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Influence of Non-ideal Voltage Measurement on Parameter Estimation in Permanent Magnet Synchronous Machines

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Abstract—This paper investigates the influence of non-ideal voltage measurements on the parameter estimation of permanent magnet synchronous machines. The influence of non-ideal voltage measurements, such as the DC bus voltage drop, zero shift in the amplifier and voltage source inverter nonlinearities, on the estimation of different machine parameters is investigated by theoretical and experimental analysis. For analysis, a model reference adaptive system based estimator is firstly derived for the parameter estimation of the q-axis inductance, stator winding resistance and rotor flux linkage. The estimator is then applied to a prototype surface-mounted permanent magnet synchronous machine to investigate the influence of non-ideal voltage measurement on the estimation of various machine parameter values. It shows that at low speed the inverter nonlinearity compensation has significant influence on both the rotor flux linkage and winding resistance estimation, while at high speed its only significant influence is on the winding resistance estimation and negligible influence on the rotor flux linkage estimation. In addition, the inverter nonlinearity compensation will not influence the q-axis inductance estimation when it is under \( i_d = 0 \) control. However, the DC bus voltage drop due to the load variation and zero shift in the amplifier will significantly influence the q-axis inductance estimation.

Index Terms— inverters, stators, rotors, magnetic flux linkage, parameter estimation, permanent magnet machines, neural networks, system identification

NOMENCLATURE

- \( R \): Stator winding resistance (Ω).
- \( L_d, L_q \): d-q-axis inductances (H).
- \( \psi_m \): Rotor flux linkage (Wb).
- \( T_s \): Sampling period (µs).
- \( i_{as}, i_{bs}, i_{cs} \): Stator abc phase currents (A).
- \( V_{as}, V_{bs}, V_{cs} \): Line-to-neutral stator abc phase voltages (V).
- \( V_{as}^*, V_{bs}^*, V_{cs}^* \): Commanded abc phase-to-center voltages (V).
- \( u_{d}^*, u_q^* \): dq-axis reference voltages measured from the PI regulators (V).
- \( V_{as}^*, V_{bs}^*, V_{cs}^* \): abc phase distorted voltages (V).
- \( \hat{R} \): Estimated stator winding resistance (Ω).
- \( \hat{\psi}_m \): Estimated rotor flux linkage (Wb).
- \( \hat{L} \): Estimated inductance value (H).
- \( E \): abc phase distorted voltage due to inverter nonlinearity (V).
- \( T_{com} \): Compensation time (µs).
- \( T_{con}, T_{off} \): Turn on/off time of IGBT (µs).
- \( T_d \): Control dead time of the switch (µs).
- \( V_{com} \): Constant of distorted voltage due to inverter nonlinearity in dq-axis reference frame (V).
- \( \omega_0 \): Electrical angular speed (rad/s) and rotor position (rad).
- \( V_{dcs} \): Actual and measured real-time DC bus voltage (V).
- \( u_{d}, u_q \): Actual dq-axis voltages (V).
- \( i_{d}, i_q \): Actual dq-axis currents (A).
- \( V^* \): Amplitude of line-to-neutral stator reference phase voltage (V).
- \( \Delta R \): Stator winding resistance estimation error due to VSI nonlinearity (Ω).
- \( \Delta \psi_m \): Rotor flux linkage estimation error due to VSI nonlinearity (Wb).
- \( U \): Distorted voltage due to the variation of DC bus voltage and zero shift in the amplifier (V).
- \( \phi, \gamma \): Angles between voltage/current vector and q-axis (rad), respectively.

I. INTRODUCTION

Permanent magnet synchronous machines (PMSM) are now widely used in many applications, ranging from industrial servo drives, automotive power trains and wind power generation, to aerospace, due to their high power/torque density, high efficiency and excellent control performance. In order to realize a high performance and reliable PMSM drive system, accurate PMSM parameters are essential. Often on-line estimation of parameters is required so the controller parameters and system conditions can be updated, especially for sensorless control [1]-[5] and optimal PI controller design. Much literature has made contributions [4]-[19] to obtain parameters with different on-line estimation strategies from the measured machine electrical signals, such as the model reference adaptive system (MRAS) [4]-[7], recursive least square algorithms (RLS) [8]-[11], Neural
Network (NN) technology [12], [15], [19], and extended Kalman filter (EKF) [13]. For example, some literature proposes the use of the MRAS estimator to identify values of PMSM parameters, then utilized to improve the performance of sensorless control [4], [5]. Similarly, other literature proposes the use of the RLS estimator to identify values of PMSM parameters for improving the performance of sensorless control [8], [9]. In fact, existing methods for identifying the values of PMSM parameters [4]-[19], irrespective of the algorithm, such as MRAS, EKF, RLS and NN, etc., are employed in these methods, the majority based on solving the PMSM equations by using measured stator currents and voltages.

