Simulation of Particle flow in an Inertial Particle Separator with an Eulerian Velocity Re-Associated Two-Node Quadrature-Based Method of Moments

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This paper presents research into practical simulations of particle flow in inertial particle separators (IPS) typically used in helicopter and tilt rotor aircraft propulsion systems. The flow field of the carrier gas is predicted by a RANS CFD method with the k-\(\varepsilon\) turbulence model. An Eulerian methodology is used to trace the trajectories of foreign particles such as droplets, ice and sand. To predict the characteristics of particle wall bouncing in dilute particle flow, the velocity re-associated two-node quadrature-based method of moments (VR-QMOM) is used. The particle distributions in the IPS are predicted for various particle sizes and these are compared with results from a Lagrangian particle tracking method. The particle-wall interactions and the separation efficiencies are studied for solid particles bouncing off perfectly elastic walls and an IPS shell coated with the M246 alloy which changes the coefficients of restitution. The simulated separation efficiencies predicted by the Eulerian method are compared with the simulation using the Lagrangian method over a range of particle sizes. The VR-QMOM method is seen to reproduce the particle bouncing and trajectory crossing behavior and to agree well with the Lagrangian method for predicted separation efficiencies. The new VR-QMOM method is shown to be an accurate and convenient alternative to established Lagrangian approaches.

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Nomenclature

\( a_{ij} \) = second-order moment of velocity deviation
\( a_{iii} \) = third-order moment of velocity deviation
\( C \) = particle concentration
\( C_D \) = particle drag coefficient
\( C \) = collision term
\( d_p \) = diameter of particles
\( e_{w,t} \) = restitution coefficient of wall particle interaction at tangential direction
\( e_{w,n} \) = restitution coefficient of wall particle interaction at normal direction
\( F \) = external force vector
\( f \) = velocity distribution function
\( m_p \) = mass of single particle
\( M_{ij..l}^{s} \) = s-order moment of velocities
\( n \) = number density of particles
\( Re_p \) = particle Reynolds number
\( s_p \) = solid phase stress
\( U_f \) = velocity vector of fluid phase
\( U \) = Velocity vector of particles
\( U_1 \) = velocity vector of node 1 particles
\( U_2 \) = velocity vector of node 2 particles
\( U_p \) = mean particle velocity vector of node 1 and node 2
\( \rho_p \) = density of particles
\( \rho_f \) = fluid density
\( \tau_p \) = characteristic time for relaxation to fluid velocity
\( \mu_f \) = kinematic viscosity of fluid phase
\( \eta \) = separation efficiency
I. Introduction

Helicopters and turbo-prop driven transport aircraft often operate in harsh environments. During take-off and landing, the downwash of the rotor can lift particles off the ground where they may be ingested by the engine. In these ‘brownout’ conditions, the pilot cannot see nearby objects and an accident may be caused. Furthermore, the engine internal components are at a high risk of being exposed to the ingested particles that leads to a reduction of the engine lifetime and an increase in fuel consumption. Sand ingestion potentially causes erosion of the compressor blades, contamination of the engine’s oil system and blockage of the cooling systems. This leads to a loss of power and a reduction in surge margins. In the 1960’s, the average operational time of helicopters in critical environments was only 300 hours due to erosion, and some helicopter engines operated in jungle environments had to be removed in 100 hours. In desert environments, such as those seen in the Gulf War, engines were removed after fewer than 100 hours and even as little as 20 hours of flight time. Although sand ingestion is mostly prevalent when helicopter and tilt rotor aircraft operate in ground effect, particles such as volcanic ash or ice crystal may be present at high altitude and pose a threat to the safe operation of commercial aircraft. The erosion from volcanic ash is four times worse than the damage from the quartz sand typically found on the desert surface. Proper design analysis, then, requires the ability to trace the particles accurately over a long distance into the engine core through 10-20 compressor stages. The present paper is focused on the more classical problem of sand ingestion in ground effect operation of helicopter and tilt rotor aircraft. Although this class of problem usually refers to a relatively coarse grain of sand, the current study also investigates ingestion of ash particles that have a finer grain, as a proof of concept for modelling volcanic ash ingestion into the engine.

Although erosion-resistant coatings for the turbomachinery blades are able to prolong the operational life of a helicopter engine in the desert, inclusion of an inertial particle separator (IPS) as part of the engine intake remains necessary, and has proven to be a successful method of protecting the engine core. These have relatively low pressure loss and low weight, making them suitable for helicopters, while preventing foreign particles from entering the gas turbine. The main geometry for the IPS design consists of the hub surface, outer surface, shroud and splitter nose. Using the inertial characteristics of particles passing through the complex geometry, the particles will leave the core and enter the scavenge flow path, and thus the foreign particles are separated from the gas flow path, as shown in Fig. 1. For a more detailed figure showing how the IPS is connected in the engine, see for example.
An IPS relies on the particle inertia in order to clean the airflow into the engine core. The inertia dominated particles impact the IPS geometry multiple times before exiting the domain. Thus, a wall interaction model is essential when designing the IPS geometry in order to manipulate the particle trajectories towards the scavenge duct. In soft surface interaction, either due to the nature of the particles (e.g. water droplets) or liquid film covered surfaces, the energy loss through deformation may lead the particle to stick or have a much weaker rebound. The interaction between sand particle and IPS surfaces, which are generally dry, has hard surface characteristics with little deformation. Therefore the rebound and incoming momenta have similar magnitude. Even if the particle shattered during the wall interaction, the smaller pieces are often flying off the wall with high momentum.
The ground sand found in brownout conditions are mostly silica (SiO₂), with a large range of particle size distribution from microns to millimeters⁹. The distributions of C-Spec (40-800µm)¹⁰ and AC-Coarse (1-200µm)¹¹ are typically used for studies of foreign particle ingestion in desert operation. In Fig. 2, the weighted distributions of particle size are shown. While these distributions are in reality continuous, here they have been split into discrete size bins¹¹ to allow their use within the Eulerian method which is monodisperse. The overall separation efficiency can be found from separate calculations using different sizes which are then appropriately weighted. The volcanic ash comprises various types of materials including rock and metal and the particle size will vary with distance from the volcanic eruption. At high altitude, the ash cloud mainly consists of fine particles, which can easily deposit on the surface of the internal components of the engine in large numbers. The blue PSD in Fig. 2 shows the fine ash distribution (1-25µm)¹² from the coal burning process which can be used a guide to that which may be expected in volcanic ash.

