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Numerical Study of Staggered Tube Bundle in Turbulent Cross Flow for an Optimum Arrangement

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A numerical study was conducted to evaluate optimum arrangement of staggered tube bundle in cross flow based on heat transfer rate, friction factor and compactness of bundle; utilizing CFD code FLUENT. The RNG (Re-Normalization Group) $k$-$\varepsilon$ turbulent model with modified constant of dissipation term in $\varepsilon$ equation was applied. The Reynolds numbers (based on maximum mean velocity inside tube bundle and hydraulic diameter of tubes) of 1000, 5000, 10000, 100000; dimensionless spacing ratios between tubes parallel and normal to flow direction ($S_p/D$ and $S_n/D$) in range of 0.6-3.0 and 1.0-2.9 respectively, and isothermal boundary condition on twelve rows of tubes inside symmetry computational domain were considered. According to the results, the model improves global heat transfer rate prediction, particularly in high Reynolds numbers, compared with standard $k$-$\varepsilon$ model in previous studies. A discussion was made in behaviour of global Nusselt number and friction factor for different tube bundle arrangements by utilizing flow stream lines and temperature contours as well as local Nusselt number on tubes. The results show that both heat transfer rate and friction factor are almost independent of $S_p/D$ for $S_p/D$ higher than specific value. Finally, by defining a performance parameter, the optimum arrangements for array of staggered tubes in cross flow was found $S_n/D=1.5$, 1.3 and $S_p/D=1.15$, 1.05 respectively.

1. INTRODUCTION

Staggered tube bundle in cross flow is used in many types of heat exchangers because of high heat transfer rate, satisfactory pressure drop, and easy manufacturing. Figure 1 shows complete and partial schematics of the compact heat exchanger designed at APU.
The external flow comprises a series of flowing inside an array of staggered tubes and impingement to the next stage. This has become possible by arranging every other one of the staggered tube arrays in opposite geometrical arrangement so that each time impinging surface is located between tubes. The internal flow comprises a series of staggered tube bundles in cross flow which are confined between two flat and parallel plates. The high heat transfer rate as a result of impinging jet and staggered tube bundle in cross flow and low pressure drop in the internal flow makes this design compact and efficient, particularly for applications such as the automotive industry in which high pressure drop in the external flow is not an important issue.

In this study, two-dimensional turbulence and incompressible flow across the staggered tube bundle is studied numerically. Then a parameter is defined as a function of global heat transfer rate, friction factor, and compactness of the tube bundle to evaluate optimum arrangement of tubes in application of compact heat exchangers.

Figure 1. Compact heat exchanger designed at APU.
The global heat transfer rate and friction factor of tube bundle in cross flow were studied by Pierson (1937), Grimson (1937), Jakob (1938), Zukauskas (1972) experimentally. An extensive study of heat transfer characteristics of tube bundle in cross flow can be found in Zukauskas and Ulinskas (1988). Hausen (1983) studied and reviewed both heat transfer rate and friction factor of tubes in cross flow, parallel flow and counter flow based on experiments. More recently, the local Nusselt number in staggered tube bundle was obtained experimentally by Meyer (1995) and global Nusselt number was compared by correlation given by Zukauskas.

The separation of flow due to adverse pressure gradient over the rear side of the tubes creates vortices which shed from tubes. This phenomenon is periodic and can not be observed with steady and time-averaged numerical turbulence models. An experimental study of vortex shedding in staggered tube array was carried out by Konstantinidis et al. (2002); using LDA measurements and visualization. An unsteady flow field and heat transfer rate in staggered tube bank was studied by Scholten and Murray (1998).

Two numerical flow field and heat transfer rate studies for tube bundle in laminar cross flow were conducted by Launder and Massey (1978) and Buyruk (2002). Antonopoulos (1985) applied finite difference method and standard $k$-$\varepsilon$ turbulence model to study axial and transverse flows as well as flow inclined to the axes of the inline and staggered tubes. The standard $k$-$\varepsilon$ model was also used in studies of Zdravistch et al. (1995), Watterson et al. (1999), and Safwat and Bassiouny (2000) to investigate the global and local heat transfer rate and friction factor for single tube, in-line, and staggered bundle in cross flow.