However, in a PMSM vector control system, the utilized voltages in the PMSM parameter estimators cannot be directly measured from the stator winding terminals and are usually measured from the output voltage of the PI regulator and specified DC bus voltage. Thus, since non-ideal voltage measurements usually exist, such as the DC bus voltage drop, zero shift in the amplifier and voltage source inverter nonlinearities, the foregoing estimation strategies [4]-[19] should cooperate with an appropriate voltage measurement scheme to achieve high accuracy of estimation results. Otherwise, the accuracy of estimated parameters may suffer from the non-ideal voltage measurements. Therefore, some papers in literature have investigated the influence of VSI nonlinearities on unbalanced/nonlinear loads [20] and sensorless control [21]. Furthermore, some papers have investigated the influence of inaccurate machine parameters on the performance of sensorless control [22], [23]. However, there is still no literature systematically investigating the influence of non-ideal voltage measurement due to VSI nonlinearity, DC bus voltage drop and zero shift in the amplifier on the estimation of different PMSM parameters. For instance, Morimoto et al. [24] proposed the estimation of distorted voltage due to VSI nonlinearity at PMSM standstill and used the estimated distorted voltage to compensate the estimated PMSM parameters when starting the PMSM. However, the method in [24] neither investigated the influence of VSI nonlinearity on the estimation of different machine parameters, nor took into account the influence of the DC bus voltage drop and zero shift in the amplifier.

In this paper, the influence of non-ideal voltage measurement on the estimation of various PMSM parameter values is systematically investigated by theory and experimental analysis. Revealing that at low speed the VSI nonlinearity compensation has significant influence on both the rotor flux linkage and stator winding resistance estimation, while at high speed it has only significant influence on the rotor flux linkage estimation. In addition, the VSI nonlinearity compensation will not influence the q-axis inductance estimation when it is under i_d=0 control. However, the DC bus voltage drop due to load variation and zero shift in the amplifier will influence the q-axis inductance estimation significantly. For analysis, a MRAS based estimator is described to estimate the q-axis inductance, stator winding resistance and rotor flux linkage of a non-salient pole SPMSM prior to the proposed investigation. Since the utilized MRAS estimator is based on the steady-state PMSM equations, which are the same as most other estimators using EKF, RLS and NN, etc., the analysis and conclusion based on the used MRAS estimator can also be applied to other estimators based on solving the steady-state PMSM equations by using the measured stator currents and voltages. Moreover, existing methods for inverter nonlinearity compensation can be categorized as, inverter model based compensation, system identification algorithm based on-line compensation and voltage harmonic analysis based compensation. However, since the system identification algorithm based on-line compensation [28]-[32] needs assistance from accurate machine parameters and the voltage harmonic analysis based compensation [33]-[35] needs high frequency AD converter sampling and usually suffers from the system noises, the inverter model based compensation [25]-[27] is preferable for compensating the inverter nonlinearity in PMSM parameter estimation. Thus, the inverter model based compensation proposed in [26] and [27] for considering all the inverter nonlinearities will be employed for this investigation.

II. MACHINE MODEL

Assuming that the employed SPMSM has negligible cross-coupling magnetic saturation, structural asymmetry, iron losses, magnet eddy current loss, and harmonics in the descriptive functions of windings, rotor anisotropy and coercive force of magnets, the dq-axis equations of the SPMSM are given by:

$$\frac{di_d}{dt} = -\frac{R}{L_d}i_d + \frac{L_q}{L_d}\omega q + \frac{L}{L_d}i_d$$ (1a)

$$\frac{di_q}{dt} = -\frac{R}{L_q}i_q - \frac{L_d}{L_q}\omega d + \frac{L}{L_q}i_q$$ (1b)

Since there is minor variation in the dq-axis inductances for the SPMSM, they will be regarded as constant (L_d=L_q=L) throughout this paper.

The steady-state discrete equations of (1) are:

$$u_q(k) = R i_q(k) - L_0 i_q(k)$$ (2a)

$$u_d(k) = R i_d(k) + L_0 i_d(k) + \psi_m$$ (2b)

where ‘k’ is the index of the discrete sampling instant. Clearly, as can be seen from (2), if the dq-axis voltages obtained from the PI regulator mismatch the actual voltages, the steady-state estimation results based on the dq-axis equation will be inaccurate.

III. ESTIMATOR DESIGN AND INVERTER COMPENSATION

A MRAS based PMSM parameter estimator is described in this section and the modeling of VSI nonlinearity proposed in [26] will be used for the analysis in section IV.

