Control of cascaded DC-DC converter-based hybrid battery energy storage systems - Part II: Lyapunov approach

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Control of Cascaded DC-DC Converter Based Hybrid Battery Energy Storage Systems – Part II: Lyapunov Approach

Nilanjan Mukherjee, Member, IEEE and Dani Strickland

Abstract - A cascaded DC-DC boost converter is one of the ways to integrate hybrid battery types within a grid-tie inverter. Due to the presence of different battery parameters within the system such as, state-of-charge and/or capacity, a module based distributed power sharing strategy may be used. To implement this sharing strategy, the desired control reference for each module voltage/current control loop needs to be dynamically varied according to these battery parameters. This can cause stability problem within the cascaded converters due to relative battery parameter variations when using the conventional PI control approach. This paper proposes a new control method based on Lyapunov Functions to eliminate this issue. The proposed solution provides a global asymptotic stability at a module level avoiding any instability issue due to parameter variations. A detailed analysis and design of the nonlinear control structure are presented under the distributed sharing control. At last thorough experimental investigations are shown to prove the effectiveness of the proposed control under grid-tie conditions.

I. INTRODUCTION

HYBRID battery integration within an energy storage system is an emerging alternative to off-the-shelf battery energy storage systems to reduce the average cost of overall energy storage systems [1] – [3]. To integrate hybrid batteries into a system requires a modular approach utilizing battery modules with sets of series connected cells per module. Unfortunately, from a reliability perspective the greater the number of series connected cells, the lower the module reliability [4]. Therefore, low number of series connected cells within a module is a preferred approach. There are two main forms of modular DC-DC converters which can integrate these low voltage batteries (e.g. <100V) to a grid-tie inverter: a) a parallel converter approach and b) a series/cascaded approach. A previous study on this area suggested a cascaded approach over the parallel approach from reliability and cost perspective [5]. Apart from the reliability/cost issues, the parallel DC-DC approach has many drawbacks in conjunction with low voltage energy sources related to the high boost ratio [6], [7]. Therefore, this paper adopts the cascaded/series approach.

However, a conventional cascaded boost converter structure is not fault-tolerant in nature which is unable to bypass a faulty battery module. Therefore, this study uses an H Bridge configuration to allow each module to handle unexpected battery failure as shown in Fig. 1. Due to the presence of different types of batteries in the system, a module based distributed power sharing strategy based on a weighting function has been presented [8].

The weighting function method helps to distribute the total power among the different battery modules according to their instantaneous battery parameters so that they aim to charge/discharge together within a charge/discharge cycle. To implement this sharing, desired module voltage or current parameter/reference of the individual module control loop is dynamically varied according to the corresponding battery parameters such as, state-of-charge/capacity to regulate the module voltage and current according to weighting function. As

Index Terms—Cascaded DC-DC converters, hybrid battery energy storage systems, lyapunov control, stability

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$\omega_i$</td>
<td>Weighting factor for $i^{th}$ module current</td>
</tr>
<tr>
<td>$V_{batt,i}$</td>
<td>Steady state battery voltage of $i^{th}$ module</td>
</tr>
<tr>
<td>$V_{batt,i}$</td>
<td>Instantaneous battery voltage of $i^{th}$ module</td>
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<tr>
<td>$i_{batt,i}$</td>
<td>Instantaneous current of $i^{th}$ battery module</td>
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<tr>
<td>$I_{batt,i}$</td>
<td>Steady state current of $i^{th}$ battery module</td>
</tr>
<tr>
<td>$V_{dc,i}$</td>
<td>Instantaneous capacitor voltage of $i^{th}$ module</td>
</tr>
<tr>
<td>$V_{dc,i}$</td>
<td>Steady state module dc-link voltage of $i^{th}$ module</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>Steady state total DC-link capacitor voltage</td>
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<tr>
<td>$V_{dc}$</td>
<td>Instantaneous inverter dc-link capacitor voltage</td>
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<tr>
<td>$I_{dc}$</td>
<td>Steady state common DC-link current</td>
</tr>
<tr>
<td>$i_{dc}$</td>
<td>Instantaneous common DC-link current</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Instantaneous duty cycle of $i^{th}$ boost converter module</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Average duty cycle of $i^{th}$ boost converter module</td>
</tr>
<tr>
<td>$C$</td>
<td>Module dc-link capacitance</td>
</tr>
<tr>
<td>$L$</td>
<td>Module boost inductance</td>
</tr>
<tr>
<td>$R_L$</td>
<td>Leakage resistance of module boost inductance</td>
</tr>
<tr>
<td>$H$</td>
<td>Weighting function for $i^{th}$ battery module</td>
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</tbody>
</table>

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a result of this control the operating point and the designed stability margin of the conventional PI-controller may vary in wide range which can hamper the stability of the overall converter as reported in [9]. To cater issue, this paper concentrates on more stable method based on Lyapunov function which helps to maintain the global asymptotic stability at the module level and the system level.

