Unsteady heat transfer in a motored internal combustion engine

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BATH UNIV.
30 JUN 1989
14 JAN 1994
1 JUL 1994
30 JUN 1995
28 JUN 1996
27 JUN 1997
UNSTEADY HEAT TRANSFER IN A MOTORED
INTERNAL COMBUSTION ENGINE

by

H. HASSAN

THESIS Submitted in fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University of Technology.

SUPERVISORS

P. H. Broadhurst
J. C. Dent
B. E. Knight

SEPTEMBER, 1968
SUMMARY

This investigation was aimed at providing a sound basis for the prediction of unsteady heat transfer in internal combustion engine cylinders. In order to obtain useful experimental data in the time available, only the convective component of the heat transfer was investigated.

In order to study the heat transfer inside the engine cylinders, it is necessary to know the flow conditions prevailing there. There have been very few quantitative measurements of the gas motion inside the engine cylinder because of the experimental difficulties. It was therefore decided to investigate the flow conditions inside a specially designed pre-combustion chamber. The constant temperature hot wire anemometer was used to measure the mean gas velocity. This was achieved by applying analytical corrections to allow for the simultaneous variation of gas temperature and pressure in addition to the changes in velocity. These corrections required a knowledge of gas temperature and pressure variations, the gas temperature was recorded by using a 5μ tungsten wire as a resistance thermometer. Pressure through the engine cycle was also recorded. The heat fluxes were computed by Fourier analysis of the metal surface temperature record obtained with a thin film type of thermocouple.

Instrumentation was developed successfully and tests were carried out on a motored engine at three different speeds and three compression ratios. Initial tests had shown that it was necessary to operate the hot wire at higher temperatures than is generally the practice in hot wire anemometry. Wind tunnel tests showed that the original analytical method of calculating the hot wire calibration curve did not agree with the experimental results when the wire operating temperature was high. This method was therefore modified. The modified method showed much better agreement with the experimental results obtained with the wire operated at high temperatures.
Tests in a high pressure and temperature wind tunnel showed that the modified method of calculation gave values to within ± 12% of the nominal velocity (which had a tolerance of ± 8%).

The computed local Nusselt and Reynolds number were correlated on the basis of flat plate heat transfer relationships. It is shown that the experimental data can be represented by the following relationship,

\[ N_u = C R_e^{0.8} \]

where 'C' varies between .0276 and .0184. These results were also checked by computing the surface heat flux using the experimental gas temperature profile. A comparison is also drawn, at two selected points in the engine cycle, between experiment and the theory for heat transfer through a developing boundary layer on a flat plate.

This investigation supplies evidence that the existing heat transfer data for pipeflow and flat plate can be used with confidence to predict the unsteady heat transfer in internal combustion engine cylinders.
PUBLICATIONS

A major portion of the work reported in Chapter 8 has been accepted for publication as a paper in the British Journal of Applied Physics (Journal of Physics D).
ACKNOWLEDGEMENTS

The author gratefully acknowledges the encouragement and advice given by Dr. P. H. Broadhurst (formerly of Loughborough University), Mr. B. E. Knight (C.A.V. Limited) and Mr. J. C. Dent (Loughborough University).

The author wishes to thank Mr. P. Norton (Department of Mechanical Engineering, Loughborough University) for his infinite patience, high degree of skill and craftsmanship, without which this project may not have been possible.

Thanks are also due to Mr. K. W. Topley for doing the photographic work involved and to the Department of Mechanical Engineering of Loughborough University of Technology for providing the facilities to carry out this work.
General Notation

A ..... Area, Empirical constant
B ..... Empirical constant
b ..... Heat penetration factor
C ..... Capacitance, Empirical constant, total heat capacity.
C_p ..... Specific heat of the gases at constant volume.
c ..... Specific heat of the wire material.
\eta_p ..... Friction coefficient
\bar{c} ..... r.m.s random velocity.
d ..... Wire diameter.
E ..... Bridge voltage
\varepsilon ..... Conversion factor to convert Joules/sec. into cal/sec.
f ..... Frequency of the fundamental wave form.
g ..... Transconductance of the amplifier system.
g_c ..... Conversion factor to convert ft-lb_m to lb_f sec^2.
h ..... Heat transfer coefficient.
H ..... Heat flux.
I ..... current flowing through the wire
k ..... Harmonic number
K ..... Thermal conductivity
\ell ..... Metal thickness at the section where heat flux is measured, half the length of sensing wire
m ..... Empirical constant
M ..... Time Constant of the hot wire system.
N ..... Engine R.P.M.
n ..... Empirical constant, number of ordinates.
\Delta P ..... Pressure drop.
P ..... Gas pressure
Q ..... Quantity of heat
q ..... Heat transfer rate.
R ...... Universal gas constant, resistance
r ...... wire radius
T ...... Temperature
u,v,w ...... Components of the velocity vector in x, y and Z direction respectively
V ...... Velocity, voltage, volume
x ...... distance
x, y, and Rectilinear coordinate system
α, β ...... Constants defining the temperature—resistivity relationship, cosine and sine coefficients
δ ...... Diffusivity of the metal wall
ε ...... Phase difference
ε ...... Fractivity
λ ...... Resistivity, molecular mean free path
τ ...... Time constant
γ ...... Stefan Boltzmann constant
μ ...... Viscosity (absolute)
θ ...... Time
η ...... Efficiency
γ ...... Isentropic index for air
ρ ...... Density.
ω ...... Angular frequency

Suffixes
a ...... Refers the properties to a reference temperature T
ag ...... Refers the temperature amplitude to the excess of gas temperature over the environment.
 gm ...... Refers the temperature to mean environment temperature
g ...... Refers the property to the gas temperature
w ..... Refers the property to the metal wall or the wire temperature.
SO ..... Refers to the temperature of the surroundings.
s ..... Refers the property to the wire support temperature.
f ..... Refers the properties to the film temperature.

**Dimensionless Groups.**

- $F_o$ ..... Fourier number
- $G_o$ ..... Grashof number
- $M_o$ ..... Mach number
- $N_u$ ..... Nusselt number
- $P_e$ ..... Peclet number
- $R_e$ ..... Reynolds number
- $K_n$ ..... Knudsen number
- $P_a$ ..... Prandtl number
- $S_t$ ..... Stanton number.
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CHAPTER 1

INTRODUCTION AND LITERATURE SURVEY
CHAPTER I

Notation

A  Area of the exposed surface, ft.²

a  Multiplying constant for convective heat transfer

b  Exponent relating $N_U$ and $R_E$

$C_p$  Specific heat of fluid at constant pressure

CHU lb⁻¹ (deg C)⁻¹

c  Multiplying constant for radiant heat transfer

CHU ft⁻² Sec⁻¹ (deg C)⁻¹

D  Bore of engine, ft.

h  Heat Transfer coefficient

CHU ft⁻² Sec⁻¹ (deg C)⁻¹

k  Thermal Conductivity of fluid

CHU ft⁻¹ Sec⁻¹ (deg C)⁻¹

L  Stroke of engine, ft.

N  Engine rotational speed, rev/min.

$N_U$  $\frac{hD}{k}$ Nusselt Number

p  pressure-atmospheres

q  Heat Transfer rate

$R_e$  $\frac{V_p \rho D}{\mu}$ Reynolds number

r  Compression ratio

s  Entropy of fluid

CHU lb⁻¹ (deg C)⁻¹

T  Gas Temperature °K

$T_w$  Wall Temperature °K

$V_p$  Mean Piston Speed, ft/sec

$\theta$  Angular movement - radians

$\mu$  Dynamic viscosity lb ft⁻¹ Sec⁻¹
INTRODUCTION

1.1. General Comments

Although the internal combustion engine has been in use as a prime mover for well over half a century, relatively little is known about the various thermodynamic and heat transfer processes associated with it. This is mainly due to the extreme complexity of the working processes involved. Because of the nature of constant volume or constant pressure cycles, on which the internal combustion engines operate, large variations in the cycle temperatures and pressures occur. Therefore the time dependent variables involved in calculations make the task of an exact analysis extremely difficult. The internal combustion engine has survived as a major prime mover because of the steady increase in its specific output and also due to its economy of operation relative to the other prime movers. The quest for higher ratings has resulted in ever increasing number of complex technical and operational problems. The increase in thermal and mechanical loading together with increased noise are some of the most important problems faced by the engine manufacturers today.

1.2. Thermal Loading Considerations

1.2.1. The engine components which are directly affected by the increased thermal loading as a result of higher engine ratings are:

(a) Piston and Piston rings
(b) Cylinder Head
(c) Cylinder liner
(d) Fuel injector nozzle
(e) Valves and valve seats.
A more complete knowledge of the laws governing the heat transfer between the working gases and the engine components will result in better design of these components.

1.2.2. Effective cooling of the cylinder walls is essential not only from consideration of stiffness and strength but also from the point of view of increased reliability. Overheated parts of the cylinder can give rise to pre-ignition or detonation which may eventually result in failure. An abnormally cooled cylinder liner can suffer corrosion due to condensation of acidic components of the combustion products.

1.3. Steady and Unsteady Heat Transfer

1.3.1. In order to understand and analyse the phenomenon of heat transfer to and from the working gases in an internal combustion engine cylinder, the heat flow can be divided into steady and unsteady components. Here and in the subsequent chapters the steady heat transfer is defined as that which does not vary with time. Similarly unsteady heat flow is time dependent.

1.3.2. It is relatively easy to obtain the steady state heat transfer in an internal combustion engine. A simple heat balance will yield this information, which can then be used to estimate the rates of coolant flow and the size of cooling equipment required for a particular installation.

1.3.3. Measurement of the unsteady heat transfer is a much more complicated affair. In order to measure this, it is necessary to measure the surface temperature fluctuations induced in the metal due to fluctuating gas temperature. Generally, the amplitude of such a fluctuation is extremely small, the measurement of which involves some fairly advanced instrumentation problems.
1.4. Importance of Unsteady Heat Transfer in I.C. Engines

Knowledge of the unsteady heat transfer is important for the following reasons.

1.4.1. Cycle Calculations and Digital Computer Simulation of Engines

With the advent of the digital computers a considerable amount of effort is being directed into the problems of the internal combustion engine design. It is aimed at reducing the development time and hence the costs, which have increased quite considerably in the past two decades, because of the higher engine ratings. The basic concept is to simulate engine performance on a digital computer and thereby cut down the actual development testing of the prototype engines. Papers by Whitehouse and others\(^1\) and Smythe and others\(^2\) are good examples of the attempts being made in this field. In order that these computer programmes simulate the engine performance realistically it is essential to supply the fundamental data on the heat transfer rates throughout a complete engine cycle. Unlike steam prime movers, where the transfer of heat is necessary to raise the temperature of the working fluid, the heat transfer in I.C. engines takes place from the working fluid to the coolant and therefore it may be considered as an unavoidable loss. This affects the rate of heat release and the cycle efficiencies.

1.4.2. Effects on Volumetric efficiency

The induced charge in the engine cylinder gains heat from the heat cylinder walls during the induction stroke; this results in increasing the charge temperature with the consequent increase in the specific volume of the gas. This can adversely affect the volumetric efficiency of the engine.
1.4.3. **Thermal Stresses**

In addition to the mechanical stresses produced in the engine components due to gas loading, thermal stresses are introduced because of the cyclic temperature variation of the metal surface. These stresses when superimposed on the mechanical stresses may cause a component to fail. If the wall surface temperature fluctuation or the unsteady heat transfer rate is known, these cyclic thermal stresses can be predicted and an allowance for the increased stress can be made at the design stage of the component.

1.4.4. **Engine Starting**

When assessing the startability of an engine under cold conditions it is necessary to know the temperature of the gases at the point in the cycle when the fuel is injected. Without any knowledge of the heat transfer rates it is not possible to calculate this gas temperature in the engine cylinder accurately.

1.5. **Review of the Literature on Unsteady Heat Transfer in I.C. Engines**

Eichelberg\(^3\)\(^4\) was the first to show that in spite of the small metal surface temperature variations, the magnitude of the heat flux fluctuation was comparatively great in the internal combustion engine components. A great deal of criticism has been levelled against his formula, but at the present moment it is very widely used in Europe and the U.S.A. Over thirty years have passed since this formula was evolved but no other significant advances have been made. This perhaps illustrates the extreme complexity of the problem and the experimental difficulties involved. Because of its historical importance and the popularity it enjoys today, it is worth discussing this formula in some detail.
Eichelberg\textsuperscript{3,4} made measurements on a large, 600 mm. bore x 1060 mm. stroke, two stroke, slow running engine (100 r.p.m.) and also on a four stroke 280 mm. bore x 420 mm. stroke engine running at 211 r.p.m. The technique used was to obtain the instantaneous metal surface temperature fluctuations with time and analyse these metal surface temperature records, using harmonic analysis, which will be described later. Thus, heat transfer rate, heat transfer coefficient and the thermal gradient at the wall surface could be calculated at any instant in the engine cycle. Based on these results Eichelberg evolved the following well known formula to give the heat transfer rate.

\[
\frac{q}{A} = 8.06 \times 10^{-5} \left( \frac{V_p}{p} \right)^{\frac{1}{3}} \left( \frac{p T}{T - T_w} \right)^{\frac{1}{2}} \quad \ldots \ldots (1.1)
\]

It is clear that the term \( V_p \), the mean piston speed, on the right-hand side of equation (1.1) is supposed to represent the effects of engine speed. Gas density and temperature variation effects are represented by the term \((pT)^{\frac{1}{2}}\). The exponents of these terms were chosen such that the formula agreed well with the experimental results. The formula is supposed to take account of radiative as well as the convective heat transfer.

The main advantage of this formula is its simplicity. The only measurement required is the cylinder pressure diagram. Knowing the cylinder geometry, perfect gas relationships and thermodynamic gas relationships for the working fluid an approximation of a spatially uniform gas temperature can be made for any instant in the engine cycle.
The main objections to this formula are as follows:

(a) Since the experimental data, on which this formula is based, was obtained on relatively slow and low rated engines, the results are not applicable to modern high specific output fast running engines.

(b) The thermocouples used for the experiments were relatively crude, which measured the temperature .25 mm. below the surface, where the temperature would be considerably damped. In order to allow for this effect, corrections were made to the recorded temperatures to yield the true metal surface temperature, but the adequacy of these corrections is questionable.

(c) The formula, as it stands, is supposed to include the effects of radiative heat transfer as well as the convective component, but there is no way of allowing for these effects separately for different engine configurations. This makes the formula less flexible for design applications.

(d) The mean piston speed, $V_p$, is supposed to characterise the air movement in the cylinder. This is not a realistic assumption.

(e) The formula is dimensionally incorrect.

In order to overcome some of the objects as outlined above, Pflaum carried out some tests on a 5.9 inch bore x 7.5 inch stroke four stroke precombustion chamber engine. He found that it was necessary to apply correction factors to the basic Eichelberg relationship to fit the results obtained. The modified formula was as follows:

$$\frac{q}{A} (T - T_w) = f(P_L) f(V_p)(p \frac{T}{2})^{\frac{1}{2}}$$ ......... (1.2)
\[
\left((V_p) = 3.0 \pm 2.57 \left[ 1 - \exp \left( 1.5 - 0.127 V_p \right) \right]\right)
\]

Positive sign for \( V_p > 11.8 \text{ ft/sec} \)
and negative sign for \( V_p < 11.8 \text{ ft/sec} \)

\[
\left((P_L) = 1.1 + 3.66 \frac{(P_L - P_o)}{P_o} \right) \quad \text{(for piston and cylinder head)}
\]

\[
\left((P_L) = 0.36 + 1.2 \frac{(P_L - P_o)}{P_o} \right) \quad \text{(for cylinder liner)}
\]

where \( P_L \) is the scavenging pressure and \( P_o \) the inlet pressure.

These factors were later modified by Pflaum to the following

\[
\left((V_p) = 6.9 - 5.9 \times 4.5^{-0.098 V_p} \right)
\]

\[
\left((P_L) = 2.3 P_L^{0.25} \right) \quad \text{(for piston and cylinder head)}
\]

\[
\left((P_L) = 0.8 P_L^{0.66} \right) \quad \text{(for cylinder liner)}
\]

The aim of these modifications was to give the formula

greater flexibility by substituting different values for the factors

given above.

Henzin\textsuperscript{7} more recently substituted a calculated instantaneous
gas velocity, instead of the mean speed, in the Eichelberg formula
and reported better agreement with his experimental results.

There was no experimental proof that the calculated velocity was
correct. A comparison of these results is shown in Fig. 1.1.

At about the same time as the earlier proposals by Eichelberg\textsuperscript{3},
Nusselt\textsuperscript{8} proposed a formula which was based on hot bomb studies.

He gave separate terms for convective and radiative heat transfer.
The proposed formula was

\[ q/A = 5.24 \times 10^{-5} (1 + 0.38 V_p)(p^2 T)(T - T_w) \]
\[ + 2.06 \times 10^{-13} (T^4 - T_w^4) \]

\[ \ldots \ldots (1.3) \]

The first term on the right-hand side gives the convective and the second term gives the radiative components of heat transfer.

Brilling\(^9\) modified the Nusselt formula by replacing the term 
\((1 + .38 V_p)\) by \((2.45 + .056 V_p)\).

These formulae due to Nusselt and Brilling are unacceptable because of the following reasons:

(a) In the radiation term, the flame radiation which is by far the more important component of the radiative heat transfer, is neglected.

(b) In the convective term, the exponent of the temperature is positive although it is well known that the heat transfer coefficient decreases with increasing temperatures.

(c) Use of the mean piston speed to characterise the gas movement inside the cylinder.

Elser\(^10\) used dimensional analysis to produce the following formula:

\[ Nu = 6.5 \left(1 + 5 \frac{\Delta S/C_p}{R_e P_r}\right)^{\frac{1}{2}} \]

\[ \ldots \ldots (1.4) \]

Here again, mean piston speed was used to calculate the Reynolds number at various points in the engine cycle. Although the thermocouples used in the experimental investigation were not true surface temperature measuring types, the experimental techniques used were better than those used by Eichelberg.
Comparison between heat transfer coefficients between Eichelberg and Henein (from Henein7)

FIGURE 1.1

Heat transfer coefficient correlation, Elser's data, two stroke engine. Different symbols indicate different test conditions. (From Annand13)
Elser reported that the equation (1.4) gave good agreement with
the results obtained on a two stroke engine but the agreement with
four stroke engine result was not satisfactory.

Oguri\textsuperscript{11} made heat transfer measurements on a four stroke
spark-ignition engine with a 114.3 mm bore x 140 mm stroke. The
thermocouples used were true surface temperature measuring types.
In the analysis of results he adopted the dimensional approach as
developed by Elser\textsuperscript{10} and suggested the following modified formula:

\[
Nu = 1.75 \left( 1 + \frac{\Delta S}{c_p} \right) \left( Re Pr \right)^{\frac{1}{2}} \left[ 2 + \cos(\theta - 2\alpha) \right] \quad \ldots \ldots (1.5)
\]

The modification consists of the multiplication of term $\left[ 2 + \cos(\theta - 2\alpha) \right]$ to the right-hand side of equation (1.4). This was intended to
represent the variation of piston speed with crank angle and hence
to some extent, the gas motion.

A detailed study of the problem of unsteady heat transfer was
carried out at the University of Wisconsin by Overbye and others\textsuperscript{12}.
Thin film type thermocouples were used for the surface temperature
measurement. The engines used for the tests were small four stroke
spark ignition engines. Motored as well as fired engine cycles
were analysed. An electronic computer was available for the
analysis of results which greatly increased the scope of the
harmonic analysis. Thus the effects of number of harmonics used
in the harmonic analysis could be investigated. Initial attempts
at analysing the results in the light of Eichelberg and Elser was
abandoned because of difficulties associated with the wide
fluctuations of the heat transfer coefficient through the engine
cycle. An empirical formula was finally produced for the motored
cases.
\[
q/L/3600A_k i T_i = (L V_p P_i C_p i/k_i)(\cdot 26 P/r P_i - 0.035)\times 10^{-4} + 1 P/r P_i - 0.02 \quad \text{(1.6)}
\]

Here the suffix 'i' refers the gas properties to the air inlet manifold conditions. It is interesting to note that although the right-hand side expression appears to be quite complex, the main variable is the pressure term, because all the other parameters will be substantially constant, for a given engine speed and load condition. This formula has also been criticised on the grounds that the wall temperature does not appear in it, hence some heat transfer rate will be predicted, for a given manifold condition, whatever the wall temperature. This criticism is only partially valid. As pointed out above, the expression on the right-hand side is pressure dependent which in turn determines the gas temperature inside the cylinder and hence the wall temperature. Therefore, although there is no direct mention of the wall temperature, it is implied by the presence of the pressure term.

To calculate the heat transfer in fired cases it was proposed by Overbye, to multiply the right-hand side by the ratio of the differences between the gas and wall temperatures for the fired and motored cases.

Annand in a recent publication has reviewed all the past significant developments in the field of unsteady heat transfer in the internal combustion engine. He concluded that all the formulae failed to meet the requirements. He criticised Eichelberg, Nusselt, Brilling, Chirkov and Stefanovski on the grounds that their formulae were dimensionally incorrect and hence could not be relied upon for extrapolation to conditions far removed from those
of the experiments upon which they are based.

Elser and Oguri were criticised on the grounds of incorrectly replacing a dimensionless group:

$$\left( \frac{q}{\rho C_p N^2} \right)$$

by

$$\Delta s / C_p$$

in their formulae. His main objection to Overbye's work was the lack of allowance for the wall temperature variation. Based on these conclusions he re-analysed Elser's experimental data and proposed a new empirical expression, which is

$$q/\Delta A = a \frac{k}{D} (Re)^b (T - T_w) + c (T^4 - T_w^4) \ldots \ldots (1.7)$$

with \( b = .7 \)

It was proposed that the factor 'a' varied with the intensity of charge motion and suggested that a value of between .35 and .8 should be selected, depending on the intensity of charge motion in the region being considered.

Suggested values of c were as follows:

During compression, \( c = 0 \)

During combustion and expansion

\( c = 1.6 \times 10^{-12} \) for diesel engines

\( c = 2.1 \times 10^{-13} \) for spark ignition engines.

The gas properties are to be evaluated at the average charge temperature which is derived from the cylinder pressure diagram.

It is clear that this was an attempt to produce a flexible and dimensionally correct formula for the unsteady heat transfer in internal combustion engines. The first term on the right-hand side is the usual Nusselt Reynolds forced convection term and the
second term gives the contribution due to flame radiation.

This formula suffers from the same weakness as the other formulae of this kind, namely, the use of mean piston speed as the velocity term in the calculation of Reynolds number. Moreover, the limits specified for the constant 'a' are quite wide, i.e., .35 to .8, which is more than can be tolerated if the formula is going to be useful.

Annand re-analysed the data of Elser and plotted it in a suitable form to draw comparison with his proposed relationship. These graphs are reproduced in Figs. 1.2 and 1.3. The value of Reynolds number exponent is fixed at .7 for the straight line shown. It is clear from an examination of these graphs that for a logarithmic plot the scatter is excessive and there is no justification in drawing the line of .7 slope to represent these results. Also, as has been pointed out by Knight in the discussion of Annand\textsuperscript{13}, the data of Taylor and Toong\textsuperscript{14} reproduced by Annand as shown in Fig.1.4, shows a variation of ±30% from the mean line. This is considered to be excessive for engine design purposes.

Another interesting approach was proposed by Alcock\textsuperscript{15}. The time mean heat transfer rate was related to the fuel flow rate by a power law. Brock and Glasspoole\textsuperscript{16} have used this relationship in correlating their experimental results and report good agreement. It is difficult to imagine how a formula of this type can be made to represent the heat transfer. The effects of poor fuel injection, inadequate airflow and the calorific value of the fuel are some of the variables which could alter the fuel flow substantially and the effect of these on the heat transfer is difficult to represent by a simple relationship as proposed in this case. Moreover, the limits of variation specified for the exponent are quite wide. For
FIGURE 1.3
Heat transfer coefficient correlation; Elser's data four stroke engine.
Different symbols indicate different test conditions.
(From Annand13)

FIGURE 1.4
Collected data of Taylor and Toong. Re-interpreted by Annand; lines drawn at a slope of 0.7.
(From Annand13)
example, French\textsuperscript{17} has suggested the use of this relationship for design purposes.

\[ \text{Heat Flux} = \text{Constant} \times (\text{Fuel Flow})^n \]

where \(n\) can vary between .7 to 1.25.

Clearly this is too wide a variation for design purposes.

Another group of workers have proposed the use of pipeflow heat transfer data in the internal combustion engine heat transfer calculations. Knight\textsuperscript{18}, in a very significant paper, outlined a method for calculating the instantaneous gas velocities in various parts of the engine cylinder and then applied the pipeflow heat transfer data for gases to calculate the heat transfer. Some initial experimental work was carried out on a \(\frac{4}{2}\) in bore x \(\frac{5}{2}\) inch stroke direct injection engine with speeds varying in the range 750-1500 r.p.m. and the load in the range 40-90 lb/sq.in B.m.p. Instantaneous heat flux measurements were not carried out but heat fluxes were multiplied by the exposed wall area and integrated throughout the engine cycle for comparison with the experimental values obtained as follows:--

\[ \text{Heat in} - \text{Heat to shaft work} - \text{Heat to friction} - \text{Heat to exhaust} \]

The results indicated that the ratio between the experimental to calculated heat fluxes varied between 1.52 to 3.06 in the extreme cases. Some later work on a .5 litre/cylinder Ricardo engine gave better agreement with a modified calculation. The modification consisted of assuming a wall roughness of about .5 mm. in the antechamber and 1 mm. in the cylinder; this resulted in enhancing the heat transfer coefficient for equivalent roughness in pipeflow heat transfer formulae.

Annand in his criticism of this work\textsuperscript{18} considered the wall roughness assumed to be excessive and suggested that the differences
could be due to a different pattern of flow than that assumed in the gas velocity calculations. Some further experimental work by Knight\textsuperscript{18}, using a British Cast Iron Research Association thermocouple (which measured the surface temperature approximately \(0.005\)" below the surface), was carried out on a Ricardo El6 research engine with 4\(\frac{1}{4}\) inch bore x 5\(\frac{1}{2}\) inch stroke and a compression ratio of 22. Both motored and fired tests were carried out. The gas temperature in the motored cases was measured by using a \(7\mu\) diameter tungsten wire as a resistance thermometer. Seventy-one harmonics were evaluated in the analysis of the surface temperature record. The experimental results agreed well with the calculated values in the motored cases but in the fired cases the agreement was not as close. This paper is considered to be a significant contribution to the vast amount of literature available on the subject of i.c. engine unsteady heat transfer. The reasons for this being so are considered here briefly.

(a) An attempt had been made at actually calculating the gas motion inside the engine cylinder. This is significantly different to the approach made by other workers which was aimed at producing empirical formulae from gas pressure measurements only.

(b) The gas temperature was directly measured in motored cases instead of being calculated from the cylinder pressure diagram as had been the practice in the previous cases. Woschin\textsuperscript{19} has also suggested the use of pipeflow heat transfer data to predict the heat transfer in the internal combustion engines. He derived the following formula for the instantaneous heat transfer coefficient.
He showed that the calculated values for the heat transfer coefficient agreed better with the experimental results of Kind\textsuperscript{20} than those of Eichelberg and Nusselt. These results are shown plotted in Fig. 1.5. The relationship defining 'W' was based on the experimental results of Ulsamer\textsuperscript{21} and Wenger\textsuperscript{22}. Ulsamer studied gas flow speeds in a low speed compressor and Wenger in the cylinder of a motored BMW-V16 aero-engine. Ulsamer gave the following relationship between the mean gas and the mean piston speed.

\[ W = 5.5 \frac{V_p}{m/sec}. \]

The experimental results of Ulsamer and Wenger are plotted in Fig. 1.6.

Although this approach by Woschani is in the right direction, i.e., using gas velocities instead of the mean piston speed, it is still very far removed from reality. If the pipeflow heat transfer data is going to be used, it is essential that correct gas velocity should be used in the relationship.

Sitkei\textsuperscript{23} also started with the basic relationship

\[ Nu = \text{const.} \times \frac{R_e}{n} \]

and obtained the following expression for the convective heat transfer coefficient:

\[ h = 0.033 (1 + b)(P^{-0.7} \times V_p^{0.7} / T^{0.2} \times d_e^{0.3}) \]
FIGURE 1.5
Curves of heat transfer coefficient due to Eichelberg Nusselt and Woschani. Comparison with Kind's experimental data.
(From Woschani\textsuperscript{19})

FIGURE 1.6
Mean gas speed during a cycle as a function of mean piston speed.
Data of Ulsamer and Wenger
(From Woschani\textsuperscript{19})
can be drawn that if the heat transfer rate is related to the gas motion the maximum velocity must also occur at this point. This fact is also supported by the velocities calculated by Knight\textsuperscript{18}, Henein\textsuperscript{6}, and measured by Semenov\textsuperscript{24}. If any function is developed with the instantaneous piston speed as one of the parameters, it is clear that at the TDC the piston is stationary, hence this function will also be zero. From the foregoing reasoning it is clear that there is a great need for fundamental data on gas motion inside the engine cylinder. Armed with this information it may be possible to adopt a more basic approach to the problem of engine unsteady heat transfer than has been the case in the past. With this information on measured velocities it may be possible to calculate the mean velocity in any given section of the engine cylinder from engine geometry. This will be a great advance on the present situation because these velocity calculations could then be carried out at the engine design stage and heat transfer to various parts estimated.

1.7. Effects of Viscosity

1.7.1. In various formulae proposed, for example, Eichelberg\textsuperscript{3+4}, Nusselt\textsuperscript{8}, Brilling\textsuperscript{9}, and others of this type, the heat transfer coefficient turns out to be proportional to the gas temperature with a positive exponent. It is well known that the viscosity of gases increases with an increase in the gas temperature. The viscosity of air may be represented by the well known Sutherland formula\textsuperscript{25}.

$$\mu = \mu_0 \left( \frac{T_0 + C}{T + C} \right)^{3/2} \left( \frac{T}{T_0} \right)$$
The factor .033 was based on results from a motored engine. The constant 'b' was intended to represent the effects of a particular engine configuration. For example:

Combustion chambers with direct injection ....... b = 0.0 - 0.15
Turbulence chamber engines ...................... b = 0.15 - 0.30
Prechamber engines .............................. b = 0.26 - 0.40

'd_e' is the 'equivalent cylinder diameter' determined by the following expression

\[ d_e = \frac{2D_h}{D + 2h_e} \]

where 'h_e' = height of the cylinder space above the piston.

Some calculations were performed for 115 mm. bore x 140 mm. stroke compression ignition engine running at 1035 r.p.m., but no experimental verification was carried out.

Separate relationships were given for gas and flame radiation.

The approach adopted by Sitkei is very similar to that of Woschini and the same criticism applies here.

1.6. On the Use of Mean Piston Speed as the Velocity Parameter

1.6.1. In the majority of formulae discussed so far with the exception of Knight¹⁸ and Henien⁶, mean piston speed has been used in one form or another to calculate the unsteady heat transfer. This is not considered to be realistic.

1.6.2. If the piston velocity has to be used, it is considered that a better, although still unsatisfactory, criterion would have been the use of instantaneous piston velocity. This again presents some difficulties because the instantaneous speed is maximum at midstroke but the maximum unsteady heat transfer generally occurs at or near the TDC on the compression stroke. This is supported by all the experimental work to date. The immediate conclusion
where \( \mu_0 \) is the viscosity at a reference temperature \( T_0 \) and \( C \) is a constant. This may be approximated by the following relationship:

\[
\frac{\mu_0}{\mu} = \left( \frac{T}{T_0} \right)^{0.71}
\]

This means that, all other parameters remaining constant, Reynolds number decreases due to an increase in the viscosity at high temperatures. Again, using the Nusselt Reynolds relationship for the heat transfer, it is clear that the heat transfer rate should also decrease. The reverse tends to happen in the formulae under discussion because of the positive exponent of the gas temperature. It is therefore concluded that the effects of viscosity on the heat transfer are not allowed for in these formulae.

1.8. Comparative Effects of Radiation and Forced Convection

1.8.1. It has generally been assumed that radiative component of the unsteady transfer compared with the convective component is of the order of 20% in large compression ignition engines, and even smaller in high speed spark ignition engines. No direct evidence is available for this assumption. Ebersole and others\(^{26}\), in a recent study, showed that the time averaged radiant heat transfer accounts for 5-10% of the total heat transfer at low engine load and as much as 35-45% at near maximum load conditions. These results are shown in Fig. 1.7. They have not as yet been verified by any other publication on this subject.

Most workers have tried to allow for the effects of radiation separately (8, 9, 13, 23\(^{85}\)). The general form is as follows:

\[
\frac{q_{\text{radiation}}}{A} = \text{Constant} \left( T^4 - T_w^4 \right)
\]
FIGURE 1.7
Heat flow ratios for diesel fuel and heptane with clean and sooty thermocouple surface
(Ebersole and others26)

FIGURE 1.8
Thermal efficiency v engine speed
(From Walker27)
Sitkei has allowed for gas and flame radiation separately. One of the practical difficulties involved in studying the radiative heat transfer is the difficulty of measuring flame temperatures and flame thicknesses. The most convenient way of studying this phenomenon would be to solve the problem of convective heat transfer and then estimate the pure radiative heat by difference from the total heat transferred. The flame radiation can be eliminated by conducting the tests on a motored engine. The gas radiation in this situation may safely be neglected because the gas temperatures in this case, compared with a fired case, will be appreciably lower.

1.9. Heat Transfer Coefficient - Effects of Gas Temperature and Pressure

Writing

$$\rho = \frac{P}{R T}$$

Various expressions for the heat transfer coefficients can be reduced to the following basic proportionality between the heat transfer coefficient and gas temperature pressure, and $V_p$ which are the main variables in a majority of the formulae discussed.

$$h \propto P^a T^b V_p^c$$

The values for $a$ and $b$ for various formulae are given in Table I. A study of this table shows that for a given mean piston speed, gas temperature and pressure the constants of proportionality for the different formulae, to give the same heat transfer coefficient, will have to differ quite considerably. It is also clear that, once these constants are fixed, the shape of heat transfer coefficient curves for a complete engine cycle will be different for these formulae. This again shows the unsatisfactory state of agreement between various existing formulae.
TABLE I

<table>
<thead>
<tr>
<th>Formula</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Eichelberg</td>
<td>.5</td>
<td>.5</td>
<td>.33</td>
</tr>
<tr>
<td>2. Pflaum</td>
<td>.5</td>
<td>.5</td>
<td>-</td>
</tr>
<tr>
<td>3. Nusselt Brilling</td>
<td>.66</td>
<td>.33</td>
<td>1</td>
</tr>
<tr>
<td>4. Elser Oguri</td>
<td>.5</td>
<td>.85</td>
<td>.5</td>
</tr>
<tr>
<td>5. Annand</td>
<td>.7</td>
<td>-1.19</td>
<td>.7</td>
</tr>
<tr>
<td>6. Woschani</td>
<td>.786</td>
<td>-.525</td>
<td>.786</td>
</tr>
<tr>
<td>7. Sitkei</td>
<td>.7</td>
<td>-.2</td>
<td>-.7</td>
</tr>
</tbody>
</table>

Comparison of Various Existing Formulae

Walker has investigated the effects of various heat transfer formulae on the cycle calculations. He compared five of the existing formulae. These are:

(i) Nusselt
(ii) Tijen
(iii) Eichelberg
(iv) Brilling 1
(v) Brilling 2

The speed range under investigation was quite wide, i.e., 500 to 6000 rev/min. Results showed that heat transfer had a significant detrimental effect on engine performance but the effect diminished at high speeds. There were appreciable differences among the performances calculated from five different equations but up to 2000 r.p.m. Nusselt, Eichelberg and Tijen gave similar results.

Thermal efficiency computed for the speed range, using five different
FIGURE 1.9
Heat transferred per cycle (Walker²⁷)

FIGURE 1.10
Heat transfer coefficient for burnt gas v crank angle at 1500 rev/min. (From Walker²⁷)

FIGURE 1.11
Heat transferred per minute during compression and expansion v engine speed - four stroke cycle (From Walker²⁷)
formulae is shown in Fig. 1.8. An examination of these graphs shows that at the lowest speed the computed thermal efficiency could vary by as much as 7% between the extreme case. This is equivalent to a variation of approximately 20% for a mean value of 30% for the thermal efficiency. Fig. 1.9 shows the heat transferred per engine cycle. Here again the discrepancy between the extreme cases at low speeds is excessive, i.e., at the lowest speed Brilling gives approximately 60 cal/cycle and the highest value at the same speed is given by Eichelberg, which is approximately 180 cal/cycle. Even at the highest speeds the difference between the total heat transferred /cycle according to Brilling is approximately 20 cal/cycle and the value obtained according to Nusselt formula is approximately 100 cal/cycle. The heat transfer coefficient for the unburnt gas is shown plotted in Fig. 1.10.

The minimum value at the TDC given by Brilling is approximately 80 cal/m² °C sec and the maximum at the same point in the engine cycle is given by Tijen, which is approximately 330 cal/in² sec °C sec. Heat transferred per minute as shown in Fig. 1.11 also varies quite considerably for various formulae. The variations of these results have illustrated quite clearly that a more basic approach to the problem of unsteady heat transfer in I.C. engines is essential in order to produce reliable methods of engine cycle calculations.

1.11. Summary of Discussion and Conclusions

In the foregoing discussion all the important existing formulae were discussed and criticised on various grounds. These formulae may be divided into three main categories.

1.11.1. Empirical Approach

Formulae due to Eichelberg, Pflaum, and other similar relationships may be classified under this heading.
1.11.2 **Dimensional Approach**

Annand, Elser and others belong to this category.

1.11.3 **Pipe Flow Heat Transfer Data**

Knight and Woschani are the two recent attempts in this field.

1.11.4 **Choice for the Present Study**

The weaknesses of the empirical formulae have already been discussed in detail. Most of these are not applicable when the type of engine is different from those engines on which the supporting experimental work for these formulae was carried out. This applies to most modern engines, which are vastly different from the large slow running engines on which original tests were carried out.

Dimensional approach appears more hopeful but the main difficulty about the lack of any realistic gas velocity to be used in the calculation of Reynolds number, remains.

Finally the approach made by Knight\(^1\), namely the calculation of gas motions in different parts of the engine cylinder and then applying the pipeflow heat transfer data, was considered to be the most promising one. Since there had been no direct attempt to verify the correctness of use of pipeflow heat transfer data, it was considered that further research work on these lines would provide information.

1.12 **Conclusions**

1.12.1 As a result of the literature survey it became clear that in order to obtain really useful experimental data and throw some more light on the complex problem of unsteady heat transfer in I.C. engines, it was necessary to obtain the following measurements for a complete engine cycle.
(a) Instantaneous gas velocity
(b) Instantaneous gas temperature
(c) True metal surface temperature
(d) Gas pressure

1.12.2. In view of (a), the difficulties involved with measurement of gas velocity and temperature in the presence of a flame, (b) the limited time available and (c), relative importance of the convective component of heat transfer, it was decided to investigate only the forced convective component of heat transfer.

1.12.3. A motored engine was considered to be most suited for this type of investigation.

1.12.4. It was decided to make heat transfer measurements in order to check the validity of pipeflow heat transfer data in relation to the I.C. engine heat transfer problems.
CHAPTER 2

THEORY OF TRANSIENT HEAT CONDUCTION
 THEORY OF TRANSIENT HEAT CONDUCTION

The equation of heat conduction in three dimensions for a solid with constant thermal properties may be derived by considering a parallelepiped of differential size. This is shown in Jakob\textsuperscript{28} to be:

\[
\frac{\partial T}{\partial \theta} = \delta \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)
\]  

..... (2.1.)

