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ON THE STRIKE NOTE OF BELLS

G. M. Swallowe*, R. Perrin+,
Department of Physics, Loughborough University, Leicestershire, LE11 3TU, UK
+ Also at the Institute of Fundamental Sciences, Massey University, Palmerston North, NZ
*G.M.Swallowe@lboro.ac.uk

Abstract
A strike note, characteristic of the particular bell, is heard when a bell is struck. It is observed that the pitch of this note sometimes does not correspond to the frequency of any one of the bell's normal modes. The origin of this strike note has been a subject of controversy for over 100 years. Previous empirical investigations have mainly made use of musically trained listeners and real or recorded bell sounds. The short duration of the actual strike note and possible influence of musical training on observations may invalidate the conclusions of previous investigators. In this work use has been made of computer generated simulated bell sounds and untrained listeners. It is demonstrated that a strike note may be isolated by beating with a pure test tone and this technique is used to investigate 28 bell-like sounds using a total of 60 listeners. It is concluded that virtual pitch theory provides the best method of predicting the presence or absence of a strike note and its frequency, but that it does not work in every case.

INTRODUCTION

When a bell is struck the sound emitted consists of a number of simultaneously produced single frequency components or "partial tones" generated by the excitation of normal modes of the particular bell. The normal modes of a bell do not in general form a harmonic series although the five lowest, and most prominent, partials are tuned by bellfounders for the production of large church and carillon bells. Higher partials are not usually tuned. After a bell is struck the first sound heard is an inharmonic one of metal striking on metal. This is due to high frequency partials and rapidly dies away leaving an audible "strike note" of definite musical pitch. This in turn decays, giving way to the more slowly decaying Hum tone. A strike note is heard whether the bell is tuned or not. Its musical pitch sometimes does not correspond to the frequency of any one of the bell's normal modes, especially in untuned cases, and its origin has been a subject of controversy for many years.

Lord Rayleigh [1] concluded that the strike note was at the frequency of the octave below the fifth partial tone (Nominal) in both tuned and untuned bells. In a tuned bell this corresponds to the Fundamental but an untuned one often has no mode at this frequency. Work by Jones [2] on a number of American carillons generally supported Rayleigh's conclusion but also concluded that the difference tone between the fifth and seventh partials may be of importance. Meyer and Klaes [3] showed that the strike note was not physically present in the sound produced by the bell but corresponded to a frequency close to the difference between the fifth and seventh partials and attributed this to a physiological effect in the ear. Arts [4], who carried out measurements on dozens of bells, concluded that an octave below the fifth partial corresponded much more closely to the strike note than did the difference between fifth and seventh partials. Later work by Schouten and 't Hart [5] and Pfundner [6] attributes the strike note to "the residue"; a tone generated by the perception of a number of frequencies in harmonic or close to harmonic relationship whose combination produces the perception of a fundamental frequency which is not physically present. Rossing [7] applied this theory to chimes, in which the strike note phenomenon is also observed, and concluded that the residue theory provided a satisfactory explanation for the strike notes. Schad and Frik [8] attribute the strike note to the combination of missing fundamentals of several harmonic series in the bell's physical frequency spectrum. However, this work is essentially theoretical and does not report on actual listener observations of strike note frequencies. Terhardt [9] has developed a version of the theory of virtual pitch and used it to attempt to explain the strike note (Terhardt and Seewann [10]). It leads to the prediction of one or more virtual pitches from a complex tonal sound which may or may not correspond to physically present frequencies. The experiments described below seek to make definite measurements of strike note frequencies and hence differentiate between the competing theories of the strike note.

EXPERIMENTAL STUDIES

In previous work most of the observers were musically trained so that their perception may well have been influenced by the tradition of musical harmonic scales and they may therefore not be considered as objective observers. This work revisits the strike note phenomenon using computer and oscillator generated tones, which could essentially generate a "strike note" of indefinite duration, and uses non musically trained listeners to determine the existence and frequency of the strike note. Two sets of experiments carried out using different sets of listeners are described. These are classified as Series 1 and Series 2. The volunteer listeners were mainly students of ages 18-22 but did also include approximately 20% of older volunteers spanning the age range 28-65.

