User collusion avoidance scheme for privacy-preserving decentralized key-policy attribute-based encryption

This item was submitted to Loughborough University's Institutional Repository by the/an author.

Citation: RAHULAMATHAVAN, Y. ... et al, 2015. User collusion avoidance scheme for privacy-preserving decentralized key-policy attribute-based encryption. IEEE Transactions on Computers, 65 (9), pp. 2939-2946.

Additional Information:

- © 2015 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

Metadata Record: [https://dspace.lboro.ac.uk/2134/23069](https://dspace.lboro.ac.uk/2134/23069)

Version: Accepted for publication

Publisher: © IEEE

Please cite the published version.
Abstract—Decentralized attribute-based encryption (ABE) is a variant of multi-authority based ABE whereby any attribute authority (AA) can independently join and leave the system without collaborating with the existing AAs. In this paper, we propose a user collusion avoidance scheme which preserves the user’s privacy when they interact with multiple authorities to obtain decryption credentials. The proposed scheme mitigates the well-known user collusion security vulnerability found in previous schemes. We show that our scheme relies on the standard complexity assumption (decisional bilinear Diffie-Hellman assumption). This is contrast to previous schemes which rely on non-standard assumption (q-decisional Diffie-Hellman inversion).

Index Terms—Attribute-based encryption, user collusion.

I. INTRODUCTION

Attribute-based encryption (ABE) supports both confidentiality and access control with a single encryption and has been identified as a potential technology for data sharing in distributed environment such as cloud [1], [2], [5]. Four parties namely data owner, users, attribute authority (AA) and storage server (i.e., cloud) involved in ABE. Users identities can be defined as a set of attributes (i.e., name, age, gender, location, passed driving test, purchased premium TV channels, etc). AA maintains encryption and decryption credentials for the users attributes. Data owner chooses a set of attributes based on the type of data and uses the corresponding encryption credentials for those attributes to encrypt the data. Then the encrypted data will be uploaded onto the cloud by the data owner. At the same time users obtain the decryption credentials for a set of attributes by proving to the AA that they are legitimate users for those attributes. If the users want to access the encrypted data, then they will first download the encrypted data from the cloud and will compare whether they satisfy the set of attributes defined by the data owner during the encryption.

The first ABE scheme was developed using single AA [2]. Later multiple AAs based ABE (MA-ABE) was proposed in [12], since single AA ABE scheme allowed too much information at single entity. In the MA-ABE scheme, there are multiple AAs responsible for disjoint sets of attributes. In conventional MA-ABE scheme, users interact with multiple AAs to obtain decryption credentials for their attributes. If there is no privacy protection for users then all AAs can share (collude) the particular user’s information (attributes) to reveal the user’s identity. Hence the focus of this paper is on achieving user privacy during the interaction with AAs (interesting readers can refer [1]–[5] for more details about ABE schemes). Let us provide concrete example applications for the proposed algorithm.

The first potential application is healthcare where ABE has been considered to provide fine-grain access control. The users of healthcare application could be staffs at hospital and patients i.e., they upload and retrieve sensitive data on daily basis [17]. Lets consider a scenario where diabetic patient wants to access his medical data which is in fact encrypted using ABE and one of the attributes is “Name of the Hospital". Unfortunately this hospital is known for diabetic clinic and obtaining a relevant key from AA (who may be a third party) responsible for "Name of the Hospital" attribute will jeopardize the patient’s privacy straightforward. However, the proposed technique allow patients to access relevant information related to their condition without revealing their attributes (i.e., type of disease, hospital, gender, age) to the AAs.

The second potential application is smart grid where smart grid’s control center can use ABE to broadcast a single encrypted message to a specific group of users (smart meters) [18]. Each user in the targeted group can individually decrypt the message based on their attributes. The attributes could be sensitive and reveal users day-to-day electricity usage e.g., attributes such as location, type of electrical equipment, subscription package can be passively leak user’s private information. Because the same user might have different homes and businesses at different locations however some attributes such name, date of birth will be same for all properties. If that is the case then, with some side information and information resides at AAs, the user’s spatial-temporal profile can be revealed by AAs. This vulnerability can be mitigated by the proposed scheme.

Within this context first known privacy-preserving (PP) decentralized ABE scheme was proposed in [6]. The scheme in [6] enables users to interact with AAs to obtain credentials for attributes via anonymous protocol. However, the solution in [6] is vulnerable to collusion attack [7] i.e., two users (colluding users) who have obtained credential for an attribute $o$ from an AA $O$ can generate credential for a third user for $o$ without involving $O$. This is a serious security issue since that third user in fact may not satisfy $o$ as part of his identity. Hence in this paper, we propose a scheme which mitigates the above user collusion attack using anonymous key issuing protocol. The main achievement of this work is on tying secret known for AA and secret known for user in a non-linear fashion. This non-linear coupling withstands the colluding users to generate new keys for unauthorized users via algebraic operations.

Related Works: There are two main types of ABE namely ciphertext-policy ABE (CP-ABE) and key-policy ABE (KP-ABE) [2], [5] and the focus of this paper is on KP-ABE. The KP-ABE scheme can be further divided into two: single AA ABE and MA-ABE. Chase et. al. presented a MA-ABE scheme [11] using trusted central AA for distributing all the keys [11]. MA-ABE scheme without central AA has been presented by Chase and Chow in [12] where, each pair of AAs securely exchange a shared secret among them during the set up process. In [11], users
must submit their global identities (GIDs) to each AA to obtain the decryption credentials. This will breach the user privacy since a set of corrupted AAs can pool together all the attributes belong to the particular GID.

In order to mitigate this privacy vulnerability, Chase and Chow proposed an anonymous key issuing protocol in [12], whereby a user can obtain the decryption keys from AAs without revealing her GID. Even though the scheme proposed by Chase and Chow eliminates the central AA, all the AAs must be online and collaborate with each other to set up the ABE system hence it is not fully decentralized. Similarly, various protocols have been proposed to decentralize the ABE scheme [9–12], however, each scheme has its own merits and demerits.