This ignores any internal heat generation in the solid considered and assumes constant thermal properties.

Where

- \( T \) is the temperature
- \( \theta \) is the time
- \( \delta \) is the thermal diffusivity of the solid

x, y and z are the distances along three mutually perpendicular axes. In the context of the present study, this coordinate system is shown in Fig. 2.1. The x-axis is chosen perpendicular to the surface, y-axis in a direction parallel to the axis of the pre-combustion chamber and z-axis is along the surface. In the case of the internal combustion engine the metal surrounding the working gases is of interest. Considering an area of metal where there are no sudden changes of cross section, the heat flow may be considered to take place in a direction perpendicular to the wall, i.e., in the direction of x-axis. In the present study the thermocouple was installed in a cross section of metal which was 1" x 1" square as shown in Fig. 2.2. Since the width of the cross section was equal to its depth, the assumption of one dimensional conduction was verified experimentally. The temperature distribution and heat flux lines are shown in Figs. 2.3 and 2.4 for the case of one dimensional and two dimensional heat conduction for the present study.
COORDINATE SYSTEM

FIG. 2.1

INSTALLATION OF THERMOCOUPLE

FIG. 2.2
It will be shown later that the heat flux in the case of transient surface temperature variation is composed of a mean heat flow and a transient component. The transient component is unaffected because the depth at which the surface temperature was measured in the present study was approximately 1/2 microns and it is reasonable to assume that the transient component of heat flux is one-dimensional at the surface. The mean heat flow may, on the other hand, be affected by two-dimensional heat conduction. In order to check the validity of one-dimensional heat conduction, metal temperatures were measured at points A and B as shown in Fig. 2.4., and the mean heat fluxes were calculated using these measured metal temperatures. If the assumption of one-dimensional heat flux is correct the mean heat fluxes calculated by using mean metal temperatures at points A and B should be the same within the limits of accuracy of the measuring thermocouples. Since under steady engine conditions the mean heat flux is given by

\[ \frac{q}{A} = \frac{K(T_1 - T_2)}{x_2} = \frac{K(T_1 - T_3)}{x_3} \quad \ldots \quad (2.2) \]

where \( x_2 \) and \( x_3 \) are the respective depths at which the metal temperatures were measured; \( T_1 \) is the mean metal surface temperature measured by the surface thermocouple.

From equation 2.2, we have

\[ \frac{T_1 - T_2}{T_1 - T_3} = \frac{x_2}{x_3} \quad \ldots \quad (2.3) \]

In the experiment \( x_2 = 1" \) and \( x_3 = 1" \). Therefore, if the assumption of one-dimensional heat flow holds the ratio \((T_1 - T_2)/(T_1 - T_3)\) should be

\[ \frac{x_2}{x_3} = \frac{1}{4} \]
ONE DIMENSIONAL HEAT CONDUCTION

FIG. 2-3

TWO DIMENSIONAL HEAT CONDUCTION

FIG. 2-4
Measured values of $T_1$, $T_2$ and $T_3$ gave the following result:

$$\frac{T_1 - T_2}{T_1 - T_3} = \frac{13}{51.3} = \frac{1}{3.95}$$

i.e., to within 1.25% of the ideal, therefore it was concluded that the assumption of one dimensional conduction was well justified.

Thus equation 2.1 reduces to the Fourier heat conduction equation in one dimension.

$$\frac{\partial T}{\partial \theta} = \delta \frac{\partial^2 T}{\partial x^2}$$

Can slaw and Jeager have treated this equation in great detail, giving solutions for numerous practical cases in which variations of $T$ with $\theta$ is a known function.

Fourier's heat conduction equation for the case of transient heat transfer in a semi-infinite plate can also be solved by the method of finite differences. Methods of Binder and Schmidt are discussed in detail in Mcadams. These methods are suitable for graphical solution but since steps in both distance and time must be taken, the procedure would be extremely tedious for the case of engine calculations.

Ideally an analogue method which directly converts the surface temperature history into heat flux history would be most suitable. This would eliminate the recording and processing of the metal surface temperature variation which can be very time consuming, if a large number of records have to be processed. Skinner has used such an analogue method of solution successfully. The electrical analogue is based on the observation that the governing equation for heat conduction in a semi-infinite solid is identical to that for a semi-infinite electrical line with distributed series resistances and shunt capacitors. Consider the similarity between Fourier's equation
\[
\frac{dT}{d\theta} = \delta \frac{d^2T}{dx^2} \\
\text{and} \quad \frac{dV}{d\theta} = \frac{1}{RC} \frac{d^2V}{dx^2}
\]

which is the equation giving the distribution of potential in an idealised transmission line.

\[V\] is the potential

\[R\] is the resistance of the line

\[C\] is the capacitance of the line.

Thus the term \(\frac{1}{RC}\) is analogous to \(\delta\) and in principle a complete simulation of heat transfer in a semi-infinite solid requires a semi-infinite line of resistances and capacitors. In practice the thermal problem can be represented adequately by a finite number of elements. This type of system is basically a filter and consequently it attenuates the input signal; hence it is more suitable for a high signal to noise ratio input. Skinner\(^3\) used the method successfully to obtain the transient heat flux directly in the case of shock tube wall heat transfer. The analogue results were compared with the digital computer solution in which the Fourier equation was numerically integrated. A comparison of the two solutions is shown in Fig. 2.5. It can be seen that agreement between the two solutions is extremely good. A five section network was used for these tests and errors of less than 1% were estimated over a frequency range of 8-8000 cps.

Signal to noise ratio was 20 for a heat flux of 20 BTU/ft\(^2\)sec. With further refinement of the circuit it was possible to measure heat flux as low as .02 BTU/ft\(^2\)sec with a signal to noise ratio of 15-20. This is reported by Hertzberg\(^3\) and others.

In the present study it was decided to use a digital computer mainly because of its ready availability. Although this proved
Comparison of 5-section Analog Network Solution with the Digital Solution (Skinner\textsuperscript{31})
satisfactory, in the absence of any digitising equipment the processing of data proved to be very laborious. Retrospectively, it is considered that development of an analogue circuit would have been well worthwhile.

In the cases where metal surface temperature variation cannot be defined by a simple function, as in the case of internal combustion engine heat transfer, it is usual practice to record the metal surface temperature and synthesise the recorded curve by the use of harmonic analysis. Sokolnikoff has dealt with the problem of fitting a finite trigonometric series to a known waveform. The recorded periodic waveform yields a set of \( n \) equally spaced ordinates. Then the temperature at the metal surface at any time defined by \( \theta \) is given by

\[
T_{o, \theta} = T_0 + \sum_{k=1}^{n/2} (A_k \cos \omega \theta) + \sum_{k=1}^{n/2} (B_k \sin \omega \theta) \ldots \quad (2.3)
\]

where

- \( T_0 \) = the mean of \( n \) temperature ordinates
- \( A_k \) and \( B_k \) are amplitudes of \( k \)th cosine and sine harmonics
- \( \omega \theta \) = The crankangle position at time \( \theta \).
- \( f \) is the frequency of the fundamental waveform.
- \( \omega \) is the angular frequency = \( 2\pi f \)

Thus:

\[
T_0 = \frac{1}{n} \sum_{j=0}^{n-1} T_j \quad \ldots \quad (2.4)
\]

\[
A_k = \frac{1}{n} \sum_{j=0}^{n-1} T_j \cos \left( j \frac{2\pi k}{n} \right) \quad \ldots \quad (2.5)
\]
38.

\[
B_k = \frac{2}{n} \sum_{j=0}^{n-1} T_j \sin \left( j \frac{k \pi}{n} \right) \quad \cdots \quad (2.6)
\]

with

\[k = 1, 2, 3, 4 \ldots \ldots \frac{n}{2} - 1\]

Here \(T_j\) signifies the \(j\)th temperature co-ordinate of \(n\) equally spaced ordinates.

Let

\[k = \frac{n}{2}\]

\[
B_k = B_{n/2} = \frac{2}{n} \sum_{j=0}^{n-1} T_j \sin j \pi \frac{n}{2} = 0 \quad \cdots \quad (2.5)
\]

Hence equation 2.3 may be written as

\[
T_{(0, \theta)} = T_0 + \sum_{k=1}^{n/2} \left( \alpha_k \cos \frac{k \pi \theta}{\ell} + \beta_k \sin \frac{k \pi \theta}{\ell} \right) \quad \cdots \quad (2.6)
\]

This form is more suitable for a digital computer solution. An efficient method of calculating \(\alpha_k\) and \(\beta_k\) on a digital has been suggested by Goertzel. The mathematical details of the method and its application to computer programming is discussed in Appendix 2.1.

If \(\ell\) is the wall thickness

\(T_{\text{out}}\) is the outside wall temperature which is assumed to be constant. Boundary conditions for the solution of equation 2.2.

become

\[x = \ell \quad ; \quad T_{(x=\ell)} = T_{\text{out}}\]

\[x = 0 \quad ; \quad T_{(x=0)} = T_0 + \sum_{k=1}^{n/2} \left( \alpha_k \cos \frac{k \pi \theta}{\ell} + \sum_{k=1}^{n/2-1} \beta_k \sin \frac{k \pi \theta}{\ell} \right)\]

Jakob gives the solution of 2.2 for the case when the surface
temperature variation is given by

\[ T(0, \theta) = \alpha_k \cos kh \omega \theta \]

The solution is

\[ T(x, \theta) = \alpha_k e^{-Fx} \cos (\frac{kh \omega \theta - Fx}{2}) \] .... (2.22)

where

\[ F = \sqrt{\frac{kh \omega}{2}} \]

Similarly, a sinusoidal surface temperature variation will induce the following temperature profile

\[ T(x, \theta) = \beta_k e^{-Fx} \sin (\frac{kh \omega \theta - Fx}{2}) \] .... (2.23)

By superposition of equations 2.22 and 2.23, the transient temperature distribution in the metal wall is given by

\[ T(x, \theta) = \sum_{k=1}^{n/2} e^{-Fx} \left[ \alpha_k \cos (\frac{kh \omega \theta - Fx}{2}) + \beta_k \sin (\frac{kh \omega \theta - Fx}{2}) \right] \] .... (2.24)

The steady state temperature distribution is given by

\[ T(x, \theta) = T_0 - (T_0 - T_{ss}) \frac{x}{\ell} \] .... (2.25)

Adding equations 2.25 and 2.24 we have the temperature distribution in the wall.

\[ T(x, \theta) = T_0 - (T_0 - T_{ss}) \frac{x}{\ell} + \sum_{k=1}^{n/2} e^{-Fx} \left[ \alpha_k \cos (\frac{kh \omega \theta - Fx}{2}) + \beta_k \sin (\frac{kh \omega \theta + Fx}{2}) \right] \] (2.25)

By examining the above equation, the following conclusions can be drawn:
(a) The basic shape of the temperature fluctuation at various values of x remains unchanged. The amplitude of the temperature fluctuation is damped due to the presence of the exponential term $e^{-Fx}$.

Since

$$F = \sqrt{\frac{k \omega}{2 \delta}} ; \quad e^{-Fx} = \frac{1}{e^{\sqrt{k \omega/2 \delta} x}}$$

This means that for a given 'x' there is greater damping of the higher harmonics. Also at higher speeds greater damping of the temperature profile will result.

(b) There is a phase lag between the true surface temperature variation and the recorded temperature. The phase lag (given by Fx) is directly proportional to the depth at which the temperature is measured.

It can be shown that at

$$x = 4.6 \sqrt{\frac{2 \delta}{k \omega}}$$

the amplitude of the fundamental harmonic will be reduced to $\frac{1}{100}$th its amplitude at the surface and for the kth harmonic, the depth at which its amplitude is reduced to $\frac{1}{100}$th is

$$x = 4.6 \sqrt{\frac{2 \delta}{k \omega}}$$

Thus substituting some typical values, for the present study

$$x = 4.6 \sqrt{\frac{2 \times 1}{2 \pi \times 10}} = 0.0148 \text{ cm.}$$

Therefore the assumption of a semi-infinite solid is well justified.

Term by term integration of a Fourier series is allowed but great caution has to be exercised in carrying out term by term differentiation because of the possibilities of slow converging or even diverging series
resulting from such differentiation. Canslaw and Jeager have observed that equation of type 2.25 may be differentiated in this manner because of the presence of term $e^{-Fx}$. This has a damping effect and acts as a converging factor. The temperature gradient is therefore obtained from equation 2.25 by differentiation.

$$\left( \frac{dT}{dx} \right)_{x=0} = - \left( \frac{T_m - T}{L} \right) + \sum_{k=1}^{n/2} F e^{-Fx} \left[ \left( \frac{\beta_k - \alpha_k}{\ell} \right) \sin k\omega \varphi + \left( \frac{\beta_k + \alpha_k}{\ell} \right) \cos k\omega \varphi \right] \ldots \text{(2.26)}$$

The instantaneous heat transfer at the surface or heat flux $H$ can be obtained by putting $x = 0$ and multiplying by $K$ the conductivity of the material.

$$\frac{q}{A} = H = K \left( \frac{T_m - T}{L} \right) + K \sum_{k=1}^{n/2} F e^{-Fx} \left[ \left( \frac{\beta_k - \alpha_k}{\ell} \right) \sin k\omega \varphi + \left( \frac{\beta_k + \alpha_k}{\ell} \right) \cos k\omega \varphi \right] \ldots \text{(2.27)}$$

The instantaneous heat transfer coefficient is then given by

$$h = \frac{H}{(T_{g\infty} - T)}$$

where $T_{g\infty}$ is the instantaneous gas temperature away from the wall.

A closer examination of equation 2.27 shows that the heat flux may be considered to have two components. The first term on the right-hand side is the steady state heat transfer. This is the value that would be obtained from a heat balance on the engine. The second term on the right-hand side is the transient component. Since the instantaneous heat transfer is the sum of these two, it may be negative, from metal to gases, or positive, i.e., from working gases to the metal. The overall contribution of the transient component is zero because $n$ is an integer. It can be seen that due to the presence of $F$ in the transient term, it is proportional to $\sqrt{k}$, the root of the harmonic number. Thus higher harmonics will contribute proportionately more than the lower harmonics. Generally the amplitude of the first
few harmonics is quite large in comparison with the higher harmonics, but due to the presence of $\sqrt{k}$ factor it is important to include higher harmonics in the analysis.

**Conclusions**

1. Assumption of one dimensional heat conduction in the metal wall was checked experimentally and found to be valid.

2. It was decided to use a digital computer for the analysis of metal surface temperature records to obtain the instantaneous heat fluxes at various points in the cycle; mainly due to its ready availability. Due to the absence of any on-line computing facilities or digitising equipment, it is considered, retrospectively, that the development of an analogue circuit would have been well worthwhile.
CHAPTER 3

THEORY OF UNSTEADY HEAT TRANSFER IN I.C. ENGINES
CHAPTER 3

THEORY OF UNSTEADY HEAT TRANSFER IN I.C. ENGINES

In Chapter 2 the problem of transient heat transfer was considered purely from the point of view of heat conduction in the solid wall. Now an overall approach to link the gas temperature variation with the wall temperature variation will be developed.

Jakob \(^2\) has treated the case of periodic change of the temperature of a medium in contact with the plane surface of an infinitely thick plate.

The temperature of the environment was assumed to fluctuate according to the following relationship:

\[
T_g = T_{ag} \cos \omega \theta 
\]

where

- \(T_g\) is the instantaneous temperature of the gas
- \(T_{ag}\) is the amplitude of the excess of the temperature of environment above the time average value \(T_{gm}\)
- \(\theta\) is the time

Defining \(b = \frac{h_0}{k_w}\)

the solution of equation \((3.2)\) gives the temperature distribution in the plate.

\[
\frac{\partial^2 T_w}{\partial \theta^2} = \delta \frac{\partial^2 T_w}{\partial x^2} 
\]

with the boundary conditions:

(i) At \(x = 0\); \(T_g = T_w\)

(ii) \(-k_w \frac{\partial T_w}{\partial x} = h_o \left( T_g - (T_w)_{x=0} \right)\)
The temperature distribution in the plate is given by

$$T_x = T_a q \eta e^{-m^2} \cos (\omega \theta - \varepsilon - mx) \quad \cdots \quad (3.3)$$

where

$$\eta = \left( \frac{1}{1 + 2m/b + 2m^2/b^2} \right)^{1/2} \quad \cdots \quad (3.4)$$

$$m = \sqrt{\frac{\omega}{2b}} \quad \cdots \quad (3.5)$$

$$\varepsilon = \tan^{-1} \frac{1}{1 + b/m} \quad \cdots \quad (3.6)$$

Jakob has tabulated the values of $\eta$ and $\varepsilon$ for various values of $b/m$. Some of these values are reproduced in Table 2.1.

<table>
<thead>
<tr>
<th>$b/m$</th>
<th>$\eta$</th>
<th>$360/(2\pi \varepsilon)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>45°</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0673</td>
<td>42° - 16°</td>
</tr>
<tr>
<td>1</td>
<td>0.447</td>
<td>26° - 34°</td>
</tr>
<tr>
<td>10</td>
<td>0.905</td>
<td>5° - 12°</td>
</tr>
<tr>
<td>100</td>
<td>0.990</td>
<td>0° - 34°</td>
</tr>
</tbody>
</table>

Since

$$b/m = \frac{b}{\sqrt{\omega/2b}} = \frac{\hbar \sqrt{2b}}{k \omega^{1/2}} \quad \cdots \quad (3.7)$$

It can be shown that for steel $b/m < 1$

\[\therefore \text{From the above equation 3.3} \quad \varepsilon = 45°.\]
**Fig-3.1 Heat Flux and Gas Temp. Curves**
(Sinusoidal Gas Temp. Variation)

**Fig-3.2 Heat Flux and Heat Transfer Coeff.**
In other words, the metal wall surface temperature leads the gas temperature by $\frac{\pi}{4}$. Also for small $\frac{b}{m}$ the value of $\eta$ is small, hence the temperature fluctuation in the metal surface is considerably damped and the amplitude of the metal surface temperature variation is $\eta$ times smaller than the environmental variations.

The equation 3.3 may be differentiated to give the surface heat transfer

$$q_v = \sqrt{2} k_w m T_{a9} \eta \cos (\eta \omega \theta - \epsilon + \frac{\pi}{4}) \quad \cdots \quad (3.7)$$

This shows that the surface heat transfer leads the metal surface temperature by $\frac{\pi}{4}$ which is equivalent to $90^\circ$ of the engine crank angle for the first harmonic. These effects are shown in Fig. 3.1 for the first harmonic. It is interesting to note that peak gas temperature occurs at $\omega \theta = 0$ for all $n$ whereas the metal surface temperature and heat transfer peaks occur at $\omega \theta = \frac{\pi}{n}$ and $\omega \theta = (\epsilon - \frac{\pi}{n})/n$.

Thus for $\epsilon = \frac{\pi}{n}$ the gas temperature and the heat transfer are in phase and for high values of $n$ the metal surface temperature peak tends to occur at the same point in the cycle. It is to be noted that the negative Fourier coefficients will cause an $180^\circ$ phase shift and will tend to make the gas temperature and heat transfer go out of phase.

Although this type of analysis does give a better understanding of the unsteady heat transfer phenomenon in I.C. engines, the theoretical model is very far removed from reality and hence the results are not directly applicable. In the foregoing analysis the effects of pressure work in the boundary layer were ignored and also no gas motion was taken into account.

**Differential Equations for Fluid Flow inside an I.C. Engine Cylinder**

The governing differential equations of fluid flow are:
1. The continuity equation
2. The momentum equation
3. The energy equation.

These equations describe the fluid flow with space co-ordinates and time as the independent variables and velocity, temperature, pressure and variable properties of the fluid as dependent variables. Most books on fluid flow give the derivation of these equations in some form or other. Here the notation used by Knudsen and Katz will be followed.

1. The Continuity Equation

The continuity equation expresses the law of conservation of mass mathematically. In three dimensional compressible flow it may be written as

\[ \text{div}(\rho \mathbf{v}) = -\frac{\partial \rho}{\partial \theta} \]  \hspace{1cm} (3.8)

where

\( V \) is the velocity vector
\( \rho \) the density and
\( \theta \) is the time.

\[ \text{div}(\rho \mathbf{v}) = \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} \]

where \( u, v \) and \( w \) are the components of the velocity vector \( V \) in the \( x, y \) and \( z \) directions respectively.

2. The Momentum Equation

The momentum equations of fluid flow are a mathematical expression of Newton's second law applied to moving masses of fluid. This involves equating the inertial forces to the external forces acting on the fluid and the viscous forces.
The general equation for 'x' direction may be written in the vector notation as:

\[ \frac{D u}{D \theta} = \frac{1}{P} \frac{DP}{dx} - \frac{1}{P} \left[ \frac{2}{3} \frac{d}{dx} \left( \mu \text{div} \mathbf{V} \right) - \text{div} (\mu \text{grad} u) \right] \] (3.9)

Where \( P \) is the pressure and \( \mu \) is the viscosity of the fluid.

Field forces are neglected in the above expression.

3. The Energy Equation

The energy equation is based on the law of conservation of energy and gives the temperature distribution in space. It may be written as

\[ \rho C_p \frac{D T}{D \theta} = k \text{div grad} T + \frac{DP}{D \theta} + q' + \phi \] (3.10)

where the total derivative

\[ \frac{D(\_)}{D \theta} = u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} + \frac{\partial}{\partial \theta} \]

\( q' \) = time rate of energy generation in the fluid per unit volume.

\( \phi \) = dissipation function. (The time rate of energy dissipated per unit volume due to viscosity of the fluid). The dissipation function may be neglected for fluids with low viscosity and at subsonic velocities.

The equation of state can be written as

\[ P = \rho RT \] (3.11)

where \( R \) is the universal gas constant.

The equations (3.8), (3.9), (3.10) and (3.11) give a description of the velocity, pressure and temperature distribution.
in the working space of an i.c. engine. This system of equations is far too complex to be solved and hence simplifying assumptions have to be made.

If a system of rectangular co-ordinates is so chosen such that the 'x' direction is perpendicular to the wall, a 'z' direction can be chosen so that the velocity component in that direction can be neglected in comparison with the other two velocity components. If it is further assumed that the heat flow is mainly in x direction due to the existence of a boundary layer on the wall; and also if it is assumed that the pressure is not space dependent, the equation 3.10 simplifies to:

\[ \rho C_p \left( \frac{\partial T}{\partial \theta} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) - \frac{\partial P}{\partial \theta} = K \frac{\partial T}{\partial x} + q' \ldots \ldots \quad (3.12) \]

Further simplification of equation 3.12 was carried out by Pfriem\(^{35}\) and Elser\(^{10}\) with the following assumptions:

1. No convective heat transfer.
2. Frictional effects neglected.
3. Constant specific heat.
4. Constant thermal conductivity.
5. One dimensional heat transfer.
6. Density independent of temperature and pressure.

It is obvious that with the exception of assumption 5, the model is not representative of engine conditions. It was assumed that the non-steady temperature distribution was produced by either:

(a) small variations of pressure with time, or
(b) time dependent small heat sources within the gas (combustion or chemical changes).

In the following case, interpretation (b) will be used.
With these simplifying assumptions equation 3.12 reduces to

\[ \rho C_p \frac{\partial T}{\partial \theta} = K \frac{\partial^2 T}{\partial x^2} + q' \]  

\[ \text{..... (3.13)} \]

Defining \( \rho q_i = q' \) where \( q_i \) is the internal heat generation per unit time and unit mass the equation 3.13 can be re-written in the form used by Elser.

\[ \rho C_p \frac{\partial T_g}{\partial \theta} = K \frac{\partial^2 T_g}{\partial x^2} + \rho q_i \]  

\[ \text{..... (3.14)} \]

Where \( T_g \) is the gas temperature above the time average value.

The above equation is linear with respect to time, the general solution can be found by the superposition of three particular integrals, these being:

1. Steady state component
2. Component due to time dependent heat sources
3. Component due to the presence of solid wall, i.e., conduction to the wall.

Since the working process involved in a motored engine is periodic it is reasonable to assume that the internal heat sources also vary periodically with a frequency \( \omega \). As the distance from the wall increases the conduction term on the r.h.s. of equation 3.14 becomes smaller and smaller. At an infinite distance from the wall this term can be neglected so that the temperature variation is only due to the time dependent heat sources.

\[ \therefore \rho C_p \frac{\partial T_g}{\partial \theta} = \rho q_i \]  

\[ \text{..... (3.15)} \]

Assuming that \( T_{ga} \), the instantaneous gas temperature at infinite distance from the wall varies according to
where $T_\infty$ is the maximum value of $T_g$. We have from equations 3.15 and 3.16:

$$q_i = \omega C_p T_\infty \cos \omega \theta$$

From equation 3.14:

$$\frac{\partial T_g}{\partial \theta} = S_g \frac{\partial^2 T_g}{\partial x^2} + \frac{\partial T_g}{\partial \theta}$$

For the wall equation 3.13 reduce to:

$$\frac{\partial T_w}{\partial \theta} = S_w \frac{\partial^2 T_w}{\partial x^2}$$

Boundary conditions for equations 3.18 and 3.19 are:

(i) at $x = 0$ (gas wall interface) \( T_g = T_w \)

(ii) Heat flow is continuous \( K_g \left( \frac{\partial T_g}{\partial x} \right)_{x=0} = K_w \left( \frac{\partial T_w}{\partial x} \right)_{x=0} \)

Solutions for equations 3.18 and 3.19 are:

$$T_w = T_\infty e^{m_w x} \sin (\omega \theta + m_w x + \phi)$$

and

$$T_g = T_\infty \sin \omega \theta - T_\infty e^{-m_g x} \sin (\omega \theta - m_g x + \phi)$$

where

$$m_w = \sqrt{\omega/2S_w} \quad \text{and} \quad m_g = \sqrt{\omega/2S_g}$$
and $T_o$, $T_w$, $\phi$ and $\psi$ are constants of integration to be found from the boundary conditions. After some algebra the above equations yield:

$$\frac{T_e}{T_0} = \frac{b_w}{b_g} ; \frac{T_2}{T_\infty} = \frac{b_2}{b_g + b_w} \text{ and } \phi = \psi = 0 \quad \ldots \quad (3.22)$$

where 'b' is the heat penetration factor

$$b = \sqrt{KC \rho}$$

the subscripts 'g' and 'w' refer the relevant properties to gas and wall respectively. Also from 3.20 and 3.22 we have:

$$\frac{T_w}{T_\infty} = \frac{b_w}{b_g + b_w} e^{m_w x} \sin (\omega \theta + m_w x) \quad \ldots \quad (3.23)$$

The temperature distribution in the gas space is given by:

$$\frac{T_2}{T_\infty} = \sin \omega \theta - \frac{b_w}{b_g + b_w} e^{m_x} \sin (\omega \theta - mx) \ldots (3.24)$$

The temperature at the wall surface is given by:

$$\frac{T_w(x=0)}{T_\infty} = \frac{b_2}{b_g + b_w} \sin \omega \theta \quad \ldots \quad (3.25)$$

The heat transfer at the wall surface is then given by:

$$\psi / A = H_o = -K_g \left( \frac{\partial T_g}{\partial x} \right)_o = -K_w \left( \frac{\partial T_w}{\partial x} \right)_o \quad \ldots \quad (3.26)$$

From equations 3.24 or 3.25 we have:

$$H_o = \frac{b_2 b_w}{b_g + b_w} \sqrt{\frac{\omega}{2}} T_\infty \left( \sin \omega \theta + \cos \omega \theta \right) \quad \ldots \quad (3.27)$$

$$H_o = \frac{b_g b_w}{b_g + b_w} \sqrt{\frac{\omega}{4}} T_\infty \left( \sin (\omega \theta + \frac{\pi}{4}) \right) \quad \ldots \quad (3.28)$$
The following conclusions can be drawn from the above analysis:

(a) The metal surface temperature is in phase with the gas temperature at a great distance from the wall.

(b) The amplitude of the wall surface temperature fluctuation, compared with the amplitude of gas temperature fluctuation, is reduced by a factor of

\[ \frac{b_g}{b_g + b_w} \]

Since \( b_w > b_g \), the reduction in the amplitude can be quite large. For instance, substituting some typical values for the case of fundamental harmonic in the present study we have

\[ \frac{(T_{\infty})_{x-o}}{T_{\infty}} = \frac{b_g}{b_g + b_w} \sin \omega \theta \]

Amplitude ratio: \( \frac{b_g}{b_g + b_w} = \frac{4.42 \times 10^{-4}}{4.42 \times 10^{-2} + 3.33 \times 10^{-1}} = \frac{1}{750} \)

(c) The heat transfer at the surface leads the gas temperature fluctuation by \( \frac{\pi}{4} \) and is proportional to the square root of the frequency. This phenomenon cannot be explained on the basis of steady heat transfer theory. Physically it may be explained by the fact that according to equation 3.17, the generation of heat in the gas space is proportional to the frequency but the heat flow ratio into the wall is proportional to the square root of the frequency. Due to the great heat capacity of the wall, the heat generated in the vicinity of the wall is immediately abstracted by it therefore with the increasing frequency of the gas temperature fluctuations an increasingly thin layer of air, in contact with the wall, is subjected to the heat abstracting action. Thus frequency dependence of the thickness of a layer
of gas in contact with the metal is implied in the analysis. Writing the heat transfer coefficient $h$ as

$$h = \frac{H_o}{(T_{g\infty} - T_{w_0})} \quad \text{..... (3.29)}$$

$T_{w_0}$ is the metal surface temperature, i.e., $T_v$ at $x = 0$. Here $H_o$ and $T_{g\infty}$ are the instantaneous values of heat flux and gas temperature respectively. Because of the phase difference between the heat transfer and gas temperature, the value of $h$ will vary with time. Also because the heat flux and gas temperature curves do not pass through zero simultaneously, the heat transfer coefficient defined by equation will have values ranging between 0 and $+\infty$. From equations 3.16 and 3.25 we have

$$T_{g\infty} - T_{w_0} = \frac{b_v}{b_y + b_w} \sin \omega \theta$$

$$\therefore \quad h = \frac{b_y}{b_y + b_w} \sqrt{\omega} \cdot \frac{\sin (\omega \theta + \pi/4)}{\sin \omega \theta} \quad \text{..... (3.30)}$$

Curves for $H_o$ and $T_{g\infty} - T_{w_0}$ have been plotted in Fig. 3.2. The heat transfer coefficient as given in equation 3.30 is also plotted. It can be seen that between $\omega \theta = 0$ and $\omega \theta = \frac{3\pi}{4}$, $H_o$ and $T_{g\infty} - T_{w_0}$ have positive values therefore the heat transfer coefficient is also positive. For $\omega \theta = \frac{3\pi}{4}$ the heat transfer is zero and hence the heat transfer coefficient is also zero but at $\omega \theta = \pi$, $H_o$ is negative, whereas the temperature difference is negative, therefore $h$ takes on a value $= -\infty$. The same pattern is repeated between $\pi$ and $2\pi$.

Although this definition of the heat transfer coefficient
is mathematically correct, considerable difficulties arise when a physical interpretation is required. On this basis most of the empirical heat transfer data which is in the form of relationships between dimensionless number would be inapplicable. To overcome this difficulty Pfriem\textsuperscript{35} gave an alternative definition of the unsteady heat transfer coefficient. A complex heat transfer coefficient is defined by

\[ h_c = h_o e^{j\alpha} \]  

..... (3.31)

where \( h_c \) is the complex heat transfer coefficient

\( h_o \) is the magnitude of the heat transfer coefficient

and \( \alpha \) is the phase angle between the maximum values of the harmonically variable temperature difference and the heat flow.

The heat flux is then given by

\[ H_c = h_o e^{j\alpha} (T_g - T_w) \]  

..... (3.32)

This definition virtually consists of making the peak gas temperature and peak heat flow coincide such that the heat transfer coefficient remains finite. Pfriem\textsuperscript{35} states that "while affording no new physical knowledge (the complex heat transfer coefficient) provides a suitable representation".

In the real engine cycle the phase differences are not as great as predicted by this analysis and the periods during which the heat transfer coefficient becomes infinite occur in those parts of the cycle when the heat transfer is not very significant, therefore during the present study the conventional definition of the heat transfer coefficient as given in equation 3.29 was used.
Although the assumptions made in the two foregoing approaches were such that the results obtained cannot apply to the conditions inside the engine cylinder, however, development of the basic theory has helped to some extent in understanding the mechanism of the unsteady transfer in i.c. engine cylinders.

The following broad conclusions may be drawn.

(1) The induced metal surface temperature variation is greatly damped compared with the gas temperature variation.

(2) There is a phase difference between the gas temperature and surface heat transfer. This phase lag is dependent on the frequency and amplitude of the gas temperature variation. In the case of engine heat transfer the phase lag will depend on all the harmonics constituting the gas temperature variation and will be extremely difficult to compute. In general the heat transfer at the wall surface leads the gas temperature.

(3) The second analysis shows that influence of the metal is mainly confined to a thin layer of gas adjacent to the wall, the thickness of this layer decreases with increasing frequency.

(4) The heat transfer coefficient concept as used in the steady heat flow problems may be used in the analysis of results, although this yields infinite heat transfer coefficient values, the physical meaning of which is not realistic.

Due to the extreme difficulties presented in formulating a model which would represent the engine conditions correctly and solving the associated equations analytically, Elser\textsuperscript{10} has concluded that valid quantitative data on the non-steady heat transfer in i.c. engines can only be obtained experimentally. Since this type of analysis could only be applied to a theoretical model which was very
far removed from reality. Elser and other workers have resorted to dimensional analysis to define relationships between the engine heat transfer and certain selected variables such as instantaneous pressure.

**Dimensional Analysis**

Dimensional analysis approach has been used by Overbye and Elser to correlate the experimental heat transfer data from i.c. engine cylinders. Overbye used the equations of fluid motion in cylindrical co-ordinates and non-dimensionalised these by referring the various quantities to the reference conditions prevailing in the inlet manifold. Two correlations were derived, the first one neglected the viscous dissipation of energy and the second took this term into account. The first relationship, i.e., without the viscous dissipation term, was used to correlate some of his motored data. The final relationship was given as

\[ Nu^* = f(P^*, Pe') \]

where \( Nu^* \) is the Nusselt number.

\[ P^* = \frac{P}{P_0} \]

\( P \) is the cylinder pressure

and \( P_0 \) the absolute manifold pressure multiplied by the engine compression ratio.

\( Pe' \) is the Peclet number based on a constant Prandtl number and a Reynolds number defined as

\[ Re = \frac{L_0 V_0 P_0}{\mu} \]

\( L_0 \) is the engine stroke

\( V_0 \) is the mean piston speed
\( \rho_o \) is the density of gases in the inlet manifold
\( \mu \) is the dynamic viscosity of the gas.

Overbye\(^{12} \) gave plots of his experimental \( N_u^* \) against \( P_e^* \) for various values of \( P_e^* \).

Elser\(^{10} \) had also earlier produced a relationship by non-dimensionalising the equations of motion discussed in the first part of this chapter. The relationship derived by him was

\[
N_u = \int_1 (P_e') \cdot \int_2 (\omega \theta)
\]

where \( N_u \) and \( P_e \) are the Nusselt and Peclet numbers and \( \omega \) is the frequency of temperature waveform and \( \theta \) is the time. He further defined

\[ \int_1 (P_e') \simeq \frac{1}{\sqrt{P_e}} \]

Although these types of correlations are extensively used in formulation of the empirical data obtained in pipes and in the cases of flow over submerged bodies of various shapes, there is an important difference between those cases and the engine heat transfer data. Whereas in the case of pipes, etc. it is quite simple to define a representative gas velocity, in the case of engine, in the absence of any reliable data on the instantaneous gas velocity, mean piston speed is generally substituted for the velocity term in the calculation of Reynolds number. This may in some cases be justified if the ultimate aim is to obtain the mean heat transfer coefficient but if such a relationship is used to predict the instantaneous heat transfer coefficient, the physical situation is very different from the theoretical model, i.e., assuming a constant piston speed to represent the gas velocity at various points in the engine cycle.
Also such a relationship would predict the same heat transfer for various parts of the engine cylinder irrespective of the local flow pattern, again a situation which is not realistic.
CHAPTER 4

METAL SURFACE TEMPERATURE MEASUREMENT
CHAPTER 4

METAL SURFACE TEMPERATURE MEASUREMENT

4.1. Description of Various Techniques Available

Knowledge of the temperature history of a metal surface is required in many fields, such as heat transfer investigations, extrusion of metals, heat treatment and satellite re-entry. In certain cases, knowledge of the surface temperature is necessary for calculation of some other quantity such as heat flux, heat transfer coefficient or the temperature gradient at the surface. In others the maximum temperature reached during a process is required because this might affect the quality or life of the end product.

A wealth of information exists on the subject of metal surface temperature measurement. Baker and others 36, Watson 37 have given a good summary of the various techniques available.

4.1.1. Requirements

Since the tests were carried out on an engine, some techniques were obviously not suited for this type of measurement. The main requirements were as follows:

(i) High Accuracy

This was considered to be very important because it is well known that the surface temperature variation in the internal combustion engines are quite small. A typical variation would be of the order of a temperature swing of 10°C (12, 18, 7, 11). Only one worker, Britain 39, has reported much higher values for the temperature swing but these measurements are considered doubtful. In the present study, the engine was going to be motored with the result that a much smaller temperature swing than 5°C was expected, hence accuracy of measurement was considered to be of prime importance.
(ii) **Size**

The size had to be as small as possible such that the heat flux path was disturbed to a minimum.

(iii) **Rapid Response**

Since the rise time of the metal surface temperature during the compression stroke is of the order of a few milliseconds, a rapid response measuring device was essential in order to avoid phase lag problems.

(iv) **True Surface Temperature**

Since the metal surface temperature gets damped very substantially even at small depths as shown by Jakob\(^2\), it was considered highly important to use, ideally, a true metal surface temperature measuring device; failing which, the depth at which the temperature was being recorded had to be kept to a minimum in order to avoid tedious corrections necessary to evaluate the true metal surface temperature.

(v) **Ruggedness and Stability**

From an operational point of view it was essential that the device used was rugged and stable.

**Choice of a Sensor**

(i) Watson\(^3\) has tabulated various techniques available for surface temperature measurement; these are given in Table 4.1. All these techniques were judged in the light of requirement criteria discussed above.

(a) Colour and melting techniques were obviously unsuitable for dynamic metal surface temperature measurement.

(b) Thermal radiation and thermographic techniques were suitable for unsteady temperatures, the range and accuracy of these techniques was also satisfactory but it was
<table>
<thead>
<tr>
<th>Technique</th>
<th>Temperature range</th>
<th>Accuracy (ºC)</th>
<th>Location of sensor</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>THERMOELECTRIC VOLTAGE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Change with temperature differential of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e.m.f. generated between dissimilar metals</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(junctions at different temperatures).</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[a] Two dissimilar wires</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[b] One wire with body as other material</td>
<td>-200/+100</td>
<td>1</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>(c) Thin film(s) deposited on surface</td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>(d) Two bodies of different metals in contact</td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>ELECTRICAL RESISTANCE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Change of resistance with temperature)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[a] Three-dimensional coil</td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>[b] Two-dimensional grid, coil or tape</td>
<td>-240/+600</td>
<td>1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>[c] One-dimensional wire</td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>[d] Thermistor (bead or block)</td>
<td>-20/+800</td>
<td>1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>[e] Thin film deposited on surface</td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>[f] Resistance of body itself</td>
<td>-270/+400</td>
<td>0.1 or 1</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>THERMAL RADIATION</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Change with temperature of heat radiated</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>from surface)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[a] Optical comparison with heated reference</td>
<td>800, up to</td>
<td>1 or 2</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>[b] Radiation emitted (total or selective)</td>
<td>20/4000</td>
<td>1 or 2</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>[c] Infra-red radiation emitted</td>
<td>100, up to</td>
<td>1 or 2</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>[d] Reflection pyrometer</td>
<td>500, up to</td>
<td>1 or 2</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>[e] Two colour pyrometer</td>
<td>200/4000</td>
<td>1 or 2</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>[f] Colour cinematography</td>
<td>500/2000</td>
<td>1</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>[g] Evaporography</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>
considered that these would be difficult to adapt for engine measurements.

