The application of finite element technique to lubrication problems

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THE APPLICATION OF FINITE ELEMENT TECHNIQUE
TO LUBRICATION PROBLEMS

A THESIS
SUBMITTED TO LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY
FOR THE DEGREE OF MASTER OF PHILOSOPHY.

DECEMBER 1985.

BY
KOJI ITO

SUPERVISORS : DR. T. P. NEWCOMB         DEPARTMENT OF
DEPARTMENT OF
TRANSPORT TECHNOLOGY

DR. R. ALI                  TRANSPORT TECHNOLOGY
Abstract

The work presented in this thesis is concerned with the application of the finite element technique to solve general lubrication problems. Incompressible isothermal condition has been considered as a first step towards the lubricant film investigation.

Fluid finite elements of triangular and rectangular planform have then been developed and incorporated in a computer program especially developed for a finite element analysis. Although the elements are primarily two dimensional in the film region, it is possible to allow for a variation in the thickness of an oil film within an element. Other parameters affecting the lubricant such as shear forces, body forces, inertia forces, squeezing velocities and diffusion velocities can also be varied at each node. These elements have been extensively tested by considering standard lubrication problems such as squeezing pad, slider bearing and step bearing and the results obtained are shown to be an excellent agreement with those derived from theoretical solutions.

The analysis has then been extended to study the behaviour of the oil film between rotating annular discs where it is known that grooving on the disc affects the pressure distribution. The results indicate that grooving reduces the disc engaging time and that the engaging speed determined by surface velocity in the $z$ direction is shown to be higher in radially grooved discs than in spirally grooved discs at a given squeezing pressure.
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### CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nomenclature</td>
<td></td>
</tr>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2. Literature Survey</td>
<td>4</td>
</tr>
<tr>
<td>2.1 General Lubrication Analysis Using the Finite Element Technique</td>
<td>4</td>
</tr>
<tr>
<td>2.2 Porous Region Analysis</td>
<td>10</td>
</tr>
<tr>
<td>2.3 Disc Problem Analysis</td>
<td>14</td>
</tr>
<tr>
<td>3. Theoretical Developments</td>
<td>17</td>
</tr>
<tr>
<td>3.1 Reynolds Equation</td>
<td>17</td>
</tr>
<tr>
<td>3.2 Incompressible Isothermal Lubrication</td>
<td>23</td>
</tr>
<tr>
<td>3.3 Inclusion of the Centrifugal Force in the Generalized Reynolds</td>
<td>25</td>
</tr>
<tr>
<td>3.4 Application to Porous Annular Discs</td>
<td>27</td>
</tr>
<tr>
<td>3.5 Friction Force, Friction Torque and Load Carrying Capacity of Bears and Discs</td>
<td>29</td>
</tr>
<tr>
<td>4. Application of the Finite Element Technique</td>
<td>32</td>
</tr>
<tr>
<td>4.1 Variational Principles</td>
<td>32</td>
</tr>
<tr>
<td>4.2 Development of Fluidity Matrices</td>
<td>33</td>
</tr>
<tr>
<td>4.2.1 Development of Fluidity Matrices for Triangular Elements</td>
<td>38</td>
</tr>
<tr>
<td>4.2.2 Development of Fluidity Matrices for Rectangular Elements</td>
<td>41</td>
</tr>
<tr>
<td>4.3 Analysis of the Porous Region</td>
<td>49</td>
</tr>
<tr>
<td>4.4 Determination of the Load Carrying Capacity of Bearings, Friction</td>
<td>52</td>
</tr>
<tr>
<td>4.5 Development of the Computer Program</td>
<td>58</td>
</tr>
<tr>
<td>4.5.1 Flow Charts</td>
<td>58</td>
</tr>
<tr>
<td>4.5.2 Data Input</td>
<td>59</td>
</tr>
</tbody>
</table>
5. Application to Standard Lubrication Problems ................................................. 62
   5.1 Rectangular Squeezing Pad ........................................................................ 62
      5.1.1 Theoretical Analysis ........................................................................ 62
      5.1.2 Finite Element Model and Results .................................................. 63
   5.2 Infinite Width Slider Bearing ...................................................................... 64
      5.2.1 Theoretical Analysis ........................................................................ 64
      5.2.2 Finite Element Model and Results .................................................. 66
   5.3 Step Bearing ............................................................................................... 67
      5.3.1 Infinite Width Step Bearing ................................................................ 67
      5.3.2 Finite Width Step Bearing .................................................................. 68
      5.3.3 Finite Element Model and Results .................................................. 69

6. Application to Annular Disc Problems ............................................................ 70
   6.1 Behaviour of the Film between Flat Discs .................................................. 70
      6.1.1 Theoretical Analysis ........................................................................ 70
      6.1.2 Finite Element Model and Results .................................................. 72
   6.2 Behaviour of the Film between Grooved Discs ........................................... 73

7. Conclusions .................................................................................................... 78
   References ...................................................................................................... 80
   Figures ........................................................................................................... 85
   Computer Programs ....................................................................................... 136
**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Page numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>area</td>
<td>5</td>
</tr>
<tr>
<td>( \hat{A} )</td>
<td>porosity</td>
<td>28</td>
</tr>
<tr>
<td>B</td>
<td>body force</td>
<td>8</td>
</tr>
<tr>
<td>( B_x, B_y )</td>
<td>body forces in ( x, y ) direction</td>
<td>19</td>
</tr>
<tr>
<td>( B_{mx}, B_{my} )</td>
<td>averaged body forces in ( x, y ) direction</td>
<td>23</td>
</tr>
<tr>
<td>( C_f )</td>
<td>centrifugal force per unit volume</td>
<td>25</td>
</tr>
<tr>
<td>( C_{fm} )</td>
<td>centrifugal force averaged along ( z ) direction</td>
<td>26</td>
</tr>
<tr>
<td>( f_i )</td>
<td>interpolation function at node ( i )</td>
<td>6</td>
</tr>
<tr>
<td>( F_x, F_y )</td>
<td>friction forces in ( x, y ) direction</td>
<td>29</td>
</tr>
<tr>
<td>( F_T )</td>
<td>total friction force</td>
<td>56</td>
</tr>
<tr>
<td>( h )</td>
<td>film thickness</td>
<td>5</td>
</tr>
<tr>
<td>( H )</td>
<td>thickness of porous region</td>
<td>29</td>
</tr>
<tr>
<td>( I )</td>
<td>functional</td>
<td>5</td>
</tr>
<tr>
<td>( \hat{i}, \hat{j} )</td>
<td>unit vectors in ( x, y ) direction</td>
<td>22</td>
</tr>
<tr>
<td>( [J] )</td>
<td>the Jacobian matrix</td>
<td>42</td>
</tr>
<tr>
<td>(</td>
<td>J</td>
<td>)</td>
</tr>
<tr>
<td>( K )</td>
<td>fluidity matrix</td>
<td>35</td>
</tr>
<tr>
<td>( L )</td>
<td>load carrying capacity</td>
<td>29</td>
</tr>
<tr>
<td>( M )</td>
<td>interpolation function</td>
<td>49</td>
</tr>
<tr>
<td>( N )</td>
<td>interpolation function</td>
<td>33</td>
</tr>
<tr>
<td>( n )</td>
<td>unit outward normal</td>
<td>5</td>
</tr>
<tr>
<td>( P_0, P_1 )</td>
<td>pressures on boundary</td>
<td>5</td>
</tr>
<tr>
<td>( p )</td>
<td>pressure in film region</td>
<td>5</td>
</tr>
<tr>
<td>( \hat{p} )</td>
<td>pressure in porous region</td>
<td>29</td>
</tr>
<tr>
<td>( Q )</td>
<td>flux component</td>
<td>33</td>
</tr>
<tr>
<td>( q )</td>
<td>volume flow rate</td>
<td>18</td>
</tr>
<tr>
<td>( q' )</td>
<td>nodal diffusion flow rate</td>
<td>49</td>
</tr>
<tr>
<td>( r )</td>
<td>polar coordinate</td>
<td>25</td>
</tr>
<tr>
<td>( r_1 )</td>
<td>inside radius of a disc</td>
<td>71</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>outside radius of a disc</td>
<td>71</td>
</tr>
<tr>
<td>( r_0 )</td>
<td>radius of flow separation</td>
<td>71</td>
</tr>
<tr>
<td>( S )</td>
<td>part of boundary</td>
<td>5</td>
</tr>
<tr>
<td>( s, t )</td>
<td>natural coordinate</td>
<td>41</td>
</tr>
<tr>
<td>( T_e )</td>
<td>friction torque of an element</td>
<td>31</td>
</tr>
<tr>
<td>( T_T )</td>
<td>total friction torque</td>
<td>57</td>
</tr>
</tbody>
</table>
\( u, v, w \) velocities in film region in \( x, y, z \) direction  
\( \hat{u}, \hat{v}, \hat{w} \) velocities in porous region in \( x, y, z \) direction  
\( \bar{u}, \bar{v}, \bar{w} \) average velocities in \( x, y, z \) direction  
\( \nu_d \) diffusion velocity  
\( V_1, V_2 \) velocities of fluid at \( z=0 \) and \( h \) in \( y \) direction  
\( W_1, W_2 \) velocities of fluid at \( z=0 \) and \( h \) in \( z \) direction  
\( x, y, z \) coordinates  
\( \alpha, \beta, \gamma \) multipliers  
\( \varepsilon \) arbitrary parameter  
\( \eta(x) \) continuous function  
\( \theta \) angle  
\( \mu \) fluid viscosity  
\( \rho \) density  
\( \tau_{xy} \) shear stress on the surface normal to \( y \) axis in \( x \) direction  
\( \phi \) permeability  
\( \omega \) angular velocity  
\( \nabla \)  
\( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \)
1. **Introduction**

The analysis of lubrication problems with a view to reduction of friction losses occurring between relatively moving surfaces have occupied researcher's attention for a long time. These problems can be recognized in almost all mechanisms which have moving parts and as such their investigation and study is important. These investigations have assumed added significance lately due to the energy crisis since the reduction of friction losses contributes greatly to energy savings.

Another aspect of lubrication analysis is the determination of ways of obtaining higher friction forces and friction torques between lubricated surfaces in wet clutches and brake discs. In this case friction has a positive influence in the engaging actions of the surfaces during operation of wet friction devices.

Behaviour of a lubricant film can be explained by the basic theory of Reynolds equation with ideal assumptions such as the lubricant is incompressible, isothermal and behaves as a Newtonian fluid and that the film thickness is known. Some simple standard problems can be solved theoretically, however, the solution of Reynolds equation for the analysis of general lubrication problems requires the use of numerical procedures such as the finite difference method and the finite element method. Furthermore, in practice, most applications encounter irregular configurations in geometry, arbitrary boundary conditions and varying film properties. Most of these difficulties can be overcome by the use of the finite element technique.
The work presented in this thesis is concerned with the application of the finite element technique to lubrication problems including friction discs. Incompressible isothermal condition has been considered as a first step towards the lubricant film investigation. The starting point is the solution of generalized Reynolds equation as stated by Huebner (11) which includes various effects such as shear force, body force, squeeze action and diffusion effect. The inertia effect and detailed consideration of the diffusion effect have been additionally included in order that rotating disc problems can be investigated.

Fluid finite elements of both triangular and rectangular planform have been developed and have been incorporated in a computer program especially formulated for the presented analysis. Although the elements are primarily two dimensional in the film region analysis, thickness of the oil film can be varied within an element. Other parameters affecting the lubricant such as shear, body and inertia forces, squeeze and diffusion velocities can also be varied at each node.

These elements have been extensively tested by their application to classical lubrication problems such as squeezing square pad, slider and step bearings and the results derived have been compared with theoretical solutions. The computed results are shown to give good agreement with the theoretical solutions in all the cases that have been studied.

The analysis has been extended to the study of the behaviour of the oil film between rotating circular discs with flat surfaces as such results will be applicable to oil immersed brakes and clutches.
The effect of inertia forces has also been considered in this analysis.

Another aspect of wet type clutches which is of interest is the effect of oil grooves on the disc surface on the behaviour of the clutches. Experimental investigations indicate that grooves greatly affect the dynamic coefficient of friction. However, only a few analytical studies have been carried out in this field so far. In the present work radial and spiral groove arrangements have been investigated to determine the pressure distributions of wet clutches.
2. Literature Survey

2.1 General Lubrication Analysis Using Finite Element Technique

Owing to the development of computer technology, numerical analyses of fluid film lubrication problems have rapidly progressed. Recently the finite element technique has been successfully applied for the solution of Reynolds equation based on classical variational principles. This has come about partly by virtue of ease of the application to cater for arbitrary boundary conditions and partly, by the ease of handling complex geometric configurations. Variations in field properties such as changes of film thickness that occur in bearing pockets and oil grooves, and also the effects of external pressures can all be investigated by this type of the analysis.

Approximate solutions of incompressible isothermal lubrication problems using the variational principle was obtained by Hays(1). His analysis is based on the following assumptions:

1. Film thickness is small compared to other system dimensions.
2. Viscosity is assumed to be constant.
3. Reynolds number is assumed to be small consequently the flow is assumed to be laminar.

Based on these assumptions, the following theorem was derived

"Of all the possible fluid motions within a region which are compatible with the equation of continuity and the prescribed boundary conditions, that motion which minimized the excess of the energy dissipation over twice the rate at which the work is being done by the specified surface tractions on the boundary, will be the true steady-state motion". Using this theorem, Hays presented the following function to be minimized:
\[ I = \int_{A} \left( \frac{h^3}{12\mu} \nabla^2 p \right) dA + 2 \int_{S} \rho \delta n \, ds \]

Subject to the following boundary conditions:

1. A specified constant pressure along the boundary or
2. If the pressure is not constant then the volume flow normal to the boundary must be zero and the normal pressure gradient across the boundary must satisfy the equation,

\[ \nabla p = \frac{6\mu}{h^2} \frac{\partial U}{\partial x} \text{ on } S \]

Subsequently the pressure distribution was assumed to be expressed in the form of an infinite series given by the equation

\[ p = p_0 + \frac{P_1 - P_0}{\pi} y + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (\sin(ny))(a_{nm} \sin(mx) + b_{nm} \cos(nx)) \]

By determining the coefficients \( a_{nm}, b_{nm} \) which minimize the function \( I \), Reynolds equation was solved. This approach was used to analyze the hydrodynamic behaviour of the finite width journal bearing and many valuable results were presented to determine such quantities as load capacity, moment, coefficient of friction, side flow, minimum film height and power loss. Hays further studied the characteristics of a finite width journal bearing under a cyclic sinusoidal load \( (2) \). Rectangular pad problems with flat and curved surfaces were also investigated by the same author \( (3) \), and results showed that the effects of curvature only become important as the film thickness decreases, and this may reduce the squeeze film capacity of the plate by several orders of magnitude.
Moore (4) experimentally verified Hays theoretical results for a flat rectangular pad and has developed an approximate analysis of the pressure distribution on a pad bearing consisting of two inclined plates. However, both studies assume that the boundary flow is zero.

An approximate hydrodynamic analysis based on variational principles has been presented by Tipei (5), though, again the limitation of the flow boundary condition is also evident in this work.

This restriction on boundary conditions has since been overcome by the application of finite element techniques. Reddi (6) applied the finite element technique to solve incompressible lubrication problems, and has investigated the effects of squeeze and shear action forces within the fluid. He stated the variational principle for an incompressible fluid to include the non-zero flow boundary condition. Advantage was then taken of the important assumption made in the application of the finite element technique, that the state of the field variable within an element may be described by values of the unknown variable at a finite number of nodes located on the boundary of an element. This implies that in lubrication problems, the pressure distribution in an element may be expressed in terms of unknown pressures at the nodes in the element. Reddi has used the triangular element for this analysis and assumed a pressure distribution of the form given below

\[ p(x,y) = \langle f_i(x,y) \rangle a_j \]

where \( f_i(x,y) \) are interpolation functions and \( a_j \) are related to the unknown pressures at the element nodes. The interpolation function
was chosen such that \( p(x,y) \) satisfies the following conditions:

1. \( f_i \) is continuous within an element;
2. pressure along any inter-element boundary should be specified completely by nodal pressures on that boundary;
3. constant pressure state is included;
4. uniform pressure gradient is included;
5. a linear transformation of the coordinate system must not change the pressure representation within the element.

This method was used to investigate the squeeze pad, slider bearing and step bearing problems and the results were compared with other theoretical solutions. Simple triangular elements and composite quadrilateral elements were used, and no special treatment was required to account for the sudden change in film thickness. These predicted results were found to be in good agreement with those obtained from classical analyses. Reddi then extended the finite element technique to analyze the compressible fluid lubrication problems using quadrilateral elements (7). Calculations of the fluid matrices were achieved by means of Gaussian integration.

Wada, Hayashi and Migita (8), (9), have derived the solutions of infinite and finite width bearing problems using an assumed pressure distribution which is expressed by a high order algebraic equation. They have examined the effects of taking a number of coefficients in the pressure function for rotating journal bearing problems and have concluded that accurate results can be obtained by using only the first few elements of the high order algebraic equation.

Allan (10) has examined the characteristics of journal bearing with externally pressured oil pockets, using an iterative method to
to determine the oil pressures and the flows through oil pockets.

In this work it is shown that the finite element technique is a powerful and flexible method capable of handling any bearing surface and that the use of simple triangular elements is adequate to illustrate the approach and provide useful results. Allan has also presented full details of the computer program required to solve journal bearing problems.

Recent works have been concentrated mainly on the generalization of the finite element technique for the analysis of lubrication problems. A detailed explanation of solution procedure of a triangular squeezing pad problem has been developed by Booker and Huebner (11), to handle effects such as shear stress, squeeze action, body force, lubricant expansion due to heating and diffusion through the pad. The functional which must be minimized for the incompressible isothermal lubrication problems is given by the equation

$$I = \int_A \left[ \left( \frac{\phi h^3}{24 \mu} V_P - \rho h U - \frac{\rho h^2}{12 \mu} \right) V_P + \left( \frac{\partial h}{\partial t} + h \frac{\partial P}{\partial t} + \rho v_3 \right) \right] dA$$

$$+ \int_{S_q} (\phi U n) p ds$$

Despite various flow action effects being taken into account in this analysis the consideration of the diffusion effect is incomplete and also the inertia forces are assumed to be negligible when compared with shear and pressure forces.

Huebner (12) has further extended this method to analyze thermo-hydrodynamic lubrication problems. The weighted residuals
associated with Galerkin's criterion is used to solve the thermal energy equation which describes the temperature distribution in the lubricant film. An iterative procedure is applied to obtain self-consistent pressure and temperature distribution results. The importance of including thermal effects in the hydrodynamic analysis has been discussed by Huebner who has concluded that the use of an isothermal analysis may lead to overestimates of calculated values of the bearing load capacity and coefficient of friction.

Stafford (13) has carried out a modification of the method and has contributed to the existing suit of finite element subroutines known as PAFEC (14). Isoparametric elements are used and the integrations of the system matrices are achieved by the Gaussian method. The effects of body force, inertia and diffusion are neglected in his analysis which was also extended to include the effect of structural distortion on the film by using an iterative approach.

The effect of pad deformation on bearing performance has been studied by Jain, Sinhasan and Singh (15) using a three dimensional finite element technique. The pressure field in the fluid film region is determined by the simultaneous solution of Reynolds equation and the relevant elasticity equations using an iterative approach.

Allaire, Nicholas and Gunter (16) have developed a systematic matrix approach using finite elements to minimize the bandwidth of the resulting algebraic equations. Also they have established an optimum method for dividing the bearing area into elements. Their error analysis indicates that the division of the bearing surface
into elements is of great importance and that the alignment of the diagonal sides of triangular elements to the expected direction of the pressure gradient provides more accurate results for the same number of elements used. The significance of variable grid spacing was also pointed out by the authors in order to reduce errors in the results.

Das and Dancer (17) have presented an analysis of the oil flow and its frictional behaviour in diesel engine bearings. Various factors influencing the bearing performance have been investigated by the finite element method based on steady state lubrication theory. It is shown that the coefficient of friction and oil flow are dependent upon the basic geometric proportions such as the length, diameter, clearance etc. of the bearing. Results also indicate that engine load and manufacturing variations such as taper and misalignment have little influence on bearing performance.

An analysis of the non-Newtonian fluid effects in a finite width journal bearing using the finite element technique has been developed by Tayal, Shinhasan and Singh (18). The non-linear behaviour of the fluid was investigated by modifying the viscosity term at each stage of an iteration process.

2.2 Porous Region Analysis

Porous materials are widely used as bearing surfaces and clutch disc facings, and many analyses have been made to predict the characteristics of the porous region. It is assumed that the oil film region satisfies the Reynolds equation and the flow in the porous region satisfies the Laplace or Poisson equation in which
are substituted Darcy's law of porous media flow. The problem is solved by coupling these equations with the associated boundary conditions. Darcy's law gives the following expressions for the flow in the porous region

$$u = - \phi \frac{\partial p}{\mu \partial x}$$

$$v = - \phi \frac{\partial p}{\mu \partial y}$$

$$w = - \phi \frac{\partial p}{\mu \partial z}$$

and if these equations are substituted in the continuity equation in the porous region, the governing Laplace or Poisson equation is obtained. Various assumptions and simplifications have been made by many investigators to solve these equations.

Wu (19) has investigated analytically the squeeze film behaviour of a porous annular disc approaching a plain disc of the same dimensions by the use of Fourier-Bessel expansions. The assumptions made in the analysis were as follows:

1. The porous facing has uniform thickness and permeability.
2. The fluid is incompressible and has constant properties.
3. The no-slip condition is applicable to all liquid-solid interfaces. Results were presented giving the pressure distribution, load-carrying capacity and film thickness as the plates come into contact.

In this problem only part of the fluid will be squeezed out and the remaining part will flow out through the porous medium.
The combined effect of these two actions will reduce the pressure in the fluid film compared with that reached in a non-porous medium. Wu also concludes that porous effects are influenced by not only the permeability of material but also by the film thickness. Later Wu also studied the effect of including rotational inertia in the analysis (20). The effects of rotational inertia are shown to further reduce the film pressure and load carrying capacity and also to shorten the time required for reducing the film thickness. Wu has further applied the same approach as used for discs to study rectangular squeeze pad problems (21).

Disc problems have also been analyzed by Ting (22) who used an expression for the average pressure through the thickness of porous region and assumed that the mean pressures in the film and porous regions are equal at any radius for small values of thickness. Good agreement between the results evaluated by the simplified method and the Fourier-Bessel solutions are reported.

Another simplification applied to the integration of Laplace's equation has been made by Prakash and Vij (23) to examine squeeze film effects in circular, annular, elliptic and rectangular porous plates.

The squeeze film behaviour in an inclined porous slider bearing has been investigated by Bhat and Patel (24) and the cause of a porous composite slider bearing by Puri and Patel (25). The analysis adopted by Prakash et al. has been extended to these studies. Results show that the response time for a composite slider bearing is greater than that for an inclined slider bearing.

In most disc problems the surfaces have been assumed to be flat,
but in practice, owing to elastic, thermal and uneven wear effects, modifications to the analysis are required to allow for plate distortion. Vora and Bhat (26), Gupta and Vora (27) have considered the effects of curvature of surfaces in their analysis of a squeeze film action between porous rotating circular plates. Expressions for the pressure and load carrying capacity of the disc are given in the form of exponential series. Again results obtained by the authors show that the effect of rotating fluid inertia is to reduce the load capacity.

Prakash and Tiwari (28) have analyzed the effects of surface roughness on the squeeze film action between rotating porous discs. They assume that the film thickness can be expressed by a combination of nominal film thickness and deviation of height from the nominal level. Their results indicate that the circumferential roughness increases while the radial roughness decreases the load carrying capacity at constant roughness values.

Application of the finite element technique to the analysis of porous regions in a variety of lubrication problems has been demonstrated by several workers. Rohde and Oh (29) have applied the technique to journal bearing problems with a compressible lubricant. They assumed the flow in the porous region only to be across its thickness, an assumption which is valid for a very thin porous region case. Eidelberg and Booker (30) have presented the technique for the analysis of squeeze films to take into account three dimensional flow in the film and porous regions. Their analysis is based on the following coupling conditions and boundary conditions:
1. The diffusion velocity in the film region interface is equal to the velocity normal to the surface in porous region, thus the model flow at the interface is zero.

2. Pressure in porous region is the same as the fluid film pressure at the interface.

3. The surrounding pressure is zero.

4. The flow at all the internal nodes is zero.

The simplest elements such as triangular elements and tetrahedra elements are used in the idealization of the film region and the porous region respectively. Applications have been made to solve problems involving irregular geometrical configurations and different material properties.

Malik, Sinhasan and Chandra (31) have recently reported the analysis of porous step bearings using rectangular and hexahedral quadratic elements. The effects of tangential velocity slip ignored previously (29), (30) have been taken into account in their results predicting load carrying capacity and coefficient of friction.

2.3 Disc Problem Analysis

The behaviour of the fluid film between annular discs has been examined both theoretically and experimentally by many investigators.

Archibald (32) has analyzed squeezing flow problems such as spherical bearings and circular plates. Jackson (33) has used an iterative procedure to solve the continuity and radial momentum equations to provide a better approximation of inertia effects that occur in the fluid film. Rotating conditions were also considered by Allen and McKillop (34) in their analysis of the problem.
normal approach of two annular surfaces, one of which is rotating with respect to the other. Consideration of centrifugal forces acting on the fluid was based upon the assumption of Couette flow in the tangential direction.

As a conclusion they have stated that the theoretical results showed that the only effect of rotation on an ideal squeeze film between parallel surfaces, is due to the centrifugal forces, which tends to increase the rate of approach of the two surfaces. The authors have also presented some empirical results using various kinds of fluid and good agreements with theoretical results have been obtained in some cases.

Ludwig (35) has analyzed the engagement characteristics of wet type clutches mathematically and experimentally. He found that the grooving pattern on the plates had a pronounced effect on the dynamic coefficient of friction and that the spiral grooves produced a higher friction than radial grooves. Wu (36) has developed a model to simulate the engagement characteristics of a single pair of wet type clutch plates used in automatic transmissions. The equations are based on those derived from his previous studies (19), (20). Expressions that enable calculation of such quantities as film thickness, transmitted torque, interface temperature, heat generation rate and engine speed have been presented. Utilizing such calculation Wu has shown that viscous shear forces can produce significant amount of the clutch torque transmitted during the squeezing motion and that most of the energy is dissipated during the squeeze film region. Results also indicate that the permeability parameter and porous facing thickness ratio are of great importance.
in engagement. The effect of surface irregularities and grooving effects were neglected in Wu's analysis. El-Serbiny and Newcomb (37) have described a general model to simulate the engagement characteristics of a wet type friction clutch using the finite element technique. Oil groove effects and the heat generated by viscous shear are taken into account in their analysis of single and repeated engagements. The predicted effects of frictional behaviour agreed with trends observed from practical investigation. Porosity effects in the clutch materials were ignored in this analysis.

In the present work inertia effects have been included in a finite element analysis of the rotating disc problems when radial and spiral grooves are incorporated in one disc surface.

Before consideration of this and other problems a theoretical development of the generalized Reynolds equation is presented in Chapter 3. A solution to this equation using a finite element analysis is then incorporated into a computer program as outlined in Chapter 4.
3. Theoretical Development

3.1. Reynolds Equation

The derivation of the generalized Reynolds equation for incompressible isothermal steady state lubrication problems is based on the following assumptions:

1. The pressure throughout the film thickness remains constant.
2. The curvature of the bearing surface is large compared to the oil film thickness.
3. No slip between the bearing surface and adjacent layers of fluid film.
4. The lubricant is considered to be a Newtonian fluid.
5. The fluid flow is laminar.
6. The viscosity is temperature dependent.

The geometry and coordinate system for a fluid film and corresponding surfaces is shown in FIG.1.

Reynolds equation is derived from the consideration of the fluid continuity of flow and the equilibrium of a fluid element.

An infinitely small element of fluid of sides $dx$, $dy$ and $dz$ is shown in FIG.2. The fluid velocities on all the faces of the element and in the orthogonal direction $x$, $y$ and $z$ are assumed to be constant.

The incoming volume flow rate is given by the equation

$$u \ dy \ dz + v \ dx \ dz + w \ dx \ dy \quad (3.1)$$

and the out flow rate by

$$\left( u + \frac{\partial u}{\partial x} \ dx \right) dy dz + \left( v + \frac{\partial v}{\partial y} \ dy \right) dx dz + \left( w + \frac{\partial w}{\partial z} \ dz \right) dx dy \quad (3.2)$$
From the continuity of flow, the net flow rate must be zero, which leads to the following relationship,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3.3)$$

Next consider a column of fluid of sides $dx$, $dy$ and height $h$ as shown in FIG.3. The fluid influx and efflux rates per unit width are shown in the figure.

**Volume flow in $x$ direction**: Influx $q_x dy$

Efflux $(q_x + \frac{\partial q_x}{\partial x} dx) dy$

**Volume flow in $y$ direction**: Influx $q_y dx$

Efflux $(q_y + \frac{\partial q_y}{\partial y} dy) dx$

In the $z$ direction, if the velocities of the lower and upper surfaces of the column are $w_0$ and $w_1$ respectively, the increase in volume can be expressed as

$$(w_0 - w_1) \, dx \, dy$$

Using the condition of continuity of flow, the following relationship is obtained,

$$(q_x dy + q_y dx) + (w_0 - w_1) \, dx dy = (q_x + \frac{\partial q_x}{\partial x} dx) dy + (q_y + \frac{\partial q_y}{\partial y} dy) dx$$

Cancelling $(dx dy)$ which is arbitrary and non-zero gives

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + (w_1 - w_0) = 0 \quad (3.4)$$

If the upper surface is permeable, the last term of eqn. (3.4) $(w_1 - w_0)$ can be explained by the squeeze and diffusion actions on the fluid then
where

\begin{align*}
\frac{\partial h}{\partial t} &: \text{the rate of change of height of the column, namely squeeze velocity} \\
v_d &: \text{diffusion velocity}
\end{align*}

Substituting eqn. (3.5) in eqn. (3.4), the following expression is obtained

\begin{equation}
\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial h}{\partial t} + v_d = 0
\end{equation}

Finally consider the equilibrium of a fluid element as shown in FIG. 4. In this case, the forces consist of viscous shear stresses, body forces and fluid pressure, and resolving in the direction of the x axis gives the equation

\begin{equation}
p dy dz + \left( \tau_{xz} + \frac{\partial \tau_{xz}}{\partial z} dz \right) dx dy + B_x dx dy dz = (p + \frac{\partial p}{\partial x} dx) dy dz + \tau_{xz} dx dy
\end{equation}

which reduces to

\begin{equation}
\frac{\partial \tau_{xz}}{\partial z} + B_x = \frac{\partial p}{\partial x}
\end{equation}

Similarly in the direction of the y axis

\begin{equation}
\frac{\partial \tau_{yz}}{\partial z} + B_y = \frac{\partial p}{\partial y}
\end{equation}

In the z direction the pressure gradient is assumed to be zero.

\begin{equation}
\frac{\partial p}{\partial z} = 0
\end{equation}

According to the Newton's law of viscosity
\( \tau_{xy} = \mu \frac{\partial u}{\partial z} \)  \hspace{1cm} (3.10)

\( \tau_{yz} = \mu \frac{\partial v}{\partial z} \)  \hspace{1cm} (3.11)

Substituting eqns (3.10) and (3.11) into (3.7) and (3.8) the following relationships are given

\[ \frac{\partial p}{\partial x} = \frac{\partial}{\partial z}(\mu \frac{\partial u}{\partial z}) + B_x \]  \hspace{1cm} (3.12)

\[ \frac{\partial p}{\partial y} = \frac{\partial}{\partial z}(\mu \frac{\partial v}{\partial z}) + B_y \]  \hspace{1cm} (3.13)

The velocity gradient is obtained by integrating eqn.(3.12) with respect to \( z \).

\[ \frac{\partial u}{\partial z} = \frac{1}{\mu} \left( \frac{\partial p}{\partial x} \right) z - \int_0^z B_x dz + \frac{C_1}{\mu} \]  \hspace{1cm} (3.14)

Integrating again gives

\[ u = \frac{\partial p}{\partial x} \int_0^z \frac{z}{\mu} dz - \int_0^z \frac{1}{\mu} \int_0^z B_x dz dz + \int_0^z \frac{C_1}{\mu} dz + C_2 \]  \hspace{1cm} (3.15)

The constants of integration \( C_1 \) and \( C_2 \) are determined by the application of the following boundary conditions

\[ u = U_1, \ v = V_1, \ w = W_1 \]  at \( z = 0 \)  \hspace{1cm} (3.16)

\[ u = U_2, \ v = V_2, \ w = W_2 \]  at \( z = h \)  \hspace{1cm} (3.17)

\[ p = P(x,y) \]  \hspace{1cm} (3.18)

where \( P(x,y) \) is a specified function on a non-vanishing segment of the boundary.

Application of the boundary condition (3.16) to eqn.(3.15) yields

\[ C_2 = U_1 \]  \hspace{1cm} (3.19)
Using boundary condition (3.17) and the value of $C_2$

$$C_1 = \frac{1}{\lambda_0} \left( U_2 - U_1 \right) \frac{\partial P}{\partial x} A_1 + \int_0^h \int_0^Z \int_0^Z B_x dz dz (3.20)$$

where

$$\lambda_0 = \int_0^h \frac{1}{\mu} dz, \quad \lambda_1 = \int_0^h \frac{z}{\mu} dz$$

Substituting values of $C_1$ and $C_2$ into eqn. (3.15), the velocity component is obtained as follows

$$u = \frac{\partial P}{\partial x} \left( \int_0^Z \frac{z}{\mu} dz - \frac{\lambda_1}{\lambda_0} \int_0^Z \frac{1}{\mu} dz \right) + U_1 + \frac{U_2 - U_1}{\lambda_0} \int_0^Z \frac{1}{\mu} dz + \bar{B}_x (3.21)$$

where

$$\bar{B}_x = \frac{1}{\lambda_0} \int_0^Z \frac{1}{\mu} dz \left( \int_0^h \int_0^Z B_x dz dz \right) - \int_0^Z \int_0^Z B_x dz dz$$

Similarly in the y direction

$$v = \frac{\partial P}{\partial y} \int_0^Z \frac{z}{\mu} dz - \frac{\lambda_1}{\lambda_0} \int_0^Z \frac{1}{\mu} dz + V_1 + \frac{V_2 - V_1}{\lambda_0} \int_0^Z \frac{1}{\mu} dz + \bar{B}_y (3.22)$$

where

$$\bar{B}_y = \frac{1}{\lambda_0} \int_0^Z \frac{1}{\mu} dz \left( \int_0^h \int_0^Z B_y dz dz \right) - \int_0^Z \int_0^Z B_y dz dz$$

The velocity in the z direction, $w$ can be obtained by substituting eqns. (3.21) and (3.22) into the continuity eqn. (3.3) and is as follows

$$w = -\int_0^Z \frac{\partial u}{\partial x} dz - \int_0^Z \frac{\partial v}{\partial y} dz + W_1 (3.23)$$

where $W_1$ is the magnitude of the velocity $w$ at $z = 0$.

