The influence of touchdown conditions on high jumping performance

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THE INFLUENCE OF TOUCHDOWN CONDITIONS ON HIGH JUMPING PERFORMANCE

BY

MATTHEW PAUL GREIG

A Master's Thesis

Submitted in partial fulfilment of the requirements for the award of Master of Philosophy of Loughborough University

March, 1998

Supervisor: Dr. M.R. Yeadon

ABSTRACT

The Influence of Touchdown Conditions on High Jumping Performance
M.P. Greig, Loughborough University, 1998

Thirty years after Dick Fosbury's triumph at the Mexico Olympic Games the Fosbury flop remains the sole technique used by the world's high jumping elite. The introduction of the flop technique was the most recent landmark in the technical evolution of the high jump. Much speculation has concerned the optimum high jumping technique.

The aim of the present study was to determine the influence of approach parameters on jump height performance, and to determine the optimum approach parameters for an elite male high jumper. This was investigated both experimentally and theoretically.

A single subject multiple trial experimental study was carried out to determine the influence on jump height of the approach speed, leg plant angle and knee angle at touchdown. Direct intervention was used to induce a greater range in approach speed than was observed for the same athlete in competition. The plant and knee angles were found to change systematically with changes in the approach speed. The interdependence of the approach parameters made it difficult to determine the influence of a single parameter on jump height. The experimental determination of optimum technique was therefore limited by a lack of experimental control. The calculated optimum approach speed and plant angle were close to the maxima of the experimental data set. Jump height was shown to be maximised with a straight leg at touchdown. The optimum approach was therefore fast, with the leg planted away from the vertical and with minimum flexion at the knee joint.

A two segment simulation model of the high jump takeoff was developed to further investigate optimum technique. The inertial characteristics of the model were made specific to the elite athlete. The muscle strength was selected to produce realistic jump height performance for given touchdown conditions. The model was evaluated using the experimental kinematic data. The simplicity of the model required an unrealistically large maximum knee torque to produce realistic jump height performance. Subsequently the optimum touchdown conditions and resulting performance were unrealistic. The optimum approach speed and plant angle exceeded the upper bounds of the experimental data. Jump height was optimised with minimum knee flexion at touchdown. As a further consequence of the large maximum knee torque the takeoff time was unrealistically short. The theoretical determination of optimum technique supported the conclusions of the experimental study, suggesting that the optimum approach was fast with a straight leg planted away from the vertical.

In order to further examine optimum high jumping technique a multi-segment simulation model is required with personalised joint torques and inertial characteristics.
PUBLICATIONS

Aspects of this work have been published as follows:

Journals


Conference Presentations


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Mike Holmes for his interest and wealth of high jump knowledge,

and

The Sports Council of Great Britain and Northern Ireland for their financial support.
DEDICATION

Mum and Dad.

Family and friends, past and present.

If the blind lead the blind, both shall fall into the ditch.
Matthew (15: 14)
# TABLE OF CONTENTS

| Abstract                                      | I       |
| Certificate of Originality                   | II      |
| Publications                                 | III     |
| Acknowledgements                             | IV      |
| Dedication                                   | V       |
| Table of Contents                            | VI      |
| List of Tables                               | X       |
| List of Figures                              | XII     |

## CHAPTER 1: INTRODUCTION

1.1 Statement of purpose

1.2 Chapter organisation

## CHAPTER 2: REVIEW OF LITERATURE

2.1 The Fosbury flop style of high jumping

2.1.1 Evolution of the Fosbury flop

2.1.2 The approach phase

2.1.3 The plant and takeoff phase

2.1.4 The flight phase and bar clearance

2.1.5 Summary

2.2 Investigations of high jumping

2.2.1 Introduction

2.2.2 Experimental studies

2.2.3 Summary of experimental studies

2.2.4 Theoretical studies

2.2.5 Summary of theoretical studies

2.3 Techniques of investigation

2.3.1 Introduction

2.3.2 Techniques of cinematographic analysis

Introduction

Three-dimensional video and cinematography

Synchronisation techniques

Three-dimensional coordinate reconstruction
CHAPTER 3: EXPERIMENTAL DETERMINATION OF OPTIMUM TECHNIQUE

3.1 Introduction 41

3.2 Pilot study: A kinematic analysis of competitive high jumping 41
  3.2.1 Introduction 41
  3.2.2 Data collection 42
  3.2.3 Data analysis 43
    Digitising procedure 43
    Calculation of performance variables 44
  3.2.4 Results 44
  3.2.5 Discussion 51
  3.2.6 Conclusions 55
  3.2.7 Future directions 55
    Data collection 55
    Data analysis 55

3.3 Single subject multiple trial study 56
  3.3.1 Introduction 56
  3.3.2 Experimental design 57
  3.3.3 Data collection 57
  3.3.4 Data analysis 60
    Digitising procedure 60
    Synchronisation of the data sets 62
    Calculation of performance variables 63
CHAPTER 4: THEORETICAL DETERMINATION OF OPTIMUM TECHNIQUE

4.1 Introduction

4.2 Development of a rigid two segment simulation model
  4.2.1 Nomenclature
  4.2.2 Model inputs
  4.2.3 Model outputs
    Optimisation criterion
    Time histories
  4.2.4 The equations of motion
  4.2.5 Assignment of model parameter values
    Segment length
    Muscle parameters
    Maximum knee torque
    The constant $k_1$
  4.2.6 Comparison of model and actual performance
  4.2.7 Optimum conditions at touchdown
  4.2.8 Discussion
  4.2.9 Conclusions

4.3 Modifying the rigid two segment model
  4.3.1 Assignment of model parameter values
    Segment length
    Muscle parameters
    Maximum knee torque
    The constant $k_1$
<table>
<thead>
<tr>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3.2</td>
</tr>
<tr>
<td>4.3.3</td>
</tr>
<tr>
<td>4.3.4</td>
</tr>
<tr>
<td>4.3.5</td>
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<tr>
<td>4.4</td>
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<td>4.4.1</td>
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<td>4.4.9</td>
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<td>4.4.10</td>
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<tr>
<td>4.4.11</td>
</tr>
<tr>
<td>4.4.12</td>
</tr>
<tr>
<td>4.4.13</td>
</tr>
<tr>
<td>4.5</td>
</tr>
</tbody>
</table>

CHAPTER 5 : SUMMARY AND DISCUSSION
5.1 Summary of findings
5.2 To the future

REFERENCES

APPENDICES
LIST OF TABLES

CHAPTER 2. REVIEW OF LITERATURE

2.3 Techniques of investigation
   Table 2.3.1. Sample population characteristics for determining inertia parameters

CHAPTER 3. EXPERIMENTAL DETERMINATION OF OPTIMUM TECHNIQUE

3.2 Pilot study: A kinematic analysis of competitive high jumping
   Table 3.2.1. Record of performance for Reilly
   Table 3.2.2. Record of performance for Smith
   Table 3.2.3. A summary of the record of performance for Reilly
   Table 3.2.4. A summary of the record of performance for Smith

3.3 Single subject multiple trial study
   Table 3.3.1. Two-dimensional digitising precision for LVC and RCC
   Table 3.3.2. Three-dimensional reconstruction accuracy
   Table 3.3.3. Record of performance
   Table 3.3.4. Summary of the performance variables determined from the video analysis
   Table 3.3.5. Summary of the performance variables determined from the cine analysis
   Table 3.3.6. Using the regression equation to predict jump height for the cine data
   Table 3.3.7. Using the regression equation to predict jump height for the video data
   Table 3.3.8. Using the regression equation to predict jump height for the cine converted video data

CHAPTER 4. THEORETICAL DETERMINATION OF OPTIMUM TECHNIQUE

4.2 Development of a rigid two segment simulation model
   Table 4.2.1. Summary of anthropometric measurements for determining model segment length
   Table 4.2.2. Summary of muscle parameters in model
Table 4.2.3. Approach parameters and jump height performance averaged over jumps br16 to br26

Table 4.2.4. A comparison of simulated and actual jump height performance

4.3 Modifying the rigid two segment model

Table 4.3.1. Summary of segmental lengths obtained from film analysis
Table 4.3.2. Approach parameters averaged over jumps br16 to br26
Table 4.3.3. The influence of the vertical mass centre touchdown velocity on the assigned value of maximum knee torque
Table 4.3.4. Determining the constant $k_i$ for varying vertical mass centre touchdown velocity
Table 4.3.5. A summary of the touchdown conditions input to the modified simulation model
Table 4.3.6. The influence of maximum knee torque on simulated jump height performance
Table 4.3.7. The influence of maximum knee torque on simulated jump height performance with zero vertical touchdown velocity
Table 4.3.8. Summary of the optimum solution for varying maximum knee torque

4.4 Development of an elastic two segment simulation model

Table 4.4.1. Summary of the inertia characteristics of the athlete
Table 4.4.2. Sequential order of muscle strength testing at the knee
Table 4.4.3. Approach parameter values used to optimise the spring parameters
Table 4.4.4. Output parameter values used to optimise the spring parameters
Table 4.4.5. Optimum spring parameters and muscle scale factor
Table 4.4.6. A comparison of simulated and actual jump height performance
Table 4.4.7. New optimum spring parameters and muscle scale factor
**LIST OF FIGURES**

**CHAPTER 1. INTRODUCTION**

| Figure 1. | The theory-prediction-experiment-modified theory cycle of scientific method (Yeadon and Challis, 1994). | 2 |

**CHAPTER 2. REVIEW OF LITERATURE**

2.1 The Fosbury flop style of high jumping

| Figure 2.1.1. | Bar height = H1 + H2 - H3; adapted from Dyson (1986). | 7 |
| Figure 2.1.2. | The ultimate high jump style; adapted from Hay (1973). | 8 |
| Figure 2.1.3. | The J-shaped approach; adapted from Dapena (1988). | 9 |
| Figure 2.1.4. | The bar clearance style used in the Fosbury flop; adapted from Doherty (1985). | 13 |

2.2 Investigations of high jumping

| Figure 2.2.1. | The simulation model of Alexander (1990). | 20 |
| Figure 2.2.2. | A contour plot of jump height, approach velocity and plant angle. | 21 |

2.3 Techniques of investigation

| Figure 2.3.1. | The collinearity condition. | 25 |
| Figure 2.3.2. | A "Sputnik" type calibration frame for three-dimensional reconstruction. | 26 |
| Figure 2.3.3. | The mathematical inertia models of Hanavan (1964) and Hatze (1980). | 29 |
| Figure 2.3.4. | The mathematical inertia model of Jensen (1978). | 30 |
| Figure 2.3.5. | The model of Yeadon (1990). | 31 |
| Figure 2.3.6. | A three-dimensional surface plot of human in-vivo muscle function; adapted from Marshall et al. (1990). | 33 |
| Figure 2.3.7. | The torque-velocity relationship at increasing joint angles; adapted from Fuglevand (1987). | 35 |
| Figure 2.3.8. | The bi-phasic force-velocity relationship observed by Edman (1988); adapted from King (1998). | 36 |
| Figure 2.3.9. | Schematic representations of a forward dynamics analysis or simulation and an inverse dynamics analysis. | 37 |
CHAPTER 3. EXPERIMENTAL DETERMINATION
OF OPTIMUM TECHNIQUE

3.2 Pilot study: A kinematic analysis of competitive high jumping

Figure 3.2.1. Camera positions relative to the high jump area. 42
Figure 3.2.2. The calibration set-up relative to the high jump uprights. 43
Figure 3.2.3. The relationship between approach speed and jump height for Reilly. 47
Figure 3.2.4. The relationship between approach speed and jump height for Smith. 48
Figure 3.2.5. The relationship between plant angle and jump height for Reilly. 49
Figure 3.2.6. The relationship between plant angle and jump height for Smith. 50
Figure 3.2.7. Competition data and the theoretical relationship between approach speed and jump height. 51

3.3 Single subject multiple trial study

Figure 3.3.1. Camera positions relative to the high jump area. 58
Figure 3.3.2. Camera locations relative to the high jump uprights. 58
Figure 3.3.3. The calibration set-up relative to the high jump uprights. 59
Figure 3.3.4. The digitising protocol for the video reference fields. 60
Figure 3.3.5. The digitising protocol for the cine reference fields. 61
Figure 3.3.6. The digitising protocol for the video movement fields. 62
Figure 3.3.7. Representation of the plant angle and the knee angle. 64
Figure 3.3.8. Jump height as a linear function of approach speed. 71
Figure 3.3.9. Jump height as a quadratic function of approach speed. 72
Figure 3.3.10. Jump height as a linear function of leg plant angle. 73
Figure 3.3.11. Jump height as a quadratic function of leg plant angle. 74
Figure 3.3.12. Jump height as a linear function of knee angle at touchdown. 75
Figure 3.3.13. Jump height as a quadratic function of knee angle at touchdown. 76
Figure 3.3.14. The relationship between jump height and approach speed linearly detrended for knee angle. 78
Figure 3.3.15. The relationship between jump height and plant angle linearly detrended for knee angle. 80
CHAPTER 4. THEORETICAL DETERMINATION OF OPTIMUM TECHNIQUE

4.1 Introduction

Figure 4.1.1. Changes in body configuration from touchdown to toe-off; adapted from Dyson (1986).

Figure 4.1.2. Location of the mass centre during the takeoff phase.

4.2 Development of a rigid two segment simulation model

Figure 4.2.1. A rigid two segment simulation model of the high jump takeoff.

Figure 4.2.2. The assumed relationship between the knee angular velocity and the torque exerted at the knee.

Figure 4.2.3. The relationship between maximum knee torque and the height the mass centre is raised in flight.

Figure 4.2.4. The relationship between jump height and approach speed at the model determined optimum knee angle and plant angle.

Figure 4.2.5. The relationship between jump height and leg plant angle at the model determined optimum approach speed and knee angle.

Figure 4.2.6. The relationship between jump height and knee angle at the model determined optimum approach speed and plant angle.

Figure 4.2.7. Time histories of the component ground reaction forces for the optimum touchdown conditions.

4.3 Modifying the rigid two segment model

Figure 4.3.1. The modified relationship between the maximum knee torque and the height the mass centre is raised in flight.

Figure 4.3.2. The influence of vertical mass centre touchdown velocity on the relationship between $T_{\text{max}}$ and $z_{b2}$.

Figure 4.3.3. The relationship between jump height and approach speed at optimum vertical velocity, knee angle and leg plant angle.
Figure 4.3.4. The relationship between jump height and leg plant angle at optimum approach speed, vertical velocity and knee angle.

Figure 4.3.5. The relationship between jump height and knee angle at optimum approach speed, vertical velocity and leg plant angle.

Figure 4.3.6. Time histories of the component ground reaction forces for optimum touchdown conditions in the modified model.

4.4 Development of an elastic two segment simulation model

Figure 4.4.1. An elastic two segment simulation model of the high jump takeoff.

Figure 4.4.2. Schematic representation and free body diagram of the elastic model.

Figure 4.4.3. Differences in the definition of the leg plant angle and knee angle between the experimental film analysis and the simulation model.

Figure 4.4.4. The knee angle time history as defined in the film analysis and as defined in the simulation model for jump br20.

Figure 4.4.5. The plant angle time history as defined in the film analysis and as defined in the simulation model for jump br20.

Figure 4.4.6. Determination of the segment orientation angles.

Figure 4.4.7. Joint angle time history for dynamometer trial brk01.

Figure 4.4.8. Joint angular velocity time history for dynamometer trial brk01.

Figure 4.4.9. The force-velocity relationships of Hill (1970) and Edman (1988).

Figure 4.4.10. General shape of the four and two parameter functions; from King (1998).

Figure 4.4.11. General shape of the six parameter function; from King (1998).

Figure 4.4.12. The torque-angular velocity relationship obtained using the isokinetic dynamometer.

Figure 4.4.13. The modified torque-angular velocity relationship.

Figure 4.4.14. Behaviour of the elastic model around optimum approach speed.
CHAPTER 5. SUMMARY AND DISCUSSION

5.3 To the future

Figure 5.3.1. Time history of the shank segment length throughout the takeoff phase. 174
Figure 5.3.2. Modelling shank segment length as a function of time. 174
Figure 5.3.3. Time history of the thigh segment length throughout the takeoff phase. 175
Figure 5.3.4. Modelling thigh segment length as a function of time. 176
CHAPTER 1

INTRODUCTION

The average person in the street becomes familiar with the sport of track and field athletics only once every four years, with the celebration of the modern Olympic games. An important feature in the more recent development of track and field athletics has been the spreading and sharing of a knowledge of the mechanical principles fundamental to its skills. The development of a new athletic technique inevitably leads to widespread discussion of its relative merits and how it compares with those techniques already in use. If the new technique is judged to be superior it is adopted by coaches and athletes and becomes accepted as the orthodox method.

The history of high jumping contains various examples of this process. The most recent landmark in the technical evolution of the high jump occurred in 1968. When Dick Fosbury of the U.S.A. won the high jump event at the Mexico Olympic games it revolutionised the high jump event. The style used was christened the "Fosbury flop". Nearly thirty years later the Fosbury flop remains the sole technique used by the world's high jumping elite.

For any given sports event we should expect to find some elements that are advantageous and therefore common to all those highly skilled in that event, and some that are peculiar to given individuals or special subsets of athletes. Dapena (1987) stated that if it were possible to identify the advantageous technique elements and to understand in detail the mechanisms through which they work to improve performance, it would then be possible to check which elements are not being used appropriately.

Investigation through careful observation alone by the experienced coach may establish how the athlete may improve personal performance. However, this becomes increasingly difficult at the elite level of competition, where even small changes in technique can have a marked influence on success.

Sports biomechanics research can attempt to increase the understanding of performance in the high jump using two approaches. Although the experimental and theoretical approaches appear to be quite different in nature, both form an integral part of the theory-experiment cycle of scientific method. Figure 1 represents just one interpretation of scientific method, but a comparison of experimental and theoretical results is a common element in the testing of a well-defined theory.

Sciences may be described as experimental sciences or theoretical sciences according to the relative degrees to which they rely on experiment and theory. Yeadon and Challis (1994) proposed that sports biomechanics should be a balanced mix of experimental data and theoretical modelling in order to obtain a realistic understanding.
The majority of the experimental literature relating specifically to high jumping technique has been observational in nature, and has therefore not attempted to determine an optimum technique. In addition the majority of studies have collected data on single jumps by multiple subjects. A comparison of techniques may give insight into which characteristics of technique most strongly influence success in the high jump. However such studies fail to determine the optimum technique for any given individual athlete. By collecting data on multiple jumps by a single subject it may be possible to determine which characteristics of the individual's technique most strongly influence success. This may give insight into how the individual can improve personal performance.

Theoretical studies, such as that of Alexander (1990), have attempted to determine the optimum technique. However there has often been a failure to make the physiological and inertial characteristics of theoretical models representative of real athletes. In addition there has been a failure to evaluate such models using data from actual performances.

The majority of the literature on high jumping technique has not attempted to determine an optimum technique. Those studies that have considered the existence of an optimum technique for an individual high jumper have failed to validate the results against real performances. Therefore a number of questions remain unanswered.

- **Question 1.**
  
  What is the optimum approach speed for an elite male high jumper?

- **Question 2.**
  
  What is the optimum leg plant angle for the same athlete?
• **Question 3.**
  What is the optimum knee angle at plant for the same athlete?

