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A coupled LES-APE approach for jet noise prediction
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ABSTRACT

Despite the continuing advances of computing power, the state-of-the-art direct numerical simulations of jet noise are still restricted to simple problems. Additionally, many established CFD codes are potentially too dissipative for propagating acoustic waves. Hence, alternative methods are needed for complex realistic configurations. Traditionally, the combination of Large-Eddy Simulations (LES) with surface integral methods has been widely used by the research community to predict jet noise in the far-field. However, its application in complex installed configurations poses quite a challenging task in defining a suitable surrounding integral surface. Furthermore, these methods only provide information of single observer locations.

In the present work, a coupled LES-APE (Acoustic Perturbation Equations) strategy is presented as an alternative to traditional methods. Two different solvers are used. The LES is performed with a density based, second order finite volume solver with the \( \sigma \)-subgrid-scale model. The APE code is a high-order Discontinuous Galerkin spectral/hp finite element solver. It solves the APE which does not suffer from instability issues related to the more common Linearised Euler Equations (LEE). Moreover, the APE is advantageous by means of a filtered source, propagating only true sound.

It is also planned to apply this methodology to more realistic configurations such as coaxial nozzles and jet-wing-flap interaction cases.

Keywords: Jet noise, Hybrid Method

1-INCE Classification of Subjects Numbers: 13.1.5.4, 21.6, 76.1

1. INTRODUCTION

The continuous advances of computing power are allowing the use of high fidelity simulations as a predictive tool for jet noise even in complex cases. Moreover, the trend in the preliminary design of engines and aircrafts will be to rely more on simulations, with a final experimental campaign to verify the results obtained.

However, the Direct Numerical Simulation (DNS) of jet noise from the near field to the far field is still unaffordable. Hence, a hybrid approach has been the main alternative used by the research community during the last 20 years. This technique consists on the use of a CFD simulation, which provides the acoustical sources, and a propagation method for the far field noise prediction. Traditionally, LES with a surface integral method has been extensively used in many studies, especially the FW-H approach (1–10). The application of this method in jet noise problems consists on placing a surface that encloses the jet plume. This surface is used to store the values of the flow variables for a certain time history. In the last step, the noise is projected to specific observer points in the far field. The main drawback of this strategy is the lack in the insight of the noise generation mechanism, and its accuracy strongly depends on the placement of the surface around the jet (4).

An alternative is the use of a propagation method (LEE, APE, etc.) that captures the noise at the source within the LES calculation, and then uses an acoustic grid for the propagation of sound to the far field. Furthermore, its application to complex cases such as installed jet is straightforward, in comparison with the integral methods. The present work uses an LES-APE framework for the prediction of jet noise, which avoids the stability issues that are present in the LEE method.

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The paper starts in section 2 with an introduction of the numerical methods used for both the LES and the APE. The Coupling Strategy is introduced in section 3. Section 4 shows some validation cases for the LES solver, the APE solver and the coupled approach. In section 5, the results obtained for a 3D jet using the LES-ape strategy are covered. Finally, section 6 contains the conclusions and the work that is planned to do in the future.

2. GOVERNING EQUATIONS AND NUMERICAL APPROACH

2.1 LES implementation

The flow field is resolved using the unsteady compressible Navier-Stokes equations filtered via a Favre-averaging procedure.

The LES solver is an industrial, density-based, cell-vertex, finite volume code that is mainly used for turbomachinery design (11). The code works with multi-block unstructured hybrid meshes, which gives a great advantage for the simulation of realistic configurations.

The calculation of the inviscid flux is based on a second-order Roe type scheme with a smoothing parameter ($\epsilon$) that acts as a blending between the central and the upwind terms in a similar way as in (6), controlling the amount of numerical dissipation. The use of a second-order spatial scheme, combined with a dissipation control technique, has demonstrated to give satisfactory results in previous studies (1,2,12). The equation of the scheme is as follows:

$$
F_{ij} = \frac{1}{2} (F(Q_i) + F(Q_j)) - \frac{1}{2} \epsilon |A_{ij}| (L_j(Q) - L_i(Q)) 
$$

(1)

Where $F$ represents the inviscid part of the flux, $Q$ is the vector of conserved variables and $L()$ the pseudo-Laplacian. The smoothing parameter varies from a very low value in the central region, to one near the boundaries. This ensures that the numerical dissipation is reduced in the plume of the jet, and increased near the boundaries to minimize the reflection of the pressure waves. Its distribution can be seen in Figure 1.