Moreover, the stability aspect of the single input cascaded two-stage DC-DC converter has also been reported in [24] using multiple Lyapunov functions.

Apart from these studies which were mainly related to power converters, some generalised investigations on stabilization of switched linear systems were reported in [25] – [27]. These studies mainly concentrate on time varying systems and focus on developing a common Lyapunov function to analyse the stability issues due to the internal time delays. Even though these studies provide an accurate analysis, those are not used in the present application because the battery state-of-charge and capacity are very slow changing variables which make the system behave similar to a time-invariant system.

There are very few research studies looking into the application of Lyapunov method on a multi-modular system especially in energy storage applications. This paper proposes such a design approach based on Lyapunov functions which operate on a module basis avoiding the traditional concept of cascaded PI-control loop per module and generates converter duty ratio directly from the global asymptotic stability criterion. As a result it overcomes any stability concern due to the battery parameter variations in the long term and also provides a more uniform dynamic response of the converter. The detailed design of the approach and limitations of this control method for the cascaded converter has been included. Moreover, the comparison with the existing controller method is also presented. At last, thorough experimental validations of the proposed approach have also been presented to show its effectiveness under various grid operating conditions.

II. DISTRIBUTED SHARING STRATEGY FOR CASCADE DC-DC CONVERTER

The distributed sharing strategy adopted in this paper of the cascaded DC-DC converter is based on the previously derived method as reported in [8]. Within a hybrid system, the charging/discharging depends purely on the module current. Therefore in order to appropriately utilise the hybrid batteries within the same converter, a current sharing strategy among the modules is necessary. The equation (1) shows the sharing scheme based on weighting factors where \( SOC_i \) and \( Q_{max,i} \) are the battery state-of-charge and maximum charge capacity.

\[
\frac{i_{bat,1}}{ω_1} = \frac{i_{bat,2}}{ω_2} = \ldots = \frac{i_{bat,n}}{ω_n} \quad \text{Where} \quad \omega_i = \frac{(1−SOC_i)Q_{max,i}}{\sum_{k=1}^{n} v_{bat,k} Q_{max,k}} \quad \text{charging, \( \forall i = 1,2, \ldots , n \)}
\]

Module power balance equation can be written from Fig. 1

\[
v_{dc,i}i_{dc} = η_i v_{bat,i}i_{bat,i} \quad \text{(2)}
\]

From the derivation of the weighting function as shown in (1);

\[
i_{bat,i} = C\omega_i \quad \text{or} \quad i_{bat,i}^* \propto \omega_i \quad \forall i = 1 \ldots n \quad \text{(3)}
\]

From the power balance equation (2) for a constant \( i_{dc} \) and \( η_i \).
\[ v_{dc,l}^* = \frac{\eta L V_{batt,0}}{i_{dc}} \quad \text{or} \quad v_{dc,l}^* \propto v_{batt,i} \omega_l \ \forall i = 1 \ldots n \]  
\[ \text{Now, } \sum v_{dc,i}^* = v_{dc} \]  
this gives the following expression;

\[ v_{dc,l}^* = v_{dc} \frac{\omega_l v_{batt,i}}{\sum_{k=1}^n \omega_k v_{batt,k}} \ \forall i = 1 \ldots n \]  

III. LYAPUNOV BASED CONTROL APPROACH

Previous attempts on Lyapunov approach was predominantly employed in non-modular DC-AC and DC-DC converters [28] – [32]. There are two Lyapunov approaches: a) direct approach e.g. as described in [31], b) indirect approach as described in [32]. The direct approach seeks for a function and aims to decrease the total system energy through a trajectory which guarantees the stability, while the indirect approach uses a linearised state-space model of the system and introduces a state-feedback control law to stabilize the system.

The direct approach is preferred because: a) the direct approach ensures a global asymptotic stability while the indirect approach only provides a local stability, b) the control design for an indirect approach requires a large computational burden because of the presence of large matrices.

There are two ways the direct approach could be applied on a converter: a) considering the full switching model and switching dynamics as reported in [24], [29] and b) focusing on the simplified averaged error dynamics. In the present case, the latter approach is considered because the stability study due to long term battery parameter variations has been looked at where the averaged error dynamics can be sufficient. The converter modelling has been performed based on Fig. 1.