  The optimum technique is defined as that which results in the greatest height cleared. High jumping is a competitive event, whereby the athlete must clear a bar height which is gradually increased throughout a competition. Although a high jumper may attempt to reproduce an optimum technique with each attempt there will be, inevitably, some variation in each aspect of technique. This will lead to a corresponding decrease in jumping performance.

  Ignoring bar clearance technique, it is the ability to raise the mass centre to the greatest possible height that ultimately determines success in the high jump. Jump height is therefore defined as the peak height reached by the mass centre, irrespective of the location of this peak relative to the bar.

• **Question 4.**
  How does jump height performance vary away from the optimum technique?

  It has been stated that optimum technique is unique to a given individual. Differences in optimum techniques between athletes may be due to inertial and physiological characteristics.

• **Question 5.**
  How sensitive is optimum technique to changes in muscular strength?

• **Question 6.**
  How sensitive is optimum technique to changes in the inertial characteristics of the body?

  There are many aspects of technique that may influence jump height performance. The influence on jump height of the changes in approach speed, plant angle and knee angle observed over a series of jumps may be quantified. If the sum of these contributions is less than the observed change in jump height then there must be other factors which influence performance.

• **Question 7.**
  What other variables may influence jump height performance?
1.1 Statement of purpose

To determine the influence of approach parameters on jump height performance, and to determine the optimum approach parameters for an elite high jumper.

The approach parameters optimised were the horizontal mass centre velocity over the final stride of the approach, the leg plant angle, and the knee angle at plant. The optimum value for each parameter was defined as that which resulted in the greatest peak mass centre height.

In order to determine the influence of each of the approach parameters on jump height performance an experimental approach was used. An elite male high jumper performed multiple jumps in a single data collection session. The experimental study was carried out in the training environment, as opposed to the competition environment, enabling greater experimental control to be exerted. Direct intervention was used to vary the length of the approach run to the bar. It was expected that this intervention would produce a greater range in approach speed than would be observed during a single competition. It was also expected that the change in approach speed would also produce changes in the leg plant angle and the knee angle at touchdown.

The approach speed, leg plant angle, knee angle at touchdown and the resulting jump height were collated for each jump. Statistical methods were used to determine the influence of each approach parameter on jump height performance and to obtain an experimentally determined optimum technique.

The extent to which a two segment model could simulate the high jump takeoff was then investigated with a view to obtaining a theoretical description of optimum technique. Initially a rigid two segment simulation model was developed, based upon the model of Alexander (1990). An elastic component was then incorporated in the development of a two segment simulation model of the high jump takeoff. The model was subject specific, with both the inertial characteristics and the mechanical properties of the muscle representation obtained directly from the elite male high jumper.

Simulations were carried out using hypothetical approach parameter values in order to determine the optimum value of each parameter. In addition, the model was used to carry out simulations using approach parameter values obtained from the experimental study. The model was then evaluated by comparing the results of the simulations with the actual performance. The extent to which variations in jump height can be accounted for by differences in approach was determined using the model in conjunction with the video data. The variation in jump height as a function of each approach parameter was obtained using the model to carry out a sensitivity analysis in the neighbourhood of optimum technique.
1.2 Chapter organisation

Chapter 2 reviews the literature on the description of high jumping techniques. First the evolution of the Fosbury flop is considered, followed by an overview of each phase of the high jump from observational studies. Investigations of high jumping technique are then considered, critically examining the results and conclusions of experimental and theoretical studies. The various investigative techniques which form the background to the following chapters are then examined.

Chapter 3 attempts to determine the optimum technique for an elite male high jumper using an experimental study. The influence of the approach on jump height performance is considered. The approach parameters considered are the horizontal velocity of the athlete's mass centre over the final approach stride, the leg plant angle and the knee angle at plant.

Chapter 4 develops the simulation model of the high jump takeoff. This model is to be subsequently used in the theoretical optimisation of those approach parameters considered in the previous chapter. The methods of obtaining the inertia and muscle parameters are described. In each case the parameter values were obtained directly from the same elite male high jumper used in the experimental study of Chapter 3. The equations of motion are presented, along with the inputs and outputs of the model and the optimisation algorithm. The simulation model is evaluated by comparing the results of the experimental and theoretical studies. The simulation model is used to optimise the three approach parameters considered in Chapter 3, with the optimum technique maximising jump height. The optimum simulation is presented and the sensitivity of the optimum technique to changes in the performance, inertia and muscle parameters is considered. The model is used to determine the variation in jump height as a function of each approach parameter in the neighbourhood of optimum technique.

Chapter 5 provides a summary of the results obtained and answers the questions posed in Chapter 1.
CHAPTER 2
REVIEW OF LITERATURE

2.1 The Fosbury flop style of high jumping

The Fosbury flop stands alone today as the sole technique used by the world's high jumping elite. This section will consider the mechanical advantages of the Fosbury flop in relation to its predecessors. The mechanical characteristics of each phase of the Fosbury flop will then be considered; the approach phase, the plant and takeoff phase, and the flight phase and bar clearance.

2.1.1 Evolution of the Fosbury flop

In considering the evolution of the Fosbury flop one must account for why earlier styles became outdated. Holmes (1985) documented the evolution of the high jump from the early days of high jumping when the quite inefficient tucked jump was the preferred method. Clearance heights below six feet were often accompanied by horizontal distances of sixteen feet.

Holmes stated that the introduction of the scissors style of high jumping permitted the athlete to make more use of the legs in attaining vertical jump height, and also allowed the athlete to be more efficient in clearing the bar than in the tucked jump. The scissors style of jumping involved kicking the inside leg up and over the bar as the bar was straddled. Jumping high by driving all parts of the body as high as possible was the crux of good technique. However, if the body is kept in the upright position during bar clearance then the mass centre must be raised considerably above the bar height to effect a successful bar clearance. The scissors remained the favoured style of the majority of jumpers until Sweeney broke the world record in 1895 with a unique style, initially christened the Sweeney twist.

Doherty (1963) proposed that Sweeney initiated attention to the lay-out phase of the high jump, with the upward leap followed by a snap of the upper body to assume a more horizontal position. The mechanical advantages of this style were related to the conversion of the horizontal approach speed into a vertical direction at takeoff. Doherty stated that this was best achieved by planting the takeoff foot well ahead of the body. This was proposed to cause a backward lean of the body at the instant of plant, which in turn served to "brake" the forward speed developed during the approach and generate vertical velocity. The vertical velocity at the end of the takeoff phase is further increased by dynamic upward motions of the free limbs during the takeoff phase. Doherty stated that these actions were an inherent part of the Sweeney twist.

The Sweeney twist was later renamed the Eastern cut-off so as to distinguish it
from the Western roll, which was described by Doherty (1963) as the next great innovation in style. The style was first developed around 1912, almost by accident. The originator was George Horine, who while attempting a scissors style jump from a restricted approach from the "wrong" side took off from the foot closest to the bar. This placed the body in a side-on position while crossing the bar so that the bar was much closer to the mass centre of the jumper. The bar clearance technique was therefore more efficient. The Eastern cut-off and Western roll dominated competitive high jumping for nearly six decades.

In 1956 Dumas of America was the first athlete to win the Olympics using the straddle technique. Paish (1976) noted that the evolution of the straddle was effected by a change in the rules of high jump competition. The new ruling allowed the head of the athletes to drop below the level of the hips, and the straddle technique allowed the jumper to drape the body over the bar in a face down position. This proved to be a more efficient style of bar clearance than adopted in either the Eastern cut-off or Western roll. Paish went so far as to state that this was the ultimate in efficiency, unless it were possible to master a dive forward roll from a single leg takeoff.

This idea was seized upon by Hay (1973a) who proposed the "ultimate high jump style" based upon those elements of the jump required to raise the mass centre, within the constraints of the rules. Hay considered that the height with which an athlete is credited in the high jump may be considered as the sum of the height (H1) of the mass centre at takeoff, the height (H2) which the mass centre is lifted during the jump and the difference (H3) between the peak height of the mass centre and the bar height. Figure 2.1.1 shows the contributions of H1, H2 and H3 to the credited bar height. Hay stated that the optimum high jump technique would simultaneously maximise H1 and H2 and optimise H3, or afford the best compromise among these three terms.

Figure 2.1.1. Bar height = H1 + H2 - H3; adapted from Dyson (1986).
Hay suggested a takeoff position with the arms fully extended overhead, the trunk erect and stretched, and the takeoff leg fully extended at the knee and ankle joints to maximise H1. To maximise the vertical impulse at takeoff and therefore the height H2, Hay suggested a fast approach run and a bent lead leg action. A front pike or jack-knife bar clearance action was proposed to optimise H3, with a frontal approach to the bar so that the front pike position could be adopted with the least difficulty. The setting of these characteristics leads to a technique as depicted in Figure 2.1.2. However, this theoretical style has remained idealised, and only been used in competition by one-legged jumpers.

![Figure 2.1.2. The ultimate high jump style; adapted from Hay (1973).](image)

In relation to its alternatives the straddle was a very advanced style. Athletes began attempting to increase the vertical takeoff velocity, concentrating on the approach, the use of the free limbs, etc. However, leading up to the 1968 Olympic games news spread of a unique style being developed by an American athlete who was to triumph in the Games using his spectacular style. The athlete was Dick Fosbury, and the style was christened the Fosbury flop. During the 1970's the world indoor or outdoor high jump record, always held by either a flopper or a straddler, was beaten by an athlete using the opposite technique on five occasions.

Ecker (1976) suggested two mechanical advantages of the flop. Firstly that the flopper need not jump as high as the straddler in order to clear a given bar height, making the flop technique more efficient in terms of bar clearance. Secondly, that the flop was more effective in creating vertical velocity at takeoff since no eccentric thrust was required to produce rotation. Straddlers were required to use precious energy to create body rotation that is already developed for floppers by virtue of the curved approach. These advantages were supported by Holmes (1985) and also the personal accounts of the high jumper Dwight Stones (in Martin, 1982).
The fact that the Fosbury flop presently dominates the world high jump scene implies that it has been found to be the superior style. Its status as the sole technique used by the world's high jumping elite attracted a great deal of attention, with research aimed at understanding the mechanics of this style of jumping for height. The Fosbury flop technique comprises three distinct phases; the approach run, the plant and takeoff, and the flight phase and bar clearance. The literature review on the mechanics of the Fosbury flop has been organised under these headings.

2.1.2 The approach phase

Woicik (1983) suggested that the approach phase in the high jump serves to bring and prepare the athlete for the most important part of the jump, the takeoff. Dapena (1988) stated that the purpose of the approach is to generate horizontal velocity which the athlete can subsequently convert into vertical velocity at takeoff, and to allow the athlete to assume the correct takeoff position. Schweigert (1992) stated that the approach taken is in the shape of a 'J'; the first part of the approach is in a straight line and is followed by a curved part. Figure 2.1.3 shows a representation of the curved approach used in the Fosbury flop.

![Figure 2.1.3. The J-shaped approach; adapted from Dapena (1988).](image)

Martin (1982) proposed that the straight part of the approach is considered to: (a) help develop the horizontal velocity needed for a successful jump, and (b) attain a consistent stride pattern and tempo. Fidler (1992) stated that the straight part of the run must be smoothly blended into the curved portion of the approach which enables the athlete to get into an ideal preparatory position for takeoff easily and naturally. The
curved portion of the approach generally constitutes only the last four or five strides prior to takeoff and should be continued right into the plant for takeoff.

Challis and Yeadon (1991) noted that during the "curved" approach the horizontal movement of the mass centre was not on a continuous curve, but rather on an alternating series of straight and curved lines. During each foot contact the ground will act against the support foot of the athlete. This reaction force pushes inwards against the support foot, changing the linear path of the mass centre to a curved path. During the non-supported phase of each stride the athlete is not acted upon by any horizontal external forces and the mass centre will travel (horizontally) in a straight path.

Martin (1982) stated that the curved approach initiates the body rotation required for the backward lay-out style of bar clearance. If an appropriate amount of rotation is imparted to the athlete, it is hypothesised that no force would be required to increase the rotary motion, such that all the vertical forces at takeoff can be used exclusively in raising the mass centre of the athlete. Dapena (1988) proposed that the lean of the athlete towards the centre of the run-up curve during the approach assists the athlete in overcoming the natural tendency to lean towards the bar at takeoff. With the inward lean prior to takeoff being distinct in the Fosbury flop, it is speculated that this body lean during the approach and at takeoff assists in the production of the rotational energy and rotational momentum required by the athlete for an effective bar clearance. If the contribution of the curved approach to the production of the angular velocity at takeoff was quantified, this may explain the use of a curved approach in the Fosbury flop technique. However, while many hypotheses have been postulated in the previous research, no quantitative information has been presented with which to formulate a sound mechanical conclusion as to how running a curved approach contributes to the angular velocity of the athlete at takeoff.

Woicik (1983) stated that during the last three or four strides of the curved approach the athlete lowers the mass centre in such a way as to maintain horizontal velocity. However, Dapena (1984) considered that the athlete may be too fast due to excessive horizontal speed developed during the approach, and too low due to an excessive plant angle for example. If the takeoff leg is not strong enough the knee will be forced to flex excessively during the takeoff phase, which may result in a less forceful extension in the final part of the takeoff phase. Many coaches describe a "buckling" of the takeoff leg if the jumper exceeds the optimum approach velocity or plant angle.

So, what is the optimum approach velocity, and what is the optimum angle of leg plant for an individual high jumper? Furthermore, what other elements of technique influence jump height performance and how does high jumping performance decrease away from the optimum technique? Such questions have not been answered in the previous literature.
2.1.3 The plant and takeoff phase

Dapena (1984) stated that the approach phase leads to the most important phase of the jump, the plant and the takeoff. The takeoff phase is defined as the period of time between the instant when the takeoff foot first touches the ground (plant) and the instant when it loses contact with the ground (toe-off). The primary objectives of the takeoff are to convert the horizontal speed developed during the approach into vertical velocity at toe-off, and to set the conditions for an effective bar clearance.

Dapena (1995) considered how the plant suddenly stops the motion of the lower part of the jumper's body, while the upper body continues to travel forward as a result of the momentum developed during the approach. The braking of the forward motion at plant imparts angular velocity to the athlete, causing changes in the body orientation. At the start of the takeoff phase the trunk normally has a backward lean and a lateral inward lean away from the bar, as a result of the curved approach. The trunk then rotates to a position past vertical by the end of the takeoff phase. Heinz (1973) stated that this change in the inward and backward lean of the body during the takeoff phase increases the range of vertical motion through which the mass centre moves during the takeoff phase. Conrad and Ritzdorf (1990) suggested that this increased the range of application of the vertical ground reaction force, thus increasing the vertical acceleration path.

After leaving the ground the athlete must rotate in order to perform an effective layout over the bar. Once airborne, the total angular momentum remains constant. The athlete must therefore develop the necessary angular momentum during the takeoff phase. Dapena (1995) stated that the lean angle of the trunk during the takeoff phase is related not only to the angular momentum produced by the athlete, but also to the vertical velocity produced at takeoff. Although large changes in body lean during the takeoff phase may lead to increased angular momentum, they may also reduce vertical velocity at the end of the takeoff phase. This was supported by Ecker (1985) who stated that the initiation of rotation from the ground automatically reduces the ground reaction forces that contribute to height. The high jump is therefore an event of compromise. The best technique requires sufficient rotation from the ground in order to rotate over the bar, but no excess rotation that will reduce the peak height of the jump more than necessary.

Schweigert (1992) stated that during the takeoff phase, the takeoff leg pushes down on the ground, and in reaction the ground pushes up on the body with an equal and opposite force through the takeoff leg. Aura and Viitasalo (1989) reported a peak ground reaction force of about 9 bodyweights at takeoff. Martin (1982) considered that during the takeoff phase the ground also pushes back on the athlete in a horizontal direction. This reduces the horizontal velocity to a residual value which gives the athlete the necessary horizontal displacement to reach the landing mat after completing the takeoff. The upward force exerted on the athlete changes the vertical velocity of the mass centre from an initial value that is close to zero at plant to a large positive (upward) vertical
velocity at toe-off. Schweigert (1992) stated that the speed of the approach plays a major role in the development of the vertical velocity at takeoff. The loss of horizontal velocity during the takeoff phase is associated with the production of vertical velocity.

With each approach stride the mass centre of the athlete normally rises slightly as the athlete takes off from the ground, and then drops again before the next foot contact. Martin (1982) proposed that at the instant the takeoff foot is planted on the ground to begin the takeoff phase, the mass centre should be comparatively low and have a large horizontal velocity. Dapena (1988) further proposed that in the final approach stride the downward vertical velocity at foot plant should be minimised, in order not to waste energy in reversing this negative downward movement. Fidler (1992) considered that an early plant of the takeoff foot may be achieved by increasing the stride cadence over the final approach strides. This may serve to initiate the takeoff phase before the mass centre develops too much downward velocity. For a given change in vertical velocity during the takeoff phase the athlete with the smallest downward vertical velocity at touchdown will jump the highest.

The peak height of the mass centre over the bar is determined by the height of the mass centre and its vertical velocity at the end of the takeoff phase. Dapena (1988) stated that the mass centre at takeoff (represented by H1 in Figure 2.1.1) is typically at a height equivalent to 70-75% of the standing height of the athlete. This height constitutes approximately 60-70% of the total jump height. The vertical velocity at the end of the takeoff phase determines how much the mass centre will be raised during flight (H2 in Figure 2.1.1), and thus the peak height (H1 + H2) of the jump.

Dapena (1988) stated that the vertical velocity at the end of the takeoff phase will be maximised when the downward vertical velocity at the start of the takeoff phase is minimised and the change in the vertical velocity during the takeoff phase is maximised. Woicik (1983) therefore considered that the actions during the takeoff phase are aimed at maximising the vertical velocity at takeoff. For example, Fidler (1992) stated that if the arms are accelerated upward during the takeoff phase the downward force exerted on the ground will increase. This will result in an increased upward vertical force on the athlete, an increased vertical velocity at the end of the takeoff phase and therefore a greater jump height. However, the takeoff phase lasts only approximately 0.16 seconds. The actions of the free limbs during the takeoff phase must therefore be dynamic in order to maximise the vertical velocity at toe-off.

In order to maximise the vertical velocity of the mass centre at toe-off the initial conditions at touchdown must be optimised. This includes the optimisation of approach parameters such as the horizontal velocity of the mass centre over the final stride, the angle of leg plant and the angle at the knee joint at touchdown.
2.1.4 The flight phase and bar clearance

Although a high vertical takeoff velocity is necessary in raising the mass centre, an effective bar clearance technique by the athlete is also required for a successful jump. In the Fosbury flop, the unique back layout has been suggested to be one of the advantages of the technique. Figure 2.1.4 shows the arched body position during the back lay-out bar clearance style characteristic of the Fosbury flop.

Back-arching in the Fosbury flop technique has been suggested by Hay (1973b) to be an efficient method of bar clearance. Hay proposed an efficiency indicator index related to the difference between the height of the bar and the peak height of the athlete's centre of mass (denoted by H3 in Figure 2.1.1). For an effective bar clearance, the efficiency index must be as small as possible. Hay proposed that, in theory, it is possible for the centre of mass of the athlete to pass below the height of the bar in the flop. However, there is no quantitative information in previous research to indicate that this occurs in practice.

