Compressibility and channel formation in sedimenting systems

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Abstract

Channelling and compression in batch and continuous gravity sedimentation systems have been investigated using an electrical impedance imaging technique. The accuracy of such a system is shown to be a function of a number of variables, in particular particle morphology. Not all materials were found to be suitable to this form of imaging. Three primary materials, aragonite, calcite and talc were used, each exhibited random channelling i.e. channel formation was not due to the presence of foreign bodies or externally induced. Channels in this type of channelling form within a zone which grows and propagates upwards into a suspension. Through visual observations this region has been sub-divided into hard and soft zones. Superposition of zone propagation on characteristic (loci of concentration) plots has yielded information on the conditions required for channel formation and growth and has led to a revised set of conclusions on the nature of random channelling.

Conventional static profile, downflow and initial rate tests were used to characterise the materials in terms of solids pressure and permeability, a novel concentration profile method was also investigated and was found to yield reliable permeability values at all concentrations in the compression regime of settling.

A compaction model which describes the compression process in terms of the variation in excess hydrostatic pressure is described. Its basis is a non-linear parabolic partial differential equation which is solved using an implicit three-time level finite difference method. Adequate representations of the systems investigated were obtained however the effects of channelling couldn’t be fully accounted for.

Other research has suggested that increased fluxes or underflow concentrations can be obtained using deep bed continuous thickeners which promote channelling. This was found not to be the case, and the operation of a continuous thickener employing a picket fence and rake mechanism was found to be more predictable and manageable.
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1. Introduction

Gravity sedimentation of suspended material from liquid is an extensively practised form of mechanical separation in many industries including: water and waste-water, minerals and chemicals. The driving potential for the separation is freely available and this has led to it being the first choice in most industrial situations, provided that sedimentation time is not a constraint. It has, however, become a poor relation for research effort compared to other separations perceived to be more modern. The relative scarcity of recent research effort into sedimentation implies that the technology is mature with little more to be gained in our understanding. This position is wholly unjustified, there are still some considerable challenges to our understanding of the sedimentation process. This thesis focuses on the less well understood areas of sedimentation, investigating some of the laboratory tests and models that can be used to provide a better awareness of the way in which a material sediments.

At low particle concentrations a settling particle has a velocity, relative to other particles of the same material, that is determined by its size. This type of settling is often used to classify materials or yield information on particle sizes and is referred to as clarification. At higher concentrations particles no longer settle individually but en-masse, this is hindered settling and is the area of most interest to researchers of sedimentation.

Two modes of settling can be identified within the hindered settling region, below a certain concentration particles settle en-masse but there is no contact between the particles, this is free settling. Above this concentration particles come into contact and form a solids structure or sediment. Stress can be transmitted through the particle contacts and depending on the nature of the discrete phase settling may occur due to breakdown of the sediment, this is compressive settling. A further sub-type of settling known as channelling also exists, flow paths with diameters significantly greater than the average pore size form and transport liquid upwards more effectively due to the reduced tortuosity.
1.1 Project Aims

It is believed that channelling augments the rate at which liquid is squeezed out of the sediment and into the supernatant and as a result increased settling rates are obtained. A design based on conditions that does not channel will, therefore, be more than adequate if channelling occurs subsequently. Since the occurrence of channelling is not detrimental to the design or operation of a thickener it has received little research attention however it is possible that many thickeners are over designed as a consequence hence the need for further study. The aims of the study were as follows.

[1] To develop an experimental sedimentation rig which included an imaging system suitable for monitoring concentration profiles in channelling compressible sedimenting systems

[2] To perform stirred and unstirred batch sedimentations using materials which exhibited channelling. These tests would allow
   (i) a greater understanding of the effects of stirring to be obtained
   (ii) the nature of settling around the free-compressive settling boundary to be investigated
   (iii) useful information on the causes and effects of channelling to be obtained from comparison between the unstirred (channelling) and stirred (channelling eradicated) tests.


[5] To compare the operation and performance of a continuous thickener in which channelling was promoted with a conventional thickener which employed a picket fence and rake which eradicated channelling.
1.2 Overview of thesis

Chapter 1 - Introduction, aims and overview of thesis

Chapter 2 - Outlines current theory in batch and continuous thickening.

Chapter 3 - Describes the experimental apparatus and methodology. The test materials used are detailed and the accuracy of the system for each material is examined.

Chapter 4 - Presents and discusses the findings of batch sedimentations conducted on a number of materials with particular emphasis on the causes and effects of channelling.

Chapter 5 - Outlines the theory behind the characterisation methods. The results of these methods are presented and mathematical expressions established.

Chapter 6 - A mathematical model of the compaction of a sediment is developed. Model results are compared with experimental data.

Chapter 7 - The results of operating a continuous thickener as a deep bed thickener are compared with results from a conventional thickener.

Chapter 8 - The main findings are reviewed and conclusions presented.
2. Literature survey

2.1 Batch thickening

Batch sedimentation tests are often performed when information on the settlement of a suspension is needed for the design of continuous gravity thickeners, or simply as a means to investigate sedimentation for process optimisation, control, knowledge, etc. The conventional method is to mix the suspension in a vessel such as a measuring cylinder and, if the concentration is sufficient, to observe the rate of descent of the suspension/supernatant interface. The height of the interface is plotted against sedimentation time and various conclusions can be drawn from the data, such as the type of sedimentation pertaining and the solids settling flux at concentrations between that of the feed suspension and that of the settled sediment.

![Diagram](image)

**Figure 2-1: Incompressible or ideal settling.**

In an ideal slurry settling particles are subject to two forces, a buoyed weight force which is determined by the solid and fluid density and a liquid drag force which is usually a function of the local solids concentration. Particles therefore settle out at a velocity which is determined solely by the local solids concentration. Eventually these particles reach a concentration at which they come into stable contact, at this point settling ceases. The build up of this concentration (or the sediment) therefore occurs at a constant rate. A typical settling curve showing the descent of the solid -
liquid interface and growth of the sediment for this type of settling behaviour is given in Figure 2-1.

This type of settling, often referred to as incompressible or ideal settling, is in general only shown by materials in which the basic settling entity is the primary particle. The particles also tend to be relatively large (>50μm). A good example of such a material is glass beads.

In reality most materials exhibit some form of compressible behaviour. That is to say when the particles come into contact they form a structure which has a yield stress. If sufficient force can be transmitted through this structure by the weight of overlying solids then the yield stress can be exceeded and further settling will occur due to the structure breaking down into a more compact form. When this does occur the settling rate is no longer a function of the effective weight and drag force but also of the solids stress gradient. The settling curve for a typical compressible material is shown in Figure 2-2. The sediment no longer builds up at constant rate and further settling can occur after the sediment reaches the solid liquid interface.

![Diagram](image)

**Figure 2-2: Typical compressible behaviour.**

Some settling curves may also exhibit an initiation period, this is shown as zone a in Figure 2-2, during this the settling rate is initially very slow but slowly increases. This is because the primary particles which would settle individually at a very slow rate because of their size are drawn together and form ‘flocs’. The size of the basic
settling unit therefore increases and hence the settling rate. Floc size is usually fairly consistent throughout a given suspension, however it is known to be very dependent on factors such as initial concentration, pH, agitation and the use of flocculants. The bonds holding flocs together are much greater than the structural strength of the solids network that is initially formed during compression. Compaction therefore initially occurs due to the breakdown of the solids network however at higher solids pressures it is possible that breakdown of the flocs occurs.

It is useful at this point to define a few terms. Below a certain concentration there is no contact between settling particles and any interactions are due to hydrodynamic effects, this is the 'free' settling regime. Zones a, b & c fall within this region. Zone a has already been mentioned, after this most suspensions exhibit a constant rate period which is representative of a suspension settling at a constant concentration (zone b). Zone c is a falling rate region, as the rate of settling decreases so the suspension thickens and higher concentrations propagate to the surface. When contact exists between particles then the suspension is in the compression regime. Further thickening can occur within the sediment (zone d), but the concentration of the sediment surface (or solid liquid interface) will not increase further as there are no overlying solids to provide the necessary stress.

Fitch's[1962] 'paragenesis' figure, reproduced in Figure 2-3, illustrates how the settling mode of a sedimentation varies. It can be seen that as the concentration of the settling suspension increases different types of sedimentation become prevalent. The mathematical description of sedimentation under conditions of zone settling and compression have been explained. However, most suspensions of interest to the process industries exhibit behaviour which lies in the region between zone settling and compression, which is a region where suspensions display channel formation and compression.

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The term free settling should not be confused with Stoke's free settling which considers only a single particle in an unbounded medium.
During "ideal" thickening liquid flows upward uniformly across the thickener cross section, when channelling occurs flow paths or channels of greater diameter than the average inter-particle (or floc) distance develop. Evidence of the effect can be observed in the shape of miniature "volcanoes" which form at the solid-liquid interface at the point where channels reach the surface. Channels offer paths of less resistance to upflowing liquid and so cause an increase in the settling rate. Such an effect is desirable in industrial thickeners (Chandler [1983]) as it reduces the area required to handle a certain flowrate, however to date there is no model which accounts for channelling so thickeners tend to be oversized or stirring is employed to negate the effect of channelling. The effect of channelling can be observed on a settling curve as a concave or accelerated settling rate period, an example is shown in Figure 2-4.
2.1.1 Mathematical models

The basis of most mathematical models is a material balance and/or a force balance on a sedimenting layer of solids. In this section two such balances will be derived and the assumptions and terms neglected by various authors in obtaining a model for sedimentation will then be considered.

2.1.1.1 Material balance

A solids material balance is easily generated by considering the flow through a differential layer (Dixon [1977]). Consider two differentially spaced, horizontal planes labelled 1 & 2, the volumetric solids balance on the layer is

\[ \text{IN} = \text{OUT} + \text{ACCUMULATION} \]

\[ c_1 \left( u_1 - \frac{dx_1}{dt} \right) = c_2 \left( u_2 - \frac{dx_2}{dt} \right) + \frac{d}{dt} \left[ \int_{x_1}^{x_2} c \, dx \right] \]  

(2.1)

Where \( u_i \) is the solids velocity through the plane and \( x_i \) is the height of the plane.
By approximating the integral using the trapezoidal rule and taking the limit as \( x_2 - x_1 \to 0 \) then:

\[
\frac{dc}{dt} = \left( \frac{\partial \phi}{\partial x} \right)_t + \frac{dx}{dt} \left( \frac{\partial c}{\partial x} \right)_t
\]

(2.2)

where \( \phi \) is the solids flux (\( \phi = c.u \)). This is a general material balance which describes a differential layer in a sedimenting suspension. Equation (2.2) can be simplified by considering the plane to be Eulerian or Lagrangian however the most widely used model in sedimentation uses an approach suggested by Kynch [1951]. The model assumes that settling rate is a function of local solids concentration only and considers the differential plane to be one which moves such that it has a constant concentration, hence \( dc/dt = 0 \) so Equation (2.2) becomes

\[
\left( \frac{\partial x}{\partial t} \right)_c = \left( \frac{\partial \phi}{\partial c} \right)_t = \nu
\]

(2.3)

i.e. the velocity at which a layer of constant concentration propagates upwards into a suspension is a function of the rate of the change of the solids flux with respect to concentration.

Figure 2-5: Kynch characteristics.

9
This result led Kynch to introduce the concept of characteristics which were simply loci of constant concentration. Characteristics emanate from the base of a suspension and propagate upwards at a velocity $v$. A typical 'fan' of Kynch characteristics is shown in Figure 2-5. The first characteristic to arise is a concentration close to the initial concentration and the final characteristic (and all points below it) is that of the sediment concentration ($C_T$). The gradient or velocity $v$ of each characteristic is a function of the rate of change of solids flux with respect to concentration. This has led many authors (Kynch [1951], Fitch [1979], Holdich [1987], Rhee [1986]) to discuss the significance of flux curve shape and the settling behaviour that it predicts.

A typical flux curve is shown in Figure 2-6, the slope of the flux curve at any point gives the propagation velocity of that concentration. The predicted behaviour of a suspension having an initial concentration $C_0$ is that a discontinuity will exist between $C_0$ and $C_b$, no intermediate concentrations can exist as they have faster propagation rates. If due to some small disturbance such an intermediate concentration were to exist it would quickly propagate to the surface and disappear. A graded zone of concentrations will exist between $C_b$ and $C_c$, as in this zone as the concentration increases so the propagation rate decreases. A second discontinuity exists (for compressible solids) between $C_c$ and $C_{\text{max}}$, this is the compression discontinuity and was proposed by Fitch [1983]. Suspensions of different initial concentrations will exhibit the same behaviour or parts of it depending on their location on the flux curve.
example an initial concentration greater than $c_b$ will settle immediately into a graded zone without first settling discontinuously.

A fundamental fault with the discontinuities discussed above, often referred to as Kynch discontinuities, is that they are mathematical ones i.e. they assume that there is a clear demarcation between the upper and lower discontinuity concentrations. Fitch [1993] proposed that to move from one concentration to another bypassing all intermediate concentrations is not possible. He suggested instead that concentration fronts ("D" fronts) must exist which have a finite distance within which all intermediate concentrations exist. Dixon [1977] also disputed the existence of Kynch discontinuities as they neglect inertial effects, which he argues is impossible as particles passing through a discontinuity must undergo some velocity change and therefore experience some sort of inertial effect.

2.1.1.1 Modified Kynch models

Flux curves are an essential part of most thickener design methods however measurement of solids flux is not always easy. The earliest technique for determining solids flux was suggested by Coe & Clevenger [1916], the method required that several batch tests at different initial concentrations be carried out. The initial settling rate of the interface was assumed to be that of the initial concentration and so the solids flux could be calculated. This was a lengthy technique as it required many batch tests, the introduction of Kynch theory reduced the number of tests required to the solids flux over a range of concentrations to a single batch test. The most widely used method which utilises Kynch theory was proposed by Talmadge & Fitch [1955], using a simple mass balance and the construction shown in Figure 2-7 solids fluxes over a range of concentrations can be obtained.
Figure 2-7: Talmadge & Fitch construction.

Kynch theory however is limited as it can only be used to predict the behaviour of incompressible suspensions or a small portion of the free settling section of a compressible sedimentation. This is because the above analysis fails to account for the presence of a rising compressible sediment. The concentration at the top of a compressible sediment is constant as it is subject to no solids stress, its rise can therefore be plotted as a characteristic (locus of concentration). Unlike a Kynch characteristic it is not a straight line, hence Kynch characteristics are impossible over a certain portion of the free settling regime as they would cross the sediment surface characteristic. A settling curve can therefore be split into four regions, these are shown in Figure 2-8. The first is a constant rate period (a) where the interface descends with a constant velocity (and therefore concentration), the second is the first falling rate period (b) over this region Kynch theory is valid. In the second falling rate period (c) straight line characteristics cannot propagate from the origin so the Kynch analysis cannot be applied. The final region, (d) is the compression zone: the falling liquid/suspension interface joins the rising sediment, the rate drops off as any further height loss is due to liquid being squeezed out of the sediment.
The problem of the origin of characteristics in region (c) was recognised by Tiller [1981] who suggested that characteristics would emanate from the sediment surface as shown by the line \((L_1, t_1)\) to \((h_2, t_2)\) in Figure 2-9. Tiller derived the following relationship to determine the concentration of a characteristic emanating from the sediment

\[
c_{s2} = \frac{c_s t_2}{h_{12} - L_1} \exp \left( \int_0^t \left( \frac{dh_2}{dt} - \frac{dL_1}{dt} \right) \right) dt_1
\]  

(2.4)

Characteristics are fitted by locating a point \((h_2, t_2)\) on the settling curve, the velocity of the interface \((u)\) at that point is then found. A value of \(c_{s2}\) is then obtained from a plot of solids velocity \((u)\) against concentration which has previously been obtained via Coe & Clevenger experiments. Values of \((L_1, t_1)\) are then chosen until the value of \(c_{s2}\) obtained via Equation (2.4) matches the experimental value.
Fitch [1983] simplified Tiller's method, by considering the solids flux at the top of the compression zone (i.e. where no solids stress acts.) Initially \((t=0)\) this is zero as no compaction zone exists but as the zone increases so does the rate at which liquid is expressed, and therefore the subsidence rate and the solids flux also increases. By plotting this on a batch flux curve he was able to argue that characteristics arising from the sediment must do so tangentially. Unlike Tiller he then used a simple material balance to arrive at a value for \(c_{s2}\), the result is shown below as Equation (2.5).

\[
c_{s2} = c_0 \left( \frac{h_i - h_j}{h_i - h_j} \right)
\]  

(2.5)

The latest work in this area has been by Font [1988]; using Fitch's presumption that characteristics arise tangentially to the sediment he modified Tiller's equation to yield the following:

\[
c_{s2} = \frac{c_o h_2}{h_{12} - L_1} \exp \left( \int_{t_2}^{t_1} \frac{1}{t_2 - t_1} dt_1 \right)
\]  

(2.6)

In presuming that characteristics arose tangentially from the sediment surface Fitch and Font eliminated the need for trial and error selection of the location of characteristics.
All of the above methods require a knowledge of the location of the sediment surface with respect to time. With some materials and at low initial concentrations all of the above authors observed that it was possible to track the rise of the sediment visually. At higher concentrations Fitch suggested use of a density scanning device developed by Gaudin and Feurstenau [1962] to follow the compression discontinuity. Where this was not possible Fitch proposed that the rise of the sediment could be obtained by plotting compression points for a series of batch settling tests made at a given initial concentration but at various initial heights (Figure 2-10). In a similar manner Font [1988] used two settling curves of different initial concentrations and joined points of equal settling rate. The sediment curve was obtained by drawing tangential to all the lines which have a positive ordinate at \( t=0 \) (Figure 2-11).

Both Font and Fitch's graphical methods require a knowledge of the location of the compression point. This is the point at which the sediment surface reaches the suspension-liquid interface, any further settling after this is due to compression. Many early studies cited that at the transition between free settling and compression that there was a large change in solids velocity due to the resistance of the solids network and that location of the compression point could be achieved via a Robert's plot (\( \ln(h) \) vs. \( \ln(t) \)) or a velocity-time plot. Subsequent studies have shown that the transition is not always accompanied by a large change in velocity, in these cases
location is best achieved through the use of an imaging system such as the X-ray method of Gaudin and Feurstenau.

![Diagram](image)

**Figure 2-11: Font's method of sediment construction.**

### 2.1.1.2 Force balance

Several researchers Fitch [1979], Auzerais [1988] and Dixon [1977] have suggested force balances which on the whole are very similar. Consider a lamina of suspension, thickness $dx$, and area $A$, assume that flow is vertical, and that slurry properties are uniform across each layer (i.e. no channelling), the forces on the layer are

\[
\text{acceleration} + \text{acceleration} = \text{fluid pressure} + \text{solids pressure}
\]

\[
\text{force on solids} \quad \text{force on fluid} \quad \text{force} \quad \text{force}
\]

The solids acceleration force is defined as \( g - \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) c \rho_s dx \). The term \( \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) \) represents local acceleration (or inertial) forces. This is the form given by Murray [1962] and was used by Fitch [1993] and Auzerais [1988], Dixon [1977] uses a comparable term in his derivation of an overall mass balance. Similarly the liquid acceleration force is defined as \( g - \left( \frac{\partial u_L}{\partial t} + u_L \frac{\partial u_L}{\partial x} \right) (1-c) \rho dx \).
The overall balance is

\[
\left( g - \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial x} \right) \right) (1 - c) \rho \ dx + \left( g - \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) \right) c \ \rho_s \ dx = \frac{\partial P}{\partial x} dx + \frac{\partial p_s}{\partial x} dx
\]

\[ (2.7) \]

\( \frac{\partial P}{\partial x} \) is the liquid pressure gradient which is the sum of a static term \( \rho g \) and a dynamic term due to fluid drag \( (1 - c) \left( u_r - u \right) \frac{\mu}{k} \). Substituting into Equation (2.7) yields

\[
g(r_s - \rho)c - (1 - c) \left( u_r - u \right) \frac{\mu}{k} = \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial x} \right) (1 - c) \rho + \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) c \ \rho_s + \frac{\partial p_s}{\partial x}
\]

\[ (2.8) \]

If the differential layer has a constant concentration then the flow of solids into the layer must be matched by an outgoing flow of liquid so

\[
(1 - c)u_r + cu = 0
\]

so Equation (2.8) simplifies to

\[
g(r_s - \rho)c - \left( 1 - c \right)u \frac{\mu}{k} = \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial x} \right) (1 - c) \rho + \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) c \ \rho_s + \frac{\partial p_s}{\partial x}
\]

\[ (2.10) \]

Equation (2.10) consists of five components, the following sections examine the individual terms contained within each.
2.1.1.2.1 Inertial terms 

\[ \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) (1 - c) \rho + \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) c \rho, \]

The significance of the inertial terms in Equation (2.10) has been subject to much debate, in most current settling models (Auzerais [1988], Font [1991], Landman [1990], Shirato [1971], Fujisaki [1993]) the inertial terms are neglected, Equation (2.10) then becomes:

\[ g(\rho_s - \rho) c - \frac{\mu}{k} \frac{\partial \rho_s}{\partial x} = \frac{\partial \rho_s}{\partial x}. \]  

(2.11)

Fitch [1979, 1993] justified such an assumption by stating "Settling velocities are very low in thickening systems. So except at true or near discontinuities, it might be anticipated that the local acceleration terms are minuscule compared to fluid drag forces and zone settling behaviour will be closely approximated. Furthermore, in a particulate system a true discontinuity cannot exist". This statement was backed up by the work of Shannon [1964] who showed that the settling curves of glass beads could be closely modelled without considering inertial terms. Only Dixon [1976, 1977, 1979] has questioned whether it is possible for inertial terms to be neglected. On the basis of results obtained from a computer model which included inertial terms he argued that expanding zones of graded concentration cannot exist in free settling. His model suggested that there is a rapid concentration change from that of the initial free settling concentration to that of the sediment in zone of constant thickness at the sediment surface. He therefore concluded that graded concentration zones could only exist in the compression region, this would appear to be a reasonable assumption as in free settling Equation (2.11) simplifies to

\[ g(\rho_s - \rho) c = \frac{\mu}{k}, \]

(2.12)

the weight term balances the drag term so there would appear to be no force which can retard the solids and so give rise to a graded zone. However as the experimental results of a great number of researchers proved the existence of graded zones in free settling Dixon later modified his conclusions by suggesting the existence of a transient compressive stress.
2.1.1.2.2 Effective weight \( g(\rho_s - \rho)c \)

This term simply represents the effect of the solids weight, the surrounding fluid provides a buoyancy effect which is allowed for by considering the difference in solid and fluid density. This form is generally accepted however Holdich [1992] has suggested that the fluid density should be the average suspension density. The basis of this suggestion came from models of fluidised beds in which the effective weight of solids \( W_{ef} \) is given by:

\[
W_{ef} = g(\rho_s - \bar{\rho})c \tag{2.13}
\]

Where the average suspension density is:

\[
\bar{\rho} = (1-c)\rho + c\rho_s \tag{2.14}
\]

Since sedimentation and fluidisation share similar constitutive equations Holdich reasoned that the above should be applicable to sedimentation. Equation (2.11), the force balance for sedimentation would then become

\[
g(\rho_s - \bar{\rho})c - \frac{u}{k} = \frac{\partial p_s}{\partial x} \tag{2.15}
\]

This form is acceptable for incompressible systems and concentrations in the free settling regime, however when in compression a portion of the solids weight is supported by the solids network. The remaining portion of the solids weight will still contribute to the liquid pressure gradient, and therefore the buoyancy, but not to the extent given in Equation (2.15). The degree to which the solids weight contributes to the buoyancy therefore becomes a function of the solid's compressibility. Holdich accounted for this by introducing a buoyancy factor \( (b_f) \)

\[
g(\rho_s - \rho)b_f c - \frac{u}{k} = \frac{\partial p_s}{\partial x} \tag{2.16}
\]

where \( 1 \geq b_f \geq (1-c) \). The first limit arises from fully settled solids, the second from fully suspended solids and the relationship \((\rho_s - \bar{\rho}) = (\rho_s - \rho)(1-c)\).
2.1.1.2.3 Solids stress $\frac{\partial p_s}{\partial x}$

In the initial stages of a sedimentation particles floc together and these flocs then settle as separate entities. As no solids stress exists under these conditions then Equation (2.12) is applicable. The settling rate under these conditions is solely a function of solids concentration. At some higher ‘critical’ concentration which is related to the floc size the flocs come into contact with each other and form a solids matrix. If the matrix structure is strong enough then no further settling will occur however in most cases the structure will exhibit some yield stress which if exceeded by the weight of solids above will lead to the matrix breaking down and repacking to a higher concentration. It is also possible at higher solids pressures for the flocs to be broken down giving rise to further compression. Many researchers have suggested relationships to describe the pressure term, some of the more popular versions are given below in Table 2-1.

<table>
<thead>
<tr>
<th>Author</th>
<th>Pressure expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shirato</td>
<td>$p_s = \left( \frac{c}{c'} \right)^{\frac{1}{a}}$ , $p_s = \left( \frac{c}{c'} \right)^{\frac{1}{b}} + b$</td>
</tr>
<tr>
<td>Tiller</td>
<td>$p_s = p_b \ln \left( \frac{c_{m} - c}{c_{m} - c_s} \right)$</td>
</tr>
<tr>
<td>Landman</td>
<td>$p_s = p_a \left( \frac{c}{c_s} \right)^x - 1$ , $p_s = p_b \left( \frac{c}{c_s} \right)^x - 1$ , $p_s = p_b \left[ \frac{c^a}{c_m - c} \right]$</td>
</tr>
</tbody>
</table>

The equations of Landman and Tiller are interesting in that they introduce the term ‘gel concentration’ ($c_s$) this is the concentration at which flocs just come into contact, i.e. below which no solids stress can exist. All of the above equations assume the solids stress to be a sole function of concentration. In his work on the continuous settling of alum sludges Kos [1978] initially fitted the following equation to his experimental data:

$$c = 8 + 0.76647 p_s^{0.578}$$  \hspace{1cm} (2.17)

This form of equation is commonly used to express the relationship between solids pressure and concentration (Font [1988], Tiller [1984]). However Kos was
dissatisfied with the relationship as there was such a large spread of experimental points. As a result he surmised that some other factor must be affecting the solids pressure and postulated that different flow conditions could cause different structures of porous medium at the same concentration, which would logically exhibit different strengths. Mathematically this means that \( p_s = f(c, (u_f - u)\varepsilon) \) where \((u_f - u)\varepsilon\) is the superficial velocity.

Kos then furthered this theory by developing a micromodel of a flocculent porous medium. The porous medium was thought to consist of imaginary capillary tubes, changes in the structure of the medium could then be characterised by changes in the capillary tube diameter. The flow conditions were determined by the superficial velocity this in turn was related to the shear stress acting on the walls of the imaginary capillary tubes. Using a capillary tube model, Kos was able to explain the spread of his experimental points and justify to some degree that \( p_s = f(c, (u_f - u)\varepsilon) \).

Another author to suggest a slightly different form for the solids stress was Dixon [1978], he defined it as

\[
p_s = p_{ss}(c) + p_{sd}(c, \frac{\partial c}{\partial t})
\]

(2.18)

The first term, \( p_{ss} \), is a static stress term, it is only applicable when particles are in contact with each other and so are exerting forces on particles above and below. To mathematically describe this component expressions similar to those suggested by Tiller and Landman were used. The second term, \( p_{sd} \), represents the "transient" or "dynamic" component of solids pressure. This term is dependent on concentration and the rate of change of concentration, \( p_{sd} \) is zero when the rate of change is zero. As discussed previously Dixon assumed that a transient force must exist if graded concentration zones were to exist in free settling. He postulated that such a force must exist to slow particles down as they approach a discontinuity and that it would arise from the excess local pressure needed to squeeze fluid out from between approaching particles.
Another aspect of compression is time dependency, the approach adopted by most researchers is to assume that the application of a certain stress to a lamina of sediment will instantly produce a corresponding concentration. Dixon however questioned whether such a change could be instantaneous hence the inclusion of the term $\partial c/\partial t$ to account for the time delay involved in the compression process.

There are other terms which possibly should be included in a solids stress expression, Fitch [1993] identified them as elastic and osmotic compression. The former would arise from random motion and collision of particles, it could be accounted for using a diffusion term such as $D (\partial c/\partial x)$. Fitch however concludes that such an effect is unlikely to be significant. Osmotic compression was suggested by Auzerais et al [1988], they assumed that a suspension could be treated as a solution, with solid particles acting like solute molecules. Fitch questioned such an assumption as it would tend to lead to disruption of flocs rather than retardation of the settling rate, and as the fraction of solids present is often small the significance of any osmotic pressure is doubtful.

In many recent sedimentation models (Landman [1994], Auzerais [1988]) a parameter has been used to describe the influence of interparticle forces, i.e. the strength of the flocculated network. Auzerais defined this as a Peclet number,

$$Pe = \frac{(\rho - \rho_f)gh_0}{\rho_s(c_0)}$$

(2.19)

For a suspension having an initial concentration within the compression region settling will only occur if $Pe > 1$. It is well known that some systems exhibit an initiation period where very little settling occurs, in these cases, because the solids network structure is only weakly flocculated many cross links may exist so the yield stress is initially high and $Pe < 1$, as time passes the solids structure changes and the yield stress weakens giving $Pe > 1$ and compression occurs.
2.1.1.2.4 Fluid drag $u \frac{\mu}{k}$

As flocs settle downwards they displace fluid in the opposite direction, the action of fluid flowing over the settling floc surface causes a retarding force to be exerted on the floc. In free settling the drag force balances the effective weight force so:

$$g(p_0 - p)c = u \frac{\mu}{k}$$  \hspace{1cm} (2.20)

An expression for permeability in terms of concentration ($k = k(c)$) can therefore be obtained by performing a series of 'Coe & Clevenger' batch experiments and measuring the initial settling rates of different initial concentrations. This technique has usually proved satisfactory, however as Talmadge & Fitch [1955] noted, initial concentration can affect the state of flocculation and therefore permeability. A suspension of initial concentration $c_o = 0.2$ may therefore not have the same permeability as a layer of concentration $c = 0.2$ propagating upwards in a suspension having an initial concentration of $c_o < 0.2$.

In the compression region calculation of permeability values is complicated by the presence of a solids stress term in the force balance (Equation (2.11)). Even if an expression for the solids stress exists such as those suggested in Section 2.1.1.2.3 a value for solids velocity is also required. Several techniques have been suggested to overcome this problem, Michaels and Bolger [1962] analysed the settling curves of suspensions having an initial concentration within the compression region. Although the settling curves were not initially linear, a fact which was attributed to the changing structure of the solids aggregate network they found a constant rate period could be identified later in the run. Runs at the same concentration but different initial heights were performed from which a plot of settling rate as a function of initial height was obtained. The data was then extrapolated to obtain a value for the settling rate at zero initial height i.e. in the absence of compression. This value was then applied to Equation (2.20) to obtain a value for the permeability. Unfortunately the settling rate of suspensions having an initial concentration in the compression region does not always show a constant rate period. Font [1990,94] suggested a number of similar techniques based on the use of Equation (2.11) for the calculation of permeability.
Font assumed tangential characteristics would emanate from the sediment surface and through a mathematical examination of the sediment arrived at an expression for solids velocity. The techniques all required a characteristic plot, concentration-time plot and solids pressure expression to be known. Results which were similar to Michaels & Bolger but over a larger concentration range were obtained. Perhaps the most popular method in recent years (Shirato [1983], Murase [1989], Massarani [1993]) has been the use of downflow cells in conjunction with free settling initial rate data. In downflow cells sediments are further compressed providing permeability values for concentrations just beyond that achieved in gravitational compaction. The permeability of concentrations in the compression regime are assumed to lie between the data of the downflow and initial rate tests.

Several expressions for permeability exist, most are one dimensional, the early equations come from work on flow through porous mediums, some typical expressions are shown in Table 2-2. The expression suggested by Kos is based on the same argument discussed in Section 2.1.1.2.3, that is to say that the permeability of a layer is determined not only by the concentration but by the flow through it.

An alternative approach to the fluid drag term has been adopted by Auzerais [1987], Landman [1990,92,94], Buscall [1987]. The effect of hydrodynamic interactions are taken into account by multiplying the drag coefficient by an interaction factor called the hindered settling factor $r(c)$. The drag term in the sedimentation equation has the form

$$-\frac{g(\rho_s - \rho)r(c)c(t)}{u_i}(u - u_f)$$

(2.21)

As $c \to 0$ then $r(c) \to 0$ and as $c \to c_f$ then $r(c) \to \infty$. Obviously a relationship between the solids velocity $u$ and $r(c)$ is required. This has usually been determined by initial rate experiments however as discussed previously there are flaws with this technique so alternate methods such as the use of vacuum filtration and downflow data for obtaining $r(c)$ have been suggested (Landman [1992]).
Table 2-2: Typical permeability relationships.

<table>
<thead>
<tr>
<th>Author</th>
<th>Permeability expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kozeny-Carman</td>
<td>[ k = \frac{(1-c)^3}{Kc^2S_v^2} ]</td>
</tr>
<tr>
<td>Carman [1937]</td>
<td></td>
</tr>
<tr>
<td>Happel Brenner</td>
<td>[ k = \frac{2 - 3c^3 + 3c^3 - 2c^5}{12c(3+2c^3)}x_{sd}^2 ]</td>
</tr>
<tr>
<td>[1965]</td>
<td></td>
</tr>
<tr>
<td>Brinkman</td>
<td>[ k = \frac{x_{sd}^2(2-3c)^2}{18c(3c+4+3(8c-3c^2)^{1/2})} ]</td>
</tr>
<tr>
<td>[1947]</td>
<td></td>
</tr>
<tr>
<td>Kos</td>
<td>[ k = k(c,u-u_f) ]</td>
</tr>
<tr>
<td>[1978]</td>
<td></td>
</tr>
</tbody>
</table>

It should be noted that any drag term should also contain a virtual-mass term. When a particle is accelerating through a fluid it experiences a greater drag at a given velocity than if it were moving constantly at that velocity, this extra drag is the virtual mass term. The same effect will occur with suspensions, however there is no knowledge of the way in which the virtual mass coefficient will vary with concentration so it has been omitted from the above balance.

2.1.2 Channelling

Channelling to date has only been investigated by means of visual observations during batch settling tests. Such observations have shown that channels require an appreciable time to form, and often disappear towards the end of the batch test. During the period in which channels are found near the surface of a settling suspension volcanoes may be observed on the surface marking the top of channels. An illustration of a sedimentation displaying channelling with surface volcanoes is given in Figure 2-12. Volcanoes are formed by the removal of solids from channel walls, the solids become entrained in the upward flow of liquid and are carried to the
suspension surface where the liquid flow subsides resulting in the deposition of solids are deposited at the mouth of channels.

Figure 2-12: Illustration of volcanoes and channels during sedimentation. Channels provide a means by which the liquid in the suspension may rise towards the supernatant without having to pass through a tortuous porous media. Thus the rate of descent of the suspension/supernatant interface often increases rapidly when channels occur at the interface. Under such circumstances the interface settling curve tends to show an acceleration period which can be identified as a convex section similar to the initiation period but later in the run.

If channelling increases the settling rate of the interface, the solid settling flux (mass per unit area and time) must also be enhanced. This important consequence has led to the conclusion that industrial thickeners, which are often continuous rather than batch in operation, may be run more intensively if channelling can be promoted. A new style of continuous thickener in which channelling is encouraged has resulted (Chandler [1983]).

The formation of channels has been attributed to many different effects. Glasrud [1993] observed the settling behaviour of iron oxide in mineral oil using a video
camera with high magnification lenses. Channelling occurred initially in the top segment of the suspension and propagated downwards slowly. The channels were formed by rising air bubbles. When degassed the settling rates were significantly retarded. Harris [1975] reported the settling of dilute phosphatic slimes in which coarser particles had been added to hasten the settling rate. The coarser particles caused tears in the suspension which subsequently developed into channels. In both of these studies channelling was artificially induced, but the results demonstrate that the suspensions investigated exhibited sufficient structure to maintain a channelling environment after the removal of a perturbing phase.

Random channelling, i.e. in the absence of materials to initiate channel formation, has been observed with many homogeneous materials and systems. Calcium carbonate suspensions were investigated by Dell [1968]: channels were observed initially in the lower regions of suspensions, followed by a period in which they propagated upwards until they met the descending suspension interface. The rate at which a channelling zone propagated upwards increased with increasing dosage of flocculant. Vesilind [1990,93] also investigated channelling during the settlement of calcium carbonate suspensions, and concluded:

i) that channels only occur at intermediate concentrations,

ii) the upper region of a suspension (i.e. near the supernatant interface) is progressively diluted,

iii) channel zone velocity is constant,

iv) the higher the initial concentration the higher the zone of channelling will appear in the suspension, and

v) channels form just above the compression region.

Dilution of the upper region of suspensions during batch sedimentations which exhibit channelling was also reported by Kos [1978]. Unlike Chandler, however, Kos observed channelling only during batch thickening; no channels were present when continuously thickening the same material. He went on to postulate that the permeability of a settling layer is a function of both the solids concentration and the rate of change of solids concentration with respect to time. The latter tends to zero.
during continuous thickening and so permeability would be a sole function of solids concentration, i.e. a Darcian flow regime would exist with no channelling.

![Vesilind Plot](image1)

![Tory Plot](image2)

- a-b constant rate
- b-c first falling rate
- c-d second falling rate
- d-e channelling
- f-g compression
- --- discontinuity

**Figure 2-13: Example of Tory and Vesilind plot.**

Often, in systems which exhibit channelling, significant changes in a batch settling curve are difficult to see and interpret directly. This problem was first recognised by Tory [1961] who suggested an alternative to the conventional interface height-time plot, the method was later revised by Fitch [1990]. Interface velocity was plotted against interface height and as a result changes in settling behaviour became more evident. Vesilind [1990] further revised the technique by plotting interface velocity against time, compared with Tory plots this has proved to be a quicker and equally illuminating technique. An example of both types of plot is given in Figure 2-13, all the zones that were identified in Figure 2-8 and also the effect of channelling are included, although it should be noted that the existence of a falling rate zone in the presence of channelling is very rare. The plots are therefore a composite of the possible behaviour of a sedimenting suspension.
2.1.3 Effect of stirring during sedimentation

When stirring, the loose aggregates which form the sediment contain trapped voids filled with water which are believed to be disrupted, causing the particles to rearrange into a closer random packing arrangement. There are several reports in the literature that suggest that stirring during the sedimentation of calcium carbonate suspensions can lead to higher sediment concentrations than in the absence of stirring (Kammermeyer [1941], Comings [1954], Scott [1968]), in accordance with the belief that loose sediment aggregates can be disrupted.

Channels can only form, or continue to exist, if the suspension or sediment can exhibit some form of structure or mechanism which prevents particles from accumulating in the channel. Stirring can therefore be used as a technique for disrupting the formation of channels as it tends to break-up any structure formation. A consequence of this is that while stirring may increase the final sediment concentration it also decreases the settling rate by reducing or eradicating channelling. In order to preserve increased settling rates and to maximise the settled sediment concentration stirring would have to be restricted to only the bottom of a thickener (Dixon [1979]).

Stirring is often employed during the settlement of biological sludges as a means of encouraging the release of gas bubbles which would otherwise adhere to the solids thus impeding the sedimentation of the biological floc. Thus in this instance stirring would increase the settling rate of the suspension. Stirring may also assist in overcoming wall support effects (Kos [1978]) when settling mineral suspensions in narrow columns. If, however, the stirring rate is too great then the particles will remain suspended. In general, gentle stirring is usually recommended during batch sedimentation tests as a means to obtain more reproducible experimental data (WPCF [1985]). However, the effect of stirring will differ according to the material and conditions under study.
2.2 Continuous thickening

At first glance the operation and concept of continuous thickening may appear relatively simple. A dilute slurry is pumped into a column through a feedwell whereupon a certain portion of the slurry's liquid phase will separate out and pass upwards into the overflow. The remaining slurry is drawn downwards to the underflow as it does so the slurry thickens so an underflow concentration which is larger than the feed suspension is obtained. In reality effects such as the action of rakes, channelling and sludge funnelling coupled with the existence of capacity limiting fluxes mean that the design and operation of thickeners is a far from simple task.

A typical thickener is shown in Figure 2-14, as with batch thickening several distinct zones can be identified. These zones have been identified by a number of authors (Hassett [1958,61], Khatib [1979], Comings [1954], Dixon [1977,81], Fitch [1966,93]). It should be noted that the zones shown were not necessarily found in every thickener examined, Dixon and Fitch in particular found no evidence of zone D.

![Figure 2-14: Possible zones of settling behaviour in a continuous thickener.](image)

Zone A is the overflow and should unless the thickener is overloaded contain a negligible amount of solids. The next zone is the feed zone, solids are pumped in via a feedwell and then spread out horizontally as a "density current". Fitch [1990]
showed that the propagation of characteristics in this zone was governed by the same mathematics as a Kynch characteristic. The type of feedwell used is important as it has been shown (Scott [1972]) to affect the state of flocculation and therefore the subsequent behaviour of the solids. Zone C is the hindered settling region, solids settle as flocs at a constant rate. The size of this zone depends on the location of the feedwell relative to the sediment surface, in a well designed thickener this zone should be as small as possible as it serves no purpose in terms of thickening the solids. Zone D is a transitional zone similar to the falling rate zones found in batch thickening, a fan of concentrations from that of zone C to that of the sediment surface arise. As noted above the existence of such a zone is not always noticed, it is more common for solids to settle discontinuously between the two concentrations or over a very small undetectable transitional zone. Zone E is the compression region, it is the size of this zone and the compressibility of the material that largely determines the underflow concentration.

2.2.1 Theory

![Diagram](image)

**Figure 2-15: Explanation of the terms used in the continuous thickener material balance.**

A material balance over a continuous thickener is very simple.

\[ F = B + Y \]  \hspace{1cm} (2.22)

Similarly a balance on the solids yields

\[ Fc_o = Bc_b + Yc_u \]  \hspace{1cm} (2.23)

If a thickener is being operated at steady state and is not overloaded then \( c_b = 0 \) so

\[ Fc_o = Yc_u \]  \hspace{1cm} (2.24)
If a system is at steady state then the total solids flux \( (\phi_t) \) must be the same at all points below the feedwell, it is therefore possible to introduce a third term into Equation (2.24) which relates the flux \( (\phi_t) \) in the thickener to the inlet and outlet conditions.

\[
Fc_o = Yc_o = A\phi_t
\]

Equation (2.25) has obvious uses in the design of thickeners, it is therefore not surprising that the majority of current design techniques rely upon the development of flux curves. It is useful at this point to outline the fluxes which exist within a thickener. In a continuous thickener there are two forces driving the solids towards the underflow, one due to gravity and the other due to the physical removal of slurry at the underflow. The total solids flux is given by

\[
\phi_t = \phi_s + \phi_u = c(u + v_s)
\]

where \( u \) is the solids settling velocity and \( v_s \) is the velocity of the suspension, if there is no further settling after the underflow then \( v_s = \frac{V}{A} \). If these fluxes are plotted as a function of concentration then a typical result would be as shown in Figure 2-16.

![Fluxes within a continuous thickener.](image)

**Figure 2-16: Fluxes within a continuous thickener.**

### 2.2.2 Design and operation

The design of thickeners is based on a postulation by Coe & Clevenger [1916] that there exists a critical concentration \( (c_c) \) at which the solids flux \( (\phi_t) \) is minimum, it is this flux \( (\phi_c) \) which dictates the capacity of thickeners and in Equation (2.25) \( \phi_c = \phi_t \).
An example of a typical flux curve showing the location of the critical flux is shown in Figure 2-16. As previously discussed in Section 2.1.1.1.1 it is the development of the solids settling flux ($\phi_s$) curve that gives rise to most errors encountered in the design of a thickeners. Whilst constructions similar to that shown in Figure 2-16 are valid a simpler graphical technique (Yoshioka [1957]) exists for locating the minimum flux. Rearrangement of Equation (2.26) yields

$$\phi_s = \phi_r - cv_s$$

(2.27)

At the underflow there is assumed to be no further settling of solids so the total flux is that due to underflow withdrawal i.e. $\phi_r = \nu_c c_s$ so

$$\phi_r = \phi_s - \left( \frac{\phi_L}{c_u} \right) c$$

(2.28)

This straight line equation can be thought of as an operating line, the maximum capacity of a thickener is the point at which this line becomes tangential to the solids settling flux as shown by line a in Figure 2-17.

![Figure 2-17: Yoshioka construction.](image)

The underflow concentration and critical flux are given by the x and y intercepts respectively, other operating conditions can be obtained by applying Equation (2.25). Line b is also a feasible operating condition, in this instance the thickener is operated below its maximum capacity. An overloaded thickener is the result of operating at conditions such as those shown by line c, the flux of solids into the thickener can not be accommodated by the solids settling flux so solids pass upwards into the overflow.

The various operating conditions and responses of thickeners to changes in these
conditions has been well documented by authors such as Tiller [1988], Dixon [1980], Khatib [1979] and Maljian [1978].

2.2.3 Sludge funnelling

![Diagram of sludge funnelling](image)

**Figure 2-18: Example of sludge funnelling.**

This is an effect which is widely accepted to occur in many thickeners it is however very rarely accounted for in the modelling of thickeners. Settled solids in cylindrical or shallow cone thickeners will often build up into a stagnant cone (see Figure 2-18), the angle of which is usually determined by the material's angle of repose or by the location of any raking mechanism. Dixon [1980] developed a model for a steep sided (60°) cone thickener and was able to show that the operating line was significantly different from that of a cylindrical thickener, in particular he found that near the bottom of the thickener the operating line tended to be much closer to the limiting flux. He suggested that while it was unlikely that any material would form such a perfect cone it was conceivable that a thickener would exhibit behaviour which lay between that of conical and cylindrical. Landman [1988] furthered Dixon's work by developing a model which could predict the behaviour of any angle of cone including diverging thickeners. As a result he was able to show that greater sediment heights were required to achieve the same underflow concentrations in converging thickeners than in cylindrical or diverging. There was also a limit to the underflow concentration which can be achieved in converging or cylindrical, the latter being greater than the former. This would seem to suggest that diverging thickeners could be advantageous,
unfortunately the construction of such a thickener particularly the rake mechanism makes this an unfeasible option.

![Diagram](image)

**Figure 2-19: Chandler's comparison of various dewatering techniques.**

### 2.2.4 Channelling in continuous thickeners

The use of pickets and rakes in most commercial thickeners means that very few are operated to take advantage of the increased flux rates that channelling promotes. This is probably also due to the relatively small sediment heights that most use. Only Chandler [1983] has discussed the advantages of operating rakeless deep bed thickeners, typically a height to diameter ratio greater than one. He reported the behaviour of 'red mud' in Alcan thickeners which had a very steep cone (60°). These thickeners showed greater throughputs and underflow concentrations than expected. Chandler commented that it was difficult to conceive that any liquid separated from the solid in the lower region could permeate through the thickness of the compacting bed above unless channels were forming. He then demonstrated the advantage of deep bed thickeners over conventional thickeners by considering the number of equilibrium stages that would be required to achieve a desired washing effect (i.e. the desired ratio of $c_o$ to $c_u$), a copy of his results is shown in Figure 2-19. It can be seen that the number of deep thickening stages required to achieve the same washing effect
is little over half the number of conventional stages. In terms of overall costs deep bed thickening will usually be lower in cost than conventional. The comparison between deep bed thickeners and filters depends on the energy and maintenance requirements of the filter and the amount of flocculant required by the thickener. The cost per stage is likely to be less for deep bed thickening but filtration usually requires fewer stages.

2.2.5 Rake mechanisms

In general most industrial thickeners employ some form of rake or picket fence or a combination of both. The purpose of the rake is that of a scraper which transports settled solids across the bottom of the thickener towards the underflow withdrawal point. To some degree this mechanism will also prevent sludge funnelling. It has been shown (Comings [1954]) that gentle stirring (typically a speed of 1 rpm) with picket fences can result in increased sediment concentrations, it also eliminates any channels making the suspension settling behaviour more predictable.

2.2.6 Depth of compaction zone

In 1954 Comings showed that higher sediments would result in higher underflow concentrations. Therefore the depth of the compression zone is often the most important factor in deciding the height of thickeners. Tiller [1980] and Kos [1978] both suggested that if Equation (2.26) is rearranged in terms of $u$

$$u = \phi \left(1 \frac{1}{c} \frac{1}{c_u} \right)$$  \hspace{1cm} (2.29)

and then substituted into the basic sedimentation Equation (2.11) then:

$$\frac{\partial p_s}{\partial x} = g(\rho_s - \rho)c - \frac{\mu}{k} \phi \left(1 \frac{1}{c} \frac{1}{c_u} \right)$$  \hspace{1cm} (2.30)

This equation can be integrated only if relationships linking $k$, $c$, $p_s$ exist. Several such expressions exist and were outlined in Section 2.1. Another approach has been adopted by Fitch [1966,90,93], he assumed Darcy's law to be applicable at low Reynolds numbers so was able to consider the compressive stress in terms of the
fluxes within a thickener by considering it to be the difference between the unbuoyed weight of solids and the fraction supported by fluid drag. He derived the following:

\[ z = \int \left( \frac{dx}{c_s c} \right) \left( \frac{\phi_s}{\phi_s - \phi_t} \right) dc \]  

(2.31)

Where \( \left( \frac{dx}{dc} \right)^r \) is the inverse of the concentration gradient that would be observed in a batch test after all subsidence is complete. This technique requires a knowledge of \( \phi_t \) which can be obtained from Equation (2.25) and \( \phi_s \), the problems in obtaining \( \phi_s \) were outlined in Section 2.1.
3. Experimental apparatus and method

The aim of the project was to study the settling behaviour of various materials using a solids imaging system. The imaging requirements were that the whole height of a suspension could be quickly scanned and at regular intervals. Versatility in terms of the materials monitored was also necessary and applicability to compressible channelling suspensions was essential. The methods available for monitoring solids concentration had been well reviewed by Williams et al. [1990] and Dickin et al [1992], as a result several imaging techniques were disregarded.

Direct sampling was disregarded as it would have been difficult to sample the whole column rapidly without causing too much disturbance and there would have been a large amount of post experiment sample analysis. Pressure measurements were not used as they are only applicable to incompressible systems or those in which the solids stress - local concentration relationship is known. The use of radiation absorption or light scattering was unfeasible as it was anticipated that some sedimentations would be relatively fast and/or settle to opaque high concentration sediments.

Ultrasonics and capacitance measurements were both possible techniques however the former is limited to materials of low density that do not cause multiple scattering and the latter is only applicable when the continuous phase is non-conducting. The procedure best suited to our requirements was the measurement of solids concentrations using an electrical impedance technique.

Imaging dispersions by electrical conductivity, or impedance, has become a popular method for the laboratory study of unit operations relevant to the process industries, including: mixing (Dicken [1993]), hydraulic conveying (Mckee [1993]), filtration (Wakeman [1981]) and various separations (Holdich [1992], Summers [1993]). Considerable effort is being made into extending the imaging to provide a two or even three dimensional map of concentration with location and processing time. In the experimental work described in this study image reconstruction was simplified by assuming a uniform dispersion concentration between the electrodes, i.e. a single
dimension system, varying with time. The use of a single dimension eliminated the errors introduced by complicated image reconstruction algorithms and, therefore, provided a means of checking some of the fundamental assumptions necessary for electrical conductivity imaging in a greater number of dimensions.

