Brittle interface crack between two dissimilar elastic layers

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Brittle Interfacial Cracking Between Two Dissimilar Elastic Layers

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18th International Conference of Composite Structures (ICCS18)
Contents

- Introduction
  - Interfacial cracks
  - Partition theories
  - Challenges with bimaterial interfacial cracks
- Analytical development
  - Partition ERR $G$ in terms of SIF-based pure modes
  - Obtain correct and accurate SIFs $K_I$ and $K_{II}$
  - Partition ERR $G$ in terms of load-based pure modes
  - Obtain ERR $G$ using shifting method
  - Use SIFs to calculate $G_I$ and $G_{II}$ for any $\delta a$
- Numerical verification
- Conclusions
Cracks tend to propagate along interfaces in laminated materials because they represent a plane of weakness.

They do not kink in order to propagate in pure mode I opening, as they would tend to in an isotropic material.

Interfacial cracks therefore propagate in a mixed-mode with a combination of mode I opening, mode II shearing, and mode III tearing.

Fracture toughness is not an intrinsic material property but depends on the fracture mode partition, i.e. it is load dependent.

Therefore, to predict fracture toughness, it is essential to know the partition of a mixed fracture mode.

It has been a particularly complex problem with many confusing aspects.
Introduction – Interfacial cracks

- There are a variety of partition theories:
  - Based on classical beam/plate theory
  - Based on shear deformable beam/plate theory
  - Based on 2D elasticity theory
- Bimaterial interfaces present two extra big challenges
Introduction – Interfacial cracks

- Similar materials
  - Energy release rates (ERR) $G_I$ and $G_{II}$ linked to corresponding stress intensity factor (SIF) $K_I$ or $K_{II}$
  - Both total ERR $G$ and its partitions $G_I$ and $G_{II}$ are independent of crack extension size, $\delta a$

- Bimaterials
  - Energy release rates (ERR) $G_I$ and $G_{II}$ are coupled with both stress intensity factors (SIFs) $K_I$ and $K_{II}$ together – *What is the mechanical meaning of this coupling?*
  - Total ERR $G$ remains constant but individual components $G_I$ and $G_{II}$ vary with crack extension size, $\delta a$ – *How to determine them for a given crack extension size $\delta a$?*
Aims: To overcome the two challenges and develop an analytical mixed-mode partition theory for brittle interfacial cracking between two dissimilar materials based on 2D elasticity.

Motivation: To provide a complete analytical tool kit for studying interfacial cracking.
Analytical Development – BDCB

- Bimaterial double cantilever beam (DCB)
- Fundamental case for in-depth study – 1D fracture
- Bending moments $M_1$ and $M_2$
- Axial forces $N_1$ and $N_2$
- “B” denotes crack tip quantity

\[ \eta = \frac{E_2}{E_1} \quad \gamma = \frac{h_2}{h_1} \]
Analytical Development – Interfacial stresses, displacements

- Suo and Hutchinson’s (1990) complex stress intensity factor (SIF):

\[
\sigma_n + i\tau_s = \frac{(K_I + iK_{II})}{\sqrt{2\pi r}} r^{i\varepsilon}
\]

- Individual real form

\[
\sigma_n = \frac{1}{\sqrt{2\pi r}} \{K_I \cos[\varepsilon \ln(r)] - K_{II} \sin[\varepsilon \ln(r)]\}
\]

\[
\tau_s = \frac{1}{\sqrt{2\pi r}} \{K_I \sin[\varepsilon \ln(r)] + K_{II} \cos[\varepsilon \ln(r)]\}
\]

- Bimaterial constant

\[
\varepsilon = \frac{1}{2\pi} \ln \left[ \left( \frac{k_1}{\mu_1} + \frac{1}{\mu_2} \right) \left( \frac{k_2}{\mu_2} + \frac{1}{\mu_1} \right)^{-1} \right]
\]

