated below.

\(^2\)The CSI is usually estimated through pilots and feedback (e.g. [72]), and the CSI estimation without feedback may also be applied (e.g. [73]). Further detail of the CSI estimation is beyond the scope of this chapter.
7.4 Secrecy Outage Probability

The secrecy outage probability is defined as $P_{out} = P(C < r_{sc})$, where $C$ is the instantaneous secrecy capacity, $r_{sc}$ is the target secrecy capacity and $P(.)$ gives the probability of the enclosed.

7.4.1 Outage event

In the max-ratio relay selection scheme, at any time, the numbers of data packets in every buffer form a “state”. Because there are $N$ available relays and every relay is equipped with a buffer of size $L$, there are $(L + 1)^N$ states in total. The $l$-th state vector is defined as

$$s_l = [\Psi_{l}(Q_1), \cdots, \Psi_{l}(Q_N)], \quad l = 1, \cdots, (L + 1)^N, \quad (7.4.1)$$

where $\Psi_{l}(Q_k)$ gives the number of data packets in buffer $Q_k$ at state $s_l$. It is clear that $0 \leq \Psi_{l}(Q_k) \leq L$. Every state corresponds to one pair of $(K_{sr}^{(s_l)}, K_{rd}^{(s_l)})$, where $K_{sr}^{(s_l)}$ and $K_{rd}^{(s_l)}$ are the numbers of available links for source-to-relay and relay-to-destination transmission, respectively. A source-to-relay link is considered available when the buffer of the corresponding relay node is not full. Thus at state $s_l$, the number of source-to-relay links $K_{sr}^{(s_l)}$ can be obtained as

$$K_{sr}^{(s_l)} = \sum_{k=1}^{N} \Phi_{s_l}^{+}(Q_k), \quad (7.4.2)$$

where $\Phi_{s_l}^{+}(Q_k) = 0$ if the buffer $Q_k$ is full and $\Phi_{s_l}^{+}(Q_k) = 1$ otherwise. On the other hand, a relay-to-destination link is available when the corresponding relay buffer is not empty, and the number of available relay-to-destination links $K_{rd}^{(s_l)}$ is given by

$$K_{rd}^{(s_l)} = \sum_{k=1}^{N} \Phi_{s_l}^{-}(Q_k), \quad (7.4.3)$$
where $\Phi_{s_l}(Q_k) = 0$ if the buffer $Q_k$ is empty and $\Phi_{s_l}(Q_k) = 1$ otherwise. It is clear from (7.4.2) and (7.4.3) that $0 \leq K_{sr}^{(s_l)} \leq N$ and $0 \leq K_{rd}^{(s_l)} \leq N$. Specifically, if none of the buffers is full or empty, all links are available such that $K_{sr}^{(s_l)} = K_{rd}^{(s_l)} = N$.

At state $s_l$, the best link is selected among all $K_{sr}^{(s_l)}$ source-to-relay and $K_{rd}^{(s_l)}$ relay-to-destination links. Depending on whether a source-to-relay or relay-to-destination link is selected, the secrecy outage event at state $s_l$ can be described as

secrecy outage event : \[
\begin{cases} 
C_{sr} < r_{sc}, & \text{if a source-to-relay link is selected,} \\
C_{rd} < r_{sc}, & \text{if a relay-to-destination link is selected,}
\end{cases}
\]

where $C_{sr}$ and $C_{rd}$ are the instantaneous source-to-relay and relay-to-destination secrecy capacities for the corresponding best selected relay, which can be obtained by (7.2.1) respectively. For later use, let $p_{s_l}^{out}$ denote as the probability that the outage event in (7.4.4) occurs at state $s_l$. Considering all possible states, the secrecy outage probability for the max-ratio relay selection is obtained as

$$P_{out} = \sum_{l=1}^{(L+1)^N} \pi_l p_{s_l}^{out}, \quad (7.4.5)$$

where $\pi_l = p(s_l)$ which is the stationary probability for state $s_l$.

### 7.4.2 State transition matrix

Suppose at time $t$, the state for the relay buffers is $s_l$. At time $(t+1)$, there is one relay selected for either receiving or transmitting a data packet, so that the number of packets in the corresponding buffer is increased or decreased by one respectively. Depending on which relay receives or transmits data, at time $(t+1)$, the buffers may move from state $s_l$ to several possible states, forming a Markov chain. Let $A$ represent as the $(L+1)^N \times (L+1)^N$ state transition matrix, where the entry $A_{n,l} = P(X_{t+1} = s_n | X_t = s_l)$ is the
transition probability that the state moves from $s_l$ at time $t$ to $s_n$ at time $(t + 1)$.

