Brittle interface crack between two dissimilar elastic layers

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Brittle Interfacial Cracking Between Two Dissimilar Elastic Layers

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18th International Conference of Composite Structures (ICCS18)
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Cracks tend to propagate along interfaces in laminated materials because they represent a plane of weakness. They do not kink in order to propagate in pure mode I opening, as they would tend to in an isotropic material. Interfacial cracks therefore propagate in a mixed-mode with a combination of mode I opening, mode II shearing, and mode III tearing. Fracture toughness is not an intrinsic material property but depends on the fracture mode partition, i.e. it is load dependent. Therefore, to predict fracture toughness, it is essential to know the partition of a mixed fracture mode. It has been a particularly complex problem with many confusing aspects.
There are a variety of partition theories:
- Based on classical beam/plate theory
- Based on shear deformable beam/plate theory
- Based on 2D elasticity theory

Bimaterial interfaces present two extra big challenges
Introduction – Interfacial cracks

- Similar materials
  - Energy release rates (ERR) $G_I$ and $G_{II}$ linked to corresponding stress intensity factor (SIF) $K_I$ or $K_{II}$
  - Both total ERR $G$ and its partitions $G_I$ and $G_{II}$ are independent of crack extension size, $\delta a$

- Bimaterials
  - Energy release rates (ERR) $G_I$ and $G_{II}$ are coupled with both stress intensity factors (SIFs) $K_I$ and $K_{II}$ together – What is the mechanical meaning of this coupling?
  - Total ERR $G$ remains constant but individual components $G_I$ and $G_{II}$ vary with crack extension size, $\delta a$ – How to determine them for a given crack extension size $\delta a$?
Introduction – Motivation and Aims

- **Aims:** To overcome the two challenges and develop an analytical mixed-mode partition theory for brittle interfacial cracking between two dissimilar materials based on 2D elasticity.

- **Motivation:** To provide a complete analytical tool kit for studying interfacial cracking.
Bimaterial double cantilever beam (DCB)

Fundamental case for in-depth study – 1D fracture

Bending moments $M_1$ and $M_2$

Axial forces $N_1$ and $N_2$

“B” denotes crack tip quantity

$$\eta = E_2/E_1 \quad \gamma = h_2/h_1$$
Suo and Hutchinson’s (1990) complex stress intensity factor (SIF):

\[ \sigma_n + i\tau_s = \frac{(K_I + iK_{II})}{\sqrt{2\pi r}} r^{i\epsilon} \]

Individual real form

\[ \sigma_n = \frac{1}{\sqrt{2\pi r}} \{K_I \cos[\epsilon \ln(r)] - K_{II} \sin[\epsilon \ln(r)]\} \]

\[ \tau_s = \frac{1}{\sqrt{2\pi r}} \{K_I \sin[\epsilon \ln(r)] + K_{II} \cos[\epsilon \ln(r)]\} \]

Relative interfacial opening and shearing displacement behind the crack tip

\[ D_n = D \cos(\xi) \sqrt{2\pi r} \{K_I \cos[\epsilon \ln(r) - \xi] - K_{II} \sin[\epsilon \ln(r) - \xi]\} \]

\[ D_s = D \cos(\xi) \sqrt{2\pi r} \{K_I \sin[\epsilon \ln(r) - \xi] + K_{II} \cos[\epsilon \ln(r) - \xi]\} \]

Bimaterial constant

\[ \epsilon = \frac{1}{2\pi} \ln \left[ \left( \frac{k_1}{\mu_1} + \frac{1}{\mu_2} \right) \left( \frac{k_2}{\mu_2} + \frac{1}{\mu_1} \right)^{-1} \right] \]

\[ \cos(\xi) = \frac{1}{(1 + 4\epsilon^2)^{1/2}} \]
### Analytical Development – Total energy release rate (ERR)

- **Total ERR in terms of stress intensity factors (SIFs):**

  \[
  G = G_I + G_{II} = \frac{D\pi}{4 \cosh(\pi\varepsilon)}(K_I^2 + K_{II}^2)
  \]

