Throughput analysis and optimization of wireless-powered multiple antenna full-duplex relay systems

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Throughout Analysis and Optimization of Wireless-Powered Multi-Antenna Full-Duplex Relay Systems

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Abstract—We consider a full-duplex (FD) decode-and-forward system in which the time-switching protocol is employed by the multi-antenna relay to receive energy from the source and transmit information to the destination. The instantaneous throughput is maximized by optimizing receive and transmit beamformers at the relay and the time-split parameter. We study both optimum and suboptimum schemes. The reformulated problem in the optimum scheme achieves closed-form solutions in terms of transmit beamformer for some scenarios. In other scenarios, the optimization problem is formulated as a semi-definite relaxation problem and a rank-one optimum solution is always guaranteed. In the suboptimum schemes, the beamformers are obtained using maximum ratio combining, zero-forcing, and maximum ratio transmission. When beamformers have closed-form solutions, the achievable instantaneous and delay-constrained throughput are analytically characterized. Our results reveal that, beamforming increases both the energy harvesting and loop interference suppression capabilities at the FD relay. Moreover, simulation results demonstrate that the choice of the linear processing scheme as well as the time-split plays a critical role in determining the FD gains.

Index Terms—Full-duplex, wireless power transfer, decode-and-forward relay, throughput, outage probability.

I. INTRODUCTION

The emergence of multimedia rich wireless services coupled with ever growing number of subscribers has placed a high demand for radio resources such as bandwidth and energy. Most wireless radios so far have adopted half-duplex (HD) operation where uplink and downlink communication are “orthogonalized” in either time or frequency domain, which leads to a loss of spectral efficiency. An attractive solution to improve the spectral efficiency is to allow full-duplex (FD) simultaneous transmission/reception at the expense of loopback interference (LI) caused by the signal leakage from the transceiver output to the input [1]–[5].

Traditionally, LI suppression has been performed using passive isolation techniques such as placing RF absorber material between antennas, deploying of directional antennas. [6]. These schemes alone are inadequate to suppress the LI below the noise floor level required in most wireless systems. To achieve more effective suppression, a FD node could apply time-domain active techniques to pre-cancel the radio frequency (RF) (analog domain) or baseband (digital domain) LI signal [5], [7]. However, such mitigation schemes require sophisticated electronic implementation [4]. With the ubiquitous use of multi-antenna wireless systems, spatial domain precoding techniques can also be deployed at MIMO FD nodes. Such techniques have received significant interest as an attractive FD solution [8], [9].

On the other hand, many contemporary communication systems are battery powered and have a limited operational lifetime. To this end energy harvesting communications is a new paradigm that can power wireless devices by scavenging energy from external resources such as solar, wind, ambient RF power etc. [10]. However, energy harvesting from such sources are not without challenges due to the unpredictable nature of these energy sources. To this end, wireless energy transfer has been touted as a promising technique for a variety of wireless applications [11]–[13].

RF signals can carry both information and energy and this fundamental tradeoff has been studied in [14], [15]. In order to address practical issues associated with simultaneous information and energy transfer (same signal can not be used for both decoding and rectifying), two practical approaches, i.e., time-switching (TS) and power-splitting (PS) point-to-point system architectures were proposed in [16]. Subsequent works have also considered wireless-powered HD relay transmission with TS and PS architectures, for example, different relay networks have been studied considering amplify-and-forward (AF) and decode-and-forward (DF) relaying [19]–[21], large scale networks [22] and multiple antenna relay systems [23], [24].

Inspired by the benefits of FD and wireless power transfer,
some recent papers have investigated the performance of wireless-powered FD point-to-point [25], [26] and relay systems [27], [28]. In [25] a wireless network model with a hybrid FD access-point (AP) that broadcasts wireless energy to a set of downlink users and at the same time receives information from the users in the uplink has been considered. In [26] software-defined radio implementation of a wireless system that transmits data and power in the same frequency has been presented. In [27], the achievable throughput of FD AF and DF relaying systems with TS has been studied. In [28] the performance of a wireless-powered AF relaying system has been also studied. The protocol in [28] considers energy harvesting from LI and therefore can recycle some part of the relay transmit energy. However, [27] and [28] only assumed single transmit/receive antennas at the relay.

Inspired by the current work on wireless-powered FD, in this paper we consider a two-hop MIMO relay system where the multiple antenna FD relay is powered via wireless energy transfer from the source. The main motivation for the adoption of multiple antennas at the relay is two-fold: (1) employment of an antenna array helps the relay to accumulate more energy (2) spatial LI cancellation techniques can be deployed. Specifically, we design receive and transmit beamformers at the relay and optimize TS parameter to characterize instantaneous as well as delay-constrained throughputs. The main contributions of the paper are summarized as follows:

1) Optimum as well as suboptimum schemes for maximizing the instantaneous throughput are proposed. In the optimum scheme, the beamformer optimization is reformulated in terms of the transmit beamformer which is shown to have closed-form solutions for some scenarios. In other scenarios where such solutions are not available, we reformulate the optimization problem as a semi-definite relaxation (SDR) problem in terms of a transmit beamforming matrix. The resulting optimization can be solved as a convex feasibility problem. We prove that the SDR problem either yields optimum rank-one solution or such solution can always be recovered from the optimum beamforming matrix solution of the SDR problem.

2) In the suboptimum schemes, we employ zero-forcing (ZF), maximum ratio combining (MRC), and maximum ratio transmission (MRT) schemes for obtaining receive and transmit beamformers. More specifically, we solve the optimization problems that arise due to the application of transmit zero-forcing (TZF)/MRC and MRT/receive zero-forcing (RZF) as transmit/receive beamformers.

3) In all of the above schemes, the optimum time-split parameter is analytically determined.

4) For the suboptimum schemes, which yield closed-form solutions, we develop new expressions for the system’s outage probability. These expressions are helpful for investigating the effects of key system parameters on performance metrics such as the outage probability and delay-constrained throughput.

5) We present simple high signal-to-noise ratio (SNR) expressions for the outage probability of suboptimum schemes which enable the characterization of the system’s diversity order and array gain.

The remainder of the paper is organized as follows: Section II presents the multiple antenna FD relay system model. Section III introduces joint transmit/receive beamforming designs. The instantaneous and delay-constrained throughput of these beamforming schemes are analyzed in Section IV and V, respectively. Numerical results are reported in Section VI. Finally, Section VII concludes the paper and summarizes the key findings.

**Notation:** We use bold upper case letters to denote matrices, bold lower case letters to denote vectors. $\| \cdot \|$, $(\cdot)^\dagger$, $(\cdot)^{-1}$ and $\text{tr}(\cdot)$ denote the Euclidean norm, conjugate transpose operator, matrix inverse and the trace of a matrix respectively; $\mathbb{E}\{\cdot\}$ stands for the expectation of the random variable $x$; $P_r(\cdot)$ denotes the probability; $f_X(\cdot)$ and $F_X(\cdot)$ denote the probability density function (pdf) and cumulative distribution function (cdf) of the random variable (RV) $X$, respectively; $\mathcal{C}(\mu, \sigma^2)$ denotes a circularly symmetric complex Gaussian RV $x$ with mean $\mu$ and variance $\sigma^2$; $\Gamma(x)$ is the Gamma function; $\gamma(a, x)$ is upper incomplete Gamma function [29, Eq. (8.350)]; $K_{\nu}(\cdot)$ is the $\nu$th order modified Bessel function of the second kind [29, Eq. (8.432)]; $\psi(x)$ is the digamma function [30, Eq. (6.3.1)]; $E_n(x)$ is the $E_n$ function [30, Eq. (5.1.4)] and $G_{pq}^{mn}(z | a_1, \ldots, a_q; b_1, \ldots, b_q)$ denotes the Meijer G-function [29, Eq. (9.301)].

---

**II. System Model and Problem Formulation**

We consider a multiple antenna DF relay system consisting of one source $S$, one relay $R$, and one destination, $D$ as shown in Fig. 1. Both $S$ and $D$ are equipped with a single antenna. To enable FD operation, $R$ is equipped with two sets of antennas, i.e., $M_R$ receiving antennas and $M_T$ transmitting antennas. We assume that the $S$ to $D$ link does not exist.

It is also assumed that $R$ has no external power supply, and is powered through wireless energy transfer from $S$ as in [19], [27]. We adopt the TS protocol [16], [19], hence the entire communication process is divided into two phases, i.e., for a transmission block time $T$, $\alpha$ fraction of the block time is devoted for energy harvesting and the remaining time, $(1-\alpha)T$, is used for information transmission. It is also assumed that the channels experience Rayleigh fading and remain constant over the block time $T$ and varies independently and identically from one block to the other.

During the energy harvesting phase, the received signal $r_e$ at the relay can be expressed as

$$ r_e = \sqrt{\frac{P_{S}}{d_1^2}} h_{S1} e^{j2\pi f_{h} t} + n_R, $$

where $P_{S}$ is the source transmit power, $d_1$ is the distance between the source and relay, $\tau$ is the path loss exponent,
the literature, see for e.g., [3]–[5], [32], [33]. Since each several residual LI channel models have been proposed in RLI can be removed effectively when a propagation-domain literature since the dominant line-of-sight component in practical design.

Moreover, as observed in single/dual antenna relay implementation [27], “re-

RVs in SINR expressions and left out as future work due to limited space. During the energy harvesting phase. Analyzing this case involves different

transmit power can be written as

\[
P_r = \frac{\kappa}{d_1^2} P_s \|h_{SR}\|^2,
\]

where \(\kappa \triangleq \frac{n_0}{\alpha}\) and \(\eta\) denotes the energy conversion efficiency. Now, let us consider the information transmission phase. The received signal at \(R\) can be expressed as

\[
r[n] = \sqrt{P_s} h_{SR} x_S[n] + H_{RR} x_R[n] + n_R[n],
\]

where \(x_S[n]\) is the source information symbol with unit energy, and \(x_R[n]\) is the transmitted relay signal satisfying \(E\{x_R[n]|x_S[n]\} = P_r\). In order to reduce the effects of LI on system performance, a spatial interference cancellation scheme is used at \(R\) and we model the \(M_R \times M_T\) residual LI channel \(H_{RR}\) as a fading feedback channel. To this end, several residual LI channel models have been proposed in the literature, see for e.g., [3]–[5], [32], [33]. Since each implementation of a particular LI cancellation scheme can be characterized by a specific residual power, the elements of \(H_{RR}\) can be modeled as independent identically distributed \(CN(0, \sigma_{RR}^2)\) RVs, which is a common assumption in the literature since the dominant line-of-sight component in LI can be removed effectively when a propagation-domain cancellation method is implemented [5], [6]. Since \(R\) adopts the DF protocol, upon receiving the signal, it first applies a linear combining vector \(w_t\) on \(r[n]\) to obtain an estimate of \(x_S\), then forwards the signal to \(D\) using the transmit beamforming vector \(w_t\). It is assumed that \(\|w_t\| = \|w_r\| = 1\).

