Analysis of a salt gradient solar pond with an internal heat exchanger

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ANALYSIS OF A SALT GRADIENT SOLAR POND

WITH AN INTERNAL HEAT EXCHANGER

by

SALAH SAOULI

A Master's Thesis submitted in partial fulfilment
of the requirements for the award of
Master of Philosophy
of Loughborough University of Technology

October 1987

Supervisor: M R Leeson
Department of Mechanical Engineering
To my mother Kalima
To my father Rabah
To my sister Nasira
To all my brothers and sister
To a special friend

V.L. G. Kamienski
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SUMMARY

Solar ponds are bodies of water in which heat convection resulting from the absorption of solar radiation is suppressed by an artificially established salt concentration gradient.

In this thesis a new method of heat extraction from the pond was investigated, both theoretically and experimentally. The method consisted of extracting heat from the whole body of the pond by means of a helical coil heat exchanger. Because of the existence of a temperature gradient through the depth, the overall efficiency of such a solar collector was expected to increase. The developed theory of this system based on the steady state heat transfer theory predicts the effect of operating parameters such as insolation, flow rate and design parameters (such as length of the heat exchanger) on the pond and outlet fluid temperature and the efficiency.

For the variation with time of the concentration of salt, the mass diffusion differential equation was solved. This shows how the gradient changes with time.

The experimental investigation was carried out in the laboratory on small scale ponds which were heated artificially. Instrumentation was developed for density and temperature measurements and data comparison between theoretical and experimental results were made and conclusions drawn concerning the performance of ponds with this configuration.

The experimental results are in agreement with the theoretical predictions and there is an increase in efficiency with the new heat extraction system.
ACKNOWLEDGEMENTS

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Special thanks go to Mr B O Mace, technician in the Department of Mechanical Engineering, for the construction of test equipment and help in setting up the experiments.

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Thanks also go to the Library staff for their assistance in searching out references and to the Computer Centre staff for their help with computing.

Many thanks to my postgraduate friends, especially Ganti, Ali Demir, Thyagarajan and Sindano. My thanks to the Algerian government for their financial support.

Finally my thanks to Janet Smith for typing this manuscript.
CHAPTER 3:

a: constant

$\alpha_i (i=1,7)$: constant coefficients

$B_{\lambda}$: Planck's function (W/m$^3$)

b: constant

C: salt concentration

$C_P$: specific heat (J/kg°C)

D: diffusion coefficient of salt (m$^2$/s)

$D_i$: inner diameter of the pipe (m)

$D_o$: outer diameter of the pipe (m)

$f_\lambda (T)$: fraction of energy transmitted between 0 and $\lambda$

$g$: acceleration of the gravitation (m/s$^2$)

$H(x)$: generated heat rate (W/m$^3$)

$H_s$: solar insolation (W/m$^2$)

$h(x)$: attenuation function

$h_i$: inner heat transfer coefficient (W/m$^2$ °C)

$h_o$: outer heat transfer coefficient (W/m$^2$ °C)

i: angle of incidence (deg)

K: thermal conductivity of water (W/m°C)

$K_t$: thermal conductivity of the pipe (W/m°C)

L: latitude (rad)

m: flow rate (kg/s)

n: index of refraction
N: day of the year ($N = 0$ for the first of January)
Q: rate of extracted heat ($W/m^3$)
$R_s$: solar radius (m)
$r$: mean earth-sun distance (m); angle of refraction (deg)
$S$: solar constant ($W/m^2$)
$dS$: path length of the pipe (m)
$T$: temperature (deg C)
$T_f$: fluid temperature (deg C)
t: time (s)
$U$: overall heat transfer coefficient ($W/m^2 \cdot \deg C$)
x: coordinate (m)
$\beta'$: thermal expansion coefficient (1/deg C)
$\delta$: angle of declination of the sun (rad)
$\delta \theta$: angular divergence (deg)
$\sigma$: Stephan Boltzmann's constant ($W/m^2 \cdot \deg K^4$)
$\tau$: transmittance
$\mu_f$: dynamic viscosity (kg/s m)
$\rho$: density (kg/m$^3$)
$\lambda$: radiation wavelength (m)

CHAPTER 4

$A$: area of the pond ($m^2$)
a: constant
$a'$: constant
$a_0$: constant
$a_n$: coefficient of the Fourier series
$a_m(m=1,5)$: constant coefficients
b: constant
b': constant
b_{m=1,4}: extinction coefficients (m^{-1})
C(t): concentration profile
C(x,0): initial concentration profile
C_0: lower convective zone concentration
C_{1,2,3}: constants of integration
D: diffusion coefficient (m^2/s); determinant
D_{1,2,3}: determinants
D_0: outer diameter of the pipe (m)
f_{m=1,4}: coefficients (deg C)
G_{1,2,3}: constants
H_s: solar insolation (W/m^2)
h(x): attenuation function
i: vector unit
j: vector unit
K: thermal conductivity of water (W/mC)
K: vector unit
L: total depth of the pond (m)
l_1: coordinate of the upper interface UCZ-NCZ (m)
l_2: coordinate of the lower interface NCZ-LCZ (m)
M(t): mass of salt in the layer above the diffuser (kg)
m: flow rate (kg/s)
n: integer number (1, =)
N: number of coils of the heat exchanger
p: pitch
R: radius of the heat exchanger coils (m)
R_{1,2}: roots of the characteristic equation
dS: path length of the heat exchanger pipe (m)
T_a: ambient temperature (deg C)
T: pond temperature (deg C)
T^*: temperature (deg C)
T_f: fluid temperature (deg C)
T_{fi}: inlet fluid temperature (deg C)
T_{fo}: outlet fluid temperature (deg C)
t: time (s)
U: overall transfer coefficient (W/m^2\cdot C)
V(t): volume above the diffuser (m^3)
x: function
x: coordinate (m)
x_d(t): position of the diffuser (m)
x_s(t): position of the water surface (m)
Y: function
y: coordinate (m)
z: coordinate (m)
\alpha: constant
\beta: constant
\Delta: discriminant of the characteristic equation
\lambda: constant different from zero
\eta: efficiency
\tau: transmittance
\theta: polar angle (deg)
CHAPTER 1

INTRODUCTION

Energy was, and still is, the main concern of all generations over the centuries. It is involved virtually in every single aspect of our lives as the primary motive power. In this respect, the history of energy is not only finding resources of coal, oil of water but how to convert these raw energy resources into a useful and suitable form for the industrialization of society. Through the years mechanical power has been substituted more and more for human physical power, leading to a shift from reliance on food energy to other sources. The energy resources are usually regarded as gasoline for transport, the production of electricity from coal and gas for homes or fuel for factories and thus there is a drive to explore new ideas able to make the energy conversion systems more efficient for these semi-traditional resources.

In primitive communities, their lives depend on energy and its use to a large extent, even without modern machines, ranging from tool making to cooking and more cooking time is absorbed in providing the necessary small quantities of energy. However in the present industrialized communities, the total energy requirement per capita is much greater; it is used in factories, transport, lighting, heating, cooling, communication and recreation. Petroleum powers cars, trucks, trains for transportation. Houses are lit, heated or cooled with oil, coal or electricity. Kitchens are equipped with goods made in factories driven by oil, coal or electric power and the appliances consume energy in other forms themselves. These material conveniences and luxuries such as packaged food, clothing and vehicles lead to a higher standard of living of the society in a sense that rather than needing to satisfy each of these needs individually, they are purchased already manufactured.
These considerations lead to another concept which can be regarded as a "feedback". The more availability of energy to the society, the higher is the standard of living providing better conditions of work which enable people to work more and develop their society more rapidly.

1.1 ACTUAL ENERGY AVAILABILITY

The importance of energy and its impact on the prosperity of a society cannot be considered in isolation. It is necessary to look at the actual energy resources available. They take on significance only when annual production rates are projected and compared to annual demand rates, because they determine energy supply and inevitably regular global energy demand.

The increase in production from conventional resources depends on the rate at which technology improves, money invested, skilled workers and economic structures implemented. At the present time, the production of energy from both renewable and non-renewable resources depends on technological developments. By improving the energy conversion systems they become commercially feasible.

As is shown in Table 1.1.1(1), the predicted production rate of primary energy in the world will require huge development and efforts in both the economic and engineering fields. The most relevant feature from this data is that all the known resources of energy will show a steadily increasing production rate up to the year 2020 except for oil and gas resources which, being limited, are predicted to reach a peak production value by the year 1995 and 2010 for oil and gas respectively and be exhausted thereafter.
<table>
<thead>
<tr>
<th>Resources</th>
<th>1972</th>
<th>1985</th>
<th>2000</th>
<th>2020</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>66</td>
<td>115</td>
<td>170</td>
<td>259</td>
</tr>
<tr>
<td>Oil</td>
<td>115</td>
<td>216</td>
<td>195</td>
<td>106</td>
</tr>
<tr>
<td>Gas</td>
<td>46</td>
<td>77</td>
<td>143</td>
<td>125</td>
</tr>
<tr>
<td>Nuclear</td>
<td>2</td>
<td>23</td>
<td>88</td>
<td>314</td>
</tr>
<tr>
<td>Hydraulic</td>
<td>14</td>
<td>24</td>
<td>34</td>
<td>56</td>
</tr>
<tr>
<td>Unconventional (oil and gas)</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>Renewable (solar, biomass, geothermal)</td>
<td>26</td>
<td>33</td>
<td>56</td>
<td>100</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>269</td>
<td>488</td>
<td>690</td>
<td>1000</td>
</tr>
</tbody>
</table>

**TABLE 1.1.1: POTENTIAL WORLD PRIMARY ENERGY PRODUCTION (EXAJOULES)**

1 EJ = $10^{18}$ J

Actually, there are three principal developed sources of energy for power: fossil fuels (coal, oil and gas), nuclear fuels and hydroelectric power. As a future energy source hydroelectricity can be taken as having only a limited potential for development.

The actual estimates of the reserves of fossil fuels have proved that coal is totally dominant with over $7.5 \times 10^{12}$ tonnes representing $10^{15}$ kWh (thermal). At the bottom of the scale, the oil shale reserves appear to be equivalent to $32 \times 10^{13}$ kWh. This represents only 0.5% of world reserves of fossil fuels, Table 1.1.2(2).
<table>
<thead>
<tr>
<th>Fuel</th>
<th>Quantity</th>
<th>Energy Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>$7.5 \times 10^{12}$ tonnes</td>
<td>$2 \times 10^{13}$ joules</td>
</tr>
<tr>
<td>Oil</td>
<td>$2 \times 10^{12}$ barrels</td>
<td>$1.2 \times 10^{22}$ joules</td>
</tr>
<tr>
<td>Gas</td>
<td>$10^{16}$ cubic feet</td>
<td>$10^{22}$ joules</td>
</tr>
<tr>
<td>Oil from tar sands</td>
<td>$3 \times 10^{11}$ barrels</td>
<td>$1.8 \times 10^{21}$ joules</td>
</tr>
<tr>
<td>Shale oil</td>
<td>$2 \times 10^{11}$ barrels</td>
<td>$1.2 \times 10^{21}$ joules</td>
</tr>
<tr>
<td>Peat</td>
<td>$3 \times 10^{11}$ tonnes</td>
<td>$7 \times 10^{22}$ joules</td>
</tr>
</tbody>
</table>

**TABLE 1.1.2: PROBABLE WORLD FOSSIL FUEL RESERVES**

Actually, these non-renewable fossil fuels provide 97% of the primary energy supply in the world. The shares of each of these energy resources in the production of the needed energy are as follows:

1. Approximately 38% comes from solid fuels (coal) (consumption increasing).
2. Another 40% comes from oil for a consumption that has been doubling every decade since 1973.
3. Nearly 19% of the actual energy supply provided comes from natural gas for a consumption doubling every seven to eight years.
4. The hydroelectricity and nuclear power produce about 3% of the world's primary energy.

This energy supply hardly seems sufficient for a demand that is exponentially increasing due to the high demographic and economic development of the world. For instance, modern agriculture already requires fuel subsidies so heavy that sometimes the supply is several times as much chemical and mechanical energy as we recover in the produced food.
The production of nitrate fertilizer, even with improved technology, requires about 2.3 kg of coal equivalent per kg of fixed nitrogen. Such a rate implies that nitrogen fertilizer factories alone in the year 2000 may need about 20% of the present total world energy use to produce $8 \times 10^{11}$ kg of nitrogen/year for agriculture.

After reviewing the energy situation in the world, an obvious question arises: how and from what will the necessary energy be provided? One of the answers to this question is solar energy as a new energy resource.

1.2 SOLAR ENERGY AS AN ALTERNATIVE

The attractiveness of solar energy as a renewable energy resource is evident in terms of its availability. Yet the exploitation of this form of energy on a large scale is confronted by two problems because of two features of solar radiation, namely the low energy-density and its intermittence. Therefore, to collect solar energy in efficient commercial quantities requires a large collector*.

As can be seen, such solar collectors need large amounts of money to be invested and materials and furthermore because of these huge dimensions, another problem arises in finding a means to bring the collected energy over that area to a central plant for use which would be expensive not only in terms of investment, but also because of the considerable losses of energy during the transportation process.

* To replace one barrel of oil per day, the collector area needed for an insolation of 200 Wm$^{-2}$ is ten hectares with a conversion efficiency of 100%.
Another crucial problem related to the use of solar energy is its irregularity due to the geographical location and the atmospheric conditions. In practice the energy demand does not match its availability, as a result of which a means of storage is required to dump the excess energy which would otherwise be wasted. Therefore, it has been a strong motivation to find a new solar collector that could be large in area and would be able to store the solar energy. The first idea in that respect was the use of oceans. However this was not successful because in the early plants the efficiency was negative since more power was needed for pumping than was being produced. Hence, that huge solar collector was substituted by a more efficient one, i.e. non-convective ponds.

Solar ponds could be considered as the simplest and least expensive device for collecting, storing and converting solar energy to thermal energy.

1.3 SOLAR PONDS

Solar ponds are simultaneous collectors of solar radiation and a large medium of heat storage and can be operated at virtually all latitudes. In some locations their surfaces may freeze in winter, yet storage temperatures remain high enough for low-temperature applications.

Solar ponds are found very advantageous rather than the other solar collectors (flat-plate, solar tower, etc) by suppressing some difficulties associated with solar devices, mainly high cost, maintenance requirements and intermittence of energy output. Typical solar ponds investments are around $10 to $40 per square metre which is relatively lower than flat-plate collectors costs(3).

For a pond situated in a region of 200 Wm$^{-2}$ average insolation, an operating efficiency of 20%, $30 per square metre construction cost and a fixed charge rate of 20%, the price of a kWh delivered would be costing around $1.7. Besides their economic feasibility, solar ponds
are more advantageous than the other solar collectors because of their capability of storing the heat collected during Summer and the rainy season over long periods to be used when needed, especially in Winter. Besides all these advantages, solar ponds are easy to construct and maintain for long term operation, since earth serves as the supporting structure and water as the collecting, storing and heat transfer medium. Therefore, the new solar collectors can be considered as a decentralized solar technology because of their relatively low-technology production and simple maintenance requirements, especially suitable for energy needs in remote areas and developing countries.
The history of the solar ponds concept began at the turn of this century when temperatures up to 70°C at a depth of 1.3 metres were recorded in Medve Lagoon, a natural salty lake in Hungary. This observed phenomenon suggested the use of stratified salt ponds as solar collectors.

In the late 1940s, the idea of using artificial stratified ponds for practical utilisation was proposed to the "Israel National Research Council" by Dr R Bloch. Some ten years later an intensive investigation was carried out at the National Physical Laboratory of Israel under the supervision of H. Tabor. A detailed picture of the problems involved and the economic prospects of solar ponds was given by H. Tabor. In his article he showed the extreme difficulty in getting competitive electric power on a large scale using conventional solar collector systems. Furthermore, he showed that solar ponds held promise for being the cheapest known device for exploiting solar energy and are particularly suitable for large installations because of their considerable built-in storage. A square kilometre in a sunny area should yield $30 \times 10^6$ kWh of power equivalent to 36,000 tons of fuel burnt at 85 per cent efficiency. On the other hand, he summed up the major questions of solar ponds and some answers to them.

A theoretical investigation of solar ponds was pioneered in 1964 by H. Weinberger from the National Physical Laboratory of Israel. He proved theoretically that solar radiation can affect a high temperature rise in non-convecting ponds of about a metre depth. The calculated efficiency of the ponds as a solar collector can be greater than 20 percent and about 4 percent of the incident solar energy can be converted to electric power by means of a Carnot engine. In his calculations, he used the meteorological data.
recorded at the Dead Sea in Israel. He found that temperatures up to 100°C can be obtained with a pond one metre deep. Furthermore, he defined a stability criterion of the pond for efficient operation. This criterion shows that a difference in concentration of 0.34 g/cm³ is required to maintain stability in a one metre deep magnesium chloride pond from which energy is not extracted and in which a 20 centimetres convecting zone is allowed at the bottom of the pond. However, a sodium chloride pond is unstable if energy is not removed from the bottom during high insolation periods.

