**Cool for cats**

This item was submitted to Loughborough University's Institutional Repository by the/an author.

**Citation:** EVERITT, M.J. ...et al., Cool for cats. arXiv:1212.4795v2 [quant-ph]

**Additional Information:**

- This pre-print was submitted to arXiv on 27 Feb 2013. It was subsequently published as “Engineering dissipative channels for realizing Schrödinger cats in SQUIDs” https://dspace.lboro.ac.uk/2134/21296

**Metadata Record:** [https://dspace.lboro.ac.uk/2134/21292](https://dspace.lboro.ac.uk/2134/21292)

**Version:** Submitted for publication

**Publisher:** arXiv.org

**Rights:** This work is made available according to the conditions of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International (CC BY-NC-ND 4.0) licence. Full details of this licence are available at: https://creativecommons.org/licenses/by-nc-nd/4.0/

Please cite the published version.
Cool for Cats

M.J. Everitt,1,∗ T.P. Spiller,2 G.J. Milburn,3 R.D. Wilson,1 and A.M. Zagoskin1
1Department of Physics, Loughborough University,
Loughborough, Leics LE11 3TU, United Kingdom
2Quantum Information Science, School of Physics and Astronomy,
University of Leeds, Leeds LS2 9JT, United Kingdom
3Centre for Engineered Quantum Systems, School of Mathematics and Physics,
The University of Queensland, St Lucia, QLD 4072, Australia

The iconic Schrödinger’s cat state describes a system that may be in a superposition of two macroscopically distinct states, for example two clearly separated oscillator coherent states. Quite apart from their role in understanding the quantum classical boundary, such states have been suggested as offering a quantum advantage for quantum metrology, communication and quantum computation. As is well known these applications have to face the difficulty that the irreversible interaction with an environment causes the superposition to rapidly evolve to a mixture of the component states in the case that the environment is not monitored. Here we show that by engineering the interaction with the environment there exists a large class of systems that can evolve irreversibly to a cat state. To be precise we show that it is possible to engineer an irreversible process so that the steady state is close to a pure Schrödinger’s cat state by using double well systems and an environment comprising two-photon (or phonon) absorbers. We also show that it should be possible to prolong the lifetime of a Schrödinger’s cat state exposed to the destructive effects of a conventional single-photon decohering environment. Our protocol should make it easier to prepare and maintain Schrödinger cat states which would be useful in applications of quantum metrology and information processing as well as being of interest to those probing the quantum to classical transition.

The development of many quantum technologies depends on an ability to engineer strongly non classical states. Such states take the form of either highly entangled states of distinct degrees of freedom or a quantum coherent superposition of macroscopically distinct states in a single degree of freedom1, known as Schrödinger’s cat states (after a well known thought experiment2). It is these cat states that we consider in this letter. There has been great progress in the production of such states as well as experimentally reconstructing such states through a series of measurements in a process known of as quantum state tomography3–5. These developments are of great importance as, in addition to their curious nature, Schrödinger cat states can be used as a resource for developing technologies such as quantum computing6,7, quantum communication8,9,10 and quantum metrology11–13. The main obstacle to deploying cat states in such applications is their fragility as they are destroyed by noise in a process termed environmental decoherence. A careful consideration of optical cat states shows that this decoherence may be interpreted as due to Poisson distributed jumps between even and odd cat states whenever a single photon is lost14–16. Their production and maintenance requires very precise quantum control as well as low dissipation. In this work we propose a protocol for double well systems to create Schrödinger cat states that actually uses the non-controllable, non-unity interaction of the system with a special kind of environment to create Schrödinger’s cat states. To be specific, we have found that for a simple double-well system system interacting with an environment comprising a bath of two-photon absorbers, for certain initial states, the system relaxed to a steady state which is close to a pure Schrödinger cat state. Such an environment when paired with a parametric photon pump is known to exhibit many interesting effects in quantum optical systems, from cats to quantum statistics19,20. Two-photon absorption has also been suggested as a powerful resource for quantum computing application21. Two-photon decay preserves parity and enables the system to relax to a steady state with same parity as the initial state. Our model is simpler than and different from other driven dissipative bistable systems (for example, the coherently driven optical cavity containing a Kerr medium22, the driven Duffing mechanical resonator23, tapered optical fibers24 and photon pumps19,20), as we do not include driving on either the cavity resonance or the coordinate degree of freedom. Our proposal opens up new opportunities for exploring quantum phenomena from the micro to macroscopic level and in fields as diverse as quantum optics25, Bose-Einstein condensates26, quantum electronics27 and nanomechanics28 (for which multi-phonon relaxation has already been proposed29) or any other system in which it is possible to generate a double well potential.