The fundamental dynamics of the multiphase flow in IPS have been investigated over decades by means of experiments and computational methods¹²,¹³,¹⁴. The particle dynamics for a specific design of IPS can be studied by experiment and the particle bounce characteristics can be investigated by testing the particle trajectories before and after collision with the wall, which are then used to generate an empirical wall bouncing model¹⁵,¹⁶,¹⁷. CFD can be a flexible and efficient tool to achieve the optimal configuration of components. The Lagrangian method with an empirical wall bouncing model is usually used within a CFD calculation to predict particle trajectories and evaluate the separation efficiency of the inertial particle separators. Calzada et al¹⁸ predicted the particle trajectories of dust at diameters from 1µm to 500µm. For the conditions they investigated, particles with a diameter larger than 20µm could be totally separated by the IPS tested. The 10µm particles partly entered the core flow path, whilst the 1µm particles followed the gas flow path and were hard to be separated.

The Lagrangian method has long been established as an accurate way of modelling dilute particle flows with relatively low computational cost. However, in practice, the total simulation cost may be high if a large number of particles must be tracked in order to build a statistically meaningful sample of particle deposition in the zone of interest. This is particularly true when the simulation is aimed at ingestion deep within the gas turbine core, for instance volcanic ash in the HP compression system, since the parallelization schemes of the Lagrangian method are not as efficient as those for Eulerian methods. Lagrangian methods are also difficult to use efficiently with multiple rotating and stationary frames of reference as found in turbomachinery. Furthermore, Lagrangian based methods
may require manual interaction by the user to position particle injectors in order to ensure enough particles are deposited in the zone of interest that lies after a series of rotor-stator blade rows. Hence it is conceivable that the total cost of performing a Lagrangian simulation can increase by more than three orders of magnitude when going from a single turbomachine zone with simple topology – representing an inlet or a single blade row – to a complete turbomachine stage, whereas the Eulerian based equivalent would increase more linearly with the number of blade rows, i.e. around one order of magnitude. For these reasons an Eulerian based multiphase method offers the potential to improve simulations of particle laden flow in aircraft engines. Traditional Eulerian methods are not able to calculate particle bouncing if only phase mean velocity is available within a cell. However, the Eulerian method of velocity-based quadrature-based method of moments (QMOM) shows the capability to simulate particle trajectory crossing and wall bouncing flow at dilute particle concentration. A new, robust QMOM method, velocity re- associated quadrature-based method of moments (VR-QMOM), has been developed in recent research and is able to predict the arbitrary crossing trajectories and wall-bouncing of a dilute particle flow. In VR-QMOM, the two-node quadrature approach used gives high computational efficiency compared to QMOM models using more nodes, and the new thirteen-moment system overcomes the problem of failed prediction at some specific conditions when using the two-node eight-moment QMOM model by Desjardins.

The aim of this research is to establish an Eulerian-Eulerian two-phase method that is capable of predicting the multiphase flow in the IPS. This is part of a wider project whose intention is to develop methods to trace the particle flow through the core of the engine using a computationally efficient Eulerian method. In this paper the Eulerian method VR-QMOM is used to investigate the particle separation behavior at various particle sizes that may damage gas turbine performance. Results are compared to those computed using a Lagrangian technique in order to demonstrate the viability of the VR-QMOM model when predicting flows with wall bouncing. Since an IPS has relatively simple topology, the cost of modelling is mainly related to the computing time, in which case the Lagrangian model is computationally cheaper than any Eulerian based method.

II. Methodologies

The dynamics of the particle flow in the inertial particle separator are determined by the inertial characteristics of the particles and their interaction with the fluid field. The simulations in this work are carried out in the Eulerian-Lagrangian and Eulerian-Eulerian multiphase framework. The gas flow field is predicted by the Reynolds Averaged Navier-Stokes equations together with a two-equation k-ε turbulence model with standard wall functions. The
particle flow under the dilute flow regime is modeled by an Eulerian method (VR-QMOM) and, for comparison, a Lagrangian method. The multiphase simulations with the Lagrangian method for particle flow are carried out with the commercial software ANSYS Fluent. The velocity re-associated two-node quadrature method of moment (VR-QMOM) is implemented into an unstructured, industrial CFD tool developed particularly for turbomachinery, the Rolls-Royce Hydra CFD code.

