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Effect of teeth micro-geometrical form modification on contact kinematics and efficiency of high performance transmissions

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Abstract:

Light weight, compactness and efficiency are key objectives in high performance vehicular transmission systems, which are subject to large variations in torque and power. Pitch line velocities of up to 52 m/s and teeth pair contact pressures of up to 3 GPa are routinely encountered under race conditions.

Contact patch asymmetry due to angular misalignments between input and output shafts leads to the generation of high edge stress discontinuities on gear flanks, inducing fatigue spalling which affects system durability. Crowning is widely used as a palliative measure to mitigate these undesired effects. These problems can be further exacerbated by contact footprint truncation.

The paper presents a new approach to modelling the kinematics and contact micro-geometry of meshing conjunctions of involute spur gears with profile and lead modifications. A time-efficient analytical method is presented to accurately determine the contact footprint and kinematics, leading to the solution of highly loaded non-Newtonian mixed thermo-elastohydrodynamic (TEHD) contact under the extreme prevalent conditions of high performance vehicular transmissions. The effect of tooth form modification on contact footprint truncation, contact kinematics and generated frictional power loss is investigated. This approach has not hitherto been reported in literature.

Keywords: High performance transmissions, Spur Gear, micro-geometrical form modification, contact kinematics, thermo-elastohydrodynamics, Non-Newtonian traction
1-Introduction

The modern light-weight and compact powertrain concept provides significant advantages in fuel efficiency, but can lead to a plethora of noise and vibration concerns. Weight reduction of rotational components in the driveline in particular tends to improve throttle response and the errant rigid body dynamics. However, this is often achieved at the expense of vibration and noise because of low structural damping, for example from hollow driveshaft tubes [1]. In transmissions, the supporting gear shafts can be made hollow in line with the light-weight concept. This is particularly true of transmissions of high performance vehicles. Shaft-integrated lubricant galleries can also be present to lubricate the bearing supports and gear contact conjunctions, but require the removal of additional material which further adds to the reduction in component rigidity. This can lead to increased structural vibration (elastodynamics). Short and stubby gear shafts and appropriate material selection mitigate the elastodynamic behaviour to a large extent. Contact loads in gear teeth meshing conjunctions in high performance vehicles can routinely exceed 20 kN. In practice, the combined torsional deflection and bending of the supporting gear shafts can cause relative angular displacement, resulting in the edge loading of teeth pair contacts with high generated localised pressures. Inspection of run-in gears has shown skewed scuffing and pitting of flank surfaces at their edges due to the presence of edge stress discontinuities, similar to those reported for misaligned rolling element bearings [2-4]. To mitigate this phenomenon teeth crowning is used as a palliative measure in a same manner as axial edge relieving of rolling element bearings highlighted in [2]. With particularly compact gears, such as in the transmissions of high performance vehicles, truncation of the contact footprint of meshing gear teeth causes high localised pressures at the loaded flank edges, which can lead to scuffing and/or fatigue spalling with appearance of pits.

Several authors have investigated improvements in the meshing contact of misaligned spur gears through crowning [5-7]. However, they have primarily focused on mitigating the effects of misalignment without regard to contact patch truncation and its effect on generated frictional losses. Harianto and Houser [8] assessed the effect of crowning and its influence upon generated contact stress distribution within an active area of the contact face-width. Variations in peak-to-peak transmission error were also presented by varying the extent of crowning and misalignment in order to assess their implications for gear dynamics. A similar analysis was conducted by Seol and Kim [9], where the effect of crowning on dynamic transmission error and the dynamic loading factor were ascertained. While truncation was
observed in the results presented by Mao [5], no further assessment seems to have been conducted to establish whether the occurrence of truncation is a hindrance to transmission efficiency.

This paper provides predictions of friction in the contact of meshing teeth pairs. Contact friction occurs as the result of shear of a thin film of lubricant as well as any direct contact of rough surfaces of mating gear teeth pairs. The thin lubricant films in the highly loaded contact of teeth pairs at high loads, representative of conditions investigated in this paper, are subjected to non-Newtonian shear as highlighted in [10-13]. In practice, the real mating surfaces are rough. Therefore, direct interaction of mating surface topography through the thin lubricant film also contributes to the generated friction. The paper presents the effect of teeth crowning upon generated friction and power loss of meshing gears under combined non-Newtonian thermal shear of thin lubricant films as well as direct interaction of real rough surface topography, an approach not hitherto reported in literature.

2- Method of Analysis

2.1- Lubricated contacts

Loaded gear teeth routinely experience contact pressures of the order of 1-3 GPa. The meshing conjunction operates under Elastohydrodynamic (EHD) regime of lubrication with Newtonian or non-Newtonian shear of a thin lubricant film, depending on the prevailing contact conditions; contact kinematics and load [10-13]. The generated contact friction comprises viscous shear of the lubricant film and any direct interaction of contiguous surfaces. For an analytical solution, such as in [10], estimation of lubricant film thickness is important. This is made through use of lubricant film thickness equations such as that originally provided by Ertel and Grubin [14]. Subsequently, many authors have provided similar expressions through regression of numerical results with different combinations of operating conditions, such as contact speed and load [15-18]. A comprehensive list of these earlier equations is provided in [19]. All these equations were for steady state conditions and do not include features such as squeeze film effect in mutual approach of surfaces or changes in the lubricant entrainment angle into the contact as the result of rolling and sliding. For the former, Jalali-Vahid et al [20] provided an equation, verified by optical interferometric studies, and Rahnejat [21] provided a squeeze film term as an addition to that of Mostofi and Gohar [22] for generalised elliptical point contact with angled entrainment flow, an approach which was also made by Chittenden et al [23]. Similar expressions exist for finite line contact
footprints [24]. The current study assumes an elliptical point contact footprint of large aspect ratio, thus the expression in [23] is used:

\[ h_c = 4.31R_x U_e^{0.68} G_e^{0.49} W_e^{-0.073} \left\{ 1 - \exp \left[ -1.23 \left( \frac{R_y}{R_x} \right)^{2/3} \right] \right\} \]  

(1)

where, the non-dimensional groups are:

\[ W_e = \frac{\pi W}{2E_r R_x^2}, \quad U_e = \frac{\pi \eta_0 U_r}{4E_r R_x}, \quad G_e = \frac{2}{\pi} (E_r \alpha) \]

where \( W \) is the instantaneous total normal contact load, \( E_r \) is the reduced elastic modulus of contact, \( R_x \) and \( R_y \) are the equivalent principal contact radii of curvature along the lubricant entrainment (semi-minor axis) and side leakage directions (semi-major axis) respectively, \( \eta_0 \) is the lubricant viscosity, \( U_r \) is the speed of lubricant entrainment, \( \alpha \) is the lubricant pressure-viscosity coefficient, and \( h_c \) is the central contact film thickness.

