NORMAL MODES OF AN 18 INCH CRASH CYMBAL

R Perrin  Institute of Fundamental Sciences, Massey University, Palmerston North, NZ
G M Swallowe  Physics Department, Loughborough University, Loughborough, LE11 3TU, UK
T R Moore and
S A Zietlow  Department of Physics, Rollins College, Winter Park, FL32789, USA

1 INTRODUCTION

The crash cymbal gets its name from the manner in which it was traditionally played. The percussionist held one cymbal in each hand, more or less vertically, and “crashed” them together in a glancing blow. To facilitate this, a central dome was included to which a handle could be attached, while the outer edges were bent slightly downwards. Today these cymbals can often also be seen mounted horizontally on stands and struck by drumsticks. Sometimes “sizzlers” are also present within the main body.

In the present work the normal modes of an 18 inch crash cymbal, with sizzlers removed, were investigated. A variety of experimental methods were used including Chladni sand patterns, electronic speckle pattern interferometry (ESPI) and laser vibrometry, as well as more conventional techniques, together with a finite-element model. Previous workers have usually restricted themselves to single experimental approaches and have not had the benefit of finite-element models. Their theoretical considerations have been restricted to comparisons with the flat circular plate and fitting data to variations on Chladni’s law. Consequently some intriguing aspects of the normal modes of this type of cymbal do not seem to have been appreciated.

2 THEORETICAL CONSIDERATIONS

Cymbals can be thought of as perturbed flat circular plates in which the axial symmetry has been retained. The consequences for the normal modes are the same as for any other axially symmetric system. Thus modes must occur in degenerate pairs, the angular parts of whose modal functions vary like sin(mθ) and cos(mθ) where m = 0, 1, 2,... Hence the nodal patterns consist of m evenly spaced diameters and n circles concentric with the rim where n = 0, 1, 2,... Exceptions are the (axisymmetric) m=0 cases which are singlets. Any degeneracies other than these are to be regarded as “accidental”, being unconnected with the symmetry, and are expected to be rare. From a group representational point of view the “symmetry type” of a mode is fixed by its m value. In practice the axial symmetry is always broken, at least slightly, so the doublets split and the absolute locations of the nodal diameters become fixed. In the case of the flat circular plate the number pair (m, n) is sufficient to identify a doublet pair uniquely. As a working hypothesis we followed earlier workers and assumed this would be true also for the crash cymbal. While this proved to be a useful working hypothesis we found the number n can sometimes lead to ambiguities, as it does in bells.

Experimental and theoretical results for flat circular plates have been summarized by Leissa. If boundary conditions do not break the axial symmetry, the radial parts of the modal functions are given by a linear combination of Bessel functions J_m(kr) and modified Bessel functions I_m(kr), both of the first kind, where k is the usual well-known parameter that is essentially the magnitude of the propagation vector.

In a previous paper one of the present authors (RP) pointed out that J_m(kr) possesses a critical point at kr = m above which it is oscillatory but below which it decays, essentially exponentially, to zero. This critical point is where, as kr decreases, the radial component of the propagation vector changes from real to imaginary as the transverse component increases beyond the value of k. Consequently the behaviour in the radial direction is technically evanescent and the size of the evanescent region will tend to increase with m. The presence of I_m(kr) in the solution complicates the situation slightly. Since these functions, for m>0, start from zero at the origin and increase very...
slowly and monotonically up to the critical point, the appearance of evanescence will be retained. Above this point the functions increase more rapidly but still monotonically. The overall solution in this region will therefore now oscillate around a rising baseline instead of around one fixed at zero. (It would also be possible for the baseline to fall, depending on signs of the mixing constants.) Nodal circles will therefore still be present but somewhat shifted from the zeros of the $J_m(kr)$.

3 THE FINITE-ELEMENT MODEL

First the main geometrical features of the cymbal were measured. There was a central hole in the middle of the dome of radius 5.9mm. The dome had radius and height of 6.4cm and 1.9cm respectively. The corresponding quantities for the complete cymbal were 22.8cm and 4.8cm while the width of the rim, measured along the surface was 3.2 cm. The thickness of the metal was measured at a selection of positions and, while there was some variation, a value of 1mm was thought to be a good overall average.

