Hierarchical risk assessment of water supply systems

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Hierarchical Risk Assessment of Water Supply Systems

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Submitted for the Degree of Doctor of Philosophy from Loughborough University

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Abstract

Water supply systems are usually designed, constructed, operated, and managed in an open environment, thus they are inevitably exposed to varied uncertain threats and conditions. In order to evaluate the reliability of water supply systems under threatened conditions, risk assessment has been recognised as a useful tool to identify threats, analyse vulnerabilities and risks, and select proper mitigation measures. However, due to the complexity and uncertainty of water supply systems and risks, consistent and effective assessments are hard to accomplish by using available risk techniques. With respect to this, the current study develops a new method to assess the risks in complex water supply systems by reconsidering the organisation of risk information and risk mechanism based on the concepts of object-oriented approach. Then hierarchical assessments are conducted to evaluate the risks of components and the water supply system.

The current study firstly adopts object-oriented approach, a natural and straightforward mechanism of organising information of the real world systems, to represent the water supply system at both component and system levels. At the component level, components of a water supply system are viewed as different and functional objects. Associated with each object, there are states transition diagrams that explicitly describe the risk relationships between hazards/threats, possible failure states, and negative consequences. At the system level, the water supply system is viewed as a network composed of interconnected objects. Object-oriented structures of the system represent the whole/part relationships and interconnections between components. Then based on the object states transition diagrams and object-oriented structures, this study develops two types of frameworks for risk assessment, i.e., framework of aggregative risk assessment and framework of fault tree analysis. Aggregative risk assessment is to evaluate the risk levels of components, subsystems, and the overall water supply system. While fault trees are to represent the cause-effect relationships for a specific risk in the system. Assessments of these two frameworks can help decision makers to prioritise their maintenance and management strategies in water supply systems.

In order to quantitatively evaluate the framework of aggregative risk, this thesis uses a fuzzy evidential reasoning method to determine the risk levels associated with components, subsystems, and the overall water supply system. Fuzzy sets theory is used to evaluate the likelihood, severity, and risk levels associated with each hazard. Dempster-Shafer theory, a typical evidential reasoning method, is adopted to aggregate the risk levels of multiple hazards along the hierarchy of aggregative risk assessment to generate risk levels of components, subsystems, and the overall water supply system. Although fuzzy sets theory and Dempster-Shafer theory have been extensively applied to various problems, their potential of conducting aggregative risk assessments is originally explored in this thesis.

Finally, in order to quantitatively evaluate the cause-effect relationships in a water supply system, fuzzy fault tree analysis is adopted in this study. Results of this analysis are likelihood of the occurrence for a specific event and importance measures of the possible contributing events. These results can help risk analysts to plan their mitigation measures to effectively control risks in the water supply system.
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Symbols used in Chapter 2

\( P_{s,sys} \) = Reliability of system
\( P_{s|Fi} \) = Conditional system reliability given that the \( i \)th component is operational
\( P_{s|Fi} \) = Conditional system reliability given that the \( i \)th component is failed
\( R \) = Risk
\( P_a \) = Probability of attack
\( P_e \) = Probability of systems effectiveness
\( C \) = Associated consequences
\( v_i \) = Vulnerability of component \( i \) in water systems
\( \alpha_i \) = Accessibility of component \( i \)
\( \gamma_i \) = Degree of exposure of component \( i \)
\( V \) = Vulnerability of the water system

Symbols used in Chapter 3

\( R \) = Risk of object
\( F_s \) = Risk associated with failure state \( s \)
\( H_{sj} \) = Risk of the \( j \)th hazard associated failure state \( s \)
\( S \) = The number of failure states
\( H_s \) = The number of hazards associated failure state \( s \)
\( f(\cdot) \) = Function of aggregating risk levels
\( WS \) = Risk of the water supply system
\( W_s \) = Risk of the subsystem \( s \)
\( s_n \) = The number of subsystems in a water supply system

Symbols used in Chapter 4

\( A \) = Fuzzy set
\( U \) = Classic set
\( A_\alpha \) = \( \alpha \)-cut set of fuzzy set \( A \)
\( \mu_{A}(x) \) = Membership function of fuzzy set \( A \)
\( \mu^L(x) \) = Membership function of likelihood of hazard
\( \mu^S(x) \) = Membership function of severity of hazard
\( \mu^R(x) \) = Membership function of risk
\( C_{A,B} \) = The intersecting degree between fuzzy numbers \( A \) and \( B \)
\( C_{A,B}^w \) = The weighted intersecting degree between fuzzy numbers \( A \) and \( B \)
\( OA_{A,B} \) = The overlap area of fuzzy numbers \( A \) and \( B \)
\( AR_B \) = Area of fuzzy number \( B \)
\( r_{ij} \) = The degree of risk \( j \) belonging to risk grade \( i \)
\( w \) = Weight factors
\( m(A) \) = Basic probability/evidence assignment to set \( A \)
\( P(X) \) = Power set of \( X \)
\( \Phi \) = Null set
\( K \) = Degree of conflict between sources
\( R \) = Risk
\( L \) = Likelihood of hazard
\( S \) = Severity of hazard
\( d(A) \) = Measure of fuzziness of fuzzy set \( A \)
\( J(A,B) \) = Degree of intersection between fuzzy numbers \( A \) and \( B \)
\( C(R_i) \) = The centre of fuzzy risk grade \( i \)

Symbols used in Chapter 5

\( P \) = Probability of top event
\( p_i \) = Probability of basic event \( i \)
\( K_i \) = The \( i \)th minimum cut set
\( N_C \) = The number of minimum cut sets in a fault tree
\( P(K_i) \) = Probability of minimum cut set \( K_i \)
\( p_a \) = Probability of \( \alpha \) cut set
\( p_a^L \) = Lower bound of \( p_a \)
\( p_a^R \) = Upper bound of \( p_a \)
\( d \) = Dependent degree
\( ED(A,B) \) = Euclidean distance between fuzzy numbers \( A \) and \( B \)
\( I \) = Importance measure
\( U \) = Uncertainty importance measure
CHAPTER 1

Introduction

1.1 Introduction

Being one of the most important fundamentals for human living and development, water supply systems have received considerable attention relating to their performance under varied conditions. Water supply systems are usually designed, constructed, operated, and managed in an open environment, thus they are inevitably exposed to varied uncertain threats and hazards (Haestad et al., 2003). Due to the requirements of system safety and reliability, risk assessment has been recognised as a useful tool to identify threats, analyse vulnerabilities and risks, and select mitigation measures for water supply systems.

Risk assessment has been highlighted world-wide by both governments and local managers of water supply systems. In the United States, on 15 July 1996, Clinton issued an Executive Order 13010 establishing the President’s Commission on Critical Infrastructure Protection (PCCIP). This order developed a national strategy for protecting these infrastructures from various threats in order to assure their continued operation (Clinton, 1996). In PCCIP, a water supply system is considered as one of the critical infrastructures. In May 1998, Clinton issued a Presidential Decision Directive 63 (PDD63) that called for a national effort to assure the security of the United States’ increasingly vulnerable and interconnected infrastructures (Clinton, 1998). The Federal Public Health Security and Bio-terrorism Preparedness and Response Act (Bio-terrorism Act) was passed on 12 June, 2002 to evaluate the vulnerability of water supply systems. It requires all community water supply systems that serve 3,300 or more persons to prepare a vulnerability assessment of their systems. The U.S. Environmental Protection Agency (USEPA) requires these water systems to conduct and submit a security assessment. In the United Kingdom, “A Guide to Risk Assessment and Risk Management for Environmental Protection” and “Guidelines for Environmental Risk Assessment and Management”, published in 1995 and 2000 respectively, view risk assessment and management as essential elements of structured decision making.
processes across government (DETR, 1995; DETR et al., 2000). The guidelines set out some basic principles which the Department of the Environment, Transport and the Regions (DETR) and Environmental Agency (EA) would normally intend to use in the assessment and management of risks and which are recommended for all public-domain risk assessments (DETR et al., 2000). The guidelines also provide decision makers, practitioners and the public with a consistent language for risk assessment.

At a local level, reactions were also taken against threats in water supply systems. The director of water services in the UK issued a statement on the request of water companies to perform disaster planning related to policies and procedures to deal with major interruptions or serious breaches in water quality or environmental performance (Vairavamoorthy and Lumbers, 1992). In this statement, it is stated that risk management will be incorporated into the design standards and operating procedures, which will form the basis for the next generation of asset management plans. Yorkshire Water in the UK carried out risk assessment in wide range of fields including risks of discoloration in pipelines, project capital investment, and asset management strategy for a water supply system (Pollard and Guy, 2001). In U.S.A, Santa Clara County’s water resources management agency and Santa Clara Valley Water District have conducted much work related to risk assessment, preparedness and protection of the county’s public water systems (Haestad et al., 2003).

Normally, an effective risk analysis requires basic knowledge about possible risks, characteristics of potential threats/hazards, and comprehensive understanding of the associated cause-effect relationships within the water system. However such an effective risk assessment method to consistently analyse risks, hazards/threats, and their relationships is unavailable so far because of the complexity and uncertainty existing in real-world water supply systems and risk assessment. Risk methods that have been developed were specific to either only subsystems/components of a water supply system, or only one aspect of risk assessment. Furthermore, the existing methods are almost application specific, which thus limits their reusability when they are applied to another system with different configuration and layout. Therefore, there is a need for research to develop a comprehensive framework according to the requirements of effective risk assessment. In the following parts, comments are given to existing risk assessment methods, and objectives of the current research are proposed.
1.2 Risk Assessment of Water Supply Systems

1.2.1 Vulnerable Points of Water Supply Systems

A general water supply system is composed of water sources, raw water transmission pipes, water treatment plants, and water distribution networks. However, these components and subsystems give the greatest opportunities for both natural and human-related influences because most of them are spatially diverse and accessible. With respect to this, researchers have identified the potential vulnerable areas during the process of delivering water from the sources to the customers as (see Figure 1.1): (1) water sources (e.g., river, reservoir, and wells); (2) water treatment plant that removes impurities and harmful agents and makes water suitable for domestic consumption and other uses; (3) water distribution pipelines that deliver clean water on demand to homes, commercial establishments, and industries; (4) storages (tanks); and (5) other facilities (transmission pipes, channels, pumps, valves, etc.) (Haestad et al., 2003). These vulnerable points are the focus of risk assessment.

![Figure 1.1 Elements and vulnerable points in a general water supply system](source Haestad et al., 2003)

1.2.2 Risk Assessment in Water Supply Systems

Risk indicates the potential damage or loss of an asset or a compromise in the function of an engineering system. Risk assessment of a water supply system is usually expressed as a process (Figure 1.2) of identifying threats/hazards, analysing vulnerabilities of components and system, and evaluating risks of components and system (revised based on Li and Vairavamoorthy, 2004a). A risk assessment would be considered effective and comprehensive if this process was conducted completely.
1.2.2.1 Basic Definitions

Although extensive applications of risk assessment have been undertaken in the water supply system, general and uniform definitions of the risk factors (i.e., hazards/threats, vulnerability, and risk) are still unavailable due to the specific characteristics of different risks. Herein, it is necessary to discuss the definitions of hazards/threats, vulnerability, and risk which are used in this thesis.

(1) Threats and hazards

Normally, a threat or a hazard means a rare or extreme event in the natural or man-made environment that has adverse effects to engineering systems, or even human life (Coburn et al., 1994). In historical risk assessment of infrastructure, natural hazards have been considered extensively. Only a limited amount of literature has been concerned with the analysis of human-related threats (Grigg, 2003). With the occurrence of more human attacks to infrastructure, human activities have been more and more highlighted in order to give more comprehensive evaluations of risks. However, in water supply systems, threats/hazards do not necessarily have to be rare or extreme type events. Risk can be high even if hazard is moderate due to high vulnerabilities of components or the system. Therefore, the current study considers threats or hazards as hazardous events that can adversely effect the performance of a water supply system, which includes both natural hazards and human-related threats.
In order to support a quantitative risk assessment of an engineering system, a threat or hazard is also defined in a more specific and mathematical sense to mean the likelihood of the occurrence and severity of the consequence. Likelihood and severity are thus two important indices representing the serious level of a hazard or threat. This indicates that given a serious level of a hazard or threat, its likelihood and severity could be also obtained. This study adopts likelihood and severity to quantitatively represent a hazard or threat to a water supply system.

(2) Vulnerability

In the Oxford English Dictionary, vulnerability is defined as the quality or state of being vulnerable in various senses. This definition has been extended to various applications in engineering systems. For example, in UNDP/UNDRO Disaster Management Manual (Coburn et al., 1994), vulnerability is defined as the extent to which a community, structure, or geographic area that is likely to be damaged or disrupted by the impact of particular damaging phenomenon with a given serious level. In industrial systems, the vulnerability is defined as the properties of an industrial system whose premises, facilities, and production equipment, including its human resources, human organisation and all its software, hardware, and net-ware, may weaken or limit its ability to endure threats and survive accidental events which originate both within and outside the system boundaries (Einarsson and Rausand, 1998). Alternatively, vulnerability analysis is defined as a simple evaluation of where exposure is greatest and access control is weakest (National Security Telecommunication Advisory Committee, 1997). The last definition has been adopted by Ezell et al. (2000a, b) to analyse vulnerability of water utilities.

In this study, vulnerability is defined as a property associated with a component, a subsystem, or the overall water supply system to represent the possibility of being influenced by hazards/threats with given likelihoods and severities. This property is determined by the attributes and conditions of a component, a subsystem, or the overall water supply system, and is varied with time and changes of hazards/threats. Detailed discussions of vulnerabilities associated with components and the water system are available in Section 3.4.1 and Section 3.4.2.

(3) Risk

Although every engineering system always involves an element of risk, definitions of risk are slightly different in different systems. The common trend is to follow Lawrence’s definition (1976) of risk by defining it as a measure of likelihood and severity of negative adverse effects, which is also accepted by ISO (2001). This risk measure thus represents the cumulative effects of frequency and severity of a hazard/threat. Normally, this risk measure is represented as

\[ Risk = \text{Likelihood} \times \text{Severity} \]  

\[ (1.1) \]
It is obvious that the above definition of risk only considers the influences of threats or hazards. However, according to the risk assessment process (Figure 1.2), vulnerabilities of assets are also playing important roles in introducing risks into the water supply system. Therefore, a modified definition of risk is formed as

\[
Risk = (\text{Likelihood} \times \text{Severity}) \times \text{Vulnerability}
\]  

(1.2)

where likelihood and severity represent the characteristics of a hazard or threat; while vulnerability represents the property of an asset that is influenced by the hazard or threat. In this definition, both hazards/threats and assets are explicitly considered.

Further, if the vulnerability was so high that it would be viewed as unity, then the second definition would be reduced to the first one. The current study adopts the second definition of risk, but views vulnerability as unity and proposes future research undertaking the detailed study on vulnerability assessment.

1.2.2.2 Risk Assessment of Water Supply Systems

Historical studies have been extensively undertaken so far on hazards identification, vulnerability assessment, and risk analysis, respectively.

Firstly, both natural hazards and human-related threats have been identified as potential negative factors compromising the performance of water supply systems. With the development of risk assessment, people have learned much from natural hazards about risks to infrastructure systems, and now they are facing new threats from wilful attacks and other human-related activities (Grigg, 2003; Mays, 2004a, b; Haimes et al., 1998).

Secondly, vulnerability assessment has been undertaken as an important part of risk assessment process. Its common elements include characterisation of the water system, identification and prioritisation of adverse consequences, determination of critical assets, evaluation of existing counter measures, and analysis of current risk and development of a prioritised plan (Mays, 2004b). With respect to these elements and complexity of water supplying, different methods have been proposed to assess the vulnerability of different systems (Mays, 2004b).

Lastly, risk assessments have been performed at different scales in water supply systems by historical studies. For example, in order to analyse the influences of contamination events, both deliberate contamination and accidental contamination have been specifically studied in water distribution systems (Mays, 2004a,b). Meanwhile, series studies have been conducted to analyse
the characteristics of pollutant agents compounds, natural hazards influences to water sources, performance of water treatment plant, and reliabilities of water distribution networks, respectively.

However, most of the existing methods consider only one aspect of risk assessment or one part of the whole water supplying process. The methods to organise those methods to give a comprehensive risk assessment have not been specifically studied. The main hurdles of this are complexity of water supply systems and uncertain information associated with risks. Therefore, this research is aimed to develop a framework that can organise various risk information by effectively dealing with complexity and uncertainty in the risk assessment of water supply systems.

1.3 Research Objectives

The general aim of this research is to develop a systematic framework and methodology for an explicit and effective risk assessment of water supply systems. This will be achieved by exploiting the synthesis of risk analysis processes with characteristics of water systems, hierarchical structure analysis, and fuzzy sets-based quantitative methods.

Detailed research objectives are:

(1) to develop qualitative frameworks for risk assessment of water supply systems including
   • a framework representing the hierarchical relationships of components and subsystems in a water supply system,
   • frameworks of aggregative risk assessment for component, subsystem, and the overall water supply system,
   • frameworks of fault trees describing the cause-effect relationships in the water supply system;

(2) to develop quantitative approaches evaluating the above qualitative frameworks. This will include
   • quantitative representation of the risks in the water supply system,
   • an approach consistently evaluating the risk levels of components, subsystems, and the overall water supply system,
   • an approach assessing fault tree for specific risk in the water supply system.
1.4 Thesis Structure

The structure of this thesis is depicted in Figure 1.3 and briefly discussed in the following:

**Chapter 1** analyses the general composition of water supply systems and possible risks associated with them. After that definitions are given to clarify basic concepts associated with risk assessment in this study including hazard/threat, vulnerability, and risk. Finally, it briefly discusses the limitations associated with historical methods, and proposes research objectives for the current study.

**Chapter 2** reviews literature on historical risk assessment techniques and methods dealing with complexity and uncertainty in water supply systems. According to the reviews, comments are obtained to express the limitations associated with historical methods and to propose possible resolutions overcoming these limitations. Then the methodology background of the current study is formed and briefly discussed at the end of the chapter.

**Chapter 3** aims to develop conceptual frameworks for aggregative risk assessment and fault tree analysis of water supply systems. Firstly, it introduces the object-oriented approach and the potential application in organising complex information in water supply systems. Then the hierarchical structure of water supply systems is developed based on the concepts of object-oriented approach. States transition diagrams are used to represent the cause-effect relationships of risks at component level. After that frameworks of aggregative risk assessment are formed based on the hierarchical whole/part relationships of water supply systems and components states transition diagrams. Frameworks of fault trees are established according to the interconnections among components and components states transition diagrams. These two frameworks can give useful information for decision makers in water supply systems.

**Chapter 4** introduces the method to quantitatively evaluate the hierarchical frameworks of aggregative risk assessment developed in Chapter 3. Fuzzy sets theory is adopted here to determine the risk levels of hazards/threats which are at the bottom level of the hierarchical structure. Fuzzy Dempster-Shafer theory, an evidential reasoning method, is adopted as an aggregative method to evaluate risk levels of components, subsystems, and the overall water supply system along the hierarchy.

**Chapter 5** applies fuzzy fault tree analysis to quantitatively evaluate the fault trees developed in Chapter 3. In fuzzy fault tree analysis method, the likelihood of top event and importance measures of contributing factors. Results of this analysis are useful to prioritise the components and hazards for specific risks and help risk analysts to make decisions.
Chapter 6 uses an assumed water supply system to illustrate the risk assessment with the methods developed in this research. Both natural hazards and human-related threats are considered in the assumed example. Risk levels of water contamination and reduced water quantity are obtained respectively for the components, subsystems, and the overall system. In addition, risk contributions and uncertainty contributions are obtained for each hazard and component. These results are useful for risk analysts to obtain a more comprehensive view of risks in the assumed water supply system.

Chapter 7 summarises the work carried out in the current study and outlines the future work. A systematic approach has been proposed and developed to assess the risk of water supply system. However, there are needs for further study to strengthen the method.

Figure 1.3 Thesis structure showing the organisation of chapters
CHAPTER 2

Literature Review

2.1 Introduction

This chapter reviews the techniques of risk assessment used in the water supply system, methods of dealing with complexity, and methods of representing uncertainty, respectively. Based on this discussion are undertaken to analyse the limitations of the existing methods and potential solutions to overcome these limitations. Then a framework is formed at the end of this chapter to illustrated the methods that are used in this study.

2.2 Literature Review

Literature review in this chapter is composed of two parts. The first part is on the methods of assessing risks in the water supply system, which includes hazards assessment, vulnerability assessment and risk analysis. The purpose of this is to identify and discuss limitations associated with these existing methods. The second part is on the methods that are able to deal with complexity and uncertainty factors in the water supply system, which is the methodology basis of this research.

2.2.1 Literature Review on Existing Risk Assessment Methods

Literature review of existing risk assessment methods is composed of three parts in this thesis as threats and hazards analysis, vulnerability assessment, and risk assessment.

2.2.1.1 Threats and Hazards to Water Supply Systems

Extensive studies show that not only natural hazards can negatively influence water supply systems, but human related threats need to be considered in the risk assessment. The frequently mentioned hazards and threats and their possible influences on the water supply system are summarised in Table 2.1.
Table 2.1 Natural hazards and human related threats to a water supply system*

<table>
<thead>
<tr>
<th>Threats and hazards</th>
<th>Consequences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural hazards</td>
<td></td>
</tr>
<tr>
<td>Earthquake</td>
<td>• Pipe breaks</td>
</tr>
<tr>
<td></td>
<td>• Loss of power</td>
</tr>
<tr>
<td></td>
<td>• Structure collapse</td>
</tr>
<tr>
<td>Flooding</td>
<td>• Loss of treatment plant</td>
</tr>
<tr>
<td></td>
<td>• Contamination of distribution system</td>
</tr>
<tr>
<td>Drought</td>
<td>• Water shortages</td>
</tr>
<tr>
<td></td>
<td>• Water quality problem</td>
</tr>
<tr>
<td>Wind</td>
<td>• Flood-induced problems</td>
</tr>
<tr>
<td></td>
<td>• Structure damage</td>
</tr>
<tr>
<td></td>
<td>• Loss of power</td>
</tr>
<tr>
<td>Water born diseases</td>
<td>• Sickness</td>
</tr>
<tr>
<td></td>
<td>• Death</td>
</tr>
<tr>
<td></td>
<td>• Loss of public confidence</td>
</tr>
<tr>
<td>Severe weather</td>
<td>• Frozen pipes,</td>
</tr>
<tr>
<td></td>
<td>• Outages and leaks</td>
</tr>
<tr>
<td></td>
<td>• High water use</td>
</tr>
<tr>
<td>Human-related threats</td>
<td></td>
</tr>
<tr>
<td>Cyber threats</td>
<td>• Physical disruption of SCADA (supervisory control</td>
</tr>
<tr>
<td></td>
<td>and data acquisition) network</td>
</tr>
<tr>
<td></td>
<td>• Attacks on central control system to create</td>
</tr>
<tr>
<td></td>
<td>simultaneous failures</td>
</tr>
<tr>
<td></td>
<td>• Electronic attacks using worms and viruses</td>
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<td></td>
<td>• Network flooding</td>
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<tr>
<td></td>
<td>• Jamming</td>
</tr>
<tr>
<td></td>
<td>• Disguising data to neutralize chlorine or add</td>
</tr>
<tr>
<td></td>
<td>no disinfectant, allowing addition of microbes</td>
</tr>
<tr>
<td>Physical threats</td>
<td>• Physical destruction of system’s assets or</td>
</tr>
<tr>
<td></td>
<td>disruption of water supply is more likely than</td>
</tr>
<tr>
<td></td>
<td>contamination</td>
</tr>
<tr>
<td></td>
<td>• Loss of water pressure compromising firefighting</td>
</tr>
<tr>
<td></td>
<td>capabilities and could lead to possible bacterial</td>
</tr>
<tr>
<td></td>
<td>build-up in the system</td>
</tr>
<tr>
<td></td>
<td>• Potential for creating a water hammer effect by</td>
</tr>
<tr>
<td></td>
<td>opening and closing major control valves and</td>
</tr>
<tr>
<td></td>
<td>turning pumps on and off too quickly, which</td>
</tr>
<tr>
<td></td>
<td>could result in simultaneous main breaks.</td>
</tr>
<tr>
<td>Chemical/Biological threats**</td>
<td>• Health problems, or death of customers</td>
</tr>
<tr>
<td></td>
<td>• Panic</td>
</tr>
<tr>
<td></td>
<td>• Loss of public confidence</td>
</tr>
</tbody>
</table>

*Source: Grigg (2003) and Mays (2004a,b)

**Detailed description of the chemical and biological contaminants are described in Appendix A.

Firstly among the natural hazards, earthquakes, floods, and droughts are three most significant hazards that can cause water utilities damage and great losses (Grigg, 2003). For example, the Kobe’s earthquake of 1995 in Japan had caused over 5,000 deaths and $100 billion in damage with main breaks and damage to pumps and treatment plants (Chung, 1996). The flood of the 1993
Midwestern had caused more than $15 billion in damage and contaminated water at 250 drinking water treatment plants (Horsley et al., 1994). In the United States alone between 1980 and 2003 extreme droughts have caused $144 billion in damages (Dai et al. 2004). Other natural disasters such as extreme weather and wind threaten water utilities as well, but not as significantly as earthquakes, floods, or droughts.

Secondly, human-related threats in water supply systems had received little attention, and the water utilities were not implementing mitigation measures to respond to them. However, the situation has changed after 11 September, 2001 (Mays, 2004b). A water supply system of pipes, pumps, storage tanks, treatment units, and the appurtenances such as various types of valves, meters, and other components offers the greatest opportunity for terrorist attacks because of its extensive, relatively unprotected and accessible nature. Some of the major human-related threats include the attacks on the Murrah Building in Oklahoma City in 1995, the U.S. Embassies in Kenya and Tanzania in 1998, the attacks on the World Trade Centre (WTC) Towers in 2001 (Gilbert et al., 2003), and the attacks on water supply system of the U.S. Embassy in Italy (Dreazen, 2003). Recently, it is shown that a concerted effort at all levels of government begin to address issues related to the threat of human-related activities. Articles on this topic also began to appear. Beiley (2001) studied the biological and toxin weapons threat to the United States. Blomgren (2002) discussed the needs of utility managers to protect water systems from cyber-terrorism. Haas(2002) analysed the role of risk assessment in understanding bioterrorism.

2.2.1.2 Vulnerability Assessment of Water Supply Systems

The common elements of vulnerability assessment in water supply systems are viewed as follows (Mays, 2004b):

- characterisation of the water system, including its mission and objectives,
- identification and prioritisation of adverse consequences to avoid,
- determination of critical assets that might be subject to malevolent acts that could result in undesired consequences,
- assessment of likelihood of such malevolent acts from adversaries,
- evaluation of existing counter measures, and
- analysis of current risk and development of a prioritised plan for risk reduction.

Normally, the complexity of vulnerability assessment ranges on the basis of the design and operation of the water systems. With respect to this, several methods have been developed to perform vulnerability assessment (Mays, 2004b).
Risk Assessment methodology for Water Utilities (RAM-W$^{SM}$) was developed in cooperation with the Energy Department’s Sandia National Laboratories with funding from USEPA. RAM-W$^{SM}$ compares system components against each other to determine which components are most critical. In this method, vulnerability is defined as an exploitable security weakness or deficiency at a facility. Vulnerability assessment is performed based on the analysis of characteristics of undesired events, accessibility of the undesired events, security features and policies, protection measures, etc. Results of the assessment are used to generate risk assessment for the components and the system.

The Vulnerability Self-Assessment Tool (VAST) was developed by the Association of the Metropolitan Sewerage Agencies (AMSA) in collaboration with two consulting firms with USEPA funding. This is a software-based system. The method is based upon a qualitative risk assessment. Vulnerability is evaluated on a qualitative scale (e.g., very high, high, moderate, and low) by considering counter measures already in place. Then the results are used to generate criticality ratings based on which the risk levels of assets can be evaluated. This method can be used to prepare for extreme events, respond should extreme events occur, and restore to normal business conditions thereafter.

The National Rural Water association (NRWA) and the Association of State Drinking Water Administrators (ASDWA) with USEPA assistance developed the security Self-Assessment Guide for Small Systems Serving between 3,300 and 10,000. This guide provides an inventory of small water system critical components and general questions of vulnerability assessment. Once the questions have been answered, then the vulnerabilities of components are obtained and prioritisation of action is developed.

ASSET (Automated Security Survey & Evaluation Tool) was developed by NEWWA in conjunction with the USEPA and other private firms. This tool is geared toward systems that serve between 3,300 and 50,000 people (small and medium systems). This is a software-based tool, which was mailed to all New England public water suppliers in June 2003. This tool is a self-guided software program designed to help drinking water systems complete a vulnerability assessment, as well as to improve their security and their responsiveness to a range of threats. The assessment is achieved by completing eight sections, i.e., information collection, identifying mission objectives, determining critical system components, threat assessment, physical security and existing countermeasures, risk analysis, prioritised plan for risk reduction, and the final report.

However, all the above methods are specific to different scales of water supply system, a more generic framework is unavailable so far to assess vulnerability consistently.
2.2.1.3 Risk Assessment of Water Supply Systems

Customers’ requirements play key roles in water management, and thus determine the process of risk assessment. According to the President’s Commission on Critical Infrastructure Protection (1997), three attributes are crucial to water users:

- There must be adequate quantities of water on demand.
- It must be delivered at sufficient pressure.
- It must be safe to use.

The above three attributes could be compromised either by damage of key physical components of the water system or by the presence of contaminants in the raw or finished water supply (Haestad et al., 2003). Consequences associated with these failures are risks of reduced water flow (i.e., risks related to water quantity) and/or contaminated water to customers (i.e., risks related to water quality). Extensive studies have been undertaken to assess the hazards/threats and reliability of components of water supply systems, respectively.

(1) Assessment of hazards/threats

In order to evaluate the influences of pollutants in a water supply system, Deininger and Meier (2000) proposed a method of ranking various agents and compounds in terms of their relative effectiveness in the system. While in order to evaluate the influences of human-related threats, Tidwell et al. (2005) proposed a method of using Markov latent effects modelling.

(2) Risk assessment of water sources

Contamination and reduced water quantity are two concerned problems associated with water sources. Contamination of source water is of concern because contaminants can enter surface or ground water sources and make it difficult removed in the treatment utilities. Reduced water quantity is of concern because customers would not have enough water if this happened. Since water sources are usually influenced by natural hazards, extensive research has been undertaken to specifically study the influences of floods (Lund, 2002; Weiler et al., 2000; Beard, 1997) and droughts (Salas et al., 2005; Shiau and Shen, 2001; Shepherd, 1998), respectively.

(3) Risk assessment of water treatment plant

Water treatment, the primary barrier to prevent contaminants from reaching the customer, may not be effective due to failures inside the treatment plant, and thus introduce contaminants to the distribution system (Haestad et al., 2003). Fujiwara and Chen (1993) analysed the reliability of water supply system by including the reliability of treatment plant operations. Eisenberg et al.
(2001) proposed a method to evaluate the reliability of treatment processes by considering both mechanical reliability and plant performance.

(4) Risk assessment of pipes

As pipelines are the key elements in delivering water, extensive research has been accomplished to study their risk mechanisms due to some specific hazardous factors. These conventional models can be classified into physical-based and statistical-based models. Physical models focus on evaluating the scope and severity of corrosion on the internal and external pipe walls and the estimation of resulting stresses from the loads applied to the water mains. Statistical models predict the likelihood and/or frequencies of pipe breakage using the past pipe breakage data. In developing a statistic model, the component (e.g., a pipe) in a water distribution system is treated as a black box or a lumped-parameter element, and its performance is observed over time (Mays, 2004a,b).

Physical models

To analyse the negative influences of hazardous factors, the following studies have been conducted. Rossum (1969) proposed a corrosion model to predict the metal pipe corrosion rate. Rajani et al. (1996) and Zhan and Rajani (1997) proposed models for estimating frost load on buried pipes in trenches and under roadways respectively. Kiefner and Vieth (1989) developed an analytical model to predict the pressure at which a pipe with a corrosion pit would fail based on experiment and tests on corroded steel pipes. The model assesses the reduction in structural resistance in the presence of corrosion pits. Rajani and Makar (2000) have conducted an experimental study on pit and spun cast iron pipe samples with and without corrosion pits. The model was based on the results from a mechanical test to establish how the dimensions and geometry of corrosion pits influence the residual strength of grey cast iron mains. Randall-Smith et al. (1992) proposed a linear model to estimate the residual service life of water mains under the assumption that a corrosion pits depth has a constant growth rate. The model was developed as a rough screening tool to identify potential problems rather than provide a means to predict a break. Seica and Packer (2004) developed a finite element model to estimate the remaining strength of water pipes based on experimentally obtained material properties.

Furthermore, Ahammed and Melchers (1994) developed a physical probabilistic-based model to estimate the failure probability of steel pipes using the Spangler-Watkins in plane pipe-soil model (Watkins and Spangler, 1958) as their underlying mechanical stress model.
Statistical models

Shamir and Howard (1979) used regression analysis to develop an exponential model for the breakage rate of a pipe as a function of time. Walski and Pelliccia (1982) enhanced the exponential model by incorporating two additional factors (ratio of break frequency with previous breaks to overall break frequency for cast iron and ratio of break frequency for 500 mm diameter to overall break frequency for pit cast pipes) in the analysis. Clark et al. (1982) observed a lag between the pipe installation year and the first break and consequently proposed to further enhance the exponential model and transform it into a two phase model comprising a linear equation to predict the time elapsed to the first break and an exponential equation to predict the number of subsequent breaks.

Kettler and Goulter (1985) found a moderate correlation between annual break rate and pipe age based on a sample of pipe installed within 10 years period, as a result, they suggested a linear relationship between pipe breaks and age. Marks et al. (1985) proposed to use a proportional hazards model (Cox, 1972) to predict water main breaks by computing the probability of the time duration between consecutive breaks. Marks et al. (1987) further developed the proportional hazard model to include a two stage pipe failure process. The early stage was observed with fewer breaks and was represented by the proportional hazard model, while the second stage was characterised by frequent breaks and was represented by a Poisson type model.

Furthermore, Copper et al. (2000) developed a drunk mains burst model to estimate the failure risk of water mains greater than 300mm in diameter. Pelletier et al. (2003) developed a model to predict the evolution of annual number of pipe breaks using survival analysis and to estimate the impact of different replacement scenarios and applied the proposed modelling approach to three case studies.

(4) Risk assessment of water distribution systems

It has become conventional wisdom that water quality can change significantly as water moves through a water distribution system. This awareness has led to the development of water quality/hydraulic models which can be used to understand the factors that affect these changes and to track and predict water quality changes in drinking water network (Mays, 2004b). Historical studies show that both deliberate (Hickman, 1999; Deininger, 2000; Clark and Deininger, 2000) and accidental factors (Craun et al., 1991; Fox and Lytle, 1996; Clark et al., 1996) can cause contaminations in water distribution systems.

Meanwhile studies were also conducted to analyse the influences of physical disruptions in water distribution networks. These studies has been analysed under different loading conditions including
fire demand, broken links, pump failures, power outages, control failure, and insufficient storage capacity (Mays, 2004a). Under such conditions, risk assessment of the distribution system is thus performed on the basis of components failure, which can be determined by the above physical or statistical models, as well as interaction and configuration of components in the water systems. As it is a complex task, various methods have been conducted on this. Some of them are summarised as follow.

**Topologic and hydraulic methods**

Most of the conventional risk or reliability assessments of water distribution network can be classified into two main categories: topological and hydraulic. Topological reliability refers to the probability that a given network is physically connected, given its components’ mechanical reliabilities or failure rates. Associated with this, there are many studies such as Wagner et al. (1988a,b), Shamir (1990) and Ostfeld (2001). In hydraulic methods, reliability refers directly to the basic function of a water distribution network: conveyance of desired water quantities at desired pressure to desired appropriated locations at desired appropriate time. Research associated with this includes Su et al. (1987), Duan and Mays, (1990a); Duan et al., (1990b); Bao and Mays, (1990), Cullinane et al. (1992), and Ostfeld (2001).

**State enumeration method**

State enumeration methods list all possible mutually exclusive states of the system components that define the state of the entire system. In general, for a water distribution system containing $M$ components, each of which can be classified into $N$ (normally $N$ equals 2 to indicate normal or failed states respectively) operating states, there will be $N^M$ possible states for the entire system. Once all the possible system states are enumerated, the states that result in successful system operation are identified and the probability of the occurrence of each successful state is computed. The last step is to sum all the successful or failure state probabilities to yield the system reliability or system risk respectively. Event tree analysis is a typical method that uses this approach (Mays, 1989, 2004a).

**Path enumeration method**

Path enumeration is a very powerful method for system reliability analysis. A path is defined as a set of components or modes of operation which lead to a certain outcome of the system. In system reliability analysis, the system outcomes of interest are those of failed state or operational state. A minimum path is one in which no component is traversed more than once in going along the path. Under this methodological category, the tie-set analysis and cut-set analysis are the two well-known techniques. Detail description of this method is available in Mays (1989, 2004a).
Conditional probability approach

This approach starts with a selection of key components and modes of operation whose states (operational or failure) would decompose the entire system into simple series and/or parallel subsystems for which the reliability or failure probability of subsystems can be easily evaluated (Mays, 1989, 2004a). Then, the reliability of the entire system is obtained by combining those of the sub-systems using conditional probability rules as

\[ P_{s,sys} = P_{s|F_i} \times P_{s,i} + P_{f|F_i} \times P_{f,i} \]  \hspace{1cm} (2.1)

In which \( P_{s,sys} \) is the reliability of system; \( p_{s,i} \) and \( p_{f,i} \) are the probabilities of system function and fail given that the \( i \)th component is operational, \( F_i \), and failed, \( F'_i \) respectively; \( p_{s|F_i} \) and \( p_{f|F_i} \) are the conditional system reliabilities given that the \( i \)th component is operational and failed respectively.

Efficient evaluation of the reliability of a complex system hinges entirely on a proper selection of key components, which generally is a difficult task when the system is large. Furthermore, this method cannot be easily coded for computerisation (Mays, 2004a).

Fault-tree analysis

The major objective of fault tree analysis is to represent the system condition, which may cause system failure, in a symbolic manner. In other words, the fault tree consists of sequences of events that lead to system failure. It is a backward analysis, which begins with a system failure and traces backward, searching for possible causes of the failure. Therefore, a fault tree is always viewed as a logical diagram representing the consequence of the component failures (basic or primary failures) on the system failure (top failure or top event). Before constructing a fault tree, engineers must thoroughly understand the system and its use. One must determine the higher-order functional events and continue the fault-tree analysis to determine their logical relationships with lower-level events. Once this is accomplished, the fault tree can be constructed and evaluated.

Fault tree analysis has been pointed by Dhillon and Singh (1981) as a technique that:

- provides insight into the system behaviour.
- requires to understand the system thoroughly and deal specifically with one particular failure at a time.
- helps ferret out failures deductively.
- provides a visible and instructive tool to designers, user, and management to justify design changes and trade off studies.
• provides option to perform quantitative or qualitative reliability analysis.
• handles complex systems well.
• has available commercial codes.

The potential application of fault tree in water distribution was considered by Mays (1989, 2004a).

2.2.2 Comments on Historical Risk Assessment Methods

Although the existing risk techniques have been successfully used in many applications, they still have different limitations when applied to a comprehensive risk assessment of a water supply system.

2.2.2.1 Limitations and Difficulties

Water supply systems usually have various configurations, scales, and uncertain operational and environmental conditions, which make them complex and introduce uncertainties in risk assessment. Complexity and uncertainty are two main hurdles that limit the extensive applications of those existing methods.

The complexity of water supply systems mainly arises from the composition of a large number of components or subsystems (including water sources, treatment, distribution, etc.) which, in turn, comprise of further sub-subsystems or components (Table 2.2). Firstly, the exact definition of components, subcomponents, and sub-subcomponents depends on the level of details of the required analysis and, to somewhat greater extent, on the level of available data. Secondly, these components depend directly upon each other and as a result effect the performance of one another (Mays, 2004b). This introduces difficulties in establishing cause-effect relationships for specific risk in water supply systems. Furthermore, as discussed in preceding sections, components in a water supply system are vulnerable to both natural hazards and human-caused threats such as extreme weather, chemical/biological contamination, etc. Therefore both knowledge of components and their relationships are important for a thorough understanding of the operation of the overall system. A risk assessment would be effective and comprehensive if it could be consistently performed at both component and the overall system levels. However, this is hard to achieve by existing methods as they usually focused on either one specific part of a water supply system (e.g., water source or water distribution network), or one aspect of risks (e.g., reduced water quantity or contamination).
Table 2.2 Common elements and their hierarchical relationships in a water supply system

<table>
<thead>
<tr>
<th>System</th>
<th>Subsystems</th>
<th>Sub-systems/Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water supply system</td>
<td>Water source</td>
<td>River</td>
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<tr>
<td></td>
<td></td>
<td>Stream</td>
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<tr>
<td></td>
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<td>Reservoir</td>
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<td></td>
<td></td>
<td>Lake</td>
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<td></td>
<td></td>
<td>Groundwater</td>
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<tr>
<td></td>
<td></td>
<td>Well</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Acquirers</td>
</tr>
<tr>
<td>Transmission pipes</td>
<td>Pipe</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Channel</td>
<td></td>
</tr>
<tr>
<td>Water treatment plant</td>
<td>Screen</td>
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<tr>
<td></td>
<td>Mixing tank</td>
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<td></td>
<td>Flocculation basin</td>
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<td></td>
<td>Settling tank</td>
<td></td>
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<td></td>
<td>Sand filter</td>
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<tr>
<td></td>
<td>Disinfection</td>
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</tr>
<tr>
<td>Distribution networks</td>
<td>Pipes</td>
<td>Main line</td>
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<td></td>
<td></td>
<td>Service line</td>
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<tr>
<td></td>
<td>Pumps</td>
<td>Demand nodes</td>
</tr>
<tr>
<td></td>
<td>Junctions/Nodes</td>
<td>Non-demand</td>
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<td></td>
<td>Fittings</td>
<td>Valves</td>
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<td></td>
<td></td>
<td>Isolation valve</td>
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<td></td>
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<td>Directional valve</td>
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<td>Altitude valve</td>
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<td></td>
<td></td>
<td>Air release valve</td>
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<td>Control valve</td>
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<td></td>
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<td>Meters</td>
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<tr>
<td></td>
<td></td>
<td>Hydrant</td>
</tr>
<tr>
<td>Storages</td>
<td>Tanks</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Water towers</td>
</tr>
</tbody>
</table>

The other factor, as important as complexity, is uncertainty of risk assessment. Normally, risk analysts are always finding difficulties in either representing risk information precisely or describing the risk mechanism of complex systems like a water supply system (Ang and Tang, 1984). In a practical water supply system, the sources of uncertainty are various and diverse. Two main uncertainties frequently mentioned by analysts are insufficient data for statistic inferences and vagueness and variations of risk information. Natural hazards usually belong to the former, while human-caused failures are the latter. Data of human error are limited, and the knowledge of analysts about this kind of error is also incomplete or in some degree vague and fuzzy. These uncertainties introduce difficulties in controlling or predicting risks with an acceptable degree of accuracy (El-Baroudy and Simonovic, 2004). Thus the probabilistic theory, which is useful to
express the former uncertainty, will be challenged and questioned when applied to deal with the latter uncertainty. Furthermore it is very difficult or even impossible in many cases to precisely determine the parameters of a probability distribution for a given hazard event due to the existing uncertainties in practice.