Due to the limitations of computational power, the early solutions assumed low to medium contact loads with fully flooded inlets and isothermal Newtonian conditions. Most gearing contact inlets are starved as some of the inlet flow is subjected to counter and swirl flows. Therefore, zero reverse flow boundary should be determined, beyond which all the entrained lubricant is drawn into the contact as shown by Tipei [25] and experimentally investigated for a circular point contact by Johns-Rahnejat and Gohar [26] and numerically verified by Mohammadpour et al [27]. This approach assumes a fully flooded inlet, which is the basis of equation (1) and film thickness is assumed not to vary along the semi-major axis which significantly reduces the computation times. With swirl and counter flows at the contact inlet, starvation occurs which reduces the lubricant film thickness in the contact (reduced supply), thus affecting friction and power loss [28].

2.2- Tooth Contact Analysis

Lubricant film thickness formulae such as equation (1) require prior knowledge of the instantaneous contact curvatures of mating teeth and their tangential sliding velocities. These are used to determine the speeds of lubricant entrainment into the contact. For ideal involute spur gears, these parameters are functions of basic formulae derived from the principles of involute geometry, as shown in [29, 30]. However, these approaches do not take into account
gear teeth modifications which are common practice by manufacturers to improve upon the baseline involute for operational integrity and durability. In cases where modifications such as profile shift (addendum modification), tip relief or crowning are applied to gear teeth, formulations such as those found in [29, 30] become inappropriate. In this paper, a methodology is developed to accurately estimate the instantaneous local surface velocities and contact curvatures for modified involute spur gears. The method is based on tracking the meshing conjunction along the Length of Contact (LOC) as illustrated in Fig.1.

Figure 1: Length of Contact (LOC) of a spur gear pair

The ideal involute gear tooth profile is first rendered for a finite number of nodes with respect to the Cartesian X-Y frame of reference for a specified base radius $r_b$, and outer radius $r_o$, where the origin of the co-ordinate set is at the gear centre (Fig.2).
Coordinates are generated for both the driving pinion gear and the driven gear wheel, for an involute roll angle in the range: \( \varphi_i \leq \varphi \leq \varphi_m \), where \( i = 1, 2, 3, \ldots, m \). \( \varphi_1 = 0. \) \( \varphi_m \) is the involute roll angle at the prescribed gear tip diameter. As a rule of thumb, the selected number of profile nodes, \( m \) should correlate positively with the gear module \( m_n \) and the required sensitivity for variations in the micro-geometry along the tooth profile. For a given base circle radius, the nodal coordinates can be determined as:

\[
x_i = r_{bp}(\cos(\varphi_i) + \varphi_i \sin(\varphi_i))
\]

\[
y_i = r_{bw}(\sin(\varphi_i) - \varphi_i \cos(\varphi_i))
\]

where the position vector of each nodal coordinate is represented as:

\[
\vec{S}_i = (x_i, y_i)
\]

\[
\theta_{\vec{S}_i} = \tan^{-1}\left(\frac{y_i}{x_i}\right)
\]

When the gears have applied profile shifts, the coordinates of the shifted profile become:

\[
x_i = \left( |\vec{S}_i| + (x_p m_n) \right) \cos(\theta_{\vec{S}_i})
\]
\[ y_i = \left( |\tilde{S}_i| + (x_p m_n) \right) \sin \left( \theta_{\tilde{S}_i} \right) \] (7)

It should be noted that profile shifts alter the working pitch and tip radii of the meshing gears, which is of consequence for the accuracy of contact tracking. The effective working pitch radii of profile-shifted pinion and wheel, \( R_p \) and \( R_w \) are obtained as:

\[ R_p = R_p' + (x_p m_n) \] (8)

\[ R_w = R_w' + (x_w m_n) \] (9)

where \( R_p' \) and \( R_w' \) are the pitch radii prior to any profile shift, and \( x_p \) and \( x_w \) are the profile shift coefficients for the pinion and wheel respectively.

If the sum of profile shifts applied to a meshing gear pair does not diminish, then the operating centre distance, \( a_c \) and consequently the working pressure angle \( \varphi \) of the gear pair alter and can be calculated as:

\[ a_c = (R_p + R_w) \] (10)

\[ \varphi = \frac{\pi}{2} - \sin^{-1} \left( \frac{r_{b,p} + r_{b,w}}{a_c} \right) \] (11)

For any subsequent tooth modifications, \( |\tilde{S}_i| \) and \( \theta_{\tilde{S}_i} \) must be recalculated prior to applying any modification when using equations (4) and (5), so that they would reflect the state of the active flank geometry.

For gears with a parabolic tip relief, the magnitude of transformation at each discrete node along the profile is determined by expressing the relief function in a vertex-form as a function of the involute roll angle and the magnitude of the local relief, \( l \) as:

\[ l = g(x - h)^2, \quad \text{where}, \ h = \varphi_{rca} \] (12)

where \( \varphi_{rca} \) is the involute roll angle at the start of relief determined, where \( |\tilde{S}_i| \) is equal to the prescribed start position of the tip relief radius \( r_{ca} \). The magnitude of the local relief at \( \varphi_m \) is taken to be the prescribed extent of tip relief, \( C_a \). Thus,
Transformation of magnitude \( l \) is applied to \( \vec{S}_i \) along a circle of radius \( |\vec{S}_i| \) for the coordinates which lie within the tip relief region of the profile (i.e. where \( \varphi_{rca} \leq \varphi_i \leq \varphi_m \)):

\[
l(\varphi_i) = g(\varphi_i - \varphi_{rca})^2
\]

\[
\theta_i(\varphi_i) = \frac{l(\varphi_i)}{|\vec{S}_i|}
\]

The resulting profile coordinates after tip relief are obtained as:

\[
x_i = |\vec{S}_i| \cos \left( \theta_{\vec{s}_i} + \theta_i(\varphi_i) \right)
\]

\[
y_i = |\vec{S}_i| \sin \left( \theta_{\vec{s}_i} + \theta_i(\varphi_i) \right)
\]