If the secrecy outage event occurs, the state $s_l$ remain unchanged at time $(t + 1)$. Otherwise, $s_l$ moves to another state. Thus the probability for $s_l$ to leave for another state is given by

$$p_{\text{leave}}^{s_l} = 1 - p_{\text{out}}^{s_l}. \quad (7.4.6)$$

It is clear from the selection rules (7.3.3) and (7.3.4) that, for both Case 1 and 2, the probabilities to select the source-to-relay and relay-to-destination transmission at any time are not the same. This is very different from the max-link approach in [33] where the selection of any available link is equally likely, making the analysis much harder. All states which can be moved from $s_l$ can be divided into two sets as:

$$U_{s_l \rightarrow R} = \left\{ \bigcup_{1 \leq n \leq (L+1)N} s_n : s_n - s_l \in Q_{s \rightarrow R} \right\},$$

$$U_{s_l \rightarrow D} = \left\{ \bigcup_{1 \leq n \leq (L+1)N} s_n : s_n - s_l \in Q_{r \rightarrow D} \right\}, \quad (7.4.7)$$

where $Q_{s \rightarrow R} \triangleq \{ \bigcup_{1 \leq k \leq K} e_k \}$, $Q_{r \rightarrow D} \triangleq \{ \bigcup_{1 \leq k \leq K} -e_k \}$, and $e_k$ is the vector where its $k$-th element is one and all other elements are zero. To be specific, if a source-to-relay link is selected, the number of packets in the selected buffer is increased by one, and then state $s_l$ moves to $s_n$ with $s_n - s_l \in Q_{s \rightarrow R}$. Therefore, the set $U_{s_l \rightarrow R}^{s_l}$ contains all states to which $s_l$ can move when a source-to-relay link is selected. Similarly, $U_{s_l \rightarrow D}^{s_l}$ contains all states to which $s_l$ can move when a relay-to-destination link is selected. Let $P_{s \rightarrow D}^{(s_l)}$ and $P_{r \rightarrow D}^{(s_l)}$ be the probabilities that the source-to-relay and relay-to-destination transmissions are selected at state $s_l$, respectively. It is clear that $P_{s \rightarrow D}^{(s_l)} + P_{r \rightarrow D}^{(s_l)} = 1$. 

On the other hand, because suppose all source-to-relay channels are i.i.d. fading and all relay-to-destination channels are also i.i.d. frequency flat fading and so are the relay-to-eavesdropper channels, the selection of one particular link within either $U_{st}^{S \to R}$ or $U_{st}^{R \to D}$ is equal likely. Therefore, the probability to select a source-to-relay or relay-to-destination link at state $s_l$ is given by

$$p_{s_l}^{out} = p_{s_l}^{\text{leave}} \left( \frac{1}{K_{sr}^{(s_l)}} p_{s_l}^{S \to R} \right) = \frac{1}{K_{sr}^{(s_l)}} (1 - p_{s_l}^{out})(1 - p_{s_l}^{R \to D}),$$

$$p_{s_l}^{-} = p_{s_l}^{\text{leave}} \left( \frac{1}{K_{rd}^{(s_l)}} p_{s_l}^{R \to D} \right) = \frac{1}{K_{rd}^{(s_l)}} (1 - p_{s_l}^{out})p_{s_l}^{R \to D},$$

(7.4.8)

respectively.

With these observations, the $(n, l)$-th entry of the state transition matrix $A$ is expressed as

$$A_{n,l} = \begin{cases} 
    p_{s_l}^{\text{out}}, & \text{if } n = l, \\
    \frac{1}{K_{sr}^{(s_n)}} (1 - p_{s_l}^{out})(1 - p_{s_l}^{R \to D}), & \text{if } s_n \in U_{s_l}^{S \to R}, \\
    \frac{1}{K_{rd}^{(s_n)}} (1 - p_{s_l}^{out})p_{s_l}^{R \to D}, & \text{if } s_n \in U_{s_l}^{R \to D}, \\
    0, & \text{elsewhere},
\end{cases}$$

(7.4.9)

For example, for a system with two relays ($N = 2$) and each relay having a buffer of size 2 ($L = 2$), there are $(L + 1)^N = 9$ states in total and the
state diagram is shown in Fig. 7.2. The transition matrix $A$ is given by

$$A = \begin{pmatrix}
p_1^{s_1} & p_2^{s_2} & 0 & p_1^{s_3} & 0 & 0 & 0 & 0 & 0 \\
p_1^{s_1} & p_2^{s_2} & 0 & p_1^{s_3} & 0 & 0 & 0 & 0 & 0 \\
0 & p_1^{s_2} & p_2^{s_3} & 0 & p_1^{s_4} & 0 & 0 & 0 & 0 \\
p_1^{s_1} & 0 & 0 & p_2^{s_4} & 0 & p_1^{s_5} & 0 & p_1^{s_6} & 0 \\
0 & p_1^{s_2} & 0 & p_2^{s_4} & p_2^{s_5} & p_2^{s_6} & 0 & p_2^{s_8} & 0 \\
0 & 0 & p_1^{s_3} & 0 & p_2^{s_5} & p_2^{s_6} & 0 & 0 & p_2^{s_9} \\
0 & 0 & 0 & p_2^{s_4} & 0 & 0 & p_2^{s_7} & p_2^{s_8} & 0 \\
0 & 0 & 0 & 0 & p_1^{s_5} & 0 & p_2^{s_7} & p_2^{s_8} & p_2^{s_9} \\
0 & 0 & 0 & 0 & 0 & p_2^{s_5} & 0 & p_2^{s_8} & p_2^{s_9} \\
0 & 0 & 0 & 0 & 0 & 0 & p_2^{s_6} & 0 & p_2^{s_8} & p_2^{s_9}
\end{pmatrix} \quad (7.4.10)$$

Figure 7.2. State diagram of the buffer-aided max-ratio relay selection system with $N = 2$ relays and $L = 2$ buffer size.