All imaging systems rely upon the conversion of an electrical signal to concentration via an empirical equation (Holdich [1992], Shirato [1971])

\[ \Sigma = \frac{\sigma_m}{\sigma_c} = (1 - c)^n \quad \text{(3.1), or} \quad c = 1 - \exp \left[ n \left( \frac{\sigma_c}{\sigma_m} - 1 \right) \right] \quad \text{(3.2)} \]

where \( c \) is the volume fraction concentration, \( n \) an empirical exponent, \( \sigma_m \) and \( \sigma_c \) are the conductivity's of the mixture and continuous phases respectively. Equation (3.1) is known in reservoir modelling as Archie's law. It is derived from considering the first two terms of a Taylor series expansion of an equation derived by Maxwell in his work on electricity and magnetism. It is assumed that the solid particles are non-conducting spheres which interfere with the electric flux field. Bruggemann [1935] deduced a theoretical value for the exponent of \( n = 1.5 \). However as most systems don't consist of non-conducting spheres it is usual to employ an empirical value for the exponent

3.1 Batch and continuous thickeners

A schematic of the system used to monitor batch sedimentations is shown in Figure 3-1. Two batch cells of different diameters (61.5 mm & 140 mm) were constructed to assess the magnitude of any wall effects. The experimental work of Kammermeyer [1941] suggested that the larger cell would show negligible wall effects therefore when the settling behaviour of a material was the same in both cells the smaller diameter rig was used as the suspension handling (mixing and metering) was considerably easier. It also reduced the quantity (and therefore cost) of materials. All the cells were constructed from clear perspex to allow visual observations of channelling and the suspension-liquid interface to be made. Pairs of electrodes were placed diametrically opposite in the cell walls at set intervals over the whole height of all the columns. Exact electrode spacings for each cell are given in Appendix A.
The smaller batch cell was mixed simply by inverting the cell by hand. The number of inversions required to obtain a uniform dispersion varied from material to material, however for a set of experiments on any one material the number of inversions was kept constant. The weight and size of the larger cell meant that the suspension had to be premixed by hand in a separate container using a small paddle and then poured through a funnel arrangement into the cell. If required, both cells had the facility to be fitted with a picket fence type stirrer which was driven by a stepper motor which was housed on the lid of the cells.

A continuous rig was also constructed (Figure 3-2) using the same materials and electronics as the batch system. The rig diameter was a 150 mm. The rig was run as a closed loop, the underflow and overflow being returned to the feed tank. The rig was fed from a continuously agitated tank, a Spirax motor fitted with a pitch blade turbine was utilised.
3.1.1 Data collection and interpretation

The mechanism for the collection of electrical conductivity readings over the height of a sedimentation column is shown in Figure 3-3, the same configuration was used on the continuous rig. Data collection was achieved using a 286 IBM personal computer fitted with a Fairchild PCL 711s labcard. The labcard provided a binary output which coupled with an arrangement of multiplexers allowed individual electrode pairs to be selected and scanned. The resulting voltage was then fed back to the labcard which converted the voltage into an A/D value which was logged by the computer.
electrode pairs are selected by a signal from the computer

![Measuring circuit diagram](image)

Figure 3-3: Measuring circuit.

A selected electrode pair became part of the measuring circuit shown in Figure 3-3, the signal from which was amplified and rectified before being fed to the labcard.

A value for the ratio of the suspension conductivity $\sigma_m$ and liquid phase conductivity $\sigma_c$ (see Equation 3.1) was obtained via the following calculations. Conductivity was defined as

$$\sigma = \frac{d}{rA_e}$$

(3.3)

where $d$ was the distance between electrodes, $A_e$ is the electrode area and $r$ is the resistance between them. The subscripts $m$ and $c$ are used to represent the mixture and continuous phase respectively so

$$\Sigma = \frac{A_c d_m}{A_m d_c} \frac{r_c}{r_m}$$

(3.4)

Since both conductivity's (mixture and continuous) were measured in the same cell it was reasonable to take the bracketed term as being equal to unity and $\Sigma$ therefore was simply the ratio of the two resistances. However calibration between electrode pairs was necessary because although ideally all the electrodes should have been identical, minute geometry and placing differences led to small variations in the values returned by
electrode pairs measuring the same conductivity. Fortunately these variations were linear, so by recording all electrode pair readings at different constant conductivity's calibration relative to one pair could be achieved. This was done by performing a regression on the conductivity’s returned by each pair and those of the calibration pair. This gave a straight line equation which converted the reading at a particular pair into that of the calibration pair, satisfactory calibrations ($r^2 > 0.999$ for each regression) were obtained.

The final output of the measuring system was in the form of a voltage the relation of this voltage to the ratio of resistances in Equation (3.4) was found by considering the potential at point A in the measuring circuit to be $V_I$ and the total voltage across the measuring circuit to be $V_a$ so

$$V_I = \left(\frac{r_k}{r_k + r_l}\right)V_a \quad (3.5)$$

where $r_k$ was a known resistance and $r_l$ the resistance across the electrodes. Rearranging this in terms of $r_l$

$$r_l = \left(\frac{V_a}{V_I} - 1\right)r_k \quad (3.6)$$

A value for $r_c$ (the resistance of the continuous phase) was obtained from an electrode pair in the liquid phase or by measuring the resistance prior to the experiment beginning. Hence

$$\Sigma = \frac{r_c}{r_m} = \frac{\left(\frac{V_a}{V_c} - 1\right)}{\left(\frac{V_a}{V_I} - 1\right)} \quad (3.7)$$

A square wave input of 5 V oscillating around earth was used, this prevented any polarising effects (Section 3.5.3, Holdich [1992]). The electrodes had machined points, a diameter of approximately 1.5 mm and were sunk 5 mm into the interior of the tube to eliminate edge effects. The most reliable electrode material, in terms of giving constant calibration constants, was found to be stainless steel.
3.1.2 Computer programs

All data logging programs were written in Turbo Pascal. The programs ran in a loop which continuously checked the computer's internal clock until a sample time was reached (sample times were pre-set depending on the duration of the experiment). The computer then activated the PCL labcard and began stepping through the channels recording A/D values from the labcard as it did so. The programs then converted the A/D values into concentrations and so provided a continuously updated concentration-height profile during the experiment which was displayed on the computer monitor. On termination of the program the raw A/D values were outputted as a text file allowing subsequent processing by other programs or import into a spreadsheet.

3.1.3 Temperature control

Temperature control was essential on all the rigs. The reason for this was twofold, it had been reported (Kos [1978]) that suspension settling behaviour could be significantly affected by a very small change in temperature (4-5 °C). Similarly it was anticipated that a small change would significantly affect the conductivity of a system. This would not be a problem if the change occurred evenly and instantaneously throughout the cell, however, this seemed unlikely. All the rigs were housed in insulating chipboard containers, temperature control of ± 0.5°C was achieved using a Smiths ETU5 thermostat coupled to two RS heating plates, or in the case of the smaller batch cell two 240V light bulbs sufficed. The temperature of the feed tank on the continuous rig was maintained by a Techne temperature control unit. All experimental runs were conducted at an operating temperature of 25°C.

3.1.4 Pumps

Two pumps were required to operate the continuous rig. A Watson Marlow 503 peristaltic pump provided a feed flowrate of up to 4 ml s⁻¹. The same type of pump was used on the underflow except in cases where a very low flowrate was required. A smaller RS peristaltic pump was then used as the larger pump could not provide a constant flowrate.
3.1.5 Sedimentation vessel stirrers

A number of factors had to be considered in designing the stirrers. The main requirement in most industrial thickening applications is that the suspensions are stirred at a very slow constant speed so at higher solid concentrations a relatively high torque would be required. Additionally, variable stirring speeds were required as the degree of disturbance to eliminate channelling from each material would probably be different. Initial studies had also shown that the position of the stirrer blade could affect the conductivity readings so a motor which could be stopped and started in a certain position to allow for scanning was required. The motor which best suited these requirements was a 1.8° RS stepper motor. Its main advantage was that it could be controlled directly by the data logging computer, this generated via the PCL labcard a square wave signal which drove the stepper motor driver board. The speed of stirring was simply controlled by varying the frequency of the pulse, and the location of the stirrer blade at the time of sampling could be controlled by counting the number of pulses. This meant that the stirrer could be stopped in the same orientation whenever a reading was required so any conductivity distortion was constant.

The stirrer blade for the batch rig was a simple picket type fence. The continuous-rig stirrer was of a similar design, except it was constructed to allow different picket heights and it also had a small rake arm located at the bottom to funnel sediment towards the underflow withdrawal point. The exact design of both stirrer blades is given in Appendix A. Ideally these stirrer blades would have been constructed from some form of stainless steel, however it was felt that the presence of a conducting medium could cause unwanted flux distortions so a non-conducting material, nylon, was used. The disadvantage of this was that in order to achieve a sufficiently rigid blade the rake arm diameter had to be somewhat larger in relation to the column diameter than would normally be found in industrial applications.
3.2 Upflow cell

This was basically a fluidisation cell, constructed and instrumented in a similar manner to the batch and continuous settling rig. A schematic of the rig is shown in Figure 3-4. Fluid was pumped from a temperature controlled bath into the base of the cell where it was dispersed by a bed of ballotini beads (>500μm) which was held in place by two 450 μm gauzes. The suspension material was introduced to the system at the top of the cell via a funnel arrangement. The system was a closed loop so overflowing liquid was returned to the bath. The purpose of this rig was to obtain additional data on the permeability of the materials investigated. Several researchers (Kos [1978], Fitch [1975]) had also reported it as a useful test in examining how the structure of a flocculent medium in compression changes according to the flow through it. It was also thought that it may prove a useful method for inducing channelling in materials which didn't normally channel because the flow of liquid up through the sediment was too small to support any channel structures.
3.3 Downflow cell

This rig was constructed to allow measurement of permeability values at higher concentrations. A schematic of the rig is shown in Figure 3-5, the downflow cell was instrumented in the same manner as all the other rigs but with the addition of two RS pressure sensors. These were located 40 mm and 70 mm from the base of the cell and gave readings which were precise to 0.001 bar. Housed at the base of the cell was a Johnson Progress JSYN32 filter cloth which was supported by a perforated perspex disc. Suspension was fed into the cell and allowed to settle to completion. Further compression of the sediment was then caused by withdrawing liquid using a peristaltic pump, this was then recycled to the top of the suspension. If insufficient compression was obtained using the pump then the air space at the top of the vessel could be pressurised. A knowledge of the sediment height, the pressure drop between the two sensors and the flowrate through the sediment allowed calculation of permeability values.

Figure 3-5: Downflow cell.
3.4 Materials

In all of the experiments conducted the fluid phase was a weak \(10^{-5}\text{mol}\) potassium nitrate solution. This very dilute electrolyte was employed to form a buffer solution; providing a constant ionic strength and enhancing the electrical conductivity of the distilled water used as the continuous phase to facilitate electrical conductivity measurement. The density of materials tested was found using a helium pycnometer or using density bottles and distilled water. A Coulter LS150 was used to obtain a size distribution. This is a device which employs two types of size detector: Fraunhofer laser light diffraction and light diffusion analyses. Images of the particle shape were obtained from a scanning electron microscope.

As discussed earlier most non-ideal materials require an empirical value for the exponent in Maxwell's equation. However very little work has been reported in which values of the exponent have been fitted and so the suitability of various materials to the electrical impedance technique was unknown. Of equal importance was that as well as being suited to this form of imaging a material also had to exhibit the required settling behaviour. An extensive part of this project was therefore spent finding materials which satisfied both criteria.

There are a number of techniques which can be used to evaluate the exponent value. The simplest method is to measure the initial conductivity \((\sigma_m)\) of a well mixed suspension of known concentration, a value for the continuous phase \((\sigma_c)\) is obtained after the suspension interface has subsided past the top most electrodes. This test is repeated over a range of concentrations and an exponent value is fitted to the data by performing a regression on the logarithmic values of \(\Sigma\) (where \(\Sigma = \sigma_m/\sigma_c\)) and \(c\), the exponent is the resulting gradient. This method is satisfactory for incompressible systems however in compressible systems it is difficult to obtain conductivity readings at known higher concentrations. Also with some materials when an initiation period was exhibited the change in structure of the solids network although at constant concentration caused conductivity readings to rapidly decrease before attaining some constant value. Initial concentration is known to affect the state of flocculation so it
would seem unwise to employ a technique which relies on initial values as whilst accurate initially it may not be accurate for a layer of concentration which has arisen from a lower initial concentration. Another technique for determining the exponent was therefore employed, although the latter was still used to give a first estimate for the following method. The method used a simple mass balance which allowed not only the exponent to be determined but also the accuracy of the system for each material. Analysis of conductivity readings using Equation (3.1) allows a concentration-height profile to be generated, the apparent volume of solids present can then be obtained from

\[
\text{Volume of solids } (V_s) = A \int_0^t c \, dx
\]  

(3.8)

This can then be compared with the actual volume of solids \( (V_{\text{act}}) \) to give a percentage error in volumes

\[
\text{Percentage error in volume (PEV)} = \frac{V_{\text{act}} - V_s}{V_{\text{act}}} \times 100
\]

(3.9)

The exponent value is varied until the smallest deviation in errors over the period of a sedimentation is obtained.

In order to do this the solids had to be compatible with the electrical impedance technique. The following is a review of the compatibility of popular sedimentation materials with the electrical impedance technique. For each material the percentage error in volumes (PEV's) were computed from the concentration at which a distinct interface could be distinguished to the concentration at which the material became impossible to mix thoroughly.

3.4.1 Ballotini

The electrical impedance technique was commissioned using a material of known settling behaviour, ballotini, which exhibits typical incompressible behaviour. A scanning electron microscope (SEM) picture of the ballotini particles is shown in Figure 3-6, a size distribution is given in Table 3-1, the ballotini used had a spherical
particle shape with a mean particle size of 44 μm and a density of 2923 kg m$^{-3}$. It was therefore expected to approximate closely to Maxwell’s theoretical model.

The PEV’s for a number of sedimentations are shown in Figure 3-7. The exponent value of 1.33 is somewhat different to Maxwell’s value of 1.5, this is attributed to the fact that Maxwell’s work was based on low concentration dispersions whereas relatively high solids concentrations were required to obtain a distinct settling interface for ballotini. The settling characteristics, in particular the final sediment concentration, and concentration profiles, closely matched that expected for an incompressible spherical material obtained. An experimental error of ±3% was quite acceptable so the system was considered to be operating adequately.

3.4.2 Kaolin

A material which has been extensively used in sedimentation research is kaolin, Michael & Bolger [1962] reported its tendency to channel at intermediate conditions and its compressive nature at higher concentrations. The kaolin used had a density of 2690 kg m$^{-3}$ and a mean particle size of 7 μm. The particle size distribution given in Table 3-2 was obtained via dilution using distilled water, no pH control was used. The smallest PEV’s were obtained using exponent value of 2.1, unfortunately as Figure 3-9 shows this still resulted in unacceptably large (±30%) errors, particularly in the early stages of the low initial concentration runs. This result can be explained by examining the result of using the initial conductivity method of fitting the exponent, as can be seen from Figure 3-10 a good fit between Archies Equation (3.1) and experimental data is obtained at higher concentrations but at lower concentrations a large deviation occurs. Better PEV’s can be obtained (Figure 3-11) if the straight line fit shown in Figure 3-10 is employed rather than Equation (3.1), however the errors are still large.
Figure 3-6: Scanning electron microscope image of ballotini.

Table 3-1: Particle size distribution of ballotini.

<table>
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<tr>
<th>Particle Diameter µm</th>
<th>14.96</th>
<th>16.38</th>
<th>17.94</th>
<th>19.65</th>
<th>21.53</th>
<th>23.58</th>
<th>25.83</th>
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</thead>
<tbody>
<tr>
<td>Cum. &lt; Volume %</td>
<td>4.7</td>
<td>4.79</td>
<td>4.9</td>
<td>5.09</td>
<td>5.44</td>
<td>6.1</td>
<td>7.3</td>
<td>9.34</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>30.99</th>
<th>33.94</th>
<th>37.18</th>
<th>40.72</th>
<th>44.6</th>
<th>48.85</th>
<th>53.51</th>
<th>58.61</th>
<th>64.20</th>
<th>70.32</th>
<th>77.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>12.6</td>
<td>17.55</td>
<td>24.68</td>
<td>34.38</td>
<td>46.70</td>
<td>60.97</td>
<td>75.47</td>
<td>88.04</td>
<td>96.66</td>
<td>99.76</td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 3-7: PEV’s for ballotini
Figure 3-8: Scanning electron microscope image of kaolin.

Table 3-2: Particle size distribution of kaolin.

<table>
<thead>
<tr>
<th>Particle Diameter µm</th>
<th>0.1</th>
<th>0.17</th>
<th>0.30</th>
<th>0.52</th>
<th>0.89</th>
<th>1.54</th>
<th>2.65</th>
<th>4.58</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cum. &lt; Volume%</td>
<td>0</td>
<td>0.05</td>
<td>0.61</td>
<td>2.43</td>
<td>5.95</td>
<td>11.25</td>
<td>19.28</td>
<td>33.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial concentration, w/v.</th>
<th>0.019</th>
<th>0.033</th>
<th>0.052</th>
<th>0.077</th>
<th>0.099</th>
<th>0.137</th>
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<tbody>
<tr>
<td>% Error in volumes</td>
<td>-40</td>
<td>-40</td>
<td>-40</td>
<td>-40</td>
<td>-40</td>
<td>-40</td>
</tr>
</tbody>
</table>

Figure 3-9: PEV's for kaolin.
It can be seen from Figures 3-9 and 3-11 that kaolin doesn't lend itself to the electrical impedance technique and in particular the application of Archie's law. There are a number of possible reasons for this; Archie's equation is derived from theoretical work which assume that the system contains ideal spheres. It can be seen from Figure 3-8 that kaolin has a platelet shape and so this may explain the failure of Archies law. It doesn't however explain why the technique fails when the straight line fit (Figure 3-10) is used. The reason for this is also shape related, because of its platelet shape kaolin particles can offer different resistances to electric current flow depending on their orientation. So the conductivity of a lamina of suspension can change if the alignment of the solid particles is altered. The orientation of particles in a suspension that has just been mixed is likely to be very random whereas a particle that has been settling for some time is likely to have a certain orientation, this will most probably be related to the mode of sedimentation (i.e. free settling or compressive). So a suspension of initial concentration $c$ will not necessarily have the same conductivity as a lamina of suspension which has thickened to a concentration $c$ because the particles maybe differently oriented. Another factor is the surface charge which kaolin is known to possess, the exact nature of any effect under the experimental conditions used is uncertain however several studies (Trusov [1993], Oakley et al. [1992], Maiti [1981]) have shown that surface charge (and particle orientation) can affect conductivity readings.

Figure 3-10: Graph to show best exponent fit to experimental data for kaolin using the initial conductivity method.
3.4.3 Calcite

Calcite has a cubic shape (Figure 3-12), a relatively large size range (Table 3-3) and a density of 2720 kg m\(^{-3}\), it channelled at low to intermediate concentrations (0.05<c<0.1) and showed slightly compressive behaviour. The range of errors obtained are shown in Figure 3-13, errors of ±10% were obtained which is acceptable although there is a tendency for the errors to move from negative values to positive as the initial concentration increases. There is no evidence of PEV's increasing during a sedimentation (i.e. as the suspension concentration increases) so this doesn't indicate a failure of Archies equation at higher concentrations. More likely is that the state of aggregation which is known to be a function of the initial concentration is changing so the size of the basic settling particle is changing which will affect the electric flux. This is a possible argument for using different exponent values not only with different materials but also different initial concentrations, however this was not done in this work.
Figure 3-12: Scanning electron microscope image of calcite.

Table 3-3: Particle size distribution of calcite.

<table>
<thead>
<tr>
<th>Particle Diameter (μm)</th>
<th>&lt;0.16</th>
<th>0.74</th>
<th>1.44</th>
<th>2.54</th>
<th>3.31</th>
<th>5.41</th>
<th>7.09</th>
<th>9.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cum. &lt; Volume%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.28</td>
<td>0.16</td>
<td>0.74</td>
<td>1.44</td>
<td>2.54</td>
<td>3.31</td>
<td>5.41</td>
<td>7.09</td>
<td>9.52</td>
</tr>
<tr>
<td>2.21</td>
<td>1.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.18</td>
<td>2.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.58</td>
<td>3.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.49</td>
<td>5.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.91</td>
<td>7.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.49</td>
<td>9.52</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Initial concentration, v/v.

Figure 3-13: PEV’s for calcite.
3.4.4 Aragonite

The other crystal form of calcium carbonate, aragonite, had a much narrower size range (Table 3-4) and a considerably smaller mean particle size (7.9 μm). Its density was 2578 kg m\(^{-3}\), the basic particle consisted of a mass of 'pine needle' structures which were arranged in a roughly spherical shape (Figure 3-14). Aragonite showed compressive attributes and channelled at intermediate to high concentrations (0.15 < \(c\) < 0.2). Good PEV's (±5%) were obtained at initial concentrations of \(c_0 > 0.075\) using an exponent value of 1.3. The change in the PEV's with respect to time was not random as would be expected (see Figure 3-15) instead it followed a definite pattern, for an initial period of 5000 seconds the PEV would remain constant after which it gradually increased. There is no obvious physical explanation for this trend the most plausible reason is that at higher concentrations aragonite suspensions deviate slightly from Archies law. The resulting error however is not large so the data was processed using Archies law and an exponent of 1.3. For initial concentrations of \(c_0 < 0.075\) the PEV's of the order -15% so the data was not used in any further analysis.

3.4.5 Talc

The degree of channelling exhibited by calcite and aragonite was mild in terms of the number of channels present, the amount of dilution experienced and the effect it had on the settling rate. A material which showed gross channelling was talc, this had a density of 2690 kg m\(^{-3}\), a mean particle size of 8.6 μm and a platelet shape (Figure 3-16). The PEV's (Figure 3-17) using an exponent value of 1.98 ranged between ±5% in the extremes but generally fell between ±2%. This was an unexpected result as talc closely resembles kaolin in its shape, i.e. it is very anisotropic. The SEM however only shows the primary particle shape which may not be that of the basic settling entity so it is conceivable that talc flocs have a more isotropic shape than their constituent particles.
Figure 3-14: Scanning electron microscope image of aragonite.

Table 3-4: Particle size distribution of aragonite.

<table>
<thead>
<tr>
<th>Particle Diameter μm</th>
<th>Cum. &lt; Volume %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.84</td>
<td>0.59</td>
</tr>
<tr>
<td>2.21</td>
<td>1.31</td>
</tr>
<tr>
<td>2.65</td>
<td>2.52</td>
</tr>
<tr>
<td>3.18</td>
<td>4.62</td>
</tr>
<tr>
<td>3.82</td>
<td>7.97</td>
</tr>
<tr>
<td>4.58</td>
<td>12.87</td>
</tr>
<tr>
<td>5.49</td>
<td>19.49</td>
</tr>
<tr>
<td>6.59</td>
<td>27.91</td>
</tr>
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</table>

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>38.41</td>
<td>51.21</td>
<td>65.61</td>
<td>79.84</td>
<td>91.40</td>
<td>98.05</td>
<td>99.90</td>
<td>100.0</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3-15: PEV's for aragonite.
Table 3-5: Particle size distribution of talc.

<table>
<thead>
<tr>
<th>Particle Diameter μm</th>
<th>0.12</th>
<th>0.17</th>
<th>0.25</th>
<th>0.36</th>
<th>0.52</th>
<th>0.74</th>
<th>1.07</th>
<th>1.54</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cum. &lt; Volume%</td>
<td>0.13</td>
<td>0.51</td>
<td>1.10</td>
<td>1.84</td>
<td>2.66</td>
<td>3.58</td>
<td>4.85</td>
<td>7.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial concentration, v/v.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.21</td>
</tr>
<tr>
<td>11.27</td>
</tr>
</tbody>
</table>

Figure 3-16: Scanning electron microscope image of talc.

Figure 3-17: PEV's for talc.
3.5 Other factors which affect the electrical impedance technique

The previous section showed that particle morphology has a significant effect on the overall accuracy of a system and also on the exponent used in Archie's equation. However this is not the only parameter which can have an effect on the electrical impedance technique, other influential factors are outlined below.

3.5.1 Stirring

In general stirring improved system accuracy. This can be seen in Figure 3-18, the PEV's for stirred and unstirred calcite sedimentations at the same initial concentrations are shown. The stirred experiments fall within a much narrower band of PEV, roughly ±3% compared with ±10% for the unstirred data. This may be due to the elimination of channels which when present provide a path of least resistance for current flow and therefore can cause unrealistic flux variations. More likely is that the effect of stirring on the initial state of aggregation predominates over the effect of initial concentration discussed in Section 2.4.3. If the result of stirring is a more isotropic aggregate at all initial concentrations then the overall PEV will be reduced as the system resembles more closely the theoretical model.

![Figure 3-18: Effect of stirring on the PEV's of calcite.](image-url)
Figure 3-19: PEV's for stirred aragonite.
Aragonite showed similar behaviour (Figure 3-19), a more constant error was obtained and there was no trend of increasing error during the run. The accuracy at low initial concentrations \( c_0 < 0.075 \) also improved. Stirring talc however led to random PEV fluctuations (Figure 3-20), this was probably due to the plastic-like behaviour of the talc which at higher concentrations became a 'dough' which moved radially. It was observed that tears would develop in the sediment structure at the electrodes when the radial movement was very large and as a result the electrodes would temporarily occupy a water filled void which affected the electric flux.

Figure 3-20: Effect of stirring on the PEV's of talc.
The speed of stirring was also an important factor in determining the exponent. The PEV's for a series of aragonite sedimentations at different stirrer speeds, in the range 0-4 rpm, are shown in Figure 3-21. The PEV's have all been calculated using the same exponent value (1.5). Whilst the spread of error for an individual sedimentation is not large the spread over all the sedimentations is quite large. If the PEV's are recalculated using exponents which give the least error for each sedimentation then a definite relationship between stirrer speed and exponent value can be seen (Figure 3-22). The exponent value jumps from 1.3 at 0 rpm to 1.47 at 0.125 rpm and then increases gradually as the stirrer speed increases.

The same trend was observed with talc, the exponent required to generate the smallest spread of PEV's in the unstirred test was 1.98, at a stirrer speed of 2 rpm an exponent of 2.3 was needed (Figure 3-20) and at 4 rpm a value of 2.35 was required to minimise the PEV's. The trend is most likely due to the effect of stirring on aggregation and the subsequent effect on the electric flux.
3.5.2 Electrode design

In the tomographic techniques which utilise the measurement of some electrical property when imaging sedimentations it is usual to use flush mounted electrodes. The argument for this is that if electrodes were to protrude into the suspension they may provide support for a solids network. It was found however that for our technique the use of flush mounted electrodes gave rise to flux tracking, the electric flux instead of passing through the centre of the suspension would tend to skirt round the walls of the container. Figures 3-23 & 3-24 show the contrast in quality of measured voltage vs. time profiles for flush mounted and protruding electrodes measuring the sedimentation of the same material at identical starting conditions. Although the profiles are essentially the same the protruding electrode profiles are noticeably smoother. The fluctuations shown by the flush mounted electrodes would make subsequent data manipulation such as the generation of characteristics difficult. Also noticeable is the difference in sensitivity between the two electrode types, the concentration change from initial to final concentration is the same for both but the change in voltage is noticeably less for the flush mounted electrodes, 0.4V compared with 0.7V. The flush mounted electrodes also don’t register the slight flattening or kink in the profiles which occurred in the protruding electrode run at 2.8V and is a
well known characteristic of aragonite sedimentations. Obviously for the electrical impedance technique it is preferable to use protruding electrodes so tests were conducted on high concentration calcite suspensions to assess the magnitude of any supporting effects. Despite the existence of a dense solids network there was little difference in settling behaviour between flush and protruding electrodes so protruding electrodes were adopted. The protruding electrodes were sharpened to points as this concentrated the electric field in the tip of the electrode and so enhanced the imaging by further reducing the chance of tracking.

3.5.3 Frequency

To measure the conductivity of a suspension a 5 volt square wave was applied to the measuring circuit described in Section 3.1.1. The frequency of this signal was of importance, below a value of 2 kHz polarisation of the electrodes was known to occur so a minimum working frequency limit of 2 kHz was defined. Above this value the effect of frequency was unknown.

In reality only pure electrical resistance is inversely proportional to conductivity however in this study it was assumed that impedance satisfied the same criterion. To examine whether such an assumption was valid it is necessary to examine the nature of impedance more carefully, the electrical impedance of a suspension is the sum of a capacitive and resistive part, the inductive part can be assumed to be negligible. Mathematically this is written:

\[ Z^2 = \left( \frac{1}{2\pi F} \right)^2 \left( \frac{1}{f} \right)^2 + R^2 \]  

(3.10)
Figure 3.23: Concentration vs. time profile (aragonite suspension $c_o = 0.075$) measured using protruding electrodes.

Figure 3.24: Concentration vs. time profile (aragonite suspension $c_o = 0.075$) for measured using flush mounted electrodes.
For impedance measurements to be valid the capacitive component has to be relatively small. Experiments conducted at a frequency of 2 kHz showed impedance was only 5% more than the resistance which was acceptable. However Equation (3.10) suggested that working at higher frequencies would further reduce the capacitive part. It therefore seemed beneficial to work at a higher frequency, however tests later showed that this was only beneficial for certain materials. Calcite for instance yielded the same PEV’s given in Figure 3-13 at 2 kHz and 16 kHz, whereas kaolin in a single stirred test showed an improvement from $\pm 10\%$ at 2 kHz to $\pm 5\%$ at 16 kHz. An operating frequency of 16 kHz (this was the maximum which could be generated) was therefore used as although not always advantageous there was no disadvantage to working at this frequency.
4. Batch settling results & discussion

The nature of the experimental apparatus used in this study means that a considerable quantity and quality of data can be presented. In most research a single experiment gives one graph or even a single point on one graph, in this study one experiment provided data linking time, concentration, interface height, channel location and the percentage error in the experimental technique. Further data processing yielded information on parameters such as permeability, solids pressure and velocity and so it has, unusually, been an arduous task to select the most appropriate data and avoiding becoming "bogged" down in graphs.

4.1 Description of the settling behaviour exhibited by the investigated materials

As outlined in Section 3.0 the experimental apparatus output was in the form of a set of voltages at set time intervals. These voltages were then calibrated and converted to concentration values using the correct concentration-conductivity relationship. The basic experimental data is therefore presented as concentration-time plots, these have been generated for all the experimental runs conducted on ballotini, calcite, aragonite and talc and are presented in the forthcoming chapters or in Appendix C. Kaolin and titanium dioxide were also investigated, data on the former has been omitted as no satisfactory concentration-conductivity relationship could be obtained and titanium dioxide has also not been included as it coated the rig in a fine coat which affected electrode calibration constants and hampered visual observations.

4.1.1 Generating concentration characteristics

Much can be learned about the settling behaviour of a material from a plot of the concentration loci that arise during a sedimentation. The shape, rate of propagation and point of origination of such lines yields information on the mode of sedimentation, the concentration at which channelling occurs and can be used to calculate solid velocities and consequently permeability values. These lines are known as characteristics, the concept of which was discussed in Section 2.1.2.1.1.
A knowledge of experimental characteristics is useful in the validation of mathematical models. Currently the merit of many models is based solely on an ability to predict the settling curve as comparative experimental data can easily be obtained. Unfortunately an accurate prediction of a settling curve doesn’t guarantee that a model is correctly predicting the behaviour of the sediment below the solid-liquid interface and this is the area of real interest for thickener designers. Comparing experimental and theoretical characteristics is a much more accurate way of evaluating a model’s worthiness however it has not been used by many researchers as they lacked the means of generating experimental characteristics. Characteristic plots have therefore been generated for all concentration-time graphs, those which are not presented in the forthcoming chapter have been included in Appendix C.

![Graphical technique for the generation of concentration characteristics.](image)

Characteristics can be developed in a number of ways, the easiest method is illustrated in Figure 4-1, a concentration is chosen on a concentration-time plot and the times $t_1$, $t_2$, $t_3$... at which concentration $c_1$ exists at the electrodes having heights $h_1, h_2, h_3$... are located. Plotting $(h_1,t_1), (h_2,t_2), (h_3,t_3)$... gives the characteristic for $c_1$. 
For experiments which show large concentration changes (i.e. there is a large number of possible characteristics) or long duration runs to generate characteristics by hand is painstaking and time consuming so the above technique was converted into a Qbasic program (CHARAC), a listing is given in Appendix B. The program differs slightly from the above method in that instead of considering a particular concentration it considers each time increment and calculates the height at which particular concentrations occur. This is only possible if the concentration change between two neighbouring measurement points can be considered to be linear. In this study the vertical gap between electrode pairs was usually 10 mm so this was not unreasonable, however with the larger diameter rig the electrode spacing was at one point 50 mm. If the measured concentration at these two electrode pairs were the same or similar then it was possible to assume a linear concentration change over 50 mm and characteristics were generated for the intermediate heights. When the concentration change was significant then such an assumption was not possible and characteristics couldn’t be generated, when this occurred the relevant area on the characteristic plot was shaded grey.

In the sedimentation of some materials dilution of the upper portion of the suspension occurred. Characteristics representing these dilution concentrations were generated using the above method however the accuracy of these ‘dilution’ characteristics relied heavily on the above assumption that there was a linear change of concentration between electrodes and that this was true for gaps up to 50 mm. This is because the density of measurement points per unit height was far greater in the lower section of the column whereas dilution occurred only in the upper portion of the suspension and therefore column. With hindsight a more uniform distribution of measurement points should have been used however at the design point of the rig it was felt that the area of interest would be the compression region hence the high density of electrodes low in the column. Nevertheless the resulting dilution characteristics did give what appeared to be a realistic representation of what was happening in the suspension.
4.1.2 Explanation of channelling observations

With the exception of ballotini all the materials exhibited some form of channelling, for some this was over a very limited concentration range while for others the effect was omnipresent. All the materials were homogeneous and so exhibited what was described in Section 2.1.3 as random channelling, i.e. there was no obvious cause of channelling such as air bubbles or coarse particles.

In accordance with Vesilind's [1990,93] observations channels when existent usually formed within an identifiable zone in the settling suspension. The growth and propagation of channel zones was recorded and superimposed on the relevant characteristic plot to enable any relationship between channelling and concentration to be established. An example of the propagation of such a zone in a talc suspension is shown in Figure 4-2 (a-e). In Figures a & b it was not possible to show the whole of the column as the channels would not be detectable, the scale on the photo gives an indication of the size and location of the zones. It can be seen that the zone grows from being approximately 15 mm in size to 100 mm over the duration of the sedimentation, the growth of individual channels in terms of their diameter is also very noticeable.

In all other studies on channelling the channel zone has always been treated as a single zone in this study the region in which channelling occurs is divided into two sub-zones, the reasons behind this will be discussed in Section 4.2.1. The two different but adjacent zones will be referred to as the "hard" and "soft" zones. In the latter the channels have a very definite structure and the surrounding suspension has a compact appearance. The soft zone immediately above is best described as a zone of lightly fluidised streams or spouting channels which emanate from the points at which channels in the hard zone reached the interface of the two zones. Unlike the hard zone there was no definite upper limit to this zone, the size of the spout was related to the size and activity of the hard zone channel below. The height of this zone was taken to be that of the highest spout. The height of the bottom of the hard zone was recorded as being the lowest point at which a channel existed, whether or not a channel was still actively transporting fluid upwards was difficult to assess.
a - duration of the sedimentation ≈ 10 minutes

b - duration of the sedimentation ≈ 30 minutes

Figure 4-2: Example of the growth and propagation of a channel zone in a talc suspension.
Figure 4-2: Example of the growth and propagation of a channel zone in a talc suspension.

c - duration of the sedimentation = 1 hour

d - duration of the sedimentation = 2 hours
Fig. 4-2: Example of the growth and propagation of a channel zone in a talc suspension.

**e** - duration of the sedimentation ≈ 3 hours 35 mins

**f** - duration of the sedimentation ≈ 5 hours 5 min
In some cases channels would collapse after a certain period whereas others retained their structure in the hard zone even after settling had subsided.

Another aspect of channelling is the appearance of volcanic like structures at the surface of the solid-liquid interface. The size and number of these did vary from material to material and with concentration so where possible rough measurements were made by eye. Exact measurements could not be made as access to a suitable photographing location was not possible due to the presence of the temperature control insulation box and cell lid. The distribution of the volcanoes on the surface was random for all the materials. There appeared to be no preferential sites of formation. Evidence of this is given in Appendix D. Sequential hand sketches of the solid surface at different times during three calcite experiments show that volcanic eruptions were equally as likely to occur at the vessel wall as they were at any other location on the surface. It was therefore reasonable to assume that visual observations made at the vessel wall of the channel zones would be an accurate resemblance of the rest of the suspension.

4.1.3 Settling behaviour of ballotini

Initial concentrations in the range of $0.25 < c < 0.5$ were used, below this the solid-liquid interface was very diffuse, and despite the narrow size range of the material it appeared that segregation was occurring.
Figure 4-3 shows the interface settling curves and also the build-up of the associated sediments. Ballotini was the only material investigated which showed a visible difference between the sediment and free settling suspension structures and so the rise of the sediment was obtained by visual observations. A very distinct interface existed at the boundary between free settling and the sediment for all the initial concentrations except $c = 0.5$. An incomplete settling curve is shown for the $c_o = 0.25$ test this is because the interface became too diffuse to follow, the sediment build-up was however visible.

At a first glance it would appear that in all the tests the interface settled at constant velocity until it collided with the rising sediment, this is not the case and is a good example of the advantage of translating settling curves into settling velocity plots. Quadratic equations were fitted to the interface data and then differentiated to give the velocity plots shown in Figure 4-4. At higher initial concentrations the interface does settle at a constant rate but for $c_o \leq 0.4$ a falling rate period is exhibited before the velocity becomes zero on interception with the sediment. The size of the falling rate zone increases with decreasing initial concentration.
Figure 4-4: Settling velocity plot for ballotini.

The presence of a falling rate zone suggests that the initial concentration doesn’t discontinuously settle to the sediment concentration but that other free settling concentrations arise. The magnitude of the velocities in the falling rate zone suggests that not all concentrations between the initial and sediment concentrations occur, for example the velocity at the end of the falling rate period for the \( c_0 = 0.3 \) test is marginally greater than the constant rate velocity of the \( c_0 = 0.35 \) test. This suggests that only concentrations between \( 0.3 < c < 0.35 \) will arise. The concentration-time plot for the \( c_0 = 0.3 \) test is shown in Figure 4-5, the behaviour shown in this plot is typical of all the initial concentrations investigated (cf. Figures C1-C6). This plot suggests that intermediate concentrations close to the initial do arise before the concentration jumps to that of the sediment ( \( c = 0.55 \) ). A plot of the characteristics gives rise to reasonable characteristic locations in terms of the sediment and in terms of the characteristic concentration values (Figure 4-6). It is likely however that the concentration and characteristic plots are slightly in error due to flux tracking which is an effect that arises due to the fact that the electrical flux between two electrodes doesn’t travel in just the horizontal plane but bows into the vertical plane as well. A lamina of suspension having a significantly different conductivity to the lamina in the plane of measurement e.g. the sediment surface can therefore affect the impedance technique if it is within the distance that the electrical flux bows in the vertical plane. This effect is discussed in more detail in Section 4.2.9.
The sediment concentration obtained in all the runs was $c = 0.55$ which is in good agreement with theoretical predictions for the packing of rigid spheres (Martin [1951]). Once formed the sediment shows no further change in concentration and so compression effects are not significant. This is the simplest form of settling encountered and is often referred to as 'ideal settling'.

Figure 4-5: Concentration-time plot for a ballotini sedimentation ($c_o = 0.3$).

Figure 4-6: Characteristic plot for a ballotini sedimentation ($c_o = 0.3$).
Essentially this material was used to commission the apparatus, the percentage error in volumes were reported in Section 3.4.2 these were of the order ± 2% which is very acceptable, the type of settling behaviour described by the electrical impedance measurements compared favourably with literature on ballotini (Holdich [1987], Shannon et al [1964], Simons [1991]).

4.1.4 Settling behaviour of aragonite

Batch sedimentations were conducted using an initial concentration range of $0.025 \leq c_o \leq 0.225$. Concentration-time graphs and characteristic plots for the initial concentration range $0.075 \leq c_o \leq 0.225$ are given in Appendix C. The lower initial concentration data is not presented because of the magnitude of the experimental errors (see Section 3.4.4).

![Figure 4-7: Test of the significance on the settling of aragonite of wall effects in different diameter vessels.](image)

Tests on high concentration suspensions were conducted to assess the magnitude of any wall effects. Figure 4-7 shows that for an initial concentration of $c_o = 0.2$ that there was very little difference in the settling behaviour in columns of different diameters, this difference decreased as the initial concentration was decreased. The effect was considered small enough to be negligible so tests were conducted in the smaller batch rig (61.5 mm cf. 150 mm) as handling and metering was considerably easier.
Figures 4.8: Settling curves for aragonite.

Settling curves for the range of concentrations considered are shown in Figure 4.8. The shape of the curves tends from a very concave shape at the lower initial concentrations to a slightly convex shape at the higher initial concentrations. More information as to the settling behaviour that these curves pertain to is obtained by replotting them as "Vesilind" interface velocity-time graphs (Figures 4.9 & 4.11). The velocities were obtained by fitting quadratics to the settling curves, in some cases more than one quadratic was required to describe the curve fully.

The gel concentration (boundary between free and compressive settling) was determined to be 0.176, the method for deducing this is given in Section 4.2.5.

Aragonite exhibited settling behaviour which conforms almost exactly with the compressible sedimentation model outlined in Section 2.1.1.1.2. The model predicts that a suspension with an initial concentration in the free settling regime will show various zones of settling behaviour which can be identified by settling velocity variations. Initially a constant rate period is shown and the suspension above the sediment settles at a constant concentration, a first falling rate period is then exhibited during which free settling concentrations develop and propagate upwards resulting in a 'Kynch' fan of characteristics, a second falling rate is then experienced during which characteristics are believed to emanate tangentially from the sediment surface.
A further falling rate period then results which is due to compression of the sediment. Depending on a suspension's initial concentration it will exhibit all or some of these velocity variations.

![Settling velocity plots for free settling initial concentrations of aragonite.](image)

**Figure 4-9: Settling velocity plots for free settling initial concentrations of aragonite.**

The velocity plots for the range of initial concentrations in the free settling regime are given in Figure 4-9. At all concentrations a falling rate is exhibited throughout however there is no rapid change in velocity and so it is difficult to determine whether two falling rate zones and a compression zone exist or just one falling rate zone and the compression period. However if the model in Section 2.1.1.1.2 is correct then a fan of free settling characteristics should arise either from the origin or sediment. The characteristic plot for the $c_o = 0.1$ test is shown in Figure 4-10, it can be seen that the model predictions are correct, a large fan of characteristics is obtained for $0.11 \leq c \leq 0.17$, above this concentration the characteristics show compression attributes.
Figure 4.10: Characteristic plot for an argonite suspension (C₀ = 0.1).

- Surface pocks
- Small volcanoes forming
Figure 4-11: Settling velocity plots for high initial concentrations of aragonite.

The velocity plots for the higher initial concentrations i.e. those in the compression region are shown in Figure 4-11. The $c_o = 0.175$ test shows a constant velocity zone proceeded by a compression period, the rate at which the interface decelerates during this period is approximately the same for all the higher concentration runs. A constant velocity represents a zone of constant concentration. Examination of the characteristic plot (Figure 4-12) for the $c_o = 0.175$ test shows that it takes approximately 15000 seconds for the $c = 0.18$ characteristic to reach the interface. This is coincident with the time that the velocity begins to decelerate in Figure 4-9. It is therefore reasonable to assume that all the suspension above the $c = 0.18$ characteristic is at $c = 0.175$ and that a constant velocity zone in the compression regime is representative of a constant concentration. In the $c_o = 0.2 & 0.225$ tests the interface attains a constant velocity after a small induction period, this constant rate is maintained in both runs for a period of 7500 seconds after which the velocity slowly increases until 15000 seconds at which point there is a rapid deceleration which represents the compression period. An increase in the interface velocity is usually indicative of channelling and also of dilution of the upper layers of a suspension to a lower and, therefore, quicker settling concentration. Channelling was observed at both concentrations and evidence of dilution can be seen on the concentration time plot for the $c_o = 0.225$ test (Figure 4-13), at $t = 5000$ seconds it can be seen that the profile lines for heights of 109 mm and above begin to move towards concentrations
lower than \( c = 0.225 \). The profiles are closely bunched and so it would appear that
dilution occurs rapidly over a considerable portion of the suspension. Approximately
90 mm of the suspension dilutes to \( c = 0.22 \) and 50 mm dilutes to \( c = 0.21 \). The limit
of the dilution is reached at \( c = 0.2 \), this could have been predicted from the velocity
profile which at its peak attains the same velocity as the constant rate zone in the \( c_o = 0.2 \) test. The same is applicable to the \( c = 0.2 \) test which diluted to \( c = 0.18 \) and at its peak had a velocity marginally less than the \( c_o = 0.175 \) test.

In the above descriptions the final phase of settling has always been the compression
period, regardless of the initial concentration the sediment build-up was the same for all the runs. Compression characteristics for \( 0.18 \leq c \leq 0.27 \) arose at regular intervals and in an approximately parallel alignment with each other. These characteristics propagated into the column at a decelerating rate for a period of 3000 seconds, a constant rate was then maintained until deceleration occurred due to the completion of compression. In runs which showed visible channelling (Figures 4-12 & 4-14) the attainment of this constant propagation velocity coincided with the first visual observations of channelling. Characteristics for \( c > 0.27 \) took longer to arise and didn’t propagate upwards at the same rate as the lower concentration compressive characteristics did. No channels were observed at \( c > 0.27 \).
Figure 4.12: Characteristic plot for an argonite sedimentation ($c = 0.175$).
$C_0 = 0.225$.

Figure 4-13: Concentration-time plot for an amphoteric separation.

Concentration, v/v.

Time, s.
Figure 4.14: Characteristic plot for an aragonite sedimentation ($c_o = 0.2$).
4.1.4.1 Channelling observations

Distinct channel zones formed in the runs $c_o = 0.175 & 0.2$ (Figures 4-12 & 4-14). In both, channel zones once formed maintained a uniform size and propagated upwards at an approximately constant rate. The channels within these zones formed and decayed but didn't appear to persist for any substantial period. The hard zone's propagation route roughly followed the $c = 0.22$ characteristic but in reality the hard zone's propagation in both runs fell within a band of concentration, $0.21 \leq c \leq 0.24$. In the case of the $c_o = 0.175$ the zone actually formed at $c = 0.18$ but then quickly propagated to a higher concentration. The bottom of the channel zone propagated upwards at a similar rate to the hard zone, channels ceased to exist above $c = 0.27$. Another feature of both runs was that prior to the formation of the channel zones the suspension structure appeared very non-homogenous, tiny tears were observed throughout the entire suspension height. These developed soon after the start and slowly decayed over a period of 3000 seconds at which point the channel zone formed. Examination of the velocity profiles show that for this period the velocity increases at a decaying rate until it reaches some constant value. This is an example of the initiation or induction period described in Section 2.1 and is attributable to flocculation.

At $c_o = 0.225$ channels were observed but no definable zone formed. Volcanoes formed and decayed on the interface throughout the experiment. The same channelling behaviour was observed at $c = 0.15$ however volcanoes only formed near the deceleration period.

At lower concentrations no channels were observed, the size and number of volcanoes decreased with initial concentration until at $c \leq 0.1$ they were better described as pock marks. The formation of volcanoes always occurred near the deceleration period (see Figures C13-C16) and close to the point where the $c = 0.18$ characteristic arrived at the interface. At all concentrations the formation of volcanoes was a gradual process, growth would occur for approximately 15 minutes after they were first observed.
4.1.4.2 Stirred Data

Stirring eradicates channels and so allows a comparison to be made between the settling behaviour of a material in the presence of, and without channelling. In Figure 4-15 the effect of varying stirrer speeds on the settling curve for a \( c_o = 0.2 \) suspension is shown. It can be seen that even a very slow stirrer speed caused a large reduction in the settling rate from that of an unstirred sedimentation but that there was very little difference over the range of stirrer speeds. For stirrer speeds between 0.125 and 1 r.p.m. there was however a clear trend as Figure 4-16, which is a magnification of Figure 4-15 shows, the settling rate decreased with increasing stirrer speed until a speed of 1 r.p.m. was reached. A similar settling velocity was obtained at 2 r.p.m. and a marginally slower settling velocity at 4 r.p.m. It was therefore decided that at and above 1 r.p.m. channelling was completely eradicated and so this speed was used in all subsequent tests on aragonite.

![Graph showing the effect of different stirrer speeds on aragonite sedimentation](image)

Figure 4-15: The effect of different stirrer speeds on an aragonite sedimentation \( (c_o = 0.2) \).
Figure 4-16: Magnification of Figure 4-15.

Stirred tests were conducted over an initial concentration range of $0.05 \leq c_o \leq 0.2$. Settling velocity plots were obtained from the settling curves (Figure C17) and are shown in Figure 4-17. The velocities shown don't decrease to zero as the tests were all terminated before settling was completed. This was because at high concentrations the stirrer would jam or began to show signs of buckling and so the tests were aborted to avoid damaging the equipment. At $c_o < 0.15$ the settling velocity curves show similar but not identical behaviour to the unstirred tests. Rapid falling rate zones are exhibited followed by a decrease in the rate of deceleration as settling becomes due to compression. The boundary between the free settling falling rate zone and compression zone is not distinct but appears to occur at roughly 6000 seconds for $c_o = 0.05$ and at 12500 seconds for $c_o = 0.1$. At $c_o \geq 0.15$ the velocity plots show constant rate zones followed by compression zones. The duration of the constant rate zones is very similar to the corresponding unstirred tests whereas the compression period is considerably longer, for the $c_o = 0.15$ test this was 12000 seconds plus compared with 7000 seconds for the unstirred test. This is attributable to the significantly higher sediment concentrations that were achieved through stirring. The sediment base concentration achieved in the stirred tests was typically $c = 0.42$ compared with $c = 0.303$ in the unstirred tests.
The characteristic plot for the \( c_o = 0.15 \) stirred test is shown in Figure 4-18, the behaviour shown is typical of all the other stirred initial concentrations (Figures C22-C24). The characteristic plots for the stirred tests were very dissimilar to their corresponding unstirred plots, the characteristics being much more akin to Kynch characteristics than to those suggested by the compressive sediment model and exhibited by unstirred aragonite. The characteristics are very linear and emanate in a fan shape until relatively high concentrations are reached. The characteristics then become more tightly bunched before spreading again. In the unstirred tests the tight parallel bunching of characteristics was indicative of the onset of compressive settling. However the gel concentration in the stirred tests was not \( c = 0.176 \) as it was in the unstirred test nor was it the same for all the stirred tests. At an initial concentration of \( c_o = 0.05 \) the gel concentration appeared to be \( c_g = 0.22 \), for \( c_o = 0.1 \) \( c_g = 0.23 \), for \( c_o = 0.15 \) \( c_g = 0.26 \), and \( c_g = 0.29 \) for \( c_o = 0.2 \).

![Figure 4-17: Settling velocity plots for stirred aragonite sedimentations](image)

(\( 0.05 < c_o < 0.2 \)).
Figure 4.18: Characteristic plot for a stirred (1 r.p.m.) aragonite sedimentation 
\( (c_0 = 0.15) \).
4.1.5 Settling behaviour of talc

Talc sedimentations were conducted over an initial concentration range of $0.05 \leq c_o \leq 0.15$. As with aragonite preliminary tests were conducted to assess the magnitude of any wall effects. Figure 4-19 shows the difference between the two cells was negligible at an initial concentration of $c_o = 0.05$ however at $c_o = 0.06$ (Figure 4-20).
there was a significant difference between the two diameters and this difference increased at higher initial concentrations. Data from tests conducted in the larger (150 mm diameter) batch cell is therefore presented.