The relay’s estimate \(\hat{x}_S[n] = w_t^\dagger r[n]\) can be expressed as

\[
\hat{x}_S[n] = \frac{P_s}{d_1^2} |w_t|^2 h_{SR} x_S[n] + \frac{1}{d_1^2} H_{RR} x_R[n] + w_t^\dagger n_R[n].
\]

The relay transmit signal is given by [4]

\[
x_R[n] = \sqrt{P_s} w_t \hat{x}_S[n - \delta],
\]

\(1\) Another design choice would be to use all receive and transmit antennas during the energy harvesting phase. Analyzing this case involves different RVs in SINR expressions and left out as future work due to limited space. Moreover, as observed in single/dual antenna relay implementation [27], “receive antennas” versus “all antennas” design options are expected to follow similar trends reported in this work and achieve comparable performance depending on the operating SNR regime.

\(2\) Perfect estimation and cancellation of the LI is not possible due to the presence of inevitable transceiver chain impairments [4], [33] and therefore, the results reported in our paper serve as useful theoretical bounds for practical design.

\(3\) Without loss of generality, we assume that \(\eta = 1\).

\[\gamma_{FD} = \min \left\{ \frac{\rho_1}{\sigma_R^2} \|w_t|^2 \|h_{SR}\|^2 \right\} \frac{\|H_{RR} w_t\|^2}{1 + \frac{\rho_2}{\sigma_{RD}^2} \|h_{SR}\|^2 \|H_{RD} w_t\|^2}, \]

where \(\rho_1 = \frac{P_s}{\sigma_R^2}\) and \(\rho_2 = \frac{P_s}{\sigma_{RD}^2}\).

Using (7), the system’s instantaneous rate is given by

\[
R(\alpha, w_t, w_r) = (1 - \alpha) \log_2 (1 + \gamma_{FD}).
\]

Our objective is to maximize the rate \(R(\alpha, w_t, w_r)\) w.r.t. \(\alpha, w_t, \) and \(w_r\). This is mathematically expressed as

\[
\max_{\|w_t\| = \|w_r\| = 1, \alpha \in [0, 1]} R(\alpha, w_t, w_r).
\]

The optimization problem (9) is nonconvex and the challenge is to obtain optimum solutions efficiently. Towards this end, in the sequel, we propose optimum and suboptimum schemes that solve (9). More specifically, in the first step, we keep \(\alpha\) fixed and propose optimum and suboptimum schemes for beamformer design. In the second step, \(\alpha\) is optimized for the obtained beamformers. Although this two-step optimization problem requires a joint optimization w.r.t. \(\alpha\) and \(w_t\) in the optimum scheme, we show that it can still be solved efficiently.

Remark 1: Apart from data-dependent transmit power, energy consumption in the power amplifier/RF circuitry and processing power of the LI cancellation technique employed could have a significant impact on FD transceiver operation [34]. Towards this end, recent advances made with very low energy consumption micro-controllers and RF circuitry is already making it possible to use harvested power in realistic applications [35]. Nevertheless, performance analysis of wireless-powered FD transceiver operation with circuit and processing power consumption is an interesting future direction worth more research.
III. JOINT RECEIVE/TRANSMIT BEAMFORMING

In this section, we consider the beamforming design problem to solve the optimization problem (9) for a given $\alpha$. In this case, (9) turns to a problem of maximizing the minimum two-hop SINR, which is expressed as

$$\max_{\|w_r\| = \|w_t\| = 1} \min \left( \frac{\kappa \rho_1}{d_1^2} \|w_t\|^2 \|h_{SR}\|^2 \|w_r^\dagger H_{rr} w_t\|^2 + 1, \frac{\kappa \rho_2}{d_2^2} \|h_{SR}\|^2 \|H_{rd} w_t\|^2 \right).$$

(10)

In the following subsections, we propose optimum as well as different suboptimum schemes for solving (10). In the optimum approach, we propose SDR problem but show that relaxation does not change the optimality of the solution, whereas in the suboptimum schemes, different linear receiver/transmitter techniques are employed at $R$.

A. Optimum Scheme

Since the second-hop SNR does not depend on $w_r$, we can maximize the first-hop SINR w.r.t $w_t$ by fixing $w_r$. In this case, the optimization problem (10) is re-formulated as

$$\max_{\|w_t\| = 1} \frac{\kappa \rho_1}{d_1^2} \|h_{SR}\|^2 \|w_t^\dagger H_{rr} w_t\|^2 + 1,$$

(11)

which is a generalized Rayleigh ratio problem [36]. It is well known that (11) is globally maximized when

$$w_r = \left( \frac{\kappa \rho_1}{d_1^2} \|h_{SR}\|^2 \|H_{rr} w_t w_t^\dagger H_{rr} + I\|^2 \right)^{-1} h_{SR}.$$

(12)

Accordingly, by substituting $w_r$ into (11) and applying the Sherman Morrison formula [37], the first term inside the $\min$ operator in (10) is obtained as

$$\psi \triangleq \frac{\rho_1}{d_1^2} \left( h_{SR}^\dagger \left( \frac{\kappa \rho_1}{d_1^2} \|h_{SR}\|^2 \|H_{rr} w_t w_t^\dagger H_{rr} + I\|^2 \right)^{-1} h_{SR} \right)$$

$$= \frac{\rho_1}{d_1^2} \left( \|h_{SR}\|^2 - \frac{\kappa \rho_1 \|h_{SR}\|^2 \|h_{SR}^\dagger H_{rr} w_t w_t^\dagger H_{rr} + I\|^2}{1 + \frac{\kappa \rho_1 \|h_{SR}\|^2}{d_1^2} \|H_{rr} w_t w_t^\dagger H_{rr}\|^2} \right).$$

(13)

Now, the optimization problem in (10) is re-expressed as

$$\max_{\|w_t\| = 1} \min \left( \frac{\rho_1}{d_1^2} \left( \|h_{SR}\|^2 - \frac{\kappa \rho_1 \|h_{SR}\|^2 \|h_{SR}^\dagger H_{rr} w_t w_t^\dagger H_{rr}\|^2}{1 + \frac{\kappa \rho_1 \|h_{SR}\|^2}{d_1^2} \|H_{rr} w_t w_t^\dagger H_{rr}\|^2} \right), \frac{\kappa \rho_2}{d_2^2} \|h_{SR}\|^2 \|H_{rd} w_t\|^2 \right),$$

(14)

which is still difficult to solve due to its nonconvex nature.

One of the key results of the optimum scheme is presented in the following proposition.

**Proposition 1:** The optimal $w_{t,o}$ is given by

$$w_{t,o} = \begin{cases} w_{m\text{SINR}}, & \text{if } f_1(w_{m\text{SINR}}) \leq f_2(w_{m\text{SINR}}) \\ w_{m\text{MRT}}, & \text{if } f_2(w_{m\text{MRT}}) \leq f_1(w_{m\text{MRT}}) \\ w_t \text{ is obtained from the feasibility problem } \mathcal{P}, & \text{otherwise} \end{cases}$$

(15)

where

$$f_1(w_t) = \rho_1 \left( \|h_{SR}\|^2 - \frac{\kappa \rho_1 \|h_{SR}\|^2 \|h_{SR}^\dagger H_{rr} w_t w_t^\dagger H_{rr}\|^2}{1 + \frac{\kappa \rho_1 \|h_{SR}\|^2}{d_1^2} \|H_{rr} w_t w_t^\dagger H_{rr}\|^2} \right)$$

$$f_2(w_t) = \frac{\kappa \rho_2}{d_2^2} \|h_{SR}\|^2 \|H_{rd} w_t\|^2$$

and $\mathcal{P}$ is

$$\mathcal{P} : \text{Find } w_t, y$$

s.t. $ty \leq \frac{\rho_1}{d_1^2} \|h_{SR}\|^2 y$

$$- \frac{\kappa \rho_1}{d_1^2} \|h_{SR}\|^2 h_{SR}^\dagger H_{rr} w_t w_t^\dagger H_{rr} w_t = \left( I + \frac{\kappa \rho_1}{d_1^2} \|h_{SR}\|^2 \|H_{rr} w_t w_t^\dagger H_{rr}\|^2 \right) \mathbf{w} \leq \mathbf{W}$$

$$t \leq \frac{\kappa \rho_2}{d_2^2} \|h_{SR}\|^2 \|H_{rd} w_t\|^2$$

$$\text{tr} (W_t) = 1, W_t \succeq 0.$$

(16)

The maximum value of $t$ for which the problem $\mathcal{P}$ is feasible gives the relaxed optimum solution $w_t$.

**Proof:** Note that $w_{m\text{SINR}}$ is the eigenvector corresponding to the minimum eigenvalue of the matrix $I + \frac{\kappa \rho_1}{d_1^2} \|h_{SR}\|^2 \|H_{rr} w_t w_t^\dagger H_{rr}\|^2 \|H_{rr} w_t w_t^\dagger H_{rr}\|^2$. The solution of $w_t$ that maximizes $f_1(w_t)$ is $w_{m\text{SINR}}$, whereas $f_2(w_t)$ is maximized by $w_{m\text{MRT}}$. Now consider the following cases:

- $f_1(w_{m\text{SINR}}) \leq f_2(w_{m\text{SINR}})$: In this case the two possible scenarios for $w_t \neq w_{m\text{SINR}}$ are $f_1(w_t) \leq f_2(w_t)$ and $f_1(w_t) > f_2(w_t)$. In the former case, $f_1(w_t) = f_1(w_{m\text{SINR}})$ but it follows that $f_1(w_t) \leq f_2(w_{m\text{SINR}})$ since $f_1(w_t) \leq f_1(w_{m\text{SINR}})$. In the latter case, $f_1(w_t) = f_1(w_{m\text{SINR}})$ but it follows that $f_2(w_t) \leq f_1(w_{m\text{SINR}}) \leq f_2(w_{m\text{SINR}})$. Therefore, $w_{m\text{SINR}}$ is the optimum precoder when $f_1(w_{m\text{SINR}}) \leq f_2(w_{m\text{SINR}})$.