Because the transition matrix $A$ in (7.4.9) is column stochastic, irreducible and aperiodic,$^3$ the stationary state probability vector is obtained as (see [49,50])

$$\pi = (A - I + B)^{-1} b, \quad (7.4.11)$$

$^3$Column stochastic means all entries in any column sum up to one, irreducible means that it is possible to move from any state to any state, and aperiodic means that it is possible to return to the same state at any steps [?, 49].
where $\pi = [\pi_1, \cdots, \pi_{(L+1)^N}]^T \in \mathbb{R}^{1 \times (L+1)^N}$, $b = (1, 1, \ldots, 1)^T \in \mathbb{R}^{1 \times (L+1)^N}$, $I$ is $(L + 1)^N \times (L + 1)^N$ identity matrix and $B_{n,l}$ is an $n \times l$ all one matrix.

Finally, substituting (7.4.9) and (7.4.11) into (7.4.5) re-formats the secrecy outage probability as

$$P_{out} = \prod_{l=1}^{(L+1)^N} \pi_l^{p_{out}^{s_l}} = \text{diag}(A)\pi$$

$$= \text{diag}(A)(A - I + B)^{-1}b,$$

(7.4.12)

where $\text{diag}(A)$ is a vector consisting of all diagonal elements of $A$.

It is clear from (7.4.12) that the secrecy outage probability $P_{out}$ is determined by $A$ which again, from (7.4.9), is determined by $p_{out}^{s_l} \text{and } p_{R\rightarrow D}^{(s_l)}$.

The secrecy outage probabilities for both Case 1 and 2 can be expressed as (7.4.12), but with different $p_{out}^{s_l}$ and $p_{R\rightarrow D}^{(s_l)}$ which are derived below.

7.5 Case 1 - exact knowledge of eavesdropping channels

In Case 1, suppose exact knowledge for the instantaneous gains for all channels, including the eavesdropping channels. This is a typical assumption in many existing approaches (e.g. [94] and [95]).

7.5.1 $p_{out}^{s_l}$: outage probability at state $s_l$ for Case 1

The relay selection rule for Case 1 is shown in (7.3.3). For better exposition, let $x = \max_{\Psi(Q_k)\neq L} \{|h_{sr_k}|^2|/|h_{se}|^2\}$, $y = \max_{\Psi(Q_k)\neq 0} \{|h_{re}\}^2 / |h_{rd}|^2\}$ and $z = \max(x, y)$. In order to concentrate on the secrecy capacity, suppose the channel SNR is high enough so that the decoding is always successfully at the relay and destination nodes$^4$. From (7.2.1) and (7.4.4), the probability

$^4$This is a typical assumption in most existing secure communications literature (e.g. [93,94]).
of outage event at state $s_l$ at the high SNR is given by

$$p_{\text{out}}^{s_l} = P(z < 2^{2r_{sc}}) = F_Z(z)|_{z=2^{2r_{sc}}}, \quad (7.5.1)$$

where $F_Z(z)$ is the CDF of $z$ which is derived below.

Recall that state $s_l$ corresponds to $(K_{sr}^{(s_l)}, K_{rd}^{(s_l)})$. From the theory of order statistics [46], if the number of available source-to-relay links is $K_{sr}^{(s_l)}$, the CDF of $x_1 = \max_{Q_k \neq L} \{|h_{sr_k}|^2\}$ is given by

$$F_{X_1}(x) = [1 - e^{-\frac{x}{\gamma_{sr}}} K_{sr}^{(s_l)}], \quad (7.5.2)$$

Noting that $x = x_1/|h_{se}|^2$, the CDF of $x$ is then obtained as

$$F_X(x) = \sum_{i=0}^{K_{sr}^{(s_l)}} C_i^{K_{sr}^{(s_l)}} (-1)^i \frac{M_1}{ix + M_1}, \quad (7.5.3)$$

where $C_i^{K_{sr}^{(s_l)}} = K_{sr}^{(s_l)}!/[i!(K_{sr}^{(s_l)} - i)!]$ which is the binomial coefficient, and $M_1 = \gamma_{sr}/\gamma_{se}$ which is the average gain ratio between the source-to-relay and source-to-eavesdropper channels.

On the other hand, because the number of available relay-to-destination links is $K_{rd}^{(s_l)}$, the CDF of $y$ is given by

$$F_Y(y) = \left[\frac{y}{M_2 + y}\right]^{K_{rd}^{(s_l)}}, \quad (7.5.4)$$

where $M_2 = \gamma_{rd}/\gamma_{re}$ which is the average gain ratio between the relay-to-destination and relay-to-eavesdropper channels.

Because $x$ and $y$ are mutually independent, from (7.5.3) and (7.5.4), the CDF of $z = \max(x, y)$ is obtained as:

$$F_Z(z) = F_X(z) F_Y(z) = \sum_{i=0}^{K_{sr}^{(s_l)}} C_i^{K_{sr}^{(s_l)}} (-1)^i \frac{M_1}{iz + M_1} \left[\frac{z}{M_2 + z}\right]^{K_{rd}^{(s_l)}}. \quad (7.5.5)$$
Substituting (7.5.5) into (7.5.1) gives

\[ p_{\text{out}}^{s_l} = \sum_{i=0}^{K_{sr}^{(s_l)}} C_i^{K_{sr}^{(s_l)}} (-1)^i \frac{M_1}{2^{2r_{sc}} + M_1} \left[ \frac{2^{2r_{sc}}}{M_2 + 2^{2r_{sc}}} \right] K_{rd}^{(s_l)}}. \] (7.5.6)

The next subsection will provide the probability of selecting the relay to destination transmission at state \( s_l \).