Gear tooth profile modifications such as profile shift and tip relief are applied prior to any lead modification. As a single cross-section of a spur gear along the X-Y plane is representative of the gear without any lead modification in three-dimensions, the profile coordinates can be replicated for the entire width of the tooth flank \( t \), along the z-direction (into the plane of paper in Fig. 2). This renders the active flank in three-dimensions. The selected number of lead nodes \( n \) should correlate positively with the required sensitivity for variations in micro-geometry along the lead direction of the flank. The system of coordinates for the resulting generated flank is represented as three coordinate matrices of dimensions \( m \times n \):

\[
X = \begin{bmatrix}
x_{11} & x_{12} & \ldots & x_{1n} \\
x_{21} & x_{22} & \ldots & x_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
x_{m1} & x_{m2} & \ldots & x_{mn}
\end{bmatrix},
\]

\[
Y = \begin{bmatrix}
y_{11} & y_{12} & \ldots & y_{1n} \\
y_{21} & y_{22} & \ldots & y_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
y_{m1} & y_{m2} & \ldots & y_{mn}
\end{bmatrix},
\]

\[
Z = \begin{bmatrix}
z_{11} & z_{12} & \ldots & z_{1n} \\
z_{21} & z_{22} & \ldots & z_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
z_{m1} & z_{m2} & \ldots & z_{mn}
\end{bmatrix},
\]

where,
The crowning modifications assessed in the current study are circular. However, the presented methodology can be readily extended to more complex forms such as parabolic and asymmetric crowning. At the edges of the tooth flank, the magnitude of crowning transformation \( v \) is taken to equate to the prescribed extent of crowning \( C_b \), while at the centre of the tooth flank \( v \) is assumed to diminish:

\[
Z = \left[ -\frac{t}{2}, 0, \frac{t}{2} \right], V = [C_b, 0, C_b]
\]

The crowning magnitude \( v \) can be determined for each nodal position along the flank by expressing \( v \), analogous to the amount of material removed from the ideal involute flank, as a circular function of the nodal position along the z-axis, thus:

\[
(z_{ij} - a)^2 + (v_{ij} - b)^2 = r^2 \tag{18}
\]

The three unknown coefficients, \( a \), \( b \) and \( r \) can be obtained simultaneously with the knowledge of the three coordinates provided above by \( Z \) and \( V \). Crowning magnitude can then be expressed in the form:

\[
v_{ij} = \left( r^2 - (z_{ij} - a)^2 \right)^{\frac{1}{2}} + b \tag{19}
\]

\[
\theta_{v,ij} = \frac{v_{ij}}{|\vec{s}_{ij}|} \tag{20}
\]

The co-ordinates on the flank post-crowning are then calculated as:

\[
x_{ij} = |\vec{s}_{ij}| \cos \left( \theta_{\vec{s}_i} + \theta_{v,ij} \right) \tag{21}
\]

\[
y_{ij} = |\vec{s}_{ij}| \sin \left( \theta_{\vec{s}_i} + \theta_{v,ij} \right) \tag{22}
\]

\[
\vec{s}_{ij} = (x_{ij}, y_{ij}) \tag{23}
\]

\[
\theta_{\vec{s}_{ij}} = \tan^{-1} \left( \frac{y_{ij}}{x_{ij}} \right) \tag{24}
\]
It should be noted that the application of crowning alters the curvature of the flank $\kappa_y$ along its width, and consequently the major axis of the contact ellipse along the width of the contact footprint. This curvature is expressed for a single mating flank as:

$$\kappa_y = \frac{1}{R_y} = \left| \frac{f''(x)}{(1 + [f'(x)]^2)^{3/2}} \right|$$

(25)

where,

$$f(x) = (r^2 - (x - a)^2)^{0.5} + b$$

The reduced radius of curvature along the major axis of the contact ellipse of a meshing conjunction becomes:

$$\frac{1}{R_y} = \frac{1}{R_{y,p}} + \frac{1}{R_{y,w}}$$

(26)

where, $R_{y,p}$ and $R_{y,w}$ are the radii of curvatures of the pinion and wheel respectively.

The length of contact (LOC) for a given gear pair, shown in Fig.1 and Fig.3, is expressed as:

$$\text{Length of Contact} = L1 + L2$$

(27)
Figure 3: Calculating the Length of Contact (LOC) of a modified gear pair

From Fig. 3, $L_1$ and $L_2$ can be derived geometrically as:

$$L_1 = \sin(\theta_{L_1}) \left( \frac{R_p}{\sin(\theta_{Rp})} \right) \quad \text{and} \quad L_2 = \sin(\theta_{L_2}) \left( \frac{R_w}{\sin(\theta_{Rw})} \right)$$ (28)

where,

$$\theta_{Rp} = \sin^{-1} \left( R_p \left( \frac{\pi}{\frac{\pi}{2} + \varphi} \right) \right), \quad \theta_{L1} = \pi - \left( \frac{\pi}{2} + \varphi \right) - \theta_{Rp}$$ (29)

and,
\[ \theta_{RW} = \sin^{-1}\left(\frac{R_w}{|\hat{S}_{m,w}|} \left(\frac{\sin\left(\frac{\pi}{2} + \phi\right)}{|\hat{S}_{m,w}|}\right)\right), \quad \theta_{L2} = \pi - \left(\frac{\pi}{2} + \phi\right) - \theta_{RW} \] (30)

where, for a given gear pair, the following condition should hold true:

\[ |\hat{S}_{m,p}| \equiv r_{o,p} \text{ and } |\hat{S}_{m,w}| \equiv r_{o,w} \] (31)

It transpires that any positional estimations of meshing conjunctions along the LOC are only accurate for a given cross-sectional profile. While this is inconsequential for gears without lead modifications, in the case of symmetrically crowned gears, it is logical to use the X-Y cross-section where \( z = 0 \) in order to estimate \( L1 \) and \( L2 \) as this is where an initial contact occurs between the two mating flanks. When this is not the case, such as with asymmetrically crowned gears, estimations of LOC should utilise datasets which are representative of the appropriate cross-sectional profiles.

For a given contact ratio \( m_p \), simultaneous contact of teeth in and out of mesh can be modelled as a set of fundamental functions, the magnitudes of which correspond to the position of the contact conjunction, \( X_i \) on the LOC relative to the pitch point (Fig.4).

![Figure 4: Contact tracking](image)
The number of fundamental functions necessary to fully define the contact between a gear pair is determined by the maximum number of simultaneous meshing contacts which can exist for a given gear pair. The leading and trailing teeth pair contacts are separated by a length along the LOC, equalling the base pitch $p_b$. In the contact tracking algorithm presented here, this length is expressed as an equivalent rotation angle $\varphi_{pb}$, about the origin of the driving pinion as:

$$\varphi_{pb} = \frac{p_b}{r_{b,p}} \quad (32)$$

Consequently, each fundamental signal has a period $\varphi_c$, of:

$$\varphi_c = [m_p] \varphi_{pb} \quad (33)$$

Thus, for a gear pair where $1 < m_p < 2$, the motion of the two resulting meshing contacts; $X_1$ and $X_2$ can be illustrated as in Fig. 5.