- **Total ERR in terms of crack tip loads:**

  \[
  G = \frac{1}{2b^2}\left[\frac{M_{1B}^2}{D_1^*} + \frac{M_{2B}^2}{D_2^*} - \frac{M_B^2}{D^*} + \frac{N_{1B}^2}{A_1^*} + \frac{N_{2B}^2}{A_2^*} - \frac{N_B^2}{A^*} - \frac{2B_1M_{1B}N_{1B}}{B_1^*} - \frac{2B_2M_{2B}N_{2B}}{B_2^*} + \frac{2BM_BN_B}{B^*}\right]
  \]

  \[
  = \{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B}\}[C]\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B}\}^T
  \]

- **ERR is in quadratic form and positive definite.**
First set of orthogonal SIF-based pure fracture modes

- Pure mode I $\theta_K$: zero effective relative shearing displacement just behind the crack tip.
- Pure mode II $\beta_K$: zero effective crack tip opening force

\[(\theta_K, \beta_K)\]

Second set of SIF-based pure fracture modes

- Pure mode I $\theta'_K$: zero effective crack tip shearing force
- Pure mode II $\beta'_K$: zero effective relative opening displacement just behind the crack tip

\[(\theta'_K, \beta'_K)\]
Analytical development – Partition G by SIF-based pure mode

- Partitions of ERR using SIF-based orthogonal pure modes

\[ G_I = \frac{D\pi}{4 \cosh(\pi\varepsilon)(1 + \beta_K^{-1}\beta_K'^{-1})}(K_I - \beta_K^{-1}K_{II})(K_I - \beta_K'^{-1}K_{II}) \]

\[ For \ G_I = 0 \Rightarrow K_{II} = \beta_K K_I \ or \ K_{II} = \beta_K' K_I \]

\[ G_{II} = \frac{D\pi}{4 \cosh(\pi\varepsilon)(1 + \theta_K^{-1}\theta_K'^{-1})}(K_I - \theta_K^{-1}K_{II})(K_I - \theta_K'^{-1}K_{II}) \]

\[ For \ G_{II} = 0 \Rightarrow K_{II} = \theta_K K_I \ or \ K_{II} = \theta_K' K_I \]

- VCCT gives 2 sets of SIF pure modes \((\theta_K, \beta_K)\) and \((\theta_K', \beta_K')\)

- However, produces 4 possible pairs of SIFs \(K_I\) and \(K_{II}\)
Analytical development – Approx. pair of SIF pure modes

- Approximate orthogonal pure modes based on $D_s(\delta a) = 0$ and $D_n(\delta a) = 0$
  \[ K_{II} = \tilde{\theta}_K K_I \quad K_{II} = \tilde{\beta}_K K_I \]
  \[ \tilde{\theta}_K = \tilde{\theta}'_K \text{ and } \tilde{\beta}_K = \tilde{\beta}'_K \]

- From which the ERR partitions become
  \[ G_I = \frac{D\pi}{4 \cosh(\pi \varepsilon)(1 + \tilde{\beta}_K^{-2})} (K_I - \tilde{\beta}_K^{-1} K_{II})^2 \]
  \[ G_{II} = \frac{D\pi}{4 \cosh(\pi \varepsilon)(1 + \tilde{\theta}_K^{-2})} (K_I - \tilde{\theta}_K^{-1} K_{II})^2 \]

- Approximate SIFs, $K_I$ and $K_{II}$ used to select only admissible pair
Variation of $\theta_K$, $\theta'_K$ and $\bar{\theta}_K$ w.r.t. crack extension size $\delta a$
Partitions of ERR using load-based orthogonal pure modes

\[
G_I = c_I \left( M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3} \right) \left( M_{1B} - \frac{M_{2B}}{\beta'_1} - \frac{N_{1B}}{\beta'_2} - \frac{N_{2B}}{\beta'_3} \right)
\]

\[
G_{II} = c_{II} \left( M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3} \right) \left( M_{1B} - \frac{M_{2B}}{\theta'_1} - \frac{N_{1B}}{\theta'_2} - \frac{N_{2B}}{\theta'_3} \right)
\]