### 7.5.2 \( p_{R \to D}^{(s_l)} \): probability of selecting the relay-to-destination transmission at state \( s_l \) for Case 1

If there are no relay-to-destination links available (or \( K_{rd}^{(s_l)} = 0 \)), it has \( p_{R \to D}^{(s_l)} = 0 \). On the other hand, if there are no source-to-relay links available (or \( K_{sr}^{(s_l)} = 0 \)), it has \( p_{R \to D}^{(s_l)} = 1 \). For other cases, \( p_{R \to D}^{(s_l)} \) is obtained below.

**Proof.** Noting the definition of \( x \) and \( y \) in Section 7.5.1, it has

\[ p_{R \to D}^{(s_l)} = P(x < y) = \int_{x < y} \int_{x}^{\infty} f_{XY}(x, y) dx dy = \int_{0}^{\infty} \int_{0}^{y} f_{XY}(x, y) dx dy, \] (7.5.7)

where \( f_{XY}(x, y) \) is the joint PDF of \( x \) and \( y \).

From the CDF-s of \( x \) and \( y \) given by (7.5.3) and (7.5.4) respectively, the PDF-s of \( x \) and \( y \) are obtained as

\[ f_X(x) = \sum_{i=0}^{K_{sr}^{(s_l)}} \frac{C_i^{K_{sr}^{(s_l)}} (-1)^i M_1 i}{(M_1 + ix)^2} \quad \text{and} \quad f_Y(y) = \frac{y^{K_{rd}^{(s_l)}-1} K_{rd}^{(s_l)} M_2}{(M_2 + y)^{K_{rd}^{(s_l)}+1}}, \] (7.5.8)

respectively. Because \( x \) and \( y \) are mutually independent, it has

\[ f_{XY}(x, y) = f_X(x) f_Y(y) = \sum_{i=0}^{K_{sr}^{(s_l)}} \frac{C_i^{K_{sr}^{(s_l)}} (-1)^i M_1 M_2 K_{rd}^{(s_l)} i y^{K_{rd}^{(s_l)}-1}}{(M_1 + ix)^2 (M_2 + y)^{K_{rd}^{(s_l)}+1}}. \] (7.5.9)
Substituting (7.5.9) into (7.5.7) gives

\[
P_{R \rightarrow D}^{(s_l)} = \int_0^\infty \int_0^y \sum_{i=0}^{K_{sr}^{(s_l)}} C_i \frac{K_i^{(s_l)} (M_1 M_2 K_{rd}^{(s_l)}) i y K_{rd}^{(s_l) - 1}}{(M_1 + ix)^2 (M_2 + y)^{K_{rd}^{(s_l)} + 1}} \, dx dy
\]

\[
= 1 + \sum_{i=1}^{K_{sr}^{(s_l)}} C_i K_i^{(s_l)} (M_1 M_2 K_{rd}^{(s_l)}) \int_0^\infty \frac{y K_{rd}^{(s_l) - 1}}{(M_1 + iy)(M_2 + y)^{K_{rd}^{(s_l)} + 1}} \, dy.
\]  

(7.5.10)

Then according to [98], if \(K_{rd}^{(s_l)} > 1\), it obtains

\[
P_{R \rightarrow D}^{(s_l)} = 1 + \sum_{i=1}^{K_{sr}^{(s_l)}} C_i K_i^{(s_l)} (M_1 M_2) \frac{K_{rd}^{(s_l)}}{M_2} B(2, K_2) \cdot \mathcal{F}_{2,1} \left([K_2 + 1, K_2], K_2 + 2, 1 - \frac{M_1}{M_2}\right); \]  

(7.5.11)

if \(K_{rd}^{(s_l)} = 1\) and \(M_1 = M_2\), it has

\[
P_{R \rightarrow D}^{(s_l)} = 1 - K_1/2 + \sum_{i=2}^{K_{sr}^{(s_l)}} C_i K_i^{(s_l)} (-1)^i \frac{i [\ln(i) - 1] + 1}{(i - 1)^2}; \]  

(7.5.12)

and if \(K_{rd}^{(s_l)} = 1\) and \(M_1 \neq M_2\), it has

\[
P_{R \rightarrow D}^{(s_l)} = 1 + \sum_{i=1}^{K_{sr}^{(s_l)}} C_i K_i^{(s_l)} (-1)^i M_1 \cdot \frac{i \ln(i) M_2 + i \ln(M_2) M_2 - i \ln(M_1) M_2 - i M_2 + M_1}{(M_1 - i M_2)^2}. \]  

(7.5.13)
In summary, it has

$$
P_{R \rightarrow D}^{(s_i)} = \begin{cases} 
0, & K_2 = 0, \ 0 < K_1 \leq N, \\
1 - K_1/2 + \sum_{i=2}^{K_1} C_i^{(s_i)} \frac{(-1)^i i^{\lceil \ln(i) - 1 \rceil + 1}}{(i-1)^2}, & 0 < K_1 \leq N, \\
1 + \sum_{i=1}^{K_2} C_i^{(s_i)} (-1)^i M_1 \\
\frac{M_2 \ln(i) + i \ln(M_2) M_2 - \ln(M_1) M_2 - M_2 + M_1}{(M_1 - i M_2)^2}, & 0 < K_1 \leq N, \\
1 + \sum_{i=1}^{K_2} C_i^{(s_i)} (-1)^i K_r^{(s_i)} \left( \frac{M_1}{M_2} \right)^{K_r^{(s_i)}} B(2, K_2) \\
\mathcal{F}_{2,1}([K_2 + 1, K_2], K_2 + 2, 1 - \frac{M_1}{M_2}), & 0 < K_1 \leq N, \\
1, & 1 < K_2 \leq N, \\
\end{cases}
$$

where $B(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt$ which is the Beta function and $\mathcal{F}_{2,1}(a, b, c, z)$ is the first hypergeometric function.