During the airborne phase, the athlete continuously changes the body configuration. Each change in body configuration brings about changes in the angular velocity of the athlete due to changes in the moment of inertia. Martin (1982) stated that the body configuration changes may explain why minor deviations occur between jumps. While the path of the mass centre cannot be changed after toe-off, the respective paths of individual body parts can be changed.

Payne (1985) noted that successful jumpers take advantage of this fact by executing a 'flip-flop' over the bar. For example, as the athlete arches the back during the bar clearance the head and shoulders are lowered toward the landing mat and the hips are raised. When the hips have crossed the bar the athlete flexes at the hips which causes the
hips to be lowered and raises the legs so that they can clear the bar. The changes in body configuration require precise timing, and if executed well it is theoretically possible for all parts of the body to pass over the bar despite the mass centre passing through or even below the bar. Successful adjustments of the body position during bar clearance can compensate for a less than optimal takeoff.

2.1.5 Summary

The high jump comprises three distinct phases; the approach run, the takeoff and the bar clearance. The potential of a high jumper is determined by the ability to raise the mass centre. The peak mass centre height is determined by the height and the vertical velocity of the athlete's mass centre at the end of the takeoff phase. The height of the mass centre at toe-off is largely dependent on the standing height of the athlete. The high jumper should therefore aim to maximise the vertical velocity of the mass centre. The initial conditions at the instant of touchdown must therefore be optimised.

The conditions at the start of the takeoff phase are a direct consequence of the approach. Woicik (1983) stated that the primary purpose of the approach is to generate horizontal velocity which the athlete can subsequently convert into vertical velocity during the takeoff phase. Many coaches advocate a fast and low approach to the high jump bar. However, Dapena (1984) considered that the athlete may be too fast and too low. If the takeoff leg is not strong excessive knee flexion will result in a less forceful extension in the final part of the takeoff phase. This in turn will reduce the peak height reached by the mass centre.

This suggests that an optimum technique exists, and many coaches describe a "buckling" of the takeoff leg if the jumper exceeds the optimum approach velocity or plant angle. So, what is the optimum technique in terms of approach velocity, leg plant angle and knee angle at touchdown and how does high jumping performance decrease away from the optimum technique? Furthermore, are there any other variables that influence jump height, and if so to what degree?

2.2 Investigations of high jumping

2.2.1 Introduction

Yeadon and Challis (1994) stated that performance related sports biomechanics research poses questions such as What? How? and Why? The answers to these questions are important to the scientist, athlete, and coach. The authors proposed that one method of seeking answers to such questions is to use an experimental approach. An alternative method of investigation is to use a theoretical approach. A theoretical model is an idealisation of the activity and may be used in specified situations to produce hypothetical data. A theoretical model may also provide a general description of movement, leading to
a more complete understanding than that provided by a number of particular examples.

Sciences may be described as experimental sciences or theoretical sciences according to the relative degrees to which they rely on experiment and theory. Both experiment and theory have posed their own particular problems in high jumping research and these will be addressed in subsequent sections.

2.2.2 Experimental studies

Dapena (1987) stated that biomechanical research aimed at the improvement of sports performance may be separated into two main forms. Applied research attempts to identify and correct the technique deficiencies of individual athletes. Basic research strives to achieve a better understanding of the mechanisms involved in a sports event and to identify advantageous elements.

The applied research studies typically filmed high jumpers during official competitions. The three-dimensional coordinates of selected body landmarks were obtained throughout each phase of the jump (Dapena, 1978, 1980a,b). The three-dimensional coordinate data were then used to produce computer graphics animation sequences which allowed the jump to be viewed from the side and rear directions. These data were subsequently used to produce reports that explained to the athletes the advantages and disadvantages of their present techniques and gave recommendations on how to correct some of their technique problems. However, such studies were essentially observational in nature, and therefore did not represent true experimental studies.

Dapena stated that the basic research stage started with a critical examination of the available literature. The studies of Ecker (1976) and Labescat (1972) were concerned with the achievement of a clearer understanding of the Fosbury flop. These studies were cinematographic in nature and generally had little or no quantitative information to support the theories proposed. In order to derive conclusions concerning the mechanics of the high jump a cinematographic study must be used in conjunction with a mechanical analysis.

Van Gheluwe and van Doninck (1979) recognised that while much had been written and speculated about the flop and straddle styles, there was very little objective comparative data on the mechanical characteristics of both styles. They subsequently carried out a comparative cinematographic study of the flop and straddle techniques. However no final conclusions were reported on the relative merits of each technique, with the number of subjects regarded as being too low to produce vital statistics.

Hunter (1974) also carried out a comparative mechanical analysis of the flop and straddle styles of jumping. Hunter questioned whether the flop was a more efficient method of high jumping, or merely a style that was more suited to certain types of jumpers. With regard to the approach phase, Hunter proposed that the greater potential approach velocity in the flop would result in a greater vertical velocity at takeoff, and so a
greater jump height. In considering the takeoff phase, Hunter (1974) concluded that the initiation of body rotation is more viably obtained in the flop than in the straddle, supporting the work of Ecker (1976). Hunter also supported the work of Ecker with regard to the mechanical advantage of the flop style in bar clearance.

An experimental study must possess control in order to distinguish itself from an observational study. However, controlled experiments impose constraints on the high jumper. If these constraints affect performance of the high jump, then the results obtained may not be applicable to a free movement. In conducting an experimental study one of the first considerations must be the type of data to be collected.

Hay (1985) used the terms cross-sectional and longitudinal to represent inter- and intra-individual studies respectively. The inter-individual approach may be used to collect data on a single jump by a number of athletes. This approach has been used in a large number of research investigations in high jumping and may be regarded as the standard research design in the field. Bruggemann and Susanka (1988) carried out a detailed kinematic analysis of the male and female finalists at the 1987 World Championships in Rome. Similarly Dapena (1988) obtained kinematic data on the top six male and female high jumpers at the 1987 Indoor World Championships. The aim of such studies was to evaluate the advantages and disadvantages of the present techniques of the athletes and also to suggest some corrective measures for technical problems. However these studies typically analysed only a single jump by each athlete. It may be argued that it would be difficult to propose corrective measures for individual athletes based on the kinematics of a single jump.

In contrast to the inter-individual approach, the intra-individual approach may be used to obtain data on multiple performances by a single athlete. This may give insight into which characteristics of the individual's technique most strongly influence success in the high jump. Suggestions may then be made as to how the athlete may improve personal performance. The intra-individual approach has been used more sparingly than the inter-individual approach in the previous research. Hay (1973b) analysed seven jumps by Pat Matzdorf at a single competition. The aim was to determine selected kinematic characteristics of the technique, and to compare the data obtained with data available on other elite high jumpers. Proposals were then made as to how Matzdorf may improve his own performance. However, proposals based on the techniques of other athletes fail to consider that technique is subject specific. The technique of the champion, for example, will not necessarily be the optimum technique for all athletes.

The intra-individual approach can only be used to obtain a fundamental understanding of technique if a large enough number of trials is accumulated. However, in a typical high jump competition each athlete may only perform around six jumps. Thus to obtain an adequate number of trials data may be amassed over several meets, a method suggested by Hay (1985). However, the physical conditioning and technical
development of the athlete will change during the competitive season. Therefore, collecting data over a series of competitions may result in clusters of jumps distinguished by changes in the conditioning of the athlete.

In addition, if an athlete is very consistent with respect to a given factor, the results of an intra-individual analysis of this athlete's performances are unlikely to reveal it as an important determinant of success. In this case intervention is required to effect a change in the performance factor under consideration. This is generally not possible during a competition. However, in the training environment more control may be exerted by the researcher. This may increase the range and amount of data that may be collected. In addition direct intervention may be used to effect a change in approach speed for example. The resulting influence on jump height may subsequently give some insight into the optimum approach speed for that athlete.

Due to the limited amount and range of data that may be obtained on a single athlete during a competition, previous research has been dominated by inter-individual studies. Dapena et al. (1990) obtained data on 77 elite high jumpers at various competitions. These data were used to carry out a regression analysis of high jumping technique and to diagnose jumpers using techniques that were below the potential of the athlete. This was one step towards optimisation, although the method used could not determine which of several corrective actions or which combination of them would be most effective for a given individual using a 'weak' technique. Also no optimum values were proposed for individual athletes. Dapena stated that it was difficult to ascertain the optimum values of the horizontal approach speed, the height of the mass centre at the end of the approach, and the activeness of the arms during the takeoff for an individual high jumper. This difficulty was due to the dependence of these factors on the individual's ability to resist buckling of the takeoff leg.

Many coaches advocate a fast and low approach to the bar. Ae et al. (1986) analysed the best jump of five athletes at a single competition in order to examine the preparatory motions for the high jump takeoff. The authors suggested that flexion at the knee and ankle joints during the takeoff phase serves to lower the athlete's mass centre. However, large flexion at either joint will result in a greater absorption of the approach speed. Considering that the optimum approach may be fast and low, Dapena (1980b) and Dyson (1986) suggested that lowering the mass centre without excessive knee flexion would prevent the loss of approach speed. However, no values were presented in the literature for parameters such as the optimal knee flexion at takeoff.

The length of the final approach stride may have a large influence on the above concepts. A long final stride may result in the mass centre dropping into the plant. Any downward movement of the mass centre at touchdown must first be overcome before positive upward vertical velocity is generated. A short final stride may maintain approach speed and catch the mass centre on the rise from the previous stride. However, a longer
final stride may result in a leg plant angle that is closer to the horizontal. Alexander (1990) proposed that this would prolong ground contact and so increase the vertical range of motion of the mass centre. Dapena (1988) proposed that a large vertical range of motion during the takeoff phase maximises the time during which the vertical ground reaction force is exerted on the athlete. This is achieved by a low mass centre height at touchdown and a high mass centre at toe-off. Increasing the time during which the vertical ground reaction force is exerted on the athlete increases the vertical velocity of the mass centre at toe-off.

Ecker (1976) considered that the only way to increase the vertical velocity at toe-off was to increase the vertical ground reaction forces, by effective use of the arms and the free leg. From the low body position at touchdown the athlete can accelerate the free limbs upward throughout the takeoff such that all are high as the foot leaves the ground. With the free limbs moving upward and the takeoff foot pushing against the ground, the effect is a marked increase in the force applied against the ground. This causes, by reaction, an increased force exerted by the ground on the athlete, and subsequently an increase in the vertical velocity of the mass centre at takeoff. However little quantitative data was provided to substantiate the speculations regarding this aspect of the technique.

2.2.3 Summary of experimental studies

The majority of research into the mechanics of the high jump has taken the form of descriptive studies as opposed to experiments designed to answer specific questions. The descriptive study was acknowledged by Yeadon and Challis (1994) as a useful preliminary step in the process of scientific investigation. However, the descriptive study should lead to more carefully designed experiments. The majority of descriptive studies have provided quantitative data on high jumping performances by elite athletes. However, few studies have attempted to explain the mechanics of the technique.

A purely experimental approach has limitations. Obtaining a sufficiently large data set has been an inherent problem in experimental studies of the high jump. Yeadon and Challis proposed that it should be a priority to incorporate a theoretical element into research studies. The form of this theoretical element would be a mechanically based mathematical model. Such models and their application in theoretical studies will be discussed in the next section.

2.2.4 Theoretical studies

Computer simulation models have been applied to investigate technique in a variety of sports. However, the application of simulation models specific to the optimisation of high jumping technique has been limited. Preiss (1985) considered whether the idealised technique proposed by Hay (1973a) was in fact the ultimate high jump style. A two-dimensional six segment model of the athlete was developed, with the airborne motion
determined by initial values of the mass centre and the time functions of joint angles. The front jackknife body position over the bar was considered optimal. However, in order to reach or leave this position it was found that the athlete must pass some less-than-optimal positions at heights below the peak height. Preiss (1985) concluded that the exact position of the body at the peak of the jump was of lesser importance for the overall performance.

The Preiss model considered only the flight phase of the high jump, and failed to consider the potential advantages of the Hay technique in raising the mass centre at takeoff, or increasing the vertical velocity of the mass centre at takeoff. This model was therefore limited in increasing the understanding of the mechanics of the high jump and provided no insight into how performance may be improved, and ultimately optimised for an individual athlete. Chen (1992) proposed that with the aid of computer simulation the individuality of the jumper could be described, weak points determined and optimal techniques developed.

The determination of the optimal Fosbury flop technique was considered by Hubbard and Trinkle (1985). A simple model for the jumper's body was used, modelling the high jumper as a single rigid rod moving only under the influence of gravity in a vertical plane. The authors attempted to show how the mechanical energy possessed by the athlete at the start of the takeoff phase should be transferred into rotation, and horizontal and vertical translation. It was assumed that the athlete had a fixed amount of initial energy, which during the takeoff was redistributed with equal facility into horizontal, vertical and rotational motion. The authors presented an expression for the height cleared above the ground given that any part of the body may dislodge the bar.

Hubbard and Trinkle concluded that the optimal flight trajectory always consisted of two brushes with the bar, near the centre of the rod so that the mass centre was above the bar at the peak of the flight. This model was clearly an over-simplification as during the bar clearance of the Fosbury flop the body is arched as opposed to straight and continuously changes its configuration so that all body segments may pass over the bar. This point was acknowledged by the authors who proposed that future work should apply similar techniques to a multi-segment model of the athlete.

As a result of the over-simplification in the structure of the model, Hubbard and Trinkle neglected many of the factors that may influence optimal Fosbury flop high jumping. Such factors include the angle of leg plant and the angle at the knee joint during the takeoff phase. The influence of the musculo-skeletal system on jumping performance was also neglected by Hubbard and Trinkle. The results presented therefore had little application in improving high jumping performance or understanding the mechanics of the optimum technique for an individual athlete.

Alexander (1990) aimed to identify the principles that govern optimum approach speed and leg plant angle in both the high jump and the long jump. A simple model was
used that considered the mechanical properties of the leg muscles. The model is shown in Figure 2.2.1. The model had a rigid trunk, and a leg formed from two rigid massless segments of equal length representing the thigh and the shank. The whole body mass was represented by a point mass located at the hip joint. A single knee extensor muscle was included, and this was assumed to be fully active throughout the period of ground contact. The control of body orientation during bar clearance was neglected by Alexander (1990), who was concerned only with finding the takeoff technique that optimised the flight trajectory of the mass centre. The optimisation function for high jumping was the peak height reached by the mass centre.

![Figure 2.2.1. The simulation model of Alexander (1990).](image)

Although outwardly simple in the number of components used Alexander's model was able to account for the relatively low approach speeds required for an optimum high jump takeoff. Alexander determined the approach velocity over the final stride and the leg plant angle that would maximise the height $h$ reached by the mass centre in high jumping. Alexander produced three-dimensional contour plots of height as a function of approach velocity $v$ and plant angle $\theta$, an example of which is shown in Figure 2.2.2. These diagrams were used to illustrate the optimum technique in terms of approach velocity and plant angle. In addition the influence of changing the approach velocity or plant angle value on jump height could be determined from the diagrams. Alexander was able to highlight the mechanical operation of the high jump takeoff and also provided some insight into the dependence on the mechanical properties of skeletal muscle.
Figure 2.2.2. A contour plot of jump height, approach velocity and plant angle.

The optimum values for approach speed and plant angle were for a given strength of the single torque generator. The representation of strength was an estimate however, and therefore the optimum technique obtained from the model was not specific to any athlete. Many of the muscle parameter values used by Alexander were based on previous literature, or chosen to obtain realistic jump height values. The torque generated was calculated as a function of the knee angular velocity. The maximum knee torque was chosen to obtain realistic ground forces. The chosen value was larger than the torques that act at the knees of athletes since it represented the muscular action at the hip, knee and ankle joints of the support leg in raising the mass centre. Despite the large maximum knee torque, the values obtained for approach speed and subsequent jump height were less than observed in real performances by elite athletes.

The Alexander model represented the total body mass as a point mass located at the hip joint. In reality the mass centre is located above the hip and its location relative to the hip changes throughout the takeoff phase. Furthermore, with the mass centre located at the hip the ground reaction force was assumed to act directly through the mass centre throughout the takeoff phase. In reality the line of action of this force is not through the mass centre during the takeoff phase since rotation is evident in high jumping. At touchdown the force acts behind the mass centre resulting in forward rotation. At toe-off this line of action will pass close to the mass centre.

2.2.5 Summary of theoretical studies

The application of computer simulation models specific to the optimisation of high jumping technique has been limited. In addition these models have typically been limited by their simplicity and therefore neglected many of the factors which may influence optimal high jumping. The results presented subsequently had little application in
improving high jumping performance or the understanding of the mechanics of optimum high jumping. Arguably the most comprehensive high jump specific computer simulation model to date was that of Alexander (1990). Although outwardly simple in the number of components used, Alexander's model was able to highlight the mechanical operation of the high jump takeoff and also provided insight into the dependence on the mechanical properties of muscle. However many assumptions were made with regard to the muscle parameters and the optimum technique was not evaluated against real performances.

Yeadon and Challis (1994) proposed that the development of a muscle-based computer simulation model which could be customised to an individual will permit the optimisation of performance. The authors further stated that the model should then be evaluated to establish whether the model predicts the behaviour of the system with acceptable accuracy. To date the evaluation of theoretical models with valid experimental data has been a common omission. This has limited the application of theoretical models in improving high jumping performance.

2.3 Techniques of investigation

2.3.1 Introduction

This section provides an insight into the component steps that were integral in the development and evaluation of a subject specific muscle driven simulation model of the high jump takeoff. Various techniques of investigation in the areas of film analysis, inertia parameters, muscle parameters and simulation models of human movement are considered and compared.

2.3.2 Techniques of cinematographic analysis

- Introduction

A quantitative mechanical analysis of human movement requires knowledge of the time histories of the joint angles that specify body configuration. In order to calculate the orientation angles of the body the spatial coordinates of the joint centres must be determined. In sports biomechanics manual digitising of film or video recordings has been the most common method for obtaining these data. The three-dimensional locations of points on the body from their recorded two-dimensional images must then be reconstructed. The rotation inherent in the high jump event moves the athlete out of a two-dimensional plane. A three-dimensional analysis is therefore required to accurately describe the motion. This section considers some of the various techniques that have been used for three-dimensional cinematography and the subsequent reconstruction of spatial coordinates.
Three-dimensional video and cinematography

The accurate identification of body landmarks during video analysis requires that the projected image be sufficiently large. Studies using fixed camera orientations require that the movement volume is small so that the image size is large throughout the movement space. If the activity occurs in a large movement volume the image may not be large enough for the accurate identification of body landmarks. A large image size may be maintained if the cameras are free to follow the activity. Yeadon (1989) used a three-dimensional reconstruction technique to study ski jumping in which the cameras were free to rotate about vertical and horizontal axes.

Studies using 16 mm film-based data collection systems generally had an advantage in terms of accuracy over video-based systems (Angulo and Dapena, 1992). The video-based systems offered lower spatial resolution since video-digitiser coordinates were limited to integral pixel values. However, Kerwin (1995) presented information on a high resolution video digitising system with enhanced colour rendition and measurement precision. Increased resolution was obtained using anti-aliasing techniques which allowed the cursor to traverse the captured video image in fractions of a pixel. In addition the image size could be increased by increasing the zoom on the captured video image.

