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Measurement of vibrational energy and point mobility of a beam subjected to moment excitation using a finite difference approximation


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Abstract:
Moment excitation is often neglected in structural vibration analysis due to difficulties in measuring the applied moment and the resulting wave motion in the structure. Further, it is often assumed that moment induced vibrational energy is only significant in the high frequency region. However, recent studies have shown that moment excitation should be included in vibrational analysis at all frequencies when the source location is in close proximity to a structural discontinuity.

In this paper a novel method is presented to measure the point mobility and resulting vibrational energy of a beam subjected to moment excitation. The proposed method utilises a finite difference approximation to calculate the rotational motion of the beam at the point of excitation. Moment excitation is induced by a specially designed impact rig which applies two equal and opposite forces on two moment arms that are
perpendicularly attached to the beam. It is shown that, using the newly developed technique, the measured point mobility follows the trend of the equivalent theoretical structure. The technique also showed good agreement over a wide frequency range between the measured input energy and measured transmitted flexural wave energy along the beam.

Keywords: structural vibration, moment excitation, flexural wave motion, finite difference approximation

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**NOTATION**

- $a$: translational acceleration
- $a_0$: approximated transverse acceleration at point of excitation
- $a_1$: measured acceleration from accelerometer one
- $a_2$: measured acceleration from accelerometer two
- $A$: cross-sectional area of beam
- $d$: distance between impact forces $F_1$ and $F_2$
- $E$: Young’s modulus
- $f$: frequency
- $F$: general impact force
- $F_1$: impact force at location one
- $F_2$: impact force at location two
- $G(a_1,a_2)$: cross-spectral density of transverse acceleration $a_1$ and $a_2$
- $G(F,F)$: auto-spectral density of impact force on moment arm
- $G(F,a)$: cross-spectral density of impact force and transverse acceleration
\( G(F_1,a_1) \) cross-spectral density of impact force \( F_1 \) and transverse acceleration \( a_1 \)
\( G(F_2,a_2) \) cross-spectral density of impact force \( F_2 \) and transverse acceleration \( a_2 \)
\( G(F,\ddot{\theta}l\ddot{t}) \) cross-spectral density of impact force and angular velocity
\( G(M,M) \) auto-spectral density of applied moment
\( G(M,\ddot{\theta}l\ddot{t}) \) cross-spectral density of moment and angular velocity
\( I \) second moment of area of the beam’s cross section
\( J \) rotary inertia of moment arms
\( k \) flexural wavenumber
\( m \) mass of moment arms
\( M \) applied moment
\( M_{\text{corrected}} \) corrected moment
\( M_0 \) moment amplitude
\( P_{\text{in}}(f)_{Fa} \) input energy due to force excitation in frequency domain
\( P_{\text{in}}(f)_{M} \) input energy due to moment excitation in frequency domain
\( (P_{\text{in}}(f)_{M})_{\text{corrected}} \) corrected input energy due to moment excitation in frequency domain
\( P_{\text{trans}}(f) \) transmitted energy in frequency domain
\( (P_{\text{trans}}(f))_{\text{corrected}} \) corrected transmitted energy in frequency domain
\( t \) time
\( u(x,t) \) spatial and temporal beam displacement
\( x \) spatial distance in x-direction along beam
\( \Delta x \) spacing between the accelerometers
\( Y(f)_{F} \) force point mobility in the frequency domain
\( Y(f)_{M} \) moment point mobility in frequency domain
\( Y_{\theta\theta\theta\ddot{t}L,F} \) cross mobility between rotational velocity and applied force
\( Y_{\ddot{u}u\ddot{t}t,M} \) cross mobility between transverse velocity and applied moment
\( (Y(f)_{M})_{\text{corrected}} \) corrected moment point mobility in frequency domain
Moment point mobility of an infinite beam

\( Y_{mm}(\omega) \)  

wavelength

\( \lambda \)  

angular displacement

\( \theta \)  

angular displacement at the excitation location

\( \theta_0 \)  

beam density

\( \rho \)  

circular frequency

\( \omega \)  

transverse velocity

\( \dot{u} \)  

angular velocity

\( \theta \)  

angular acceleration

\( \frac{\partial^2 \theta}{\partial t^2} \)  

