Active suspension applied to railway trains

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Active suspension applied to railway trains

by

Ian Pratt

A doctoral thesis submitted in partial fulfilment of the requirements
for the award of Doctor of Philosophy of
Loughborough University

December 1996

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To my
Mother
Acknowledgements

I would like to acknowledge a variety of people who have provided both technical, financial, and moral support during the course of my Ph.D studies at Loughborough.

A large debt of gratitude is owed to my supervisor Prof. Roger M Goodall, he encouraged the research to have a practical focus and developed the industrial links so crucial to the quality of this thesis. Some of subtle points raised in this thesis about active suspension are not commonly aired in the literature, a direct result of some of the discussions we held, and a reflection of Roger's experience in the field.

As I've already mentioned the research project had strong industrial links. I would like to thank all my colleagues from the collaborative companies involved in this research, they are: Dave Hatt, Hung Hua, Mike Kellet, Andy Powell, Eddie Searancke, Clive Walker, and Prof. Alan Wickens. Their combined knowledge in railway dynamics and active suspension is vast, and I'm grateful to them for the pointers and corrections they have applied to my research.

I would also like to acknowledge the members and ex-members of the Control & Systems Research Group in the Department of Electronic & Electrical Engineering at Loughborough University. They've been technically supportive and also humorous friends, an important relief during some of the more staid moments of my research. They are: Mustafa Abuzeid, Dr. Colin MacLeod, Dr. Abdullah El-Allabar, Andy Fox, Dr. Malcolm Fraser, John Gow, Dr. Pete Holme, Hong Li, Dr. Kenneth Nai, Mike Oliver, Dr. Jonathon Paddison, Dr. John Pearson, and Gang Shen. Other members of the department have played a part in this research, John Rowbottom in particular deserves praise for his work on the active suspension test-rig.

I am grateful for the funding provided both by Bombardier Prorail Ltd and EPSRC.

Finally I would like to thank Yvonne for all her patience and understanding, particularly over recent months when I've been engrossed in the completion of my research.
Publications

The work contained in this thesis has generated several papers which may be found in the literature and are listed below. It is hoped that some of ideas and results presented in these papers and this thesis will be utilised by potential exploiters of active suspension technologies.

I would like to express my gratitude to the IEE, EPSRC, the Royal Academy of Engineering, and the UKACC for their financial support in assisting the dissemination of this information.


Synopsis

There has been an impetus in recent years to increase railway train speeds and reduce journey times. As train speeds have increased, other problems have manifested themselves, in particular the consequent deterioration in ride quality at these higher operating speeds. Improvement in suspension design is one option which can circumvent this problem. Suspension design for a modern high-speed train has hitherto been a heuristic procedure directed towards optimising the passive components of the suspension. Performance limits are now being reached with passive suspensions due to the inherent trade-offs which need to be made in the design process. Active suspension, which eases this inherent trade-off, has received a great deal of interest in both academia and industry over recent years. A number of theoretical and experimental studies have highlighted the potential benefits of active suspension technology. Theoretical studies have concentrated on using simple vehicle models and although providing the initial impetus to active suspension they have not given the industry full confidence in them. In contrast, experimental studies have highlighted a number of problems, most notably the significant effect actuators can have on the overall performance.

This thesis addresses a number of the issues just raised, and makes contributions in four key areas to the current knowledge about active suspension and its application to railway vehicles, these are:

- Theoretical investigation of active suspension applied to a full vehicle model.
- Theoretical verification of the observed impact of actuator technologies.
- Investigation of train-wide active suspension.
- The novel idea of having active inter-vehicle suspension.

The first area adapts active suspension control laws developed by other researchers for the simplified two-mass model to a full vehicle model. This raises a number of issues which are accounted for in this thesis. It also gives confidence to the industry in applying active suspension
to full scale vehicles. This investigation is performed in both the vertical and lateral directions.

The second area investigates the effect actuator technologies and their internal dynamics have on the overall operation of an active suspension. This has been noted in experimental studies and a theoretical basis for this is given here.

The third area investigates the potential for train-wide active suspension. Preview control in active suspension applied to automobiles is known to afford significant benefits. A railway train with its elongated structure naturally offers scope for preview control and other control methods which draw upon train-wide information as opposed to local vehicle control.

The final area investigated also illustrates the wider scope for applying active suspension to railway trains. It is the notion of having an active inter-vehicle suspension. This concept reduces the number of actuators required with a resulting improvement in system reliability. Quantitative benefits of such an arrangement are given in this thesis.

This thesis concludes that active suspension applied to a single railway vehicle can offer a 30% improvement in ride quality in the vertical direction, and 45% in the lateral direction. Utilisation of train-wide control offers another 10% improvement in ride quality. It also shows that these figures are optimistic when taking into account the effects of internal actuator dynamics. The potential ride quality benefits of active suspension may be degraded by as much as 15% with certain types of actuators. The use of inter-vehicle actuators does not provide ride quality improvements for every vehicle in the train, but a control law has been developed which suggests a 30% vertical ride improvement and 40% lateral ride improvement for vehicles in the central portion of a train.
Notation

\( a_a \) .................................................. Electrohydraulic actuator annulus area
\( a_b \) .................................................. Electrohydraulic actuator bore area
\( A \) .................................................. State-space model matrix
\( B \) .................................................. State-space model matrix, actuator inputs
\( c_{bd} \) ............................................ Body roll damping
\( c_{by} \) ............................................. Bogie yaw damping
\( c_{cy} \) ............................................. Coupler lateral damping constant
\( c_{cv} \) ........................................... Coupler vertical damping constant
\( c_{bh} \) ........................................ Anti-hunting damping constant
\( c_{ps} \) ........................................ Primary suspension longitudinal damping
\( c_{pz} \) ........................................ Primary damping constant
\( c_{rz} \) ........................................ Secondary damping constant
\( c_{sy} \) ........................................ Secondary lateral damping constant
\( C \) .................................................. State-space model matrix
\( C_{SKY} \) ........................................ Skyhook damping coefficient
\( d \) .................................................. Suspension deflection
\( D_B \) ........................................ State-space model matrix, actuator inputs
\( D_w \) ........................................ State-space model matrix, track inputs
\( \zeta \) ................................................ General track input
\( f_{act} \) ........................................ Actuator force
\( f_{bml} \) ........................................ Active lateral inter-vehicle force, leading
\( f_{bm2} \) ........................................ Active lateral inter-vehicle force, trailing
\( f_s \) .............................................. Suspension force
\( f_t \) ................................................ Temporal frequency
\( f_{um1} \) .......................................... Active secondary lateral force, front left-hand corner
\( f_{um2} \) .......................................... Active secondary lateral force, front right-hand corner
\( f_{um1} \) .......................................... Active secondary lateral force, rear left-hand corner
\( f_{um2} \) .......................................... Active secondary lateral force, rear right-hand corner
\( f_{11} \) ........................................ Parameter used in linear model
\( f_{22} \) ........................................ Parameter used in linear model
\( f_w \) .............................................. Wheelset force
f_{zm1} \quad \text{Vertical inter-vehicle force on leading end of } m^{th} \text{ vehicle}

f_{zm2} \quad \text{Vertical inter-vehicle force on trailing end of } m^{th} \text{ vehicle}

F_{ACTY} \quad \text{System matrix relating to lateral actuator control inputs}

F_{ACTZ} \quad \text{System matrix relating to vertical actuator control inputs}

F_{TRACKY} \quad \text{System matrix relating to lateral track inputs}

F_{TRACKZ} \quad \text{System matrix relating to vertical track inputs}

i \quad \text{Current}

i_p \quad \text{Truck/bogie pitch inertia}

i_r \quad \text{Truck/bogie roll inertia}

i_y \quad \text{Truck/bogie yaw inertia}

i_{bp} \quad \text{Body pitch inertia}

i_{br} \quad \text{Body roll inertia}

i_{by} \quad \text{Body yaw inertia}

i_{wy} \quad \text{Wheelset yaw inertia}

I_{DISPL} \quad \text{Integral displacement gain}

k_{az} \quad \text{Secondary change of area stiffness}

k_{bdr} \quad \text{Body roll stiffness}

k_{bgx} \quad \text{Bogie yaw stiffness}

k_c \quad \text{Stiffness between track and wheelset}

k_{cy} \quad \text{Coupler/gangway lateral spring constant}

k_{cz} \quad \text{Coupler/gangway vertical spring constant}

k_{rz} \quad \text{Secondary reservoir stiffness}

k_{px} \quad \text{Primary longitudinal spring constant}

k_{py} \quad \text{Primary lateral spring constant}

k_{pz} \quad \text{Primary suspension spring constant}

k_{ry} \quad \text{Secondary lateral spring constant}

k_{sz} \quad \text{Air spring stiffness}

k_v \quad \text{Spool valve flow gain}

K \quad \text{Stiffness matrix}

K_B \quad \text{Controller bounce skyhook gain}

K_\phi \quad \text{Controller pitch skyhook gain}

l_{a_{rm}} \quad \text{Motor armature inductance}

l_{asy} \quad \text{Airspring lateral spacing}


- \( l_b \) : Bogie to bogie spacing
- \( l_{bw} \) : Semi-bogie width
- \( l_{bz} \) : Body-bogie vertical spacing
- \( l_s \) : Wheelset gauge length
- \( l_{sz} \) : Semi vehicle to vehicle spacing
- \( l_{px} \) : Distance from bogie centre to axle
- \( l_{pp} \) : Primary suspension lateral position
- \( l_p \) : Semi wheel to wheel spacing
- \( l_t \) : Semi bogie to bogie spacing
- \( l_v \) : Vehicle length + gangway length
- \( l_w \) : Wheel to wheel spacing
- \( l_{wd} \) : Semi-vehicle width
- \( \lambda \) : Wavelength of first symmetric bending mode
- \( m_b \) : Body mass
- \( m_{mp} \) : Airspring mid-point mass
- \( m_t \) : Truck/bogie mass
- \( m_{w} \) : Wheelset mass
- \( M \) : Mass matrix
- \( \omega_n \) : First symmetric bending mode natural frequency
- \( \omega_s \) : System natural frequency
- \( \Omega_v \) : Vertical track roughness
- \( \Omega_L \) : Lateral track roughness
- \( p_a \) : Annulus side pressure
- \( p_b \) : Bore side pressure
- \( p_o \) : System poles
- \( \phi_{bm} \) : Body 'm' pitch displacement
- \( \phi_{tm1} \) : Leading wheelset pitch on 'mth' vehicle
- \( \phi_{tm2} \) : Trailing wheelset pitch on 'mth' vehicle
- \( \psi_{bm} \) : Yaw motion of body 'm'
- \( \psi_t \) : Track incidence angle
- \( \psi_{tm1} \) : Leading bogie yaw motion of 'mth' vehicle
- \( \psi_{tm2} \) : Trailing bogie yaw motion of 'mth' vehicle
- \( \psi_w \) : Wheelset yaw angle
\[ \Psi_{wml} \quad \text{Leading wheelset yaw angle} \]
\[ \Psi_{wml2} \quad \text{Second wheelset yaw angle} \]
\[ \Psi_{wm21} \quad \text{Third wheelset yaw angle} \]
\[ \Psi_{wm22} \quad \text{Trailing wheelset yaw angle} \]
\[ q_a \quad \text{Anulus side oil flow} \]
\[ q_b \quad \text{Bore side oil flow} \]
\[ r_{arm} \quad \text{Motor armature resistance} \]
\[ r_0 \quad \text{Wheelset radius} \]
\[ \rho_{bm} \quad \text{Body roll angle, } 'm^{th} \text{ vehicle} \]
\[ \rho_{m1} \quad \text{Roll motion of leading bogie, } 'm^{th} \text{ vehicle} \]
\[ \rho_{m2} \quad \text{Roll motion of trailing bogie, } 'm^{th} \text{ vehicle} \]
\[ \rho_{wm11} \quad \text{Leading wheelset, track roll motion} \]
\[ \rho_{wm12} \quad \text{Second wheelset, track roll motion} \]
\[ \rho_{wm21} \quad \text{Third wheelset, track roll motion} \]
\[ \rho_{wm22} \quad \text{Trailing wheelset, track roll motion} \]
\[ S_S \quad \text{Track spatial spectrum} \]
\[ S_T \quad \text{Track temporal spectrum} \]
\[ t, T \quad \text{Time} \]
\[ t_m \quad \text{Motor torque} \]
\[ t_w \quad \text{Wheelset torque} \]
\[ \theta_m \quad \text{Motor rotation angle} \]
\[ v \quad \text{Vehicle speed} \]
\[ V_{act} \quad \text{Actuator input voltage} \]
\[ V_{YN} \quad \text{Signal flow vector on lateral model} \]
\[ V_{ZN} \quad \text{Signal flow vector on vertical model} \]
\[ W \lambda \quad \text{New wheel conicity} \]
\[ W \quad \text{State-space model matrix, track inputs} \]
\[ X_{act} \quad \text{Actuator extension} \]
\[ X_m \quad \text{Linear motor extension} \]
Electrohydraulic valve extension

Vertical model state vector

Lateral model state vector

Vertical system matrix

Lateral motion of body 'm'

Track lateral motion

Leading bogie lateral motion of 'm\textsuperscript{th} vehicle

Trailing bogie lateral motion of 'm\textsuperscript{th} vehicle

Wheelset lateral motion

Leading wheelset lateral motion

Second wheelset lateral motion

Third wheelset lateral motion

Trailing wheelset lateral motion

Lateral track inputs

Output of vertical model

Body vertical motion

Body 'm' centre of gravity vertical displacement

Vertical body displacement above leading bogie on 'm\textsuperscript{th} vehicle

Vertical body displacement above trailing bogie on 'm\textsuperscript{th} vehicle

Vertical body displacement at lead end of 'm\textsuperscript{th} vehicle

Vertical body displacement at trailing end of 'm\textsuperscript{th} vehicle

Truck/bogie vertical motion

Leading bogie vertical displacement on 'm\textsuperscript{th} vehicle

Trailing bogie vertical displacement on 'm\textsuperscript{th} vehicle

Wheelset vertical motion

Leading wheelset vertical displacement on 'm\textsuperscript{th} vehicle

Second wheelset vertical displacement on 'm\textsuperscript{th} vehicle

Third wheelset vertical displacement on 'm\textsuperscript{th} vehicle

Trailing wheelset vertical displacement on 'm\textsuperscript{th} vehicle

Transmission zero

Vertical track inputs

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Chapter 1: Introduction

Chapter 1
Introduction

1.1 - Preface

This thesis is the culmination of 4 years of research at Loughborough University investigating and attempting to extend current knowledge in the field of active suspensions. During this period I have constantly been intrigued by the interaction between man and the environment he lives in. Active suspension is a good example of man attempting to control the environment within the constraints imposed by nature. Throughout this body of work extensive use has been made of the theories of: Newton, d'Alembert, Lagrange, Taylor, Euler, and many others. It will hopefully become apparent as the reader delves deeper into the thesis how physical limitations on suspensions may be explained through the physical principles and laws laid down by them. In active suspensions the constraint of "invariant properties" and the minimum-phase nature of mass-spring-damper arrangements results in known limitations in performance. Some of the mechanisms involved in passive and active suspension design are quite involved, but these can be traced back to more fundamental principles.

This chapter is aimed at giving the reader a foundation of suspension knowledge and other associated issues from which comprehension of subsequent chapters will be made easier. This thesis is concerned with applying active suspension techniques specifically to high-speed passenger rail vehicles. The motivation behind this is the ever increasing need to improve vehicle speeds and reduce journey times. Associated with this speed up is the need to improve braking (if we speed vehicles up we also need to slow them down!), improvements in wheelset curving/stability, reduction of aerodynamic effects, and maintaining an acceptable ride quality.
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when operating at these higher speeds. This thesis is concerned with the question of maintaining and improving ride quality at higher speeds.

Limitations imposed by passive suspensions have meant that in recent years suspension designers have looked towards active suspensions to achieve this goal of improving ride quality at higher speeds. A modern high speed passenger vehicle is shown in Figure 1.1. The suspension of this vehicle, a British Rail Mk IV coach, is purely passive. It is capable of operating up to 225 km hr$^{-1}$ (140 mph or 63 ms$^{-1}$), and at these levels can provide a reasonably good ride quality.

At these sort of speeds the vehicle is excited by track wavelengths of around 70m in wavelength. As we require vehicles to travel faster, the dominant track wavelength responsible for the excitation increases since the vehicle modes remain constant and the excitation frequency is related to speed and frequency. It becomes increasingly difficult to eliminate long track wavelengths through track maintenance machines, and hence the onus is placed on suspension designers rather than improvements by track civil engineers to ensure that ride quality is maintained at higher speeds.

A high-speed passenger rail vehicle is dynamically complex. From Figure 1.1 a variety of dampers and stiffness around the bogie can be seen. Many of the other suspension elements remain secluded. However, in Figure 1.2 a more detailed view of the suspension can be seen. This figure also introduces some terminology used to classify different parts of the suspension. The suspension between the wheelsets and the bogie (or truck in the USA) is known as the 'primary suspension' and that between the bogies and the body is known as the 'secondary suspension'.
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Figure 1.1 - Typical high-speed passenger rail vehicle
Figure 1.2 is again not a complete inventory of suspension components but serves to illustrate the complex dynamic nature. The system contains a number of bodies: wheelsets, bogies and the vehicle itself; some of which are flexible, linked together by springs, dampers, and airsprings. Bogies are required on lengthy vehicles to allow the vehicle to negotiate a curve while maintaining relatively small wheelset attack angles, particularly small radii curves. A variety of rail vehicles have now been constructed without bogies, but these tend to be low speed, and vehicles of reduced length. Each body in this system has six degrees of freedom: roll, pitch, yaw, longitudinal, vertical and lateral motions. Many of the dynamic modes of this system are coupled together; lateral motions of the body result in roll motions of the body, etc. All of the links - springs, dampers, and airsprings - possess some degree of non-linearity in their behaviour. There are often parasitic effects affecting the dynamics of a vehicle in unexpected ways, anti-roll bars for instance introduce vertical stiffness. It is clear that a railway vehicle is dynamically a highly complex, non-linear, coupled system.

Figure 1.2 - Dynamic elements in a high-speed rail vehicle
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Irregularities in the track surface will result in excitation of the dynamic modes, consequent accelerations on board the vehicle which will be perceived by the vehicle occupants.

The obvious approach in analysing such a complex system is to break it down into its constituent parts. The vertical and lateral suspensions are not strongly coupled and for the purpose of dynamic analysis can be segregated. This together with the elimination of modes which are not viewed as having a significant effect on the dynamics is the approach taken by most researchers and suspension designers. It is fair to say that early research has concentrated on modelling vehicle dynamics in an over-simplified manner; in part due to the simulation tools available at the time, but also because it was natural to gain a simple analytical understanding of active suspensions before looking at some of the more detailed problems. Accurate modelling of such a system is important if the designer is to determine accurately the ride quality to which the vehicle’s occupants will be subjected. Advances in computing power have meant that more detailed models can be developed, an important factor since a number of crucial features are over-looked with simple models. The methodology of developing accurate computer models, and from there developing active suspension control laws based upon and extending the groundwork techniques developed by other researchers, is the one adopted here. This approach is due to this piece of research having a strong industrial focus, it is important for the industry to gain acceptance of the conclusions and have confidence in the models and techniques used here.

Much of this groundwork has already been established by other researchers in the field. It is the aim here to extend this groundwork to look at other avenues which have not previously been investigated. The literature review in Chapter 2 outlines much of the work by other researchers. Although several authors have attempted to design and analyse active suspensions on full vehicle automotive models, the focus of this thesis will be to apply active suspension to railway vehicles and to investigate whether or not active suspension may be applied to an entire train of vehicles, and if so what the potential benefits might be. This thesis is also concerned with implementation issues related to active suspensions, and in particular which actuator technologies are best suited to this type of application. This aspect of active suspension requires further investigation after
a number of experimental studies highlighted problems with actuators when attempting to build an active suspension.

The structure of the thesis will be as follows:

- Actuator modelling (Chapter 3)
- Train modelling (Vertical - Chapter 4, Lateral - Chapter 5)
- Active suspension control methods (Secondary - Chapter 7, Inter-vehicle - Chapter 8)
- Practical implementation issues - Chapter 9

1.2 - Fundamental suspension requirements

A basic introduction to the requirements of a suspension, an explanation of the terminology "passive" and "active" suspension, together with a brief overview now follows.

A vertical suspension is required to provide two main functions. These are:

- Isolation of the vehicle body from the track irregularities
- Following of the low frequency track components (i.e. hills, and gradients)

It is important to gain an appreciation of the suspension requirements. The simple one-mass model shown in Figure 1.3 provides a basic suspension function and is a good starting point in the explanation of these requirements.
Chapter 1: Introduction

Figure 1.3 - Passive one-mass model

The transfer function between the track and the body is given by equation (1.1).

\[
\frac{\ddot{z}_b}{\ddot{z}_t} = \frac{c_m s + k_m}{m_j s^2 + c_m s + k_m}
\] 

... (1.1)

The frequency response of this suspension for a variety of damping levels is shown in Figure 1.4.

Figure 1.4 - One mass model frequency response
Chapter 1: Introduction

The designer has the freedom to vary stiffness and damping in the design procedure. The natural frequency and system damping will obviously vary with these parameters and are given by the pair of equations (1.2) and (1.3).

\[
\omega_s = \sqrt{\frac{k}{m_s}} \quad \ldots (1.2)
\]

\[
\zeta = \frac{c_s}{2m_s} \sqrt{\frac{k}{m_s}} \quad \ldots (1.3)
\]

Human anatomical response and the general suspension requirements dictate that we must maintain the natural response of the suspension to be around 1 Hz. Significant variations from this frequency result in passengers feeling either "sea-sickness" if the frequency is set too low without significant damping, and "harshness" if it is set too high. In effect the designer is constrained to maintain the ratio of mass to stiffness. Variation in the vehicle mass between unladen and laden conditions can be significant, the designer needs to reach a median in the choice of suspension stiffness, even then it is possible for a passenger to feel the difference in ride quality between laden and unladen. The designer is left with variations in the damper $c_s$ to achieve the objective of providing acceptable ride quality. Referring back to the frequency response of Figure 1.4 we can see the effect of variations in the damper $c_s$. The system being second order clearly has a single resonance peak, and because of the unity pole excess, has an asymptotic roll-off of 20 dB/decade. The range of frequencies considered in the response 0.1\to 10\, Hz are the most acute in terms of human perceptibility. Frequencies higher than above 30 Hz are known to affect vision but the attenuation provided by the suspension at these frequencies eliminates any cause for concern. In our assessment of ride quality frequencies in the range 0.1\to 30 \, Hz are the critical ones. Increases in the levels of damping reduce the resonance peak at the expense of increasing the transmissibility at higher frequencies. This "high frequency transmissibility" is perceived as harshness by the vehicle occupants; an unacceptable feature. The designer is left with a trade off between resonance attenuation and reduction in the high
frequency transmissibility. This is only one of the trade-offs the designer has to make.

Suspension design is a multi-objective procedure, and as is the case with many engineering designs, the problems are not as clear-cut as previously outlined.

Re-iterating the suspension requirements, we have to provide high frequency isolation and follow the low frequency undulation in the track. We can clearly see that the one-mass model can perform this role: it functions as a low-pass filter eliminating the high frequencies, and at low frequencies the response of equation (1.1) has a magnitude of 1.

However, transiently, with certain low frequency features the suspension may behave poorly. On steps and gradients for instance, the value of the suspension deflection 'd' may become unacceptably large or small. It is important to remember that in reality 'd' is bounded. Violation of the bound on 'd' will result in high vehicle jerk levels due to bumpstop contact. It is also important to note that variations in the vehicle payload will also affect the available suspension workspace.

The track profile is designed and laid out so that the transition to the more severe vertical gradients will be a constant sustained vertical acceleration, with the length of the transition determined by the steepness of the gradient. This is a worst case scenario, and is only occasionally encountered in practice. It does however give the designer confidence in the suspension's performance even in the most arduous conditions. The vertical superimposed acceleration for a railway is usually specified to be 0.4 ms$^{-2}$.

The transfer function between the suspension deflection 'd' and \( \ddot{z} \) is given by equation (1.4).

\[
\frac{d}{\ddot{z}} = \frac{m_s}{m_s s^2 + c_s s + k_s} \quad \ldots (1.4)
\]
From equation (1.4) it can be seen that the quasi-static suspension deflection (i.e. on the longer transitions will be \(-0.4m_{s}/k_{s}\) as shown in equation (1.5).

\[
\lim_{t \to 0} d(t) = \lim_{s \to 0} s\frac{d}{\ddot{s}(s)} = \frac{-0.4 m_{s}}{k_{s}}
\] \hspace{1cm} \ldots (1.5)

The negative sign indicates suspension compression. It is clear that, if there is a bound on 'd', a similar bound must be applied to \(-0.4m_{s}/k_{s}\). This is only a necessary condition, because the transient response of the suspension deflection as the vehicle enters the gradient also comes into play, and the designer must ensure that the suspension deflection remains bounded during this transient period. A rigorous mathematical proof of this point is not given here but the problem of maintaining suspension deflection under these conditions is illustrated by Figure 1.5, which shows the suspension deflection of the one-mass model as it encounters a transition to a gradient.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{one_mass_model_transient_response.pdf}
\caption{One-mass model transient response}
\end{figure}

This serves to illustrate the disparity between the quasi-static suspension deflection and the true maximum, it also illustrates the effect of variations in the damper setting. Although the damper has not affected the quasi-static deflection it has had a significant effect on the maximum suspension deflection during the transient period.
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Throughout this thesis extensive use has been made of computer simulations to evaluate the maximum suspension deflection on deterministic features. The complex dynamic nature of suspension systems means that traditionally designs have proceeded in a rather heuristic manner; 'rules of thumb' from previous designs have resulted in an ad-hoc approach to choice of suspension parameters. The design techniques used within this thesis are a fusion of analytical and numerical techniques, an approach which is not uncommon in a complex multi-objective design process.

The suspension requirements cited earlier may be translated into the following pair of requirements:

- Achieve good ride quality
- Maintain adequate suspension clearances

These requirements are synonymous with the earlier statement. The coalescence is merely disguised by practical constraints. The suspension requirements are conflicting; attempts to improve one generally has a detrimental impact on the other.

Separation of the main causes of ride degradation and suspension deflection excitation is achieved by analysing the vehicle response to two categories of track excitation. These are:

- Stochastic (Random excitation)
- Deterministic (Track design features, i.e. gradients, steps, etc.)

Stochastic response is due to track irregularities. These irregularities result in excitation of the suspension with consequent ride degradation and small-scale movements in the suspension deflection. Deterministic features are track components which are known and designed by track civil engineers, e.g. a gradient of 1 in 40 and its associated transition. Railway track contains both stochastic and deterministic components. The stochastic track component is frequently modelled...
Chapter 1: Introduction

Statistically as a random white noise process and as such the resulting suspension deflection is not known precisely although confidence limits may however be placed on the excursions of the suspension. This is used in conjunction with the deterministic suspension deflection to evaluate the worst case suspension deflection. A rigorous mathematical explanation of this technique is given in Chapter 6. Suffice it for now to say that the predominant cause of ride perception by the vehicle occupant is due to the track irregularities and although they will feel quasi-static accelerations on deterministic features, this perception is not dominant primarily because the track has been laid out with passenger comfort in mind. Ride quality is highly subjective and the methods used throughout this thesis parallel and extend those in the literature. Improvement in ride quality is the focus of this thesis, therefore a clear understanding of ride perception is paramount and an entire chapter is dedicated to the topic. Chapter 6 gives further discussion and a more rigorous definition of ride quality and suspension deflection performance indices.

1.3 - Active suspensions

It will hopefully be apparent from the previous discussion that a properly tuned passive suspension can provide a suspension function and passive suspensions are fitted to nearly all current rolling stock. The limitations imposed by the inherent trade-off between ride quality and suspension deflection are now being reached. Active suspensions ease this inherent trade-off. It is important to emphasise the word "ease", the trade-off still exists in an active suspension but the overall performance of an active scheme may be enhanced in comparison to an optimised passive suspension. The scheme of an active suspension is shown in Figure 1.6. In addition to the passive components, an active force-injecting element is added. The spring is an essential passive component providing support for the static weight of the vehicle. The active element can add additional suspension forces, forces which can be chosen to have a beneficial effect on the vehicle ride quality. In practice the actuator is limited in the magnitude and bandwidth of forces it can produce. This is a significant problem with an active suspension. Further investigation of actuator technologies and their performance in an active suspension environment is the focus.
of Chapter 3.

The damper shown in Figure (1.6) is not an essential component; the damping function could be provided by the actuator, the spring could also be eliminated if a sufficiently large actuator could be found to support the static vehicle load, but the sensible approach is to add an active element in parallel with an already optimised passive suspension, thereafter designing the active suspension to improve performance. The argument against having a lone actuator to provide the suspension function is that it would be impractically large. The argument for retaining the passive damper is that under failure conditions retention of the damper would mean that at worst the vehicle would resort to the performance of the passive system. Active suspension actuators in this role need only provide a force modulating function with the passive spring providing the static force required to support the vehicle. The force generated by the actuator may be a function of any of the vehicle's state variables, as well as other variables such as vehicle speed, track data, etc. Of course these variables must be measurable or derivable through some form of estimator.

Figure 1.7 illustrates the structure of an active scheme. An active suspension is a truly mechatronic system which from its inception integrates vehicle measurements (sensors), processing electronics, actuators that interface with the mechanical world, and the vehicle itself.
Chapter 1: Introduction

[Karnopp - 1978] views active suspension as giving the designer additional freedom to interconnect mechanical devices in a manner which would be impossible through purely mechanical means. For example, attaching a damper between the track and the vehicle body becomes a reality with an active suspension. As shown in Figure 1.7, the choice of sensing devices is most commonly accelerometers and displacement transducers (potentiometric, inductive, etc.), and in practice we are constrained by what we can sensibly measure. Preview control for instance requires knowledge of the track profile in advance of the train's arrival, which is a demanding sensing requirement. Computational demands on the processing hardware also need to be addressed. The recent explosion in the use of DSPs and microcontrollers and their successful introduction into engine management and ABS systems has provided impetus and confidence towards their introduction into active suspensions. The bandwidth of the suspension, and the on-line mathematical operations impose requirements on the processing hardware and also on the actuator technology used to effect the control. It will hopefully become clear how this thesis attempts to address the various facets of active suspension recognised here, and extends the knowledge further.

A simple active suspension control method which has received a great deal of attention in the literature is the so-called "skyhook damping" scheme visualised in Figure 1.8.
Implementation of such a control strategy is virtually impossible through purely mechanical means. The anchor to the inertial frame could be achieved through some form of aerofoil but this would not function properly at low speeds and would probably violate the kinematic gauge of the train. Such a scheme could however be implemented through the scheme illustrated in Figure 1.6. The control action required from the actuator is given by equation (1.6).

\[ f_{\text{act}} = -c_{\text{sky}} \ddot{z}_b \]  \hspace{1cm} (1.6)

The benefits of such a control action will now be demonstrated. Figure 1.9 shows the frequency response of the purely passive one-mass model. Also plotted is the response of the system shown in Figure 1.4. It is clear that the trade-off between resonance attenuation and high frequency transmissibility has been eased.
The reason for this is apparent from a simple analysis of the transfer function of the system shown in Figure 1.8. This is given by equation (1.7).

\[
\frac{\ddot{z}_s}{\ddot{z}_i} = \frac{c_n s^2 - k_n}{m_s s^2 \cdot (c_n + c_{nv}) s + k_n} \quad \ldots (1.7)
\]

For this system the undamped natural frequency and damping are given by equations (1.8) and (1.9).

\[
\omega_n = \sqrt{\frac{k_n}{m_b}} \quad \ldots (1.8)
\]

\[
\zeta = \frac{c_n}{2 m_b \sqrt{k_n/m_b}} \quad \ldots (1.9)
\]
Chapter 1: Introduction

It is apparent from these equations that the natural frequency remains unaltered. System damping is increased by the introduction of the "skyhook" damper, thus reducing the level of resonance. High frequency transmissibility is not exacerbated by the introduction of the skyhook damper because the zero in the transfer function remains stationary; increases in the damper \( c_{\alpha} \) would however bring the zero into effect at a lower frequency giving rise to an increased transmission at higher frequencies.

Variations in both the skyhook damper and \( c_{\alpha} \) do not affect the system natural frequency and have only a slight effect on the damped frequency. There is therefore little movement of the system pole pair, equations (1.10) and (1.11), consequently the system zero is dominant in the response and responsible for the high frequency transmissibility with increases in the passive damper.

\[
p_0 = \frac{-(c \cdot c_{\alpha}) - \sqrt{(c \cdot c_{\alpha})^2 - 4m_b k_m}}{2m_b} \quad \ldots (1.10)
\]

\[
p_1 = \frac{-(c \cdot c_{\alpha}) - \sqrt{(c \cdot c_{\alpha})^2 - 4m_b k_m}}{2m_b} \quad \ldots (1.11)
\]

The benefits of the skyhook damping scheme previously outlined have to be tempered. There are known problems with the performance of such a scheme in response to deterministic track inputs. Using the same assessment undertaken for the passive system, the response of the active skyhook system to a deterministic track input may be evaluated through equations (1.12) and (1.13).

\[
\frac{d}{dt} \frac{x}{x} = \frac{m_b s \cdot c_{\alpha s}}{m_b s + (c_{\alpha} \cdot c_{\alpha s}) s^2 + k_{\alpha} s} \quad \ldots (1.12)
\]
Equation (1.13) together with Figure 1.10 highlight one of the problems with this particular type of active suspension.

\[ \lim_{s \to 0} \frac{d}{Z(s)} = \lim_{s \to 0} \frac{-0.4(m \cdot c_{dy})}{m \cdot s^4 + (c_{as} \cdot c_{dy})s^3 \cdot k_{as}^2} = \ldots (1.13) \]

Clearly the suspension deflection in this situation would become unacceptably large. This is not an unsurmountable problem, a variety of techniques may be used to overcome this problem.

The simple outline just given serves to demonstrate the potential benefits of active suspension, but it has also highlighted certain difficulties which need to be overcome if a realistic active suspension is to be developed. The benefits are obviously a reduction in the suspension resonance without an adverse effect on the high frequency transmissibility, although the basic skyhook damping form does give rise to suspension deflection problems on deterministic features.

A word of caution needs to be noted with respect to high frequency transmissibility. Certain texts
Chapter 1: Introduction

correctly describe the scheme shown in Figure 1.8 as a skyhook damping scheme with a $\frac{2}{\pi}$ roll off rate of 40 dB/decade provided the damper $c_w$ is set to zero. In reality an active suspension will be implemented as previously shown in Figure 1.6. Even with the passive damper removed from the arrangement, the actuator in reality would provide damping resulting in a roll-off rate of only 20 dB/decade. The bandwidth of the actuator and its ability to avoid having any effect on the system at higher frequencies is therefore a crucial issue. There is a spectrum where the actuator can still control the transmissibility but ultimately all actuators have a finite bandwidth and will provide some damping effect at a high enough frequency. In terms of passenger perception, the attenuation of the suspension at the frequencies where the actuator is at the limits of its operation may be sufficient to avoid concern about the impact on vehicle ride quality.

A more accurate model of the vertical suspension of a railway vehicle is shown in Figure 1.11. The actuator is shown to allow the possibility of some form of active suspension.

![Figure 1.11 - Two-mass model](image)

This model has received a great deal of attention in the literature, or rather a slight variation which is more specific to the automobile has received the attention. The analogy between the two is quite strong, and many of the conclusions drawn from automobile work naturally extend into the railway field. Several researchers have concentrated specifically on the rail application and
many references to this work may be found in this thesis. The main differences between the rail and automotive suspensions are twofold. Firstly rail vehicles use secondary suspension airsprings with surge reservoirs, as opposed to coil springs and dampers in the automotive field and these two devices are sufficiently dissimilar to warrant different modelling. Secondly, the primary suspension in a railway vehicle is quite often formed through a parallel spring-damper combination held by a swing-arm mechanism, in the automotive field the primary suspension is provided by the tyre. The tyre provides low level damping which cannot be varied, and it is common place to see automotive primary suspensions modelled as a single spring, whereas in the railway field the primary suspension can provide a large amount of controllable damping; also the stiffness of a tyre is much higher than that normally provided by rail primary suspensions.

The model shown in Figure 1.11 attempts to parallel directly the vertical dynamics of the single vehicle shown in Figure 1.2. It is an extreme simplification of what in reality is a highly complex dynamic system. Nevertheless it can demonstrate important facts about active suspension. Many of the degrees of freedom have been ignored in this simplified model, along with structural modes which would also influence vehicle ride quality. The most important elements influencing vertical ride quality are shown. The body is modelled as a solid lumped mass. The airsprings have been modelled as a combination of springs and dampers. Justification for the airspring model is given in Chapter 4. The bogie is also modelled as a lumped mass. The vertical primary suspension is generally formed of laminated rubber in parallel with a damper. This construction has dynamic properties similar to a parallel spring-damper combination. The vertical primary is generally designed to be quite stiff and well damped. This ensures good bogie damping and very little dynamic variation of the vertical wheelset force. It also provides a first stage of suspension. The overall response of the passive two-mass model is shown in Figure 1.12, which for comparison also shows the response of the one-mass model. Above the primary suspension frequency the roll-off rate of the two-mass model is over 20dB/decade higher than that of the one-mass model. The two-mass model has two-stages of suspension. At high frequencies the primary suspension provides isolation and works to take up most of the random variations in the
Chapter 1: Introduction

track. At the body bounce frequency (0.9 Hz) both the body and bogie move in phase with large amplitude motions. At the bogie bounce frequency (10 Hz) the two bodies move out of phase, again with a relatively high amplitude. Both of these system resonances lie within the range of human perception, it is therefore important to control these resonances while satisfying the suspension deflection constraints.

The conclusions drawn from investigation of the two-mass model will have important connotations in the development of active suspension control laws for a real-life railway vehicle. The frequency response of the two-mass active skyhook scheme of Figure 1.11 is also shown in Figure 1.12. As with the one-mass model this demonstrates the significant benefits that skyhook damping has on resonance attenuation. However, closer inspection of the response reveals that the active suspension cannot improve over the passive system around the bogie bounce mode. This is a well known result; the bogie bounce frequency is known as an "invariant point" in the acceleration transfer function. An invariant point also exists in the suspension deflection transfer function $d/\ddot{z}$, but at a frequency between the body and bogie bounce...
frequencies. These invariant points arise because of the reactive nature of the forces applied through the actuator.

Figure 1.13 illustrates the role a suspension should perform, it should supply a constant suspension force when travelling over random track, this force needs to be modulated in order that the body may negotiate the deterministic features.

Without a disturbing force the vehicle body will remain stationary in space or travel longitudinally along the track, the weight of the vehicle exactly balancing the suspension force. In order to climb deterministic features the suspension force needs to be increased. Obviously using springs and dampers to realise the suspension means that it is not practically feasible to realise a constant force suspension, however, active suspensions allow for a more controllable suspension force; a distinct advantage over their passive counterparts.

1.4 - Active suspension terminology

It is interesting at this point to introduce and clarify some of the terminology used in this field of research. Classifications of suspensions fall into one of the four categories:
Chapter 1: Introduction

- Passive
- Semi-passive and Adaptive
- Semi-Active
- Active - Slow active
  - Fully active

We have already talked about passive and active suspensions. The earlier confusion about classifying suspension types may be clarified by stating that passive suspensions do not permit energy injection into the suspension via an external actuator.

An 'adaptive' system is one whereby the parameters of the system may be changed, taking more time to do so than the slowest dynamic mode of the system, a 'semi-passive' suspension is one in which the characteristics of passive components vary. For example, it may be advantageous to switch damper settings between high or low settings depending upon whether the train is on smooth or rough track.

A 'semi-active' suspension is one which is based upon passive components, most commonly switched or continuously variable dampers. There is a subtle difference between the use of switched dampers in a semi-passive suspension and their usage in a semi-active suspension. The difference arises in the time-scales in which changes to the damper settings occur. In a 'semi-passive' suspension the changes occur in a slow manner, whereas in a semi-active the changes occur at the dynamic frequencies of the suspension. The name semi-active arises from the fact that the control action of the switched or continuously variable damper attempts to emulate a fully active control law. Obviously there are occasions when the active control law demands energy injection into the suspension, a requirement which a damper cannot perform and in this situation the damper is switched to a lower setting. During the periods when the damper can dissipate energy and at the same time implement a fully active control law it is switched to a high setting if it is a switched type damper, or alternatively is made to generate the equivalent.
active control force if the damper is of the continuously variable type.

Fully active suspensions have already been discussed, and a particular type of active suspension, namely the skyhook damping approach has been investigated. A subdivision of active suspensions into 'slow-active' and 'fully active' also exists. The skyhook damping strategy would fall into the fully-active category, the reason behind this is due to the form of actuation technology used to implement the control law. In this instance the actuator must provide skyhook damping over the entire frequency range. As we will see in Chapter 3 this is often an onerous demand on an actuator, for at higher frequencies the actuator is required not to transmit any force to the body but simply to take up the suspension motions. The actuator needs to provide full control around the body bounce frequency and hence an alternative to a fully-active strategy is to incorporate some compliance in series with the actuator. This allows the actuator to control the body bounce mode but does not transmit any significant force to the body at higher frequencies.

1.5 Actuator technologies

Actuator technologies, their properties and performance have a significant impact on the overall performance of an active suspension. A more complete explanation of this statement and a more in depth study of actuator technologies is given in Chapter 3. The three actuator technologies deemed most suitable to an active suspension application are:

- Electrohydraulic
- Electromechanical
- Electromagnetic

Figure 1.14 shows an electrohydraulic actuator implementing the skyhook damping control function previously described.
Chapter 1: Introduction

The hardware involved in implementing such a scheme would obviously require some form of sensing, processing hardware, and some form of actuation device, as discussed earlier and shown in Figure 1.7. The electrohydraulic actuator shown in Figure 1.14 will have its own internal dynamics.

![Electrohydraulic actuator implementing skyhook damping](image)

Figure 1.14 - Electrohydraulic actuator implementing skyhook damping

Demands on the actuator to generate the skyhook damping force demanded by the processing hardware will not result in an immediate response from the actuator. In the case of an electrohydraulic actuator there are dynamics associated with the inertia of the spool valves, or the time delay in operating a switched valve. There are also dynamics associated with the inertia and compressibility of the hydraulic oil. Friction and ram inertia also play a role in the internal dynamics of the actuator. Several experimental studies revealed the considerable effect actuator dynamics can have on the overall performance of the active suspension. In this thesis the internal dynamics of the three actuator technologies previously listed will be modelled and controlled. The overall performance of these various actuators needs to be analysed in conjunction with the vehicle and controller dynamics in order to make a judgement on the performance of each of the actuator technologies in an active suspension scenario.
1.6 - Vertical active suspensions

A more accurate model than the two-mass model in representing the dynamics of the vertical suspension is shown in Figure 1.15.

![Figure 1.15 - Sideview vertical model](image)

As with the move from the one-mass model to the two-mass, this move to a more complex and accurate model reveals further detail about the performance of an active suspension. Clearly the invariant properties described for the two-mass model still apply although in a somewhat modified form. The forces generated by the actuator are still reacted by the bogie but the issue is complicated by the presence of the pitching degree of freedom. The vehicle occupants well away from the vehicle centre will perceive the effects of both bounce and pitch accelerations in their judgement of ride quality.

Another issue which is important in ride assessment is the phenomenon known as 'geometric filtering'. This arises because unlike the two-mass model, the sideview model has geometrically
Chapter 1: Introduction

spaced wheelsets. The interaction between the vehicle's dynamic modes, the geometric spacing of the wheelsets, and certain classes of track wavelengths, means that at certain frequencies the occupants will feel no excitation in one particular mode, say bounce, and in the pitch they will perceive a very large excitation at the same frequency. This is clearly an important point in the determination of vehicle ride quality. The introduction of another degree of body freedom may appear to complicate issues, but it also gives the suspension designer greater freedom over the modes that need to be controlled.

Another aspect of the work contained in this thesis is an investigation of the benefits of train-wide active suspension control laws, an example of which is given in Figure 1.16.

In this instance the train model contains 3 vehicles. Each of these three vehicles will have a dynamic interaction with its neighbours through the inter-vehicle coupling. A natural extension of the development of active suspension control laws to improve the ride quality of a single vehicle is the concept of having a train-wide active suspension control. Preview control and a knowledge of the track profile ahead of the train is known to have benefits, and the degree by
which we can improve ride quality is greatly improved. The problems in applying preview control to an automobile, namely the sensing of the road 1 to 2 metres ahead, is a significant practical problem, whereas railway trains with their elongated nature readily allow for the possibility of track preview. Sensors may be placed on the power car, the ride quality of which is not so important, and this information may then be fed to trailing vehicles. The dynamic properties of the inter-vehicle coupling result from a combination of parasitic and design components. The parasitic components are: the drawbar, and the gangway connection. On a variety of high-speed trains, inter-vehicle dampers are intentionally placed in between the vehicles, because they are known to have a beneficial effect on vehicle ride quality.

The previous discussion on the beneficial effect the use of inter-vehicle dampers has on ride quality naturally gives rise to the question of what ride quality improvements may be obtained through an active inter-vehicle element. Figure 1.17 shows the topology of an active inter-vehicle connection applied to a train of 3 vehicles.

The motivation behind consideration of such an arrangement is twofold. These are:
Chapter 1: Introduction

- No transmission path between bogies and body
- A lower actuator count

The discussion earlier on the impact which actuator technologies have on the performance of an active secondary suspension system is due in part to the finite force control bandwidth of the actuators, but also because they are in a position which provides a vibration transmission path from the bogies to the body. The bogie bounce mode is usually around 10Hz, much higher than the bounce and pitch modes of the body which are usually just below 1Hz. An actuator placed across the secondary suspension will obviously provide a transmission path for frequencies around the bogie bounce mode. Inter-vehicle actuators would operate between two vehicles both of which possess low frequency modes, and hence the bandwidth requirement placed on them would be reduced.

A second motivation behind the interest in inter-vehicle active suspension is the reduced actuator count. A major concern about active suspensions and the reservation behind implementation is the question of reliability. Passive suspensions have been the bedrock of suspension design and any change from this status quo is naturally going to generate some nervousness. Active suspensions require sensors and processing hardware. Placing sensors in harsh environments makes them vulnerable to damage with consequent failure of the suspension system as a whole. The component count is increased in an active suspension reducing the reliability. Any attempt to reduce this component count would increase the overall reliability. The scheme shown in Figure 1.17 clearly has a lower actuator count than the secondary active scheme of Figure 1.18, but the question of how much performance we can achieve through active inter-vehicle connections as opposed to a secondary active scheme is an unanswered question.

In the systems we have recently outlined, some form of global controller would be required, together with a communication bus to retrieve signals from the sensors and to send signals to drive the actuators. In my view this arrangement is simply a 'suspension-by-wire' arrangement very similar to the 'fly-by-wire' systems which incorporate redundancy and duplication and have
gained wide acceptance in the aerospace industry. The questions about reliability of fly-by-wire systems in safety critical operation have been resolved. All that is required in the rail industry is the confidence in the reliable operation of active-suspension systems and attempts to develop active suspensions with a reliability close to that currently provided by passive suspensions.

The methods used to model the vertical dynamics of both a single vehicle and a train of vehicles are given in Chapter 4. The active secondary and active inter-vehicle techniques used to control the vehicles are discussed in more depth in Chapters 7 and 8.

1.7 - Lateral active suspensions

The final area of work investigated in this thesis is similar to the two aspects we have just discussed. These are the investigation of an active lateral suspension applied to a single vehicle and also to a train of vehicles: again with a train of vehicles the possibility of an active lateral inter-vehicle control law also arises.

Figure 1.18 shows a planview model of a single vehicle. This model is clearly a lot more complicated than the sideview vertical model discussed earlier, possessing a larger number of degrees of freedom, and most importantly the introduction of wheelset dynamics. The coning of the wheelsets gives rise to an intrinsic wheelset mode as discussed in Chapter 5. The principles used to apply active suspension bear similarities to the methods used with the active secondary vertical suspension, but clearly the different modes and the degree of coupling between them gives rise to other questions. The coupling between body roll and lateral motion is strong in the lateral direction and this is an important issue with an active lateral suspension. The active secondary and active inter-vehicle control methods are discussed in more detail in Chapters 7 and 8.
Chapter 1: Introduction

Figure 1.18 - Secondary active lateral suspension

1.8 - Summary

This chapter introduces railway vehicles as complex, dynamic entities. It describes the fundamental suspension requirement. This is the need to isolate the vehicle from high-frequency irregularities and at the same time negotiate low-frequency track features. Early work on active suspension has concentrated on simplifying a vehicle down to a 'two-mass' model. This model has provided many fundamental results and enlightened current understanding of active suspension limitations. The widely acclaimed 'skyhook' damping control law is introduced with respect to the two-mass model and the means by which active suspensions can outperform a passive suspension is demonstrated.

The focus of this thesis is emphasised as being the extension of simple active suspension control laws to vehicle and train-wide models. The novel concept of active inter-vehicle control is introduced.
2.1 - Introduction

There exists a considerable body of knowledge relating to active suspension and its application to railway vehicles. Academic literature is easily accessible with many of the fundamental studies already in the public domain. Results from many of the industrial studies are less available. Industry, quite naturally due to the commercial nature of active suspension, tends to be more reticent about the publication of their work. This review has nevertheless uncovered several industrial studies. The majority of the published work covered in this review consequently has an academic origin.

Active suspension is a diverse topic in its own right with many sub-divisions. A distinct boundary exists between active suspension applied to automobiles and active suspension applied to railway vehicles. Many of the ideas naturally have crossover with each other and this review covers many papers which have an automotive focus. Another sub-division was noted in Chapter 1, that is the categorisation of different classes of suspension, see [Crolla - 1988] for an alternative summary. Terminology exists which categorises active suspension according to the control action used in its implementation: 'preview', and 'optimal' for instance. 'Active' suspension is an umbrella term which is widely accepted as being the covering term for all these classifications. This suspension classification and other related topics form the basis of this review, the exact topics covered are:

- Review papers
Chapter 2: Literature Review

- Modelling papers
- Passive suspension optimisation papers
- Adaptive/semi-passive control papers
- Semi-active control papers
- Active suspension papers
  - Low-bandwidth active control
  - Active suspension with body flexibility
  - Optimal control
  - Preview control
  - Incorporating time delays
  - Other control methods
- Actuator technology papers
- Ride/track quality and suspension performance assessment papers
- Analysis of long trains
- Implementation of active suspension

This review does not purport to provide an exhaustive coverage of all the literature relating to active suspensions. It gives a brief coverage of some key papers which are of relevance to this thesis. It is important to bear in mind the focus of this thesis, which is the essentially the evaluation of the performance of active suspension control laws applied to trains, rather than just to single vehicles. In this context the categories listed above contribute in some degree towards this end. For a more in depth inventory of active suspension literature the reader is referred to [Elbeheiry et al - 1995].

2.2 - Theoretical studies

A number of review papers [Goodall and Kortüm- 1983], [Hedrick - 1981] relating specifically to active suspension applied to railway vehicles may be found, both papers highlighting the potential benefits of exploiting active suspension technology.
Chapter 2: Literature Review

A number of more general discussion papers also exist [Appleyard et al - 1995], [Goodall - 1990], [Karnopp - 1978, 1991], [Wickens - 1992], [Williams - 1994]. [Appleyard et al - 1995], [Williams - 1994] serve as an introduction to the basic concepts of active suspension. In [Goodall - 1994] an executive overview of active suspension is given with the terminology 'dimensions of control' being introduced to emphasise three issues which are important in the choice of active suspension, these 'dimensions' are the 'degree of control', 'scope of control', and the 'technology of control'. The first, the degree of control, relates to the choice of suspension, the second, to the mode being controlled on the vehicle, and the third concerns the feasibility of implementation bearing in mind a variety of practical issues. In the same paper a vision of the progress of active suspension is formed based upon these three dimensions. The author envisages three generations of active suspension, the first having limited scope of control, restricted frequencies of operation, and utilising only local measurements and control methods. The second, having a greater functionality, a higher frequency of operation, and utilising more sophisticated measurement and control techniques. The third, possessing a high degree of integration with other vehicle dynamic systems (e.g. traction/braking), with links to track profile databases, and satellite positioning systems, etc.

A number of more technical papers highlighting the fundamental limitations of active suspension also exist [Goodall - 1993], [Hedrick & Butsuen - 1990]. [Goodall - 1993] explains the 'geometric filtering' phenomenon due to the interaction between the vehicles' dynamic modes, the geometric spacing of the wheelsets, and certain classes of track wavelengths, as detailed in Chapter 1. [Hedrick et al - 1990] authored a classic paper demonstrating another phenomenon illustrated in Chapter 1, the concept of 'invariant properties' which results in limitations in the achievable performance of an active suspension.

Computer modelling of vehicle dynamics for the subsequent design and analysis of active suspension is now the norm. This is not surprising considering the costs and speculative nature in applying potential designs to a full-scale railway vehicle and the inaccuracies generally present in small-scale models of vehicles. [Multibody codes - 1993] is an account of a variety of
software packages which could potentially be used to model train dynamics.

A number of papers have concentrated on passive suspension optimisation, [Redfield & Karnopp - 1989], [Sharp & Hassan - 1986], for example, highlight some of the variables important in suspension design (ride quality, and suspension deflection) and show the effect variations in the passive suspension parameters have on these two quantities. Both papers highlight the problems of 'high frequency transmissibility' and the consequent ride degradation which is a result of too high a damping setting as demonstrated in Chapter 1. [Sharp & Hassan - 1986] give arguments for the optimum choice of suspension stiffness and damping parameters. Both papers conclude that a passive suspension may not always be the most suitable given differing terrains and variations in the vehicle payload, and intimate that a better form of suspension in this case would be a 'semi-passive' suspension in which the suspension parameters may be varied according to terrain. An investigation of the limitations of a passive suspension is an important part of any active suspension study, it provides a justification for active suspension which is a more complicated and arguably less reliable form of suspension.

'Semi-passive' suspensions may be classified under the umbrella of 'adaptive' suspensions. There is a subtle difference between the two in that although the parameters in a semi-passive suspension may be adapted to the terrain, the parameters in a fully-active suspension are also open to adaptation and so the term 'adaptive' applies equally to both categories. These types of suspension have been studied by a number of authors [Alleyene & Hedrick - 1995], [Burton, Truscott, and Wellstead - 1995], [Gordon, Marsh, and Milsted - 1991], [Truscott and Wellstead - 1995]. These papers have employed a number of textbook control system design techniques (Generalised minimum variance control, sliding mode control, and optimal control) to suit the active suspension application. The general conclusion from all the papers is that suspension benefits are achievable, with the adaptive control algorithms being shown to adapt adequately to system and environmental changes.

'Semi-active' suspensions have received a great deal of attention in the literature, in part due to
Chapter 2: Literature Review

the positive reputation for reliability they have developed. A large number of researchers have proposed control laws and performed theoretical studies to evaluate achievable performance. A mixture of some of the fundamental papers, theoretical studies and some which are more up to date is now given: [Allen & Karnopp - 1975], [Crolla, Firth, Hyne, and Pearce - 1990], [Crolla & Abdel Hady - 1991], [Horton & Crolla - 1986], [Karnopp - 1990], [Margolis - 1982], [O'Neill & Wale - 1994], [Tseng & Hedrick - 1994], and [Venhovens - 1994]. [Allen & Karnopp - 1975] give a mathematical justification for the use of semi-active control laws, this paper dates from an era when this method of control was in its infancy, Karnopp having made contributions in the two years prior to this paper and having proposed the original concept of semi-active suspension and filed for a patent on the design. Karnopp however concedes that a contemporary by the name of Witt filed the patent before him but was not granted the patent until after Karnopp's successful application. [Karnopp - 1990] in a more recent paper illustrated the simplicity of semi-active suspension and how this type of suspension could be implemented using hydraulic cylinders in conjunction with valve switching logic to implement the control law. [Horton & Crolla - 1986] performed a theoretical analysis on what was a very faithful model of a real-life vehicle, incorporating a gas-spring suspension, and applying semi-active suspension by modulation of flows through a switched valve. Semi-active control may be applied using either a switched or continuously variable damper, a point noted by [O'Neill & Wale - 1994] in a paper which gave results from a laboratory test-rig and showed that semi-active suspension can indeed have a beneficial effect on ride quality. [Tseng & Hedrick - 1994], and [Venhovens - 1994] are two examples of recent papers which have attempted to extend the basic mathematical examination of semi-active control laws in order to improve performance.

The term 'active' suspension encompasses a wide range of topics. The categorisation outlined earlier in this chapter, and reiterated below, will now be followed. This categorisation combines a number of general active suspension topics, as well as topics which are of specific to the interests of this thesis.

- Low-bandwidth active control
Chapter 2: Literature Review

- Active suspension with body flexibility
- Optimal control
- Preview control
- Incorporating time delays
- Other control methods

'Low-bandwidth' active suspension gained popularity because of the onerous demands placed on actuator technologies when attempting to implement fully active control strategies, resulting in suspension harshness at higher frequencies. Low-bandwidth active methods attempt to control only the low frequency vehicle modes and provide passive isolation at higher frequencies. [Redfield - 1991], [Williams, Best, and Crawford - 1993], [Prokop and Sharp - 1995], and [Williams and Best - 1994] are examples of theoretical and practical studies in this area. [Redfield - 1991], and [Prokop and Sharp - 1995] are both theoretical studies of low-bandwidth active suspensions. The methodologies used to implement low-bandwidth active suspension are not dissimilar to those used in fully active and semi-active suspension designs, an important difference being the incorporation of some form of series compliance between the actuator and the vehicle. Both these studies are based on this principle. [Williams, Best, and Crawford - 1993], and [Williams and Best - 1994] are both practical studies of low-bandwidth active suspension applied to automobiles. The conclusions of which show that there are ride benefits in low-bandwidth active suspension.

Flexibility in the vehicle body is known to cause problems in the design of an active suspension. It is possible for an active suspension designed on the basis of body rigidity alone to in practice exhibit excitation of the body flexible mode. Several studies, [Hać - 1986], and [Capitani, and Tibaldi - 1988] for example, have specifically considered this problem. Both studies having modelled the body flexibility and applied standard optimal control and a variety of optimal control variants to control the rigid modes of the body while avoiding excitation of the body flexible mode.
'Optimal control' applied to active suspension has received a considerable amount of attention in recent years. The optimal control problem formulation finds favour with suspension designers due its inherent ability to perform trade-offs in system performance. Optimal control has been around for many hundreds of years, and can be traced by to calculus of variations developed by Newton and his contemporaries. Several authors have been instrumental in the development of these original theories, most notably: [Anderson and Moore - 1990], and [Kwakernaak and Sivan - 1972]. Optimal control received considerable impetus in the 1960's in missile and aerospace control where the problem may frequently be stated as a 'minimum error on target' or Mayer problem. This formulation does not naturally suit suspension design. Problems in suspension design are more suited to a 'minimum fuel consumption/accumulation of a performance index', or Lagrange problem as it is known in optimal control, which provides a natural way to perform the inherent trade-off between ride quality and suspension deflection which needs to be made as an integration over time rather than an error minimisation problem. The quantity of material published relating to optimal active suspension warrants a review applied solely to this topic: nine papers are cited here crediting the authors who are prominent in this area: [Hać - 1985], [Hrovat - 1993], [Karnopp - 1986], [Krtolica, and Hrovat - 1992], [Ray - 1992], [Thompson - 1976], [Thompson and Davis - 1988], [Ulsoy, Hrovat, and Tseng - 1994], [Wilson, Sharp, and Hassan - 1986].

[Hać - 1985] has been instrumental in adapting standard optimal control theory to the specific needs of suspension design. [Hrovat - 1993] applied optimal control theory to simple car models and extended this work to look at more complicated models which represent the vehicle dynamics more accurately and the work utilised realistic road spectra drawn from measurements of road profiles made by the Ford Motor Company. [Karnopp - 1986] investigated optimal active suspensions primarily using frequency domain techniques, which has followed on from earlier work he performed in the late 1960's with Bender also utilising frequency domain techniques. [Krtolica, and Hrovat - 1992] investigated optimal control theory specifically applied to a half-car vehicle model. [Thompson - 1976], and [Thompson and Davis - 1988] address many pragmatic issues relating to the application of optimal control theory to vehicle suspension
Chapter 2: Literature Review

design, most notably the need to be concerned with a sensible choice of measured variables, and avoid the use of variables which are practically unmeasurable. [Wilson, Sharp, and Hassan - 1986] show the need to perform a coordinate transformation which makes the system controllable and observable when applying optimal control to active suspension problems, failure to do so rendering optimal control theory inapplicable to this problem. A frequent argument made against optimal control, which again is a by-product of the earlier work on missile control performed during the 1960's, is the lack of robustness guarantees on performance. [Ray - 1992], and [Ulsoy, Hrovat, and Tseng - 1994] investigated a variety of robustness issues relating to optimal control applied to active suspensions. [Ulsoy, Hrovat, and Tseng - 1994] conclude that robustness issues can be a problem in active suspension applications if the design or the road model deviate significantly, and suggest the application of $H^\infty$ theory to the problem. [Ray - 1992] applies loop transfer recovery to improve robustness.

'Preview control' first gained consideration in the 1960's, [Bender - 1968] having performed some pioneering work in the field and demonstrated the benefits of an advanced knowledge of the track or road profile. This original piece of work used frequency domain techniques, later work tends to utilise state-space techniques to a larger extent. [Hač - 1995], and [Hač - 1992] are two examples of preview control utilising state-space theory. These papers make extensive use of optimal control theory and adapt it to incorporate preview information, demonstrating the advantages of preview through computer simulation. [Jezequel and Roberti - 1996] is a recent example of the continued interest in preview control and serves to illustrate the crossover which is frequently made between various branches of suspension design, in this instance a study of preview control applied to a semi-active design. Experimental studies of preview control are limited, the sensing requirements being quite demanding. The consensus of opinion is that preview control will gain popularity once active suspensions have gained general acceptance.

It is intuitive that if a knowledge of track or road profile can have a beneficial effect on ride quality, then a control law which utilises information gained from leading wheelsets or tyres should also offer benefits. This is indeed the case and has been demonstrated so for vehicles with
a pair of axles. In the context of active suspension applied to railway trains, the benefits would appear to be amplified owing to the elongated nature of a train possessing numerous axles. Several studies have addressed this issue [Frühau, Kasper, and Lückel - 1986], [Hač, and Youn - 1993], [Sharp, and Wilson - 1990], and [Louam, Wilson, and Sharp - 1988] for example. [Frühau, Kasper, and Lückel - 1986] gives a descriptive account of the design process, the method in this paper utilises Padé approximations to model the speed dependant time delay between axles, which is used in conjunction with optimal control theory to form an active suspension controller, the overall system demonstrating performance improvements. [Hač, and Youn - 1993] design a controller based upon optimal control techniques which again incorporates the time delay between axles, the resulting system has a feedforward path directly between the road measurements and the actuation. [Sharp, and Wilson - 1990] evaluate the optimal performance index for a system with time delays, a controller parameterisation is, however, not addressed. An earlier paper [Louam, Wilson, and Sharp - 1988] by the same authors re-formulates the problem in the discrete domain and provides a solution based on optimal control theory. The time delay between axles in this instance needs to be a multiple of the sampling frequency.