A. Lyapunov Based Design for Modular DC-DC Converter

There are two state variable per converter module according to Fig. 1: a) \( i_{batt,i} \) and b) \( v_{dc,i} \). the dynamic equations per module can be expressed in (6) – (7).

\[ \frac{d(i_{batt,i})}{dt} + R_L i_{batt,i} + (1 - D_i)v_{dc,i} = v_{batt,i} \ \forall i = 1 \ldots n \]  
\[ \frac{dv_{dc,i}}{dt} - (1 - D_i)i_{batt,i} = -i_{dc} \ \forall i = 1 \ldots n \]  

The reference values of these states are \( i_{batt,i}^* \) and \( v_{dc,i}^* \). Therefore, the dynamic equations at the reference point become:

\[ \frac{d(i_{batt,i}^*)}{dt} + R_L i_{batt,i}^* + (1 - D_i)v_{dc,i}^* = v_{batt,i} \ \forall i = 1 \ldots n \]  
\[ \frac{dv_{dc,i}^*}{dt} - (1 - D_i)i_{batt,i}^* = -i_{dc} \ \forall i = 1 \ldots n \]  

The following error functions can be defined for the states:

\[ x_{ji} = i_{batt,i} - i_{batt,i}^* \quad \text{and} \quad x_{2i} = v_{dc,i} - v_{dc,i}^* \ \forall i = 1 \ldots n. \]

Substituting, \( x_{ji} = x_{ji} + i_{batt,i} \) and \( x_{2i} = x_{2i} + v_{dc,i} \) in (6), (7)

\[ \frac{d(x_{ji}+i_{batt,i})}{dt} + R_L (x_{ji} + i_{batt,i}^*) + (1 - D_i)(x_{2i} + v_{dc,i}^*) = v_{batt,i} \]  
\[ \frac{dx_{2i}+v_{dc,i}^*}{dt} - (1 - D_i)(x_{2i} + v_{dc,i}^*) = -i_{dc} \]

\( d_t \) is the control input of the converter, therefore, it can be written as a combination of reference and perturbed points \( d_t = D_t + \tilde{d}_t \). Substituting \( d_t \) in (10) and (11) gives:

\[ \frac{L d(x_{ji}+i_{batt,i})}{dt} + R_L (x_{ji} + i_{batt,i}^*) + (1 - D_i)(x_{2i} + v_{dc,i}^*) = v_{batt,i} \]  
\[ \frac{d(x_{2i}+v_{dc,i}^*)}{dt} - (1 - D_i)(x_{2i} + v_{dc,i}^*) = -i_{dc} \]

Now substituting, (14), (15) in (17) and rearranging:

\[ \frac{dL(x)}{dt} = x_{ji} L \frac{dx_{ji}}{dt} + x_{2i} L \frac{dx_{2i}}{dt} \]  

According to Lyapunov’s stability theorem, any linear or nonlinear system is globally asymptotically stable if a function termed the Lyapunov function, \( L(x) \) satisfies the following properties [32].

1) \( L(0) = 0; \)
2) \( L(x) > 0 \) for all \( x \neq 0; \)
3) \( \frac{dL(x)}{dt} < 0 \) for all \( x \neq 0; \)
4) \( L(x) \rightarrow \infty \) as \( ||x|| \rightarrow \infty. \)

A suitable Lyapunov function for use in this application has been chosen similar to that previously reported [18]:

\[ L(x) = \frac{1}{2} L x_{ji}^2 + \frac{1}{2} C x_{2i}^2 \]

Taking the derivative,

\[ \frac{dL(x)}{dt} = x_{ji} L \frac{dx_{ji}}{dt} + x_{2i} L \frac{dx_{2i}}{dt} \]

According to the criterion listed above, it requires \( \frac{dL(x)}{dt} < 0 \) for the stability. Therefore, select \( \tilde{d}_t = K (x_{2i} i_{batt,i} - x_{ji} v_{dc,i}^*) \) and substituting in (18)

\[ \frac{dL(x)}{dt} = -R_L x_{ji}^2 - K (x_{2i} i_{batt,i} - x_{ji} v_{dc,i}^*)^2 \]

Therefore, the necessary and sufficient condition for submodule stability becomes \( K > 0 \) but it plays an important role in the performance of the Lyapunov control. Moreover, the design of \( K \) could be different in charging and discharging because the control references \( i_{batt,i} \) and \( v_{dc,i} \) are different as explained in section II.