Special symbols

\( \Re \)  

real part

\( \Im \)  

imaginary part

\( j \)  

complex unity \( \sqrt{-1} \)

1 INTRODUCTION

Moment excitation has often been neglected within the field of structural analysis in the past as it was assumed that moment excitation was only significant in the high frequency region. Practical difficulties in measuring the applied moment and resulting rotational displacement have also been a reason to exclude this subject. However, recently it has been shown that moment induced vibration at low frequencies is important and should be included in the analysis when the source of excitation is in close proximity to discontinuities [1,2] and especially when the transverse motion at a discontinuity is constrained [3,4].
In references [1,2] Yap and Gibbs measured moment induced energy at the interface of a machine and the supporting structure using the reciprocal method, an approach which made it unnecessary to measure directly the applied moment at the point of contact between the machine and the receiver. To verify the approach measurements on a centrifugal fan connected to a concrete plate were made for single-point force and moment contact.

Vibrational energy transmissions to machinery supporting structures subjected to multi-point force and moment excitation systems were studied by Koh and White in references [5,6], where driving point mobility functions of uniform beam and rectangular plates were derived. Also in references [5,6] measured mobilities on a clamped-simply supported beam and clamped-free-simply supported-free rectangular plate were compared to the plate’s theoretical response determined by the Raleigh-Ritz method. However, difficulties occurred with measurement of the rotational response at lower frequencies and measurement of the applied moment excitation.

An investigation of moment excited beam structures using T-and I-shape exciter configurations was presented in references [7,8]. Specifically it was shown that errors in the measurement of moment mobility arise in two ways: firstly errors in the rotation caused by unwanted excitation; and secondly errors in the measurement of the moment due to the rotational inertia of the moment exciter. The unwanted excitations arise because of the mass of the moment exciter and any differences between the two forces used to create the moment. It was suggested that the I-shape exciter configuration is the better choice when exciting structures by an applied moment because of the configuration’s lower mass and because the line of action of the applied force couple being in the direction of low mobility.

Champoux et al. [9] achieved almost pure moment excitation of a simply supported aluminium plate. Two impact hammers acting parallel to each other and in opposite directions were employed to generate a force couple separated by a certain distance. With
this method, different force separation distances were used to measure a wide range of frequencies. Direct measurement of moment mobility using a prototype moment actuator was reported by Petersson in reference [10]. Potential sources of error due to the presence of a non-zero cross mobility when using a pair of matched shakers, a shaker lever system or the Petersson moment actuator are considered by Jianxin and Gibbs in reference [11].

The aim of this paper is to present a novel finite difference based technique to measure the input energy and point mobility of a straight rectangular section beam subjected to moment excitation. In Section 2 existing theoretical expressions for the mobility of an “infinite” beam subjected to moment excitation are presented. From these equations a set of expressions are derived which enable measurement of the input energy and moment point mobility using a finite difference approximation for the rotational velocity at the excitation location. In Section 3 the experimental apparatus and measurement method are described. In particular, the specially designed moment excitation rig is described as well as the experimental “infinite” beam used for the measurements. In Section 4 a comparison is shown between the measured moment point mobility of the beam and the corresponding theoretical value over a wide range of frequencies. A comparison is also shown between the measured energy input to the structure due to moment excitation and the measured energy transmitted away from the excitation location by the resulting flexural wave motion. Section 5 summarises the main findings of the research.

