The mechanics of interface fracture in layered composite materials: (2) cohesive interfaces

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THE MECHANICS OF INTERFACE FRACTURE IN LAYERED COMPOSITE MATERIALS: (2) COHESIVE INTERFACES

Simon S. Wang\textsuperscript{1, 2}, Christopher M. Harvey\textsuperscript{1}, Liangliang Guan\textsuperscript{3} and Hezong Li\textsuperscript{2}

\textsuperscript{1}Department of Aeronautical and Automotive Engineering, Loughborough University, Loughborough, Leicestershire LE11 3TU, United Kingdom
E-mails: s.wang@lboro.ac.uk, c.m.harvey@lboro.ac.uk

\textsuperscript{2}School of Mechanical and Equipment Engineering, Hebei University of Engineering, Handan, China
E-mail: Lhzong@126.com

\textsuperscript{3}Department of Mechanical Engineering, Jinzhou University of Engineering, Jinzhou, China

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ABSTRACT

The author’s mixed-mode partition theories [1-9] for rigid interfaces are extended to non-rigid cohesive interfaces for one dimensional (1D) interface fracture. In the absence of crack tip through thickness shear forces both classical and shear deformable partition theories have identical mode I and II energy release rate (ERR) partitions which are the same as those of shear deformable partitions for a mixed mode at rigid interfaces and independent of interface cohesive laws. Consequently, the mode mixity remains constant during fracture evolution. In the case of interface fracture in the layered isotropic materials, the pure modes in 2D elasticity partition theory only depend on the ratio between the penalty stiffness to the Young’s modulus of the materials and are independent of the shape of the cohesive laws. A mixed fracture mode can be readily partitioned by using the pure modes and a constant mode mixity is shown.

1 INTRODUCTION

Cohesive interfaces are different from the brittle rigid interfaces by allowing non-negligible relative interface displacements. They are called non-rigid interfaces in this study. Partition of mixed-fracture modes at non-rigid interfaces is usually carried out by using cohesive zone modelling (CZM) in conjunction with the finite element method (FEM). Various CZMs exist. The present study aims to develop an analytical theory to achieve a more in-depth understanding of cohesive interface fractures.

2 THEORY

For hard but non-rigid interfaces [10] the total ERR $G$ for 1D interface fractures in layered composite materials in the absence of crack tip through thickness shear forces is the same as that for rigid interfaces. That is,

$$ G = \begin{bmatrix} M_{1b} & M_{2b} & N_{1b} & N_{2b} \end{bmatrix} \left[C\right] \begin{bmatrix} M_{1b} & M_{2b} & N_{1b} & N_{2b} \end{bmatrix}^T $$

(1)

It is a quadratic form non-negative definite in terms of the crack tip bending moments per unit width $M_{1b}, M_{2b}$ and axial forces per unit width $N_{1b}, N_{2b}$, with subscripts 1 and 2 denoting the layers above and below the crack.

By using the powerful orthogonal pure-mode partition theory (OPPT) [1-9], the total energy release rate (ERR) $G$ can be partitioned into mode I and II ERR components based on both classical and shear deformable partition theories.
which are the same as those of shear deformable partitions for a mixed mode at rigid interfaces and independent of interface cohesive laws. Consequently, the mode mixity remains constant during fracture evolution. This is due to the absence of any crack tip stress singularity for a non-rigid interface. When crack tip through thickness shear forces are in existence, the partitions will be affected by interface cohesive law. Details of the effects can be found in the study [10]. For layered isotropic materials, Eqs. (2) and (3) become

\[ G_i = c_i \left( M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\theta_1} \right)^2 \]  

(4)

\[ G_{II} = c_{II} \left( M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} \right)^2 \]  

(5)

where \( N_{1Be} = N_{1B} - N_{2B}/\gamma \) with \( \gamma = h_2/h_1 \) is the thickness ratio. In the above equations

\[ \theta_1 = -\gamma^2, \quad \theta_2 = -\frac{6}{h_1}, \quad \beta_1 = \frac{\gamma^2(3 + \gamma)}{1 + 3\gamma}, \quad \beta_2 = \frac{2(3 + \gamma)}{h_1(\gamma - 1)} \quad \text{for} \quad \gamma \neq 1, \quad \beta_2 = \infty \quad \text{for} \quad \gamma = 1 \]  

(6)

\[ c_i = G_{\theta_i} \left( 1 - \frac{\theta_i}{\beta_i} \right)^2, \quad c_{II} = G_{\beta_{II}} \left( 1 - \frac{\beta_2}{\theta_2} \right)^2 \]  