A. MRAS Based Estimator Design

Take (1) as the reference model, it can be transformed into:

$$\dot{X} = AX + Br + Cl$$ (3)

where

$$A = \begin{bmatrix} -\alpha & \omega \\ -\omega & -\alpha \end{bmatrix}, \quad B = \frac{1}{L}, \quad C = -\frac{\psi_m}{L}, \quad X = \begin{bmatrix} X_d \\ X_q \end{bmatrix},$$

$$r = \begin{bmatrix} u_d \\ u_q \end{bmatrix}, \quad l = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Similarly, the variable model can be shown as follows:

$$\dot{\hat{X}} = \hat{A} \hat{X} + \hat{B} r + \hat{C} l$$ (4)
where \( \dot{X} = \begin{bmatrix} -\hat{\alpha} & \omega \\ \omega & -\hat{\alpha} \end{bmatrix}, \quad \hat{\alpha} = \frac{R}{L}, \quad \hat{B} = \frac{1}{L}, \quad \hat{C} = \frac{\psi_m}{L} \).

\( u_d \) and \( u_q \) are the inputs of the SPMSM and the variable model whose outputs are currents. The parameters and variables with '^' are obtained from the variable model.

By using Lyapunov second theorem on stability, the stator inductance, winding resistance and rotor flux linkage estimators are shown as follows and the design processes are shown in Appendix A:

\[
\frac{1}{L} \dot{L} = \frac{1}{T_i} \int_0^t \left[ u_d (i_d - \hat{i}_d) + u_q (i_q - \hat{i}_q) \right] dt + \dot{i}_d (i_d - \hat{i}_d) + \dot{i}_q (i_q - \hat{i}_q) dt
\]

\[
\hat{R} = R_0 - \dot{L} \left[ \dot{i}_d (i_d - \hat{i}_d) + \dot{i}_q (i_q - \hat{i}_q) \right] dt
\]

\[
\hat{\psi}_m = \psi_{m0} - \dot{L} \left( \phi (i_d - \hat{i}_d) dt \right)
\]

where \( L_0, R_0 \) and \( \psi_{m0} \) are the initial values of estimated inductance, winding resistance and rotor flux linkage.

**B. Compensation of Inverter Nonlinearity Voltage**

Cited from [26], Fig. 1 shows the three-phase PWM inverter with a PMSM load and Fig. 2 shows the ideal and actual IGBT drive signals for phase A. The reference voltage is the area between \( t_1 \) and \( t_2 \) while the actual voltage, considering the control dead-time and conducting time delay of the switch, is the area between \( t_3 \) and \( t_4 \) \((i_d > 0)\) or \( t_5 \) and \( t_6 \) \((i_d < 0)\). From Fig. 2(c), the voltage error depends on the direction of the phase current and the length of \( T_{on}, T_{off} \) and \( T_{on} \).

Thus, it is proposed in [26] that the actual phase A output voltage can be obtained by:

\[
V_{an} = V_{an}^* + \text{sign}(i_d) E
\]

where \( \text{sign}(i) = \begin{cases} 1, & i > 0 \\ -1, & i < 0 \end{cases} \)

\( E \) represents the error due to non-ideal switching and the method proposed in [26] will be applied to obtain the value of \( E \). As detailed in [26], for phase A winding voltage generation, the relationship between the commanded and actual time duration of PWM wave can be:

\[
T_{on} = T_{on}^* + \text{sign}(i_d) M
\]

where \( T_{on}^* \) is the commanded time and \( T_{on} \) is the actual time. \( M \) is the total time error.

\[
M = T_{on} + T_{off} - T_{on}^* - T_{d}
\]

In [26], it is suggested that the length of \( T_{on} \) may be adjusted to eliminate the distorted voltage \( E \) and the actual phase A voltage by:

\[
V_{an} = V_{an}^* + \frac{1}{2} (r_{on} + r_{d}) i_d
\]

where \( r_{on} \) and \( r_d \) are the on-state slope resistances of the active switch and freewheeling diode, respectively. In real applications, \( \frac{1}{2} (r_{on} + r_{d}) i_d \) acts as a part of the stator winding resistance. \( V_{an}^* \) is the distorted voltage and can be shown as follows:

\[
V_{an}^* = \frac{1}{6} (V_{dc} - M \frac{V_{ce} - V_{co}}{T_s} (2 \text{sign}(i_d) - \text{sign}(i_q) - \text{sign}(i_r)) \}
\]

where \( V_{dc} = 36 \) V in this paper, \( V_{ce} \) and \( V_{do} \) are the threshold voltages of the active switch and freewheeling diode, respectively. When \( V_{an}^* = 0 \), \( T_{on} \) can be obtained as follows:

\[
T_{on} = T_{d} - T_{off} + T_{on} + T_{d} \frac{V_{ce} + V_{do}}{V_{dc}}
\]

Therefore, \( E \) can be obtained as:

\[
E = \frac{T_{on} - V_{dc}}{2T_s}
\]

Similarly, the voltages for the other two phases can be compensated in the same way.

**IV. EXPERIMENTAL INVESTIGATION**

In this section, the hardware platform for the PMSM drive system is firstly introduced. Then, the MRAS based estimator is implemented in the PMSM drive system and the influence of non-ideal voltage measurement on different PMSM parameters is investigated through theoretical and experimental analysis.