The settling curves for all the sedimentations conducted are shown in Figure 4-21, the curves have a convex shape and the degree of convexity increases with increasing initial concentration. The convex shape of these curves indicates an increase in the settling velocity of the solid-liquid interface, the degree of acceleration achieved can be seen in the velocity plots (Figure 4-22). At low initial concentrations a constant rate period is exhibited followed by a rapid acceleration and then deceleration period. As the initial concentration is increased the width of this peak increases i.e. acceleration is less sudden and the size of the constant rate zone decreases until at $c_o = 0.15$ the interface velocity increases from the onset. The magnitude of the initial velocity in each run showed the expected trend in that it decreased with increasing initial concentration, the peak velocity pattern however was less predictable. The highest peak velocity achieved was $0.012 \text{ mm s}^{-1}$ in the $c_o = 0.052$ test, the peak velocity then decreased with increasing initial concentration to a value of $0.006 \text{ mm s}^{-1}$ in the $c_o = 0.09$ test before increasing again to a value of $0.01 \text{ mm s}^{-1}$ in the $c_o = 0.15$ test. The duration of the experiments showed the converse trend reaching a peak at $c_o = 0.09$ of 45000 seconds and then decreasing to 30000 seconds in the $c_o = 0.15$ test.
The settling data of aragonite showed that constant rate zones can occur both in free settling and compression, the absence of any falling rate zones other than the compression periods in the settling velocity plots of talc suggests that talc possesses a structure at concentrations as low as $c = 0.052$. The actual gel concentration was determined to be between $0.05 < c < 0.06$ (Section 4.2.4). It is therefore not surprising that the characteristic plots for talc (Figures 4·24 to 4·29) show no 'Kynch' like characteristics at any concentration. All the characteristics were either the result of compression or dilution. The compression characteristics were similar to aragonite in that at the lower compression concentrations ($0.05 \leq c \leq 0.13$) they arose at regular
intervals and in an approximately parallel alignment with each other, concentrations above \( c = 0.13 \) took longer to form and the characteristics although still parallel became progressively further apart. The compression characteristics propagated into the column at a decelerating rate. The degree of deceleration was more significant at higher initial concentrations.

Dilution occurred in the upper regions of settling suspensions and can be observed on the concentration-time plots, for example in Figure 4-23, the \( c_o = 0.113 \) test if we follow the concentration changes at 130 mm from the base we can see a steadily dilution from \( c = 0.11 \) to \( c = 0.1 \) over a period of 15000 seconds after which the sediment reaches that height and the concentration rapidly rises. The solid-liquid interface height at this time is 270 mm so over half the suspension is in dilution and from the close packing of the measurement lines at this point it is reasonable to assume that the majority of the suspension above 130 mm is also at \( c = 0.1 \). More extreme dilution occurs higher and later in the run, at a height of 170 mm and after 26000 seconds the suspension has diluted to \( c = 0.08 \), the solid-liquid interface height is 225 mm.
Figure 4.23: Concentration-time plot for a talc sedimentation (C_0 = 0.12).
4.1.5.1 Channelling observations

As with aragonite the propagation of the hard and soft channel zones was recorded and superimposed on the characteristic plot for each sedimentation (Figures 4-24 to 4-28). The channelling behaviour could broadly be split into two groups, below an initial concentration of $c_o = 0.1$ the channel zone formed after approximately 30 minutes and at a height of roughly 10 mm from the base. The hard zone would then propagate upwards at a rate which was identical to the characteristic of the concentration at which it formed. This was not the same concentration for all the initial concentrations rather there was a trend of increasing concentration of channel formation with increasing initial concentration. This is shown in Figures 4-24 to 4-26, at $c_o = 0.052$ the hard zone propagates up at a rate equivalent to a characteristic having a concentration of $c = 0.07$, at $c_o = 0.072$ it is $c = 0.085$ and at $c_o = 0.099$ it is $c = 0.105$. In each case large evenly spaced channels formed and were interspersed by smaller ones. The soft zone's propagation path followed no definite characteristic.

At $c_o > 0.1$ (Figures 4-27 to 4-28) the channel zone took progressively longer to form (1 hour plus) and appeared higher in the column. In the early stages of these sedimentations ($t > 10$ min) the suspension would develop large tears throughout its height, some of which would subsequently develop into channels. These tears and channels would gradually decay away over a period of an hour except for ones which fell within the eventual channel zone. The channel zones were noticeably larger at the higher initial concentrations and neither zone's propagation followed closely any concentration characteristic. The general trend was for the hard zone to form at a certain concentration and then to propagate upwards through characteristics of lower concentration which had arisen through dilution.
Figure 4.24: Characteristic plot for a talc sedimentation ($c_s = 0.052$).

- **channel zones**
  - hard
  - soft
  - bottom

- **Y-axis (Height, mm)**:
  - 0 to 350

- **X-axis (Time, s)**:
  - 0 to 40,000

- Heights and times marked on the graph for various channel zones.
Figure 4.25: Characteristic plot for a talc sedimentation ($c_n = 0.073$).
Figure 4.26: Characteristic plot for a talc sedimentation ($c_0 = 0.09$).
Figure 4-27: Characteristic plot for a talc sedimentation ($C_0 = 0.113$).
Figure 4-28: Characteristic plot for a lake sedimentation ($C_0 = 0.127$).

- Channel zones
- Bottom
- Hard
- Soft
Figure 4.29: Characteristic plot for a talc sedimentation ($c_s = 0.15$).
At $c_o \geq 0.15$ the tear effect described previously persisted throughout the sedimentation and channels were perceived to be occurring randomly rather than within some definable zone.

In all of the runs the bottom of the channel zone would propagate upwards at a slower rate than the other zone boundaries. The upper concentration limit for the existence of channels fell between $0.17 < c < 0.18$.

![Settling curves for stirred talc sedimentations](image)

**Figure 4-30:** Settling curves for stirred talc sedimentations (stirrer speed = 2 r.p.m.).

### 4.1.5.2 Stirred data

Stirring talc had a large effect on its settling rate and as a result the stirred talc tests were very long, typically 24 hours. As a result the optimum stirrer speed was not determined through a series of tests but was based on the experimental findings of the calcite and aragonite stirred tests. For both these materials a stirrer speed of 1 r.p.m. was used however it was found in the first talc test ($c_o = 0.052$) that this was insufficient to destroy all the channels formed. The stirrer velocity was therefore increased to 2 r.p.m., this speed proved sufficient at $c_o = 0.052$ & 0.112 however at the intermediate concentrations $c_o = 0.073$ & 0.09 channels still evolved and a stirrer speed of 4 r.p.m. was required to completely eradicate channelling.
Figure 4-31: Velocity plots for stirred talc sedimentations (stirrer speed = 2 r.p.m.).

The settling curves and velocity plots for the 2 r.p.m. test are shown in Figures 4-30 & 4-31 respectively, the acceleration segments on the velocity plots for the $c_o = 0.072$ & 0.09 tests is evidence of the failure of stirring to remove channelling. It does however reduce the amount of channelling, the maximum velocity obtained at $c_o = 0.072$ in the unstirred test was 0.011 mm s$^{-1}$ whereas in the stirred test it was 0.006 mm s$^{-1}$.

At 4 r.p.m. channelling was completely eradicated, the velocity plots for $c_o = 0.072$ & 0.09 (Figure 4-32) show no acceleration periods instead they consist of constant rate periods followed by compression zones.

The quality of the concentration-time graphs (Figures C30-33 & C36-37) and as a result the characteristic plots (Figures C34-36 & C38-39) for stirred talc is not as good as for other materials. The profiles show small random fluctuations and as the suspension thickens the size of these fluctuations increases. The reason for this is due to talc acting as a dough and moving radially in the direction of the stirrer. The suspension is dragged across the protruding electrodes and tears develop in the suspension. The tip of the electrode is therefore in a water filled void and this appears to have a detrimental effect on the readings. At a stirrer speed of 2 r.p.m. at $c_o = 0.09$ & 0.112 the fluctuations are so large that generation of characteristic plots was not possible.
The sediment base concentration obtained in the stirred tests was marginally greater than in the unstimred, the concentration achieved varied between \( 0.22 < c < 0.28 \) in comparison with \( c = 0.22 \) for the unstimred tests.

![Graph showing velocity plots for stirred talc sedimentations](image)

**Figure 4-32: Velocity plots for stirred talc sedimentations**

(stirrer speed = 4 r.p.m.).

### 4.1.6 Settling behaviour of calcite

An initial concentration range of \( 0.025 < c_0 < 0.15 \) was investigated. The quantity of calcite available was limited so tests were restricted to the smaller batch rig.

The settling curves for the full range of initial concentrations is shown in Figure 4-33. As with aragonite at lower initial concentrations the curves have a concave shape which tends towards a convex shape as the concentration increases. Unlike aragonite however as the concentration further increases the curves then straighten so the settling rate is constant for the majority of the experiment.
Figure 4-33: Settling curves for calcite sedimnetations ($0.025 < c_0 < 0.15$).

Figure 4-34: Velocity plots for calcite sedimnetations.

Velocity profiles are shown in Figure 4-34. At $c_0 = 0.025$ the velocity behaviour is considerably different to that of higher initial concentrations. The velocity initially decreases rapidly for a period of 200 seconds, the rate of deceleration then decreases for a period of 250 seconds before increasing again. This change in behaviour is not discernible on the settling curve and highlights the advantage of velocity-time plots. The three different periods of velocity behaviour can be identified as the first falling rate, second falling rate and compression periods respectively. The first falling rate period should contain 'Kynch' like characteristics, the second falling rate, tangential
characteristics emanating from the sediment surface. These are not observable on the characteristic plot (Figure 4-35) but this is not surprising as the gel concentration had been determined as \( c = 0.04 \), the two zones must therefore lie between \( 0.025 < c < 0.04 \) and so lie beyond the current scale of scrutiny. Whether such velocity behaviour does result in the predicted characteristics will be investigated in Section 4.2.7.

For the runs \( 0.05 \leq c_o \leq 0.1 \) the velocity profiles consist of a constant rate period followed by an acceleration period and then compression period. The acceleration in the \( c_o = 0.05 \) and \( c_o = 0.1 \) tests is small, in the \( c_o = 0.075 \) test the velocity increases from 0.1 to 0.2 mm/s. This is consistent with channelling observations, channels were only observed in tests at these initial concentrations and most predominantly in the \( c_o = 0.075 \) test.

At \( c_o > 0.1 \) the velocity profiles consist of a constant rate period followed by a compression period suggesting a constant concentration zone followed by compression characteristics, the characteristic plots show that this is true. It should be noted that the concentration-time graphs for \( c_o \geq 0.125 \) showed a peculiar trend of decreasing concentration throughout the whole suspension for a period of approximately 250 seconds immediately after the start of each run. This is not a true physical representation of what was happening, most likely the decrease was due to a change in the suspension structure that affected the electric flux rather than a change in concentration. The resulting variation in the percentage error in volumes was discussed in Section 3.4.3.

Dilution occurred in the runs \( c_o = 0.075 \) & \( c_o = 0.1 \), this can be seen on the concentration-time plot (Figures 4-36 & 4-37). At \( c_o = 0.075 \) the suspension dilutes to \( c = 0.055 \) and at \( c_o = 0.1 \) to a concentration of \( c = 0.09 \). These have been plotted as dilution characteristics (Figure 4-39 & Figure C44) and are very similar to those exhibited by talc.
Figure 4.35: Characteristic plot for a calcite sedimentation ($c_0 = 0.025$).
Figure 4-36: Concentration-time plot for a calcite sedimentation \((c_0 = 0.075)\).

Figure 4-37: Concentration-time plot for a calcite sedimentation \((c_0 = 0.1)\).
The characteristic plots for calcite showed, with the exception of the aforementioned dilution characteristics, compression characteristics. These were very similar to the compression characteristics of talc and aragonite, although the calcite characteristics didn’t exhibit the initial decelerating propagation rate that was characteristic of the other two materials. The rate of propagation of the calcite characteristics was constant until the completion of compression was approached when the propagation rate decreased.

4.1.6.1 Channelling observations

Channels were observed to form at $0.05 \leq c_o \leq 0.1$ however distinguishable zones could only be identified at $c_o = 0.05$ & $0.075$ (Figure 4-38 & 4-39). These observations were made prior to the identification of multiple channel zones and so only one zone is shown however a subsequent one-off test has shown that calcite does exhibit both types of channelling. The zone’s propagation path didn’t follow any single characteristic in either run but did reach the solid liquid interface at the compression point. The size of the zone was approximately constant for the $c_o = 0.05$ test whereas in the $c_o = 0.075$ the zone size increased with time and the bottom of the zone maintained a constant height after an initial growth period of 250 seconds.

Volcanoes were observed in the early stages of all the sedimentations, the size of these increased with the initial concentration. For $c_o \leq 0.075$ the volcanoes would disappear after a period of approximately 10 minutes, an explosion of volcanoes would then occur just prior to the channel zone reaching the surface. These volcanoes were greater in number and size than those which formed earlier in the run. For $c_o \geq 0.1$ volcanoes appeared and disappeared throughout the run, these peaked in size and number close to the mid-point of the sedimentation.
Figure 4.38: Characteristic plot for a calcite sedimentation ($c_0 = 0.05$).
Figure 4.39: Characteristic plot for a calcite sedimentation ($C_s = 0.075$).
4.1.6.2 Stirred data

As with aragonite preliminary tests were conducted to find the optimum stirrer speed, the results of these tests are shown in Figures 4-40 & 4-41. In Figure 4-40 it can be seen that there was no difference between using different stirrer speeds for the majority of a sedimentation’s duration. The only difference arose near the completion of settling when the settling curves diverged, the slower the stirrer speed the quicker the interface settled. This was true up to a stirrer speed of 1 r.p.m., at which point the trend was reversed (Figure 4-41) and quicker stirrer speeds resulted in higher settling rates. This is the same trend that was observed with aragonite although it was not as noticeable as testing was only conducted up to a stirrer speed of 4 r.p.m.. A stirrer speed of 1 r.p.m. was used as it caused the most reduction in the settling rate.

![Figure 4-40: Settling curves for calcite sedimentations at low stirrer speeds (0 → 1 r.p.m.).](image)

Tests were conducted over the range of concentration at which channelling was observed ($c_o = 0.05 \& 0.075$) and at a considerably higher concentration ($c_o = 0.15$). The sediment formed by stirring calcite proved to be very strong and the stirrer jammed before the completion of settling in all runs except $c_o = 0.05$. The effect of stirring on the settling velocity for $c_o = 0.05 \& 0.075$ was to remove the acceleration periods observed in the unstirred runs (Figure 4-34) and so the settling velocity plot for the stirred tests (Figure 4-42) consists of a constant rate period followed by a
compression period. The velocities of the constant rate zone in corresponding stirred and unstirred tests are similar.

**Figure 4-41**: Settling curves for calcite sedimentations at high stirrer speeds

(1 → 16 r.p.m.).

**Figure 4-42**: Settling velocity plots for stirred (1 r.p.m.) calcite sedimentations

(0.05 < c₀ < 0.15).
Figure 4.43: Characteristic plot for a stirred (1 r.p.m.) calcite sedimentation

$C_0 = 0.075$. 

Height, mm. 

Time, s.
A longer compression period was exhibited by the stirred tests and as a result considerably higher sediment base concentrations were obtained, $c = 0.38$ compared with $c = 0.18$ in the unstimred tests. Figure 4-43 shows the characteristic plot for the $c_o = 0.075$ test, it can be seen that the increased sediment concentration results in a greater density of characteristics in the sediment and that they are more evenly spread. The characteristics are very linear and appear to emanate in a fan shape at all concentrations. The $c_o = 0.05$ & 0.15 tests showed similar behaviour (Figures C50-C51).

4.2 Discussion

4.2.1 The existence of channel sub-zones

In the experimental observations on talc and aragonite two different zones of channelling behaviour were reported, the hard and soft zones. This is a novel approach in the study of the channelling phenomenon, the justification for the existence of two zones instead of one was that previously when there was considered to be only one zone channels formed not at the top of the zone but somewhere lower in the zone. If two or more channels formed at the same instance they would do so at the same height. The fact that channels didn’t form at the top of the zone is not on its own indicative of anything as it is possible that channel formation could occur over a range of heights. However channels didn’t form throughout the height of the zone but at one particular height suggesting that channel formation was limited to certain conditions. The fact that once formed a channel could exist above and below the point of formation suggested two different regimes of channelling. Closer examination of the height revealed that it formed a boundary between two different physical structures of channels. The difference between the compact, well defined channel structures of the hard zone and the lightly fluidised streams of the soft zone is illustrated in Figure 4-44.
It was also observed that a hard zone channel could exist without a soft zone channel existing above, however the opposite did not occur. This suggests that although a channel may not be actively transporting fluid in the hard zone the surrounding medium has sufficient structural strength to support channels, the same cannot be said of the soft zone. The soft zone would therefore appear to be a product of the hard zone.

4.2.2 Relationship between channel zones' and characteristics' propagation paths

In the previous section it was stated that if two channels formed they would do so at the same height suggesting that channels only form under certain conditions. In order to ascertain whether these conditions are concentration related the propagation of
channel zones was superimposed on the relevant characteristic plots. In particular the propagation of the hard-soft zone boundary is of interest as this is where channels form. The following is a summary of the experimental observations, with aragonite the formation of channels occurred at and followed closely the characteristic for \( c = 0.22 \). For low initial concentrations of talc channels formed and followed closely the characteristic of a concentration marginally larger than the initial concentration. At higher initial concentrations channels also formed at some marginally greater concentration but then propagated to lower concentrations which had arisen through dilution.

If we were to consider the aragonite data only then it would appear that channel formation is solely dependent on concentration however the talc data indicates that this is not the case. A reasonable postulation is that, in a similar manner to Kos’ permeability and solids pressure relationships, the formation of channels is a function of concentration and also the fluid velocity relative to the solids, i.e. \( \text{channel formation} = f( c, u_f - u) \). The premise for this is that a certain concentration must be ascertained before a suspension structure can support the formation of channels and for a random disturbance to grow into a channel there must be a sufficient flow of fluid into that area. If the above is assumed then the differences in the aragonite and talc data can be explained.

![Graph](image)

**Figure 4-45:** Comparison between the propagation rate of a compression characteristic at different initial concentrations of aragonite.

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With aragonite channel formation occurred at the same concentration in both runs which showed distinct channel zones \( (c_o = 0.175 \& 0.2) \), if the above assumption is correct then \( u_f - u \) must be the same in both. Proof of this can be obtained by comparing the propagation of the same compression concentration \( (c = 0.25) \) in both tests. As can be seen in Figure 4-45 the two characteristics don’t emanate from the same point on the x-axis but they do have the same rate of propagation upwards. It is not surprising that in the \( c_o = 0.2 \) test the characteristic arises earlier than in the \( c_o = 0.175 \) test as it requires less thickening of the initial concentration to achieve \( c = 0.25 \). The fact that in both runs the concentration arises at the same rate suggests using basic Kynch theory (Section 2.1.1.2, Equation 2.3) that the solids are settling through the characteristic at the same rate and therefore the upflow of liquid is the same i.e. \( c (u_f - u) \) is the same for both runs and so channelling will occur at the same concentration in both runs.

![Figure 4-46](image)

**Figure 4-46: Comparison between the propagation rate of a compression characteristic at different initial concentrations of talc.**

In the case of talc, channel formation occurred over a concentration range of \( 0.07 < c < 0.13 \), as the initial concentration was increased so did the concentration of channel formation. For channel formation to occur at progressively higher concentrations requires that there be a greater upflow of liquid as a greater pressure will be required to break the solid’s structure, i.e. as \( c \) increases a greater value of \( (u_f - u) \) is required. This can be proved by, as with aragonite, comparing the propagation of the same concentration compression characteristic for all the runs...
which showed distinct channel zones. It can be seen (Figure 4-46) that the rate of propagation of the $c = 0.15$ characteristic increases with increasing initial concentration so the upflow of liquid through the $c = 0.15$ layer is increasing with increasing initial concentration. The greater flow of liquid at the higher concentrations means that the condition for channel formation at lower concentrations is satisfied and so when lower concentrations arise through dilution the hard zone moves towards these lower concentrations. This effect is illustrated on the $c_o = 0.113$ and $c_o = 0.127$ characteristic plots (Figures 4-27 & 4-28).

4.2.3 Revision of Vesilind’s conclusions on channelling

Vesilind [1993] conducted a large number of calcium carbonate sedimentations in which solid’s concentration was measured using a grab technique. As a result Vesilind was able to draw a number of conclusions about the channelling behaviour of calcium carbonate. These were detailed in Section 2.1.2. In general these conclusions agree with the observations made of aragonite in that

1) channelling occurred only at intermediate concentrations
2) the upper layers of the suspension were progressively diluted
3) the channel zone velocity was constant for a given initial concentration.
4) the higher the initial concentration the higher the zone of channelling appeared in the suspension.
5) the top of the channel zone had a constant concentration.

However Vesilind also stated that channels occur in a region just above the compression zone. This statement is in direct contrast to the results of this study which showed the top of the hard zone to occur at $c = 0.22$ and the top of the soft zone at $c = 0.19$, the sediment surface concentration was $c = 0.176$.

Whilst the majority of Vesilind’s observations are valid for aragonite, a material which closely resembles Vesilind’s calcium carbonate, they are not generally applicable to all materials which exhibit channelling. Immediate discrepancies with the data of this study are that the channel zone was not subdivided into a hard and soft zone; both talc and calcite channeled at low concentrations; channels formed at
different concentrations; and the velocity of the channel zone for talc suspensions at high initial concentrations was not constant.

The data collected in this study, in particular the characteristic plots with superimposed channel zones, allowed Vesilind's conclusions on channelling to be revised so that they were applicable to the materials of this study and also Vesilind's calcium carbonate. The revised conclusions were

1] A channel zone can be subdivided into a hard and soft zone.
2] Channels form at the boundary between these two zones, this occurs in the compression zone at a concentration marginally greater than the sediment surface concentration.
3] The soft zone can exist in both regimes of settling, (compression and free settling).
4] The higher the initial concentration the quicker the channel zone propagates upwards.
5] The propagation of a channel zone although not identical is similar in shape to the rise of the concentration characteristics at which the channel zone initially forms
6] The higher the initial concentration the higher the zone of channelling appears.
7] There is an upper limit to the concentration at which channels can exist, for aragonite this was \(c = 0.27\), talc \(c = 0.17\) and calcite \(c = 0.15\).
8] The diameter of channels increases with time and with initial concentration.

The above observations allowed a qualitative model for the formation, growth and subsistence of channels to be formed. Firstly the process of compression was defined. When particles settle they form flocs which are not necessarily of a uniform shape or size. As a result when these flocs come into contact they do not immediately pack in an orderly fashion as ballotini did, instead they pack loosely in a random manner. The resultant weak structures are broken down as the weight of solids supported increases, i.e. as more flocs settle into the compression zone. So as the compression zone builds up the flocs in the lower part of the sediment are packed together more closely. Eventually further compression will only be possible if the inter-floc bonds can be broken. The strength of these bonds is likely to be dependent on factors such as initial
concentration, pH, surface charges and primary particle morphology and so different materials show varying degrees of compressibility.

Figure 4-47: Diagrammatic representation of channel formation.
The channel model is illustrated in Figure 4-47. The random inhomogeneity which acts as a seed for channel formation arises from the random packing of flocs in the early stages of compression. A pocket of fluid is formed due to the random packing which is larger than the average local porosity, Figure 4-47a. A greater liquid pressure will therefore act on the solids structure above as it will have a lesser porosity than that of the void. If the liquid velocity relative to the solids velocity is great enough then the liquid pressure will be enough to break the solids structure above and the channel begins to form, Figure 4-47b. The flow of liquid into the channel actually acts to destroy the channel as it exerts an inward drag force on the flocs that form the channel wall. Hence it is not possible for channels to form in free settling or at lower compression concentrations as a certain amount of compressive stress is required to hold the flocs in place. Liquid preferentially flows into the channel as it offers a path of least resistance and so as the channel grows the volume of liquid transported increases. Eventually this flow of liquid is great enough that the flow at the top of the channel breaks through more than just the immediate structure above. The structure
above will not have the same structural strength and so particles will be dragged into the flow creating the lightly fluidised streams which constitute the soft zone. At the top of this zone the structure is so broken up that there is no longer a preferential flow path, Figure 4-47c. When the hard zone reaches the solid-liquid interface small particles which are dragged from the channel walls and carried upwards are deposited at the mouth of the channel forming the characteristic volcano. As the channel grows in size the solids pressure at the bottom will increase, eventually the channel wall will ‘buckle’ or the floc structure will break and the bottom of the channel will be destroyed, Figure 4-47d.

4.2.4 Dilution characteristics

Dilution of the upper layers of settling suspensions was an effect shown by all the materials investigated with the exception of ballotini. Dilution only occurred in tests which exhibited channelling, and the degree of dilution was proportional to the amount of channelling. The portion of suspension that was subject to dilution included the upper regions of the soft channelling zone and the suspension above.

The most extreme dilution was shown by talc, in the case of the \( c_0 = 0.127 \) run (Figure 4-28 & Figure C28) the suspension progressively diluted to \( c = 0.08 \). The height of the segment of suspension that diluted to \( c = 0.08 \) was 25 mm, the height that diluted to the intermediate \( c = 0.1 \) was 130 mm. The height of the overall suspension when dilution to \( c = 0.1 \) had occurred was 290 mm and so it can be seen that dilution was occurring over a significant portion of the overall suspension. At lower initial concentrations of talc \( (c_0 = 0.052 \& 0.073) \) dilution occurred to a concentration at which the solid-liquid interface became indistinguishable as the suspension was diffuse.

The characteristic plots showed that dilution occurred in both free settling and compression. A consequence of this is that fully settled sediments can contain concentrations lower than the initial so long as the initial and lower concentrations are in the compression regime. It should be noted that thickening still occurs in the lower regions of a suspension showing dilution in its upper portion. What effect dilution
will have on the design of industrial thickeners (continuous or batch) is uncertain. The same degree of thickening will be possible however a greater depth of thickener may be required to accomplish it. The problem of dilution most likely will not be of great concern to an engineer, it does however present problems to the theoreticians looking to model batch sedimentations.

![Diagram](image)

**Figure 4-48: Vesilind's visualisation of the nature of characteristics in a compressible sedimentation.**

The dilution characteristics for all of the materials in this study emanated from the solid-liquid interface and propagated downwards into the suspension at a rate which was related to the degree of overall dilution experienced i.e. if a large degree of dilution occurred then the characteristics propagated downwards quicker. The rate was also related to the dilution concentration, early dilution characteristics (those close to the initial concentration) propagated downwards at a quicker rate and to a greater depth than later concentrations. Eventually the characteristics reached a depth which corresponded with the height of the rising sediment at this point the dilution characteristics showed behaviour similar to the characteristics of the suspension below.

In all current thickening models the concentration of the solid-liquid interface is taken to be that of the initial concentration or higher. To date no one has offered any theory to accommodate the dilution effect and only Vesilind [1993] has suggested what form
dilution characteristics should take. Figure 4-48 shows Vesilind's characteristic plot for a typical compressible sedimentation, the dilution characteristics lie in the region FCDG. It can be seen that Vesilind's theoretical characteristics don't match well with the experimental dilution characteristics generated for talc, calcite and aragonite. The experimental dilution characteristics for all these materials were remarkably consistent in their origin and propagation paths and so it is reasonable to assume that they are a reasonable representation of the propagation of a dilution characteristic.

It is logical that dilution characteristics should emanate from the solid liquid interface as if they were to arise from any other point in the suspension then a situation would exist where a lower concentration formed below higher concentrations. The lower concentration would have a quicker settling rate and therefore would settle away from the above suspension leaving a void in the suspension. This situation has never been observed.

In Vesilind's work dilution occurred before channels were observed and so Vesilind deduced that dilution must be caused by the squeeze of liquid from the compressive sediment. In this study the opposite was observed, dilution tended to occur in the upper region of the soft zone and above it would therefore appear to be a direct result of the channelling below. In the previous section it was suggested that in the soft zone the flow of liquid was so great that the solids structure in the region of the channel was broken and the flow of liquid was as a result redistributed across the suspensions area. If the flow of liquid is so great that the condition

\[ cu + (1-c)u_f = 0 \]  

(4.1)

is not satisfied then dilution will occur in order to accommodate the flow of liquid. Equation 4.1 is used in the derivation of all current model to remove the liquid velocity term hence it is not surprising that dilution is not predicted.

4.2.5 Locating the gel concentration

Locating the boundary concentration between free settling and compressive settling is useful as it yields information on the nature of characteristics in each regime, which regime channelling occurs in and often it is a parameter in concentration-pressure laws
laws (see Section 2.1.1.2.3). Traditionally the gel concentration is deduced by locating the compression point on the relevant settling curve. This is the height at which the sediment reaches the solid-liquid interface and therefore the time at which there is a rapid change in the velocity of the interface. The velocity at this height is then compared with initial rate data to give a value for the gel concentration. Unfortunately this method didn't prove compatible with the materials used for the following reasons.

1) With aragonite no rapid change in the settling velocities was observed for suspensions with initial free settling concentrations, it was therefore impossible to locate the compression point. Initial concentrations in compression did show what could be conceived to be a compression point however as very little dilution occurred the velocity at this point is likely to be that of the initial concentration or similar and not the gel concentration.

2) From the above definition of the compression point it would seem reasonable to assume that the compression point for materials such as talc is the peak of the acceleration period on a settling velocity curve. However as can be seen from Figure 4-22 these peaks don't always occur at the same height. One could tacitly assume that the highest peak represents the gel concentration velocity, however it was observable in some tests that dilution was occurring to such an extent that the interface became diffuse i.e. dilution was occurring well into the free settling regime and so this assumption is questionable as the velocity is most likely that of a free settling concentration settling discontinuously into the sediment.

3) The use of initial rate experiments was considered questionable, i.e. there was evidence to suggest that the initial settling rate of a suspension was not the same as the settling rate of a lamina of suspension of the same concentration that had arisen through thickening (see Section 2.1.1.2.4).

The gel concentration was instead established using combination of basic theory coupled with a knowledge of the settling velocity profiles and of the nature of the characteristics for that material. The value of the gel concentration was therefore obtained through logical reasoning as opposed to analytical mathematical examination. The reasoning for each material will now be outlined.
The gel concentration of aragonite was deduced to be $c = 0.176$. This was determined from examination of the concentration-time and settling velocity plots and from application of the tangential characteristics theory (Section 4.2.7). The settling velocity profiles for $c_o < 0.175$ (Figure 4-22) were similar to those predicted by the ideal compressible sedimentation model (Section 2.1.1.1.2) in that they appeared to show first and second falling rate zones followed by a compression zone. It was therefore reasonable to assume that the characteristics of aragonite would be similar to those suggested by the model and that tangential characteristics may be present. The topic of tangential characteristics is dealt with in Section 4.2.7 where it is shown that the presence of tangential characteristics can be detected on concentration-time plots as a kink or step in the profile. The concentration at the top of such a kink is the gel concentration. The concentration profiles (Figures C7-10) showed such kinks and they ended at approximately $c = 0.176$. This fitted well with the nature of the characteristics either side of $c = 0.176$, the lower concentration characteristics emanated in a ‘Kynch’ like fan whereas the higher concentrations propagated upwards in a near parallel manner to each other which is indicative of compressive behaviour.

To the sceptical eye it may appear that no tangential characteristics emanate from $c = 0.176$ on any of the characteristic plots (Figure 4-10 & Figures C13-15), this is because of a flaw in the method of generating characteristics and with the electrical impedance technique (see Section 4.2.7). It may also appear that for $0.18 \leq c \leq 0.23$ the characteristics reach the interface and disappear suggesting that these characteristics actually represent free settling concentrations however this is not the case as these characteristics have compressive attributes. The disappearance of the characteristics is explained by the fact that it was impossible to have an electrode positioned exactly at the interface height and by the susceptibility of the electrical impedance technique to errors near interfaces (see Section 4.2.10). A small zone (approximately 8 mm) of immeasurable suspension therefore existed just below the interface in each experiment, the concentrations $0.18 \leq c \leq 0.23$ occur within.

The difficulty in interpreting the settling velocity plots of talc was discussed above and as a result deduction of the gel concentration was based on the shape and emanation point of the characteristics and on evidence provided by wall effect tests.
The characteristics for tale with the exception of dilution characteristics all showed compressive attributes in that they arose from the origin or at some later time and propagated upwards parallel to each other. It would therefore appear that the suspension was in compression at concentrations as low as $c = 0.06$. Further evidence is gained from wall effect tests, for wall effects to be large enough to significantly influence the rate of settling then the effect must occur across the diameter of the cell and not just for particles near the wall. Therefore there must be some form of structure present before wall effects are noticeable i.e. the suspension has to be in compression. From Figures 4-19 & 4-20 it can be seen that there was a very significant difference between the $c_o = 0.05$ and $c_o = 0.06$ tests in terms of the variation in settling rate shown between the two different column sizes. The retardation experienced due to using the smaller cell in the $c_o = 0.05$ test was minimal and only occurred towards the end of the test whereas the retardation in the $c_o = 0.06$ test occurred almost from the onset of settling and had a very significant overall effect. It therefore seemed reasonable to assume that the boundary between free and compressive settling was approximately $c = 0.05$.

Investigation of the settling velocity curves (Figure 4-34) for calcite located the gel concentration as being between $0.025 < c_o < 0.05$. At $c_o = 0.025$ the settling velocity curve shows a first and second falling rate zone and a compression zone i.e. typical free settling behaviour. At $c_o = 0.05$ a constant rate zone is followed by a small acceleration period and then a deceleration period, constant rate zones can occur in both regimes however if $c_o = 0.05$ was in the free settling regime then falling rate zones similar to those shown by $c_o = 0.025$ velocity plot would be observed. This is not the case so $c = 0.05$ is deduced to be a compression concentration. Examination of the characteristic plot for the $c_o = 0.025$ test shows the characteristics for $c \geq 0.05$ to be compressive in nature whereas the characteristic for $c = 0.04$ is more similar to a Kynch characteristic. So the gel concentration was deduced to be between $0.04 < c < 0.05$. 
Several authors have suggested methods for predicting the rise of the sediment surface. These methods were outlined in Section 2.1.1.2. Fitch’s [1983] method required several sedimentations to be made at different initial heights. The settling curve for each initial height was plotted and the compression point on each curve was located. A line was then drawn from the origin through each compression point to give the rise of the sediment for the highest initial height. Whilst relatively simple this method is time consuming and as previously discussed it was not possible to locate any compression points on the settling curves of the materials used in this study. The validity of this method therefore could not be evaluated. Font’s [1988] method required only two tests on either different initial heights or concentrations. Lines which join points with the same settling velocity and intersect with the y-axis are constructed. The shape of the sediment can then be deduced, this method was applied to the aragonite tests $c_o = 0.075 \& 0.1$. As can be seen from Figure 4-49 the method proved useful in predicting the initial rise of the sediment but failed after approximately 3000 seconds. This method therefore has some merit as it can be used to locate the characteristic which represents the gel concentration, it is also not as time consuming as Fitch’s technique.
4.2.6 Effect of stirring

Two types of settling behaviour resulted from stirring - that shown by calcite and aragonite and that shown by talc.

In the former it was found that a stirrer speed of 1 r.p.m. caused the most retardation of the settling rate and that the initial velocity was the same regardless of the stirrer speed. The fact that stirrer speeds above 1 r.p.m. caused an increase in settling rate was surprising. It had been anticipated that an increased stirrer speed would lead to increased retardation of the settling rate as the stirrer action would as well as destroying channels act to resuspend particles. This however was not the case, no plausible explanation can be offered to suggest why this was. The most noticeable aspect of stirring calcite and aragonite was however the increase in sediment concentration, significantly higher concentrations were achieved at both the base and the surface of the sediment. For concentrations higher than the gel concentration determined in the unstirred tests to be obtained at the sediment surface required a force to act on the particles at the surface. The stirrer action cannot provide any vertical compressive force and so the increase was attributed to the radial action of the stirrer breaking the solids structure and allowing it to repack to a higher concentration. This occurs throughout the height of the sediment. The compaction process in a stirred system is therefore a function of two variables, the weight of overlying solids i.e. the traditional compressive force and the reordering effect of the stirrer.

The increase in sediment concentration due to stirring presents a dilemma to the industrial engineer as to how to best operate a continuous thickener. An increase in underflow concentration may be desirable as it could reduce the amount of downstream processing required however the throughput of the thickener would be reduced due to the decrease in settling rate and a powerful stirring system would be required to move the sediment to the underflow without jamming. A thorough investigation of the material to be thickened would be required to establish which was most economically viable.
Talc in contrast to the other two materials showed only a nominal increase in sediment concentration but a large reduction in settling rate as the stirrer speed was increased. The stirrer speed required to completely eradicate channelling was significantly larger than that required by calcite and aragonite (4 r.p.m. cf. 1 r.p.m.). The initial velocities at the same concentration but different stirrer speeds were also significantly different suggesting that stirring may have been affecting the initial state of aggregation. The reason for the dissimilarity between talc and the other materials lies in the sediment structure of talc. In Section 4.1.5.2 this was described as 'dough' like and that unlike other materials' sediment structures which broke and reformed as the stirrer rotated it would be dragged in the direction of stirring i.e. the interparticle forces in the radial plane must be very strong. Talc is known to possess variable surface charges and so it is possible that under the correct conditions a strong sediment structure could form.

There would therefore appear to be little advantage in stirring a material such as talc in a continuous thickener as the benefit of a marginally greater sediment concentration would be outweighed by the decrease in solids flux through the thickener.

4.2.7 Tangential emanation of characteristics from the sediment

To date the only theory which accounts for the nature and origin of characteristics in the second falling rate period (i.e. the area where characteristics cannot emanate from the origin to the solid liquid surface in a linear fashion as they would intercept with the sediment surface curve) is that of tangential characteristics. This theory was first suggested by Tiller [1981] and was later revised by Fitch [1983] and Font [1988]. These papers were predominantly theoretical and offered very little experimental evidence of tangential characteristics. Early studies using aragonite led this author (Butt et al. [1993]) to question the validity of tangential theory, however a subsequent more vigorous examination of data has led to a revised conclusion.

At a first glance it may seem relatively simple when in-situ measurement of solids concentration is possible and the gel concentration is known to confirm the existence of tangential characteristics however this was found not to be the case. A number of problems were encountered, these are detailed below.
The theory of tangential characteristics states that characteristics of progressively higher concentrations will arise tangentially from a rising sediment surface. At the point where such a characteristic starts there is a discontinuity between the characteristic concentration and the gel concentration. The size of this discontinuity is not large and decreases as the tangential characteristic concentration increases. Such discontinuities couldn’t be measured using electrical impedance measurements (or indeed with any other known sampling method) as the technique was not sensitive enough to detect such a small concentration discontinuity. This was due to the fact that electric flux does not travel only in the horizontal plane of measurement but bows a small distance either side in the vertical direction. Any impedance measurement was therefore an average of the resistance offered by a segment of finite depth, i.e. if it contained a discontinuity a value representing the average resistance of the two concentrations is returned. This effect is discussed fully in Section 4.2.9.

The possible existence of a small discontinuity also gave rise to an error in the method used to generate characteristics (Section 4.1.1). The method used located the heights of certain concentrations in the suspension at every time increment. These heights were calculated by comparing the required concentration with the concentrations registered by adjacent electrode pairs until the desired concentration fell within those limits. The height of that concentration was then determined by assuming that the concentration varied linearly between the two electrodes. The flaw in this technique was that it assumed that all concentrations between the concentrations registered by the electrodes existed, obviously this was not the case when a discontinuity was present and therefore a degree of error was introduced.

Experimental characteristics pertaining to be of a tangential nature can therefore only be considered valid if they are a significant distance away from the sediment i.e. the distance between electrodes.
Figure 4-50: Plot of theoretical characteristics emanating from the experimental sediment profile for the $c_o = 0.1$ aragonite test.

The problem of generating tangential characteristics was overcome by examining how this type of settling is portrayed on a concentration-time plot. To do this it was necessary to work backwards and generate a concentration-time plot from a theoretical plot of tangential characteristics. The rise of the sediment in the $c_o = 0.1$ aragonite test was described by the $c = 0.176$ characteristic, an exponential curve was fitted to this characteristic and the methods of Font and Fitch (Section 2.1.1.1.2) were applied to generate the characteristics shown in Figure 4-50. The characteristic concentration values are given in Table 4-1. It is interesting to note that both methods gave identical results at lower concentrations and varied only slightly as the concentration approached that of the sediment. The values generated were realistic for the system considered. The times at which a characteristic concentration occurred at set heights could now be deduced which allowed a concentration-time plot to be constructed (Figure 4-51). This was compared with the relevant section of the experimental concentration plot for $c_o = 0.1$ test (Figure 4-52), it can be seen that both graphs predict very similar behaviour. A ‘kink’ in the concentration time profile was shown by all the aragonite runs having an initial concentration of $c_o < 0.175$. The same was also observed at a concentration of $c = 0.04$ in the $c_o = 0.025$ calcite run.
Table 4-1: Theoretical concentration values for the characteristics shown in Figure 4-50.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Height of emanation (mm)</th>
<th>Characteristic concentration</th>
</tr>
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<tbody>
<tr>
<td>a</td>
<td>0.061</td>
<td>0.114 0.114</td>
</tr>
<tr>
<td>b</td>
<td>0.414</td>
<td>0.124 0.124</td>
</tr>
<tr>
<td>c</td>
<td>0.888</td>
<td>0.130 0.130</td>
</tr>
<tr>
<td>d</td>
<td>1.584</td>
<td>0.134 0.134</td>
</tr>
<tr>
<td>e</td>
<td>3.958</td>
<td>0.142 0.142</td>
</tr>
<tr>
<td>f</td>
<td>7.054</td>
<td>0.149 0.149</td>
</tr>
<tr>
<td>g</td>
<td>10.799</td>
<td>0.154 0.154</td>
</tr>
<tr>
<td>h</td>
<td>17.955</td>
<td>0.161 0.161</td>
</tr>
<tr>
<td>i</td>
<td>27.290</td>
<td>0.167 0.167</td>
</tr>
<tr>
<td>j</td>
<td>41.872</td>
<td>0.173 0.174</td>
</tr>
</tbody>
</table>

Figure 4-51: Theoretically generated concentration time plot over the tangential characteristic concentration range for the aragonite $c_n = 0.1$ test.

Although the theory predicted that a kink in the profiles should exist it didn’t correctly predict the size of the kink. The effect of this is shown in Figure 4-53 in which the theoretical characteristics of Figure 4-50 are compared with corresponding experimental characteristics. When examining this figure the above comments on the validity of experimentally determined characteristics should be considered and as such only the top segment of characteristics should be considered. It can be seen that a
A good match is achieved at low concentrations but as the concentration increases a noticeable difference arises between the theoretical and experimental characteristics. This may be due to a flaw in the theory however it is also possible that it is due to the inadequacy of the measurement technique near discontinuities. It would appear however that overall the theory of tangential characteristics is a reasonable approximation for the settling behaviour encountered in the second falling rate zone.

**Figure 4-52:** Experimental concentration-time plot corresponding to the plot in Figure 4-51.

**Figure 4-53:** Comparison between theoretical and experimental tangential characteristics.
4.2.8 Nature of free-settling characteristics

In Section 2.1.1.2 a mathematical description of the process of sedimentation was developed in which the free settling regime was represented by

\[ c(\rho_s - \rho)g = \frac{\mu}{k} \]  \hspace{1cm} (4.2)

and the compression regime by

\[ \frac{\partial p_s}{\partial x} = c(\rho_s - \rho)g - \frac{\mu}{k} \]  \hspace{1cm} (4.3)

These equations form the basis of all mathematical models of sedimentation, however Equation (4.2) is flawed as it contains no component which will cause a lamina of suspension with a free settling concentration in an infinite fluid to thicken to a higher free settling concentration. Hence use of Equation (4.2) will not generate a fan of characteristics such as those shown by aragonite at concentrations of \( c < 0.18 \). It instead predicts that a suspension will settle at its initial concentration until it reaches the sediment at which point Equation (4.3) becomes valid and thickening occurs because the solids stress gradient is present. This fact is often overlooked by thickener designers for whilst Kynch theory doesn’t predict why a fan of characteristics should arise it does accommodate the fact that they do (i.e. is a material balance).

The only author to suggest what force may give rise to a change in concentration in the free settling regime is Dixon [1978] who suggested the presence of a transient solids stress. He suggested that this would arise because as particles move closer together, liquid is ‘squeezed out’ from between them, and the flow resistance of the liquid gives rise to a force resisting the movement of particles towards each other. This seems to be a plausible reason for the resistance which gives rise to fans of free settling characteristics. If we further Dixon’s suggestion and consider the case of three identical spheres settling in a vertical line. If the subscripts 1,2,3 are used to describe the bottom, middle and top spheres respectively then when \( u_1 = u_2 = u_3 \) the net force on sphere 2 is zero, however when \( u_2 - u_1 > u_3 - u_2 \) sphere 2 experiences a greater force from below than above causing the particle to decelerate. In a well mixed suspension all particles will initially settle at the same velocity and so the first
case cited above will apply except at the base of the vessel where $u_1$ can be considered to be zero, in this case the second case is satisfied as $u_2 - u_1 > u_3 - u_2$ reduces to $u_2 > 0$. The resulting reduction in $u_2$ causes the particles to move closer together and hence thickening occurs. Once established this condition will propagate upwards into the rest of the suspension. This reasoning that the transient stress gradient originates from the base of a suspension is consistent with the fact that characteristics in the first falling rate zone emanate from the origin i.e. the base of the suspension. This transient solids stress can exist in both regimes of settling however it is unlikely to be a significant force in compression as the static solids stress will predominate. In the previous section it was shown that the rise of the sediment surface is not linear rather it is an exponential shape. The significance of this is that the velocity of solids settling through the sediment characteristic will differ as the sediment grows and so therefore will the value of the transient solids stress hence it is possible for different free settling concentrations to settle into the sediment. This strengthens the case for characteristics emanating from the sediment although it does not indicate that they will do so in a tangential manner. As yet no mathematical relationship has been proposed for the solids transient stress, the linear nature of the free settling characteristics in the aragonite tests suggests that it may be a simple function of concentration. A method of determining values for the transient stress from experimental data needs to be developed before this can be verified.

4.2.9 Flux Tracking

As already noted the electrical impedance technique was subject to distortion near boundaries of significant conductivity change e.g. the solid-liquid interface or the free settling-compression interface when the sediment has a considerably higher concentration than the free settling concentration above. This was because the electric flux didn’t transverse the distance between electrodes only in the horizontal plane but also bowed into the vertical plane. It was therefore possible for a higher (or lower) conductivity region to affect the flux even though it was not level with the electrodes. A result of this was that it was difficult for the electrical impedance technique to register a step change in conductivity.
Flux tracking was evident in many of the conducted experiments an example is shown in Figure 4-54. As the solid-liquid interface of the ballotini suspension settled past sets of electrodes step changes in conductivity from that of the suspension to that of the clear phase should have been registered however the change was more gradual. This was because the interface affected the electric flux as it approached the plane of measurement and for some time after it had settled past. The effect was most pronounced with ballotini because of the high concentrations attained which resulted in large conductivity changes. Experiments with calcite and aragonite in which impedance measurements at short time intervals were made in conjunction with interface height observations showed that the interface only affected conductivity measurements within 8 mm of a measurement plane. The concentration at electrodes within 8 mm of the interface was therefore taken to be an extrapolation of the concentrations registered at the two heights of measurement immediately below. The extent to which flux tracking occurred within a suspension i.e. in the case of the sediment surface was less easy to measure. The concentration changes and therefore conductivity changes would be much smaller than experienced at the solid-liquid interface and so any effect was likely to be considerably smaller.
5. Characterising the sediment

In order to be able to model the consolidation process expressions were needed which described the variation in solids pressure and permeability. Traditionally both are considered to be functions of local concentration only and were treated as such in this study. This proved satisfactory for the materials used however other researchers should keep in mind the Kos’ postulation (Section 2.1.1.2, [1981]) that they are not unique functions rather a combination of local concentration and some other parameter probably fluid velocity. The following section describes how permeability and solid pressure values were derived from experimental data and details the function which gave the best fit to the data.

5.1 Measurement of solids pressure

Measuring the total pressure in a sedimenting suspension is relatively simple, the use of pitot tubes or pressure transducers are both proven methods. However in-situ measurement of the solids pressure is less easy. Very few researchers have addressed this problem, the only method available in the literature was suggested by Kos [1978]. Using pressure transducers the total pressure and the hydrostatic pressure in a continuous thickener were measured, the hydrostatic pressure was obtained by destroying the solids structure in the vicinity of the transducer. The solids pressure was taken to be the difference between the total and hydrostatic pressures. Consistent results were only achieved by taking a large number of readings averaged over time hence this method is only applicable to steady state systems and so could not be applied to the batch systems used in this study.

In the absence of any reliable in-situ method for measuring solids pressure during batch sedimentations it was necessary to employ a technique which obtained solids pressures via the examination of fully consolidated sediments. The technique had been used previously, and successfully, by a number of authors (Tiller [1983], Shirato [1983], Massarani [1993]).
When a sediment has completely consolidated there is no flow of liquid through the sediment so \( u_f = 0 \), likewise the solids velocity \( u = 0 \) so the basic force balance (Equation 2.11) reduces to
\[
\frac{dp_s}{dx} = cg(p_s - \rho) \quad \text{or} \quad p_s = g(p_s - \rho) \int c \, dx
\] (5.1)

If the concentration-height relationship of a sediment is known then the solids pressure at any given height can be obtained by integrating the sediment profile above that point. The concentration at the given height is uniquely associated with that value of solids pressure. This technique therefore assumes that solids pressure is a unique function of concentration however as already noted this has been observed not to always be true. In this study the assumption was tested by plotting and hence comparing the solids pressure values from a range of different initial concentration runs (and consequently different flow histories). If the data fell within a narrow band then solids pressure was considered to be a unique function of concentration. The assumption was found to be true for all the materials investigated.

Final sediment profiles for all of the runs detailed in Chapter 4 were obtained via electrical impedance measurements. These were integrated using the trapezium rule, the differential strip width was taken to be the distance between electrodes.

The solids pressure-concentration relationships shown in Table 5-1 were fitted to the experimental data. In the case of the Shirato, Landman 1 and Landman 2 expressions this was achieved by a regression on the logarithmic values of solids pressure and the term raised to a power. With the Tiller equation a regression was performed on the solids pressure and logarithmic term. The Landman 3 coefficients were determined by trial and error.
Table 5-1: Solids pressure-concentration relationships.

<table>
<thead>
<tr>
<th>Shirato</th>
<th>Tiller</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_s = \left( \frac{c}{c'} \right)^{1} )</td>
<td>( p_s = p_h \ln \left( \frac{c_m - c}{c_m - c_0} \right) )</td>
</tr>
<tr>
<td>( p_s = p_h \left[ \left( \frac{c}{c_g} \right)^{\phi} - 1 \right] )</td>
<td>( p_s = p_h \left[ \left( \frac{c}{c_g} \right)^{\phi} - 1 \right] )</td>
</tr>
</tbody>
</table>

5.1.1 Experimentally derived solids pressures for aragonite

Figure 5-1 shows a plot of solids pressure in terms of solids concentration. Experimentally deduced values are shown as single points and the fitted expressions as solid lines. The experimental values were taken from the final profiles of a number of sedimentations ranging in initial concentration from \( c_o = 0.075 \) to \( c_o = 0.2 \), these are shown in Figure 5-2. It can be seen from this figure that in all the runs a final concentration of \( c = 0.306 \) was obtained at the base of the sediment and that this concentration extended upwards to a depth which was dependent on the initial concentration. This is the maximum concentration that could be obtained through gravitational compaction, a sediment structure which was strong enough to resist any further compression had formed and this could only be disrupted by some external force such as stirring. The degree of compressibility shown was small, only a small depth (~ 4 cm) of sediment was required to attain the final sediment concentration indicating that a very small force was required to reduce the sediment surface concentration \( c = 0.176 \) to that of \( c = 0.306 \) hence the band of pressures shown in Figure 5-1 initially has a very shallow slope which increases rapidly near the final concentration.
Figure 5-1: Solids pressure plot for aragonite.

Figure 5-2: Fully consolidated sediment profiles of aragonite.
The coefficients required to fit the pressure laws defined in Table 5-1 to the experimental data shown in Figure 5-1 are given in Table 5-2.

