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HIGH RESOLUTION ARRAY SIGNAL PROCESSING

by

BAYAN MAHDI SABBAR, B.Sc, M.Sc.

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy of the University of Technology, Loughborough.

1987

Supervisor: Professor J.W.R. Griffiths

Department of Electronic and Electrical Engineering
Loughborough University
England

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TO MY PARENTS

WIFE

AND CHILDREN
ABSTRACT

This study is concerned with the processing of signals received by an array of sensor elements which may range from acoustic transducers in a sonar system to microwave horns in a radar system. The main aim of the work is to devise techniques for resolving the signals arriving from closely spaced sources in order to determine the presence and direction of these sources.

A standard method for separating signals arriving from differing directions is to form a conventional beam, but although this well tried technique is very robust it is very limited in its capability particularly for arrays which are not long in terms of the wavelength of operation. In the thesis the many alternative methods which have been proposed in the literature are discussed in some depth and in particular a class of methods based on eigenvector decomposition. A substantial number of simulation programs have been written to compare the methods and in addition the eigenvector method has been extended to deal with the near-field sources. Difficulties arise with the application of these methods when the sources are correlated and a new method based on the combination of arrays is proposed and evaluated by computer simulation.

Some practical work has been carried out to show the capability of these algorithms using acoustic signals in a sonar test tank.
ACKNOWLEDGEMENTS

The author would like to thank his supervisor professor J.W.R Griffiths for his encouragement and help throughout the research, and Dr. J.Hudson who contributed many useful ideas and discussions.

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Finally I would like to acknowledge the support in my life from my parents and brothers and also sincere thanks to my wife Samira and my children who suffered a lot.
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<th>Description</th>
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<tbody>
<tr>
<td>( A )</td>
<td>Signal amplitude</td>
</tr>
<tr>
<td>( r )</td>
<td>Range from source to sensor</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>Phase angle of ( i )-th source</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Arriving angle</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Wavelength</td>
</tr>
<tr>
<td>( k )</td>
<td>( 2\pi/\lambda )</td>
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<tr>
<td>( N )</td>
<td>Number of sensors</td>
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<tr>
<td>( MP )</td>
<td>Number of sources</td>
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<td>( d )</td>
<td>Distance between sensors</td>
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<tr>
<td>( d_s )</td>
<td>Euclidean distance</td>
</tr>
<tr>
<td>( \ell )</td>
<td>Subscript for source index</td>
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<td>( x )</td>
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<td>( n )</td>
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<td>( T )</td>
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<td>( \sigma_n^2 )</td>
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<td>Covariance matrix</td>
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<td>( R_s )</td>
<td>Signal covariance matrix</td>
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<tr>
<td>( P )</td>
<td>Number of snapshots</td>
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<tr>
<td>( C(\ ) )</td>
<td>Steering vector</td>
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<tr>
<td>( \psi_i )</td>
<td>Eigenvector</td>
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<td>( Z )</td>
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<td>( U )</td>
<td>Eigenvectors matrix</td>
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<td>( P_1 )</td>
<td>Total number of elements in the subset</td>
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<td>( q )</td>
<td>Index of subarray</td>
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<tr>
<td>( t_{de} )</td>
<td>Time delay</td>
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<tr>
<td>( t )</td>
<td>Time</td>
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<tr>
<td>( \Delta )</td>
<td>Distance between arrays</td>
</tr>
<tr>
<td>( \Delta )</td>
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<td>( y(\theta) )</td>
<td>The beampattern of a linear array</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>--------</td>
<td>-------------</td>
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<tr>
<td>Q</td>
<td>Total number of arrays</td>
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<td>Phase angle</td>
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<td>$X_s$</td>
<td>Signal vector</td>
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<td>$\Sigma$</td>
<td>Summation sign</td>
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<td>$/$</td>
<td>Division sign</td>
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<td>Frequency</td>
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<td>Spatial frequency</td>
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<td>$h$</td>
<td>Focusing vector</td>
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<td>Parameters of Byrne and Steele method.</td>
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<td>Maximum likelihood</td>
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<td>$W$</td>
<td>Weighting matrix</td>
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<td>$w_i$</td>
<td>Weight value</td>
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<td>$H(\cdot)$</td>
<td>Cost function</td>
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<td>Noise eigenvectors matrix</td>
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<td>$E_S$</td>
<td>Signal eigenvectors matrix</td>
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<td>$v$</td>
<td>Vector in the noise subspace</td>
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<tr>
<td>$g$</td>
<td>First row of $E_S$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>First row of $E_N$</td>
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<td>$E_S'$</td>
<td>Signal eigenvectors matrix with first element deleted</td>
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<tr>
<td>$E_N'$</td>
<td>Noise eigenvectors matrix with first element deleted</td>
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<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<tr>
<td>$\ln$</td>
<td>$\log_e$</td>
</tr>
<tr>
<td>$v$</td>
<td>Wave velocity</td>
</tr>
<tr>
<td>AIC</td>
<td>Akaike Information Criterion</td>
</tr>
<tr>
<td>AMC</td>
<td>Akaike Modified Criterion</td>
</tr>
<tr>
<td>CBM</td>
<td>Conventional Beamformer Method</td>
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<tr>
<td>EVD</td>
<td>Eigenvector Decomposition</td>
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<tr>
<td>MDL</td>
<td>Schwartz and Rissanen criterion</td>
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<tr>
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<td>Maximum Entropy Method</td>
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<td>MLM</td>
<td>Maximum Likelihood Method</td>
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<td>MNM</td>
<td>Minimum Norm Method</td>
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<td>MUSIC</td>
<td>Multiple Signal Classification</td>
</tr>
<tr>
<td>SEVM</td>
<td>Signal eigenvector method</td>
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CHAPTER 1

INTRODUCTION

1.1 The problem and the scope of the thesis

Arrays of sensors are used in many applications such as radar, underwater acoustics, radio direction finding and geophysics. Array processing deals with the processing of the signals received by such an array of sensors and the principle aim is to process the signals from the array so as to determine the characteristics of the field of interest as accurately as possible. The two major problems common to all of these applications are the poor resolution obtainable from receiving arrays that are physically small in terms of the wavelength, and the ambiguities that arise due to the sidelobes of the beampattern. This is the reason for the search for array signal processing techniques which exhibit high angular resolution capability without ambiguity.

One of the most commonly used arrays is the linear array, i.e., a set of equally spaced sensors in a straight line, and the research described in this thesis is mainly concentrated on this type of array. It may be shown fairly simply that the distribution of sources in the far-field, the so-called angular spectrum, is related to the complex amplitude distribution across the aperture by the Fourier Transform. Thus, there is a very close analogy between the problem of obtaining the angular spectrum from a measured aperture distribution and that of obtaining the frequency spectrum from a time waveform.

Normal beamforming corresponds in fact to taking the Fourier Transform of the aperture distribution. The beampattern of
the array is its response to a single point source in the far-field plotted against the angle of the source. This is akin to testing the frequency response of a linear system by using a single sinusoid of a varying frequency.

A commonly used measure of an array's resolution is the Rayleigh criteria, which states that two sources are resolvable if their angular separation is at least half the BWFN [ref.46] [BWFN is the beamwidth between first nulls or the two points of zero response nearest to the direction angle of the mainlobe].

Since the beamwidth is inversely related to the length of the aperture then to improve resolution the size of the array must be increased. This solution, however, is of limited value, as the size of the array is often restricted by physical considerations. As an example, in sonar it is necessary to use low frequencies because of the high attenuation of acoustic waves as frequency increases. A typical frequency for a long range passive sonar may be as low as 100 Hz and at a velocity of 1500 m/s this corresponds to a wavelength of 15 m. Thus to obtain resolution by conventional beam processing would require a very long array [ref.28].

This lack of resolution is the main limitation of the traditional methods but in addition the relatively high sidelobe responses (equivalent to so-called leakage in frequency analysis) also causes difficulty. For a uniform linear array, the first sidelobe is only about 13 dB below the main lobe. Thus, a strong interference arriving from a direction corresponding to one of the sidelobes might easily cause an output which is greater than the signal coming from the direction to which the beam is pointed. This results in ambiguity in determination of the direction of the signal. Various shading techniques to weight the aperture distribution have been developed and these can help to
alleviate the problem of the sidelobes but only at the expense of resolution.

Over the last decade or so, there has been considerable interest in so-called modern spectral analysis techniques and these have been the subject of intensive investigation among many researchers in various fields. An excellent review of the application of these techniques in the frequency domain was published by Kay and Marple [ref.40] and a good survey of the application to array processing can be found in the books published by Childers [ref.15], Hudson [ref.33] and Haykin [ref.30]. All these works contain extensive references. These new algorithms offer the possibility of obtaining much better angular resolution together with good sidelobe suppression when compared to that obtainable by the more traditional approaches using the Fourier Transform and the classical "lag window" methods [ref.53]. It must be said however that most of these new methods require a considerable amount of computation.

More and more these days the processing of the received signals is carried out by digital methods. The nature of arrays is such that spatial sampling is already discrete but in addition the incoming signal waveform on each element is sampled, giving a so-called snapshot of the signal being received by the array at a particular instant in time. This thesis is concerned with the processing of such snapshots in a digital fashion and in particular the use of the covariance matrix which relates the signals from the various combinations of elements.

Recently there has been a considerable interest in a group of techniques which depend upon the properties of the eigenvectors of this covariance matrix and it is mainly on these methods that the thesis concentrates [ref.11,12,35,41,47,59,69].
1.2 Thesis organisation

Chapter 2 is a preliminary phase providing the necessary modelling of sources in the far-field and the different methods used to obtain the covariance matrix.

Chapter 3 reviews current adaptive techniques and gives a description of a number of different algorithms, although a direct comparison of the methods is not attempted at this stage. This problem is treated in later chapters for selected algorithms.

Chapter 4 examines the performance of some adaptive algorithms in resolving correlated sources using a linear array of sensors. The task of partitioning of the signal eigenvalues from the noise eigenvalues is studied and a new method is proposed for separating the two groups. The fully correlated sources and multipath phenomena are dealt with using a new technique of subgrouping of arrays and applying the two eigenvector methods, Multiple Signal Classification Method (MUSIC) proposed by Schmidt and Minimum Norm Method (MNM) proposed by Tufts and Kumaresan.

Chapter 5 examines the resolution of narrowband signals from near-field sources impinging on an array of sensors and discusses the application of eigenvector methods such as MUSIC and MNM algorithms to separate these sources. Two fairly simple methods are developed to obtain the same type of performance in the near-field as is obtained in the far-field.

Chapter 6 is concerned with some experimental work and the application of the high resolution techniques to the practical measurements. The basic experiments involved the transmission of a series of sonar pulses within a large
water tank from two active sources and reception of these pulses by a receiving transducer moving along the array line. The real data was used as a common input to most of the algorithms mentioned in chapter 3 to show their performance in resolving these two sources. Valuable results are shown to provide a solid base for choosing the proper algorithms to be used in real time.

Finally chapter 7 has some conclusions on the results obtained in the research and some suggestions for further work.
2.1 Introduction

This chapter introduces the fundamental model of an array of sensors receiving signals from a number of sources which are located in the far-field. The array under consideration is a discrete equally spaced linear array of isotropic receiving sensors as shown in figure (2.1). The output of the sensors is considered as the input to the covariance matrix processor. Two main approaches to obtain the covariance matrix are presented. The first is the theoretical representation and the second is concerned with the simulation approach. Both representations are derived for correlated and uncorrelated sources.

2.2 System modelling

We assume that time series data received by an array, is transmitted from a number of sources located in the far-field. Although the general case in practice may often be broadband, the system assumed here is narrowband. However, the broadband system can be reduced to a number of narrowband systems [ref. 20, 26] and hence the assumption is not too restrictive. In the narrowband system, we can use band pass sampling techniques whereas in the broadband system, the sampling rate has to be greater than twice the highest frequency in the band. Another advantage of the narrowband system is that the array can be steered simply by phase changes, whereas in the broadband system physical delay is required. The source signal is assumed to be a
sinusoid waveform defined by:

\[ a_2(t) = A_2(t) \exp(i\omega t). \]  

(2.1)

for \( t = 1, 2, \ldots, M \).

Where \( A_2(t) \) is the random complex amplitude of the exciting source.

2.2.1 Theoretical model for uncorrelated sources.

Figure (2.1) depicts a linear array of \( N \) sensors uniformly spaced \( d \) units apart. A plane wave is impinging upon the array with an incident angle \( \theta \). A basic derivation of the relationship between the signal direction \( \theta \) and the wavenumber \( k \) has been expressed by many researchers (e.g. Bronez and Cadzow[ref.8]). By defining our origin at sensor one, the \( i \)-th sensor will sample the wave at the point \((i-1)d\). Hence at any particular instant in time the array output is[ref.26].

\[ s_{2i}(t) = a_2(t)c_{2i}. \]  

(2.2)

where

\[ c_{2i} = \exp[j(1-1)u_2] \]

and

\[ u_2 = kds\sin(\theta_2). \]  

(2.3)

\( k = 2\pi/\lambda \) wavenumber or spatial frequency.

As the system is assumed to be narrowband, then a phasor notation can be used. Thus, \( x_{2i} \) is assumed to be the random complex amplitude of the signal at the \( i \)-th sensor due to \( \lambda \)-th source. Equation(2.2) can be rewritten as a multiplication of complex amplitude \( A_2 \) and the effect of propagation \( c_{2i} \) as follows.
or

\[ x_{k_i} = A_k \cdot e_{k_i} \] (2.4)

\[ x_{k_i} = A_k \cdot \exp[j(k-1)u_k] \] (2.5)

We now extend our model to include multiple plane waves incident on the array of \( N \) sensors. Thus the output of each sensor will be:

\[ s_i(t) = \sum_{k=1}^{MP} s_{k_i}(t) \] (2.6)

where \( MP \) is the total number of sources.

The complex signal \( x_i \) at the output of each sensor due to these sources can be expressed as

\[ x_i = \sum_{k=1}^{MP} x_{k_i} = \sum_{k=1}^{MP} A_k \cdot e_{k_i} \] (2.7)

Let \( S^T(t) \) is the signal vector at the output of \( N \) sensors as

\[ S^T(t) = [s_1(t), s_2(t), \ldots, s_N(t)] \] (2.8)

\[ C_k = \begin{bmatrix}
1 \\
\exp[ju_k] \\
\exp[j2u_k] \\
. \\
. \\
. \\
\exp[j(N-1)u_k]
\end{bmatrix} \] (2.9)
\[ A^T(t) = [a_1(t), a_2(t), \ldots, a_{MP}(t)] \]

and

\[
C = \begin{bmatrix}
\cdot & \cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdot & \cdots & \cdot \\
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\cdot & \cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdot & \cdots & \cdot \\
\end{bmatrix}
\]

Thus

\[ S(t) = A(t) \cdot C \quad (2.10) \]

In practice, the sensor indications are contaminated by white noise \( n_i \) which we will assume has zero mean and power \( \sigma_i^2 \). Thus, the complex amplitude \( x_i \) at the output of \( i \)-th sensor will be:

\[
x_i = n_i + \sum_{k=1}^{MP} x_{i_k} \exp[j(i-1)u_{ik}],
\]

or

\[
x_i = n_i + \sum_{k=1}^{MP} A_{ik} \exp[j(i-1)u_{ik}], \quad (2.11)
\]

In a narrowband system we can simply multiply the sensor signal \( x_i \) by a complex weight which would affect its amplitude and phase. The output of the system will then be

\[ y = w_1 x_1 + w_2 x_2 + \ldots + w_N x_N \quad (2.12) \]

Define a weight vector \( W \), where

\[ W^T = [w_1, w_2, \ldots, w_N] \]

and a vector
where $X$ is regarded as a 'snapshot' of the outputs of the various sensors at a particular instant in time. The output $y$ is given by the inner product of these two vectors, namely

$$y = y^T X$$  \hspace{1cm} (2.13)$$

$y$ is a scalar but may, of course, be complex. The average output power ($P$) is given by the expected value of the modulus of $y$, i.e.

$$P = E[|y|^2] = E[|y|^2] = y^T R y$$  \hspace{1cm} (2.14)$$

where

$$R = E[X^* X^T]$$  \hspace{1cm} (2.15)$$

Thus the task in the following stages is to obtain, theoretically and by simulation, the covariance matrix $R$ for the uncorrelated and correlated sources. The components of this covariance matrix $R$ represents the correlation between the different channel signals. Thus $R$ can be written as:

$$R = E \begin{bmatrix} x_1^* x_1 & x_1^* x_2 & \cdots & x_1^* x_N \\ x_2^* x_1 & x_2^* x_2 & \cdots & x_2^* x_N \\ \vdots & \vdots & \ddots & \vdots \\ x_N^* x_1 & x_N^* x_2 & \cdots & x_N^* x_N \end{bmatrix}$$  \hspace{1cm} (2.16)$$

It is clear that the components of matrix $R$ may be obtained from the channel signals defined in equation (2.11) whereupon these averaged correlations are derived as follows:
The assumption made here is that the signals and noise are independent, and the noise from one sensor to another is also independent. Moreover, in this stage the signals are also independent (i.e., the correlation coefficient is zero). Thus,

\[ E[n_i^* \cdot n_m] = 0 \quad \text{for } i \neq m \]

or

\[ E[n_i^* \cdot n_m] = E[|n_i|^2] = \sigma_i^2 \quad \text{for } i = m \] (2.18)

Substituting equation (2.2) in equation (2.20) and let the source power \( P_x = E[|A_x|^2] \) then,

\[ \sum_{k=1}^{MP} E[x_i^* \cdot x_m] = \sum_{k=1}^{MP} E(A_k^* \exp[-j(i-1)u_k]) \]

\[ = \sum_{k=1}^{MP} P_k \exp[-j(i-m)u_k] \quad \text{for } i \neq m \]

or

\[ = \sum_{k=1}^{MP} P_k \quad \text{for } i = m \] (2.21)

The assumption that \( E[A_x] = |A_x| \) and the source power defined before are only true if the source random complex amplitude \( A_x = |A_x| \exp(j\phi_x) \) and \( \phi_x \) is the random variable.
Now by substituting equations (2.18), (2.19) and (2.21) in equation (2.17) then it will be:

\[ E[x_i x_m] = \sum_{k=1}^{MP} P_k \exp[-j(i-m)\omega_k] \]

for \( i=m \)  \hspace{1cm} (2.22)

and

\[ E[x_i x_m] = |x_i|^2 = \sigma_i^2 + \sum_{k=1}^{MP} P_k \]

for \( i=m \)  \hspace{1cm} (2.23)

Thus, the covariance matrix can be considered as the sum of the noise matrix \( R_N \) and individual interference source matrices \( R_k \) as follows:

\[ R = R_N + \sum_{k=1}^{MP} R_k \]  \hspace{1cm} (2.24)

where

\[
R_N = \begin{bmatrix}
\sigma_1^2 & 0 & 0 & \cdots & 0 \\
0 & \sigma_2^2 & 0 & \cdots & 0 \\
0 & 0 & \sigma_3^2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \sigma_N^2 \\
\end{bmatrix}
\]

(2.25)

The signal covariance matrix \( R_k \) is,

\[
R_k = P_k \begin{bmatrix}
1 & \exp[j\omega_k] & \cdots & \exp[j(N-1)\omega_k] \\
\exp[-j\omega_k] & 1 & \cdots & \exp[j(N-2)\omega_k] \\
\exp[-j2\omega_k] & \exp[-j\omega_k] & \cdots & \cdots \\
\exp[-j(N-1)\omega_k] & \exp[-j(N-2)\omega_k] & \cdots & 1 \\
\end{bmatrix}
\]

(2.26)

Note that \( R \) is a positive definite Hermitian matrix and can be expressed as:

\[
R = \begin{bmatrix}
b_{11} & b_{12} & b_{13} & \cdots & b_{1N} \\
b_{21} & b_{22} & b_{23} & \cdots & b_{2N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
b_{N1} & b_{N2} & b_{N3} & \cdots & b_{NN} \\
\end{bmatrix}
\]

(2.27)
Where

\[ b_{im} = b_{im}(\text{real}) + jb_{im}(\text{imag}) \quad \text{for} \quad i \text{ and } m = 1, N \]  
\[ (2.28) \]

\[ b_{im}(\text{real}) = \sum_{k=1}^{r} P_k \cos[(i-m)u_k] \quad \text{for} \quad m \neq 1 \]  
\[ (2.29) \]

\[ b_{im}(\text{real}) = \sigma_i^2 + \sum_{k=1}^{r} P_k \quad \text{for} \quad m = 1 \]  
\[ (2.30) \]

\[ b_{im}(\text{imag}) = -\sum_{k=1}^{r} P_k \sin[(i-m)u_k] \quad \text{for} \quad m \neq 1 \]  
\[ (2.31) \]

\[ b_{im}(\text{imag}) = 0.0 \quad \text{for} \quad m = 1 \]  
\[ (2.32) \]

Equations (2.29) to (2.32) are simplified version of equations (2.22) and (2.23).