- Relative interfacial opening and shearing displacement behind the crack tip

\[
D_n = D \cos(\xi) \sqrt{2\pi r} \{K_I \cos[\varepsilon \ln(r) - \xi] - K_{II} \sin[\varepsilon \ln(r) - \xi]\}
\]

\[
D_s = D \cos(\xi) \sqrt{2\pi r} \{K_I \sin[\varepsilon \ln(r) - \xi] + K_{II} \cos[\varepsilon \ln(r) - \xi]\}
\]

\[
\cos(\xi) = \frac{1}{(1 + 4\varepsilon^2)^{1/2}}
\]
Total ERR in terms of stress intensity factors (SIFs):

\[ G = G_I + G_{II} = \frac{D\pi}{4\cosh(\pi\varepsilon)}(K_I^2 + K_{II}^2) \]

Total ERR in terms of crack tip loads:

\[ G = \frac{1}{2b^2} \left[ \frac{M_{1B}^2}{D_1^*} + \frac{M_{2B}^2}{D_2^*} - \frac{M_B^2}{D^*} + \frac{N_{1B}^2}{A_1^*} + \frac{N_{2B}^2}{A_2^*} - \frac{N_B^2}{A^*} \right] \frac{2B_1 M_{1B} N_{1B}}{B_1^*} - \frac{2B_2 M_{2B} N_{2B}}{B_2^*} + \frac{2B M_B N_B}{B^*} \]

\[ = \{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B}\}[C]\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B}\}^T \]

ERR is in quadratic form and positive definite.
First set of orthogonal SIF-based pure fracture modes
- Pure mode I $\theta_K$: zero effective relative shearing displacement just behind the crack tip.
- Pure mode II $\beta_K$: zero effective crack tip opening force

$(\theta_K, \beta_K)$

Second set of SIF-based pure fracture modes
- Pure mode I $\theta'_K$: zero effective crack tip shearing force
- Pure mode II $\beta'_K$: zero effective relative opening displacement just behind the crack tip

$(\theta'_K, \beta'_K)$
Analytical development – Partition G by SIF-based pure mode

- Partitions of ERR using SIF-based orthogonal pure modes

\[ G_I = \frac{D\pi}{4 \cosh(\pi \varepsilon)(1 + \beta_K^{-1}\beta_K'^{-1})}(K_I - \beta_K^{-1}K_{II})(K_I - \beta_K'^{-1}K_{II}) \]

*For* \( G_I = 0 \) \( \Rightarrow K_{II} = \beta_K K_I \) or \( K_{II} = \beta_K' K_I \)

\[ G_{II} = \frac{D\pi}{4 \cosh(\pi \varepsilon)(1 + \theta_K^{-1}\theta_K'^{-1})}(K_I - \theta_K^{-1}K_{II})(K_I - \theta_K'^{-1}K_{II}) \]

*For* \( G_{II} = 0 \) \( \Rightarrow K_{II} = \theta_K K_I \) or \( K_{II} = \theta_K' K_I \)

- VCCT gives 2 sets of SIF pure modes \((\theta_K, \beta_K)\) and \((\theta_K', \beta_K')\)

- However, produces 4 possible pairs of SIFs \( K_I \) and \( K_{II} \)
Analytical development – Approx. pair of SIF pure modes

- Approximate orthogonal pure modes based on $D_s(\delta a) = 0$ and $D_n(\delta a) = 0$

  \[ K_{II} = \tilde{\theta}_K K_I \quad K_{II} = \tilde{\beta}_K K_I \]

  \[ \tilde{\theta}_K = \tilde{\theta}'_K \text{ and } \tilde{\beta}_K = \tilde{\beta}'_K \]

- From which the ERR partitions become

  \[ G_I = \frac{D\pi}{4 \cosh(\pi \varepsilon)(1 + \tilde{\beta}_K^{-2})} \left( K_I - \tilde{\beta}_K^{-1} K_{II} \right)^2 \]

  \[ G_{II} = \frac{D\pi}{4 \cosh(\pi \varepsilon)(1 + \tilde{\theta}_K^{-2})} \left( K_I - \tilde{\theta}_K^{-1} K_{II} \right)^2 \]