**Table 5-2 : Coefficients required to fit pressure laws to the aragonite data.**

<table>
<thead>
<tr>
<th></th>
<th>Shirato</th>
<th>Landman 1</th>
<th>Landman 2</th>
<th>Landman 3</th>
<th>Tiller</th>
</tr>
</thead>
<tbody>
<tr>
<td>c'</td>
<td>0.22206</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p_b</td>
<td>0.01177</td>
<td>5075.8</td>
<td>3000000</td>
<td>-140</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>0.04966</td>
<td>19.4475</td>
<td>7.42703</td>
<td></td>
<td>0.306</td>
</tr>
<tr>
<td>c_m</td>
<td></td>
<td></td>
<td></td>
<td>0.306</td>
<td>0.306</td>
</tr>
<tr>
<td>c_g</td>
<td>0.176</td>
<td>0.176</td>
<td></td>
<td></td>
<td>0.176</td>
</tr>
</tbody>
</table>

The Shirato, Landman 1 and Landman 2 expressions gave an adequate fit to the data at solids pressures less than 400 Pa, at higher pressures concentrations greater than the maximum attainable through compaction \((c > 0.306)\) were predicted. The Tiller and Landman 3 expressions didn’t give good fits to the data. The Tiller expression however could be manipulated to predict high solids pressures \((p_s > 400)\). The best solids pressure-concentration relationship for aragonite was therefore found to be a combination of the Shirato and Tiller expressions

\[
\begin{align*}
  c \leq 0.30007, \quad p_s & \leq 448.21 \\
  p_s & = \left( \frac{c}{0.22206} \right)^{\frac{1}{0.04966}} \\
  c > 0.30007, \quad p_s & > 448.21 \\
  p_s & = -140 \ln \left( \frac{0.306 - c}{0.306 - 0.176} \right)
\end{align*}
\]  

(5.2)

### 5.1.2 Experimentally derived solids pressures for calcite

The final sediment profiles for the calcite runs detailed in the previous chapter are shown in Figure 5-4. In most of the tests a base sediment concentration of \(c = 0.181\) was obtained, the exceptions to this were the \(c_o = 0.025\) and \(c_o = 0.15\) tests. In the former this was because an insufficient height of sediment existed to achieve compaction to \(c = 0.181\). The latter showed a large discrepancy with the other tests suggesting a base sediment concentration of \(c = 0.19\) was achieved, the reason for this was assumed to be the large % error in volumes recorded in the \(c_o = 0.15\) run. Due to the magnitude of the discrepancy the data from this test was not utilised to generate solids pressure values.
Figure 5-3: Solids pressure plot for calcite.

Figure 5-4: Fully consolidated sediment profiles of calcite.
Although the maximum attainable sediment concentrations of aragonite and calcite were very dissimilar (0.306 cf. 0.181) the magnitude of the compression concentration range and the pressure required to achieve the same degree of compression were very similar. Hence the band of experimental solids pressure data points for calcite (Figure 5-3) have the same shape as the aragonite data in Figure 5-1.

The coefficients required to plot the fitted expressions are given in Table 5-3.

**Table 5-3: Coefficients required to fit pressure laws to the calcite data.**

<table>
<thead>
<tr>
<th></th>
<th>Shirato</th>
<th>Landman 1</th>
<th>Landman 2</th>
<th>Landman 3</th>
<th>Tiller</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c')</td>
<td>0.094976</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P_b)</td>
<td>0.003553</td>
<td>0.336333</td>
<td></td>
<td>90</td>
<td>-89.5</td>
</tr>
<tr>
<td>(a)</td>
<td>0.108466</td>
<td>7.579303</td>
<td>5.450599</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td>(c_s)</td>
<td>0.183</td>
<td>0.181</td>
<td></td>
<td>0.181</td>
<td>0.045</td>
</tr>
<tr>
<td>(c_c)</td>
<td>0.045</td>
<td>0.045</td>
<td></td>
<td>0.045</td>
<td></td>
</tr>
</tbody>
</table>

As with aragonite the Shirato, Landman 1 and Landman 2 expressions adequately fitted the experimental data at low concentrations but over predicted concentrations at higher pressures. The best data fit was achieved using the Landman 3 expression or a combination of the Shirato and Tiller expressions. The solids pressure-concentration relationship for calcite is therefore best described by

\[
\begin{align*}
  c & \leq 0.17569, \quad p_s \leq 290.31 & \quad p_s &= \left( \frac{c}{0.09498} \right)^{1/0.108466} \\
  c & > 0.17569, \quad p_s > 290.31 & \quad p_s &= -140 \ln \left( \frac{0.181 - c}{0.181 - 0.045} \right) \\
  \end{align*}
\]

or

\[
\begin{align*}
  p_s &= 90 \left[ \frac{c^{2.2}}{0.181 - c} \right] 
\end{align*}
\]

**5.1.3 Experimentally derived solids pressures for talc**

The final profile plot for the talc investigated is shown in Figure 5-6. It can be seen that only a small height of sediment was required before a decrease in concentration from that of the base sediment concentration occurred and there was a gradual
decrease in concentration towards the sediment surface concentration. This was indicative of a highly compressive material, a fact that was confirmed by the solids pressure plot (Figure 5-5). Whereas aragonite and calcite required only a very small pressure to reach a concentration near the maximum attainable through compaction, talc required a significant solids pressure to achieve the intermediate compression concentrations. The solids pressure-concentration plot therefore showed less of a shoulder, the increase in pressure with respect to concentration being more gradual.

The best fit with the experimental data was achieved using Shirato’s expression, the Landman 1 and Landman 2 expressions also gave satisfactory fits. The Tiller and Landman 3 expressions couldn’t be fitted. The solids pressure-concentration relationship was therefore best described by

\[ p_s = \left( \frac{c}{0.096428} \right)^{0.134897} \]  

(5.5)

<table>
<thead>
<tr>
<th>Table 5-4: Coefficients required to fit pressure laws to the talc data.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shirato</td>
</tr>
<tr>
<td>c'</td>
</tr>
<tr>
<td>p_b</td>
</tr>
<tr>
<td>a</td>
</tr>
<tr>
<td>c_a</td>
</tr>
<tr>
<td>c_g</td>
</tr>
</tbody>
</table>

The best fit with the experimental data was achieved using Shirato’s expression, the Landman 1 and Landman 2 expressions also gave satisfactory fits. The Tiller and Landman 3 expressions couldn’t be fitted. The solids pressure-concentration relationship was therefore best described by
Figure 5-5: Solids pressure plot for talc.

Figure 5-6: Fully consolidated sediment profiles of talc.
5.1.4 Summary of solids pressure measurement

The solids pressure measurements showed that aragonite and calcite were slightly compressive, only a very small force (~ 10 Pa) was required to achieve a concentration close to that of the maximum attainable concentration. The best solids pressure concentration relationship for these materials was a combination of the Shirato and Tiller expressions. Talc was found to be a more compressive material i.e. a greater solids pressure was required to achieve the same degree of compression. The Shirato equation gave the best fit to the experimental data of talc. The recently suggested Landman expressions for solids pressure offered no advantage over the older and mathematically simpler Shirato expression. However the Landman 3 and Tiller expressions proved useful for predicting high solids pressures at concentrations close to the maximum attainable.

It can be seen from Figures 5-1, 5-3 & 5-5 that experimental solids pressure values were not obtained, for any material, over the whole of the compression region. Low compression concentrations i.e. those situated close to the sediment-liquid surface could not be measured because of electrical flux tracking (see Section 4.2.9). The problem of measuring concentrations close to the solid-liquid interface appears to be inherent to most sampling techniques, for example Tiller [1983] used grab tubes to measure the concentration profiles and reported that the sediment structure near the solid-liquid interface was too unstable to obtain reliable results. Although a sizeable portion of the concentrations in the compression range could not be measured the pressure ranges involved were relatively small (approximately 0-10 Pa) compared to the overall ranges (approximately 0-600 Pa) and so any error introduced was likely to be small.
5.2 Permeability measurement techniques

The model presented later in this thesis predicts the settling behaviour of suspensions having an initial concentration greater or equal to the gel concentration i.e. in compressive settling. It was therefore necessary to deduce a permeability function for the compression concentration range of each material. Initial rate and downflow (or compression-permeability cell) tests provide permeability data for suspensions in free settling and for highly compacted, pressurised sediments respectively. Previous studies (Shirato [1983], Murase [1989]) have shown that reasonable correlations can be obtained between downflow and initial rate data and as a result the authors suggested that the permeability of the intermediate concentrations could be estimated via a straight line extrapolation on a logarithmic plot from the free settling data to the downflow data. It should however be noted that no rigorous modelling work was performed in either study to validate this assumption. Until recently the above method was the only available method for estimating permeability values at intermediate compression concentrations. An alternative, the concentration profile method, was suggested by Font [1994] but as yet the method is relatively untested. The data taken in this study allowed further evaluation of the method.

Additionally permeability variation in the free settling regime was also investigated, this was achieved using initial rate and upflow tests. The following is an outline of the theory for each measurement technique discussed above.

5.2.1 Initial rate

It has been shown (Tiller[1981], Shirato [1983], Murase [1989]) that the permeability of concentrations within the free settling region can be determined by initial rate experiments provided the incipient settling rate of the suspension is constant. The initial gradient of the settling curve provides a value for solids velocity at the starting concentration, as no solids stress exists the permeability is calculated from
\[ k = \frac{\mu \xi}{c g(\rho_s - \rho)} \]  

(5.6)

This technique assumes that a layer of concentration \( c \) which arises during an experiment will approximate to the permeability of a suspension of initial concentration \( c \) and that any transient solids stress is negligible compared with the drag forces present.

Table 5-5: Examination of the relative magnitudes of the solids pressure gradient and fluid drag terms.

<table>
<thead>
<tr>
<th>Concentration</th>
<th>Velocity ( \text{v/v} )</th>
<th>Permeability ( \text{m}^2 )</th>
<th>( \frac{dc}{dx} ) ( \text{m}^{-1} )</th>
<th>( \frac{dp_s}{dc} ) ( \text{Pa} )</th>
<th>( \frac{\mu}{k} ) ( \text{Pa} \text{m}^{-1} )</th>
<th>( \frac{dp_s}{dx} ) ( \text{Pa} \text{m}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aragonite</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>5.86E-06</td>
<td>1.86E-12</td>
<td>0.01</td>
<td>12.24</td>
<td>3084.26</td>
<td>0.12</td>
</tr>
<tr>
<td>0.225</td>
<td>4.01E-06</td>
<td>1.13E-12</td>
<td>0.01</td>
<td>116.63</td>
<td>3469.79</td>
<td>1.16</td>
</tr>
<tr>
<td>0.258</td>
<td>3.14E-06</td>
<td>7.74E-13</td>
<td>0.01</td>
<td>1600.42</td>
<td>3978.70</td>
<td>16.00</td>
</tr>
<tr>
<td>Calcite</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0491</td>
<td>1.9E-04</td>
<td>2.46E-10</td>
<td>0.01</td>
<td>0.42</td>
<td>757.42</td>
<td>0.004</td>
</tr>
<tr>
<td>0.0779</td>
<td>9.99E-05</td>
<td>8.15E-11</td>
<td>0.01</td>
<td>19.06</td>
<td>1201.56</td>
<td>0.19</td>
</tr>
<tr>
<td>0.102</td>
<td>6.18E-05</td>
<td>3.83E-11</td>
<td>0.01</td>
<td>180.83</td>
<td>1579.83</td>
<td>1.80</td>
</tr>
<tr>
<td>0.128</td>
<td>3.95E-05</td>
<td>1.95E-11</td>
<td>0.01</td>
<td>1169.87</td>
<td>1982.72</td>
<td>11.69</td>
</tr>
<tr>
<td>0.157</td>
<td>2.5E-05</td>
<td>1.01E-11</td>
<td>0.01</td>
<td>6214.46</td>
<td>2429.40</td>
<td>62.14</td>
</tr>
<tr>
<td>0.175</td>
<td>2.05E-05</td>
<td>7.46E-12</td>
<td>0.01</td>
<td>14746.70</td>
<td>2698.73</td>
<td>147.46</td>
</tr>
<tr>
<td>Talc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.052</td>
<td>7.36E-06</td>
<td>8.35E-11</td>
<td>0.01</td>
<td>86.31</td>
<td>86.31</td>
<td>0.01</td>
</tr>
<tr>
<td>0.0728</td>
<td>5.05E-06</td>
<td>4.09E-11</td>
<td>0.01</td>
<td>120.83</td>
<td>120.83</td>
<td>0.12</td>
</tr>
<tr>
<td>0.09</td>
<td>3.95E-06</td>
<td>2.59E-11</td>
<td>0.01</td>
<td>149.38</td>
<td>149.38</td>
<td>0.49</td>
</tr>
<tr>
<td>0.113</td>
<td>2.74E-06</td>
<td>1.43E-11</td>
<td>0.01</td>
<td>187.56</td>
<td>187.56</td>
<td>2.12</td>
</tr>
<tr>
<td>0.126</td>
<td>2.26E-06</td>
<td>1.06E-11</td>
<td>0.01</td>
<td>209.14</td>
<td>209.14</td>
<td>4.27</td>
</tr>
<tr>
<td>0.15</td>
<td>1.05E-06</td>
<td>4.13E-12</td>
<td>0.01</td>
<td>248.97</td>
<td>248.97</td>
<td>13.07</td>
</tr>
</tbody>
</table>

In this study the technique was extended to concentrations within the compression region. The justification for this was that for some materials at lower compression concentrations the solids pressure gradient is negligible compared to the fluid drag i.e. \( \frac{\mu \xi}{k} \frac{dp_s}{dx} \) and so Equation (5.6) can still be considered valid. The range over which this supposition could be considered true was determined for each material using the
analysis shown below in Table 5-5. The fluid drag term was calculated via Equation (5.6) using the initial settling rate. The pressure gradient was determined using the chain rule, values of $\frac{dp_s}{dc}$ were obtained from the solids pressure concentration relationships developed in the previous section. A value of 0.01 was used for $\frac{dc}{dx}$ this represents a concentration change of 0.0001 over a distance of 1 cm. The electrical impedance techniques suggested that for all the materials that there was initially no or very little concentration variation near the sediment surface the value of 0.01 seemed realistic.

In the case of aragonite it seemed reasonable to use the initial rate permeability data for all of the considered initial concentrations as in all cases the drag force was significantly larger than the pressure gradient. For talc and calcite use of the initial rate method was considered to be valid for initial concentration tests of $c_o < 0.126$ and $c_o < 0.157$ respectively, above this the pressure gradient was significant. It should be noted that the method of analysis was very dependent on the value of $\frac{dc}{dx}$ used, if a significant concentration gradient had quickly formed in any of the tests then the pressure term would have been considerably larger and the method would have been invalid.

### 5.2.2 Upflow tests

Sedimentation and fluidisation are very similar processes physically and in their mathematical treatment. In sedimentation liquid is squeezed upwards by descending particles, in fluidisation the particles are suspended and the liquid is forced upwards by some external force. In both cases the same forces (buoyancy and drag) are acting and so the permeability of a fluidised layer may be considered to be the same as that of a settling layer having the same concentration.

The upflow rig detailed in Section 3.2 was used, the concentration of a fluidised suspension was deduced from knowing the initial volume of solids present and
measuring the height of suspension. Conductivity readings were used to ensure that a uniform concentration existed. The flowrate through the suspension was found by measuring the volume of fluid entering the overflow. Given that the concentration and flowrate were known the fluid velocity was calculated from

\[ u_f = \frac{Q}{A(1-c)} \]  

(5.7)

Inertial terms and the presence of a solids pressure gradient were neglected, at steady state the solids velocity is zero so Equation (2.8) can be rearranged to give permeability in terms of fluid velocity and concentration.

\[ k = \frac{\mu u_f (1-c)}{c^2 g(\rho_f - \rho)} \]  

(5.8)

Permeability values were therefore found for a range of concentrations by altering the rate at which fluid was pumped into the system.

5.2.3 Downflow tests

In a downflow test a fully settled sediment is further compressed using some form of pressure filtration or centrifugation, the pressure between two points in the sediment is either measured or in the case of centrifuges can be deduced. The concentration between the two measurement points is taken to be constant and Darcian flow is assumed so

\[ k = \frac{\mu u_{\text{sup}} L}{\Delta P} \]  

(5.9)

where \( u_{\text{sup}} \) is the superficial velocity

\[ u_{\text{sup}} = \frac{Q}{A} \]  

(5.10)

This method provides permeability values for concentrations marginally greater than that achieved through gravity settling and so as a stand alone test can only provide an estimate of the permeability at the higher concentration range end of compressive settling. However if free settling permeability data is available then a line between the last measurable permeability in the free settling range and values obtained via downflow tests can be fitted to give an approximate function for permeability in the
compressive settling regime.

In this work the downflow cell described in Section 3.3 was used. This had two modes of operation it could either be used as a vacuum filter or when this technique failed it could be used as pressure filter. In both cases the flow through the cell was measured using a type 15 rotameter and the pressure drop was measured using two RS pressure transducers.

5.2.4 Concentration profile technique

If the concentration profile of a suspension in compression can be measured (using electrical impedance measurements or any other reliable method) and if a relationship between solids pressure and concentration exists then it is possible to deduce the permeability at different heights (and therefore concentrations) in a sediment using a rearranged form of Equation (2.11).

\[ k = \frac{u \mu}{(\rho_s - \rho)gc - \left( \frac{dp_s}{dc} \right)_s \frac{dc}{dx}} \]  

(5.11)

To be able to utilise Equation (5.11) requires knowledge of the solids velocity at different heights (and therefore concentrations) in a sediment. Two methods for obtaining the solids velocity were used, the first assumed Kynch theory to be valid so a value for solids velocity was obtained from a rearranged version of Kynch’s material balance (Equation (2.3))

\[ u = \frac{1}{c_0} \int_0^t \frac{dc}{dx} \, dx \]  

(5.12)

The second method was suggested by Font [1993], by considering material balances over the compression region and around the sediment surface he deduced that

\[ -u = \frac{1}{c(z,t)} \left[ c_{z2} (-u_{z2}) - \frac{dL}{dt} \left[ c(z,t) - c_{z2} \right] + \int_0^z \left( \frac{\partial c(z,t)}{\partial t} \right) dz \right] \]  

(5.13)

where is the z distance from the sediment surface i.e. z = L - x.
The concentration profile technique calculation method was as follows:

1) Permeabilities were calculated at three times during an experiment. Suitable time intervals were chosen and the concentration height profile at each was plotted.

2) Values of \( \frac{dc}{dx} \) and \( x \) were obtained at concentration intervals of \( dc = 0.01 \) from the profiles generated in step 1.

3) Values of \( \frac{dp}{dc} \) were obtained at the same concentrations using a differentiated form of the relevant solids pressure relationship.

4) \( \frac{dp}{dx} \), the product of \( \frac{dp}{dc} \frac{dc}{dx} \) was calculated at each concentrations.

5) Values of \( \frac{dc}{dt} \) were obtained for each concentration by measuring the gradient of the profile at \((c, t)\) on the concentration-time plot.

6) The integral \( \int_0^t \frac{dc}{dt} \, dx \) (Kynch) or \( \int_0^t \frac{dc}{dt} \, dz \) (Font) was then evaluated for each concentration using the trapezium rule.

7) If the solids velocity was being calculated via Font's method then values for \( \frac{dL_1}{dt} \) (the velocity of the sediment surface), \( c_{s2} \) (the concentration of the suspension immediately above the sediment), \( u_{s2} \) (the velocity of the interface at its point of intersection with the \( c_{s2} \) characteristic) were obtained from the characteristic plot.

8) the solids velocity at each concentration was calculated using Equation (5.12) or (5.13).

9) Permeability values were obtained via Equation (5.11)

### 5.2.5 Experimental derived permeability values for concentrations in the compression regime

Permeability values for concentrations in the compression regime were deduced for each material using the previously outlined methods. As a result of the analysis presented in Section 5.2.1 initial rate data was considered at low to medium compression concentrations. The permeability data for each material is presented on
two graphs, on each graph initial rate, downflow and concentration profile data is presented. The difference between the graphs lies in the Equation (5.12 or 5.13) used to generate the solids velocity values used in the concentration profile technique.

The permeability data for aragonite, calcite and talc are shown in Figures 5-7, 5-8 & 5-9 respectively. It can be seen that talc and aragonite had a permeability range of $1 \times 10^{-11} < k < 1 \times 10^{-12}$ m$^2$ in the compression regime whereas calcite had a range of $1 \times 10^{-9} < k < 1 \times 10^{-11}$ m$^2$. All the materials showed a good correlation between the downflow and initial rate data i.e. had the initial rate data been extrapolated it would have intersected or passed close to the downflow data. The data obtained via the downflow tests conducted on aragonite and talc tended to be over a relatively small concentration range. In the case of aragonite a terminal concentration appeared to have been reached, the solids structure resisted any further deformation despite large increases in the applied pressure. The concentration achieved however was not as great as that achieved through stirring. In stirred systems the direction of the disturbance caused to the solids structure is in a radial direction which indicates that the solids structure of aragonite has a greater strength in the longitudinal plane than the radial. When greater liquid flows were forced through the talc sediments further compression occurred, however, a stable height was never established and the concentration between the pressure transducers varied greatly so Darcian flow could not be assumed. Calcite showed a greater concentration range and a trend which corresponded well with the initial rate data.

The permeability data generated via the concentration profile method using Font's solids velocity (Equation (5.13)) showed good agreement with the initial rate data. With the exception of the $c_o = 0.025$ calcite run there was good agreement between the data of tests of different initial concentrations suggesting that as with solids pressure it was reasonable to assume that permeability was a sole function of concentration. The data generated at each of the three times in each test in general gave similar results although there was a tendency for slightly higher permeabilitys to be predicted at the later times.
a) using Font's solids velocity in the concentration profile method.

b) using Kynch's solids velocity in the concentration profile method.

Figure 5-7: Experimental permeability values for aragonite.
Initial concentration of test

\[ c = 0.025 \quad c = 0.05 \quad c = 0.075 \]

a) using Font's solids velocity in the concentration profile method.

b) using Kynch's solids velocity in the concentration profile method.

Figure 5-8: Experimental permeability values for calcite.
Figure 5-9: Experimental permeability values for talc.
Permeability data obtained from the concentration profile method using Kynch's solids velocity gave a less satisfactory agreement with the downflow/initial rate data. A good match was obtained at concentrations near the initial value but as the concentration increased the permeability rapidly decreased and tended away from the downflow data. A much larger spread of data across the initial concentration range was also exhibited. This was most pronounced in the talc runs (Figure 5.9 b)

The concentration profile method coupled with Font's solids velocity expression would appear to offer a reliable means of calculating the permeability of all concentrations in the compression region. The average of two or three tests gives values similar to the alternative initial rate/downflow method with significantly less experimentation being required to obtain the former data. Kynch theory failed to predict reasonable permeability values for concentrations in the compression regime and this would, as other researchers have reported, indicate that it's application is limited to the free settling regime where all the solids characteristics emanate from the origin.
Figure 5-10: Averaged permeability values for aragonite.

Figure 5-11: Averaged permeability values for calcite.
5.2.5.1 Development of an expression to fit the experimental permeability data

In Figures 5-10, 5-11 & 5-12 the average permeability based on the downflow, initial rate and concentration profile technique (using Font’s solids velocity) data for each material has been compared with three commonly used permeability expressions (Table 5-6).

Table 5-6: Commonly used permeability expressions.

<table>
<thead>
<tr>
<th>Author</th>
<th>Permeability expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kozeny-Carman</td>
<td>$k = \frac{(1-c)^3}{Kc^2S_v^2}$</td>
</tr>
<tr>
<td>Carman [1937]</td>
<td></td>
</tr>
<tr>
<td>Happel &amp; Brenner [1965]</td>
<td>$k = \frac{\left(2 - 3c^3 + 3c - 2c^2\right)x_{id}^2}{12c\left(3 + 2c^3\right)}$</td>
</tr>
<tr>
<td>Brinkman [1947]</td>
<td>$k = \frac{x_{id}^2(2 - 3c)^2}{18c\left(3c + 4 + 3\left(8c - 3c^2\right)^{\frac{1}{2}}\right)}$</td>
</tr>
</tbody>
</table>
The Kozeny expression was fitted using a Kozeny constant (K) of 5 (Carman [1937]). The specific surface was calculated using the size distributions given in Section 3.4 and

\[
S_v = 6 \int \frac{n_f(x_p)}{x_p} \, dx
\]  

(5.14)

where \( n_f(x_p) \) was the volume fraction of solids in a size range having an average particle size of \( \bar{x}_p \). The resultant specific surfaces were:

- aragonite: 805222 m\(^{-1}\)
- calcite: 1870643 m\(^{-1}\)
- talc: 313689 m\(^{-1}\)

The Sauter mean diameter was used to plot the Happel and Brinkman expressions where

\[
x_{sd} = \frac{6}{S_v}
\]  

(5.15)

Equations (5.14) & (5.15) are derived by assuming that the particles are spherical in shape and as such a degree of error was introduced due to the non-spherical nature of the particles used in this study.

For all the materials the permeability predicted by the three expressions was higher than the experimental values. The best approximation was attained at lower compressive concentrations. The over prediction was not due to particles floculating and forming larger particles as better fits were obtained if the Sauter mean diameter was reduced or the specific surface increased.

The Happel and Brinkman equations contain two parameters, concentration and particle size. As such they couldn’t justifiably be manipulated to fit the experimental data as particle sizes smaller than the primary particle would have been required to fit the data at high concentrations. The Kozeny Carman equation is a transformation of the Hagen-Poiseuille equation and contains several factors which are often for simplicity considered to be constant. Historically (Carman [1937]), and initially in this study the value of K was taken to be constant and equal to five. However, subsequent to Carman’s work several authors (Coulson [1949], Davies [1975], Shirato [1970])...
et al. [1983], Wasan [1976], Wyllie et al. [1955]) have reported that K is not a constant but a function of concentration and shape and therefore could reasonably be expected to be variable during a sedimentation. Variations in specific surface during sedimentations were reported by Edmundson & Toothill [1963] who showed that as the concentration of a suspension increases so does the specific surface. In flocculated systems this was attributed to the fact that at high concentrations fluid is forced through the floc rather than around it hence the specific surface available for fluid flow increases. The breakdown of flocs at high solids pressures also contributed to the effect. In view of the above evidence the Kozeny Carman equation was fitted to the experimental data by regarding the Kozeny constant as a variable factor which accounted for particle shape, and changes in the specific surface with respect to the local concentration. As changes in specific surface were accounted for by the Kozeny factor the specific surface term was retained as a constant and the values calculated via Equation (5.14) were used.

The variations in the resultant Kozeny factors with respect to concentration are shown in Figure 5-13. It can be seen that in general there was an increase in the value of K with concentration, this was consistent with the findings of Edmundson and Toothill as it is equivalent to an increasing specific surface. Expressions which described the variation of K with concentration were deduced by fitting quadratics to the data shown in Figure 5-13, these are given below in Table 5-7. As the Kozeny factor accounted for particle shape any error introduced by considering particles to be spherical in the calculation of the specific surface was negated.
Figure 5-13: Kozeny factors required to fit experimental data.

Table 5-7: Functions describing the variation in Kozeny factors.

<table>
<thead>
<tr>
<th>Material</th>
<th>Kozeny factor function</th>
</tr>
</thead>
<tbody>
<tr>
<td>aragonite</td>
<td>K = -57.06 + 1276.76c - 8629.04c^2 + 23508.96c^3 - 19301.56c^4</td>
</tr>
</tbody>
</table>
| calcite  | c ≤ 0.17 K = 12.5 + 65.98c  
|          | c > 0.17 K = 122355 - 2780374c + 2.4E+7c^2 - 9E+7c^3 + 1.3E+8c^4 |
| talc     | K = 23.7 - 535.64c + 7595.75c^2 - 42459.7c^3 + 85458.16c^4 |

5.2.5.2 Alternative method for the prediction of the Kozeny factor (K)

In the previous section Kozeny factor functions were derived by selecting K values which fitted the Kozeny Carman permeability expression to experimentally derived values. A simpler method for predicting the variation of K in terms of concentration which bypasses the necessity of deducing experimental permeability values is the Davies & Dollimore [1980] method, an outline of which is given below. A common form of the Kozeny-Carman equation for a bed of sedimenting particles is

\[
u = \frac{(1-c)^3 \Delta P}{c^2 S_i^2 K \mu} \frac{\Delta P}{L}
\]  

(5.16)
Darcian flow is assumed\(^1\) so the pressure drop across a bed depth (L) can be written as

\[
\frac{\Delta P}{L} = c(\rho_s - \rho)g
\]

(5.17)

The particles are assumed to be spherical so the free fall velocity \((u_t)\) can be given by Stokes velocity

\[
u_t = \frac{x_p^2(\rho_s - \rho)g}{18\mu}
\]

(5.18)

and the specific surface

\[S_v = \frac{6}{x_p}
\]

(5.19)

Substituting Equations (5.17), (5.18) & (5.19) into Equation (5.16) yields

\[
u = \frac{1}{2K} \frac{(1-c)^3}{c} u_t
\]

(5.20)

Richardson & Meikle [1961] showed that many suspensions in sedimentation or fluidisation obey the empirical equation

\[u = u(1-c)^m
\]

(5.21)

Comparing Equations (5.20) & (5.21) gives K in terms of the Richardson & Meikle exponent and concentration

\[
K = \frac{1}{2(1-c)^m} \frac{c}{3}
\]

(5.22)

Equation (5.22) is the Davies & Dollimore equation, prediction of the Kozeny factor at any concentration is possible provided a value for the Richardson exponent \((m)\) is known. Values of \(m\) are obtained from the slope of a plot of \(ln(u)\) vs. \(ln(1-c)\). The following values were obtained using initial settling rates as the values of \(u\)

<table>
<thead>
<tr>
<th>material</th>
<th>(m)</th>
<th>(r^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>aragonite</td>
<td>11.244</td>
<td>0.9753</td>
</tr>
<tr>
<td>calcite</td>
<td>15.603</td>
<td>0.9868</td>
</tr>
<tr>
<td>talc</td>
<td>16.816</td>
<td>0.9756</td>
</tr>
</tbody>
</table>

\(r^2\) denotes the accuracy of the regression fit, a value of unity indicates a perfect fit.

Using Equation (5.22) and the exponent values given above it was possible to compare the K values predicted by the Davies & Dollimore method with the experimentally derived values. The result is shown in Figure 5.14, a relatively good

---

\(^1\)only truely valid in free settling but used as an approximation for compression concentrations in the following analysis
match was obtained for calcite however Equation (5.22) tended to over predict K values for aragonite and talc.

![Graph showing the comparison between experimental and Davies & Dollimore Kozeny factors.](image)

**Figure 5-14**: Comparison between experimental and Davies & Dollimore Kozeny factors.

An improvement to this result was made by revising the technique for deriving the exponent \( m \). In accordance with Richardson’s method \( \ln(u) \) had been plotted against \( \ln(1-c) \) and hence it was assumed that the value of the \( y \)-intercept was the free settling velocity \( v_s \). In the revised method \( \ln(u/v_s) \) was plotted against \( \ln(1-c) \), values of \( v_s \) were derived using Equation (5.18) and the regression was forced to zero. The resultant exponent values are given below and the revised K values are shown in Figure 5-15.

<table>
<thead>
<tr>
<th>material</th>
<th>( m )</th>
<th>( r^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>aragonite</td>
<td>9.166</td>
<td>0.9397</td>
</tr>
<tr>
<td>calcite</td>
<td>15.256</td>
<td>0.9862</td>
</tr>
<tr>
<td>talc</td>
<td>11.215</td>
<td>0.8546</td>
</tr>
</tbody>
</table>

It can be seen that although there was a small decrease in the accuracy of the regression fits the K value prediction (Figure 5-15) for all three materials was much more in line with the experimental data. The exponent value for calcite didn’t change greatly however the aragonite and talc values both decreased.
The Davies and Dollimore equation for predicting $K$ is a parabolic and as such suggests that a minimum $K$ value will exist and that it will occur at the maximum solids flux. For most materials the maximum solids flux occurs in the free settling regime so it is not surprising that no minimum was shown in the experimental data presented above. The ballotini which was used to commission the electrical impedance technique showed a large free settling concentration range and so was used to investigate the existence of a minimum. The settling data for the ballotini runs was treated as detailed above, an exponent value of $m = 4.80$ at a regression accuracy of $r^2 = 0.9999$ was obtained. Experimental permeability values were obtained via the initial rate method and the Kozeny equation was fitted to the data. The $K$ values required to achieve the fit and those predicted by the Davies and Dollimore equation are shown in Figure 5-16. It can be seen that a minimum does occur in the experimental data and that it coincides closely with the minimum predicted by the Davies and Dollimore equation.
The magnitude and shape of Kozeny factor functions predicted via the Davies and Dollimore method compared favourably for all the materials with the experimentally determined functions. The Davies and Dollimore method is therefore worthy of some merit, experimentally all that is required is a series of initial rate tests and analytically it is only necessary to derive a value for the exponent in Equation (5.21). The revised regression method seems to offer better results to the traditional reported method however more tests on different materials would be required to prove this conclusively.

5.2.6 Permeability measurement in the free settling regime - upflow tests

The use of initial settling rates to determine the permeability of concentrations in the free settling regime is a well established method, less widely used are upflow tests. The equipment used to conduct the upflow tests in this study was detailed in Section 3.2 and the theory was covered in Section 5.2.2. Tests were conducted on talc and aragonite but not on calcite as an insufficient amount of the material was available.
Aragonite

The upflow column was charged with 200 g of aragonite, an initial liquid flowrate of $Q = 0.9 \text{ ml s}^{-1}$ was used which resulted in a bed height of 555 mm and a uniform bed concentration of $c = 0.0449$. The uniformity of the bed was confirmed by electrical impedance measurements. The flowrate through the column was then systematically reduced, the resultant concentration profiles and associated flowrates are shown in Figure 5-17. It can be seen that uniform concentration profiles were obtained at flowrates of $Q \geq 0.4 \text{ ml s}^{-1}$, when the flowrate was reduced from 0.4 to 0.32 ml s$^{-1}$ the bed collapsed from a height of 319 mm to 141 mm in 360 seconds and then rose again and adopted a fairly constant height of 151 mm after 1000 seconds. The bed collapse was accompanied by the formation of sizeable channels approximately 40 mm high and 1 mm wide. These formed 90 seconds after the flowrate change and persisted for a further 210 seconds, disappearing just before the minimum height was reached. Further reductions in the flowrate showed the same fall and rise in the solid-liquid interface but were not accompanied by channelling. In each case a constant sediment height and steady concentration profile were obtained. The base sediment concentration attained was significantly higher than that achieved through normal compaction ($c = 0.35$ cf. $c = 0.306$). This was a surprising result as it had been anticipated that the upward drag of the liquid would result in lower concentrations. It was therefore assumed that at low flowrates the drag force was negligible but the motion of the liquid flowing through the sediment had a 'jiggling' effect similar to stirring, which weakened the solids structure allowing further compaction to occur.

During the upflow tests the fluidisation ensured that there were no solids concentration or stress gradients at concentrations of $c \leq 0.076$ this allowed Equation (5.8) to be employed to determine the permeability values shown in Figure 5-18. The permeability values deduced from the downflow tests agree well with initial rate data at $c < 0.05$ but show slightly greater values at higher concentrations.
Figure 5-17: Concentration profiles obtained in upflow tests on aragonite.

Figure 5-18: Comparison between permeability values predicted by upflow and initial rate tests.
A bed of uniform concentration can only be maintained in a fluidised state when no stress gradients exist. In Section 4.1.4 it was deduced that the gel concentration of aragonite was $c = 0.176$ and therefore static compression (the transmission of stress through a solids network) could only occur at higher concentrations. The above data however suggests the existence of a stress gradient at $c > 0.076$ and hence strengthens the argument for the existence of a transient solids stress (see Section 3.2.8). If a transient solids stress existed the observed bed collapse could therefore be explained by assuming that at $c > 0.076$ the proximity of particles to each other is such that a force which is transmitted through the liquid acts and hence a stress gradient is established which initiates the collapse.

An alternative argument was offered by Holdich & Butt [1997], considering the solids velocity curve shown in Figure 5-19 it was observed that the velocity-concentration curve was very shallow at concentrations greater than $c = 0.08$. Thus it was concluded that it would be extremely difficult, or impossible, to control the fluidisation between $c = 0.08$ and $c = 0.29$ as a small change in velocity would considerably alter the bed concentration. This explanation whilst plausible is flawed in that it suggests that stable bed heights and concentration profiles would not be obtained at flowrates less than $Q = 0.32$ ml s$^{-1}$ (the flowrate at which the bed collapsed). Such conditions were obtained and are shown in Figure 5-17, thus the transient stress explanation seems more likely.

The results of upflow tests such as those conducted on aragonite also have implications in the design of continuous thickeners. The conventional method of estimating a continuous thickener's area is from a graphical construction on a batch flux curve. The limiting flux value comes from the intercept, on the flux axis, of a line drawn as a tangent to the batch flux curve and going through the underflow solid concentration. The product of this limiting flux and the thickener area must be equal to the product of the volumetric feed rate to the thickener and the feed volume fraction concentration, assuming all the solids leave the thickener by the underflow. The batch flux curve for aragonite is shown in Figure 5-19. From the point of view of equipment design the important part of the batch flux curve is that illustrated to the
right of the minimum fluidised velocity, as the tangent to the batch flux curve required to fix the limiting thickener flux is most likely to be drawn in this region. Industrial continuous thickeners have a reputation for being difficult to design and to control in operation if they are operating close to their limiting flux capacity. Some of the reasons behind this may be seen by reference to Figure 5-19. The terms minimum and maximum 'fluidised' velocity and concentration, respectively, are used. These values are not the same as those required to achieve fluidisation: for this material the minimum 'fluidising' velocity would be the value required to fluidise solids at $c = 0.29$. Thus the maximum 'fluidised' concentration is the value which provides stable conditions of fluidisation or sedimentation ($c \leq 0.076$). The up-flow velocity at this concentration is the minimum 'fluidised' velocity.

![Figure 5-19: Batch flux curve and settling velocities for aragonite.](image)

The maximum fluidised concentration is the point at which the existence of a stress gradient becomes significant and often coincides with the point of inflection on the batch flux curve. The maximum fluidised concentration also represents the maximum point at which a continuous thickener can be designed in the conventional manner employing the flux curve: in Figure 5-19 a tangent to the batch flux curve going through the maximum fluidised concentration provides an underflow concentration of $c = 0.12$. It may be possible to operate a thickener to achieve a greater concentration
than this, but the operating conditions should not be obtained from the conventional flux curve as the batch settling flux at these concentrations is time dependent.

**Talc**

The gel concentration of talc had been determined to be $c = 0.05$ and so it was expected that unless a transient solids stress was exhibited that concentrations up to $c = 0.05$ might be attained. In contrast to the aragonite tests the flowrate was incrementally increased, sample profiles and flowrates are shown in Figure 5-20. A constant concentration profile was not achieved at any flowrate, at flows of $Q > 0.38$ ml s$^{-1}$ segregation occurred and fines were carried out into the overflow. At lower flowrates a very obvious structure change from a very compact sediment to a dilute suspension of discrete free settling particles could be observed.

![Figure 5-20: Concentration profiles obtained in upflow tests on talc.](image)

In contrast to the aragonite tests the concentration achieved at the base of the fluidisation column didn’t exceed that achieved through normal compaction. The increase in aragonite concentration was attributed to the ‘jiggling’ effect of the upflowing liquid and compared with the action of stirring. Talc when stirred didn’t show any noticeable increase in base sediment concentration and so it was reasoned
that the above assumption for aragonite was still correct and that the talc solids structure was strong enough to resist any such action.

Unlike aragonite channelling was observed at all changes of flowrate. The channels that formed were considerable wider (approximately 4 mm wide) than the ones which were observed during talc sedimentations. Channels were also observed at steady state fluidisation, the channel size was noticeably smaller than those which formed initially after a flowrate change.

5.2.7 Channel formation ‘paragenesis’

The observations made during the upflow tests on both materials indicate that the formation and size of channels is directly related to fluid velocity and allowed the following conclusions to be made:

1] The mechanism by which channels form during an upflow test is the same as that suggested in Section 4.2.3 to explain channel formation during sedimentations i.e. if a localised anomaly in the solids structure exists and the fluid velocity into the area is great enough then channel formation will occur.

2] Channelling will therefore only occur when compressive concentrations are present

3] When the flowrate in an upflow test is incrementally increased channelling will occur provided a compressive solids network exists. This is because a greater fluid velocity than is required to maintain steady state is forced through the solids structure, if any weaknesses exist then liquid will preferentially flow to these areas and channels will form.

4] When fluid velocity is incrementally decreased channels will form only at the flowrate reduction which results in the transition from uniform to graded concentration profiles i.e. the flowrate at which the bed collapses. The channels will only form after a period of thickening as a solids network has to be established and a concentration at which the reduced flowrate can cause channel formation has to be
achieved. Subsequent reductions in flowrate are unlikely to result in channel formation as the correct conditions will not exist as the liquid velocity will be too low and a stronger solids structure will have formed.

5] Channelling is more predominant in step up experiments as the liquid flow available to exploit weaknesses in the solids structure is greater.

The above observations coupled with those made in Section 4.2 allowed a 'paragenesis' plot similar to Fitch's (Section 2.1) to be developed which instead of predicting the type of settling which would occur at certain conditions predicted under what conditions channel formation would occur. The plot is shown in Figure 5-21.

![Figure 5-21: Conditions under which channelling will occur.](image)

It can be seen that below a certain fluid velocity channel formation will not occur as the liquid flow will be insufficient to develop the pressure required to deform the solids structure and then maintain channel formation. This is the minimum channelling velocity ($u_{mc}$). Above this velocity at low concentrations the distance between settling particles is sufficiently large enough to accommodate the upflow of liquid without channel formation. At greater concentrations soft zone channel formation will be supported, however, as stated in Section 3.2 this is unlikely to occur
unless the sediment below is satisfying the conditions required for hard zone channelling. It should be noted that the boundary between hard and soft zone channelling will not necessarily lie on the boundary between free and compressive settling (eg), as at lower compressive concentration the solids structure may not be strong enough to sustain a channel structure. Hard zone channelling occurs over a range of fluid velocities and concentrations, as the concentration of a sediment increases the fluid velocity required for channel formation increases. At velocities greater than the minimum fluidising velocity defined in Section 5.2.6 any suspension will have a uniform concentration the magnitude of which is determined by its location on the thick line which represents fluidised conditions. The above is by no means a complete picture of the conditions under which channelling will occur it does however provide a platform for further investigations.
6. Consolidation model

Knowledge of the sediment characteristics discussed in Chapter 5 enabled the mechanism of consolidation to be mathematically analysed by considering the variation in excess hydraulic pressure. The result was a non-linear parabolic partial differential equation which was numerically solved using a finite difference approximation. The following section outlines the development of the model and its numerical solution.

6.1 Material co-ordinates

Working in material co-ordinates rather than cartesian co-ordinates proved advantageous as it removed the need to consider the suspension interface as a moving boundary condition. In material co-ordinates the mass (or volume) of solids per unit area below a Lagrangian datum plane is considered. In simpler terms this means that the total mass of solids in a sedimentation vessel can be split into a number of elements, each containing the same mass of material. At the start of a sedimentation the elements are of equal volume, i.e. area and height, as consolidation occurs the height of the more concentrated elements becomes less in order to maintain the same overall mass of solids in each element. Figure 6-1 defines the material co-ordinate system that was employed in the model.

![Figure 6-1: Definition of the material co-ordinate system.](image)
The mass of solids per unit area below the datum plane is defined as

$$w = \rho_s \int_0^c dx \quad \text{or} \quad \frac{dw}{dx} = \rho_s c \quad (6.1)$$

### 6.2 Mathematical development of a consolidation model

In Section 2.1 the process of sedimentation was shown to be described by a force balance and a continuity equation. As stated previously use of material co-ordinates was desirable so the first stage of modelling was to convert the force balance and continuity equation to the correct co-ordinate system. The continuity equation was developed in Section 1.1.1.1 and is written as

$$\frac{dc}{dt} = -\left( \frac{\partial \phi}{\partial x} \right) + \frac{dx}{dt} \left( \frac{\partial c}{\partial x} \right) \quad (6.2)$$

When working in material co-ordinates the mass below a Lagrangian plane is considered, the plane velocity \(\frac{dx}{dt}\) is the same as the local solids velocity so \(\frac{dx}{dt} = u\), substituting this into Equation (6.2) yields

$$\frac{dc}{dt} = -\left( \frac{\partial \phi}{\partial x} \right) + u \left( \frac{\partial c}{\partial x} \right) \quad (6.3)$$

The solids flux (\(\phi\)) is the product of the solids velocity and concentration (\(\phi = c \cdot u\)) so the first term on the right side of Equation (6.3) can be expanded using the product rule

$$\left( \frac{\partial c u}{\partial x} \right) = c \left( \frac{\partial u}{\partial x} \right) + u \left( \frac{\partial c}{\partial x} \right) \quad (6.4)$$

Substituting (6.4) into (6.3) and cancelling

$$\left( \frac{\partial c}{\partial t} \right)_w = -c \left( \frac{\partial u}{\partial x} \right)_t \quad (6.5)$$

taking the differential form of (6.1) and substituting for \(dx\) in (6.5) yields

$$\left( \frac{\partial c}{\partial t} \right)_w = -c^2 \rho_s \left( \frac{\partial u}{\partial w} \right)_t \quad (6.6)$$

This is the continuity equation in material co-ordinates, it is comparable with that derived by Shirato [1970] and Fujisaki [1993].
Equation (6.7) is the force balance derived in Section 2.1.2.2 for a suspension settling in the compressive regime in the absence of inertial effects.

\[
\left( \frac{\partial p_i}{\partial x} \right)_{t} = c g (\rho_s - \rho) + u \frac{\mu}{k} \tag{6.7}
\]

The dynamic term \(u \frac{\mu}{k}\) of the liquid pressure gradient is the liquid pressure gradient in excess of the static gradient \(\left( \frac{\partial p_i}{\partial x} \right)_{t}\), substituting this into Equation (6.7) yields

\[
\left( \frac{\partial p_i}{\partial x} \right)_{t} + \left( \frac{\partial p_s}{\partial x} \right)_{t} = c g (\rho_s - \rho) \tag{6.8}
\]

or

\[
p_i + p_s = g (\rho_s - \rho) \int_{x}^{c} dx \tag{6.9}
\]

Combining (6.1) & (6.9) relates the pressure terms to the mass of solids per unit area

\[
p_i + p_s = \frac{\rho_s}{\rho} \tag{6.10}
\]

Differentiating with respect to \(w\)

\[
\frac{\partial p_i}{\partial w} + \frac{\partial p_s}{\partial w} = \frac{g (\rho_s - \rho)}{\rho_s} \tag{6.11}
\]

Using the chain rule \(\frac{\partial p_s}{\partial w} = \frac{\partial p_s}{\partial x} \frac{\partial x}{\partial w}\) Equation (6.11) becomes

\[
\frac{\partial p_i}{\partial w} + \frac{\partial p_s}{\partial x} \frac{\partial x}{\partial w} = \frac{g (\rho_s - \rho)}{\rho_s} \tag{6.12}
\]

Expressions for \(\frac{\partial p_s}{\partial x}\) and \(\frac{\partial x}{\partial w}\) are obtained from Equations (6.7) & (6.1) respectively, substitution into Equation (6.12) yields

\[
u = - \frac{\partial p_i}{\partial w} \frac{c \rho_s k}{\mu} \tag{6.13}
\]

If \(K_i = \frac{c \rho_s k}{\mu}\) then the differential form of (6.13) via the product rule in terms of \(w\) is

\[- \frac{\partial u}{\partial w} = K_i \frac{\partial^2 p_i}{\partial w^2} + \frac{\partial p_i}{\partial w} \frac{\partial K_i}{\partial w} \tag{6.14}\]
Equations (6.6) and (6.14) represent the continuity equation and the force balance in terms of material co-ordinates, to obtain an expression in terms of the excess hydrostatic pressure the two equations are combined

\[
\frac{1}{c^2 \rho_s} \frac{\partial c}{\partial t} = K_i \frac{\partial^2 p_t}{\partial w^2} + \frac{\partial p_t}{\partial w} \frac{\partial K_i}{\partial w} \quad (6.15)
\]

Equation (6.15) is rearranged into a format more amenable to solution. Multiplying through by \( \frac{\partial p_s}{\partial c} \)

\[
\frac{1}{c^2 \rho_s} \frac{\partial p_s}{\partial t} = \frac{\partial p_s}{\partial c} \left[ K_i \frac{\partial^2 p_t}{\partial w^2} + \frac{\partial p_t}{\partial w} \frac{\partial K_i}{\partial w} \right] \quad (6.16)
\]

The bracketed term is simplified using the product rule

\[
\frac{1}{c^2 \rho_s} \frac{\partial p_t}{\partial t} = \frac{\partial p_t}{\partial c} \frac{\partial}{\partial w} \left[ K_i \frac{\partial p_t}{\partial w} \right] \quad (6.17)
\]

Equation (6.17) can be rewritten in terms of \( \frac{\partial p_t}{\partial w} \). If Equation (6.10) is differentiated in terms of time then

\[
\frac{\partial p_t}{\partial t} + \frac{\partial p_s}{\partial t} = g \left( p_s - \rho \right) \frac{\partial w}{\partial t} \quad (6.18)
\]

\( w \) is defined as the mass per unit area, the height required to maintain \( w \) varies with time but the overall mass doesn’t change so \( w \) is not a function of time i.e. \( \frac{\partial w}{\partial t} = 0 \) so Equation (6.18) simplifies to

\[
\frac{\partial p_t}{\partial t} = -\frac{\partial p_s}{\partial t} \quad (6.19)
\]

and Equation (6.17) therefore becomes

\[
-\frac{\partial p_t}{\partial t} = -c^2 \rho_s \frac{\partial p_s}{\partial c} \frac{\partial}{\partial w} \left[ K_i \frac{\partial p_t}{\partial w} \right] \quad (6.20)
\]

Equation (6.20) is the required result, a parabolic equation describing the consolidation process in terms of the variation in excess hydrostatic pressure.

**6.3 Creating a non-dimensional form of the model**

The above equation can be solved for a given set of conditions, however it is more useful to create a non dimensional formulation of the model as this allows a single
solution of the corresponding non-dimensional equation to be solved for a wide variety of conditions. The following non-dimensional variables are therefore defined

\[ p_i^* = \frac{p_i}{p_0} \]  
\[ p_s^* = \frac{p_s}{p_0} \]  
\[ w^* = \frac{w}{w_0} \]  
\[ t^* = \frac{t}{t_0} \]

where

\[ w_0 = c_0 h_0 \rho_x \]  
\[ p_0 = w_0 \left( \frac{p_s - p}{\rho_x} \right) g \]  
\[ t_0 = \frac{h_0}{u_0} \]

Using the above substitutions in Equation (6.20) the governing equation of the consolidation model can be expressed in a dimensionless form

\[ \left[ \frac{dc}{dp_i^*} \nabla \frac{p_i^*}{c^3} \right] \frac{\partial p_i^*}{\partial t^*} = - \frac{\partial}{\partial w^*} \left[ t_0 p_0 \rho_x K, \frac{\partial p_i^*}{w_0} \frac{\partial w^*}{\partial w^*} \right] \]  

(6.28)

### 6.4 Incorporation of solids pressure and permeability expressions

The final modelling stage requires the solids pressure and permeability expressions which were developed in Section 5.0 to be linked to Equation (6.28). This is achieved by defining two non-linear coefficients \( a(p_i^*) \) and \( b(p_i^*) \) in Equation (6.28)

\[ b(p_i^*) \frac{\partial p_i^*}{\partial t^*} = - \frac{\partial}{\partial w^*} \left[ a(p_i^*) \frac{\partial p_i^*}{w_0} \right] \]  

(6.29)

where

\[ a(p_i^*) = \frac{t_0 p_0 \rho_x K_1}{w_0} \]  

(6.30)
\[ b(p_i^*) = \left[ \frac{\partial c}{\partial p_s^*} \frac{1}{c^2} \right] \]  

Substituting for \( t_o, p_o \) and \( K_i \) in (6.30) yields an expression containing the permeability term

\[ a(p_i^*) = \frac{(p_s - p)gh_kcp_s}{w_uu_\mu} \]  

In Section 5.2.5.1 the Kozeny-Carman expression for permeability was found to best describe the experimental data

\[ K = \frac{(1-c)^3}{c^2S_v^2K} \]  

\( K \), the Kozeny factor, was found not to be constant and was shown to vary with concentration and from material to material, so \( K = K(c) \). Defining \( u_o \) as the initial solids velocity in the absence of any solids stress and substituting Equation (6.33) into the relevant force balance (Equation (5.7)) yields

\[ u_o = \frac{(p_s - p)(1-c_o)^3 g}{c_o\mu K_o S_v^2} \]  

Substituting (6.33) & (6.34) into (6.32)

\[ a(p_i^*) = \frac{K_o}{(1-c_o)^3} \left[ \frac{(1-c)^3}{Kc} \right] \]  

The bracketed term is concentration dependent and therefore variable whereas the rest of the expression is constant.