- $f_2(w_{m\text{MRT}}) \leq f_2(w_{m\text{MRT}})$: The two possible scenarios are $f_2(w_t) \leq f_1(w_t)$ and $f_2(w_t) > f_1(w_t)$ where $w_t \neq w_{m\text{MRT}}$. In the former case, $f_1(w_t) = f_1(w_{m\text{MRT}})$ but it follows that $f_1(w_t) \leq f_2(w_{m\text{MRT}})$ since $f_2(w_t) \leq f_2(w_{m\text{MRT}})$. In the latter case, $f_1(w_t) = f_1(w_{m\text{MRT}})$ but it follows that $f_2(w_t) \leq f_1(w_{m\text{MRT}}) \leq f_1(w_{m\text{MRT}})$. Therefore, $w_{m\text{MRT}}$ is the optimum precoder when $f_2(w_{m\text{MRT}}) \leq f_1(w_{m\text{MRT}})$.

- For remaining cases, to the best of our knowledge, there is no closed-form solution for the optimization problem (14). Introducing an auxiliary variable $t \geq 0$, (14) is
expressed as
\[
\begin{align*}
\max & \quad t \\
\text{s.t.} & \quad t \leq \frac{\rho_1}{d_1^2} \left( \|h_{SR}\|^2 - \frac{\sigma_{n,r}^2}{d_1} \|h_{SR}\|^2 \|H_{RR}w_t\|^2 \right) \\
& \quad \leq \frac{K \rho_2}{d_1 d_2^2} \|h_{SR}\|^2 \|h_{RD}w_t\|^2.
\end{align*}
\]
(17)

This is a nonconvex quadratic optimization problem with nonconvex constraint. To solve the problem in (17), we first apply SDR technique by using a positive-semidefinite matrix \(W_t = w_t w_t^H\) and relaxing the rank-constraint on \(W_t\). Moreover, define the auxiliary variable
\[
y = \text{tr} \left( W_t \left( I + \frac{K \rho_1}{d_1^2} \|h_{SR}\|^2 H_{RR}^H H_{RR} \right) \right).
\]

The relaxed optimization problem (17) in terms of \(W_t\), \(y\), and \(t\) is
\[
\begin{align*}
\max & \quad t \\
\text{s.t.} & \quad ty \leq \frac{\rho_1}{d_1^2} \|h_{SR}\|^2 y \\
& \quad - \frac{\rho_1}{d_1^2} \|h_{SR}\|^2 h_{SR}^H H_{RR} W_t H_{RR}^H h_{SR} \\
& \quad = \text{tr} \left( W_t \left( I + \frac{K \rho_1}{d_1^2} \|h_{SR}\|^2 H_{RR}^H H_{RR} \right) \right) \\
& \quad \leq \frac{K \rho_2}{d_1 d_2^2} \|h_{SR}\|^2 \|h_{RD}w_t\|^2 \\
& \quad \text{tr}(W_t) = 1, W_t \succeq 0.
\end{align*}
\]
(18)

The optimization problem (18) is still nonconvex. However, for a given \(t\), the problem turns to a convex feasibility problem given in (16). We show that there is no need to solve this feasibility problem for any possible values of \(t\). This can be demonstrated as follows. Note that the constraints of the problem (18) show that \(t\) can be upper bounded as
\[
t \leq \min \left( \frac{\rho_1}{d_1^2} \|h_{SR}\|^2, \frac{\rho_2}{d_1 d_2} \|h_{SR}\|^2 \|h_{RD}\|^2 \right).
\]
Consequently, we can start solving the feasibility problem (16) in the decreasing order for \(t\). The largest value of \(t\) for which the problem is feasible yields the optimum \(W_t\). If the optimum \(W_t\) is rank-one, then the relaxed problem (18) is equivalent to the original problem.

In the following, we show that the relaxed optimization (18) or convex feasibility problem (16) provides an optimum rank-one solution. To this end, we present another key result of the optimum scheme in the following proposition.

**Proposition 2:** The rank-one optimum \(W_t\) can always be guaranteed in (18).

**Proof:** The proof is based on Karush-Kuhn-Tucker (KKT) conditions and given in Appendix I.

**B. TZF Scheme**

We now present some suboptimum beamforming solutions. The first is the TZF scheme, where relay takes advantage of the multiple transmit antennas to completely cancel the LI [8]. To ensure this is feasible, the number of the transmit antennas at relay should be greater than one, i.e., \(M_R > 1\). In addition, MRC is applied at the relay input, i.e., \(w_r = \frac{h_{SR}}{\|h_{SR}\|}\). After substituting \(w_r\) into (10), the optimal transmit beamforming vector \(w_t\) is obtained by solving the following problem:
\[
\begin{align*}
\max & \quad \|h_{RD}w_t\|^2 \\
\text{s.t.} & \quad h_{SR}^H H_{RR} w_t = 0.
\end{align*}
\]
(19)

We know that \(A \triangleq H_{RR}^H h_{SR}^H H_{RR}\) is a rank-one Hermitian matrix with eigenvalue \(\lambda \triangleq \|h_{SR}\|^2\|H_{RR}\|^2\) and eigenvector \(x \triangleq \frac{H_{RR} h_{SR}}{\|H_{RR} h_{SR}\|}\). Consequently, the eigenvalue decomposition of \(A\) can be given by \(x^H (I - \frac{1}{\lambda} A) x = 0\) which implies that \(x^H (I - \frac{1}{\lambda} A) w_t = 0\) for all \(w_t \neq 0\). Comparing this with the ZF constraint in (19), it is clear that we can take \(w_t = B w_t\), where \(B \triangleq I - \frac{H_{RR} h_{SR}^H h_{SR}^H H_{RR}}{\|H_{RR} h_{SR}\|^2}\). Consequently, the transpose beamformer for the TZF scheme is given by
\[
w_t^{ZF} = \frac{B h_{RD}^H}{\|B h_{RD}\|}.
\]
(20)

**C. RZF Scheme**

As an alternative solution, the transmit beamforming vector can be set using the MRT principle, i.e., \(w_t = \frac{h_{RD}}{\|h_{RD}\|}\), and \(w_r\) is designed with the ZF criterion \(w_r^H H_{RR} w_t = 0\). To ensure feasibility of RZF, \(R\) should be equipped with \(M_R > 1\) receive antennas. Substituting the MRT solution for \(w_t\) into (10), the optimal receive beamforming vector \(w_r\) is the solution of the following problem:
\[
\begin{align*}
\max & \quad \|w_r\|^2 \|h_{SR}\|^2 \\
\text{s.t.} & \quad w_r^H H_{RR} w_r = 0.
\end{align*}
\]
(21)

Using similar steps as in the TZF scheme, the optimal combining vector \(w_r\) is obtained as
\[
w_r^{ZF} = \frac{D h_{SR}^H}{\|D h_{SR}\|},
\]
(22)
where \(D \triangleq I - \frac{H_{RR} h_{SR}^H h_{RD}^H H_{RR}^H}{\|H_{RR} h_{RD}\|^2}\).

**D. MRC/MRT Scheme**

Finally, we consider the MRC/MRT scheme, where \(w_r\) and \(w_t\) are set to match the first hop and second hop channel, respectively. Hence,
\[
w_r^{MRC} = \frac{h_{SR}^H}{\|h_{SR}\|}, \quad w_t^{MRT} = \frac{h_{RD}^H}{\|h_{RD}\|}.
\]
(23)

It is worthwhile to note that the optimum, TZF, and RZF schemes reduce to the MRC/MRT scheme in the absence of LI. Although the MRC/MRT scheme is not optimal in the presence of LI, it could be favored in situations where compatibility with HD systems is a concern. Moreover, as we
show in Section VI, the MRC/MRT scheme exhibits a very
good performance as compared to other schemes under mild
LI effect. Note that the MRC/MRT scheme requires only
the knowledge of $h_{SR}$ and $h_{RD}$, whereas the other three
schemes require the knowledge of $h_{SR}$, $h_{RD}$, and $H_{RR}$.

IV. OPTIMIZING $\alpha$ FOR INSTANTANEOUS THROUGHPUT

In this section, we optimize $\alpha$ for the FD beamforming
schemes proposed in the previous section and maximize the
instantaneous throughput which is given by [27]

$$R_{0}(\alpha) = (1 - \alpha) \log_2 (1 + \gamma_{FD}).$$

(24)

This expression reveals an interesting trade-off between the
duration of energy harvesting and the instantaneous through-
put. A longer energy harvesting time increases the harvested
energy and consequently the second hop SNR, however
decreases the available time for information transmission
and vice-versa. Therefore, an appropriate system design can
optimize the instantaneous throughput by adjusting $\alpha$.

A. Optimum Scheme

Substituting the optimum $w_{t,o}$, the instantaneous rate as a
function of $\alpha$ is

$$R_{Opt}(\alpha) = (1 - \alpha) \log_2 \left( 1 + \|h_{SR}\|^2 \right)$$

$$\times \min \left( \frac{\rho_1}{d_1^2} \left( 1 - \frac{\kappa h_{SR}^T h_{SR} w_{t,o}}{1 + \frac{\rho_1}{d_1^2} \|h_{SR}\|^2 \|H_{RR} w_{t,o}\|^2} \right), \right.$$

$$\left. \frac{\kappa p_2}{d_2^2} \|H_{RD} w_{t,o}\|^2 \right).$$

Let $b_0 = \frac{\rho_1}{d_1^2} \|h_{RD} w_{t,o}\|^2$, $b_1 = \frac{\rho_1}{d_1^2} \|h_{SR}^T h_{SR} w_{t,o}\|^2$, and $b_2 = \frac{\rho_1}{d_1^2} \|h_{SR}\|^2 \|H_{RR} w_{t,o}\|^2$. Hence, (25) is written as

$$R_{Opt}(\alpha) = (1 - \alpha) \log_2 \left( 1 + \frac{\rho_1}{d_1^2} \|h_{SR}\|^2 \right)$$

$$\times \min \left( 1 - \frac{\alpha b_0}{1 + \frac{\rho_1}{d_1^2} \|h_{SR}\|^2 \|H_{RR} w_{t,o}\|^2}, \frac{\kappa p_2}{d_2^2} \|H_{RD} w_{t,o}\|^2 \right).$$

(26)

Thus, the optimal $\alpha$ is obtained by solving

$$\alpha_{Opt}^* = \arg \max_{0 \leq \alpha < 1} R_{Opt}(\alpha).$$

(27)

Denote $f = \frac{\rho_1}{d_1^2} \|h_{SR}\|^2$ and $\tilde{f} = \rho_1 b_0$. The above optimization
problem can be solved analytically as shown in the
following proposition.