The average velocities in the $x$ and $y$ directions can be expressed as
\[ \bar{u} = \frac{1}{h} \int_0^h u \, dz \quad (3.24) \]

\[ \bar{v} = \frac{1}{h} \int_0^h v \, dz \quad (3.25) \]

and the volume flows as

\[ q_x = h \cdot \bar{u} = \int_0^h u \, dz \quad (3.26) \]

\[ q_y = h \cdot \bar{v} = \int_0^h v \, dz \quad (3.27) \]

Substituting these values in the continuity eqn. (3.6), the following expression is obtained

\[ \frac{\partial}{\partial x} \left( \int_0^h u \, dz \right) + \frac{\partial}{\partial y} \left( \int_0^h v \, dz \right) + \frac{\partial h}{\partial t} + v_d = 0 \quad (3.28) \]

Substituting the values of velocities from eqn. (3.21) and (3.22) in the above expression, the generalized Reynolds equation in vector form is obtained

\[ - \nabla G \cdot \bar{V} = \nabla(h \, \bar{U}) + \nabla(\Delta U A_2) + \nabla B + \frac{\partial h}{\partial t} + v_d \quad (3.29) \]

where

\[ \bar{V} = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \]

\[ G = \int_0^h \int_0^z \frac{z}{u} \, dz \, dz - \frac{A_1}{A_0} \int_0^h \int_0^z \frac{1}{u} \, dz \, dz \]

\[ U = u_1 \hat{i} + v_1 \hat{j} \]

\[ \Delta U = (u_2 - u_1) \hat{i} + (v_2 - v_1) \hat{j} \]

\[ A_2 = \frac{1}{A_0} \int_0^h \frac{A_1}{A_0} \, dz \, dz \]
3.2 Incompressible Isothermal Lubrication

In this section development of the incompressible form of the Reynolds equation is presented. The viscosity of the lubricant is assumed to be constant. Then eqn.(3.15) can be expressed as

\[ u = \frac{1}{\mu} \int_{0}^{h} B_x dz + \frac{1}{\mu} \int_{0}^{h} B_y dz + C_1 dz + C_2 \]  

(3.30)

The body forces are expressed by averaged values in z direction to simplify the analysis.

\[ B_{mx}(x,y) = \frac{1}{h} \int_{0}^{h} B_x dz \]

\[ B_{my}(x,y) = \frac{1}{h} \int_{0}^{h} B_y dz \]

(3.31)

Using these averaged values for the body forces in eqn.(3.30), the following form can be obtained

\[ u = \frac{z^2}{2\mu} \frac{\partial p}{\partial x} - \frac{z^2}{2\mu} B_{mx} + \frac{C_1}{\mu} z + C_2 \]  

(3.32)

Applying the boundary conditions (3.16) and (3.17), the velocity component of the fluid in incompressible condition is given in the equation

\[ u = \frac{z(z-h)}{2\mu} \left( \frac{\partial p}{\partial x} - B_{mx} \right) + \frac{z}{h} (U_2 - U_1) + U_1 \]  

(3.33)

Similarly

\[ v = \frac{z(z-h)}{2\mu} \left( \frac{\partial p}{\partial y} - B_{my} \right) + \frac{z}{h} (V_2 - V_1) + V_1 \]  

(3.34)
The corresponding volume flows are

\[ q_x = \int_0^h u \, dz = - \frac{h^3}{12 \mu} \left( \frac{\partial p}{\partial x} - B_{mx} \right) + \frac{h}{2} \left( U_1 + U_2 \right) \quad (3.35) \]

\[ q_y = \int_0^h v \, dz = - \frac{h^3}{12 \mu} \left( \frac{\partial p}{\partial y} - B_{my} \right) + \frac{h}{2} \left( V_1 + V_2 \right) \quad (3.36) \]

The volume flow gradients are obtained by differentiating eqns. (3.35) and (3.36) with regard to \(x\) and \(y\)

\[ \frac{\partial q_x}{\partial x} = - \frac{1}{12 \mu} \frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} - h \frac{\partial}{\partial x} \left( \frac{h(U_1 + U_2)}{2} \right) \right) \quad (3.37) \]

\[ \frac{\partial q_y}{\partial y} = - \frac{1}{12 \mu} \frac{\partial}{\partial y} \left( h^3 \frac{\partial p}{\partial y} - h \frac{\partial}{\partial y} \left( \frac{h(V_1 + V_2)}{2} \right) \right) \quad (3.38) \]

Finally substituting these gradients in eqn. (3.28), the generalized Reynolds equation for incompressible isothermal steady state is obtained in the form

\[ \frac{1}{12 \mu} \left[ \frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( h^3 \frac{\partial p}{\partial y} \right) \right] = \frac{1}{\partial x} \left( h \bar{u} \right) + \frac{1}{\partial y} \left( h \bar{v} \right) \]

\[ + \frac{1}{12 \mu} \left[ \frac{\partial}{\partial x} \left( h B_{mx} \right) + \frac{\partial}{\partial y} \left( h B_{my} \right) \right] + \frac{\partial h}{\partial \tau} + \nu_d \quad (3.39) \]

where

\[ \bar{u} = \frac{U_1 + U_2}{2} \quad , \quad \bar{v} = \frac{V_1 + V_2}{2} \]

This Reynolds equation is solved with boundary conditions such that along part of the boundary \(S_p\) shown in FIG. 8., the pressure is specified by

\[ p = P(x,y) \quad \text{on} \quad S_p \quad (3.40) \]
and along the remainder of the boundary \( S_q \), the volume flow per unit boundary length is specified by:

\[
q = q_x \hat{i} + q_y \hat{j} \quad \text{on} \quad S_q
\]  

(3.41)

### 3.3 Inclusion of the Centrifugal Force in the Generalized Reynolds Equation

Normally inertia effects in fluids are of little significance and as such are neglected in Reynolds equation. Since in this project, discs are to be analyzed and Reynolds' equation is applied to disc problems, the inertia effects are of interest and the theory has been extended to include these.

A typical arrangement is shown in FIG. 5, where the lower surface has an angular velocity of \( \omega_1 \) and the upper surface has an angular velocity of \( \omega_2 \). The centrifugal force per unit volume is

\[
C_f = \rho r \omega^2
\]  

(3.42)

Assuming that the angular velocity varies linearly with height

\[
\omega = \omega_0 z + \omega_1
\]  

(3.43)

where

\[
\omega_0 = \frac{\omega_2 - \omega_1}{h}
\]

The centrifugal force can now be expressed as

\[
C_f = \rho r (\omega_0 z + \omega_1)^2
\]  

(3.44)

The average centrifugal forces are given by the equations
And the average centrifugal forces in x and y direction are as follows

\[ C_{fmx} = \frac{\partial r \cos \theta}{3h\omega_o} \left[ \left( \omega_0 h + \omega_1 \right)^3 - \omega_1^3 \right] \]  (3.46)

\[ C_{fmy} = \frac{\partial r \sin \theta}{3h\omega_o} \left[ \left( \omega_0 h + \omega_1 \right)^3 - \omega_1^3 \right] \]  (3.47)

Considering the equilibrium of a fluid element

\[ \frac{\partial p}{\partial x} = \frac{\partial \tau_{xz}}{\partial x} + p_{mx} + C_{fmx} \]  (3.48)

\[ \frac{\partial p}{\partial y} = \frac{\partial \tau_{yz}}{\partial x} + p_{my} + C_{fmy} \]  (3.49)

Proceeding as before and using continuity equation, Reynolds equation including the inertia terms is obtained as follows

\[ \frac{1}{12\mu} \left[ \frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} + h^3 \frac{\partial p}{\partial y} \right) \right] = \frac{\partial}{\partial x} \left( h\bar{u} \right) + \frac{\partial}{\partial y} \left( h\bar{v} \right) + \frac{1}{12\mu} \left[ \frac{\partial}{\partial x} \left( h^3 p_{mx} \right) + \frac{\partial}{\partial y} \left( h^3 p_{my} \right) \right] \]

\[ + \frac{1}{12\mu} \left[ \frac{\partial}{\partial x} \left( h^3 C_{fmx} \right) + \frac{\partial}{\partial y} \left( h^3 C_{fmy} \right) \right] + \frac{\partial h}{\partial t} + \nu_d \]  (3.50)
3.4 Application to Porous Annular Discs

The flow field in the porous region of a disc is governed by the Laplace equation which is coupled to the Reynolds equation governing the film region. In the film region, the pressure distribution can be expressed in a two dimensional form as before. However, in the porous region of the disc, the pressure changes across the thickness and accordingly a three dimensional approach has to be made. A three dimensional finite element technique would be suitable for the solution of the three dimensional Laplace equation and would overcome the complicated surface configuration.

As a primary analysis, a simplified investigation of the porous region is presented here. The investigation is confined to squeezing pads, one of which has a porous facing. This model has been adopted, because it is nearest to automotive applications such as clutch plates and oil immersed disc brakes. The model under consideration is shown in FIG.6.

The analysis is based on the following assumptions:

1. The porous facing has constant permeability.
2. The pressure in surrounding field is zero.
3. Squeezing action on the fluid is the most dominant effect and other effects are neglected.

These assumptions are in addition to those applicable to the film region.

The fluid velocities for a porous region can be derived from Darcy's law (38).
\[ \dot{u} = -\frac{\rho}{\mu} \frac{\partial p}{\partial x} \]
\[ \dot{v} = -\frac{\rho}{\mu} \frac{\partial p}{\partial y} \]  
(3.51)
\[ \dot{w} = -\frac{\rho}{\mu} \frac{\partial p}{\partial z} \]

The three dimensional continuity equation can be expressed as follows (39)

\[ \nabla (\rho \dot{u}) + \dot{A} \frac{\partial p}{\partial t} = 0 \]  
(3.52)

where

\[ \dot{U} = \dot{u} \hat{i} + \dot{v} \hat{j} + \dot{w} \hat{k} \]

\[ \dot{A} = \text{porosity} \]

Using Darcy's equation (3.51), the continuity equation becomes

\[ \nabla \left( \frac{\rho \dot{u}}{\mu} \right) \nabla p = \dot{A} \frac{\partial p}{\partial t} \]  
(3.53)

The associated boundary conditions between the film region and porous region are

\[ v_d(x,y) = \dot{u}(x,y,h) \hat{n} \]  
(3.54)
\[ p(x,y) = \dot{p}(x,y,h) \]  
(3.55)

where

\[ \hat{n} = \text{unit normal vector to a boundary surface} \]
Also the boundary conditions between the porous region and the surrounding field are given by

\[
\begin{align*}
  p &= 0 & \text{on boundary } S \\
  \dot{p} &= 0 & \text{on boundary } S \\
  \dot{U} &= (x,y,h) \hat{n} = 0 & \text{on } h = H
\end{align*}
\]  

(3.56)\hspace{1cm} (3.57)

3.5 Friction Force, Friction Torque and Load Carrying Capacity of Bearings and Discs.

The theory developed so far can be extended to assess the performance of bearings and discs in so far as their load and torque carrying capacity is concerned. Friction forces and torques indicate the power loss for bearings. However for discs these values are more significant because they govern their engagement capacity.

The load carrying capacity \( L \) can be expressed as

\[
L = \int_{A} p(x,y) \, dA
\]  

(3.58)

and the friction forces by

\[
\begin{align*}
  F_x &= \int_{A} \tau_x \big|_{z=0,h} \, dA \\
  F_y &= \int_{A} \tau_y \big|_{z=0,h} \, dA
\end{align*}
\]  

(3.59)\hspace{1cm} (3.60)

where \( A \) is the area concerned.
From eqns. (3.10) and (3.11) the shear stresses are
\[ \tau_x = \mu \frac{\partial u}{\partial z} \]
\[ \tau_y = \mu \frac{\partial v}{\partial z} \]
and the velocity gradients are derived from eqns. (3.33) and (3.34)
\[ \frac{\partial u}{\partial z} = \frac{(2z-h)}{2\mu} \left( \frac{\partial p}{\partial x} - B_{mx} - C_{fmx} \right) + \frac{(U_2 - U_1)}{h} \] (3.61)
\[ \frac{\partial v}{\partial z} = \frac{(2z-h)}{2\mu} \left( \frac{\partial p}{\partial y} - B_{my} - C_{fmy} \right) + \frac{(V_2 - V_1)}{h} \] (3.62)
Substituting these values into eqns. (3.10) and (3.11), the components shear stresses are now expressed as
\[ \tau_x = \frac{(2z-h)}{2} \left( \frac{\partial p}{\partial x} - B_{mx} - C_{fmx} \right) + \frac{\mu}{h} U \] (3.63)
\[ \tau_y = \frac{(2z-h)}{2} \left( \frac{\partial p}{\partial y} - B_{my} - C_{fmy} \right) + \frac{\mu}{h} V \] (3.64)
where
\[ U = U_2 - U_1 \quad , \quad V = V_2 - V_1 \]
The shear stresses for the lower surface, \( z = 0 \) are given by the equations
\[ -\tau_{x1} = -\frac{h}{2} \left( \frac{\partial p}{\partial x} - B_{mx} - C_{fmx} \right) + \frac{\mu}{h} U \] (3.65)
\[ -\tau_{y1} = -\frac{h}{2} \left( \frac{\partial p}{\partial y} - B_{my} - C_{fmy} \right) + \frac{\mu}{h} V \] (3.66)
and the shear stresses for the upper surface, \( z = h \) by

\[
\tau_{x2} = \frac{h}{2} \left( \frac{\partial P}{\partial x} - B_{mx} - C_{fmx} \right) + \frac{U}{h} u
\]  
(3.67)

\[
\tau_{y2} = \frac{h}{2} \left( \frac{\partial P}{\partial y} - B_{my} - C_{fmy} \right) + \frac{U}{h} v
\]  
(3.68)

The friction forces are obtained by substituting eqns. (3.67) and (3.68) into eqns. (3.59) and (3.60)

\[
F_{x1} = \int_A \frac{h}{2} \left( \frac{\partial P}{\partial x} - B_{mx} - C_{fmx} \right) - \frac{U}{h} u \, dA
\]  
(3.69)

\[
F_{x2} = \int_A \frac{h}{2} \left( \frac{\partial P}{\partial x} - B_{mx} - C_{fmx} \right) + \frac{U}{h} u \, dA
\]  
(3.70)

\[
F_{y1} = \int_A \frac{h}{2} \left( \frac{\partial P}{\partial y} - B_{my} - C_{fmy} \right) - \frac{U}{h} v \, dA
\]  
(3.71)

\[
F_{y2} = \int_A \frac{h}{2} \left( \frac{\partial P}{\partial y} - B_{my} - C_{fmy} \right) + \frac{U}{h} v \, dA
\]  
(3.72)

For disc problems, determination of friction torque is required as a measure of their performance. The torque field for a disc is shown in FIG. 7. The friction torque of the area \( \Delta A \) is expressed as

\[
\Delta T_e = r \left( \tau_x \sin \theta + \tau_y \cos \theta \right) \Delta A
\]  
(3.74)

The total friction torques of the lower and upper surfaces can be obtained by integrating eqn. (3.73) over the appropriate area and take the form

\[
T_{e1} = \int_A \left( r \left( \tau_{x1} \sin \theta + \tau_{y1} \cos \theta \right) \right) \, dA
\]  
(3.74)

\[
T_{e2} = \int_A \left( r \left( \tau_{x2} \sin \theta + \tau_{y2} \cos \theta \right) \right) \, dA
\]  
(3.75)
4. Application of The Finite Element Technique

4.1 Variational Principles

The generalized Reynolds equation (3.50) describes the behaviour of film lubrication when the film thickness and other variables such as body forces, surface velocities, centrifugal forces, squeezing velocities and the diffusion velocities together with appropriate boundary pressure and flow conditions (3.40), (3.41) are known. Variational principles can be applied for the solution of this equation.

The integral $I$ is a functional which has independent variables $x, y$ and unknown function $p(x, y)$.

$$I(p) = \iint_{A} F(x, y, p, \frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial^2 p}{\partial x^2}, \frac{\partial^2 p}{\partial y^2}, \frac{\partial^2 p}{\partial x \partial y}) \, dx \, dy = \text{constant} \quad (4.1)$$

Variational calculus is used for the determination of function $p$ which minimizes $I(p)$.

Let the function

$$p = p^* + \varepsilon \eta(x)$$

where $\varepsilon$ is an arbitrary parameter and $\eta(x)$ is a continuous function having zero values on the boundaries. In order that function $I(p)$ be a minimum at $p = p^*$, the following conditions must be satisfied

$$\left. \frac{dI}{d\varepsilon} \right|_{\varepsilon=0} = 0$$

$$\left. \frac{d^2 I}{d\varepsilon^2} \right|_{\varepsilon=0} > 0 \quad (4.2)$$
Substituting eqn. (4.2) into (4.1) a Euler-Lagrange form of equation is obtained,

\[
\frac{\partial^2}{\partial x^2} \left( \frac{\partial F}{\partial p_{xx}} \right) + \frac{\partial^2}{\partial x \partial y} \left( \frac{\partial F}{\partial p_{xy}} \right) + \frac{\partial^2}{\partial y^2} \left( \frac{\partial F}{\partial p_{yy}} \right)
- \frac{\partial}{\partial x} \left( \frac{\partial F}{\partial p_x} \right) - \frac{\partial}{\partial y} \left( \frac{\partial F}{\partial p_y} \right) + \frac{\partial F}{\partial p} = 0
\]  

(4.3)

The function \( p \) that minimizes this functional satisfies the Reynolds equation (3.50) and the boundary conditions (3.40) and (3.41) given by the equation

\[
I(p) = \int_A \left\{ \frac{h^2}{24\mu} \nabla p - hU \frac{h^2}{12\mu} (B_m + C_{mn}) \nabla p + \left( \frac{\partial h}{\partial t} + v_d \right) p \right\} \partial A + \int_{S_d} Q p \, ds \quad (4.4)
\]

4.2 Development of Fluidity Matrices

The finite element technique has been applied for the determination of an approximate pressure distribution in a two dimensional field. Initially the field under consideration is subdivided into smaller elements having a finite number of nodes. Approximate interpolation functions are chosen to express pressure and other variable variations within these elements. These functions satisfy the boundary continuity criteria.

The approximate variation of various variables can be expressed as

\[
p = N \{ p \} = \sum_{i=1}^{n} N_i(x, y) p_i
\]  

(4.5)
\[
\begin{align*}
U_x &= N \{ U_x \} = \sum_{i=1}^{r} N_i(x,y) U_{x_i} \\
U_y &= N \{ U_y \} = \sum_{i=1}^{r} N_i(x,y) U_{y_i} \\
B_{mx} &= N \{ B_{mx} \} = \sum_{i=1}^{r} N_i(x,y) B_{mx_i} \\
B_{my} &= N \{ B_{my} \} = \sum_{i=1}^{r} N_i(x,y) B_{my_i} \\
C_{fmx} &= N \{ C_{fmx} \} = \sum_{i=1}^{r} N_i(x,y) C_{fmx_i} \\
C_{fmy} &= N \{ C_{fmy} \} = \sum_{i=1}^{r} N_i(x,y) C_{fmy_i} \\
\frac{\partial h}{\partial t} &= N \{ \frac{\partial h}{\partial t} \} = \sum_{i=1}^{r} N_i(x,y) \frac{\partial h}{\partial t_i} \\
v_d &= N \{ v_d \} = \sum_{i=1}^{r} N_i(x,y) v_{di}
\end{align*}
\]

The field values (4.5) and (4.6) are substituted into the functional of eqn. (4.4) which is then minimized with respect to the nodal pressure \( p_i \) of the element

\[
\frac{\partial I}{\partial p_i} = 0 \quad i = 1, 2, \ldots, r \quad (4.7)
\]
and considering the whole domain

\[ \sum_{i=1}^{N} \frac{\partial I_i}{\partial p_i} = 0 \]  
(4.8)

Substituting (4.5) and (4.6) into (4.4) each derivatives of \( p \) becomes

\[ v_p = \frac{\partial p}{\partial x} + \frac{\partial p}{\partial y} \]

\[ = \left[ \sum_{i=1}^{\xi} \frac{\partial N_i}{\partial x} \right] p_j + \left[ \sum_{i=1}^{\xi} \frac{\partial N_i}{\partial y} \right] p_j \]

\[ \nabla v_p = \left( \frac{\partial p}{\partial x} \right)^2 + \left( \frac{\partial p}{\partial y} \right)^2 \]

\[ = \left[ \sum_{i=1}^{\xi} \frac{\partial N_i}{\partial x} \right] \left[ \sum_{j=1}^{\xi} \frac{\partial N_j}{\partial x} p_j \right] + \left[ \sum_{i=1}^{\xi} \frac{\partial N_i}{\partial y} \right] \left[ \sum_{j=1}^{\xi} \frac{\partial N_j}{\partial y} p_j \right] \]

and on differentiation with respect to \( p_i \),

\[ \frac{\partial}{\partial p_i} (v_p) = \frac{\partial N_i}{\partial x} + \frac{\partial N_i}{\partial y} \]

\[ \frac{\partial}{\partial p_i} \nabla v_p = \left( \frac{\partial N_i}{\partial x} \right) \left[ \sum_{j=1}^{\xi} \frac{\partial N_j}{\partial x} p_j \right] + \left( \frac{\partial N_i}{\partial y} \right) \left[ \sum_{j=1}^{\xi} \frac{\partial N_j}{\partial y} p_j \right] \]

\[ \frac{\partial}{\partial p_i} (p) = N_i \]

Thus equation (4.8) can be expressed, using the fluidity matrices and nodal values as follows

\[
\begin{align*}
\mathbf{K}_p \{ p \} & = -\mathbf{K}_{u_x} \{ u_x \} - \mathbf{K}_{u_y} \{ u_y \} - \mathbf{K}_{B_{mx}} \{ B_{mx} \} - \mathbf{K}_{B_{my}} \{ B_{my} \} \\
& \quad - \mathbf{K}_{C_{fmx}} \{ C_{fmx} \} - \mathbf{K}_{C_{fmy}} \{ C_{fmy} \} - \mathbf{K}_{n} \{ \frac{\partial h}{\partial t} \} - \mathbf{K}_{v_d} \{ v_d \} + \{ q \} \quad (4.9)
\end{align*}
\]
where matrices $[K_p], [K_{Ux}], [K_{Uy}], \ldots$ are of size $r \times r$ and matrices $\{p\}, \{U_x\}, \{U_y\}, \ldots$ are of size $r \times 1$

Pressure: \[ K_{Pij} = \mathbf{F} \left( \frac{h^3}{12\mu} \left( \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) \right) dA \] (4.10)

Shear: \[ K_{Uxij} = \int_A h \left( \frac{\partial N_i}{\partial x} N_j \right) dA \] \[ K_{Uyij} = \int_A h \left( \frac{\partial N_i}{\partial y} N_j \right) dA \]

Body force: \[ K_{Bmxij} = \int_A \left( \frac{h^3}{12\mu} \frac{\partial N_i}{\partial x} N_j \right) dA \] \[ K_{Bmyij} = \int_A \left( \frac{h^3}{12\mu} \frac{\partial N_i}{\partial y} N_j \right) dA \] (4.11)

Centrifugal force: \[ K_{Cmxij} = K_{Bmxij} \] \[ K_{Cmyij} = K_{Bmyij} \]

Squeeze: \[ K_{Ni} = \int_A N_i N_j dA \]

Diffusion: \[ K_{dij} = K_{Ni} \]

Flow: \[ q_i = \int_{S_q} Q N_i dA \]

where $q_i$ is the outward flow across the boundary $S_i$ associated with the node $i$. The half-boundary $S_i$ is either side of the node as shown in FIG.8.
Equation (4.9) can be solved provided \( n_1 \) nodal pressures and flows of the rest of the nodes \( (N - n_1) \) are both known. All other nodal forcing values such as body forces, must also be known in order to solve this equation.

Equation (4.9) can be expressed in matrix form as

\[
\begin{bmatrix} \mathbf{K}_p \end{bmatrix} \{p\} = \{q\} - \begin{bmatrix} \mathbf{K}_a \end{bmatrix} \{a\} \tag{4.12}
\]

These matrices can be partitioned and rearranged as follows into known and unknown value matrices

\[
\begin{bmatrix} \mathbf{K}_{p11} & \mathbf{K}_{p12} \\ \mathbf{K}_{p21} & \mathbf{K}_{p22} \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} - \begin{bmatrix} \mathbf{K}_{a11} & \mathbf{K}_{a12} \\ \mathbf{K}_{a21} & \mathbf{K}_{a22} \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} \tag{4.13}
\]

Equation (4.13) can then be subdivided into two separate matrix equations

\[
\begin{align*}
\{q_1\} &= \begin{bmatrix} \mathbf{K}_{p11} \end{bmatrix} \{p_1\} + \begin{bmatrix} \mathbf{K}_{p12} \end{bmatrix} \{p_2\} - \begin{bmatrix} \mathbf{K}_{a11} \end{bmatrix} \{a_1\} - \begin{bmatrix} \mathbf{K}_{a12} \end{bmatrix} \{a_2\} \tag{4.14} \\
\{q_2\} &= \begin{bmatrix} \mathbf{K}_{p21} \end{bmatrix} \{p_1\} + \begin{bmatrix} \mathbf{K}_{p22} \end{bmatrix} \{p_2\} - \begin{bmatrix} \mathbf{K}_{a21} \end{bmatrix} \{a_1\} - \begin{bmatrix} \mathbf{K}_{a22} \end{bmatrix} \{a_2\} \tag{4.15}
\end{align*}
\]

Rearranging eqn. (4.14) gives the following equations

\[
\begin{align*}
\{q_1\} &= \begin{bmatrix} \mathbf{K}_{p11} \end{bmatrix} \{p_1\} \\
\{p_1\} &= \begin{bmatrix} \mathbf{K}_{p11} \end{bmatrix}^{-1} \{q_1\} \tag{4.16}
\end{align*}
\]

where

\[
\{q_1\} = \{q_1\} - \mathbf{K}_{p12} \{p_2\} + \mathbf{K}_{a11} \{a_1\} + \mathbf{K}_{a12} \{a_2\}
\]
Since all nodal pressures become known from eqn. (4.16), these can be substituted into eqn. (4.15) to obtain the corresponding flows.

Once the nodal pressure at each node has been determined, the component flows (flows in an element) can be obtained by applying the pressures to the original equation (4.7).

4.2.1 Development of Fluidity Matrices for Triangular Elements

In this project the triangular element system is presented as a primary development of F.E. Technique for lubrication problems.

The elements are connected at the nodes which are located on the corners of triangles as shown in FIG. 8, and are assumed to have linear variation of states. This variation is represented by a linear interpolation polynomial of the form

\[ N_i(x, y) = a_i + b_i x + c_i y \]  \hspace{1cm} (4.17)

The constants \( a_i, b_i \) and \( c_i \) are chosen so that \( N_i = 1 \) at node \( i \) and \( N_i = 0 \) at the other two nodes; that is

\[ a_i = \frac{(x_i y_k - x_k y_i)}{2A} \]

\[ b_i = \frac{(y_i - y_k)}{2A} \]  \hspace{1cm} (4.18)

\[ c_i = \frac{(x_k - x_j)}{2A} \]

where

\[ A = \frac{1}{2} \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = \text{(area of a triangle)} \]
The pressure distribution $p$ can be expressed as

$$p = \sum_{i=1}^{3} N_i(x,y) p_i$$

$$= \sum_{i=1}^{3} (a_i + b_i x + c_i y) p_i \quad (4.19)$$

The other field values are defined in a similar manner. The film thickness variation can also be expressed by using an interpolation function such as

$$h = N\{h\} = \sum_{i=1}^{3} N_i(x,y) h_i \quad (4.20)$$

The fluidity matrices are then described as follows. Since the derivations of $N$ in eqn.(4.10) can be expressed as

$$\frac{\partial N_i}{\partial x} \frac{\partial N_i}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_i}{\partial y} = (b_i b_j + c_i c_j) = \text{const}$$

and

$$h^3 = \left(\sum_{i=1}^{3} N_i h_i\right)^3 = (N_1 h_1 + N_2 h_2 + N_3 h_3)^3$$

Hence the pressure matrix assumes the following form

$$K_{pi,j} = -\frac{\rho}{12\mu} (b_i b_j + c_i c_j) \int_A (N_1 h_1 + N_2 h_2 + N_3 h_3)^3 dA$$

The integrated result of interpolation functions over the area of a triangular element is presented by Zienkiewicz (4) and is as follows.
\[ \int_{A} N_{1}^{\alpha} N_{2}^{\beta} N_{3}^{\gamma} \, dA = \frac{\alpha! \beta! \gamma!}{(\alpha + \beta + \gamma + 2)!} \frac{1}{2A} \] (4.21)

Accordingly the fluidity matrix is given by the following expression

\[ K_{pij} = -\frac{1}{480\mu} \left( b_{i} b_{j} + c_{i} c_{j} \right) B \] (4.22)

where \( B = \left[ \frac{3}{2} \sum_{k=1}^{3} h_{k} \right] \left[ \frac{3}{2} \sum_{k=1}^{3} h_{k} \right] + h_{1} h_{2} h_{3} \)

Other fluidity matrices can be expressed in a similar manner by the equations

\[ K_{Ux_{ij}} = \frac{b_{i}}{24} \sum_{k=1}^{3} h_{k} (1 + \delta_{kj}) \] (4.23)

\[ K_{Uy_{ij}} = \frac{c_{i}}{24} \sum_{k=1}^{3} h_{k} (1 + \delta_{kj}) \] (4.24)

\[ K_{Bmx_{ij}} = \frac{b_{i}}{1440 \mu} G \] (4.25)

\[ K_{Bmy_{ij}} = \frac{c_{i}}{1440 \mu} G \] (4.26)

\[ K_{Cfmx_{ij}} = K_{Bmx_{ij}} \] (4.27)

\[ K_{Cfmy_{ij}} = K_{Bmy_{ij}} \] (4.28)

\[ K_{n_{ij}} = -\frac{1}{12} A (1 + \delta_{ij}) \] (4.29)

\[ K_{vd_{ij}} = K_{n_{ij}} \] (4.30)
where

\[ \delta_{ij} = 1 \quad \text{when} \quad i = j \]
\[ \delta_{ij} = 0 \quad \text{when} \quad i \neq j \]

\[
G = \left[ \sum_{k=1}^{3} h_k \left( \sum_{k=1}^{3} h_k \right) + 2h_j \left( \sum_{k=1}^{3} h_k \right) + h_j \left( \sum_{k=1}^{3} h_k^2 \right) + 2h_1h_2h_3 \right]
\]

4.2.2 Development of Fluidity Matrices of Rectangular Elements

In this derivation of fluidity matrix a different type of natural coordinate system has been used. FIG.9 shows the two coordinate systems. The Cartesian coordinates are expressed in terms of the natural coordinate as follows

\[
x = \frac{1}{4} \left[ (1-s)(1-t)x_1 + (1+s)(1-t)x_2 \right. \\
+ (1+s)(1+t)x_3 + (1-s)(1+t)x_4 \left. \right]
\]

\[
y = \frac{1}{4} \left[ (1-s)(1-t)y_1 + (1+s)(1-t)y_2 \right. \\
+ (1+s)(1+t)y_3 + (1-s)(1+t)y_4 \left. \right]
\]

and the interpolation function for a linear rectangular element is

\[
N_i(x,y) = \frac{1}{4} (1+ss_i)(1+tt_i) \]

Hence the pressure distribution can be described as

\[
p = \sum_{i=1}^{4} \frac{1}{4} (1+ss_i)(1+tt_i)p_i
\]
Transformation of the coordinates from Cartesian to natural is carried out as follows

\[ \frac{\partial N}{\partial x} = \frac{\partial N}{\partial s} \frac{\partial s}{\partial x} + \frac{\partial N}{\partial t} \frac{\partial t}{\partial x} \]  
\[ (4.34) \]

\[ \frac{\partial N}{\partial y} = \frac{\partial N}{\partial s} \frac{\partial s}{\partial y} + \frac{\partial N}{\partial t} \frac{\partial t}{\partial y} \]  
\[ (4.35) \]

\[ \frac{\partial s}{\partial x} = \frac{\partial y}{\partial t} \frac{1}{|J|} \]

\[ \frac{\partial s}{\partial y} = -\frac{\partial x}{\partial t} \frac{1}{|J|} \]

\[ \frac{\partial t}{\partial x} = \frac{\partial y}{\partial s} \frac{1}{|J|} \]

\[ \frac{\partial t}{\partial y} = -\frac{\partial x}{\partial s} \frac{1}{|J|} \]

Also

\[ dx\,dy = |J| \, ds\,dt \]

where \( J \) is the determinant of the Jacobian matrix \([J]\) given by

\[ [J] = \begin{bmatrix} \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} \end{bmatrix} \]

Eqns. (4.34) and (4.35) can be expressed as

\[ \frac{\partial N}{\partial x} = \frac{1}{|J|} \left( \frac{\partial N}{\partial s} \frac{\partial y}{\partial t} - \frac{\partial N}{\partial t} \frac{\partial y}{\partial s} \right) \]  
\[ (4.36) \]

\[ \frac{\partial N}{\partial y} = -\frac{1}{|J|} \left( \frac{\partial N}{\partial s} \frac{\partial x}{\partial t} - \frac{\partial N}{\partial t} \frac{\partial x}{\partial s} \right) \]  
\[ (4.37) \]
The derivatives of $N$ in eqn.(4.10) can now be expressed as

\[
\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} = \frac{1}{|J|} \left( \frac{\partial N_i}{\partial s} \frac{\partial N_j}{\partial t} - \frac{\partial N_i}{\partial s} \frac{\partial N_j}{\partial t} \right) \frac{\partial N_j}{\partial s} \frac{\partial N_j}{\partial s} \frac{\partial y}{\partial s} \frac{\partial y}{\partial s} (4.38)
\]

\[
\frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} = \frac{1}{|J|} \left( \frac{\partial N_i}{\partial s} \frac{\partial N_j}{\partial t} - \frac{\partial N_i}{\partial s} \frac{\partial N_j}{\partial t} \right) \frac{\partial N_j}{\partial s} \frac{\partial N_j}{\partial s} \frac{\partial y}{\partial s} \frac{\partial y}{\partial s} (4.39)
\]

From eqn.(4.31) and eqn.(4.32)

\[
\frac{\partial N_i}{\partial s} = \frac{s_i}{4} (1+t_{i1})
\]

\[
\frac{\partial N_i}{\partial t} = \frac{t_i}{4} (1+s_{ii})
\]

also

\[
\frac{\partial x}{\partial s} = \frac{1}{4} \{(x_2-x_1) + (x_3-x_4)\} = a_1
\]

\[
\frac{\partial x}{\partial t} = \frac{1}{4} \{(x_4-x_1) + (x_3-x_2)\} = a_2
\]

\[
\frac{\partial y}{\partial s} = \frac{1}{4} \{(y_2-y_1) + (y_3-y_4)\} = a_3
\]

\[
\frac{\partial y}{\partial t} = \frac{1}{4} \{(y_4-y_1) + (y_3-y_2)\} = a_4
\]

Hence the determinant $|J|$ becomes

\[
|J| = \frac{\partial x}{\partial s} \frac{\partial y}{\partial t} - \frac{\partial x}{\partial t} \frac{\partial y}{\partial s} = a_1a_4 - a_2a_4 (4.40)
\]

Rearranging eqns.(4.38) and (4.39)
If the film thickness and viscosity are constant, the integration of eqn. (4.10) takes the following form after substituting eqns. (4.40) and (4.41),

\[
K_{p_{ij}} = - \int_{A} \frac{h^3}{12 \mu} \left( \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) dA
\]

\[
= \frac{-h^3}{192 \mu |J||J'|} \int_{-1}^{1} \int_{-1}^{1} \left[ a_4 s_i (1+tt_i) - a_3 t_i (1+ss_i) \right] \\
+ \left[ a_4 s_j (1+tt_j) - a_3 t_j (1+ss_j) \right] + \left[ a_2 s_i (1+tt_i) - a_1 t_i (1+ss_i) \right] \\
+ \left[ a_2 s_j (1+tt_j) - a_1 t_j (1+ss_j) \right] |J| d\sigma dt
\]