For the results presented in this paper we have used as an example system a superconducting quantum interference device (SQUID) ring. Our reason for choosing SQUIDs is that these devices are routinely fabricated and their theory is very well understood. We note that we have investigated a number of other systems (but do not include results here) and our analysis indicates that the key feature of the ring is that it can be made to form a double well potential. Moreover, nonlinear systems derived from the Josephson junction in circuit QED exhibit multi photon resonance when driven by an external field30 and thus we expect two-photon decay to be present.
in such systems. The real difficulty is making it dominate over single photon effects. We will return to this later. Beyond these considerations we believe there is nothing particularly special about the exact form of the potential needed to realise our protocol. Subject to being able to engineer an appropriate dissipative channel we therefore believe that the methodology that we propose for generating cat states will, as previously mentioned, find wide application. The potential energy of the SQUID comprising a thick superconducting ring enclosing a Josephson junction weak link takes the form of a harmonic oscillator perturbed by a cosine

\[ U(\Phi_x) = \frac{(\Phi - \Phi_0)^2}{2 \Lambda} - \frac{h \omega_c}{2e} \cos \left( 2\pi \frac{\Phi_x}{\Phi_0} \right) \]

where the coordinate \( \Phi \) is the total magnetic flux in the ring and \( \Phi_0 = h/2e \) is the superconducting flux quantum. We have chosen example circuit parameters that are in line with modern fabrication techniques and suited to experimental realisations: \( \Lambda = 3 \times 10^{-10} \text{H} \) for the ring’s inductance and \( I_c = 2 \mu \text{A} \) as the critical current of the weak link (although not in the above formula we also chose a capacitance \( C = 5 \times 10^{-15} \text{F} \)).

We set the externally applied magnetic flux \( \Phi_x = 0.5\Phi_0 \) so that the ring’s potential forms a degenerate double well. It is also convenient to introduce the bosonic annihilation \( a \) and creation \( a^\dagger \) operators \( \Phi = \sqrt{2\omega_{LC}} (a + a^\dagger) \) where \( \omega_{LC} = 1/\sqrt{\Lambda C} \). In Fig. 1 we show the potential energy of the ring as well as the energy of the ring’s stationary states. It is worth noting that the ground state and first excited state approximately, respectively, symmetric and anti-symmetric superpositions of two coherent states centred at the bottom of each well. These two states have very nearly the same energy and the difference in their energy has been exaggerated in this plot (as have those for the second and third excited states).

We model the effect of the environment on the system using the master equation in the Lindblad form\(^{31}\)

\[ \frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] + \frac{1}{2} \sum_j \left\{ \left[ L_j, \rho L_j^\dagger \right] + \left[ L_j^\dagger, L_j \rho \right] \right\} \]

where \( \rho \) is the density matrix describing the state of the system (initially \( \rho = |\psi(t = 0)\rangle \langle \psi(t = 0)| \)) and \( H \) is the system’s Hamiltonian. The non-unitary effect of the environment on the system is contained in the Lindblad operators \( L_j \) with each describing a possible environment. For example the usual Ohmic (i.e. analogous to friction proportional to velocity) bath, at zero temperature, would be described by a Lindblad proportional to the annihilation operator. For an undriven system the master equation has steady state solution that, in the presence of an environment, is usually a density operator in a mixed state. In certain circumstances, at zero temperature, these solutions may be pure states such as the vacuum state of the harmonic oscillator. In these circumstances the solutions will not exhibit features such as superpositions of macroscopically distinct states and are relatively uninteresting. It is precisely this process where the environment essentially removes the system’s quantum coherence from de-localised, or more generally non-Gaussian, states of which is known as environmental decoherence.

The density matrix for a decohered system without these quantum correlations represents a statistical mixture of possible states of the system and, for a single quantum object, can be directly compared with classical probability density distributions\(^{32}\). It should be noted however that there are driven dissipative systems, for example dispersive bistability, for which the steady state is a mixed state with a considerable amount of quantum coherence in the limit of large Kerr nonlinearity\(^{31,32,33}\).

