Influence of clutch lining frictional characteristics upon cold and hot take-up judder

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Influence of Clutch Lining Frictional Characteristics upon Cold and Hot Take-Up Judder

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Abstract

Clutch take-up judder is a torsional vibration phenomenon induced by stick-slip oscillations at the friction lining interfaces between the clutch friction disc and the flywheel and pressure plate surfaces during the process of clutch engagement. This is short-lived transient phenomenon affected by the clutch lining material’s friction characteristics, topography of the mating-sliding surfaces and operational conditions during engagement such as contact pressure, interfacial slip speed and contact temperature. The phenomenon leads to driver discomfort as well as gradual wear of contacting surfaces, even if short-lived. Its frequency is usually reported to be in the range 5-20Hz depending on the vehicle type. The friction characteristics alter in a transient manner during engagement and it should be measured under controlled conditions.

In this paper the measured interfacial friction characteristics together with clamp load variation (contact pressure) under different surface temperatures are included in a multi-degree of freedom dynamic analysis to obtain torsional vibrations of the system, pertaining to take up judder conditions. Such an in-depth investigation has not hitherto been reported in literature. The paper shows that take up stick-slip judder is omnipresent under all clutch engagement conditions, but its poignancy is most evident at cold surface temperatures. It is also shown that the transient judder response has a broader spectral content that is generally understood.

Keywords: Dry automotive clutch; Torsional vibrations; Take-up judder; Stick-slip vibration; Friction lining characteristics

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Effective vehicle frontal area</td>
</tr>
<tr>
<td>$a$</td>
<td>Wavelet scaling parameter</td>
</tr>
<tr>
<td>$b$</td>
<td>Position parameter</td>
</tr>
<tr>
<td>$c$</td>
<td>Damping coefficient</td>
</tr>
</tbody>
</table>
$C_d$  
Aerodynamic drag coefficient

$d_{\text{air}}$  
Density of air

$F_{\text{drag}}$  
Aerodynamic drag

$F_n$  
Normal (clamp) load

$F_{\text{rot}}$  
Rolling resistance

$I$  
Polar mass moment of inertia

$k$  
Stiffness

$m$  
Wave number

$m_v$  
Vehicle mass

$n$  
Engine order harmonic number

$R$  
Effective radius

$R_w$  
Wheel radius

$t$  
Time

$T_e$  
Engine torque

$T_f$  
Friction torque

$T_m$  
Mean engine torque

$T_{\text{pm}}$  
Pulsating engine torque (unbalanced torque)

$T_0$  
Resistive torque

**Greek symbols**

$\eta$  
Non-dimensional time parameter

$\theta$  
Angular displacement

$\mu$  
Coefficient of friction

$\mu_{\text{rot}}$  
Coefficient of rolling resistance

$\phi_{\text{pn}}$  
Initial phase of the nth engine order

$\psi$  
Mother wavelet function
1. Introduction

Self-exited clutch system vibrations or judder are the first rigid body low frequency (3-20Hz) torsional mode of the clutch system during the clutch engagement [1]. These are caused by stick-slip oscillations at the clutch friction lining interfaces with the flywheel and the pressure plate. Jarvis and Mills [2] showed that clutch judder is primarily vibration induced by friction in dry clutches due to the stick-slip phenomenon. The same phenomenon also occurs in wet clutches and is not confined to dry clutches only [3]. The studies by Centea et al [4,5] show that clutch take-up judder is influenced by the gradient of coefficient of friction variation with the relative speed at the clutch lining interfaces (i.e. the \( \mu \)-v characteristics). A negative slope for these characteristics promotes the propensity to judder with a greater degree of severity which is discernible by the vehicle occupants [4-7]. Another issue is hurried release of clutch pedal by the driver, leading to loss of clamp load during the engagement process [4,5,7].