A variety of other control laws have received acclaim, many of which are variations on a theme, or may be traced back to more fundamental studies. [Truscott - 1994], [Williams - 1994], [Crolla and Abdel-Hady - 1991], and [Yue, Butsuen, and Hedrick - 1989] highlight this collection of control techniques. A prime example is the well known 'skyhook damping' scheme. This term dates back to early investigations by Karnopp and Bender who took a generic approach to the consideration of the ideal suspension, and through the use of Wiener filtering theory discovered that an 'almost passive' system could be arranged to have what they termed to be an optimal suspension in terms of optimisation of a performance index which trades off ride quality and suspension deflection. The only problem with this scheme being that it required absolute body damping as opposed to the more orthodox relative damping. The term 'skyhook' damper was coined to reflect the fact that absolute damping could in theory be applied by attaching a damper between the body and the sky. Further studies of skyhook damping have demonstrated that when
other suspension issues are addressed it does not provide an adequate form of suspension. Additional filtering is required to form a realistic design [Yue, Butsuen, and Hedrick - 1989], and [Williams - 1994]. Feedback of suspension deflection also has beneficial effects [Truscott - 1994]. Development of active suspension control laws frequently comprise a number of these features.

Actuator technologies and the bandwidth over which they are capable of controlling vehicle modes are issues in active suspension design. [Williams and Miller - 1994] appreciate the importance of consideration of actuators in an active suspension and have performed a theoretical analysis of an electrohydraulic actuator in this application. The study concludes that as much as 20% of the energy required to provide an active suspension is dissipated in the actuator. A number of actuator technologies are deemed suitable for consideration in this thesis:

- Electrohydraulic actuator [Cottell - 1996]
- Electromechanical actuator [Friar - 1996]
- Electromagnetic [Denne - 1993]

Ride quality and suspension performance evaluation is clearly an important aspect of the work in this thesis. A number of standards have been defined which quantify the impact of vibrations on the human body, [ISO 2631 - 1978] and [BS 6841 - 1987] for example. These standards have been drawn from empirical test results and debate about their applicability still abounds due to the subjective nature of ride quality. [Pollard and Simon - 1984] gives an account of the application of these standards to railway vehicles and gives an overview of the causality between track spectra and ride quality. [Tanifuji - 1988] gives a mathematical account of ride assessment methods applied to railway vehicles, and extends this to a full train model. [Hedrick and Firouztash - 1974] and [Karnopp - 1978] apply an alternative assessment method generally known as covariance analysis to suspension problems. [LaBarre, Forbes, and Andrew - 1969], and [Corbin and Kaufman - 1975] give accounts of how track and roadway may be modelled, these models are used in conjunction with vehicle models to assess ride quality.
Chapter 2: Literature Review

Research into dynamics associated with long trains has been limited. [Kamopp - 1968] establishes a train model and evaluates the optimal path the centre of gravity of each vehicle must follow in order to minimise a performance index which weights ride quality and suspension deflection. [Tanifuji - 1988] performed ride quality tests on the Japanese Shinkansen trainsets and compared these results with more formally derived predictions. His experimental results showed ride degradation towards the rear of a train, and in particular towards the ends of vehicles. [Masfrand, Ravalard, and Coutellier - 1994] analysed the changes in vehicle modes due to coupling for a variety of train lengths. Many of the aforementioned papers have considered only the vertical suspension. However, the ideas espoused in these papers may be extended to the lateral case.

2.3 - Applications

Application of active suspension technology to railway vehicles has been particularly intensive in Europe and Japan. In Europe, ABB have tested a semi-active lateral suspension for the X2000 trains [Roth and Lizell - 1995]; this uses a novel semi-active damper which has been specially developed. ABB also offer an active lateral 'hold-off' device, mainly for use on more conventional trains in the U.K. SGP, Bombardier Eurorail, and GEC-Alsthom all have either ongoing tests on active control, or offer some form of semi-active damper for current rolling stock. In Japan a wide range of studies are happening, partly with their WIN 350 vehicle [Higaki - 1995] which is being used for extending Shinkansen operating speeds up to 350 kmh\(^1\), but also with a 'Try-Z' test vehicle on which a number of active suspension concepts are being developed and tested.

2.4 - Summary

The review covers various facets of active suspension: 'modelling', 'passive suspension optimisation', 'semi-active', 'real actuators', 'ride quality', and 'train-wide behaviour'. Papers are predominantly sourced from academia although some industrial studies are also referenced.
Chapter 3
Actuator modelling

3.1 - Overview

An essential component of any active suspension is an actuator (force generating device). It is the mechanism whereby force and hence energy injection can complement the existing passive suspension function. Previous studies [Goodall, Pearson, and Pratt - 1993] have shown that actuator performance can have a significant impact on the overall success of an active suspension implementation. The traditional notion of an actuator as a 'force injecting' element needs to be reviewed in the context of an active suspension application. The introduction given in Chapter 1 stated the fundamental suspension requirements as being the need to provide high frequency isolation, while at the same time maintaining the ability to follow the low frequency track undulations. An actuator implementing an active suspension function would need to provide negligible force at these higher frequencies, but would need to be receptive to fluctuations in the displacement across the actuator, a role which many actuator technologies cannot readily accommodate. For this reason, a considerable emphasis is placed throughout this thesis on the integration of actuator performance with vehicle dynamics in the overall assessment of active suspension performance.

This chapter will firstly discuss the actuators deemed most suited to the active suspension role. It will then compare the general properties of these actuators with their counterparts. Dynamic models for each of the technologies will be developed, followed by a discussion on the procedure used to evaluate the parameters for each of these models. It will then conclude by illustrating the impact of the internal dynamic behaviour on the active suspension as a whole.
Three types of actuator technology are deemed most suitable to the active suspension application, a number can already be found on active suspension test vehicles, and the others have considerable potential.

They are:

- Electrohydraulic actuation [Cottell - 1996]
- Electromechanical actuation [Friar - 1996]
- Electromagnetic actuation [Denne - 1993]

Although other technologies are worthy of consideration, at the time of writing, these three are deemed the most suitable contenders for an active suspension implementation. Other actuation methods include: electro-strictive, magneto-strictive, thermo-strictive, and piezo-electric, which are based upon the application of electrical, magnetic, and thermal fields to susceptible materials which expand or contract in sympathy with the excitation. The main drawback in many of these technologies is the short stroke; ongoing research and development of these technologies may ultimately lead to devices which are suited to active suspension.

Pneumatic actuation is omitted from the list of most suitable actuators, which is not because it is deemed an unsuitable form of actuation. On the contrary, it is a most appropriate form, being a well established technology, used extensively in food processing and other industries with exacting demands on contamination-free actuation. This form of actuation finds favour with a number of researchers who assert that the airsprings themselves can be adapted to form a pneumatic actuator. This is a very pragmatic assertion, because current airspring designs already require a compressor and valves for self-levelling. The compressor would not be able to supply the increased operational duty imposed on it by an active suspension, and would require at least an upgrade of the existing system. Pneumatic actuation is difficult to control, because of the high compressibility of air requiring large pressures and energy dissipative air-flows to achieve the bandwidth of control envisaged for the type of active suspension being considered in this thesis.
Chapter 3: Actuator modelling

For these reasons an active pneumatic suspension is deemed to be a less attractive option than the three technologies selected for consideration in this thesis.

Electrohydraulic actuation [Cottell - 1996], as its name suggests, utilises a combination of electrical and hydraulic means to generate force, as shown in Figure (3.1). The electrical component provides the primary stage of actuation, which is frequently realised by a torque motor; this is a device which provides a torque proportional to the current delivered to it, and having a limited angle of movement. The secondary hydraulic stage consists of a hydraulic cylinder connected to hydraulic pressure via a throttling spool-valve. The spool-valve is spring-loaded and actuated by the torque motor. Controlled oil flows into the cylinder generate pressure and hence output force because of the relative incompressibility of the hydraulic oil.

The term 'servo-hydraulic' is synonymous with 'electro-hydraulic' actuation. The term 'electro-hydraulic actuation' is also used to cover systems which do not necessarily possess spool-valves, they may in fact be: 'flapper', or 'poppet' types. The term 'electrohydraulic actuator' is used throughout this thesis to describe the system shown in Figure (3.1). Electrohydraulic actuators are now being introduced in self-contained units which require only electrical connection to the 'outside world'. The hydraulics are self-contained, fluid power being developed via an internal electric motor and hydraulic pump combination. This has obvious implications on the overall cleanliness, with the possibility of oil leakage being considerably reduced. The term 'electrohydraulic actuator' covers both these autonomous units as well as systems which are composed of discrete components, although the models developed herein apply to the more conventional type of system.

Electromechanical actuators [Friar - 1996] are formed from a combination of an electrical motor with some form of rotary to linear motion conversion. The electric motor can be one of a number of types, although a brushless DC motor is an attractive option due to its high reliability and appropriate characteristics. The conversion is generally in the form of a screw-thread, whether this be of a 'recirculating ball-screw', 'roller-nut', or simple 'screw-jack' form. Figure (3.6)
Chapter 3: Actuator modelling

illustrates the structure of a simple 'screw-jack' mechanism, 'roller-nut' type actuators are similar in construction but differ in that they do not have an outer screw but rather a planetary system which contains a series of rollers running along the main thread. 'Recirculating -ball screw' mechanisms are different, they consist of a central pre-stressed screw thread, this thread forms a race on which ball-bearings run. The outer race reacts the force generated because of the motion of these ball-bearings; this being the output force of the actuator. The ball-bearings are recirculating once they reach the end of the screw thread. Electromechanical actuators have recently been introduced into railway tilting systems and it is envisaged that they will do the same in active suspension.

Electromagnetic actuators [Denne - 1993] have not traditionally been deemed suitable for railway vehicles. 'Maglev' vehicles however, are reliant on electromagnetic actuation to provide the basic suspension function. The magnetic circuitry required to suspend a vehicle of this magnitude is bulky, not a profound problem in a Maglev vehicle in which the entire suspension and guidance system are provided by the electromagnets. In railway vehicles, wheelsets and a primary suspension are still required to guide the vehicle, meaning that electromagnets would need to be installed in addition to the existing suspension, a difficult problem considering the bulk of such an actuator and the restraints imposed by the kinematic gauge. Recent improvements in magnetic circuit design and reductions in actuator size has led a few researchers to investigate the possibility of having an electromagnetic stage of suspension in a railway vehicle. The active suspensions developed in this thesis all operate in parallel with the existing passive suspension. This ensures that the passive suspension carries the static weight of the vehicle, the actuators providing a force modulation role. In this scenario the demands placed on the actuator are less demanding than those placed on a Maglev's electromagnets. A novel electromagnetic actuator design constructed in a self-contained unit, and known as the Pemram® [Denne -1993], has recently entered the market. In the author's opinion such a device may be used as an active suspension actuator in parallel with an existing passive suspension. It is compact and can be located across the secondary suspension with minimal interference and negligible impact on the kinematic gauge.
3.2 - General actuator properties

Each of the three actuator technologies discussed in section 3.1 will have its own merits which is the reason why these differing technologies find themselves fulfilling different roles in practice. Electrohydraulic actuators are frequently found in enclosed areas, where space constraints are an important issue: aircraft flight surfaces, and active suspension on racing cars for example. Electromechanical actuators are frequently found in environments where the presence of hydraulic oil would be hazardous, or where hydraulic power is not readily available, steel furnace and garage door control for example. Autonomous electromagnetic actuators have only recently been developed and therefore still need to prove themselves. Their behaviour should of course reflect the excellent controllability and reliability of their maglev counterparts.

Some of the beneficial properties which an actuator should possess have just been given, a more detailed, but by no means complete list of properties is given below:

- Maintainability
- Reliability
- Achievable force bandwidth
- Force to size ratio
- Stroke
- Availability of power source
- Efficiency
- Cost

These properties are important in the context of an active railway suspension. Other properties such as their cleanliness, noise production, and hazardous nature have been omitted since it is assumed that these problems may be eliminated through the design of self-contained actuator units.
Chapter 3: Actuator modelling

The maintainability of an actuator is an important property. Maintenance of the suspension should involve the minimum of incumbrance to maintenance personnel. An intricate, difficult to maintain actuator is likely to create problems for even the most highly skilled maintenance technicians. Table 3.1 gives a qualitative performance comparison for each of the actuators. All three technologies rank very similarly in performance.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Electrohydraulic</th>
<th>Electromechanical</th>
<th>Electromagnetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maintainability</td>
<td>Average</td>
<td>Average</td>
<td>Average</td>
</tr>
<tr>
<td>Reliability</td>
<td>Good</td>
<td>Good</td>
<td>Excellent</td>
</tr>
<tr>
<td>Force bandwidth</td>
<td>Moderate</td>
<td>Moderate</td>
<td>Very high</td>
</tr>
<tr>
<td>Force/size ratio</td>
<td>Excellent</td>
<td>Average</td>
<td>Poor</td>
</tr>
<tr>
<td>Stroke</td>
<td>High</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Power Availability</td>
<td>Low</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Efficiency</td>
<td>Low</td>
<td>Average</td>
<td>High</td>
</tr>
<tr>
<td>Cost</td>
<td>Low</td>
<td>Average</td>
<td>High</td>
</tr>
</tbody>
</table>

Table 3.1 - Various actuator technology properties

The question of reliability is of paramount importance if active suspensions are to be widely accepted by the railway industry. A high-speed passenger vehicle can easily complete 100,000 km between services. A noticeable number of suspension failures during this period will result in a lack of confidence in this technology. Active suspensions are competing against passive suspensions with a proven reliability record. It is not necessary for active suspensions to be able to match this reliability, but it is important that they have a very good reliability and that the possibility of any failure lies within the capabilities of an appropriate maintenance programme. The reliability of each of the three actuator technologies is nevertheless important in terms of the reliability of the overall system. Table 3.1 compares the reliability of each of the technologies. The electromagnetic actuator has an excellent reliability due to the absence of contacting parts,
which is one of the main reasons for its inclusion within this thesis despite the fact it is still a developing technology.

The achievable force control bandwidth of an actuator is an understated issue in active suspensions and is frequently the reason behind the optimistic theoretical predictions of active suspension performance when compared with experimental assessments. A more in-depth explanation of this issue is given later in this chapter, and is one of the key issues addressed in this thesis. Suffice it for now to say that an actuator needs to control the dynamic modes of the vehicle, some of which lie at the extremity of the achievable force control bandwidth of many actuators. Bandwidth limitations arise because of the inertial and resistive nature of the internal components of the actuator. These dynamic components differ between the various technologies and hence the differing bandwidths possessed by each of the actuators. Table 3.1 preempts results from section 3.5 and compares the three actuator technologies. The second reason for the inclusion of the electromagnetic actuator is the very high force control bandwidth it possesses in comparison with the other two technologies. The reason behind this is explained in detail later in this chapter, but is largely due to the lack of internal inertial dynamics.

The force to size ratio for each of the actuators needs to be addressed to ensure that an actuator which is sized to provide the active suspension forces and power levels can fit within the confined space available between the body and the bogie. The philosophy used throughout this research is to augment existing rolling stock with active devices rather than re-engineering the entire vehicle to incorporate active suspension. Even if the latter philosophy were applied, significant space savings between the body and bogies might be difficult to make. The actuator must therefore be a relatively compact device. Table 3.1 compares this property. The electromagnetic actuator has a clear failing here. However, recent improvements in this type of actuator are increasing the force to size ratio. The electrohydraulic actuator is clearly the best in respect of this property. The electromechanical actuator is not a particularly bulky device but is not in the same league as the electrohydraulic actuator.
Chapter 3: Actuator modelling

The stroke capability of the various actuator technologies also needs to be considered. During the operation of an active suspension on a railway vehicle, an actuator may need to provide a stroke capability of at least \( \pm 80 \) mm. This is within the capabilities of both the electrohydraulic and electromechanical actuators as indicated in Table 3.1. The electromagnetic actuator in a direct 'pull-pull' configuration, as shown in Figure (3.12), is limited in the available stroke. The novel electromagnetic actuator, the Pemram\(^{\circledR}\) [Denne - 1993] mentioned earlier, operates in a similar manner to a linear motor and has an improved stroke in comparison with the basic 'pull-pull' arrangement.

The availability of a suitable power source is an important issue. High-speed passenger vehicles are not normally fitted with hydraulic power packs, and consequently electrohydraulic actuation fares worse in the categorisation given in Table 3.1. However, if a self-contained electrohydraulic actuator, having only electrical connections to the 'outside world' were to be used, this would improve the rating of this type of actuation. Electrical supplies are always available on high-speed passenger vehicles making the electromechanical and electrohydraulic option more readily acceptable; the British Rail Mk IV coaches for instance, possess a 110 volts DC supply and a 240 V a.c. supply. The power drawn from this supply is also an issue, active suspensions are widely expected to require up to 8kW per vehicle if both a vertical and lateral active suspension is to be installed. This figure is dependent upon the choice of actuator technology, some actuators of course having a higher efficiency, consequently this figure may be reduced to a lower level.

The question of actuator efficiency was referred to previously. The electrohydraulic actuator fares badly in this respect, as shown in Table 3.1. This is due to the large energy losses inherent in the throttling of hydraulic fluid. The electromechanical actuator fares much better, but still suffers losses due to motor heating and friction. The electromagnetic actuator is an efficient device, with a small amount of loss due to electrical heating and friction.

The cost of each of the actuators is important considering the number of actuators required in a train, and the need to avoid significant cost increases over existing rolling stock. Electrohydraulic
actuation has gained popularity in recent years, this popularity has led to a significant drop in the cost of this form of actuation. Electromechanical actuation ranks slightly higher due to the more demanding production requirements with this device. Electromagnetic actuation is still in its infancy and consequently the costs of such an actuator reflect this. Mass production of these devices will inevitably lead to a drop in price.

3.3 - Dynamic models of actuators

The internal dynamics of the three actuator technologies discussed in section 3.1 have a notable impact on the overall performance of an active suspension as will be demonstrated in section 3.5. The three actuator technologies: electrohydraulic, electromechanical, and electromagnetic will now be analysed in turn, and models representing their behaviour will be developed. The models developed are all linear, which is naturally a significant approximation, particularly for actuators possessing notable non-linearities. Linear models are developed because of the ease with which they may be interfaced with the vehicle dynamics, and because it is not an unrealistic assumption in an active suspension to assume that the actuator operates about a nominal operating point, making only small-scale excursions away from this point.

3.3.1 - Electrohydraulic actuator model

The structure of an electrohydraulic actuator, shown in Figure (3.1), was explained briefly in section 3.1. For modelling purposes a distinct segregation exists between the electrical and hydraulic aspects of the actuator. The link between the two is made through the positioning of the servo-valve. The hydraulic aspects of the modelling procedure will be developed first followed by those of the electrical circuitry.

The actuator generates force by compression of the hydraulic fluid on either side of the cylinder. Compression or de-compression of the cylinder chamber is achieved via a throttled connection to either a hydraulic pressure or to tank pressure.
Chapter 3: Actuator modelling

Figure 3.1 - Electrohydraulic actuator

The oil flow into the cylinder and the oil compressibility are the root causes of pressure development. Equation (3.1) represents the oil flow into the annulus side of the cylinder.

\[ q_a = \frac{v_a}{\beta} \frac{dp_a}{dt} + a_a \frac{dx_{act}}{dt} \]  

... (3.1)

In the equation, \( q_a \) is the flow into the annulus side, \( p_a \) is the oil pressure in that side of the cylinder, \( v_a \) is the oil volume on the annulus side with the piston in its nominal position, and \( x_{act} \) is the extension of the actuator. The equation relates oil flow \( q_a \) to two components: the flow created because of actuator extension, and that due to the compressibility of the fluid \( \beta \).

Equation (3.2) below applies the same principles as described previously, but in this case the variables are associated with the bore side of the cylinder.

\[ q_b = \frac{v_b}{\beta} \frac{dp_b}{dt} + a_b \frac{dx_{act}}{dt} \]  

... (3.2)
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The flows expressed by equations (3.1), and (3.2) are controlled by the spool-valve. Throttling of fluid flow through the spool-valve is non-linear and follows a 'square-root' characteristic as given below by equation (3.3) for the bore side of the cylinder.

\[ q_s = k_x \sqrt{p_t - p_b} \]  \hspace{1cm} (3.3)

Equation (3.3) applies to oil flow into the bore side, at the same time flow from the annulus side of the cylinder is throttled back to tank pressure, as given by equation (3.4).

\[ q_s = k_x \sqrt{p_t} \]  \hspace{1cm} (3.4)

\( q_b \) being the bore side oil flow, \( p_b \) the bore side cylinder pressure, \( p_t \) the hydraulic supply pressure, \( x \) the spool-valve displacement, and \( k \) a flow-coefficient.

The flow characteristic described by equations (3.3), and (3.4) may be linearised to form equations (3.5) and (3.6) respectively.

\[ q_s = \frac{\partial q_s}{\partial x} x + \frac{\partial q_s}{\partial p_s} p_s \]  \hspace{1cm} (3.5)

\[ q_s = \frac{\partial q_b}{\partial x} x + \frac{\partial q_b}{\partial p_b} p_b \]  \hspace{1cm} (3.6)

Equating the cylinder oil flows given by equations (3.1), and (3.2) with the spool-valve oil flows given by equations (3.5), and (3.6) gives rise to equations (3.7), and (3.8).

\[ \frac{v_s}{\beta} \frac{dp_s}{dt} + a_s \frac{dx_s}{dt} = \left( \frac{\partial q_s}{\partial x} x + \frac{\partial q_s}{\partial p_s} p_s \right) \]  \hspace{1cm} (3.7)

\[ \frac{v_b}{\beta} \frac{dp_b}{dt} + a_b \frac{dx_b}{dt} = \left( \frac{\partial q_b}{\partial x} x + \frac{\partial q_b}{\partial p_b} p_b \right) \]  \hspace{1cm} (3.8)
Chapter 3: Actuator modelling

Equations (3.7), and (3.8) give the cylinder pressures in terms of the spool-valve setting, and the actuator extension. It is a simple matter therefore to evaluate the force developed by the cylinder, given the differential pressures acting on the two piston areas. Other authors frequently model oil mass inertia, and pipe expansion in the derivation of a model, these are ignored here since it is usual to locate the servo-valve very close to the actuator in order to achieve a good bandwidth of operation. This completes the discussion on the hydraulic dynamics.

The spool-valve and torque motor may be described by equation (3.9).

\[
x_v = \frac{k_i i}{m_s s^2 + c_s s + k_s}
\]

... (3.9)

This equation gives the spool displacement for a given input current 'i'. The torque motor develops a torque proportional to the input current, this torque is converted into a linear force and drives a simple second order mass-damper-stiffness arrangement which represents the mechanical dynamics of the spool-valve and its centring spring.

Equations (3.7), (3.8), and (3.9) may be placed in a more compact state-space form as given by equations (3.10), and (3.11).

\[
\begin{bmatrix}
\dot{x}_v \\
\dot{x}_v' \\
\dot{p}_s \\
\dot{p}_s'
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
\frac{-k_i}{m_s} & \frac{-c_i}{m_s} & 0 & 0 \\
\frac{B \frac{\partial q_s}{\partial p_s}}{v_s} & 0 & \frac{B \frac{\partial q_s}{\partial x_v}}{v_s} & 0 \\
\frac{B \frac{\partial q_s}{\partial x_v}}{v_s} & 0 & \frac{B \frac{\partial q_s}{\partial p_s}}{v_s} & 0
\end{bmatrix} \begin{bmatrix}
x_v \\
x_v' \\
p_s \\
p_s'
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{k_i}{m_s} \\
\frac{B a_e}{v_e} \\
\frac{-B a_e}{v_e}
\end{bmatrix} i + \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} \dot{x}_v \quad ... (3.10)
\]
A force feedback control loop, as shown in Figure (3.2), is required around the actuator in order to improve the bandwidth and disturbance rejection of the system. A standard PID controller is generally used to fulfill this role.

The parameters used in this electrohydraulic actuator are derived in section 3.4. Suffice it for now to say that the parameters given in Table 3.2 are representative of an actuator operating in a railway active suspension.
Figure 3.2: Linearised model of an electrohydraulic actuator

Chapter 3: Actuator modelling
### Chapter 3: Actuator modelling

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spool mass</td>
<td>$m_s$</td>
<td>0.075 (kg)</td>
</tr>
<tr>
<td>Spool stiffness</td>
<td>$k_s$</td>
<td>185000 (Nm$^{-1}$)</td>
</tr>
<tr>
<td>Spool damping</td>
<td>$c_s$</td>
<td>471 (Nsm$^{-1}$)</td>
</tr>
<tr>
<td>Torque motor gain</td>
<td>$k_l$</td>
<td>142 (NmA$^{-1}$)</td>
</tr>
<tr>
<td>Oil supply pressure</td>
<td>$p_l$</td>
<td>2x10$^7$ (Nm$^{-1}$)</td>
</tr>
<tr>
<td>Linearised parameter</td>
<td>$dq_{addxv}$</td>
<td>0.00935</td>
</tr>
<tr>
<td>Linearised parameter</td>
<td>$dq_{bddxv}$</td>
<td>0.01218</td>
</tr>
<tr>
<td>Linearised parameter</td>
<td>$dq_{addpa}$</td>
<td>2.022x10$^{-12}$</td>
</tr>
<tr>
<td>Linearised parameter</td>
<td>$dq_{bdddpb}$</td>
<td>-1.9232x10$^{-12}$</td>
</tr>
<tr>
<td>Oil compressibility</td>
<td>$\beta$</td>
<td>1.38x10$^9$</td>
</tr>
<tr>
<td>Annulus cylinder volume</td>
<td>$v_a$</td>
<td>5.063x10$^{-6}$ (m$^3$)</td>
</tr>
<tr>
<td>Bore cylinder volume</td>
<td>$v_b$</td>
<td>9.039x10$^{-6}$ (m$^3$)</td>
</tr>
<tr>
<td>Annulus side area</td>
<td>$a_a$</td>
<td>2.25x10$^{-4}$ (m$^2$)</td>
</tr>
<tr>
<td>Bore side area</td>
<td>$a_b$</td>
<td>4.02x10$^{-4}$ (m$^2$)</td>
</tr>
<tr>
<td>Ram damping</td>
<td>$c_r$</td>
<td>1.2x10$^9$ (Nsm$^{-1}$)</td>
</tr>
</tbody>
</table>

**Table 3.2 - Electrohydraulic actuator parameters**

The PID controller parameters for this actuator were tuned to achieve a gain margin of 20 dB, and phase margin of 80° in the force loop as shown by the Nichols chart of Figure (3.3). The crossing frequency is around 20 Hz and with a phase margin of 80° the bandwidth is also 20 Hz. Section 3.5 will show that even though the vehicle modes are well below 20 Hz, the actuator does not control very well in an active suspension environment above 5 Hz.
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Figure 3.3 - Electrohydraulic actuator controller design

The closed loop performance of this actuator is shown in Figure (3.4), and (3.5). Figure (3.4) illustrates the actuator's ability to follow a 1000 N 'square-wave' force input at 10 Hz.

Figure 3.4 - Electrohydraulic actuator force following

Figure (3.5) illustrates its ability to reject a 5mm 'square-wave' disturbance at 10 Hz on the
extension of the actuator given a zero force demand. Clearly the first test is performed on a stalled actuator, and is therefore not fully representative of a real-life scenario, but serves to illustrate some degree of model validity. The achievable bandwidth of this actuator is given in section 3.5 when it is tested in situ in a vehicle model.

![Graph showing 0.1 Hz disturbance rejection](image)

**Figure 3.5** - Electrohydraulic actuator disturbance rejection

### 3.3.2 - Electromechanical actuator model

Figure (3.6) gives a crude illustration of the structure of an electromechanical actuator. A more complex 'recirculating ball-screw', 'roller-nut' arrangement is generally used in practice but the simple arrangement shown in Figure (3.6) is dynamically similar to these devices; the representation of the 'gearing' during rotary to linear motion conversion is adequately described by this arrangement.
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Force is generated by compression of the screw as the motor is turned; although the screw thread will normally be stiff it still possesses finite stiffness and damping. An equivalent schematic representation of the actuator which makes deduction of the model much simpler is given in Figure (3.7).

Figure 3.6 - Electromechanical actuator

Figure 3.7 - Equivalent electromechanical actuator model
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The torque generated by the electric motor is given by equation (3.12).

\[
i_m = \frac{r_{arm}}{l_{arm}} i_m + \frac{k_r}{l_{arm}} \theta_m - \frac{k_t}{l_{arm}} \theta_m
\] ... (3.12)

The electrical time constant of the winding is taken into account in this equation; \(r_{arm}\) and \(l_{arm}\) being the winding resistance and inductance respectively. \(v_{arm}\) is the applied motor voltage, \(\theta_m\) being the angular rotation of the motor, and \(i_m\) being the torque generated by the motor. \(k_r\) and \(k_t\) are fixed motor parameters. The mechanical inertia \(J_m\) and inherent damping \(c_m\) of the motor are accounted for in the dynamic equation (3.13).

\[
\dot{\theta}_m = \frac{t_m}{J_m} - \frac{c_m}{J_m} \dot{\theta}_m - \frac{k_s (\theta_m n^2 - x_m n)}{J_m}
\] ... (3.13)

The interconnected system of masses, dampers, and springs shown in Figure (3.7) may be represented by the equation (3.14). \(n\) being the rotary to linear 'gearing', and \(m_t\) being the mass of the screw thread.

\[
\ddot{x}_m = \frac{(k_s - k_t)}{m_s} x_m - \frac{c_s}{m_s} \dot{x}_m - \frac{c_s}{m_s} \dot{x}_m + \frac{k_t}{m_s} \dot{x}_m + \frac{k_t}{m_s} \dot{x}_m
\] ... (3.14)

The force developed by this system is due to compression of the lead screw and is given by equation (3.15).

\[
f_{act} = k_s x_m + c_s \dot{x}_m - k_t \dot{x}_m - c_t \dot{x}_m
\] ... (3.15)

The state equation (3.16), and the output equation (3.17) are an amalgamation of equations (3.12) to (3.15) and are a concise representation of an electromechanical actuator. The motor voltage \(v_m\), and the actuator extension \(x_{act}\) are system inputs, the output is of course actuator force \(f_{act}\).
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As was the case with the electrohydraulic actuator discussed in section 3.3.1, a force feedback loop, as shown in Figure (3.8), is required to improve the performance of the actuator.
Figure 3.8 - Electromechanical actuator linear model
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A PID controller is again used to fulfill this task, the parameters being tuned to give acceptable gain and phase margins. The Nichols chart of Figure (3.9) shows these quantities for the open-loop response of the actuator in conjunction with the PID controller. The bandwidth in this instance is 15 Hz although the actuator is again incapable of controlling modes higher than 5 Hz as will be seen in section 3.5. These gain and phase margins ensure acceptable closed loop time domain behaviour of the actuator.

![Open-loop controller & actuator](image)

**Figure 3.9 - Electromechanical actuator controller design**

The parameters used in the model are derived later in this chapter, but the values which are representative of this type of actuator fulfilling an active suspension role in a railway vehicle are given in Table 3.3.
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor torque constant</td>
<td>k_t</td>
<td>0.108 (NmAmp⁻¹)</td>
</tr>
<tr>
<td>Motor back-emf gain</td>
<td>k_v</td>
<td>0.108 (NmAmp⁻¹)</td>
</tr>
<tr>
<td>Winding inductance</td>
<td>l_w</td>
<td>0.1 (mH)</td>
</tr>
<tr>
<td>Winding resistance</td>
<td>r_w</td>
<td>0.95 (Ω)</td>
</tr>
<tr>
<td>Motor inertia</td>
<td>j_m</td>
<td>1.2×10⁻⁴ (kgm²)</td>
</tr>
<tr>
<td>Motor damping</td>
<td>c_m</td>
<td>0.012 (Nmsrad⁻¹)</td>
</tr>
<tr>
<td>Motor series stiffness</td>
<td>k_s</td>
<td>1×10⁷ (Nm⁻¹)</td>
</tr>
<tr>
<td>Screw pitch</td>
<td>n</td>
<td>3.82×10⁴ (Nm⁻¹)</td>
</tr>
<tr>
<td>Screw mass</td>
<td>m_s</td>
<td>2 (kg)</td>
</tr>
<tr>
<td>Screw stiffness</td>
<td>k_s</td>
<td>1.8×10⁴ (Nm⁻¹)</td>
</tr>
<tr>
<td>Screw damping</td>
<td>c_s</td>
<td>1.2×10⁹ (Nsm⁻¹)</td>
</tr>
</tbody>
</table>

Table 3.3 - Electromechanical actuator parameters

A pair of validation tests are now performed on the actuator model. These are identical to the tests performed in section 3.3.1 for the electrohydraulic actuator. The first, shown in Figure (3.10), is the output force response to 'square-wave' demand input of 1000 N at 10 Hz. Figure (3.11) illustrates this actuator's ability to reject a 10 Hz, 5mm 'square-wave' disturbance on the extension of the actuator ends given a zero force demand. These responses indicate that the actuator is representative of the real-life equivalent, and more revealing tests are conducted in section 3.5 with the actuator in situ in the vehicle.
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Figure 3.10 - Electromechanical actuator force following

Figure 3.11 - Electromechanical actuator disturbance rejection
3.3.3 - Electromagnetic actuator model

A simplified structure of an electromagnetic actuator is given in Figure (3.12).

![Electromagnetic actuator diagram](image)

**Figure 3.12 - Electromagnetic actuator**

This is distinctly different from the Pernram® actuator mentioned earlier. The actuator possesses an inner and an outer frame. A pair of electromagnets, both operating in attraction, are mounted on these frames, one half mounted on the inner frame, the other on the outer frame. Energising one set of electromagnets causing the actuator to extend, the other resulting in contraction.

Modelling of this actuator is based on work performed by [Williams - 1986]. The force generated by the actuator is a non-linear dynamic function of the input current and the actuator extension. The modelling techniques used here assume a linearisation of these characteristics. The output force may therefore be represented by equation (3.18) where 'k_i' and 'k_x' are constants.

\[ f_{act} = k_i \cdot i \cdot k_x \cdot x_{act} \]  \[ ... (3.18) \]

The current in the coil 'i' is a combination of two currents 'i_1' and 'i_2' as shown by equation (3.19).
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\[ i = i_1 + i_2 \] ... (3.19)

The two components of current: 'i_1' and 'i_2' are due to the applied voltage and actuator extension respectively. The first 'i_1' is given by equations (3.20) and (3.21) as a function of applied voltage.

\[ v_1 = v_{act} - l_m \frac{di_1}{dt} \] ... (3.20)

\[ i_1 = v_1 - \frac{dl}{dt} \frac{i_1}{r} \] ... (3.21)

Mutual 'l_m', and self inductances 'i_1' and 'l_1' as well as coil resistance 'r' are necessary in this evaluation. The second current component 'i_2' is a linear function of actuator extension rate as given by equation (3.22).

\[ i_2 = k_1 x_{act} \] ... (3.22)

The dynamics described by equations (3.18) to (3.22) may be placed in a more compact state-space model as given by equations (3.23) and (3.24).

\[
\begin{bmatrix}
\dot{x}_{act} \\
\dot{i}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 \\
-\frac{k_1}{l_m} & -\frac{r}{l_m}
\end{bmatrix}
\begin{bmatrix}
x_{act} \\
i
\end{bmatrix} +
\begin{bmatrix}
1 \\
-\frac{l_m k_1}{l_m}
\end{bmatrix}
\begin{bmatrix}
\dot{x}_{act} \\
\dot{i}
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{1}{l_m}
\end{bmatrix}
\begin{bmatrix}
v_{act}
\end{bmatrix}
\] ... (3.23)

\[ f_{act} = k_g k_i \begin{bmatrix}
x_{act} \\
i
\end{bmatrix} - \begin{bmatrix}0 \end{bmatrix} x_{act} - \begin{bmatrix}0 \end{bmatrix} v_{act} \] ... (3.24)

The system inputs are partitioned into actuator extension 'x_{act}' and applied voltage 'v_{act}'. The system output is actuator force 'f_{act}'.

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A force feedback control loop is placed around the actuator as was the case for the electrohydraulic and electromechanical actuators. A PID controller is again used as shown in Figure 3.13.

The actuator parameters are derived in section 3.4 and are dependant upon the force and power requirements placed on the device. The parameters shown in Table 3.4 are typical of an actuator with the force and power capabilities required by a railway active suspension.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of turns</td>
<td>N</td>
<td>477</td>
</tr>
<tr>
<td>Pole Area</td>
<td>a</td>
<td>0.008 (m^2)</td>
</tr>
<tr>
<td>Coil resistance</td>
<td>r</td>
<td>4.214 (Ω)</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>l_m</td>
<td>0.216 (H)</td>
</tr>
<tr>
<td>Self inductance</td>
<td>l_i</td>
<td>0.120 (H)</td>
</tr>
<tr>
<td>Linearised parameter k</td>
<td>k_x</td>
<td>4×10^5 (Nm^-1)</td>
</tr>
<tr>
<td>Linearised parameter k</td>
<td>k_i</td>
<td>107.8 (NAmp^-1)</td>
</tr>
<tr>
<td>Nominal airgap</td>
<td>x_m</td>
<td>0.015 (m)</td>
</tr>
</tbody>
</table>

Table 3.4 - Electromagnetic actuator parameters
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The PID parameters for this actuator were tuned to give acceptable gain and phase margins as shown by the Nichols chart of Figure 3.14. This ensures that the transient response of the actuator will have a fast rise time with minimal overshoot. The bandwidth is clearly very high and hence this actuator performs very well in active suspension applications as will be demonstrated in section 3.5.

![Nichols chart of Figure 3.14](image)

**Figure 3.14 - Electromagnetic actuator controller design**

Validation tests on this model are the same as those applied to the electrohydraulic and electromechanical actuators. Figure 3.15 shows the output force response due to a 'square-wave' force demand of 1000 N applied at 10 Hz.
Figure 3.15 - Electromagnetic actuator force following

Figure 3.16 shows the disturbance rejection of the actuator in response to a 5mm, 10 Hz 'square-wave' disturbance on the actuator extension, the force command in this instance being set to zero. Both plots show acceptable performance. The resultant forces in the disturbance rejection response are very much lower than those encountered with the previous actuator types.

Figure 3.16 - Electromagnetic actuator disturbance rejection
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3.4 - Evaluation of actuator parameters

The parameters used for the three actuator technologies outlined in section 3.3 are based upon typical railway active suspension requirements. These parameters will of course vary with the size of the vehicle and the demands made upon them by the active suspension control law. Typical demands on an actuator would be an r.m.s. force requirement of 1000 N, an r.m.s. actuator extension velocity of 40 mms⁻¹, and an r.m.s. stroke of 5mm. These figures are obviously different for an active lateral as opposed to an active vertical suspension. The general design procedure used in this thesis for an active suspension incorporating actuator dynamics is shown in Figure 3.17.

![Diagram](image)

**Figure 3.17** - General design including actuator dynamics

The procedure involves designing the active suspension based on the assumption that the actuator will have an 'ideal' characteristic; an 'ideal' actuator being one which is capable of generating any demanded force irrespective of the imposed extension of the actuator. The r.m.s. force, stroke velocity, and stroke demands on the actuator are then translated into a set of parameters for an
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actuator capable of supplying these demands.

Each of the actuator technologies will now be considered in turn and a methodology outlined for each technology whereby the parameters may be evaluated given a specification of the requirements imposed on the actuator.

3.4.1 - Electrohydraulic actuator parameter evaluation

Figure 3.18 is a design flow-chart for evaluating the parameters of this particular actuator given a specification of force, stroke velocity, and stroke. The Mathcad® script listed in Appendix I is a useful tool in evaluating these parameters and may be used as a worked example in conjunction with the account that now follows. This Mathcad script operates as a spreadsheet; differing actuator requirements may be entered at the beginning of the script, appropriately sized parameters are returned.

![Figure 3.18 - Electrohydraulic actuator parameter evaluation](image)

A number of basic assumptions are made when sizing this type of actuator. These are not unreasonable assumptions which include: the compressibility of hydraulic oil, the hydraulic...
supply pressure, the maximum spool current, and the spool movement this creates.

The initial r.m.s. specification of force, stroke velocity, and stroke are converted into maximum requirements, a process which assumes these signals are random and have a Gaussian distribution. On this assumption, a large confidence limit can be placed on the maximum values these signals will have, this confidence limit is generally 3 times the r.m.s. values. From the maximum force requirement, the cylinder cross-sectional area may be found assuming that this will be achieved at ¾ supply pressure as shown in Figure 3.18. This cross-sectional area in conjunction with the maximum stroke velocity gives the maximum oil flow into the cylinder. This maximum oil flow may be used together with the maximum spool current and displacement to evaluate the flow gains \( k_{\text{annulus}} \) and \( k_{\text{bore}} \) for the annulus and bore sides of the cylinder. The oil flow characteristic is 'square-root' in nature and shown in the Mathcad script of Appendix I. This characteristic is linearised at the operating point in order to evaluate the parameters required by the linear model developed in section 3.3. The other parameters, for example the swept oil volume, is evaluated from the known cylinder cross-sectional area and the maximum stroke requirements. The reader is referred to Appendix I for a more in depth evaluation of the other parameters.

3.4.2 - Electromechanical actuator parameter evaluation

The evaluation of parameters for the electromechanical actuator model developed in section 3.3 is based on the same r.m.s. force, stroke velocity, and stroke specifications given in section 3.4.1 for the electrohydraulic actuator.

The flowchart of Figure 3.19, and the worked example given in Appendix II, aid the explanation of the sizing methodology used for this particular actuator.
The maximum speed of the motor is not unreasonably assumed to be 3000 r.p.m. From the specification of actuator stroke velocity, the lead screw pitch may be evaluated knowing the maximum motor speed. The motor torque may be evaluated from the known maximum force and the recently derived screw pitch. The rated motor power and peak torque naturally follow from the previous speed and torque evaluations. An appropriate motor may then be selected fulfilling these torque and power requirements. The characteristics of this motor, namely: armature resistance and armature inductance may be found from manufacturer's data and used within the linear model given in section 3.3. For a more in depth account, the reader is referred to Appendix II.

3.4.3 - Electromagnetic actuator parameter evaluation

The parameters of the electromagnetic actuator developed in section 3.3 are again based on a specification of r.m.s. force, stroke velocity, and stroke. The evaluation procedure is given by the flowchart of Figure 3.20, together with a worked example given by the Mathcad script in Appendix III.
The assumptions made in the evaluation are: the maximum coil current, flux density, and airgap. The magneto-motive force 'NI' is evaluated from the maximum stroke and coil current. From the m.m.f., the pole area of the electromagnet may be evaluated knowing the maximum force required from the actuator. Assumptions for the coil current density and packing factor leads to the dimensions of the actuator being found. From these dimensions, the coil resistance, mutual inductance, and self inductance may be found. These parameters are required by the linear model developed in section 3.3. For a more in depth account, the reader is referred to Appendix III, and the work of [Williams - 1986].

3.5 - Performance implications of 'real' actuator dynamics

The effects of 'real' actuator dynamics on the overall performance of an active suspension are quite profound. This section will demonstrate these effects through the simple two-mass model described in Chapter 1. This model is shown in Figure 1.11, and represented by the state-space model given in equations (3.23), and (3.24).
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These may be placed in a more concise structure as given by equations (3.25), and (3.26). This is a general format used throughout this thesis to represent vehicle dynamics. The input is segregated into 'track-based' inputs, and 'actuator' based inputs.

The actuator models developed in section 3.3 need of course to be linked with this vehicle model and the active suspension control law in order to evaluate the overall system performance. The actuator models have the general structure shown in Figure 3.27.
Two inputs are required: the force command, and the differential velocity movements of the attachment points to the vehicle which it is connected to. A single output force is generated by the model.

\[
\dot{X}_{\text{ACT}} = A_{\text{ACT}}X_{\text{ACT}} + B_{\text{ACT}}U_{\text{CUT}} + B_{\text{ACT}}Y_{\text{ACT}} \quad \ldots (3.27)
\]

\[
U_{\text{ACT}} = C_{\text{ACT}}X_{\text{ACT}} + D_{\text{ACT}}U_{\text{CUT}} + D_{\text{ACT}}Y_{\text{ACT}} \quad \ldots (3.28)
\]

The three items: the vehicle dynamic model, the active suspension controller, and the actuator dynamics may be viewed as being linked with the structure shown in Figure 3.28. The active suspension control law provides force commands to the 'real' actuator model, which in turn influences vehicle motions through the output force, the motions of the attachment points at the same time being sent back to the actuator dynamic block.

This is a general arrangement used throughout this thesis. It applies even with more complicated vehicle models having numerous track and actuator inputs.
Chapter 3: Actuator modelling

The example of 'skyhook' damping, together with the two-mass model recently outlined, may be used to demonstrate the impact of 'real' actuators on the overall performance of an active suspension. The control law shown in Figure 3.28 has a general state-space structure as given by equations (3.29) and (3.30).

\[ \dot{X}_C = A_C X_C + B_C Y \]  \hspace{1cm} (3.29)

\[ U_{\text{ACT}} = C_C X_C + D_C Y \]  \hspace{1cm} (3.30)

The skyhook damping control law may readily represented in this state-space form. The output vector 'Y' from the vehicle model contains redundant information: body accelerations, bogie motions, etc., but these are simply ignored by the control law. The actuator dynamic block also utilises information within the output vector 'Y' in order to extract the extension rates across the actuators. This is performed via the simple mapping shown by equation (3.31). The output vector therefore remains the same irrespective of the control law, or the internal actuator dynamics. This
Chapter 3: Actuator modelling

means that a suite of analysis programs may be developed in order to analyse suspension performance given this fixed input-output structure,

\[ Y_{ACT} = K_Y Y \]  \hspace{1cm} \text{...(3.31)}

This methodology applies to more complex models developed later in this thesis. Each of the three actuator technologies: 'electrohydraulic', 'electromechanical', and 'electromagnetic' were tested with identical skyhook damping on the two-mass model. Figure 3.29 compares the power spectral densities of the body accelerations for all three actuator technologies, together with the 'ideal' type of actuator which is not influenced by internal dynamics.

![Body acceleration PSD](image)

**Figure 3.29 - Effect of 'real' actuators on ride quality**

The areas underneath the p.s.d. plots are representative of vehicle ride quality with each of the different types of actuator. The ride quality shows notable deterioration when 'real' actuator dynamics are included. Clearly it is a serious omission to neglect these effects from a study of active suspension.

The various technologies of course fare very differently in the degree by which they influence
ride quality. This is due to the differing bandwidth limitations of each of the technologies. Figure (3.30) to (3.32) show the closed-loop force bandwidth of each of the actuators operating with the active two-mass model. These figures show the ratios of the actuator force outputs between the 'ideal' and 'real' actuator force outputs. The degradation in ride quality shown in Figure (3.29) is due to the inability of the relevant actuators to supply appropriate control effort at higher frequencies.

**Figure 3.30 - Electrohydraulic actuator force response**

**Figure 3.31 - Electromechanical actuator force response**
The ratios are around unity for very low frequencies; the 'real' actuators are able to track these low frequency demands without difficulty. At higher frequencies the 'real' actuators begin to lose control. The active control law does not perform well at these frequencies, resulting in higher body velocities and hence higher actuator force demands; the skyhook damping control law still attempting to apply absolute damping. The actual force output from the 'real' actuators is higher than the 'ideal' case, the ratios shown in Figures (3.30) to (3.32) are between 'ideal' and 'real' and are generally less than unity. Internal actuator resonances may be seen with two of the actuators.

3.6 - Summary

The actuator technologies described within this chapter are subsequently linked to more complex vehicle and train models developed in Chapters 4 and 5. The electromagnetic actuator has a very high bandwidth (50Hz), the other two technologies: electrohydraulic and electromechanical (5Hz) are less able to control higher frequencies. The ability to control the vehicle modes through each of the actuator technologies is also important with the more complex train models and active suspension control laws developed later in this thesis; some of the trends observed in section 3.5 for the simple two-mass model may also be seen with these more complex models.
4.1 - Overview

This chapter will develop software models representing the sideview vertical dynamics of a train. The vertical dynamics are essentially decoupled from the lateral dynamics and although the modelling methods used for both bear similarity, they are dissimilar enough to warrant separate treatment. Chapter 5 develops the lateral train dynamic models. A good model is one which accurately represents the real-life equivalent. As was intimated in the introduction, the dynamic properties of a railway vehicle are complex, possessing non-linearities, and high coupling in certain modes. The models developed in this chapter are all linear and time-invariant. The time-invariant property is not dissimilar from the real-life equivalent; various suspension components do age with time and therefore alter the overall dynamic behaviour, but it is true to say that this variation is small and corrected with regular maintenance. Railway dynamics are non-linear, airspring behaviour, damper blow-offs, and bumpstop contact are all significant non-linearities. Linear models are developed because of the wealth of design and analysis techniques available which are all based upon linear systems theory, and also because railway suspensions operate about a static equilibrium position, and for small perturbations about this point their behaviour can be quite accurately represented with linear models. It is only extreme movements which need non-linear representation. For this reason the models developed in this chapter are all linear, but in subsequent chapters in which active suspension is applied and investigated, further verifications of suspension performance will be carried out with a software package which represents non-linear model dynamics.
Chapter 4: Modelling sideview vertical train dynamics

Figure 4.1 shows a flowchart summarising the procedure used to design and analyse active suspensions throughout this thesis. The iterative nature of suspension design mentioned in the introduction is clearly visible.

Figure 4.1 - Active suspension design flowchart

In-house modelling of train dynamics is performed first, followed by development of linear active suspension control laws. These are then combined and suspension performance is assessed using linear techniques: eigenvalues, power spectral densities, etc. If the performance is inadequate, the active suspension is modified and re-assessed. Once a satisfactory design is achieved, the active suspension control law is tested on the more accurate non-linear model, and as before if the performance is unacceptable the design is re-iterated. Assessment of the non-linear models may only be carried out through time simulation, because all the linear systems techniques such as frequency domain analysis can not be used. Evaluation of the two main criteria - vehicle acceleration levels, and suspension deflections - are readily obtainable from time history simulations.

The in-house development of linear models may initially appear redundant because of the more fundamental assessments which are later made with the more accurate non-linear models. This begs the question as to why we do not simply include all the non-linearities and have a single
Chapter 4: Modelling sideview vertical train dynamics

iterative loop containing non-linear design, non-linear analysis, and re-design. One reason is of course the large foundation linear systems theory provides, but the other reason is due to the lack of 'off the shelf' modelling and design software which fully represents the non-linearities, and is capable of modelling long train lengths, crucial to the focus of this thesis. A probable reason for this is the high dynamic order involved in modelling such a system imposing demanding computational requirements. For this reason the train length models have been modelled as linear time invariant systems in order to get a simple representation of the long train lengths and one which is not too computationally intensive.

At the time of writing a variety of software packages existed to enable the suspension designer to analyse railway vehicle dynamic behaviour, most notably:

- Vampire®
- Voco®
- Nucars®

A good account of these software packages, and their capabilities is given in [Multibody codes - 1993]. These packages are specific to railway dynamics. These software packages can describe vehicle dynamics quite accurately, but in the context of the primary aim of the work performed in this thesis they possess a major drawback: they do not readily permit the analysis of long trains of vehicles. It is possible to implement secondary active suspension control laws, and even an active inter-vehicle control law with these packages; this has indeed been performed in this thesis and is illustrated in Chapters 7 and 8 through the use of Vampire. However, the Vampire analysis is performed for a 3 vehicle train, the longest train length permitted, the motivation for the use of Vampire being its wide acceptance by the railway suspension design community and its accurate description of dynamic elements in the vehicle. The implementation of an active suspension control law in Vampire is performed through an additional library of FORTRAN subroutines, the control law developed therefore needs to be expressed at a low level in terms of FORTRAN code. Subsequent chapters will illustrate that a control law may incorporate several
filters or a high order state-space representation, and implementation of such a controller in FORTRAN code is an arduous task prompting the use of simulation tools that can handle filters and other controller structures at a higher level.

The flowchart of Figure 4.1 needs to be qualified. The more accurate non-linear analysis is only performed for trains containing up to 3 vehicles. In this thesis the software of choice used to perform this task is Vampire. For trains of higher lengths a single iterative loop utilising linear models, designs, and analysis techniques is the method of choice. Vampire is not used for this latter task.

Apart from the specific railway dynamics packages stated previously, other more general modelling, control, and dynamic analysis packages also exist, a few of these are:

- Medyna®
- LMS Cadex®
- ADAMS®
- Matrix X®
- Matlab®

The principle simulation tool used to analyse linear vehicle dynamic behaviour in this thesis is Matlab 4.2c together with its graphical interface: Simulink 1.3a. It was felt that Matlab with its wealth of control and signal processing functions would suit the overall demands of this research project. A major drawback in the choice of Matlab is that much of the modelling needs to be performed by hand, ultimately giving more flexibility but arguably giving rise to the possibility of modelling errors. The approach taken in this thesis is to minimise the potential for errors by thorough verification of the linear models, and contrasting results with those generated by Vampire.

Analysis of long train effects motivates the need for a generic piece of software that generates
Chapter 4: Modelling sideview vertical train dynamics

a model of an 'n' vehicle train as shown in Figure 4.2. Several researchers have investigated the dynamics and built up models for short train lengths. The focus of this thesis is the investigation of active suspension applied to long train lengths; it was therefore prudent from the outset to develop quite lengthy train models. The possibility of secondary suspension and inter-vehicle actuators also needs to be included but is not shown in this figure purely for simplicity. Allied to this demand for long train lengths is the need to have flexibility in the modelling; it may be necessary for instance to investigate the difference between having an electrohydraulic actuator as opposed to an electromechanical one between the vehicles. It may also be interesting to incorporate an active secondary suspension at the same time, and to incorporate body structural modes into the model.

![Figure 4.2 - Generic 'n' vehicle train](image)

It is intuitively obvious that vehicles a long distance away from a vehicle under consideration will have a diminishing effect on its behaviour. The exact nature of this effect depends upon the type of interconnection, therefore it is important to model the entire train and not simply assume behaviour from a short train length.
From the outset it was envisaged that a piece of software would be required that would permit generic changes to any of the following items:

- The dynamics of an individual vehicle (i.e. changes in masses, spring/damper rates)
- The dynamics of any type of secondary suspension actuator (i.e. electrohydraulic, etc.)
- The dynamics of any type of inter-vehicle actuator

These three components form the library of building blocks which are used to construct a train length. The procedure adopted is to connect repetitively items from this standard library and form a train of the required length. The drawback in using this procedure is the fact that all of the vehicles are identical in nature, whereas in some instances (for example when considering locomotives at the ends of a trainset) it is important to have vehicles with different dynamical properties. One could also argue this point for the actuators in the model, for example it may be desirable to have different sizes or technologies of actuators on different vehicles. Many of these issues are however secondary: our primary aim here is to form models of the train and later use these to investigate the potential for train-wide active suspension control and to assess the novel idea of having active elements between the vehicles in a train.

Matlab® possesses facilities for connecting a system of dynamic sub-blocks into an overall system. Intrinsic Matlab functions 'mvblk.m' and 'mvcon.m' respectively unify the sub-blocks and then connect them together. The principal sub-blocks in the construction of this model are the choice of: 'secondary vertical actuator', 'inter-vehicle vertical actuator', and the 'single vehicle vertical dynamics'. Figure 4.3 is a schematic illustrating how these sub-blocks interact, and how they may be connected together to form the full train model.

The actuator technologies considered most effective in active suspension were outlined in Chapter 3. Each of the three technologies: 'electrohydraulic', 'electromechanical', and 'electromagnetic', will have different sizes and ratings according to their use in either the secondary or inter-vehicle suspension, and will also vary with vehicle size and the demands
made upon them by the active suspension control law. The procedure used is to make an initial estimate of actuator force, extension rate, and stroke based upon the performance of a particular control law when using an 'ideal' actuator. (An 'ideal' actuator is one which develops any force required of it, irrespective of external influence. It is therefore an essential part of the actuator library when constructing the train.)

Figure 4.3 shows the signal flows between the various sub-blocks. These signals are vectors of information. The information passed between blocks is in the form of forces, displacements, and velocities, etc. For example, an inter-connector situated between two vehicles, say between vehicle 'm' and 'm-1', requires the displacements and velocities of the attachment points to both these vehicles, and from these a force may be derived depending on the values of inter-vehicle stiffness and damping. In the case of an inter-vehicle actuator, a drive signal is required for this block, and the force developed by the actuator may be a dynamic function of the displacements and velocities at the attachment points. This is possible with the structure shown in Figure 4.3 in which the sub-blocks may contain their own internal dynamics. More detail about the signal flows and a more in depth examination of Figure 4.3 follows later in this chapter. First a dynamic model of a single vehicle will be developed. A slightly more complex model which incorporates body flexibility will also be developed, these two models may easily be swapped within the structure shown in Figure 4.3 giving this piece of software its generic nature. Afterwards an explanation of how the actuator technologies developed in Chapter 3 may be linked in to the structure shown in Figure 4.3 will be given. There are obviously differences in the way an inter-vehicle actuator as opposed to a secondary actuator links in to this structure. The differences are, however, only in the protocol of the signal flow; the internal actuator structure remains the same in both cases.
Figure 4.3 - Schematic structure of a railway train model
4.2 - Single vehicle model

The sideview structure of a single vehicle is shown in Figure 4.4. The model has eight degrees of freedom, two associated with the body's bounce and pitch motions, four associated with the bounce and pitch motions of the leading and trailing bogies, and two associated with the internal dynamics of the airspring. All the masses are considered rigid in the first instance. The external forces and components influencing motion are all shown. The vehicle body is influenced by the forces generated by the passive secondary suspension components: $k_{z1}$, $k_{z2}$, $k_{z3}$, $c_{rz}$, and by the active forces $f_{m1z}$ and $f_{m2z}$ if an active suspension is installed. Inter-vehicle forces $f_{bm1z}$ and $f_{bm2z}$ also influence the body motion. The bogie motions are obviously influenced by forces from both the secondary and passive suspension as well as the reaction of any active secondary forces $f_{im1z}$ and $f_{im2z}$ which may be present. Primary suspension forces are generated because of the disturbances in the track. Modelling such a system is simply a matter of looking at each mass in isolation and balancing the inertial forces with the external forces, according to d'Alembert's principle, and thus creating an equation of motion for that particular mass.