In this study RNG $k$-$\varepsilon$ turbulence model with standard wall function was applied to evaluate optimum arrangement of staggered tube bundle in cross flow. The constant of dissipation term in $\varepsilon$ equation was modified to make the model sensitive to high strain flows such as stagnation area in front of tubes. The wide ranges of Reynolds number and tube bundle arrangement, particularly the range of compact tube arrays were considered.

2. NUMERICAL MODELLING

In this section governing equations of flow, energy, and two turbulence transport equations were presented. The features of computational domain, grid consideration and solution parameters were explained.

2.1 Governing Equations

The continuity, Navier-Stokes, and energy equations for steady state and incompressible flow with variable fluid properties may be written as:
2.2 RNG k-ε Turbulent Modelling

The two transport equations in RNG k-ε model for turbulence kinetic energy (k) and its dissipation rate (ε) respectively are

\[ \frac{\partial u_i}{\partial x_i} = 0 \quad \text{Continuity Equation} \]

\[ \rho \frac{\partial}{\partial x_j} (u_j \mu_j) = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] \quad \text{Momentum Equations} \]  

\[ \rho \frac{\partial}{\partial x_j} (c_i \rho j T) = \frac{\partial}{\partial x_j} (\lambda_j \frac{\partial T}{\partial x_j}) \quad \text{Energy Equation} \]

\[ \frac{\partial k}{\partial t} + \nabla \cdot (\rho \mathbf{u} k) = \nabla \cdot (\Gamma k \nabla k) + G_k - \rho \varepsilon \]

\[ \frac{\partial \varepsilon}{\partial t} + \nabla \cdot (\rho \mathbf{u} \varepsilon) = \nabla \cdot (\Gamma \varepsilon \nabla \varepsilon) + C_{1\varepsilon} \frac{\varepsilon}{k} G_k - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} \]

where \( G_k \) (turbulence energy generation) = \( 2 \mu_s \sum_i \frac{\partial u_i}{\partial x_i} \)

In the above equations the right hand side involves diffusion, generation, and dissipation rate terms and left hand side is convection term. The turbulent viscosity is modeled as \( \mu_t = C_{\mu} \rho k^2 / \varepsilon \) where from RNG theory \( C_{\mu} = 0.0845 \). The dissipation constants, \( C_{1\varepsilon} = 1.42 \) and \( C^*_{2\varepsilon} \) is calculated from

\[ C^*_{2\varepsilon} = C_{2\varepsilon} + \frac{C_{\mu} \rho \eta^3 (1 - \eta / \eta_c)}{1 + \beta \eta} \]

where \( C_{2\varepsilon} = 1.68, \beta = 0.012, \eta_c = 4.38 \omega c \delta, \eta = S_k / \varepsilon \). In the low strain region (\( \eta < \eta_c \)), effect of equation (3) is to increase \( C^*_{2\varepsilon} \) and the result is comparable to standard k-ε model. In high strain region (\( \eta > \eta_c \)), \( C_{2\varepsilon} \) decreases as a consequence of equation (3). The less destruction of \( \varepsilon \), augments dissipation and thus reduces turbulence kinetic energy and effective viscosity. As a result prediction of heat transfer rate in stagnation region in front of tubes is expected to be improved by RNG k-ε model compared to standard k-ε model which over predicts turbulence kinetic energy and heat transfer rate in stagnation region (Durbin, 1996).
The RNG $k - \varepsilon$ model in comparison with standard $k - \varepsilon$ model takes about 10-15\% extra computational time due to additional term and a greater degree of non-linearity. Since in high strain rates turbulent viscosity is reduced in the RNG $k - \varepsilon$ model and because diffusion has stability effect, the RNG model is more instable compared to standard $k - \varepsilon$. This characteristic also makes RNG $k - \varepsilon$ model more responsive to physical instabilities such as time dependent turbulent vortex shedding.