This expression can be simplified by expressing it as a sum of the following terms

\[
A = s_i s_j (a_2^2 + a_4^2) \int_{-1}^{1} \int_{-1}^{1} (1+tt_i) (1+tt_j) d\sigma dt
\]

\[
B = t_i t_j (a_1^2 + a_3^2) \int_{-1}^{1} \int_{-1}^{1} (1+ss_i) (1+ss_j) d\sigma dt
\]

\[
C = s_i t_j (a_1 a_2 + a_3 a_4) \int_{-1}^{1} \int_{-1}^{1} (1+ss_j) (1+tt_i) d\sigma dt
\]

\[
D = -s_j t_i (a_1 a_2 + a_3 a_4) \int_{-1}^{1} \int_{-1}^{1} (1+ss_i) (1+tt_j) d\sigma dt
\]
Integrating these terms, the following results are obtained

\[ A = 4s_i s_j (a_1^2 + a_4^2) \left( \frac{1}{3} t_i t_j + 1 \right) \]

\[ B = 4t_i t_j (a_1^2 + a_3^2) \left( -\frac{1}{3} s_i s_j + 1 \right) \]  
\[ C = -4s_i t_j (a_1 a_2 + a_3 a_4) \]

\[ D = -4s_j t_i (a_1 a_2 + a_3 a_4) \]  

The fluidity matrix \( K_{ pij } \) therefore becomes

\[ K_{ pij } = -\frac{h^3}{192\mu |J|} (A + B + C + D) \]  
\[ (4.43) \]

Similarly, other fluidity matrices for the rectangular element can be expressed as

\[ K_{ U_{ X_{ ij} } } = \frac{h}{4} \left\{ s_i a_4 \left( \frac{1}{3} t_i t_j + 1 \right) - t_i a_3 \left( \frac{1}{3} s_i s_j + 1 \right) \right\} \]

\[ K_{ U_{ Y_{ ij} } } = \frac{h}{4} \left\{ s_i a_2 \left( \frac{1}{3} t_i t_j + 1 \right) - t_i a_1 \left( \frac{1}{3} s_i s_j + 1 \right) \right\} \]

\[ K_{ B_{ mX_{ ij} } } = \frac{h^3}{48\mu} \left\{ s_i a_4 \left( \frac{1}{3} t_i t_j + 1 \right) - t_i a_3 \left( \frac{1}{3} s_i s_j + 1 \right) \right\} \]

\[ K_{ B_{ mY_{ ij} } } = \frac{h^3}{48\mu} \left\{ s_i a_2 \left( \frac{1}{3} t_i t_j + 1 \right) + t_i a_1 \left( \frac{1}{3} s_i s_j + 1 \right) \right\} \]  
\[ (4.44) \]

\[ K_{ C_{ fX_{ ij} } } = K_{ B_{ mX_{ ij} } } \]

\[ K_{ C_{ fY_{ ij} } } = K_{ B_{ mY_{ ij} } } \]
If the film thickness is variable in an element, it is assumed that the thickness can also be expressed using the interpolation function $N_i$, as follows

$$h = \sum_{i=1}^{4} N_i(x,y) h_i = \frac{1}{4} \sum_{i=1}^{4} (1+s_{i1})(1+s_{i2}) \, h_i$$  \hspace{1cm} (4.45)

The matrix $K_{Pij}$ in this case is given by the equation

$$K_{Pij} = - \frac{1}{192u J1} (A'+B'+C'+D')$$  \hspace{1cm} (4.46)

where

$$A' = s_i s_j (a_2^2 + a_4^2) \int_{-1}^{1} \int_{-1}^{1} h^3 (1+tt_i)(1+tt_j) \, ds dt$$

$$B' = t_i t_j (a_1^2 + a_3^2) \int_{-1}^{1} \int_{-1}^{1} h^3 (1+s_{i1})(1+s_{j1}) \, ds dt$$

$$C' = -s_i t_j (a_1 a_2 - a_3 a_4) \int_{-1}^{1} \int_{-1}^{1} h^3 (1+s_{j1})(1+tt_j) \, ds dt$$

$$D' = -s_j t_i (a_1 a_2 - a_3 a_4) \int_{-1}^{1} \int_{-1}^{1} h^3 (1+s_{i1})(1+tt_i) \, ds dt$$

Substituting eqn.(4.45) into eqn.(4.46) and rearranging terms, the quantities $A'$ to $D'$ can be expressed in the form

$$A' = 4s_i s_j (a_2^2 + a_4^2) \sum_{k=1}^{4} \sum_{m=1}^{4} \sum_{n=1}^{4} \left\{h_k h_m h_n \left(\frac{s_{12}+1}{3} \left(\frac{t_{21}+1}{3} t_{22}+1\right) \right) \right\}$$
\[ B' = 4\sum_{t=1}^{4} t_{ij}(a_{1}^{2} + a_{2}^{2}) \sum_{m=1}^{4} \sum_{n=1}^{4} \left( h_{i} h_{m} h_{n} \left( \frac{1}{3} t_{12} + 1 \right) \left( \frac{1}{2} s_{21} + \frac{1}{3} s_{22} + 1 \right) \right) \]

\[ C' = -4s_{t} t_{ij}(a_{1} a_{2} - a_{3} a_{4}) \sum_{m=1}^{4} \sum_{n=1}^{4} \left[ h_{i} h_{m} h_{n} \left( \frac{1}{3} s_{i} s_{j} + \frac{1}{3} (s_{12} + s_{13} s_{j}) + 1 \right) \right] \]

\[ + \left( \frac{1}{5} t_{1} t_{1} t_{1} + \frac{1}{3} (t_{12} + t_{13} t_{1}) + 1 \right) \]

\[ D' = -4s_{t} t_{ij}(a_{1} a_{2} - a_{3} a_{4}) \sum_{m=1}^{4} \sum_{n=1}^{4} \left[ h_{i} h_{m} h_{n} \left( \frac{1}{3} s_{i} s_{j} + \frac{1}{3} (s_{12} + s_{13} s_{j}) + 1 \right) \right] \]

\[ + \left( \frac{1}{5} t_{1} t_{1} t_{1} + \frac{1}{3} (t_{12} + t_{13} t_{1}) + 1 \right) \] (4.47)

where

\[ t_{11} = t_{m,n} \]

\[ t_{12} = t_{m,n} + t_{m,n} + t_{m,n} \]

\[ t_{13} = t_{m,n} + t_{m,n} \]

\[ s_{11} = s_{m,n} \]

\[ s_{12} = s_{m,n} + s_{m,n} \]

\[ s_{13} = s_{m,n} + s_{m,n} \]

\[ t_{21} = t_{11} t_{i} + (t_{11} + t_{12} t_{i}) t_{i} \]

\[ t_{22} = (t_{12} + t_{13} t_{i}) + (t_{13} + t_{12} t_{i}) t_{i} \]

\[ s_{21} = s_{11} s_{j} + (s_{11} + s_{12} s_{j}) s_{i} \]

\[ s_{22} = (s_{12} + s_{13} s_{j}) + (s_{13} + s_{12} s_{j}) s_{i} \]

Similarly other matrices are derived as

\[ K_{u} x_{1} = \frac{s_{a} t_{i} t_{j}}{16} \sum_{\ell=1}^{4} \left[ (s_{a} + 1) \left( \frac{1}{3} t_{s} t_{t} + \frac{1}{3} t_{t} t_{s} + \frac{1}{3} t_{t} t_{s} + 1 \right) \right] \]

\[ - \frac{t_{a} s_{j}}{16} \sum_{\ell=1}^{4} \left[ (s_{a} t_{i} t_{j} + 1) \left( \frac{1}{3} s_{a} s_{j} + \frac{1}{3} s_{a} s_{j} + \frac{1}{3} s_{a} s_{j} + 1 \right) \right] \] (4.48)
\[ K_{ij} = \frac{s_1 s_2}{16} h \sum_{\ell=1}^{4} \left[ \frac{1}{3} s_\ell \cdot s_j + 1 \right] \left( t_{k} t_{i} + t_{i} t_{j} + t_{j} t_{k} + 1 \right) \]

\[ = -\frac{t_{i} a_{\ell}}{16} h \sum_{\ell=1}^{4} \left( \frac{1}{3} t_{k} \cdot t_{j} + 1 \right) \left( \frac{1}{3} s_\ell \cdot s_i + s_\ell \cdot s_j + s_\ell \cdot s_k + 1 \right) \]  

(4.49)

\[ K_{m,xij} = \frac{1}{768\mu} \sum_{\ell=1}^{4} h \sum_{m=1}^{h} \sum_{n=1}^{h} \left( s_{\ell} a_{s_{23} t_{24}} - t_{i} a_{s_{24} t_{23}} \right) \]  

(4.50)

\[ K_{m,yij} = \frac{1}{768\mu} \sum_{\ell=1}^{4} h \sum_{m=1}^{h} \sum_{n=1}^{h} \left( s_{\ell} a_{s_{24} t_{23}} - t_{i} a_{s_{24} t_{23}} \right) \]  

(4.51)

\[ K_{h,ij} = \frac{-|J|}{h} \left( \frac{1}{3} s_i \cdot s_j + 1 \right) \left( \frac{1}{3} t_i \cdot t_j + 1 \right) \]  

(4.52)

\[ K_{v,ij} = K_{h,ij} \]  

(4.53)

where

\[ s_{23} = \frac{1}{5} s_{\ell} m_{n} s_{j} + \frac{1}{3} \left( s_m s_{m} \cdot s_n s_{n} \cdot s_j s_{j} \cdot s_{j} \right) + 1 \]

\[ s_{24} = \frac{1}{5} s_{25} + \frac{1}{3} s_{26} + 1 \]

\[ s_{25} = s_m s_{m} s_{m} + s_m s_{m} s_{i} + s_m s_{m} s_{j} + s_m s_{n} s_{m} + \ldots \]

\[ s_{26} = s_m s_{m} + s_m s_{i} + s_m s_{j} + s_{n} s_{m} + \ldots \]

\[ t_{23} = \frac{1}{5} t_{\ell} m_{n} t_{j} + \frac{1}{3} \left( t_{\ell} m_{n} t_{n} t_{j} + t_{\ell} t_{i} + t_{\ell} t_{j} + t_{\ell} t_{k} + 1 \right) \]

\[ t_{24} = \frac{1}{5} t_{25} + \frac{1}{3} t_{26} + 1 \]

\[ t_{25} = t_{\ell} m_{n} + t_{\ell} m_{i} + t_{\ell} m_{j} + t_{\ell} m_{k} + t_{\ell} n_{i} + \ldots \]

\[ t_{26} = t_{\ell} m_{n} + t_{\ell} n_{i} + t_{\ell} i_{i} + t_{\ell} j_{j} + t_{\ell} m_{n} + \ldots \]
4.3 Analysis of The Porous Region

The pressure distribution for a surface with a porous region can be found by solving equation (4.9) together with the Laplace equation (3.53) and the associated boundary conditions (3.54)-(3.57). Fig.10 shows the finite element idealization of a porous region.

By expressing the diffusion term \([K_{vd}]\{v_d\}\) as the nodal diffusion flow \(\{q'\}\) in equation (4.9) and by expressing the fluidity matrix terms \([K_a]\\{a\}\), except the pressure term, the rearranged form of the equation is obtained

\[
\{q\} = [K_p]\{p\} + [K_a]\{a\} + \{q'\}
\] (4.54)

The Laplace equation (3.53) is solved in a similar manner to the Reynold's equation using variational principles. The functional to be minimized is given by

\[
\bar{I}(\bar{p}) = \int_\Omega (\frac{\partial}{\partial t} + \nabla \cdot \bar{v}) \bar{p} \, d\Omega + \int_\Gamma \bar{p} \, d\Gamma + \int_s \bar{q} \, dS
\] (4.55)

Hence

\[
\frac{\partial \bar{I}}{\partial \bar{p}_i} = 0 \quad i = 1,2,\ldots,N
\] (4.56)

The pressure distribution as before is assumed to be linear within an element. For the analysis of the three dimensional porous region tetrahedral elements are used. For these elements the pressure distribution is expressed as

\[
\bar{p} = \sum_{i=1}^N M_i(x,y,z) \bar{p}_i
\] (4.57)

\[
M_i = a_i + b_i x + c_i y + d_i z
\] (4.58)
where

\[
\begin{bmatrix}
  a_1 & b_1 & c_1 & d_1 \\
  a_2 & b_2 & c_2 & d_2 \\
  a_3 & b_3 & c_3 & d_3 \\
  a_4 & b_4 & c_4 & d_4
\end{bmatrix}
= \begin{bmatrix}
  1 & 1 & 1 & 1 \\
  x_1 & x_2 & x_3 & x_4 \\
  y_1 & y_2 & y_3 & y_4 \\
  z_1 & z_2 & z_3 & z_4
\end{bmatrix}
\]

\[
6V = \begin{vmatrix}
  1 & x_1 & y_1 & z_1 \\
  1 & x_2 & y_2 & z_2 \\
  1 & x_3 & y_3 & z_3 \\
  1 & x_4 & y_4 & z_4
\end{vmatrix} = 6 \quad \text{(Volume of the tetrahedron defined by nodes 1,2,3,4.)}
\]

Substituting eqn.(4.57) into eqn.(4.56)

\[
\frac{\partial T}{\partial \mathbf{p}_1} = \int_{\Omega} \frac{\partial \mathbf{u}}{\partial x} \left[ \sum_{j=1}^{4} \left( \frac{\partial M_i}{\partial x} (\mathbf{r}) \right) + \sum_{j=1}^{4} \left( \frac{\partial M_i}{\partial y} (\mathbf{r}) \right) + \sum_{j=1}^{4} \left( \frac{\partial M_i}{\partial z} (\mathbf{r}) \right) \right] \, d\Omega \\
+ \int_{\Omega} M_i \sum_{j=1}^{4} (M_j)(\mathbf{r}) \, d\Omega + \int_{S} QN_1 \, dS = 0 \quad (4.59)
\]

Using the interpolation function (4.58) with eqn.(4.59) and the integration formula (40)

\[
\int_{\Omega} M_i M_j M_k \, d\Omega = \frac{\alpha^i \beta^j \gamma^k}{(\alpha+\beta+\gamma+3)!} \quad 6V \quad (4.60)
\]

The flow equation for a porous region is obtained as follows

\[
\{q\} = [\mathbf{K}_p] \{\mathbf{p}\} + \{\mathbf{q}'\} \quad (4.61)
\]
where

\[ X_{p_{ij}} = -\frac{\rho \phi v}{\mu} (b_i b_j + c_i c_j + d_i d_j) \]

\[ \bar{q}_i = \int_S Q N_i \, ds \quad \text{(on surfaces)} \]

\[ q'_1 = \int_\Omega M_1 \sum_{j=1}^4 [M_j] \{\bar{U}\} d\Omega \]

To couple the analysis pertaining to the film and porous region conditions, equations (4.54) and (4.61) are reordered and partitioned into submatrices as follows

\[
\begin{bmatrix}
\bar{q}_1 \\
\bar{q}_2
\end{bmatrix} =
\begin{bmatrix}
\bar{K}_{11} & \bar{K}_{12} \\
\bar{K}_{21} & \bar{K}_{22}
\end{bmatrix}
\begin{bmatrix}
\bar{p}_1 \\
\bar{p}_2
\end{bmatrix} -
\begin{bmatrix}
\bar{q}'_1 \\
0
\end{bmatrix} \quad (4.62)
\]

where \( \bar{q}_1, \bar{p}_1, \bar{q}'_1 \) share nodes with the film region. The associated boundary condition (3.54) gives the following relationship for the flow

\[ \{q'_1\} + \{\bar{q}'_1\} = \{0\} \quad (4.63) \]

The pressures at the common boundary of the film and the porous regions (3.55) is given by

\[ \{p\} = \{\bar{p}_1\} \quad (4.64) \]

The matrix equation (4.62) can be rewritten as

\[ \{\bar{q}_1\} = [\bar{K}_{11}] \{\bar{p}_1\} + [\bar{K}_{12}] \{\bar{p}_2\} - \{\bar{q}'_1\} \quad (4.65) \]

\[ \{\bar{q}_2\} = [\bar{K}_{21}] \{\bar{p}_1\} + [\bar{K}_{22}] \{\bar{p}_2\} \quad (4.66) \]
Equations (4.54), (4.63), (4.64) and (4.65) yield

\[ \{ \overline{q}_1 + q \} = \left[ K_{11} + K_p \right] \{ \overline{p}_1 \} + \left[ K_{12} \right] \{ \overline{p}_2 \} + [K_a] \{ a \} \]  

(4.67)

and combining eqn. (4.67) and eqn. (4.66) the following result is obtained

\[ \begin{bmatrix} \overline{q}_1 + q \\ \overline{q}_2 \end{bmatrix} = \begin{bmatrix} K_{11} + K_p & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \overline{p}_1 \\ \overline{p}_2 \end{bmatrix} + \begin{bmatrix} K_a & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ 0 \end{bmatrix} \]  

(4.68)

Equation (4.68) can be expressed in a simplified form as

\[ \{ \overline{q} \} = [K] \{ \overline{p} \} + [K_a] \{ a \} \]  

(4.69)

This equation can be solved in the same way as that described in Section 4.2.

4.4 Determination of the Load Carrying Capacity of Bearings, Friction Force and Friction Torque

The load carrying capacity of bearings \( L_e \) was written in the form of an integral (3.58) in Section 3.5. The calculation can by finalized by substituting the pressure values (4.5) into equation (3.58).

\[ L_e = \int_A \sum_{i=1}^{r} N_i(x,y) p_i \, dA \]

For the triangular element, the interpolation function is

\[ N_i(x,y) = a_i + b_i x + c_i y \]

and using the integration formulae
\[
\int_A N_i dA = \frac{1}{3} A
\]

the load carrying capacity for the triangular element can be expressed as

\[
L_e = \frac{1}{3} A \sum_{i=1}^{3} P_i
\]

Similarly the load carrying capacity \(L_e\) for a rectangular element is given by

\[
L_e = \frac{1}{4} A \sum_{i=1}^{4} P_i
\]

Total load carrying capacity of the domain is

\[
L = \sum_{e=1}^{E} L_e \quad \text{(E : number of elements)}
\]

The friction forces in an element are given by eqns. (3.69) - (3.72). Substituting the pressure and other field values from eqns. (4.5), (4.6) into eqns. (3.69) - (3.72), the friction forces in the form of an interpolation function and nodal values can be obtained as follows:

\[
F_{x1} = -\frac{1}{2} \int_A \left( \sum_{i=1}^{r} N_i h_i \right) \left( \sum_{j=1}^{r} \frac{\partial N_i}{\partial x} p_j \right) dA - \frac{1}{2} \int_A \left( \sum_{i=1}^{r} N_i h_i \right) \left( \sum_{j=1}^{r} N_j B_{mxj} \right) dA
\]

\[
- \frac{1}{2} \int_A \left( \sum_{i=1}^{r} N_i h_i \right) \left( \sum_{j=1}^{r} N_j C_{fmxj} \right) dA + u \int_A \sum_{i=1}^{r} N_i h_i dA
\]

\[
F_{x2} = \frac{1}{2} \int_A \left( \sum_{i=1}^{r} N_i h_i \right) \left( \sum_{j=1}^{r} \frac{\partial N_i}{\partial x} p_j \right) dA + \frac{1}{2} \int_A \left( \sum_{i=1}^{r} N_i h_i \right) \left( \sum_{j=1}^{r} N_j B_{mxj} \right) dA
\]

\[
+ \frac{1}{2} \int_A \left( \sum_{i=1}^{r} N_i h_i \right) \left( \sum_{j=1}^{r} N_j C_{fmxj} \right) dA + u \int_A \sum_{i=1}^{r} N_i h_i dA
\]
To simplify the integration of the last term an average thickness $\bar{h}$ is used in place of $N_i h_i$.

For the triangular element, substituting the interpolation function (4.17) into eqns. (4.73) - (4.76), the friction forces derived are

$$F_{x_1} = -\frac{1}{12} \sum_{i=1}^{3} b_i p_i - \frac{A}{6} \sum_{i=1}^{3} \sum_{j=1}^{3} (h_i (1+\delta_{ij}) (B_{myj} + C_{fmyj})) + \frac{UA}{3h} \sum_{i=1}^{3} U_i$$

$$F_{x_2} = \frac{1}{12} \sum_{i=1}^{3} b_i p_i + \frac{A}{6} \sum_{i=1}^{3} \sum_{j=1}^{3} (h_i (1+\delta_{ij}) (B_{myj} + C_{fmyj})) + \frac{UA}{3h} \sum_{i=1}^{3} U_i$$

$$F_{y_1} = -\frac{1}{12} \sum_{i=1}^{3} c_i p_i - \frac{A}{6} \sum_{i=1}^{3} \sum_{j=1}^{3} (h_i (1+\delta_{ij}) (B_{myj} + C_{fmyj})) + \frac{UA}{3h} \sum_{i=1}^{3} V_i$$

$$F_{y_2} = \frac{1}{12} \sum_{i=1}^{3} c_i p_i + \frac{A}{6} \sum_{i=1}^{3} \sum_{j=1}^{3} (h_i (1+\delta_{ij}) (B_{myj} + C_{fmyj})) + \frac{UA}{3h} \sum_{i=1}^{3} V_i$$

For the rectangular element, substituting the interpolation function (4.32) into eqns. (4.73) - (4.76), and using transformations (4.31) - (4.37) the friction forces for a rectangular element can be expressed as
\begin{align*}
F_{x1} &= -c_{100} - c_{101} - c_{102} + c_{103} \\
F_{x2} &= c_{100} + c_{101} + c_{102} + c_{103} \\
F_{y1} &= -c_{100} - c_{104} - c_{105} + c_{106} \\
F_{y2} &= c_{100} + c_{104} + c_{105} + c_{106}
\end{align*}

where

\begin{align*}
c_{100} &= 2|J|\left(\frac{1}{3} c_3 c_6 + (c_1 c_5 + \frac{1}{3} c_4 c_7)\right) \\
c_{101} &= 2|J|\left((c_1 c_8 + \frac{1}{3} c_4 c_{11}) + \frac{1}{3} (c_3 c_{10} + \frac{1}{3} c_2 c_9)\right) \\
c_{102} &= 2|J|\left((c_1 c_{12} + \frac{1}{3} c_4 c_{15}) + \frac{1}{3} (c_3 c_{14} + \frac{1}{3} c_2 c_{13})\right) \\
c_{103} &= \frac{\mu|J|}{h} \sum_{i=1}^{4} u_i \\
c_{104} &= 2|J|\left((c_1 B_8 + \frac{1}{3} c_4 B_{11}) + \frac{1}{3} (c_3 B_{10} + \frac{1}{3} c_2 B_9)\right) \\
c_{105} &= 2|J|\left((c_1 B_{12} + \frac{1}{3} c_4 B_{15}) + \frac{1}{3} (c_3 B_{14} + \frac{1}{3} c_2 B_{13})\right) \\
c_{106} &= \frac{\mu|J|}{h} \sum_{i=1}^{4} v_i \\
c_1 &= \frac{1}{4} \sum_{i=1}^{4} h_i \\
c_2 &= \frac{1}{4} \sum_{i=1}^{4} s_i t_i h_i \\
c_3 &= \frac{1}{4} \sum_{i=1}^{4} s_i h_i \\
c_4 &= \frac{1}{4} \sum_{i=1}^{4} t_i h_i \\
c_5 &= \frac{1}{4} \sum_{i=1}^{4} (s_i a_i - t_i a_3) p_i \\
c_6 &= -\frac{1}{4} \sum_{i=1}^{4} a_i s_i t_i p_i \\
c_7 &= \frac{1}{4} \sum_{i=1}^{4} a_i s_i t_i p_i
\end{align*}
This leads to the determination of friction forces in an element.

Furthermore, the total friction forces are obtained by summing these values over the domain area, as follows:

\[
\begin{align*}
F_{T_{x1}} &= \sum_{i=1}^{E} F_{x1i} \\
F_{T_{x2}} &= \sum_{i=1}^{E} F_{x2i} \\
F_{T_{y1}} &= \sum_{i=1}^{E} F_{y1i} \\
F_{T_{y2}} &= \sum_{i=1}^{E} F_{y2i}
\end{align*}
\]

(4.79)

For the rotational disc problems, the evaluation of friction torque becomes necessary, since it is a measure of the disc performance.
The friction torque $T_e$ for an element as shown in FIG.7 is given by

$$T_e = (F_x \sin \theta + F_y \cos \theta) \cdot r \cdot A \quad (4.80)$$

the coordinates of the point $C$ are

$$x_c = \frac{1}{n} \sum_{i=1}^{n} x_i$$
$$y_c = \frac{1}{n} \sum_{i=1}^{n} y_i$$

where $(x_i, y_i)$: the coordinates of each node in the element

$n$ : number of nodes in the element

and the radius $r$ can be expressed as

$$r = \sqrt{(x_c^2 + y_c^2)} \quad (4.81)$$

By substituting eqn.(4.81) into eqn.(4.80) the elemental torque is given by the following expression

$$T_e = (F_x \sin \theta + F_y \cos \theta)(x_c^2 + y_c^2)^{\frac{1}{2}} \cdot A \quad (4.82)$$

The total friction torque $T_T$ is then calculated by summing the elemental torques over the domain area and is found to be

$$T_T = \sum_{e=1}^{E} T_e \quad (4.83)$$

where $E$ : number of elements in the domain area
4.5 Development of the Computer Program

In this section listings of the computer program developed for the lubrication analysis is presented. The program was written in standard FORTRAN for the PRIME 400 Computer of Loughborough University of Technology.

4.5.1 Flow Charts

The computer program developed in this work consists of several subroutines. FIG.11 contains the main flow chart required in the calculations and the subroutine system is listed in FIG.12. The purpose of each subroutine is also presented in TABLE.1, and details of the important subroutines are shown in FIG.13 to 17. The complete program is presented from page 136 onwards.
4.5.2 Data input

As a guide for using the program, details of input data are presented here.

1. RESULT; Are details of calculations needed? YES or NO
2. COOR; Coordinate system Cartesian or polar coordinate? XY or PO
3. UNIT1; Only when polar coordinate is used, input the unit of angle. DEG or RAD
4. UNIT2; Which velocity unit is used, XY or angular vel.? Input 1 for XY, or 2 for angular vel.

5. TEST NAME; Restricted to 80 characters

6. ELEMENT DETAILS; Input NOCE, NODES, NELE, NNEL.

where NOCE: Number of dimension (two-dimension: 2)
NODES: Total number of nodes
NELE: Total number of elements
NNEL: Number of nodes per element

7. DENVIS; Is density and viscosity constant throughout the system? YES or NO

8. INPUT DENSITY AND VISCOsITY OF EACH ELEMENT

<table>
<thead>
<tr>
<th>ELEMENT No.</th>
<th>ELEMENT TYPE</th>
<th>NNEL</th>
<th>DENSITY</th>
<th>VISCOSITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>(example)</td>
<td></td>
<td></td>
<td>1.0E-10</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>1.3E-9</td>
<td>1.7</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>1.1E-10</td>
<td>1.0</td>
</tr>
</tbody>
</table>

(ELEMENT TYPE)

3: triangular
4: rectangular

9. INPUT NODE No. AND THICKNESS OF FILM AT EACH NODE

<table>
<thead>
<tr>
<th>ELEMENT No.</th>
<th>NODE No.</th>
<th>THICKNESS OF FILM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(note) Change in thickness is described in the change of upper surface, therefore lower surface is treated flat.
10. LTYPE; Choose the type of mesh generation. 1 or 2 or 3
where
LTYPE 1 Circular or sector shape system meshed uniformly

LTYPE 2 Parallelogram shape system meshed equally

LTYPE 3 Arbitrary shape system

11. Only when LTYPE = 1

\[
\begin{array}{cccccc}
R_1 & R_2 & \Theta_1 & \Theta_2 & N \\
\text{(inner radius)} & \text{(outer radius)} & \text{(angle of system)} & \text{(angle of division)} & \text{(number of division through radius)}
\end{array}
\]

when circular shape is used, input \((360-\Theta_2)/\Theta_1\) for \(\Theta_1\)

12. Only when LTYPE = 2

\[
\begin{array}{ccccccc}
X_1, Y_1 & X_2, Y_2 & X_3, Y_3 & X_4, Y_4 & N_1 & N_2 \\
\text{(X4, Y4)} & \text{(X3, Y3)} & \text{(X2, Y2)} & \text{(X1, Y1)} & \text{N2=3} & \text{N1=6}
\end{array}
\]

13. Only when LTYPE = 3

\[
\begin{array}{ccc}
\text{ELEMENT No.} & X & Y \\
\text{(ex.) 1 2.0 1.0} & \text{2 1.5 1.0} & \text{3 1.0 1.0} & \ldots \\
\text{(NELE)}
\end{array}
\]

14. BOUNDARY CONDITION NQ and NP

where
NQ : Number of nodes where flow values are known as boundary conditions
NP : Number of nodes where pressure is known as a boundary condition
(note. NQ + NP = NODES)
15. BC ; Are values of boundary conditions same throughout the whole system?  **YES** or **NO**

16. Only when BC = **YES** , input the numbers of node where pressures are known as boundary condition

<table>
<thead>
<tr>
<th>NODE No.</th>
<th>BC TYPE</th>
<th>VALUE OF BC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ex.)</td>
<td>1 2 4 6 7</td>
<td></td>
</tr>
</tbody>
</table>

Then input the values of boundary conditions

<table>
<thead>
<tr>
<th>value of flow</th>
<th>value of pressure</th>
</tr>
</thead>
</table>

17. Only when BC = **NO** , input BC type and value at each node

<table>
<thead>
<tr>
<th>NODE No.</th>
<th>BC TYPE</th>
<th>VALUE OF BC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ex.)</td>
<td>1 2 0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>2 1 0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>3 2 1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

18. BCA ; Are flow action values (shear action, body force etc) same throughout the system?  **YES** or **NO**

19. BCAC ; Input the values of flow actions at each node

<table>
<thead>
<tr>
<th>NODE No.</th>
<th>UX1</th>
<th>UX2</th>
<th>UY1</th>
<th>UX2</th>
<th>EX1</th>
<th>EX2</th>
<th>BY1</th>
<th>BY2</th>
<th>AH</th>
<th>Vd</th>
</tr>
</thead>
<tbody>
<tr>
<td>where</td>
<td>UX1 : velocity of lower surface in X-direction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>UX2 : velocity of upper surface in X-direction</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>UY1 : velocity of lower surface in Y-direction</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>UY2 : velocity of upper surface in Y-direction</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>EX1 : body force of lower surface in X-direction</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \dot{H} ) : squeeze velocity in Z-direction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Vd : diffusion velocity in porous surface</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

when angular velocity is used, input format is as follows

<table>
<thead>
<tr>
<th>NODE No.</th>
<th>( \omega_1 )</th>
<th>( \omega_2 )</th>
<th>0.0</th>
<th>0.0</th>
<th>EX1</th>
<th>EX2</th>
<th>BY1</th>
<th>BY2</th>
<th>( \dot{H} )</th>
<th>Vd</th>
</tr>
</thead>
<tbody>
<tr>
<td>where</td>
<td>( \omega_1 ) : angular velocity of lower surface</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \omega_2 ) : angular velocity of upper surface</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Application to Standard Lubrication Problems

The validation of the finite element analysis outlined in Chapter 4 was established by its application to standard lubrication problems and making a comparison with the results obtained from analytical solutions of Reynolds equation. These finite element idealizations and the associated software were used to solve such problems as the rectangular squeezing pad, slider bearing, step bearing etc. The squeezing rectangular pad problem was first investigated to determine the accuracy of including the squeezing effect in the analysis and afterwards the slider bearing and step bearing problems were considered to deal with lubricated surfaces of both infinite and finite width.

For all the analyses discussed in this chapter, the film viscosity is taken as

\[ \mu = 0.102 \text{ kg·sec} / \text{m}^2 \]

5.1 Rectangular Squeezing Pad

5.1.1 Theoretical Analysis

The geometry of a rectangular pad is shown in FIG.18 and the pressure distribution in such pads has been determined by the solution of Reynolds equation using variational techniques (3). The pressure on the pad surface can be expressed by the equation

\[ P = \frac{uA^2}{H} \frac{\partial h}{\partial t} \sum_{k=1}^{\infty} \sum_{l=1,3,5,...} P_k \sin k\phi \cos \theta \]  

(5.1)
where

\[
\phi = \frac{\pi X}{A} \\
\theta = \frac{\pi Y}{B} \\
\frac{B_k}{k} = \frac{192R^2}{\pi k \phi (R^2k^2 + k^2)} \\
R = \frac{B}{A}
\]

and the corresponding load carrying capacity is obtained by integrating the pressure over the area to give the infinite series

\[
L = \frac{4\mu A^2 B \Delta h}{\pi h^3} \frac{\partial}{\partial t} \sum_{k} \sum_{k} \frac{1}{k_k} B_k
\]

5.1.2 Finite Element Model and Results

A finite element model was developed for the determination of pressure distribution in the pad. Owing to the symmetry of the bearing, only a quarter of the pad was idealized. Triangular fluid finite elements were used for this idealization which are shown as (a) to (e) in FIG.19. The properties of the pad and fluid used in the calculation were as follows

\[
A = 1.0 \text{ m} \\
B = 1.0 \text{ m} \\
R^2 = 1.0 \\
\mu = 0.102 \times 10^{-5} \text{ kg} \cdot \text{sec} / \text{mm}^2 \quad \text{(viscosity)} \\
h = 0.01 \text{ mm} \quad \text{(film thickness)} \\
\frac{\partial h}{\partial t} = 10 \text{ mm} / \text{sec} \quad \text{(squeezing velocity)}
\]
The circumferential pressure was assumed to be zero in this case.

The pressure distribution in the pad at \( y = 0.0 \) along the direction and along the diagonal as predicted by the theoretical approach and the finite element method are shown in FIG's 20, 21 and 22. The effect of the finite element mesh size on the convergence of results is shown in TABLE 2 for the maximum pressure and TABLE 3 for the load carrying capacity of the pad. In spite of the assumption of linear pressure within the elements, very satisfactory agreement between the two approaches was obtained as is obvious from the figures. The maximum error obtained was of the order of 6.20% when using 8 elements. It appears that the maximum pressure at the midpoint is overestimated for the coarse mesh elements. This is to be expected because of the nature of the formulation. However these results can be considerably improved to give less than 1% error by utilizing a finer finite element mesh.

5.2 Infinite Width Slider Bearing

5.2.1 Theoretical Analysis

As one of the various analyses of standard lubrication problems, the case of an infinite width slider bearing shown in FIG.23 is studied in this section. The properties of the bearing and the fluid used in this analysis are as follows:

- \( B = 0.5 \) m
- \( h_0 = 1.0 \times 10^{-5} \) m
- \( U_x = -15.0 \) m/sec (velocity)
\[ \mu = 0.102 \text{ kg sec} / \text{m}^2 \] (viscosity)

The pressure distribution has been given in many references (38), (39), and can be expressed in the form

\[ p = \frac{6}{h_o} \mu U B K_p \] (5.3)

where

\[
K_p = \frac{1}{m} \left\{ \frac{2}{2+2} + \frac{1}{(1+m B)^2} \right\} - \frac{1}{2+m}
\]

\[ m = \frac{h - h_o}{h_o} \]

Again the corresponding load carrying capacity is obtained by integrating the pressure over the plate area of unit width and is given by the equation

\[ L = \frac{6 U W B^2}{h} K_p \] (5.4)

and the friction force at the two bearing surfaces is expressed by the equation

\[
P_h = -\frac{\mu U}{h_o B} \frac{\log_2(1+m)}{m} + \frac{W m h_o}{2 B} \] (at \( h=h \)) (5.5)

\[
P_o = -\frac{\mu U}{h_o B} \frac{\log_2(1+m)}{m} - \frac{W m h_o}{2 B} \] (at \( h=0 \))
5.2.2 Finite Element Model and Results

Finite element layouts of a slider bearing are shown as (a) to (c) in FIG. 24. Number of elements was chosen to be 20, 60 and 240 respectively in order to examine the effects of the mesh size on the convergence of results. The calculation is made by specifying zero oil flow in the y direction as the width in y direction is assumed to be infinite and the circumferential pressure is assumed to be zero. The effect of graded mesh is also studied in this section using 20 triangular elements. The graded mesh patterns are shown in TABLE 8. Five patterns of grading are studied.