![Figure 4.4 - Sideview of a single vehicle](image-url)
Chapter 4: Modelling sideview vertical train dynamics

The model shown in Figure 4.5 is drawn from the work of [Williams - 1986], and is widely considered to be an acceptable representation of the airspring dynamics. It is important to note the internal mass \( m_{mp} \), which is not shown in Figure 4.4, this mass is small and is included only for modelling convenience. This is a linear model; a more accurate model would contain a non-linearity due to the nature of airflows within the airspring, but the one used is an accurate representation for small suspension motions. The values of the sideview parameters for a typical high-speed train with an airspring suspension are given in Table 4.1. It is important to note that the values given in Table 4.1 are specific to the sideview vertical model. For example, looking from the sideview there are in fact two airsprings per bogie. Airspring parameters therefore need duplication in the sideview.
Chapter 4: Modelling sideview vertical train dynamics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body mass</td>
<td>( m_b )</td>
<td>38000 (kg)</td>
</tr>
<tr>
<td>Truck/bogie mass</td>
<td>( m_t )</td>
<td>2500 (kg)</td>
</tr>
<tr>
<td>Airspring midpoint mass</td>
<td>( m_{mp} )</td>
<td>5 (kg)</td>
</tr>
<tr>
<td>Body pitch inertia</td>
<td>( i_{bp} )</td>
<td>2310000 (kg)</td>
</tr>
<tr>
<td>Bogie pitch inertia</td>
<td>( i_{p} )</td>
<td>2000 (kgm²)</td>
</tr>
<tr>
<td>Airspring stiffness</td>
<td>( k_{sz} )</td>
<td>508000 (Nm⁻¹)</td>
</tr>
<tr>
<td>Change of area stiffness</td>
<td>( k_{az} )</td>
<td>1 (Nm⁻¹)</td>
</tr>
<tr>
<td>Reservoir stiffness</td>
<td>( k_{rz} )</td>
<td>254000 (Nm⁻¹)</td>
</tr>
<tr>
<td>Primary stiffness</td>
<td>( k_{pz} )</td>
<td>2500000 (Nm⁻¹)</td>
</tr>
<tr>
<td>Secondary damping</td>
<td>( c_{az} )</td>
<td>30000 (Nsm⁻¹)</td>
</tr>
<tr>
<td>Primary damping</td>
<td>( c_{pz} )</td>
<td>17900 (Nsm⁻¹)</td>
</tr>
<tr>
<td>Bogie-bogie spacing</td>
<td>( l_{b} )</td>
<td>19 (m)</td>
</tr>
<tr>
<td>Semi bogie-bogie spacing</td>
<td>( l_{t} )</td>
<td>9.5 (m)</td>
</tr>
<tr>
<td>Wheel - wheel spacing</td>
<td>( l_{w} )</td>
<td>2.5 (m)</td>
</tr>
<tr>
<td>Semi wheel-wheel spacing</td>
<td>( l_{s} )</td>
<td>1.25 (m)</td>
</tr>
<tr>
<td>Vehicle+gangway length</td>
<td>( l_{v} )</td>
<td>27 (m)</td>
</tr>
<tr>
<td>Semi vehicle-vehicle spacing</td>
<td>( l_{hv} )</td>
<td>13.5 (m)</td>
</tr>
</tbody>
</table>

Table 4.1 - Sideview vertical model parameters

Listings of the entire software code are given in Appendix IV, the reader is to referred to these for component values specific to the sideview vertical model, for example, '\( k_{az} \)' in Table 4.1 needs to be doubled with the sideview model. The approach to holding individual component values in software is a sensible one, it will become apparent throughout this thesis that all modelling programs access the same parameter file and the onus is then placed on these
modelling programs to adjust parameters according to their own needs. The variables $z_b$ and $z_i$ are used in the same context as they were used for the two-mass model in Figure 1.11, the equation developed for the airspring may then be adapted for the sideview model with airspring attachments above the leading and trailing bogies. The expression between $z_b$, $z_{mp}$ and $z_i$ is given in equation (4.1).

\[
\ddot{z}_{wp} - \frac{c_{rz}}{m_{mp}} \dot{z}_{i} - \frac{k_{rz}}{m_{mp}} z_{i} - \frac{c_{rz}}{m_{mp}} \dot{z}_{wp} - \frac{k_{rz}}{m_{mp}} z_{wp} - \frac{k_{sz}}{m_{mp}} z_{b} \rightarrow (4.1)
\]

Figure 4.5 - Airspring model

The equations of motion may now developed and in conjunction with the expression given by equation (4.1) we may develop a model of the system. The body bounce degree of freedom is expressed in equation (4.2) by considering the vehicle body in isolation and balancing suspension and inertial forces.
Chapter 4: Modelling sideview vertical train dynamics

\[ m_b \ddot{z}_{bw} = k_{st} z_{amp1} + k_{st} z_{om1} + k_{st} \dot{z}_{om1} + k_{st} \ddot{z}_{om1} - \cdots \]
\[ 2(k_m + k_n) z_{bw} + f_{s\pi} + f_{s}\dot{z}_{om1} - f_{\dot{s}\pi} \]

... (4.2)

The body pitch degree of freedom is expressed in equation (4.3).

\[ i_b \dot{\phi}_{bw} = k_{st} z_{om1} + k_{st} l\dot{z}_{om1} - k_{st} l\ddot{z}_{om1} - k_{st} l\dot{z}_{om1} - 2(k_m + k_n) l^2 \dot{\phi}_{bw} - \cdots \]
\[ l\dot{f}_{s\pi} + l f_{\dot{s}\pi} + l f_{\ddot{s}\pi} + l f_{s\ddot{z}_{om1}} \]

... (4.3)

Similarly equations (4.4) through to (4.7) are expressions for the bounce and pitch degrees of freedom of the leading and trailing bogies.

\[ m_{f\pi} \ddot{z}_{om1} = -k_{st} z_{om1} - k_{st} \dot{z}_{om1} - c_{st} \dot{z}_{om1} + k_{st} \ddot{z}_{om1} + k_{st} \dot{z}_{om1} + k_{st} \dot{z}_{om1} - \cdots \]
\[ k_{st} z_{om1} - c_{st} \dot{z}_{om1} - k_{st} \ddot{z}_{om1} + k_{st} l \dot{\phi}_{bw} - \cdots \]

... (4.4)

\[ i_{f\pi} \dot{\phi}_{om1} = -2l^2 k_p \dot{\phi}_{om1} + 2l^2 c_p \dot{\phi}_{om1} + l u k_p \dot{z}_{om1} - l u c_p \ddot{z}_{om1} - l u c_p \dot{z}_{om1} - \cdots \]

... (4.5)

\[ m_{f\pi} \ddot{z}_{om2} = -k_{st} z_{om2} - k_{st} \dot{z}_{om2} - c_{st} \dot{z}_{om2} + k_{st} \ddot{z}_{om2} + k_{st} \dot{z}_{om2} + k_{st} \dot{z}_{om2} - \cdots \]
\[ k_{st} z_{om2} - c_{st} \dot{z}_{om2} - k_{st} \ddot{z}_{om2} + k_{st} l \dot{\phi}_{bw} - \cdots \]

... (4.6)

\[ i_{f\pi} \dot{\phi}_{om2} = -2l^2 k_p \dot{\phi}_{om2} + 2l^2 c_p \dot{\phi}_{om2} + l u k_p \dot{z}_{om2} - l u c_p \ddot{z}_{om2} - l u c_p \dot{z}_{om2} - \cdots \]

... (4.7)

Equation (4.1) which gave a general expression for the internal dynamics of the airspring may now be adapted to fit within the framework of the sideview vertical model, giving rise to equations (4.8) and (4.9) which apply to the leading and trailing airsprings respectively.

\[ \ddot{z}_{amp1} = \frac{c_{st}}{m_{amp}} \dot{z}_{om1} - \frac{k_{st}}{m_{amp}} z_{om1} + \frac{k_{st}}{m_{amp}} \dot{z}_{om1} - \frac{k_{st}}{m_{amp}} \ddot{z}_{om1} + \frac{k_{st}}{m_{amp}} z_{bw} + \frac{k_{st}}{m_{amp}} l \dot{\phi}_{bw} \]

... (4.8)

\[ \ddot{z}_{amp2} = \frac{c_{st}}{m_{amp}} \dot{z}_{om2} - \frac{k_{st}}{m_{amp}} z_{om2} + \frac{k_{st}}{m_{amp}} \dot{z}_{om2} - \frac{k_{st}}{m_{amp}} \ddot{z}_{om2} + \frac{k_{st}}{m_{amp}} z_{bw} + \frac{k_{st}}{m_{amp}} l \dot{\phi}_{bw} \]

... (4.9)
Chapter 4: Modelling sideview vertical train dynamics

The airspring model midpoint mass $m_{mp}$ is considered to be very small with respect to the rest of the system. Ideally it should be set to zero but this leads to a breakdown of this modelling procedure. The introduction of such a small mass can also lead to simulation problems due to the imbalance in the mixture of the modes present in the system. There are ways to circumvent these problems by rewriting equation (4.1) and moving from there directly to a state-space model.

The 8 equations (4.2) through to (4.9) fully describe the system shown in Figure 4.4. They can be rearranged into a more concise matrix form as given by equation (4.10).

$$MX_{2}^{\cdot} + CX_{2}^{\cdot} + KX_{2}^{\cdot} - F_{ACTZ} U_{ACTZ} + F_{TRACKZ} \dot{Z}_{TRACK} + F_{TRACKZ} Z_{TRACK} \quad \ldots \quad (4.10)$$

This is a standard format for modelling dynamics of mechanical systems and is slightly modified to adapt it to the specific needs of this chapter. It equates forces, the main difference lying in the r.h.s of the equation which contains the external influences on the system. These external influences are separated into those caused by track motions and those caused by active secondary and inter-vehicle forces. This separation is beneficial in our later development of active suspensions. The expanded forms of the input vectors $U_{ACTZ}$ and $Z_{TRACK}$ are given by equations

$$U_{ACT} = \begin{bmatrix} f_{av1} \\ f_{av2} \\ f_{av12} \\ f_{av22} \end{bmatrix} \quad \ldots \quad (4.11)$$

$$Z_{TRACK} = \begin{bmatrix} z_{wm11} \\ z_{wm13} \\ z_{wm21} \\ z_{wm23} \end{bmatrix} \quad \ldots \quad (4.12)$$

(4.11) and (4.12). The $Z_{TRACK}$ vector shown in equation (4.10) is split into a direct and a derivative form, which is necessary when the input is connected straight to a damper, as is the
case here between the wheelset inputs and the bogies.

The $X_z$ vector shown in equation (4.10) is given by equation (4.13).

\[ X_z = \begin{bmatrix} z_{lm} \\ z_{amp1} \\ z_{im1} \\ z_{amp2} \\ z_{im2} \\ \phi_{lm} \\ \phi_{im1} \\ \phi_{im2} \end{bmatrix} \quad \ldots (4.13) \]

The mass, damper, and stiffness matrices $M$, $C$, and $K$ may be written in a more concise form based upon equations (4.2) to (4.9), these matrices are given by equations (4.14) to (4.16) respectively. There are 8 degrees of freedom in this modelling procedure, but the use of the airspring midpoint mass may be eliminated as was mentioned before giving some redundancy in the model. However, this is not a significant problem.

\[ M = \begin{bmatrix} m_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{mp} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{mp} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & i_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i_i \end{bmatrix} \quad \ldots (4.14) \]
Chapter 4: Modelling sideview vertical train dynamics

\[ C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{rs} & -c_{rs} & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_{rs} & (c_{rs} - 2c_{pr}) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{rs} & -c_{rs} & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_{rs} & (c_{rs} - 2c_{pr}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

... (4.15)

\[ K = \begin{bmatrix} 2(k_{as} - k_{as}) & -k_{as} & -k_{as} & -k_{as} & -k_{as} & 0 & 0 & 0 \\ -k_{as} & (k_{as} - k_{as}) & -k_{as} & 0 & 0 & -l_t k_{as} & 0 & 0 \\ -k_{as} & -k_{as} & (k_{as} - k_{as} - 2k_{pr}) & 0 & 0 & -l_t k_{as} & 0 & 0 \\ -k_{as} & 0 & 0 & (k_{as} - k_{as}) & -k_{as} & l_t k_{as} & 0 & 0 \\ -k_{as} & 0 & 0 & -k_{as} & (k_{as} - k_{as} - 2k_{pr}) & l_t k_{as} & 0 & 0 \\ 0 & -l_t k_{as} & -l_t k_{as} & l_t k_{as} & l_t k_{as} & 2(k_{as} - k_{as}) l_t^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

... (4.16)

The three input matrices $F_{ACTZ}$, $\hat{F}_{TRACKZ}$, and $F_{TRACKZ}$, are given below by equations (4.17) to (4.19) respectively.

\[ F_{ACTZ} = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

... (4.17)

The necessity to have two vectors representing the track input will eventually be eliminated when the model is placed in a state-space format.
Equation (4.10) may be modified to represent the model in a state-space format with a single vector representing the track input disturbance as shown in equations (4.20) and (4.21).

$$
\begin{bmatrix}
X_z \\
\dot{X}_z \\
\ddot{Z}_{\text{track}}
\end{bmatrix}
= \begin{bmatrix}
0 & I & 0 \\
-M^{-1}K & -M^{-1}C & M^{-1}F_{\text{track}} \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
X_z \\
\dot{X}_z \\
\ddot{Z}_{\text{track}}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
-M^{-1}F_{\text{actz}} \\
0
\end{bmatrix} U_{\text{actz}}
+ \begin{bmatrix}
0 \\
0 \\
I
\end{bmatrix} \ddot{Z}_{\text{track}}
\ldots (4.20)
$$

$$
\begin{bmatrix}
X_z \\
\dot{X}_z \\
\ddot{Z}_{\text{track}}
\end{bmatrix}
= \begin{bmatrix}
I & 0 & 0 \\
0 & I & 0 \\
-M^{-1}K & -M^{-1}C & M^{-1}F_{\text{track}}
\end{bmatrix}
\begin{bmatrix}
X_z \\
\dot{X}_z \\
\ddot{Z}_{\text{track}}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
-M^{-1}F_{\text{actz}}
\end{bmatrix} U_{\text{actz}}
+ \begin{bmatrix}
0 \\
0 \\
-M^{-1}F_{\text{track}}
\end{bmatrix} \ddot{Z}_{\text{track}}
\ldots (4.21)
$$
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Equation (4.20) is the state equation, and equation (4.21) is the system output equation. They can be written in a more compact form as given by equations (4.22) and (4.23).

\[
\dot{X}_v = AX_v + BU_{ACT} + W\dot{Z}_{TRACK} \quad \ldots (4.22)
\]

\[
Y_v = CX_v + D_B U_{ACT} + D_{W}\dot{Z}_{TRACK} \quad \ldots (4.23)
\]

This is a general state-space representation of the dynamics of the single vehicle shown in Figure 4.4. It has two input vectors, one from the track disturbances, the other from the actuator forces which then provides a mechanism whereby active suspension control laws may be implemented. The track input vector is in a derivative form owing to the presence of primary suspension dampers. The output vector contains displacements, velocities, and accelerations which will be useful in our later assessment of the performance of the suspension. This same structure is used to represent the dynamics of a single vehicle as a sub-block within the structure of Figure 4.3 which is used to generate the full train models. The full train model itself has the state-space structure given by equations (4.22) and (4.23). A similar approach is used with the lateral models which are developed in Chapter 5.

As was stated at the beginning of this chapter, model verification is important due to all the system modelling being performed 'in house.' Two tests are performed to validate the model, these are:

- Mode inspection
- Transient response tests

The first, 'mode inspection' simply involves evaluating the eigenvalues of the vehicle and inspecting their frequency and damping. Table 4.2 shows the modes present within the single vehicle sideview vertical model using the parameters listed in Table 4.1, and it was confirmed
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that these bear close similarity to widely accepted industry norms. Note that the airspring mode is an artificial mode, the airspring mass must be kept low, but its frequency must be kept low enough to avoid simulation difficulties. A mass of 5kg in this instance is negligible in comparison with the bogie and body mass and results in a 63 Hz mode, this frequency permits acceptable simulation times without affecting the validity of the model.

The second test involves injecting inputs into the model and investigating the resulting motions; in particular the steady-state values of certain variables. The model responses obviously need to parallel what we would expect from the full-scale equivalent. Six tests are performed on this model and are shown in Figures 4.6 to 4.11.

The first test shown in Figure 4.6 shows the response of the leading bogie $z_{wm1}$ to a unit step input on the leading wheelset $z_{wm1}$. It is apparent that the transient dies off very quickly illustrating the well damped higher frequency bogie mode. The bogie bounce settles at $\frac{1}{2}$ p.u, this is due to the bogie behaving as a semi-divider due to its wheelset $z_{wm12}$ remaining in the same position. Figure 4.7 shows the body bounce response $z_{bm}$ to the same input. This shows the lower frequency and damping of the body mode. It also shows the body bounce displacement settling at $\frac{1}{4}$ p.u. This is due to both the body and the bogie behaving as semi-dividers in response to the unit step on the leading wheelset. This geometric property has significance in our later assessment of vehicle ride quality.
Chapter 4: Modelling sideview vertical train dynamics

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body bounce</td>
<td>0.71</td>
<td>15.8</td>
</tr>
<tr>
<td>Body pitch</td>
<td>0.84</td>
<td>19.1</td>
</tr>
<tr>
<td>Lead bogie bounce</td>
<td>10.58</td>
<td>22.7</td>
</tr>
<tr>
<td>Lead bogie pitch</td>
<td>14.06</td>
<td>31.6</td>
</tr>
<tr>
<td>Trail bogie bounce</td>
<td>10.58</td>
<td>22.7</td>
</tr>
<tr>
<td>Trail bogie pitch</td>
<td>14.06</td>
<td>31.6</td>
</tr>
<tr>
<td>Lead airspring</td>
<td>62</td>
<td>100</td>
</tr>
<tr>
<td>Trail airspring</td>
<td>62</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 4.2 - Single vehicle dynamic modes

Figures 4.8 and 4.9 show the response to a 1 ton step on the secondary actuator force \( f_{m2} \). The body bounce response is shown in Figure 4.8. This movement is logical since the lead bogie will have no steady-state movement and so the actuator force is reacted by displacement of the passive secondary suspension resulting in body motion. The body pitch as shown in Figure 4.9 would therefore also be expected to be positive.

Figures 4.10 and 4.11 show the response to a 1 ton inter-vehicle force input on \( f_{m2z} \). The body bounce shown in Figure 4.10 will obviously be negative, the reaction force provided by both the secondary and primary suspension forces. Figure 4.11 shows the bounce response of the trailing bogie in response to this change in primary suspension force.

Further validation tests may be performed on the model to give full confidence in its accuracy. The program may be generated in the state-space format given by equations (4.22) and (4.23) through the Matlab program ‘rcarmek.m’ listed in Appendix IV.
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Figure 4.6 - Lead bogie bounce

Figure 4.7 - Body bounce

Figure 4.8 - Body bounce

Figure 4.9 - Body pitch

Figure 4.10 - Body bounce

Figure 4.11 - Bogie bounce
Chapter 4: Modelling side view vertical train dynamics

4.3 - Single vehicle model with body flexibility

Body flexibility can have a significant impact on vehicle ride quality, a problem which has been exacerbated by trends in the industry to reduce vehicle weight and thereby energy costs for the train operator. Figure 4.12 shows the first symmetric bending mode, the dominant mode of vibration, and consequently the one we are most concerned with for ride assessment.

![Figure 4.12 - Simulation Model](image)

This section will develop a method for incorporating equations describing the first symmetric bending mode into the rigid body model which was developed in section 4.2. The bending mode is excited by forces from the secondary suspension and inter-vehicle connection. In the context of the work performed in this thesis it is therefore crucial to perform an assessment of its effects. The model developed here is interchangeable with the rigid body model with respect to the construction of the overall train model as shown in Figure 4.3 and a software listing ‘rcarflxv.m’ is given in Appendix IV.

As an approximation the body of the vehicle may be considered to be a solid beam. It is not difficult, using Bernoulli-Euler beam theory, to show by separating the beam into an infinite
number of small masses, that a partial differential equation (4.24) may be established to describe its motion. In the equation, 'x' is the position along the beam, 'z' is the vertical motion away from a point of equilibrium, 'E' is the Young's modulus, 'A_c' is the beam cross-sectional area, 'I' is the moment of inertia, 'ρ' is the material density, and 't' is of course time.

\[
\frac{\partial^2 z}{\partial t^2} + \frac{EI}{A_c \rho} \frac{\partial^4 z}{\partial x^4} = 0 \quad \ldots (4.24)
\]

The standard method to solve this equation is to establish functions which describe the spatial shape of the vibration mode, and functions which describe the beam's temporal behaviour. The overall solution is a combination of these two functions as shown in equation (4.25).

\[
z = Z(x)T(t) \quad \ldots (4.25)
\]

The spatial description is therefore given by equation (4.26).

\[
\frac{d^4 z}{dx^4} + \frac{A_c \rho}{EI} z = 0 \quad \ldots (4.26)
\]

The temporal description is given by equation (4.27).

\[
\frac{d^2 T}{dt^2} + \frac{EI}{A_c \rho} T = 0 \quad \ldots (4.27)
\]

The first symmetric mode solution to equation (4.26) with free ends is given by equation (4.28).

\[
z(x) = \cosh(\lambda x) \cdot \cos(\lambda x) - \frac{\cosh(\lambda l) - \cos(\lambda l)}{\sinh(\lambda l) - \sin(\lambda l)} (\sinh(\lambda x) + \sin(\lambda x)) \quad \ldots (4.28)
\]

The solution to the temporal description of equation (4.27) is given by equation (4.29).

\[
T(t) = A \cos(\omega t) + B \sin(\omega t) \quad \ldots (4.29)
\]

'lv' is the beam length, 'ω' and 'λ' are given by equations (4.30) and (4.31) respectively.
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\[
\omega = \sqrt{\frac{EI}{A_c \rho}} \quad \ldots (4.30)
\]

\[
\lambda^t = \frac{A_c \rho}{EI} \quad \ldots (4.31)
\]

The amplitude of the oscillation and consequently 'A' and 'B' would have to be evaluated through other boundary conditions. For the purposes of introducing the flexible mode into the rigid body model of section 4.2 we can use the idea of a receptance function [Capitani and Tinbald - 1988] which allows forcing functions to be used to excite the flexible mode described by equation (4.24). A receptance function for this mode is shown in equation (4.32).

\[
\alpha(s) = \frac{z(x_c)z(x_p)}{M(s^2 + 2\zeta\omega s + \omega^2)} \quad \ldots (4.32)
\]

This is a more convenient description of the bending mode, for it is not easy to quantify quantities such as the density, and Young's modulus of a railway vehicle, whereas in practice we do know the frequency, damping, and amplitude of the mode. Equation (4.32) may be expressed in a more appropriate state-space form, equations (4.33) and (4.34), and then incorporated in the state-space model generated in section 4.2.

\[
\begin{bmatrix}
  \dot{x}_r \\
  \ddot{x}_r
\end{bmatrix} =
\begin{bmatrix}
  0 & 1 \\
  -\omega^2 & -2\zeta\omega
\end{bmatrix}
\begin{bmatrix}
  x_r \\
  \ddot{x}_r
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  \frac{z(x_c)}{M}
\end{bmatrix} [f] \quad \ldots (4.33)
\]

\[
\begin{bmatrix}
  y_r \\
  \dot{y}_r
\end{bmatrix} =
\begin{bmatrix}
  z(x_c) & 0 \\
  0 & z(x_p)
\end{bmatrix}
\begin{bmatrix}
  x_r \\
  \ddot{x}_r
\end{bmatrix} \quad \ldots (4.34)
\]

Equations (4.33) and (4.34) give the displacement '\(y_r\)' given a bending force 'f' at the appropriate positions along the beam. This can be incorporated within the rigid body model since there are
four points where force is transmitted to the vehicle body, a pair at the vehicle ends, and a pair at the secondary suspension attachment points. These displacements are then superimposed on the displacement generated because of the rigid modes.

Validation tests were performed on the flexible vehicle model. The transient tests performed for the rigid body model shown in Figures 4.6 to 4.11 were also performed on the flexible model. They all resulted as expected in similar steady-state responses. The only difference between the two is the addition of the flexible mode with a frequency of 8.43 Hz and 5% damping.

The model is generated in the same state-space format given by equations (4.22) and (4.23). The input and output signals within these vectors are given by equation (4.35). This is a general format for vehicle 'm', and applies to both the rigid and flexible models. The input variables are the wheelset motions, the inter-vehicle forces, and the secondary active forces. The output signals are bounce and pitch accelerations, end-movements of the vehicle, and movements across the secondary suspension. It will become apparent that this is a prudent choice of signals for both the analysis of suspensions and the inter-connection of vehicles into a train model.

The flexible model may be generated by the Matlab program 'rcarflxv.m' in Appendix IV.
4.4 - Generation of sideview train model

The building blocks required to generate the sideview train model shown in Figure 4.3 have now been established. In chapter 3, a variety of actuator technologies were investigated and models were developed which had the interface structure shown in Figure 3.19. The force developed by the actuator is a dynamic function of the force command, and the velocity across its end. The single vehicle models derived in sections 4.2 and 4.3 had an interface structure which naturally caters for the inter-connection of these vehicles both to actuators, and to other vehicles, and also provides signals which are useful in the analysis of suspension performance.

This section gives a detailed explanation of the signal flows involved in Figure 4.3, and how through the use of Matlab's block building facility [Matlab manual - 1992] a full train model may be generated. This is a useful facility allowing state-space models, transfer-functions, and pure gains to be integrated and formed into a single model. The main difficulty in the use of this facility is the need to have a predetermined protocol for signal flow around Figure 4.3. A number of the blocks shown in this figure may initially appear confusing, but the sole purpose of many of these blocks is to format signals in order to establish the interface between blocks. This section will consider the signals for an arbitrary vehicle 'm' and its connection to vehicle 'm+1' in a train of length 'n'. Figure 4.3 refers to some vehicles as being 'odd' or 'even', vehicle 'm' in this instance is considered even although this is an arbitrary convention. The terminology 'odd/even' is again due to the nature of the inter-connection. The following blocks are local to vehicle 'm'.

- Pre-format block m1
- Pre-format block m2
- Railway vehicle m
- Secondary actuator m
- Inter-vehicle actuator/connection m
- Post-format block m
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The functionality of each sub-block is described in sections 4.4.1 to 4.4.5.

4.4.1 - Pre-format blocks 'm1'

The input block in Figure 4.3 is simply a unity gain matrix, which is required as an input interface by the Matlab programs 'mvcon' and 'mvblk' which are used to connect multivariable dynamic blocks together. The system input vector 'V_{zi}' contains all the vertical wheelset velocities, and actuator forces in a stacked form as shown in equation (4.36). These inputs are repeated from vehicle 1 through to 'n'. The function of the pre-format block m1 is to extract the signals required for vehicle 'm' thus creating the mapping shown in equation (4.36) and producing vector 'V_{za}' in this instance.

\[ V_{zi} = \begin{bmatrix} z_{w11} \\ z_{w12} \\ z_{w13} \\ \vdots \\ f_{a11} \\ f_{a12} \\ \vdots \\ f_{c1m1}/f_{c2m1}/2_{wm2} \end{bmatrix} \quad \rightarrow \quad V_{za} = \begin{bmatrix} z_{wm11} \\ z_{wm12} \\ z_{wm13} \\ \vdots \\ f_{cmb}/0 \\ 0 \\ 0 \end{bmatrix} \quad \ldots (4.36) \]

The inputs to the secondary actuators 'f_{c1m2}' and 'f_{c2m2}' are command forces and are ignored in this mapping. These signals are sent through pre-format block m2 to the secondary actuator dynamics block, and the output forces from this block are then sent on to influence the vehicle. The force command force 'f_{cmb}' is passed through to vector 'V_{za}' but this is fed straight through the vehicle dynamic block and enters the inter-vehicle actuator dynamic block via vector 'V_{zb}'. In this way the dynamics of the inter-vehicle connector are thereby taken into account, and the force generated thereby influences the vehicle. This may appear a rather obscure method but given the topology of Figure 4.3 it is the one deemed most suitable to this structure. It is important to
appreciate the differences between the command forces \( f_{cm1z} \), \( f_{cm2z} \), \( f_{cbnz} \) and the forces generated after being processed by the internal actuator dynamics.

### 4.4.2 - Pre-format blocks 'm2'

The function of this block is to select the secondary actuator command forces from the input vector \( V_{zi} \) as shown by equation (4.37).

\[
V_{zi} = \begin{bmatrix}
\dot{z}_{umin1} \\
\dot{z}_{umin2} \\
\dot{z}_{umin3} \\
f_{cm1z} \\
f_{cm2z} \\
f_{cbnz} \\
\vdots \\
f_{cm1z}/f_{cm2z} \\
f_{cbnz}/\dot{z}_{umin22}
\end{bmatrix}
\quad \rightarrow \quad
V_{zi} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\vdots \\
0 \\
0 \\
0
\end{bmatrix}
\quad \& \quad
V_{zi} = \begin{bmatrix}
f_{cm1z}/0 \\
f_{cm2z}/0 \\
0 \\
0 \\
0 \\
0 \\
\vdots \\
0 \\
0 \\
0
\end{bmatrix}
\quad \ldots (4.37)
\]

As stated in section 4.4.1 vector \( V_{zi} \) contains the wheelset vertical velocities and actuator
command forces for all the vehicles in the train. The function of pre-format block m2 is to select
the secondary actuator command forces which are intended for vehicle 'm'. The mapping of this
block is shown in equation (4.37). It is immediately apparent that the output vector does not only
contain the signals 'f_{cm1}' and 'f_{cm2}' which are of interest, but also a large number of zeros. The
presence of these zeros is to ensure compatibility in the addition of vectors 'V_{z1}', and 'V_{z9}'. This
other vector contains secondary suspension deflection information which is also required by the
secondary actuator dynamics block. The contents of vector 'V_{z9}' are given in section 4.4.3 where
the output of the vehicle dynamic block is derived.

4.4.3 - Railway vehicle blocks 'm'

Equations (4.22) and (4.23) are a state-space description of the dynamics of a single vehicle. The
input and output vectors given by equation (4.35) need slight adaptation to ensure the correct
format is used when connecting the vehicles to the full train model with the structure assumed
in Figure 4.3.

The inputs and output vectors presented to the 'railway vehicle m' block are given by equation
(4.38). The input vector contains the vertical velocities of the wheelsets, the external forces after
being processed by the actuator dynamics, and the inter-vehicle command vector 'f_{cm1}' which
needs to be fed straight through the railway vehicle block to the output. The output vector
contains displacements, velocities, and commands which are useful in the connection to other
blocks, and from which the system performance may be comprehended. The large number of
zeros present are again there to ensure compatibility in the addition of vectors. The reasoning
behind having a different output for odd and even vehicles is again associated with the protocol
used to connect the system together. Considering for example if vehicle 'm' is the second vehicle
(and hence even): the output vector 'V_{z9}' would pass all its output information on the top portion
of the output vector; conversely if it were an odd vehicle it would pass information on the bottom
portion. The system is constructed in this manner to ensure that the outputs of adjacent vehicles
may be added together, the resultant vector will then contain information about two vehicles,
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information that is required by the inter-connector sub-blocks. Clearly the state-space model of equations (4.22) and (4.23) needs some dissection to provide a direct feed-through path for \( f_{\text{chem}} \), and the reader is referred to the program listings in Appendix IV for further information.

\[
\begin{align*}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
&= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
&= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
V_{Z4} \\
V_{Z15} \\
V_{Z19} \\
V_{Z19}
\end{bmatrix}
&= 
\begin{bmatrix}
f_{\text{chem}}/0 \\
f_{\text{em}1}/0 \\
f_{\text{em}2}/0 \\
f_{\text{km}1}/0 \\
f_{\text{km}2}/0
\end{bmatrix}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
&= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
&= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
V_{Z4} \\
V_{Z15} \\
V_{Z19} \\
V_{Z19}
\end{bmatrix}
&= 
\begin{bmatrix}
f_{\text{chem}}/0 \\
f_{\text{em}1}/0 \\
f_{\text{em}2}/0 \\
f_{\text{km}1}/0 \\
f_{\text{km}2}/0
\end{bmatrix}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
&= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
&= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
V_{Z4} \\
V_{Z15} \\
V_{Z19} \\
V_{Z19}
\end{bmatrix}
&= 
\begin{bmatrix}
f_{\text{chem}}/0 \\
f_{\text{em}1}/0 \\
f_{\text{em}2}/0 \\
f_{\text{km}1}/0 \\
f_{\text{km}2}/0
\end{bmatrix}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
&= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
&= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
V_{Z4} \\
V_{Z15} \\
V_{Z19} \\
V_{Z19}
\end{bmatrix}
&= 
\begin{bmatrix}
f_{\text{chem}}/0 \\
f_{\text{em}1}/0 \\
f_{\text{em}2}/0 \\
f_{\text{km}1}/0 \\
f_{\text{km}2}/0
\end{bmatrix}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
&= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
&= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
V_{Z4} \\
V_{Z15} \\
V_{Z19} \\
V_{Z19}
\end{bmatrix}
&= 
\begin{bmatrix}
f_{\text{chem}}/0 \\
f_{\text{em}1}/0 \\
f_{\text{em}2}/0 \\
f_{\text{km}1}/0 \\
f_{\text{km}2}/0
\end{bmatrix}
\begin{bmatrix}
\dot{z}_{\text{chem}} \\
\dot{z}_{\text{em}1} \\
\dot{z}_{\text{em}2} \\
z_{\text{km}1} \\
z_{\text{km}2}
\end{bmatrix}
\end{align*}
\]
4.4.4 - Secondary actuator blocks 'm'

The signal flows for the 'secondary actuator blocks m' are given by equation (4.39).

\[
\begin{align*}
\text{EVEN} & : 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\ddots \\
0 \\
0
\end{bmatrix} - V_{212} - \\
\Phi_{bm} & : 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\ddots \\
0 \\
0
\end{bmatrix} - V_{212} - \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\ddots \\
0 \\
0
\end{bmatrix} - V_{212} - \\
\text{ODD} & : 
\begin{bmatrix}
\ddots \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\ddots \\
0 \\
0
\end{bmatrix} - V_{212} - \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\ddots \\
0 \\
0
\end{bmatrix} - V_{212} - \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\ddots \\
0 \\
0
\end{bmatrix} - V_{212} - \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\ddots \\
0 \\
0
\end{bmatrix} - \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\ddots \\
0 \\
0
\end{bmatrix} - V_{212} - \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\ddots \\
0 \\
0
\end{bmatrix} - V_{212} - \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\ddots \\
0 \\
0
\end{bmatrix} - V_{212} - \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\ddots \\
0 \\
0
\end{bmatrix}.
\end{align*}
\]

This actuator block receives information from 'V_{Z12}' which is a combination of the vectors 'V_{Z5}' and 'V_{Z6}' with reference to Figure 4.3. Vector 'V_{Z5}' contains secondary suspension information.
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which is utilised by the secondary actuator block to determine the generated actuator force. The other vital component, the actuator command force, is obtained from \('V_{ZS}'\) which contains the command signals for the pair of secondary actuators associated with vehicle \('m'\). These signals are obtained through 'pre-format block m2' as described in section 4.4.2.

There are obviously two secondary actuators per vehicle, all the information required by both the leading and trailing bogie actuators is contained within the unified vector \('V_{ZI}'\). Two actuators are drawn from the secondary actuator library and appended to form the sub-block shown in Figure 4.3. This task is facilitated by other Matlab utilities, the details of which are given in the program listings of Appendix IV.

4.4.5 - Inter-vehicle actuator/connections 'm'

The signal mapping for this particular block is shown in equation (4.40). The input vectors are an amalgamation of adjacent railway vehicle output information. In this instance the vectors for vehicles \('m'\) and \('m+1'\), contains signals such as: \('i_{endm2}'\) - the trailing end movement of vehicle \('m'\), \('i_{end(m+1)}'\) - the leading end movement of vehicle \('m+1'\), and \('f_{chm2}'\) - the inter-vehicle actuator demand signal. These variables are obviously important to an inter-connector, whether it is simply a passive interconnection or an actuator.

The exact nature of the signal flow again depends upon whether the vehicle has an odd or an even number. This follows naturally from the precedence set by the railway vehicle block which stipulates that even number vehicles pass information on the top section of a vector, and odd numbered vehicles pass information on the bottom.

The outputs from this block are inter-vehicle forces which are then passed on to influence the dynamic behaviour of the vehicle.
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\[
\begin{align*}
V_{z1} \cdot V_{z10} & \rightarrow V_{z19} = V_{z20} \cdot V_{z10} = \rightarrow V_{z19} \cdot V_{z10} = \ldots (4.40)
\end{align*}
\]
4.4.6 - Post-format blocks 'm'

The mapping provided by this block is given by equation (4.41). The post-format blocks are intended to re-construct the output information provided by the railway vehicle blocks in a format suitable for overall system output. For the purposes of suspension analysis, it is useful to
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have accelerations (ride quality), displacements (suspension deflections), and actuator forces as outputs. Although this information is contained within the railway vehicle output vector ('V_{29}', in the case of vehicle 'm') it requires some tidying up, and the function of the post-format blocks is to strip off many of the zeros and to eliminate variables which are of no interest to the future analysis of the system.

The output block is used to amalgamate the output vectors: in the example shown in Figure 4.3, vectors: 'V_{221}', 'V_{223}', and 'V_{224}', from each of post-format blocks.

The program 'railtmv.m' in Appendix IV is the main program, it is responsible for formatting all the sub-blocks in the appropriate manner as outlined in sections 4.4.1 to 4.4.6. It then proceeds to construct the overall system. This suite of programs requires the 'signal', and 'mfd' Matlab toolboxes. The example shown below shows the construction of a 3 vehicle train. The vehicle bodies in this example do not have a flexible mode present. There are 'ideal' secondary actuators, and 'electrohydraulic' inter-vehicle actuators.

```matlab
[a,b,c,w,db,dw] = railtmv(3,'rigid','ideal','elhyd');
```

***** IDEAL SECONDARY VERTICAL ACTUATORS *****

***** VEHICLE #: 1 *****

***** ELECTROHYDRAULIC VERTICAL INTER-CONNECTORS *****

***** VEHICLE #: 1 *****

***** IDEAL SECONDARY VERTICAL ACTUATORS *****

***** VEHICLE #: 2 *****

***** ELECTROHYDRAULIC VERTICAL INTER-CONNECTORS *****

***** VEHICLE #: 2 *****
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***** IDEAL SECONDARY VERTICAL ACTUATORS *****
***** VEHICLE #: 3 *****

***** ELECTROHYDRAULIC VERTICAL INTER-CONNECTORS *****
***** VEHICLE #: 3 *****

***** BLOCKBUILD SYSTEM *****
***** CONNECT SYSTEM TOGETHER *****

Warning in MVCON the output of block 15 is not used.
MVCON Number of Gain blocks found was 14
  Gain block 19 is an input
  Gain block 20 is an output

This illustrates the flexibility in the software, which can be used to construct a train model of length 'n', possessing a mixture of different types of secondary or inter-vehicle actuators. The train model is returned in a state-space format as detailed previously in equations (4.22) and (4.23). The inputs and outputs of this model parallel those given in equation (4.35). The wheelset vertical velocities are contained within the vector $Z_{\text{TRACK}}$ and the actuator forces within the vector $U_{\text{ACTZ}}$. The only difference being that the vectors for the train model are stacked according to the number of vehicles in the train. The system output vector is simply a stacked version of the post-format block outputs.

4.5 - Train model validation

As with the validation tests performed in section (4.2) for the single vehicle model, a number of tests have been performed for the train model. These tests are predominantly transient tests. The modes of the full train model are not vastly dissimilar from the modes of the individual vehicles, there is some change of course due to the inter-vehicle connection, but this information is not
useful for validating a full train model. Two sets of tests are performed for different train lengths, one train containing 3 vehicles, the other 6.

Figures 4.13 to 4.18 show the tests for a 3 vehicle train. Figures 4.13 and 4.14 show responses for a unity wheelset displacement on 'z_{w2|2}', the second wheelset on vehicle 2. Figure 4.13 shows the bounce motion of vehicle 3. This response is to be expected as the wheelset displacement will produce positive pitching of vehicle number 2, the inter-vehicle connection will then push vehicle 3 down slightly. Figure 4.14 shows the bounce response of vehicle 1. This shows a positive steady-state displacement, which is due to the lifting of the front end of vehicle 2, and once again through the actions of the inter-vehicle connection the leading vehicle will be raised slightly.

Figures 4.15 and 4.16 show the response to a 1 ton force step on the leading secondary actuator of vehicle 2, 'f_{21|2}'. Figure 4.15 shows the pitching response of vehicle 1 due to the front end of vehicle 2 being raised because of the secondary actuator force. The actions of the inter-vehicle connection between vehicles 1 and 2 mean that vehicle 1 will have a negative steady-state pitch. Figure 4.16 shows the bogie displacement associated with the leading actuator of vehicle 2, which shows a net negative displacement. For a single vehicle this response would have a zero steady-state value, however, in a train there are inter-vehicle reaction forces. These forces acting on vehicle 2 in this instance are balanced by compression of the primary suspension resulting in a downward motion of the leading bogie of vehicle 2.

The final pair of tests for the 3 vehicle model are inter-vehicle actuator force tests. A 1 ton force step input between vehicles 2 and 3 is applied ( 'f_{62|2}'). The pitching response for vehicle 1 is shown in Figure 4.17, which can be explained with reference to Figure 4.2. A 1 ton step input would result in negative pitching of vehicle 2, which would result in a downward motion of the front end of vehicle 2 and positive pitching of vehicle 1 because of the inter-vehicle connection. Figure 4.18 shows the pitch response of vehicle 3, which is negative due to the reaction of the active inter-vehicle force between vehicles 2 and 3.
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Figure 4.13 - 3 vehicle train/vehicle 3 bounce

Figure 4.14 - 3 vehicle train/vehicle 1 bounce

Figure 4.15 - 3 vehicle train/vehicle 1 pitch

Figure 4.16 - 3 vehicle train/lead bogie veh 2

Figure 4.17 - 3 vehicle train/vehicle 1 pitch

Figure 4.18 - 3 vehicle train/vehicle 3 pitch
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The tests performed on the longer 6 vehicle trainset are similar to those performed on the 3 vehicle train. The first test was a unity step input on wheelset 'z_{w312}'. The bounce response of vehicle 3 is shown in Figure 4.19, the steady-state response is somewhat lower than the 1/4 predicted for the single vehicle model as was shown in Figure 4.8. The movement is lower due to the actions of the inter-vehicle connectors, because these connections attempt to restrain the movement of adjacent bodies. Figure 4.20 shows the bounce response of vehicle 2, which has positive steady-state displacement due to the front end of vehicle 3 rising, although the value is somewhat lower than that for vehicle 3.

Figures 4.21 and 4.22 show the pitch response of vehicle 2, and the bounce motion of lead bogie 'z_{b1}', respectively for a 1 ton force step input on 'f_{b1}'. Figure 4.21 has the negative pitch as expected because of the inter-vehicle connection, the bogie associated with the secondary actuator drops to react forces through the primary suspension.

Figures 4.23 and 4.24 illustrate the diminishing effect on variables which are remote from the point of disturbance in a train. Figure 4.23 shows the pitching response of vehicle 1 with a 1 ton force step input applied between vehicles 2 and 3, 'f_{b2}'. Figure 4.24 shows the pitching response of vehicle 6 with the same input. Both vehicle react but their responses are very small.

4.6 - Summary

The vertical train models developed in this chapter clearly resemble their full-scale counterparts. They are used in subsequent chapters to develop train-wide control laws. The models are presented in a format (separate inputs for track velocities and actuator forces) which eases their integration with active suspension control laws. The model output vector contains accelerations and displacements which are both useful quantities in suspension performance assessment.
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Figure 4.19 - 6 vehicle train/vehicle 3 bounce

Figure 4.20 - 6 vehicle train/vehicle 2 bounce

Figure 4.21 - 6 vehicle train/vehicle 2 pitch

Figure 4.22 - 6 vehicle train/bogie bounce

Figure 4.23 - 6 vehicle train/vehicle 1 pitch

Figure 4.24 - 6 vehicle train/vehicle 6 pitch
Chapter 5
Modelling Lateral Train Dynamics

5.1 - Overview

The modelling procedure outlined in this chapter bears similarities to the methods used in Chapter 4. This chapter is dedicated to the construction of models which represent the lateral dynamics of a railway train. As stated in the introduction, the lateral and vertical dynamics do not possess significant coupling and can be modelled as separate entities. The use of a library of building blocks containing vehicle, secondary actuator, and inter-vehicle connector dynamics, which are then repetitively connected to form a train, is the method of choice in this chapter. As stated in Chapter 4, modelling from first principles can be an error prone method but is the option chosen due to the flexibility given when subsequently developing train-wide secondary active, and active inter-vehicle control laws. The prospect of verifying the accuracy of the lateral model is much more daunting than that for its vertical counterpart. The lateral model of a single vehicle is shown in Figures 5.3 and 5.4, from which it is clear that the number of degrees of freedom has increased considerably. This complexity is aggravated by the introduction of four coned wheelsets and anti-roll bars on the leading and trailing bogies.

The software used to generate the lateral train model is again Matlab 4.2c, together with the 'signal', and 'mfd' toolboxes. Similar arguments to those used for the vertical model are used here to advance the choice of Matlab in preference to other modelling software. The points relating to Vampire and its deficiency in modelling long train lengths also applies to the lateral direction.
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Indeed the only restriction on the number of vehicles that may be connected which applies to this
generic method of modelling trains is memory limitations on the hardware used.

Analysis of long train effects motivates the need for a generic piece of software that generates
a model of an 'n' vehicle train as shown in Figure 5.1, which in this instance has an active inter-
vehicle connection.

![Generic connection of lateral vehicle models](image)

Figure 5.1 - Generic connection of lateral vehicle models

As with the vertical model, three components are important when assembling the model, each
of these components having a pre-defined interface structure but possibly having different
internal dynamics, varying according to the requirements of the designer. For example it may be
interesting to create a model of a laden and also an unladen train. The three generic modelling
components are:

- The dynamics of an individual vehicle (i.e. allowing changes in masses, spring/damper rates)
- The dynamics of any type of secondary suspension actuator (i.e. electrohydraulic, etc.)
- The dynamics of any type of inter-vehicle actuator

Figure 5.2 is a schematic illustrating how these components interact, and how they may be
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connected together to form the full train model. It is similar to Figure 4.3 which applied to the vertical model. Matlab functions 'mvblk.b.m' and 'mvecon.m' from the multivariable frequency domain toolbox are used to connect the structure and form the overall model.

The structure of a single vehicle in the lateral direction as shown by Figures 5.3 and 5.4 is essentially fixed, in contrast to the methods used in Chapter 4, which considered the development of a single vehicle having purely rigid body modes and also a vehicle incorporated body flexibility. Body flexibility is not a significant problem in the lateral direction. It is possible with this piece of software to vary the topology of Figures 5.3 and 5.4. For example, to incorporate wheelset steering, it would be a simple matter to adapt the equations and generate a new vehicle model which incorporated wheelset steering, and the generic piece of software could then link these vehicles together to form a train model provided the interface protocol between different components was preserved.

The possibility of having four secondary actuators per vehicle is apparent from Figure 5.3. Each of the three technologies considered for the vertical direction: 'electrohydraulic', 'electromechanical', and 'electromagnetic', will also be applied to the lateral direction. The possibility of having an 'ideal' actuator with infinite bandwidth also applies here. The structures of each of these actuator models are the same as for the vertical direction and are detailed in Chapter 3. Obviously each of the actuator technologies will have different sizes and ratings according to their use in the lateral suspension; the size varying with vehicle mass and the demands made upon them by the active suspension control law. The procedure for sizing the actuator is given in Chapters 3 and 4. The actuators work against an existing passive lateral suspension.

Inter-vehicle actuators are drawn from a library containing the appropriately sized actuator technology. An 'ideal' actuator form may also be used as the secondary actuator. The inter-vehicle connection may also be a nominal parallel spring-damper combination which is an acceptable dynamic representation of the combined effects of the gangway and coupler.
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Clearly in the lateral direction, an inter-vehicle connection whether it be active or passive would need to know the motion across its ends. This requires a knowledge of: the lateral displacements and velocities, yaw displacements and velocities, and possibly the roll displacements and velocities for each of the adjacent vehicles. The procedure is similar to that used in Chapter 4 for the vertical model but is clearly complicated by the additional degrees of freedom.

Similarly the secondary actuators need to interface with individual vehicle models. A more noticeable difference between the vertical and lateral models is the fact that 4 actuators per vehicle are incorporated in the lateral model as opposed to 2 in the vertical case. This is not a significant problem since the secondary actuator dynamics are appended within a single dynamic block. The lateral model is constructed with 4 actuators to permit some degree of control over the bogie yaw motions when the actuators are placed off the c.o.g line of the bogie. A pair of actuators per bogie ensures that the bogie lateral and yaw motions may be decoupled. Locating the actuators off the bogie c.o.g. line is simply a matter of changing a length parameter within the main parameter file 'railprmt.m'.
Figure 5.2 - Schematic structure of a lateral railway train model
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The suspension requirements stated in the introduction for a vertical model also apply to the lateral direction. Clearly the requirement to support the vehicle weight does not apply in the lateral direction but the need to provide isolation from the track irregularities while maintaining adequate suspension deflections still applies. The measurement variables stated earlier are clearly a prudent choice in this respect.

The validity of the train model comes into question when subjected to extreme perturbations. The dynamic behaviour of the wheelsets is highly non-linear in reality. The most significant non-linearity being the transition from running on coned tread to flange contact. The model used to describe wheelset motion for the lateral model assumes only tread running and hence is only valid for quite shallow curves with no flange contact, but this is recognised to be an acceptable approach for ride quality assessment.

Each of the blocks contained within Figure 5.2 will now be analysed in turn, these are:

- Pre-format block m1
- Pre-format block m2
- Single vehicle lateral dynamic blocks
- Secondary actuator dynamic blocks
- Inter-vehicle actuator dynamic blocks

The functionality of each block together with the protocol used to interface them are the issues which are addressed in the subsequent sections. A model of the lateral dynamics of a single vehicle has not yet been developed and consequently this will be performed first in section 5.2 before moving on to consider interfacing each of the blocks in section 5.3.

5.2 - Single vehicle lateral dynamics

The development of a model representing the lateral dynamics of a single vehicle follows a
similar procedure to that used in section 4.2 for the vertical train model. Each mass is considered in isolation and by equating inertial forces with spring and damper forces using d'Alembert's principle an equation of motion may be developed for each degree of freedom. Figures 5.3 and 5.4 are an endview and a planview respectively for the lateral model. They show the various mass elements and their interconnection. The rotational stiffness and damping shown in Figure 5.3 represents the roll-bar connection between the body and the bogie, and the arrangement is of course repeated for each bogie. A pair of airsprings per bogie is shown in this figure illustrating the faithful representation this model provides to the full-scale equivalent. However, the airsprings only contribute to the roll motion of the vehicle, the vertical degrees of freedom are ignored. A parallel spring-damper combination is used to represent the lateral secondary suspension, which is an accurate representation of the lateral behaviour of an airspring in conjunction with other dampers and lateral parasitic effects that are present. The existence of coupling between roll and lateral motion is clearly evident from this figure.

Figure 5.3 - Endview lateral model
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The vertical primary suspension for the trailing wheelset is also shown in Figure 5.3. This again resembles the full-scale equivalent with a parallel spring-damper being located at each end of the axle. Swing-arms are frequently used in practice but simple scaling of the parameters in order to cater for the levering effect of the swing-arm results in exactly the same effect. Rubber chevron vertical primary suspensions are sometimes used and may also be represented by a parallel spring-damper combination. Vertical degrees of freedom are obviously omitted in this figure, however the vertical suspension clearly influences bogie and body roll and is therefore included.

Figure 5.4 is a planview of the lateral vehicle model. This emphasises the points made earlier about the need to repeat roll-bars and airsprings for the leading bogie. Also shown in this figure is the longitudinal and lateral primary suspension, which is frequently formed through rubber bushing but is shown here as a parallel-spring damper combination.

![Figure 5.4 - Planview lateral model](image)

Anti-hunting dampers are included to give yaw damping to the bogies, a fairly common feature
on high speed railway vehicles. Additional rotational stiffness and damping is placed between
the body and the bogies in order to cater for parasitic effects. The 4 secondary actuators are
placed in parallel with the nominal passive secondary suspension, it may be beneficial to remove
the secondary damping when developing an active suspension, but this can easily be performed
by simply setting various components to zero in the global parameter file.

Each of the degrees of freedom will now be accounted for, and equations of motion developed
to describe their behaviour.

For the body lateral d.o.f., the body is influenced by motions of the bogies and perhaps external
forces from secondary or inter-vehicle actuators as shown by equation (5.1).

\[
\begin{align*}
\ddot{y}_{\text{b,m}} &= m_f y_{\text{b,m}} - 4k_f y_{\text{b,m}} - 4k_{l,bs} y_{\text{b,m}} - 2k_g y_{\text{m,1}} - 2k_g y_{\text{m,2}} - 4c_f y_{\text{b,m}} - 4c_{l,bs} y_{\text{b,m}} - 2c_f y_{\text{m,1}} - 2c_f y_{\text{m,2}} - f_{\text{m,1y}} - 2f_{\text{m,2y}} - f_{\text{m,3y}} - f_{\text{m,4y}}. \\
\end{align*}
\]

The body yaw degree of freedom is expressed by equation (5.2).

\[
\begin{align*}
\dot{\psi}_{\text{b,m}} &= -4k_{l,bs} y_{\text{b,m}} - 2k_g y_{\text{m,1}} - 2k_g y_{\text{m,2}} - 4c_{l,bs} y_{\text{b,m}} - 2c_f y_{\text{m,1}} - 2c_f y_{\text{m,2}} - f_{\text{m,1y}} - f_{\text{m,2y}} - f_{\text{m,3y}} - f_{\text{m,4y}}. \\
\end{align*}
\]

The body roll degree of freedom is given by equation (5.3).

\[
\begin{align*}
\dot{\psi}_{\text{b,m}} &= 4k_f y_{\text{b,m}} - 4k_{l,bs} y_{\text{b,m}} - 4k_{l,as} y_{\text{m,1}} - 4k_{l,as} y_{\text{m,2}} - 2k_{g,bs} y_{\text{b,m}} - 2k_{g,as} y_{\text{m,1}} - 2k_{g,as} y_{\text{m,2}} - k_{l,as} z_{\text{m,1}} - k_{l,as} z_{\text{m,2}} - k_{l,as} z_{\text{m,3}} - k_{l,as} z_{\text{m,4}} - 2k_{h,b} y_{\text{m,1}} - (2k_{l,as} y_{\text{m,1}} + k_{h,b}) y_{\text{m,2}} - 2k_{l,as} y_{\text{m,1}} + (2k_{l,as} y_{\text{m,1}} + k_{h,b}) y_{\text{m,2}} - 2k_{g,bs} y_{\text{b,m}} - 2c_{l,bs} y_{\text{b,m}} - 2c_{l,bs} y_{\text{b,m}} - 2c_{l,bs} y_{\text{b,m}} - 2c_{l,bs} y_{\text{b,m}} - 2c_{l,bs} y_{\text{b,m}} - 2c_{l,bs} y_{\text{b,m}} - f_{\text{m,1y}} - f_{\text{m,2y}} - f_{\text{m,3y}} - f_{\text{m,4y}}. \\
\end{align*}
\]

The equations of motion are developed in conjunction with equation (4.1) from the previous
chapter which describes the internal dynamics of a general airspring. Equations of motion for
the four airsprings in this model are given by equations (5.4) to (5.7).
The equations of motion describing the lateral, yaw, and roll degrees of freedom of the leading bogie are given by equations (5.8), (5.9), and (5.10) respectively.

\[
\begin{align*}
m_{\psi_{\text{sm}}} & = 2k_{pr}y_{\text{sm}} + 2k_{pr}l_{\psi_{\text{sm}}} + 2l_{\psi_{\text{py}}}\rho_{\text{sm}} - \{2k_{pr} + 4k_{ip}\}y_{\text{sm}} + 2k_{ip}y_{\text{sm}l} + \
& 2k_{ip}\psi_{\text{sm}l} + 2c_{ip}\psi_{\text{sm}} + 2c_{ip}\rho_{\text{sm}} - 2c_{ip}\psi_{\text{sm}l} - f_{\text{sm}l}, \quad \ldots (5.8) \\
i_{\psi_{\text{sm}}} & = \{4k_{pr}l_{\psi_{\text{py}}}^2 + 2k_{pr}l_{\psi_{\text{py}}}, \quad \ldots (5.9) \\
i_{\rho_{\text{sm}}} & = \{2k_{pr}l_{\psi_{\text{py}}}^2 + 2k_{pr}l_{\psi_{\text{py}}}, \quad \ldots (5.10) \\
\end{align*}
\]

Similar equations of motion may be expressed for the degrees of freedom of the trailing bogie, as expressed by equations (5.11) through to (5.13).
The equations of motion for the wheelsets need to incorporate some form of creep model in order to evaluate the forces acting on them and hence be able to predict the motions they will exhibit. A general model of a wheelset is shown in Figure 5.5.

Two linear equations (5.14) and (5.15) may be used to describe the motion of this wheelset when
running on its tread [Vampire manual - 1989], Although as mentioned earlier this model is not valid under extreme conditions when flange contact is likely. The first equation (5.14) gives the force \( f_w \), and the torque \( t_w \) generated as a result of changes in the wheelset position and orientation.

\[
\begin{bmatrix}
  f_w \\
  t_w
\end{bmatrix} = \frac{1}{V} \begin{bmatrix}
  -2f_{11} & 0 \\
  0 & -2f_{11} l_1^2
\end{bmatrix} \begin{bmatrix}
  y_w \\
  \psi_w
\end{bmatrix} \begin{bmatrix}
  -k_c & 2f_{22} \\
  -2f_{11} \lambda l_1 \\ r_0
\end{bmatrix} \begin{bmatrix}
  y_w \\
  \psi_w
\end{bmatrix} \quad \ldots (5.14)
\]

\[
\begin{bmatrix}
  f_i \\
  t_i
\end{bmatrix} = \frac{1}{V} \begin{bmatrix}
  0 & 2f_{22} r_0 \\
  0 & 0
\end{bmatrix} \begin{bmatrix}
  y_i \\
  \psi_i
\end{bmatrix} \begin{bmatrix}
  k_c & r_0 k_c \\
  2f_{11} \lambda l_1 \\ r_0
\end{bmatrix} \begin{bmatrix}
  y_i \\
  \psi_i
\end{bmatrix} \quad \ldots (5.15)
\]

The parameters within the equation are: the wheelset velocity, the nominal rolling radius, and a variety of coefficients associated with the wheelset conicity and its material properties. For a more in-depth analysis, the reader is referred to [Vampire manual - 1989]. The second equation (5.15) gives the force \( f_i \), and the torque \( t_i \) developed as a result of changes in the track position and orientation. The parameters in this equation may be evaluated using the same set of variables used in the previous equation.