One year later, in 1965, H Tabor and R Matz(7) published a paper giving an account of some of the more important investigations that have been carried out between 1959 and 1964, the results obtained and the guidelines for future work. Among some of the results reported in their paper is that it has been demonstrated practically that a density gradient can be set up that will suppress convection for bottom temperatures reaching boiling point. Laboratory and field tests showed that at least for distances of 55 metres, layer flow across a pond having a density gradient is possible for both surface and submerged layers. It was also reported that the diffusion of salt upwards can be controlled so that no salt needs to be added to the pond after the initial filling and that waves at the surface cause limited mixing of the upper zones to a depth that is not prohibitive to an operational pond.

In 1970, J R Hirschmann(8) made a detailed study of some physical characteristics associated with the operation of solar ponds. His work was undertaken on the Andean salt flats situated in the north of Chile, the north-west of Argentina and the south-west of Bolivia. He suggested the use of these salt flats for industrial purposes like desalination or salt production besides electricity generation (Figures 2.1 and 2.2).

Since 1969 a group of Russian scientists at the Uzbek Society Socialist Republic Academy of Sciences have been actively engaged in very detailed and complicated research of the relevant physics of
FIGURE 2.1: SCHEME OF DUAL PLANT OF SOLAR PONDS TO DISTILL SALINE WATER AND PRODUCE PURE SALT (8)

FIGURE 2.2: SCHEME OF A DUAL PLANT OF SOLAR PONDS TO DISTILL SALINE WATER AND PRODUCE ELECTRICITY (8)
solar ponds. In their paper, V N Eliseev et al\(^{(9)}\) proposed an elaborate calculation of the thermal regime of a solar pond by adding some other considerations on Weinberger's model. They assumed in their study that the incident solar radiation, the transmittance and the spectral intensity depend on the astronomic parameters of the sun and the wavelength. Moreover, they assumed that the wavelength and the spectral absorbance were functions of the salt concentration which is specified in the form of a certain function of the depth of the pond. Using the method of finite difference to solve the general heat diffusion equation, they concluded that the temperature of the bottom layer oscillates during the heating process because of the alternation of night and day and that the amplitude of the temperature fluctuations decreases with increase in the depth of the pond. Moreover, they showed that the temperature of the bottom depends strongly on the depth and that for a given pond there is a definite value of the depth to which the bottom layer can be heated.

Two years later, V N Eliseev et al\(^{(10)}\) evaluated the efficiency of a solar pond and how it depends on the fundamental factors of operation. They considered the solar pond as a heat machine and by writing a heat balance equation for the pond in the transit region, they concluded that the efficiency of a solar pond depends on time and that to ensure a stable year round operation of the pond, the reduced solar radiation during the Winter should be compensated by the energy stored during intense insolation periods. Furthermore they showed that in the case of the steady state operation, the efficiency is close to the maximum thermodynamic efficiency which is about 70%. This overestimation is justified by the fact that in their calculations they neglected that some of the energy accumulated by the pond is lost through the walls and the bottom of the pond.

The possibility of using a solar pond in space heating was evaluated for the first time by A Rabl and C E Nielson\(^{(11)}\) for different locations in the United States of America. By taking into account the amount of heat that can be stored in the ground underneath the pond and assuming a heating demand of 25,000 Btu/degree day
characteristic of a 2000 ft$^2$ house, they found that solar ponds can supply adequate heating even in regions near the Arctic circle.

As far as the economic aspects of the use of the pond for space heating is concerned, the authors found that this new collector is very competitive with heating by conventional fuels since the annual heating cost is $0.41 \text{ kWh}^{-1}$ compared with the oil heating cost which is about $0.128/\text{gallon}$ with an efficiency of 75%.

The possibility of using a solar pond as a heat source for low-temperature multi-effect distillation plants was investigated by H Tabor$^{(12)}$. For instance, in Israel a distillation plant operating with a solar pond reduces the cost of desalinated water by 18% per m$^3$ when compared with the same system using fuel.

Besides the use of solar ponds in power generation, heating and distillation, their use in agriculture as a source of heat for greenhouses has also been experienced. S Shah, T H Short and R P Fynn$^{(13,14)}$ reported that a 5 metres deep pond with a one metre gradient zone would be a significant improvement over the present system, provided that the greenhouse heating system was operated in conjunction with a heat pump.

On studying the performance and the main characteristics of solar ponds, different approaches may be adopted: experimental, theoretical or a combination of both. However, in view of the complexity of most real systems, a complete experimental analysis is costly, unreliable, and scale effect results in some cases in large discrepancies. Therefore theoretical methods of investigation have assumed great importance and are subject to continual development.

Over approximately the last decade, different researchers have attempted theoretical studies of the performance of solar ponds. Because of the importance of the salt gradient and its maintenance in the solar ponds, C E Nielsen et al$^{(15)}$ proposed a flow system for the maintenance of the salt concentration. The system operates by
pumping water from the UCZ to be purified from salt and returned fresh. However, in the LCZ the brine was pumped through a salt input where more salt could be added to the brine before it was returned to the LCZ.

This method was slightly costly because of the use of the pump. An improved method to establish a salt gradient in solar ponds was reported by F Zangrando(16), this method is called the filling by redistribution. It consists of partially filling the pond with high salinity brine; fresh water is then pumped through a diffuser which is immersed in the upper portion of the existing solution. The diffuser is subsequently raised to the surface, either in a continuous motion or in discrete steps. The brine above the diffuser will be progressively diluted while the diffuser, as well as the water level, rise. Timing must be such that the diffuser reaches the surface at the predetermined final level of the pond. This method was found sufficient to establish or modify the existing gradient without the need for additional space or mixing tanks.

The absorption of solar radiation in solar ponds has been studied theoretically by means of the radiative heat transfer theory. The absorption of solar radiation in pond water takes on importance because it is the generator of heat in the solar pond.

The first analysis related to this field was first carried out by R Viskanta and J S Toor(17). Their analysis, based on radiative heat transfer theory, was able to predict the local rate of solar energy absorption in a pond. The model considers absorption and scattering by the water and internal reflection of radiation from the air-water interface as well as the bottom. They investigated the effects of the directional distribution of solar radiation incident on the water surface, the attenuation of solar radiation by the atmosphere during the diurnal cycle and the modification of the spectral radiation characteristics of water by impurities and additives on the absorption and distribution of the absorbed energy in the pond. They concluded that the use of a reflecting bottom for a pond increases
the total reflection from the pond, but at the same time causes a more uniform absorption of solar radiation throughout the pond and particularly increases the absorption near the bottom. The radiative characteristics of a pond bottom must be chosen with care for optimum absorption and distribution of the absorbed energy. Moreover, they found that impurities and additives to water to increase absorption of solar radiation will result in a more non-uniform deposition of the absorbed energy. A similar analysis of the absorption of the solar radiation in solar ponds was reported by Y A Cenget and M N Ozisik\(^{(18)}\) by considering the local rate of absorption of the solar radiation in a solar pond and the determination of the direct component at angles of incidence from \(0^\circ\) to \(75^\circ\) as well as of the diffuse component. The analysis also includes the effects of bottom reflection, the pond depth and the type of radiation on the thermal efficiency of the pond. By means of computation, the local rate of solar energy absorption at any depth and at any incidence angle was found and the relative coefficients of this function were tabulated for different incidence angles and bottom reflectivities.

They concluded that the index of refraction may be assumed constant but the variation of absorption coefficient with wavelength cannot be neglected. The absorption rate of direct radiation is virtually independent of the angle of incidence for angles up to about \(45^\circ\), and diffuse beams can be treated as direct radiation with an incidence angle of \(60^\circ\), with a relatively small error. The highly reflecting bottom may in some cases double and even triple the amount of solar radiation absorbed in the bottom half of the pond and therefore painting the bottom surface black may not be the best solution and more research is needed to understand better the physics of the solar radiation absorption in salt solutions.

The interest in using solar ponds on a large scale has required more advanced and precise simulation models to be carried out in order to predict the general thermal behaviour and efficiency of such solar collectors.
Basing their numerical simulation on Weinberger's solar pond model, A Akbarzadeh and G Ahmadi (19) developed a numerical model of a solar pond to be built in the southern part of Iran. By using the daily solar insolation, ambient temperature and evaporation rate data, their model could optimise the performance of the actual solar pond by evaluating the temperature rise of the bottom for different rates of energy removal. The simulation showed that solar ponds are reliable solar collectors for the production of hot water all year round with such climatic conditions (Iran) and that in the Summer heat can be extracted at a rate of 50% of the annual average solar radiation. However, in Winter the rate of heat removal should be reduced to 10% in order to maintain the temperature at the bottom above 80°C.

A similar numerical analysis of solar ponds based on the dynamic equations was developed by M N A Hawlader and B J Brinkworth (20) using meteorological data for a site in southern England. They reported that the pond temperatures are strongly dependent on the effective extinction coefficient for solar radiation and the heat losses from the pond bottom. Although the steady state regime of the thermal behaviour of the pond is reached within two to three years of operation, modest loads (around 10% of the annual average insolation) can be served in this climate at temperatures appropriate for practical uses.

J F Atkinson and D R F Harlaman (21) elaborated a computer model to predict transient salinity and temperature profiles in a salt gradient solar pond by using a wind-mixing layer in contrast to earlier solar pond models which have been generally considered with a constant upper convecting layer.

A one year simulation of a large hypothetical pond in Virginia (USA) showed that some measures should be taken to counteract the mixing due to wind stress and keep the upper mixing layer depth to an acceptable level in order to maintain the pond stability for better and more efficient operation. The description of the time-dependent
behaviour of the interfaces between the convecting and non-convecting regions of a salt-gradient solar pond, salinity and temperature profiles were determined by a numerical model developed by K.A. Meyer(22). The model utilises empirical correlations from oceanographic literature that describes the heat and salt fluxes in double diffusive systems. The model was found to be useful to determine pond performance under various operating conditions.

The examination of the pond stability criteria and the overall efficiency in terms of the upper convective layer, the optimization of the thickness of the non-convective zone in terms of net energy transmission to the lower convective zone, and the overall efficiency of the pond as a function of the loading rate for a particular depth of storage zone was carried out by Z.Panali, J.C.Batty and J.P Riley(23) by using a one-dimensional mathematical model. The model showed that the pond surface temperature is not seriously affected by pond depth when only the lower convective layer depth changes. For a three metre deep pond, a uniform heat extraction of 25% of the annual average insolation was found to be appropriate and that the elimination of wind effects reduces the evaporation rate by 18% and increases heat extraction by 1.8% which represents 0.36% of the annual average insolation.

The effects of friction and extraction on the stability of solar ponds were investigated by Y.S Cha, W.T Sha and S.L Soo(24). They reported that withdrawal of hot water in the bottom convective layer and wind-waves at the surface of the pond can lead to instability in the non-convective layer. However, longitudinal stratification can be maintained by using suitable distributors for extracting hot water from the pond and by returning cold brine to the pond. Such a method provides high availability of energy from the solar pond.

Besides the sophisticated numerical methods to study the thermal behaviour and the overall efficiency of solar ponds, the integration method of the heat diffusion equation has been successful in investigating both the thermal behaviour and performance of solar
ponds. The first attempt was carried out by C F Kooi\cite{25} who used the Hottel-Whittier-Bliss (HWB) formula to analyse a salt gradient solar pond as a steady-state flat-plate solar collector. He found that the common quantities that occur in the HWB formula, the effective absorptivity-transmissivity product, the loss factor, the heat extraction factor and the incidence angle modifier are related to the physical properties and dimensions of the pond. The efficiency was found to be a function of the thickness of the non-convective zone and therefore steady-state salt gradient solar ponds are less efficient than flat-plate collectors at high insolation, but they are more efficient at low insolation.

A further study by the same author was undertaken\cite{26} to investigate the behaviour of a saturated pond. He reported that the precipitation of salt due to temperature fluctuations can increase the reflectivity of the base which reduces the thickness of the non-convective layer and seriously degrades the performance of the pond. Moreover, he showed that the boundary between the non-convective zone and the lower convective zone will move to its maximum temperature position if the solubility of the used salt is a strongly increasing function of temperature such as in the case of magnesium chloride salt.

In order to understand the solar pond behaviour, Y F Wang and A Akbarzadeh\cite{27} carried out a parametric study based on a steady-state analysis of heat transfer. They presented a linear relation between the efficiency and the temperature difference between the pond bottom and the ambient divided by the average insolation. Moreover, the derived equation incorporates design parameters such as the thickness of the upper convective zone, the thickness of the gradient zone, the thermal properties of the ground beneath the pond and the depth of underground water table. They concluded that the existence and the depth of the upper convective layer has a negative effect on the output of solar ponds and because of ground losses, in the case of wet soil where the level of the underground water level is high, the pond should be thermally insulated.
When designing the heat extraction system of a salt gradient solar pond, two major approaches can be followed either by submerging the heat exchanger into the lower convective zone, or by withdrawing the hot brine from the lower convective zone and passing it through an external heat exchanger and then returning it to the bottom of the pond. In fact, the in-pond heat exchanger has been tried before and abandoned because of the excessive corrosion occurring during the pond operation due to electrochemical processes between the different design materials. However, the latter is still competitive with the external heat exchanger which is considerably costlier as a consequence of the use of pumps which are specially anti-corrosion designed (pumps made of fibre glass). The analysis of an internal heat exchanger was carried out by F Sabetta et al (28) by considering an in-pond heat exchanger made of a reinforced polyethylene pipe. Studying the cost of such heat exchangers they found that the present system is 40% less expensive than an external heat exchanger.

Later on, in order to improve the overall pond efficiency, a repaired in-pond heat exchanger was proposed by P J Unsworth, N Al-Saleh and V Phillips (29). The in-pond heat exchanger was designed in such a way that heat could be extracted from both the non-convective zone and the lower convective zone. They concluded that the overall efficiency is much higher than the common efficiency, while extracting heat from the lower convective zone only either by means of an internal heat exchanger or an external one.

In the present work, a more elaborate study has been carried out both experimentally or theoretically, by extracting heat from the upper convective zone and by giving a more realistic theoretical analysis of the pond with the proposed type of heat exchanger.
CHAPTER 3

SOLAR RADIATION AND HEAT TRANSFER EQUATIONS

In this chapter, general considerations of the solar radiation, its absorption, and the transmittance of the air-water interface are made. In addition, the general mass transfer equation and heat transfer equation and the heat exchange theory are also presented.

3.1 SOLAR RADIATION SPECTRUM

The sun is considered as the main origin of energy actually available on earth. This includes the direct heat, as well as wind energy, hydropower and fossil fuels. The interior of the sun is inaccessible for direct experimentation. Nevertheless, based on astronomic observations of the solar surface and theoretical considerations, it is estimated that the surface of the sun is at an effective temperature of 5700-5800°K(30), while in the central interior regions the temperature is variously estimated at 8 to 40 million Kelvins and the density is about 80 to 100 times that of water.

As far as the chemical composition of the sun is concerned, it is believed that the main elements are hydrogen and helium in the proportion of 96 to 99% of the total mass of the sun. These gases are under enormous pressure and the large gravitational pull of the sun keeps this mass together. The fusion reactions which have been suggested as applying the energy radiated by the sun, have been several. The one considered the most relevant is a process in which four protons combine to form helium (Beth's cycle) and the mass difference between the four protons and the helium nucleus is converted into energy. This energy finds its way to the surface and is eventually emitted into space primarily in the form of electromagnetic radiation.
If the sun is considered as a black body at a constant temperature of \( T = 5760^\circ K \), then the radiant flux emitted at the solar surface can be represented by Planck's distribution.

The characteristic wavelength of the solar spectrum can be determined from Wien's displacement law:

\[
\lambda_{\text{max}} = \frac{2898}{T}
\]

(3.1.1)

For \( T = 5760^\circ K \) gives \( \lambda_{\text{max}} = 502 \text{ nm} \), which corresponds to the green light.

The value of the sun's radiation flux received at the outer atmosphere is called the "solar constant". It is not actually a constant, but varies with the season. This constant is determined by assuming the sun as a black body at a temperature of \( 5760^\circ K \) and by using Stefan-Boltzmann's law:

\[
S = \int_0^\infty \frac{R_s^2}{x^2} B_\lambda \, d\lambda
\]

(3.1.2)

where

- \( S \) is the solar constant
- \( R_s \) is the solar radius = \( 6.95 \times 10^8 \text{ m} \)
- \( r \) is the mean earth-sun distance which equals \( 1.5 \times 10^{11} \text{ m} \)
- \( \lambda \) is the wavelength (m)

and \( B_\lambda \) is the Planck's function given by:

\[
B_\lambda(T, \lambda) = \frac{a}{\lambda^5 \left(e^{b/\lambda T} - 1\right)}
\]

(3.1.3)

where

- \( a = 3.74 \times 10^{-16} \text{ Wm}^{-2} \text{m}^4\)
- \( b = 1.43 \times 10^{-2} \text{ m}^2\text{K} \)
FIGURE 3.1.1: DIVERGENCE OF SUN'S RADIATION BEAM

FIGURE 3.1.2: COMPARISON BETWEEN THE SPECTRAL DISTRIBUTION OF THE FLUX EMITTED FROM THE SUN'S SURFACE AND THAT OF BLACK BODY AT $T = 5800^\circ$ K (31)
The evaluation of the integral in equation 3.1.2 yields the following value of the solar constant $S$:

$$S = 1352 \text{ Wm}^{-2}$$

Therefore, the spectral distribution of the flux as it reaches the top of the earth's atmosphere, is substantially the same as that emitted by the sun. However, each spectral component has been attenuated equally during transit.