(c) The choice was therefore narrowed down to resistance thermometry or thermocouples. These will be discussed in greater detail.

(ii) Resistance Thermometry

Resistance elements designed for attaching to a surface to measure the surface temperatures, can be obtained commercially. These are generally in the form of wire grids and are attached to the surface by cement. If a very short response time is required, then metallic films are used. In the case of conducting bodies a thin film of insulating substrate is applied; this, of course, interferes with the heat flow to the body and increases the response time. These films are also sensitive to mechanical strain and pressure. The films are applied by evaporation, or by 'sputtering on' in the form of paint. A gold plated resistance thermometer was used by Meier[^39] to measure the engine cylinder wall temperature but the transfer measurements were doubtful due to the presence of an insulating layer below the gold film.

(iii) Thermocouples

Bead type thermocouples were ruled out due to their slow response. Film type thermocouples are available commercially. These are very sensitive, have a high response, are easy to install, are rugged and can be made such that no appreciable disturbance is caused to the path of heat flow. The only disadvantage is that they do not measure the true surface temperature but with modern techniques it is possible to manufacture these thermocouples so that the temperature is measured only a
fraction of a microne below the surface. Moreover, Overbye\textsuperscript{12} had demonstrated that these types of thermocouple are satisfactory for the engine heat transfer studies. It was therefore decided to use these for the surface temperature measurement. The importance of the true surface temperature measurement is illustrated by a comparison made by Overbye\textsuperscript{12}, between the film type (approximately 1 microne thick) thermocouple and a sub-surface thermocouple due to Schmidt\textsuperscript{40}. The hot junction in this case was .0005 inches below the surface. The metal surface temperature records obtained by these two thermocouples are shown in Fig. 4.1. It is clear that an analysis of these two temperature records to obtain the heat transferred will give widely different results. This is because the thermal gradient at the surface is very sensitive to the rate of temperature rise of the surface.

4.1.2. Description of the Surface Thermocouple Chosen

The basic design of the film type thermocouple is due to Hackemann\textsuperscript{41}, who produced it to measure gun barrel bore temperatures during firing. Bendersky\textsuperscript{42} reproduced a similar thermocouple and measured gun barrel temperatures successfully. The development of this type of thermocouple is described in detail by Moeller\textsuperscript{43}. As a result of improved manufacturing techniques these thermocouples became commercially available in America in the early 'Sixties.

A sectional sketch of the thermocouple is shown in Fig. 4.2. This was produced by MO-RE Inc., U.S.A., to a special design so that the main steel plug was manufactured from EN8 British Standard steel. This was necessary because the cylinder head material was EN8 and it was decided to minimise the disturbance caused to the path of heat flow by thermocouple installation.
Comparison of two different types of thermocouple

**Fig 4.1**

**Fig 4.7**
SURFACE THERMOCOUPLE

FIG 4.2

TIME CONSTANT

FIG 4.5
The constantan wire shown in Fig. 4.2. is insulated from the main body of the thermocouple by producing a layer of oxide on the wire by heating it in a gas flame. The coated wire is then inserted in the iron body and the end is polished to a fine finish. A thin film of constantan is then deposited on the polished surface by a vacuum deposition technique. The thickness of the film can be controlled. In this case a thickness of \( \frac{1}{2} \) micron was quoted by the manufacturers. The hot junction of the thermocouple is formed in a plane parallel to the polished face and at a depth equal to the film thickness as shown in Fig. 4.2.

4.1.3. Calibration of Thermocouple

Since the thermocouple consists of two thermocouple materials, constantan and iron in this case, the manufacturers recommended the use of standard Iron-Con temperature tables. However, the thermocouple was checked by immersing the thermocouple in hot oil and comparing the readings with those obtained by NPL calibrated mercury in glass thermometers. These results are plotted in Fig. 4.3. The agreement is quite good. It was therefore decided to use the standard tables for the engine test records. The thermocouple was always operated with a cold junction kept at the temperature of melting ice, as shown in Fig. 4.4.

4.1.4. Thermocouple Time Constant

In transient solutions the exponential term of type \( e^{pt} \) gives the decay of the transient component. Chestnut and Moyer\(^4\)\(^4\) give the following definitions of the time constant:

(i) It is the time in seconds for a transient exponential term to be reduced to \( e^{-1} = 0.368 \) of its initial value.

(ii) It is the time that would be required for the transient to disappear completely if its rate of decay continued at its initial value. Both these definitions are illustrated in Fig. 4.5.
CALIBRATION OF SURFACE THERMOCOUPLE

VALUES FROM Fe-Con Tables.

EXPERIMENTAL VALUES

TEMPERATURE °C

TEMPERATURE °C

FIG. 4.3

SURFACE T/COPPE CIRCUIT

FIG 4.4
Chart 1. Temperature response, temperature gradient and heating rate in a semi-infinite solid, \( x \geq 0 \), after sudden change in surface temperature from \( T_s \) when \( \theta < 0 \) to \( T_s \) for \( \theta \geq 0 \). (*

*Italicized letters in parentheses indicate whether curves are based on exact, integral or numerical solutions.

FIG - 4.6 Temperature response chart for a semi-infinite solid.

(Schneider 45*)
Schneider has given a large number of charts which represent analytical solutions for transient one-dimensional heat conduction in bodies of fundamental geometric shape. In the present case the response of a semi-infinite solid to a step change in the surface temperature applies. Fig. 4.6 has been replotted from Chart I given in Schneider, which shows the response of a semi-infinite solid for $x > 0$ after a sudden change in surface temperature from $T_1$ at time $t < 0$ to $T_2$ at time $t > 0$. The abscissa is $X = \frac{x}{T}$, where $X = \frac{x}{T}$ = depth ratio; 'x' being the distance into the wall, and 'T' the plate thickness.

$$F_0 = \text{Fourier number} = \frac{\delta^2}{T_2}$$

$\delta$ is the thermal diffusivity of the material.

Putting the values of $X$ and $F_0$ we have

$$\frac{x}{2\sqrt{\delta T}} = \frac{x}{2\sqrt{F_0}}$$

From Fig. 4.6, for a value of .368 for $\frac{T - T_1}{T_2 - T_1}$ we have

$$\frac{x}{2\sqrt{\delta T}} = .64$$

Substituting the value for $\delta$ for constantan of .06 cm$^2$/sec we have

$$\theta = \frac{x^2}{1.64 \times .06} \text{ secs.}$$

which shows that the time constant varies as a square of the distance into the wall and hence the importance of using a very thin film.

The film thickness of ½ to 1 microne was quoted by the manufacturers for the thermocouples used in the present study, which gives a time constant of

$$\theta = \frac{(0.0001)^2}{1.64 \times .06} = .102 \times 10^{-6} \text{ secs.}$$
Since the highest cycle frequency used during the present tests was 9.15 cycles/sec at 1100 r.p.m., the thermocouple should have adequate response even for $10^6$th harmonic of the fundamental.

### 4.1.5 Amplifier System

The output signal from the surface thermocouple is composed of a large standing voltage, due to the mean metal temperature, and a superimposed fluctuating component due to the surface temperature variation. The typical values for these voltages in a motored case for an Iron-Constantan thermocouple are about 4 to 5 millivolts for the standing voltage and .15 millivolts for the fluctuating component. The fluctuating voltage is of main interest for analysis of the temperature record. In order that the recorded trace of the fluctuating temperature may be analysed accurately, a very high gain amplifier is required. Another requirement from the point of view of analysis is to have a high signal to noise ratio. Since harmonic analysis of the temperature record had to be carried out to obtain heat fluxes, no attenuation of the signal at any frequency could be tolerated. RC-coupled amplifiers have attenuation characteristics at low frequencies, therefore a DC-amplifier would be more suitable, but these amplifiers are generally unstable. Overbye$^{46}$ designed a DC amplifier and claimed that it was stable. This circuit is shown in Fig. 4.7. In order to save time it was decided to construct a similar amplifier, but it proved impossible to operate the system satisfactorily. Dry batteries were used for power supplies and as the batteries were drained there was considerable drift; it was therefore decided to discard this particular amplifier. Next, a Hewlett Packard type DX-2461A-M1 amplifier was tried. This proved to be quite stable and satisfactory results were achieved. Details
of this amplifier are given in Appendix 4.1. It is a wide-band high-gain, chopper-stabilized amplifier of all transistor design. Since the maximum frequency of the temperature wave was about 10 c.p.s. (1100 r.p.m. engine speed) and the chopper frequency was high, there was not any likelihood of the recorded wave form being affected by it.

In the earlier stages of work, considerable trouble was experienced due to high noise level. After putting in a great deal of effort by thoroughly screening all parts of the circuit, this was, however, reduced to an acceptable level. Some attempts were made to eliminate the high frequency hash present on the trace by filtering but this resulted in attenuating the signal, hence it was decided to use unfiltered traces for analysis. Filtered and unfiltered traces are shown in Fig. 4.8. A special circuit was designed to bias off the large standing voltage appearing at the amplifier output stage due to the D.C. component of the input signal. This circuit is shown in Fig. 4.9.

Initially some time was spent in obtaining a dynamic calibration of the surface temperature trace. Microswitches operated by the engine crankshaft were tried but these proved to be too slow. High speed teleprinter relays were tried next; these proved successful, but disturbed the temperature trace, as shown in Fig. 4.10. In view of the fact that there was virtually no drift of the amplifier system it was decided to record the calibration voltages after the surface temperature trace had been obtained. A calibration circuit to give finely controlled voltages was produced and connected to the main circuit in such a way that the calibration signal was in parallel with the thermocouple output. This circuit is also shown in Fig. 4.9.
METAL SURFACE TEMP. RECORD — FILTERED

FIG. 4-8A

$\gamma = 4.86$

METAL SURFACE TEMP. UNFILTERED SIGNAL

FIG. 4-8B
Surface Temperature Thermocouple Circuit

Fig 4.9
4.1.6. Harmonic Analysis of the Input and Output Waveforms

Since the surface temperature fluctuation was of the order of 5°C, the resulting signal from the thermocouple was quite small. Therefore it was necessary to have a large amplification in order to produce a temperature record which could be analysed accurately. At such large amplification, significant distortion of the input waveform can occur. The usual method of checking the effects of the amplification is to feed known waveforms into the amplifier system and then compare the output with the input waveform. The most stringent criterion in this respect is a square waveform.

Tests were carried out by feeding square and sine waves of the same frequency as expected in the engine tests, 10 c.p.s. at 1200 r.p.m. in this case; both input and output waveforms were recorded. This is shown in Fig. 4.10. It can be seen, from an examination of these records, that even with the square wave the distortion is not significant. In addition to this check, Fourier analysis of the output sine waveform was carried out. The sine and cosine sufficients for thirty-six harmonics are given in Table 4.2. For a pure sine wave all Fourier coefficients except the first sine coefficient should be zero. The first Beta coefficient (i.e., the first sine coefficient) should be unity. A study of Table 4.2, shows that this coefficient is very nearly unity and all the other Fourier coefficients are insignificant. This showed that no significant distortion of the waveform was occurring due to amplification.

4.2. Conclusions

4.2.1. Of all the techniques discussed for surface temperature measurement, the film type thermocouple was considered to be the most suitable because of its ruggedness, stability, high response
AMPLIFIER input-output (square wave) Fig 4.10(a)

D.C. Amplifier sine wave input-output Fig 4.10(b)
### TABLE 42: Pure Sine Wave Input

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>$\beta_k$</th>
<th>$\alpha_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1.00000000E+00$</td>
<td>$-0.00000000E+00$</td>
</tr>
<tr>
<td>2</td>
<td>$-2.0711884E-02$</td>
<td>$1.0133319E+04$</td>
</tr>
<tr>
<td>3</td>
<td>$-1.3843939E+03$</td>
<td>$1.0065797E+03$</td>
</tr>
<tr>
<td>4</td>
<td>$-8.639721E-02$</td>
<td>$-8.615133E-03$</td>
</tr>
<tr>
<td>5</td>
<td>$-1.6879905E-02$</td>
<td>$1.6946523E-02$</td>
</tr>
<tr>
<td>6</td>
<td>$-9.1540726E-03$</td>
<td>$-3.945418E-03$</td>
</tr>
<tr>
<td>7</td>
<td>$-9.4210500E-03$</td>
<td>$-5.1268917E-03$</td>
</tr>
<tr>
<td>8</td>
<td>$9.1486059E-03$</td>
<td>$4.7414467E-04$</td>
</tr>
<tr>
<td>9</td>
<td>$4.2485284E-03$</td>
<td>$2.1915886E-03$</td>
</tr>
<tr>
<td>10</td>
<td>$-6.0798128E-03$</td>
<td>$1.5915793E-03$</td>
</tr>
<tr>
<td>11</td>
<td>$-1.4856860E-03$</td>
<td>$-6.2057594E-04$</td>
</tr>
<tr>
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<td>$-1.9624484E-03$</td>
<td>$7.0665781E-04$</td>
</tr>
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<td>13</td>
<td>$7.067741E-03$</td>
<td>$-4.3331856E-03$</td>
</tr>
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<td>14</td>
<td>$-3.7814545E-04$</td>
<td>$5.499926P-04$</td>
</tr>
<tr>
<td>15</td>
<td>$-5.0865115E-03$</td>
<td>$-5.052877E+00$</td>
</tr>
<tr>
<td>16</td>
<td>$7.2477514E-03$</td>
<td>$-7.4723284E-03$</td>
</tr>
</tbody>
</table>

### Amplifier Output Wave Analysis

**Ideal Sine Wave Coefs.** —

\[ \beta_i = 1.0 \]

\[ \alpha_i = 0.0 \]
and easy installation.

4.2.2. It was shown that this type of thermocouple has a sufficiently small time constant to record the highest harmonics likely to be present in the metal surface temperature variation, without any significant attenuation.

4.2.3. Since the metal surface temperature could be recorded at only 1/2 to 1 micron below the surface, no correction to the recorded metal surface temperature was considered necessary to evaluate the true metal surface temperature.

4.2.4. Separate calibration lines rather than dynamic calibration marks were found to be satisfactory.

4.2.5. A suitable amplification system was developed. Harmonic analysis of the output waveforms for known input waveforms did not indicate any significant distortion of the input waveform.
CHAPTER 5

GAS TEMPERATURE MEASUREMENT
5. GAS TEMPERATURE MEASUREMENT

5.1. General Comments

One of the important parameters required to explain the working processes of any type of engine is the knowledge of the temperature of gases taking part in the process. In the internal combustion engines the temperature of the working gas is fluctuating which makes the precise measurement of this temperature very difficult.

There are various techniques available to obtain the compression temperatures. Some of the important ones are discussed here.

5.1.1. Thermodynamic Calculations

The cylinder pressure can be generally measured with reasonable accuracy. With the knowledge of the instantaneous cylinder volume and the universal gas law the cylinder gas temperature can be computed. One of the major difficulties in this method is the lack of certainty about the trapped mass. Simplifying assumptions have to be made which might result in unrealistic gas temperature. Small errors in the estimate of suction temperature can result in quite large errors in the estimated temperature.

5.1.2. Velocity of Sound Method

The velocity of sound inside the engine cylinder depends on the gas properties, i.e., composition, temperature and pressure of the gases. If the composition and pressure are known, the gas temperature can be measured by experimentally determining the time it takes a sound wave to travel a given distance in the engine cylinder. If gas temperatures at various points in the engine cycle are required, a point by point technique has to be adopted. Livengood and others have claimed an accuracy of ±1% for this method.
5.1.3. **Optical Methods**

These methods have the advantage of instantaneous response and a wide range of temperatures can be investigated. The main disadvantages are the practical difficulties involved in installing these systems on an engine. Also, certain corrections have to be made in order to allow for the effects of pressure variation.

5.1.4. **Resistance Thermometry**

This technique has been successfully used to measure rapidly changing gas temperatures. The main disadvantage is the finite time lag involved even when extremely fine wires are used as sensing elements. Since this method has been used to measure the engine cylinder gas temperatures by Lyn, Aftalion and many others, it was decided to adopt it in the present study.

Resistance thermometry was employed as long ago as the end of last century to record the fluctuating temperatures. Callender and Nicolson used a wire of .001" diameter to record steam chest temperature variation. Similar work was carried out by Callender and Dalby. Coker and Scable and Burstall measured gas temperatures in gas engines. Petersen did useful work on the thermal lag of the sensing elements and pointed out that the thermal lag of a resistance thermometer was less than that of a thermocouple of the same size because of the additional thermal capacity of the thermocouple due to the joint.

As reported by Benson and Brundrett in their comparatively recent survey of this field, Groeber considered the theoretical aspects of the response time of fine wire thermometers. He proposed a method whereby corrections could be made for the time lag but this was of limited use because heat transfer coefficient to the wire had to be used in the calculation of the correction. In most cases the
wire operates in an environment where the heat transfer coefficient is unknown, therefore this method of correction can only be applied in very limited cases. Pfriem evolved a technique to correct the observed temperature to obtain the true gas temperature. This technique required the measurement of rate of temperature rise with time at small intervals. The heat transfer coefficient was estimated from the heat transfer data on cylinders in crossflow of gases. In the cases where the heat transfer coefficient could not be estimated Pfriem used two wire temperature records. One with no significant heating current and the second one with heat generation in the wire. He set up two simultaneous equations and eliminated the heat transfer coefficient. The same could be achieved by using two wires of different diameter or different materials. This method was successfully applied by Ghoneim to a low speed air compressor. Schultz used a 2-micron wire to record the gas temperature in the cylinder of an engine motored at 3000 r.p.m. and calculated a phase lag of 1.5 to 2.5 engine crank degrees, which was considered to be small enough to be ignored. He also suggested that by having a sensing element with a length of at least 2000 times the diameter the end conduction losses may be ignored.

Scadran and Warshawsky measured the time constants of the sensing elements by heating them to a given temperature and recording the rate of cooling. They also discussed the end conduction and radiation errors in thermocouples. Another interesting approach for correction is the electrical compensation for transient gas temperature measurement suggested by Shepard and Warshawsky. Neglecting the radiation and end losses, and in an environment where the heat transfer coefficient is not changing, the true gas temperature is proportional to the rate of temperature rise of the
wire. A differentiating circuit would give output which is proportional to the rate of change of input temperature record. Because the variation of heat transfer cannot be allowed for in this method its application is limited. Benson and Brundrett have developed a three-wire method for the correction of thermal lag. Neglecting the radiation and conduction errors and also the recovery temperature of the wire, the difference between the recorded gas temperature and the true gas temperature is shown to be a function of the wire diameter and rate of recorded temperature rise with time. Using three different wire diameters, graphs are plotted for each point in time and the true gas temperature evaluated. Wienke and others used a resistance thermometer for engine compression temperature measurement. A wide range (.00015 inch-.0010 inch diameter) of wire diameters was used at various engine speeds. Empirical relationships were produced for determining the true gas temperatures but this type of relationship cannot be used with confidence for engines other than the one on which these relationships were established. Lyn and Valdmanis applied the theory developed by Scadron and Warshawsky to engine compression temperature measurement using .0015" diameter wires. Some useful relationships between wire time constants and temperature phase lag were evolved. These will be discussed in greater detail later. In a recent theoretical study of the measurement of rapidly varying gas temperatures in an unsteady flow Chomiak and Niedzialek have produced exact solutions for an assumed sinusoidal variation of gas temperature with an assumed time dependent heat transfer coefficient variation.
FIG - 5.1 Gas temperature measurement — Circuit and the block diagram of the apparatus.

FIG - 5.2 Hot oven test apparatus.
Wheatstone Bridge Circuit

A simple Wheatstone bridge shown in Fig. 5.2. was used to record the fluctuating temperature. Compensating leads were used to balance out the probe lead resistances. Dymec D.C. amplifier as described in Chapter 4 was used to amplify the signal. The multi-decade box shown in Fig. 5.1 could vary the resistance down to .001 ohms. Unbalanced bridge operation, which is suitable for electronic indication of rapidly changing temperature, was used. Resistances $r_1$ and $r_2$ were made equal but 'n' times larger than $r_3$. The sensitivity of the bridge is given by the following relationship:

$$\text{Bridge Sensitivity} = \alpha \frac{n}{n + 1}$$

This will tend to 1 for 'n' very large but for $n=10$, the factor $\frac{n}{n + 1}$ attains ten-eleventh of its maximum value and there is no advantage to be gained by further increasing 'n'. In the bridge shown in Fig. 5.1, the value of 'n' is much higher; this was necessary to reduce the current flowing in the probe to such a value that the self-heating effects of the probe would be negligible. The current flowing through the sensing element could have been reduced by reducing the supply voltage instead of increasing the bridge resistances as discussed. This method was not selected because the output would have been affected by the variation in the supply voltage.

Determination of Temperature Coefficient of Resistance

In resistance thermometry the sensing probe resistance $R_T$ at a temperature $T$ is related to its resistance $R_0$ at a datum temperature $T_0$ by the following relationship:

$$R_T = R_0 (1 + \alpha T + \beta T^2 + \gamma T^3) \quad (5.3.1)$$
For the low temperatures, typically up to 400°C, it is usual to modify the relationship to:

\[ R_T = R_0 (1 + \alpha T) \]

i.e., terms containing higher powers of the temperature are ignored without introducing serious errors. It is highly important to determine the value of temperature coefficient of resistance \( \alpha \), as accurately as possible. The published values of \( \alpha \) can only be relied upon in the case of noble metals. In other cases the values of \( \alpha \) can differ quite significantly from the published data. In the case of Tungsten this variation could be quite appreciable from batch to batch or even in the same batch, because of the method of manufacture employed and further changes introduced during the etching or drawing process to produce the small diameter wires. It was therefore decided to determine the temperature coefficient of resistance for all the materials used.

The experimental set up to achieve this is shown in Fig. 5.2. The sensing probe was mounted inside an enclosed porcelain tube. A 10% Pt-Rh-PT thermocouple was mounted near the probe inside the tube. This thermocouple was chosen because of its high accuracy. The tube assembly was placed inside a pyrex tube containing granulated porcelain and the whole assembly was placed inside an electrically heated oven. Originally the eddies produced inside the oven due to gases coming into contact with the hot walls produced erratic signals from the resistance thermometer. This was eliminated by placing the porcelain tube inside a pyrex tube described above. This resulted in a virtually steady signal. The resistance of the sensor could be measured down to 0.001 ohms and the output of the thermocouple was measured on a precision potentiometer which could measure down to 1 μV. The results of
Determination of $\alpha$

$\left(\frac{R_w}{R_0} - 1\right)$

$P_t/30\% Ir$

$10\,\mu\text{m Dia Wire}$

1st Test

2nd Test

Slope = 0.00073

Fig. 5.3
DETERMINATION OF TEMPERATURE COEFFICIENT OF RESISTIVITY FOR TUNGSTEN.

**First Run**

**Second Run**

![Graph showing temperature coefficient of resistivity for tungsten with two lines representing first and second runs.](image-url)
The results of these tests are plotted in Figs. 5.3. and 5.4. An examination of Figs. 5.4. and 5.5. shows that there is a permanent increase in the resistance of both tungsten and PT/30% IR wires although there is not any appreciable change in the value of temperature coefficient of resistivity given by the first and second runs. Since the tests showed that PT/30% IR wire followed the linear law up to the highest temperature used in the tests (approximately 600°C) there was no need to fit a quadratic to the test points. These experimentally determined values of the temperature coefficient were used for all the tests. Value of $a = 0.00073$ for PT/30% IR agreed reasonably well with the published values (Ref. 64) but in the case of tungsten the experimentally determined value was about 20% lower than the published values for annealed tungsten and about 6% lower than the published values for hard tungsten.

5.4. Errors and Corrections

5.4.1. Thermal Lag

All resistance thermometer elements have a thermal inertia, therefore these elements only indicate a true static gas temperature or a very low frequency fluctuating gas temperature. As the frequency of gas temperature variation increases the temperature indicated by the resistance thermometer lags more and more behind the true gas temperature. This also results in the attenuation of the amplitude of gas temperature variation. In order to simplify this complex phenomenon, as a first approximation, it may be assumed that there is only a phase lag present between the recorded and true gas temperatures. Let this phase lag be represented by a time lag $\tau$. This means that the recorded temperature at time $t + \tau$ is what the
true gas temperature was at a time \( t \). We shall now look at this time lag in two cases:
(a) without any end conduction from the sensing wire, and
(b) with end conduction.

5.4.2. Thermal Lag Without End Conduction

Neglecting the radiation and end conduction losses, consider the transfer of heat in a small element of time \( \delta \theta \).

Heat transferred from the gas = Heat absorbed by the wire

\[
\text{Heat transferred} = hA (T_g - T_w) \delta \theta 
\]

where
\( h \) is the heat transfer coefficient
\( T_g \) is the true gas temperature
\( T_w \) is the wire temperature
\( A \) is the exposed area of the wire.

Heat absorbed by the gas = \( C_w \frac{d T_w}{d \theta} \delta \theta \) .......... (5.2)

\( C_w \) is the total heat capacity of the wire.

\[
C_w = \rho_w V_w c 
\]

where
\( \rho_w \) is the density of the wire material
\( V_w \) is the volume of the element under consideration
\( c \) is the specific heat of the wire.

Combining equations (1) and (2) we have

\[
hA (T_g - T_w) \delta \theta = C_w \frac{d T_w}{d \theta} \delta \theta
\]
or

\[
(T_g - T_w) = \left( \frac{C_w}{hA} \right) \frac{d T_w}{d \theta}
\]

... (5.4)
\[
T_g = T_w + \frac{C_w}{\frac{k}{A}} \frac{dT_w}{d\theta}
\]

Let \( \frac{C_w}{\frac{k}{A}} = \tau \) .... (5.5)

\[
T_g = T_w + \tau \frac{dT_w}{d\theta}
\] .... (5.6)

This equation gives the response of the wire in the absence of any conduction or radiation losses.

5.4.3. Thermal Lag with End Conduction

In the previous section the thermal lag was considered in the absence of end conduction. Now the problem of thermal lag with end conduction losses will be considered. The constant current hot wire anemometer theory can be applied to the resistance thermometer because the current in the sensing probe is constant. First, consider the case of thermal lag, without end losses, in the light of this theory. Hinze\(^6\) gives the following expression for the time constant of a constant current hot wire anemometer

\[
\tau = \frac{e C_w}{\alpha R_0 \left[ I^2 + (A + B\sqrt{V}) \right]} \] .... (5.7)

where

- \( I \) is the current flowing through the wire
- \( \alpha \) is the temperature coefficient of resistance.
- \( A \) and \( B \) are constants
- \( V \) is the mean velocity of gases
- \( e \) is a conversion factor to convert Joules/sec into cals/sec.

In the case of resistance thermometer the current flowing through the sensing element is usually so small that there are no
significant self-heating effects. A current of 3 mAmps was used in the present study which gives

$$I^2 = 9 \times 10^{-6} \text{ (Amps}^2)$$

The values of constants A and B are generally as follows:

$$A = .24 \quad ; \quad B = .56$$

Thus in equation (5.8), $I^2$ term may be neglected in comparison with A and B. The time constant for the resistance thermometer is then given by the following expression:

$$\tau = \frac{e C_w}{\alpha R_0 [A + B\sqrt{\cdot}]} \quad \ldots \quad (5.8)$$

Many workers have collected heat transfer data for the case of gas flow in a direction perpendicular to the wire. This is summarised in Moadams$^{30}$. The following empirical relationship which gives satisfactory results in the case of many gases and liquids was proposed by Kramers$^{66}$.

$$\frac{h_d}{K_f} = N_u = .42 P_r^{\cdot 2} + .57 P_r^{\cdot 33} R_e^{\cdot 5} \quad \ldots \quad (5.9)$$

Here

$K_f$ is the thermal conductivity of gases at film temperature

$T_f$ is the film temperature given by $\frac{W + T}{2}$

$P_r$ is the Prandtl number $\frac{C_p \mu_f}{K_f}$

$R_e$ is the Reynolds number $\frac{Upd}{\nu_f}$

$N_u$ is the Nusselt number $\frac{h_d}{K_f}$

Doing a heat balance on a hot wire anemometer element of length $l$, we have

heat generated in the wire = heat transferred
or

\[ I^2 R_w = e K_f \pi d l (T_w - T_g) \left[ .42 P_r^2 + .57 P_r^{.33} R_e^{.5} \right] \] .... (5.10)

Making use of the following relationships

\[ R_w = R_o \left[ 1 + \alpha (T_w - T_o) \right] \]

\[ \therefore T_w - T_g = \frac{R_w - R_g}{\alpha R_o} \]

where the suffix 'g' refers to the gas temperature, the equation (5.10) reduces to

\[ \frac{I^2 R_w}{R_w - R_g} = \left( A + B \sqrt{\nu} \right) \] .... (5.11)

where

\[ A = e \pi K_f l \left[ .42 P_r^2 \right] \]

and

\[ B = \frac{e \pi K_f l}{\alpha R_o} \left[ .57 P_r^{.33} R_e^{.5} \right] \]

Combining equations (5.9) and (5.11) we have

\[ A + B \sqrt{\nu} = \frac{e \pi l h d}{\alpha R_o} \] .... (5.12)

Substituting this in equation (5.8) we have

\[ \tau = \frac{C_w}{\pi l h d} = \frac{C_w}{A h} \]

which is the same as equation (5.5).

Now consider the case with end losses. A heat balance on the wire yields:
\[ I^2 R_w = (R_w - R_g) (A + B \sqrt{V}) = -e \frac{\pi}{4} d^2 K_w \frac{d^2 T_w}{d x^2} \]

Here, \( R_w \) and \( R_g \) are resistances for length \( l \) of the wire. It is shown in Appendix 6.1 that in this case the time constant for the wire is given by the following expression:

\[ \tau = \frac{e C_w}{\alpha R_0 [A + B \sqrt{V}]} \left[ \frac{R_w - \tanh l^*}{R_g - \tanh l^*/l^*} \right] \]

with \( l^* = l/2 \ell_c^* \)

\( \ell_c^* \) is the 'cooling length' given by the following expression:

\[ \ell_c^* = \frac{d}{2} \sqrt[\sqrt{\alpha R_0 (A + B \sqrt{V} - l^2)}] \]

In the case of resistance thermometer, the current \( I \) is generally very small; therefore, the \( I^2 \) term can be neglected.

\[ \therefore \ell_c^* = \frac{d}{2} \sqrt[\sqrt{\alpha R_0 (A + B \sqrt{V})}] \]

A comparison of equations (5.15) and (5.9) shows that the time constant for the case where the end losses are taken into account is enhanced by a factor

\[ \left[ \frac{(R_w/R_g) - (\tanh l^* / l^*)}{1 - (\tanh l^*/l^*)} \right] \]

\[ \left[ (R_w/R_g) - (\tanh l^* / l^*) \right] \left[ 1 - (\tanh l^*/l^*) \right] \]

\[ \left[ (R_w/R_g) - (\tanh l^* / l^*) \right] \left[ 1 - (\tanh l^*/l^*) \right] \]

As the ratio \( \frac{R_w}{R_g} \rightarrow 1 \), this factor also tends to unity. In other
words, if the temperature of the wire is not very different from the temperature that the wire would have attained if placed in the gas flow without any current, the effect on the time constant, calculated for the case of no end losses, is negligible.

Consider the case of 5 microne tungsten wire of length .16 cm.

\[ l_c = \frac{d}{2} \frac{e\pi K_w}{\alpha R_o (A + B\sqrt{V})} \]

but

\[ A + B\sqrt{V} = \frac{e\pi l h d}{\alpha R_o} \]

\[ \therefore l_c = \frac{d}{2} \sqrt{\frac{K_w}{\ell h d}} \]

\[ l^* = \frac{l}{2l_c} = \frac{l}{d\sqrt{K_w/\ell h d}} = \frac{.16}{.0005\sqrt{K_w/(16x3x.0005)}} \]

here a typical value of \( h = .3 \) cals/sq cm sec \( ^\circ \)C is assumed (Spangenberg 69)

\[ K_w = .35 \text{ cals/cm}^2/\text{cm/}^\circ \text{C/sec.} \]

\[ \therefore l^* = 2.66 \]

A substitution in equation (5.17) yields the value of multiplying factor.

\[ \text{Multiplying factor} = \frac{1 - .372 R_g/R_w}{.628} \]

For a heating current of 4 mA

power dissipation in the wire = \( I^2R_w = 20 \times 10^{-6} \) watts. This means that the wire is approximately .8\( ^\circ \)K above its surroundings (Davies 67).
\[ R_w = R_g (1 + .0033 \times .8) = 1.0026 \, R_g \]

here \( \alpha = .0033 \)

\[ \text{Multiplying factor} = \frac{629}{626} = 1.001 \]

This shows that the time constant will increase by \( .1\% \) which is small enough to be neglected.

5.4.4. Thermal Lag of Engine Compression Temperature Measurement

If the time constant of the wire \( '\tau' \) is known corrections can be made to the measured temperature to take account of the thermal lag effect but on engine measurements the picture is not so simple. It has been shown that

\[ \tau \propto \frac{1}{h} \]

The heat transfer coefficient inside the engine cylinder depends on the following parameters which vary with the conditions in the cylinder.

(a) Pressure or density of gases
(b) Gas temperature
(c) Gas viscosity which can be related to temperature changes
(d) Thermal conductivity of the gases, which again is temperature dependent
(e) Specific heat of gases at constant pressure.

All the above parameters are time dependent, therefore the heat transfer coefficient \( 'h' \) will have to be estimated for different parts of the engine cycle using a point by point method. This process will be extremely laborious in practice. An 'average delay' could be estimated and the temperature record modified accordingly but, again, this will not get rid of the distortion completely. In order to eliminate the effects of thermal lag completely, correction...
methods have been devised. Benson has discussed these methods in detail. A brief account of these methods is given.

(a) Single Wire Method

From the consideration of heat balance on the wire element the following relationship is developed:

\[ T_g = T_w - \frac{V^2}{2gJc_p} + \frac{1}{h} \frac{\rho d c}{4} \frac{dT_w}{d\theta} + \sigma \varepsilon (T_w^4 - T_{so}^4) \]

where

- \( J \) - Joules equivalent
- \( \varepsilon \) - Emissivity
- \( \sigma \) - Stefan-Boltzmann constant
- \( T_{so} \) - Temperature of the surrounding walls

The above expression contains the effects of temperature recovery (second term on the r.h.s.) and radiation (last term in the bracket).

The value of \( \frac{dT_w}{d\theta} \) is measured graphically from the temperature record and a reasonable value of 'h' is assumed initially. The true gas temperature is computed after a few iterations involving new values of the heat transfer coefficient 'h' for the computed values of \( T_g \).

(b) Two Wire Method

This method consists of using two wires of different diameters; the same results may be achieved by using two wires of different material, or operating the same wire with a certain amount of heat dissipation. The basic idea is to eliminate the unknown heat transfer coefficient. In the case of two wires of different diameters, the final expression is
\[ T_g - T_{w_1} = (T_{w_1} - T_{w_2}) \times \left[ \frac{1}{(d_2)^{i-m}} \left( \frac{p c d_2}{4} \right) \frac{dT_{w_2}}{d \theta} + \sigma \varepsilon (T_{w_2}^{4} - T_{so}^{4}) \right] - 1 \]

where 'm' is a constant defined by the following relationship:

\[ h = \text{constant} \times R_{e}^{m} \]

In the case where two wires of different diameter and different materials are used the above relationship is modified to:

\[ T_g - T_{w_1} = (T_{w_1} - T_{w_2}) \left[ \frac{1}{(K_1/K_2)(d_2)^{i-m}} \left( \frac{p c d_2}{4} \right) \frac{dT_{w_2}}{d \theta} + \sigma \varepsilon (T_{w_2}^{4} - T_{so}^{4}) \right] - 1 \]

\[ \ldots \text{(5.19)} \]

Temperature records for a constant engine speed were obtained with a 5 μ tungsten and 7 μ tungsten wires. This calculation for a point on the compression curve has been carried out in Appendix 5.2.

(c) **Three Wire Graphical Method**

This method is due to Bensen and Brundrett. Radiation and conduction errors are neglected but recovery temperature is included. From the usual heat balance considerations and Nusselt-Reynold number relationship for heat transfer, the following relationship is developed between the wire temperature and the rate of change of wire temperature with time.
FIG 5.5 Response of a 6.9 μm diameter wire compared to the gas temperature. (Benson and Brandt, 1968)

FIG 5.6 Effect of heat transfer coefficient upon the maximum recorded temperature in a 130 cycles per second fluctuation (Benson and Brandt, 1968)
Fig. 5.6 (b) Effect of heat transfer coefficient upon phase lag.
(Benson and Bondrell)
\[ T_w = -B d^{1-m} \frac{dT_w}{d\theta} - \left( T_g + \frac{V^2}{2gJc_p} \right) \] ........................ (5.21)

where \( B \) is a constant.

This relationship may be written as an equation for a straight line.

\[ Y = -B X - C \] ........................ (5.22)

where \( Y = T_w \) and \( X = d^{1-m} \frac{dT_w}{d\theta} \)

If a graph of this function is plotted the intercept will give \( 'C' \).

\[ \text{Intercept} = C = T_g + \frac{V^2}{2gJc_p} \] ........................ (5.23)

from which \( T_g \) can be calculated.

In principle at least three wires are required to give three points for the straight line but greater accuracy will result if greater number of wires is used. A comparison of the three methods discussed may be drawn from the results obtained by Benson and Brundrett\(^55\) shown in Fig. 5.5. The differences between the three methods are not great. The second method shows the greatest scatter but this could be due to the difficulty of measuring \( \frac{dT}{d\theta} \) accurately and also due to the fact that the temperature records for the different wires were not obtained in the same cycle.

Another alternative is to make the time constant so small that ignoring its effects will not introduce serious errors. To attain this object, it is necessary to make the diameter extremely small.

Data on the heat transfer from cylinder in cross flow of gases
obtained by Hilpert is correlated by the following relationship.

\[ N_u = C \left( \frac{R_e}{\frac{d}{3}} \right)^{\frac{1}{2}} P_r^{n} \]  \hspace{1cm} (5.24)

where 'n' varies between .33 and .6 depending on the value of Reynolds number. In the Reynolds number range of interest of between 1 to 100 it is reasonable to assume \( n = .4 \), which gives

\[ h \propto (d)^{-0.6} \]

but

\[ \tau \propto \frac{1}{h} \]

\[ \therefore \tau \propto d^{1.6} \]  \hspace{1cm} (5.25)

Benson and Brundrett plotted the relationship between the wire diameter, phase lag and the heat transfer coefficient for the case of 130 c/sec fluctuation. This is shown in Figs. 5.6a and 5.6b. An examination of these figures shows that even at very low Reynolds numbers or heat transfer coefficient, the phase lag for the smallest diameter (6.9μ) wire is only a fraction of a millisec.