The results of pressure distribution are shown in TABLE 4 and these values are also compared with those derived from the theoretical solution and another finite element solution (13). The percentage errors in the pressures at different values of x are also given in TABLE 5 and it can be seen that if the number of elements is around 20 the percentage error is less than 1%.

The magnitude of errors of load carrying values and friction forces are presented in TABLEs 6 and 7 and again accurate results are obtained when the number of elements is approximately 20. The effect of the grading of meshes is also examined and the results are shown in TABLE 8. Five grading patterns were decided as follows:

(a) : regular mesh

(b) : converged mesh around the area where maximum pressure is assumed to be obtained and roughly meshed in other area
(c): converged more intensely around the area of maximum pressure from the result of (b)
(d): converged more intensely from the result of (c)
(e): converged more intensely from the result of (d)

It can be seen from the results in TABLE 8 that the grading of meshes is extremely effective method of getting more accurate results using a certain elements. The percentage error of pressure in grading pattern (c) is less than 3 % while that of regular meshing pattern (a) is 12 %. However, extreme convergence of grading mesh gives poorer accuracy which can be seen in the results of patterns (d) and (e).

5.3 Step Bearing

The significance of the finite element analysis of a step bearing lies in its ability to study the effects of abrupt changes in the film thickness, which leads to the investigation of the effect of oil grooves of discs on the moving surfaces. The configuration of groove surfaces can be assumed to be the combination of two step bearings whose geometry is illustrated in FIG.25.

5.3.1 Infinite Width Step Bearing

The theoretical solutions of an infinite width step bearing (40) to predict the pressure distribution in the two flow regions are given by the equations

\[ p = \frac{P^*}{c_1} x \quad \text{ (region (I): } 0 \leq x < c_1 \text{ )} \]

\[ p = \frac{P^*}{c_2 - c_1} (c_2 - x) \quad \text{ (region (II): } c_1 \leq x \leq c_2 \text{ )} \]

where

\[ P^* = \frac{6 h U}{h_1^2} (h^* - 1) \left/ \left( \frac{h^*^3}{c_2 - c_1} + \frac{1}{c_1} \right) \right. \quad (= \text{maximum pressure}) \]

\[ h^* = \frac{h_2}{h_1} \]
and the corresponding load carrying capacity per unit width can be determined from the relationship

\[
L = \frac{1}{2} P_o c_2
\]  
(5.7)

5.3.2 Finite Width Step Bearing

Theoretical solutions of the pressure distributions are obtained by using a Fourier sine series expansion (40) and is described in series form as

\[
p = \sum_{n=1,3,5,\ldots}^{\infty} P_n \left( \frac{\sin \frac{n\pi z}{b}}{\sinh \frac{n\pi c_1}{b}} + \frac{\sinh \frac{c_2 - x}{b}}{\sinh \frac{n\pi (c_2 - c_1)}{b}} \right)
\]

: region (I) (5.8)

\[
p = \sum_{n=1,3,5,\ldots}^{\infty} P_n \left( \frac{\sin \frac{n\pi z}{b}}{\sinh \frac{n\pi (c_2 - c_1)}{b}} \right)
\]

: region (II) (5.9)

where

\[
P_n = \frac{24 \mu U b (h_2 - h_1)}{n^2 \pi^2 \left[ h_1 \coth \frac{n\pi c_1}{b} + h_2 \coth \frac{n\pi (c_2 - c_1)}{b} \right]} - \frac{n\pi (c_2 - c_1)}{b}
\]

The corresponding load carrying capacity of the bearing is given by the equation

\[
L = \sum_{n=1,3,5,\ldots}^{\infty} 2 b^2 P_n \left( \frac{\cosh \frac{n\pi c_1}{b} - 1}{\sinh \frac{n\pi c_1}{b}} + \frac{\cosh \frac{n\pi (c_2 - c_1)}{b} - 1}{\sinh \frac{n\pi (c_2 - c_1)}{b}} \right)
\]

(5.10)
5.3.3 Finite Element Models and Results

Bearing size and film properties used in this analysis were taken as follows:

\[ c_1 = 0.5 \text{ m} \]
\[ c_2 = 0.6 \text{ m} \]
\[ b = 1.1 \text{ m} \quad \text{(for a finite width bearing)} \]
\[ h_1 = 1.7 \times 10^{-5} \text{ m} \]
\[ h_2 = 1.0 \times 10^{-5} \text{ m} \]
\[ U = 15.0 \text{ m/sec} \]

and the element layouts used in this bearing analysis are shown as (a) to (c) in FIG.26.

Results of the pressure distribution of an infinite width bearing are shown graphically in FIG.27 and since the pressure of an infinite width bearing linearly distributed in the x direction, the computed results show good agreement with those obtained theoretically.

Results of a finite width step bearing are also presented in FIG.28 and these results show that as finite element mesh size approaches that taken in FIG.26 (c) the pressure distribution is within two or three percent of that calculated from the theoretical solution.

In summarizing the above the finite element technique is particularly amenable for the solution of lubrication problems and is a very versatile tool as it is capable of dealing with complex geometrical shapes that cannot be readily solved by conventional analytical methods.
6. Application to Annular Disc Problems

The finite element technique developed in this work was applied to the annular disc problems as a first step to studying the performance of oil-immersed brakes. Most investigations (32), (33), (34), (35), (36) and (37) of the behaviour of an oil film between discs consider rotating and squeezing effects between flat surfaces. However, most brake discs used in practice have grooves cut in on the surfaces in order to supply cooling oil efficiently over the surfaces especially during long brake applications. Various kinds of grooving patterns have been experimented with but most popular are radial and spiral grooves. The effects of the grooves on the hydrodynamic behaviour have been studied by only a few investigators (35), (37), and little details of the pressure distribution on the grooved surfaces have been presented.

Extending our knowledge of the flow behaviour between rotating discs is the purpose of this chapter which is divided into two sections. In the first section characteristics of the film between flat discs have been examined and results have been compared with the theoretical solutions, and in the second section the effects of grooves on the pressure distribution have been studied. Typical radial and spiral groove patterns were chosen for this investigation.

6.1 Behaviour of the Film between Flat Discs

6.1.1 Theoretical Analysis
FIG. 29 shows the geometry of a disc. The pressure distribution of the fluid film between such two flat discs with rotating and squeezing motions has been investigated both theoretically and experimentally (34), and the derivative of the pressure has been expressed in the form

$$\frac{\partial p}{\partial r} = \frac{6\mu \frac{\partial h}{\partial t}}{h^3} \left( r - \frac{r_o^2}{r} \right) + 0.3 \rho \omega^2 r \quad (6.1)$$

with boundary conditions

$$p = 0 \quad \text{at} \quad r = r_1, r_2 \quad (6.2)$$

where

$$r_o$$ denotes a radius of flow separation.

Integrating eqn (6.1) with regard to r enables the pressure distribution to be determined from the equation

$$p = \left( \frac{3\mu}{h} - \frac{0.15 \rho \omega^2}{h^3} \right) r^2 - \frac{6\mu \frac{\partial h}{\partial t}}{h^3} \frac{r_o^2 \log|r|}{r} + c \quad (6.3)$$

The radius of flow separation $r_o$ can be obtained by substituting the boundary conditions (6.2) into eqn (6.3) and

$$r_o^2 = \left( 0.5 + 0.025 \frac{\rho \omega^2 h^3}{\mu} \right) \frac{(r_1^2 - r_2^2)}{\log|r_1| - \log|r_2|} \quad (6.4)$$

Also the integration constant $c$ is obtained by substituting eqns. (6.2) and (6.4) into eqn. (6.3) so that

$$c = \left( \frac{3\mu}{h^3} + 0.15 \rho \omega^2 \right) \left[ (r_1^2 - r_2^2) \frac{\log|r|}{\log|r_1| - \log|r_2|} - r_1 \right] \quad (6.5)$$
A maximum pressure $P_{\text{max}}$ is calculated at $\frac{\partial P}{\partial r} = 0$ and

$$P_{\text{max}} = \left( 3\mu \frac{\partial h}{\partial t} + 0.15\rho \omega^2 \right) r^2 = \frac{6\mu}{h^3} \frac{\partial h}{\partial t} r_0^2 \log |r^2| + c \quad (6.6)$$

where $r^*$ is the radius where $\frac{\partial P}{\partial r} = 0$ and $r^{*2}$ is expressed as

$$r^{*2} = \frac{6\mu}{h^3} \frac{\partial h}{\partial t} r_0^2 \left( \frac{6\mu}{h^3} \frac{\partial h}{\partial t} + 0.3\rho \omega^2 \right) \quad (6.7)$$

Calculations have been made using the above equation where the properties of the discs and the fluid are taken as follows

- $r_1 = 1.0 \text{ m}$
- $r_2 = 2.0 \text{ m}$
- $h = 1.0, 0.5 \text{ mm}$
- $\mu = 1.0 \text{ kg sec / m}^2$
- $\frac{\partial h}{\partial t} = -1.0 \text{ m / sec}$
- $\omega = 0.0, 1.0, 2.0, 3.0, 4.0 \text{ rad / sec}$

### 6.1.2 Finite Element Model and Results

Two types of finite element layout shown in FIG.30 and FIG.31 have been used for the analyses of disc problems.

A 60 degree sector shown in FIG.30 can be applied to the non-rotating disc problems, however, when the discs rotate, neither oil pressure nor oil flow can be determined at the boundaries denoted by the edge nodes (numbered 1, 2, 3, 4, 5, 31, 32, 33, 34, 35). The pressures have to be calculated at those nodes. And at the node 2 in FIG.30 for example, the flow $q_2$ is expressed as follows
\[ q_2 = (q_2 \text{ in the element } E_1) + (q_2 \text{ in the element } E_2) + (q_2 \text{ in the element } E_3) \]

as each flow in each element at node 2 can not be specified, total flow at node 2 \( q_2 \) remains unknown. However, at node 7, the total flow \( q_7 \) can be expressed as follows

\[ q_7 = (q_7 \text{ in the element } E_2) + (q_7 \text{ in } E_3) + (q_7 \text{ in } E_4) + (q_7 \text{ in } E_5) + (q_7 \text{ in } E_6) + (q_7 \text{ in } E_1) \]

\[ = 0 \]

so the boundary condition can be specified as \( q_7 = 0 \). When applying symmetrical abbreviation to the element layout this kind of consideration has to be included.

Results of both non-rotating and rotating discs with flat surfaces are presented in FIG.32.

For the squeeze motion analysis both the theoretical result and that given by the finite element method show a very good agreement at low speeds of rotation, the effect of inertia appears to be rather larger at high rotational speeds.

6.2 Behaviour of the Film between Grooved Discs

Many groove patterns, some of which are presented in FIG.33, have been practically used for brake and clutch discs. Radial, spiral and waffle patterns are largely adopted both for paper-
composited discs and for sintered alloy discs. However, in many cases the depth and size of the groove configurations have been chosen on the basis of experimental investigations.

The finite element technique developed in this work can readily be applied to investigate the pressure distribution of the film between grooved discs. Two typical groove patterns, namely, radial and spiral patterns have been chosen for this investigation. Finite element idealizations for these patterns using 320 triangular elements are shown in FIG.34, and FIG.35. General data for the calculation are given below.

\[
\begin{align*}
\text{inside radius of a disc} & = 1.0 \text{ m} \\
\text{outside radius of a disc} & = 2.0 \text{ m} \\
\text{depth of groove} & = 0.5 \text{ mm} \\
\text{film thickness} & = 1.0 \text{ mm} \\
\text{viscosity} & = 1.0 \text{ kg sec} / \text{ m}^2 \\
\text{squeezing speed} & = -1.0 \text{ m/sec} \\
\text{angular velocities} & = 0.0, 1.0 \text{ rad/sec}
\end{align*}
\]

These values are chosen to enable a comparison to be made with results of the flat surface disc problem analyzed in the previous section.

Calculated results of pressure distributions are presented in FIGS.36 to 43. FIGS.36 and 37 show the effects of groove pattern to the pressure distribution of discs with simple squeezing motion and complex squeezing and rotating motions respectively. FIGS.38 and 39 show the effects of rotating motion on radially and spirally grooved discs respectively. The contour diagrams of the pressure
distribution of radially grooved discs are shown in FIGS. 40 and 41, and those of spirally grooved discs are shown in FIGS. 42 and 43. These four figures are presented in order that the distortion of the pressure distribution throughout the disc surface should be understood.

Results of pressure distributions, that is, variation of pressure with $\theta$, of radially and spirally grooved discs during single squeezing motion compared with the results of the grooveless disc are shown in FIG. 36. Maximum pressures are seen at the center of disc facings and minimum pressures are seen at the center of grooves. It is found that grooves on the disc surface reduce the pressure greatly and that radial grooves cause a higher maximum pressure, a lower minimum pressure and more drastic change of the pressure over the surface than spiral grooves which give a more even pressure over the surface.

Pressure distributions during squeezing and rotating motions are presented in FIG. 37. These curves indicate that the pressure decreases because of the centrifugal force and grooves affect the drop of pressures more than flat surface. The maximum pressure decreases in the radially grooved discs with increase of rotating speed. Differences between maximum and minimum pressures become greater in both radially and spirally grooved discs when discs rotate.

In the radially grooved disc the pressure at the groove is lower since the length of the groove is shorter and the width of the groove in the oil flow direction is wider, both of which reduce the resistance to flow pass more in radial groove than in spiral groove.
Rearrangement of results in accordance with the rotating motion effect on radially grooved disc is shown in FIG.38 and that on spirally grooved disc is shown in FIG.39. It is more clearly seen that the effect of the rotating motion reduces both maximum and minimum pressures in radially grooved discs, however, in spirally grooved discs the maximum pressure keeps the same level but only minimum pressure reduces.

FIGS.38 and 39 also indicate that the positions of peak pressures are moved in the circumferential direction in accordance with the rotating motion. The peak points move about three degrees in radially grooved discs, while ten degrees in spirally grooved discs.

Each value of calculated maximum and minimum pressures is presented in TABLE.9. It is found that grooves on the disc surface reduce the pressure greatly to about 80% of that of the flat surface and the effect of rotating motion on pressure distributions is greater when using grooved discs.

The distortion of the pressure distribution due to the pattern of grooves and the rotating motion throughout the disc surface can be understood more clearly by using contour plots which are shown in FIGS.40 to 43. By comparing FIG.40 and FIG.41, it is found that the pressure distributes very simply and with less distortion on radially grooved discs, and that the points of maximum pressure move only in circumferential direction and not in radial direction when discs rotate.

Contours of the pressure distribution of the spirally grooved discs shown in FIGS.42 and 43 indicate that spiral grooves cause the greater distortion of pressure distribution than radial grooves. The points of maximum pressure are also found to move only in the
circumferential direction when discs rotate.

It is of great interest that the pressure gradient at the area $A_p$ shown in FIG.43 is very large while this phenomenon is not found in the result of radially grooved discs in FIG.41. This indicates that in spiral grooves pressure goes down to the atmospheric pressure even inside the grooves. Contour diagrams are very useful figures for researchers to understand the overall pressure distribution of complicated configurations.
7. Conclusions

A finite element application to lubrication problems which includes rotating annular discs has been successfully developed. Although limited to incompressible isothermal conditions, the solution of the generalized Reynolds equation developed in this work includes various effects such as shear force, body force, squeeze and diffusion effects. Furthermore, inertia effects have also been considered as these are essential when investigating disc problems. Thickness of the oil film can be varied within an element, which overcomes irregular configurations of the film thickness such as grooving.

The finite element technique has been validated by solving standard lubrication problems such as squeezing pad, slider bearing and step bearing. The comparison with the theoretical results presented in chapter 5 shows very satisfactory agreements between two methods. The increasingly finer grading of meshes generally provides a better accuracy and this has also been established in this work.

The results of flat disc problems show that the finite element technique developed here can be a powerful tool for the investigation of clutch disc problems, however, irregular configuration of surfaces such as groovings requires the finite element idealization of the whole disc instead of considering symmetrical sections of the disc. The inertia effect is found to be greater than theoretical results which are based on the assumption of the Couette flow in the tangential direction.
The investigation of the grooved disc problems by using the finite element technique presented in chapter 6 leads to the following conclusions.

1. Radial grooves cause higher pressure and greater pressure gradients than spiral grooves.

2. Rotating motion affects the oil film pressure more in radially grooved surfaces than in spirally grooved surfaces. The pressure decreases more rapidly in radially grooved discs than in spirally grooved discs as the discs increase in speed. This indicates that the engaging speed, which is represented by the surface velocity in z direction, is higher in radially grooved discs than in spirally grooved discs for a given squeezing pressure.

As for future research, wider and more intensive investigations of discs are worthwhile for the analyses of brake discs. Also the iterative calculation will enable the thermal analyses of dynamic engaging characteristics of wet type clutch discs to be made.
REFERENCES


(39) FULLER, "Theory and Practice of Lubrication for Engineers", (1970), pp166

FIG. 1 Geometry and coordinate system for a fluid film and corresponding surfaces.
FIG. 2  Continuity of flow of a fluid element.
FIG. 3  Continuity of flow of a column of fluid.
FIG. 4 Equilibrium of a fluid element.
FIG. 5 Description of Centrifugal force action.
FIG. 6  Geometry of a porous system.
FIG. 7 Representation of a torque field.
FIG. 8 Lubricant domain and F.E. idealization.
FIG. 9 Natural coordinates for a quadrilateral element.

(a) Cartesian Coordinates  (b) Natural Coordinates.
FIG. 10  F.E. idealization of a porous region.
START

SELECT OUTPUT FORM, UNITS AND COORDINATE SYSTEM

READ NO. OF ELEMENTS, NODES, FILM PROPERTIES AND BOUNDARY CONDITIONS FOR FILM REGION ANALYSIS

PRINT INPUT DATA

POROUS REGION ANALYSIS REQUIRED?

READ POROUS REGION INPUT NO. OF ELEMENTS, NODES, PERMEABILITIES, ETC.

NO

CALCULATE PRESSURE MATRIX "MTKP"

CALCULATE OTHER FLUIDITY MATRICES

CENTRIFUGAL FORCE REQUIRED?

YES

CALCULATE CENTRIFUGAL FORCE

NO

POROUS REGION ANALYSIS REQUIRED?

YES

SOLVE NON-POROUS SYSTEM EQUATION (4.12)

SOLVE POROUS SYSTEM EQUATION (4.69)

NO

CALCULATE LOAD CARRYING CAPACITY, FRICTION FORCE AND FRICTION TORQUE

PRINT RESULTS

END

FIG. 11 MAIN FLOWCHART
FIG. 12 SUBROUTINE SYSTEM
<table>
<thead>
<tr>
<th>ROUTINE</th>
<th>PURPOSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTKPF</td>
<td>General pressure matrix routine</td>
</tr>
<tr>
<td>MTKP</td>
<td>Pressure matrix routine</td>
</tr>
<tr>
<td>MTKP3</td>
<td>for triangular element</td>
</tr>
<tr>
<td>MTKP4</td>
<td>for rectangular element</td>
</tr>
<tr>
<td>MTKUX</td>
<td>X-direction shear action matrix routine</td>
</tr>
<tr>
<td>MTKUX3</td>
<td>for triangular element</td>
</tr>
<tr>
<td>MTKUX4</td>
<td>for rectangular element</td>
</tr>
<tr>
<td>MTKUY</td>
<td>Y-direction shear action matrix routine</td>
</tr>
<tr>
<td>MTKUY3</td>
<td>for triangular element</td>
</tr>
<tr>
<td>MTKUY4</td>
<td>for rectangular element</td>
</tr>
<tr>
<td>MTKUX</td>
<td>X-direction body force action matrix routine</td>
</tr>
<tr>
<td>MTKUX3</td>
<td>for triangular element</td>
</tr>
<tr>
<td>MTKUX4</td>
<td>for rectangular element</td>
</tr>
<tr>
<td>MTKY</td>
<td>Y-direction body force action matrix routine</td>
</tr>
<tr>
<td>MTKY3</td>
<td>for triangular element</td>
</tr>
<tr>
<td>MTKY4</td>
<td>for rectangular element</td>
</tr>
<tr>
<td>MTKH</td>
<td>Squeeze action matrix routine</td>
</tr>
<tr>
<td>MTKH3</td>
<td>for triangular element</td>
</tr>
<tr>
<td>MTKH4</td>
<td>for rectangular element</td>
</tr>
<tr>
<td>SOLVE</td>
<td>Routine for solving the system equation (4.12), (4.69) and for the calculation of friction forces, torques and load capacity</td>
</tr>
<tr>
<td>PRINT</td>
<td>Routine for listing the global matrix</td>
</tr>
<tr>
<td>COOR</td>
<td>Routine for arranging the coordinate system from various type of input</td>
</tr>
<tr>
<td>MB02A</td>
<td>Routine for calculating the inverse of a matrix</td>
</tr>
<tr>
<td>CFORCE</td>
<td>Routine for the calculation of centrifugal forces</td>
</tr>
<tr>
<td>FFORCE</td>
<td>Routine for the calculation of friction forces and torques</td>
</tr>
<tr>
<td>output</td>
<td>Routine for printing pressures, flows and other flow variables</td>
</tr>
<tr>
<td>MTKV</td>
<td>General routine for porous region analysis</td>
</tr>
<tr>
<td>PREAD</td>
<td>Routine for reading input for porous analysis</td>
</tr>
<tr>
<td>PMATP</td>
<td>Pressure matrix routine for porous region</td>
</tr>
<tr>
<td>PMATPL</td>
<td>Other fluidity matrix routine for porous region</td>
</tr>
<tr>
<td>PFLON</td>
<td>Routine for arranging boundary conditions for porous region</td>
</tr>
<tr>
<td>PACT</td>
<td>Routine for arranging fluidity action values for porous region</td>
</tr>
<tr>
<td>CHANGE</td>
<td>Routine for changing the system from film to porous</td>
</tr>
<tr>
<td>FRRETURN</td>
<td>Routine for returning the system from porous to film</td>
</tr>
</tbody>
</table>
START

ALL INITIAL VALUES = 0.0

CARTESIAN COORDINATE?

no

yes

X(I) = GCOE(K)
Y(I) = GCOE(K+NODES)

RAD(I) = GCOE(K)
ANG(I) = GCOE(K+NODES)

UNIT=DEG?

no

yes

ANG=(3.1415/180.0)·ANG

X(I)=RAD(I)·cos(ANG(I))
Y(I)=RAD(I)·sin(ANG(I))

A(I,J) = X(I)·Y(J)

NNEL=2?

no

yes

CALL MTKP2

NNEL=3?

no

yes

CALL MTKP3

NNEL=4?

no

yes

CALL MTKP4

STOP

PRINT RESULT MKP

PRINTMTKP

RETURN

FIG. 13 SUBROUTINE MTKPF
START

SET INITIAL VALUES

CALCULATION OF A, B, C

PRINT A, B, C

\[ KP = B(I)B(J) + C(I)C(J) \]

\[ TH1 = TH1 + TH(I)TH(J) \]

\[ TH2 = TH(1)TH(2)TH(3) \]

CALCULATION OF ELEMENT THICKNESS

CALCULATION OF ELEMENT AREA

\[ VIS \neq 0.0 \? \rightarrow \text{ERROR MESSAGE} \rightarrow \text{STOP} \]

\[ AREA \neq 0.0 \? \rightarrow \text{ERROR MESSAGE} \rightarrow \text{STOP} \]

CALCULATION OF CNST(LE)

PRINT AREA(LE), THICK(LE), CNST(LE)

CALCULATION OF MATRIX

\[ MKP(I,J) = CNST \times KP \]

\[ MTKP = MTKP + MKP \]

RETURN

FIG. 14 SUBROUTINE MTKP3
Calculation of common constants

START

CALL CNST4

PRINT CONSTANTS

IS THICKNESS CONSTANT WITHIN AN ELEMENT? no *1 yes

TERM C21 ≠ 0.0? no TERM C22 ≠ 0.0? no yes yes

CALL TERMA CALL TERMA1 CALL TERMA3
CALL TERMB CALL TERMB1 CALL TERMB3
CALL TERMC CALL TERMC1 CALL TERMC3

CALCULATION OF TERM D

MKP1 = A + B - C - D
MKP = CNST * MKP1
MTKP = MTKP + MKP

RETURN

FIG. 15 SUBROUTINE MTKP4
FIG. 15 SUBROUTINE MKP4 (continued)
START

CALCULATE $b_i, c_i, d_i,$
AND VOLUME OF ELEMENT

PRINT $b_i, c_i, d_i, V$

CALCULATE MATRIX

$K_{pij}$ BY EQN. (4.61)

AT EACH ELEMENT

PRINT $K_{pij}$ AT EACH ELEMENT

ASSEMBLE LOCAL MATRICES INTO
GLOBAL MATRIX

CHANGE FILM REGION FLUIDITY MATRICES
TO THE WHOLE DOMAIN REGION MATRICES

CHANGE NODE NO. OF FLOW ACTIONS AND
BOUNDARY CONDITIONS OF FILM REGION
INTO THE WHOLE DOMAIN NODE NO.

PRINT RESULTS OF NEW FLUIDITY MATRICES,
ACTION VALUES, AND BOUNDARY CONDITIONS

END

FIG. 16 SUBROUTINE MTKVD
START

CALCULATE FLOW VALUES UX, UY, BX, •••

NO I=NODES?

YES PRINT FLOW VALUES

IF NQ+NP=NODES NO ERROR MESSAGE END

YES CLASSIFY ALL NODES INTO Q-KNOWN NODES AND P-KNOWN NODES

DEFINE ALL THE BOUNDARY CONDITION VALUES AS EITHER Q OR P, AND MAKE BOUNDARY CONDITION MATRICES

ALL Q VALUES KNOWN?

YES CALCULATE RIGHT HAND SIDE OF EQN.( CALL MB02A )

NO ALL P VALUES KNOWN?

YES CALCULATE RIGHT HAND SIDE OF EQN.( RETURN )

NO CALCULATE PRESSURES AT EACH NODE

RETURN

RETURN

FIG. 17 SUBROUTINE SOLVE
DEVIDE EACH FLOW MATRIX INTO SUB-MATRICES CONSIST OF NODES WHICH HAVE KNOWN P AND Q IN EQN.(

CALL MB02A

FIND UNKNOWN PRESSURE VALUES BY SOLVING EQN.(

OBTAIN CORRESPONDING FLOWS BY SOLVING EQN.(

CALL MB02A

CALCULATE FRICTION TORQUE

CALCULATE ELEMENT FLOWS

RETURN

FIG. 17 SUBROUTINE SOLVE (continued)
FIG. 18  Geometry of a rectangular squeezing pad.
FIG. 19 Finite element layouts of a rectangular pad.
FIG. 20: Finite element solution of a rectangular squeezing pad. (Section $Y = 0$)

Finite Element (a)

Finite Element (b)

Finite Element (c)

Finite Element (d)

Exact Solution from ref. [3]
Finite element solution of a rectangular squeezing pad. (Section $Y = 0$)
FIG. 22  F.E. Solution of a rectangular pad (diagonal direction)
### TABLE 2 Maximum pressure values for rectangular squeezing pad at \((x,y) = (0.0, 0.0)\)

<table>
<thead>
<tr>
<th>Analysis Methods</th>
<th>Finite Element Technique</th>
<th>Exact Solution (ref. 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of elements</td>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>% Errors</td>
<td>+6.20</td>
<td>+3.60</td>
</tr>
</tbody>
</table>

### TABLE 3 Percentage errors of load carrying capacities of rectangular squeezing pad

<table>
<thead>
<tr>
<th>Analysis Methods</th>
<th>Finite Element Technique</th>
<th>Exact Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of elements</td>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>Load carrying capacity</td>
<td>4.6990</td>
<td>4.4996</td>
</tr>
<tr>
<td>% Errors</td>
<td>9.23</td>
<td>4.60</td>
</tr>
</tbody>
</table>
FIG. 23  Geometry of a slider bearing
FIG. 24 Finite element layouts of a slider bearing.
### TABLE 4 Pressure profiles for infinitely wide slider bearing (regular meshes)

<table>
<thead>
<tr>
<th>x (m)</th>
<th>Exact Solution</th>
<th>Number of Element in Flow Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LUB6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>0.0</td>
<td>14552.2</td>
<td>-----</td>
</tr>
<tr>
<td>0.05</td>
<td>16557.1</td>
<td>-----</td>
</tr>
<tr>
<td>0.10</td>
<td>15091.2</td>
<td>-----</td>
</tr>
<tr>
<td>0.15</td>
<td>12571.3</td>
<td>11628.0</td>
</tr>
<tr>
<td>0.20</td>
<td>10105.7</td>
<td>9854.6</td>
</tr>
<tr>
<td>0.25</td>
<td>7665.3</td>
<td>7169.8</td>
</tr>
<tr>
<td>0.30</td>
<td>5432.8</td>
<td>5344.5</td>
</tr>
<tr>
<td>0.35</td>
<td>3420.9</td>
<td>3396.5</td>
</tr>
<tr>
<td>0.40</td>
<td>1616.9</td>
<td>1594.3</td>
</tr>
<tr>
<td>0.45</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

### TABLE 5 Percentage error of pressure values from the exact solution (regular meshes)

<table>
<thead>
<tr>
<th>x (m)</th>
<th>Exact Solution</th>
<th>Number of Element in Flow Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LUB6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>0.0</td>
<td>14552.2</td>
<td>-----</td>
</tr>
<tr>
<td>0.05</td>
<td>16557.1</td>
<td>-----</td>
</tr>
<tr>
<td>0.10</td>
<td>15091.2</td>
<td>-----</td>
</tr>
<tr>
<td>0.15</td>
<td>12571.3</td>
<td>-----</td>
</tr>
<tr>
<td>0.20</td>
<td>10105.7</td>
<td>-----</td>
</tr>
<tr>
<td>0.25</td>
<td>7665.3</td>
<td>6.46</td>
</tr>
<tr>
<td>0.30</td>
<td>5432.8</td>
<td>-----</td>
</tr>
<tr>
<td>0.35</td>
<td>3420.9</td>
<td>5.43</td>
</tr>
<tr>
<td>0.40</td>
<td>1616.9</td>
<td>-----</td>
</tr>
<tr>
<td>0.45</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
### TABLE 6 Percentage errors of load carrying capacities from the exact solution

<table>
<thead>
<tr>
<th>Analysis methods</th>
<th>Finite Element Technique</th>
<th>Exact Solution (ref. [38])</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of elements</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Load carrying capacity ($10^{-7}$)</td>
<td>0.3667</td>
<td>0.4221</td>
</tr>
<tr>
<td>% Error</td>
<td>17.93</td>
<td>5.54</td>
</tr>
</tbody>
</table>

### TABLE 7 Percentage errors of friction force values from the exact solution

<table>
<thead>
<tr>
<th>Analysis methods</th>
<th>Finite Element Technique</th>
<th>Exact Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of elements</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Friction force</td>
<td>upper surface</td>
<td>3340.18</td>
</tr>
<tr>
<td></td>
<td>lower surface</td>
<td>3340.32</td>
</tr>
<tr>
<td>% Error</td>
<td>upper surface</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>lower surface</td>
<td>0.40</td>
</tr>
<tr>
<td>Graded Meshes</td>
<td>Values and Percentage Errors from the Exact Solution Values</td>
<td></td>
</tr>
<tr>
<td>-------------</td>
<td>--------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Maximum Pressure</td>
<td>Load Carrying Capacity</td>
</tr>
<tr>
<td>(a)</td>
<td>14614.0</td>
<td>0.3667x10^-7</td>
</tr>
<tr>
<td></td>
<td>(11.74%)</td>
<td>(17.93)</td>
</tr>
<tr>
<td>(b)</td>
<td>15239.2</td>
<td>0.3962</td>
</tr>
<tr>
<td></td>
<td>(7.96)</td>
<td>(11.33)</td>
</tr>
<tr>
<td>(c)</td>
<td>16081.9</td>
<td>0.4199</td>
</tr>
<tr>
<td></td>
<td>(2.87)</td>
<td>(6.02)</td>
</tr>
<tr>
<td>(d)</td>
<td>15802.1</td>
<td>0.4181</td>
</tr>
<tr>
<td></td>
<td>(4.56)</td>
<td>(6.41)</td>
</tr>
<tr>
<td>(e)</td>
<td>15058.7</td>
<td>0.3944</td>
</tr>
<tr>
<td></td>
<td>(9.05)</td>
<td>(11.72)</td>
</tr>
<tr>
<td>Exact Solution</td>
<td>16557.1</td>
<td>0.4468x10^-7</td>
</tr>
</tbody>
</table>
FIG. 25  Geometry of a step bearing
FIG. 26  Finite element layouts of a step bearing.
FIG. 27 F.E. Solution of an infinite width step bearing.
FIG. 28 F.E. Solution of a finite width step bearing.
FIG. 29  Geometry of discs.
FIG. 30

Finite Element Layout of a 60 degree sector
FIG. 31 Finite element layout of a rotating disk
FIG. 32 Pressure Distribution of Disc
FIG. 33  Examples of groove patterns
FIG. 34  Finite element layout of a radial grooved disk
FIG. 35 Finite element layout of a spiral grooved disk
upper disc

lower disc

grooveless

spiral groove

radial groove

FIG. 36 Pressure distribution between grooved discs (at r=1.5 m)
\[ r_1 = 1.0 \text{ m} \]
\[ r_2 = 2.0 \text{ m} \]
depth of groove = 0.5 mm
film thickness = 1.0 mm
squeezing speed = -1.0 m/sec
angle velocity = 1.0 rad/sec

**FIG.37** Pressure distribution between grooved disc with rotation (at \( r = 1.5 \text{ m} \))
groove pattern = radial
\[ r_1 = 1.0 \text{ m} \]
\[ r_2 = 2.0 \text{ m} \]
depth of groove = 0.5 mm
film thickness = 1.0 mm
squeezing speed = -1.0 m/sec
angle velocity = 0, 1.0 rad/sec

FIG. 38 Pressure distribution of radial grooved disc (at r=1.5m)
groove pattern = spiral
$r_1 = 1.0 \text{ m}$
$r_2 = 2.0 \text{ m}$
depth of groove = 0.5 mm
film thickness = 1.0 mm
squeezing speed = -1.0 m/sec
angle velocity = 0, 1.0 rad/sec

**FIG. 39** Pressure distribution of spiral grooved disc (at r=1.5m)
### TABLE 9

Maximum, minimum pressures of grooved discs and flat disc at the radius \( r = 1.5 \, \text{m} \)