- Approximate SIFs, $K_I$ and $K_{II}$ used to select only admissible pair
Variation of $\theta_K$, $\theta'_K$ and $\tilde{\theta}_K$ w.r.t. crack extension size $\delta a$
Analytical Development – Partition G by load-based pure mode

- Partitions of ERR using load-based orthogonal pure modes

\[
G_I = c_I \left( M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3} \right) \left( M_{1B} - \frac{M_{2B}}{\beta'_1} - \frac{N_{1B}}{\beta'_2} - \frac{N_{2B}}{\beta'_3} \right)
\]

\[
G_{II} = c_{II} \left( M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3} \right) \left( M_{1B} - \frac{M_{2B}}{\theta'_1} - \frac{N_{1B}}{\theta'_2} - \frac{N_{2B}}{\theta'_3} \right)
\]

- 2 sets of load type pure modes: \((\theta_i, \beta_i)\) and \((\theta'_i, \beta'_i)\) with \(i = 1, 2, 3\)

\[
\beta_i = \text{orthogonal}(\theta_i)
\]

\[
\beta'_i = \text{orthogonal}(\theta'_i)
\]

- Pure modes are \(\delta a\) dependent, therefore consider bending moments
Analytical Development – Partition G by load-based pure mode

\[ \log_{10} \left| \frac{M_2}{M_1} \right| \]

\[ \log_{10} (\delta a) \]

\[ \delta a = 0.05 \]

\[ \theta_1, \varepsilon = 0.0786 (\gamma = 1, \eta = 10, \nu = 0.29) \]
\[ \theta_1, \varepsilon = 0.0786 (\gamma = 10, \eta = 10, \nu = 0.29) \]
\[ \theta_1, \varepsilon = 0 (\gamma = 10, \eta = 1, \nu = 0.29) \]
\[ \theta_1, \varepsilon = -0.0786 (\gamma = 10, \eta = 0.1, \nu = 0.29) \]
\[ \theta_1, \varepsilon = -0.0786 (\gamma = 1, \eta = 0.1, \nu = 0.29) \]
In second region where $\delta \alpha = 0.05$

$\theta_i = \theta_i'$ and $\beta_i = \beta_i'$ (with $i=1, 2, 3$

Therefore

$G_I = c_I \left( M_{1B} - \frac{M_{2B}}{\beta_1} \right)^2$

$G_{II} = c_{II} \left( M_{1B} - \frac{M_{2B}}{\theta_1} \right)^2$

By considering $\delta \alpha = 0.05$, can find $\theta_1$ (pure mode I) by using 1 loading condition

Obtain other pure modes ($\beta_1, \theta_2, \beta_2, \theta_3, \beta_3$) through orthogonality condition

Now $G_I$ and $G_{II}$ are known for all loading conditions
Consider bending moments only

One loading condition:
- \( \frac{M_{2B}}{M_{1B}} = 0 \) with \( \delta a = 0.05 \)

Correct \( \theta_1 \) can then be used for other loading conditions

Other pure modes obtained using orthogonality condition between pure modes
Analytical Development – Shifting technique

- \( \frac{M_{2B}}{M_{1B}} = 0 \)
- \( \frac{1}{10} \leq \gamma \leq 10 \)
- \( \frac{1}{100} \leq \eta \leq 100 \)
- \( \nu_1 = \nu_2 = 0.29 \)
- \( \delta a = 0.05 \)
- Plane strain

\[ \eta = \frac{E_2}{E_1} \quad \gamma = \frac{h_2}{h_1} \]
Analytical Development – Obtain pure mode I $\theta_1$

\[ G_I = c_I \left( M_{1B} - \frac{M_{2B}}{\beta_1} \right)^2 \]

\[ G_{II} = c_{II} \left( M_{1B} - \frac{M_{2B}}{\theta_1} \right)^2 \]

\[ c_I = G_{\theta_1} \left( 1 - \frac{\theta_1}{\beta_1} \right)^{-2} \]

\[ c_{II} = G_{\beta_1} \left( 1 - \frac{\beta_1}{\theta_1} \right)^{-2} \]

\[ M_{2B} = \theta_1 M_{1B} \]

\[ \{1 \quad \theta_1\}[C]\left\{\begin{array}{c} 1 \\ \beta_1 \end{array}\right\} = 0 \]
Numerical verification – Pure mode \((\theta_1, \beta_1)\)