The present simulation is of dilute particle concentration, where the inter-particle collisions can be neglected and one-way coupling is assumed for the interaction between the gas and particle phase. It also assumed that the particles in the simulation are uniform and spherical. The derivation of the methodology of VR-QMOM and the comparison of the new model with the other QMOM models are presented elsewhere. The mathematical description of the basic Eulerian method and the new VR-QMOM model for particles applied to the current simulation are presented here.

A. Basic Eulerian method

The basic Eulerian method cannot predict bouncing flow at dilute particle concentration. Taking the dispersed particle phase as a continuum, the particle concentration \( C \) is the variable used to represent the percentage of the particles dispersed locally in the fluid phase. The flux transportation of the particle phase is expressed by the mean velocity of particles. The transport equations for the particle phase are presented briefly by the continuity equation and momentum equation, seen as follows.

\[
\frac{\partial (\rho_p C)}{\partial t} + \nabla \cdot (\rho_p C \mathbf{U}_p) = 0 \tag{1}
\]

\[
\frac{\partial (\rho_p C \mathbf{U}_p)}{\partial t} + \nabla \cdot (\rho_p C \mathbf{U}_p \mathbf{U}_p) = \nabla \cdot \mathbf{s}_p + \mathbf{F} \tag{2}
\]

where \( \mathbf{U}_p \) is the mean velocity vector of the dispersed particle phase, \( \mathbf{s}_p \) is the solid phase stress, which can be neglected in dilute flow, \( \mathbf{F} \) is the external force vector, may including the effect of gravity, drag forces, electrostatic forces, lift force, thermophoresis and turbophoresis.

The Neumann boundary condition can be used at the wall for the simulation of solid particles. This boundary treatment represents particle deposition on the wall, which is largely representative for ‘softer’ particles such as liquid droplets. This boundary is not easily extensible to hard particle simulation where wall bouncing is a key phenomenon, even when an impact source term is added to the momentum equation in order to model the change of particle momentum at the wall.
B. VR-QMOM for solid particles

The new VR-QMOM model used here is a two-node approach based on the velocity-distributed quadrature moment method. This section presents the details of the implementation used in this work. The transport equations are first established by the kinetic theory of gases

\[ \frac{\partial f}{\partial t} + \mathbf{U} \cdot \frac{\partial f}{\partial x} + \frac{\partial f}{\partial x} \left( f \frac{\mathbf{F}}{m_p} \right) = \mathcal{C} \]  

(3)

where \( f \) is the velocity distribution function which gives the possibility (density) of the particles near a specific velocity. \( \mathbf{U} \) is the particle velocity vector. \( \mathcal{C} \) is the inter-particle collision term, where \( \mathcal{C} = 0 \) for dilute particle flow and hence is ignored in this work. \( \mathbf{F} \) in the last term on the LHS is the external force vector, which for this study is the drag force,

\[ \mathbf{F}(\mathbf{U}_f, \mathbf{U}) = \frac{m_p}{\tau_p} (\mathbf{U}_f - \mathbf{U}) \]  

(4)

where \( \tau_p \) is the characteristic time for relaxation of particle velocity to fluid velocity due to aerodynamic drag,

\[ \frac{1}{\tau_p} = \frac{3 \rho_f}{4 \rho_p d_p} C_D |\mathbf{U}_f - \mathbf{U}| \]  

(5)

where \( \rho_f \) is the fluid density, \( \rho_p \) is the particle density, \( d_p \) is the particle diameter, and \( \mathbf{U}_f \) is the fluid velocity vector. \( C_D \) is the particle drag coefficient, which in this work is given by the Schiller-Nauman correlation for small spheres

\[ C_D = \frac{24}{Re_p} \left( 1 + 0.15 Re_p^{0.87} \right) \]  

(6)

and \( Re_p = \rho_f d_p |\mathbf{U}_f - \mathbf{U}|/\mu_f \) is particle Reynolds number, where \( \mu_f \) is the kinematic viscosity of the fluid phase.

In the two-node quadrature approach, the distribution of the velocity fields can be denoted by the number density \( n \) and velocity \( \mathbf{U} \), which are the weights \( (n_1, n_2) \) and abscissas \( (\mathbf{U}_1, \mathbf{U}_2) \) of the two nodes. Using delta-functions to represent the dispersed distribution of particle velocity,

\[ f = n_1 \delta(\mathbf{U} - \mathbf{U}_1) + n_2 \delta(\mathbf{U} - \mathbf{U}_2) \]  

(7)

The arbitrary moment of velocity can be expressed as,

\[ M_{ij...l}^k = n_1 U_{1i} U_{1j} \cdots U_{1l} + n_2 U_{2i} U_{2j} \cdots U_{2l} \]  

(8)