2 sets of load type pure modes: \((\theta_i, \beta_i)\) and \((\theta'_i, \beta'_i)\) with \(i = 1, 2, 3\)

\[
\beta_i = \text{orthogonal}(\theta_i)
\]

\[
\beta'_i = \text{orthogonal}(\theta'_i)
\]

Pure modes are \(\delta a\) dependent, therefore consider bending moments.
Analytical Development – Partition G by load-based pure mode

\[\log_{10} \left| \frac{M_2}{M_1} \right|\]

\[\log_{10} (\delta a)\]

\[\delta a = 0.05\]

- \(\theta_1, \varepsilon = 0.0786 \ (\gamma = 1, \eta = 10, \nu = 0.29)\)
- \(\theta_1, \varepsilon = 0.0786 \ (\gamma = 10, \eta = 10, \nu = 0.29)\)
- \(\theta_1, \varepsilon = 0 \ (\gamma = 10, \eta = 1, \nu = 0.29)\)
- \(\theta_1, \varepsilon = -0.0786 \ (\gamma = 10, \eta = 0.1, \nu = 0.29)\)
- \(\theta_1, \varepsilon = -0.0786 \ (\gamma = 1, \eta = 0.1, \nu = 0.29)\)

- \(\theta'_1, \varepsilon = 0.0786\)
- \(\theta'_1, \varepsilon = 0.0786\)
- \(\theta'_1, \varepsilon = 0\)
- \(\theta'_1, \varepsilon = -0.0786\)
- \(\theta'_1, \varepsilon = -0.0786\)
Analytical Development – Coincident pure modes

\[ G_I = c_I \left( M_{1B} - \frac{M_{2B}}{\beta_1} \right) \left( M_{1B} - \frac{M_{2B}}{\beta'_1} \right) \]

\[ G_{II} = c_{II} \left( M_{1B} - \frac{M_{2B}}{\theta_1} \right) \left( M_{1B} - \frac{M_{2B}}{\theta'_1} \right) \]

- In second region where \( \delta a = 0.05 \)
  \[ \theta_i = \theta'_i \quad \text{and} \quad \beta_i = \beta'_i \quad \text{(with i=1, 2, 3)} \]
- Therefore
  \[ G_I = c_I \left( M_{1B} - \frac{M_{2B}}{\beta_1} \right)^2 \]
  \[ G_{II} = c_{II} \left( M_{1B} - \frac{M_{2B}}{\theta_1} \right)^2 \]
- By considering \( \delta a = 0.05 \), can find \( \theta_1 \) (pure mode I) by using 1 loading condition
- Obtain other pure modes \( (\beta_1, \theta_2, \beta_2, \theta_3, \beta_3) \) through orthogonality condition
- Now \( G_I \) and \( G_{II} \) are known for all loading conditions
Consider bending moments only

One loading condition:
\[ \frac{M_{2B}}{M_{1B}} = 0 \text{ with } \delta a = 0.05 \]

Correct \( \theta_1 \) can then be used for other loading conditions

Other pure modes obtained using orthogonality condition between pure modes
- $M_{2B}/M_{1B} = 0$
- $\frac{1}{10} \leq \gamma \leq 10$
- $\frac{1}{100} \leq \eta \leq 100$
- $\nu_1 = \nu_2 = 0.29$
- $\delta a = 0.05$
- Plane strain

$$\eta = E_2/E_1 \quad \gamma = h_2/h_1$$
Analytical Development – Obtain pure mode I $\theta_1$

\[ G_I = c_I \left( M_{1B} - \frac{M_{2B}}{\beta_1} \right)^2 \]

\[ G_{II} = c_{II} \left( M_{1B} - \frac{M_{2B}}{\theta_1} \right)^2 \]

\[ c_I = G_{\theta_1} \left( 1 - \frac{\theta_1}{\beta_1} \right)^{-2} \]

\[ c_{II} = G_{\beta_1} \left( 1 - \frac{\beta_1}{\theta_1} \right)^{-2} \]

\[ M_{2B} = \theta_1 M_{1B} \]

\[ \{1 \quad \theta_1\}[C]\{\frac{1}{\beta_1}\} = 0 \]
Numerical verification – Pure mode \((\theta_1, \beta_1)\)