We found very different behaviour if one chooses a different environment comprising two-photon absorbers, described by a Lindblad proportional to the square of the annihilation operator. In Fig. 2 we show the energy expectation values and von-Neumann entropy, \( S = -\text{Tr}[\rho \ln \rho] \) as functions of time for solutions of the master equation for the ring in the presence of such an environment. We used as initial conditions the first twenty energy eigenstates of the ring Hamiltonian. In these plots the energy behaves just as one would expect the energy of an undriven open quantum system to do – it settles to a single value. When one inspects the dynamics of the entropy however the story is quite different. One usually expects the entropy to grow from zero to some asymptotic value as the system evolves into a mixed state. While we see that this is the initial behaviour the entropy does not monotonically increase, instead it decreases until the entropy is nearly negligible. It appears that the system
has to a significant extent recohered and the final density matrix is very nearly that of a pure state. While this is not the usual behaviour of an open quantum system it is in-line with our expectations of an environment that “decoheres” a system to an almost pure state that is a very good approximation to a Schrödinger cat state\textsuperscript{18}.

In order to demonstrate that the system does indeed decay to a Schrödinger cat state we will make use of the Wigner function. These pseudo probability density functions in phase space have been of great utility in demonstrating that some quantum states are Schrödinger cats\textsuperscript{3}. The Wigner function is

\[ W(\Phi, Q) = \frac{1}{2\pi\hbar} \int \langle \Phi + \zeta | \rho | \Phi - \zeta \rangle \exp \left( \frac{-2iQ\zeta}{\hbar} \right) \, d\zeta \]

where \( Q \) is the charge variable that is conjugate to the magnetic flux \( \Phi \). In Fig. 3 we show three Wigner functions. Fig. 3a shows the initial state and is a coherent state centred at the origin. This is clearly recognizable as the expected Gaussian bell shape associated with coherent states. We have solved the master equation for the ring in a lossy bath, with a Lindblad of \( L = \sqrt{0.2a} \) and allowed the system to reach its steady state to obtain Fig. 3b. This is the Wigner function of a statistical mixture of two macroscopically distinct states and is in-line with expectations of the effect of a decohering environment on such a device\textsuperscript{34}. In Fig. 3c we show the Wigner function that we obtain by solving the master equation, as for (b), but replacing the damping term with a bath of two-photon absorbers, with \( L = \sqrt{0.2a^2} \). We notice two things: firstly that the state has rotated which we believe to be a consequence of a squeezing action associated with the bath and secondly that there are interference terms between the distinct states of the system. These interference terms, indicating quantum coherence, confirm that this state state is indeed a very good approximation to a Schrödinger cat. In order to examine quantitatively the emergence of this cat from the initial coherent state we introduce, following\textsuperscript{35,36}, a measure of how de-localised the system is in phase space that is the integral of negative parts of the Wigner function

\[ N(\rho) = \frac{1}{2} \int \left\{ |W(\Phi, Q)| - W(\Phi, Q) \right\} \, d\Phi dQ. \]

In absolute terms this is a useful measure, but when we know (by inspecting the Wigner function) that the states we are examining are cat-like a more useful measure may well be a relative cattiness to some reference Schrödinger cat state. Hence we define:

\[ \text{Cat}(\rho, \rho_{\text{ref}}) = \frac{N(\rho)}{N(\rho_{\text{ref}})} \]

which quantifies the ratio of the de-localisation of one cat state against a reference cat and enables us to quantify if one is more \([\text{Cat}(\rho, \rho_{\text{ref}}) > 1]\), less \([\text{Cat}(\rho, \rho_{\text{ref}}) < 1]\) or just as \([\text{Cat}(\rho, \rho_{\text{ref}}) = 1]\) catty than the other. In Fig. 4 we show the dynamics of this quantity for comparison with the results presented in Fig. 3 using as a reference state \( \rho_{\text{ref}} \) the final cat state shown in Fig. 3c. Here we can clearly see that the cattiness of the system subject to an environment of two-photon absorbers monotonically increases and asymptotically converges to a steady state.