2 THEORY

2.1 Point mobility of an “infinite” beam due to moment excitation
Real, finite structures are typically subjected to both force and moment excitation. Thus, in general coupling occurs between the transverse velocity, \( \partial u / \partial t \), and the applied moment, \( M \), as well as coupling between the rotational velocity, \( \partial \theta / \partial t \), and the applied force, \( F \). In matrix form this can be expressed as:

\[
\begin{bmatrix}
\frac{\partial u}{\partial t} \\
\frac{\partial \theta}{\partial t}
\end{bmatrix} =
\begin{bmatrix}
Y_F & Y_{uM} \\
Y_{\theta F} & Y_M
\end{bmatrix}
\begin{bmatrix}
F \\
M
\end{bmatrix}
\]  

(1)

where, \( Y_{uM} \), and, \( Y_{\theta F} \), are the cross mobility terms. Hence, the rotational velocity has components arising from both the applied force, \( F \), and the applied moment, \( M \):

\[
\frac{\partial \theta}{\partial t} = Y_{\theta F} F + Y_M M
\]  

(2)

It was shown in reference [11] that an estimate of the moment point mobility formed from the ratio, \( \partial \theta / \partial t / M \), will be in error due do the presence of the cross mobility term, \( Y_{\theta F} \), and any unwanted force, \( F \). Thus,

\[
\frac{\left( \frac{\partial \theta}{\partial t} \right)}{M} = Y_{\theta F} \frac{F}{M} + Y_M
\]  

(3)

It was noted in reference [12] that the coupling cross mobility terms vanish in only a few special cases, one of which is the moment excited infinite beam. Thus, for a moment excited infinite beam the point mobility, \( Y_{\alpha M} \), can be formed using the measured rotational velocity, \( \partial \theta / \partial t \), and the applied moment, \( M \). Hence,

\[
Y_{\alpha M} = \frac{\left( \frac{\partial \theta}{\partial t} \right)}{M}
\]  

(4)
Reference [12] also derives the point impedance of a moment excited infinite beam. In terms of mobility this can be expressed as:

$$Y_{M}(\omega) = \frac{(1+j)\omega}{4Elk}$$  \hfill (5)

### 2.2 Finite difference approximation of the angular velocity

Measurement of the moment mobility as well as moment induced energy input to the beam relies upon accurate measurement of the applied moment, $M$, and the resulting angular velocity, $\partial \theta / \partial t$, where, $\theta$, is the angular displacement. In principle, an applied moment can be calculated directly by multiplying the measured forces, $F_1$, and, $F_2$, by half the perpendicular length, $d$, the forces are separated by. However, the angular velocity is not measured directly but approximated by using a finite difference technique. In the proposed method, the measured accelerations, $a_1$, and, $a_2$, from two closely spaced accelerometers around the point of moment excitation (see Figure 1) are used to estimate the translational acceleration, $a_0$, at a point midway between the accelerometers. For small displacements, the angular acceleration, $\partial^2 \theta / \partial t^2$, is approximately equal to the first spatial derivative of the transverse acceleration, $a$, hence,

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{\partial a}{\partial x}$$  \hfill (6)

Time integration of the spatially differentiated transverse acceleration, $\partial a / \partial x$, yields the angular velocity, $\partial \theta / \partial t$. The spatial derivative of the measured acceleration, $\partial a / \partial x$, is approximated by using the finite difference of two closely spaced accelerometer signals. Applying a forward difference of the first order to the two accelerometer signals, $a_1$, and, $a_2$, gives the angular acceleration at a point midway between the two transducers, i.e.
\[
\frac{\partial^2 \theta_0}{\partial t^2} = \frac{\partial a_a}{\partial x} \approx \frac{a_2 - a_1}{\Delta x}
\]

where, \(\Delta x\), is the spacing between the two acceleration sensors. Multiplying by \(1/j\omega\) for integration purposes, the angular velocity, \(\partial \theta_0/\partial t\), at the point of excitation is given by:

\[
\frac{\partial \theta_0}{\partial t} \approx \text{Re} \left\{ \frac{1}{j\omega} \left( \frac{a_2 - a_1}{\Delta x} \right) \right\}
\]

Figure 1 displays the configuration of the two closely spaced accelerometers, \(a_1\), and, \(a_2\), located at the point of excitation. The spacing of the accelerometers is crucial to the range of frequencies that can be measured when using a finite difference approximation. Redman-White [13] suggested that the accelerometers should be placed within a range of 0.15-0.2 of the wavelength, \(\lambda\), of the frequency to be considered.