Then the moment transport equations for particles can be obtained from Eq (3) as,

\[ \frac{\partial M^0}{\partial t} + \frac{\partial M^1_k}{\partial x_k} = 0 \]
\[
\frac{\partial M_1^1}{\partial t} + \frac{\partial M_2^1}{\partial x_k} = \frac{n_1}{m_p} F_{1,i} + \frac{n_2}{m_p} F_{2,i}
\]

\[
\frac{\partial M_3^1}{\partial t} + \frac{\partial M_4^1}{\partial x_k} = \frac{n_1}{m_p} U_{1,i} F_{1,i} + \frac{n_1}{m_p} U_{1,j} F_{1,j} + \frac{n_2}{m_p} U_{2,j} F_{2,j} + \frac{n_2}{m_p} U_{z,j} F_{z,j}
\]

\[
\frac{\partial M_1^2}{\partial t} + \frac{\partial M_3^2}{\partial x_k} = \frac{n_1}{m_p} U_{1,i} F_{1,i} + \frac{n_1}{m_p} U_{1,j} F_{1,j} + \frac{n_2}{m_p} U_{2,j} F_{2,j} + \frac{n_2}{m_p} U_{z,j} F_{z,j}
\]

\[
\frac{\partial M_3^2}{\partial t} + \frac{\partial M_4^2}{\partial x_k} = \frac{n_1}{m_p} (U_{1,i})^2 F_{1,i} + \frac{n_2}{m_p} (U_{2,j})^2 F_{2,j}
\]

The terms on the right hand side represent interactions due to the influence from the interactions with fluid phase. In VR-QMOM, the moment equations (9) are solved for 13 moments in a three dimensional system, these are

\[
W^3 = (M^0, M_1^1, M_1^2, M_1^3, M_1^4, M_2^1, M_2^2, M_2^3, M_3^1, M_3^2, M_3^3) \in \mathbb{R}^{13}.
\]

Based on the two-node definition of the moments given in Equation (7), the transport equation for an arbitrary one of these 13 moments \( M_{ab} \) can be written in terms of the velocity fields of the two nodes

\[
\frac{\partial M_{ab}}{\partial t} + \frac{\partial n_1 U_{1a} U_{1b} U_{ii}}{\partial x_i} + \frac{\partial n_2 U_{2a} U_{2b} U_{ii}}{\partial x_i} = S
\]

where \( S \) is the source term such as drag. The key to the QMOM approach is that once the moments have been updated it is necessary to recover the updated nodal velocity fields and their weightings. In the closure approach of VR-QMOM, the first step is to calculate the abscissas and weights with the product-difference (PD) algorithm, as in the established third-order QMOM model.\(^{19}\) The mean velocity of particles in each direction is found by \( U_{pi} = \frac{M_{pi}^1}{M^0} \). This can be used to define the covariance matrix and the third order velocity variance vector

\[
[a_{ii}] = \begin{bmatrix}
M_{i1}^2 / M^0 - U_{pi} U_{pi} & M_{i2}^2 / M^0 - U_{pi} U_{pj} & M_{i3}^2 / M^0 - U_{pi} U_{pk} \\
M_{i2}^2 / M^0 - U_{pi} U_{pj} & M_{i1}^2 / M^0 - U_{pj} U_{pj} & M_{i4}^2 / M^0 - U_{pj} U_{pk} \\
M_{i3}^2 / M^0 - U_{pi} U_{pk} & M_{i4}^2 / M^0 - U_{pj} U_{pk} & M_{i1}^2 / M^0 - U_{pk} U_{pk}
\end{bmatrix}
\]

\[
[a_{iii}] = \begin{bmatrix}
\frac{1}{M_0} M_{iii} - U_{pi}^3 - 3 a_{pi}^2 U_{pi} \\
\frac{1}{M_0} M_{jjj} - U_{pj}^3 - 3 a_{pj}^2 U_{pj} \\
\frac{1}{M_0} M_{kkk} - U_{pk}^3 - 3 a_{pk}^2 U_{pk}
\end{bmatrix}
\]

The node weights can be calculated using

\[
x = \frac{a_{iii}}{2 \sqrt{a_{iii}^2 + 4 a_{ii}^3}}, \quad a_{ii} \neq 0
\]

\[
n_{1,2} = \left(\frac{1}{2} \pm x\right) M_0
\]

The velocity components in the \( i \)-direction for the two nodes are then found from
\[ U_{1i} = U_{pi} - \left( \sqrt{a_{ii}} \right) \sqrt{\frac{n_2}{n_1}} \] (16)

\[ U_{2i} = U_{pi} + \left( \sqrt{a_{ii}} \right) \sqrt{\frac{n_1}{n_2}} \] (17)

The components in the \( j \) and \( k \) directions can be found by the same procedure. However, in order to ensure correct physical behavior for all trajectory crossing cases it is necessary to ensure that the velocity components in each node are correctly associated with each other. Therefore, the second step is to set up the correlation between these abscissas and weights in order to set the final two velocity components for each flow field. The key issue of the model is to re-associate the relationship between \((U_{-j}, U_{j}, U_{-k}, U_{+k})\) and \((n_1, n_2, U_{1i}, U_{2i})\) using the second order cross moments of the velocity of particles. Therefore, the value of the remaining four velocities can be recalculated by the second order cross moment of velocity derivation.

\[ U_{1j} = U_{pj} - \left( \frac{a_{ij}}{\sqrt{a_{ii}}} \right) \sqrt{\frac{n_2}{n_1}} \]

\[ U_{2j} = U_{pj} + \left( \frac{a_{ij}}{\sqrt{a_{ii}}} \right) \sqrt{\frac{n_1}{n_2}} \] (18)

\[ U_{1k} = U_{pk} - \left( \frac{a_{ik}}{\sqrt{a_{ii}}} \right) \sqrt{\frac{n_2}{n_1}} \]

\[ U_{2k} = U_{pk} + \left( \frac{a_{ik}}{\sqrt{a_{ii}}} \right) \sqrt{\frac{n_1}{n_2}} \]

A unique pair of weighted velocity fields is thus obtained from the set of moments. Thus, the velocities of the two nodes are re-associated by the cross-moments of second-order.