It is interesting to consider what would happen to a ring that was initially in a Schrödinger cat state under...
the influence of a bath of two-photon absorbers. For systems with deep enough double well potentials such as the one considered here the ground and first excited energy eigenstates are both Schrödinger cats. The ground state is, to good approximation, an even superposition of two macroscopically distinct coherent states while the first excited state is an odd superposition as can be seen from their Wigner functions in Fig. 5a and c respectively. The even and odd nature of these superpositions is reflected in the Wigner function by the phase of the interference terms between the two Gaussian’s of the cat. It is known that such states would decohere under the environment of a lossy bath to a statistical mixture\(^{34}\). The dynamics of the system coupled to an environment comprising a bath of two-photon absorbers are, once more, found by solving the master equation with an \( L = \sqrt{0.2a} \), until an approximate steady state is reached. The Wigner function of these states is then shown with Fig. 5a evolving to b and Fig. 5c to d. We observe that the phase in the final cat reflects that of the initial cat and the system has not simply decohered to the same steady state. The environment thus seems to preserve some of the symmetry of the initial state. We have checked the first twenty stationary states all of which decay to one of these cats or the other. Moreover, the pattern that was observed from the ground and first excited state persists and all even and odd states seem to evolve to cats of the same form as those shown Fig. 5b and Fig. 5d that are out of phase with each other.

Our protocol seems all very well and good but an environment of two-photon absorbers is very special. It would be hard to construct such an environment without having any other source of decoherence present. We therefore need to verify that the effects of a two-photon absorbing environment cannot be completely destroyed.
There are two phenomena that embody quantum mechanics, namely entanglement and the Schrödinger’s cat.

In order to make our above discussion a reality we need to engineer a dissipative quantum channel that acts as a two-photon absorber. Here we suggest a concrete realisation that, whilst not perfect, still retains the key feature of environmentally induced “decoherence” to a Schrödinger cat state. Our proposal makes use of non-linearly coupled electromagnetic fields and SQUIDs. Such quantum electrodynamic circuits have already been investigated in the context of weak nondemolition measurement\(^{37,38}\). One example comprises two microwave superconducting resonators coupled via a SQUID which in addition to a cross Kerr effect also manifests two photon conversion terms if the cavities are resonant\(^{37}\). Such systems can be quantised\(^{39-42}\) and with a suitable arrangement and choice of circuit parameters can be reduced\(^{43,44}\) to the form of a double well system subject to a two-photon absorbing environment (see supplementary material for details). Unavoidably, this process also brings with it an additional dephasing term, that adds to the master equation another Lindblad proportional to \(a^\dagger a\). Nevertheless, we can report that whilst the dephasing term smears out the Gaussian peaks in the cat the interference terms in the Wigner function representing quantum coherence between the cat states remains strong. The fact that this dephasing term preserves parity is once more the key factor in ensuring the steady state of our engineered dissipative channel is still a Schrödinger cat state. Our proposal could lead to an initial realisation of a two-photon absorbing environment and concomitant interesting effects. The engineering of improved dissipative channels, without additional and unwanted decoherence effects, remains an open and interesting problem.

There are two phenomena that embody quantum mechanics, namely entanglement and the Schrödinger’s cat
thought experiment\(^2\). The latter was proposed to highlight the difficulties we have connecting quantum mechanics with everyday experience it neatly demonstrates the problems of understanding the emergence of the classical world from quantum theory and the measurement of quantum systems. Schrödinger’s cat has become the icon of the subject and evolved to have a well defined meaning. It is an accepted explanation within the popular literature that the reason the original thought experiment does not translate into reality (if conducted with a real cat in a box etc.) is that the environment to the radiation source (which included the cat itself) deletes the quantumness connecting the two states in a process known as decoherence. As such environmental decoherence is something that many deem to be a crucial element in the the quantum to classical transition\(^{32,45–48}\). We have shown that some environments may have a dramatically different effect on double well systems producing very quantum states as a result of “decoherence”. It may well be that system and environment such as the one we have used here could play an interesting role in quantum mechanically enhanced metrology probing foundational aspects of quantum mechanics associated with realising macroscopic quantum phenomena and the quantum to classical transition. Furthermore, it is known that open quantum systems can be used to model the measurement process. Although it is beyond the scope of the current paper, we conjecture that it may well be possible to make use of an environment to measure a system into a Schrödinger cat state.

- MJE, RDW, and AMZ thank the Templeton Foundation for their generous support. GJM acknowledges the support of the Australian Research Council Centre of Excellence for Engineered Quantum Systems grant CE110001013. We would like to thank Peter Knight for interesting and informative discussions. MJE would like to thank Andrew...
Archer, Gerry Swallowe and Richard Giles for their help with the preparation of our manuscript.

- The authors declare that they have no competing financial interests.

- Correspondence and requests for materials should be addressed to M.J. Everitt (email: m.j.everitt@physics.org).