2.3 Measurement of the moment point mobility of a beam using spectral density functions

Using the well known \(H_1(f)\) estimator the moment point mobility can be calculated from the ratio of the cross-spectral density between the applied moment, \(Y(f)_M\), and the resulting rotational velocity, \(G(M,\partial \theta/\partial t)\), and the power spectral density of the applied moment, \(G(M,M)\), thus,

\[
Y(f)_M = \frac{G\left(M, \frac{\partial \theta}{\partial t}\right)}{G(M,M)}
\]

The power spectral density of the applied moment can be calculated from the two impact force signals and the distance of the moment arms, hence,

\[
M = \frac{d}{2}(F_1 + F_2)
\]
If it is assumed that the magnitude of both forces of the applied moment are equal, then $F_1 = F_2 = F$, and the power spectral density of the applied moment is given by:

$$G(M, M) = d^2 G(F, F) \quad (11)$$

Thus, the cross-spectral density between the applied moment and the resulting rotational velocity, $G(M, \partial \theta / \partial t)$, at the excitation location can be calculated using the cross-spectral density between the applied forces and the resulting velocity $\partial \theta / \partial t$, hence,

$$G \left( M, \frac{\partial \theta}{\partial t} \right) = dG \left( F, \frac{\partial \theta}{\partial t} \right) \quad (12)$$

In the approach adopted in this paper the rotational velocity, $\partial \theta / \partial t$, is estimated using a finite difference approximation between two closely spaced accelerometers, given by equation (8). Applying equation (8) to equation (12) above, gives:

$$G \left( M, \frac{\partial \theta}{\partial t} \right) = \frac{d}{j 2 \pi f \Delta x} \left\{ G(F, a_2) - G(F, a_1) \right\} \quad (13)$$

In the experimental method described in section 3 later, the cross-spectral density functions are calculated using the physically closest force transducer to each response accelerometer, thus,

$$G \left( M, \frac{\partial \theta}{\partial t} \right) = \frac{d}{j 2 \pi f \Delta x} \left\{ G(F_2, a_2) - G(F_1, a_1) \right\} \quad (14)$$

The moment point mobility is then estimated by substituting equation (11) and (14) into equation (9) to give:

$$Y(f)_{M} = \frac{I}{j 2 \pi f d \Delta x} \frac{G(F_2, a_2) - G(F_1, a_1)}{G(F, F)} \quad (15)$$

One error when using the finite difference approximation is the underestimation of the true value due to the first spatial derivative approximation of the transverse acceleration, $a$. In reference [13], Redman-White suggests a correction factor for vibrational power
measurements, $k \Delta x / \sin(k \Delta x)$, which compensates for the finite difference approximation. When applied to equation (15) above, this correction factor leads to the following expression for a corrected measurement of the moment point mobility:

$$
(Y(f)_m)_{corrected} = \frac{k}{2 \pi d \sin(k \Delta x)} \frac{G(F_2, a_z) - G(F_1, a_z)}{G(F, F)}
$$

### 2.4 Measurement of the energy input to a beam subjected to moment excitation

One method used to determine the energy input to a beam by an impact, $P_{in}(f)_F$, is to measure the real part of the cross-spectral density between the applied force and resulting velocity, $P_{in}(f)_F = \Re \{G(F, v)\}$ [14,15]. Using the applied force signal and the time integrated transverse acceleration signal, vibrational energy input to a beam due to force excitation can be written as [15]:

$$
P_{in}(f)_F = \frac{1}{2 \pi f} \Im \{G(F, a)\}
$$

For moment excitation of the structure the corresponding quantities are applied moment and rotational velocity at the excitation location. Hence, the energy input to a beam by moment excitation can be calculated by taking the real part of equation (12), thus,

$$
P_{in}(f)_M = \Re \left\{ G \left( M, \frac{\partial \theta}{\partial t} \right) \right\}
$$

Applying the finite difference correction factor, $k \Delta x / \sin(k \Delta x)$, equation (18) can be written as:

$$
(P_{in}(f)_M)_{corrected} = \Re \left\{ \frac{kd}{2 \pi f \sin(k \Delta x)} \{G(F_2, a_z) - G(F_1, a_z)\} \right\}
$$
2.5 Measurement of the vibrational energy transmitted away from the excitation location