Additionally, the effect of engine torque fluctuations on judder vibrations has also been investigated [8, 9]. These investigations have concluded that judder vibrations exacerbate when the engine angular velocity coincides with the natural frequency of the clutch system. Rabieh and Crolla [9] investigated the coupling of clutch torsional vibration with the fore and aft longitudinal oscillations of the vehicle, termed as shunt. This effect occurs during the clutch engagement and is distinct from the more generally accepted definition of the shunt phenomenon which is as the result throttle tip-in or back-out or sudden release of the clutch pedal with the vehicle in motion [10]. This is as opposed to the longitudinal oscillations accompanying take-up judder in vehicle sf. Shunt is often coupled with first rigid body mode of the entire driveline system, known as shuffle, which is at a lower frequency than clutch judder.

Heat is generated in the contact due to friction, often in a localised manner, referred to hot spotting [7], causing uneven thermal deformation of the clutch friction disc. This effect exacerbates stick-slip vibration and is generally termed as hot judder [11]. Panier et al [12] experimentally investigated hot spotting in train disc brakes, using a full-scale rig. It was determined that the main cause of hot spotting was the gradual distortion of the surface waviness due to high thermal loading. Graf and Ostemeyer [13] studied the effect of material thermal properties such as thermal conductivity, thermal diffusivity and wear on the
formation of hot spots. It was found that in order to reduce the formation of hot spots, thermal conductivity needs to be decreased or alternatively thermal diffusivity increased.

Pin-on-disc tribometer is often used to obtain frictional characteristics of clutch/brake lining materials at different applied pressure, sliding velocity and bulk surface temperature. Ost et al [14] used a pin-on-disc tribometer to study the frictional characteristics of paper-based friction material used in wet clutches. They concluded that the kinetic coefficient of friction increases with decreased contact pressure, but reduces with increased sliding speed. Öztürk et al [15] also used a pin-on-disc for friction and wear tests of dry friction composite materials. Their results showed that the coefficient of friction decreased with both rising slip speed and applied load, but increased with disc temperature up to 300 °C.

It is important to validate the measurements obtained with pin-on-disc with those more representative of friction lining material in situ within a clutch or brake system, often better represented by friction disc test rig as set out in SAE J661a. Marklund and Larsson [16] and Bezzazi et al [17] showed good comparison between pin-on-disc and friction disc test rig frictional characteristics for clutch lining materials.

This paper presents a 4 degree of freedom torsional dynamics model of the clutch system, incorporating the clutch lining interfacial friction characteristics with measured coefficient of friction variations under vehicle clutch-representative slip speed, applied pressures and surface temperatures obtained using a pin-on-disc set up. Engine order vibrations in the form of torque fluctuations are also included in the model.

### 2. Dynamics Model

A multi-degree of freedom lumped parameter model is developed, comprising various inertial components of the clutch assembly. The model for the dry friction single clutch disc is shown schematically in Figure 1. This torsional model comprises 4 degrees of freedom (DOF).

One inertial component represents the engine (inertia $I_1$). The second inertia is that of flywheel and friction disc $I_2$, simplifying the engagement process to that between the friction disc and the pressure plate $I_3$. The vehicle inertia, including the remainder of the drivetrain system is represented by $I_4$.

The engine/flywheel and the clutch input are connected through an equivalent shaft of known stiffness and damping. Similarly, the same provision is made for the output part of the clutch and the driveline/vehicle. The shaft connecting the flywheel/engine with the input part of the clutch is representative of the crankshaft.
The equations of motion for the 4-DOF model are:

\[ I_1 \ddot{\theta}_1 + k_1 (\theta_1 - \theta_2) + c_1 (\dot{\theta}_1 - \dot{\theta}_2) = T_e \]  
(1)

\[ I_2 \ddot{\theta}_2 - k_1 (\theta_1 - \theta_2) - c_1 (\dot{\theta}_1 - \dot{\theta}_2) = -T_f \]  
(2)

\[ I_3 \ddot{\theta}_3 + k_2 (\theta_3 - \theta_4) + c_2 (\dot{\theta}_3 - \dot{\theta}_4) = T_f \]  
(3)

\[ I_4 \ddot{\theta}_4 - k_2 (\theta_3 - \theta_4) - c_2 (\dot{\theta}_3 - \dot{\theta}_4) = -T_0 \]  
(4)