It is now a simple matter of incorporating this pair of equations to fit within the framework of the lateral vehicle model. The equations for the four wheelsets in this model will obviously have a similar form. The motions of each of them being effected by similar creep equations and possessing similar primary suspension connections to the bogie. These equations (5.16) to (5.23) are given below:

\[
\begin{align*}
  m_w \ddot{y}_{wm} - 2k_{py} y_{wm} - 2k_{pp} l_1^2 \dot{\psi}_{wm} - (2k_{py} + k_c) y_{wm} - 2f_{22} \dot{\psi}_{wm} & \\
  2f_{22} \dot{y}_{wm} & + k_c y_{wm} + r_0 k_c \dot{\phi}_{wm} & + 2f_{22} r_0 \dot{\phi}_{wm} & & & & & & & & & & & & & & & & & & \quad \ldots (5.16)
\end{align*}
\]
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\[ l_{y_1} \Psi_{wm1} = -(2 c_{pp} f_{pp}^2 - \frac{2 f_{11} f_{12}^2}{V}) \Psi_{wm1} + 2 k_{pp} f_{pp}^2 \Psi_{wm1} - \frac{2 f_{11} \lambda l_1}{r_o} y_{wm1} - \ldots \]  

\[ 2 k_{pp} f_{pp}^2 \Psi_{wm1} - \frac{2 f_{11} \lambda l_1}{r_o} y_{wm1} - 2 f_{11} \lambda l_1 \rho_{wm1} \]  

\[ m_w \ddot{y}_{wm1} = 2 k_{pp} y_{wm1} + 2 k_{pp} f_{pp}^2 \Psi_{wm1} - (2 k_{pp} + k_c) y_{wm1} - 2 f_{22} \Psi_{wm1} - \ldots \]  

\[ \frac{2 f_{22}}{V} \ddot{y}_{wm1} + k_c y_{wm1} + r_o k_c \rho_{wm1} - \frac{2 f_{22} r_o}{V} \rho_{wm1} \]  

\[ l_{y_2} \Psi_{wm2} = -(2 c_{pp} f_{pp}^2 - \frac{2 f_{11} f_{12}^2}{V}) \Psi_{wm2} + 2 k_{pp} f_{pp}^2 \Psi_{wm2} - \frac{2 f_{11} \lambda l_1}{r_o} y_{wm2} - \ldots \]  

\[ 2 k_{pp} f_{pp}^2 \Psi_{wm2} - \frac{2 f_{11} \lambda l_1}{r_o} y_{wm2} - 2 f_{11} \lambda l_1 \rho_{wm2} \]  

\[ m_w \ddot{y}_{wm2} = 2 k_{pp} y_{wm2} + 2 k_{pp} f_{pp}^2 \Psi_{wm2} - (2 k_{pp} + k_c) y_{wm2} - 2 f_{22} \Psi_{wm2} - \ldots \]  

\[ \frac{2 f_{22}}{V} \ddot{y}_{wm2} + k_c y_{wm2} + r_o k_c \rho_{wm2} - \frac{2 f_{22} r_o}{V} \rho_{wm2} \]  

\[ l_{y_3} \Psi_{wm3} = -(2 c_{pp} f_{pp}^2 - \frac{2 f_{11} f_{12}^2}{V}) \Psi_{wm3} + 2 k_{pp} f_{pp}^2 \Psi_{wm3} - \frac{2 f_{11} \lambda l_1}{r_o} y_{wm3} - \ldots \]  

\[ 2 k_{pp} f_{pp}^2 \Psi_{wm3} - \frac{2 f_{11} \lambda l_1}{r_o} y_{wm3} - 2 f_{11} \lambda l_1 \rho_{wm3} \]  

\[ m_w \ddot{y}_{wm3} = 2 k_{pp} y_{wm3} + 2 k_{pp} f_{pp}^2 \Psi_{wm3} - (2 k_{pp} + k_c) y_{wm3} - 2 f_{22} \Psi_{wm3} - \ldots \]  

\[ \frac{2 f_{22}}{V} \ddot{y}_{wm3} + k_c y_{wm3} + r_o k_c \rho_{wm3} - \frac{2 f_{22} r_o}{V} \rho_{wm3} \]  

\[ i_y \Psi_{wm22} = -(2 c_{pp} f_{pp}^2 - \frac{2 f_{11} f_{12}^2}{V}) \Psi_{wm22} + 2 k_{pp} f_{pp}^2 \Psi_{wm22} - \frac{2 f_{11} \lambda l_1}{r_o} y_{wm22} - \ldots \]  

\[ 2 k_{pp} f_{pp}^2 \Psi_{wm22} - \frac{2 f_{11} \lambda l_1}{r_o} y_{wm22} - 2 f_{11} \lambda l_1 \rho_{wm22} \]  

\[ m_w \ddot{y}_{wm22} = 2 k_{pp} y_{wm22} + 2 k_{pp} f_{pp}^2 \Psi_{wm22} - (2 k_{pp} + k_c) y_{wm22} - 2 f_{22} \Psi_{wm22} - \ldots \]  

\[ \frac{2 f_{22}}{V} \ddot{y}_{wm22} + k_c y_{wm22} + r_o k_c \rho_{wm22} - \frac{2 f_{22} r_o}{V} \rho_{wm22} \]  

\[ i_y \Psi_{wm23} = -(2 c_{pp} f_{pp}^2 - \frac{2 f_{11} f_{12}^2}{V}) \Psi_{wm23} + 2 k_{pp} f_{pp}^2 \Psi_{wm23} - \frac{2 f_{11} \lambda l_1}{r_o} y_{wm23} - \ldots \]  

\[ 2 k_{pp} f_{pp}^2 \Psi_{wm23} - \frac{2 f_{11} \lambda l_1}{r_o} y_{wm23} - 2 f_{11} \lambda l_1 \rho_{wm23} \]  

\[ m_w \ddot{y}_{wm23} = 2 k_{pp} y_{wm23} + 2 k_{pp} f_{pp}^2 \Psi_{wm23} - (2 k_{pp} + k_c) y_{wm23} - 2 f_{22} \Psi_{wm23} - \ldots \]  

\[ \frac{2 f_{22}}{V} \ddot{y}_{wm23} + k_c y_{wm23} + r_o k_c \rho_{wm23} - \frac{2 f_{22} r_o}{V} \rho_{wm23} \]
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The equations of motion (5.1) to (5.13), and (5.16) to (5.23) represent the 21 degrees of freedom in the lateral single vehicle model. They may be expressed in a more compact, mass, damper, and stiffness matrix format as shown below in equation (5.24).

\[
M \ddot{X}_y + C \dot{X}_y + K X_y - F_{ACTY} U_{ACTY} - F_{\text{TRACTY}} \dot{Y}_{\text{TRACTY}} + F_{\text{TRACTY}} Y_{\text{TRACTY}} = \ldots (5.24)
\]

\(X_y\) is a vector representing the degrees of freedom, and M, C, K, F_{ACTY}, and F_{\text{TRACTY}} are matrices representing the equations of motion which have just been derived. \(X_y\) is shown by equation (5.25).

\[
X_y = \begin{bmatrix}
Y_{km} \\
\Psi_{km} \\
\phi_{km} \\
y_{mm} \end{bmatrix} \ldots (5.25)
\]
Chapter 5: Modelling lateral train dynamics

The inputs to the system in equation (5.24) are track disturbances 'Y_TRACK', and external forces 'U_ACTY'. These inputs are given in equations (5.26) to (5.28). The derivative of 'Y_TRACK' is required here, but the need for this is eliminated in the final state-space representation.

\[
\dot{Y}_{\text{TRACK}} = \begin{bmatrix}
\dot{y}_{\text{om11}} \\
\dot{y}_{\text{om11}} \\
\dot{y}_{\text{om12}} \\
\dot{y}_{\text{om12}} \\
\dot{y}_{\text{om21}} \\
\dot{y}_{\text{om22}} \\
\dot{\rho}_{\text{om11}} \\
\dot{\rho}_{\text{om11}} \\
\dot{\rho}_{\text{om12}} \\
\dot{\rho}_{\text{om12}} \\
\end{bmatrix}
\quad \ldots (5.26)
\]

\[
Y_{\text{TRACK}} = \begin{bmatrix}
y_{\text{om11}} \\
y_{\text{om11}} \\
y_{\text{om12}} \\
y_{\text{om12}} \\
y_{\text{om21}} \\
y_{\text{om22}} \\
\rho_{\text{om11}} \\
\rho_{\text{om11}} \\
\rho_{\text{om12}} \\
\rho_{\text{om12}} \\
\end{bmatrix}
\quad \ldots (5.27)
\]

\[
U_{\text{ACTY}} = \begin{bmatrix}
f_{\text{om11}} \\
f_{\text{om11}} \\
f_{\text{om12}} \\
f_{\text{om12}} \\
f_{\text{om21}} \\
f_{\text{om22}} \\
f_{\text{om11}} \\
f_{\text{om11}} \\
f_{\text{om12}} \\
f_{\text{om12}} \\
\end{bmatrix}
\quad \ldots (5.28)
\]
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The matrices $M$, $C$, $K$, $F_{\text{ACTY}}$, $F_{\text{TRACKY}}$, and $F_{\text{TRACKY}'}$ are given in equations (5.29) to (5.34) respectively.

$$
M = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{m_{r}}
\end{bmatrix}
$$
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\[
\begin{bmatrix}
4c_\nu & 0 & 4c_y^l & 0 & 0 & 0 & 0 & -2c_\nu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4c_y^l & 0 & 0 & 0 & 0 & 0 & -2c_y^l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4c_y^l & 0 & 4c_y^l & 2c_\nu & 0 & 0 & 0 & -2c_y^l & 0 & 0 & -2c_\nu & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c_\nu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & c_\nu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c_\nu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2c_\nu & -2c_y^l & -2c_y^l & 0 & 0 & 0 & 0 & 2c_\nu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -c_\nu & -c_y^l & -c_y^l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2c_\nu & 2c_y^l & -2c_y^l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -c_\nu & 0 & -c_y^l & -c_y^l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(5.30)
Chapter 5: Modelling lateral train dynamics

\[
\begin{array}{ccccccccccccc}
48_n & 0 & 48_m^{l,0} & 0 & 0 & 0 & 0 & -28_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 48_m^{l,1} & 0 & 0 & 0 & 0 & 0 & -28_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
48_m & 0 & 48_m^{l,0} & 6(8_m^{l,0})^{2} & 28_m & -8_m^{l,0} & 8_m^{l,0} & -28_m & 0 & -28_m & 0 & -28_m & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -8_m^{l,0} & 8_m^{l,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -8_m^{l,0} & 8_m^{l,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -8_m^{l,0} & 8_m^{l,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -28_m & -28_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
28_m & -28_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
28_m & -28_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[\ldots (5.31)\]
Chapter 5: Modelling lateral train dynamics

\[ F_{\text{max}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \ldots (5.32) \]

\[ \frac{2f_{\text{in}}w_{\text{in}}}{r_o} \begin{bmatrix} 
2f_{\text{in}}w_{\text{in}}l_{\text{in}} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} \]

\[ \begin{bmatrix} 
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \ldots (5.33) \]
Equation (5.24) may be rearranged to represent the dynamics in a state-space format as performed in equations (5.35) and (5.36) below.

\[
\begin{bmatrix}
  \dot{X}_r \\
  \dot{X}_y \\
  \dot{Y}_{\text{track}}
\end{bmatrix} =
\begin{bmatrix}
  0 & I & 0 \\
  -M^{-1}K & -M^{-1}C & M^{-1}F_{\text{track}} \\
  0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
  X_r \\
  X_y \\
  Y_{\text{track}}
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  M^{-1}F_{\text{act}} \\
  0
\end{bmatrix} U_{\text{act}} +
\begin{bmatrix}
  0 \\
  M^{-1}F_{\text{track}} \\
  I
\end{bmatrix} \dot{Y}_{\text{track}} \quad \ldots (5.35)
\]

\[
\begin{bmatrix}
  \dot{X}_r \\
  \dot{X}_y \\
  \dot{X}_z
\end{bmatrix} =
\begin{bmatrix}
  I & 0 & 0 \\
  0 & I & 0 \\
  -M^{-1}K & -M^{-1}C & M^{-1}F_{\text{track}}
\end{bmatrix}
\begin{bmatrix}
  X_r \\
  X_y \\
  Y_{\text{track}}
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  0 \\
  M^{-1}F_{\text{act}}
\end{bmatrix} U_{\text{act}} +
\begin{bmatrix}
  0 \\
  0 \\
  M^{-1}F_{\text{track}}
\end{bmatrix} \dot{Y}_{\text{track}} \quad \ldots (5.36)
\]

The inputs to this state-space model are segregated into forces associated with actuators, either secondary or inter-vehicle, lateral wheelset and track cross-level inputs. The outputs from the system are displacements, velocities, accelerations of the vehicle body, and displacements and velocities of both the leading and trailing bogies. These variables are useful in the later interconnection of vehicles into a train model, and in the assessment of suspension performance.
Chapter 5: Modelling lateral train dynamics

Performance assessment methods for the lateral model bear some similarity to its vertical counterpart. It is still paramount to maintain adequate suspension deflection clearances and provide acceptable ride quality, hence the choice of model output variables. A more in depth analysis of suspension performance assessment methods is given in Chapter 6. It is assumed in Chapter 6 that models for both the single vehicle and train models are represented in the general state-space format shown in equations (5.37) and (5.38).

\[
\dot{X}_L = AX_L + BU_{ACTY} + W\dot{Y}_{TRACK} \quad \ldots (5.37)
\]

\[
Y_L = CX_L + D_\alpha U_{ACTY} + D_\omega \dot{Y}_{TRACK} \quad \ldots (5.38)
\]

The single vehicle parameters for the lateral model used throughout this thesis are shown in Table (5.1). These parameters are typical of a modern high-speed passenger coach, and although they are not specific to any particular rolling stock they are based on a best current practice study performed by an industrial partner, Bombardier Prorail Ltd, involved in this collaborative research project. The derivation of many of these parameters highlights the historically heuristic nature of suspension design. Improvements are based on notional modifications to existing practice using rules of thumb and analysis of trends which ultimately leads to an optimised suspension. Trade-offs and engineering design decisions need to be made in the choice of these parameters. The ride quality-suspension deflection trade-off discussed in Chapter 1 also applies in the lateral direction. It is generally considered that lateral ride quality is now the most difficult suspension design problem for railway vehicles. The softer the suspension the better the isolation but this is obtained at the expense of increased static and dynamic deflections. These deflections must be limited to keep the vehicle within the kinematic gauge. A number of design decision therefore need to be made regarding the lateral suspension parameters. Decisions relating to the bogie weights, primary suspension settings, and wheelset conicities all need to be made; the possibility of kinematic instability of wheelsets arises if these parameters are not designed properly. The trend is to reduce bogie mass and inertia because of its beneficial impact on vehicle speed.
**Table 5.1 - Parameters of the lateral model**
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<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bogie yaw damping</td>
<td>$c_{bgy}$</td>
<td>10000 (Nmsrad$^{-1}$)</td>
</tr>
<tr>
<td>Bogie-bogie spacing</td>
<td>$l_b$</td>
<td>19 (m)</td>
</tr>
<tr>
<td>Semi bogie-bogie spacing</td>
<td>$l_t$</td>
<td>9.5 (m)</td>
</tr>
<tr>
<td>Wheel-wheel spacing</td>
<td>$l_w$</td>
<td>2.5 (m)</td>
</tr>
<tr>
<td>Semi wheel-wheel spacing</td>
<td>$l'_w$</td>
<td>1.25 (m)</td>
</tr>
<tr>
<td>Vehicle + gangway length</td>
<td>$l_v$</td>
<td>27 (m)</td>
</tr>
<tr>
<td>Semi vehicle - vehicle spacing</td>
<td>$l_{hz}$</td>
<td>13.5 (m)</td>
</tr>
<tr>
<td>Bogie centre to axle</td>
<td>$l_{pa}$</td>
<td>1.25 (m)</td>
</tr>
<tr>
<td>Semi vehicle width</td>
<td>$l_{wd}$</td>
<td>2 (m)</td>
</tr>
<tr>
<td>Semi bogie width</td>
<td>$l_{bw}$</td>
<td>1.3 (m)</td>
</tr>
<tr>
<td>Primary lateral position</td>
<td>$l_{py}$</td>
<td>1 (m)</td>
</tr>
<tr>
<td>Airspring lateral spacing</td>
<td>$l_{asy}$</td>
<td>0.9 (m)</td>
</tr>
<tr>
<td>Actuator 1 longitudinal spacing</td>
<td>$l_{s1}$</td>
<td>9.5 (m)</td>
</tr>
<tr>
<td>Actuator 2 longitudinal spacing</td>
<td>$l_{s2}$</td>
<td>9.5 (m)</td>
</tr>
<tr>
<td>Body- bogie vertical spacing</td>
<td>$l_{ox}$</td>
<td>1.15 (m)</td>
</tr>
<tr>
<td>Wheel conicity</td>
<td>$w_\lambda$</td>
<td>0.05</td>
</tr>
<tr>
<td>Wheelset radius</td>
<td>$r_0$</td>
<td>0.46 (m)</td>
</tr>
<tr>
<td>Wheelset model parameter</td>
<td>$f_{11}$</td>
<td>90400000</td>
</tr>
<tr>
<td>Wheelset model parameter</td>
<td>$f_{22}$</td>
<td>78400000</td>
</tr>
<tr>
<td>Wheelset gauge length</td>
<td>$l_g$</td>
<td>1.4 (m)</td>
</tr>
<tr>
<td>Gravitational stiffness</td>
<td>$k_c$</td>
<td>126000000 (Nm$^{-1}$)</td>
</tr>
</tbody>
</table>

**Table 5.1 - Parameters of the lateral model**

The limiting factor on bogie weight is the need to provide a robust frame capable of enduring shock loads. The primary suspension needs to ensure running stability of the wheelset, but also
Chapter 5: Modelling lateral train dynamics

needs to be compliant to ensure good curving performance. Reductions in wheelset conicity will also permit higher vehicle speeds but at the expense of worse curving performance. These are just a few of the many design decisions which need to be made. Table (5.1) reflects these design decisions and represents a best current design for a high-speed passenger vehicle.

5.2.1 - Validation of the lateral vehicle model

Due to the 'in-house' construction of the single vehicle model, a number of validation tests need to be performed on the model to verify its accuracy. As was the case for the vertical model, two types of test are performed. The first is simply an analysis of the system modes to ensure compatibility with the modes of the full-scale vehicle. The second is a series of transient tests aimed at ensuring that the vehicle model behaves in a manner which emulates the real vehicle. The tests are based on injecting secondary and inter-vehicle active forces and noticing the direction and degree of the resulting body and bogie motions. Lateral wheelset displacements are also applied to the model and resulting wheelset and vehicle motions are again observed to ensure parity with the full-scale equivalent.

The modes present in the model are shown in Table (5.2). These modes are all very close to the expected industry norms.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body upper sway</td>
<td>1.5</td>
<td>29.0</td>
</tr>
<tr>
<td>Body lower sway</td>
<td>0.6</td>
<td>12.0</td>
</tr>
<tr>
<td>Body yaw</td>
<td>1.0</td>
<td>24.0</td>
</tr>
<tr>
<td>Bogie lateral</td>
<td>25.2</td>
<td>10.0</td>
</tr>
<tr>
<td>Bogie yaw</td>
<td>45.6</td>
<td>14.0</td>
</tr>
<tr>
<td>Bogie roll</td>
<td>14.7</td>
<td>20.0</td>
</tr>
</tbody>
</table>

Table 5.2 - Single vehicle dynamic modes
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Transient tests for the single vehicle model are shown in the series of figures (5.6) - (5.11). The first pair, figures (5.6) and (5.7), show the leading wheelset lateral displacement and yaw given a step lateral misalignment in the track of 10mm at a speed $55\text{ms}^{-1}$. The inputs to the four wheelsets are time-delayed according to the vehicle speed and its internal geometry. Figure (5.6) shows the leading wheelset ultimately moving laterally 10mm to run along the misaligned track. Figure (5.7) shows the intrinsic steering ability of a coned wheelset as it steers to null the rolling radius difference between the axle ends.

The second pair of figures, (5.8) and (5.9), shows the vehicle response due to a 1 ton secondary actuator force being applied to $f_{\text{uml1}}$. This actuator is on the front left-hand corner of the vehicle as shown by Figure (5.3). Figure (5.8) shows the body yaw motion as a result of this input. It has a positive steady-state response due to the extension of the passive suspension across the actuator. Figure (5.9) shows the lateral bogie response $y_{\text{uml}}$, which has a nearly zero steady-state response. This is not surprising owing to most of the actuator force being reacted by the lateral secondary suspension; very little force is reacted by the primary suspension and hence there is only transient motion.

The final pair of tests shows responses to a 1 ton inter-vehicle actuator force $f_{\text{dmt1}}$ being applied to the front end of the vehicle. Figure (5.10) shows the positive steady-state response of body yaw, which is due to the lateral suspension on the leading bogie having a greater deflection, a consequence of the need to provide a reaction torque to counter the torque generated by the inter-vehicle force. Reaction of the inter-vehicle force requires that the body as a whole moves in the direction of this force in order to generate a reaction force from the secondary suspension, and this results in a positive steady-state response for body displacement as shown in Figure (5.11).
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Figure 5.6 - Wheelset displacement
Figure 5.7 - Wheelset yaw

Figure 5.8 - Body yaw
Figure 5.9 - Bogie displacement

Figure 5.10 - Body yaw
Figure 5.11 - Body lateral
5.3 - Lateral train modelling

As stated earlier, the method used to construct the lateral train model is similar to the vertical model developed in Chapter 4. The single vehicle lateral model recently developed in section 5.2 is used in conjunction with a library of secondary and inter-vehicle actuators to assemble a full train model having 'n' repetitively connected vehicles.

Figure (5.2) showed the interconnection of the component sub-blocks. Some of the anomalies in the structure of this figure. For example the need to have formatting blocks in addition to the standard library of building blocks, are due to the prerequisites of the Matlab functions: 'mvcon', and 'mvblk' which are used to interconnect the various sub-blocks. For further information the reader is referred to [Matlab manual - 1992] in association with some of the issues raised in Chapter 4.

The sub-blocks in Figure (5.2) fall into the following categories:

- Pre-format block m1
- Pre-format block m2
- Railway vehicle block m
- Secondary actuator block m
- Inter-vehicle actuator block m

The input and output blocks are simply unity gain blocks and are essential to the functioning of the aforementioned Matlab utilities. Each of the sub-blocks will now be discussed and an account of their functionality given.
Chapter 5: Modelling lateral train dynamics

5.3.1 - Pre-format blocks 'm1'

This block is responsible for selecting the appropriate input signals required by vehicle 'm' from the combined input vector 'V_Y1', which contains all the input signals for the train model. The mapping performed by this block is shown in equation (5.38). Primarily it selects track lateral and roll velocities, but it also selects the inter-vehicle command signal 'f_{comy}'. This signal is later fed straight through the 'railcar' dynamic block for later manipulation by the inter-vehicle actuator dynamic block.

\[ V_{Y1} = \begin{bmatrix} \dot{y}_{vml1} \\ \dot{p}_{vml1} \\ \dot{y}_{vml2} \\ \dot{p}_{vml2} \\ \dot{y}_{vml21} \\ \dot{p}_{vml21} \\ \dot{y}_{vml22} \\ \dot{p}_{vml22} \\ f_{com11} \\ f_{com12} \\ f_{com21} \\ f_{com22} \\ f_{com2m} \\ \end{bmatrix} = V_{Y4} = \begin{bmatrix} \dot{y}_{vml1} \\ \dot{p}_{vml1} \\ \dot{y}_{vml2} \\ \dot{p}_{vml2} \\ \dot{y}_{vml21} \\ \dot{p}_{vml21} \\ \dot{y}_{vml22} \\ \dot{p}_{vml22} \\ f_{com11} \\ f_{com12} \\ f_{com21} \\ f_{com22} \\ f_{com2m} \\ f_{comy} \\ \end{bmatrix} \quad \ldots (5.38) 

5.3.2 - Pre-format blocks 'm2'

This block selects the secondary actuator command signals from the global vector 'V_Y1'. The mapping is shown in equation (5.39). The mapping is different for 'odd' and 'even' numbered vehicles. The reasoning behind this terminology is related to the protocol used to transfer signals between blocks; 'odd' numbered vehicles passing information on the top rungs of a vector, 'even' numbered vehicles passing information on the bottom rungs. Equation (5.39) illustrates how in
both cases, the four secondary actuator command signals associated with vehicle 'm' are sent to the dynamic actuator block associated with this particular vehicle.

\[
\begin{align*}
\begin{bmatrix}
  \dot{\gamma}_{wm11} \\
  \dot{\gamma}_{wm12} \\
  \dot{\gamma}_{wm21} \\
  \dot{\gamma}_{wm22} \\
  f_{cm1y} \\
  f_{cm1z} \\
  f_{cm2y} \\
  f_{cm2z} \\
  \vdots \\
  f_{cm2y} + \dot{\rho}_{wm22}
\end{bmatrix} & \quad \rightarrow \quad
\begin{bmatrix}
  \dot{\gamma}_{wm11} \\
  \dot{\gamma}_{wm12} \\
  \dot{\gamma}_{wm21} \\
  \dot{\gamma}_{wm22} \\
  f_{cm1y} \\
  f_{cm1z} \\
  f_{cm2y} \\
  f_{cm2z} \\
  \vdots \\
  f_{cm2y} + \dot{\rho}_{wm22}
\end{bmatrix} & \text{\textit{EVEN}} \\
\begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix} & \quad \& \quad \\
\begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix} & \text{\textit{ODD}}
\end{align*}
\]
5.3.3 - Railway vehicle blocks 'm'

The lateral dynamics of a single vehicle were modelled and represented in a state-space form as outlined in section 5.2. The outputs from that model are useful in the development of a full train model. The inputs and outputs of this model need only slight adjustment in order to adapt themselves to the protocol used in Figure 5.2. This adaptation varies according to whether the vehicle is 'even' or 'odd' numbered. Equation (5.40) gives the modified mapping in both cases. The large number of zeros in the output vector is to ensure compatibility with other sub-blocks in the structure. Additional to this readjustment of the input and output vectors is the direct feed through of the command signal 'f_{emly}' which is subsequently used by the inter-vehicle actuator block.
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\[
V_{T4} \cdot V_{T15} \cdot V_{T11} = V_{yy} \quad \text{&} \quad \begin{bmatrix}
\dot{y}_{\text{even}} \\
\dot{y}_{\text{odd}} \\
\dot{\theta}_{\text{even}} \\
\dot{\theta}_{\text{odd}} \\
\dot{\phi}_{\text{even}} \\
\dot{\phi}_{\text{odd}} \\
\dot{\psi}_{\text{even}} \\
\dot{\psi}_{\text{odd}} \\
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} \quad \text{&} \quad \begin{bmatrix}
\psi_{\text{even}} \\
\psi_{\text{odd}} \\
\psi_{\text{even}} \\
\psi_{\text{odd}} \\
\psi_{\text{even}} \\
\psi_{\text{odd}} \\
\psi_{\text{even}} \\
\psi_{\text{odd}} \\
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

(5.40)
5.3.4 - Secondary actuator blocks 'm'

Equation (5.41) shows the mapping between vectors $V_{Y12}$ and $V_{Y15}$ with reference to Figure (5.2). It is apparent that the signals in vector $V_{Y12}$ contain variables which are required by secondary lateral actuators, namely: body lateral and yaw velocities, bogie lateral velocities, and actuator force command signals. These signals are then processed by the secondary actuator dynamic block and four actuator force outputs associated with vehicle 'm' are generated. The format is again different for 'odd' and 'even' numbered vehicles. The internal dynamic of this sub-block is a simple uncoupled amalgamation of the dynamics of four individual actuators. These may be chosen from any of the actuator technologies outlined in Chapter 3, or the sub-block may be set to a 'null' block if no secondary actuators are required in the train model.
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\[
\begin{bmatrix}
V_{y13} \\
V_{y31} \\
V_{y33}
\end{bmatrix}
= \begin{bmatrix}
\gamma_{bn} \\
\Psi_{bn} \\
\Phi_{bn} \\
\hat{\gamma}_{bn} \\
\hat{\Psi}_{bn} \\
\hat{\Phi}_{bn} \\
\gamma_{m1} \\
\Psi_{m1} \\
\Phi_{m1} \\
\hat{\gamma}_{m1} \\
\hat{\Psi}_{m1} \\
\hat{\Phi}_{m1} \\
\gamma_{m2} \\
\Psi_{m2} \\
\Phi_{m2} \\
\hat{\gamma}_{m2} \\
\hat{\Psi}_{m2} \\
\hat{\Phi}_{m2} \\
f_{\text{chmp}}/0 \\
f_{m1y}/0 \\
f_{m1y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0 \\
f_{m2y}/0
\end{bmatrix}
\]

\[\ldots(5.41)\]
5.3.5 - Inter-vehicle actuator blocks 'm'

This sub-block utilises the same actuator technologies outlined previously in section 5.3.4. The actuators may of course have different sizes. This sub-block requires only one actuator for each sub-block. The inputs and outputs for this particular sub-block are shown in equation (5.42). The inter-vehicle actuator obviously requires information about adjacent vehicle motions together with a force command signal, which are all contained within the input vector. The output vector contains the true force generated by the actuator. The mapping is once again different for 'even' and 'odd' numbered vehicles.
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\[
\begin{align*}
V_{y1} + \dot{V}_{y1,0} &= 0 & V_{y1} - \dot{V}_{y1,0} &= \sum f_{\text{cham}}/0 \\
\ddot{y}_b + \dot{\psi}_b &= 0 & \dot{\ddot{y}}_b &= \psi_b \\
\ddot{y}_1 + \dot{\psi}_1 &= 0 & \dot{\ddot{y}}_1 &= \psi_1 \\
\ddot{y}_m + \dot{\psi}_m &= 0 & \dot{\ddot{y}}_m &= \psi_m \\
\dot{f}_{\text{cham}}/0 &= 0 & \ddot{f}_{\text{cham}}/0 &= 0
\end{align*}
\]

... (5.42)
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5.3.6 - Post-format blocks 'm'

The post format blocks strip off redundant information from the outputs of the railway dynamic sub-blocks. The resulting output forms the basis for the overall system output. The mapping is shown in equation (5.43) for 'even' and 'odd' numbered vehicles. The final output consists of vehicle body displacements and accelerations together with bogie displacements. These are useful in assessing suspension performance. The output vector $V_{Y25}$ also contains the secondary and inter-vehicle actuator forces which are useful in assessing actuator force and power requirements.
### Chapter 5: Modelling lateral train dynamics

\[
\begin{align*}
  \begin{bmatrix}
    \mathbf{V}_{yy} \\
    \mathbf{V}_{yy}' \\
    \mathbf{V}_{yy}'' \\
    \mathbf{V}_{yy}''' \\
    \mathbf{V}_{yy}'''' \\
    \mathbf{V}_{yy}''''
  \end{bmatrix}
  & \rightarrow
  \begin{bmatrix}
    \mathbf{V}_{yy} \\
    \mathbf{V}_{yy}' \\
    \mathbf{V}_{yy}'' \\
    \mathbf{V}_{yy}''' \\
    \mathbf{V}_{yy}'''' \\
    \mathbf{V}_{yy}''''
  \end{bmatrix}
  \quad \text{EVEN} \\
  \begin{bmatrix}
    \mathbf{Y}_{km} \\
    \mathbf{Y}_{km}' \\
    \mathbf{Y}_{km}'' \\
    \mathbf{Y}_{km}''' \\
    \mathbf{Y}_{km}'''' \\
    \mathbf{Y}_{km}''''
  \end{bmatrix}
  & \rightarrow
  \begin{bmatrix}
    \mathbf{Y}_{km} \\
    \mathbf{Y}_{km}' \\
    \mathbf{Y}_{km}'' \\
    \mathbf{Y}_{km}''' \\
    \mathbf{Y}_{km}'''' \\
    \mathbf{Y}_{km}''''
  \end{bmatrix}
  \quad \text{ODD}
\end{align*}
\]
5.4 - Validation of the lateral train model

Analysis of the train modes does not reveal a significant amount of useful information in terms of the overall validation of the train model. The train modes are basically a repetition of the individual modes; there is of course some difference due to the coupling, but this does not disclose any discernible information. Instead a series of transient tests is used to reveal how well the model emulates the full-scale equivalent. Three tests were applied to the full train model. The first is a sudden misalignment in the lateral position of the track. The other two are force injection tests on the secondary and inter-vehicle actuators. All the tests are performed on a 3 vehicle train with 'ideal' secondary and inter-vehicle actuators.

Figures (5.12) and (5.13) are responses to a sudden track misalignment of 10mm when travelling at 55ms\(^{-1}\). Figure (5.12) shows the lateral body motion of vehicle 2. The misalignment input is time-delayed to each wheelset and it is interesting to note this time-delay in the response of the second vehicle. It ultimately settles with a net displacement of 10mm. Figure (5.13) is a plot of body yaw for the same vehicle subject to the same input. The body exhibits a transient negative yaw due to vehicle 1 initially having positive yaw as it negotiates the misalignment, the response then has a positive excursion as vehicle 2 negotiates the misalignment and ultimately settles with a zero steady-state value, with the whole train having moved laterally 10mm.

Figures (5.14) and (5.15) are responses to a 1 ton force step input on the secondary actuator \(f_{axy}\), attached to the trailing bogie of the second vehicle. Figure (5.14) shows the yaw motion of the second vehicle. This response is predictable: the body will have a net positive yaw due to the location of this particular actuator and the associated extension of the lateral passive suspension. Figure (5.15) shows the negative steady-state response of the lateral motion of vehicle 3, which is caused by the inter-connection between vehicles 2 and 3, and has a negative value because of the negative movement of the end of vehicle 2.
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The final test is a 1 ton force step on the inter-vehicle actuator $e_{oz_2}$. Figures (5.16) and (5.17) show the yaw response of vehicles 3 and 1 respectively. The arguments justifying these responses are akin to those given for similar tests on the vertical model of Chapter 4. Figure (5.16) shows the large negative yaw of vehicle 3, the inter-vehicle actuator force being reacted by extension of the spring-damper combination in parallel with the actuator as well as secondary suspension forces from adjacent vehicles, the net effect being an extension of the actuator with resulting negative yaw of vehicle 3. Figure (5.17) shows the steady-state negative yaw of vehicle 1, which is due to vehicle 2 having a positive steady-state yaw angle and hence the leading end of that vehicle moving in a positive lateral direction. The inter-vehicle connection between vehicles 1 and 2 force vehicle 1 to have a negative a steady-state yaw angle.

![Figure 5.12 - Train model/vehicle 2 lateral response](image1)

![Figure 5.13 - Train model/vehicle 2 yaw response](image2)
5.5 - Summary

This verified lateral train model is used in subsequent chapters to design and analyse secondary and inter-vehicle active suspension control laws. The models are in a convenient form, actuator forces and track velocities are separate inputs permitting easy integration with active suspension control laws.
Chapter 6
Suspension performance assessment methods

6.1 - Overview

Ride quality and suspension performance are the nucleus of this thesis, and acceptable performance assessment methods play a pivotal role throughout. A number of issues have been addressed in preceding chapters: the potential for improvements in suspension performance through active suspension, the influence of 'real' actuators, and the development of vertical, lateral and train models which are to be subsequently used in the development of active suspension controllers. All of these aspects of the work are reliant upon accurate assessment of the performance of the suspension. The introduction given in Chapter 1 gave a brief overview of the two important aspects of suspension performance, these being:

- Ride quality
- Suspension deflection

The one-mass and two-mass models outlined in the introduction used raw body acceleration as a measure of ride quality. This is not an unreasonable choice of assessment variable, having been used in the past by a number of authors, [Crolla and Abdel-Hady - 1991] for example. The human body does not perceive the broad range of acceleration frequencies equally, and for this reason a more refined approach involves the use of a frequency weighting function as shown in Figure 6.1 to ascertain a more accurate index of ride quality [Tanifuji - 1988]. This function is
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drawn from [ISO 2631 - 1978] and is based upon empirical data collected from human endurance tests. The tests were conducted to obtain the average acceleration levels at a number of frequencies which were just comfortable for human subjects to abide for periods of 45 minutes to 4 hours as shown in Figure 6.2. Clearly the time factor plays a key role in the perception; lower levels of acceleration are required for a longer journey time. These results may be inverted to form a weighting function which heavily weights the frequencies which are less tolerable as shown in Figure 6.1.

![Figure 6.1 - Ride quality weighting function](image-url)
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Figure 6.2 - Ride endurance tests
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Many authors have quoted active suspension results based on this weighting function [O’Neill and Wale - 1994]. Ride quality is a highly subjective quantity, frequently confused by passengers with the appearance of a vehicle, i.e. cleanliness, décor, and noise levels. In the author’s opinion the raw accelerations give a good comparative measure of vehicle ride quality without the additional complication of weighting factors. For these reasons the ride assessment methods described in this chapter will all be based upon the use of raw body accelerations.

Ride quality in a real-life railway vehicle is not a simple matter of measuring a single acceleration point and from this deducing the ride quality of the entire vehicle. Several authors have defined methodologies for incorporating the extra degrees of freedom within a ride index [Hač - 1986], a more rigorous definition of ride assessment is defined in this thesis for both the vertical and lateral directions, and for both an isolated vehicle and an entire train of vehicles.

Suspension deflection is another important issue in the overall performance. As highlighted in the introduction, this has an inherent trade-off with ride quality. It is a constraint rather than a continuous variable as in the case of the ride quality index. A number of authors have treated it as the latter purely since in many cases it suits the active control law design methodology being used at the time. Suspension deflection is a constraint imposed because of the detrimental effect which contact with the limits of suspension travel would have upon ride quality. The best suspension designs are those which utilise the available suspension deflections to the full when subjected to the most demanding terrain - bumpstop contact must be avoided even in these demanding situations. Throughout this thesis, suspension deflection is treated as a constraint. A performance index can not be ascribed to suspension deflection, it is a quantity that is either violated or not under demanding circumstances. This procedure obviously necessitates the demand to have some predefined form of ‘worst terrain’ which each vehicle whether it be active, or passive, must satisfy. The methodology used throughout this thesis is to ascertain whether the suspension design violates this criteria, and if so, a reiteration of the design process must be performed. Suspension deflection for the one-mass and two-mass models detailed in Chapter 1 was a clearcut issue, defined as being the maximum secondary suspension excursion. In terms
of the vertical and lateral models developed in Chapters 4 and 5 it is complicated by the number
of bogies present. The methodology used in this case is to simply take the worst case deflection
of any particular bogie given that the vehicle or train is negotiating this worst case track
topography. A more rigorous mathematical definition of the previous statement is given in the
subsequent sections.

The mathematical techniques used to assess suspension performance in this chapter bear
similarity to the techniques used by other researchers. They are adapted to suit the input and
output format of the train models developed in Chapters 4 and 5. Figure 6.3 shows the general
structure which applies to both the vertical and the lateral directions. The random track
irregularities are the causative agents in vehicle excitation. Track design features namely curves
and vertical transitions, also give rise to vehicle accelerations but are not used in ride quality
assessment. The outputs of the vehicle or train models have a fixed format, which is naturally
different between the vertical and lateral models, the output format is of course pre-defined for
both cases. This pre-specified output format gives rise to a suite of analysis programs which can
interface with the models developed in previous chapters. This input and output format is
independent of whether an active suspension is used in conjunction with the vehicle model as
illustrated in Figure 6.3. The active suspension in this scenario plays an internal role.

![Figure 6.3 - Assessment path](image)

"Chapter 6: Suspension performance assessment methods"
Chapter 6: Suspension performance assessment methods

The structure of this chapter will be as follows:

- An account of track irregularities (vertical and lateral)
- An account of track design features (vertical and lateral)
- Frequency domain assessment techniques
- Time domain assessment techniques
- Time history methods

As recently discussed, track irregularities are the primary cause of ride degradation on a train and therefore some time is spent discussing the different track qualities on which high speed trains run, and the mathematical techniques used to describe them. Track design features (curves and vertical transitions) are obviously required and an account of some of these features is then given, in particular the worst case scenario a train must negotiate, which is necessary in determining the suspension response to this feature. Two analysis techniques are used to assess performance, these being frequency domain methods [Tanifuji - 1988], and time domain methods [Müller et al - 1980]. Both of these techniques require linear models of the combined vehicle and active suspension. A small scale test rig is developed in Chapter 9, which produces 'time histories' of various signals, for this reason the two techniques described previously need to be abandoned in favour of 'time history' techniques. Many of these techniques may be found in the literature, but are adapted to suit the train models addressed in this thesis. There is a dearth of information relating to the assessment of ride quality on a train wide basis. This thesis proposes a methodology to fulfill this task.

6.2 - Track irregularities

Track irregularities are the primary cause of ride degradation, although irregularities on the wheels will also give rise to vehicle excitation. A number of features may be identified as being localised rail irregularities, and the term 'stochastic' or 'random' will be used to describe these features. This is because they are unintentional features resulting from badly rolled track, or
misalignment in their installation. This term is used in a liberal sense as many of these features are in the strictest sense not stochastic: jointing between rail sections, sleeper spacing, and rail corrugation are not stochastic but fall into the broad category of randomly occurring track irregularities. Continuously welded rail or rail without ballast of course does not have some of these features, nevertheless all track contains some degree of random irregularity.

Significant differences exist between vertical and lateral irregularities. This section will give separate accounts for both cases.

6.2.1 - Vertical track quality

Track recording vehicles have tested track irregularity on a number of high-speed routes. These vehicles record data with respect to distance travelled. Figure 6.4 shows spectra from a variety of routes produced in this way. The test runs in this instance were 70km test runs performed on both the 'East coast main-line', and the 'Midland main-line', (U.K.), measured every 0.25m using a 70m high-pass filter. Filtering the output of such measurements is usual; long wavelength measurements begin to lose accuracy due to limitations in the measuring equipment. Also long wavelength inputs are largely determined by the intended track design features.

The plots shown in Figure 6.4 show a number of peaks due to repetitive features such as rail joints. Lower amplitudes are seen at higher spatial frequencies due to the ease with which these may be reduced by track maintenance, rail straightening and grinding. The plots are frequently approximated by the fourth order relationship shown by equation (6.1) where \( \Omega_i \) is the vertical track roughness and \( f_i \) is the spatial frequency.

\[
S_i(f) \;
\frac{\Omega_i}{f_i^2 \cdot 5.86 f_i^2 \cdot 17.29 f_i} \quad \text{m}^2 \text{(cycle/m)}^i
data (6.1)
\]
Equation (6.1) has break frequencies at 3m and 6m track wavelengths. These cut-offs obviously attenuate the higher frequencies. A less accurate 'best-fit' fit to this experimentally derived data is obtained if the denominator of this equation is a simple inverse-square relationship as shown by equation (6.2). Equation (6.2) is widely accepted as being an appropriate expression of the vertical spatial spectrum of railway track.

\[ S_s(f) = \frac{\Omega}{f_i^2} \quad m^2 (\text{cycle/m})^4 \quad \ldots (6.2) \]

The information presented by equation (6.2) needs to be converted into a temporal form if it is to be used in dynamic analysis. The dynamic modes of a railway vehicle lie between 0.1 and 20 Hz, it will therefore be excited by track inputs in this frequency range. The relationship
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between the spatial wavelengths expressed by equation (6.2) and the temporal excitation is velocity dependent; a 10m track wavelength would produce a 1 Hz excitation if the vehicle travelled at 10ms\(^{-1}\). The same wavelength would excite the vehicle with a different frequency at a different speed. The relationship between spatial and temporal frequency is given by equation (6.3).

\[
f_s \, (cycles \, m^{-1}) = \frac{f_t \, (cycles \, s^{-1})}{v \, (m \, s^{-1})} \quad \ldots (6.3)
\]

Conversion of the spatial expression (6.2) to a temporal one is given by the series of equations (6.4) to (6.9). Equation (6.4) is a modification of equation (6.2) to express the track wavelengths in terms of the temporal frequency 'f_t'.

\[
S_s(f) = \frac{\Omega v^2}{f_t^2} \quad m^2(\text{cycle}/m)^1 \quad \ldots (6.4)
\]

In order to convert this spectrum to one with a temporal base, the division shown by equation (6.5) needs to be performed, giving rise to the expression shown by equation (6.6).

\[
S_r(f) \quad m^2(\text{cycle}/s)^1 = \frac{S_s(f)}{v \, (m \, s^{-1})} \quad m^2(\text{cycle}/m)^1 \quad \ldots (6.5)
\]

\[
S_r(f) = \frac{\Omega v}{f_t^2} \quad m^2(\text{Hz})^1 \quad \ldots (6.6)
\]

The vehicle models developed are based upon the input being a velocity rather than a displacement. This is due to the wheelsets being connected directly to dampers, which necessitates track velocities as an input if the models are to be expressed in a state-space form. In order to convert the vertical track displacement spectrum given by equation (6.6) into its
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derivative a number of operations need to be carried out. The first involves converting equation (6.6) so that it is expressed in terms of 'radians' rather than 'cycles' as shown by equation (6.7).

\[ S_r(f) = \frac{\Omega_r v}{2\pi f^2} \text{ m}(rad/s)^{-1} \]  \hspace{1cm} \ldots (6.7)

A simple multiplication is required to convert the spectrum into an expression for its derivative as shown by equation (6.8).

\[ \hat{S}_r(f) \text{ (ms}^{-1})(rad/s)^{-1} = S_r(f) \text{ m}(rad/s)^{-1} \times (2\pi f)^2 \]  \hspace{1cm} \ldots (6.8)

The expression for the spectrum of the vertical track velocity is derived from equations (6.7) and (6.8) and is given by equation (6.9).

\[ \hat{S}_r(\omega) = 2\pi \Omega_r v \text{ (ms}^{-1})(rad/s)^{-1} \]  \hspace{1cm} \ldots (6.9)

The final adjustment required to equation (6.9) is the conversion of the expression back to representation in terms of 'cycles' rather than 'radians'. The final expression is given in equation (6.10).

\[ \hat{S}_r(f) = (2\pi)^2 \Omega_r v \text{ (ms}^{-1})(Hz)^{-1} \]  \hspace{1cm} \ldots (6.10)

The vertical track spectrum is 'flat' over all frequencies, and it is in essence 'white noise' with a Gaussian distribution. The approximations made earlier ignoring the fourth order expression in favour of the simpler inverse-square relationship give rise to the infinite bandwidth of this white noise. If the other terms were included, a high frequency roll-off of the expression given by equation (6.10) would occur giving rise to a finite bandwidth velocity spectrum as shown by Figure 6.5.
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Figure 6.5 - Vertical track spectra (Temporal description)

The spectra shown in Figure 6.5 were calculated at 55ms\(^{-1}\) (200kmh\(^{-1}\) or 125 mph) which is typical of a high-speed passenger train. The dynamic modes of the vehicle are between 0.1 and 20 Hz, and even the more accurate expression shown in Figure 6.5 is relatively constant over this region, thus justifying the use of the simple expression given by equation (6.10) in the assessment of vehicle behaviour. If the vehicle travelled faster this expression would be more accurate; the parity between the two expressions given by equations (6.1) and (6.2) is better at lower spatial frequencies, the frequencies responsible for vehicle excitation at higher speeds. The worsening parity at lower speeds is neglected since ride quality is only critical at the higher speeds.

6.2.2 - Lateral track quality

The lateral track irregularities have a different structure to the vertical irregularities described in section 6.2.1. Similar techniques are used to convert a spatial description of track irregularities into a temporal form. This can then be used to analyse the response of the vehicle. Figure 6.6 shows empirical test data for both the 'East Coast main-line', and the 'Midland main-line', 
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(U.K.), measured every 0.25m using a 70m filter.

Figure 6.6 - Lateral track spectra

A 'best-fit' curve is again used to approximate the measured spectra. An expression for this curve is given by equation (6.11), where the symbols have the usual meaning.

\[ S_r(f_r) = \frac{\Omega v^2}{f_r} \quad \text{m}^2\text{(cycle/s)}^{1} \quad \ldots (6.11) \]

The frequency-dependance of the spatial spectrum of the lateral irregularities is noticeably different from that of the vertical irregularities. An identical procedure is used in the derivation of the temporal lateral velocity spectrum as used in section 6.2.1 for the vertical case. Equations (6.12) to (6.16) show this evaluation. A number of steps are repetitions of equations (6.3), (6.5), and (6.8) and are omitted for brevity.

\[ S_s(f_s) = \frac{\Omega v^3}{f_s} \quad \text{m}^2\text{(cycle/m)}^{1} \quad \ldots (6.12) \]
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The temporal expression given by equation (6.16) is the final result. The lateral track velocity spectrum is not 'flat' as was the case in the vertical direction, rather it has a steady roll-off with increasing frequency.

\[ S_{x}(f) = \frac{\Omega_{r}v^{2}}{f_{r}^{2}} \quad m^{2} \text{(cycle/s)}^{3} \quad \ldots (6.13) \]

\[ S_{x}(f) = \frac{\Omega_{r}v^{2}}{2\pi f_{r}^{2}} \quad m^{2} \text{(rad/s)}^{3} \quad \ldots (6.14) \]

\[ \hat{S}_{x}(\omega) = \frac{2\pi \Omega_{r}v^{2}}{f_{r}} \quad (m \text{s}^{-1})(\text{rad/s})^{4} \quad \ldots (6.15) \]

\[ \hat{S}_{x}(f) = \frac{(2\pi)^{3} \Omega_{r}v^{2}}{f_{r}} \quad (m \text{s}^{-1})^{3} \text{(Hz)}^{3} \quad \ldots (6.16) \]

The vertical and lateral spectra developed here will be used later in sections 6.4, and 6.5 to determine the response of the vehicle models to these random track irregularities.

6.3 - Track design features

High-speed railway track contains 'design' features which are intentionally installed by the track civil engineers. These features include: the maximum curvature for a given line speed, the amount of installed track cant, and the profile of a vertical transition. These criteria are often set in response to the need to limit track shifting forces and the need to ensure passenger comfort.

Track design features will now be discussed for both the vertical and lateral cases.
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6.3.1 - Vertical design features

Figure 6.7 shows the criteria by which a vertical transition is designed. A curved profile is used to form the junction between 'flat' running and a constant gradient. The maximum vertical acceleration endured by a high-speed vehicle during such a transition is around 0.4 ms\(^{-2}\). A vehicle travelling at 55ms\(^{-1}\) may take up to 4 or 5 seconds (250 metres) to negotiate the transition depending upon the steepness of the gradient, this is a sufficiently long period to ensure settling of the dynamics of the suspension. For the purposes of dynamic analysis it makes no difference if the vehicle is subjected to a sustained vertical acceleration of 0.4ms\(^{-2}\) or to a transient one, the motive behind performing such a test being to establish the worst suspension deflection while negotiating such a feature. This would occur within the suspension's settling time. The worst case design feature used throughout this thesis for the vertical direction is the sustained acceleration profile shown in Figure 6.8.

The term 'deterministic' is used throughout this thesis to refer to the vertical response of the vehicle to the feature shown in Figure 6.8, which represents a worst case situation. Since such a feature is rarely encountered in reality this may appear conservative, but relaxation of this criteria would result in suspension designs which would in general perform better but would behave badly if subjected to this worst case input. The choice of the worst case deterministic feature is crucial to the overall design, the deterministic feature being the main cause of suspension deflection violation, an effect which must be avoided as stated in Chapter 1. It is important to subject all designs to this input, which provides standardisation and a mechanism whereby different designs may be compared.
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Max. vertical acceln. in vertical curve
0.2 to 0.6 \(^{\circ}\)/
(higher values are usually tolerated in hollows than in humps for reasons both of comfort and wheel-loading)

Minimum radius of vertical curve 500 to 1000m
(165 to 400m for LRV systems)

Vertical Transition usually provided by a constant radius vertical curve

Steepest gradient 1 in 35 approx.
(1 in 10 for LRV systems)

Figure 6.7 - Vertical design feature

Figure 6.8 - Vertical test profile
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An account of the mathematical technique used to evaluate the deterministic response of the train models developed in Chapter 4 to the worst case vertical track input is now given. Subsequent active suspension designs will have the same input and output formats as the models developed in Chapter 4 and can therefore also be used with this technique. Equations (6.17) and (6.18) summarise the format used for vertical train models irrespective of the number of vehicles.

\[
\dot{X}_v = AX_v \cdot BU_{ACTZ} \cdot W_\text{TRACK} \\
Y_v = CX_v \cdot DU_{ACTZ} \cdot D_\omega \ddot{Z}_\text{TRACK}
\]  

\ldots (6.17)  
\ldots (6.18)

The output vector '\(Y_v\)' contains displacements, velocities, and accelerations of various masses within the train model as outlined in Chapter 4. It is possible to pre-multiply this vector by the matrix '\(C_{DDV}\)' in order to get the output vector shown by equation (6.19), which contains all the suspension deflections associated with each bogie of the train.

\[
\begin{bmatrix}
Z_{n1} & lt \phi_{n1} - Z_{n1} \\
Z_{n2} & lt \phi_{n2} - Z_{n2} \\
\vdots & \vdots \\
Z_{n5} & lt \phi_{n5} - Z_{n5} \\
Z_{n6} & lt \phi_{n6} - Z_{n6}
\end{bmatrix} \cdot C_{DDV} \quad Y_v 
\]  

\ldots (6.19)

The input vector '\(Z_{\text{TRACK}}\)' is simply a vector of the four wheelset vertical motions, appended for each vehicle within the train. It is possible to construct a time delayed input vector for each wheel of the train as given by equation (6.20) in which the vector is formed from a standard function '\(\zeta(t)\)' which is the velocity input corresponding to the acceleration profile shown in Figure 6.8. This input is time-delayed according to the wheel spacing and vehicle speed, resulting in the input vector shown in equation (6.20).
The worst case suspension deflection may be found by performing the integration shown in equation (6.21). This simulates the vehicle behaviour in response to the track input described by equation (6.20), and outputs a positive value for the maximum suspension deflection experienced by any bogie within the train.

\[
DEFL_{DV} = \int_{-\infty}^{\infty} C_{DV}C_{e}^{4(t-\tau)}W\xi(\tau) \, d\tau
\]  
\[\ldots (6.21)\]

6.3.2 - Lateral design features

Figure 6.9 shows the criteria used to design a lateral track feature, which are set on the basis of providing a smooth transition between straight line and curved running. Limits are imposed because of restrictions on the maximum rate of cant, maximum rate of cant deficiency, and maximum curvature. The construction is formed from an initial transition having a linearly increasing curvature, followed by constant curvature. High-speed track usually has installed cant which again increases linearly through the transition to a steady value on the curve. The lateral train models developed in Chapter 5 were all based upon the use of linear wheelset creep models which are not valid during flange contact, for this reason only a shallow curve with no installed cant is used to test the lateral suspension. This is nevertheless a demanding test which must be endured by the lateral suspension.

Figure 6.10 shows the lateral acceleration profile applied to each wheelset of the vehicle, similar
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to the methods used previously with the vertical model. This steady rise to a lateral acceleration of 0.6ms\(^{-2}\) is typical of a real-life situation and corresponds to a vehicle running at 55ms\(^{-1}\) onto a steady 5km radius curve. The maximum rate of cant deficiency means that the transition to steady curving takes about 2½ second; in other words a 150m transition onto steady curving.

Figure 6.9 - Lateral design feature
For long curves the feature shown in Figure 6.10 typically persists for longer than the 5 seconds shown in this example. An 8 vehicle train can span over 200m in length requiring an observation of the dynamic behaviour of the train for well over 10 seconds after the first wheelset has reached the transition. The profile shown in Figure 6.8 can be applied to the lateral vehicle models developed in Chapter 5 in a similar manner to methods described previously in section 6.3.1.

The lateral model structure is similar to that given by equations (6.17) and (6.18), the deterministic excitation vector in this instance is different as shown by equation (6.22) which contains a series of zeros illustrating the absence of a track cant input. A selection matrix 'C_{DDL}' is used to construct the lateral suspension deflections from the large array of data contained within the system output vector.
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\[
\begin{bmatrix}
\zeta(t)
\end{bmatrix} = \begin{bmatrix}
0 \\
\zeta(t - \frac{L}{v}) \\
0 \\
\zeta(t - \frac{L}{v}) \\
0 \\
\vdots
\end{bmatrix} \quad \ldots \quad (6.22)
\]

An integration, shown by equation (6.23), is performed to determine the worst case lateral suspension deflection experienced while negotiating the aforementioned feature.

\[
DEFL_{L} = \int_{t_{0}}^{t} C_{DL} e^{\mu(t) - \tau} W(t) \zeta(\tau) d\tau. 
\]

\[
\ldots \quad (6.23)
\]

The integration time being of sufficient value to permit the train to negotiate the deterministic feature and allow settling of the internal states of the model.

6.4 - Frequency domain analysis techniques

The behaviour of a railway train may be separated into causes due to deterministic track features as outlined in section 6.3, and responses to stochastic track features as detailed in section 6.2. The ride quality may be inferred from responses to straight-line stochastic track despite the fact that vehicle accelerations will also arise as a result of deterministic features, largely because these features have been designed not to cause discomfort to the passengers. Suspension deflection arises because of both the stochastic and deterministic track features, although deterministic features are the predominant cause of the larger suspension excursions. This section will utilise the stochastic track descriptions given in section 6.2 and apply this excitation to both the vertical and lateral train models. The resulting body accelerations and suspension deflections will be
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evaluated. The stochastic body accelerations will be used to form a ride performance index. The
stochastic suspension deflections will be used in conjunction with the deterministic deflections
evaluated using the procedures outlined in section 6.3 to form an overall suspension deflection
performance index for the train model.

The vertical and lateral models will now be analysed in turn.

6.4.1 - Vertical analysis

Section 6.2.1 stated that vertical track could be accurately represented as a random white noise
process with an amplitude dependent upon track roughness and vehicle speed. A simple
frequency domain technique may therefore be used to evaluate the body accelerations and
suspension deflections due to this random process. Taking the two-mass model shown in Figure
1.11 and described by the state-space model given by equations (3.23) and (3.24), an account of
this technique may be given. This involves converting the state-space model into the frequency
domain as shown by equation (6.24).

\[
H(s) = C(sI-A)^{-1} W \cdot D_w
\]  \hspace{1cm} (6.24)

This is a multi-variable transfer function, from which frequency responses between the wheelset
vertical velocity input \(z_w\) and both the body acceleration \(z_b\), and the suspension deflection \(z_{SD}\),
\(H_{SR}(j\omega)\) and \(H_{SD}(j\omega)\) may be derived as given by equations (6.25) and (6.26).

\[
H_{SR}(j\omega) = C_{SR}H(j\omega)
\]  \hspace{1cm} (6.25)

\[
H_{SD}(j\omega) = C_{SD}H(j\omega)
\]  \hspace{1cm} (6.26)
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Parseval's theorem may be used to evaluate the r.m.s. values of both body acceleration and suspension deflection using equations (6.25) and (6.26) together with the track p.s.d. given by equation (6.10). Equation (6.27) gives the r.m.s. value of body acceleration, and equation (6.28) gives the r.m.s. value of suspension deflection for the given track input.

\[
\text{RIDE} \cdot \sqrt{\int_{0}^{\infty} |H_y(2\pi f)|^2 \hat{S}_f(f) \, df} \quad \ldots (6.27)
\]

\[
\text{DEFL} \cdot \sqrt{\int_{0}^{\infty} |H_y(2\pi f)|^2 \hat{S}_f(f) \, df} \quad \ldots (6.28)
\]

A maximum frequency of 300 Hz is used, there is little to be gained at frequencies higher than this level due to the natural roll-off of the system. The p.s.d.s of both body acceleration and suspension deflection may be obtained prior to the integrations performed in equations (6.27) and (6.28). It is important to note that the integrations performed in equations (6.27) and (6.28) are performed on a digital computer and are hence discrete integrations.

This procedure may be extended to the vertical models developed in Chapter 4. The only complication being the number of wheelset inputs, which of course have the same basic input profile, but time-delayed according to the vehicle speed and geometry.
Assuming the Laplace transform of the input to the leading wheelset is $\zeta(s)$, the input to the other wheelsets is given by equation (6.29).

$$\begin{bmatrix}
\zeta(s) \\
e^{-sT_1} \zeta(s) \\
e^{-sT_1} \zeta(s) \\
e^{-sT_1} \zeta(s) \\
\ddots
\end{bmatrix} \quad \ldots \quad (6.29)$$

There is in effect only a single input to the vehicle model in spite of the fact that the models developed in Chapter 4 have an input for every wheelset present in the train. It is possible to combine the state-space model description given by equations (4.22) and (4.23) with the recent frequency domain conversion performed for the two-mass model example and the time-delayed inputs described by equation (6.29) to evaluate the ride quality and suspension deflection for the
vertical train models. The ride performance is given by the pair of equations (6.30) and (6.31).

\[
H_{SR}(j\omega) = C_{SRY}(C(j\omega I - A)^{-1}) \sum_{i=1}^{4n} W_i e^{j\omega T_i} \sum_{i=1}^{4n} D_{i} e^{j\omega T_i} \qquad \ldots (6.30)
\]

\[
RIDE_{SV} = \frac{1}{3n} \int_{0}^{\infty} (2\pi)^2 \Omega_s v [H_{SR}(2\pi f) H_{SR}(2\pi f)] df \qquad \ldots (6.31)
\]

The suspension deflection is given by equations (6.32) and (6.33).

\[
H_{SD}(j\omega) = C_{SDY}(C(j\omega I - A)^{-1}) \sum_{i=1}^{4n} W_i e^{j\omega T_i} \sum_{i=1}^{4n} D_{i} e^{j\omega T_i} \qquad \ldots (6.32)
\]

\[
DEFL_{SV} = \int_{0}^{\infty} (2\pi)^2 \Omega_s v [H_{SD}(2\pi f) H_{SD}(2\pi f)] df \qquad \forall 1 \leq i \leq 2n \qquad \ldots (6.33)
\]

The ride performance is an average taken over the entire train, the measurement points being the vertical accelerations experienced at 3 points on each vehicle: \(\ddot{z}_{bogm1}\), \(\ddot{z}_{bogm}\), and \(\ddot{z}_{bogm2}\) as shown in Figure 6.11. This averaging technique takes into account contribution from both the bounce and pitch motions.

The stochastic suspension deflection evaluation performed in equations (6.32) and (6.33) generates the maximum suspension deflection experienced by any bogie within the train in response to the random track input.

Both ride and suspension deflection evaluations require the selection matrices \(C_{SRV}\), and \(C_{SDV}\) to select the appropriate measurements from the global output vector \(Y_v\).
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An important phenomenon arises in the analysis of the vertical train models, which is not seen in the simple two-mass analysis of active suspensions frequently seen in the literature. This phenomenon is known as 'geometric filtering' and leads to significantly different ride predictions when compared with the equivalent simple two-mass model and vindicates the decision to analyse active suspensions applied to more realistic models of vehicles. Figure 6.12 shows the cause of this phenomenon. Track irregularities are spread over a broad range of frequencies. Figure 6.12 shows the response of a simple two-axle vehicle to a pair of these track wavelengths. The geometric spacing between the wheelsets results in pure pitch excitation in response to one of these track wavelengths whereas a different wavelength will produce pure bounce excitation. Harmonics of these track wavelength will give rise to similar excitations of one mode at the expense of nulling the other. The degree of excitation is of course speed dependent. The exact temporal position of these nulls is a complex interaction between vehicle speed, a wide range of track wavelengths, and the wheelbase, and this is further complicated for a 4 axle vehicle.

![Figure 6.12 - Vertical geometric filtering](image)

Figures 6.13 and 6.14 are p.s.d. plots of body bounce and body pitch respectively for the rigid single vehicle model. The nulls in both plots are interspersed at frequency antinodes; a null in one mode corresponding to a maximum in the other. The first null in the bounce response
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corresponds to a track wavelength of 19x2=38m, 19m being the leading to trailing bogie spacing, a 38m track wavelength would of course produce a 55ms⁻¹/38m=1.4 Hz null in the bounce response. A 19m track wavelength would of course produce a 55ms⁻¹/19m=2.9 Hz null in the pitch mode. The other nulls are due to other track wavelengths and wheel spacings.

Figure 6.13 - Bounce mode (55m/s - 0.7Hz)

Figure 6.14 - Pitch mode (55 m/s - 0.8 Hz)
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The position of these nulls is speed dependent, Figures 6.15 and 6.16 show the response of the vehicle at 35\(\text{ms}^{-1}\) (approximately 125\(\text{kmh}^{-1}\) or 80mph) and 25\(\text{ms}^{-1}\) (approximately 90\(\text{kmh}^{-1}\) or 55 mph) respectively. The second plot in particular shows interaction with the body bounce mode and serves to emphasise the significant impact which geometric filtering can play in the determination of ride quality.

**Figure 6.15 - Bounce mode (35 m/s - 0.7 Hz)**

**Figure 6.16 - Bounce mode (25 m/s - 0.7 Hz)**
6.4.2 - Lateral analysis

The lateral analysis techniques described here bear similarity to the frequency domain methods used for the vertical models and described in section 6.4.1. The underlying principles remain the same, the only difference being the application of a different track description and of course a different model as developed in Chapter 5. Lateral track irregularities were developed earlier and are represented by equation (6.16). The input vector to the lateral model contains both track lateral as well as track cant inputs, as discussed in section 6.3.2 the deterministic test is an uncanted 5km radius curve, this means that only the model inputs corresponding to lateral track motion need to be driven as can be seen with reference to equation (6.34) where \( \zeta(s) \) represents the lateral track irregularity and the other inputs are simply time-delayed versions of this input.

\[
\begin{bmatrix}
\zeta(s) \\
0 \\
\frac{d}{dt} \zeta(s) \\
0 \\
\frac{d}{dt} \zeta(s) \\
0 \\
\cdots \\
\end{bmatrix}
\]

\[
\cdots \quad (6.34)
\]

Using a similar procedure to that used for the vertical model, equation (6.35) may be developed. The main difference between this equation and equation (6.30) for the vertical case is the need to select only lateral track inputs to drive the model input. The selection matrix \( C_{SRL} \) of course selects different variables, in this instance they are lateral body accelerations at three points on each of the vehicles, one located above the leading bogie, another above the trailing bogie, and the final one at the c.o.g. of the vehicle.

\[
H_{SR}(j\omega) = C_{SRL} \{ C(j\omega I - A) \}^{4n} \sum_{s=1}^{4n} W_{s1} e^{-j\omega T_s} \sum_{s=1}^{4n} D_{W(s-1)} e^{-j\omega T_s} \]  \( \cdots \quad (6.35) \)
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A ride quality index for the lateral model may be developed by integration over a variety of track wavelengths, followed by averaging of the lateral body accelerations calculated at a variety of measurement points throughout the train as shown by equation (6.36).

\[
RIDE_{\alpha} = \sum_{i=1}^{3n} \int_{0}^{(2\pi)^2 \Omega_{i} v^2} \frac{\{H_{sr}(2\pi f_j)H_{sr}^*(2\pi f_j)\}}{f} df
\]

\[
\ldots (6.36)
\]

The stochastic suspension deflection may be calculated using a similar frequency domain technique. The selection matrix \( C_{SDL} \), as shown in equation (6.37), constructs the suspension deflections associated with each bogie from the global output vector of the model \( Y_v \). The maximum value may then be determined via equation (6.38).

\[
H_{sd}(j\omega) = C_{SDL} \{C(j\omega I-A)^{1/2} W_{2n-1} e^{j\omega T_{2n-1}} W_{2n} e^{j\omega T_2} \}
\]

\[
\ldots (6.37)
\]

\[
DEFL_{\alpha} = \left[ \int_{0}^{(2\pi)^2 \Omega_{i} v^2} \frac{\{H_{sr}(2\pi f_j)H_{sr}^*(2\pi f_j)\}}{f} df \right]_{i=1}^{2n} \ldots (6.38)
\]

The geometric filtering phenomenon described in section 6.4.1 also applies to the lateral model as shown in Figure 6.17. Nulls arise in both the lateral and yaw responses.
6.5 - Covariance analysis techniques

The frequency domain techniques described in section 6.4 are used to evaluate vehicle ride quality and suspension deflection in response to track irregularities, and they are also useful in evaluating p.s.d plots of signals of interest. The integration process involved in this evaluation is computationally intensive particularly for the lateral models and long train lengths in general. An alternative method is used throughout this thesis in the evaluation of ride quality and suspension deflection where the use of frequency domain techniques would be impractical. This assessment technique is based in the time domain and is widely known as 'covariance analysis'. Close correlation between the results produced by covariance analysis and the frequency domain techniques outlined previously in section 6.4 has been noted throughout this thesis, however, a number of qualifying statements need to be made about this technique. The models generated in Chapter 4 and 5 both contain pure integrations associated with the wheelset velocity inputs, but these are unacceptable with covariance analysis, consequently the integrations associated with the vehicle models need to be modified to low-pass filters, and the movement of system poles into the l.h.p. makes the covariance analysis technique workable. This involves slight modification to the models developed previously. The corner frequency of the low-pass filter
must be selected according to the overall bandwidth of the system, a frequency of about 0.01 Hz has been found to be appropriate. The covariance analysis technique is nevertheless a computationally fast method of assessing suspension performance.

The covariance analysis theory will now be given in general terms, for a more detailed account see [Karnopp - 1978], it will then be applied to either the vertical or lateral models.

The vehicle models, whether they be vertical or lateral, containing an active suspension or not, may be placed in the state-space form shown by equations (6.39) and (6.40).

\[
\begin{align*}
\dot{X} &= AX + W\xi \\
Y &= CX + D_w \xi
\end{align*}
\]

(6.39) \hspace{2cm} (6.40)

The models do not contain direct feedthrough (i.e. they are all strictly proper) and hence the matrix 'D_w' will be dropped from now onwards.

Section 6.2.1 described vertical track as being closely represented by a random white noise process. The autocorrelation function of the track input in this instance being given by equation (6.41) which is derived from the track p.s.d. function given by equation (6.10).

\[
E(\xi(t)\xi'(t-\tau)) = \int S_r(\omega)e^{-j\omega\tau} d\omega
\]

(6.41)

The autocorrelation may be stated more simply as given by equation (6.42) after performing the integration given by equation (6.41).

\[
E(\xi(t)\xi'(t-\tau)) = (2\pi)^3 \Omega \cdot v \delta(\tau) \cdot q(\tau)
\]

(6.42)
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The variance, 'q(0)' is simply the value of the autocorrelation function at zero. The system given by equation (6.39) is subjected to this random track input. It must have only one input and hence the two-mass model is applicable in this instance. The theory will later be extended to include time-delayed wheel input. This is then used in the Lyapunov equation (6.43) to determine the covariance matrix '\( P_{xx} \)' of the system state.

\[
AP_{\alpha} - P_{\alpha} A^T - WqW^T = 0 \quad \ldots (6.43)
\]

The covariance of the output vector may be derived via equation (6.44).

\[
P_{yy} = CP_{\alpha} C^T \quad \ldots (6.44)
\]

The variances of each of the relevant output quantities, for example body acceleration, may be selected from the output covariance matrix '\( P_{yy} \)'.