To get an accurate profile a plaster cast of the inside of the cymbal was made. This was cut diametrically and photographs of the cross-section taken with a camera with a long focal length lens. The (inner) profile of the cymbal was then digitized from the photograph, with the origin of coordinates arranged to be at the centre of the top of the dome. The y-axis was taken to run vertically downwards. These data points were then fitted with appropriately chosen curves. The region from the bottom of the dome to the inside of the rim was covered extremely well by a single straight line. Likewise for the rim region. The dome required a mixture of straight lines and circular arcs. These curves were used to generate the model using the Lusas 13.5 software package.

The set of profile curves was used to generate a surface making up a thin pie slice of the cymbal. This was done by rotating the curves through 11.25° about the y-axis. This surface was meshed in such a way as to produce four-sided elements without any of their aspect ratios becoming excessive. In this respect the inclusion of the central hole in the model was helpful. The number of elements used was chosen with the intention to be sufficient to mimic the modal functions with the maximum numbers of nodal diameters and circles expected to be of interest. A rigid constraint was placed on the line bounding the central hole. This both accounted for the fact that the cymbal was to be bolted down during the experiments and had the advantage of suppressing rigid body modes. The thin slice of surface was then copied 31 times by rotation about the y-axis in order to complete the full cymbal geometry. The result can be seen in Figure 1. Thin shell elements were used all with thickness 1mm with standard material properties for bronze.

The Lusas software lists the frequencies of the modes found in increasing order. To identify them one needs to compare the distorted and undistorted meshes. The number and locations of nodal circles and diameters are then obvious, provided different colours are used for the two meshes. The results obtained agreed well with the requirements of group theory in terms of both the types of nodal patterns found and of degeneracy structure. A small number of accidental degeneracies were also predicted. Some examples are given in Figure 2 where most views are shown vertically downwards but one is from the side.
Overall the nodal patterns predicted are much as one might expect from the flat circular plate. Examining the complete set shows that, although the modes clearly fall into families which more or less correspond to fixed values of n, this is an oversimplification. For example, what should be the (2, 0) mode, has an unexpected circle near the dome bottom. There is no other candidate for the (2, 0) slot and there is a genuine (2,1) with its circle out towards the rim, as expected. The same phenomenon occurs for (3, 0) but by the time (5,0) has been reached the extra circle has disappeared as the evanescent region moves outwards. An extra circle can also appear at higher values of n in much the same way as has previously been reported for handbells$^5$.

The frequency predictions of the model are shown in Figure 3, where families of modes at “fixed” n are shown. It was decided to cut off the results at 2kHz because the number of modes had already become very large and m was becoming too great to expect accuracy without a significant increase in the numbers of elements and a corresponding one in computational time.

**Figure 2. Predicted modal forms for selected cases**

**Figure 3. Lusas Crash Cymbal Predictions**
4 MECHANICAL DRIVE AND PICKUP METHODS

The cymbal was attached horizontally to a massive support by a screw passing through the central hole and protected by washers above and below. In these preliminary experiments only mechanical drive was used. A drive point was selected and a piece of radio metal attached. Excitation was affected through a magnetic transducer (B&K type MM0002) driven by an oscillator (B&K type 1022). Initially a spectrum was obtained by sweeping the oscillator frequency slowly up to 5 kHz. To minimize damping, pickup was with a capacity transducer (B&K type MM0004) going via an amplifier (B&K type 2606) to an oscilloscope and a chart recorder (B&K type 2305). The frequencies of the peaks on the chart were noted from a timer-counter during the sweep. This was repeated for other drive and pickup positions. A total of about 80 possible modes were found, although many later proved to be partners in split doublets.

The next step was to measure the frequencies more accurately and identify the nodal patterns. Two approaches were used. Firstly the well-known Chladni patterns were employed. Fine dry sand was scattered evenly over the surface and the frequency was fine-tuned until a clear pattern emerged. Perhaps 30 modes were identified in this way, not counting split partners. The method was quick and easy in those cases where sufficient amplitude could be generated to make the particles dance. However some of the patterns were too complicated to identify the components of what were assumed to be mixed modes.

A second method enabled more modes to be identified, although it was tedious to use. The drive was as before, but detection was via an accelerometer (B&K type 4344) connected to a double beam oscilloscope through a preamplifier (B&K type 2625). After fine tuning the frequency by reference to the magnitude of the output signal, the accelerometer was held against the surface and moved by hand. Comparing the signal with that of the drive, which was fed to the other beam of the oscilloscope, nodes could be identified by watching for phase changes of π as the accelerometer crossed them.

The results from both of these methods are combined in Figure 4. Values of n greater than 2 are excluded from the figure because there were rather few of them. Clearly there is good qualitative agreement with Lusas. Quantitatively agreement is also good for n = 0 and n = 1. This will be discussed in more detail after the holographic results have been presented.