During the changeover between charging to discharging or vice-versa the duty ratio \( \tilde{d}_t \) of the converter is dynamically adjusted using the changeover command from the line side inverter. As a result of this dynamic changeover the control
parameter ‘$K$’ in (19) needs to be adjusted at the time of switching the operating mode to guarantee the stability. The difference between the charging and discharging mode is reflected through the formulation of derivative of Lyapunov function or the duty ratio (expression (19)) as the current and voltage references ($i_{\text{batt},i}^*$ and $v_{dc,i}$) are function of $\omega$.

B. Significance of ‘$K$’ in Proposed Control Design

In order to study the importance of $K$, let us substitute $\delta_i = K(x_{2i}i_{\text{batt},i}^* - x_{1i}v_{dc,i}^*)$ in (14) and (15) and rearranging,

$$L \frac{d(x_{1i})}{dt} = -(1 - D_i - K_{i_{\text{batt},i}}v_{dc,i}^*)x_{2i} + \left(x_{1i}(R_L - KV_{dc,i}^*)^2\right) + \omega_{av}x_{1i}x_{2i} (\omega_{av})^2$$ \forall i = 1 \ldots n \tag{20}$$

$$C \frac{d(x_{2i})}{dt} = (1 - D_i + K_{i_{\text{batt},i}}v_{dc,i}^*)x_{1i} + K(x_{1i})^2v_{dc,i}^* - Kx_{1i}x_{2i} i_{\text{batt},i}^* - Kx_{1i}(i_{\text{batt},i})^2$$ \forall i = 1 \ldots n \tag{21}$$

Now linearizing (20) and (21) by substituting $x = \hat{x} + X$,

$$L \frac{d(\hat{x}_{1i})}{dt} = -(1 - D_i - K_{i_{\text{batt},i}}v_{dc,i}^*)\hat{x}_{2i} - \hat{x}_{1i}(R_L - KV_{dc,i}^*)^2$$ \forall i = 1 \ldots n \tag{22}$$

$$C \frac{d(\hat{x}_{2i})}{dt} = (1 - D_i + K_{i_{\text{batt},i}}v_{dc,i}^*)\hat{x}_{1i} - K\hat{x}_{2i}(i_{\text{batt},i})^2$$ \forall i = 1 \ldots n \tag{23}$$

Converting into the matrix form,

$$\begin{bmatrix} \frac{d(\hat{x}_{1i})}{dt} \\ \frac{d(\hat{x}_{2i})}{dt} \end{bmatrix} = \begin{bmatrix} -(1 - D_i - K_{i_{\text{batt},i}}v_{dc,i}^*) & \omega_{av}\hat{x}_{2i} \\ \omega_{av}\hat{x}_{1i} & -(1 - D_i + K_{i_{\text{batt},i}}v_{dc,i}^*) \end{bmatrix} \begin{bmatrix} \hat{x}_{1i} \\ \hat{x}_{2i} \end{bmatrix} + \begin{bmatrix} (R_L - KV_{dc,i}^*)^2 \\ 0 \end{bmatrix}$$ \forall i = 1 \ldots n \tag{24}$$

Averaging the matrix around the frequency $\omega$, allows the expression (24) to be further simplified.

$$\begin{bmatrix} \frac{d(\bar{x}_{1av})}{dt} \\ \frac{d(\bar{x}_{2av})}{dt} \end{bmatrix} = \begin{bmatrix} -(R_L - KV_{dc,i}^*)^2 \\ 0 \end{bmatrix} \begin{bmatrix} \bar{x}_{1av} \\ \bar{x}_{2av} \end{bmatrix}$$ \forall i = 1 \ldots n \tag{25}$$

Solving the average value of $\bar{x}_{1av}$ and $\bar{x}_{2av}$ from (25),

$$\frac{d(\bar{x}_{1av})}{dt} = -\frac{K(R_L - KV_{dc,i}^*)^2}{L} \bar{x}_{1av} \rightarrow \bar{x}_{1av}(t) = e^{-\frac{(R_L - KV_{dc,i}^*)^2}{L}t}$$ \forall i = 1 \ldots n \tag{26}$$

$$\frac{d(\bar{x}_{2av})}{dt} = -\frac{K(i_{\text{batt},i})^2}{C} \bar{x}_{2av} \rightarrow \bar{x}_{2av}(t) = e^{-\frac{K(i_{\text{batt},i})^2}{C}}$$ \forall i = 1 \ldots n \tag{27}$$

These equations are important because they contain the explicit expressions of the error dynamics. These error dynamics are important to predict the steady-state errors and dynamics responses of their individual states. It can be seen from (26) and (27) that the average values of steady state errors asymptotically go to zero for any positive values of $K$ which guarantees the stability. A higher value of $K$ provides a faster rate of convergence. Therefore, the individual control bandwidth of module voltage ($BW_{c,i}$) and current ($BW_{v,i}$) can be taken proportional to these values as shown in (28).

