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Partition of Mixed-Mode Fractures in 2D Elastic Beams with Through-Thickness Shear Forces

Joe Wood, Chris Harvey, Simon Wang
J.Wood@lboro.ac.uk

Department of Aeronautical & Automotive Engineering
Loughborough University, LE11 3TU, UK

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Interfacial cracks

• Cracks tend to propagate along interfaces in laminated materials because they represent a plane of weakness.

• They do not kink in order to propagate under pure mode I opening conditions, as they would tend to in an isotropic material.

• Interfacial cracks therefore propagate in a mixed-mode with a combination of mode I opening, mode II shearing, and/or mode III tearing.
Fracture toughness

- Fracture toughness depends on the fracture mode partition.
- Predicting fracture toughness requires the knowledge of the partition of a mixed-mode fracture.
- Essential to have a correct analytical partition theory to predict the fracture toughness.
One-dimensional fractures

- Delamination during drilling
- Thermal barrier cracking
- Helicopter blade delamination
  Robert Davies, Dreamstime.com
- Needle puncture of red blood cell/IVF treatment
  Alamy
Mixed-mode interfacial fracture

- 1D fracture of DCB is fundamental case for study
  - Bending moments $M_1$ and $M_2$
  - Axial forces $N_1$ and $N_2$
  - Shear forces $P_1$ and $P_2$
  - $\eta = E_2/E_1$, $N = \nu_2/\nu_1$, $\gamma = h_2/h_1$
Total energy release rate (ERR)

- Quadratic form and non-negative definite
- Partition total ERR $G$ into its pure mode components, $G_I$ and $G_{II}$
- Use the orthogonal pure fractures modes

$$G = \begin{bmatrix} M_{1B} \\ M_{2B} \\ N_{1B} \\ N_{2B} \end{bmatrix}^T \begin{bmatrix} M_{1B} \\ M_{2B} \\ N_{1B} \\ N_{2B} \end{bmatrix}$$

$$C_{ij} = f(E_1, E_2, \nu_1, \nu_2, h_1, h_2, b)$$

Mixed Mode = Mode I + Mode II + Mode III
Pure fracture modes

- The inner product matrix transforms the $\{M_{1B} \ M_{2B}\}$ vectors into ERR space.
- In ERR space, orthogonality between two $\{M_{1B} \ M_{2B}\}$ vectors means
  $$\{M_{1B} \ M_{2B}\}_1 [C] \{M_{1B} \ M_{2B}\}_2^T = 0$$
- Orthogonal pairs of $\{M_{1B} \ M_{2B}\}$ vectors exist that represent pure fracture modes:
  - Denote pure mode I as $\{1 \ M_{2B}/M_{1B}\} = \{1 \ \theta_1\}$
  - Denote pure mode II as $\{1 \ M_{2B}/M_{1B}\} = \{1 \ \beta_1\}$, etc.
  - With $\theta_i$ and $\beta_i = f(E_1, E_2, v_1, v_2, h_1, h_2, b)$

Contours of ERR with $E = 1$, $b = 1$, $h = 1$, $\gamma = 1$, $\eta = 1$
ERR partitions general theory

- Euler beam partitions:
  \[ G_{IE} = c_{IE} \left( M_{1B} \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3} \right) \left( M_{1B} \frac{M_{2B}}{\beta_1'} - \frac{N_{1B}}{\beta_2'} - \frac{N_{2B}}{\beta_3'} \right) \]
  \[ G_{IIE} = c_{IIE} \left( M_{1B} \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3} \right) \left( M_{1B} \frac{M_{2B}}{\theta_1'} - \frac{N_{1B}}{\theta_2'} - \frac{N_{2B}}{\theta_3'} \right) \]

- Timoshenko beam partitions:
  \[ G_{IT} = c_{IT} \left( M_{1B} \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3} - \frac{P_{1B}}{\beta_4} - \frac{P_{2B}}{\beta_5} \right)^2 \]
  \[ G_{ITT} = c_{ITT} \left( M_{1B} \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3} - \frac{P_{1B}}{\theta_4} - \frac{P_{2B}}{\theta_5} \right)^2 \]

- 2D elasticity partitions:
  \[ G_{I-2D} = c_{I-2D} \left( M_{1B} \frac{M_{2B}}{\beta_{1-2D}} - \frac{N_{1B}}{\beta_{2-2D}} - \frac{N_{2B}}{\beta_{3-2D}} - \frac{P_{1B}}{\beta_{4-2D}} - \frac{P_{2B}}{\beta_{5-2D}} \right)^2 \]
  \[ G_{II-2D} = c_{II-2D} \left( M_{1B} \frac{M_{2B}}{\theta_{1-2D}} - \frac{N_{1B}}{\theta_{2-2D}} - \frac{N_{2B}}{\theta_{3-2D}} - \frac{P_{1B}}{\theta_{4-2D}} - \frac{P_{2B}}{\theta_{5-2D}} \right)^2 \]
General 2D elasticity partition theory

- Bending moments $M_{1B}$ and $M_{2B}$ and axial forces $N_{1B}$ and $N_{2B}$
- Revisit the orthogonal pure fracture modes $(\theta_i, \beta_i)$
  - Condition using beam theories does not produce the same stress distribution in 2D elasticity theory
  - Apply a correction factor for 2D elasticity to the part of the condition that represents the intact portion of the beam
  - Calibrate correction factor for $\theta_{1-2D}$ using $\theta_1 \leq \theta_{1-2D} \leq \theta_1'$
  - Obtain other pure modes $(\theta_{2-2D}, \beta_{1-2D}, \beta_{2-2D}, \text{etc.})$ using orthogonality
Timoshenko beam partition theory