The first known fully decentralized (i.e., without central AA) MA-ABE scheme was proposed in [9]. In [9], any party can become an AA and there is no requirement for any global coordination other than the creation of an initial set of common reference parameters. Crucially [9] overcomes the collusion vulnerability without any coordination between AAs by proposing novel techniques to tie key components together and prevent collusion attacks between users with different global identifiers. However, the scheme in [9] does not preserve the user privacy i.e., attributes of users can be collected by AAs by tracing users’ GIDs. The scheme in [12] considers privacy but not fully decentralized. For the first time, Han et al. proposed a PP decentralized scheme for KP-ABE in [6]. In contrast to the existing decentralized ABE schemes, the scheme in [6] preserves the user privacy and relies only on decisional bilinear Diffie-Hellman (DBDH) standard complexity assumption.

In [6], the GID of the user is used to tie all the decryption keys together, where blind key generation protocol has been used to issue the decryption keys. Hence, corrupted AAs cannot pool the users’ attributes by tracing the GIDs’ of the users from the decryption keys. Unfortunately, the scheme in [6] cannot prevent user collusion, hence, two users can pool their decryption keys to generate decryption keys for an unauthorized user [7]. This is due to the weak bind between users’ GID and the decryption keys.

In this paper, contrasting to all the works in literature, we propose a PP decentralized KP-ABE scheme in order to mitigate the known user collusion security vulnerability [7]. We exploit the anonymous key issuing protocol in [12] to strengthen the bind between decryption keys and GID as well as to preserve the user privacy. In order to incorporate the anonymous key issuing protocol, we modify the PP decentralized KP-ABE scheme in [6]. We prove this by contradiction that the proposed scheme is secure i.e., we reduce the DBDH standard complexity assumption to show that an adversary who can break the proposed scheme can be exploited to break the DBDH assumption. We also proved that the anonymous key issuing protocol is free from leak, selective-failure and avoid user collusion.

II. DECENTRALIZED KEY-POLICY ATTRIBUTE-BASED ENCRYPTION

The following notations are used throughout this paper: we use $x \overset{R}{\rightarrow} X$ to denote that $x$ is randomly selected from $X$. Suppose $Z_p$ is a finite field with prime order $p$, by $Z_p[x]$, we denote the polynomial ring on $Z_p$. We used the following well-known techniques as building blocks for our scheme: Laguez Interpolation, bilinear groups, decisional bilinear Diffie-Hellman complexity assumption, commitment scheme, proof-of-knowledge and access tree (refer [2], [6] for more details).

In a decentralized scheme, any AA can join and/or leave the system at any time without rebooting the system. First of all, we will explain sub-algorithms and security game, original scheme in [6] followed by our PP decentralized KP-ABE. In Section III, we incorporate the anonymous key issuing protocol to strengthen the bind between user GID and decryption keys.

A. Sub-algorithms

Global Setup: In general decentralized schemes contain the following five sub-algorithms:

Global Setup: This algorithm takes a security parameter as input and output system parameters. These system parameters can be used by AAs who join the system.

AA Setup: Each AA uses the system parameters obtained from the global setup to generate public and private keys for the attributes it maintains.

Key Issuing: User and AA interact via anonymous key issuing protocol (see Section III) in order to determine attributes belong to the user. Then AA generates decryption credentials for those attributes and send them to the user.

Encryption: The encryption algorithm takes a set of attributes maintained by AA and the data as input. Then it outputs the ciphertext of the data.

Decryption: The decryption algorithm takes the decryption credentials received from AAs and the ciphertext as input. The decryption will be successful if and only if the user attributes satisfy the access structure.
B. Security Game

In order to avoid the security vulnerabilities, ABE schemes should be proven to be secure against the selective identity (ID) model [1]. In the selective ID model, the adversary should provide the identities of the AAs (challenge identities) he wishes to challenge the challenger with. Then challenger (i.e., the system) will generate necessary parameters corresponding to the challenge identities and send them to the adversary. Then the adversary is allowed to make secret queries about the challenge identities. If the adversary cannot decrypt the encrypted message at the end with non-negligible advantage then the proposed scheme is secure against the selective ID model. Formally, this is represented by the following game between the adversary and the challenger:

**Setup** - Adversary sends a list of attribute sets and AAs including corrupted AAs to the challenger. Now the challenger generates public and private keys corresponding to the attributes and AAs provided by the adversary. Challenger provides public and private keys corresponding to the corrupted AAs to the adversary while only public keys corresponding to the remaining AAs to the adversary.

**Secret Key Queries** - The adversary is allowed to make any number of secret key queries as long as he satisfies the requirement given earlier.

**Guess** - Now the adversary guesses which message was encrypted by the challenger. The adversary is said to be successful if he guesses the correct message with probability $\frac{1}{2} + \epsilon$ whereby $\epsilon$ is a non-negligible function.

C. Construction of our new algorithm

Let us consider $N$ number of AAs and denote them as $A_1, \ldots, A_N$. The attribute set managed by the AA $A_k$ is denoted as $A_k = \{a_{k,1}, a_{k,2}, \ldots, a_{k,n_k}\} \forall k$. Let us first describe the original algorithm in [6] in Fig. 1. Then in Fig. 2 we show how our algorithm differs from [6]. Let us explain the important steps involved in Fig. 1 and how our algorithm in Fig. 2 differs from [6].

Initially, for a given security parameter $\lambda$, global setup algorithm (GSetup) generates the bilinear groups $G_1$ and $G_2$ with prime order $p$ i.e., $\{G_1, G_2\} \leftarrow \varepsilon \widetilde{G}(1^\lambda)$. The AA setup algorithm (ASetup) is executed by each AA to randomly generate public keys (PK) and the corresponding secret keys (SK). The public-secret key pairs for $A_k$ is given as $\{(Y_k, Z_k), \{T_{k,1}, \ldots, T_{k,n_k}\}\}$. Let us denote the attribute set belong to user $u$ as $A_u$ and the common attribute set between user $u$ and AA $k$ as $A_u^k$ i.e., $A_u^k = A_u \cap A_k$. Key generation (KG) algorithm will be used to issue decryption keys to the user $u$ with a set of attributes $A_u$. The algorithm outputs a key that enables the user to decrypt a message encrypted under a set of attributes $A_u^k$ based on threshold policy i.e., if users satisfy more attributes than AA's defined threshold then the user can successfully generate decryption credential. Since threshold policy is very basic, we use access tree based access structure in our scheme.