Under clutch clamped conditions, these become:

\[ I_1 \ddot{\theta}_1 + k_1 (\theta_1 - \theta_2/3) + c_1 (\dot{\theta}_1 - \dot{\theta}_2/3) = T_e \]  
(5)

\[ (I_2 + I_3) \ddot{\theta}_2/3 - k_1 (\theta_1 - \theta_2/3) - c_1 (\dot{\theta}_1 - \dot{\theta}_2/3) = -k_2 (\theta_2/3 - \theta_4) - c_2 (\dot{\theta}_2/3 - \dot{\theta}_4) \]  
(6)

\[ I_4 \ddot{\theta}_4 - k_2 (\theta_3 - \theta_4) - c_2 (\dot{\theta}_3 - \dot{\theta}_4) = -T_0 \]  
(7)

Rewriting equations (1)-(4) under sliding condition:

\[ I \ddot{\theta} + c \dot{\theta} + k \theta = T \]  
(8)

The equation set in matrix form becomes:
\[
\begin{bmatrix}
I_1 & 0 & 0 & 0 \\
0 & I_2 & 0 & 0 \\
0 & 0 & I_3 & 0 \\
0 & 0 & 0 & I_4 \\
\end{bmatrix} \dot{\theta} + \begin{bmatrix}
c_1 & -c_1 & 0 & 0 \\
-c_1 & c_1 & 0 & 0 \\
0 & 0 & c_2 & -c_2 \\
0 & 0 & -c_2 & c_2 \\
\end{bmatrix} \dot{\theta} + \begin{bmatrix}
k_1 & -k_1 & 0 & 0 \\
-k_1 & k_1 & 0 & 0 \\
0 & 0 & k_2 & -k_2 \\
0 & 0 & -k_2 & k_2 \\
\end{bmatrix} \theta = \begin{bmatrix}
T_e \\
-T_f \\
T_f \\
T_0 \\
\end{bmatrix}
\]

The state space representation of the above equations is of the form:

\[
\dot{x}(t) = Ax(t) + By(t)
\]

The state variables are:

\[
x_1(t) = \theta_1 \\
x_2(t) = \dot{\theta}_1 \\
x_3(t) = \theta_2 \\
x_4(t) = \dot{\theta}_2 \\
x_5(t) = \theta_3 \\
x_6(t) = \dot{\theta}_3 \\
x_7(t) = \theta_4 \\
x_8(t) = \dot{\theta}_4
\]

And the state variable derivatives are:

\[
\dot{x}_1(t) = \dot{\theta}_1(t) = x_2(t) \\
\dot{x}_2(t) = \ddot{\theta}_1(t) = (1/I_1)(T_e - c_1(x_2(t) - x_4(t)) - k_1(x_1(t) - x_3(t))) \\
\dot{x}_3(t) = \dot{\theta}_2(t) = x_4(t) \\
\dot{x}_4(t) = \ddot{\theta}_2(t) = (1/I_2)(-T_f + c_1(x_2(t) - x_4(t)) + k_1(x_1(t) - x_3(t))) \\
\dot{x}_5(t) = \dot{\theta}_3(t) = x_6(t) \\
\dot{x}_6(t) = \ddot{\theta}_4(t) = (1/I_3)(T_f - c_2(x_6(t) - x_8(t)) - k_2(x_5(t) - x_7(t))) \\
\dot{x}_7(t) = \dot{\theta}_4(t) = x_8(t) \\
\dot{x}_8(t) = \ddot{\theta}_4(t) = (1/I_4)(-T_0 + c_2(x_6(t) - x_8(t)) + k_2(x_5(t) - x_7(t)))
\]

\[
\dot{x}(t) = Ax(t) + By(t), \text{ where:}
\]
The above system of equations describes the system only when the clutch faces are sliding relative to one another. Post engagement the system is modelled as a 3-DOF system (Figure 2). This is because the flywheel/friction disc/pressure plate assembly is clamped (stick state).