Proposition 3: The optimal $\alpha_{Opt}^*$ is given by

$$\alpha_{Opt}^* = \begin{cases} \frac{\rho_1}{d_1^2} W \left( \frac{\tilde{f}}{1 + \alpha} \right) + 1, & \text{if } e^{W \left( \frac{\tilde{f}}{1 + \alpha} \right)} + 1 < \rho_1 \tilde{f}; \\ \frac{\rho_1}{d_1^2}, & \text{otherwise}, \end{cases}$$

(28)

where $W(x)$ is the Lambert W function in which $W(x)$ is
the solution of $W \exp(W) = x$ and noting that $b_1 \leq b_2$ from
Cauchy-Schwarz inequality, $\alpha_0$ is obtained by

$$\alpha_0 = \frac{(b_2 - b_1 - b_0) + \sqrt{(b_2 - b_1)^2 + 2b_0(b_1 + b_2)}}{2b_0 b_2}.$$ 

Proof: The following two cases are considered:

1) if $\frac{\alpha b_0}{1 + \alpha} < 1 - \frac{\alpha b_0}{1 + \alpha}$ or $\alpha < \frac{\alpha b_0}{1 + \alpha}$, we have

$$R_{Opt}(\alpha) = (1 - \alpha) \log_2 \left( 1 + \frac{\alpha}{1 - \alpha} \tilde{f} \right).$$

Therefore, taking the first order derivative of $R_{Opt}(\alpha)$
with respect to $\alpha$, and using the procedure described
in [27], the optimal time portion $\alpha$ can be obtained as

$$\alpha_{Opt}^* = \frac{\rho_1}{d_1^2} W \left( \frac{\tilde{f}}{1 + \alpha} \right) + 1.$$ 

(29)

2) Otherwise, if $\alpha > \frac{\alpha b_0}{1 + \alpha}$, the instantaneous throughput
in (26) is given by

$$R_{Opt}(\alpha) = (1 - \alpha) \log_2 \left( 1 + f \left( 1 - \frac{\alpha b_1}{1 - \alpha + \alpha b_2} \right) \right).$$

Taking the first derivative of $R_{Opt}(\alpha)$ with respect to $\alpha$
yields

$$\frac{dR_{Opt}(\alpha)}{d\alpha} = -\log_2 \left( 1 + f \left( 1 - \frac{\alpha b_1}{1 - \alpha + \alpha b_2} \right) \right)$$

$$- \frac{(1 - \alpha) f b_1}{1 + f \left( 1 - \frac{\alpha b_2}{1 - \alpha + \alpha b_2} \right)},$$

which is strictly smaller than zero. Therefore, $R_{Opt}(\alpha)$
is a decreasing function with respect to $\alpha$, and hence
the optimal $\alpha$ is given by

$$\alpha_{Opt}^* = \frac{\alpha_0}{1 + \alpha_0}.$$ 

(30)

B. TZF Scheme

Substituting $w_{t,e}^{ZF}$ into (7), the instantaneous throughput
of the TZF scheme is given by

$$R_{1,TZF}(\alpha) = (1 - \alpha) \log_2 \left( 1 + \min \left( \frac{\rho_1}{d_1^2} \|h_{SR}\|^2, \frac{\kappa p_2}{d_2^2} \|h_{SR}\|^2 \|H_{RD}\|^2 \right) \right).$$

(31)

Hence, the optimal $\alpha$ can be obtained by solving the following
optimization problem

$$\alpha_{Opt}^{*ZF} = \arg \max_{0 \leq \alpha < 1} R_{1,TZF}(\alpha).$$

(32)

In (32), since $R_{1,TZF}(\alpha)$ is concave with respect to $\alpha$, $\alpha_{Opt}^{*ZF}$
can be obtained by solving the equation $\frac{dR_{1,TZF}(\alpha)}{d\alpha} = 0$. Using
the similar approach as in the optimum scheme, $\alpha_{Opt}^{*ZF}$
can be derived as

$$\alpha_{Opt}^{*ZF} = \begin{cases} \frac{\rho_1}{d_1^2} W \left( \frac{\alpha_1}{1 + \alpha_1} \right) + 1, & \text{if } e^{W \left( \frac{\alpha_1}{1 + \alpha_1} \right)} + 1 < \rho_1 \alpha_1; \\ \frac{\alpha_1}{1 + \alpha_1}, & \text{otherwise}, \end{cases}$$

(33)

where $\alpha_1 = \frac{\rho_2 p_2}{d_2^2} \|h_{SR}\|^2 \|H_{RD}\|^2$, $a_2 = \frac{d_1}{\rho_1 \|H_{SR}\|^2}$, and

$$\alpha_1 = a_1 a_2.$$ 

C. RZF Scheme

Substituting $w_{t,e}^{ZF}$ into (7), the instantaneous throughput
of the RZF scheme can be written as

$$R_{1,RZF}(\alpha) = (1 - \alpha) \log_2 \left( 1 + \min \left( \frac{\rho_1}{d_1^2} \|H_{SR}\|^2, \frac{\kappa p_2}{d_2^2} \|h_{SR}\|^2 \|H_{RD}\|^2 \right) \right).$$

(34)
Accordingly, the optimal \( \alpha \) can be obtained as
\[
\alpha^*_\text{RZF} = \begin{cases} 
\frac{e^{\rho_1^2} \rho_2 \|h_{SR}\|^2}{\rho_1^2 \rho_2 \|h_{RD}\|^2} + 1, & \text{if } e^{\frac{\rho_2}{\rho_1^2}} + 1 < \frac{\alpha_1}{\alpha_2} + 1; \\
\frac{1}{1+\alpha_2}, & \text{otherwise},
\end{cases}
\]
where \( \alpha_3 = \frac{\rho_2}{\rho_1^2} \|h_{SR}\|^2 \|h_{RD}\|^2, \) and \( \alpha_2 = \alpha_3 \alpha_4. \)

**D. MRC/MRT Scheme**

Substituting \( w_{\text{MRC}}^* \) and \( w_{\text{MRT}}^* \) into (7), the instantaneous throughput of the MRC/MRT scheme can be expressed as
\[
R_{\text{MRC}}^* = (1-\alpha) \log(1 + \eta_{\alpha} \|h_{SR}\|^2 \|h_{RD}\|^2).
\]

Taking the first derivative of \( R_{\text{MRC}}(\alpha) \) with respect to \( \alpha \) yields
\[
\frac{dR_{\text{MRC}}(\alpha)}{d\alpha} = -\log(1 + \frac{\eta}{\beta} \|h_{SR}\|^2 \|h_{RD}\|^2) \frac{1}{1-\alpha} + \frac{\eta}{\beta} \|h_{SR}\|^2 \|h_{RD}\|^2 - \log(1 + \frac{\eta}{\beta} \|h_{SR}\|^2 \|h_{RD}\|^2) \frac{1}{1-\alpha}.
\]

We end this section with the following remarks. In contrast to the suboptimum schemes, the obtained solution of \( w_t \) in optimum scheme depends on \( \alpha \). As a consequence, joint optimization w.r.t. \( \alpha \) and \( w_t \) is required in the latter scheme. There are two ways to solve this joint optimization. The first way is to find \( w_t \) by following Proposition 1 and next performing a one-dimensional line search over \( 0 < \alpha < 1 \). This guarantees the global optimum solutions for \( \alpha \) and \( w_t \). Another way is to employ an iterative approach where each iteration step consists of a two-step optimization, i.e., optimizing \( w_t \) for a given \( \alpha \) and vice-versa. In either way, the computational complexity of the proposed optimum scheme is limited due to the following facts. As seen from Proposition 1, there is a need to solve SDR problem only for a specific scenario. Moreover, the rank-one optimum solution can always be recovered from optimum \( W_t \) which is obtained by solving the convex feasibility problem \( \mathcal{P} \) in a few number of iterations. Also \( \alpha \) can be obtained analytically for a given \( w_t \) or only a one-dimensional search is required for finding the optimum \( \alpha \). Despite these facts, the computational complexity of the optimum scheme is higher than that of the suboptimum schemes. This may be justified since, depending on the scenarios, the optimum scheme significantly outperforms the suboptimum schemes (see Fig. 2).

We outline the proposed optimum scheme for the instantaneous throughput maximization problem in Algorithm 1.
\begin{align*}
F_{\gamma_{TZF}}(z) &\approx \left\{ 
\begin{aligned}
\frac{1}{\Gamma(M_R+1)} + \frac{1}{\Gamma(M_T-1)\Gamma(M_R)} \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k!(M_T+M_R-k+1)} \left( \frac{\sigma_2^2}{\sigma_1^2} \right)^{k+1} \left( \frac{M_T+1}{M_T+M_R-k+1} \right) \left( \frac{d_z}{\rho_1} \right)^{M_R} , & M_T > M_R + 1, \\
\frac{1}{\Gamma(M_R+1)} \left( 1 + \frac{1}{\Gamma(M_R)} \ln \left( \frac{\rho_1}{\rho_2} \right) - \ln \left( d_T^2 + \psi(1) \right) \right) \left( \frac{\sigma_2^2}{\sigma_1^2} \right)^{M_R} \left( \frac{d_z}{\rho_1} \right)^{M_R} , & M_T = M_R + 1, \\
\frac{1}{\Gamma(M_T+M_R)} \left( \frac{d_T^2}{\eta^2} \right)^{M_T-1} \left( \frac{d_z}{\rho_2} \right)^{M_T-1} , & M_T < M_R + 1.
\end{aligned}
\right.
\end{align*}

\textbf{Algorithm 1} The proposed optimum scheme for the instantaneous throughput maximization.

\textbf{Step 1:} Initialize \( \alpha \):

Choose \( \alpha \) from its grid (Line search Method -LS) or generate an initial point for \( \alpha \) (Alternating optimization method-AO) where \( \alpha \in [0, 1] \).