Let us briefly explain the access tree. First choose a polynomial $q_x$ for each node $x$ (including the leaves) in the tree $T$. These polynomials are chosen in the following way in a top-down manner, starting from the root node $N$, the tree, set the degree $d_x$ of the polynomial $q_x$ for each node $x$ (including the leaves) $T$. For each node $x$, the degree $d_x$ of the polynomial $q_x$ is $d_x = k_x - 1$ where $k_x$ is threshold value of that node. Now, for the root node $r$, set $q_r(0) = \tau_k$. For any other node $x$, set $q_x(0) = q_{parent(x)}(\text{index}(x))$. Now decryption keys for the user $u$ is generated as follows:

$$D = D_{k,u} = g^{-x_u} \cdot h^{\sum_{v \in \Delta_{x_u}} \beta_v}, D^1_{k,u} = h^{\sum_{v \in \Delta_{x_u}}} \cdot D^3_{k,u} = h^{\sum_{v \in \Delta_{x_u}} \beta_v} \cdot \forall x_u \subseteq \hat{A}_u.$$  

Let us consider the adversary $A_u$; $A_u \cap A_k = A_u^k \forall k$. AA $k$ generates $r_{k,u} \in_R \mathbb{Z}_p$ and polynomial $q_x$ for each node $x$ (including the leaves) $T$. For each node $x$, the degree $d_x$ of the polynomial $q_x$ is $d_x = k_x - 1$ where $k_x$ is threshold value of that node. Now, for the root node $r$, set $q_r(0) = \tau_k$. For any other node $x$, set $q_x(0) = q_{parent(x)}(\text{index}(x))$. Now decryption keys for the user $u$ is generated as follows:

$$D = D_{k,u} = g^{-x_u} \cdot h^{\sum_{v \in \Delta_{x_u}} \beta_v}, D^1_{k,u} = h^{\sum_{v \in \Delta_{x_u}}} \cdot D^3_{k,u} = h^{\sum_{v \in \Delta_{x_u}} \beta_v} \cdot \forall x_u \subseteq \hat{A}_u.$$  

**Encryption** (E) - This exactly the same as the original scheme.

**Decryption** (D) - In order to decrypt $C$, the user $u$, computes $X$, $Y$ and $S_k$ as follows: $X = \prod_{k \in T} e(C_2,D_{k,u}), Y = e(C_3,D^1_{k,u})$ and $S_k = \prod_{a_{k,j} \in A_k^u} e(C_3,D^1_{k,u})^{\Delta_{a_{k,j},A_k^u}(0)}$. User then decrypts the message $m$ as follows: $m = \frac{C_1}{C_3} \cdot X$.  

**Fig. 2.** The proposed decentralized KP-ABE scheme. This shows the difference between the original and the proposed algorithm.
and uses $D_{\omega,\nu}^j, C_{k,j}, \forall a_{k,j} \in \overline{A}_m$ and polynomial interpolation to get $r_{k,u}$ as

$$S_k = \prod_{a_{k,j} \in \overline{A}_m} e \left( C_{k,j}, D_{k,u}^j \right)_{a_{k,j} \in \overline{A}_m, \overline{A}_m(0)} = \prod_{a_{k,j} \in \overline{A}_m} \left( e(g,h_1)_{a_{k,j} \in \overline{A}_m, \overline{A}_m(0)} \right)^{q_{\text{parent}}(a_{k,j})(\text{index}(a_{k,j}))},$$

where $q_{\text{parent}}(0) = q_{\text{parent}}(a_{k,j})(\text{index}(a_{k,j}))$. Hence, $S_k = e(g,h_1)^{q_{\text{parent}}(a_{k,j})(\text{index}(a_{k,j}))}$. Now user can get the message $m$ using $C_1$ and pre-computed values $X, Y, S_k, \forall k$ as follows:

$$C_1X \left( \prod_{k \in I_C} \prod_{\ell \in I_C} e(g,g)^{\nu_{\ell,k}} \prod_{k \in I_C} \prod_{\ell \in I_C} e(g,h)^{\nu_{\ell,k}} \prod_{k \in I_C} e(g,h_1)^{\nu_{\ell,k}} \right) \times \left( \prod_{k \in I_C} \prod_{\ell \in I_C} e(g,g)^{\nu_{\ell,k}} \prod_{k \in I_C} \prod_{\ell \in I_C} e(g,h)^{\nu_{\ell,k}} \prod_{k \in I_C} e(g,h_1)^{\nu_{\ell,k}} \right) = m.$$

### D. Security Analysis

**Theorem 1.** The proposed scheme is semantically secure against chosen plain text attack (CPA) in the selective ID model, if there exist negligible function $\nu$ such that, any adversary will succeed the security game explained earlier with probability at most $\frac{1}{2} + \nu$.

**Proof.** Suppose if there is a probabilistic polynomial time adversary who can break our algorithm then there will be a challenger who can break the DBDH assumption by exploiting the adversary. Let $A$ be the challenger and is provided with $[g^n, g^g, g, Z]$ and if the challenger wants to break the DBDH assumption then he needs to determine whether $Z = e(g,g)^{abc}$ or not with at least $\frac{1}{2} + \nu$ probability.

Let us assume that there is an adversary who can break the proposed algorithm. In this section, we will show that the challenger can use such an adversary to break the DBDH assumption. In order to exploit such an adversary, the challenger needs to incorporate the given $[g^n, g^g, g, Z]$ within the proposed algorithm (i.e., Fig. 2). First of all, let us explain how the challenger incorporates $[g^n, g^g, g, Z]$ within the global setup, AA setup, and key generation sub-algorithms. We stress here that this incorporation is indistinguishable from the steps provided in Fig. 2.

Initially, as explained in the security game, the adversary must submit a set of attributes and a set of AAs he wants to challenge. Let us denote the set of attributes provided by the adversary as $A_C = \{A_1, A_2, \ldots, A_n\}$ where $A_2 = A_C \cap \overline{A}_m$ and a list of corrupted AAs as $C_A$. One of the conditions as given in security game is that at least there will be one honest AA for each user whereby the adversary can get sufficient number of decryption credentials [11]. Let us denote GID of a particular user as $\omega$ and the corresponding honest AA as $\kappa$, and it’s access structure $\mathbb{T}^\omega$ hence $\mathbb{T}^\omega(A_C \cap \overline{A}_m) = 0$. Hence, we can divide the AAs who maintain attributes $A_C$ into three: corrupted AAs, AAs who are not corrupted and not $\kappa$, and $\kappa$.