**A. Machine Drive System**

The described MRAS based parameter estimation method is applied on a DSP (TMS320LF2812) based vector control PMSM drive system. All the experimental results are on-line commissioning results, which are recorded in external RAM of the DSP control board. The sampling period \( T_s \) is set to 83.3 \( \mu \)s. The employed VSI is a Mitsubishi PS21867 whose typical electrical characters (from the Mitsubishi official datasheet) are shown in Table I. Fig. 3 shows the prototype motor drive system and the SPMSM design parameters shown in Table II are the actual measured machine parameters. The 12-slot/10-pole PMSM used is a fractional slot motor, equipped with surface-mounted NdFeB magnets, previously developed for servo applications and has a rated torque of 5.5Nm at 400r/min. A control diagram of the whole
system is shown in Fig. 4. In the following experiments the DC link is connected with a commercial DC power source whose output is fixed to 36V. The rotating electrical angular speed and load are constant via the current loop PI controller. In addition, the waveforms of the PMSM three-phase currents are shown in Fig. 5.

B. Influence from Voltage Measurement Non-idealities

As detailed in [12], under \( i_d=0 \) control, it is impossible to solve for stator winding resistance and rotor flux linkage simultaneously due to rank deficient problems, while the \( q \)-axis inductance can be individually estimated from the \( d \)-axis equation. Therefore, in the following investigation, the \( q \)-axis inductance will be individually estimated at \( i_d=0 \) control. Then, similar to the schemes proposed in [13]-[15], a flux weakening current (\( i_q=2A \)) is transiently injected to activate the winding resistance term in the \( d \)-axis equation, and the \( dq \)-axis inductances are regarded to be \( L_d=L_q \) so that the stator winding resistance and rotor flux linkage can be simultaneously solved from the steady-state \( dq \)-axis equations. Although this assumption that \( L_d=L_q \) will cause an error in the estimation of rotor flux linkage and stator winding resistance, due to variation in \( dq \)-axis inductance, it will not inconvenience the analysis of the parameter estimation error caused by voltage measurement non-idealities because we only need to analyze the variation of estimated results, with or without taking into account the non-ideal voltage measurement.

In the following investigations, the DC bus voltage will be measured in real-time using a DC voltage sensor to avoid the influence due to the on-load voltage drop. The schematic diagram of the proposed investigation is shown in Fig. 6. Under ideal conditions, the real-time DC bus voltage \( V_m \) will be equal to the actual DC bus voltage \( V_{dc} \) and is measured using a DC voltage sensor. However, it is known that there may be a zero shift in the amplifier, which will introduce a non-zero DC offset \( V_{offset} \). Therefore, as detailed in Fig. 6, this DC offset \( V_{offset} \) will be measured with the PMSM at standstill and will be utilized to compensate the measured real-time DC bus voltage \( V_m \) when the PMSM is started. Finally, the actual DC bus voltage \( V_{dc} \) can be obtained by \( V_m - V_{offset} \). In addition, the method detailed in section III B will be utilized for the compensation of VSI nonlinearity.

At \( \omega=157 \text{rad/s} \), Fig. 7(a) shows the estimated \( L_q \) under \( i_d=0 \) control without taking into account all the non-ideal voltage measurements. For comparison, Fig. 7(b) shows the estimated \( L_q \) under \( i_d=0 \) control, which takes into account the VSI nonlinearity compensation, measurement of the DC bus voltage variation and the DC offset due to zero shift. Comparing Fig. 7(a) with Fig. 7(b), it is evident that the estimated result in Fig. 7(b) is much closer to the nominal value of the \( q \)-axis inductance shown in Table II due to its consideration of non-ideal voltage measurement. Further, from Fig. 7(b), it shows that the influence from VSI nonlinearity compensation on the \( q \)-axis inductance estimation is negligible, which can be explained as follows:

As detailed in literature [30], the steady-state \( dq \)-axis equations can be shown as follows:

\[
\begin{align*}
\dot{u}_q(k) + D_d(k)V_{con} & = -L_q i_q(k) + R_i q(k) + U \cos(\phi) \\
\dot{u}_d(k) + D_q(k)V_{con} & = L_d i_d(k) + \psi_m(k) - U \sin(\phi)
\end{align*}
\]

As detailed in [28]-[32], the value of \( V_{con} \) can be regarded as a constant when the PMSM is in steady state, and it varies as the working condition of PMSM changes. In addition, as detailed in [30], \( U \) is the function of actual DC bus voltage \( V_{dc} \) and measured real-time DC bus voltage \( V_{meas} \), which is expressed as follows:
which can be expressed as follows:

$$ U = V_{dc} (V_m - V_c + V_{co} - V_{do}) $$

In reality, since there exists a zero shift in the amplifier and DC bus voltage drop when on load, $V_m$ will differ slightly from $V_{dc}$. In addition, $Dd$ and $Dq$ are functions of rotor position $\theta$ and the directions of the three phase currents, which can be expressed as follows:

$$ \begin{bmatrix} Dd \\ Dq \end{bmatrix} = 2 \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} \text{sign}(i_e) \\ \text{sign}(i_d) \end{bmatrix} $$