Consider the case of 5μ tungsten wire used in the present study.

\[ A = \text{Surface for a length of } .16 \text{ cm} = .8 \pi \times 10^{-4} \text{ cm}^2 \]

\[ V_w = \text{Volume} = \frac{\pi (5 \times 10^{-4})^2 \times .16}{4} \]

\[ \therefore \tau = \frac{C_w}{hA} = \frac{\rho_w V_w c}{A h} \]
Assuming a value of 0.25 cals/cm²/C⁰/sec (for \( k' \))

\[
V_w = 19.3 \text{ g/cm}^3 \text{ for tungsten}
\]

\[
c = 0.034 \text{ cals/g/C}^0
\]

\[
\therefore \tau = 3.3 \times 10^{-4} \text{ sec} = 0.33 \text{ m sec}
\]

At an engine speed of 700 r.p.m. this is equivalent to 1.35 crank angle degrees. Even at the highest engine speed used during the tests, i.e., 1100 r.p.m., the phase lag is 2.2 crank angle degrees. In reality the phase lag will be even smaller than this. Scadran and Warshawsky⁶⁰ have shown that for thin wire in the cross flow of air

\[
\tau = 4.05 \times \frac{P_w C_w D^{1.5}}{(M^2 P)^{\frac{1}{2}}} \left(1 + 2M^2\right)^{\frac{1}{4}} \quad (5.26)
\]

where

\[
\tau = \text{ time constant in seconds}
\]

\[
M = \text{ Mach number of the flow}
\]

\[
P = \text{ Pressure in atmospheres}
\]

\[
T_t = \text{ Total temperature of the gas } \text{°R}
\]

The value of Mach number term \((1 + 2M^2)\) can be assumed unity when Mach number is low.

Therefore

\[
\frac{\tau}{\tau_z} = \left(\frac{P_z}{P_i}\right)^{\frac{1}{2}} \left(\frac{T_z}{T_i}\right)^{0.18} \left(\frac{M_{az}}{M_{az'}}\right)^{\frac{1}{2}}
\]

\[
= \left(\frac{P_z}{P_i}\right)^{\frac{1}{2}} \left(\frac{T_1}{T_2}\right)^{0.7} \left(\frac{V_z}{V_i}\right)^{\frac{1}{2}}
\]

\[
\therefore \frac{M_{az}}{M_{az'}} = \left(\frac{V_z/a_z}{V_i/a_i}\right) = \left(\frac{V_z}{V_i}\right)\left(\frac{T_1}{T_2}\right)^{\frac{1}{2}}
\]

\[
\]
If it is further assumed that the gas velocity is proportional to the engine speed, the following relationship results:

$$\frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^\frac{1}{4} \left(\frac{N_2}{N_1}\right)^\frac{1}{4} \left(\frac{T_1}{T_2}\right)^{0.7}$$

...... (5.20)

Typical values for a point in the engine cycle where the temperature is changing rapidly is

- $P_2 = 7$ atmospheres, $T_2 = 500 \, ^oK$
- for $P_1 = 1$ atmosphere, $T_1 = 293 \, ^oK$

the value of $\tau$ calculated was .33 m sec.

$$\therefore \tau_2 = 0.33 \times \left(\frac{1}{7}\right)^\frac{1}{2} \left(\frac{500}{293}\right)^{0.7} = 0.33 \times 0.38 = 0.129 \, m\, sec.$$

This corresponds to .8 engine crank degrees at 1100 r.p.m.

Lyn and Valdmanis\(^{48}\) carried out tests to measure compression temperatures in an I.C. engine. Using the relationship established by Scadron and Warshawsky they produced theoretical curves for compression temperature in an engine running at 250 r.p.m. These curves are shown in Figs. 5.7.

A study of these curves in Fig. 5.7, shows that for all but 5 ms time constant curve serious distortion of the temperature curve has taken place. With the 5 msec curve there is a reduction of approximately 2.5% in the measured temperature. The cross plot of time constant against the % drop of temperature showed that at 1100 r.p.m. using a wire having .129 m sec time constant there will be less than 1.5% drop in the measured temperature. This also agrees with the order of temperature drop estimated from Fig. 5.6 (Benson and Brundrett \(^{55}\)). In this case, for the smallest wire (6.9 $\mu$ diameter), at moderate heat transfer coefficient, the predicted drop is approximately 3%.
FIG - 5.3 Cooling curves for Tungsten wire - sweep rates of 2, 5, 10 and 20 m/sec/cm.

FIG - 5.9 Cooling curves for 10 PT/304LR wire - sweep rates of 2, 5, 10 and 20 m/sec/cm.

FIG - 5.10 Graphical interpretation of the cooling curves.
Time Constant Determination for 10.4 µm PT/30% IR and 5.4 µm Tungsten Wires.

FIG 5.11 Cooling curves plotted on a logarithmic scale.

FIG 5.12 Variation of time constant of a thin thermocouple with Reynolds Number. (Solders and Warshawsky)
It is therefore concluded that even at the highest engine speed during the present study, the lack of correction to the compression temperature curve will result in an error not exceeding 2-3% in the worst case. At low speeds better accuracy will be attained.

5.5. Experimental Determination of Time Constants

In order to check the accuracy of calculations it was decided to measure the wire time constants experimentally.

Five microne tungsten and ten microne PT/30% IR wires were used for tests. The test procedure was to heat the wire to a predetermined temperature and record its rate of cooling.

The wire was heated by placing it in the hot wire anemometer circuit (described in the next chapter) and its temperature was set by setting the hot resistance to the required value. A fast acting double pole double throw switch was used to disconnect the wire from this circuit and connect it to the resistance measuring circuit. The resistance bridge was left in 'cold balance' position, thus on throwing the switch 'S', an out of balance signal was produced due to the wire being at a higher temperature. The rate of decrease of the wire resistance, which is the same as its cooling rate, was recorded on a storage oscilloscope for 2, 5, 10 and 20 msec/cm sweep rates. These records are shown in Figs. 5.8 and 5.9. A graphical interpretation of the temperature records is given in Fig. 5.10. Cooling curves were plotted on a logarithmic scale to show first order relationship. These plots are shown in Fig. 5.11. Since by definition, the time constant is the time required for an element to reach .632 of the final temperature after a step change in temperature, it can be directly measured on the curves shown in Fig. 5.8. For better accuracy, Fig. 5.10 was used. The time
constants measured for 5 microme tungsten and 10 microme PT/30% IR wires in still air were:

Tungsten ................ 1.75 m secs.
PT/30% IR .................. 4.4 m secs.

Comparing the 5 μ tungsten wire time constant with that calculated in section 5.4.4., the experimental values appear to be high. The variation of the time constant with Reynolds number for a chromel-constantan thermocouple has been plotted by Scadron and Warshawsky ⁶⁰, this is shown in Fig. 5.12. It shows clearly that the time constant increases with a decrease in the Reynolds number. The slope of lines is approximately -0.5. Therefore

\[ \tau \propto R_e^{-0.5} \]

which shows that a time constant measured under free convection conditions will have a much higher value than the calculated value in section 5.4.4. with an assumed 'h' of .25 cals/cm²°C/sec which implies quite a large value for the velocity of gases.

Looking at the experimental time constants, in a little more detail, for the case of free convection on horizontal cylinders Mcadams³⁰ has given a logarithmic plot of Nusselt number against the product of Grashof and Prandtl numbers.

Defining Grashof number as

\[ G_r(f) = \frac{\rho_f g \beta_f \Delta T D^3}{\mu_f^2} \]  \hspace{1cm} (5.29)

where the suffix 'f' refers to the film temperature,

ΔT is the temperature potential

\( \beta \) is the coefficient of volumetric expansion.

For a film temperature of 650°C used in the experiments
Using the plot given by Mcadams we have

\[ \phi(t) = \frac{\rho C_p}{\mu k} = 4.5 \times 10^5 \text{ /ft}^2 \text{c} \]

\[ G_{T_f} \times P_{r_f} = \phi(t) D^3 \Delta T \]

\[ \log_{10} G_{T_f} \times P_{r_f} = -5.8356 \]

\[ \log_{10} N_u = -6, \quad \text{or} \quad h = 0.6 \text{ cal/s/cm}^2/\text{C}/\text{sec}. \]

for a 5 micron diameter wire.

Since

\[ \tau \propto \frac{1}{h} \]

On the basis of foregoing analysis, the ratio of two time constants should be as follows:

\[ \frac{\tau_{\text{experiment}} (\text{free convection})}{\tau_{\text{calculated}} (\text{assumed } h)} = 4 \]

The actual ratio is approximately 5, which is considered to be of the right order.

5.6. End Conduction Errors

Scadron and Warshawsky\(^6\) and Benson and Brundrett\(^5\) have discussed these errors in great detail. The relationship involved is discussed briefly. Assuming the following relationship for the time dependent \( T_g \)

\[ T_g = T e^{i\omega \theta} \]

and using the usual heat balance considerations we have for a wire
of length 'L' the following expression for the difference between the indicated temperatures and the temperature without end conduction.

\[
\Delta T_{\text{cond}} = \frac{T_0}{L} \left[ \begin{array}{c}
\left\{ \cos(\frac{\theta}{2}) \sinh \theta a + \sin(\frac{\theta}{2}) \sin \frac{2b}{2} \right\} \left( \sin \omega \theta - \omega \tau \cos \omega \theta \right) \\
- \left\{ \sin \frac{2b}{2} \cos(\frac{\theta}{2}) - \sin(\frac{\theta}{2}) \sinh \theta a \right\} \left( \cos \omega \theta + \omega \tau \sin \omega \theta \right)
\end{array} \right] \\
\left( 1 + \omega^2 \tau^2 \right) \left( \ell \sqrt{2} h + \kappa \right) \left( 1 + \omega^2 \tau^2 \right)^{\frac{1}{4}} \left( \cosh \theta a + \cos \theta a \right)
\]

with

\[
a = L \sqrt{2} h / \kappa \left( 1 + \omega^2 \tau^2 \right)^{\frac{1}{4}} \cos \left( \frac{\theta}{2} \right)
\]

\[
b = L \sqrt{2} h / \kappa \left( 1 + \omega^2 \tau^2 \right)^{\frac{1}{4}} \sin \left( \frac{\theta}{2} \right)
\]

\[
\phi = \arctan \left( \omega \tau \right)
\]

\[r = \text{wire radius}
\]

\[\omega = \text{angular velocity}
\]

Since \(a \propto \tau^{-\frac{1}{2}}\), for small diameter wires \(a \gg 2\)

Equation 5.30 simplifies to

\[
\Delta T_{\text{cond}} = \frac{T_0}{L} \sqrt{2} h / \kappa \left( 1 + \omega^2 \tau^2 \right)^{\frac{1}{4}} \left[ \omega \theta \cos \left( \omega \theta + \phi \frac{1}{2} \right) - \sin \left( \omega \theta - \phi \frac{1}{2} \right) \right] \\
\left( 1 + \omega^2 \tau^2 \right)^{\frac{1}{4}} \sin \left( \phi \frac{1}{2} \right)
\]

for maximum error

\[
\omega \theta = \arctan \left[ - \frac{\cos \left( \phi \frac{1}{2} \right)}{\sin \left( \phi \frac{1}{2} \right)} \right]
\]

Based on these equations, maximum conduction error for the case of 5 \(\mu\) tungsten wire is approximately 1.5% of the temperature fluctuation for 1100 r.p.m. case.
5.9.5 No allowances were made for the temperature recovery because this tends to reduce the thermal lag and end conduction errors. Thus the overall accuracy of the measured temperature will be approximately 6\% in the worst case at the highest engine speed of 1100 r.p.m. Better accuracy can be expected at lower engine speeds.

5.9.6 The time constant of 5 μ wire was estimated for a typical case and also determined experimentally. The agreement between the measured and estimated time constants was considered reasonable.

5.9.7 The time constant of 5 μ wire was considered small enough to give a faithful reproduction of the engine cycle temperature variation. A phase lag of approximately 0.8 engine crank degrees at 1100 r.p.m. was estimated.
5.7. Recovery Temperature

Since the wire records the recovery temperature, allowance should be made for this. For a gas velocity of 200 ft/sec, assuming a recovery factor of unity, the wire will record a temperature approximately 3°C higher than the true gas temperature. This error results in recording a higher temperature than the true gas temperature, therefore it tends to cancel out some of the thermal lag and end conduction errors. Because of this, no allowances were made for this error.

5.8. Measurement Errors

These errors are introduced mainly due to the following factors.

5.8.1. Tolerances on the temperature coefficient of resistance.
5.8.2. Measurement of the wire resistance.
5.8.3. Contact resistances, etc.

These errors are discussed in detail in Appendix 5.1. It is shown that the overall error will be approximately ± 6% in the worst case.

5.9. Conclusions

5.9.1. Resistance thermometry was considered to be a suitable method for engine gas temperature measurement.
5.9.2. It was found necessary to establish the temperature coefficient of resistance of the materials used.
5.9.3. It was shown that a 5 μ wire will give a reasonable record of gas temperature changes even at the highest engine speed of 1100 r.p.m. It was calculated that in the absence of any correction technique for the thermal lag, the recorded temperature would be 1.5% lower than the actual gas temperature at the highest engine speed.
5.9.4. Neglecting the end conduction error correction results in an error of approximately 1.5%.
CHAPTER 6

GAS VELOCITY MEASUREMENT IN I.C. ENGINE CYLINDERS
6. GAS VELOCITY MEASUREMENT IN I.C. ENGINE CYLINDERS

6.1. General Comments

The flow inside an internal combustion engine is non-steady because of the periodic motion of the piston. This fluctuation of flow is also accompanied by large changes in the density and temperature of the working gases which result in complications to the problem of gas velocity measurement inside the engine cylinder.

Methods for the non-steady flow measurements may be divided into two broad categories:

(a) Optical Techniques

(b) Methods involving the use of a flow detector within the flow under investigation.

(a) Optical Methods

(i) Methods involving the use of a tracer or other indicator introduced into the flow to make the flow pattern visible or observable by a suitable detecting apparatus outside the field of flow. These methods include the use of smoke for flow visualization. The results obtained by these methods are generally qualitative.

(ii) Methods making use of changes in physical properties of the medium by detecting these changes with the passage of a light beam through the medium.

These methods may depend on one of the following physical phenomena:

(a) The speed of light is related to the refractive index of the medium through which it is transmitted. The refractive index of the gases can be related to the density, therefore this phenomenon can be used to detect small changes in the density of the medium.
(b) Light passing through a density gradient in a gas is deflected as though by a prism.

The interferometer is based on the first phenomenon. It measures the density changes directly and is most suited to quantitative determination of the density field, from which the velocity pattern can be derived.

The Schlieren technique is based on phenomenon (b) and measures the changes in the density gradient in the fluid. It is theoretically possible to interpret the results quantitatively but its main use is to obtain a qualitative picture of the flow pattern.

The Shadowgraph method measures the second gradient of density, i.e., the first derivative of the density gradient. It is therefore suitable for use in a situation where large variations of the density gradient occurs, i.e., study of shock waves, etc.

Of these three methods, the interferometer gives the most useful results but it is expensive to manufacture and difficult to operate. In the case of engine measurements, this method requires expensive plate glass windows and would be difficult to apply.

The optical methods have been widely used in the study of the combustion process in I.C. engines. These will not be discussed in detail. Most of the results obtained to date are qualitative, mainly because it is difficult to interpret the results quantitatively due to the three-dimensional nature of the fluid motions. These methods are also difficult and expensive when applied to the I.C. engine problems, mainly due to the necessity of having precision ground glass or quartz
windows. It was therefore decided to apply some other method of measuring the gas velocity inside the engine cylinder.

6.2. Requirements for a Flow Sensor inside the Engine

(i) The sensor has to be sufficiently rugged to withstand the arduous conditions of operation inside the engine cylinder.

(ii) It should cause minimum possible disturbance to the flow by its presence.

(iii) It should be of such a size that the velocity measurement may reasonably be assumed to be a point measurement and not a space average measurement.

(iv) It must have a very rapid response to changes in flow velocity so that the measured velocity may be regarded as an instantaneous velocity without any serious phase lag or attenuation.

(v) It must have a high sensitivity.

(vi) It must give repeatable results over a period of time. This implies the stability of the sensing element.

Owen\(^1\) has given various designs of pilot tubes which meet some of the requirements listed above but there are serious objections to the use of pilot tube for the measurement of a rapidly fluctuating flow. The main objection is the low response of the pilot tubes which introduces phase lag and attenuation of the basic flow pattern being recorded. In the application of pitot tubes to indicate flow fluctuations, the pitot static differential pressure is fed to the opposite sides of a sensing diaphragm of an electronic transducer. It is therefore necessary to have a relatively long length of narrow tubing connecting the tube to the transducer. The combination of a long narrow tube and a comparatively large chamber at the diaphragm can lead to serious errors. Moreover, the disturbance caused to the
Comparison of the hot-wire anemometer and the Pitot-tube for measurement of mean velocity

**Fig. 6-1**

**Fig. 6-2**

*Constant current and constant temperature operation of a 5 diameter 0.024 in. dia. tungsten hot wire (Davis)*
flow due to presence of even a small hypodermic pitot tube is not acceptable. The hot wire anemometer meets most of the requirements listed above. The main drawback is the fragile nature of the sensing element and its weakness in flows containing dust or oil particles. Another difficulty is the interpretation of results. Apart from these drawbacks, it has an excellent response, sensitivity and the sensing probes can be made of extremely fine wires which introduce very little disturbance in the flow regime. Also, it can easily be adapted for use on an engine.

It must be mentioned that normally for the steady state measurements, the hot wire anemometer for the mean gas velocity measurement is inferior to the pitot tube, when sensitivity and reproducibility are considered. The response curves for the hot wire anemometer and the pitot tube are shown in Fig. 6.1. It is clear from a study of these curves that for low mean velocities (5-60 m/sec) the hot wire anemometer is more sensitive but at the higher velocities the pitot tube is far more sensitive. When studying unsteady phenomenon, the hot wire anemometer with electronic thermal lag compensation is far superior. From the foregoing discussion it is clear that the hot wire anemometer was considered a suitable tool for velocity measurement inside the engine cylinder.

6.3. Constant Current and Constant Temperature Hot Wire Anemometers

6.3.1. Basically, there are two methods of operation of the hot wire anemometer. In one mode of operation a constant current is supplied to the wire and the changes in resistance of the wire, produced by the cooling effect of the flow, are measured and related to the flow velocity. This is called the 'constant current hot wire anemometer'. The other mode of operation is the constant temperature or constant resistance; in this mode of operation the temperature of the sensing
wire is kept constant by using a high gain feedback amplifier. The subsequent changes in the heating current give a measure of the flow velocity. The operating characteristics of the two systems are shown in Fig. 6.2. It can be seen, from these curves, that in the case of constant temperature operation the rate of heat supply to the wire increases as the velocity increases and in the case of constant current operation the power dissipation decreases as the velocity increases. Therefore it is sometimes necessary, in the case of constant current operation, to increase the current at high velocities to obtain improved sensitivity. This means that the current has to be decreased when the velocity decreases otherwise serious damage to the sensing element may occur, due to overheating. In spite of this disadvantage, in the absence of a reliable means of thermal lag compensation for the constant temperature anemometer, the constant current anemometer has been used widely in the past. The constant current anemometer can give better sensitivity at low gas velocities but as the velocity rises this advantage is lost. This result is an apparent contradiction of the theoretical analysis by Davis which suggests a better sensitivity for the constant current anemometer if the wire is operated at a resistance ratio of 1.5. However, the effects of end supports and the variation of heat transfer coefficient is ignored in this analysis and these tend to reverse the trend at higher flow velocities.

6.3.2. In general, the constant current hot wire anemometer is unsuitable for an application where large fluctuations in the mass flow rate are likely to occur. Therefore in the case of gas velocity measurement inside the engine cylinder, where such fluctuations are present, it was decided to use the constant
temperature hot wire anemometer.

6.3.3. It is normal practice to make the sensing element one arm of a Wheatstone bridge circuit. The bridge balance can then be adjusted (manually in the case of constant current anemometer) electronically for the constant temperature circuit. If the velocity fluctuation frequency is high, the effects of thermal inertia of the wire become significant and these have to be compensated electronically. Until the advent of the transistor devices, such thermal lag compensation circuits were expensive and operationally complex, but with the general availability of such devices the task has been made much simpler. This was an additional reason for choosing the constant temperature hot wire anemometer.

6.3.4. In an environment where gas velocity fluctuations are also accompanied by gas temperature changes, for instance inside an internal combustion engine cylinder, the wire also responds to the gas temperature fluctuations. The effect of an increase in the gas temperature is to reduce the output signal from the circuit. The sensitivity of the output signal being roughly inversely proportional to the temperature difference between the gas and the wire. This presents great problems in the interpretation of results. This effect can be compensated electronically as suggested by Semenov\(^4\), or by calculations, as was done in the present study. These two methods of compensation will be discussed later.

6.3.5. An understanding of the operation of the constant temperature hot wire anemometer requires an understanding of laws of heat transfer to and from the wire, variation of gas properties with temperature and pressure, and the physical and electrical properties of the wire material. On the practical side a knowledge of probe manufacture, knowledge of operation, calibration and performance of the practical systems is essential.
6.4. The Constant Temperature Hot Wire Anemometer

6.4.1. General Comments

L. V. King\textsuperscript{75} in his classic paper, published in 1914, laid down the foundations of hot wire anemometry. In the same year Schrödt published his thesis and produced empirical calibration graphs for heat losses from a thin wire.

Originally constant current system gained popularity because of its simplicity. In order to improve its response to high frequency fluctuations electrical compensation of the lag was required. Dryden and Kuethe\textsuperscript{76} used such compensation to correct the high frequency attenuation of the signal. Kovasznay\textsuperscript{74} described the equipment to cover the frequency range from 2 to 70 k cycles/sec.

Constant temperature system increases the frequency of the system to several times its range at constant current operation. Ziegler\textsuperscript{77} suggested this in 1934. The idea was further developed by Weske\textsuperscript{78} and Ossafsky\textsuperscript{79} derived the requirements of direct coupled amplifier system so as to avoid high frequency oscillations. In France a carrier-wave system was developed which gave stable and reliable operation compared with the earlier systems. More recently Davies and Fisher\textsuperscript{67} used a solid state D.C. amplifier system for constant temperature operation. This system gave satisfactory response up to 40 kc/sec which was well above the required level in the present study. The circuit details were given in the published paper. Since it was a simple system which could be produced cheaply and it had given satisfactory results, it was decided to use this in the present study.

6.4.2. Response of the Sensing Element to Flow Parameters

King\textsuperscript{75} derived the following theoretical relationship between
the power dissipated and the mass flow rate

\[ I^2 R_w = e K_g l (T_w - T_g)(1 + \sqrt{2 \pi \rho_g c_p V_d / \kappa_g}) \] .... (6.1.)

This may also be expressed as

\[ N_u = A + B R_e^{1/2} \] .... (6.2.)

Where

- \( I \) is the current flowing through the probe
- \( R_w \) is the 'hot resistance' of the element
- \( e \) = conversion factor to convert heat units into electrical units
- \( K_g \) = Thermal conductivity of the fluid
- \( l \) = Length of the sensing wire
- \( T_w \) = Wire temperature (hot)
- \( T_g \) = Gas Temperature
- \( d \) = Wire diameter
- \( \rho_g \) = Density of the fluid
- \( \mu_g \) = Fluid viscosity (absolute)
- \( C_p \) = Specific heat of the fluid at constant pressure
- \( V \) = Free stream velocity
- \( h d \over K_g \) = Nusselt number
- \( R_e = {U \rho_g d \over \mu_g} \) = Reynolds number

The equations 6.1. and 6.2. may also be written in the following form:-
LINEAR RELATIONSHIP BETWEEN $I^2$ AND $\sqrt{V}$ FOR A HOT WIRE

FIG. 6.3

HOT WIRE IN GAS FLOW

FIG. 6.4
\[ \frac{I^2 R_w}{R_w - R_g} = A + B \sqrt{V} \] ...... (6.3.)

Therefore, by plotting \( I^2 \) against \( \sqrt{V} \) the intercept will give \( A \left( \frac{R_w - R_g}{R_w} \right) \) and the slope of the straight line will give \( B \left( \frac{R_w - R_g}{R_w} \right) \). This is shown in Fig. 6.3.

In the compressible flow equation 6.2. may be modified to:

\[ Nu = A g(M_a) + B f(M_a) \frac{R_e^{1/2}}{P_r} \] ...... (6.4.)

where \( g(M) \) and \( f(M) \) are functions of Mach number \( \frac{v}{a_g} \), where \( a_g \) is the velocity of sound in the fluid.

Also the coefficient 'B' varies with the Prandtl number.

These features of the basic equation 6.2. are discussed in greater detail in Appendix A - 6.1.

Since King assumed potential flow round the cylinder, which ignores the formation of a boundary layer around the cylinder, the results are not strictly correct but his form of correlation has been widely used by other workers who have produced a wealth of empirical data and presented it in the form of equation 6.2.

Consider the case of an electrically heated wire in cross flow of air as shown in Fig. 6.4. Let the origin be at the midpoint of the wire and the total length of the wire be '2\( \ell \)'. Consider the thermal equilibrium of a unit length of wire. The radial temperature gradient in the wire may be ignored because the wire length is generally about \( 10^3 \) times that of its diameter and also the thermal conductivity of the wire material is about \( 10^4 \) times that of the fluid.
The conduction component of the heat transfer is

\[ Q_k = -A \kappa \frac{d^2 T}{dx^2} + \frac{dK}{dT} \left( \frac{dT}{dx} \right)^2 \]  \hspace{1cm} (6.5)

The heat loss due to radiation is

\[ Q_r = \pi d \sigma (T_w^4 - T_s^4) \]  \hspace{1cm} (6.6)

The heat input due to the electrical energy supplied to the wire is

\[ Q_e = \frac{I^2 \lambda(T_w)}{A} \]  \hspace{1cm} (6.7)

The heat loss due to convection is

\[ Q_c = \pi d h (T_w - T_g) \]  \hspace{1cm} (6.8)

where

- \( \sigma \) is the radiation constant
- \( h \) is the convective heat transfer coefficient
- \( \kappa \) is the thermal conductivity of the wire material
- \( \lambda \) is the electrical resistivity of the wire material
- \( A \) is the cross sectional area of the wire

\[ Q_e = Q_k + Q_c + Q_r \]  \hspace{1cm} (6.9)

which gives

\[ KA \frac{d^4 T}{dx^4} + A \frac{dK}{dT} \left( \frac{dT}{dx} \right)^2 + \frac{I^2 \lambda(T)}{A} - \pi d h (T_w - T_g) + \sigma (T_w^4 - T_g^4) = 0 \]  \hspace{1cm} (6.10)
This is a non-linear equation which can be solved on a digital computer using step by step methods. Davies and Fisher\(^7\) have made simplifying assumptions and neglected the radiation losses thereby linearizing it.

In the linearized form the equation 6.10 is as follows:

\[ K_A \frac{d^2 T}{dx^2} + \frac{I^2 \lambda_0 \alpha}{A} - \pi d \frac{h}{A} (T_w - T_o) + \frac{I^2 \lambda_0}{A} = 0 \ldots \ldots (6.11) \]

Simplifying assumptions were:

(i) No radiation losses
(ii) Constant thermal conductivity \(K\)
(iii) The electrical resistivity and temperature relationship.

\[ \lambda_T = \lambda_o (1 + \alpha T) \] where '\(\alpha\)' is the temperature coefficient of resistance. \(T_o\) is a reference temperature and \(\lambda_o\) is the resistivity of the wire material at the reference temperature. \((T_o)\)

The equation 6.11 reduces to:

\[ \frac{d^2 T}{dx^2} + K_1 T_1 + K_2 = 0 \ldots \ldots (6.12) \]

where

\[ T_1 = (T_w - T_o) \]
\[ K_1 = \frac{I^2 \lambda_0 \alpha}{K A^2} - \frac{\pi d h}{K A} \]
\[ K_2 = \frac{I^2 \lambda_0}{K A} \]

If some mean value of '\(h\)' is assumed \(K_2\) is a constant. For most values of '\(h\)', \(K_2\) is negative. The solution of 6.12 is then given by

\[ T_1 = \frac{K_2}{K_1} \left[ \cosh \left( \frac{K_1}{K_2} \frac{x}{\cosh \left( \frac{K_1}{K_2} \ell \right) } \right) \right] \ldots \ldots (6.13) \]
If equation 6.13 is integrated for the whole length of the wire, i.e., between the limits \(-l\) to \(+l\) we get the mean temperature of the wire

\[ T_w' = \frac{K_w}{|K_w|} \left[ \tanh \sqrt{|K_w|} \frac{l}{|K_w|} - 1 \right] \]

where \( T_w' \) is the integrated temperature difference between the wire and the gas. It includes the effects of end conduction.

The conductive heat loss to the supports can be expressed as

\[ \frac{dT_i}{dx} = \frac{K_s}{|K_s|} \tanh \sqrt{|K_s|} \frac{l}{|K_s|} \]

defining \( \frac{Nu_w}{Nu_m} \) as the Nusselt number in the case of infinite wire, i.e., without end losses and \( Nu_m \) as the measured Nusselt number we have

\[ \frac{Nu_w}{Nu_m} = \frac{1}{2} \frac{R \omega}{A} \frac{1}{2} \frac{\lambda y \alpha / A}{2 \pi l K_w T_i} = Nu_w \]

Substituting \( \frac{R_w - R_g}{2 \pi l K_w T_i} \) for \( \lambda y \alpha / A \)

and

\[ Nu_m = \frac{1}{2} \frac{R_w}{2 \pi l K_w T_i} \]

we have

\[ K_s = \frac{4K_s}{K_s d_s^2} Nu_m \left[ Nu_r - \frac{R_w - R_g}{R_w} \right] \]

\[ \ldots \ldots \ldots (6.14) \]

where \( K_w \) is the conductivity of gas at wire temperature and \( K_s \) is the thermal conductivity of the support material at the support temperature. Equation 6.14 can be solved by iterative techniques.

Davis and Fisher have solved the equation 6.10 ('exact solution') and equation 6.14 ('approximate analytic solution'). The temperature distribution on a wire calculated according to equations 6.10 and 6.14 is shown plotted in Fig. 6.5. The difference between the two solutions is small. Davis and Fisher have quoted a figure of 2 to 3% difference between the two solutions. Further development of this approach, which was used in the present
study, will be discussed later. First a brief discussion of heat transfer to cylinders in cross flow of air will be carried out.

6.4.3. Heat Transfer to Cylinders in Cross Flow of Gases

McAdams has summarised the work of numerous workers and has suggested the following relationship for general use:

\[ N_u = A + B \Re^n \]

which is the same as King's relationship. Values of A, B and n depend on the range of Reynolds' number under consideration. For a Reynolds' number range of .1 to 1000 the recommended values are

\[ N_u = 0.32 + 0.42 \Re^{0.52} \]

(6.15)

Collis and Williams have recommended the following relationship

\[ N_u = \left[ 0.24 + 0.56 \Re^{0.45} \left( \frac{T_M}{T_m} \right)^{0.17} \right] \]

(6.16)

for \( 0.02 < \Re < 44 \) and

\[ N_u = 0.48 \Re^{0.45} \left( \frac{T_M}{T_m} \right)^{0.17} \]

(6.17)

for \( 44 \Re > 140 \).

Where \( T_m \) is the mainstream gas temperature and \( T_M \) is the arithmetic mean of gas and wire temperature.

The discontinuity at \( \Re = 40 \) is supposed to be due to Von Karman "streets" forming in the wake of the cylinder.

Hilpert obtained the heat transfer data over a wide range of Reynolds number. His results are very consistent and are widely used.

He proposed the following power law relationship:

\[ N_u = C \left[ \Re \left( \frac{T_m}{T_M} \right) \right]^{1/4} \]

(6.18)
Computed temperature distributions on a 0.3 cm long, 5 μm diameter tungsten wire for a range of air speeds. (From Davis and FIGURE 6-5)

Comparison of Nusselt number measurements. x, 2.5 μm, platinum wire; +, 4.86 μm, 'pure' tungsten; O, 5 μm tungsten; •, Spengenberg (corrected); □, Collis & Williams. Estimates: (a) McAdams; (b) Collis & Williams; (c) equation (1.3). FIG-6-6
Values of $C$ and $n$ depend on $Re$ range.

McAdams has summarised the values of $C$ and $n$ over a wide range of Reynolds number.

Hinze recommends the following relationship

$$N_u = -0.42 Pr^{0.2} + 0.57 Pr^{0.35} R_e^{0.5} \quad \ldots \quad (6.19)$$

Davies and Fisher have replotted Nusselt number against Reynolds number for cylinders; this graph is reproduced in Fig. 6.6. The results fall into two distinct groups. Group I contains the data summarised by McAdams and data due to Collis and Williams. Group II includes the data due to Davies and Fisher, Lowell, Laufer, and high wire temperature data of Spangenberg.

It is clear that the differences between the two groups at $Re = 40$ is more than 200%. Davies and Fisher have pointed out that some of the difference is due to the practice of evaluating the fluid properties at different temperatures. Nusselt numbers in Group II were evaluated at the wire temperatures whereas in Group I the fluid properties were evaluated at the film temperature. Collis and Williams' results were corrected for this effect and there was a significant change in the plotted curve shown in Fig. 6.6., but even after this correction the differences between the corrected curve and Group II is too large. This discrepancy could be due to the difficulty of arriving at the true temperature of the sensing elements. Large variations in the values of temperature coefficient of resistance can occur (Spangenberg) which result in incorrect estimate of the wire temperature.

The foregoing discussion has made it clear that due to inconsistency of the available data on heat transfer from cylinders,
it cannot be used directly in hot wire anemometry. It is usual practice to calibrate each probe separately. Generally this task is straightforward, but in special cases, for example measurements in an engine cylinder, direct calibration presents difficult problems.

6.5. Mean Gas Velocity Measurement Inside an I.C. Engine Cylinder

6.5.1. Very few attempts have been made to obtain the mean gas velocity inside the I.C. engine cylinders. Only five publications could be traced. Wenger used a constant current probe of 15 μdiameter tungsten wire in a 190 mm. bore x 160 mm. stroke motored single cylinder engine. Mean velocities for the suction and exhaust stroke were obtained. End conduction errors were ignored in this analysis. Tindal in 1963 used the hot wire anemometer in a single stroke apparatus. PT/10% IR probes were used in preference to the tungsten probe because of drift in the calibration of tungsten probes. These tests were however for low pressure and temperature conditions. Lob in 1966 used a constant temperature hot wire anemometer inside the cylinder of a motored engine but here again the difficulties associated with the high temperatures and pressures were avoided by replacing the air inlet valve by an orifice. Thus the tests were not truly motored tests. Semenov used the constant temperature hot wire anemometer in a truly motored engine. He used separate wire to cancel the effects of gas temperature changes. This technique will be discussed in greater detail later.

6.5.2. Analytical Correction for the Effects of Time Dependent Parameters in the Engine Cycle

During the engine cycle gas temperature, gas pressure and hence the density vary over a wide range. Since the heat transfer from the wire is greatly affected by all these parameters, it is necessary to apply analytical or electrical corrections to the output signal
from the anemometer bridge. These methods of correction will now be discussed separately.

(i) **Analytical Corrections**

At the time when the present project was started, in early 1965, the author was unaware of any attempt in this field. It was therefore decided to investigate the possibilities. Starting with the usual Nusselt-Reynolds relationship for fine wires, we have

\[
Nu = \frac{h}{K(T_w - T_f)} = A + B \frac{Re^\frac{1}{2}}{\cdots \cdots \cdots (6.20)}
\]

Variation of thermal conductivity of air and its viscosity with temperature may, with reasonable accuracy, be approximated by the following relationships

\[
\frac{K}{K^*} = \left(\frac{T}{T^*}\right)^{0.86} \quad \text{and} \quad \frac{\mu}{\mu^*} = \left(\frac{T}{T^*}\right)^{0.71} \cdots \cdots \cdots (6.21)
\]

Also the density ratio may be expressed as

\[
\frac{\rho}{\rho^*} = \left(\frac{T}{T^*}\right) \left(\frac{P}{P^*}\right) \cdots \cdots \cdots (6.22)
\]

where \(T, \mu, \rho\) and \(P\) are at some reference point and \(T^*, \mu^*, \rho^*, \text{\ and } P^*\) are the instantaneous values at a given point in the engine cycles. Combining 6.20, 6.21 and 6.22 we have

\[
h = \Delta T \left[ AK^* \left(\frac{T}{T^*}\right)^{0.86} + B \left(\frac{T}{T^*}\right) \left(\frac{P}{P^*}\right) \frac{\frac{1}{2} \frac{1}{2}}{\mu^*} \right] \cdots \cdots \cdots (6.23)
\]

There are two unknown quantities in the above equation, \(h'\) and \(V\). Starting with a reasonable value of \(h'\), it can be solved using an iterative technique to yield \(V\).
Fig. 6.7: Logarithmic variation of Nusselt number with Mach number for infinitely long wires. (Spangenberg^68^)

FIG 6.8 Heat transfer data for cylinders in air by Spangenberg. (Benson^68^)
However there are some serious shortcomings which make this method unacceptable. Spangenberg has made a thorough study of the effects of density and velocity on Nusselt number in the case of fine electrically heated wires. He has concluded that "In the subsonic range, large differences occur in the response to velocity and density. The parameters governing this behaviour are Mach number $\frac{d}{\lambda}$ and the temperature difference $\Delta T$. Fine wires, in general, exhibit low sensitivity to velocity, again depending on $\frac{d}{\lambda}$ and $M_a$".

Here the dimensionless parameter $\frac{d}{\lambda}$ (reciprocal of Knudsen number) represents the effect of density.

$\lambda$ is the molecular mean free path.

It can be established from the kinetic theory that the product

$$\rho \lambda = \text{constant}$$

$$\therefore \rho_d = \frac{d \times \text{constant}}{\lambda} = a \frac{d}{\lambda}$$

Figures 6.7 and 6.8 are reproduced from Spangenberg's paper to illustrate this important point in detail. Nusselt number variation with Mach number is plotted on logarithmic scale in Fig. 6.7, for various values of $\frac{d}{\lambda}$ ranging between 4.9 to 1470. Taking some typical values for 5μ wire we have

$$\frac{d}{\lambda} = \frac{5 \times 10^{-4}}{6.3 \times 10^{-6}} = 79.3$$

here a value of $\lambda = 6.3 \times 10^{-6}$ cm at 760 mm. of mercury and 15°C is taken. Referring to Fig. 6.7 and Mach number of .4, point A gives the value of $N_{u\infty}$, the Nusselt number without end conduction.

$$N_{u\infty} = 3.5 \quad \text{point A}$$
Now assume that the density of the gas decreases such that 
\[
\frac{d}{\lambda} = 20, \text{ i.e., one quarter its previous value; in this case}
\]
point B gives the \(N_{\infty}\)

\[
N_{\infty} = 1.7 \quad \ldots \ldots \text{point B}
\]

Since Mach number gives the change in velocity of gases it also gives the variation of Reynolds number at a constant density. Since reducing the parameter \(\frac{d}{\lambda}\) resulted in a Reynolds number \(\frac{1}{4}\) its previous value; same effect can be achieved by keeping \(\frac{d}{\lambda}\) constant and reducing the Mach number to \(\frac{1}{4}\) its previous value, i.e., \(M = 1\). Point C is shown on the graph for this condition.

\[
N_{\infty} = 2
\]

This shows that for the same Reynolds number two different Nusselt numbers can be obtained depending on whether the change in the Reynolds number was brought about by a change in density or the velocity of the gas. Since the velocity is proportional to the square of the Nusselt number, the two velocities obtained by the use of this method will have the following ratio:

For a constant \(R_e\)

\[
\frac{\text{Velocity (due to change in density)}}{\text{Velocity (due to change in velocity)}} = \left(\frac{1.7}{2}\right)^2 = 0.72
\]

or a difference of almost 30%.