Firstly, challenger generates two random values $\gamma, \eta \overset{R}{\leftarrow} \mathbb{Z}_p$ and sets $h = g^\gamma$, and $h_1 = g^{\gamma}$. For the corrupted AAs $A_{\kappa} \in C_A$. The challenger generates $\omega_k, \beta_k, w_{k,\kappa} \overset{R}{\leftarrow} \mathbb{Z}_p$ and sets $Y_k = e(g,g)^{\nu_{k}}$, $Z_k = g^\eta$ and $(T_{k,j} = g^{\eta_{k,j}})_{a_{k,j} \in \overline{A}_m}$. The public-secret key pairs for $a_{k,j} \in C_A$ is given as

$$\{v_k, \beta_k, \{w_k, 1, \ldots, w_k, n\}, (Y_k, Z_k, \{T_k, 1, \ldots, T_k, n\})\}.$$ challenger provides the secret-public key pairs of the corrupted AAs to the adversary. Hence, the adversary can compute $D_{\omega,\nu}^j, D_{\omega,\nu}^{-},$ and $D_{\omega,\nu}^\kappa, \forall a_{k,j} \in A_k^\kappa$ himself for user $\omega$ without interacting with the challenger.

For the AAs who are not corrupted and not $\kappa$ (i.e., $A_k \notin C_A \cup \kappa$): Challenger generates $\omega_k, \beta_k, w_{k,\kappa} \overset{R}{\leftarrow} \mathbb{Z}_p$ and sets $Y_k = e(g,g)^{\nu_{k}}$, and $Z_k = g^\eta$. For the attributes $a_{k,j} \in A_C \cap \overline{A}_m$, the challenger sets $T_{k,j} = g^{\eta_{k,j}}$. Other attributes i.e., $s_{k,j} \in \overline{A}_k - A^\kappa_C$, the challenger sets $T_{k,j} = h_{1}^{\nu_{k,j}} = g^{a_{\nu_{k,j}}}$, Hence, the public-secret key pairs for $A_k \notin C_A \cup \kappa$ is given as $\{(v_k, \beta_k, w_{k,1}, \ldots, w_{k, n_k}), (Y_k, Z_k, T_k, 1, \ldots, T_k, n_k)\}$. Challenger sends public keys $(Y_k, Z_k, T_k, 1, \ldots, T_k, n_k)$ to the adversary. Now AA $A_k$ assigns a polynomial $q_\omega$ with degree $d_x$ for every node in it’s access tree $\mathbb{T}^\omega$. It first set up a polynomial $q_\omega$ of degree $d_x$ for the root node $x$. It sets $q_x(0) = r_{k,x}$, and then sets the rest of the points to randomly to completely fix $q_x$. Now it sets polynomials for each child node $x'$ of $x$ as follows: $q_x(0) = q_x(index(x'))$. Then computes $D_{\omega,\nu}^{x'} = g^{\nu_{k,j}}$, $Y_{\omega,\nu}^{x'} = g^{\nu_{k,j} \nu_{\omega,\nu}^{x'}}$, and $D_{\omega,\nu}^{x'} = g^{\nu_{k,j} \nu_{\omega,\nu}^{x'}}$. For each child $s_{k,j} \in \overline{A}_k \cap \overline{A}_m$ or $s_{k,j} \in \overline{A}_k \cap \overline{A}_m$, the challenger generates $\nu_{\omega,\nu}^{x'} = \nu_{\omega,\nu}^{x'} \nu_{\omega,\nu}^{x'}$. Thus, the challenger computes $\nu_{\omega,\nu}^{x'} = \nu_{\omega,\nu}^{x'} \nu_{\omega,\nu}^{x'}$. For the attributes $a_{k,j} \in A_C \cap \overline{A}_m$, the challenger sets $T_{k,j} = g^{a_{\nu_{k,j}} \nu_{\omega,\nu}^{x'}}$. Hence, the public-secret key pairs for $A_k \in \overline{A}_k$ is given as $\{(v_k, \beta_k, w_{k,1}, \ldots, w_{k, n_k}), (Y_k, Z_k, T_k, 1, \ldots, T_k, n_k)\}$. challenger sends public keys $(Y_k, Z_k, T_k, 1, \ldots, T_k, n_k)$ to the adversary.

Since $\mathbb{T}^\omega(A_C \cap \overline{A}_m) = 0$, it should be stressed that maximum $d_x$ - 1 number of children of node $x$ could be satisfied. It first defines a polynomial $q_\omega$ of degree $d_x$ for the root node $x$ such that $q_x(0) = r_x$. For each satisfied child $x'$ of $x$, the procedure chooses a random point $e_{x} \nu_{\omega,\nu}^{x'}$ and sets $q_x(index(x')) = e_{x} \nu_{\omega,\nu}^{x'}$. We will show later in this section that it is possible to implicitly define remaining children nodes using polynomial interpolation. Now it recursively defines polynomials for the rest of the nodes in the tree as follows. For each child node $x' \in x$, $q_x(0) = q_x(index(x'))$. Computes

$$D_{\omega,\nu} = g^{\nu_{\omega,\nu}^{x'}}, D_{\omega,\nu} = g^{\nu_{\omega,\nu}^{x'}}, D_{\omega,\nu} = g^{\nu_{\omega,\nu}^{x'}}, \forall a_{k,j} \in A_C \cap \overline{A}_k$$

or $D_{\omega,\nu} = g^{\nu_{\omega,\nu}^{x'}}, D_{\omega,\nu} = g^{\nu_{\omega,\nu}^{x'}}, \forall a_{k,j} \in \overline{A}_k - A_C \cap \overline{A}_m$.

The following lemma proves $D_{\omega,\nu}$ and $D_{\omega,\nu}$ are correct and the challenger can generate these without knowing $a, b, c$ from $g, g^g, g^g$.

**Lemma 1.** $D_{\omega,\nu}$ and $D_{\omega,\nu}$ are correct and they can be generated by challenger without knowing $a, b, c$ from $g, g^g, g^g$.