As seen from (15), it is evident that the distorted voltage due to VSI nonlinearity in the $dq$-axis reference frame is decomposed into $V_{com}Dd$ and $V_{com}Dq$. Assuming $\gamma$ is the angle between the current vector and $q$-axis, the simulated waveforms of $Dd$ and $Dq$ are 6th order harmonics and will vary with $\theta = \theta + \pi/2$. For example, the first mode, shown in Table III, $\text{sign}(i_e)=1$, $\text{sign}(i_d)=\text{sign}(i_d)=-1$, $i_d=0$ and $\gamma=0$. Therefore, $Dd$ will vary from $-2$ ($\theta_e=\pi/6$) to $2$ ($\theta_e=\pi/6$) while $Dq$ will vary from $3.46$ ($\theta_e=\pi/6$) to $4$ ($\theta_e=0$). This is the reason why waveforms for $Dd$ and $Dq$ are so different in Fig. 8. Further, it is evident that the average value of distorted voltage in the $d$-axis reference frame is zero and the distorted voltage in the $q$-axis reference frame will introduce a DC voltage offset into the $q$-axis equation. This is the reason why the compensation of $V_{com}Dd$ has no contribution to the estimation of $L_q$ and its influence on the estimation of $L_q$ can be neglected. In addition, as can be seen from the $d$-axis equation of (15), the term $U\cos(\phi)$ will influence the estimation of $L_q$. Thus, from Fig. 7(a), it is evident that the estimated $L_q$ (2.33mH) is far away from the nominal value (3.24mH) because $U$ is relatively large. However, from Fig. 7(b), it is evident that the estimated $L_q$ (3.1mH) will be much closer to the nominal value (3.24mH) because the measured $V_m$ is used for the real-time update of $V_{dc}$, which can minimize the term $U$ to a point at which it is negligible.

**Table III**

<table>
<thead>
<tr>
<th>$\theta_e=\theta + \pi/2$</th>
<th>$\text{sign}(i_e)$</th>
<th>$\text{sign}(i_d)$</th>
<th>$V_{com}Dd$</th>
<th>$V_{com}Dq$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\pi/6$</td>
<td>1</td>
<td>-1</td>
<td>$4V_{com}\sin(\theta_e)$</td>
<td>$4V_{com}\cos(\theta_e)$</td>
</tr>
<tr>
<td>$-\pi/3$</td>
<td>-1</td>
<td>1</td>
<td>$4V_{com}\sin(\theta_e/2)$</td>
<td>$4V_{com}\cos(\theta_e/2)$</td>
</tr>
<tr>
<td>$-\pi/2$</td>
<td>-1</td>
<td>-1</td>
<td>$4V_{com}\sin(\theta_e/3)$</td>
<td>$4V_{com}\cos(\theta_e/3)$</td>
</tr>
<tr>
<td>$0$</td>
<td>1</td>
<td>1</td>
<td>$4V_{com}\sin(\theta_e/4)$</td>
<td>$4V_{com}\cos(\theta_e/4)$</td>
</tr>
<tr>
<td>$\pi/2$</td>
<td>1</td>
<td>-1</td>
<td>$4V_{com}\sin(\theta_e/5)$</td>
<td>$4V_{com}\cos(\theta_e/5)$</td>
</tr>
<tr>
<td>$\pi/3$</td>
<td>-1</td>
<td>1</td>
<td>$4V_{com}\sin(\theta_e/6)$</td>
<td>$4V_{com}\cos(\theta_e/6)$</td>
</tr>
</tbody>
</table>

At $\omega=157\pi$/s, Fig. 9(a) shows the estimated stator winding resistance and rotor flux linkage without taking into account the influence of non-ideal voltage measurement, which also shows that the estimated parameter values are far from their nominal values. Similarly, Fig. 9(b) shows the estimated stator winding resistance and rotor flux linkage, which have taken into account the non-ideal voltage measurement, the measurement of DC bus voltage variation and the DC offset due to zero shift.

**Fig. 8.** Relationship between current directions and $dq$-axis distorted voltage.

At $\omega=157\pi$/s, the corresponding $dq$-axis voltages and currents with or without VSI nonlinearity compensation are shown in Fig. 10. The VSI typical electrical parameters in

---

**Fig. 9(a)**

**Fig. 9(b)**

**Fig. 10.**
Table I will be used to compute the distorted voltage \((E)\). In Fig. 10, there is no inverter nonlinearity compensation before 1.5s and the compensation is added to the drive system after 1.5s. From Fig. 10, it is evident that the \(dq\)-axis voltages, obtained from the PI regulator, will slightly change following the addition of compensation. However, the \(dq\)-axis currents are maintained almost constant all the time since the load and specified \(d\)-axis current do not change.