\textbf{Step 2:} Obtain the transmit beamformer \( w_{t,o} \) using (15), if (15) requires solving the feasibility problem \( \mathcal{P} \) of (16) then

if \( w_t \) is rank-one then

Take \( w_{t,o} \) as the eigenvector corresponding to non-zero eigenvalue of \( w_t \).

else

Take \( w_{t,o} \) as given in \textit{Case b} of the Appendix I ( see (28))

end if

end if

\textbf{Step 4:}

If AO method then

while not converged do

Obtain \( \alpha_{Opt} \) from (28) using \( w_{t,o} \) obtained in \textbf{Step 2}.

Update \( \alpha \) as \( \alpha = \alpha_{Opt} \) and go to \textbf{Step 2}.

end while

Save \( w_{t,o}, \alpha, \) and the objective function.

else if LS method then

Save \( w_{t,o}, \alpha, \) and the objective function.

Take another \( \alpha \) from its grid, and repeat \textbf{Step 1}.

end if

\textbf{Step 3:} Choose the \( w_{t,o} \) and \( \alpha \) that give maximum objective value.

where \( \gamma_{th} = 2^{R_c} - 1 \). Therefore in order to find the delay-constrained throughput, the remaining key task is to characterize the exact outage probability of the system. In the sequel, we investigate the outage probability of the considered TZF, RZF, and MRC/MRT schemes. In addition, simple high SNR approximations are presented, which provide a concise characterization of the diversity order, and enable a performance comparison of these three schemes. Derivation of the outage probability of the optimum scheme is difficult. Hence we have resorted to simulations for evaluating the delay-constrained throughput of the optimum scheme in Section VI.

\textbf{A. TZF Scheme}

Substituting the \( \mathbf{w}_t^{TZF} \) and \( \mathbf{w}_t^{MRC} \) into (7), the end-to-end SNR \( \gamma_{TZF} \) can be expressed as

\begin{align*}
\gamma_{TZF} = \frac{\rho_1}{\rho_2} \frac{\| \mathbf{h}_{SR} \|^2}{\| \mathbf{h}_{RD} \|^2} \min \left( 1 - \alpha, \frac{\rho_2}{\rho_1} \frac{\eta \alpha}{\rho_2} \| \mathbf{h}_{RD} \|^2 \right),
\end{align*}

where \( \mathbf{h}_{RD} \) is an \( (M_T - 1) \times 1 \) vector. \( \| \mathbf{h}_{RD} \|^2 \) follows the chi-square distribution with \( (M_T - 1) \) degrees of freedom, denoted as \( \| \mathbf{h}_{RD} \|^2 \sim \chi^2_{2(M_T-1)} \) [38]. Let

\[ Y_1 = \min \left( 1 - \alpha, \frac{\rho_2}{\rho_1} \frac{\eta \alpha}{\rho_2} \| \mathbf{h}_{RD} \|^2 \right). \]

The cdf of \( Y_1 \) is given by [24, Appendix II]

\begin{align*}
F_{Y_1}(y) = \left\{ \begin{array}{ll}
1 & y > 1 - \alpha, \\
1 - \frac{\gamma_{TZF}\left(M_T-1,\frac{\rho_1}{\rho_2}\frac{\eta \alpha}{\rho_2}\| \mathbf{h}_{RD} \|^2\right)}{\Gamma(M_T-1)} & y < 1 - \alpha.
\end{array} \right.
\end{align*}

Therefore, the cdf of \( \gamma_{TZF} \) can be obtained as

\begin{align*}
F_{\gamma_{TZF}}(z) = 1 - \frac{1}{\Gamma(M_T)} \int_{\frac{d_z}{\rho_1}}^{\infty} Q(1 - \frac{d_T^2}{\eta^2}, \frac{\rho_1^2}{\rho_2^2}, z) x^{M_T-1} e^{-x} dx,
\end{align*}

where \( Q(a, x) = \Gamma(a, x)/\Gamma(a) \).

To the best of the authors’ knowledge, the integral in (46) does not admit a closed-form expression. However, (46) can be evaluated numerically.

To gain further insights, we now look into the high SNR regime and derive a simple approximation for the outage probability, which enables the characterization of the achievable diversity order of the TZF scheme.

\textbf{Proposition 5}: In the high SNR regime, i.e., \( \rho_1, \rho_2 \to \infty \), the outage probability of the TZF scheme can be approximated as (47) at the top of the page.

\textbf{Proof}: See Appendix B.

By inspecting (47), we see that the TZF scheme achieves a diversity order of \( \min(M_R, M_T - 1) \). This is intuitive since one degree of freedom is used for interference cancellation. Moreover, we notice that for the case \( M_R + 1 = M_T \), \( F_{\gamma_{TZF}}(z) \) decays as \( \rho_1^{-M_R} \ln(\rho_1) \) rather than \( \rho_1^{-M_R} \) as in the conventional case, which implies that in the energy harvesting case the slope of \( F_{\gamma_{TZF}}(z) \) converges much slower compared with that in the constant power case.

\textbf{B. RZF Scheme}

Invoking (7), and using \( \mathbf{w}_t^{TZF} \) and \( \mathbf{w}_t^{MRT} \), the end-to-end SNR \( \gamma_{RZF} \) can be expressed as

\begin{align*}
\gamma_{RZF} = &\min \left( \frac{\rho_1}{\rho_2} \| \mathbf{h}_{SR} \|^2, \frac{\rho_2}{\rho_1} \frac{\| \mathbf{h}_{SR} \|^2}{\| \mathbf{h}_{RD} \|^2} \right) \\
= &\min \left( \frac{\rho_2}{\rho_1} \| \mathbf{h}_{SR} \|^2, \frac{\rho_2}{\rho_1} \frac{\| \mathbf{h}_{SR} \|^2}{\| \mathbf{h}_{RD} \|^2} \right) \\
= &\min \left( \frac{\rho_1}{\rho_2} \| \mathbf{h}_{SR} \|^2, \frac{\rho_2}{\rho_1} \frac{\| \mathbf{h}_{SR} \|^2}{\| \mathbf{h}_{RD} \|^2} \right),
\end{align*}

where \( \mathbf{h}_{SR} = \Phi \mathbf{h}_{SR} \) with \( \Phi \) is a unitary matrix, and \( \mathbf{h}_{SR} \) is a \( (M_R - 1) \times 1 \) vector, consisting of the \( M_R - 1 \)
last element of \( \mathbf{h}_{SR} \). In (47), the first equality follows from the fact that \( \mathbf{D} \) is idempotent and the second equality is due to the eigen decomposition. Let us denote \( Z_1 \triangleq 1 \frac{1}{d_T^2} \left( \| \mathbf{h}_{SR} \|^2 + \| h_1 \|^2 \right) \) where \( h_1 \) is the first element of the \( \mathbf{h}_{SR} \) and \( X_1 = \frac{1}{\| \mathbf{h}_{SR} \|^2 + \| h_1 \|^2} \). Therefore, the end-to-end SINR can be re-expressed as

\[
\gamma_{RZF} = Z_1 \min \left( \rho_1 X_1, \frac{\kappa \rho_2}{d_T^2} \| h_{RD} \|^2 \right). \tag{48}
\]

It is well known that \( Z_1 \) follows central chi-square distribution with \( 2M_R \) degrees-of-freedom, denoted as \( Z_1 \sim \chi^2_{2M_R} \) and that \( X_1 \) follows a beta distribution with shape parameters \( M_R - 1, 1 \), denoted as \( X_1 \sim \text{Beta}(M_R - 1, 1) \), with [38]

\[
F_{X_1}(x) = x^{M_R - 1}, \quad 0 < x < 1.
\]

Moreover, let \( Y_2 = \frac{\kappa \rho_2}{d_T^2} \| h_{RD} \|^2 \); we have

\[
F_{Y_2}(y) = P \left( M_T, \frac{d_T^2}{\kappa \rho_2} y \right),
\]

where \( P(a, x) = \gamma(a, x)/\Gamma(a) \). With \( F_{X_1}(x) \) and \( F_{Y_2}(y) \) in hand, \( F_{\gamma_{RZF}}(z) \) can be expressed as

\[
F_{\gamma_{RZF}}(z) = 1 - Q \left( M_R, \frac{d_T^2 z}{\rho_1} \right) + \frac{1}{\Gamma(M_R)} \int_0^\infty P \left( M_T, \frac{d_T^2}{\kappa \rho_2} z \right) x^{M_R - 1} e^{-x} dx + \frac{d_T^2}{\rho_1} \int_0^\infty Q \left( M_T, \frac{d_T^2}{\kappa \rho_2} z \right) e^{-x} dx.
\]

(50)

Although (50) does not admit a closed-form solution, it can be efficiently evaluated numerically. Now, we look into the high SNR regime, and investigate the diversity order.

**Proposition 6:** In the high SNR regime, i.e., \( \rho_1, \rho_2 \to \infty \), the outage probability of the RZF scheme can be approximated as

\[
F_{\gamma_{RZF}}(z) \approx \begin{cases} 
\frac{1}{\Gamma(M_R)} \left( \frac{d_T^2 z}{\rho_1} \right)^{M_R - 1}, & M_R < M_T + 1, \\
\frac{1}{\Gamma(M_T + 1)} \left( \frac{d_T^2}{\kappa \rho_2} \right)^{M_T} \left( \frac{d_T^2 z}{\rho_1} \right)^{M_T}, & M_R = M_T + 1, \\
\Gamma(M_R - M_T) \frac{d_T^2}{\rho_1} \left( \frac{d_T^2}{\kappa \rho_2} \right)^{M_T} \left( \frac{d_T^2 z}{\rho_1} \right)^{M_T}, & M_R > M_T + 1.
\end{cases}
\]

(51)

**Proof:** See Appendix C.

Proposition 6 indicates that the RZF scheme achieves a diversity order of \( \min(M_R - 1, M_T) \). This result is also intuitively satisfying since one degree-of-freedom should be allocated for LI cancellation at the receive side of \( R \).

C. MRC/MRT Scheme

The outage probability analysis of the MRC/MRT scheme for arbitrary \( M_T \) and \( M_R \) appears to be cumbersome. Therefore, we now consider two special cases as follows: Case-1 \( M_T = 1, M_R \geq 1 \) and Case-2 \( M_T \geq 1, M_R = 1 \).