- Authors’ contributions: TS and MJE formulated the original problem and choice of the system (choice of system was independently corroborated by AZ); GJM proposed the form of the environment and its possible realisation; MJE directed and RDW performed numerical calculations and analysis of results; All authors contributed to the technical discussion; MJE and RDW prepared the manuscript with help from all other authors.

* Electronic address: m.j.everitt@physics.org


22 Walls, D. & Milburn, G. Quantum Optics. Springer e-Books (Springer, 2008). URL http://books.google.co.uk/books?id=L1Hwsc3N1f0KC.


Cool for cats Supplementary Information:

Engineering two photon decay in circuit QED.

M. J. Everitt\textsuperscript{1}, T.P. Spiller\textsuperscript{2}, G. J. Milburn\textsuperscript{3}, R.D. Wilson\textsuperscript{1} and A.M. Zagoskin\textsuperscript{1}

\textsuperscript{1}Department of Physics, Loughborough University, Loughborough, Leicestershire LE11 3TU, United Kingdom,
\textsuperscript{2}Quantum Information Science, School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, United Kingdom
\textsuperscript{3}Centre for engineered Quantum Systems, School of Mathematics and Physics, The University of Queensland, St Lucia, QLD 4072, Australia.

We show that a microwave superconducting cavity can be engineered to have a dominant two photon decay term using two cavities coupled by a SQUID.

PACS numbers:

There are a number of models\cite{1, 2} whereby two microwave superconducting cavities can be nonlinerally coupled using SQUIDs. We will base our discussion on Kumar and Divincenzo\cite{1}. In that model, the hamiltonian describing two microwave cavities, a probe (p) cavity and a signal (s) cavity, coupled with a SQUID is

\begin{equation}
H = E_{cp}n_p^2 + E_{lp}\phi_p^n + E_{cs}s^2 + E_{ls}\phi_s^n + A\left[ E_{lp}\phi_p^n \cos^2 \beta + E_{ls}\phi_s^n \sin^2 \beta \right] + 6E_{lp}E_{ls}\phi_p^n \phi_s^n \sin^2 \beta \sin^2 \beta \cos^2 \beta \sin^2 \beta
\end{equation}

where $n_a, \phi_a$ are the standard charge and phase conjugate variables describing the collective electrical degree of freedom in each cavity and $A = 16\pi^2L_1/\Phi_0^2$ with $L_1$ defined as the coefficient of the leading non-linear current term of the SQUID inductance. We will set $\cos^2 \beta = \sin^2 \beta = 1/2$.

The system can be quantised in the usual way in terms of the bosonic annihilation and creation operators $b,b^\dagger$ for the probe and $a,a^\dagger$ and for the signal cavity defined by\cite{3}

\begin{align}
\phi_p &\rightarrow \left( \frac{E_{cp}}{4E_{lp}} \right)^{1/4} (b + b^\dagger) \\
n_p &\rightarrow -i \left( \frac{E_{lp}}{4E_{cp}} \right)^{1/4} (b - b^\dagger) \\
\phi_s &\rightarrow \left( \frac{E_{cs}}{4E_{ls}} \right)^{1/4} (a + a^\dagger) \\
n_s &\rightarrow -i \left( \frac{E_{ls}}{4E_{cs}} \right)^{1/4} (a - a^\dagger)
\end{align}

The Hamiltonian may then be written as

\begin{equation}
H = \hbar\omega_p b^\dagger b + \hbar\omega_a a^\dagger a + \hbar\chi_b b^\dagger b^2 + \hbar\chi_a a^\dagger a^2 + \hbar\sqrt{\chi_a\chi_b}\beta_0 (b^\dagger a + \bar{a}^\dagger b)
\end{equation}

Unlike Kumar and Divincenzo\cite{1} we have not neglected the terms like $b^2a^\dagger b^2$ as we will choose $\omega_p = \omega_s$ so that these terms are resonant.