In order to determine three-dimensional spatial coordinates within the movement volume, information must be obtained from the recorded image. This data reduction is often performed manually. The frame-by-frame identification and digitisation of selected body landmarks is however very time consuming. This has led to the development of automatic motion analysis systems which have the potential to greatly reduce the time required for analysis. However, the need for the attachment of markers to the athlete limits the applicability of such systems in competition for example.

Yeadon and Challis (1994) noted that since the markers are placed on the surface of the body segments, the subsequent analysis provides the locations of the surface placed markers. In contrast joint centre locations may be estimated using manual digitisation, and the locations of obscured points may be estimated. Automatic tracking systems have difficulty in identifying obscured markers. This becomes an increasing problem as the complexity of the movement is increased. Consequently, it is proposed that in the three-dimensional analysis of the high jump, manual digitising may be the best way to determine the three-dimensional spatial coordinates from the recorded video or film data.

During data collection and the subsequent analysis noise may be generated which influences the results obtained. Yeadon and Challis (1994) stated that the sampled signal may be considered to be a summation of the true signal, systematic noise and random noise. The incorrect identification of body landmarks is an example of the systematic noise that may be generated during the digitising process. Sources of systematic noise should be identified and removed wherever possible. Error propagation during numerical differentiation means that random noise must be reduced if the derivatives are to be close
to the true value. There are a number of techniques available for the reduction of noise so that an estimate of the true signal may be obtained. Wood (1982) considered that the need for an estimating function that can fit data with varying curvature has lead to the use of spline functions.

Spline functions are piecewise polynomials of low order n which are joined at knots to form a continuous function with n-1 continuous derivatives. The piecewise nature of spline functions allows them to adapt quickly to changes in curvature. The curvature in the experimental data is divided into sections, each of which begins and ends at an inflection point. Each section is fitted separately. An advantage of spline functions is that the experimental data need not be equispaced. Challis and Kerwin (1988) showed that the quintic spline gives greater accuracy in the first and second derivatives than other commonly used techniques. Yeadon (1990) applied repeated digitisation of points to determine the closeness of fit for a quintic spline.

- **Synchronisation techniques**
  
  In order to define the location of a given point in three dimensions it is necessary to first simultaneously obtain the two-dimensional image coordinates of that point from two or more camera views. Yeadon (1990) stated that synchronisation of the camera views may be achieved physically where the original data sets are obtained at the same times, or analytically where time-matched data sets are interpolated from the original data. Physical synchronisation of the recorded video images may be achieved by genlocking the video cameras, which simultaneously opens the shutter of each genlocked camera so that the exposures made with each camera are simultaneous. However, synchronisation is also possible by analytical means if environmental constraints exist so that the cameras may not be genlocked.

  Yeadon and Challis (1994) cited two unpublished doctoral dissertations which interpolated separately recorded data sets over the same time base. Walton (1981) used a timer in the field of view of two cameras to establish the times for each frame from both cameras. The data from the faster camera was then linearly interpolated over the time base of the slower camera. Dapena (1979) also used a timing device to establish the start and end times of the activity from both cameras. The framing rate between these two points was assumed to be constant, giving frame times for each camera.

  During many activities it is not possible to place a timing device in the view of each camera. Yeadon (1989) used the actual digitised film data to synchronise camera views of ski jumping by determining the time of a particular field in the time scale of the other camera. Similarly, digitised film data of the high jump may be synchronised by identifying the instances of foot contact. The accuracy of the synchronisation using the digitised video data is dependent on the accuracy of the three-dimensional reconstruction and camera calibration. Inaccurate estimates of each camera location may subsequently
result in poor synchronisation. Accuracy is therefore required in the three-dimensional reconstruction procedure.

- **Three-dimensional coordinate reconstruction**

  The two-dimensional image coordinates from two synchronised recordings are used to reconstruct the key body landmarks in a three-dimensional reference frame. Direct linear transformation (DLT) is the most common reconstruction technique used in the analysis of sporting activities. Abdel-Aziz and Karara (1971) stated that the transformation from comparator coordinates into object space coordinates was conventionally performed in two stages; a transformation from comparator-to-image coordinates, and from image-to-object space coordinates. The DLT method is a simultaneous solution, directly transforming comparator coordinates into object space coordinates.

  The DLT arises as a result of the collinearity condition shown in Figure 2.3.1. Yeadon (1996) stated that for an ideal camera-digitiser lens system the digitised and spatial coordinates of points are related by the collinearity condition. That is, that a point P in space, the centre C of the lens and the image I are collinear.

  ![Figure 2.3.1. The collinearity condition.](image)

  Van Gheluwe (1978) stated that the DLT procedure permits arbitrary camera locations so that it can be used in field work, such as recording during a high jump competition where there may be restrictions on camera locations. However, control points with known locations must be distributed throughout the movement volume in order to calibrate the movement volume (Abdel-Aziz and Karara, 1971). In the analysis of sporting activities the movement volume is often large. This presents difficulties in constructing such a large calibration structure. However, if a small calibration frame is used the reconstruction of points lying outside of the calibration volume will be
inaccurate. The design of the calibration frame may also produce problems. Yeadon and Challis (1994) stated that the control points must be distributed evenly throughout the calibration volume since this distribution influences the reconstruction accuracy.

Figure 2.3.2 shows a representation of the "Sputnik" calibration frame used to calibrate the movement volume in a study by Angulo and Dapena (1992). The advantages of this calibration frame are that it is easily constructed and portable. However, the calibration frame of the Sputnik contains a disproportionately large number of control points close to the central hub. This clustering results in an increased reconstruction accuracy in this region, but a less accurate reconstruction away from the central hub.

![Figure 2.3.2. A "Sputnik" type calibration frame for three-dimensional reconstruction.](image)

In contrast to the Sputnik's large number of centrally clustered control points it may be advantageous to determine the location of fewer points which are more evenly distributed throughout the movement volume. The calibration frame must comprise sufficient control points in order to fully determine the complete set of DLT parameters. In order to determine each of the 11 parameters that define the DLT and calibrate a camera-digitiser system, the locations of at least six digitised control points must be known. This results in at least 12 equations since each control point is defined by a pair of digitised coordinates (u, v). This overdetermined system is sufficient to determine the 11 unknown DLT parameters using a least squares formulation.

Solution of the 11 DLT parameters allows the determination of the location and orientation of the camera. The camera location is defined by the three coordinates \( (x_c, y_c, z_c) \) which describe the principal point of the lens. The camera orientation is defined by three angles describing the pan, roll and tilt of the camera. Solution of the 11 DLT
parameters also allows the researcher to establish how the digitiser coordinates may be transformed into the image coordinates through translation, scaling and shear. The image centre is described by two coordinates ($u_0, v_0$), there are two scale factors ($f_u, f_v$) related to the focal lengths and a shear factor ($k$) which is typically zero.

If the digitised coordinates of a point in space from two camera views are known, then the three-dimensional coordinates may be determined. Four equations are generated to describe each of the two coordinates from each of the two cameras. These four equations can be used to determine the unknown coordinates of the point in three dimensions. A least squares formulation is applied to these four equations to determine the three-dimensional coordinates of the point closest to the four planes defined by each of the equations.

### 2.3.3 Determination of segmental inertia parameters

**Introduction**

Inertia may be defined as the reluctance of a body to alter its state of motion. Moment of inertia describes the distribution of mass with respect to a particular axis, and quantifies the resistance of a body to rotational motion. Mass distribution is characterised with reference to three orthogonal principal axes of rotation. Human motion is based on the rotation of body segments about one another, and therefore the distribution of mass in a segment is as important as the absolute segment mass.

Quantitative mechanical analyses of human movement using simulation models require estimates of the inertia parameters of the body segments. For each segment values may be determined for the mass, distance of the mass centre from each joint and moments of inertia. This section considers both experimental and theoretical methods that have been used to determine segmental inertia parameters.

**Experimental techniques**

Traditional experimental methods involved the direct measurement of inertia parameters using dissected cadavers. Dempster (1955) estimated segment mass centre using a balance plate and moments of inertia using a compound pendulum technique in 8 male subjects. However, Dempster only considered parameters about the transverse axis. Clauser et al. (1969) used a total of 73 measured anthropometric variables in a step-wise regression to determine the best predictor equations for segment mass and mass centre location in 13 male subjects. However this study failed to measure segmental moments of inertia. Chandler et al. (1975) did measure the segmental moments of inertia of six embalmed cadavers from male subjects. Linear regression equations were used to predict principal moments of inertia from segment volume and body mass.

The inertia parameter set derived from cadaver studies may only provide accurate results within the particular cadaver sample used. Details on the sample populations for
each of the above cadaver studies are shown in Table 2.3.1. The cadavers were derived from an elderly male caucasian population and are therefore not representative of other sample groups, such as elite athletes. The cadaver studies were further limited by the small sample sizes. Furthermore, variations in measurement procedures and sectioning of segments between studies limited the possibility of combining samples or comparing results across studies.

Also shown in Table 2.3.1 are the characteristics of the sample population used by Zatsiorsky and Seluyanov (1983). More recently developed imaging and scanning procedures were used to determine body segment parameters *in situ*. The Gamma Mass Scanning technique was used to determine various body segment properties from the measured intensity of a gamma-radiation beam before and after it is passed through a segment. Zatsiorsky and Seluyanov performed this procedure on 100 subjects and derived regression equations with which to estimate body segment parameters for other subjects. However, the collection of scanned data over the whole body is time consuming, limiting the applicability of such procedures.

<table>
<thead>
<tr>
<th>Author</th>
<th>Subjects</th>
<th>Age [years]</th>
<th>Sex</th>
<th>Body Mass [kg]</th>
<th>Stature [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dempster</td>
<td>8 cadaver</td>
<td>52-83</td>
<td>M</td>
<td>49-72</td>
<td>1.59-1.86</td>
</tr>
<tr>
<td>Clauser et al.</td>
<td>13 cadaver</td>
<td>28-74</td>
<td>M</td>
<td>54-88</td>
<td>1.62-1.85</td>
</tr>
<tr>
<td>Chandler et al.</td>
<td>6 cadaver</td>
<td>45-65</td>
<td>M</td>
<td>51-89</td>
<td>1.64-1.81</td>
</tr>
<tr>
<td>Zatsiorsky and Seluyanov</td>
<td>100 live</td>
<td>19-35</td>
<td>M</td>
<td>55-91</td>
<td>1.68-1.80</td>
</tr>
</tbody>
</table>

Direct experimental techniques have also been used to determine segment inertia parameters of living subjects *in situ*. Segment volumes have been determined by the process of water immersion (Dempster, 1955) using the Archimedes principle. Mass centre locations have been determined using reaction board measurements (Hay, 1973c) and moments of inertia have been obtained using oscillation techniques (Hatze, 1975). However, such methods do not permit access to the complete range of axes and body segments, most commonly failing to determine the inertia of the central segments such as the torso.

Segment inertia parameters have also been determined from anthropometric measurements using regression analyses and scaling techniques (Hinrichs, 1985).
Yeadon and Morlock (1989) generated linear and non-linear equations for estimating segmental inertia parameters from anthropometric parameters. It was found that non-linear equations, based upon theoretical considerations, were most appropriate when the subject's anthropometric characteristics were outside the range of the sample. However, the derivation of regression equations based on the extrapolation of cadaver data limits their applicability.

• **Theoretical techniques**

  Mathematical modelling procedures have been used to represent the irregular body segment shapes with standard geometric forms. These geometric solids are capable of simple mathematical description, and their dimensions may be determined from anthropometric measurements of an individual. Yeadon, Challis and Ng (1993) stated that in applications such as the analysis of individual sports performances, it is desirable to have a parameter set for the particular individual under study. Geometrical approximation allows subject specific inertia data to be derived by mathematical procedures. The application of mathematical inertia models in computer simulation studies enables the determination of segmental inertia parameters for various body configurations. One of the first to use this approach was Hanavan (1964). The Hanavan model, shown in Figure 2.3.3(a), divided the body into 15 simple geometric solids of uniform density.

![Figure 2.3.3. The mathematical inertia models of Hanavan (1964) and Hatze (1980).](image)

The main advantage of the Hanavan model was that it required only 25 simple anthropometric measurements to personalise the model and to predict the mass centre location and moment of inertia for each segment. However, the accuracy was limited by
three assumptions typically used in modelling body segments. Segments were assumed to be rigid, with distinct boundaries and uniform density. In reality there may be substantial displacement of the soft tissue during movement, segmental boundaries are not distinct and the density varies between and within segments. Ackland et al. (1988) observed variations in cross-sectional density throughout the length of the leg segment. However, the errors arising from the assumption of uniform density were shown to be minor in comparison with the errors resulting from the inaccurate estimations of segment volumes.

Figure 2.3.3(b) shows the more detailed human body model of Hatze (1980). Hatze's hominoid consisted of 17 body segments, each subdivided into small mass elements of different geometrical structures. This allowed for greater detail in modelling the fluctuations in segmental density and shape. Furthermore, no assumptions were made with regard to bilateral symmetry of segments. The Hatze model provided accurate estimates of volume, mass, mass centre location, and the moments of inertia for the identified body segments. However, while the model is applicable to many different subject types in terms of body shape for example, the acquisition of the 242 anthropometric measurements required for input to the model is very time consuming. This may prove to be a major limitation when working with elite athletes.

Anthropometric measurements may be calculated from the digitisation of photographic images (Jensen, 1978), or taken directly from a subject (Yeadon, 1990). The anthropometric measurements allow the determination of segmental volumes. Using the calculated volumes and previously reported density values (Dempster, 1955) the mass and moments of inertia of body segments can be determined. Figure 2.3.4 shows the Jensen model which sectioned 16 body segments into elliptical zones 20 mm thick.

Figure 2.3.4. The mathematical inertia model of Jensen (1978).
In the Jensen (1978) model the dimensions of each elliptical zone were obtained by digitising photographic records of the side and front view of the subject. The volume of each zone was calculated geometrically, with the segmental volume expressed as the summation of the zone volumes.

The mathematical model presented by Yeadon (1990) was also based upon simplifying assumptions. However, the effect of any potential systematic errors could be evaluated since the calculated inertia values were used as input to a 11 segment simulation model (Yeadon et al., 1990).

In the inertia model of Yeadon (1990) all body segments, with the exception of the cranium, were represented by a number of stadium solids or truncated cones. The cranium was modelled as a semi-ellipsoid of revolution. The body segments were sectioned into 40 solids. This segmentation was obtained using planes perpendicular to the longitudinal axes of the segments as shown in Figure 2.3.5. It was assumed that the solids comprising a segment shared a coincident longitudinal axis. In order to specify each of the 40 geometric solids a total of 95 anthropometric measurements were required. This method allows inertia parameter values to be obtained for body models of up to 20 segments. The method was evaluated through application to three subjects. The maximum error of the total body mass estimate was 2.3%.

In order to determine the mass and moments of inertia of body segments the segmental volumes calculated using mathematical inertia models are used in conjunction with previously reported density values (Dempster, 1955; Clauser et al., 1969; Chandler et al., 1975). The use of previously reported density values makes the majority of
mathematical inertia models reliant in part upon cadaver data. However this dependence is much less pronounced than in the experimental methods.

Mathematical models of the human body may be used to obtain a full set of segmental inertia parameters from anthropometric measurements of an individual. This method is more time-effective and of the methods available is unique in providing a convenient means of accurately determining a complete and personalised set of segmental inertia parameters for a given individual.

2.3.4 Determination of muscle parameters

• Introduction

Muscle models allow more specific questions regarding human movement to be addressed. However, this requires that the muscle model be a valid representation of the system. In addition the muscle parameters in the model should be subject specific, allowing the simulation model to be customised to an individual athlete and the accuracy of the model to be evaluated. This section reviews the methods by which subject specific muscle parameters may be obtained for input to a muscle driven simulation model.

• Isokinetic dynamometry

The mathematical modelling of the forces exerted by human skeletal muscles requires input of in-vivo muscle properties, such as force-velocity and length-tension relationships. These muscular forces may be quantified using the joint moments recorded by an isokinetic dynamometer. Muscles change length at varying velocities and with varying amounts of tension. An isokinetic dynamometer allows the measurement of muscle torque at a constant velocity of joint motion. Perrine and Edgerton (1978) proposed that an isokinetic dynamometer allows for a direct measurement of at least the lower-velocity portions of the force-velocity relationships of human muscles in-vivo.

Considerable attention has been given to the force-velocity relationship during knee extension under conditions of isokinetic loading. Froese and Houston (1985) noted that two major approaches have been adopted using isokinetic measuring devices for determining torque production during dynamic knee extension. One approach has been to determine the torque produced at a specified knee angle for multiple velocities.

Gregor et al. (1979) and Perrine and Edgerton (1978) recorded the maximal voluntary joint torques developed by the knee extensors at 30° from full extension at controlled velocities ranging from zero to 288°/s. A specific joint angle was used to minimise the effect of changing moment arms and muscle lengths on extensor torque production. It was proposed that only if velocity specific torque measurements were taken at a specific joint angle could the relative torque outputs be compared with force-velocity relationships in isolated and in-vivo preparations. However, this method does
not necessarily provide the maximum torque value for the knee extensors. The anglespecific torque measured at $30^\circ$ may be exceeded at different joint angles.

An alternative approach for determining torque production during knee extension has been to measure the peak torque generated at each testing velocity, without considering the knee angle at which this peak value occurs. Thorstensson et al. (1976) tested the dynamic strength of the knee extensor in 25 male subjects at preset velocities of 15, 30, 60, 90, and 180°/s. Peak torque was reached at knee angles between $55^\circ$ and $66^\circ$ from full extension, with a shift toward smaller angles at the higher velocities. The peak torque was found to decrease with increased velocity. This second method does provide a maximum torque value for the knee extensors since it is not angle-specific.

It has been well documented that the force capability of muscle is dependent on the length and the shortening velocity. Marshall et al. (1990) stated that while considerable research has reported separately length-tension and force-velocity data, limited research has considered the interaction between force, velocity and length. Marshall et al. suggested that three-dimensional surfaces, as shown in Figure 2.3.6, allowed for a more accurate interpretation of muscle function. The three-dimensional presentation also eliminates the methodological issue as to whether to record peak torque or angle-specific torque data.

![Figure 2.3.6. A three-dimensional surface plot of human in-vivo muscle function; adapted from Marshall et al. (1990).](image)

Herzog (1988) noted that the resultant joint moments have often been implicitly assumed to be equivalent to the recorded moments (Thorstensson et al., 1976; Perrine and Edgerton, 1978). However, the moments obtained from the dynamometer are not equivalent to the resultant joint moments. In order to calculate the resultant knee joint moment Herzog modelled a system of two rigid bodies: the dynamometer arm, and the
human shank-foot segment. An equation was derived to express the difference between the resultant joint moment and the moment recorded by the dynamometer. The reason for the difference was expressed due to gravitational effects, inertial effects, and the non-rigidity of the dynamometer arm and shank-foot segment.