(7)

\[ G_{\theta_i} = \frac{24 \gamma}{Eh_1^3(1 + \gamma)}, \quad G_{\beta_{II}} = \frac{72 \gamma(1 + \gamma)}{Eh_1^3(1 + 3\gamma)^2} \]  

(8)

Based on 2D elasticity partition theory, Eqs. (4) and (5) become

\[ G_i = c_i \left( M_{1B} - \frac{M_{2B}}{\beta_{1NR}} - \frac{N_{1B}}{\beta_{2NR}} \right)^2 \]  

(9)

\[ G_{II} = c_{II} \left( M_{1B} - \frac{M_{2B}}{\theta_{1NR}} - \frac{N_{1B}}{\theta_{2NR}} \right)^2 \]  

(10)

The subscript NR denotes for non-rigid interfaces. The constants \( c_i \) and \( c_{II} \) are

\[ c_i = G_{\theta_{iNR}} \left( 1 - \frac{\theta_{iNR}}{\beta_{iNR}} \right)^2, \quad c_{II} = G_{\beta_{II}} \left( 1 - \frac{\beta_{2NR}}{\theta_{2NR}} \right)^2 \]  

(11)

with

\[ G_{\theta_{iNR}} = \frac{6}{Eh_1^2} \left( 1 + \frac{\theta_{iNR}^2}{\gamma^3} - \frac{(1 + \theta_{iNR})^2}{(1 + \gamma^3)} \right), \quad G_{\beta_{II}} = \frac{6}{Eh_1^2} \left( 1 + \frac{\beta_{2NR}^2}{\gamma^3} - \frac{(1 + \beta_{2NR})^2}{(1 + \gamma^3)} \right) \]  

(12)

The pure mode I \( \theta_{iNR}(\gamma, k_{\theta}) \) is found as
\[ \theta_{1\text{NR}}(\gamma, k_r) = \theta_a + 1/2(\theta_a - \theta_c) \log_{10} k_r + 1/2(\theta_a - 2\theta_h + \theta_c)(\log_{10} k_r)^2 \]  
where \( k_r = k / E \) is the ratio between the interface penalty stiffness \( k \) and the Young’s modulus \( E \) and the other parameters are

\[ \theta_{1c}(\gamma) = (\theta_1 + 3\theta_1')/4 \quad , \quad \theta_{1b}(\gamma) = (\beta_1 + 3\beta_1')/4 \quad , \quad \theta' = -1 \quad , \quad \beta' = \gamma^3 \]  
\( \theta(\gamma) \) can be determined by using the orthogonality condition, i.e.

\[ \{1 \quad \beta_c \quad 0 \quad 0\}^T [c|1 \quad \theta_c \quad 0 \quad 0] = 0 \]  
Or simply, \( \theta(\gamma) \) is expressed as

\[ \theta_c(\gamma) = \text{orthogonal}(\beta_c) \]  
\( \theta_b(\gamma) \) is given as

\[ \theta_b(\gamma) = (\theta_a + \theta_c)/2 \]  
Then \( \theta_{2\text{NR}}, \beta_{1\text{NR}} \) and \( \beta_{2\text{NR}} \) are determined using the orthogonal condition, that is

\[ \theta_{2\text{NR}}(\gamma, k_r) = \text{orthogonal}(\beta_{1\text{NR}}) = \text{orthogonal}(\beta_{2\text{NR}}) \]  
\[ \beta_{1\text{NR}}(\gamma, k_r) = \text{orthogonal}(\theta_{1\text{NR}}) = \text{orthogonal}(\theta_{2\text{NR}}) \]  
\[ \beta_{2\text{NR}}(\gamma, k_r) = \text{orthogonal}(\theta_{1\text{NR}}) = \text{orthogonal}(\theta_{2\text{NR}}) \]

Note that these pure modes only depend on the thickness ratio \( \gamma \) and the ratio of interface penalty stiffness to the Young’s modulus \( k_r \). They are independent of the shape of softening part of the cohesive laws. Constant mode mixity is expected in the fracture evolution. Note that the assumed units of \( k_r \) are \( \text{m}^{-1} \). When \( k_r \) is inside the range from 0.1 to 10, the quartic approximation in Eq. (13) gives excellent agreement with FEM simulations.