$$BW_{c,i} \propto \frac{(R_L - KV_{dc,i}^*)^2}{L} \quad \text{and} \quad BW_{v,i} \propto \frac{K(i_{\text{batt},i})^2}{C}$$ \forall i = 1 \ldots n \tag{28}$$

Now, substituting $v_{dc,i}^*$ and $i_{\text{batt},i}^*$ from (5) and (1) in (29),

$$v_{dc,i}^* = \frac{v_{dc}^*}{\sum_{k=1}^{\infty} k^2 \omega_{av} R_{k_{\text{batt}}} L_{k_{\text{batt}}} k} \quad \text{And} \quad i_{\text{batt},i}^* = P \frac{v_{dc}^*}{\sum_{k=1}^{\infty} k^2 \omega_{av} R_{k_{\text{batt}}} L_{k_{\text{batt}}} k}$$ \forall i = 1 \ldots n \tag{29}$$

To understand the variation of the relative bandwidth derived in (30), a comparative study has been presented in Fig. 2 where the variation of $BW_{c,i}$ for the existing cascaded PI and Lyapunov approach has been shown for a 12V battery. It can be found that relative control bandwidth remains flat in the Lyapunov approach because $V_{\text{batt}}$ does not vary in wide range. For this reason, the Lyapunov method can provide a more uniform dynamic response compared to conventional method.
C. Design Guidelines for the Control Parameter $K$

To provide a design guideline for the control parameter $K$, it is necessary to investigate the effect of system parameter changes in control stability because any error in the measurement and/or estimation process can result in inaccurate references. These inaccurate references may make the derivative of the Lyapunov function non-negative according to (31) in which the Lyapunov function can give rise to the stability issue.

Assume the inaccurate references due to measurement and estimation processes, are $\hat{I}_{batt,ic}$ instead of $I_{batt,ic}^*$ and $V_{dc,ic}^*$ instead of $V_{dc,ic}^*$. Under these conditions, the derivative $\frac{dL(x)}{dt}$ becomes:

$$\frac{dL(x)}{dt} = -K(x_2i_{batt,ic} - x_3i_{batt,ic}^*)(x_2i_{batt,ic}^* - x_4i_{batt,ic}^*) - R_2x_4^2 \quad (31)$$

This expression can be written in the form $X^TQX$ for convenience of analysis where $X = [x_1 \ x_2]$ and $Q$ is the following matrix:

$$Q = \begin{pmatrix} P & Q \\ Q^T & R \end{pmatrix}$$

where

$$P = -(Kv_{dc,i}^*v_{dc,ic}^* + R_2)$$

$$Q = \frac{k}{2}(i_{batt,ic}^*v_{dc,ic}^* + i_{batt,ic}v_{dc,i}^*)$$

$$R = -K(i_{batt,ic}^*i_{batt,ic}^*)$$

In order to fulfill the criterion $\frac{dL(x)}{dt} < 0$, the matrix $Q$ has to be negative definite which means $(Kv_{dc,i}^*v_{dc,ic}^* + R_2) > 0$ and $\det(Q) < 0$. The expression $(Kv_{dc,i}^*v_{dc,ic}^* + R_2) > 0$ if $K > 0$ as $v_{dc,i}^*$, $R_2$ and $v_{dc,ic}^*$ all are positive. $\det(Q)$ is derived below.

$$\det(Q) = -\frac{k^2}{4}
\begin{bmatrix}
  a^2v_{dc,i}^* + b^2i_{batt,ic}^* - 2abi_{batt,ic}^*v_{dc,i}^* - 4\frac{b}{K}ai_{batt,ic}^*
\end{bmatrix}
$$

Where

$$a = i_{batt,ic}^* \text{ and } b = v_{dc,ic}^* \quad (32)$$

Rearranging (31) provides

$$\det(Q) = -\frac{k^2}{4}
\begin{bmatrix}
  (av_{dc,i}^* - bi_{batt,ic}^*)^2 - 4\frac{b}{K}ai_{batt,ic}^*
\end{bmatrix}
$$

Therefore, necessary condition for which $\det(Q) < 0$ will be:

$$(av_{dc,i}^* - bi_{batt,ic}^*)^2 < 4\frac{b}{K}ai_{batt,ic}^*$$

or

$$K > \frac{4\frac{b}{K}ai_{batt,ic}^*}{(av_{dc,i}^* - bi_{batt,ic}^*)^2} \quad (33)$$

It can be seen from (33) that if there is an error in $v_{dc,i}^*$ and $i_{batt,ic}^*$, $\frac{dL(x)}{dt}$ is not always negative. Therefore, the stability is not guaranteed if references are not accurate enough. This is a practical scenario because measurements and estimations will not be accurate. Therefore, the expression (33) provides the minimum value of $K$ which can be treated as the design value.