Finally, substituting (7.5.6) and (7.5.14) into (7.4.12) gives the secrecy outage probability at the high SNR for Case 1.

### 7.6 Case 2 - knowledge of the average channel gains for eavesdropping channels

In case 2, the destination node only have knowledge of the average gains for eavesdropping channel, while the exact information for all other channels are still available.
7.6.1 $p_{\text{out}}^{s_l}$: outage probability at state $s_l$ for Case 2

Let $u_{ab} = |h_{ab}|^2/\gamma_{ab}$ be the normalised gain for channel $h_{ab}$, where $\{ab\} \in \{sr_k, se, r_kd, r_kee\}$. Because of the normalisation, all $u_{ab}$ have the same PDF as

$$f_U(u) = e^{-u}, \quad (7.6.1)$$

The selection rule in (7.3.4) can then be expressed as

$$R^{(2)}_b = \arg \max_{R_k: \Psi(Q_k) \neq L} \left\{ \max_{R_k: \Psi(Q_k) \neq 0} \{ M_1 u_{sr_k} \} , \max_{R_k: \Psi(Q_k) \neq 0} \{ M_2 u_{r_kd} \} \right\}, \quad (7.6.2)$$

where $M_1$ and $M_2$ are defined in (7.5.3) and (7.5.4), respectively.

Let $f = \max_{R_k: \Psi(Q_k) \neq L} \{ M_1 u_{sr_k} \}$, $g = \max_{R_k: \Psi(Q_k) \neq 0} \{ M_2 u_{r_kd} \}$, $w = \max\{f, g\}$ and $v = w/u$, where $u$ is normalised exponentially distributed with PDF given by (7.6.1). Then from (7.2.1) and (7.4.4), the probability of outage event for state $s_l$ at the high SNR for Case 2 is given by

$$p_{\text{out}}^{s_l} = P(v < 2^{2r_{sc}}) = F_V(v)|_{v=2^{2r_{sc}}}. \quad (7.6.3)$$

The CDF-s of $f$ and $g$ are given by

$$F_F(f) = \left[ 1 - e^{-f/M_1} \right] K_{sr}^{(s_l)} \quad \text{and} \quad F_G(g) = \left[ 1 - e^{-g/M_2} \right] K_{rd}^{(s_l)}, \quad (7.6.4)$$

respectively. Because $f$ and $g$ are mutually independent, the CDF of $w = \max\{f, g\}$ is obtained as

$$F_W(w) = F_F(w)F_G(w) = \left[ 1 - e^{-w/M_1} \right] K_{sr}^{(s_l)} \left[ 1 - e^{-w/M_2} \right] K_{rd}^{(s_l)}. \quad (7.6.5)$$
Then the CDF of \( v = w/u \) is

\[
F_V(v) = \int_0^\infty \left[ 1 - e^{-uv/M_1} \right] K_{sr}^{(s_1)} K_{rd}^{(s_1)} e^{-u} du
\]

\[
= \sum_{i=0}^{K_{sr}^{(s_1)}} \sum_{j=0}^{K_{rd}^{(s_1)}} C_i^{(s_1)} C_j^{(s_1)} (-1)^{i+j} M_1 M_2 \frac{v(M_2 i + M_1 j) + M_1 M_2}{2^{2s_r} (M_2 i + M_1 j) + M_1 M_2}.
\]

Substituting (7.6.6) into (7.5.1) gives

\[
P_{out}^{(s_1)} = \sum_{i=0}^{K_{sr}^{(s_1)}} \sum_{j=0}^{K_{rd}^{(s_1)}} C_i^{(s_1)} C_j^{(s_1)} (-1)^{i+j} M_1 M_2 \frac{v(M_2 i + M_1 j) + M_1 M_2}{2^{2s_r} (M_2 i + M_1 j) + M_1 M_2}.
\]

The next subsection will provide the probability of selecting the relay to destination transmission at state \( s_1 \).

**7.6.2 \( p_{R \rightarrow D}^{(s_1)} \): probability of selecting the relay-to-destination transmission at state \( s_1 \) for Case 2**

Similar to that in Case 1, it has \( p_{R \rightarrow D}^{(s_1)} = 0 \) for \( K_{rd}^{(s_1)} = 0 \), and \( p_{R \rightarrow D}^{(s_1)} = 1 \) for \( K_{sr}^{(s_1)} = 0 \). But otherwise, it has

\[
p_{R \rightarrow D}^{(s_1)} = P(f < g) = \int \int f_{FG}(f, g) df dg,
\]

where \( f_{FG}(f, g) \) is the joint PDF of \( f \) and \( g \).