The function \( b(p_i^*) \) is manipulated in similar manner, in Section 5.1 the expression \( c = c'p_s^a \) was shown to adequately describe the relationship between solids pressure and concentration at low solids pressures, differentiating in terms of \( p_s \)

\[ \frac{dc}{dp_s} = ac'p_s^{a-1} \]  

Rewriting in terms of non-dimensional solids pressure

\[ \frac{dc}{dp_s^*} = ac'p_s^{a-1}p_o \]  

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Solids pressure is considered to be a function of local concentration only so \( \frac{\partial c}{\partial p^*_s} = \frac{dc}{dp^*_s} \), substituting (6.37) into (6.30) yields.

\[
b(p^*_s) = \frac{ac^* p_s^{\alpha-1} p_o}{c^2}
\]  

(6.38)

At higher solids pressures Tiller's expression gave the best fit with experimental data, following the same method

\[
\frac{dc}{dp^*_s} = -\frac{(c_m - c_s)}{p_b} \exp \left( \frac{p_s}{p_b} \right) p_o
\]  

(6.39)

so

\[
b(p^*_s) = -\frac{(c_m - c_s)}{c^2 p_b} \exp \left( \frac{p_s}{p_b} \right) p_o
\]  

(6.40)

6.5 Numerical solution using a three time-level finite difference method

Equation (6.28) is a parabolic equation with non-linear coefficients \((a(p^*_l) & b(p^*_l))\). This was numerically solved using an implicit finite difference three time-level method (Smith [1979]). The following central-difference approximation to Equation (6.29) was used

\[
b(p^*_l, j) = \frac{1}{2k} \left[ p^*_l, j+1 - p^*_l, j-1 \right] = \frac{1}{h^2} \left\{ a \left( p^*_l, j \right) \left[ p^*_l, j+1 - p^*_l, j-1 \right] - a \left( p^*_l, j+1 \right) \left[ p^*_l, j+1 - p^*_l, j \right] \right\}
\]  

(6.41)

From Equation (6.29) it can be deduced that \( p^*_l \) is a function of \( t^* \) and \( w^* \), therefore the finite difference grid is a \( w^*-t^* \) plane which is subdivided into sets of equal rectangles of sides \( \partial w^* = h \) and \( \partial t^* = k \).
Equation (6.40) could be solved explicitly providing \( \frac{2k}{h^2} < 1 \). However it was found that for many conditions that the solution was unstable, stability was achieved by making the following replacements

\[
\begin{align*}
    p_{i+1}^{*} & = \frac{1}{3} \left( p_{i}^{*} + p_{i+1}^{*} + p_{i-1}^{*} \right) \\
p_{i}^{*} & = \frac{1}{3} \left( p_{i+1}^{*} + p_{i-1}^{*} + p_{i}^{*} \right) \\
p_{i-1}^{*} & = \frac{1}{3} \left( p_{i+1}^{*} + p_{i-1}^{*} + p_{i}^{*} \right)
\end{align*}
\]

and

\[
\begin{align*}
    a \left( p_{i-\frac{1}{2}}^{*} \right) & = a \left( \frac{1}{2} \left( p_{i-1}^{*} + p_{i}^{*} \right) \right) \\
a \left( p_{i+\frac{1}{2}}^{*} \right) & = a \left( \frac{1}{2} \left( p_{i}^{*} + p_{i+1}^{*} \right) \right)
\end{align*}
\]

Substituting the above into Equation (6.41) and rearranging to obtain a solution for \( p_{i,j}^{*} \) yields
To obtain a solution for \( p_{i,j+1}^* \) (point X) it is necessary to know values for \( p_i^* \) at all of the labelled grid references in Figure 6·2, the process is therefore implicit and several iterations are required to achieve a satisfactory solution.

### 6.5.1 Initial and boundary conditions

The initial distribution of the excess hydrostatic pressure is obtained by converting Equation (6.9) into a non-dimensional form. The result is an expression which describes the distribution of \( p_i^* \) with respect to \( w^* \) at \( t^* = 0 \).

\[
p_i^* = 1 - w^* - p_{iso}^*
\]

where \( p_{iso}^* \) is a non-dimensionalised form of the initial solids stress.

The boundary conditions used were

\[
\begin{aligned}
\frac{\partial p_i^*}{\partial w} &= 0 \quad &\text{at} \quad w^* &= 0 \\
p_i^* &= 0 \quad &\text{at} \quad w^* &= 1
\end{aligned}
\]

(6.44)

As described above to obtain a solution for \( p_{i,j+1}^* \) requires values of \( p_i^* \) on the \( j^\text{th} \) and \( j-1 \) rows of the mesh to be known. Two rows of initial conditions were therefore required to initiate solution of Equation (6.42). This was achieved by assuming that the \( j^\text{th} \) row corresponds to \( t = 0 \) and that the \( j-1 \) row is some imaginary negative time which has the same \( p_i^* \) distribution as the \( j^\text{th} \) row.
6.5.2 Calculation procedure

A sample mesh is shown in Figure 6-3, the bold lines denote values of $p_i^*$ which are known as a consequence of the boundary and initial conditions. Unknown values are set initially to that of the initial conditions.

Boundary condition

\[
\frac{dp_i^*/dw}{d\omega} = 0
\]

known values of $p_i^*$

Figure 6-3: Sample mesh.

To obtain a solution, starting at point A, the following calculations are performed

1) The solids pressure is calculated by converting $p_i^*$ to its dimensional form ($p_i = p_i^* p_s$) and then applying this to Equation (6.10).

2) The concentration at point A is calculated using the appropriate solids pressure-concentration relationship e.g. $c = c' p_i^*$

3) A value for $a \left[ \frac{1}{2} \left( p_{i,i+1}^* + p_{i,i}^* \right) \right]$ is obtained by finding the average value of $p_i^*$ between points B and A, applying steps 1 & 2 gives values for $p_i$ and $c$ which are then applied to Equation (6.35).

4) Likewise a value for $a \left[ \frac{1}{2} \left( p_{i-1,i}^* + p_{i,i}^* \right) \right]$ is found by taking the average value of $p_i^*$ at points C and A and then performing the same calculations as step 3.

5) The function $b(A)$ is evaluated using Equation (6.38) or (6.40) depending on which solids pressure relationship is being used.

6) A value for $p_i^*$ at point A is calculated using Equation (6.42).
The procedure is repeated for points B,D,...etc. on the first row and then on the second row beginning with point E. The number of rows required depends on the length of the sedimentation and the value of $\Delta t^*$ that is used.

In the first instance the whole of the grid is set to the initial conditions of $p_i^*$ as defined by Equation (6.43), thus the first calculation of $p_i^*$ at A is an estimate as the values at B,C,D,E are simply initial guesses. Corrected values of B,C,D,E are dependent on the value of A calculated. The process is therefore iterative and must be repeated many times until the values of $p_i^*$ stabilise.

Stability was determined by measuring the change in the interfacial height, when the difference between successive iterations dropped below 0.00001 m stability was assumed. A minimum of five iterations were carried out before this criteria could be satisfied. The height of the interface at each time interval and each $w^*$ increment was calculated using Equation (6.44), the integral was approximated using the trapezoidal rule so

$$x_i = x_{i-1} + \frac{2w}{\rho_s (c_{i-1} + c_i)}$$

(6.45)

where $i=1,2,3...1/dw$ and $x_0 = 0$.

### 6.5.3 Computer model

The model was initially set up on a Quattro Pro spreadsheet, the multiple pages with identical cell addresses of the spreadsheet enabled concentration, solids pressure, excess hydrostatic pressures etc. to be displayed on individual pages and allowed easy identification of any errors. The spreadsheet was implemented on a Pentium coprocessor, for small grid sizes (10 weight divisions by 200 time intervals) iteration times were of the order of three to four hours. To be able to use increased grid sizes, variable Kozeny factors and multiple solids pressure expressions it was necessary to translate the method to a more versatile programming base, Qbasic. This gave increased versatility but iteration times tended to be of the same order or longer, visualisation of the iterative process was also lost as this greatly increased the run
time. These disadvantages were overcome by a further translation into Turbo Pascal, run times including visualisation were reduced to approximately 15 minutes and versatility was retained.

The Turbo Pascal program ‘MODEL.pas’ is given in Appendix B, to achieve stability double precision data types are used which require an 8087 processor or the 8087 emulation mode to be set. A number of inputs are required, these are

- solid properties: density, specific surface
- fluid properties: density, viscosity

Coefficients for the Kozeny factor function
Coefficients for the solids pressure function
Grid dimensions: number of weight fractions, dimensionless time interval, duration
Starting conditions: initial height and concentration

Default values for the materials used in this study are resident in the program and can be chosen. If this option is chosen the Kozeny factor functions and solids pressure relationships detailed in Section 5 are automatically implemented. If the user chooses to model another material then the default values are set to those of aragonite and coefficients will be requested for Shirato’s solids pressure law. Coefficients will also be requested for the Kozeny factor function which is assumed to be a straight line relationship \( K = ac + b \). If more complicated relationships are required then these must be entered directly into the program code.

The program output is in the form of a characteristic plot which can be compared with experimental plots. The characteristic plot is saved as a text file in a format which allows it to be easily imported into a spreadsheet. The interface height, concentration, solids pressure and excess hydrostatic pressure data is also saved in separate files.

### 6.5.4 Optimum grid set-up

Initially when using the spreadsheet version of the model the grid was split into 10 weight fractions however subsequent simulations in Turbo Pascal showed that different solutions were obtained at 25 and 50 weight fractions. Grid sizes greater and
including 50 weight fractions gave the same result and so fifty was used as the optimum number of weight fractions as a grid larger than fifty required more computational time. Additionally the profiles obtained at grid sizes of fifty were smoother than those generated by smaller grids.

The time interval was set so that $\frac{2k}{h^2} = 0.75$, for a grid of 50 weight fractions the dimensionless time interval was therefore $dt^* = 0.00015$.

### 6.6 Model results

Computer simulations were performed to allow comparison with the experimental data of aragonite, calcite and talc. Simulations at three different initial conditions were performed for each material. The initial conditions used corresponded to the following experimental runs:

#### Aragonite

$c_o = 0.175$, $h_o = 0.329$ m; $c_o = 0.20$, $h_o = 0.0329$ m; $c_o = 0.225$, $h_o = 0.323$ m

#### Calcite

$c_o = 0.05$, $h_o = 0.324$ m; $c_o = 0.075$, $h_o = 0.323$ m; $c_o = 0.125$, $h_o = 0.326$ m

#### Talc

$c_o = 0.052$, $h_o = 0.0323$; $c_o = 0.0728$, $c_o = 0.0320$ m; $c_o = 0.0113$, $h_o = 0.313$ m

The solids pressure and permeability relationships derived in Section 5 were used to describe the settling behaviour. The computer simulations were compared with experimental data by superimposing the simulated data onto an experimental characteristic plot. The results of the above simulations are shown in Figures 6-4 to 6-13, the thick lines denote simulated characteristics and the thin lines experimental characteristics.
6.6.1 Aragonite

Figure 6-4 shows the result of the $c_o = 0.175$ simulation. It can be seen that the interface descent is well predicted however there is a discrepancy in the final heights of 6.7 mm. At $c < 0.28$ the characteristics correspond very well and the simulation accurately predicts the rate at which concentrations will propagate up into the suspension. The exception to this is the $c = 0.22$ characteristic, the simulated characteristic initially predicts the same rate of propagation as the experimental data however as the sedimentation progresses the experimental characteristic shows a decreasing propagation rate which is not matched by the simulated data. This behaviour was also shown by characteristics of concentrations $c \leq 0.25$ in the $c_o = 0.2$ & 0.225 simulations (Figures 6-5 & 6-6). As a result the characteristic match was not as good as the $c_o = 0.175$ simulation. In both simulations the initial rate of descent of the interface was well predicted but the final simulated heights were 7.5 mm ($c_o = 0.2$) and 8.0 mm ($c_o = 0.225$) greater than the actual heights.
Figure 6.4: Aragonite simulation, c₀ = 0.175.
Figure 6.5: Aragonite simulation, $c_0 = 0.2$. 
Figure G-6: Argonite simulation, $c_0 = 0.225$. 
The failure of the model to predict the decrease in propagation rates for concentrations of $0.21 < c < 0.25$ could be attributed to channelling. This would be justifiable as the channel zone was observed to occur over the concentration range $0.21 < c < 0.24$, if this was the case then it suggests that the amount of channelling was decreasing as the sedimentation reached completion. This is entirely feasible, as a sediment nears the completion of compaction the amount of liquid expelled will decrease and so less liquid flow is available to create or sustain channels. From a modelling point of view it may be possible to account for the variation in the amount of channelling by applying some factor to the permeability function. Such a factor was used by Font [1991] to account for the lack of channelling at the start of a sedimentation, there was no observed decrease in propagation rate towards the end of Font’s experiments. The limitation of Font’s factor was that it was solely time dependent and so its use was restricted to a single initial concentration as different initial concentrations would have different sedimentation times. A better approach would be to consider how close the completion of compaction is, for example the difference in height between the top of a characteristic and solid-liquid interface could be used. Ideally the amount of channelling and hence any correction factor could be related to the fluid velocity as this would appear to be the determining factor as to the amount of channelling. This is not currently possible as there is no established method for measuring fluid velocity.

Although channelling seems to be the most likely cause of the decrease in characteristic propagation rate other factors which could contribute or cause such a variation do exist and should be considered. The solids pressure could not be determined because of flux tracking over the range $c < 0.25$ so it is possible that there was an error in the solids pressure expression over this range, this suggestion is strengthened by the failure of the model to correctly predict the final height of the sediment. It may also be that the consolidation process is time dependent i.e. the breakdown of the solids structure from one concentration to another when a certain solids pressure is exerted is not instantaneous.
Figure 6.7: Talc simulation, \( c_0 = 0.052 \).
Figure 6.8: Talc simulation, $c_0 = 0.073$. 
Figure 6.9: Talc simulation, $c_0 = 0.113$. 
6.6.2 Talc

The characteristics generated by the $c_0 = 0.052 \& 0.073$ simulations (Figures 6-7 & 6-8) gave a good match with experimental data at all concentrations. The interface descent was also accurately predicted except near the completion of settling when the experimental data showed in both cases acceleration periods which could not be predicted by the model because of the boundary conditions imposed on it. The final simulated sediment heights were also very similar to the actual values.

![Graph showing revised talc permeability values calculated using Font's profile method.](image)

**Figure 6-10**: Revised talc permeability values calculated using Font’s profile method.

When an initial concentration of $c_0 = 0.113$ was used the simulation gave a poor comparison with experimental data (Figure 6-9). The initial descent rate of the interface was accurately predicted however the actual interface soon showed an increasing rate period and as a result there was a significant difference in the predicted and actual sedimentation times, and in the simulated and real characteristics. The latter emanated earlier and showed a much more concave propagation path than their simulated counterparts. It was accepted that the model could not predict an
accelerating interface or dilution concentrations however these properties were shown at the lower initial concentrations and an accurate prediction of non dilution characteristics was still obtained. The only difference between the higher and lower initial concentration runs was the amount of channelling and dilution exhibited, the effect being greatest at the higher initial concentration. The final predicted height and actual height were not too dissimilar and so it would appear that failure of the simulation to correctly predict characteristic propagation rates was due to an error in the permeability expression. This was developed from data taken from experimental runs having the same initial concentrations as the simulated runs. A good match was obtained between the data of different initial concentrations however a re-examination of the method showed that permeability values derived using the solids profile method were done at sample times of 15000 and 25000 seconds. It can be seen from Figure 6-9 that the propagation rates of experimental characteristics at and beyond 15000 seconds are decreasing, hence permeability values were not obtained at the greater propagation rates encountered in the earlier stages of the run. The propagation rates of the characteristics in the \( c_o = 0.113 \) run at the sample times are similar to those exhibited by characteristics in the \( c_o = 0.052 \) & \( 0.073 \) runs and so the match in data is not surprising.

To obtain a fuller picture the data was analysed at more sample times and the variation of permeability at set concentrations was plotted as a function of time (Figure 6-10). The analysis showed no variation in permeability at any concentration between 5000 - 10000 seconds, after which there was a rapid increase in permeability between 10000 - 15000 seconds the rate of increase then subsided and levelled out at 25000 seconds and showed a slight decrease thereafter. The increase between 10000 - 15000 seconds didn’t coincide with the appearance of channels (\( t = 3600 \) seconds) as expected however it did occur at the point at which the solid liquid interface descent rate began to increase. This suggested that in the presence of channelling permeability could not be regarded as a unique function of concentration. An approach similar to Font [1991 & Section 6.5.2] was therefore followed. The permeability of concentrations at \( t = 10000 \) seconds were taken to be the base values of permeability, the factors required to increase these values to those shown later in the run were then
calculated and are shown in Table 6-1. It can be seen that the magnitude of the factors required varied with concentration and suggested that channelling had a greater effect at higher concentrations. In Font’s work the variation in permeability across all concentrations could be accounted for by a single time dependent function, this was not possible in this study because of the variation in magnitude across the concentration range. It would appear, as already postulated, that the degree of channelling that occurs in a compacting sediment is a function of concentration and some other factor most probably fluid velocity.

Table 6-1: Permeability correction factors

<table>
<thead>
<tr>
<th>Correction factors</th>
<th>15000</th>
<th>20000</th>
<th>25000</th>
<th>30000</th>
</tr>
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<tr>
<td>Concentration</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.18</td>
<td>3.7</td>
<td>4.3</td>
<td>4.65</td>
<td>3.9</td>
</tr>
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<td>3.2</td>
<td>3.6</td>
<td>3.75</td>
<td>3.3</td>
</tr>
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<td>3.1</td>
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<td>3.38</td>
<td>2.8</td>
</tr>
<tr>
<td>0.15</td>
<td>2.83</td>
<td>3.2</td>
<td>3.1</td>
<td>2.47</td>
</tr>
<tr>
<td>0.14</td>
<td>2.18</td>
<td>2.43</td>
<td>2.46</td>
<td>1.83</td>
</tr>
<tr>
<td>0.13</td>
<td>1.56</td>
<td>1.66</td>
<td>1.78</td>
<td>1.28</td>
</tr>
</tbody>
</table>

6.6.3 Calcite

The calcite simulations are shown in Figures 6-11 - 6-13, in general the characteristics generated in the \( c_o = 0.05 \) simulation gave a good match with experimental characteristics in terms of their rate of propagation although at concentrations of \( c = 0.06 \) and \( 0.1 \) there was a tendency for the rate to be slightly over predicted in the later stages of the sedimentation. In the \( c_o = 0.075 \) simulation the opposite trend was exhibited (Figure 6-11), the rate of propagation of the simulated characteristics at all concentrations was less than the rate of the real characteristics. At the highest initial concentration simulation (\( c_o = 0.125 \)) the trend was once again reversed the predicted rates of propagation being greater than the true rates. At \( c_o = 0.05 \) and \( c_o = 0.075 \) the initial rate of descent of the interface was well predicted however at \( c_o = 0.125 \) the rate was slightly over predicted. Conversely the \( c_o = 0.05 \) and \( 0.125 \) simulations over estimated the final sediment height whereas the \( c_o = 0.075 \) simulation gave good agreement with the true final height.
Figure 6.11: Calcite simulation, c₀ = 0.05.
Figure 6.12: Cattle simulation, $c_i = 0.075$. 

Height, mm

Time, s.
The calcite simulations were difficult to interpret as there were no obvious trends in the data. The cause of this anomalous behaviour was difficult to identify, any variation in the experimental temperature or amount of initial agitation was minimal and so initial conditions were unlikely to have been the cause. The only possible variant was the amount of time suspensions were left to acclimatise and consolidate after a change in concentration. The minimum period was ten hours and the maximum twenty hours, further research would be required to determine whether this was the cause.

6.6.4 Manipulation of the boundary conditions to allow for dilution effects

All of the materials investigated showed at some concentration an accelerating interface descent and/or dilution concentrations. The model could not predict such changes as the boundary condition imposed at the solid liquid interface (i.e. \( w^* = 1 \)) was that \( p_l^* = 0 \) and so it was assumed that the concentration of the solid liquid interface would remain constant at the initial concentration throughout a sedimentation. However it was subsequently reasoned that a change in interface concentration would not necessarily invalidate the boundary condition \( p_l^* = 0 \). When dilution occurs in a sedimentation the interface concentration is reduced, the rate at which this occurred in this study was obtained from the concentration-time plots and developed into mathematical functions. The model was therefore modified so that the concentration at \( w^* = 1 \) (the solid -liquid interface) was predicted by a time dependent function. However when the model was run the simulation crashed. The reason for this was when the concentration of the interface was reduced the model in order to accommodate the same amount of solids in the \( n^{th} \) weight fraction (i.e. the uppermost weight fraction) increased the concentration at the interface with the \( n-1 \) weight fraction. Likewise the \( n-1 \) weight fraction to contain the same weight of solids reduced the concentration at its interface with the \( n-2 \) weight fraction and so an oscillating concentration profile resulted which led to the model failing. It would appear therefore that the process of dilution cannot be accounted for simply by imposing a different boundary condition to that used in conventional sedimentation
models. The problem is more fundamental and it would seem likely that the dilution process is governed by different force balance to that of sedimentation.

6.6.5 Summary

In general the model provided a reasonable representation of the build up of compressible sediments for all the materials, this included sediments in which channelling was occurring. The most accurate simulations were obtained at low initial compression concentrations which showed moderate channelling. The methods utilised for deriving permeability and solids pressure expressions can therefore be regarded as sufficient for predicting the behaviour of low initial compression concentration sediments. For higher initial concentrations the variation in permeability due to channelling must be accommodated, the concentration profile method provides a means of measuring the variation a suitable mathematical treatment must now be developed.
7. Continuous Thickening

In Section 2.2.4 an analysis by Chandler [1982] was presented in which it was shown that a greater washing effect (ratio of underflow to feed concentration) could be achieved by operating a thickener as a deep bed thickener in which channelling was promoted. Deep bed thickeners employ sediment height to thickener width ratios in excess of one and contain no moving parts i.e. no rake mechanisms. This is contrary to the operation of conventional thickeners which use small sediment heights and employ raking and stirring mechanisms which eradicate channelling.

In order to assess the possible advantage of deep bed thickening a laboratory scale continuous thickener was constructed (see Section 3.1 for a detailed description). The thickener was equipped with a rake mechanism and picket fence (see Appendix A) which were employed in stirred experiments to move solids towards the central underflow removal point and disrupt channelling.

![Figure 7-1: Operating line construction for limiting flux](image)

In Section 2.2.1 the concept of flux curves and their significance in the design and operation of thickeners was outlined. The primary result was that a continuous thickener operating at steady state can be described by

\[ Fc_u = Yc_u = A\phi, \]  

(7.1)

where \( \phi \) is the solids flux in the thickener. Obviously \( \phi \) is a function of the feed conditions imposed on a thickener however there is an upper limit to the flux that can be accommodated by a given thickener area the value of which is defined by the
required underflow concentration. This flux can be determined by constructing an operating line on a solids flux-concentration plot from the required underflow concentration tangential to the batch settling solids flux curve, an example is shown in Figure 7-1. The intercept with the y-ordinate gives a value for $\phi$, this is the maximum possible value of $\phi$, that can be accommodated at that underflow concentration. Knowledge of this allows suitable feed conditions to be determined and the underflow withdrawal rate set.

Aragonite and talc were both investigated using stirred and unstirred modes of continuous thickening. A comparison between the unstirred and stirred batch settling fluxes of aragonite and talc are shown in Figure 7-2. If the theory presented in Section 2.2.1 is correct, then there should be little difference between stirred and unstirred continuous thickening for this material. Conversely the stirred and unstirred batch flux curves for talc are very dissimilar and suggest that much greater fluxes should be possible in unstirred continuous thickening.

![Figure 7-2: Comparison between unstirred and stirred flux curve for aragonite and talc.](image-url)
7.1 Experimental method

Feed conditions and underflow withdrawal rates were chosen such that a range of underflow concentrations and thickener solids fluxes ($\phi_t$) were obtained. In stirred tests a stirrer speed of 1 r.p.m. was used for aragonite and 2 r.p.m. for talc.

In all experiments the underflow withdrawal rate was set such that the thickener would be operating at its maximum capacity. In some cases this resulted in the thickener overloading, when this occurred the thickener could no longer accommodate the flux imposed on it so the solid-liquid interface which at steady state had a constant height at that of the feedwell would rise upwards and if undetected overflow out of the thickener. Overload conditions were countered by increasing the underflow withdrawal rate (or depending on the pumps' range decreasing the feed flowrate) incrementally until steady state was re-established.

The thickener was operated in a closed loop and therefore the feed concentration was subject to variation depending on the depth and concentration of sediment present in the thickener. A larger feed tank could have alleviated this effect however heating and mixing considerations meant that this was not possible.

In all of the experiments conducted the feed suspension concentration was relatively low, the suspension at the feedwell height was diffuse and appeared to settle discontinuously into an easily distinguishable sediment. Steady state was therefore assessed by monitoring the variation in sediment height. When steady state was achieved samples of the feed and underflow were taken, these were weighed, dried and then re-weighed to allow the respective concentrations to be calculated. The volume of fluid dispensed by the underflow and overflow over a certain period was also measured. The feed flowrate was deduced from

$$ F = B + Y $$

(7.2)

A check on the steady state was obtained by comparing the measured feed concentration with a value determined from Equation 7.1, the steady state balance:

$$ c_o = \frac{Yc_s}{F} $$

(7.3)
7.2 Experimental results

The stirred and unstirred data for the continuous thickening of aragonite is shown in Tables 7-1 & 7-2 respectively and for talc in Tables 7-3 & 7-4 respectively. A row break in a table indicates where the feed tank concentration was externally altered to allow a greater range of feed concentrations to be used. Figures 7-3 to 7-6 show some of the operating lines that were used, not all operating lines are shown as some were very similar.

The continuous thickener was initially run in stirred mode with aragonite as the discrete phase, the greater number of tests conducted relative to the other mode of operation and also to talc reflect a period of familiarisation with the equipment and its operation.

Feed flowrates and concentrations in the ranges $1.53 \leq F \leq 2.1 \text{ ml s}^{-1}$ and $0.012 \leq c_o \leq 0.061$ were used. The resulting underflow concentrations for an underflow withdrawal rate range of $0.0625 \leq Y \leq 0.667 \text{ ml s}^{-1}$ were $0.175 \leq c_u \leq 0.316$. A satisfactory steady state was obtained at all sample times except for tests AS5 and AS11. The maximum concentration of $c_u = 0.316$ was greater than was achieved in unstirred batch thickening but was considerably less than in stirred batch thickening where the sediment base concentration was typically $c \approx 0.42$. It is however unlikely that the peristaltic underflow pump could have effectively transported such concentrations had they been achieved.

Unstirred continuous thickening of aragonite resulted in similar underflow concentrations $0.168 \leq c_u \leq 0.306$ over an underflow withdrawal rate range of $0.036 \leq Y \leq 0.622 \text{ ml s}^{-1}$, the minimum flowrate was considerably less than in the stirred tests. The maximum possible feed flowrate was also less ($1.477 \leq F \leq 1.613 \text{ ml s}^{-1}$) although the feed concentration range was comparable ($0.00725 \leq c_o \leq 0.067$). Good steady state checks were obtained in all tests except A4.
Table 7-1: Experimental results for the stirred continuous thickening of aragonite.

<table>
<thead>
<tr>
<th>Test</th>
<th>Overflow</th>
<th>Underflow</th>
<th>Feed</th>
<th>Solids flux ( \phi )</th>
<th>Sediment height Co</th>
<th>Steady state check Co (v/v)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B (m/s)</td>
<td>Y (m/s)</td>
<td>Cu (v/v)</td>
<td>F (m/s)</td>
<td>Co (m/s)</td>
<td>(m)</td>
</tr>
<tr>
<td>AS1</td>
<td>1.425</td>
<td>0.358</td>
<td>0.188</td>
<td>1.783</td>
<td>0.037</td>
<td>3.8E-06</td>
</tr>
<tr>
<td>AS2</td>
<td>1.483</td>
<td>0.292</td>
<td>0.183</td>
<td>1.775</td>
<td>0.028</td>
<td>3E-06</td>
</tr>
<tr>
<td>AS3</td>
<td>1.583</td>
<td>0.258</td>
<td>0.169</td>
<td>1.842</td>
<td>0.026</td>
<td>2.8E-06</td>
</tr>
<tr>
<td>AS4</td>
<td>1.625</td>
<td>0.183</td>
<td>0.195</td>
<td>1.808</td>
<td>0.021</td>
<td>2E-06</td>
</tr>
<tr>
<td>AS5</td>
<td>1.750</td>
<td>0.063</td>
<td>0.210</td>
<td>1.813</td>
<td>0.012</td>
<td>7.4E-07</td>
</tr>
<tr>
<td>AS6</td>
<td>1.292</td>
<td>0.511</td>
<td>0.176</td>
<td>1.603</td>
<td>0.051</td>
<td>5.1E-06</td>
</tr>
<tr>
<td>AS7</td>
<td>1.300</td>
<td>0.511</td>
<td>0.183</td>
<td>1.811</td>
<td>0.050</td>
<td>5.3E-06</td>
</tr>
<tr>
<td>AS8</td>
<td>1.617</td>
<td>0.511</td>
<td>0.190</td>
<td>2.128</td>
<td>0.046</td>
<td>5.5E-06</td>
</tr>
<tr>
<td>AS9</td>
<td>1.889</td>
<td>0.113</td>
<td>0.316</td>
<td>2.001</td>
<td>0.019</td>
<td>2E-06</td>
</tr>
<tr>
<td>AS10</td>
<td>1.833</td>
<td>0.158</td>
<td>0.301</td>
<td>1.992</td>
<td>0.024</td>
<td>2.7E-06</td>
</tr>
<tr>
<td>AS11</td>
<td>1.687</td>
<td>0.284</td>
<td>0.195</td>
<td>1.971</td>
<td>0.034</td>
<td>3.1E-06</td>
</tr>
<tr>
<td>AS12</td>
<td>1.296</td>
<td>0.667</td>
<td>0.176</td>
<td>1.963</td>
<td>0.061</td>
<td>6.6E-06</td>
</tr>
<tr>
<td>AS13</td>
<td>1.429</td>
<td>0.567</td>
<td>0.187</td>
<td>1.996</td>
<td>0.051</td>
<td>6E-06</td>
</tr>
<tr>
<td>AS14</td>
<td>1.625</td>
<td>0.373</td>
<td>0.213</td>
<td>1.998</td>
<td>0.041</td>
<td>4.5E-06</td>
</tr>
<tr>
<td>AS15</td>
<td>1.180</td>
<td>0.368</td>
<td>0.206</td>
<td>1.548</td>
<td>0.052</td>
<td>4.3E-06</td>
</tr>
<tr>
<td>AS16</td>
<td>1.267</td>
<td>0.271</td>
<td>0.226</td>
<td>1.538</td>
<td>0.043</td>
<td>3.5E-06</td>
</tr>
<tr>
<td>AS17</td>
<td>1.373</td>
<td>0.169</td>
<td>0.282</td>
<td>1.542</td>
<td>0.032</td>
<td>2.7E-06</td>
</tr>
<tr>
<td>AS18</td>
<td>1.305</td>
<td>0.183</td>
<td>0.279</td>
<td>1.488</td>
<td>0.036</td>
<td>2.9E-06</td>
</tr>
</tbody>
</table>

Table 7-2: Experimental results for the unstirred continuous thickening of aragonite.

<table>
<thead>
<tr>
<th>Test</th>
<th>Overflow</th>
<th>Underflow</th>
<th>Feed</th>
<th>Solids flux ( \phi )</th>
<th>Sediment height Co</th>
<th>Steady state check Co (v/v)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B (m/s)</td>
<td>Y (m/s)</td>
<td>Cu (v/v)</td>
<td>F (m/s)</td>
<td>Co (m/s)</td>
<td>(m)</td>
</tr>
<tr>
<td>A1</td>
<td>0.979</td>
<td>0.622</td>
<td>0.168</td>
<td>1.601</td>
<td>0.067</td>
<td>5.9E-06</td>
</tr>
<tr>
<td>A2</td>
<td>1.067</td>
<td>0.540</td>
<td>0.175</td>
<td>1.607</td>
<td>0.063</td>
<td>5.3E-06</td>
</tr>
<tr>
<td>A3</td>
<td>1.125</td>
<td>0.488</td>
<td>0.169</td>
<td>1.613</td>
<td>0.052</td>
<td>4.7E-06</td>
</tr>
<tr>
<td>A4</td>
<td>1.213</td>
<td>0.320</td>
<td>0.180</td>
<td>1.533</td>
<td>0.048</td>
<td>3.3E-06</td>
</tr>
<tr>
<td>A5</td>
<td>1.317</td>
<td>0.293</td>
<td>0.194</td>
<td>1.610</td>
<td>0.036</td>
<td>3.2E-06</td>
</tr>
<tr>
<td>A6</td>
<td>1.383</td>
<td>0.229</td>
<td>0.195</td>
<td>1.613</td>
<td>0.030</td>
<td>2.5E-06</td>
</tr>
<tr>
<td>A7</td>
<td>1.450</td>
<td>0.163</td>
<td>0.223</td>
<td>1.613</td>
<td>0.022</td>
<td>2.1E-06</td>
</tr>
<tr>
<td>A8</td>
<td>1.467</td>
<td>0.105</td>
<td>0.223</td>
<td>1.571</td>
<td>0.015</td>
<td>1.3E-06</td>
</tr>
<tr>
<td>A9</td>
<td>1.367</td>
<td>0.112</td>
<td>0.222</td>
<td>1.479</td>
<td>0.017</td>
<td>1.4E-06</td>
</tr>
<tr>
<td>A10</td>
<td>1.367</td>
<td>0.112</td>
<td>0.245</td>
<td>1.479</td>
<td>0.020</td>
<td>1.6E-06</td>
</tr>
<tr>
<td>A11</td>
<td>1.425</td>
<td>0.056</td>
<td>0.305</td>
<td>1.481</td>
<td>0.012</td>
<td>9.7E-07</td>
</tr>
<tr>
<td>A12</td>
<td>1.442</td>
<td>0.036</td>
<td>0.306</td>
<td>1.478</td>
<td>0.007</td>
<td>6.3E-07</td>
</tr>
</tbody>
</table>
Table 7-3: Experimental results for the stirred continuous thickening of talc

<table>
<thead>
<tr>
<th>Test</th>
<th>Overflow B (ml/s)</th>
<th>Y (ml/s)</th>
<th>Cu (v/v)</th>
<th>Feed F (ml/s)</th>
<th>Co (v/v)</th>
<th>Solids flux ( \phi ) (m/s)</th>
<th>Sediment check height Co (m)</th>
<th>Steady state check Co (v/v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST1</td>
<td>0.079</td>
<td>0.113</td>
<td>0.106</td>
<td>0.192</td>
<td>0.062</td>
<td>6.8E-07</td>
<td>0.253</td>
<td>0.062</td>
</tr>
<tr>
<td>ST2</td>
<td>0.117</td>
<td>0.075</td>
<td>0.134</td>
<td>0.192</td>
<td>0.051</td>
<td>5.7E-07</td>
<td>0.307</td>
<td>0.052</td>
</tr>
<tr>
<td>ST3</td>
<td>0.099</td>
<td>0.053</td>
<td>0.147</td>
<td>0.151</td>
<td>0.053</td>
<td>4.4E-07</td>
<td>0.292</td>
<td>0.051</td>
</tr>
<tr>
<td>ST4</td>
<td>0.078</td>
<td>0.047</td>
<td>0.158</td>
<td>0.124</td>
<td>0.060</td>
<td>4.2E-07</td>
<td>0.232</td>
<td>0.059</td>
</tr>
<tr>
<td>ST5</td>
<td>0.079</td>
<td>0.038</td>
<td>0.173</td>
<td>0.116</td>
<td>0.059</td>
<td>3.7E-07</td>
<td>0.233</td>
<td>0.056</td>
</tr>
<tr>
<td>ST6</td>
<td>0.087</td>
<td>0.030</td>
<td>0.191</td>
<td>0.117</td>
<td>0.056</td>
<td>3.2E-07</td>
<td>0.245</td>
<td>0.049</td>
</tr>
</tbody>
</table>

Table 7-4: Experimental results for the unstirred continuous thickening of talc

<table>
<thead>
<tr>
<th>Test</th>
<th>Overflow B (ml/s)</th>
<th>Y (ml/s)</th>
<th>Cu (v/v)</th>
<th>Feed F (ml/s)</th>
<th>Co (v/v)</th>
<th>Solids flux ( \phi ) (m/s)</th>
<th>Sediment check height Co (m)</th>
<th>Steady state check Co (v/v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>0.079</td>
<td>0.113</td>
<td>0.114</td>
<td>0.193</td>
<td>0.064</td>
<td>7.3E-07</td>
<td>0.129</td>
<td>0.067</td>
</tr>
<tr>
<td>T2</td>
<td>0.134</td>
<td>0.053</td>
<td>0.109</td>
<td>0.188</td>
<td>0.035</td>
<td>3.3E-07</td>
<td>0.216</td>
<td>0.031</td>
</tr>
<tr>
<td>T3</td>
<td>0.203</td>
<td>0.076</td>
<td>0.114</td>
<td>0.279</td>
<td>0.034</td>
<td>4.9E-07</td>
<td>0.257</td>
<td>0.031</td>
</tr>
<tr>
<td>T4</td>
<td>0.194</td>
<td>0.114</td>
<td>0.104</td>
<td>0.309</td>
<td>0.040</td>
<td>6.8E-07</td>
<td>0.264</td>
<td>0.039</td>
</tr>
<tr>
<td>T5</td>
<td>0.034</td>
<td>0.017</td>
<td>0.097</td>
<td>0.051</td>
<td>0.037</td>
<td>9.5E-08</td>
<td>0.244</td>
<td>0.033</td>
</tr>
</tbody>
</table>
Another aspect of the tests was the start-up times and equilibrium achieving times, the former was 18 hours in stirred mode whereas in unstirred it was 57 hours. The time taken to reach equilibrium after a change in either the underflow withdrawal rate or feed conditions ranged between 4.5 hours and 17 hours in both modes.

In Figure 7-7 the sediment depths attained in both the stirred and unstirred tests are plotted as a function of underflow concentration. It can be seen that the unstirred tests showed predictable behaviour, the sediment height increasing with increases in the underflow concentration. The stirred tests show an increasing sediment height with increasing underflow concentration up to a concentration of \( c = 0.22 \), above this concentration the height required levelled out.

In the talc tests a good underflow concentration range \( 0.1 < c_u < 0.19 \) was obtained in stirred mode using underflow withdrawal rates in the range \( 0.0375 < Y < 0.11278 \) \( ml/s \). In contrast a poor range was obtained in unstirred mode \( 0.0966 < c_u < 0.114 \) over a withdrawal range of \( 0.0338 < Y < 0.2033 \) \( ml/s \). The ranges of the feed conditions used in stirred mode, \( (0.051 < c_o < 0.062, 0.116 < F < 0.19 \) \( ml/s \)\) were considerably smaller than in unstirred mode \( (0.034 < c_o < 0.064, 0.05 < F < 0.3 \) \( ml/s \)\). The steady state checks showed greater deviations than in the aragonite tests but were still acceptable except for test ST6. The start-up times were 83 and 117 hours for the unstirred and stirred tests respectively. The time required to recover equilibrium after a condition change ranged from 23 to 85 hours for both modes. The sediment heights used showed no discernible trends in either mode of operation but were typically of the order 0.2 to 0.32 m.

Channelling was not observed at steady state with any of the materials or in either mode, a short period of channelling was observed in the start-up of the talc unstirred tests but it didn’t persist.
Figure 7-3: Operating line plot for continuously thickened stirred aragonite tests.

Figure 7-4: Operating line plot for unstirred continuously thickened aragonite tests.
Figure 7-5: Operating line plot for stirred continuously thickened talc tests.

Figure 7-6: Operating line plot for unstirred continuously thickened talc tests.
Figure 7-7: Sediment height as a function of underflow concentration for stirred and unstirred continuous thickening of aragonite.

Figure 7-8: Sediment height as a function of underflow concentration for stirred and unstirred continuous thickening of talc.
7.3 Discussion

To achieve the same underflow concentrations lower feed and underflow flowrates were used in the unstirred continuous thickening of aragonite than in the stirred mode suggesting that the capacity of the thickener was less in unstirred mode. A thickener operating at its maximum capacity has an operating line which passes tangential to the batch settling flux curve, the above assumption can therefore be confirmed by examination of the operating line plots (Figures 7-3 & 7-4) for both modes.

The operating lines for a sample of the stirred tests are shown in Figure 7-3, it can be seen that all the lines pass close and tangential to the batch settling flux which indicate that the thickener was operating at its maximum capacity and in accordance with the theory presented in Section 2.2.1. This is not the case in Figure 7-4, the unstirred operating lines describe a curve similar to the batch settling flux curve in shape but at lower fluxes i.e. the thickener was operating below its theoretical capacity.

The stirred results showed that it was possible to operate the thickener at its maximum possible capacity so some effect was causing the thickener to under perform in unstirred mode. The effect was greatest at high underflow concentrations, at $c_u = 0.305$ a flux of $9.7 \times 10^7 \text{ ms}^{-1}$ was obtained in unstirred mode whereas a flux of $2 \times 10^6 \text{ ms}^{-1}$ was possible at $c_u = 0.316$ in stirred mode. At an underflow concentration of $c_u = 0.176$ the unstirred mode handled $5.3 \times 10^6 \text{ ms}^{-1}$ compared with $6.6 \times 10^6 \text{ ms}^{-1}$ in stirred mode. At higher underflow concentrations two unstirred thickeners would therefore be required to process the same load as a single stirred thickener, whilst this may seem to indicate that stirring is beneficial it has been noted by Chandler [1983] that the additional capital and operating costs of a stirring mechanism may be greater than the construction cost of a second unstirred thickener.

In the case of talc the effect was more extreme, lower underflow rates and reduced feed fluxes were used in the unstirred tests but a maximum underflow concentration of $c_u = 0.116$ was the best obtained compared with $c_u = 0.19$ in the stirred tests. The operating line plots (Figures 7-5 & 7-6) illustrate the difference in the size of the
ranges of feed fluxes used and the underflow concentrations achieved. The stirred mode operating lines pass close to the stirred batch settling flux and in some cases cross the flux curve. Theoretically this is not possible however this result is most likely due to an error in the batch flux curve which was determined from initial rate analysis. The unstirred mode operating lines show the same disparity, the reason for this is most likely the same. The concave shape of the unstirred batch settling curve means that tangential operating lines weren’t possible, it was however possible to construct an operating line which allowed higher underflow concentrations higher than \( c_u = 0.116 \) to be attained, however physically this was not achieved.

The effect that causes the unstirred mode of continuous thickening to under perform relative to stirred thickening is most likely sludge funnelling. Chandler in his description of deep bed thickeners reported that a cone of more or less stagnant slurry builds up to an angle corresponding to the natural angle of repose and that this angle is maintained throughout the life of the thickener. When the thickener used in this study was drained such cones were observed to have formed. The effect of coning or sludge funnelling is to reduce the effective area available for thickening. Dixon (1980) mathematically examined this effect and concluded that the operating line equation used to describe stirred or cylindrical thickening and that used in this study was invalid for conical thickening or cylindrical thickeners with stagnant funnels as it was based on the assumption that the flux at all points in the thickener was the same. The equation was derived in Section 2.2.2 and is given below

\[
\phi_t = \phi_r - \left( \frac{\phi_r}{c_u} \right) c
\]  

Dixon defining the flux in the thickener as the volumetric flowrate of particles divided by the cross sectional area

\[
\phi_r = \frac{Q_t}{A}
\] 

and argued that since the area of a thickener would change due to its conical nature so must the flux within. Hence Equation (7.4) no longer described a straight line as \( \phi_t \) was not constant rather the operating lines were of the form shown in Figure 7-9.
Dixon’s work was furthered by Landman [1988] who performed a very detailed analysis of all shapes of thickeners and concluded that the greatest fluxes could be accommodated by thickeners which had a divergent shape and that additionally there would be no problem with build-up of solids on the walls (i.e. sludge funnelling). It was readily accepted however that underflow removal could be difficult and so a comprise design in which the thickener diverged over most of its height and then converged rapidly was suggested.

Whilst sludge funnelling explains the reduced capacity behaviour of unstirred continuous thickening it doesn’t completely explain the low underflow concentrations obtained for talc in unstirred mode. The only feasible explanation is that an insufficient height of sediment existed, the variation in sediment height with regards to underflow concentration is shown in Figure 7-8. It can be seen that no obvious trend existed, the heights ranging from 0.2 to 0.32 m. The reason for this may also lie with sludge funnelling, if as the papers of Dixon and Chandler suggest it is possible to obtain cones with 60° angles to the horizontal then it is possible for a thickener of diameter 0.15 m to support a cone 0.26 m high. This is the mid point of the heights obtained and could also explain the absence of any channelling and also the spurious impedance measurements obtained. Experimentation with a higher feedwell hence allowing greater sediment heights would be required to verify this.

In general the stirred mode of continuous thickening gave greater underflow concentrations at larger feed fluxes relative to the unstirred mode, it also showed
behaviour easily predictable by the theory presented in Section 2.2 i.e. making the thickener more easy to operate. The unstirred mode was less predictable a fact which was attributed to sludge funnelling and the more complex behaviour associated with conical thickening. For the materials used in this study it would therefore not seem beneficial to work in unstirred mode although it is accepted that on scale-up the rake and picket fence arrangement used in this study may not perform so well.
8. Conclusions

8.1 Use of electrical impedance measurements as a means of monitoring suspension concentration

Electrical conductivity measurements were used to map the variation in solids concentration throughout a sedimentation. The variation of suspension conductivity with solids concentration was described by a simple power law known as Archie's law. Theoretically for a suspension of rigid spheres the exponent in Archie's law is 1.5 however this value was known not to be accurate for all materials. By employing a simple mass balance over the height of a suspension it was possible to determine the best possible exponent value and the accuracy of the system for each material. A summary of the exponents used is given below

<table>
<thead>
<tr>
<th>Material</th>
<th>Archie's law exponent</th>
<th>maximum percentage error in volumes</th>
</tr>
</thead>
<tbody>
<tr>
<td>ballotini</td>
<td>1.33</td>
<td>3%</td>
</tr>
<tr>
<td>aragonite</td>
<td>1.30</td>
<td>5%</td>
</tr>
<tr>
<td>talc</td>
<td>1.98</td>
<td>5%</td>
</tr>
<tr>
<td>calcite</td>
<td>1.50</td>
<td>8%</td>
</tr>
<tr>
<td>kaolin</td>
<td>2.10</td>
<td>80%</td>
</tr>
</tbody>
</table>

The exponent used in Archie’s equation was found to be strongly related to particle morphology but was also affected by factors such as the speed of stirring, initial concentration, particle orientation and the frequency of the impedance measurement. The exponent value used wasn’t affected by the type of electrodes employed, however, it was found that greater resolution was obtained using protruding electrodes rather than conventional flush mounted electrodes. Kaolin was found to be a material which didn’t conform to Archie’s law: better percentage errors (PEV < 20%) were obtained using a simple straight line relationship instead of Archie’s law.
A flaw with the electrical impedance technique was found to be its lack of sensitivity near concentration discontinuities. Electric flux does not travel only in the horizontal plane of measurement but bows a small distance either side in the vertical direction therefore any impedance measurement was an average of the resistance offered by a segment of finite depth, i.e. if it contained a discontinuity a value representing the average resistance of the two concentrations was returned.

8.2 Batch sedimentation

Stirred and unstirred batch sedimentations were carried out using calcite, aragonite and talc over a range of initial concentrations. Aragonite and calcite were found to be slightly compressive and showed behaviour which conformed well with the compressible model presented in Section 2.1, both showed mild channelling at intermediate concentrations. Talc was a more compressive material which channeled at all initial concentrations and showed gross dilution at high initial concentrations. Interface velocity and characteristic plots were found to give the best insight into the settling behaviour of a material.

8.2.1 Effect of stirring

Stirred sedimentations were carried out at a variety of stirrer speeds, the stirrer speed required to eliminate channelling was found to be dependent on the material and initial concentration used. The effect of stirring on a sedimentation which would otherwise have channeled was examined by comparing characteristic plots, the stirred characteristics were straight and emanated in a 'Kynch' like fan shape whereas the unstirred sedimentation characteristics were convex in shape and conformed to the compressible sedimentation model outlined in Section 2.1. It was expected that stirring would simply eradicate channelling and so allow a direct comparison between a channelling and non-channelling sedimentation. Stirring however also affected the compression process, resulting in greater sediment concentrations. The extent to which this occurred varied from material to material and was inversely proportional to the degree of retardation experienced as a result of stirring.
8.2.2 Channelling

A type of channelling known as 'random' channelling was investigated. The term random refers to the fact that the channels occur naturally and are not induced by external means. Through visual observations it was found for all the materials that the zone in which channelling occurred could be sub-divided according to the channel structure into two zones termed the hard and soft zones. The hard zone of channelling always occurred within the compressive region of settling, the soft zone was a consequence of the hard zone and as such occurred immediately above. Channel formation occurred at the boundary between the two zones, this was easily identifiable and usually occurred at a concentration marginally greater than the sediment surface. The soft zone existed in both free and compressive settling but predominantly in the former.

The propagation path of the hard zone in aragonite and low initial concentrations of talc suspensions was found to closely follow the characteristic of the concentration at which channels first formed. At higher initial concentrations of talc the propagation rate of the hard zone was quicker than the characteristic at which it formed. This behaviour was explained by examining the propagation rates of characteristics of the same concentration but from different initial concentration tests. This led to the conclusion that the formation of channels is a function of concentration and also the fluid velocity relative to the solids, i.e. channel formation = f(c, u_f - u). The premise for this is that a certain concentration must be ascertained before a suspension structure can support the formation of channels and for a random disturbance to grow into a channel there must be a sufficient flow of fluid into that area.

Vesilind's conclusions on channelling were revised into a more universal form, the conclusions are given below
1] A channel zone can be subdivided into a hard and soft zone.
2] Channels form at the boundary between these two zones, this occurs in the compression zone at a concentration marginally greater than the sediment surface concentration.
3] The soft zone can exist in both regimes of settling, (compression and free settling).
4] The higher the initial concentration the quicker the channel zone propagates upwards.
5] The propagation of a channel zone although not identical is similar in shape to the rise of the concentration characteristics at which the channel zone initially forms.
6] The higher the initial concentration the higher the zone of channelling appears.
7] There is an upper limit to the concentration at which channels can exist.
8] The diameter of channels increases with time and with initial concentration.

The batch sedimentation data and also observations made during upflow tests allowed a qualitative model for the formation, growth and subsistence of channels to be formed in which a random inhomogeneity which arises from the random packing of flocs in the early stages of compression acts as the seed for channel formation. Additionally a 'paragenesis' plot similar to Fitch's (Section 2.1) has been developed which instead of predicting the type of settling which would occur at certain conditions predicts under what conditions channel formation occurs. The plot is shown in Figure 5-21.