Additionally, as engineering risk analysis is a general methodology for the quantification of uncertainty and evaluation of its consequences (Ganoulis, 1994), the first step in any risk analysis is to identify the risk, clearly detailing all sources of uncertainty that may contribute to the risk of failure. Then quantification of the risk is second step by analysing the risk levels of each component and/or subsystem and their contribution to the overall system. For the first step, probability theory alone in traditional risk analysis has limited applicability in representing all types of the uncertain information. For the second step, risk of the overall system is not easily obtained by considering contributions from components and subsystems because of few specific models have been proposed. Therefore, there is a need to propose a new risk assessment framework that is able to overcome these limitations.

2.2.2.2 Proposed Resolutions

(1) Resolutions to complexity

To deal with the complexity, hierarchical structure analysis is one of the promising methods (Haines, 2004). Since the composition of water supply systems is hierarchical in nature (Table 2.2), risk assessment of such systems is also driven by this hierarchical structure reality. Furthermore, the hazards and potential consequences associated with each component can also be simulated in a similar hierarchy. In this kind of hierarchical structure, risk levels of components/systems at a higher level are contributed by risk levels of components/subsystems at relative lower levels. The risk evaluation of the overall water supply system can then be obtained by knowing both the risk information of each basic element at the lowest level and their combination rules. This is one of the methodology basis of aggregative risk assessment in this thesis. Furthermore, as fault tree analysis is a special case of hierarchical analysis and can represent the cause-effect relationships explicitly, it is also adopted in this research.

(2) Resolutions to uncertainty

Two types of uncertainties are considered in this study. One is uncertainty with random characteristics, and the other is uncertainty with vague, fuzzy, and incomplete properties. For the uncertainty introduced by random variables, probability theory is applicable and practical methods are available. For uncertainty brought by vagueness or incomplete data, applications of probability theory are challenged. An alternative method, fuzzy sets theory, introduced by Zadeh (1965), can
be adopted to give a fundamental support for risk analysis (Lee, 1996; Chen, 2003; Sadiq and Husain, 2005). In fuzzy sets theory, the vague information is described by fuzzy number, and risk evaluation thus becomes a process of dealing with fuzzy numbers rather than normal probabilistic numbers from the mathematic point of view. Furthermore, fuzzy sets had been used to effectively represent and analyse human reliability or subjective risk analysis in many studies (Onisawa, 1988, 1996; Utkin and Gurov, 1998; Konstandinidou et al., 2005).

Based on the above comments, there is a need for literature review on application of hierarchical analysis, probability theory, and fuzzy sets theory in risk assessment of water supply systems, respectively.

### 2.2.3 Literature Review on Methods Dealing with Complexity and Uncertainty

#### 2.2.3.1 Methods of Dealing with Complexity

Hierarchical framework method is considered to be straightforward and effective to deal with complexity by many researchers. Hierarchy is so common in practice that General Systems Theory (GST) regards it as an essential organizational principle among all types of systems (Whyte et al., 1969; von Bertalanffy, 1968; Iberall, 1972). In risk assessment, one of the most valuable and critical contributions of hierarchical framework is its ability to facilitate the evaluation of subsystems risks and their corresponding contributions to the risks of the overall system (Haines and Tarvainen, 1981). Particularly, its ability to model the intricate relationships among the various subsystems and to account for all relevant and important elements of risk and uncertainty renders the risk assessment process more tractable, representative, and encompassing (Haines, 2004). In the following sections, literature is reviewed on applications of hierarchical structure to different aspects of risk assessment of water supply systems.

1. **(1) Threats assessment of water supply system**

   To assist water utility industry in vulnerability assessments, a methodology, RAM-W™, is developed (AWWA, 2002) to evaluate risks, $R$, by

   \[ R = P_a (1 - P_e) C \]  

   (2.2)

   where $P_a$ is the probability of attack, $P_e$ is the probability of system effectiveness, and $C$ is the associated consequence. Risk assessment is performed primarily through a process of expert elicitation in which values for $P_a$, $P_e$, or $C$ are quantified according to a structured and defined scale of high, medium, or low.
Herein $P_d(1-P_a)$ is viewed as the part of threat assessment. Although the governing relation for threat assessment is simple, defining the associated terms is difficult, particularly human-related wilful attacks. Therefore, this method is in limited use by utilities because of the lack of data on threats (Grigg, 2003). Furthermore, insufficient experience and data also make it harder to quantify the associated probabilities, particularly in the case of human wilful attacks (Tidwell et al., 2004, 2005). With respect to this, Tidwell et al. (2004, 2005) proposed a Markov Latent Effects modelling method, in which the complex threats in water supply systems are decomposed into more manageable subsystems or decision elements (Figure 2.1). Each decision element identified in the decomposition process represents a single factor influencing the likelihood that a threat will yield its intended consequence. Characteristics of decision elements are evaluated by experts or analysts using qualitative attributes which can be transferred to quantitative measures (e.g., scales from 0, very weak, to 1, very strong). Then weighted aggregation scheme is adopted to obtain the ultimate threats assessment at the top level of the hierarchical structure.

This method can help risk analysts to evaluate system effectiveness (i.e., $P_e$) under the threat of human wilful attacks. However, its applications to evaluations of natural hazards are not explicitly explained. Furthermore, the factor of probability of attack, $P_a$, is not explicitly included in Tidwell’s method.

![Figure 2.1 Hierarchical structure for threat assessment in water systems (Tidwell et al. 2005)](image-url)
(2) Vulnerability assessment

**Vulnerability assessment to external threats or hazards**

To give a risk analysis of municipal water distribution system, Ezell et al. (2000a,b) proposed a method based on evaluation of component vulnerabilities which are assessed in terms of exposure and access control. Vulnerability of a water system is defined as

\[
V = \sum_{i=1}^{n} v_i
\]

where \(v_i\) denotes the vulnerability of a component in the water system, which is determined by

\[
v_i = \alpha_i \gamma_i
\]

where \(\alpha_i\) is accessibility and subjectively scaled in [0,1]; \(\gamma_i\) is the degree of exposure and subjectively scaled in [0,1]. A low vulnerability score for a component is an advantage.

In this method, vulnerabilities for specific components are subjective (0 to 1) and constructed from an attribute scale. For example, in a distribution system, tank 1 is very exposed, say \(\gamma_1=0.7\), due to its prominent location on the installation. However, tank 2 is in a remote area and has significantly less exposure, say \(\gamma_2=0.2\). Tank 1 only has a simple lock preventing attackers access, hence, its accessibility is \(\alpha_1=0.8\).

![Hierarchical structure of vulnerability assessment of water supply system](Ezell et al., 2000a)
In order to quantify the vulnerabilities, a detailed decomposition of a water distribution system is developed (Figure 2.2) using hierarchical methods proposed by Haimes (1981). Based on this hierarchical structure, vulnerability of each component are first analysed by identifying the values of access and exposure respectively with respect to given threats (e.g., human physical damage). Then vulnerability of the subsystem or the overall system is calculated by Equation (2.3). With the values of vulnerabilities, a rank order is obtained for a water supply system.

In this method only access and exposure are identified as the contributing factors to vulnerability, which makes it more suitable to analyse the vulnerability related to external hazards. However, for internal factors like deterioration of pipes due to changes of surrounding conditions, access and exposure might not be the proper indicators to value vulnerability.

_Vulnerability assessment of pipes_

In order to assess the current condition of pipes in water distribution systems, Yan and Vairavamoorthy (2003) introduced a hierarchical model for vulnerability assessment of water pipes. This model explicitly considers most of the indicators affecting water pipes and was established from historical data available in Lei and Saegrov (1998), FCM and NRC (2002), and AWWSC (2002). A hierarchical assessment framework was developed as shown in Figure 2.3, by grouping these factors with respect to their characteristics. In this hierarchical structure, two groups of water pipe deterioration indicators, i.e., physical and environmental indicators, have been selected in this simple example. For physical indicators, basic properties of pipes are considered, which include age, diameter, and material. For environmental factors, surrounding condition, soil condition, and road loading are considered as contributing factors. Result of this method is a fuzzy number representing the condition of a pipe.

![Hierarchical framework for vulnerability assessment of pipes in water distribution network (Yan and Vairavamoorthy, 2003)](image-url)
In this method, some indicators (e.g., pipe age and diameter) are represented as crisp numbers, while other factors (e.g., surroundings, soil, etc.) are described as fuzzy numbers due to their vague nature. Then a fuzzy composite programming approach, which has been used in various areas as a multi-criteria decision making tool (Bardossy and Duckstein, 1992; Hagemeister et al., 1996; Jones and Barnes, 2000; Bender and Simonovic, 2000; Akter and Simonovic, 2005), was used to generate relative pipe condition.

This method proposed a way to assess current conditions in the water pipelines, and has been applied to the assessment of contaminant intrusion into water pipes (Yan, 2006).

(3) Evaluation of risks

**Risks of water supply systems**

Following on president Clinton’s commission (1998), Haimes et al. (1998) reviewed needs and opportunities to reduce risks in water systems to wilful attacks. They developed a hierarchical holographic model (Haimes, 1981), based on the overall risks to water supply systems. 15 major categories are envisioned in this study (Haimes et al., 1998). The categories represent risks to water systems from different perspectives, and each of the categories is further divided into detailed components which are more manageable and easy to evaluate. These 15 categories cover aspects of physical, scope, temporal, maintenance, organisational, management, resource allocation, SCADA, system configuration, hydrology, geography, external factors, system buffers, contaminants, and quality of surface and ground water. Detailed explanations of these categories are available in Haimes et al. (1998). In Haimes’s framework, different categories are considered as different views of the same water supply system. Each category is one perspective of the water supply system. Thus this method identifies as more as possible risks by checking different categories. However, the number of risks is usually too large for a moderate system, which thus requires extra effort to filter unimportant risks.

**Risk of water quality in water distribution systems**

Water quality failures in distribution networks can generally be classified into five major categories (Kleiner, 1998): intrusion of contaminants, regrowth of microorganism, microbial/chemicals breakthrough, by-products and residual chemicals from water treatment plants, leaching of pollutants from system component into water, and permeation of organic compounds from the soil through system components into water.
However quantitative analysis of risks is a difficult task because of different pipes with varied ages, various materials, variable operational and environmental conditions, pipes buried structures, and limited understanding of failure processes. With respect to this, Sadiq et al. (2004), based on the above categories of water quality failures (Kleiner, 1998), proposed a hierarchical model for the evaluation of an aggregative (cumulative) risk of water quality failure in distribution network (Figure 2.4). In this structure, each risk item is partitioned into its contributory factors, which are also risk items, and each of these can be further portioned into lower level contributory factors. The unit consisting of a risk factor is called “parent”, and its contributory factors are called “children”. A risk element with no children is called a “basic risk item” which is evaluated in terms of likelihood of failure event and its consequence. Both the likelihood and the consequence are defined using fuzzy numbers to capture vagueness in the qualitative definitions. These fuzzy numbers were subjectively determined in Sadiq et al.’s (2004) study. Then a multi-stage aggregation was conducted to obtain the risks of water quality.

(4) Hierarchical organisation of simulation models in water system

To reduce the complexity in water distribution system simulation models, White et al. (1999) used a hierarchical method to organise the models (Figure 2.5). In this hierarchical structure, the entire system is viewed as a collection of subsystems. Each of these subsystems is called a ‘Region’, i.e., a collection of continuous water pressure zones. A specific region is further decomposed into
‘Areas’. When a specific area is investigated, it is decomposed into its parts, called the ‘Locals’. Similarly, locals can be further divided into more detailed components such as pipe, pump, valve, etc. which are at the lowest level of the hierarchical structure.

![Hierarchical structure of organising simulations models in a regional water system](white.png)

Different time steps for simulations also change with the hierarchical levels. At the whole system level, its behaviour (e.g., flow rate and water demand) is always observed based on time step of years. Across the levels, weeks are set for ‘Region’, days are set to ‘Area’, and minutes or hours are set for ‘Local’. Based on this, water systems can be analysed at different levels and different time ranges. This hierarchical structure allows detailed modelling of a small portion (component or subsystem in a water distribution network) within the larger system without representing the entire system at the same level of detail.

(5) Other applications of hierarchical method

Beside the above applications, other research has also been extensively performed by using hierarchical structure methods to solve the problems in water systems. For example, fault tree analysis was adopted to study the performance in wastewater treatment plant (Kelley and Allison, 1979), to analyse risks in water treatment plant (Egerton, 1996), and to model the constraints to urban stream enhancements (Hess and Johnson, 2001). Xu and Powell (1991) used a hierarchical framework of supply and storage facilities to analyse the reliability of water supply systems. Qian et al. (2004) used a hierarchical modelling approach to estimate national distribution of chemicals in public drinking water systems.
2.2.3.2 Methods of Dealing with Uncertainty

As pointed by Ang and Tang (1984), there is uncertainty in all engineering-based systems because these systems rely on the modelling of physical phenomena that are either inherently random or difficult to model with a high degree of accuracy. This is certainly applied to water supply systems. Both nature hazards and human-caused threats, as discussed in the preceding chapter, can cause serious damages or the total failures of a water supply system. However, the likelihood of occurrence and potential severity of a hazard or threat is quite uncertain and hard to quantitatively represent. Risk assessment itself is thus a process of analysing uncertain information so that evaluations of system performance can be obtained under different uncertain conditions. With respect to this, probability theory and fuzzy sets theory have been highlighted by many researchers.

(1) Probability theory

The problem of engineering system reliability, including water supply system, has received considerable attention from statisticians and probability scientists. The probabilistic reliability analysis has been extensively studied to deal with the problem of statistic uncertainty in water supply systems.

Mays and Tung (1989) analysed component reliability by using probabilistic failure rate, repair rate, and loading-resistance methods. Probabilistic failure rates, combined with hydraulic theory, provide the basis for reliability analysis of water pipe networks. Major achievements of this type of methods include Su et al. (1987), Bao and Maya (1990), Gupta and Bhave (1994), Yang et al. (1996), Xu and Goulter (1998), and Shinstine et al. (2002).


(2) Fuzzy sets theory

In practice, most of the hazards cannot be controlled or predicted with an acceptable degree of accuracy. Uncontrollable external factors (natural or human-related) also affect the performance of water supply systems. Therefore, the determination of the uncertainties using probabilistic distribution is neither a easy nor a straightforward task. In order to deal with this, fuzzy sets theory has been proposed as an alternative measure.
The applications of fuzzy sets theory have covered various aspects of water supply systems. For water quality assessment and management, Sadiq et al. (2004) used fuzzy sets to represent the likelihood and severity associated water quality risk in water distribution in order to give an aggregative risk analysis for the overall system. Liou et al. (2003) used fuzzy sets theory to evaluate river quality by representing standard river pollution index indicators (e.g., DO, BOD₅, SS, etc.) in terms of fuzzy numbers. Mujumdar and Sasikumar (2002) and Mujumdar et al. (2004) developed a fuzzy optimisation model for the seasonal water quality management of river systems by viewing the occurrence of low water quality as a fuzzy event.

For evaluations of water supplies, Sherstha and Duckstein (1998) and El-Baraoudy and Simonovic (2004) suggested one measure of fuzzy reliability that can be used when both capacity and demand in water resources systems are fuzzy variables. Lee et al. (2000) proposed a method using fuzzy sets to assist decision makers in evaluating various water supply lines with uncertain information and deciding a proper line.


### 2.2.4 Comments on Methods Dealing with Complexity and Uncertainty

Two comments are summarised as follow about the methods dealing with complexity and uncertainty. Firstly, hierarchical framework has been extensively studied and applied in every part of risk assessment of water supply systems, such as threats identification, vulnerability assessment, risk evaluation, etc. Hierarchical framework is effective because it decomposes the complex problem into more manageable subsystems or components, and represents the contributions to overall system by its components and subsystems. Thus it has the ability to perform risk evaluations at both component and system levels.

Secondly, probability theory is more suitable to represent the uncertainties that can be statistically analysed. It can effectively express the statistical uncertainties in water systems. Fuzzy sets theory is more effective to represent the uncertainties introduced by incomplete, vague, ambiguous, descriptive, and subjective data or knowledge. The literature review shows that fuzzy sets theory has been accepted by more and more researchers of water systems due to two reasons. Firstly, it is not always possible to have enough data and historical records for a complex water system in practice. Then risk analysts can only represent some parameters by approximate probability distribution functions. While in other cases such as related to human activities (e.g., wilful attacks on the tank in a water distribution network), it is usually impossible or improper to use probabilities...
representing likelihoods of event occurrences or relative consequences. As an alternative option, fuzzy sets theory is always adopted in these cases. Further, people’s understanding of the system is not complete in most cases. There are not always mathematical models that can be followed, which thus limit the effective use of probabilistic measures. In this case, fuzzy sets based analysis also plays important roles.

However, limitations are still existing in the developed methods, which are summarised as follow:

- The developed methods always focus on one aspect of risk assessment of water supply system. For example, Tidwell’s (2004, 2005) method is on threats assessment of human-related wilful attack; Yan’s (2003) method is only on condition assessment of pipes; while Sadiq’s (2004) method is specifically developed to water quality problems in water distribution networks. All these methods are unable to give a comprehensive framework for risk assessment of water supply systems.

- The developed methods only considers one type of risk in water supply systems. For example, Sadiq’s method (2003) can only analyse water quality problems, but is not able to evaluate the problem with reduced water quantity. Tidwell’s method (2004,2005) is not able to analyse natural hazards in the same framework.

- Some methods that consider system complexity do not consider uncertainty. For example, both Haines (1998) method and White’s (1999) method only effectively deal with complexity of the water supply system. They did not explicitly consider uncertain information associated with the basic parameters in their frameworks.

### 2.3 Framework of Methodology in This Study

With the awareness of the effectiveness of hierarchies in dealing with complexity, this study adopts hierarchies, but based on an object-oriented approach, to represent the relationships in water supply systems, and to develop frameworks for risk assessment. Meanwhile, fuzzy sets theory, evidence reasoning, and fault tree analysis are integrated with these hierarchies to generate quantitative results. This method is composed of four parts (Figure 2.6): (1) hierarchical representation of water supply systems, (2) vulnerability assessment of components and system, (3) aggregative risk assessment, and (4) risk propagations in water systems. These four parts are described briefly as follow.

#### 2.3.1 Object-Oriented Framework for Water Supply Systems

Firstly, object-oriented approach (OOA) is proposed in this research to deal with complexity of water supply systems and to generate hierarchical structure for risk assessment. Object-oriented
approach is a method that represents engineering systems in terms of objects. Every component in a water supply system is viewed as an object, and the overall water supply system is viewed as a network composed of sets of objects that are interconnected with each other. All risk factors about the components are considered as attributes or behaviours of objects. Furthermore, with the generalization and aggregation relationships, object-oriented hierarchical structures can be easily formed to represent the whole/part relationships and interconnections between objects in the water supply system.

### 2.3.2 Object-Oriented Model for Vulnerability Assessment

Practically, the influence of threats to component in water systems is a process in which the component changes its state from normal to failure due to its vulnerability. Vulnerability is the degree that a component can be influenced by threats. This mechanism can be easily described by objects states transition diagrams in object-oriented environment. Each object in a water supply system has different failure states due to the influences of different hazards or threats. Normally the vulnerabilities of an object are different according to the failure states and can be explicitly represented by its states transition diagrams. Then the vulnerabilities of objects can be integrated by following the whole/part structure to produce vulnerabilities of subsystems or the overall water system.

For a quantitative analysis, fuzzy sets theory and aggregation methods can be an option to produce quantitative results for vulnerabilities. However, study on vulnerabilities exceeds the scope of this thesis, and is proposed in further work. According to the risk definition in Chapter 1, vulnerability of any component, subsystem, or the overall water supply system is assumed unity in the following part of this thesis, which will not compromise the usefulness of the current study.

### 2.3.3 Object-Oriented Model for Aggregative Risk Assessment

According to Figure 2.6, aggregative risk assessment is composed of two stages, i.e., the component level and the system level.

Firstly, states transition diagrams of objects describe the relationships between hazards, object failure states, and object risks, which thus provides a hierarchical framework for risk assessment at components level. In this hierarchical framework, risk of object is at top level followed by relative failure states that are at its immediate lower level. Hazards or threats are the bottom level in this framework. This indicates that risks of object are determined by its failure states, which are in turn determined by the threats or hazards directly related to them. This research represents each hazard or threat in terms of its likelihood of occurrence and consequence that are represented by fuzzy
numbers. Risk of a component is thus an aggregative measure that is determined by aggregating the risks of threats or hazards along the hierarchical structure.

Secondly, for the risk assessment at the system level, object-oriented whole/part relationship structure is used to determine aggregative risks of water supply systems. In this hierarchical framework, the water supply system is at the top level, its subsystems and components are at relative lower levels. Therefore, risk of the overall system is an aggregative measure which is contributed by the risks of its subsystems and components along the hierarchical structure.

After the conceptual framework for aggregative risk assessment has been developed, fuzzy sets theory and aggregation method (i.e., evidence theory) are used to produce quantitative evaluations.

![Diagram](image)

**Figure 2.6 Research compositions of the present study**

### 2.3.4 Object-Oriented Fault Tree Analysis

Fault tree analysis is considered in this study to represent the cause-effect relationships in the water supply system. Fault tree analysis, a deductive reliability and risk analysis technique, can answer the question of how the system could produce a failure X (e.g., contamination at some demand nodes). With the help of fault tree analysis, risk analysts will know which component in the system
is more critical and which risk scenario is more possible. Meanwhile risk contributions and uncertainty contributions can also be easily obtained to support selection of mitigation measures and asset management.

However the development of fault trees is still as much an art as a science. This research uses an object-oriented approach to generate fault tree structures via two steps. Firstly, object states transition diagram is used to generate the fault trees at component level. Then, interconnections between components in a water supply system are used to develop fault trees at system level. After fault trees have been constructed, fuzzy fault tree analysis is adopted to obtain quantitative results.

### 2.4 Summary

This chapter proposes the methodology used in this study on the basis of literature review about historical risk assessment, methods dealing with complexity, and methods dealing with uncertainty in water supply systems. An object-oriented approach is adopted in this study to analyse water supply systems and represent the hierarchical structure of risk assessment.

![Figure 2.7 Logic relationships among the methods used in this research](image)
Four frameworks, i.e., hierarchical representation of water supply systems, vulnerability assessment, aggregative risk assessment, and cause-effect relationships, can be developed based on concepts of object-oriented approach. Fuzzy sets theory, aggregation method, and fuzzy fault tree analysis are integrated with these frameworks to generate quantitative evaluation of risks. A diagram illustrated in Figure 2.7 explicitly shows the relationships among object-oriented approach, fuzzy sets theory, aggregation method (i.e., Dempster-Shafer theory), and fuzzy fault tree analysis in this study.
CHAPTER 3

Object-Oriented Risk Assessment of Water Supply Systems

3.1 Introduction

Frameworks of risk assessment are developed in this chapter based on concepts of object-oriented approach and characteristics of water supply systems (Figure 3.1). Firstly, Section 3.2 introduces the basic concepts of object-oriented approach and their potential in dealing with complexity of water supply systems. Then in Section 3.3, object-oriented hierarchy is developed to represent the relationships among components, subsystems, and the overall water supply system. Furthermore, for a component at the lowest level in the hierarchical structure, object states transition diagrams are used to describe its states transitions due to the influences of multiple hazards or threats. After that two kinds of frameworks are developed for risk assessment on the basis of the above object-oriented structure and object states transition diagrams. One is for aggregative risk assessment and discussed in Section 3.4; and the other is for fault tree analysis and discussed in Section 3.5. Aggregative risk assessment is to analyse the risk levels of different objects in a water supply system, i.e., a component, a subsystem, and the overall system. While fault trees are used to describe the cause-effect relationships for a given risk in the system. In this thesis, these frameworks are developed at both component and system levels in order to meet the requirements of a comprehensive risk assessment. Figure 3.1 gives an explicit illustration of the structure of this chapter.
Figure 3.1 Structure of developing conceptual frameworks for risk assessment based on concepts of object-oriented approach
3.2 Object-Oriented Approach

3.2.1 Basics Concepts

Object-oriented approach is a method that can naturally represent real-world entities and phenomena in terms of objects and classes (Booch, 1994; Martin and Odell, 1995, 1998; Embley et al., 1992). In an object-oriented modelling paradigm, object and class are two key concepts with which analysts can effectively manage complex engineering systems. These two concepts are also effective to organise risk information in a water supply system.

(1) Objects

Objects are model constructs used to represent real-world entities which can communicate with one another (Booch, 1994; Martin and Odell, 1998). This communication consists of messages exchanged between objects. Messages represent the transfer of information, materials, or energy. Components (e.g., pipe, pump, etc.) and subsystems (e.g., water source, water distribution, etc.) in a water supply system can be viewed as different objects. These objects are interconnected together to form a system of supplying water to customers. Water flow is the message exchanged between any two objects in a water supply system.

When an object receives a message, it gives responses by altering its internal state and important characteristics or attributes, and generating output messages to other objects in the model. The way in which the object responds to messages depends on its internal processes and states. For a pipe in a water supply system, road loading can be viewed as an external message/hazard which is potentially influencing the pipe. If the road loading is too high, the pipe will be broken and change its state from normal operation to failure state, which consequently introduces risk to the water system.

One of the most important characteristics of object is encapsulation. This means that the attributes and behaviours of a component or subsystem are entirely encapsulated within the confines of a self-contained object. Attributes define an object’s state and behaviours describe an object’s functionality. The entire system is thus viewed as the combination of individual objects with different functionalities. Meanwhile the individual objects communicate with one another in a way that faithfully replicates their interactions in the real-world (Booch, 1994). In a water supply system, for example, a river can be viewed as an individual object encapsulating the attributes of raw water. A water treatment plant can be viewed as an another object which encapsulates the behaviour of removing the impurities from raw water in order to meet the standards of drinking water. Similarly, a pipe in the water system can be viewed as an object which encapsulates the
attributes, such as length, diameter, age, and roughness factor, and behaviours of delivering water and reducing water head due to roughness. The overall water supply system is thus a composite object composed of interconnected individual objects (including river, water treatment plant, pipes, etc.). In the object-oriented environment, the system object has functions (e.g., hydraulic models) that control the communications among individual objects in a way that faithfully replicates the interactions in the real-world water supply system.

(2) Classes

In a real engineering system, there are many objects of a specific kind. It would be extremely inefficient to redefine the same methods in every single occurrence of that object. Thus the concept of class is proposed in the object-oriented approach. A class is a template or blueprint that defines the methods and variables included in a particular kind of objects. The methods and variables that make up the object are defined only once in the definition of the class. The objects that belong to a class, commonly called instances of the class, contain only their own particular values for the variables. This concept is also applied to a water distribution network with large numbers of pipes. Although different pipes have different values of length, diameter, and material, all the pipes share some common attributes (e.g., diameter, length, age, material, etc.) and behaviours (e.g., delivering water, introducing head loss, deteriorating with age, etc.). With respect to this, mechanical engineers always use the same template/blueprint to produce pipes that have similar properties; and water engineers use the concept of pipe class to represent different pipe instances in the system by extracting their common features. Normally associated with the pipe class, many models are developed to describe its behaviours in real water supply processes. This process of obtaining classes is usually called abstraction or generalisation in practice. Even though it is only an abstract concept which has no physical counterpart in real world, class plays important roles in helping people organise complex information in the system.

Inheritance is an important characteristic of class. It is one of the fundamental rules supporting abstraction and generalisation in object-oriented approach. During the generalizing process, the more general class is called base class, and the relative less general class is viewed as derived class or instance. Inheritance allows the derived class or instance to inherit the attributes and behaviours defined within the base class. In a distribution network, pipe instances are usually viewed as derived classes, and the general pipe class is viewed as their base class. Attributes (i.e., diameter, length, age, material, roughness coefficient, etc.) and behaviours (i.e., introducing head loss, deteriorating with time, etc.) are defined within the base class. These attributes and behaviours are inherited by the pipe instances. This mechanism of inheritance facilitates the process of risk assessment in water supply system by developing common risk models for pipe class, while repeatedly reusing these models in different pipe instances.
(3) Applications of object-oriented approach

Applications of object-oriented approach have covered various areas in practice. Object-oriented paradigm has represented a major achievement in software engineering that facilitates modelling complex real-world problems (Martin and Odell, 1998). When properly applied, it can yield robust models consisting of reusable, easy-to-maintain components in different kinds of engineering systems (Booch, 1994; Solomatine, 1996; Ross et al., 1992; Black and Megabith, 1995). Meanwhile, object-oriented approach has also been used to solve engineering problems, which includes development of framework for decision making (Liu and Stewart, 2003), surface water quality management (Elshorbagy and Ormsbee, 2006), management of river system and water resources (McKinney and Cai, 2002; Simonovic, et al., 1997, Reitsha and Carron, 1997; Tisdale, 1996), reliability and risk assessment (Wyss, et al., 2004; Wyss and Durán, 2001; Black and Megabith, 1995; Matsinos, et al., 1994), material failures (Roberge, 1996), uncertainty of early design (Crossland, et al., 2003) etc. The effectiveness of using object-oriented approach to deal with complexity is also specifically illustrated by many researchers (Booch, 1994; Wyss et al, 1999; Weber and Joffè, 2006;). However, its potential in risk assessment of complex system has not been specifically considered in the existing research.

From the above discussion and applications of object-oriented approach, it is identified that one of most important power of objects and classes is their effectiveness in organising complicated information of engineering system (Martin and Odell, 1998). Firstly, all engineering systems, including water supply systems, are designed, constructed, operated and managed in terms of objects. For a water supply system, its performance is determined by the performance of the components/objects. That is risks of individual objects contributing to the risks of the overall water system. Secondy, most of the knowledge about the engineering system focuses on objects. For an example of analysing pipe deterioration, many models (including physical and statistical models) have been developed to represent the deteriorating process with time. Even though these models are different with their applications, they are all related to specific pipe objects in a water system. These models can be viewed as the behaviours or methods of pipe objects. Thirdly, generalisation of classes is a straightforward way to avoid repeated work, which thus makes it effective in managing the common features in a complex system.

The above discussion shows the possibility of using object-oriented approach as an effective tool to organise complex risk information in water supply systems. This awareness motivates this study to adopt object-oriented approach to develop frameworks of risk assessment.
3.2.2 Complexity of Water Supply Systems and Object-Oriented Approach

Since complexity is one of the hurdles limiting historical risk assessment methods, it is necessary to explicitly discuss the potential of object-oriented approach in dealing with complexity of the water supply system.

In order to effectively analyse complex systems, many researchers have carried out extensive studies on the characteristics of complex systems. Courtois (1985) suggests five attributes common to all complex systems by building upon the work of Simon (1982). The characteristics of object-oriented approach make it possible to deal with the complexities effectively. Being one of the complex engineering systems, a general water supply system inherently has these five attributes. The following discussion is about the effectiveness of using object-oriented approach to deal with the five attributes of water supply systems.

(1) “Frequently, complexity takes the form of a hierarchy, whereby a complex system is decomposed of interrelated subsystems that have in turn their own subsystems, and so on, until some lowest of elementary components is reached.” (Courtois, 1985).

Based on above discussions, this attribute is obvious for water supply systems and could be easily represented by object-oriented approach. This can be shown by a simple example in Figure 3.2. The object-oriented hierarchical structure depicts the whole/part relationships in the water supply system, which enables us to understand, describe, and see the system and its parts better (Booch, 1994).

![Figure 3.2 Hierarchical structure of a simple water supply system](image)

Furthermore this hierarchical structure also provides a possible framework for risk assessment. It is obvious within this hierarchy that risk levels of the water system are governed by the risk levels of its subsystems (i.e., river, treatment unit, and distribution network) as well as the hierarchical relationships among them. Risk levels of the subsystem, e.g., distribution network, are further
determined by risk levels of its own components, e.g., a set of pipes. Normally risks of elementary components such as river, treatment unit, and pipes in this example are not difficult to obtain as many existing models can be used to evaluate them. Once risks of these components have been determined, aggregation can then be conducted along the hierarchy to generate risks of subsystems and the overall system.

(2) “The choice of what components in a system are primitive is relatively arbitrary and is largely up to the discretion of the observer of the system.”

Primitive elements in this study are viewed as the components that are indecomposable and at the lowest level of the hierarchical structure. Above discussion shows that they play important roles in risk assessment. However, the determination of primitive elements are arbitrary and depends a lot on the observer of the system because they have different choices of what components are primitive in practical risk assessment. As the example shown in Figure 3.2, a project manager is usually interested in the risks of the overall system, i.e., the risks associated the object at the top of the hierarchical structure. While a water treatment manager is interested in the risks within the water treatment plant, i.e., object at the second level in the figure. Pipe engineers are always interested in the risks associated with different pipes which are at the bottom level of the hierarchical structure. With the different interests of stakeholders, difficulties are consequently introduced in risk assessment of the water supply system.

In addition, available risk information also determines what components in a water supply system are primitive. Normally more risk information will support risk analysts to derive the hierarchical structure to more detailed or micro levels, while limited risk information will limit the risk assessment at relative higher or macro levels.

No matter what components are primitive, they are directly related to specific objects in the object-oriented hierarchical structure. This structure can be either truncated at higher levels or extended to lower levels with the changes of primitive elements. This therefore provides a consistent and flexible way of developing hierarchies for different users.

(3) “Intra-component linkages are generally stronger than inter-component linkages. This fact has the effect of separating the high-frequency dynamics of the components, involving the internal structure of the components, from the low-frequency of dynamics involving interaction among components.”

This difference between intra- and inter-component interactions provides a clear separation of concerns among the various parts of the water supply system, which makes it possible to study risk levels of each part in relative isolation. Object-oriented hierarchical structure (Figure 3.2) is
developed with respect to this attribute. For a primitive element in this structure, its risk is usually determined by its own internal state and current condition, that is, its states transition diagrams. However, the influences from other elements are relative smaller and could be neglected in many cases. For the composite object (e.g., distribution network) in this structure, its risk is more directly affected by the components that it is composed of (i.e., pipes) rather than other components (such as river). Actually object-oriented structure is developed by implicitly considering intra-component linkages in the water supply systems.

(4) “Hierarchic systems are usually composed of only a few different kinds of subsystems in various combinations and arrangement.”

This attribute indicates that complex systems have common patterns (Booch, 1994). This is also obvious in object-oriented structure of water supply systems. A general water supply system is composed of some common elements such as river, reservoir, channel, treatment facility, pump, etc. All these elements are further abstracted as fewer common element types or classes like water source, water treatment plant, pipe, pump, valve, junction, and storage. The overall system is thus a specified arrangement of these different objects or classes. Identification of these basic components is obvious and explicit in an object-oriented approach.

(5) “A complex system that works is invariably found to have evolved from a simple system that worked...A complex system designed from scratch never works and cannot be patched up to make it work.” (Gall, 1986)

This attribute indicates that a water supply system will work normally if all its components and subsystems are working normally. A water supply system will fail to supply water to consumers, if some components or subsystems have failed. However direct determination of risk levels of a complex water supply system is difficult or nearly impossible. A possible solution to this is indirect evaluation by aggregating the risks of its subsystems (i.e., water source, water treatment, etc.) because they are less complex. These less complex objects, in turn, are composed of much less complex objects such as river, pipe, etc. Therefore, risk information can be obtained for a complex water supply system by studying risks of simple objects in an object-oriented hierarchical structure.

The above discussions not only demonstrate the potential abilities to deal with all the five attributes of complex systems, but also support the development of risk frameworks.
3.3 Object-Oriented Representation of Water Supply Systems

Now it is obvious that the similarities exist between objects in object-oriented approach and components in water supply systems. Objects are the focus of object-oriented approach. An object is an abstraction of real world entity described by attributes and methods. Each object can be influenced by the environment or external factors and interact with other objects by receiving and sending messages. Similarly, in risk assessment of a water supply system, the focus is the components in the system. Risks of water system are introduced by failures of one or more components. External hazards or threats, or environmental factors, can only compromise the functions of a water system by failing its components. With respect to these similarities, is straightforward to use object-oriented structures to represent the risk assessment process of water supply systems.

3.3.1 Object-Oriented Hierarchical Structure

The hierarchy of a water supply system is constructed (Figure 3.3) by viewing all the physical elements in a water supply system as objects that encapsulate specific attributes and behaviours and interact with one another (Li and Vairavamoorthy, 2004a; Li et al., 2005). There are two kinds of relationships, that is, aggregation relationship and generalisation relationship, represented in this hierarchy. These two relationships not only represent the water supply system from different point of view, but also are useful for risk assessment.

(1) Aggregation relationship

Aggregation represents the “is composed of” or “whole/part” relationship, e.g., a water distribution network is composed of pipes, pumps, tanks, etc. In object-oriented approach, the structure of aggregation is also called object/component structure (Booch, 1994) because it represents the relationships among objects/components. This aggregation relationship also provides a framework for aggregative risk assessment of water supply systems. This indicates that risk of water supply system, which is at the top level, is determined by risks of the objects at its immediate lower level (i.e., water source, transmission pipe, water treatment plant, and water distribution system) which are in turn determined by risks of objects at their immediate lower levels (i.e., well, river, etc.) respectively. This research views the real physical elements in a water supply system as primitive objects and put them at the lowest levels in the hierarchical structure (Figure 3.3). For each of these primitive objects, states transition diagrams are used to represent its responses to hazards or threats, which is discussed later in the following section.
(2) Generalisation relation

Generalisation represents the “is kind of” relationship, e.g., a river is kind of surface water source, and a well is kind of ground water source. All river objects belong to river class which in turn belongs to surface water class. Similarly, all pipes belong to pipe class, and all pumps belong to pump class in a water supply system. With the help of this generalisation process, all elements of a system are grouped with respect to their “likeliness of behaviour” into classes; these classes are grouped into larger classes, and so on (Solomatine, 1996). Finally another hierarchical structure is formed to express the class structure in water systems (Figure 3.3).

In the class hierarchical structure, the number of classes are much smaller than that of objects in object structure, which thus simplifies the work of assessing risks for primitive objects (Figure 3.3). For risk assessment, a risk analyst needs not develop different models for every object in a
water supply system, rather he/she can develop risk analysis models only for the types of objects (or class), and reuse them in the object instances. Actually this is also in compliance with the common sense of people in risk modelling and analysis.

By combining the concept of the aggregation with generalisation relationships, this study develops frameworks of risk assessment at both component and system levels (Figure 3.4). Firstly, basic classes are identified on the basis of class structure of a water supply system. Associated with each class are the states transition diagrams representing the relationship between hazards, its failure states, and potential risks. By extracting risk information from the states transition diagrams, analysts can develop the frameworks of risk assessment (i.e., aggregative risk and fault trees) at component level. Secondly, aggregation relationship provides a framework of
aggregative risk assessment at system level. While interconnections between objects provides a framework describing the cause-effect relationships at a system level. By considering the frameworks at both components and system levels, risk analysts can obtain a more comprehensive view of risks in the water supply system. Furthermore, this is a general method and can be applied to various water supply systems.

3.3.2 Object States Transition Diagram

Figure 3.4 shows that states transition diagrams are an important step to develop frameworks of risk assessment at component level. Therefore it is necessary to discuss them specifically in this thesis.

In an object-oriented environment, a state transition diagram shows the state space of a given class or object type, the events that cause a transition from one state to another, and the actions that result from a state change (Booch, 1994). Associated with each object in a water supply system, there are inputs, outputs, states, and methods (Figure 3.5). Inputs include water flow, external hazards (e.g., traffic loads, flood, etc.), and internal failures (e.g., deterioration, etc.); outputs denote the outflow of water; states represent the possible states of the component such as normal, failure, etc.; methods are the relative responses of the object to input information. Within each object, the states are stored as its attributes, and methods represent its behaviours in varied conditions. Inputs and/or internal state drive the methods of object to produce the associated outputs and alter state of object.

![Figure 3.5 States transition diagram of an object due to hazards or threats](image-url)
An explicit explanation of states transition is illustrated by a pipe example (Figure 3.6). In this example, inputs are external load and water with flow rate ($Q_{in}$) and pressure ($H_{in}$); outputs is water with flow rate ($Q_{out}$) and pressure ($H_{out}$); the possible states are normal and leakage; and the methods or behaviours are delivering water (Figure 3.6). There are three operational scenarios associated with the pipe that are normally considered in practice. In the first scenario, the pipe works normally (Figure 3.6a) and delivers normal water flow to other components with which it is connected. In the second scenario, the pipe changes its state from normal to leakage due to some internal factors such as deterioration with time (Figure 3.6b). Water quantity and pressure of outflow is consequently reduced. In the last scenario, the pipe changes its state from normal to leakage due to the overloading from external factors (e.g., road traffic, etc.). Excessive loading force tends to cause longitudinal cracks and results in dramatic reduction of water quantity. Usually the first scenario represents the normal operation of pipes, while the last two scenarios are specifically applied to a risk or reliability study of water distribution systems due to the influences of external or internal threats/hazards.

It is obvious that hazards, failure states, and possible risks are important factors to develop the states transition diagrams of a specific primitive object. According to the object-oriented hierarchical structures (Figure 3.3), this study identifies five classes as primitive, which includes water source, water treatment plant, pipe, pump, and storage. Although water distribution networks and water supply systems are also objects, they are usually viewed as composite objects because they are composed of more basic objects or components. Risk assessment of these objects is not easily accomplished by directly using states transition diagrams, but can be obtained by using aggregative method on primitive objects.
Associated with each primitive object, the following discussions identify the hazards, its failure states, and relative risks.

### 3.3.2.1 Hazards, Failure States, and Risks of Each Primitive Object

According to the literature review, possible risks in a water supply system are reduced water quantity, reduced pressure, and water contamination. Normally water quantity and pressure are two closely related factors because occurrence of one of them will result in the occurrence of the other. Considering this and in order to simplify the risk analysis, this thesis only considers risks of reduced water quantity and contamination specifically. The methods developed based on these two risks will also be applied to the risk assessment of pressure reduction.