The emitted flux leaving the solar surface is diffuse, however it becomes monodirectional by the time it arrives at the atmosphere. As a result of the large distance between the sun and the earth, the sun appears as a disc, therefore the radiation is not a perfectly parallel beam like, but diverges slightly. From Figure 3.1.1 the divergence is determined by the ratio of the solar diameter to the mean earth-sun distance and thus the angular divergence $\delta \theta$ is:

$$\delta \theta = \frac{2 R_s}{x} = 9.28 \times 10^{-3} \text{ rad} = 32'$$

(3.1.4)

This is slightly more than half a degree, hence, the solar flux making up the solar constant can be approximated to a monodirectional radiation.

In addition to the total energy (solar constant) in the solar spectrum, it is useful to know the spectral distribution of this radiation.

The fraction of the energy $f$ transmitted by those wavelengths between 0 and $\lambda$ is proportional to the area under the black-body curve between these limits, Figure 3.1.3. The fraction $f$ can be evaluated by:
By setting $x = \lambda T$ and using Stefan-Boltzmann's law:

$$\int_{0}^{\infty} B_\lambda(T, \lambda) \, d\lambda = \sigma T^4 \quad (3.1.6)$$

Equation 3.1.5 can be written as:

$$f(x) = \int_{0}^{x} \frac{a \, dx}{a x^5 (e^{b/x} - 1)} \quad (3.1.7)$$

where $a$ is a constant.

This integral can be tabulated for black body curves at any temperature\(^{(32)}\).

The energy fraction for the wavelengths between 0 and 0.4 μm (lower limit of the visible spectrum) for a black body at a temperature of 5760°K is 13% and the fraction of energy for wavelengths between 0.4 μm and 0.75 μm (upper limit of the visible spectrum) is 40%\(^{(32)}\), the remaining fraction carried by those wavelengths longer than 0.75 μm is $1 - 0.53 = 0.46$ which is 46%.

Therefore, if the sun is approximated as a black body at a temperature of 5760°K, approximately 13% of the energy is transmitted in the ultraviolet region. The visible fraction of the solar spectrum contains 40% of the energy and 46% is emitted in the infrared range of the spectrum.

As can be seen from the above, the solar radiation reaching the earth's atmosphere contains a harmful percentage of ultraviolet and
FIGURE 3.1.3: RELATIONSHIP BETWEEN THE TOTAL RADIANT FLUX AND THE WAVELENGTH FOR A BLACK BODY AT A TEMPERATURE T

FIGURE 3.1.4: PROPORTIONS OF ULTRAVIOLET, VISIBLE AND INFRARED RADIATIONS IN THE SOLAR SPECTRUM IF THE SUN IS ASSIMILATED AS BLACK BODY AT THE TEMPERATURE OF 5760°K
FIGURE 3.1.5: SOLAR SPECTRUM (33)

FIGURE 3.1.6: SOLAR SPECTRUM (32)
even a few gamma and X rays. Fortunately, while passing through the earth's atmosphere, these undesirable rays are largely absorbed along with some wavelengths of visible light. Figure 3.1.5 shows the solar spectrum at sea level compared with the solar spectrum outside the atmosphere\(^{(33)}\). This filtration of the solar radiation through the atmosphere is due to the absorption and scattering by dust, water vapour molecules, ozone, oxygen and carbon dioxide, Figure 3.1.6.

### 3.2 TRANSMITTANCE OF SOLAR RADIATION BY AIR-WATER INTERFACE

Using the electromagnetic theory of light, Fresnel has derived the following expression for the transmittance of air-water interface\(^{(32)}\):

\[
\tau(i) = 1 - 0.5 \frac{\sin^2(i-r) + \tan^2(i-r)}{\sin^2(i+r) + \tan^2(i+r)} \quad (3.2.1)
\]

where \(i\) is the angle of incidence

\(r\) is the angle of refraction.

These two angles, \(i\) and \(r\), are related to the index of refraction \(n\) of the water by Snell's law:

\[
\sin i = n \sin r \quad (3.2.2)
\]

with \(n = 1.33\).

In the case of radiation falling at normal incidence \((i=0)\), the above equations can be combined to yield the following expression of the transmittance:

\[
\tau(0) = 1 - \left(\frac{n-1}{n+1}\right)^2 \quad (3.2.3)
\]
Figure 3.2.1 shows the variation of both the transmittance and the reflectance $\rho$ (i.e. $\rho = 1 - r$) for different angles of incidence $i$.

As far as solar radiations are concerned, the angle of incidence $i$ of direct sunlight varies with the time of day and year according to the formula (27):

$$\cos i = \cos L \cos \left(\frac{t}{24}\right) + \sin L \sin \delta$$

(3.2.4)

where $L$ is the latitude of the location in radians.

$t$ is the hour of day, noon is zero, morning positive, and afternoon negative

$\delta$ is the angle of declination of the sun in radians given by the expression (34):

$$\delta = a_1 + a_2 \cos \left(\frac{2\pi (N-1)}{365}\right) + a_3 \sin \left(\frac{2\pi (N-1)}{365}\right)$$

$$+ a_4 \cos \left(\frac{4\pi (N-1)}{365}\right) + a_5 \sin \left(\frac{4\pi (N-1)}{365}\right)$$

$$+ a_6 \cos \left(\frac{6\pi (N-1)}{365}\right) + a_7 \sin \left(\frac{6\pi (N-1)}{365}\right)$$

(3.2.5)

where: $a_1 = 0.006918$

$a_2 = -0.39912$

$a_3 = 0.070257$

$a_4 = -0.006758$

$a_5 = 0.000907$

$a_6 = -0.002697$

$a_7 = 0.00148$

and $N$ is the day of the year, with $N = 1$ for the first day of January.
FIGURE 3.2.1: VARIATION OF THE TRANSMITTANCE WITH THE ANGLE OF INCIDENCE

![Graph of transmittance variation with angle of incidence.]

<table>
<thead>
<tr>
<th>Angle of Incidence</th>
<th>Angle of Refraction</th>
<th>Reflectance $(1 - r(i))$</th>
<th>$1/cosr$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>7.50</td>
<td>0.02</td>
<td>1.01</td>
</tr>
<tr>
<td>20</td>
<td>14.90</td>
<td>0.02</td>
<td>1.04</td>
</tr>
<tr>
<td>30</td>
<td>22.08</td>
<td>0.021</td>
<td>1.08</td>
</tr>
<tr>
<td>40</td>
<td>28.90</td>
<td>0.024</td>
<td>1.15</td>
</tr>
<tr>
<td>50</td>
<td>35.17</td>
<td>0.033</td>
<td>1.23</td>
</tr>
<tr>
<td>60</td>
<td>40.63</td>
<td>0.059</td>
<td>1.32</td>
</tr>
<tr>
<td>70</td>
<td>44.95</td>
<td>0.133</td>
<td>1.41</td>
</tr>
<tr>
<td>80</td>
<td>47.77</td>
<td>0.347</td>
<td>1.49</td>
</tr>
<tr>
<td>90</td>
<td>48.75</td>
<td>1.00</td>
<td>1.52</td>
</tr>
</tbody>
</table>

TABLE 3.2.1: ANGLE OF REFRACTION, REFLECTANCE AND $1/cosr$ FOR VARIOUS ANGLES OF INCIDENCE (11)
FIGURE 3.2.2(a)
Transmittance coefficient of the water surface of a pond situated in Loughborough, latitude 52.47N

b) Minimal transmittance coefficient
c) Maximal transmittance coefficient
FIGURE 3.2.3(a)

Transmittance coefficient of the water surface of a pond situated in London, latitude 51.32N

b) Minimal transmittance coefficient  

c) Maximal transmittance coefficient
FIGURE 3.2.4(a)
Transmittance coefficient of the water surface of a pond situated in Tamanrasset, latitude 22.47N

b) Minimal transmittance coefficient
The variation of the transmittance coefficient $T(i)$ of the water surface of a pond situated in Loughborough (UK latitude 52.47 N) during the year at noon-time is shown in Figure 3.2.2a. It increases during the first six months of the year from 0.777 on the 1 January to reach a maximum of 0.9790296 = 0.979 on the 22 June which corresponds to the summer solstice, then decreases thereafter to reach a minimum of 0.768 on the 22 December, corresponding to the winter solstice, and then increases again to reach 0.775 on the 31 December. An error of 0.11% occurs in the transmittance value due to errors in the evaluation of the sun's declination (equation 3.2.5). The latter error is less than 3 minutes if the declination is in degrees and less than 12 minutes if the two last terms are omitted in this equation.

For London (latitude 51.32 N), the transmittance increases from 0.800 on the 1 January to reach a maximum of about 0.979 on the 22 June and drops after that to a minimum on the 22 December, this minimum is 0.793 and increases again to 0.779 on the 31 December.

However, for Tamanrassat (Algeria latitude 22.47 N), the transmittance varies from 0.972 on the 1 January to a maximum of 0.980 on the summer solstice, and decreases to a minimum slightly less than 0.972 in the winter solstice and increases again to reach 0.972 on the 31 December.

These results show how the transmittance varies from one location to another; it is higher at lower latitudes than at higher latitudes. However, for practical calculations, the transmittance coefficient is taken equal to its value evaluated at the summer solstice since more radiation is received during the summer when the transmittance is higher than in winter, such an approximation can only overestimate the performance of the pond.
3.3 SOLAR RADIATION ABSORPTION IN PONDS

The transmitted portion of the solar radiation by the air-water interface of the pond is attenuated throughout the depth. The absorption of the solar radiation as it passes through the pond water cannot be described by a single exponential function since different wavelengths have widely differing absorption coefficients (35) (see Table 3.3.1):

<table>
<thead>
<tr>
<th>Thickness of Water</th>
<th>0</th>
<th>1 cm</th>
<th>10 cm</th>
<th>1 m</th>
<th>10 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>(µm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2-0.6</td>
<td>76.3</td>
<td>76.4</td>
<td>76.4</td>
<td>77.1</td>
<td>82.8</td>
</tr>
<tr>
<td>0.6-0.9</td>
<td>64.0</td>
<td>64.7</td>
<td>69.5</td>
<td>87.1</td>
<td>99.1</td>
</tr>
<tr>
<td>0.9-1.2</td>
<td>82.1</td>
<td>87.7</td>
<td>99.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Over 1.2</td>
<td>91.3</td>
<td>98.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

TABLE 3.3.1: THE ABSORPTION DISTRIBUTION IN PERCENT OF THE SOLAR SPECTRUM AFTER PASSING THROUGH WATER LAYER
(adapted from (35))

As shown in Table 3.3.1, the short wavelengths of the solar radiation spectrum are absorbed less and penetrate deeper up to several metres, whereas the near infrared wavelengths are rapidly absorbed within the first centimetres and even millimetres, which is the reason why solar ponds can function as heat traps. On this basis, the performance of solar ponds largely depends on the amount of radiation that reaches the bottom of the pond. It is obvious that a correct evaluation of the absorption of solar radiation throughout its path in the pond water is important for studying the thermal behaviour of such a solar collector.
In order to fit the observed data given by Defant of the attenuation function $h(x)$, many empirical functions have been suggested\(^{(36,37,38,11)}\). In Figure 3.3.3 a comparative study of these expressions with the experimental function is made\(^{(38)}\).

Those semi-empirical expressions of the attenuation function $h(x)$ are as follows:

1. $h(x) = 0.36 - 0.08 \ln(x)\(^{(36)}\)

2. $h(x) = 0.276 \exp(-2.81 \times)x\(^{(37)}\)

3. $h(x) = \sum_{m=1}^{m=4} a_m e^{-b_mx} \quad (11)\(^{(11)}\)

4. $h(x) = \sum_{m=1}^{m=5} a_m e^{-b_mx} \quad (38)\(^{(38)}\)

where $x$ is the depth in metres from the pond surface downwards, $a_m$ and $b_m$ are constant coefficients given by:

- $a_1 = 0.237$
- $a_2 = 0.193$
- $a_3 = 0.167$
- $a_4 = 0.179$
- $a_5 = 0.224$
- $b_1 = 0.032 \quad m^{-1}$
- $b_2 = 0.450 \quad m^{-1}$
- $b_3 = 3.000 \quad m^{-1}$
- $b_4 = 35.00 \quad m^{-1}$
- $b_5 = 255.00 \quad m^{-1}$

Figure 3.3.4 shows the variation of the amount of heat generated $\tau(i) \cos(i) H_s \frac{dh(x)}{dx}$ through the depth of the pond for three different angles of incidence $i$. Here $\tau(i)$ is the transmittance coefficient of the air-water interface, $H_s$ is the solar insolation and $h(x)$ is the attenuation function expressed by\(^{(11)}\):

$$h(x) = \sum_{m=1}^{m=5} a_m e^{-b_mx} \quad (3.3.2)$$

where $x$ is the depth from the pond surface downwards.

Here the $\cos(i)$ takes into account the fact that the solar radiation per unit area falling on the pond surface is reduced by the factor
FIGURE 3.3.3: COMPARATIVE STUDY OF THE EMPIRICAL EXPRESSION OF $h(x)$ WITH THE EXPERIMENTAL DATA (38)
cosi, where $i$ is the incidence angle which is the angle between the sun's direction and the normal to the pond surface. Moreover, as the solar radiation penetrates to the pond water, the light has to travel a factor $1/\cos(r)$ further to reach a given depth $x$ of the pond ($r$ is the angle of refraction). Therefore, to incorporate this variation, the depth $x$ in the function $h(x)$ should be replaced by $x/\cos(r)$ (see the figure below).

\[ G(x, i) = \frac{r(i) \cos i}{\cos r} \sum_{m=1}^{m=4} a_m b_m e^{-b_m x/\cos r} \]  

Here $H_s$ has been taken as unity ($H_s = 1 \text{ Wm}^{-2}$). It can be seen from Figure 3.3.6 that the amount of heat generated decreases along the depth of the pond as a result of the decrease in the amount of solar radiation reaching a depth $x$. This is due to the
FIGURE 3.3.6: AMOUNT OF HEAT GENERATED PER UNIT VOLUME IN THE POND AT A DEPTH X FOR THREE ANGLES OF INCIDENCE $i$

a) $x = 0, 1.2$ m; b) $x = 0, 0.5$ m; c) $x = 0.5, 0.9$ m; d) $x = 0.9, 1.2$ m
fact that the light is strongly absorbed within the first few centimetres from the pond surface, especially as far as the red and infrared radiations are concerned. For example, the amount of heat generated at a depth 0.50m is 0.18206 W/m\(^3\) \((H_g = 1W/m^2)\) and is 0.08452 W/m\(^3\) at a depth of 1 metre for the same insolation \(H_g\) and the same angle of incidence, in this case is ten degrees, this represents a decrease of about 54%.

Moreover because of the movement of the sun, the direct solar radiations do not keep a constant angle of incidence relative to the pond water surface. As can be seen the incidence angle of the solar radiations has a marked effect on the amount of heat generated in the pond. As the angle of incidence increases, the amount of solar radiation per unit area falling on the pond surface is reduced by a factor \(\cos \theta\). Furthermore, the air-water interface transmittance coefficient decreases as the angle of incidence increases. Besides these two reducing effects, a third effect comes to reduce the amount of heat generated in the pond. This is the path length of the light.

As the angle of incidence increases, the path length increases by a factor \(1/\cos \theta\). For instance, if the solar insolation falling on the pond surface is 1 W/m\(^2\) and the angle of incidence varies from ten degrees to seventy degrees, the solar insolation drops from 0.98 W/m\(^2\) at 10 degrees to 0.34 W/m\(^2\). Secondly the transmittance coefficient decreases from 0.98 to 0.867 and thirdly, the factor \(1/\cos \theta\) increases from 1.01 to 1.41. These three effects combined cause the amount of heat generated at a depth of 0.5m to be 0.18206 W/m\(^2\) at an incidence angle of 10 degrees and drops to 0.05482 at the same depth of 0.5m. This is a reduction of about 70% which is significant.

Therefore the first effect indicates the reason why deep ponds take a long time to heat up, whereas the second indicates why ponds situated at high latitudes also take time to heat up and operate.
3.4 EFFECT OF DIFFUSION ON CONCENTRATION PROFILES

In order to maintain stability in the pond, it is necessary to create a density gradient by dissolving salt to counterbalance the unstable density gradient due to the thermal expansion effect. This can be achieved by producing a linear density gradient in the pond with high salt concentration at the bottom and a low concentration at the top.