![Figure 5: A snap-shot of meshing pattern for a gear pair of $1 < m_p < 2$](image)

The progress of the gear pair along this recurring contact cycle is expressed as:

$$A = \begin{bmatrix} \varphi_p \\ \varphi_c \end{bmatrix} \quad (34)$$

The instantaneous contact location $X_1$ can then be calculated as:
\( X_1 = -L_2 + \varphi_p r_p - A \varphi_c r_p \) \hspace{1cm} (35)

1. Depending on \( X_2 \) trailing or leading \( X_1 \) can instantaneously be obtained as:

\[
X_2 = \begin{cases}
-L_2 + (\varphi_p - \varphi_{p_b}) r_p - A \varphi_c r_p; & \varphi_p > A \varphi_c + \varphi_{p_b} \\
-L_2 + (\varphi_p + \varphi_{p_b}) r_p - A \varphi_c r_p; & \varphi_p < A \varphi_c + \varphi_{p_b}
\end{cases}
\hspace{1cm} (36)

2. While \( X_i \) lies on the LOC: \(-L_2 < X_i < L_1\), the distances of the meshing conjunctions \( X_i \) to the pinion and wheel centres \( O_p \) and \( O_g \) can be expressed as:

\[
|O_p X_i| = \begin{cases}
\left( R_p^2 + X_i^2 - \left(2 R_p |X_i| \cos \left( \frac{\pi}{2} + \theta \right) \right) \right)^{0.5}; & 0 \leq X_i \leq L_1 \\
\left( R_p^2 + X_i^2 - \left(2 R_p |X_i| \cos \left( \frac{\pi}{2} - \theta \right) \right) \right)^{0.5}; & -L_2 \leq X_i < 0
\end{cases}
\hspace{1cm} (37)

\[
|O_w X_i| = \begin{cases}
\left( R_w^2 + X_i^2 - \left(2 R_w |X_i| \cos \left( \frac{\pi}{2} - \theta \right) \right) \right)^{0.5}; & 0 \leq X_i \leq L_1 \\
\left( R_w^2 + X_i^2 - \left(2 R_w |X_i| \cos \left( \frac{\pi}{2} + \theta \right) \right) \right)^{0.5}; & -L_2 \leq X_i < 0
\end{cases}
\hspace{1cm} (38)

3. As the pinion and wheel profile coordinates have been previously expressed as functions of \( \vec{S}_{ij,p} \) and \( \vec{S}_{ij,w} \), the nodal coordinates of the meshing conjunctions for a given cross-sectional profile can be obtained relative to the coordinate systems attached to the pinion and the wheel origins as:

\[
|\overline{O_p X}| \equiv |\vec{S}_{ij,p}|, \text{ and } |\overline{O_w X}| \equiv |\vec{S}_{ij,w}|
\hspace{1cm} (39)

4. Where,

\[
\overline{O X} \equiv \vec{S}_{ij} = (x_{ij}, y_{ij}, z_{ij})
\hspace{1cm} (40)

5. However, equations (39) and (40) do not hold true along the width of the flank for a given pinion angle \( \varphi_p \) when the gears are crowned. Thus, the orientation of the instantaneous
position vectors $\vec{O_pX_i}$ and $\vec{O_wX_i}$ which remain constant along the tooth width for a given pinion angle are utilised to obtain the nodal coordinates of the meshing conjunction along the width of the tooth as:

$$\theta_{\vec{O_pX_i}} \equiv \theta_{\vec{S}_{ij}} = \tan^{-1}\left(\frac{y_{ij}}{x_{ij}}\right)$$

(41)

By subsequently determining the nodal coordinates on the either sides along the profile of the instantaneous meshing conjunction, the position $X_i$, the instantaneous curvature along the minor axis $R_x$ of the elliptical contact footprint can be determined at each nodal coordinate for the width of each tooth. This also serves to calculate the velocity component tangential to this curvature (i.e. the local surface velocity). Thus, for a specified contact location along the pinion tooth width, instantaneous coordinate sets $X_p$, $Y_p$ and $Z_p$ are given as:

$$X_p = \begin{bmatrix} x_i \cdots x_{i+1} \end{bmatrix}; \quad |\vec{O_pX}| < |\vec{S}_{m,p}|$$

$$Y_p = \begin{bmatrix} y_i \cdots y_{i+1} \end{bmatrix}; \quad |\vec{O_pY}| < |\vec{S}_{m,p}|$$

$$Z_p = \begin{bmatrix} z_i \cdots z_{i+1} \end{bmatrix}$$

where,

$$z_{i-1} = z_i = z_{i+1}$$

As in equation (18), the centre $C_{R_x}(a, b)$ and radius of the circle, corresponding to the local radius of curvature $R_x$, formed by the coordinate sets $X_p$, $Y_p$ and $Z_p$, can be calculated. This procedure is replicated for the wheel such that the instantaneous reduced radius of curvature of the meshing contact becomes:
\[
\frac{1}{R_x} = \frac{1}{R_{x,p}} + \frac{1}{R_{x,w}} \tag{42}
\]

The instantaneous surface velocities of the pinion and wheel teeth in mesh can be resolved along the contact tangential plane (i.e. along the minor axis of the contact ellipse) as:

\[
v_{p,\text{minor}}^t = |v_p \cos \left( \pi - \left| \frac{\theta_{c_{R_{x,p}X_i}} - \theta_{o_pX_i}}{2} \right| \right)\]  \tag{43}
\]

\[
v_{w,\text{minor}}^t = |v_w \cos \left( \pi - \left| \frac{\theta_{c_{R_{x,w}X_i}} - \theta_{o_wX_i}}{2} \right| \right)\]  \tag{44}
\]

where, the velocity of any point on the pinion and gear teeth in contact may be obtained as,

\[
v_p = |O_pX_i| \omega_p, \text{and } v_w = |O_wX_i| \omega_w \tag{45}\]

And for the pinion:

\[
\theta_{c_{R_{x,p}X_i}} = \tan^{-1}\left( m_{c_{R_{x,p}X_i}} \right) \tag{46}\]

\[
\theta_{o_pX_i} = \tan^{-1}\left( m_{o_pX_i} \right) \tag{47}\]

where \( m_{c_{R_{x,p}X_i}} \) and \( m_{o_pX_i} \) are the slopes of the vectors tangential to the local contact curvature, and the contact position respectively:

\[
m_{c_{R_{x,p}X_i}} = -\frac{1}{m_{o_pX_i}} \text{ and, } \quad m_{c_{R_{x,p}X_i}} = \frac{Y_p[2] - b}{X_p[2] - a} \tag{48}\]

\[
m_{o_pX_i} = -\frac{1}{m_{o_pX_i}} \text{ and, } \quad m_{o_pX_i} = \frac{Y_p[2]}{X_p[2]} \tag{49}\]

Replicating equations (46)-(49) for the wheel yields its corresponding instantaneous local surface velocities. Note that with an assumed no side-leakage flow of the lubricant across the teeth flanks, the components of the pinion and gear surface velocities along the major axis of the contact ellipse becomes:

\[
v_p^{t,\text{major}} = v_w^{t,\text{major}} = V = 0
\]
This assumption is reasonable due to the thinness of the prevailing lubricant film. Therefore, the lubricant entraining velocity along the minor axis of the contact footprint is used in equation (1) at any instant of time during meshing of a gear teeth pair:

\[ U_r = \frac{1}{2} (v_p^{t,\text{minor}} + v_w^{t,\text{minor}}) \]  

(50)

Similarly, the tangential components are used to obtain the instantaneous contact sliding velocity as:

\[ U_s = |v_p^{t,\text{minor}} - v_w^{t,\text{minor}}| \]  

(51)

As this methodology is sensitive to variations in contact geometry along the flank, variations in contact and curvatures at the discrete nodes along the major axis of the prevailing contact patch are considered.