- \(M_{2B}/M_{1B} = -1\)
- \(N_{1B} = N_{2B} = 0\)
- \(\nu_1 = \nu_2 = 0.29\)
- \(\delta a = 0.05\)
- Plane strain

\[ G/G_0 = \frac{M_{2B}}{M_{1B}} = -1 \]
\[ N_{1B} = N_{2B} = 0 \]
\[ \nu_1 = \nu_2 = 0.29 \]
\[ \delta a = 0.05 \]

FEM = Markers
Analytical = Line

\[ \log_{10}(1/\eta) \]
Numerical verification – Pure mode ($\theta_2, \beta_2$)

- $N_{1B}/M_{1B} = 10$
- $M_{2B} = N_{2B} = 0$
- $\nu_1 = \nu_2 = 0.29$
- $\delta a = 0.05$
- Plane strain
Numerical verification – Pure mode ($\theta_3, \beta_3$)

- $N_{2B}/M_{1B} = 10$
- $M_{2B} = N_{1B} = 0$
- $\nu_1 = \nu_2 = 0.29$
- $\delta a = 0.05$
- Plane strain

FEM = Markers
Analytical = Line
Numerical verification – SIF calculation

- $N_{1B}/M_{1B} = 10$
- $M_{2B} = N_{2B} = 0$
- $\nu_1 = \nu_2 = 0.29$
- $\delta a = 0.05$
- Plane strain
Numerical verification – SIF calculation

- $N_{1B}/M_{1B} = 10$
- $M_{2B} = N_{2B} = 0$
- $\nu_1 = \nu_2 = 0.29$
- $\delta a = 0.05$
- Plane strain

![Graph showing numerical verification - SIF calculation]
Analytical means of obtaining SIFs $K_I$ and $K_{II}$ for bimaterial interfacial crack with $\delta a = 0.05$

- SIFs are independent on crack extension size $\delta a$

- Use SIF to obtain ERR $G_I$ and $G_{II}$ for any crack extension size $\delta a$
Numerical verification – $G_I$ and $G_{II}$

- $-20 \leq M_{2B}/M_{1B} \leq 20$
- $\frac{1}{10} \leq \gamma \leq 10$
- $\frac{1}{100} \leq \eta \leq 100$
- $\nu_1 = \nu_2 = 0.29$

$\eta = E_2/E_1 \quad \gamma = h_2/h_1$
Numerical verification – $G_I$ and $G_{II}$

- $-20 \leq N_{1B}/M_{1B} \leq 20$
- $-20 \leq N_{2B}/M_{1B} \leq 20$
- $\frac{1}{10} \leq \gamma \leq 10$
- $\frac{1}{100} \leq \eta \leq 100$
- $\nu_1 = \nu_2 = 0.29$

$\eta = E_2/E_1 \quad \gamma = h_2/h_1$
Conclusion

- Material mismatch causes the existence of 2 sets of SIF-based pure modes \((\theta_K, \beta_K)\) and \((\theta'_K, \beta'_K)\) which are \(\delta a\) dependent and can be used to partition ERR

- Coincident approximate pure modes \((\tilde{\theta}_K, \tilde{\beta}_K)\) and \((\tilde{\theta}'_K, \tilde{\beta}'_K)\) enable the calculation of approximate SIFs which accurately aid in the selection of the mechanically admissible pair of SIFs

- Brittle interface causes 2 distinct sets of orthogonal pure modes \((\theta_i, \beta_i)\) and \((\theta'_i, \beta'_i)\) which coincide when \(\delta a = 0.05\) and separate again as \(\delta a\) decreases.

- When \(\delta a = 0.05\), \(\gamma = 1\) and \(\nu = 0.29\), the pure modes produced by 2D FEM are approximately equal to that from Timoshenko beam partition.

- Shifting technique and orthogonality produce correct pure modes for axial forces and bending moments, enabling accurate calculation of \(G_I/G\) when \(\delta a = 0.05\) for any combination of axial forces and bending moments.