6.5.1 - Vertical analysis

For the vertical models the general theory recently outlined needs slight adaptation. The autocorrelation function of the track input may be written as given by equation (6.45) in which the correlation between each pair of wheel inputs is dependent upon the time-delay that exists between them.

\[
Q(\tau) = E\{(\zeta(t)\zeta(t-\tau)) = \begin{bmatrix}
q\delta(\tau) & q\delta(\tau-T_1) & q\delta(\tau-T_2) & \ldots \\
q\delta(\tau-T_1) & q\delta(\tau) & q\delta(\tau-T_2;T_1) & \ldots \\
q\delta(\tau-T_2) & q\delta(\tau-T_2;T_1) & q\delta(\tau) & \ldots \\
\vdots & \vdots & \vdots & \ddots \\
\end{bmatrix} \quad \ldots (6.45)
\]

This modified autocorrelation function may then be incorporated within the mathematical theory [Hedrick and Firouztash - 1974] giving rise to equation (6.46) which gives the covariance matrix
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'P_{xx}' for a general length of train 'n', including the time-delay between wheelsets.

\[ 0 \cdot A P_{xx} \cdot P_{xx} A^T \cdot WQ(0)W^T \cdot \sum_{i=2}^n \sum_{i=2}^n e^{AT_i} W Q(T_i) W^T \cdot \sum_{i=2}^n e^{AT_n} W Q(T_n) W^T ]^T \ldots (6.46) \]

The ride index for the train is the average body acceleration measured at a number of points along the train length. The selection matrix 'C_{SRV}' may be used to select these measurement points and form an overall index as given by equation (6.47).

\[ \text{RIDE}_{SV} = \frac{\sum {C_{SV} C_{PS} C_{CSV} C_{SV}}}{3n} \ldots (6.47) \]

The stochastic deflection is simply the maximum experienced by any bogie within the train and is given by equation (6.48).

\[ \text{DEFL}_{SV} = \frac{\|C_{SV} C_{PS} C_{CSV} C_{SV}^T\|}{\forall 1 \leq 2n} \ldots (6.48) \]

6.5.2 - Lateral analysis

The analysis for the lateral models is similar to the methods described in section 6.5.1 for the vertical model, the obvious difference being the selection matrices. These are however identical to those given for the frequency domain analysis of the lateral model as performed in section 6.4.2. A more crucial difference lies in the different track spectrum used to describe the lateral track irregularities, which has a roll-off with increasing frequency. A number of authors have used 'coloured' noise inputs to represent track that is a filtered version of the nominal white noise excitation [Crolla and Abdel-Hady - 1991]. This method could indeed be used with the lateral models, however, the method used throughout this thesis is simply to include a first order filtering stage within the vehicle models, this does not accurately represent the lateral spectrum but the overall method is useful in obtaining comparative results between designs. It is not
difficult to incorporate this filtering particularly in view of the fact that some form of filter is required to replace the wheelset integration and thus making the Lyapunov equations (6.43) and (6.46) soluble.

6.6 - Time history analysis

The track data recordings used to develop the vertical and lateral track spectra, shown in Figures 6.4 and 6.6, may also be used to evaluate vehicle response without resorting to frequency domain techniques. This is particularly useful with non-linear systems. The frequency domain techniques described in section 6.4, and the covariance analysis methods described in section 6.5, both require the system under investigation to be represented in a linear state-space format. This is quite acceptable if the linear models developed in Chapters 4 and 5 are to be used in isolation or in conjunction with some form of linear active suspension, because in either case the resulting system would be linear and could therefore be analysed using either frequency domain or covariance analysis techniques. The possibility of non-linear active suspension control which could have additional benefits over a linear type and the test-rig assessment methods mean that some other analysis technique must be found to assess suspension performance.

The simplest method of analysing such a system is to take the track history data and simulate the response of the non-linear model on a computer. The model outputs will naturally be time histories from which the r.m.s. values may be obtained directly, or a *Fast Fourier Transform* performed to analyse the frequency content of the resulting signal.

Figure 6.18 shows 10 seconds worth of vertical track data. The track measured in this instance is a section of the 'East Coast main-line', (U.K.), measured near North Allerton, Yorkshire.

The methods used to assess the vertical and lateral suspension performance are similar, and so a generic description of the analysis techniques will be given rather than giving separate treatment for both.
A general model description was given by the state-space equations (6.39) and (6.40). This form is arrived at after incorporating the active suspension into the model. The format remains the same even if a purely passive arrangement is to be analysed. In both cases there is only one excitation path, that between the track input and the vehicle response. The vehicle response contains different variables for the vertical and lateral cases, hence the need for differing selection matrices 'C_{SRV}' and 'C_{SRL}'. Both selection matrices return the suspension performance indices, namely: body accelerations at a number of measurement points, and suspension deflections associated with each bogie in the train. These are then arranged to form an overall ride index, and a maximum suspension deflection.

![Vertical track time history](image)

**Figure 6.18 - Vertical track history data (55m/s)**

The state-space model equations (6.39) and (6.40) may of course contain non-linearities. The ride index for the vertical train model is given by equation (6.49). This evaluates the system output at each instant between 5 and 35 seconds, the accelerations are then averaged in time and averaged across all the measurement points along the train length.
The maximum suspension deflection for the vertical model is given by equation (6.50).

\[
RIDE_{sy} = \frac{\sum_{k=1}^{n} C_{spr}C \int_{\frac{35}{2}}^{\frac{35}{2}} e^{-a(t-\alpha)} W_{\zeta}(t) dt \, dt}{90n}
\]  

... (6.49)

Similar equations apply to the lateral models. The ride and suspension deflection in this case are given by equations (6.51) and (6.52).

\[
RIDE_{sl} = \frac{\sum_{k=1}^{n} C_{spr}C \int_{\frac{35}{2}}^{\frac{35}{2}} e^{-a(t-\alpha)} W_{\zeta}(t) dt \, dt}{90n}
\]  

... (6.51)

\[
DEFL_{sy} = \left| C_{spr}C \int_{\frac{35}{2}}^{\frac{35}{2}} e^{-a(t-\alpha)} W_{\zeta}(t) dt \, dt \right|
\]  

... (6.50)

\[
DEFL_{sl} = \left| C_{spr}C \int_{\frac{35}{2}}^{\frac{35}{2}} e^{-a(t-\alpha)} W_{\zeta}(t) dt \, dt \right|
\]  

... (6.52)

It is important to note the simulation process used in equations (6.49) to (6.52). The first few seconds are ignored because of the decaying transient response of the vehicle. There must also be sufficient measurement points contained within the track data \(\zeta(t)\). A good average is achieved after approximately 15 cycles of the slowest mode. In vehicle dynamics this mode is around 0.5 Hz and hence approximately 30 seconds of measurement is used throughout this thesis. This time requirement means that at least 2km of measured data is required to cope with higher speeds. Track data used throughout this thesis is measured every 0.25m. At 55ms\(^{-1}\) the time resolution is around 5ms, high enough for even the fastest dynamic mode of the vehicle.
Figures (6.19) and (6.20) show the time response of the leading bogie suspension deflection, and the lateral model body c.o.g. acceleration respectively. Maxima and averaging need to be applied to achieve performance indices. In both cases these are single vehicle models.

**Figure 6.19** - Vertical model suspension deflection

**Figure 6.20** - Lateral model c.o.g. acceleration
Chapter 6: Suspension performance assessment methods

As mentioned earlier, FFTs of the resulting responses may be evaluated to determine the frequency content of these signals. Figure (6.21) shows the frequency content of the lateral c.o.g. acceleration signal shown in Figure (6.20).

![Lateral model c.o.g. acceleration](image)

**Figure 6.21 - Lateral model c.o.g. acceleration**

The spectrum is not as revealing as that obtained previously using frequency domain techniques since it does not give precise nulls arising from geometric filtering. The reason for this being that the FFT requires a number of responses on different track, together with averaging if it is to reveal useful information. Nevertheless it does reveal system resonances. Throughout this thesis, 1024 point FFTs are performed with a folding frequency of 30Hz, which gives a frequency resolution of about 0.05 Hz. The FFT produces a linear frequency ordinate axis and hence the only way to improve this resolution is to increase the number of points used in the FFT, which naturally increases the computation required. The time history response of course needs to be decimated to achieve this 1024 point FFT. Despite a number of drawbacks, FFTs can give comparative performance indices for active suspension models.
Chapter 6: Suspension performance assessment methods

6.7 - Overall performance indices

The ride performance indices developed in sections 6.4, 6.5, and 6.6 for both the vertical and lateral models are all based upon the response to straight track containing random irregularities. The ride performance index is an average over a number of measurement points on the vehicle body. The suspension deflection constraint is to avoid bumpstop contact, whether this be in the vertical or the lateral directions. The suspension travel on a high-speed vehicle is typically limited to ±35mm in the vertical direction, and ±80mm in the lateral direction. The methodology used throughout this thesis to assess whether a particular design would violate these constraints is to use a combination of the vehicle's response to a deterministic feature, and its response to random track irregularities. This is not an unreasonable proposition: deterministic features would result in a quasi-static suspension compression, but irregularities are also present on vertical transitions and curves and hence it is necessary to add on the deflection due to the irregularities onto the quasi-static deflections. Equations (6.53) and (6.54) give the criteria which the vertical and lateral suspensions must comply with. The r.m.s. stochastic suspension deflection component is trebled to produce a maximum value (99% confidence levels, a value frequently used by railway dynamicists).

\[ 35 \geq \text{DEFL}_{yr} + 3 \times \text{DEFL}_{sv} \quad \ldots \ (6.53) \]

\[ 80 \geq \text{DEFL}_{yl} + 3 \times \text{DEFL}_{sl} \quad \ldots \ (6.54) \]

6.8 - Summary

The suspension analysis techniques outlined in this chapter relate to the evaluation of stochastic and deterministic ride quality and suspension deflections for both the vertical and lateral models outlined in Chapters 4 and 5. These techniques are used in subsequent chapters to evaluate the performance of a variety of active suspension designs.
Chapter 7

Active Secondary Suspensions

7.1 - Overview

Active secondary suspensions have been investigated by a number of researchers. Theoretical and experimental studies have demonstrated the benefits of active suspension. The vast majority of studies have concentrated on applying active suspension to a simple two-mass model. Chapter 6 highlighted some deficiencies, most notably the inaccurate prediction of ride quality when using over-simplified models. The studies performed on the two-mass model have espoused a number of active suspension control techniques which may be translated to fit more complex and accurate vehicle models. The two-mass control laws clearly lack the extra degrees of freedom which are present in a real vehicle. It is therefore crucial that these simple control laws be extended so that realistic control laws may be developed. A number of studies have adapted the simple two-mass control laws and provided computer simulation or experimental results to highlight the benefits of active suspension. This thesis will attempt to extend the current knowledge of active secondary suspensions applied to railway vehicles by making the following contributions:

- Theoretical investigation of 'real' actuator dynamics in active secondary suspension
- Extension of control strategies based on vehicle measurements to train-wide measurements

Active secondary suspensions will be designed for both the vertical and the lateral directions based on the train models developed in Chapters 4 and 5. The control techniques used to
implement active suspension controllers are not new. Three basic types are identified in this chapter which need slight adaptation in order that they may be applied to single vehicles or a train of vehicles. The three categories of controller are:

- Modified skyhook damping
- Optimal control
- Classical control

The first of these categories 'skyhook damping' was introduced in its basic form in Chapter 1. A number of problems were identified which make this basic form unacceptable in practice, and so the basic form of skyhook damping is modified and applied to the more complex vertical and lateral models already developed. A key feature in the design process is the suspension assessment methodology outlined in Chapter 6, which is used in conjunction with the design flowchart of Figure 3.17 which includes the effect of 'real' actuator dynamics. The methodology is to design an active suspension based on the use of 'ideal' actuators, a design which must satisfy the suspension deflection criteria laid down in section 6.7 which stipulates that the total suspension deflection due to both 'stochastic' and 'deterministic' causes must be limited. If this criteria is violated the design process must be repeated. Once a successful design is achieved the effects of 'real' actuator dynamics may then be addressed. The considerable constraint imposed by meeting suspension deflection limits in response to deterministic inputs is frequently ignored in the literature, but is fundamental to the overall design procedure.

'Optimal control' techniques have received considerable attention in both automotive and railway active suspension research. They are applied here using the same methodology as outlined for the modified skyhook damper. Optimal control falls into a unique niche in that it intrinsically permits train-wide active suspension. It has been extensively used in conjunction with preview control to attain further benefits, it will be seen how train-wide optimal active suspensions offer a similar advantage due to the elongated nature of a train.
'Classical control' is used to classify active suspensions which utilise intuitively-formulated transfer function filter structures to implement the active suspension. In some senses the modified skyhook damping strategy recently outlined can be classified this way, but is used here in active suspension design to denote more sophisticated 'complementary filter' types of controller structures [Williams - 1986].

These three basic forms of secondary active suspension will now be applied to both the vertical and lateral directions.

**7.2 - Vertical active secondary suspensions**

The three basic types of control law have already been outlined. This section will apply these control laws to the vertical vehicle and train models outlined in Chapter 4. It will be noted that the application of some of the control laws on a train-wide basis is an intractable problem, but in these instances the individually controlled vehicles may still be coupled together and their overall behaviour analysed.

The abstraction of simple two-mass control laws to the design of secondary vertical active suspension controllers is shown in the series of Figures (7.1) to (7.3). They introduce the concepts of 'local design', 'vehicle design', and 'train-wide design', each being an incremental advance in the application of simpler control techniques. This categorisation of design methods is based on the fundamental unit by which the design is conceived. For instance, in 'local design' the design model is the two-mass model, although the active suspension is applied to a full vehicle. The three controller categorises may be abstracted to produce vehicle and train-wide designs, although as stated previously this is intractable in several cases.

The behaviour of the three types of actuator technology outlined in Chapter 3 will be addressed for each of the control laws developed. The effects of body flexibility when subjected to active suspension forces will be assessed. Neither actuator dynamics nor body flexibility are used in the
initial design process; their effects are analysed after the initial design phase.

**Figure 7.1 - Local design method**

**Figure 7.2 - Vehicle Design Model**
Each of the three controller categories will now be addressed in turn.

7.2.1 - **Modified skyhook damping**

The basic skyhook damping strategy which was outlined in Chapter 1 failed to meet the deterministic suspension deflection criteria as shown by the response of Figure 1.10 which showed an ever increasing suspension deflection as the vehicle negotiated the constant acceleration vertical transition. There are ways to circumvent the large deterministic suspension deflections experienced with the basic form of skyhook damping, and at the same time improve straight line ride quality, and Figure 7.4 shows how the basic form of skyhook damping needs to be modified if a viable active suspension is to be developed.
Chapter 7: Active Secondary Suspensions

Figure 7.4 - Skyhook damping with integral displacement feedback

It shows the basic passive secondary suspension, provided by an airspring, together with a parallel active element which is driven by control signals derived from acceleration and suspension displacement measurements. This control structure is of course associated with a two-mass model, and will subsequently be developed to form a full vehicle controller. The body acceleration is integrated to produce absolute velocity. This is filtered to eliminate low frequencies, usually less than 0.1 Hz, which are responsible for the deterministic suspension compression. Ride improvement results from skyhook damping being provided around and upwards from the body frequencies (0.7 Hz). This filter therefore has a beneficial effect on the quasi-static suspension deflection without detrimentally affecting the ride improvements associated with skyhook damping. Integrated suspension deflection also has beneficial effects in limiting the suspension deflection experienced on deterministic features. The integration process is relatively slow in its initial response but ultimately responds to null the suspension deflection. The combined filtering and integration processes are required if a practical implementation of skyhook damping is to be developed.
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The structure shown in Figure 7.4 applies to only one end of the vehicle model. 'z_bog' represent the motion of the body just above the bogie, 'z,' represent the vertical motion of the bogie itself. The simple two-mass model control law may be applied at each of the vehicle ends, in the process ignoring the body pitch degree of freedom and assuming that each end is an autonomous entity. The control design must be performed on a half-car model to ensure that the control law is consistent with the associated mass, and all the stiffnesses and dampings also need to be scaled to ensure this. Figure 7.5 shows the structure of the aforementioned 'local modified skyhook damping' active control strategy. The force commands 'f_{ctm1}' and 'f_{ctm2}' are used to drive the leading and trailing actuators and are associated with vehicle 'm'. The control laws clearly act independently.

![Diagram](image)

Figure 7.5 - Local modified skyhook damping
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The local control strategy may be adapted to include the pitch degree of freedom. This type of control strategy is shown in Figure 7.6 and is termed: ‘vehicle modified skyhook damping’ control, this terminology arising because the resultant controller is based on an observation of the vehicle as a whole. The structure is very similar to the local strategy shown in Figure 7.5, integral displacement feedback is used across each of the bogies, filtered skyhook damping is applied in an integrated manner to both the bounce and pitching motions, rather than acting independently as was the case in Figure 7.5. The important feature is that different filtering and levels of damping may be applied in the two modes, which is beneficial to ride quality.

![Figure 7.6 - Vehicle modified skyhook damping](image)

This control law may of course be extended on a train-wide basis. Each of the vehicles would have the control structure shown in Figure 7.6 and would simply be coupled together, but the control law in this instance is not ‘train-wide’ because it does not utilise information from adjacent vehicles. Whereas it is relatively straightforward to develop the local control law into a vehicle controller, it is an intractable problem developing it on a train wide basis. [Karnopp - 1968] gives the the optimum path a train should follow if it is to improve ride quality and
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maintain suspension deflection, but it is nevertheless difficult to substantiate this idea and produce drive signals which could apply modified skyhook damping on a train-wide basis. Optimal control methods are more readily suited to this task and are analysed in subsequent sections, but a control strategy acting on individual vehicles in a coupled train is a realistic option and is therefore included here.

An initial estimate of the skyhook gain may be made by selecting the gain which moves the body bounce mode damping to 0.7, which [Karnopp - 1978] argues is the optimum value of damping. The control gains 'c_xy' and 'i_depl', (or 'c_xe', and 'c_pl' in the vehicle controller), are incremented or decremented, depending upon whether or not the design violates the suspension deflection criteria outlined in Chapter 6. Figure 7.7 shows the suspension deflections for each bogie in a passive 3 vehicle train. This represents a 0.4ms⁻² sustained vertical acceleration feature traversed at 55ms⁻¹.

![Deterministic deflection for 3 coupled vehicles](image)

Figure 7.7 - Passive 3 vehicle train (deterministic)

The stochastic ride averaged across all vehicles in this case is 0.98 %g, and the worst stochastic suspension deflection associated with any of the six bogies is 4.0mm (r.m.s.) - see Figure 7.8.
following the procedures outlined in Chapter 6.

The total deflection in this case is 22mm (deterministic) + 3 x 4mm (stochastic) = 34mm, just under the total allowable excursion of 35mm in the vertical direction. The active modified skyhook control strategies outlined earlier must also adhere to this maximum deflection constraint. The active suspension designs must also improve the straight line ride quality using the passive vehicle ride evaluation as a yardstick of measurement.

As an example of the ride quality achieved by active suspension and for reasons of brevity, only p.s.d. plots of the ‘vehicle modified skyhook control’ applied to a single vehicle will be given. Quantitative results for the other two strategies will be given for other cases. Figure 7.9 compares the body bounce p.s.d for both the passive and actively controlled vehicles.

The pitch acceleration p.s.d.s comparing the same pair of systems are shown in Figure 7.10. The pitch acceleration is scaled so that it represents the equivalent acceleration above the bogies to allow comparison with the bounce acceleration.
Figure 7.9 - Single vehicle modified skyhook control (Ideal actuator)

Figure 7.10 - Single vehicle modified skyhook control (Ideal actuator)
Both Figures (7.9) and (7.10) contribute to the overall vehicle ride, both show a 35% ride improvement with respect to the passive vehicle. This must of course be achieved while maintaining suspension deflection clearances. Figure (7.11) shows the deterministic suspension deflection at the leading and trailing bogies of the actively controlled system when negotiating the same feature as the passive vehicle was exposed to in Figure (7.7). It can be seen that the trailing bogie deflection lags the leading bogie by about 0.4 seconds. The maximum deflection is slightly larger than the original passive system, the trailing bogie deflection in this instance reaches just over 26mm, but this is allowable due to the reduction in the stochastic component of suspension deflection. Provided the two components of suspension deflection are within the permitted 35mm allowance the design is deemed to be acceptable. This reduction in stochastic deflection does not always occur, in many active suspension designs the stochastic deflection worsens with improvements in ride.

![Deterministic deflection for a single vehicle](image)

**Figure 7.11 - Active (modified skyhook control) single vehicle**

### 7.2.1.1 - Real actuators

The behaviour of the system with the inclusion of 'real' actuator dynamics will now be addressed. Figure 7.12 compares the bounce acceleration p.s.d's between the 'ideal' actuator and
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the 'electrohydraulic' actuator.

Figure 7.12 - Electrohydraulic vs Ideal actuation

The response shows similarities to the two-mass simulations performed in Chapter 1. The electrohydraulic actuator deteriorates the ride quality at higher frequencies, which is due to the inability of the actuator to follow the active control forces demanded of it. The actuator provides a high-frequency transmissibility path which result in ride deterioration, and similar behaviour may be seen in the pitch motion. An internal actuator resonance may be seen in Figure 7.12, at around 200Hz, but the response is severely attenuated at this frequency and does not have a significant impact on the overall ride quality. This resonance is due to the spool-valve. The poor tracking ability of this actuator at higher frequencies is due to the oil compressibility and flow-rate restrictions.

Figure 7.13 emphasises the difference between ideal and electrohydraulic actuator control forces. The increased actuator force at lower frequencies results in a worsening of the ride quality, and is due to the inability of the actuator to maintain the required force in the presence of suspension
movement. 'Geometric filtering' nulls are present in the response for the same reason.

![Electrohydraulic vs Ideal actuation](image)

**Figure 7.13 - Electrohydraulic vs Ideal actuation**

Comparative results will now be given for the electromechanical and electromagnetic actuators. These p.s.d. plots are all for speeds of 55 ms\(^{-1}\), but ride performance indices are also given without reference to p.s.d.'s for a variety of vehicle speeds. This is necessary since geometric nulls may cancel system resonances as shown by the example given in Chapter 6.

The bounce acceleration spectrum for 'vehicle modified skyhook damping' using an electromechanical actuator is shown in Figure 7.14. The 'real' actuator again deteriorates the vehicle ride quality when compared with the behaviour of the 'ideal' actuator, but the effect is less extreme than previously observed with the electrohydraulic actuator. This type of actuator begins to lose force control due to its inability to accelerate the internal motor inertia, and the deviation in force control effort is shown by Figure 7.15. The electromechanical actuator also has an internal resonance due to the lead-screw stiffness, but this is at a high frequency and usually well damped.
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Figure 7.14 - Electromechanical vs Ideal actuation

Figure 7.15 - Electromechanical vs Ideal actuation
The equivalent results for the electromagnetic actuator are finally presented in Figures 7.16 and 7.17, which show the bounce acceleration and actuator force respectively. The response shown in Figure 7.16 is virtually identical to the response of the 'ideal' actuator. There is a slight difference which is more prominent in the actuator force p.s.d. shown in Figure 7.17. The deviation is again due to internal actuator dynamics. This actuator nevertheless has an excellent performance.

**Figure 7.16 - Electromagnetic vs Ideal actuation**
It is important to test the vehicle over a wide range of speed to ensure acceptable performance. Figure 7.18 shows the variation of ride quality with vehicle speed for each of the different actuator technologies. All show a monotonic decrease in ride quality with vehicle speed. The relationship is not proportional to speed in all 4 cases due to a geometric null coinciding with the system resonances at around 25ms⁻¹, but it should be stressed that the high-speed results are the most crucial as this is where the ride quality is a significant problem.
The modified skyhook damping control law may also be applied on a train-wide basis. As mentioned before this simply involves replicating the control law for each vehicle. Figures 7.19 and 7.20 show the ride quality for 3 and 4 vehicle train lengths. The ride index for each vehicle is averaged over 3 measurement points on the vehicle body. The index for the entire train is obtained using Equation (6.31) and is the average of all measurement points. Both figures show a deterioration in ride down the length of the train, similar behaviour is seen when coupling purely passive vehicles. The reason for this deterioration down the vehicle length is due to particular track wavelengths interacting with train modes. The 38m wavelengths excite pitch motion, the vehicles are spaced at 26m intervals which means that the pitch excitation of each vehicle is out of phase with its neighbour. This results in the inter-vehicle force of a leading vehicle exciting downstream vehicles and at the same time stabilising its own motion.
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Figure 7.19 - Train-wide performance (3 vehicle train)

Figure 7.20 - Train-wide performance (4 vehicle train)
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The coupling of vehicles tends to worsen the average ride of the train by allowing vibration transfer between them as shown by Figure 7.21. This is not the entire story however, because if the damping of the coupling is increased this tends to improve the overall ride quality. The results in Figure 7.21 illustrate the effect of vehicle coupling on ride quality for a variety of train lengths. On average the ride quality tends to worsen as more vehicles are coupled; the reason for this being the vibration transmission path arising from the inter-vehicle stiffness.

![Ride vs Train length](image)

**Figure 7.21 - Coupling effect**

A complete summary of the computer simulation results for this section is given by the two Tables 7.1 and 7.2. These show the behaviour of 'local modified skyhook damping, 'vehicle modified skyhook damping', and 'train-wide modified skyhook damping', with a variety of actuator technologies, and vehicles possessing purely rigid modes as well as those possessing body flexibility.
## Chapter 7: Active Secondary Suspensions

### Table 7.1 - Modified skyhook control (Rigid bodies - 55 m/s)

<table>
<thead>
<tr>
<th>Control method</th>
<th>Ride quality (% improvement over passive)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ideal</td>
</tr>
<tr>
<td>Local control</td>
<td>30.5</td>
</tr>
<tr>
<td>Vehicle control</td>
<td>35.7</td>
</tr>
<tr>
<td>Train-wide (2 vehicles)</td>
<td>34.2</td>
</tr>
<tr>
<td>Train-wide (3 vehicles)</td>
<td>33.3</td>
</tr>
<tr>
<td>Train-wide (4 vehicles)</td>
<td>33.2</td>
</tr>
<tr>
<td>Train-wide (5 vehicles)</td>
<td>32.7</td>
</tr>
<tr>
<td>Train-wide (6 vehicles)</td>
<td>32.5</td>
</tr>
<tr>
<td>Train-wide (7 vehicles)</td>
<td>32.6</td>
</tr>
<tr>
<td>Train-wide (8 vehicles)</td>
<td>32.6</td>
</tr>
</tbody>
</table>

### Table 7.2 - Modified skyhook control (Flexible bodies - 55 m/s)

<table>
<thead>
<tr>
<th>Control method</th>
<th>Ride quality (% improvement over passive)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ideal</td>
</tr>
<tr>
<td>Local control</td>
<td>28.3</td>
</tr>
<tr>
<td>Vehicle control</td>
<td>33.1</td>
</tr>
<tr>
<td>Train-wide (2 vehicles)</td>
<td>31.8</td>
</tr>
<tr>
<td>Train-wide (3 vehicles)</td>
<td>31.0</td>
</tr>
<tr>
<td>Train-wide (4 vehicles)</td>
<td>30.9</td>
</tr>
<tr>
<td>Train-wide (5 vehicles)</td>
<td>30.5</td>
</tr>
<tr>
<td>Train-wide (6 vehicles)</td>
<td>30.3</td>
</tr>
<tr>
<td>Train-wide (7 vehicles)</td>
<td>30.4</td>
</tr>
<tr>
<td>Train-wide (8 vehicles)</td>
<td>30.4</td>
</tr>
</tbody>
</table>
7.2.2 - Complementary filter control

The complementary filter control strategy has been espoused by a number of researchers [Williams - 1986]. As its name suggests it is composed of pair of filters, one providing a high-pass function, the other providing a low-pass function. These two filters are used to fulfil the fundamental suspension requirements. The low-pass filter acts to minimise suspension excursion which is predominantly a low frequency effect, the high-pass filter provides the ride improvement function. The cut-off frequencies of these filters lie just below the body modes, allowing the high-pass filter to provide some degree of control over these modes, and the low-pass filter acts on low frequencies including quasi-static components. The complementary filter approach may be applied on a 'local', 'vehicle', and 'train-wide' basis as was performed in section 7.2.1 for the modified skyhook damping controller. Figure 7.22 shows the structure of 'local complementary filter control'.

![Diagram of Local Complementary Filter Control]

Figure 7.22 - Local complementary filter control

The structure of 'local' control here is similar to the structure shown previously in Figure 7.5. The
control law as a whole applies independent control actions to each of the leading and trailing bogie actuators. Accelerometer and displacement measurements are required at each end of the vehicle. A simple adaptation of the structure shown in Figure 7.22 in which both pitch and bounce motions are filtered results in 'vehicle complementary filter' control analogous to the methods used in Figure 7.6, and shown below in Figure 7.23.

\[ Z_{\text{bogm1}} - Z_{\text{tm1}} \]

\[ \dot{Z}_{\text{bm}} \quad \int \quad \dot{\phi}_{\text{bm}} \quad \int \quad \int \]

\[ Z_{\text{bogm2}} - Z_{\text{tm2}} \]

This control law may be extended on a train-wide basis by simply connecting the actively controlled vehicles together, although this control action is not truly train-wide since it doesn't utilise information from neighbouring vehicles. It will be seen later how optimal control more readily copes with train-wide information and can produce a truly 'train-wide' control strategy.

The low-pass filter shown in Figures 7.22 and 7.23 in this instance is chosen to be a third order type with an additional zero as given by equation (7.1).
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\[
LP(s) = \frac{1 + \frac{2}{\omega_f} s}{\left(1 + \frac{2}{\omega_f} s + \frac{2}{\omega_f^2} s^2 + \frac{1}{\omega_f^3} s^3 \right)} \quad \ldots (7.1)
\]

The high-pass filter is the complement of equation (7.1) as given by equation (7.2).

\[
HP(s) * 1 - LP(s) = \frac{\frac{2}{\omega_f^2} s^2 + \frac{1}{\omega_f^3} s^3}{\left(1 + \frac{2}{\omega_f} s + \frac{2}{\omega_f^2} s^2 + \frac{1}{\omega_f^3} s^3 \right)} \quad \ldots (7.2)
\]

The corner frequency of the filter is chosen to be around 0.3 Hz, just below the body modes. There are also gains associated with these filters which are selected using the procedure outlined in Chapter 6; the design must ensure maximum ride improvement while maintaining the suspension deflection constraint.

The performance of complementary filter type control applied to the vertical model will now be assessed using a variety of 'real' actuator technologies, with both rigid and flexible bodies. In order to give some variation in the presentation of the secondary vertical control laws, only a subset of the p.s.d.s given in section 7.2.1 will be shown for this type of control law, these will be shown at a different operating speed. A complete listing of the performance of this type of control law is given in Tables 7.3 and 7.4. A number of results will be included to illustrate how close the Matlab simulations are to their Vampire equivalent; this test of course only applies to a single vehicle because Vampire can not be used to simulate long train lengths.

Figure 7.24 shows the bounce acceleration p.s.d.s for both the passive and an active vehicle using complementary filter control. This test was performed at 40ms\(^{-1}\) (144 kmh\(^{-1}\) or 90 mph), the geometric nulls in these cases are at 1, 3Hz, etc, a consequence of the spacing between the bogies. Another interesting plot is shown in Figure 7.25, which shows the body acceleration just
above the leading bogie at the same speed, for both the passive and actively controlled vehicles. The geometric nulls caused by the bogie axle spacing are at different frequencies, in this case 8, 11.5Hz, etc. This is solely due to bogie geometric filtering, the wheelset spacing being 2.5m, the wavelength causing the first null will be $2 \times 2.5 = 5$ m, which at $40 \text{ms}^{-1}$ produces an 8Hz null. The 3 point averaging process described in Chapter 6 which is used to determine a ride index for the vehicle as a whole overcomes this problem of different ride spectra at different measuring points along the vehicle length.

![Diagram of Bounce Acceleration for Vehicle 8: 1 in a 1 Vehicle Train](image)

**Figure 7.24 - Local complementary filter control (c.o.g. measurement)**
The procedure used in this thesis is to verify the results obtained through Matlab simulations by contrasting them with equivalent Vampire results. An example of this verification will now be given with regard to the deterministic suspension deflection behaviour of the active complementary filter type control strategy. Figure 7.26 shows the suspension compression for both the leading and trailing bogies when negotiating the predefined vertical deterministic feature.
The results shown in Figure 7.26 may be contrasted against the purely passive response shown by Figure 7.7. The suspension in the active case settles at a lower quasi-static level due to a steady active force being generated when the vehicle achieves constant vertical acceleration of 0.4\(\text{ms}^{-2}\); this force being transmitted through the low-pass filter. This is different from the 'modified skyhook damping strategy' deflection shown previously in Figure 7.11; this demonstrated the deflection 'zeroing' nature of integral displacement feedback which is absent with complementary filter control. The worst suspension deflection between either the leading or trailing bogies is shown by the Figure 7.27 for both the active and passive cases. This is a Vampire simulation of exactly the same systems, the complementary filter control law in this case is written in FORTRAN and is linked into the main Vampire simulation routines. There is clearly close parity between the Vampire results and those shown in Figures 7.7 and 7.26. Gravitational effects are included within Vampire meaning that the quasi-static deflection does not reach a constant level, this leads to unloading of the leading suspension.
A complete set of performance figures for the complementary filter type of control strategy is given in Tables 7.3 and 7.4, the former for purely rigid vehicle bodies, the latter for vehicles with flexibility. The tables give results which are on a par with the results obtained for the modified skyhook damping strategy given in section 7.2.1. The same trends appear in the results: the differing actuator technologies maintain the same ranking as before, the electromagnetic type giving the best performance, the electrohydraulic the worst. The motivation behind using Matlab as opposed to Vampire is the ability to analyse long train lengths, although there is a constraint imposed by the computational demands of longer train lengths, particularly when complex active control is applied to the model. (The simulation results throughout have been performed on a Sun Sparc 10 workstation, memory constraints mean the results for 7 and 8 vehicle train lengths are omitted in this case)
### Table 7.3 - Complementary filter control (Rigid bodies & differing actuator technologies)

<table>
<thead>
<tr>
<th>Control method</th>
<th>Ride quality (% improvement over passive)</th>
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<td>Vehicle control</td>
<td>36.7</td>
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<td>Train-wide (2 vehicles)</td>
<td>35.4</td>
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<tr>
<td>Train-wide (3 vehicles)</td>
<td>34.3</td>
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### Table 7.4 - Complementary filter control (Flexible bodies & different actuators)

<table>
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</thead>
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<td>-</td>
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<tr>
<td>Train-wide (7 vehicles)</td>
<td>-</td>
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</tbody>
</table>
7.2.3 - Optimal control

Optimal control has been extensively applied to the active suspension problem, particularly with respect to automobiles [Wilson et al - 1986], [Thompson - 1976], and [Gordon et al - 1991] for example, but also to railway vehicles [Williams - 1986], and [Pratt and Goodall - 1994]. The reason for its favourable status is due to the formulation of the optimal control problem. Optimal control inherently provides a mechanism for permitting trade-offs in system performance [Anderson and Moore - 1990], and [Kwakernaak and Sivan - 1972], suspension design being a trade-off between minimising body accelerations and minimising suspension deflections. A number of studies have only addressed this problem with respect to improvements on random surfaces, whereas this thesis proclaims the importance of addressing deterministic suspension performance. It argues that a realistic optimal active suspension controller may still be constructed, but this will have a significantly different structure from the simpler forms which only consider random track inputs. As mentioned previously, optimal control may be used to 'weight' the improvements that are required; for ride improvement a heavy weighting must be given to body accelerations, but the suspension deflection is however a constraint and the optimal control formulation is not ideally suited to such a problem. The iterative design process involves modifying 'weights' in the design formulation, then analysing the system to ensure that it meets the suspension deflection constraint. If it does not, the 'weights' must be adjusted to ensure that the suspension deflection constraint is met. The optimal design is the one which minimises body accelerations, and at the same time utilises the available suspension working space.

Optimal control will be applied on a 'local', 'vehicle', and 'train-wide' basis, as demonstrated in Figures 7.1 to 7.3. Each of these will now be discussed in turn.

7.2.3.1 - Local optimal control

'Local optimal control' is an active suspension applied to a single vehicle. The control law has
two autonomous components driving the leading and trailing actuators independently. The control law is designed assuming the vehicle to be constructed from a pair of two-mass models as shown in Figure 7.1. Figure 7.28 shows the general structure of an optimal controller. This structure is not specific to 'local optimal control' and it will be used again in the discussion of 'vehicle optimal' and 'train-wide optimal' control. In the case of 'local optimal control', the system matrices shown in Figure 7.28 are for a two-mass model rather than a vehicle model.

The optimal control problem is solved by using the separation principle [Kwakernaak and Sivan - 1972]. The problem of estimating system states is distinct from the feedback used to move system poles. The overall compensator is a combined 'estimator & controller' as shown in Figure 7.28. The gains $K_{LQG}$ and $L_{LQG}$ need to be calculated. The estimator itself uses the system matrices in the estimation of the state variable. The determination of these gains involves formulating a performance index which weights ride quality and suspension deflection as shown by equation (7.3).

\[ J = \lim_{T \to \infty} \int_0^T E(Y^T Q Y + U^T R U) dt \]  

\[ \ldots (7.3) \]

The system output vector 'Y' contains body accelerations and suspension displacements, it is not difficult therefore to form a symmetric matrix 'Q' which weights these two quantities. Weighting of the control effort is inherent in the optimal LQG formulation but is not strictly necessary in active suspension design, actuator forces of course need to be limited in a realistic design but the weighting 'R' is chosen to be very low in this formulation, checks are performed after the design to ensure that the actuator forces are not too large, and if they are a re-iteration of the design is performed with a new 'Q' weighting matrix.
Figure 7.28 - Optimal control structure

The estimator gain $L_{\text{est}}$ may be found through the matrix Riccati equation (7.4) and equation (7.5).
The controller gain $K_{LQG}$ may be found through the matrix Riccati equation (7.6) and equation (7.7).

\[
0 = D_u^T Q D_u - P_c B_u R^{-1} B_u^T P_c - A^T P_c - P_c A
\]

\[
K_{LQG} = R^{-1} B^T P_c
\]

In the 'local optimal control' case the overall controller would be formed from a pair of 'estimators and controllers' for each end of the vehicle. In addition to the nominal optimal control action, integral displacement feedback needs to be incorporated to ensure that the deterministic suspension deflection constraint is met. This gives rise to an augmented control action as shown by Figure 7.29. A number of authors have attempted to include the integral action with the state-space model thereafter solving the LQG problem, the approach taken here is to separate the integral control action from the optimal controller.
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![Diagram](image)

**Figure 7.29** - Optimal control strategy with integral displacement feedback

The ride quality improvement of 'local optimal' control is shown by the bounce acceleration p.s.d. shown in Figure 7.30. The integral control action tends to worsen the low frequency ride quality; in fact if the integral gain is increased too far it can even destabilise the system. The overall ride quality is nevertheless improved significantly.

As mentioned earlier, an inspection of the secondary actuator force is necessary to ensure that it is of realistic magnitude. Figure 7.31 shows the response for one of these actuators.
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Figure 7.30 - Local optimal control bounce acceleration p.s.d

Figure 7.31 - Random actuator force demand
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The r.m.s. actuator force is around 600 N, which is a realistic force level. The most severe demand on an actuator is while negotiating a deterministic feature, and Figure 7.32 shows the worse response, in this case the leading actuator. There is initially a negative force due to the optimal control action which attempts to improve ride quality. After a while the controller realises that the vehicle is negotiating a deterministic feature, so the integral displacement control action comes into play and ultimately gives a steady force which is required to accelerate the mass of the body vertically. Both of these force levels are realistic values. They are then used together with: actuator extension, and extension rate requirements to give an appropriately sized 'real' actuator which can then be used with this type of control law.

![Figure 7.32 - Deterministic actuator force demand](image)

The performance of 'local optimal' control with a variety of 'real' actuators and a comparison between the use of purely rigid body modes as well as flexible ones given later in the summary in Tables 7.5 and 7.6.
7.2.3.2 - Vehicle optimal control

The general structure of vehicle optimal control is given by Figure 7.2. The design method is very similar to the methods used in section 7.2.3.1 for 'local optimal control'. The problem formulation is again described by equations (7.3) to (7.7), and shown in Figure 7.28, the only difference with 'vehicle optimal control' is the fact that the vehicle model is that of a single vehicle as opposed to a pair of two-mass models. The performance index, and in particular the 'Q' weighting matrix is different, the reason for this being that some form of weighting needs to be applied to body pitch acceleration. The design procedure is again an iterative one: 'weighting' values are selected and a controller designed. If the performance of the controller is poor or it violates suspension deflection constraints, the design is repeated. In order to avoid repetition, p.s.d plots will not generally be reproduced here. It is however interesting to note the bounce acceleration p.s.d. with body flexibility included, and such a response is shown in Figure 7.33 The final ride quality results for this control strategy are shown in Tables 7.5 and 7.6.

![Figure 7.33 - Vehicle optimal control with flexible mode](image)
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7.2.3.3 - Train-wide optimal control

The 'vehicle optimal' control strategy outlined in section 7.2.3.2 may be used on a train-wide basis simply by connecting actively-controlled vehicles together. Alternatively a feedforward arrangement may be augmented with the standard LQG optimal controller to give the structure shown in Figure 7.34. The time-delays between the wheelsets are then also taken into account and a superior control law is developed. Several researchers have addressed this time delay problem, some using the method used here which involves modelling the time-delays by Padé approximation [Krtolica and Hrovat - 1992], others initially converting the models to the discrete domain which is more accommodating towards time-delays, thereafter solving the optimal control problem [Prokop and Sharp - 1995], others have reworked the optimal control problem to incorporate time-delays [Sharp and Wilson - 1990], and [Hać - 1993].

The feedforward gain matrix 'F_{LQG}' may be obtained by solution of equations (7.8) and (7.9).

\[ 0 = P_cD_o \cdot [A - BP_c]^TP_f \cdot P_fA_o \quad \ldots (7.8) \]

\[ F_{LQG} = R^{-1}B^TP_f \quad \ldots (7.9) \]

The time-delay dynamics are in a standard state-space form 'A_o,B_o,C_o,D_o'. This system simply models the time-delay to each axle through a Padé approximation. The other matrices in equation (7.8) and (7.9) are acquired through the previous solutions of equations (7.3) to (7.4).

Figure 7.35 shows the body bounce acceleration of the middle vehicle in a 3 vehicle train using time-delay optimal control with a travelling speed of 80ms\(^{-1}\) (290 kmhr\(^{-1}\) or 180 mph). Body flexibility is included in the analysis although it is important to emphasise that it is not incorporated in the design model of the optimal controller. The ride performance of the train-wide optimal control strategy which does not take time-delays into account is given in Tables 7.5 and 7.6, the ride performance using the track feedforward network of Figure 7.34 is given in Tables 7.7 and 7.8.
Figure 7.34 - Optimal control with track feedforward

Figure 7.35 - Train-wide optimal control with time-delays and flexibility
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<table>
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<tr>
<th>Control method</th>
<th>Ride quality (% improvement over passive)</th>
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**Table 7.5** - Optimal control (Rigid bodies & differing actuator technologies)

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**Table 7.6** - Optimal control (Flexible bodies & differing actuator technologies)
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### Table 7.7 - Optimal control (with time-delays / rigid modes only)

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### Table 7.8 - Optimal control (with time-delays / flexible modes only)

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</table>
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The three control strategies investigated have all demonstrated the improvements offered by active suspensions. In each case the actuator technologies had the same performance ranking, the electrohydraulic faired worst, followed by the electromechanical and then the electromagnetic. There are only minor differences in the improvements offered by the three control strategies, these differences are around 5%. However, if time-delays are accounted for in the controller design, a further 10% improvement may be achieved. *Modified skyhook damping* and *complementary filter control* had a worsening ride index with increasing train length whereas the optimal control designs tended to cater for the coupling effect much better and actually improved ride quality.

7.3 - Lateral train models

Secondary active suspension may also be applied to the lateral direction using the models developed in Chapter 5. Ride quality in the lateral direction is more acute than in the vertical, hence the larger concentration of studies applied to this area [Roth and Lizzell - 1995], [Higaki et al - 1995], and [O'Neill and Wale - 1994]. This is confirmed by the fact the passive ride quality calculated for a single vehicle is 2.2 %g in this study, and reflects the industrial demand for a realistic secondary active lateral suspension. Particular emphasis has been concentrated on semi-active lateral suspensions, but there has also been an emergence of interest in fully active lateral suspensions, which is an inevitable progression which can be attributed to the tentative approach taken by the railway industry in view of the safety critical nature of suspension design.

The same subset of control laws which were applied to vertical direction, namely: *modified skyhook damping*, *complementary filter control*, and *optimal control* will also be applied to the lateral direction. The train models developed in Chapter 5 possessed four actuators per vehicle as shown by Figure 7.36. The control laws clearly need some modification if they are to applied to the lateral direction.
Each of the three control laws will now be addressed in turn. Many of the control laws bear close resemblance to their vertical counterparts and consequently only a brief outline is given for each.

7.3.2 - Modified skyhook control

The modified skyhook damping control law described in section 7.2.1 was composed of two components, the first being high-pass filtered body velocities, the second being integral displacement feedback. A similar structure may be applied to each of the four actuators shown in Figure 7.37. The control structure for each actuator would be as shown in Figure 7.38.
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Figure 7.38 - Lateral local modified skyhook control

Clearly this control law uses measurement in the vicinity of the actuator itself, but it may also be adapted to form a 'vehicle modified skyhook damping' control law which utilises body lateral velocity, yaw velocity, and roll velocity rather than body velocity at the actuator attachment in order to implement the ride improvement function in a similar manner to the methods used in section 7.3.1 for the vertical direction.

A comparison between the passive p.s.d. of body lateral acceleration and the active local modified skyhook damping control law is given in Figure 7.39. A comparison between this active strategy using an 'ideal' actuator and an electromechanical one is shown in the lateral acceleration p.s.d. of Figure 7.40.
A complete set of results for this control strategy implemented on a 'local', 'vehicle', and 'train-wide' basis, together with the impact of 'real' actuator technologies is given in Table 7.9. The
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longer train lengths are omitted because of simulation difficulties.

<table>
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<tr>
<th>Control method</th>
<th>Ride quality (% improvement over passive)</th>
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<td>41.8</td>
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<tr>
<td>Train-wide (4 vehicles)</td>
<td>40.7</td>
</tr>
</tbody>
</table>

Table 7.9 - Modified skyhook control (Lateral direction)

7.3.2 - Complementary filter control

The complementary filter control strategy may also be applied to the lateral direction on a 'local', 'vehicle', and 'train-wide' basis, although the train-wide control law does not fully utilise the potential benefits of train-wide knowledge but is formed from coupled vehicles having a 'vehicle' control structure. The overall structure is shown by Figure 7.41, in which high-pass and low-pass filters (3rd order) are applied to body displacements and suspension deflections respectively. The need to maintain suspension deflection clearances is paramount, Figure 7.42 shows the suspension deflection associated with each of the four corners of a single passive vehicle when traversing the deterministic feature defined in Chapter 6 at 55ms\(^{-1}\), i.e. a transition followed by a constant curvature of 5km on uncanted track, this gives a steady body acceleration of 0.6ms\(^2\) which is a typical of what a railway vehicle is subjected to.
The active suspension design must not violate the suspension deflection constraint. The worst deterministic deflection in Figure 7.43 combined with the maximum suspension deflection due to stochastic features must not exceed 80mm in the lateral case. All active suspensions are designed to give maximum ride quality while fulfilling the suspension deflection requirement.
The final results for this control strategy with a variety of actuator technologies is shown in Table 7.10.

![Complementary filter suspension deflection](image)

**Figure 7.43 - Complementary filter suspension deflection**

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**Table 7.10 - Complementary filter control (Lateral direction)**
7.3.3 - Optimal control

Optimal control may also be applied to the lateral direction. The application in this instance does not include the feedforward network described in section 7.2.3.3. The structure of optimal control applied to the lateral direction is shown in Figure 7.44, and it is very similar to the methodology used in the vertical case, with integral displacement feedback included to meet the deterministic deflection constraint.

![Figure 7.44 - Optimal control (Lateral direction)](image)

The application of 'local optimal' control to the lateral direction is theoretically possible but not included here since a half-car lateral model has not been developed. The design procedure involves 'tuning' the weights applied to body lateral and yaw accelerations. Analysis is performed to see if the design meets the suspension deflection constraints and if it does not, the design is re-iterated. Figure 7.45 shows the worst stochastic actuator force of vehicle 2 in a 3 vehicle train,
this is used to size the 'real' actuators. The final ride quality performance of an optimal lateral suspension for a variety of train lengths is shown in Table 7.11.

![Stochastic actuator force graph]

**Figure 7.45 - Straight line actuator force**

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<tr>
<th>Control method</th>
<th>Ride quality (% improvement over passive)</th>
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<tr>
<td>Train-wide (3 vehicles)</td>
<td>52.8</td>
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</tbody>
</table>

**Table 7.11 - Optimal control (Lateral direction)**
Chapter 7: Active Secondary Suspensions

7.4 - Summary

Three active secondary suspension control laws: 'modified skyhook damping', 'complementary filter control', and 'optimal control' were applied to the vertical and lateral directions. These control laws were applied on a 'local', 'vehicle', and 'train-wide' basis. Optimal control permits the development of a truly 'train-wide' control law which caters for the time-delays between axles at the design stage, 'modified skyhook damping', and 'complementary filter' control may also be applied to a train of vehicles, however, in these cases the actively-controller vehicles are simply coupled together to form a train of vehicles.

In the vertical direction, optimal control, which caters for the time-delays between axles gave approximately a 39% improvement in ride quality over a passive train. Modified skyhook damping and complementary filter control produced approximately a 35% improvement in ride. Local control approaches produced a 30% improvement. These ride calculations were made using an 'ideal' actuator. The electrohydraulic actuator produced the most severe degradation in these ride quality predictions, followed by the electromechanical actuator, and finally the electromagnetic actuator which produced results on par with those seen with an 'ideal' actuator.

In the lateral direction, the relative ranking of the various actuator technologies remained the same as the vertical direction. Optimal control produced the best results with a 50% improvement in ride quality, 'modified skyhook damping', and 'complementary filter control' gave approximately a 45% improvement.
Chapter 8

Active Inter-Vehicle Connections

8.1 - Overview

The train models developed in Chapters 4 and 5 included the possibility of having inter-vehicle actuation. This opens up the prospect of having some form of active inter-vehicle connection whereby ride quality may be improved on a train-wide basis. Figure 8.1 shows a sideview vertical model of a three vehicle railway train and illustrates this novel method of applying active suspension to a train of vehicles. Active control is applied via actuators fitted between the vehicles, rather than across the secondary suspension which is the more orthodox approach. This is a natural extension of the use of inter-vehicle dampers which have gained popularity on high-speed trains because of their beneficial impact on ride quality.

![Diagram of Inter-vehicle active suspension](image-url)
This is a new, unfamiliar idea and hence the prospect of developing an active control law is quite daunting, nevertheless some of the basic control laws developed for secondary active suspensions may be extended to form inter-vehicle ones. The motivation behind considering an active inter-vehicle suspension is twofold.

Firstly a barrier exists to the application of secondary active suspensions because of the perceived degradation in overall reliability.

Resistance to secondary active suspension arises partly because of questions about their reliability. Active suspensions are reliant upon environmentally-vulnerable sensors and failure-prone actuators. Clearly any attempt to reduce the number of components would have a beneficial impact on system reliability and also the overall cost. Processing hardware, sensors, and actuators are still required in an active inter-vehicle suspension, as shown by Figure 8.1. For an eight vehicle train for example, only seven actuators are necessary. For a secondary active suspension using actuators in the conventional position across the secondary suspension, sixteen actuators are needed. Another advantage of such a scheme is the more accommodating environment for fitting the actuators; neither end of the actuator is now connected to the harsh vibration environment encountered on the bogie frame. There is a similar benefit for the sensors, active inter-vehicle control requires only body-mounted absolute accelerometers and inter-vehicle displacement transducers. The normal form of active secondary suspension requires displacement measurements to be made across the harsh environment of the secondary suspension.

Secondly, the force control bandwidth requirements placed on a secondary active suspension are demanding, as demonstrated in Chapter 7, resulting in lower performance than that predicted with an 'ideal' actuator. Figure 8.2 shows the frequency content of both the inter-vehicle and secondary suspension deflections for a purely passive 3 vehicle train; these deflections will be 'seen' across the actuator ends and will therefore have an impact on the performance of the actuator.
It is clear that the inter-vehicle deflections contain a much lower high frequency content than the secondary deflections. This higher frequency content in the secondary deflection arises because of the higher bogie modes, and the inter-vehicle deflections are simply the difference between two adjacent vehicle motions which have already been 'filtered' by the primary and secondary suspensions of the vehicles and hence do not contain the same high frequency content. This higher frequency content of secondary deflections gives rise to high-frequency transmissibility from track to body, and hence a worsened ride quality if the actuator is unable to provide control at these frequencies.

The rationale behind the move to an active inter-vehicle connection as opposed to a secondary active suspension may be summarised in two points.

- The improved reliability because of a lower component count and more favourable environment.
- The lower bandwidth requirement placed upon actuators.

The first point can only be proved once a train with active inter-vehicle control has been
developed and is in revenue generating service. The second point will be investigated here with active inter-vehicle control being applied to both the vertical and lateral directions. These will now be considered in turn.

8.2 - Vertical train model

The overview cited the example of a 3 vehicle vertical train model with active inter-vehicle control. Two control strategies will be developed for this model, the first being the application of optimal control techniques, the second being an extension of skyhook damping applied on an inter-vehicle basis. These control strategies will be investigated for train lengths containing between 2 and 8 vehicles. The impact of active inter-vehicle control on body flexibility will also be investigated. This is an important point because the active control forces are injected into the system at locations which are more likely to excite the flexible modes, and there is therefore the possibility that such control forces could excite body flexibility to such a degree that any benefits obtained from improving the basic suspension response would be lost.

8.2.1 - Optimal control

Optimal control was introduced in Chapter 7 for an active secondary suspension. The performance index, equation (8.1), used here 'weights' the body accelerations and suspension deflections in a similar manner to the formulation used previously. 'n' being the number of vehicles, 'i' the bogie number, and 'j' the vehicle number.

\[ J = \lim_{\tau \to \infty} \int_0^\tau E \left( \sum_{i=1}^n d_i (z_{bog_i} - \dot{z}_{bog_i})^2 + \sum_{i=1}^n d_i (z_{bog_i} - \ddot{z}_{bog_i})^2 + \sum_{i=1}^n \rho (\dot{z}_{bog_i} - z_{bog_i})^2 \right) dt \quad \ldots \quad (8.1) \]

The models developed for active inter-vehicle control are of course different in that they contain a lower number of input channels. It is also interesting to note that the system is less controllable than was previously the case with secondary active control. In the inter-vehicle case the actuators have no control over the bogie bounce and pitch modes, whereas the secondary actuators at least
had some degree of control over the bogie bounce modes. This lack of controllability makes the application of optimal control to active inter-vehicle connections a cumbersome task involving the removal of uncontrollable modes in a similar manner to that performed by [Wilson, Sharp, and Hassan -1986]. The performance index given by equation 8.1 is minimised to obtain estimator and control matrices which are then used to form an overall controller. The overall structure of an optimal controller applied to the active inter-vehicle problem is shown in Figure 8.3. The performance assessment techniques outlined in Chapter 6 are used here in preference to the optimal control index as a measure of performance. The optimal control formulation is used as if it were to be implemented only on straight track: the performance index 'J' and its minimisation are used to obtain gain matrices and obtain a straight track optimal controller, but an additional component of integral inter-vehicle displacement feedback is also required if a realistic system is to be constructed. The need for integral displacement feedback is borne out of the need to keep the inter-vehicle displacements constrained to a reasonable level, as will be demonstrated in subsequent simulations.

![Figure 8.3- Optimal control implementation of active inter-vehicle suspension](image-url)
Each iteration of design is used to select an appropriate set of optimal control 'weights', which are then used to solve the optimal control problem and obtain a closed loop active inter-vehicle system. The integral displacement feedback gains are also 'tuned' iteratively to ensure compliance with secondary suspension and inter-vehicle deflection constraints.

Figures 8.4 and 8.5 contrast the achievable performance of a 3 vehicle train with an optimal active secondary suspension and an optimal inter-vehicle suspension. The plots show stochastic (straight-line) performance with increasing ride weighting ($p$ in equation 8.1). The deterministic secondary suspension must also be checked to ensure no violation of its constraints.

![Optimal control (Secondary vs Inter-vehicle)](image)

Figure 8.4 - Ride quality with increasing 'weight'
In both cases there is found to be a monotonic improvement in ride quality, but at the same time a minimum in stochastic deflection is obtained, which is usually the point that many researchers have termed the 'optimal active' suspension. The deterministic constraint is difficult to maintain with increasing ride weighting and is ultimately violated. The secondary active suspension with six actuators offers around a 30% improvement over the passive, whereas the inter-vehicle active with only two actuators offers around 15%. This comparison is performed relative to a single passive vehicle with a ride quality of 0.98 %g at 55ms\(^{-1}\); the ride index of a passive 3 vehicle train is slightly higher at 1.02 %g due to the coupling, but to avoid confusion comparisons are made with respect to an uncoupled vehicle unless stated otherwise. Figure 8.6 shows the relative response of the bounce and pitch acceleration between the active inter-vehicle and purely passive system with a typical passive inter-vehicle connection. Figure 8.7 shows the effect of coupling in a passive 3 vehicle train. The ride indices shown in this figure are slightly different from the figures previously quoted, the reason being that ride quality in this instance is measured via averaged time history responses on 'real' track, previous results assume a 'flat' track spectrum. Nevertheless, the relative trends remain the same. Increasing inter-vehicle stiffness tends to
Chapter 8: Active inter-vehicle connections

worsen the overall train ride quality; some inter-vehicle stiffness is inevitable since the vehicles need a coupler/gangway if they are to form a train. Increasing the inter-vehicle damping ultimately improves ride quality, however, a plateau is reached after which no improvement is seen.

The average improvement of around 15% is spread across all three vehicles. If the passive inter-vehicle damping between three isolated vehicles (each having a ride quality of 0.98 %g) is increased an improvement of 10% in the overall train ride quality may be obtained. The argument for introducing a more complex active scheme with only a marginal improvement over this figure then begins to falter. It should be stressed that the figure of 1.02 %g stated earlier for a coupled 3 vehicle train arises because of the low level of nominal inter-vehicle damping as well as the inter-vehicle stiffness inherent in a coupler/gangway connection.

Figure 8.6 - Active inter-vehicle vs Passive response (3 vehicle train)
The possibility of improving train ride quality on a global basis will now be dropped. An alternative to train-wide control is to concentrate on improving only a small section of the train. High-speed train sets usually have power cars at each end, the ride quality in which is not important, and hence the original optimal control formulation given by equation (8.1) may be modified to give equation (8.2) which applies 'weighting' only to the central portion of a train of arbitrary length 'n'.

\[
J \to \lim_{\rho \to \infty} \int_0^\tau \left[ E \left( \sum_{i=3}^{n-2} d(z_{beg1} - z_{end})^2 \right) \right. \\
&\left. + \sum_{i=3}^{n-2} d(z_{beg2} - z_{end})^2 \right) + \sum_{j=2}^{n-1} \rho (\frac{\dot{x}_j}{\rho} - \frac{\dot{z}_{beg1} + \dot{z}_{beg2}}{2})^2) \, dt \quad \ldots (8.2)
\]

The design in this instance again involves tuning the optimal control weights and analysing the system to ensure that suspension deflection constraints are met. Taking the example of the 3...
vehicle train shown in Figure 8.3, significant ride improvements may be obtained with this formulation. The integral displacement component of control shown in Figure 8.3 is vital and the reason for its inclusion will be explained in section 8.2.2, suffice it for now to say that it is required if all the suspension deflection constraints are to be met.

The histogram of Figure 8.8 shows the performance of optimal control applied to only the central portion of a 3 vehicle train, presented relative to the passive response. The ride of the centre vehicle may be improved quite considerably. In fact if it were not for the deterministic constraint it could be improved by over 90%, but the deterministic constraint was found to limit the improvement to around 30%. There is degradation of the adjacent vehicle ride quality, which tends to worsen the train-wide ride index with respect to its passive counterpart, but as stated earlier if these are power cars the control strategy as a whole is viable. There is also the possibility mentioned earlier of using conventional secondary actuators on the end vehicles, but this has not been investigated.

The p.s.d. plot of Figure 8.9 shows the ride improvement of vehicle 2 with this form of active inter-vehicle control applied.
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Figure 8.8 - Optimal control of centre portion (3 vehicle train)

Figure 8.9 - Passive vs Inter-vehicle optimal control
There was an initial worry concerning the impact that active inter-vehicle control would have on the ride quality if body flexibility were included; it could be that the control action would cause excitation of this mode and for the benefits gained by improving the rigid body modes to be engulfed by the excitation of the flexible mode. This has been confirmed to be false through computer simulation. Figure 8.10 includes body flexibility and compares passive and active optimal control strategies. There is a slight increase around 8.5Hz, but not a noticeable impact.

The degradation of the adjacent vehicle ride is shown by the pitch acceleration p.s.d. of Figure 8.11. Figure 8.11 is the response of vehicle 3. The degradation tends to occur around the resonance frequency and below.

The effect of 'real' actuator dynamics was found to be less profound than that observed with secondary active control strategies. Figure 8.12 compares the behaviour of an 'ideal' actuator with an electromechanical one by showing the differing bounce acceleration p.s.d's produced in each case. This is exactly what would be expected from the earlier discussion, the reason being that the inter-vehicle actuators do not provide a high-frequency transmission path from the bogie to
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the body once they begin to lose control. This response can be compared with a secondary electromechanical actuator by reference to Figure 7.14.

Figure 8.11 - Ride quality degraded on adjacent vehicle

Figure 8.12 - Comparison of 'Ideal' and 'Electromechanical' IV - actuators
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The application of optimal control to the central portion of a train will now be extended to consider longer train lengths. Figure 8.13 shows the improvement of the middle 3 vehicles in a 5 vehicle train set. Similar trends to the 3 vehicle train model recently analysed still apply, the ride quality of the central portion may be improved but at the expense of worse ride at the train ends.