To simulate near wall turbulence, the standard wall function of Launder and Spalding (1974) is applied which has been used widely in previous numerical studies of tube bank in cross flow. By using standard wall function in FLUENT, the mean velocity is applied with linear law for the near wall laminar sub-layer for $y^* < 11.225$ and with logarithmic law where $y^* > 11.225$. The $y^*$ is obtained by

$$y^* = \frac{\rho C_{\mu}^{1/4} k^{1/2} y}{\mu}$$

(4)

where $k$, and $y$ are turbulent kinetic energy and distance from wall of specific point respectively, $\mu$ is viscosity of the fluid, $\rho$ is density, and $C_{\mu}=0.09$ is constant.

The law of wall for temperature comprises linear law for thermal conduction sub-layer and logarithmic law for turbulent region. The linear and logarithmic equations are as:

$$T^* = \left( \frac{T_s - T_p}{ Pr_t} \right) \rho C_{\mu}^{1/4} k^{1/2} \frac{\dot{q}^*}{\rho} = \begin{cases} Pr \frac{y^*}{Pr} + P & \left( y^* < y_T^* \right) \\ Pr_t \left( \frac{1}{\kappa} \ln \left( E y^* \right) + P \right) & \left( y^* > y_T^* \right) \end{cases}$$

(5)

where $P$ is empirical function (Djilali et al. 1989) as:

In equations (5) and (6), $T_p$ and $k_p$ are the temperature and turbulent kinetic energy in point $P$, $T_s$ and $\dot{q}^*$ are the wall surface temperature and heat flux, $Pr$ and $Pr_t$ are molecular and turbulent Prandtl numbers ($Pr_t=0.85$), $E=9.793$ is constant of wall function and $y_T^*$ is dimensionless thermal sub-layer thickness which is computed as $y^*$ value of intersection of linear and logarithmic laws.

2.3 Computational Domain

The two dimensional computational domain consists of twelve rows of half tubes in staggered arrangement with inlet, outlet, and symmetry boundaries at the top and bottom (Figure 2). Twelve rows of tubes is considered in this research to improve accuracy of
results for global Nusselt number and pressure drop values which are important in the heat exchanger studies. The diameter of tubes is 1 cm and dimensionless parallel and normal spacing inside the array of tubes are $S_p/D$ and $S_n/D$, respectively, where $D$ is hydraulic diameter of tubes.

In the inlet boundary condition velocity normal to the flow was zero ($v=0$) and in flow direction there was a uniform velocity profile $u=U_{\text{avg}}$. The average inlet velocity ($U_{\text{avg}}$) was calculated based on $U_{\text{max}}$ (mean velocity in smallest surface between tubes, Grimson, 1937). The Reynolds number was defined as

$$Re_{\text{max}} = \frac{\rho U_{\text{max}} D}{\mu}$$

where $\rho$ and $\mu$ are density and viscosity calculated in the film temperature (average of inlet flow and tube surface temperatures). Inlet temperature was assumed constant 300 K. To model low turbulence inlet flow comparable with most experiments, the turbulent intensity and length scale in the inlet were set to 3% and 0.1 mm respectively.

In the outlet boundary, pressure was constant ambient pressure and other parameters were obtained by assuming zero diffusion flux.

All the tubes were imposed to no-slip boundary condition and constant temperature of 360 K. In the symmetry boundary condition the gradient of all flow and turbulence parameters and temperature were set to zero ($\partial \phi / \partial y = 0$).

The fluid used in this study was air which its density was assumed constant however conduction, specific heat capacity in constant pressure, and viscous coefficients were varied with temperature utilizing kinematic theory.

The global Nusselt number was based on log mean temperature difference (Hausen, 1983) was defined as

![FIGURE 2. Computational domain and boundary conditions.](image)
\[ \Delta T_{\text{lm}} = \left( \frac{T_s - T_{\text{in}}}{\ln \left( \frac{T_s - T_{\text{in}}}{T_s - T_{\text{out}}} \right)} \right) \] (8)

where \( \Delta T_{\text{lm}}, T_{\text{in}}, T_{\text{out}}, T_s \) are log mean temperature difference, inlet and outlet temperatures, and constant temperature on tubes respectively.

The bulk temperature in definition of local Nusselt number for each tube was mass weighted average of fluid temperature passing the surface normal to the flow at side of tube \( \theta = 90^\circ \).