The gravitational effects represent the error due to the weight of the dynamometer arm and the shank-foot segment. This error increases towards the end of the knee extension exercise where the smallest moments are obtained. The difference between the resultant joint moment and the dynamometer moment due to the weights of the lever arm and the shank-foot segment will therefore be largest toward the end of the knee extension, that is at the largest dynamometer angles. The inertial effects of the dynamometer arm and the shank-foot segment would be small since only the isokinetic part of the exercise was examined. The error associated with the non-rigidity of the dynamometer arm/shank-foot segment is due to relative angular movement between the mechanical arm and the human segment. In most instances Herzog observed that the moments recorded by the dynamometer underestimated the actual resultant joint moment.

Froese and Houston (1985) considered that the force-velocity relationships observed from isokinetic dynamometer experiments should not be expected to be purely demonstrated in the results of experiments on isolated muscle. There are many factors which contribute to force development during isokinetic exercise that do not affect the force-velocity relation in isolated muscle. There may be a contribution from subject motivation, and also a learning experience of exerting force on an isokinetic dynamometer. In addition intrinsic factors such as lever arm length and factors controlling muscle recruitment may vary.

Fuglevand (1987) investigated the torque-angular velocity and torque-joint angle relationships of the intact human system during knee extension. Fuglevand found that the joint angle at which peak torque occurred increased with increased angular velocity. This result is inconsistent with the length-tension relationship, which would predict a single muscle length of peak tension independent of shortening velocity. However, at the higher speeds the muscles in the intact system may not be fully activated throughout the movement.

The classic hyperbolic torque-velocity relationship observed in isolated muscle studies was evident only at the smaller joint angles. As the joint angle was increased the deviation from the classic hyperbolic relationship became more pronounced. Figure 2.3.7 shows the torque-velocity relationship for three different joint angles. Fuglevand suggested that this deviation was the result of interaction between muscle length and the torque-velocity relationship. However, no conclusions were made regarding the agents responsible for the deviation.
Perrine and Edgerton (1978) also observed a significant departure of the torque-velocity curve of intact human muscle from the hyperbolic relationship of isolated muscle. The in-vivo muscle relationship was observed to follow a curve similar to the isolated muscle hyperbola over the highest test velocities (192 - 288°/s). However, at approximately 192°/s the torque-velocity curve of intact human muscle was observed to depart from the isolated muscle hyperbola and follow a distinctly different curve. Below 192°/s the relationship showed a sharply diminishing rate of rise in torque production as velocities continued to decrease. On the low velocity end Perrine and Edgerton suggested that the departure of the in-vivo curve from the higher forces that would be expected from the hyperbolic relationship may be the result of a neural regulatory mechanism.

Wickiewicz et al. (1984) also observed a departure from in-vivo muscle force-velocity relationships and the hyperbolic relationship of isolated muscle at the higher forces and lower velocities. Again a neural inhibitory or disfacilitating mechanism was proposed to be responsible for the difference in response from isolated muscle preparations, limiting the maximum forces that could be produced under optimal stimulating conditions.

The hyperbolic force-velocity relationship of Hill (1970) for concentric muscular action in isolated muscle preparations has been used extensively. The previous discussion has shown that this classical relationship has not been observed in-vivo in many isokinetic studies. However, Edman (1988) also showed deviation from the classical force-velocity relationship using isolated muscle preparations. Edman found that this relationship was not a simple hyperbolic function in determining the force-velocity relation of frog striated muscle fibres. Edman observed two distinct curvatures on either side of a breakpoint located at approximately 78% of the isometric force as shown in Figure 2.3.8. At this point the velocity of shortening was approximately 10% of the maximum.
Figure 2.3.8. The bi-phasic force-velocity relationship observed by Edman (1988); adapted from King (1998).

The same bi-phasic shape of the force-velocity curve was evident at the same relative values of maximum isometric force and velocity of shortening when the isometric force was reduced to 80% of the control value. This finding suggested that the breakpoint evident in the force-velocity curve was related not to the force level but to the speed of shortening of the contractile system.

2.3.5 Simulation models of human movement

* Introduction

Vaughan (1984) stated that computer simulation refers to the use of a validated computer model to carry out 'experiments' under carefully controlled conditions, on the real-world system that has been modelled. Computer modelling was said to refer to the derivation of mathematical equations to describe the system of interest, the gathering of appropriate input data, and the incorporation of these equations and data into a computer program.

A theoretical model that can predict the behaviour of a system is a requisite element of scientific method. Yeadon and Challis (1994) stated that the strength of the theoretical approach is that it can go beyond answering questions such as "How is it done?" The authors proposed that the theoretical approach is ideally suited to answering "How should it be done?" Furthermore, it may also be possible to provide insight into how the system works.

Vaughan stated that the major advantage of the theoretical approach is the exploratory nature of computer simulation, that is the ability to answer the question "What if?" Vaughan also proposed four other advantages of the computer simulation approach over the experimental approach. The first was safety, since the athlete is
represented on the computer and is thus saved from having to perform potentially hazardous experiments. The second was time, since many different computer simulations can be performed in a matter of minutes whereas the corresponding experiments may take days to conduct. A third advantage was the saving in expense, with Vaughan comparing the cost of running various simulations with the need to produce various physical models. The fourth advantage was the potential for predicting optimal performance. The researcher is able to isolate and control selected variables independently, and is therefore able to highlight the technique which results in the best performance.

- Simulation models

Simulation models of human movement have been developed with varying complexity. However, the most fundamental understanding of human movement may come from the simplest models. Alexander (1992) stated that biomechanists often try to reproduce as much of the complexity of the human body as possible, in the belief that more is better. However, Yeadon and Challis (1994) noted that if the model is too simple then its area of applicability will be restricted. In a simple model care must be taken that important features are not omitted, so that the accuracy is not reduced. The application of biomechanical models may be subdivided into forward dynamics and inverse dynamics analyses as represented by simplified chains in Figure 2.3.9.

![Diagram showing forward dynamics and inverse dynamics](Figure 2.3.9. Schematic representations of a forward dynamics analysis or simulation and an inverse dynamics analysis.)

Yeadon and Challis (1994) stated that the inverse dynamics approach has useful applications in the field of sports injuries, while forward dynamics simulations have their use in theoretical studies of sports performance. Therefore a theoretical model employing forward dynamics would be required to investigate the mechanics of optimum high jumping technique. The application of theoretical muscle models to the high jump has been limited. Furthermore, the rare exceptions have typically been over simplified as discussed previously. Yeadon and Challis proposed that the development of a muscle-
based computer simulation model which may be customised to an individual will permit the optimisation of athletic performance.

• **Modelling muscles**

  During the high jump takeoff the ground exerts force on the athlete. In order to model the forces that can be exerted by the athlete a model of the high jump takeoff must incorporate muscles. A major limitation in the analysis of muscular action during athletic movements is that there are more muscles acting at a joint than there are degrees of freedom at the joint (Alexander and Vernon, 1975). Therefore for a known set of external forces acting at a joint it is not possible to determine individual muscle forces. Alexander (1990) used a simple model of jumping comprising a thigh and shank with a single extensor muscle located at the knee. The action of the single muscle at the knee was considered to be equivalent to that of all the muscles crossing the joint. The force-producing-capability of the muscle was based on the model of Hill (1970). The model was used to show the operation of the mechanics of high jumping technique and the dependence on the properties of skeletal muscle.

  Another feature of the muscle representation in the model of Alexander (1990) was the representation of muscle control. The control of the muscles was assumed to be bang-bang, so that the muscles were considered to be 'on' or 'off'. The 'on' state means that the muscle is maximally active, while the 'off' state means that the muscle is producing no force. The transition between the two extreme states is immediate. In reality the muscle control during human movement is not bang-bang. However, Yeadon and Challis (1994) suggested that for dynamic activities such as the high jump takeoff finer control may not be required. The authors further stated that the advantage of bang-bang control is that it reduces the number of parameters required to describe muscle activation, and therefore the complexity of the model. This type of control may therefore be a useful model simplification. Yeadon and Challis stated that the simplest valid model should be used to model muscles, with as few parameters describing the model as possible. Furthermore it was stated that these parameters must be measurable using reasonable tests on the experimental subjects.

• **Optimisation**

  A simulation model may be used to determine the outcome of a given set of conditions. It is therefore possible to determine the conditions which result in the best or optimum performance. Alexander (1990) optimised the approach velocity and the plant angle to maximise the height reached by the mass centre in high jumping. Contour maps were subsequently produced of height as a function of approach velocity and plant angle. Yeadon and Challis (1994) stated that this allowed an overview of the solution space as well as locating an optimum.
The process of optimisation may be described as the search for the global maximum or minimum of a function which is dependent on one or more independent variables. The global maximum represents the real highest function value. However, local maxima may exist which represent the highest function value in the vicinity of the global maximum. Optimisation requires the evaluation of numerous functions and the manipulation of the function variables so that the global maximum or minimum of the function is obtained.

Alexander (1990) optimised only two parameters; approach velocity and plant angle. Hubbard (1984) stated that as the number of parameters to be optimised is increased so the number of simulations required to determine a global maximum increases dramatically. Mathematical techniques may then be used to locate an optimum solution. Yeadon and Challis (1994) stated that these optimisation methods typically iterate to a solution by using a gradient estimate to move 'uphill' at each step. To avoid locating only a local maximum the optimisation should be started from a number of different positions so that a number of local maxima are obtained. If the process is repeated sufficiently the highest valued maxima may be accepted as the global maxima.

The simulated annealing process uses the Metropolis algorithm to continuously search for the global maximum. Goffe et al. (1994) compared the simulated annealing algorithm with other minimisation algorithms and found it to be superior in its ability to determine global optima. An advantage of the simulated annealing algorithm is its ability to accept both 'uphill' and 'downhill' moves throughout the optimisation.

Yeadon and Challis stated that having determined an optimum a sensitivity analysis should be carried out to investigate the effect of uncertainties in the values assigned to the model parameters not included in the optimisation procedure. If performance is not particularly sensitive to changes in these parameter values then techniques close to the optimum technique will still produce good performances. However, if the optimum technique is very sensitivity to changes in the model parameters then even slight deviations from the optimum technique will cause marked decreases in performance.

• Evaluation

Yeadon and Challis proposed that theoretical studies should be combined with an experimental approach to gain most insight into the system. They proposed that sports biomechanics should be a balanced mix of experimental data and theoretical modelling in order to obtain a realistic understanding. Evaluation of the model requires a comparison of the predictions with experimental data. In order to draw conclusions about the mechanics of an activity, the model must be a valid representation of the system. Model evaluation is required to establish whether the model predicts the behaviour of the system with acceptable accuracy. The evaluation of theoretical models with valid experimental data has been a common omission in previous research. This has limited the application of theoretical models to improving sports performance.
2.4 Summary of review of literature

Both experimental and theoretical studies have been used to investigate the mechanics of the takeoff technique in the high jump. The majority of experimental studies have collected kinematic data on a single competition performance by multiple athletes. However, this method does not allow the determination of optimum technique for an individual athlete. The experimental approach has also been limited by the amount and range of data that is observed in competition. Yeadon and Challis (1994) proposed that a theoretical element in the form of a mechanically based mathematical model should be incorporated into research studies. The application of mathematical models specifically to the optimisation of high jumping performance has been limited. Furthermore, the rare exceptions have failed to determine the optimum technique for an individual athlete. Subsequently, while both experimental and theoretical studies have acknowledged the existence of an optimum takeoff technique, many questions remain unanswered regarding the optimum takeoff technique for an individual athlete.
CHAPTER 3

EXPERIMENTAL DETERMINATION OF OPTIMUM TECHNIQUE

3.1 Introduction

The previous experimental research into the mechanics of the high jump may be split into two main divisions; those studies analysing single jumps by multiple athletes, and those studies analysing multiple jumps by a single athlete. The single subject approach has been used more sparingly in the previous research. By obtaining data on multiple trials by a single athlete it may be possible to determine which elements of the individual's technique most strongly influence success. Proposals may then be made as to how the individual may improve personal performance. As the technique is improved so it becomes closer to the optimum technique, i.e. the technique which will result in the best possible performance for a given individual.

Much speculation has concerned the optimum high jumping technique. Alexander (1990) used a theoretical simulation model of the high jump takeoff to show that the best jump heights are obtained at intermediate values of approach speed and leg plant angle. Optimal values for such approach parameters and so an optimum takeoff technique must therefore exist for an individual high jumper. However, Alexander did not attempt to predict the optimum technique for an individual athlete. The aim of the present study was to show how selected approach parameters influence jump height performance, and to determine the optimum technique for a single athlete. The intra-individual approach will be used, whereby data are obtained on multiple performances by the same athlete.

3.2 Pilot study: A kinematic analysis of competitive high jumping

3.2.1 Introduction

It may be expected that high jumpers will perform differently in the competitive environment than they do in training. However, the number of jumps performed by a single high jumper in competition may be limited to around six and so quantitative analyses of data collected in competition will often be limited by the amount of data available. The present study aimed to determine the optimum approach speed and leg plant angle for two elite male high jumpers, Brendan Reilly and Steve Smith. The optimum technique was defined as that which resulted in the greatest jump height. Jump height was defined as the peak height reached by the mass centre, irrespective of its location relative to the bar. Kinematic data were collected over three competitions, a method proposed by Hay (1985) as a means to amass multiple trials by a single athlete.
3.2.2 Data collection

Data were collected over three competitions (Tan, 1993). In each case a Panasonic F15 video camera with a Panasonic AG-5745 video recorder, and a Panasonic MS2 (SVHS) video camera were used to record all the jumps performed by two elite male athletes. The cameras recorded the activity at 50 fields per second, with a shutter speed of 1/250.

At each competition the camera locations were fixed. The field of view from each camera included the final three strides of the high jump approach and the entire flight phase. A similar camera placement was used for each competition, with the cameras positioned on the perimeter of the track so as not to obstruct any athletes. The approximate positions of the video cameras relative to the high jump uprights are shown in Figure 3.2.1. The optical axes of the two video cameras were approximately perpendicular.

![Figure 3.2.1. Camera positions relative to the high jump area.](image)

Markers were placed at known locations on the high jump uprights and six accurately constructed calibration posts placed in the field of view of both video cameras at precisely measured locations relative to the high jump uprights. The calibration system is shown in Figure 3.2.2. The placement and the set-up of the calibration system was carried out with great care so as to ensure that accurate three-dimensional coordinate data were obtained from the subsequent Direct Linear Transformation (DLT) procedure. The area of activity with an unobstructed view of the high jump uprights and the calibration poles was recorded by both cameras prior to the start of each competition so that the movement area and the camera locations could be calibrated.
KEY:
- Calibration posts

Markers at \( z = 0.13, 1.13 \) m

Upright markers at \( z = 0.10, 2.40 \) m

Figure 3.2.2. The calibration set-up relative to the high jump uprights.

3.2.3 Data analysis

- Digitising procedure

The video images of the recordings from both cameras were viewed prior to any digitising. This ensured that sequences of sufficient quality for analysis were identified. The calibration procedure involved digitising both markers on each of the six calibration posts and both markers on each of the high jump uprights, giving a total of 16 digitised points. This procedure was repeated for five consecutive video fields where there was an unobstructed view of the calibration structure and the high jump uprights. The calibration procedure was carried out for both camera views to determine the three-dimensional coordinates of each digitised point and so calibrate the movement volume.

For each jump analysed two consecutive video fields were selected before the athlete came into the camera view. For each of these reference fields the four upright markers and both ends of the bar were digitised twice to check the accuracy of reconstruction and to determine the bar height. The repeated digitising of points provided a check for any camera movement that may have occurred between jumps.

The third field digitised for each jump was identified as two fields prior to the fourth last foot contact. Digitising then continued to include the final three approach strides, the takeoff phase, and the flight phase of the jump until the field prior to contact with the high jump mat. Every other video field was digitised giving a data sampling rate of 25 Hz. For each digitised field 15 body landmarks were identified; the wrist, elbow, shoulder, hip, knee, ankle and toe of the left and right sides, and the centre of the head. The two camera views were synchronised by determining the offset between the field numbers at which touchdown and toe-off occurred for each of the four foot contacts.
• **Calculation of performance variables**

Direct linear transformation (Yeadon, 1990a) was used to convert the two-dimensional digitised coordinates from each camera view into three-dimensional coordinate locations in real space for each digitised point. Personalised segmental length and mass ratios were calculated using the mathematical inertia model of Yeadon (1990b). Moments were calculated for each segmental mass about three orthogonal axes, allowing the calculation of the whole body mass centre location. Internal body angles were calculated using the known locations of joint centres from the three-dimensional reconstruction (Yeadon, 1990a).

In the present study three performance variables were identified. The jump height \( h \) was defined as the peak height reached by the athlete's mass centre. This peak height was identified irrespective of its location relative to the high jump bar, since the present study was concerned only with the influence of selected approach parameters on the athlete's ability to raise the mass centre. The approach speed \( v \) was defined as the mean horizontal velocity of the athlete's mass centre over the final stride of the approach. The leg plant angle \( \phi \) was defined at the instant of touchdown for the takeoff phase as the angle between the vertical and a straight line joining the ankle and hip joints of the support leg.

### 3.2.4 Results

A summary of the jumps analysed and the values recorded for each performance variable are presented in Tables 3.2.1 and 3.2.2 for Reilly and Smith respectively. The jump code indicates at which competition each jump was performed.