Figure 6.8 shows Spangenberg data replotted by Benson. Here again the slope of constant Knudsen number lines and constant Mach number lines are different. Points 'X' and 'Y' are plotted by taking two sets of values for Knudsen and Mach numbers such that Reynolds number remains unchanged.
Point X \[ K_n = 0.2 \]
\[ M = 0.4 \]
Point Y \[ K_n = 0.05 \]
\[ M = 0.1 \]

The values of Nusselt number obtained for the two cases are

\[ \begin{align*}
X & \quad N_u = 0.68 \\
Y & \quad N_u = 1.2
\end{align*} \]

Velocity ratio in this case is

\[ \frac{V_X}{V_Y} = \left(\frac{0.68}{1.2}\right)^2 = 0.32 \]

The above reasoning shows that this method will not give reliable results.

(ii) Corrsin Method

Another method for the correction of temperature effects is due to Corrsin. Starting with

\[ \frac{\int R_w}{R_w - R_s} = A + B \sqrt{\theta} \]

where

\[ A = \alpha \frac{l K}{R_0 \theta} \]
\[ B = \frac{b l}{R_0 \alpha} \sqrt{\frac{d c_p K}{\rho}} \]

\[ \alpha \text{ and } b \text{ are constants} \]

At any other gas temperature \( T_r \)

\[ \frac{\int R_w}{(R_w - R_s)} = A_r + B_r \sqrt{\theta} \]

\[ \therefore \quad \frac{A}{A_r} = \frac{K}{K_r} \quad ; \quad \frac{B}{B_r} = \sqrt{\frac{K \rho}{K_r \rho_r}} \]

with

\[ \frac{K}{K_r} = \frac{T_r + 1.25}{T_r + 1.25} \left( \frac{T_r}{T_r} \right)^{3/2} \]
Hot-wire calibration in heated air, $20 \leq T \leq 37 \text{ C (o.o...)}$

Fig 6.10 Spankenberg's reported heat loss measurements from electrically heated wires in air. (From Chord and Fronek 60)

Fig 6.11 Convective heat loss from electrically heated wires in air. (From Davis and Fisher 60)
and

\[ R_y = R_0 (1 + \alpha (T_y - 273)) \]

\[ R_x = R_0 (1 + \alpha (T_x - 273)) ; \text{i.e.} \ T_o = 273 \text{ °K} \]

we have

\[
\frac{A}{A_f} = \left( \frac{T_r + 125}{T + 125 + \phi(T_r, R_i/R_f)} \right)^{3/2} \tag{6.24}
\]

and

\[
\frac{B}{B_f} = \left[ \frac{T_r + 125}{T + 125 + \phi(T_r, R_i/R_f)} \right]^{1/2} \left[ \frac{T_r + \phi(T_r, R_i/R_f)}{T_r} \right]^{1/4} \tag{6.25}
\]

\[
\phi (T_r, R_i/R_f) = \frac{1}{\alpha} + \left( T_r - 273 \right) \left( \frac{R_i}{R_r} - 1 \right) \tag{6.26}
\]

Using this approach some experimental work was carried out by Corrsin over a limited gas temperature range of 20°C < T_g < 87°C. Figure 6.9. shows a plot of A-A_r experimental against A - A_r theoretical. The agreement is quite poor, therefore it was decided not to adopt this approach although some calculations were performed for comparison with the correction method used in the present study.

(iii) Semenov Method

Semenov\textsuperscript{24} used electronic means for compensating the effects of temperature and density changes. Assuming a Nusselt-Reynolds relationship of the form

\[
N_u = C R_e^m \tag{6.27}
\]

\[
h = C k d^{n-1} \left( \frac{V_0}{\mu} \right)^m \tag{6.28}
\]

If some velocity 'V' corresponding to initial conditions temperature and density conditions of T_o and \( \rho_o \) is recorded
in terms of the probe current $I_o$, Semenov assumed that this velocity could be recorded by a current $I$ for any $T$ and and he defined the relationship between $I$ and $I_o$ as

$$I_o = \varphi_{PT} I \quad \cdots \quad (6.29)$$

where $\varphi_{PT}$ is called the coefficient of reduction to initial conditions.

Using equation 6.28 and heat balance on the wire (ignoring the conduction and radiation losses) the following expression is obtained for $\varphi_{PT}$:

$$\varphi_{PT} = \left( \frac{\rho_o}{\rho_T} \right)^{\frac{n}{2}} \left( \frac{K_o}{K_T} \right)^{\frac{1}{2}} \left( \frac{\mu_T}{\mu_o} \right)^{\frac{n}{2}} \quad \cdots \quad (6.30)$$

also

$$\frac{T}{T_o} = \left( \frac{\nu}{\nu_o} \right)^{n-1}$$

for the case of engine gases,

where $\nu$ denotes the volume and 'n' is the polytropic index of compression

$$\varphi_{PT} = \left[ \frac{1}{\eta_v} \left( \frac{T}{T_o} \right)^{\frac{1}{n-1}} \right]^{\frac{n}{2}} \left( \frac{K_o}{K_T} \right)^{\frac{1}{2}} \left( \frac{\mu_T}{\mu_o} \right)^{\frac{n}{2}} \quad \cdots \quad (6.31)$$

where $\eta_v$ is the volumetric efficiency.

Thus $\varphi_{PT}$ is shown to be proportional to the gas temperature.

A signal proportional to $\varphi_{PT}$ is produced by using a thin wire as a resistance thermometer. This signal and the signal from the hot wire anemometer circuit are passed through a cascade signal multiplier and the resulting signal was then linearized to find the gas velocity. No experimental
verification of equation 6.28 was presented. Because $\varphi_{pT}$ is based on equation 6.27, objections applied to the first method also apply here.

(iv) Method of Davies and Fisher

Davies and Fisher\textsuperscript{67} have recently proposed a method of calculations to obtain the calibration curve of a constant temperature hot wire anemometer from a knowledge of gas properties and physical and electrical properties of the sensing material. Since it was decided to use this method in the present study, it will be discussed here in some detail.

It is assumed that 'pVd' represents the rate of arrival of mass per unit length in the neighbourhood of the cylinder. Later arguments depend on the hypothesis that the coefficient of frictional drag $c_f$ expresses the proportion of this mass flow which takes part in the transport of properties at the surface of the cylinder. It is also assumed that $c_f$ remains the same when heat transfer is occurring between the fluid and the surface although the local properties of the fluid will change with the change in temperature. This is based on the fact that the wall shear stress $\tau_0$ is proportional to $(\rho u)^{\frac{1}{2}}$ at constant pressure, also in the range of interest product $\rho u$ varies slowly with temperature for most gases.

For a cylinder

$$N_u = \frac{Q}{\pi \ell K \Delta T}$$

where 'Q' is the quantity of heat transferred and

$$\Delta T = T_W - T_g$$

Kinetic energy of the gas molecules colliding with the hot surface = $c_v T_g$ (per unit mass).
'\( c_v \)' is the specific heat of the gases at constant volume.

Net heat transfer from the cylinder/unit mass = \( c_v (T_v - T_g) \)

Heat to the gas/unit mass = \( \rho V_d c_v \Delta T_1 = Q/\ell \) where \( c_v \Delta T_1 \)

is the rise in internal energy of the gas per unit mass.

This expression for \( Q/\ell \) gives

\[
N_u = \frac{\rho V_d c_v \Delta T_1}{\pi K_w \Delta T}
\]

where \( 'K_w' \) is the thermal conductivity of the fluid at the surface temperature.

The term \( \frac{\Delta T_1}{\Delta T} \) is equivalent to the proportion of mass flow taking part in the heat transfer. If the fluid and the wire are at the same temperature this proportion will be given by \( \zeta_f \). When the fluid and the wire are at different temperatures the proportion of mass taking part in the heat transfer remains the same but more kinetic energy per unit mass is taken away from the surface. The ratio of kinetic energies per unit mass can be expressed as \( K_w/K_g \) because

\[
K = \frac{1}{3} \rho c \lambda \bar{\tau} \quad \text{(from the kinetic theory of gases)}
\]

(Ref. 86)

where

'\( \lambda \)' is the mean free path

\( \bar{\tau} \) is the r.m.s. random velocity.

\( K_w/K_g \) is roughly proportional to \( T_v/T_g \). The rate of heat transfer from the surface can therefore be expressed as being proportional to \( \zeta_f K_w/K_g \), which will replace the ratio \( \frac{\Delta T_1}{\Delta T} \) in equation 6.32.
\[ \text{Nu} = \frac{c_p \rho \nu c_p K_w}{\pi K_w K_y} \]

\[ = \frac{c_p \rho \nu c_p}{\gamma \pi K_y} \]  \hspace{1cm} (6.33)

where

- \( c_p \) is the specific heat of the gas at constant pressure.
- \( \gamma \) is the ratio of specific heats \( \frac{c_p}{c_v} \).

Equation 6.33 can also be written as

\[ \text{Nu} = \frac{c_p R_e P_r}{\gamma \pi} \]

In this equation the fluid properties are evaluated at the temperature of gas and are independent of the wire temperature.

From equation 6.33 the heat transfer coefficient may be defined as

\[ h = \frac{c_p \rho c_v K_w}{\pi K_y} \]  \hspace{1cm} (6.34)

Further defining the Nusselt number with the fluid properties evaluated at the wire temperature we have

\[ \text{Nu} = \frac{h \cdot d}{K_w} = \frac{Q}{\pi \ell K_w \Delta T} \]  \hspace{1cm} (6.35)

Davis and Fisher\(^6^9\) used 6.35 to replot the original data of Spangenberg\(^6^8\). Fig. 6.10 shows the curves plotted by Spangenberg and Fig. 6.11 shows the same data replotted by Davis and Fisher according to equation 6.11. A comparison shows that the systematic variation of Nusselt number with Mach number at a constant Reynolds number has been reduced quite appreciably in the replotted data. Thus the dominant
Fig. 6.12 VARIATION OF VISCOSITY AND DENSITY OF AIR WITH TEMPERATURE.
effect of density changes has been reduced. Therefore in
a situation where gas density and velocity are changing
simultaneously, more consistent results will be obtained by
the use of equations 6.34 and 6.35. There is still some
appreciable scatter in Fig. 6.10 but most of the points
which show this scatter are from high Mach number tests.
In the engine the gas velocities should be in the subsonic
region therefore better agreement can be expected.

Now the assumptions made during this analysis will be
examined more closely.

\[ \left( \rho u \right)^{\frac{1}{2}} = \text{it was assumed that} \left( \rho u \right)^{\frac{1}{2}} \text{is}
\]

independent of gas temperature. Variation of \( \rho \) and \( u \) with
temperature were plotted on a logarithmic scale with the values
computed from Reference(87). The slope of viscosity
temperature curve shown in Fig. 6.12 was found to be .683
and that of density temperature to be -1. These values give

\[ \left( \rho u \right)^{\frac{1}{2}} \alpha \left( \frac{T}{T_{\text{w}}} \right)^{0.683} \alpha T^{-1.15} \]

Heat leaving the surface, which was expressed as

\[ c_f \frac{T_w}{T_{\text{e}}} \text{is proportional to} \left( \frac{T_w}{T_{\text{e}}} \right)^{0.85} \]

Second assumption was to replace \( c_f \frac{T_w}{T_{\text{e}}} \) by \( c_f \frac{K_w}{K_{\text{e}}} \). McAdams(30)
gives values of the thermal conductivity of air at various
temperatures, this can be approximated by the following
expression

\[ K \alpha T^{0.846} \]

Therefore

\[ \frac{K_w}{K_{\text{e}}} \alpha \frac{T_w}{T_{\text{e}}}^{0.846} \]
Hence it is justified to replace $C_f \frac{T_w}{T_G}$ by $C_f \frac{K_w}{K_G}$ with $C_f$ constant, because it has been shown that the heat transferred i.e., $C_f \frac{T_w}{T_G}$ is proportional to $C_f \frac{T_w}{T_G}$.

Of all the methods discussed, the method proposed by Davies and Fisher appeared most promising for the application of hot wire anemometer to mean velocity measurement in the engine cylinder, i.e., in an environment with simultaneous variation of gas velocity and density. It was therefore decided to adopt this approach in the present study.

Since the difference in the 'exact solution' of equation 6.10 and 'approximate analytic solution' of equation 6.11 was of the order of 2-3%, it was decided to use the 'approximate analytic solution', for the analysis of the gas velocity records, due to its comparative simplicity. This method and its application to the engine records will be discussed in the chapter on analysis of results.

6.6. Time Constant and Frequency Response of the Constant Temperature Anemometer

Theory of the constant temperature hot wire anemometer response in unsteady flow is given in Appendix 6.1. It will be discussed here briefly. Hinze\textsuperscript{65} has discussed this topic in detail. The sensitivity of the hot wire depends on the frequency of the velocity fluctuations, due to the finite thermal inertia of the hot wire. At large values of frequency, there can be a significant loss of response. The value of frequency above which this effect becomes noticeable is dependent on the 'time constant' $\mu$ of the system. Here the word system includes the electronic backfeed as well as the hot wire itself.
The value of 'M' is given by the following expression

\[ M = \frac{e C_w}{\alpha R_s \left[ \frac{R_g}{(R_v - R_j)} \right]^{1/2} \left[ 1 + \frac{2}{(R_v - R_j)} R_w g_{tr} \right]} \]  

(6.36)

where

- \( C_w \) = Total heat capacity of the wire
- \( e \) = Conversion factor
- \( c_v \) = Specific heat of the wire material
- \( g_{tr} \) = Transconductance of the amplifier system

In the constant current system of operation the time constant \( M_c \) is given by

\[ M_c = \frac{e C_w (R_v - R_j)}{\alpha I^2 R_s R_j} \]  

(6.37)

or

\[ M_c = (\pi/4) \int c_w \rho (T_v - T_j)/(\lambda I^2) \]  

(6.38)

The time constant in the above equations is defined as the reciprocal of the angular frequency at which the wire output signal is \( \frac{1}{\sqrt{2}} \) times its value without attenuation.

Combining equations (6.37) and (6.36) we have

\[ M = \frac{M_c}{\left[ 1 + \frac{2}{(R_v - R_j)} R_w g_{tr} \right] R_w g_{tr}} \]  

(6.39)

i.e., the time constant under constant temperature operation is a fraction of the time constant for constant current operation. By making the transconductance \( g_{tr} \) large \( M \) can be made very small.

In the present investigation, constant temperature circuit devised by Davies and Fisher was used. Frequency response of this system with a 5\( \mu \) tungsten wire 2 m.m. long was measured with a
transconductance of 60 mhos and was found to be 40 Kc/sec. This was considered to be high enough for the engine mean velocity measurements. Value of 'M' for some typical values of $R_w$, $R_g$ and $g_{tr}$ is worked out in Appendix 6.1.

### 6.7. Effects of Turbulence on Mean Velocity Measurement

Corrsin° has assessed the effects of turbulence on the mean gas velocity measurement. Assuming $u_1$ to be the turbulence-velocity component in the direction of flow and $u_2$ to be the lateral turbulence-velocity component perpendicular to the wire, the relationship between the measured mean gas velocity and the actual mean gas velocity is as follows:

$$V_{\text{measured}} = V_{\text{actual}} \left(1 - \frac{1}{4} \frac{u_1^2}{V_{\text{actual}}} + \frac{1}{2} \frac{u_2^2}{V_{\text{actual}}} \right) \quad \ldots (6.40)$$

The correction can therefore be either negative or positive depending on the magnitude of the ratios $\frac{u_1}{V_{\text{actual}}}$ and $\frac{u_2}{V_{\text{actual}}}$. If it is assumed that $u_1 = u_2$ the correction is negative, i.e., the measured velocity is smaller than the actual velocity. Assuming a value of $\frac{u_1}{V} = .2$, there will be an error of 10 per cent on the basis of the above analysis. This applies for the constant current hot wire anemometer. Hinz65 has shown that for the same conditions, the constant temperature system gives a much smaller distortion. The error of 10 per cent mentioned above will be reduced to approximately one-third its value with constant temperature operation. No turbulence measurements were made inside the engine cylinder and on the basis of the above reasoning it was assumed that the error due to ignoring this factor will be of the order of 2-3%. 
6.8. **Linearizing the Bridge Output**

The output of a constant temperature hot wire anemometer is a non-linear function of the flow velocity. At high velocities the changes in the output voltage are small and hence the sensitivity of the system is low. Using electronic means the bridge output can be made linear with the changes of gas velocity. Since some attempts were made to linearize the output, in the present study, the principle of operation will be discussed here briefly.

The relationship between the flow velocity $\mathbf{V}$ and the bridge output voltage is as follows:

$$
E = A + B \mathbf{V}^n
$$  \hspace{1cm} (6.41)

where

$E$ is the bridge voltage

$A$, $B$ are empirical constants.

Let $E_{\text{in}}$ be the input voltage to the linearizer and $E_{\text{out}}$ the output voltage from it.

Let $E^2 = E_{\text{in}}^2$ and $E_{\text{out}}$ be defined by the following linearizer transfer function.

$$
E = K_s (E_{\text{in}}^2 - E_{\text{ino}}^2)^m
$$

where $K_s$, $E_{\text{ino}}$ and $m$ are constants.

or

$$
E_{\text{out}} = K_s (A + B \mathbf{V}^n - E_{\text{ino}}^2)^m
$$  \hspace{1cm} (6.42)

Let $E_{\text{ino}} = A$

$$
E_{\text{out}} = K_s (B \mathbf{V}^n) = K'_s \mathbf{V}^q
$$  \hspace{1cm} (6.43)

where $K'$ and $q$ are constants.
6.9. **Conclusions**

6.9.1. Hot wire anemometer with constant temperature operation is a suitable instrument for recording the mean gas velocity inside an internal combustion engine.

6.9.2. Due to the difficulty of obtaining a direct calibration at high gas densities and gas temperatures it was decided to resort to analytical methods for applying corrections to the recorded signal to compensate for the effects of gas density and gas temperature changes.

6.9.3. Of the three analytical methods discussed, Davis and Fisher method was considered to be most promising and was adopted for the present study.

6.9.4. Errors in the measured mean gas velocity due to ignoring the effects of turbulence will be of the order of 2 to 3%.

6.9.5. Time constant of 10 μ PT/30 % I.R. wire and the constant temperature anemometer circuit used in the present study is small enough for satisfactory operation in the engine tests.
CHAPTER 7

MANUFACTURE OF PROBES
7.

MANUFACTURE OF PROBES

7.1. Choice of Materials

The requirements of resistance thermometry and hot wire anemometry are highly exacting as far as the sensing elements are concerned. Materials commonly used are listed in Table 7.1., together with some of their mechanical and electrical properties.

7.1.1. High Mechanical Strength

This is most important in the context of present investigation. The conditions under which the wire has to operate inside an engine cylinder are very arduous. In spite of effective filtering and stringent measures to stop the lubricating oil being carried into the gases, the probe breakage can be a serious time consuming factor. Assuming that the wire hangs in the shape of a catenary the tension $T$ is given by

$$T = \text{Constant} \times \frac{1}{2} C_D \rho \frac{V^2}{d} L$$

where

$C_D$ is the drag coefficient

$\rho$ is the density

$V$ is the air velocity

$d$ is the diameter of the wire

$L$ is the length of the wire.

In the range of interest of Reynolds number of 10 to 100

$$C_D \propto (Re)^{-5}$$

...... (7.2)
(a) It oxidises at temperatures above 500°C.

(b) It is not available commercially in pure form.

Wide variations of electrical properties can occur from batch to batch. This is a serious fault. Unless these properties are established accurately for each batch separately, reliance on the published values can give inaccurate results.

Tungsten wire was available in 5, 7 and 11 micron diameter sizes. Unfortunately, except for Platinum, none of the materials listed in Table 7.1 are available in Wollaston wire form. A different technique for attaching the sensing wire to the electrodes had to be adopted. Tungsten wire cannot be soft-soldered directly to the electrodes but must be either copper-plated first or welded. A capacitor discharge method of welding was used by Van der Hegge Zijnen. Benson and Brundrett tried various techniques including lead and silver base solders, spot welding and spot welding with a nickel sleeve. The last method produced a satisfactory solder type joint with nickel as the bonding medium. Davies and NPL have described a technique for copper plating the wire to make it ready for a soldered joint. The wire is first cleaned by making it the anode in a strong solution of sodium carbonate and passing a current of approximately 2 000 amp/sq.ft. surface area, to clean the wire. The wire is then washed in clean water and copper plated in a saturated solution of copper sulphate containing 10% by volume of concentrated sulphuric acid. Pure copper acts as the anode and a low current is passed for several hours at a current density of about 50 amp/ft². It was the author's experience that lower current at the start of the plating procedure produced more even plating. Opinions differ about the best procedure for obtaining the required sensing length of the wire. NPL workers prefer to coat the whole of the wire and then etch the copper to produce a tungsten
Fig. 7.1. The Copper Plating Rig

Fig. 7.2. The Etching Rig
element of required length. Davies\textsuperscript{30} on the other hand prefers to leave the sensing length unplated so that no etching is required. In the present investigation etching technique was used.

7.2.3. Description of Copper Plating and Etching Rigs

A special copper plating rig was made; this is shown in Fig. 7.1. An adjustable potentiometer was incorporated in the plating circuit to control the plating current.

A special etching rig was constructed, this is shown in Fig. 7.2. It consisted of a burette containing 10% nitric acid. The jet from the burette is directed on to the copperplated wire and the copper coating is etched away. A potential of 9-12 volts was applied through a platinum electrode immersed in the acid, this accelerated the etching process quite considerably. The wire could be observed, during the etching process, through a 50 x 1 magnification microscope shown in Fig. 7.2. This microscope had a graduated graticule so that the etched length could be measured while the etching process was in progress. The graduations measured the length of the wire to .05 mm. A special light bulb, as shown in Fig. 7.2, was fitted to give better visibility.

Originally ordinary soft solder was used to attach the copper plated wire to the .015" diameter platinum electrodes, but tests in the engine showed that due to the electrodes attaining high temperatures and because of relatively high gas temperatures in the top compression ratio cases, the solder became plastic and the electrical joint was damaged. This was overcome by using a special high melting point solder. This proved very successful and no further trouble was experienced.

7.2.4. Description of the Probe

The probe is shown in Fig. 7.3.

The main body of the probe was made out of \( \frac{1}{4} \) inch diameter Pyrotenax, normally used for household electric supply lines; this was found to be
Fig. 7.3 - The Hot Wire Probe

Sensing Element

Platinum .015" Dia.

Insulation

Araldite Seal

Copper Electrodes
cheap and functional. Two .015 inch diameter platinum wires about 1" long were silver soldered to the end of the copper conductors. In order to exclude moisture the exposed ends of the Pyrotenax were sealed with Araldite. The sensing wire was soldered across these two electrodes. Because of the reasons discussed later (Chapter 9) a bent probe had to be used in the present tests. The centre line of the probe carrier was offset from the centre line of the pre-combustion chamber by 1/4 inch, therefore the platinum electrodes were bent by this amount and this resulted in giving a probe traverse in a direction perpendicular to the precombustion chamber wall.

7.2.5. Low Resistance Measuring Circuit

Typical cold (ambient temperature) resistances of the finished probes was 3-6 ohms. While measuring the resistance of fine wires it is important not to pass too high a current which might result in heating the wire, thereby giving a high resistance. As an example, for 2 mm. long, 5 micrometer diameter wire the calculated resistance at 20°C is 5.5 ohms. With a current of about 3 mA flowing through it, the wire is only about .2°C above its surroundings in a vacuum. If the power dissipation is increased by a factor of 100 the wire will be approximately 50°C higher than its surrounding, which would cause a significant error in the value of measured resistance. A special circuit was produced to check the cold resistance of the manufactured probe; this way faulty probes could be detected and discarded. This circuit is shown in Fig. 7.4. A current of the order of 10 μA was necessary to measure the resistance. A plot of current through the probe and resistance is shown in Fig. 7.5. This procedure proved extremely useful in obtaining consistent probes.

7.3 Conclusions

A satisfactory method of making the hot wire anemometer probe was developed. After attaining proficiency in the method of manufacture of these probes, a probe could be made in approximately 20 minutes.
CHAPTER 8

WIND TUNNEL TESTS
The constant temperature hot wire anemometer circuit used in the present study was the same as used by Davies and Fisher in their study of the heat transfer from electrically heated wire. This circuit is shown in Fig. 8.1. Care was taken to mount the two power transistors (OC24) on a large copper plate which acted as a heat sink. The two 12 Ω resistors shown in the circuit were rated at 5 watts; this was necessary because of the high current flowing through that branch of the circuit which seriously overheated the lower rated resistors. The circuit is designed to operate with a low impedance input source therefore a 12 volt automotive battery was used as power supply. This proved to be quite adequate. The 'hot resistance' $R_B$ shown in Fig. 8.1 was a multidecade resistance box on which the operating resistance of the probe could be set down to a minimum step of 0.001Ω. All the resistors used in the circuit, with the exception of 12 Ω resistors mentioned above, were high stability carbon type. As can be seen from Fig. 8.1, the bridge is symmetrical with two identical amplifiers such that a balanced operation of each half can be obtained. Since the bridge is symmetrical, the current can be made to flow in the probe in either direction. This is achieved by altering the setting of the bias potentiometer such that the bridge is correctly balanced in free convection and this bias signal is termed an 'offset'. Let the bridge voltage output under these conditions be $E_0$. The bridge voltage output $E$ at any other gas velocity is given by

$$E = E_0 + Ae$$

(8.1)

where

$A$ is the amplifier gain, and

$e$ is the unbalance signal or the input signal to the amplifier.
Hot Wire Anemometer Circuit

**FIG 8-1**
It can be shown that

\[
\frac{R_w}{R_B} = \frac{A - 2 \left(1 - \frac{E_0}{E}\right)}{A + 2 \left(1 - \frac{E_0}{E}\right)} \quad \ldots (8.2)
\]

Davies and others have given a plot of \( \frac{R_w}{R_B} \) for various values of \( A \), this is shown in Fig. 8.2. It shows that apart from the free convection conditions, the bridge is never really in perfect balance but for a high enough value of \( A \), the amplifier gain, \( \frac{R_w}{R_B} \) is practically constant at a value very close to unity. The effects of bias control setting are illustrated in Fig. 8.3. Some trouble with probe breakage was experienced if the setting was in the high frequency oscillation band. This resulted in high amplitude oscillation which was sometimes in audible frequency range, this tended to seriously weaken the probe which resulted in ultimate failure. Care was taken to operate the hot wire probe out of this range. Ideal position of the bias control setting is indicated in Fig. 8.3., in order to achieve this the bias control potentiometer was replaced by a ten turn precision potentiometer.

8.2. DISA Calibration Wind Tunnel

In order to check the validity of Davies and Fisher method of obtaining the wire calibration curve analytically, it was necessary to carry out controlled calibration of the wire. Tests were carried out using 5 μ tungsten, 10 μ iridium, 10 μ PT/20% IR and 10μPT/30% IR wires of known dimensions. The wind tunnel used was a DISA type 55 D41/42 calibration tunnel and is shown in Fig. 8.4. The tunnel consisted of a constricted nozzle through which air, at atmospheric pressure and temperature conditions, was fed via a turbulence filter array. The measuring section was located at the throat of a nozzle which was the highest velocity section in the tunnel. The other side of the measuring section was connected to a variable speed motor.
Fig. 8.2 Operating resistance of 'constant' resistance anemometer
Optimum performance

High frequency oscillation at high amplitude.

Operating range for lineariser

Low frequency oscillation

-1.5   centre   Right
Balance potentiometer setting, arbitrary scale.

Fig-8.3 Effect of balance setting adjustment on bridge output.
TABLE 7.1
ELECTRICAL AND THERMAL PROPERTIES OF METALS USED

Their mechanical and electrical properties

(Ref. 64' )

<table>
<thead>
<tr>
<th>Metal or Alloy</th>
<th>Resistivity ((\Omega/cm\cdot{^\circ}C))</th>
<th>Temperature Coefficient of Resistance (0-100(^\circ)C)</th>
<th>Tensile Strength (psi x 1000)</th>
<th>Elongation %</th>
<th>Melting Point (Solidus) (\circ)C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commercial Platinum</td>
<td>63 61 .00375 .00386</td>
<td>65 24</td>
<td>2 38</td>
<td>1769</td>
<td></td>
</tr>
<tr>
<td>Tungsten</td>
<td>42 33 .0036 .0048</td>
<td>320 160</td>
<td>1.5 16</td>
<td>3410</td>
<td></td>
</tr>
<tr>
<td>Iridium</td>
<td>36 28.3 .0031 .0042</td>
<td>340 180</td>
<td>2 16</td>
<td>2443</td>
<td></td>
</tr>
<tr>
<td>Platinum/ 10% Iridium</td>
<td>154 150 .0012 .0013</td>
<td>120 55</td>
<td>2 24</td>
<td>1800</td>
<td></td>
</tr>
<tr>
<td>Platinum/ 30% Iridium</td>
<td>218 210 .0005 .0006</td>
<td>270 160</td>
<td>1.5 18</td>
<td>1880</td>
<td></td>
</tr>
</tbody>
</table>

Substituting Equation 7.1 in Equation 7.2 we have

\[ T \propto V^{1.5} d l \]

Therefore the stress produced in the wire is

\[ \text{Stress} \propto \frac{V^{1.5} d^{1.5} l}{\pi d^{3/4}} \]

For a given resistance

\[ R \propto \frac{l}{d^2} \]

which is a constant.

\[ \therefore \text{Stress} \propto V^{1.5} d^{1.5} \]
This shows that for a given resistance, the stress produced in a smaller diameter wire will be smaller.

7.1.2. High Resistivity

For a given resistance and diameter the resistivity of the material used determines the dimensions of the probe. When working in confined spaces like the clearance volume of an engine it may not be possible to use very long wires; therefore, high resistivity is desirable.

7.1.3. High Temperature Coefficient of Resistance

The wire sensitivity for a given heating current depends on its temperature coefficient of resistance. Pure metals like Platinum and Tungsten have high coefficient of resistance. Platinum alloys generally have a low temperature coefficient of resistance, therefore it would be preferable to use a pure metal.

7.1.4. Resistance to Oxidation

Most metals oxidise at high temperatures. In hot wire anemometry it is sometimes necessary to operate the sensing element at a high temperature in order to maintain a sufficiently high temperature difference between the wire and the gases flowing over it. This is the case when the hot wire is operated in the engine cylinder; it is therefore necessary to use a metal or alloy which is not affected by high temperature operation. A reference to Table 7.1 shows that Tungsten has about 7 times the tensile strength of platinum, but it cannot be used for long periods above 400°C. This limits its use in engine investigations where it may be necessary to operate the wire at high temperatures.

7.1.5. Ductility

This is important, because it affects the minimum size of the wire that a material can be drawn to.
7.2. Different Methods of Probe Manufacture

7.2.1. General Comments

Although sensing probes for various applications are available commercially, their physical characteristics vary quite widely for the same nominal dimensions. Also these probes are quite expensive.

Since it was anticipated that probe breakage will be quite common in engine tests and a special configuration of probe was required, it was decided to make these probes.

7.2.2. Different Methods of Probe Manufacture

Since platinum is the most commonly used metal in resistance thermometry and hot wire anemometry, this was the first metal tried. Platinum is available in Wollaston wire form. Wollaston process consists of covering the platinum by a silver rod about ten times the diameter and drawing the assembly as a solid billet to the smallest size practicable. In this form, platinum wire with an outer silver casing, the wire can be handled easily in the initial stages of probe manufacture and then the silver casing can be etched to expose the platinum sensing element, which remains unaffected by the acid.

Wollaston wire with 4 microne core of platinum was welded to the platinum electroded by the fusion method described in Ewer\textsuperscript{71}, which proved quite simple, and a satisfactory electrical joint was produced. The outer silver casing was etched by dipping the probe in concentrated nitric acid, the joints being protected from the acid by coating them with perspex cement. In order to test the probe for mechanical strength, it was inserted in the exhaust pipe of an engine. Immediate failure of the probe under these conditions proved that it was too fragile for engine tests. New materials were therefore considered.

Tungsten was the next material tested. It meets most of the requirements but has two disadvantages, namely:
coupled to a suction fan via a diffuser. The flow velocity in the measuring section was controlled by varying the speed of the motor by controlling the supply voltage through a transformer. A difference manometer was used to measure the pressure drop between points A and B shown in Fig. 8.4.

The flow velocity in the measuring section is given by

\[ V^2 = \frac{2 \gamma}{\gamma - 1} R T_o \left[ 1 - \left( 1 - \frac{\Delta P}{P_o} \right)^{\frac{\gamma - 1}{\gamma}} \right] \] .... (8.3)

where
\[ \gamma \] is the isentropic exponent for air (= 1.4)
\[ R \] is the gas constant
\[ T_o \] and \[ P_o \] are the ambient temperature and pressures
\[ \Delta P \] is the pressure drop

A calibration curve for the wind tunnel for standard temperature and pressure conditions is shown in Fig. 8.5. If the ambient pressure and temperature differed from the standard conditions, the following corrections were employed

\[ \Delta P' = \Delta P \frac{760}{P_o} \]

and

\[ V' = \sqrt[\gamma - 1]{\frac{T_o}{293}} \]

8.3. Wire Physical Dimensions

It is highly important to know the wire physical dimensions accurately to be able to obtain the analytical calibration curve. Of the two basic wire dimensions, i.e., length and diameter, the
Sectional View

Type 55D41 Calibration Unit with Nozzle, Diffuser and Airshunt

Fig. 8.4 DISA Calibration Wind Tunnel

Pressure difference $\Delta p$ versus flow velocity $v$ for air ($k=1.4$).

Reservoir conditions: $P_o$ mm Hg, $T_o$ °K.

Fig. 8.5 Calibration Curve for the Wind Tunnel
Fig. 8.6. Electron microscope photograph of 5 micron tungsten wire

Fig. 8.7. Electron microscope photograph of 7 micron tungsten wire
Fig. 8.8 Hot Wire Anemometer Calibration Curves.
calculation is extremely sensitive to the diameter. If a very low current is passed through the wire its resistance can be established accurately. Knowing the length, the diameter may then be calculated. Since with the etching process some uncertainty exists about the actual etched length and also the published resistivity data cannot be entirely relied upon in the case of tungsten probes, some other means of measuring the wire diameter is desirable. Specially in the case of 5 µ tungsten wires which are produced by an etching process the diameter may vary appreciably from the nominal diameter quoted by the manufacturers. In order to resolve these uncertainties, electron microscope photographs of the 5 µ and 7 µ tungsten wires were obtained, and are shown in Figs. 8.6 and 8.7. These photographs showed that the diameter was very close to the nominal values quoted by the manufacturers. No attempt was made to ascertain the diameter of platinum alloy probes because with these the published data is reliable and the wires are produced by a wire drawing process thereby giving greater consistency of manufacture. The effects of assuming different diameters for the 10 µ iridium wire is shown in Fig. 8.8, which shows the different analytical calibration curves computed with five different diameters ranging from 11 µ to 10.5 µ. In order to standardise a procedure for tests any probe which gave a different measured cold resistance than obtained by calculations using nominal diameter, measured length and published resistivity data was discarded. All the wires were annealed before use by heating to a dull red state for a few minutes. This tended to stabilise the probes. Cold resistance of the probe was checked from time to time and if there was an appreciable change, the probe was rejected for any further tests. PT/30% IR probes proved to be more stable than tungsten probes in this respect and therefore most of the engine tests were
carried out with these probes.

8.4. Hot Wire Calibrations

Results of the wind tunnel tests are plotted in Figs. 8.9 and 8.10. Fig. 8.10 shows the linearised output. It can be seen that agreement between the calculated points and the experimental points is quite good in all the cases. The wire operating temperature in all the cases was approximately 570°K, which is of the same order as the high temperature cases investigated by Davies and Fisher. Since in the engine tests it was necessary to operate the wire at higher temperature than those used by Davies and Fisher. This is necessary because at high gas temperatures, the differential between the gas and wire temperature has to be maintained in order to obtain sensitive operation of the hot wire anemometer circuit. Tests were carried out with the sensing wire operated between 600°C and 800°C in order to ascertain the effects of high wire temperature on the calculated calibration curves, these results showed that the Davies and Fisher method did not predict the calibration curve when the sensing element was operated at high temperatures. The basic method of calculating the calibration was therefore modified. It is shown that this modified method of calculation gives better agreement with experimental results at high wire temperatures.

8.5. Method of Calculating the Calibration Curve

It was shown in Chapter 6 that the mean temperature difference between the wire and gas is given by the following equation:

\[ T_w' = \frac{K_2}{K_1} \left[ \frac{\tanh \sqrt{1K_2|l|}}{1K_2|l|} - 1 \right] \]  \hspace{1cm} (8.4)

Since the value of \( T_w' \) is known from the temperature resistivity relationship, equation 8.4 can be solved on a digital computer using
Voltage Velocity Calibration

(DISA TUNNEL 1st. Sept. 1966)

Experimental Points.
Calculated Points.

5AU Tungsten Wire
1.6 mm long.
Hot Resistance = 15 ohms.

Gas Velocity = m/sec.
Hot Wire Anemometer

Calibration

Experimental Points
Calculated Points

Lineariser Transfer Function

\[ V_{out} = C \left[ \left( \frac{V_{in}}{V_{ref}} \right)^2 - 1.0 \right]^m \]

Voltage Setting = 1.44 volts

\[ m = 2.5 \]

\[ C = 0.329 \]

Fig 8.10
iterative techniques to yield the unknown heat transfer coefficient \( h \).

The heat transfer coefficient is related to the gas velocity by the following relationship.

\[
\frac{h}{D} = \frac{c_f \rho V_c \nu}{\pi K_o} 
\]

\[ \ldots \ldots (8.5) \]

Where \( h_i \) is the heat transfer coefficient as defined by Davies and Fisher.

In this present study the relationship between \( c_f \) and \( R_e \) defined by Davies and Fisher and valid in the range \( 0 < R < 50 \) was used

\[
c_f = 2.6 \left( \frac{R_e}{2} \right)^{\frac{2}{3}} \]

\[ \ldots \ldots (8.6) \]

Property values for the calculation of Reynolds number were evaluated at free stream conditions.

8.6. The Physical Properties of the Wire Material and the Air

In order to calculate the calibration curves for the hot wire anemometer using the relationships described above it is necessary to know the electrical properties of the wire material and the physical properties of the air. Platinum is the most commonly used metal in hot wire anemometry because it is commercially available in very pure form and therefore the electrical properties in the published literature can be relied upon. In certain applications where strength and ruggedness of the sensing element are the over-riding factors, as in this case, use of Pt/IR alloys has been suggested by Kovasznay (1955). Pt/IR probes were used successfully by Tindal (1959). In such cases the published values are doubtful, particularly when the material is maintained at high temperatures over long periods (Darling 193). The Iridium component of the alloy can evaporate or oxidise thus altering its composition which in turn affects the temperature coefficient of
resistance and the resistivity of the material. Due to these objections to the use of PT/30% IR at high temperatures, both PT/30% IR and pure platinum wires were used in the tests.

The electrical properties of the wire material required for the calculation of the calibration curve are as follows:

(i) Electrical resistivity of the wire
(ii) Temperature coefficient of resistance

In the case of pure platinum, Davies and Fisher have pointed out that the published data from the International Critical Tables can be used, therefore in the present investigation the following temperature resistivity relationship from that source was used.