From (7.6.4), the PDF of \( f \) and \( g \) are obtained as

\[
f_F(f) = \frac{K_{sr}^{(s_1)}}{M_1} e^{-\frac{f}{M_1}} (1 - e^{-\frac{f}{M_1}}) K_{sr}^{(s_1)} - 1,
\]

\[
f_G(g) = \frac{K_{rd}^{(s_1)}}{M_2} e^{-\frac{g}{M_2}} (1 - e^{-\frac{g}{M_2}}) K_{rd}^{(s_1)} - 1,
\]

respectively. Because \( f \) and \( g \) are mutually independent, it has \( f_{FG}(f, g) = f_F(f) f_G(g) \).
Substituting (7.6.9) into (7.6.8) gives

\[
\begin{align*}
    p_{R\rightarrow D}^{(s_l)} &= \int_{0}^{\infty} \int_{y}^{\infty} \frac{K_{sr}^{(s_l)} K_{rd}^{(s_l)}}{M_1 M_2} e^{-\frac{x}{M_1}} e^{-\frac{y}{M_2}} (1 - e^{-\frac{x}{M_1}}) K_{sr}^{(s_l)} - 1 (1 - e^{-\frac{y}{M_2}}) K_{rd}^{(s_l)} - 1 dx \ dy \\
    &= \sum_{i=0}^{K_{sr}^{(s_l)} - 1} \sum_{j=0}^{K_{rd}^{(s_l)} - 1} C_i^{(s_l)} C_j^{(s_l)} (-1)^{i+j} \frac{M_1 K_{rd}^{(s_l)}}{M_1 + M_1 j + M_2 i}.
\end{align*}
\]

(7.6.10)

Therefore, it has

\[
\begin{align*}
    p_{R\rightarrow D}^{(s_l)} &\begin{cases} 
        0, & K_2 = 0, \ 0 < K_1 \leq N, \\
        \sum_{i=0}^{K_{sr}^{(s_l)} - 1} \sum_{j=0}^{K_{rd}^{(s_l)} - 1} C_i^{(s_l)} C_j^{(s_l)} (-1)^{i+j} \frac{M_1 K_{rd}^{(s_l)}}{M_1 + M_1 j + M_2 i}, & 0 < K_1 \leq N, \ 0 < K_2 \leq N, \\
        1, & K_1 = 0, \ 0 < K_2 \leq N.
    \end{cases}
\end{align*}
\]

(7.6.11)

Finally, substituting (7.6.7) and (7.6.11) into (7.4.12) gives the secrecy outage probability at the high SNR for Case 2.

### 7.7 Discussion

In this section, a specific scenario that the average gain ratios for the source-to-relay and relay-to-destination transmissions are the same, or \(M_1 = M_2\) is considered. Of particular interest is the outage performance for \(L \rightarrow \infty\) which shows the best potential performance for the max-ratio scheme.

First the Case 2 is considered. Because of \(M_1 = M_2\), the probabilities to select a source-to-relay and relay-to-destination link are the same, and both equal \(1/(K_{sr}^{(s_l)} + K_{rd}^{(s_l)})\). This is similar to the max-link scheme in [33]. Thus building upon the analysis in [33], it shows that, if \(L \rightarrow \infty\), the stationary probability for any state corresponding to \(K_{sr}^{(s_l)} \neq N\) or \(K_{rd}^{(s_l)} \neq N\) must be
0, or it has

\[ \sum_{s_l:K_{sr}^{(s_l)}=K_{rd}^{(s_l)}=K} P(s_l) = 1. \] (7.7.1)

This implies that, if \( L \to \infty \), at any time, the best link is always selected from \( K_{sr}^{(s_l)} + K_{rd}^{(s_l)} = 2N \) available channels with the max-ratio scheme in Case 2.

In Case 1, unfortunately, a simple form as in (7.7.1) for \( L \to \infty \) is not that easy to have, because the probabilities to select a source-to-relay and relay-to-destination link are not equal, even with \( M_1 = M_2 \). In order to illustrate the performance for \( L \to \infty \), the original scheme can be assumed to be equivalent to a virtual relay section scheme having equal probability to select a source-to-relay and relay-to-destination link, or \( P(x < y) = 0.5 \) where \( x \) and \( y \) are defined in Section 7.5.1. Then in the virtual scheme with \( L \to \infty \), the number of available source-to-relay and relay-to-destination links are always \( K'_1 \) and \( K'_2 \), respectively, or \( \sum_{s_l:K_{sr}^{(s_l)}=K_{rd}^{(s_l)}=K'} P(s_l) = 1 \).

If let \( K'_1 = K \), then \( K'_2 \) can be obtained by solving \( P(x < y) = 0.5 \) as

\[
P(x < y) = \sum_{i=1}^{K'_1} C_{K'_1}^i (-1)^{i+1} K'_2 \left( \frac{M_1}{M_2^i} \right) K'_2 B(2, K'_2) \cdot F_{2,1} \left( [K'_2 + 1, K'_2], K'_2 + 2, 1 - \frac{M_1}{M_2^i} \right) = 0.5,
\] (7.7.2)

where \( P(x < y) \) is given by (7.5.11) in the Appendix I.

Since \( K'_2 \) obtained from (7.7.2) is usually not an integer, the virtual selection scheme cannot be realised in practice. However, it provides an interest insight in understanding the secrecy performance of the original max-ratio scheme in Case 1 for \( L \to \infty \). Normally, it has \( K'_2 < K \), which implies that, on the average, the best link is always selected from \( K'_1 + K'_2 \) available channels at any time and \( K \leq K'_1 + K'_2 \leq 2K \).