### 3 VALIDATIONS

This group of numerical tests aimed to validate that the four pure modes, that is the \( \theta_{1\text{NR}}, \theta_{2\text{NR}}, \beta_{1\text{NR}} \) and \( \beta_{2\text{NR}} \) modes given in Eqs. (13) and (18) to (20), are independent of the type of interface constitutive laws. The double cantilever beam is considered to represent a 1D interface fracture. The total thickness is \( h = h_1 + h_2 = 5 \text{ mm} \). Two thickness ratios, \( \gamma = h_2 / h_1 = 2, 6 \), and five stiffness ratios, \( k_{r} = k / E = 0.1, 0.5, 1, 5 \) and 10 were considered. The linear, bi-linear and exponential interface constitutive laws in Abaqus were used. The total fracture ERR was set to be 200 kN/mm, which in the cases of the bi-linear and exponential interface constitutive laws, was equally divided between the linear elastic part and the softening part. The number of finite elements through the thickness of the top beam was fixed to 30 with a uniform mesh distribution. The number of evenly distributed elements through the thickness of the bottom beam was 17 and 22 for \( \gamma = 2 \) and 6 respectively. That is, the finite element size was always smaller than 0.2 mm. Table 1 shows the excellent pureness of the pure mode pair \( \theta_{1\text{NR}} \) and \( \beta_{1\text{NR}} \), while Table 2 shows the excellent pureness of the pure mode pair \( \theta_{2\text{NR}} \) and \( \beta_{2\text{NR}} \). The labels ‘Li’, ‘Bi’, ‘Ex’ and ‘An’ in Tables 1 and 2 denote the numerical results from the linear, bi-linear and exponential interface constitutive laws in Abaqus and the results from the analytical partition theory for 2D elasticity, respectively.

The second group of numerical tests aimed to investigate the accuracy of the analytical mixed-
mode partition theory for 2D elasticity when the interface is described by the bilinear interface constitutive law in Abaqus. The quadratic stress criterion in Abaqus was used to determine the onset of damage, as given by Eq. (21).

\[
\left(\frac{\sigma_{xx}}{\sigma_{yy}}\right)^2 + \left(\frac{\tau_{xy}}{\tau_{yz}}\right)^2 = 1
\]  

(21)

Two values for the mode-independent total fracture ERR were specified: firstly \( G_c = 1000 \text{kN/mm} \) and secondly \( G_c = 4000 \text{kN/mm} \). In both cases, 200 kN/mm of the total fracture ERR was from the linear elastic part of the interface constitutive law, and the remaining amount was from the softening part. This condition provided the cracking stresses \( \sigma_{xx}^0 \) and \( \tau_{xy}^0 \) in Eq. (21) as \( \sigma_{xx}^0 = \tau_{xy}^0 = \sqrt{400k} \). Three different values for \( k_{w} = k/E \) were used, which were \( k_{w} = 0.1 \text{m}^{-1}, 1 \text{m}^{-1} \) and \( 10 \text{m}^{-1} \). One bending moment \( M_i \) was applied to the upper beam to produce the total fracture ERR. Two methods were used to calculate the ERR partition. The first method considers the stresses and relative displacements at the crack tip over the loading history, called here, a ‘crack tip analysis’. The second considers the stresses and relative displacement ahead of the crack tip over the damaged region, called here, a ‘spatial analysis’. The two methods are expected to give the same partitions of ERR. Note that the first group of numerical tests, the crack tip analysis was used to calculate the ERR. This is consistent with the theoretical developments in Section 2.