We now include the dissipative channels for this model in the usual way. The density operator for the total system, in the interaction picture, satisfies

\begin{equation}
\frac{d\rho}{dt} = -i[H_I,\rho] + \kappa_a D[a]\rho + \kappa_b D[b]\rho
\end{equation}

where $D[L]\rho = L\rho L^\dagger - \frac{1}{2}(L^\dagger L\rho + \rho L^\dagger L)$ and

\begin{equation}
H_I = \hbar\chi_b b^\dagger b^2 + \hbar\chi_a a^\dagger a^2 + \hbar(\epsilon b + \epsilon b^\dagger) + \hbar\sqrt{\chi_a\chi_b} (b^\dagger a^\dagger a^\dagger b + b^\dagger a^\dagger a^\dagger b)
\end{equation}

and $\kappa_a, \kappa_b$ are the decay rates of the photon number in the signal and probe cavity respectively and we have included a resonant coherent driving of the probe cavity with $\epsilon = \sqrt{\kappa_b\epsilon_b}$ where $|\epsilon_b|^2$ is the photon flux of the driving field. We have also assumed that each cavity sees a zero temperature environment.

In the absence of the SQUID mediated interactions the probe cavity will relax to a coherent state with the steady state amplitude

\begin{equation}
\beta_0 = \frac{-2i\epsilon}{\kappa_b}
\end{equation}

We will chose the phase of the probe driving as a reference phase and set $\beta_0$ to be real. If we make a canonical transformation to the displaced picture by

\begin{equation}
b = \bar{b} + \beta_0
\end{equation}

we can linearise the Hamiltonian, Eq. 8, in $\bar{b}, \bar{b}^\dagger$ to obtain

\begin{equation}
H_I = H_a + 4\hbar\sqrt{\chi_a\chi_b}\beta_0 (b^\dagger a^\dagger a + 2a^\dagger b\bar{b} + \bar{b}^\dagger b^\dagger a^\dagger a^\dagger b^\dagger b)
\end{equation}

where the effective Hamiltonian for the signal mode alone is

\begin{equation}
H_a = \hbar\chi_a a^\dagger a^2 + 4\hbar\sqrt{\chi_a\chi_b}\beta_0^2 a^\dagger a + \hbar\sqrt{\chi_a\chi_b}\beta_0^2 (a^\dagger a^2 + a^\dagger a^\dagger b^\dagger b)
\end{equation}

which is equivalent to a parametrically driven Kerr nonlinear cavity. This model was considered by Wielinga and Milburn\cite{4}. It is equivalent to a double well system with a hyperbolic fixed point at the origin in phase space and

two elliptic fixed points symmetrically displaced from the origin. The second and third terms in Eq. (11) can be given a familiar interpretation. The second term is of the same form as the radiation pressure interaction between a mechanical resonator $(\hat{h}, \hat{b}^\dagger)$ and a cavity field $(\hat{a}, \hat{a}^\dagger)$. The last term is equivalent to the quantum derivation of sub/second harmonic generation considered by Drummond et al. [5].

We now assume that $\kappa_b$, the line width of the probe cavity is large, $\kappa_b >> \kappa_a, \sqrt{\chi_a \chi_b}$ and we adiabatically eliminate it from the dynamics. In that case from the point of view of the signal mode, the first term in Eq. (11) looks like a fluctuating cavity detuning while the last arms looks like a two photon loss term. This can be verified by explicit adiabatic elimination of the probe cavity field. We assume that the probe cavity, in the displaced picture, remains very close to its steady state of zero photons. The method is described in [6]. The effective master equation for the signal cavity is

$$\frac{d\rho_s}{dt} = -\frac{i}{\hbar} [H_s, \rho_s] + \Gamma_2 D[a^\dagger a] \rho_s + \Gamma_\perp D[a^\dagger a]|0\rangle \langle 0| + \kappa_a D[a] \rho_s \tag{13}$$

where the two photon decay rate $\Gamma_2$ and dephasing rate $\Gamma_\perp$ are given by

$$\Gamma_2 = \frac{16 \chi_a \chi_b \beta_0^2}{\kappa_b}, \quad \Gamma_\perp = \Gamma_2 / 4 = \frac{4 \chi_a \chi_b \beta_0^2}{\kappa_b} \tag{14}$$

A peculiar feature of using SQUID coupled cavities is that the price paid for two-photon decay is an additional dephasing term on the signal cavity field. Using the strong dependance on the steady state amplitude $\beta_0$ in the two photon rate we can make this term dominate over the single photon decay of the signal cavity over the time scales of interest. In Fig. 1 we show that the dephasing term that is introduced in the above (undamped, $\kappa_a = 0$) master equation has little effect on the Schrödinger cat nature of the steady state solution associated with the two-photon absorbing bath. We therefore believe that the discussion in the main article is in-line with the behaviour of realistic environments.

---