To give the desired surface bouncing behavior it is necessary to specify appropriate boundary conditions for the dispersed phase. In this research, we assume that the particle bouncing can be described by deterministic coefficients of restitution in the normal and tangential directions. This can be applied for both nodal velocity fields.

\[ \begin{bmatrix} n_a \\ U_{an} \\ U_{at} \end{bmatrix}_{bc} = \begin{bmatrix} n_a/e_t \\ -e_{wn}U_{an} \\ e_{wt}U_{at} \end{bmatrix}_f \] (19)

Where the subscript \( n \) is for normal direction of the wall and \( t \) is for tangential direction. \( e_t \) is the total velocity coefficient of restitution defined as the ratio of velocity of reflected particles to injecting particles. This method allows the present model to utilize existing empirical wall bouncing models that have been devised primarily for Lagrangian based particle simulations.

The VR-QMOM model is implemented into the Rolls-Royce Hydra CFD code\(^{24} \) which uses an unstructured mesh formulation with median dual control volumes. This uses a second order MUSCL type upwind scheme for
spatial discretization, a fourth order Runge-Kutta scheme for temporal advancement and multigrid for convergence acceleration. The transport step of the QMOM requires the moments to be updated using the weighted velocity nodes. A Finite Volume method is used to construct the fluxes in the unstructured discretization scheme. Currently, for robustness, a first-order upwind scheme is used in the simulation with VR-QMOM \( M_{a,f} = M_{a,up} \).

### III. Simulation and Results

#### A. Particle wall bouncing prediction

Figure 3 shows the particle concentration predicted by the existing basic Eulerian method introduced in section II-A. When the solid particles hit the surface of the IPS, the solid particles should bounce off the surface. But with the basic Eulerian method, particle bouncing cannot be predicted and they incorrectly lose all the momentum in the normal direction. Therefore in the inlet region, the particle concentration builds up at the wall of the hub and the particles flow along the surface. As they are collecting in a region one cell thick they cannot be seen in the contour plot. They then leave the wall due to the curvature of the hub at its widest point. The particles then incorrectly collect on the outer casing before releasing into the scavenge duct. Hence, in this simulation, the basic Eulerian method is unable to predict the particle wall interaction correctly.

![Figure 3. Particle concentration of 200µm sand in IPS as predicted by the basic Eulerian method showing surface particle collection.](image)

The reason for this incorrect prediction can be analysed by the simplified test case of particle wall collision shown in Figure 4. In Fig. 4(a), the particle concentration is simulated by the basic Eulerian method. The single velocity field averages the particle velocities of injecting particles and reflected particles, making the mean velocity near to zero normal to the wall, such that the particles will flow along the surface. Figure 4(b) shows a simulation using the new VR-QMOM method. As two velocity fields or nodes are used to represent the incoming and outgoing particles at the surface, the bouncing flow is able to be predicted successfully. There is some slight numerical dispersion that spreads the contours after bouncing.
**Figure 4.** Concentration of particles bouncing elastically from a solid surface, as predicted by (a) basic Eulerian method and (b) VR-QMOM method. With the basic Eulerian method the particles are seen to incorrectly collect on the surface.

The particle-surface interaction mechanism is defined with the boundary conditions and the restitution coefficients as described in Eq (19). In VR-QMOM, the particle velocities at the surface are described by the two velocity fields. Therefore, the rebounded particle velocities are calculated directly from the injection particles. Figure 5 shows the prediction of the particle wall bouncing flow with varying restitution coefficients. The particle beams can be seen to exactly follow the expected analytical rebound angles as shown by the dotted lines.

**Figure 5.** Particle paths predicted by VR-QMOM at various normal restitution coefficients. Expected rebound path shown as dotted line.
B. Geometry and gas phase of a generic IPS

The generic IPS geometry used in the simulation is based upon the axisymmetric advanced IPS configuration that was studied by Hamed et al.\textsuperscript{15} The IPS was modelled as a 3D sector domain (15°) of the full annulus with symmetry side walls. The unstructured computational mesh was generated by processing a structured multi-block body-fitted grid in order to minimize grid related numerical diffusion. There were 231 nodes along the top and bottom walls, 81 nodes across the inlet and 41 nodes across each of the outlets. For convenience the 15° sector was resolved with 16 nodes (15 cells). This gives a total of 300,00 nodes for the whole domain. The mesh used has a similar radial resolution and near wall spacing, and an improved axial resolution as compared to that used by Hamed et al.\textsuperscript{15} who reported satisfactory results for the same aerodynamic flow. Figure 6 shows the grid and the aerodynamic flow in the IPS. The flow enters the gas path, which is bounded by the hub (lower radius wall) and casing (higher radius wall), from the left boundary and then split into the engine core flow (lower outlet) and scavenge (upper outlet) on the right. The inflow has. Figure 6 shows the pressure and Mach number contour of the gas phase as computed by the CFD simulation using a RANS k-\(\varepsilon\) model. A subsonic inlet is used in the simulation, with an average axial velocity of 41 m/s, and the exit boundaries are set such that 25.8% of the mass flow passes through the scavenge flow path to match the flow field reported by Hamed et al\textsuperscript{15}. The flow-field was chosen as representative air flow through an IPS that is characterised by a recirculating low momentum region on the casing side near the splitter nose that is the inlet to the scavenge geometry. The gas phase simulation is based on the second order spatial numerical scheme while the VR-QMOM equations are solved using a first order spatial scheme. Hamed et al.\textsuperscript{15} previously reported a study of the sensitivity of the aerodynamics flow to computational grid. The flow field was chosen prior to any particle simulations. As such, it was not intended to highlight certain behaviours other than being representative of a realistic sand particle flow in an IPS. In this paper the particle trajectory in the IPS are predicted by the new Eulerian method VR-QMOM and, for comparison, with a Lagrangian method. The gas phase simulation details, particle properties and the drag law used in the Lagrangian method are, as far as possible, the same as those used in the VR-QMOM calculations to make sure the simulated particle trajectories are comparable. It is worth noting that the simple axisymmetric nature of this simplified IPS model means a straightforward Lagrangian model can be performed, which consumes one order of magnitude less CPU time than that of VR-QMOM. For the results shown here each VR-QMOM calculation take up to 12 hours on a single processor core. However, this simplified case neglected the complexity of a real IPS geometry, in which the scavenge duct is no
longer axisymmetric and also made up of a torturous flow path that would require a more complicated Lagrangian model in order to ensure enough sand particle tracks arrive in the duct in order to obtain a statistically conclusive result.