A well known technique to measure bending wave vibrational energy in the far field of a beam is the so called two-accelerometer technique [16]. In this technique, the cross-spectral density between two accelerometers, $G(a_1,a_2)$, is measured and the transmitted vibrational energy, $P_{trans}(f)$, is then calculated from the imaginary part using the following expression:

$$P_{trans}(f) = 2 \sqrt{\rho AEI} \Delta x \frac{\Im[G(a_1,a_2)]}{(2\pi)^2 f^2}$$

(20)

where, $A$, is the cross-sectional area, $\rho$, the density, $EI$, the bending stiffness of the beam, and, $\Delta x$, is the spacing between the two accelerometers. To compensate for the finite-difference approximation, the above introduced correction factor, $k\Delta x / \sin(k\Delta x)$, may be applied [13]. The corrected transmitted vibrational energy, $(P_{trans}(f))_{corrected}$, is given by:

$$(P_{trans}(f))_{corrected} = P_{trans}(f) \left( \frac{k\Delta x}{\sin(k\Delta x)} \right)$$

(21)

3 EXPERIMENTAL APPARATUS AND MEASUREMENT METHOD

3.1 Moment excitation rig

Based on previous investigations at Loughborough University [17,18] a specially designed moment excitation rig was constructed. A photograph of the moment excitation rig is shown in Figure 2. This rig is predominately made from 1 inch square steel bar to ensure the rig has sufficient mass to avoid any undesired movement when impacting the beam. The impact fork is welded to a shaft which runs in self aligning bearing units. The use of an impacter held within a static frame, ensures that the direction of application of the impact forces is consistent between tests, something which poses difficulty when using an impact
hammer. The two arms of the impact fork are offset slightly to ensure perpendicular impact of the fork’s arms onto the beam. One force transducer is placed on each of the fork’s arms to measure the applied forces. Steel tips, screwed onto the force transducers, allow point force impact onto the moment arms of the beam and provide an even distribution of the force over the force transducer.

3.2 Experimental “infinite” beam

To enable moment impact, two moment arms 60 mm in length, 50 mm deep, and 6 mm thick were welded to the centre of the beam as shown in Figure 1. The mass of the moment arms was kept to a minimum to avoid bias errors in measuring the moment mobility when using an I-shaped exciter configuration \[7,8\]. It was shown in references \[7,8\] that an I-shaped exciter configuration will induce additional force and moment contributions at the excitation location due to the translational and the rotational inertia of the I-piece. Hence, the true moment, \(M_{\text{corrected}}\), applied at the excitation location, \(x_0\), was shown to consist of the measured force couple, \(M\), given by equation (10) less a contribution from a rotary inertia term, \(J\frac{\partial^2 \theta}{\partial t^2}\). Thus,

\[
M_{\text{corrected}} = M - J \frac{\partial^2 \theta}{\partial t^2}
\]

where, \(J\), is the rotary inertia of the I-piece.

In references \[7,8\] this correction was applied directly. However, an alternative approach adopted in this paper is to calculate the magnitude of this proposed correction using the finite difference and spectral density technique outlined in Sections 2.2 and 2.3 and, hence, determine the frequency range where the measured moment, \(M\), is a good approximation to the actual moment, \(M_{\text{corrected}}\). Thus, from equation (22) the following ratio can be formed:
\[ \frac{M_{\text{corrected}}}{M} = 1 - \frac{J}{M} \frac{\partial^2 \theta}{\partial t^2} \]  

(23)

The quantity, \( \frac{\partial^2 \theta}{\partial t^2} / M \), is now evaluated using the \( H_1(f) \) estimator with the rotational acceleration approximated using the finite difference expression given by equation (7). Hence, since equation (15) represents, \( \frac{\partial \theta}{\partial t} / M \), the quantity, \( \frac{\partial^2 \theta}{\partial t^2} / M \), can be calculated by multiplying equation (15) by \( j2\pi f \). The ratio of actual moment to measured moment, \( M_{\text{corrected}} / M \), is formed using equation (23). This ratio is used to assess the likely error in using the measured moment, \( M \), as an approximation for the actual moment, \( M_{\text{corrected}} \).