<table>
<thead>
<tr>
<th>Groove Patterns</th>
<th>Motions</th>
<th>Radial</th>
<th>Spiral</th>
<th>Flat</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Max.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressure</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Squeezing</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>1.25</td>
<td>1.17</td>
<td>1.52</td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>0.91</td>
<td>1.02</td>
<td>0.34</td>
<td>0.15</td>
</tr>
<tr>
<td>Max.-Min.</td>
<td>0.34</td>
<td>0.15</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Squeezing + Rotating</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>1.15</td>
<td>1.17</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>0.70</td>
<td>0.85</td>
<td>0.45</td>
<td>0.37</td>
</tr>
<tr>
<td>Max.-Min.</td>
<td>0.45</td>
<td>0.37</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Effect of Grooves (percentage change)</th>
<th>Radial</th>
<th>Spiral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>82 %</td>
<td>77 %</td>
</tr>
<tr>
<td>Minimum</td>
<td>60 %</td>
<td>67 %</td>
</tr>
<tr>
<td>Max.-Min.</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Effect of Rotating Motion change of pressure</th>
<th>Maximum</th>
<th>Radial</th>
<th>Spiral</th>
<th>Flat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>-0.1</td>
<td>0.0</td>
<td>-0.07</td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.21</td>
<td>-0.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max.-Min.</td>
<td>0.11</td>
<td>0.22</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 41: Pressure Distribution of Squeeze Disc (Radial Grooves) (Rotating)
Figure 44: Pressure Distribution of Squeeze Disc (Spiral Grooves) (Non-Rotating)
FIG. 43 PRESSURE DISTRIBUTION OF SQUEEZE DISC (SPIRAL GROOVES) (rotating)
** THE APPLICATION OF FINITE ELEMENT METHOD TO LUBRICATION **

*** Nomenclature ***

- **NOOE**: No. of coordinate
- **NNEL**: No. of node per element
- **NELE**: Total no. of element
- **IBPS(I,1)**: Element sequence counter
- **IBPS(I,2)**: Element type code
  - 3: Triangular element
  - 4: Quadrilateral element
  - 5: Parallelogram, Rectangle
- **IBPS(I,3)**: No. of nodes per element
- **IBPS(I,4)**: No. of data item
- **SECPR(I,1)**: Fluid film density (DEN)
- **SECPR(I,2)**: Fluid film viscosity (VIG)
- **TH(L2,NEL)**: Fluid film thickness
- **NMOD**: Nodal No.
- **NDF**: Type of known value
  - 1: Flow value
  - 2: Pressure value
- **BCVL**: Known value of flow or pressure
- **BCAC(I,1)**: Shear action in X-direction: (UX1)
  - Lower surface velocity (H=0, θ)
  - Angular velocity of the lower surface
  - When using UNIT2=2
- **BCAC(I,2)**: Shear action in X-direction: (UX2)
  - Upper surface velocity (H=H)
  - Angular vel. of the upper surface
  - When using UNIT2=2
- **BCAC(I,3)**: Shear action in Y-direction: (UY1)
  - (θ, 0 in polar coordinate)
- **BCAC(I,4)**: Shear action in Y-direction: (UY2)
  - (θ, 0 in polar coordinate)
- **BCAC(I,5)**: Body force action in X-direction (BX1)
- **BCAC(I,6)**: Body force action in X-direction (BX2)
- **BCAC(I,7)**: Body force action in Y-direction (BY1)
- **BCAC(I,8)**: Body force action in Y-direction (BY2)
- **BCAC(I,9)**: Squeeze action (H)
- **BCAC(I,10)**: Diffusion action (VD)
- **NODES**: Total no. of nodes
- **CFX(I)**: Centrifugal force in X-direction
- **CFY(I)**: Centrifugal force in Y-direction
IMPLICIT REAL*(8(A-H,M,0-Z))
INTEGER R,W

COMMON/BLK1/ NODES, NODEL, NEL, NP, NME(400, 4), NMOD(400),
1 NDF(400), IBPS(400, 4), SECE(400, 4), DEN, TH(400, 4), VIS,
2 GILCO(400, 2), GCCB(800), BCVL(400), BCAC(400, 10),
3 A(4, 4), AN(400, 4), B(400, 4), C(400, 4), AREA(400), R, COOR,
4 RESULT, UNIT1, UNIT2
COMMON/BLK2/MKP(200, 200), MKP(200, 200), MKP1(200, 200), MP(400, 4, 4)
COMMON/BLK3/MTKUX(200, 200), MTKUX(400, 4, 4)
COMMON/BLK4/MTKUY(200, 200), MTKUY(400, 4, 4)
COMMON/BLK5/MTKEX(200, 200), MTKEX(400, 4, 4)
COMMON/BK6/MTKEX(200, 200), MTKEX(400, 4, 4)
COMMON/BLK7/MTKEX(200, 200), MTKEX(400, 4, 4)
COMMON/BLK8/MTKVD(200, 200), MTKVD(400, 4, 4)
COMMON/BLK9/Q(400), P(400), Q1, Q2, W, QEL(400, 4)
COMMON/BLK10/TEST
COMMON/BLK11/DATA(4, 4)
COMMON/BLK12/COOR(400), EFY(400)
COMMON/BLK13/CRH(400), NDQ(400), NDP(400)

C
C________READ INPUT DATA

C
C
R=S
W=S
WRITE(6, 1000)
1000 FORMAT('DO YOU NEED TO PRINT THE DETAILS OF CALCULATIONS?',
1 'YES...OR...NO...OR...XX (XX: ONLY THE FINAL RESULT)')
READ(S, 1000) RESULT
WRITE(6, 1011) RESULT
1011 FORMAT(1H+,T55, ',',A4)
1012 WRITE(6, 1032)
1002 FORMAT('CHOOSE THE COORDINATE SYSTEM.',
1 'NOTICE:Only when the polar coordinate is used.',
2 'the centrifugal force will be considered...',
3 'IF YOU USE X-Y COORDINATE...........PLEASE KEY IN :XY',
4 'IF YOU USE POLAR COORDINATE...........PLEASE KEY IN :P0')
READ(S, 1002) COOR
WRITE(6, 1012) COOR
1012 FORMAT(1H+,T55, ',',A3)
IF(COOR.NE.'XY') GO TO 1004
UNIT2=1
GO TO 1009
1004 IF(COOR.NE.'P0') GO TO 1001
WRITE(6, 1035)
1005 FORMAT('CHOOSE THE UNIT OF ANGLE.',
1 'IF YOU USE UNIT DEGREE...........PLEASE KEY IN ,DEG',
2 'IF YOU USE UNIT Radian...........PLEASE KEY IN ,RAD')
READ(5, 1005) UNIT1
WRITE(6, 1013) UNIT1
1013 FORMAT(1H+,T48, ',',A3)
WRITE(6, 1037)
1007 FORMAT('WHICH VELOCITY DO YOU USE, X-Y VEL. OR ANGULAR?',
1 'IF X-Y VEL........PLEASE KEY IN 1',
2 'IF ANGULAR VEL........PLEASE KEY IN 2')
READ(5, 1007) UNIT2
1009 FORMAT(1H)
WRITE(6, 1014) UNIT2
1014 FORMAT(1H,103, 'XX')
C---READ ELEMENT PROPERTIES

1090 WRITE(6,6)
   6 FORMAT(///,'Calculation Start.',
       1///,'READING TEST NAME ')
      READ(R,1019) TEST
      WRITE(6,19)
19 FORMAT('READING NCOE, NODES, NELE, NNEL')
      READ(R,*) NCOE, NODES, NELE, NNEL
      WRITE(6,30)
30 FORMAT('READING IBPS(I,J), SECPR(I,K)')
      READ(R,1010) DENVIS
      IF(DENVIS.EQ.'YES') GO TO 21
      DO 20 I=1,NELE
20 READ(R,*) (IBPS(I,J),J=I,4), (SECPR(I,K),K=1,2)
      GO TO 51
21 READ(R,*) (IBPS(I,J),J=I,4), (SECPR(I,K),K=1,2)
      DO 22 I=2,NELE
22   SECPR(I,1)=SECPR(I,1)
      SECPR(I,2)=SECPR(I,2)
      DO 22 J=I,4
22 READ(R,*) (IBPS(I,J),J=I,4),(SECPR(I,K),K=1,2)
      CONTINUE
51 WRITE(6,59)
59 FORMAT('READING IBPS(I,I),GELCO(I,J), TH(I,K)')
      DO 40 I=1,NODES
40 READ(R,*) IBPS(I,I), (GELCO(I,J),J=1,4), (TH(I,K),K=1,NNEL)
      CONTINUE
C---READ COORDINATES OF NODES
      IF(COOR.NE.'PO') GO TO 73
71 WRITE(6,72)
72 FORMAT('READING I, GELCO(I,J)',
      1 ' IN POLAR COORDINATE')
      GO TO 74
73 WRITE(6,73)
73 FORMAT('READING I, GELCO(I,J)')
74 READ(5,*) LTYPE
      IF(LTYPE.EQ.1) GO TO 61
      DO 60 I=1,NODES
60 READ(R,*) I, (GELCO(I,J),J=1,NCOE)
      GO TO 65
61 READ(5,*) RA1, RA2, THITA1, THITA2, ND
      DO 62 I=1,NODES
62 NA=1+ND
      NB=(I-1)/NA
      GELCO(I,1)=RA1+(I-1-NA*NB)*(RA2-RA1)/ND
      GELCO(I,2)=THITA2*NB
      CONTINUE
65 NNO=2*NNODES
      DO 75 L=1,NNO
      GCOE(L)=.0
      CONTINUE
75 CONTINUE
      DO 80 I=1,NELE
50 READ(R,*) (NME(I,J),J=1,4), (TH(I,K),K=1,NNEL)
      CONTINUE
C——READ BOUNDARY CONDITIONS

WRITE(6, 941)
901 FORMAT('READING NQ,NP')
READ(R,*), NQ,NP
WRITE(6, 942)
902 FORMAT('READ BC YES OR NO')
READ(5, 1100) BC
WRITE(6, 941)
91 FORMAT('READING NNOD(I),NDF(I),BCVL(I)')
IF(BC.EQ. 'YES') GO TO 906
DO 92 I=1, NODES
92 READ(R,*), NNOD(I),NDF(I),BCVL(I)
   J=1
   K=0
   DO 905 I=1,NODES
      IF(NDF(I).NE.1) GO TO 904
      K=K+1
      NDQ(K)=NNOD(I)
   GO TO 905
904 J=J+1
      NDP(J)=NNOD(I)
CONTINUE
GO TO 911
906 READ(5,*), (NDP(I),I=1,NP)
   K=0
   DO 907 I=1,NODES
      NNOD(I)=I
      NDF(I)=2
   DO 908 J=1,NP
      IF(I.EQ.NDP(J)) GO TO 907
   CONTINUE
   K=K+1
      NDQ(K)=I
      NDF(I)=1
CONTINUE
907 READ(5,*), BCVL,BCVLQ
DO 909 I=1,NP
      BCVL(NDP(I))=BCVL
CONTINUE
DO 910 I=1,NP
      BCVL(NDQ(I))=BCVLQ
CONTINUE
913 WRITE(6, 93)
911 FORMAT('READING NNOD(I),BCAC(I,J)')
READ(3,1130) BCA
IF(BCA.EQ. 'YES') GO TO 93
DO 94 I=1,NODES
94 READ(R,*), NNOD(I),(BCAC(I,J),J=1,10)
GO TO 96
98 READ(R,*), NNOD(I),(BCAC(I,J),J=1,10)
DO 99 I=2,NODES
   DO 99 J=1,10
      BCAC(I,J)=BCAC(I,J)
CONTINUE
1010 FORMAT(AG3)
1130 FORMAT(A3)
C
C——WRITE THE TITLE AND INPUT DATA TO CHECK
C
96 WRITE(W,100)
100 FORMAT(/'*'1RTIE 1E TITL: MID IIPUI mTA',
1/,'C',48X,'C',,
2/,'C THE APPLICATION OF F.E.M. TO THE LUBRICATION C',
3/,'C',48X,'C',
4/,'C',48X,'C',
5/,'C')
WRITE(W,101) TEST
101 FORMAT(//'TEST NAME',3X,A99)
   IF(RESULT.NE.'YES') GO TO 106
WRITE(W,105)
105 FORMAT(/'*'1RTIE(W,101)
   TEST
106 FORMAT(//,'TI!t NAME',3X,JL0)
   IF(RESULT.NE.'YES') GO TO 106
WRITE(W,201)
201 FORMAT(/'*'1RTIE(1-I, 2032) NCOE, NODES, NELE, NNEL
202 FORMAT(A15)
   NN=NELE
   IF(DENVIS.EQ.'YES') NN=1
   DO 2033 I=1,NN
2033 WRITE(W,2034) (IBPS(I,J),J=1,NNEL), (SECPR(I,K),K=1,2)
2034 FORMAT(4I5,2E10.3)
   IF(NNEL.EQ.4) GO TO 2035
   NNOD=1
2035 WRITE(W,2036) IBPS(I,1), (NME(I,J),J=1,NNEL), (TH(I,K),K=1,NNEL)
2036 FORMAT(14,2X,4I4,4F10.5)
   GO TO 3013
3013 DO 3011 I=1,NELE
3011 WRITE(W,3012) IBPS(I,1), (NME(I,J),J=1,NNEL), (TH(I,K),K=1,NNEL)
3012 FORMAT(14,2X,4I4,4F10.5)
3013 DO 3017 I=1,NELE
3017 WRITE(W,3018) I, (GELCO(I,J),J=1,NCOE)
3018 FORMAT(14,2X,2F10.5)
   DO 3019 I=1,NELE
3019 WRITE(W,3020) NMOD(I),NDF(I),BCVL(I)
3020 FORMAT(2I4,2X,3I4,3F10.5)
   MOD=NODES
   IF(BCA.EQ.'YES') MOD=1
   DO 3021 I=1,MOD
3021 WRITE(W,3022) (BCAC(I,J),J=1,19)
3022 FORMAT(14,19(2X,F6.3))
   WRITE(W,3023) NP
3023 FORMAT(2I4)
106 WRITE(W,110)
110 FORMAT(/,'ELEMENT NO.',3X,'EL-TYPE',3X,'NODE NO.',
1/,'14X,'FILM THICKNESS')
   IF(NNEL.EQ.4) GO TO 121
   DO 120 I=1,NELE
120 WRITE(W,120) IBPS(I,1), IBPS(I,2), (NME(I,J),J=1,NNEL),
1/,(TH(I,K),K=1,NNEL)
120 FORMAT(1H ,5X,I3,7X,I2,4X,3I4,3(3X,EL4.3))
   GO TO 123
121 WRITE(W,121) IBPS(I,1), IBPS(I,2), (NME(I,J),J=1,NNEL),
1/,(TH(I,K),K=1,NNEL)
121 FORMAT(1H ,5X,I3,7X,I2,4X,4I4,4(3X,EL4.3))
122 IF(CORR.EQ.'PO') GO TO 141
IF(COOR.EQ.'PO') GO TO 141
WRITE(W,140)
140 FORMAT(/, 'COORDINATE ARRAY OF NODAL POINT',
     1///, 'NODE NO.', 3X, 'X-COORDINATE', 2X, 'Y-COORDINATE')
GO TO 143
141 WRITE(W,142)
142 FORMAT(/, 'COORDINATE ARRAY OF NODAL POINT',
     1///, 'NODE NO.', 3X, 'RADIAL', 2X, 'ANGLE')
143 DO 150 I=1,NODES
150 WRITE(W,164) I,GCOC(I),GCOC(I+NODES)
160 FORMAT(1H,12X,13,6X,13,9X,3X,E10.3)
WRITE(W,161)
161 FORMAT(/, 'THE FLEM PROPERTIES',
     1///, 'ELEMENT NO.', 10X, 'DEN', 12X, 'VIS')
IF(DENVIS.EQ.'YES') GO TO 164
DO 162 I=1,NELE
162 WRITE(W,163) I,(SECPR(I,K),K=1,2)
163 FORMAT(1H,12X,13,1X,2(S1C,E10.3))
GO TO 169
164 WRITE(W,165) NELE,(SECPR(I,K),K=1,2)
165 FORMAT(1H,12X,13,1X,2(S1C,E10.3))
169 WRITE(W,170) NQ,NP
170 FORMAT(/, 'I==-----------------------------------I',
     1///, ' Boundary Condition Value I',
     2///, '----------------------------------I',
     3///, 'NO=', 13, 5X, 'NP=', 13,
     4///, 'BCVL TYPE 1: FLOW VALUE (Q)',
     5///, '6X', 'TYPE 2: PRESSURE (P)',
     6///, 'IX', 'NODE', 4X, 'TYPE', 7X, 'BCVL')
DO 175 I=1,NODES
175 WRITE(W,181) NNOD(I),NDF(I),BCVL(I)
180 FORMAT(1H,12X,13,4X,13,3X,E10.3)
IF(UNIT2.EQ.2) GO TO 181
WRITE(W,181)
181 FORMAT(/, 'C', 21(' '), 'C', '/C VALUES OF ACTIONS C',
     1///, 'C', 21(' '), 'C', /9X, 'UX1', 'UX2', '9X', 'UY1', '9X', 'UY2',
     2///, 'BX1', 'BX2', '9X', 'BY1', '9X', 'BY2')
GO TO 1183
182 WRITE(W,182)
1182 FORMAT(/, 'VALUES OF ACTIONS',
     1///, '9X', 'W1', '10X', 'W2', '9X', '---', '9X', '---',
     2///, '9X', 'BX1', '9X', 'BX2', '9X', 'BY1', '9X', 'BY2')
1183 NOD=NODES
IF(BCA.EQ.'YES') NOD=1
DO 182 I=1,NOD
182 WRITE(W,183) NNOD(I),(BCAC(I,J),J=1,8)
183 FORMAT(13,8(2X,E10.3))
WRITE(W,187)
187 FORMAT(/,11X, 'H', 11X, 'V')
DO 188 I=1,NOD
188 WRITE(W,189) NNOD(I),(BCAC(I,J),J=9,19)
189 FORMAT(13,2(3X,E10.3))
WRITE(6,190)
190 FORMAT(/, 'Finished Reading and Writing Input Data')
C
DO 300 I=1,NEEL
DO 300 J=1,NEEL
DELTA(I,J)=9.9
DELTA(I,I)=1.7
300 CONTINUE
C
CALL MTXPF
WRITE(1,191)

IF(INEL-3.0) 410,420,430
410 GO TO 500
420 CALL MTUX3
WRITE(1,192)
CALL MTUY3
WRITE(1,193)
CALL MTXK3
WRITE(1,194)
CALL MTKX3
WRITE(1,195)
CALL MTXH3
WRITE(1,196)
CALL MTND3
WRITE(1,197)
GO TO 450

CALL MTUX4
WRITE(1,192)
CALL MTUY4
WRITE(1,193)
CALL MTXK4
WRITE(1,194)
CALL MTKX4
WRITE(1,195)
CALL MTXH4
WRITE(1,196)
CALL MTVD4
WRITE(1,197)

CALL SOLVE
WRITE(1,198)
CALL OUTPUT
WRITE(1,199)
GO TO 500

191 FORMAT('[' Finished Calculation of MTXPF ']')
192 FORMAT('[' Finished Calculation of MTUXF ']')
193 FORMAT('[' Finished Calculation of MTUYF ']')
194 FORMAT('[' Finished Calculation of MTXKF ']')
195 FORMAT('[' Finished Calculation of MTKXF ']')
196 FORMAT('[' Finished Calculation of MTXHF ']')
197 FORMAT('[' Finished Calculation of MTXDF ']')
198 FORMAT('[' Finished Calculation of SOLVE ']')
199 FORMAT('[' Finished Calculation of OUTPUT ']')

500 WRITE(1,200)
200 FORMAT('Your Calculation has been finished.')

CALL EXIT
END
SUBROUTINE MKPF

*** THIS SUBROUTINE MAKES MATRIX OF PRESSURE ***

IMPLICIT REAL*8(A-H,M-O-Z)
INTEGER W

COMMON/BUCK/ XOE,NODES,NELE,NNEL,NP,NO,IME(400,4),ANDD(400),
1 NDF(400),IPBS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
2 CEL(400,2),GCOE(400),BCUL(400),BCAC(400,10),
3 A(4,4),AA(A400,4),B(400,4),C(400,4),AREA(400),W,COOR,
4 RESULT,INT1,INTIT2

COMMON/BK12/MKP(200,200),MKP(200,200),MKP1(200,200),MP(400,4,4)
COMMON/BK15/X(4),Y(4)
DIMENSION RAD(400),ANG(400)

WRITE(W,50)
50 FORMAT(///,2X,'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX',
1 /,2X,'C',15X,'C',
2 /,2X,'C',15X,'C',/2X,'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX')
WRITE(W,100)
100 FORMAT(///,'I',45('..'),'I',
1 /,'I',1. CALCULATION OF ELEMENT PRESSURE MATRIX I',
2 /,'I',45('..'),'I'///)

LET ALL INITIAL VALUES ZERO.
DO 140 I=1,NNEL
RAD(I)=3.0
ANG(I)=0.0
X(I)=3.0
Y(I)=0.0
DO 140 LE=1,NELE
B(LE,I)=3.0
C(LE,1)=0.0
AA(LE,1)=3.0
DO 140 J=1,NNEL
A(I,J)=0.0
140 CONTINUE
DO 150 I=1,NODES
DO 150 J=1,NODES
MKP(I,J)=0.0
MKP(I,J)=3.0
150 CONTINUE
DO 200 LE=1,NELE
IF(RESULT.lE.'YES') GO TO 201
WRITE(H,210) LE
201 FORMAT(///,2X,'_ELEMENT NO.',13)
200 FORMAT(///,2X,'ELEMENT NO.',13)

ARRANGEMENT OF COORDINATE
201 IF (COORD.EQ.'PO') GO TO 221
DO 220 I=1,NNEL
X(I)=GCOE(NME(LE,I))
Y(I)=GCOE(NME(LE,I)+NODES)
220 CONTINUE
DO 227 I=1,NNEL
RAD(I)=GCOE(NME(LE,I))
ANG(I)=GCOE(NME(LE,I)+NODES)
227 CONTINUE
GO TO 227
221 DO 226 I=1,NNEL
X(I)=GCOE(NME(LE,I))
Y(I)=GCOE(NME(LE,I)+NODES)
ANG(I)=GCOE(NME(LE,I)+NODES)
226 CONTINUE
IF(UNIT1.EQ.'DEG') GO TO 222
IF(UTLT.EQ.'DEG') GO TO 222
IF(UTLT.EQ.'RAD') GO TO 223

222 ANG(I)=3.141592654/180.*ANG(I)
X(I)=RAD(I)*DCOS(ANG(I))
Y(I)=RAD(I)*DSIN(ANG(I))
IF(RESULT.NE.'YES') GO TO 226
WRITE(W,225) NME(LE,I),UNITL,RAD(I),ANG(I),X(I),Y(I)

225 FORMAT(6X,'NODE NO.' ,13,3X,(RAD,ANG(' ,A3,' ))= (',E10.3,
1 1X,1E10.3,')',3X,'(X,Y)= (',E10.3,1X,E10.3,')')
CONTINUE

226 CONTINUE

C--CALCULATION OF A(I,J)

227 DO 230 I=1,NNEL
   DO 230 J=1,NNEL
      A(I,J)=X(I)*Y(J)
   CONTINUE

230 CONTINUE

C--CALCULATION OF PRESSURE MATRIX MTKP OF ALL TYPES OF ELEMENT

IF(NNEL.EQ.2) GO TO 1
IF(NNEL.EQ.3) GO TO 2
IF(NNEL.EQ.4) GO TO 3
WRITE(W,1200)
1200 FORMAT('/ERROR...PLEASE USE NNEL=2,3 OR 4 OTHERWISE '
1 'CALCULATION CANNOT BE DONE.')
   CALL EXIT
   1 GO TO 240
   2 CALL MTKP3(LE)
   GO TO 240
   3 CALL MTKP4(LE)
   GO TO 240

C--PRINT THE RESULT OF MTKP

240 IF(RESULT.NE.'YES') GO TO 200
WRITE(W,1000)
1000 FORMAT(4X,'ELEMENT PRESSURE MATRIX')
   DO 250 I=1,NNEL
      WRITE(W,1100)(NME(LE,I),NME(LE,J),MTKP(NME(LE,I),
1 NME(LE,J)),J=1,NNEL)
   CONTINUE

1100 FORMAT(6X,4('MKP(' ,I2,IX,')=' ,E10.3,2X)
   CONTINUE

200 CONTINUE

C

IF(RESULT.EQ.'XX') GO TO 270
WRITE(W,260)
260 FORMAT('///I',38('-')','I',
1 '/I 2. RESULT OF GLOBAL MATRIX (MTKP) I ',
2 '/I',38('-')','I',')/)

C

CALL PRINT(NODES,MTKP,W)

C

270 RETURN
END
SUBROUTINE MTKP3(ILS)

THIS ROUTINE IS TO CALCULATE PRESSURE MATRIX MTKP
OF TRIANGLE ELEMENT

IMPLICIT REAL*8(A-H,K-M,O-Z)

INTEGER W

COMMON/BLK1,AXCO,NODES,NELE,NNEL,IP,NQ,NME(400,4),NXQD(400),
1 NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
2 GELCO(400,2),GCX(300),BCVL(400),BCAC(400,10),
3 A(4,4),MA400,4),B(400,4),C(400,4),AREA(400),W,COOR,
4 RESULT,UNIT1,UNIT2

COMMON/BLK2/MTKP(200,200),MP(400,200),MTKP1(200,200),MP(400,4,4)

COMMON/BLK15/X(4),Y(4)
DIMENSION KP(4,4),CNST(400),THICK1(400)

AA(ILS,1)=A(2,3)-A(3,2)
AA(ILS,2)=A(3,1)-A(1,3)
AA(ILS,3)=A(1,2)-A(2,1)
B(ILS,1)=Y(2)-Y(3)
B(ILS,2)=Y(3)-Y(1)
B(ILS,3)=Y(1)-Y(2)
C(ILS,1)=X(3)-X(2)
C(ILS,2)=X(1)-X(3)
C(ILS,3)=X(2)-X(1)

IFRESULT.NE.'YES') GO TO 35
WRITE(W,10)
10 FORMAT(4X,'CONSTANT A,B,C')
DO 20 I=1,NNEL
20 WRITE(W,30)LE,I,AA(ILS,I),LE,I,B(ILS,I),LE,I,C(ILS,I)
30 FORMAT(1H,5X,'A(',12,12,')=',EL0.3,5X,'B(',12,12,')=',
1 EL0.3,5X,'C(',12,12,')='E10.3)

C——— ELEMENT PRESSURE MATRIX KP CALCULATION

35 AREA(ILS)=0.0
CNST(ILS)=0.0
DEN=SECPR(ILS,1)
VIS=SECPR(ILS,2)
THICK1(ILS)=0.0
TH1=0.0
TH2=0.0
DO 50 I=1,NNEL
50 J=I,NNEL
KP(1,J)=(B(ILS,I)*B(ILS,J)+C(ILS,I)*C(ILS,J))
TH1=TH1+TH(ILS,I)**2.0*TH(ILS,J)
TH2=TH2+TH(ILS,I)*TH(ILS,J)**2.0
TH1=TH1+TH(ILS,I)**2.0
TH2=TH2+TH(ILS,J)**2.0
C——— CALCULATION OF AREA

AREA(ILS)=DABS(AR)

IF(VIS.NE.0.0) GO TO 55
WRITE(1,51)
55 CONTINUE

C
WRITE(1,51)
GO TO 100
55 IF(AREA(LE).NE.0) GO TO 56
WRITE(1,52)
GO TO 100
56 CNST(LE)=(-1.0)*DEN*THICK(LE)/(480.0*VIS*AREA(LE))
IF(RESULT.NE.'YES') GO TO 65
WRITE(6,69) AREA(LE),THICK(LE),CNST(LE)
60 FORMAT(/,4X,'AREA(LE)='E10.3,3X,'THICK(LE)='E10.3,
1 4X,'CNST(LE)='E10.3)
65 DO 70 I=1,NODES
DO 70 J=1,NODES
MKP(I,J)=9.9
70 CONTINUE
C———CALCULATION OF ELEMENT MATRIX MKP, AND ASSEMBLY MATRIX MKP
DO 80 I=1,NNEL
DO 80 J=1,NNEL
IN=M(E(LE,I))
JN=M(E(LE,J))
MKP(IN,JN)=CNST(LE)*KP(I,J)
MP(LE,I,J)=MP(IN,JN)
MKP(IN,JN)=MKP(IN,JN)+MKP(IN,JN)
80 CONTINUE
C
51 FORMAT('ERROR: As VIS=4.9, CNST(LE) will be infinite.',
1 'Please check the value of VIS.')
52 FORMAT('ERROR: As AREA(LE)=9.0 CNST(LE) becomes infinite.',
1 'Please check the value of AREA(LE).')
100 RETURN
END
SUBROUTINE MTKP4(LE)

C--THIS ROUTINE IS TO CALCULATE THE PRESSURE MATRIX MTKP OF QUADRILATERAL ELEMENT
IMPLICIT REAL*8(A-H,M,O-Z)
INTEGER W
COMMON/BLKL/ NCOE, NODES, NELE, NNEL, NP, NQ, ALPHA(400,4), BETA(400,4),
1 MDF(400), IBPS(400,4), SECP1(400,4), TH1(400,4), VIS,
2 CBL(400,4), CDE(300,4), FCUL(400,4), FCD(400,4),
3 A(4,4), AA(400,4), B(400,4), C(400,4), AREA(400), W, COOR,
4 RESULT, UNIT1, UNIT2
COMMON/BLK2/MTKP(200,200), MTKP1(200,200), MTKP2(200,200), MTKP3(200,200), MTKP4(200,4,4)
COMMON/BLK15/X(4), Y(4)
COMMON/BLK16/T(4), S(4)
COMMON/BLK17/THICK(400)
COMMON/BLK18/C1(400), C2(400), C3(400), C4(400), C5(400), C6(400), C8(400)
DIMENSION TA(4,4), TB(4,4), TC(4,4), TD(4,4), MTKP(4,4)

C--CALCULATION OF EACH CONSTANT VALUE
C
CALL C1ST4(X,Y,A,LE,C1L,C2L,C3L,C4L,CC6,CC8, 1
C1L(LE)=CC1
C2L(LE)=CC2
C3L(LE)=CC3
C4L(LE)=CC4
C6L(LE)=CC6
C8L(LE)=CC8
DEN=SECP1(LE,1)
VIS=SECP1(LE,2)
WRITE(W,100) LE
100 FORMAT(/'','LE=',I2,/)  

C--CALCULATION OF EACH TERM A, B, C, D
C
C-- 1 WHEN THICKNESS IS CONSTANT WITHIN AN ELEMENT
C
CALL DINTEG(2,2,2,ITGA)
CALL DINTEG(2,2,2,ITGB)
CALL DINTEG(2,2,2,ITGC)
CALL DINTEG(2,2,2,ITGD)
CALL DINTEG(2,2,2,ITGE)

WRITE(W,101) C21, C40, C21, C41
101 FORMAT(/'','C21,C40,C21,C41'/'.,4(E10.3,3X))
WRITE(W,102) ITGA, ITGB, ITGC, ITGD, ITGE
102 FORMAT(/'','ITGA ITGB ITGC ITGD ITGE'/'.,5(E10.3,3X))

CONST=(-1.0)*DEN*TH1(LE,1)/(12.0*VIS)
DO 11 I=1,4
DO 11 J=1,4
IF(IBPS(LE,2).EQ.5) GO TO 15
IF(C21.EQ.0.0) GO TO 15
11 CONTINUE

C-- Routines for General Quadrilateral Element
CALL TERM4(C10,C11,C12,C13,C14,C15,C16,C17,C18,C19, 1
C10,C11,C12,C13,C14,C15,C16,C17,C18,C19,C21,C22,C23,C40,C41,T,S,TA,W)
CALL TERM4(C13,C14,C15,C16,C17,C18,C19,C21,C22,C23,C40,C41,T,S,TB,W)
CALL TERM4(C10,C17,C18,C19,C21,C22,C23,C40,C41,T,S,TC,W)

GO TO 10
C
14 IF(C22.EQ.0.0) GO TO 15
14 IF(C22.EQ.0.0) GO TO 15

C——ROUTINES FOR TECTONIC ELEMENT
   CALL TEM1(C10,C11,C12,C22,C23,T,S,TA)
   CALL TERM1(C13,C15,C22,C23,T,S,TB)
   CALL TERM1(LE,TH,C10,C17,C18,C19,C22,C23,T,S,TC)
   GO TO 16

C——ROUTINES FOR RECTANGLE AND PARALELOGRAM ELEMENTS
15 CALL TERM3(C10,C12,C23,T,S,TA)
   CALL TERM3(C13,C15,C23,S,T,TB)
   CALL TERM3(C13,C17,C18,C19,C23,T,S,TC)

C——CALCULATION OF TERM D
16 TD(I,J)=TC(I,J)

C——CALCULATION OF MKP AND GLOBAL MATRIX MKP
   IN=LOC(LE,I)
   JN=LOC(LE,J)
   MKP(I,J)=TA(I,J)+TB(I,J)-TC(I,J)-TD(I,J)
   MKP(IN, JN)=CONST*MKP(I,J)
   MP(LE,I,J)=MKP(IN, JN)
   MKP(IN, JN)=MKP(IN, JN)+MKP(IN, JN)
   CONTINUE

C IF(Result.NE.'YES') GO TO 12

C——PRINT THE RESULT OF MKP,MTKP
   WRITE(W,7) LE,TH(LE,1),DEN,VIS,CONST
7 FORMAT('/ ELEMENT NO.=',I5,/,5X,'THICK4, DEN, VIS, CONST',
   1 /,5X,4(E10.3,2X))
   DO 6 I=1,4
   WRITE(W,9) TA(I,J),TB(I,J),TC(I,J),TD(I,J),MKP(I,J)
9 FORMAT('A B C D MKP1 ',/5(E10.3,3X))
   CONTINUE
12 RETURN

END
SUBROUTINE MKUX3

C——-THIS SUBROUTINE MAKES MATRIX OF SHEAR ACTION IN X-DIRECTION

IMPLICIT REAL*3(A-H,M,O-Z)
INTEGER W

COMMON/BLK1/ NOCE,NODES,NELE,NNEL,MP,NO,NME(400,4),NNOD(400),
1 NS(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
2 GCLO(400,2),GCOE(300),BCVL(400),BCAC(400,1),
3 A(4,4),NA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
4 RESULT,UNIT1,UNIT2
COMMON/BLK2/MKUX(200,200),MKUX(400,4,4)
COMMON/BLK1/MKUX(400,4)
COMMON/BLK1/DELTA(4,4)

C——-LET ALL INITIAL VALUES ZERO.
DO 100 I=1,NODES
DO 100 J=1,NODES
MKUX(I,J)=.0
100 CONTINUE
DO 200 I=1,NODES
IF(BCAC(I,1).NE.0.0) GOTO 300
200 IF(BCAC(I,2).NE.0.0) GOTO 300
RETURN
300 WRITE(W,800)

C——-CALCULATION OF ELEMENT MATRIX MKUX AND GLOBAL MATRIX MKUX

DO 30 LE=1,NELE
DO 40 J=1,NNEL
THCK=TH(LE,J)
DO 400 K=1,NNEL
THCK=THCK+TH(LE,K)*(1.0+DELTA(K,J))
400 CONTINUE

THCK2=THCK*(LE,J)*(1.0+DELTA(K,J))

C——-PRINT THE RESULT

IF(RESULT.NE.'YES') GO TO 700
WRITE(W,600) LE
600 FORMAT('MKUX MATRIX ELEMENT NO.',I2)
DO 610 J=1,NNEL
610 WRITE(W,620) ((LE,NME(LE,I),NME(LE,I),MKUX(LE,I,J)),J=1,NNEL)
620 FORMAT(3X,'MKUX(','I3',',',I2',',',I3',',',I3',',')=','E10.3',2X))
700 CONTINUE