Figure 6. Contours of pressure and Mach number of gas phase in the IPS model
C. Sand distribution in a generic IPS with elastic surfaces

For the particle phase, it is assumed that the sand particles in the IPS consist of hard silica particles with density of 2444 kg/m$^3$. In this section, wall boundary conditions for the particles are set by assuming perfectly elastic collisions. More realistic inelastic particle-wall interactions are considered in the next section to assess the ability of the VR-QMOM method to predict the changed behaviour this causes.

The particle motions in the IPS are mainly affected by the gas phase carrier and the inertial characteristics of the particle wall bouncing mechanics. Smaller particles will tend to follow the flow path of the gas phase carrier, while larger particles are separated from the gas flow by the inertia of the particles and the confinement of the surface. In this case, the particle concentration is assumed small and the particle-particle interaction can be neglected. This means that the actual concentration of particles at the inlet does not affect the flow and all concentrations are calculated as being relative to the inlet value. The flow rates of the particles in all the following studies are the same, with inlet particle velocity set to be equal to the gas inlet velocity. In this study, we assume the particle size is uniform in each simulation and different simulations are run for different particle sizes. A final simulation result in terms of separation efficiency can be integrated from the prediction at each size weighted by the realistic particle distribution function, shown in Fig 2. Four sizes of particles are simulated in this section: 3μm, 10μm, 200μm and 1000μm. Particle trajectories and separation efficiency are investigated from the simulations.

In Fig. 7(a), the particle trajectories are predicted by the Lagrangian method and the particle flow trajectories are coloured by the local particle concentration normalized by the concentration at the inlet. With the Lagrangian method, the particle trajectory shows the flow path of each particle injected from the inlet normal to the inlet surface. For 3μm particles, the particle flow is dominated by the gas phase, so the particles follow along the flow path of the gas phase. Some of the small particles leave the IPS from the scavenge path. From the particle trajectories of the 200μm particles, the particles bounce off the surface and the trajectories cross in the centre of the flow domain. The particle inertia controls the flow of the particles. Almost all the 200μm particles can be separated. The simulated separation efficiency is about 100.0% of the IPS for 200μm particles by the Lagrangian method. It should be noted that large particle trajectory, and hence the separation efficiency, is predominantly determined by a combination of wall bouncing characteristics and the wall countouring since these particles behave mostly in a ballistic manner. Since the IPS geometry was not designed to have an elastic wall, the IPS contour actually failed to separate all of the 1000 μm particles. The 1000μm particles rebound in the path of IPS and bounce into the engine through the core.
path. For the 10 µm case, the inertia of the particles is seen to carry most of them directly into the scavenge duct. High concentrations are observed on the lower surface of the scavenge intake and in the low flow velocity regions near the splitter nose, both at the edge of the re-circulation bubble and the upper radius wall of the splitter. Some particles impact the nose of the splitter and rebound into the core stream.

These features are largely seen in both Lagrangian and Eulerian results in Figure 7 although there are some slight discrepancies observed in the predicted distribution of 200 µm and 1000 µm particles. Also some particle concentration is seen in the recirculation zone above the splitter nose for 10 µm which is not seen in the Lagrangian

![Particle trajectories and normalized instantaneous particle concentrations of sand in the IPS with elastic bouncing](image)

**Figure 7. Particle trajectories and normalized instantaneous particle concentrations of sand in the IPS with elastic bouncing (from top to bottom 3µm, 10µm, 200µm, 1000µm)**

![Normalized particle concentrations](image)
simulation. This is possibly due to the numerical dispersion in the recirculation zone caused by the first order numerical method used in VR-QMOM.