To achieve moment excitation, the moment excitation rig is aligned carefully such that the moment arms are struck by the impact fork at the same point in time, ensuring that the impact forces are equal and simultaneous. The outline of the impact fork is shown in dotted outline over the moment arms of the beam in Figure 1. The I-shape exciter arrangement has an advantage over the T-shape exciter in that any extraneous force due to an impure force couple is in the direction of low mobility. This advantage also applies when comparing excitation via the moment arms with direct application of the force couple onto the beam.

A schematic representation of the experimental beam is shown in Figure 1. The “infinite” beam which is comprised of a mild steel beam, 50 mm x 6 mm in cross section and 3 m in length was suspended on two thin wires from a rigid support. Each end of the beam was embedded in an anechoic termination to minimize reflections in the wave motion at the beam’s end. The anechoic termination was constructed using plastic boxes that contained two triangular foam wedges and sand to dissipate the vibrational energy. The length of the anechoic termination restricted the lowest frequency to be measured to approximately 100 Hz.
3.3 Measurement method

Measurement of the moment point mobility of the beam was undertaken by implementing equation (16) using the transducer signals $F_1$, $F_2$, $a_1$, and $a_2$, as shown in Figure 1. The accelerometer spacing, $\Delta x$, was chosen to be $0.15\lambda$, where $\lambda$ was the wavelength of the highest frequency to be measured, in this case, 1.6 kHz. It was noted in Section 2.1 that the cross mobility, $Y_{\partial\theta/\partial t,F}$, of a force and moment excited infinite beam will be zero. However, in general a moment mobility measurement formed by measuring, $\frac{\partial\theta}{\partial t}/M$, as implemented in equation (16) will contain errors due to a non-zero cross mobility and extraneous force, $F$, as shown in equation (3). Therefore, a calculation of the force to moment ratio, $F/M$, will indicate the quality of the applied moment [7,8,11]. In cases of non-zero cross mobility this force to moment ratio can also be used to calculate the true moment point mobility of the structure, $Y_M$, using equation (3). The cross mobility, $Y_{\partial\theta/\partial t,F}$, being obtained from theoretical or numerical data or in a separate measurement using pure force excitation.

Assuming that any extraneous force due to an impure force couple acts in the longitudinal direction of the beam and, thus, can be neglected, the unwanted force, $F$, acting in the transverse direction at the measurement location, $x_0$, will be due solely to the translational inertia of the I-piece [7,8]. Hence,

$$F = m \frac{\partial^2 u}{\partial t^2}$$  \hspace{1cm} (24)

where, $m$, is the mass of the I-piece. Thus, the force to moment ratio, $F/M$ is given by:

$$\frac{F}{M} = \frac{m \frac{\partial^2 u}{\partial t^2}}{M \frac{\partial u}{\partial t}}$$  \hspace{1cm} (25)
The transverse acceleration, $\partial^2 u / \partial t^2$, can be calculated using the finite difference approximation as:

$$\frac{\partial^2 u}{\partial t^2} \approx \frac{a_2 + a_1}{2}$$

(26)

Thus, the transverse acceleration to moment ratio, $\frac{\partial^2 u}{\partial t^2} / M$, can be calculated using the $H_1(f)$ estimator in a manner analogous to that described in Sections 2.2 and 2.3 for the rotational velocity to moment ratio, $\frac{\partial \theta}{\partial t} / M$. Hence,

$$\left( \frac{\partial^2 u}{\partial t^2} \right) \frac{1}{M} = \frac{1}{2d} \frac{G(F_2, a_2) + G(F_1, a_1)}{G(F, F)}$$

(27)

And, thus, the force to moment ratio, $F/M$, can be calculated using equation (25).