![Graph showing bounce and pitch response of controlled to passive system](image)

Figure 8.13 - Control applied to a 5 vehicle train

A comprehensive set of results for optimal control applied to the central portion of a train is given in Tables 8.1 and 8.2 for both rigid and flexible vehicles. The effects of different actuator technologies is given in terms of their impact on ride quality. The ride indices in this case are not taken on a train wide basis as outlined in Chapter 6, the indices in this case are averages taken for all the acceleration measurement points within the central portion in the train, for a four vehicle train the average of \( z_{bog1}, z_{bog2}, z_{bog31}, z_{bog32}, \) and \( z_{bog32} \) are taken, the ride qualities of the two outer vehicles are ignored.
### Chapter 8: Active inter-vehicle connections

#### Table 8.1 - Vertical optimal control of central portion (Rigid bodies)

<table>
<thead>
<tr>
<th>Number of vehicles</th>
<th>Ride quality (% improvement over passive)</th>
<th>Ideal</th>
<th>Electro-hydraulic</th>
<th>Electro-mechanical</th>
<th>Electro-magnetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 vehicles</td>
<td></td>
<td>29.6</td>
<td>27.1</td>
<td>28.9</td>
<td>29.6</td>
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<tr>
<td>4 vehicles</td>
<td></td>
<td>27.4</td>
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<td>26.5</td>
<td>27.4</td>
</tr>
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<td>25.6</td>
<td>22.1</td>
<td>24.3</td>
<td>25.6</td>
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<tr>
<td>6 vehicles</td>
<td></td>
<td>23.2</td>
<td>20.6</td>
<td>22.7</td>
<td>23.2</td>
</tr>
<tr>
<td>7 vehicles</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 8.2 - Vertical optimal control of central portion (Flexible bodies)

<table>
<thead>
<tr>
<th>Number of vehicles</th>
<th>Ride quality (% improvement over passive)</th>
<th>Ideal</th>
<th>Electro-hydraulic</th>
<th>Electro-mechanical</th>
<th>Electro-magnetic</th>
</tr>
</thead>
<tbody>
<tr>
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<td>27.2</td>
<td>28.4</td>
</tr>
<tr>
<td>4 vehicles</td>
<td></td>
<td>26.8</td>
<td>24.7</td>
<td>25.5</td>
<td>26.8</td>
</tr>
<tr>
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<td></td>
<td>24.5</td>
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</tr>
<tr>
<td>6 vehicles</td>
<td></td>
<td>22.2</td>
<td>19.7</td>
<td>20.3</td>
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<td>7 vehicles</td>
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<td>-</td>
</tr>
</tbody>
</table>
8.2.2 - Absolute damping of centre portion

The successful application of optimal control to the central portion leads naturally to investigating if absolute damping may be applied in the same manner. A modified form of skyhook damping is shown in Figure 8.14. This applies skyhook damping to the central portion of the train, in this instance with a 3 vehicle train, only vehicle 2 is controlled. The high-pass filtering and integral displacement feedback are similar to the methods used in section 7.2.1 for the vertical secondary active suspension, and are retained here for identical reasons, i.e. to achieve a practical control law.

![Figure 8.14 - Controlling the central vehicle](image)

The control law structure shown in Figure 8.14 contains $\dot{z}_{\theta_2}$ and $\phi_{\theta_2}$ which are the vertical and pitch velocities respectively of the body of vehicle 2, and $d_{12}$ and $d_{23}$ which are relative displacements between vehicle ends. The controller attempts to apply absolute damping to the centre vehicle, in the process reacting the actuator forces on the outer vehicles. In the case of longer train lengths, absolute damping is applied to the central portion of the train, reacting forces
away from the centre of the train. Skyhook damping is filtered, and integral displacement is included to avoid excessive suspension travel on deterministic features. The control law is in effect composed of two components as shown by equation (8.3), which gives the inter-vehicle actuator forces for a train of arbitrary length 'n'. The first component is filtered skyhook damping and the second is integral displacement feedback.

\[
\begin{pmatrix}
F_1 \\
F_2 \\
\vdots \\
F_n \\
\end{pmatrix} = F \begin{pmatrix}
s^2 \cdot 2 \omega_n \omega_a \\
-s^2 \cdot 2 \omega_n \omega_a^2 \\
\vdots \\
-s^2 \cdot 2 \omega_n \omega_a^n \\
\end{pmatrix} F_{n-1} + \begin{pmatrix}
F^{(1)} \\
F^{(2)} \\
\vdots \\
F^{(n)} \\
\end{pmatrix} \quad \ldots (8.3)
\]

The skyhook damping component is given by equations (8.4a) and (8.4b). Equation (8.4a) applies to inter-vehicle actuators upstream from the centre of the train, whereas equation (8.4b) applies to downstream actuators, clearly for even numbered train lengths the inter-vehicle actuator lies on the centre of the train and by convention, equation (8.4a) applies.

\[
F^{(1)} = -\frac{\dot{Z}_{BOG(i-1)} \cdot \dot{Z}_{BOG(i+1)}}{2} C_{SKY} \cdot \frac{\dot{Z}_{BOG(i-1)} - \dot{Z}_{BOG(i+1)}}{l_a} C_{SKY} \quad \forall \ i = \frac{n}{2} \quad \ldots (8.4a)
\]

\[
F^{(2)} = -\frac{\dot{Z}_{BOG(1)} \cdot \dot{Z}_{BOG(n)}}{2} C_{SKY} \cdot \frac{\dot{Z}_{BOG(1)} - \dot{Z}_{BOG(n)}}{l_a} C_{SKY} \quad \forall \ i = \frac{n}{2} \quad \ldots (8.4b)
\]

The integral displacement feedback component of the control force is given by equation (8.5), which simply infers movement of the vehicle ends by measurements of body displacement just above the leading and trailing bogies, although in practice a displacement transducer would probably be placed across the inter-vehicle actuator and hence integral displacement would be more readily attainable. The method shown by equation (8.5) is more convenient for the control
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formulation used in computer simulation and is hence the method of choice.

\[
F^{\alpha} = \int_{0}^{\tau} K_{\text{pr}} \left[ \frac{\{Z_{\text{BGF1}} \cdot Z_{\text{BGF2}}\}}{2} \cdot \frac{\{Z_{\text{BGF1}} - Z_{\text{BGF2}}\}^{l_{\text{R}}} + \cdots}{l_{\text{R}}} \right] dt
\]

This control law will now be assessed in terms of ride quality improvement over a conventional passive train. The methods of assessment used in section 8.2.1 will also apply here; ride indices will be evaluated for only the central portion of the train. The p.s.d. plots given in section 8.2.1 will not be reproduced here to avoid repetition, instead Vampire plots are given to confirm the findings using the Matlab models. Two plots are given here, the first, Figure 8.15 shows the pitch acceleration p.s.d. for vehicle 2 in a 3 vehicle train, this confirms the ride improvement offered by absolute damping control of the central portion. The second plot, Figure 8.16 shows the pitch acceleration p.s.d. for vehicle 1, this demonstrates the degradation in ride of the outer vehicles.

The overall results for this particular control law using both rigid and flexible bodies with a variety of actuator technologies are given in Tables 8.3 and 8.4.
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Figure 8.15 - Vampire pitch acceleration p.s.d of vehicle 2 (3 vehicle train)

Figure 8.16 - Vampire pitch acceleration p.s.d of vehicle 1 (3 vehicle train)
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The need for high-pass filtering and integral displacement feedback as shown in Figure 8.14 will now be demonstrated through a series of time domain responses. A 3 vehicle train possessing neither high-pass filtering nor integral displacement feedback will be compared with a train possessing these two features. The time response will be due to the standard $0.4\text{ms}^{-2}$ vertical acceleration feature outlined in Chapter 6. Figure 8.17 shows how each of the 3 vehicles respond to this feature.

![Body accelerations graph](image)

**Figure 8.17** - Body bounce accelerations for 3 vehicle train

There is clearly a delay before each of the three vehicles reach a constant acceleration of $0.4\text{ms}^{-2}$. The suspension deflections of the first vehicle are shown in Figures 8.18 and 8.19. Figure 8.18 is for a train with pure skyhook damping, Figure 8.18 is the modified form. In Figure 8.18, the lead bogie deflection compresses to react the vertical acceleration force, and the trailing bogie deflection surprisingly extends, the reason for this being that the active inter-vehicle force between vehicles 1 and 2 attempts to provide absolute damping to the centre vehicle. The ever-increasing bounce velocity of vehicle 2 results in an ever-increasing active force, a force which is reacted by vehicle 1 and causes the trailing bogie's suspension to extend. This diagram
indicates unacceptable violations of the maximum 35mm vertical deflection constraint, but this effect is limited by providing high-pass filtering and integral displacement, as demonstrated by Figure 8.19.

**Figure 8.18** - Basic damping (3 vehicle train)

**Figure 8.19** - With integral displacement and filtering (3 vehicle train)
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The response of vehicle 2 is somewhat different, as shown in Figure 8.20 for the basic form of skyhook damping. This vehicle does not need to react active inter-vehicle forces and hence the suspension deflections are both compressions, with ever-increasing inter-vehicle skyhook damping force resulting in large violations of the suspension limits.

![Vehicle 2 suspension deflections](image)

**Figure 8.20 - Basic damping (3 vehicle train)**

High-pass filtering and integral displacement feedback reduce these suspension excursions as shown by Figure 8.21.

The behaviour of vehicle 3 is akin to that of vehicle 1, it also needs to react the inter-vehicle actuator forces. The leading and trailing suspension deflection responses are of course reversed.
The inter-vehicle displacement has limits on its maximum excursion; these are generally less than 100mm. Figure 8.22 shows how the modified skyhook damping scheme achieves this objective for both inter-vehicle displacements, these would of course be violated if high-pass filtering and integral displacement feedback were not included.
Figure 8.22 - With integral displacement and filtering (3 vehicle train)

Performance results are presented in Tables 8.3 and 8.4 for this modified form of absolute damping applied to the central portion of a train. A variety of train lengths and actuator technologies are examined for both rigid and flexible bodies.
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<table>
<thead>
<tr>
<th>Number of vehicles</th>
<th>Ride quality (% improvement over passive)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ideal</td>
</tr>
<tr>
<td>3 vehicles</td>
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<td>4 vehicles</td>
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</tr>
<tr>
<td>8 vehicles</td>
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</tr>
</tbody>
</table>

Table 8.3 - Absolute damping control of central portion (Rigid bodies)

<table>
<thead>
<tr>
<th>Number of vehicles</th>
<th>Ride quality (% improvement over passive)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ideal</td>
</tr>
<tr>
<td>3 vehicles</td>
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<td>19.3</td>
</tr>
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</table>

Table 8.4 - Absolute damping control of central portion (Flexible bodies)
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8.3 - Lateral train model

The two control laws applied to the vertical train model in section 8.2 will now be adapted and applied to the lateral direction. The application will concentrate purely on improving the ride quality of the central portion of a train. This is not a difficult task; in general body bounce is analogous to the lateral displacement, and yaw is analogous to pitch in the vertical direction. The formulation of optimal control weighting functions and the application of skyhook damping follow on naturally from the work performed in section 8.2. However, the lateral model is a much more complicated entity because of the greater number of degrees of freedom and the inclusion of wheelset creep dynamics. It becomes increasingly difficult to simulate reliably the behaviour of such an unwieldy model, particularly in a long train set and when 'real' actuator dynamics are included. For this reason, the results presented here are limited to the analysis of a simple 3 vehicle train without including 'real' actuator dynamics. Nevertheless, the results from applying 'optimal control', and 'absolute damping of the centre portion' to this restricted train length are extremely promising. It is important to emphasise that these simulation restrictions only arise because of computational demands, because the train models developed in Chapter 5 are capable of representing a train of arbitrary length with a multitude of 'real' actuator dynamics. The restriction is primarily due to the intensive demands placed on computer memory when simulating such a system, although computation time is also a secondary issue.

A preliminary statement needs to be made concerning the roll degree of freedom. The roll degree of freedom is included in the model for the wheelsets, bogies, and the body. In terms of ride quality analysis, the roll degree of freedom is simply neglected. It is inevitable that roll impacts on the perceived ride quality but for simplicity it is neglected here.

The two active inter-vehicle control strategies will now be analysed in turn. In order to avoid repetition, p.s.d. ride results will be given for the optimal control method, and time history plots will be given for the absolute damping method, for the same reasons given in section 8.2.
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8.3.1 - Optimal control

Optimal control may be applied to the lateral model in a similar manner to that performed for the vertical model in section 8.2.1. The model must again be adapted to extract the uncontrollable modes and hence ensure solution of the Riccati equations. The performance index in the lateral case 'weights' the body accelerations at a number of measurement points. For the 3 vehicle train under consideration here, the weighting is applied to the lateral accelerations above the leading and trailing bogies as well as the lateral acceleration at the c.o.g., which ensures that yaw acceleration is also taken into consideration when designing the controller. The secondary suspension deflections are also taken as part of the formulation.

Deterministic constraints are again of key importance, and the design procedure attempts to find the largest improvement in ride quality but without violating the secondary suspension deflection constraint. Integral displacement feedback is again used to supplement the basic optimal controller. This forms a more realistic controller which satisfies these objectives. The suspension constraints will be analysed in more detail in section 8.3.2. The ride performance will be addressed here through a series of p.s.d. plots. Figure 8.23 demonstrates the improvement in lateral acceleration using optimal control to improve the centre portion of the train.

![Figure 8.23 - Centre vehicle lateral acceleration](image-url)
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Yaw acceleration is also of importance, and Figure 8.24 shows the improvement in yaw acceleration for the centre vehicle. The trend of deteriorating the ride quality of the outer vehicles, as shown for the vertical models, also applies here, and Figure 8.25 shows the deterioration in yaw acceleration of the leading vehicle. The ride quality of the centre vehicle is improved by around 40%.

Figure 8.24 - Centre vehicle yaw acceleration (equivalent)

Figure 8.25 - Lead vehicle yaw acceleration
8.3.2 - Absolute damping of centre portion

The absolute damping control strategy outlined in section 8.2.2 for the vertical model used a combination of filtered skyhook damping of the bounce and pitch modes together with integral displacement feedback. This ensures ride improvement of the centre vehicle and at the same time limits suspension excursion. This control law may be readily extended to the lateral direction. Filtered lateral and yaw velocities are used to improve ride quality, integral displacement feedback is used to limit suspension travel particularly on deterministic features. Obviously in the lateral direction there are 4 actuators per body and integral displacement feedback needs to be applied to each of them. The skyhook damping role would be shared by all four.

This type of control law was found again to improve the centre vehicle ride by around 40%. At the same time deterioration in the ride quality of the outer vehicles was noted. A time-history demonstration of this control law will be used to demonstrate the benefits of filtering and integral displacement feedback. Many of these results are similar to those seen for the optimal control law developed previously. The deterministic test case in the lateral direction is a 0.6ms\(^2\) curve as given in Chapter 6. Figure 8.26 shows how an ordinary passive train would respond to this feature.

![Figure 8.26 - Constant 5km radius curve (55 m/s)](image-url)
Chapter 8: Active inter-vehicle connections

This same feature will be used as a test case for the active inter-vehicle control law. The yaw velocity response of this passive train is shown in Figure 8.27. The suspension deflections response is shown in Figure 8.28. The yaw response will later be used in an account of the behaviour of the active system. The maximum deflections shown in Figure 8.28 must also be satisfied by the active system.

![Figure 8.27 - Steady yaw velocity on the curve](image)

![Figure 8.28 - Secondary suspension deflections](image)
Chapter 8: Active inter-vehicle connections

The vehicle yaw motion is responsible for a number of minor movements which occur as each vehicle enters the feature, which can be seen in Figures 8.26 and 8.28 and also appear in the active strategy but are not as obvious due to other effects coming into play. The small negative accelerations seen in Figure 8.26 for vehicles 2 and 3 are due to the vehicle ahead having a positive yaw as it enters the curve, which tends to push the trailing vehicle in a negative direction. This effect is seen to a lesser extent in Figure 8.28, the predominant effect here is the outward movement of the body. The inter-vehicle displacement for this train is shown in Figure 8.29 - both the secondary suspension displacement and inter-vehicle movement are maintained at reasonable levels.

![Inter-vehicle displacement graphs](image)

Figure 8.29 - Passive inter-vehicle displacements

Two contrasting plots, Figure 8.30 and 8.31 show the secondary suspension and inter-vehicle displacements for the active inter-vehicle control strategy. The responses are somewhat different from the passive case. The difference is naturally due to the active inter-vehicle forces. As the leading vehicle enters the curve, the displacements behave in a very similar manner to their passive counterparts. The 'near-side' suspensions compress and the 'off-side' extends in order to generate the centripetal force required to make the leading vehicle follow the curve. However, as soon as the second vehicle enters the curve it develops absolute lateral and yaw velocity, and the inter-vehicle control law attempts to keep the second vehicle moving in a straight-line. The
reaction of the inter-vehicle actuator force on vehicle 1 causes the trailing end of vehicle 1 to require less centripetal force, which causes the secondary displacement of the trailing end to reverse as the body moves inwards. The integral displacement eventually comes into play and nulls the quasi-static suspension deflections.

**Figure 8.30 - Suspension deflections with inter-vehicle control**

**Figure 8.31 - Inter-vehicle displacement with active control**
Chapter 8: Active inter-vehicle connections

The inter-vehicle deflections shown in Figure 8.31 are higher than for the passive scheme as shown by Figure 8.29. The maximum value for the active scheme is however within acceptable limits.

8.4 - Summary

In summary, two active inter-vehicle control strategies: 'optimal control', and 'absolute damping of the central portion' were applied to both the vertical and lateral train models. Both strategies can provide similar performance levels when compared with an active secondary suspension (30%) but only for the central portion of the train. This result is not surprising considering that active inter-vehicle controllers do not have a force datum to react applied forces, this suggests that a better overall control strategy would be to incorporate some form of secondary active suspension on the outer vehicles to provide this force datum. This is left as an extension of this work.
Chapter 9

Implementation - Test Rig

9.1 - Overview

A small-scale test rig has been developed to demonstrate the potential benefits of active inter-vehicle connections as outlined in Chapter 8. There are a number of limitations in using a small-scale test rig to represent full-scale dynamics, but a practical implementation can be useful when validating purely theoretical results. It also provides suggestions on how to implement a full-scale version.

This chapter is divided into a number of areas: firstly the layout of the rig is described and a number of key components are identified, secondly the inter-vehicle actuators are discussed, and finally the control software is discussed and test results are given.

9.2 - Rig layout

The test-rig is composed of 3 vehicles slung underneath a conveyor belt, which is the shortest train length that may be used to study active inter-vehicle connections. There are 12 rollers acting as 'wheelsets' connected via a pushrod to the bogie frames. A spring and damper mechanism is used to connect the centre point of the bogie frame with an attachment on the body. The vehicles are inverted, which provides an inherent method of centring the vehicles through gravitational means. The conveyor belt is profiled to represent the track irregularities. In this instance the irregularity is a 200mm wavelength 'square-wave' profile. The bounce and pitch modes of the vehicles were both set at around 1Hz by varying the vehicle mass, and with a belt speed of
around 280mms\(^{-1}\) and a bogie spacing of around 350mm this ensures that both the bounce and pitch modes are excited. The secondary vertical damping is quite low, in fact the measurement of suspension displacement is amalgamated with the provision of vertical damping through the use of a potentiometric rotary displacement transducer which also provides frictional damping. There are also two accelerometers located on the centre vehicle close to the leading and trailing secondary suspensions. These measurements are sufficient to implement the 'modified absolute damping applied to the centre vehicle' control strategy outlined in Chapter 8.

Figures 9.1 to 9.3 show the arrangement of the test-rig from a number of perspectives with the final construction shown by the photographs of Figures 9.4 to 9.7. Figure 9.4 shows the rig in its entirety, Figure 9.5 is a closer view of the centre vehicle which also shows the inter-vehicle actuator and Figure 9.6 is a close-up view of the secondary suspension rotary displacement transducer discussed previously. Figure 9.7 shows the conveyor belt and track irregularity described previously.
Figure 9.1 - Sideview of test-rig
Figure 9.2 - Planview of test-rig
Chapter 9: Implementation - Test Rig

Figure 9.4 - Photograph of the test-rig in its entirety

Figure 9.5 - Close-up of the centre-vehicle
Figure 9.6 - Rotary displacement transducer & secondary damping

Figure 9.7 - Track/conveyor belt profile
Chapter 9: Implementation - Test Rig

Figure 9.8 is a photograph of the motor-drive, inter-vehicle actuator drive unit, and the Motorola 96000 control unit. This control unit was developed by Lamerholm Fleming, and supplied with software which permits the development of control laws on a PC before downloading to the control unit.

![Figure 9.8 - Photograph of the drive units and controller](image)

Figure 9.9 shows the scheme of the rig, including the motor drive unit for the conveyor belt. The overall controller is a Motorola 96000 DSP driven by PC software, which provides commands to the drive-unit and also commands the inter-vehicle actuator drive-units. The DSP unit reads in all the accelerometer and displacement measurements, and from these signals generates two inter-vehicle drive signals after processing the input signals in accordance with the active inter-vehicle control law.
Chapter 9: Implementation - Test Rig

Figure 9.9 - Overall scheme of the test-rig
Chapter 9: Implementation - Test Rig

9.3 - Inter-vehicle actuators

A form of electromagnetic actuation was chosen in preference to electrohydraulic or electromechanical, the reason for choosing electromagnetic actuation is obvious when considering the performance obtained in Chapters 7 and 8. The force levels involved in active inter-vehicle control, particularly at the test-rig level were predicted to be very small, less than 1 Newton, and it is difficult to obtain electromechanical and electrohydraulic actuators at these force levels. An actuator with the mechanical arrangement shown in Figure 9.10 was constructed. It consists of a pair of solenoids placed back to back to give bi-directional force capability, with an outer frame to link the two armatures together to form one end of the actuator. The other end of the actuator is connected to a base plate which supports the actuator coils. A photograph of the actuator is shown in Figure 9.11.

The pair of solenoids is force-controlled using measurements from a strain gauge bridge. The strain gauges are mounted on a thin aluminium plate. The strain is small, particularly for forces less than 1 Newton, and hence needs a lot of magnification.
This force-feedback is used to improve the bandwidth of the actuator and generate a more predictable output force. The force feedback loop shown in Figure 9.12 contains both a PID controller and a notch filter in cascade to provide compensation. The notch filter was needed to counteract the effect of a resonance at 25Hz caused by the flexibility of the strain-gauge beam. The PID controller was designed to give a bandwidth of around 150Hz for the force control loop.
The control of current supplied to the solenoids is obtained through pulse width modulation, and Figure 9.13 shows how these drive signals are obtained. A 200kHz clock is used to drive an 8-bit up-down synchronous counter. 512 clock pulses are required to form each modulation cycle, and the frequency of the output triangle wave is therefore around 400Hz. The triangle wave is compared with the output from the PID compensator and notch filter to give a pulse width modulated waveform. This pulse width modulated waveform is complemented to form another signal which is used in driving the two solenoids.
Figure 9.13 - Operation of solenoids and drive unit
Chapter 9: Implementation - Test Rig

A more detailed circuit diagram for each of the actuator drive units is given in Figures 9.14a-9.14c. A power bus and a control circuitry bus are generated from a coarse input supply. The power bus operates at 0-18 volts and control bus operates at ±9 and 0 volts. CMOS technology is used to implement the synchronous up-down counter. The basic synchronous counter chip is a 4 bit unit, two of which are cascaded to form an 8 bit counter, and a flip-flop is used to toggle between counting up and counting down, as the maximum or minimum count is reached.

The clock is generated from a pair of 'NOT' gates, this proved to be a very poor method of generating a clock frequency due to practical limits on the maximum frequency, the 200kHz clock should ideally be higher, and hence the pulse width modulation frequency would be a lot higher, the frequency of 400Hz used here is the lowest frequency that should be used. Frequencies lower than this level would tend to interfere with the mechanical modes of the system, however, the noise generated by higher frequencies also needs to be taken into consideration.

Operational amplifiers are used to perform the difference operation shown in Figure 9.12, the comparisons required between the triangle wave and the signal from the strain gauge measurements, and the PID controller. The strain gauge measurements are converted into a force signal by a high gain, low drift, dedicated amplifier.

Each solenoid is driven by a MOSFET, which simply converts the pulse width modulated gate drive signal generated previously into a high current signal for each solenoid. A flywheel diode is used to ensure that no damage is caused to the MOSFET when it turns off. The current into each solenoid reaches a maximum of around 3 Amps, limited by a shunt resistor. With zero output force each solenoid draws about 1.5 Amps, which keeps the actuator as a whole in tension, avoiding backlash and improving controllability. The overall combination of mechanical configuration and high performance force control overcomes the essentially non-linear nature of the electromagnetic system to produce a device suitable for inter-vehicle actuation.
Figure 9.14b - Inter-vehicle actuator drive-unit circuit diagram
Figure 9.14c - Inter-vehicle actuator drive-unit circuit diagram
9.4 - Control software

The control software used to implement the active suspension control law is developed on a PC and then cross-compiled to run on the target Motorola 96000 DSP. All the source code is written in 'C' and with the development tools provided it is not necessary to resort to low level routines to implement the control function. Literature on the implementation of active suspension is scarce, not surprising considering its commercial value, however [Majeed - 1990] describes the application of DSP technology to the control of a small-scale two-mass model. [Hanselmann - 1987] is a useful summary on the implementation of digital control. The control methodology is shown by Figure 9.15. Accelerometers are placed close to the secondary suspension attachment points on the centre vehicle, and these signals are integrated to produce absolute velocities and hence the skyhook damping forces. It is a simple matter to convert the acceleration measurements at the suspension attachment points into bounce and pitch accelerations. Rotary displacement transducers are placed across the secondary suspension of each vehicle. These provide not only a damping role as mentioned earlier, but also some means of measuring the suspension displacements. These displacements are not used within the control law but are used to assess how the train is behaving, a simpler and cheaper method than attaching accelerometers to all three vehicles.

The acceleration signals naturally need to be integrated to form velocities. A 'self-zeroing' type of integration is used to avoid drift in the velocity signal. This self-zeroing integrator has a transfer function as shown in Figure 9.15, which is basically an integrator with a low-frequency roll-off (set to 0.35Hz). Equation (9.1) is a continuous description of self-zeroing integrator.

\[
I(s) = \frac{\frac{1}{\omega^2}}{\left(1 + \frac{\sqrt{2}}{\omega_f} s + \frac{1}{\omega_f^2} s^2\right)} \quad \text{(9.1)}
\]
Chapter 9: Implementation - Test Rig

The control law is implemented by digital means and hence the continuous filter needs to be converted before it can be coded. The 'z' - transfer function is used to describe all transfer functions and is the universal method used when developing the 'C' code. A sampling frequency of around 500Hz is used, which is at least 10 times higher than the highest dynamic mode of the system. It is a simple matter to convert this transfer function to its discrete equivalent in this case using the bilinear transform of equation (9.2). Equation (9.3) is a discrete representation of equation (9.1).

\[
\frac{1}{s} = \frac{T}{2} \left( \frac{1 - z^{-1}}{1 - z^{-1}} \right) \quad \ldots (9.2)
\]

\[
I(z) = \frac{\left( -\frac{2}{T \omega_f} \right) z^{-2} + \left( \frac{2}{T \omega_f^2} \right)}{\left( 1 - \frac{2 \sqrt{2}}{T \omega_f} - \frac{4}{T^2 \omega_f^2} \right) z^{-2} + \left( 2 - \frac{8}{T^2 \omega_f^2} \right) z^{-1} + \left( 1 - \frac{2 \sqrt{2}}{T \omega_f} - \frac{4}{T^2 \omega_f^2} \right)} \quad \ldots (9.3)
\]
Chapter 9: Implementation - Test Rig

The DSP control unit code is called at every sample instant, it is therefore not difficult to implement the transfer function given by equation (9.3) provided a rolling record is kept of variables at previous sample times. The Motorola 96000 is a floating point processor with the capability of performing the numerous multiplications and additions involved in the control well within the allotted 20 ms sampling period. The 'C' source code for this control formulation is given in Appendix V. The operation may be segregated into supervisory control and dynamic control. The control operates in 4 modes which supervise the operation. The first involves ramping the belt up to full speed, and the second involves running the train as a purely passive vehicle, next active control is applied for another specified period, and finally the belt speed is ramped down to conclude the operation of the rig. The transition through the different modes is achieved by counting the sampling frequency to give an indication of time. Prespecified transition points are defined to mark the transition to a different operating mode, and a buzzer is sounded to mark this transition. While the supervisory control caters for the transition between modes, the dynamic control is turned on only during the third phase when active control is applied; otherwise the inter-vehicle actuator demand is set to zero. The transition times are constants and set within the 'C' source code. The controller and development software does however permit the gains within Figure 9.15 to be changed on-line.

There is continuous communication between the DSP control unit and the PC, this also allows variables to be observed on-line. Figures 9.16 to 9.18 show the user interface: the belt speed is represented by an analogue meter, whereas accelerations and displacements of a variety of variables are plotted as graphs. These facilities are provided with the development tool, and are distinct from the 'C' source code which aims simply to manipulate internal variables at each sample instant.

The assessment of ride quality is calculated a short time after each transition, which allows the transient response to decay. Measurement of ride quality is therefore derived only from the forced response, because inclusion of the transient response would lead to erroneous answers.
Chapter 9: Implementation - Test Rig

9.5 - Results

Before any analysis of the behaviour of the inter-vehicle test rig is performed it is important to ensure that the model possesses the correct modes of vibration. As stated earlier, both the bounce and pitch modes were set at around 1Hz, which is slightly higher than in a full-scale vehicle. Observation of the system frequencies may be performed by observing the natural response of the vehicle. For example, if the train is given an equal initial displacement to both the leading and trailing secondary suspensions of each vehicle and released, the bounce mode can be observed. An example of this test is shown in Figure 9.16. If the central vehicle is excited in pitch, the pitch mode may be observed as shown in Figure 9.17.

A practical proof of active inter-vehicle connections involves comparing results for the passive and active trains and observing the ride quality of the centre vehicle. The actuators in a purely passive mode are given a zero force command. Figure 9.18 shows a time recording of displacement and accelerometer time history measurements during a passive run. The time of the run is set to approximately 30 seconds, this ensures a large number of 1 Hz cycles are observed. The accelerometer measurements are used in formulating the ride index. The ride quality is assessed by recording the accelerometer time histories and performing an FFT on this data at the end of the run. This result is compared with a run using active inter-vehicle control. Figure 9.19 shows 4 FFT's. Two sets of results are given for the train running in a purely passive mode. The leading and trailing accelerometer FFT's are shown. Two contrasting results are given for the actively controlled run; the 1 Hz mode has been significantly improved for the centre vehicle.

9.6 - Summary

A small-scale test rig has been developed to further prove the concept of active inter-vehicle connections. The model does not contain roll-bars and a number of other more realistic features, however, the control law does demonstrate an improvement in the ride quality of the centre vehicle.
Figure 9.16 - Natural response (Bounce mode)
Figure 9.17 - Natural response (Pitch mode)
Figure 9.18 - Typical time history running as a purely passive train
Figure 9.19 - FFT comparisons
Chapter 10: Conclusions & Future work

Chapter 10
Conclusions & Future Work

This thesis addresses 4 key areas:-

- Theoretical investigation of 'full-vehicle' railway active suspension
- Development of 'train-wide' active suspension control laws
- Theoretical investigation of the effect of 'real' actuator dynamics on the overall active suspension performance
- Development of the novel idea of active inter-vehicle suspension
- Practical implementation issues related to active suspension

10.1 - Vehicle and train modelling

This thesis develops full-vehicle models for both the vertical and lateral directions, extending the basic two-mass models which are frequently considered by other workers in order that they are more representative of real vehicles. A crucial aspect which is frequently under-stated in the literature is the need to constrain suspension excursions, not only on random track, but also on deterministic features. This has been shown generally to be the limiting factor in the achievable performance of an active suspension.

The computer simulation results for a vertical secondary active suspension indicate that a 30% improvement in ride quality is likely with body flexibility deteriorating this by around 3% on average. A 45% improvement in the lateral ride quality through a secondary active suspension is possible. Control strategies utilising train-wide information offer an average 7% improvement...
over these figures. It must be stated that in both the vertical and lateral cases the deterministic suspension deflection associated with gradients and curves was found to be the limiting factor in improving ride quality, which will obviously depend upon the characteristics chosen for such track features.

10.2 - Real actuators

Experimental investigations of both semi-active and fully active strategies have been initiated by several manufacturers, but frequently the experimental benefits of active suspension have been lower than predicted by theoretical studies, one reason for which is the limited bandwidth of the actuator used to implement the active suspension. This thesis models a variety of actuator technologies and analyses the degradation in ride quality due to this finite bandwidth of each of the actuator types. Three actuator technologies deemed most suited to the active suspension role have been considered:

- Electrohydraulic actuation
- Electromechanical actuation
- Electromagnetic actuation

The limited force control bandwidth of each of the technologies is due to their internal dynamics. The electrohydraulic actuator is incapable of providing control at higher frequencies because of oil compressibility and flow limitations into the hydraulic cylinder. The electromechanical actuator is limited by the stiffness of the lead-screw and the high currents required to accelerate the motor armature inertia. The electromagnetic actuator has no such shortcomings, electrical and magnetic time constants being the main restriction. However these are small and hence this actuator is capable of controlling all the vehicle modes and is ideally suited to the active suspension role. The performance of each of the three actuator technologies has been assessed in terms of the deterioration in ride quality which they exhibit when replacing an 'ideal' actuator. The electrohydraulic actuator fared worst with up to a 15% deterioration, the electromechanical
Chapter 10: Conclusions & Future work

actuator ranked second with roughly a 7% deterioration, whereas the electromagnetic actuator fared best with a negligible deterioration with respect to the 'ideal' actuator.

10.3 - The use of inter-vehicle actuators

Two types of inter-vehicle control strategy were applied both vertically and laterally to a variety of train lengths. If ride improvements are sought for each vehicle in the train, a 15% improvement may be obtained over the train-wide ride quality of a similar train length possessing no inter-vehicle connection. This compares with a 13% improvement over the train-wide ride quality of a similar train length possessing no inter-vehicle connection which is attainable simply by providing a large amount of inter-vehicle damping. This thesis investigates the effect of inter-vehicle damping and stiffness on ride quality. Increasing the inter-vehicle stiffness initially tends to worsen the overall ride quality, this effect is more pronounced with low levels of inter-vehicle damping. Inter-vehicle damping tends to improve ride quality. However, with low levels of stiffness there can be an initial deterioration in ride quality after which there is a continual improvement. A plateau is reached with sufficiently high damping beyond which no further benefits are noticed. The train-wide active strategy does not provide significant benefits over those achievable by purely passive means. Other active control strategies have been developed which concentrate on improving the ride quality of just the central portion of a train, and improvements in ride quality by 30% vertically and 40% laterally have been demonstrated for a range of train lengths up to 8 vehicles, significantly higher than that achievable by purely passive means, although the ride quality of the outer vehicles is deteriorated slightly with this form of active inter-vehicle control.

10.4 - Practical implementation

Practical issues need to be addressed if active suspensions are to receive wide acceptance, and a test-rig demonstrating a 3-vehicle active inter-vehicle control strategy using electromagnetic actuation has been constructed. An active suspension requires sensing, computation, and
Chapter 10: Conclusions & Future work

actuation, each of which is important to the overall operation, and the test-rig demonstrated a number of issues relating to each of these requirements. The rig uses a conveyor belt to represent track inputs caused by moving vehicles, a feature which is essential if active suspensions on trains of vehicles are to be effectively addressed. An active suspension using electromagnetic inter-vehicle actuators has been implemented and tested, showing that improvements to the central vehicle(s) in trains is possible.

10.5 - Future Work

At the start of this thesis it was stated that suspension design was a heuristic procedure based on historical design rules. The 'optimal control' and 'modified skyhook damping control strategies' outlined in this thesis still possess this design procedure; the optimal control 'weights', 'skyhook damping gains', and 'integral displacement feedback gains' all need to be tuned to ensure an acceptable active suspension design. Current passive suspension design methodologies rely heavily on computer simulations and further design iterations to achieve an acceptable design. The active suspension design methodologies in this thesis are no different in that respect. This methodology is not surprising considering the highly complex systems involved; computer models are becoming more complex and accurate, reliance on iterative computer designs is a natural trend. However, in the author's opinion there is still a need for analytical solutions to suspension problems without the need for iterative design procedures. The analytical solutions must attempt to optimise ride quality and take account of the fact that suspension deflection is a constraint. Given a model of the vehicle and of the stochastic and deterministic track features it must be possible to arrive at an optimal suspension design; a solution which takes account of a number of suspension objectives. In the author's opinion, it is envisaged that covariance control [Skelton and Ikeda - 1989] could provide a framework for achieving this objective - 'optimal control' in its current textbook formulation is fundamentally an iterative procedure. The application of optimal control to the active suspension problem could also be advanced. The development of optimal control laws which truly cater for the time-delays between axles deserves further work, a number of authors [Hać and Youn - 1993], and [Sharp and Wilson - 1990] for
example have reformulated the optimal control problem, whereas others have used Padé approximations [Krtolica and Hrovat - 1992], and others have reformulated the problem in the discrete domain [Louam et al - 1988], the application of frequency domain based optimal control [Gupta - 1980] to the active railway suspension problem is another interesting area of work. The frequency domain provides a more accommodating environment for representing pure time-delays. These points are left as theoretical extensions of the work contained within this thesis.

In terms of practical extensions, the inter-vehicle test-rig could be developed to have some form of displacement centring of the inter-vehicle actuators. The actuators currently installed utilise force-feedback only, incorporating an outer displacement feedback loop would make them less prone to drift. Clearly further inter-vehicle control laws may be developed, which can subsequently be investigated on the test-rig. The final stage of work is the development of an active inter-vehicle strategy on a full-scale vehicle.
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References


Appendices

Appendix I - Parameter evaluation for the electrohydraulic actuator (Mathcad ®)

Appendix II - Parameter evaluation for the electromechanical actuator (Mathcad ®)

Appendix III - Parameter evaluation for the electromagnetic actuator (Mathcad ®)

Appendix IV - Train model software listings, vertical and lateral (Matlab ®)

Appendix V - Inter-vehicle actuator test-rig software ('C' code)
Appendix I - Parameter evaluation for the electrohydraulic actuator (Mathcad ®)
Evaluation of electrohydraulic actuator parameters for the linear model

An electrohydraulic actuator throttled through a servo-valve is shown below. It is convenient and realistic for the designer to assume a standard ram area, this parameter is solely effected by the maximum force required out of the actuator, and therefore this should cover a wide range of actuators, with an upper bound on their maximum force capability. This text does not aim to design for all possible actuators, but rather a reasonable subset of these, the above point should be noted when designing an actuator.

![Diagram of electrohydraulic actuator](image)

**Initial actuator specification**

We must specify three parameters when attempting to size an electrohydraulic actuator, these are RMS actuator force, RMS ram velocity, and RMS ram displacement. These are given below:

\[
\begin{align*}
F_{\text{rms}} &= 1000 \text{ N} \\
F_{\text{max}} &= 3 \cdot F_{\text{rms}} \\
V_{\text{rms}} &= 40 \text{ mms}^{-1} \\
V_{\text{max}} &= 3 \cdot V_{\text{rms}} \\
D_{\text{rms}} &= 5 \text{ mm} \\
D_{\text{max}} &= 3 \cdot D_{\text{rms}}
\end{align*}
\]

**Assumptions**

Supply pressure, and spool mass are assumed, and given below:

Assuming a supply pressure of

\[
P_s := 3000 \text{ PSI}
\]

\[
P_s := P_s \frac{100000}{15} \text{ Note the unit change}
\]

for the hydraulic power pack

\[
\beta = 1.38 \cdot 10^{-9} \text{ Oil compressibility}
\]

Assuming a spool mass of

\[
M_s := 0.075 \text{ kg}
\]

Assuming a maximum spool movement of

\[
Spool_{\text{max}} := 5 \text{ mm} \\
Spool_{\text{max}} := \frac{Spool_{\text{max}}}{1000} \text{ Note unit change}
\]
Assuming a maximum servo-valve current of

\[ \text{Currentmax} := 0.025 \text{ Amps} \]

Torque motor gain

\[ K_i := 142 \text{ NmAmp}^{-1} \]

Ram damping

\[ C_{	ext{damp}} := 50 \text{ Nsm}^{-1} \]

**Calculating the cylinder areas**

The basis for calculating the cylinder areas is the requirement to generate the maximum force. We know that both sides of the cylinder are pressurised under equilibrium conditions, and these pressures are known, it is a simple matter therefore to calculate the areas.

The equilibrium annulus pressure is assumed to be

\[ \text{Apres} := \frac{2}{3} \text{Ps} \]

This means that \( \frac{1}{3} \) \( \text{Ps} \) is available to drive oil into the cylinder.

For the annulus side of the cylinder

\[ \text{Area} := \frac{\text{Fmax}}{\text{Apres}} \]

Assuming a ram diameter of 15 mm

\[ D := 0.015 \text{ m} \]

For the bore of the cylinder

\[ \text{Barea} := \text{Area} + \frac{\pi D^2}{4} \]

**Calculating the servo-valve flow characteristics**

Under equilibrium conditions there is a force balance between both sides of the cylinder.

\[ \text{Bpress} := \frac{\text{Area} \cdot \text{Apres}}{\text{Barea}} \]

We need to calculate the non-linear flow characteristic of the cylinder first.

The flow drawn by the annulus side is

\[ \text{Aflow} := \text{Vmax} \cdot \text{Area} \text{ m}^3\text{s}^{-1} \]

Thus giving a flow gain of

\[ K_{qa} := \frac{\text{Aflow}}{\text{Spoolmax} \cdot \sqrt{\text{Ps} - \text{Apres}}} \text{ Flow gain } K_{qa} \]

The maximum oil flow is drawn by the bore side

\[ \text{Bflow} := \text{Vmax} \cdot \text{Barea} \text{ m}^3\text{s}^{-1} \]

Thus giving a flow gain of

\[ K_{qb} := \frac{\text{Bflow}}{\text{Spoolmax} \cdot \sqrt{\text{Ps} - \text{Bpress}}} \text{ Flow gain } K_{qb} \]

\[ \text{Pl} := 0, \frac{\text{Ps}}{30} \text{ Range of load pressures} \]

\[ q_a(\text{Pl}) := K_{qa} \cdot \text{Spoolmax} \cdot \sqrt{\text{Ps} - \text{Pl}} \text{ Annulus flow characteristic} \]

\[ q_b(\text{Pl}) := K_{qb} \cdot \text{Spoolmax} \cdot \sqrt{\text{Ps} - \text{Pl}} \text{ Bore flow characteristic} \]
This non-linear (square-root characteristic) is now linearised about a certain operating point in order to derive the linear model. This operating point is at the equilibrium force point.

\[ q_{\text{addp}}(P_I) := \frac{d}{dP_I} q_a(P_I) \quad \text{Derivative of annulus flow characteristic} \]

\[ q_{\text{bddp}}(P_I) := \frac{d}{dP_I} q_b(P_I) \quad \text{Derivative of bore flow characteristic} \]

\[ dq_{\text{addps}} := q_{\text{addp}}(\text{Apres}) \text{Value at linearisation point for the annulus side} \]

\[ dq_{\text{bddps}} := q_{\text{bddp}}(\text{Bpress}) \text{Value at linearisation point for the bore side} \]

\[ q_{a}(P_I) := dq_{\text{addps}} \cdot P_I + q_a(\text{Apres}) - dq_{\text{addps}} \cdot \text{Apres} \quad \text{Tangent for the annulus side} \]

\[ q_{b}(P_I) := dq_{\text{bddps}} \cdot P_I + q_b(\text{Bpress}) - dq_{\text{bddps}} \cdot \text{Bpress} \quad \text{Tangent for the bore side} \]

### Servo-valve characteristics

![Graph of oil flow vs load pressure](image)

**Calculating the spool-valve characteristics**

The spool-valve characteristics are calculated by linearising the flow to spool-valve displacement characteristic. This is again performed at different pressures for the bore and annulus sides.

\[ q_a(x_v) := x_v \cdot K_q a \cdot \sqrt{P_S} \quad \text{Characteristic for annulus side} \]
\[
q_b(x) = x \cdot K_b \sqrt{P_s}
\]
Characteristics for bore side

\[
q_{add}(x) := \frac{d}{dx} q_a(x)
\]
Derivative of flow characteristic

\[
q_{bdd}(x) := \frac{d}{dx} q_b(x)
\]
Derivative of flow characteristic

\[
dq_{add} := q_{add} \left( \frac{2}{3} \cdot \text{Spoolmax} \right)
\]
Value at linearization point for the annulus side of the cylinder

\[
dq_{bdd} := q_{bdd} \left( \frac{2}{3} \cdot \text{Spoolmax} \right)
\]
Value at linearization point for the bore side of the cylinder

Choosing a range of spool position settings

\[
x = 0, \frac{\text{Spoolmax}}{30} \quad \text{Spoolmax}
\]

Flow to spool valve displacement for linearization points

\[
\text{Oil flow (m}^3\text{s}^{-1})
\]

Spool displacement (m)

**Calculating the spool-valve dynamics**

For the spool dynamics we have to make certain assumptions. These assumptions are based upon the known damping and natural frequencies of these devices. There are many unknowns in our modelling (e.g. friction) so this assumption based upon empirical data is an acceptable one.

We know the natural frequency of the spool is around 250 Hz

\[
K_s := (2 \cdot \pi \cdot 250)^2 \cdot M_s
\]

We know the damping should be in the region of 200% (uncompensated)

\[
C_s := 2 \cdot 2 \cdot \pi \cdot 250 \cdot \frac{200}{100} \cdot M_s
\]

**Calculating the cylinder volumes**

Cylinder volumes may be calculated from the previous area calculations as well as the given specification for the ram displacement. A design tolerance of 50% is used to avoid the ram taking up the maximum cylinder stroke.
For the annulus side of the cylinder

\[ A_{\text{vol}} = D_{\text{max}} \cdot A_{\text{area}} \cdot 1.5 \text{ Note 50\% factor to avoid taking maximum stroke} \]

For the bore of the cylinder

\[ B_{\text{vol}} = D_{\text{max}} \cdot B_{\text{area}} \cdot 1.5 \text{ Note 50\% factor to avoid taking maximum stroke} \]

**Summary of calculated parameters**

A list of all the parameters required to represent an electrohydraulic servo-valve are given below:

- Beta = \( 1.38 \cdot 10^9 \text{ Dimensionless} \)
- \( A_{\text{area}} = 2.25 \cdot 10^{-4} \text{ m}^2 \)
- \( B_{\text{area}} = 4.01715 \cdot 10^{-4} \text{ m}^2 \)
- \( A_{\text{vol}} = 5.0625 \cdot 10^{-6} \text{ m}^3 \)
- \( B_{\text{vol}} = 9.03858 \cdot 10^{-6} \text{ m}^3 \)
- \( dq_{addpa} = -2.02193 \cdot 10^{-12} \text{ m}^5 \text{N}^{-1} \text{s}^{-1} \)
- \( dq_{bdp} = -1.92329 \cdot 10^{-12} \text{ m}^5 \text{N}^{-1} \text{s}^{-1} \)
- \( dq_{addxv} = 0.00935 \text{ m}^2 \text{s}^{-1} \)
- \( dq_{bdxv} = 0.01218 \text{ m}^2 \text{s}^{-1} \)
- \( C_{\text{damp}} = 50 \text{ Nsm}^{-1} \)
- \( M_s = 0.075 \text{ kg} \)
- \( C_s = 471.2389 \text{ Nsm}^{-1} \)
- \( K_s = 1.85055 \cdot 10^5 \text{ Nm}^{-1} \)
- \( K_i = 142 \text{ NmAmp}^{-1} \)
Appendix II - Parameter evaluation for the electromechanical actuator (Mathcad ®)
Evaluation of parameters for linear electromechanical actuator model

An electromechanical actuator normally consists of an electric motor, in this case we will consider only a permanent magnet DC motor, but other variations are possible. Some mechanism of translating the motor's rotational motion into linear motion is required, this normally takes the form of a standard screw thread, or a recirculating ball type screw thread. A cut through view of such an actuator is shown below:

Initial actuator specification

We must specify three parameters when attempting to size an electromechanical actuator, these are RMS actuator force, RMS ram velocity, and RMS ram displacement. These are given below:

\[
\begin{align*}
F_{\text{rms}} &= 1000 \text{ N} \\
F_{\text{max}} &= 3 \cdot F_{\text{rms}} \\
V_{\text{rms}} &= 40 \text{ mm/s} \\
V_{\text{max}} &= 3 \cdot V_{\text{rms}} \\
D_{\text{rms}} &= 5 \text{ mm} \\
D_{\text{max}} &= 3 \cdot D_{\text{rms}}
\end{align*}
\]

Assuming random Gaussian signals for F, V, and D.

Assumptions

Assuming a maximum motor speed of 3000 RPM

\[
\omega_{\text{max}} = 3000 \text{ RPM} \quad \omega_{\text{max}} = \frac{2 \pi \cdot \omega_{\text{max}}}{60} \text{ rad/s}
\]

Derivation of screw parameters

\[
pitch = \frac{V_{\text{max}}}{\omega_{\text{max}}} \quad \text{The actuator must give maximum linear speed for maximum rotary speed}
\]

\[
n = pitch
\]

Derivation of motor rating

\[
\omega_{\text{rms}} = \omega_{\text{max}} \frac{V_{\text{rms}}}{V_{\text{max}}} \quad \text{Assuming no backlash, the rotary speed is related to the linear speed}
\]

\[
T_{\text{rms}} = \frac{F_{\text{rms}} \cdot V_{\text{rms}}}{\omega_{\text{rms}}} \quad \text{Assuming negligible power loss in the conversion between linear and rotary motion}
\]
Prms := Trms-watts  Rated motor power
Tpeak := 4 Trms  The peak torque is 4 times the continuous torque

Summary of motor and screw parameters

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>pitch</td>
<td>$3.82 \times 10^{-4}$ m Screw pitch</td>
</tr>
<tr>
<td>Prms</td>
<td>40 Watts</td>
</tr>
<tr>
<td>Trms</td>
<td>0.382 Nm</td>
</tr>
<tr>
<td>Tpeak</td>
<td>1.528 Nm</td>
</tr>
</tbody>
</table>

With this information we can locate an appropriate motor and screw pair, and from manufacturers data obtain motor inertias, torque gains, etc.

This is now performed using a Printed Motors Ltd catalogue. Please note that if the motor rating and maximum torques are larger than the catalogue caters for (> 1.5kW and > 0.4 Nm) then this program will default to the largest motor in the catalogue, which may be inadequate!

PML catalogue

$i := 0..16$

<table>
<thead>
<tr>
<th>Model</th>
<th>GR12C</th>
<th>GR12CH</th>
<th>GR16C</th>
<th>GR16CH</th>
<th>GR19CH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>270</td>
<td>420</td>
<td>720</td>
<td>1050</td>
<td>1000</td>
</tr>
<tr>
<td>Rated torque</td>
<td>86.1</td>
<td>133.4</td>
<td>229.5</td>
<td>334</td>
<td>320</td>
</tr>
<tr>
<td>Rated speed</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>44.5</td>
<td>63.8</td>
<td>86.9</td>
<td>128.7</td>
<td>83</td>
</tr>
<tr>
<td>Rated current</td>
<td>8.62</td>
<td>8.36</td>
<td>10.34</td>
<td>9.55</td>
<td>14.4</td>
</tr>
<tr>
<td>Max pulse torque</td>
<td>860</td>
<td>1358</td>
<td>2380</td>
<td>3747</td>
<td>4500</td>
</tr>
<tr>
<td>Max current</td>
<td>5.2</td>
<td>6.2</td>
<td>5.7</td>
<td>8.6</td>
<td></td>
</tr>
<tr>
<td>Torque constant</td>
<td>10.8</td>
<td>17.0</td>
<td>23.7</td>
<td>37.3</td>
<td>24.0</td>
</tr>
<tr>
<td>EMF constant</td>
<td>11.3</td>
<td>17.8</td>
<td>24.8</td>
<td>39.0</td>
<td>25.0</td>
</tr>
<tr>
<td>Damping constant</td>
<td>1.16</td>
<td>1.95</td>
<td>3.57</td>
<td>6.44</td>
<td>7.76</td>
</tr>
<tr>
<td>Friction torque</td>
<td>4.2</td>
<td>4.2</td>
<td>7.7</td>
<td>7.7</td>
<td>9.8</td>
</tr>
<tr>
<td>Terminal resistance</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.65</td>
</tr>
<tr>
<td>Armature Inductance</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>Total inertia</td>
<td>1.2</td>
<td>1.2</td>
<td>5.93</td>
<td>5.93</td>
<td>12.71</td>
</tr>
<tr>
<td>Mechanical time constant</td>
<td>9.8</td>
<td>3.9</td>
<td>9.9</td>
<td>4.0</td>
<td>8.4</td>
</tr>
<tr>
<td>Regulation</td>
<td>7.71</td>
<td>3.12</td>
<td>1.6</td>
<td>0.65</td>
<td>1.07</td>
</tr>
<tr>
<td>Power rate</td>
<td>616</td>
<td>1536</td>
<td>955</td>
<td>2367</td>
<td>1593</td>
</tr>
</tbody>
</table>

Locate an appropriate motor

index1 := if(Prms ≥ GR19CH, 0, 0)
index2 := if(Prms ≥ GR16CH, 1, 0)
index3 := if(Prms ≥ GR16C, 1, 0)
index4 := if(Prms ≥ GR12CH, 1, 0)
index5 := if(Prms ≥ GR12C, 1, 0)
index := index1 + index2 + index3 + index4 + index5
Group together in one variable

\[
\begin{align*}
\text{Parameters}_{1,0} & := \text{GR12C}_i \\
\text{Parameters}_{1,1} & := \text{GR12CH}_i \\
\text{Parameters}_{1,2} & := \text{GR16C}_i \\
\text{Parameters}_{1,3} & := \text{GR16CH}_i \\
\text{Parameters}_{1,4} & := \text{GR12CH}_i
\end{align*}
\]

Evaluate parameters and scale to the correct units

\[
\begin{align*}
\text{Parameters}_{7,\text{index}} & := \frac{K_t}{100} \\
\text{Parameters}_{8,\text{index}} & := \frac{K_c}{1000 \cdot 2 \cdot \pi} \\
L_m & := \text{Parameters}_{12,\text{index}} \\
R_m & := \text{Parameters}_{11,\text{index}} \\
\text{Parameters}_{13,\text{index}} & := \frac{J_m}{10000} \\
C_m & := \frac{1000 \cdot J_m}{\text{Parameters}_{14,\text{index}}}
\end{align*}
\]

Screw parameter evaluation

A typical recirculating ball-screw is found to have these properties:

The connection between motor and screw stiffness

\[K_m := 10000000 \text{ Nm}^{-1}\]

A typical screw mass for up to 1 kW of power would be

\[M_s := 2 \text{ kg} \quad \text{The screw mass}\]

The natural frequency and damping of a screw thread are usually

\[w_n := 1000 \text{ Hz}\]

\[\zeta := 1.0 \quad 100\% \text{ damped}\]

Therefore

\[K_s := M_s w_n^2 \quad \text{Screw stiffness}\]

\[C_s := M_s \cdot 2 \cdot \zeta w_n \quad \text{Screw damping}\]
Summary of parameters

The parameters important for the linear actuator model are given below:

- Motor torque constant: $K_t = 0.108 \ \text{Nm/Amp}^{-1}$
- Motor back-EMF gain: $K_e = 0.108 \ \text{Vs}$
- Winding inductance: $L_m = 1 \cdot 10^{-4} \ \text{Henries}$
- Winding resistance: $R_m = 0.95 \ \text{Ohms}$
- Motor inertia: $J_m = 1.2 \cdot 10^{-4} \ \text{kgm}^2$
- Motor damping: $C_m = 0.012 \ \text{Nms}$
- Motor series stiffness: $K_m = 1 \cdot 10^7 \ \text{Nm}^{-1}$
- Screw pitch: $n = 3.82 \cdot 10^{-4} \ \text{m}$
- Screw mass: $M_s = 2 \ \text{kg}$
- Screw stiffness: $K_s = 2 \cdot 10^6 \ \text{Nm}^{-1}$
- Screw damping: $C_s = 4 \cdot 10^3 \ \text{Ns/m}^{-1}$
Appendix III - Parameter evaluation for the electromagnetic actuator (Mathcad ®)
Evaluation of parameters for a linear electromagnetic actuator model

The electromagnetic actuator shown below consists of an outer shell containing a pair of magnets, and an inner armature containing a pair of magnets. These magnets are set in a so-called push-pull arrangement, that is to say they can develop a bi-directional force depending upon the direction in which current is fed through the coils.

Initial actuator specification

We must specify three parameters when attempting to size an electromechanical actuator, these are RMS actuator force, RMS ram velocity, and RMS ram displacement. These are given below:

\[
\begin{align*}
F_{\text{rms}} &= 1000 \text{ N} \quad F_{\text{max}} = 3 \cdot F_{\text{rms}} \\
V_{\text{rms}} &= 40 \text{ mms}^{-1} \quad V_{\text{max}} = 3 \cdot V_{\text{rms}} \\
D_{\text{rms}} &= 5 \text{ mm} \quad D_{\text{max}} = 3 \cdot D_{\text{rms}}
\end{align*}
\]

Assumptions

We must assume that a typical actuator will operate with a maximum gap flux density of 1 Tesla.

\[
B_{\text{max}} = 1 \text{ Tesla}
\]

A typical actuator will generate this force when at maximum extension with a coil current of approximately 50 Amps.

\[
\begin{align*}
\text{Gap} &= 15 \text{ mm} \quad \text{Gap} = \frac{\text{Gap}}{1000} \text{ m} \quad \text{Nominal airgap} \\
I_{\text{max}} &= 50 \text{ Amps} \\
\mu &= 4 \cdot \pi \cdot 10^{-7} \quad \text{Magnetic permeability of air}
\end{align*}
\]
Derivation of MMF and number of turns

We need to calculate the magneto-motive force required to create a flux density of 1 Tesla at the maximum working gap:

$$\text{MMF} = 2 \cdot D_{\text{max}} \cdot B_{\text{max}}$$

We can calculate the number of turns required from the previous assumption of the maximum coil current

$$N = \text{floor} \left( \frac{\text{MMF}}{\text{Imax}} \right) \text{TURNS} \quad \text{Number of turns on the coil}$$

Calculation of the magnetic pole area

The force generated by a magnetic actuator is given by:

$$P_{\text{area}} = \frac{2 \cdot F_{\text{max}} \cdot \mu}{B_{\text{max}}^2} \text{m}^2$$

The pole diameter is therefore

$$D_{\text{pole}} = \frac{4 \cdot P_{\text{area}}}{\pi} \text{m} \quad R_1 := \frac{D_{\text{pole}}}{2} \text{m}$$

Calculation of the conductor window dimensions

A dimensioned view of the magnets is given in the figure below. The coils are circular in nature and this view is a cut through of the centre of the coils.

The RMS current in the coils is

$$I_{\text{rms}} = \frac{8 \cdot D_{\text{rms}}}{N^2 \cdot P_{\text{area}} \cdot \mu} \text{Amps}$$

Assuming the coils can carry a current density of

$$J = 2000000 \text{ Amps m}^{-2}$$

The copper area is therefore

$$A_c := \frac{I_{\text{rms}}}{J} \text{m}^2$$
Assuming a packing factor of 0.7 gives us the conductor window area

\[ PF = 0.7 \quad \text{Window packing factor} \]

\[ Aw = \frac{Ac}{PF} \text{ m}^2 \quad \text{Window area} \]

\[ d = \sqrt{Aw + Ac} \quad \text{m} \quad \text{Assuming a square window} \]

\[ w = d \quad \text{m} \quad \text{Outer pole width} \]

\[ R2 = R1 + w + a \quad \text{m} \quad \text{Entire magnet radius} \]

**Calculation of the self and mutual inductances**

The mutual flux density is given by

\[ \Phi_m = \frac{\mu N \text{Irms} \cdot R1^2}{2 \cdot \text{Drms}} \quad \text{Weber} \]

The mutual coil inductance is given by

\[ L_m = \frac{N \Phi_m}{\text{Irms}} \quad \text{Henries} \quad \text{Coil mutual inductance} \]

The leakage flux density is given by

\[ \Phi_l = N \text{Irms} \cdot \mu \left( \frac{d}{R2} + 2 \cdot R2 \ln \left( 1 + \frac{2 \cdot a}{R2 - R1} \right) \right) \]

The self inductance is given

\[ L_1 = \frac{N \Phi_l}{\text{Irms}} \quad \text{Henries} \quad \text{Coil self inductance} \]

**Calculation of the coil resistance**

The resistivity of copper is known to be

\[ \rho = 1.67 \cdot 10^{-8} \quad \text{Ohm m} \]

\[ R = \frac{N^2 \cdot \pi \cdot \rho}{d} \left( 1 + \frac{2 \cdot R1}{R2 - R1} \right) \quad \text{Ohm} \]

**Calculation of the linearised model parameters**

For the linear model we need to calculate the gains Kg and Ki

\[ Kg = \frac{-2 \cdot \text{Frms}}{\text{Drms}} \quad \text{Nm}^{-1} \quad \text{Gain} \]

\[ Ki = \frac{2 \cdot \text{Frms}}{\text{Irms}} \quad \text{NAmp}^{-1} \quad \text{Gain} \]
Summary of parameters

The parameters required by the linear and non-linear models are given below:

\( \mu = 1.257 \times 10^{-5} \) Magnetic permeability of air

\( N = 477 \) TURNS Number of turns

\( A = 0.008 \) m² Pole Area

\( A = 0.008 \) m² Pole Area

\( R = 4.214 \) Ohms Coil resistance

\( L_{m} = 0.216 \) Henries Coil mutual inductance

\( L_{1} = 0.12 \) Henries Coil self inductance

\( K_{g} = -4 \times 10^{-5} \) Nm⁻¹ Gain

\( K_{i} = 107.79 \) NAmp⁻¹ Gain

\( \text{Gap} = 0.015 \) m
Appendix IV - Train model software listings, vertical and lateral (Matlab ®)
RAILPRMT

% Define parameters of a Bombardier Prorail railway vehicle

% Masses
mb = 3.8e+04; (kg) body mass
mt = 2500; (kg) truck/bogie mass
mmp = 5; (kg) airspring mid-point mass
mw = 1900; (kg) wheelset mass

% Inertias
ibp = 2.31e+06; (kgm^2) body pitch inertia
ibr = 5e+04; (kgm^2) body roll inertia
iby = 2.31e+06; (kgm^2) body yaw inertia
itp = 2000; (kgm^2) truck/bogie pitch inertia
itr = 1500; (kgm^2) truck/bogie roll inertia
ity = 3100; (kgm^2) truck/bogie yaw inertia
iw = 1100; (kgm^2) wheelset yaw inertia

% Stiffness
ksz = 5.08e+05; (N/m) air spring stiffness
kaz = 1; (N/m) secondary change of area stiffness
krz = 2.54e+05; (N/m) secondary resevoir stiffness
kpz = 2.5e+06; (N/m) primary suspension spring constant
kcz = 1.1e+05; (N/m) bellow vertical spring constant
kcy = 1.65e+04; (N/m) bellow lateral spring constant
ksy = 2.6e+05; (N/m) secondary lateral spring constant
kbdr = 2.4e+06; (Nm/rad) body roll stiffness
kpx = 1.75e+07; (N/m) primary longitudinal spring constant
kpy = 1.75e+07; (N/m) primary lateral spring constant
kby = 1000; (Nm/rad) bogie yaw stiffness

% Damping
crz = 3e+04; (Nsm/rad) secondary damping constant
cpz = 1.79e+04; (Nsm/rad) primary damping constant
ccs = 6000; (Nsm/rad) bellow vertical damping constant
csv = 6000; (Nsm/rad) bellow lateral damping constant
csy = 1.65e+04; (Nsm/rad) secondary lateral damping constant
chx = 2e+05; (Nsm/rad) anti-hunting damping constant
cpx = 1.5e+04; (Nsm/rad) primary suspension longitudinal damping
cbdr = 1e+05; (Nms/rad) body roll damping

cby = 1e+04; (Nms/rad) bogie yaw damping

% Dimensions
lb = 19; (m) bogie to bogie spacing
lc = 9.5; (m) semi bogie to bogie spacing
lw = 2.5; (m) wheel to wheel spacing
lr = 1.25; (m) semi wheel to wheel spacing
lv = 27; (m) vehicle length + gangway length
lh = 15; (m) semi vehicle to vehicle spacing
lp = 1.25; (m) distance from bogie centre to axle
lad = 2; (m) semi-vehicle width
lbw = 1.3; (m) semi-bogie width
lpy = 1; (m) primary suspension lateral position
lasy = 0.9; (m) airspring lateral spacing
ls1 = 9.5; (m) first actuator longitudinal location
ls2 = 9.5; (m) second actuator longitudinal location
lbs = 1.15; (m) body-bogie vertical spacing

% Bending mode parameters
lambda = 35; (m) wavelength of first symmetric bending mode
bendw = 53.41; (rad/s) first symmetric bending mode natural frequency
benddamp = 0.05; % first symmetric bending mode damping

% Wheel-rail contact parameters
wlambda = 0.05; % new wheel conicity
r0 = 0.46; (m) wheelset radius
f1 = 9.04e+06; % parameter used in linear model
f2 = 7.84e+06; % parameter used in linear model
lg = 1.4; (m) wheelset gauge length
kc = 1.28e+07; (N/m) stiffness between track and wheelset

% Bending mode parameters
amplitude = 1; % (m) maximum amplitude of first symmetric bending mode
function [a,b,c,w,db,dw] = railcarv

RAILCARV = function [a,b,c,w,db,dw] = railcarv;

Generate sideview vertical state-space model of a railway carriage; data taken from a Bombardier Prorail best current passive suspension. The force inputs 'Fm1z' and 'Fm2z', are applied to the vehicle body and reacted on the bogie. The force inputs 'Fhrnlz' and 'Fbm2z' are end connection force inputs. The model does not use the small mass approximation in the airspring.