<table>
<thead>
<tr>
<th>Jump</th>
<th>Bar height [m]</th>
<th>Approach speed [m.s(^{-1})]</th>
<th>Plant angle [°]</th>
<th>Jump height [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B21</td>
<td>2.15 O</td>
<td>7.18</td>
<td>36.9°</td>
<td>2.22</td>
</tr>
<tr>
<td>B22</td>
<td>2.20 X</td>
<td>7.44</td>
<td>36.8°</td>
<td>2.35</td>
</tr>
<tr>
<td>B23</td>
<td>2.25 X</td>
<td>7.21</td>
<td>38.6°</td>
<td>2.30</td>
</tr>
<tr>
<td>B24</td>
<td>2.25 X</td>
<td>7.15</td>
<td>37.8°</td>
<td>2.33</td>
</tr>
<tr>
<td>B25</td>
<td>2.25 X</td>
<td>7.23</td>
<td>37.1°</td>
<td>2.28</td>
</tr>
<tr>
<td>B31</td>
<td>2.15 O</td>
<td>7.22</td>
<td>39.9°</td>
<td>2.20</td>
</tr>
<tr>
<td>B32</td>
<td>2.15 O</td>
<td>7.14</td>
<td>41.3°</td>
<td>2.22</td>
</tr>
<tr>
<td>B33</td>
<td>2.20 O</td>
<td>6.95</td>
<td>38.2°</td>
<td>2.25</td>
</tr>
<tr>
<td>B34</td>
<td>2.25 X</td>
<td>7.29</td>
<td>38.7°</td>
<td>2.25</td>
</tr>
<tr>
<td>B35</td>
<td>2.25 X</td>
<td>7.25</td>
<td>37.2°</td>
<td>2.19</td>
</tr>
</tbody>
</table>

Table 3.2.1. Record of performance for Reilly
Table 3.2.2. Record of performance for Smith

<table>
<thead>
<tr>
<th>Jump</th>
<th>Bar height [m]</th>
<th>Approach speed [m.s⁻¹]</th>
<th>Plant angle [°]</th>
<th>Jump height [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S21</td>
<td>2.25</td>
<td>7.35</td>
<td>36.7°</td>
<td>2.27</td>
</tr>
<tr>
<td>S22</td>
<td>2.28</td>
<td>7.36</td>
<td>35.4°</td>
<td>2.26</td>
</tr>
<tr>
<td>S23</td>
<td>2.28</td>
<td>7.43</td>
<td>33.3°</td>
<td>2.33</td>
</tr>
<tr>
<td>S24</td>
<td>2.31</td>
<td>7.51</td>
<td>37.1°</td>
<td>2.32</td>
</tr>
<tr>
<td>S31</td>
<td>2.15</td>
<td>7.36</td>
<td>35.3°</td>
<td>2.21</td>
</tr>
<tr>
<td>S32</td>
<td>2.25</td>
<td>7.47</td>
<td>35.7°</td>
<td>2.20</td>
</tr>
<tr>
<td>S33</td>
<td>2.25</td>
<td>7.33</td>
<td>37.2°</td>
<td>2.24</td>
</tr>
<tr>
<td>S34</td>
<td>2.25</td>
<td>7.57</td>
<td>38.4°</td>
<td>2.21</td>
</tr>
<tr>
<td>S41</td>
<td>2.20</td>
<td>7.42</td>
<td>36.3°</td>
<td>2.25</td>
</tr>
<tr>
<td>S42</td>
<td>2.26</td>
<td>7.44</td>
<td>38.7°</td>
<td>2.27</td>
</tr>
<tr>
<td>S43</td>
<td>2.30</td>
<td>7.69</td>
<td>39.6°</td>
<td>2.33</td>
</tr>
<tr>
<td>S44</td>
<td>2.34</td>
<td>7.53</td>
<td>41.0°</td>
<td>2.24</td>
</tr>
<tr>
<td>S45</td>
<td>2.34</td>
<td>7.68</td>
<td>40.6°</td>
<td>2.31</td>
</tr>
<tr>
<td>S46</td>
<td>2.34</td>
<td>7.62</td>
<td>39.4°</td>
<td>2.24</td>
</tr>
</tbody>
</table>

Jump code:

B3  Brendan Reilly 1993 Loughborough University
S2  Steve Smith    1992 U.K. A.A.A. Championships
S3  Steve Smith    1993 Loughborough University
S4  Steve Smith    1993 U.K. A.A.A. Championships

Tables 3.2.3 and 3.2.4 show the ranges observed in the jump height h, the approach speed v and the plant angle φ for Reilly and Smith respectively. The minimum, maximum and mean values are presented for each approach variable. Also included is a measure of the standard deviation and the standard error about the mean value. The data are presented for all competitions treated as a single data set for each athlete and also for each separate competition for each athlete.
Table 3.2.3. A summary of the record of performance for Reilly

<table>
<thead>
<tr>
<th>Competition</th>
<th>min.</th>
<th>max.</th>
<th>range</th>
<th>mean</th>
<th>s.d.</th>
<th>s.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reilly h [m]</td>
<td>2.19</td>
<td>2.35</td>
<td>0.16</td>
<td>2.26</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>v [m.s⁻¹]</td>
<td>6.95</td>
<td>7.44</td>
<td>0.49</td>
<td>7.21</td>
<td>0.12</td>
<td>0.04</td>
</tr>
<tr>
<td>φ [°]</td>
<td>36.8</td>
<td>41.3</td>
<td>4.5</td>
<td>38.3</td>
<td>1.45</td>
<td>0.46</td>
</tr>
</tbody>
</table>

n = 10

| B2 h [m] | 2.22 | 2.35 | 0.13  | 2.30 | 0.05 | 0.02 |
| v [m.s⁻¹] | 7.15 | 7.44 | 0.29  | 7.24 | 0.11 | 0.05 |
| φ [°]        | 36.8 | 38.6 | 1.8   | 37.4 | 0.76 | 0.34 |

n = 5

| B3 h [m] | 2.19 | 2.25 | 0.06  | 2.22 | 0.03 | 0.01 |
| v [m.s⁻¹] | 6.95 | 7.29 | 0.34  | 7.17 | 0.13 | 0.06 |
| φ [°]        | 37.2 | 41.3 | 4.1   | 39.1 | 1.59 | 0.71 |

n = 5

Table 3.2.4. A summary of the record of performance for Smith

<table>
<thead>
<tr>
<th>Competition</th>
<th>min.</th>
<th>max.</th>
<th>range</th>
<th>mean</th>
<th>s.d.</th>
<th>s.e.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith h [m]</td>
<td>2.20</td>
<td>2.33</td>
<td>0.13</td>
<td>2.27</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>v [m.s⁻¹]</td>
<td>7.33</td>
<td>7.69</td>
<td>0.36</td>
<td>7.48</td>
<td>0.12</td>
<td>0.03</td>
</tr>
<tr>
<td>φ [°]</td>
<td>33.3</td>
<td>41.0</td>
<td>7.7</td>
<td>37.5</td>
<td>2.23</td>
<td>0.60</td>
</tr>
</tbody>
</table>

n = 14

| S2 h [m] | 2.26 | 2.33 | 0.07  | 2.30 | 0.04 | 0.02 |
| v [m.s⁻¹] | 7.35 | 7.51 | 0.16  | 7.41 | 0.07 | 0.04 |
| φ [°]        | 33.3 | 37.1 | 3.8   | 35.6 | 1.71 | 0.86 |

n = 4

| S3 h [m] | 2.20 | 2.24 | 0.04  | 2.22 | 0.02 | 0.01 |
| v [m.s⁻¹] | 7.33 | 7.57 | 0.24  | 7.43 | 0.11 | 0.05 |
| φ [°]        | 35.3 | 38.4 | 3.1   | 36.7 | 1.43 | 0.71 |

n = 4

| S4 h [m] | 2.24 | 2.33 | 0.09  | 2.27 | 0.04 | 0.02 |
| v [m.s⁻¹] | 7.42 | 7.69 | 0.27  | 7.56 | 0.12 | 0.05 |
| φ [°]        | 36.3 | 41.0 | 4.7   | 39.3 | 1.68 | 0.68 |

n = 6
Figure 3.2.3 shows the relationship between the approach speed \( v \) and the jump height \( h \) for Reilly. In total 10 jumps were analysed for Reilly. Each point in Figure 3.2.3 is labelled with the jump code. It is therefore possible to determine the plant angle corresponding to each jump from Table 3.2.1. The results of both linear and quadratic regression analyses of \( h \) against \( v \) for all 10 jumps follow Figure 3.2.3.

\[
\begin{align*}
  h &= 1.255 + 0.1393 \, v \\
  h &= 47.02 - 12.59 \, v + 0.8849 \, v^2
\end{align*}
\]

\[
\begin{align*}
  \text{s.e.} &= 0.055 \, m & r^2 &= 0.100 & p &= 0.374 \\
  p_c &= 0.273 & p_v &= 0.374 \\
  \text{s.e.} &= 0.053 \, m & r^2 &= 0.256 & p &= 0.355 \\
  p_c &= 0.253 & p_v &= 0.270 & p_{v^2} &= 0.265
\end{align*}
\]

where: \( \text{s.e.} \) = standard error in dependent variable
\( r^2 \) = coefficient of determination
\( p \) = significance level of regression equation
\( p_c \) = significance level of constant term
\( p_v \) = significance level of term in \( v \) (similarly for \( v^2, \phi, \phi^2 \))
Figure 3.2.4 shows the relationship between the approach speed \( v \) and the jump height \( h \) for Smith. In total 14 jumps were analysed for Smith. Each point in Figure 3.2.4 is labelled with the jump code so that the plant angle corresponding to each jump can be determined from Table 3.2.2. The results of both linear and quadratic regression analyses of \( h \) against \( v \) for all 14 jumps follow Figure 3.2.4.

![Figure 3.2.4. The relationship between approach speed and jump height for Smith.](image)

**Regression analyses:**

\[
\begin{align*}
  h &= 1.3588 + 0.1208 v \\
  s.e. &= 0.044 \\
  r^2 &= 0.108 \\
  p &= 0.252 \\
  p_c &= 0.096 \\
  p_v &= 0.252
\end{align*}
\]

In the quadratic regression \( v^2 \) was highly correlated with \( v \) and was removed from the equation.
Figure 3.2.5 shows the relationship between the leg plant angle $\phi$ and the jump height $h$ for Reilly. Each point in Figure 3.2.5 is labelled with the jump code so that the approach speed corresponding to each jump can be determined from Table 3.2.1. The results of both linear and quadratic regression analyses of $h$ against $\phi$ for all 10 jumps follow Figure 3.2.5.

Regression analyses:

\[
\begin{align*}
    h &= 2.8022 - 0.0142 \phi & \text{s.e.} &= 0.054 \text{ m} & r^2 &= 0.142 & p &= 0.283 \\
    h &= -0.61 + 0.1615 \phi - 0.002258 \phi^2 & \text{s.e.} &= 0.057 \text{ m} & r^2 &= 0.149 & p &= 0.569
\end{align*}
\]

$\rho < 0.001$  $\rho_\phi = 0.283$

$\rho = 0.967$  $\rho_\phi = 0.832$  $\rho_\phi^2 = 0.818$
Figure 3.2.6 shows the relationship between the leg plant angle $\phi$ and the jump height $h$ for Smith. Each point in Figure 3.2.6 is labelled with the jump code so that it is possible to determine the approach speed corresponding to each jump from Table 3.2.2. The results of both linear and quadratic regression analyses of $h$ against $\phi$ for all 14 jumps follow Figure 3.2.6.

Regression analyses:

\[
\begin{align*}
  h &= 2.2212 + 0.001113 \phi \\
  h &= 6.548 - 0.2309 \phi + 0.003099 \phi^2
\end{align*}
\]

- $s.e. = 0.046$ m $r^2 = 0.003$ $p = 0.850$ $\rho_{\phi} = 0.850$
- $s.e. = 0.045$ m $r^2 = 0.131$ $p = 0.461$ $\rho_{\phi} = 0.231$ $\rho_{\phi^2} = 0.229$
3.2.5 Discussion

The theoretical approach of Alexander (1990) allowed a wide range in the values of approach speed and plant angle to be considered. Alexander found that the best jump heights were obtained at intermediate values of approach speed and plant angle. Correspondingly, Nigg (1974) suggested that for an individual athlete the relationship between the velocity of the approach and the jump height would be as represented by the theoretical data points in Figure 3.2.7. The competition data points show how the data collected for a single athlete during a competition would be clustered near the peak of the theoretical curve. In competition it would be expected that the athlete is performing close to the optimum technique on every jump. Therefore only a small range in parameters such as the approach speed and plant angle would be expected during a competition.

![Figure 3.2.7. Competition data and the theoretical relationship between approach speed and jump height.](image)

In the present study all of the jumps analysed were performed by elite athletes in competition. Reilly performed 10 jumps over two competitions, and Smith performed 14 jumps over three competitions. Figures 3.2.3-3.2.6 show how the approach speed and plant angle influenced jump height for each athlete. These figures give an indication of the consistency of the technique used by both athletes. This consistency was generally evident for each competition and over all competitions for each athlete. Tables 3.2.3 and 3.2.4 show the small range in the observed values of approach speed and leg plant angle. Consequently the relationships between jump height and these performance variables showed little of the curvature presented by Alexander (1990). This lack of curvature was evident in Figures 3.2.3-3.2.6 and was supported by the results of the regression analyses.

Figure 3.2.3 showed the relationship between approach speed $v$ and jump height $h$ for the ten jumps by Reilly. The linear regression of $h$ against $v$ produced a small coefficient of determination ($r^2 = 0.100$), and a high $p$ value ($p = 0.374$). These results
show a lack of linear correlation between approach speed and jump height, and a lack of significance in the regression equation. The lack of linear correlation between approach speed and jump height may have been expected since the theoretical work of Alexander (1990) showed that it is not simply a case of the faster the approach the higher the jump.

The inclusion of a quadratic term in a regression analysis gives an indication of the amount of curvature evident in the relationship. A significant quadratic relationship allows the calculation of an optimum value. For the relationship between approach speed and jump height for Reilly the inclusion of a quadratic term $v^2$ did slightly increase the coefficient of determination ($r^2 = 0.256$) and the standard error was slightly reduced. However, the coefficient of determination was still small and the $p$ values for each of the coefficient terms in the quadratic equation were high. These results show a lack of quadratic correlation between approach speed and jump height for Reilly, and a lack of significance in the regression equation. Jump height was therefore found to be not significantly correlated to either a linear or quadratic function of the approach speed over ten jumps by Reilly. It was therefore not possible to determine an optimum approach speed for Reilly.

The relationship between jump height and approach speed for Reilly was also examined for each competition. Table 3.2.3 shows that a small range in approach speed was observed for each competition. There was no significant linear correlation for either competition. In the quadratic regression analyses $v^2$ was found to be highly correlated with $v$ and was subsequently removed from the regression equations. Jump height was therefore found to be not significantly correlated to a quadratic function of approach speed for either competition. It was therefore not possible to determine an optimum approach speed for either competition treated separately.

Figure 3.2.4 showed the relationship between approach speed $v$ and jump height $h$ for the fourteen jumps by Smith. The linear regression of $h$ against $v$ produced a small coefficient of determination ($r^2 = 0.108$), and a high $p$ value ($p = 0.252$). These results show a lack of linear correlation between approach speed and jump height, and a lack of significance in the regression equation. As previously stated the lack of linear correlation between approach speed and jump height may have been expected. However, when the quadratic term was included in the regression, the term in $v^2$ was found to be highly correlated with $v$ and was subsequently removed from the equation. Jump height was therefore found to be not significantly correlated to either a linear or quadratic function of the approach speed over fourteen jumps by Smith. It was therefore not possible to determine an optimum approach speed for Smith.

The relationship between jump height and approach speed for Smith was also examined for each competition. Table 3.2.4 shows that over the fourteen jumps performed by Smith a range of only 0.36 m.s$^{-1}$ was observed in the approach speed. The majority (0.27 m.s$^{-1}$) of this range was obtained from competition S4. For each
competition no significant linear correlation was observed between jump height and approach speed. In addition no quadratic relationship was evident in each case, with the term in $v^2$ being removed from the regression equation. It was therefore not possible to determine an optimum approach speed for either competition treated separately.

In Figure 3.2.5 the relationship between plant angle $\phi$ and jump height $h$ for Reilly was shown. The linear regression equation produced a low coefficient of determination ($r^2 = 0.142$) and a high $p$ value ($p = 0.283$). These results show a lack of linear correlation between plant angle and jump height, and a lack of significance in the regression equation. Alexander (1990) showed that the best jump heights were observed at intermediate values of leg plant angle, so that it is not simply a case of the greater the plant angle the higher the jump. The inclusion of a quadratic term $\phi^2$ did slightly increase the value of the coefficient of determination ($r^2 = 0.149$) although the strength of the correlation remained low. In addition the $p$ values for each of the coefficients in the regression equation were large, indicating a lack of correlation between jump height and a quadratic function of leg plant angle and a lack of significance in the regression equation. It was therefore not possible to determine an optimum leg plant angle for Reilly.

Table 3.2.3 showed that over the ten jumps a range of 4.5° was observed in the leg plant angle for Reilly. The majority (4.1°) of this total range was due to the competition coded B3. For each competition no significant linear or quadratic correlation was observed between jump height and leg plant angle. It was therefore not possible to determine an optimum leg plant angle for either competition treated separately.

The range in $\phi$ recorded for Smith over fourteen jumps was 7.7°. The relationship between $h$ and $\phi$ for Smith was shown in Figure 3.2.6. The linear regression produced no evidence of any significant linear correlation ($r^2 = 0.003$, $p = 0.850$). The inclusion of the quadratic term $\phi^2$ did slightly increase both the strength ($r^2 = 0.131$) and the significance ($p = 0.461$) of the relationship. However the value of the coefficient of determination was still small and the $p$ values of each of the coefficients in the regression equation remained large. These results indicate a lack of quadratic correlation between plant angle and jump height for Smith and a lack of significance in the regression equation. It was therefore not possible to accurately determine an optimum leg plant angle for Smith.

Each competition was treated separately to determine the optimum plant angle computed for each competition for Smith. For each competition no significant linear correlation was observed between jump height and approach speed. In addition no quadratic relationship was evident in each case. It was therefore not possible to determine an optimum leg plant angle for either competition treated separately.

No significant linear or quadratic correlations were observed between jump height and approach speed or leg plant angle for either athlete. This was true when treating each competition separately and when all jumps were grouped to form a single data set. It may be argued that the sample sizes were too small to obtain any statistically significant
correlations when treating each competition separately. The lack of significant correlations observed when all jumps were combined to form a single data set for each athlete may have been due to systematic differences between competitions.

The ten jumps by Reilly were performed over two competitions. Each competition was treated as a separate data group, each containing five jumps. The Student's $t$ test was used to determine whether the two competitions were significantly different in terms of the values recorded for the jump height, the approach speed and the leg plant angle. It was found that the recorded values for approach speed and leg plant angle were not significantly different between the two competitions ($p = 0.05$). However, the jump height values between the two competitions were significantly different ($p = 0.05$).

The fourteen jumps by Smith were performed over three competitions. Each competition was treated as a separate data group and analysis of variance was used to determine whether the values recorded for jump height, approach speed and plant angle were significantly different between competitions. Analysis of variance assumes normality of distribution and homogeneity of variance between groups. A Chi-squared test and a variance-ratio test were carried out to ensure that these assumptions were not violated. Approach speed was found to be not significantly different between competitions ($p = 0.05$). However the values recorded for leg plant angle and jump height were found to be significantly different between competitions ($p = 0.05$). A post hoc comparison using the Tukey honestly significant difference test showed that the significant ($p = 0.05$) difference in plant angle over the three competitions was due to the differences between the competitions coded S2 and S4. Similarly it was shown that the significant ($p = 0.05$) difference in the jump height values over the three competitions was due to the differences between competitions S2 and S3.

Over the fourteen jumps performed by Smith there was a linear relationship between approach speed and plant angle. Regressing $v$ against $\phi$ over all fourteen jumps produced the following equation:

$$v = 6.0385 + 0.03854 \phi$$

$$s.e. = 0.089 \text{ m.s}^{-1} \quad r^2 = 0.502 \quad p = 0.005$$

$$p_v < 0.001 \quad p_\phi = 0.005$$

The value of the coefficient of determination and the low $p$ value suggest a fairly strong and significant linear relationship so that as the approach speed is increased so there is a systematic increase in the leg plant angle. Although this relationship was not evident for Reilly it does highlight the potential limitations in examining the influence of selected approach parameters on jump height performance. If approach parameters are highly correlated with each other then they will not vary independently. This makes it difficult to isolate the effect of a single approach parameter on jump height performance.
3.2.6 Conclusions

It may be expected that during competition the athlete would perform close to the optimum technique on every jump. In the present study the observed range in approach speed and plant angle was small for each competition and also over all competitions for both elite athletes. There was subsequently little evidence of any curvature when examining the influence of approach speed and leg plant angle on jump height performance. Consequently it was not possible to determine optimum values for the approach speed or the plant angle for either athlete.