The state space representation of the above equations for the engaged state are given by the equations:

\[ \dot{x}(t) = Ax(t) + By(t) \]

The state variables are:

\[ x_1(t) = \theta_1 \]
\[ x_2(t) = \dot{\theta}_1 \]
\[ x_3(t) = \theta_{2/3} \]
\[ x_4(t) = \dot{\theta}_{2/3} \]
\[ x_5(t) = \theta_4 \]
\[ x_6(t) = \dot{\theta}_4 \]

And the state variable derivatives are:

\[ \dot{x}_1(t) = \dot{\theta}_1(t) = x_2(t) \]
\[ \dot{x}_2(t) = \dot{\theta}_1(t) = (1/I_1)(T_e - c_1(x_2(t) - x_4(t)) - k_1(x_1(t) - x_3(t)) \]

\[ \dot{x}_3(t) = \dot{\theta}_2/3(t) = x_4(t) \]

\[ \dot{x}_4(t) = \dot{\theta}_2/3(t) = (1/(I_2 + I_3))(c_1(x_2(t) - x_4(t)) + k_1(x_1(t) - x_3(t)) - c_2(x_4(t) - x_6(t)) - k_2(x_3(t) - x_5(t)) \]

\[ \dot{x}_5(t) = \dot{\theta}_4(t) = x_6(t) \]

\[ \dot{x}_6(t) = \dot{\theta}_4(t) = (1/I_4)(-T_0 + c_2(x_4(t) - x_6(t)) + k_2(x_3(t) - x_5(t)) \]

\[ \dot{x}(t) = Ax(t) + By(t), \text{ where:} \]

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-k_1/I_1 & -c_1/I_1 & k_1/I_1 & c_1/I_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
k_1/(I_2 + I_3) & c_1/(I_2 + I_3) & (-k_1 - k_2)/(I_2 + I_3) & -c_2/(I_2 + I_3) & k_2/(I_2 + I_3) & c_2/(I_2 + I_3) \\
0 & 0 & k_2/I_4 & c_2/I_4 & -k_2/I_4 & -c_2/I_4 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
T_e/I_1 & 0 & 0 & 0 & 0 & -T_0/I_4 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

2.1 Resistive torque

In the above model the resisting torque, \( T_0 \) is calculated as the sum of rolling resistance (\( F_{rot} \)) and aerodynamic drag (\( F_{drag} \)) [18]:

\[
T_0 = R_w(F_{rot} + F_{drag}) \tag{11}
\]

where, \( R_w \) is the wheel radius.

Rolling resistance, \( F_{rot} \) depends on the coefficient of rolling resistance, \( \mu_{rot} \) and the vehicle weight, \( W_v \):

\[
F_{rot} = \mu_{rot} W_v \tag{12}
\]

The aerodynamic drag is:

\[
F_{drag} = \frac{d_{air} V^2 C_d A}{2} \tag{13}
\]
2.2 Engine torque

The engine power torque is subject to fluctuations according to the combustion process (4 or 2 stroke) and engine configuration (number of cylinders and crankshaft flexibility). The resulting vibration superimposed upon steady engine speed yield engine order vibration [19] which should be included in any practical analysis [8]. Engine harmonics are used to describe the variation of the engine torque in time. Therefore, the engine output torque \( T_e \) comprises a mean component \( T_m \) and a fluctuating part \( T_p \) (imbalanced torque) with time. Therefore, the torque is presented by a Fourier series as:

\[
T_e = T_m + \sum_n T_{pn} \sin(n\omega_e t + \varphi_{pn}) \tag{14}
\]

where, \( \omega_e = \dot{\theta}_1 \) and \( n \) is the harmonic order of imbalanced torque, \( \omega_e \) is the nominal angular velocity of engine crankshaft, and \( \varphi_{pn} \) is the initial phase of the \( n^{th} \) order which for a 4-stroke combustion process is [19]: \( n = 0.5, 1, 1.5, 2, 2.5, 3 \ldots \). The engine modelled in this analysis is a 4-stroke 4-cylinder diesel engine. Engine time variable torque resonance occurs when time \( n^{th} \) engine order harmonic \( (n\omega_e) \) for any harmonic order equates natural frequency of the piston-connecting rod- crankshaft sub-system \( (\omega_n) \). In resonance the engine speed is regarded as critical speed \((s_n)\), defined as:

\[
s_n = \frac{s_n}{n} = \frac{60}{2\pi} \frac{\omega_n}{n} \tag{15}
\]