2.2.2 Theoretical model for correlated sources

As shown in equation (2.17) the elements of the covariance matrix are calculated as:

\[ b_{im} = E[x_i^*x_m] \]  
\[ (2.33) \]

The source signals are assumed to be correlated while the noise has the same assumption made in equations (2.18) and (2.19). Thus,

\[ b_{im} = E[(A_1^*e^{j(1-i)u_1} + A_2^*e^{j(1-i)u_2})(A_1e^{j(m-1)u_1} + A_2e^{j(m-1)u_2})] \]

or

\[ b_{im} = E[A_1^*A_1] e^{j(m-i)u_1} + E[A_1^*A_2] e^{j((m-1)u_2-(i-1)u_1)} + E[A_2^*A_1] e^{j((m-1)u_1-(i-1)u_2)} + E[A_2^*A_2] e^{j(m-i)u_2} \]  
\[ (2.34) \]
In general \( \mathcal{E}[\Lambda_i^* \Lambda_j] \) is the expectation of the inner product of the complex sources amplitudes of the \( i \)-th and the \( j \)-th sources. Thus the correlation coefficient \( c_f \) will be

\[
c_f = \frac{\mathcal{E}[\Lambda_i^* \Lambda_j]}{\sqrt{\mathcal{E}[|\Lambda_i|^2] \mathcal{E}[|\Lambda_j|^2]}}
\]

Using the assumptions about \( \Lambda_i \) used in section (2.2.1) then the following relations are true:

\[
\mathcal{E}[\Lambda_1^* \Lambda_1] = p_1, \quad \mathcal{E}[\Lambda_2^* \Lambda_2] = p_2
\]

and

\[
\mathcal{E}[\Lambda_1^* \Lambda_2] = \mathcal{E}[\Lambda_2^* \Lambda_1] = |\Lambda_1| \Lambda_2 |c_f|
\]  \hspace{1cm} (2.35)

In the case where two sources are fully correlated \( c_f \) is one and \( c_f \) is zero when the sources are independent.

Equation (2.34) can be rewritten as:

\[
b_{i \text{m}}(\text{real}) = p_1 \cos[(m-1)u_1] + p_2 \cos[(m-1)u_2] + |\Lambda_1||\Lambda_2|c_f \cos[(m-1)u_2-(1-1)u_1]
\]

\[
+ |\Lambda_2||\Lambda_1|c_f \cos[(m-1)u_1-(1-1)u_2] \hspace{1cm} (2.36)
\]

\[
b_{i \text{m}}(\text{imag}) = p_1 \sin[(m-1)u_1] + p_2 \sin[(m-1)u_2] + |\Lambda_1||\Lambda_2|c_f \sin[(m-1)u_2-(1-1)u_1]
\]

\[
+ |\Lambda_2||\Lambda_1|c_f \sin[(m-1)u_1-(1-1)u_2] \hspace{1cm} (2.37)
\]

In the presence of noise, it is obvious that the noise variance is added only to the real diagonal values according to the assumptions made earlier.

\[
b_{i j}(\text{real}) = b_{i j}(\text{real}) + \sigma_i^2
\]  \hspace{1cm} (2.38)
For MP correlated sources equation (2.34) becomes:

\[
\begin{align*}
b_{lm} &= \mathbb{E}\left[ (A_1 e^{j(m-1)\phi} + A_2 e^{j(m-1)\phi})^* (A_1 e^{j(m-1)\phi} + A_2 e^{j(m-1)\phi})^* \right] + \ldots + A_{MP}^* e^{j(m-1)\phi}\right]
\end{align*}
\] (2.39)

### 2.2.3 Simulation model

In practice an estimate of the covariance matrix is calculated from the data received by the array sensors. The sampled data is measured from \( P \) snapshots at each sensor and it can be represented as

\[
X_k^T = [x_{1k}, x_{2k}, \ldots, x_{Nk}].
\] (2.40)

where \( k \) is the snapshot instant. The "snapshot" is defined as one simultaneous sampling of the aperture signals at all array sensors. The data matrix can be formed from a given sequence of snapshot vectors as:

\[
D = [x_1, x_2, \ldots, x_P].
\] (2.41)

Substituting equation (2.40) in equation (2.41) then the data matrix \( D \) will be:

\[
D = \begin{bmatrix}
x_{11} & x_{12} & \cdots & \cdots & x_{1P} \\
x_{21} & x_{22} & \cdots & \cdots & x_{2P} \\
\vdots & \vdots & \ddots & \cdots & \vdots \\
\vdots & \vdots & & \ddots & \vdots \\
x_{N1} & x_{N2} & \cdots & \cdots & x_{NP}
\end{bmatrix}
\] (2.42)
Thus,

\[ R = \frac{1}{p} \left[ D^* D^T \right] \]

or

\[ R = \frac{1}{p} \sum_{k=1}^{p} [x_k^* x_k] \quad (2.43) \]

Substituting equation (2.40) in equation (2.43) then,

\[ b_{lm} = \frac{1}{p} \sum_{k=1}^{p} [x_k^* x_{mk}] \quad (2.44) \]

Using the same assumptions of \( \lambda_k \) used in the preceding sections then the source amplitude samples \( \Lambda_{k,k} \) can be defined by:

\[ \Lambda_{k,k} = |\Lambda_k| \exp[j \phi_{k,k}] \]

\( \phi_{k,k} \) = random phase of \( k \)-th sample from \( k \)-th source.

From the above relation, it is clear that \( \Lambda_{k,k} \) is random complex because \( \phi_{k,k} \) is randomly generated for each source. So the sources are uncorrelated. However, equation (2.11) can be rewritten in the following form to simulate the samples \( x_{ik} \)’s.

\[ x_{ik} = n_{ik} + \sum_{k=1}^{M_p} \Lambda_{k,k} \exp[j (1-1) u_k], \]

or

\[ x_{ik} = n_{ik} + \sum_{k=1}^{M_p} |\Lambda_k| \exp[j (1-1) u_k + \phi_{k,k}], \quad (2.45) \]

where

\( u_k \) is defined in equation (2.3).
Fig. 2.1 Representation of linear array receiving signals coming from the far-field (regular spacing of sensors)
3.1 Introduction

A review of several spatial resolution techniques in published literature reveals that they can be classified into two categories: a) the direct methods and b) the indirect methods. In the direct methods the power and direction of the received signal can be obtained directly from a spectral beam such as the conventional beamformer, whereas in the indirect methods, the power is calculated indirectly from the detected direction of arrival as in the methods based on the eigenvector decomposition technique.

Conventional beamformers require less computation particularly when the bandwidth is wide, and are widely used in practice [ref.17]. The indirect methods are used in high resolution systems but at a price because of their computational complexity [ref.17]. The resolution advantages of indirect methods, in particular eigenvector decomposition methods, are most significant when processing short data sets.

In general, the high resolution techniques can now provide asymptotically unbiased estimates of the number of sources, source directions, and source strengths [ref.52].

Using the newer techniques in processing short data sets, implies a potential for improved resolution for arrays with few elements, and this chapter gives a short background on some of these algorithms. Also the derivation, advantages and disadvantages of the some of the algorithms are discussed.
3.2 Literature survey

In the late 1960s, the maximum entropy method (MEM) introduced by Burg [ref.9,10,51] and the maximum likelihood method introduced by Capon [ref.10,14,44,48], attracted the attention of scientists and engineers. They have been used in high resolution temporal frequency analysis and spatial angular power estimation.

In particular, it would appear that the desirable stability of the maximum likelihood wavenumber estimator can be balanced against the desirable high resolution and undesirable instability [ref.30] of the MEM by using a combination of both methods [ref.10].

A third method which has been given much attention in 1973 and after, is the so-called Pisarenko harmonic retrieval method [ref.37]. This method is based on the use of the smallest eigenvector of the correlation matrix obtained from the observed random process. A practical difficulty related to the Pisarenko method is that the number of sinusoids is usually not known in advance [ref.37]. Thus, the correct length of data vector is determined by increasing the number of sinusoids till the smallest eigenvalue remains unchanged. The Pisarenko harmonic retrieval method for the one dimension was improved later by Schmidt in 1979 [ref.59]. The improvement presented by Schmidt for multiple signal classification (MUSIC) is very general and has wide application.

Another method for the one dimensional problem, which can be considered as an extension of the Pisarenko method in the frequency domain, yields asymptotically unbiased estimates of the direction of arrival (DOA) of multiple narrowband sources. This method, which was pioneered by Bienvenu in
1979 [ref.5], is based on the eigenvector decomposition of the spectral density matrix of the 'snapshot' vector and is the exact analogue of Schmidt's method in the frequency domain [ref.37]. However, the average of noise eigenvector beams [ref.5,59] and that of the signal eigenvectors used by Barlett [ref.41], has become the more interesting work in high resolution techniques in the last decade. They seem to provide a fundamental approach to the DOA estimation problem and have yielded the most promising results among all the high resolution methods available today [ref.65].

The eigenvector method proposed by Johnson and DeGraaf [ref.35] is similar to MUSIC except that the beam eigenvector is weighted by its eigenvalue. They produce quite similar results in the spatial spectrum. The main difficulty of the eigenvector method [ref.35], and MUSIC [ref.59] is the number of computations required by the full eigenvector analysis and the evaluation of the spatial spectrum using all the noise eigenvectors. Therefore, another two methods have been proposed, the first by Bronez and Cadzow [ref.8], which has a difficulty similar to the Pisarenko method, and the second one by Tufts and Kumaresan which is able to reduce the computation time of the spatial domain spectrum.

The eigenvector decomposition techniques (EVD) have widely been applied recently in adaptive signal processors. These methods have attracted considerable interest since they seem to provide a fundamental approach to the DOA estimation problem.

The present adaptive processors fail to operate where the desired signal and interference are coherent. Coherent interference can arise when multipath propagation is present, or when 'smart' jammers deliberately introduce coherent interference, e.g. by retrodirectivity of the signal energy to the receiver. Coherence can completely
destroy the performance of adaptive array systems [ref.64]. Shan and Kailath [ref.64] suggested an adaptive processor system which is able to overcome this degradation of performance in coherent input environments, without considerably increasing the complexity of the system structure or the computational burden. This new adaptive processor is also based on the eigenstructure of the covariance matrix obtained from the average of covariance matrices resulting from subarrays. The algorithm gives nulls in the interference directions. The idea of subarrays was used by Tang and others in the late 1960s [ref.71]. The coherence problem has also been discussed by other researchers using the eigenvector decomposition techniques [ref.12,19,29].

There is another ill-condition in the adaptive beamforming methods when the number of array elements are greater than the number of sources which amplifies estimation, arithmetic, and other system errors[ref.3]. It is defined as the ratio between the maximum eigenvalue and minimum eigenvalue which is occasionally very large in practical and simulated systems. In the numerical analysis literature it is well known that eigenvalue decomposition is the only reliable method for detection and correction of the above ill-conditioning [ref.43]. Sibul [ref.62] has shown how this ill-conditioning arises in adaptive beam processing, and how the eigenvalue preprocessor was used to correct it by reducing the dimensionality of the adaptive beamformer to correct dimensions required by the number of uncorrelated sources.

The performance of EVD has been compared, by many researchers, with the conventional beamformer method (CBM), MLM, and MEM in resolving targets for real passive sonar data. Schneider [ref.57], Scholz and Kroll [ref.58], Lucas and LeCadre [ref.49], Pignon and Rithouey [ref.56], and Gray [ref.24] compared the performance of these methods and have
shown that MLM, MEM and EVD give a better separation than CBM. The EVD seems to have the best detection ability compared with MLM and MEM [ref. 24, 49, 56, 57, 58].

The eigenstructure methods developed so far require that the additive sensor noise be spatially white, i.e., of equal power and uncorrelated from sensor to sensor. The problem of non-white noise that has a known covariance (or equivalently, a known angular power spectrum) can be tackled either through prewhitening or by solving a generalized eigenvalue problem [ref. 5]. But in the case of unknown noise covariance problem, Paulraj and Kailath [ref. 55] presented a technique which is applicable to situations where it is possible to obtain two estimates of the array covariance in which the unknown noise field remains invariant while the signal field undergoes some change. Their method is based on computing the difference of the two measured covariances, thus subtracting out the unknown noise covariance and leaving only the difference matrix of the two covariances.

Finally the eigenvector decomposition techniques are also applied to solve the problem of high frequency radio direction finding [ref. 36]. This section has covered the applications and performance of some eigenvector decomposition techniques in narrowband systems. But these newer techniques can also be applied in resolving targets in the broadband systems by reducing the broadband system into narrowband systems. The reduction can be done by two main methods either by the Frost method [ref. 20] or by carrying out an FFT at each sensor and then processing the resulting narrowbands individually. These newer techniques are not viewed as a "superresolution" replacement for more conventional estimation methods, but rather, the technology is considered complementary to the other methods and best used in parallel [ref. 23].
3.3 The Spatial resolution techniques

The following algorithms will be explained in detail in this section.

1. The classical method (Conventional beamformer)
2. The maximum likelihood method (Capon method)
3. The maximum entropy method (Burg method)
4. The eigenvector decomposition methods
   a. Principal components method (Barlett method)
   b. MUSIC method (Schmidt’s method)
   c. Wax and Kailath methods
   d. DeGraaf and Johnson method
   e. Byrne and Steele method
   f. Minimum Norm Method MNM (Kumaresan & Tufts method)
   g. Cantoni and Godara method
   h. The generalised approach

3.3.1 The classical method (The conventional beamformer)

Given a covariance matrix $R$, which is the correlation matrix of the output of a linear array, the system can be represented as a bank of matched filters, which are called spatial filters. The steering beams $C(\theta)$ can be stored in the processor memory and the task of this processor is to deal the covariance matrix with each entry from these stored steering beams. Then the output of this process can be displayed as a beamformer and it is presented mathematically by:

$$P_{CBM}(\theta)=C^H(\theta)RC(\theta) \quad (3.1)$$

Where
The coefficients $c_i$'s are calculated in chapters 2 and 5, depending upon whether the field is far-field or near-field.

The direction, $\theta$, or the wanted spatial frequency, is taken as the one where the maximum value of $P_{CBM}(\theta)$ occurs. This will be the direction of strongest signal, and the next maxima will be considered as the second source/target and so on. This simple beamformer can be applied in real time because it does not need a diagonalization process to the covariance matrix [ref.17].

The disadvantages of this technique are:

1-Producing unwanted outputs from signals arriving from directions other than the wanted directions, i.e., sidelobe effects [ref.31].

2-Resolution performance between two close sources separated by less than the standard beamwidth is very poor due to the broadness of the maxima.

3.3.2 The maximum likelihood method (Capon method)

The maximum likelihood method (MLM) was originally developed by J. Capon [ref.13] as a tool for analysing the data from a very large seismic array. Thus, the original use of the technique was for frequency-wavenumber analysis. Capon [ref.13] states that the wavenumber resolution achievable by MLM is much greater than by conventional methods and is limited primarily by the signal to noise ratio. The maximum likelihood spectral estimate is defined as a filter designed to pass the power in a narrowband about the signal frequency of interest, and to minimize or reject all other frequency components in an optimal manner [ref.13,48].
Gabriel [ref.21], in his description of a type of adaptive array whose operation is equivalent to an MLM spectral estimation, listed several advantages of this technique:

1-The output power is directly referenced to receiver noise power, thus permitting calibration and measurement of relative source strength. Other methods (MEM and eigenvector methods) do not give a direct estimate of signal strength.

2-A pseudolinear superposition holds at the peaks, if the sources can be resolved, giving true relative source strengths among several signals. Again, some other methods cannot do this.

3-The output of the filter is a real, physical signal. If the filter is steered to a particular source, one can monitor it at full array gain while rejecting all others. Other techniques do not provide physical signals. (A null, for example, can give a very good indication of angle of arrival, but the signal coming in at that angle cannot be monitored at full gain.)

4-The residual background ripple is low and well behaved.

5-The array sensors need not to be equally spaced or spaced near λ/2. Thus, one can spread the elements in a wider aperture and the resolution for a given number of elements. This technique is used in the geophysics field [ref.13].

The frequency mentioned in the definition of MLM is of general form, but in the spatial domain it is equal to:

\[ f_s = 2\pi d/\lambda \sin(\theta_s) . \]
e. is the only variable in this relation. The coefficients of the mentioned filters are chosen so that the response at the direction of interest is unity and the output variance is minimized. The filters are adjusted to reject power from other frequencies (directions) not near the frequency (direction) of interest in an optimal, adaptive manner.

This representation is similar to the conventional beamformer explained in section 3.1. The difference between the conventional method and MLM is that the shape of the MLM narrowband filters changes, in general, for each frequency to optimally reject out-of-band signals. In the conventional method, the shape of the filter is fixed [ref.40].

The MLM filters are finite impulse response (FIR) types with N weights \( w_i \), \( i=0, 1, 2, \ldots, N-1 \).

\[
W^T = [ w_0, w_1, \ldots, w_{N-1} ]
\]  

(3.3)

The derivation of the method involves minimizing the output variance (output power)

\[
P = W^H R_s W
\]

(3.4)

where \( P \) is the output power and \( R_s \) is the signal covariance matrix. If the system is constrained to the unity spatial angle response (frequency response \( f_s \)) the condition at \( \theta \) (\( f_{s_0} \)) is,

\[
W^H C = 1
\]

(3.5)

Here, the \( H \) indicates Hermitian transpose. \( C \) is the direction vector as defined in chapter 2. Using the Lagrange method [ref.26], the average power output is subject to the above constraint by defining a cost function.
\[ H(w) = P + z(1 - W^H C). \]  

(3.6)

where \( z \) is an arbitrary constant. Differentiating with respect to the weight vector \( W \) and equating to zero gives the optimum weight vector as

\[ W = R_s^{-1} C^* / C^T R_s^{-1} C^* \]  

(3.7)

and the minimum output power (variance) is then,

\[ P = 1 / C^T R_s^{-1} C^* \]  

(3.8)

In practice the covariance matrix \( R \) is estimated from the measured array data output which is a combination of signal data and noise as in equation (2.45). Now we replace the direction vector \( C \) by the steering vector \( C(\theta) \) and \( P \) by the spectral power \( P_{\text{MLM}}(\theta) \). Thus,

\[ P_{\text{MLM}}(\theta) = 1 / C^T(\theta) R_s^{-1} C^*(\theta) \]  

(3.9)

The derivation for the above equations is explained in Appendix A.

Despite the advantages of MLM cited above, the method has several serious disadvantages which have discouraged its use. The first problem with the method is that it has less resolution than the autoregressive estimate (AR or MEM) though it has better resolution than the conventional beamformer [ref.40].