3.2.7 Future directions

• Data collection

To examine the influence of variables such as approach speed on jump height performance direct intervention is required to obtain a wider range of values than was observed in competition. An example of such intervention would be to vary the approach speed by changing the length of the approach run. This form of direct intervention is obviously not possible during a competition. However, in the training environment the biomechanist has more control over the data collection procedure. In addition, without the constraints imposed by the nature of competition a greater number of jumps may be performed by the athlete. A greater amount and range of data may therefore be collected if the jumps are performed in the training environment as opposed to during competition.

In the present study multiple trials for each subject were amassed over three competitions, a method suggested by Hay (1985). However, there are many variables which may change between competitions. For example, the athlete may be at varying levels of physical conditioning throughout the competitive season. This approach may then produce only clusters of jumps at varying levels of physical conditioning. In addition, no account could be made for injuries that may influence performance at one competition but not the next, or the motivation of the athlete which may vary depending on the status of the competition. Climatic conditions may also vary between competitions, so that jump height performance may be reduced at a certain competition. The pilot study showed that jump height performance varied significantly \( p = 0.05 \) between competitions for two elite high jumpers. For one of these athletes the values recorded for the leg plant angle also varied significantly \( p = 0.05 \) between competitions. Therefore treating jumps recorded over a series of competitions may produce serious logistical problems in the subsequent analysis. It is therefore suggested that the data collection should involve jumps performed in a single session.

• Data analysis

In the present study every other video field was digitised, with the video cameras recording the activity at 50 fields per second. Therefore coordinate data were obtained at
time intervals of 0.04 s. The takeoff phase lasts only about 0.16 s, so that only four fields of data would be analysed during the takeoff phase. Approach parameters such as the leg plant angle change rapidly during the takeoff phase and it is therefore essential that the data are obtained at the exact instant of plant. The accuracy in locating the instant of plant, and therefore in the recorded value of variables such as the plant angle would increase with a higher framing rate. It is therefore proposed that high speed cinematography be used in a kinematic analysis of the high jump takeoff. With an increased framing rate it may also be possible to determine the influence of the knee angle at plant on jump height performance.

3.3 Single subject multiple trial study

3.3.1 Introduction

Dapena (1987) stated that some elements of the high jump are advantageous and will therefore be common to all elite high jumpers. There are also likely to be elements unique to given individuals. The multiple subject single trial design has dominated research into the mechanics of the Fosbury flop. Dapena et al. (1990) stated that a data set consisting of many jumps by a single athlete would face serious methodological problems. The authors proposed that a high jumper cannot make many maximum effort jumps in a single day. However, no evidence was cited to support this proposal.

Smith (1988) stated that the case study refers to investigations of individuals, groups, or social systems in the absence of experimental control. Where individual subjects are exposed to specific and definable experimental conditions this is termed a single-subject experimental design. Here more experimental control is exercised and so single-subject experimental designs were proposed to provide less ambiguity in making causal inferences than uncontrolled case studies. The present study may therefore be described as a single subject multiple trial experimental design.

An analysis of the technique used in competition may be more useful since high jumpers may perform differently in competition than they do in training (Hay, 1985). The pilot study, however, showed that data collected in the competition environment is invariably limited by the amount and range of data that may be collected. The small range observed in approach parameters such as the approach speed makes it difficult to examine their influence on jump height performance. For example, if the approach speed is very consistent over a number of competition jumps then Hay stated that the results of an intra-individual analysis will be unlikely to reveal it as an important determinant of success. The pilot study also showed that jump height performance varied significantly between competitions. Amassing data over multiple competitions is therefore not a valid means of obtaining a large amount of data.
3.3.2 Experimental design

The present study was carried out in the training environment. The location chosen was familiar to the subject, Brendan Reilly, an elite high jumper. The athlete's coach was also present throughout the data collection session. A single session was filmed to ensure that the physical conditioning of the athlete was consistent for all of the jumps analysed. The single session also ensured that climatic conditions did not vary greatly, since this external variable might also have affected performance. In the training environment a complete record of the athlete's performance was ensured as data collection was free from the logistic issues present when filming at a competition. In addition the subject had complete freedom as to when he would jump, and direct intervention could be used to instruct the athlete regarding the length and speed of the approach.

The aim of the study was to induce a change in technique that would highlight the influence of approach speed, leg plant angle and knee angle on jump height. The approach speed was targeted as the performance variable which would be manipulated. It was expected that changes in the approach speed would induce changes in the leg plant angle and the knee angle at plant. It was decided through consultation with the coach and athlete that the athlete could perform a maximum effort Fosbury flop takeoff from a minimum of five strides.

The approach speed was varied by gradually increasing the length of the approach from five strides to the full length approach used by the athlete in competition. For a given number of approach strides the athlete was also instructed to run at a fast, medium or slow pace. In each case the aim for the athlete was to jump as high as possible. For each trial a high jump bar was included to give the athlete a realistic target and to make the task more familiar. The bar height was set by the athlete's coach according to the conditions of the approach. From a five stride approach the bar height was initially set at 2.00 m. The bar height then increased gradually to 2.20 m for the jumps performed from a full length approach.

3.3.3 Data collection

The data collection was carried out at the Loughborough University athletics track on 18 September 1995. Three cameras were used, whose approximate positions relative to the high jump uprights are shown in Figure 3.3.1. The left sided video camera (LVC in Figure 3.1.1) was a Panasonic F15 video camera with a Panasonic AG-5745 video recorder. The right sided video camera (RVC in Figure 3.1.1) was a Panasonic MS2 video camera. The two video cameras were gen-locked for synchronisation purposes. In addition to the two video cameras a right sided Locam IT 16 mm high speed cine camera (RCC in Figure 3.1.1) was used. The cine camera was placed close to the right video camera so that it could be used in a three-dimensional reconstruction of the activity by combining it with the left video camera.
The two video cameras were operated manually allowing sufficient time to record the last two strides prior to takeoff and the entire flight phase. LVC and RVC recorded the activity at 50 fields per second. The cine camera was also operated manually, recording the entire takeoff phase and the flight phase past the peak of the jump at approximately 200 frames per second. The faster framing rate and greater image size from RCC were required to obtain accurate joint angle data during the takeoff phase. The locations of all three cameras relative to the high jump uprights are shown in Figure 3.3.2.

Figure 3.3.1. Camera positions relative to the high jump area.

Figure 3.3.2. Camera locations relative to the high jump uprights.
Markers were placed at known locations on nine accurately constructed calibration poles and the high jump uprights. The nine poles were placed in the field of view of both video cameras and at precisely measured positions relative to the left high jump upright as shown in Figure 3.3.3. Since the subject used a left sided approach the calibration structure was chosen so that its volume encompassed the final two approach strides and the bar clearance.

In contrast to the video cameras, the cine camera had a smaller field of view so that only six of the calibration poles were visible. These are marked with an asterisk (*) in Figure 3.3.3. In addition only the left high jump upright was visible in the cine camera field of view.

**KEY:**

- **Markers at** $z = 0.13, 1.13, 2.13$ m
- **Markers at** $z = 0.13, 1.13$ m
- **Upright markers at** $z = 0.10, 2.40$ m
- **Calibration poles visible in cine field of view**

![Figure 3.3.3. The calibration set-up relative to the high jump uprights.](image)

The placement and the set-up of the calibration system was carried out with great care. Accuracy was required at this stage to ensure that accurate three-dimensional coordinate data were obtained from the Direct Linear Transformation (DLT) procedure. The area of activity with an unobstructed view of the high jump uprights and the calibration poles was recorded by all cameras prior to any jumps to allow the calibration of the movement area and the camera locations.
3.3.4 Data analysis

- Digitising procedure

The images of the recordings from all three cameras were viewed prior to any digitising. This was done to ensure that all sequences were of sufficient quality for analysis. The video film was digitised using the Apex Target high resolution video digitising system (Kerwin, 1995). The 16 mm cine film was digitised by projecting it onto a HR48 TDS digitising tablet using a NAC analysis projector (DF16C).

Prior to digitising the jump sequences the calibration set-up was digitised. Five consecutive fields with an unobstructed view of the calibration structure were selected from each camera view. The DLT procedure allowed the calibration of the cameras in order to reproduce the spatial coordinates of each digitised point.

For each field from the video cameras a total of 24 points on the calibration poles and an additional 4 points on the high jump uprights were digitised. Figure 3.3.4 shows the digitising protocol used for the reference fields from the two video cameras.

![Digitising protocol for the video reference fields.](image-url)
As mentioned previously, only six of the calibration poles and the left upright were visible in the cine camera field of view. For the cine camera a total of 16 points on the calibration poles and 2 additional points on the left upright were digitised. Figure 3.3.5 shows the digitising protocol used for the reference fields from the cine camera.

![Digitising protocol for cine reference fields](image)

Figure 3.3.5. The digitising protocol for the cine reference fields.

For each jump analysed two consecutive reference fields were digitised before the athlete came into view. In each of the reference fields from the video cameras, 4 markers on the uprights and the end points of the high jump bar were digitised. From the cine camera the two markers on the left high jump upright and the end point of the bar were digitised. This allowed the bar height to be determined and also permitted any camera movement that may have occurred between jumps to be identified and corrected.

In the movement fields 15 body landmarks were digitised for each camera view. The image locations of the wrists, elbows, shoulders, hips, knees, ankles, toes and the mid-head were obtained. For the video cameras the marker near the top of each high jump upright was also digitised and for the cine camera both markers on the left upright were digitised. The digitising of the upright markers allowed a check for any camera movement that may have occurred during a jump. A total of 17 points were digitised in the movement fields for each camera. Figure 3.3.6 shows the digitising protocol used for the video cameras.
Figure 3.3.6. The digitising protocol for the video movement fields.

The digitising procedure for the two video cameras began five fields prior to foot contact to give the last two approach strides. Every subsequent field was digitised until one field before contact with the landing mat. The cine camera digitising began ten fields prior to the final foot contact. Every subsequent field was digitised until just after the peak of the jump.

- Synchronisation of the data sets

Synchronisation of the data sets between the left and right cameras was obtained using the method of King (1998). This iterative mathematical approach was based upon the DLT reconstruction technique and used the global RMS distance to synchronise the data sets. The global RMS distance represented an overall error estimate of the reconstruction accuracy of the digitised body landmarks. This distance is smallest when the digitised data sets are synchronised. The digitised data sets from LVC and RVC, and from LVC and RCC (Figure 3.1.1) were synchronised by varying the time offset between the data sets to minimise the global RMS distance. A least squares quadratic was fitted, whose minimum point was calculated to give the offset between the time scales of the digitised data.

The King method was used to check the gen-lock synchronisation of the data sets from the two video cameras. The same method was applied to the synchronisation of the cine camera and the left video camera where there were differences in the framing rate at which each camera recorded the activity. The video cameras operated at a framing rate of 50 Hz whereas the cine camera was set to operate at a framing rate of approximately 200 Hz.
The exact framing rate of the cine camera was determined using a timing device based on a Darlington Decade Counter. Four related sequences of light emitting diodes strobe at 1.0, 0.1, 0.01 and 0.001 seconds. The timing device was recorded in a darkened laboratory both before and after the jumps session. The framing rate of the cine camera was determined as 198 Hz both before and after the jumps session. It was therefore assumed that the framing rate of the cine camera had remained constant at 198 Hz throughout the jumps session.

- Calculation of performance variables

Three-dimensional direct linear transformation (DLT) was used to convert the two-dimensional digitised coordinates from each camera view into three-dimensional coordinate locations in real space for each digitised point. This procedure involved the use of a battery of five video analysis programs developed by Yeadon (1984).

Program \textit{wob5} used the five fields of the digitised calibration set-up together with the two reference fields from each movement sequence to detect for camera movement or changes in focal lengths of the cameras during data collection.

Program \textit{framer} was used to convert the two-dimensional digitised coordinates of the calibration points into pseudo-scaled values by assuming a 10 m wide field of view. Program \textit{calib} used the pseudo-scaled data along with the known locations of the digitised calibration points to determine the 11 DLT parameters for each camera.

Program \textit{framew} was used to convert the two-dimensional digitised coordinates into pseudo-scaled values for each point digitised in the movement fields. Program \textit{framew} was also used to correct for any camera movement and changes in focal length of the cameras during the data collection.

Program \textit{film} used the 11 DLT parameters for each camera and the two-dimensional pseudo-scaled data to perform the three-dimensional DLT reconstruction of the digitised points in the movement fields. Using the DLT reconstruction the least squares simultaneous solution method was used to determine the three-dimensional location of each digitised point. Personalised segmental length, mass and mass centre location parameters were calculated using the mathematical inertia model of Yeadon (1990).

The full set of anthropometric data required by the inertia model is shown in Appendix 1a. These anthropometric measurements were taken directly from the subject. The whole body mass centre location was determined from the 3D locations of body landmarks and the segmental masses and relative mass centre locations. Internal body angles were calculated using the known locations of joint centres from the 3D reconstruction.

Four performance variables were identified for the present study. The jump height \( h \) was defined as the peak height reached by the whole body mass centre of the athlete, irrespective of its location relative to the high jump bar.
The approach speed \( v \) was defined as the average magnitude of the horizontal velocity of the mass centre over the final approach stride. In order to calculate the approach speed, the field prior to the penultimate foot contact and the field prior to the final foot contact were identified during the digitising procedure. The location of the mass centre at these two fields and the time difference between the two fields were then identified. The displacement of the mass centre over the final stride and the time taken allowed the calculation of the average speed of the mass centre over the final stride.

The leg plant angle and the knee angle were defined at the instant of touchdown of the final foot contact. The plant angle \( \phi \) was defined as the angle between the vertical and a straight line joining the ankle to the hip of the support leg. The knee angle \( \gamma \) was defined as the angle between the shank and the thigh segment of the support leg at the moment of plant. The shank segment was defined by a straight line joining the ankle to the knee, and the thigh segment by a straight line joining the knee to the hip. A value of 180° represented a straight leg, i.e. zero flexion at the knee joint. Figure 3.3.7 shows a diagrammatical representation of both the plant angle \( \phi \) and the knee angle \( \gamma \).

![Diagram of plant angle and knee angle](figure3.3.7)

Figure 3.3.7. Representation of the plant angle and the knee angle.

The 3D coordinate data obtained from the DLT reconstruction involving LVC and RCC were available at time intervals of approximately 0.005 s. (framing rate = 198 Hz). The data obtained from the DLT reconstruction involving LVC and RVC were available at time intervals of 0.02 s. (framing rate = 50 Hz).

The data obtained from the analysis using LVC and RCC were therefore considered to have greater accuracy in terms of defining the instant of plant, and therefore in the
values of the leg plant angle and the knee angle at touchdown. From the increased amount of data over a given time the determination of jump height was also considered to be more accurate from the analysis using LVC and RCC than from the analysis using LVC and RCC.

3.3.5 Evaluation of the film data

• Digitising precision

For each jump analysed an average precision estimate in both the horizontal (x) and vertical (z) directions was obtained for each of the 15 digitised body landmarks. The precision estimates represent the error estimates of each landmark used in the quintic spline fitting of the displacement data in the film program. The quintic spline fitting required that error estimates were obtained for the displacement data. Each body landmark was only digitised once however. Error estimates were therefore obtained by generating an estimated data set using the mean value of the coordinates from plus and minus one field.

The digitising precision estimates (x,z) averaged over the 15 body landmarks are shown in Table 3.3.1 for those jumps analysed using LVC and RCC.

Table 3.3.1. Two-dimensional digitising precision for LVC and RCC

<table>
<thead>
<tr>
<th>Trial</th>
<th>LVC</th>
<th>RCC</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x [mm]</td>
<td>z [mm]</td>
<td>x [mm]</td>
</tr>
<tr>
<td>br05</td>
<td>11</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>br06</td>
<td>14</td>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>br07</td>
<td>11</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>br08</td>
<td>11</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>br09</td>
<td>12</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>br10</td>
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<tr>
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<td>14</td>
<td>6</td>
</tr>
<tr>
<td>br14</td>
<td>11</td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>br16</td>
<td>11</td>
<td>13</td>
<td>6</td>
</tr>
<tr>
<td>br18</td>
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</tr>
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</tr>
<tr>
<td>mean</td>
<td>11</td>
<td>13</td>
<td>6</td>
</tr>
</tbody>
</table>
It can be seen that the mean digitising precision was 9 mm. This value represents the digitising precision averaged over the horizontal and vertical directions for the 16 jumps analysed using LVC and RCC. These trials and an additional 9 jumps were also analysed using LVC and RVC. The corresponding mean digitising precision for the trials analysed using LVC and RVC was 11 mm.

- **Three-dimensional reconstruction accuracy**

The overall three-dimensional reconstruction errors of the digitised markers on the calibration poles and high jump uprights (Figures 3.3.4 and 3.3.5) are presented in Table 3.3.2. These errors indicate the accuracy of the reconstructed spatial coordinates of the digitised markers. The errors presented were calculated as the root-mean-square error averaged over each direction (x,y,z), each calibration point and each reference field.

Also presented in Table 3.3.2 is the three-dimensional reconstruction accuracy of the digitised body landmarks (Figure 3.3.6). The errors were calculated as the root-mean-square error averaged over each direction (x,y,z), each digitised point and each movement field. The overall 3D reconstruction accuracy for both the reference and movement fields are presented for the data obtained from LVC and RVC, and the data obtained from LVC and RCC.

<table>
<thead>
<tr>
<th>camera</th>
<th>overall r.m.s. [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>reference</td>
</tr>
<tr>
<td>LVC and RVC</td>
<td>6</td>
</tr>
<tr>
<td>LVC and RCC</td>
<td>9</td>
</tr>
</tbody>
</table>

- **Error estimate in the jump height**

The precision estimate of the peak mass centre height was equivalent to the sum of systematic and random errors. The systematic error in locating the mass centre was equivalent to the overall r.m.s. error in digitising the calibration set-up (Table 3.3.2). There was an additional random error arising from the digitising of the movement. Table 3.3.2 showed that the overall r.m.s. error from the movement fields was 11 mm for both camera systems. This error was obtained by averaging over the 15 body landmarks. The error in locating the mass centre may therefore be calculated as 3 mm (11/15 = 0.733 mm). The sum of the systematic and random errors in locating the mass centre using LVC and RVC was therefore 9 mm. The corresponding error using LVC and RCC was 12 mm.
• Error estimate in the approach speed

The approach speed was defined as the magnitude of the average horizontal velocity of the whole body mass centre over the final stride of the approach. The field of view of camera RCC was insufficient to analyse the whole of the final approach stride. The reported values for approach speed were therefore determined from the analysis using LVC and RVC at a framing rate of 50 Hz. The calculation of approach speed required that the field prior to the penultimate foot contact and the field prior to the final foot contact were identified during the digitising procedure. The displacement of the mass centre over the final stride and the time taken allowed the calculation of the average speed of the mass centre over the final stride.

The error in locating the mass centre using LVC and RVC was previously reported as 9 mm. The precision estimate in the horizontal displacement of the mass centre over the final approach stride was therefore calculated as 0.013 m \(= (0.009^2 + 0.009^2)^{1/2} \). The time between the field prior to the penultimate foot contact and the field prior to the final foot contact averaged over all the jumps analysed was 0.19 s. The precision estimate in the approach speed was therefore calculated as 0.07 m.s\(^{-1}\) \(= 0.013/0.19\).