- Crack tip through-thickness shear forces $P_{1B}$ and $P_{2B}$ only
  \[ M_{1B} = M_{2B} = N_{1B} = N_{2B} = 0 \]

- \[ G_{\theta P-T} = \frac{1}{2b^2 h_1 \kappa \mu} \left(1 + \frac{\theta_{P-T}^2}{\gamma}\right) \quad G_{\beta P-T} = \frac{1}{2b^2 h_1 \kappa \mu} \left(1 + \frac{\beta_{P-T}^2}{\gamma} - \frac{(1 + \beta_{P-T})^2}{1 + \gamma}\right) \]

- \( (\theta_{P-T}, \beta_{P-T}) = (-1, \gamma) \therefore G_{II} = 0 \)

- Shear correction factor \( \kappa = 5/6 \)
2D elasticity partition theory

- Crack tip through-thickness shear forces $P_{1B}$ and $P_{2B}$ only
  - $M_{1B} = M_{2B} = N_{1B} = N_{2B} = 0$

\[
G_{\theta_{P-2D}} = \frac{1}{2b^2 h_1 \kappa(\gamma) \mu} \left( 1 + \frac{\theta_{P-2D}^2}{\gamma} \right),
G_{\beta_{P-2D}} = \frac{1}{2b^2 h_1 \kappa(\gamma) \mu} \left( 1 + \frac{\beta_{P-2D}^2}{\gamma} - \frac{(1 + \beta_{P-2D})^2}{1 + \gamma} c(\gamma) \right)
\]

- $(\theta_{P-2D}, \beta_{P-2D}) = (??, ??)$
- Shear correction factor now $\gamma$ dependent $\kappa(\gamma)$
- $G_{II} \neq 0$ and introduce pure-mode-II correction factor $c(\gamma)$
Shear Force Pure Modes

- \((\theta_{P-2D}, \beta_{P-2D})\)
- FEM simulations
- \(-1.7 \leq \log_{10}(1/\gamma) \leq 1.7\)

- Pure mode I \(\theta_{P-2D}\)
  - \(G_{II} = 0, \ \theta_{P-2D} = -1\)
  - \(\therefore P_{2B} = -P_{1B}\)

- Pure mode II \(\beta_{P-2D}\)
  - \(G_I = 0\)
  - \(\beta_{P-2D} = \gamma \exp(-1.986060 \text{ atanh}(0.563483\gamma_I))\)
Shear & Pure Mode II Correction Factors

- FEM Simulations
- $M_{1B} = M_{2B} = N_{1B} = N_{2B} = 0$
- $-1.7 \leq \log_{10}(1/\gamma) \leq 1.7$

- Shear Correction Factor
  - $\kappa(\gamma)$
  - $P_{2B}/P_{1B} = \theta_{P-2D} = -1$

- Pure-mode-II ERR Correction Factor
  - $c(\gamma)$
  - $P_{2B}/P_{1B} = \beta_{P-2D}$
Numerical Verification

\[
\frac{1}{10} \leq \gamma \leq 10
\]
\[ \frac{1}{10} \leq \gamma \leq 10 \]
Blister Test

- Interface fracture toughness

Image from Koenig (2011)
Adhesion of graphene membranes

\[ \gamma = \frac{h_2}{h_1} \rightarrow \infty \]

\[ G_I = \frac{6M_{Be}^2}{Eh^3} (1 - \nu^2) \left( 1 - \frac{N_{Be}h}{4.450M_{Be}} - \lambda \right)^2 0.6227 \]

\[ G_{II} = \frac{6M_{Be}^2}{Eh^3} (1 - \nu^2) \left( \frac{N_{Be}h}{2.697M_{Be}} \right)^2 0.3773 \]
• Pressure loaded blister test
  – Linear failure criterion
  – $G_{Ic} = 0.226 \, J/m^2$
  – $G_{IIc} = 0.683 \, J/m^2$
Adhesion of graphene membranes

- Pressure loaded blister test
  - Linear failure criterion
  - $G_{Ic} = 0.226 \text{ J/m}^2$
  - $G_{IIc} = 0.683 \text{ J/m}^2$
  - $\rho_{mono} = G_I/G_{II} = 0.431$
  - $\rho_{multi} = G_I/G_{II} = 0.764$
Experimental validation

• Pressure loaded blister test – Koenig et al. (2011)
  – Linear failure criterion
  – $G_{Ic} = 0.226 \, J/m^2$ and $G_{IIc} = 0.683 \, J/m^2$

• Point loaded blister – Zong et al. (2012)
  – Experimental Results
  – $\delta/R_B = 0.2309$, $E = 1TPa$, $nt = 1.7nm$ and $n = 5$.
  – $G_{exp} = 0.438 \, J/m^2$
  – Mode mixity $\rho_{th} = G_I/G_{II} = 0.381$
  – Linear failure criterion $G_{th} = 0.438 \, J/m^2$
Conclusion

• 2D elasticity partition theory
  – Developed for general loading conditions (bending moments, axial forces and shear forces).
  – Numerically verified for a number of loading conditions

• Application to:
  – Adhesion of graphene membranes
  – Adhesion energy has been explained and well-predicted
Thank you very much for your attention

Questions are now welcome

• Submitted for publication at Composite Structures
  – Partition of mixed-mode fractures in 2D elastic orthotropic laminated beams under general loading (2016).