IF(RESULT.EQ.'XX') GO TO 900
800 FORMAT///5X,43(''),1/5X,'RESULT OF SHEAR ACTION MATRIX (MKUX) I',
2/5X,43(''))
WRITE(W,900)
900 FORMAT///31(''),1/','TOTAL GLOBAL MATRIX MKUX I',
2/31(''))

CALL PRINT(NODES,MKUX,W)

1000 RETURN
END
SUBROUTINE MKUY3

--- THIS SUBROUTINE MAKES MATRIX OF SHEAR ACTION IN Y-DIRECTION

IMPLICIT REAL*8(A-H,M,O-Z)
INTEGER I,J
COMMON/BLKL/ NODE,NODES,NELE,NNEL,MP,NQ,NME(400,4),NNOD(400),
1 NDF(400), IBPS(400,4), SECPR(400,4), DEN, TH(400,4), VIS,
2 Gelco(400,2), Gelce(300), BCVL(400), BCAc(400,10),
3 A(4,4), AA(400,4), B(400,4), C(400,4), AREA(400), W, COOR,
4 RESULT, UNIT1, UNIT2
COMMON/BLK4/MKUY(200,200), MKUY(400,4,4)
COMMON/BLK11/THICK2(400,4)
COMMON/BLK13/Delta(4,4)

LET ALL INITIAL VALUES ZERO.
DO 100 I=1,NODES
  MKUY(I,J)=0.0
100 CONTINUE
DO 200 I=1,NODES
  IF (UNIT2.EQ.2) GO TO 101
  IF (BCAC(I,3).NE.0.0) GO TO 300
  IF (BCAC(I,4).NE.0.0) GO TO 300
101 IF (BCAC(I,1).NE.0.0) GO TO 300
  IF (BCAC(I,2).NE.0.0) GO TO 300
200 CONTINUE
RETURN

CALCULATION OF ELEMENT MATRIX MKUY AND GLOBAL MATRIX MKUY
DO 700 LE=1,NELE
  DO 500 I=1,NNEL
    DO 500 J=1,NNEL
      THCK=0.0
      DO 400 K=1,NNEL
        THCK=THCK+THICK(LE,K)*(1.0+Delta(K,J))
        400 CONTINUE
      END=THICK4TH(LE,J)*C(LE,I)/24.0
      MKUY(LE,I,J)=DEN*THICK2(LE,J)*C(LE,I)/24.0
      MKUY(IN,IN)=MKUY(IN,IN)+MKUY(LE,I,J)
      500 CONTINUE
PRINT THE RESULT
  IF (RESULT.EQ."YES") GO TO 700
  WRITE(W,500) LE
500 FORMAT('MKUY MATRIX ELEMENT NO.',12)
  WRITE(W,600) ((LE,NME(LE,I),NME(LE,J),MKUY(LE,I,J)),J=1,NNEL)
600 FORMAT(3X,'MKUY(''I3","',I2","',I3,"X')="',ELI3,3X))
  700 CONTINUE
IF (RESULT.EQ."XX") GO TO 1000
800 FORMAT('TOTAL GLOBAL MATRIX(MKUY) I',3X,'RESULT OF GLOBAL MATRIX(MKUY) i',3X)'!,
1/5X,'RESULT OF GLOBAL MATRIX(MKUY) I',3X,'RESULT OF GLOBAL MATRIX(MKUY) i',3X)'!
2/5X,'RESULT OF GLOBAL MATRIX(MKUY) I',3X,'RESULT OF GLOBAL MATRIX(MKUY) i',3X)'!
900 FORMAT('TOTAL GLOBAL MATRIX I',3X,'TOTAL GLOBAL MATRIX i',3X)'!
1/5X,'TOTAL GLOBAL MATRIX I',3X,'TOTAL GLOBAL MATRIX i',3X)'!
2/5X,'TOTAL GLOBAL MATRIX I',3X,'TOTAL GLOBAL MATRIX i',3X)'!
RETURN
END
SUBROUTINE MKBX3

C-- THIS SUBROUTINE MAKES MATRICE OF BODY FORCE ACTION
C-- IN X-DIRECTION
C
C IMPLICIT REAL*8(A-H,M-O-Z)
INTEGER W
C
COMMON/BK1/ NOE,NODES,NELE,NNEL,NP,NO,E,ME(400,4),NMOD(400),
1 NDF(400),IPPS(400,4),SECPr(400,4),DEN,TH(400,4),VIS,
2 GELC(400,2),GCOE(300),BCVL(400),BCAC(400,1),
3 A(4,4),AA(400,4),B(400,4),C(4,4),AREA(400),W,COORD,
4 RESULT,UNIT1,UNIT2
COMMON/BK5/MKBX(200,200),MKBX(400,4,4)
COMMON/BK12/THICK3(400,4)
C-- LET ALL INITIAL VALUES ZERO.
DO 100 I=1,NODES
MKBX(I,J)=0.0
100 CONTINUE
DO 200 I=1,NOE
IF(BCAC(I,5).NE.0.0) GO TO 300
IF(BCAC(I,6).NE.0.0) GO TO 300
IF(IUNIT2.NE.2) GO TO 300
IF(BCAC(I,1).NE.0.0) GO TO 300
IF(BCAC(I,2).NE.0.0) GO TO 300
200 CONTINUE
RETURN
300 WRITE(W,700)
C-- CALCULATION OF ELEMENT MATRIX MKBX AND GLOBAL MATRIX MKBX
DO 600 LE=1,NELE
G1=TH(LE,1)+TH(LE,2)+TH(LE,3)
G2=TH(LE,1)**2+TH(LE,2)**2+TH(LE,3)**2
G3=TH(LE,1)*TH(LE,2)*TH(LE,3)
DO 400 J=1,NNEL
IN=NME(LE,I)
JN=NME(LE,J)
THICK3(LE,J)=G1*G2+2.0*G1*(TH(LE,J)**2)+G2*TH(LE,J)+2.0*G3
VISCO=VIS*1440.0
MKBX(LE,I,J)=DEN*THICK3(LE,J)*B(LE,I)/VISCO
MTKBX(I,J)=MTKBX(I,J)+MKBX(LE,I,J)
400 CONTINUE
C-- PRINT THE RESULT
IF(RESULT.NE.'YES') GO TO 600
WRITE(W,500) LE
500 FORMAT('MKBX MATRIX ELEMENT NO.' ,I2)
DO 510 I=1,NNEL
510 WRITE(W,520) (LE,NME(LE,I),NME(LE,J),MKBX(LE,I,J)),J=1,NNEL
520 FORMAT(3X,3('MKBX(',I3,',',I2,'','I2,'','I3,1X,'')=','E10.3,2X))
600 CONTINUE
IF(RESULT.EQ.'XX') GO TO 1000
700 FORMAT(///5X,'I','44('-'),'I','
1/5X,'I' RESULT OF BODY FORCE MATRIX (EX) I ',
2/5X,'I','44('-'),'I'
WRITE(W,500)
800 FORMAT(///31(''-'),
1 /*I' TOTAL GLOBAL MATRIX MKBX I ',
2 /*31(''-'))
C CALL PRINT(NODES,MTKBX,W)
1000 RETURN
END
SUBROUTINE MTKYBY

C --- THIS SUBROUTINE MAKES MATRIX OF BODY FORCE ACTION
C --- IN Y-DIRECTION
C
C IMPLICIT REAL*8(A-H, I-O, Z)
INTEGER W
C
COMMON/BLK1/ NCOE, NODES, NELE, NNEL, NP, NME(400, 4), NNODE(400),
1 NDF(400, 4), IBPS(400, 4), SECPR(400, 4), LEN, TH(400, 4), VIS,
2 GELCO(400, 2), GOCO(400, 4), C(400, 4), AREA(400), W, COOR,
3
RESULT, UNIT1, UNIT2
COMMON/BLK6/MTKYBY(200, 200), MGBK(400, 4, 4)
C
C --- LET ALL INITIAL VALUES ZERO.
DO 100 I=1, NODES
MTKYBY(I, J)=0.0
100 CONTINUE
DO 200 I=1, NODES
IF (BCAC(I, 1).NE.0.0) GOTO 300
IF (BCAC(I, 2).NE.0.0) GOTO 300
IF (UNIT2. NE. 2) GO TO 200
IF (BCAC(I, 1).NE.0.0) GO TO 300
IF (BCAC(I, 2).NE.0.0) GO TO 300
200 CONTINUE
CONTINUE
RETURN
300 WRITE(W, 700)
DO 600 LE=1, NELE
G1=TH(LE, 1)+TH(LE, 2)+TH(LE, 3)
G2=TH(LE, 1)**2+TH(LE, 2)**2+TH(LE, 3)**2
G3=TH(LE, 1)*TH(LE, 2)*TH(LE, 3)
DO 400 J=1, NNEL
IN=NE(LE, I)
JN=NE(LE, J)
THICKJ(LE, J)=G1*G2+2.0*G1*(TH(LE, J)**2)+G2*TH(LE, J)+2.0*G3
VISCO=VIS*1.44*0.6
MGKY(LE, I, J)=LEN*THICKJ(LE, J)*C(LE, I)/VISCO
MTKYBY(IN, JN)=MTKYBY(IN, JN)+MGKY(LE, I, J)
400 CONTINUE
C ---- PRINT THE RESULT
IF (RESULT. NE. 'YES') GO TO 600
WRITE(W, 500) LE
500 FORMAT ('MTKY MATRIX ELEMENT NO.', I2)
WRITE(W, 500) I, J
510 FORMAT (I2, 1X, 3(E10.3, 2X))
600 CONTINUE
IF (RESULT. EQ. 'XX') GO TO 1000
700 FORMAT (I, 1X, 3(E10.3, 2X))
701 FORMAT (I, 1X, 3(E10.3, 2X))
702 FORMAT (I, 1X, 3(E10.3, 2X))
800 FORMAT ('TOTAL GLOBAL MATRIX MTKYBY')
CALL PRINT (NODES, MTKYBY, W)
1000 RETURN
END
SUBROUTINE MKH3

C C——-THIS SUBROUTINE MAKES MATRIX OF SQUEEZE ACTION
C C——-** MKH=(-1)*AREA*DEN*(1.0+DELTA)/12.0 ***
C
C IMPLICIT REAL*3(A-H,M,O-Z)
INTEGER W

C COMMON/BLX1/ NNODE, NODES, NELE, NNEL, NP, HQ, NME(400, 4), NNO(400),
1 NDF(400), IBPS(400, 4), SECPR(400, 4), DEN, TH(400, 4), VIS,
2 GELO(400, 2), CCME(990), BCVL(400), BCAE(400, 19),
3 A(4, 4), AA(400, 4), B(400, 4), C(400, 4), AREA(400), N, COOR,
4 RESULT, UNIT1, UNIT2
COMMON/BLX7/MKH(200, 200), MKH(400, 4, 4)
COMMON/BLX13/DELTA(4, 4)

C C——-LET ALL INITIAL VALUES ZERO.
DO 100 I=1, NODES
DO 100 J=1, NODES
MKH(I, J)=0.0
100 CONTINUE
DO 200 I=1, NODES
200 IF(ECAC(I, 9) .LT. 0.0) GOTO 300
RETURN
300 WRITE(*, 700)

C C———CALCULATION OF ELEMENT MATRIX MKH AND GLOBAL MATRIX MKH
DO 600 LE=1, NELE
DO 400 I=1, NNEL
DO 400 J=1, NNEL
IN=NME(LE, I)
IN=NME(LE, J)
MKH(LE, I, J)=-1.0*AREA(LE)*DEN*(1.0+DELTA(I, J))/12.0
MKH(IN, JN)=MKH(IN, JN)+MKH(LE, I, J)
400 CONTINUE

C C———PRINT THE RESULT
IF(Result .NE. 'YES') GO TO 600
WRITE(*, 500) LE
500 FORMAT('MKH MATRIX ELEMENT NO.', I2)
DO 510 I=1, NNEL
510 WRITE(*, 520) ((LE, NME(LE, I), NME(LE, J), MKH(LE, I, J)), J=1, NNEL)
520 FORMAT(3X, I('MKH(' ,13, ',' ,12, ',' ,13, I3, ')=', I9.3, 2X)
600 CONTINUE
IF(Result .EQ. 'XX') GO TO 1000
700 FORMAT(//'I', 33('-'), 'I',
1 '/' , 'I RESULT OF SQUEEZE MATRIX (MKH) I ',
2 '/' , 'I 33('-'), 'I')
WRITE(*, 800)
800 FORMAT(//'I', 33('-'),
1 ' /', 'I TOTAL GLOBAL MATRIX MKH I ',
2 ' /', 39('-'))

C CALL PRINT (NODES, MKH, 0)
1000 RETURN
END
SUBROUTINE MKVD3
C
C-----THIS SUBROUTINE MAKES MATRIX OF DIFFUSION ACTION
C
C     MKVD=(-1.0)*AREA*DEN*(1.0+DELTA)/12.0
C
IMPLICIT REAL*(A-H,M-O-Z)
INTEGER W
COMMON/BUK1/ NCOD, NODES, NELE, NNEL, NQ, NME(400,4), NMOD(400),
1     NDF(400), IBPS(400,4), SECPR(400,4), DEN, TH(100,3), VIS,
2     CECO(400,2), GOEO(500), BCVL(400), BCAC(400,10),
3     A(4,4), AA(400,4), B(400,4), C(400,4), AREA(400), W, COOR,
4     RESULT, UNIT1, UNIT2
COMMON/BUK13/DELTA(4,4)
C
C-----LET ALL INITIAL VALUES ZERO.
DO 100 I=1,NODES
DO 100 J=1,NODES
MKVD(I,J)=0.0
100 CONTINUE
DO 200 I=1,NODES
200 IF(BCAC(I,10).NE.0.0) GOTO 300
RETURN
300 WRITE(W,700)
C
C-----CALCULATION OF ELEMENT MATRIX MKVD AND GLOBAL MATRIX MKVD
DO 400 LE=1,NELE
DO 400 J=1,NNEL
IN=NME(LE,I)
JN=NME(LE,J)
DELTA(I,J)=0.0
DELTA(I,I)=1.0
MKVD(LE,I,J)=-1.0*AREA(LE)*DEN*(1.0+DELTA(I,J))/12.0
MKVD(IN,JN)=MKVD(IN,JN)+MKVD(LE,I,J)
400 CONTINUE
C
C-----PRINT THE RESULT
IF(RESULT.NE.'YES') GO TO 600
WRITE(W,500) LE
500 FORMAT('MKVD MATRIX ELEMENT NO.',I2)
WRITE(W,510) IN
510 FORMAT('I',NME(LE,I),NME(LE,J),MKVD(LE,I,J)),J=1,NNEL)
520 FORMAT(3X,3('MKVD(',I3,',',',I2,'',',I3,IX,'')=','13',3,2X))
600 CONTINUE
IF(RESULT.EQ.'XX') GO TO 700
700 FORMAT(3X,'I RESULT OF GLOBAL MATRIX (MKVD) I',
1/,5X,'I RESULT OF GLOBAL MATRIX (MKVD) I',
2/,5X,'I RESULT OF GLOBAL MATRIX (MKVD) I',
WRITE(W,800)
800 FORMAT(31(''),1/,31(''))
C
CALL PRINT(NODES,MKVD,W)
1000 RETURN
END
SUBROUTINE MKUX4

C——THIS ROUTINE IS TO CALCULATE THE MATRIX MKUX

IMPLICIT REAL*8(A-H,N,O-Z)

INTEGER W

COMMON/BK1, V1, V2, NODES, NEL, NNEL, NP, QM, QM0(400, 4), NNODE(400)

1 NDF(400), IBPS(400, 4), SECPR(400, 4), DEN, TH(400, 4), VIS,

2 GCLO(400, 2), CDE(800), ECVL(400), BCAC(400, 10),

3 A(4, 4), AA(400, 4), B(400, 4), C(400, 4), AREA(400), N, COOR,

4 RESULT, INT1, INT2

COMMON/BK3/MKUX(200, 200), MKUX(400, 4, 4)

COMMON/BK16/T(4), S(4)

COMMON/BK17/T1IF4(400)

COMMON/BK18/C1(400), C2(400), C3(400), C4(400), C5(400), C6(400), C8(400)

DO 10 I=1, NODES
DO 10 J=1, NODES
MKUX(I, J)=0.0
10 CONTINUE
DO 30 IE=1, NNEL

C———CALCULATION OF MKUX AND MKUX

DO 20 LE=1, NNEL

C———CALCULATION OF MKUX AND MKUX

DO 20 LE=1, NNEL

CALL TERM3(T, S, TH, LE, DEN, C3, C4, C8, MKUX)

DO 20 IE=1, 4
DO 20 J=1, 4

IF(BCAC(I, 1).NE.0.0) GO TO 5
IF(BCAC(I, 2).NE.0.0) GO TO 5
1 CONTINUE
RETURN
5 WRITE(V1, 100)
100 FORMAT(/5X, '*******************************************'

C———CALCULATION OF MKUX AND MKUX

DO 30 LE=1, NNEL

C———CALCULATION OF MKUX AND MKUX

DO 30 LE=1, NNEL

CALL TERM3(T, S, TH, LE, DEN, C3, C4, C8, MKUX)

DO 20 IE=1, 4
DO 20 J=1, 4

IF(RESULT.NE.'YES') GO TO 39
WRITE(W, 110) LE
110 FORMAT(/5X, 'ELEMENT MATRIX MKUX, /, 2X, ELEMENT NO. ', I2)

DO 120 I=1, NNEL
DO 130 J=1, NNEL
WRITE(W, 130) ((LE, NME(LE, I)), NME(LE, J), MKUX(LE, I, J)).', J=1, NNEL)
130 FORMAT(2X, 4('MKUX(', I3, ',', I3, ',', I3, ',', I3, ')=' ', E16.3, 2X))

39 CONTINUE
IF(RESULT.EQ.'XX') GO TO 150
WRITE(W, 140)
140 FORMAT(/5X, 'TOTAL GLOBAL MATRIX MKUX')

CALL PRINT(NODES, MKUX, W)

150 RETURN
END
SUBROUTINE MKUY4

C——THIS ROUTINE IS TO CALCULATE THE MATRICES OF MKUY AND MKUY

IMPLICIT REAL*8(A-H,M-O-Z)
INTEGER W
COMMON/BK4/A,COE, NODES, NELE, NIEL, NP, NP, NM(400, 4), MND(400),
1 NDF(400, 4), IBPS(400, 4), SECPR(400, 4), DEN, TH(400, 4), VIS
2 GENCO(400, 2), GCCE(900), BCVL(400, 4), BCAC(400, 10),
3 A(4, 4), A(400, 4), B(400, 4), C(400, 4), AREA(400), W, COOR.
4 RESULT, UNIT1, UNIT2
COMMON/BK4/MKUY(200, 200), MKUY(400, 4, 4)
COMMON/BK6'/T(4),5(4)
COMMON/BK7/THICK4(400)
COMMON/BK18/C1(400), C2(400), C3(400), C4(400), CS(400), CS(400)

DO 10 I=1,NODES
DO 10 J=1,NODES
MKUY(I,J)=0.0
10 CONTINUE
IF(IUNIT2.EQ.2) GO TO 11
IF(BCAC(I,3).NE.0.0) GO TO 5
IF(BCAC(I,4).NE.0.0) GO TO 5
11 IF(BCAC(I,1).NE.0.0) GO TO 5
IF(BCAC(I,2).NE.0.0) GO TO 5
1 CONTINUE
RETURN
5 WRITE(W,100)
100 FORMAT(///,5X,'**************************************************************************',
1 //,5X, '**RESULT OF SHEAR ACTION MATRIX (MKUY) **',
2 //,5X,'**************************************************************************')</n
C——CALCULATION OF MKUY AND MKUY
DO 30 LE=1,NELE
CALL TERMU3(T,S,TH,LE,DEN,C1,C2,C6,MKUY)
DO 20 I=1,4
DO 20 J=1,4
IN=NAME(LE,I)
JN=NAME(LE,J)
MKUY(IN,JN)=MKUY(IN,JN)+MKUY(LE,I,J)
20 CONTINUE
IF(RESULT.NE.'YES') GO TO 39
WRITE(W,110) LE
110 FORMAT(///,'ELEMENT MATRIX MKUY,/,2X,ELEMENT NO.',I2)
DO 120 I=1,NIEL
120 WRITE(W,130) ( (LE,NAME(LE,I),NAME(LE,J),MKUY(LE,I,J)),J=1,NIEL)
130 FORMAT(9X,'(MKUY('''i3'' ',',',i3,1X,')=' ',E13.3,2X))
30 CONTINUE
WRITE(W,140)
140 FORMAT(///,'TOTAL GLOBAL MATRIX MKUY')

C——CALL PRINT(NODES,MKUY,W)

RETURN
END
SUBROUTINE MTKBX4
C——— THIS ROUTINE IS TO CALCULATE THE MATRIX OF MTKBX
IMPLICIT REAL*8(A-H,M-O-Z)
INTEGER N
COMMON/BLK1/NECE,NODES,NELE,HNEL,IP,KD,ME(400,4),NNOD(400),
  1 NDF(400),IBPS(403,4),SECPR(409,4),DEN,TH(400,4),VIS,
  2 GELCO(409,2),GCOE(300),BCVL(400),BCAC(400,10),
  3 A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COORD,
  4 RESULT,UNIT1,UNIT2
COMMON/BLK5/MKBX(200,200),MTKBX(400,4,4)
COMMON/BLK16/T(4),S(4)
COMMON/BLK17/THICK4(400)
COMMON/BLK18/C1(400),C2(400),C3(400),C4(400),C5(400),C6(400)
C
DO 10 I=1,NODES
DO 10 J=1,NODES
MTKBX(I,J)=0.0
10 CONTINUE
DO 1 I=1,NODES
IF(BCAC(I,5).NE.0.0) GO TO 5
IF(BCAC(I,6).NE.0.0) GO TO 5
IF(UNIT2.NE.2) GO TO 1
IF(BCAC(I,1).NE.0.0) GO TO 5
IF(BCAC(I,2).NE.0.0) GO TO 5
1 CONTINUE
RETURN
5 WRITE(W,100) LE
100 FORMAT('五/ ,130 scanf(13,13,2X,RESULT) ..',I2)
C——— CALCULATION OF MKBX AND MTKBX
DO 20 LE=1,NELE
CALL TEMB3(T,S,TH,LE,C3,C4,C8,DEN,VIS,MKBX)
DO 20 I=1,4
DO 20 J=1,4
IN=MME(LE,I)
JN=MME(LE,J)
MTKBX(IN,JN)=MTKBX(IN,JN)+MKBX(LE,I,J)
20 CONTINUE
IF(RESULT.NE.'YES') GO TO 30
WRITE(W,110)
110 FORMAT('YES')
WRITE(W,110) I2
DO 120 I=1,NNEL
120 WRITE(W,130) ((LE,NME(LE,I),NME(LE,J),MKBX(LE,I,J)),J=1,NNEL)
130 FORMAT(3X,4,'MTKBX(',I3,',',I2,',',I2,',',I2,',',I2,')=',E19.3,2X))
30 CONTINUE
WRITE(W,140)
140 FORMAT('TOTAL GLOBAL MATRIX MTKBX')
C
CALL PRINT(NODES,MTKBX,W)
C
RETURN
END
SUBROUTINE MTKBY4

C---THIS ROUTINE IS TO CALCULATE MKBY AND MTXY
IMPLICIT REAL*8(A-H,M,O-Z)
INTEGER W
COMMON/BLK1/NODES,NELE,NNEL,NEQ,NAME(400,4),INDEX(400),
1 NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
2 GELC(400,2),GCOE(300),BCVL(400),BCAC(400,1),
3 A(4,4),M(400,4),N(400,4),AREA(400),W,COOR,
4 RESULT,UNIT1,UNIT2
COMMON/BLK6/MKBY(200,200),MKBY(400,4,4)
COMMON/BLK16/T(4),S(4)
COMMON/BLK17/THICK(400)
COMMON/BLK18/C1(400),C2(400),C3(400),C4(400),C5(400),C6(400),C7(400)

DO 10 I=1,NODES
DO 10 J=1,NODES
MKBY(I,J)=0.0
10 CONTINUE
DO 1 I=1,NODES
IF(BCAC(I,7).NE.0.0) GO TO 5
IF(BCAC(I,8).NE.0.0) GO TO 5
IF(UNIT2.NE.2) GO TO 1
IF(BCAC(I,1).NE.0.0) GO TO 5
IF(BCAC(I,2).NE.0.0) GO TO 5
1 CONTINUE
RETURN
5 WRITE(W,100)
100 FORMAT(/,5X,'****************************************************',1
1 /,5X,'** RESULT OF BODY FORCE MATRICE MKBY **',2
2 /,5X,'****************************************************')
DO 30 LE=1,NELE
CALL TERTMB(S,T,LE,C1,C6,C2,DEN,VIS,MKBY)
DO 20 I=1,4
DO 20 J=1,4
IN=NAME(LE,I)
JN=NAME(LE,J)
MKBY(IN,JN)=MKBY(IN,JN)+MKBY(LE,I,J)
20 CONTINUE
IFRESULT.NE.'YES') GO TO 30
WRITE(W,110) LE
110 FORMAT('ELEMENT MATRIX MKBY ELEMENT NO. ',I2)
DO 120 I=1,NEL
120 WRITE(W,130) ((LE,NAME(LE,I),NAME(LE,J),MKBY(LE,I,J)),J=1,NEL)
130 FORMAT(3X,4('MKBY(',I3,','I2,',',I3,1X,')=',E9.3,2X))
30 CONTINUE
WRITE(W,140)
140 FORMAT('TOTAL GLOBAL MATRIX MKBY')
RETURN
END

C
SUBROUTINE MTHK4

C---THIS ROUTINE IS TO CALCULATE MKH AND MTHK OF QUADRILATERAL

IMPLICIT REAL*(A-H,M-O-Z)
INTEGER W

COMMON/BLK1/IXOE, NODES, NELE, NNEL, NP, NQ, NME(400, 4), NND(400),
  1 NDE(400), IBPS(400, 4), SECPR(400, 4), DEN, TH(400, 4), VIS,
  2 GEE(400, 2), OCCE(800), BC2L(400), BC2C(400, 4),
  3 A(4, 4), FA(400, 4), B(400, 4), C(400, 4), AREA(400), V, COOR,
  4 RESULT, UN1, UN12

COMMON/BLK7/MI'H(201, 200), MKH(400, 4, 4)
COMMON/BLKn/T(4), S(4)

C

DO 10 I=1, NODES
DO 10 J=1, NODES
MKH(I, J)=0.0
10 CONTINUE
DO 19 I=1, NODES
IF(EAC(I, 9).NE.0.0) GO TO 5
1 CONTINUE
RETURN
5 WRITE(W, 100)
100 FORMAT(///, 5X, 44('**'),
  1 /, 5X, '*** RESULT OF SQUEEZE ACTION MATRIX MTHK***',
  2 /, 5X, 44('**'))

C---CALCULATION OF MKH AND MTHK

DO 30 LE=1, NELE
DO 29 I=1, NNEL
DO 29 J=1, NNEL
IN=NME(LE, I)
JN=NME(LE, J)
T1=T(I)*T(J)
T2=T(I)*T(J)
S1=S(I)*S(J)
S2=S(I)*S(J)
C21=C1(LE)*C8(LE)-C3(LE)*C6(LE)
C22=C2(LE)*C3(LE)-C1(LE)*C4(LE)
C23=C2(LE)*C8(LE)-C4(LE)*C6(LE)
C24=0.9*(T1*C22/3.0+C22)
C25=0.9*(T1*C21+T2*C23)/3.0+2.0*C23
H1=(-1.0)*DEN/B, 9
H2=(C24*S1+C25*S2)/3.0*C25
MKH(IN, JN)=H1*H2
MARK(IN, JN)=MKH(IN, JN)*MKH(LE, I, J)
WRITE(W, 14) LE, I, J, T1, T2, S1, S2
14 FORMAT(///, 'LE, I, J, T1, T2, S1, S2', //, 'I3, 4X, 2I4, 5X, 4(F5.2, 2X))
WRITE(W, 11) C21, C22, C23, C24, C25
11 FORMAT('C21, C22, C23, C24, C25', '/, 5(E19.3, 3X))
WRITE(W, 12) H1, H2
12 FORMAT('H1, H2', '/, 2(E19.3, 5X))
20 CONTINUE
IF(Result, NE. 'YES') GO TO 30
WRITE(W, 110) LE
110 FORMAT('ELEMENT MATRIX MKH ELEMENT NO.', '12)
DO 120 I=1, NNEL
120 WRITE(W, 130) (LE, NME(LE, I), NME(LE, J), MKH(LE, I, J)), J=1, NNEL
30 CONTINUE
WRITE(W, 140)
140 FORMAT(///, 'TOTAL GLOBAL MATRIX MTHK')

C

CALL PRINT(NODES, MKH, W)

C

RETURN
END
SUBROUTINE MI'KVD4

C---THIS ROUTINE IS TO CALCULATE MKVD AND MTKVD

INTEGER W

COMMON/BLK1/ND1, NODES, NELE, NEL, NP, NO, NME(400, 4), NDD(400),
1 NDF(400), IBES(400, 4), SECFR(400, 4), DEN, TH(400, 4), VIS,
2 CELCO(400, 2), GCCE(200), PCVL(400), BCAC(400, 10),
3 A(4, 4), AA(400, 4), B(400, 4), C(400, 4), AREA(400), W, COOR,
4 RESULT, UNIT1, UNIT2

COMMON/BLK8/MKVD(200, 200), MKVD(400, 4, 4)

COMMON/BLK16/T(4), S(4)

COMMON/BLK18/C1(400), C2(400), C3(400), C4(400), C6(400), C8(400)

DO 1 1 = 1, NODES
DO 1 J = 1, NODES
1 MKVD(I, J) = 1.0

1 CONTINUE

RETURN

5 WRITE(W, 100)

100 FORMAT(//'10 I = 1, NODES'10 J = 1, NODES' MKVD(I, J) = 1.0')

C-----CALCULATION OF MKVD AND MTKVD

DO 30 LE = 1, NELE
DO 20 I = 1, NNEL
DO 20 J = 1, NNEL
IN = NME(LE, I)
JN = NME(LE, J)
T1 = T(I) + T(J)
T2 = T(I) * T(J)
S1 = S(I) + S(J)
S2 = S(I) * S(J)
C21 = C1(LE) * C2(LE) * C3(LE) * C6(LE)
C22 = C2(LE) * C3(LE) * C4(LE)
C24 = 2.0 * C21 * (S2 / 3.0 + 1.0)
C25 = 2.0 / 3.0 * (S1 * C22 + S2 * C23) + 2.0 * C23
VD1 = (-1.0) * DEN / 8.0
VD2 = (T1 * C24) / 3.0 * C25 + C25
MKVD(IN, JN) = VD1 * VD2
MTKVD(IN, JN) = MKVD(LE, I, J)

30 CONTINUE

IF (RESULT.NE. 'YES') GO TO 30

WRITE(4, 110) LE

110 FORMAT('MTKVD MATRIX ELEMENT ID.' I2)

DO 129 I = 1, NNEL
129 WRITE(4, 130) ((LE, NME(LE, I), NME(LE, J), MKVD(LE, I, J)), J = 1, NNEL)

130 FORMAT(3X, 4('MTKVD(', I3, ', ', I2, ', ', I3, ', ', I3, 1X, ')=' I10.3, 2X))

30 CONTINUE

WRITE(4, 140)

140 FORMAT(//'TOTAL GLOBAL MATRIX MTKVD')

C

CALL PRINT(NODES, MKVD, W)

C

RETURN

END
SUBROUTINE PRINT(NODES,MTK,W)

C--- THIS SUBROUTINE IS TO PRINT OVERALL MATRIX

IMPLICIT REAL*3(A-H,M,O-Z)
INTEGER W

DIMENSION JJ(200,5),MTK(200,200)

C---DEFINITION OF WRITING FORM.
ICASE=0
DO 500 IN=1,NODES
IF(5*IN.GE.NODES) GO TO 250
290 CONTINUE
250 ICASE=IN
IF(ICASE.EQ.1) GO TO 550
IN=ICASE-1
DO 300 J=1,IN
DO 300 K=1,5
JJ(J,K)=S*(J-1)+K
300 CONTINUE
DO 500 J=1,IN
WRITE(W,350) (JJ(J,K),K=1,5)
350 FORMAT(1H1,14X,5(I3,12X))
DO 400 I=1,NODES
400 FORMAT(W,450) I, (MTK(I,JJ(J,K)),K=1,5)
450 FORMAT(1H1,3X,I3,3X,5(E10.3,4X))
500 CONTINUE
550 IK=NODES-5*(ICASE-1)
DO 560 K=1,5
JJ(ICASE,K)=5*(ICASE-1)+K
560 CONTINUE
WRITE(W,600) (JJ(ICASE,K),K=1,IK)
600 FORMAT(W,650) I, (MTK(I,JJ(ICASE,K)),K=1,IK)
650 FORMAT(W,700) I, (MTK(I,JJ(ICASE,K)),K=1,IK)
700 FORMAT(W,850) I, (MTK(I,JJ(ICASE,K)),K=1,IK)
RETURN
END
SUBROUTINE SOLVE

THIS SUBROUTINE FIND THE UNKNOWN VALUES OF FLOW AND PRESSURE.