#### (1) Water source

The variations of water quantity and quality are important characteristics of a water source since they influence the operations of the following water treatment and the finished water supplied to the customers ([WHO, 2004](#)). A water source is vulnerable to multiple external hazards or threats, and would thus be contaminated or in the condition of reduced capacity. Generally, the potential hazards or threats are categorised as natural and human-caused factors. Important natural factors include drought, flood, underground minerals, etc. Human-related factors include sewage discharge, industrial discharge, wilful chemical/biological contamination, etc. For example, discharge of municipal wastewater can be a major source of pathogens; urban runoff and livestock can contribute substantial microbial load; and serious drought can dramatically reduce the quantity supplied to customers, etc. ([WHO, 2004](#)) Associated with these hazards, relative failure states and risks are identified for the water source object and summarised in Table 3.1.

#### (2) Water treatment plant

Water treatment plant is the most important facility in a water supply system to remove contaminants in raw water, disinfect treated water, and produce drinkable water to consumers. However, hazards may be introduced during the process of treatment, or hazardous circumstances may allow contaminants to pass through treatment in significant concentrations. Constituents of drinkable water can be introduced through the treatment process, including chemical additives used in the treatment process or products in contact with water. Furthermore, suboptimal filtration following filter backwashing can lead to the introduction of pathogens into the distribution system. Meanwhile extreme natural hazards, wilful human attacks, or interdependency failures (e.g., power failures) can all introduce risks in water treatment process.
<table>
<thead>
<tr>
<th>Basic components</th>
<th>Failure states</th>
<th>Hazards/Threats</th>
<th>Relative risk</th>
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<tbody>
<tr>
<td>Water source</td>
<td>Natural hazards failure</td>
<td>Drought</td>
<td>Reduced water quantity</td>
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<td>Natural hazards failure</td>
<td>Flood</td>
<td>Water contamination</td>
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<td>Human-caused threat</td>
<td>Sewage discharge</td>
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<td>Chemical/biological</td>
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<td>Interdependence failure</td>
<td>Spills</td>
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<td>Contaminated site</td>
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<td>Water treatment plant</td>
<td>Natural hazards failure</td>
<td>Earthquake</td>
<td>Reduced water quantity and water contamination</td>
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<td>Flood</td>
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<td></td>
<td>Human-caused threat</td>
<td>Chemical/biological</td>
<td>Water contamination</td>
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<td>Operational failure</td>
<td>Process control</td>
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<td>Equipment failure</td>
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<td>Alarm and monitoring</td>
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<td>Inadequate backup</td>
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<td>Inappropriate treatment</td>
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<td>Interdependence failure</td>
<td>Power failure</td>
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<td>Reduced water quantity and water contamination</td>
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<td>Interdependence failure</td>
<td>Contaminated material</td>
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<td>Water contamination</td>
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<td>Pipe</td>
<td>Natural hazards failure</td>
<td>Earth movement</td>
<td>Reduced water quantity</td>
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<td>Flood</td>
<td>Reduced water quantity and water contamination</td>
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<td>Operational failure</td>
<td>External load</td>
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<td>Natural deterioration</td>
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<td>Operational failure</td>
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<td>Leaching of chemicals</td>
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<td>Contaminated water</td>
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<td>Contaminated soil</td>
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<td>Pump</td>
<td>Natural hazards failure</td>
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<td>Human-caused threat</td>
<td>Bombing</td>
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<td>Operational failure</td>
<td>Control failure</td>
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<td>Interdependence failure</td>
<td>Power failure</td>
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<td>Reduced water quantity</td>
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<td>Storage</td>
<td>Natural hazards failure</td>
<td>Animal</td>
<td>Water contamination</td>
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<td>Rainfall</td>
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<td>Human-caused threat</td>
<td>Disruption of structure</td>
<td>Reduced water quantity</td>
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<td>Human-caused threat</td>
<td>Chemical/biological</td>
<td>Water contamination</td>
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<td>Contaminated water</td>
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</table>
Generally, potential hazards and hazardous events that can have an impact on the performance of water treatment have been classified as the following (WHO, 2004):

- Natural disasters such as flood, earthquake, etc.
- Human–related threats, such as accidental and deliberate pollution;
- Inappropriate or insufficient treatment processes, including disinfection;
- Inadequate backup (including infrastructure and human resources);
- Process control and equipment failure;
- Failures of alarm and monitoring equipment;
- Power failures.

According to the above discussion, hazards are identified for water treatment plant as shown in Table 3.1. In addition, the relative failure states and possible consequences are also identified and summarised in this table.

However, it is obvious that water treatment itself is also a complex system due to its complicated composition and process. Normally, a water treatment plant includes many subunits such as pre-treatment, coagulation, flocculation, sedimentation, filtration, and disinfection. Each of the subunits has attributes and behaviours and is connected to one another in order to effectively remove different types of contaminants from raw water. From the object-oriented point of view, all the subunits can be viewed as different objects, and water treatment plant is viewed as an composite object composed of these objects. Therefore object-oriented hierarchical structures as well as state transition diagrams can also be applied to the treatment plant by following the similar steps which we have done for the water supply system. This research views water treatment plant as a primitive component in water supply process. That is a self-contained object, or “black function block. Detailed analysis within the water treatment plant is beyond the scope of this research and proposed to future study.

(3) Pipe

Pipelines are the most important part in a water distribution network. Extensive research has been performed to analyse the risks associated them among which are leakage, deterioration, corrosion, contamination intrusion, etc.

Leakages of pipes can occur from multiple reasons such as slow deterioration caused by mechanical cycling of pipes, joints, and fittings; corrosion of the internal or external surfaces of
network components; specific events and situations such as ground movement, stresses from road traffic, excessive water pressure; and/or faulty workmanship or construction (Smith et al., 2000).

Leakage rates can range from a slow leak or “drip” to large compromise of integrity called a “main break”. Examples of typical drips include loose joints, gaskets, or service connections. Typical examples of a break include a longitudinal crack in a pipe body or end bell, a circumferential crack in a pipe body, or a through-wall penetration of a network component. Drips usually result in lost of water. While main breaks in large transmission lines could result in wide spread outages and/or present the potential for contamination.

Normally, water contamination events in the pipelines in distribution system can be influenced by five major categories as (Kleiner 1998):

- Intrusion of contaminants into the distribution system through system components whose integrity was compromised or through misuse;
- Regrowth of micro-organisms in the distribution network;
- Microbial/Chemical breakthrough and by-products and residual chemicals from water treatment plant;
- Leaching of chemicals and corrosion products from system components into the water; and
- Permeation of organic compounds from the soil through system components into the water supplies.

Based on the above discussion, Table 3.1 summarises the hazards, relative failure states and risks associated with pipes in the water supply systems.

(4) Pump

Pumps also play the important roles in the process of delivering water. However, hazards or threats sometimes more easily affect pumps than pipes because it is visible and its normal operation depends highly on control, equipment reliability, and human proper activity. In risk analysis of pumps, analysts have to consider all the four failure states, i.e., natural hazard failure, human-caused failure, operational failure, and interdependence failure. Hazards to pumps in a water supply system are listed in Table 3.1.
(5) Storage

Water storages are another important part contributing to water quality problems in distribution systems besides pipes. Contamination can occur within the distribution system through open or insecure treated water storage reservoirs and tanks, as they are potentially vulnerable to surface runoff from the land and to attracting animals and waterfowl as faecal contamination sources and may be insecure against vandalism and tampering (WHO, 2004). Meanwhile, water quantity can also be decreased due to cracks of storage facilities. Table 3.1 lists the hazards, failure states, and relative risks of a storage in water supply systems.

3.3.2.2 States Transition Diagrams of Primitive Objects in Water Supply Systems

Based on the information identified in Table 3.1, states transition diagrams are developed to represent the influences of hazards or threats for water source, water treatment plant, pipe, pump, and storage respectively (Figure 3.7 through Figure 3.11).

State transition diagrams depict the relationships among hazards, failure states, and relative risks. Hazards are viewed as input information in this diagrams, to which an object gives its responses by changing its state from normal to one of the failure states and produces risk as output information. The hazards and internal state (i.e., vulnerability) of the object drive the states changes and produce risks, which is in accordance with the definition of risk in Section 1.2.2.1.

In the following states transition diagrams, Figure 3.7 shows that water source can change its state from normal to natural hazard failure state due to the influence of drought, and consequently produces reduced capacity. Accidental sewage or industrial discharge can change the water source to human-caused failure state and contaminates raw water quality. Similarly in Figure 3.8, power supplying failures will negatively influence the performance of water treatment plant and make it in an interdependence failure state. While in Figure 3.9, equipment failure can make pump station in an operational failure state and reduce the water quantity lifted by the pumps.

Furthermore, it is also shown that states transition diagrams used here focus on the logic relationships among hazards/threats, failure states, and risks rather than analysing the likelihood or consequence of a specific hazard or threat. Therefore, they can represent the influences of both natural hazards and human-caused threats in a consistent way. Specific methods of analysing likelihood and consequence of a hazard or threat are either available from historical achievements, or proposed to the future research.
Figure 3.7 States transition diagrams of water source
Figure 3.8 States transition diagrams of water treatment plant
Chapter 3 Object-Oriented Risk Assessment of Water Supply Systems

Figure 3.9 States transition diagrams of pipe
Figure 3.10 States transition diagrams of pump
3.3.3 Information Interacted between Objects

In order to describe the cause-effect relationships for specific risk, it is also necessary to discuss the interconnections between components in a water supply system. States transition diagrams only provide an explicit illustration of risk scenarios at components level. While for the cause-effect relationships at system level, the interconnections are important because they actually describe how water flow and risk information is delivered and propagated in the water supply process.

Information interconnection in a water supplying process is explained by a simple example (Figure 3.12). In the water supplying process, the water source (i.e., river A) is the first object whose outputs information will be passed to downstream component/object (i.e., transmission pipe B) in terms of “message”. Message in this example is water with certain degrees of turbidity
and organic pollutants. Then the “message” receiving object (transmission pipe B) treats the message as an input, makes responses to it by delivering this “message” to its intermediate downstream object C (i.e., water treatment). Object C removes the contaminants and turbidity in the water, and delivers it to its downstream object D (deliver pipe). Then object D receives the message and deliver the water to users that the end of the process.

However, in a risk scenario, the above process will be integrated with objects states transition diagrams. For example, the river is influenced by high contaminated flood, while all other components are working normally. Because of the flood, the river object (A) is at a state with high pollutants (Figure 3.7). Then transmission pipe (object B) delivers the highly contaminated raw water to water treatment object (C). Since the pollutant levels of raw water exceeds the limits of normal treatment, the outputs from water treatment would be contaminated water (Figure 3.8) that will be delivered and distributed by pipelines in water distribution system (Object D). In this process, object A changes its state from normal to natural hazard failure due to the influence of flood. Other objects are working normally and do not have states change. The cause of contamination at users side is therefore flood influences to river in this example.

![Figure 3.12 Information interaction between components in a simple water supply process](image)

The above example indicates there are two sources of hazards in a water supply process. One is the directly hazards from external factors, the other is the hazards delivered with the water flow. This is the basis of fault tree analysis of this study, which is specially discussed in the following section.

### 3.4 Object-Oriented Frameworks for Aggregative Risk Assessment

Based on the above discussions, frameworks of aggregative risk assessment are developed at component and system levels, respectively, in this study.
3.4.1 Aggregative Risk Assessment for Basic Components

(1) General framework of aggregative risk assessment at component level

Framework of aggregative assessment is developed at component level by extracting the risk information from object states transition diagrams (Figure 3.13). In the states transition diagram, failure states are directly related to negative consequences or risks. Thus the risk levels of an object is determined directly by the risk levels of its failure states. Further, the change from normal state to a failure state of an object is directly related to and driven by specific hazard or threat. That is the hazards directly contributing to the risk levels of the failure states. A specific hazard is usually evaluated in terms of its likelihood of occurrence and possible consequence. Then risks can be determined for hazards, failure states, and risk of object respectively by following the hierarchy of aggregative risk assessment.

![Figure 3.13 Framework of aggregative risk assessment at component level which is developed on the basis of object states transition diagrams](image)

According to the above hierarchical structure, conceptual equations for quantitative analysis can be expressed as:

\[
R = f(F_1, \ldots, F_s, \ldots, F_S)
\]

\[
F_s = f(H_{s1} \cdot V_{s1}, \ldots, H_{sH_s} \cdot V_{sH_s}); s = 1, \ldots, S
\]

\[
H_{sj} = H_{sj}^L \times H_{sj}^S; s = 1, \ldots, S; j = 1, \ldots, H_s
\]

where \(R\) denotes the risk of the object; \(F_s\) denotes the risk of failure state \(s\); and \(H_{sj}\) denotes the risk of \(j\)th hazard associated failure state \(s\), which is determined by the production of likelihood, \(H_{sj}^L\), and severity, \(H_{sj}^S\), of the hazard. \(V_{sj}\) denotes the vulnerability of the object to the hazard \(H_{sj}\). \(S\) is the number of failure states associated the object, and \(H_s\) is the number of hazards associated failure state \(s\).
In order to quantitatively perform Equation (3.1), it is important to consider the following aspects specifically:

- Firstly a mathematical methods, \( f(\cdot) \), is necessary to generate quantitative results for this framework, which is specifically studied in Chapter 4.

- Secondly, likelihood and severity are two important factors to evaluate the serious level of a hazard or threat. However, the study on methods determining the likelihood and severity of a hazard exceeds the scope of this research. This thesis assumes that both of these two factors can be predetermined either objectively by existing methods (e.g., those reviewed in Chapter 2) or subjectively by experts’ opinion.

- Furthermore, vulnerability, a property of the component/object, represents the degree that the object can be influenced by the hazard. However, it is not unusually difficult to determine the vulnerability of an object due to the influence of a hazard or threat. Detailed study on vulnerability is also beyond the scope of current study. Therefore, this thesis assumes the vulnerability to be unity and proposes detailed study of vulnerability to future work.

**2) Frameworks of aggregative risk assessment for primitive objects in the water system**

With respect to Table 3.1, negative consequences of the risks are reduced water quantity and/or water contamination in water supply systems. For each primitive object frameworks are developed to represent risk contributions from hazards to risk of the object (Figure 3.14 through Figure 3.18).

![Figure 3.14 Frameworks of aggregative risk assessment of water source](image-url)
Figure 3.15 Frameworks of aggregative risk analysis of water treatment plant
Figure 3.16 Frameworks of aggregative risk analysis of pipe

(a) Risk of reduced water quantity

(b) Risk of water contamination

Figure 3.17 Framework of aggregative risk analysis of pump
3.4.2 Aggregative Risk Assessment of Subsystems and the Overall System

Framework of risk assessment of subsystems and the overall water supply system is determined by the whole/part relationships represented in object-oriented structure (Figure 3.19). In this framework primitive components are at the bottom level whose risk levels are determined by the framework proposed above (Figure 3.13). This aggregative process explicitly shows that risk of the overall system is determined by risks of its subsystems, which are in turn determined by risks of their components.

According this structure, conceptual equations for quantitative analysis can be expressed as:

\[
WS = f(W_1 \cdot V_{W_1}, \ldots, W_s \cdot V_{W_s}, \ldots, W_m \cdot V_{W_m})
\]

\[
W_s = f(s_1 \cdot V_{s_1}, \ldots, s_n \cdot V_{s_n})
\]  

(3.2)
where $WS$ is the risk of the water supply system; $W_i$ is the risk of subsystem $s_i$; and $s_i (i=1, \ldots, s_n)$ is the risk of object $s_i$ which is determined by Equation (3.1). $V_{W_i}$ denotes the vulnerability of the overall system due to the failure of subsystem $W_i$. $V_{s_i}$ denotes the vulnerability of subsystem $W_i$ due to the failure of component/object $s_i$.

In order to quantitatively perform Equation (3.2), it is important to consider the following two aspects:

- Firstly it is necessary to use proper mathematical methods, $f(\bullet)$, to solve this equation. This is specifically studied in Chapter 4.

- Secondly, vulnerability is important to determine the final risk assessment of the water supply system. The vulnerabilities of system and subsystem are determined by multiple factors such as the composition and layout the water supply system, changes of water demands, various requirements of different users, etc. Specific study of this is beyond the scope of this study. In the following parts, this thesis assumes these vulnerabilities to be unity and propose the detailed study to future work.

Figure 3.20 gives an example illustrating the framework of aggregative risk assessment at the system level. In this framework, risks of the water supply system are determined by its immediate subsystems including water source, transmission pipes, water treatment plant, and water distribution network. For the subsystem water source, its risks are further determined by risks of river, wells, and the reservoir. For the transmission pipe, its risks are contributed by a channel and two pipes. While for the water distribution network, its risks are contributed by its components such as a pump, a tank, and a set of distribution pipes. Subsystem of water treatment plant is viewed as an primitive component whose risk is directly determined by its states transition diagrams.

Finally, by combining the frameworks at component and system levels, a general framework is formed for aggregative risk assessment of water supply systems. This hierarchical assessment is believed effective as it has the ability to model the intricate relationships among components and subsystems and to account for all relevant and important elements of risk and uncertainty, which thus renders the assessing process more tractable and representative (Haimes, 2004). Furthermore, since both of the frameworks are developed from a general point of view, they can be applied to specific applications in various water supply systems. A possible quantitative evaluation of these aggregative frameworks is particularly studied in the following chapter.
3.5 Object-Oriented Fault Tree Analysis

3.5.1 Basics of Fault Tree Analysis

Fault tree analysis was developed by H.A. Waston of the Bell Telephone Laboratories in 1961-62 during an Air Force study contract for the Minuteman Launch Control System. The first published papers were presented at the 1965 Safety Symposium sponsored by the University of Washington and the Boeing Company, where a group had been applying and extending the technique. After that, this method was extensively used in every engineering system, especially with the
Fault tree is a backward analysis tool which begins with a system risk and traces backward, searching for possible causes of the risk. Thus it can identify the causal relationships in an engineering system. In practice, a fault tree is used to provide a logical and hierarchical description of a risk (top event) in terms of sequences and combinations of malfunctions of individual components. Then the reliability or risk of a complex system can be computed in terms of the given probabilities of the components failures.

Even though it has been used widely, the construction of fault trees for an engineering system is still as much as an art as a science. To give an consistent risk analysis based on fault trees, the construction process is required to perform at both component and the system levels. Encouraged by the fact that effective fault tree analysis is determined by component interrelationships in system and component failure characteristics (Henley and Kumamoto, 1981), the construction of fault trees is studied, particularly in this section, based on object-oriented concepts. Similar with the development of aggregative risk frameworks discussed in preceding sections, two steps are also proposed here to develop the structure of fault trees, i.e., (1) fault trees at component level; and (2) fault trees at a system level.

### 3.5.2 Hazardous Influences based on Object-Oriented Concepts

Before the construction of fault trees, it is necessary to analyse the hazardous influences from object-oriented point of view. Based on the discussion in Section 3.3.3, the unsatisfied outflow of a component in water systems is mainly caused by two ways: (1) hazards/threats directly affecting on the component itself; and/or (2) abnormal water flow delivered from its upstream components (Figure 3.21). The first hazardous scenario is also called primary hazards in this research. The mechanism of risks introduced by this hazards can be analysed based on state transition diagrams of the objects. On the contrary, in the second hazardous scenario, the component/object itself is not directly affected by hazards/threats and could be working in a normal state. However what is abnormal in this case is its inflow water transmitted from upstream component/object which might be directly influenced by hazards/threats and in a failure state. For the instance in Figure 3.12, the outflow of a water treatment plant is unsatisfied even though the water treatment plant works normally. The cause of the contaminated outflow of water treatment plant is not the failures of treatment plant itself, but the contaminated inflow coming from its upstream, i.e., river. Risks belonging to this situation are called inflow hazards in this study. Characteristics of inflow
hazards are related to the interconnections and interdependences between objects in a water supply system.

![Diagram of object-oriented risk assessment](image)

**Figure 3.21** Diagrams representing the influences to an object by primary and inflow hazards

The above analysis of hazardous influences therefore leads to the development of fault trees on the basis of objects state transition diagrams and interconnections between objects.

### 3.5.3 Development of Fault Trees based at Component Level

This study applied object states transition diagrams to develop frameworks of fault trees at component level by extracting the logic relationships between negative consequences, failure states and hazards. With respect to this, three steps are identified by the current study to develop fault tree structures for objects in a water supply system (Figure 3.22). Step 1 is to precisely represent risk such as what the risk is and when it occurs, and to describe the risks in terms of three risk scenarios (i.e., reduced water quantity, contamination, or contamination and reduced water quantity); step 2 is to identify what failure states can possibly cause the risk; and step 3 is to extract the potential hazards/threats that can possibly alter the object state from normal to the failure states identified in step 2. These above three steps will produce a hierarchical fault tree in which hazards are at the bottom level and viewed as basic events, and risk of object is the top event. This therefore provides a cause-effect relationship for a given risk of object.
By repeating the above three steps, fault trees are formed for all the primitive objects in the water supply system (Figure 3.23 through Figure 3.27). In the fault trees, $R_1$ denotes the risk of reduced water quantity; $R_2$ denotes the risk of contamination; and $R_3$ denotes the risk of contamination and reduced water quantity. $F_1$, $F_2$, $F_3$, and $F_4$ represent the failure states of natural hazard failure, human-caused failure, operational failure, and interdependence failure, respectively; $H_{ij}$ is the hazard $j$ associated with $i$th failure state as shown by the relative states transition diagrams.

![Figure 3.22](image)

**Figure 3.22** Steps of developing fault trees of an object due to its primary hazards by extracting risk information from objects state transition diagrams

![Fault Tree Diagrams](image)

**Figure 3.23** Structures of fault trees for water source
Figure 3.24 Structures of fault trees for water treatment plant

Figure 3.25 Structures of fault trees for pipe

Figure 3.26 Structure of fault tree for pump
3.5.4 Development of Fault Trees at System Level

This study develops fault trees at a system level based on interconnections between components in the water supply system. This indicates that inflow hazards of an object are from its immediate upstream objects, whose inflow hazards, in turn, are from their immediate upstream objects. A three-step method is proposed in this study as follow:

- To obtain the flow directions in a water supply system with the help of simulation models or software (e.g., EPANET, KYPE, etc.), and to record information of upstream and downstream components as attributes of a specific object.
- To find the object that is directly related the risk under study, and to identify its upstream objects based on the results obtained in step 1.
- To set one of the upstream object as current object and repeat step 2 until all the upstream objects have been studied.

In order to facilitate the construction of fault trees, a notional diagram is provided in Figure 3.28 based on which computer programming are coded in this study.

The above diagram is general and applicable to construct fault trees of serial, parallel, and complex water systems.
Figure 3.28 Diagrams of constructing fault trees at system level based on interconnections among the components in a water supply system
(1) Serial connections of objects

A serial water supply system means that its components are serially interconnected (Figure 3.29). For the risk occurs at user point (e.g., reduced water quantity, contamination, or reduced water quantity and contamination), a fault tree is formed by following the above diagram (Figure 3.28). The result shows that risk at user point can be caused by primary hazard from object C (pipe) or by inflow hazard which is resulted from object B (pump). Then for the risk at outflow of B, it can be caused by primary hazard from object B or by inflow hazard from object A (storage). This structure is equivalent to the fault tree developed by using conventional techniques on serial systems.

The fault tree of serial water system can be represented in terms “and” (\( \cap \)) and “or” (\( \cup \)) relations as

\[ R = A \cup B \cup C \]

where \( R \) denotes the risk of reduced water, contamination, or reduced water and contamination at the user point; \( A, B, \) and \( C \) are primary risk events of pipe, pump, and storage respectively.

Figure 3.29 Construction of fault tree for a serial water system
(2) Parallel connections of objects

A simple example of parallel system means is given in Figure 3.30 where fault trees are developed to represent risk of contamination \( R_2 \), reduced water quantity \( R_1 \), and reduced water quantity and contamination \( R_3 \) respectively.

i. Risk of water contamination

The risk of water contamination \( R_2 \) at user point can be caused either by contamination failures of object B (pipe 1) or object C (pipe 2), or by inflow hazards from their upstream object, i.e., object A (storage). Therefore, the fault tree can be formed to represent this risk as

\[
R_2 = A \cup B \cup C
\]

where \( R_2 \) denotes the top risk event of contamination; \( A, B, \) and \( C \) respectively represent primary risk event associated with different objects.

ii. Risk of reduced water quantity

Analysis of the reduced water quantity \( R_1 \) at user point is not straightforward because different reduced quantities might have different cause-effect relationships. In order to explain this, this risk is classified to three scenarios (Figure 3.30). In the first possible scenario, the possible range of reduced quantity \( \Delta Q \) at user point satisfies \( 0 \leq \Delta Q \leq \min(Q_{B_{\text{max}}}, Q_{C_{\text{max}}}) \) and \( \Delta Q \neq Q \). Here \( Q_{B_{\text{max}}} \) and \( Q_{C_{\text{max}}} \) denote maximum flow delivered by pipes B and C respectively; \( Q \) is water flow demanded at user point. This indicates that the water reduction could be caused either directly by primary failures of pipes B or C, or indirectly by inflow hazards from their upstream object storage A (Figure 3.30(a)). The top event is represented as

\[
R_1 = R_2 = A \cup B \cup C
\]

where \( R_1 \) denotes risk event in the first possible scenario; \( A, B, \) and \( C \) are primary failures of pipe 1, pipe 2, and storage respectively.

In the second possible scenario, the possible range of reduced quantity \( \Delta Q \) at user point is constrained by \( \min(Q_{B_{\text{max}}}, Q_{C_{\text{max}}}) \leq \Delta Q \leq \max(Q_{B_{\text{max}}}, Q_{C_{\text{max}}}) \) and \( \Delta Q \neq Q \). Then the risk could be caused directly by primary failure of the pipe with larger flow rate or primary failures of two pipes simultaneously. The indirect cause is resulted by inflow hazards from their upstream object storage A. Then the fault tree can also be simplified to a simpler fault tree from the logic point of views (Figure 3.30b). Here we assume that \( Q_{B_{\text{max}}} \) is larger than \( Q_{C_{\text{max}}} \). Thus the top event is finally represented as
Figure 3.30 Construction of fault trees for a parallel water system
\[ R_1^2 = (B \cup (B \cap C)) \cup A = B \cup A \]

where \( R_1^2 \) denotes top event in the second possible case.

In the third possible scenario, the possible range of reduced quantity (\( \Delta Q \)) at user point satisfies \( \Delta Q \geq \max(Q_{B_{\text{max}}}, Q_{C_{\text{max}}}) \) or \( \Delta Q = Q \), which indicates that the risk could be caused either directly by simultaneous primary failures of pipes B and C, or indirectly by inflow hazard from their upstream object storage A (Figure 3.30(c)). The top event is represented as

\[ R_1^3 = (B \cap C) \cup A \]

where \( R_1^3 \) denotes top event in the third possible case.

Results of the above three scenario are summarised in Table 3.2.

<table>
<thead>
<tr>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 \leq \Delta Q \leq \min(Q_{B_{\text{max}}}, Q_{C_{\text{max}}})</td>
<td>\min(Q_{B_{\text{max}}}, Q_{C_{\text{max}}}) \leq \Delta Q \leq \max(Q_{B_{\text{max}}}, Q_{C_{\text{max}}})</td>
<td>\Delta Q \geq \max(Q_{B_{\text{max}}}, Q_{C_{\text{max}}})</td>
</tr>
<tr>
<td>A \cup B \cup C</td>
<td>B \cup A</td>
<td>(B \cap C) \cup A</td>
</tr>
</tbody>
</table>

It is obvious that \( A \cup B \cup C \geq B \cup A \geq (B \cap C) \cup A \), which indicates that likelihood of the first scenario is larger than the second scenario which is larger than the third scenario. This is in compliance with the common sense of risk knowledge about the water systems because minor water reduction is more frequent than major water reduction. However in practical risk or reliability assessment of water distribution system, only no water flow at user points (i.e., \( \Delta Q = Q \)) is more frequently considered by analysts. With respect to this, current study therefore only considers the construction of fault trees for risk of no water flow at user points in the following parts of this thesis.

**iii. Risk of water contamination and reduced water quantity**

The risk of reduced water quantity and contamination (\( R_3 \)) at user point is an intersection events of reduced water quantity (\( R_1 \)) and contamination (\( R_2 \)). Thus it is represented as:

\[ R_3 = R_1 \cap R_2 \]
Figure 3.31 Construction of fault tree of no water flow delivered at user point of a complex water system
(3) Complex water supply systems

Most practical water supply systems, particular the distribution networks, have no serial-parallel configuration and the development of fault trees are much more difficult. Backward analyses, such as fault tree analysis, are believed to be effective for identifying potential causes for a specific problem, by starting with a system risk and tracing backward to search possible causes of the hazard (Mays and Tung, 1989). The diagram proposed above (Figure 3.28) is in accordance with the concept of backward analysis, which makes it applied to generate fault trees complex water supply systems.

An example is adopted here to demonstrate the construction of fault trees by using the proposed method (Figure 3.31). This example is a distribution network composed of seven pipes (i.e., P1,…,P7). The flow directions among the network are also pre-determined and represented in the figure. Because of the complicated interconnections among pipes, the network is neither serial nor parallel. Suppose that there is no water delivered to user in this example. Fault tree is developed (Figure 3.31) to depict the cause effect relationships for this problem.

Fault tree shows that the top event can be caused by failures pipe 1, pipe 2 and pipe 3, pipe 2 and pipe 6, pipe 5 and pipe 6, pipe 3 and pipe 4 and pipe 5, or pipe 7. This means that single failures of pipe 1 or pipe 7, or some combination failures of the pipes (e.g., pipes 2 and 3, etc.) will result in no water at user side. This can be simply represented by

\[ R_i = P1 \cup (P2 \cap P3) \cup (P2 \cap P6) \cup (P5 \cap P6) \cup (P3 \cap P4 \cap P5) \cup P7 \]

However, the above analysis only considers the normal flow direction in the pipelines (Figure 3.31). In practice water flow in pipe 4 can be in the reverse direction as shown in Figure 3.31. By considering these two possible directions in the pipelines, the result of fault tree analysis will be

\[ R_i = P1 \cup (P2 \cap P3) \cup (P2 \cap P4 \cap P6) \cup (P5 \cap P6) \cup (P3 \cap P4 \cap P5) \cup P7 \]

It is obvious that the former result is a conserve approximation of the latter as it ignores the other possible delivering path from the source to the users. Even the result of the former is slightly higher than the real value, it is sufficient to represent the cause-effect relationships and much easier to be developed. This will be more advantageous in complex water supply systems. Therefore, this thesis only considers the development of fault trees based on normal water flow directions in a water supply system.

3.6 Summary and Comments

This study has discussed that object-oriented approach is effective in dealing with complexity in a water supply system, and can be used to develop frameworks of risk assessment for water supply
systems. Two types of frameworks, i.e., aggregative risk assessment and fault trees, have been developed on the basis of object-oriented structure of the water supply system. Framework of aggregative risk assessment is used to evaluate the risks associated with components, subsystems, and the overall system. While fault trees are developed to represent the cause-effect relationships for specific risk in the water system. By combining these two frameworks, risk analysts can obtain a more comprehensive view of the risks in a water supply system.

Frameworks of risk assessment developed on the basis of object-oriented approach are more suitable for a comprehensive risk assessment than most existing methods from several aspects:

• It can be used by different users. The frameworks can be flexibly established at different hierarchical levels according to the requirements of system observers and/or available information.

• It can be reused in different water supply systems. The frameworks are developed from a general point of view, which encapsulate the common features of various water supply systems and can be reused in any specific application.

• It can evaluate of risks by considering multiple hazards. The frameworks can aggregate both natural hazards and human-related threats along a consistent hierarchy to generate useful risk information for decision makers.

• It is flexible to real application of risk assessment. Even though only reduced water quantity and water contamination are considered as risks in this study, the method developed here can be easily reused to other risks (such as low pressure failures, etc.).

However, there are still further work required to improve the frameworks developed in this study, which can be summarised briefly as follow.

• The framework for vulnerability assessment is required to be further studied as vulnerability also plays important role in introducing risk into the water supply system. This study assumes the values of vulnerabilities to be unity, which makes the evaluation of this study more conservative. However, detailed study is required in order to make the assessment be closer to the real situation.

• The process of generating fault tree structure at system level is based on the normal flow directions of a water supply system, which is a conserve approximate of the real cases. However, the flow directions might change in real cases of failures. Therefore, further study is necessary to improve the generation of fault tree at system level in the future so that more reasonable results can be obtained.
Chapter 4 Quantitative Aggregative Risk Assessment of Water Supply Systems

CHAPTER 4

Quantitative Aggregative Risk Assessment of Water Supply Systems

4.1 Introduction

This chapter studies a method that can quantitatively evaluate the frameworks of aggregative risk assessment for water supply system proposed in Chapter 3. There are two aspects required to be mathematically represented for these frameworks. One is mathematical evaluation of risks associated with hazards. The other is the mathematical method that can aggregate risk along the hierarchical structure to obtain risks of objects, subsystems, and the overall water supply system.

Fuzzy sets theory and Dempster-Shafer theory are adopted in this study to perform quantitative evaluations of aggregative risks (Figure 4.1). Firstly, fuzzy sets theory is used to represent the characteristics of a hazard such as likelihood of the occurrence and relative severity. Fuzzy sets theory, an effective method of representing imprecise, vague, and fuzzy information, has been effectively applied to many engineering problems. Usually, there are multiple types of hazards in a water supply system that are described in various forms including probability distribution, numerical intervals, linguistic variables, etc. Its application of representing risks is specifically considered in this chapter.

Secondly, combination rule of Dempster-Shafer theory is used in this study as an aggregative method to integrate risk information from lower levels to higher levels along the hierarchical assessment structure. Dempster-Shafer theory is one of the evidential reasoning methods that can effectively aggregate or fuse information from multiple sources. It has been extensively applied to many engineering systems like AI and expert systems, engineering design assessment, safety analysis, environmental assessment, etc. This study adopts this method, coupled with fuzzy sets theory, to generate aggregative risks of the water supply system.
Chapter 4 Quantitative Aggregative Risk Assessment of Water Supply Systems

Figure 4.1 Quantitative methods used to evaluate the framework of aggregative risk assessment for the water supply system

Among the following parts of this chapter, Section 4.2 explains the basics of fuzzy sets theory and its ability of representing uncertain information in a risk assessment. Section 4.3 introduces the basics of Dempster-Shafer theory and its effectiveness of aggregating/integrating information from multiple sources. After that Section 4.4 illustrates the process of aggregative risk assessment for water supply systems based on the two theories. Then Section 4.5 gives a brief discussion about the potential usefulness of aggregative risks in water supply system. Lastly, Section 4.6 summarises the proposed quantitative method and also gives comments on it. Figure 4.2 explicitly depicts the structure of this chapter.
Figure 4.2 Structure of developing quantitative methods for aggregative risk assessment of water system

4.2 Fuzzy Sets Theory and Its Potential in Risk Assessment

This section firstly discusses the motivations of using fuzzy sets theory in risk assessment, then introduces its basic concepts that are used in the current study.

4.2.1 Motivations to Use Fuzzy Sets Theory in Risk Assessment

In a risk assessment, likelihood and severity associated with a hazard or threat are not unusually difficult to be evaluated by probability theory. Firstly some hazards are always related to many uncertain factors which are difficult to be expressed in terms of probabilities. For example, human-related attack or contamination to water service components is influenced by several
uncertain factors like the ability of human approaching the component, the ability of human transporting and implanting explosives, the ability of human obtaining sufficient quantities, and the extent the terrorist willing to risk their lives, etc. (Haimes et al., 1998). All of these factors are subjective in nature and hard to be represented by a single precise probability distribution function. Secondly, historical records of several risk scenarios, particularly extreme hazardous events (e.g., extreme flood, human explosive, etc.), are not complete and sufficient. An analyst always has difficulties in developing proper probability distribution functions with these limited data. Thirdly, in many cases, risk analysts are more confident with linguistic representations (such as very high, slightly low, etc.) and logic cause-effect relationships rather than precisely numerical representations and physical or mathematical models to give risk evaluations. However, probabilistic variables have limited ability to represent these linguistic or descriptive information. Fourthly, an effective risk assessment is always based on the consideration of multiple hazards which are always represented in various forms such as probabilistic data, experts opinions, linguistic representation, etc. This is another hurdle that limits the effectiveness of probability theory in risk assessment. Therefore, there is a need to find an effective method to meet the above characteristics of risk assessment.

Alternative to probabilistic theory, fuzzy sets theory, introduced by Zadeh (1965) to deal with problems in which vagueness is presented, could be used to represent subjective, vague, linguistic, and imprecise data and information effectively. Applications of fuzzy sets theory have been extensively studied to handle the ambiguity and vagueness involved in the risk analysis in different engineering areas (Lee, 1996; El-Baroudy and Simonovic, 2004; Sadiq and Husain, 2005).

4.2.2 Basics of Fuzzy Sets Theory

To get a brief view of fuzzy sets theory, it is necessary to introduce several basic concepts, i.e., fuzzy sets, fuzzy numbers, $\alpha$-cut sets, and extension principle, as they are frequently used in engineering problems.

(1) Fuzzy sets

Fuzzy sets are a mathematical tool used to deal with real-world-sense fuzziness (Cai, 1996). In a traditional set, “membership” stipulation, or characteristic function, is what actually defines a set whose boundaries are precisely defined: e.g., “a set of pipes whose failure probability is 0.1.” For each element, characteristic function, can be either 1, if its failure probability is 0.1, or 0, if its failure probability is not 0.1. On the contrary, a fuzzy set is defined as a set of elements that belong to a space (the universe) whose boundaries are not precisely defined. Rather than
represents the likelihood of a hazard or threat in precise probability, fuzzy sets theory uses a range or a set of probabilistic values to represent the probability. Associated with each probability, \( p \), in the range or the set, a membership function is defined to express the grade, between 0 and 1, with which the analyst believes that the likelihood of the hazard is \( p \). For example, a fuzzy set \( A \) can be defined as “the set of pipes whose failure probabilities are about 0.1.” Because of the subjective restriction ‘about’, the membership of an element \( x \) to the set \( A \) is no longer a binary type (i.e., the element belongs or does not belong to the set), but rather is defined by its membership function \( \mu_{A}(x) \) (Figure 4.3).

\[
\mu_{A}(x) = \begin{cases} 
20x - 1 & x \in [0.05, 0.1] \\
3 - 20x & x \in [0.1, 0.15] \\
0 & \text{otherwise}
\end{cases}
\]

Figure 4.3 An example of using fuzzy set to represent failure probability of pipes

In Figure 4.3, \( x \) denotes the probability of pipe failure; \( \mu_{A}(x) \) denotes the membership of \( x \). \( \mu_{A}(x) \) establishes how much the element \( x \) belongs to the set \( A \) (i.e., if \( \mu_{A}(x) \) is close to 1, then \( x \) more strongly belongs to set \( A \); on the contrary, if \( \mu_{A}(x) \) is close to 0, \( x \) then more weakly belongs to set \( A \)). Thus it is shown that elements whose failure probabilities are in the range of [0.05, 0.15] belong to the fuzzy set \( A \) with different grades from 0 to 1. Pipes with failure probability 0.1 have the highest grade, i.e. 1. The grade levels are gradually decrease with the change of failure probability from 0.1 to 0.05 or from 0.1 to 0.15. Thus fuzzy sets explicitly represent the subjective representation, ‘about’, in a mathematical way.

Formally, let \( U \) be a classic set, whose generic element is denoted \( x \). A fuzzy set \( A \) defined on \( U \) is a mapping from \( U \) to the unit interval \([0, 1]\), denoted by (Dubois and Prade, 1980; Zimmermann, 1991)

\[
A = \{(x, \mu_{A}(x)) : x \in U\} \tag{4.1}
\]
or simply by

\[
A = (x, \mu_{A}(x)) \tag{4.1'}
\]

where \( \mu_{A}(x) \) is referred to as the membership function, whose value at \( x \) signifies the grade of membership of \( x \) of the fuzzy set \( A \) and may vary from 0 to 1.
(2) Fuzzy numbers

In addition to the above representation of fuzzy sets, fuzzy numbers are also frequently used to represent imprecise variables in practical applications. In terms of mathematical definition, a fuzzy number is a convex and normal fuzzy set, i.e., a special case of fuzzy sets. Normally there are two simple membership functions frequently used for a fuzzy number: one is triangular function, the other is trapezoidal function. The triangular function could correspond to an inaccuracy of the type “the variable is certainly included between $x_a$ and $x_b$ and close to $x_c$”, “the variable is about $x_c$”, or “the variable is close to $x_c$”, etc. While trapezoidal function translates information such as “the variable is certainly included between $x_a$ and $x_b$ and is likely around a value between $x_c$ and $x_d$”. The key fuzzy or subjective words are “certainly”, “close”, “about”, and “likely around”. These words constitute the only real information that any risk analyst would have about the risk. The application of triangular and trapezoidal functions can be illustrated by two examples respectively: (1) “the failure probability of pipe A is about 0.1.” (Figure 4.4(a)); and (2) “the failure probability of pipe B is included between 0.05 and 0.25 and is most likely around value between 0.1 and 0.2.” (Figure 4.4(b)).

![Figure 4.4](image_url)

**Figure 4.4** Examples of using triangular and trapezoidal fuzzy numbers to represent failure rates of pipes

It is stated, from above example, that a membership function simply indicates how much a given value belongs to a fuzzy set/number or how close it is to the most likely value. Thus it is advantageous to use a fuzzy number to represent imprecise or inaccuracy information in terms of mathematical language, which is useful for quantitative risk assessment.
Once the imprecise data are changed into fuzzy numbers, the aim of risk assessment is to
determine the final risk based on these fuzzy numbers. To accomplish this, the concepts, \( \alpha \)-cut set
and/or extension principle, are frequently used in quantitative analysis with fuzzy sets theory.

(3) \( \alpha \)-cut set

For any \( \alpha \in [0,1] \), an \( \alpha \)-cut set of fuzzy set \( A \), denoted by \( A_\alpha \), is a classic set defined by

\[
A_\alpha = \{ x \in U, \mu_A(x) \geq \alpha \} \tag{4.2}
\]

That is, the \( \alpha \)-cut set of fuzzy set \( A \) is a crisp set that contains all the elements of the universe set
whose membership grades are greater than or equal to the specified value of \( \alpha \). This is indicated in
Figure 4.4. When \( \alpha = 0 \) the corresponding interval is called the “support” of the fuzzy set (the
interval [0.05, 0.15] in Figure 4.4 (a)) while for \( \alpha = 1 \), and when the membership function is
triangular, the interval comes down to one crisp value only, that is, the “the most likely value”
(i.e. 0.1 in Figure 4.4 (a)).