Burg [ref.10] explained the above problem very neatly when he showed analytically that the MLM is actually an average of MEM spectra of order one to \( N \):

\[ 1 / P_{\text{MLM}}(\theta) = \frac{1}{N} \sum_{i=1}^{N} 1 / P_{\text{MEM}}(1, \theta) \]  

(3.10)
where $P_{MEM}(i,e)$ is the MEM output power for $i$-th model, both $P_{MLM}(e)$ and $P_{MEM}(i,e)$ being based upon a known covariance matrix of order $N$. Thus, the low order, low resolution MEM spectra combine with the high order, high resolution MEM spectra to produce a smoothened, lower resolution result. However, MLM variance is less than that of the MEM estimate [ref.48]. The second problem which leads to doubts about its use is that ill-conditioning might arise. Sibel [ref.62] showed how the problem of large ratio between maximum and minimum eigenvalues arises in adaptive beam processing using the inverse of a correlation matrix which is estimated from noisy experimental data. In the numerical analysis literature, it is generally acknowledged that the eigenvector decomposition is the only reliable method for detection and correction of this ill-conditioning [ref.43]. The third most serious problem, is that the resolution may deteriorate very seriously if the actual data structure does not fit with the assumed model. The derivation of MLM requires the assumption that the noise must be a Gaussian distribution and variance must be greater than zero [ref.63]. The performance of MLM can be quite poor when there is correlated interference [ref.18,70].

3.3.3 Maximum Entropy Method (Burg technique)

The maximum entropy method was introduced in 1967 by Burg [ref.9] for the processing of geophysics data. This method was developed originally to find the spectral estimate of time series data [ref.40]. MEM can be thought of also as a discrete filter which adjusts itself to be least distributed by power at frequencies (spatial frequencies) different from the one to which it is tuned [ref.31].

The MEM has several advantages which make it well-suited to be used in high resolution direction finding. The first advantage is that it can be applied to short data records
The second one is its high resolution performance compared with the conventional method with relatively extra computation [ref.32]. Another advantage of the MEM is the absence of sidelobes and the ambiguity of the array is low [ref.9].

The operation of the maximum entropy filter may be considered as minimizing the output power, subject to the constraint that one array element should have unity weighting. Thus, the convenient way to estimate MEM spectra is to determine a whitening filter since the generating filter transfer function is simply the inverse of the whitening filter. This yields the spectrum estimate [ref.18].

\[ P_{MEM}(\theta) = |C^H(\theta) W|^2, \]  

(3.11)

where \( W \) is the whitening filter of length \( N \) obtained from the sampled array signals and it is defined in equation (3.3) with first element equal to 1. By substituting the vector \( C \) in equation (3.5) by a constraint vector \( Z \), then

\[ W^H Z = 1 \quad \text{and} \quad Z^T = (1,0,0,...). \]

Now equation (3.7) can also be rewritten by substituting \( R^{-1}_s \) by \( R^{-1} \), then

\[ W = R^{-1} z^* / Z^T R^{-1} z^* \]

The denominator in this equation can be reduced by simple calculations to a constant \( z \) which is equal to \( (1,1) \) element of \( R^{-1} \) and it is real value for Hermitian covariance matrix. The \( Z \) vector is a real vector. Thus,

\[ W = R^{-1} z / z \]  

(3.12)

Finally, by substituting \( W \) in equation (3.11), then
Despite the advantages of the maximum entropy method, there are several disadvantages. The first problem, which is stated by Lacoss [ref.48], is the poor performance of MEM in measuring sources power, i.e., the amplitude of the response peak is not a good measure of power. However, the peak's location gives a source direction. The second problem stated by Kesler and Haykin [ref.31], is considered as the most serious one. This problem is called line-splitting, which occurs when a valid single spectral peak splits into two closely spaced peaks due to phase and noise sensitivity.

3.3.4 The eigenvector decomposition methods

An appropriate procedure to find the unknown frequencies and powers is the Pisarenko spectral decomposition procedure when a signal is known to consist of pure sinusoids in white noise [ref.37]. Pisarenko's method has found widespread use and it has provided a fundamental eigenvalue basis for several closely related techniques which have demonstrated excellent performance in resolving close sources (targets) [ref.22]. Most of these techniques have been reviewed in section (3.2), but this section will provide a detailed explanation of some of the methods.

The geometric relationships between the spatial source vectors and the eigenvectors of the sample covariance matrix obtained from the output of an array of sensors, is the key in all of these techniques. From the theory of matrices the eigenvalue and eigenvector of matrix $R$ are related by

$$Rv_i = \lambda_i v_i$$

for $i=1,2, \ldots , N \ (3.14)$

The eigenvectors, which are normalised to unit length and
because $R$ is Hermitian, are orthogonal to one another. They can be used as the columns of the matrix defined as the $U$ matrix

$$U = \begin{bmatrix}
  \cdot & \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot & \cdot \\
  v_1 & v_2 & v_3 & \cdots & v_N \\
 \end{bmatrix} \quad (3.15)$$

where

$$v_i^T = [v_{i1} \quad v_{i2} \quad \cdots \quad v_{iN}].$$

The matrix $R$ and $U$ are related by the following expressions [ref. 25, 67]

$$RU = \Lambda \quad (3.16)$$

Pre multiplication by $U^{-1}$ gives

$$U^{-1}RU = \Lambda,$$

but because $R$ is Hermitian $U^{-1}$ can be shown to be equal to $U^H$. Hence

$$U^HRU = \Lambda$$

$$= \begin{bmatrix}
  \lambda_1 & 0 & \cdots & 0 \\
  0 & \lambda_2 & \cdots & 0 \\
  0 & 0 & \cdots & 0 \\
  \cdots & \cdots & \cdots & \cdots \\
  0 & 0 & \cdots & \lambda_N \\
 \end{bmatrix} \quad (3.16)$$
From the above relations the covariance matrix $R$ can be written as

$$R = UAU^{-1}$$

or

$$= UAU^H$$

Thus,

$$R = \sum_{i=1}^{N} \lambda_i v_i v_i^H$$

In chapter 2, it was shown that the covariance matrix $R$ can be written as the sum of the noise covariance matrix and the signal covariance matrix. The signal covariance matrix is a matrix of rank $MP$ and hence will have only $MP$ non-zero eigenvalues. Then,

$$R_s = \sum_{i=1}^{MP} \lambda_i v_i v_i^H$$

Where $MP$ is the number of sources.

Under ideal conditions, the noise covariance matrix $R_N = \sigma^2 I$.

Thus the covariance matrix $R$ can be written as:

$$R = \sum_{i=1}^{MP} \lambda_i v_i v_i^H + \sigma^2 I$$

(3.17a)

In practice the noise eigenvalues may not be equal to the noise power and hence,

$$R_N = \sum_{i=1}^{N} \sigma^2_{i} v_i v_i^H$$

and therefore the covariance matrix $R$ can be written as

$$R = \sum_{i=1}^{MP} (\lambda_i + \sigma^2_{i}) v_i v_i^H + \sum_{i=MP+1}^{N} \sigma^2_{i} v_i v_i^H$$

(3.17b)

The decomposition of $R$ into its eigenvalues and their associated eigenvectors can be done by using QL or QR algorithms [ref.54]. The resulting eigenvalues are distributed from minimum value to maximum value. However, the following relations will show the eigenvalues...
distribution in practice. The noise and signal eigenvalues are

\[ \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \ldots \leq \lambda_{N-MP} \]  
(3.18a)

\[ \lambda_{N-MP+1} \leq \lambda_{N-MP+2} \leq \ldots \leq \lambda_N \]  
(3.18b)

Therefore \( R \) in equation (3.17b) can be rewritten as:

\[ R = \sum_{i=1}^{N-MP} \lambda_i v_i v_i^H + \sum_{i=N-MP+1}^{N} \lambda_i v_i v_i^H \]  
(3.19)

This equation shows that there are two subspaces: signal subspace and noise subspace. The separation task of these subspaces will be dealt with in detail in chapter 4.

Now the task of the eigenvector processor is to deal with the eigenvectors with the steering beams to find the estimate of signal directions. The relationship between the signal direction vectors and signal eigenvectors will be explained in Appendix B. The signal direction vectors and the noise eigenvectors are orthogonal. However, these relations provide the basis for a number of powerful techniques for determining the directions of sources. These techniques are discussed in greater detail in the following sections.

3.3.4.1 Principal components method.

Juha Karhunen [ref.37] presented a historical view about the principal components analysis development. Thus, in this research short notes are given.

Barlett [ref.41] obtained the inverse covariance matrix in MLM only from the signal eigenvectors as:

\[ R^{-1} = \sum_{i=N-MP+1}^{N} v_i v_i^H \]  
(3.20)

Thus, the angular spectrum will be:
The disadvantage of this method is the inability to discriminate between the very close targets due to the broadness of maxima. The performance of signal eigenvectors will be shown in chapter 6.

3.3.4.2 The MUSIC(MUltiple Signal Classification) method

This algorithm was suggested by Schmidt [ref.59,60] to determine, both theoretically and experimentally, some of the important parameters of multiple wavefronts arriving at an antenna array. The parameters include number of signals, direction of arrivals, strength and cross correlations among the directional wavefronts, polarisation and strength of noise/interference. This algorithm uses measurements made on the signals received by an array of sensors as in the geometric model in chapter 2 and the model explained in some detail in chapter 5 for the near-field.

3.3.4.2.1 The Algorithm

If $E_N$ is defined to be the $M \times N$ matrix whose columns are the $N$ noise eigenvectors, and the ordinary Euclidean distance (squared) from a vector $Y$ to the signal subspace is:

$$d_s^2 = Y^H E_N E_N^H Y.$$ 

$1/d_s^2$ can be plotted for the points along the steering vector $C(\theta)$ continuum as a function of $\theta$. That is,
\[ P_{MU}(\theta) = \left[ C^H(\theta) E_N E_N^H C(\theta) \right]^{-1} \]  

(3.22)

If the covariance matrix \( R \) is asymptotically perfectly measured, then and only then will \( E_N \) be perfectly measurable. Under this condition MUSIC is asymptotically unbiased even for multiple incident wavefronts. \( C(\theta) \) is not data dependent and the MUSIC beamformer gives nulls in the direction of the incident wavefronts. Hence, a new matrix \( A \) can be formed from the direction of arrival of MP incident signals, which can be used to compute the parameters of incident signals. The solution for the power matrix \( P \) is direct and can be expressed in terms of \( (R - \lambda_{\text{min}} R^*) \) and \( A \).

That is, since

\[ APA^H = R - \lambda_{\text{min}} R^*, \]

\[ P = (AA)^{-1} A^H (R - \lambda_{\text{min}} R^*) A (A^H A)^{-1} \]  

(3.23)

\( R^* \) is the identity matrix and can be replaced by \( I \) and \( \lambda_{\text{min}} R^* \), as \( I \).

3.3.4.2.2 Special specification of MUSIC

By comparing MUSIC with conventional beamformer, maximum likelihood and maximum entropy methods, Schmidt [ref.59,60] in his description of the MUSIC algorithm, listed several advantages of this technique.

1-It can resolve very close targets while the conventional beamformer cannot.

2-\( P_{MLM}(\theta) \) calculates the maximum likelihood function under the assumption that the measured data vector has zero mean, multivariate Gaussian distribution and variance that must be greater than zero while MUSIC does not
assume these conditions.

3-MLM and MEM exhibit an ambiguity and wrong peak in power estimation spectrum while MUSIC does not.

4-MUSIC spatial spectrum estimation has no bias error as in MLM and MEM.

5-In general terms, MUSIC minimizes the distance from the \( C(e) \) continuum to the signal subspace whereas MLM minimizes weighted combinations of component distances.

The array elements may be arranged in a variety of patterns, i.e., there are no geometrical restrictions, provided that their polarisation characteristics are all identical.

Gabriel [ref.22] mentioned that the MUSIC algorithm does indeed produce very large peaks in the actual locations in the angular spectra for good covariance matrix estimates. This is because of the orthogonality of the noise eigenvectors to the source vector space. Despite the above advantages of MUSIC there are some disadvantages which can be overcome such as:

1-Tufts and Kumaresan [ref.47] concluded that the individual eigenvectors in the noise subspace of the covariance matrix \( R \) exhibit spurious peaks in the angular spectra. This disadvantage is improved by calculating the average of all individual eigenvector spectra.

2-The time consumed in calculating the average of all individual eigenvector spectra, is considered as the main disadvantage of the MUSIC method [ref.8]. This can be overcome by either using parallel fast processing or by using the minimum norm technique.
3.3.4.3 Wax and Kailath methods

Wax and Kailath [ref.69] introduced two different methods. The first method is based on the eigenvalues and their associated eigenvectors corresponding to the signal subspace. The other, referred to as the noise estimator, is based on the eigenvalues and eigenvectors corresponding to the noise subspace. Suppose that the estimated sampled covariance matrix $\hat{R}$ is the same as the deterministic one. Thus,

$$ R = \sum_{i=1}^{N} \lambda_i v_i v_i^H $$

(3.25)

As far as the covariance matrix is estimated for $P$ snapshots then the eigenvalues are ordered as in equation (3.18). Wax and Kailath maximized the maximum likelihood estimator by taking the metric as the difference between the noise eigenvalues and the mean of the signal eigenvalues. The estimation scheme is then to determine the direction of arrivals as the number of sources that maximize the following pseudo-likelihood expression

$$ P_{Wn}(\theta) = \frac{|C^H(\theta) v_i|^2}{\sum_{i=1}^{N-MP} (\lambda_{ms} - \lambda_i)} $$

(3.26)

where $\lambda_{ms}$ is the mean of the signal eigenvalues

$$ \lambda_{ms} = \frac{1}{MP} \sum_{i=N-MP+1}^{N} \lambda_i $$

$C(\theta)$ is the steering vector.

Equation (3.25) represents the first estimator proposed by Wax and Kailath. This estimator is optimal in the case of one source but not in the case of multiple sources and also its computational load is less than that of maximum likelihood estimators because the maximization problem involved is only one dimensional.
As the sample size increases, the noise eigenvalues approach the actual value of the noise power $\sigma_i^2$. Thus, in an asymptotic case, all the weights become equal and therefore this estimator is reduced to the one proposed by Schmidt and Bienvenu and Kopp.

The second estimator which is an alternative one to the above estimator, is expressed by the following equation

$$ P_{WS}(\theta) = \sum_{i=N-MP+1}^{N} \frac{\left| C^H(\theta) v_i \right|^2}{\sum_{i=1}^{N-MP} \lambda_i - \lambda_{mn}} $$

(3.27)

where $\lambda_{mn}$ is the mean of the noise eigenvalues

$$ \lambda_{mn} = \frac{1}{N-MP} \sum_{i=1}^{N-MP} \lambda_i $$

In the case of linear array the multiplication of steering vector by its transpose conjugate is a constant. Thus the denominator of equation (3.25) and (3.26) can be ignored.

The performance of these two estimators will be examined in chapter 6.

3.3.4.4 Johnson and DeGraaf method [ref. 35]

Johnson and DeGraaf proposed a method for improving the resolution of bearing using the eigenvalues decomposition method. The inverse covariance matrix can be obtained as

$$ R^{-1} = \sum_{i=1}^{N} \lambda_i^{-1} v_i v_i^H $$

(3.28)

The maximum likelihood estimator can be written as a function of eigenvalues and eigenvectors as in the following expression
Johnson and DeGraaf used the above expression to evaluate the spectral beam from only the noise subspace as,

\[ P_{\text{MLM}}(\theta) = \frac{1}{\sum_{i=1}^{N} \lambda_i^{-1} |C^H(\theta) v_i|^2} \]  

(3.29)

This estimator is equivalent to the MUSIC algorithm which assumes that all the noise eigenvalues are equal, except that the eigenvector beams are weighted by their eigenvalues. In the asymptotic case, both MUSIC and eigenvector methods produce quite similar results in the spatial spectrum. However, they differ in at least two respects, i.e.:

1-The sensitivity to the right choice of the number of assumed or estimated sources. If the number of assumed sources is not close to the actual value, the spectra obtained differ from those obtained with the proper value. Spurious peaks appear and/or peaks can be missed. In contrast, the eigenvector method is less sensitive to the choice of assumed sources than MUSIC method [ref.35].

2-The shape of spectrum of the background noise is drastically altered in the MUSIC method while this is much less pronounced in the eigenvector method [ref.35]. The disadvantage of the latter method is the sensitivity to ill-conditioning.

3.3.4.5 Byrne and Steele method [ref.11]

Byrne and Steele proposed a technique which is an extension to MLM method and is more dependent on the stable eigenvectors. They show that by using simulated data, the smallest eigenvector is unstable within the visible region
in the presence of phase errors, while those corresponding to the higher eigenvalues are stable within this region. The signal eigenvector provides broad bandwidth peaks which degrade resolution of close sources. Thus, Byrne and Steel concluded that in order to obtain a stable estimator, a set of those eigenvectors whose null structures are essentially unaffected by the phase errors should be chosen. The stable nonlinear method has a similar form to the MLM estimator expressed in equation (3.29), but involves a different weighting of the terms to emphasize the eigenvectors with stable null structures. Also this estimator deemphasizes the eigenvectors with unstable null structures and signal eigenvectors (which degrade resolution). The SNLM estimator is given by

\[
P_{\text{SNLM}}(\theta) = \frac{1}{N} \sum_{i=1}^{N} \frac{\lambda_i^{\beta-1}}{(\lambda_i^2 + \delta^2)^\beta} |C^N(\theta) v_i|^2 \tag{3.31}
\]

where \(\beta\) and \(\delta\) are parameters to be chosen by the user. It is important to choose \(\delta\) to be greater than the eigenvalues of the unstable eigenvectors, but not greater than maximum signal eigenvalue. When \(\beta = 0\) the SNLM estimator will be the same as MLM estimator. The choice of \(\delta\) is very important. If \(\delta\) is too large, then the resolution will be lost due to the increased weighting of the terms for \(i < N\); if \(\delta\) is too small, the stability will be lost due to the increased weighting of the unstable eigenvectors. Thus, the effect of those stable noise eigenvectors with eigenvalues equal to \(\delta\) can be enhanced by the SNLM estimator. In order to achieve the same sort of performance as in the Johnson and DeGraaf method the \(\beta\) value has to be increased. The disadvantage of this estimator is the computation time compared to MUSIC or the Johnson and DeGraaf method. The advantage of this estimator is that it can be used even in the presence of unknown correlated noise.
3.3.4.6 Minimum Norm Method (MNM) [ref.47]

This method was suggested by Kumaresan and Tufts to provide asymptotically unbiased estimates of the number of signal sources, directions of arrival, strengths and cross correlations among the directional waveforms and strength of noise/interference. In actual fact, MNM is similar to MUSIC except that one vector (v-vector) is used in obtaining the angular spectra. This vector is orthogonal to signal subspace and can be generated from either the signal eigenvector subspace or the noise one.

3.3.4.6.1 The advantages of MNM

Kumaresan and Tufts, in their description of the MNM method, listed two advantages of this technique over all eigenvector decomposition methods:

1-The estimate of sources direction are more accurate even at relatively low SNR values compared with other methods.

2-The N-MP extraneous zeros of the newly obtained vector polynomial tend to be uniformly distributed within the unit circle and have less tendency for "false sources" or spurious estimates.

In addition to these two advantages, one more attractive point is that the time consumed for calculating the angular spectra is less than in the MUSIC technique. This is because there is only one vector rather than N-MP beams in MUSIC.
3.3.4.6.2 The MNM technique.