From the results above we see that the new Eulerian method is able to predict the trajectory crossing and the wall-bouncing of particles. The advantage of the Eulerian simulation is that the particle concentration can be read directly from the simulation as a variable in the transport equations. However, when using the Lagrangian method the particle concentration has to be calculated from the particle trajectory and numbers of particles in each cell using post-processing. These concentrations will depend on the number and injection location of particles used in the simulation. As mentioned earlier, Lagrangian simulations may require large numbers of injections to achieve statistical convergence. As can be seen in Fig. 7, the particle concentration displayed by a finite number of particles in the Lagrangian figures cannot represent the local information continuously. Therefore, a large number of particles have to be injected into the flow domain in order to give sufficient statistical information for the particle distribution in the whole domain.16

Table 1. Separation efficiency of particles in generic IPS with elastic surface

<table>
<thead>
<tr>
<th>Separation efficiency</th>
<th>3μm</th>
<th>10μm</th>
<th>200μm</th>
<th>1000μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagrangian</td>
<td>32.29%</td>
<td>76.06%</td>
<td>100.0%</td>
<td>83.20%</td>
</tr>
<tr>
<td>VR-QMOM</td>
<td>34.72%</td>
<td>74.70%</td>
<td>100.0%</td>
<td>94.12%</td>
</tr>
</tbody>
</table>

The separation efficiency is calculated by the mass flow rate of the scavenge particle stream to the inlet particle stream. In the simulations in this study, all the particles complete their path and reach the outlet in Lagrangian method, either by scavenge outlet or to the engine core path.

\[
\eta = \frac{\text{mass flow rate of particles at scavenge path}}{\text{mass flow rate of particles at inlet}}
\] (19)

In table 1, the separation efficiencies are listed from both Lagrangian method and Eulerian VR-QMOM simulations. Again it should be noted that while the exact value found from the Lagrangian method will be affected by the number of particles used (this has been studied for the present case by Hamed et al.15) the VR-QMOM method will not. The separation efficiencies for large particles such as 200μm diameter are predicted to be 100% with both methods. However, in the case of elastic collisions, as in this section, for 1000μm particles the separation efficiency reduces as particles can now bounce multiple times to reach the core. Fine particles are unlikely to be totally separated by the IPS. The particles of 3μm are only about 30% separated, which is close to the air mass flow.
split of 25.8%. About 80% of 10\(\mu\)m particles leave the engine through the scavenge path with those entering the core having bounced from the splitter nose. The prediction efficiencies of the Lagrangian method and VR-QMOM method are quite close to each other. Although the coefficient of restitution based model and the particle momentum equations are equivalent between the Eulerian and Lagrangian method, some differences between the two is to be expected due to the different numerical treatments that must necessarily be used and the different sources of error. For example the Lagrangian result will be affected by the number of particles while the Eulerian method will have errors due to numerical diffusion, which, for example, can be observed in the recirculating zone of the 10 \(\mu\)m particle distribution in Fig 7. While taking these into consideration the agreement between the two methods is still highly promising, especially compared to previous Eulerian methods.

D. Sand particle distribution in a generic IPS coated with M246 alloy

The collision mechanisms and rebound data of the particle surface interaction has been previously investigated by experiments\textsuperscript{8,9}. The restitution coefficient of particle-wall interactions is affected by various elements such as the particle properties of density, diameter, shape, material and hardness as well as the geometry and surface characteristics of the IPS. These, together with the flow conditions of the incoming particles such as impingement angles and velocities, affect how the particles behave. In this work we neglect the effect of particle size and speed on restitution coefficient but consider the effect of restitution coefficient being a function of impingement angle. 

![Figure 8. Restitution coefficient variation with impingement angle \(\beta_{im}\)](image)

Hamed and Tabakoff\textsuperscript{12} studied the restitution coefficient by measuring particle bouncing in a high temperature facility for different alloys and coatings. Figure 8 shows the rebound data of sand particles from a M246 alloy
coating as a function of the impingement angle. The restitution coefficient $e_{wn}$ is the ratio of normal velocities of the injecting particles to reflected particles, and $e_{wt}$ is the ratio of tangential velocities. Zero degree impingement represents particles sliding along the surface. As such, both restitution coefficients are unity at zero degree impingement.

Figure 9 shows the particle trajectories modeled by the Lagrangian method and the normalized particle concentration of sand calculated by the VR-QMOM. In these simulations the particle-wall coefficients of restitution vary according to the impingement angle using the data in Fig. 8. The restitution coefficients are less than unity and the rebound angles of the particle changes accordingly. The speed of the sand flow is reduced in the collision with the coating surface and the reduction enhances the influence of the gas carrier on the particle flow. As shown earlier for the elastic case, a very good agreement is seen between Lagrangian and Eulerian simulations for all particle sizes. The VR-QMOM method has also successfully reproduced the changes to the particle flow caused by the changed surface properties. For example, in the concentration distribution of 200µm particles, the location where the particle stream generated from the hub surface collides with the outer surface has moved downstream. The 30µm case is particularly interesting as it shows that the particles bounce off the early part of the hub surface with a low trajectory before collecting in the low speed re-circulation region located above the splitter where a high concentration is observed.

The surface bouncing mechanisms are different between large particles and small particles. For small particles, whose particle flow path are more closely following the gas phase, the impingement angle will be lower meaning that particles are more likely to be directed towards the flow path of the gas phase. This will reduce the separation efficiency of small particles as seen in 3µm case. For the large particles whose flow path is mainly controlled by the particle inertia, the effect of the inelastic surface provides a better match to the IPS wall contour. For the large particles over 500µm which were hard to remove totally when a fully elastic wall was used, the separation efficiency may be increased. For particles in 1000µm diameter, the separation efficiency was raised to 100% in the IPS coated with M246.
Figure 9. Particle trajectories and normalized instantaneous concentrations of sand in the IPS with M246 alloy coating (from top to bottom 3µm, 10µm, 30µm, 200µm, 1000µm).