MODEL DESCRIPTION

INPUTS:

OUTPUTS:

STATES:

% Create 'a' matrix
a = [

% Create 'b' matrix
b = [

% Create 'c' matrix
c = [

% Create 'w' matrix
w = [

% Create 'db' matrix
db = [

% Create 'dw' matrix
dw = [zzeros(1,4)];

% Load vehicle parameters (NOTE: adjust to full sideview values)
railprmt
mb = mb; % (kg) body mass
mt = mt; % (kg) truck/bogie mass
ibp = ibp; % (kgm^2) body pitch inertia
itp = itp; % (kgm^2) truck/bogie inertia

ksz - 2*ksz; % (N/m) air spring vertical series stiffness
kaz - 2*kaz; % (N/m) secondary vertical change of area stiffness
krz - 2*krz; % (N/m) secondary vertical resevoir stiffness
kpz - 2*kpz; % (N/m) primary suspension vertical spring constant
crz - 2*crz; % (N/m) secondary vertical damping constant
cpz - 2*cpz; % (N/m) primary vertical damping constant

% Develop by Ian Pratt, Loughborough University.
function [a,b,c,w,db,dw] = rcarflxv
%RCARFLXV
FUNCTION [a,b,c,w,db,dwJ : rcarflxv;

•

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•

Generate sideview vertical state-space model of a railway
carriage; data taken from an Bombardier Prorail best current
passive suspension. The force inputs 'Fmtlz' and 'Fmt2z', are
applied to the vehicle body and reacted on the bogie. The
force inputs 'Fbmlz', and 'Fbm2z' are end connection force
inputs. The model does not use the small mass approximation
in the airspring.
MODEL DESCRIPTION :
INPUTS ;
w =

I

Zwmll'
Zwml2'

Zwm2l'
Zwm22 ,

I

STATES :
Zbm

Zbm'
Zf
Zf'

Ztml
Ztml'
ZmpI

b

OUTPUTS

Fmtlz
Fmt2z
Fbmlz
Fbm2z

Zbm' ,
Phibm' ,
Zendml
Zendml'
Zbogml
Zbogml'
Zbogm2
Zbogm2'
Zendm2
Zendm2 '
Ztml
Ztml'
Ztm2
Ztm2'

Ztm2
Ztm2'
Zmp2
Phibm
Phibm'
Phitml
Phitml'
Phitm2
Phitm2 '
Zwmll
Zwml2
Zwm21
Zwm22

'a', 'b', 'c', 'w', 'db', 'dw' is the returned state-space model.
Developed by Ian Pratt , Loughborough University.

, Load vehicle parameters (NOTE: adjust to full sideview values)
railprmt
mb = mb; % (kg) body mass
mt = mt; \ (kg) truck/bogie mass
ibp = ibp; \ (kgm~2) body pitch inertia

itp
ksz
kaz
krz
kpz
crz
cpz

itp; \
2*ksz;
2*kaz;
2*krz;
2*kpz;
2*crz;
2*cpz;

(kgm~2) truck/bogie inertia
\ (N/m) air spring vertical series stiffness
\ (N/m) secondary vertical change of area stiffness
\ (N/m) secondary vertical resevoir stiffness
\ (N/m) primary suspension vertical spring constant
\ (Ns/m) secondary vertical damping constant
\ (Ns/m) primary vertical damping constant

, Create 'a' matrix
a = I a 1 zeros(1,l8)
-2* (kaz+ksz)/mb a -(kaz+ksz) * (bendmode(lv, lambda,amplitude,
(lhz-lt))+bendmode(lv,lambda,&mplitude, ...
(lhz+lt)))/mb a kaz/mb 0 ksz/mb kaz/mb 0 ksz/mb zeros(l,lO)
zeros(l,3) 1 zeros(l,16)
-(kaz+ksz)*(bendmode(lv, lambda, amplitude, (lhz-lt»)+ '"
bendmode(lv, lambda, amplitude, (lhz+lt)I)/mb 0 ...
-bendwn~2 -2*benddamp*bendwn kaz*bendmode(1v,1ambda,amplitude,
(1hz-1t)l/rob a ksz*bendmode(lv,1ambda.amp1itude.
(lhz-lt))/mb kaz*bendmode(lv, lambda, amplitude • . . .
(lhz+lt))/mb 0 ksz*bendmode(1v,lambda,amplitude • . "
(lhz+lt))/mb -1t*kaz* (bendmode(lv, lambda,
amplitude. (lhz-lt))-bendmode(1v,lambda,amplitude,
(1hz+lt)) )/mb*a zeros(l.9)
0000 a 1 zeros(l,l4)
(kaz+ksz)/mt a (kaz+ksz)*bendmodellv,lambda,amp1itude,
11hz-1t))/mt 0 -(2*kpz+kaz)/mt -2*cpz/mt -ksz/mt ...
zerosll,3) (kaz+ksz)*lt/mt zeros(l.51 kpz/mt kpz/mt a 0
ksz/crz 0 ksz/crz*bendmode(1v,lambda,amp1itude, (1hz-It))
o krz/crz 1 -(ksz+krz)lcrz 0 0 0 ksz*1t/crz zeros(l,9)
zeros(l,B) 1 zeros(l,ll)
(kaz+ksz)/mt a (kaz+ksz) *bendmode(lv, 1ambda, amplitude.
(1hz+1t))/mt zeros(l,4) -(kaz+2*kpz)/mt -2*cpz/mt ...
-ksz/mt -lkaz+ksz)*1t/mt zeros(l,7) kpz/mt kpz/mt
ksz/crz a ksz/crz*bendmode(1v,lambda,amp1itude, (1hz+1t)) ...
o 0 a 0 krz/crz 1 -(ksz+krzl/crz -ksz*1t/crz zeros(l,9)
zeros 11. 11) 1 zeros(l,B)
00 It* (kaz+ksz) * (bendmode(lv, lambda,amplitude, (lhz-lt))bendmode(lv,lambda,amplitude, (lhz+lt))/ibp ...
o kaz*lt/ibp 0 ksz*lt/ibp -kaz*lt/ibp 0 -ksz*lt/ibp '"
-2*(kaz+ksz)*lt A 2/ibp zeros(l.9)
zeros(l,l3) 1 zeros(l,6)
zeros(l,l2) -2·kpz·lr~2/itp -2*cpz*1r~2/itp 0 a kpz*lr/itp -kpz*1r/itp 0 0
zeros(l,15) 1 zeros(l,4)
zeros(l,14) -2*kpz*lr A 2/itp
zeros(4,2a) ];

-2*cpz*lr~2/itp

a 0 kpz*lr/itP -kpz*lr/itP

\ Create 'b' matrix
b::: I zerostl,4) ; 11mb 11mb 11mb -11mb; zeros(l,4)
bendmode(lv.lambda.amplitude,O)/mb ...
bendmode(lv, lambda. amplitude. (1hz-It) 11mb .. .
bendmode(lv,lambda.amplitude, (lhz+lt»)/mb .. .
-bendrnode(lv, lambda, amplitude. IVl/mb; zeros(l,4)
-l/mt zeros(l,3) ; zerosl2,4) ; a -l/mt 0 0
zeros(2,4) ; It/ibp -lt/ibp lhz/ibp 1hz/ibp ; zeroslB,41 ];
, Create 'c' matrix
c ::: I -2*(kaz+ksz)/mb 0 -bendwn~2*bendmode(lv,1ambda,amplitude,
1hz) -2*bendwn*benddamp*bendmodellv,1ambda,amp1itude,
1hz) kaz/mh 0 ksz/mb kaz/rnb 0 ksz/mb zerosll.10)
o 0 0 0 kaz*lt/ibp 0 ksz*lt/ibp -kaz*lt/ibp 0 -ksz*lt/ibp

,J, ."

7>'~i: ':~~ ::;~,+ .
/~ ., :7' J~ 'di~':v~~~~-i:::~::'

J


Create 'w' matrix
\[
\begin{bmatrix}
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\end{bmatrix}
\]

Create 'db' matrix
\[
\begin{bmatrix}
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\end{bmatrix}
\]

Create 'dw' matrix
\[
\begin{bmatrix}
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) & \text{zeros}(S,4) \\
\end{bmatrix}
\]
Generate sideview vertical state-space model of a railway carriage; data taken from a Bombardier Prorail best current passive suspension. The force inputs 'Fmt1z' and 'Fmt2z', are applied to the vehicle body and reacted on the bogie. The force inputs 'Fbm1z', and 'Fbm2z' are end connection force inputs. The model is developed through the 'MASS-DAMPER-SPRING' method of formulation:

$$M X + C X + K X = B f F + B w Zw$$

Developed by Ian Pratt, Loughborough University.
\[ c = \{ \text{zeros}(1,16) \ 1 \text{zeros}(1,7) \} \text{zeros}(1,21) \ 1 \ 0 \ 0 \\
\text{zeros}(1,4) \ \text{hz} \ \text{zeros}(1,8) \ \text{zeros}(1,8) \ \text{zeros}(1,4) \ \text{hz} \ \text{zeros}(1,10) \\
1 \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
\text{zeros}(1,8) \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
\text{zeros}(1,8) \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
0 \ 0 \ 1 \ \text{zeros}(1,21) \\
\text{zeros}(1,10) \ 1 \ \text{zeros}(1,13) \\
\text{zeros}(1,4) \ 1 \ \text{zeros}(1,19) \\
\text{zeros}(1,12) \ 1 \ \text{zeros}(1,11) \ \}^*c; \]

\[ db = \{ \text{zeros}(1,16) \ 1 \text{zeros}(1,7) \} \text{zeros}(1,21) \ 1 \ 0 \ 0 \\
1 \ \text{zeros}(1,4) \ \text{hz} \ \text{zeros}(1,10) \\
\text{zeros}(1,8) \ \text{zeros}(1,4) \ \text{hz} \ \text{zeros}(1,10) \\
1 \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
\text{zeros}(1,8) \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
\text{zeros}(1,8) \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
1 \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
\text{zeros}(1,8) \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
0 \ 0 \ 1 \ \text{zeros}(1,21) \\
\text{zeros}(1,10) \ 1 \ \text{zeros}(1,13) \\
\text{zeros}(1,4) \ 1 \ \text{zeros}(1,19) \\
\text{zeros}(1,12) \ 1 \ \text{zeros}(1,11) \ \}^*db; \]

\[ dw = \{ \text{zeros}(1,16) \ 1 \text{zeros}(1,7) \} \text{zeros}(1,21) \ 1 \ 0 \ 0 \\
1 \ \text{zeros}(1,4) \ \text{hz} \ \text{zeros}(1,10) \\
\text{zeros}(1,8) \ \text{zeros}(1,4) \ \text{hz} \ \text{zeros}(1,10) \\
1 \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
\text{zeros}(1,8) \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
1 \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
\text{zeros}(1,8) \ \text{zeros}(1,4) \ \text{lt} \ \text{zeros}(1,10) \\
0 \ 0 \ 1 \ \text{zeros}(1,21) \\
\text{zeros}(1,10) \ 1 \ \text{zeros}(1,13) \\
\text{zeros}(1,4) \ 1 \ \text{zeros}(1,19) \\
\text{zeros}(1,12) \ 1 \ \text{zeros}(1,11) \ \}^*dw; \]
function [a,b,c,w,db,dw] = railtrn(n,model,actuator,intcon)

%RAILTRN

FUNCTION [a,b,c,w,db,dw] = railtrn(n,model,actuator,intcon);

Generate sideview vertical state-space model of an 'n'
carriage train of railway vehicles. 'model' may be set
to one of the following options:
1. 'rigid' - rigid body modes only present ('railcarv.m')
2. 'flexible' - flexible body modes present ('rcarflxv.m')
3. 'mck' - use mass-damping-stiffness model ('rcarmckv.m')

The model may have a variety of secondary actuators
('actuator') or active inter-vehicle connections
('intcon'), from one of the following:
1. 'passive' - purely passive device
2. 'ideal' - infinite bandwidth actuator
3. 'e/hyd' - electro-hydraulic actuator
4. 'e/mech' - electro-mechanical actuator
5. 'e/mag' - electro-magnetic actuator

MODEL DESCRIPTION:

INPUTS:

w = Zwll
    2Zml1
    2Zml2
    2Zml'n
    2Zmn1
    2Zmn2
    2Zmn'n

STATES:

railcarv2-1
    intcon-1
    railcarv2-n
    intcon-n

b = Fml1z/-
    Fml2z/-
    Fmlz/-
    Fmnlz/-

OUTPUTS:

Zbl'
    Pbl'
    Zemb1
    Zemb1'
    Zbom1
    Zbom1'
    Zbom2
    Zbom2'
    Zemb2
    Zemb2'
    Ztn1
    Ztn1'
    Ztn2
    Ztn2'

% Returned is the train state-space model 'a', 'w', 'c', 'dw'.

Developed by Ian Pratt, Loughborough University.
elseif strcmp(intcon,'e/hyd'),
tcon = 3;
eif strcmp(intcon,'e/mech'),
tcon = 4;
eif strcmp(intcon,'e/mag'),
tcon = 5;
else
error('Inter-connector actuator loading error!');
end;

"Loop through number of vehicles."

"Assign blocks associated with vehicle:
1. pre format block m1
2. pre format block m2
3. railway carriage
4. actuator block
5. inter-connector
6. post format block
for blocks = 1:6,
Cases 1,2,3,4,5,6 above :
if (blocks == 1), % pre format block m1
% Secondary actuator inputs required or not
% Purely passive secondary suspension
if (actuator == 1),
% Inter-connector input required or not
if (intcon == 1),
% Purely passive inter-connector
else % inter-connector input required
% Only 6 inputs from the last vehicle
if (vehicle == n),
else
end; % if
end; % if

elseif (blocks == 2), % pre format block m2
% Secondary actuator inputs required or not
% Purely passive secondary suspension
if (actuator == 1),
% Inter-connector input required or not
if (intcon == 1),
% Purely passive inter-connector
else % inter-connector input required
% Only 6 inputs from the last vehicle
if (vehicle == n),
else
end; % if
end; % if
else
% Secondary actuator inputs required
% Inter-connector input required or not
if (intcon == 1),
% Purely passive inter-connector

end; % if
else
% Secondary actuator inputs required
if (intcon == 1),
% Purely passive inter-connector
% Even model
if (round(vehicle/2) == vehicle/2),
eval(['num', num2str(vehicle*6-1), ' = [ zeros(36,vehicle*6-6) ... '
  zeros(16,6) ; zeros(2,4) eye(2) ; zeros(18,6) ] ; ... '
  zeros(36,6*(n-vehicle)) ;']);
end; % odd model

else % odd model

  eval(['num', num2str(vehicle*6-1), ' = [ zeros(36,vehicle*6-6) ... '
    zeros(16,6) ; zeros(2,4) eye(2) ; zeros(18,6) ] ; ... '
    zeros(36,6*(n-vehicle)) ;']);
end; % if odd/even

else % inter-connector input required

% Even model
  if (round(vehicle/2) == vehicle/2),
    eval(['num', num2str(vehicle*6-1), ' = [ zeros(36,vehicle*6-6) ... '
      zeros(16,6) ; zeros(2,4) eye(2) ; zeros(18,6) ] ; ... '
      zeros(36,6*(n-vehicle)) ;']);
  end; % if odd/even

% Remove Fcbn on last vehicle
  if (intcon == 1),
    eval(['num', num2str(vehicle*6-1), ' = num', num2str(vehicle*6-1), ... '
      zeros(1,1;n-1);']);
  end; % if if inter-connector
end; % end if actuator

eval(['conden', num2str(vehicle*6-1), ' = 1;']); % Incorporate in blkbb & mvcon variables : railway carriage

mvblks = [ ambalks vehicle*6-4 ];

% Incorporate in connection matrix
  if (vehicle == 1),
    % No inter-connector on vehicle 1 (i.e there is no vehicle 0!)
    q = [ q ; vehicle*6-4 vehicle*6-5 vehicle*6-2 vehicle*6-3 ];
    elseif vehicle == n,
      % No inter-connector on vehicle n (i.e. there is no vehicle n+1!)
      q = [ q ; vehicle*6-9 vehicle*6-5 vehicle*6-2 vehicle*6-3 ];
    else
      % Both inter-connectors are connected
      q = [ q ; vehicle*6-4 vehicle*6-9 vehicle*6-5 vehicle*6-2 vehicle*6-3 ];
  end; % end if vehicle

% Map to mvblkb representation
  eval(['ac', num2str(vehicle*6-1), ' = a rc']);
  eval(['bc', num2str(vehicle*6-1), ' = c rc']);
  eval(['dc', num2str(vehicle*6-1), ' = dw rc']);

elseif (blocks == 4), % actuator block

% Load appropriate actuator
  if (actuator == 1),
    % Create a dummy actuator
    [a_ac, b_ac, c_ac, d_ac] = shydacv;
    a_ac = zeros(size(a_ac));
    b_ac = zeros(size(b_ac));
    c_ac = zeros(size(c_ac));
    d_ac = zeros(size(d_ac));
  end; % end if (actuator == 1)

% Adjust 'railcarv' model for odd/even format
  if (round(vehicle/2) == vehicle/2),
    % Even models
    wb_rc = [ w_rc b/rc/); dbw_rc = [ dw rc db rc ];
  else
    % Odd models
    wb_rc = [ w_rc b/rc/); dbw_rc = [ dw rc db rc ];
  end; % end if odd/even

% Incorporate in mvblkb & mvcon variables : railway carriage

mvblks = [ ambalks vehicle*6-4 ];

% Incorporate in connection matrix
  if (vehicle == 1),
    % No inter-connector on vehicle 1 (i.e there is no vehicle 0!)
    q = [ q ; vehicle*6-4 vehicle*6-5 vehicle*6-2 vehicle*6-3 ];
    elseif vehicle == n,
      % No inter-connector on vehicle n (i.e. there is no vehicle n+1!)
      q = [ q ; vehicle*6-9 vehicle*6-5 vehicle*6-2 vehicle*6-3 ];
    else
      % Both inter-connectors are connected
      q = [ q ; vehicle*6-4 vehicle*6-9 vehicle*6-5 vehicle*6-2 vehicle*6-3 ];
  end; % end if vehicle

% Load railway carriage
  if (blocks == 1),
    [ac/rc, b/rc, c_rc, w_rc, db rc, dw rc] = railcarv;
    eval(['ac', num2str(vehicle*6-4), ' = a rc']);
    eval(['bc', num2str(vehicle*6-4), ' = c rc']);
    eval(['dc', num2str(vehicle*6-4), ' = dw rc']);
  elseif (model == 2),
    [ac/rc, b/rc, c_rc, w_rc, db rc, dw rc] = railroad;
    eval(['ac', num2str(vehicle*6-4), ' = a rc']);
    eval(['bc', num2str(vehicle*6-4), ' = c rc']);
    eval(['dc', num2str(vehicle*6-4), ' = dw rc']);
  else
    [ac/rc, b/rc, c_rc, w_rc, db rc, dw rc] = railroad;
    eval(['ac', num2str(vehicle*6-4), ' = a rc']);
    eval(['bc', num2str(vehicle*6-4), ' = c rc']);
    eval(['dc', num2str(vehicle*6-4), ' = dw rc']);
  end; % end else
disp('***** PURE PASSIVE SECONDARY VERTICAL SUSPENSION *****');
disp(['***** VEHICLE #: ',num2str(vehicle),'.*****']);
end;

elseif (actuator == 2);
% Ideal actuator
[a_ac, b_ac, c_ac, d_ac] = idealactv;
disp('***** IDEAL SECONDARY VERTICAL ACTUATORS *****');
disp(['***** VEHICLE #: ',num2str(vehicle),'.*****']);
disp('');
end; % if actuator == 2

elseif (actuator == 3);
% Electrohydraulic actuator
[a_ac, b_ac, c_ac, d_ac] = eydhactv;
disp('***** ELECTROHYDRAULIC SECONDARY VERTICAL ACTUATORS *****');
disp(['***** VEHICLE #: ',num2str(vehicle),'.*****']);
disp('');
end; % if actuator == 3

elseif (actuator == 4);
% Electromechanical actuator
[a_ac, b_ac, c_ac, d_ac] = emachactv;
disp('***** ELECTROMECHANICAL SECONDARY VERTICAL ACTUATORS *****');
disp(['***** VEHICLE #: ',num2str(vehicle),'.*****']);
disp('');
end; % if actuator == 4

elseif (actuator == 5);
% Electromagnetic actuator
[a_ac, b_ac, c_ac, d_ac] = emagactv;
disp('***** ELECTROMAGNETIC SECONDARY VERTICAL ACTUATORS *****');
disp(['***** VEHICLE #: ',num2str(vehicle),'.*****']);
disp('');
end; % if actuator == 5

end; % if actuator

% Even/odd models
if (round(vehicle/2) == vehicle/2),
% Even model
b_ac = b_ac* [ zeros(1,22) 1 zeros(1,13) ; zeros(3,23) 1 zeros(1,12);
zeros(1,28) 1 zeros(1,7) ; zeros(1,29) 1 zeros(1,8);
zeros(1,24) 1 zeros(1,11) ; zeros(2,25) 1 zeros(1,10);
zeros(1,30) 1 zeros(1,5) ; zeros(1,31) 1 zeros(1,6);
zeros(1,16) 1 zeros(1,9) ; zeros(1,17) 1 zeros(1,8);];
c_ac = [zeros(5,size(c_ac,2)) ; c_ac ; zeros(2,size(c_ac,2))];
d_ac = d_ac* [ zeros(1,22) 1 zeros(1,13) ; zeros(3,23) 1 zeros(1,12);
zeros(1,28) 1 zeros(1,7) ; zeros(1,29) 1 zeros(1,8);
zeros(1,24) 1 zeros(1,11) ; zeros(1,25) 1 zeros(1,10);
zeros(1,30) 1 zeros(1,5) ; zeros(1,31) 1 zeros(1,6);
zeros(1,16) 1 zeros(1,9) ; zeros(1,17) 1 zeros(1,8);];
d_ac = [ zeros(5,36) ; d_ac ; zeros(2,36)];
else
% Odd model
b_ac = b_ac* [ zeros(1,4) 1 zeros(1,13) ; zeros(1,5) 1 zeros(1,10);
zeros(1,10) 1 zeros(1,25) ; zeros(1,11) 1 zeros(1,24);
zeros(1,6) 1 zeros(1,29) ; zeros(1,7) 1 zeros(1,28)];
end; % if odd/even

else
% Ideal actuator connection
elseif (intcon
endif

elseif (intcon

% Modify model for blanks on the input & zeros on the output
if (round(vehicle/2) == vehicle/2),
% Even model
b_ic = b_ic* (zeros(1,26) ; zeros(1,9) ; zeros(1,27) ; zeros(1,8)
zeros(1,14) ; zeros(1,20) ; zeros(1,15)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)

end; % if odd/even
else (intcon == 3),
% Electrohydraulic type interconnector
[a_ic,b_ic,c_ic,d_ic] = ehhydcv(kcz.ccz);
disp('****** ELECTROHYDRAULIC VERTICAL INTER-CONNECTORS *****');
disp(['****** VEHICLE #: ' num2str(vehicle), ' *****']);
disp('');

% Modify model for blanks on the input & zeros on the output
if (round(vehicle/2) == vehicle/2),
% Even model
b_ic = b_ic* (zeros(1,26) ; zeros(1,9) ; zeros(1,27) ; zeros(1,8)
zeros(1,14) ; zeros(1,20) ; zeros(1,15)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)

end; % if odd/even
else (intcon == 5),
% Electromagnetic inter-connector
[a_ic,b_ic,c_ic,d_ic] = emagcv(kcz.ccz);
disp('****** ELECTROMAGNETIC VERTICAL INTER-CONNECTORS *****');
disp(['****** VEHICLE #: ' num2str(vehicle), ' *****']);
disp('');

% Modify model for blanks on the input & zeros on the output
if (round(vehicle/2) == vehicle/2),
% Even model
b_ic = b_ic* (zeros(1,26) ; zeros(1,9) ; zeros(1,27) ; zeros(1,8)
zeros(1,14) ; zeros(1,20) ; zeros(1,15)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)

end; % if odd/even
else (intcon == 4),
% Electromechanical inter-connector
[a_ic,b_ic,c_ic,d_ic] = emechcv(kcz.ccz);
disp('****** ELECTROMECHANICAL VERTICAL INTER-CONNECTORS *****');
disp(['****** VEHICLE #: ' num2str(vehicle), ' *****']);
disp('');

% Modify model for blanks on the input & zeros on the output
if (round(vehicle/2) == vehicle/2),
% Even model
b_ic = b_ic* (zeros(1,26) ; zeros(1,9) ; zeros(1,27) ; zeros(1,8)
zeros(1,14) ; zeros(1,20) ; zeros(1,15)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)

end; % if odd/even
else (intcon == 6),
% Ideal vertical inter-connector
[a_ic,b_ic,c_ic,d_ic] = idealv(kcz.ccz);
disp('****** IDEAL VERTICAL INTER-CONNECTOR *****');
disp(['****** VEHICLE #: ' num2str(vehicle), ' *****']);
disp('');

% Modify model for blanks on the input & zeros on the output
if (round(vehicle/2) == vehicle/2),
% Even model
b_ic = b_ic* (zeros(1,26) ; zeros(1,9) ; zeros(1,27) ; zeros(1,8)
zeros(1,14) ; zeros(1,20) ; zeros(1,15)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)

end; % if odd/even
else (intcon == 7),
% Ideal horizontal inter-connector
[a_ic,b_ic,c_ic,d_ic] = ideahcv(kcz.ccz);
disp('****** IDEAL HORIZONTAL INTER-CONNECTOR *****');
disp(['****** VEHICLE #: ' num2str(vehicle), ' *****']);
disp('');

% Modify model for blanks on the input & zeros on the output
if (round(vehicle/2) == vehicle/2),
% Even model
b_ic = b_ic* (zeros(1,26) ; zeros(1,9) ; zeros(1,27) ; zeros(1,8)
zeros(1,14) ; zeros(1,20) ; zeros(1,15)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)

end; % if odd/even
else (intcon == 8),
% Real horizontal inter-connector
[a_ic,b_ic,c_ic,d_ic] = realhcv(kcz.ccz);
disp('****** REAL HORIZONTAL INTER-CONNECTOR *****');
disp(['****** VEHICLE #: ' num2str(vehicle), ' *****']);
disp('');

% Modify model for blanks on the input & zeros on the output
if (round(vehicle/2) == vehicle/2),
% Even model
b_ic = b_ic* (zeros(1,26) ; zeros(1,9) ; zeros(1,27) ; zeros(1,8)
zeros(1,14) ; zeros(1,20) ; zeros(1,15)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)

end; % if odd/even
else (intcon == 9),
% Real vertical inter-connector
[a_ic,b_ic,c_ic,d_ic] = realv(kcz.ccz);
disp('****** REAL VERTICAL INTER-CONNECTOR *****');
disp(['****** VEHICLE #: ' num2str(vehicle), ' *****']);
disp('');

% Modify model for blanks on the input & zeros on the output
if (round(vehicle/2) == vehicle/2),
% Even model
b_ic = b_ic* (zeros(1,26) ; zeros(1,9) ; zeros(1,27) ; zeros(1,8)
zeros(1,14) ; zeros(1,20) ; zeros(1,15)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)

end; % if odd/even
else (intcon == 10),
% Ideal inter-connector
[a_ic,b_ic,c_ic,d_ic] = idealcv(kcz.ccz);
disp('****** IDEAL INTER-CONNECTOR *****');
disp(['****** VEHICLE #: ' num2str(vehicle), ' *****']);
disp('');

% Modify model for blanks on the input & zeros on the output
if (round(vehicle/2) == vehicle/2),
% Even model
b_ic = b_ic* (zeros(1,26) ; zeros(1,9) ; zeros(1,27) ; zeros(1,8)
zeros(1,14) ; zeros(1,20) ; zeros(1,15)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)

end; % if odd/even
else (intcon == 11),
% Real inter-connector
[a_ic,b_ic,c_ic,d_ic] = realcv(kcz.ccz);
disp('****** REAL INTER-CONNECTOR *****');
disp(['****** VEHICLE #: ' num2str(vehicle), ' *****']);
disp('');

% Modify model for blanks on the input & zeros on the output
if (round(vehicle/2) == vehicle/2),
% Even model
b_ic = b_ic* (zeros(1,26) ; zeros(1,9) ; zeros(1,27) ; zeros(1,8)
zeros(1,14) ; zeros(1,20) ; zeros(1,15)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)
zeros(1,1) ; zeros(1,32) ; zeros(1,33)

end; % if odd/even

else

% Odd model
b_ic = b_ic* [ zeros(1, 8) 1 zeros(1, 27) 1 zeros(1, 9) 1 zeros(1, 26)
zeros(1, 14) 1 zeros(1, 20) 1 zeros(1, 15)
zeros(1, 21) 1 zeros(1, 14)];
c_ic = [ zeros(7, size(a_ic, 2)) ; c_ic ; zeros(1, size(a_ic, 2)) ];
d_ic = d_ic* [ zeros(1, 8) 1 zeros(1, 27) 1 zeros(1, 9) 1 zeros(1, 26)
zeros(1, 14) 1 zeros(1, 20) 1 zeros(1, 15)
zeros(1, 21) 1 zeros(1, 14)];
d_ic = [ zeros(7, size(d_ic, 2)) ; d_ic ; zeros(1, size(d_ic, 2)) ];
end; % if odd/even
end; % if inter-connector

% On the last inter-connector transmit a null force
if (vehicle == n),
a_le = zeros(size(a_ic));
b_ic = zeros(size(b_ic));
c_ic = zeros(size(c_ic));
d_ic = zeros(size(d_ic));
end;

% Incorporate in mwbk & connect representation: inter-connector
all carriages except the last one, make this a null block
snblks = [ snblks vehicle*6-3 ];
else % Inter-connector
if vehicle < n,
q = [ [q; vehicle*6-3 vehicle*6-4 vehicle*6+2 0 0 0];
else
q = [ [q; vehicle*6-3 vehicle*6-4 vehicle*6 0 0 0];
end; % if

% Map to mwbk & mvcon representation
eval(["a",num2str(vehicle*6-3)," = a_ic;");
end;

% Post format block
% Select odd/even model
if (round(vehicle/2) == vehicle/2),

% Select even
eval(["num",num2str(vehicle*6)," = [ zeros(vehicle*17-17,36) ... zeros(14,14) eye(14) zeros(14,14); ... zeros(1,3), eye(3), zeros(n-vehicle*17,36);];");
else
% Select odd
eval(["num",num2str(vehicle*6)," = [ zeros(vehicle*17-17,36) ... eye(14) zeros(14,22), zeros(3,15) eye(3) zeros(3,18); ... zeros(n-vehicle*17,36);];");
end; % if

% Incorporate in mwbk & connect: post format block
mvnblks = [ mvnblks vehicle*6 ];
q = [ [q; vehicle*6 vehicle*6-6 0 0 0];
end % if blocks
end % for blocks
outblock = [ [outblock vehicle*6];
end % vehicle

% Create input & output blocks
% Input block
% Secondary actuator input required or not
if (actuator == 1),
% Purely passive secondary actuator
% Inter-connector input required or not
if (intcon == 1),
eval(["num",num2str(n*6+1)," = eye(4*n);]");
else % We have an inter-connector input
eval(["num",num2str(n*6+1)," = eye(3*n-1);");
end; % if
else % We have a pair of secondary actuator inputs
% Inter-connector input required or not
if (intcon == 1),
eval(["num",num2str(n*6+1)," = eye(6*n);");
else % We have a pair of secondary actuator inputs
eval(["num",num2str(n*6+1)," = eye(7*n-1);");
end; % if
end; % if

% Incorporate in mwbk & mvcon format
mvnblks = [ mvnblks n*6+1 ];
end

% Output block
eval(["num",num2str(n*6+2)," = eye(17*n);]");
eval(["condn",num2str(n*6+2)," = 1;']);
end;
else % we have a pair of secondary actuator inputs
% Inter-connector input required or not
if (intcon == 1),
eval(["num",num2str(n*6+1)," = eye(6*n);");
else % We have a pair of secondary actuator inputs
eval(["num",num2str(n*6+1)," = eye(7*n-1);");
end; % if
else
end % if

% Incorporate output block connections in q, resize if required
if (size(outblock,2) > size(q(1,:),2)),
q = [ [q; zeros(size(q,1),size(outblock,2)-size(q(1,:),2)); outblock];
elseif (size(outblock,2) < size(q(1,:),2)),
q = [ [q; outblock zeros(size(q(1,:),2)-size(outblock,2))];
else
q = [ [q; outblock];
end; % if

% Blockbuild railway train mvblkb
disp('***** BLOCKBUILD SYSTEM *****');
mvblkb

% Connect system together to produce a,b,c,d
disp('····· CONNECT SYSTEM TOGETHER ·····');
[a,b,c,d] = mvcon(a,b,c,d,q,iu,iy,si);

% Offset the effect of passive inter-vehicle connections on active force
offset = eye(size(c,l));
if (n=l), % only apply offset when an inter-connector exists
for vehicle = 2:n,
    offset(17*vehicle-17,:) = zeros(1,17*vehicle-34) zeros(1,8) kcz ccz ...
    zeros(1,6) -1 0 0 -kcz -ccz zeros(1,13) zeros(1,17*(n-vehicle)) ;
end; % for

end; % if

% Even vehicle
offset(17*vehicle-17,:) = zeros(1,17*vehicle-34) zeros(1,8) kcz ccz ...
zeros(1,6) -1 0 0 -kcz -ccz zeros(1,13) zeros(1,17*(n-vehicle)) ;

end; % for

% Extract track/actuator inputs
windex = [1,bindex = [1];
% Secondary actuator input required or not
if (actuator == 1),
% Purely passive secondary actuator
% Inter-connector input required or not
if (intcon == 1),
    w = b; dw = d; b = [1]; db = [1];
else % We have an inter-connector input
    for vehicle = 1:n,
        windex = [ windex [6*vehicle-4:1:6*vehicle-1] ];
        if (vehicle==n),
            bindex = [ bindex 5*vehicle ];
        end; % if
        for w = b(:,windex); b = b(:,bindex); dw = d(:,windex); db = d(:,bindex);
        end; % if
    else % we have a pair of secondary actuator inputs
        % Inter-connector input required or not
        if (intcon == 1),
            for vehicle = 1:n,
                windex = [ windex [6*vehicle-5:1:6*vehicle-2] ];
                bindex = [ bindex 6*vehicle-1 6*vehicle ];
            end; % for
            for w = b(:,windex); b = b(:,bindex); dw = d(:,windex); db = d(:,bindex);
        end; % for
    else % We have an inter-connector input
        % Inter-connector input required or not
        if (vehicle==n), % no inter-connector on last vehicle
            bindex = [ bindex 7*vehicle-2 7*vehicle-1 7*vehicle ];
            else
                bindex = [ bindex 7*vehicle-2 7*vehicle-1 ];
            end; % if
        end; % if
    end; % for
    w = b(:,windex); b = b(:,bindex); dw = d(:,windex); db = d(:,bindex);
end; % if

end; % if

% Strip of last vehicle inter-connector output signal
% c = c(1:size(c,l)-1,:);
% db = db(1:size(db,l)-1,:);
% dw = dw(1:size(dw,l)-1,:);
function phi = bendmode(lv, lambda, amplitude, x)

% Define vehicle bending mode shape, of wavelength 'lambda'
% and size 'amplitude', at a position 'x' from the right-hand
% end of the vehicle. 'lv' is the vehicle length + gangway
% spacing.

% Developed by Ian Pratt, Loughborough University.

% Error conditions
error(nargchk(nargin, 4, 4));

% Mode shape
for loop = 1:length(x),
    phi(loop) = amplitude*cos(pi*(lambda+lv)/lambda)*...  
               cos(2*pi*x(loop)/lambda) + ...  
               amplitude*sin(pi*(lambda+lv)/lambda)*...  
               sin(2*pi*x(loop)/lambda);
end; % for
function [a,b,c,d] = sprdmpv(kc,cc); 
%SPRMPV
FUNCTION [a,b,c,d] = sprdmpv(kc,cc);

Create model of inter-connector between 2 carriages.  
A parallel spring/damper arrangement with inter-vehicle  
stiffness 'kc' and damping 'cc' are used to model a bellow  
connection between 2 vehicles. This is shown below:

---------- |
| Spring   |
| Damper  |
---------- |
Vehicle (m+1) | Vehicle (m) |
---------- Fb(m+1)z  
\ or Fbm2z

MODEL DESCRIPTION:

INPUTS:
Zend
Zend'
Zend(m+1)2
Zend(m+1)2'

STATES: GAIN BLOCK

Zend1
Zend1'
Zend(m+1)2
Zend(m+1)2'  

OUTPUTS:
Fbm2z

Fbm2z is the vertical force applied by body m on body m+1.

Developed by Ian Pratt, Loughborough University.

% Create state-space model
a = 0;
b = zeros(4,4);
c = 0;
d = [ kc cc -kc -cc ];
function [a,b,c,d] = idealcv(kcz,ccz)

% IDEALCV
FUNCTION [a,b,c,d] = idealcv(kcz,ccz);

% Locate an ideal inter-connector between 2 carriages.
% A parallel spring/damper arrangement with inter-vehicle
stiffness 'kcz' and damping 'ccz' are used to model a bellow
connection between 2 vehicles. This is described by
the diagram shown below:

+----+        +----+
|     |        |     |
|     |        |     |
|     |        |     |
+----+        +----+

Vehicle m+1        Vehicle m

+-------------+     +-------------+
| SPR          |     | DMP          |
|---------------|     |---------------|
| Fb(m+1)z      |     | Fbm2z        |
+-------------+     +-------------+

Fb(m+1)z is the vertical force applied by body m on body m+1.

% Form state-space model
a = 0;
b = zeros(1,5);
c = 0;
d = [ kcz ccz -1 -kcz -ccz ];

% DEVELOPED BY IAN PRATT, LOUGHBOROUGH UNIVERSITY.
function [a,b,c,d] = ehydactv(kcz,ccz)

FUNCTION [a,b,c,d] = ehydactv(kcz,ccz);

% Locate electrohydraulic inter-connector between
% 2 carriages. A parallel spring/damper arrangement with
% inter-vehicle stiffness 'kcz' and damping 'ccz' are used
to model a bellow connection between 2 vehicles. This is
described by the diagram shown below:

-------
|
SPR
|
-------

Vehicle m+1

Fb(m+1)z
Vehicle m

Model Description:

Inputs: Zendm Zendm'

Zends Zendm+1 Zendm+1'

Fb(m+1)z

States: Gain Block

Outputs: Fbm2z

Fbm2z is the vertical force applied by body m on body m+1.

% Look up actuator model
[a,bf,byd,c,df,dyd] = ehydactv:

% Create interface matrix
afc = zeros(1,5);
bfc = zeros(2,1);
cfc = zeros(1,2);
dfc = [0 1 0 0 -1 zeros(1,2)];

% Create spring/damper model
abl = zeros(1,5);
bbbl = zeros(1,5);
cbl = zeros(1,2);
dbl = [kcz ccz 0 -kcz -ccz];

% Generate state-space model
[astemp,btemp,ctemp,dtemp] = series(afc,bfc,cfc,dfc,a,bf,bf,c,bf);
[a,b,c,d] = parallel(abl,bbbl,cbl,dbl,astemp,btemp,ctemp,dtemp);

Developed by Ian Pratt, Loughborough University.
function [a,b,c,d] = emchicV(kcz,ccz)

% Locate electromechanical inter-connector between
% 2 carriages. A parallel spring/damper arrangement with
% inter-vehicle stiffness 'kcz' and damping 'ccz' are used
% to model a bellow connection between 2 vehicles. This
% is described by the diagram shown below:

% MODEL DESCRIPTION:
% INPUTS:
% Zend(m+1)2  Zend(m+1)2'  Zend(m+1)2  Zend(m+1)2'  Zend(m+1)2  Zend(m+1)2'
% OUTPUTS:
% Fbm2z
% Fbm2z is the vertical force applied by body m on body m+1.

% Developed by Ian Pratt, Loughborough University.

% Look up actuator model
% [a,bf,by,c,d,f] = emechacV;

% Create interface matrix
aifc = 0;
bifc = zeros(1,5);
cifc = zeros(2,1);
difc = [0 1 0 0 1
         zeros(1,2) 1 zeros(1,2)];

% Create spring/damper model
ablw = 0;
bblw = zeros(1,5);
cblw = 0;
dblw = [kcz ccz 0 -kcz -ccz];

% Generate state-space model
[atemp,btemp,ctemp,dtemp] = series(aifc,bifc,cifc,difc,a,bf,by,c,d,f);
[a,b,c,d] = parallel(ablw,bblw,cblw,dblw,atemp,btemp,ctemp,dtemp);
function [a,b,c,d] = emagicv(kcz,ccz)
%EMAGICV

FUNCTION [a,b,c,d] = emagicv(kcz,ccz);

% Locate electromagnetic inter-connector between
% 2 carriages. A parallel spring/damper arrangement with
% inter-vehicle stiffness 'kcz' and damping 'ccz' are used
% to model a bellow connection between 2 vehicles. This is
% described by the diagram shown below :

% MODEL DESCRIPTION :

% INPUTS :  
%  Zend m1  Zend m
%  Zend m1' Zend m'
%  Fcbr nz

% OUTPUTS :  
%  Fm2z
%  Fm2z

Fm2z is the vertical force applied by body m on body m+1.

% Developed by Ian Pratt, Loughborough University.

% Look up actuator model
[a,bf,byd,c,df,dydl] = emagact;
% Create interface matrix
aifc = 0;
bifc = zeros(1,5);
cifc = zeros(2,1);
difc = [ 0 1 0 0 -1 zeros(1,2) 1 zeros(1,2) ];
% Create spring/damper model
ablw = 0;
bblw = zeros(1,5);
cblw = 0;
dblw = [ kcz ccz 0 -kcz -ccz ];
% Generate state-space model
[astemp,btemp,ctemp,dtemp] = series(aifc,bifc,cifc,difc,a, [byd bf], -c, -[dyd df]);
[a,b,c,d] = parallel(ablw,bblw,cblw,dblw,astemp,btemp,ctemp,dtemp);
function [a,b,c,d] = idealacv

IDEALACV

Create model of an ideal actuator to be situated in parallel
with the secondary suspension. The arrangement is shown
diagramatically below:

```
\[\text{Vehicle m}\]
```

```
\[\text{SPR}\] \[\text{DMP}\] \[\text{Fctmlz}\]
```

```
\[\text{Bogie/Truck}\]
```

MODEL DESCRIPTION:

INPUTS:

\[Z_{bog1}\]
\[Z_{bog1}^{'}\]
\[Z_{tm1}\]
\[Z_{tm1}^{'}\]
\[Z_{bog2}\]
\[Z_{bog2}^{'}\]
\[Z_{tm2}\]
\[Z_{tm2}^{'}\]
\[F_{ctmlz}\]
\[F_{ctm2z}\]

STATES: GAIN BLOCK

\[F_{ctmlz}\]
\[F_{ctm2z}\]

\[F_{ctmlz}, \text{ and } F_{ctm2z}\] are forces generated by the actuators

Developed by Ian Pratt, Loughborough University.

% Spring/damper combination
\[a = 0;\]
\[b = \text{zeros(1,10)};\]
\[c = \text{zeros(2,1)};\]
\[d = [\text{zeros(1,8) 1 0 ; zeros(1,9) 1 }];\]
function [a,b,c,d] = ehydactv

Create model of an electrohydraulic actuator to
be situated in parallel with the secondary suspension.
The arrangement is shown diagramatically below:

```
Vehicle m

SPR ____________

DMP _________

Fctrlz        Ftmzlz

Bogie/Truck

Ml
```

MODEL DESCRIPTION:

INPUTS:

\[ \begin{array}{c}
Zbogm1 \\
Zbogm1' \\
Ztm1 \\
Ztm1' \\
Zbogm2 \\
Zbogm2' \\
Ztm2 \\
Ztm2' \\
Fctnlz \\
Fctm1z \\
\end{array} \]

OUTPUTS:

\[ \begin{array}{c}
Fctm1z \\
Ftm2z \\
\end{array} \]

**Gain Block**

Developed by Ian Pratt, Loughborough University.

% Look up actuator model
[a,bf,byd,df,dyd] = ehydactv;

% Leading bogie displacement rate and drive signal
afbl = 0;
bfbl = zeros(1,10);
cfb1 = zeros(2,1);
dfb1 = [ 0 0 0 zeros(1,6) zeros(1,9) 1 ];

% Force applied to leading bogie
afbl = 0;
bfbl = zeros(2,1);
dfb1 = [1,0];

% Force applied to trailing bogie
afb4 = 0;
bfb4 = 0;
cfb4 = zeros(2,1);
dfb4 = [0,1];

% Generate state-space model
[atempl,btempl,ctempl,dtempl] = series(afbl,bfbl,cfbl,dfbl,...
a,[byd,bfJ,c,[dyd,df]);
[atempl,btempl,ctempl,dtempl] = series(afbl,bfbl,cfbl,dfbl,...
a,[byd,bfJ,c,[dyd,df]);
[atempl,btempl,ctempl,dtempl] = series(afb2,bfb2,cfb2,dfb2,...
a,[byd,bfJ,c,[dyd,df]);
[atempl,btempl,ctempl,dtempl] = series(afb2,bfb2,cfb2,dfb2,...
a,[byd,bfJ,c,[dyd,df]);

{a,b,c,d] = parallel([atempl,btempl,ctempl,dtempl,atempl,btempl,...
c,templ,btempl,dtempl]);
function [a,b,c,d] = emchactv

FUNCTION [a,b,c,d] = emchactv;

Create model of an electromechanical actuator to
be situated in parallel with the secondary suspension.
The arrangement is shown diagrammatically below:

-----------------------------------------------
Vehicle n
    |
    |       |
    |       |
    |       |
    | SFR /\  |
    | EMP /\  |
    |       |
    |       |
    |       |
    |       |
    |       |
    |       |
    |       |
    |       |
    |       |
    |       |
    |-------|
    |       |
    |       |
    |       |
    |       |
    |       |
    |       |
    |       |
    |       |
    |-------|

Bogie/Truck

-----------------------------------------------

MODEL DESCRIPTION:

INPUTS :

Zbogml
Zbogl'1
Ztm1'
Zbogm2
Zbogl'2
Ztm2'
Zbogl'3
Ztm3'
Fctmlz
Fctm2z

OUTPUTS :

Ftm1z
Ftm2z

Ftm1z, and Ftm2z are forces generated by the actuators.

% Force applied to leading bogie
afbl = zeros(2,1);
bfb1 = zeros(1,10);
cfb1 = zeros(2,1);
dfb1 = [ 0 1 0 -1 zeros(1,6)
        zeros(1,6) 1 ];
% Force applied to trailing bogie
afb2 = zeros(2,1);
bfb2 = zeros(1,10);
cfb2 = zeros(2,1);
dfb2 = [ zeros(1,5) 1 0 -1 0 0
        zeros(1,9) 1 ];

% Generate state-space model
[atempl,btemp1,ctemp1,dtemp1] = series(afbl,bfb1,cfb1,dfb1, ...  
    a,[hyd bf].c.[dyd df]);
[atempl,btemp1,ctemp1,dtemp1] = series(atempl,btemp1,ctemp1,dtemp1, ...  
    afbl,bfb1,cfb1,dfb1);
[atempl,btemp2,ctemp2,dtemp2] = series(afbl,bfb1,cfb1,dfb1, ...  
    a,[hyd bf].c.[dyd df]);
[atempl,btemp2,ctemp2,dtemp2] = series(atempl,btemp1,ctemp1,dtemp1, ...  
    afbl,bfb1,cfb1,dfb1);
[atempl,btemp1,ctemp1,dtemp1] = parallel(atempl,btemp1,ctemp1,dtemp1,atempl,btemp2, ...  
    ctemp2,dtemp2);

Developed by Ian Pratt, Loughborough University.
Create model of an electromagnetic actuator to be situated in parallel with the secondary suspension. The arrangement is shown diagramatically below:

INPUTS:

Zbogm1
Zbogm2
Ztm1
Ztm2
Zbogm1'
Zbogm2'
Ztm1'
Ztm2'

OUTPUTS:

Ftmzl
Ftmz2

Ftmzl, and Ftmz2 are forces generated by the actuators.

Developed by Ian Pratt, Loughborough University.
function [a,b,c,w,db,dw) = railcarl(v)

RAILCARL

Generate lateral state-space model of a railway carriage;
data taken from a Bombardier Prorail best current passive
suspension. The model does not use the small mass
approximation in the airspring.

MODEL DESCRIPTION:

INPUTS:

w = Ym11

STATES:

Ym

OUTPUTS:

Ym

Load vehicle parameters

railcarl

% create 'a' matrix

a = [ zeros(1,2) 1 zeros(1,43)
     zeros(1,3) 1 zeros(1,42)
     -4*csy*lbz/ibr zeros(l,2) zeros(l,1)
     zeros(l,1)
     zeros(l,1)]

% Developed by Ian Pratt, Loughborough University.
Create c matrix:

c = { eye(4) zeros(4,42)
  -4*kay/mb 0 -4*csy/mb 0 -4*ksy/mb zeros(1,4) ...
  2*kay/mb kay(lal-la2-2*1t)/mb 0 2*csy/mb csy(lal-la2-2*1t)/mb ...
  0 2*kay/mb kay(lal-la2-2*1t)/mb 0 2*csy/mb ...
  -csey(lal-la2-2*1t)/mb zeros(1,25) ...
  0 -2*kay(la2-2*1t)/mb 0 -2*csy(la2-2*1t)/mb zeros(1,6) ...
  kay(la2-2*1t)/iby kay(la2-2*1t)/iby 0 ...
  csey(la2-2*1t)/iby csey(la2-2*1t)/iby 0 ...
  -kay(la2-2*1t)/iby kay(la2-2*1t)/iby 0 ...}

Create d matrix:

dw = zeros(14,8);
function \([a, b, c, w, db, dw] = \text{rcarmckl}(v)\)

\% \text{RCARMCKL} \quad \text{FUNCTION} \quad [a, b, c, w, db, dw] = \text{rcarmckl}(v);

MODEL DESCRIPTION:

INPUTS :

\[ w = \begin{bmatrix}
    \text{Ywmt11}' \\
    \text{Rhwm11}' \\
    \text{Rhwm12}' \\
    \text{Ywmt12}' \\
    \text{Ywmt21}' \\
    \text{Ywmt22}'
\end{bmatrix}\]

STATES :

\[ \begin{bmatrix}
    \text{Ybm}' \\
    \text{Psiwm1}' \\
    \text{Psiwm2}' \\
    \text{Ywm1}' \\
    \text{Ywm2}' \\
    \text{Psiwm1l}' \\
    \text{Psiwm2l}'
\end{bmatrix}\]

OUTPUTS :

\[ \begin{bmatrix}
    \text{Ybmy}' \\
    \text{Psiwml}' \\
    \text{Psiwm2l}' \\
    \text{Ywmtl1}' \\
    \text{Ywmtl2}' \\
    \text{Psiwm1l}' \\
    \text{Psiwm2l}'
\end{bmatrix}\]

\% 'a', 'b', 'c', 'w', 'db', 'dw' is the returned state-space model.

\% Developed by Ian Pratt, Loughborough University.

\% Load vehicle parameters

railprm

\% Define mass matrix 'M'

\[ M = \text{diag}(\{mb, iby, mrb, mmp, mmp, mmp, mmp, mmp, mmp\}); \]

\% Define damping matrix 'C'

\[ C = \begin{bmatrix}
    4*csy & 4*csy*lbz & \text{zeros}(1,4) & -2*csy & -(lal+la2+2*lt)*csy & \ldots \\
    0 & -2*csy & (lal+la2-2*lt)*csy & \text{zeros}(1,4) & \ldots \\
    0 & 2*csy & \text{zeros}(1,4) & -2*csy & \text{zeros}(1,4) & \ldots \\
    \text{zeros} & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{bmatrix}; \]

\% Define stiffness matrix 'K'

\[ K = \begin{bmatrix}
    4*csy & 4*csy*lbz & \text{zeros}(1,4) & \ldots & \ldots & \ldots \\
    0 & -2*csy & (lal+la2+2*lt)*csy & \ldots & \ldots & \ldots \\
    0 & 2*csy & \text{zeros}(1,4) & \ldots & \ldots & \ldots \\
    \text{zeros} & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{bmatrix}; \]

Developed by tan Pratt, Pratt, Pratt, ...
Define input matrix for $B_t$

$$B_t = \begin{bmatrix}
0 & 0 & -k_{rz} \cdot lasy & k_{sz} \cdot krz \cdot zeros(1,5)
\end{bmatrix} - k_{rz} \cdot lasy \cdot zeros(1,11)
$$

Define wheel input matrix for $B_{td}

$$B_{td} = \begin{bmatrix}
0 & 0 & -k_{rz} \cdot lasy & k_{sz} \cdot krz \cdot zeros(1,5)
\end{bmatrix} - k_{rz} \cdot lasy \cdot zeros(1,11)
$$

Define wheel input matrix for $B_f$

$$B_f = \begin{bmatrix}
1 & -1 & 1 & -1 & 1 & -1
\end{bmatrix} \cdot \begin{bmatrix}
l_{1a} & -l_{2a} & -l_{1a} & l_{2a} & l_{iv} & l_{iv}
\end{bmatrix} + \begin{bmatrix}
l_{b} & -l_{b} & -l_{b} & l_{b} & l_{b} & l_{b}
\end{bmatrix} \cdot \begin{bmatrix}
l_{v} & -l_{v} & -l_{v} & l_{v} & l_{v} & l_{v}
\end{bmatrix} + \begin{bmatrix}
l_{v} & -l_{v} & -l_{v} & l_{v} & l_{v} & l_{v}
\end{bmatrix} \cdot \begin{bmatrix}
l_{b} & -l_{b} & -l_{b} & l_{b} & l_{b} & l_{b}
\end{bmatrix}
$$

Form state-space model

$$a = \begin{bmatrix}
0 & 0 & -k_{rz} \cdot lasy & k_{sz} \cdot krz \cdot zeros(1,5)
\end{bmatrix} - k_{rz} \cdot lasy \cdot zeros(1,11)
$$

Define output variables

$$c = \begin{bmatrix}
0 & 0 & -k_{rz} \cdot lasy & k_{sz} \cdot krz \cdot zeros(1,5)
\end{bmatrix} - k_{rz} \cdot lasy \cdot zeros(1,11)
$$

Generate output variables

$$d = \begin{bmatrix}
0 & 0 & -k_{rz} \cdot lasy & k_{sz} \cdot krz \cdot zeros(1,5)
\end{bmatrix} - k_{rz} \cdot lasy \cdot zeros(1,11)
$$
function [a,b,c,w,db,dw] = railtrnl(n,v,model,actuator,intcon)

% RAILTRNL
% Generate lateral state-space model of an 'n' carriage
% train of railway vehicles travelling at 'v' m/s. 'model'
% may be set to one of the following options:
% 1.) 'sspace' - use state-space formulated model ('railcarl.m')
% 2.) 'mck' - use mass-damping-stiffness model ('rcarmckl.m')
% This sets the type of railway carriage to be used. These may
% have a variety of secondary lateral actuators ('actuator')
% or active lateral inter-vehicle connections ('intcon'), which
% may be one of the following:
% 1.) 'passive' - purely passive device
% 2.) 'ideal' - infinite bandwidth actuator
% 3.) 'e/hyd' - electro-hydraulic actuator
% 4.) 'e/mech' - electro-mechanical actuator
% 5.) 'e/mag' - electro-magnetic actuator

% MODEL DESCRIPTION:
% INPUTS:
% w = [Yw11; Rhw11; Yw11; Rhwmt11; Yw12; Rhwmt22;]
% STATES:
% railcar12-1; intcon-1
% railcar12-m; intcon-m
% railcar12-n; intcon-n
% OUTPUTS:
% Ybl
% Psibl
% Ybl'
% Psibl'
% Yt11
% Psit11
% Yt11'
% Psit11'
% Yt12
% Psit12
% Yt12'

% Returned is the train state-space model 'a', 'w', 'c', 'dw'.

% Developed by Ian Pratt, Loughborough University.