Now, if there is a $\varepsilon_1$% and $\varepsilon_2$% error assumed in $i_{batt,ic}^*$ and $v_{dc,ic}^*$, then the minimum $K$ needed from (33) can be further modified as below.

$$K_{min,i} = \frac{4\frac{b}{K}ai_{batt,ic}^*}{(av_{dc,i}^* - bi_{batt,ic}^*)^2}$$

Now, if we assume $v_{dc,i} = 50V$, $R_2 = 0.05\Omega$, $\varepsilon_1 = 10\%$ and $\varepsilon_2 = 5\%$, the calculated $K_{min} = 0.0352$ therefore, $K > 0.0352$.

The following conclusions can be drawn about the proposed Lyapunov based control:

- A minimum value of $K$ is necessary to guarantee the Lyapunov function (34)
- A higher value of $K$ provides greater stability, fast convergence or provides better control bandwidth from (28) and (29).
- An excessive value of $K$ can increase noise and ripple in the module voltage and current because it enhances the perturbation part of the duty cycle ($\Delta_i$) as $d_i = D_i + \Delta_i$ which can also cause improper voltage and current sharing among the modules.
- Inappropriate choice of the control parameter $K$ can make $\frac{dL(x)}{dt}$ in (19) near to zero or more than zero, in which case, the system can enter into the oscillatory region.
- The parameter $K$ can be fixed for a particular design because the relative bandwidth does not vary significantly for the battery application as demonstrated in Fig. 2. However, an adaptive $K$ can also be used to obtain a uniform dynamic response throughout the operating cycle of the energy storage system (i.e. for the SOC 0 – 100% range).

D. Proposed Control Structure for Cascaded DC-DC Converter using Lyapunov Method

The requirements of the control system remain unchanged as earlier; control each converter module (this time using the Lyapunov function) and to maintain the central dc-link voltage constant so that the stability and dynamic response are not sacrificed at a module level. This control approach requires individual references for the system states to be generated independently unlike in the cascaded control approach (based on PI-controller) where each outer voltage loop generates the reference for the inner current. The proposed control structure is presented in Fig. 3. It consists of four different stages: a) reference generation for module voltages, b) reference generation for module currents, c) reference generation for module duty ratio, and d) actual control logic.

The module dc-bus voltage references can be generated using the central dc-link voltage reference and weighting factors as shown in Fig. 3(a). Module current references are generated from the output of an overall dc-link controller which helps to maintain the central dc-link voltage as shown in Fig. 3(b). The output of that controller generates the reference for the common dc-link current ($I_{dc}$) which in turn generates the power reference for each module. These power references are then converted to the individual current references dividing by their module input voltages. Fig. 3(c) shows the reference generation for the module duty ratio through equation (21). A LPF (low pass filter) has been employed to eliminate the high frequency noise generated from the differentiation. The switching signals for the converter are generated using functions in Fig. 3(d).
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6

Fig. 3 Proposed control structure: a) voltage reference generation, b) current reference generation, c) duty ratio reference generation, d) control logic

E. Advantages and Disadvantages of the Proposed control

The main advantages of the proposed Lyapunov based approach over the conventional PI control approach are as follows:

- Provides more stable response because the converter duty ratio is directly generated from the derivative of the energy function which provides a guaranteed stability at a module level. This method suits the modular converter structure because it is important to maintain stability for all the modules within the converter.

- Relative bandwidth between the control variables remains nearly constant which helps to provide more uniform dynamic response

- Implementation does not involve integrators therefore, it is straightforward to implement

- It is particularly suitable for the application where the system parameters are subjected to variations during operation similar to this application

- It is also suitable where a large number of cascaded control loops could have been needed and the relative dependency of the control bandwidth is critical.

This approach also suffers from some drawbacks:

- Design method is more complicated and dependent on the choice of Lyapunov function because there is no specific design method for the Lyapunov approach

- Control references needs to be generated independently from the control loops using the system equations

- Inappropriate selection of the control parameter can cause slow convergence of the steady-state error.

IV. COMPARISON BETWEEN THE EXISTING APPROACH AND PROPOSED APPROACH

Cascaded DC-DC converter used in previous applications such as in [7], [8] uses predominantly cascaded PI control approach with an outer PI and an inner proportional or a hysteresis controller per module basis. An alternative Lyapunov control strategy has been compared with the cascaded PI control approach. The comparison between the existing PI approach and the proposed Lyapunov based approach is presented from three aspects such as: a) stability issue, b) design difficulty and c) computation requirements.