However, as a result of the concentration gradient of the solution in the pond, molecular diffusion of salt will occur, consequently erasing the density gradient. Hence, it is necessary to find out the evolution of the initial density distribution to the final uniform concentration state when the density through the pond is uniform.

In the one-dimensional problem with constant diffusion coefficient $D$, the governing differential equation is (40):

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

(3.4.1)

where $C$ is the concentration

$t$ is the time

$x$ is the depth

and $D$ is the diffusion coefficient of the salt.

This equation can be solved by the method of separation of variables for given boundary conditions and initial condition.

3.5 HEAT TRANSFER AND HEAT EXCHANGER EQUATIONS

The thermal behaviour of a salt gradient solar pond can be studied by using the general one-dimensional heat conduction equation (41):

$$\rho C_p f \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( K \frac{\partial T}{\partial x} \right) + H(x) - Q$$

(3.5.1)
where \( \rho \) is the density
\[ C_{pf} \] is the specific heat
\[ K \] the thermal conductivity
\[ T \] is the temperature
\[ t \] is the time
\[ x \] is the coordinate

\( H(x) \) is the generated heat rate per unit of volume expressed by:

\[
H(x) = - \tau H_S \frac{dh(x)}{dx}
\]  \hspace{1cm} (3.5.2)

where \( \tau \) is the transmittance
\( H_S \) the solar insolation
and \( h(x) \) is the attenuation function
and finally \( Q \) is the rate of the extracted heat.

If a heat exchanger is used to extract heat from the pond, the transfer rate of heat from the pond into the heat exchanger fluid is (42):

\[
dQ = m C_{pf} \Delta T_f
\]  \hspace{1cm} (3.5.3)

where \( m \) is the flow rate
\[ C_{pf} \] is the specific heat of the fluid
and \( \Delta T_f \) is an increase in the fluid temperature

Over a length \( dS \) of tubing of the heat exchanger, the transfer rate \( dQ \) can also be expressed as:

\[
dQ = \pi D_0 U (T - T_f) \, dS
\]  \hspace{1cm} (3.5.4)
where $T$ and $T_f$ are the pond and cooling fluid temperature, $D_o$ is the outer diameter of the tubing and $U$ is the overall heat transfer coefficient for unit area of the heat exchanger given by:

$$\frac{1}{U} = \frac{D_o}{D_i h_i} + \frac{D_o}{2K_t} \ln \left( \frac{D_o}{D_i} \right) + \frac{1}{h_o}$$

(3.5.5)

where $D_i$, $K_t$, $h_i$ and $h_o$ are the inner diameter, the thermal conductivity, the inner surface heat transfer coefficient and the outer surface heat transfer coefficient of the heat exchanger piping respectively. The inner and outer surface heat transfer coefficients are given by (43):

$$h_i = 0.027 \frac{\mu_f^{0.8}}{D_i^{1.8}} (\frac{C_p f}{\rho})^{0.4} K^{0.6}$$

(3.5.5)

$$h_o = 0.47 \frac{K^{0.75}}{D_o^{0.25}} \rho^{0.5} (g \beta' C_p f)^{0.25}$$

(3.5.6)

where $\mu_f$, $g$, $\beta'$ are the dynamic viscosity, gravitational constant and thermal expansion coefficient of the fluid (water) respectively.

* $\beta'$ here substitutes the product thermal expansion coefficient $\beta$ by the temperature difference $\Delta T$. 

* Note: The $\beta'$ symbol represents the product of thermal expansion coefficient $\beta$ and the temperature difference $\Delta T$. This is a common practice in heat exchanger design to account for the temperature difference effect on the thermal expansion. 

* $S'$ stands for the product thermal expansion coefficient $S$ adjusting for the temperature difference $\Delta T$. This is often used to correct the thermal expansion effect in practical applications where temperature differences are not negligible. The correction factor $S'$ is typically calculated as:

$$S' = S \left(1 + \frac{\Delta T}{T_0} \right)$$

where $T_0$ is the reference temperature. This expression assumes a linear relationship between the temperature and the product thermal expansion coefficient.
In this chapter, the theory of filling by redistribution of the initial salt concentration profile in the pond is described, and the linear profile determined.

The time wise concentration gradient variation as a result of the diffusion of salt is found by solving the differential partial equation of mass transfer for given boundary conditions.

Finally, a theory based on the steady state analysis theory of heat transfer is developed. The pond with the heat exchanger system is described by a system of two differential equations. Their solutions can predict the thermal behaviour of both the pond solution and the fluid in the heat exchanger and their variation with some operating and design parameters.

4.1 ESTABLISHMENT OF THE INITIAL SALT CONCENTRATION PROFILE IN THE SALT GRADIENT SOLAR POND

Among many ways of establishing a salt gradient in the pond, there is one simple method called filling by redistribution. The procedure consists of partially filling the pond with high concentration brine, followed by the addition of fresh water through a diffuser which is immersed in the upper part of the existing solution.

The injected water does not mix with the solution below which is unchanged, but mixes with the solution above, progressively diluting it. In order to evaluate the concentration profile by this method of redistribution consider a one-dimensional pond of constant area A and a total depth L divided into a lower convective zone of thickness
(L-1_2) and a concentration C_0 and a non-convecting zone of thickness (l_2-1_1) and concentration C(x), Figure 4.1.

Supposing now that initially the pond is filled to a depth \( L = \frac{l_2-1_1}{2} \) with a solution of maximum salt concentration C_0 by dissolving the required amount of salt, but a volume \( \frac{l_2-1_1}{2} A \) of fresh water is still missing. If the diffuser D is now immersed in the solution just at the interface LCZ-NCZ and raised at a constant velocity, the fresh water will mix with the solution above and progressively dilute it. Therefore the diffuser should be raised at a velocity proportional to the surface motion and reaches the surface at \( x = 1_1 \). If at any time \( t \) the position of the diffuser is \( x_d(t) \) and the position of the surface is \( x_s(t) \), the proportionality between \( x_d(t) \) and \( x_s(t) \) can be stated as follows(16):

\[
x_d(t) = \alpha x_s(t) + \beta \quad (4.1.1)
\]

where \( \alpha \) and \( \beta \) are unknown constants that can be found by applying the following boundary conditions:

\[
t = 0 \quad x_d(0) + l_2, \quad x_s(0) = \frac{l_2-1_1}{2} \quad (4.1.2)
\]

\[
t = t_f \quad x_d(t_f) = x_s(t_f) = 1_1
\]

\[
\alpha = \frac{2(l_2-1_1)}{l_2-3} 1_1 \quad \beta = -\frac{l_1(l_2+1_1)}{l_2-3} 1_1
\]

Therefore:

\[
x_d(t) = \frac{1}{l_2-3} 1_1 \left(2(l_2-1_1) x_s(t) - l_1(l_2+1_1)\right) \quad (4.1.3)
\]

and the velocities of both the surface and the diffuser are related by the following equation:
FIGURE 4.1.1: GEOMETRY DESCRIBING THE FILLING OF A POND BY THE METHOD OF FILLING BY REDISTRIBUTION
\[
\frac{dx_d(t)}{dt} = \frac{2(l_2 - l_1)dx_s(t)}{l_2 - 3l_1} \quad (4.1.4)
\]

The concentration in the fully-mixed layer above the diffuser is:

\[
C(t) = \frac{M(t)}{V(t)} \quad (4.1.5)
\]

where \(C(t)\) is the concentration, \(M(t)\) the mass of salt in the layer above the diffuser, and \(V(t)\) is the volume above the diffuser given by:

\[
V(t) = A(x_s(t) - x_d(t))
\]

\[
= \frac{A(l_2 + l_1)}{2(l_2 - l_1)} (l_1 - x_d(t)) \quad (4.1.6)
\]

since there is no salt transport into the volume \(V(t)\). However its concentration is reduced to \(C(t)\) as the diffuser is raised at the rate \(dx_d/dt\). Therefore one can write:

\[
\frac{dM(t)}{dt} = -C(t)A \frac{dx_d(t)}{dt} \quad (4.1.7)
\]

Differentiating equation (4.1.5) with respect to the time \(t\), the following equation is obtained:

\[
\frac{dC(t)}{dt} = \frac{1}{V(t)} \frac{dM(t)}{dt} - C(t) \frac{dV(t)}{dt} \quad (4.1.8)
\]
Using equations 4.1.6 and 4.1.7, the following differential equation is obtained:

\[ \frac{dC(t)}{dt} = -\frac{(l_2 - 3l_1)}{(l_2 + l_1)} C(t) \frac{dx_d(t)/dt}{l_1 - x_d(t)} \] (4.1.9)

Taking out the term \( dt \) and integrating, the concentration is then:

\[ C = \frac{(l_2 - 3l_1)}{(l_2 + l_1)} I (l_1 - x_d) \] (4.1.10)

where \( I \) is a constant of integration, by applying the following boundary condition:

\[ C = C_0 \text{ for } x_d = l_2 \]

Then \( I = C_0 \frac{(l_2 + l_1)}{(l_2 - 3l_1)(l_1 - l_2)} \) (4.1.10')

and therefore

\[ C(x) = C_0 \left( \frac{x-1}{l_2-1} \right) \] (4.1.11)

where \( C(x) \) is the final profile built within the non-convective zone.

4.2 TIME-VARIATION OF THE SALT GRADIENT IN THE SOLAR POND

The solution of the equation (3.4.1) can be found by writing \( C(x,t) = X(x) Y(t) \) and equation (3.4.1) yields the following ordinary differential equations (44):
\[
\frac{d^2X}{dx^2} + \lambda^2 X = 0
\]  \hspace{1cm} (4.2.1)

\[
\frac{dy}{dt} + D \lambda^2 y = 0
\]

Therefore \( C(x,t) \) is given by the expression:

\[ C(x,t) = e^{-\lambda^2 D t} \left( a \sin \lambda x + b \cos \lambda x \right) \]  \hspace{1cm} (4.2.2)

if \( \lambda \) is different from zero and \( a, b \) are arbitrary constants.

The constants \( \lambda, a \) and \( b \) are determined from the boundary conditions which in this case (salt gradient solar pond) are:

\[
\frac{\partial C(x,t)}{\partial x} = 0 \quad \text{for} \quad x = 0
\]  \hspace{1cm} (4.2.3)

\[
\frac{\partial C(x,t)}{\partial x} = 0 \quad \text{for} \quad x = L
\]

These boundary conditions are reasonable because at the surface and the bottom of the pond there is no salt transfer.

These boundary conditions require that "\( a \)" is equal to zero and \( \lambda \) equal to \( n \pi / L \) for \( n = 1, 2, 3, \ldots \). Furthermore, if \( \lambda = 0 \), the differential equation (3.4.1) has the solution:

\[ C(x,t) = a' + b'x \]  \hspace{1cm} (4.2.4)
in this case the boundary conditions (4.2.3) require that $b'$ is nil. Hence a fundamental solution is

$$C = \frac{a_0}{2}$$

and

$$C_n(x, t) = a_n e^{-n^2 \pi^2 \frac{Dt}{L^2}} \cos \frac{n\pi}{L} x$$

The general solution of the equation (3.4.1) can be written as a linear combination of the fundamental solution:

$$C(x, t) = \sum_{n=0}^{n=\infty} C_n(x, t)$$

which is:

$$C(x, t) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n e^{-n^2 \pi^2 \frac{Dt}{L^2}} \cos \frac{n\pi}{L} x$$ \hspace{1cm} (4.2.5)

The different coefficients of $a_n$ are determined by the requirement that:

$$C(x, t) = \frac{C(x, 0)}{2} \sum_{n=1}^{n=\infty} a_n \cos \frac{n\pi x}{L}$$

Using Fourier cosine series for period $2L$, one will have:

$$a_n = \frac{2}{L} \int_{0}^{L} C(x, 0) \cos \frac{n\pi x}{L} dx \hspace{1cm} n = 1, 2, \ldots$$ \hspace{1cm} (4.2.6)

and

$$a_0 = \frac{2}{L} \int_{0}^{L} C(x, 0) dx$$

where $C(x, 0)$ is the initial profile of salinity at time $t = 0$. 
In the case of the salt gradient solar pond, the initial profile of salt gradient is given by equation (4.1.1):

\[
C(x, o) = \begin{cases} 
0 & \text{if } 0 \leq x \leq l_1 \\
C_0 \left( \frac{x - l_1}{l_2 - l_1} \right) & \text{if } l_1 \leq x \leq l_2 \\
C_0 & \text{if } l_2 \leq x \leq L 
\end{cases} \tag{4.2.7}
\]

The coefficients \(a_n (n = 1, 2, \ldots)\) are given by using equations (3.3.5) and (3.3.6):

\[
a_n = \frac{2}{L} \left\{ \int_0^{l_1} 0 \, x \cos \frac{n\pi x}{L} \, dx + \int_1^{l_2} C_0 \left( \frac{x - l_1}{l_2 - l_1} \right) \cos \frac{n\pi x}{L} \, dx 
+ \int_{l_2}^L C_0 \cos \frac{n\pi x}{L} \, dx \right\}, \quad n = 1, 2, \ldots
\]

Thus:

\[
a_n = \frac{2C_0 L}{n^2 \pi^2 (l_2 - l_1)} \left( \cos \frac{n\pi l_2}{L} - \cos \frac{n\pi l_1}{L} \right) \tag{4.2.8}
\]

and

\[
a_0 = \frac{2}{L} \left\{ \int_0^{l_1} 0 \, dx + \int_1^{l_2} C_0 \left( \frac{x - l_1}{l_2 - l_1} \right) \, dx + \int_{l_2}^L C_0 \, dx \right\}
- \frac{L}{L} \quad (4.2.9)
\]

Finally injecting equations (4.2.8) and (4.2.9) in equation (4.2.5) and rearranging the terms, the concentration profile \(C(x,t)\) at any time is then:
As can be seen, the concentration expression consists of two terms, the steady state expression \( C_0 \left( 1 - \frac{l_2 + l_1}{2L} \right) \) which is independent of the time and represents an average along the depth plus the transient term \( \sum_{n=1}^{n=\infty} \frac{\exp(-\frac{n^2 \pi^2 \Delta t}{L^2})}{n^2} \) which vanishes with time. Table 4.2.1 and Figure 4.2.3 show how the initial concentration gradient of salt through the depth of the pond vanishes with time. This erosion of the concentration profile is very slow. It takes about twenty-one years for the salt concentration gradient to become uniform through the depth of a pond one metre in depth. This is due to the low value of the diffusion coefficient of salt \( (D = 1.5 \times 10^{-5} \text{ cm}^2/\text{s}) \).

The migration of salt from the lower layers of the pond where the concentration is high, towards the surface of the pond where the layers are mainly fresh water, is responsible for the destruction of the initial concentration gradient. The slow process of the diffusion of salt makes solar ponds very stable. Yet, in reality, in addition to the effect of the salt diffusion on the concentration gradient, there is another effect called the Soret effect. It states that in reality especially in the solar ponds, the destruction of the initial concentration profile is accentuated by the diffusion of salt under the effect of the gradient of temperature* as experienced in reality because of the absorption of the solar radiation. However, the Soret effect contributes to about 4% of the diffusion of salt if the temperature between the bottom and the top of the pond is approximately 40°C. Whereas, it is 85% if the temperature difference between the bottom and the top of the pond is doubled.

\[
C(x,t) = C_0 \left(1 - \frac{l_2 + l_1}{2L}\right) - \frac{4 C_0 L}{\pi^2 (l_2 - l_1)} \sum_{n=1}^{n=\infty} \exp\left(-\frac{n^2 \pi^2 \Delta t}{L^2}\right) \frac{\sin \left(\frac{n\pi (l_2 - l_1)}{2L}\right)}{n^2} \sin \left(\frac{n\pi l_1}{2L}\right) \cos \left(\frac{n\pi L}{L} x\right) (4.2.10)
\]

* A general form of the diffusion equation \( J_x = -D \frac{dC}{dx} \) is replaced by

\( J_x = -D \frac{dC}{dx} - D S_T C \frac{dT}{dx} \), where \( S_T \) is the Soret coefficient.
FIGURE 4.2.3: Variation of the relative concentration $C/C_0$ along the relative depth $x/L$ of the pond for various relative times $T = Dt/L$. $t_1 = 10$, $t_2 = 80$, $t = 100$ cm.

<table>
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<th>$T$</th>
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<th>0.001</th>
<th>0.01</th>
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<th>1</th>
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<tr>
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<td>$C/C_0$</td>
<td>$C/C_0$</td>
<td>$C/C_0$</td>
<td>$C/C_0$</td>
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<td>5.70E-2</td>
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</tr>
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<td>0.28</td>
<td>0.29</td>
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</tr>
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<td>0.55</td>
</tr>
<tr>
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<td>1</td>
<td>0.99</td>
<td>0.98</td>
<td>0.73</td>
<td>0.55</td>
</tr>
</tbody>
</table>

TABLE 4.2.1: Variation of the relative concentration $C/C_0$ along the pond depth for various relative times $T = Dt/L^2$ in the case of $t_1 = 10$, $t_2 = 80$ and $L = 100$ cm.
4.3 ANALYTICAL MODEL OF THE SALT GRADIENT SOLAR POND WITH THE INTERNAL HEAT EXCHANGER

There are two methods which have been employed for heat extraction from solar salt ponds. The first is based on the selective withdrawal of the bottom layer of brine from the pond. The second method makes use of a second fluid in a heat exchanger piping placed in the bottom region of the pond. The heat exchanger fluid, usually water, while flowing through these pipes, is heated and transfers the heat to an external system for use.