- Above method allows \(G_I\) and \(G_{II}\) to be accurately calculated for any crack extension size \(\delta a\).
Thank you very much for your attention

Questions are now welcome

- Submitted for publication at Composite Structures
  - Brittle interfacial cracking between two elastic layers:
    - Part 1 – Analytical development, Composite Structures 2015 (submitted)
    - Part 2 – Numerical verification, Composite Structures 2015 (submitted)
Numerical verification – SIF calculation

- \( M_{2B}/M_{1B} = 0 \)
- \( N_{1B} = N_{2B} = 0 \)
- \( \nu_1 = \nu_2 = 0.29 \)
- \( \delta a = 0.05 \)
- Plane strain

![Graph showing FEM and Analytical results for SIF calculation](image)
Numerical verification – SIF calculation

- \( M_{2B} / M_{1B} = 0 \)
- \( N_{1B} = N_{2B} = 0 \)
- \( \nu_1 = \nu_2 = 0.29 \)
- \( \delta a = 0.05 \)
- Plane strain
Numerical verification – SIF calculation

- \( N_{2B}/M_{1B} = 10 \)
- \( M_{2B} = N_{1B} = 0 \)
- \( \nu_1 = \nu_2 = 0.29 \)
- \( \delta a = 0.05 \)
- Plane strain

Graph showing FEM markers and Analytical line with log10(1/\( \eta \)) on the x-axis and values of \( \log_{10}(1/\gamma) \) from -0.8 to 1.0.
Numerical verification – SIF calculation

- \( \frac{N_{2B}}{M_{1B}} = 10 \)
- \( M_{2B} = N_{1B} = 0 \)
- \( \nu_1 = \nu_2 = 0.29 \)
- \( \delta \alpha = 0.05 \)
- Plane strain

\[ K_{II} \]

FEM = Markers
Analytical = Line

\[ \log_{10} (1/\gamma) \]

- \( \log_{10} (1/\gamma) = -0.8 \)
- \( \log_{10} (1/\gamma) = -0.4 \)
- \( \log_{10} (1/\gamma) = -0.2 \)
- \( \log_{10} (1/\gamma) = 0.2 \)
- \( \log_{10} (1/\gamma) = 0.4 \)
- \( \log_{10} (1/\gamma) = 0.6 \)
- \( \log_{10} (1/\gamma) = 0.8 \)
- \( \log_{10} (1/\gamma) = 1 \)
Orthogonality of pure modes

\[ \beta_1 = \text{orthogonal}(\theta_1) \text{ or } 0 = \{1 \quad \theta_1 \quad 0 \quad 0\}^T [C] \{1 \quad \beta_1 \quad 0 \quad 0\}^T \]

\[ \theta_2 = \text{orthogonal}(\beta_1) \text{ or } 0 = \{1 \quad 0 \quad \theta_2 \quad 0\}^T [C] \{1 \quad \beta_1 \quad 0 \quad 0\}^T \]

\[ \theta_3 = \text{orthogonal}(\beta_1) \text{ or } 0 = \{1 \quad 0 \quad 0 \quad \theta_3\}^T [C] \{1 \quad \beta_1 \quad 0 \quad 0\}^T \]

\[ \beta_2 = \text{orthogonal}(\theta_1) \text{ or } 0 = \{1 \quad 0 \quad \beta_2 \quad 0\}^T [C] \{1 \quad \theta_1 \quad 0 \quad 0\}^T \]

\[ \beta_3 = \text{orthogonal}(\theta_1) \text{ or } 0 = \{1 \quad 0 \quad 0 \quad \beta_3\}^T [C] \{1 \quad \theta_1 \quad 0 \quad 0\}^T \]

\[ G_I = c_I \left( M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3} \right)^2 \]

\[ G_{II} = c_{II} \left( M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3} \right)^2 \]
Numerical verification – ν

- $N_{1B} = N_{2B} = 0$
- $\delta a = 0.05$
- Plane strain

$\eta = E_2/E_1 \quad \gamma = h_2/h_1$