Because it is an interval and easier to be analysed, \( \alpha \)-cut set is important to determine the
operations on fuzzy sets, fuzzy arithmetic, and fuzzy relations, which is discussed in details in
Appendix B.

(4) Extension principle (Zimmermann, 1991; Dubois and Prade, 1980)

Let \( f \) be a mapping from \( U_1 \times \cdots \times U_n \) to a universe \( Y \) such that

\[
f = f(x_1, \ldots, x_n)
\]

The extension principle allows us to induce from fuzzy sets \( A_1 \times \cdots \times A_n \) to a fuzzy set \( B \) on \( Y \)
through \( f \) such that

\[
\mu_B(y) = \begin{cases} 
\sup_{x_1, \ldots, x_n} \min \{ \mu_{A_1}(x), \ldots, \mu_{A_n}(x) \}, & f^{-1}(y) \neq \Phi \\
0, & f^{-1}(y) = \Phi \end{cases} \tag{4.3}
\]

where \( f^{-1}(y) \) is the inverse image of \( y \), \( \mu_B(y) \) the greatest among the membership values
\( \mu_{A_1 \times \cdots \times A_n}(x_1, \ldots, x_n) \) of the realization of \( y \) using \( n \)-tuples \( (x_1, \ldots, x_n) \).

For \( n=1 \), the extension principle reduces to
Extension principle is one of the most important fundamentals in fuzzy sets theory to perform fuzzy calculations by using functions on fuzzy sets or numbers. Details about extension principle can be found in Appendix B. Although conceptual elegant, extension principle is difficult to use and the mathematical tools required to are still scarce (Revelli and Ridolfi, 2002). Alternatively, α-cut sets method is easier to be computerised. Therefore, this research adopts α-cut sets method to conduct fuzzy calculations.

In this research, likelihood and severity of a hazard are represented in terms of fuzzy numbers. Risk of the hazard is then a function of these two fuzzy parameters. Detailed discussions about this are conducted in Section 4.5.

4.3 Dempster-Shafer Theory and Its Potential in Risk Assessment

4.3.1 Methods of Aggregating Information

Once the risks of hazards in a water supply system have been determined, aggregation methods are then necessary for risk evaluations of components, subsystems, and the whole water system. Normally aggregation is used as an assessment method to meaningfully summarise and simplify a set of data, whether the data come from a single source or multiple sources. In aggregative risk assessment, for example, the determination of components risks is a process of aggregating risks from multiple hazards. Similarly determination of system risks is a process of aggregating risks from its subsystems and components.

Familiar examples of aggregation techniques include arithmetic averages, geometric averages, harmonic averages, maximum values, and minimum values (Ayuub, 2001). Among them, combination rules are special types of obtaining data from multiple sources. From a set theoretic standpoint, combination rules can potentially occupy a continuum between conjunction (AND-based on set intersection) and disjunction (OR-based on set union) (Dubois and Prade, 1992). In the situation of risk analysis where all sources of input information are considered equally reliable, a conjunctive operation is appropriate (i.e., \( A \) and \( B \) and \( C \)…). In the case where there is only one reliable source among many risk inputs, a disjunctive combination operation (i.e., \( A \) or \( B \) or \( C \)…) is used. However, many combination operations lie between these two extremes (i.e., \( A \) and \( B \) or \( C \), \( A \) and \( C \) or \( B \), etc.). In this thesis, the Dempster-Shafer theory is used to aggregate risks along the hierarchical structure of aggregative risk assessment.

\[
\mu_{\beta}(y) = \begin{cases} 
\sup_{x \in f^{-1}(y)} \mu_{\beta}(x); & f^{-1}(y) \neq \Phi \\
0; & f^{-1}(y) = \Phi 
\end{cases} \quad \text{(4.3')}
\]
4.3.2 Basics of Dempster-Shafer Theory

The Dempster-Shafer theory was first developed by Dempster (1967) in the 1960s and later extended and refined by Shafer (1976) in the 1970s. It offers an alternative to traditional probabilistic theory for the mathematical representation of uncertainty. The significant innovation of this method is that it allows for the allocation of a probability to sets or intervals. The Dempster-Shafer theory does not require an assumption regarding the probability of the individual constituents of the set or interval. This is potentially valuable for the evaluation of risk in engineering applications where it is not possible to obtain a precise measurement from experiments, or where knowledge is obtained from expert elicitation. Several concepts of this theory, believed to be useful to risk assessment in this thesis, are the basic probability assignment (bpa) and its combination rule of multiple sources. More details about the Dempster-Shafer theory can be found in Appendix C.

(1) Basic probability assignment function

The basic probability assignment (bpa), also called basic evidence assignment, is an important concept in Dempster-Shafer theory. The term “basic probability assignment”, usually represented by \( m \), does not generally refer to probability in the classical sense. It defines a mapping from the power set to the interval between 0 and 1, where its value of the null set is 0 and the summation of all subsets of the power set is 1. The value of the basic probability assignment for a given set \( A \), \( m(A) \), expresses the proportion of all relevant and available evidence that supports the claim that a particular element of \( X \) (the universal set) belongs to the set \( A \) but no particular subset of \( A \) (Klir and Wierman, 1998). That is the value of \( m(A) \) pertains only to the set \( A \) and makes no additional claims about any subsets of \( A \). Any further evidence on the subsets of \( A \) would be represented by another bpa (i.e. \( B \subset A \), where \( m(B) \) would be the bpa for subset \( B \)). Formally, this description of \( m \) can be represented with the following equations:

\[
m : P(X) \rightarrow [0,1] \\
m(\Phi) = 0 \\
\sum_{A \in P(X)} m(A) = 1
\]

where \( P(X) \) represents the power set of \( X \), \( \Phi \) is the null set, and \( A \) is a subset of the power set (Klir and Wierman, 1998).

For example, the failure frequency of a pipe in a distribution system is represented in terms of different levels—Low (L), Medium (M), and High (H). Thus in this case \( X=\{L, M, H\} \) there is a power set which contains 8 subsets: \( \Phi, \{L\}, \{M\}, \{H\}, \{L, M\}, \{L, H\}, \{M, H\}, \{L, M, H\} \).
In an assessment of the pipe failure, an analyst may report with 50% confidence that the likelihood is high, \{H\}, and with 30% confidence that the likelihood is medium or high, \{M, H\}. The basic probability assignment of this assessment \(A\) can be written as: \(m(A)_{H} = 0.5\), \(m(A)_{M, H} = 0.3\), and \(m(A)_{P(A)} = 0.2\) because \(m(A)_{H} + m(A)_{M, H} + m(A)_{P(A)} = 1\). In this example, \{H\} is a subset of \{M, H\}, but their basic assignments, i.e., 0.5 and 0.3 respectively, are different.

(2) Combination rule

One of the kernels in the Dempster-Shafer theory is its combination rule by which the evidence/information from different sources are combined. The original combination rule of multiple basic probability assignments is a generalisation of Baye’s rule (Dempster, 1967). This rule strongly emphasises the agreement between multiple sources and ignores all the conflicting evidences through a normalisation factor. This can be considered as a strict AND-operation. The rule assumes that the information sources are independent and use the orthogonal sum to combine multiple sources. Formally, the combination of two sources \(m_{12}\) is calculated from the aggregation of two basic probability assignments \(m_1\) and \(m_2\) in the following manner:

\[
m_{12}(A) = \frac{\sum_{B \subseteq A} m_1(B)m_2(C)}{1 - K}
\]

where \(K = \sum_{B \subseteq \emptyset} m_1(B)m_2(C)\) (4.5)

Here \(K\) is called the degree of conflict. It represents the basic probability associated with the two conflicting sources (i.e., source 1 and source 2). This is determined by summing the products of the basic probability assignments of all sets where the intersection is null. This rule is commutative, associative, but not idempotent or continuous.

To continue the example illustrated above, another analyst gives a second assessment on the pipe failure probability with 80% confidence that it is high and with 20% confidence that it is medium. By aggregating the assessments of two analysts, the overall probability levels of the pipe can be obtained (Figure 4.5). Details of performing this combination are available in the Appendix C. The combined results show that failure probability of the pipe is Medium with 11% confidence and High with 89% confidence. These are in accordance with the two analysts evaluations.
Although the original combination rule discussed above can give reasonable results in most cases, it also receives serious criticism when there is significant conflict between the information sources. Consequently, other researchers (Zadeh, 1986; Yager, 1987; Inagaki, 1991; Zhang, 1994; Dubois and Prade, 1986, 1992) have developed modified Dempster rules that attempt to represent the degree of conflict in the final result. In this research, weights factors are considered for each elements and weighted combination rule is used to discount the conflicts among multiple sources. This method has also been applied to other studies (Yang et al., 2006; Yang and Singh, 1994).

### 4.4 Aggregative Risk Assessment of Water Supply Systems

#### 4.4.1 Applications of Fuzzy Sets in Risk Assessment

With respect to the definitions in Section 1.2.2.1 and given that vulnerability is unity, risk is then a measure determined by likelihood and severity of a hazard, i.e.,

\[
R = L \otimes S
\tag{4.6}
\]

where \(R\) is the risk associated with a hazard or threat; \(L\) represents the likelihood of the hazard; and \(S\) represents the severity or consequence of the hazard; \(\otimes\) denotes the multiplication relationship between likelihood and severity. This definition has been applied to risk assessment in many engineering systems such as software (Lee, 1996), environment (Sadiq et al., 2004), mechanical (Wang et al., 1995), process industries (Khan, et al., 2002, 2004; Khan and Haddara, 2003; Krishnasamy, et al., 2005), water pipe deterioration analysis (Kleiner et al., 2006a,b), etc.

In this thesis, Equation (4.6) is used to describe the risk levels associated with each hazard in the water supply system.

This definition indicates that if likelihood \((L)\) and/or severity \((S)\) are represented by fuzzy numbers, \(R\) will be a fuzzy number as well. The operator \(\otimes\) is thus representing the
multiplication of two fuzzy numbers. This can be performed by using fuzzy operation rules. Specifically, if both likelihood and severity are represented as crisp numbers, risk will be also a crisp number. The operator $\otimes$ reduces to normal multiplications between two real numbers.

According to the above discussions, the application of fuzzy sets theory in this study is depicted by Figure 4.6.

4.4.1.1 Linguistic risk levels

To discuss the levels that are used to represent likelihood, severity, and risk in this study

4.4.1.2 Fuzzy representations of risk factors

To discuss the calculation of fuzzy risk based on fuzzy likelihood and severity

4.4.1.3 Determination of risk levels

To determine the relative risk levels (or risk contributions) of a specific risk

Figure 4.6 Structure of applications of fuzzy sets theory to aggregative risk assessment

4.4.1 Linguistic Risk Levels

In risk assessment, it is not unusual that analysts prefer to describe risks in terms of different levels, such as high, medium, low, etc., rather than absolute values. This is necessary for several reasons.

Firstly, risk is not absolutely objective in nature, but rather relative and subjective. It is usually a fuzzy concept in the sense that there does not exist a unique risk associated with a hazardous event occurring in a given period (Karwowski and Mital, 1986; Feagans and Biller, 1980). Then risk assessment deals with quantities which are inherently imprecise and whose future values are uncertain. Usually, linguistic categories or levels (e.g., very high, high, medium, low, very low, etc), instead of absolute numbers, are usually adopted because each linguistic category or level can deal with the various and uncertain risk values by including a range or set of numbers.

Secondly, the real meaning of risk in practice is varied and application-specific; risks are thus measured in different units; and even similar risk values may indicate different levels of influences in different applications. Furthermore, risks with different units are difficult to compare or aggregate in a risk assessment. Alternative to the absolute numerical values, risk levels, one of the relative measures, can be more easily used by risk analysts, system managers, policy makers, and general users.
Lastly, it is not unusual that analysts have more confident to give risk evaluations in terms of risk levels rather than numerical values in certain circumstances. Risk of human wilful attack on water supply assets is one of the examples.

Therefore, this thesis uses risk levels to represent the risk items. Furthermore, the numbers of risk levels are also studied by many researchers as an important factor in practical risk assessment. Experiments by psychologists, such as those of Miller (1956), suggest that the maximum number of chunks of information is on the order of seven, plus or minus two. With respect to this, it is often recommended that the number of categories be restricted to no more than seven (Karwowski and Mital, 1986). Normally, too few levels will not be adequate to represent the real knowledge of analysts, while too many levels will bring extra difficulties in the following assessment. Therefore, seven categories of linguistic representations are adopted in this paper to express the degrees of likelihood and severity of hazards (Table 4.1). The seven categories or grades are extremely low, very low, slightly low, middle, slightly high, very high, and extremely high.

Based on the definition of risk and seven grades for likelihood and severity, the relative grades of risk are generated based on seven rules (Table 4.2). In this research, seven basic categories (i.e., extremely low, very low, slightly low, middle, slightly high, very high, and extremely high) are defined as standard risk grades or categories, and the risk distribution of each hazard can be evaluated on the seven grades.

Table 4.1 Linguistic levels and explanations used to evaluate likelihood and severity of a hazard

<table>
<thead>
<tr>
<th>Linguistic representation</th>
<th>Descriptions of the linguistic values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Likelihood</td>
<td></td>
</tr>
<tr>
<td>Extremely low (EL)</td>
<td>Practically impossible during the lifetime of the component</td>
</tr>
<tr>
<td>Very low (VL)</td>
<td>Conceivable, but highly unlikely</td>
</tr>
<tr>
<td>Slightly low (SL)</td>
<td>Only remotely possible</td>
</tr>
<tr>
<td>Middle (M)</td>
<td>Possible</td>
</tr>
<tr>
<td>Slightly high (SH)</td>
<td>Unusual but possible</td>
</tr>
<tr>
<td>Very high (VH)</td>
<td>Quite possible</td>
</tr>
<tr>
<td>Extremely high (EH)</td>
<td>Might well be expected</td>
</tr>
<tr>
<td>2. Severity*</td>
<td></td>
</tr>
<tr>
<td>Extremely low (EL)</td>
<td>Influences to the users are minor and ignored</td>
</tr>
<tr>
<td>Very low (VL)</td>
<td>Few users are influenced</td>
</tr>
<tr>
<td>Slightly low (SL)</td>
<td>A small part of users are influenced</td>
</tr>
<tr>
<td>Middle (M)</td>
<td>Part of the users are influenced</td>
</tr>
<tr>
<td>Slightly high (SH)</td>
<td>Many users are influenced</td>
</tr>
<tr>
<td>Very high (VH)</td>
<td>A large part of the users are influenced</td>
</tr>
<tr>
<td>Extremely high (EH)</td>
<td>Catastrophic risks, or most users are influenced</td>
</tr>
</tbody>
</table>

*Descriptions of severity levels are only a reference here, which can be changed when applied to according to the requirements of practical risk assessment.
Table 4.2 Linguistic levels and explanations used to evaluate risks

<table>
<thead>
<tr>
<th>Linguistic representation</th>
<th>Descriptions of the linguistic values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely low (EL)</td>
<td>If both likelihood and severity are extremely low</td>
</tr>
<tr>
<td>Very low (VL)</td>
<td>If both likelihood and severity are very low</td>
</tr>
<tr>
<td>Slightly low (SL)</td>
<td>If both likelihood and severity are slightly low</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>If both likelihood and severity are medium</td>
</tr>
<tr>
<td>Slightly high (SH)</td>
<td>If both likelihood and severity are slightly high</td>
</tr>
<tr>
<td>Very high (VH)</td>
<td>If both likelihood and severity are very high</td>
</tr>
<tr>
<td>Extremely high (EH)</td>
<td>If both likelihood and severity are extremely high</td>
</tr>
</tbody>
</table>

### 4.4.1.2 Fuzzy Representations of Risk Factors

After the determination of linguistic levels for likelihood, severity, and risk, it is required to determine the relative mathematical expressions by using membership functions of fuzzy numbers. However, the determination of membership function is also hard and complicated. Any shape of a membership function is possible, but the selected shape should be justified by available information. Ross (2004) and Bilgic and Turksen (1999) discussed several methods of determining the membership functions. Meanwhile, it is also believed, in some cases, that the expressions of membership functions are not the dominant factors in engineering applications (Klir and Yuan, 1995). Therefore, this research assumes that the forms of membership functions would not influence the analysis results dramatically, and adopts the simple triangular fuzzy numbers to represent the basic risk factors.

#### (1) Fuzzy representation of likelihood

Triangular membership functions are adopted to mathematically represent the likelihood levels of hazards in this research (Table 4.1), which are determined by the following equations and illustrated in Figure 4.7.

\[
\mu^L(x) = \begin{cases}
1 - 6x, & 0 \leq x \leq \frac{1}{6} \\
0, & \frac{1}{6} \leq x \leq 1.0
\end{cases}
\]

\[
\mu^L(x) = \begin{cases}
0, & 0 \leq x \leq \frac{k - 2}{6} \\
6x - (k - 2), & \frac{k - 2}{6} \leq x \leq \frac{k - 1}{6} \\
k - 6x, & \frac{k - 1}{6} \leq x \leq \frac{k}{6} \\
0, & \frac{k}{6} \leq x \leq 1.0
\end{cases} \quad (k=2,3,\ldots,6) \quad (4.7)
\]
\[ \mu^L(x) = \begin{cases} 0, & 0 \leq x \leq \frac{5}{6} \\ 6x - 5, & \frac{5}{6} \leq x \leq 1.0 \end{cases} \]

where \( \mu^L(x) \) denotes the membership function of likelihood; \( x \) denotes the values of likelihood.

These above equations and Figure 4.7 show that likelihood is defined between 0 and 1. An extremely low (EL) fuzzy number is defined in the range \([0,1/6]\), and each value of likelihood in this interval has a different membership value to represent the grade or confidence that the likelihood is viewed as extremely low. Similarly, other likelihood levels, i.e. very low (VL), slight low (SL), medium (M), slightly high (SH), very high (VH), and extremely high (EH), are defined as fuzzy numbers on the intervals \([0,1/3]\), \([1/6,1/2]\), \([1/3,2/3]\), \([1/2,5/6]\), \([2/3,1]\), and \([5/6,1]\) respectively.

(2) Fuzzy representation of severity

In practice, severity can be represented by different measures such as monetary loss, populations influenced by the risk, mean time to failure, etc. These measures are valued with different dimensions, which thus introduce difficulties in comparing risks by using Equation (4.6). To overcome this difficulty, severity is usually normalised to a dimensionless measure defined in the interval \([0,1]\). The normalisation process is always performed by dividing the severity value with the maximum value of possible severities. For example, a water distribution is composed of a pump and a set of pipes that supplies water to 10,000 persons. However, during a failure of the pump, there were about 2000 persons (Figure 4.8a) who were influenced and had no accessible water. By using fuzzy numbers, the severity of this pump failure is illustrated in Figure 4.8b.

From the above example, a normalisation process can therefore be represented by
$S \in [S_{\min}, S_{\max}] \Leftrightarrow S \in [S_{\min} / S_{\max}, S_{\max} / S_{\max}] \Leftrightarrow S \in [0,1]$ \hspace{1cm} (4.8)

where $S$ denotes the severity value, which can be measured by monetary loss, population of influenced people, mean time to failure, etc.; $S_{\min}(=0)$ and $S_{\max}$ denote the possible minimum and maximum values of severities, respectively.

Figure 4.8 An example of normalizing severity measure

After normalisation, severity levels become dimensionless variables and can be represented by fuzzy numbers defined in $[0,1]$. With respect to the normalised severity, triangular fuzzy functions are determined by the following equations and illustrated in Figure 4.9.

\[
\mu^S(x) = \begin{cases} 
1 - 6x, & 0 \leq x \leq \frac{1}{6} \\
0, & \frac{1}{6} \leq x \leq 1.0 
\end{cases}
\]

\[
\mu^S(x) = \begin{cases} 
0, & 0 \leq x \leq \frac{k - 2}{6} \\
6x - (k - 2), & \frac{k - 2}{6} \leq x \leq \frac{k - 1}{6} \\
k - 6x, & \frac{k - 1}{6} \leq x \leq \frac{k}{6} \\
0, & \frac{k}{6} \leq x \leq 1.0 
\end{cases} \hspace{1cm} (k=2,3,\ldots,6) \hspace{1cm} (4.9)
\]

\[
\mu^S(x) = \begin{cases} 
0, & 0 \leq x \leq \frac{5}{6} \\
6x - 5, & \frac{5}{6} \leq x \leq 1.0 
\end{cases}
\]

where $\mu^S(x)$ denotes the membership function of severity; $x$ denotes normalised values of severity.
These equations and Figure 4.7 show that severity is defined between 0 and 1. A fuzzy number of extremely low (EL) is defined in the range [0,1/6], and each value of severity in this interval has a different membership value to represent the grade or confidence that the severity is viewed as extremely low. Similarly, other severity levels, i.e. very low (VL), slight low (SL), medium (M), slightly high (SH), very high (VH), and extremely high (EH), are defined as fuzzy numbers on the intervals [0,1/3], [1/6,1/2], [1/3,2/3], [1/2,5/6], [2/3,1], and [5/6,1] respectively.

![Figure 4.9 Fuzzy representation of severity levels used in this study](image)

### (3) Fuzzy representation of risk

According to the definitions in Table 4.2, the standard categories of risk levels can thus be determined by

\[
R_{\text{Extremely Low}} = L_{\text{Extremely Low}} \otimes S_{\text{Extremely Low}} \\
R_{\text{Very Low}} = L_{\text{Very Low}} \otimes S_{\text{Very Low}} \\
R_{\text{Slightly Low}} = L_{\text{Slightly Low}} \otimes S_{\text{Slightly Low}} \\
R_{\text{Medium}} = L_{\text{Medium}} \otimes S_{\text{Medium}} \\
R_{\text{Slightly High}} = L_{\text{Slightly High}} \otimes S_{\text{Slightly High}} \\
R_{\text{Very High}} = L_{\text{Very High}} \otimes S_{\text{Very High}} \\
R_{\text{Extremely High}} = L_{\text{Extremely High}} \otimes S_{\text{Extremely High}}
\]

where \( R \) denotes fuzzy risk variable; \( L \) and \( S \) denote fuzzy variables of likelihood and severity respectively.

This study adopts the \( \alpha \)-cut sets method that was proposed by Kaufmann and Gupta (1991) to derive the equations for risks. The results are shown as below, while the detail process of the derivation can be found in Appendix B.
Chapter 4 Quantitative Aggregative Risk Assessment of Water Supply Systems

\[ \mu^R(x) = \begin{cases} 
1 - 6\sqrt{x}, & 0 \leq x \leq \left(\frac{1}{6}\right)^2 \\
0, & \left(\frac{1}{6}\right)^2 \leq x \leq 1.0 
\end{cases} \]

\[ \mu^R(x) = \begin{cases} 
6\sqrt{x} - (k - 2), & 0 \leq x \leq \left(\frac{k - 1}{6}\right)^2 \\
(k - 6\sqrt{x}), & \left(\frac{k - 1}{6}\right)^2 \leq x \leq \frac{k}{6} \\
0, & \frac{k}{6} \leq x \leq 1.0 
\end{cases} \quad (k=2,3,\ldots,6) \quad (4.11) \]

\[ \mu^R(x) = \begin{cases} 
0, & 0 \leq x \leq \left(\frac{5}{6}\right)^2 \\
6\sqrt{x} - 5, & \left(\frac{5}{6}\right)^2 \leq x \leq 1.0 
\end{cases} \]

where \( \mu^R(x) \) denotes the membership function of risk.

The above equations show that risks are still fuzzy numbers, but not triangular as fuzzy numbers for likelihood or severity (Figure 4.10). Risks determined by the above equations are served as standard risk grades with which a calculated risk can be compared in order to obtain its relative risk levels (or risk distributions) for the calculated risk.

![Figure 4.10 Fuzzy representation of risk categories used in this study](image-url)
4.4.1.3 Determinations of Risk Levels

Normally a calculated risk is not always limited to the seven standard risk developed in the above section, however, it could be intersected with several risk grades with different degrees. This can be illustrated by an example as “The likelihood of the flood occurrence during summer time is very high, while its relative influences to the quality of river A is slightly low, how about the risk of flood to river A?” By using Equation (4.6), risk of the flood (hazard) can be obtained. However, it is not feasible to simply conclude that the risk is completely belonging to any one of the seven grades as it does not completely match with any single risk grade which has been determined by Equation (4.11) (Figure 4.11). The calculated risk intersects with several risk grades like Slightly low (SL), Medium (M), Slightly high (SH), and Very high (VH) with different degrees. The figure shows that the flood risk is close to Medium risk level and slightly moves to the direction of Slightly high. To give a reasonable evaluation, it is thus necessary to determine the degrees of intersections or compatibility between the calculated risk and the seven standard risk categories. Based on the degree of intersection, risk levels of calculated risks can then be easily determined.

![Figure 4.11 Intersections between fuzzy numbers](image)

(1) The degree of intersection between fuzzy numbers

In order to determine the intersection or compatibility between risks, this research supposes two axioms as follow:

Axiom 1. If the calculated risk $R$ is entirely included within a predefined fuzzy risk grade $R_i$ ($i=1,\ldots,7$), then it should completely belong to $R_i$.

Axiom 2. If $R$ intersects with several predefined fuzzy risk grades at the same time but it is not entirely included in any one of them, then it should belong to each of them to a certain degrees.

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Several methods are available, but none of them can be described as the best method (Chen, 2003). Extensive studies have been performed to determine the intersections of fuzzy numbers. Similarity or compatibility is one of most important factors that has been proposed in the fuzzy risk analysis (Chen, 1996; Kangari et al.) and fuzzy decision making (Chen and Chen, 1998; Hsu and Chen, 1996; Lee, 1999). However, the existing methods could not perfectly meet the requirements of the above two axioms. Therefore, this research develops two methods, named overlap area method and weighted overlap area method, respectively, to determine the intersecting degrees between two fuzzy numbers.

### i. Overlap Area (OA) method

The overlap area (shaded area in Figure 4.12) between a calculated risk and a predefined risk, as a fraction of the total area of the total area of the calculated risk, can represent the degree of their intersections; that is,

\[
C_{A,B} = \frac{OA_{A,B}}{AR_B} \tag{4.12}
\]

\[
OA_{A,B} = \int_0^1 \min(\mu_A(x), \mu_B(x)) dx
\]

\[
AR_B = \int_0^1 \mu_B(x) dx
\]

where \(C_{A,B}\) is the intersecting degree between fuzzy numbers predefined risk \((A)\) and calculated risk \((B)\); \(OA_{A,B}\) is the overlap area of \(A\) and \(B\); and \(AR_B\) is the area of calculated risk \(B\); \(\mu_A(x)\) and \(\mu_B(x)\) are the membership functions of risk \(A\) and risk \(B\) respectively.

Alternatively, Equation (4.12) can also be written in terms of \(\alpha\)-cuts as

\[
C_{A,B} = \frac{\int_0^1 \min(0, (a_R^\alpha - b_L^\alpha)) d\alpha}{\int_0^1 (b_R^\alpha - b_L^\alpha) d\alpha} = \frac{\int_0^1 \min(0, (a_R^\alpha - b_L^\alpha)) d\alpha}{\int_0^1 b_R^\alpha - b_L^\alpha d\alpha} \tag{4.13}
\]

where \(a_L^\alpha, a_R^\alpha, b_L^\alpha,\) and \(b_R^\alpha\) are the bounds of \(\alpha\)-cuts of \(A\) and \(B\) respectively.

Equations (4.12) and (4.13) are identical from mathematical point of view and produce the same results for compatibility between risks \(A\) and \(B\). The difference between the two equations is that the latter is based on \(\alpha\)-cuts of fuzzy numbers and easier to code in a programming language.
Figure 4.12 Compatibility analyses to quantify the degrees of intersection between two fuzzy numbers

**ii. Weighted Overlap Area (WOA) method**

In the example shown in Figure 4.13, if the overlap areas, $OA_{A,B}$ and $OA_{A,C}$, are the same and areas of calculated risks $B$ and $C$ (i.e., $A_B$ and $A_C$) are also same, then the intersecting degrees of $C_{A,B}$ and $C_{A,C}$, determined by Equation (4.13), are identical. However, risk analysts might be prone to believe that calculated risk $C$ is closer to predefined risk ($A$) than risk $B$ because the most possible value of $C$ is closer to $A$ than that of $B$ (Figure 4.13). This indicates that an overlap in higher membership values is more preferable to an overlap in a low membership values area.

Figure 4.13 Overlap analyses between fuzzy numbers

To account for this, an modified compatibility or intersecting degree measure is proposed here. This is achieved by considering the weights of membership values in Equation (4.13)

$$C_{A,B}^{\alpha} = \int_0^1 \frac{\min\{0,(a_R^\alpha - b_L^\alpha)\}}{b_R^\alpha - b_L^\alpha} \alpha d\alpha = 2 \int_0^1 \frac{\min\{0,(a_R^\alpha - b_L^\alpha)\}}{b_R^\alpha - b_L^\alpha} \alpha d\alpha$$  \hspace{1cm} (4.14)
where $C^{w}_{A,B}$ is the weighted intersecting degree between fuzzy numbers predefined risk ($A$) and calculated risk ($B$); $\alpha$ is used as a weighted factor in this equation.

(2) Determination of risk levels

When applying the overlap area method or weighted overlap area method, the degrees of intersection between calculated risk and each of the seven predefined risk grades, denoted as $C_{Ri,R}$ ($i=1,\ldots,7$), are determined respectively. The closer the calculated risk ($R$) is to the $i$th risk grade ($R_i$), the larger $C_{Ri,R}$ is. More specifically, if $R$ satisfies the requirement of Axiom 1, $C_{Ri,R}$ is equal to 1 and $C_{Rj,R}$ is 0 for any $j=1,\ldots,7$ and $j\neq i$. Therefore, degree of the calculated risk belonging to a risk grade $i$ ($i=1,\ldots,7$) can thus be obtained by

$$r_i = \frac{C_{Ri,R}}{\sum_{j=1}^{7} C_{Rj,R}} (j=1,2,\ldots,N) \quad (4.15)$$

where $r_i$ denotes the degrees of calculated risk $j$ belonging to risk grade $i$. $r_i$ can also be viewed as a kind of risk distribution of risk $j$ to risk grade $i$. Normally, in probability theory, risk distribution is defined with respect to a set of points, while risk distribution, in this fuzzy environment, is defined on a set of intervals (i.e., risk grades EL,….EH). This is closer to the real situation of risk assessment and can cover the uncertainty due to incomplete data.

For the example shown in Figure 4.11, risk levels of the flood during summer time are obtained by using overlap area method and weighted overlap method respectively. The results are listed in Table 4.3.

<table>
<thead>
<tr>
<th>Risk levels</th>
<th>EL</th>
<th>VL</th>
<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
</tr>
</thead>
<tbody>
<tr>
<td>OA</td>
<td>0</td>
<td>0</td>
<td>0.128</td>
<td>0.563</td>
<td>0.288</td>
<td>0.021</td>
<td>0</td>
</tr>
<tr>
<td>WOA</td>
<td>0</td>
<td>0</td>
<td>0.046</td>
<td>0.796</td>
<td>0.157</td>
<td>0.001</td>
<td>0</td>
</tr>
</tbody>
</table>

OA—Overlay Area method; WOA—Weighted Overlay Area method;
The above analyses are used in this study to determine the risk levels of hazards to a water supply system as shown by Figure 4.1.

**3) Fuzziness of the risk level**

It is shown that risk levels are defined on seven grades. A risk may have different degrees, between 0 and 1, to multiple risk grades simultaneously. This can also be viewed as a fuzzy number defined on discrete set, i.e., seven risk grades. In order to evaluate the degree of fuzziness of this fuzzy number, a measure proposed by de Luca and Termini (1972) and Zimmermann (1991) is adopted in this thesis:

\[
\sum_{i=1}^{n} K \left( -\mu_A(x_i) \ln \mu_A(x_i) - (1 - \mu_A(x_i)) \ln (1 - \mu_A(x_i)) \right)
\]

where \(d(A)\) is the measure of fuzziness; \(\mu_A(x_i)\) is the degree to risk grade \(i\); \(i\) denotes the number of risk grades, i.e., \(i=1,\ldots,7\); and \(K\) is assumed to be 1 in this thesis.

Based on this equation, measures of fuzziness can be determined for the risk levels shown in Table 4.3. The measure of fuzziness of the risk level determined by overlay area method is 1.770, while the measure of fuzziness determined by weighted overlay area method is 1.135. This shows that the latter evaluation is less uncertain compared with the former. Application of this measure of fuzziness will be specifically discussed in Chapter 6.

### 4.4.2 Risk Aggregation Using Dempster-Shafer Theory and Fuzzy Sets

After risk levels of hazards have been determined, this study adopts combination rule of Dempster-Shafer theory to aggregate these results on the framework of aggregative risk assessment. The main task of Dempster-Shafer theory is to infer the risk of some hypotheses by collecting and combining relevant evidence for or against these hypotheses. This study views risk
levels of hazards as evidence, and risk levels of an object as a hypothesis. The determination of
the risk levels of an object is thus a process of combining or aggregating the evidence from its
multiple hazards. Similarly, risk levels of objects are further viewed as evidence to determining
risk levels of subsystems and the overall water supply system. Therefore, by repeatedly using the
combination rule on aggregative hierarchical structure, risk levels can be obtained for objects,
subsystems, and overall system, respectively.

However, modifications are needed for original combination rule of Dempster-Shafer theory
before it is applied to this study due to two reasons. Firstly, this study represents basic evidences
(i.e., risk levels of each hazard) in terms of fuzzy evaluating numbers rather than probabilistic
numbers. However, original combination is usually conducted on probability numbers. Secondly,
risk grades are required to be mutually exclusive and exhaustive in Dempster’s original
combination rule. This requires intervals representing risk grades either in complete conflict (i.e.,
the intersection in null) or not in conflict at all. However, this is not the case in this study. For
example, risk grades Very low (VL) and Slightly low (SL), described by two fuzzy numbers, are
neither completely conflicting nor completely intersected, but partially intersected. Special study
is therefore required to include the degrees of conflict between fuzzy numbers into the
combination of Dempster-Shafer theory.

Considering the fuzzy application of Dempster-Shafer theory, many researchers (Zadeh, 1979;
Ishizuka et al., 1982; Yager, 1982; and Ogawa and Fu, 1985) have conducted the generalisations
in different ways. However, there are four problems with these extensions recognised by Yen
(1990) who developed a new approach based on the concept of $\alpha$-cuts in fuzzy sets theory by
developed a method of managing complex fuzzy information with Dempster-Shafer theory by
analysing relationships between the probability of fuzzy event and its membership functions.
Yang et al. (2006) also developed a generalised method which can deal with triangular or
trapezoidal fuzzy numbers. This research adopts critical results of Ishizuka and Yen’s methods to
modify the original combination rule Dempster-Shafer theory. In the following part, this
modification is discussed in two parts: (1) degrees of conflict between fuzzy numbers, and (2)
combination in fuzzy sets theory.

4.4.2.1 Degrees of Conflict between Fuzzy Numbers

Normally the intersection between two different fuzzy numbers is a subnormal fuzzy set (Figure
4.15). It is this subnormal fuzzy set that introduces partial conflict or partial intersection between
two fuzzy numbers.
Ishizuka et al. (1982) proposed a measure to evaluate the degree of conflict between these two fuzzy sets (i.e., A and B). The measure is determined by

\[ 1 - J(A, B) = 1 - \frac{\max_x \mu_{A \cap B}(x)}{\min_x [\max_x \mu_A(x), \max_x \mu_B(x)]} \]  
(4.17)

where \(A \cap B\) denotes the intersection of A and B by following fuzzy intersection rule; \(J(A, B)\) represents the degree of intersection between them. If sets A and B are normal fuzzy sets, that is \(\max_x \mu_A(x) = \max_x \mu_B(x) = 1\), then

\[ 1 - J(A, B) = 1 - \max_x \mu_{A \cap B}(x) \]  
(4.18)

which is the degree of conflict proposed by Yen (1990).

### 4.4.2.2 Combination Rule in Fuzzy Set Theory

By considering the degree of intersection between two fuzzy numbers, the combination of risk levels from two hazards can be determined by

\[ m_{12}(R_i) = m_1(R_j) \oplus m_2(R_k) = \frac{\sum_{R_j \cap R_k = R_i} J(R_j, R_k) m_1(R_j) m_2(R_k)}{1 - \sum_{R_j \cap R_k} [1 - J(R_j, R_k)] m_1(R_j) m_2(R_k)} \quad (i,j,k=1,\ldots,14) \]

\[ m_1(R_j) = w_1 r_{i,j} \quad \text{and} \quad m_2(R_k) = w_2 r_{i,k} \]  
(4.19)

where \(R_i, R_j, \) and \(R_k\) represent predefined risk grades \(i, j,\) and \(k\) respectively if \(i, j,\) and \(k=1,\ldots,7\); \(R_{14}\) represents the degree to \(\Theta\); \(R_i (i=8,\ldots,13)\) represents the partial intersections between two adjacent predefined risk categories respectively; \(m_{12}(R_i)\) denotes the combined degree to risk level \(i\) from two hazards 1 and 2; \(m_1(R_j)\) denotes degree to risk category \(j\) from hazard 1; and \(m_2(R_k)\) denotes degree to risk category \(i\) from hazard 2; \(w_1\) and \(w_2\) are the weights of hazards 1 and 2.
respectively; $r_{j1}$ is the degree to risk category $j$ of the risk introduced by hazard 1; and $r_{k2}$ is the degree to risk category $k$ of the risk introduced by hazard 2.

Detailed derivation of the above equation is available in Appendix C.

The application of the above equation is illustrated by a simple example: “In a water supply system, quality of the river is potentially threatened by two hazards, i.e., flood and accidental industrial discharge. According to the analysis based on historical data, flood with the return period of 10 years has very high influences on the river quality. The possibility of industrial charge failure is slight low, while its possible consequence will be extremely high according to the industry’s record and experts’ opinion. With these data, what about the risk levels of the river under the influences of these two hazards?”

Firstly likelihoods and severities of the hazards (i.e., flood and industrial discharge) are represented in terms of probability number and fuzzy number, respectively (Table 4.4). In this example, likelihood of flood is represented by a probability number 0.1; likelihood of accidental industrial discharge is represented by a fuzzy number. Risks of these two hazards are obtained (Figure 4.16) as the multiplication of relative likelihoods and severities.

<table>
<thead>
<tr>
<th>Hazards</th>
<th>Likelihood</th>
<th>Severity</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood</td>
<td>0.1</td>
<td>Very high (2/3,5/6,1)</td>
<td>(0.067, 0.083, 0.1)</td>
</tr>
<tr>
<td>Industrial discharge</td>
<td>Slight low (1/6,1/3,0.5)</td>
<td>Extremely high (5/6,1,1)</td>
<td>(0.14, 0.33, 0.5)</td>
</tr>
</tbody>
</table>

Figure 4.16 Risks caused by hazards of flood and industrial discharge and their relationships with predefined risk categories
Figure 4.16 shows that both of the risks introduced by flood and industrial discharge do not completely intersect with the seven predefined risk grades. Weighted overlay area method is adopted here to calculate the risk levels of these two hazards (Table 4.5). Results show that risk of flood is less uncertain and belongs to risk levels VL and SL with degrees of 0.04 and 0.96 respectively, while risk of industrial discharge is more uncertain and has degrees of 0.027, 0.598, 0.372, and 0.02 to risk levels SL, M, SH, and VH respectively. Suppose weights of flood and industrial discharge are 0.4 and 0.6 in this example. Then the combined results are obtained by using Equation (4.19). The results are given in Table 4.5 and Figure 4.17. It is shown that the river example has degrees of 0.01, 0.359, 0.428, 0.202, and 0.001 to risk levels VL, SL, M, SH, and VH, respectively. These values can also be viewed as a risk distribution defined on the seven predefined risk grades. For the river, it has highest possible density in risk level M, then are the risk levels of SL, SH, VL, and EH in a decreasing order. This also indicates that more evidence from hazards support the risk level M, and no evidence supports risk levels of EL and EH.

Table 4.5 Results of example showing the combination of risk levels from flood and industrial discharge

<table>
<thead>
<tr>
<th>Risk levels</th>
<th>Weight</th>
<th>EL</th>
<th>VL</th>
<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood</td>
<td>0.4</td>
<td>0</td>
<td>0.04</td>
<td>0.96</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Industrial</td>
<td>0.6</td>
<td>0</td>
<td>0</td>
<td>0.027</td>
<td>0.598</td>
<td>0.373</td>
<td>0.002</td>
<td>0</td>
</tr>
<tr>
<td>Result</td>
<td>0</td>
<td>0</td>
<td>0.010</td>
<td>0.359</td>
<td>0.428</td>
<td>0.202</td>
<td>0.001</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 4.17 Results of combining risks introduced by flood and industrial discharge
4.4.3 Aggregative Risk Levels of Objects

(1) Determination of risk levels of objects

Based on above discussions and the framework of aggregative risk assessment (Figure 4.1), a process is formed to evaluate risk levels of objects (Figure 4.18).

This process firstly determines the risks levels of hazards and then aggregates these values to obtain risk levels of failure states and the object by using modified combination rule of Dempster-Shafer theory.
Shafer theory. This is also a general process that can be applied to evaluate the aggregative risk levels of all basic components (i.e., water source, water treatment plant, pipe, pump, or tank) in a water supply system. This study also codes the process (Figure 4.18) in an object-oriented programming language, Visual C++, to facilitate the calculation.

In order to illustrate the application of the above process, a river under threat of multiple hazards is considered here as an example. “There are several hazards that influence the quality of river as a water source. Firstly, since the river is in a flooding plain, it is influenced by floods with low frequencies (i.e., lower than 1/50) according to historical records and experience. Furthermore, an industrial discharging point and a sewage discharge point are along the bank of the river. Their improper treatments or operations are two potential hazards to the river quality and consequently influence the treating process. In addition, human-related wilful chemical/biological contamination is also a potential threat to river quality as suggested by experts and managers. Lastly, there are also some regular transporting vehicles on this river. Historical studies show that pollutant spills of these vehicles are potentially contaminating the river as well. With respect to these above hazards, how about the risk levels of the river?”

Figure 4.19 Aggregative risk assessment of a river example

Based on the states transition diagrams of river object in Chapter 3, a framework is formed to represent aggregative risk assessment of the river example (Figure 4.19). This framework shows that flood is related to natural hazard failure state; human-related chemical/biological contamination, industrial, and sewage discharges are hazards related to state of human-caused failures; and spills from vehicles are related to interdependence failure state of the river. In this structure, \( R \) is used to denote the risk levels of the river contamination; \( F_1, F_2, \) and \( F_3 \) are risk
levels associated with failure states of natural hazards failure state, human-caused failure state, and interdependence respectively; $H_{ij}$ denotes the risk levels of hazard $j$ which is related to failure state $i$.