<table>
<thead>
<tr>
<th>( \gamma = 2 )</th>
<th>( \frac{M_2}{M_1} = \theta_{1SR} )</th>
<th>( \frac{M_2}{M_1} = \beta_{1SR} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_w ) (l/m)</td>
<td>0.1 0.5 1 5 10</td>
<td>0.1 0.5 1 5 10</td>
</tr>
<tr>
<td>( M_1 ) (Nm)</td>
<td>11.30 11.05 10.95 10.70 10.60</td>
<td>5.85 6.31 6.45 6.80 7.00</td>
</tr>
<tr>
<td>( M_2 ) (Nm)</td>
<td>-14.58 -16.11 -16.65 -17.84 -18.55</td>
<td>39.28 8.82 38.50 37.94 37.67</td>
</tr>
<tr>
<td>Li ( G_1 ) (kN/mm)</td>
<td>199.9 199.5 199.5 199.5 198.9</td>
<td>194.5 196.4 195.4 195.1 195.5</td>
</tr>
<tr>
<td>( G_1/G ) (%)</td>
<td>100 100 100 100 100</td>
<td>100 100 100 100 100</td>
</tr>
<tr>
<td>Bi ( G_1 ) (kN/mm)</td>
<td>199.7 199.3 199.0 198.0 198.5</td>
<td>193.4 195.9 194.2 194.1 194.7</td>
</tr>
<tr>
<td>( G_1/G ) (%)</td>
<td>100 100 100 100 100</td>
<td>99.9 99.9 99.9 99.9 99.9</td>
</tr>
<tr>
<td>Ex ( G_1 ) (kN/mm)</td>
<td>199.7 199.4 199.0 198.2 198.5</td>
<td>193.7 195.6 194.4 194.2 194.2</td>
</tr>
<tr>
<td>( G_1/G ) (%)</td>
<td>100 100 100 100 100</td>
<td>99.9 99.9 99.9 99.9 99.9</td>
</tr>
<tr>
<td>An ( G_1 ) (kN/mm)</td>
<td>199.4 199.1 198.7 197.5 198.3</td>
<td>196.5 198.0 197.1 197.1 197.6</td>
</tr>
<tr>
<td>( G_1/G ) (%)</td>
<td>100 100 100 100 100</td>
<td>100 100 100 100 100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \gamma = 6 )</th>
<th>( \frac{M_2}{M_1} = \theta_{1SR} )</th>
<th>( \frac{M_2}{M_1} = \beta_{1SR} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_w ) (l/m)</td>
<td>0.1 0.5 1 5 10</td>
<td>0.1 0.5 1 5 10</td>
</tr>
<tr>
<td>( M_1 ) (Nm)</td>
<td>3.45 3.36 3.33 3.20 3.15</td>
<td>0.503 0.875 1.04 1.36 1.50</td>
</tr>
<tr>
<td>( M_2 ) (Nm)</td>
<td>-6.27 -15.31 -19.26 -27.37 -30.71</td>
<td>83.63 82.43 81.80 79.07 78.37</td>
</tr>
<tr>
<td>Li ( G_1 ) (kN/mm)</td>
<td>199.1 197.6 199.5 198.4 199.8</td>
<td>195.4 195.2 195.7 194.2 196.2</td>
</tr>
<tr>
<td>( G_1/G ) (%)</td>
<td>100 100 100 100 100</td>
<td>99.6 99.6 99.6 99.6 99.6</td>
</tr>
<tr>
<td>Bi ( G_1 ) (kN/mm)</td>
<td>198.6 196.5 199.0 198.0 199.9</td>
<td>195.0 194.5 195.4 193.7 195.8</td>
</tr>
<tr>
<td>( G_1/G ) (%)</td>
<td>100 100 100 100 100</td>
<td>99.5 99.5 99.5 99.5 99.5</td>
</tr>
<tr>
<td>Ex ( G_1 ) (kN/mm)</td>
<td>198.6 196.9 199.1 198.0 199.0</td>
<td>194.9 194.4 195.1 193.6 195.7</td>
</tr>
<tr>
<td>( G_1/G ) (%)</td>
<td>100 100 100 100 100</td>
<td>99.5 99.5 99.5 99.5 99.5</td>
</tr>
<tr>
<td>An ( G_1 ) (kN/mm)</td>
<td>198.6 196.9 198.7 197.7 198.8</td>
<td>197.5 197.4 198.3 196.5 199.0</td>
</tr>
<tr>
<td>( G_1/G ) (%)</td>
<td>100 100 100 100 100</td>
<td>100 100 100 100 100</td>
</tr>
</tbody>
</table>

Table 1: Validation of the pureness of the \( \theta_{1SR} \) and \( \beta_{1SR} \) modes being independent of the type of interface constitutive law.
\[
\begin{array}{c|c|c|c|c|c|c}
\gamma = 2 & N_1 / M_1 = \theta_{2NR} & N_1 / M_1 = \beta_{2NR} \\
\hline \hline k_{\nu} (l/m) & 0.1 & 0.5 & 1 & 5 & 10 & 0.1 & 0.5 & 1 & 5 & 10 \\
M_1 (Nm) & 13.10 & 13.00 & 13.00 & 12.90 & 12.90 & 9.4 & 14.5 & 16.4 & 20.6 & 22.9 \\
N_1 (kN) & -19.65 & -21.64 & -22.42 & -24.03 & -25.01 & 530.2 & 521.5 & 520.6 & 512.7 & 508.4 \\
\hline \hline Li & G_1 (kN/mm) & 198.9 & 197.2 & 197.8 & 196.7 & 198.0 & 195.4 & 195.0 & 196.9 & 196.5 & 196.3 \\
G_1/G (%) & 100 & 100 & 100 & 100 & 100 & 99.9 & 99.9 & 99.9 & 99.9 & 99.9 \\
\hline \hline Bi & G_1 (kN/mm) & 198.7 & 197.1 & 197.7 & 196.5 & 197.8 & 195.1 & 195.0 & 196.4 & 196.0 & 196.0 \\
G_1/G (%) & 100 & 100 & 100 & 100 & 100 & 99.9 & 99.9 & 99.9 & 99.9 & 99.9 \\
\hline \hline Ex & G_1 (kN/mm) & 198.7 & 197.1 & 197.7 & 196.5 & 197.7 & 195.2 & 194.9 & 196.5 & 196.2 & 196.1 \\
G_1/G (%) & 100 & 100 & 100 & 100 & 100 & 99.9 & 99.9 & 99.9 & 99.9 & 99.9 \\
\hline \hline An & G_1 (kN/mm) & 198.7 & 197.1 & 197.8 & 196.6 & 197.8 & 196.5 & 196.0 & 197.7 & 197.4 & 197.5 \\
G_1/G (%) & 100 & 100 & 100 & 100 & 100 & 100 & 100 & 100 & 100 & 100 \\
\end{array}
\]