**Proof.** In the beginning of the access structure we choose $e_1, e_2, \ldots, e_{d_x-1}$ random values for children nodes of root node and if we set $q_x(0) = r_x = r + b \frac{1}{d_x-1}$. Hence, using the polynomial interpolation technique we can implicitly assign values to other children nodes of root using the following valid polynomial function:
\[ q_e = r' \Delta_0, S(x) + \sum_{i=1}^{d_e-1} e_i \Delta_i, S(x), S \in \{0, 1, 2, \ldots, d_e - 1\}. \]

During the key extraction, for the correctness, the challenger hides \( r' \) in attribute \( m_{e,j} \in \{A_k - A_k^2\} \). Since, \( T_{e,j} = h^{-1}_{m_{e,j}} \), the \( D_{e,0,\omega} \) is given as

\[
D_{e,0,\omega} = g^{\frac{\sum_{i=1}^{d_e-1} e_i \Delta_i, S(m_{e,j})}{m_{e,j}}} = g^r \Delta_0, S(m_{e,j}) + \sum_{i=1}^{d_e-1} e_i \Delta_i, S(m_{e,j}),
\]

which is valid and identical to that in the original scheme. Now we will prove that the challenger can generate \( D_{e,0,\omega} \) and \( D_{e,0,\omega} \) for \( a_{e,j} \in \{A_k - A_k^2\} \) without knowing \( a, b, \) and \( c \).

\[
D_{e,0,\omega} = g^{\frac{\sum_{i=1}^{d_e-1} e_i \Delta_i, S(m_{e,j})}{m_{e,j}}} \prod_{i=1}^{d_e-1} g^{\frac{e_i \Delta_i, S(m_{e,j})}{m_{e,j}}},
\]

and

\[
D_{e,0,\omega} = g^{\frac{\sum_{i=1}^{d_e-1} e_i \Delta_i, S(m_{e,j})}{m_{e,j}}} g^{\frac{e_i \Delta_i, S(m_{e,j})}{m_{e,j}}},
\]

Once adversary received all the credential from the challenger, he will send two messages, \( m_0 \) and \( m_1 \) to the challenger. Now the challenger randomly chooses one of the messages and encrypts it and sends the encrypted message back to the adversary. Let us denote the message chosen by the challenger as \( m_0 \) where \( b \) is 0 or 1 and the encrypted message as \( C = \{C_1 = m_0, Z, C_2 = g^r, C_3 = \prod_{k \in \mathbb{A}_k} (g^i)^{S_k}\} \).

We stress here that \( C \) is a valid encryption of the message \( m_0 \) if \( Z = e(g, g)^{abc} \). Hence, the adversary should have his usual non-negligible advantage \( \epsilon \) of correctly identifying the message \( m_0 \). However, when \( Z \neq e(g, g)^{abc} \), then \( C \) is just random value, hence, the adversary can have no more than \( \frac{\epsilon}{2} \) probability of guessing correctly. Hence, if the adversary guesses correctly then challenger guesses that \( Z = e(g, g)^{abc} \) and if adversary is wrong then challenger guesses that \( Z \neq e(g, g)^{abc} \). Hence, the challenger has an advantage of \( \frac{\epsilon}{2} \) in distinguishing whether \( Z = e(g, g)^{abc} \). Hence, an adversary who breaks our scheme with advantage \( \epsilon \) implies an algorithm for breaking DBDH assumption with non-negligible advantage \( \frac{\epsilon}{2} \). We can conclude that the proposed scheme is selective ID secure.

### III. ANONYMOUS KEY ISSUING PROTOCOL

In order to issue keys to the users, Han et al. [6] used key extraction protocol. The key extraction protocol joins secrets known to the user and AA via simple operation which let the users to collude successfully. To avoid the user collusion, we have to use schemes which support non-linear coupling of secrets, hence we are considering anonymous key issuing protocol based on anonymous credential system (See Section III-C for more details).

In an anonymous credential system [14], the users can obtain and prove the possession of credentials while remaining anonymous. In such work it is assumed that each user has a unique secret key i.e., GIUs. Then the user can interact with each AA under a different pseudonym (generated using GIU) in such a manner that it is impossible to link multiple pseudonyms belonging to the same user. In the proposed work, we exploited the anonymous credentials to allow users to obtain decryption keys from the AAs without revealing their GIUs.

In particular, in the proposed key issuing protocol, we coupled the properties of anonymous credential system with ABE. Hence, the user can obtain a set of decryption keys for his secret ID without revealing any information about that GIU to the AA (preserve the privacy). At the same time, the AA is guaranteed that the attack agreed upon decryption-keys are the only information that the user learns from the transaction (i.e., provide security by mitigating user collusion). This is derived from blind IBE schemes [15]. We define this algorithm as \([U(\text{params, PK}_u, \text{GIU}, \text{decom}) \xrightarrow{\text{\text{Commit}}} \text{AA}(\text{params, PK}_u, \text{SK}_u) \xrightarrow{\text{\text{DecryptionKeysForUser}}. \text{User}}] \).

In the algorithm, the AA program \text{Commit} and sends the decom to AA and keeps the decom [6]. Then the user and AA interact with each other to generate decryption keys. The decryption keys for the user will be generated only if the output of \text{Decommit} is 1. From [15], the anonymous key extraction protocol should be leak-free and selective-failure blindness. Definition and security games for leak-free and selective-failure blindness can be found in [6], hence we jump straight into the formulation.

#### A. Construction of anonymous key issuing protocol

Fig. 3 shows the anonymous key issuing protocol where user with private value \( u \in \mathbb{Z}_p \) and \( A_k \) with private values \( r_{k,u}, b_k \) and \( a_k \in \mathbb{Z}_p \) jointly computing decryption keys for the user. The decryption keys for \( u \) are \( D_{k,u}, D_{k,u}^2, D_{k,u}^3, \ldots, D_{k,u}^{\text{a}u,j} \). In order to obtain these keys, first user and AA interact with each other using two-party protocol (2PC). The 2PC protocol can be done via a general 2PC protocol for a simple arithmetic computation. Alternatively, we can do this more efficiently using the construction in [8]. The 2PC protocol takes \((u, r_1, \beta_1) \in \mathbb{Z}_p \) from user and \((r_2, b_2, \beta_2) \in \mathbb{Z}_p \) from \( A_k \) and returns \( x = (u + \beta_1 + \beta_2) \) mod \( p \) and \( y = (u + r_1) \) mod \( p \).