The latter method has been extended by introducing additional heat exchanger piping to the other layers of the pond in order to obtain a higher efficiency, unlike the conventional method, by extracting heat from the whole body of the pond through which a temperature gradient exists. This method of heat extraction improves the thermodynamic efficiency of the pond because the heat flow in the non-convective layer which would normally be lost at the surface, is intercepted and extracted by means of the fluid flowing in the heat exchanger.

4.3.1 The Model and Approximations

The thermal behaviour investigation of the salt gradient solar pond with the internal heat exchanger is carried out. A one-dimensional three layered pond of area A and total depth L is considered in which an helical coil heat exchanger of height L twisted into N coils of diameter 2R made of piping of $D_0$ outer diameter and $D_1$ inner diameter is inserted, see Figure 4.3.1.1.

Instead of using the general equation (3.5.1) applicable to the pond, here a heat balance for an element of volume of the pond has been chosen from which to develop the following relationship. For an element of volume $dV = Adx$, where $dx$ is an element of length of the pond, see Figure 4.3.1.2.
FIGURE 4.3.1.1: THE SOLAR POND MODEL WITH THE HELICOIL HEAT EXCHANGER INSTALLED
FIGURE 4.3.1.2: Element volume $dV = Adx$ in which a heat balance is evaluated

FIGURE 4.3.1.3: Geometry of an helix of diameter $2R$
Defining \( \partial Q_1, \partial Q_2, \partial Q_3 \) and \( \partial Q_4 \) by:

\[
\partial Q_1 = KA \frac{dT}{dx} \bigg|_x, \quad \partial Q_2 = \tau H_S A h(x)
\]

\[
\partial Q_3 = KA \frac{dT}{dx} \bigg|_{x+dx}, \quad \partial Q_4 = \tau H_S A h(x + dx)
\]

The rate of heat transfer, \( \partial Q \), which represents the difference between the entering flow of heat \( \partial Q_2 \) and \( \partial Q_3 \) and the leaving flow of heat \( \partial Q_1 + \partial Q_4 \), is carried out by the fluid in the heat exchanger for an element length \( ds \) of the piping, therefore:

\[
\partial Q = \tau H_S A h(x) + KA \frac{dT}{dx} \bigg|_{x+dx} - \tau H_S A h(x + dx) - KA \frac{dT}{dx} \bigg|_x
\]

(4.3.1.1)

Using equation (2.6.4) for \( \partial Q \) and Taylor's theorem to the first order applied to \( \frac{dT}{dx} \bigg|_{x+dx} \) and \( h(x + dx) \) one gets:

\[
\partial Q = (T - T_f) U \pi D_0 \, ds
\]

\[
\frac{dT}{dx} \bigg|_{x+dx} = \frac{dT}{dx} \bigg|_x + \frac{d^2T}{dx^2} \, dx
\]

and

\[
h(x + dx) = h(x) + \frac{dh}{dx} \, dx
\]

Then equation (4.3.1.1) is reduced to:

\[
U \pi D_0 (T - T_f) \, ds = KA \frac{dT}{dx} \bigg|_x - A \tau H_S \frac{dh}{dx}
\]

(4.3.1.2)
As it can be seen, this equation contains two terms $dS$ and $dx$ that can be expressed by another parameter which makes the equation homogeneous, that parameter is the polar angle $\theta$. The heat exchanger is a helix of $N$ turns of diameter $2R$ and height $L$ (Figure 4.3.1.3), then any point $M$ of the helix can be expressed by (47):

$$\vec{OM} = \vec{zi} + \vec{yj} + \vec{xk}$$

(4.3.1.3)

where:

$$z = R \cos \theta, \quad y = R \sin \theta, \quad x = \frac{P\theta}{2\pi}$$

(4.3.1.4)

and

$$dx = \frac{P}{2\pi} \, d\theta$$

where $p$ is the pitch, which is the distance between two successive coils defined by:

$$p = \frac{L}{N}$$

So, any element of length $dS$ of the helix is:

$$dS = (dx^2 + dy^2 + dz^2)^{\frac{1}{2}}$$

which becomes by substituting $dx$, $dy$ and $dz$ from (4.3.1.4):

$$dS = \left(R^2 + \frac{L^2}{4\pi^2N^2}\right)^{\frac{1}{2}} \, d\theta$$

(4.3.1.5)

Instead of $x$, $\theta$ can be introduced in equation (4.3.1.2) by differentiating $T$ and $h$ with respect to $\theta$ to have:
Taking the expression of \( dx \) from equation (4.3.1.4) and after injecting those terms in equation (4.3.1.2) and dividing through by \( d\theta \), the following differential equation is obtained:

\[
\frac{d^2 T}{d\theta^2} = \left( \frac{2N \pi}{L} \right)^2 \frac{d^2 T}{d\theta^2} \quad \text{and} \quad \frac{d\theta}{dx} = \frac{2N \pi}{L} \frac{d\theta}{dx}
\]

Taking the expression of \( dx \) from equation (4.3.1.4) and after injecting those terms in equation (4.3.1.2) and dividing through by \( d\theta \), the following differential equation is obtained:

\[
\frac{d^2 T}{d\theta^2} - \frac{U \pi D}{2NKA} \left( R^2 + \frac{L}{4 \pi^2 N^2} \right)^{\frac{1}{2}} (T-T_f) = - \frac{\tau H S L^2}{4 \pi^2 N^2} \sum_{m=1}^{m=4} a_m h_m e^{\frac{b_m L}{2\pi N} \theta}
\]

The term \( h(\theta) \) was substituted by \( \sum_{m=1}^{m=4} a_m e^{\frac{b_m L}{2\pi N} \theta} \) taken from (11) after replacing \( x \) by \( \frac{L \theta}{2\pi N} \).

The second differential equation is obtained by combining equations (2.6.3), (2.6.4) and (4.3.2.5) and is written as:

\[
\frac{dT_f}{d\theta} - \frac{U \pi D}{m C_{pf}} \left( R^2 + \frac{L^2}{4 \pi^2 N^2} \right)^{\frac{1}{2}} (T-T_f) = 0
\]

Consequently, by solving those two differential equations simultaneously, the thermal behaviour of the pond with the heat exchanger can be studied because both the pond temperature \( T \) and the fluid temperature \( T_f \) are expressed by the operating parameters such as the flow rate, the insolation and the design parameters such as the length of the heat exchanger and the total depth of the pond.

### 4.3.2 Resolution of the Equations

Taking \( T_f \) from equation (4.3.1.6) and injecting it in equation (4.3.1.7), the following third order homogeneous equation is obtained:
A general solution to this equation is written as:

\[ T(\theta) = C_1 + C_2 \exp (2R_1 N \frac{\pi}{L}) + C_3 \exp (2R_2 N \frac{\pi}{L}) \]  

(4.3.2.2)

Here \( C_1, C_2 \) and \( C_3 \) are constants of integration that are found from the boundary conditions. \( R_1 \) and \( R_2 \) are the roots of the characteristic equation of the differential equation (4.3.2.1) and expressed by:

\[ R_1 = \frac{1}{2} \left\{ - \frac{U_D}{m C_{pf}} \left( R^2 + \frac{L^2}{4\pi^2N^2} \right)^{\frac{1}{2}} + \sqrt{\Delta} \right\} \]  

(4.3.2.3)

\[ R_2 = \frac{1}{2} \left\{ - \frac{U_D}{m C_{pf}} \left( R^2 + \frac{L^2}{4\pi^2N^2} \right)^{\frac{1}{2}} - \sqrt{\Delta} \right\} \]

Here

\[ \Delta = \left( \frac{U_D}{m C_{pf}} \right)^2 \left( R^2 + \frac{L^2}{4\pi^2N^2} \right) + \frac{2U_D}{NK} \left( R^2 + \frac{L^2}{4\pi^2N^2} \right) \]  

A particular solution of equation (4.3.2.1) is obtained by considering a solution of the form

\[ T_0 = \sum_{m=1}^{m=4} \frac{b_m}{m} e^{-\frac{2\pi n}{L}} \]
which after being inserted again in (4.3.2.1) and after identification of the terms, the coefficients \( f_m \) are found to be:

\[
 f_m = \frac{\tau H S \ a_m}{2\pi N} \left( \frac{b_m L}{2\pi N} - \frac{U_m D}{\pi C P_f} (R^2 + \frac{L^2}{4\pi^2 N^2})^{\frac{1}{2}} \right) \\
+ \frac{U_m D}{A} (R^2 + \frac{L^2}{4\pi^2 N^2})^{\frac{1}{2}} + \frac{U_m K D}{\pi C} (R^2 + \frac{L^2}{4\pi^2 N^2})^{\frac{1}{2}} b_m - \frac{X L}{2\pi N} \ b_m^2 
\]

(4.3.2.4)

Therefore the respective forms of \( T \) and \( T_f \) are given as follows:

\[
 T(x) = C_1 + C_2 \ e^{(2RN\pi \ X \ L)} + C_3 \ e^{(2R2 \ N\pi \ X \ L)} + \sum_{m=1}^{m=4} f_m \ e^{-b_m L} 
\]

(4.3.2.5)

\[
 T_f(x) = C_1 + C_2 (1 - \frac{2N K A}{ULD_0} (R^2 + \frac{L^2}{4\pi^2 N^2})^{\frac{1}{2}} R_1^2) \ \exp(2RN\pi \ X \ L) \\
+ C_3 (1 - \frac{2N K A}{ULD_0} (R^2 + \frac{L^2}{4\pi^2 N^2})^{\frac{1}{2}} R_2^2) \ \exp(2R2 \ N\pi \ X \ L) \\
+ \sum_{m=1}^{m=4} f_m - \frac{K A L}{2\pi^2 N U D_0} \ \frac{b_m^2}{2\pi^2 N U D_0} \ \frac{\tau H S \ A_m b_m}{2\pi^2 N U D_0} \ \exp(R^2 + \frac{L^2}{4\pi^2 N^2})^{\frac{1}{2}} e^{-b_m L} 
\]

(4.3.2.6)

The corresponding boundary conditions for \( T(x) \) and \( T_f(x) \) are:

\[
 i) \ x = 0 \rightarrow \begin{cases} 
 T(0) = T_a \\
 T_f(0) = T_{fi} 
\end{cases} 
\]

(4.3.2.7)

where \( T_a \) is the ambient temperature and \( T_{fi} \) is the fluid inlet temperature.
The second boundary condition for the pond temperature $T(x)$ is that the incoming solar radiation that reaches the blackened bottom of the pond is absorbed there completely, therefore:

$$
ii) \left. K \frac{dT}{dx} \right|_{x=L} = \tau H_s \sum_{m=1}^{m=4} \alpha_m e^{-b_m L} \quad (4.3.2.8)
$$

Using equations (4.3.2.7) and (4.3.2.8), the constants of integration $C_1, C_2$ and $C_3$ are given by:

$$
C_{1,2,3} = \frac{D_{1,2,3}}{D} \quad (4.3.2.9)
$$

where $D_{1,2,3}$ and $D$ are:

$$
D_1 = \{1 - \frac{2NKA}{U_0 L} (R^2 + \frac{L^2}{4\pi^2N^2})^{-\frac{1}{2}}R_1^2\} \{G_1R_2e^{2\pi NR_2} - G_3\} \\
+ \{1 - \frac{2NKA}{U_0 L} (R^2 + \frac{L^2}{4\pi^2N^2})^{-\frac{1}{2}}R_2^2\} \{G_1R_1e^{2\pi NR_1} + G_3\} \\
+ G_2 \{R_1 e^{2\pi NR_1} - R_2 e^{2\pi NR_2}\} \quad (4.3.2.10)
$$

$$
D_2 = \{G_2-G_1\} R_2 e^{2\pi NR_2} + \frac{2NKA}{U_0 L} (R^2 + \frac{L^2}{4\pi^2N^2})^{-\frac{1}{2}}G_3 R_2^2 \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (4.3.2.11)
$$

$$
D_3 = -\{G_2-G_1\} R_2 e^{2\pi NR_2} - \frac{2NKA}{U_0 L} (R^2 + \frac{L^2}{4\pi^2N^2})^{-\frac{1}{2}}G_3 R_1^2 \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (4.3.2.12)
$$

and

$$
D = \frac{2NKA}{U_0 L} (R^2 + \frac{L^2}{4\pi^2N^2})^{-\frac{1}{2}} R_1 R_2 \{R_2 e^{2\pi NR_2} - R_1 e^{2\pi NR_1}\} \quad (4.3.2.13)
$$
where the different terms $G_1$, $G_2$ and $G_3$ are expressed in terms of $T_a$ and $T_{fi}$:

$$G_1 = T_a - \sum_{m=1}^{m=4} f_m$$  \hfill (4.3.2.14)

$$G_2 = T_{fi} - \sum_{m=1}^{m=4} \left( f_m - \frac{\frac{\text{KAL}}{2\pi^2 \text{NUD}_o}}{(R^2 + \frac{L^2}{4\pi^2 N_e})^{-\frac{1}{2}}} f_m b_m^2 \right)$$

$$- \frac{\frac{\text{rS}}{2\pi^2 \text{NUD}_o}}{(R^2 + \frac{L^2}{4\pi^2 N_e})^{-\frac{1}{2}} a_m b_m}$$  \hfill (4.3.2.15)

$$G_3 = \sum_{m=1}^{m=4} \left( -\frac{\frac{\text{rS}}{2\pi N K_L}}{a_m + \frac{L}{2\pi N} f_m b_m} \right) e^{-b_m L}$$  \hfill (4.3.2.16)

The outlet temperature of the fluid is evaluated by setting $x$ to $L$ in equation (4.3.2.6) which yields:

$$T_{fo} = C_1 + C_2 \left( 1 - \frac{2\text{NKA}}{\text{ULD}_o} (R^2 + \frac{L^2}{4\pi^2 N_e})^{-\frac{1}{2}} R_1^2 \right) \exp(2\pi N R_1)$$

$$+ C_3 \left( 1 - \frac{2\text{NKA}}{\text{ULD}_o} (R^2 + \frac{L^2}{4\pi^2 N_e})^{-\frac{1}{2}} R_2^2 \right) \exp(2\pi N R_2)$$

$$+ \sum_{m=1}^{m=4} \left( f_m - \frac{\frac{\text{KAL}}{2\pi^2 \text{NUD}_o}}{(R^2 + \frac{L^2}{4\pi^2 N_e})^{-\frac{1}{2}}} f_m b_m^2 \left( R^2 + \frac{L^2}{4\pi^2 N_e} \right)^{-\frac{1}{2}} - \frac{\frac{\text{rS}}{2\pi^2 \text{NUD}_o}}{a_m b_m} \right)$$

$$\left( R^2 + \frac{L^2}{4\pi^2 N_e} \right)^{-\frac{1}{2}} e^{-b_m L}$$  \hfill (4.3.2.17)

As can be seen, the outlet temperature is only a function of the design and operating parameters.
The thermodynamic efficiency of such a system is defined by the ratio of the extracted heat over the input heat. Therefore, since the extracted heat is \( Q = \dot{m} C_{pf} (T_{fo} - T_{fl}) \) and the input heat is \( AH_s \), the efficiency \( \eta \) is given by:

\[
\eta = \frac{\dot{m} C_{pf} (T_{fo} - T_{fl})}{AH_s}
\]  

(4.3.2.9)

4.3.3 Numerical Computations and Results

Numerical computation has been carried out for:
- The overall heat transfer coefficient \( U \)
- The bottom pond temperature
- The fluid outlet temperature \( T_{fo} \)
- The rate of heat extracted and the efficiency.

The properties of the other parameters involved in the equations is listed below:

- \( K = 0.556 \text{ Wm}^{-1}\text{C}^{-1} \)
- \( \rho_f = 1000 \text{ kg m}^{-3} \)
- \( g = 9.81 \text{ ms}^{-2} \)
- \( \beta = 0.00729 \text{ C}^{-1} \star \)
- \( C_{pf} = 4185 \text{ J/kg}^{-1}\text{ C}^{-1} \)
- \( \mu_f = 9.71 \times 10^{-4} \text{ kg s}^{-1} \text{ m}^{-1} \)
- \( K_f = 110.72 \text{ Wm}^{-1}\text{ C}^{-1} \)

The overall heat transfer coefficient which characterises the rate of heat transferred from the pond water to the fluid in the heat exchanger has been evaluated for different flow rates and for copper pipe diameters.
It will be observed from Table 4.3.3.1 that the overall heat transfer coefficient increases as the flow rate increases corresponding to $m^{0.8}$. However, it decreases with an increase in the diameter of the pipe. The computation of coefficient has been carried out on a model pond of ten square metres and one and a half metres deep. The heat exchanger is made of copper tube of 15 millimetres and 13 millimetres outer and inner diameter.