For the temporal discretisation, a second-order, four-stages Runge-Kutta explicit algorithm is employed. The size of the time step is chosen to keep the CFL number around one near the nozzle exit.

![Figure 1](image)

Figure 1 – Distribution of the smoothing parameter on the entire jet domain.
In this code, the evaluation of the eddy viscosity of the unresolved scales is done by an implicit filter based on the local grid refinement. A general equation for the subgrid-scales model eddy-viscosity is:

\[ \nu_{SGS} = (C_m \Delta)^2 D_m(u) \]  

(2)

Where, \( C_m \) is the constant of the model used, \( \Delta \) the subgrid characteristic length scale and \( D_m \) is the differential operator of the model selected. In any LES calculation one of the most relevant aspects is the definition of this operator, since it should represent the underlying physical problem correctly, especially in coarse meshes in which more turbulent scales are modelled. The model chosen in the present work is the \( \sigma \)-model (13), that defines the differential operator as:

\[ D_m = \frac{\sigma_3(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3)}{\sigma_1^2} \]  

(3)

Where \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0 \) are the eigenvalues of the velocity gradient tensor. The constant of the model used in this study is the same as in (13), \( C_m = 1.35 \). The \( \sigma \)-model has the advantage over more traditional models like Smagorinsky of vanishing in any two-dimensional or two-component flow field. It also has the cubic behaviour near the wall, and it is more stable in complex cases than a dynamic approach. The use of this model has already given encouraging results for jet noise prediction, even in different solvers and with different meshes (12).

2.2 APE implementation

The acoustic solver is based on the Acoustic Perturbation Equations proposed by Ewert and Schröder (14). These equations apply a specific filtering procedure that ensures the sole propagation of acoustic modes. This represents an advantage over the traditional Linearised Euler Equations (LEE), which suffer from stability problems because of the inclusion of vortical non-acoustic modes. Version 4 of the equations is used since the sources are captured from a compressible simulation. In this version, all the nonlinear terms are included as additional source terms. The final formulation of APE-4 system is

\[ \frac{\partial p'}{\partial t} + \rho' \left( \rho \mathbf{u} + \mathbf{u}' \right) = c^2 q_c \]  

(4)

\[ \frac{\partial \mathbf{u}'}{\partial t} + \nabla (\mathbf{u} \cdot \mathbf{u}') + \nabla \left( \frac{p'}{\rho} \right) = q_m \]  

(5)

Where the source terms \( q_c \) and \( q_m \) are

\[ q_c = -\nabla \cdot (\rho' \mathbf{u}') + \frac{\bar{\rho} B s'}{c_p D t} \]  

(6)

\[ q_m = -(\mathbf{\omega} \times \mathbf{u})' + T'\nabla s' - s'\nabla T' - \frac{(\nabla (u')^2)}{2} + \frac{(\nabla \cdot \mathbf{\tau})'}{\rho} \]  

(7)

In the present work, only the Lamb vector \( (\mathbf{L}' = -(\mathbf{\omega} \times \mathbf{u})') \) is considered, since in isothermal applications with strong vortical motions, i.e. shear layers and wakes, it represents the major sound contribution, as demonstrated in (14–16).
The solver used is the open source code Nektar++ (17) that incorporates an application called APESolver, which computes the APE-4 formulation. The code is based on a high order, spectral/hp element method with a Discontinuous Galerking (DG) formulation (18). The DG method uses the variational form of the underlying problem, multiplying the governing system, in this case the APE-4 system, by a test function and integrating it over the partitioned domain. The solution is then represented locally in each element with a basis of polynomials and a set of quadrature points (Gauss-Legendre, Gauss-Radau-Legendre or Gauss-Lobatto-Legendre). After this, the different local elements of the domain are coupled to get the solution of the global problem. This coupling involves the definition of numerical fluxes that propagates the solution from adjacent elements. The method ensures the high spatial resolution required for the propagation of the waves in an efficient way. For the calculation of the fluxes the APESolver applies a local Lax-Friedrichs scheme similar to (19).