Table 2. Separation efficiency of particles in the IPS with M246 alloy coating

<table>
<thead>
<tr>
<th>Separation efficiency</th>
<th>3µm</th>
<th>10µm</th>
<th>200µm</th>
<th>1000µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagrangian</td>
<td>30.00%</td>
<td>75.67%</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>VR-QMOM</td>
<td>31.02%</td>
<td>76.55%</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>
In Tables 1 and 2, the separation efficiencies of particles are listed for the IPS assumed with elastic surface and the IPS coated with M246 alloys. The separation efficiencies of 3µm, 10µm, 200µm and 1000µm are calculated with both simulation methods for fully elastic and M246 alloy bouncing conditions. For the elastic case the flow path of the 3µm particles are close to the gas phase and about 32% particles leave through the scavenge path, this being close to the splitting ratio of the gas phase (25.8%). Coated with M246 alloy, the separation of 3µm particles has been reduced to around 30%. The largest particles, of 1000µm, are now 100% separated.

Comparing the simulations of particle distribution in the IPS with elastic surface in Fig. 7 and in the IPS with M246 alloy coating in Fig 9, better agreement is seen from the simulation with a realistic coefficient of restitution than the simulation with perfect elastic particle wall bouncing. The combined normal and tangential coefficients of restitution for the M246 alloy have the effect of favouring a low-trajectory bounce off the alloy surface. This in turn can be seen, in both Lagrangian and Eulerian results, to cause the particles to form into coherent ‘streams’ of particles to a greater extent than they do for the elastic bounce case. This can be seen most clearly when comparing the 1000 µm cases in Figs 7 & 9. It is likely that finite volume discretisation used in the Eulerian calculations is more suited to simulating these broad streams than the case where there are multiple rebounding streams. A higher mesh density or a higher order numerical scheme may be required to improve results for the elastic wall case.

Figure 10 shows a summary all the separation efficiency of particles in the IPS with M246 alloy coating at the whole range of particle size distributions listed in the Fig. 2. The separation efficiency of particles larger than 20 µm are 100%. As expected the smaller particles have a lower separation efficiency. The separation efficiency of 1µm particles is near to the splitting ratio of the gas phase.
In order to fully assess the output of the VR-QMOM method a comparison with Lagrangian results for realistic particle size distributions was made. Here two standard sand specifications that are commonly used for certification in desert environment operations – namely C-Spec and AC Coarse (ACC) – and a widely used ash representation that is described in Fig. 2 are used. The aim is to assess whether the differences that have been observed before for single particle sizes would lead to a different prediction of the overall IPS efficiency. The total efficiency is calculated by the monodisperse separation efficiencies weighted by the particle size distribution. The C-Spec particle spectrum includes particles in the diameter range from 40-800 \( \mu \text{m} \). Calculated total efficiencies by both Lagrangian and VR-QMOM methods are equal to 100%. The ash model consists of fine particles with diameter in the range 1-25 \( \mu \text{m} \), which can represent the most damaging fine volcanic ash encountered at high altitude for commercial aircraft operation. The calculated efficiencies are near to 50%. The total efficiency of separation simulated by the Lagrangian method is \( \eta_{\text{ash,Lag}}=48.60\% \), and \( \eta_{\text{ash,VR-QMOM}}=50.21\% \) calculated by the VR-QMOM. AC Coarse covers a large range of particles from 1-200 \( \mu \text{m} \), including both fine particle and coarse particles. The calculated separation efficiency is 86.45% by Lagrangian method and 86.56% by VR-QMOM. It is observed that the separation efficiency simulated by the Eulerian VR-QMOM is in all cases very close to the Lagrangian method, which has previously been shown to give accurate predictions of separation efficiency for IPS of helicopters. Hence the new VR-QMOM method is able to accurately predict particle flows dominated by wall-particle interactions.

### IV. Conclusion

In this research, the multiphase dynamics of an inertial particle separator have been investigated for foreign particle ingestion. The motivation of the research is to develop an Eulerian method for multiphase flow which can predict the interactions between the particles and surfaces. If the foreign particles are hard solids such as sand or dust, the basic Eulerian method fails to describe the particle trajectories and distributions as it cannot predict bouncing. A new Eulerian method VR-QMOM model is used here to predict the dilute particle flow in the IPS. The particle flow stream bouncing and crossing in a generic IPS are successfully modeled. The distributions of particles are seen to match those calculated by Lagrangian simulation. The trend with particle size and with surface contact behavior is correctly reproduced.
The particle distribution in an IPS coated with M246 alloy are predicted by both VR-QMOM and Lagrangian method. The separation efficiency predicted by the VR-QMOM is very close to the efficiency found by the Lagrangian method in the whole range of particle spectrum, from fine particles of about 1μm to coarse particles of 800μm. This enables separation efficiencies for particle size distributions to be found using the monodisperse efficiencies weighted by the size distribution. The ability of the Eulerian VR-QMOM method to predict the behavior of dilute particle flows in an IPS is thus demonstrated by the calculation separation efficiency which is seen to be very close to those found with established Lagrangian methods. This gives confidence that the VR-QMOM method can be successfully applied in other environments where the availability of Eulerian methods will offer significant advantages over Lagrangian methods. These include turbomachinery within the engine core where multiple rotating and stationary frames of reference make efficient use of Lagrangian methods difficult.

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References


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