Through subsequent application of a quasi-static finite element technique, the Tooth Contact Analysis (TCA) software (CALYX, Advanced Numerical Solutions) employed in this study allows for accurate representation of instantaneous load distribution across the flank of the modified spur gear teeth [30].

The contact load applied per teeth pair is a function of the dynamic response of the system. The ratio of the applied load \( W_i \) on a given flank under consideration to the total transmitted load \( W_T \) [10, 11] is known as the load factor, \( lf \). This is a function of the pinion angle. Therefore, the load per pair of contacting teeth is obtained as:

\[ lf = \frac{W}{W_T} \]  

(52)

where, the total load on the gear pair is obtained from the applied torque.

Time varying contact stiffness resulting from the variation in the meshing contact location and simultaneous load sharing between multiple teeth pairs is taken into account through TCA to acquire representative individual tooth loading distributions.

To observe the crowning-induced variations in the localised contact pressures along the semi-major axis of the elliptical footprint, the instantaneous contact ellipse is discretised into a
number of finite equivalent rectangular strips (similar to the contact of slender cylindrical rollers). The semi-major and semi-minor half-widths of the prevailing contact ellipse, \( a \) and \( b \) can be calculated as [18]:

\[
a = \left( \frac{6\bar{k}^2 \varepsilon W R'}{\pi E'} \right)^{1/3}
\]

\[
b = \left( \frac{6\varepsilon W R'}{\pi k E'} \right)^{1/3}
\]

where, \( R' \) is the reduced contact radii of the curvature:

\[
\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y}
\]

\( E' \) is the reduced elastic modulus, \( \bar{k} \) is the ellipticity parameter given as:

\[
\bar{k} = 1.0339 + \left( \frac{R_y}{R_x} \right)^{0.636}
\]

and,

\[
\bar{\varepsilon} = 1.0003 + \frac{0.5968R_x}{R_y}
\]

The resulting contact ellipse is discretized into \( n \) individual rectangular contact strips, where the semi-major and semi-minor half-widths of each strip, \( a_j \) and \( b_j \) are:

\[
a_j = \frac{a}{n} \text{ where } j = 1,2,3 \ldots n
\]

\[
b_j = \left( \frac{4WR'}{\pi a_j E'} \right)^{1/2}
\]

and the contact area of each strip becomes:

\[
A_j = 4a_j b_j
\]

The distance of the centre-point of a strip \( j \) from the centre-point of the contact ellipse along the semi-major axis is:
\[ x_j = -a + \left( \frac{2a}{\pi} (j - 1) \right) + a_j \]  \hspace{1cm} (59)

For instances, where the contact ellipse is truncated at the gear teeth flank edges, the total length of the contact semi-major axis is limited to the length of the gear flank \( t \). The semi-major axis of each individual discretized strip then becomes:

\[ a_j = \frac{t}{2} \]  \hspace{1cm} (60)

and \( x_j \) is given as:

\[ x_j = -\frac{t}{2} + \left( \frac{t}{n} (j - 1) \right) + a_j \]  \hspace{1cm} (61)

The local load acting over each discretised strip \( W_j \) is estimated using knowledge of the load intensity distribution \( Q(x) \) acquired through TCA (Fig. 6).

\[ W_j = \int_{x-a}^{x+a} Q(x) \]  \hspace{1cm} (62)

where, the average (Pascal) contact pressure acting at each discretised strip becomes:

\[ \bar{p}_j = \frac{W_j}{A_j} \]  \hspace{1cm} (63)

Figure 6: Instantaneous flank load intensity distribution - TCA
2.3- Viscous friction

The conditions investigated in the current analysis pertains to transmissions of high performance vehicles at high contact loads and shear rates, leading to thin non-Newtonian thermo-elastohydrodynamic films. Evans and Johnson [31] modified Crook’s [32] original thermal analysis of Newtonian fluids to account for discrepancies between theoretical and observed values of viscous traction in elastohydrodynamic contacts. They provided an analytical expression for coefficient of friction under TEHD conditions subject to non-Newtonian shear of a thin film, which is utilised in this analysis [31]:

\[
\mu_j = 0.87 \alpha \tau_0 + 1.74 \frac{\tau_0}{\bar{p}_j} \ln \left[ \frac{1.2}{\tau_0 \frac{K}{\bar{h}_c} \left( \frac{2K_0}{1 + 9.6 \xi_j} \right)} \right] \tag{64}
\]

Note that the coefficient of friction is calculated for each discretised strip of the instantaneous contact. Therefore, an average of these can represent the value at any instant of time during the meshing cycle. \( \tau_0 \) is the lubricant Eyring shear stress, \( K \) is its thermal conductivity, and \( \xi_j \) is:

\[
\xi_j = \frac{4}{\pi} \frac{K}{\bar{h}_c / R_{x,j}} \left( \frac{\bar{p}_j}{E' R_{x,j} K' \rho' c' U_{r,j}} \right)^{1/2} \tag{65}
\]

where, \( R_{x,j} \) is the local reduced contact radius of curvature in the direction of lubricant entrainment at position \( x_j \), and \( K', \rho', \) and \( c' \) are the thermal conductivity, density, and specific heat capacity of the contacting solids respectively.

The generated friction due to viscous shear of the lubricant film is then expressed as

\[
f_{v,j} = \mu_j W_j \tag{66}
\]

2.4- Flash surface contact temperature

Crook [32] showed that heat generated due to viscous friction is transferred across the film through conduction to the solid surfaces, which in turn rapidly convects away. Through the reasonable assumption that the shear stress \( \tau \) varies parabolically along the direction of lubricant entrainment, Crook showed that the temperature rise of the solid surfaces in elastohydrodynamic conjunctions, from bulk temperature \( \theta_O \), is obtained as:
\[ \theta_{s,j} - \theta_{0,j} = + \frac{0.5T_j\Delta U_j}{(\pi K'\rho'c'b_j\bar{U}_j)^{1/2}} \] (67)

where, \( T \) is the traction per unit width:

\[ T_j = \frac{2b_jf_{v,j}}{A_j} \] (68)

\( \Delta U \) is the sliding velocity, and \( \bar{U} \) is the rolling velocity.