- \(M_{2B}/M_{1B} = -1\)
- \(N_{1B} = N_{2B} = 0\)
- \(\nu_1 = \nu_2 = 0.29\)
- \(\delta a = 0.05\)
- Plane strain
Numerical verification – Pure mode \((\theta_2, \beta_2)\)

- \(N_{1B}/M_{1B} = 10\)
- \(M_{2B} = N_{2B} = 0\)
- \(\nu_1 = \nu_2 = 0.29\)
- \(\delta a = 0.05\)
- Plane strain
Numerical verification – Pure mode ($\theta_3, \beta_3$)

- $N_{2B}/M_{1B} = 10$
- $M_{2B} = N_{1B} = 0$
- $\nu_1 = \nu_2 = 0.29$
- $\delta a = 0.05$
- Plane strain

FEM = Markers
Analytical = Line

\[ \frac{N_{2B}}{M_{1B}} = 10 \]
\[ M_{2B} = N_{1B} = 0 \]
\[ \nu_1 = \nu_2 = 0.29 \]
\[ \delta a = 0.05 \]
\[ \text{Plane strain} \]
Numerical verification – SIF calculation

- \( \frac{N_{1B}}{M_{1B}} = 10 \)
- \( M_{2B} = N_{2B} = 0 \)
- \( \nu_1 = \nu_2 = 0.29 \)
- \( \delta a = 0.05 \)
- Plane strain
Numerical verification – SIF calculation

- $N_{1B}/M_{1B} = 10$
- $M_{2B} = N_{2B} = 0$
- $\nu_1 = \nu_2 = 0.29$
- $\delta a = 0.05$
- Plane strain
Analytical means of obtaining SIFs $K_I$ and $K_{II}$ for bimaterial interfacial crack with $\delta a = 0.05$

SIFs are independent on crack extension size $\delta a$

Use SIF to obtain ERR $G_I$ and $G_{II}$ for any crack extension size $\delta a$
Numerical verification – $G_I$ and $G_{II}$

- $-20 \leq \frac{M_{2B}}{M_{1B}} \leq 20$
- $\frac{1}{10} \leq \gamma \leq 10$
- $\frac{1}{100} \leq \eta \leq 100$
- $\nu_1 = \nu_2 = 0.29$

$\eta = \frac{E_2}{E_1}$ \hspace{1cm} $\gamma = \frac{h_2}{h_1}$
Numerical verification – $G_I$ and $G_{II}$

- $-20 \leq N_{1B}/M_{1B} \leq 20$
- $-20 \leq N_{2B}/M_{1B} \leq 20$
- $\frac{1}{10} \leq \gamma \leq 10$
- $\frac{1}{100} \leq \eta \leq 100$
- $\nu_1 = \nu_2 = 0.29$

$$\eta = E_2/E_1 \quad \gamma = h_2/h_1$$
Conclusion

- Material mismatch causes the existence of 2 sets of SIF-based pure modes $(\theta_K, \beta_K)$ and $(\theta_K', \beta_K')$ which are $\delta a$ dependent and can be used to partition ERR.

- Coincident approximate pure modes $(\tilde{\theta}_K, \tilde{\beta}_K)$ and $(\tilde{\theta}_K', \tilde{\beta}_K')$ enable the calculation of approximate SIFs which accurately aid in the selection of the mechanically admissible pair of SIFs.

- Brittle interface causes 2 distinct sets of orthogonal pure modes $(\theta_i, \beta_i)$ and $(\theta_i', \beta_i')$ which coincide when $\delta a = 0.05$ and separate again as $\delta a$ decreases.