Figure 4.3.3.1 shows the variation of the bottom temperature of the pond versus the solar insolation $H_s$ for three different flow rate values $m$. The bottom temperature increases with an increase in the insolation linearly. This is due to the fact that when the insolation increases, the amount of radiation received by the pond surface will also increase. This results in a rise in the amount of heat generated per unit volume of the pond solution. Here the bottom temperature indicates how the temperature profile throughout the depth varies with the insolation, since the bottom is the hottest region of the pond. However, as the flow rate of the cooling water increases the bottom temperature decreases and thus the temperature profile. This is attributed to the effect of increase in flow depth because flow rate increases, the rate on the overall heat transfer.

<table>
<thead>
<tr>
<th>Flow Rate x $10^{-2}$</th>
<th>Overall Heat Transfer Coefficient $D_o = 0.5$ cm</th>
<th>Overall Heat Transfer Coefficient $D_o = 1$ cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>146.23</td>
<td>117.74</td>
</tr>
<tr>
<td>3.0</td>
<td>146.66</td>
<td>118.76</td>
</tr>
<tr>
<td>3.5</td>
<td>146.98</td>
<td>119.53</td>
</tr>
<tr>
<td>4.0</td>
<td>147.23</td>
<td>120.13</td>
</tr>
<tr>
<td>4.5</td>
<td>147.43</td>
<td>120.61</td>
</tr>
<tr>
<td>5.0</td>
<td>147.59</td>
<td>121.01</td>
</tr>
<tr>
<td>5.5</td>
<td>147.73</td>
<td>121.34</td>
</tr>
<tr>
<td>6.0</td>
<td>147.84</td>
<td>121.63</td>
</tr>
<tr>
<td>6.5</td>
<td>148.03</td>
<td>122.09</td>
</tr>
<tr>
<td>7.0</td>
<td>148.11</td>
<td>122.28</td>
</tr>
<tr>
<td>7.5</td>
<td>148.17</td>
<td>122.45</td>
</tr>
</tbody>
</table>

**TABLE 4.3.3.1: VARIATION OF THE OVERALL HEAT TRANSFER COEFFICIENT WITH THE FLOW RATE FOR A COPPER PIPE ($k_t = 110.72$ W/m°C)**
coefficient between the pond water and the cooling water. There is an increase in the heat transfer, resulting in an increase in the rate of heat transferred from the pond to the cooling water. Therefore the pond becomes cooler.

As far as the effect of the insolation on the outlet temperature of the cooling water is concerned, Figure 4.3.3.2 illustrates the dependence of the cooling water outlet temperature versus the solar insolation. Since the temperature profile along the depth of the pond increases, the outlet temperature increases too. Yet, it decreases as the flow rate increases, since with a greater flow rate there will be a higher mass of water flowing in the heat exchanger pipe.

The rate of heat extracted from the pond by the cooling water as a function of the flow rate is represented in Figure 4.3.3.4 for $H_s = 300 \text{ W/m}^2$. The rate of heat extracted increases as the flow rate rises because of the increase in the overall heat transfer coefficient with the flow rate. In addition, the same increase is seen with a rise in the solar insolation heat input. The efficiency shows a similar incremental trend as the rate of heat extracted. The efficiency is directly proportional to the heat extracted (e.g. $\text{eff} = \frac{Q_{\text{ext}}}{A_{\text{hs}}}$). It varies between 58% and 59% when the flow rate changes from 0.03 to 0.21 kg/s. This slow variation is attributed to the way the overall heat transfer coefficient varies with the flow rate ($\sim m^{0.8}$). However, it is surprising to find the variation of the efficiency with the insolation is constant. This is due to the fact that the pond with fixed design parameters has only one efficiency. Any variation of the efficiency should involve the change of another parameter, such as the length of the pipe or the flow rate.

In Figure 4.3.3.6 is the effect of the number of coils on the length of the pipe of the heat exchanger on the bottom temperature or the temperature profile throughout the depth of the pond. By increasing the number of coils, the bottom temperature decreases and the outlet temperature of the cooling water increases, Figure 4.4.3.7. This is a result of the effect of the number of coils on the area of exchange
between the pond water and the cooling fluid. The area of the heat exchanger increases as the length of the pipe increases as a result of the increase in the number of turns making the heat exchanger. Therefore more heat is transferred from the pond water to the cooling water. This results in a decrease in the temperature profile of the pond through its depth and an increase in the outlet temperature. Figure 4.4.3.8 illustrates the variation of the rate of heat extracted with the number of coils of the heat exchanger for \( H_S = 100 \text{ W/m}^2 \). As far as the efficiency is concerned, it does vary between 52\% and 59\% while the number of coils goes from 5 to 45.

This study gives a view of the behaviour of the proposed solar pond model with the internal helicoil heat exchanger. First, it is interesting to note that the efficiency is increased compared with the traditional method of heat extraction from solar ponds, where heat is just extracted from the bottom of the pond. The efficiency in this model was at most 40\%, whereas in the proposed model, the efficiency is theoretically more than 50\% giving an improvement of about 10\% which is considerable.

This theory can predict the efficiency of such solar ponds with an internal heat exchanger. Depending on the location and the rate of heat extraction needed, the pond and its heat exchanger can be designed in such a way that the pond operates at its highest efficiency.
FIGURE 4.3.3.1: Variation of the bottom temperature with the insolation for different flow rates

Flow rates: 0.03 kg/s, 0.12 kg/s, 0.21 kg/s

FIGURE 4.3.3.2: Variation of the outlet temperature with the insolation for different flow rates

Flow rates: 0.03 kg/s, 0.12 kg/s, 0.21 kg/s
FIGURE 4.3.3.3: Variation of the overall heat transfer coefficient with the flow rate

FIGURE 4.3.3.4: Variation of the rate of heat extracted with the flow rate
FIGURE 4.3.3.5: Effect of the flow rate on the efficiency
FIGURE 4.3.3.6: Variation of the bottom temperature with the insolation for different numbers of coils

FIGURE 4.3.3.7: Variation of the outlet temperature with the insolation for different numbers of coils
FIGURE 4.3.3.8: Variation of the rate of heat exchanged with the number of coils

FIGURE 4.3.3.9: Variation of the efficiency with the number of coils
CHAPTER 5

LABORATORY TESTS ON THE SOLAR POND WITH THE HELICOIL HEAT EXCHANGER INSTALLED

These tests were carried out in order to compare the theoretical predictions of the effect of flow rate, solar insolation and the total length of the heat exchanger pipe on the thermal behaviour of the solar pond. These experiments were carried out on small scale pond with constant illumination in the laboratory.

The instrumentation consisted of three microprocessor controlled thermometers and two chart recorders which were used to measure and record the temperature changes during the tests (see Appendix 5). In addition, measurements of the salt concentration gradient profile were made and the results compared with the corresponding theoretical profile.

5.1 LABORATORY POND MODEL

The experimental pond model consisted of a plastic cylindrical tank of twenty five centimetres height and twenty centimetres diameter.

The bottom of the tank was fitted with a thin circular black plate to avoid the use of paint which did not resist the corrosive action of the saline water.

The heat exchanger consisted of a copper tube of four millimetres for inner and five millimetres outer diameter. This was formed into an helix twenty centimetres high and thirteen centimetres mean diameter.

The tank was insulated with a fibre glass wool jacket around the sides and the bottom and placed inside a cardboard box.

The artificial daylight illumination was provided by an incandescent lamp of one kilowatt, mounted above the pond surface.
The measurement of the temperature profiles throughout the depth was obtained with six copper - copper nickel thermocouple probes (see Appendix 5) fixed in the side of the tank through rubber plugs at four centimetres intervals. Two other thermocouple probes were fixed in the heat exchange piping at the upper and lower ends in order to measure the inlet and outlet temperatures of the cooling water.

The thermocouples measuring the inlet, outlet and bottom temperature were each connected to a separate microprocessor controlled type Comark 6200 thermometer (see Appendix 5).

The Comark thermometer analogue output voltages were fed into the input modules of the chart recorders to enable continuous recordings to be taken (see Appendix 5).

The temperature profile through the depth of the pond was obtained from five additional thermocouples connected to one of the Comack thermometer instruments. In combination with the sixth thermocouple measuring the bottom temperature, the profile was obtained at regular intervals.

The density profile of the pond solution was measured by weighing with a digital balance small samples drawn from the solution at six levels throughout the depth, where six tappings were fixed at four centimetre intervals.

The first thermocouple probe and tapping were at surface level of the solution, whereas the sixth probe and needle were a few millimetres above the base of the tank.

The cooling water flowing into the heat exchanger was tap water fed from a reservoir placed at a height of two metres above the tank. The water flows from the reservoir through a pipe connected to a floating ring flowmeter. It is fitted with a valve that may be adjusted to enable the flow rate to be set to the desire value. From
the flowmeter, the water enters the upper end of the heat exchanger and leaves the other end to waste.

5.2 ESTABLISHMENT OF THE SALT CONCENTRATION GRADIENT

The salt concentration gradient in the laboratory pond model was created by the method of filling by redistribution. The pond solution was made by dissolving sodium hexacyanoferrate II (Na₄Fe(CN)₆) in a given volume of hot water to give a saturated solution.

![Table 5.2.1: Solubility of Sodium Hexacyanoferrate II in Water and Density of the Solution Obtained versus the Temperature of the Water (49)*](image)

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Solubility (wt %)</th>
<th>Density d₄/w</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.65</td>
<td>10.23</td>
<td>1.0689</td>
</tr>
<tr>
<td>16.70</td>
<td>14.69</td>
<td>1.1079</td>
</tr>
<tr>
<td>25.35</td>
<td>17.63</td>
<td>1.1303</td>
</tr>
<tr>
<td>35.75</td>
<td>21.54</td>
<td>1.1572</td>
</tr>
<tr>
<td>43.65</td>
<td>26.20</td>
<td>1.1921</td>
</tr>
<tr>
<td>58.75</td>
<td>30.35</td>
<td>1.2180</td>
</tr>
</tbody>
</table>

As in the laboratory tests the pond model consisted of three layers: the upper, lower and non-convective zones. The thickness of the lower and the non-convective zones was seven and thirteen centimetres respectively, whereas, the upper convective zone was just a few millimetres. The required mass of salt to make up the solution was one and a half kilograms dissolved into five litres of hot water.

The prepared solution was then poured into the tank up to a height of thirteen and a half centimetres which is the thickness of the lower

* Solubility is defined by the ratio: 100 x grams of solute/grams of solvent + grams of solute
convective zone plus half the thickness of the non-convective zone. Then the diffuser was placed at the interface LCZ/NCZ and raised two centimetres each time the solution surface rose by one centimetre as the water flowed from the diffuser outlet diluting the brine and thus creating the salt concentration gradient.

5.3 PROCEDEURE

The first three experimental tests performed on the laboratory pond model concerned the effect of the flow rate of the cooling water on the behaviour of the pond. In each of the tests, the heat exchanger consisted of a six turns helicoil pipe made of copper. The heating lamp was mounted forty five centimetres above the pond solution surface. In each test, the pond solution was replaced by a new solution that had been prepared twenty four hours before and the salt concentration gradient established.

The flow rates used in these three tests were 50, 90 and 155 cm$^3$/min respectively.

In order to simulate experimentally the effect of the solar insolation on the pond, another three tests were performed on the laboratory pond. Using the same heat exchanger with a constant flow rate of 90 cm$^3$/min the light intensity was changed. The pond solution was replaced each time by a new solution that had been prepared in advance and the salt concentration gradient established. A lightmeter was used to measure the variation of the light intensity at the pond surface as shown in the table below:

<table>
<thead>
<tr>
<th>Distance between lamp and pond surface (cm)</th>
<th>25</th>
<th>35</th>
<th>45</th>
<th>55</th>
<th>65</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light intensity x 10$^3$ (lux)</td>
<td>48</td>
<td>23</td>
<td>15</td>
<td>10.5</td>
<td>7.4</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Table 5.3.1: Variation of the light intensity of 1 kW lamp with the distance separating the lamp and the pond surface.
It can be seen the light intensity increases as the distance decreases. In the tests, the lamp was mounted 35, 45 and 55 centimetres above the pond which caused the light intensity at the surface to be 23000, 15000 and 10500 luxes.

Finally, the effect of the length of the heat exchanger was investigated experimentally by changing the heat exchanger length. In each of the tests concerning the effect of the heat exchanger, the flow rate was constant at 90 cm³/min and the lamp fixed at 45 centimetres above the pond surface. The heat exchanger was made of copper tube of four and five millimetres inner and outer diameter respectively. However, the number of coils was changed. Three coils were tested of 2, 6 and 12 turns respectively.
PHOTO (A)

DIFFUSER USED TO CREATE THE SALT CONCENTRATION GRADIENT
PHOTO (B)

LABORATORY POND WITH THERMOCOUPLES AND CONCENTRATION TAPINGS
PHOTO (C)

EXPERIMENTAL POND AND INSTRUMENTS
Water reservoir

Pipe

Heating lamps (4 KW)

Frame

Pond rig

Floating ring flowmeter

Chart recorder

Comark 6200 Thermometer
PHOTO (D)

HEAT EXCHANGERS USED IN THE EXPERIMENTS
Heat exchanger of 2 coils

Heat exchanger of 12 coils
The experimental results obtained during the tests to investigate the effects of the flow rate, the incoming light intensity and the length of the heat exchanger are presented and discussed in the following sections. These results are in agreement with the theoretical predictions for the effects of the flow rate and light intensity. However, the tests on the influence of the length of the heat exchanger failed to show the expected theoretical predictions.

6.1 EFFECTS OF THE FLOW RATE

The figure 6.1.1 a,b,c show the outlet temperature profile and its corresponding inlet temperature profile that were recorded in each of the three experimental tests performed on the laboratory pond model to study the effect of the flow rate.

The inlet temperature profile is approximately constant during each test, however, the outlet temperature increase during the heating process. The trend followed by the outlet profiles is characteristic of the time dependent heating phenomenon involving a constant heat source.

The outlet temperature appears to increase exponentially in the early stages of the heating process. This variation characterizing the transient process is followed by the steady state profile. Moreover, the time taken by the outlet temperature to reach the stationary state seems to depend on the flow rate. Another observation deserving comment is the overall outlet temperature profile depends on the flow rate with which the cooling water flows into the heat exchanger. As it can be seen, the higher the flow rate, the lower is the outlet temperature profile. This would explain why the time taken by the outlet temperature to stabilize is higher in the case of
FIGURE 6.1.1: Inlet and outlet temperature profiles recorded during the three tests on the effect of the flow rate.
light intensity = 15000 luxes, number of coils = 6

FIGURE 6.1.2: Bottom temperature profiles recorded during the three tests on the effect of the flow rate

FIGURE 6.1.3: Initial and final temperature profiles along the depth of the pond
FIGURE 6.1.4: Variation of the rate of heat extracted from the pond with the flow rate

FIGURE 6.1.5: Variation of the outlet temperature with the flow rate
a lower flow rate. As the flow rate increases so the outlet temperature profile decreases. Thus, from the same initial temperature it will take a longer period to reach the higher temperature.

The same feature characterizing the outlet temperature profiles are observed in the evaluation of the bottom temperature profile as it can be seen from figure 6.1.2. From this figure it is possible to work out the time taken by the pond to reach the steady state. For instance it takes thirteen hours for the pond to reach the steady state in the case where the flow rate is 50 cm$^3$/min, and it falls by approximately 70% if the flow rate increases to 155 cm$^3$/min. Moreover, the bottom temperature profile decreases as the flow rate increases.

On Figure 6.1.3 are the temperature profiles through the depth measured initially, and at the end of each experiment. Although, the initial temperature profiles are more or less the same, the final profiles vary depending on the flow rate with which the cooling water flows into the heat exchanger. The higher the flow rate, the lower is the final or steady state temperature profile through the depth.

Examining Figure 6.1.4 where the dependence of the rate of heat extracted from the pond by the cooling water against the flow rate is illustrated, it is seen this increase of the rate of heat extraction with the augmentation of the flow rate, indicates the reason why the pond is cooler in the case where the flow rate increases.

The variation of the outlet temperature of the cooling water versus the flow rate is shown on figure 6.1.5. As the flow rate increases, the outlet temperature decreases. This can be attributed to the increase of the mass of the water flowing into the heat exchanger per unit of time.
6.2 EFFECT OF THE LIGHT INTENSITY

The effect of the light intensity on the outlet temperature is shown on figure 6.2.1 a,b,c. The outlet temperature increases as the pond is heated following always the exponential law of heating involving a constant source of heat. Then it tends to stabilize indicating that the pond starts reaching the steady state. Moreover, as the light intensity increases, the outlet temperature profile moves towards a higher temperature.