Let $E_N$ be the matrix constructed with the noise subspace eigenvectors

$$E_N = [v_1 \ v_2 \ \ldots \ \ldots \ \ldots \ v_{N-MP}]$$  \hfill (3.32)

where $N$ is the total number of elements, $MP$ total number of sources, and $E_S$ the matrix constructed with the signal subspace eigenvectors:

$$E_S = [v_{N-MP+1} \ v_{N-MP+2} \ \ldots \ \ldots \ \ldots \ v_N]$$  \hfill (3.33)

Kumaresan and Tufts [ref.47] proposed the following method to calculate the $v$ vector from $E_S$ or $E_N$. These two matrices can be rewritten as:

$$E_S = \begin{bmatrix} g^T \\ \hline \\ E'_S \\ \hline \\ E_S \\ \end{bmatrix}$$  \hfill (3.34)

and

$$E_N = \begin{bmatrix} \eta^T \\ \hline \\ E'_N \\ \hline \\ E_N \\ \end{bmatrix}$$  \hfill (3.35)

where $g$ and $\eta$ have the first elements of the signal and noise subspace eigenvectors, respectively, and $E'_S$ and $E'_N$ have the same elements as in $E_S$ and $E_N$ with the first row deleted. Since $v$ is needed to be in the space of $E_N$, therefore $v$ will be orthogonal to $E_S$. $v$-vector has the first element equal to unity and the rest of the elements $v_2, v_3, \ldots, v_N$ are calculated by using the
appropriate pseudoinverse of $E_s'$.

$$v = \begin{bmatrix}
1 \\
-E_s' g^*/(1-g^* g)
\end{bmatrix}$$

(3.36)

Due to the complementary nature of the signal and noise subspaces [ref.7], then the vector $v$ can also be obtained from the noise subspace eigenvectors. The following equation expresses the vector $v$ in terms of noise subspace.

$$v = \begin{bmatrix}
1 \\
E_n' \eta^*/\eta^* \eta
\end{bmatrix}$$

(3.37)

The minus sign in equation (3.36) was omitted in the original reference [ref.47] and hence there was no consistency in the results. This technique can be expressed in the following expression:

$$P_{\text{MNM}}(\theta) = \left| C^H(\theta) v \right|^2$$

(3.38)

However, at lower SNR values, the MNM method starts to exhibit spurious peaks and merging of spectral peaks [ref.47]. At low Signal to Noise Ratio, the N-MP extraneous zeros of $v$-vector polynomial are not guaranteed to be inside the unit circle, and at very low SNR they might fall close to the unit circle causing spurious estimates. But it appears that the tendency for this to happen in MNM or for the spectral peaks to merge is at lower SNR values in comparison to other methods [ref.47]. MNM starts to resolve two equipowered emitters spaced 0.1 BW apart before MUSIC at a lesser SNR [ref.39].
3.3.4.7 Cantoni and Godara method

This method is proposed by Cantoni and Godara [ref.12] to obtain the angular spectral estimation using the eigenvector associated with the smallest eigenvalue. From the angular spectra, one can pinpoint the signals (sources) direction.

\[
P_{\text{CG}}(\theta) = \left| C^{H}(\theta) v_{1} \right|^{2}^{-1} \tag{3.39}
\]

The angular spectra exhibits spurious peaks and shallows which could be misunderstood as additional sources. The level of the spurious peaks is high, and this is the main disadvantage of this method.

3.3.4.8 Generalised approach

Kates [ref.38] proposed a generalised high resolution spectral estimator:

\[
P_{\text{K}}(\theta) = \sum_{i=1}^{N} \left( \frac{1}{\lambda_{i}} \right)^{m} \left| C^{H}(\theta) v_{i} \right|^{2}^{-1} \tag{3.40}
\]

The parameter \( m \) controls the resolution of the estimator and is continuously adjustable between zero and infinity. Setting \( m \) to zero, for example, gives the same estimated power for all possible steering vectors, resulting in no target resolution. Choosing \( m \) to be equal to 1 gives MLM and setting \( m \) equal to 2 gives a procedure with resolution similar to MEM. As \( m \) is increased, the procedure approaches the method of Cantoni and Godara since the eigenvector having the smallest associated eigenvalue will make an increasingly important contribution to the spectral estimates.
RESOLUTION OF SOURCES IN THE FAR-FIELD

4.1 Introduction

This chapter uses the concepts discussed in chapter 2, of modelling of sources located in the far-field of arrays. Many algorithms have been proposed for high resolution beamforming. Methods with the best performance are often based on the idea of decomposition of the covariance matrix into its eigenvectors and their associated eigenvalues. Two methods, in particular, the well known MUSIC and MNM methods are dealt with in depth, using a linear array to find the direction of two sources which are correlated to some extent. These two methods are compared with the MLM and conventional beamformer. Some attention is also given to the partitioning of eigenvalues into two subspaces, the signal subspace and noise subspace. A new criterion is proposed for separating the signal eigenvalues from the noise eigenvalues. Difficulties arise with the application of high resolution techniques when the sources are fully correlated. Thus, a new method based on the combination of arrays is proposed and evaluated by computer simulation. By using this method it is observed that the MNM and MUSIC algorithms give a superior performance in the direction finding of fully correlated sources. An extension to the problem of multipath is also made. This arrangement is explained in section(4.5) and the analysis here is not exhaustive. Thus, under alternative conditions the discrimination performance of these two eigenvector methods could be totally different from the obtained results.
4.2 Assessment of systems

4.2.1 Computer simulation test using linear array

The information summarized in the preceding chapter provided a reasonably good idea of how well each algorithm would work in the narrowband direction finding application. The model explained in chapter 2 is used for a linear array assuming that the noise is uncorrelated from sensor to sensor. Thus, the noise covariance matrix is assumed as an identity matrix $I$. To determine and compare their performance, more specifically, several of the algorithms were coded in FORTRAN or BASIC and run on a Honeywell main frame and BBC microcomputer. Three algorithms were tested and compared with the conventional beamformer. Two of these are based on the idea of eigenvector decomposition (MUSIC and MNM methods), while the third one is the MLM method. The performance of all the eigenvector methods mentioned in chapter 3 has been tabulated in chapter 6, using real data.

4.2.2 Comparison of results using linear array

The program is developed to make comparison, and provides a plot of relative power received as a function of the incidence angle with respect to the line of the array. All algorithms share a common input routine. In this routine two incident plane waves have been specified. The input routine requires amplitude and bearing of signals. Figures (4.1) to (4.8) show the spectral beam of different algorithms for the same input data. From the figure (4.1) it is clear that the MNM and MUSIC algorithms produce deep nulls while the MLM algorithm has acceptable peaks when the correlation coefficient is zero. The correlation coefficient has the significant effect of introducing a loss of resolution with the MLM algorithm as shown in figures (4.3) to (4.5), i.e., the height of peaks in the MLM beam are
reduced as the correlation coefficient is increased. However, these two algorithms, the MNM and MUSIC, produce deep nulls even when the correlation coefficient is 0.99. In the case of fully correlated signals all the algorithms mentioned here produce ambiguities in resolving sources. But the MUSIC algorithm performs better than the other techniques when the two sources are separated by 0.7BW or over as in the examples given in figures (4.7) and (4.8). Although, the depth of nulls obtained by using the MNM technique appear deeper than the ones produced by the MUSIC algorithm as shown in figures (4.1) to (4.5), this is due to small computational errors.

Finally the important conclusion which can be stated at this stage is that the eigenvector methods tested here are promising in resolving two closely correlated sources using a linear array provided that the correlation coefficient is less than one. This problem is quite possible to occur in real time when the measured data matrix is obtained from reflected signals from a number of targets in a non-stationary environment. However this case is not always guaranteed. Thus, a new technique will be discussed in section(4.5) to resolve the fully correlated signals using these two eigenvector algorithms (MUSIC and MNM).
FIG. 4.1: SPATIAL SPECTRAL .... 2 SOURCES OF 20dB EACH & DIRECTIONS ARE 0 AND 5 DEG. CORRELATION COEFFICIENT=0 SENSORS=10
FIG. 4.2: SPATIAL SPECTRAL... 2 SOURCES OF 20dB EACH & DIRECTIONS ARE 0 AND 5 DEG. CORRELATION COEFFICIENT=0.5 SENSORS=10
FIG. 4.3: SPATIAL SPECTRAL ... 2 SOURCES OF 20dB EACH & DIRECTIONS ARE 0 AND 5 DEG. CORRELATION COEFFICIENT=0.8 SENSORS=10
FIG. 4.4: SPATIAL SPECTRAL ... 2 SOURCES OF 20dB EACH & DIRECTIONS ARE 0 AND 5 DEG. CORRELATION COEFFICIENT=0.95 SENSORS=10
FIG. 4.5: SPATIAL SPECTRAL .... 2 SOURCES OF 20dB EACH & DIRECTIONS ARE 0 AND 5 DEG. CORRELATION COEFFICIENT=0.99 SENSORS=10
FIG. 4.6: SPATIAL SPECTRAL ... 2 SOURCES OF 20dB EACH & DIRECTIONS ARE 0 AND 5 DEG. CORRELATION COEFFICIENT=1.0 SENSORS=10
### Legend

<table>
<thead>
<tr>
<th>Method</th>
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<tbody>
<tr>
<td>Conventional method</td>
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<tr>
<td>MLM method</td>
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<tr>
<td>MUSIC method</td>
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<tr>
<td>MNM method</td>
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**FIG.4.7:** SPATIAL SPECTRAL .... 2 SOURCES OF 20dB EACH & DIRECTIONS ARE 0 AND 7 DEG. CORRELATION COEFFICIENT=1.0  SENSORS=10
FIG. 4.8: SPATIAL SPECTRAL... 2 SOURCES OF 20db EACH & DIRECTIONS ARE 0 AND 9 DEG. CORRELATION COEFFICIENT=1.0 SENSORS=10
4.3 Partitioning of eigenvalues

The eigenvector decomposition methods are based mainly on the right partitioning of the eigenvalues of the covariance matrix with their associated eigenvectors into two subspaces. These subspaces are the signal subspace and the noise subspace. Wrong partitioning can introduce additional or spurious, possibly shallow, nulls and there is also a possibility of missing some signal sources. The overestimation of the number of sources has less effect than the underestimation, but may cause small error in signal direction. The additional source which may be seen in overestimation will be neglected if its amplitude is in the noise level. Thus, a knowledge of the noise floor is required for the field of interest. If more noise components are included then null shapes suffer to the detriment of the discrimination ability [ref.16].

In section(4.3.1) some characteristics of the signal and noise eigenvalues using a linear array of elements are presented. These characteristics will help in choosing the proper criteria for partitioning in certain numbers of snapshots and signal to noise ratio. This matter is dealt with fairly comprehensively in reference 33.

4.3.1 Eigenvalues characteristics

1-The noise eigenvalues approach the power of the noise if the number of snapshots increases.

2-The signal eigenvalues increases linearly with the signal powers provided the signals are not correlated.

3-Correlation between signals affects the distribution of power between the eigenvalues.
4-The spacing of sensors and sources produces also a spatial
correlations between the signal eigenvalues.

Computer simulation on the behaviour of eigenvalues have
been carried out and their results are plotted in figures
(4.9) to (4.17). The results will now be analysed to
justify the above mentioned characteristics and some more.

a-Figure (4.9) shows the noise eigenvalues variation with up
to a maximum of 500 snapshots. From this test it is
seen that the noise eigenvalues become nearly constant
after 50 snapshots. Figure (4.10) shows the noise
eigenvalues variation with up to a maximum of 50
snapshots. The results in figures (4.9) and (4.10)
confirm the statement mentioned in point 1.

b-Because of the large signal to noise ratio, it is clear
that quite a good estimate of the signal eigenvalues
can be made with relatively few snapshots.

c-Figure (4.13) shows the signal eigenvalues as a function
of signal power. From figure (4.13) it is clear that
the signal eigenvalues are linearly related to the
signal power. This figure confirms the characteristic
of signal eigenvalues mentioned in point 2.

d-Figure (4.14) shows the distribution of signal eigenvalues
as a function of variation in elements (sensors)
spacing. From this figure, it can be concluded that
for equal sources the signal eigenvalues are equal in
certain spatial frequencies because the two sources
become uncorrelated at these frequencies.

e-Figure (4.15) shows the distribution of signal eigenvalues
as a function of spacing between two equal sources and
the sensors are separated by \( d = \lambda/2 \). It is seen that
as a function of spacing between two equal sources and the sensors are separated by $d = \lambda/2$. It is seen that the two signal eigenvalues will be equal at certain locations for the same reason mentioned in point (d). Figure (4.16) shows the same result for the non-equal sources. However figures (4.14) and (4.16) confirm the statement mentioned in point 4.

Figure (4.17) shows the distribution of signal eigenvalues as a function of the correlation coefficient between the two sources. These two sources are separated by half of the beamwidth. It is seen that one signal eigenvalue converges to zero while the second signal eigenvalue diverges towards the sum of the powers of the two sources. The signal eigenvalues variation is non-linear with the correlation coefficient. Thus this figure confirms what is mentioned in point 3. The important statement from this figure is that, by using eigenvector techniques, the two sources of correlation coefficient less than one can be resolved by using a linear array of sensors. Figure (4.5) gives evidence to the above statement. The MLM and conventional beamformer cannot resolve two sources of 0.99 correlation coefficient, but the MNM and MUSIC algorithms have a significant performance in resolving these two sources in the mentioned conditions as shown in figure (4.5).

Finally, the above characteristics have a significant effect on the separation of the signal eigenvalues from the noise eigenvalues. The effect due to sources power, number of sensors and number of snapshots will be shown directly in the partitioning expressions. An indirect effect on the partitioning due to sources spacing will be shown in the results of this task which will be examined and discussed in section (4.3.3).
4.3.2 Similarity between beampattern and signal eigenvalues

Signal on array due to a plane wave at an angle $\theta$.

$$X^T = [1, e^{-j\psi}, \ldots, e^{-j(N-1)\psi}], \text{ where } \psi = k d \sin(\theta)$$

If $C = [X]$ (at $\psi = \phi_\ast$) , where $\phi_\ast$ is the wanted direction and

$$W^T C = 1$$

Thus

$$W^T = \frac{1}{N} [1, e^{j\phi_\ast}, \ldots, e^{j(N-1)\phi_\ast}]$$

The beam pattern of a linear array is given by

$$y(\theta) = W^T X$$

$$y(\theta) = \frac{1}{N} [1 + e^{-j\psi} + e^{-j2\psi} + \ldots + e^{-j(N-1)\psi}] \quad (4.1)$$

where $\psi = \phi - \phi_\ast$.

Using a computer program the beampattern was calculated by setting $\phi_\ast$ to zero value. The result is shown in figure (4.18). A similarity exists between the maximum signal eigenvalue distribution shown in figure (4.15) and the beampattern shown in figure (4.18).
FIG. 4.10: Noise eigenvalues as a function of snapshots. Two sources of 20 dB each, sensors=10.
FIG. 4.11: SIGNAL EIGENVALUES AS A FUNCTION OF SNAPSHOTS TWO SOURCES OF 20 dB EACH AND DIRECTIONS ARE 0 AND 10 DEG. _SENSORS=10
Figure 4.12: Signal eigenvalues as a function of snapshots. Two sources of 20 dB each and directions are 0 and 10 deg. Sensors = 10.
FIG. 4.13: SIGNAL EIGENVALUES AS A FUNCTION OF POWER SOURCES DIRECTIONS ARE 0 AND 10 DEG. _SENSORS=10_SNAPSHOTS=32
FIG. 4.14: SIGNAL EIGENVALUES AS A FUNCTION OF SPACING OF ELEMENTS TWO SOURCES OF 20dB EACH AND DIRECTIONS ARE 0 AND 10 DEG. _SENSORS=10, SNAPSHOTS=32

CHAPTER 4
CHAPTER 4

FIG. 4.15: SIGNAL EIGENVALUES AS A FUNCTION OF SEPARATION OF SOURCES. TWO SOURCES OF 20dB EACH, SENSORS=10, SNAPSHOTS=32
FIG. 4.16: SIGNAL EIGENVALUES AS A FUNCTION OF SEPARATION OF SOURCES—TWO SOURCES OF 20 dB AND 14 dB
SENSORS = 10, SNAPSHOTS = 32
FIG. 4.17: SIGNAL EIGENVALUES AS A FUNCTION OF CORRELATION COEFFICIENT. TWO SOURCES OF 20dB EACH AND DIRECTIONS ARE 0.0, 5.0 DEG. SENSORS=10, SNAPSHOTS=32
FIG. 4.18: Beam pattern of linear array of 10 sensors half wave length separated.
4.3.3 Partitioning methods

4.3.3.1 Wax and Kailath methods

Two main criteria have been discussed in detail by Wax and Kailath [ref. 68]. The first one was introduced by Akaike and yields an inconsistent estimate, i.e. in the large sample limit, it tends to overestimate the number of signals. The second one is introduced by Schwartz and Rissanen and yields a consistent estimate of the number of signals (MDL)

\[
\text{AIC}(m) = -2 \log \left( \frac{\text{Maximum}}{\text{Likelihood}} \right) + 2m(2N-m) + 1 \quad (4.2)
\]

\[
\text{MDL}(m) = -\log \left( \frac{\text{Maximum}}{\text{Likelihood}} \right) + 0.5m(2N-m)\log(P) \quad (4.3)
\]

\[
\text{Maximum Likelihood} = \left[ \frac{\prod_{i=m+1}^{N} \lambda_i}{\left( \sum_{i=m+1}^{N-m} \lambda_i \right)^{N-m}} \right]^P
\]

where \( N = \) number of elements, \( P = \) number of snapshots and \( m = 0, 1, \ldots, N-1 \).

The number of sources \( M \) is taken at the value of \( m \) which minimises the AIC\((m)\) or MDL\((m)\). These criteria have been investigated for a different number of sensors and a different number of snapshots. Examples are given in tables (4.1) and (4.2).

4.3.3.2 The proposed new method

A new method is proposed in this research to separate the signal eigenvalues from the noise eigenvalues. This method
can be summarised by the following points:

1-The eigenvalues are ordered from maximum value to minimum.

2-Scale the eigenvalues by the maximum eigenvalue.

3-Maximum likelihood is calculated from

\[ ML = \sum_{j=m+1}^{N} \frac{\lambda_j}{\lambda_{\text{max}}} \]

4-Number of free adjusted parameters = \( (m+1)(1+1/N) \)

The advantages of this proposed method are the simplicity in calculation and the quickness compared to ACI and MDL methods. An intensive measurement is made to show the performance of this method. This idea is based on the criterion suggested by Akaike to determine the model order by minimising an information theoretic function [ref.40] which was expressed by the following expression.

\[ AIC(m) = \ln(\varepsilon_m) + \frac{2(m+1)}{N} \]

Where \( \varepsilon_m \) is the predicted error value at \( m \)-th sample. The proposed criterion can be expressed in the following form:

\[ AMC(m) = \ln(ML) + (m+1)(1+1/N) \]

ML is calculated according to point 3. The estimated number of sources can be taken corresponding to the value of \( m \) which minimizes the value of AMC. This proposed criterion is compared in relation to Wax and Kailath methods. Tables (4.3) to (4.7) show the behaviour of this new proposed criterion in separating the two groups having different numbers and amplitudes of sources, different numbers of snapshots, and sensors. Tables (4.3) and (4.4) show the positive behaviour of this method using two linear arrays, respectively. The arrays are of seven and ten sensors.
receiving two, three, and four sources. The parameters of sources such as amplitude and directions are shown in the tables. Two of these sources are very close in the spatial domain. It is evident from table (4.3) that two sources can be detected correctly even at a low number of snapshots (e.g., P=10). But in the case of three sources with low amplitudes and a low number of snapshots, the detected number of sources is two. This is improved when the number of snapshots is increased to 20. A similar test is repeated in table (4.4) for a higher number of sensors (e.g., N=10) and a different number of sources. Tables (4.5) to (4.7) show the separation capability of the new criterion at a low signal to noise ratio (SNR), low number of snapshots and sensors. Two of these sources are also separated by 3 degrees. This method provides underestimation for the number of sources at a very low SNR when the sources are very close. This is evident from the results in table (4.6) when the number of snapshots and sensors is low. But at higher numbers of snapshots and sensors, this criterion is able to separate the signal eigenvalues from noise eigenvalues correctly with the same parameters shown in table (4.7).