Dilution of the upper regions of sedimenting suspensions was found to be a major effect of channelling. The characteristic plots showed that dilution occurred in both free settling and compression. An interesting consequence of this is that fully settled sediments can contain concentrations lower than the initial so long as the initial and lower concentrations are in the compression regime. The dilution characteristics for all of the materials in this study emanated from the solid-liquid interface and propagated downwards into the suspension at a rate which was related to the degree of overall dilution experienced. Eventually the characteristics reached a depth which corresponded with the height of the rising sediment at this point the dilution characteristics showed behaviour similar to the characteristics of the suspension below. The experimentally generated dilution characteristics were found to be different to Vesilind's theoretical suggestions and should assist in future modelling of the dilution process.
8.2.3 Locating the gel concentration

Several methods for locating the gel concentration through geometric constructions were detailed in the theory section none were found to be applicable to the materials used in this study. The gel concentration was instead established using a combination of basic theory coupled with a knowledge of the settling velocity profiles and of the nature of the characteristics for that material. The value of the gel concentration was therefore obtained through logical reasoning as opposed to analytical mathematical examination. The gel concentrations were found to be

\[
\begin{align*}
\text{aragonite} & \quad c_g = 0.176 \\
\text{talc} & \quad c_g = 0.05 \\
\text{calcite} & \quad c_g = 0.045
\end{align*}
\]

8.2.4 Tangential characteristic theory

Tiller's proposal that characteristics emanate tangentially from the sediment surface was investigated using the settling data of aragonite. The result was contrary to an earlier paper by the author (Butt[1993]) as experimental evidence was found to support Tiller's theory. Two methods of generating tangential characteristics (Fitch[1983] & Font [1988]) were compared, both methods gave similar values. Fitch's method however was considerably simpler. A good match was achieved between experimental and predicted characteristics at low concentrations but at concentrations close to the sediment surface concentration a noticeable difference arose between the theoretical and experimental characteristics. This may point to a flaw in tangential theory however it is also possible that it was due to the inadequacy of the measurement technique near discontinuities. Overall the theory of tangential characteristics appears to be a reasonable approximation for the settling behaviour encountered in the second falling rate zone.

8.2.5 Transient solids stress

Upflow tests were conducted which yielded permeability values for free settling concentrations. These were compared with values derived from initial rate tests and were found to be very similar. Not all concentration in free settling could be achieved
in the upflow test, a minimum fluidising velocity was identified and the concentration at which this occurred was concluded to be that at which transient solids stress became important.

8.3 Sediment characterisation

It was found that all the materials used in this study could be characterised by considering solids pressure and permeability to be sole functions of the solids concentration. Experimental solids pressure values were derived from analysing fully consolidated sediment profiles. The Shirato equation gave the best fit to the experimental data of talc whereas the best solids pressure concentration relationship for calcite and aragonite was a combination of the Shirato and Tiller expressions. The recently suggested Landman expressions for solids pressure offered no advantage over the older and mathematically simpler Shirato expression. However the Landman 3 and Tiller expressions proved useful in predicting high solids pressures at concentrations close to the maximum attainable. Permeability measurements were achieved using a combination of initial rate tests, downflow tests and a concentration profile technique. The latter method is relatively novel in that there is very little published literature on its use. When used with Font's solids velocity expression it was found to give accurate permeability values across the whole of the compression concentration range. The average of two or three tests gave values similar to the initial rate/downflow data with significantly less experimentation being required to obtain the former data. It is possible that the technique may remove the need for downflow and initial rate tests. The solids velocity expression derived from Kynch theory failed to predict reasonable permeability values for concentrations in the compression regime and this would, as other researchers have reported, indicate that it's application is limited to the free settling regime.

Use of the initial rate method for generating permeability values for concentrations in the compression regime can be justified by comparing the magnitude of the drag force with that of the solids pressure gradient. For the materials used in this study it was found that initial rate data at the following initial concentrations could be used
aragonite \( c_o < 0.27 \)
calcite \( c_o < 0.157 \)
talc \( c_o < 0.126 \)

This, of course, is subject to there being an identifiable constant rate zone which is not always the case.

The Kozeny Carman equation for permeability was fitted to the experimental data by considering the Kozeny 'constant' to be a variable factor. The Davies & Dollimore method of generating Kozeny factor functions was investigated and the method of obtaining the exponent in Richardson & Meikle's equation was revised. The Davies & Dollimore method offers an approximate but quick method of generating Kozeny functions, experimentally all that is required is a series of initial rate tests and analytically it is only necessary to derive a value for the exponent in Richardson & Meikle's equation.

8.4 Compression Model

The consolidation of a sediment can be mathematically represented by a non-linear partial differential equation which describes the process in terms of the variation in excess hydrostatic pressure. The equation can be solved using an implicit finite difference three time level method. The relationships developed in the characterisation work were applied to the model and experimental characteristic plots were compared with model plots. In general the model provided a reasonable representation of the build-up of compressible sediments for all the materials, this included sediments in which channelling was occurring. The most accurate simulations were obtained at low initial compression concentrations which showed moderate channelling. The methods utilised for deriving permeability and solids pressure expressions can therefore be regarded as sufficient for predicting the behaviour of low initial compression concentration sediments. For higher initial concentrations the variation in permeability due to channelling must be accommodated, the concentration profile method provides a means of measuring the
variation a suitable mathematical treatment must now be developed. Attempts to modify the model to account for dilution failed.

8.5 Continuous thickening

Tests were carried out in a continuous thickener using two modes of operation, in stirred mode a picket fence and rake were employed to eradicate channelling and move solids to the underflow, in unstirred mode no agitation was employed. The stirred mode of continuous thickening gave greater underflow concentrations at larger feed fluxes relative to the unstirred mode, it also showed behaviour which conformed to cylindrical thickener theory. The reduced capacity of the unstirred mode was attributed to the build-up of sludge funnels which meant that the thickener in effect operated as a conical thickener. The behaviour of conical thickeners is more complex than cylindrical thickeners as the flux in the thickener is no longer constant and as a result the operating line is curved rather than straight. The low underflow concentrations achieved using talc in the unstirred mode were attributed to an insufficient height of sediment.

8.6 Recommendations for future work

[1] The possibility of using the concentration profile technique as a means of measuring the magnitude of channelling in terms of permeability at different concentrations and times during a sedimentation should be investigated.

[2] A permeability model which accounts for channelling needs to be developed - this will most probably be a function of concentration and some parameter which accounts for the change in the degree of channelling throughout an experiment. The liquid flow or height from the solid-liquid interface are possibilities.
[3] Dilution of the upper layers of a suspension has been shown to occur in almost all channelling suspensions. Investigation into the conditions required for dilution should be undertaken and consideration given to how dilution can be mathematical modelled.


[5] Points [2],[3],[4] need to be mathematically coupled together to give a complete model for batch sedimentation.

[6] The materials used in this study suggested that deep bed thickeners offer no advantages over conventional stirred thickeners. To confirm this more materials need to be investigated including highly flocculent materials and a greater examination of economic factors such as the costs (operating and capital) of picket fences and rakes relative to the capital cost of unstirred thickeners should be conducted.
References


Nomenclature

\( a \) exponent in solids pressure expressions
\( A \) cross sectional area of settling column (m\(^2\))
\( A_e \) cross sectional area of electrode (m\(^2\)).
\( B \) overflow flowrate of continuous thickener (m\(^3\)s\(^{-1}\))
\( c \) solids concentration (v/v).
\( c_c \) critical concentration at which \( \phi_i \) is minimum (v/v).
\( c_o \) initial solids concentration and feed concentration to continuous thickener (v/v).
\( c_{s2} \) solids concentration of a characteristic (v/v).
\( c_b \) solids overflow concentration (v/v).
\( c_f \) final base sediment concentration (v/v).
\( c_g \) gel concentration (v/v).
\( c_u \) solids underflow concentration (v/v).
\( c' \) coefficient in Shirato solids pressure expression
\( c_m \) maximum concentration achievable through compaction (v/v).
\( d \) distance between electrodes (m).
\( F \) feed flowrate to continuous thickener (m\(^3\)s\(^{-1}\))
\( g \) acceleration due to gravity (ms\(^{-2}\))
\( h_{0,1,12,4} \) heights of interface (m)
\( k \) permeability (m\(^2\))
\( L \) sediment height from base (m)
\( p \) hydrostatic pressure (Pa)
\( p_b \) coefficient in solids pressure expressions
\( p_e \) excess hydrostatic pressure (Pa)
\( p_s \) solids pressure (Pa)
\( p_{sd} \) transient component of solids pressure (Pa)
\( p_{ss} \) static component of solids pressure (Pa)
\( p_y \) solids yield pressure (Pa)
\( n \) Maxwell's exponent
\( n_3(x_p) \) volume fraction of solids in a size range having an average particle size \( \bar{x}_p \).
\( m \) Richardson & Meikle exponent
\( Q \) flowrate through upflow cell (m\(^3\)s\(^{-1}\))
\( r \) electrical resistance (ohm)
\( S_v \) specific surface (m\(^{-1}\))
\( t \) time (s)
\( u \) solids settling velocity (ms\(^{-1}\))
**Greek**

- \( \varepsilon \): porosity (v/v).
- \( \mu \): viscosity (Pa s).
- \( K \): Kozeny factor.
- \( \rho \): liquid density (kg \( m^3 \)).
- \( \rho_s \): solids density (kg \( m^3 \)).
- \( \bar{\rho} \): mean suspension density (kg \( m^3 \)).
- \( \phi_s \): capacity limiting solids flux (ms\(^{-1}\)).
- \( \phi_a \): solids flux due to gravity (ms\(^{-1}\)).
- \( \phi_u \): total solids flux (ms\(^{-1}\)).
- \( \phi_d \): solids flux due to underflow removal (ms\(^{-1}\)).
- \( \sigma_c \): conductivity of continuous phase (ohm\(^{-1}\)m\(^{-1}\)).
- \( \sigma_d \): "dispersed" (ohm\(^{-1}\)m\(^{-1}\)).
- \( \sigma_m \): "suspension" (ohm\(^{-1}\)m\(^{-1}\)).

**Subscripts**

- \( * \): dimensionless variable.
- \( o \): initial condition.
Appendix A

Schematics and relevant dimensions of experimental apparatus

Electrode heights from the column base (mm)

Batch rig (61.5mm diameter) - 9, 19, 29, 39, 49, 59, 69, 79, 89, 99, 109, 119, 129, 139, 149, 159, 169, 179, 189, 199, 219, 239, 259, 279, 299.

Batch rig (150mm diameter) - 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 230, 280, 300.

Continuous rig* (150mm diameter) - 10, 20, 30, 40, 50, 70, 90, 110, 130, 150, 170, 190, 210, 230, 250, 270, 310, 350, 390, 430, 470.

* denotes the distance to the base of the perspex column, not the bottom of the conical section, see following diagrams
Diagram of continuous rig (1/4 scale)
Diagram of continuous rig's conical base section (1/2 scale)

- perspex tube
- nylon base piece
- 18 mm
- 40 mm
- 15°
- locating holes
- stirrer locating piece
- "O" ring seal
Diagram of scraper / stirrer base section (full scale)

8 mm diameter nylon rods

16 mm

scraper section
Appendix B

Computer program listings

Contents

CHARAC.bas - Qbasic program which generates a file of characteristics for import into a spreadsheet. The characteristics are generated from experimental data in the form of two text files which are inputted at the start of the program.

MODEL.pas - Turbo Pascal program based on the consolidation model outlined in Section 6.0. Using inputted expressions for permeability, solids pressure and specific surface the program uses a finite difference method to model the consolidation process producing characteristics which can be compared with experimentally generated characteristics.
PROGRAM CHARAC
******************
This program generates a file of characteristics - these can be for concentrations above and below that of the initial concentration but lower dilution concentrations must be specified. The program requires that two text files are prepared beforehand - a height file which contains the interface height-time data and a concentration-time file containing the concentrations at all electrode heights at all times. These files should have the same filename preceded by a h or c. If the electrode heights on the rig are different to those given in the DATA section then this section should be revised accordingly.

DECLARE SUB velocity (count60, hei(), concsed(), solcon(), height(), timht(), height2(), time(), correction, count, count2)
DECLARE SUB datainp (timchar(), charlow%, Cchar%, exponent, number, Co, Cinitial%, kc, paf$, paf2$, dato$, dato2$, tch$)
DECLARE SUB fileconc (time(), solcon(), hei(), elecnumber, count, number, Cinitial%, Cfinal%, paf$, dato$)
DECLARE SUB fileht (timht(), height(), count2, paf2$, dato2$)
DECLARE SUB dataout (height(), solcon(), hei(), elecnumber, count, number, Cinitial%, Cfinal%, paf$, dato$)
DECLARE SUB characteristic (concsed(), height2(), count60, time(), solcon(), hei(), charht(), lowtime(), highconc(), lowconc(), hightime(), marker, Cinitial%, Cfinal%, count, count2, Cchar%, gap%)
DECLARE SUB characteristic (concsed(), height2(), count60, time(), solcon(), hei(), charht(), lowtime(), hightime2(), hightime(), lowcon(), highconc(), count, Cinitial%, Cfinal%, Cchar%, elecnumber, gap%)
CLS
DIM time(300), solcon(300, 26), height(300), timht(300), hei(26), timchar(30)
DIM x(6), y(6)

FOR loopy = 1 TO 26 ' convert electrode heights to millimeters
READ hei(loopy)
hei(loopy) = hei(loopy) * 10
NEXT loopy

elecnumber = 25 'number of electrodes

choose between the different size rigs (different electrode positions)
PRINT "input rig diameter in inches (5.5 or 2.5)"
INPUT dia
IF dia > 4 THEN
elecnumber = 21
FOR loopy = 1 TO 22
READ hei(loopy)
hei(loopy) = hei(loopy) * 10
NEXT loopy
ELSE
FOR loopy = 1 TO 22
READ rubish
NEXT loopy
END IF

for high concentration changes it may be advisable to generate characteristics for every other concentration otherwise the characteristic plot becomes very crowded.
PRINT "input gap between characteristics 1 = all, 2 = everyother etc"
INPUT gap%
CALL datainp(timcharO, charlow%, Cchar%, exponent, number, Co, Cinitial%, kc, paf$, paf2$, dato$, dato2$, tech$)
CALL fileconc(time(), solcon(), hei(), elecnumber, count, number, Cinitial%, Cfinal%, paf$, dato$)
CALL fileht(timht(), height(), count2, paf2$, dato$)
PRINT count, Cchar%, Cfinal%
DIM height2(count + 1), lowconc(count + 1), highconc(count + 1), charht(count + 1, Cchar% TO Cfinal%), timelow(Cchar% TO Cfinal%), concsed(count + 1)
DIM hightime2(Cchar% TO Cfinal%), hightime(Cchar% TO Cfinal%), timelow(Cinitial% - 1 TO Cfinal%), count6(count + 1)
CLS
marker = (number + 1) / 2  'number of point either side of regression pt
CALL velocity(count60, heiO, concsedO, solconO, heightO, timhtO, height2O, timeO, correction, count, count2)
CALL characteristic(concsedO, height2O, count60, timeO, solconO, heiO, charhtO, lowtimeO, hightime2O, hightimeO, lowconcO, highconcO, count, Cinitial%, Cfinal%, Cchar%, elecnumber, gap%)
CALL dataout(heightO, timhtO, charhtO, timeO, lowtimeO, highconcO, 10wtimeO, hightimeO, marker, Cinitial%, Cfinal%, count, count2, Cchar%, gap%)
END

DATA 29.85, 27.875, 25.9, 23.9, 21.9, 19.9, 18.9, 17.9, 16.9, 15.9, 14.9, 13.9
DATA 12.9, 11.9, 10.9, 9.9, 8.9, 7.9, 6.9, 5.9, 4.9, 3.9, 2.9, 1.9, 0.9, 0
DATA 30, 28, 23, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0
SUB characteristic (concsedO, height2O, count60, timeO, solconO, heiO, charhtO, lowtimeO, hightime2O, hightimeO, lowconcO, highconcO, count, Cinitial%, Cfinal%, Cchar%, elecnumber, gap%)
' scans each time increment and calculates the height of each characteristic
' concentration at that time. If the concentration exists at two heights
' the lowest height is taken
DIM check(300)
FOR b% = Cchar% TO Cfinal% STEP gap%
bob% = 0
FOR a% = 1 TO count
bob2% = 0
FOR elec% = count6(a%) TO elecnumber
SELECT CASE elec%
CASE count6(a%)
solcon = concsed(a%)
hei = height2(a%)
CASE ELSE
solcon = solcon(a%, elec%)
hei = hei(elec%)
END SELECT
IF solcon > Cchar% / 100 THEN
IF (b% / 100) < solcon(a%, elec% + 1) AND (b% / 100) > solcon THEN
  IF bob2% = 0 THEN bob% = bob% + 1
  bob2% = bob2% + 1
  IF bob% = 1 THEN
    lowtime(b%) = a%
    bob% = a%
  END IF
  IF bob2% = 1 AND check(a%) = 0 THEN
    lowconc(a%) = b%
    check(a%) = 1
  END IF
END IF
END FOR
END FOR
charht(bob%, b%, 1) = hei(elec% + 1) + ((hei - hei(elec% + 1)) / (solcon - solcon(a%, elec% + 1)))
* ((b% / 100) - solcon(a%, elec% + 1)))
charht(bob%, b%, 2) = time(a%)
highht(b%) = bob%
hightime(b%) = a%
hightime2(b%) = a%
hightime(a%) = b%
END IF
NEXT elec%
NEXT a%
NEXT b%
END SUB

SUB datainp (timcharO, charlow%, Cchar%, exponent, number, Co, Cinitial%, kc, paf$, paf2$, dato$, dato2$, tech$)
' the subroutine asks what file you want to generate characteristics for
' and over what concentration range. The paths for the files have been
' preset- see paf$ & paf2$
PRINT "Input number of bits of data to be regressed 3 or 5"
INPUT number
paf$ = "a:\conv\"
PRINT "input filename"
INPUT dato$
paf2$ = "a:\height\"
dato2$ = "h" + dato$
dato$ = "c" + dato$
PRINT "input initial concentration"
INPUT Co
PRINT "Some suspensions exhibit channeling which causes dilution"
PRINT "if you wish to produce characteristics for concentrations"
PRINT "less than the intial then enter the no. of % below Co which"
PRINT "you wish to consider (default = 0)"
INPUT dil%
Cinitial% = INT((Co * 100) + .5)
Cchar% = INT((Co * 100) + .5) + dil%
PRINT "input first sensible characteristic"
INPUT charlow%
PRINT "input time at which lower (dilution) characteristics become valid"
PRINT ";if never enter 10000"
FOR b% = Cchar% TO charlow% - 1
PRINT "time for characteristic", b%
INPUT timchar(b%)
NEXT b%
END SUB

SUB dataout (heightO, timhtO, charhtO, timeO, lowconcO, highconcO, lowtimeO, hightimeO, marker, Cinitial%, Cfinal%, count, count2, Cchar%, gap%)
'generates the text file of the characteristics for import into a spreadsheet
CLS
PRINT "Name of results file : ",
INPUT "", full$
path$ = "c:\temp\"
'path$ = "a:\"
OPEN "o", #1, path$ + full$ + ".prn"

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PRINT #1,
FOR b% = Cchar% TO Cfinal% STEP gap%
FOR a% = lowtime(b%) TO hightime(b%)
  PRINT #1, USING "###.#"; b%;
  PRINT #1, ";
  PRINT #1, USING "####.#"; charht(a%, b%, 2);
  PRINT #1, ";
  PRINT #1, USING "####.#"; charht(a%, b%, 1);
  PRINT #1,
  NEXT a%
PRINT #1,
NEXT b%
FOR a% = 1 TO count2
  PRINT #1, USING "###.#"; timht(a%);
  PRINT #1, USING "####.#"; height(a%);
  PRINT #1,
  NEXT a%
CLOSE
END SUB

SUB fileconc (timeO, solconO, heiO, elecnumber, count, number, Cinitial%, Cfinal%, paf$, dato$)
  'inputs the concentration-time data
  chicha = 1 'counter
  count = 1 'number of time increments
  PRINT paf$ + dato$ + ".pm"
  OPEN "i", #1, paf$ + dato$ + ".pm"
  INPUT #1, time(count)
  FOR i = 1 TO elecnumber
    INPUT #1, solcon(count, i)
  NEXT
  PRINT count
  INPUT # I, time(count)
  LOCATE 1, 1
  PRINT count, time(count)
  FOR i = 1 TO elecnumber
    INPUT #1, solcon(count, i)
  NEXT
  PRINT time(count), solcon(count, 1), solcon(count, elecnumber)
WEND
CLOSE

  'only consider every "numb" time increment - use with long files
numb = 2

WHILE NOT EOF(1)
  IF chicha / numb = INT(chicha / numb) THEN count = count + 1
  chicha = chicha + 1
  PRINT count
  INPUT #1, time(count)
  LOCATE 1, 1
  PRINT count, time(count)
  FOR i = 1 TO elecnumber
    INPUT #1, solcon(count, i)
  NEXT
  PRINT time(count), solcon(count, 1), solcon(count, elecnumber)
WEND
CLOSE

  'calculates concentration at the botttom (h=0)
FOR a% = 1 TO count
  solcon(a%, elecnumber + 1) = solcon(a%, elecnumber) + ((solcon(a%, elecnumber) - solcon(a%,
      elecnumber - 1)) / (hei(elecnumber) - hei(elecnumber - 1)) * (hei(elecnumber + 1) - hei(elecnumber)))
  IF a% >= 2 THEN
    IF solcon(a%, elecnumber + 1) > solcon(a% - 1, elecnumber + 1) THEN
      Cfinal% = INT(solcon(a%, elecnumber + 1) * 100)
    END IF
  END IF
NEXT a%
END SUB
SUB fileht (timht(), height(), count2, paf2$, dato2$)

'input the height file
count2 = 0
PRINT paf2$ + dato2$ + ".prn"
OPEN "i", #1, paf2$ + dato2$ + ".prn"

WHILE NOT EOF(1)
  count2 = count2 + 1
  INPUT #1, timht(count2)
  INPUT #1, height(count2)
  'if height file is not in mm the insert the correct multiplier
  'in the line below
  height(count2) = height(count2)
WEND
CLOSE
END SUB

SUB velocity (count60, heiO, concsedO, solconO, heightO, timhtO, height20, timeO, correction, count, count2)

'calculates the height of the interface at each time increment

FOR a% = 1 TO count
  count3 = count2 + 1
  count4 = 0
  height(count3) = height(count2)
  timht(count3) = time(count)
  DO
    count3 = count3 - 1
  LOOP UNTIL timht(count3) <= time(a%)
  PRINT height(count3), time(a%)
  height2(a%) = height(count3) + ((time(a%) - timht(count3)) * (height(count3) - height(count3 + 1)) / (timht(count3) - timht(count3 + 1)))
  PRINT height2(a%)
  count5 = 0
  'calculates the concentration at the interface by interpolating
  'from the nearest electrodes. If the nearest electrode is less than 8 mm
  'then the next two sets are consider. This allows for flux distortions near
  'the interface
  DO
    count5 = count5 + 1
  LOOP UNTIL hei(count5) <= height2(a%)
  count6(a%) = count5
  correction = (height2(a%) - hei(count5))
  IF correction < 8 THEN
    m = (solcon(a%, count5 + 1) - solcon(a%, count5 + 2)) / (hei(count5 + 1) - hei(count5 + 2))
    solcon(a%, count5) = solcon(a%, count5 + 1) + (m * (hei(count5) - hei(count5 + 1)))
  END IF
  IF solcon(a%, count5) > solcon(a%, count5 + 1) THEN solcon(a%, count5) = solcon(a%, count5 + 1)
  concsed = solcon(a%, count5) + (m * correction)
  IF concsed > solcon(a%, count5) THEN concsed = solcon(a%, count5)
  concsed(a%) = concsed
NEXT a%
END SUB
Program Model;

("*****************Gareth Butt*****************)
("This program solves a non linear parabolic equation.")
("using a three time level finite difference method.")
("The solution gives a complete description of the")
("settling of a suspension having properties as")
("defined by the user. the compiler must be in")
("8087 mode to support the double variable type")
("*****************version 1.1*****************)

uses crt,dos,graph;

type

grid = array[1..3,0..50] of double;
clock = array[1..3] of longint;
high = array[0..50] of double;

var

ldensity,sdensity,left,right,top,bot,count,
graphmode,nfractions,min_iterate,timesave,
length,i3 : integer;
viscosity,kozeny_a,kozeny_b,gravity,newvalue,dw,dt,

rsquared, heightdiff,oldheight,uo,too,pso,amultiply,
kozeny1,po,wo,densconst,ho,co,max_diff,
coeff_pressure,exponent_pressure : double;
spsurface,duration,count2,xold,yold,x2,y2,oldtime : longint;
material_name,filename,path,path2,file2 : string;
ans : shortint;
textfile,chfile : text;
saveresults,compare,material,dilute,whichtest : char;
conc,pl,ps,fun_a_minus,fun_a_plus,func_b : grid;
time : clock;
height,charht : high;
first,firsttime,start : boolean;
oldchar : array[0..50] of longint;
checker : array[0..50] of boolean;

function I2S(Integ:longint):string; {converts integers into strings}
var

s : string;
begin
  str(integ,s);
  I2S:= s;
end;

function r2S(lnteg:real;digit,decimal:integer):string; {converts real data types into strings}
var

s : string;
begin
  str(lnteg:digit:decimal,s);
  r2S:= s;
end;

procedure Titlescreen;

var
graphdriver, errorcode : integer;

begin
  graphdriver := detect;
  Initgraph(graphdriver, graphmode, 'c:\tp6\bgi');
  {Initgraph(graphdriver, graphmode, 'z:\site\tp6\bgi');}
  errorcode := graphresult;
  if errorcode <> grOk then begin
    writeln('Graphics error; ', grapherrorMsg(errorcode));
    writeln('program aborted ... .');
    halt (1);
  end;
  bot := round(getmaxy * 0.3);
  top := round(getmaxy * 0.7);
  right := round(getmaxx * 0.9);
  left := round(getmaxx * 0.1);
  setfillstyle(solidfill, 15);
  rectangle(left, bot, right, top);
  floodfill(getmaxx div 2, getmaxy div 2, 15);
  setcolor(0);
  settextjustify(centertext, centertext);
  settextstyle(defaultfont, horizdir, 2);
  outtextxy(getmaxx div 2, getmaxy div 2, 'CONSOLIDATION MODEL');
  delay(3000);
  restoreCrtmode;
end;

procedure material properties;
begin
  clrscr;
  writeln('CHOOSE YOUR MATERIAL');
  writeln;
  writeln('Choose the material you wish to model- this automatically');
  writeln('presets some variables although you have the option to change');
  writeln('them later. Enter the correct letter');
  writeln('[a] Aragonite [c] calcite [t] talc [n] other material');
  repeat
    readln(material);
    case material of
      'a': begin
        sdensity := 2578; spsurface := 805222; kozeny_a := -80.91; kozeny_b := 41.72;
        exponent_pressure := 0.049661; coeff_pressure := 0.22206;
        material_name := 'aragonite';
      end;
      'c': begin
        sdensity := 2720; spsurface := 313689; kozeny_a := 0; kozeny_b := 20;
        exponent_pressure := 0.108457; coeff_pressure := 0.09498;
        material_name := 'calcite';
      end;
      't': begin
        sdensity := 2690; spsurface := 1870643; kozeny_a := 0; kozeny_b := 14;
        exponent_pressure := 0.134897; coeff_pressure := 0.096428;
        material_name := 'talc';
      end;
      'n': begin
        sdensity := 2690; spsurface := 1875000; kozeny_a := -80.91; kozeny_b := 41.72;
      end;
    end;
  end;
end;
exponent_pressure:=0.044292; coeff_pressure:=0.226574;
writeln('enter material name');
readln(material_name);
end;
end;
until (material='a')or (material='c')or (material='t')or (material='n');

[preset other variables with typical values]
density := 1000; viscosity:= 0.001; gravity := 9.81; dilute := 'n';
repeat
  clrscr;
  ans:=0;
go toxy(25,1);
writeln(Material Properties);
writeln;
writeln([1] Specific surface of solid is - ',spsurface:8,' m-1');
writeln([2] solid density is - ',sdensity:8,' kg/m3');
writeln([3] liquid density is - ',ldensity:8,' kg/m3');
writeln([4] liquid viscosity is - ',viscosity:8:6,' N/m2s');
writeln;
writeln([5] Include dilution effects (preset materials only) - ',dilute);
writeln;
writeln(Pressure and Permeability Expressions');
writeln;
case material of
  'a','c':begin
    writeln('Pressure law is a combination of c =cn P^n and Tillers law');
  end;
  't','n':begin
    writeln('Pressure law is of the form c =cn P^n');
    writeln([6] value of cn is - ',coeff_pressure:8:7);
    writeln([7] value of n is - ',exponent_pressure:8:7);
  end;
case material of
  'a','c','t':begin
    writeln('Kozeny constant is defined by a preset function');
  end;
  'n':begin
    writeln('Kozeny constant has the form K=a*c+b');
    writeln([8] value of a is - ',kozeny_a:8:2);
    writeln([9] value of b is - ',kozeny_b:8:3);
  end;
case ans of
  2:sdensity:=round(newvalue);
  3:ldensity:=round(newvalue);
  4:viscosity:=newvalue;
  6:coeff_pressure:=newvalue;
  7:exponent_pressure:=newvalue;
  8:kozeny_a:=newvalue;

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9:kozeny_b:=newvalue;
end;
end;
if ans=1 then
begin
  writeln('enter new value');
  readln(spsurface);
end;
if ans=5 then
begin
case material of
  't':begin
    repeat
      writeln('Use data from which test [a] talc5 [b] talc7 [c] talc 11');
      readln(whichtest);
      until (whichtest = 'a')or (whichtest='b') or (whichtest='c');
      if dilute='n' then dilute:= 'y' else dilute:= 'y';
    end;
  'a','c','n':begin
    writeln('no preset dilution function');
    delay(3000);
    end;
  end;
end;
until ans=0;
end;

{ function to calculate the Kozeny constant
  { input variable = concentration value
}

function kozconst(var concen:double):double;
var
  con2,con3,con4 :double;
begin
  con2:=exp(ln(concen)*2);
  con3:=exp(ln(concen)*3);
  con4:=exp(ln(concen)*4);
  case material of
    'a':begin
      kozconst:= -57.0599 + concen*1276.76 + con2*-8629.04 + con3*23508.96 +
                   con4*19301.56;
      end;
    'c':begin
      if concen<0.17 then
        begin
          kozconst:=12.497+concen*65.98123;
        end else
        begin
          kozconst:=122355.3+concen*-2780374+2.4e7*con2+9e7*con3+1.3e8*con4;
        end;
      end;
    't':begin
      kozconst:=23.70478+concen*-535.636+con2*7595.751+con3*-
                   42459.7+con4*85458.16;
      end;
    'n':begin
      kozconst:=kozeny_a*concen+kozeny_b;
    end;
  end;
function pressure(concen:double):double;
begin
  case material of
  'a':begin
    if concen<0.301418 then
      begin
        pressure:=exp(ln(concen/0.22206)*(1/0.049661));
      end else
      begin
        pressure:=140*ln((0.306-concen)/(0.306-0.176));
      end;
  end;
  'c':begin
    if concen<0.175693 then
      begin
        pressure:=exp(ln(concen/0.09498)*(1/.108467));
      end else
      begin
        pressure:=ln((0.181-concen)/(0.181-0.045))*89.5;
      end;
  end;
  't':begin
    pressure:=exp(ln(concen/coefCpressure))*(1/exponent_pressure));
    end;
  'n':begin
    pressure:=exp(ln(concen/coefCpressure))*(1/exponent_pressure));
    end;
end;
end;

function concentration(press:double):double;
begin
  case material of
  'a':begin
    if press<470 then
      begin
        concentration:=0.22206*exp(ln(press)*0.049661);
      end else
      begin
        concentration:=0.306-(0.306-0.176)*exp(press/-140);
      end;
  end;
end;

{function to calculate concentration}
{input variable = pressure value }

function concentration(press:double):double;
begin
  case material of
  'a':begin
    if press<470 then
      begin
        concentration:=0.22206*exp(ln(press)*0.049661);
      end else
      begin
        concentration:=0.306-(0.306-0.176)*exp(press/-140);
      end;
  end;
end;
concentration:=0.09498*exp(ln(\text{press}))*0.108467;
end else
begin
concentration:=0.181-(0.181-0.045)*exp(\text{press}/-89.5);
end;
't':begin
concentration:=\text{coefCpressure}\cdot\exp(ln(\text{press})\cdot\text{exponent_pressure});
end;
'n':begin
concentration:=\text{coefCpressure}\cdot\exp(ln(\text{press})\cdot\text{exponent_pressure});
end;
end;

\text{(function to calculate } b(pl^*) \text{ )}
\text{(input variables = concentration + pressure )}

\text{function } \text{functb(} \text{concen, press:double):double;}
\text{var}
\text{derivative :double;}
\text{begin}
\text{case material of}
'a':begin
if press<470 then
begin
\text{derivative:}=0.049661\cdot\text{concen/press};
end else
begin
\text{derivative:}=-(0.306-0.176)/-140\cdot\exp(\text{press}/-140);
end;
end;
'c':begin
if press<210 then
begin
\text{derivative:}=0.108467\cdot\text{concen/press};
end else
begin
\text{derivative:}=-(0.183-0.04)/-65.37\cdot\exp(\text{press}/-65.37);
end;
end;
't':begin
\text{derivative:}=\text{exponent_pressure}\cdot\text{concen/press};
end;
'n':begin
\text{derivative:}=\text{exponent_pressure}\cdot\text{concen/press};
end;
end;
\text{functb:=derivative*po/sqr(concen);}
\text{end;
}

\text{(function to calculate solids pressure )}

\text{function } \text{spressure(} \text{var weightstep:real;xpl:double }):double;}
\text{begin}
\text{spressure := ((wo - (weightstep * wo/\text{fractions}) \cdot \text{densconst}) - (xpl \cdot po);)
end;
function func_a( concen,concen3 :double):double;
begin
func_a:= amultiply * exp(ln(l - concen)*3) / (concen * kozconst(concen3));
end;

function aplushalf(var weightstep:integer;layer:integer):double;
var
avgpl,temp :double;
modwstep :real;

begin
avgpl:=(pl[layer,weightstep]+pl[layer,weightstep+1])/2;
modwstep:=weightstep +0.5;
temp:=spressure(modwstep,avgpl);
aplushalf:=concentration(temp);
end;

function aminushalf(var weightstep:integer;layer:integer):double;
var
avgpl,temp :double;
modwstep :real;

begin
avgpl:=(pl[layer,weightstep]+pl[layer,weightstep-1])/2;
modwstep:=weightstep -0.5;
temp:=spressure(modwstep,avgpl);
aminushalf:=concentration(temp);
end;

procedure initialconstants; {initial constant values}
begin
height[0]:=0;
densconst:= (1 - ldensity / sdensity) * gravity;
wo:= co * ho * sdensity;
po := wo * densconst;
kozeny l:= kozconst(co);
uo:=exp(ln(1 - co)*3) * gravity * (sdensity - ldensity) /kozenyl /co / spsurf/spsurf/viscosity;
too:= ho/uo;
pso:= pressure(co)/po;
amultiply:= kozenyl/exp(ln(1 - co)*3);
end;

{procedure to define the finite difference grid properties}
{and the location of any results files}

procedure finitediffproperties;

var
  ans2 :char;
  check :boolean;

begin
  co:=0.05;ho:=0.33;duration:=30000;nfractions:=50;dt:=0.00015;
  saveresults:='y';path:='c:\model\results\';filename='test'
  timesave:=120;max_diff:=0.00001;
repeat
  rsquared:=2*dt/sqr(1/nfractions);
  clrscr;
  ans:=0;
  gotoxy(25,1);
  writeln('Calculation Parameters');
  writeln;
  writeln(['1'] Initial solids concentration - ',co:8:3,' volume fraction');
  writeln(['2'] Initial suspension height - ',ho:8:6,' m');
  writeln(['3'] Sedimentation duration - ',duration:8,' s');
  writeln;
  writeln('Define finite difference grid');
  writeln;
  writeln(['4'] Column split into ',nfractions: weight fractions (1/dw)');
  writeln(['5'] Time increment (dt) is - ',dt:S:6);
  writeln(' this gives a value for 2k1hA2 of - ',rsquared:9:7);
  writeln(' ***acceptable value is < 1 ****');
  writeln;
  writeln(['6'] Save results - ',saveresults);
  writeln(['7'] filename is [max 6 letters] - ',filename);
  writeln(['8'] path is - ',path);
  writeln(['9'] save every - ',timesave:S: seconds');
  writeln('iterate until height difference is- ',max_diff:9:7,' m');
  writeln;
  writeln('To change any of the above simply type the relevant number [1],');
  writeln('and wait for the prompt, enter 0 to continue');
  readln(ans);
if ans >0 then begin
  case ans of
    1:begin
      writeln('enter the new value');
      readln(co);
      end;
    2:begin
      writeln('enter the new value');
      readln(ho);
      end;
    3:begin
      writeln('enter the new value');
      readln(newvalue);
      duration:=round(newvalue);
      end;
    4:begin
      writeln('enter the new value');
      readln(newvalue);
      nfractions:=round(newvalue);
  end;
end;
5: begin
    writeln('enter the new value');
    readln(dt);
end;

6: if saveresults='y' then saveresults:=n' else saveresults:=y';
7: begin
    writeln('enter new filename');
    readln(filename);
end;

8: begin
    writeln('enter new path');
    readln(path);
end;

9: begin
    writeln('enter the new value');
    readln(timesave);
end;

10: begin
    writeln('enter the new value');
    readln(max_diff);
end;

end;
end;

until (ans=0);

if saveresults <> 'n' then begin
    repeat
        clrscr;
        assign(textfile,path+filename+'.prn ');
        {$I-} reset (textfile); {$I+}
case ioresult of
0: begin
    writeln('file already exists, overwrite y/n');
    repeat
        readln(ans2);
    until (ans2='y') or (ans2='n');
    check:=ans2='y';
end;
2: check:=true;
3: writeln('invalid path try again');
end;
if not check then begin
    writeln('enter new filename');
    readln(filename);
end;
until check;
assign(textfile,path+filename+'.prn');
rewrite(textfile);
writeln(textfile,'Sedimentation Data');
writeln(textfile,'material = ' + material_name);
writeln(textfile,'initial concentration = ' + co:8:3,' volume fraction');
writeln(textfile,'initial height = ' + ho:8:6,' m');
writeln(textfile,'Pressure law c = cm^n');
writeln(textfile,'cn = ' + coeff_pressure:8:6,'  n = ',exponent_pressure:8:6);
writeln(textfile,'Kozeny constant K = a c + b');
writeln(textfile,'a = ',kozeny_a:8:4,'  b = ',kozeny_b:8:4);
writeln(textfile,'Specific surface = ',sp.surface:9);
writeln(textfile,'Densities');
writeln(textfile,'fluid = ',ldensity:6);
writeln(textfile,'particle (contaminant) = ',sdensity:6);
writeln(textfile,'fluid viscosity = ',viscosity:6:5);
close(textfile)
end;
end;

{procedure to assign concentration, pressure and excess pressure}
{values to the first three rows of the grid}

procedure initialgrid;
var
i2 :integer;
begin
for i2:= 0 to nfractions do
begin
p[1,i2):=1 -(i2/nfractions)-pso;
p[2,i2):=p[1,i2);
p[3,i2):=p[1,i2);
end;
for i2:= 0 to nfractions do
begin
ps[1,i2):=((wo - (i2*wo/nfractions)) * densconst) - (p[1,i2]*po);
ps[2,i2):=((wo - (i2*wo/nfractions)) * densconst) - (p[2,i2]*po);
ps[3,i2):=((wo - (i2*wo/nfractions)) * densconst) - (p[3,i2]*po);
conc[1,i2):=concentration(ps[1,i2]);
conc[2,i2):=concentration(ps[2,i2]);
conc[3,i2):=concentration(ps[3,i2]);
func_b[1,i2):=func_b(conc[1,i2],ps[1,i2]);
func_b[2,i2):=func_b(conc[2,i2],ps[2,i2]);
func_b[3,i2):=func_b(conc[3,i2],ps[3,i2]);
if (i2 < nfractions) and (i2>0) then
begin
fun_aplus[1,i2):=func_a(aplushalf(i2,1),aplushalf(i2,3));
fun_aplus[2,i2):=func_a(aplushalf(i2,2),aplushalf(i2,3));
fun_aplus[3,i2):=func_a(aplushalf(i2,3),aplushalf(i2,3));
fun_aminus[1,i2):=func_a(aminushalf(i2,1),aminushalf(i2,3));
fun_aminus[2,i2):=func_a(aminushalf(i2,2),aminushalf(i2,3));
fun_aminus[3,i2):=func_a(aminushalf(i2,3),aminushalf(i2,3));
end;
end;
end;

{finite difference scheme - solution to non-linear}
{parabolic equation (see thesis modelling section )}

procedure finite_scheme;
var
i2 :integer;
procedure dilution;  \{conc-time data - set interface boundary condition\}

var
t1,t2,t3,t4 :double;

begin
t1:=count2*dt*too;t2:=t1*t1;t3:=t1*t2;t4:=t3*t1;

case material of
\text{\texttt{\textapos{}t}}}:
  begin
    case whichtest of
      \text{\texttt{\textapos{}a}}}:
        conc[3,nfractions]:=0.052084+t1*-4.93285e-7+
        t2*4.06841e-11+t3*-2.78030e-15+t4*4.55049e-20;
      \text{\texttt{\textapos{}b}}}:
        if t1<=17500 then
          conc[3,nfractions]:=0.0728;
        end else
          conc[3,nfractions]:=0.276423+t1*-3.81508e-5+
          t2*2.64754e-9+t3*-7.95620e-14+t4*8.49307e-19;
      \text{\texttt{\textapos{}c}}}:
        conc[3,nfractions]:=0.11274+t1*-3.85018e-7+
        t2*6.42277e-11+t3*-4.90857e-16+t4*1.00699e-20;
    end;
  end;
\text{\texttt{\textapos{}a''}},\text{\texttt{\textapos{}c''}},\text{\texttt{\textapos{}n}}}:
  begin
    writeln('no preset dilution function - program halted');
    delay(3000);
    halt(0);
  end;
end;

ps[3,nfractions]:=pressure(conc[3,nfractions]);
pl[3,nfractions]:=ps[3,nfractions]/po;
end;

begin
pl[3,0]:=(pl[2,0]+pl[2,1])/2;
\{variation of excess pressure at base - essential to start iteration\}
pl[3,nfractions]:=-pso; \{boundary condition\}
if dilute='y' then dilution; \{includes a dilution effect - must be preset\}

For i2 := 1 to (nfractions-1) do \{finite difference scheme\}
begin
  i:=i2;
  c1 := 3 * func_b[2,i2] / rsquared;
  c2 := pl[1,i2+1] + pl[2,i2+1] + pl[3,i2+1];
  c3 := pl[1,i2-1] + pl[2,i2-1] + pl[3,i2-1];
  c4 := (pl[1, i2] * c1) + (fun_aplus[2,i2] * (c2 - pl[1, i2] - pl[2, i2]))
  pl[3, i2] := c4/(c1 + fun_aplus[2,i2] + fun_aminus[2,i2]);
  ps[3,i2] := spressure(i,pl[3,i2]);
  conc[3,i2] := concentration(ps[3, i2]);
  func_b[3,i2] := functb(conc[3,i2],ps[3,i2]);
\begin{verbatim}
end;
i:=nfractions;
ps[3, nfractions] := spressure(i, p[3, nfractions]);
i:=0;
ps[3, 0] := spressure(i, p[3, 0]);
conc[3, nfractions] := concentration(ps[3, nfractions]);
conc[3, 0] := concentration(ps[3, 0]);

FOR i2 := 1 TO nfractions do
begin
  if (i2 < nfractions) then
    begin
      fun_aplus[3, i2] := func_a(aplushalf(i2, 3), aplushalf(i2, 3));
      fun_aminus[3, i2] := func_a(aminushalf(i2, 3), aminushalf(i2, 3));
    end;
  concint := (1 / conc[3, i2] + 1 / conc[3, i2 - 1]) / 2;
  height[i2] := concint* w0/nfractions_density + height[i2 - 1];
end;
end;

{ saves the third line of the grid every time the }
{ iteration criteria is satisfied }

procedure outputdata;
var
  nfiles, loopy, loopy2, mark2, mark1 : integer;
  concfile, pressfile, highfile : text;
begin
IF (round(nfractions/25) < 1) THEN
begin
  nfiles := 1;
end else begin
  nfiles := trunc(nfractions/25)+1;
end;
if first then
begin
  FOR loopy := 1 TO nfiles do
  begin
    assign(concfile, path+'c'+filename+i2s(loopy)+'.prn');
    rewrite(concfile);
    close(concfile);
    assign(pressfile, path+'p'+filename+i2s(loopy)+'.prn');
    rewrite(pressfile);
    close(pressfile);
    assign(highfile, path+'h'+filename+'.pm');
    rewrite(highfile);
    close(highfile);
    first:=false;
  end;
end;

FOR loopy := 1 TO nfiles do
begin
  assign(concfile, path+'c'+filename+i2s(loopy)+'.prn');
end;
\end{verbatim}
assign(pressfile,path+'p'+filename+i2s(loopy)+'.prn');
assign(highfile,path+'h'+filename+'.prn');

IF loopy= 1 THEN
begin
append(concfile);
write(concfile,time[3],',');
close(concfile);
append(pressfile);
write(pressfile,time[3],',');
append(highfile);
write(highfile,time[3],',');
writeln(highfile,height[nfractions]:6:4);
close(highfile);
end;

IF loopy < nfiles THEN
begin
mark1:= loopy * 25;
end else
begin
mark1:=nfractions;
end;
IF loopy= 1 THEN
begin
mark2:= 0;
end else
begin
mark2:= (loopy- 1) * 25 + 1;
end;

FOR loopy2 := mark2 TO mark1 do
begin
if loopy2 < mark1 then
begin
append(concfile);
write(concfile,conc[3,loopy2]:6:4,',');
close(concfile);
append(pressfile);
write(pressfile,ps[3,loopy2]:6:1,',');
close(pressfile);
end else
begin
append(concfile);
writeln(concfile,conc[3,loopy2]:6:4);
close(concfile);
append(pressfile);
writeln(pressfile,ps[3,loopy2]:6:1);
close(pressfile);
end;
end;
END;
{set up screen to show characteristic development}
procedure screensetup;
begin
setgraphmode(graphmode);
setcolor(white);

B-19
\texttt{length:=round(0.7*getmaxy);} \\
\texttt{bot:=round(getmaxy *0.1);} \\
\texttt{top:=round(getmaxy *0.1+1.2*length);} \\
\texttt{right:=round(getmaxy *0.3+1.2*length);} \\
\texttt{left:=round(getmaxy *0.3);} \\
\texttt{setviewport(left,bot,right,top,clipon);} \\
\texttt{rectangle(0,0,round(1.2*length),round(1.2*length));} \\
\texttt{left:=round(0.1*length);} \\
\texttt{right:=round(1.1*length);} \\
\texttt{top:=round(1.1*length);} \\
\texttt{bot:=round(0.1*length);} \\
\texttt{line(left,bot,left,top);} \\
\texttt{line(left,top,right,top);} \\
\texttt{settextjustify(centertext,centertext);} \\
\texttt{settextstyle(2,horizdir,S);} \\
\texttt{outtextxy(round(1.2*length/2),round(1.15*length), 'Time (s).');} \\
\texttt{outtextxy(left,round(1.15*length),'0');} \\
\texttt{outtextxy(round(0.05*length),top,'0');} \\
\texttt{outtextxy(round(0.05*length),bot,'2S(ho,5,2));} \\
\texttt{outtextxy(right,round(1.15*length),12S(duration));} \\
\texttt{settextstyle(2,vertdir,5);} \\
\texttt{outtextxy(round(0.05*length),round(1.2*length/2),'height (m)');} \\
\texttt{x2:=left;y2:=top;yold:=top;xold:=left;} \\
\texttt{end;} \\

\{ calculates the location of characteristics at set time \} \\
\{ intervals and then plots the result on the screen \} \\

\textbf{procedure updatescreen;} \\
\{ searches the iterated row for integer concentration values \} \\
\{ and calculates the height of such concentrations. These \} \\
\{ values are then saved to file \} \\

\textbf{procedure characteristic;} \\

\textbf{var} \\
\hspace{1em} conclow,conchigh,b,elec,loops :integer; \\
\hspace{1em} y3 :longint; \\
\hspace{1em} found :boolean; \\
\hspace{1em} charfile:text; \\
\hspace{1em} charac :real; \\

\textbf{begin} \\

\hspace{1em} conclow:=round(conc[3,nfractions]*100+0.5); \\
\hspace{1em} if conclow=trunc(co*100) then conclow:=conclow + 1; \\
\hspace{1em} conchigh:=trunc(conc[3,0]*100); \\
\hspace{1em} if firsttime then begin \\
\hspace{2em} assign(charfile,path+'ch'+filename+'.prn'); \\
\hspace{2em} rewrite(charfile); \\
\hspace{2em} close(charfile); \\
\hspace{2em} for loops:=0 to 50 do \\
\hspace{3em} begin \\
\hspace{4em} checker[loops]:=true; \\
\hspace{3em} end; \\
\hspace{2em} firsttime:=false; \\
\hspace{2em} end;}
FOR b := conclow TO conchigh do
begin
elec:=0;found:=false;
repeat
IF (b / 100 > conc[3,elec+ 1]) AND (b / 100 < conc[3,elec]) THEN
begin
found:=true;
if checker[b] then
begin
oldchar[b]:=top;
checker[b]:=false;
end;
y3:=round(top-charht[b]/ho*length);
line(xold,oldchar[b],x2,y3);
oldchar[b]:=y3;
end;
elec:=elec+1;
until (elec = nfractions) or (found)
end;
assign(charfile,path+'ch'+filename+'.pm');
append( charfile);
write(charfile,time[3],' ,');
FOR b := conclow TO conchigh-1 do
begin
charac:=charht[b ]*1000;
write(charfile,charac:5:1,' ,');
end;
charac:=charht[ conchigh]*1000;
writeln( charfile,charac:5:1);
close(charfile);
end;

begin
x2:=round(left+time[3]/duration*length);
y2:=round((top-height[nfractions])/ho*length);
line(xold,yold,x2,y2);
yold:=y2;
setcolor(black);
outtextxy(round(1.2*3*length/4),round(0.05*length), 'Time ='+i2s(oldtime));
setcolor(white);
outtextxy(round(1.2*3*length/4),round(0.05*length), 'Time ='+i2s(time[3]));
characteristic;
xold:=x2;
end;

{ compares the computer generated characteristics with experimental }
{ characteristics-these are read from a text file which must have the }
{ correct format eg }
{ first characteristic value separation between characteristics }
{ time height (1st characteristic) }
{ time height }
{ time height }
{ -1 -1 }
{ time height (2nd characteristic) }
procedure model_vs_exptl;

var

ans2,ans3 :char;
ok,check :boolean;

begin
ok:=false;check:=false;
compare:='y';path2:='c:\model\data\';file2:='cal7S';
crscr;
gotoxy(1,1);
writeln('Compare model characteristics with experimental y/n?');
repeat
readln(compare);
until (compare='y') or (compare='n');
if compare='y' then
begin
repeat
clrscr;
ans:=0;
writeln('[1] path is - ',path2);
writeln('[2] select choice from - [a] cal5 [b] cal75 [c] cal15');
writeln('[d] arag17 [e] arag20 [f] arag22');
writeln('[g] talc5 [h] talc7 [i] talc11');
writeln;
writeln('[3] load new characteristic file.');
writeln;
writeln('Select either [2] or [3], [1] is optional');
readln(ans);
if ans>0 then begin
case ans of
1:begin
writeln('enter the new path');
readln(path2);
end;
2:begin
writeln('Enter the letter which corresponds to the file you wish to load');
repeat
readln(ans3);
until (ans3 <');
case ans3 of
'a':file2:='cal5';
b':file2:='cal75';
c':file2:='cal15';
d':file2:='arag17';
e':file2:='arag20';
f':file2:='arag22';
g':file2:='talc5';
h':file2:='talc7';
i':file2:='talc11';
end;
ok:=true;
end;
end;
end;
3:begin
   writeln('Enter the new filename');
   readln(file2);
   ok:=true
end;
end;
end;
assign(chfile,path2+file2+'.prn ');
{$I-) reset (chfile); {$I+}
case ioresult of
0: begin {file exists}
   check:=true;
   end;
2: check:=false;
3: begin
   writeln('invalid path try again');
   check:=false
end;
end;
if not check then begin
   writeln;
   writeln("**** The file has not been located, try again ****");
   delay(3000);
end;
until (ans=0) and (ok) and (check);
close(chfile);
end;
end;
begin {main program}
titlescreen;
materialproperties;
finitediffproperties;
initialconstants;
model_vs_exptl;
screensetup;
initialgrid;
time[1]:=-1;time[2]:=0;time[3]:=round(1*dt*too);
oldheight:=0;min_iterate:=5;count2:=1;first:=true;firsttime:=true;
oldtime:=0;[xold]:=left;yold:=top-length;}
settextstyle(2,horizdir,5);
selcolor(white);
outtextxy(round(1.2*3*lengthl4 ),round(O.05*length), 'Time ='+i2s(oldtime));
repeat {loop-continues until the time exceeds 'duration'}
count:=0;
repeat {loop-does 'min_iterate' iterations and then
   continues until heightdiff is less than max_diff}
   finite_scheme;
   heightdiff:=oldheight-height[nfractions];
   count:=count+1;
   oldheight:=height[nfractions];
   until (count>min_iterate) and (heightdiff<max_diff);
   if (time[3]-oldtime>timesave) or (count2=1) then
begin
updatescreen;
if saveresults='y' then outputdata;
oldtime:=time[3];
end;

for i3 := 0 TO nfractions do {moves the grid up one}
begin
pl[1,i3] := pl[2,i3];
ps[1,i3] := ps[2,i3];
conc[1,i3] := conc[2,i3];
fun_aminus[1,i3] := fun_aminus[2,i3];
fun_aplus[1,i3] := fun_aplus[2,i3];
func_b[1,i3] := func_b[2,i3];
pl[2,i3] := pl[3,i3];
ps[2,i3] := ps[3,i3];
conc[2,i3] := conc[3,i3];
fun_aminus[2,i3] := fun_aminus[3,i3];
fun_aplus[2,i3] := fun_aplus[3,i3];
func_b[2,i3] := func_b[3,i3];
end;

count2:=count2+1;
time[1] := round((count2 - 2)*dt*too);
time[2] := round((count2 - 1)*dt*too);
time[3] := round(count2*dt*too);

until time[3] > duration;
setcolor(white);
outtextxy(round(1.2*length/4),round(0.05*length), 'return to finish');

if compare='y' then
begin
reset(chfile);
setcolor(red);
readln(chfile,firstchar,gaps);
settextjustify(righttext,centertext);
outtextxy(round(1.2*0.95*length),round(0.1*length), 'First characteristic ='+r2s(firstchar,4,2));
outtextxy(round(1.2*0.95*length),round(0.15*length), 'gap between ='+r2s(gaps,2,1));
readln(chfile,timechar,htchar);
xold:=round(left+timechar/duration*length);
yold:=round(top-htchar/1000*length);
start:=true;
while not eof(chfile) do
begin
readln(chfile,timechar,htchar);
if timechar< 0 then
begin
start:=false;
readln(chfile,timechar,htchar);
xold:=round(left+timechar/duration*length);


yold:=round(top-htchar/ho/1000*length);
end;

if start then
begin
    x2:=round(left+timechar/duration*length);
    y2:=round(top-htchar/ho/1000*length);
    line(xold,yold,x2,y2);
    xold:=x2; yold:=y2;
    end;
    start:=true;
end;
end;
readln;
outputdata;
end.
Appendix C

Supplementary Experimental data

This appendix contains experimental data which has not been included in the main body of text but which was used to either provide general observations on the settling behaviour of a material over a wide range of initial concentrations or to produce further results such as permeability or solid's pressure values. Additionally the data maybe of use to other researchers in the area of sedimentation.