Measurement of the energy input to the structure was calculated between the applied moment and resulting rotational velocity using equation (19). To verify the moment input energy measurement two accelerometers, $a_3$ and $a_4$, were also placed opposite the force impact points on the moment arm, as shown in Figure 1. The energy input to the moment arms was then calculated using the measured force and resulting transverse velocity on the tip of the moment arms by employing equation (17). Two accelerometers pairs, $(a_5, a_6)$ and $(a_7, a_8)$ were also placed in the vibrational far field of the beam shown also in Figure 1. These pairs of accelerometers were used to measure transmitted energy along the beam by employing the two-accelerometer power flow technique implemented in equations (20) and (21).
The acquired signals were processed using a multi-channel spectrum analyzer with the FFT folding frequency set at 1.6 KHz. The record length used for each acquired signal was 2 seconds and an average of 10 impacts was used to form the relevant auto- and cross-power spectral density functions which had a frequency resolution bandwidth of 0.5 Hz. Thus, the spectral density data presented in Section 4 were scaled by the record length and the resolution bandwidth to obtain energy data in units of Joule/Hz [19].

4 RESULTS AND DISCUSSION

Figure 3(a) shows typical time histories of the two measured force signals. It can be seen in Figure 3(a) that the force signals on each moment arm occur simultaneously and are of almost equal amplitude. Figure 3(b) shows the auto-spectral density of the two impact force signals, $F_1$ and $F_2$. It is apparent from Figure 3(b) that the first zero in the applied force spectra occurs at approximately 470 Hz and that both force spectra are almost identical, particularly below the first zero frequency at 470 Hz.

Figure 4 displays on a logarithmic frequency scale from 100 Hz to 1000 Hz the amplitude ratio of actual moment to measured moment signals calculated using equation (23). It can be seen in Figure 4 that below the first zero in the applied force spectra, at 470 Hz, this ratio is approximately 1.0 indicating that the measured moment, $M$, is a good approximation to the actual moment, $M_{corrected}$. However, above 470 Hz, the trend is for the moment ratio to increase with increasing frequency. Thus, from equation (22) it is clear that as the impact forces shown in Figure 3(b) reduce in amplitude the applied moment, $M$, will reduce in amplitude and, hence, the rotary inertia, $J$, of the moment arms will become a more significant contributor to the actual moment, $M_{corrected}$. 


Figure 5 displays logarithmically the modulus of the moment point mobility of the experimental “infinite” beam over the frequency range of interest, 100 Hz to 1000 Hz. The “corrected” measured point mobility data calculated using equation (16), $Y_{\text{meas}}$, are shown as a solid line. Also shown in Figure 5 is the modulus of the moment point mobility, $Y_{\alpha M}$, calculated using equation (5). It can be seen that the measured point mobility data from the experimental beam follow the trend of the equivalent theoretical infinite structure. The fluctuations of the measured mobility curve over frequency are due to the fact that the experimental anechoic terminations do not work perfectly and, hence, there is some wave reflection from the ends of the beam. The measured moment point mobility data shown in Figure 5 appear to show a resonance at approximately 470 Hz. However, since this frequency coincides with the first zero frequency in the applied force signal, this effect is probably not due to a resonance in the structure but rather is attributable to the biasing effect of noise in the force signal when using the $H_1(f)$ estimator.

Figure 6 shows the magnitude of amplitude ratio of extraneous force to applied moment, $|F/M|$, calculated using equation (25). It can be seen in Figure 6 that this ratio remains less than 0.75 up to the first zero in the applied force signals at 470 Hz. From equation (25) it is clear that as the impact forces, and hence, the measured moment, $M$, reduce in amplitude with increasing frequency then the transverse inertia term, $m\ddot{u}\ddot{t}\ddot{t}$, will become increasingly significant compared to the measured moment. For the experimental “infinite” beam studied in this paper the extraneous force, $F$, is assumed to have no effect upon the measured results since the cross mobility of an infinite beam is zero. However, Figure 6 indicates that a correction to the measured moment mobility, as expressed in equation (3), may become necessary if the current technique is applied to practical finite structures with non-zero cross mobility terms.