IMPLICIT REAL*8(A-H,I-M,O-Z)
INTGTER W

C--NOMENCLATURE--
NO. OF KNOWN P VALUE——NP
NO. OF KNOWN Q VALUE——NQ
NODE NO. OF KNOWN P——NDP
NODE NO. OF KNOWN Q——NDQ
MEAN SURFACE VELOCITY——UX, UY
MEAN BODY FORCE——BX, BY

COMMON/BLK1/ NCOE, NODES, NELE, NNEL, NP, NQ , NNE(400, 4), NNOD(400),
1 NDP(400), TRBS(400, 4), SECPR(400, 4), DEN, TH(400, 4), VIS,
2 GEICO(400, 2), COE(400), BCVL(400), BCAC(400, 10),
3 A(4, 4), AA(400, 4), B(400, 4), C(400, 4), AREA(400), H, COOR,
4 RESULT, UNIT1, UNIT2
COMMON/BLK2/ MKP(200, 200), MK2(200, 200), MKP1(200, 200), MP(400, 4, 4)
COMMON/BLK3/ MAX(400, 4, 4)
COMMON/BLK4/ MKUY(200, 200), MKY(400, 4)
COMMON/BLK5/ MKUX(200, 200), MKX(400, 4, 4)
COMMON/BLK6/ MUY(200, 200), MUY(400, 4)
COMMON/BLK7/ MXH(200, 200), MXH(400, 4, 4)
COMMON/BLK8/ MXVD(200, 200), MXC(400, 4, 4)
COMMON/BLK9/ Q(400), P(400), QM, QM, MQ, QEI(400, 4)
COMMON/BLK14/ CFY(400), CFY(400)
COMMON/BLK20/ UX(400), UY(400), EX(400), BY(400), H(400), JD(400)
COMMON/BLK21/ EX1, EX2, FY1, FY2, RF1, RF2, TF1, TF2, TQ1, TQ2
COMMON/BLK22/ TFX1, TFY1, TFY2, TRF1, TRF1, TRF2, TORQ1, TORQ2
COMMON/BLK23/ NDQ(400), NDP(400)
DIMENSION Q01(400), Q02(400), Q03(400), Q04(400),
1 Q05(400), Q06(400), Q07(400)
DIMENSION Q12(400), Q13(400), Q14(400), Q15(400),
1 Q16(400), Q17(400)
DIMENSION Q21(400), Q22(400), Q23(400), Q24(400),
1 Q25(400), Q26(400)
DIMENSION Q27(400), Q1(400), Q2(400), P1(400)

IP=3

C--CALCULATION OF FLOW ACTION VALUES--
IF(UNIT2.EQ.2) GO TO 20
DO 10 I=1, NODES
10 CONTINUE
GO TO 35

20 DO 30 I=1, NODES
RAD=GEICO(I)
ANG=C0E(I)+NOD
IF(C0E, EQ., 'RAD') GO TO 21
ANG=3.141592654/180.0*ANG
21 SN=DOSIN(ANG)
CS=DCOS(ANG)
UX1=RAD*BCAC(I, 1)*SN
UX2=RAD*BCAC(I, 2)*SH
UX2=RAD*BCAC(I,2)*SN
UY1=RAD*BCAC(I,1)*CS
UY2=RAD*BCAC(I,2)*CS
UX(I)__((UX1-UX2)/2.0
UY(I)=(-1.0)*(UY1-UY2)/2.0

30 CONTINUE
35 DO 49 I=1,NODES
H(I)=BCAC(I,9)
VD(I)=BCAC(I,10)
40 CONTINUE

C-----------------------------------------------C
C       CALCULATION OF BODY FORCES              C
C-----------------------------------------------C

DO 41 IE=1,NELE
DO 41 I=1,IREL
IN=INE(IE,I)
BX(IN)=(BCAC(IN,5)+BCAC(IN,6))/2.0
BY(IN)=(BCAC(IN,7)+BCAC(IN,8))/2.0

41 CONTINUE

C-----------------------------------------------C
C       CALL CFORCE                               C
C-----------------------------------------------C

DO 50 I=1,NODES
BX(I)=BX(I)+CFX(I)
BY(I)=BY(I)+CFY(I)
50 CONTINUE

C-----------------------------------------------C
C       PRINT ALL THE FLOW ACTION VALUES CALCULATED. C
C-----------------------------------------------C

WRITE(W,51)
51 FORMAT('I',40('-')','I','I',
1 //I FLOW ACTION VALUES (MEAN VALUES) I',
2 //I INCLUDING CENTRIFUGAL FORCES. I',
3 //I',40('-')','I',
4 //5X, 'SHEAR ACTION (X-DIRECTION) ...........UX',
5 //5X, 'SHEAR ACTION (Y-DIRECTION) ...........UY',
6 //5X, 'CENTRIFUGAL FORCE (X-DIRECTION) ......CFX',
7 //5X, 'CENTRIFUGAL FORCE (Y-DIRECTION) ......CFY',
8 //5X, 'TOTAL BODY FORCE (X-DIRECTION) ......BX (=8X+CFX))',
9 //5X, 'TOTAL BODY FORCE (Y-DIRECTION) ......BY (=8Y+CFY))',
1 //5X, 'SQUEEZE ACTION (Z-DIRECTION) .........H',
2 //5X, 'DIFFUSION ACTION (Z-DIRECTION) .........VD',
1, //I, 'NODE',6X,'UX',10X,'UY',9X,'CFX',9X,'CFY',
2 //I, 'BX',10X,'BY',1IX,'H',10X,'VD')
DO 52 IE=1,NODES
52 WRITE(W,53) I,UX(I),UY(I),CFX(I),CFY(I),BX(I),BY(I),H(I),VD(I)
53 FORMAT(2X,I3,6(2X,E10.3))
C-----------------------------------------------C
C       Q(I)=MTKP(I,J)*P(J) I,J=1,NODES          C
C-----------------------------------------------C

WRITE(W,69) NODES, NP, NQ
69 FORMAT(1HL,1/16X,** SOLVE THE EQUATIONS **',
1//,16X,'NODES=',I4,10X,'NP=',I4,10X,'NQ=',I4,
2//,16X,'*** INCONSISTENT ***STOP***)
STOP
70 CONTINUE
CLASSIFICATION OF CONSTRAINED NODES

J=0
K=0
DO 100 I=1,NODES
Q(I)=0.0
P(I)=0.0
IF(NDF(I).EQ.1) GO TO 110
IF(NDF(I).EQ.2) GO TO 120
110 J=J+1
Q(I)=BCVL(I)
GO TO 100
120 K=K+1
P(I)=BCVL(I)
100 CONTINUE
DO 250 I=1,NODES
LET ALL INITIAL VALUES ZERO.
Q01(I)=0.0
Q02(I)=0.0
Q03(I)=0.0
Q04(I)=0.0
Q05(I)=0.0
Q06(I)=0.0
Q07(I)=0.0
250 CONTINUE
IF(NP.NE.0) GO TO 500
CASE-I IF ALL FLOW VALUES ARE KNOWN
DO 350 I=1,NODES
DO 390 J=1,NODES
Q01(I)=Q01(I)+MTKP(I,J)*P(J)
Q02(I)=Q02(I)+MTKUX(I,J)*UX(J)
Q03(I)=Q03(I)+MTKUY(I,J)*UY(J)
Q04(I)=Q04(I)+MTKEX(I,J)*EX(J)
Q05(I)=Q05(I)+MTKBY(I,J)*BY(J)
Q06(I)=Q06(I)+MTKHX(I,J)*H(J)
Q07(I)=Q07(I)+MTKVD(I,J)*VD(J)
300 CONTINUE
Q(I)=Q(I)+Q01(I)-(Q02(I)+Q03(I)+Q04(I)+Q05(I)+Q06(I)+Q07(I))
350 CONTINUE
CALCULATE THE INVERSED MATRIX OF MTKP.
CALL MB02A(MTKP,MTK,NODES,IP)
DO 400 I=1,NODES
DO 400 J=1,NODES
THE MATRIX MTKP HAS BEEN INVERSED.
P(I)=P(I)+MTKP(I,J)*Q(J)
400 CONTINUE
GO TO 955
IF(NQ.NE.4) GO TO 720
CASE-2  IF ALL PRESSURE VALUES ARE KNOWN.

DO 650 I=1,NOD
DO 660 J=1,NOD

CASE 1

600 CONTINUE
Q(I)=Q(I)+QQ1(I)+(QQ2(I)+QQ3(I)+QQ4(I)+QQ5(I)+QQ6(I)+QQ7(I))

CASE 3  FORM STANDARD PROBLEM

C1  DIVIDE ALL Q AND P INTO KNOWN Q1,P2 AND UNKNOWN Q2,P1
C  1  FIND UNKNOWN VALUE P1(I)
C Q1(KNOWN)=MTKP11*P1(UNKNOWN)+MTKP12*P2(KNOWN)

CASE 2

C2(I)=QQ2(I)+QQ3(I)+QQ4(I)+QQ5(I)+QQ6(I)+QQ7(I)

C CASE-3  IF ALL PRESSURE VALUES ARE KNOWN.

Sll DO 830 I=1,NQ

C1  LET ALL INITIAL VALUES ZERO.
C Q01(I)=0.0
C Q02(I)=0.0
C Q03(I)=0.0
C Q04(I)=0.0
C Q05(I)=0.0
C Q06(I)=0.0
C Q07(I)=0.0

Sll DO 820 J=1,NQ
MTKP1(I,J)=MTKP(NDQ(I),NDQ(J))

CASE 3

Sll DO 819 I=1,NP

C3(I)=Q03(I)+Q04(I)+Q05(I)+Q06(I)+Q07(I)

CASE 2

C2(I)=Q02(I)+Q03(I)+Q04(I)+Q05(I)+Q06(I)+Q07(I)

Sll DO 819 I=1,NP

C4(I)=Q02(I)+Q03(I)+Q04(I)+Q05(I)+Q06(I)+Q07(I)

CASE 2

C2(I)=Q02(I)+Q03(I)+Q04(I)+Q05(I)+Q06(I)+Q07(I)

01(I)=Q1(NDQ(I))-(Q01(I)+Q02(I))

CASE 3  IF ALL PRESSURE VALUES ARE KNOWN.

C CASE 3  IF ALL PRESSURE VALUES ARE KNOWN.

Sll DO 830 I=1,NQ

C1  LET ALL INITIAL VALUES ZERO.
C Q01(I)=0.0
C Q02(I)=0.0
C Q03(I)=0.0
C Q04(I)=0.0
C Q05(I)=0.0
C Q06(I)=0.0
C Q07(I)=0.0

Sll DO 820 J=1,NQ
MTKP1(I,J)=MTKP(NDQ(I),NDQ(J))

CASE 3

Sll DO 819 I=1,NP

C3(I)=Q03(I)+Q04(I)+Q05(I)+Q06(I)+Q07(I)

CASE 2

C2(I)=Q02(I)+Q03(I)+Q04(I)+Q05(I)+Q06(I)+Q07(I)

Sll DO 819 I=1,NP

C4(I)=Q02(I)+Q03(I)+Q04(I)+Q05(I)+Q06(I)+Q07(I)

CASE 2

C2(I)=Q02(I)+Q03(I)+Q04(I)+Q05(I)+Q06(I)+Q07(I)

01(I)=Q1(NDQ(I))-(Q01(I)+Q02(I))
C2—ALL FLOW VALUES Q1 HAVE BEEN CALCULATED.
C——FIND UNKNOWN PRESSURE PI.
C——MAKE INV. MATRIX OF MTKP1(NDQ(I),NDQ(I))
   IF(UQ-1)950,840,950
840 MTKP1(I,1)=1.0/MTKP1(I,1)
   GO TO 850
C
850 CALL MB82A(MTKP1,MTK,NDQ,IP)
   IF(RESULT.NE.'YES') GO TO 860
WRITE(W,851)
851 FORMAT(//'2X,37('C'),
   1 '2X,'C',35X,'C',)
   2 '2X,'C' RESULT OF INVERTED MATRIX MTKP1 C',
   3 '2X,'C',35X,'C',
   4 '2X,37('C'),
   5 '2X,NDQ(I')
WRITE(W,852) (NDQ(I),I=1,NDQ)
852 FORMAT(5X,NQ)
C
CALL PRINT(NDQ,MTKP1,IP)
C—THE MATRIX MTKP1 HAS BEEN INVERTED.
860 CONTINUE
   DO 900 I=1,NDQ
   P1(I)=0.0
   DO 900 J=1,NDQ
      P1(I)=P1(I)+MTKP1(I,J)*Q1(J)
   900 CONTINUE
   DO 901 I=1,NDQ
      P(NDQ(I))=P1(I)
   901 CONTINUE
C
C3—ALL PRESSURE VALUES ARE KNOWN NOW.
C——THEN FIND UNKNOWN FLOW VALUES Q2.
C
   DO 950 I=1,NDP
   Q2(I)=0.0
   Q21(I)=0.0
   Q22(I)=0.0
   Q23(I)=0.0
   Q24(I)=0.0
   Q25(I)=0.0
   Q26(I)=0.0
   Q27(I)=0.0
C
   C3-1 LET ALL INITIAL VALUES ZERO.
   Q2(I)=0.0
   Q21(I)=0.0
   Q22(I)=0.0
   Q23(I)=0.0
   Q24(I)=0.0
   Q25(I)=0.0
   Q26(I)=0.0
   Q27(I)=0.0
C
   C3-2 CALCULATION OF Q2
   C——Q2(I)=Q21*P1+Q22*P2
   DO 910 J=1,NDP
      910 Q21(I)=Q21(I)+MTKP(NDP(I),J)*P1(J)
      Q22(I)=Q22(I)+MTKUX(NDP(I),J)*UX(J)
      Q23(I)=Q23(I)+MTKUY(NDP(I),J)*UY(J)
      Q24(I)=Q24(I)+MTKBX(NDP(I),J)*BX(J)
      Q25(I)=Q25(I)+MTKBY(NDP(I),J)*BY(J)
      Q26(I)=Q26(I)+MTKVD(NDP(I),J)*VD(J)
      Q27(I)=Q27(I)+MTKVD(NDP(I),J)*VD(J)
   CONTINUE
   Q2(I)=Q21(I)+Q22(I)+Q23(I)+Q24(I)+Q25(I)+Q26(I)+Q27(I)
   Q(NDP(I))=Q2(I)
950 CONTINUE
C——ALL FLOW VALUES HAVE BEEN CALCULATED
C  CALCULATION OF INWARD and OUTWARD FLOW (Qin, Qout)  C

955  QIN=0.0
    QOUT=0.0
    DO 970 I=1,NODES
      IF(Q(I).LT.0.0) GOTO 960
    QOUT=QOUT+Q(I)
    GO TO 970
960  QIN=QIN+Q(I)
    970  CONTINUE

955  CONTINUE

C5——CALCULATION OF LOAD CAPACITY (W)

955  W=0.0
    DO 990 LE=1,NELE
      PP=0.0
      DO 980 I=1,INEL
        PP=PP+P(NHE(LE,I))/INEL
      980  CONTINUE
      W=W+PP*AREA(LE)
    990  CONTINUE

C  CALCULATION OF FRICTION FORCE AND TORQUE.  C

DO 1000 I=1,NODES
  DO 1000 J=1,4
    IF(SCAC(I,J).NE.0.0) GO TO 1001
  1000  CONTINUE
  GO TO 1000
1001  WRITE(W,1002)
1002  FORMAT(///,**,60('**','**',
  1///**  'CALCULATION OF FRICTION FORCE AND TORQUE OF EACH ELEMENT **,**
  2 /**,'**','**','**')
  WRITE(W,1003)
1003  FORMAT(///,15X,'LOWER SURFACE',35X,'UPPER SURFACE',
  1//,'ELEMENT',2X,'X-Direction',2X,'Y-Direction',3X,
  2'RESULTANT',5X,'F.TORQUE',4X,'X-Direction',2X,'Y-Direction',
  3 3X,'RESULTANT',4X,'F.TORQUE',//,)
    TXF1=0.0
    TFY1=0.0
    TRF1=0.0
    TXF2=0.0
    TFY2=0.0
    TRF2=0.0
    TFJG=0.0
    TFY1=0.0
    TRF1=0.0
    TXF2=0.0
    TRF2=0.0
    IF(CCOR.NE.'PO') GO TO 1004
    1004  WRITE(W,1005) LE,FX1,FY1,FX2,FY2,TRF1,TRF2
1005  FORMAT(3X,13,3X,4(E14.3,3X),3X,4(E14.3,3X))
    1009  CONTINUE

C  CALL PFORCE(LE)

C  TOTAL FRICTION FORCE CALCULATION
  TEX1=TEX1+FX1
  TEX2=TEX2+FX2
  TFY1=TFY1+FY1
  TFY2=TFY2+FY2
  TRF1=TRF1+RF1
  TRF2=TRF2+RF2
  IF(CORR.NE.'PO') GO TO 1004

C  TOTAL TORQUE CALCULATION
  TORQ1=TORQ1+TQ1
  TORQ2=TORQ2+TQ2

1004  WRITE(W,1005) LE,FX1,FY1,FX2,FY2,TRF1,TRF2
CALCULATION OF ELEMENT FLOWS

IF (RESULT.EQ.'XX') GO TO 1096

1103 DO 2001 LE=1,NELE

1104 DO 2003 I=1,NEI

1105 INITIAL VALUES ARE ZERO

1106 EP=0.0

1107 EUX=0.0

1108 EUY=0.0

1109 EYL=0.0

1110 EV=0.0

1111 DO 2001 J=1,NEJ

1112 JN=QEJ(LE,J)

1113 EP=EP+EPJ(LE,I,J)*PJ(JN)

1114 EUX=EU+EKUX(LE,I,J)*UX(JN)

1115 EUY=EU+EKUY(LE,I,J)*UY(JN)

1116 BEX=BEX+EBEX(LE,I,J)*BX(JN)

1117 BYE=BY+EBYE(LE,I,J)*BY(JN)

1118 E=EH+EHE(LE,I,J)*H(JN)

1119 EV=E+EKV(LE,I,J)*VD(JN)

1120 DO 2001 J=1,NEJ

2001 CONTINUE

2002 CONTINUE

2010 CONTINUE

2011 CONTINUE

IF (RESULT.EQ.'XY') GO TO 1096

WRITE(W, 2010) LE, (NEI(LE,I),I=1,3), (QEI(LE,I),I=1,3)

2011 FORMAT(/,'ELEMENT NO.',I3,3X,'NODE NO.',I3,3X,'NO.',I3),

1 /19X,'ELEMENT FLOW',3X,E12.5)

GO TO 2015

2012 WRITE(W, 2013) LE,(NEJ(LE,J),J=1,4), (QEL(LE,J),J=1,4)

2013 FORMAT(/,'ELEMENT NO.',I3,5X,'NODE NO.',I3,5X,'NO.',I3),

1 /5X,'ELEMENT FLOW',4X,E12.5)

2015 CONTINUE

1096 RETURN

END
SUBROUTINE MB02A(A, C, M, IP)

IMPLICIT REAL*8(A-H, O-Z)
DIMENSION A(200, 200), C(200, 200)
DIMENSION D(400), IND(400), JND(400)

MAX=3.0
DO 2 I=1, M
IND(I)=I
IND(I)=I
DO 2 J=1, M
IF(DABS(A(I, J))=MAX)2, 2, 3
2 AMAX=DABS(A(I, J))
I4=I
J4=J
2 CONTINUE
D(1)=1.0
M4=M-1
DO 11 J4=1, M4
STO=IND(J4)
IND(J4)=IND(I4)
IND(I4)=STO
DO 5 K=1, M
STO=A(I4, K)
A(K, J4)=A(K, J)
A(J4, K)=STO
5 CONTINUE
6 IF(J4=J)8, 8, 9
9 D(1)=D(1)
STO=JND(J)
JND(J)=JND(J4)
JND(J4)=STO
DO 12 K=1, M
STO=A(K, J4)
A(K, J4)=A(K, J)
A(J, J4)=STO
12 CONTINUE
8 AMAX=3.0
J1=J+1
DO 11 I4=1, J1
STO=A(I4, J4)/A(J, J)
DO 10 K=1, M
A(I4, K)=A(I4, K)+STO*A(J, K)
10 IF(K<J)10, 10, 15
15 IF(DABS(A(I4, K))=AMAX)19, 19, 17
17 AMAX=DABS(A(I4, K))
I4=I
J4=K
10 CONTINUE
19 CONTINUE
A(I,J)=STO
11 CONTINUE
DO 18 I=1,M
D(I+1)=D(I)*A(I,I)
18 CONTINUE
DET=DA(M)*A(M,M)
PROD=1.0
IF(IP-2)>99,19,16
16 PROD=1.0/DET
19 DO 20 J=1,M
DO 21 K=1,J
C(K,J)=0.0
21 CONTINUE
DO 22 K=J,M
C(K,J)=A(K,J)
22 CONTINUE
C(J,J)=1.0
PROD=PROD1
DO 30 I=1,M
I2=M-I
II=I2+1
STO=C(II,J)
C(II,J)=D(II)*STO*PROD
IF(DABS(STO)-DABS(A(II,II)))>25,25,26
25 STO=STO/A(II,II)
DO 27 K=1,II
C(K,J)=C(K,J)-STO*A(K,II)
27 CONTINUE
PROD=PROD*A(II,II)
GO TO 30
26 STO=A(II,II)/STO
DO 28 K=1,II
C(K,J)=A(K,II)-STO*C(K,J)
28 CONTINUE
PROD=PROD*STO
39 CONTINUE
C(1,J)=D(1)*C(1,J)*PROD
39 CONTINUE
DO 49 I=1,M
K=IND(I)
DO 49 J=1,M
L=IND(J)
A(L,K)=C(J,I)
49 CONTINUE
99 CONTINUE
RETURN
END
SUBROUTINE OUTPUT

IMPLICIT REAL*3(A-H,M-O-Z)
INTEGER W

COMMON/BLK1/ NCOE, NODES, JNELE, INEL, UP, NO, NME(400,4), NNOD(400),
1 NDF(400), IBPS(400,4), SECPR(400,4), DEH, TH(400,4), VIS,
2 GELCO(400,2), GCOE(900), BCLL(400), BCAC(400,10),
3 A(4,4), NA(400,4), B(400,4), C(400,4), AREA(400), W, COOR,
4 RESULT, UNIT1, UNIT2
COMMON/BLK9/Q(400), P(400), QIN, QOUT, WEL, CEL(400,4)
COMMON/BLK10/TEST
COMMON/BLK14/ CFX(400), CFY(400)
COMMON/BLK22/ TPX, TFX2, TFY1, TFY2, TF1, TF2, TORQ1, TORQ2

WRITE(W,10)
10 FORM7(I,27('C'))
1 1 FORM7(I,25X, 'C')
2 1 FORM7(I,27('C'))
3 WRITE(W,190) TEST
190 FORMAT(/, 'RESULT OF CALCULATION C'
1 1, 'RESULT OF SYSTEM FLOWS AND PRESSURES X',
2 1, 'RESULT OF INWARD FLOWS X',
4 1, 'RESULT OF OUTWARD FLOWS X',
5 1, 'TEST NAME... A000',
6/ 1, 'NODE X, GLOBAL CO-ORDINATES', 13X, 'FLOW', 1X, 'PRESSURE')
7 IF (COORD.EQ. 'PO') GO TO 120
8 WRITE(W,119)
9 GO TO 140
120 WRITE(W,130)
130 FORMAT(2X, NUMBER', 6X, 'RADIUS', 8X, 'ANGLE', 14X, 'Q(I)', 9X, 'P(I)')
140 DO 200 I=1, NODES
200 WRITE(W, 300) I, (GELCO(I,J), J=1, NCOE), Q(I), P(I)
400 FORMAT(/, 'NUMBER X', 1X, 'CENTRIFUGAL FORCE HAS BEEN CONSIDERED.')
500 FORMAT(/, 'END OF PRINT')
600 DO 750 I=1, NODES
700 IF(CFX(I).LE.0.0) GO TO 800
800 IF(CFY(I).LE.0.0) GO TO 810
900 CONTINUE
PRES=1.0
GO TO 905
905 DO 910 I=1, NODES

WRITE(W,600)
600 FORMAT(/, 'CENTRIFUGAL FORCE HAS BEEN CONSIDERED.')
700 WRITE(W,690)
800 WRITE(W,690)
900 FORMAT(/, 'CENTRIFUGAL FORCE HAS BEEN CONSIDERED.')
910 WRITE(W,690)
DO 910 I = 1, NODES
   DO 915 J = 1, 4
   IF (DCAC(I, J) .NE. 0.0) GO TO 950
910 CONTINUE
   GO TO 1002
C——PRINT THE RESULT OF FRICTION FORCES AND FRICTION TORQUES
950 WRITE (W, 1900) TPX1, TPY1, TRF1, TPX2, TPY2, TRF2
1900 FORMAT ('/X', 2X, 29('X'), '/2X, 'X 4. TOTAL FRICTION FORCE X',
1 //2X, 29('X'), '/16X, 'LOWER SURFACE', 29X, 'UPPER SURFACE',
2 //4X, 'X-Direction', 3X, 'Y-Direction', 3X, 'RESULTANT', 7X,
3 'X-Direction', 2X, 'Y-Direction', 3X, 'RESULTANT',
4 'X-Direction', 2X, 'Y-Direction', 3X, 'RESULTANT',
5 'X-Direction', 2X, 'Y-Direction', 3X, 'RESULTANT',
6 //, 3(E10.3, 3X), 3(3X, E10.3))
   IF (PRM0 .EQ. 1.0) GO TO 1002
   WRITE (W, 1900) TORQ1, TORQ2
1901 FORMAT ('/X', 2X, 30('X'), '/2X, 'X 5. TOTAL FRICTION TORQUE X',
1 //2X, 30('X'), '/2X, 'TOTAL TORQUE ON LOWER SURFACE. TQ1=', E12.5,
2 //2X, 'UPPER SURFACE. TQ2=', E12.5)
1002 WRITE (W, 600)
   RETURN
END
SUBROUTINE CFORCE

THIS ROUTINE IS TO CALCULATE THE CENTRIFUGAL FORCES
BECAUSE OF THE TREATMENT OF MASS.

IMPLICIT REAL*S(A-H,M-O-Z)
INTEGER W

COMMON/BLK1/ NODE, NODES, NELE, NNEL, NP, NQ, NME(400, 4), NNOD(400),
1 NDF(400), IBPS(400, 4), SECPR(400, 4), DEN, TH(400, 4), VIS,
2 CELCO(400, 2), COOE(400), BCVL(400), BCAC(400, 10),
3 A(4, 4), AA(400, 4), R(400, 4), C(400, 4), AREA(400), W, COOR,
4 RESULT, UNIT1, UNIT2
COMMON/BLK14/ CFX(400), CFY(400)
DIMENSION RAD(400), ANG(400), AG(400), S(400)
DIMENSION CF(400), FC(400), FX(400), FY(400)

IF(COOR.NE.'PO') GO TO 1000
IF(UNIT2.NE.2) GO TO 1000

10 DO 10 I=1,NODES
IF(BCAC(I,1).NE.0.0) GO TO 20
IF(BCAC(I,2).NE.0.0) GO TO 20
10 CONTINUE
RETURN

20 WRITE(W,193)
C--- ALL INITIAL VALUES ZERO
FCX=0.0
FCY=0.0
DO 30 I=1,NODES
CFX(I)=0.0
CFY(I)=0.0
RAD(I)=0.0
ANG(I)=0.0
S(I)=0.0
FC(I)=0.0
FX(I)=0.0
FY(I)=0.0
30 CONTINUE

C--- ARRANGEMENT OF COORDINATE
DO 40 I=1,NODES
ANG(I)=COOE(I)
IF(UNIT1.EQ.'RAD') GO TO 45
ANG(I)=ANG(I)*3.141592654/180.0
40 CONTINUE

45 DO 50 LE=1,NELE
DO 50 I=1,NNEL

C--- CALCULATION OF CENTRIFUGAL FORCE
K=NME(LE,I)
AG(K)=COOE(K+NODES)
AVL=BCAC(K,1)
AV2=BCAC(K,2)
IF(AVL-AV2.EQ.0.0) GO TO 69
AV3=(AV2-AVL)/TH(LE,I)
F1=(AV3*TH(LE,I)+AV1)**3-AV1**3
F2=ANG(K)**DEH/(3.8*TH(LE,I)**AV3)
F=F1+F2
GO TO 61
69 F=ANG(K)**DEH*AV1**2
60 F=RAD(K)*DEN*AV1**2
C---ASSEMBLY STEP
61 FC(K)=FC(K)+F
S(K)=S(K)+1.0
IF(UNIT1.EQ.'DEG') GO TO 62
IF(RESULT.NE.'YES') GO TO 50
IF(LE.GT.100) GO TO 50
WRITE(W,100) LE,K,RAD(K),AG(K)
100 FORMAT(/,2X,'ELEMENT NO.=',I3,3X,'NODE NO.=',I3,1//6X,'RADIUS=',E10.3,5X,'AXLE=',E10.3,'RADIANS')
GO TO 63
62 IF(RESULT.NE.'YES') GO TO 50
WRITE(W,101) LE,K,RAD(K),AG(K)
101 FORMAT(/,2X,'ELEMENT NO.=',I3,3X,'NODE NO.=',I3,1//6X,'RADIUS=',E10.3,5X,'AXLE=',E10.3,'DEG')
63 WRITE(W,102) AV1,AV2,F1,F2,F
102 FORMAT(/,2X,'LOWER SURFACE ANGULAR VELOCITY...'),'E10.3,1//6X,'UPPER SURFACE ANGULAR VELOCITY...','E10.3,1//6X,'( = F1*F2 )')
50 CONTINUE
DO 79 I=1,NODES
IF(S(I).EQ.0.0) GO TO 79
FX(I)=FC(I)*DCOS(AG(I))
FY(I)=FC(I)*DSIN(AG(I))
CF(I)=FC(I)/S(I)
CFX(I)=FX(I)/S(I)
CFY(I)=FY(I)/S(I)
79 CONTINUE
C
103 FORMAT(/,5X,'C--CALCULATION OF CENTRIFUGAL FORCE--C',1//5X,'C','45X,'C',1//5X,'C','CALCULATION OF CENTRIFUGAL FORCE C',1//5X,'C',1//5X,'C','(MEAN FORCE VALUE)',1//5X,'C',1//5X,'C')
WRITE(W,104)
104 FORMAT(/,2X,'MEAN CENTRIFUGAL FORCE AT EACH NODE.'//)
DO 80 I=1,NODES
80 WRITE(W,105) I,S(I),FC(I),CFX(I),CFY(I)
105 FORMAT(6X,'NODE NO.=',I3,3X,'S(I)=','E10.3,5X,1//6X,'FC(I)=','E10.3,3X,'CFX='E10.3,3X,'CFY='E10.3)
1000 RETURN
END
SUBROUTINE FFORCE(LE)

IMPLICIT REAL*8(A-H,M-O-Z)
INTEGER N

COMMON/BLK9/NCOE,NODES,NELE,NNEL,TP,NQ,NME(400,4),NNOD(400),
1 NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
2 GELC(400,2),GCOE(800),BCVL(400),BCAC(400,1),
3 A(4,4),AN(400,4),B(400,4),C(400,4),AREA(400),N,COORD,
4 RESULT,UNIT1,UNIT2
COMMON/BLK9/G/400),P(400),QIN,OUT,NNW,CEL(400,4)
COMMON/BLK3/DELTA(4,4)
COMMON/BLK20/UX(400),UY(400),EX(400),BY(400),H(400),VD(400)
COMMON/BLK21/FX1,FY1,FY2,F1,F2,SQ1,TQ2

TX1=0.0
TX2=0.0
TY1=0.0
TY2=0.0
UX1=0.0
UY1=0.0
CX=0.0
CY=0.0
THICKM=0.0

DO 1001 I=1,NNEL
IN=2ME(LE,I)
TX1=TX1+B(LE,I)*P(IN)
TY1=TY1+C(LE,I)*P(IN)
UXI=2.0*UX(IN)
UYI=2.0*UY(IN)
UXI=UX+UXI
UYI=UY+UYI
THICKM=THICKM+THICKM

DO 1001 J=1,NNEL
IN=2ME(LE,J)
TX2=TX2+TH(LE,I)*(DELTA(I,J)+1.0)*EX(JN)

1001 CONTINUE
THICKM=THICKM/3.0
TX1=TX1/12.0
TX2=TX2/12.0
TX1=TX1/6.0
TX2=TX2/6.0

IF(UX1.NE.0.0) GO TO 120
FX1=0.0
FX2=0.0
GO TO 130
120 FX1=(-1.0)*((TX1+TX2)+TX31
FX2=TX1+TX2+TX31

130 IF(UY1.NE.0.0) GO TO 140
FY1=0.0
FY2=0.0
GO TO 150
140 FY1=(-1.0)*((TY1+TY2)+TY31
FY2=TY1+TY2+TY31

C——CALCULATION OF FRICTION TORQUE
150 RF1=SQRT((FX1**2+FY1**2)
RF2=SQRT((FX2**2+FY2**2)
DO 150 J=1,NNEL
C=C+RF1*FX2*(UX1,UY1)
\begin{align*}
\text{CX} &= \text{GELCO}(\text{NME}(\text{LE}, I), 1) + \text{CX} \\
\text{CY} &= \text{GELCO}(\text{NME}(\text{LE}, I), 2) + \text{CY}
\end{align*}

1092 \text{CONTINUE}
\begin{align*}
\text{CX} &= \text{CX}/3.0 \\
\text{CY} &= \text{CY}/3.0 \\
\text{TQ1} &= (-1.0) * \text{CY} * \text{FX1} + \text{CX} * \text{FY1} \\
\text{TQ2} &= (-1.0) * \text{CY} * \text{FX2} + \text{CX} * \text{FY2}
\end{align*}
\text{RETURN}
\text{END}
SUBROUTINE C1ST4(X,Y,A,LE,C1,C2,C3,C4,C6,C8,
1	C10,C11,C12,C13,C14,C15,C16,C17,C18,C19,
2	C21,C22,C23,C49,C41,AREA,T,S)

C
C----THIS ROUTINE IS TO CALCULATE THE COMMON CONSTANTS FOR MATRICES
C----OF PRESSURE AND OTHER ACTIONS.
C
IMPLICIT REAL*8(A-H,M-O-Z)
DIMENSION X(4),Y(4),A(4,4),XX(4,4),YY(4,4),AREA(40),T(4),S(4)
C
C
DO 1 I=1,4
DO 1 J=1,4
XX(I,J)=0.0
YY(I,J)=0.0
XX(I,J)=X(I)-X(J)
YY(I,J)=Y(I)-Y(J)
1 CONTINUE
AREA(LE)=0.0
AREA1=ABS(A(1,2)+A(2,3)+A(3,1)-(A(2,1)+A(3,2)+A(1,3)))
AREA2=ABS(A(1,3)+A(3,4)+A(4,1)-(A(3,1)+A(4,3)+A(1,4)))
AREA(LE)=5.5*(AREA1+AREA2)
C
C1=(XX(3,4)-XX(2,1))/4.0
C2=(XX(3,4)+XX(2,1))/4.0
C3=(YY(3,4)-YY(2,1))/4.0
C4=(YY(3,4)+YY(2,1))/4.0
C5=(XX(3,4)+XX(4,4))/4.0
C6=(XX(3,4)+XX(4,1))/4.0
C7=(YY(3,4)+YY(4,1))/4.0
C
C10=C1*C1+C3*C3
C11=2.0*(C1*C6+C3*C8)
C12=C6*C6+C8*C8
C13=C10
C14=2.0*(C1*C2+C3*C4)
C15=C2*C2+C4*C4
C16=C10
C17=C1*C6+C3*C3
C18=C14/2.0
C19=C2*C6+C4*C8
C21=C1*C8-C3*C6
C22=C2*C3-C4*C1
C23=C2*C8-C4*C6
C40=C23+C21
C41=C23-C21
T(1)=1.0
T(2)=1.0
T(3)=1.0
T(4)=1.0
S(1)=1.0
S(2)=1.0
S(3)=1.0
S(4)=1.0
RETURN
END
SUBROUTINE TERM(C10, C11, C12, C21, C22, C23, C40, C41, T, S, A, W, RESULT)
C
C---- THIS ROUTINE IS TO CALCULATE THE TERI 'A' IN MATRIX OF MKP
C---- FOR QUADRILATERAL ELEMENT (C21.NE.9.9)
C-- IMPLICIT REAL*8(A-H, M-Q, Z)
REAL*8 ITGA, ITGB, ITGF, ITGC, ITGH
DIMENSION A(4,4), T(4), S(4)
INTEGER W
C
C---- CALCULATION OF TERM A
C
CALL INTEGA(C21, C40, C21, C41, ITGA)
CALL INTEGB(C21, C40, C21, C41, ITGB)
CALL INTEGF(C21, C40, C21, C41, ITGF)
CALL INTEGG(C21, C40, C21, C41, ITGC)
CALL INTEGH(C21, C40, C21, C41, ITGH)

WRITE(W, 2) C21, C40, C21, C41
2 FORMAT(//'C21,C40,C21,C41',//,4(E10.3))
WRITE(W, 3) ITGA, ITGB, ITGF, ITGC, ITGH
3 FORMAT(//'ITGA, ITGB, ITGF, ITGC, ITGH',//,5(E10.3, 3X))
DO 1 I=1, 4
DO 1 J=1, 4
T1=T(I)*T(J)
T2=T(I)*T(J)
S1=S(I)+S(J)
S2=S(I)+S(J)
A1=T2/C21
C24=(-1.0)*A1*C22/C21
C25=(T1-A1*C23)/C21
C26=(-1.0)*C22*C24
C27=(-1.0)*(C22*C25+C23*C24)
C28=1.0*C23*C25