![Figure 4. Preliminary Crash Cymbal Results](image-url)
5 ELECTRONIC SPECKLE PATTERN INTERFEROMETRY

Operational deflection shapes of the cymbal were obtained by electronic speckle pattern interferometry. The interferometer was made from discrete components mounted on an optical table that was actively isolated from vibrations, and has been described in detail by one of the present authors (TRM). The cymbal was rigidly mounted at the centre, and placed on the same table as the interferometer, with the plane of the cymbal perpendicular to the table top. The entire apparatus was enclosed in an anechoic room. Vibrations of the cymbal were induced acoustically by a speaker placed approximately one meter from the cymbal, or mechanically by a piezoelectric disk attached to the edge of the cymbal. The bandwidth of the driving signal was less than one hertz, and the total harmonic distortion was less than 0.1%.

Even with active vibration isolation, ambient vibrations of the order of one hertz were transmitted to the table from the floor and produced a slow motion of the cymbal. Usually, this vibration is not visible in an interferogram when the interferometer and the object are mounted on the same table; however, because the cymbal was large and flimsy, this motion caused decorrelation of the speckle on a time scale of approximately one second. This problem of decorrelation was overcome by modifying the method of analyzing the images from the interferometer.

Typical images are shown in Figures 5 and 6. Nodes are shown in white, while dark lines represent lines of equal amplitude. The amplitude of vibration at any point can be deduced by counting lines of equal amplitude from a nodal point. The absolute amplitude can be calculated using the theory described by one of the present authors (TRM).

Figure 5. ESPI images of (m, 0) modes for m = 2 – 12 and 23. m=9 – 12 cases show mixing with (m, 1) modes.
Shown in Figure 5 are the ESPI images for the modes \((m, 0)\) for a range of \(m\) values. They illustrate very well how the evanescent region grows as \(m\) increases. It’s easy to see why some of our students have referred to them as “rim” modes. It should be noticed that both \((2, 0)\) and \((3,0)\) have circles close to the dome bottom in line with the Lusas predictions discussed in section 3.

Also of note is that modes claimed to be rim ones with \(m\) values of 9, 10, 11 and 12 appear mixed with \((2, 1)\), \((4, 1)\), \((5,1)\) and \((6,1)\) respectively. The mixing is so complete that it’s not at once obvious which of the two modes is dominant. It proved almost impossible to detect any of these four \((m,1)\) modes, or most of the higher \((m,1)\) cases either, without an accompanying rim mode being mixed in. This was also the case for the large majority of other higher order “main body” modes. Sometimes an additional main body mode, usually again neighbouring in frequency, was also mixed in, making the patterns exceedingly difficult to decipher. An example is given in Figure 6. Also shown are the true \((m,1)\) modes for low values of \(m\), still mixed with the same rim modes as in Figure 5. How it was decided which frequency was the correct one for each of the mixed mode components will be discussed in section 7.

6 FURTHER EXPERIMENTS

In order to try to get further insight into how to interpret the ESPI images two further sets of experiments were undertaken. Firstly a new set of Chladni pattern investigations was undertaken, but this time driving at the centre of the dome. Our hope was to remedy the scarcity of \(m=0\) data in all the previous experiments. The cymbal was mounted horizontally, with convex face upwards, using an 18mm diameter washer and a screw through the 11.8mm diameter central hole of the cymbal to fix it to an 18mm diameter cylindrical mount. This in turn was fixed to a vibrator (B&K type 4812). This mounting arrangement covered approximately the same area of the crown as is covered when the cymbal is used normally. The cymbal was excited into vertical motion using an oscillator (B&K type 1022) and an amplifier. Frequencies could be controlled to about 0.1Hz. The surface of the cymbal was dusted with fine dry sand and the Chladni patterns observed at resonance frequencies. These agreed in virtually all cases with the patterns obtained using laser speckle on the cymbal when in a vertically mounted position. A few extra modes were found including several with \(m=0\). An example is shown in Figure 7.
In a second additional experiment a Polytec Compact Laser Vibrometer was used to investigate the resonant vibrational motion at selected places on the surface of the cymbal. The laser sampled an area of about 4mm² so could be considered as taking a “point” measurement. The Vibrometer output was recorded on a PC and a Fourier transform, implemented using MATLAB, performed to determine the frequencies present. The cymbal was excited by gentle impact at various places. The resulting spectra were extremely detailed and showed the resonant peaks all to be extremely sharp. A number of the peaks found did not correspond to any modes detected in our earlier experiments. These enabled some gaps in our overall mode data to be filled, although the nodal patterns had to be inferred. This work will be discussed in detail in a future report.