With the assumption that heat generation occurs locally at the centre-plane of the lubricant film and that the separated solid surfaces have the same temperature, Johnson and Greenwood [33] derived formulae, estimating the temperature rise across the lubricant film. The resulting estimate is the local temperature rise, averaged across the semi-minor axis of the elliptical contact footprint at any instant of time. With the assumption that the lubricant thermal conductivity remains constant, whilst its dynamic viscosity reduces exponentially with any rise temperature and the lubricant’s temperature-viscosity coefficient \( \beta \), they were able to accurately predict the prevailing lubricant film centre-plane temperature as:

\[ \frac{T_j\Delta U_jh_c\beta_L}{16b_jK} = \frac{(1 + X_j^2)^{1/2}}{X_j \sinh^{-1}X_j} \] (69)

\[ X_j = \sqrt{e^{\beta(\theta_{c,j} - \theta_{s,j})} - 1} \] (70)

The work in [33] further led to the derivation of equation (64) in Evans and Johnson [31] presented in section 2.3. While these formulations serve to predict the temperature at the centre plane of the contact, it is merely used to observe temperature variation on the active teeth flank area. Thermal predictions do not serve to vary rheological parameters (provided in Table 2) during the course of the simulation as they may do in reality.

2.5- Boundary Friction

The thin lubricant films in the meshing contacts of loaded gear teeth pairs in high performance transmissions are comparable in magnitude to the roughness of the teeth flanks. Consequently, asperity interactions and therefore boundary friction is to be expected. Figure 7 is an image of a patch of a tooth flank obtained through use of white light interferometry.
with a vertical resolution (in the z-direction) of 10 nm and 0.175 µm in the contacting xy plane. The gear considered has been subjected to severe race conditions for a distance of 4000 km, well past its running-in state.

Figure 7: Surface Roughness of gear tooth flank centre after 4000 km on a high performance racing drive cycle

Greenwood and Tripp [34] developed a method to evaluate the generated boundary friction as the result of direct interaction of asperities on the counter face contacting surfaces. The method assumes a Gaussian distribution of surface height asperities. When mixed or boundary regimes of lubrication occur, Stribeck’s oil film parameter: \(1 < \lambda = \frac{h_c}{\sigma} < 2.5\), specifies the fraction of the load carried by the asperities in each discretized contact area, \(A_j\) as:

\[
W_{a,j} = \frac{16\sqrt{2}}{15\pi} (\xi \beta \sigma)^2 \left( \frac{\sigma}{\beta} \right)^{E' A_j F_{5/2}(\lambda)}
\]  

where, \(\beta\) is the average asperity tip radius, \(\sigma\) is the composite RMS surface roughness of the contacting surfaces, and the statistical function \(F_{5/2}(\lambda)\) for a Gaussian distribution of asperities can be represented by a polynomial fit function as [35]:
\[ F_{s/2} = \begin{cases} 0; & \text{for } \lambda < 2.5 \\ -0.004\lambda^5 - 0.06\lambda^4 - 0.3\lambda^3 - 0.8\lambda^2 - 0.8\lambda - 0.6 & \text{for } \lambda \geq 2.5 \end{cases} \] (72)

The roughness parameter; \( \xi \beta \sigma \) for steel surfaces is generally in the range of 0.01–0.07 [35]. The average asperity slope; \( \sigma / \beta \) is in the range of 10^{-4}-10^{-2} [24]. Surface measurements of the load bearing flank centre of the gear considered in this study, using focus variation imaging technique yielded: \( \xi \beta \sigma = 0.011 \) and \( \sigma / \beta = 0.0194 \).

Asperity friction should be considered in mixed and boundary regimes of lubrication. A thin adsorbed film (a tribo-film) exists at the summit of the asperities and/or is entrapped in their inter-spatial valleys. This thin tribo-film is subjected to non-Newtonian shear, thus boundary friction \( f_{b,j} \) at each discretised strip is given as:

\[ f_{b,j} = \tau_L A_{a,j} \] (73)

where, the asperity contact area \( A_{a,j} \) [34] is:

\[ A_{a,j} = \pi^2 (\xi \beta \sigma)^2 A_f F_2(\lambda) \] (74)

and the lubricant’s limiting shear stress \( \tau_L \) is given by [36]:

\[ \tau_{L,j} = \tau_0 + \varepsilon P_{m,j} \] (75)

where, \( \varepsilon \) is the slope of the lubricant limiting shear stress-pressure dependence, and the mean (Pascal) pressure \( P_{m,j} \) is:

\[ P_{m,j} = \frac{W_{a,j}}{A_{a,j}} \] (76)

and the statistical function \( F_2(\lambda) \) is expressed as [35]:

\[ F_2(\lambda) = \begin{cases} -0.002\lambda^5 - 0.03\lambda^4 - 0.2\lambda^3 + 0.5\lambda^2 - 0.8\lambda - 0.5 & \text{for } \lambda < 2.5 \\ 0; & \text{for } \lambda \geq 2.5 \end{cases} \] (77)

In this study, the topographical properties of the contacting teeth surfaces (i.e. surface roughness, roughness parameter, and average asperity slope) are assumed constant both along and across the flank. However, values used in this study are based on measurements sampled over multiple areas of the flank, thus it is unlikely to significantly affect the results of the analysis.
2.6- Power Loss

The total instantaneous friction in each discretised element is as the results of combined viscous and boundary friction contributions:

\[ f_{T,j} = f_{v,j} + f_{b,j} \]  \hspace{1cm} (78)

The instantaneous power loss per instantaneous contact strip is determined as:

\[ P_{\text{loss},j} = f_{T,j}U_{s,j} \]  \hspace{1cm} (79)

where, \( U_{s,j} \) is the local sliding velocity, acting at the centre of the discretised contact strip, \( j \).

3-Results and Discussion

The effect of symmetric crowning (Fig. 8), and contact ellipse truncation on contact efficiency in spur gears is studied.

![Symmetric gear teeth crowning modification (plan view)](image)

The simulated conditions are typical of high performance transmissions and are listed in Table 1, along with the relevant design parameters of the gear pair and the operating conditions.
Table 1: Pinion and gear parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module (mm)</td>
<td>3.6</td>
</tr>
<tr>
<td>Number of teeth (pinion:gear)</td>
<td>27:27</td>
</tr>
<tr>
<td>Pitch diameter (pinion:gear) (mm)</td>
<td>97.97</td>
</tr>
<tr>
<td>Normal pressure angle (°)</td>
<td>25</td>
</tr>
<tr>
<td>Face width (mm)</td>
<td>13.5</td>
</tr>
<tr>
<td>Pinion speed (RPM)</td>
<td>9500</td>
</tr>
<tr>
<td>Pinion torque (Nm)</td>
<td>700</td>
</tr>
<tr>
<td>Bulk solid temperature (°C)</td>
<td>130</td>
</tr>
</tbody>
</table>

Table 2 lists the relevant data for the solid surfaces and the lubricant rheological properties.