Case-1): In this case \( \| \mathbf{h}_{RR} \|^2 \) is given by

\[
X_2 \triangleq \| \mathbf{w}_t^{MRC} \mathbf{h}_{RR} \|^2 = \frac{1}{\| \mathbf{h}_{SR} \|^2} \| h_{RR} \|^2.
\]

(52)

For notational convenience, we define \( c_1 = \frac{\kappa \rho_2}{d_T^2} \), \( c_2 = \frac{\kappa \rho_2}{d_T^2} \), then, the end-to-end SINR can be re-expressed as

\[
\gamma_{MRC} = \min \left( c_1 \| \mathbf{h}_{SR} \|^2, c_2 \| \mathbf{h}_{SR,1} \|^2 \| h_{RD} \|^2 + 1, c_3 \| \mathbf{h}_{SR} \|^2 \| h_{RD} \|^2 \right).
\]

(53)

Let us denote \( X = c_1 / \left( c_2 X_2 + \frac{1}{c_2} \right) \) where \( X_2 = \| \mathbf{h}_{SR,1} \|^2 \| h_{RD} \|^2 \) and \( Y = c_3 Y_3 Y_4 \), with \( Y_3 = \| \mathbf{h}_{SR} \|^2 \) and \( Y_4 = \| h_{RD} \|^2 \). Accordingly, the cdf of \( \gamma_{MRC} \) in (53) can be expressed as

\[
F_{\gamma_{MRC}}(z) = \text{Pr}(\min(X, Y) < z),
\]

(54)

Conditioned on \( Y_3 \), the RVs \( X \) and \( Y \) are independent and hence we have

\[
\text{Pr}(X > z, Y > z) = \int_{d_T^2 z}^{\infty} (1 - F_Y(z))(1 - F_X(z)) f_Y(z) dY,
\]

(55)

where \( f_Y(z) \) is the cdf of the RV \( Y \). Note that \( \| \mathbf{h}_{RR} \|^2 \sim \chi^2_{2M_R} \), \( Z_2 \triangleq \| \mathbf{h}_{SR,1} \|^2 \) is distributed as \( Z_2 \sim \text{Beta}(1, M_R - 1) \), and the cdf of \( X_2 \) can be readily evaluated as

\[
F_{X_2}(t) = \int_0^1 F_{\| \mathbf{h}_{RR} \|^2} \left( \frac{t}{z} \right) f_Z(z) dz,
\]

(56)

where \( \gamma(a, x) = \Gamma(a) - \Gamma(a, x) \). The equality (a) in (56) follows by substituting \( x = 1/z \) and (b) is obtained by using the identity \( \gamma(c, x) = G_{12}^{11} \left( 1, 0 | \begin{array}{c} x \ 0 \end{array} \right) \) and applying the integral identity [29, Eq. (7.811.3)]. Now, using the cdf of RV \( Y \), and substituting (56) into (55) we obtain

\[
F_{\gamma_{MRC}}(z) = 1 - \frac{1}{\Gamma(M_R)} \Gamma(M_R - 1) \int_1^\infty y^{M_R - 1} e^{-y - \frac{1}{c_2}} dy.
\]

(57)

To the best of the authors’ knowledge, the integral in (57) does not admit a closed-form solution. However, (57) can be evaluated numerically. We now look into the high SNR regime to gain more insights. To this end, neglecting the noise term of the first term inside in minimum function in (53), we write

\[
\gamma_{low} > \gamma_{MRC} = \min \left( \frac{1}{\kappa X_2} \frac{\kappa \rho_2}{d_T^2} \| \mathbf{h}_{SR} \|^2 \| h_{RD} \|^2 \right).
\]

(58)

We now present the following proposition.

**Proposition 7:** In the high SNR regime, i.e., \( \rho_1, \rho_2 \to \infty \), with \( M_T = 1 \) the outage probability of the MRC/MRT
scheme can be approximated as
\[
F_{\gamma_{\text{MRC}}}^\infty(z) = 1 - \frac{2}{\Gamma(M_R)} \left( \frac{1}{\kappa T z} \right)^{\frac{1}{\kappa T z}} M_R \left( \frac{d_1 t_2}{\rho_2 \kappa} \right)^{\frac{1}{\kappa T z}} F_{M_R} \left( 2 \left( \frac{d_1 t_2}{\rho_2 \kappa} \right)^{\frac{1}{\kappa T z}} \right).
\]

In most cases, \( k = 0 \) is sufficient for accurate results leading to the following compact expression
\[
F_{\gamma_{\text{MRC}}}^\infty(z) \approx 1 - \left( 1 - e^{-\frac{1}{\kappa T z}} \right) \left( e^{\frac{d_1 t_2}{\rho_2 \kappa}} - \frac{1}{\Gamma(M_T + 1)} \right) \left( \frac{d_1 t_2}{\rho_2 \kappa} \right)^{M_T + k} E_{M_T + k} \left( \frac{d_1 t_2}{\rho_2 \kappa} \right),
\]
while in other cases it can be truncated using few terms up to 10.

At this point, it is important to determine the value of \( \alpha \) that maximizes the throughput. We note that delay-constrained throughput should converge to the ceiling value of \( R_c(1-\alpha) \) when \( P_{\text{out}} \to 0 \). For each beamforming scheme we observe that \( P_{\text{out}} \) is a complicated function of \( \alpha \) and it decreases as the value of \( \alpha \) is increased. However, this will lead to the decrease of the term \( (1-\alpha) \) at the same time. Therefore, an optimal value of \( \alpha \) that maximizes the delay-constrained throughput exists and it can be found by solving the following optimization problem [27, Eq. (18)]
\[
\alpha^* = \arg\max_{0<\alpha<1} R(\alpha).
\]

Given (46), (50), (57) and (63), unfortunately the optimization problem in (67) does not admit closed-form solutions. However, the optimal \( \alpha^* \) can be solved numerically.

VI. NUMERICAL RESULTS AND DISCUSSION

We now present numerical results based on analytical expressions developed and investigate the impact of key system parameters on the performance. The simulations adopt parameters of the 3GPP LTE for small cell deployments [39]. The maximum transmit power of the source node is set to 26 dBm. The energy conversion efficiency is set to be \( \eta = 0.5 \).\(^3\)

A. Instantaneous Throughput

We consider the influence of optimal time-split \( \alpha \) and LI effect on the instantaneous throughput of different beamforming schemes. Fig. 2 shows the instantaneous throughput versus \( \alpha \) of the beamforming schemes for a single time frame channel realizations. There are two groups of curves: the OPA (dashed line) and EPA (solid line) curves. OPA and EPA refer to the cases where source allocates different and equal power levels for energy harvesting phase and the information transmission phase respectively. The results for OPA have been obtained using a two dimensional grid search strategy over \( \alpha \) and the splitting factor that controls the source power for the energy harvesting phase and the information transmission phase. It is clear that the OPA scheme achieves a higher throughput than the EPA scheme over the entire range of the time-split. We can see that the values of the optimal \( \alpha \) calculated by (28), (33), (35), and (39) coincide with the corresponding ones obtained via simulation. Also, as expected, the optimum scheme outperforms all other schemes on all time-split values. In addition, simulation results, not

\(^3\)We note that the typical values for practical parameters used in EH systems will depend on both the system application and specific technology used for implementation of RF energy harvesting circuits.
shown in the figure to avoid clutter, reveal that the values of the optimal $\alpha$ decrease as either the number of relays' receive antennas or the sources' transmit power increases. This is because in these cases the relay node can harvest the same amount of energy in a shorter time. Therefore, more time must be allocated to the information transmission phase in order to improve the system throughput.

B. Outage Probability

Fig. 3 shows the outage probability for the ZF based beamforming schemes with different antenna configurations and for a specific $\alpha = 0.5$ and $d_1 = d_2 = 10$ m. The asymptotic results based on (47) and (51) are also presented. Since the relay is capable of canceling LI, we see that the outage probability of the TZF and RZF beamforming schemes decays proportionally to the diversity orders reported in Proposition 5 and 6, respectively. Comparing the TZF and RZF schemes with the same diversity orders and different receive antenna numbers (i.e., TZF, with $M_R = 1$ and $M_T = 3$, and RZF with $M_R = 3$ and $M_T = 1$) we see that the additional receive antenna could harvest more energy to increase the second-hop SNR and to facilitate information transfer. Moreover, for the case where $M_R = M_T$, RZF achieves a higher array gain. This observation demonstrates that while under some configurations ($M_T = 1$) or ($M_R = 1$) only one form (receive or transmit) of beamforming design can be implemented, when both designs can be applied, the system designer has to carefully decide on the configuration as well as the beamforming design.

Fig. 4 compares the outage probability of the optimum, TZF, RZF, and MRC/MRT schemes with different antenna configurations and for $\alpha = 0.5$ and $d_1 = d_2 = 10$ m. The residual LI strength at the relay is set to be $-50$ dBm. Asymptotic results in (59) and (66) are also provided for the MRC/MRT scheme. The outage performance of the MRC/MRT scheme is almost identical to the optimum scheme at low SNRs, while the ZF-based schemes can achieve almost the same performance of the optimum scheme in the high SNR regime. When the transmit power increases and $\alpha$ remains fixed, an excessive amount of energy will be collected at the relay, which is detrimental for the MRC/MRT scheme since it results in a strong LI effect. Therefore, the outage probability of the MRC/MRT scheme shows an outage floor at high SNRs. However, to some level a proper choice of $\alpha$ can improve the outage performance of the MRC/MRT scheme. Fig. 3 and 4 illustrate that the outage probability of the system depends on the amount of energy harvested through the receive antenna numbers at the relay, source-relay link distance, and the energy conversion efficiency of the deployed energy harvester at the relay. Moreover, it is significantly influenced by the transmit/receive beamforming design at the relay.

C. Delay-Constrained Throughput

Fig. 5 shows the impact of optimal $\alpha$ on the delay-constrained throughput. As expected, the optimum scheme exhibits the best throughput among all beamforming schemes. The superior performance of the optimum scheme is more pronounced especially between 0.4 and 0.8 values of $\alpha$. The highest throughput with optimized $\alpha$ for the
The optimal, RZF, MRC/MRT and TZF schemes are given by 0.557, 0.549, 0.453 and 0.404, respectively. Moreover, we see that each one of the TZF, RZF and MRC/MRT schemes can surpass other beamforming schemes depending on the value of $\alpha$. This observation reveals the existence of various design choices when performance-complexity tradeoff is considered.