For the selected engine that the \( n^{th} \) engine order harmonic is described as:

\[
\frac{60}{2\pi} \frac{\omega_n}{s_{max}} \leq n \leq \frac{60}{2\pi} \frac{\omega_n}{s_{min}} \tag{16}
\]

The speed range of this diesel engine is 700-3000 rpm and knowing that the natural frequency of the of the system is \( \omega_n = 77.5 \frac{rad}{s} \) and substituting this into equation (16), the engine orders will be \( n = 0.24 \sim 1.06 \), hence \( n_1 = 0.5 \) and \( n_2 = 1 \). As this is a 4 cylinder 4-stroke engine the cylinder firing phase variation is \( \frac{2\pi}{4} \) so the initial phase for the first order \( (\varphi_{p1}) \) is set to zero and the second order \( (\varphi_{p2}) \) to \( \frac{\pi}{2} \). The mean component \( (T_m) \) of the torque is assumed to be constant at a value of 300 Nm and the fluctuating parts \( T_{n1} = T_{n2} = 50Nm \) and therefore equation (14) can be rewritten as:

\[
T_e = 300 + 50 \sin(0.5\omega_e t) + 50 \sin \left( \omega_e t + \frac{\pi}{2} \right) \tag{17}
\]

2.3 Friction torque

During the transient sliding phase of the clutch engagement different interfacial slip velocities occur between the friction lining and the flywheel and pressure plate. This leads to the
generation of friction. The friction torque calculated in the area of an annular surface similar to that of a clutch pad:

$$T_f = \iint_A \frac{\tau_f}{A} dA = \iint_A \frac{\tau F_n \mu}{\pi(r_o^2 - r_i^2)} r dr d\theta = \frac{F_n \mu}{\pi(r_o^2 - r_i^2)} \int_{r_i}^{r_o} \int_0^{2\pi} r^2 dr d\theta =$$

$$= \frac{2F_n \mu (r_o^2 - r_i^2)}{3(r_o^2 - r_i^2)} = \frac{2}{3} R F_n \mu$$  \hspace{1cm} (18)

where, \(R = \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\) is the effective radius of the clutch \([4, 5]\), \(f\) is friction, \(A\) the area of the clutch, \(r_o\) and \(r_i\) are the inner and outer radii of the friction lining. \(\mu\) is the coefficient of friction and \(F_n\) is the applied normal load (generally termed as clutch clamp load) (Figure 3).

![Figure 3: Normalized clutch pressure during clutch engagement](image)

2.4 Stick-slip algorithm

The generated clutch friction torque varies transiently according to the stick-slip oscillatory behaviour during the clutch engagement process. This is of course a function of clamp load, contact area and coefficient of friction, which itself is a function of lining material, contact pressure, generated temperature and contact kinematics. Therefore, the coefficient of friction must be determined through experimentation. Under sliding conditions kinetic coefficient of friction varies transiently with in situ conditions. Static coefficient of friction occurs at the onset of sliding post stiction, whilst at stiction there is no generated friction. Thus:

$$T_f = \begin{cases} 
\frac{2}{3} R F_n \mu & \text{if } (\omega_2 - \omega_3) > 0 \\
-\frac{2}{3} R F_n \mu & \text{if } (\omega_2 - \omega_3) < 0 \\
0 & \text{if } (\omega_2 - \omega_3) = 0 
\end{cases}$$  \hspace{1cm} (19)

where, \(\omega_2 = \dot{\theta}_2\) and \(\omega_3 = \dot{\theta}_3\) are the angular velocities of the clutch disc and the pressure plate respectively. \(\mu=0\) under stiction condition and varies transiently with clutch interfacial conditions under slip condition. During the engagement process the coefficient of friction alters until a full clamp load is achieved.
In addition:

\[ T_2 = k_1(\theta_2 - \theta_1) + c_1(\dot{\theta}_2 - \dot{\theta}_1) \]  

(20)

\[ T_3 = k_2(\theta_3 - \theta_4) + c_2(\dot{\theta}_3 - \dot{\theta}_4) \]  

(21)

\( T_2 \) is the applied torque at input to the clutch and \( T_3 \) is the transmitted torque from the clutch to the transmission system.