Figure 7 shows a comparison between the vibrational energy input to the beam due to moment excitation, $(P_{\text{in}}(f)M)_{\text{corrected}}$, calculated using equation (19), and the sum of direct
vibrational energy input to the beam’s two moment arms by the two impact forces, 
\( P_{in}(f)_{E1} + P_{in}(f)_{E2} \), calculated using the well known cross-spectral density method, equation (17), for both force signals. It can be seen in Figure 7 that both methods indicate similar values of input energy over a wide frequency range. Since both methods used the same force signals and differ only in the response accelerometers employed, it can be assumed that the finite difference approximation of the rotational velocity incorporated into equation (19) can be used to successfully measure the energy input to a beam by moment excitation.

Figure 8 shows a comparison of the measured input energy due to moment excitation, \((P_{in}(f)_{M})_{corrected} \), and the transmitted vibrational energy, \((P_{trans}(f)_{r} + P_{trans}(f)_{l})_{corrected} \), calculated using the two-accelerometer technique, equation (21). The transmitted energy was measured on both sides of the excitation location, as shown in Figure 1, and then summed to provide the total transmitted energy in the structure, \((P_{trans}(f)_{r} + P_{trans}(f)_{l})_{corrected} \). Figure 8 indicates a close match between the measured input and transmitted energies below 470 Hz. The discrepancy between the energy values at approximately 470 Hz can be attributed to noise on the input force signal.

Above 470 Hz it can be seen in Figure 8 that the measured transmitted vibrational energy is slightly higher than the moment input energy, which is in conflict to the energy conservation law. One explanation may be that different accelerometers pairs were used on either side of the beam. During the experiment a pair of lightweight ICP accelerometers was employed to measure the energy flowing to the right of the excitation location, however a pair of conventional charge type accelerometers was used to measure the energy flowing to the left of the excitation location. The charge type accelerometers are much heavier than the ICP accelerometers and so may have mass loaded the structure. For these particular transducers the ICP accelerometers were a more closely phase matched pair than the charge type accelerometers. Hence, the ICP accelerometers can be
expected to produce a more accurate measurement of transmitted energy. Assuming that
the transmitted energy, flowing to either side of the excitation location is half the input
energy, the doubled transmitted energy to the right of the beam, $2(P_{\text{trans}}(f)_{r})_{\text{corrected}}$, is
shown versus the moment input energy in Figure 9. It can be seen that both sets of data
are in excellent agreement with each other, particularly at frequencies below the first zero
in the applied force spectrum at 470 Hz.

5 SUMMARY

In this paper a technique to measure the moment point mobility and energy input to a
beam by moment excitation has been introduced. The novelty of this technique is the use
of a specially designed moment excitation rig to generate an impulsive moment combined
with a finite difference based spectral density measurement method. Preliminary
measurements made upon an experimental “infinite” beam have shown this technique can
be used to accurately measure the moment point mobility and energy input to the
structure. In particular, the measured moment point mobility of the experimental “infinite”
beam was seen to follow the trend line of the theoretically infinite structure. Further, the
energy input to the beam by moment excitation measured with the new technique was
shown to match (i) conventional input energy measurements made using the cross-
spectral density between applied force and transverse acceleration signals at the points of
impact on the moment arms of the beam; and (ii) conventional transmitted energy
measurements made using the two accelerometer technique in the vibrational far field of
the beam.

However, it is clear that the duration of the moment impact should be kept as small
as possible so as to avoid a zero in the force spectra at low frequency. At frequencies
above the first zero in the applied force spectra it was shown that the effect of translational
inertia and rotational inertia due to the attachment of the moment arms becomes significant. A further disadvantage of the proposed technique is the requirement to rigidly attach a moment arm to the structure and the need for sufficient access to carefully locate the moment exciter. Despite these practical limitation, initial measurement results are sufficiently encouraging to propose applying the measurement technique to more realistic beam-type structures, in particular, finite structures where the cross mobility terms are non-zero. Since, for this type of structure a correction to the measured moment mobility using the estimated unwanted force to applied moment ratio will probably be required.

REFERENCES


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