Finally, from the results obtained it can be concluded that this new method has a very good capability in separating signal eigenvalues from noise eigenvalues at low SNR, low number of snapshots and low numbers of sensors. This method will underestimate the number of sources at a very low SNR, very low numbers of snapshots, and low numbers of sensors if the sources are very close.
<table>
<thead>
<tr>
<th></th>
<th>m=</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
<tbody>
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<td>AIC</td>
<td>262.6</td>
<td>102.8</td>
<td>47.0</td>
<td><strong>46.2</strong></td>
<td>49.3</td>
<td>49.7</td>
<td>49.0</td>
</tr>
<tr>
<td></td>
<td>MDL</td>
<td>130.8</td>
<td>50.9</td>
<td>23.0</td>
<td><strong>22.6</strong></td>
<td>24.1</td>
<td>24.3</td>
<td>24.0</td>
</tr>
<tr>
<td></td>
<td>AMC</td>
<td>1.020</td>
<td>-1.891</td>
<td>-2.724</td>
<td>-2.283</td>
<td>-1.883</td>
<td>-1.851</td>
<td>-1.875</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>AIC</th>
<th>468.1</th>
<th>179.3</th>
<th>37.0</th>
<th>39.4</th>
<th>42.7</th>
<th>46.6</th>
<th>49.0</th>
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<td>MDL</td>
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<td>91.1</td>
<td><strong>21.6</strong></td>
<td>24.1</td>
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Table (4.1) Criterion values for an array of seven sensors

\[ A_1 = A_2 = 10 \]  Directions 20°, 23°

P = snapshot number  
A = source amplitude  

CHAPTER 4
<table>
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<th>0</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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</thead>
<tbody>
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<td>AIC</td>
<td>P = 10</td>
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<td>256.7</td>
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<td>124.7</td>
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<tr>
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<td>62.5</td>
<td>65.4</td>
<td>67</td>
<td>65.8</td>
<td>61.8</td>
<td>58.9</td>
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<th>85.3</th>
<th>86.4</th>
<th>90.9</th>
<th>92.3</th>
<th>95.0</th>
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<td>-0.211</td>
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Table (4.2) Criterion values for an array of 10 sensors

$A_1 = A_2 = 10$ Directions $20^\circ, 23^\circ$

P = snapshot number

A = source amplitude
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<th>( MP = 4 )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
<th>( m = 0 )</th>
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<th>( 2 )</th>
<th>( 3 )</th>
<th>( 4 )</th>
<th>( 5 )</th>
<th>( 6 )</th>
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<td>1.02</td>
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<td>-2.724</td>
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<td>-1.851</td>
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<td>0</td>
<td></td>
<td></td>
<td>1.019</td>
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<td>-1.577</td>
<td>-1.54</td>
<td>-1.57</td>
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<td>-1.783</td>
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<td>-0.855</td>
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<td></td>
<td>1.302</td>
<td>0.958</td>
<td>-1.053</td>
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<td>-1.340</td>
<td>-1.010</td>
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<td>-1.099</td>
<td>-0.910</td>
<td>-0.578</td>
<td>-0.612</td>
</tr>
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</table>

Table (4.3) The performance of the proposed method in separating signal eigenvalues from noise eigenvalues using an array of 7 sensors and directions of 20°, 23°, 5°, -5°.
<table>
<thead>
<tr>
<th>m=0</th>
<th>A_1</th>
<th>A_2</th>
<th>A_3</th>
<th>A_4</th>
<th>A_5</th>
<th>A_6</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<th>8</th>
<th>9</th>
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<tr>
<td>P = 10</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>-1.804</td>
<td>-1.369</td>
<td>-0.922</td>
<td>-0.759</td>
<td>-0.605</td>
<td>-0.301</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
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<td>-1.474</td>
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</tr>
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<td>1.046</td>
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<td>-1.896</td>
<td>-1.356</td>
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<td>-0.244</td>
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<td>0.754</td>
</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1.039</td>
<td>-1.229</td>
<td>-2.586</td>
<td>-1.937</td>
<td>-1.347</td>
<td>-0.763</td>
<td>-0.210</td>
<td>0.290</td>
<td>0.594</td>
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<tr>
<td>MP=2</td>
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<td>-0.047</td>
<td>0.086</td>
<td>0.586</td>
<td>0.889</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.039</td>
<td>-1.23</td>
<td>-2.587</td>
<td>-1.937</td>
<td>-1.347</td>
<td>-0.763</td>
<td>-0.210</td>
<td>0.280</td>
<td>0.595</td>
</tr>
<tr>
<td>P = 20</td>
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<td>0</td>
<td>1.525</td>
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<td>0.810</td>
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<td>1.325</td>
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<td>10</td>
<td>10</td>
<td>0</td>
<td>10</td>
<td>1.911</td>
<td>2.397</td>
<td>2.972</td>
<td>3.363</td>
<td>3.21</td>
<td>1.045</td>
<td>0.351</td>
<td>0.868</td>
<td>1.106</td>
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<td>5</td>
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<td>2.466</td>
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<td>0.699</td>
<td>1.214</td>
<td>1.434</td>
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</table>

Table (4.4) The performance of the proposed method in separating signal eigenvalues from noise eigenvalues using array of 10 sensors and directions 20°, 23°, -20°, -10°, 0°, 10°.
Table (4.5)  Test for the new proposed method in separating signal eigenvalues from noise eigenvalues using an array of 7 sensors and directions $-1.5^\circ, 1.5^\circ, 20^\circ, -10^\circ$ at high, low and very low SNR.
Table (4.6) Test for the new proposed method in separating signal eigenvalues from noise eigenvalues using an array of 7 sensors and directions -1.5°, 1.5°, 20°, -10° at high, low and very low SNR for low snapshots.
Table (4.7) Test for the new proposed method in separating signal eigenvalues from noise eigenvalues using an array of 10 sensors and directions $-1.5^\circ, 1.5^\circ, 20^\circ, -10^\circ$ at high, low and very low SNR for low snapshots.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$m$</th>
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<th>1</th>
<th>2</th>
<th>3</th>
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<th>8</th>
<th>9</th>
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<tbody>
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<td>1.635</td>
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<td>5</td>
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<td>2.514</td>
<td>2.895</td>
<td>2.617</td>
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4.4 Linear array with subgrouplng of elements

4.4.1 System modelling

Figure (4.19) shows the general distribution of sensors. The resultant covariance matrix is the average of subcovariance matrices. Given N sensors, the output at time instant $X(t)$ is

$$X^T(t) = [x_1(t) \ x_2(t) \ \ldots \ \ldots \ x_N(t)]$$

Define $P_1$ subsets, then the subset output will be

$$X_{1}^T(t) = [x_1(t) \ x_2(t) \ \ldots \ \ldots \ x_K(t)]$$

$$X_{2}^T(t) = [x_2(t) \ x_3(t) \ \ldots \ \ldots \ x_{K+1}(t)]$$

$$\vdots$$

$$X_{P_1}^T(t) = [x_{P_1}(t) \ x_{P_1+1}(t) \ \ldots \ \ldots \ x_N(t)]$$

Where $K=N-P_1+1$

The system is assumed to be narrowband as mentioned earlier in chapter 2. Thus a phasor notation can be used and the covariance matrix of each subset can be obtained by equations (2.15) and (2.43) i.e.

$$R_q = E[X_q^* X_q^T]$$

and the spatial smoothed correlation matrix:

$$R = \sum_{q=1}^{P_1} R_q$$

(4.5)

The assumptions concerning the noise and source signals used in equation (2.45) are also applied in this section.
4.4.2 Algorithms performance

The resultant covariance matrix in equation (4.5) is considered as an input to the different algorithm processors, i.e. MLM processor and eigenvector decomposition processors. The main advantage of this technique is to obtain the required smoothing for the resultant covariance matrix. The smoothing can be done either by taking a high number of snapshots as mentioned before or by this technique with a lesser number of snapshots. The results shown in figures (4.20) to (4.23) and the ones in tables (4.5) and (4.6) show that the resolution capability of the MNM algorithm is higher than that of the MLM and the MUSIC algorithms. The results presented in tables (4.8) and (4.9) are obtained for three trials, and the noise generator has different start values in each trial. Thus, the values of noise samples are different from one trial to another. The results in these tables show the separation times of these two sources out of three trials, despite the biasing effect.

Finally from the results presented in this section, it can be concluded that the change of noise has a significant effect in finding the direction of close sources under similar conditions. The second point which can be mentioned is that the subgrouping technique provides a high performance in separating close sources with a lower number of snapshots. This technique is applicable even for correlated signals. A more efficient technique to resolve the fully correlated signals is developed in the next section based on the idea of subgrouping the arrays in such a manner that an extremely high performance is obtained.
Fig. 4.19 Representation of subgrouping of sensors in a linear array
FIG. 4.20: MLM SPECTRAL USING SUBGROUPING OF ELEMENTS. TWO SOURCES 20dB EACH & DIRECTIONS ARE 0.0deg. 2.0deg. SENSORS=20, SUBGROUPING OF SENSORS=11.
FIG. 4.21: MLM SPECTRAL USING SUBGROUPING OF ELEMENTS. TWO SOURCES 20 dB EACH & DIRECTIONS ARE 0.0 deg. 4.0 deg. SENSORS = 20, SUBGROUPING OF SENSORS = 11.

Legend

Snapshots = 32
Snapshots = 8
Snapshots = 2
FIG. 4.22: MUSIC SPECTRAL ... USING SUBGROUPING OF ELEMENTS ... TWO SOURCES 20dB EACH & DIRECTIONS ARE 0.0deg, 2.0deg. SENSORS=20, SUBGROUPING OF SENSORS=11
FIG. 4.23: MNN SPECTRAL USING SUBGROUPING OF ELEMENTS TWO SOURCES 20dB EACH & DIRECTIONS ARE 0.0deg. 2.0deg.
SENSORS=20, SUBGROUPING OF SENSORS=11
Table (4.8) Number of resolving times obtained from 3 trials for different sets of noise using linear array.

<table>
<thead>
<tr>
<th>Angle separation of sources</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
<th>snapshots number</th>
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<td>0</td>
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<td>2</td>
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<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>MNM</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
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<td>3</td>
<td>3</td>
<td>3</td>
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</tr>
<tr>
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<td>3</td>
<td>3</td>
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<td>MNM</td>
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<td>3</td>
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<td></td>
</tr>
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</tr>
</tbody>
</table>

sensors = 10

Two equal sources of 20 dB each
### Table (4.9)

Number of resolving times obtained from 3 trials for different sets of noise using subgrouping of elements

<table>
<thead>
<tr>
<th>Angle separation of sources</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
<th>snapshots number</th>
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<tbody>
<tr>
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<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
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<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>MNM</td>
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<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>MLM</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>MUSIC</td>
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<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>MNM</td>
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<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
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<tr>
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<td>0</td>
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<td>MNM</td>
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<td>3</td>
<td>3</td>
<td>3</td>
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</tbody>
</table>

sensors = 20
subgroups = 11
Two equal sources of 20 dB each
4.5 The subgrouping of linear arrays

A new method based on the combination of arrays is proposed and discussed in this section. The main advantage of this technique is to resolve very close and fully correlated sources using the eigenvector decomposition techniques. Therefore, a study of fully correlated signals eigenvalue behaviour is presented in section (4.5.2). Some of the proposed array combinations are shown in figures (4.24) to (4.26). Taking figures (4.24) and (4.25) as examples for the purpose of modelling, the time delay between the subarrays due to \( q \)-th source, is calculated as

\[
t_{d_q} = \frac{\Delta_q \sin(\theta_q)}{v}
\]  

(4.6)

\( v \) is the signal velocity in the field of interest. This is the delay when the arrays are in the same line, but when they are in parallel, as shown in figure (4.25), the delay will be:

\[
t_{d_q} = \frac{\Delta_q \cos(\theta_q)}{v}
\]  

(4.7)

The source signal is also assumed as a sinusoid waveform given by equation (2.1) and the fundamental model was developed in chapter 2. Thus, the vector representation of the subarrays output will only be explained in the next section.

4.5.1 Theoretical signal vector representation:

The signal vector at the output of the first subarray can be calculated as,

\[
X_{s_1} = X_{s_1} (1) + X_{s_1} (2) + \ldots + X_{s_1} (MP)
\]

where \( X_{s_1} (q) \) is the signal vector at the output of the first subarray from \( q \)-th source. The first approach is to deal with the problem of two sources. Thus,
\[ X_{s_1} = X_{s_1}(1) + X_{s_1}(2) \]

where

\[
X_{s_1}(1) = [A_1 \ e^{jU_1} \ A_1 \ e^{j2U_1} \ . \ . \ . \ A_1 \ e^{j(N-1)U_1}]^T
\]

\[
X_{s_1}(2) = [A_2 \ e^{jU_2} \ A_2 \ e^{j2U_2} \ . \ . \ . \ A_2 \ e^{j(N-1)U_2}]^T
\]

In general the signal vector at the output of the \(q\)-th subarray from the \(l\)-th source is

\[
X_{s_1}(l) = [A_l \ e^{jU_l} \ A_l \ e^{j2U_l} \ . \ . \ . \ A_l \ e^{j(N-1)U_l}]^T \ e^{j\varpi_l(q)} \quad (4.8)
\]

\(\varpi_l\) is defined in equation (2.3) and \(\varpi_l(q)\) is the phase due to time delay between subarrays.

\[
T_l(q) = k\Delta_q \cos(\theta_l) \quad \text{Parallel arrays}
\]

or

\[
T_l(q) = k\Delta_q \sin(\theta_l) \quad \text{Arrays in one line}
\]

The signal vector at the output of the \(q\)-th subarray from two sources is

\[
X_{s_q} = [A_1 e^{jT_1(q)} + A_2 e^{jT_2(q)}, \]

\[
A_1 e^{j(U_1 + T_1(q))} + A_2 e^{j(U_2 + T_2(q))}, \ldots , \]

\[
A_1 e^{j((N-1)U_1 + T_1(q))} + A_2 e^{j((N-1)U_2 + T_2(q))}]
\]

Thus, the covariance matrix at the output of the \(q\)-th subarray is

\[
R_q = E[X_{s_q}^* \ X_{s_q}^T]
\]

and the average covariance matrix will be
\[ R = \sum_{q=1}^{Q} R_q \]  

(4.9)

For simplicity the \( b_{im} \) value of the \( q \)-th subarray matrix can be derived by the same way developed in section (2.2.2). Thus,

\[ b_{im} = E[\mathbf{A}_1^* e^{j((1-i)u_1-T_1(q))} + \mathbf{A}_2^* e^{j((1-i)u_2-T_2(q))}] \]

\[ \cdot (\mathbf{A}_1 e^{j((m-1)u_1+T_1(q))} + \mathbf{A}_2 e^{j((m-1)u_2+T_2(q))}) \]

or

\[ b_{im} = E[\mathbf{A}_1^* A_1 e^{j((m-1)u_1)} + E[\mathbf{A}_2^* A_2] e^{j((m-1)u_2)}] \]

\[ E[\mathbf{A}_1^* A_2^*] e^{j((m-1)u_1-(1-i)u_1+T_2(q)-T_1(q))} \]

\[ E[\mathbf{A}_2^* A_1^*] e^{j((m-1)u_1-(1-i)u_2+T_1(q)-T_2(q))} \]

\( E[\mathbf{A}_1^* A_2^*] \) has the form of relations in equation (2.35). Thus,

\[ b_{im}(\text{real}) = P_1 \cos((m-1)u_1) + P_2 \cos((m-1)u_2) + \]

\[ |A_1| |A_2| c_f \cos((m-1)u_2-(1-i)u_1+T_2(q)-T_1(q)) + \]

\[ |A_2| |A_1| c_f \cos((m-1)u_1-(1-i)u_2+T_1(q)-T_2(q)) \quad (4.10a) \]

\[ b_{im}(\text{imag}) = P_1 \sin((m-1)u_1) + P_2 \sin((m-1)u_2) + \]

\[ |A_1| |A_2| c_f \sin((m-1)u_2-(1-i)u_1+T_2(q)-T_1(q)) + \]

\[ |A_2| |A_1| c_f \sin((m-1)u_1-(1-i)u_2+T_1(q)-T_2(q)) \quad (4.10b) \]

Note that equations (4.10a) and (4.10b) are similar to equations (2.36) and (2.37), the difference in phase being due to the subarray effect. This covariance matrix is also generated from a simulated model by averaging the subcovariance matrices resulting from \( P \) snapshots of measurements. This task will be derived and discussed in section (4.5.3).
4.5.2 Signal eigenvalues characteristics.

The covariance matrix calculated in the preceding subsection is fully decomposed into its eigenvalues with its associated eigenvectors. The two signal eigenvalues are calculated for fully correlated signals by considering the first source at zero direction with respect to the first array and moving the second source from $-90^\circ$ to $+90^\circ$. The distribution of signal eigenvalues is taken as a function of $\theta$. The idea behind this technique is to decorrelate the fully correlated sources in such a way that the two eigenvalues are easy to separate. This statement is evident from figures (4.27) to (4.30) which show the eigenvalues distribution. The main limitation of this technique is the grating lobe effect which produces nulls in the first signal eigenvalue. However, this effect can be overcome by irregular distribution of arrays, i.e., the separation distance between the arrays is irregular with respect to wavelength. Figures (4.31) and (4.32) show the distribution of signal eigenvalues using irregular array spacing. The irregular distribution can be done by the trial and error method when the number of arrays is small as is the case in the examples used here, but when the number is large then one of the sparse array techniques has to be used [ref.2,42,66]. In the case of uncorrelated signals received by the technique of subarrays, the distribution of signal eigenvalues is the same as in the case of one linear array as shown in figure (4.15).

Finally, the summary of this section, is that the signal eigenvalues of fully correlated sources can be separated easily using the new technique of subarrays. In the case of using one linear array receiving fully correlated signals, the maximum signal eigenvalue will be equal to the sum of all signal power and the rest of the signal eigenvalues will be in the noise level. Thus, the signal eigenvalues cannot be separated from noise level correctly.
Fig. 4.24 Distribution of apertures of N Sensors each - in line
Fig. 4.25  Distribution of apertures of N Sensors each - in parallel
$R = \sum_{q=1}^{Q} R_q$

Fig. 4.26 Distribution of apertures of $N$ sensors each - combined arrays
FIG. 4.27: SIGNAL EIGENVALUES OF TWO FULLY CORRELATED SOURCES OF 20dB EACH, TWO ARRAYS IN LINE, SENSORS=10.
FIG. 4.28: SIGNAL EIGENVALUES OF TWO FULLY CORRELATED SOURCES OF 20dB EACH, FOUR ARRAYS IN LINE, SENSORS=10
FIG. 4.29: SIGNAL EIGENVALUES OF TWO FULLY CORRELATED SOURCES OF 20dB EACH, FOUR ARRAYS IN PARALLEL, SENSORS = 10
FIG. 4.30: SIGNAL EIGENVALUES OF TWO FULLY CORRELATED SOURCES OF 20dB EACH, FOUR COMBINED ARRAYS, SENSORS=10

DELTA=2 λ

DELTA=4 λ

DELTA=6 λ

DELTA=8 λ
FIG. 4.31: SIGNAL EIGENVALUES OF TWO FULLY CORRELATED SOURCES OF 20dB EACH AS A FUNCTION OF THEIR SEPARATION _ARRAYS=4 IN LINE, IRREGULAR DELTA=9.75 18.5 26.25
FIG. 4.32: SIGNAL EIGENVALUES OF TWO FULLY CORRELATED SOURCES OF 20dB EACH AS A FUNCTION OF THEIR SEPARATION. ARRAYS=4, COMBINED IRREGULAR DELTA=4.5, 6.5 λ
4.5.3 Simulation of subarrays technique

The simulation of this system is based on equation (2.45) where for fully correlated signals, \( \Phi_k \) is the same for the two signals. The only modification in this equation is the phase due to the time delay between the arrays. Therefore, equation (2.45) can be written as,

\[
x_{ik}(q) = n_{ik}(q) + \sum_{k=1}^{M_p} A_k \exp(j(1-1)u_k + \Phi_k + \tau_k(q)) \quad (4.11)
\]

where

- \( x_{ik}(q) \) = k-th snapshot at i-th element of q-th subarray.
- \( n_{ik}(q) \) = noise of k-th snapshot at i-th element of q-th subarray.
- \( u_k \) is defined in equation (2.3).
- \( \Phi_k \) = random phase at k-th sample (common for fully correlated sources).
- \( \tau_k(q) \) = phase of k-th source due to time delay of q-th subarray.