The definition of friction factor was

\[ f = \frac{\Delta p}{n \cdot d \cdot \rho U_{\text{max}}^2} \] (9)

where \( f, \Delta p, d \) are friction factor, pressure drop, and tube diameter respectively, and \( \rho \) is fluid density at film temperature.

A ten layer boundary layer grid was set on all tubes with height of \( 5 \times 10^{-5} \, \text{m} \) in the first row and growing factor of 1.15. Unstructured quadrilateral grid was used to pave the entire domain. Two grid independency tests (parallel and normal to the tubes) were conducted to evaluate the sensitivity of Nusselt number to grid size. These tests were carried out for arrangement of \( S_p/D = 2.0, S_n/D = 1.5 \), and \( \text{Re}_{\text{max}} = 5000 \) and on the seventh row of the tube bundle. The local Nusselt numbers for different numbers of grids on the tube (parallel) are shown in Figure 3. The results confirm that in the range of this test the model is independent of grid size parallel to the tubes. Figure 4 presents local Nusselt numbers related to three different sizes of boundary layer grid on the wall shown in Table 1 and for two dissimilar Reynolds numbers. In the stagnation region in front of tube, course grid leads to underestimation of local Nusselt number and this grows in high Reynolds numbers. The grid size vertical to tubes may be assumed independent when dimensionless wall coordinate \( (y^+) \) is about 1.0 at the stagnation region and 2.0 apart from stagnation region.

<table>
<thead>
<tr>
<th>First Row Height</th>
<th>Growing Factor</th>
<th>Number of Rows</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.L. Grid 12x10-4</td>
<td>1.15</td>
<td>10</td>
</tr>
<tr>
<td>B.L. Grid 25x10-5</td>
<td>1.15</td>
<td>10</td>
</tr>
<tr>
<td>B.L. Grid 31x10-5</td>
<td>1.15</td>
<td>10</td>
</tr>
</tbody>
</table>
FIGURE 3. Grid independency test parallel to the wall.

FIGURE 4. Grid independency test normal to the wall.
2.4 Solution Parameters

A segregated and explicit method was considered to solve flow, energy, and two turbulence equations in two dimensions. For the pressure-velocity coupling, the SIMPLE algorithm of Patankar (1980) was used. Since flow stream lines are not align with grid, second order discretization for pressure and third order interpolation scheme (QUICK, see Leonard 1979) for momentum, turbulence kinetic energy and dissipation rate, and energy equations were applied.

A common value of underrelaxation factor was set 0.3 for pressure and 0.8 for other parameters. The solution was considered converged when the scaled residual reduced to $10^{-6}$ for energy and $10^{-4}$ for other equations.

3. RESULTS AND DISCUSSIONS

The numerical results of global and local Nusselt numbers and friction factor were compared with previous experimental studies and correlations as well as standard $k$-$\varepsilon$ model in the study of Safwat and Bassiouny (2000).

Comparisons of global Nusselt number between experimental correlations of Hausen (1983) and Zukauskas et al. (1988), standard $k$-$\varepsilon$ turbulent model, and RNG $k$-$\varepsilon$ model in the present study for arrangements of $S_\theta/D = 2.0$, $S_\theta/D = 1.5$ and $S_\theta/D = 1.5$, $S_\phi/D = 1.2$ are shown in Figure 5. Using standard $k$-$\varepsilon$ model leads to overestimation of Nusselt number in high Reynolds numbers. The RNG $k$-$\varepsilon$ model shows a good agreement for former arrangement however in the latter arrangement; there is slightly underestimation of Nusselt number in high Reynolds numbers. Since in tighter arrangements local Reynolds

![Figure 5. Comparison of global Nusselt number in different studies.](image)
number inside the bundle is higher, the underestimation may be solved by applying finer boundary layer grid (see grid independency test). The global friction factor in the present study was compared by experimental studies of Jakob (1938) and Zukauskas (1972), and standard $k$-$\varepsilon$ model for arrangements of $S_n/D = 1.5$, $S_p/D = 2.0$ and $S_n/D = 1.5$, $S_p/D = 1.2$ (Figure 6). Despite considerable difference between two experimental data, both standard and RNG $k$-$\varepsilon$ models show acceptable agreement in the range of Reynolds number in this study.