Stability: This section shows the stability comparison between the PI approach and the Lyapunov approach using Lyapunov energy function as shown below. The stability can be judged using the derivative of the Lyapunov function. It is derived for the two control approaches here. It can be seen from Fig 3 that the duty ratio is generated from output of the current controller which means the duty ratio can be expressed as below using its error dynamics.

\[
\dot{d}_i = K_{ci} \left( k_{vi}(v_{dc_i} - v_{dc,i}) + \frac{k_{vi}}{\tau_v}(v_{dc,i} - v_{dc,i}) - i_{batt,i} \right) \quad \forall \ i = 1 ... n \quad (35)
\]

\[
\dot{x}_i = -K_{ci} \left( x_{2i} k_{vi} - x_{3i} \frac{k_{vi}}{\tau_v} - x_{1i} - i_{batt,i} \right) \quad \text{Where} \quad (36)
\]

\[
x_{3i} = f(v_{dc,i} - v_{dc,i}^*), \ x_{2i} = (v_{dc,i} - v_{dc,i}^*) x_{1i} = (i_{batt,i} - i_{batt,i}^*)
\]
For stability purposes, \( \frac{dt(x)}{dt} \) is derived below by substituting \( \hat{a}_i \) in (14) and (15)

\[
\frac{dt(x)}{dt} = -\left( R_L + k_{ci} v_{dci} \right) x_{1i}^2 + \left( k_{ci} k_{vi} \right) x_{2i}^2 - \left( k_{ci} k_{vi} \right) x_{1i} x_{2i} - \left( k_{ci} \frac{k_{vi}}{T_p} v_{dci} \right) x_{1i} x_{3i} + \left( k_{ci} i_{batt,i} \right) x_{2i} x_{3i} + \left( k_{ci} v_{dci} i_{batt,i} \right) x_{1i} + \left( k_{ci} i_{batt,i} \right) x_{2i} \forall i = 1 \ldots n \tag{37}
\]

Note the expression in (37) is of third order because of the presence of an integrator in the PI controller. Moreover, it can be noted that some of the terms e.g. the coefficient of \( x_{3i}^2 \) are negative in (37) and some of them are strictly positive e.g. coefficient of \( x_{2i}^2 \) which means \( \frac{dt(x)}{dt} \) is strictly \(<0\) for all values of \( i_{batt,i} \) and \( v_{dci} \). Therefore, the stability is not guaranteed using the cascaded PI control approach.

On the other hand, the expression of the duty ratio for the Lyapunov approach is given in (38) which provide the expression of \( \frac{dt(x)}{dt} \) as derived earlier in (19). Note \( \frac{dt(x)}{dt} < 0 \forall i_{batt,i} \) and \( v_{dci} \) for a minimum \( K \) which provides a stable response in case of Lyapunov approach.

\[
\hat{a}_i = K(x_{2i} i_{batt,i} - x_{1i} v_{dci}) \tag{38}
\]

**Design issues:** Lyapunov control design predominantly depends on the choice of appropriate Lyapunov function and accurate design of a nonnegative control parameter \( K \). The design of the control parameter is directly related to the accurate reference values of the system states (e.g. voltage and current). Therefore, there is no direct design formula for the Lyapunov method. However, the Lyapunov design does not depend on the design of individual control loop and also does not involve integration which simplifies the computation.

On the other hand, PI control loop approach has multiple design methods which make it straightforward and widely accepted method.

**Computation Requirements:** The hardware implementation is one of the important criterions for power electronic applications because the overall control algorithm needs to be implemented by a digital controller which is normally expensive. It can be seen from Fig. 14 that Lyapunov control does involve only algebraic calculation and comparisons which can be implemented through an inexpensive digital controller even if there is a large number of modules. It only requires an overall PI controller to generate references for all the modules. However, the PI control approach requires multiple integrators both in inner and outer loop per module which puts slightly higher complexity and computation burden on the controller compared to the proposed approach especially in a multi-modular system. However, such difference is not significant because both approaches use the same number of sensors and I/O’s to implement the distributed sharing. The summary of the overall comparison has been presented in Table 1 for completeness of the study. It is can be seen from the table that the proposed Lyapunov control method is a preferred method in this application where parameters prone to vary.