In the process of determining aggregative risk of the river, risks associated with these hazards are firstly determined in terms of their likelihoods and severities. Here without compromising the applicability of proposed method, likelihoods and severities are subjectively assumed and given in Table 4.6. In addition, weights factors are also subjectively assumed in this example.

### Table 4.6 Inputting data for the river example

<table>
<thead>
<tr>
<th>Object</th>
<th>State</th>
<th>Hazard</th>
<th>Weight</th>
<th>Likelihood</th>
<th>Severity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>$w_1=0.5$</td>
<td>$H_{21}$</td>
<td>$w_{21}=0.4$</td>
<td>Slightly low</td>
<td>Very high</td>
</tr>
<tr>
<td>$F_2$</td>
<td>$w_2=0.2$</td>
<td>$H_{22}$</td>
<td>$w_{22}=0.3$</td>
<td>Slightly low</td>
<td>Extreme high</td>
</tr>
<tr>
<td>$F_3$</td>
<td>$w_3=0.3$</td>
<td>$H_{23}$</td>
<td>$w_{23}=0.3$</td>
<td>Very low</td>
<td>Slightly low</td>
</tr>
<tr>
<td>$F_3$</td>
<td>$w_3=0.3$</td>
<td>$H_{31}$</td>
<td>$w_{31}=1$</td>
<td>Very low</td>
<td>Medium</td>
</tr>
</tbody>
</table>

Risk levels of hazards are determined (Table 4.7) by using the method discussed in Section 4.4.1.3. It is obvious that industrial discharge has the highest risk levels compared with other hazards, which is followed by sewage discharge, flood, spills, and human-related chemical/biological contamination in a decreasing order of risk levels. Then the aggregation is conducted to (Figure 4.19) to produce the risk levels of failure states. It is shown that failure state related to human factors has the highest levels, which indicates that mitigating measures on reducing human-related failures will be more effective to control the river quality than other measures. Lastly, by repeating the combination rules, risk levels of river are ultimately obtained. It belongs to the risk grades, from EL to SH, with degrees of 0.1%, 12.9%, 75.3%, 9.7%, and 1.9%, respectively.

Further, risk levels of the river distributed on more ranges of risk grades than those of failure states and the hazards, which indicates that the evaluation of river is more uncertain than that of each failure state or hazard. This is obvious and in accordance with the common sense of risk assessment because that there are always more data and information directly supporting the evaluations of hazards rather than the evaluations of the river in a practical risk assessment. There are two sources contributing to the uncertainty of risk evaluation of the river. One is from the incomplete information of hazards; the other is from the conflict multiple sources during the process of aggregation. The former uncertainty can be reduced by obtaining more data about the hazards, while the latter uncertain can be expected to reduce by modifying the aggregation methods. Therefore, the aggregative risk can not only provide general risk levels, but also,
indicate uncertainties associated with the evaluations. Both of them are very useful for decision makers to optimise their managing and maintenance plans in water supply systems.

Table 4.7 Risk levels associated with the river example

<table>
<thead>
<tr>
<th>Risk levels</th>
<th>EL</th>
<th>VL</th>
<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Representative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{11} )</td>
<td>0</td>
<td>0.119</td>
<td>0.881</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.101</td>
</tr>
<tr>
<td>( H_{21} )</td>
<td>0</td>
<td>0</td>
<td>0.046</td>
<td>0.796</td>
<td>0.158</td>
<td>0</td>
<td>0</td>
<td>0.274</td>
</tr>
<tr>
<td>( H_{22} )</td>
<td>0</td>
<td>0</td>
<td>0.026</td>
<td>0.598</td>
<td>0.374</td>
<td>0.002</td>
<td>0</td>
<td>0.320</td>
</tr>
<tr>
<td>( H_{23} )</td>
<td>0.015</td>
<td>0.510</td>
<td>0.464</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.068</td>
</tr>
<tr>
<td>( H_{31} )</td>
<td>0.005</td>
<td>0.191</td>
<td>0.753</td>
<td>0.051</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.102</td>
</tr>
<tr>
<td>( F_1 )</td>
<td>0</td>
<td>0.119</td>
<td>0.881</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.101</td>
</tr>
<tr>
<td>( F_2 )</td>
<td>0.003</td>
<td>0.119</td>
<td>0.163</td>
<td>0.546</td>
<td>0.168</td>
<td>0</td>
<td>0</td>
<td>0.233</td>
</tr>
<tr>
<td>( F_3 )</td>
<td>0.005</td>
<td>0.191</td>
<td>0.753</td>
<td>0.051</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.102</td>
</tr>
<tr>
<td>River (R)</td>
<td>0.001</td>
<td>0.129</td>
<td>0.753</td>
<td>0.097</td>
<td>0.019</td>
<td>0</td>
<td>0</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Figure 4.20 Risk levels of the river example

(2) Representative values of risk levels

Results of aggregative risk assessment provide a kind of risk distribution on predefined risk categories for hazards, failure states, or an object (Figure 4.20). With respect to the risk distribution, a representative value, similar to expected value in probability distribution, can also be obtained by using centroid method proposed by Yager (1980) as

\[
\text{Representative value of risk} = \sum_{i=1}^{7} C(R_i) m(R_i) 
\]

(4.20)

where \( C(R_i) \) is the centre of fuzzy risk category \( i \); \( m(R_i) \) is the degree to risk category \( i \). With both the final representative value and risk distributions, risk analysts can obtained a more comprehensive view about risks at object levels. Representative risk values of the river example
are shown in Table 4.7. The representative value can be used to rank the risk distributions, and has been applied in many engineering problems (Sadiq et al., 2004).

Additionally, since this method can consistently produce risk levels of the elements that are at different hierarchical levels, it facilitates different analysts to make risk evaluations on object, its failure states, or hazards respectively according to their different requirements.

### 4.4.4 Aggregative Risk Levels of Subsystems and System

Similar to the process of determining risk levels of objects, a process is also formed to determine the aggregative risk at system level (Figure 4.21).

Considering that, the exact hierarchy of whole/part relationship of a water system is closely related to its configuration and layout, this study uses a simple distribution network to illustrate the process of obtaining aggregative risks at system level (Figure 4.22).

![Figure 4.21 Process of aggregative risk assessment at system level based on combination rule of Dempster-Shafer theory](image)

The water distribution network in this example is composed of seven pipes that are under corrosion. The hazard causing pipe corrosion is assumed to be the surrounding soil because of its high moisture content, chemical content, and electrical resistivity. Although likelihood and severity of this hazard can be obtained from physical and statistical models that have been developed (Kleiner and Rajoni, 2001; and Rajani and Kleiner, 2001), this thesis subjectively
assume these data for this example (Table 4.8) in order to simply the process of obtaining inputs data. Here severity denotes the scope of soil corrosion on the external pipe walls or contaminants that will be introduced, and likelihood means the frequency of pipe failure due to the soil corrosion.

Figure 4.22 Application of Dempster-Shafer theory in a simple distribution system

Furthermore, since failure of each pipe has different influence to the users in this network, a weight factor is used to describe the degrees of this influence. In this example, the importance factor of a pipe is determined by the ratio between the reduced water quantity due to the pipe failure and the total demand quantity required at user point. The water demanded at the user point is assumed 14 L/s in this example network. Water flow rates of the seven pipes are given in Table 4.8. Then weight of each pipe is determined by

\[ w_i = \frac{C_i}{\sum_{j=1}^{7} C_j}; \quad i = 1,...,7 \]

where \( w_i \) denotes the weight of pipe \( i \), and \( C_i \) is the importance factor of pipe \( i \). Therefore weight factor is a normalised importance factor in this example. There are certainly other methods that are applicable to determine weights for the pipes in a distribution network, however, this study adopts the above equation to simplify the process of determining the weights. This simplification will not compromise the value or usefulness of the aggregation method proposed in this thesis.
The linguistic representations about severities (i.e., medium, extremely low, etc.) are represented by the fuzzy numbers discussed in Section 4.4.1.2. The fuzzy representations “about 0.1”, “about 0.05”, and “about 0.2” are represented by triangular fuzzy numbers \((0.05,0.1,0.15)\), \((0.0,0.05,0.1)\), and \((0.15,0.2,0.25)\) respectively. Then risks associated with pipe are obtained as the multiplication of fuzzy likelihoods and severities (Table 4.9). Finally by aggregating these risks along the hierarchical structure (Figure 4.22), risk levels are obtained for the distribution network.
The results (Figure 4.23) show that the water distribution system has different degrees, i.e., 25.4%, 55.1%, and 19.5%, to three risk grades, i.e., EL, VL, and SL respectively, because of the risk evidence from its pipes. The distribution system has the highest degree to risk grade VL, then are grades of EL and SL. Furthermore, since pipes 1, 6, and 7 have higher weights, that is higher influences on users, their risk levels are more dominant in determining the risk levels of the distribution system. This thus makes the network have higher degrees to risk grade VL than risk grade EL.

4.5 Potential Applications of Aggregative Risks

4.5.1 Aggregative Risk Assessment with Time

In the preceding discussions, the proposed risk assessment only performed static evaluations for hazards, objects, subsystems and system so far. However, in practical plan or management of water system, managers and engineers would like to know the risk levels of the system at different period of time beside current risk levels, which requires a dynamic risk assessment considering the time factor. If the changes of likelihoods and/or severities of hazards over time could be predetermined by proper models, then the relative risk changes of objects, subsystems and the overall water supply system would be determined by repeatedly using the above aggregation method (Figure 4.24).

![Diagram of dynamic risk analysis process](image.png)

Figure 4.24 Process of dynamic risk analysis of object, subsystem, and the overall water supply system
4.5.2 Assessment of Water Supply System Using Aggregative Risks

Aggregative risks have been applied in many problems including risk assessment of software (Lee, 1996 and 1999; Chen, 2001; Lee et al., 2003), environmental risk assessment (Sadiq et al., 2005), water quality failure in water distribution systems (Sadiq et al., 2004), etc. These applications imply the potential of aggregative risk in the future research.

Aggregative risks can also be used as abstract measures to compare water supply systems, subsystems, or components from the risk perspective. With the risk distributions and the relative representative risk values, risk analysts can have a clear view of which part of the system has higher risk levels than others. This will be especially useful in project design and system maintenance and rehabilitations with limited budgets.

Furthermore, aggregative risk can also be viewed as a surrogate measure of representing the status of the water system. With this surrogate measure and dynamic analyses, detailed analysis can be conducted to study the dynamic characteristics of the system, such as deterioration rate, availability of the system, failure rate at a specific time of period. In long terms, aggregative risk can serve a basis for benching marking acceptable risk levels in water systems (Sadiq et al., 2004).

4.6 Summary and Comments

4.6.1 Summary

This chapter discusses the quantitative evaluation of aggregative risk assessment of water supply system by using fuzzy sets theory and Dempster-Shafer theory. The proposed method has the following characteristics:

- The method can deal with data from multiple sources that are represented in various forms including probabilistic data, linguistic variables, etc. by converting them to fuzzy numbers. With fuzzy representation of these input data, aggregative risk assessment can be conducted to evaluate the risk levels of components, subsystem, and overall water supply system respectively.

- Direct results of this method is a kind of risk distribution which describes the degrees to predefined risk grades. Associated with this distribution, a representative aggregative risk value is also proposed in this study. The risk distribution and representative value together give a more comprehensive view of risk levels.
Chapter 4 Quantitative Aggregative Risk Assessment of Water Supply Systems

- The aggregative risk assessment can be performed at different hierarchical levels with this method. The combination rule is applied consistently at both object and system levels to obtain the risk levels. Different users can easily obtain the risk information according to the specific concern and requirements.

4.6.2 Comments

However there are several points needed to studied further in the future research:

- The methods of determining likelihood and severity are required to specifically study for each hazard in the water supply system so that the inputting data for this method are more reliable and suitable for the real cases.

- Weights also requires specific study by using proper methods. In practice, many factors such as costs, maximum flow rate, objectives of the water usage, hydraulic behaviours of the system, etc. will influence the determination for the weights factors for hazards, components, and subsystems. Therefore, the determination of weights can be performed in further work based on methods such as hydraulic simulation, cost analysis, etc.

- The risk assessment is limited to single component failure or hazard occurring at a time. However, the assessment of simultaneous failures of components and hazards should be further studied in the future work because the consequences of simultaneous failures are usually more serious.

- Risk assessment proposed in this thesis is limited to steady state condition. However, time dependent analysis is required in the further study.

- Results of this chapter is only approximate because the vulnerabilities or the influences of component failures were not considered using hydraulic and other simulation models, which makes assessment rather subjective. Therefore, further work is required to overcome this limitations by including vulnerabilities explicitly.

- Based on the object-oriented concepts, complex objects such as subsystems and the overall water supply system can be viewed as an self-contained unit, i.e., they can be viewed as abstract functional components as well. The aggregative risk results obtained in this research can be viewed a surrogate measure representing the risk property of the abstract component/object. Therefore, extensive applications of this surrogate measure is deserved .a further study.
Chapter 5 Quantitative Fault Tree Analysis of Water Supply Systems

CHAPTER

5

Quantitative Fault Tree Analysis of Water Supply Systems

5.1 Introduction

This chapter studies the quantitative solution of the fault trees developed in Chapter 3 in order to quantitatively evaluate the cause-effect relationships in a water supply system (Figure 5.1). However, conventional probabilistic methods cannot be directly adopted in this study because the inputting data are not only represented in terms of probabilistic numbers but also fuzzy numbers. Therefore, fuzzy fault tree analysis is discussed in this thesis. Fuzzy numbers are used to represent the likelihood of occurrence of basic events which are at the bottom level of the fault tree. After that fuzzy fault tree analysis is performed to generate the quantitative results for the likelihood of occurrence of the top event.

Even fuzzy fault tree analysis has been applied extensively in many engineering problems, it is still necessary for this study to consider special characteristics of water supply systems. One of the most important characteristics is failure dependencies in a water system. This indicates that the failure of a component in a water supply system can be either independent or dependent on the failure of another component. Therefore, the fault tree analyses are required to simultaneously consider independency and dependency in the process of risk assessment. However, few studies have ever considered the fuzzy information and dependencies so far. This study proposes a method that can deal with both fuzzy information and dependencies.

Firstly, in risk scenarios where component failures are independent, a fuzzy evaluation method is formed by extending conventional probabilistic techniques to fuzzy domain. A fuzzy number, instead of a normal probability number is ultimately obtained for the top event of a fault tree. Secondly, in risk scenarios where component failures are dependent in a certain degree, a dependency factor is introduced to simply and approximately evaluate this dependency in a fault tree analysis.
Chapter 5 Quantitative Fault Tree Analysis of Water Supply Systems

Figure 5.1 Quantitative methods used to evaluate the fault trees of the water supply system

Besides the likelihood of the top event, another useful result of fault tree analysis is importance measure for basic event that represents the risk or uncertainty contribution of the basic event to the risk or uncertainty of the top event. This importance measure is useful to rank the basic risk factors and therefore support the risk analysts to make decision in water supply systems. This study considers both risk and uncertainty importance measures associate with each basic event.

In this chapter, Section 5.2 introduces basic concepts and reviews the methods about fuzzy fault tree analysis. After that, it proposes a method of fuzzy fault tree analysis based on $\alpha$-cut sets. Section 5.3 discusses the methods of analysing failure dependencies among basic events in a fault tree by considering dependence factor. Section 5.4 discusses the methods of determining the fuzzy importance and fuzzy uncertainty importance of a basic event in a fault tree. Based on the methods proposed above, Section 5.5 illustrates the application of fuzzy fault tree analysis at both component and system levels. Lastly, Section 5.6 gives the summaries and comments. An explicit illustration of the structure of this chapter is available in Figure 5.2.
Figure 5.2 Structure of developing quantitative methods for fault trees of water systems

5.2 Basics of Fuzzy Fault Tree Analysis

5.2.1 Traditional Fault Tree Analysis

Traditionally, it is always assumed that the basic events contained in a fault tree are independent and could be represented as probabilistic numbers. With this assumption, quantitative analysis of fault trees are usually performed by considering two cases: (1) fault trees without repeated event, and (2) fault trees with repeated events (Andrews and Moss, 2002; Henley and Kumamoto, 1981).
(1) Fault trees without repeated events

In the event that the fault tree for a top event $T$ contains independent basic events which appear only once in the tree structure, the top event probability can be obtained by working the basic event probabilities up through the tree. In doing this, intermediate gate event (“and” or “or”) probabilities are calculated by starting at the base of the tree and working upwards until the top event probability is obtained.

![ Fault tree with gate events ]

Figure 5.3 Symbols representations of “and” and “or” events in fault trees

For an “and” gate event, its probability is determined by

$$ P = \prod_{i=1}^{n} p_i $$

(5.1)

where $P$ is the probability of “and” event; $p_i$ denotes the failure probability of basic event $i$; and $n$ is the number of basic events associated with the “and” gate.

For an “or” gate event, its probability is determined by

$$ P = 1 - \prod_{i=1}^{n} (1 - p_i) $$

(5.2)

where $P$ is the probability of “or” event; $p_i$ denotes the failure probability of basic event $i$; and $n$ is the number of basic events associated with the “or” gate.

(2) Fault trees with repeated events

When fault trees have basic events which appear more than once, the methods most often used to obtain the top event probability utilise the minimal cut sets. A minimal cut set is a collection of basic events. If all these events occur, the top event is guaranteed to occur; however, if any basic event does not occur, the top event will not occur. Therefore, if a fault tree has $n_C$ minimal cut sets $K_i$, $i=1, \ldots, n_C$ then the top event exists if at least one minimal cut set exists, i.e.,

$$ T = K_1 + K_2 + \ldots + K_{n_C} = \bigcup_{i=1}^{n_C} K_i $$
An approximate evaluation of this is determined by

\[ P(T) \leq P(T)_{\text{max}} = 1 - \prod_{i=1}^{n_c} \left[ 1 - P(K_i) \right] \]  

(5.3)

where \( P(K_i) \) is the occurrence probability of minimal cut set \( i \). This equation gives a conservative approximation of the likelihood of top event. Detailed derivation of Equation (5.3) can be found in Appendix D.

### 5.2.2 Fuzzy Fault Tree Analysis

However, it is often difficult to precisely describe the likelihoods of all hazards in terms of probability number in practice. Both probabilistic and fuzzy variables are inevitably used in a risk assessment. Such a situation challenges the application of conventional fault tree analysis which is based on probabilistic theory. Therefore a new formalism is required to capture the fuzzy and imprecision of likelihoods of multiple hazards.

With respect to this inadequacy of the conventional fault tree analysis, extensive research has been performed by using fuzzy sets theory in fault tree analysis. The pioneering work on this belongs to Tanaka et al. (1983), which treated probabilities of basic events as trapezoidal fuzzy numbers, and applied the fuzzy extension principle to determine the probability of top event. Based on Tanaka’s work, further extensive research has been performed (Misra and Weber, 1990; Liang and Wang, 1993). Another variation of fuzzy fault tree analysis was given by Misra and Weber (1989). Their analysis is based on possibility distribution associated with the basic events and a fuzzy algebra for combining these events. Parallel with this, Singer (1990) analysed fuzzy reliability by using \( L-R \) type fuzzy numbers. In order to facilitate the calculation of Singer’s method, Cheng and Mon (1993) and Chen (1994) proposed revised methods to analyse the fault trees by specifically considering the failure probabilities of basic events as triangular fuzzy numbers. In addition to the above studies, Onisawa (1988) proposed a method of using error possibility to analyse human reliability in a fault tree. By combining with Onisawa’s work, Lin and Wang (1997) developed a hybrid method which can simultaneously deal with probability and possibility measures in a fault tree analysis. Dong and Yu (2005) used the hybrid method to analyse failure probability of oil and gas transmission pipelines. Sawyer and Rao (1994), applied \( \alpha \)-cuts to determine the failure probability of the top event in fuzzy fault trees of mechanical systems. Cai et al. (1991) and Huang et al. (2004) adopted possibility theory to analyse fuzzy fault trees. Shu et al. (2006) used intuitionistic fuzzy methods to analyse fault trees on printed circuit board assembly.
Meanwhile, different methods have also been proposed to determine the importance of each basic event in a fuzzy fault tree by using fuzzy importance measures (Tanaka et al., 1983; Furuta and Shiraishi, 1984; Suresh et al., 1996; Guimarães and Ebecken, 1999).

It is obvious from the above reviews that fuzzy fault tree analysis has been extensively studied for a long time and effectively applied to many engineering problems. However, its application in risk assessment of water system is still scarce and rarely considered. This research specifically discusses the application of fuzzy fault tree analysis in risk assessment of water supply systems.

(1) Fuzzy fault trees without repeated events

To obtain a quantitative evaluation, α-cuts are adopted to extend methods of conventional fault tree to the fuzzy fault trees. This method has several advantages to analyse fuzzy information in fault trees: (1) it is a kind of extensive method of many existing methods (such as Tanaka’s and Singer’s methods), which makes it easily used in many practical engineering systems including water supply systems; (2) it can deal with basic events with different membership functions (Mon and Cheng, 1994); and (3) it is easy to code into computer programmes, which thus facilitates the calculations on complex fault trees.

As mentioned in Chapter 4, every fuzzy number can be equivalently represented by its α-cut sets. Given each α, there is an interval of confidence for the fuzzy number (Kaufmann and Gupta, 1991), and calculation of fuzzy numbers can be thus based on this interval of confidence. For example, if a parallel system has n components whose failure probabilities are represented by n fuzzy numbers, then there are n intervals associated with each value of α: [p_{IA}^L, p_{IA}^R],..., and [p_{IN}^L, p_{IN}^R]. Extending conventional fault tree analysis to fuzzy analysis is thus a process of extending the conventional calculation on probability numbers to calculation on probability intervals. Details of calculation on intervals are available in Appendix B. This extension is illustrated by the following parallel and serial systems.

i. Parallel system

Since the failure of the parallel system occurs when all the n component fail, the failure probability of the system is determined by performing the multiplication of n intervals

\[ p_{\alpha} = \prod_{i=1}^{n} p_{\alpha}^{L} \times \prod_{i=1}^{n} p_{\alpha}^{R} \] (5.4)

where \( p_{\alpha} \) is the α-cut set of the parallel system; \( p_{\alpha}^{i} \) is the α-cut set of component i in the system. \( p_{\alpha}^{L} \) and \( p_{\alpha}^{R} \) denote the lower bound and upper bound of the interval of the system, respectively;
and $p_{ia}^L$ and $p_{ia}^R$ denote the lower bound and upper bound of the interval of component $i$ respectively.

The difference between Equation (5.1) and Equation (5.4) is that the former is calculated on probability numbers, while the latter is calculated on intervals. By performing the calculation on all intervals’ values, i.e., $\alpha$ from 0 to 1, the fuzzy failure probability of the parallel system is ultimately obtained. This is equivalent to the “and” gate in the fault trees (Figure 5.3).

**ii. Serial system**

Similarly a serial system composed of $n$ components will be failed if any one of the components fails. The failure probability of the system can be represented as an “or” gate in fault tree analysis. By performing calculations on intervals, the system failure probability can be represented as

$$p_{\alpha}^L = 1 - \prod_{i=1}^{n} (1 - p_{ia}^L); p_{\alpha}^R = 1 - \prod_{i=1}^{n} (1 - p_{ia}^R)$$  \hspace{1cm} (5.5)

After determining the intervals of all values of $\alpha$, i.e., from 0 to 1, the fuzzy failure probability of an “or” gate in a fuzzy fault tree is obtained.

Since the failure likelihoods of basic events are represented by fuzzy numbers, it is obvious that the output (i.e., failure likelihood of top event) is also a fuzzy number according to equations (5.4) and (5.5). This is different from the probabilistic values determined by equations (5.1) and (5.2) in conventional fault tree analysis.

**(2) Fuzzy fault trees with repeated events**

Even fuzzy fault tree analysis has been studied for decades of years, the repeated events have been rarely considered in fuzzy fault trees. In order to deal with this repeated basic events, this study develops a method applicable to this problem by extending conventional analysis to fuzzy domain. In conventional analysis, failure likelihood of the top event in a fault tree with repeated events is approximately evaluated by its upper bound (Equation (5.3)). Therefore, it is straightforward that, failure likelihood of top event in fuzzy fault trees with repeated events can also be approximately evaluated by its maximum value. Here the difference with conventional methods is that the maximum value is also a fuzzy number rather than a crisp probability. Therefore, based on $\alpha$–cut sets Equation (5.3) can be extended to fuzzy sets and represented by intervals as

$$[p_{\alpha}^L, p_{\alpha}^R] \subseteq [p_{ua}^L, p_{ua}^R]$$  \hspace{1cm} (5.6)
where $p_{a}^{L}$ and $p_{a}^{R}$ denote the interval of the top event associated with each confidence level $\alpha$; $p_{wa}^{L}$ and $p_{wa}^{R}$ represent an interval which is the upper bound of the top event. $p_{ua}^{L}$ and $p_{ua}^{R}$ are determined by

$$p_{ua}^{L} = 1 - \prod_{i=1}^{N_{C}} (1 - p_{ia}^{L}) \text{ or } p_{ua}^{L} = \sum_{i=1}^{N_{C}} p_{ia}^{L}$$

$$p_{ua}^{R} = 1 - \prod_{i=1}^{N_{C}} (1 - p_{ia}^{R}) \text{ or } p_{ua}^{R} = \sum_{i=1}^{N_{C}} p_{ia}^{R}$$

(5.7)

where $N_{C}$ denotes the number of the minimal cut sets in the fuzzy fault tree; $p_{ia}^{L}$ and $p_{ia}^{R}$ denote the interval which is the upper bound of the basic event $i$.

**Figure 5.4** Examples of fuzzy fault trees without and with repeated events

**Figure 5.5** Results of top events with and without repeated basic events by using fuzzy fault tree analysis

Here, two simple examples (Figure 5.4) are given to explicitly explain the evaluations of fault trees without and with repeated events respectively. Suppose that failure probabilities of all basic
events are “about 0.5” and represented by triangular fuzzy number (0.4, 0.5, 0.6). The analysed results of these two examples are shown in Figure 5.5. In the fuzzy fault tree without repeated events, events B and C are connected by an “and” gate, which is in turn connected with event A by an “or” gate. By using the equations (5.4) and (5.5), the likelihood of the top event, also a fuzzy number, is obtained. In the fuzzy fault tree with repeated events (i.e., event B in this example), its top failure likelihood is approximately determined by its maximum value. In this example, there are 3 minimal cut sets (i.e., A, B “and” C, and B “and” D). By using equations (5.6) and (5.7), the conservative approximation is obtained to represent the failure likelihood of top event in a fault tree with repeated event.

5.3 Dependency in Fuzzy Fault Tree Analysis

In some risk assessment of water supply systems, failure dependency is very important because it reflects the real situations of risk scenarios. For example, in a water supply system with multiple sources, say a river and a reservoir, contamination of the river will influence the contamination of the reservoir during flood season. Therefore, contamination of the river would increase the likelihood of contamination of the reservoir. With respect to this, dependency should be considered during the fault tree analysis of contamination of water sources in the water supply system.

Failure dependency has been considered in other engineering problems in the literature. Onisawa (1988) and Misra and Weber (1989) proposed two different methods of using conditional possibility to solve this problem respectively. In Onisawa’s method, error possibility was used to represent the human error, based on which the dependency between consecutive human tasks is considered for both parallel and serial tasks. The fuzzy causal relations, which represent the dependence between human tasks, are required to be explicitly known before analysis with this method. In Misra’s method, a factor for modelling dependency between events is used. Based on this factor and possibility operations, the conditional possibility was simply determined to represent the dependences in a fault tree.

In this research, dependence factor, \( d \), is adopted and the calculation is based on \( \alpha \)-cut sets of fuzzy numbers. \( d \) can be either a crisp number or a fuzzy number in practical applications. If \( d \) equals zero, it means that no dependencies exist among basic events. If \( d \) equals 1, it indicates complete dependencies or perfect dependencies. With dependence factor \( d \), conjunction of two events, say \( A \) and \( B \), can be determined by

\[
p(A \cap B) = p(A \mid B) p(B)
\]

\[
p(A \mid B) = d + (1 - d) p(A)
\]
Therefore,

\[ p(A \cap B) = [d + (1 - d)p(A)]p(B) = [1 - (1 - d)(1 - p(A))]p(B) \quad (5.8) \]

where \( d \) is the dependence of event \( A \) occurs given that event \( B \) has happened. This equation represents a simplified approximation of real-world dependencies among failure events.

For disjunction of two events \( A \) and \( B \), in which occurrence of \( A \) depends on \( B \) with a degree of \( d \), its probability can be derived as follow

\[
\begin{align*}
p(A \cup B) &= p(A) + p(B) - p(A \cap B) \\
&= p(A) + p(B) - p(B)[d + (1 - d)p(A)] \\
&= 1 - [1 - p(A)][1 - (1 - d)p(B)]
\end{align*}
\]

Based on equations (5.8) and (5.9), \( p(A \cap B) \) is equal to \( p(A)p(B) \) if \( d \) is zero and \( p(A \cup B) \) is equal to \( 1 - (1 - p(A))(1 - p(B)) \), which is the case that \( A \) and \( B \) are independent with each other. If \( d \) increases to unity, \( p(A \cap B) \) will be \( p(B) = \min(p(A), p(B)) \) and \( p(A \cup B) \) will be \( p(A) = \max(p(A), p(B)) \), which is the case of complete dependency. This therefore indicates that the value of failure likelihood of top event changes with the variations of dependence factor and is constrained in the following intervals.

\[
\begin{align*}
p(A \cap B) &\in \left[p(A)p(B), \min(p(A), p(B))\right] \\
p(A \cup B) &\in \left[\max(p(A), p(B)), 1 - (1 - p(A))(1 - p(B))\right] \quad (5.10)
\end{align*}
\]

Since probability of basic events \( A, B \), and dependence factor can be fuzzy numbers, equations (5.8), (5.9), and (5.10) should be determined by using fuzzy arithmetic and fuzzy sets operations. The relative equations based on \( \alpha \)-cuts are summarised in Table 5.1.

The above method can be demonstrated by examples in two possible cases, i.e., the dependence degree can be predetermined and cannot be predetermined. In the first case, Figure 5.4a is taken as an example in which the failure of event \( B \) has influences on the failure of event \( C \). The dependence degree is assumed high and represented by a fuzzy number \( (0.6, 0.7, 0.8) \). Failure probabilities of basic event are “about 0.5” and represented by a fuzzy number \( (0.4, 0.5, 0.6) \). The analysing results of fault trees both without and with dependency are shown in Figure 5.6 to give a comparison. As failure of event \( C \) is partially dependent on event \( B \), the intersection of these two events will be increased based on equation (5.10). Thus the ultimate result of the fault tree is also increased. Analysed result (Figure 5.6a) agrees with this, which shows that the proposed method in this study reasonably reflects the dependencies in fault trees.
Table 5.1 Basic equations representing dependencies in fuzzy fault trees

<table>
<thead>
<tr>
<th>Dependence degree</th>
<th>Event</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependence degree is predetermined ( A \cap B )</td>
<td>( p_a^L = [1 - (1 - a^L_a)(1 - p_{Aa}^L)]p_{Ba}^L )</td>
<td>( p_a^R = [1 - (1 - a^R_a)(1 - p_{Aa}^R)]p_{Ba}^R )</td>
</tr>
<tr>
<td>( A \cup B )</td>
<td>( p_a^L = 1 - (1 - p_{Aa}^L)[1 - (1 - a^L_a)p_{Ba}^L] )</td>
<td>( p_a^R = 1 - (1 - p_{Aa}^R)[1 - (1 - a^R_a)p_{Ba}^R] )</td>
</tr>
<tr>
<td>Dependence degree is not predetermined ( A \cap B )</td>
<td>( [p_{IA}^L, p_{IA}^R] \leq p_a \leq [p_{UA}^L, p_{UA}^R] )</td>
<td>( p_{IA}^L = p_{AA}^L p_{BA}^L; \quad p_{IA}^R = p_{AA}^R p_{BA}^R )</td>
</tr>
<tr>
<td>( p_{UA}^L = \min(p_{AA}^L, p_{BA}^L); \quad p_{UA}^R = \min(p_{AA}^R, p_{BA}^R) )</td>
<td>( p_{UA}^L = 1 - (1 - p_{AA}^L)(1 - p_{BA}^L) )</td>
<td>( p_{UA}^R = 1 - (1 - p_{AA}^R)(1 - p_{BA}^R) )</td>
</tr>
<tr>
<td>( A \cup B )</td>
<td>( [p_{IA}^L, p_{IA}^R] \leq p_a \leq [p_{UA}^L, p_{UA}^R] )</td>
<td>( p_{IA}^L = \max(p_{AA}^L, p_{BA}^L); \quad p_{IA}^R = 1 - (1 - p_{AA}^L)(1 - p_{BA}^L) )</td>
</tr>
<tr>
<td>( p_{UA}^L = \max(p_{AA}^L, p_{BA}^L); \quad p_{UA}^R = 1 - (1 - p_{AA}^R)(1 - p_{BA}^R) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the second case, Figure 5.4b is selected as an example in which the failure of event B has influence on event C. But the dependence degree is hard to be determined, i.e., dependence degree cannot be represented by a crisp or fuzzy number. Failure probabilities of basic events are also assumed to be “about 0.5” in this case. To obtain the final results of the top event, equations in Table 5.1 are applied. An interval of fuzzy numbers is obtained to represent the failure likelihood of top event (Figure 5.4b). The real likelihood of the top event is in the area constrained by the lower and upper bounds.

![Figure 5.6 Results of fuzzy fault trees with dependent events](image_url)

From the object-oriented point of view, dependencies among components in a water supply system indicate the degrees of interrelationships between different objects. Higher dependencies
show closer interrelationships, or higher likelihood of inflow hazards introduced by other components/objects. Even though inter-component linkages are not specially considered in object-oriented whole/part relationships, fault trees are able to analyse the influences of inter-component linkage explicitly. Therefore, considering both aggregative risk and fault tree analysis together is expected to give a more comprehensive view of the risks in a water supply system.

5.4 Importance Measures in Fuzzy Fault Trees

Besides the likelihood of top event, another useful result of fault tree analysis is the importance of each basic event. The importance is used to represent the risk contribution of each basic event to the occurrence of the top event, and thus viewed as useful information for risk-related decision makings. Extensive research has been carried out to determine the importance of basic events in fuzzy fault trees. Tanaka et al. (1983) defined an improvement index to evaluate the importance of each basic event. Furuta and Shiraishi (1984) used representative value of fuzzy membership function to calculate the importance. Liang and Wang (1993) used ranking values to evaluate fuzzy importance index. Suresh et al. (1996) applied Euclidean distance to determine fuzzy importance measure and fuzzy uncertainty importance measure, which was further improved by Guimarães and Ebecken (1999).

This research adopts the method based on Euclidean distance to obtain fuzzy importance measure and fuzzy uncertainty importance measure for the basic event in a fault tree. Fuzzy importance measure is used to evaluate the contribution of a basic event to the top event in a fuzzy fault tree. It is useful for the analysts to identify the priorities of components from the point of view of fault tree structure. While fuzzy uncertainty importance measure is used to evaluate the contribution of uncertainty from a basic event to the top event in a fuzzy fault tree. It plays an important role in the reduction of uncertainty because it identifies the sources of uncertainty having greatest impact on the uncertainty of the top event.

5.4.1 Fuzzy Importance Measure

Suppose that the likelihood of top event in a fault tree can be represented by

\[ p = f(p_1, \ldots, p_{i-1}, p_i, p_{i+1}, \ldots, p_n) \]

where \( p \) denotes the likelihood of top event in a fault tree; \( p_i \) \((i=1, \ldots, n)\) represents the failure likelihood of basic event in a fault tree; \( n \) is the number of basic components; \( f(\cdot) \) denotes the structure function of the fault tree to determine the likelihood of top event.

The likelihood of top event by making the component “\( i \)” fully unavailable (i.e., \( p_i = 1 \)) is
Similarly when the component “i” is fully functioning, the likelihood of the top event is

\[ p_{i=0} = f(p_1, \ldots, p_{i-1}, 0, p_{i+1}, \ldots, p_n) \]  \hspace{1cm} (5.12)

The total contribution of component “i” to the top event is thus the difference between \( p_{i=1} \) and \( p_{i=0} \) and is called Birnbaum importance (Suresh et al., 1996; Andrews and Moss, 2002). However, in fuzzy fault tree analysis where the likelihoods of basic events are fuzzy numbers, point estimate that is used in conventional fault trees is challenged and not easy to be applied. In order to deal with this, Euclidean distance is proposed (Suresh et al., 1996) to determine the importance measure of components in a fuzzy fault tree analysis, which is defined as

\[ I_i = ED(p_{i=1}, p_{i=0}) \]

\[ ED(p_{i=1}, p_{i=0}) = \sum_\alpha \left[ (p_{i=1,\alpha}^L - p_{i=0,\alpha}^L)^2 + (p_{i=1,\alpha}^R - p_{i=0,\alpha}^R)^2 \right]^{1/2} \]  \hspace{1cm} (5.13)

where \( I_i \) is the importance measure of components \( i \); \( ED(p_{i=1}, p_{i=0}) \) is the Euclidean distance between two fuzz numbers \( p_{i=1} \) and \( p_{i=0} \); \( p_{i=1,\alpha}^L \) and \( p_{i=1,\alpha}^R \) are the lower and upper bounds of the interval associated with confidence level \( \alpha \) when component “i” is fully unavailable; \( p_{i=0,\alpha}^L \) and \( p_{i=0,\alpha}^R \) are the lower and upper bounds of the interval associated with confidence level \( \alpha \) when component “i” is fully functioning.

Furthermore, if the fuzzy fault trees have dependent events but the dependence degrees cannot be predetermined, the relative Euclidean distances can be determined for the upper and lower bounds of the top event respectively. Then the final importance measure is calculated as the average value of these two Euclidean distances in this research. Therefore, it is determined by

\[ I_i = (ED(p_{u,i=1}, p_{u,i=0}) + ED(p_{l,i=1}, p_{l,i=0}))/2 \]  \hspace{1cm} (5.14)

where \( ED(p_{u,i=1}, p_{u,i=0}) \) and \( ED(p_{l,i=1}, p_{l,i=0}) \) are the Euclidean distances associated with the upper and lower bounds of top event respectively.

### 5.4.2 Fuzzy Uncertainty Importance Measure

Fuzzy uncertainty importance measure is adopted to identify the components which contribute maximum uncertainty to the uncertainty of the top event and is defined as

\[ U_i = ED(p, p) \]  \hspace{1cm} (5.15)
where $U_i$ is the uncertainty importance measure of component $i$, $p_i$ is the likelihood of top event if the failure probability of component $i$ is a crisp value, i.e., no uncertainty with this value.

If the fuzzy fault trees have dependent events but the dependence degrees cannot be predetermined, uncertainty measures can be determined for upper and lower bounds of the top event respectively. Then the final uncertainty importance measure is calculated as the average value of these two distances in this research. Therefore, it is determined by

$$U_i = \frac{(ED(p_{u}, p_{u,i}) + ED(p_{l}, p_{l,i}))}{2}$$

(5.16)

where $ED(p_{u}, p_{u,i})$ and $ED(p_{l}, p_{l,i})$ are the uncertainty importance measures associated with upper and lower bounds of top event respectively.

The applications of the above importance measures can be illustrated by several examples: (1) Example 1 (Figure 5.4a) in which basic events are independent; (2) Example 2 (Figure 5.4b) in which basic events are independent, but repeated events are included; (3) Example 3 (Figure 5.4a) in which event $C$ is partially dependent on event $B$ with a fuzzy degree of high ($0.6, 0.7, 0.8$); and (4) Example 4 (Figure 5.4b) in which event $C$ is partially dependent on event $B$, but the dependence degree cannot be predetermined. The analysed results of the four examples are listed in Table 5.2.

**Table 5.2 Importance measures for four examples**

<table>
<thead>
<tr>
<th>Example</th>
<th>Events</th>
<th>Fuzzy importance</th>
<th>Rank</th>
<th>Fuzzy uncertainty importance</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td>A</td>
<td>11.65</td>
<td>1</td>
<td>0.58</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>3.83</td>
<td>2</td>
<td>0.19</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>3.83</td>
<td>2</td>
<td>0.19</td>
<td>2</td>
</tr>
<tr>
<td>Example 2</td>
<td>A</td>
<td>8.83</td>
<td>1</td>
<td>0.44</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>5.76</td>
<td>2</td>
<td>0.28</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>2.87</td>
<td>3</td>
<td>0.14</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>2.87</td>
<td>3</td>
<td>0.14</td>
<td>3</td>
</tr>
<tr>
<td>Example 3</td>
<td>A</td>
<td>9.00</td>
<td>1</td>
<td>0.45</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>6.55</td>
<td>2</td>
<td>0.33</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1.17</td>
<td>3</td>
<td>0.06</td>
<td>3</td>
</tr>
<tr>
<td>Example 4</td>
<td>A</td>
<td>7.40</td>
<td>1</td>
<td>0.37</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>5.76</td>
<td>2</td>
<td>0.26</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>2.87</td>
<td>3</td>
<td>0.15</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>2.40</td>
<td>4</td>
<td>0.12</td>
<td>4</td>
</tr>
</tbody>
</table>

Results show that in Example 1 event $A$ has the highest importance, and events $B$ and $C$ have the same importance values which are lower than that of event $A$. In Example 2, event $A$ has the highest importance, event $B$ has the second highest importance, while event $C$ and $D$ have the lowest importance value. Since event $B$ is a repeated event in this example, its importance value is
higher than those of events $C$ and $D$. In Example 3, event $A$ has the highest importance, event $B$ is in the second place while event $C$ is the lowest in importance. Analysed results of this example are different from those of example 1 because of the failure dependency between basic events $B$ and $C$ in this example. Considering that event $B$ influences the occurrence of event $C$ with certain degree, importance of event $B$ is higher than that of event $C$ as shown by the results. In Example 4, event $A$ has the highest importance value; event $B$ is in second place followed by event $C$, and event $D$ has the lowest value. The importance value of event $C$ in Example 4 is higher that in Example 2 because of the failure dependency.

### 5.5 Applications of Fuzzy Fault Trees in Water Supply Systems

With the above quantitative methods and fault tree structures developed in Chapter 3, it is thus possible to evaluate the cause-effect relationships of risks in water supply systems. The whole evaluation process is divided into two steps: (1) fuzzy fault tree analysis for components; and (2) fuzzy fault tree analysis for system, which is discussed in detail in the following part.

Furthermore, considering that both repeated events and dependencies influence the evaluation of fuzzy fault trees, it is necessary to identify them before the quantitative evaluation. A diagram is therefore formed to show the process of fuzzy fault tree analysis (Figure 5.7). Associated with each situation in the diagram, different equations discussed above are used to produce quantitative evaluations for the fault tree. By following this diagram, fuzzy fault tree can be performed at both component and system levels.