Fig. 6 shows the effect of the LI strength on the delay-constrained throughput when optimum $\alpha$ is used. The total number of antennas is $M_R + M_T = 4$. The ZF-based schemes do not suffer from LI, therefore the delay-constrained transmission throughput remains constant. On the contrary, as expected, the delay-constrained transmission throughput of the MRC/MRT scheme decreases as $\sigma^2_{RR}$ and consequently the LI strength increases. When the LI strength is low the ZF-based schemes become inferior as compared to the MRC/MRT scheme. In this region the combination $(M_R = 2, M_T = 2)$ with MRC/MRT scheme exhibits a near optimum performance. Results, not shown for the configuration $(M_R = 3, M_T = 1)$, showed inferior performance as compared to the configuration $(M_R = 2, M_T = 2)$ in all schemes while they showed a superior performance as compared to the configuration $(M_R = 1, M_T = 3)$. Therefore, performance enhancements can be achieved through equal transmit and receive antenna deployment.

As a final cautionary note we would like to express that the above main findings and insights can be further examined by applying specific RF circuitry and power amplifier model. For example, low efficiency of a RF amplifier used for digital communications can be accounted in a detailed analysis. An implication of such use would be that, often the relay will be unable to operate and the price paid would be a high outage probability due to communication blackout periods. Nevertheless, our results provide useful theoretical performance bounds for the studied system and motivates practical interest from the perspective of wireless-powered full-duplex system implementation.

VII. CONCLUSION

In this paper, we studied the instantaneous and delay-constrained throughput of a wireless-powered FD MIMO relay system. We designed optimum linear processing at the relay as well as investigated several suboptimum schemes. The optimal time-split for instantaneous throughput maximization of all schemes was derived. We also presented exact and asymptotic closed-form expressions for the outage probability useful to characterize the delay-constrained throughput. We found that the MRC/MRT scheme can offer a higher instantaneous/delay-constrained throughput as compared to the RZF and TZF schemes, when the LI is significantly canceled, and vice versa. The MRC/MRT scheme can provide a better outage performance at low-to-medium SNRs, while the ZF precoders outperform the former at high SNRs.

APPENDIX A
PROOF OF PROPOSITION 2

Proof: We show that (18) leads to optimum rank-one solution or such solution can be recovered from the optimum solution that has rank higher than one. First note that instead of solving (16) for a given $t$, (18) can be solved for a given $y = \bar{y}$. Such optimization problem is expressed as

$$\max_{W_t, \bar{t}} \quad \bar{t}$$

s.t. \[ \frac{d_1^2}{d_1^2} t \bar{y} \leq ||h_{SR}||^2 \bar{y} - \frac{\kappa \rho_1}{d_1^2} ||h_{SR}||^2 h_{SR}^\dagger h_{RR} W_t h_{RR}^\dagger h_{SR} \]

\[ \bar{y} = \text{tr} \left( W_t \left( I + \frac{\kappa \rho_1}{d_1^2} ||h_{SR}||^2 h_{RR}^\dagger h_{RR} \right) \right) \]

\[ t \leq \frac{\kappa \rho_2}{d_1^2 d_2^2} ||h_{SR}||^2 h_{RD} W_t h_{RD}^\dagger \]

\[ \text{tr} \left( W_t \right) = 1, \quad W_t \succeq 0 \quad (68) \]

where $\bar{y} \in [1, y_{up}]$. Here, $y_{up}$ is the maximum value of $\bar{y}$ which is the maximum eigenvalue corresponding to $R \equiv I + \frac{\kappa \rho_1}{d_1^2} ||h_{SR}||^2 h_{RR}^\dagger h_{RR}$. The optimum $W_t$ is the solution that maximizes $t$ for all $y \in [1, y_{up}]$ for which (68) is feasible.
The optimum \( W_t \) thus obtained will be the same as that obtained by solving (16). The Lagrangian multiplier function for (68) is expressed as
\[
\mathcal{L} (W_t, t, \{\lambda_i\}_{i=1}^{4}) = -t + \lambda_1 \left( \frac{d_t}{\rho_1} \tilde{y} - ||h_{SR}||^2 \right) + \lambda_2 (\text{tr}(W_t R) - \tilde{y}) + \lambda_3 \left( \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 \right) + \lambda_4 (\text{tr}(W_t - 1)) - \text{tr}(Y W_t),
\] (69)
where \( y \geq 0 \) is the matrix dual variable associated with the constraint \( W_t \geq 0 \). Since (68) is convex and Slater condition holds true, the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality. The KKT conditions for (68) are
\[
\frac{\partial \mathcal{L}}{\partial W_t} = 0 \rightarrow Y = \lambda_1 \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 R_t h_{SR} h_{SR}^\dagger H_{RR} + \lambda_2 R + \lambda_4 I - \lambda_3 \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 h_{RD}^\dagger h_{RD} \tag{70}
\]
\[
\frac{\partial \mathcal{L}}{\partial t} = 0 \rightarrow t \frac{d_t}{\rho_1} \tilde{y} + \lambda_3 = 1 \tag{71}
\]
\[
\lambda_1 \left( \frac{d_t}{\rho_1} \tilde{y} - ||h_{SR}||^2 \right) + \lambda_2 (\text{tr}(W_t R) - \tilde{y}) + \lambda_3 \left( \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 \right)
\times \left( ||h_{SR}||^2 R_t h_{SR} h_{SR}^\dagger H_{RR} ||h_{SR}||^2 R_t h_{SR} h_{SR}^\dagger H_{RR} \right) = 0 \tag{72}
\]
\[
\lambda_3 \left( t - \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 \right) h_{RD}^\dagger h_{RD} = 0 \tag{73}
\]
\[
\text{tr}(W_t R) - \tilde{y} = 0 \tag{74}
\]
\[
\text{tr}(W_t - 1) = 0 \tag{75}
\]
\[
\text{tr}(Y W_t) = 0 \rightarrow Y W_t = 0. \tag{76}
\]

The complementary slackness condition (76) means that at KKT optimality, the optimum \( W_t \) lies in the null-space of \( Y \). This means that the rank of \( W_t \) is the nullity of \( Y \). In the following, we analyze the cases in which the optimum \( W_t \) is rank-one (Cases a and c) and not rank-one but the rank-one optimum solution can be recovered from optimum \( W_t \) (Case b).

- **Case a**: \( \lambda_1 = 0, \lambda_3 \neq 0 \) - When \( \lambda_1 = 0 \), the inequality constraint corresponding to (72) is not satisfied with equality, whereas the inequality constraint corresponding to (73) is satisfied with equality since \( \lambda_3 = 0 \) leads to \( \lambda_3 = 1 \) (see (71)). In this case, \( Y \) reduces to
\[
Y = (\lambda_2 + \lambda_4) I + \lambda_3 \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 H_{RR}^\dagger H_{RR} - \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 h_{RD}^\dagger h_{RD}. \tag{77}
\]
It is clear that both \( \lambda_2 \) and \( \lambda_3 \) cannot be equal to zero at optimality. Otherwise, \( Y \) turns to a negative semi-definite matrix contradicting the fact that \( Y \succeq 0 \). In all other possible values of \( \lambda_2 \) and \( \lambda_4 \), it is seen that \( Z \triangleq (\lambda_2 + \lambda_4) I + \lambda_3 \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 H_{RR}^\dagger H_{RR} \) is a full-rank matrix. Now, we can show that the nullity of \( Y \) cannot be greater than one by contradiction. Assume that \( \{u_{y,q}, q = 1, 2\} \in Ns(Y) \) where \( Ns(Y) \) denotes null-space of \( Y \) and let \( aq \| h_{SR} \|^2 h_{RD}^\dagger h_{RD} \). Then,
\[
Y u_{y,q} = Z u_{y,q} - aq u_{y,q} \rightarrow u_{y,q} = Z^{-1} aq u_{y,q}, \forall q. \tag{78}
\]
which shows that \( u_{y,q} \) is an eigenvector of \( Z^{-1} aq \) corresponding to eigenvalue 1. Since \( \text{rank}(Z^{-1} aq) = 1 \), it turns out that \( \tilde{y} \) cannot take a value greater than 1. This shows that the dimension of null space of \( Y \) is 1, and therefore, the rank of \( W_t \) is one.

- **Case b**: \( \lambda_1 \neq 0, \lambda_3 = 0 \) - When optimum \( \lambda_3 = 0 \), the inequality constraint associated with (72) will be satisfied with equality, whereas that associated with (73) will not be satisfied with equality. Note that \( \lambda_3 = 0 \) leads to \( \lambda_1 = \frac{\rho_1}{d_1^2} \). In this case, \( Y \) reduces to
\[
Y = \frac{\rho_1}{d_1^2} \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 H_{RR}^\dagger h_{SR} h_{SR}^\dagger H_{RR} + (\lambda_2 + \lambda_4) I + \lambda_3 \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 H_{RR}^\dagger h_{SR} h_{SR}^\dagger H_{RR}. \tag{79}
\]
Note that a feasible \( Y \) is the one which has at least a nullity of 1, since the optimum \( W_t \) lies in the null-space of \( Y \). Therefore, at optimality both \( \lambda_2 \) and \( \lambda_4 \) should be zero, otherwise \( Y \) in (79) turns to a full-rank matrix which is not feasible. Consequently, \( Y \) reduces to
\[
Y = \frac{\rho_1}{d_1^2} \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 H_{RR}^\dagger h_{SR} h_{SR}^\dagger H_{RR} \text{ which is a rank-one matrix}. \text{ Therefore, the optimum \( W_t \) may not be rank-one. However, we show that optimum rank-one matrix can be recovered from optimum \( W_t \) without loss of optimality.}