\[ F^R = \frac{1}{2} [ F(U_L) + F(U_R) - |\alpha|_{max} (U_R - U_L) ] \]  

Where \( |\alpha|_{max} \) is the maximum absolute eigenvalue of the jacobian of \( F \), and \( U_L \) and \( U_R \) the values of the left and right hand side at the interface. Figure 2 shows a diagram of the flux across two elements. The temporal discretisation is performed using a fourth order Runge-Kutta scheme that reduces the dispersion errors.

**Figure 2** – Representation of the numerical flux across two elements including the global numbering system for a nodal expansion of polynomial order 3. Adapted from (18,20)

### 3. COUPLING PROCEDURE

The main advantage of the LES-APE approach compared to an isolated LES for the flow and acoustic fields is the use of different grids for each solver. For low Mach number cases this is even more effective due to the disparity between the length scales of the acoustic and the hydrodynamic field (14). Using this advantage, the LES-APE method has been successfully applied by Ewert and Schroder (21) and Ewert (22) to predict trailing edge noise; and Bui et al. (23,24) and Lackhove et al. (19) for the prediction of combustion noise. However, as mentioned by Koh et al. (25), at transonic speeds this argument is no longer applicable. On the other hand, the extremely fine grid resolution that an LES of jets requires near the nozzle exit results in a very small time step. In addition, the extraction of the sources from the LES gives an idea of the location of the major noise sources, which could be used to modify the jet flow in a way that the noise is reduced. However, the use of different grid resolutions for each solver requires the implementation of an interpolation procedure that ensures that pseudo noise sources are not generated.

In the present work, the coupling of the LES solution to the APE solver consists in the following steps:

1) The LES calculation is initialised until the flow field reaches a statistically stationary state.
2) The average of the flow field is stored. This average is performed over the primitive variables and the mean Lamb vector.
3) The LES is run again to store the perturbed Lamb vector.
4) The files where the sources are stored are converted to the Nektar++ reading format.
5) The APESolver is run for the time steps stored in the LES calculation.
The type of coupling between the two solvers can then be classified as a file-based one way coupling. Figure 3 shows a sketch of the overall procedure. The sources are captured for the first 17 diameters, and a damping zone is applied at the boundaries of the capturing box to avoid the generation of spurious sound. The right image of Figure 3 shows the propagation of a monopole source similar to those encountered at the end of the potential core. The LES mean flow is included as a baseflow to include the convection effect over the waves.

4. VALIDATION OF THE SOLVERS

4.1 LES of a High Reynolds Number Jet

The case selected for the validation of the LES code is that of Set Point 7 of Tanna (26). This case corresponds to a cold jet at acoustic Mach number Ma=0.9 and Reynolds number $10^6$ condition. In previous studies by Angelino et al. (1,12) and Mahak et al. (2) the influence of different SGS models and mesh resolutions has already been tested. In the present work, a comparison between a 5 and a 20 Million meshes is shown.

Figure 4 illustrates the vorticity magnitude for both cases in an xy-plane and a yz-plane. It is clear from the contours that the 20 Million case presents an earlier transition to turbulent regimen in the shear layer. This earlier transition has an important effect on the averaged behaviour of the jet, as shown in Figure 5. The centreline of the 20 Million case compares better with the experiments than the 5 Million. In addition, the peak at $x/D=2$ in the radial profiles is stronger in the 5 Million case due to this delayed transition.
These results demonstrate that the LES solver is capable of resolving cases of engineering interest (high Reynolds number and high speed) even with relatively coarse meshes.

### 4.2 APE with Analytical Sources

#### 4.2.1 Monopole in a sheared mean flow

The first case is the propagation of a monopole in a shear mean flow. This case reproduces the typical conditions of a turbulent jet where the main sources come from mixing layer flows within a sheared mean flow.