#### 5.5.1 Quantitative Analysis of Fault Trees for Components

The fault tree structures, developed in Chapter 3, can now be quantitatively evaluated by using fuzzy fault tree analysis. The river example considered in Section 4.4.3 is also used here to demonstrate the process of fuzzy fault tree analysis at component level (Figure 5.8). In this figure, $R$ denotes the contamination of the river, which is the top event of the fault tree. $F_1$, $F_2$, and $F_3$ denote the failure states of natural hazard, human-caused threat, and interdependencies respectively. $H_{11}$ is the hazard of flood; $H_{21}$, $H_{22}$, and $H_{23}$ are hazards of sewage discharge, industrial discharge, and human wilful chemical/biological contamination respectively; and $H_{31}$ is the hazard of pollutant spills from vehicles on the river.
Chapter 5 Quantitative Fault Tree Analysis of Water Supply Systems

Figure 5.7 Diagram of fuzzy fault trees at both component and water system levels

Figure 5.8 Fuzzy fault tree analysis for contamination of the river example
The fault tree of river contamination does not contain any repeated events and failure dependencies, therefore, Equations (5.4) and (5.5) are used according the process in Figure 5.7. Inputs of the analysis, i.e., likelihoods of these hazards, are given in Table 4.6, and the result is shown in Figure 5.9. Importance measures are calculated for this fault tree by using Equations (5.13) and (5.15).

![Figure 5.9 Fuzzy likelihood of the top event shown in Figure 5.8](image)

**Table 5.3 Importance measures of the river example in Figure 5.8**

<table>
<thead>
<tr>
<th>Hazards</th>
<th>Fuzzy importance</th>
<th>Rank</th>
<th>Fuzzy uncertainty importance</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{11}$</td>
<td>5.87</td>
<td>3</td>
<td>0.16</td>
<td>5</td>
</tr>
<tr>
<td>$H_{21}$</td>
<td>6.57</td>
<td>1</td>
<td>0.59</td>
<td>1</td>
</tr>
<tr>
<td>$H_{22}$</td>
<td>6.57</td>
<td>1</td>
<td>0.59</td>
<td>1</td>
</tr>
<tr>
<td>$H_{23}$</td>
<td>5.34</td>
<td>4</td>
<td>0.48</td>
<td>3</td>
</tr>
<tr>
<td>$H_{31}$</td>
<td>5.34</td>
<td>4</td>
<td>0.48</td>
<td>3</td>
</tr>
</tbody>
</table>

The results show that the likelihood of top event is about 0.75 and constrained in the range of [0.41, 0.92] (Figure 5.9). Among the basic events, failures of sewage discharge and industrial discharge (i.e., $H_{21}$ and $H_{22}$ respectively) have the highest importance values (Table 5.3), which indicates that they contribute most to river contamination from the perspective of likelihood. Vehicle spills ($H_{31}$) and human chemical/biological ($H_{23}$) contamination have the lowest importance values and contribute least to the river contamination. For the uncertainty associated the top event, sewage and industrial discharges give the highest contributions, while flood has the lowest contribution to the uncertainty. With respect these results, risk analysts can have clear idea that effective mitigation measures should be those that can reduce the likelihoods of sewage and industrial charges so that the river contamination can be effectively controlled.
5.5.2 Quantitative Analysis of Fuzzy Fault Trees for System

After the failure likelihoods of components have been determined, the likelihood of a subsystem or the overall water supply system risk can be obtained by analysing the fault trees associated with subsystems and the overall system. In the fault trees for water systems, failure likelihoods of components are viewed as basic events in this research. This can be explained by an example.

![Fault Tree Diagram](image)

Figure 5.10 Fuzzy fault tree analysis for a small water distribution network with no water delivered to user

The small water distribution network is composed of seven pipes (Figure 5.10). Fault tree structure of no water flow at user side can be obtained from Figure 3.31. In this fault tree, the failure likelihoods of pipes are evaluated by fuzzy probabilities as (0.15, 0.2, 0.3), i.e., the failure likelihood of each pipe is about 0.2 and limited in 0.15 and 0.3. Meanwhile failure of pipe 5 is assumed to depend on pipe 6 with a degree of about 0.4 (0.3, 0.4, 0.5). Since there are repeated events in this fault trees (such as failures of pipe 2, pipe 3 and pipe 5), approximation equations (equations (5.6) and (5.7)) are used in this example to determine the risk likelihood of the top event. From the fault tree structure, six minimum cut sets are identified, i.e., \{1\}, \{2,3\}, \{2,6\}, \{5,6\}, \{3,4,5\}, and \{7\}, based on which the likelihood of top event (Figure 5.11) and importance of each basic event can be obtained (Table 5.4).

<table>
<thead>
<tr>
<th>Pipe</th>
<th>Fuzzy importance</th>
<th>Rank</th>
<th>Fuzzy uncertainty importance</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.92</td>
<td>1</td>
<td>0.35</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3.12</td>
<td>4</td>
<td>0.13</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1.97</td>
<td>5</td>
<td>0.08</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>0.35</td>
<td>7</td>
<td>0.02</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>1.39</td>
<td>6</td>
<td>0.06</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>5.60</td>
<td>3</td>
<td>0.24</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>9.92</td>
<td>1</td>
<td>0.35</td>
<td>1</td>
</tr>
</tbody>
</table>
The results show that the likelihood of top event is about 0.48 and constrained in the range of [0.35, 0.68] (Figure 5.11). Among the basic events, failures of pipes 1 and 7 have the highest importance values, which indicates that pipes 1 and 7 are ranked as most important components in the distribution network. Pipes 6, 2, 3, 5, and 4 are ranked as second, third, fourth, fifth, and sixth most important components to the risk of no water at user point respectively. These results also indicate that mitigation measure should be taken on pipes 1, 7, and 6 so that the likelihood of the top event can be effectively reduced. Since pipe 4 has little influence on the top event, mitigation measures on this pipe will not improve the reliability of the network dramatically.

5.6 Summary and Comments

5.6.1 Summary

This chapter discusses the methods of quantitative fault tree analysis based on which the cause-effect relationships of risks are assessed. Fuzzy sets theory is adopted in this research to analyse the issues associated with fault trees such as repeated events and dependencies among basic events. The proposed in this chapter has the following characteristics:

- The method can deal with both precise data and fuzzy data which are usually used to represent likelihood of different hazards in practice. Therefore it is applicable to assess the risks of water supply systems influenced by hazards with multiple forms.
- Repeated events in fault trees are considered in the proposed method. In a fault tree with repeated events, the likelihood of top event is approximately evaluated by its upper bounds.
- Dependencies among failure evens are considered in the proposed method. A simple and approximate method is discussed in this research by using dependence degrees.
• Both fuzzy importance measure and fuzzy uncertainty importance measure are considered in the proposed method. Fuzzy importance measure is used to evaluate the contributions of basic events. Fuzzy uncertainty importance measure is used to evaluate the contributions of uncertainties from basic events.

5.6.2 Comments

However there are several points needed to studied further:

• The inputs likelihood of basic events are assumed in this study. In the future research, it is required to develop the methods of determining the likelihood of each hazard.

• The fault tree structure developed so far are not reflecting the vulnerability of each component in water supply systems. The influence of vulnerability to fault tree analysis is expected to be studied in the future.
6.1 Introduction

This chapter discusses the applications of the method developed in this study to an assumed water supply system. Since the main purpose of this chapter is to demonstrate the risk assessment of the developed methods, it is feasible to adopt a virtual water supply system to illustrate the process of aggregative risk assessment and fault tree analysis. Although most of the inputs are subjectively assumed in this study, it is developed based on the consideration of real water supply systems. All the information provided in this example (i.e., data of nodes and links in Section 6.1.1) operates normally in the simulation software (i.e., EPANET), which indicates that the assumptions in this case are realistic.

There are three important aspects that need to be discussed before conducting the risk assessment, i.e., basic information of the water supply system, potential hazards, and objectives of the risk assessment. In the following parts, Section 6.1.1 introduces the basic information of the system. Then Section 6.1.2 analyses the potential hazards associated with each component in the system. At last, Section 6.1.3 states the objectives of risk assessment for this assumed water supply system.

6.1.1 Assumed Water Supply System

The assumed water supply system is composed of water sources, water treatment plant, and water distribution network (Figure 6.1). There are three water sources, i.e., a river, a reservoir, and a group of wells supplying raw water to a water treatment plant by transmission pipes or channel. After raw water is treated in the water treatment plant, it is pumped to a water distribution network which is composed of 29 pipes and 22 water demand nodes.
In this example, water distribution network is also a complex system that is neither serial nor parallel (Figure 6.2). To analyse this complex network, basic data about nodes and pipes are given and listed in Table 6.1 and Table 6.2, respectively. Then, hydraulic simulation is performed in EPANET (Rossman, 2000) to determine the flow directions in the distribution network (Figure 6.2). These flow directions will be used to identify the cause-effect relationships for specific risk in the system.
Table 6.1 Information of nodes in the water distribution network

<table>
<thead>
<tr>
<th>Node</th>
<th>Elevation (m)</th>
<th>Demand (L/s)</th>
<th>Node</th>
<th>Elevation (m)</th>
<th>Demand (L/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200*</td>
<td>790.73</td>
<td>12</td>
<td>182</td>
<td>263.72</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>790.73</td>
<td>13</td>
<td>181</td>
<td>263.72</td>
</tr>
<tr>
<td>3</td>
<td>181</td>
<td>440.71</td>
<td>14</td>
<td>183</td>
<td>158.15</td>
</tr>
<tr>
<td>4</td>
<td>181</td>
<td>263.72</td>
<td>15</td>
<td>181</td>
<td>114.34</td>
</tr>
<tr>
<td>5</td>
<td>179</td>
<td>349.59</td>
<td>16</td>
<td>182</td>
<td>114.34</td>
</tr>
<tr>
<td>6</td>
<td>179</td>
<td>105.58</td>
<td>17</td>
<td>182</td>
<td>142.38</td>
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<td>7</td>
<td>179</td>
<td>224.73</td>
<td>18</td>
<td>183</td>
<td>99.01</td>
</tr>
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<td>8</td>
<td>178</td>
<td>91.12</td>
<td>19</td>
<td>182</td>
<td>114.34</td>
</tr>
<tr>
<td>9</td>
<td>180</td>
<td>226.92</td>
<td>20</td>
<td>183</td>
<td>79.29</td>
</tr>
<tr>
<td>10</td>
<td>182</td>
<td>224.73</td>
<td>21</td>
<td>184</td>
<td>71.41</td>
</tr>
<tr>
<td>11</td>
<td>184</td>
<td>69.65</td>
<td>22</td>
<td>183</td>
<td>103.39</td>
</tr>
</tbody>
</table>

*This value is the water level at the outlet of pump.

Table 6.2 Information of pipes in the water distribution network

<table>
<thead>
<tr>
<th>Pipe</th>
<th>Terminal node</th>
<th>Material</th>
<th>Age (yr)</th>
<th>Diameter (mm)</th>
<th>Length (m)</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,2</td>
<td>CI</td>
<td>5</td>
<td>600</td>
<td>3050</td>
<td>140</td>
</tr>
<tr>
<td>2</td>
<td>2,3</td>
<td>CI</td>
<td>10</td>
<td>450</td>
<td>1520</td>
<td>110</td>
</tr>
<tr>
<td>3</td>
<td>2,5</td>
<td>CI</td>
<td>10</td>
<td>450</td>
<td>1980</td>
<td>110</td>
</tr>
<tr>
<td>4</td>
<td>5,9</td>
<td>CI</td>
<td>10</td>
<td>400</td>
<td>670</td>
<td>110</td>
</tr>
<tr>
<td>5</td>
<td>5,6</td>
<td>CI</td>
<td>10</td>
<td>300</td>
<td>1070</td>
<td>110</td>
</tr>
<tr>
<td>6</td>
<td>5,7</td>
<td>CI</td>
<td>10</td>
<td>400</td>
<td>120</td>
<td>110</td>
</tr>
<tr>
<td>7</td>
<td>6,8</td>
<td>CI</td>
<td>10</td>
<td>150</td>
<td>760</td>
<td>110</td>
</tr>
<tr>
<td>8</td>
<td>9,8</td>
<td>CI</td>
<td>10</td>
<td>300</td>
<td>1370</td>
<td>110</td>
</tr>
<tr>
<td>9</td>
<td>9,4</td>
<td>CI</td>
<td>10</td>
<td>350</td>
<td>1675</td>
<td>110</td>
</tr>
<tr>
<td>10</td>
<td>9,14</td>
<td>CI</td>
<td>10</td>
<td>350</td>
<td>1375</td>
<td>110</td>
</tr>
<tr>
<td>11</td>
<td>3,10</td>
<td>CI</td>
<td>10</td>
<td>400</td>
<td>1520</td>
<td>110</td>
</tr>
<tr>
<td>12</td>
<td>3,4</td>
<td>CI</td>
<td>10</td>
<td>350</td>
<td>1520</td>
<td>110</td>
</tr>
<tr>
<td>13</td>
<td>4,13</td>
<td>CI</td>
<td>10</td>
<td>350</td>
<td>910</td>
<td>110</td>
</tr>
<tr>
<td>14</td>
<td>10,11</td>
<td>PVC</td>
<td>10</td>
<td>150</td>
<td>300</td>
<td>140</td>
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<tr>
<td>15</td>
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<td>350</td>
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<td>250</td>
<td>440</td>
<td>110</td>
</tr>
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<td>22</td>
<td>18,17</td>
<td>CI</td>
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<td>250</td>
<td>210</td>
<td>110</td>
</tr>
<tr>
<td>23</td>
<td>18,21</td>
<td>CI</td>
<td>10</td>
<td>250</td>
<td>300</td>
<td>110</td>
</tr>
<tr>
<td>24</td>
<td>21,20</td>
<td>PVC</td>
<td>10</td>
<td>200</td>
<td>210</td>
<td>140</td>
</tr>
<tr>
<td>25</td>
<td>17,19</td>
<td>PVC</td>
<td>10</td>
<td>250</td>
<td>140</td>
<td>140</td>
</tr>
<tr>
<td>26</td>
<td>16,22</td>
<td>PVC</td>
<td>10</td>
<td>250</td>
<td>180</td>
<td>140</td>
</tr>
<tr>
<td>27</td>
<td>19,16</td>
<td>PVC</td>
<td>10</td>
<td>250</td>
<td>240</td>
<td>140</td>
</tr>
<tr>
<td>28</td>
<td>20,19</td>
<td>PVC</td>
<td>10</td>
<td>200</td>
<td>150</td>
<td>140</td>
</tr>
<tr>
<td>29</td>
<td>20,22</td>
<td>PVC</td>
<td>10</td>
<td>200</td>
<td>240</td>
<td>140</td>
</tr>
</tbody>
</table>
6.1.2 Hazards to Components in the Water Supply System

In this assumed water supply system, as most components are constructed and operated in an open environment, multiple hazards are potentially threatening the functionalities of components with different degrees. Identification of the potential hazards for each component is necessary and a critical step for risk assessment of the system.

For the river in the water supply system, its service is potentially compromised by multiple sources (Figure 6.3). Firstly, since the river is within a flooding plain, it is inevitably influenced by floods. It is shown by historical data that floods, especially those with low frequencies (i.e., lower than 1/50), can affect its quality significantly because they have the ability to carry various and a large amount of pollutants. Secondly, there is a chemical factory along the west bank of the river, and a sewage discharge point along the east side. Both of these discharge points are at upstream part of the raw water in-take point. Thus improper treatments of the chemical waste water or sewage will negatively effect the river quality and hence influence the treating process. In addition, there are some regular transporting vehicles on this river. Data from the environmental agencies prove that pollutants from spills of the vehicles are another contributing factor to the river contamination. Figure 6.3 explicitly depicts the above four hazards (i.e., flood, sewage discharge point, industrial discharge point, and spills from vehicles) associated with the river.

![Figure 6.3 Potential hazards to water sources of the assumed water supply system](image)

The reservoir in this example locates at a rural area that is in the northwest part of the system. Livestock is always a hazardous factor that contributes substantial microbial load. Furthermore, non-point pollutant sources scattered around the reservoir have contaminated the site and hence
compromise its water quality. Meanwhile, as shown by Figure 6.3, the reservoir is within the flooding area and potentially contaminated by surface runoff with high pollutants. Therefore, flood, livestock, and contaminated site are viewed as three possible hazards to the reservoir contamination.

Another water source, wells, has high iron concentration (e.g., Fe, Mn, etc.) because of the high content of minerals in the areas where the underground water flows. This has brought great difficulties of treatment process, and thus is viewed hazardous in this example. Meanwhile this area has been developed to a recreation area for human beings, which has made it influenced by human activities and site contaminations of non-point pollutant sources. Therefore, underground mineral, human activity, and site contamination of non-point sources are the three hazards needed to be considered for the wells (Figure 6.3).

Even though drought is important to cause water shortage in a general water supply system, it is assumed that it is not serious and can be neglected in this example. The following analysis, therefore, will not consider the influence of drought.

Being laid in an open area, the transmission aqueducts (i.e., pipes and channels) are potentially affected by several factors including extreme floods (or surface runoff), earth movement, extreme temperatures, and human wilful attacks. Consequence associated these hazards is dramatic decrease of water quantity supplied to the water treatment plant and the ultimate users.

For the water treatment plant in the water supply system, its potential hazards are various as shown by its historical records of operation and management. These factors include flood influences on the constuctures, process control failures, equipment failures, inadequate backup of main equipments, failures of alarm and monitoring system, power supply failures, and the human wilful chemical/biological contamination to the clean water tank at the end of the treatment process. These factors have different likelihoods of occurrences and possible consequences, and contribute substantially to the risk of water treatment plant.

The pump station in the water supply system is potentially influenced by various types of hazards according to its historical records. Natural hazards that can affect the pump performance include flood and earth movement. Hazards that can make it in an operational failure state are control failure, equipment failure, failures of alarm and monitoring, and inadequate backup of the main pumps. In addition, power outage is also a significant hazard that will interrupt its normal operation due to its high dependence on power supply. Furthermore, with the highlight of human-related threats, experts also suggest that threats such as bombing and physical disruption are also important and required in a comprehensive risk assessment of the pump.
Due to the surrounding conditions of the pipes and historical records, earth movement, soil, sewer leakage, and external loads are identified by the analysts as potential hazardous factors that would comprise the normal function of pipes (Figure 6.4). Firstly, in the area where pipes are buried, the volume of soil expands during wetting and shrinks during drying, which consequently introduces cyclic displacement and produces extra loads to damage the distribution pipes. Secondly, soils surrounding the pipes are with high concentrations of moisture content, chemical and microbiological contents, redox potential, etc., which thus increase the rate of soil-pipe interactions and pipe deterioration. Thirdly, there are many open drains in the distribution areas. Surrounding soil can be contaminated by sewer leakage from these open drains and hence has negative effects to the under buried pipes. Lastly, over burden of external loads of pipes, including earth weight and vehicle weight, is a possible reason of crushing force to cause longitudinal cracks of the pipes.

By summarising the above discussions, potential hazards associated with the components are given in Table 6.3.

### 6.1.3 Objectives of Risk Assessment of the Assumed Water Supply System

Two concerns are identified for the risk assessment of the assumed water supply system:

- to evaluate the risk levels of each component, subsystem, and the overall water system due to the influences of multiple potential hazards that are identified in Table 6.3, and
- to identify the potential causes and their relative contributions to specific risk at the user demand points (Figure 6.2).

With the results from the first issue, risk analysts can obtain useful risk information such as which kind of risk and which component risk levels are higher due to the influences from multiple hazards. While for each of the identified risk, the second issue can provide information of what
are the possible causes and how about their contributions to this risk. Both of two issues are important for a risk assessment. This study will use demand point 8 as an example to illustrate the process of analysing cause-effect relationships based on fault tree method. Similar analyses could be conducted to other user points, but will not be specifically considered in this thesis.

Table 6.3 Potential hazards associated with the assumed water supply system

<table>
<thead>
<tr>
<th>Component or Object</th>
<th>Function</th>
<th>Potential hazards</th>
</tr>
</thead>
<tbody>
<tr>
<td>River</td>
<td>Water source</td>
<td>• Flood</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Sewage discharge</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Industrial discharge</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Spills</td>
</tr>
<tr>
<td>Reservoir</td>
<td>Water source</td>
<td>• Flood</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Livestock</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Contaminated site</td>
</tr>
<tr>
<td>Wells (including well 1, 2, and 3)</td>
<td>Water source</td>
<td>• Underground mineral</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Human activity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Contaminated site</td>
</tr>
<tr>
<td>Transmission</td>
<td>Deliver water</td>
<td>• Flood</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Earth movement</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Extreme temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Human sabotage</td>
</tr>
<tr>
<td>Channel</td>
<td>Deliver water</td>
<td>• Flood</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Earth movement</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Extreme temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Human sabotage</td>
</tr>
<tr>
<td>Water treatment plant</td>
<td>Treat water</td>
<td>• Flood</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Human chemical/biological contamination</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Process control failure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Equipment failure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Alarm and monitoring failure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Inadequate backup</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Power failure</td>
</tr>
<tr>
<td>Pump</td>
<td>Pump water</td>
<td>• Flood</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Earth movement</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Bombing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Control failure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Equipment failure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Alarm and monitoring</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Inadequate backup</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Power failure</td>
</tr>
<tr>
<td>Pipes in water distribution network</td>
<td>Deliver water</td>
<td>• Earth movement</td>
</tr>
<tr>
<td>(including 29 pipes)</td>
<td></td>
<td>• External loads (e.g., soil load and traffic load)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Soil</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Sewer leakage from open drains</td>
</tr>
</tbody>
</table>
To analyse the two issues above, an object-oriented framework of risk assessment is firstly formed in Section 6.2. Following this, Section 6.3 describes the aggregative risk assessment process to evaluate the risk levels of components, subsystems, and the system, respectively. Section 6.4 analyses the cause-effect relationships for specific risk at demand point 8 by using fuzzy fault tree analysis. Meanwhile priorities of risk contributions are identified for potential causes. Finally, Section 6.5 summarises the applications of risk assessment.

### 6.2 Object-Oriented Framework of Risk Assessment

#### 6.2.1 Whole/Part Relationships of the Water Supply System

According to the configuration of the water supply system, object-oriented hierarchy is developed to represent the aggregation and generalisation relationships among components, subsystems, and the overall system (Figure 6.5). Aggregation relationship depicts the whole/part relationships in the water supply system, while generalization relationship provides the abstraction between objects and classes.

![Object and class structures of the assumed water supply system under study](image)

Figure 6.5 Object and class structures of the assumed water supply system under study
In this hierarchy, basic elements are river, reservoir, wells, water treatment plant, raw water transition pipes and channel, pump, and the distribution pipes. They are further classified to four basic classes, i.e., water source, water treatment plant, pipe, and pump, as shown by the class structure for this water supply system. Therefore, states transition diagrams of water source, water treatment plant, pipe, and pump developed in Chapter 3 are used here to represent the risk mechanism for each of the basic elements respectively. This is basis of risk assessment at component level for this example water supply system.

Aggregation relationships provide the frameworks for aggregative risk assessment of water supply system. Risk level of the overall water supply system is determined by risk levels of its subsystems, which are, in turn, determined by risk levels of their sub-subsystems or components. If risk levels of components at the bottom level of aggregation structure have been determined, risk levels of subsystems or system can thus be obtained by integrating the risks levels along the aggregation structure.

### 6.2.2 Interconnections Relationships between Objects

Interconnections between components are determined by the water flow directions in the water supply system (Figure 6.1). To facilitate the automated evaluations by computer programming, the results are summarised in a table by viewing them as attributes associated with different components/objects (Figure 6.4). Although nodes (e.g., river, reservoir, etc.) and links (e.g., pipe and pump) are different for object-oriented programming, it is not necessary to consider them separately in risk assessment. Interconnections among these components provide the information of developing fault trees to represent cause-effect relationships of specific risk at system level.

### 6.3 Aggregative Risk Assessment

Aggregative risk assessment is composed of two parts: (1) frameworks of risk assessment of objects/components and system, and (2) quantitative evaluations of the frameworks developed in step 1, which is illustrated in detail as follow.

#### 6.3.1 Frameworks of Aggregative Risk Assessment

**(1) Frameworks of aggregative risk assessment at component level**

States transition diagrams proposed in Chapter 3, in collaboration with potential hazards (Table 6.3) and object-oriented hierarchy (Figure 6.5), are used here to develop frameworks of aggregative risk assessment for basic elements of the system, respectively (Figure 6.6 to Figure 6.12).
Table 6.4 Interconnections among components in the assumed water supply system

<table>
<thead>
<tr>
<th>Object</th>
<th>From</th>
<th>To</th>
<th>Object</th>
<th>From</th>
<th>To</th>
</tr>
</thead>
<tbody>
<tr>
<td>River</td>
<td>-</td>
<td>TP1</td>
<td>Pipe 12</td>
<td>Pipe 2</td>
<td>Pipe 13</td>
</tr>
<tr>
<td>Reservoir</td>
<td>-</td>
<td>Channel</td>
<td>Pipe 13</td>
<td>Pipe 9</td>
<td>Pipe 17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pipe 12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wells</td>
<td>-</td>
<td>TP2</td>
<td>Pipe 14</td>
<td>Pipe 11</td>
<td>-</td>
</tr>
<tr>
<td>TP1</td>
<td>River</td>
<td>WTP</td>
<td>Pipe 15</td>
<td>Pipe 11</td>
<td>Pipe 16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pipe 11</td>
<td></td>
<td>Pipe 18</td>
</tr>
<tr>
<td>TP2</td>
<td>Well</td>
<td>WTP</td>
<td>Pipe 16</td>
<td>Pipe 15</td>
<td>Pipe 21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pipe 17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Channel</td>
<td>Reservoir</td>
<td>WTP</td>
<td>Pipe 17</td>
<td>Pipe 13</td>
<td>Pipe 16</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Pipe 17</td>
<td></td>
<td>Pipe 18</td>
</tr>
<tr>
<td>WTP</td>
<td>TP1</td>
<td>Pump</td>
<td>Pipe 18</td>
<td>Pipe 15</td>
<td>Pipe 19</td>
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<td></td>
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<td>Pipe 17</td>
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<tr>
<td>Pump</td>
<td>WTP</td>
<td>Pipe 1</td>
<td>Pipe 19</td>
<td>Pipe 18</td>
<td>Pipe 26</td>
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<td></td>
<td></td>
<td></td>
<td>Pipe 20</td>
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<tr>
<td>Pipe 1</td>
<td>Pump</td>
<td>Pipe 2</td>
<td>Pipe 20</td>
<td>Pipe 22</td>
<td>Pipe 19</td>
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<tr>
<td>Pipe 2</td>
<td>Pipe 1</td>
<td>Pipe 11</td>
<td>Pipe 21</td>
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<td>Pipe 12</td>
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<td>Pipe 23</td>
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<td>Pipe 3</td>
<td>Pipe 1</td>
<td>Pipe 4</td>
<td>Pipe 22</td>
<td>Pipe 21</td>
<td>Pipe 20</td>
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<td>Pipe 5</td>
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<td>Pipe 25</td>
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<td>Pipe 6</td>
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<td>Pipe 4</td>
<td>Pipe 3</td>
<td>Pipe 8</td>
<td>Pipe 23</td>
<td>Pipe 21</td>
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<td>Pipe 9</td>
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<td>Pipe 10</td>
<td></td>
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<tr>
<td>Pipe 5</td>
<td>Pipe 3</td>
<td>Pipe 7</td>
<td>Pipe 24</td>
<td>Pipe 23</td>
<td>Pipe 28</td>
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<td></td>
<td></td>
<td></td>
<td>Pipe 25</td>
<td></td>
<td>Pipe 29</td>
</tr>
<tr>
<td>Pipe 6</td>
<td>Pipe 3</td>
<td>-</td>
<td>Pipe 25</td>
<td>Pipe 22</td>
<td>Pipe 27</td>
</tr>
<tr>
<td>Pipe 7</td>
<td>Pipe 5</td>
<td>-</td>
<td>Pipe 26</td>
<td>Pipe 19</td>
<td>-</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Pipe 27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pipe 8</td>
<td>Pipe 4</td>
<td>-</td>
<td>Pipe 27</td>
<td>Pipe 25</td>
<td>Pipe 26</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Pipe 28</td>
<td></td>
<td></td>
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<tr>
<td>Pipe 9</td>
<td>Pipe 4</td>
<td>Pipe 13</td>
<td>Pipe 28</td>
<td>Pipe 24</td>
<td>Pipe 27</td>
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<tr>
<td>Pipe 10</td>
<td>Pipe 4</td>
<td>Pipe 21</td>
<td>Pipe 29</td>
<td>Pipe 24</td>
<td>-</td>
</tr>
<tr>
<td>Pipe 11</td>
<td>Pipe 2</td>
<td>Pipe 14</td>
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<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Pipe 15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.6 Framework of aggregative risk assessment of river contamination
Figure 6.7 Framework of aggregative risk assessment of reservoir contamination

Figure 6.8 Framework of aggregative risk assessment of wells contamination

Figure 6.9 Framework of aggregative risk assessment of transmission pipe and channel

Figure 6.10 Framework of aggregative risk assessment of water treatment plant object
(2) Framework of aggregative risk assessment at system level

Frameworks of aggregative risk assessment at system level are developed on the basis of object-oriented whole/part relationships. This framework shows that risk of the water supply system is contributed by the risk of its subsystems including water sources, water treatment plant, raw water transmission pipes, and water distribution network. Risks of these subsystems are further determined by their sub-systems and components. Inputs data for this framework are the risk levels of basic elements. There are two risks considered in this thesis, i.e., reduced water quantity and water contamination. Two frameworks are thus needed at system level to evaluate the aggregative levels of these two risks (Figure 6.13). Even though shortage of water sources also influences the reduced water quantity in a general water supply system, it is not applied to this assumed example. This is because that drought, the possible hazard that will introduce water shortage, is not possible here according to the hazards analysis in Section 6.1.2.
6.3.2 Inputs Information of Aggregative Risk Assessment

In order to evaluate the above frameworks of risk assessment quantitatively, it is necessary to firstly determine the inputs data including likelihood and severity of each hazard and weights factors of elements at the different hierarchical levels.

(1) Likelihood and severity of hazard

Determination of risks caused by multiple hazards is an uncertain process because their likelihoods and severities are related to many uncertain factors and cannot be represented precisely. This thesis uses multiple forms, including probability number, fuzzy number, and linguistic variables, to represent these multiple uncertainties associated hazards (Table 6.6).

Firstly, flood is one of the most important hazards because of its influences on many components of the water supply system. Historical records and experiences are available to analyse the relationships between its likelihood and relative severity (Table 6.5). It is shown that floods with high frequencies (i.e., 1/10) have minor influences to the components of the system. However, floods with low frequencies, or extreme floods, have the ability to carry various and a large amount of pollutants, and hence can introduce more serious influences on water sources and other
components. Therefore, this study specifically considers the flood of low frequency (i.e., likelihood of 1/50 which is highlighted in Table 6.5) and analyses its influences to the risk of the overall system.

Table 6.5 Relationships between likelihood and severity of flood hazards

<table>
<thead>
<tr>
<th>Likelihood</th>
<th>Consequence*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>River</td>
</tr>
<tr>
<td>1/10</td>
<td>M</td>
</tr>
<tr>
<td>1/50</td>
<td>VH</td>
</tr>
<tr>
<td>1/100</td>
<td>VH</td>
</tr>
</tbody>
</table>

* All the evaluations about consequences are based on experts’ opinions or engineers quantitative evaluations.

For other hazards, there may not be sufficient records that can be used to evaluate their likelihoods and consequences. However, they can be approximated by considering the limited records, coupled with experiences of engineers and experts’ opinion. For example, from the records of chemical factory and relative records of other factories, the failure likelihood can be estimated approximately. While, its relative influences to the river (i.e., severity) is determined by the quality simulation models of the river. Similar evaluations can also be performed on other hazards including sewage discharge, pollutant spills, livestock, human activity, underground minerals, and contaminated site.

While for the hazards related to water treatment plant and the pump station, linguistic and descriptive evaluations of their hazards can be easily obtained based on operational records coupled with managing experiences in other similar plants and pump stations. Even these linguistic evaluations are vague and uncertain in nature, they can normally be represented in terms of fuzzy numbers and valuable for a quantitative analysis.

Finally, for the pipes in the water distribution network, the likelihoods of earth movement and temperature can be estimated based on the historical records, and their consequences can be subjectively described by fuzzy numbers. The influences of hazards like external loads, soil, and sewer leakage can be analysed by various statistical and/or physical models that have been well developed. However, in order to simply the process of determining these data, they are subjectively assumed and represented by fuzzy numbers in this thesis.
According to the above discussion, likelihoods and severities of hazards are summarised in Table 6.6.

Table 6.6 Inputs of likelihood and severity of hazards to the assumed water supply system

<table>
<thead>
<tr>
<th>Element</th>
<th>Failure state</th>
<th>Hazards</th>
<th>Likelihood</th>
<th>Severity</th>
</tr>
</thead>
<tbody>
<tr>
<td>River</td>
<td>NH</td>
<td>Flood</td>
<td>0.02</td>
<td>VH</td>
</tr>
<tr>
<td></td>
<td>HC</td>
<td>Sewage discharge</td>
<td>SL</td>
<td>VH</td>
</tr>
<tr>
<td></td>
<td>IN</td>
<td>Industrial discharge</td>
<td>SL</td>
<td>EH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Spill</td>
<td>VL</td>
<td>M</td>
</tr>
<tr>
<td>Reservoir</td>
<td>NH</td>
<td>Flood</td>
<td>0.02</td>
<td>SH</td>
</tr>
<tr>
<td></td>
<td>HC</td>
<td>Livestock</td>
<td>SH</td>
<td>VL</td>
</tr>
<tr>
<td></td>
<td>IN</td>
<td>Contaminated site</td>
<td>SH</td>
<td>SL</td>
</tr>
<tr>
<td>Wells (1,2 and 3)</td>
<td>NH</td>
<td>Underground mineral</td>
<td>VH</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td>HC</td>
<td>Human activity</td>
<td>SL</td>
<td>SL</td>
</tr>
<tr>
<td></td>
<td>IN</td>
<td>Contaminated Site</td>
<td>SH</td>
<td>SH</td>
</tr>
<tr>
<td>Transmission (Pipe)</td>
<td>NH</td>
<td>Flood</td>
<td>0.01</td>
<td>EH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Earth movement</td>
<td>About 0.01</td>
<td>EH</td>
</tr>
<tr>
<td></td>
<td>HC</td>
<td>Sabotage</td>
<td>VL</td>
<td>SH</td>
</tr>
<tr>
<td></td>
<td>OP</td>
<td>Temperature</td>
<td>About 0.2</td>
<td>VH</td>
</tr>
<tr>
<td>Transmission (Channel)</td>
<td>NH</td>
<td>Flood</td>
<td>About 0.02</td>
<td>VH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Earth movement</td>
<td>About 0.01</td>
<td>VH</td>
</tr>
<tr>
<td></td>
<td>HC</td>
<td>Sabotage</td>
<td>VL</td>
<td>SH</td>
</tr>
<tr>
<td></td>
<td>OP</td>
<td>Temperature</td>
<td>About 0.02</td>
<td>SL</td>
</tr>
<tr>
<td>Water treatment plant</td>
<td>NH</td>
<td>Flood</td>
<td>0.02</td>
<td>SH</td>
</tr>
<tr>
<td></td>
<td>HC</td>
<td>Chemical/biological</td>
<td>VL</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td>OP</td>
<td>Process</td>
<td>VL</td>
<td>VH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Equipment</td>
<td>SL</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Alarm and monitor</td>
<td>SL</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Backup</td>
<td>VL</td>
<td>SH</td>
</tr>
<tr>
<td></td>
<td>IN</td>
<td>Power</td>
<td>SL</td>
<td>EH</td>
</tr>
<tr>
<td>Pump</td>
<td>NH</td>
<td>Flood</td>
<td>0.02</td>
<td>SH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Earth movement</td>
<td>About 0.01</td>
<td>EH</td>
</tr>
<tr>
<td></td>
<td>HC</td>
<td>Bombing</td>
<td>EL</td>
<td>EH</td>
</tr>
<tr>
<td></td>
<td>OP</td>
<td>Control</td>
<td>SL</td>
<td>SH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Equipment</td>
<td>SL</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Alarm and monitor</td>
<td>SL</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Backup</td>
<td>SL</td>
<td>SL</td>
</tr>
<tr>
<td></td>
<td>IN</td>
<td>Power</td>
<td>VL</td>
<td>VH</td>
</tr>
<tr>
<td>Pipe (1-13, 15-23)</td>
<td>NH</td>
<td>Earth movement</td>
<td>About 0.01</td>
<td>VH</td>
</tr>
<tr>
<td></td>
<td>OP</td>
<td>External load</td>
<td>M</td>
<td>SH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Soil</td>
<td>SL</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td>IN</td>
<td>Sewerage leakage</td>
<td>SL</td>
<td>M</td>
</tr>
<tr>
<td>Pipe (14, 24-29)</td>
<td>NH</td>
<td>Earth movement</td>
<td>About 0.01</td>
<td>VH</td>
</tr>
<tr>
<td></td>
<td>OP</td>
<td>External load</td>
<td>M</td>
<td>VH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Soil</td>
<td>VL</td>
<td>SL</td>
</tr>
<tr>
<td></td>
<td>IN</td>
<td>Sewerage leakage</td>
<td>SL</td>
<td>M</td>
</tr>
</tbody>
</table>

* NH—natural hazard failure state; HC—human-caused threat failure state; OP—operational failure state; IN—independence failure state.

**EH—extremely high; VH—very high; SH—slightly high; M—medium; SL—slightly low; VL—very low; EL—extremely low. These values can be represented by fuzzy numbers based on the discussion in Chapter 4.
(2) Determination of weight factors

Weight factors represent the importance of risk contribution from basic factors. In practice, there are many methods that can be used to obtain these values, such as experts opinions, analytical hierarchy process (AHP), methods based on hydraulic simulations, etc. Detailed discussion of these methods exceeds the scope of this study. To simplify the process of demonstrating the proposed method, it is assumed that the relative hazards, associated with each failure state, equally contribute to the risk values of failure state. Furthermore, for each object, its failure states also have identical weights in risk assessment. According to different risk (i.e., water contamination or reduced water quantity), weights are subjectively assumed for pipe network, pump, water treatment plant, transmission pipes, channel, river, reservoir, and wells, respectively. Table 6.7 and Table 6.8 list the weights for the two risk scenarios.

Table 6.7 Weight factors for aggregative risk assessment of water contamination

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
<th>Failure state</th>
<th>Hazards</th>
</tr>
</thead>
<tbody>
<tr>
<td>WSS</td>
<td>WSS</td>
<td>River</td>
<td>NH</td>
<td>Flood</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td>$w=0.2$</td>
<td>$w=0.5$</td>
<td></td>
<td>HC</td>
<td>Sewage discharge</td>
<td>$w=0.5$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IN</td>
<td>Spill</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td>Reservoir</td>
<td></td>
<td></td>
<td>NH</td>
<td>Flood</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td>$w=0.2$</td>
<td></td>
<td></td>
<td>HC</td>
<td>Livestock</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IN</td>
<td>Contaminated site</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td>Wells</td>
<td></td>
<td></td>
<td>NH</td>
<td>Natural chemical</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td>$w=0.3$</td>
<td></td>
<td></td>
<td>HC</td>
<td>Human activity</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IN</td>
<td>Contaminated Site</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td>WTP</td>
<td>WTP</td>
<td></td>
<td>NH</td>
<td>Flood</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td>$w=0.5$</td>
<td></td>
<td></td>
<td>HC</td>
<td>Chemical/Biological</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OP</td>
<td>Process</td>
<td>$w=0.25$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Equipment</td>
<td>$w=0.25$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Alarm and monitor</td>
<td>$w=0.25$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Backup</td>
<td>$w=0.25$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IN</td>
<td>Power</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td>WDN</td>
<td>Pipes</td>
<td>Pipe 1-29</td>
<td>IN</td>
<td>Sewerage leakage</td>
<td>$w=1.0$</td>
</tr>
<tr>
<td>$w=0.3$</td>
<td>$w=1.0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*WSS—water supply system; WSS—water source; WTP—water treatment plant; WDN—water distribution network; NH—natural hazard failure state; HC—human-caused threat failure state; OP—operational failure state; IN—independence failure state.
Table 6.8 Weight factors for aggregative risk assessment of reduced water quantity

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
<th>Failure state</th>
<th>Hazards</th>
</tr>
</thead>
<tbody>
<tr>
<td>WSS</td>
<td>Transmission</td>
<td>Pipe 1-2</td>
<td>NH</td>
<td>Flood</td>
<td>w=0.5</td>
</tr>
<tr>
<td></td>
<td>w=0.3</td>
<td>w=1/3</td>
<td>w=0.5</td>
<td>Earth movement</td>
<td>w=0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HC</td>
<td>Sabotage</td>
<td>w=1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OP</td>
<td>Temperature</td>
<td>w=1.0</td>
</tr>
<tr>
<td>Channel</td>
<td></td>
<td></td>
<td>NH</td>
<td>Flood</td>
<td>w=0.5</td>
</tr>
<tr>
<td></td>
<td>w=0.2</td>
<td></td>
<td>w=1/3</td>
<td>Earth movement</td>
<td>w=0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HC</td>
<td>Sabotage</td>
<td>w=1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OP</td>
<td>Temperature</td>
<td>w=1.0</td>
</tr>
<tr>
<td>WTP</td>
<td></td>
<td></td>
<td>NH</td>
<td>Flood</td>
<td>w=1.0</td>
</tr>
<tr>
<td></td>
<td>w=0.2</td>
<td></td>
<td>w=1/3</td>
<td>Process</td>
<td>w=0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OP</td>
<td>Equipment</td>
<td>w=0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IN</td>
<td>Alarm and monitor</td>
<td>w=0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Backup</td>
<td>w=0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Power</td>
<td>w=1.0</td>
</tr>
<tr>
<td>WDN</td>
<td></td>
<td></td>
<td>NH</td>
<td>Flood</td>
<td>w=0.5</td>
</tr>
<tr>
<td></td>
<td>w=0.5</td>
<td></td>
<td>w=0.25</td>
<td>Earth movement</td>
<td>w=0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>HC</td>
<td>Bombing</td>
<td>w=1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OP</td>
<td>Control</td>
<td>w=0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Equipment</td>
<td>w=0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Alarm and monitor</td>
<td>w=0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Backup</td>
<td>w=0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Power</td>
<td>w=1.0</td>
</tr>
<tr>
<td>Pipes</td>
<td></td>
<td></td>
<td>NH</td>
<td>Earth movement</td>
<td>w=1.0</td>
</tr>
<tr>
<td></td>
<td>w=0.5</td>
<td></td>
<td>w=0.5</td>
<td>External load</td>
<td>w=0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>OP</td>
<td>Soil</td>
<td>w=0.5</td>
</tr>
</tbody>
</table>

For the pipes in water distribution network, their weight factors are determined by their capacities of carrying water flow. Carrying capacity is determined by \( (Wagner et al., 1988) \)

\[
Q = 0.2795CD^{2.63}S^{0.54}
\]

where \( Q \) is the flow capacity (million gallons per day (mgd)), \( C \) is the Hazen-Williams coefficient, \( D \) is the internal pipe diameter (feet, \( 1\text{ft}=0.3048\text{m} \)), and \( S \) is the maximum hydraulic gradient. Values of \( C \) and \( D \) are given in Table 6.2. \( S \) is assumed to be 0.01 in this example. Based on the values of carrying capacity, weight of each pipe is determined by

\[
w_i = \frac{Q_i}{\sum_{i=1}^{n} Q_i}
\]

where \( Q_i \) is the carrying capacity of pipe \( i \); \( w_i \) is the weight of pipe \( i \) in aggregative risk assessment; \( i \) is the index of pipe number; \( n \) is the number of pipes in the distribution network. The results are summarised in Table 6.9 and applied to both risk of reduced water quantity and risk of water contamination.
Table 6.9 Weights factors of pipes in the water distribution network

<table>
<thead>
<tr>
<th>Pipe No.</th>
<th>Capacity (mgd)</th>
<th>Weights</th>
<th>Pipe No.</th>
<th>Capacity (mgd)</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.15</td>
<td>0.20</td>
<td>16</td>
<td>2.56</td>
<td>0.03</td>
</tr>
<tr>
<td>2</td>
<td>7.43</td>
<td>0.07</td>
<td>17</td>
<td>2.56</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>7.43</td>
<td>0.07</td>
<td>18</td>
<td>1.58</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>4.60</td>
<td>0.05</td>
<td>19</td>
<td>1.58</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>2.56</td>
<td>0.03</td>
<td>20</td>
<td>1.58</td>
<td>0.02</td>
</tr>
<tr>
<td>6</td>
<td>5.45</td>
<td>0.05</td>
<td>21</td>
<td>1.58</td>
<td>0.02</td>
</tr>
<tr>
<td>7</td>
<td>0.41</td>
<td>0.00</td>
<td>22</td>
<td>1.58</td>
<td>0.02</td>
</tr>
<tr>
<td>8</td>
<td>2.56</td>
<td>0.03</td>
<td>23</td>
<td>1.58</td>
<td>0.02</td>
</tr>
<tr>
<td>9</td>
<td>3.84</td>
<td>0.04</td>
<td>24</td>
<td>1.12</td>
<td>0.01</td>
</tr>
<tr>
<td>10</td>
<td>3.84</td>
<td>0.04</td>
<td>25</td>
<td>2.01</td>
<td>0.02</td>
</tr>
<tr>
<td>11</td>
<td>5.45</td>
<td>0.05</td>
<td>26</td>
<td>2.01</td>
<td>0.02</td>
</tr>
<tr>
<td>12</td>
<td>3.84</td>
<td>0.04</td>
<td>27</td>
<td>2.01</td>
<td>0.02</td>
</tr>
<tr>
<td>13</td>
<td>3.84</td>
<td>0.04</td>
<td>28</td>
<td>1.12</td>
<td>0.01</td>
</tr>
<tr>
<td>14</td>
<td>0.53</td>
<td>0.01</td>
<td>29</td>
<td>1.12</td>
<td>0.01</td>
</tr>
<tr>
<td>15</td>
<td>3.84</td>
<td>0.04</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.3.3 Quantitative Results of Aggregative Risk Assessment

The method of aggregative risk assessment have been developed in Chapter 4 and coded with Visual C++ in this study. By using this method and the above inputs, quantitative results of aggregative risk assessment are obtained and discussed below.