### Table 1: Comparison Between the Existing Approach and the Proposed Approach

<table>
<thead>
<tr>
<th>Control method</th>
<th>Applicability in hybrid battery energy storage</th>
<th>Stability</th>
<th>Design difficulty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lyapunov method</td>
<td>Yes</td>
<td>Guaranteed</td>
<td>High</td>
</tr>
<tr>
<td>Existing PI controller approach</td>
<td>Yes</td>
<td>Not guaranteed</td>
<td>Low</td>
</tr>
</tbody>
</table>

### V. Experimental Validation of the Proposed Approach

Three different battery types were used in the experimental implementation to prove the effectiveness of the Lyapunov approach: Module – 1: 12V, 10Ah lead acid (SOC\(_{max} = 13.8V\) OCV\(_{min} = 9.6V\)) Module – 2: 24V, 16Ah lead acid (SOC\(_{max} = 27V\) OCV\(_{min} = 18V\)), Module – 3: 7.2V, 6.5Ah NiMH (SOC\(_{max} = 8.5V\) OCV\(_{min} = 5.5V\)). The entire validation has been performed at two different dc-link voltages and power levels connecting to a 100V, 50Hz grid system through a Variac in the laboratory. The overall control system shown in Fig. 3 has been implemented in OP5600 based Opal-rt controller.

The first stage of experiment is performed at dc-link voltage \( v_{dc} = 150V \) and power level \( P = 500W \). Fig. 4 and Fig. 6 shows the battery current responses under with the Lyapunov function based control present. The starting SOC are set to e.g. SOC\(_{o,1} = 10\%\), SOC\(_{o,2} = 45\%\) and SOC\(_{o,3} = 8.0\%\) during discharging and SOC\(_{o,1} = 96\%\), SOC\(_{o,2} = 90\%\) and SOC\(_{o,3} = 86\%\) during charging. Smooth and fast dynamic response even at the extreme conditions is possible using this controller. Fig. 8 shows a longer term charge using the Lyapunov based control strategy. A stable current sharing was achieved both during the charging as well as in discharging mode and no stability problem has been found while switching the mode.

![Lyapunov control in discharging at 500W power level](image)

The second stage of experiment is performed at a reduced dc-link voltage \( v_{dc} = 120V \) and power level \( P = 250W \). Similar set of results have been presented at extreme conditions as before. Fig. 5 and Fig. 7 shows the battery current response at SOC\(_{o,1} = 15\%\), SOC\(_{o,2} = 40\%\) and SOC\(_{o,3} = 10.0\%\) during discharging and SOC\(_{o,1} = 91\%\), SOC\(_{o,2} = 86\%\) and SOC\(_{o,3} = 80\%\) during charging. Note the current responses are quite similar to Fig. 4 and Fig. 6.

A smooth dynamic response has been achieved in both cases even at reduced voltage and power levels. On the other hand, a slow acquisition result has also been presented to validate the
long term effect as shown in Fig. 8 and Fig. 9 at different power levels. Moreover, an effect due to dynamic change in power has also been presented in Fig. 10 to understand the transient performance of the proposed controller. Note module currents show a smooth dynamic response when changing the power levels. The overall system response time of the energy storage system was found to be around 10 – 20 ms.

The effect of variation of the control parameter has also been investigated experimentally. It was found in section III.B that the value of the control parameter $K$ plays an important role in the proposed control. An effect of variation in the control parameter, $K$, in the proposed control has also been experimentally validated. The validation has been performed in two stages: a) effect of very low value of $K$ and b) effect of very high value of $K$.

In the first case, the value of $K$ was reduced from the designed value online to see how this affects stability as shown in Fig. 11. It was found that a low value of $K$ creates stability problem. The value of $K$ of module – 3 has been reduced from 0.015 (designed value) to 0.005 to prove this. It can be observed from Fig. 11 that the system tends to get oscillatory as $K$ moves towards zero because the derivative of the energy function in (19) tends to zero at this value because the leakage resistor of the boost inductor ($R_L$) is generally quite small. This validates that a minimum value of $K$ is required to ensure the system stability. In the second case, the value $K$ of module – 2 was increased from the designed value 0.01 to 0.04 online to see how this affects stability as shown in Fig. 12. Module – 2 is chosen to demonstrate this effect because it carries a higher share of current compared to other modules. It can be seen that module – 2 current slightly reduces while the module – 1 current slightly increases due to this variation.

However, this is undesired because the battery weighting factor has not been modified significantly. Therefore, it can be concluded that a high value of the control parameter $K$ does not create any stability issue but increases noise and causes improper sharing among the modules or creates steady state errors. This result shows a reasonable match with the explanation presented in section III.C.
thorough experimental validations have been presented under different grid operating conditions to show the effectiveness of the proposed control solution. The Lyapunov solution is found to be the preferred method compared to the conventional control approach under varying parameter conditions which enables the use of cascaded DC-DC converter successfully in hybrid energy storage systems.

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