7 INTERPRETATION OF THE RESULTS

We were concerned primarily with understanding the ESPI data as they were the most extensive and detailed. The other results were all helpful in doing this. Those modes that occurred unmixed were easily identified as already discussed in section 5. It always had to be remembered that nearly all the modes have doublet partners, and in analyzing the situation we always tried to use the higher frequency members of pairs in our considerations.

The first problem faced was how to handle the widespread mode mixing. Even in cases where the component modes could be identified care was required. Take as a simple example the case of the [(2, 1) + (9, 0)] included in Figures 5 and 6. The same mixture occurred at the two different frequencies of 344 Hz and 355 Hz as shown in the figures. We had to decide which frequency belonged to which mode. Here the preliminary experiments came to the rescue. They indicated, without showing any mixing, that the (9, 0) was at 340 Hz and the (2, 1) at 355Hz. Hence the frequencies quoted in Figures 4 and 5. In those experiments the (9, 0) was produced alone by driving on the rim, while the (2, 1) was obtained alone by driving on the main body, the detection methods being relatively insensitive compared to ESPI. In a high proportion of these “clear” mixed cases, modes could be sorted out in a similar way because the preliminary experiments had given reasonably accurate frequencies for nearly all the (m, 0) and (m, 1) modes up to quite high values of m. Classification of modes beyond 3kHz or above n=5 was not attempted because identification then became extremely difficult.

We took all the modes we had definitely identified and plotted them on a frequency versus m graph, looking for families at fixed values of n. There were a few gaps and some could be filled by previously unidentified modes whose frequencies fitted them comfortably onto interpolation curves.
The few still remaining gaps were then able to be filled by previously unallocated peaks in the vibrometer spectrum. The final results are shown in Figure 8.

![Figure 8. Overall summary of experimentally measured data](image)

8 COMPARISON WITH THE FINITE ELEMENT PREDICTIONS

Comparison of Figure 8 with Figure 3 shows at once that, at least qualitatively, agreement between the experimental results and the finite element predictions is good. In order to compare them more quantitatively they are shown superimposed in Figures 9 and 10. Overall the agreement is very good. The failure of the model for high m in the n=0 curves is due to the number of elements in the azimuthal direction becoming insufficient, and could be corrected at the expense of extra machine time. A similar problem could also be expected to arise for larger values of n. The predictions for n>0 are all low by about 10%. Rerunning the model after modestly changing the thicknesses of the elements in various regions of the dome and main body showed that such variations could easily account for this level of discrepancy, while leaving the n=0 results almost unchanged.

![Figure 9. Comparison of theory and experiment for n=0, 2 and 4](image)
9 DISCUSSION AND CONCLUSIONS

While there can be confidence that the modes of the crash cymbal are reasonably well understood themselves, the way in which they couple remains unclear. Looking at the n=0 curve in Figure 8 it is seen that, as m increases beyond its lowest value, new modes appear at regular small frequency intervals. The “density” of modes is unusually high and, as frequencies reach points where higher n values occur, then further similar sets of contributions to this density begin to appear. The fact that every mode is a split doublet will make the density higher still. The chance of any given mode having a reasonably near neighbour is therefore high, at least after the lower n=0 modes have been passed, and gets progressively higher as frequency increases. The obvious explanation for the large number of coupling cases is therefore that the modes are close enough in frequency for them to couple directly by their spectral curves overlapping. This would be good enough, even though the large number of them would be a surprise, for group theory to regard them as accidentally degenerate and therefore able to mix, even though they may be of different symmetry types. Unfortunately measurements of Q by mode decay, made during the preliminary experiments, suggested values of the order of $10^3$, which is far too high to be consistent with this explanation. The same conclusion followed from the laser vibrometer experiments which showed that most of the resonant peaks were extremely narrow.

The high density of modes can probably be put down to the fact that the crash cymbal was large and flimsy. This also made it a prime candidate for non-linear behaviour. However this would only be expected to produce coupling if the modes were harmonics or subharmonics. This was not the case because it was usually only near neighbours that were involved. Also the results in the ESPI experiments all had amplitudes so low that the non-linear regime should not have been entered. It is felt that measurements of Q are now required on as wide a range of modes to as high an accuracy as possible in order to try to solve this problem.
10 REFERENCES