The subarray covariance matrix is obtained using equation (2.43) and the average covariance matrix is calculated by applying equation (4.9).

4.5.3.1 Discrimination performance of the MNM and MUSIC

Figure (4.33) shows the performance of the MNM and MUSIC algorithms in resolving two equal and fully correlated sources as a function of the sources power. From this figure, it is clear that MNM technique starts to resolve the two sources at a lower power than the MUSIC algorithm with some biasing in the detected direction when the power is less than 45 dB. The two sources are very close together and are separated by 0.1 of the beamwidth. Figure (4.34) shows the error in resolution as a function of the sources power. From this figure the MNM technique has the same error as the MUSIC algorithm but at a lower power. After 45 dB both algorithms have no error in resolution.
4.5.3.2 Results for different number of sources.

This section presents representative results of the direction finding beams of two or three fully correlated sources. Figures (4.35) and (4.36) show the spatial spectral beams using the MUSIC and MNM algorithms at the conditions mentioned in the figures. The spacing between arrays is regular as indicated in the figures, i.e., of the form used in figure (4.28). From the figures (4.35) and (4.36), it is clear that the combination of arrays, shown in figure (4.28), is able to decorrelate the fully correlated sources. Thus, the arrival directions can be found easily from the position of nulls as indicated in the figures. The nulls in the MNM beam are deeper than those in the MUSIC. Figures (4.37) and (4.38) show the performance capability of the MNM and MUSIC algorithms in resolving two sources separated by 0.1BW in the same configuration as shown in figure (4.36) for less number of snapshots. These two figures (4.37) and (4.38) show that the MUSIC algorithm has an ambiguity while the MNM technique has clear separation between the two sources of 34 dB and 40 dB, respectively. The above results can also be obtained by using the irregular combination of arrays that gives the signal eigenvalues distribution as shown in figure (4.32).

4.5.3.3 Results for multipath model.

The multipath is modelled by two sources and their images when the array is vertically mounted. The two sources are 20 dB each and their images are 14 dB. Two algorithms have been examined. The first one is the well known MUSIC algorithm and the second is the MNM technique. The main contribution of this section is to show the performance of MUSIC and MNM algorithms in resolving the two correlated
sources and their multipath as a function of snapshots. It is evident from figures (4.39) to (4.42), that the MUSIC algorithm has less biasing effect than the MNM technique when the number of snapshots is low. When the number of snapshots is increased, both MUSIC and MNM algorithms provide exactly the same performance in estimating the direction of received wavefronts. In other word, the two algorithms are unbiased when the covariance matrix is asymptotically estimated. Figures (4.41) and (4.42) show the effect of noise sequences (sets) on the biasing. From these two figures, it is clear that the noise effect appears only when the number of snapshots is not very large.

Finally, some conclusions can be made from the results obtained in this section:

1-Both MUSIC and MNM algorithms are asymptotically unbiased when they are used to resolve fully correlated sources by applying this new technique.

2-MUSIC algorithm has less biasing effect than MNM technique at a lower number of snapshots.

3-The biasing in the detected directions is sensitive to the noise at a low number of snapshots.

4-MUSIC and MNM algorithms are asymptotically insensitive to noise sequence.
FIG. 4.33: DISCRIMINATION PERFORMANCE FOR TWO FULLY CORRELATED SOURCES SENSORS=10 ARRAYS=4 DELTA=8 λ
FIG. 4.34: DISCRIMINATION ERROR FOR TWO FULLY CORRELATED SOURCES
SENSORS = 10 ARRAYS = 4 DELTA = 8 λ
FIG. 4.35: SPATIAL SPECTRAL USING SUBARRAYS FOR THREE FULLY CORRELATED SOURCES 20 dB, 32 dB, 30 dB & DIRECTIONS ARE -10, 0, 1.5 deg. SENSORS = 10 SNAPSHOTS = 32 ARRAYS = 4 DELTA = 8 λ.
FIG. 4.36: SPATIAL SPECTRAL USING SUBARRAYS FOR TWO FULLY CORRELATED SOURCES 34dB EACH & DIRECTIONS ARE 0, 1.0 deg. SENSORS=10 SNAPSHOTS=32 ARRAYS=4 DELTA=8λ
FIG. 4.37: SPATIAL SPECTRAL USING SUBARRAYS FOR TWO FULLY CORRELATED SOURCES 34 dB EACH & DIRECTIONS ARE 0, 1.0 deg. SENSORS=10 SNAPSHOTS=7 ARRAYS=4 DELTA=8 λ
FIG. 4.38: SPATIAL SPECTRAL USING SUBARRAYS FOR TWO FULLY CORRELATED SOURCES 40 dB EACH & DIRECTIONS ARE 0, 1, 0 deg. SENSORS = 10 SNAPSHOTS = 7 ARRAYS = 4 DELTA = 8 λ
FIG. 4.39: DISCRIMINATION PERFORMANCE FOR TWO FULLY CORRELATED SOURCES AND THEIR TWO IMAGES (MULTIPATH) SENSORS = 10 ARRAYS = 4 DELTA = 6λ - SIGNALS POWER 20 dB EACH AND MULTIPATH SIGNAL POWER 14 dB USING MUSIC ALGORITHM
FIG. 4.40: DISCRIMINATION PERFORMANCE FOR TWO FULLY CORRELATED SOURCES AND THEIR TWO IMAGES (MULTIPATH) SENSORS = 10, ARRAYS = 4, DELTA = 6λ, SIGNALS POWER 20 dB EACH AND MULTIPATH SIGNAL POWER 14 dB, USING MNM ALGORITHM.
FIG. 4.41: DISCRIMINATION PERFORMANCE FOR TWO FULLY CORRELATED SOURCES AND THEIR TWO IMAGES (MULTI-PATH) SENSORS = 10 ARRAYS = 4 DELTA = 6λ - SIGNALS POWER 20dB EACH AND MULTI-PATH SIGNAL POWER 14dB USING MUSIC ALGORITHM
FIG. 4.42: DISCRIMINATION PERFORMANCE FOR TWO FULLY CORRELATED SOURCES AND THEIR TWO IMAGES (MULTIPATH) SENSORS = 10, ARRAYS = 4, DELTA = 6\degree, SIGNALS POWER 20 dB EACH AND MULTIPATH SIGNAL POWER 14 dB, USING MNM ALGORITHM.
5.1 Introduction

In this thesis so far the high resolution methods have mainly been applied to the separation of sources in the far-field of the array, but as arrays become larger it is quite possible for the sources to be in the near-field [ref.27]. The performance of two of these high resolution techniques, MUSIC and MNM, in resolving close sources in the near-field is evaluated by computer simulation. Two fairly simple methods have been developed to obtain the same type of performance in the near-field as is obtained in the far-field. The first method [ref.27] has been derived on the basis that the measured data matrix has to be modified by a focusing vector which is a function of range and focusing angle. The second method is based on the idea of considering the steering vector as a function of range and steering angle. A detailed derivation will be shown in section (5.3). Some statistical studies of using these two methods and applying MUSIC and MNM algorithms are included.

5.2 System modelling

The system being considered here comprises a linear array of N equally spaced sensors receiving signals generated from a number of sources located at unknown positions. Figure (5.1) shows the system model. The sources are assumed to be sinusoidal and are given by the same equation(2.1) . However some of the relations mentioned in chapter 2 are repeated for the purpose of near-field modelling. Thus,
\[ a_\ell(t) = A_\ell(t) \exp(i\omega t) \]  

(5.1)

for \( \ell = 1, 2, \ldots \), MP

Where \( A_\ell(t) \) is the random complex amplitude of the \( \ell \)-th source. The signal at the \( i \)-th element in figure (5.1) due to \( \ell \)-th source will be

\[ s_{\ell i}(t) = a_\ell(t) \cdot c_{\ell i} \]  

(5.2)

Where \( c_{\ell i} \) is a factor which represents the effect of the propagation and is given by

\[ c_{\ell i} = \frac{1}{r_{\ell i}} \exp(jkr_{\ell i}) \]  

(5.3)

\[ k = 2\pi/\lambda \]

where \( r_{\ell i} \) is the range from \( \ell \)-th source to element \( i \). In practice the values of \( r_{\ell i} \) are fairly similar and although small differences have a very significant effect on the phase, their effect on amplitude can be ignored. We can therefore re-define \( c_{\ell i} \) as

\[ c_{\ell i} = \exp(jkr_{\ell i}) \]  

(5.4)

If there are MP sources then the total signal received by the \( i \)-th element is given by

\[ s_i(t) = \sum_{\ell=1}^{MP} s_{\ell i}(t) \]  

(5.5)

It is convenient to express the signals received by the array at a particular instant as a column vector viz:
The space effect on each source can be considered as \( N \) dimensional column vector:

\[
\mathbf{c}_l = \begin{bmatrix}
\exp(jkr_{l1}) \\
\exp(jkr_{l2}) \\
\vdots \\
\exp(jkr_{ln})
\end{bmatrix}
\]  

(5.7)

Similarly the source signals can be represented as a signal vector

\[
\mathbf{a}_l(t) = [a_1(t), a_2(t), \ldots, a_{MP}(t)]
\]  

(5.8)

and the space vectors can be combined to form a matrix,

\[
\mathbf{C} = [\mathbf{C}_1, \mathbf{C}_2, \mathbf{C}_3, \ldots, \mathbf{C}_{MP}]
\]  

(5.9)

The vector \( \mathbf{S}(t) \) represents a snapshot at a particular instant in time

\[
\mathbf{S}(t) = \mathbf{A}(t) \cdot \mathbf{C}
\]  

(5.10)

In practice since the bandwidth is usually small compared with the centre frequency, the received signal snapshot vector can be expressed as a column vector of complex values representing the amplitude and phase of the received signal on each element at instant \( t \):

\[
\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_N(t)]
\]  

(5.11)
Of course in addition to the signals there is almost certainly some accompanying noise. The data matrix $D$ is formed from a succession of snapshots and it can be derived in the same way discussed in chapter 2. However, the form of the complex amplitude at the output of each sensor due to $l$-th source is similar to the one in equation (2.5). The covariance matrix $R$ is also the same as in equation (2.43) and can be derived for this case (near-field) as follows:

$$x_{ki} = A_k \exp[j \ u_{ki}]$$  \hspace{1cm} (5.12)

where

$$u_{ki} = k r_{ki}$$  \hspace{1cm} (5.13)

Figure (5.1) shows the geometrical model of a source in the near-field. Applying geometrical analysis to figure (5.1), the following relations can be obtained,

$$r_{ki} = \sqrt{(d_{ki} \text{Imag}) + (i-1) d^2 + r_{ki}^2 \text{Imag}}$$

$$d_{ki} \text{Imag} = r_{ki} \sin(\theta_k)$$

$$r_{ki} \text{Imag} = r_{ki} \cos(\theta_k)$$

Thus,

$$r_{ki} = \sqrt{(r_{ki} \sin(\theta_k) + (i-1) d)^2 + (r_{ki} \cos(\theta_k))^2}$$

or

$$r_{ki} = \sqrt{r_{ki}^2 + 2(i-1) d r_{ki} \sin(\theta_k) + (i-1) d^2}$$  \hspace{1cm} (5.14)

When all the sources have range $r_1$ from element number one of the array, then equation (5.14) can be rewritten as:

$$r_{ki} = \sqrt{r_{ki}^2 + 2(i-1) d r_1 \sin(\theta_k) + (i-1) d^2}$$  \hspace{1cm} (5.15)
As we are concerned with finding the direction of sources, \( \mathbf{C}_t \) presented in equation (5.7) can be rewritten as a vector relative to element number one. Thus \( \mathbf{C}_t \) will be

\[
\mathbf{C}_t = \begin{bmatrix}
\exp(jkr_1 - jkr_1) \\
\exp(jkr_2 - jkr_1) \\
\vdots \\
\exp(jkr_{N-1} - jkr_1) \\
\end{bmatrix} = \begin{bmatrix}
1 \\
\exp(jk(r_2 - r_1)) \\
\vdots \\
\exp(jk(r_{N-1} - r_1)) \\
\end{bmatrix}
\]

(5.16)

When \( r_1 \) is very large then equation (5.15) can be approximated to

\[
r_{x_1} = r_1 + (i-1)d \sin(\theta_x),
\]
or

\[
r_{x_1} - r_1 = (i-1)d \sin(\theta_x),
\]

and

\[
\mathbf{C}_t = \begin{bmatrix}
1 \\
\exp(jkds \sin(\theta_x)) \\
\vdots \\
\exp(jk(N-1)ds \sin(\theta_x)) \\
\end{bmatrix}
\]

(5.17)

The elements of vector \( \mathbf{C}_t \) are the same ones used in equation (2.9) which is mainly representing the model of far-field system. However, the covariance matrix \( R_f \) can be evaluated theoretically as in the far-field.
where \( P_t \) is the source power and it is equal to \( E[|\lambda_t|^2] \). \( R_k \) is the covariance matrix due to \( k \)-th source, and for independent sources

\[
R^H = \sum_{k=1}^{MP} R_k
\]

(5.18)

The data matrix \( D \) for this field is constructed by a similar method in equation (2.42). The sample \( x_{ik} \) is simulated by the following formula,

\[
x_{ik} = n_{ik} + \sum_{k=1}^{MP} |\lambda_k| \exp[j(u_{ki} + \phi_{ik})]
\]

(5.19)

\( u_{ki} \) is defined in equation (5.13), \( n_{ik} \) and \( \phi_{ik} \) are the same random variables used in equation (2.45). Thus, the covariance matrix will be

\[
R = \frac{1}{P} \sum_{k=1}^{P} [X_k X_k^T]
\]

(5.20)

and the elements of the covariance matrix \( b_{im} \)'s are calculated in the same expression of equation (2.44).

5.3 Compensating the effect of near-field

5.3.1 Compensation by method 1.

As mentioned in section (5.1) that the data matrix \( D \) in this method is modified by a focusing vector \( h \). This vector \( h \) can be calculated from figure (5.1) by making \( \theta_f \) = focusing angle (\( \theta_f \)). Thus, equation (5.15) can be rewritten as,

\[
r_i = r_{f_i} = \sqrt[r_{i}^2+2(i-1) d r_i \sin(\theta_f)+((i-1) d)^2]
\]

(5.21a)

or for the special case where \( \theta_f = 0 \), then
In equation (5.7) will be

\[ C_t = t \exp(jkr_1) \exp(Jkr_2) \exp(jkr_2) \]

Then the focusing vector \( h \) is given by

\[ h = C_t^* = \begin{bmatrix} \exp(-jkr_1) \\ \exp(-jkr_2) \\ \vdots \\ \exp(-jkr_N) \end{bmatrix} \]

In this method, each of the columns of data matrix is multiplied by the focusing vector \( h \). The steering vector used in obtaining the angular spectrum is the same one as used in the far-field, which is a function of \( \theta \) only.

5.3.2 Compensation by method 2.

In this method, the data matrix \( D \) is not modified by any vector and the covariance matrix is calculated as in
equation (5.20). The covariance matrix $R$ is decomposed as in the far-field system into its eigenvalues with their associated eigenvectors. Another difference between this method and method 1 is that the steering vector $C(\theta)$ is a function of range and steering angle $\theta$. In general $r_i$ in equation (5.21a) can be written as

$$r_i = \sqrt{r_i^2 + 2(l-1) d r_i \sin(\theta) + ((l-1) d)^2} \quad (5.24)$$

Thus, the $i$-th element of steering vector will be,

$$c_i = \exp(jkr_i) \quad (5.25)$$

and as $r_i$ is a function of $\theta$, then the steering vector will be a function of $\theta$. Therefore $C(\theta)$ is given by:

$$C(\theta) = \begin{bmatrix} \exp(jkr_1) \\ \exp(jkr_2) \\ \vdots \\ \exp(jkr_N) \end{bmatrix} \quad (5.26)$$

Equation (5.26) is similar to equation (5.22), but the values of $c_i$'s are different as expressed in equations (5.24) and (5.25).

5.4 Computer simulation

The MUSIC and MNM techniques are the main algorithms used in direction finding of MP sources located in the near-field. The theoretical background of these two algorithms has been dealt with in chapter 3. However this chapter concentrates
on the obtained results when the two algorithms were simulated. The whole system can be represented by a flow chart diagram shown in figures (5.2a) and (5.2b). The main difficulty of direction finding in this system is that both the range and direction of the sources are unknown. Thus, angular spectrums are obtained at different range values. This task is made by two methods, method 1 and method 2. In method 1 as mentioned earlier the data matrix D is modified by a focusing vector h which is a function of range and focusing angle. In method 2 there is no modification of D matrix and it is less time consuming than method 1. This is because the decomposition of the covariance matrix is needed just once in method 2, while in method 1 it is needed as many times as the data modifications are made. The results for the applications of these two methods, method 1 and 2, are presented and discussed in section(5.5).

5.5 Results and discussion

The results in this chapter provide some general guidelines and specific recommendations for using the MUSIC and MNM algorithms. These advantages can be noted from the following descriptions and discussions of the available results.

I. Direction finding by MUSIC and MNM algorithms

The high resolution properties of the eigenvector techniques have been investigated, in particular, the MUSIC and the MNM where applied in the spatial domain to simulate array data. These tests are done for three or two closely spaced sources in the near-field of receiving array sensors. Each figure has five graphs obtained at different values of focusing range. The first two are made at a focusing range less than
the actual range, while the last two are made at a focusing range higher than the actual one. The third graph is obtained by using the actual range as a focusing range. Figures (5.3) to (5.10) are arranged according to the near-field compensation methods and number of sources. When method 1 is employed, both MUSIC and MNM algorithms produce ambiguities when the sources are very close together, as shown in figures (5.5) and (5.9), respectively. However, by using method 2, both MUSIC and MNM algorithms are able to discriminate clearly these sources as it is evident from figures (5.6) and (5.10), respectively. Moreover, the nulls are also deeper when method 2 is employed with MNM technique than with the MUSIC algorithms shown in figures (5.6) and (5.10), respectively.

Figures (5.11) and (5.12) show the discrimination performance of the MUSIC and MNM algorithms using the proposed two methods. In general, both algorithms give a high performance in separating two close sources. These two figures also show the biasing effect on the actual directions of arrival as a function of signal to noise ratio (SNR). However, the capability of the MUSIC algorithm in separating close sources is worse than the MNM technique using method 1. In general, more accurate estimation of directions can be obtained by using method 2 rather than method 1. This is because of the focusing angle used in vector h which is taken between the actual direction of sources in method 1. But in method 2, the focusing angle is the steering angle, so that the spatial beam can be focused independently from one source to another. Thus, there is no biasing effect on the detected directions by method 2. This method will be explained in detail in point II.

Figures (5.13) and (5.14) show the separation performance of two close sources (separated by 0.18W) using the MUSIC and MNM algorithms with a comparison to the MLM and conventional methods. The conventional methods cannot resolve two close
sources as is evident from figures (5.13) and (5.14), while the MLM technique can resolve them with a significant effect of biasing. Apart from having deeper nulls, the MNM algorithm provides a superior separation of these sources. This property can be seen from figures (5.13) and (5.14), respectively. Figure (5.13) shows the effect of increasing the number of snapshots, which gives deeper nulls compared to that of figure (5.14).