C31=C10*C25+C11*C24
C32=C12*C25
C34=C10*C26+C21
C35=(C10*C27+C11*C26)/C21
C36=(C13*C28+C11*C27+C12*C26)/C21
C37=(C11*C28+C12*C27)/C21
C38=C12*C28/C21

A1=S2*(C31/3.0+C33)/4.0
A2=2/16.0
A(I, J)=A1+A2
IF(RESULT.NE. 'YES') GO TO 20
WRITE(W, 9) I, J
9 FORMAT(//'I, J',//,2I5)
WRITE(W, 10) C10, C11, C12, C21, C22, C23, C40, C41
10 FORMAT(//'TERM-A C10, C11, C12, C21, C22, C23, C40, C41',//,6(E10.3, 2X),//,1
2(E10.3, 2X))
WRITE(W, 11) C24, C25, C26, C27, C28, C31, C33, C34, C35, C36, C37, C38
11 FORMAT(//'C24, C25, C26, C27, C28, C31, C33, C34, C35, C36, C37, C38
1',//,5(E10.3, 3X),//,7(E10.3, 3X))
WRITE(W, 12) A11, A21, A22, A2, A(I, J)
12 FORMAT(//'A11, A21, A22, A2, A(I, J)',//,4(E10.3, 2X), 5X, E10.3)
1 CONTINUE
20 RETURN
END
SUBROUTINE TEMB(C13, C14, C15, C21, C22, C23, C40, C41, T, S, B, V, RESULT)

C—THIS ROUTINE IS TO CALCULATE THE TERM 'B' IN MATRIX MKP FOR

C—QUADRILATERAL ELEMENT (C21.NE.3.0)

IMPLICIT REAL*8(A-H,M-O-Z)
REAL*8 ITGA, ITGB, ITGF, ITG, ITGH
DIMENSION B(4, 4), T(4), S(4)
INTEGER W

C—CALCULATION OF TERM B

CALL INTEGA(C21, C40, C21, C41, ITGA)
CALL INTEGB(C21, C40, C21, C41, ITGB)
CALL INTEGF(C21, C40, C21, C41, ITGF)
CALL INTEGRG(C21, C40, C21, C41, ITG)
CALL INTEGH(C21, C40, C21, C41, ITGH)

DO 1 I=1, 4
DO 1 J=1, 4
T1=T(I)*T(J)
T2=T(I)**T(J)
S1=S(I)+S(J)
S2=S(I)**S(J)
B1=C13/C21
C24=(-1.0)*B1*C22/C21
C25=(C14-B1*C23)/C21
C26=(-1.0)*C22*C24
C27=(-1.0)*(C22*C25+C23*C24)
C28=C15-C23*C25
C31=C24*S1+C25*S2
C33=C25
C34=S2*C26/C21
C35=(S1*C26+S2*C27)/C21
C36=(S1*C27+S2*C28)/C21
C38=C28/C21
B11=T2*(C31/3.0+C33)/4.0
B21=T2*ITGA+C35*ITGB+C36*ITGF+C37*ITG+B38*ITGH
B22=T2/16.0
B(I,J)=B11+B21*B22

IF(RESULT.NE.'YES') 00
WRITE(W,9) I, J
9 FORMAT(//,'I,J',4X,2I5)
WRITE(W,10) C13, C14, C15, C21, C22, C23, C40, C41
10 FORMAT(//, 'TERM-B C13, C14, C15, C21, C22, C23, C40, C41',
1 /, /6(E10.3,3X),/2(E10.3,3X)
WRITE(W,11) C24, C27, C28, C31, C33,
1 C34, C35, C36, C37, C38
11 FORMAT(// , 'C24, C27, C28, C31, C33',/,
1 /C34, C35, C36, C37, C38*/5(E10.3,3X),/5(E10.3,3X)
WRITE(W,12) B11, B21, B22, B(I,J)
12 FORMAT(//, 'B11, B21, B22, B(I,J)',/3(E10.3,3X),5X,E10.3)
1 CONTINUE
29 RETURN
END
SUBROUTINE TERM(C10, C17, C18, C19, C21, C22, C23, C40, C41, T, S, C, W, RESULT)
   
   C-- THIS ROUTINE IS TO CALCULATE THE TERM 'C' IN MATRIX MKP FOR
   C-- QUADRILATERAL ELEMENT (C21 .NE. *1, 9)
   IMPLICIT REAL*8(A-H, M-O, Z)
   REAL*8 ITGA, ITGB, ITGF, ITGH
   DIMENSION T(4), S(4), C(4, 4)
   INTEGER W
   
   CALL INTEGA(C21, C40, C21, C41, ITGA)
   CALL INTEGR(C21, C40, C21, C41, ITGB)
   CALL INTEGF(C21, C40, C21, C41, ITGF)
   CALL INTEGH(C21, C40, C21, C41, ITGH)
   
   DO 1 I=1, 4
   DO 1 J=1, 4
   C60=C17*T(I)/C21
   C61=C17*T(I)/C21
   C62=(-1.0)*C22*C60/C21
   C63=(C18+C19*T(I)-C21*C60)/C21
   C64=(C10+C19*T(I)-C21*C61)/C21
   C65=(-1.0)*C22*C62
   C66=(-1.0)*C22*C63+C23*C62
   C67=C18+C64-C23*C63
   C68=C19-C23*C64
   C70=C62*C63*S(J)
   C71=C64
   C72=C65*S(J)/C21
   C73=C65*S(J)/C21
   C74=(C65+C65*S(J))*C21
   C76=C69/C21
   
   CN=S(I)*T(J)*(C70/3.0+C71)/4.0
   CB1=S(I)*T(J)/16.0
   C72=C72*ITGH+C73*ITGF+C74*ITGB+C76*ITGA
   C5=C81*CB2
   C(I, J)=CA+CB
   IF(RESULT .NE. 'YES') GO TO 20
   WRITE(W, 9) I, J
   9 FORMAT('//' , 'I, J' , / , 215)
   WRITE(W, 10) C60, C61, C62, C63, C64, C65, C66, C67, C68, C70, C71, C72,
   1 C73, C74, C75, C76
   10 FORMAT('//' , 'TERM-C C60, C61, C62, C63, C64, C65, C66, C67, C68, C70, C71, C72,
   1 ' C73, C74, C75, C76',
   2 5(E19.3, 3X) , / , 4(E19.3, 3X) , / , 2(E19.3, 3X))
   WRITE(W, 11) CA, CB1, CB2, CB, C(I, J)
   11 FORMAT('//' , 'CA, CB1, CB2, CB, C(I, J)' , / , 5(E19.3, 3X))
   1 CONTINUE
   20 RETURN
   END
SUBROUTINE INTEGA(A, B, C, D, INT)

IMPLICIT REAL*8(A-H, M-O-Z)
REAL*8 INT, INT1, INT2
AA=A+B
BB=A-B
CC=C+D
DD=C-D
INT1=A*DLOG(DABS(AA))/A+BB*DLOG(DABS(BB))/A
INT2=CC*DLOG(DABS(CC))/DD*DLOG(DABS(DD))/C
INT=INT1+INT2
RETURN
END

SUBROUTINE INTEGB(A, B, C, D, INT)

IMPLICIT REAL*8(A-H, M-O-Z)
REAL*8 INT, INT1, INT2, INT3
AA=A+B
BB=A-B
CC=C+D
DD=C-D
INT1=(2/A-D/C)
INT2=AA*BB*DLOG(DABS(AA/BB))/(2*A**2)
INT3=CC*DD*DLOG(DABS(CC/DD))/(2*C**2)
INT=INT1+INT2-INT3
RETURN
END

SUBROUTINE INTEGC(A, B, C, D, INT)

IMPLICIT REAL*8(A-H, M-O-Z)
REAL*8 INT, INT1, INT2
T1=-1.0
T2=1.0
X1=A*T1+B
X2=A*T2+B
AA=A/C
BB=-A*D/C
R=A/BB
AR=DABS(R)
IF(DABS(X1+X2)) 1, 2, 2
1 MAX=X2
XMIN=X1
GO TO 3
2 MAX=X1
XMIN=X2
3 IF(DABS(MAX)-AR) 19, 20, 20
19 CALL INTG01(AA, BB, X1, X2, INT)
RETURN
20 IF(DABS(XMIN)-AR) 39, 40, 40
39 CALL INTG02(AA, BB, X1, X2, INT)
RETURN
40 IF((X1+X2).LT.0.0) 50, 60, 70
50 R=(-1.0)*R
60 CALL INTG01(AA, BB, XMINT, R, INT1)
CALL INTG02(AA, BB, R, XMAX, INT2)
GO TO 199
70 CALL INTG01(AA, BB, XMAX, R, INT1)
CALL INTG02(AA, BB, R, XMIN, INT2)
199 INT=INT1+INT2
RETURN
SUBROUTINE INTEG1(A, B, X1, X2, INT)
IMPLICIT REAL*3(A-H, M, O-Z)
REAL*3 INT, INT1, INT2, INT21, INT22
INT1=DLOG(DABS(A))*DLOG(DABS(X2))-DLOG(DABS(X1)))
DO 1 N=1,50
INT21=((-1.0)*B*X2/A)**N)/(N*N)
INT22=((-1.0)*B*X1/A)**N)/(N*N)
INT2=INT21-INT22
INT1=INT1+INT2
IF(INT2/INT1<0.001) 2,2,1
1 CONTINUE
2 INT=INT1
RETURN
END

SUBROUTINE INTEG2(A, B, X1, X2, INT)
IMPLICIT REAL*8(A-H, M, O-Z)
REAL*8 INT, INT1, INT2, INT21, INT22
INT1=(DLOG(DABS(B*X2))*2)-DLOG(DABS(B*X1))*2)/2.0
DO 1 N=1,50
INT21=((-1.0*A)/(B*X2)**N)/(N*N)
INT22=((-1.0*A)/(B*X1)**N)/(N*N)
INT2=INT21-INT22
INT1=INT1+INT2
IF(INT2/INT1<0.001) 2,2,1
1 CONTINUE
2 INT=INT1
RETURN
END

SUBROUTINE INTEGR(A, B, C, D, INT)
IMPLICIT REAL*8(A-H, M, O-Z)
REAL*8 INT, INT1, INT2, INT21, INT22
AA=A+B
BB=A-B
CC=C+D
DD=C-D
INT1=DLOG(DABS(BB))/(C*DD)-DLOG(DABS(AA))/(C*CC)
INT2=DLOG(DABS(BD))/(C*DD)-DLOG(DABS(CC))/(C*CC)
A1=(1.0-1)/(1.0+A)
B1=(1.0-1)/(1.0+B)
INT3=DLOG(DABS(A1))-DLOG(DABS(A1))
INT4=C*(C*E-D*A)
INT=INT1+INT2+INT3+INT4
RETURN
END

SUBROUTINE INTEG(A, B, C, D, INT)
IMPLICIT REAL*8(A-H, M, O-Z)
REAL*8 INT, INT1, INT2, INT3, INT4
AA=A+B
BB=A-B
CC=C+D
DD=C-D
INT1=(DLOG(DABS(BB))/(C*DD)-DLOG(DABS(AA))/(C*CC))/(2.0*C)
INT2=2.0*C*(A*D-B*C)**2)
INT3=DLOG(DABS(BD)/(AA*DD))
INT4=2.0*C*(B*C-A*D)/(A*CC*DD)
INT=INT1+INT2+INT3+INT4
RETURN
END

SUBROUTINE INTEG(A, B, C, D, INT)
REAL*3 INT, INT1, INT2

A1=((A**3+B**3)*DLOG(DABS(A+B)))/(3.0*A**3)
A2=((A**3-B**3)*DLOG(DABS(A-B)))/(3.0*A**3)
A3=2.0*(B/A)**2/3.0
INT1=A1+A2-A3

C1=((C**3+D**3)*DLOG(DABS(C+D)))/(3.0*C**3)
C2=((C**3-D**3)*DLOG(DABS(C-D)))/(3.0*C**3)
C3=2.0*(C/D)**2/3.0
INT2=C1+C2-C3

INT=INT1-INT2
RETURN

SUBROUTINE lllTEm(A,B,C,D,INT)

IMPLICIT REAL*8(A-H,M-O-Z)
REAL*3 INT, INT1, INT2

A1=(A**4-B**4)/(4.0*A**4)
A12=DLOG(DABS((A+B)/(A-B))
A13=B/(2.0*A)
A14=1.0/3.0+(B/A)**2
INT1=A1*INT1+A12+A13*A14

C11=(C**4-D**4)/(4.0*C**4)
C12=DLOG(DABS((C+D)/(C-D))
C13=D/(2.0*C)
C14=1.0/3.0+(D/C)**2

INT2=C11+C12+C13*C14
INT=INT1-INT2
RETURN
END

SUBROUTINE INTEG(A,B,C,D,INT)

IMPLICIT REAL*8(A-H,M-O-Z)
REAL*8 INT, INT1, INT2

A1=(A**5+B**5)/(5.0*A**5)
A12=DLOG(DABS(A+B))
A13=(A**5-B**5)/(5.0*A**5)
A14=DLOG(DABS(A-B))
A15=2.0*((B/A)**2/3.0+(B/A)**4)/5.0
INT1=A1*INT1+A12+A13*A14-A15

C11=(C**5+D**5)/(5.0*C**5)
C12=DLOG(DABS(C+D))
C13=(C**5-D**5)/(5.0*C**5)
C14=DLOG(DABS(C-D))
C15=2.0*((D/C)**2/3.0+(D/C)**4)/5.0

INT2=C11+C12+C13*C14-C15
INT=INT1-INT2
RETURN
END

THESE ROUTINE HAVE NOT COMPLETED YET.
SUBROUTINE TRM1(LE, TH, C13, C14, C15, C17, C18, C19, T, S, A)
IMPLICIT REAL*3(A-H, O-Z)
DIMENSION A(4, 4), T(4), S(4), TH(4*3, 4)

C----CALCULATION OF TERM-A WHEN C21=1.0

DO 1 I=1,4
DO 1 J=1,4
A(I, J)=1.0
TA1=1.0
TA=1.0
C1=T(I)*T(J)
C2=T(I)+T(J)
C3=1.0
DO 2 L=1,4
DO 2 N=1,4
C
T1=T(L)*T(H)*T(N)
T2=T(L)*T(H)*T(N)*T(H)*T(N)*T(L)
T3=T(L)*T(H)*T(N)
S1=S(L)*S(H)*S(N)
S2=S(L)*S(H)+S(N)*S(H)+S(N)*S(L)
S3=S(L)+S(H)*S(N)
THICK=TH(LE,H)*TH(LE, M)*TH(LE, N)

CALL DINT61(C1, C2, C3, T1, T2, T3, TA1)
CALL DINT62(C22, C23, C13, C14, C15, S1, S2, S3, TA2)
TA=TA1*TA2
TA=TA+THICK*TA1
2 CONTINUE
A(I, J)=S(I)*S(J)*TA/1024.0
1 CONTINUE
RETURN
END

SUBROUTINE TRM2(LE, TH, C21, C22, C23, C13, C14, C15, C22, C23, T, S, B)
IMPLICIT REAL*3(A-H, O-Z)
DIMENSION B(4, 4), T(4), S(4), TH(4*3, 4)

C----CALCULATION OF TERM-B WHEN C21=4.9

DO 1 I=1,4
DO 1 J=1,4
B(I, J)=1.0
TB1=1.0
TB2=1.0
TB1=1.0
TB2=1.0
C1=S(I)*S(J)
C2=S(I)+S(J)
C3=1.0
DO 2 L=1,4
DO 2 N=1,4

T1=T(L)*T(H)*T(N)
T2=T(L)*T(H)*T(N)*T(H)*T(N)*T(L)
T3=T(L)*T(H)*T(N)
S1=S(L)*S(H)*S(N)
S2=S(L)*S(H)+S(N)*S(H)+S(N)*S(L)
S3=S(L)+S(H)*S(N)
THICK=TH(LE, L)*TH(LE, M)*TH(LE, N)

CALL DINT61(C13, C14, C15, T1, T2, T3, TR1)
CALL DINT62(C20, C21, C22, C23, C13, C14, C15, T1, S0, S2, TR2)
SUBROUTINE TR4C1(LE,TH,C1,C2,C3,C4,C22,C23,T,S,C)
IMPLICIT REAL*8(A-1,0-9)
DIMENSION C(4,4),T(4),S(4)

C--CALCULATION OF TERMC WHEN C21=9.0

DO 1 I=1,4
DO 2 J=1,4
C(I,J)=9.0
TC=9.0
A1=C1*T(I)
A2=C2*T(I)
A3=C3*T(I)+C1
A4=C4*T(I)+C2
A5=C3
A6=C4

DO 2 L=1,4
DO 2 N=1,4
DO 2 N=1,4
T1=T(L)*T(M)*T(N)
T2=T(L)*T(M)*T(N)+T(N)*T(L)
T3=T(L)+T(M)+T(N)
S1=S(L)*S(H)*S(J)
S2=S(L)*S(M)*S(J)+S(J)*S(L)
S3=S(L)*S(H)*S(M)
THJK=T(LE,L)*TH(LE,J)*TH(L,J)
CALL INTEG2(C22,C23,C19,C11,C12,S1,S2,S3,TC1)
TC=TC+THICK*TC1

2 CONTINUE
C(I,J)=S(I)*T(J)*TC/1024.0
1 CONTINUE
RETURN
END

SUBROUTINE TR4A3(T,S,TH,LE,C11,C12,C13,C14,C15,
C17,C18,C19,C23,A,B,C,D)

C--THIS SUBROUTINE IS FOR (C21=4.0,C22=7.0) CASE
C--THICKNESS CHANGES WITHIN AN ELEMENT

IMPLICIT REAL*8(A-1,0-9)
DIMENSION A(4,4),B(4,4),C(4,4),D(4,4),T(4),S(4),TH(4),4,4)

DO 1 I=1,4
DO 1 J=1,4
A1=T.I
B1=T.J
C1=T.I
A1=C11
B1=C12
C1=C13
D1=C14
D2=C15
D3=C16
D4=C17
D5=C18
D6=C19
D7=C20
D8=C21
D9=C22
D10=C23
D11=T(I)*T(J)*T(I)*T(I)*T(J)*T(J)
1 CONTINUE
RETURN
END
C

C

THICK = TH(L,E,L) * TH(L,E,N)

CALL INTEG5(I, J, T1, T2, T3, TA1)

CALL INTEG7(C10, C11, C12, S, S1, S2, S3, TA2)

A1 = A1 + THICK * TA1 * TA2

CALL INTEG5(I, J, S, S1, S2, S3, TB1)

CALL INTEG7(C13, C14, C15, T, T1, T2, T3, TB2)

B1 = 31 * THICK * TB1 * TB2

C24 = C13 * T2 * (T1 + T1) + C18 * T1 * T1

C25 = C17 * T2 * (T1 + T1) + C19 * T1 * T1

C26 = C13 * (T1 + T3) + C18 * (T1 * T3 + T2)

C27 = C17 * (T1 + T3) + C19 * (T1 * T3 + T2)

C28 = C24 / 5.9 / C26 / 3.6 + C18

C29 = C25 / 5.9 / C27 / 3.6 + C19

CALL INTEG6(J, C28, C29, S, S1, S2, S3, TC1)

C1 = C1 + THICK * TC1

CONTINUE

A(I, J) = G1 * 5(J) * A1 / (256.0 * C23)

B(I, J) = R1(I) * T(J) * B1 / (256.0 * C23)

C(I, J) = S(I) * T(J) * C1 / (256.0 * C23)

D(J, I) = C(I, J)

1 CONTINUE

RETURN

END

SUBROUTINE TERM1(C14, C11, C12, C22, C23, T, S, A)

IMPLICIT REAL*3(A-H, O-Z)

DIMENSION A(4, 4), T(4), S(4)

C----CALCULATION OF TERM-A

C

C24 = C10 / C22

C25 = (C11 - C23 * C24) / C22

C26 = C12 - C23 * C25

C42 = C23 + C22

C43 = C23 - C22

DO 1 I = 1, 4

DO 1 J = 1, 4

T2 = T(I) * T(J)

S2 = S(I) * S(J)

A1 = S2 / T2 * (T2 / 3.6 * 4.1.9)

A2 = 2.9 * C25 * C26 / C22 * DBLSUM(DABS(C42 / C43))

A(I, J) = A1 * A2

1 CONTINUE

RETURN

END

SUBROUTINE TERM2(C13, C15, C22, C23, T, S, B)

IMPLICIT REAL*3(A-H, O-Z)

DIMENSION B(4, 4), T(4), S(4)

C----CALCULATION OF TERM-B

C

C42 = C23 + C22

C43 = C23 - C22

DO 1 I = 1, 4

DO 1 J = 1, 4

T2 = T(I) * T(J)

S2 = S(I) * S(J)

A1 = S2 / T2 * (T2 / 3.6 * 4.1.9)

A2 = 2.9 * C25 * C26 / C22 * DBLSUM(DABS(C42 / C43))

A(I, J) = A1 * A2

1 CONTINUE

RETURN

END
```plaintext
SUBROUTINE TERM1(C10, C12, C19, C22, C23, T, S, C)
IMPLICIT REAL*3(A-H, M-O, Z)
DIMENSION C(4,4), T(4), S(4)

C42 = C23 + C22
C43 = C23 - C22
DO 1 I=1,4
DO 1 J=1,4
C24 = 2.0*(C10*T(I)/3.0 + C18)
C25 = 2.0*(C17*T(I)/3.0 + C19)
C1 = C24*S(J)/C22
C2 = (C24 + C25*S(J) - C1*C23)/C22
C3 = C25 - C24*C23
C1 = 3.0*I*T(J)/16.0
C2 = 2.0*C24 + C23/16.0
C2 = C22*(C42/C43)
C(I,J) = C1*C2
1 CONTINUE
RETURN
END

SUBROUTINE TERM2(C10, C17, C19, C22, C23, T, S, C)
IMPLICIT REAL*9(A-H, M-O, Z)
DIMENSION C(4,4), T(4), S(4)

C42 = C23 + C22
C43 = C23 - C22
DO 1 I=1,4
DO 1 J=1,4
T2 = T(I)*T(J)
S2 = S(I)*S(J)
A1 = T2/(4.0*C23)
A2 = T2/3.0 + 1.0
A3 = C17/3.0 + C12
1 CONTINUE
RETURN
END

SUBROUTINE TERM3(C19, C17, C19, C23, T, S, C)
IMPLICIT REAL*3(A-H, M-O, Z)
DIMENSION C(4,4), T(4), S(4)

C42 = C23 + C22
C43 = C23 - C22
DO 1 I=1,4
DO 1 J=1,4
T2 = T(I)*T(J)
S2 = S(I)*S(J)
A1 = T2/(4.0*C23)
A2 = T2/3.0 + 1.0
A3 = C17/3.0 + C12
1 CONTINUE
RETURN
END
```

C24 = 2.7 * (C17 * T(1)/3.7 + C16)
C25 = 2.7 * (C17 * T(1)/3.7 + C16)
C1 = 5(I) * T(I)/(3.7 * C23)
C2 = C24 * S(J)/3.7 + C25
C(I, J) = C1 * C2
1 CONTINUE
RETURN
END

SUBROUTINE INT25(I, J, T, T1, T2, T3, ITG5)
C——INTEGRATION OF (1+T*T1)(1+T*T2)(1+T*T3)(1+T*T4)
C
IMPLICIT REAL*3(A-H, I-O, Q-Z)
REAL*3 ITG5
DIMENSION T(4)

T11 = T1 * T(J) + (T1 + T2) * T(J) * T(I)
T12 = (T2 + T3) * T(J) + (T3 + T(J)) * T(I)

ITG5 = T11 / 5.0 + T12 / 3.0 + 1.0
RETURN
END

SUBROUTINE INT26(J, C1, C2, T, T1, T2, T3, ITG6)
C——INTEGRATION OF (1+T*T1)(1+T*T2)(1+T*T3)(1+T*T4)(C1*T+C2)
C
IMPLICIT REAL*3(A-H, I-O, Q-Z)
REAL*3 ITG6
DIMENSION T(4)

T11 = T1 * (T1 + T2 * T(J)) + C2 * T1 * T(I)
T12 = C1 * (T3 + T(J)) + C2 * (T3 + T(J))

ITG6 = T11 / 5.0 + T12 / 3.0 + 1.0
RETURN
END

SUBROUTINE INT27(C1, C2, C3, T, T1, T2, T3, ITG7)
C——INTEGRATION OF (1+T*T1)(1+T*T2)(1+T*T3)(C1*T+C2*T+C3)
C
IMPLICIT REAL*3(A-H, I-O, Q-Z)
REAL*3 ITG7
DIMENSION T(4)

T11 = C1 * T3 * C2 * T1
T12 = C1 + C2 * T3 + C3 * T2.

ITG7 = T11 / 5.0 + T12 / 3.0 + 1.0
RETURN
END

SUBROUTINE INT28(C, C2, T, T1, T2, T3, C3, C4, C5, Y)
IMPLICIT REAL*3(A-H, I-O, Q-Z)
DIMENSION T(4), S(I, J), S(4, 4), T(I, J, 4, 4)
C
If, I, J, L = 1, 4

UA1 = (C1 * (T(J) + (T(I) + S(J))/T(L))) / 3.9 + C2
UA2 = (C3 * S(J) * S(L) + (S(J) + S(L)) * C3) / 3.9 + C2
UA = IN + TH(LE, L) * UA1 * UA2
UB1 = (S(I) + S(J)) / 3.9 + C2
UB2 = (C4 * T(J) + (T(J) + T(L)) * C3) / 3.9 + C4
UB = IN + TH(LE, L) * UB1 + UB2

1 CONTINUE
A = 1, 4
B = 1, 4
C = S(I) * D/I16.9
C2 = T(I) * D/I16.9
A = C * UA
B = C * UB
A B = LE, L, J = A - B

2 CONTINUE
RETURN
END

SUBROUTINE TERN3(T, S, T1, LE, D1, C1, C2, C3, AP)

IMPLICIT REAL*3(A-L, I-0-Z)
DIMENSION T(4), S(4), A(4), D(4)
DO 1 I = 1, 4
DO 1 J = 1, 4
UA = 1.9
UB = 1.9
DO 1 L = 1, 4
UA1 = (C1 * (T(L) * T(J)) + C2 * (T(L) * T(J))) / 3.9 + C2
UA2 = (S(I) * S(J) + S(I) * S(J)) / 3.9 + C2
UB1 = (C1 * (S(I) * S(J)) + C2 * (S(I) * S(J))) / 3.9 + C2
UB2 = (T(L) * T(J) + T(J) * T(J)) / 3.9 + C2
UA = UA + TH(LE, L) * UA1 * UA2
UB = UB + TH(LE, L) * UB1 + UB2

1 CONTINUE
C = T(I) * D/I16.9
C2 = T(I) * D/I16.9
A = C * UA
B = C * UB
A B = LE, L, J = A - B

2 CONTINUE
RETURN
END

SUBROUTINE TERN3(T, S, T1, LE, C3, C4, C5, D1, D2, D3, D4, D5)

IMPLICIT REAL*3(A-L, I-0-Z)
DIMENSION T(4), S(4), A(4), D(4)
DO 1 I = 1, 4
DO 1 J = 1, 4

T1 = T(L) * (T(J) + T(J) * T(L)) / 3.9 + C2
T2 = T(L) * T(J) * T(J) * T(J) * T(J) * T(J)
T3 = T(L) * T(J) * T(J)
S1 = S(I) + S(J)
S2 = S(I) * S(J) / 3.9 + C2
S3 = S(I) * S(J) / 3.9 + C2
CALL RTN3S(L, J, C1, C2, C3, D1, D2, D3, D4, D5)
CALL INTEG1(J,C3,C4, C5,T1,T2,T3,T9)
B1=Q1+CHICX1*T2
2 CONTINUE
CST1=5(J)*(DT3)/(3772.*VIS)
CST2=5(J)*(DT9)/(3772.*VIS)
A(LE,L,J)=(LE,L,J)*CST1*CST2*C1
1 CONTINUE
RETURN
END

SUBROUTINE INTEG1(CL9,C11,C12,T1,T2,T3,EN1)
C--INITIATION OF (1+TTL) (1+TM1) (1+TM2) (C19*T**2+C11*T+C12)
C
IMPLIED REAL'S(A=H,M,O-Z)
REAL'S EN1
T12=CI9*T2+C11*T1
T14=CI9*C11*T3+C12*T2
EN1=3.*T12/5.9+T14/3.2+C12
RETURN
END

SUBROUTINE INTEG11(CL, C2, C3, C4, C5, C6, T1, T2, T3, C15, C16)
C--INITIATION OF (1+TTL) (1+TM1) (1+TM2) (C19*T**2+C11*T+C12)
C--REM C19=C19*5+C2 C11=C11*5+C4 AND C13=C5*5+C6
IMPLIED REAL'S(A=H,M,O-Z)
C
C23=C1*T2+C3*T1
C21=C2*C2+C4*T1
C22=C1*C3*T3+C5*T2
C23=C2*C4*T3+C6*T2
C15=2.*T12/5.9+C23/1.9+C6
C16=2.*T21/C21/5.9+C23/1.9+C6
RETURN
END

SUBROUTINE INTEG2(CL, C2, C10, C11, C12, T1, T2, T3, EN2)
C--INITIATION OF (1+TTL) (1+TM1) (1+TM2) (C19*T**2+C11*T+C12)/(C19+C2)
C
IMPLIED REAL'S(A=H,M,O-Z)
REAL'S EN2
T13=C19*T1/C1
T11=(C19*T2+C11*T1-T13*C2)/C1
T12=(C19*T3+C11*T2+C12*T1-T13*C2)/C1
T13=(C19*C11*T3+C12*T2-T13*C3)/C1
T14=(C11*C12*T3-T13*C2)/C1
T15=C12-T14*C2
T16=2.*T13/5.9+T14/3.2+T12
T12=2.5*C1*DLOG(DP,~S*(C1+22)/(C1-~S))
EN2=TT1+TT2
RETURN
END

SUBROUTINE INTEG21(CL, C3, C4, C5, C6, C7, C10, C11, C12, T1, T2, T3, C15, C16)
C--INITIATION OF (1+TTL) (1+TM1) (1+TM2) (C19*T**2+C11*T+C12)/(C19+C2)
C--REM C19=C19*5+C2 C11=C11*5+C4 AND C13=C5*5+C6
IMPLIED REAL'S(A=H,M,O-Z)
C

C1^2 = C17 * C1 / C1
C21 = (-1.0) * T1^2 + C3 / C1
C22 = (C1^2 * T2 + C11 * T1 - C17 * C4) / C1
C23 = (-1.0) * C3 / C1
C24 = (-1.0) * (C3 * C22 + C4 * C21) / C1
C25 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C26 = (-1.0) * C3 / C1
C27 = (-1.0) * (C3 * C22 + C4 * C21) / C1
C28 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C29 = (-1.0) * C3 / C1
C30 = (-1.0) * (C3 * C22 + C4 * C21) / C1
C31 = (-1.0) * (C3 * C22 + C4 * C21) / C1
C32 = (-1.0) * (C3 * C22 + C4 * C21) / C1
C33 = (-1.0) * (C3 * C22 + C4 * C21) / C1
C34 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C35 = (-1.0) * C3 / C1
C36 = (-1.0) * (C3 * C22 + C4 * C21) / C1
C37 = (-1.0) * (C3 * C22 + C4 * C21) / C1
C38 = (-1.0) * (C3 * C22 + C4 * C21) / C1
C39 = (-1.0) * (C3 * C22 + C4 * C21) / C1
C40 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C41 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C42 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C43 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C44 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C45 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C46 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C47 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C48 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C49 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C50 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C51 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1
C52 = (C1^2 * T3 + C11 * T2 + C12 * T1 - C3 * C22) / C1

C13 = 3 * T1 / C1
C14 = 4 * T1 / C1
C15 = (-1.0) * C13 * A1 / C1
C16 = (A3 * T2 + A5 * T1 - C13 * A2 - C14 * A1) / C1
C17 = (A4 * T2 + A6 * T1 - C14 * A2) / C1
C18 = (-1.0) * C15 * A1 / C1
C19 = (-1.0) * (C15 * A2 + C16 * A1) / C1
C20 = (A3 * T2 + A5 * T1 - C16 * A2 - C17 * A1) / C1
C21 = (A4 * T2 + A6 * T1 - C17 * A2) / C1
C22 = (-1.0) * C18 * A1 / C1
C23 = (-1.0) * (C18 * A2 + C19 * A1) / C1
C24 = (-1.0) * (C19 * A2 + C20 * A1) / C1
C25 = (A3 * A5 * T3 + A5 * T2 + A5 * T2 + C20 * A2 - C21 * A1) / C1
C26 = (A4 * A6 * T3 + A6 * T2 + A6 * T2 - C21 * A2) / C1
C27 = (-1.0) * C22 * A1 / C1
C28 = (-1.0) * (C22 * A2 + C23 * A1) / C1
C29 = (-1.0) * (C23 * A2 + C24 * A1) / C1
C30 = (-1.0) * (C24 * A2 + C25 * A1) / C1
C31 = (A5 * A7 * T3 - C25 * A2 - C26 * A1) / C1
C32 = (A6 * A0 * T3 - C35 * A2) / C1
C33 = (-1.0) * C37 * A1
C34 = (-1.0) * (C37 * A2 + C36 * A1)
C35 = (-1.0) * (C29 * A2 + C28 * A1)
C36 = (-1.0) * (C30 * A2 + C31 * A1)
C37 = (-1.0) * (C32 * A2 + C31 * A1)
C38 = (-1.0) * (C32 * A2 + C31 * A1)
C39 = (-1.0) * (C32 * A2 + C31 * A1)
C40 = (-1.0) * (C32 * A2 + C31 * A1)
C41 = (-1.0) * (C32 * A2 + C31 * A1)
C42 = (-1.0) * (C32 * A2 + C31 * A1)
C43 = (-1.0) * (C32 * A2 + C31 * A1)
C44 = (-1.0) * (C32 * A2 + C31 * A1)
C45 = (-1.0) * (C32 * A2 + C31 * A1)
C46 = (-1.0) * (C32 * A2 + C31 * A1)
C47 = (-1.0) * (C32 * A2 + C31 * A1)
C48 = (-1.0) * (C32 * A2 + C31 * A1)
C49 = (-1.0) * (C32 * A2 + C31 * A1)
C50 = (-1.0) * (C32 * A2 + C31 * A1)
C51 = (-1.0) * (C32 * A2 + C31 * A1)
C52 = (-1.0) * (C32 * A2 + C31 * A1)
C CONTINUE
ITGI=2.0*ITG2/(A1+1.0)
ITG3=(C1**(N+1)-C2**(N+1))/((N+1)*C1**(N+1))
ITG2=ITG3*DLOG(DABS(C1)/C11)
ITG=ITG1+ITG2
GO TO 17

C—WHEN N IS EVEN NUMBER
173  ITGI=1.0
    ITG2=1.0
    K2=(N+2)/2
    DO 2 I=1,K2
    ITGL=(C2/C1)**(2*I-2)/(A1+1.0-2.0**(I-1))
    ITG2=ITGL+ITG2
2 CONTINUE
    ITG1=2.0*ITG2/(A1+1.0)
    ITG3=(C1**(N+1)+C2**(N+1))/((A1+1.0)*C1**(N+1))
    ITG4=(C1**(N+1)+C2**(N+1))/((A1+1.0)*C1**(N+1))
    ITG2=ITG3*DLOG(DABS(C1))
    ITG3=ITG4*DLOG(DABS(C11))
    ITG=ITG2+ITG3-ITG1

15 RETURN
END

BOTTOM