The estimation results with or without compensation of VSI nonlinearity at \(\omega = 157 \text{rad/s}\) are compared in Fig. 9(c). It shows that the inverter nonlinearity compensation has significant influence on the winding resistance estimation but negligible influence on the rotor flux linkage estimation.

This can be analyzed as follows:

(a). By using typical electrical parameters in Table I and (14), the compensated voltage \(E\) should be \(-1.9\text{V}\).

(b). The amplitude of compensated inverter nonlinearity voltage \((E = -1.9\text{V})\) is significant (larger than 100%) compared with \(R_i = 1.6\text{V}\) and \(R_d = -0.7\text{V}\). Further, \(E\) is also significant compared with \(R_i + \text{Loi} \approx 0.6\text{V}\) and \(R_d \sim -2.9\text{V}\). Therefore, after Clarke and Park transforms, the \(dq\)-axis voltage variation due to \(E\) will influence the winding resistance estimation significantly.

\[
\begin{align*}
E &= -1.9\text{V} \\
R_i + \text{Loi} &\approx 0.6\text{V} \\
R_d &\sim -2.9\text{V}
\end{align*}
\]

In addition, since the estimation is based on the \(dq\)-axis reference frame, the influence of \(E\) on the estimation of \(R\) and \(\psi_m\) can be further analyzed by using the steady-state \(dq\)-axis equations for a PMSM at \(i_d < 0\). As can be seen from (15), in the \(dq\)-axis reference frame, \(E\) can be decomposed into \(\text{D}d \text{V}_{\text{comp}}\) and \(\text{D}q \text{V}_{\text{comp}}\). Thus, the steady-state equations for a PMSM, including all the voltage measurement non-idealities, can be shown as follows:

\[
\begin{align*}
\dot{u}_d^* + \text{D}d(k) \text{V}_{\text{comp}} &= R_i(k) - L_q \rho (k) \dot{i}_q(k) + U \cos(\phi) \quad (16a) \\
\dot{u}_q^* + \text{D}q(k) \text{V}_{\text{comp}} &= R_i(k) + L_d \dot{i}_d(k) \rho (k) + U \sin(\phi) \quad (16b)
\end{align*}
\]

Assuming accurate measurement of the DC bus voltage, \((16)\) can be simplified to \((17)\):

\[
\begin{align*}
\dot{u}_d^* + \text{D}d(k) \text{V}_{\text{comp}} &= R_i(k) - L_q \rho (k) \dot{i}_q(k) \quad (17a) \\
\dot{u}_q^* + \text{D}q(k) \text{V}_{\text{comp}} &= R_i(k) + L_d \dot{i}_d(k) \rho (k) + U \psi_m o(k) \quad (17b)
\end{align*}
\]

Assuming the VSI nonlinearity is compensated properly, and the DC bus voltage is accurately measured, \((17)\) can be simplified to \((18)\):

\[
\begin{align*}
\dot{u}_d^* &= R_i(k) - L_q \rho (k) \dot{i}_q(k) \quad (18a) \\
\dot{u}_q^* &= R_i(k) + L_d \rho (k) \dot{i}_d(k) + U \psi_m o(k) \quad (18b)
\end{align*}
\]

As can be seen from \((16)-(18)\), the estimation of \(R\) mainly relies on the \(d\)-axis equation, and \(R\) can be independently estimated from the \(d\)-axis equation if \(i_d = 0\). Furthermore, as can be seen from \((16)-(18)\), the estimation of \(\psi_m\) mainly relies on the \(q\)-axis equation and the parameter value of \(R\) should be estimated prior to the estimation of \(\psi_m\). Since the DC bus voltage is measured, the influence of \(\text{D}d \text{V}_{\text{comp}}\) and \(\text{D}q \text{V}_{\text{comp}}\) on the estimation of stator winding resistance and rotor flux linkage can be further analyzed as follows:

![Fig. 10. Dq-axis currents and voltages with/without inverter compensation.](image)

![Fig. 9. Estimated stator winding resistance and rotor flux linkage.](image)
Assuming that \( R=R_a+\Delta R \), \( R_a \) is the actual stator winding resistance and \( R \) is the estimation error due to VSI nonlinearity. Since the DC bus is accurately measured, \( \Delta R \) can be derived from (17a) and shown as:

\[
\Delta R = \frac{Dd(k)V_{\text{com}}}{i_d(k)}
\]  

(19)

If the estimation is with compensation, the DC components of \( DdV_{\text{com}} \) and \( DqV_{\text{com}} \) will be minimized to negligible values and (17) will be simplified to (18). Based on (18a), the estimated \( R \) will be close to \( R_a \) because its accuracy mainly relies on the \( d \)-axis equation. Therefore, by using the estimated \( R \) from (18a), the \( \psi_m \) can be estimated from (18b) then and the estimated \( \psi_m \) will be close to its actual value because \( R \) is close to \( R_a \) and \( DqV_{\text{com}} \) becomes negligible.