The series labels on concentration-time plots denote the height of the measurement point in millimetres from the base of the sedimentation column, on the characteristic plots the labels are the value of concentration (volume fraction) that the characteristic represents.
Figure C 1: Concentration-time plot for a ballotini sedimentation ($c_0 = 0.25$)

Figure C 2: Concentration-time plot for a ballotini sedimentation ($c_0 = 0.3$).
Figure C 3: Concentration-time plot for a ballotini sedimentation ($c_o = 0.35$)

Figure C 4: Concentration-time plot for a ballotini sedimentation ($c_o = 0.4$).
Figure C 5: Concentration-time plot for a ballotini sedimentation ($c_0 = 0.45$).

Figure C 6: Concentration-time plot for a ballotini sedimentation ($c_0 = 0.5$)
Figure C.7: Concentration-time plot for aragonite sedimentation ($C_{0} = 0.075$).
Figure C.8: Concentration-time plot for an agarose gel sedimentation (c0 = 0.1).
Figure C.9: Concentration-time plot for an aragonite sedimentation ($c_o = 0.125$).
Figure C10: Concentration-time plot for an argonite sedimentation ($c = 0.15$).
Figure C11: Concentration-time plot for an aragonite sedimentation ($c_0 = 0.175$).
Figure C.12: Concentration-time plot for an aragonite sedimentation ($c_0 = 0.2$)

Concentration, w/v.

Time, s.
Figure C.13: Characteristic plot for an aragonite sedimentation ($c_0 = 0.075$).

Appearance of pock marks

Height, (mm).

Time, (s).
Figure C.14: Characteristic plot for an aragonite sedimentation \( C_p = 0.125 \).
Figure C15: Characteristic plot for an argonite sedimentation (c° = 0.15)
Figure C.16: Characteristic plot for an aragonite sedimentation \( (c = 0.255) \).
Figure C 17: Settling curves for stirred (1 r.p.m.) aragonite sedimentations.
Figure C.18: Concentration-time plot for a stirred (1 r.p.m.) aragonite sedimentation ($C_0 = 0.05$).
Figure C.19: Concentration-time plot for a stirred (1 r.p.m.) aragonite sedimentation ($C_0 = 0.1$).
Figure C.20: Concentration-time plot for a stirred (1 r.p.m.) aragonite sedimentation (C₀ = 0.15).
Figure C.21: Concentration-time plot for a stirred (1 R.P.M.) aerogel mixture (C_0 = 0.2).
Figure C.22: Characteristic plot for a stirred (1 r.p.m.) aragonite sedimentation ($c_0 = 0.05$).

- Height, mm.
- Time, s.
- Lines represent various concentrations: 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18, 0.20, 0.24, 0.28, 0.30, 0.34, 0.38.
Figure C.23: Characteristic plot for a stirred (1 r.p.m.) aragonite sedimentation
(c₀ = 0.1).
Figure C.24: Characteristic plot for a stirred (1 r.p.m.) aragonite sedimentation ($c_0 = 0.2$).
Figure C.25: Concentration-time plot for a lake sedimentation ($c_0 = 0.052$)
Figure C 26: Concentration-time plot for a talc sedimentation \( (c_i = 0.073) \).
Figure C.27: Concentration-time plot for a tale sedimentation (c0 = 0.09)
Figure C 28: Concentration-time plot for a talc sedimentation ($c_0 = 0.127$).
Figure C-30: Concentration-time plot for a stirred (2 r.p.m.) lake sedimentation.
Figure C31: Concentration-time plot for a stirred (2 r.p.m.) tale sedimentation

$C_s = 0.073$
Figure C32: Concentration-time plot for a stirred (2 r.p.m.) lake sedimenteration (c0 = 0.09).
Figure C.34: Characteristic plot for a stirred (2 r.p.m.) tail-sedimentation
(c0 = 0.05).

Height, mm.

Time, s.
Figure C.35: Characteristic plot for a stirred (2 r.p.m.) lake sedimentation

\( c_0 = 0.073 \)
Figure C 36: Concentration-time plot for a stirred (4 r.p.m.) talc sedimentation (c = 0.072).
Figure C.38: Characteristic plot for a slotted (4 rpm) take sedimentation

\( c_0 = 0.073 \)

- Height, mm.
- Time, s.
Figure C.39: Characteristic plot for a stirred (4 rpm) talc sedimentation 
\((c_t = 0.09)\).
Figure C 40: Concentration-time plot for a calcite sedimentation ($c_0 = 0.025$).

Figure C 41: Concentration-time plot for a calcite sedimentation ($c_0 = 0.05$).
Figure C 42: Concentration-time plot for a calcite sedimentation ($c_o = 0.125$).

Figure C 43: Concentration-time plot for a calcite sedimentation ($c_o = 0.15$).
Figure C.44: Characteristic plot for a calcite sedimentation ($C_0 = 0.1$).
Figure C.47: Concentration-time plot for a stirred (1 rpm) calcite sedimentation ($C_o = 0.05$).
Figure C 48: Concentration-time plot for a stirred (1 RPM) calcite sedimentation (c0 = 0.075).

- Concentration, \( \text{wt.\%} \)
- Time, s.

Time intervals: 0, 400, 800, 1200, 1600, 2000, 2400 s.

Concentration values: 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35.
Figure C.49: Concentration-time plot for a stirred (1 r.p.m.) calcite sedimentation ($C^* = 0.15$).
Figure C-50: Characteristic plot for a stirred (1 rpm) calcite sedimentation.

(εo = 0.05).

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Appendix D

This appendix contains hand sketches made during 4 calcite sedimentations. The sketches show the approximate size and location of the volcanoes on the suspension surface at different times during the sedimentation. Two experiments were conducted at an initial concentration of $c_0 = 0.125$ (Cal125 & Cal125a) and two at $c_0 = 0.15$ (Cal15 & Cal15a).
Cal125

Time: 26:00

Time: 30:00

Time: 32:00

Time: 38:00
Time: 39:00
Ca15a

Time: 16:30

Time: 20:30

Time: 24:30

Time: 29:30

Time: 35:00
Solid/liquid separation by sedimentation

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Abstract: The analysis of batch sedimentation tests performed for the purpose of continuous sedimentation vessel design is described. Conventional design techniques employ the concept of settling flux with, under certain conditions, a constitutive equation linking solid concentration and pressure in a unique and time-independent manner.

Experimental studies employing measurement of local concentration and the liquid pressure gradient during sedimentation were used to determine the limits of the conventional design techniques. A 'maximum fluidized concentration' is defined which is the value above which it is difficult to maintain stable flux conditions and is the limit of applicability of conventional design methods based on settling flux. It is shown that the suspended solids contribute towards the liquid pressure gradient and, therefore, to the buoyancy experienced by the settling solids. However, during settlement the liquid pressure gradient reverts to the hydrostatic gradient alone; hence the buoyancy effect is a function of sedimentation time. Additional considerations also suggest that a unique relation between concentration and pressure should only be used as a constituent term in a time-dependent consolidation model. Current research effort includes a suitable method of linking time-dependent consolidation theory, under conditions of extremely low applied pressure, and sedimentation flux analysis.

Keywords: sedimentation, consolidation, buoyancy, flux, thickener, pressure

NOTATION

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>C</td>
<td>solid concentration by volume fraction</td>
</tr>
<tr>
<td>C_max</td>
<td>maximum solid concentration by volume fraction</td>
</tr>
<tr>
<td>h</td>
<td>height of sediment (m)</td>
</tr>
<tr>
<td>k</td>
<td>hydraulic permeability (m²)</td>
</tr>
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<td>n</td>
<td>exponent on equation (8)</td>
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<tr>
<td>P_L</td>
<td>liquid pressure (Pa)</td>
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<tr>
<td>P_s</td>
<td>solids pressure (Pa)</td>
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<tr>
<td>S</td>
<td>specific surface area per unit volume of particles (m²/m³)</td>
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<tr>
<td>t</td>
<td>sedimentation time (s)</td>
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<tr>
<td>T_o</td>
<td>dimensionless consolidation time</td>
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<td>U</td>
<td>propagation velocity of concentration characteristic (m/s)</td>
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<td>U_o</td>
<td>consolidation ratio</td>
</tr>
<tr>
<td>v</td>
<td>solids settling velocity (m/s)</td>
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<td>v_f</td>
<td>fluidizing or up-flow test velocity (m/s)</td>
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<td>v_L</td>
<td>liquid velocity (m/s)</td>
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<td>x</td>
<td>distance from base of column (m)</td>
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Greek

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<tbody>
<tr>
<td>µ</td>
<td>liquid viscosity (Pa s)</td>
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<tr>
<td>ρ</td>
<td>liquid density (kg/m³)</td>
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<tr>
<td>ρ_s</td>
<td>suspension density (kg/m³)</td>
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<tr>
<td>ρ_s</td>
<td>solid density (kg/m³)</td>
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<td>ν</td>
<td>consolidation behaviour index</td>
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1 INTRODUCTION

Gravity sedimentation of suspended material from liquid is an extensively practised form of mechanical separation in many industries including: water and waste-water, minerals and chemicals. The driving potential for the separation is freely available and this has led to it being the first choice in most industrial situations, provided that sedimentation time is not a constraint. It has, however, become a poor relation for research effort compared to other separations perceived to be more modern. The relative scarcity of recent research effort into sedimentation implies that the technology is mature with little more to be gained in understanding. This position is wholly unjustified; there are still some considerable challenges to understanding the sedimentation process. This paper focuses on the less well understood areas of sedimentation, describing some of the laboratory tests.

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that can be used to provide a better awareness of the way in which a material sediments. The relevance of the tests to the design of commercial sedimentation vessels that often employ continuous feed and continuous discharge of clear liquid and settled sediment and are called simply 'thickeners' will also be discussed.

Firstly, the description of the sedimentation process needs to be split into two: 'clarification' at very low solid concentration and 'gravity thickening' or 'hindered settling' at higher values. The hindered settling region needs further subdivision (1), as illustrated in Fig. 1.

There are four distinct regions illustrated in Fig. 1, each region requiring a separate mathematical approach. In the clarification region particles are settling freely without hindrance from each other. The particle sedimentation rate is that of a single particle suspended in a liquid. The mathematical description of this region is well known (2, 3) and so this type of sedimentation will not be further discussed here. When the particle concentration is sufficiently great the particles do not settle individually but en masse. Hence an interface between the clear liquid at the top of the vessel, or supernatant, and the settling suspension can be observed moving slowly down the sedimentation vessel. If the suspension was homogeneous initially, the rate of descent of this interface is constant. In this instance the suspension velocity is a unique function of solid concentration: the greater the concentration the lower the velocity.

In a one-dimensional analysis of sedimentation the concentration at any height is uniform. In a homogeneous suspension the first layer at a concentration slightly greater than the original is formed at the base of the vessel; in the next instant in time the layer next to the base has increased by a further increment in concentration. The position of the layer at the same concentration that was just above the base in the first time increment is now higher up in the vessel. In this way concentrations between the initial value and the final settled value are perceived to move upwards in the sedimentation vessel. Of course, all the solids are moving downwards; it is the position at which a certain concentration of solids exists that is moving upwards, until that position coincides with the settling interface. At this point in time the solid concentration at the interface is greater than that of the initial suspension and the interface settles more slowly. From this point onwards the concentration at the interface is continually increasing; hence the settling rate continually decreases, supporting the belief that the sedimentation rate is a unique function of solid concentration. The perceived upward motion of the intermediate concentrations is often referred to as the concentration 'characteristic', which has a fixed 'propagation' velocity from the origin. The interface settling curve and concentration characteristics for the settling of a suspension of ballotini (glass beads) at an initial solid concentration of 30 per cent by volume in water are illustrated in Fig. 2.

In compression or consolidation settling the suspension has a resistance to motion due to the mechanical strength of the compact. In this instance the suspension does not exist as discrete particles suspended in a liquid, but more like a network of solid particles or porous media with liquid in the voids. These can be displaced by the application of a suitable pressure on the solid network, the analogy with a saturated sponge sometimes being made. Between zone and compression settling a further subdivision into 'channelling settling' is visible in Fig. 1. This is a very ill defined region in which the suspension displays the properties of a connected network but may also show evidence of zones of constant concentration (at the initial value), variable concentration due to the propagation of concentration characteristics and sediment, similar to the description of zone settling. This region has not received much research.

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Fig. 1 Types of sedimentation behaviour according to Fitch (1)

Fig. 2 Solid concentration characteristics during settling of an incompressible solid (ballotini)
attention as the belief is that channelling augments the rate at which liquid is squeezed out of the sediment and into the supernatant. It is argued that channelling is a useful phenomenon for increasing settling rates and a design based on conditions that does not channel will, therefore, be more than adequate if channelling occurs subsequently.

1.1 Analytical techniques

The following mathematical analysis includes the constituent equations to describe the sedimentation in the hindered settling and compression regions described above. All the models are one-dimensional, assuming a concentration gradient in the vertical direction only. The equations are also the most simplified forms that are available. Forms employing substitutions for constituent terms or unusual coordinate systems or rendering variables dimensionless have been avoided in order to reduce the complexity of the analysis. In most instances there are two independent variables: time and height.

The laboratory tests employed to obtain design data for continuous thickeners are usually of the batch type and often employ a simple measuring cylinder. Observation of the settling interface height with time provides the settling velocity at the initial solid concentration. This can then be converted into a settling flux (solid mass per unit time and area). In continuous thickening an additional flux is due to the removal of material from the base of the vessel; this underflow withdrawal flux is usually added to the batch settling flux to give a composite value. The underflow withdrawal flux depends on how the process is operated whereas the batch settling flux is a property of the solids; hence most of the laboratory and mathematical analytical techniques investigate the batch settling velocity or flux. The following analysis refers, therefore, to the batch settling tests.

The mathematical description of batch sedimentation has been given by many research workers. The usual approach is to consider a material and a force, or momentum, balance. A material balance on the solids settling in a differential layer provides (4)

\[ v = \frac{1}{C} \int \frac{\partial C}{\partial t} \, dx \]  

(1)

where \( C \) is the solid concentration of the differential layer in volume fraction terms, \( v \) is the solids settling velocity, \( x \) is position and \( t \) is settling time. A force balance (5) on the solids, neglecting inertial terms (6), provides

\[ 0 = C(p_s - p_m)g - CF_a - \frac{\partial P_s}{\partial x} \]  

(2)

where \( CF_a \) is the liquid drag force on the solids in the differential layer, \( \frac{\partial P_s}{\partial x} \) is the solids stress gradient and \( p_s \) and \( p_m \) are the densities of the solids and suspending medium respectively. There is some controversy over the density of the suspending medium: whether to use the density of the liquid or that of the mean suspension density including the solids present. This will be considered later; for the present moment the suspending medium density will be taken to refer to either option. Using Darcy’s law for the dynamic liquid pressure gradient \( \frac{\partial P_s}{\partial x} \) and combining with a force balance on the liquid gives

\[ CF_a = \frac{\partial P_s}{\partial x} = -\mu \frac{k (1 - C)(v_L - v)}{k} \]  

(3)

where \( v_L \) is the liquid velocity, \( \mu \) is the viscosity and \( k \) is the hydraulic permeability. Combining equations (2) and (3) gives

\[ \frac{\partial P_s}{\partial x} = C(p_s - p_m)g + \mu \frac{k (1 - C)(v_L - v)}{k} \]  

(4)

Now, in batch settling the combined continuity equations for the solid and liquid in a layer is

\[ Cv + (1 - C)v_L = 0 \]

Therefore,

\[ (1 - C)(v_L - v) = - v \]

Hence, equation (4) can be simplified to

\[ \frac{\partial P_s}{\partial x} = C(p_s - p_m)g - \mu \frac{k v}{k} \]  

(5)

Equation (5) considers only three forces to be relevant: the buoyed weight of the solids in the layer, the liquid drag as a result of particle layer motion and the solids stress gradient which can transmit particle weight by point contact through a network of contacts. When fully settled only the weight and stress term are appropriate.

Equation (5) is relevant to the analysis of batch sedimentation which was qualitatively described earlier. It is also relevant to another experimental technique based on the ‘up-flow’ test in which liquid is passed upwards through a porous plate and the otherwise settling suspension. During the up-flow test, or fluidization, the intention is for the solid concentration to be constant throughout the vessel height; thus no solids stress gradient can exist. The only relevant forces in this instance are buoyed weight and liquid drag.

In batch sedimentation the solids material balance [equation (1)], can be rearranged as follows:

\[ v = \frac{1}{C} \int \frac{\partial C}{\partial t} \, dx = \frac{1}{C} \int \frac{\partial P_s}{\partial x} \, dx \]  

(6)

using the chain rule. The term \( \frac{\partial x}{\partial t} \) is the velocity at which a layer of constant concentration propagates upwards within the batch sedimentation vessel, i.e. the velocity of the solid characteristic. These are illustrated in Fig. 2. Equation (6) can be rearranged to provide an analytical expression for the characteristic velocity
If the solid settling velocity is a unique function of concentration, equation (7) becomes a full differential. The product of settling velocity and concentration is often called the batch settling flux and has the units of velocity. Further multiplication by solid density provides the settling flux in terms of the more usual units: mass of solids settling per unit area and time. However, solid density is independent of solid concentration and is, therefore, conventionally not used in the flux expression. The propagation velocity of the characteristic is, therefore, the differential of the batch flux plotted against the solid concentration, according to equation (7). Hence, if it is possible to obtain a unique batch settling flux curve by observing the subsidence of the suspension/supernatant liquid interface with time, then the propagation velocities of the solid characteristics can be determined. The height of a layer at any concentration between that of the initial and final values at any instant in time therefore results, i.e. the settling concentration profile at any time can be predicted. This situation represents the simplest form of sedimentation analysis that is valid for incompressible materials ($\partial P_s/\partial x = 0$).

For compressible sediments there are several equations that relate the solid concentration to the solid pressure, such as

$$C = C_0 P_s^n$$  \hspace{1cm} (8)

where $C_0$ and $n$ are empirical constants. The chain rule can be applied at any instant in time to provide

$$\frac{dC}{dx} = \frac{dC}{dP_s} \frac{dP_s}{dx}$$

Hence, with compressible materials the concentration gradient may be obtained from equation (5) and the differential form of equation (8), provided the solid settling velocity can be estimated or determined by iteration. The solid concentration profile then follows by integration.

During sedimentation the mean suspension density can be obtained from the component densities and concentrations:

$$\rho_s = C \rho_s + (1 - C) \rho$$  \hspace{1cm} (9)

Hence, the solid density minus the mean suspension density is as follows:

$$\rho_s - \rho_m = (1 - C)(\rho_s - \rho)$$  \hspace{1cm} (10)

The term $(1 - C)$ can be thought of as the buoyancy correction term on the density difference. A brief discussion on this controversial topic follows. At low solid concentration the buoyancy correction is simply that of the density difference between the solids and liquid. However, at higher values of $C$ the solids may be fully suspended in the liquid. The suspended solids will, therefore, contribute to the hydrostatic liquid pressure gradient which is the force giving rise to buoyancy. If the hydrostatic liquid pressure gradient is increased the buoyancy force will also increase. It is this principle that enables the hydrometer and dense medium separation in mineral processing to function. Hence, the buoyancy term should be the solid density minus the suspension density, or the buoyancy correction term $(1 - C)$ is required for the difference between the solid and liquid densities. However, when the solids are fully settled they will no longer contribute to the liquid pressure gradient and the buoyancy correction term becomes unity. Between being fully suspended and fully settled the buoyancy correction term must have some finite, and fractional value. In the literature most mathematical analyses of sedimentation use a buoyancy correction term of unity. The following experimental work includes some investigation into the buoyancy correction term.

One means of investigating how the suspended solids contribute to the liquid pressure gradient is to measure the hydrostatic pressure difference between two locations in a suspension. The ‘excess’ liquid pressure is the pressure in excess of the pure hydrostatic head, and is a measure of the amount of suspended solids present. It is derived from equation (9) and the static component of Bernoulli’s equation. The excess liquid pressure ($\Delta P_e$) is, therefore,

$$\Delta P_e = \Delta h g C (\rho_s - \rho)$$  \hspace{1cm} (11)

where $\Delta h$ is the distance between the two pressure transducers. The excess hydrostatic pressure should become less significant as solids form a network that can transmit particle weight, i.e. compression becomes important.

A means of predicting the hydraulic permeability of a suspension and how it varies with solid concentration would enable an analytical solution of equation (5). Two of the more common permeability models are due to Kozeny–Carman [for moving beds the Kozeny constant is 3.36 (7)]

$$k = \frac{(1 - C)^3}{3.36C^2 S^2}$$  \hspace{1cm} (12)

and Happel and Brenner (8)

$$k = \frac{2 - 3C^{1/3} + 3C^{5/3} - 2C^2}{3 + 2C^{5/3} \frac{3}{CS^2}}$$  \hspace{1cm} (13)

where $S$ is the specific surface area per unit volume of the solids making up the size distributed material. The experimental procedures enabled permeability to be measured under conditions when a transient solids stress gradient may be present, and in the absence of such a stress. The procedures are described below.

Finally, techniques from the consolidation analysis of soils (9) have found some application in solid/liquid separation (10, 11). There are three significant constituent equations. Firstly, the dimensionless consolidation...
ratio \((U_c)\) is

\[ U_c = \frac{h_1 - h}{h_1 - h_{\infty}} \]  

(14)

where \(h_1\) is the height at which consolidation of a sediment becomes significant, \(h\) is the sediment height and \(h_{\infty}\) is the final settled height. The dimensionless consolidation time \((T_c)\) is (12)

\[ T_c = \frac{\nu_f}{h_0} \]  

(15)

where \(\nu_f\) is the fluidization velocity at the feed concentration and \(h_0\) is the initial height of the sludge. The Terzaghi constitutive equation linking the consolidation ratio and the consolidation time is

\[ U_c = \frac{\sqrt{4T_c / \pi}}{[1 + (4T_c / \pi)]^{1.5}} \]  

(16)

where \(\nu\) is the consolidation behaviour index, which is often assumed to have a constant value of 2.85 (11).

2 EXPERIMENTAL

The solid materials reported in this study were ballotini, calcium carbonate in the crystal form of aragonite and talc. The true solid densities were 2923, 2590 and 2690 kg/m\(^3\) and the particle diameters that have the same specific surface area per unit volume as the full size distribution were 40, 7.6 and 4.5 \(\mu\)m respectively.

Two experimental sedimentation techniques are briefly described: batch tests in vessels 350 mm high and an 'up-flow' test in a vessel 1 m high. Two sedimentation vessels were used; the internal diameters were 65 and 150 mm. These were used to check on the effect of wall support forces; the results provided here are only those that showed insignificant wall support. The up-flow test is a liquid fluidization, in which the intention is to maintain a constant concentration throughout the vessel by pumping clean liquid upwards through a porous plate at the bottom of the vessel; the 65 mm diameter vessel was used in these tests. The local solid concentration during both batch sedimentation and up-flow testing was recorded by means of pairs of electrodes positioned at 10 mm intervals on all the vessels employed. The local electrical resistance was measured using a square wave of 2000 Hz frequency and converted into a solid concentration via a calibration. Scanning the columns took approximately 6 seconds and the data were stored on a computer. The experimental equipment is illustrated schematically in Fig. 3. Further details of the equipment can be found elsewhere (13).

A third experimental technique involved the use of two pressure transducers positioned within the sedimentation column, in order to investigate liquid pressure gradients. Two very sensitive diaphragm pressure transducers were used. Both faced downwards to avoid material settling on to the face of the transducer. The bottom transducer was positioned 48 mm from the base of the column and the top transducer was 289 and later 252 mm from the base. In all the tests it was the difference between the pressures read by these two transducers that is reported. External factors such as suspension height and atmospheric pressure are not relevant as they are assumed to have a similar effect on both transducers. The pressure tests were conducted in two ways: mixing known masses of solids into suspension and measuring the pressure difference between the transducers immediately, and a similar technique followed by continuing the measurements during the ensuing sedimentation until all movement had ceased. In the latter case the concentration profile of solids in the vessel was also measured, using the technique described earlier. In all instances the pressure transducers registered the liquid pressure in the vessel which was, at first, greater than the pure hydrostatic head due to the presence of suspended solids.

3 RESULTS AND DISCUSSION

The batch sedimentation rig employing determination of local solid concentration was used in the production of Fig. 2. The material settled was ballotini. This material behaves in a highly incompressible manner and displays classic zone settling behaviour. A variable concentration zone, below that of the initial solid concentration zone of just over 29 per cent by volume, was measured with characteristics emanating from the origin towards the settling interface. The characteristics were close together and only six of those measured are recorded in Fig. 2. At approximately 300 seconds the settling interface slows slightly, coinciding with the first characteristic (not shown in Fig. 2) reaching the interface. Setting continued until the sediment reached a uniform concentration of 53–54 per cent by volume. At any instant in time a concentration profile, giving the local

![Fig. 3 Schematic diagram of sedimentation equipment](image-url)
concentration as a function of height, can be drawn as illustrated in Fig. 4.

The incompressible nature of this material is evident from the way in which the sediment builds up. The sediment has a consistent concentration, increasing only in height at each time scan reported. The final sediment concentration is consistent with a value obtained from the random packing of a spherical material. The aragonite, however, settled in a considerably different fashion (see Fig. 5).

The aragonite sediment increased in concentration at each recorded time, eventually reaching a value of 30 per cent by volume, as well as increasing in depth. This type of behaviour might be assumed to be consistent with sedimentation in a compressible way. Finite time is required before sufficient solid mass exists in network contact to squeeze the water trapped in the voids of the sediment below. The compressible nature of this material was investigated using the equipment incorporating pressure transducers. The excess hydrostatic pressure head was calculated at a range of solid concentrations, in accordance with equation (11). The results are plotted in Fig. 6, together with the experimental measurements. The measured excess pressures match the predicted values using the true solid density of aragonite in equation (11) for all concentrations. Thus all the solid weight appears to be supported by the liquid at all concentrations investigated, i.e. there is no evidence of a solid network, or compression, at concentrations up to 30 per cent by volume. This result indicates that aragonite should sediment in an incompressible manner similar to that of the ballotini. Clearly, comparison of Figs 4 and 5 shows that this is not immediately apparent.

Figure 6 supports the concept that suspended solids add to the liquid pressure (hence the liquid pressure gradient and buoyancy) but does not support the concept of a unique concentration at which compressive forces become significant.

If the solids stress gradient is negligible (i.e. incompressible settling) and the concentration, densities, viscosity and settling velocity are known then equation (5) can be used to calculate the permeability. A series of batch tests at known initial concentrations were undertaken with the aragonite and the settling velocities observed. The permeability determined via equation (5) is given in Fig. 7, together with the two permeability models represented by equations (12) and (13). Also shown in Fig. 7 are the permeabilities determined from the up-flow tests. During the up-flow test the fluidization

![Fig. 4 Solid concentration profile during sedimentation of an incompressible solid (ballotini)](image)

![Fig. 5 Aragonite solid concentration profile during sedimentation](image)

![Fig. 6 Buoyancy test data for aragonite suspensions—excess hydrostatic liquid pressure as a function of suspended solid concentration](image)

![Fig. 7 Hydraulic permeability determined during aragonite sedimentations](image)
ensured that there were no solids concentration or stress gradients. The analysis of the up-flow test data also employs equation (5), but with a greater degree of certainty about the absence of a stress gradient.

The uniformity of the solid concentration during the up-flow test was confirmed by the local concentration data from the electrically scanned vessels. It was not possible to run the up-flow test with aragonite concentrations greater than 7.6 per cent by volume as the fluidized column collapsed; i.e. the superficial velocity to maintain concentrations of more than 7.6 per cent by volume was less than the apparent minimum fluidizing velocity. For solid concentrations greater than 5 per cent by volume, the solids settling velocities determined from the batch sedimentations were slightly less than the superficial velocity during the up-flow test. Hence, the permeabilities calculated from the up-flow tests are slightly greater in this region. This experimental data supports the argument that a solids stress does exist at concentrations in excess of 5 per cent by volume. Thus the experimental evidence from Figs 5 and 7 indicates significant compression. The evidence from Fig. 6, however, indicates that no significant solids network exists, at least at the start of the sedimentation, at concentrations up to 30 per cent by volume. Also shown in Fig. 7 are the permeability models represented by equations (12) and (13), which do not appear to fit the experimental data. However, the batch sedimentation permeabilities are close to the Kozeny–Carman model at low solid concentrations, and approaches the Happel and Brenner model at higher values not illustrated in Fig. 6.

The conventional method of estimating a continuous thickener's area is from a graphical construction on a batch flux curve. The limiting flux value comes from the intercept, on the flux axis, of a line drawn as a tangent to the batch flux curve and going through the underflow solid concentration. The product of this limiting flux and the thickener area must be equal to the product of the volumetric feed rate to the thickener and the feed volume fraction concentration, assuming that all the solids leave the thickener by the underflow. The batch flux curve for aragonite is shown in Fig. 8. The data were based on the batch sedimentations of known initial solid content and measured settling velocity. The batch flux curve can be used to infer the characteristic velocities by means of differentiation, in accordance with equation (7). Thus an illustration such as Fig. 2 can be inferred from the data provided in Fig. 8.

From the point of view of equipment design the important part of the batch flux curve is that illustrated to the right of the minimum fluidized velocity, as the tangent to the batch flux curve required to fix the limiting thickener flux is most likely to be drawn in this region. Industrial continuous thickeners have a reputation for being difficult to design and to control in operation if they are operating close to their limiting flux capacity. Some of the reasons behind this may be seen by reference to Fig. 8. The terms minimum and maximum 'fluidized' velocity and concentration, respectively, are used. These values are not the same as those required to achieve fluidization; for this material the minimum 'fluidizing' velocity would be the value required to fluidize solids at approximately 29 per cent solids (see Fig. 5). However, the velocity-concentration curve is very shallow until a concentration of 8 per cent solids. Thus it is extremely difficult, or impossible, to control the fluidization between 8 and 29 per cent solids; a small change in velocity considerably alters the bed concentration. This was observed in the up-flow tests. Thus the maximum 'fluidized' concentration is the value that provides stable conditions of fluidization or sedimentation. The up-flow velocity at this concentration is the minimum 'fluidized' velocity. The lack of bed stability is not immediately apparent from looking at the batch flux curve alone; the settling velocity curve is important for this assessment.

The maximum fluidized concentration is the point at which compressive forces become significant and often coincides with the point of inflection on the batch flux curve. The maximum fluidized concentration also represents the maximum point at which a continuous thickener can be designed in the conventional manner employing the flux curve. In Fig. 8 a tangent to the batch flux curve going through the maximum fluidized concentration provides an underflow concentration of 12 per cent by volume. It may be possible to operate a thickener to achieve a greater concentration than this, but the operating conditions should not be obtained from the conventional flux curve as the batch settling flux at these concentrations is time dependent.

The maximum fluidized concentration for the highly compressible suspensions formed from the mineral talc is illustrated in Fig. 9. Four up-flow liquid rates were employed: the suspended bed concentration reached 2.5 per cent by volume as the flowrate was decreased and then collapsed into a non-fluidized form. Hence for talc suspensions compressive forces and time-dependent fluxes are important from a very low solid concentration.
Talc suspensions were also used to further investigate the apparent lack of agreement between the expected and observed behaviour illustrated in Fig. 6. Again a series of experiments at various initial solid concentrations employing the liquid pressure measurement indicated that there was no significant variation between the excess hydrostatic head given by equation (11) and that measured, despite the apparent maximum fluidized concentration for this material being 2.5 per cent by volume. A further sedimentation test using an initial concentration of 17.7 per cent by volume, including both electrical scanning for solid concentration and liquid pressure measurement, was conducted. The sedimentation was, however, allowed to proceed to completion. At the end of the sedimentation both pressure transducers were still resident in the sediment. The result of the measured excess hydrostatic pressure and the observed interface settling height are illustrated in Fig. 10. There is no obvious relation between the interface settling curve and the pressure curve, and it should be remembered that the pressure curve is based on the liquid pressure difference between two points within the suspension. The lack of agreement between excess hydrostatic pressure and compact deformation, as provided by settling height, has been noted before (14), and will lead to serious errors in attempts to infer local concentration from pressure measurement.

The initial excess hydrostatic pressure is within 5 per cent (30 Pa below) of that predicted by equation (11). After 1.5 days, at the end of the sedimentation the excess hydrostatic pressure was 30 Pa below the expected value of zero. The closeness with which the pressure curve approaches the expected excess hydrostatic pressure of zero, while residing in the fully settled sediment, confirms that the pressure transducers were measuring the liquid pressure alone and not the combined liquid and solid pressures. The liquid pressure variation is in agreement with the earlier discussion just prior to equation (11). The concentration profile during the sedimentation was substantially constant throughout, increasing from an initial concentration of 17.7 per cent to a final concentration of just over 22 per cent solids by volume. Thus, during the sedimentation the vessel contained concentrations between 17.7 and 22 per cent by volume but the excess hydrostatic pressure was not provided by equation (11) for these intermediate concentrations. Hence equation (11) is only valid for fully suspended materials or for compressible suspensions prior to settlement.

During the sedimentation of compressible materials both the liquid and solid pressures are dependent on the treatment history of the compressible compact; i.e. a single pressure value cannot be defined for a given solid concentration. The data illustrated in Fig. 10 can be used to illustrate this point: after 59 000 seconds the concentration of the compact was approximately 20 per cent by volume and the excess liquid pressure was approximately 300 Pa. If a suspension at an initial concentration of 20 per cent by volume was settled the excess liquid pressure would be 677 Pa prior to the consolidation period beginning. The variable nature of the pressures pertaining during sedimentation imply that equation (8), which states that the concentration is a unique function of the applied pressure, can only be true under certain circumstances, i.e. when there is negligible resistance to that concentration being formed. Otherwise, the concentration resulting from an applied load is both a function of pressure and time in a manner similar to conventional consolidation theory.

Conventional consolidation theory, the Terzaghi–Peck model, was provided in equations (14) to (16). The model relies essentially on establishing the concentrations at which consolidation becomes significant and at which no further consolidation with time occurs. The concentration that exists in a consolidating compact is then a function of the consolidation time and 'consolidation constants', giving rise to values between these two concentration limits.

Some research workers have combined consolidation and sedimentation models (14, 15). Most of these models have been applied to suspensions of very fine particles, forming highly compressible compacts that are
normally associated with consolidation theory. By contrast the aragonite data reported here appear to display significant sedimentation as well as consolidation behaviour under conditions of very low applied stress, i.e. overlying solid weight. Published consolidation constants for aragonite (11), obtained under compressive pressures of 2–10 bar, have been used as a first approximation to assess the significance of the consolidation of aragonite. During aragonite sedimentation, compressive pressures of 100–1000 Pa are typical, but consolidation times of up to 10 minutes still result from the model using the published consolidation constants. Hence, when dealing with inorganic precipitates that are in themselves incompressible a significant part of the settling process is due to the time consolidation of the compact rather than the sedimentation of the particles; i.e. the sediment concentration does not instantly jump to the equilibrium concentration that will be ultimately achieved in accordance with equation (8).

The greatest challenge in sedimentation research at present is in an appropriate method to link the classical consolidation and sedimentation flux methods in a unified model to describe the time-dependent behaviour of slightly compressible sediments. Associated with this is the time-dependent nature of solid settling velocity and batch flux. This facet affects the method of solution of equation (5) and the conventional method of designing thickeners from the batch settling flux curve.

4 CONCLUSIONS

The established method of describing thickener performance is by means of the batch solids flux curve, which is augmented by the underflow withdrawal flux line in continuous thickeners. It is not immediately apparent from the batch flux curve whether operating problems, such as maintaining a stable bed concentration, will be encountered in continuous thickeners. The solids settling velocity plotted against the solid concentration curve is a far better indicator of potential control problems. The up-flow test is also useful in this context as the solid concentration in the vessel is uniform during the test until bed collapse occurs. The collapse is very noticeable if the fluidizing velocity is reduced in very small increments until a massive change in bed height is given by a small change in liquid velocity. This test helps to identify the 'maximum fluidized concentration', which is the concentration above which it becomes difficult or impossible to maintain equilibrium conditions. The conventional method of calculating the area of a thickener from the batch flux curve should only be used for critical concentrations at, or below, the maximum fluidized value. At concentrations above the maximum fluidized value the batch flux is unstable and time dependent.

When the fluxes and velocities are time-dependent variables a simple constitutive relation between concentration and pressure is no longer valid. This situation is similar to that of conventional consolidation theory. The consolidation pressures are extremely small, typically of the order of 1000 Pa, and significant time may be required in order to compress the compact to its equilibrium concentration. For the aragonite sedimentation illustrated consolidation times of 10 minutes were calculated using the Terzaghi–Peck model. The consolidation time is distinct from the sedimentation time: the former is due to the reduction in excess liquid pore pressure in the surrounding porous medium, the latter is governed only by the local conditions of concentration and, therefore, buoyed weight and permeability and hence liquid drag. However, both sedimentation and consolidation are concurrent processes and it is impossible to separate them without recourse to careful analytical techniques. Furthermore, sedimentation behaviour is linked to consolidation performance through the buoyancy correction term in the basic force balance. When fully suspended all the particle weight contributes to the liquid pressure gradient and, therefore, the suspension buoyancy. When fully settled and consolidated the particle weight acts through point contact on the surrounding particles and the buoyancy is simply due to the presence of liquid within the pores. Between these two extremes the particle weight must contribute by some fraction to the net hydrostatic liquid pore pressure and, therefore, buoyancy effect of the suspension. The excess hydrostatic liquid pore pressure during the late sedimentations demonstrated that this pressure did not simply coincide with solids movement; i.e. there was a significant time difference between movement and consequent liquid pressure reading.

Current research into sedimentation fundamentals includes coupling consolidation theory with conventional batch sedimentation theory, with particular reference to materials that would not normally be regarded as demonstrating significant consolidation behaviour, such as inorganic and incompressible precipitates, and adapting the models to facilitate continuous thickener design or control including these insights.

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AN EXPERIMENTAL STUDY OF CHANNELLING AND SOLID CONCENTRATION DURING THE BATCH SEDIMENTATION OF CALCITE SUSPENSIONS

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During gravity sedimentation of finely divided precipitates and flocculated materials liquid channels free of solids may be observed within the suspension or sediment. Some considerable time may pass before the top of the channels reaches the batch settling suspension/supernatant interface. During this time fronts of increasing solid concentration, or characteristics, propagate from the base of the batch settling column. Solid concentration measurements and observations during the sedimentation of calcite suspensions showed that the top of the channel region coincided with a solid concentration characteristic. The solid concentration of this characteristic represented the first significant increase in concentration over that of the original suspension, and not a unique characteristic value, when performing sedimentations at different starting concentrations. The region above the top of the channels diluted, and this increased the suspension/supernatant interface velocity. Further rapid acceleration occurred when the top of the channels reached the interface, prior to the consolidation period when the interface slowed and compression resulted in a maximum sediment concentration of 18% by volume. Stirring the suspension at a speed of 1 rpm completely suppressed the channels and, under these conditions, the maximum sediment concentration was 34% by volume. The sedimentation velocity and, therefore, flux at the interface was up to 26% greater in the presence of channels. However, for concentrations between 10 and 33% v/v stirring improved the batch sedimentation flux. Hence, if the flux limiting concentration during continuous thickening of this material is between 10 and 33% by volume, conditions that do not promote channels should be employed for maximum settling capacity. Increased underflow concentration would also be possible.

Keywords: sedimentation; channeling; compression; stirring

INTRODUCTION

Batch sedimentation tests are often performed when information on the settlement of a suspension is needed for the design of continuous gravity thickeners, or simply as a means to investigate sedimentation for process optimization, control, knowledge, etc. The conventional method is to mix the suspension in a vessel such as a measuring cylinder and, if the concentration is sufficient, to observe the rate of descent of the suspension/supernatant interface. The height of the interface is plotted against sedimentation time and various conclusions can be drawn from the data, such as the type of sedimentation pertaining and the solid settling flux at concentrations between that of the feed suspension and that of the settled sediment. The classical mathematical description of the batch settlement of discrete particles, i.e. zone settling, is due to Kynch, and has been revised by several workers to take into account the effect of the rising sediment in which the particles are in compression.

In many instances a sedimentation may originally be of one type, such as free settling or clarification, and as the concentration of the settling suspension increases another type of sedimentation may become prevalent. This is illustrated in Fitch's 'paragenesis' figure, reproduced in Figure 1. The mathematical description of sedimentation under conditions of zone settling and compression, in the absence of channels, is reasonably well advanced. However, most suspensions of interest to the process industries exhibit behaviour which lies in the region between zone settling and compression, which is a region where suspensions display channel formation and compression.

Channels have previously been investigated by means of observations during batch settling tests. Such observations have shown that channels require an appreciable time to form, and often disappear towards the end of the batch test. During the period in which channels are found at the surface of the settling suspension, volcanoes may be observed that mark the top of the channel. An illustration of a sedimentation displaying channelling with surface volcanoes is given in Figure 2. It is believed that the volcano is formed by solids that fall into a channel and then become discharged from the
suspension surface. The channel provides a means by which the liquid in the suspension may rise towards the supernatant without having to pass through the porous media constituting the settling suspension. Thus the rate of descent of the suspension/supernatant interface often increases rapidly when channels occur at the interface. Under such circumstances the interface settling curve is no longer linear, as is observed for a significant period in time when sedimenting under zone settling conditions.

If channelling increases the settling rate of the interface, the solid settling flux (mass per unit area and time) must also be enhanced. This important consequence has led to the conclusion that industrial thickeners, which are often continuous rather than batch in operation, may be run more intensively if channelling can be promoted. A new style of continuous thickener in which channelling is encouraged has resulted.

The formation of channels has been attributed to many different effects. Glasrud observed the settling behaviour of iron oxide in mineral oil using a video camera with high magnification lenses. Channelling occurred initially in the top segment of the suspension and propagated downwards slowly. The channels were formed by rising air bubbles. When degassed the settling rates were significantly retarded. Harris reported the settling of dilute phosphatic slimes in which coarser particles had been added to hasten the settling rate. The coarser particles caused tears in the suspension which subsequently developed into channels. In both of these works channelling was artificially induced, but the results demonstrate that the suspensions investigated exhibited sufficient structure to maintain a channelling environment after the removal of a perturbing phase.

Random channelling, i.e. in the absence of materials to initiate channel formation, has been observed with many homogeneous materials and systems. Calcium carbonate suspensions were investigated by Dell, channels were observed initially in the lower regions of the suspension, followed by a period in which they propagated upwards until they met the descending suspension interface. The rate at which the channeling zone propagated upwards increased with increasing dosage of flocculant. Vesilind also investigated channelling during the settlement of calcium carbonate suspensions, and concluded:

(i) that channels only occur at intermediate concentrations,
(ii) the upper region of the suspension (i.e. near the supernatant interface) is progressively diluted,
(iii) the channel zone velocity was constant,
(iv) the higher the initial concentration the higher the zone of channelling appeared in the suspension, and
(v) the vertical channels were formed just above the compression region.

In systems which exhibit channelling, an alternative method to represent the batch settling data from the conventional plot of the interface height against time is the 'Tory' plot, which was later revised by Fitch. In this figure the interface velocity is plotted against interface height; if a distinct maximum exists then channelling is a likely cause. Vesilind further revised the technique by plotting interface velocity against time.

Dilution of the upper region during batch sedimentation with channelling was also reported by Kos. Unlike Chandler, however, Kos observed channelling only during batch thickening; no channels were present when continuously thickening the same suspension. He went on to postulate that the permeability of a settling layer is a function of both the solids concentration and the rate of change of solid concentration with respect to time. The latter tends to zero during continuous thickening and the permeability is thus given by the solid concentration only, i.e. Darcian flow regime with no channelling.

Solid Concentration and Electrical Conductivity

There are many techniques that can be employed to measure the in situ solid concentration of a dispersion. The methods most suitable to concentrated mineral suspensions such as those employed in this study are radiation or electrical conductance. The latter technique was favoured because of its simplicity of use and low cost.

When the solid in the dispersion has negligible electrical conductivity the relation between the solid concentration and the ratio of conductivities of the

Figure 1. Types of sedimentation behaviour according to Fitch.

Figure 2. Illustration of volcanoes and channels during sedimentation.
Effect of Stirring During Sedimentation

When stirring, the loose aggregates of calcium carbonate forming the sediment containing trapped voids filled with water are believed to be disrupted, causing the particles to rearrange into a closer random packing arrangement. There are several reports in the literature that suggest that stirring during the sedimentation of calcium carbonate suspensions can lead to higher concentrations than in the absence of mixing\textsuperscript{17,18,19}, in accordance with the belief that loose sediment aggregates can be disrupted.

Channels can only form, or continue to exist, if the suspension or sediment can exhibit some form of structure or mechanism which prevents particles from accumulating in the channel. Stirring can be used, therefore, as a technique to break-up any structure during sedimentation, disrupting the formation of channels. Thus it has been concluded that stirring may increase the final sediment concentration, but decrease the settling rate. In order to preserve increased settling rates and to maximise the settled sediment concentration, stirring would have to be restricted to only the bottom of a thickener\textsuperscript{20}.

Stirring is often employed during the settlement of biological sludges as a means of encouraging the release of gas bubbles which would otherwise adhere to the solids thus impeding the sedimentation of the biological floc. Thus, in this instance stirring would increase the settling rate of the suspension. Stirring may also assist in overcoming wall support effects\textsuperscript{12} when settling mineral suspensions in narrow columns. If, however, the stirring rate is too great then the particles will remain suspended. In general, gentle stirring is usually recommended during batch sedimentation tests as a means to obtain more reproducible experimental data\textsuperscript{24}. However, the effect of stirring will differ according to the material and conditions under study.

Mass Balance on a Layer During Sedimentation

A material balance on a settling lamina layer of solids of depth $\delta x$ provides the following result

$$-rac{\partial C\nu}{\partial x}\bigg|_t \delta x = \frac{\partial C}{\partial t} \bigg|_x \delta x$$

where $\nu$ is the solid velocity into the lamina layer. Cancelling $\delta x$, considering a fixed moment in time and transferring to material co-ordinates using the chain rule provides the following expression for solid settling velocity of a concentration between the maximum value within the sedimentation column ($C_m$), i.e. at the base of the column, and an arbitrary value $C$ within the column

$$v = -\frac{1}{C} \int_{C_m}^C \frac{dx}{d\tau}$$

where the values of $dx/d\tau$ are the characteristic velocities for concentrations between $C_m$ and $C$, at a fixed instance in time, i.e. the gradient of a characteristic on a plot of height of concentration characteristic against time—such as those provided later. Equation (3) is valid during sedimentation with channelling, as well as during stirred tests, provided:

- the amount of solids entering the channels and carried upwards is small,
- the reduction in vessel area due to channel formation is also small, and
- the suspension in the lamina layer is assumed to be distributed uniformly outside of the channels.

In sedimentation it is conventional to report settling rates in terms of flux ($\Phi$), in the units of solid mass per unit area and time, which is related to solid concentration and velocity as follows

$$\Phi = \nu \rho_s C$$

where $\rho_s$ is solid density.