% Check arguments
error(nargchk(1,5,nargin));

% Check if n is zero
if (n<1),
    error('At least one vehicle must be specified!');
    end;

% if n = round(n);

% Initialise block build & connect variables
mvnblks = [ ];
ssnblks = [ ];
u = [ ];
y = [ ];
q = [ ];

% Initialise output block
outblock = [ ];

% Check for correct 'model' data format
if strcmp(model, 'sspace'),
    model = 1;
elseif strcmp(model, 'mck'),
    model = 2;
else
    error('Model loading error!');
end;

% if
% Check for correct secondary 'actuator' data format
if strcmp(actuator, 'passive'),
    actuator = 1;
elseif strcmp(actuator, 'ideal'),
    actuator = 2;
elseif strcmp(actuator, 'e/hyd'),
    actuator = 3;
elseif strcmp(actuator, 'e/mech'),
    actuator = 4;
elseif strcmp(actuator, 'e/mag'),
    actuator = 5;
else
    error('Secondary actuator loading error!');
end;
% Check for correct inter-connector 'actuator' data format
if strcmp(intcon,'passive'),
    intcon = 1;
elseif strcmp(intcon,'ideal'),
    intcon = 3;
elseif strcmp(intcon,'e/hyd'),
    intcon = 4;
elseif strcmp(intcon,'e/mech'),
    intcon = 5;
else
    error('Inter-connector actuator loading error!');
end;

% Loop through number of vehicles.
for vehicle = 1:n,

    % Assign blocks associated with vehicle:
    % 1. pre format block ml
    % 2. pre format block m2
    % 3. railway carriage
    % 4. actuator block
    % 5. inter-connector
    % 6. post format block

    % Cases 1,2,3,4,5,6 above:
    if (blocks == 1), % pre format block ml
        % Secondary actuator inputs required or not
        % Purely passive secondary suspension
        if (actuator == 1),
            % Purely passive inter-connector
            eval(['num',num2str(vehicle*6-5),' = [ zeros(15,12*vehicle-12), ... 
                [ eye(8) zeros(8,6) ; zeros(7,12) ]', ' 
                zeros(15,12*(n-vehicle)) ]']);
        else % inter-connector input required
            eval(['num',num2str(vehicle*6-5),' = ( 
                zeros(15,9*vehicle-9), 
                [ eye(8) zeros(6,9) ; zeros(7,12) ]', ' 
                zeros(15,9*(n-vehicle)-1) ]');
        end;
    end;

    % Only 12 inputs from the last vehicle
    if vehicle == n,
        eval(['num',num2str(vehicle*6-5),' = ( 
                zeros(15,9*vehicle-9), 
                [ eye(8) zeros(6,9) ; zeros(7,12) ]', ' 
                zeros(15,9*(n-vehicle)-1) ]');
    end;

    if inter-connector
        eval('comden',num2str(vehicle*6-5),' = 1;');
    end;

    % Incorporate in mvblkbld & connect variables: pre format block ml
    mvnb1ks ~ [ mvnb1ks vehicle*6-5 ];

    % Incorporate in connection matrix
    q = [ q ; vehicle*6-5 n*6+1 0 0 0 ];

    elseif (blocks == 2), % pre format block m2
        % Secondary actuator inputs required or not
        if (actuator == 1),
            % Purely passive secondary suspension
            if (actuator == 1),
                % Purely passive inter-connector
                eval(['num',num2str(vehicle*6-6),' = [ zeros(15,9*vehicle-9), ... 
                    [ eye(8) zeros(6,9) ]', ' 
                    zeros(15,9*(n-vehicle)-1) ]']);
            else % last vehicle
                eval(['num',num2str(vehicle*6-6),' = [ zeros(15,9*vehicle-9), ... 
                    [ eye(8) zeros(7,8) ]'; 
                    ' ]);'
            end;
        end;

        % Only 8 inputs from the last vehicle
        if vehicle == n,
            eval(['num',num2str(vehicle*6-6),' = [ zeros(15,9*vehicle-9), ... 
                [ eye(8) zeros(6,9) ]', ' 
                zeros(15,9*(n-vehicle)-1) ]']);
        else % last vehicle
            eval(['num',num2str(vehicle*6-6),' = [ zeros(15,9*vehicle-9), ... 
                [ eye(8) zeros(7,8) ]'; 
                ' ]);'
        end;
    end;

    % Secondary actuator inputs required or not
    if (actuator == 1),
        % Purely passive secondary suspension
        if (actuator == 1),
            % Purely passive inter-connector
            eval(['num',num2str(vehicle*6-7),' = [ zeros(40,8*n) ];']);
        else % inter-connector input required
            eval(['num',num2str(vehicle*6-7),' = [ zeros(40,8*n-1) ];']);
        end;
    end;

    % if inter-connector

else
    % Secondary actuator inputs required
    % Inter-connector input required or not
    if (intcon == 1),
        % Purely passive inter-connector
        eval(['num',num2str(vehicle*6-5),' = [ zeros(15,12*vehicle-12), ... 
            [ eye(8) zeros(8,6) ; zeros(7,12) ]', ' 
            zeros(15,12*(n-vehicle)) ]']);
    else % inter-connector input required
        % Only 12 inputs from the last vehicle
        if vehicle == n,
            eval(['num',num2str(vehicle*6-5),' = ( 
                zeros(15,12*vehicle-12), 
                [ eye(8) zeros(8,6) ; zeros(7,12) ]', ' 
                zeros(15,12*(n-vehicle)-1) ]');
        else % last vehicle
            eval(['num',num2str(vehicle*6-5),' = ( 
                zeros(15,12*vehicle-12), 
                [ eye(8) zeros(8,6) ; zeros(7,12) ]', ' 
                zeros(15,12*(n-vehicle)-1) ]');
        end;
    end;

    % Secondary actuator inputs required or not
    if (actuator == 1),
        % Purely passive secondary suspension
        if (actuator == 1),
            % Purely passive inter-connector
            eval(['num',num2str(vehicle*6-6),' = [ zeros(40,8*n) ];']);
        else % last vehicle
            eval(['num',num2str(vehicle*6-6),' = [ zeros(40,8*n-1) ];']);
        end;
    end;

    % if inter-connector

else
% Secondary actuator inputs required
if (intcon == 1),
  % Purely passive inter-connector
  % Even model
  if (round(vehicle/2) == vehicle/2),
    eval({'num',num2str(vehicle*6-1), ' = [ zeros(40,vehicle*12-12) ; zeros(16,13); zeros(4,1) ; zeros(20,12) ] ;', ...'
      'zeros(40,12*(n-vehicle)) ; zeros(40,12*(n-vehicle)) ]');
  else
    odd model
    eval({'num',num2str(vehicle*6-1), ' = [ zeros(20,7) ; zeros(6,5) ; zeros(4,1) ; zeros(16,13) ] ;', ...'
      'zeros(40,12*(n-vehicle)) ; zeros(40,12*(n-vehicle)) ]');
  end;

% Adjust 'railcarl' model for odd/even format
if (round(vehicle/2) == vehicle/2),
  % Even models
  wb_rc = wb_rc*([ eye(8) zeros(8,7) ; zeros(6,5) ; zeros(4,1) ; zeros(20,13) ];
  c_rc = [ c_rc ; zeros(6,5) ; zeros(4,1) ; zeros(20,13) ];
  dwb_rc = dwb_rc*([ eye(8) zeros(8,7) ; zeros(6,5) ; zeros(4,1) ; zeros(20,13) ];
  dwb_rc = [ dwb_rc ; zeros(6,5) ; zeros(4,1) ; zeros(20,13) ];
else
  % Odd models
  wb_rc = wb_rc*([ eye(8) zeros(8,7) ; zeros(6,5) ; zeros(4,1) ; zeros(20,13) ];
  c_rc = [ c_rc ; zeros(6,5) ; zeros(4,1) ; zeros(20,13) ];
  dwb_rc = dwb_rc*([ eye(8) zeros(8,7) ; zeros(6,5) ; zeros(4,1) ; zeros(20,13) ];
  dwb_rc = [ dwb_rc ; zeros(6,5) ; zeros(4,1) ; zeros(20,13) ];
end;
% If vehicle
% Incorporate in mvblkb & mvcon variables : railway carriage
% ssnnblks = [ ssnnblks vehicle*6-4 ];
% Incorporate in connection matrix
q = [ q ; vehicle*6-4 vehicle*6-5 vehicle*6-6 vehicle*6-7 vehicle*6-8 vehicle*6-9 vehicle*6-10 vehicle*6-11 ];
% Both inter-connectors are connected
q = [ q ; vehicle*6-4 vehicle*6-5 vehicle*6-6 vehicle*6-7 vehicle*6-8 vehicle*6-9 vehicle*6-10 vehicle*6-11 ];
% If vehicle
% Map to mvblkb representation
% eval({'a',num2str(vehicle*6-1), ' = a_rc;'});
% eval({'b',num2str(vehicle*6-4), ' = b_rc;'});
% eval({'c',num2str(vehicle*6-5), ' = c_rc;'});
% eval({'d',num2str(vehicle*6-6), ' = d_rc;'});
% eval({'e',num2str(vehicle*6-7), ' = e_rc;'});
else (blocks == 4), % actuator block
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Incorpore an actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
  % Load appropriate actuator
  if (actuator == 1),
    % Create a dummy lateral actuator
    [la_ac,b_ac,c_ac,d_ac] = ehydactl;
elseif (actuator == 2),
    if (round(vehicle/2) == vehicle/2).
        a_ac = zeros(size(a_ac));
        b_ac = zeros(size(b_ac));
        c_ac = zeros(size(c_ac));
        d_ac = zeros(size(d_ac));
        disp('**** PURE PASSIVE SECONDARY LATERAL SUSPENSION *****');
        disp('**** VEHICLE #: ' + num2str(vehicle) + '*****');
        disp(' '); 
    elseif (blocks == 1),
        a_ac = zeros(size(a_ac));
        b_ac = zeros(size(b_ac));
        c_ac = zeros(size(c_ac));
        d_ac = zeros(size(d_ac));
        disp(' **** IDEAL SECONDARY LATERAL ACTUATORS *****');
        disp(' **** VEHICLE #: ' + num2str(vehicle) + '*****');
        disp(' '); 
    elseif (actuator == 4),
        % Electrohydraulic lateral actuator
        [a_ac, b_ac, c_ac, d_ac] = ehydactl;
        disp(' **** ELECTROHYDRAULIC SECONDARY LATERAL ACTUATORS *****');
        disp(' **** VEHICLE #: ' + num2str(vehicle) + '*****');
        disp(' '); 
    elseif (actuator == 5),
        % Electromechanical lateral actuator
        [a_ac, b_ac, c_ac, d_ac] = emchactl;
        disp(' **** ELECTROMECHANICAL SECONDARY LATERAL ACTUATORS *****');
        disp(' **** VEHICLE #: ' + num2str(vehicle) + '*****');
        disp(' '); 
    end; if even/odd
        % Map to mbblk representation
        eval('a', num2str(vehicle*6-2), ' = a_ac');
        eval('b', num2str(vehicle*6-2), ' = b_ac');
        eval('c', num2str(vehic1e*6-2), ' = c_ac');
        eval('d', num2str(vehic1e*6-2), ' = d_ac');
        disp(' '); 
        % Incorporate in mbblk & mvec variables: actuator
        ssnblks = [ ssnblks vehicle*6-2 ];
    end; % if even/odd
    % Incorporate in connection matrix
    q = [ q : vehicle*6-2 vehicle*6-1 vehicle*6-4 0 0 ];
else (blocks == 5), % inter-connector
    % Load appropriate inter-connector if (intcon == 1),
    % Spring damper lateral connection
    [a_ic, b_ic, c_ic, d_ic] = sprdmpl(key, ccy);
    disp('**** INTER-CONNECTORS : LATERAL SPRING DAMPERS *****');
    disp(' **** VEHICLE #: ' + num2str(vehicle) + '*****');
    disp(' '); 
    % Modify model for blanks on the input & zeros on the output
    if (roundvehicle/2) == vehicle/2,
        % Even model
        b_ac = b_ac';
        zeros(1,16) 1 zeros(1,15) 1 zeros(1,17) 1 zeros(1,18);
        zeros(1,19) 1 zeros(1,20);
        b_ac = zeros(size(b_ac));
        c_ac = zeros(size(c_ac));
        d_ac = zeros(size(d_ac));
        disp('**** VEHICLE #: ' + num2str(vehicle) + '*****');
        disp(' '); 
    elseif (actuator == 2),
        if (round(vehicle/2) == vehicle/2).
            a_ac = zeros(size(a_ac));
            b_ac = zeros(size(b_ac));
            c_ac = zeros(size(c_ac));
            d_ac = zeros(size(d_ac));
            disp('**** PURE PASSIVE SECONDARY LATERAL SUSPENSION *****');
            disp('**** VEHICLE #: ' + num2str(vehicle) + '*****');
            disp(' '); 
        elseif (blocks == 1),
            a_ac = zeros(size(a_ac));
            b_ac = zeros(size(b_ac));
            c_ac = zeros(size(c_ac));
            d_ac = zeros(size(d_ac));
            disp(' **** IDEAL SECONDARY LATERAL ACTUATORS *****');
            disp(' **** VEHICLE #: ' + num2str(vehicle) + '*****');
            disp(' '); 
        elseif (actuator == 4),
            % Electrohydraulic lateral actuator
            [a_ac, b_ac, c_ac, d_ac] = ehydactl;
            disp(' **** ELECTROHYDRAULIC SECONDARY LATERAL ACTUATORS *****');
            disp(' **** VEHICLE #: ' + num2str(vehicle) + '*****');
            disp(' '); 
        elseif (actuator == 5),
            % Electromechanical lateral actuator
            [a_ac, b_ac, c_ac, d_ac] = emchactl;
            disp(' **** ELECTROMECHANICAL SECONDARY LATERAL ACTUATORS *****');
            disp(' **** VEHICLE #: ' + num2str(vehicle) + '*****');
            disp(' '); 
        end; if even/odd
            % Map to mbblk representation
            eval('a', num2str(vehicle*6-2), ' = a_ac');
            eval('b', num2str(vehicle*6-2), ' = b_ac');
            eval('c', num2str(vehicle*6-2), ' = c_ac');
            eval('d', num2str(vehicle*6-2), ' = d_ac');
            disp(' '); 
            % Incorporate in mbblk & mvec variables: actuator
            ssnblks = [ ssnblks vehicle*6-2 ];
        end; % if even/odd
        % Incorporate in connection matrix
        q = [ q : vehicle*6-2 vehicle*6-1 vehicle*6-4 0 0 ];
    else (blocks == 5), % inter-connector
        % Load appropriate inter-connector if (intcon == 1),
        % Spring damper lateral connection
        [a_ic, b_ic, c_ic, d_ic] = sprdmpl(key, ccy);
        disp('**** INTER-CONNECTORS : LATERAL SPRING DAMPERS *****');
        disp(' **** VEHICLE #: ' + num2str(vehicle) + '*****');
        disp(' '); 
        % Modify model for blanks on the input & zeros on the output
        if (roundvehicle/2) == vehicle/2,
            % Even model
            b_ac = b_ac';
            zeros(1,16) 1 zeros(1,15) 1 zeros(1,17) 1 zeros(1,18);
elseif (intcon
Zero: 1 zeros(1,19); zeros(1,21); 1 zeros(1,19); zeros(1,21); 1 zeros(1,19); zeros(1,19); 1 zeros(1,19); zeros(1,19); 1 zeros(1,19)
end; % if odd/even

else (intcon != 3),

% Electrohydraulic type lateral inter-connector
[a_ic,b_ic,c_ic,d_ic] = ehdyolicl(kcy,ccy);
disp('% ELECTROHYDRAULIC LATERAL INTER-CONNECTORS ......');
disp('% vehicle/2); ',num2str(vehicle),'
end; % if odd/even
else (intcon == 4),

% Electromechanical lateral inter-connector
[a_ic,b_ic,c_ic,d_ic] = electromechanicl(kcy,ccy);
disp('% ELECTROMECHANICAL LATERAL INTER-CONNECTORS ......');
disp('% vehicle/2); ',num2str(vehicle),'
end; % if odd/even

elseif (intcon
% odd model
b_ic = [ zeros(1,39) ; zeros(1,2) 1 zeros(1,17)
zeros(1,3) 1 zeros(1,36) ];
c_ic = [ zeros(14,size(a_ic,2)) ; c_ic ];
d_ic = d_ic(1) ; zeros(1,19) ; zeros(1,20) 1 zeros(1,19)
zeros(1,21) 1 zeros(1,22) 1 zeros(1,17)
zeros(1,23) 1 zeros(1,36) ];
d_lc = [ zeros(13,size(a_ic,2)) ; c_ic ; zeros(1,size(a_ic,2)) ];
d_lc = d_lc(1) ; zeros(1,19) ; zeros(1,20) 1 zeros(1,19)
zeros(1,21) 1 zeros(1,22) 1 zeros(1,17)
zeros(1,23) 1 zeros(1,36) ];
else
% Electromagnetic lateral inter-connector
[a_ic,b_ic,c_ic,d_ic] = emsfig(kcy,cyy);
disp('**** ELECTROMAGNETIC LATERAL INTER-CONNECTORS ****');
disp('***** VEHICLE #: ' num2str(vehicle) ' *****');
disp('****');
% Modify model for blanks on the input & zeros on the output
if (round(vehicle/2) == vehicle/2),
% Even model
b_ic = b_ic(:)
zeros(1,20) 1 zeros(1,19) ; zeros(1,21) 1 zeros(1,18)
zeros(1,22) 1 zeros(1,17) ; zeros(1,23) 1 zeros(1,16)
zeros(1,34) 1 zeros(1,35) ; zeros(1,19) 1 zeros(1,36)
zeros(1,3) 1 zeros(1,18) ; zeros(1,2) 1 zeros(1,17)
zeros(1,23) 1 zeros(1,16) ];
c_ic = [ zeros(14,size(a_ic,2)) ; c_ic ];
d_ic = d_ic(1) ; zeros(1,19) ; zeros(1,20) 1 zeros(1,19)
zeros(1,21) 1 zeros(1,22) 1 zeros(1,17)
zeros(1,23) 1 zeros(1,36) ];
d_lc = [ zeros(14,size(d_ic,2)) ; d_ic ];
else
% Odd model
b_ic = b_ic(:)
zeros(1,19) ; zeros(1,2) 1 zeros(1,17)
zeros(1,21) 1 zeros(1,22) 1 zeros(1,18)
zeros(1,23) 1 zeros(1,36) ];
c_ic = [ zeros(13,size(a_ic,2)) ; c_ic ];
d_ic = d_ic(1) ; zeros(1,19) ; zeros(1,20) 1 zeros(1,19)
zeros(1,21) 1 zeros(1,22) 1 zeros(1,17)
zeros(1,23) 1 zeros(1,36) ];
d_lc = [ zeros(13,size(d_ic,2)) ; d_ic ];
end
% if odd/even
end
end
% if inter-connector
% On the last inter-connector transmit a null force
if (vehicle == n),
a_ic = zeros(size(a_ic));
b_ic = zeros(size(b_ic));
c_ic = zeros(size(c_ic));
d_ic = zeros(size(d_ic));
end
% if
% Incorporate in mbkbl & connect representation : inter-connector
% All carriages except last one - make this a null block
mbkbl = [ mbkbls vehicle*6-3 ];
if (q == [ q ; vehicle*6-3 vehicle*6-4 vehicle*6-2 0 0 0 ]);
else
g = [ q ; vehicle*6-3 vehicle*6-4 0 0 0 ];
end
% if
% Map to mbkbl & mvcn representation
eval(['a.',num2str(vehicle*6-4) , ' = a_ic;']);
eval(['b.',num2str(vehicle*6-3) , ' = b_ic;']);
eval(['c.',num2str(vehicle*6-2) , ' = c_ic;']);
eval(['d.',num2str(vehicle*6-1) , ' = d_ic;']);
else % Post format block
% Select odd/even model
if (round(vehicle/2) == vehicle/2),
% Select even
end
% Select odd
end
% Incorporate in mbkbl & connect : post format block
mbkbl = [ mbkbls vehicle*6 ];
g = [ q ; vehicle*6 vehicle*6-4 0 0 0 ];
end if blocks
end for blocks
outblock = [ outblock vehicle*6 ];
end

% Create input & output blocks
% Input block
% Secondary actuator input required or not
if (actuator == 1),
% Purely passive secondary actuator
% Inter-connector input required or not
if (intcon == 1),
eval(['num', num2str(n*6+1), ' = eye(2*n);']);
else
% We have an inter-connector input
eval(['num', num2str(n*6+1), ' = eye(9*n-l);']);
end;
% if
else
% we have secondary actuator inputs
% Inter-connector input required or not
if (intcon == 1),
eval(['num', num2str(n*6+1), ' = eye(12*n);']);
else
% We have an inter-connector input
eval(['num', num2str(n*6+1), ' = eye(13*n-l);']);
end;
% if
end;
% if

% Incorporate in mvblkb & mvccon format
mvnblks = [ mvnblks n*6+1 ];
q = [ q; n*6+1 0 0 0 0 ];
u = [ u; n*6+1 ];

% Output block
eval(['num', num2str(n*6+2), ' = eye(19*n);']);
eval(['comden', num2str(n*6+2), ' = 1;']);

% Incorporate output block connections in q, resize if required
if (size(outblock,2) > size(q,1)),
q = [ q zeros(size(q,1),size(outblock,2)-size(q,1,1,2)); outblock ];
else if (size(outblock,2) < size(q,1,1,2)),
q = [ q; outblock zeros(1,size(q,1,1,2)-size(outblock,2)) ];
else
q = [ q; outblock ];
end;
% if

% Blockbuild railway train mvblkb
disp('***** BLOCKBUILD SYSTEM *****');
mvblkb
% Connect system together to produce a,b,c,d
[a,b,c,d] = mvccon(a,b,c,d,q,lu,ly,sk);

% Offset the effect of passive inter-vehicle connections on active force
offset = eye(size(c,1));
if (n==1),
% only apply offset when an inter-connector exists
for vehicle = 2:n,
% Even vehicle
offset(19*vehicle-19,:) = [ zeros(1,19*vehicle-38) key -lhz*key ...
ccy -lhz*ccy zeros(1,14) -key -lhz*key -ccy ...
zeros(1,15) zeros(1,19*(n-vehicle)) ];
end;
% for
end;
% if

c = offset*c;
d = offset*d;

% Extract track/actuator inputs
windex = [ 1; bindex ];
% Secondary actuator input required or not
if (actuator == 1),
% Purely passive secondary actuator
% Inter-connector input required or not
if (intcon == 1),
for vehicle = l:n,
windex(:,windex); b = [I; db];
else
% We have an inter-connector input
for vehicle = l:n,
% even vehicle
windex(:,windex); b = [I; db];
dw d(:,windex); db
else
% we have a pair of secondary actuator inputs
% Inter-connector input required or not
if (intcon == 1),
for vehicle = l:n,
windex = [ windex 9*vehicle-3:12*vehicle-4 ];
if (vehicle==n),
bindex = [ bindex 9*vehicle ];
end;
% if
else
% we have an inter-connector input
for vehicle = l:n,
if (intcon == 1),
for vehicle = l:n,
end;
% if
end;
% if

% Incorporate in mvblkb & mvccon format
mvnblks = [ mvnblks n*6+1 ];
q = [ q; n*6+1 0000 ];
u = [ u; n*6+1 ];

% Output block
eval(['num', num2str(n*6+2), ' = eye(19*n);']);
eval(['comden', num2str(n*6+2), ' = 1;']);

% Incorporate output block connections in q, resize if required
if (size(outblock,2) > size(q,1)),
q = [ q zeros(size(q,1),size(outblock,2)-size(q,1,1,2)); outblock ];
else if (size(outblock,2) < size(q,1,1,2)),
q = [ q; outblock zeros(1,size(q,1,1,2)-size(outblock,2)) ];
else
q = [ q; outblock ];
end;
% if

% Blockbuild railway train mvblkb
disp('***** BLOCKBUILD SYSTEM *****');
mvblkb
% Connect system together to produce a,b,c,d
[a,b,c,d] = mvccon(a,b,c,d,q,lu,ly,sk);

% Offset the effect of passive inter-vehicle connections on active force
offset = eye(size(c,1));
if (n==1),
% only apply offset when an inter-connector exists
for vehicle = 2:n,
% Even vehicle
offset(19*vehicle-19,:) = [ zeros(1,19*vehicle-38) key -lhz*key ...
ccy -lhz*ccy zeros(1,14) -key -lhz*key -ccy ...
zeros(1,15) zeros(1,19*(n-vehicle)) ];
end;
% for
end;
% if

c = offset*c;
d = offset*d;

% Extract track/actuator inputs
windex = [ 1; bindex ];
% Secondary actuator input required or not
if (actuator == 1),
% Purely passive secondary actuator
% Inter-connector input required or not
if (intcon == 1),
for vehicle = l:n,
windex(:,windex); b = [I; db];
else
% We have an inter-connector input
for vehicle = l:n,
% even vehicle
windex(:,windex); b = [I; db];
dw d(:,windex); db
else
% we have a pair of secondary actuator inputs
% Inter-connector input required or not
if (intcon == 1),
for vehicle = l:n,
for vehicle = l:n,
end;
% if
end;
% if
end; % for
w = b(:,windex); b = b(:,:,bindex); dw = d(:,windex); db = d(:,:,bindex);
end; % if
end; % if

% Strip of last vehicle inter-connector output signal
c = c(1:size(c,1)-1,:);
db = db(1:size(db,1)-1,:);
dw = dw(1:size(dw,1)-1,:);
function [a,b,c,d] = sprdmpl(kcy,ccy)

% Create model of inter-connector laterally between 2 carriages.
% A parallel spring/damper arrangement with inter-vehicle
% stiffness 'kcy' and damping 'ccy' is used to model a bellow
% connection between 2 vehicles. This is shown below:

% PLANVIEW OF ADJACENT VEHICLES

% Vehicle m+1
% ---------------
% | Bogie m+1 |
% | SPR | DMP |
% | Fb(m+1)y |

% Vehicle m
% ---------------
% | Bogie m |
% | SPR | DMP |
% | Fb(m)y |

% MODEL DESCRIPTION:
% INPUTS:
% Ybm, Psibm, Ybm', Psibm', Yb(m+1), Psib(m+1), Yb(m+1), Psib(m+1)
% STATES:
% 1 null state
% OUTPUTS:
% Fb(m+1)y, Fbm2y

% Fbm2y is the lateral force applied by body m on body m+1.

% Developed by Ian Pratt, Loughborough University.

% Load vehicle parameters
railprmt

% Form state-space model
a = 0;
b = zeros(1,8);
c = 0;
d = [kcy, -lhz*kcy, ccy, -lhz*ccy, -kcy, -lhz*kcy, -ccy, -lhz*ccy];
function [a,b,c,d] = idealicl(kcy,ccy)

% IDEALICL
FUNCTION [a,b,c,d] = idealicl(kcy,ccy);

Locate ideal inter-connector laterally between 2 carriages. A parallel spring/damper arrangement with inter-vehicle stiffness 'kcy' and damping 'ccy' is used to model a bellow connection between 2 vehicles. This is described by the diagram shown below:

--- PLANVIEW OF ADJACENT VEHICLES --

Vehicle \( m + 1 \)

\[ \begin{align*}
\text{Bogie}^{m+1} & \quad \text{SPR} \quad \text{DAMP} \quad \text{Fbb(m+1)y} \\
\text{Vehicle} & \quad \text{Bogie}^{m} \\
\end{align*} \]

MODEL DESCRIPTION:

INPUTS:
- \( Y_{bm} \)
- \( \psi_{lbm} \)
- \( Y_{bm}' \)
- \( \psi_{lbm}' \)
- \( F_{cbm} \)
- \( Y_{h(m+1)} \)
- \( \psi_{lb(m+1)} \)
- \( Y_{b(m+1)} \)
- \( \psi_{lb(m+1)}' \)

OUTPUTS:
- \( F_{bm2y} \)

\( F_{bm2y} \) is the lateral force applied by body \( m \) on body \( m+1 \).

Developed by Ian Pratt, Loughborough University.
function [a,b,c,d] = ehydicl(key,eey)

% EHYDICL
FUNCTION [a,b,c,d] = ehydicl(key,eey);

Locate electrohydraulic inter-connector laterally between
2 carriages. A parallel spring/damper arrangement with
inter-vehicle stiffness 'key' and damping 'eey' is used to
model a bellow connection between 2 vehicles. This is
described by the diagram shown below:

PLAN VIEW OF ADJACENT VEHICLES

Vehicle m+1

Bogie
m+1

SPR

DMP

Fb(m+1)ly
or Fbmy

Vehicle m

Bogie
m

Fb(m)ly

MODEL DESCRIPTION:

INPUTS:

Ybm
Psibm
Ybm'
Psibm'
Fcbm
Yb(m+1)
Psib(m+1)
Yb(m+1)'
Psib(m+1)'

OUTPUTS:

Fbm2y

Fbm2y is the lateral force applied by body m on body m+1.

% Developed by Ian Pratt, Loughborough University.

% Load vehicle parameters
railprm

% Look up actuator model
[a,bf,byd,c.df,dyd] = ehydact;

% Create interface matrix
aifc = 0;
bifc = zeros(1,9);
cifc = zeros(2,1);
difc = [0 0 0 1 -lhz 0 0 0 -1 -lhz zeros(1,4) 1 zeros(1,4) ];

dblw = zeros(1,9);
dblw = [-key -lhz key -lhz -ccy 0 -kc -lhz*key -ccy -lhz*ccy ];

dblw = [0 0 0 -key -lhz*key ccy -lhz*ccy 0 -kc -lhz*key -ccy -lhz*ccy ];

% Generate state-space model
atemp, btemp, ctemp, dtemp) = series(aifc, bifc, cifc, difc,a, [byd bf1,-c,-(dyd df)];
[a,b,c,d] = parallel(ablw, bb lw, cblw, db lw, atemp, btemp, ctemp, dtemp);
function [a,b,c,d] = emchicl(key,ccy)
EMCHICL

Locate electromechanical inter-connector laterally between 2 carriages. A parallel spring/damper arrangement with inter-vehicle stiffness 'key' and damping 'ccy' is used to model a bellows connection between 2 vehicles. This is described by the diagram shown below:

PLAN VIEW OF ADJACENT VEHICLES

Vehicle m+1

-----------------

Bogie

m+1

-----------------

SPR

///

FSCP

---

Bogie

n

-----------------

Vehicle m


\% Create spring/damper model
ablw = 0;
bblw = zeros(1,9);
cblw = 0;
dblw = [ kcy -1hz*key ccy -1hz*ccy 0 -key -1hz*key -ccy -1hz*ccy ];
\%
\% Generate state-space model
[latemp,btemp,ctemp,dtemp] = series(aifc,bifc,cifc,difc,a,byd bf1,c,-[dyd df]);
[a,b,c,d] = parallel(ablw,bblw,cblw,dblw,atemp,btemp,ctemp,dtemp);

Fbm2y is the lateral force applied by body m on body m+1.

Developed by Ian Pratt, Loughborough University.
function [a,b,c,d) = emagicl(kcy,ccy)

% EMAGICL
% FUNCTION (a,b,c,d) = emagicl(kcy,ccy);

Locate electromagnetic inter-connector laterally between 2 carriages. A parallel spring/damper arrangement with inter-vehicle stiffness 'kcy' and damping 'ccy' is used to model a bellows connection between 2 vehicles. This is described by the diagram shown below:

**PLAN VIEW OF ADJACENT VEHICLES**

Vehicle m+1

```
Bogie
m+1
```

---

**SPR**  
**DMP**  
---

Fb(m+1)y or Fbm2y

---

Bogie m

---

**MODEL DESCRIPTION**:

**INPUTS**:

Ybm  
Psbm  
Ybm'  
Psbm'  
Fbmy

**STATES**:

Actuator states

**OUTPUTS**:

Fbm2y

---

Fbm2y is the lateral force applied by body m on body m+1.

% Load vehicle parameters
railprmt

% Look up actuator model
[a,bf,byd,c,df,dyd] = emagact;

% Create interface matrix
aifc = zeros(1,9);  
bHc = zeros(2,1);  
difc = [1 0 0 1 -1h -1h -1h zeros(1,4)];

df = [1 zeros(1,4) 1 zeros(1,4)];

d = parallel(ablw,bblw,cblw,dblw,atemp,btemp,ctemp,dtemp);

df = series(aifc,bifc,cifc,difc,a,byd bf,-c,-dyd df);

% Create spring/damper model
ablw = zeros(1,9);  
cblw = zeros(1,9);  
dblw = [kcy -lhz*kcy ccy -lhz*ccy 0 -kky -lhz*kky -ccy -lhz*ccy];

dblw = [kcy -lhz*kcy ccy -lhz*ccy 0 -kky -lhz*kky -ccy -lhz*ccy];

% Generate state-space model
latemp,btemp,ctemp,dtemp = series(aifc,bifc,cifc,difc,a,byd bf,-c,-dyd df);

[a,b,c,d) = parallel(ablw,bblw,cblw,dblw,atemp,btemp,ctemp,dtemp);
function [a,b,c,d] = idealacl

IDEALAACL

Create model of an ideal actuator to be situated in parallel with the secondary lateral suspension. The arrangement is shown diagramatically below:

```
 Leading
 bogie
 planview

Bogie/Truck

Fctm1y

SPR [///]--
DMP [\///]--
SPR [///]--

Fctm2y
```

MODEL DESCRIPTION:

INPUTS:

```
Ybm
Psibm
Ybm'
Psibm'
Ytm1
Psitm1
Ytm1'
Psitm1'
Ytm2
Psitm2
Ytm2'
Psitm2'
Fctm1y
Fctm12y
Fctm21y
Fctm22y
```

STATES: 1 null state

OUTPUTS:

```
Ftm1y
Ftm12y
Ftm21y
Ftm22y
```

Ftm1y, Ftm12y, Ftm21y, and Ftm22y are forces generated by the actuators.

Developed by Ian Pratt, Loughborough University.

% Apply force only
a = 0;

b = zeros(1,16);
c = zeros(4,1);
d = [zeros(1,12) 1 0 0 0; zeros(1,13) 1 0 0;
    zeros(1,14) 1 0; zeros(1,15) 1];
function [a,b,c,d) = ehydactl

Create model of an electrohydraulic actuator to be situated in parallel with the secondary lateral suspension. The arrangement is shown diagramatically below:

```
 Leading bogie planview

SPR    [///][/]
DMP    [///]
        [///] Fctml1y

Bogie/Truck

SPR    [///][/]
DMP    [///]
        [///] Fctml2y
```

MODEL DESCRIPTION:

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>STATES</th>
<th>OUTPUTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vbm</td>
<td>2 Actuator states</td>
<td>Fctml1y, Fctml2y, Ftm12ly, Ftm21y</td>
</tr>
<tr>
<td>Vbm'</td>
<td></td>
<td>Fctml1y</td>
</tr>
<tr>
<td>Ytm1</td>
<td></td>
<td>Fctml2y</td>
</tr>
<tr>
<td>Ytm1'</td>
<td></td>
<td>Ftm12ly</td>
</tr>
<tr>
<td>Ytm2</td>
<td></td>
<td>Ftm21y</td>
</tr>
<tr>
<td>Ytm2'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fctml1y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fctml2y</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fctml1y, Fctml2y, Ftm12ly, and Ftm21y are forces generated by the actuators.

Developed by Ian Pratt, Loughborough University.

% Look up actuator model
[a,b,byd,c,dyd] = ehydactl;

% Actuator 1ly displacement rate and drive signal
afb11 = 0;
bfb11 = zeros(1,16);
cfb11 = zeros(2,1);
dfb11 = [ 0 0 1 1 0 0 -1 1 1 1 zeros(1,8) zeros(1,12) 1 0 0 0 ];

% Actuator 12y displacement rate and drive signal
afb12 = 0;
bfb12 = zeros(1,16);
cfb12 = zeros(2,1);
dfb12 = [ 0 0 -1 -1 1 1 1 zeros(1,8) zeros(1,13) 1 0 0 ];

% Actuator 21y displacement rate and drive signal
afb21 = 0;
bfb21 = zeros(1,16);
cfb21 = zeros(2,1);
dfb21 = [ 0 0 -1 zeros(1,6) 1 1 zeros(1,4) zeros(1,14) 1 0 ];

% Actuator 22y displacement rate and drive signal
afb22 = 0;
bfb22 = zeros(1,16);
cfb22 = zeros(2,1);
dfb22 = [ 0 0 -1 zeros(1,6) 1 1 zeros(1,4) zeros(1,15) 1 ];

% Force applied to leading bogie (1ly actuator)
afb112 = 0;
bfb112 = 0;
cfb112 = zeros(4,1);
dfb112 = [1 0 0 0];

% Force applied to leading bogie (12y actuator)
afb122 = 0;
bfb122 = 0;
cfb122 = zeros(4,1);
dfb122 = [0 1 0 0];

% Force applied to trailing bogie (2ly actuator)
afb212 = 0;
bfb212 = 0;
cfb212 = zeros(4,1);
dfb212 = [0 1 0 0];

% Force applied to leading bogie (22y actuator)
afb222 = 0;
bfb222 = 0;
cfb222 = zeros(4,1);
dfb222 = [0 0 1 0];

% Generate state-space model
[atempl1,btempl1,ctempl1,dtempl1] = series(afb11,bfb11,cfb11,dfb11, ... 
a, [byd bf] c, [dyd df]);
[atempl2,btempl2,ctempl2,dtempl2] = series(afb12,bfb12,cfb12,dfb12);
(atemp12, btemp12, ctemp12, dtemp12) = series(afb121, bfb121, cfb121, dfb121, ...
  a, (byd bfl), c, (dyd dfl));
(atemp12, btemp12, ctemp12, dtemp12) = series(atemp12, btemp12, ctemp12, dtemp12, ...
  a, (byd bfl), c, (dyd dfl));
(atemp21, btemp21, ctemp21, dtemp21) = series(afb211, bfb211, cfb211, dfb211, ...
  a, (byd bfl), c, (dyd dfl));
(atemp21, btemp21, ctemp21, dtemp21) = series(atemp21, btemp21, ctemp21, dtemp21, ...
  a, (byd bfl), c, (dyd dfl));
(atemp22, btemp22, ctemp22, dtemp22) = series(afb221, bfb221, cfb221, dfb221, ...
  a, (byd bfl), c, (dyd dfl));
(atemp22, btemp22, ctemp22, dtemp22) = series(atemp22, btemp22, ctemp22, dtemp22, ...
  a, (byd bfl), c, (dyd dfl));
(a, b, c, d) = parallel(atemp11, btemp11, ctemp11, dtemp11, atemp12, btemp12, ...
  ctemp12, dtemp12);
(a, b, c, d) = parallel(a, b, c, d, atemp21, btemp21, ctemp21, dtemp21);
(a, b, c, d) = parallel(a, b, c, d, atemp22, btemp22, ctemp22, dtemp22);
function [a,b,c,d] = emchact

Create model of an electromechanical actuator to be situated in parallel with the secondary lateral suspension. The arrangement is shown diagramatically below:

MODEL DESCRIPTION:

INPUTS: Ybm, Ybm', Ycnl, Ycnl', Ycnl1, Ycnl1', Ycnl2, Ycnl2', Pctm1y, Pctm2y, Pctm31y, Pctm32y

OUTPUTS: Fctm1y, Fctm2y, Fctm31y, Fctm32y

Fctm1y, Fctm2y, Fctm31y, and Fctm32y are forces generated by the actuators

`Look up actuator model
[a,bf,bf1,bf2,cf,c1,c2,df1,df2,d1,d2] = emchact;

% Actuator 11 y displacement rate and drive signal
afb11 = 0; bfb11 = zeros(1,16); cfb11 = zeros(2,1); dfb11 = [0 0 1 1 a(1) -1 a(1) zeros(1,8) zeros(1,12) 1 0 0 0];

% Actuator 12 y displacement rate and drive signal
afb12 = 0; bfb12 = zeros(1,16); cfb12 = zeros(2,1); dfb12 = [0 0 1 -a(1) 0 0 1 a(2) -1 zeros(1,8) zeros(1,12) 1 0 0 0];

% Actuator 21 y displacement rate and drive signal
afb21 = 0; bfb21 = zeros(1,16); cfb21 = zeros(2,1); dfb21 = [0 0 1 -a(1) zeros(1,6) -1 a(2) -1 zeros(1,4) zeros(1,12) 1 0 0 0];

% Actuator 22 y displacement rate and drive signal
afb22 = 0; bfb22 = zeros(1,16); cfb22 = zeros(2,1); dfb22 = [0 0 1 -a(1) zeros(1,6) -1 a(2) -1 zeros(1,4) zeros(1,12) 1 0 0 0];

% Force applied to leading bogie (11y actuator)
afb1l1 = 0; bfb1l1 = zeros(1,16); cfb1l1 = zeros(2,1); dfb1l1 = [0 0 1 -a(1) zeros(1,6) -1 a(2) -1 zeros(1,4) zeros(1,12) 1 0 0 0];

% Force applied to leading bogie (12y actuator)
afb1l2 = 0; bfb1l2 = zeros(1,16); cfb1l2 = zeros(2,1); dfb1l2 = [0 0 1 -a(1) zeros(1,6) -1 a(2) -1 zeros(1,4) zeros(1,12) 1 0 0 0];

% Force applied to trailing bogie (21y actuator)
afb2l1 = 0; bfb2l1 = zeros(1,16); cfb2l1 = zeros(2,1); dfb2l1 = [0 0 1 -a(1) zeros(1,6) -1 a(2) -1 zeros(1,4) zeros(1,12) 1 0 0 0];

% Force applied to trailing bogie (22y actuator)
afb2l2 = 0; bfb2l2 = zeros(1,16); cfb2l2 = zeros(2,1); dfb2l2 = [0 0 1 -a(1) zeros(1,6) -1 a(2) -1 zeros(1,4) zeros(1,12) 1 0 0 0];

% Generate state-space model
[tec(1),btem1,ctem1,
te(1),dtem1] = series(afb1l1,bfb1l1,ctem1,dtem1); [ad,bd,cd,dd] = a,b,c,d;

% Load vehicle parameters
railprmt
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([atemp12,btemp12,ctemp12,dtemp12) = series(atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([a,b,c,d) = parallel([atemp12,btemp12,ctemp12,dtemp12, a,b,c,d)];
([a,b,c,d) = parallel([a,b,c,d,atemp12,btemp12,ctemp12,dtemp12);
function [a,b,c,d] = emagactl

EMAGACTL

Create model of an electromagnetic actuator to be situated in parallel with the secondary lateral suspension. The arrangement is shown diagramatically below:

<table>
<thead>
<tr>
<th>Leading bogie planview</th>
<th>Bogie/Truck mount</th>
</tr>
</thead>
</table>

MODEL DESCRIPTION:

INPUTS:

<table>
<thead>
<tr>
<th>Ybrn</th>
<th>Ybm</th>
<th>Ybrm'</th>
<th>Ybm'</th>
</tr>
</thead>
</table>

OUTPUTS:

<table>
<thead>
<tr>
<th>Fctmllyy</th>
<th>Fctm12y</th>
<th>Fctm21y</th>
<th>Fctm22y</th>
</tr>
</thead>
</table>

Fctmllyy, Fctm12y, Fctm21y, and Fctm22y are forces generated by the actuators.

Developed by Ian Pratt, Loughborough University.

% Look up actuator model
[a,bf,byd,c,df,dyd] = emagact;

% Actuator 1ly displacement rate and drive signal
afb11  = 0;
bfb11 = zeros(1,16);
cfb11 = zeros(2,1);
dfb11 = [0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1];

% Actuator 2ly displacement rate and drive signal
afb12  = 0;
bfb12 = zeros(1,16);
cfb12 = zeros(2,1);
dfb12 = [0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1];

% Actuator 2ly displacement rate and drive signal
afb21  = 0;
bfb21 = zeros(1,16);
cfb21 = zeros(2,1);
dfb21 = [0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1];

% Force applied to leading bogie (1ly actuator)
afb112 = 0;
bfb112 = 0;
cfb112 = zeros(4,1);
dfb112 = [1;0;0;0];

% Force applied to leading bogie (2ly actuator)
afb122 = 0;
bfb122 = 0;
cfb122 = zeros(4,1);
dfb122 = [0;1;0;0];

% Force applied to trailing bogie (1ly actuator)
afb212 = 0;
bfb212 = 0;
cfb212 = zeros(4,1);
dfb212 = [0;0;1;0];

% Force applied to leading bogie (2ly actuator)
afb222 = 0;
bfb222 = 0;
cfb222 = zeros(4,1);
dfb222 = [0;0;0;1];

% Generate state-space model
[atempl1,btempl1,ctempl1,dtempl1] = series(afb111,bfb111,cfb111,dfb111,...
    a,[byd byd byd byd]);

[atempl1,btempl1,ctempl1,dtempl1] = series(atempl1,btempl1,ctempl1,dtempl1,...
    afb112,bfb112,cfb112,dfb112);
{atemp12, btemp12, ctemp12, dtemp12} = series{afb121, bfb121, cfb121, dfb121, ...
  a, (byd bf), c, (dyd df));
{atemp12, btemp12, ctemp12, dtemp12} = series{atemp12, btemp12, ctemp12, dtemp12, ...
  a, (byd bf), c, (dyd df));
{atemp21, btemp21, ctemp21, dtemp21} = series{afb211, bfb211, cfb211, dfb211, ...
  a, (byd bf), c, (dyd df));
{atemp21, btemp21, ctemp21, dtemp21} = series{atemp21, btemp21, ctemp21, dtemp21, ...
  a, (byd bf), c, (dyd df));
{atemp22, btemp22, ctemp22, dtemp22} = series{afb222, bfb222, cfb222, dfb222, ...
  a, (byd bf), c, (dyd df));
{atemp22, btemp22, ctemp22, dtemp22} = series{atemp22, btemp22, ctemp22, dtemp22, ...
  a, (byd bf), c, (dyd df));
{atemp22, btemp22, ctemp22, dtemp22} = series{atemp22, btemp22, ctemp22, dtemp22, ...
  a, (byd bf), c, (dyd df));

{a, b, c, d} = parallel{atemp11, btemp11, ctemp11, dtemp11, atemp12, btemp12, ...
  ctemp12, dtemp12);
{a, b, c, d} = parallel{a, b, c, d, atemp21, btemp21, ctemp21, dtemp21};
{a, b, c, d} = parallel{a, b, c, d, atemp22, btemp22, ctemp22, dtemp22};
% Define parameters of an electrohydraulic actuator

% Spool parameters
Ms = 0.075; % (kg) spool mass
Ks = 1.85e6; % (N/m) spool mass stiffness
Cs = 471; % (Ns/m) spool mass damping
Ki = 142; % (Nm/A) torque motor gain

% Servo-valve & cylinder flow characteristics
Ps = 2e7; % pressure (N/m²) type
Kqa = 2.091e-6; % (N²/m³) annulus flow constant
Kqb = 2.723e-6; % (N²/0.5s/m³) bore flow constant
dqaddx = 0.00935;
dqaddx = 0.01218;
dqaddp = 2.022e-12;
dqaddp = 1.923e-12;

% Oil parameters
B = 1.38e9; % oil compressibility

% Cylinder parameters
Va = 5.06e-6; % (m³) annulus cylinder volume
Vb = 9.03e-6; % (m³) annulus cylinder volume
Aa = 2.25e-4; % (m²) annulus side area
Ab = 4.02e-4; % (m²) bore side area
Cdamp = 50; % (Ns/m) ram damping

% PID controller parameters
FRP = 5e-3; INT = 5e-2; DRV = 0;
function [a,bf,byd,df,dyd] = ehydact

% EHDACT

FUNCTION [a,bf,byd,df,dyd] = ehydact;

Create model of an electrohydraulic actuator, all the essential dynamics are included in this model, namely, oil compressibility, flow accumulation, ram damping, and spool dynamics. The arrangement is shown diagramatically below:

\[
\begin{array}{c|c|c}
F & \text{\textbf{INPUTS}} & \text{\textbf{OUTPUTS}} \\
\hline
y' & xv & y \\
\hline
\text{\textbf{RAM DISPLACEMENT:}} & \text{\textbf{STATES:}} & \text{\textbf{Fcommand}} \\
\text{\textbf{FORCE DEMAND:}} & \text{\textbf{INPUTS}} & \text{\textbf{Xc}} \\
\text{\textbf{INPUTS}} & \text{\textbf{OUTPUTS:}} & \text{\textbf{P}}
\end{array}
\]

A force control loop is positioned around the actuator dynamics. The states of the actuator are present: 'xv' (spool displacement), 'Pa' the annulus area, and 'Pb' the bore area. 'Xc' represents the internal states of the force loop controller.

F is the force generated by the actuator, given a force command 'Fcommand', and a ram displacement rate 'y'.

% Load electrohydraulic actuator parameters

ehydprmt

% Create 'aact' matrix
aact = [ 0 1 0 0 -Ks/Ms -Cs/Ms 0 0 -B/VA*dqaddxv 0 0 -B/VA*dqaddpa 0 B/VA*dqaddpa 0 0 0 0 0 ];
% Create 'bact' matrix
bact = [ 0 X1/Ms 0 0 ];
% Create 'bactyd' matrix
bactyd = [ 0 0 0 0 0 ];
% Create 'cact' matrix
cact = [ 0 0 -As Ab ];

% Create 'dacti' matrix
dacti = 0;
% Create 'dactyd' matrix
dactyd = -Cdamp;
% Form PID controller
numpid = [ DRV PRP INT ];
denpid = [ 0.0001 1 0 ];
[apid,bpid,cpid,dpidl] = tf2ss(numpid,denpid);
% Form closed loop system
I = eye(size(dacti,1));
a = [ (aact-bacti*dpid*inv(I+dacti*dpid)*cact) ...
    bacti*dpid-bacti*dpid*inv(I+dacti*dpid)*dacti*cpid ...]
    bacti*dpid-bacti*dpid*inv(I+dacti*dpid)*dacti*cpid ];
bf = [ bacti*dpid-bacti*dpid*inv(I+dacti*dpid)*dacti*cpid ...]
    bacti*dpid-bacti*dpid*inv(I+dacti*dpid)*dacti*cpid ];
byd = [ bacti*dpid-bacti*dpid*inv(I+dacti*dpid)*dacti*cpid ...]
    bacti*dpid-bacti*dpid*inv(I+dacti*dpid)*dacti*cpid ];
c = [ inv(I+dacti*dpid)*cact inv(I+dacti*dpid)*dacti*cpid ];
df = [ inv(I+dacti*dpid)*dacti*cpid ];
dyd = [ inv(I+dacti*dpid)*dacti*cpid ];

% Developed by Ian Pratt, Loughborough University.
% Define parameters of an electromechanical actuator

% Motor electrical parameters
Kt = 0.108; (% (Nm/Amp) motor torque constant
Km = 0.108; (% (Vs) motor backemf gain
Im = 1e-4; (% (H) winding inductance
Rm = 0.95; (% (Ohms) winding resistance
Imax = 8.62; (% (Amps) maximum motor current
Vmax = 44.5; (% (Volts) maximum motor voltage

% Motor mechanical parameters
Jm = 1.2e-4; (% (kgm^2) motor inertia
Cm = 0.012; (% (Nm/rad) motor damping
Km = 1e7; (% (N/m) motor series stiffness
bsh = 0.0001; (% (rad) motor backlash
cfrc = 0.042; (% (Nm) motor coulomb friction
frcgn = 0.004; (% (Nms/rad) motor friction

% Screw parameters
n = 3.82e-4; (% (m/rad) screw pitch
Ms = 2; (% (kg) screw mass
Ks = 2e6; (% (N/m) screw stiffness
Cs = 4e3; (% (Ns/m) screw damping

% PID controller parameters
PRP = 0.08; INT = 0.5; DVR = 1e-6;
function [a, b, c, d, e, f, g, h] = emechact

Create model of an electromechanical actuator, all the essential dynamics are included in this model, namely, motor electrical/mechanical time constants, screw mass, stiffness and damping. The arrangement is shown diagrammatically below:

\[
\begin{align*}
F &\quad \leftrightarrow \quad F_{\text{command}} \\
\dot{y}' &\quad \rightarrow \quad \theta \\
\theta &\quad \rightarrow \quad \theta' \\
x_m &\quad \rightarrow \quad x_c \\
F &\quad \rightarrow \quad F
\end{align*}
\]

MODEL DESCRIPTION:

<table>
<thead>
<tr>
<th>RAM DISPLACEMENT:</th>
<th>( y' )</th>
<th>( \theta )</th>
<th>( \theta' )</th>
<th>( x_m )</th>
<th>( x_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>STATES:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INPUTS:</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>FORCE DEMAND:</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( F_{\text{command}} )</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>INPUTS:</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>OUTPUTS:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F )</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

A force control loop is positioned around the actuator dynamics. The states of the actuator are present: 'Xc' represents the internal states of the force loop controller.

\( F \) is the force generated by the actuator, given a force command 'Fcommand', and a ram displacement rate \( y' \).

Developed by Ian Pratt, Loughborough University.
%EMAGPRMT  SCRIPT emagprmt
% Define parameters of an electromagnetic actuator

% Permeability of air
mu0 = pi*4e-7;

% Coil parameters
N = 477; % turns
A = 0.008; % (m^2) pole area
R = 4.214; % (Ohms) coil resistance
Lm = 0.218; % (H) mutual inductance
Li = 0.120; % (H) self (leakage) inductance
Kg = 4e5; % (N/m)
Ki = 107.8; % (N/Amp)
Gap = 0.015; % (m) nominal airgap

% PID controller parameters
PRP = 7; INT = 10; DRV = 0.0000001;
function [a,bf,byd,c,df,dyd] = emagact

%EMAGACT

Create model of an electromagnetic actuator, all the essential
dynamics are included in this model, namely, winding
dynamics (coil resistance and inductance), amplifier
gains, and variable reluctance effects. The arrangement is shown
diagramatically below:

\[ F \]
\[ \begin{bmatrix} \frac{-Rkg}{(Lm+Ll)*Kl} & -R/(Lm+Ll) \\ 1 & 0 \end{bmatrix} \]
\[ y' \rightarrow \text{Fcommand} \]
\[ F \]

MODEL DESCRIPTION:

<table>
<thead>
<tr>
<th>RAM DISPLACEMENT:</th>
<th>y'</th>
<th>STATES:</th>
<th>x</th>
<th>INPUTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>FORCE DEMAND:</td>
<td>Fcommand</td>
<td>Xc</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INPUTS</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>OUTPUTS:</td>
<td>--</td>
<td>--</td>
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</tbody>
</table>

A force control loop is positioned around the actuator
dynamics. The states of the actuator are present: actuator
extension, and coil current. 'Xc' represents the internal
states of the force loop controller.

F is the force generated by the actuator, given a force
command 'Fcommand', and a ram displacement rate 'y'.

% Load electromagnetic actuator parameters
emagprmt

% Create 'aact' matrix
aact = [ 0 0 ; -Rkg/(Lm+Ll)*Kl -R/(Lm+Ll) ];
% Create 'bacti' matrix
bacti = [ 0 ; 1/(Lm+Ll) ];
% Create 'bactyd' matrix
bactyd = [ 1 ; -(Ll)*Kg/(Lm+Ll)*Kl ];
% Create 'cact' matrix
cact = [ 0 0 K ];
% Create 'dacti' matrix
dacti = [ 0 ];
dactyd = [ 0 ];
% Create PID controller
numpid = [ 0.0001 1 0 ];
denpid = [ 1 ];
[apid,bpid,cpid,dpid] = tf2ss(numpid,denpid);
% Form closed loop system
I = eye(size(dacti,1));
a = [ aact-bacti*dpid*inv(I+dacti*dpid)*cact ... bacti*dpid-bacti*dpid*inv(I+dacti*dpid)*dacti*cpid ... bacti*bacti*dpid*inv(I+dacti*dpid)*dacti*dpid ... (apid-bpid*inv(I+dacti*dpid)*dacti*dpid) ];
b = [ bacti*dpid-bacti*dpid*inv(I+dacti*dpid)*dacti*dpid ... bacti*bacti*dpid*inv(I+dacti*dpid)*dacti*dpid ... bacti*bacti*dpid*inv(I+dacti*dpid)*dacti*dpid ... bacti*bacti*dpid*inv(I+dacti*dpid)*dacti*dpid ];
bdatey = [ -bacti*dpid*inv(I+dacti*dpid)*dacty ... bacti*dpid*inv(I+dacti*dpid)*dacty ... bacti*dpid*inv(I+dacti*dpid)*dacty ];
c = [ inv(I+dacti*dpid)*cact inv(I+dacti*dpid)*dacti*cpid ... inv(I+dacti*dpid)*dacti*cpid inv(I+dacti*dpid)*dacti*cpid ];
df = [ inv(I+dacti*dpid)*dacti*cpid ];
dydy = [ inv(I+dacti*dpid)*dacti*dpid ];
dydy = [ inv(I+dacti*dpid)*dacti*dpid ];
dydy = [ inv(I+dacti*dpid)*dacti*dpid ];

Developed by Ian Pratt, Loughborough University.
Appendix V - Inter-vehicle actuator test-ing software (C)


```c
#include "c:\dsp\include\stdlib.h"
#include "c:\dsp\include\math.h"
#include "c:\rd230ccp\r230defn.h"

void main (void)
{
    /* this C compiler expects this function */
    UserFunction(int Vars); /* User coded C function */
    Initialise(int Vars); /* Initialisation function */
    initialiseparam(int Vars, float demandpsd); /* rate limited ramp function */
    int alarm(int Vars); /* the C compiler expects this function */
    UserFunction(int Vars); /* User coded C function */
}

static float dencoef2 =
static float dencoefl
static float accelzb21init, accelzb22init, displzb21init, displzb22init;
static int sampleT = 0.002 /* Seconds */
static START 0 /* Seconds */
static WAIT 1.5 /* Seconds */
static RAMPUP 6.5 /* Seconds (End of phase) */
static UNCONTROL 35 /* Seconds (End of phase) */
static CONTROL 65 /* Seconds (End of phase) */
static RAMPDOWN 70 /* Seconds (End of phase) */
static WAITSTATE 1
static RAMPDUPSTATE 2
static UNCTRLSTATE 3
define CYRLLSTATE 4
define RAMPDOWNSTATE 5
define ACCELB21GN 1.0
define ACCELB22GN 1.0
#define PI 3.1416
#define Hertz 0.05 /* Roll-off low frequency cut-off */
#define Omega 2*PI*Hertz
#define ETA 0.7 /* Damping of filter frequency */
#define Minspd 0.0
#define Maxspd 9.5
#define Rate 0.005 /* Rate of rise/fall of motor speed */
#define Alarmdur 1.5 /* duration of buzzer */
#define Alrmprd 0.1 /* period of buzzer */
#define Volumem 9.3 /* buzzer volume on */
define Volumef 0.0 /* buzzer volume off */
define float mtrspd;
define static float pitstiff = 0.0, bncdamp = 0.0, bncvelv2 = 0.0, bncv2 = 0.0;
#define static float pitstiff = 0.0, bncdamp = 1000.0, bncvelv2 = 0.0, bncv2 = 0.0;
define static int sampleT = 0, state = RAMPDUPSTATE, oldstate = RAMPDOWNSTATE, alarmflag = 0;
define static float accelexb21init, accelexb22init, displxb21init, displxb22init;
define static float dencoef2 = 4.4*sampleT, dencoef1 = 2.0*Omega*Omega*sampleT, dencoefO =
define static float dencoef1 = 2.0*Omega*Omega*sampleT, dencoefO = 2*Omega*Omega*sampleT, bnczero1 =
define static float bnczero1 = 2*Omega*Omega*sampleT, bnczero0 = 2*Omega*Omega*sampleT, bnczero2 =
define static float bnczero2 = 2*Omega*Omega*sampleT, bnczero1 = 2*Omega*Omega*sampleT, bnczero0 =
define static float inbnc2, inbnc1, inbnc0, outbnc2, outbnc1, outbnc0;
define static float inpit2, inpit1, inpit0, outpit2, outpit1, outpit0;

void UserFunction(int Vars) { /* the C compiler expects this function */
    void Initialise(int Vars); /* User coded C function */
    void initialiseparam(int Vars, float demandpsd); /* rate limited ramp function */
    int alarm(int Vars); /* the C compiler expects this function */
    void UserFunction(int Vars); /* User coded C function */
}

#define lb 0.35 /* m bogie - bogie spacing */
define Hertz 0.32 /* m semi vehicle spacing + gangway */
define sampleT = 0.002 /* Seconds */
define START 0 /* Seconds */
define WAIT 1.5 /* Seconds */
define RAMPUP 6.5 /* Seconds (End of phase) */
define UNCONTROL 35 /* Seconds (End of phase) */
define CONTROL 65 /* Seconds (End of phase) */
define RAMPDOWN 70 /* Seconds (End of phase) */
define WAITSTATE 1
#define RAMPDUPSTATE 2
#define UNCTRLSTATE 3
define CYRLLSTATE 4
define RAMPDOWNSTATE 5
define ACCELB21GN 1.0
define ACCELB22GN 1.0
#define PI 3.1416
#define Hertz 0.05 /* Roll-off low frequency cut-off */
#define Omega 2*PI*Hertz
#define ETA 0.7 /* Damping of filter frequency */
#define Minspd 0.0
#define Maxspd 9.5
#define Rate 0.005 /* Rate of rise/fall of motor speed */
#define Alarmdur 1.5 /* duration of buzzer */
#define Alrmprd 0.1 /* period of buzzer */
#define Volumem 9.3 /* buzzer volume on */
define Volumef 0.0 /* buzzer volume off */
define float mtrspd;
define static float pitstiff = 0.0, bncdamp = 0.0, bncvelv2 = 0.0, bncv2 = 0.0;
#define static float pitstiff = 0.0, bncdamp = 1000.0, bncvelv2 = 0.0, bncv2 = 0.0;
define static int sampleT = 0, state = RAMPDUPSTATE, oldstate = RAMPDOWNSTATE, alarmflag = 0;
define static float accelexb21init, accelexb22init, displxb21init, displxb22init;
define static float dencoef2 = 4.4*sampleT, dencoef1 = 2.0*Omega*Omega*sampleT, dencoefO =
define static float dencoef1 = 2.0*Omega*Omega*sampleT, dencoefO = 2*Omega*Omega*sampleT, bnczero1 =
define static float bnczero1 = 2*Omega*Omega*sampleT, bnczero0 = 2*Omega*Omega*sampleT, bnczero2 =
define static float bnczero2 = 2*Omega*Omega*sampleT, bnczero1 = 2*Omega*Omega*sampleT, bnczero0 =
define static float inbnc2, inbnc1, inbnc0, outbnc2, outbnc1, outbnc0;
define static float inpit2, inpit1, inpit0, outpit2, outpit1, outpit0;

void main (void) { /* the C compiler expects this function */
    void Initialise(int Vars); /* User coded C function */
    void initialiseparam(int Vars, float demandpsd); /* rate limited ramp function */
    int alarm(int Vars); /* the C compiler expects this function */
    void UserFunction(int Vars); /* User coded C function */
}
```

Listing for Ian Pratt


Page 1
switch(state) {
  case UNCTRLSTATE: /* Uncontrolled mode of operation */
    Initialise(Vars); /* Read in sensor offsets */
    break;
  case WAITSTATE: /* Wait before starting cycles */
    if (alrmflag == 0) alrmflag = alarm(Vars); /* Apply buzzer once at start of phase */
    Initialise(Vars); /* Read in sensor offsets */
    break;
  case RAMPUPSTATE: /* Ramp up to maximum speed */
    if (alrmflag == 0) alrmflag = alarm(Vars); /* Apply buzzer once at start of phase */
    softramp(Vars, MAXSPD);
    break;
  case UNCTRLSTATE: /* Run as passive vehicle */
    if (alrmflag == 0) alrmflag = alarm(Vars); /* Apply buzzer once at start of phase */
    /* Apply zero actuator forces */
    force12 = 0.0;
    force23 = 0.0;

    if (alrmflag == 0) alrmflag = alarm(Vars); /* Apply buzzer once at start of phase */
    /* Calculate actuator forces & apply */
    force12 = -pitdamp*pitchvelv2 + pitstiff*pitchv2 - bncdamp*bncvelv2 - bncstiff*bnvelv2;
    force23 = -pitdamp*pitchvelv2 + pitstiff*pitchv2 - bncdamp*bncvelv2 - bncstiff*bnvelv2;
    if (alrmflag == 0) alrmflag = alarm(Vars); /* Apply buzzer once at start of phase */
    /* Activate inter-vehicle control */
    /* Apply buzzer once at start of phase */
    softramp(Vars, MINSPD);
    break;
  case RAMPDOWNSTATE: /* Ramp track speed down at end of demo */
    if (alrmflag == 0) alrmflag = alarm(Vars); /* Apply buzzer once at start of phase */
    softramp(Vars, MINSPD);
    break;
}

/* Acquisition of parameters */
bnv2 = sample+bnvelv2; /* bounce velocity scaling */
pitchv2 = sample[pitchvelv2]; /* pitch displacement */
pitchdamp = fGetYf(int*Vars[BNC DAMP]);
pitstiff = fGetYf(int*Vars[PITSTIFF]);
/* Determine mode of operation */
switch(state) {
void Initialise(int Vars)
{
    int iA;
    /* Acquisition of initial sensor information */
    iA = iGet(iA, (int*)Vars+ACCELZB21); /* get address of accelerometer zb21 */
    accelzb21init = fGetX(accelzb21init, (int*)iA); /* get data from accelerometer zb21 */
    iA = iGet(iA, (int*)Vars+ACCELZB22); /* get address of accelerometer zb22 */
    accelzb22init = fGetX(accelzb22init, (int*)iA); /* get data from accelerometer zb22 */
    iA = iGet(iA, (int*)Vars+DISPLZB21); /* get address of displacement zb21 */
    displzb21init = fGetX(displzb21init, (int*)iA); /* get data from displacement zb21 */
    iA = iGet(iA, (int*)Vars+DISPLZB22); /* get address of displacement zb22 */
    displzb22init = fGetX(displzb22init, (int*)iA); /* get data from displacement zb22 */
}
void softramp(int Vars, float demandspd)
{
    int iA;
    if (mtrspd > demandspd) mtrspd -= RATE; /* apply control action */
    else mtrspd += RATE; /* constrain control action */
    if (mtrspd > MAXSPD)
        mtrspd = MAXSPD;
    if (mtrspd < MINSPD)
        mtrspd = MINSPD;
    fPutY(mtrspd, (int*)Vars+MTRSPDMON);
    iA = iGet(iA, (int*)Vars+MOTORSPD); /* get address of Motor speed */
    fPutX(mtrspd, (int*)iA); /* copy data to output */
}
int alarm(int Vars)
{
    int iA;
    float buzzvol;
    if (tremain -= sampleT) > 0)
    {
        if (buzzcount++ > ALRMPRD/sampleT)
            buzzvol = VOLUMEOFF;
        else buzzvol = VOLUMON;
        iA = iGet(iA, (int*)Vars+BUZZER); /* get address of OutputA */
        fPutX(buzzvol, (int*)iA); /* set buzzer on */
        return 0;
    }
    else
    {
        iA = iGet(iA, (int*)Vars+BUZZER); /* get address of OutputA */
        fPutX(VOLUMOFF, (int*)iA); /* set buzzer off */
        remain = ALRMDUR;
    }
CERTIFICATE OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this thesis, that the original work is my own except as specified in acknowledgments or in footnotes, and that neither the thesis nor the original work contained therein has been submitted to this or any other institution for a higher degree.

(Signed)

(Date)

20 November 96