Suppose the optimum \( W_t \) has a rank \( r \) where
\[
W_t = \sum_{q=1}^{r} \sigma_q u_{q} u_{q}^\dagger, \sigma_q \text{ and } u_{q}, (q = 1, \cdots, r) \text{ are, respectively, the eigenvalues and eigenvectors of the matrix } W_t. \text{ Furthermore, due to the equality constraint (75), } \sum_{q=1}^{r} \sigma_q = 1. \text{ Substituting eigenvalue decomposition of } W_t \text{ into the condition (76), we find that}
\]
\[
\sum_{q=1}^{r} \sigma_q u_{q}^\dagger \text{Yu}_{q} = 0 \rightarrow u_{q}^\dagger H_{RR}^\dagger h_{SR} h_{SR}^\dagger H_{RR} \text{ Yu}_{q} = 0, \forall q \tag{80}
\]
where the last step is due to the fact that \( Y \succeq 0 \). Moreover, following is due to (80)
\[
\frac{d_t}{\rho_1} = ||h_{SR}||^2 - \frac{1}{\tilde{y}} \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 H_{RR}^\dagger H_{RR} W_t H_{RR}^\dagger h_{SR} \rightarrow t = \frac{\rho_1}{d_1^2} ||h_{SR}||^2. \tag{81}
\]
On the other hand, using the eigenvalue decomposition of \( W_t \), the inequality constraint associated with (73) and equality constraint (74), respectively, yield
\[
t < \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 \sum_{q=1}^{r} \sigma_q u_{q}^\dagger h_{RD} h_{RD} u_{q} \tag{82}
\]
\[
1 + \frac{\kappa_\rho}{d_1^2} ||h_{SR}||^2 \sum_{q=1}^{r} \sigma_q u_{q}^\dagger H_{RR}^\dagger H_{RR} u_{q} = \tilde{y}. \tag{83}
\]
It can be observed from (81) that the optimum value of \( t \) does not depend on \( \tilde{y} \) in the underlying case. An arbitrary \( \tilde{y} \) where \( \tilde{y} > 1 \) (see (83) ) remains optimum.
from a set \( \{ u_i \}_{i=1}^r \) and a specific value of \( \sigma_q \), (82) is not violated. Consider that \( \tilde{q} = \max_q u_i^h b_{Ri} h_{RD} u_i \). Then, the maximum of (82) is achieved by choosing \( u_i \) with \( \sigma_q = 1 \). This choice does not affect the optimum objective value which remains (81). As such, the optimum rank-one matrix recovered from \( W_t \) turns to \( \sigma_q u_i b_{Ri} \).

- Case c: \( \lambda_1 \neq 0, \lambda_3 \neq 0 \) - In this case, both the inequality constraints ((72) and (73)) will be satisfied with equality. As such, \( Y \) is given by (70). As long as at least one of \( \lambda_2 \) and \( \lambda_4 \) is non-zero, \( \lambda_i \frac{\mu_i}{\sigma_i^2} \| h_{SR} \|^2 | h_{Ri} b_{SR} | h_{Ri} h_{RR} + \lambda_2 R + \lambda_4 I \) is a full-rank matrix. On the other hand, \( \lambda_i \frac{\mu_i}{\sigma_i^2} \| h_{SR} \|^2 | h_{Ri} b_{RD} | h_{RD} \) is a rank-one matrix. Consequently, as in Case a, the optimum \( W_t \) can be shown to be a rank-one matrix. Therefore, it is sufficient to show that both \( \lambda_2 \) and \( \lambda_4 \) cannot be zero at the optimality. Towards this end, we use the method of contradiction. Assume that \( \lambda_2 = 0 \) and \( \lambda_4 = 0 \). Then, \( Y \) reduces to

\[
Y = \lambda_1 \frac{\mu_1}{\sigma_1^2} \| h_{SR} \|^2 | h_{Ri} b_{SR} | h_{Ri} h_{RR} \]
\[
- \lambda_3 \frac{\mu_3}{\sigma_3^2} \| h_{SR} \|^2 | h_{Ri} b_{RD} | h_{RD}.
\] (84)

Note that \( Y \) should be positive-semidefinite. Since \( \lambda_1 \) in (84) is the difference between two rank-one matrices, it can be readily shown from Weyl’s inequalities for eigenvalues of sum of Hermitian matrices that \( \lambda_1 \) cannot remain positive-semidefinite except in the case with \( \lambda_3 \frac{\mu_3}{\sigma_3^2} \| h_{SR} \|^2 | h_{Ri} b_{RD} | h_{RD} = 0 \), i.e., \( \lambda_3 \neq 0 \) for non-zero \( h_{SR} \) and \( h_{RD} \). This contradicts the assumption \( \lambda_3 \neq 0 \), which consequently contradicts the assumption that both \( \lambda_2 \) and \( \lambda_4 \) are zero. This completes the proof of the proposition.

**APPENDIX B**

**PROOF OF PROPOSITION 5**

Applying the series expansion of \( \gamma(a, x) \) and \( \Gamma(a, x) \) [29, Eq. (8.354.1) and (8.354.2)] we have

\[
F_{\gamma_{SR}}(z) = 1 - \frac{1}{\Gamma(M_R)} \int_{z_1}^{\infty} \left( 1 - \frac{1}{\Gamma(M_T-1)} \right) x^{M_T-1} e^{-x} dx,
\]

\[
= 1 - \frac{1}{\Gamma(M_R)} \Gamma(M_T-1) + \frac{1}{\Gamma(M_T-1)} \Gamma(M_R)
\]

\[
\sum_{k=0}^{\infty} \left( \frac{d_1 z}{(R_1) \delta} \right)^k \frac{d_2 z}{\kappa} x^{M_T-k} e^{-x} dx,
\]

where \( \mathcal{I}(k) = \int_{z_1}^{\infty} x^{M_T-M_T-k} e^{-x} dx \), that has closed form solution given by

\[
\mathcal{I}(k) = \left\{ \begin{array}{ll}
\frac{d_1 z}{(R_1) \delta} \frac{d_1 z}{(R_1) \delta} - M_T-M_T-k+1 \frac{d_1 z}{(R_1) \delta}, & M_T > M_T-k \\
E_{M_T-M_T-k+1} \left( \frac{d_1 z}{(R_1) \delta} \right), & M_T > M_T-k \\
\frac{d_1 z}{(R_1) \delta} \frac{d_1 z}{(R_1) \delta} - M_T-M_T-k+1 \frac{d_1 z}{(R_1) \delta}, & M_T \
\end{array} \right.
\] (86)

where \( \alpha_n(x) = \int_{0}^{\infty} x^n e^{-x} dx \) [30, Eq. (5.1.5)]. By substituting (86) into (85), and then applying [30, Eq. (5.1.8)] and [30, Eq. (5.1.12)], we get (87) at the top of the next page where \( \psi(1) = -0.57721 \) and \( \psi(n) = \psi(1) + \sum_{m=1}^{n-1} \frac{1}{m} \), for \( n > 1 \) [29, Eq. (9.73)]. In the high SNR regime, i.e., \( \rho_1, \rho_2 \to \infty \), omitting higher order items of the series expansion in (87), the desired result follows after some simple algebraic manipulations.

**APPENDIX C**

**PROOF OF PROPOSITION 6**

Using the series expansion of \( \gamma(a, x) \) and \( \Gamma(a, x) \), we get

\[
F_{\gamma_{SR}}(z) = 1 - \frac{\Gamma(M_R, \frac{d_1 z}{(R_1) \delta})}{\Gamma(M_R)} + \frac{1}{\Gamma(M_R) \Gamma(M_T)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+M_T)} \left( \frac{d_1 d_2 z}{(R_1 \delta)} \frac{z}{\kappa} \right)^{M_T+k} \mathcal{I}_1(k) + \frac{1}{\Gamma(M_R)} \left( \frac{d_1 d_2 z}{(R_1 \delta)} \frac{z}{\kappa} \right)^{M_T-1} \frac{1}{\Gamma(M_T)} \mathcal{I}_2(k),
\] (88)

where \( \mathcal{I}_1(k) = \int_{z_1}^{\infty} x^{M_T-M_T-k-1} e^{-x} dx \) is evaluated as

\[
\mathcal{I}_1(k) = \left\{ \begin{array}{ll}
\left( \frac{d_1 z}{(R_1) \delta} \right)^{M_T-M_T-k} \frac{E_{M_T-M_T-k+1} \left( \frac{d_1 z}{(R_1) \delta} \right),}{M_T > M_T-k + 1} \\
\left( \frac{d_1 z}{(R_1) \delta} \right)^{M_T-M_T-k} \frac{\alpha_{M_T-M_T-k-1} \left( \frac{d_1 z}{(R_1) \delta} \right),}{M_T \
\end{array} \right.
\] (89)

and \( \mathcal{I}_2(k) = \int_{z_1}^{\infty} x^{M_T-M_T-k-1} e^{-x} dx \) is solved as

\[
\mathcal{I}_2(k) = \left( \frac{d_1 z}{(R_1) \delta} \right)^{M_T-M_T-k+1} \frac{E_{M_T-k} \left( \frac{d_1 z}{(R_1) \delta} \right),}{M_T > M_T-k + 1} \] (90)

Substituting (89) and (90) into (88), and expanding the result with the help of [30, Eq. (5.1.8) and Eq. (5.1.12)], and then only selecting the sufficient and ignoring the higher order terms, we arrive at (51).

**APPENDIX D**

**PROOF OF PROPOSITION 7**

Let \( Z_3 = \| h_{SR} \|^2 | h_{RD} |^2 \), and observing that \( X_2 \) and \( Z_3 \) are independent, we have

\[
F_{\gamma_{MC}}(z) = 1 - F_{X_2} \left( \frac{1}{\kappa R_2 z} \right) \left( 1 - F_{Z_3} \left( \frac{d_1 d_2 z}{(R_1 \delta)} \right) \right),
\] (91)

where \( F_{X_2}(\cdot) \) is given in (56) and the cdf of \( Z_3 \) can be derived as

\[
F_{Z_3}(z) = \int_{0}^{\infty} \text{Pr} \left( | h_{SR} |^2 \leq \frac{z}{x} \right) f_{| h_{SR} |^2}(x) dx,
\]

\[
= 1 - \frac{2}{\Gamma(M_R) \kappa} \cdot 2^{\frac{M_R}{2}} K_{M_R}(2\sqrt{z}),
\] (92)

where we used [29, Eq. (3.471.9)] to derive (92). Finally, substituting (56) and (92) into (91) yields the desired result.
\[
F_{\gamma_{\text{TE}}}(z) = 1 - \frac{\Gamma \left( M_R, \frac{d^2 z}{\rho_1} \right)}{\Gamma(M_R)} + \frac{1}{\Gamma(M_R-1) \Gamma(M_R)} \left( \frac{d^2 z}{\rho_1} \right)
\]
\[
\times \left\{ \begin{array}{l}
\frac{1}{(M_R-M_T-k+1)\Gamma(M_R-M_T-k+1)} + \ln \left( \frac{d^2 z}{\rho_1} \right)
\end{array} \right. 
\]
\[
\times e^{-\frac{d^2 z}{\rho_1}(1+\frac{d^2 z}{\rho_1}+\cdots+\frac{d^2 z}{\rho_1})^{M_R-M_T-k}}, \quad M_T \leq M_R-k
\]
\[(87)\]