If the compensation of VSI nonlinearity is not taken into account, it is evident from (19) that the estimated \( R=R_a+\Delta R \) will be significantly larger than \( R_a \), which is experimentally shown in Fig. 9(b). Furthermore, since \( \Delta R \) is not equal to zero, the term \( DqV_{\text{com}} \) in (17b) will be partly counteracted by \( \Delta R_iq \) because the estimated \( R=R_a+\Delta R \) is used to assist the estimation of \( \psi_m \), which is a reason why the variation of estimated \( \psi_m \) with or without the compensation is quite small. Assuming that \( \Delta \psi_m \) is the estimation error due to \( DqV_{\text{com}} \) and \( \Delta R \), \( \Delta \psi_m \), it can be derived from (17b) and shown as follows:

\[
\Delta \psi_m = \frac{Dq(k)V_{\text{com}}-\Delta R_i(q)}{\alpha(k)} = \frac{Dq(k)V_{\text{com}}}{\alpha(k)}\left(1-\frac{Dq(k)V_{\text{com}}}{\alpha(k)}\right)
\]  

(20)

From Table III, the DC components of \( Dd(k) \) and \( Dq(k) \) are close to \( 4\sin(\gamma) \) and \( 4\cos(\gamma) \), respectively. In addition, \( \cot(\gamma)\approx i_d(k)/i_d(k) \). Therefore, assuming that \( \Delta \psi_m \) is the DC component of (20), it can be expressed as:

\[
\Delta \psi_m = \frac{\sin(\gamma)\cos(\gamma)}{\alpha(k)}\left|4V_{\text{com}}\right| = 0
\]  

(21)

(21) is the expression for the rotor flux linkage estimation error, due to VSI nonlinearity, which shows that there is cancellation in the numerator and the DC component of \( \Delta \psi_m \) is close to 0. Therefore, this is the reason why the variation of \( \psi_m \) with or without the compensation is negligible compared with \( \psi_m \). Thus, although the computed inverter nonlinearity voltage \( E \) is as large as -1.9, it has negligible influence on the estimation of \( \psi_m \) but significant influence on the estimation of \( R \). Fig. 11 shows the characteristics of the IGBT collector-emitter saturation voltage and the diode’s forward voltage, which is cited from the datasheet of Mitsubishi ps21867-p. At \( \omega=52.3\text{rad/s} \), the corresponding \( dq \)-axis voltages and currents with or without compensation are shown in Fig. 12. From Fig. 11 and Fig. 12, the computed \( E \) will be \(-1.3V\) at \( \omega=52.3\text{rad/s} \). In Fig. 13, there is no inverter compensation before 1.5s and the compensation added to the drive system after 1.5s. At \( \omega=52.3\text{rad/s} \), \( \omega \psi_m \approx 4 \) and it is obvious that \( E=-1.3V \) is not negligible compared with \( \omega \psi_m \). Further, since \( \Delta \psi_m \) can not be exactly equal to 0 and the voltage drop of \( \omega \psi_m \) at 52.3rad/s is much smaller than the voltage drop at 157rad/s, the variation of estimated \( \psi_m \) with or without the compensation of VSI nonlinearity will be significant. Therefore, the estimated rotor flux linkage and stator winding resistance in Fig. 13 is different from Fig. 9(c), and both vary significantly.

In addition, for most PMSM, the voltage drop due to the stator winding resistance (\( R_d \)) is usually quite small compared with the back EMF voltage (\( \omega \psi_m \)) when operating at rated speed. This is because a large stator winding resistance will increase the heat losses due to the stator current and will cause the power efficiency to decrease. Although some special PMSM, for high speed applications, have small \( \psi_m \), its back EMF voltage at rated speed is still much larger than \( R_i \). Thus, in this paper, the conclusion based on the example PMSM in Table II is suitable for most PMSM in real applications. Furthermore, for those special PMSM, the PMSM with a large stator winding resistance and small back EMF voltage (\( \omega \psi_m \)), the VSI nonlinearity compensation will have significant influence on both the rotor flux linkage and winding resistance estimation irrespective of low or high speed operation.

V. CONCLUSION

This paper systematically investigates the influence of non-ideal voltage measurements on the parameter estimation for PMSM. From the experimental results and theoretical analysis, it is evident that the VSI nonlinearity compensation has negligible influence on the estimation of rotor flux linkage, but significant influence on the estimation of winding resistance at high speeds. However, at low speeds, the VSI nonlinearity compensation has significant influence on both the rotor flux linkage and winding resistance estimation. Further, the estimation accuracy of PMSM parameters depends on their \( dq \)-axis voltage drop compared with the distorted inverter voltage, if the VSI nonlinearity compensation is not employed. In addition, under \( i_d=0 \) control, the estimation of the \( q \)-axis inductance will not be influenced by the VSI nonlinearity compensation but will be significantly influenced by the non-ideal measurement of varying the DC bus voltage and zero shift in the amplifier. In addition, the analysis and conclusion based on the used
MRAS estimator can be applied to other estimators based on solving the steady-state PMSM equations using the measured stator currents and voltages.