Since, \( r_1 \) and \( r_2 \) were randomly generated by user, the AA \( A_k \) cannot extract the user identity from \( x \) and \( y \). After executing the 2PC protocol, the user now computes \( P = g^{\frac{x+y}{2p}}, Q = h^{\frac{x+y}{2p}} \) and \( R = h^{-1} \) and send those values to \( A_k \). Now \( A_k \) computes \( D_{k,u}, D_{k,u}^2, D_{k,u}^3, \ldots, D_{k,u}^{(\text{a}u,j)} \) which is \( \text{i.e., randomized decryption credentials} \) using \( P, Q, R, x \) and sends them back to the user. Now the user exponentiates the obtained values by \( \rho_1, \rho_2 \) to get the decryption keys. Note that, since \( u \) coupled non-linearly within the decryption keys. We show in Section III-C that this approach avoid user collusion.

#### B. Security Analysis

To obtain the decryption credential blindly from \( A_k \), the user needs to prove that he holds the identifier \( u \) in zero knowledge. As shown in Fig. 3, the user randomly generates \( \rho_1, \rho_2 \in \mathbb{Z}_p \) and computes \( \Psi_1 = g^{\rho_1}, \Psi_2 = g^{\rho_2}, \Psi_3 = g^{\rho_2}, \) and \( \Psi_4 = g^{\rho_2} \) as commitments. At the end of the 2PC protocol, the AA obtains \( x = (r_1 + u) \rho_1 \) mod \( p \), \( y = (\beta_1 + b_2) \rho_2 \) mod \( p \). Then AA verifies \( g^x = A^{\rho_1}_{x,u} \) and \( g^y = C^{\rho_2}_{x,u} \). If they are correctly verified then the AA continues otherwise it aborts.

Now AA needs to prove that he knows \((a_1, a_2, \ldots, r_{k,u})\) in zero knowledge to the user. This will be done using the following steps:

1. \( A_k \) randomly generates \( b_1, b_2, b_3 \in \mathbb{Z}_p \) and computes \( D_{k,u} = P^{-1} Q^{b_3} R^{b_2} \) and sends \( D_{k,u} \) and \( D_{k,u}^{-1} \) to the user.

2. User generates \( c_1 \in \mathbb{Z}_p \) and sends it to the AA.

3. AA computes \( b_1' = b_1 - c_1 a_k, b_2' = b_2 - c_1 a_2, b_3' = b_3 - c_1 a_3, \) and sends \( b_1', b_2', b_3' \) to the user.

4. User verifies \( \widetilde{D_{k,u}} = P^{-1} Q^{b_3} R^{b_2} \) and otherwise aborts.
We ignored the zero-knowledge proofs for $\frac{1}{x}$ and $\frac{1}{y}$ for brevity since they are similar to the above proof.

**Theorem 3.** The proposed anonymous key issuing protocol is both leak-free and selective-failure blind.

**Proof.** Leak freeness. Suppose there exists an adversary $\mathcal{U}$ in the real experiment (where $\mathcal{U}$ is interacting with an honest AA $A_k$ running the anonymous key issuing protocol) and a simulator $\mathcal{S}$ in the ideal experiment (where $\mathcal{S}$ can access the trusted AA running the key issuing protocol without privacy preservation) such that no efficient distinguisher $\mathcal{D}$ can distinguish the real experiment from the ideal experiment. The simulator $\mathcal{S}$ communicates the distinguisher between the black box and the adversary $\mathcal{U}$ by passing the output of $\mathcal{D}$ to $\mathcal{U}$ and the output of $\mathcal{U}$ to $\mathcal{D}$. The simulator $\mathcal{S}$ works as follows:

1. $\mathcal{U}$ sends the adversary $\mathcal{A}$ the public-key $PK_k$ of $A_k$.
2. The adversary $\mathcal{A}$ must proof the possession of $u$ in zero-knowledge to $\mathcal{U}$. If proof is successful then $\mathcal{U}$ obtains $(u,p_1,p_2)$ using rewinding technique.
3. $\mathcal{U}$ sends $u$ to the trusted party. The trusted party runs KeyGen to generates $(D_{k,u},D^1_{k,u},D^2_{k,u})$ and sends to $\mathcal{A}$.
4. Now $\mathcal{U}$ computes $\left(D^\ast_{k,u},\frac{1}{\sqrt{\tau}}D^1_{k,u},\frac{1}{\sqrt{\tau}}D^2_{k,u}\right)$ and sends them to $\mathcal{A}$

If $(D^\ast_{k,u},D^1_{k,u},D^2_{k,u})$ are correct keys from the trusted AA in the ideal experiment, then $\left(D^\ast_{k,u},\frac{1}{\sqrt{\tau}}D^1_{k,u},\frac{1}{\sqrt{\tau}}D^2_{k,u}\right)$ are the correct keys from $A_k$ in the real experiment. Hence, $(D_{k,u},D^1_{k,u},D^2_{k,u})$ and $\left(D^\ast_{k,u},\frac{1}{\sqrt{\tau}}D^1_{k,u},\frac{1}{\sqrt{\tau}}D^2_{k,u}\right)$ are correctly distributed and no efficient distinguisher can distinguish the real experiment with the ideal experiment.

**Proof.** Selective-failure blindness. The adversarial AA $A_k$ submits the public key $PK_k$ and two GIDs $u_0$ and $u_1$. Then, a bit $b \in \{0,1\}$ is randomly selected. $A_k$ can have a black box access to $u_0$’s and $u_1$’s parameters i.e., $U(\text{params},PK_k,u_0)$ and $U(\text{params},PK_k,u_1)$. Then, $U$ executes the anonymous key issuing protocol with $A_k$ and outputs secret keys for $u_0$ and $u_1$ respectively, i.e., $SK_{u_0}$ and $SK_{u_1}$. A batch...