For both Case 1 and 2, with a limited buffer size \( L \), it always has \( K_{sr}^{(s_l)} + K_{rd}^{(s_l)} \geq K \) at any time. On the contrary, in the max-min-ratio
selection scheme, the best link is selected from only $K$ links. Therefore, it follows that the max-ratio scheme has significantly better secrecy outage performance than the benchmark max-min-ratio scheme, and the best potential performance for the max-ratio scheme is reached when $L \to \infty$. This will be well verified in the simulation below.

### 7.8 Numerical Results

In this section, simulation results are given to verify the secrecy outage probabilities for the proposed max-ratio relay selection scheme. Both Case 1 and 2 are considered. In the simulation, all noise variance and transmission powers are normalised to unity, and the average channel gains for source-to-relay and relay-to-destination transmissions are set as $\gamma_{sr} = \gamma_{rd} = 30$ dB. Thus the corresponding average channel SNR is also 30 dB which is high enough to guarantee successful decoding at the relays and destination. The average eavesdropping channel gains, i.e. $\gamma_{se}$ and $\gamma_{re}$, are determined through the settings of the average gain ratios $M_1 = \gamma_{sr}/\gamma_{se}$ and $M_2 = \gamma_{rd}/\gamma_{re}$, where let $M_1 = M_2$ for various values in the simulations.

In every simulation below, both theoretical and simulated secrecy outage probabilities are shown to essentially perfect match, where the theoretical results are based on (7.4.12) and simulation results are obtained by averaging 5,000,000 independent runs. This verifies well the novel closed-form secrecy outage probabilities obtained in this chapter.

Fig. 7.3 compare the secrecy outage performance of the proposed max-ratio scheme with those for the no relay selection and the benchmark max-min-ratio schemes, where the relay number is set as $N = 5$ in all relay selection schemes. It is clearly shown that the no relay selection scheme has the worst outage performance and the proposed max-ratio has the best. It is interesting to observe that, for both Case 1 and 2, with buffer size $L = 1$, the
Figure 7.3. The secrecy outage probabilities for different relay selection schemes.

max-ratio still performs significantly better than the max-min-ratio scheme. It worth highlight that the max-min-ratio scheme is effectively also equipped with a buffer of size 1, because the relays need to store the decoded data at
one time and transmit out at the next time. However, even with $L = 1$, the max-ratio does not reduce to the max-min-ratio scheme. This is because, for any relay receiving a data packet in the max-ratio scheme, it has the choice of whether to transmit the packet out at the next time or not. On the contrary, the relays in the max-min-ratio schemes do not have such choices. Therefore, the max-ratio scheme has “coding gain” over the max-min-ratio approach. On the other hand, with $L \rightarrow \infty$, the best potential performance of the max-ratio is reached, which is clearly shown in Fig. 7.3 (a) and (b) for both Case 1 and 2 respectively.

Fig. 7.4 compares the secrecy outage probabilities of the max-ratio scheme for Case 1 and 2, where the buffer size is fixed at $L = 3$, the average gain ratios are set as $M_1 = M_2 = 2$ or 5, and the relay number is set as $N = 2$ or 5. It is clearly shown that the max-ratio scheme in Case 1 has consistently better performance than that in Case 2.

Fig. 7.5 compares the secrecy outage probabilities of the max-ratio scheme for different numbers of relays, where the buffer size is fixed as $L = 5$ and the average gain ratios are set as $M_1 = M_2 = 5$. It is clearly shown that,
in both cases, the increase of the relay number can significantly improve the secrecy outage performance. For example, for the target secrecy capacity $\gamma_{sc} = 0.5$, when the relay number is increased from $N = 2$ to $N = 5$, the secrecy outage probability decreases by approximately 10 dB in Case 1, and by almost 6 dB in Case 2. This verifies the effectiveness of using the max-ratio relay selection in improving the secrecy performance.

Fig. 7.6 compares the secrecy outage probabilities of the max-ratio scheme for different relay buffer sizes, where the relay number and average gain ratio are fixed at $N = 4$ and $M_1 = M_2 = 5$, respectively. While both the theoretical and simulation results are obtained and perfectly match each other, only the theoretical results are shown in Fig. 7.6 as otherwise it would be too congested for illustration. Particularly, for $L \to \infty$ in Case 1, $K'_{1}$ and $K'_{2}$ for the virtual selection scheme (described in Section 7.7) are obtained as $K'_{1} = N = 4$ and $K'_{2} \simeq 2.23$ respectively. The theoretical result for Case 1 is then obtained by using $K'_{1} = 4, K'_{2} \simeq 2.23$ to calculate the outage probability in (7.4.12). On the other hand, the theoretical result for $L \to \infty$ in Case 2 is obtained by using $K^{(s_l)}_{sr} = K^{(s_l)}_{rd} = N = 4$ in the corresponding outage probability expression. In both Case 1 and 2, for $L \to \infty$, the simulation results are actually obtained by using a very large buffer size $L = 500$. It is clearly shown that, for both Case 1 and 2, the secrecy outage probability decreases with the increase of the buffer size $L$, but the improvements becomes less significant as $L$ increases. In fact, when $L = 50$, the secrecy outage probability is already very close to that for $L \to \infty$.

Fig. 7.7 shows the secrecy outage probabilities of the max-ratio scheme for different average gain ratios $M_1$ and $M_2$. It is clear that, in both Case 1 and 2, the secrecy outage probability becomes less with higher $M_1$ and $M_2$. This is because, for a larger $M_1$ and $M_2$, the eavesdropping channels become
Figure 7.5. The secrecy outage probabilities of the max-ratio scheme for different relay numbers $N$. 
Figure 7.6. The secrecy outage probabilities of the max-ratio scheme for different relay buffer sizes $L$. 