Table 2: Lubricant rheology and surface data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure viscosity coefficient (Pa-1)</td>
<td>$1.05 \times 10^{-8}$</td>
</tr>
<tr>
<td>Lubricant dynamic viscosity at atmospheric pressure at 130°C (mPa.s)</td>
<td>4.04</td>
</tr>
<tr>
<td>Lubricant Eyring stress (MPa)</td>
<td>2</td>
</tr>
<tr>
<td>Thermal conductivity of fluid (W/mK)</td>
<td>0.137</td>
</tr>
<tr>
<td>Modulus of elasticity of contacting solid (GPa)</td>
<td>206</td>
</tr>
<tr>
<td>Poisson’s ratio of contacting solids (-)</td>
<td>0.3</td>
</tr>
<tr>
<td>Density of contacting solids (kg/m3)</td>
<td>7800</td>
</tr>
<tr>
<td>Thermal conductivity of contacting solids (W/m.K)</td>
<td>46.7</td>
</tr>
<tr>
<td>Heat capacity of contacting solids (J/kg K)</td>
<td>460</td>
</tr>
<tr>
<td>RMS composite Surface roughness (μm)</td>
<td>0.2</td>
</tr>
<tr>
<td>Roughness parameter ($\xi \beta \sigma$)</td>
<td>0.011</td>
</tr>
<tr>
<td>Average asperity slope ($\sigma / \beta$)</td>
<td>0.0194</td>
</tr>
</tbody>
</table>

Table 3 lists the amount of crowning applied for each studied case. All crowning are symmetric.

Table 3: Amount of crowning and semi-major axis curvatures

<table>
<thead>
<tr>
<th>Case</th>
<th>Crowning Amount (μm)</th>
<th>Contact radii of curvature (along semi-major axis) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.5</td>
<td>9.12</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>4.56</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>2.28</td>
</tr>
<tr>
<td>D</td>
<td>20</td>
<td>1.12</td>
</tr>
<tr>
<td>E</td>
<td>30</td>
<td>0.76</td>
</tr>
</tbody>
</table>

A complete meshing cycle is simulated through quasi-static Tooth Contact Analysis (TCA) at 150 discrete contacting locations from root to tip of the active flank area for each case listed in Table 3. The prevailing local load intensity, contact curvature, and rolling and sliding velocities are calculated for 251 equally-spaced locations (strips) of interest along the contact’s semi-major axis. The resulting 251-by-150 data arrays form the input to the
analytical thermal elastohydrodynamic (TEHL) model. The computation time for each discrete location (i.e. instant of meshing is approximately 2 min.

The size of each discretised cell (strip) in the TEHL model was selected through iterative trial-and-error, allowing appropriate compromise between computational effort and any loss of necessary resolution to observe the effects of contact ellipse truncation at the edges of contacting flanks. Consequently, the analytical TEHL model discretises the prevailing contact width (along the semi-major axis of the contact footprint) into 128 equally-spaced sampling points. Variations in the tribological parameters at each discretised location are acquired for a complete meshing cycle, simulated in 100 discrete contacting locations from root to tip of the active flank area.

For the purposes of estimating the instantaneous film thickness (Equation (1)), the contact geometry and kinematics are taken as those at the centre of the instantaneous contact footprint. Figure 9 shows the variation of the central lubricant film thickness, as a pair of meshing teeth contact progresses from the root to the tip.

![Figure 9: Central contact lubricant film thickness variation in a meshing cycle](image)

The vertical axes in Figures 10-12 and 14 have been normalised to represent the length of the active tooth flank in the direction of the tooth profile. Similarly, the horizontal axis represents the length along the flank width from one edge to the other (i.e. the lead direction).
Hereinafter, figure suffixes correspond to the scenario studied. Figures 10(a-d) show the variation of contact footprint geometry at seven discrete locations on the active flank, as a single teeth meshing contact progresses from the tooth root to the tooth tip for scenarios A-D under the loading conditions given in Table 1. Case E is not shown for sake of rationalised presentation, but follows the same trends as the others.

Figure 10: Variation of contact footprint geometry in a meshing cycle for: a) Case, A b) Case B, c) Case C, and d) Case D

With sufficient load or small amount of crowning, any crowning-induced curvature is flattened. If the semi-major width of the resulting contact ellipse is larger than the available tooth width, the contact footprint is truncated along the edges of the gear tooth flank width.
This is observed for the total duration of contact from its root to its tip in Figures 10a and 10b. Figure 10c shows that the truncation only occurs when the contact is approximately half-way up the flank. This is because while the contact on the active flank remains in the vicinity of the flank tip and root, leading and trailing teeth are still in contact with their conjugate pairs. Thus, the load is shared between them and the individual tooth loads remain lower, subject of course to the instantaneous load-share ratio: $l_f$. However, as the meshing contact passes through the central region of the flank, the load is no longer shared among multiple teeth pairs. It is entirely borne by a single instantaneous contact footprint. As contact truncation occurs, stress discontinuities create pressure concentrations at the edges of the flank. This is observed in Figure 11a and to a lesser extent in Figure 11b.

A crowning magnitude of 10µm (Figure 11c) is found to be sufficient to mitigate these pressure concentrations at the contact edges. This is illustrated by the uniform pressure fields on the flank edges in Figure 11c. However, the redistribution of load on an active tooth flank creates areas of significantly higher pressures towards the flank centre, even though the total active flank area remains largely unchanged (Figure 11a-11c). Regions, where contact does not occur are illustrated in black. This trend of increased pressures at the flank centres is further exaggerated in Figure 11d, where the extent of crowning is higher and the active contact area is reduced, as would be expected.
Figure 11: Contact Pressure distribution on an active flank (GPa) – complete meshing cycle:
a) Case A, b) Case B, c) Case C, and d) Case D

The elimination of stress discontinuities reduces the onset of contact fatigue and wear at the edges of flank and lubricant depletion there. These issues are adequately described elsewhere with regard to lubricated contacts [4, 19, 37]. This study focusses on the resulting effects on power losses.