Table 1: Frequencies (Hz) and frequency ratios relative to the strike note of bell-like sounds derived from the modified Chladni Law. Conventional partial numbers are listed in the first row. The observed strike notes listed in the final column have an accuracy of ±3 Hz

Sound	1st	3rd	4th	5th	7th	8th	Strike note
	Frequency (Hz) and frequency ratios (in brackets) relative to strike note						
1	103 (0.52)	249 (1.25)	297 (1.49)	399 (2.00)	597 (2.99)	816 (4.08)	200
2	87 (0.44)	235 (1.19)	282 (1.43)	384 (1.95)	626 (3.18)	765 (3.88)	197
3	90	202	275	455	562	809	
4	141 (0.47)	373 (1.24)	453 (1.51)	597 (1.99)	915 (3.05)	1197 (3.99)	300
5	154 (0.51)	384 (1.27)	440 (1.46)	626 (2.07)	915 (3.03)	1161 (3.84)	302
6	141	360	455	562	950	1265	
7	204 (0.51)	481 (1.19)	597 (1.48)	816 (2.02)	1197 (2.97)	1614 (4.00)	403
8	193 (0.49)	499 (1.26)	626 (1.58)	765 (1.93)	1161 (2.93)	1621 (4.09)	396
9	202 (0.50)	455 (1.13)	562 (1.39)	809 (2.00)	1265 (3.13)	1624 (4.02)	404
10	249 (0.50)	597 (1.20)	751 (1.51)	1018 (2.04)	1497 (3.00)	1982 (3.97)	499
11	235 (0.46)	626 (1.23)	765 (1.50)	994 (1.95)	1524 (2.99)	2034 (3.99)	510
12	275	562	809	950	1439	2029	
13	297 (0.49)	719 (1.19)	915 (1.52)	1197 (1.99)	1816 (3.01)	2415 (4.00)	603
14	282 (0.48)	694 (1.18)	915 (1.55)	1161 (1.97)	1823 (3.09)	2369 (4.02)	590
15	275	680	950	1265	1821	2478	
16	347 (0.50)	848 (1.22)	1052 (1.51)	1381 (1.99)	2109 (3.03)	2778 (4.00)	695
17	331 (0.46)	839 (1.18)	1076 (1.51)	1430 (2.00)	2144 (3.00)	2848 (3.99)	714
18	360 (0.52)	809 (1.17)	1102 (1.60)	1439 (2.09)	2029 (2.94)	2720 (3.94)	690

Series 1

Previous work (Swallowe et al [11], Perrin et al [12]) has shown that the frequency spectra of bell-like sounds can be approximately described by a modified Chladni law of the form

$$f_{m.n} = c(m + bn)^p$$

where m and n are non negative integers while b, c and p are constants such that $1 \leq b \leq 2$ and $1.4 \leq p \leq 2.4$. Eighteen groups of six partials which approximated to frequency ratios in a bell were produced using this formula. The frequencies and frequency ratios of these tones are set out in Table 1. The partials were of equal intensity. A combination of an AMIGA 500 computer (which can generate 4 simultaneous tones) and two Bruel and Kjaer type 1022 signal generators were used to produce the tones. They were mixed in an audio mixer and fed to speakers via an audio amplifier. Since the strike note is observed in the region of the second partial this frequency was not produced and the 6 frequencies in the sounds can be approximated to 1st, 3rd, 4th, 5th, 7th, and 8th partials of a conventional bell enabling both the octave below the fifth and the difference tone between 5th and 7th to be investigated. The conventional 6th partial was not used because it is relatively unimportant in the sound of a conventional bell (Schad and Warlimont [13]). All of the eighteen groups investigated produced pleasant bell-like sounds. An additional test frequency produced by a further Bruel and Kjaer type 1022 oscillator was fed into the sound mixer together with the tones listed in Table 1 and an observer in a sound proof booth scanned the test frequency between 100 Hz and the upper frequency of the sound under investigation, and listened for beats. When beats were found the frequency was noted. All of the tones present in the sounds were readily identified by prominent beats with the test tone. Weak but audible beats due to difference tones between most of the pairs on tones in each sound were also detected. In addition to these, other beats of a different character, could be prominently heard in 14 of the 18 sounds and they are indicated as strike notes in Table 1. A total of 12 volunteer listeners took part in these tests and all but 2 could identify the strike note. The data showed that the strike note is different in character and much more easily observed than difference tones. Difference tones were weak, occured in all the sounds and required only the presence of the two relevant tones to be detected. The strike note was not consistently found at either the Rayleigh criterion frequency of an octave below the fifth or close to the difference tone between the fifth and seventh of a conventional bell. Removal of *any* of the other frequencies either diminished or removed the strike note. The strike note appears therefore to be produced by the totality of the physically present tones.