The figure 6.2.2 illustrates the dependence of the bottom temperature development on the light intensity. The higher the light intensity received by the pond surface, the higher is the bottom temperature profile; especially as far as the steady state profile is concerned. In addition, the time taken by the bottom temperature profile to stabilize is about eleven and a half hours when the light intensity is 23000 luxes and drops by 21% as the light intensity becomes 10500 luxes. The rate of heat extracted by the cooling water from the pond increases as the light intensity increases as illustrated on figure 6.2.3. Since the outlet temperature is proportional to the rate of heat extracted and since the flow rate is constant; the outlet temperature increases with an increase of the light intensity as is seen in figure 6.2.4. As far as the temperature profiles through the depth are concerned, figure 6.2.5 shows the plot of the measured initial and final temperature profiles during the three tests. It is clear that the final temperature profile through the depth of the pond depends on the light intensity, since it is higher when the light intensity is higher. These results can be explained by the fact that as the light intensity increases, the amount of light absorbed and therefore the rate of heat generated inside the pond increases. This results in a higher temperature profile through the depth resulting in a higher rate of heat extracted by the cooling water flowing into the heat exchanger.
flow rate=90 cm$^3$/min, number of coils=6

FIGURE 6.2.1: Inlet and outlet temperature profiles recorded during the three tests on the effect of the light intensity
FIGURE 6.2.2: Bottom temperature recorded during the three tests on the effect of the light intensity of the heating lamp

FIGURE 6.2.3: Variation of the rate of heat extracted from the pond with the light intensity of the heating lamp
FIGURE 6.2.4: Variation of the outlet temperature with the light intensity of the heating lamp

FIGURE 6.2.5: Initial and final temperature profiles along the depth of the pond
6.3 EFFECT OF THE LENGTH OF THE HEAT EXCHANGER

The results concerning the effect of the length or the number of coils of the heat exchanger on the outlet temperature, the rate of heat extracted, the bottom temperature profile and the steady state temperature profile through the depth are presented in Figures 6.3.1, 6.3.2, 6.3.3, 6.3.4 and 6.3.5.

Although it seems that there is an effect but it is not as marked as it should be to indicate the expected profile. In fact, as it can be seen on figure 6.3.1 a, b, c the increase of the outlet temperature characterizing the heating process is present. But, if figure 6.3.2 is considered where the bottom temperature profiles are plotted, it is not clear to see the effect of the length of the heat exchanger as expected. By looking at figure 6.3.3, one can see that the heat exchanger length has an effect, an increase of the number of coils leads to a decrease of the temperature profile of the pond throughout its depth. The decrease of the pond temperature profile through the depth is due to the increase of the area of exchange between the cooling water and the pond solution as the number of coils increases. In addition to this effect, another effect is present experimentally and is difficult to predict theoretically. This secondary effect is the heat capacity of the heat exchanger. As the length of the heat exchanger increases the mass of metal constituting the pipe increases. This leads in addition to a higher heat capacity of the pond. Being a metallic tube, the heat exchanger absorbs some of the heat present in the pond and moreover, the volume of the pond solution changes as the length of the heat exchanger changes. It therefore appears that these secondary effects are of sufficient size to mask the effect of the length of the heat exchanger.
Inlet and outlet temperature profiles recorded during the three tests on the effect of the number of coils of the heat exchanger.
light intensity = 15000 luxes, flow rate = 90 cm³/min

FIGURE 6.3.2: Bottom temperature profiles recorded during the tests on the effect of the number of coils of the heat exchanger

FIGURE 6.3.3: Initial and final temperature profiles along the depth of the pond
FIGURE 6.3.4: Variation of the rate of heat extracted from the pond with the number of coils

FIGURE 6.3.5: Variation of the outlet temperature with the number of coils
6.4 DENSITY PROFILE THROUGH THE POND DEPTH

The measured density profiles through the depth of the pond solution are plotted in figure 6.4.1 a,b,c and compared to the corresponding theoretical density profiles predicted by equation 4.2.10.

An important point deserving comment, is the initial salt concentration gradient or density gradient can be established in the pond using the diffuser (see photograph 1, CH 5). Moreover, as it can be seen, the density profile throughout the depth of the pond does not change sufficiently, this shows how stable is the salt concentration gradient created through the depth of the pond. In addition, the agreement between the measured and the theoretical density profiles is good despite some discrepancies due to errors in the measurement manipulation. However, these discrepancies are very small, in fact the error between the measured and the calculated values of the density does not exceed 5% in the worst case.
FIGURE 6.4.1: Experimental and theoretical density profiles along the depth of the pond
a) Initial, b) 5 hours later, c) 19 hours later
CHAPTER 7

CONCLUSIONS AND SUGGESTIONS

The present research concerns a new method of heat extraction from a path gradient solar pond. Unlike the conventional way of heat extraction where heat is extracted only from the lower convective zone, either by a selective withdrawal of the hot brine or by using an internal heat exchanger placed in the lower convective zone. The present method consists of using an internal helicoil heat exchanger throughout the depth of the pond in order to increase the efficiency of the salt gradient solar pond.

This idea is motivated by the fact that some of the heat flux flowing upwards from the bottom to the surface of the pond due to the temperature gradient resulting from the absorption of the solar radiations and maintained by the salt concentration gradient will be lost to the surroundings.

However by using the proposed method, the heat flux is intercepted and extracted and this will result in an increase in the efficiency of such a solar collector.

The method of creating the initial salt concentration gradient was described and the time dependent mass transfer differential equation was solved analytically. The theory showed the stability of the initial salt concentration gradient in the course of time although the solution does not take into account the effect of the soret effect due to the additional diffusion of salt under the influence of a temperature gradient as can be met in a solar pond. However the effect taken on significance only when the temperature difference between the bottom and surface of the pond is about 80°C that happens when no heat is extracted from the pond. Yet, as the use of the solar pond is to deliver heat, the soret effect would not have a significant effect.
Experimentally, the initial concentration gradient profile through the depth was successfully established using the diffuser designed for this purpose (Chapter 5). Moreover, the salt concentration profiles through the depth were measured and compared with the theoretical profiles and it was found that the discrepancies did not exceed 5%.

As far as the thermal behaviour of the salt gradient solar pond with the internal helicoil heat exchanger is concerned, a mathematical model based on the steady state heat transfer differential equation governing the solar pond combined with the basic equations of the heat exchanger theory was developed.

In addition to providing a good first approximation to the thermal performance of the solar pond, the steady state analysis provides physical insight into the effects of parameter variation for a given design. The predicted bottom temperature, outlet temperature, rate of heat extracted and the efficiency are the same as the mean values predicted by a model driven by sinusoidal inputs. The computation was carried out with the model to study the effect of three parameters, the flow rate of the cooling water, the solar insolation and the length of the heat exchanger.

The choice of these three parameters is justified by the fact that:

1. the flow rate is an important parameter influencing the heat transfer mode between the pond solution and the cooling water in the heat exchanger;

2. the solar insolation is the driving energy of the solar pond;

3. the length of the heat exchanger as it is proportional to the area of heat exchange between the pond solution and the cooling water in the heat exchanger.
The increase of the flow rate and the length of the heat exchanger yields an increase in the rate of heat extracted from the pond and thus an increase of the efficiency. Moreover, for a fixed pond design the rate of heat extracted increases with the insolation. It has been found that the efficiency is greater than 50% according to the proposed model, however, since the model does not include the effect of heat losses, the efficiency is a little overestimated. Comparing this efficiency with the efficiency of a salt gradient from which heat is extracted only from the lower convective zone, an improvement of about 10% is found.\(^{[50]}\)

The theoretical model was verified experimentally on a small scale pond heated artificially with an incandescent lamp.

The experimental results were in good agreement with the theoretical predictions, except in the case of the length of the heat exchanger where it was not easy to gain insight into the effect because of the high heat capacity of the heat exchanger, and the small size of the pond, this secondary effect masked the expected effect.

However, qualitatively, the experimental results followed the same trend of variation according to the theoretical model.

We do believe according to the theoretical and experimental investigations, that the proposed method of heat extraction improves significantly the performance of the salt gradient solar pond. Also we suggest that more research should be focused on this method of heat extraction both theoretically and experimentally especially in the following directions:

a) Improving the theoretical model by taking into account the heat losses through the sides, the bottom and from the surface of the pond

b) Simulating the theoretical model by using more realistic climatological data of the solar insolation, ambient temperature
c) Testing a full size pond model outdoors for longer periods to study the effect of exterior conditions on the system.

d) Investigating the effect of the new method of heat extraction on the stability of the pond.

e) Testing different materials for the heat exchanger piping able to withstand corrosion by the brine according to the experiments, (copper pipes are believed to be good candidates for this purpose).
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APPENDIX

1. COMPUTER PROGRAM FOR THE MATHEMATICAL MODEL

10 REM "POND"
20 INPUT "DO="DO, "DI="DI, "L="L, "N="N, "R="R, "A="A
30 REM"DD: IS THE OUTER DIAMETER OF HEAT EXCHANGER:m"
40 REM"DI: IS THE INNER DIAMETER OF HEAT EXCHANGER:m"
50 REM"L: IS THE TOTAL POND DEPTH:m"
60 REM"N: IS THE NUMBER OF COILS OF HEAT EXCHANGER"
70 REM"R: IS THE RADIUS OF COILS OF HEAT EXCHANGER:m"
80 REM"A: IS THE AREA OF THE POND:m2"
90 INPUT "HO=HO"
100 REM"HO: IS THE SOLAR INSOLATION:W/m2"
110 INPUT "TA=TA, "TFI=TFI"
120 REM"TA: IS THE AMBIENT TEMPERATURE:degC"
130 REM"TFI: IS THE INLET FLUID TEMPERATURE:degC"
140 INPUT "M="M
150 PRINT
160 PRINT
140 PRINT: TAB(1): "***************************************************"
180 PRINT: TAB(1): "***************************************************"
190 REM"m: IS THE FLUID FLOW RATE IN THE HEAT EXCHANGER:KG/S"
200 PRINT: TAB(1): "THIS PROGRAM COMPUTES THE THERMAL BEHAVIOUR OF A"
210 PRINT: TAB(20): "SALT GRADIENT SOLAR POND"
220 PRINT: TAB(1): "***************************************************"
230 PRINT: TAB(1): "***************************************************"
240 PRINT
250 PRINT
260 KF=0.556
270 REM"KF: IS THE WATER THERMAL CONDUCTIVITY:W/m degC"
280 PF=1000
290 REM"PF: IS THE FLUID DENSITY:KG/m3"
300 G=9.81
310 REM"G: IS THE GRAVITATION ACCELERATION:m/s2"
320 BF=0.00729
330 CF=4185
340 REM"CF: IS THE FLUID SPECIFIC HEAT:J/KG degC"
350 KT=110.72
360 REM"KT: IS THE HEAT EXCHANGER PIPING MATERIAL THERMAL"
370 REM"CONDUCTIVITY:W/m degC"
370 MF=9.71E-4
380 REM"mf: IS THE WATER DYNAMIC VISCOSITY:KG/mS"
390 ho=0.47*(KF*0.75)*(DO^(-0.25))*((PF^0.5)*((G*BF*CF)^0.25)
400 hi=0.0160*(m^0.8)*((DI^(-1.8))*((CF/MF)^0.4)*((KT^0.6)
410 hc=((2*DO)/KT)*LN(DO/DI)
420 U=(DO/(DI*hi)+hc+1/ho)^(-1)
430 PT=(R^2+(L/(2*PI*n))^2)*(0.5)
440 IPT=PT^(-1)
450 DEL=1/(1+(2*U*PI*DO*PT)/(M*CF))^2+(2*U*PI*DO*PT)/(M*CF)
460 R1=-0.5*((1+2*U*PI*DO*PT)/(M*CF)-DEL^0.5)
470 R2=-0.5*((1+(2*U*PI*DO*PT)/(M*CF)+DEL^0.5)
480 a1=0.237
490 a2=0.193
500 a3=0.167
510 a4=0.179
520 b1=0.032
530 b2=0.45
540 b3=3
550 b4=35
560 REM"ai: ARE CONSTANT COEFFICIENTS(REF )"
570 REM"b1: ARE EXTINCTION COEFFICIENTS(REF ):1/m"
580 t=0.96
590 REM"t: IS THE WATER TRANSMITTANCE AT NORMAL INCIDENCE"
600 W1=L/(2*PI*N)
610 W2=(U*PI*DO*PT)/(M*CF)
620 W3=PI*U*PI*DO*PT
630 f1=(t*HO*a1*(W1*b1-W2))/((W3/A+W2*KF*b1-KF*W1*(b1^2))
640 f2=(t*HO*a2*(W1*b2-W2))/((W3/A+W2*KF*b2-KF*W1*(b2^2))
650 f3=(t*HO*a3*(W1*b3-W2))/((W3/A+W2*KF*b3-KF*W1*(b3^2))
660 f4=(t*HO*a4*(W1*b4-W2))/((W3/A+W2*KF*b4-KF*W1*(b4^2))
670 S=f1+f2+f3+f4
680 Q1=(KF*A*L*PI^2)/(2*(PI^2)*N*KF*DO)
690 Q2=(t*HO*L*A*PI^2)/(2*(PI^2)*N*KF*DO)
700 Q3=(2*N*KF*A*PI^2)/(U*L*DO)
710 Q4=((2*R1*N*KF*pi)/L)*EXP(2*R1*N*PI)
720 Q5=((2*R2*N*KF*pi)/L)*EXP(2*R2*N*PI)
730 D=((t*HO*a1)/KF-f1)*EXP(-b1*L)+((t*HO*a2)/KF-f2)*EXP(-b2*L)+
740 ((t*HO*a3)/KF-f3)*EXP(-b3*L)+((t*HO*a4)/KF-f4)*EXP(-b4*L)
750 E=TFI-S+Q1*(f1*b1^2+f2*b2^2+f3*b3^2+f4*b4^2)+Q2*(a1*b1+a2*b2+a2
760 *b3+a4*b4)
770 C3=((E-TA+S)*Q4*Q3*D*(R1^2))/((Q3*(Q5*(R1^2)-Q4*(R2^2))))
780 C2=(D-C3*C5)/C4
790 C1=TA-S-C2-C3
800 PRINT TAB(1):"FLOW RATE,m(KG/S)="M
PRINT TAB(1):"INNER HEAT TRANSFER COEFF,hi(W/M2 degC)="hi
PRINT TAB(1):"OUTER HEAT TRANSFER COEFF,ho(W/M2 degC)="ho
PRINT TAB(1):"CONDUCTIVE HEAT TRANSFER COEFF,hc(W/M2 degC)="hc
PRINT TAB(1):"OVERALL HEAT TRANSFER COEFF,U(W/M2 degC)="U
FOR X=0 TO 1.1 STEP 0.1
TP=C1+C2*EXP(2*R1*N*PI*(X/L))+C3*EXP(2*R2*N*PI*(X/L))+f1*EXP(-b1*X)+f2*EXP(-b2*X)+f3*EXP(-b3*X)+f4*EXP(-b4*X)
TF=C1+C2*(1-Q3*(R1-2))*EXP(2*R1*N*PI*(X/L))+C3*(1-Q3*(R2-2))*EXP(2*R2*N*PI*(X/L))+(f1-Q1*f1*b1-2-Q2*a1*b1)*EXP(-b1*X)+f2-Q1+f2*b2-2-Q2*a2*b2)*EXP(-b2*X)+(f3-Q3*f3*b3-2-Q2*a3*b3)*EXP(-b3*X9+(f4-Q4*f4*b4-2-Q2*a4*b4)*EXP(-b4*X)
TFO=C1+C2*(1-Q3*(R1-2))*EXP(2*R1*N*PI*(X/L))+C3*(1-Q3*(R2-2))*EXP(2*R2*N*PI*(X/L))+(f1-Q1*f1*b1-2-Q2*a1*b1)*EXP(-b1*X)+f2-Q1+f2*b2-2-Q2*EXP(-b2*X)+(f3-Q3*f3*b3-2-Q2*a3*b3)*EXP(-b3*X9+(f4-Q4*f4*b4-2-Q2*a4*b4)*EXP(-b4*X)
Q=M*CF*(TFO-TFI
REN=(M*CF*(TFO-TFI)/(A*HO)
PREN=100*REN
PRINT TAB(1);X;TAB(10);TP;TAB(29);TP;TAB(10);TF
PRINT TAB(1);X;TAB(10);TP;TAB(29);TF
PRINT TAB(1);"HEAT EXTRACTED,Q(W)="Q
PRINT TAB(1);"---------------------"
PRINT TAB(1);"EFFICIENCY,Q/A*HO(%)="PREN
PRINT TAB(1);"---------------------"
END
2. COMPUTER PROGRAM FOR THE CONCENTRATION PROFILE