The shear mean flow and the source term are expressed by the following equations:

\[
\bar{u} = \frac{1}{2} \tanh \left( \frac{2y}{\delta_w} \right), \quad q_e = \exp \left[ -\ln(2) \frac{x^2 + y^2}{\sigma^2} \right] \cos(\omega t), \quad q_m = 0 \tag{9}
\]

Where \( \delta_w = 10 \) defines the shear layer thickness, \( \sigma^2 = 9 \) and the angular frequency is \( \omega = 0.5 \). For the simulation, a domain of size 200 x 200 is divided into 200 x 200 elements with a polynomial degree of one. Figure 6 shows the pressure contour obtained with the APESolver, and a comparison with the LEE and APE-2 results obtained by Ewert and Schröder (14). As can be seen in Figure 6 right, there is almost a perfect match between the three solutions, which means that the APESolver can be used to describe the acoustic propagation in arbitrary mean flows.

Figure 6 – Pressure contour and comparison of the present result with an LEE and an APE-2 solutions (14)
4.2.2 **Spinning vortex**

The sound field generated by a pair of vortices has a quadrupolar nature. For this case, the flow field is considered inviscid and incompressible with a separation between the vortices of \(2r_0\) and a circulation \(\Gamma\). The rotation period is defined as \(T = \frac{8\pi^2 r_0^2}{\Gamma}\). The angular velocity is \(\omega = \frac{\Gamma}{4\pi r_0^2 c}\). An analytical solution of the perturbation pressure was obtained by Müller and Obermeier (29). By using a Gaussian vorticity distribution, Ewert and Schröder (14) found the source term based on the Lamb vector that represents the acoustic field for this case

\[
q_m = -\frac{\Gamma^2 e_r(t)}{8\pi^2 \sigma^2 r_0} \sum_{i=1}^{2} (-1)^i \exp\left(-\frac{|\mathbf{r} + (-1)^i \mathbf{r}_0(t)|^2}{2\sigma^2}\right), \quad \sigma \approx r_0
\]

Where \(\mathbf{r} = (x, y)\), \(\mathbf{r}_0 = r_0 \mathbf{e}_r\) and \(\mathbf{e}_r = (\cos \theta, \sin \theta)\), \(\theta = \omega t\).

Figure 7 – Pressure contour and comparison of the present result with the analytical solution of Müller and Obermeier (29).

In this case, the domain has an extension of 100 x 100 and it is divided in 141 x 141 points. Figure 7 shows the contour of instantaneous pressure and the comparison of the APESolver result with the analytical solution. The present results match perfectly the analytical solution, which proves that the solver is capable of resolving the propagation of noise generated by vortical structures define by the Lamb vector.

### 4.3 LES-APE of a 2D Cylinder

For the validation of the coupled approach, the flow around a cylinder at \(M=0.3\) and \(Re=200\) has been used. The domain extends up to 80 cylinder diameters from the origin, with a mesh resolution of 600 x 576 points in the circumferential and radial direction respectively. This resolution ensures that the acoustic waves are propagated in the CFD simulation until the far field without any non-physical dissipation. To avoid any interpolation problems, the resolution of the APE mesh for this case is the same as in the CFD. Since this problem presents a time periodicity the source is stored for one of the period being repeatedly read by the APESolver.

As mentioned in section 2, the main source term for this problem is the Lamb vector, being the rest of them negligible. For this reason, only the Lamb vector is stored during the CFD simulation. Figure 8 shows the contour of the two components of the Lamb vector. From the two components is very clear that most of the noise comes from the near cylinder region.
Figure 8 – Contour of the Lamb vector components ($q_{m1}$ and $q_{m2}$).

Figure 9 – Contour perturbation pressure for the DNS and LES-APE coupling approach.