3. Experimental Setup and Results

The interfacial coefficient of friction varies according to material of lining type, surface topography of the contacting pair as well as the operating conditions; clamp load, interfacial slip speed and contact temperature. Therefore, for a given lining material, its topography and that of the pressure plate, the variables affecting the kinetic coefficient of friction are the operating conditions. Therefore, it is important to measure the coefficient of friction, using a rig set up under representative vehicle clutch conditions. In this paper an appropriately pin-on-disc tribometer is used for this purpose. Pin-on-disc is a widely used standard apparatus for tribological assessment of material pairs in counterformal contact. Measurements are carried out in line with standard test procedures highlighted in ASTM G99 or DIN 50324.

Figure 3 shows the experimental set up, comprising a pin-on-disc apparatus. The pin-on-disc comprises a disc, made of the same stock as the pressure plate and subjected to the same surface treatment (i.e. yielding the same surface topography as a new pressure plate). The disc is rotated by an AC induction motor. The pin is attached to an arm instrumented with a strain gauge rosette, calibrated to measure the tangential applied force transmitted from the pin-disc contact. Signal conditioning and data acquisition from the strain gauges is carried out using a National Instrument cDAQ-9178 chassis, housing a specialised NI-9237 module with a 4-channel analogue input device with USB PC connectivity.

![Figure 3: Fully instrumented pin-on-disc rig](image-url)
A square piece of new friction lining material, with precisely measure topography is cut and firmly horizontally attached to the pin, making contact with the rotating surface of the disc. A vertical load is applied onto the pin, resulting in the same contact average (Pascal) pressure as the clamp load variation during the clutch engagement process. The pin contacts the surface of the rotating disc at various track radii in order to obtain representative slip speeds, occurring during the clutch engagement process. Finally, the disc is heated through use of a copper disc placed underneath disc with three 100W heating cartridges. The temperature of the contact is monitored by a K-type thermocouple and maintained steadily by a temperature control unit. Heat is conducted through to the test disc so that its bulk surface temperature can be varied according to different interfacial designed temperatures for different vehicle conditions.

National Instruments LabView is used to process and record friction measurements made during testing to spread sheet-friendly data files for post-processing and analysis.

### 3.1 Experimental measurements

The test conditions were so chosen in order to represent the actual applied pressure, sliding velocity and temperature in a real clutch system during engagement. The manufacturer provided information about the maximum clamp pressure during clutch engagement (Figure 3). Based on that the size of specimen, this was adjusted so that it would replicate similar contact pressures as encountered in real clutch applications. It was assumed that the sliding velocity reduces linearly with respect to the slipping time, \( t_s \). This is expressed as:

\[
\omega_s = \omega_0 \left( 1 - \frac{t}{t_s} \right)
\]  

(22)

This allows calculation of the changes in flywheel speed during the engagement process. It is in all cases that the sliding time (time of engagement process/clutch pedal actuation) to be 0.5 seconds. Equation (22) can be adjusted to simulate different clutch actuation times by varying \( t_s \). The starting flywheel speed (\( \omega_0 \))is the idling speed of the engine to replicate idle to first gear transition at an initial angular velocity of 100 rad/s. Temperature range was selected based on manufacturer’s recommendations for optimum clutch lining performance.

Measurements were performed to determine the clutch lining frictional characteristics under various operating conditions. Figure 4 shows the variation of coefficient of friction with sliding velocity at different disc bulk temperatures. It should be noted that the contact temperature (flash temperature) would be in excess of the measured bulk temperature. However, this is a good indication of the clutch lining material performance at nominal working temperatures of the clutch.
Figure 4: Coefficient of friction variation with interfacial slip speed various disc temperatures: (a) 20°C, (b) 40°C and (c) 60°C and (d) 90°C

The results in Figure 5 are indicative of the various stages of the engagement process. As the two surfaces are clamped the initial temperature is considered to be 20°C as shown in Figure 5(a). However, as the engagement proceeds further, the temperature in the clutch contact rapidly rises. The results in Figures 5 (b) to (d) show the variation of coefficient of friction at other clutch operating temperatures, which are typical of temperatures of the clutch system under consideration.