In the anonymous key issuing protocol, $U$ sends four random parameters $\Psi_1 = g^{u_0,p_1} \in G_1$, $\Psi_2 = g^{u_1,p_1} \in G_1$, $\Psi_3 = g^{u_2,p_2} \in G_1$, and $\Psi_4 = g^{u_3,p_2} \in G_1$. If $A_k$ sets $SK_{u_0}$, $SK_{u_1}$ and proves $PoK(\mathcal{u}_0,p_1,p_2)$. Now it is $A_k$’s turn to respond. So, $A_k$’s view on the two black boxes is computationally indistinguishable. Otherwise, the hiding property of the commitment scheme and the witness undistinguishability property of the zero-knowledge proof will be broken. Suppose that $A_k$ uses any computing strategy to output secret keys $\{D_{k,u_0},D^1_{k,u_0},D^2_{k,u_0}\}$ for the first black box. In the following, we will show that $A_k$ can predict $SK_{u_0}$ of $U$ without interacting with the two black boxes:

1. $A_k$ checks $\mathcal{P}oK\left(\mathcal{a}_k,\mathcal{b}_k,\mathcal{c}_k,\mathcal{d}_k\right)$:

If proof fails, $A_k$ uses $SK_{u_0}$ and proceeds for the second black box and a proof of knowledge:

2. $A_k$ generates different secret keys $\{D_{k,u_1},D^1_{k,u_1},D^2_{k,u_1}\}$ for the second black box and a proof of knowledge:

If proof fails, $A_k$ sets $SK_{u_1} = \bot$.

3. Finally $A_k$ outputs his prediction on $(u_0,u_1)$ with $\{SK_{u_0},SK_{u_1}\}$ if $SK_{u_0} \neq \bot$ and $SK_{u_1} \neq \bot$; $(\bot,\bot)$ if $SK_{u_0} = \bot$ and $SK_{u_1} \neq \bot$; $(\bot,\bot)$ if $SK_{u_0} \neq \bot$ and $SK_{u_1} = \bot$; $(\bot,\bot)$ if $SK_{u_0} = \bot$ and $SK_{u_1} = \bot$.

The predication on $(u_0,u_1)$ is correct, and has the identical distribution with the black box. Because $A_k$ performs the same check as the honest $U$, it outputs the valid keys as $U$ obtains from anonymous key issuing protocol. Hence, if $A_k$ can predict the final outputs of the two black boxes, the advantage of $A_k$ in distinguishing the two black boxes is the same without the final outputs. Therefore, the advantage of $A_k$ should come from the received $\Psi_1,\Psi_2,\Psi_3,\Psi_4 \in G_1$ and $PoK(u_0,p_1,p_2)$. From the hiding property of the commitment scheme and witness undistinguishability property of the zero-knowledge proof, $A_k$ cannot distinguish one from the other with nonnegligible advantage. Hence our anonymous key issuing protocol is secure against selective-failure.

C. User collusion

In this section we demonstrate the user collusion vulnerability in [6] identified by [7]. We will use a simple example for this, however, a generic scenario is provided in [7]. Later we show...
that the proposed scheme is free from the identified vulnerability and discuss the reason. Let us consider a scenario with two AAs $A_1$ and $A_2$. Let $A_1$ monitors attribute $\{a_1,1\}$ and $A_2$ monitors $\{a_2,1\}$. Hence according to the original scheme presented in Fig. 1, the public keys and secret keys of $A_k$ are $PK_k = \{Y_k = e(g,g)^{a_k}, T_{k,j} = g^{h_{k,j}}\}$ and $SK_k = \{\alpha_k, \beta_k, t_{k,1}\}$ for $k = \{1, 2\}$. Suppose the encryption algorithm specifies the attribute set for message $m$ is $A_m = \{a_1, a_2\}$ while the users $U_1$ and $U_2$ with identifiers $u_1$ and $u_2$ only have the decryption credential for the attribute $\{a_1\}$ and the user $U_3$ with identifier $u_3$ only has the credential for attribute $\{a_2\}$. Note that none of these three users can decrypt the ciphertext alone. However, due to the vulnerability in [6], these users can collude together and generate decryption credentials to decrypt the ciphertext encrypted by attribute set $A_m = \{a_1, a_2\}$ as follows: according to Fig. 1, credential issued to $U_1$ and $U_2$ are

$$D_{1,u_1} = g^{\alpha_1}h^{r_{1,u_1}}h_1^{u_{1,1}}, \quad D_{1,u_2} = g^{\alpha_1}h^{r_{1,u_2}}h_2^{u_{1,2}}, \quad U_1 \text{ and } U_2 \text{ can collude to compute the secret key corresponding to the user } U_3 \text{ as follows:}$$

$$D_{1,u_3} = D_{1,u_1} \cdot \frac{D_{1,u_2}}{D_{1,u_2}} = g^{\alpha_1}h^{r_{1,u_1}}h_1^{u_{1,1}} = h^{r_{1,u_3}}h^{r_{1,1} - \beta_1}h^{u_{1,3}}h^{u_{1,2}},$$

$$D_{3,u_3} = D_{1,u_1} \cdot \frac{D_{1,u_2}}{D_{1,u_2}} = h^{r_{1,u_3}}h^{r_{1,u_2} - \beta_1}h^{u_{1,3}}h^{u_{1,2}} = h^{r_{1,u_3}}.$$ 

Note that $D_{1,u_3}$ and $D_{3,u_3}$ are valid credentials for $U_3$ for attribute $a_1$ since according to the power of $h_1$ (i.e., $\beta_1$) and power of $h_1$ (i.e., random value $r_{1,u_1}$), $r_{1,u_1} - \beta_1$ or $(u_{1,3} - u_{1,2})$ are correctly produced in the credentials $D_{1,u_3}$ and $D_{3,u_3}$ due to the collusion.

Let us analyse the reason behind this successful collusion. If we look at the variables (i.e., $\beta_1$ and $u_3$) in the power of $h_1$ in $D_{1,u_3}$, $\beta_1$ is known to AA and $u_3$ is known to the user. However, the power $\beta_1$ is just a multiplication (it is very naive) of both the variables. Hence the users can remove $u_1$ and $u_2$ while keeping $\beta_1$ using the above simple algebraic operations to produce decryption credential for $U_3$.