EXPERIMENTAL

The calcium carbonate used in this study was supplied by Merck Ltd, Poole, UK, and was Analar grade. The density of the material was found to be 2590 kg m\textsuperscript{-3}, from a series of tests using density bottles and distilled water. Suspensions of various known initial masses and, therefore, concentrations were made in a 10\textsuperscript{-3} molar potassium chloride solution. The initial solid concentrations varied from 5 to 15% by volume. The very dilute electrolyte was employed to form a buffer solution; providing a constant ionic strength during the sedimentations and to enhance the electrical conductivity of the distilled water used as the continuous phase to facilitate electrical conductivity measurement. The particle size distribution of the solids is given in Table 1. A Coulter LS150 was used to obtain the distribution. This is a device which employs two types of size detector: Fraunhofer laser light diffraction and light diffusion
analyses. The particle diameter distribution is broad, but with only a small amount of material less than 10 μm. This is typical of the type of material which gives rise to channel formation. The zeta potential on the particles was measured by a Malvern Instruments Zeta Master at −10 mV, in a similar ionic strength solution used for the sedimentations. The pH of all the suspensions was 8.6.

The sedimentations took place in an acrylic batch settling column of total length and internal diameter of 350 and 65 mm respectively. Electrodes were positioned every 10 mm from the base of the column up to a height of 200 mm. The electrode spacing was 20 mm from a height of 200 to 300 mm. The mixture height was maintained at 330 mm during all the sedimentations. The sedimentation column was placed in a constant temperature box maintained at 25 ± 0.5°C. The sequence adopted for the tests was to start with the lowest initial concentration and to replace supernatant at the end of the test with dry solids ready for the next test. After the addition of solids at least 12 hours at constant temperature and with mixing was allowed in order to allow the system to come to thermal and ionic equilibrium. The sedimentation tests in the absence of stirring were performed three times as a check on reproducibility.

During the sedimentation, the height of the visible supernatant/suspension interface was recorded, and observations were made on the height of channels through the clear sides of the sedimentation column. Observations were also made of the top surface, noting the occurrence of volcanoes. In addition to the observations the electrical resistance, or conductivity, within the sedimentation column at the position of the electrodes was measured. The technique used for conductivity scanning has been described elsewhere and will be only briefly reported here. A square wave AC signal of 2000 Hz and 5 Volts was produced by an audio generator board (Radio Spares, based on an ICL 8038), and passed through a ballast resistor of 2180 Ohms. The signal was then multiplexed to one of 25 pairs of electrodes in the sedimentation column, and the potential difference between the source electrode and earth was measured via a diode pump connected to a Fairchild 12 bit Analogue to Digital converter in an IBM compatible PC.

The reading given by the A/D converter was calibrated with true rms voltage using a Black Star 4.5 digit multimeter. The AC signal oscillated either side of Earth, to prevent any net polarization effect, and both the source and earth sides of each electrode pair were switched at the same time to prevent any current leakage to Earth through neighbouring electrodes. A schematic diagram of the experimental equipment is provided in Figure 3.

Calibration was needed between all the electrode pairs so that any effect due to slight variation of distance between electrodes or performance was removed. All the electrodes were calibrated against the third electrode pair from the top of the suspension, i.e. at a depth of 70 mm from the air/suspension interface. The supernatant/suspension interface always settled past this electrode pair, which resulted in a means to quantify the continuous phase conductivity during the sedimentation. This confirmed that the continuous phase conductivity did not vary significantly during the experiment, as would be expected when employing a constant temperature environment. The A/D readings taken during the experiment were converted to values consistent with electrode 3, via the calibration, and then converted into rms voltages. The conductivity ratio was then calculated according to

$$K_c = \frac{(V_1/V_2 - 1)}{(V_3/V_2 - 1)}$$

where $V_3$, $V_1$ and $V_2$ are the voltages applied to the ballast resistor and sedimentation cell, the potential between the cell and earth with the mixture between the electrodes, and the potential between the cell and earth with the continuous phase between the electrodes respectively.

Sedimentations were performed at initial concentrations of: 5, 7.5, 10, 12.5 and 15% by volume. Considerable channelling was observed during the sedimentation of both the 5 and 7.5% suspensions. The sedimentations were repeated with the addition of a stirrer in the sedimentation vessel. The central shaft of the stirrer was 12 mm in diameter, and two additional stirring bars of 340 x 6 x 4 mm, length x width x thickness, were positioned either side of the central shaft at a distance of 8 mm from the shaft. A series of stirring speeds during sedimentation were investigated, and it was found that at 1 rpm no perceptible channelling was observed during any of the sedimentations. Hence the sedimentation of a material with and without channelling was investigated by comparing the behaviour during the stirred and unstirred tests.

**RESULTS AND DISCUSSION**

The 5 and 7.5% initial solid concentration by volume sedimentations exhibited channelling visible through the sides of the vessel, and provided a settling interface height with time figure which accelerated rapidly when the top of the channels met the settling interface surface.

**Table 1. Particle size distribution of calcite used.**

<table>
<thead>
<tr>
<th>Cumulative mass undersize (%)</th>
<th>100</th>
<th>90</th>
<th>80</th>
<th>70</th>
<th>60</th>
<th>50</th>
<th>40</th>
<th>30</th>
<th>20</th>
<th>10</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle diameter (μm)</td>
<td>62</td>
<td>37</td>
<td>32</td>
<td>27</td>
<td>23</td>
<td>19</td>
<td>17</td>
<td>14</td>
<td>9.8</td>
<td>3.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

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The triplicate unstirred sedimentations showed that the sedimentation behaviour did not change significantly with time; each batch sedimentation followed a very similar interface height with time relation despite the apparent randomness with which channels appeared. Thus the occurrence of channels may be a random process, but the number and effect of channelling on the sedimentation performance of a material is itself a function of the material and prevailing test conditions. The visible channels at the cylinder walls were assumed to be representative of those within the sediment and only visible from the above. The justification for this assumption is that the formation of volcanoes on the settling sediment was random; without any preference in speed of formation or size, to volcano formation at the walls or within the sediment. Sedimentations employing initial solid concentrations of 10% by volume, or more, did not exhibit significant channelling and so will not be considered in detail.

For the two sedimentation concentrations that displayed channelling, the 'Tory' plot as modified by Vesilind is shown in Figure 4. Also included on this figure is the same data for the sedimentations with stirring at 1 rpm. The interface velocity is approximately constant for the initial part of all the sedimentations, followed by a period of rapid acceleration in the unstirred and, therefore, channelling systems. On stirring, the interface velocity did not accelerate and it is noticeable that the initial settling velocity is the same as that for the unstirred test before channels had broken through to the settling surface. This confirms that the stirring did not either assist or hinder the sedimentation at these concentrations for this initial period. Stiring hindered the rate of sedimentation at the later stages due to the suppression of channelling. A comparison of the stirred and unstirred settling rates provides a means of quantifying the effect of channel formation on the sedimentation flux. The hypothesis that all channels were destroyed by stirring was confirmed by visual observation during the experiment.

Before equation (2) can be used to convert the conductivity ratio into local concentration the value of the empirical exponent \( n \) must be determined. It is probable that the exponent is influenced by many variables such as particle shape, size, operating conditions and even possibly container geometry. The last effect may be important because of the finite volume that an electric flux field occupies between two electrodes, rather than following a direct line between two points. The effect of the shape of the flux field and the possibility that it may significantly distort to a region of higher conductivity, i.e. lower solids content, has been investigated. It was concluded that the distortion was not significant provided that readings were taken at a significant distance from the suspension/water or water/air interface. The mass present in the sedimentation column may be calculated by integration of the solid concentration profile thus

\[
A \rho \int_0^{0.33} C \, dx
\]

which can be compared with the known mass of material added to the column. The upper limit of the integral in equation (6) is the initial suspension height. In an earlier work, mass balances performed during the sedimentation of several materials ranging from highly isotropic ballotini to anisotropic platelet suspensions have shown that the mean error in solids content is less than 5%.

The error depended on the value chosen for the empirical exponent, hence the value taken for this work was fixed by calculating the value that provided the lowest mass balance error for all sedimentations of a given type as a function of time. This resulted in one empirical value for the stirred work, \( n = 1.55 \), and a slightly different one for the unstirred work, \( n = 1.50 \). The slightly different values of the exponent are attributed to the difference in vessel...
geoemetry; during the stirred tests the central stirrer shaft would have impeded the electric flux field. Also, whilst stirring the particles would have become re-oriented in the flux field. The difference in exponent value is slight, and Maxwell’s equation (equation (1)) appears to be valid for this material for solid concentrations up to 34% by volume.

Figures 5(a) and (b) show the solid concentration characteristics which exist between the initial concentration of 5% by volume and the maximum sediment concentration; 34 and 17% by volume for the stirred and unstirred tests respectively. Figures 6(a) and (b) show the solid concentration characteristics which exist between the initial concentration of 7.5% by volume and the maximum sediment concentration, which were 34 and 18%. During both stirred tests, the characteristics appear to propagate uniformly from the origin in accordance with the mathematics of Kynch. The only exceptions to this are at concentrations approaching the maximum, where a finite time appears to be required before the concentration exists.

However, after a higher concentration has been formed, the propagation is still reasonably uniform with respect to sedimentation time. Figures 5(b) and 6(b) show characteristics that may also be traced back to the origin, but the higher concentrations appear to accelerate into the column. When sedimenting these suspensions there was no evidence of characteristics starting from the top surface of the rising sediment, as would be required for the analyses of Tiller² or Font². There must, therefore, be different types of compression in sedimenting systems depending on the behaviour of the rising sediment surface. The stirred tests provide solid concentration characteristics in considerable excess to that given by the unstirred tests. Two consequences of this are the longer times to final settlement and lower final interface height of the stirred tests compared with the unstirred.

Also marked on Figures 5(b) and 6(b) are the visible channel top and bottom heights measured throughout the sedimentation. Considering the two figures, the top of the channel zone does not appear to correspond uniquely to a solid concentration. It is, however, noticeable that the top of the channel zone appears to closely follow the first characteristic illustrated in both figures, i.e. the first significant concentration increase over the initial suspension concentration. The bottom of the channel zone appears to be fairly constant in height until the interface settling curve becomes very shallow, corresponding to the point at which the first characteristic meets the interface curve. The channels then disappeared.

When settling the 10% suspension no consistent channel zone developed. During the sedimentation a few volcanoes could be observed on the surface of the settling suspension, but these would disappear again after only a few minutes. The sedimentations at 12.5 and 15% initial solid content showed no indication of channelling at all.

The solid settling velocities of concentrations formed during the batch settling of a 5% suspension, according to equation (3), are provided as a function of settling time in Figures 7(a) and 7(b), for the stirred and unstirred tests respectively. Also included on both figures are the observed interface settling velocities. The integration described by equation (3) uses the gradient of the solid characteristics displayed in Figure 5. Characteristics which accelerate into the sedimentation column cause an increase in the term dx/dt and, therefore, an increase in the integral for any fixed value of concentration. Hence, the solid settling velocity given by equation (3) also increases. Figure 7(b) demonstrates a consistent increase in solid settling velocity, with respect to time, for all the solid concentrations occurring below the settling interface. The stirred tests, shown in Figure 7(a), provide a more consistent solids settling velocity with negligible variation with sedimentation time. The sedimentations conducted at an initial concentration of 7.5% provided a very similar result to that illustrated in Figures 7(a) and (b).

The solid velocity data provides a means of assessing the state of aggregation of the particles during the stirred sedimentation. The solid velocities illustrated on Figure 7(a) are substantially constant with respect to sedimentation time, and decrease with increasing solid concentration. The equivalent spherical settling diameter of the particles, or aggregates, can be calculated via a linear regression on a logarithmic plot of solid velocity against porosity, in accordance with the Richardson and Zaki equation²

\[ v = v_s (1 - C)^n \]  

(7)
where \( m \) is an empirical exponent and \( v_0 \) is the free settling velocity of the primary particle or aggregate. Such a linear regression resulted in a correlation coefficient greater than 0.99, and an apparent free settling velocity of 0.000251 m s\(^{-1}\) by extrapolating velocity from the free settling range to \( C = 0 \). Equation (7) appears to provide an adequate fit to the data and, therefore, has not been modified by any empirical factor on the concentration term. The equivalent spherical settling diameter equal to this settling velocity according to Stokes’ settling equation is 17 \( \mu m \). This is the primary particle, or aggregate, settling diameter under these settling conditions. The expected settling diameter can also be estimated from the particle size distribution provided in Table 1. The settling diameter \( (x_{\text{st}}) \) is related to the mean volume diameter \( (x_v) \) as follows\(^6\)

\[
x_{\text{st}} = x_v^4 \sqrt{\frac{\Psi}{\pi}}
\]

where \( \Psi \) is the sphericity of the particle. The calcite particle shape is close to that of a cube, which has a sphericity of 0.806, and the mean volume diameter is 21 \( \mu m \). The settling diameter of the primary particles should, therefore, be 20 \( \mu m \) by equation (8). The inferred settling diameter is close to, and even slightly smaller than, what would be expected from the size distribution data assuming no aggregation of the particles. Hence, it is reasonable to assume that no aggregation took place during the stirred sedimentation. The situation during the unstirred sedimentations is much less certain. A structure must have existed in the suspension in order to support the existence of channels, and for the final sediment concentration to be considerably less than that achieved during stirring. However, no flocs or aggregates were visible during the sedimentation. It is likely that the region above the channel zone was in a different structural state to that in and below the channel zone, for the reasons described below.

The solid concentration measurements showed some evidence of dilution of the layers between the top of the channels and the settling interface, to values less than the starting mixture concentration. Further evidence for dilution can be deduced from Table 2. In the table batch solid fluxes during the stirred and unstirred tests are compared. For solid concentrations between 6 and 10% by volume the settling flux is enhanced in the channelling system. For concentrations greater than 10% settling flux is enhanced by stirring, a consequence of the greater sediment concentration achieved by stirring. The 5% stirred settling flux ranges between 0.025 and 0.022 kg m\(^{-2}\) s\(^{-1}\). During channelling the settling flux of both the 6 and 7% solid concentration lamina layers can be in excess of this value, for a substantial part of the sedimentation. Thus material would apparently settle out of these layers faster than it could settle into the layers, if a layer of 5% solid content without channels existed above them. This is not a realistic proposition and can only be explained by dilution of the region above the layer at 6% concentration during the channelling sedimentation. It is also intuitively reasonable to propose that if channels are discharging clear liquid into a zone of suspension of a fixed initial concentration, i.e. 5% in Figure 5(b), then that zone will become diluted with the liquid from the channel. The dilution will continue until the top of the channels meet up with the surface of the suspension/supernatant interface. A consequence of this dilution is that the visible settling interface rate increases significantly, prior to channels joining the settling interface. This could not occur if the 5% settling flux is a fixed material property without the dilution of this zone.
Dilution of the region above the channel zone depends upon that zone following the characteristic representing the first significant increase in concentration over that of the original suspension. Other materials are being studied to discover the general applicability, or otherwise, of this observation.

Finally, the data displayed in Table 2 indicates that the batch settling flux can be increased by up to 26% by the presence of channels, compared with the stirred system. However, for solid concentrations in excess of 10% by volume stirring provides greater flux values. In this case, and similar such circumstances, it may be beneficial to stir a continuous thickener at a speed much greater than that producing the characteristic showing the first significant increase in concentration over that of the initial suspension. This may be an important design implication for continuous thickening and is being studied further. The added benefit of an increased underflow concentration may also result, as indicated in the work reported here and elsewhere.

CONCLUSIONS

The calcite mineral used in this study was adequately described by Maxwell’s equation between the electrical conductivity ratio of the dispersion and the solid concentration, up to a volume fraction of 34% solids. The resulting measurements of local solid concentration during sedimentation, both with and without channeling, were investigated together with observations taken during the settlement. Stirring at a rotational speed of 1 rpm was sufficient to eliminate any channel formation. As the initial sedimentation rates when stirring or not were the same, it can be concluded that stirring at this speed did not provide sufficient turbulence for axial mixing of the solids. It was sufficient for radial mixing of the solids, however, and this led to sediment concentrations twice that obtained when settling in the absence of stirring. The occurrence of channeling led to a more rapid sedimentation rate at a fixed initial solid concentration; the rate increasing until solid characteristics propagating from the origin became substantially horizontal. Channels did not need to be present on the surface of the sedimenting suspension for the rate of descent of the settling interface to accelerate. Thus the presence of channels below the interface apparently caused the interface settling flux to increase. This is attributed to dilution of the zone between the top of the rising channels and the settling interface. In the absence of channeling this region is the zone of constant concentration, which displays constant settling velocity and, therefore, flux.

The top of the channel zone corresponded to the characteristic showing the first significant increase in solid concentration over that of the initial suspension, rising from the bottom of the sedimentation column. It did not correspond to a unique solid concentration for both sedimentations at 5 and 7.5% solids. The bottom of the channel zone remained fairly constant in height throughout the sedimentation, until the sedimentation was substantially complete. The channels then filled in with solids and ceased to exist.

All the solid characteristics propagated from the base of the column, but some solid concentrations required finite time before they appeared during the unstirred sedimentation. These concentrations then accelerated into the column. Their occurrence and acceleration into the column led to an increase in the settling velocity of the solids lying above, and may be one cause of channel formation.

For solid concentrations less than 10% by volume the greater solids flux was given by the unstirred system, due to the presence of channels. However, for concentrations greater than 10% stirring improved the settling flux. This concentration is also the threshold at which channels were no longer observed when sedimenting from a uniform suspension at this value. In the absence of stirring, the final maximum sediment concentration was 18% by volume, whereas with stirring it reached 34% by volume. Thus, channeling may improve the interface settling velocity at low to intermediate concentrations, but adequately stirred systems may be preferential as settling fluxes of the higher solids concentrations are increased and greater sediment concentrations result.

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NOTATION

\( A \) area of sedimentation column, \( m^2 \)
\( C \) solid concentration by volume fraction,
\( C_m \) maximum solid concentration by volume fraction,
\( K_m \) mixture phase conductivity, \( S m^{-1} \)
\( K_c \) continuous phase conductivity, \( S m^{-1} \)
\( m \) exponent on Archie’s law—equation (1),
\( n \) exponent on Archie’s law—equation (1),
\( t \) sedimentation time, \( s \)
\( V_s \) voltage applied to sedimentation cell and ballast resistor, Volts
\( V_1 \) potential between cell and earth with clear liquid in the cell, Volts
\( V_2 \) potential between cell and earth with mixture in the cell, Volts
\( v \) solids settling velocity, \( m s^{-1} \)
\( x \) distance from base of column, \( m \)
\( x_m \) settling diameter, \( m \)
\( x_v \) mean volume diameter of size distribution, \( m \)

Greek letters

\( \rho_s \) solid density, \( kg m^{-3} \)
\( \Phi \) solid settling flux, \( kg m^{-1} s^{-1} \)
\( \psi \) Wadell's particle sphericity

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Experimental and Numerical Analysis of a Sedimentation Forming Compressible Compacts

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ABSTRACT

Batch sedimentations of the mineral talc suspended in water at various initial concentrations resulted in compacts that displayed compression, and compression with channel formation. During the experiments the local concentration was deduced by means of local electrical resistance measurement. The technique provided concentrations that integrated throughout the vessel to give masses that matched the known initial mass employed to within ±5%. Two types of channel zones were observed; soft and hard, the former appeared to be due to the liquid inertia of water discharging from the latter. The region within and above the soft channel zone diluted from the initial concentration, and this caused the visible interface between the suspension and the supernatant to accelerate. The top of the hard channel zone followed the line of constant solids concentration representing the first significant increase in concentration over the initial suspension. A finite difference numerical model of sedimentation matched the experimental data, including the data determined below the visible interface, with very high precision for the talc suspensions exhibiting compression with insignificant channeling. The implicit model was implemented on a conventional computer spreadsheet package and rapidly converged. The model did not employ a function for hydraulic permeability, instead a linear function between the so-called Kozeny "constant" (or coefficient) and concentration was used. In order to provide an accurate numerical model for compressible sedimentation with significant channel formation, the hydraulic permeability needs to be augmented, or the Kozeny coefficient reduced, and the dilution above the channel zone must be predicted. These should be achieved in a way that is general to all sedimentations of a given type of material, and not specific to only one starting concentration. Experimental and numerical results also indicate that the buoyancy force experienced by the solids is adequately described by the density difference between the solids and the suspending liquid, and not the density difference between the solids and the suspension.
Gravity sedimentation is a widely practiced operation in many industries. Process scale sedimentation equipment is generally designed after performing tests on a small laboratory scale. The laboratory tests identify the appropriate settling regime, and can also generate operating data suitable for use in the design. Laboratory tests may also be employed when investigating the performance of existing process scale sedimentations; identifying possible operating conditions and settling behavior. The different settling regimes have been described by Fitch (1), and are illustrated in Fig. 1.

Clarification, or free settling, is restricted to conditions of low solid concentration, and particles may settle as individual units or agglomerates unaffected by the presence of other particles. At solid concentrations greater than 1 or 2% by weight, hindered settling occurs, and particles settle en-masse rather than as individual units. However, when sedimenting in the hindered settling regime, further subdivision into zone settling, compression, and compression with channeling must be made. In the instance of zone settling alone the particles form incompressible suspensions and sediments where the settling velocity is a unique function of solid concentration, and may be described by well-known correlations (2). When undergoing compression, the particle settling velocity is a function of solid concentration and other local conditions such as the compressive stress gradient within the sediment.

The mathematical description of sedimentation, leading to design procedures for batch and continuous thickeners, started with the work of Coe and Clever (3) and was advanced in the 1950s by Kynch (4), Talmage and Fitch (5), and Yoshikawa (6). The designs were based on the assumption of incompressible discrete particles, where settling velocity is a unique function of local concentration. Mathematical techniques for the design of thickeners with due account of solid compact compression were advanced in the 1960s, 1970s, and 1980s (7-13). However, in most instances the occurrence of channels within the compressing sediment was neglected, as a simple means to incorporate channeling into the thickening model was not possible. The significance of channels in the overall sedimentation process has been recognised (1, 10, 14–18), and some thickener geometries have been proposed to encourage this phenomenon (19).

In recent years several research workers have used numerical methods to solve the differential equations for sedimentation of materials giving rise to compressible sediments (9, 20-23). The usual method of assessing the relevance of the numerical model was to compare the numerically predicted interface settling curve, i.e., the height of the interface between the supernatant clear liquid and the settling suspension with respect to time, with the experimentally observed settling curve. This is not a satisfactory test of the applicability of a numerical model as the constituent equations for the model often include terms that have to be determined from such settling tests, and an element of curve fitting may be suggested in some instances.

Obtaining data to investigate the constituent terms is a frequently occurring problem. In the absence of detailed data on local solid concentrations underneath the settling interface, the only concentrations and velocities that appear to be reliably known are those of the initial suspension concentrations. However, the occurrence of channels can change these values significantly by dilution of the suspension just below the settling interface (17, 18). One of the most difficult to assess constituent equations is the variation of permeability with solid concentration, as the presence of channels will have a considerable influence on results deduced by applying a force balance to data taken from the interface settling plot. The usual approach in this instance is to neglect compression and channeling, and simply to equate the solid weight and liquid drug terms in the force balance, rearrange for hydraulic permeability, and substitute in the known initial concentration and measured settling velocity together with other known physical constants. Permeability can be deduced from data taken from the settling curve only when that curve does not display significant
compression or dilution; indicated by a curve bending toward or away from the origin, respectively.

The above restrictions limit the calculation of hydraulic permeability from interface settling data to suspensions of low initial concentration, where channeling and compression are not significant for a substantial part of the settling curve. However, in a numerical model of compressible sedimentation the permeability at moderate and high concentrations is very important for the solution. The approach adopted in this work was to use the settling data from sedimentations at low initial concentrations, in the absence of significant compression and channeling, to use a technique discovered by Font (14) to calculate permeability at higher concentrations. On comparison of these two methods for permeability analysis with the Kozeny relation, it was discovered that if the Kozeny constant was allowed to vary linearly as a function of concentration, the Kozeny permeability relation gave an exact match to the permeability measured by the two experimental methods. The approach of allowing the so-called Kozeny "constant" to vary with concentration in sedimentation analysis has been suggested before (24). The advantage of using this approach here was that the linear variation of the Kozeny constant, at least over the range of concentrations of interest in this study, made extrapolating the permeability data to high concentration values more reliable than the use of any alternative permeability expression. Also, the application of the Kozeny permeability expression provided, after canceling, a much simplified coefficient in the partial differential equation describing the sedimentation. This coefficient contained only terms for solid concentration and the Kozeny "constant," as described later.

The modeling work was also used to investigate the varying contribution from the suspended solids to the liquid pressure gradient surrounding the particle, and hence the buoyancy force. In sedimentation the buoyed weight force per unit area within a force balance conducted on settling particles is

\[ C_g(\rho_s - \rho) dx \]

where \( C \) is the solid concentration in volume fraction, \( g \) is the acceleration due to gravity, \( x \) represents distance from the base of the column, and \( \rho_s \) and \( \rho \) are the solid and liquid densities, respectively. A particle in suspension experiences a buoyancy force because of the surrounding liquid pressure gradient, and the surrounding liquid pressure gradient is increased due to the presence of particles. Thus, the buoyed weight term may be written as

\[ C_g(\rho_s - \rho_m) dx \]

where \( \rho_m \) is the mean density of the suspension including the suspended particles. If the mean suspension density is given by its component densities in appropriate volume ratios, the buoyed weight term becomes, after simplification,

\[ C_g(\rho_s - \rho)(1 - C) dx \]

It has been argued (8, 25) that when the particles are fully suspended the above equation is valid, but when the particles are fully settled the equation must revert to that given initially. Hence the term \( (1 - C) \) is the buoyancy correction term varying from a fractional value to unity during sedimentation. For the sedimentation of largely incompressible materials, the change from a fractional value to unity is sudden; it occurs at the end of the settling period. However, for the sedimentation of compressible compacts a network of solids that can transmit the suspended particle weight to the base of the vessel will slowly build up, thus the buoyancy correction term will slowly vary from \( (1 - C) \) at the start of the sedimentation to unity at the end. This makes the buoyancy correction term a function of the sedimentation history, and the numerical modeling of the process will also become complicated. The numerical and experimental investigations of talc sedimentations also considered the significance, or otherwise, of the change in the buoyancy correction term.

**MODEL DEVELOPMENT**

A force balance on a settling layer of suspension during batch sedimentation in Cartesian coordinates gives

\[ \frac{\partial P_s}{\partial x} = C_g(\rho_s - \rho) - \frac{\mu}{k} v \]

where \( \frac{\partial P_s}{\partial x} \) is the solids compressive stress gradient, \( k \) is the hydraulic permeability, \( v \) is the solid velocity, and \( \mu \) is the liquid viscosity. Defining the mass of solids per unit area to be \( w \), then

\[ w = C_g \rho_s \delta x \]

The total mass of solids up to a height \( x \) is

\[ w_x = \int_0^x C_g \rho_s dx \]

Equation (2) can be used to transfer the system to material coordinates. In this system the total mass of solids in a batch sedimentation vessel is split into a number of elements, each containing the same mass of material.
Only at the start of the sedimentation are the elements of equal volume, i.e., area and height. The material coordinate system is illustrated schematically in Fig. 2.

Darcy's law, written in a material coordinate system and combined with a liquid force balance, gives

\[ v = \frac{kC_{ps} \frac{dP_L}{\mu}}{\frac{d\nu}{d\nu}} \]  

(3)

where \( P_L \) is the liquid pressure in excess of the hydrostatic pressure due to the presence of suspended solid material. The continuity equation in material coordinates is

\[ \frac{1}{\rho_s} \frac{\partial C}{\partial t} = \frac{\partial v}{\partial \nu} \]  

(4)

Combining Eq. (1) to (4), employing the Chain Rule, and rearranging provides the following equation for the variation in the excess liquid pressure, as a function of time, due to the settlement of a compressible compact:

\[ \frac{\partial P_L}{\partial t} = -\frac{\rho_sC^2}{dC/dP_s} \frac{\partial v}{\partial \nu} \left[ \frac{kC_{ps} \frac{dP_L}{\mu}}{\frac{d\nu}{d\nu}} \right] \]  

(5)

which is a parabolic equation in excess liquid pressure. A suitable choice of coefficients can be made to make Eq. (5) nondimensional. These are

\[ v_0 = H_0C_0\rho_s \]  

(6)

\[ P_0 = w_0(\rho_s - \rho)g/\rho_s \]  

(7)

\[ P_L^* = P_L \left[ \frac{\rho_s}{g(\rho_s - \rho)w_0} \right] \]  

(8)

\[ P_s^* = P_s/P_0 \]  

(9)

\[ w_0^* = w/w_0 \]  

(10)

\[ t^* = \frac{t}{H_0} \]  

(11)

where the nondimensional terms have asterisks, and the nondimensionalizing time function \( t_0 \) is defined as

\[ t_0 = H_0w_0 \]  

(12)

The nondimensionalizing function of solids settling velocity \( (w_0) \) is defined as

\[ v_0 = \left( 1 - C_0 \right) \frac{g(\rho_s - \rho)}{K_0C_0S^2} \]  

(13)

where \( S \) is the specific surface area per unit volume of the solid particles, and \( K_0 \), the Kozeny constant in the Kozeny–Carman permeability expression, typically has the value of 5 for static beds and 3.36 for moving beds. It is often found that the Kozeny permeability expression provides a good representation of the functional relation between hydraulic permeability and solid concentration provided an empirical value is used for the Kozeny constant (24). Furthermore, the Kozeny constant is a term that includes a tortuosity factor and various other numbers that have their origins in the transformation of the Hagen–Poiseuille equation into the Kozeny–Carman equation. There is no reason why these factors and numbers should be constant with variable concentration. Hence it is a reasonable approach to allow the Kozeny “constant” to become a variable dependent on solid concentration (24). Under these circumstances it is more meaningful to use the phrase Kozeny “coefficient” rather than constant. The correct Kozeny coefficient to use in Eq. (13) is the value appropriate to the concentration \( C_0 \). Equation (13) physically represents the hindered settling velocity when there is no stress gradient \( (\partial P_s/\partial x = 0) \), and it can be derived from Eq. (1) by using the Kozeny permeability expression.

After making the appropriate substitutions, Eq. (5) becomes

\[ \left( dC/dP_s^* \right) \left( \frac{1}{C^2} \right) \frac{\partial P_L^*}{\partial t^*} = -\frac{\partial}{\partial \nu} \left[ \rho_sP_wu_0kC_{ps} \frac{dP_L^*}{\mu w_0^*} \right] \]  

(14)
where the subscript "o" refers to nondimensionalizing functions for pressure, time, and mass of solids per unit area. Equation (14) is a parabolic equation with nonlinear coefficients and is of the form

$$b(P^{*}) \frac{\partial P^{*}}{\partial t} = \frac{\partial}{\partial H} \left[ a(P^{*}) \frac{\partial P^{*}}{\partial H} \right]$$

and can be solved numerically, under appropriate boundary and initial conditions, by a finite difference method (26). The function $b(P^{*})$ can be assessed once the functional relation between solid compressive pressure and concentration has been determined, and in combination with Eq. (6), (7), and (9). The experimental equipment provided a means to measure the final settled sediment concentration profiles. Thus, these profiles were used in the analysis of solid concentration as a function of compressive pressure.

When the sedimentation is complete, Eq. (1) is simplified as the settling velocity is zero, and the local solids pressure can be obtained by integrating the concentration profile between the height under investigation and the surface of the sediment. Thus the total solid pressure within the vessel is

$$P_{S} = (\rho_{S} - \rho) g \int_{0}^{H} C dx$$

where $H$ is the sediment height. The functional relation between concentration and solid pressure was described adequately by

$$C = C'P_{S}^{n}$$

(15)

where $C'$ and $n$ are empirical constants and had values of 0.088 and 0.144, respectively, for the talc mineral employed in this study.

Using Eq. (6), (7), (9), and (15) gives the following equation for the function $b(P^{*})$ after suitable simplification:

$$b(P^{*}) = \frac{a(H_{0} C_{0} (\rho_{S} - \rho) g)}{C_{0}}$$

The function $a(P^{*})$ contains the hydraulic permeability which is a function of local solid concentration. However, if the Kozeny permeability expression is used, i.e.,

$$k = \frac{(1 - C)^{3}}{KC_{S}^{2}}$$

where $K$ is the Kozeny coefficient, and Eq. (12) and (13) are substituted into the function, then the permeability term cancels from the expression together with the specific surface area per unit volume of the solids. The resulting expression for the function $a(P^{*})$ is

$$a(P^{*}) = \frac{K_{0}}{1 - C_{0}^{3}} \left[ \frac{(1 - C)^{3}}{KC_{S}^{2}} \right]$$

(16)

where $K_{0}$ is the Kozeny coefficient at the concentration $C_{0}$. The term within the square bracket on the right side of Eq. (16) contains the local concentration and the Kozeny coefficient, which is also dependent on the local concentration. If the Kozeny coefficient was, in fact, a constant, Eq. (16) would simplify considerably. Such an approach may be valid as a first approximation in a sedimentation analysis, but in this study Eq. (16) was used after tests to evaluate the concentration dependency of the permeability and, thereby, the Kozeny coefficient. The functional relation between permeability and solid concentration was deduced following the technique described by Font (14), applied to one set of the experimental results. The analysis requires the local solids settling velocity calculated from a modified material balance:

$$v = \frac{1}{C} \left[ C^{*} \frac{dH}{dt} (C - C^{*}) + \int_{0}^{H} \frac{dC}{dx} dx \right]$$

(17)

where $C^{*}$ is the concentration at the top of the sediment (in this analysis the concentration of the first line of constant concentration from the base of the vessel), $C^{*}$ is the settling velocity of that concentration when it reaches the settling interface, and $dH/dt$ is the rate of propagation, upward from the base, of the sediment concentration at time $t$. A knowledge of the local concentration with time is needed in order to apply Eq. (17). According to Font's technique, the local velocity obtained from Eq. (17) can then be used in Eq. (18) to provide a value for permeability:

$$k = \frac{\mu v}{C_{0}(\rho_{S} - \rho) - \left( \frac{dP^{*}}{dC} \right) \left( \frac{\partial C}{\partial x} \right)}$$

(18)

Again, a knowledge of the local concentration, this time with respect to distance within the column, is required in order to apply Eq. (18), and the functional relation between solid pressure and concentration must be known. The original reference contained a typographical error in the reproduction of this equation: Eq. (30) of Ref. 14, but Eq. (18) above is the correct form of this expression.

Given the appropriate constituent equations, an explicit finite difference solution to Eq. (14) was attempted but found to be unstable. Thus an
The implicit three time level method (26) was employed as follows:

\[ P_{t_{j+1}} = \frac{1}{3} (P_{t_{j+1,j+1}} + P_{t_{j+1,j}} + P_{t_{j+1,j-1}}) \]  
(19.1a)

\[ P_{t_{j,j}} = \frac{1}{3} (P_{t_{j+1,j}} + P_{t_{j,j}} + P_{t_{j-1,j}}) \]  
(19.1b)

\[ P_{t_{j-1,j}} = \frac{1}{3} (P_{t_{j-1,j-1}} + P_{t_{j-1,j}} + P_{t_{j-1,j+1}}) \]  
(19.1c)

where \( i \) and \( j \) represent dimensionless material and time coordinates. The finite difference solution to Eq. (14) using this approach is:

\[ P_{t_{j+1}} = \frac{3M(C_{l,j})}{2 \Delta x^2} P_{t_{j-1}} + a \left( \frac{1}{2} [C_{i+1,j}; C_{l,j}] \right) (P_{t_{j+1,j+1}} + P_{t_{j+1,j}} + P_{t_{j+1,j-1}} - P_{t_{j-1,j+1}} - P_{t_{j-1,j}} - P_{t_{j-1,j-1}}) \]

\[ \times \left[ \frac{3M(C_{l,j})}{2 \Delta x^2} + a \left( \frac{1}{2} [C_{i+1,j}; C_{l,j}] \right) + a \left( \frac{1}{2} [C_{i-1,j}; C_{l,j}] \right) \right] \]

(20)

The initial condition used in the solution was

\[ P_{t_{0}} = 1 - w^* - P_{80} \]

where \( P_{80} \) is the nondimensionalized form of the initial solid stress. The boundary conditions were

\[ \partial P_{t_{j}} / \partial w^* = 0, \quad \text{at} \quad w^* = 0 \]

\[ P_{t_{j}} = 0, \quad \text{at} \quad w^* = 1 \]

The use of a three time level method necessitated a pseudoinitial row before the row at \( j = 0 \). This pseudoinitial row was assumed to have the same values as the true initial condition row. The model was solved iteratively on a standard computer spreadsheet package using the cells within the spreadsheet pages to represent the elemental cells for the numerical model. The solution used 6 spreadsheet pages for values of

\[ P_{t_{i,j}}; C_{i,j}; a \left( \frac{1}{2} [C_{i+1,j}; C_{l,j}] \right); a \left( \frac{1}{2} [C_{i-1,j}; C_{l,j}] \right); P_{80}; b(C_{l,j}) \]

Each spreadsheet cell location on each page represented the value of the appropriate function under the same conditions of \( i \) and \( j \). The values used in the solution for \( \Delta w^* \) and \( \Delta t^* \) were 0.05 and 0.000125, respectively. The spreadsheet function of iterative calculation of the cell values was employed until convergence of the cell values was observed. The model ran on a Pentium IBM compatible PC at 90 MHz, with 16 MB of RAM, and was generally completed within 5 minutes. A further page was used for the calculation of the height from the base corresponding to each value of \( i \) and \( j \). The total height of the suspension undergoing compression was calculated from

\[ h = \sum_{n=1}^{25} \Delta w n \]

EXPERIMENTAL TECHNIQUES

Two batch-settling vessels were used with internal diameters of 61.5 and 140 mm. Both sedimentation vessels were 408 mm in height, and the initial suspension height was 331 mm in all of the experiments. The equipment was housed in an insulated wooden box to provide a constant temperature environment. Temperature controlled to \( 25 \pm 0.5 \) °C was maintained by a thermostat connected to two Radio Spares heating mats, or two 240 V light bulbs for the smaller rig. In all the tests the local concentration was measured by using vessels equipped with electrodes to record the local electrical resistance and hence concentration (see Fig. 3). Full details of the experimental equipment have been described before (27).

The calibration of electrical conductivity ratio (ratio of the continuous phase and mixture phase conductivity) to solid concentration by volume fraction requires knowledge of one empirical constant (m):

\[ C = 1 - m \frac{\kappa_m}{\sqrt{\kappa_e}} \]

where \( \kappa_m \) and \( \kappa_e \) are the mixture and continuous phase conductivities, respectively. The experimental equipment provided values of the local conductivity ratio throughout the sedimentation. The empirical constant that provided the lowest error in the mass balance on the talc solids for
all the sedimentations and at all sedimentation times was used to convert the conductivity ratios to concentrations. The mass balance compared the known original mass of solids added to the vessel with that obtained by integrating the concentration profile at all settling times

$$\int_0^H A \frac{\partial s}{\partial y} \, dy$$

where $A$ is the area of the sedimentation vessel and $H$ is the height of the suspension. The percentage error in the mass balance using an exponent value of 1.98 ranged between ±5% at the extremes, but was generally within the range ±2%.

The variation in the liquid pressure gradient due to solids settlement was measured by placing two very sensitive pressure transducers 204 mm apart in the larger diameter sedimentation vessel. The transducers were Druck-type PDCR 810, with a full-scale reading of 70 mbarg corresponding to a 17 mV output. The outputs from the transducers were measured using two digital volt meters (DVM) with 4.5 digit resolution (Black Star model 4310). The outputs from the transducers measured by the DVMs were calibrated against a column of water set to various heights. Other experimental conditions were similar to those described, except that the initial concentration was very high at 17.5% by volume. The use of such a high starting concentration ensured that channeling was not significant during the sedimentation. The final sediment concentration was at a reasonably uniform value of 22.5%. During the sedimentation, and after settlement, the suspension height was always well above the top transducer. The settlement of solids reduced the readings from both transducers. There was also a reduction in pressure difference between these two transducers, which was due to the solid weight becoming transmitted to the base of the vessel by a developing solid network. This experiment was designed to test the theory that buoyancy correction may be a complicated function of sedimentation history, as described in the Introduction.

In all of the experiments the liquid phase was a weak (10⁻⁵ M) potassium nitrate solution. This very dilute electrolyte was employed as a buffer to provide a constant ionic strength against any slight dissolution of ionic material from the surface of the talc mineral. It also enhanced the conductivity of the distilled water making up the continuous phase, and thus facilitated the use of electrical conductivity measurement. The mineral density was determined by using density bottles and distilled water, and the size distribution of the solids was obtained on a Coulter Electronics LS 130, a device that uses both Fraunhofer laser light diffraction and light diffusion analyses. The particle shape was discovered to be a platelet type, similar to that of a clay mineral, from images obtained by a scanning electron microscope. The physical properties of the talc mineral employed in this study are given in Table 1.

### DISCUSSION OF EXPERIMENTAL RESULTS

Channels were observed during the sedimentations. The most significant channeling occurred when settling suspensions with initial concentrations between 7% and 15% by volume were used. In all instances it was possible to deduce two distinct channel zones: hard and soft. These zones are illustrated in Fig. 4. The lower hard zone had distinct channel walls that stayed in the same location for a significant length of time. The suspension phase surrounding the hard channels appeared to have a more cohesive structure than the suspension surrounding the soft channels.

### TABLE I

<table>
<thead>
<tr>
<th>Density (kg·m⁻³)</th>
<th>100</th>
<th>90</th>
<th>70</th>
<th>50</th>
<th>30</th>
<th>10</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talc 2690</td>
<td>40</td>
<td>17</td>
<td>11</td>
<td>7.4</td>
<td>4.6</td>
<td>2.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

FIG. 3 Schematic diagram of sedimentation equipment.
The channels within the soft zone were smaller and prone to move while maintaining a base positioned directly above a hard channel. The general appearance of the suspension surrounding the soft channels was that of a fluidized system. Thus it may be concluded that the soft channels are due to liquid inertia caused by discharge from the hard channels. In all these respects the talc mineral behaved in a similar way to the sedimentation of calcite described elsewhere (18).

Figure 5 shows the interface height with time, together with the iso-concentration curves or "lines of constant solids concentration" formed during the batch sedimentation of a suspension at an initial uniform concentration of 7.2%. The coincidence between the top of the hard channel zone and the line of constant solids concentration representing the first significant increase in concentration over the feed is evident. This observation was found to be true for all the initial suspension concentrations investigated. The electrical conductivity scanning equipment indicated that the concentration above the channel zone but still below the settling interface diluted to values below the starting concentration. Thus the interface velocity increased during sedimentation because of three factors: a lowering of the solid concentration below the interface, on breakthrough of soft channels to the surface, and on breakthrough of hard channels to the surface. On the occurrence of these last two phenomena, the interface curve accelerates very rapidly.

The settled sediment concentration profiles for all the tests were used to provide values for the constants in Eq. (15), following the procedure in the section on Model Development detailed just prior to Eq. (15). This resulted in values of 0.088 and 0.144 for the coefficient and exponent, respectively. Consideration of these values leads to the conclusion that compressive stress or solid pressure is insignificant at concentrations below 4% by volume, and this value represents the limit of applicability of Eq. (15). Observations made of the settling behavior (e.g., the concentration profiles) of suspensions just above and below this value also suggested that the critical solids concentration, marking the boundary between the hindered settling zone and the compression zone, is close to a concentration of 4% by volume.

Hydraulic permeability was estimated from the initial settling rates of suspensions at concentrations of 2 and 5.2% by volume by equating the solid weight and liquid drag terms in the force balance, rearranging for hydraulic permeability, and substituting in the known initial concentrations and measured settling velocities together with other physical con-
stants. This was not attempted for higher concentrations as channeling and compression became increasingly significant. The permeability resulting from the analysis using an initial concentration of 5.2% by volume is likely to be slightly in error due to the presence of small compressive forces at this concentration. However, permeability was also determined by the method of Font (14), and the resulting values were consistent with those given by the earlier method. Font's technique was applied to the experimental data obtained from the sedimentation at an initial concentration of 7.2% by volume, and the procedure given in Eqs. (17) and (18) was used to provide permeability values up to a concentration of 18% by volume. For the purpose of numerical modeling, permeability values of up to 22% by volume were required, hence the Kozeny permeability expression was compared to the deduced permeability values. When using a linearly varying Kozeny coefficient, the Kozeny permeability expression matched the values provided by the two experimental techniques. The variation in the Kozeny coefficient with solid concentration is illustrated in Fig. 6. The consistency of the data obtained from the initial settling rates and the technique of Font can be seen from this figure. The linear function between the Kozeny coefficient and concentration was deduced in Fig. 6. The consistency of the data obtained from the initial settling rates and the technique of Font can be seen from this figure. The linear function between the Kozeny coefficient and concentration was deduced in Fig. 6.

Equation (20) was solved, using the spreadsheet finite difference model, for an initial solid concentration of 5.2% by volume. The result of the solution is compared to the experimental measurements in Fig. 7. At this concentration only limited channeling was observed, and the very close agreement between the model and measured values indicates that compression was significant for all the concentrations present during sedimentation, but that the channeling can be ignored. Figure 8 illustrates the concentration profile within the batch sedimentation vessel after settling for 22,000 seconds. Again the agreement between the experimentally measured values of concentration with height and those provided by the numerical model is excellent. The comparison between the model and experimental data illustrated in Figs. 7 and 8 is regarded as a more rigorous test of a numerical compression model than is given by simply comparing the interface settling height with respect to time; both the interface and local concentrations underneath the interface are correctly positioned. Thus it may be concluded that the numerical model accurately represents the compression of the talc mineral in the absence of significant channeling effects. It is worth noting that the permeability data used to provide one of the constituent equations used in the model were derived empirically, but from a test using a different initial concentration to the test numerically modeled.

Figure 9, however, illustrates the significance of channeling on the compressible talc system. In this instance the initial suspension concentration

![FIG. 6 Variation in Kozeny coefficient for talc with solid concentration obtained from permeability data using two different experimental techniques.](image)

![FIG. 7 Measured and predicted lines of constant solids concentration for a talc sedimentation at an initial concentration of 5.2% v/v.](image)
Experimental and not by the term. 

FIG. 9 Measured lines of constant solids concentration position during a tare sedimentation at an initial concentration of 5.2% v/v.

was 11.2% by volume, and significant channeling was observed. The initial rate of descent of the settling interface, and the final sediment height, are correctly predicted by the numerical model considering compression only, but there is considerable discrepancy between all the predicted and measured lines of constant solids concentration and with the interface settling zone. 

The uniform increase in lines of constant solids concentration position during channeling sedimentation supports the concept of augmenting the permeability, or decreasing the Kozeny coefficient, during the channeling part of the sedimentation by a time-dependent function only (13). However, such a time-dependent function will only be valid for the sedimentation under consideration and is not applicable to ones starting at a different concentration, etc. Thus, augmenting permeability in a numerical model is a useful approach to account for the existence of channels, but the augmentation must be based on a more fundamental basis than a time function. Such a basis does not yet exist and may provide a fruitful area of research.

The experimental results illustrated in both Figs. 5 and 9 show some interesting results worthy of further mention: the divergent lines of constant solids concentration in the compression zone. These are evident in Fig. 5 from a time of 30,000 seconds (note the 0.17 to 0.19 isoconcentration curves), and to a lesser extent in Fig. 9 at concentrations between 0.20 and 0.22. Curves of such shape have been reported before (18, 28) with other mineral suspensions. The shape of these curves cannot be explained by compression alone: they appear to accelerate into the column and are associated with sedimentation displaying channeling.

The last objective of this work was to investigate the varying contribution from the suspended solids to the liquid pressure gradient, and hence the buoyancy force experienced by a particle during sedimentation of a compressible compact. The origin of a variable buoyancy force was described in the Introduction. The numerical model was formulated to calculate the liquid pressure in excess of the hydrostatic at any height within the vessel. The calculation was based on the assumption that the buoyancy term is given by the expression \( P_b = \rho_b - \rho \) and not by the term \( \rho_b - \rho_w \). Thus, if the liquid pressure in excess of the hydrostatic (excess pore pressure) should be calculated using the latter rather than the former approach, then the experimentally measured pressures will be significantly different from those predicted by the numerical model. For this part of the work an initial solid concentration that displayed insignificant channeling but significant compression was required. Adequate mixing of the
initial suspension imposed the upper limit of solid concentration usable at 17.2%. The suspension subsided slowly to provide a sediment with an average concentration between the two sensitive pressure transducers of 22.5%. Before sedimentation the excess pore pressure between these two transducers corresponded to approximately 600 Pa, and is in accordance with the equation

$$ΔP = Δh(p_s - \rho)g$$

where $Δh$ is the height difference between the transducers and

$$p_m = C_{ps} + (1 - C)\rho$$

These two equations are only applicable prior to compression taking place.

During sedimentation the excess pore pressure reduced to zero; i.e., liquid pressure reduced to that of the hydrostatic head alone, as solids settled out of suspension and all the particle weight became supported by a solid network from the base of the vessel. The numerical model predicts this behavior assuming that the buoyancy term remains $(p_s - \rho)$ throughout the process, and is not given by the term $(p_s - p_m)$ or the term $(p_s - p_a)$ together with a time-dependent function taking a value between $(1 - C)$ and unity as explained earlier. The results of this experiment and the numerical simulation, up to a time of 75,000 seconds, are given in Fig. 10.

The excellent agreement between the predicted and measured excess pore pressure indicates that the numerical model was correctly formulated, and concerns over the use of any function for buoyancy correction

![Graph](image)

**FIG. 10** Hydrostatic pressures and interface height for a talc sedimentation at an initial concentration of 17.2% w/w.

other than $(p_s - \rho)$ appears unfounded, at least in systems similar to that described here. This is a reassuring result as a varying buoyancy correction term would otherwise need to be incorporated into the numerical model, and it is not immediately apparent what sort of function the correction term would take.

**CONCLUSIONS**

Numerical models of sedimentation of compressible compacts have been available for some time. However, most of the experimental work reported with these models has considered only the rate of fall of the settling interface, and has used these measurements to obtain data for application in the model. Often interface settling curves, bending away from the origin, were observed. They are an indication of significant dilution below that interface and, therefore, will reduce the reliability of data deduced from the settling interface. In the study reported here an experimental technique provided a comprehensive set of results on the local conditions below the settling interface. Hydraulic permeability at low concentration was deduced by two experimental techniques, and the method of Font (14) was discovered to be reliable for low and intermediate concentrations. A variable Kozeny coefficient, which displayed a linear function with solid concentration, was apparent. It was used to extrapolate to high concentrations. The linear Kozeny coefficient function was incorporated into the numerical model, resulting in an expression that was dependent only upon concentration and the Kozeny coefficient, and not explicitly on permeability.

For stable numerical solutions the finite difference numerical model required an implicit (iterative) solution technique, and it was implemented on a conventional computer spreadsheet. Convergence occurred in only a few minutes. The model successfully predicted the sedimentation behavior of talc suspensions at initial concentrations below those at which significant channeling was observed. However, in cases of significant channeling the experimentally measured lines of constant solids concentration were positioned considerably higher in the vessel than the predicted ones, even at concentrations below the channel zone. Hence channeling increases the speed with which all concentrations form and propagate during sedimentation. Augmentation of the hydraulic permeability, or reduction of the Kozeny coefficient, may be one method to account for channeling within the numerical model, but this could only be achieved by "curve-fitting" the predicted and measured lines of constant solids concentration. The function chosen to augment the permeability should be of general applicability to sedimentations at different starting concentrations rather
than a time-dependent function that will be applicable to sedimentation at only one initial concentration. Such an approach does not yet exist.

A further complication to the numerical modeling of sedimentation with channeling is the dilution that occurs below the settling interface. Dilution below the interface is one of the main causes of the curvature away from the origin that the interface displays, and an accurate numerical model should incorporate this phenomenon. At the bottom of the region in which dilution occurred, "soft" channels exist. These appeared to be channels that owed their existence to the discharge of liquid from the underlying "hard" channels. The suspension surrounding the soft channels was in a semifluidized, and well-mixed, state. This condition will assist the uniform dilution of the suspension below the interface.

Liquid pressure measurements compared with the model prediction showed that the buoyancy correction within the weight term of the force balance is correctly given by the expression $\Delta \rho = \rho_b$ and not by the term $\Delta \rho = \rho_a$. Thus there does not appear to be a functional relation between buoyancy force and compressive pressure during sedimentation, at least for the material and conditions reported here.

REFERENCES