Figure 7 compares local Nusselt number on fourth row, in the arrangement of $S_n/D = 2.0$, $S_p/D = 2.0$ and $Re = 34100$, between different turbulent models and experimental study of Meyer (1995). The termis based on correlation of Zukauskas and Ulinskas (1988). According to the results, the RNG $k$-$\varepsilon$ model leads to more accurate results for the local Nusselt number on front part of tube where maximum heat transfer rate happens. The overestimation of local Nusselt number by standard $k$-$\varepsilon$ model is due to overestimation of turbulence kinetic energy in stagnation and high strain regions.

In Figure 8, the global Nusselt number versus dimensionless parallel spacing ratio is shown for different dimensionless normal spacing ratios and Reynolds numbers. According to the results for all Reynolds numbers there is specific value of the $S_n/D$ in which the global Nusselt number is independent of $S_p/D$. By increasing of $S_p/D$, below the specific $S_n/D$ the global Nusselt number first decreases and then almost remains constant, however above the specific $S_n/D$ the global Nusselt number first increases and then remains almost constant. The specific value for $Re = 1000$, $5000$, $10000$, $100000$ is almost $S_n/D = 2.1$, $1.9$, $1.7$, $1.5$ respectively; in other words the range of sensibility of heat transfer rate to $S_n/D$ reduces in high Reynolds numbers. Figure 8 also shows almost independency of heat transfer rate from $S_p/D$ where $S_p/D > 1.2$. 
Study of staggered tube bundle in turbulent cross flow for an optimum arrangement

**Figure 7.** Comparison of local Nusselt number between different turbulence models and experiment.

**Figure 8.** Global Nusselt number versus $S_p/D$ for different $S_\alpha/D$. 
The flow stream lines for arrangements of $S_n/D = 1.7$, $S_p/D = 2.9$ and $S_n/D = 2.1$, $S_p = 0.8$ and $Re = 1000$ are presented in Figure 9. The separation of flow because of staggered arrangement of the bundle happens in higher angles where $S_n/D = 1.7$; therefore the heat transfer rate improves in compare with arrangements with higher $S_n/D$. Increase of $S_p/D$ when $S_n/D = 1.7$, reduces flow momentum on the rear portion of tubes and the global Nusselt number decreases.

When $S_n/D = 2.1$, increase of $S_p/D$ reduces local heat transfer in rear portion of tubes because of high $S_n/D$ however local Nusselt number in front of tubes increases as a result of increasing of impinging surface. The result is almost constant global Nusselt number in all range of $S_p/D$.

In higher $S_n/D$ (2.9), there is just vortex flow between tubes parallel to flow and the flow momentum influences small portion of tubes in side; thus a lower global heat transfer rate is predictable. Increase of $S_n/D$ in this case causes increasing of flow momentum in front portion of tubes and the global heat transfer rate increases.

The flow streamlines for constant $S_n/D = 1.9$ and different $S_p/D = 1.5, 3.0$ in Figure 9 shows no difference in mechanism of heat transfer rate in front and back portions of tubes so that the global Nusselt number for $S_p/D > 1.3$ is almost independent of $S_p/D$.

Figure 10 presents the global Nusselt number versus $S_n/D$ for different $S_p/D$ and Reynolds numbers. According to the results, for $S_p/D > 1.2$ and $Re = 1000$, the global
Nusselt number is independent of $S_p/D$ however by increase of Reynolds number global heat transfer rate is increased by increasing of $S_p/D$. In other words, in this range of $S_p/D$ by increasing of Reynolds number the heat transfer rate from front portion of tubes (impinging surface) becomes more dominant. Figure 10 shows that in lower $S_p/D$ and Reynolds number, the global Nusselt number by increasing of $S_p/D$ decreases however in high Reynolds numbers increase of $S_p/D$ leads to increase of global Nusselt number.