- When $\delta a = 0.05$, $\gamma = 1$ and $\nu = 0.29$, the pure modes produced by 2D FEM are approximately equal to that from Timoshenko beam partition.

- Shifting technique and orthogonality produce correct pure modes for axial forces and bending moments, enabling accurate calculation of $G_I/G$ when $\delta a = 0.05$ for any combination of axial forces and bending moments.

- Above method allows $G_I$ and $G_{II}$ to be accurately calculated for any crack extension size $\delta a$. 
Questions are now welcome

- Submitted for publication at Composite Structures
  - Brittle interfacial cracking between two elastic layers:
    - Part 1 – Analytical development, Composite Structures 2015 (submitted)
    - Part 2 – Numerical verification, Composite Structures 2015 (submitted)

Thank you very much for your attention
Numerical verification – SIF calculation

- $M_{2B}/M_{1B} = 0$
- $N_{1B} = N_{2B} = 0$
- $v_1 = v_2 = 0.29$
- $\delta a = 0.05$
- Plane strain
Numerical verification – SIF calculation

- \( M_{2B} / M_{1B} = 0 \)
- \( N_{1B} = N_{2B} = 0 \)
- \( \nu_1 = \nu_2 = 0.29 \)
- \( \delta a = 0.05 \)
- Plane strain

Plane strain FEM = Markers
Analytical = Line

\[ K_I = \log_{10}(1/\eta) \]

Markers and lines represent numerical and analytical solutions, respectively.
Numerical verification – SIF calculation

- \( N_{2B}/M_{1B} = 10 \)
- \( M_{2B} = N_{1B} = 0 \)
- \( \nu_1 = \nu_2 = 0.29 \)
- \( \delta a = 0.05 \)
- Plane strain
Numerical verification – SIF calculation

- \( N_{2B} / M_{1B} = 10 \)
- \( M_{2B} = N_{1B} = 0 \)
- \( \nu_1 = \nu_2 = 0.29 \)
- \( \delta a = 0.05 \)
- Plane strain

![Graph showing FEM and Analytical results](image)
Orthogonality of pure modes

\[ \beta_1 = \text{orthogonal}(\theta_1) \text{ or } 0 = \begin{bmatrix} 1 & \theta_1 & 0 & 0 \end{bmatrix} C \begin{bmatrix} 1 & \beta_1 & 0 & 0 \end{bmatrix}^T \]

\[ \theta_2 = \text{orthogonal}(\beta_1) \text{ or } 0 = \begin{bmatrix} 1 & 0 & \theta_2 & 0 \end{bmatrix} C \begin{bmatrix} 1 & \beta_1 & 0 & 0 \end{bmatrix}^T \]

\[ \theta_3 = \text{orthogonal}(\beta_1) \text{ or } 0 = \begin{bmatrix} 1 & 0 & 0 & \theta_3 \end{bmatrix} C \begin{bmatrix} 1 & \beta_1 & 0 & 0 \end{bmatrix}^T \]

\[ \beta_2 = \text{orthogonal}(\theta_1) \text{ or } 0 = \begin{bmatrix} 1 & 0 & \beta_2 & 0 \end{bmatrix} C \begin{bmatrix} 1 & \theta_1 & 0 & 0 \end{bmatrix}^T \]

\[ \beta_3 = \text{orthogonal}(\theta_1) \text{ or } 0 = \begin{bmatrix} 1 & 0 & 0 & \beta_3 \end{bmatrix} C \begin{bmatrix} 1 & \theta_1 & 0 & 0 \end{bmatrix}^T \]

\[ G_I = c_I \left( M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3} \right)^2 \]

\[ G_{II} = c_{II} \left( M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3} \right)^2 \]
Numerical verification –

- $N_{1B} = N_{2B} = 0$
- $\delta a = 0.05$
- Plane strain

\[ \eta = \frac{E_2}{E_1} \quad \gamma = \frac{h_2}{h_1} \]