Figure 10 – Distribution of perturbation pressure along x=0 for DNS, LES-APE and Curle’s analogy.
In Figure 9, a qualitative comparison between the APE and the DNS contours is presented. The images show that the propagation of the waves is almost identical in the two cases. This agreement is further analysed plotting a line along x=0 and comparing the two results with Curle’s analytical analogy in Figure 10. Both the APE and CFD solutions compare well with Curle’s equation. This result is encouraging for the final application of the coupling approach in jet noise prediction cases.

5. RESULTS

The results obtained for a 3D jet at Ma=0.9, a constant stagnation temperature (Tj/T∞=0.86) and Re=3600 are presented here. First, some instantaneous contours and mean profiles are shown comparing them with the LES results obtained by Freund (30). Then, the far-field noise results obtained using the LES-APE approach are shown, including a comparison with an LES-FWH calculation and the experiment performed by Stromberg (31).

The LES domain has an extent of 70 diameters in the axial direction and 15 diameters in the radial direction. However, from x/Dj=25 the growth rate rapidly increases to dissipate the pressure waves avoiding any reflections from the outlet boundary. The domain is discretised using 380×90×45 points in the axial, radial and azimuthal directions respectively. The acoustic domain has an extension of 60 diameters in every direction using 170×120×120 with a polynomial degree of p=2. The acoustic mesh is refined near the jet plume region to improve the interpolation of the sources from the flow simulation.

5.1 Flow field

Figure 11 shows contours of non-dimensional temperature and vorticity magnitude. Since the Reynolds number is quite low, there is a long laminar region before the transition to turbulent regime which occurs at around x/Dj=5. This transition is driven by strong pairing vortices that collapse the two shear layers only a couple of diameters downstream from the transition point.

Further analysis of the mean flow in Figure 12 shows that the present study compares well with the LES results of Freund (30), and that it has the decay rate and the linear spreading that any turbulent jet must have.
Figure 13 – Radial axial and shear stresses at x/D_j=0, 2.1, 4.2, 6.25, 8.3, 10.4, 12.5 and 14.5. Solid lines: present study; dashed lines correspond to Freund (30).

This agreement is also verified by looking at the components of the Reynolds stresses. As can be seen in Figure 13, after the initial laminar region there is a rapid increase of the peak value of the stresses near the end of the potential core. The discrepancy between the present result and that of Freund is almost insignificant for all the positions except for x/D_j=6.25.

5.2 Acoustic field

Once the flow field is established, the sources for the APE-4 are stored for a time history of 100 non-dimensional time units. The sources are then transferred to the APESolver and the noise is propagated for the entire time history. In order to minimize the reflections, a sponge zone is applied near the outlet boundaries of the acoustic domain.

The comparison between the propagation of the sound waves in the LES mesh and the acoustic mesh used in this study can be seen in Figure 14. As mentioned in the introduction, the high dissipation of the LES code makes the pressure waves to dissipate rapidly. At only 15 diameters most of the noise frequencies have vanished. On the other hand, a greater range of frequencies is propagated from the jet plume to the far-field without excessive numerical dissipation. From the image, it is also clear the noise has a clear directivity towards the 30º angle from the centerline.

Figure 14 – Contours of perturbation pressure (p’) for the LES (left) and the APE calculations.
In the present study, a set of pressure probes was located at $x/D_j=20$ to predict the noise in the far field. The OASPL is shown in Figure 15. The APE results obtained in the present study has been properly scaled to matched the other two. The directivity of most of the angles has been accurately predicted and further work is being carried out to improve the prediction for all the angles.

6. CONCLUSIONS

An LES-APE coupling strategy for the prediction of jet noise has been presented. The method used in this work combines a state-of-the-art LES code, which main application is the industrial design of turbomachinery components, with an open source spectral/hp high order finite element APE code.

The LES code has demonstrated to correctly predict the flow physics of jets at transonic speed and high Reynolds numbers. In addition, a thorough validation of the APE code is done, using both analytical and numerical cases.

Finally, preliminary results for a 3D jet has been shown giving the expected low dissipation of sound waves in the acoustic solver.

The next step of the project will be the quantitative validation of the 3D jet using experimental and FWH results. Once the validation is done the methodology will be applied to installed jet-wing cases.

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