For \( T_w < 400 \degree C \); \( T = T_w - T_o \); \( T_o = 273 \degree K \).

\[
\lambda_w = \lambda_o \left( 1 + 3.52 \times 10^{-3} - 55 \times 10^{-6} \right) \quad \ldots \ldots \quad (8.7)
\]

and for \( 400 < T < 1000 \degree C \)

\[
\lambda_w = \lambda_o \left( 1 + 3.98 \times 10^{-3} - 585 \times 10^{-6} \right) \quad \ldots \ldots \quad (8.8)
\]

An electrical resistivity-temperature calibration for the pure platinum wire was carried out in an oven with temperatures up to \( 200 \degree C \). These tests showed close agreement with the values obtained for \( \lambda_w \) from equation 8.7, thereby justifying the use of the International Critical Tables equations for \( \lambda_w \). It was therefore considered reasonable to use equation 8.8 for the higher wire temperature range.

Since the published data on the temperature coefficient of resistance for PT/30% IR is not as reliable as the data for pure platinum, this was determined experimentally. The wire was heated to about \( 600 \degree C \) in an oven over a period of five hours; the oven was allowed to cool slowly. A second run was then carried out by heating
the wire again to 600°C. The results of these tests are shown plotted in Fig. 8.11. A study of these curves shows that there was a permanent increase in the resistance after the first run (curve 1), but there was no indication of any further increase in resistance after the second run. It can be seen that the curve II in Fig. 8.11 does not depart from the linear law significantly hence a polynomial fit of the type

\[ R_w = R_0 (1 + \alpha T + \beta T^2 + \cdots) \]

was not considered necessary. The measured value of \( \alpha \) was used to calculate the wire temperature.

The final physical property of the wire material required for the evaluation of equation 8.4 is the thermal conductivity. The expression for the variation of thermal conductivity with temperature used by Davies and Fisher, for Platinum, was based on the International Critical Tables. This expression is:

\[ K_{PT} = 0.646 + 0.03 \times 10^{-4} (T_w - 31) \text{ watts/cm}^2 \text{K} \ldots \ldots (8.9) \]

here \( T \) is expressed in °K.

In the case of PT/30% IR Lorenz Constant was used to relate the thermal conductivity to the electrical resistivity of the PT/30% IR wire, i.e.,

\[ K_{PT/30\%IR} = \frac{2.23 \times T_w}{\lambda_w} \text{ watts/cm} \text{K} \ldots \ldots (8.10) \]

here again \( T \) is expressed in °K.

The thermal conductivity of air used in equation 6 to calculate the calibration curve was evaluated by the expression used by Davies and Fisher in their work.
Determination of $'\alpha'$

$P_{T}/30\%\text{Ir}$

10.46 Dia Wire

1st Test

2nd Test

Slope = 0.00073

Fig. 8-11
This expression was stated to be within ± 2% of the published data in the temperature range of 0-400°C. Since higher wire temperatures (up to 800°C) were used during the experiments reported here, a logarithmic plot of thermal conductivity of air against temperature was carried out using published data from Tables of Thermal Properties of Gases. No significant deviations from a linear law were observed at higher temperatures; therefore the relationship given by equation 8.11 was used for all the calculations.

8.7. Experimental Procedure

Since the hot oven tests with PT/30% IR probes had shown that the probe resistance increases after being maintained at high temperature all the probes used in the tests were stabilised by heating the wire electrically (over a period of five minutes) to the highest wire temperature used in the tests and kept at that temperature for a period of approximately 4 minutes. This resulted in a slight increase, approximately 2%, in the cold resistance (resistance at room temperature — approximately 20°C) of the wire. A similar procedure was used by Spangenberg to stabilise his PT/Rh probes at high temperatures. A close check was kept on the value of cold resistance of the probe during the tests and any probe which showed an increase of more than 2% in resistance was rejected. In order to further safeguard the probe against damage during operation at high temperatures, the heating current was only switched on for a few seconds while obtaining a reading at any given air velocity. This procedure ensured that the cumulative operating time for PT/30% IR wires did not exceed about one hour.

Although the platinum probes were also subjected to the same treatment, no increase in the cold resistance was observed in their case.
Experimental results and the computed calibration curves are shown plotted in Figs. 8.12, 8.13, 8.14 and 8.15. The calibration curves shown were obtained by first calculating the wire temperature from the known temperature-resistivity relationships such as equations 8.7 and 8.8 and then solving equation 8.4 making use of equations 8.5 and 8.6 to calculate the heat transfer coefficient for a given value of the gas velocity. Solution of equation 8.4 yielded the unknown $|K_1|$ and hence the value of heating current $I$ which then yielded the hot wire anemometer bridge output voltage.

8.8. Discussion of Results and Modifications to the Basic Method of Calculation

The tests, with PT and PT/30% IR wires gave identical results, thus any uncertainty that may have existed if the tests had only been carried out with PT/30% IR probes was resolved. Had there been any significant deterioration of the PT/30% IR wires the results would have differed quite considerably because of the steep increase in the temperature coefficient of resistance of this alloy (Darling$^{93}$), when the Iridium content decreases due to effects of oxidation and evaporation at high temperatures. Also had there been any significant change in the surface finish of the PT/30% IR wire due to oxidation, the trend for PT/30% IR and PT would have been different.

A comparison of the experimental and computed calibration curves shows that agreement between the measured and computed values is good when the wire operating temperature is low but for high wire temperatures the differences are quite appreciable in all the cases. This showed that the effects of high wire temperature were not adequately allowed for in the analysis of Davies and Fisher.

Spangenberg$^{69}$ has investigated the temperature dependence of the Nusselt number in detail, under the heading of temperature loading effects.
Hot wire anemometer calibration curve --- experimental and calculated - wire temperature 572 K.
Hot wire anemometer calibration curves - experimental and calculated - wire temperature 889 K.
Hot wire anemometer calibration curves - experimental and calculated - wire temperature 984 K.

- Experimental
- ∇ Hilpert correction to Davies and Fisher calculation
- △ Collis and Williams correction to Davies and Fisher calculation
Hot wire anemometer calibration curve - experimental and calculated - wire temperature 381 K.
Hot wire an

Pt. 100m Dia. Wire.
Wire Temp. $T_w = 381^\circ K$
Gas Temp. $T_g = 295^\circ K$
Hot wire anemometer calibration curves - experimental and calculated - wire temperature 1056 K.
He found that while the King's Law states that the rate of heat loss from a heat wire is proportional to its temperature rise, deviations occur from this law. Nusselt number was found to depend on the temperature loading. The temperature loading was defined as the difference between the mean wire temperature and recovery temperature of the stream, i.e., $T_w - T_r$. The ratio $\Delta T/T_s$ was used as a criterion to measure the distortion of flow around the wire caused by the addition of heat. Here $T_s$ is the stagnation temperature. Heat transfer data from a single wire was plotted for three different temperature loadings of $67^\circ$, $204^\circ$ and $511^\circ$ C respectively. Spangenberg states that "... it is seen that lines of constant Mach number are always straight when Nusselt number is plotted against Reynolds number, but a differing line results for each constant temperature loading. A decrease of Nusselt number results as the temperature loading is increased, except at lower Mach numbers, where Nusselt number may either increase or decrease with temperature loading depending upon the density".

Because the temperature loading effect has important bearing on the hot wire anemometer performance, many workers have included the wire temperature in some form in their formulation of heat transfer data for cylinders in crossflow of air. Allowances for differences between surface and gas temperatures have been included in the correlation of data on heat transfer from heated cylinders in a crossflow of air by (Hilpert, 1933) and (Collis and Williams, 1959).

The following empirical correlation was obtained by (Hilpert, 1933):

$$Nu_f = C \left[ Re_f \left( \frac{T_w}{T_g} \right)^{\frac{1}{2}} \right]^n \left( Pr_f \right)^m \quad \ldots \quad (8.12)$$
If Prandtl number is constant the above relationship reduces to:

\[ \text{Nu}_f = \left[ \text{Re}_f \left( \frac{T_w}{T_g} \right)^{\frac{1}{4}} \right]^n \]  \hspace{1cm} \ldots \ldots (8.13)

Where the suffix \( f \) refers the property values to the film temperature and \( n \) varies between .33 and .805 depending on the value of Reynolds number. Thus in addition to the temperature dependence of \( h \) through gas properties there is a direct temperature loading factor of \( \left( \frac{T_w}{T_g} \right)^{\frac{1}{4}} \).

This direct dependence of \( h \) on the wire temperature may be expressed as:

\[ h \propto \left( \frac{T_w}{T_g} \right)^{\frac{1}{4}} \]  \hspace{1cm} \ldots \ldots (8.14)

In the Reynolds number range of interest here, between 5 and 50, it is reasonable to assume \( n = .4 \)

\[ h \propto \left( \frac{T_w}{T_g} \right)^{1} \]  \hspace{1cm} \ldots \ldots (8.15)

Collis and Williams have obtained the following correlation for electrically heated wires in crossflow of air:

\[ \text{Nu}_f \left( \frac{T_f}{T_g} \right)^{-1.17} = \left[ A + B \text{Re}_f^n \right] \]  \hspace{1cm} \ldots \ldots (8.16)

Here again there is a direct temperature dependence of \( h \) given by the factor \( \left( \frac{T_f}{T_g} \right)^{-1.17} \). From equation 17 we have:

\[ h \propto \left( \frac{T_f}{T_g} \right)^{-1.17} \]  \hspace{1cm} \ldots \ldots (8.17)

The Davies and Fisher definition of \( h_p \) as given in equation 8.5 includes the temperature dependence of \( h_p \) by the presence of \( K_w \) on the right-hand side of the equation 8.5. Thus there is no direct temperature dependence in this case. The experimental results at high wire
temperature show that the relationship given by equation 6 does not give satisfactory results for those conditions.

It is clear, therefore, that in order to produce better agreement at high wire temperatures the Davies and Fisher method of calculating the calibration curve has to be modified. A modified heat transfer coefficient \( h_1 \) may be defined such that it includes the effects of high temperature loading.

\[
h_1 = h_0 x \quad \text{(Temperature loading factor)}
\]

If the temperature loading factors used by Hilpert and Collis and Williams are used we have

\[
h'_1 = h_0 \left( \frac{T_w}{T_g} \right)^{1.7} \quad \text{(Collis and Williams correction) \quad (8.19)}
\]

\[
h''_1 = h_0 \left( \frac{T_g}{T_w} \right)^{1.7} \quad \text{(Hilpert Correction) \quad (8.18)}
\]

Using the Davies and Fisher relationship between the heat transfer coefficient and the gas velocity defined by equations 8.5 and 8.6, we have

\[
V = h_0^3 \frac{d^2}{d} \left( \frac{\pi K_g}{2 \delta K_w} \right)^3 \quad \text{\quad (8.20)}
\]

Again using equations 8.5 and 8.6, the modified heat transfer coefficient \( h_1 \) gives a modified velocity \( V_1 \)

\[
V_1 = \left( \frac{T_w}{T_g} \right)^3 V \quad \text{\quad Hilpert correction \quad (8.21)}
\]

and

\[
V_1 = \left( \frac{T_w}{T_g} \right)^{5.1} V \quad \text{\quad Collis and Williams correction \quad (8.22)}
\]

The modified calibration curves obtained by including the
TABLE 8.1

RESULTS OF HIGH DENSITY AND HIGH TEMPERATURE FLOW TESTS

<table>
<thead>
<tr>
<th>TEST CONDITIONS</th>
<th>GAS VELOCITY FT/SEC</th>
<th>GAS PRESSURE psia</th>
<th>GAS TEMPERATURE °K</th>
<th>WIRE TEMPERATURE °K</th>
<th>COMPUTED VELOCITY WITH HILPERT TEMPERATURE CORRECTION FT/SEC</th>
<th>% VARIATION FROM THE NOMINAL GAS VELOCITY</th>
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</thead>
<tbody>
<tr>
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<td></td>
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</tr>
<tr>
<td>70</td>
<td>54.7</td>
<td>474</td>
<td></td>
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<td>62.0</td>
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</tr>
<tr>
<td></td>
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<td></td>
<td>1147</td>
<td>66.6</td>
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<tr>
<td>67</td>
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<td>67.1</td>
<td>+ .15</td>
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<td>+ .89</td>
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<td>503</td>
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<td>75</td>
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<td>+12.0</td>
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<tr>
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<td>474</td>
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<td></td>
<td>1147</td>
<td>65.0</td>
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</tbody>
</table>
FIG - 8.16 Hot wire anemometer calibration rig for high gas temperature and pressure tests.
temperature correction factors such as used by Hilpert and Collis and Williams show much better agreement with the experimental results. This correction was therefore used for evaluating the engine results.

8.9. High Gas Temperature and Density Tests

It was decided to carry out some calibration runs at high gas temperature and density, i.e., the conditions under which the hot wire anemometer would operate in the engine. The high gas temperature and density flow rig used for the tests is shown in Fig. 8.16. It consisted of a bank of air bottles changed to about 2000/\text{lb/sq.in.} which discharged through a reducing valve into the metering and calibration sections as shown in Fig. 8.16. A hydrogen burner was incorporated to raise the temperature of the gases to any desired value. The flow thus obtained in the test section was liable to fluctuation, a tolerance of $\pm 5\%$ being specified for the mean flow velocity. The gas pressure used during the tests was 104 psia and the gas temperature 562°K. A 10\text{\mu m} PT/30\% IR wire was used for the tests and was operated at two temperatures. The results of these tests are tabulated in Table 8.1. There is a reasonable agreement between the calculated and experimental values. The calculated mean velocity agreed to within $\pm 12\%$ of the nominal mean velocity.

8.10. CONCLUSIONS

The results of the tests showed that a calibration curve for a constant temperature hot wire anemometer operating at high wire temperature in a high temperature and high density subsonic gas flow can be calculated with reasonable accuracy using the basic relationship evolved by Davis and Fisher; provided that a correction based on Hilpert or Collis and Williams is incorporated in the basic calculation.
CHAPTER 9

THE EXPERIMENTAL RIG
9. THE EXPERIMENTAL RIG

9.1. Description of the Rig

9.1.1. Engine Specifications

A Ruston and Hornsby WB engine was available in the department. This engine had previously been used for combustion photography and was already rigged up for motoring tests. Also, the fact that the engine had a side valve arrangement meant that a special cylinder head could easily be fitted. A general arrangement of the engine with the motor and generator is shown in Fig. 9.1. The specifications of the engine were as follows:

Four stroke, air-cooled, side valve.

Bore ........................................ 3 inches
Stroke ...................................... 3.25 inches
Piston Displacement .................... 376.5 cc.
Piston Speed at 3000 r.p.m. .......... 1625 ft/min.
Inlet Valve Opening ................. 25° B.T.D.C.
Inlet Valve Closing ................. 42° A.B.D.C.
Exhaust Valve Opening .......... 42° B.B.D.C.
Exhaust Valve Closing ........ 15° A.T.D.C.

The only modification necessary to the basic engine was to fit an oversize piston with special downward scrapping ring arrangement. The reason for this modification was to minimise the amount of lubricating oil reaching the piston crown. The fine wires used for the gas temperature and velocity measurement were extremely fragile and could easily be damaged by the lubricating oil droplets. This modification proved to be very successful and the breakage of sensing probes due to oil particles did not occur.

Since it was necessary to know the specific heat thermal
Fig. 9.1. General Arrangement of the Engine Rig
conductivity and density of the metal for heat transfer calculations, it was decided to fabricate the cylinder head out of a British Standard steel. EN 8 was considered quite suitable. Typical composition and the physical properties of this steel are given in Appendix 9.1. These properties have been taken from Woolman and Mottram.

9.1.2. Engine Motoring Arrangements

The engine was directly coupled to a D.C. motor. The speed of the motor could be controlled by a fine vernier control of the field current and varying the supply voltage. The only limitation imposed by the motor was that of the top speed limit of 1100 r.p.m.

9.1.3. Degree Markers

A Southern Instruments type M 738 degree (Ref. 101) marker was used. This degree marker gave marks at every $20^\circ$ of the engine crank angle which could easily be sub-divided into $10^\circ$ intervals. This was considered adequate for the analysis of all but surface temperature records.

A special degree marker was constructed to divide the engine cycle into 73 parts. This was dictated by the fact that 73 ordinates were required for carrying out the harmonic analysis using 36 harmonics.

9.1.4. Recording Equipment

A Southern Instruments two-beam Oscilloscope was used to display and record all the traces. A M 731 Universal Oscillograph recording camera was used to record the traces. The record was done on RP 30 Kodak paper. The recorded length for each trace was 20", which proved quite adequate for the purposes of analysis.

9.2. Design of a Special Cylinder Head

9.2.1. Since very little was known about the pattern of airflow in the engine it was considered necessary to design a cylinder head in which
Fig. 9.3. Close-up View of the Cylinder Head

Fig. 9.4. Cylinder Head with the Side Plates taken off
gas temperature and velocity measurements could be made such that there was minimum, or no, cycle to cycle variation.

9.2.2. Cycle to cycle variations could be minimised by creating 'ordered flow' in a precombustion chamber such as used by Lyn et al\textsuperscript{96} for schlieren photography of airflow and combustion. These photographs clearly show that the gas in the precombustion chamber rotates in a solid mass and does not change its direction of flow with the reversal of piston motion. This design of the combustion chamber was not intended to represent any particular type of engine but was aimed at producing controllable gas temperature, pressure and air swirl. The chamber used by Lyn was of flat disc shape, approximately 1 inch thickness and 2\frac{1}{2} inch diameter. The direction and speed of the air swirl could be varied by changing the shape of the throat at the bottom of the detachable combustion chamber.

9.2.3. A similar precombustion chamber was designed for the present study. This design of head is shown in Fig. 9.2. A close up view of the head is shown in Fig. 9.3.

9.2.4. The gas was transferred from the cylinder to the swirl chamber via the transfer port shown in Fig. 9.2. Since the port was tangential to the circular chamber, the transfer of gas from the cylinder to the chamber tended to produce a swirling motion of the gas. The other bounding surfaces of the chamber were formed by the two side plates shown in Fig. 9.4. These side plates could be moved in or out, thereby altering the compression ratio. This feature of the cylinder head design was essential because tests were planned with the compression ratio as one of the variables.

Since the distance between the two plates was never more than \frac{3}{8} inch (at the lowest expression ratio) in a direction along the axis of the swirl chamber circle, the gas flow could be regarded as one-dimensional. This meant that, by obtaining the gas flow measurements
ranging in position from as near the wall as possible to the centre of the swirl chamber, a good picture of gas flow pattern could then be formed.

9.2.5. Design of a Probe Carrier and Traversing Mechanism

Since the sensing probes for the measurement of gas temperature and velocity were extremely fragile it was necessary to design a special probe carrier so that the probes could be inserted into the cylinder head without damaging them. After two unsuccessful designs, a satisfactory design of the probe carrier was produced and this is shown in Figs. 9.5 and 9.6.

This probe carrier satisfied the following basic requirements.

(1) No interference with the flow. Since the sensing probe was bent so that the sensing element was upstream of the hole through which the probe was fed, no disturbance was caused to the flow. This is shown clearly in Fig. 9.6. The surface of the probe carrier and the circumference of the antichamber were carefully blended by polishing it in position such that there were no ridges along the surface.

(2) The probe could be traversed by the help of a micrometer. Thus a traverse could be carried out from as near the wall as possible to the centre of the chamber. The probe could also be locked in any desired position.

(3) The gases from the chamber were sealed by using asbestos and an ordinary gas nipple. This arrangement proved satisfactory and no leakage of gases was observed even at the highest compression ratio.

(4) In order to ascertain the distance of the sensing wire from the metal surface, prongs of the probe could be observed visually and also checked electrically before inserting the probe into the chamber.
Fig. 9-5. The Probe Carrier

Fig. 9-6. The Sensing Probe Inside The Precombustion Chamber
CHAPTER 10

ANALYSIS OF RESULTS AND CONCLUSIONS
10. ANALYSIS OF RESULTS AND CONCLUSIONS

10.1. Test Procedure

The engine was warmed up by motoring at the required speed for approximately one hour before each run. All the electronic equipment was also switched on well before the time the readings were recorded. It was ensured that the thermocouple cold junctions were immersed in melting ice. The engine speed was checked before each recording of traces. In the case of the hot wire anemometer circuit the current was switched on only for a brief period during which the gas velocity trace was recorded. This was necessary to safeguard the probe. The gas temperature bridge was balanced before starting the engine and the ambient temperature was recorded during the test run. Static calibration of the metal surface temperature record was obtained by impressing two known voltages through the same circuit as the surface thermocouple signal. A base line for each record was also impressed on all the traces. The gas temperature scale was obtained by bringing the gas temperature bridge to balance at the peak gas temperature by the use of a multi-decade box in one arm of the bridge. The gas velocity scale was obtained by impressing a known voltage (mercury cell 1.35 volts) onto the trace. The outside metal surface temperature of the cylinder head was obtained by a copper constantan thermocouple. All the thermocouple voltages were measured by a vernier potentiometer. Since the same probe was used to obtain the gas temperature and gas velocity measurements, these measurements could not be obtained on the same engine cycle. It will be shown later that there was not any significant cycle to cycle variation in these records hence this procedure did not result in unrepresentative results. The recording camera speed was adjusted to give at least two complete engine cycles
FIG-10.2

HOT WIRE ANEMOMETER OUTPUT — WIRE TEMP. 260°C

FIG-10.3

HOT WIRE ANEMOMETER OUTPUT — WIRE TEMP. 565°C

FIG-10.4

HOT WIRE ANEMOMETER OUTPUT — WIRE TEMP. 610°C
at any engine test speed. Cylinder pressure was recorded with a strain gauge pick-up described by Lyn and others. Details of the pressure pick-up and electrical circuit used are described in Appendix 10.1. The pick-up was air-cooled during operation and no drift of the trace was observed. The sensing face of the pick-up was mounted flush with the surface of the pre-combustion chamber such that there were no 'passage effect' associated with the pressure measurement in i.c. engines.

The metal surface thermocouple was mounted as shown in Fig. 10.1. The alignment of the thermocouple face with the pre-combustion chamber metal face was carried out optically. All the thermocouple leads were carefully screened to reduce the electronic noise to a minimum.

10.2. Gas Velocity and Temperature Traces

Originally a 7 μ tungsten probe was used to investigate the effects of wire temperature and repeatability of the gas velocity through a complete engine cycle. Fig. 10.2. shows the velocity record obtained at a temperature of approximately 260°C. Since the probe also responds to the gas temperature, there is a pronounced drop in the bridge voltage in the high temperature region of the engine cycle. It was therefore decided to increase the wire temperature gradually to the maximum temperature that the probe would stand. In order to minimise the oxidation of the tungsten probe at high temperatures, the heating current was only switched on for a very brief period during which the trace was photographed. Probe temperatures of 783°K, 858°K and 910°K were tried. The traces obtained with these probe temperatures are shown in Figs. 10.3, 10.4 and 10.5. It is clear from a study of these preliminary traces that as the probe gets hotter the drop in the bridge output near the compression TDC gradually tends to disappear. Thus it was clear that in order to minimise the gas
temperature effects the probe had to be run with as high a temperature as possible.

In order to check the repeatability of the gas velocity traces, five traces were superimposed to highlight the cycle to cycle variation of the gas velocity. This is shown in Fig. 10.6; a single trace is also shown to provide comparison. The superimposed traces show that there is very little cycle to cycle variation of gas velocity. Thus the main aim of designing a special pre-combustion chamber to produce repeatable gas motion from cycle to cycle was achieved. Had this not been the case some statistical method of averaging the gas velocity over a number of engine cycles would have been necessary.

The next problem was to define a representative main stream velocity. Lyn and Valdmanis\(^\text{96}\) used a similar chamber for schlieren studies of the flow pattern and concluded that taking the wall as the origin, the gas velocity increases gradually with the distance from the wall until a maximum is reached and any further increase in the distance, i.e., towards the centre of the chamber, the gas velocity tends to fall. Traverses were carried out with the probe positioned at intervals of .1", starting from as near the wall as possible (about .004") to the centre of the chamber. The bridge output was found to increase up to .05" from the wall and then it was almost constant up to .2" from the wall. Any further movement showed a fall in the bridge output. Results of these tests are plotted in Fig.10.7. In order to further establish the region of constant velocity, traverses were carried out between .025" to .200" from the wall at intervals of .025". Results of these tests are plotted in Figs. 10.8 and 10.9. for various points in the engine cycle. A study of these graphs shows that between .075" and .125" from the wall maximum variation in the bridge output for any point in the engine cycle is of
FIG - IO.6 Repeatability of gas velocity traces.

FIG - IO.10 Repeatability of gas temperature traces.

FIG - IO.11 Metal surface temperature -- Engine motored 900 R.P.M.
VOLTAGE V. POSITION.  APRIL 25th.

5 μ DIA WIRE
17A BRIDGE RESISTANCE

FIG. 11-7
Variation of Hot Wire Anemometer Voltage with the Distance from Wall.

Maximum Variation in this Band: 5.5%
FIG 10-9

MAX VARIATION APPROX 35%

COMPRESSIO N MAXIMUM VARIATION

30

27

24

21

18

15

12

9

COMPRESSIO N EXPANSION

MAXIMUM VARIATION

3% <

2%
Fig-10.12(a) Metal Surface and Gas Temperature Traces — Engine Motored 600 R.P.M.

Crank Degree Marker

Calibration Voltage 12

Metal Surface Temp. Trace Scale: 635°C/cm

GAS VELOCITY AND PRESSURE TRACES  ENGINE MOTORED 600 R.P.M.

Fig-10.12(b) Gas Velocity and Pressure Traces — Engine Motored 600 R.P.M.
ENGINE MOTORED 600 RPM.
COMPRESSION RATIO 8.5

FIG. 10·13
ENGINE MOTORED 600 RPM.
COMPRESSION RATIO 8.5
O — GAS VELOCITY
— ANEMOMETER OUTPUT VOLTAGE

FIG. 10.14
the order of 3\%. It was therefore decided to select a point in the middle of this region, i.e., 0.010" from the wall, as the point of measurement for the main stream velocity. All further tests were carried out with the probe positioned at this point.

In order to check the repeatability of gas temperature traces, similar tests were carried out. Fig. 10.10 shows five gas temperature traces superimposed and also a single trace. The difference between the two traces is only the thickness of a line. This showed that the gas temperature traces were repeatable and no significant error would result when the gas velocity and gas temperature traces were obtained during different engine cycles.

10.3. Metal Surface Traces

Some difficulty was experienced in the initial stages to obtain this measurement. Since the thermocouple junction was grounded a differential input to the oscilloscope was necessary. Also because the temperature fluctuation in the motored case was only of the order of 2\,-\,3\,^\circ\,C this resulted in an extremely small fluctuating signal superimposed on a large standing voltage due to mean temperature of the surface; this standing voltage was biased off as described in Chapter 4. It was necessary to use maximum amplification (x 1000) on the Dymec amplifier and maximum sensitivity of the oscilloscope to obtain a usable trace. At such large overall amplification it was absolutely vital to meticulously screen all the circuiting associated with this measurement. The trace as obtained with all these precautions is shown in Fig. 10.11. It can be seen that there is some high frequency noise present but the amplitude is not excessive. Since filtering resulted in the attenuation of the signal and also because the signal had to be harmonically analysed, it was decided to record the trace unfiltered and smooth out the noise.
at the signal processing stage.

Dynamic calibration of the metal surface temperature trace was attempted in the earlier stages. Microswitches operated from the engine crankshaft were first tried to break the thermocouple circuit for a small part of the engine cycle and impress a known voltage on to the trace but this proved unsuccessful due to slow speed of operation of these switches. High speed teleprinter switching relays were next tried. These could be made to work but again the resultant disturbance caused to the signal was not acceptable. In the final form, the calibration lines were impressed on the paper record separately from the trace; these are shown in Fig. 10.11.

10.4 Tests at Different Compression Ratios and Speeds

Most of the initial work of perfecting the instrumentation was carried out at 700 r.p.m. and 8.5 compression ratio. In order to investigate the widest possible speed range, without altering the rig, tests were carried out at three engine speeds; 600 r.p.m., 900 r.p.m. and 1100 r.p.m. The first and the last speeds being the minimum and maximum speeds that the rig was suited to run at without altering the set up. Three different compression ratios, 8.5, 9.8 and 11.6 were obtained by inserting distance pieces between the side plates and the pre-combustion chamber. Four traces were obtained at each speed and compression ratio. These were:

(a) Metal surface temperature record
(b) Gas temperature record
(c) Gas velocity record
(d) Gas pressure record.

A complete set of traces for one speed and compression ratio is shown in Fig. 10.12. Plots of gas temperature, surface temperature, gas velocity bridge voltages and pressure against the engine crank angle
obtained from the above traces are shown in Figs. 10.13 and 10.14.

10.5. Processing of the Traces

Since there was no digitising equipment available the processing of traces proved to be a very cumbersome and time consuming business. Thirty-six harmonics were used in the analysis of the metal surface temperature record which necessitated reading 73 ordinates on each metal surface temperature record. The output from the computer was programmed to give the various quantities at $10^0$ for a complete engine cycle which required 72 ordinates from gas temperature record. Gas velocity was calculated at $20^0$ interval which required 36 ordinates from each of the pressure, gas temperature and gas velocity records. Thus reading off ordinates for nine different cases required a considerable amount of effort. It was therefore considered that in any future work automatic digitising of such data would be very desirable.

10.6. Discussion of Computer Calculations

These may be divided into two parts. Part I consisted of Fourier analysis of the metal surface temperature record to give instantaneous values of heat flux, heat transfer coefficient, Nusselt number and gas and metal surface temperature differences at various points in the engine cycle. Part II consisted of computing the gas velocity from the hot wire anemometer output and hence the Reynolds number at various points in the engine cycle. A full account of the computer programme is given in Appendix 10.1. Only the essential points will be discussed here.

Part I of the computer programme consisted of fitting a Fourier series to the measured temperature as discussed in Chapter 2. Thirty-six harmonics were used in the analysis of the metal surface temperature record. The next step was to differentiate the series with respect
to \( x \), the distance into the wall, to obtain the temperature gradient at the wall surface. This yielded the instantaneous heat transfer and Nusselt number at various points in the engine cycle. In order to check the cross-over point of the temperature difference between the gas and metal, this quantity was also computed.

Part 2 of the computer calculations consisted of computing a gas velocity from the hot wire anemometer output record. This basically involved solving the following equation which has previously been developed in Chapter 6.

\[
|K_i| = \frac{4K_w}{K_h d^2} \frac{Nu_m}{N_{um}} \left[ Nu_m - \frac{R_w - R_0}{R_w} \right] .... (10.1)
\]

Conductivity of air as a function of temperature was represented by a relationship used previously by Davis and Fisher:

\[
K_g = (2.56 + (7.3(T_g - 54)/1000)) \times 10^{-4} \text{ watts cm}^{-1}\text{\degree K}^{-1} \ldots (10.2)
\]

The resistivity of wire material was represented by the following linear law.

\[
\lambda_w = \lambda_0 (1 + \alpha(T_w - T_0)) \Omega \text{ cm} \ldots (10.3)
\]

with \( \lambda_w \) - resistivity at temperature \( T_w \)

\( \lambda_0 \) - resistivity at temperature \( T_0 \)

\( \alpha \) - temperature coefficient of resistance.

Thermal conductivity of the wire material was calculated by using Lorenz Constant, which relates the electrical resistivity to the thermal conductivity. Lorenz Law (Ref. 25) states that

\[
\frac{\lambda K_w}{T_w} = \text{Lorenz Constant}
\]
The value of constant was taken as $2.23 \times 10^{-8}$ (Ref. 25)

\[ K_w = \frac{T_w \times 2.23 \times 10^{-8}}{\lambda_w} \text{ watts cm}^{-1} \text{ deg K}^0 \ldots (10.4) \]

Density of air was expressed as a function of instantaneous cylinder pressure and temperature

\[ \rho_g = \frac{2.31 P_g}{96.5 x T_g} \text{ g/m}^3 \ldots (10.5) \]

where the pressure $P_g$ is in lb/sq.in. and $T_g$ is in K.

The specific heat of gases at constant volume was expressed as (Ref. 87).

\[ c_v = (1.715 + 0.2788 \rho_g)^{4.1813} \text{ watts/cm m K}^0 \ldots \]

Viscosity of gas was expressed by Sutherland formula (Ref. 25).

\[ \mu_g = \frac{\mu_0 \times 390 (T_g/273)^{2/3}}{(T_g + 117)} \text{ g/m cm sec.} \]

In evaluating Reynolds number all the properties were taken at the film temperature.

Each calculation was started at the beginning of the suction stroke. Seventy-three ordinates were read in for the surface temperature and gas temperature for the calculation of Nusselt number at 10 crank degree intervals. For the velocity calculation hot wire anemometer voltage was read in and the heating current calculated for each case. The program is described in detail in Appendix A 10.f.

10.7. Comparison of Heat Flux Curves with Different Number of Harmonics Used

It was observed in Chapter 2 that due to the presence of term $\sqrt{\frac{n \omega}{25}}$ on the right-hand side of equation 10.4, the higher harmonics take on
a new significance. The choice of number of harmonics included in
the analysis of the metal surface temperature is a compromise between
the amount of labour involved and the accuracy required. If too few
ordinates are taken the metal surface temperature will not be
represented by the Fourier series with sufficient accuracy and the
fine detail of the recorded temperature will be lost. Shannon's
rule quoted by Overbye states that sampling frequency to completely
define the trace is twice the highest frequency present in the trace,
but here a further difficulty presents itself. In the presence of
high frequency noise it is extremely difficult to decide about the
value of highest significant frequency. Therefore in practice the
deciding factor is the amount of labour involved and the accuracy
required. Since the number of ordinates required for the analysis is
equal to $2n + 1$ and the computational labour increases as the square
of the number of ordinates, a reasonable number of harmonics should
be chosen for the analysis. Eichelberg and Elser used 12 harmonics
whereas Overbye used 71 harmonics. In the present study most of
the analysis was carried out with 36 harmonics included. Some initial
work was done using 22 harmonics. The 22 and 36 harmonic heat flux
curves are shown in Fig. 10.15. It can be seen that difference between
the two curves is quite small.

10.8. Comparison of the Present Study with Existing Formulae

In order to compare the heat fluxes obtained during the present
study with the existing formulae, heat flux calculations were performed
using the following formulae.

Eichelberg

$$\frac{q}{A} = 8.06 \times 10^{-5} (V_p) \left( \frac{\frac{1}{3}}{T_g} \right) \left( T_g - T_w \right) \text{ CHU/sq.ft/s}$$  \hspace{1cm} (10.6)
Nusselt (convection only)

$$\frac{q}{A} = 5.27 \times 10^{-5} (1 + 0.38 V_p) \left( P^2 T_g \right) \left( T_g - T_w \right) \quad \text{CHU/ft/sec.} \quad \cdots (10.7)$$

Annand (convection only)

$$\frac{q}{A} = 0.49 \frac{K_t}{D} \left( R_e \right) \left( T_g - T_w \right) \quad \text{CHU/ft/sec.} \quad \cdots (10.8)$$

Overbye

$$\frac{q}{A} = \left( 3600 K_i T_i \right) \left[ \left( L V_p \rho_i C_p i \right) \left( \frac{26 P}{k_i r P_i} - 0.35 \right) \times 10^{-4} + 0.1 \frac{P}{r P_i} - 0.2 \right] \quad \cdots (10.9)$$

Here the following units were used,

- $V_p$ - Piston velocity ft/sec.
- $P$ - Cylinder Pressure - atmospheres
- $T$ - Temperature °K
- $K$ - Thermal conductivity CHU/ft.sec. °K
- $\rho$ - Density of gases - lb/cubic ft.
- $C_p$ - Specific heat of gases - CHU/lb °K
- $r$ - Compression ratio
- $L$ - Stroke of the engine - ft.

The suffix 'i' refers the gas properties to the conditions in the air inlet pipe. Results of these calculations are plotted in Fig. 10.16, together with the instantaneous heat fluxes obtained by the analysis of the metal surface temperature record. A study of these curves shows that the instantaneous heat flux calculated according to the above mentioned formulae is lower than the experimental
HARMONICS
CALCULATED FROM SMOOTHED METAL SURFACE TEMPERATURE CURVE

FIG. 10-15
OVERBYE
• ANNAND (CONVECTION ONLY)
• NUSSELT (CONVECTION ONLY)
• EICHELBERG
• PRESENT STUDY
(CALCULATED FROM THE RECORDED
METAL SURFACE TEMPERATURE.)

700 R.P.M
8.5 c.r.

FIG 10-16
values in all the cases. Confining our attention to the TDC position a comparison between the various formulae is given in Table 10.1.

<table>
<thead>
<tr>
<th></th>
<th>Experimental Value</th>
<th>OVERBYE Eq. (10.9)</th>
<th>ANNAND Eq. (10.8)</th>
<th>EICHELBERG Eq. (10.6)</th>
<th>NUSSELT Eq. (10.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat Flux CHU/sq.ft.sec.</td>
<td>11.5</td>
<td>7.3</td>
<td>5.2</td>
<td>2.5</td>
<td>1.8</td>
</tr>
<tr>
<td>Ratio H (Formula) / H (Experiment)</td>
<td>-</td>
<td>.635</td>
<td>.45</td>
<td>.218</td>
<td>.157</td>
</tr>
</tbody>
</table>

**TABLE 10.1**

Comparison of four existing formulae with the experimental values at the TDC position

A comparison of these figures clearly supports the remarks made in the discussion about those formulae in Chapter I. The two latest formulae, i.e., Overbye and Annand, are the nearest to the experimental values. The difference between these two formulae may be explained by the fact that the Overbye formula was obtained from the results which included the effects of radiation heat transfer thus for a purely convective case it will tend to predict slightly higher values when compared with the results obtained with the convection term in Annand's formula. The greatest discrepancy between the predicted and experimental values exists in the case of Nusselt formula but as commented in Chapter 1, this formula was based on hot bomb tests in which case the amount of heat transferred due to forced convection would be expected to be small due to low gas velocities in the hot bomb compared with an engine cylinder. The overall comparison between the predicted values and the experimental values is quite poor, which
again justifies the critical remarks made in Chapter 1.

10.9. Variation of the Instantaneous Heat Transfer Coefficient through the Engine Cycle

The instantaneous heat transfer coefficient was computed according to the definition given in Chapter 3, i.e.,

\[ h = \frac{H}{(T_g - T_w)} \]  \hspace{1cm} (...) (10.10)

where \( H \) is the instantaneous heat flux and \( T_g \) is the instantaneous gas temperature. Whereas this definition does not pose any problems in the steady heat transfer in the case of unsteady heat transfer, as discussed in Chapter 3, unrealistic values of the heat transfer coefficient can result. A typical case (600 r.p.m. with 8.5 C.R.) is shown in Fig. 10.17. Instantaneous heat flux, instantaneous heat transfer coefficient and the instantaneous temperature difference between the gas and metal are plotted against the engine crank degrees. These curves show that the points at which the instantaneous heat flux and the temperature difference pass through zero do not coincide. These points are shown on the graph (B and D for the heat flux, A and C for the temperature difference). Thus the heat transfer coefficient has infinite values at points A and C. This phenomenon has previously been observed by other workers. The concept of infinite heat transfer coefficient is physically unrealistic, but fortunately in the relevant part of the cycle, i.e., the part in which a large proportion of unsteady heat transfer takes place, around TDC, the heat transfer coefficient has finite values. Therefore the steady heat transfer approach of relating the Nusselt and Reynolds number to produce an empirical relationship can be used. Attention was therefore concentrated on this portion of the engine cycle for further analysis of results.
Discussion of Results for Different Compression Ratios and Speeds

Instantaneous heat fluxes and temperature difference computed for three different compression ratios (8.5, 9.8 and 11.5) and three different engine speeds (600, 900 and 1100) are plotted in Figs. 10.13 and 10.18 to 10.25. Preliminary work had shown that there was no significant cycle to cycle variation in the computed heat flux curves, because of this repeatability of results statistical averaging of results from a number of cycles was considered to be unnecessary, however, the results plotted in the figures 10.13 and 10.18 to 10.25 were obtained from an average of 3 cycles for each case. Metal surface temperature for 600 r.p.m. and 8.5 compression ratio case is plotted in Fig. 10.13. It can be seen that the metal surface temperature rises to its peak value in about 70° crank degrees. This is comparable to 60° reported by Overbye and about 50° reported by Knight. Since the speeds in all the three cases were of the same order, i.e., 700, 850 and 500 r.p.m., these values indicate similar rates of metal surface temperature rise. The magnitude of metal surface temperature for the case shown in Fig. 10.13 is approximately 1.5°C compared with 4°C reported by Knight for his motored tests but the compression ratio used in his case (22:1) was much higher than that used in the case shown in Fig. 10.13 (8.5:1) therefore the difference between the magnitudes of the two temperatures is not unexpected. A better comparison would be 11.5 compression ratio at 600 r.p.m. in the present study which gave a metal surface temperature rise of approximately 2.5°C. Further examination of the metal surface temperature and gas temperature traces shows that there is no appreciable phase difference between them which is contrary to the results obtained with the theoretical model in Chapter 3; however, these results were in line with the results obtained by other workers who also failed to find any significant phase difference between the gas and metal surface temperatures. This again shows that the theoretical model
discussed in Chapter 3 is not realistic.