(a) Case 1

(b) Case 2
weaker.

Figure 7.7. The secrecy outage probabilities for different channel SNR, where $\tau_{se} = \tau_{te} = 30$ dB and target capacity is one.
This chapter proposed a new max-ratio relay selection policy for secure buffer-aided cooperative DF networks. With the help of the buffers at the relays, the best relay is selected with the largest gain ratio among all available source-to-relay and relay-to-destination links. Both cases for knowledge of exact and average gain for eavesdropping links are considered, and for the first time, in the challenging secure transmission context, closed-form expressions for the secrecy outage probabilities for both cases have been derived. Both analysis and simulations show that the proposed max-ratio relay selection has significantly better performance in secrecy outage probability than the benchmark max-min-ratio scheme, and provides an attractive way to realise and improve secure transmission at the physical layer of wireless communications.
Chapter 8

SUMMARY, CONCLUSION
AND FUTURE WORK

In this chapter, the contributions of this thesis and the conclusions that can be drawn from them are summarised. A discussion on possible future work is also included.

8.1 Summary and Conclusions

The research in this thesis has focused on applying buffer on relay in order to improve the performance of wireless cooperative networks. A scheme with AF type buffer-aided relay network and a scheme with reduced average packet delay for buffer-aided relay network have been presented. Buffer-aided relay were also extended to applications including multi-hop networks, cognitive networks and security networks. Outage probability, throughput and average delay analysis was used for performance assessment. Considering the chapters in detail:

Chapter 1 introduced cooperative communication systems was presented. Two cooperative communication protocols and relay selection scheme were described and the network’s performance measurements was also introduced of applying buffer on relays.

In Chapter 2, a literature review on current buffer-aided relay system was presented. A three-node buffer-aided relay system was first introduced,
followed by two buffer-aided relay selection schemes. A brief introduction of the analysis tools, including Markov Chain, Little’s Law, Trellis Diagram and queue, for buffer-aided network were presented.

In Chapter 3, the AF buffer-aided max-link relay selection was investigated. Both symmetric and asymmetric source-to-relay and relay-to-destination channel configurations were considered. The outage performance of the buffer-aided AF relay selection was analysed and the results of numerical simulations show that the outage improvement of AF max-link scheme over the traditional max-SNR scheme was more significant in symmetric channels. The average packet delays for both asymmetric and symmetric channels were also analysed. Both the diversity order and coding gain of AF max-link scheme were investigated.

In Chapter 4, a novel buffer-aided relay selection scheme which can significantly reduce packet delay was proposed. The reduced packet delay was achieved by giving higher priority to select the relay-to-destination than the source-to-relay links, so that the data queuing lengths at the relay buffers were minimised. Both the closed-form expression for outage probability and average packet delay were derived and verified with numerical results.

In Chapter 5, a novel multi-hop link selection scheme which seamlessly integrates max-link selection and physical layer network coding was proposed. In this approach, the average throughput was significantly improved over both low and high SNR ranges. A new analysis tool to obtain the average throughput of the proposed scheme was described. The closed-form expression of the average packet delay of the proposed scheme was derived and verified with simulations.

In Chapter 6, a buffer-aided max-ratio relay selection in the underlay cognitive relay network was proposed. The proposed scheme provides a more efficient way to handle the correlation among relay selection candidates than existing approaches. The closed-form expression of the outage probability
for the proposed relay selection scheme was derived. The analysis not only provides a deep insight in understanding the proposed scheme but also shows a potential approach to analyses similar systems in the future.

In Chapter 7, a novel max-ratio relay selection scheme was proposed for secure transmission in decode-and-forward relay networks with an eavesdropper which can intercept signals from both the source and relay nodes. Two cases were considered, which either the exact, or the average gain, for the eavesdropping channel was available. The closed-form expressions for the secrecy outage probability of the max-ratio scheme for both cases were derived.

In summary, in this thesis firstly both the symmetric and asymmetric channel configuration AF buffer-aided relay selection scheme was provided. Secondly, a significant reduced average packet delay buffer-aided relay selection scheme was investigated. Finally, the utilisation of buffer-aided relay selection was examined in multi-path networks, cognitive network and security network.

8.2 Future Work

There are several directions in which the research presented in this thesis could be extended. First, the solutions presented in this thesis were for flat fading Rayleigh channels. It is of interest to consider a wider class of fading channel conditions, modeled by for example the Nakagami-m distribution [99]. This fading distribution has gained much attention lately since the Nakagami-m distribution often gives the best fit to land-mobile and indoor mobile multi-path propagation environments as well as scintillating ionospheric radio links [100].

Secondly, in Chapter 3, 5, 6 and 7, the average packet delay were only analysis as the performance factor. It is interesting to analyse the delay
distribution, so that the outage can include both the channel realisation and delays. Furthermore, in Chapters 5, the links selection was decided by central node. It’s of interest to consider distributed link selection.

Finally, fifth generation (5G) wireless backhaul traffic is becoming a topical area of research and engineering [101] and [102]. In particular, the energy efficiency of wireless backhaul networks is becoming an important research area. The possibility of achieving economical and highly energy-efficient 5G wireless backhaul networks among the intended network nodes could be considered. How to use a buffer-aided relay selection scheme to improve the energy efficiency of wireless backhaul networks is a challenging problem that should be studied in the future.
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