(1) Aggregative risk assessment at component level

With the above inputs and aggregative assessment frameworks, risk levels are obtained for different components in the water supply system, which includes river, reservoir, wells, transmission pipes, channel, water treatment plant, pump, and pipes. Table 6.10 list the results of aggregative risk assessment of the river, while risk levels of other components can be found in Appendix E.

Here risk levels are represented by seven predefined risk grades from extremely low (EL) to extremely high (EH). Value associated with each risk grade denotes the degrees that the risk item belongs to it. All the values of the seven grades therefore represent a risk distribution of the risk item under study. Meanwhile, associated with each risk distribution are representative value and fuzziness measure that represent the aggregative risk value and relative uncertainty with this evaluation, respectively.
Table 6.10 Results of aggregative risk assessment of river contamination

<table>
<thead>
<tr>
<th>Risk levels</th>
<th>EL</th>
<th>VL</th>
<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Fuzziness</th>
<th>Representative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood</td>
<td>0.079</td>
<td>0.921</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.555</td>
<td>0.027</td>
</tr>
<tr>
<td>Sewage discharge</td>
<td>0</td>
<td>0</td>
<td>0.046</td>
<td>0.796</td>
<td>0.158</td>
<td>0.001</td>
<td>0</td>
<td>1.134</td>
<td>0.275</td>
</tr>
<tr>
<td>Industrial discharge</td>
<td>0</td>
<td>0</td>
<td>0.026</td>
<td>0.598</td>
<td>0.373</td>
<td>0.002</td>
<td>0</td>
<td>1.471</td>
<td>0.320</td>
</tr>
<tr>
<td>Spills</td>
<td>0.005</td>
<td>0.191</td>
<td>0.753</td>
<td>0.051</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.279</td>
<td>0.102</td>
</tr>
<tr>
<td>Natural hazard</td>
<td>0.079</td>
<td>0.921</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.555</td>
<td>0.027</td>
</tr>
<tr>
<td>Human-caused</td>
<td>0</td>
<td>0</td>
<td>0.031</td>
<td>0.718</td>
<td>0.249</td>
<td>0.001</td>
<td>0</td>
<td>1.303</td>
<td>0.295</td>
</tr>
<tr>
<td>Interdependence</td>
<td>0.005</td>
<td>0.191</td>
<td>0.753</td>
<td>0.051</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.279</td>
<td>0.102</td>
</tr>
<tr>
<td>River</td>
<td>0.029</td>
<td>0.427</td>
<td>0.301</td>
<td>0.189</td>
<td>0.054</td>
<td>0</td>
<td>0</td>
<td>2.121</td>
<td>0.117</td>
</tr>
</tbody>
</table>

For example, due to the influences of multiple hazards (i.e., flood, sewage discharge, industrial discharge, and spills pollutant from vehicles) the river has different degrees to several risk grades from EL to SH (Table 6.10). It has the highest degrees on risk grade VL, followed by risk grades SL, M, SH, and EL. The representative risk value of this risk distribution is approximate 0.117 and the uncertainty or fuzziness with this distribution is 2.121. This uncertainty or fuzziness comes from the fact that risk is represented in terms of a distribution on several risk grades rather than a crisp value. These numbers can be explained more clearly in the following ways:

- Risk levels or risk distributions are a type of measure based on evidence. Higher values to specific risk grade indicates more or stronger evidence supporting the evaluation on this risk grade. On the contrary, lower values indicates weaker evidence. The river, for instance, has the highest degree to risk grade VL and zero to risk grade EH because of stronger evidence supporting risk grade VL, while no evidence supporting its belonging to risk level EL.

- There are uncertainties with the evaluations of risk levels or risk distributions. Uncertainty associated with a hazard is caused by incomplete data or approximate evaluation of its likelihoods of occurrence and relative severity. While the uncertainty of aggregative risk, e.g., risk of the river, is introduced by two sources. One is the lack of information about hazard; the other is the conflicting evidence from multiple hazards. For example, hazard of flood provide evidence supporting the risk grades of EL and VL (Table 6.10), while industrial discharge supports risk grades from SL to VH. It is obvious that there are conflict between the information provided by these two hazards. It is the conflict that introduces extra uncertainty to the evaluation of river risk. The uncertainty of the evaluation of river (i.e., 2.121) is therefore larger than that of a single hazard (e.g., 0.555 of flood). However, if multiple sources of evidence were not conflict, the uncertainty with the aggregative risk would not necessary increase. For example, sewage
Chapter 6 Applications of Object-Oriented Risk Assessment in Water Supply Systems

discharge and industrial discharge (Table 6.10) support the same risk grades from SL to 
VH, the uncertainty of their aggregation (i.e., 1.303 of human-caused failure state) is not 
increased by comparing with the uncertainties of the two hazards (i.e., 1.134 and 1.471 
respectively). With these results, risk analysts can identify the main reasons of uncertainty 
associated specific risk assessment and make reasonable decisions in the water systems.

- Aggregative risk also gives a way of ranking components in terms of their risk levels. 
Among the three water sources, for instance, risk associated with wells is higher than that 
of reservoir, which is in turn higher than that of the river (Figure E.1 to Figure E.3 in 
Appendix E). This comparison can help system managers to adjust the management 
policies and maintenance measures among the water sources.

- If the analysts or managers have their acceptable level for each risk grade, this 
aggregative risk distribution can explicitly show whether the hazard or risk of a 
component is acceptable or not. For example, if the acceptable level for risk grade SL is 
20%, then it is obvious that risk of the river is unacceptable as it has 30.1% degrees to the 
SL.

Risk levels of all the element are also depicted in Figure 6.14 where risk distributions and their 
uncertainties are obviously expressed.

(2) Aggregative risk assessment at system level

After the determination of risk levels of components, it is easy to obtain risk levels of subsystems 
and the overall water supply system by aggregating these risks along the hierarchical structures 
(Figure 6.13).

Firstly, contributing factors to reduced water quantity are transmission pipes, water treatment 
plant, and components of water distribution network. Results of this aggregation are summarised 
in Table 6.11.

- Risk levels of transmission pipes, belonging to four risk grades from EL to M, are 
obtained by aggregating the risks of the pipes and channel that connected the water 
sources and water treatment plant.

- Risk of water distribution network is determined by repeating the risk aggregation on the 
pump and pipes. It has degrees to six risk grades from EL to VH, which indicates that it 
has higher uncertainty (i.e., 2.295) than that of transmission pipes (i.e., 1.878).

- Finally, by aggregating risks of transmission pipes, water treatment plant (WTP), and 
water distribution network (WDN), risk levels of reduced water quantity in the water 
supply system are ultimately obtained. Its is shown that it has the highest degree to risk 
grade SL which is followed by risk grades VL, M, EL, SH, and VH.
• Representative risk values show that water treatment plant has the highest risk degrees, then are the transmission pipes and water distribution network. While fuzziness measures show that the risk evaluations of water treatment plant are most uncertain because conflicting among its contributing factors. The risk levels of water distribution network is also with high uncertain, even its representative risk value is low, because the conflict between risks of pump and pipes. These results can be used by risk analysts to select mitigation measures and determine the further detailed risk studies.

Secondly, for the risk of water contamination, its contributing sources are water sources, water treatment plant, and pipe lines in the distribution network. Results of this aggregation are listed in Table 6.12.

• The risk levels of contamination of water source are obtained by aggregating the risks from river, reservoir, and wells. For the water distribution network, its risk of contamination are evaluated by doing the aggregation on its pipes. Risk of pump is not considered in this assessment because consequence of pump failures is only reduced water quantity rather than contamination.

• The water supply system has the highest degree to SL that is followed by M, VL, SH, EL, and VH. Among the contributing subsystems, water sources have a risk value that is slightly higher than that of water distribution network. Both of them are higher than that of water treatment plant. Meanwhile, due to consistency between multiple contributing elements, uncertainty of risks of the whole system is not increased.

The results of above aggregative risks are depicted in Figure 6.15 and Figure 6.16 respectively.

Table 6.11 Results of aggregative risk assessment of reduced water in water supply system

<table>
<thead>
<tr>
<th>Risk levels</th>
<th>EL</th>
<th>VL</th>
<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Fuzziness</th>
<th>Representative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission pipes</td>
<td>0.075</td>
<td>0.182</td>
<td>0.574</td>
<td>0.170</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.878</td>
<td>0.113</td>
</tr>
<tr>
<td>WTP</td>
<td>0.060</td>
<td>0.323</td>
<td>0.170</td>
<td>0.308</td>
<td>0.138</td>
<td>0.001</td>
<td>0</td>
<td>2.337</td>
<td>0.168</td>
</tr>
<tr>
<td>WDN</td>
<td>0.188</td>
<td>0.386</td>
<td>0.229</td>
<td>0.157</td>
<td>0.039</td>
<td>0.001</td>
<td>0</td>
<td>2.295</td>
<td>0.097</td>
</tr>
<tr>
<td>WSS</td>
<td>0.122</td>
<td>0.314</td>
<td>0.327</td>
<td>0.191</td>
<td>0.046</td>
<td>0.001</td>
<td>0</td>
<td>2.298</td>
<td>0.116</td>
</tr>
</tbody>
</table>

Table 6.12 Results of aggregative risk assessment of water contamination in water supply system

<table>
<thead>
<tr>
<th>Risk levels</th>
<th>EL</th>
<th>VL</th>
<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Fuzziness</th>
<th>Representative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>WS</td>
<td>0.022</td>
<td>0.272</td>
<td>0.315</td>
<td>0.165</td>
<td>0.212</td>
<td>0.015</td>
<td>0</td>
<td>2.278</td>
<td>0.189</td>
</tr>
<tr>
<td>WTP</td>
<td>0.042</td>
<td>0.287</td>
<td>0.347</td>
<td>0.232</td>
<td>0.092</td>
<td>0</td>
<td>0</td>
<td>2.271</td>
<td>0.147</td>
</tr>
<tr>
<td>WDN</td>
<td>0</td>
<td>0.012</td>
<td>0.502</td>
<td>0.475</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>1.511</td>
<td>0.180</td>
</tr>
<tr>
<td>WSS</td>
<td>0.020</td>
<td>0.162</td>
<td>0.424</td>
<td>0.333</td>
<td>0.060</td>
<td>0.001</td>
<td>0</td>
<td>2.086</td>
<td>0.162</td>
</tr>
</tbody>
</table>
Figure 6.14 Risk levels of components in the assumed water supply system
Figure 6.14 (Cont.) Risk levels of components in the assumed water supply system
Figure 6.15 Risk levels reduced water quantity in the assumed water supply system

Figure 6.16 Risk levels of contamination in the assumed water supply system
6.4 Fault Tree Analysis

Fault tree analysis is conducted here to assess the cause-effect relationships for a specific risk in the water supply system. Fault tree analyses in this application is composed of two steps: (1) construction of fault trees; and (2) quantitative analysis of fault trees developed in step 1. This is illustrated in detail in the following parts.

6.4.1 Fault Tree Structures

To analyse the risk of no water flow and water contamination at demand point 8, fault trees are required to be constructed at both component and system levels. Fault trees at the component level identify the potential hazards or threats which can make the specific component fail, while those at system level identify the potential components which will influence or contribute to the risk at user point.

(1) Fault Trees of Components

As discussed in Chapter 3, fault trees of components are developed in three steps based on the relationships among object, failure states, and potential hazards or threats. To take the contamination of river as an example, there are three possible failure states according to the hazards identified above. Each of these failure states, i.e., natural hazards failure state, human caused failure state, or interdependence failure state, can cause contamination to the river component. Furthermore the natural hazard failure state is caused by hazard flood. While human-caused failure state is caused by either industrial discharge or sewage discharge, and interdependence failure state is caused by hazard of pollutant spills. By viewing those hazards as basic events, fault tree for contamination of the river is thus constructed (Figure 6.17). Similarly, fault trees for risks of other components are also developed in Figure 6.18 through Figure 6.23. Meanings of the symbols used in the following fault tree can be referred to figures shown in Figure 6.7 through Figure 6.12, respectively.

![Fault Tree Diagram](image)

Figure 6.17 Structure of fault tree representing the potential causes to river contamination
Figure 6.18 Structure of fault tree representing the potential causes to reservoir contamination

Figure 6.19 Structure of fault tree representing the potential causes to well contamination

Figure 6.20 Structure of fault tree representing the potential causes to transmission pipes and channel
Figure 6.21 Structure of fault tree representing the potential causes to water treatment plant

Figure 6.22 Structure of fault tree representing the potential causes to pump failure

Figure 6.23 Structure of fault tree representing the potential causes to of pipe
(2) Fault Trees of the System

In order to develop fault trees for risk of the water supply system, the interconnection relationships are required (Table 6.4). Two fault trees are developed for the risk of no water flow and contamination at user point 8, respectively (Figure 6.24 and Figure 6.25).

![Fault Tree Diagram](image)

Figure 6.24 Fault tree representing the potential causes to no water flow at user point 8
The developed fault tree for no water at user point 8 (Figure 6.24) indicates that this risk is potentially introduced by failures of pipes 1, 3, 4, 5, 7, and 8, pump, water treatment plant, failures of transmission aqueducts. While for the risk of contamination at user point 8 (Figure
6.25), failures of pipes 1, 3, 4, 5, 7, or 8, water treatment plant, river, reservoir, or wells are identified as the potential contributing factors. The following quantitative risk assessment is able to describe the cause-effect relationships more clearly.

6.4.2 Inputs Information for Fault Tree Analysis

The input data for the quantitative fault tree analysis are the likelihoods of hazards which are the basic event and at the bottom level of the tree structures. These inputs data are available in Table 6.6. The method to evaluate the fault trees quantitatively have been discussed in Chapter 5 and coded with Visual C++ in this study. Results of these fault trees are specifically discussed below.

6.4.3 Quantitative Results of Fault Tree Analysis

(1) Fault tree analysis at component level

Based on the fault tree structures (Figure 6.17 to Figure 6.23) and evaluation method proposed in Chapter 5, quantitative results of fault trees are obtained for risks of components of the water supply system (Figure 6.26). As most of the input data are fuzzy numbers, the obtained results are also fuzzy numbers. Meanwhile, the importance measures of hazards are also analysed for each component in order to evaluate the risk contributions from basic event to the top event in the fault tree structures (Table 6.13). Several conclusions are obtained from the fault tree analysis as follow:

- Fault tree analysis represents the cause-effect relationships to specific risk of each component in terms of likelihood of occurrence. Because the differences of hazards and fault tree structures, the likelihood of top event of different component is also different. Therefore, they will have different contributions to the relative risk in the water supply system, which are discussed in the following part. Additionally, the likelihoods of risk of different components have different uncertainties. For example, the possible likelihood of river contamination is with more uncertainty compared with that of reservoir contamination as shown by Figure 6.26.

- Different hazards have different contributions to the likelihood of component risk. The importance measures (including fuzzy importance and fuzzy uncertainty importance) of each hazards are determined to describe the above differences of contributions explicitly (Table 6.13). For the river, sewage and industrial discharges have the highest importance measures, which indicates that they contribute more to occurrence of river contamination. Thus river quality would be effectively controlled if these two hazard were controlled at an acceptable level. Furthermore, fuzzy uncertainty importance measure provides a
ranking of reducing uncertainty of the top event in a fault tree analysis, which is useful to guide the further data collection or detailed analysis about the hazards.

Figure 6.26 Fuzzy representations of failure likelihoods of components in the assumed water supply system.
Figure 6.26 (Cont.) Fuzzy representations of failure likelihoods of components in the assumed water supply system
## Table 6.13 Importance measures of fault trees at component levels

<table>
<thead>
<tr>
<th>Risk of elements*</th>
<th>Threats</th>
<th>Fuzzy importance</th>
<th>Rank</th>
<th>Fuzzy uncertainty importance</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>River (R_2)</td>
<td>Flood</td>
<td>6.52</td>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Sewage discharge</td>
<td>8.9</td>
<td>1</td>
<td>0.762</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Industrial discharge</td>
<td>8.9</td>
<td>1</td>
<td>0.762</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Spills</td>
<td>7.2</td>
<td>3</td>
<td>0.62</td>
<td>3</td>
</tr>
<tr>
<td>Reservoir (R_2)</td>
<td>Flood</td>
<td>2.117</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Livestock</td>
<td>5.296</td>
<td>1</td>
<td>0.451</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Human activity</td>
<td>5.296</td>
<td>1</td>
<td>0.451</td>
<td>1</td>
</tr>
<tr>
<td>Well (R_2)</td>
<td>Underground mineral</td>
<td>3.902</td>
<td>1</td>
<td>0.346</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Human activity</td>
<td>1.239</td>
<td>3</td>
<td>0.12</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Contaminated site</td>
<td>2.234</td>
<td>2</td>
<td>0.209</td>
<td>2</td>
</tr>
<tr>
<td>Transmission aqueducts</td>
<td>Flood</td>
<td>10.452</td>
<td>3</td>
<td>0.053</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Earth movement</td>
<td>10.347</td>
<td>4</td>
<td>0.052</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Sabotage</td>
<td>12.094</td>
<td>2</td>
<td>1.009</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Temperature</td>
<td>12.701</td>
<td>1</td>
<td>0.319</td>
<td>2</td>
</tr>
<tr>
<td>Water treatment plant (R_1)</td>
<td>Flood</td>
<td>4.37</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Process control</td>
<td>4.693</td>
<td>4</td>
<td>0.434</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Equipment</td>
<td>5.755</td>
<td>1</td>
<td>0.528</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Alarm and monitoring</td>
<td>5.755</td>
<td>1</td>
<td>0.528</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Inadequate backup</td>
<td>4.693</td>
<td>4</td>
<td>0.434</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Power failure</td>
<td>5.755</td>
<td>1</td>
<td>0.528</td>
<td>1</td>
</tr>
<tr>
<td>Water treatment plant (R_2)</td>
<td>Flood</td>
<td>4.019</td>
<td>7</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Chemical/Biological</td>
<td>4.282</td>
<td>4</td>
<td>0.41</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Process control</td>
<td>4.282</td>
<td>4</td>
<td>0.41</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Equipment</td>
<td>5.24</td>
<td>1</td>
<td>0.498</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Alarm and monitoring</td>
<td>5.24</td>
<td>1</td>
<td>0.498</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Inadequate backup</td>
<td>4.282</td>
<td>4</td>
<td>0.41</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Power failure</td>
<td>5.24</td>
<td>1</td>
<td>0.498</td>
<td>1</td>
</tr>
<tr>
<td>Pump (R_1)</td>
<td>Flood</td>
<td>3.54</td>
<td>6</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Earth movement</td>
<td>3.486</td>
<td>7</td>
<td>0.02</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Bombing</td>
<td>3.484</td>
<td>8</td>
<td>0.061</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>4.636</td>
<td>1</td>
<td>0.43</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Equipment</td>
<td>4.636</td>
<td>1</td>
<td>0.43</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Alarm and monitoring</td>
<td>4.636</td>
<td>1</td>
<td>0.43</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Inadequate backup</td>
<td>4.636</td>
<td>1</td>
<td>0.43</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Power failure</td>
<td>3.785</td>
<td>5</td>
<td>0.354</td>
<td>5</td>
</tr>
<tr>
<td>Pipes (R_1)</td>
<td>Earth movement</td>
<td>5.617</td>
<td>3</td>
<td>0.029</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>External loads</td>
<td>10.396</td>
<td>1</td>
<td>0.872</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Soil</td>
<td>7.865</td>
<td>2</td>
<td>0.663</td>
<td>2</td>
</tr>
<tr>
<td>Pipes (R_2)</td>
<td>Sewer leakage</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Pipes (R_3)</td>
<td>Earth movement</td>
<td>1.604</td>
<td>4</td>
<td>0.008</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>External loads</td>
<td>3.307</td>
<td>2</td>
<td>0.27</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Soil</td>
<td>2.428</td>
<td>3</td>
<td>0.196</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Sewer leakage</td>
<td>5.404</td>
<td>1</td>
<td>0.868</td>
<td>1</td>
</tr>
</tbody>
</table>

*R_1, R_2, and R_3 denote risks of reduced water quantity, water contamination, and water contamination and reduced quantity, respectively.*
Therefore, the fault tree analysis can produce information of which component has higher likelihood of occurrence, and which hazard contribute most to this occurrence. They are useful for risk management of components in the water supply system.

(2) Fault tree analysis at system level

Based on analysis at component level, quantitative results are obtained for the risks of reduced water and contamination at demand point 8 respectively (Figure 6.27). It is shown that both the risk of reduced water quantity and risk of contamination have very high probability to occur. Thus, under the current situation, the hazards can very possibly influence the water supply system by decreasing the water quantity of contaminating the water quality delivered to users at demand point 8. Therefore, mitigation measures should be taken to reduce the risks caused by the hazards. This can be achieved with the help of analysing the importance as follow.

Figure 6.27 Fuzzy representations of failure likelihoods of risks in the water supply system

Table 6.14 summarises the fuzzy importance measures and fuzzy uncertainty measures of the components. Associated with the risk of reduced water quantity, failure pumps has the highest contribution, which is followed by failures within water treatment plant and distribution pipes 1 and 3. Therefore, in order to reduced the likelihood of reduced water quantity, effective migration measure should focus on controlling the occurrences of pump failure, water treatment failure, or failures of pipes 1 and 3 in the distribution network. While in order to mitigate water contamination, effective measures should be on controlling the failure of water treatment plant, wells contamination and river pollution. These conclusions can be further integrated with the conclusions achieved at component levels. For example, in order to reduce pump failure, the effective measures should be those that can reduced the risks caused by control failure, equipment
failure, alarm and monitor failure, and inadequate backup as shown by Table 6.13. Similarly, for the river pollution, the effective measures should be able to mitigate the occurrences of sewage and industrial discharge because they influence more to risk of the river.

Table 6.14 Results of importance measures of fault trees at system level

<table>
<thead>
<tr>
<th>Elements</th>
<th>Fuzzy importance</th>
<th>Rank</th>
<th>Fuzzy uncertainty importance</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission pipes</td>
<td>0.108</td>
<td>5</td>
<td>0.017</td>
<td>5</td>
</tr>
<tr>
<td>Transmission channel</td>
<td>0.108</td>
<td>5</td>
<td>0.017</td>
<td>5</td>
</tr>
<tr>
<td>Water treatment plant</td>
<td>0.186</td>
<td>2</td>
<td>0.05</td>
<td>2</td>
</tr>
<tr>
<td>Pump</td>
<td>0.225</td>
<td>1</td>
<td>0.051</td>
<td>1</td>
</tr>
<tr>
<td>Pipe 1</td>
<td>0.171</td>
<td>3</td>
<td>0.03</td>
<td>3</td>
</tr>
<tr>
<td>Pipe 3</td>
<td>0.171</td>
<td>3</td>
<td>0.03</td>
<td>3</td>
</tr>
<tr>
<td>Pipe 4</td>
<td>0.073</td>
<td>7</td>
<td>0.013</td>
<td>7</td>
</tr>
<tr>
<td>Pipe 5</td>
<td>0.073</td>
<td>7</td>
<td>0.013</td>
<td>7</td>
</tr>
<tr>
<td>Pipe 7</td>
<td>0.073</td>
<td>7</td>
<td>0.013</td>
<td>7</td>
</tr>
<tr>
<td>Pipe 8</td>
<td>0.073</td>
<td>7</td>
<td>0.013</td>
<td>7</td>
</tr>
</tbody>
</table>

Contamination at user point 8

<table>
<thead>
<tr>
<th>Elements</th>
<th>Fuzzy importance</th>
<th>Rank</th>
<th>Fuzzy uncertainty importance</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>River</td>
<td>0.368</td>
<td>3</td>
<td>0.066</td>
<td>2</td>
</tr>
<tr>
<td>Reservoir</td>
<td>0.23</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Well</td>
<td>0.494</td>
<td>2</td>
<td>0.061</td>
<td>3</td>
</tr>
<tr>
<td>Water treatment plant</td>
<td>0.546</td>
<td>1</td>
<td>0.134</td>
<td>1</td>
</tr>
<tr>
<td>Pipe 1</td>
<td>0.284</td>
<td>4</td>
<td>0.036</td>
<td>4</td>
</tr>
<tr>
<td>Pipe 3</td>
<td>0.284</td>
<td>4</td>
<td>0.036</td>
<td>4</td>
</tr>
<tr>
<td>Pipe 4</td>
<td>0.284</td>
<td>4</td>
<td>0.036</td>
<td>4</td>
</tr>
<tr>
<td>Pipe 5</td>
<td>0.284</td>
<td>4</td>
<td>0.036</td>
<td>4</td>
</tr>
<tr>
<td>Pipe 7</td>
<td>0.284</td>
<td>4</td>
<td>0.036</td>
<td>4</td>
</tr>
<tr>
<td>Pipe 8</td>
<td>0.284</td>
<td>4</td>
<td>0.036</td>
<td>4</td>
</tr>
</tbody>
</table>

Contamination at user point 8 (With dependent event)

<table>
<thead>
<tr>
<th>Elements</th>
<th>Fuzzy importance</th>
<th>Rank</th>
<th>Fuzzy uncertainty importance</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>River</td>
<td>0.375</td>
<td>3</td>
<td>0.085</td>
<td>2</td>
</tr>
<tr>
<td>Reservoir</td>
<td>0.224</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Well</td>
<td>0.478</td>
<td>2</td>
<td>0.06</td>
<td>3</td>
</tr>
<tr>
<td>Water treatment plant</td>
<td>0.523</td>
<td>1</td>
<td>0.132</td>
<td>1</td>
</tr>
<tr>
<td>Pipe 1</td>
<td>0.276</td>
<td>4</td>
<td>0.035</td>
<td>4</td>
</tr>
<tr>
<td>Pipe 3</td>
<td>0.276</td>
<td>4</td>
<td>0.035</td>
<td>4</td>
</tr>
<tr>
<td>Pipe 4</td>
<td>0.276</td>
<td>4</td>
<td>0.035</td>
<td>4</td>
</tr>
<tr>
<td>Pipe 5</td>
<td>0.276</td>
<td>4</td>
<td>0.035</td>
<td>4</td>
</tr>
<tr>
<td>Pipe 7</td>
<td>0.276</td>
<td>4</td>
<td>0.035</td>
<td>4</td>
</tr>
<tr>
<td>Pipe 8</td>
<td>0.276</td>
<td>4</td>
<td>0.035</td>
<td>4</td>
</tr>
</tbody>
</table>

This thesis also considers the failure dependency in the water supply system. For example, during the flooding season, the contamination of the reservoir is possibly influenced by the contamination of the river as well. With respect to this, the likelihood of reservoir contamination due to flood is dependent on the likelihood of river contamination during flood time. It is assumed
that the dependency is about 0.5 and can be represented by a triangular fuzzy number (0.4, 0.5, 0.6) in this application. The results are listed in Table 6.14. It is shown that the importance value of river is increased from 0.368 to 0.375 by considering the failure dependence, while importance measures of other components are decreased. This indicates that river contamination is more important in this situation than that in the former situation.

6.5 Summary

The applications of the risk assessment methods developed in this research are demonstrated by an assumed water supply example. From this example application, the following conclusions can be made:

- Risk levels of each element (including pipe, pump, river, well, etc.), each subsystem (i.e., water source, transmissions, water treatment, and water distribution), and the overall water supply system are obtained by using the aggregative risk assessment method proposed in Chapter 4. Risk levels are defined on seven grades (i.e., EL, VL, SL, M, SH, VH, and EH) in this application. The risk levels are a set of numbers which represent the degrees belonging to different risk grades. From these numbers, the risk distribution are directly illustrated. Meanwhile, the representative risk value, similar to the expected value in probability theory, and fuzziness of the risk distribution are also obtained to produce more useful information for risk decision making.

- For each risk, the potential hazards or threats and their risk contributions are identified and analysed by using the fuzzy fault tree analysis proposed in Chapter 5. The analysis are performed at both component and system levels. At the component level, the mechanism of hazards resulting in different risk are analysed. At the system level, fault trees are constructed to represent the logic relationships for risk at user demand point. In these fault trees, failures of components and their contribution to the top event are identified and analysed. Based on these analyses, analysts can easily identify the critical assets and hazards for the specific risk at user demand points.
Conclusions and Recommendations for Further Work

Water supply systems are one of most important fundamentals for human living and development. Risk assessment has been recognised as an important part of water management. Risks within a water supply system are mainly related to water quality and/or water quantity. Potential factors that can cause these risks are generally associated with complex internal and external activities during the process of delivering water from sources to customers. An effective risk assessment thus requires the consideration of the characteristics of potential hazards and threats, risk levels of components, and comprehensive understanding of cause-effect relationships for specific risk in the system. With respect to this requirement, several conclusions are obtained from the current study as follow.

7.1 Conclusions

7.1.1 General Conclusions

The literature review, conducted in Chapter 2, highlights the limitations associated with the existing methods to generate a comprehensive risk assessment for water supply systems. Most of the previous studies were specific to either subsystems/components, or one aspect of risk assessment. Furthermore, they were developed for specific applications, which thus limits their reusability if applied to other systems with different configurations and layouts. The important hurdles of conducting a comprehensive risk assessment are identified as complexity of water supply systems and uncertainty of risk analysis.

In order to overcome the limitations of existing methods, the current study adopts an object-oriented approach to develop hierarchical frameworks for risk assessment of water supply systems, and applies fuzzy sets-based methods to evaluate these frameworks. Two types of frameworks are developed in this thesis, i.e., (1) framework for aggregative risk assessment to
evaluate risks of components, subsystems, and overall water supply system, and (2) framework for fault tree analysis to represent the cause-effect relationships of specific risk.

Results generated from these frameworks are useful for decision makers to prioritise their mitigations strategies for water supply systems. Firstly, the results of aggregative risk assessment are risk levels for components, subsystems, and the overall water supply system, respectively. The application in Chapter 6 shows that risk analysts can use this information to compare components and to identify the uncertainties contributing to the aggregative risk. Secondly, the results of the fault tree analysis are the likelihood of occurrence for a specific risk and importance measures of the potential contributing factors. The application in Chapter 6 demonstrates that they are useful to identify the critical components and hazards for a specific risk. Based on this analysis, decision makers can select effective mitigation measures to control and manage risks.

7.1.2 Specific Conclusions

The method developed in thesis can better meet the requirements of comprehensive risk assessments than existing methods in the following ways.

(1) This study proposes a new method of developing hierarchical frameworks for risk assessment of water supply system. These frameworks can provide a comprehensive view of the risks in the system, and also be easily used by different users.

Firstly, hierarchical frameworks of risk assessment have been developed at both component and system levels in Chapter 3. These frameworks provide a comprehensive view of risks associated components, subsystems, and the overall water supply system, and explicitly represent the cause-effect relationships for specific risk. However, this would be difficult to obtain by using the existing methods that are reviewed in Chapter 2.

Secondly, the frameworks can be flexibly established at different hierarchical levels according to the requirements of system observers or available information at hand. The discussion in Section 3.2.2 shows that system observers can truncate the hierarchy at higher/micro levels or extend it to lower/micro levels dependent on the their specific requirements and/or available information of hazards. However, existing methods can hardly meet the requirements of multiple users, as they were originally developed for some specific users.

(2) The frameworks developed in thesis can be easily reused in different water supply systems because of its new way of organising risk information.
Chapter 3 of this thesis discusses the organisation of risk in the water supply system around objects, and based on this, has developed general frameworks for risk assessment. These general frameworks can be easily reused in specific water supply system as shown in Chapter 6. However, most of the existing methods consider the risk information according to specific application or hazards, which limits their reusability in different situations.

(3) The framework developed in this thesis can give evaluations of risks by considering multiple hazards that are represented in varied forms.

The frameworks developed in this thesis have the ability to deal with varied number and types of hazards. The examples in Chapter 4 and application in Chapter 6 show that multiple hazards, both natural and human-related, can be simultaneously considered in the same framework. Furthermore, the number of hazards associated with an object can also be varied in water supply systems. Although hazards may be varied in different water supply systems, the ability of the frameworks will not be compromised. However, most of the existing methods can only deal with specific hazards, which thus limits their ability to generate comprehensive views of risk assessment.

(4) The fuzzy sets-based methods used in this study can cover more uncertainties than probabilities can in most historical methods.

Hazards in the water supply system are usually represented in varied forms. Probability is suitable to describe those that are statistic in nature, but has difficulties in representing others that are vague and descriptive. Fuzzy sets theory can effectively represent the latter uncertainties. In addition, probability can also be transferred to fuzzy numbers (Kaufmann and Gupta, 1991), while the reverse transformation is not unusually difficult. Therefore, fuzzy sets-based methods, used in this thesis, can consider wider range of uncertainties to generate a more comprehensive view of risk.

7.2 Further Work

This study has developed a new approach for risk assessment of water supply systems. However, there is work that should be undertaken to further strengthen the approach developed here. This includes extra conceptual frameworks to meet the requirements of comprehensive risk assessment, and quantitative analysis methods to produce more reasonable and useful risk information for decision makings.
Chapter 7 Conclusions and Recommendations for Further Work

7.2.1 Further Conceptual Frameworks

The present study only considers aggregative risk assessment and fault tree analysis in water supply systems. However, more frameworks are required for a comprehensive risk assessment.

(1) A framework for vulnerability assessment of water supply system is required. By considering the vulnerability and hazards simultaneously, risk analysts can have better understanding on the likelihood and severities associated risks. In this study, vulnerability is assumed to be unity for all the components, which is not in accordance with the real case. Therefore there is a need to develop a framework for vulnerability assessment.

(2) In order to describe the cause-effect relationships more comprehensively, a framework is also required for forward event tree analysis to analyse the potential consequences for some specific risks. With this framework, risk analysts can predict possible risk scenarios associated with specific hazardous event, which will be of benefit for planning and management of water supply system.

7.2.2 Further Revision of the Quantitative Analysis Methods

Further work is required to enhance the applicability of the developed risk assessment method by water utilities. Firstly for the aggregative method, further research should focus on.

(1) Developing proper membership functions of likelihood and severity for each hazard in the water supply system. Likelihoods of hazards can be analysed by using different existing models based on historical data, data from similar systems, experts’ opinion, experiences of engineers, etc. Severity of each hazard can either be objectively studied with help of developed models, or subjectively evaluated by the risk analyst, experts, etc. However, how to represent likelihoods and severities with proper fuzzy membership functions is still a important task requiring further studies. Ross (2004) and Bilgic et al. (1999) have discussed several methods of determining membership functions, which could be the basis for future work.

(2) Determining weights factors properly. Weights are only subjective simple heuristics in this research. However, more detailed research is required to study proper methods of determining weight factors, as they significantly influence the results of aggregative risk assessment. In practice, weights of a component might be related to many factors such as cost, location, hydraulic behaviours, etc., which may bring difficulties in evaluating weights precisely. Therefore, proper consideration of these factors is important for the method developed in this thesis to generate more reasonable and useful results.
Secondly for fault tree analysis, there are also several aspects that would benefit from further study:

(1) Degrees of dependencies are assumed in this study. However, more reasonable values can be used if they can be determined more precisely.

(2) The fault tree structure developed in this study does not recognise the vulnerability of each component in water supply systems. Therefore, vulnerability factors should be included in the assessment of fault trees in future.

7.3 End Point

This thesis has made a contribution to the understanding of risk assessment for water supply system and new ideas of organising complex risk information based on concepts of object-oriented approach. It is hoped that this work will increase the efficiency of water utilities to manage risks in a complex water system that is affected by multiple hazards.

It is emphasised that this work is the first step in the formulation of object-oriented frameworks for risk assessment of water supply systems. It is anticipated that future research will result in a better understanding of this method, and overall improved risk assessment.
References


References


References


Potential Chemical and Biological Hazards/Threats to Water Supply Systems

The following table lists the chemicals that are effective in drinking water. The list includes both chemical warfare agents and industrial chemical poisons, which could be used a short checking list for contamination assessment in water supply systems.

Table A.1 Potential chemicals effectiveness in water supply systems (Mays, 2004b)

<table>
<thead>
<tr>
<th>Chemical agents (mg/L unless otherwise noted)</th>
<th>Acute concentration</th>
<th>Recommended guidelines</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5L</td>
<td>5L/day</td>
</tr>
<tr>
<td><strong>Chemical warfare agents:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrogen cyanide</td>
<td>25</td>
<td>6.0</td>
</tr>
<tr>
<td>Tabun (GA, µg/L)</td>
<td>50</td>
<td>70.0</td>
</tr>
<tr>
<td>Sarin (GB, µg/L)</td>
<td>50</td>
<td>13.8</td>
</tr>
<tr>
<td>Soman (GD, µg/L)</td>
<td>50</td>
<td>6.0</td>
</tr>
<tr>
<td>VX (µg/L)</td>
<td>50</td>
<td>7.5</td>
</tr>
<tr>
<td>Lewisite (arsenic fraction)</td>
<td>100-130</td>
<td>80.0</td>
</tr>
<tr>
<td>Sulfer Mustard (µg/L)</td>
<td>140.0</td>
<td></td>
</tr>
<tr>
<td>3-quinuclidinyl benzilate (BZ, µg/L)</td>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>Lysergic acid diethylamide (LSD)</td>
<td>0.050</td>
<td></td>
</tr>
<tr>
<td><strong>Industrial chemical poisons:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cyanides</td>
<td>25</td>
<td>6.0</td>
</tr>
<tr>
<td>Arsenic</td>
<td>100-130</td>
<td>80.0</td>
</tr>
<tr>
<td>Fluoride</td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>Cadmium</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>75-300</td>
<td></td>
</tr>
<tr>
<td>Dieldrin</td>
<td>5000</td>
<td></td>
</tr>
<tr>
<td>Sodium fluoroacetate</td>
<td>None provided</td>
<td></td>
</tr>
<tr>
<td>Parathion</td>
<td>None provided</td>
<td></td>
</tr>
</tbody>
</table>
The following table lists several pathogens and biotoxins that can potentially cause problems in drinking water production and distribution. This list includes important characteristics of agent such as type, possibility of being weaponized, stable in water, and chlorine tolerance. This could be used as a checking list for biological hazards to water supply systems in practice.