Series 2

The Series 1 experiment used theoretically predicted bell-like sounds generated from 6 physical frequencies. It could be argued that the strike note phenomenon produced was artificial in that it was not generated from a set of partial frequencies and amplitudes present in a real bell. In this set of experiments two AMIGA computers were used to generate a total of 8 tones (4 from each computer) which were mixed and fed to a speaker via an audio amplifier. The output amplitude at each frequency was adjusted so that it contributed a sound intensity approximately equal to the measured relative intensities of the bells in Schad and Warlimont [13] whose published values of the frequencies of ideal and experimentally determined bell partials were used to simulate 10 bells as listed in Table 2. The simulated bell sounds were played to ten volunteer listeners. A Bruel and Kjaer type 1022 oscillator connected to another speaker was provided to the volunteers and they were asked to identify all the tones that they could find in the sound by slowly scanning in the frequency range ±100 Hz around the octave below the fifth partial and listening for beats. The listeners found all the frequencies in the range tested that were physically present in the sound. In addition some observers found an additional frequency in the expected region of the strike note and this is recorded in Table 2 as the strike note. Where no additional frequency was observed the 2nd partial is recorded, in brackets in Table 2, as the strike note. All of the simulated bell sounds were fed to a PC running a Fourier analysis programme. This, as expected, identified all the partials in the sounds and showed that none of the additional frequencies recorded as a strike note were physically present in the sounds.

Table 2: Partial frequencies and strike note of simulated bells.

	Partial Frequencies (Hz)								Strike
Bell	First	Second	Third	Fourth	Fifth	Sixth	Seventh	Eighth	
1	293	586	693	883	1172	1200	1394	1470	(585)
2	145	290	360	432	580	690	766	854	(290)
3	102	202	240	323	403	479	538	603	(201)
4	208	392	494	623	832	989	1111	1245	416
5	220	365	462	660	779	1093	1173	1296	390
6	135	262	343	392	535	548	637	671	268
7	127	254	302	381	508	640	678	761	(253)
8	142	284	338	425	568	716	758	851	(284)
9	266	543	643	856	1142	1322	1418	1500	571
10	276	531	647	856	1134	1314	1410	1471	568

DISCUSSION

Sounds 1,4,7,9,10,11,13,16,17 of *Series 1* have two or more harmonics of the reported strike note in their spectrum so the strike note can be explained as a missing fundamental. Additionally sounds 5,8,14,and 18 have two or more frequency components whose ratios to the recorded strike are close to integer ratios and could therefore be explained as beats of mistuned consonances. However, sounds 6,12 and 15 also have such ratios but no strike note was observed which suggests that beats of mistuned consonances cannot offer an explanation for the strike note.

Bells 1,2,3,7 and 8 of *Series 2* have a second partial which coincides with an octave below the 5th and therefore the strike note is expected to be at the same frequency as the second partial and thus not be independently detectable. In bells 4 and 5 the octave below the 5th differs significantly from the second partial but second and third harmonics of this frequency are present in the physical spectrum so strike notes at this frequency are expected as a missing fundamental. In bells 9 and 10 the octave below the 5th again differs greatly from the second partial but only the second harmonic of this frequency is physically present so it cannot correspond to a missing fundamental.

The concept of the strike note being due to the 'residue' of higher frequencies is not supported by these results. Only the lower partials of simulated bells were used in this study but the strike note was clearly perceived in the expected region. Pfundner's [6] conclusion of the strike note being due to a missing fundamental of frequencies starting with the fifth or higher partials therefore cannot be correct. Schlad and Frik's [8] concept of a strike note being due to a combination of several missing fundamentals also does not explain the results of these experiments.

An application of a simplified form of Terhardt's virtual pitch theory [9] based on matching subharmonics to the sounds and taking into account the most significant virtual pitches leads to the correct prediction of the strike note, or lack of one, in 15 of the 18 sounds of *Series 1* and 7 of the 10 sounds of *Series 2*. In addition it predicts strike notes in bells 3 and 6 of *Series 2* where none was found. However, weak beating was heard at these predicted frequencies so the theory could be said to have worked. In bells 9 and 10 of *Series 2* this approach failed to predict the strong strike notes heard. It also predicted a strike note for sound 12 of *Series 2* but none was heard. The virtual pitch theory could be said to have worked in 86% of the tests. The possibility that the unpredicted strike notes were due to a superposition of several difference tones at almost the same frequency was considered. A calculation of the possible difference tones note of *Series 2* bells 9 and 10 shows that only one difference tone has a frequency close to the observed strike of bell 9 and none of bell 10. Superposition of several difference tones does not therefore offer an explanation for these strike notes.

CONCLUSIONS

The results of these investigations can be summarised as
a) Strike notes *can* be detected by beating with a test frequency.
b) A strike note can be produced even in the absence of overtones to generate a missing fundamental.
c) The Rayleigh rule provides a good estimate of the frequency of a strike note, when one exists, but cannot predict the absence of one.
d) Virtual pitch theory is the best method of predicting the presence or absence of a strike note as well as its frequency. However, it does not work in every case.

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