II. The depth of focusing:

This task is done for the two proposed methods, to show the freedom of choosing the focusing range. This provides a general idea of how the arrival angle of each source can be affected by the range. Figures (5.16) to (5.19) show the depth of nulls and the detected arrival angles versus the focusing range, using the two methods of compensating near-field effect. From figure (5.16), it is clear that the deeper nulls are not in the actual range of sources. The biasing effect on detected directions by method 1 can be seen from figure (5.17). This distortion is caused by having a common focusing angle. On the other hand, the near-field effect compensation by method 2 is quite efficient because the steering angle is used as a steering and focusing angle, i.e. the steering vector is a function of range and focusing angle (steering angle). Figures (5.18) and (5.19) show the performance of method 2 in producing a reasonable depth of focusing about the actual range of sources. The error in the detected direction of sources is due to the range of steering vector, signal to noise ratio, and the number of snapshots.

III. The effect of the using subgrouping techniques

The detected direction of a source by a subarray of sensors
in the near-field differs from that detected by another subarray located in different position. Thus, the detected number of sources by a combination of subarrays is the resultant effect of these sources on each subarray. Table(5.1) shows that the number of signal eigenvalues are more than the actual number of sources. This is not applied in the far-field condition since all the subarrays receive parallel signals from each source. Thus, the detected source by each subarray will coincide. From table(5.1), it can be noted that the number of signal eigenvalues is increased as the number of subgroupings is increased. In this case, the subarrays have to be focused in a proper way to get rid of the above problem. Thus, the focusing vector \( h \) will be more complicated than the vector \( h \) developed in section(6.3.1).

IV. The contributions of the results

1-The results of a detailed analysis of the basic limitations associated with the two methods proposed in this chapter were discussed. These results provided a clear explanation of spatial peak separation errors and anomalies observed when using MUSIC or MNM algorithms. The results were obtained for different conditions concerning the focusing range of steering vector, equal, greater, and smaller than the actual range. The depth of focusing was found for the two methods using these two eigenvector algorithms.

2-The biasing effect in the detected directions using the MUSIC and MNM algorithms was tested as a function of SNR applying the two proposed methods for compensating the near-field effect.
3-The resolution performance of the MUSIC and MNM algorithms was compared with the conventional and the MLM techniques using method 2. Moreover, the effect of varying number of snapshots was presented using method 2.

4-A test of the behaviour of signal eigenvalues was done using the subarray technique.

V. Conclusions of the results

Finally, the following points can be concluded from the results under the mentioned conditions:

1-The obtained results verify that the superresolution capabilities of MUSIC and MNM algorithms compared well to conventional beamformer and MLM techniques.

2-All the sources must be at the same range or within the depth of focusing for method 1. However for method 2 this is not necessary.

3-The focusing angle in method 1 has to be chosen in the centre of expected region of source directions.

4-The subgrouping technique of sensors is not easy to use in the near-field, applying the eigenvector decomposition methods, unless the very complicated focusing and steering vectors are modified.
Fig. 5.1 Layout of the near field system
Generating of Data Matrix

Generating of vector $h$

$D = D_{\text{mod.}}$

$R = D^* D^T$

Eigen Decomposition

Finding of MP sources

Angular spectrum $P_{\text{MUSIC}}(\theta)$ or $P_{\text{MNIM}}(\theta)$

Deepest nulls

No

Yes

Plotter display

Finding of estimated directions

Variation of steering range

Fig. 5.2a Flow chart of processing sources in the near field using method 1
Generating of Data Matrix

\[ R = D^* D^T \]

Eigen Decomposition

Finding of MP sources

Steering vector Range Coefficients

Angular spectrum \( P_{\text{MUSIC}}(\theta) \) or \( P_{\text{MNN}}(\theta) \)

Deepest nulls

Variation of steering range

Yes

Plotter display

Finding of estimated directions

No

Fig. 5.2b - Flow chart of processing sources in the near field using method 2
FIG. 5.3: RESOLUTION OF THREE SOURCES OF 30 dB EACH IN THE NEAR FIELD BY MUSIC USING METHOD 1. DIRECTIONS ARE -7.5, -2.5, 2.5°. SENSORS = 10, SNAPSHOTS = 32.
FIG. 5.4: RESOLUTION OF THREE SOURCES OF 30 dB EACH IN THE NEAR FIELD BY MUSIC USING METHOD 2. DIRECTIONS ARE -7.5, -2.5, 2.5 DEG. SENSORS = 10 SNAPSHOTS = 32.
Fig. 5.5: Resolution of two sources of 30 dB each in the near field by MUSIC using Method 1. Directions are -1, 1 deg. Sensors = 10, Snapshots = 32.
Fig. 5.6: Resolution of two sources of 30 dB each in the near field by MUSIC using Method 2. Directions are -1, 1 deg. Sensors = 10  Snapshots = 32.
FIG. 5.7: Resolution of three sources of 30 dB each in the near field by MNM using method I. Directions are -7.5, -2.5, 2.5 deg. Sensors = 10, Snapshots = 32.
FIG. 5.8: RESOLUTION OF THREE SOURCES OF 30 dB EACH IN THE NEAR FIELD BY MMN USING METHOD 2 DIRECTIONS ARE -7.5, -2.5, 2.5 DEG.
SENSORS = 10 SNAPSHOTS = 32
FIG. 5.9: RESOLUTION OF TWO SOURCES OF 30dB EACH IN THE NEAR FIELD BY MPM USING METHOD 1. DIRECTIONS ARE -1.1 DEG.
SENSORS = 10 SNAPSHOTS = 32
FIG. 5.10: RESOLUTION OF TWO SOURCES OF 30 db EACH IN THE NEAR FIELD BY MNN USING METHOD 2 DIRECTIONS ARE -1,1 deg.
SENSORS=10 SNAPSHOTS=32
**Legend**

- Source1 by method1
- Source2 by method1
- Source1 by method2
- Source2 by method2

**FIG. 5.11: DISCRIMINATION PERFORMANCE OF MUSIC FOR TWO SOURCES IN THE NEAR FIELD. DIRECTIONS ARE -0.5°, 0.5°.**

SENSORS = 10  SNAPSHTS = 100  RANGE = 2*L
FIG. 5.12: DISCRIMINATION PERFORMANCE OF MNN FOR TWO SOURCES IN THE NEAR FIELD, DIRECTIONS ARE \(-0.5\)\,deg,\,0.5\,deg.
SENSORS=10, SNAPSHOTS=100, RANGE=2*L
FIG. 5.13: SPATIAL SPECTRAL FOR 2 SOURCES IN THE NEAR FIELD OF 45 dB EACH & DIRECTIONS ARE -0.5, 0.5 DEG. USING METHOD2, SENSORS=10, SNAPSHOTS=32, RANGE=2*L
FIG. 5.14: Spatial spectral for 2 sources in the near field of 45 dB each & directions are -0.5, 0.5 deg. Using method 2: sensors = 10, snapshots = 10, range = 2*L
Fig. 5.15: Discrimination performance of MNM and MUSIC for two sources in the near field as function of snapshots using Method2. Directions are -0.5, 0.5 deg., sensors=10, range=2*L.
FIG. 5.16: DETECTED NULLS OF 2 SIMULATED SOURCES OF 30 dB IN THE NEAR FIELD AT RANGE 2*L USING METHOD 1 DIRECTIONS ARE -2.5, 2.5 DEG. _SENSORS=10 _SNAPSHOTS=32
FIG. 5.17: DETECTED DIRECTIONS OF 2 SIMULATED SOURCES OF 30dB IN THE NEAR FIELD AT RANGE 2*L USING METHOD1 DIRECTIONS ARE -2.5, 2.5 DEG. _SENSORS=10 _SNAPSHOTS=32
FIG. 5.18: DETECTED NULLS OF 2 SIMULATED SOURCES OF 30dB IN THE NEAR FIELD AT RANGE 2*L USING METHOD2 DIRECTIONS ARE -2.5,2.5 DEG. _SENSORS=10 _SNAPSHOTS=32
FIG. 5.19: DETECTED DIRECTIONS OF 2 SIMULATED SOURCES OF 30dB IN THE NEAR FIELD AT RANGE 2*L USING METHOD2. DIRECTIONS ARE -2.5, 2.5 DEG. _SENSORS=10 _SNAPSHOTS=32
<table>
<thead>
<tr>
<th>No. of Sensors</th>
<th>Subgrouping</th>
<th>Eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>3</td>
<td>0.6575 0.8915 1.0753 1.2769 1.3689 1.5638 2.0158 40.178 467.434 1454.720</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>0.6827 0.8538 0.9476 1.1563 1.2465 1.3203 3.1952 72.585 506.732 1379.803</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
<td>0.6633 0.7560 0.8382 0.9629 1.2178 1.4788 13.7446 156.984 606.862 1208.829</td>
</tr>
<tr>
<td>20</td>
<td>11</td>
<td>0.7762 0.8981 0.9317 1.0763 1.6835 16.5931 123.304 382.657 605.006 797.179</td>
</tr>
</tbody>
</table>

Table (5.1) Eigenvalues resulting from a covariance matrix at the output of N-sensors using a subgrouping technique in the near field at a range of 11\(\lambda\)
Two sources of Amplitude = 10, and directions -3°, 3°
Snapshots = 32
6.1 Introduction

The data used for the analysis was collected from a series of practical experiments using equipment originally designed for work on synthetic apertures. The experimental set up used to collect the data was designed to gather unprocessed wavefront information for subsequent analysis. The basic experiment involved the transmission and reception of a series of sonar pulses within a large water tank. For this work two active sources with uncorrelated phases were deployed at a short distance away from the line of the synthetic aperture array as shown in figure (6.1). The measured data consists of 'In phase' (IP) and 'Quadrature' (QU)** samples of the arriving wavefront at the transducer.

This chapter examines the use of synthetic aperture equipment as a controlled testbed for exploring the high resolution performance of some of the eigenvector algorithms mentioned in chapter 3. Two special algorithms, MUSIC and MNM, are investigated widely using real data and simulated data for the same model used in these experiments. The practical work is not exhaustive and the results in this chapter will be limited to the particular environment of the practical work. However, the analysis made here provides practical performance limits of the used algorithms for resolving underwater sources.

** I and Q symbols are replaced by IP and QU because of their use for different purposes
6.2 The Synthetic Aperture Equipment

The water tank dimensions are approximately 8 metres long by 6 metres wide and the water depth is normally 2 metres. Across the 6 metres width at one end is a rigid (RSJ) assembly supporting a rack drive 'railway' on which runs the synthetic aperture measuring head. The movement of the sampling head is a stepper motor controlled by a microcomputer. A preprogrammed sampling rate stores 12 bits of digitised 'IP' and 'QU' samples onto disc from each position of the transducer as it traverses the array aperture.

6.3 Simulation model

The experimental set-up consists of two sources and a receiving transducer moving along the array line. The transmitting array consists of two identical elements which are separated by 4 degrees. The total physical receiving aperture is 36.1 cm divided into subapertures of 17.1 cm each. Figure (6.1) shows the general system configuration. The l-th source range to the I-th sensor can be expressed from figure (6.1) as:

\[ r_{2l} = \sqrt{r^2 + ((l_{\text{ref}} - 1)d + \varphi/2)^2} \]
and
\[ r_{1l} = \sqrt{r^2 + ((l_{\text{ref}} - 1)d - \varphi/2)^2} \]  
(6.1)

Where \( \varphi \) is the distance between the sources.

The sources used in the experimental set-up are sinusoid waveforms of equal frequency. Thus, the source signal is

\[ a_l(t) = A_l \cos(\omega t + \psi_l) \]  
(6.2)

Where \( A_l \) and \( \psi_l \) are the amplitude and initial phase of the l-th source, respectively.
By using equations (2.2), (5.2) and (5.3), the signal received by the $i$-th sensor from $l$-th source is

$$s_{li}(t) = a_{li}(t).a_{rli}$$

Where $a_{rli}$ is the propagation effect which is

$$a_{rli} = \exp(jkr_{li}) = \exp(ju_{li}).$$

$k = 2\pi/\lambda$

We assume $A_1 = A_2 = B$. However, the total signal received by the $i$-th sensor from these two sources is

$$s_i(t) = B[\cos(\omega t + \psi_1).\exp(ju_{1i}) + \cos(\omega t + \psi_2).\exp(ju_{2i})]$$

Now this signal is multiplied by a complex reference signal which has the following form

$$a_{ref}(t) = \Lambda_{ref} \exp(j\omega t)$$

Thus output sample of the $i$-th sensor is a complex value and can be expressed as

$$x_i(t) = s_i(t).a_{ref}(t)$$

$$= s_i(t).\Lambda_{ref} \exp(j\omega t)$$  \hspace{1cm} (6.3)

Note that $\Lambda_1$, $\Lambda_2$, $\Lambda_{ref}$ and $B$ are real values.

By filtering out the high frequency components, and because the system is narrowband, then equation (6.3) can be reduced to the following phasor notations:

$$x_i(\text{real}) = A \left[ \cos(\psi_1 - u_{1i}) + \cos(\psi_2 - u_{2i}) \right]$$  \hspace{1cm} (6.4)

and

$$x_i(\text{imag}) = A \left[ -\sin(\psi_1 - u_{1i}) - \sin(\psi_2 - u_{2i}) \right]$$  \hspace{1cm} (6.5)

Where $A = B.\Lambda_{ref}/2$.  

These two equations (6.4) and (6.5) show the nature of variation of real and imaginary components with the incident angles of the two sources with respect to the i-th sensor. The experiments were done by adjusting the initial phase of the first source to zero value while the initial phase of the second source is changed from zero to $2\pi$ in step of $\pi/16$ every snapshot. The maximum number of snapshots was 32. However the noise components are also added to the signal components either because of the medium of measurement or the sensors. In the simulation the noise is generated from a Gaussian noise random generator. The general form of snapshot values can be written as:

$$x_{ik} = x_{ik}(\text{real}) + jx_{ik}(\text{imag})$$  \hspace{1cm} (6.6)  \\
$$x_{ik}(\text{real}) = A[\cos(-u_{1i}) + \cos(\pi(k-1)/16 - u_{2i})] + n_{ik}(\text{real})$$  \hspace{1cm} (6.7) \\
and

$$x_{ik}(\text{imag}) = A[-\sin(-u_{1i}) - \sin(\pi(k-1)/16 - u_{2i})] + n_{ik}(\text{imag})$$  \hspace{1cm} (6.8)

$x_{ik}$ is the i-th sensor sample at the k-th snapshot. The elements of covariance matrix $R$ are calculated as in equation (2.34) for theoretical evaluation or as in equation (2.44) for simulation. From figure (6.1), the signal at the output of each sensor can be considered as a vector sum of the two source signals. Thus the $b_{im}$ in equation (2.34) for this particular case will be

$$b_{im} = E[(A_1 \exp(ju_{1i}) + A_2 \exp(ju_{2i})) \cdot (A_1 \exp(-j u_{1m}) + A_2 \exp(-j u_{2m}))]$$  \hspace{1cm} (6.9)

This equation can also be simplified in the same way in the equations (2.34) to (2.37). In the case of noise presence, equation (2.37) is also used.
Fig. 6.1 Layout of practical experiments performed in the Tank Room
6.4 The results and performance of algorithms

This section attempts to show by using real data, the performance of the different methods considered in chapter 3. The data as mentioned earlier was collected from an emulator of an array of sensors, which were spaced by half wavelength approximately. The receiving sensor beamwidth was 60° horizontal and 16° vertical. It is convenient to produce a so-called spectrum for each of the optimisation techniques considered in chapter 3. The spectrum considered here is the display of power versus the steering angle. The steered input is the covariance matrix or its replacement obtained from real data. This section will be divided into two subsections. The first involves mainly the performance of MUSIC and MNM algorithms using real data in comparison with the simulated data. The second subsection involves examining the rest of techniques mentioned in chapter 3.

6.4.1 Performance of MUSIC and MNM

Figures (6.2) and (6.3) show the spatial spectrum of the two sources described in the preceding section. Two main algorithms, MUSIC and MNM were used. The analysis here is on a theoretical covariance matrix obtained from two uncorrelated sources. The main advantage of these two figures is, the pinpointing of the arrival direction of the two sources when the receiving array is moving along the aperture. This spatial spectrum in figures (6.2) and (6.3) also gives an idea of the overall depth of nulls that can be obtained in practice and in simulation. Figures (6.3) and (6.4) show the ability of MNM and MUSIC in resolving two real sources which are horizontally separated by 4°. Three curves are presented in each figure as indicated in the Legend boxes. The most important observation made was that the depth of nulls was greater when choosing 12 consecutive
snapshots than when choosing 32 consecutive snapshots or when selecting snapshots randomly. Moreover, a significant biasing effect appears in detected directions. These two problems have led to the search for the reasons of such unacceptable behaviour. Figures (6.6) and (6.7) show the simulation model spectrum for the same test procedure as in figures (6.4) and (6.5). Figures (6.8) and (6.9) show that in the case of 12 snapshots, the choice of noise set added to the simulated signals has a significant effect on the depth of nulls and biasing. Sometimes the presence of noise and its variation will distort the nulls in a low number of snapshots as shown in set 1 (figure (6.8)). These problems can be overcome by taking more snapshots, for example 32, which cover the total phase difference between the two sources (2\pi). Figure (6.9) shows that the depth of nulls is almost the same using different sets of noise in the case of 32 snapshots. The biasing effect is decreased as the number of snapshots is increased and the SNR also affects biasing as explained in chapter 4 and 5. Figures (6.10) and (6.11) show the solution of the above problems using the MUSIC algorithm. Figures (6.12) and (6.13) show the resolution ability and performance of the MUSIC and MNM algorithms in comparison with the MLM and conventional beamformer. These two sources can be easily resolved in the case of experimental and simulated data using the MUSIC and MNM algorithms, while they cannot be resolved by the conventional beamformer. There is also an ambiguity and biasing effect in the case of the MLM method as is evident from figure (6.13). These tests confirm the facts mentioned in the preceding chapters about these algorithms.
FIG. 6.2: Spatial spectral ... 2 sources theoretically represented 5 apertures in line using MNM method. Sensors = 10
FIG. 6.3: SPATIAL SPECTRAL CHARACTERISTICS OF 2 SOURCES THEORETICALLY REPRESENTED USING 5 APERTURES IN LINE USING MUSIC METHOD WITH 10 SENSORS.
FIG 6.4: _SPATIAL SPECTRAL..._ 2 REAL SOURCES USING MNN FOR DIFFERENT NUMBER OF SNAPSHOTS
SENSORS=10  _P=SNAPSHOTS

**Legend**

- P=32 Regular
- P=12 Regular
- P=12 Random
FIG. 6.5: SPATIAL SPECTRAL ANALYSIS FOR 2 REAL SOURCES USING MUSIC FOR DIFFERENT NUMBER OF SNAPSHOTS
SENSORS = 10 _P = SNAPSHOTS

Legend
P=32 Regular
P=12 Regular
P=12 Random
FIG. 6.6: SPATIAL SPECTRAL ANALYSIS OF TWO SIMULATED SOURCES USING MNM FOR DIFFERENT NUMBER OF SNAPSHOTS.
SENSORS = 10, P = SNAPSHOTS.
FIG. 6.7: SPATIAL SPECTRAL FOR DIFFERENT NUMBER OF SNAPSHOTS USING MUSIC.
SENSORS = 10, P = SNAPSHOTS.