<table>
<thead>
<tr>
<th>Agent</th>
<th>Type</th>
<th>Weaponized</th>
<th>Water threat</th>
<th>Stable in water</th>
<th>Chlorine tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anthrax</td>
<td>Bacteria</td>
<td>Yes</td>
<td>Yes</td>
<td>2 years (spores)</td>
<td>Spores resistant</td>
</tr>
<tr>
<td>Brucellosis</td>
<td>Bacteria</td>
<td>Yes</td>
<td>Probable</td>
<td>20-72 days</td>
<td>Unknown</td>
</tr>
<tr>
<td>C. perfringens</td>
<td>Bacteria</td>
<td>Probable</td>
<td>Probable</td>
<td>Common in sewage</td>
<td>Resistant</td>
</tr>
<tr>
<td>Tularemia</td>
<td>Bacteria</td>
<td>Yes</td>
<td>Yes</td>
<td>Up to 90 days</td>
<td>Inactivated, 1ppm, 5min</td>
</tr>
<tr>
<td>Glanders</td>
<td>Bacteria</td>
<td>Probable</td>
<td>Unlikely</td>
<td>Up to 30 days</td>
<td>Unknown</td>
</tr>
<tr>
<td>Meliodosis</td>
<td>Bacteria</td>
<td>Possible</td>
<td>Unlikely</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td>Shigelllosis</td>
<td>Bacteria</td>
<td>Unknown</td>
<td>Yes</td>
<td>2-3 days</td>
<td>Inactivated, 0.05ppm, 10min</td>
</tr>
<tr>
<td>Cholera</td>
<td>Bacteria</td>
<td>Unknown</td>
<td>Yes</td>
<td>“Survives well”</td>
<td>Easily killed</td>
</tr>
<tr>
<td>Salmonellos</td>
<td>Bacteria</td>
<td>Unknown</td>
<td>Yes</td>
<td>8 days, fresh water</td>
<td>Inactivated</td>
</tr>
<tr>
<td>Plague</td>
<td>Bacteria</td>
<td>Probable</td>
<td>Possible</td>
<td>16 days</td>
<td>Unknown</td>
</tr>
<tr>
<td>Q fever</td>
<td>Rickettsia</td>
<td>Yes</td>
<td>Possible</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td>Typhus</td>
<td>Rickettsia</td>
<td>Probable</td>
<td>Unlikely</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td>Psittacosis</td>
<td>Rickettsia-like</td>
<td>Possible</td>
<td>Possible</td>
<td>18-24 h, sea water</td>
<td>Unknown</td>
</tr>
<tr>
<td>Encephalomyelitis</td>
<td>Virus</td>
<td>Probable</td>
<td>Unlikely</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td>Hemorrhagic fever</td>
<td>Virus</td>
<td>Probable</td>
<td>Unlikely</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td>Variola</td>
<td>Virus</td>
<td>Possible</td>
<td>Possible</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td>Hepatitis A</td>
<td>Virus</td>
<td>Unknown</td>
<td>Yes</td>
<td>Unknown</td>
<td>Inactivated, 0.4ppm, 30min</td>
</tr>
<tr>
<td>Cryptospridiosis</td>
<td>Protozoan</td>
<td>Unknown</td>
<td>Yes</td>
<td>Stable for days or more</td>
<td>Oocysts resistant</td>
</tr>
<tr>
<td>Botulinum toxins</td>
<td>Biotoxin</td>
<td>Yes</td>
<td>Yes</td>
<td>Stable</td>
<td>Inactivated, 6ppm, 20min</td>
</tr>
<tr>
<td>T-2 mycotoxin</td>
<td>Biotoxin</td>
<td>Probable</td>
<td>Yes</td>
<td>Stable</td>
<td>Resistant</td>
</tr>
<tr>
<td>Aflatoxin</td>
<td>Biotoxin</td>
<td>Yes</td>
<td>Yes</td>
<td>Probably stable</td>
<td>Probably tolerant</td>
</tr>
<tr>
<td>Ricin</td>
<td>Biotoxin</td>
<td>Yes</td>
<td>Yes</td>
<td>Unknown</td>
<td>Resistant at 10ppm</td>
</tr>
<tr>
<td>Staph enterotoxins</td>
<td>Biotoxin</td>
<td>Probable</td>
<td>Yes</td>
<td>Probably stable</td>
<td>Unknown</td>
</tr>
<tr>
<td>Microcystins</td>
<td>Biotoxin</td>
<td>Possible</td>
<td>Yes</td>
<td>Probably stable</td>
<td>Resistant at 100ppm</td>
</tr>
<tr>
<td>Anotoxin A</td>
<td>Biotoxin</td>
<td>Unknown</td>
<td>Probable</td>
<td>Inactivated in days</td>
<td>Unknown</td>
</tr>
<tr>
<td>Tetrodotoxin</td>
<td>Biotoxin</td>
<td>Possible</td>
<td>Yes</td>
<td>Unknown</td>
<td>Inactivated, 50ppm</td>
</tr>
<tr>
<td>Saxitoxin</td>
<td>Biotoxin</td>
<td>Possible</td>
<td>Yes</td>
<td>Stable</td>
<td>Resistant at 10ppm</td>
</tr>
</tbody>
</table>
Fuzzy Sets Theory

For the convenience of the reader to understand both basics of fuzzy theory and its application in this thesis, we divide this appendix to two parts. The first part is to briefly summarise some basic concepts of fuzzy sets theory, which includes fuzzy sets, fuzzy numbers, fuzzy operations, fuzzy arithmetic. The second part introduces the derivations of equations used in Chapter 4.

B.1 Basics of Fuzzy Sets Theory

B.1.1 Fuzzy Sets and Fuzzy Numbers

Definitions of fuzzy sets and numbers have been provided in Chapter 4. The main purpose of this appendix is to introduce the operations and arithmetic on fuzzy sets.

B.1.2 Operations of Fuzzy Sets

The fuzzy sets-theoretic operations are defined via membership functions. Here we present the concepts suggested by Zadeh (1965).

Definition 1: Intersection. The membership function \( \mu_c(x) \) of the intersection is pointwise defined by

\[
\mu_c(x) = \min(\mu_A(x), \mu_B(x)), x \in X \tag{B.1}
\]
Definition 2: Union. The membership function $\mu_c(x)$ of the union $C = A \cup B$ is pointwise defined by

$$\mu_c(x) = \max(\mu_A(x), \mu_B(x)), x \in X$$

(B.2)

Definition 3: Complement. The membership function of the complement of a fuzzy set $A$, $\mu_A(x)$, is defined by

$$\mu_A(x) = 1 - \mu_A(x)$$

(B.3)
B.1.3 Extension Principle

Mathematical definition has been found in Chapter 4. The application of extension principle can be illustrated by an example: “if a pump is about 10 years old, how about its failure probability?” Normally, failure probability of a pump with time can be determined by \( p(t) = e^{-\lambda t} \). However, in this example, \( t \) (about 10 years old) is represented as a fuzzy number (Figure B.4), which thus requires extension principle to obtain the membership function for the failure probability \( p(t) \). The result of failure probability of pump is also a fuzzy number due to the fuzzy time inputs.

![Figure B.4 An example of explaining fuzzy extension principle](image)

Extension principle is one of the most important fundamentals in fuzzy sets theory to perform fuzzy calculations by using functions on fuzzy sets or numbers. However, it is difficult to apply to practical problem. An alternative to this is \( \alpha \)-cut as discussed below.

B.1.4 Fuzzy Arithmetic

Since \( \alpha \)-cut of fuzzy number is always an intervals of confidence. Fuzzy number can be considered as a generalisation of interval of confidence. Therefore, the arithmetic operation of fuzzy number can use the same process as intervals of confidence operation, but level by level (Kaufmann and Gupta, 1991).

**Definition 4: Interval of confidence arithmetic.** For any two intervals of confidence, \([a, b]\) and \([c, d]\), the arithmetic operation are performed in the following way:

**Addition:** \([a, b]+[c, d]=[a+c, b+d]\)
Subtraction: \([a, b] - [c, d] = [a-d, b-c]\)

Multiplication: \([a, b] \times [c, d] = [\min(ac, ad, bc, db), \max(ac, ad, bc, db)]\)

Division: \([a, b] / [c, d] = [\min(a/c, a/d, b/c, b/d), \max(a/c, a/d, b/c, b/d)]\)

Power: \([a, b]^{[c, d]} = [\min(a^c, a^d, b^c, b^d), \max(a^c, a^d, b^c, b^d)]\)

**Definition 5: Triangular fuzzy number.** The membership function \(\mu_T\) of triangular fuzzy number \(T\) on \(R = (-\infty, +\infty)\) is \(\mu_T : R \to [0,1]\):

\[
\mu_T(x) = \begin{cases} 
\frac{x-l}{m-l} & \text{if } l < x \leq m \\
\frac{u-x}{u-m} & \text{if } m < x \leq u \\
0 & \text{otherwise}
\end{cases}
\] (B.4)

where \(l \leq m \leq u\), and \(l\) and \(u\) stand for the lower and upper values of the support of the fuzzy number \(T\), respectively, and \(m\) for the modal value. A triangular fuzzy number is also denoted by \((l, m, u)\).

![Figure B.5 Triangular fuzzy number](image)

**Definition 6: Operations of triangular fuzzy number.** For any two triangular fuzzy numbers, \(T_1 = (l_1, m_1, u_1)\), and \(T_2 = (l_2, m_2, u_2)\), the arithmetic operations are performed in the following way (van Laarhoven et al., 1983):

**Addition:** \(T_1 + T_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2)\)

**Subtraction:** \(T_1 - T_2 = (l_1 - u_2, m_1 - m_2, u_1 - l_2)\)

**Multiplication:** \(T_1 \times T_2 \equiv (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2)\)
Appendix B

Division: \[ T_1 + T_2 \equiv (l_1 / u_2, m_1 / m_2, u_1 / l_2) \]

Power: \[ T_1^{T_2} \equiv (l_1^{l_2} m_1^{m_2} u_1^{u_2}) \]

The above operations are suitable to be programmed, therefore they are used to automatically perform the operations of fuzzy numbers in this study.

B.2 Calculation of Risk

Because of the simple representations of triangular fuzzy numbers, it is easy to derive mathematical representations of risks in terms of the production of likelihood and severity that are represented by triangular fuzzy numbers (Kaufmann and Gupta, 1991). For any hazard, its likelihood of occurrence and relative severity are represented by two triangular fuzzy number \( L=(l_1, m_1, u_1) \) and \( S=(l_2, m_2, u_2) \). Therefore, \( L \) and \( S \) can be mathematically represented by

\[
\mu_L(x) = \begin{cases} \frac{x-l_1}{m_1-l_1} & l_1 < x \leq m_1 \\ \frac{m_1-x}{u_1-m_1} & m_1 < x \leq u_1 \\ 0 & \text{otherwise} \end{cases}
\]

\[
\mu_S(x) = \begin{cases} \frac{x-l_2}{m_2-l_2} & l_2 < x \leq m_2 \\ \frac{m_2-x}{u_2-m_2} & m_2 < x \leq u_2 \\ 0 & \text{otherwise} \end{cases}
\]

For a confidence level \( \alpha \), the \( \alpha \)-cut sets can be determined for \( L \) and \( S \) respectively as

\[
\alpha = \mu_L(x) = \begin{cases} \frac{L_1^\alpha - l_1}{m_1-l_1} & \text{and} \end{cases} \alpha = \mu_S(x) = \begin{cases} \frac{S_1^\alpha - l_1}{m_2-l_2} \\ \frac{m_2-l_2}{u_1-m_1} \end{cases}
\]

\[
R_1^\alpha = L_1^\alpha \cdot S_1^\alpha \\
R_2^\alpha = L_2^\alpha \cdot S_2^\alpha
\]

Figure B.6 Determination of risk in terms of the production of likelihood and severity
Therefore,

\[
\begin{align*}
L_1^\alpha &= (m_1 - l_1)\alpha + l_1 \\
L_2^\alpha &= u_1 - (u_1 - m_1)\alpha \\
S_1^\alpha &= (m_2 - l_2)\alpha + l_2 \\
S_2^\alpha &= u_2 - (u_2 - m_2)\alpha
\end{align*}
\]

By using production of intervals, then

\[
R_1^\alpha = L_1^\alpha \cdot S_1^\alpha = (m_1 - l_1)(m_2 - l_2)\alpha^2 + [(m_1 - l_1)l_2 + (m_2 - l_2)l_1]\alpha + l_1l_2
\]

\[
R_2^\alpha = L_2^\alpha \cdot S_2^\alpha = (u_1 - m_1)(u_2 - m_2)\alpha^2 - [(u_1 - m_1)u_2 + (u_2 - m_2)u_1]\alpha + u_1u_2
\]

Thus by solving the equations,

\[
\alpha = \mu(x) = \begin{cases} 
- \frac{b_1 + \sqrt{b_1^2 - 4a_1(c_1 - x)}}{2a_1} & l_1l_2 < x \leq m_1m_2 \\
\frac{b_2 + \sqrt{b_2^2 - 4a_2(c_2 - x)}}{2a_2} & m_1m_2 < x \leq u_1u_2 \\
0 & \text{otherwise}
\end{cases}
\]

where

\[
\begin{align*}
a_1 &= (m_1 - l_1)(m_2 - l_2) \\
b_1 &= (m_1 - l_1)l_2 + (m_2 - l_2)l_1 \\
c_1 &= l_1l_2 \\
a_2 &= (u_1 - m_1)(u_2 - m_2) \\
b_2 &= (u_1 - m_1)u_2 + (u_2 - m_2)u_1 \\
c_2 &= u_1u_2
\end{align*}
\]

Applying the above results to the likelihood and severity levels defined in Chapter 4, risk analysts can obtain the seven risk levels respectively.
Dempster-Shafer Theory

For the convenience of the reader to understand both basics of Dempster-Shafer theory and its application in this thesis, we divide this appendix to two parts. The first part is to briefly summarise some basic concepts of Dempster-Shafer theory, which includes basic probability assignment, belief, plausibility, and combination rule. The second part introduces the combination rules used in Chapter 4.

C.1 Basics of Dempster-Shafer Theory

C.1.1 Basic Probability Assignment, Belief, and Plausibility

(1) Basic Probability Assignment
The definition of basic probability assignment is available in Chapter 4.

(2) Belief function (Bel) and plausibility (Pl)
From the basic probability assignment, the upper and lower bounds of an interval can be defined. The interval is bounded by two non-additive continuous measures called belief and plausibility respectively. The lower bound belief (Bel) for a set $A$ is defined as the sum of all the basic probability assignments of the proper subsets ($B$) of the set of interest $A$, i.e., $B \subseteq A$. The general relation between bpa and Bel can be defined as

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{C.1}$$

The upper bound, plausibility (Pl), is the summation of basic probability assignment of the sets $B$ that intersect with the set of interest $A$, i.e., $B \cap A \neq \Phi$ and therefore, it can be defined as

$$Pl(A) = \sum_{B \cap A \neq \Phi} m(B) \tag{C.2}$$
In Dempster-Shafer theory, basic probability assignment (bpa), belief (Bel), and plausibility (Pl) are in one-to-one correspondence, they can be seen as three facets of the same piece of information. In Chapter 4, it is introduced how to use bpa to derive Bel and Pl. However, it is also possible to obtain the basic probability assignment (bpa) from the belief measure with the following inverse function:

$$m(A) = \sum_{B \subseteq A} (-1)^{|A-B|} \text{Bel}(B)$$  \hspace{1cm} (C.3)

where $|A-B|$ is the difference of the cardinality of the two sets.

In addition, plausibility can be determined by

$$\text{Pl}(A) = 1 - \text{Bel}(\tilde{A})$$  \hspace{1cm} (C.4)

where

$$\text{Bel}(\tilde{A}) = \sum_{B \subseteq A} m(B) = \sum_{B \subseteq A, B \neq \emptyset} m(B)$$  \hspace{1cm} (C.5)

### C.1.2 Combination Rule of Dempster-Shafer Theory

To continue with the introduction in Chapter 4, the combination can be shown by an example (Figure C.1). In this example, there are two analysts to give their assessments of failure probability of a pipe in terms of linguistic levels such as high (H), medium (M), and low (L). The process of combining the two assessments is explicit shown by a computation table (Table C.1).

![Figure C.1 An example of combining evidences in Dempster-Shafer theory](image-url)
Table C.1 Computation table of combining evidences in Dempster-Shafer theory

<table>
<thead>
<tr>
<th>$m_2$</th>
<th>$m_1$</th>
<th>L</th>
<th>M</th>
<th>H</th>
<th>L,M</th>
<th>L,H</th>
<th>M,H</th>
<th>L,M,H</th>
</tr>
</thead>
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<td>{L}</td>
<td>{Φ}</td>
<td>0</td>
<td>{L}</td>
<td>{Φ}</td>
<td>0</td>
<td>{L}</td>
</tr>
<tr>
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<td>{Φ}</td>
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<td>{Φ}</td>
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<td>{H}</td>
<td>0.4</td>
<td>{Φ}</td>
<td>0</td>
<td>{H}</td>
</tr>
<tr>
<td>L,M</td>
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<td>{L}</td>
<td>{M}</td>
<td>0</td>
<td>{L}</td>
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<td>{L}</td>
<td>0</td>
<td>{H}</td>
<td>0</td>
<td>{L}</td>
<td>0</td>
<td>{H}</td>
</tr>
<tr>
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<td>0.16</td>
<td>{M}</td>
</tr>
</tbody>
</table>

Although the original combination rule discussed above can give reasonable results in lots of cases, it also receives serious criticism when significant conflict in the information sources. Consequently, other researchers have developed modified Dempster rules that attempt to represent the degree of conflict in the final result. This will be discussed in the following part.

C. 2 Combination Rule in Fuzzy Sets Theory

C.2.1 Two Fuzzy Combination Rules

This appendix only reviews the methods proposed by Ishizuka et al. (1982) and Yen (1990) as they are closely related to the derivation the combination rule used in this thesis.

(1) Ishizuka et al.’s (1982) combination rules

In order to combine two evidences within fuzzy sets, i.e., risk levels used in this study, Ishizuka et al. (1982) extended Dempster’s rule by taking into account the degree of intersection of two sets, $J(A,B)$.

$$m_{12}(C) = \frac{\sum_{A_i \cap B_j = C} J(A_i, B_j) m_1(A_i) m_2(B_j)}{1 - \sum_{i,j} [1 - J(A_i, B_j)] m_1(A_i) m_2(B_j)} \quad (C.6)$$

where

$$J(A, B) = \frac{\max_x [\mu_{A \cap \cup B}(x)]}{\min [\max_x \mu_A(x), \max_x \mu_B(x)]} \quad (C.7)$$
The denotations of the parameters used in the equations can be explicitly described by a figure shown above.

(2) Yen’s (1990) combination rule

An important assumption in this study is that all risk levels are normal fuzzy sets. Therefore, their intersections will be subnormal fuzzy set as shown in Figure C.2. Yen (1990) dealt with these subnormal fuzzy sets by normalising them to normal fuzzy sets in the process of deriving combination rule.

It is straightforward to normalise the subnormal fuzzy set, say $A$, by

$$
\mu_{\tilde{A}}(x) = \frac{\mu_A(x)}{\max_{x} \mu_A(x)} = k \times \mu_A(x)
$$

where $k$ is the normalisation factor.

The criterion for normalising the probability mass of a subnormal fuzzy set, say $A$, is that the probabilistic constraints imposed by the subnormal fuzzy set should be preserved after the normalisation. Since the $a_i$ cut of the subnormal fuzzy set becomes $ka_i$ cut of the normalised fuzzy set, the probability assigned to them should be the same:

$$
m(A_{a_i}) = m(\tilde{A}_{a_i})
$$

Since

$$
m(A_{a_i}) = m(A)(\alpha_i - \alpha_{i-1})
$$

$$
m(\tilde{A}_{a_i}) = m(\tilde{A})(k\alpha_i - k\alpha_{i-1}) = km(\tilde{A})(\alpha_i - \alpha_{i-1})
$$

Therefore,
\[ m(\overline{A}) = m(A) / k = m(A) \max_x \mu_A(x) \]

The remaining mass \((1-1/k)m(A)\) is the amount assigned to the empty set the subnormal fuzzy set and, hence, should be part of the normalisation factor in generalised Dempster’s rule.

Therefore, the combination rule proposed by Yen (1990) is

\[ m_{12}(C) = \sum_{A',B'\in C} \max_x \mu_{A'\cap B'}(x) m_1(A)m_2(B) \]

\[ 1 - \sum_{A,B} (1 - \max_x \mu_{A'\cap B'}(x)) \max_x \mu_{A'\cap B'}(x) \]

\[ m_{12}(C) = \frac{\sum_{A',B'\in C} \max_x \mu_{A'\cap B'}(x) m_1(A)m_2(B)}{1 - \sum_{A,B} (1 - \max_x \mu_{A'\cap B'}(x)) \max_x \mu_{A'\cap B'}(x)} \quad \text{(C.9)} \]

It is also obvious that Equation (C.7) can be derived by setting fuzzy sets \(A\) and \(B\) normal in Ishizuka et al.’s method. Equation (C.7) is used the current study as the combination rules applied to fuzzy risk levels.

**C.2.1 Combination Rule Used in This Thesis**

(1) **Combination rule**

By considering the confliction measure of fuzzy sets, the risk extents to intersection of two fuzzy sets can be determined by

\[ J(A,B)m_1(A)m_2(B) \]

where \(m_1(A)\) and \(m_2(B)\) are the basic probabilities assigned to \(A\) and \(B\) respectively which are the values of extents to risk levels in this research.

Normally, if two sets are conflict, their intersection will be empty set. If two sets are partially conflict, their intersection will belong to empty set with a certain degree. Therefore, degree of conflicting between two fuzzy sets is viewed as the measure of representing the degree to empty set. Therefore,

\[ m(\Phi) = (1 - J(A,B))m_1(A)m_2(B) \quad \text{(C.10)} \]

\[ m(A \cap B) = J(A,B)m_1(A)m_2(B) \quad \text{(C.11)} \]

where \(m(\Phi)\) denotes the degree to empty set; and \(m(A \cap B)\) denotes the degree of intersection between \(A\) and \(B\). Therefore, the combination of extents to risks from two hazards can be thus derived in this research as
\[ m_{ij}(R_j) = m_i(R_j) \oplus m_k(R_k) = \frac{\sum_{R_j, R_k} J(R_j, R_k) m_i(R_j) m_k(R_k)}{1 - \sum_{R_j, R_k} (1 - J(R_j, R_k)) m_i(R_j) m_k(R_k)} \quad (i,j,k=1,\ldots,14) \]

\[ m_i(R_j) = w_1 r_{j1} \text{ and } m_k(R_k) = w_2 r_{k2} \quad (C.12) \]

where \( R_i, R_j, \) and \( R_k \) represent predefined risk levels \( i, j, \) and \( k \) respectively if \( i, j, \) and \( k = 1,\ldots,7; R_{14} \) represents the degree to \( \Theta; R_i \) (\( i=8,\ldots,13) \) represents the partial intersections between two predefined risk levels \( i-8 \) and \( i-7 \) (i.e., risk levels 1 and 2, 2 and 3,\ldots, 6 and 7; \( m_{12}(R_i) \) denotes the combined degree to risk level \( i \) from two hazards 1 and 2; \( m_i(R_j) \) denotes degree to risk level \( j \) from hazard 1; and \( m_k(R_k) \) denotes degree to risk level \( i \) from hazard 2; \( w_1 \) and \( w_2 \) are the weights of hazards 1 and 2 respectively; \( r_{j1} \) is the extent to risk level \( j \) of the risk introduced by hazard 1; and \( r_{k2} \) is the extent to risk level \( k \) of the risk introduced by hazard 2.

(2) Reassignment

From the above equations, extent to the intersection area between two adjacent risk levels (i.e., level \( i \) and level \( i+1 \)), \( m_{12}(R_{7i+1}) \), is also determined. Since \( R_{7i+1} \) is not an predefined standard fuzzy evaluation grade for risk, however, the extent to it should be reassigned back to \( R_i \) and \( R_{i+1} \). The ratios of assignments to \( R_i \) and \( R_{i+1} \) can be obtained by using the concepts of compatibility discussed above:

\[ Ra_i = \frac{C_{R_i, R_{7i+1}}}{C_{R_i, R_{7i+1}} + C_{R_{i+1}, R_{7i+1}}} \quad \text{and} \quad Ra_{i+1} = \frac{C_{R_{i+1}, R_{7i+1}}}{C_{R_i, R_{7i+1}} + C_{R_{i+1}, R_{7i+1}}} \quad (C.13) \]

where \( Ra_i \) and \( Ra_{i+1} \) denote the ratios of reassignment to risk levels \( R_i \) and \( R_{i+1} \) respectively; \( C_{R_i, R_{7i+1}} \) and \( C_{R_{i+1}, R_{7i+1}} \) denote the compatibilities between the intersected area, a subnormal fuzzy number, and predefined risk levels \( R_i \) and \( R_{i+1} \) respectively.

A combination table is used here to help the explanation of combination rules (Table C.2). This table illustrates the process of combining risk extents from two sources. For each source, its risk extents are represented for seven risk levels and intersections between these risk levels. It shows that complete intersection, complete confliction, and partial confliction exist between two sources. For example, if the risk levels of two sources are \{EL\}, then their intersection will be complete intersection and resulted risk level is also \{EL\}. While if the risk levels of two sources are EL and VL respectively, their intersection will be partial intersection. Then two results, \{EL\} and \{Φ\}, are obtained. \{EL\} represents that risk level fuzzy numbers EL and VL are partially intersected, while \{Φ\} denotes partial conflict between the two sources. If risk levels of two sources are \{EL\} and \{M\}, they don’t intersect with each other. Thus complete conflict is
introduced and represented as \( \{ \Phi \} \). According to the fuzzy risk levels in this research (Figure 4.10) and Ishizuka’s intersection measure, \( J(A,B) \) is equal to 1 for complete intersection, 0.5 for partial section, and 0 for complete conflict. Therefore the combined risk levels of two sources are determined by

\[
m'_{12}(R_i) = m_1(R_i)m_2(R_i) + m_1(R_i)m_2(R_{14}) + m_1(R_{14})m_2(R_i) \quad ; \; i = 1, \ldots, 7
\]

\[
m'_{12}(R_i) = m_1(R_i)m_2(R_i) + 0.5[m_1(R_{i-7})m_2(R_{i-6}) + m_1(R_{i-6})m_2(R_{i-7})] \\
+ m_1(R_i)[m_2(R_{14}) + m_2(R_{i-6}) + m_2(R_{i-7})] \\
+ m_2(R_i)[m_1(R_{14}) + m_1(R_{i-6}) + m_1(R_{i-7})] \quad ; \; i = 8, \ldots, 13
\]

\[
m'_{12}(R_{14}) = m_1(R_{14})m_2(R_{14})
\]

\[
m_{12}(\Phi) = 1 - \sum_{i=1}^{14} m'_{12}(R_i)
\]

\[
m_{12}(R_i) = \frac{m'_{12}(R_i)}{1 - m_{12}(\Phi)} \quad \text{(C.14)}
\]
### Appendix C

#### Table C.2 Combination table of Dempster-Shafer in fuzzy sets theory

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<tr>
<th></th>
<th>EL</th>
<th>VL</th>
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<th>M</th>
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<td>{Φ}</td>
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<td>{SH∩VH}</td>
<td>{SH∩VH}</td>
<td>{SH∩VH}</td>
<td>{SH∩VH}</td>
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</tr>
<tr>
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<td>Θ</td>
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</tbody>
</table>
Fault Tree Analysis

This appendix gives the derivation of approximate determination of fault trees with repeated events.

When fault trees have basic events which appear more than once, the methods most often used to obtain the top event probability utilize the minimal cut sets. A minimal cut set is a collection of basic events. If all these events occur, the top event is guaranteed to occur; however if any basic event does not occur, the top event will not occur. Therefore, if a fault tree has \( n_c \) minimal cut sets \( K_i, i=1,\ldots, n_c \) then the top event exists if at least one minimal cut set exists, i.e.,

\[
T = K_1 + K_2 + \ldots + K_{n_c} = \bigcup_{i=1}^{n_c} K_i
\]

Thus,

\[
P(\text{system failure}) = P(\text{at least 1 minimal cut set occurs})
\]

\[
= 1 - P(\text{no minimal cut set occurs})
\]

Since

\[
P(\text{no minimal cut set occurs}) \leq \prod_{i=1}^{n_c} P(\text{minimal cut set } i \text{ does not occur})
\]

(Equality being when no event appears in more than one minimal cut set), therefore

\[
P(\text{system failure}) \leq 1 - \prod_{i=1}^{n_c} P(\text{minimal cut set } i \text{ does not occur})
\]

i.e.,
\[ P(T) \leq P(T)_{\text{max}} = 1 - \prod_{i=1}^{n_c} [1 - P(K_i)] \]  \hspace{1cm} (D.1)

where \( P(K_i) \) is the occurrence probability of minimal cut set \( i \). This equation gives a conservative approximation of the likelihood of top event.
## APPENDIX E

### Tables of Aggregative Risk Assessments for Chapter 6

#### Table E.1 Results of aggregative risk assessment of river contamination

<table>
<thead>
<tr>
<th>Risk levels</th>
<th>EL</th>
<th>VL</th>
<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Fuzziness</th>
<th>Representative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood</td>
<td>0.079</td>
<td>0.921</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.555</td>
<td>0.027</td>
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<tr>
<td>Sewage discharge</td>
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<td>0</td>
<td>0.046</td>
<td>0.796</td>
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<td>0.001</td>
<td>0</td>
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<td>0.275</td>
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<td>0.598</td>
<td>0.373</td>
<td>0.002</td>
<td>0</td>
<td>1.471</td>
<td>0.320</td>
</tr>
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<td>Spills</td>
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<td>0.191</td>
<td>0.753</td>
<td>0.051</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.279</td>
<td>0.102</td>
</tr>
<tr>
<td>Natural hazard</td>
<td>0.079</td>
<td>0.921</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.555</td>
<td>0.027</td>
</tr>
<tr>
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<td>0</td>
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<td>0.718</td>
<td>0.249</td>
<td>0.001</td>
<td>0</td>
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<td>0.191</td>
<td>0.753</td>
<td>0.051</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.279</td>
<td>0.102</td>
</tr>
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<td>0.427</td>
<td>0.301</td>
<td>0.189</td>
<td>0.054</td>
<td>0.000</td>
<td>0</td>
<td>2.121</td>
<td>0.117</td>
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#### Table E.2 Results of aggregative risk assessment of reservoir contamination

<table>
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<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Fuzziness</th>
<th>Representative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood</td>
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<td>0.834</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.899</td>
<td>0.027</td>
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<tr>
<td>Livestock</td>
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<td>0.059</td>
<td>0.861</td>
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<td>0</td>
<td>0.913</td>
<td>0.117</td>
</tr>
<tr>
<td>Contaminated site</td>
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<td>0.001</td>
<td>0.122</td>
<td>0.821</td>
<td>0.056</td>
<td>0</td>
<td>0</td>
<td>1.060</td>
<td>0.244</td>
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<td>0.834</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.899</td>
<td>0.027</td>
</tr>
<tr>
<td>Human-caused</td>
<td>0.001</td>
<td>0.059</td>
<td>0.861</td>
<td>0.079</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.913</td>
<td>0.117</td>
</tr>
<tr>
<td>Interdependence</td>
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<td>0.056</td>
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<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Fuzziness</th>
<th>Representative value</th>
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<td>0.439</td>
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<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0.100</td>
<td>0.845</td>
<td>0.055</td>
<td>0</td>
<td>0.971</td>
<td>0.439</td>
</tr>
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<td>1</td>
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<td>0</td>
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### Table E.4 Results of aggregative risk assessment of transmission pipe

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<th>SL</th>
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<th>SH</th>
<th>VH</th>
<th>EH</th>
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<th>Representative value</th>
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<tr>
<td>Earth movement</td>
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<td>0.660</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1.282</td>
<td>0.025</td>
</tr>
<tr>
<td>Sabotage</td>
<td>0.001</td>
<td>0.059</td>
<td>0.861</td>
<td>0.079</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.910</td>
<td>0.117</td>
</tr>
<tr>
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<td>0</td>
<td>0.563</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>1.305</td>
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<tr>
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<td>0.059</td>
<td>0.861</td>
<td>0.079</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.910</td>
<td>0.117</td>
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<td>0</td>
<td>0.563</td>
<td>0.437</td>
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<td>1.371</td>
<td>0.172</td>
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### Table E.5 Results of aggregative risk assessment of channel

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<th>SL</th>
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<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Fuzziness</th>
<th>Representative value</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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<tr>
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<td>0.611</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>1.337</td>
<td>0.025</td>
</tr>
<tr>
<td>Sabotage</td>
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<td>0.059</td>
<td>0.861</td>
<td>0.079</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.910</td>
<td>0.117</td>
</tr>
<tr>
<td>Temperature</td>
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<td>0.908</td>
<td>0.071</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.665</td>
<td>0.034</td>
</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0.026</td>
</tr>
<tr>
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<td>0.059</td>
<td>0.861</td>
<td>0.079</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.910</td>
<td>0.117</td>
</tr>
<tr>
<td>Operational</td>
<td>0.021</td>
<td>0.908</td>
<td>0.071</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0.665</td>
<td>0.034</td>
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<td>Transmission channel</td>
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<td>0</td>
<td>1.650</td>
<td>0.061</td>
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Table E.6 Results of aggregative risk assessment of water treatment plant

<table>
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<th>EL</th>
<th>VL</th>
<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Fuzziness</th>
<th>Representative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood</td>
<td>0.166</td>
<td>0.834</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.899</td>
<td>0.027</td>
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<td>0.753</td>
<td>0.051</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0.102</td>
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<td>0.253</td>
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<td>0.180</td>
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<td>0.502</td>
<td>0.475</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>1.511</td>
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</tr>
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<td>0.079</td>
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<td>0</td>
<td>0</td>
<td>0.910</td>
<td>0.117</td>
</tr>
<tr>
<td>Power</td>
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<td>0.598</td>
<td>0.373</td>
<td>0.002</td>
<td>0</td>
<td>1.471</td>
<td>0.320</td>
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<td>0</td>
<td>0</td>
<td>0.899</td>
<td>0.027</td>
</tr>
<tr>
<td>Human-caused</td>
<td>0.005</td>
<td>0.191</td>
<td>0.753</td>
<td>0.051</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.279</td>
<td>0.102</td>
</tr>
<tr>
<td>Operational</td>
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<td>0.029</td>
<td>0.657</td>
<td>0.308</td>
<td>0.005</td>
<td>0</td>
<td>0</td>
<td>1.431</td>
<td>0.153</td>
</tr>
<tr>
<td>Interdependence</td>
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<td>0</td>
<td>0.026</td>
<td>0.598</td>
<td>0.373</td>
<td>0.002</td>
<td>0</td>
<td>1.471</td>
<td>0.320</td>
</tr>
<tr>
<td>WTP (R1)*</td>
<td>0.060</td>
<td>0.323</td>
<td>0.170</td>
<td>0.308</td>
<td>0.138</td>
<td>0.001</td>
<td>0</td>
<td>2.337</td>
<td>0.168</td>
</tr>
<tr>
<td>WTP (R2)*</td>
<td>0.042</td>
<td>0.287</td>
<td>0.347</td>
<td>0.232</td>
<td>0.092</td>
<td>0</td>
<td>0</td>
<td>2.271</td>
<td>0.147</td>
</tr>
</tbody>
</table>

*R1 and R2 denote the risk levels of reduced water quantity and water contamination associated with water treatment plant (WTP) respectively.

Table E.7 Results of aggregative risk assessment of pump failure

<table>
<thead>
<tr>
<th>Risk levels</th>
<th>EL</th>
<th>VL</th>
<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Fuzziness</th>
<th>Representative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood</td>
<td>0.166</td>
<td>0.834</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.899</td>
<td>0.027</td>
</tr>
<tr>
<td>Earth movement</td>
<td>0.340</td>
<td>0.660</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.282</td>
<td>0.025</td>
</tr>
<tr>
<td>Bombing</td>
<td>0.301</td>
<td>0.529</td>
<td>0.166</td>
<td>0.004</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.781</td>
<td>0.040</td>
</tr>
<tr>
<td>Control</td>
<td>0</td>
<td>0.001</td>
<td>0.122</td>
<td>0.821</td>
<td>0.056</td>
<td>0</td>
<td>0</td>
<td>1.060</td>
<td>0.244</td>
</tr>
<tr>
<td>Equipment</td>
<td>0</td>
<td>0.012</td>
<td>0.502</td>
<td>0.475</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>1.511</td>
<td>0.180</td>
</tr>
<tr>
<td>Alarm and monitor</td>
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<td>0.012</td>
<td>0.502</td>
<td>0.475</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>1.511</td>
<td>0.180</td>
</tr>
<tr>
<td>Backup</td>
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<td>0</td>
<td>0.026</td>
<td>0.598</td>
<td>0.373</td>
<td>0.002</td>
<td>0</td>
<td>1.463</td>
<td>0.143</td>
</tr>
<tr>
<td>Power</td>
<td>0.001</td>
<td>0.059</td>
<td>0.680</td>
<td>0.253</td>
<td>0.006</td>
<td>0</td>
<td>0</td>
<td>1.463</td>
<td>0.143</td>
</tr>
<tr>
<td>Natural hazard</td>
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<td>0.765</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.092</td>
<td>0.026</td>
</tr>
<tr>
<td>Human-caused</td>
<td>0.301</td>
<td>0.529</td>
<td>0.166</td>
<td>0.004</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.781</td>
<td>0.040</td>
</tr>
<tr>
<td>Operational</td>
<td>0</td>
<td>0.005</td>
<td>0.538</td>
<td>0.442</td>
<td>0.015</td>
<td>0</td>
<td>0</td>
<td>1.487</td>
<td>0.177</td>
</tr>
<tr>
<td>Interdependence</td>
<td>0.001</td>
<td>0.059</td>
<td>0.680</td>
<td>0.253</td>
<td>0.006</td>
<td>0</td>
<td>0</td>
<td>1.463</td>
<td>0.143</td>
</tr>
<tr>
<td>Pump</td>
<td>0.130</td>
<td>0.349</td>
<td>0.368</td>
<td>0.149</td>
<td>0.004</td>
<td>0</td>
<td>0</td>
<td>2.138</td>
<td>0.092</td>
</tr>
</tbody>
</table>
### Table E.8 Results of aggregative risk assessment of pipes (CI)

<table>
<thead>
<tr>
<th>Risk levels</th>
<th>EL</th>
<th>VL</th>
<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Fuzziness</th>
<th>Representative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth movement</td>
<td>0.389</td>
<td>0.611</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.337</td>
<td>0.025</td>
</tr>
<tr>
<td>External load</td>
<td>0</td>
<td>0</td>
<td>0.012</td>
<td>0.498</td>
<td>0.479</td>
<td>0.012</td>
<td>0</td>
<td>1.512</td>
<td>0.347</td>
</tr>
<tr>
<td>Soil</td>
<td>0</td>
<td>0.012</td>
<td>0.502</td>
<td>0.475</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>1.511</td>
<td>0.180</td>
</tr>
<tr>
<td>Sewage leakage</td>
<td>0</td>
<td>0.012</td>
<td>0.502</td>
<td>0.475</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>1.511</td>
<td>0.180</td>
</tr>
<tr>
<td>Natural hazard</td>
<td>0.389</td>
<td>0.611</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.337</td>
<td>0.025</td>
</tr>
<tr>
<td>Operational</td>
<td>0</td>
<td>0.005</td>
<td>0.234</td>
<td>0.534</td>
<td>0.222</td>
<td>0.005</td>
<td>0</td>
<td>1.824</td>
<td>0.262</td>
</tr>
<tr>
<td>Interdependence</td>
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<td>0.475</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>1.511</td>
<td>0.180</td>
</tr>
<tr>
<td>Pipe (R1)</td>
<td>0.245</td>
<td>0.398</td>
<td>0.093</td>
<td>0.186</td>
<td>0.077</td>
<td>0.002</td>
<td>0</td>
<td>2.302</td>
<td>0.109</td>
</tr>
<tr>
<td>Pipe (R2)</td>
<td>0</td>
<td>0.012</td>
<td>0.502</td>
<td>0.475</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>1.511</td>
<td>0.180</td>
</tr>
</tbody>
</table>

* $R_1$ and $R_2$ denote the risk levels of reduced water quantity and water contamination associated with CI pipes respectively.

### Table E.9 Results of aggregative risk assessment of pipes (PVC)

<table>
<thead>
<tr>
<th>Risk levels</th>
<th>EL</th>
<th>VL</th>
<th>SL</th>
<th>M</th>
<th>SH</th>
<th>VH</th>
<th>EH</th>
<th>Fuzziness</th>
<th>Representative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth movement</td>
<td>0.511</td>
<td>0.489</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.386</td>
<td>0.024</td>
</tr>
<tr>
<td>External load</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.100</td>
<td>0.845</td>
<td>0.055</td>
<td>0</td>
<td>0.971</td>
<td>0.439</td>
</tr>
<tr>
<td>Soil</td>
<td>0.015</td>
<td>0.510</td>
<td>0.464</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.522</td>
<td>0.069</td>
</tr>
<tr>
<td>Sewage leakage</td>
<td>0</td>
<td>0.012</td>
<td>0.502</td>
<td>0.475</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>1.511</td>
<td>0.180</td>
</tr>
<tr>
<td>Natural hazard</td>
<td>0.511</td>
<td>0.489</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.386</td>
<td>0.024</td>
</tr>
<tr>
<td>Operational</td>
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<td>0.251</td>
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<td>0.063</td>
<td>0.417</td>
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<tr>
<td>Interdependence</td>
<td>0</td>
<td>0.012</td>
<td>0.502</td>
<td>0.475</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>1.511</td>
<td>0.180</td>
</tr>
<tr>
<td>Pipe (R1)</td>
<td>0.328</td>
<td>0.435</td>
<td>0.081</td>
<td>0.019</td>
<td>0.129</td>
<td>0.008</td>
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</tr>
<tr>
<td>Pipe (R2)</td>
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<td>0.012</td>
<td>0.502</td>
<td>0.475</td>
<td>0.011</td>
<td>0</td>
<td>0</td>
<td>1.511</td>
<td>0.180</td>
</tr>
</tbody>
</table>

* $R_1$ and $R_2$ denote the risk levels of reduced water quantity and water contamination associated with PVC pipes respectively.
Basic Concepts about Risk Assessment

**Threats/Hazards:** are natural or human-related events that adversely influence normal performance of water supply system. In this study, both natural hazards and human-related threats are represented in terms of their likelihoods (occurrence) and severities (consequences) in order to support a quantitative analysis.

**Vulnerability:** is defined in this study as a property associated with a component, a subsystem, or the overall water supply system to represent the possibility of being influenced by hazards/threats with given likelihoods and severities. This property is determined by the attributes and conditions of a component, a subsystem, or the overall water supply system, and is varied with time and changes of hazards/threats.

**Risk:** is a cumulative measure that is determined by likelihood and severity of hazard and vulnerability of asset in a water supply system.

**Object:** is an abstract of a components, a subsystems, and the overall water supply system in object-oriented representation.

**Class:** is a template or blueprint that defines the methods and variables included in a particular kind of objects.

**States transition diagram:** is a diagram that shows the states space of a given object, the event that cause a transition from normal state to failure state, and reaction that result from a state change.