Figures 12a-12d show the contact power loss per unit length (W/mm), the integral of which along the tooth flank width yields the total instantaneous power loss. The contact losses are
highest at the start and at the end of the meshing cycle, where the relative sliding velocities between the contacting teeth pairs is the highest. This corresponds to the tooth root and the tooth tip contacting regions. Similarly, power losses are the lowest where the gear contact passes through the pitch point (approximately half-way between the flank root and tip) and the contact experiences pure rolling condition for an involute spur gear pair. This trend is observed in Figures 12a-12d.

The crowning-induced curvature along the semi-major axis of the contact causes slight variations in the local surface geometry and induces some changes in the sliding velocities along the semi-major axis. Though this variation is small, its effects are exaggerated as the sliding velocity tends to diminish as the meshing contact approaches the pitch point. The influence on power losses can be seen as undulations in the contours of Figures 12a-12d.
With increasing crowning, Figures 12a-12d show a gradual shift and an increase in the contact losses towards the centre of the flank; a consequence of the pattern observed in the pressure isobars of Figures 11a-11d. When contact truncation occurs (Figures 12a-12b), the power losses are higher in the localised regions along the edges of the flank which correlate to the areas of pressure concentrations due to stress discontinuity. However, even though the active flank area remains largely unchanged as in Figures 12a-12c, the distribution of contact losses is noticeably less severe with lesser crowning. This remains the case when considering the magnitude of the total contact power losses incurred for a complete meshing cycle.
Figure 13 shows a larger percentage and magnitude of contact losses with increasing crowning. While crowning is quite important in mitigating fatigue due to edge loading and thus enhances reliability, the results show how in some cases crowning can have a detrimental effect on efficiency.

Figure 13: Percentage variation in contact losses relative to Case A (‘A’ in figure) – stated values are for a single active flank

Figures 14a-d show the lubricant centreline temperatures in the active flank area. Contact temperatures are highest at the root and at the tip as there is higher relative sliding velocities of the surfaces in these regions. Mid-meshing cycle, where the contact is in the region of the flank centre and sliding velocity is at its lowest, temperature rise is minimal as temperatures remain closer to the bulk temperature of 130°C.
With increased crowning, Figures 14a-14d show a gradual increase in the maximum contact temperatures near the root and the tip of the flank. With contact truncation (Figure 14a), contact temperatures are observably higher in the localised regions along the edges of the flank. Mid-meshing cycle where the contact is in the vicinity of the flank centre, the temperatures at the edges of the flank rise by approximately 8°C more than at the contact centre (Figure 14a). However, this variation becomes less pronounced with a slight increase in crowning, even when truncation and the stress discontinuity are still present (Figure 14b). When crowning sufficiently mitigates the edge pressure concentrations (Figure 14c), the...
temperature rises by approximately 15°C more than in the case of Figure 14a, even though
the active flank area remains largely unchanged. This trend is further pronounced in Figure
14d.

4- Conclusions

The high loading conditions experienced in compact high performance transmissions can
cause contact footprint truncation in the meshing gear teeth pairs. This phenomenon causes
stress discontinuities and generated high edge pressures. These pressure concentrations can
be detrimental to system durability. They can also act to inhibit lubricant flow into these
regions of the contact when lubricant nozzles are directed onto the side wall of the meshing
gears. High pressure spikes have been shown to inhibit lubricant entrainment, resulting in
very thin lubricant films in rolling element bearings [4] as well as cam-tappet contacts [38].

Crowning is used primarily as a palliative measure for misalignment issues, which
exacerbates the effect of edge pressure spikes. Crowning reduces the magnitude of high
pressure spikes at gear flank edges and its associated undesirable repercussions. While the
reduction of contact area, some as the result of contact truncation, implies lowered contact
friction, the redistribution of pressure as the result of crowning can increase the average
contact pressures over the contact footprint and thus increase the frictional power loss. The
effect of starvation and cavitation is not included in the current analysis, both of which would
have important repercussions as well.

Thermal analysis has shown that for the gears, lubricant and operating conditions considered
in this study, peak contact temperatures rise by approximately 15°C when crowning is
introduced to reduce the edge pressure concentrations.

Nomenclature

\(A_j\) Area of a discretised cell
\(a\) Semi-major half-width of contact ellipse
\(a_c\) Gear pair centre distance
\(a_j\) Semi-major half-width of a discretised cell j
\(b\) Semi-minor half-width of contact ellipse
\(b_j\) Semi-minor half-width of a discretised cell j
\(c'\) Specific heat capacity of solid surfaces
\(C_\alpha\) Amount of tip relief
\(C_b\) Amount of crowning
\(E'\) Reduced elastic modulus of the contact
\(E_r\) Reduced (effective) Young’s modulus of elasticity
\(f_{b,j}\) Boundary friction at a discretised cell \(j\)
\(f_{v,j}\) Viscous friction at a discretised cell \(j\)
\(h_c\) Central lubricant film thickness
\(K\) Lubricant thermal conductivity
\(K'\) Solid thermal conductivity
\(m_n\) Gear module
\(m_p\) Gear contact ratio
\(n\) Number of discretised cells along the semi-major axis of the contact footprint
\(\bar{p}_j, P_{m,j}\) Mean pressure in a discretised cell \(j\)
\(R'\) Reduced radius of a counterformal contacting pair
\(R_p\) Pinion pitch radius
\(R_w\) Wheel pitch radius
\(R_x\) Principal radius of curvature along the semi-minor axis (direction of lubricant entrainment)
\(R_y\) Principal radius of curvature along the semi-major axis (side leakage direction)
\(r_{b,p}\) Pinion base radius
\(r_{b,w}\) Wheel base radius
\(r_{o,p}\) Pinion outer radius
\(r_{o,w}\) Wheel outer radius
\(T\) Traction per unit width of contact
\(t\) Tooth flank width
\(U_{r,\bar{U}}\) Rolling velocity (Speed of lubricant entrainment)
\(U_{S, \Delta U}\) Sliding velocity
\(x_p\) Pinion profile shift coefficient
\(x_w\) Wheel profile shift coefficient
\(W\) Normal contact load

**Greek Letters**

\(\alpha\) Lubricant pressure-viscosity coefficient
\(\beta\) Average asperity tip radius
\(\beta_L\) Thermal conductivity of lubricant
\(\varepsilon\) Slope of the lubricant limiting shear stress-pressure dependence
\(\eta_0\) Lubricant viscosity at atmospheric pressure
\(\theta_0\) Bulk solid temperature
\(\theta_c\) Contact centre-plane temperature
\(\theta_s\) Solid surface flash temperature
\(\kappa\) Curvature
\(\xi\) Asperity density
\(\rho'\) Density of solids
\(\sigma\) Composite Surface roughness
\(\tau_0\) Eyring shear stress
\(\tau_L\) Limiting shear stress
Abbreviations

\[ TCA \] Tooth Contact Analysis
\[ TEHL \] Thermal Elastohydrodynamic Lubrication

References