In the proposed scheme, if we compute $D_{1,u_3} = \frac{D_{1,u_2}}{D_{1,u_2}} \cdot \frac{D_{1,u_1}}{D_{1,u_1}} \cdot \frac{D_{1,u_2}}{D_{1,u_2}}$ then the power of the generator $h$ becomes $r_{1,u_1} + r_{1,u_2} - r_{1,u_1} + r_{1,u_2}$ and the power of generator $h_1$ becomes $r_{1,u_1} + r_{1,u_2} - r_{1,u_1} + r_{1,u_2}$ which are known only to the AA (i.e., $r_{1,u_1} + 2r_{1,u_2}$). Hence, $u_1$ and $u_2$ cannot be replaced with $u_3$ without knowing $\beta_1$, $r_{1,u_1}$ and $r_{1,u_2}$. Hence our method withstands the collusion found in [6]. In general, since the variable known to user and the variables known to the AA are non-linearly coupled in our scheme, it is impossible for two users to generate decryption credential for third user without involving AA.

IV. COMPLEXITY COMPARISON

In this section, we compare the computational complexity of our scheme against the following KP-ABE schemes: Han et. al decentralized KP-ABE scheme [6], Chase MA-ABE KP-ABE scheme with central AA [11], and Chase et.al MA-ABE KP-ABE scheme without central AA [12]. Let us assume there are $N$ number of AAs and each AA monitors $n$ number of attributes. In order to compare the complexity at worst case scenario, let us assume that the ciphertext encrypted using all the attributes in the system (i.e., $nN$) and user has decryption credentials for all the attributes. Let us denote the computational time (in ms) for one multiplication, one exponentiation, and one pairing as $C_m$, $C_e$, and $C_p$, respectively.

In order to simulate, we use the benchmark time values given in the popular pairing-based cryptography library namely jPBC in [16]. Table I shows the time values (in ms) for $C_m$, $C_e$, and $C_p$ for two different testbeds: testbed 1 uses Intel(R) Core(TM) 2 Quad CPU Q6600 with 2.40GHz and 3 GB memory running on Ubuntu 10.04 and testbed 2 uses HTC Desire HD A9191 smart phone running on Android 2.2. The time values given in Table I are for a symmetric elliptic curve called $\alpha$-curve, where the base field size is 512-bit and the embedding degree is 2. The $\alpha$-curve has a 100-bit group order.

We consider the healthcare scenario described in Section I as an example to simulate the comparison of four different schemes. Without loss of generality lets assume $N = 2$ and $n$ varies from 1 to 10. As shown in Figures 1 and 2, each algorithm consists of four sub algorithms. Fig. 4 compares the performance of four schemes across four sub algorithms. In Fig. 4(a), the time complexity for setting up the ABE system for all four algorithms increase linearly with same order against the number of attributes per AA. Setup cost for [11] is better than other schemes. It is due to a simplest problem formulation without privacy consideration (i.e., central AA knows secret keys of all AAs).

In Fig. 4(b), the key generation time complexity for [12] increases quadratically with number of attributes due to the interaction between pair of AAs to agree on a shared secret per

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Testbed 1 (ms)</td>
</tr>
<tr>
<td>$C_m$</td>
</tr>
<tr>
<td>$C_e$</td>
</tr>
<tr>
<td>$C_p$</td>
</tr>
</tbody>
</table>
attribute. However, complexity of other schemes increase linearly with almost same order. Complexity of the proposed scheme is high when $n = 1$ due to generation of other independent credential $D_{k,u}$. Han et al. scheme performs better than the proposed scheme due to the difference in number of credentials generated in both the schemes.

In Fig. 4(c), the time complexity for encryption increases linearly with same order for all schemes similar to the setup. Han et al.'s scheme and the proposed scheme show equal performance while other two schemes also show equal performance. This clearly shows that there is no significant difference in Encryption sub algorithm and new schemes are built on top of the previous schemes.

Finally the time complexity of decryption shown in Fig. 4(d) is slightly different compared to other three sub algorithms. Even though the time complexity of all four algorithms increase linearly, the orders are different for all four algorithms. In fact Han et al. perform better than [12] after $n = 10$. The proposed algorithm takes longer (roughly $10ms$ longer for 10 attributes) to decrypt messages than other algorithms. This is solely due to the additional operations involved for the privacy protection. In overall, Chase’s scheme [11] performs better than other three schemes due to its simplicity. However, the algorithms in [11], [12] need a central AA and/or interaction among AAs; hence [11], [12] are vulnerable for single point of failure. The proposed scheme and Han et al.’s scheme are performing equally well as each other. However, the proposed algorithm mitigates the user collusion vulnerability compared to [6]. Due to page limitation, we restricted to $N = 2$ case, however we observed the similar trend for $N > 2$ cases.

Let us now compare the ciphertext length and performance of the proposed scheme on mobile platform. Fig. 5(a) shows ciphertext length for all schemes against the number of attributes. The ciphertext length is increasing linearly with number of attributes at same order for all schemes. This is one of the drawbacks of these schemes; hence, developing a novel scheme which ensures the user’s privacy with constant size ciphertext could be a potential future work. Recent trend of Internet of Things replaces PCs with smart devices and it is anticipated that decryption part of the proposed scheme could be done on mobile device i.e., users want to download and read data on their smartphones. To compare the performance on smartphones against PC, we simulate the time complexity of decryption sub algorithm on both the test beds given in Table I. As expected, Fig. 5(b) shows that the decryption process on smartphone is much slower than PC. More importantly, even though the time complexity on both the test beds is increasing linearly, the complexity on test bed 2 increases with higher order than PC. The main reason behind this is that the mobile device used in test bed 2 is equipped with one processor. This led the non-linear relationship of $C_p$ between both the test beds (in test bed 1, $C_p \approx 5C_m$ but in test bed 2 $C_p \approx 14C_m$).

V. CONCLUSIONS

In this paper, we proposed a PP KP-ABE scheme for a distributed data sharing environment. The proposed scheme enables users to download and decrypt the data from online such as cloud without revealing their attributes to the third-parties. The novelty of the work is to mitigate the user collusion attack in the existing scheme. We used anonymous key issuing protocol to strengthen the bind between user identity and decryption keys; hence, two or more users cannot pool their keys to generate decryption keys for an unauthorized user. We validated the security of the proposed scheme using the decisional bilinear Diffie-Hellman standard complexity assumption.

REFERENCES

[16] The Java Pairing-Based Cryptography Library (JPBC), http://tinyurl.com/12p359