Table 2: Validation of the purity of the \( \theta_{2NR} \) and \( \beta_{2NR} \) modes being independent of the type of interface constitutive law.

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
\gamma = 6 & N_1 / M_1 = \theta_{2NR} & N_1 / M_1 = \beta_{2NR} \\
\hline \hline k_{\nu} (l/m) & 0.1 & 0.5 & 1 & 5 & 10 & 0.1 & 0.5 & 1 & 5 & 10 \\
M_1 (Nm) & 3.48 & 3.44 & 3.40 & 3.33 & 3.29 & 0.116 & 0.495 & 0.655 & 1.00 & 1.13 \\
\hline \hline Li & G_1 (kN/mm) & 199.0 & 198.1 & 196.9 & 198.4 & 198.9 & 196.4 & 196.7 & 195.1 & 197.0 & 195.0 \\
G_1/G (%) & 100 & 100 & 100 & 100 & 100 & 99.9 & 99.9 & 99.9 & 99.9 & 99.9 \\
\hline \hline Bi & G_1 (kN/mm) & 199.0 & 198.1 & 196.8 & 198.4 & 198.7 & 195.7 & 196.4 & 194.7 & 196.4 & 194.6 \\
G_1/G (%) & 100 & 100 & 100 & 100 & 100 & 99.9 & 99.9 & 99.9 & 99.9 & 99.9 \\
\hline \hline Ex & G_1 (kN/mm) & 199.0 & 198.0 & 196.8 & 198.4 & 198.7 & 195.7 & 196.5 & 194.9 & 196.5 & 194.5 \\
G_1/G (%) & 100 & 100 & 100 & 100 & 100 & 99.9 & 99.9 & 99.9 & 99.9 & 99.9 \\
\hline \hline An & G_1 (kN/mm) & 198.7 & 197.9 & 196.4 & 198.0 & 198.5 & 198.1 & 198.7 & 197.5 & 198.9 & 196.5 \\
G_1/G (%) & 100 & 100 & 100 & 100 & 100 & 100 & 100 & 100 & 100 & 100 \\
\end{array}
\]

Table 3: Comparison between the analytical model and FEM simulations for the ERR partition of a DCB with a single bending moment \( M_1 \) with a bilinear constitutive interface law.

Table 3 compares the results from the two methods when there is a bilinear interface constitutive law. Generally, the results from the spatial analysis agree very well with the analytical partitions and are much closer than the crack tip analysis. The analytical partitions work equally well for \( G_\nu = 1000 \text{ kN/mm} \) as for \( G_\nu = 4000 \text{ kN/mm} \), showing that the size of the damaged region does not have a significant effect for the ranges of \( G_\nu \) and \( k_{\nu} \) examined here.
4 CONCLUSIONS

For non-rigid interfaces both classical and shear deformable partition theories have identical mode I and II energy release rate (ERR) partitions which are the same as those of shear deformable partitions for a mixed mode at rigid interfaces and independent of interface cohesive laws in the absence of crack tip through thickness shear forces. The mode mixity remains constant during fracture evolution. Moreover, in the case of interface fracture in the layered isotropic materials, the pure modes in 2D elasticity partition theory only depend on the ratio between the penalty stiffness to the Young’s modulus of the materials and are independent of the shape of the cohesive laws. FEM results show excellent agreement with theoretical predictions. Partitions of mixed-mode fracture with bi-linear interface constitutive laws by using the pure modes are accurately obtained. The analytical theory provides valuable benchmarks for current CZM and FEM modelling.

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