VI. APPENDIX A

From (3) and (4), assuming that $e = X - \hat{X}$, an error state equation can be obtained as follows:

$$\dot{e} = AX - \hat{A} \hat{X} + Br - \hat{B}r + Cl - \hat{C}l$$

$$= AX - A\hat{X} + A\hat{X} - \hat{A}\hat{X} + Br - \hat{B}r + Cl - \hat{C}l$$

$$= A(e - \hat{e}) - \hat{A}\hat{e} + (B - \hat{B})r + (C - \hat{C})l$$

(A1)

Let $\hat{\alpha} = a$, $A - \hat{A} = (\hat{\alpha} - \alpha)I = \alpha I$, $b = B - \hat{B} =$$

$$\left(1 - \frac{1}{L}\right) = \Psi_m = \frac{\psi_m}{L} = g$ , $\phi^T = [a, b, g]$ , $s =$$

$$\left[\dot{X} \ r \ l\right]^T$$.  

Then, equation (A1) can be transferred into (A2):

$$\dot{e} = Ae + \phi^T s$$

(A2)

Design a Lyapunov function below, which is positive definite.

$$V(X, t) = \frac{1}{2}(e^TPe + \phi^T \Gamma \phi)$$

(A3)

where $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\Gamma = \Gamma^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $V(X, t)$ is positive definite. Lyapunov second theorem on stability is employed here to ensure the global asymptotic stability of the system and $e = \phi^T = 0$ is set as the stable equilibrium point. The theorem is quoted as follows:

1: $V(X, t)$ is positive definite.

2: $\dot{V}(X, t)$ is negative definite.

3: $V(X, t)$ is infinite when $|X| \rightarrow \infty$.

It is obvious that conditions (1) and (3) are achieved and the condition (2) is discussed as follows:

$$\dot{V}(X, t) = \frac{1}{2}\left(\dot{e}^T Pe + e^T \dot{P}e + \phi^T \Gamma \phi + \phi^T \Gamma \phi\right)$$

$$= \dot{e}^T Pe + e^T \dot{P}e = e^T (PA + A^T P)e + s^T \phi \dot{e} + e^T P(\phi^T s)$$

$$\phi^T \Gamma \phi + \phi^T \Gamma \phi = 2(aa + bb + gg)$$

$$\Rightarrow \dot{V}(X, t) = \frac{1}{2} e^T (PA + A^T P)e + \frac{1}{2} (s^T \phi Pe + e^T P(\phi^T s))$$

$$+ bb + gg, PA + A^T P =$$

$$\begin{bmatrix} -2a & -2a \\ -2a & -2a \end{bmatrix}$$

where $\alpha = \frac{R}{L} > 0$, it is obvious that $\frac{1}{2} e^T (PA + A^T P)e$ is negative definite.

$$\frac{1}{2} (s^T \phi Pe + e^T P(\phi^T s))$$

$$= \dot{a}i_s (i_r - \hat{i}_r) + \dot{a}i_s (i_r - i_s) + \dot{b}u_s (i_r - i_s) + (bu_s + oog)(i_r - i_s)$$

Therefore,

$$\dot{V}(X, t) = \frac{1}{2} e^T (PA + A^T P)e + a \dot{i}_s (i_r - \hat{i}_r) + (bu_s + oog)(i_r - i_s)$$

Then, the MRAS based stator inductance, winding resistance and rotor flux linkage estimators can be designed by following steps:

Let

$$b(u_s (i_r - \hat{i}_r) + u_s (i_r - i_s)) = 0$$

(A5)

$$a(i_r - \hat{i}_r) + i_r (i_r - i_s) + \dot{a} = 0$$

(A6)

Then it is obvious that $\dot{V}(X, t)$ is negative definite and condition (2) is satisfied. Hence, the SPMSM stator inductance, winding resistance and rotor flux linkage can be obtained from (A5), (A6) and (A7), respectively:

$$\frac{1}{L} = \frac{1}{t_0} + \int_0^t (b(u_s (i_r - \hat{i}_r) + u_s (i_r - i_s)) dt$$

(A9)

$$\dot{\psi_m} = \psi_m - \frac{1}{L} \int_0^t (a(i_r - \hat{i}_r) + i_r (i_r - i_s) dt$$

(A10)

where $t_0, R_0$ and $\psi_m$ are the initial values of estimated inductance, winding resistance and rotor flux linkage.

VII. REFERENCES