The coefficients of friction are shown in Table 1 for various stages of the engagement process as the slip speed alters with increasing levels of clamp load to fully locked condition. This is condensed version specifically extracted for the conditions of the clutch dynamics encountered in the current analysis. A regression analysis is used to for the data presented in Table 1.

Table 1: Variation of coefficient of friction with slip speed, $v$ at different temperatures

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Coefficient of friction extrapolated equation ($\mu$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20°C</td>
<td>$0.48 - 0.021v$</td>
</tr>
<tr>
<td>40°C</td>
<td>$0.49 - 0.0191v$</td>
</tr>
<tr>
<td>60°C</td>
<td>$0.48 - 0.0178v$</td>
</tr>
<tr>
<td>90°C</td>
<td>$0.47 - 0.0167v$</td>
</tr>
</tbody>
</table>
In all these formulae of the form: \( \mu = \mu_s - mv \), where the intercept \( \mu_s \) is the static coefficient of friction and \( m \) is the gradient of kinetic coefficient of friction with slip speed. Two important observations can already be made. Firstly, the static coefficient of friction hardly alters with temperature for the fresh friction lining material. This of course will alter with wear. Secondly, the slope of kinetic coefficient of friction is negative with slip speed under all conditions. According to the findings of Centea et al [4, 5] this indicates a propensity to judder. However, as they showed take-up judder is only discernible with significant torsional oscillations of the clutch system.

4. Numerical Results

The system of the differential equations corresponding to the sliding (1- 4) and for the fully locked condition (5-7) are solved numerically using the ode45 solver in MATLAB. The condition simulated is at an engine speed of 960 rpm or the initial angular velocity of 100 rad/sec. This is the idle engine speed at the instance of clutch engagement. During the engagement process, the engine speed drops (inertial components \( I_1 \) and \( I_2 \) as that of the driven inertial components; \( I_3 \) and \( I_4 \) increase, ideally to conform at full clutch clamp load.

The experimentally measured coefficients of friction (Table 1) are used to determine the instantaneous friction torque \( T_f \) during the engagement process as the slip speed alters with increasing clamp loading and at different assumed aforementioned bulk clutch disc temperatures. All model parameters are listed in Table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torsional inertia, ( I_1 )</td>
<td>1.5</td>
<td>kg/m²</td>
</tr>
<tr>
<td>Torsional Inertia, ( I_2 )</td>
<td>0.15</td>
<td>kg/m²</td>
</tr>
<tr>
<td>Torsional Inertia, ( I_3 )</td>
<td>0.015</td>
<td>kg/m²</td>
</tr>
<tr>
<td>Torsional Inertia, ( I_4 )</td>
<td>8</td>
<td>kg/m²</td>
</tr>
<tr>
<td>Damping coefficient, ( c_1 )</td>
<td>100</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>Damping coefficient, ( c_2 )</td>
<td>5</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>Stiffness coefficient, ( k_1 )</td>
<td>40000</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>Stiffness coefficient, ( k_2 )</td>
<td>10000</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>Wheel radius, ( R_w )</td>
<td>0.34</td>
<td>m</td>
</tr>
<tr>
<td>Vehicle mass, ( m_v )</td>
<td>2500</td>
<td>kg</td>
</tr>
</tbody>
</table>
The table below lists the values of various parameters used in the analysis:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of rolling resistance, $\mu_{rol}$</td>
<td>0.015</td>
</tr>
<tr>
<td>Density of air, $d_{air}$</td>
<td>1.2922 kg/m$^3$</td>
</tr>
<tr>
<td>Effective frontal area</td>
<td>2.8 m$^2$</td>
</tr>
<tr>
<td>Aerodynamic drag coefficient, $C_d$</td>
<td>0.37</td>
</tr>
<tr>
<td>Inner radius of clutch, $r_i$</td>
<td>0.0925 m</td>
</tr>
<tr>
<td>Outer radius of clutch, $r_o$</td>
<td>0.135 m</td>
</tr>
</tbody>
</table>

Figure 5 shows the variation of angular velocities of the engine/flywheel/friction disc assembly (driver in this model), and the pressure plate and vehicle inertia (the driven) during the process of clutch engagement. The oscillations prior to the take-up are due to the stick-slip oscillations of the driven inertia as the result of instantaneous changes in the kinetic coefficient of friction.