The friction factor versus $S_p/D$ in dissimilar $S_n/D$ and Reynolds numbers is shown in Figure 11. The trends of the friction factor when $S_p/D<1.2$ is same as global Nusselt number in Figure 8 and can be explained by Figure 9. In high $S_n/D$ and low $S_p/D$, the fluid flows with low resistance from side of tubes so that friction factor is low, however by increasing of $S_p/D$ flowing of the fluid between tubes parallel to flow results increasing of impingement surface and friction factor increases. In low $S_n/D$ and $S_p/D$, there are both impingement to front portions of tubes as well as flow momentum on back portion of tube.
because of delay in flow separation, therefore both friction factor and heat transfer rate are high. Increase of $S_p/D$ in this case results reduction of both heat transfer rate and friction factor.

In the correlation of friction factor given by Jakob (1938), friction factor is independent of $S_p/D$ however in study of Zukauskas (1972), the friction factor depends to $S_p/D$ in high Reynolds numbers when $S_p/D$ is higher compare to $S_n/D$ and in low Reynolds numbers when $S_p/D$ is lower compare to $S_n/D$. Figure 12 almost confirms results of Zukauskas (1972). For $S_p/D < 1.4$, when almost $S_n/D = 2.5$, in all range of Reynolds numbers, the friction factors depends on the $S_p/D$ however when $S_n/D = 1.5$ dependency of the friction factor on $S_p/D$ occurs mostly in high Reynolds numbers. In the range of $S_p/D < 1.4$, flowing of the fluid between tubes parallel to the flow, and therefore amount of impingement portion of tubes affects friction factor. For $S_p/D < 1.4$, $S_n/D = 1.5$ is an arrangement that change of $S_p/D$ has its maximum influence on flow momentum on rear portion of tubes so that there is a small change of friction factor with change of $S_p/D$.

![Figure 11. Total friction factor versus $S_p/D$ in different $S_n/D$.](image)
The local Nusselt numbers for five cases of Figure 9 on the seventh row are shown in Figure 13. The behavior of local Nusselt number confirms explanation of streamlines in Figure 9 in which the global Nusselt number in low $Sn/D$ arrangements is influenced by two mechanisms, flow impingement in front portion and flow momentum on back of tubes. In constant $Sp/D$, by increasing of $Sn/D$ and development of vortex flow in front of tubes in Figure 9, impingement portion moves to higher angles and becomes weaker. According to figure 13 maximum Nusselt number moves forward and also is reduced. By increasing of $Sn/D$ because of reduction in flow momentum on rear portion of tubes, the local Nusselt number reduces in this portion as seen in Figure 13. The local Nusselt number for constant $Sp/D$ and different $Sn/D$ in figure 13 confirms that for $Sp/D$ larger than specific value, the mechanism of heat transfer rate is almost same and the global Nusselt number remains constant.
The temperature contours around 6th and 7th rows for different arrangements are shown in Figure 14. For arrangements of constant $S_p/D=0.8$ and dissimilar $S_n/D$, temperature is higher in low $S_n/D$ where the global Nusselt number is higher. The thickness of high temperature layers are more in front and back portions of tubes and minimum temperature is occurred where velocity is maximum inside the arrangement. By comparing constant $S_p/D$ arrangements in Figure 14, because the global Nusselt number is almost same, the identical temperature values are observed however in high $S_p/D$ arrangement the difference between near wall high temperature and free flow low temperature is lower. In high $S_p/D$, since the length of arrangement is higher, mixing of high and low temperature fluids is increased.

In the final step, a performance parameter ($1/\eta_{compact}$) as function of global Nusselt number, friction factor, and compactness of tube bundle is defined as:

$$\frac{1}{\eta_{\text{Compact}}} = n \frac{f}{S_n} \frac{S_n}{N_{\text{Nu}} D}$$  \hspace{1cm} (10)

By assuming that pressure loss is independent of $S_p/D$, $1/\eta_{\text{compact}}$ is friction factor per bundle length times dimensionless surface area of bundle divided by the global Nusselt number.