An examination of heat flux curves shows that in general the peak heat flux values coincide with the peak of temperature difference values or in other words there is no significant phase difference between the heat flux and gas temperature variation through the engine cycle. This again is contrary to the analysis carried out in Chapter 3 but is in line with the findings of other workers (Overbye\textsuperscript{12}, Knight\textsuperscript{18} and Heinen\textsuperscript{7}). A further examination of these curves shows that the major portion of unsteady heat transfer takes place between the interval 70\textdegree BTDC (compression) and 60\textdegree ATDC (expansion) this is of the same order for the respective values obtained by Overbye (80\textdegree BTDC to 80\textdegree ATDC) and Knight (60\textdegree BTDC to 60\textdegree ATDC) for the motored cases.

The foregoing discussion has made it clear that rates of metal temperature rise and hence the heat transfer through the engine cycle, observed in the present study, are in line with the findings of other workers in this field of research.

10.11. Variation of Peak Heat Flux with Engine Speed

It was shown in Chapter 2 that the instantaneous heat flux is proportional to the square root of fundamental frequency or is directly proportional to the square root of the engine speed. Therefore if the heat flux values at any given point in the engine cycle, TDC for example, are plotted against the engine speed on a logarithmic scale the slope of the resulting line should be equal to .5. The experimental values for the three different compression ratios were plotted for three different speeds; these are shown in Fig. 10.26. It can be seen that the slopes of the lines are:
Although it is considered that three points are not sufficient for a graph of this type, the resulting slopes are of the right order of magnitude. This shows that the Eichelberg formula which relates the heat flux to \( \frac{1}{N} \) would tend to reduce the effects of speed variations on a given engine and the values predicted for high speeds will be lower than the experimental values. This is further supported by Henien's results shown in Fig. 1.1, where the calculated points at 1231 r.p.m. according to the Eichelberg formula show greater disagreement with the experimental results than the calculated points at 840 r.p.m. In the light of results from the present study, Annand's relationship of 

\[
\frac{\dot{q}}{A} \propto N^{0.7}
\]

is considered to be comparatively better.

10.12. Gas Temperature Profile

Further examination of the heat flux and temperature difference \( (T_g - T_w) \) curves in Figs. 10.18 to 10.25 shows that in all the cases, the temperature difference assumes a negative value around 70° ATDC on the expansion stroke and remains negative until about 120° BTDC on the compression stroke. The heat flux on the other hand is generally marginally positive in this part of the engine cycle even when the gas temperature is lower than the metal temperature, which should produce a negative heat flux (heat transfer from the metal to gases). The reverse phenomenon, i.e., negative heat flux in the presence of a positive temperature difference, is also shown to take place in the early part of the compression stroke, as is shown in Figs. 10.20 to
10.25. In order to investigate this phenomenon it was decided to obtain the gas temperature profiles, experimentally. Some of these temperature profiles are shown plotted in Fig. 10.27. Gas temperature traces were obtained at intervals of .005" from the wall, the curves shown in Fig. 10.27 are plotted at selected points in the engine cycle to show the development of the gas temperature profile through the engine cycle. In order to relate these temperature profiles to the heat flux variation, the gas temperature trace obtained at .005" from the wall and the main stream gas temperature (.1" from the wall) are plotted in Fig. 10.28 together with the heat flux variation through a complete engine cycle. Referring to Fig. 10.27 curves (a), (b) and (c) in the compression part of the engine cycle show the development of the gas temperature profile as the gases are compressed and heat transfer starts to take place from hot gases to the metal wall. Starting with curve (c) it is clear that at this point in the engine cycle the temperature gradient at the wall surface has a very low value signifying a low positive heat flux. The heat flux curve in Fig. 10.28 shows a negative heat flux of .5 CHU/sq.ft sec. The discrepancy between the heat flux indicated by the temperature profile, i.e., a small positive heat flux and that obtained experimentally is within the margin of experimental error if it is assumed that the heat flux calculations give the heat flux values to within ± .75 CHU/sq.ft.sec which corresponds to about ± 6% at the TDC condition. Curves (b) and (a) in Fig. 10.27 indicate a substantial positive heat flux which is also supported by the heat flux curve in Fig. 10.28 which gives heat flux values of 1.5 CHU/sq.ft.sec and 11.5 CHU/sq.ft.sec respectively. Curve (d) in Fig. 10.27 was plotted at 80° ATDC, a point in the engine cycle where the heat flux calculated is -1.5 CHU/sq.ft.sec. The temperature profile also indicates a negative
heat flux.

An attempt was made to calculate the instantaneous heat fluxes using the gas temperature profiles. Since the nearest point to the wall at which the gas temperature was measured was .005", this was not considered sufficiently close to justify the extrapolation of the temperature curve by hand, it was decided to use numerical extrapolation using Newton's forward interpolation formula (Ref. Nielsen94).

Nusselt number was calculated according to the definition due to Martinelli (Ref. Jakob28), this graphical definition of Nusselt number is illustrated in Fig. 10.29. Using Newton's interpolation formula and the experimental gas temperature profile, the profile curve was constructed up to .001" from the surface for the TDC case. The Nusselt number obtained thus was compared with the experimental value. The respective values were

\[
\begin{align*}
\text{Nu (From the gas temperature profile)} &= 870 \\
\text{Nu (From experimental results)} &= 2000
\end{align*}
\]

This discrepancy was considered to be due to the unrepresentative character of the gas temperature profile near the wall because there was only one point, i.e., .005", near the wall whereas majority of the points were farther removed from the wall thus making the extrapolation of the curve near the wall unrealistic. It was therefore decided to manufacture a special gas temperature probe which would allow the sensing wire to approach the wall to within .001". The new and the old design of the probe are shown in Fig. 10.30. In order to eliminate the copper plating thickness and the solder which prevented the old design of probe from approaching the wall nearer than .005", the new probe was manufactured by spot welding the sensing wire directly on to the platinum electrodes as shown in Fig. 10.30.
Although attempts were made to ascertain the minimum distance from the wall that the new design of probe could approach without getting damaged, it was extremely difficult to arrive at an exact figure due to fragility of the sensor. A figure of between .001" to .002" was finally arrived at. Gas temperature traces were obtained at intervals of .002" up to a maximum distance of .024" from the wall. A sixth order polynomial was then fitted to the experimental points for various points in the engine cycle. The Nusselt number was obtained by differentiating the series and obtaining the value of the temperature gradient at the wall. Thermal conductivity of air was calculated at the wall temperature for this calculation. In order to ascertain the effects of minimum distance from the wall, calculations were performed with distance assumed as .001" and .002". The results were tabulated in Table 10.2 together with the Nusselt number values computed from the heat flux measurements obtained from the metal surface temperature record.
ENGINE MOTORED 600 RPM
COMPRESSION RATIO 8.5

- HEAT FLUX
- TEMPERATURE DIFFERENCE
- HEAT TRANSFER COEFFICIENT

FIG. 10-17
ENGINE, MOTORED, 900 RPM
COMPRESSION RATIO 9:5

- HEAT FLUX
- TEMPERATURE DIFFERENCE

FIG. 10:18
FIG - 10.26 Variation of heat flux with engine speed.

FIG - 10.27 Gas temperature profiles at various points in the engine cycle.
FIG - 10.26 Heat flux and gas temperature curves — Engine motored 700 R.P.M.

FIG - 10.31 Gas temperature profiles at BDC and 140° ATDC (expansion).
2.33

\[ \text{Nu} = \left( \frac{\gamma}{\partial (\theta/L)} \right)_{\gamma=0} \]

FIG - 10.29 Graphical interpretation of Nusselt number.

FIG - 10.30 Original and modified designs of the sensing probe for gas temperature profiles.
A comparison of heat flux values in columns 2, 3 and 4 shows that in general the heat fluxes are of the same order. Since the gas temperature profile method requires the mean surface temperature only, it was an independent check on the results obtained by the metal surface temperature measurement. Referring to Figs. 10.18 to 10.25 and Table 10.2, it can be seen that the part of engine cycle...
investigated so far lies in the internal 60° BTDC to 60° ATDC. In order to investigate the existence of positive heat fluxes in the presence of a negative temperature difference and vice versa the gas temperature profiles at 180° BTDC and 140° ATDC are shown plotted in Fig. 10.31. Curve (a) shows the temperature profile at the BDC at the start of the compression stroke. It can be seen that the gas temperature up to approximately 0.014" from the wall is higher than the wall temperature by approximately 10°C, any further movement away from the wall shows gas temperatures which are lower than the metal temperature thus suggesting the existence of a cushion of hot air near the wall. A similar effect is produced by aerodynamic heating at high gas velocities discussed by Schlichting. It is shown that when Eckert number \( E = \frac{V^2}{g C_p (T_w - T_{gas})} \) exceed 2.4 a heat cushion is created which prevents the transfer of heat. Eckert number was calculated for this case but this was found to be quite low because although the temperature difference \( (T_w - T_{gas}) \) is not great the gas velocity is not sufficiently high to give a high Eckert number. A temperature difference of the order of 5% (i.e., 5°C at 100°C) would be attributed to the instrumentation although under the conditions prevailing in this part of the engine cycle the error due to the thermal inertia of the wire will be very small because the gas temperature is relatively steady.

Curve (b) in Fig. 10.31 shows a negative temperature gradient but the calculated heat flux is marginally positive (+ 0.05 CHU/sq.ft.sec°C) but this is so small that the discrepancy could be due to slight over-estimate of heat flux. Although the effects discussed above only exist in those parts of the engine cycle where the heat transfer is generally not high, nevertheless it is considered that further work is required to explain these phenomena in more detail. Due to lack of
time it was not possible to devote any further effort to find a satisfactory explanation.

10.13 Choice of Characteristic Length and Property Values for the Calculation of Reynolds and Nusselt Numbers

It has been the usual practice in heat transfer work of this nature to take the piston diameter as the characteristic length. This choice has been arbitrary and was selected in the absence of any more relevant characteristic length. In the present study the choice of characteristic length was limited to the following:

(i) Piston diameter ..... 3"
(ii) Pre-combustion chamber diameter ..... 2"
(iii) Developed length of the pre-combustion chamber from the point of entry of the gases to the point of measurement ..... 6"

Since the results were being analysed with a view to investigating the suitability of using the existing heat transfer data, it was decided to use the third characteristic length, namely the developed length of the pre-combustion chamber. Using this length as the entry lengths on a flat plate the experimental results could be compared directly with the wealth of empirical data which is available for the cases of heat transfer from flat plates.

Choice of the temperature at which the property values were calculated was also dictated by this choice. In the case of flat plate it is usual practice to use the fluid properties at a temperature which is a function of gas and wall temperatures. Many workers have suggested the use of following temperature for property evaluation.

\[ T^* = T_{g\infty} - \frac{Pr + 40}{Pr + 72} (T_{g\infty} - T_w) \]  \hspace{1cm} \text{(10.11)}

but Eckert and Drake\textsuperscript{29} have suggested that this temperature can be
replaced by the film temperature, i.e., the arithmetic mean between the fluid and the wall temperatures, for the case of gases with a Prandtl number of unity. In the case of air \( \text{Pr} = 0.7 \) therefore all the property values were computed at the film temperature in the present study.

10.14 Variation of Nusselt and Reynolds Number through the Engine Cycle

Plots of Nusselt and Reynolds numbers for the relevant part of the engine cycle (Compression BDC to expansion 100° ATDC) are shown in Figs. 10.32 to 10.37. First the effect of compression ratio on Reynolds number will be discussed.

An examination of Figs. 10.32, 10.34 and 10.36 shows that for a given speed as the compression ratio increases, there is a reduction in the value of Reynolds number through the engine cycle. In order to explain this phenomenon consider the two extreme cases of compression ratio, i.e., 8.5 and 11.5 at 1100 r.p.m. The total mass of gas in the pre-combustion chamber at the end of the compression stroke may be expressed as a fraction of the total trapped mass by the following relationships.

For 8.5 compression ratio:

\[
M_1 = \frac{\text{Mass trapped in the pre-combustion chamber}}{\text{total trapped mass}} = \frac{v_1}{v_{c_1}} = 0.39
\]

where \( v_1 \) is the pre-combustion chamber volume and \( v_{c_1} \) is the total clearance volume. Similarly for the 11.5 compression ratio case

\[
M_2 = \frac{v_2}{v_{c_2}} = 0.185
\]

where \( v_2 \) is the pre-combustion chamber volume in this case and \( v_{c_2} \) is the total clearance volume. Since the engine speed is the same in both the cases the time available for the transfer of gases from the
VARIATION OF REYNOLDS NUMBER THROUGH ENGINE CYCLE.

A5 COMPRESSION RATIO.

- 180 R.P.M.
- 200 R.P.M.
- 220 R.P.M.
- 260 R.P.M.
- 280 R.P.M.
- 300-320 R.P.M.
- 340-360 R.P.M.
- 380-400 R.P.M.
- 420-440 R.P.M.
- 460-480 R.P.M.

ENGINE CRANK DEGREES

FIG. 10.32
VARIATION OF KINEMATIC NUMBER THROUGH ENGINE CYCLE

9:1 COMPRESSION RATIO

1100 R.P.M.
920 R.P.M.
600 R.P.M.

ENGINE CRANK DEGREES

Fig-10-34
VARIATION OF REYNOLDS NUMBER THROUGH ENGINE CYCLE:

11.5 COMPRESSION RATIO

- 1100 R.P.M.
- 900 R.P.M.
- 600 R.P.M.

Fig 10.36
VARIATION OF NUSSELT NUMBER THROUGH ENGINE CYCLE

11:5 COMPRESSION RATIO

FIG-10:37(a)

FIG-10:37(b)

FIG-10:37(c)

ENGINE CRANK DEGREES
FIG. 10.36 Variation of $k_e$ with engine crank-angle at 8.5 and 11.5 compression ratios.
main chamber to the pre-combustion chamber is the same in both the cases, i.e., the time from inlet valve closing to top dead centre position. Now if an average flow rate is assumed for this time, a corresponding value of an average Reynolds number can be defined such that

\[ \text{Re}_1^{\text{(average)}} \times \theta \propto M_1 \]  

..... (10.12)

and

\[ \text{Re}_2^{\text{(average)}} \times \theta \propto M_2 \]  

..... (10.13)

where suffixes 1 and 2 signify the same compression ratio conditions as defined above. Equations (10.12) and (10.13) give

\[ \frac{\text{Re}_1^{\text{(average)}}}{\text{Re}_2^{\text{(average)}}} = \frac{M_1}{M_2} = \frac{9}{18.5} = 2.1 \]  

..... (10.14)

Since the instantaneous Reynolds number values are known the average Reynolds number can be found by the following relationship

\[ \text{Re}^{\text{(average)}} = \frac{1}{\theta} \int_0^\theta \text{Re}^{(i)} d\theta \]  

..... (10.15)

where \( \text{Re}^{(i)} = \text{Instantaneous Reynolds number} = f(\theta) \)

If it is assumed that

\[ \text{Re}^{(i)} = m \theta^n \]  

..... (10.16)

where 'm' and 'n' can be obtained from the experimental results by plotting \( \text{Re}^{(i)} \) against \( \theta \) (or since the speed is constant - the engine crankangle).
From equations \((10.15)\) and \((10.16)\) we have

\[
\text{Re (average)} = \frac{1}{\theta} \int_{\theta_0}^{\theta} m \theta^n d\theta = \frac{1}{\theta} m \theta^{n+1}
\]

\[
\therefore \frac{M_1}{M_2} = \frac{\text{Re}_1}{\text{Re}_2} = \left(\frac{m_1}{m_2}\right) \left(\frac{n_2^{n_2+1}}{n_1^{n_1+1}}\right) \left(\theta^{n_1-n_2}\right) \ldots \ldots (10.17)
\]

A plot of instantaneous Reynolds number against crank degrees is shown in the Fig. 10.38 for the two cases under discussion. The values of \(m\) and \(n\) obtained from these plots were substituted in equation 10.17 which gave a ratio \(M_1/M_2\) of 2.04, which agrees quite well with the value of 2.11 calculated in equation 10.14. This simple calculation has shown that it is reasonable to expect the Reynolds number to decrease with an increase in the compression ratio in the present case.

### 10.15: Correlation of Nusselt and Reynolds Numbers

Reynolds and Nusselt numbers were plotted on a logarithmic scale to compare with existing data on heat transfer. Figs. 10.39 to 10.41 show all the different compression ratio and speed cases. In order to highlight any differences that may exist between the compression and expansion strokes, these have been plotted separately for each compression ratio case. Finally all the compression stroke points and all the expansion stroke points are shown plotted in Figs. 10.42 and 10.43. Heat transfer data from flat plates has been correlated by various workers. References to various formulae may be found in numerous textbooks on the subject. Some of the more commonly used relationships for relating the local Nusselt number \(\text{Nu}_x\) to the local Reynolds number \(\text{Re}_x\) are as follows.
$P_r$ assumed constant = .7 for air

$Nu_x = .0236 \text{Re}_x^8$ (Prandtl Taylor Analogy)

$Nu_x = .0241 \text{Re}_x^8$ (Von Karman)

$Nu_x = .0258 \text{Re}_x^8$ (Colburn Analogy)

$Nu_x = .0238 \text{Re}_x^8$ (Disseler - Experimental Results)

A study of above relationships shows that the main difference between these formulae is in the value of constant on the right-hand side, the slope remains unchanged. In order to compare the experimental results with the above mentioned formulae a line to represent

$$Nu_x = .023 \text{Re}_x^8$$

(10.18) has been drawn in all the cases. Consider the case of compression stroke first. In almost all the cases the experimental values are higher than those predicted by equation 10.18. But the discrepancy can be accounted for by 20% increase in the value of the constant in equation 10.18, i.e.,

$$Nu_x = .0276 \text{Re}_x^8$$

(10.19)

All the compression points are shown plotted in Fig. 10.42, it can be seen that a majority of the experimental points lie between the two lines representing equations 10.18 and 10.19. Now consider the expansion stroke. Except for the 8.5 compression ratio case here again the majority of points lie between the lines represented by equations (10-18) and (10-19). Only in the case of the lowest compression ratio there are some points which lie outside this range. If the constant
.023 is reduced by 20% to yield the following relationship.

\[ N_u \times = 0.0184 \, R_{e_x}^{0.8} \quad \text{..... (10.20)} \]

All the expansion stroke points will then lie between the lines represented by equations 10.19 and 10.20. This is shown in Fig. 10.43, where only expansion stroke points are plotted.

Thus a comparative study of the compression and expansion strokes has shown that in general the heat transfer in the parts of the engine cycle analysed in the present study can be represented by the existing data on heat transfer from a flat plate in turbulent flow. The experimental results can be correlated by the following equation:

\[ N_u \times = C \, R_e^{0.8} \]

where \( C \) varies between .0276 to .0184. One of the reasons for the difference between the compression and expansion stroke could be a slight underestimation of the gas temperature on the compression stroke.

In Chapter 4 it was shown that the temperature measurement error was approximately \( \pm 6\% \), this error will be negative in the cases where the gas temperature is increasing and positive where the gas temperature is decreasing. This is due to the finite thermal inertia of the temperature measuring sensor. Thus for the hot wire anemometer the temperature difference for the wire and gas will be overestimated in the case of compression stroke and underestimated in the case of expansion stroke, resulting in underestimation and overestimation of Reynolds number in compression and expansion strokes respectively.

This point can be resolved by the use of a thinner wire (2 \( \mu \) m diameter) to measure the gas temperature but due to the fragility of such a sensor the experimental difficulties would be increased.
In order to check this hypothesis Reynolds and Nusselt number were calculated for the 1100 r.p.m. and 8.5 compression ratio case. The gas temperature was increased by 6% during the compression stroke and reduced by 6% during the expansion stroke. Reynolds and Nusselt numbers obtained as a result of this calculation are shown plotted in Figs 3.4(a) and 3.4(b) for the compression and expansion strokes respectively. An examination of these graphs shows that in both the cases there is a better agreement between the modified points and the ideal relationship. The overall scatter in this case can be allowed for by approximately ±10% tolerance on the value of C instead of ±20% as was the case previously.

Error Analysis

The error analysis has been carried out in Appendix 10.3. It is shown that the tolerance in the calculation of Reynolds and Nusselt number is ±13% and ±8% respectively. The tolerance limit for C, based on these figures, is ±19%.

The above results are based on ±12% error in gas velocity measurement (reference Chapter 6) and ±6% error in the gas temperature measurement (reference Chapter 6).
FIG - IO.39(a) Plot of the local Nusselt number against the local Reynolds number - compression stroke only - 8.5 compression ratio.

FIG - IO.39(b) Plot of the local Nusselt number against the local Reynolds number - expansion stroke only - 8.5 compression ratio.
Fig - 10.40(a) Plot of the local Nusselt number against local Reynolds number - compression stroke only - 9.3 compression ratio.

Fig - 10.40(b) Plot of the local Nusselt number against local Reynolds number - expansion stroke only - 9.3 compression ratio.
FIG - 10.41(a) Plot of the local Nusselt number against the local Reynolds number - compression stroke only - II.5 compression ratio.

FIG - 10.41(b) Plot of the local Nusselt number against the local Reynolds number - expansion stroke only - II.5 compression ratio.
FIG - 10.42(a) Plot of the local Nusselt number against the local Reynolds number all cases compression stroke only.

FIG - 10.42(b) Plot of the local Nusselt number against the local Reynolds number all cases expansion stroke only.
FI: - 10.43(a) Plot of Nusselt and Reynolds numbers. Gas temperature increased by 6%. Compression stroke only.

FIG - 10.43(b) Plot of Nusselt and Reynolds numbers. Gas temperature reduced by 6%. Expansion stroke only.
Figure 10-44: Thermal Boundary Layer at TDC.
Fig. 1045 Thermal Boundary Layer at 40° ATDC.
10.16. **Boundary Layer Approach**

Since the experimental results gave a reasonable agreement with the flat plate and pipeflow heat transfer data, it was decided to compare the thermal boundary layer profiles discussed in section 10.12 with the theoretical profiles for thermal boundary layer on flat plate in order to provide a check on the experimental results which would be independent of the surface temperature measurements.

Kays' has discussed thermal boundary layers in detail giving references to the latest published papers on this subject. Here the approach developed by Kays will be followed.

Following notation will be used:

- \( y \) ...... Distance from the wall
- \( T \) ...... Gas temperature
- \( T_w \) ...... Wall temperature
- \( T_l \) ...... Gas temperature at the edge of laminar sub-layer
- \( T_b \) ...... Gas temperature at the edge of buffer layer
- \( T_c \) ...... Gas temperature in the turbulent core
- \( T_g \) ...... Gas temperature away from the wall
- \( \tau_w \) ...... Shear stress at the wall
- \( C_p \) ...... Specific heat of the gas at constant pressure
- \( \rho \) ...... Gas density
- \( H \) ...... Heat flux at the wall
- \( u \) ...... Gas velocity in direction x
- \( u' \) ...... Dimensionless velocity = \( u/\sqrt{g_c T_o / \rho} \)
- \( y' \) ...... Dimensionless distance from the wall = \( y\sqrt{g_c T_o / \rho} / \nu \)
- \( h_x \) ...... Heat transfer coefficient

\( \nu \) is the kinematic viscosity of the gas.

The critical Reynolds number as a criterion for transition from the laminar to turbulent boundary layer was based on local momentum thickness.
\[ \text{Re}_{\text{c}k(\text{crit})} = 360 \]

where \( \delta_2 \) is the local momentum thickness given by

\[ \delta_2 = 0.664 \sqrt{\frac{\gamma x}{u_\infty}} \]

\( u_\infty \) is the free stream velocity.

Shear stress at the wall was calculated from the following relationship

\[ \tau_o = f(x) \frac{p u_\infty^2}{2 \frac{\partial u}{\partial y}} \]

where \( f(x) \) is the local friction factor obtained from the following relationship

\[ f_x = 0.059 \left( \text{Re}_x \right)^{-0.2} \]

which is in good agreement with experiments for Reynolds number up to several million. The following relationships were used to construct the theoretical thermal boundary layers.

**Laminar Boundary Layer**

For laminar sub-layer 0 < \( y^+ < 5 \)

\[ T - T_w = \frac{H}{\rho c_p} \frac{P_r y^+}{\sqrt{\frac{\partial u}{\partial y}}} \quad \ldots (10.21) \]

at the edge of the sub-layer \( y^+ = 5 \) giving

\[ T_e - T_w = \frac{H}{\rho c_p} \frac{P_r 5}{\sqrt{\frac{\partial u}{\partial y}}} \]

For buffer layer \( 5 < y^+ < 30 \)

\[ T - T_e = \frac{5H}{\rho c_p} \frac{1}{\sqrt{\frac{\partial u}{\partial y}}} \log \left( \frac{y^+ P_r}{5} - P_r + 1 \right) \quad \ldots (10.22) \]

at the edge of the buffer layer \( y^+ = 30 \) giving
\[ T_b - T_e = \frac{5H}{\rho C_p \sqrt{g_c \tau_o / \rho}} \log \left( 5Pr + 1 \right) \]

for the turbulent core \( y^+ > 30 \)

\[ T_c - T_b = \frac{H}{\rho C_p \sqrt{g_c \tau_o / \rho}} \log \frac{y^+}{30} \]

**Turbulent Boundary Layer**

Equations (10.21 and 10.22) giving the temperature profile for the laminar sub-layer and the buffer layer for the case of laminar boundary layer are also valid for the turbulent boundary layer but the expression for the turbulent core changes to

\[ dT = \frac{H}{\rho C_p \sqrt{g_c \tau_o / \rho}} \frac{du^+}{u^+} \]

using the one seventh power law we have

\[ u^+ = 8.7 \left( y^+ \right)^{\frac{1}{7}} \quad y^+ = 30 \quad , \quad u^+ = 14 \]

therefore we have

\[ \int_{T_b}^{T} dT = \int_{14}^{u^+} \frac{H}{\rho C_p \sqrt{g_c \tau_o / \rho}} \frac{du^+}{u^+} \]

or

\[ T_c - T_b = \frac{H}{\rho C_p \sqrt{g_c \tau_o / \rho}} \left( 8.7 \left( y^+ \right)^{\frac{1}{7}} - 14 \right) \quad \ldots \ldots \quad (10.23) \]

\[ T_c - T_b = \frac{H}{\rho C_p \sqrt{g_c \tau_o / \rho}} \left( u^+ - 14 \right) \]

This temperature profile also gives rise to the following relationship between the dimensionless heat transfer coefficient, i.e., Stanton number and Reynolds number.
The theoretical and experimental values for the 700 r.p.m. and 8.5 compression at TDC are given in Table 10.3. Theoretical curves and experimental points are shown plotted in Figs. 10.44 & 10.45. It can be seen from an examination of Table 10.3 and Fig. 10.44 that the agreement between the theoretical and experimental temperature profiles is reasonable. The agreement in the case of Fig. 10.45 showing the theoretical and experimental boundary layers at 40° ATDC is not as good as the agreement in the first case, although the differences in this case are not very large. Since there were only four points available near the wall it was not considered worthwhile to fit a polynomial to the experimental points.

**TABLE 10.3**

Reₜ = 3.5 x 10⁶ (experimental Uₜ = 280 ft/sec; P = 184 psia

T = Tₜ - Tₚ = 168 °K; x = 6"

hₜ (experimental) = .059 CHU/sq.ft. sec °K

hₜ (theoretical; equation 10.24) = .0694 CHU/sq.ft. sec °K

<table>
<thead>
<tr>
<th>Y Inches x 10⁻³</th>
<th>Y⁺</th>
<th>T - Tₚ</th>
<th>T - Tₚ ( \frac{T - Tₚ}{T_c - Tₚ} )</th>
<th>Experimental ( \frac{T - Tₚ}{T_c - Tₚ} )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>.21</td>
<td>5</td>
<td>32.5</td>
<td>.194</td>
<td>-</td>
<td>Edge of sub-layer (Eq. 10.21)</td>
</tr>
<tr>
<td>1.26</td>
<td>30</td>
<td>102.5</td>
<td>.61</td>
<td>-</td>
<td>Edge of Buffer layer (Eq. 10.22)</td>
</tr>
<tr>
<td>2</td>
<td>47.5</td>
<td>118.8</td>
<td>.71</td>
<td>.76</td>
<td>Turbulent Core (Eq. 10.23)</td>
</tr>
<tr>
<td>4</td>
<td>95</td>
<td>134.4</td>
<td>.80</td>
<td>.83</td>
<td>ditto</td>
</tr>
<tr>
<td>6</td>
<td>142.5</td>
<td>138.2</td>
<td>.860</td>
<td>.84</td>
<td>ditto</td>
</tr>
<tr>
<td>8</td>
<td>168</td>
<td>150.3</td>
<td>.895</td>
<td>.89</td>
<td>ditto</td>
</tr>
</tbody>
</table>
This approach does not depend on the wall heat flux measurements but on the gas temperature and velocity measurements only. Since the difference between the experimental and theoretical boundary layer profiles are within the limits of experimental accuracy, these results confirmed the correlation obtained in the previous section. It is worth mentioning that since Prandtl number for air is very near to unity the differences between the thermal and velocity boundary layers will be small, hence it was not considered necessary to establish the velocity boundary layer thickness experimentally.

10.17 Conclusions

Correlation of Nusselt and Reynolds numbers showed that the experimental results at three different compression ratios and three different speeds can be represented by the following relationship

\[ N_u = C R_e^{0.8} \]

where 'C' varies between 0.0276 to 0.0184. Thus it has been shown that the convective heat transfer in the engine cylinders can be predicted by using the existing heat transfer data for flat plates.

It has also been shown that agreement between experiment and theory, for heat transfer through a developing boundary layer on a flat plate, is within the limits of experimental accuracy.

In the course of this investigation it was necessary to obtain the instantaneous gas velocity, metal surface temperature and gas temperature. The reader is referred to the relevant chapter for detailed conclusions.
CHAPTER 11

FUTURE WORK
CHAPTER 11

11. Future Work

Since the purpose of the present study was to investigate the use of existing heat transfer data for unsteady heat transfer prediction in I.C. engines, it is obvious that such a system can only be used if the gas motion in various parts of the engine cylinder can be calculated reliably. If the gas motion can be related to the engine geometry and running conditions, this would result in providing a method whereby the unsteady heat transfer calculations can be carried out at design stage of the engine. Also such a method of calculation could be incorporated in engine cycle calculations and in the simulation of engine performance on digital computers with more reliable estimates of the instantaneous heat transfer.

Quantitative information on the gas motion inside the engine cylinder is rather scant at the present moment. This is mainly due to the difficulty of making quantitative measurements inside the engine cylinder. In the present study a method of measuring the mean gas velocity in a simplified pre-combustion chamber was developed. The next step should be to apply this method inside the engine cylinder on a motored engine. A qualitative picture of the gas flow inside the engine can be obtained by the use of Schlieren photography as demonstrated by Taylor. A study of these pictures should indicate the position and orientation of the hot wire anemometer probe to measure the mean velocity in a given direction. It can reasonably be assumed that the flow inside the engine cylinder would be three-dimensional, therefore it may be necessary to use three mutually perpendicular hot wire anemometer probes to measure it. Gas velocities thus obtained can then be compared with those obtained by analytical methods such as proposed by Knight and modifications made to the analytical method if found necessary. All these tests can be carried out on a motored engine because the task of gas velocity measurement is greatly simplified in the absence of combustion. The next step
would be to assess the effects of combustion on the mean gas velocities, both qualitatively and quantitatively. Evidence exists that, on the engine used in the present study, the flow regime in the cylinder remains unaltered in the presence of combustion. This is based on a study of Schlieren photographs of motored and fired cycles. A study of these photographs has shown that although the basic pattern of flow in the cylinder remains unaltered there may be a difference in magnitude of the mean gas velocity. This phenomenon can be studied by the use of hot wire anemometers to measure the gas velocities in the presence of flames. The main difficulty would be to produce a suitably rugged sensing probe which would withstand the high temperatures, but here quartz coated platinum or platinum film on quartz may be experimented with.

Once a reasonable estimate of gas motion in the presence of combustion is obtained, the forced convective heat transfer may be calculated by the use of existing heat transfer data on pipes or flat plate. Where appropriate the existing heat transfer data in the case of swirling flow in pipes or jet impingement may be utilized for the prediction of heat transfer in the engine cylinder. The next step would be to investigate the radiative component of the heat transfer. Thus a comprehensive picture of the unsteady heat transfer in I.C. engines can be built up. It is realised that the amount of work involved is very great therefore it is very important to resort to automatic data logging or computer on-line operation of the engine.

In the present study all the traces were evaluated by photographing the required traces and reading off the ordinates. This involved some 250 ordinates for one set of results. Since it was necessary to analyse a large number of traces, this proved extremely laborious. One further disadvantage of this procedure is to introduce unavoidable
measurement errors. Ideally an on-line system similar to the one proposed by Smythe would be extremely useful but the cost of installing such a system would be too high, therefore a simplified system of multi-track magnetic tape recording may be used. The recorded traces could then be digitised by playing back at low speed.

A further improvement as regards heat flux calculation could be the development of an electrical analogue network as discussed in Chapter 2 to give the instantaneous heat flux directly from the metal surface temperature. This would accelerate the process of obtaining the instantaneous heat fluxes considerably. A further extension of the heat transfer investigation could be to measure the unsteady heat flow to those engine components which are normally subjected to high thermal loading, for instance piston, fuel injector nozzles, exhaust valves and valve seats. The information thus obtained would be of great value for engine design.

Some aspects of the work reported here could be investigated further. For instance the difference in heat transfer during compression and expansion stroke could be investigated further. The phenomena of negative heat transfer in the presence of a positive temperature difference and vice versa should be investigated further. This can only be achieved if the accuracy of heat flux measurement is known. This would involve testing the surface thermocouple in known transient heat fluxes.

The boundary layer and gas temperature profile approaches discussed in the last Chapter should yield useful results to give a better understanding of flow inside the engine cylinder.
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APPENDICES
APPENDIX 4.1

Details of DYMEC - DY 2460 A Amplifier (Ref. )

**GAIN** 10, 30, 100, 300, 1000 fixed settings.

Adjustment ± 2%

**DC Gain Accuracy**

- X 30 ... ± 0.5%
- X 100 ... ± 0.5%
- X 300 ... ± 1.0%
- X 1000 ... ± 1.0%

**DC Gain Stability**

± 0.01% per week at constant temp.

± 0.01%/°C max. temp. Coeff.

**DC Linearity**

± 0.01% at any gain setting

**Zero Drift**

- Constant temp ... 1 μV/week max.
- Temp. Coeff. ... 0.5 μV/°C max.

(2 hr. warm up)

**DC Output Capability**

Voltage ± 10 V ... dc to 10 kc

**Chopper Frequency**

190-300 c.p.s.
CALCULATION OF END CONDUCTION ERROR

(a) 1100 r.p.m. case for 5 µ Tungsten wire

\[ \omega = \frac{1100 \times 2\pi}{60 \times 2} = \frac{36.6\pi}{2} \text{ rads/sec.} \]

\[ = 18.3\pi \text{ rads/sec} \]

\[ \tau = \text{Time constant} = \frac{\rho_w \cdot d \cdot c}{4h} \]

\[ L = \text{length of the wire in cm} = 16 \text{ cm.} \]

\[ \rho_w = \text{density of the wire material} = 19.3 \text{ g/cm} \]

\[ d = \text{diameter of the wire} = .0005 \text{ cm.} \]

\[ c = \text{Specific heat of tungsten} = .034 \text{ cals/cm}^0/\text{C} \]

\[ h = \text{Heat transfer coefficient} = .25 \text{ cals/cm}^2/\text{C}^0/\text{sec} \ (\text{assumed}) \]

\[ = \frac{19.3 \times .005 \times .034}{4 \times .25} = .33 \text{ m sec.} \]

\[ \phi = \arctan (\omega \tau) = \arctan (18.3 \times .33) \times 10^{-3} \]

\[ = \arctan (18.9) \times 10^{-3} = \arctan (.0189) \ 1^0 \]

End conduction according to Bensen\(^{(4)}\) is

\[ \Delta T_{\text{cond}} = \frac{T_0}{\sqrt{2\ln (1 + \omega^2 \tau^2)}} \left[ \omega \tau \cos (\omega \theta + \phi/2) - \sin (\omega \theta - \phi/2) \right] \]

\[ \omega \theta = \arctan \left[ -\frac{\cos \phi/2}{\sin (3\phi/4)} \right] \]

\[ = \arctan \left[ -\frac{\cos \frac{1}{3}}{\sin \frac{3}{4}} \right] \]

\[ = 180^0 \]
\[
\Delta T_{\text{cond}} = \frac{T_0}{12.1} \left( 0.188 \right) \\
= T_0 \times 0.155
\]

or \( \Delta T_{\text{cond}} = 1.55\% \) of the temperature fluctuation.

(b) **Measurement Errors**

The temperature of the recording element is calculated from the following relationship:

\[
T = \frac{R_w}{R_0} - 1 / \alpha
\]

where

- \( T_w \) is the temperature of the wire
- \( T_0 \) is a reference temperature
- \( T = (T_w - T_0) \)
- \( \alpha \) is the temperature coefficient of resistance. The resistance could be measured down to .001 ohms. Contact resistances were compensated. Therefore the maximum variation in the values of \( R_w \) and \( R_0 \) was .001 ohms. Maximum error in the measured value of \( T \) will occur when this is positive for \( R_w \) and negative for \( R_0 \). Therefore the recorded temperature \( T' \) is given by

\[
T' = \left[ \frac{R_w + .001}{R_0 - .001} - 1 \right] / \alpha
\]
\[ T' - T = \frac{1}{\alpha} \left[ \left( \frac{R_w + R_0}{R_o - 0.001} \right) - \left( \frac{R_w - R_0}{R_o} \right) \right] \]

\[ = \frac{0.001 (R_o + R_w)}{\alpha} \]

Substituting some typical values we have

\[ R_o = 5.876 \Omega \quad \quad \quad \quad \quad \quad R_w = 11.700 \Omega \]

\[ (T' - T) = \frac{0.001 (5.876 + 11.700)}{\alpha} = \frac{0.001 \times 17.576}{\alpha} = \frac{0.0033}{\alpha} \]

\[ < 0.5^\circ C \]

Magnitude of error at TDC temperature of 380°C

\[ = \frac{0.5}{380} \times 100 = 0.17\% \]

Error Due to Tolerance on the Temperature Coefficient of Resistance

It has been shown earlier that there was a tolerance of approximately ± 1% on the value of measured temperature coefficient of resistance.