![Figure 5](image)

**Figure 5:** Clutch engagement process for coefficient of friction of four different temperatures:
(a) 20°C, (b) 40°C and (c) 60°C and (d) 90°C
The oscillations are better observed in the inset to all the figures, and reduce significantly with increasing clutch surface temperature. This is in line with experience of increased take-up judder under cold vehicle conditions. Post clutch take-up judder (with fully clamped clutch), the angular velocities of the driving and driven inertias coincide, with smaller oscillations as the result of transmitted engine order vibrations.

The clutch engagement duration is approximately 0.55 s (time to reach full clamp load with full release of the clutch pedal). If the driver releases the clutch in a shorter period (hurried clutch pedal release), full clamp load is not achieved, resulting in clutch judder as shown by Centea et al [4,5]. All the simulation carried out follow the recommended clutch pedal movement for the simulated vehicle.

There is lower frequency content at smaller amplitudes of oscillation than those highlighted in the insets to figure 5. Therefore, to observe the broader band response of the system during the engagement process a time-frequency analysis is the most appropriate. For this purpose continuous wavelet transform (CWT) is carried out in order to ascertain the transient spectral response during the engagement process. The function used for the CWT:

$$W_\psi(a, b; f(t), \psi) = \int_{-\infty}^{\infty} f(t) \sqrt{a} \psi^* \left( \frac{t-b}{a} \right) dt$$

where, $\psi$ is the mother wavelet and $*$ denotes the complex conjugate of the function. $a$ is the wavelet scale parameter, $b$ the position parameter and $f(t)$ is the analysed data series. The mother wavelet used in this analysis is the Morlet or Gabor wavelet function:

$$\psi(\eta) = \pi^{-1/4} e^{imb} e^{-\eta^2/2}$$

where, $m$ is the wavenumber and $\eta$ is a non-dimensional time parameter. The input parameters are $m$ and $b$ which in this case are 8 and 35 respectively.

Figure 6 shows the CWT of the oscillations prior to the fully clamped clutch at 0.55 s. Firstly, it can be seen that the amplitude of torsional vibrations reduce with increased bulk surface temperature of the clutch. Secondly, an interesting observation can be made, indicating that unlike the widely reported (predicted or measured) spectral content of judder being in the range 5-20 Hz, there is a broader range of frequencies, particularly at the lower clutch surface temperatures. At the normal steady state clutch surface temperature of 90°C, the spectrum of vibration is dominated by the frequency range below 25 Hz. At the low surface temperature of 20°C there are momentary sharp oscillations at higher frequencies as well. These are discerned poignantly by the driver under cold vehicle start and particularly at partial clamp load (also observed in the inset to figure 5(a)). An important point to note is that the driver discerns the clutch judder response due to fore and aft motion of the vehicle. This is because of coupling between the torsional oscillations of the clutch and the axial movements of the powertrain system. Not all the torsional oscillation frequencies couple with the fore and aft motion of the vehicle. However, the current model does not include the longitudinal degree of
freedom to note this point. Centea et al [4,5] show this with a more complex constrained multi-body dynamic analysis.

![Continuous Wavelet Time-Frequency Spectrum](image1)

![Continuous Wavelet Time-Frequency Spectrum](image2)

Figure 6: Continuous wavelet spectrum of the torsional vibrations during clutch engagement for two temperatures: (top) 20°C, and (bottom) 90°C

5. Conclusions

The paper presents a dynamic model for the investigation of clutch take up judder phenomenon. The interfacial friction of the clutch lining material in interactions with the surface of pressure plate is obtained under various bulk surface temperatures and with clutch clamp load variation. The resistive forces on the vehicle during clutch actuation are also included to represent realistic vehicle conditions. The input engine torque, including its imbalance due to engine order oscillations are also included in the analysis. Therefore, a comprehensive analysis, representative of various vehicle conditions is carried out. The results show the inherent nature of judder phenomenon with a greater propensity to stick-slip variations at lower bulk clutch face temperatures. It also shows the transient broader-band spectral content at lower clutch face temperatures during the take up engagement.
Acknowledgement

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References