Figure 15 shows $1/\eta_{\text{compact}}$ for different $S_p/D$ and $S_n/D$ and two dissimilar Reynolds numbers. According to the results the worst performance of staggered arrangement (higher $1/\eta_{\text{compact}}$) is obtained when one of the spacing ratios is high and other one is low. The high performance of staggered tube bundle in cross flow occurs in the range of $S_n/D = 1.3-1.5$ and $S_p/D = 1.05-1.15$ respectively. The optimum arrangement is constant for the range of Reynolds number in this study however results show that performance of higher $S_n/D$ arrangements is slightly better in low Reynolds numbers.

4. CONCLUSION

The RNG $k-\varepsilon$ turbulence model with standard wall function was used to predict the global and local Nusselt numbers and friction factor for staggered tube bundle in cross flow in range of turbulent and incompressible flow. By modifying constant of dissipation term in $\varepsilon$ equation, the RNG $k-\varepsilon$ model overcomes overestimation of the global Nusselt number with standard $k-\varepsilon$ model yet leads to acceptable results for friction factor. The influence of different arrangements and Reynolds numbers on the global heat transfer rate, total friction factor, local Nusselt number, and temperature contours was discussed. A parameter was defined as function of the global Nusselt number, friction factor and compactness of tube bundle to evaluate arrangements of optimum performance for different Reynolds numbers. The optimum geometry was obtained in range of $S_n/D=1.3-1.5$ and $S_p/D=1.05,1.15$ respectively and almost independent of Reynolds number.
Study of staggered tube bundle in turbulent cross flow for an optimum arrangement

FIGURE 13. Local Nusselt number on seventh row for different arrangements.

FIGURE 14. Temperature contours for different arrangements around 6th and 7th rows.
NOMENCLATURE

- $c_p$: specific heat at constant pressure
- $d$: diameter of tubes (m)
- $D$: hydraulic diameter of tubes (m)
- $f$: friction factor (1)
- $G_k$: turbulent energy generation rate
- $h$: average convection heat transfer coefficient (W/m²K)
- $k$: turbulent kinetic energy (m²/s²)
- $n$: number of rows in tube bundle (1)
- $Nu$: local Nusselt number (1)
- $\overline{Nu}$: average Nusselt number (1)

Figure 15. Efficiency parameter versus $S_u/D$ and $S_p/D$. 
Study of staggered tube bundle in turbulent cross flow for an optimum arrangement

\begin{itemize}
\item \(p\) pressure drop (kg/ms²)
\item \(Pr\) Prandtl number
\item \(Pr_t\) turbulent Prandtl number
\item \(q_s^*\) heat flux rate from wall surface
\item \(Re\) maximum Reynolds number inside tube bundle (1)
\item \(S\) strain rate
\item \(S_{ij}\) mean strain tensor
\item \(S_n\) distance between tubes normal to flow (m)
\item \(S_p\) distance between tubes parallel to flow (m)
\item \(T\) temperature (K)
\item \(u\) velocity parallel to flow (m/s)
\item \(u_i\) Cartesian velocity component (m/s)
\item \(U_{avg}\) uniform inlet velocity (m/s)
\item \(U_{max}\) maximum average velocity in tube bundle (m/s)
\item \(v\) velocity normal to flow (m/s)
\item \(x_i\) spatial coordinate (m)
\item \(y\) distance from wall (m)
\item \(y^*, y^+\) dimensionless wall coordinates (1)
\item \(y^*_T\) dimensionless thermal sublayer thickness
\end{itemize}

\textit{Greek}

\begin{itemize}
\item \(\delta_{ij}\) Kronecker delta
\item \(\varepsilon\) turbulent dissipation rate (m²/s³)
\item \(\eta = Sk/\varepsilon\)
\item \(\eta_{\text{compact}}\) efficiency defined for compact H.E. (1)
\item \(\theta\) (Theta) angle on tubes (degree)
\item \(\lambda\) conduction coefficient of fluid (W/mK)
\item \(\mu\) viscosity (kg/ms)
\item \(\mu_t\) turbulent viscosity (kg/ms)
\item \(\rho\) fluid density (kg/m³)
\end{itemize}

\textit{Subscripts}

\begin{itemize}
\item \(\text{in}\) inlet
\item \(n\) normal to the flow
\item \(\text{out}\) outlet
\item \(p\) parallel to the flow
\item \(s\) surface of tubes
\end{itemize}
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