10 PRINT TAB(1);"*******************************************************************************" 
20 PRINT TAB(1);"*******************************************************************************" 
30 PRINT TAB(1);"THIS PROGRAM EVALUATES THE CONCENTRATION PROFILES VERSUS RELATIVE TIME T" 
40 PRINT TAB(1);"*******************************************************************************" 
50 PRINT TAB(1);"*******************************************************************************" 
60 PRINT 
70 PRINT 
80 INPUT "11=":11 
90 REM "11: IS THE THICKNESS OF UPPER CONVECTIVE ZONE:M" 
100 INPUT "12=":12 
110 REM "12: IS THE THICKNESS OF THE NON-CONVECTIVE ZONE:M" 
120 INPUT "L=":L 
130 REM "L: IS THE TOTAL DEPTH OF THE POND:m" 
140 INPUT "N=":N 
150 REM "N: IS AN INTEGER NUMBER" 
160 INPUT "C0=":C0 
170 REM "C0: IS THE INITIAL LCZ CONCENTRATION" 
180 INPUT "T=":T 
190 REM "T: IS DIMENSIONLESS RELATIVE TIME=D*t/L2" 
200 D=54E-7 
210 REM "D: IS THE SALT DIFFUSIVITY:m2/S" 
220 PRINT 
230 PRINT TAB(1);"DEPTH m";TAB(20);"CONCENTRATION" 
240 PRINT TAB(1);"********";TAB(20);"***************" 
250 FOR X=0 TO L+0.1 STEP 0.1 
260 SUM=0 
270 FOR M=1 TO N STEP 1 
280 A=C0*(1-(12+11)/2) 
290 B=(4*C0*L)/((PI^2)*(12-11) 
300 C1=((M^2)*pi^2)*D*T/(L^2) 
310 D1=M*(-2) 
320 E=SIN((M*PI*(11+12))/(2*L)) 
330 F=SIN((M*PI*(12-11))/(2*L)) 
340 G=COS((M*PI*X)/L9 
350 SUM=SUM+D1*E*F*G*EXP(-C1) 
360 NEXT M 
370 C=A-B*SUM 
380 PRINT TAB(1);X;TAB(20);C 
390 NEXT X 
400 END
APPENDIX

3. COMPUTER PROGRAM TO EVALUATE THE TRANSMITTANCE COEFFICIENT IN ANY LOCATION IN THE WORLD

```fortran
open(unit=7, file='YA', form='formatted')

This program calculates the transmittance and the reflectance of a air-water interface at any location in the world

a, are constant coefficients
a1=0.006918
a2=0.397912
a3=0.070257
a4=0.006758
a5=0.000907
a6=0.002697
a7=0.00148

d=3.1415927*52xL/180
xL is the latitude of the location (deg)
d is the latitude of the location (rad)

write(7,7)
write(0,10)
format(5x,'Number of day',10x,'Reflec',20x,'Transm/')

do100 n=1,360
  g=6.3830*(n-1)/365
  p=a1-a2*cos(g)+a3*sin(g)-a4*cos(2*g)+a5*sin(2*g)
  %-a6*cos(3*g)+a7*sin(3*g)
  n is the day number(1st of January is zero), p is the sun declination (rad)

  f=acos(cos(d)*cos(p)+sin(d)*sin(p)

  f is the angle incidence of the sun's radiations (rad)

  format(17x,'input data')
  r0=asin(sin(f)/1.33)
  r1=sin(f-r0)
  r2=sin(f+r0)
  r3=tan(f-r0)
  r4=tan(f+r0)
  r=0.5*(r1**2/r2**2+r3**2/r4**2)
  t=1.0-r

  t and r are the transmittance and reflectance of air-water surface situated at a latitude d

write(0,20)n,xi,t
format(5x,i4,10x,f5.2,10x,f5.3)

xi is the angle of incidence of the sun's radiations (deg)
write(7,50)n,t
format(5x,i4,10x,f9.7)

continue
write(7,300)

format(7x,'eod.')
stop
end
```
APPENDIX

4. COMPUTER PROGRAM TO EVALUATE THE RATE OF HEAT GENERATED PER UNIT VOLUME IN THE POND

```fortran
open(unit=7, file='GEN', form='formatted')

c ***************************************************************
c This program calculates rate of heat generated at a depth x in the pond as a result of the absorption of solar radiations
 c ***************************************************************

do 100 i = 10, 90, 30
   c i is the angle of incidence
   write(7, 7)
7 format(7x, 'input data')
f = 3, 1415927*i/180
r0 = asin(sin(f)/1.33)
r = (180*r0)/(3.1415927)
   c r is the of refraction, r = asin(sin(i)/n)
   write(0, 50)i, r
50 format(i2, 3x, f5.2)
rl = sin(f-r0)
r2 = sin(f+r0)
r3 = tan(f-r0)
r4 = tan(f+r0)
t = 1.0 - 0.5*(rl**2.0/r2**2.0 + r3**2.0/r4**2.0)
   c t is the transmittance coefficient of pond surface
   do 200 j = 0, 120
      y = j/100.
x = y/cos(r0)
   c x is the depth of the pond
   g = 0.0076*exp(-0.032*x)+0.08685*exp(-0.45*x)
   &+0.501*exp(-3*x)+6.265*exp(-35*x)
   D = (g*t)/cos(r0)
   s = D*cos(f)
m = 4
   c s: the generated heat: t(i)*cos(i)/cos(r) \sum_{m=1}^{m} \exp(-b_m x/cosr)
   write(7, 10) y, s
10 format(f4, 2, 2x, f10.5)
   write(0,30)y, x, s
30 format(f4, 2, 2x, f6, 4, 5x, f10.5)
   continue
   100 continue
   write(7, 20)
20 format(7x, 'eod.')
stop
end
```
5. OPERATION OF THE COMARK 6200 AND THE PEN RECORDERS

A) MICROPROCESSOR CONTROLLED THERMOMETER COMARK 6200

The Comark 6200 comprises the latest technology in microprocessor application. The versatility of such a technology has produced in one instrument the facilities which previously would have required several instruments. For instance, six thermocouple types are accommodated within one appropriate instrument and can readily be selected on the touch sensitive panel. Cold function compensation is provided for all inputs. In addition there are three scales of temperature available, being absolute Kelvin, Celsius, Fahrenheit degrees. All these ranges are autozeroing and autocalibrating under a microprocessor control. The instrument offers also the advantage of an analogue output, switchable in three ranges to enable the selection of a suitable resolution appropriate to the temperature range required.

A useful feature of the instrument is the multiplying factor facility which enables calibration of the instrument.

When the instrument is plugged into the AC supply, the following start-up sequences will result:

a) All indicator lamps.
b) All digital system display checks.
c) Measure mode: display reading and indicator lamps will indicate the input selected by the front panel rotary switch.

Indication lamps will light up indicating setting of:

a) High and low alarm if operating.
b) Multiplying factor FCTR if set to another value than 1,0000.
c) Scale $A$, $C$, $F$ or $\mu V$.


The instrument must be left for at least fifteen minutes to warm up before any reading would be carried out.

To see what values reside in the various functions, it is only necessary to press the appropriate button function. The key must be pressed until the scale indicator blinks showing that the key has been pressed correctly and therefore read.

In order to enter any data into the instrument, the corresponding key should be pressed then the value of the scale is selected. To make this value on this scale operative, the key ENT RUN is then pressed. The same operation is followed in the case of the thermocouple material type. This is achieved by using the upper case characters in background colour green. The function key material/scale is first pressed, this will put the instrument in the selection mode, if the thermocouple is of type $K$ for instance, it is necessary to press the data button marked $K$ and then the key ENT RUN. The lower right four buttons on the data block will change the scale, between $A$, $C$, $F$ or $\mu V$ to select the desired scale.

The instrument has a resolution of $\pm 0.1$ degrees on the temperature ranges and switches up automatically to $\pm 1$ degree above 700 degrees. However, on the microvolt range, the resolution is $\pm 1$ mV.

The multiplying factor (FCTR) gives an opportunity for the user to calibrate the instrument to his own setting. To enter the desired value of the multiplying factor FCTR, it is necessary to press the button MATERIAL/SCALE, enter the desired value which should range from 0.0001 to 9.0000, then press the button ENT RUN and the lamp above the FCTR key will light up if the entered value is other than the unity (1.0000).
Figure A: Display and operating keys of the Comark 6200
The FCTR key also used to scale the analogue output. This function multiplies the measurement showed on the display and it also multiplies the analogue output by the same value which appears at the rear output of the instrument.

This instrument provides an output which is the analogue representation of the digital display, enabling the use of chart recorders, flat bed recorders, etc. with the instrument. The connection is made by means of a miniature jack-plug on the rear panel. The analogue signal is provided in the form of a ramping direct voltage that can be scaled by a slide-switch on the rear panel. The positions of the slide switch are marked 1, 5 and 10 corresponding to 1, 5 and 10 mV per excursion, referred to the front panel reading (100Ω = 1 volt). For instance, if the temperature read by the instrument is 20 degrees celsius, the analogue output voltage will be 200 mV if the slide-switch is 1, 40 mV if the slide-switch is on the mark 5 and becomes 20 mV if finally the slide-switch is on the 10 mark.

The analogue output facility is a bipolar direct voltage, with a resolution of ± 0.1% on full scale (1 volt). The output impedance is less than one ohm and a current ranging from -8 to 8 milliampers.

B) THERMOCOUPLe TYPES USED WITH THE COMARK 6200

The Comark 6200 operates with six thermocouple types which are K(NiCr-AlNi), J(Fe-CuNi), T(Cu-CuNi), R(Pt-13% Rh/Pt); S(Pt-10% Rh/Pt) and E(NiCr-CuNi). Any one of these six thermocouple types may be selected but they may not be mixed. That is, all inputs to the rear panel must be of the same type K, J, T, R, S or E. All thermocouples must have compatible or compensating materials for leads and plug components. If for instance, an extension lead is to be used, it should be made of compatible components otherwise the lead or the plug will create another thermocouple. Table B gives the ranges and accuracy values of the six thermocouple types that operate with the Comark 6200.
<table>
<thead>
<tr>
<th>Thermocouple type</th>
<th>range (°C)</th>
<th>maximum error ± °C(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>-200 500 1150 1372</td>
<td>&lt;15 5 &lt;10</td>
</tr>
<tr>
<td>J</td>
<td>-200 -140 700 1200</td>
<td>&lt;10 5 &lt;15</td>
</tr>
<tr>
<td>E</td>
<td>-200 883 1000</td>
<td>&lt;15 &lt;24</td>
</tr>
<tr>
<td>T</td>
<td>-200 0 400</td>
<td>&lt;14 5</td>
</tr>
<tr>
<td>R</td>
<td>200 1100 1767</td>
<td>&lt;5 &lt;15</td>
</tr>
<tr>
<td>S</td>
<td>200 1000 1767</td>
<td>&lt;5 &lt;15</td>
</tr>
</tbody>
</table>

Table B: Thermocouple Types, Operative Temperature Ranges and Accuracy
In the experiment tests, the T thermocouple types have been used because of two main reasons. First, the temperature range is most suitable for the purpose. Secondly, the error is less than 5%. The maximum error on the temperature measurement should not exceed \((0.1 + 0.05) = 0.15^{\circ}\text{C}\), therefore the relative error will be approximately 31\% in the temperature range up to 50 degrees celsius.

C) CHART RECORDERS

The recording of temperature profiles against time was achieved by using two chart recorders, one with two input channels and the second, a single input recorder.

The two pen recorder is compactly designed on one chart two independent variables that occur simultaneously against time. For use in laboratory, plant or field, the recorder not only aids in analysing the interrelation of the two simultaneous variables but eliminates the time consuming of comparing the two charts and collecting data.

This recorder plots voltage or current variations against time and thus enables measurement and recording of any physical or chemical variable which can be converted into a voltage or a current within the scale range of the recorder, such as temperature, pH, vibration etc.

C.1: Principle

The principle on which the chart recorder works is as follows:

A potentiometer measures an electromotive force (EMF) of an unknown value by comparing it with a known EMF. The unknown EMF is balanced electrically against the known EMF, provided in this recorder for each channel by stable Zener diode controlled reference supply. When the two EMFs are brought to balance, no current flows in the detector circuit, the balancing action is automatically achieved by a
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Figure C: Principle and design of the chart recorder

1. Input terminal
2. Span adjustor
3. G (ground) terminal (for each channel)
4. Fuse
5. G (ground) terminal
6. Switch for chart feed control of external and internal
7. Connector for external signal of chart drive
8. AC Power supply cord
9. Attenuator
10. Input module
11. Gain adjustor
12. Vernier (Calibration)
13. Damping adjustor
14. Zero point adjustor
15. Scale up lever
16. Scale
17. Chart drive power switch
18. Pen drive power switch
19. Main power switch
20. Main power display lamp
21. Pen
22. Pen up lever
23. Chart speed select dial
24. Chart rewind (for roll chart)
25. Chart speed change over switch
26. Chart "Drive" or "Free" change lever
27. Chart
28. Chart presser
servo-motor driven contact on the precision measuring slide wire. The potentiometer circuit is supplied from a precise source of direct voltage, and consists of a slide wire and fixed resistors. The unknown EMF is connected into the potentiometer circuit in opposition to the value of the slide wire voltage. A zero adjust control is provided in the potentiometer circuit to allow positioning of the recorded trace. An attenuation is provided between the unknown EMF and the potentiometer circuit to allow signals larger than that of the one referenced to be measured.

The balance action is started by a change in the voltage output from the circuit under measurement. This unbalances the potentiometer circuit and produces a direct perturbation current signal. This signal is then converted to an alternating current signal by the solid state field effect transistor FET chopper (input converter). The output voltage of the input converter is amplified electronically.

The amplifier is a plug-in solid state integrated circuit characterised by a 120 db gain. It increases the AC perturbation signal to a power level sufficient to operate the servo-motor. The amplifier output voltage drives the servo-motor in a direction that restores the potentiometer circuit to an electrical balance. The indicating pointer and recording mechanisms are mechanically coupled and simultaneously positioned to the new value of the measured variable. This rebalancing action is continuous and begins immediately upon the slightest change in the input signal. The direction of the rebalancing action of the recorder is based on the phase relationship between the AC perturbation signal from the input converter and the AC supply voltage fed to the reference winding voltage of the servo-motor. This phase relationship depends upon the direction of unbalance in the potentiometer circuit. when it has been rebalanced, the entire potentiometer remains motionless until another change in the measured variable occurs. The chart is driven by a pulse motor and controlled by a solid state electronic circuitry, as well as by an external frequency signal less than 250
Hertz.

C.2: Operation

The recorder should be installed in a place where there is as little dust as possible. In a dusty environment the pen knib is liable to become blocked and the pen will move erratically due to increased friction in the course of time. Moreover, being a sensitive electronic device, the recorder is sensitive to sudden changes of the ambient temperature and should not be exposed to sun light.

The recorder's power supply should be connected into a convenient AC line of the same voltage and frequency as marked on the name plate mounted on the rear side of the instrument, that is 240 Volts and 50 Hertz.

The analogue output voltage of the microprocessor controlled thermometer is fed into the recorder input module by means of two 'banana' plugs. The positive plug to the positive terminal and the negative plug to the negative terminal of the input module of the recorder. The negative terminal is connected to the ground terminal by a shorting strip.

In order to make sure that the recorder operates successfully, it is necessary to check that the ink reservoirs of the pens and the chart paper are enough for the duration of the test which in this case was up to 48 hours.

Therefore, the ink reservoir of each unit is filled with a syringe through the filling hole. Then, the rubber cap is pressed to force the ink to the pen point. The two pen traces can be differentiated by using two ink colours, such as black and red ink.

The pen carriage shafts should be cleaned and lubricated in order to insure a smooth movement with minimum friction on the supporting rod.
The chart paper roll is placed under the chart plate between the supports often inserting the stock flanges into the ends of the paper roll. The paper is then fitted over the pins at the end of the sprocket after making sure that the horizontal line is paralleled to the sprocket.

The last step before the recorder is ready for operation is the calibration. By putting the attenuation to the desired value, it should be greater than the maximum input voltage to ensure the signal will be within the plotting range. Using a DC input, the calibration is made step by step. After adjusting the zero position of the pen, the battery is connected to the input module and by delivering a given voltage and checking that the pen moves to the corresponding position without oscillation. If it does vibrate, the damping adjuster should be used to increase the damping. The chart power is then switched on and the desired chart speed is selected. Now, the recorder is ready, the input lead is connected to the input module and operation can start successfully.
ERRATUM

Page 70: For a practical application the number of coils will be constrained by the physical dimensions.

Page 74: These curves are in agreement with the inverse square law for light.

Page 75: The intensity of a 1000 Watt lamp is not ten times the intensity of a 100 Watt lamp due to Joule's law.

Page 81: For an exact comparison, the inlet temperature should be kept constant.

Pages 82, 88 and 91: As the pond depth increases the temperature profile becomes fully defined and the LCZ temperature reaches a constant and optimum value.

Page 97: It should be noted that the model does not include heat losses which could be considerable and practically reduce the 50% efficiency predicted by the theoretical model.

The movement of the sun in practical applications must also be included in calculating the overall efficiency. Published experimental results suggest efficiencies around 20%.