Legend

P = 12 Regular
P = 12 Random
P = 32 Regular
FIG. 6.8: SPATIAL SPECTRAL ... 2 SIMULATED SOURCES USING MNM FOR DIFFERENT SETS OF NOISE
SENSORS=10, SNAPSHOTS=12
FIG. 6.9: SPATIAL SPECTRAL ..... 2 SIMULATED SOURCES USING MFM FOR DIFFERENT SETS OF NOISE
SENSORS=10 ... SNAPSHOTS=32
FIG. 6.10: SPATIAL SPECTRAL SIMULATED SOURCES USING MUSIC FOR DIFFERENT SETS OF NOISE
SENSORS=10 _SNAPSHOTS=12
FIG. 6.11: SPATIAL SPECTRAL SIMULATED SOURCES USING MUSIC FOR DIFFERENT SETS OF NOISE
SENSORS=10, SNAPSHTS=32
FIG. 6.12: SPATIAL SPECTRAL ... 2 SIMULATED SOURCES USING DIFFERENT METHODS _SENSORS=10 _SNAPSHOTS=32
FIG. 6.13: SPATIAL SPECTRAL ... 2 REAL SOURCES USING DIFFERENT METHODS _SENSORS=10 _SNAPSHOTS=32
6.4.2 Performance of alternative methods

The high resolution properties of the eigenvector techniques were discussed in chapter 3. The performance of MUSIC and MNM algorithms was examined by simulation in chapter 4 for the correlated and uncorrelated sources using the new technique of combining the subarrays of sensors. Also, an examination was made in the preceding section of their performance in separating two sources compared with the conventional and MLM methods using real data. This section attempts to compare MUSIC and MNM algorithms with some alternative spatial techniques by real data studies.

The results of the Cantonl and Godara, Barlett, Burg, Johnson and DeGraaf, Wax and Kallath, and Byrne and Steele methods will be discussed. The theoretical derivation of these methods were mentioned in chapter 3. It should be noted that the real data study can never be exhaustive and the results in this section will be limited to the particular environment of the practical system. These limitations will be summarised in section 6.5.

6.4.2.1 Results and discussions

This subsection concentrates on a discussion of the results obtained in figures (6.14) to (6.22) using real sonar data. A test of the performance of some of these algorithms has been carried out by Jefferies [ref.34] using simulated data. The real data was measured at the output of an aperture of 10 sensors and a maximum of 32 snapshots.

It is convenient to produce a so-called spectrum for each of the six optimisation techniques considered in this section. These results show the advantages and disadvantages of these algorithms in comparison with the conventional beamformer.
The conventional beamformer is not able to resolve the two sources as shown in figure (6.14). The three beams shown in each figure in this section were calculated for a different number of snapshots, i.e., 32, 12 consecutive and 12 random snapshots.

Figure (6.15) shows the performance of the MLM estimator using the above three conditions. The case of 32 consecutive snapshots provides better resolution performance than 12 consecutive snapshots. The power of signals can be obtained from the MLM beamformer directly when the number of snapshots is high enough. This statement is evident from the figure where the detected source power is degraded as the number of snapshots is low. The depth of dip between the two adjacent peaks is not deep enough compared to MUSIC and MNM algorithms as shown earlier in section (6.4.1).

Figure (6.16) shows the performance of the Burg method. The MEM provides a high resolution performance, but the disadvantage of this method is that it produces extra high peaks which can be considered as extra sources while they are not. The biasing effect is relatively high compared to MUSIC and MNM algorithms at a low number of snapshots.

The eigenvector method (EVM) proposed by Johnson and DeGraaf was examined on the same real data used in the preceding figures. Figure (6.17) shows the performance of this method which is better than MLM and less effective than MUSIC and MNM algorithms. The reason for degradation of EVM is the weighting of the beam of each eigenvector by its eigenvalue which is very small in real time.

Bartlett's signal eigenvector method was also examined on the same data and figure (6.18) shows its performance. Although the SEVM method contains the same information, it is more difficult to resolve two close sources which are separated by less than a half beamwidth, because of the broadness of the maxima. The appearance of sidelobes in
this estimator is considered as another disadvantage.

Figure (6.19) shows the performance of the Byrne and Steele estimator (SNLM) using $\delta=0.005$ and $\beta=4$. The value of $\delta$ is chosen within the stable noise eigenvalues. This method provides a good performance in resolving these two real sources. The sensitivity to the number of snapshots is very high which can be noticed from the figure when the number of snapshots is low, i.e. the biasing is relatively high at a low number of snapshots. The main difficulty is the choice of $\delta$ and $\beta$ which will be examined in the next subsection (6.4.2.2).

Wax and Kailath's methods were also examined. Figure (6.20) shows that the noise eigenvector method provides a performance similar to the MUSIC algorithm. Figure (6.21) shows the second estimator proposed by Wax and Kailath which produces a performance similar to the conventional beamformer shown in figure (6.14). This estimator is unable to resolve very close sources.

The Cantoni and Godara estimator was also examined and its performance is shown in figure (6.22). This estimator gives high performance in pinpointing the direction of one source and an ambiguity in the second source direction. This estimator has a high biasing effect when the number of snapshots is low. Also, this estimator creates extra nulls which may be considered as extra sources. In other words, this estimator's performance is worst when compared with other noise eigenvector estimators due to instability of the smallest noise eigenvector.

Finally, it can be concluded that the MUSIC and MNM estimators provide superb performance in resolving these two real sonar sources with comparison to all other high resolution estimators discussed in this research.
Fig. 6.14: Spatial spectral.... 2 real sources using CBM for different number of snapshots. Sensors=10, P=Snapshots.
FIG. 6.15: SPATIAL SPECTRAL PSD OF 2 REAL SOURCES USING MLM FOR DIFFERENT NUMBER OF SNAPSHOTS
SENSORS=10 _P=SNAPSHOTS

Legend
P=32 Regular
P=12 Regular
P=12 Random
LEGEND

P=32 Regular
P=12 Regular
P=12 Random

FIG. 6.16: SPATIAL SPECTRAL DEGRADATION USING MEM FOR DIFFERENT NUMBER OF SNAPSHOTS. SENSORS=10, P=SNAPSHOTS.
FIG. 6.17: Spatial spectral... 2 real sources using EVD for different number of snapshots. Sensors = 10, P = Snapshots.
FIG. 6.18: SPATIAL SPECTRAL .... 2 REAL SOURCES USING SEVD FOR DIFFERENT NUMBER OF SNAPSHOTS SENSORS=10, _P_=SNAPSHOTS
CHAPTER 6

Legend

- P=32 Regular
- P=12 Regular
- P=12 Random

FIG. 6.19: SPATIAL SPECTRAL ... 2 REAL SOURCES USING SNLM FOR DIFFERENT NUMBER OF SNAPSHOTS
SENSORS=10, P=SNAPSHOTS
FIG. 6.20: SPATIAL SPECTRAL .... 2 REAL SOURCES USING WK1 FOR DIFFERENT NUMBER OF SNAPSHOTS
SENSORS=10 _P=SNAPSHOTS

Legend
P=32 Regular
P=12 Regular
P=12 Random
CHAPTER 6

FIG. 6.21: SPATIAL SPECTRAL .... 2 REAL SOURCES USING WK2 FOR DIFFERENT NUMBER OF SNAPSHOTS SENSORS=10 _P=SNAIPSHOTS

Legend
P=32 Regular
P=12 Regular
P=12 Random
FIG. 6.22: SPATIAL SPECTRAL ANALYSIS OF 2 REAL SOURCES USING CGM FOR DIFFERENT NUMBER OF SNAPSHOTS
SENSORS=10, P=SNAPSHOTS
6.4.2.2 Performance of SNLM

The data collected in this chapter was examined also to show how the SNLM estimator would work as a function of $\delta$ and $\beta$. This function can be simplified to show the effect of trivial solution when the denominator approaches zero value, when $\beta$ is very large and $\delta$ is within the stable region of noise eigenvalues. Thus, this statement can be explained mathematically as:

$$\frac{\lambda_i^{\beta-1}}{(\lambda_i^2 + \delta^2)^{\beta}}$$

$$= \lambda_i^{-1} \left[ \frac{1}{\lambda_i^2 + \delta^2} \right]^\beta$$

This function will be very high using real sonar data, or in other words when the noise eigenvalues are very small. The other difficulties, which are noticed, are in the choice of $\beta$ and $\delta$. The choice of $\delta$ has a severe effect on the resolution capability of this estimator. Figures (6.23) and (6.24) show the effect for two choices of $\beta$, for example 4 and 8. When the $\delta$ lies within the region of small noise eigenvalues, this estimator provides an acceptable performance, but when $\delta$ lies within the highest noise eigenvalues, the performance of SNLM estimator is degraded. On the other hand when $\delta$ lies within the signal eigenvalues the performance of the SNLM estimator is completely corrupted as shown in figures (6.23) and (6.24). The choice of $\beta$ has an effect on separation capability of source peaks as well. When $\beta$ is zero, then the SNLM estimator behaves
exactly the same as MLM, but when $\beta$ increases, the SNLM estimator provides better performance as shown in figure (6.25). The computation performance of this estimator using real data is not comparable to MUSIC and MNM algorithms. Figures (6.23) to (6.25) show the degraded performance and the ill-conditioning effect when $\beta$ is high.

6.5 Limitations of practical experiments

1-A low pulse width of 10 cycles was chosen in order to avoid interference with the multipath signal. However, as can be seen in figure (6.27), which shows the transmitted and received pulses, the received one has very small pulse width. As a result if the snapshots are not timed properly, the resultant amplitude of the snapshots will not be accurate. The data which was processed in these experiments was the only sample in the centre of the received pulse.

2-Samples away from the pulse centre are liable to produce biasing in the resolved directions of the two sources. This is due to phase distortion of the transducer when sampling is done in the region of building up of the pulse. Figure (6.26) shows this problem using MUSIC and MNM algorithms as examples.

3-The array of sensors was implemented by moving one transducer along a straight line and switching on at every 1.9 cm step. The actual array should be at least 10 sensors focused mechanically in such a way as to get a full far-field representation of sources.

4-Moving of the receiving transducer has an indirect effect if it generates unexpected or unwanted noise.
5-The limited physical dimensions of the water tank are a significant reason for the production of a multipath phenomena.

6-The 12 bits Analogue to Digital converter used to convert the output of sensors to digital form may introduce inaccuracies due to the quantisation error. If this is so, then the actual estimation of the covariance matrix will be affected. Thus, the eigenvalue and eigenvector calculations will be affected and may cause a biasing error.
FIG. 6.23: SPATIAL SPECTRAL ..... 2 REAL SOURCES USING SNLM FOR DIFFERENT VALUES OF $\delta$ _SENSORS=10 $\beta=8$
FIG. 6.24: SPATIAL SPECTRAL 2 REAL SOURCES USING SNLM FOR DIFFERENT VALUES OF \( \delta \) _SENSORS=10  \( \beta =4 \)
FIG. 6.25: SPATIAL SPECTRAL ... 2 REAL SOURCES USING SNLM FOR DIFFERENT VALUES OF $\beta$  _SENSORS=10  $\delta =0.010$
FIG. 6.26a: SPATIAL SPECTRAL ... 2 REAL SOURCES USING MNM FOR DIFFERENT LOCATIONS OF SNAPSHOTS IN THE RECEIVED PULSE
SENSORS=10 _SNAPSHOTS=32

Legend
Centre Sample
Right Sample
Left Sample
FIG. 6.26b: SPATIAL SPECTRAL ... 2 REAL SOURCES USING MUSIC FOR DIFFERENT LOCATIONS OF SNAPSHOT IN THE RECEIVED PULSE
SENSORS=10 SNAPSHOTS=32
a) Transmitted pulse 10 cycles 40kHz 2 volt/cm

b) Transmitted and received pulses of one source 40 kHz
   transmitted 2 volt/cm
   received 0.2 volt/cm
   time base 0.5 msec/cm
c) Received pulse of two sources 40 kHz 0.2 volt/cm: Time base 0.5 msec/cm

Fig. 6.27. Transmitted and received pulses at an element of the array

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Fig. 6.28: Part of the set up and processing equipment used in the practical experiments
CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

7.1 Conclusions

A number of high resolution signal processing algorithms have been examined during this study using simulated data for correlated and uncorrelated sources, and real sonar data for uncorrelated sources. Eigenvector techniques have been discussed and shown to have superior resolution properties over the conventional beamformer method. A new criterion has been proposed to separate signal eigenvalues from noise eigenvalues. Also, a new technique has been worked out for combining linear arrays of sensors to resolve fully correlated sources and the multipath problem using MUSIC and MNM estimators. The detected number of sources by eigenvector decomposition methods in the near-field are more than the actual number using a subgrouping of sensors unless a very complicated focusing and steering vectors are developed as explained in section(5.5).

The MUSIC and MNM estimators offer superior performance in resolving two correlated sources of 0.99 correlation coefficient and separated by half the conventional beamwidth. As a result of the practical experiments and computer simulation of algorithms used in this thesis, the following conclusions can be drawn.

1-The conventional beamformer is less complex in calculations, but it does not resolve close sources.

2-The MLM estimator provides direct power estimation and is able to resolve close sources. Disadvantages of this
estimator are that it cannot resolve highly correlated sources and the biasing is high in both the near-field and far-field.

3-An intensive study was made of the MUSIC algorithm which provides superior performance in resolving correlated or uncorrelated sources using the proposed techniques of subarrays of sensors. It is less sensitive to noise at a low number of snapshots than MNM algorithm. At a very high number of snapshots both MUSIC and MNM algorithms are unbiased.

4-Another intensive study was made of the MNM algorithm which performs well in separating very close sources. It was noticed that the MNM algorithm is sensitive to the noise variation sequence at low number of snapshots but at very high number of snapshots, it is unbiased. One more disadvantage is that it is liable to produce many spurious nulls.

5-The eigenvector method proposed by Johnson and DeGraaf has the same sort of performance as MUSIC algorithm at a very high number of snapshots. Its performance is degraded and biased at low number of snapshots.

6-A test for the performance of the signal eigenvector method suggested by Barlett has been made, which is similar to that of the conventional beamformer. The main advantage of this estimator is the identity of peaks for equal and non equal sources in the spatial spectrum.

7-Wax and Kailath's method using the signal eigenvectors is similar to the method suggested by Barlett in resolving close sources. But the performance of the noise eigenvector method is similar to MUSIC algorithm but its gain depends on the signal eigenvalues.
8-The resolution capability of the Byrne and Steele estimator (SNLM) is good, but this method is computationally complex and requires several of its parameters to be optimised. It is liable to produce a very low spatial spectral beams which are unacceptable in the processing of real data as discussed in section(6.4.2.2).

9-Finally the MEM (Burg method) was also tested in resolving two real sonar sources. This method is highly efficient in resolving close sources. However it is liable to produce extra peaks which can be considered as extra sources, it is not good for power estimation and it is biased at a low number of snapshots.

7.2 Further work

Over the past one or two decades there has been considerable advances in high resolution techniques for array processing. What is disappointing is that there has not been a substantial number of applications of these new techniques. Part of this can be explained by the complexity of most of the algorithms and the consequent demand on computation. However over a similar period there has been a revolution in the capabilities of processors and several very powerful reasonably priced devices have appeared on the market. It would appear that perhaps now is the time for significant effort to be concentrated on the realisation of some of the more powerful algorithms in a form that would make real time operation an economic possibility.
APPENDIX A

Capon estimator [ref. 25]

Solution using constraints

Complex variables

We require the same constraint to keep the gain unity in the direction of C

\[ W^T C = 1 \] \hspace{1cm} (A-1)

However since in general C and hence W may be complex this implies two constraints

\[ \text{Re}[W^T C] = 1 \]
\[ \text{Im}[W^T C] = 0 \]

Let \( W = W_r + j W_i \) and \( C = C_r + j C_i \)

\[ W^T C = (W_r^T C_r - W_i^T C_i) + j(W_r^T C_i + W_i^T C_r) \] \hspace{1cm} (A-2)

We wish to minimise the output subject to this constraint. The language multiplier is going to be defined. We first generate a "cost function".

\[ H(W) = P + 2z(1-W_r^T C_r + W_i^T C_i) + 2z(W_r^T C_i + W_i^T C_r) \] \hspace{1cm} (A-3)

where \( P \) output power

\( z \) arbitrary value

\[ \frac{\partial H(W)}{\partial W_r} = 2 \text{Re}[R_r W] - 2z_r C_r + 2z_r C_i \] \hspace{1cm} (A-4)

\[ \frac{\partial H(W)}{\partial W_i} = 2 \text{Im}[R_r W] + 2z_i C_i + 2z_i C_r \] \hspace{1cm} (A-5)

Equating equations (A-4) and (A-5) to zero, then

\[ \text{Re} [R_r W_0] = z_r C_r - z_i C_i \] \hspace{1cm} (A-6)
Combining (A-6) and (A-7) then

\[
R_0 \omega = z_r C_r - z_i C_i - j z_r C_i - j z_i C_r
\]

\[
= z_r (C_r - j C_i) - j z_i (C_r - j C_i)
\]

\[
R_0 \omega = (z_r - j z_i) C^*
\]

\[
\omega = (z_r - j z_i) R^{-1} C^*
\]

\[
c^T \omega = 1 = (z_r - j z_i) C^T R^{-1} C^*
\]

\[
(z_r - j z_i) = \frac{1}{C^T R^{-1} C^*}
\]

\[
\omega = \frac{R^{-1} C^*}{C^T R^{-1} C^*}
\]

\[
\omega = \frac{W^T C^*}{C^T R^{-1} C^*} = \frac{1}{C^T R^{-1} C^*}
\]

This equation (A-11) is the maximum likelihood estimator, and the weight coefficients vector is

\[
\omega = \omega^T R \omega = \frac{W^T \omega}{C^T R^{-1} C^*} = \frac{1}{C^T R^{-1} C^*}
\]

\[
\omega = \omega^T R^{-1} C^*
\]
APPENDIX B

As shown earlier in Chapter 2 the covariance matrix \( R \) can be written as the sum of the noise covariance matrix plus the signal covariance matrix, thus \( R \) is [ref. 28]

\[
R = \sum_{m=1}^{MP} A_m C_m C_m^H A_m^* + \sigma^2 I \quad \ldots \quad (B-1)
\]

or

\[
R_s = \sum_{m=1}^{MP} A_m C_m C_m^H A_m^* = R - \sigma^2 I \quad \ldots \quad (B-2)
\]

where \( A_m \) is the \( m \)-th signal vector

\[
A_m = [a_{m1} \ a_{m2} \ a_{m3} \ldots \ldots \ a_{mN}] \quad \ldots \quad (B-3)
\]

and \( C_m \) is a factor which represents the effect of propagation and is given by

\[
C_m = [c_{m1} \ c_{m2} \ c_{m3} \ldots \ldots \ c_{mN}] \quad \ldots \quad (B-4)
\]

\( c_{mi} \) is calculated in Chapter 2 for far-field and Chapter 5 for near-field, which is a function of signal direction. The matrix \( R_s \) in Equation (B-2) is a matrix of rank \( MP \) and hence will have only \( MP \) non-zero eigenvalues.

Therefore

\[
\sum_{m=1}^{MP} A_m C_m C_m^H A_m^* = \sum_{m=1}^{MP} \lambda_m U_m U_m^H \quad \ldots \quad (B-5)
\]

Hence the signal direction vectors are contained in the signal subspace and hence are orthogonal to the noise eigenvectors. That is,
\[ C_m U_j = 0 \quad m = 1 \text{ to } MP \]
\[ j = MP + 1 \text{ to } N \]

This provides the basis for a number of very powerful techniques for determining the directions of the signals.


[17]. Clarke I.J, "Comparison of advanced signal processing algorithms" electronics division, Colloquium on "Direction finding systems in their operating environment" Jan. 21, 1986.


[19]. Farrier D.R., and Mcleod F.N., "Direct estimation of multipath signals" ICASSP86 vol.4, pp.52.9_1_52.9_4, 1986.


[45]. Kopp L., and Bienvenu G., "Multiple detection using eigenvalues when the noise spatial coherence is partially unknown", NATO ASIS on underwater signal processing in 1984.