Error due to this on the measured temperature is as follows:

\[ T = \left( \frac{R_w}{R_o} - 1 \right) / \alpha \]

\[ T' = \left( \frac{R_w}{R_o} - 1 \right) / \alpha' \]

\[ T' - T = \alpha' \left( \frac{R_w}{R_o} - 1 \right) - \left( \frac{R_w - 1}{R_o} \right) \alpha \]

\[ = \frac{\alpha' (R_w - 1) - (R_w - 1) \alpha}{\alpha \alpha'} \]
\[ T' = \alpha \times 1.01 \left( 1 \pm 1\% \right) \]

\[ \therefore T' - T = \alpha \left[ 1.01 \left( \frac{R_w}{R_o} - 1 \right) - \left( \frac{R_w}{R_o} - 1 \right) \right] \]

\[ = 0.01 \left( \frac{R_w}{R_o} - 1 \right) / 1.01 \times \alpha \]

Inserting the value of \( R_w \) and \( R_o \) for \( T = 51^\circ C \)

\[ T' - T = \frac{0.01 \left( \frac{7.551}{6.512} - 1 \right)}{1.01 \times 0.0033} \]

\[ = 0.482 \, ^\circ C \]

or

\[ \% \text{ error} = \frac{0.482 \times 100}{51} = 9.35\% \]

\[ \therefore \text{Total error due to measurement} = \pm 0.17 + 0.93 = \pm 1\% \]

\[ \therefore \text{Final error} = \text{Measurement errors} + \text{System errors} \]

System errors = Thermal lag + End conduction + Radiation + Recovery temperature

For a 5 \( \mu \) tungsten wire

\[ \therefore \text{System errors} = -3 - 1.55 = -4.55 \]

\[ \therefore \text{Total error} = (\pm 1 - 4.55) \% \]

\[ = \text{Maximum of 6\%.} \]
**APPENDIX A5.2**

It has been shown in chapter 5 that the equation giving the temperature difference between the true gas temperature and the recorded gas temperature is

\[
T_g - T_{w_1} = (T_{w_1} - T_{w_2}) \left[ \frac{1}{\left( \frac{D_2}{D_1} \right)^{1-m} \left( \frac{PCD_2}{4} \right) T_{w_2}^\varepsilon} \frac{\sigma \varepsilon (T_{w_2} - T_{50})}{\left( \frac{PCD_1}{4} \right) T_{w_1}^\varepsilon + \sigma \varepsilon (T_{w_2} - T_{50})} - 1 \right]
\]

Ignoring the radiation term the above equation reduces to

\[
T_g - T_{w_1} = (T_{w_1} - T_{w_2}) \left[ \frac{1}{\left( \frac{D_2}{D_1} \right)^{1-m} \left( \frac{PCD_2}{4} \right) T_{w_2}^\varepsilon} \frac{1}{\left( \frac{PCD_1}{4} \right) T_{w_1}^\varepsilon} \right]
\]

Inserting some typical values for the 5 and 7 micron wires the two wire method give the following value for the true and recorded temperature difference;

\[
(T_g - T_{w_1}) = (640 - 635) \left[ \frac{1}{\left( \frac{7}{5} \right)^{5/4} \left( \frac{7}{5} \right)^{0.9}} - 1 \right] = 11.0 K
\]

Thus it is shown that the 5 micron wire will record a temperature which is approximately 3% lower than the true gas temperature.
A number of workers have presented the heat transfer data for cylinders in cross flow of air in the following form originally proposed by King:

\[ \text{Nu} = A + B \left( \frac{1}{R_e} \right)^{1/2} \]  \hspace{1cm} (A 6.1)

For very large values of \( R_e \) the above relationship may be modified to:

\[ \text{Nu} = B \left( \frac{1}{R_e} \right)^{1/2} \]

The limiting behaviour at large \( R_e \) is characteristic of thin boundary layers where \( Pr \) is constant. The effect of \( Pr \) on the slope of King's law is shown in Fig. A 6.1.

At very low velocities the second term on the r.h.s. of equation A 6.1 may be neglected such that heat loss is solely due to conduction through ends and radiation. This is dependent on the length to diameter ratio of the probe. This dependence is shown in Fig. A 6.2.

In the case of compressible flow the equation A 6.1 is modified to include the effects of Mach number

\[ \text{Nu} = A + B \left( f(M_a) \right) \left( \frac{1}{R_e} \right)^{1/2} \]

The function \( f(M_a) \) has been plotted against \( M_a \) 'the Mach number' by Grant and Kronauer based on the work carried out by Spangenberg. This is shown in Fig. A 6.3.

In order to investigate the response of the hot wire it is necessary to go into the theory of hot wire anemometry. Here the approach used by Hinze will be followed. The sensitivity of a hot wire is defined in the case of constant current system as:
Effect of $Pr$ on King’s Law Slope

$\text{FIG A 6-1}$

Dependence of Conduction $Nu$ on Wire Geometry

$\text{FIG A 6-2}$

$M$ Dependence of King’s Law Intercept

$\text{FIG A 6-3}$

$M$ Dependence of King’s Law Slope

$\text{FIG A 6-4}$
\[ S = \frac{(R_w - R_g)^2}{2IR_g} \frac{B\sqrt{V}}{V} \]  

... (A 6.2)

where

- \( R_w \) is the 'hot' resistance of the wire
- \( R_g \) is the resistance of wire at gas temperature
- \( \bar{V} \) is the mean gas velocity
- \( I \) is the current flowing through the wire.

In deriving the above relationship it was assumed that

1. The wire has no thermal inertia
2. There are no end conduction losses.

In practice the response of the wire is modified because these assumptions are not realistic. First consider the case of a wire with finite thermal inertia. The temperature along the wire is assumed to be uniform or in other words, there is no conduction of heat through the ends. A heat balance on a wire of length 'l' yields:

\[ \dot{1}R_w = (R_w - R_g)(A + B\sqrt{V}) + e \rho c_w \frac{\pi d^2}{4} l \frac{dT_w}{d\theta} \]  

... (A 6.3)

The second term on the r.h.s. gives the heat stored in the wire.

Writing

\[ C_w = \text{Total heat capacity of the wire} = \frac{\rho c_w \pi d^2 l}{4} \]

we have

\[ \dot{1}R_w = (R_w - R_g)(A + B\sqrt{V}) + e C_w \frac{dT_w}{d\theta} \]

\( \frac{dT_w}{d\theta} \) is the rate of change of wire temperature with time.

\( e \) is a conversion factor.

In order to study the response of wire to unsteady conditions, let

\[ V = \bar{V} + v \quad ; \quad R_w = \bar{R}_w + \tau_w \quad ; \quad T_w = \bar{T}_w + \epsilon_w \]

where \( v, \tau_w \) and \( \epsilon_w \) are the fluctuating components.
also assuming that
\[ \frac{v}{V} \ll 1 \quad \text{and} \quad \frac{\tau}{R_0} \ll 1 \]
we have
\[ I \tau = (A + B\sqrt{V}) \tau + (\bar{R}_w - R_0) B\sqrt{V} \frac{v}{2V} + \frac{e C_w}{\alpha R_0} \frac{d \tau}{d\theta} \] \[ \text{.... (A 6.4)} \]

\[ \therefore \tau = R_0 \alpha T_w \]

where
\[ R_0 \] is a reference resistance at \( \theta_0 \)
\[ \alpha \] is the temperature coefficient of resistance

The equation A 6.4 may be written as:
\[ \frac{d \tau}{d\theta} + \frac{1}{M_0} \tau = \phi(\theta) \] \[ \text{.... (A 6.5)} \]

where
\[ \phi(\theta) = -\frac{\alpha R_0 (\bar{R}_w - R_0) B\sqrt{V}}{e C_w} \frac{v}{2V} \]
\[ M_0 = \frac{e C_w}{\alpha R_0 - 1^2 + (A + B\sqrt{V})} \]

\[ = \frac{e C_w (\bar{R}_w - R_0)}{\alpha 1^2 R_0 R_0} \] \[ \text{.... (A 6.6)} \]

The solution of equation A 6.5 is:
\[ \int^{\theta}_{-\infty} \phi(\tau) \exp \left\{-\frac{1}{M_0}(\theta - \tau)\right\} \, d\tau \]
Assuming

\[ V(\theta) = \phi^* \exp(i \omega r) \]

i.e., periodic fluctuations at a frequency \( \omega \) and \( \phi(\theta) = \phi^* \exp[i(\omega \theta - \pi)] \)

The solution then gives

\[ r_w = -\phi^* M \frac{\exp(i \omega \theta)}{1 + i \omega M_v} = r_w^* \exp[i(\omega \theta - \gamma)] \]

\[ \gamma = \arctan(-\omega M) \]

From this it can be shown that sensitivity 's' is given by

\[ S(\omega) = \frac{1}{1 + \omega^2 M_0^2} \frac{1}{2 IR_y} \frac{\sqrt{1 + \omega^2 M_0^2}}{\sqrt{1 + \omega^2 M^2}} \]

\[ \ldots (A 6.7) \]

A comparison between equations A 6.2 and A 6.7 shows that the sensitivity in the second case, i.e., with thermal inertia, is reduced by a factor:

\[ \frac{1}{\sqrt{1 + \omega^2 M_0^2}} \]

Thus for a high \( M_0 \) there will be a greater reduction in the sensitivity of the wire. \( M \) has units of time is known as the 'time constant' of the wire.

An examination of equation A 6.6 shows that \( M \) is directly proportional to the total heat capacity of the wire. From equation A 6.7 it is clear that in order to have high sensitivity either the factor:

\[ \left( \frac{R_w - R_y}{2 IR_y} \right)^2 \]

has to be large or

\[ \frac{1}{\sqrt{1 + \omega^2 M_0^2}} \]

has to be large or \( M_0 \) has to be small.

The factor

\[ \frac{(R_w - R_y)^2 B}{2 IR_y} \]
is dependent on
\[ \frac{\bar{R}_w}{\bar{R}_i} \text{ or } \frac{T_w}{T_i} \text{ and } \ell \bar{R}_d \]; since the heating ratio \( \frac{T_w}{T_i} \)
is limited from considerations of the wire material used the only alternative is
to increase \( \ell \bar{R}_d \), but
\[ \ell \bar{R}_d \propto \frac{l^2}{d} \]
therefore a greater \( \frac{l}{d} \) ratio will give higher sensitivity or a small diameter
wire will be more sensitive. This also gives a smaller heat capacity and hence
a small 'M' which is directly proportional to the heat capacity \( (\alpha l d^2) \).
Therefore it is clear that a reduction in diameter of the sensing wire will greatly
improve the response of the element.

The Cooling Effect of the Wire Supports

The supports are generally large in comparison with the wire and since
these are at a lower temperature than the centre of the wire, heat flows from
the centre to the supports by conduction. This results in a non-uniform
temperature distribution on the wire, consequently modifying the behaviour of
the wire.

Time Constant for the Case with End Conduction and Thermal Inertia

Following Hirpe:
\[
\bar{R}' = (\bar{R}'_w - \bar{R}'_g) (A + B \sqrt{\nu}) + \rho c_w \frac{\pi d}{4} \frac{d^2}{d \theta} e K_\pi \frac{d}{d x^2} \frac{\partial T}{\partial x^2}
\]
\[ \text{..... (A 6.8)} \]
where \( \bar{R}'_w \) and \( \bar{R}'_g \) are the resistance of a unit length of wire at temperatures
\( T_w \) and \( T_g \). Writing
\[ \bar{R} = \bar{R}' + r' \]
and \( V = \overline{V} + v \) the equation reduces to

\[
\frac{d\nu'}{d\theta} = -eK_w \frac{\partial^2 \nu'}{\partial \chi^2} + \alpha R_w \frac{A + B \sqrt{V} - I}{R_w \frac{P_c}{4} d^4} \nu' = - \frac{\alpha R_w (R_w' - R_w)}{R_w} B \sqrt{V} \frac{\nu}{2V} \quad \text{..... (A 6.9)}
\]

Betchov has studied the above equation in great detail and gives the following time constant for this case:

\[
M = \frac{e C_w}{\alpha R_w (A + B \sqrt{V})} \frac{R_w'}{R_w} \left[ 1 - \frac{(R_w'/R_w)}{(\tanh \ell^*)/\ell^*} \right] \quad \text{..... (A 6.10)}
\]

where

\[
\ell^* = \frac{\ell}{\ell_c} = \frac{\ell}{2 \ell_c} = \frac{\ell}{\frac{2 \frac{d}{2} e \pi K}{\ell}} \quad \text{..... (A 6.11)}
\]

\( \ell_c \) is defined as the cooling length and takes into account the end effects.

It can also be shown that

\[
\ell = \ell \sqrt{\frac{P_c d}{2 \pi d K M}}
\]

Response of a Wire in Constant Temperature Operation

When this circuit is operated with a transconductance of \( g_{tr} \) any small change \( r_w \) in the resistance of the probe causes a compensating current \( i \) to flow through the circuit. This gives the following relationship:

\[
i = -g_l I + \nu \quad \text{..... (A 6.12)}
\]

Assuming

\[ I = \overline{I} + i \quad \text{and} \quad V = \overline{V} + v \]

Assuming
From the above equation it can be shown that the sensitivity of the circuit is given by the following expression

\[ S_{ct} = \frac{R_w - R_f}{4i} \frac{B \sqrt{\lambda}}{V} \]  \hspace{1cm} (A 6.14)

Thus

\[ S_{ct} = S \left( \frac{1}{2} \frac{R_t}{(R_w - R_f)} \right) \]  \hspace{1cm} (A 6.15)

i.e., the sensitivity under constant temperature operation is \( \frac{1}{2} \frac{R_w}{R_f} \) times the sensitivity under constant current operation, which shows that for \( \frac{R_w}{R_f} \geq \frac{3}{2} \) the sensitivity of constant temperature operation is lower than the sensitivity under constant current conditions.

It can also be shown that the time constant (with end losses reflected) under constant temperature operation is related to the time constant under constant current operation \( M_o \) by the following relationship:

\[ M_{ct} = \frac{M_o}{1 + 2 \left[ (R_w - R_f)/R_f \right] R_w g_{tr}} \]  \hspace{1cm} (A 6.16)

For the present study some of the typical values are:

\[ g_{tr} = 100 \text{ mho} \]

\[ R_w = 6 \Omega \]

\[ R_f = 3.67 \Omega \]

we have:

\[ M_{ct}/M_o = \frac{1}{760} \]
APPENDIX 9.1

PHYSICAL AND THERMAL PROPERTIES OF BRITISH STANDARD EN 8 STEEL

<table>
<thead>
<tr>
<th>Specific Heat</th>
<th>Temp. Range °C</th>
<th>( C_p ) (Mean value)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20-100</td>
<td>.115</td>
</tr>
<tr>
<td></td>
<td>20-200</td>
<td>.119</td>
</tr>
<tr>
<td></td>
<td>20-300</td>
<td>.122</td>
</tr>
<tr>
<td></td>
<td>20-400</td>
<td>.127</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Thermal Conductivity</th>
<th>Temperature °C</th>
<th>Thermal Conductivity cals/cm sec deg C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td>.124</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>.123</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>.115</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>.109</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>.100</td>
</tr>
</tbody>
</table>

Density = 7.8 gms/cc.

Typical Composition of EN 8 Steel

<table>
<thead>
<tr>
<th>Element</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>.35 - .45</td>
</tr>
<tr>
<td>Silicon</td>
<td>.05 - .35</td>
</tr>
<tr>
<td>Manganese</td>
<td>.60 - 1.0</td>
</tr>
<tr>
<td>Sulphur</td>
<td>.06</td>
</tr>
<tr>
<td>Potassium</td>
<td>.06</td>
</tr>
</tbody>
</table>
Gas Pressure Measurement

The pressure pick-up used in the present study has been described fully by Lyn and others. It was a C.A.V. strain-gauge transducer. It consists of a steel strain tube around which are placed the strain and temperature windings. The strain assembly is supported by a thin diaphragm which reacts to the pressure being recorded. The whole system is air cooled by air at a pressure of 40 lb/sq.in.

The pick-up was mounted such that the sensing diaphragm was flush with the face of the pre-combustion chamber side plate. Static calibration of the pressure pick-up was carried out by using a Budenberg dead weight tester. The calibration curve is shown in Fig. A.10.1.
PRESSURE PICK-UP CALIBRATION CURVE.

FIG. A-10-1
APPENDIX 10.2

COMPUTER PROGRAMME SYMBOL

Input Data for Heat Flux Calculations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>Constant = 2.0 for four stroke cycle</td>
</tr>
<tr>
<td>X</td>
<td>Distance into the wall, i.e., wall thickness (cm)</td>
</tr>
<tr>
<td>CON</td>
<td>Thermal conductivity of metal</td>
</tr>
<tr>
<td>D</td>
<td>Diffusivity of metal</td>
</tr>
<tr>
<td>OD</td>
<td>Output interval in degrees</td>
</tr>
<tr>
<td>R</td>
<td>Engine R.P.M.</td>
</tr>
<tr>
<td>Tx</td>
<td>Metal temperature on the outside °C</td>
</tr>
<tr>
<td>TMIN</td>
<td>Minimum temperature on the inside surface °C</td>
</tr>
<tr>
<td>SC</td>
<td>Metal surface temperature scale °C/cm</td>
</tr>
<tr>
<td>GTS</td>
<td>Gas temperature scale °C/cm</td>
</tr>
<tr>
<td>TMIN</td>
<td>Minimum gas temperature (cold balance temperature) °C</td>
</tr>
<tr>
<td>COMP</td>
<td>Compression ratio</td>
</tr>
<tr>
<td>KORD</td>
<td>Number of ordinates of the surface temperature trace</td>
</tr>
<tr>
<td>MRT</td>
<td>Constant = 1 if reconstituted surface temperature desired in the output; = 2 if not desired</td>
</tr>
<tr>
<td>MVEL</td>
<td>Constant = 1 if gas velocity calculation desired; = 2 if not desired</td>
</tr>
<tr>
<td>MLIN</td>
<td>Constant = 1 if lineariser used for the gas velocity output record; = 2 if not used</td>
</tr>
<tr>
<td>NRUN</td>
<td>No. of runs required for calculation</td>
</tr>
</tbody>
</table>
NVEL

Constant = 1 if heat flux and gas velocity calculation desired; = 2 if only gas velocity calculation desired

### Gas Velocity Calculation Input Data

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Diameter of wire in cm.</td>
</tr>
<tr>
<td>Z</td>
<td>Length of wire - cm.</td>
</tr>
<tr>
<td>VIS</td>
<td>Viscosity of air at 273 °K</td>
</tr>
<tr>
<td>TO</td>
<td>Ambient temperature °C</td>
</tr>
<tr>
<td>RW</td>
<td>Hot resistance of the wire in Ω</td>
</tr>
<tr>
<td>ELP</td>
<td>Temperature coefficient of resistivity Ω/°C</td>
</tr>
<tr>
<td>M</td>
<td>Number of ordinates fed in for the gas velocity trace.</td>
</tr>
<tr>
<td>SP</td>
<td>Scale for the pressure diagram lb/sq.in./cm.</td>
</tr>
<tr>
<td>ST</td>
<td>Scale for the gas temperature diagram °C/cm</td>
</tr>
<tr>
<td>SV</td>
<td>Scale for the gas velocity output record volts/cm</td>
</tr>
<tr>
<td>DP</td>
<td>Characteristic length dimension for the calculation of dimensionless numbers</td>
</tr>
<tr>
<td>TS</td>
<td>Temperature of wire supports</td>
</tr>
<tr>
<td>TGM</td>
<td>Gas temperature at cold balance of the wire °C</td>
</tr>
<tr>
<td>MLIN</td>
<td>Constant = 1 if lineariser used; = 2 if not used</td>
</tr>
<tr>
<td>VOS</td>
<td>Voltage for lineariser zero output - volts</td>
</tr>
<tr>
<td>VINS</td>
<td>Lineariser input voltage</td>
</tr>
<tr>
<td>VO</td>
<td>Lineariser reference voltage</td>
</tr>
<tr>
<td>AXP</td>
<td>Exponent for the lineariser</td>
</tr>
<tr>
<td>TG</td>
<td>Gas temperature record ordinates cm</td>
</tr>
<tr>
<td>P</td>
<td>Pressure trace ordinates cm</td>
</tr>
<tr>
<td>BV</td>
<td>Voltage drop ordinates cm</td>
</tr>
</tbody>
</table>
COMPUTER PROGRAM FOR THE CALCULATION OF NUSSELT AND REYNOLDS NUMBERS THROUGH THE ENGINE CYCLE.

*FORTRAN NO5, HASAN
MASTER NO5

C HASSAN DEPARTMENT OF MECHANICAL ENGINEERING

C HEAT FLUX AND HEAT TRANSFER COEF. PROG H2

DIMENSION T(73), ALP(36), BET(36), ET(73), H(73), Q(73), UN(73), TDIF(73)
DIMENSION V(38), HU(38), BV(38), CUR(38), TG(38), DT(58), P(38), RT(73)

INTEGER KORD, MRT, NVEL, MLIN

EXP(X) = EXP(X)
LOG(X) = ALOG(X)
SIN(X) = SIN(X)
COS(X) = COS(X)
ATAN(X) = ATAN(X)

C COUNT = 0
C COUNT = COUNT + 1
READ (1, 40) KORD, MRT, NVEL, MLIN
WRITE (2, 40) KORD, MRT, NVEL, MLIN

DO FORMAT(14, 51X)
READ (1, 6500) KORD, NVEL

DO FORMAT(213)
C READ TO (400, 6503), NVEL

DO FORMAT(15F5.2/1SF5.2/15F5.2/15F5.2/13F5.2)
READ (1, 1000) (CT(I), I=1,73)
WRITE (2, 1000) (CT(I), I=1,73)

DO FORMAT(15F5.2/15F5.2/15F5.2/15F5.2/13F5.2)
READ (2, 402) R, COMP

DO FORMAT(5X, F7.1, 12X, 10HR, P, M CASE, 5X, F4.1, 2X, 17HCOMPRESS RATIO//1)

C CALCULATION OF METAL SURFACE TEMP. FROM ORDINATES.
DO 700 1=1, KORD

700 CONTINUE
DO 17 1=1,73

17 CT(I) = CT(I) + GTS + TMI + SC * CT(I)

SUM = 0.0

C CALCULATION OF MEAN METAL SURFACE TEMP.
DO 2 I=1, KORD

2 SUM = SUM + CT(I)

SMEAN = SUM / KORD

WRITE (2, 15) SMEAN

C FORMAT(10X, 13HMEAN TEMP. = E14.7)
INT = 360. / 90
AK = 2. / E
OMEGA = AK * 3.1415927
C1 = COSF(OMEGA)
SK = SINF(OMEGA)
C = C1
S = SK

LH = (KORD - 1) / 2
WRITE (2, 1100)

1100 FORMAT(43X, 16HFOURIER COEFS. // 21X, 8NHARMONIC, 12X, 4HALPA, 7X, 4HALPHA, 7X, 2HALPHA, 7X, 4HALPHA, 7X, 2HALPHA, 7X, 4HALPHA, 7X, 2HALPHA, 7X, 4HALPHA, 7X, 2HALPHA, 7X, 4HALPHA, 7X, 2HALPHA, 7X, 4HALPHA, 7X, 2HALPHA, 7X, 4HALPHA, 7X, 2HALPHA, 7X, 4HALPHA, 7X, 2HALPHA, 7X, 4HALPHA, 7X, 2HALPHA, 7X, 4HALPHA, 7X, 2HALPHA, 7X, 4HALPHA, 7X, 2HALPHA)
CALCULATION OF FOURIER COEFFICIENTS FOR THE METAL SURFACE TEMPERATURE.

DO 6 L=1, LHR
  U2=0.
  U1=0.
  T=KORD
  DO 4 K=2, KORD
  U0=(T(I)+2.*U1*C-U2)
  U2=U1
  U1=U0
  I=I-1
  ALP(L)=(T(I)+U1*C-U2)*AK
  BET(L)=UT*AK*U
  C2=C1*C-S1*C
  C=C2
  6 WRITE(2,13)L, ALP(L), BET(L)
  13 FORMAT(21X, I3, 2(10X, E14.7))
ENDFILE 2
GO TO (27, 29), MRT
27 THET=0.
WRITE(2, 32)
32 FORMAT(79X1H-)
WRITE(2, 32)
30 FORMAT(20X, 13HSURFACE TEMP., 8X, 19HR Reconstituted Temp.)
RECONSTITUTION OF THE METAL SURFACE TEMPERATURE.
25 DO 21 L=1, 73
  RT(L)=0.
  22 DO 23 K=1, LHR
  K=K
  23 RT(L)=RT(L)+ALP(K)*COSF(THET+W)+BET(K)*SINF(THET+W)
  THET=THET+OMEGA
  RT(L)=RT(L)+SMean
  21 CONTINUE
  29 WRITE(2, 26) I, RT(L)
  26 FORMAT(17X, 2(10X, E14.7))
ENDFILE 2
2A CONTINUE
  INT=INT+1
  40=73
WRITE(2, 1201)
201 FORMAT(79X1H-)
WRITE(2, 1200)
200 FORMAT(10X, 10HCrank Deg., 10X, 9HHeat Flux, 12X, 15HHeat Tran. Coef., 9X
  110HNum. Number, 17X, 10HTemp. Diff.)
  U=4.2831853*(P/60.)
CALCULATION OF THE METAL SURFACE TEMPERATURE GRADIENT, HEAT FLUX, HEAT TRANSFER
COEFFICIENT, NUSSELT NUMBER AND GAS METAL TEMPERATURE DIFFERENCE.
DO 9 J=1, INT
  IF (J-400) 300, 300, 400
  300 AM=1
  A=OD3.1415927*A/(180.*F)
  W(J)=0.
  30 DO 10 K=1, LHR
  10 AP(K)=ALP(K)+BET(K)
  BF(K)=BET(K)-ALP(K)
  W(J)=H(J)+CGH*U*(AP(K)*COSF(B)+BF(K)*SINF(B))
H(J) = H(J) * CON * (SMEAN - TX) / X
H(J) = H(J) * 2.04
3(J) = (H(J) / GT(J)) - SMEAN
UN(J) = Q(J) * .50 / (5.87 * (GT(J) * SMEAN) / 2., * 273.) * .748 / 10. * .8
V = A * 0
TDIF(J) = GT(J) - SMEAN
WRITE(2, 14) Y, H(J), O(J), U(J), TDIF(J)
14 FORMAT(11X, F6.1, 4(10X, E14.7))
ENDFILE 2
400 CONTINUE
HOT ANEMOMETER CALIBRATION UNDER OPERATING CONDITION
PROG NO. V
001 READ (1, 6002) D, VEL, TO, RW, EL, M, SP, ST, SV, DP
WRITE(2, 6002) D, VEL, TO, RW, EL, M, SP, ST, SV, DP
002 FORMAT(2F6.4, E12.4, F6.1, F6.2, E12.4, F6.1, 2F5.2)
READ (1, 6003) TS, VEL, HE, TGM
WRITE(2, 6003) TS, VEL, HE, TGM
003 FORMAT(2F6.1, F5.2, F5.1, F6.1)
READ (1, 6004) P(I), I = 1, M
WRITE(2, 6004) P(I), I = 1, M
004 FORMAT(15F5.2/15F5.2/7F5.2)
READ (1, 6005) TG(I), I = 1, M
WRITE(2, 6005) TG(I), I = 1, M
005 FORMAT(15F5.2/15F5.2/7F5.2)
READ (1, 6006) RV(I), I = 1, M
WRITE(2, 6006) RV(I), I = 1, M
006 FORMAT(15F5.2/15F5.2/7F5.2)
WRITE (2, 7001)
7001 FORMAT(/)
GO TO (6007, 6010), MLIN
6007 READ (1, 6008) VS, VINS, VO, AXP
6008 FORMAT(4F6.2)
CONST = VS / ((VINS / VO)**2 - 1.)**AXP
DO 6009 N = 1, M
CS(N) = VS(N) * SV
6009 CONTINUE
7 = Z2 / 2.
U06 = 34.9 / 10. ** 6
NC 6200 I = 1, M
P(I) = P(I) * S, T 14.7
TG(I) = TG(I) * ST + 273. + TGM
GO TO (6200, 6011) MLIN
6011 BV(I) = BV(I) * SV
6200 CONTINUE
WRITE (2, 6012) Z, D
6012 FORMAT(10X, OHL) LENGTH = F6.4, 6HDIA = F9.7
A = 2. / 3.
R = (3.1415927 * (D ** 2.)) / 4.
ROD = (215. * 2 - (12. * 4.3)) * (.00254 / D) ** 2
ROD*R = (1. + EL * (TO - 273.))
WRITE (2, 6014) ROW, RO, RW
6014 FORMAT(10X, 16HRSITY = E14.7, 5X, 5HR0 = F6.2, 5X, 10HNOT RES = F6.2)
6015 TWE = ((RO - RO) / (ROD = EL)) + TO
CAW = (2.56 + (7.3 * (TW - 54.) / 1000.)) / 10. ** 4.
CSUS = ROWO * (1. + EL * (TS - 273.))
TCWT = (2.23 * TW) / (ROWS * 10. ** 8)
WRITE (2, 6017) CAW, TCWS
```
018 FORMAT(10X,AHCAW = E14.7,10X,HTCWS = E14.7)
WRITE(2,6019) TCWS
019 FORMAT(10X,12H12MIRE TEMP = F6.1///)
WRITE(2,6020)
020 FORMAT(10X,2YN2,4X,8HGAS TEMP,8X,8HGAS PRES,8X,8HR VOLTS,8X,8HT)
12R COEF.,8X,8HGAS VELY)
MAIN CALCULATION FOR THE GAS VELOCITY.EVALUATION OF THE HEAT TRANSFER COEFF.
DO 6026 N=1,N
QG=POD+(1.+ELP*(TG(N)-273.))  
CAG=(2.56+(7.3*(TG(N)-54.)/1000.))/10.**4
N(T(N)=TW-TG(N)
CUR(N)=BV(N)/(2.*RW)
DEN=N(N)+2.31/(96.5*TG(N))
VSY=(VIS-390.)(TG(N)/273.)**1.5)/(TG(N)+117.)
CV=(.1715+.0278B*DEN)**4,1813
HPW=6.
HWN=0
V1=1+(.4*CAW)/(TCWT*D**2.)
MAIN ITERATION,
SOLVING FOR K1.
004 CAYP=HPW*D/CAW-CONST
CAYM=HWN*D/CAW-CONST
CAYP3=ABS(CAY1=CAYP)
CAYM3=ABS(CAY1=CAYM)
CAYP4=(SQRT(CAYP3))**2
CAYM4=(SQRT(CAYM3))**2
01F1=CUR(N)**2/(2.*,1415927*Z*CAW*DT(N))
02=(EXP(2.*CAYP4)-1.)/EXP(2.*CAYP4)+1.)
03=(EXP(2.*CAYM4)-1.)/EXP(2.*CAYM4)+1.)
04=1,-(PG+TCWS*EP)/(RW*TCWT*CAYP4)
05=1,-(PG+TCWS*EP)/(RW*TCWT*CAYM4)
FUP=HPW-01F1-DIFP2
IF (FUP-.000001)8001,8001,8010
8010 CONTINUE
FUP=HWN-01F1-DIFN2
IF (ABS(FUN)-.0000001)8011,8011,8012
8011 H(N)=HWN
GO TO 8013
8012 CONTINUE
HWN=HWN*FUP-HWP*FUN)/(FUP-FUN)
CAYM=HWM*D/CAW-CONST
CAYM3=ABS(CAY1=CAYM)
CAYM4=(SQRT(CAYM3))**2
FM=(EXP(2.*CAYM4)-1.)/EXP(2.*CAYM4)+1.)
DIFM2=1,-(PG+TCWS*EP)/(RW*TCWT*CAYM4)
FM=HWM-DIF1-DIFM2
IF (FUN)8000,8001,8002
8000 HWN=HWM
GO TO 8003
8002 HPW=HWN
8003 IF (ABS(HWP-HWN)-.001)8001,8001,8005
8005 GO TO 8004
8004 H(N)=HWP
8013 V(N)=(H(N)-3.1415927*CAG/(2.6+CV+CAW))**3/(DVSY)**2/(DEN)
6025 V(N)=V(N)+(TW/TG(N)**.33/(2.54*12.)
6026 WRITE(2,6027)N,TG(N),P(N),BV(N),H(N),V(N)
6027 FORMAT(10X,12,2X,4(E12.4,4X),E12.4)
6999 CONTINUE
WRITE(2,6028)
```
**C**

**CALCULATION OF REYNOLDS NUMBER AND ECKERT NUMBER.**

NO 6029 **NM=1,M**

**SHIFT=SMALL+273.-TG(N)**

**DEN=P(N)+2.31/(96.5+TG(N))**

**TG(N) = (TG(N)+SMEAN)/2.**

**VSY=VNS**

**300.*(TG(N)/273.)***

**RE(N)**

**1.5/(TG(N)+117.)**

**ECK(N)=%.5*V(N)**

**2./32.2+DIFT+1400.**

**6029 WRITE(2,6030)N,V(N),RE(N),ECK(N)**

**6030 FORMAT(14X,13.3E14.7,7X)**

**IF (NCOUNT-NRUN)6501,6900,6100**

**6100 STOP 01**

**C INPUT DATA FOR THREE COMPRESSION RATIOS AND THREE DIFFERENT SFPEDS.**

**C DATA 8.5 COMPRESSION RATIO AND 1100 R.P.M.**

**C HEAT FLUX CALCULATION DATA.**

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**C GAS VELOCITY CALCULATION DATA.**

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**C DATA 8.5 COMPRESSION RATIO AND 900 R.P.M.**

**C HEAT FLUX CALCULATION DATA.**

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**DATA 8.5 COMPRESSION RATIO AND 1100 R.P.M.**

**C HEAT FLUX CALCULATION DATA.**

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<td>.121</td>
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</tr>
<tr>
<td>Pressure (lbf/in²)</td>
<td>Flow Rate (gpm)</td>
<td>Power (hp)</td>
</tr>
<tr>
<td>-------------------</td>
<td>----------------</td>
<td>------------</td>
</tr>
<tr>
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<td>600</td>
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</tbody>
</table>

**Notes:**
- Flow rate and power values are approximate and may vary depending on specific operating conditions.
- The pressure values are given in pounds per square inch (lbf/in²).
- The flow rate is measured in gallons per minute (gpm).
- Power consumption is measured in horsepower (hp).

**Additional Information:**
- The data is provided for engineering applications, particularly in the context of fluid dynamics and machinery performance.
### Table 1: Gas Velocity Calculation Data

<table>
<thead>
<tr>
<th>Volumetric Flow Rate (m³/min)</th>
<th>Gas Velocity (m/s)</th>
<th>Gas Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>2.54</td>
<td>0.12</td>
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<td>3.6</td>
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### Table 2: Heat Flux Calculation Data

<table>
<thead>
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<th>Heat Flux Density (W/m²)</th>
<th>Temperature (°C)</th>
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</thead>
<tbody>
<tr>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>250</td>
<td>150</td>
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<tr>
<td>300</td>
<td>200</td>
</tr>
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### Table 3: Gas Velocity Calculation Data

<table>
<thead>
<tr>
<th>Gas Velocity (m/s)</th>
<th>Temperature (°C)</th>
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</thead>
<tbody>
<tr>
<td>1.5</td>
<td>90</td>
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<td>2.0</td>
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<td>2.5</td>
<td>150</td>
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### Table 4: Gas Velocity Calculation Data

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<th>Gas Velocity (m/s)</th>
<th>Temperature (°C)</th>
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</thead>
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<td>3.0</td>
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<td>200</td>
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<td>4.0</td>
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### Table 5: Gas Velocity Calculation Data

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<th>Gas Velocity (m/s)</th>
<th>Temperature (°C)</th>
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<tbody>
<tr>
<td>4.5</td>
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### Table 6: Gas Velocity Calculation Data

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<th>Gas Velocity (m/s)</th>
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<td>6.0</td>
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<td>6.5</td>
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<td>7.0</td>
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### Table 7: Gas Velocity Calculation Data

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<td>7.5</td>
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<td>8.0</td>
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### Table 8: Gas Velocity Calculation Data

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<th>Temperature (°C)</th>
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<td>9.0</td>
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<td>9.5</td>
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<td>10.0</td>
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### Table 9: Gas Velocity Calculation Data

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<th>Temperature (°C)</th>
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<td>10.5</td>
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### Table 10: Gas Velocity Calculation Data

<table>
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<tr>
<th>Gas Velocity (m/s)</th>
<th>Temperature (°C)</th>
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<td>12.0</td>
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### Table 11: Gas Velocity Calculation Data

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<th>Temperature (°C)</th>
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### Table 12: Gas Velocity Calculation Data

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### Table 13: Gas Velocity Calculation Data

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### Table 14: Gas Velocity Calculation Data

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<th>Temperature (°C)</th>
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### Table 15: Gas Velocity Calculation Data

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### Table 16: Gas Velocity Calculation Data

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### Table 17: Gas Velocity Calculation Data

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</tbody>
</table>

- 301 -
APPENDIX 10.3

Error Analysis

Expression generally used for the propagation of errors is as follows:

\[ (\sigma_G)^2 = \left( \frac{\partial f}{\partial M_1} \sigma_{M_1} \right)^2 + \left( \frac{\partial f}{\partial M_2} \sigma_{M_2} \right)^2 + \left( \frac{\partial f}{\partial M_3} \sigma_{M_3} \right)^2 \ldots + \left( \frac{\partial f}{\partial M_r} \sigma_{M_r} \right)^2 \]  \hspace{1cm} (All.2.1.)

where \( G \) is a function of \( (M_1, M_2, \ldots, M_r) \); \( \sigma_G, \sigma_{M_1}, \sigma_{M_2}, \ldots, \sigma_{M_r} \) etc. are the standard deviations of \( G, M_1, M_2 \) and \( M_r \). Assuming a rectangular distribution, i.e., in which no readings occur outside certain limit, + \( \ell \) say, but all readings within these limits are equally probable, we have

\[ \sigma = \frac{\ell}{2\sqrt{3}} = \approx 0.29 \ell \]

In assessing the effect of experimental error on results, consider the final relationship correlating the Nusselt and Reynolds numbers

\[ Nu = \frac{C}{Re} \]

Since all the lines were drawn at a slope of \( \beta \), the effect of the experimental error will show itself in \( C \)

\[ C = f(Re, Nu) \]

\[ \therefore (\sigma_C)^2 = \left( \frac{\partial f}{\partial Re} \sigma_{Re} \right)^2 + \left( \frac{\partial f}{\partial Nu} \sigma_{Nu} \right)^2 \]

thus in order to compute the error in \( C \) it is necessary to know the standard deviations of \( Re \) and \( Nu \). Equation (All.2.1.) can also be applied to calculate the standard deviation in these cases. Since \( Re \) and \( Nu \) are dependent on the measured values of the gas velocity, gas
temperature and surface heat flow the error on these values has to be specified first. It was shown in Chapter 8 that the gas velocity could be measured within ± 12%. The estimated error for the gas temperature measurement was ± 6%. It is difficult to specify the error on surface heat flux measurement and a search of literature did not show any work in which the accuracy of this method of heat flux calculation had been investigated. If it is assumed that heat flux is measured to within ± 1 CHU/sq.ft.sec., the resultant error at the TDC flux values will be approximately ± 7%.

With these figures and application of equation All.2.1. it can be shown that the error in Re and Nu will be ± 13 and ± 8% respectively. For a typical value of Re = 7.3 x 10^6, the value for the standard deviation for C works out to be

\[ \sigma_C = 0.013 \]

which gives the tolerance limits for C to be ± 19%.