• Error estimates in the plant angle and knee angle

The precision in the reported values for the plant angle and the knee angle at touchdown was dependent on the ability to determine the exact instant of touchdown. It was assumed that there was a half-frame error in determining the exact instant of touchdown. The plant and knee angles were initially determined at the nearest frame, noted during the digitising process. The plant and knee angles were also determined in the frames immediately before and immediately after the best estimate of touchdown. The difference in the plant and knee angles between consecutive frames was calculated and halved for the frames before and after touchdown. These half-frame values were then averaged to obtain the half-frame error.

The precision estimate in plant angle averaged over all jumps analysed (Table 3.3.3) using LVC and RCC was 0.6°. The precision estimate in plant angle averaged over all jumps analysed using LVC and RVC was 2.1°.

The precision estimate in knee angle averaged over all jumps analysed (Table 3.3.3) using LVC and RCC was 0.8°. The precision estimate in knee angle averaged over all jumps analysed using LVC and RVC was 3.8°.

3.3.6 Results

A summary of all the jumps attempted by the athlete in the single data collection session is shown in Table 3.3.3. This includes a brief description of the approach used in each jump and shows which jumps resulted in a successful bar clearance.
Table 3.3.3. Record of performance

<table>
<thead>
<tr>
<th>Trial</th>
<th>Approach</th>
<th>Bar height</th>
<th>Clear/Fail</th>
<th>Video</th>
<th>Cine</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2 strides</td>
<td>1.80 m</td>
<td>O</td>
<td></td>
<td></td>
</tr>
<tr>
<td>br02</td>
<td>2 strides</td>
<td>1.90 m</td>
<td>O</td>
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<td></td>
</tr>
<tr>
<td>br03</td>
<td>2 strides</td>
<td>1.95 m</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>br04</td>
<td>2 strides</td>
<td>2.00 m</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>br05</td>
<td>5 strides slow</td>
<td>2.00 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br06</td>
<td>5 strides medium</td>
<td>2.00 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br07</td>
<td>5 strides fast</td>
<td>2.00 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br08</td>
<td>5 strides slow</td>
<td>2.10 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br09</td>
<td>5 strides medium</td>
<td>2.10 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br10</td>
<td>5 strides fast</td>
<td>2.10 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br11</td>
<td>5 strides fast</td>
<td>2.10 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br12</td>
<td>9 strides medium</td>
<td>2.15 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br13</td>
<td>9 strides fast</td>
<td>2.15 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br14</td>
<td>9 strides fast</td>
<td>2.15 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br15</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br16</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br17</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br18</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br19</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br20</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br21</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br22</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br23</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br24</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br25</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br26</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br27</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br28</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br29</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br30</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br31</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br32</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br33</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br34</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br35</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br36</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br37</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br38</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br39</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br40</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br41</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br42</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>br43</td>
<td>13 strides fast</td>
<td>2.20 m</td>
<td>O</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
In Table 3.3.3 all of the jumps that were analysed are marked with a tick (✓). A total of 16 jumps were analysed using the cine camera and the left video camera. An additional 9 jumps were analysed using the two video cameras. As mentioned previously the analysis from the video cameras included the whole of the final two approach strides and the entire flight phase. The video analysis therefore allowed the determination of the jump height h, the approach speed v, the leg plant angle \( \phi \) and the knee angle \( \gamma \) at touchdown.

Table 3.3.4 shows a summary of the results obtained from the video analysis of each jump analysed. Also included for each variable is the mean value, the standard deviation and the standard error about the mean value.

Table 3.3.4. Summary of the performance variables determined from the video analysis

<table>
<thead>
<tr>
<th>Trial</th>
<th>Jump height [m]</th>
<th>Approach speed [m.s(^{-1})]</th>
<th>Leg plant angle [°]</th>
<th>Knee angle [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>br05</td>
<td>2.102</td>
<td>5.73</td>
<td>33.7°</td>
<td>156.1°</td>
</tr>
<tr>
<td>br06</td>
<td>2.129</td>
<td>6.29</td>
<td>33.5°</td>
<td>157.5°</td>
</tr>
<tr>
<td>br07</td>
<td>2.162</td>
<td>6.50</td>
<td>32.9°</td>
<td>164.1°</td>
</tr>
<tr>
<td>br08</td>
<td>2.063</td>
<td>5.87</td>
<td>30.0°</td>
<td>157.2°</td>
</tr>
<tr>
<td>br09</td>
<td>2.118</td>
<td>6.28</td>
<td>34.6°</td>
<td>161.9°</td>
</tr>
<tr>
<td>br10</td>
<td>2.195</td>
<td>6.68</td>
<td>34.9°</td>
<td>171.6°</td>
</tr>
<tr>
<td>br11</td>
<td>2.192</td>
<td>6.62</td>
<td>34.1°</td>
<td>169.4°</td>
</tr>
<tr>
<td>br12</td>
<td>2.130</td>
<td>6.13</td>
<td>30.9°</td>
<td>163.0°</td>
</tr>
<tr>
<td>br13</td>
<td>2.194</td>
<td>6.63</td>
<td>33.8°</td>
<td>171.9°</td>
</tr>
<tr>
<td>br14</td>
<td>2.206</td>
<td>6.73</td>
<td>35.4°</td>
<td>169.7°</td>
</tr>
<tr>
<td>br16</td>
<td>2.203</td>
<td>7.22</td>
<td>35.7°</td>
<td>170.4°</td>
</tr>
<tr>
<td>br18</td>
<td>2.200</td>
<td>7.16</td>
<td>34.9°</td>
<td>169.4°</td>
</tr>
<tr>
<td>br20</td>
<td>2.168</td>
<td>7.26</td>
<td>36.5°</td>
<td>164.1°</td>
</tr>
<tr>
<td>br22</td>
<td>2.171</td>
<td>6.88</td>
<td>34.2°</td>
<td>168.8°</td>
</tr>
<tr>
<td>br24</td>
<td>2.172</td>
<td>6.77</td>
<td>33.5°</td>
<td>166.7°</td>
</tr>
<tr>
<td>br26</td>
<td>2.169</td>
<td>6.81</td>
<td>34.6°</td>
<td>168.4°</td>
</tr>
<tr>
<td>br28</td>
<td>2.183</td>
<td>7.01</td>
<td>36.0°</td>
<td>169.8°</td>
</tr>
<tr>
<td>br30</td>
<td>2.176</td>
<td>7.26</td>
<td>32.7°</td>
<td>170.1°</td>
</tr>
<tr>
<td>br32</td>
<td>2.132</td>
<td>6.78</td>
<td>31.7°</td>
<td>167.9°</td>
</tr>
<tr>
<td>br34</td>
<td>2.131</td>
<td>6.58</td>
<td>32.0°</td>
<td>163.4°</td>
</tr>
<tr>
<td>br36</td>
<td>2.206</td>
<td>6.86</td>
<td>33.6°</td>
<td>168.9°</td>
</tr>
<tr>
<td>br38</td>
<td>2.199</td>
<td>7.13</td>
<td>34.3°</td>
<td>171.4°</td>
</tr>
<tr>
<td>br40</td>
<td>2.158</td>
<td>6.89</td>
<td>33.1°</td>
<td>168.3°</td>
</tr>
<tr>
<td>br42</td>
<td>2.177</td>
<td>6.92</td>
<td>34.4°</td>
<td>171.8°</td>
</tr>
<tr>
<td>br43</td>
<td>2.232</td>
<td>6.93</td>
<td>34.8°</td>
<td>169.7°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th></th>
<th>mean value</th>
<th>mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.167</td>
<td>6.72</td>
<td>33.74°</td>
<td>166.71°</td>
</tr>
<tr>
<td>s.d.</td>
<td>0.039</td>
<td>0.41</td>
<td>1.71°</td>
<td>4.87°</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.008</td>
<td>0.08</td>
<td>0.34°</td>
<td>0.97°</td>
</tr>
</tbody>
</table>
Digitising of the movement from the cine camera started just before the instant of touchdown of the final foot contact, and continued until the flight path of the mass centre had passed its peak. From the analysis of the cine data and the data from the left video camera it was therefore possible to determine \( h, \phi \) and \( \gamma \), but not the approach speed \( v \).

A summary of the results obtained for each jump from the analysis of the cine data and the data from the left video camera is shown in Table 3.3.5. Also included in Table 3.3.5 are the approach speed values (presented in Table 3.3.4) for the 16 digitised jumps recorded using the cine camera. The data shown in Table 3.3.5 were selected for use in determining how the approach influences jump height performance.

Table 3.3.5. Summary of the performance variables determined from the cine analysis

<table>
<thead>
<tr>
<th>Trial</th>
<th>Jump height [m]</th>
<th>Approach speed [m.s(^{-1})]</th>
<th>Leg plant angle [°]</th>
<th>Knee angle [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>br05</td>
<td>2.113</td>
<td>5.73</td>
<td>33.8°</td>
<td>158.9°</td>
</tr>
<tr>
<td>br06</td>
<td>2.129</td>
<td>6.29</td>
<td>33.7°</td>
<td>163.7°</td>
</tr>
<tr>
<td>br07</td>
<td>2.172</td>
<td>6.50</td>
<td>33.2°</td>
<td>168.0°</td>
</tr>
<tr>
<td>br08</td>
<td>2.069</td>
<td>5.87</td>
<td>29.7°</td>
<td>159.0°</td>
</tr>
<tr>
<td>br09</td>
<td>2.136</td>
<td>6.28</td>
<td>33.8°</td>
<td>172.9°</td>
</tr>
<tr>
<td>br10</td>
<td>2.207</td>
<td>6.68</td>
<td>34.3°</td>
<td>172.4°</td>
</tr>
<tr>
<td>br11</td>
<td>2.204</td>
<td>6.62</td>
<td>34.0°</td>
<td>171.9°</td>
</tr>
<tr>
<td>br12</td>
<td>2.140</td>
<td>6.13</td>
<td>30.9°</td>
<td>166.6°</td>
</tr>
<tr>
<td>br13</td>
<td>2.208</td>
<td>6.63</td>
<td>34.4°</td>
<td>171.4°</td>
</tr>
<tr>
<td>br14</td>
<td>2.224</td>
<td>6.73</td>
<td>35.5°</td>
<td>174.4°</td>
</tr>
<tr>
<td>br16</td>
<td>2.221</td>
<td>7.22</td>
<td>35.8°</td>
<td>177.2°</td>
</tr>
<tr>
<td>br18</td>
<td>2.215</td>
<td>7.16</td>
<td>35.0°</td>
<td>175.6°</td>
</tr>
<tr>
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<td>2.174</td>
<td>7.26</td>
<td>37.3°</td>
<td>176.1°</td>
</tr>
<tr>
<td>br22</td>
<td>2.179</td>
<td>6.88</td>
<td>33.6°</td>
<td>172.6°</td>
</tr>
<tr>
<td>br24</td>
<td>2.182</td>
<td>6.77</td>
<td>33.9°</td>
<td>170.5°</td>
</tr>
<tr>
<td>br26</td>
<td>2.180</td>
<td>6.81</td>
<td>35.9°</td>
<td>173.5°</td>
</tr>
</tbody>
</table>

| Mean  | 2.172          | 6.60                          | 34.05°            | 170.29°       |
| S.D.  | 0.044          | 0.45                          | 1.83°             | 5.64°         |
| S.E.  | 0.011          | 0.11                          | 0.46°             | 1.41°         |

The aim of the direct intervention was to induce a change in the approach speed so that the range would be greater than that observed in competition. Figure 3.3.8 shows the relationship between the approach speed \( v \) and the jump height \( h \) for the 16 jumps analysed (Table 3.3.5). In Figure 3.3.8 the data points have been fitted with a linear function. This linear function was defined by an equation obtained by performing a linear regression of \( h \) against \( v \) for the 16 jumps.
From Figure 3.3.8 and the corresponding regression equation it can be seen that there was a fairly strong ($r^2 = 0.647$) and significant ($p < 0.001$) linear relationship between the approach speed and the jump height. This relationship implies that the faster the approach the greater the jump height. The optimum approach speed determined from Figure 3.3.8 would therefore be located at the maximum of the experimental approach speeds. From Table 3.3.5 it can be seen that the maximum recorded approach speed was 7.26 m.s$^{-1}$. However, from the linear function in Figure 3.3.8 it would be difficult to predict the jump height outside of the experimental range of approach speed with any certainty.

In contrast to the linear relationship shown in Figure 3.3.8, Alexander (1990) showed that the relationship between approach speed and jump height is quadratic in nature. The best jump heights were shown to be obtained at intermediate values of approach speed. In Figure 3.3.9 the same data points are fitted with a quadratic function.
of approach speed. The inclusion of the quadratic term also increases the confidence in extrapolating outside of the experimental range. The function in Figure 3.3.9 was defined by the regression equation obtained from a quadratic regression of jump height against approach speed.

\[ h = -0.836 + 0.8449v - 0.05871v^2 \]

\[ \text{s.e.} = 0.024 \text{ m} \quad r^2 = 0.734 \quad p < 0.001 \]

\[ p_c = 0.502 \quad p_v = 0.041 \quad p_{v'} = 0.060 \]

Figure 3.3.9. Jump height as a quadratic function of approach speed.

It can be seen that there was a strong \((r^2 = 0.734)\) and significant \((p < 0.001)\) quadratic relationship between jump height and approach speed. The inclusion of the quadratic term also decreased the standard error in the jump height \(h\). The regression equation was differentiated with respect to \(v\) and set equal to zero in order to determine the optimum approach speed. The value of the optimum approach speed from this equation was 7.20 m.s\(^{-1}\). The maximum recorded approach speed was 7.26 m.s\(^{-1}\).

It was expected that the change in approach speed as a result of the direct intervention would also bring about changes in the leg plant angle and the knee angle at touchdown. Figure 3.3.10 shows the influence of the leg plant angle on jump height.
Figure 3.3.10. Jump height as a linear function of leg plant angle.

In Figure 3.3.10 the data points have been fitted with a linear function, defined by an equation obtained by linearly regressing \( h \) against \( \phi \) for the 16 jumps analysed.

\[ h = 1.6127 + 0.016429 \phi \quad \text{s.e.} = 0.033 \text{ m} \quad r^2 = 0.468 \quad p = 0.003 \quad p_c < 0.001 \quad p_\phi = 0.003 \]

It can be seen that there was a fairly strong \((r^2 = 0.468)\) and significant \((p = 0.003)\) linear relationship between the leg plant angle and the jump height. This relationship implies that the greater the angle of leg plant away from the vertical the greater the jump height. The optimum plant angle would therefore be located at the maximum of the experimental plant angles, with the maximum recorded plant angle being 37.3°. However, it would be difficult to predict with any certainty the jump height outside of the experimental range of plant angle.

In contrast to the linear relationship shown in Figure 3.3.10, Alexander (1990) showed that the best jump heights were obtained at intermediate values of plant angle. In Figure 3.3.11 a quadratic function has been fitted to the data points.
The inclusion of the quadratic term allowed the determination of an optimum plant angle and also increased the confidence in extrapolating outside of the experimental range. The quadratic relationship between plant angle and jump height in Figure 3.3.11 was defined by the regression equation obtained from a quadratic regression of jump height against plant angle.

\[ h = -1.112 + 0.1801 \phi - 0.002449 \phi^2 \]

s.e. = 0.032 m  \( r^2 = 0.542 \)  \( p = 0.006 \)
\( p_c = 0.567 \)  \( p_\phi = 0.137 \)  \( p_{\phi^2} = 0.173 \)

From this equation it can be seen that the inclusion of a quadratic term slightly improved the strength (\( r^2 = 0.542 \)) of the correlation and also caused a slight decrease in the standard error in the jump height \( h \). From the regression equation the optimum plant angle was calculated as 36.8°, which is little different from the maximum recorded value of 37.3°. However the \( p \) values for each of the coefficient terms in the regression equation were not small, implying a lack of a significant quadratic relationship between jump height and plant angle, and subsequently little confidence in the optimum.
The influence of the knee angle $\gamma$ at touchdown on jump height performance was also examined. Figure 3.3.12 shows the influence of the knee angle on jump height. The data points have been fitted with a linear function, defined by an equation which was obtained by a linear regression of $h$ against $\gamma$ for the 16 jumps analysed.

$$h = 1.0493 + 0.006593 \gamma$$

s.e. = 0.024  $r^2 = 0.715$  $p < 0.001$

$P_c < 0.001$  $P_\gamma < 0.001$

**Figure 3.3.12.** Jump height as a linear function of knee angle at touchdown.

From Figure 3.3.12 and the corresponding regression equation it can be seen that there was a strong ($r^2 = 0.715$) and significant ($p < 0.001$) linear relationship between the knee angle at touchdown and the jump height. In addition the $p$ values for each of the coefficients in the regression equation were very small. The strong linear relationship between knee angle and jump height implies that the straighter the leg at touchdown the greater the jump height. The optimum knee angle determined from Figure 3.3.12 would therefore be located at the maximum of the experimental data set. From Table 3.3.5 it can be seen that the maximum recorded knee angle at touchdown was $177.2^\circ$. 
A linear function makes it difficult to extrapolate outside of the experimental range and predict jump height with any certainty. However, the maximum knee angle value is $180^\circ$ which equates to a straight leg. There is therefore only a small range of values that may exist which are greater than the maximum recorded value. In Figure 3.3.13 a quadratic term is included to investigate the influence of any curvature in the relationship between the knee angle at touchdown and the jump height. The quadratic relationship between knee angle and jump height in Figure 3.3.13 was defined by the regression equation obtained from a quadratic regression of jump height against knee angle.

$$h = -3.964 + 0.06645 \gamma - 0.0001784 \gamma^2$$

s.e. = 0.025 m  $r^2 = 0.730$  $p < 0.001$

$p_c = 0.522$  $p_\gamma = 0.372$  $p_{\varphi} = 0.420$

Figure 3.3.13. Jump height as a quadratic function of knee angle at touchdown.

The inclusion of a quadratic term did slightly improve the strength ($r^2 = 0.730$) of the correlation. However the standard error in the jump height increased slightly. From the quadratic regression equation the optimum knee angle was calculated as $186.2^\circ$. This value is greater than the maximum recorded value of $177.2^\circ$, and the maximum value of $180^\circ$. This result does however suggest that jump height is optimised by having a straight
leg at touchdown. The optimum value must be treated with considerable reservation since the p values for each of the coefficient terms were not small. This indicates a lack of a significantly quadratic relationship between jump height and knee angle at touchdown.

The change in approach speed as a result of the direct intervention was also expected to cause changes in the leg plant angle and the knee angle at touchdown. In the above regressions it was implicitly assumed that there was no influence on jump height other than that from the variable which was included in the regression. This assumption would only be valid if the approach parameters were completely independent of each other. To test the interdependence of the approach parameters they were regressed against each other.

\[
\phi = 13.671 + 3.0885 v \\
\phi = -6.971 + 0.24088 \gamma \\
\gamma = 95.071 + 11.400 v
\]

\[
s.e. = 1.24^\circ \quad r^2 = 0.576 \quad p = 0.001 \\
s.e. = 1.27^\circ \quad r^2 = 0.550 \quad p = 0.001 \\
s.e. = 2.43^\circ \quad r^2 = 0.827 \quad p < 0.001
\]

It can be seen that there was a strong and significant linear relationship between each of the approach parameters. The strongest relationship \((r^2 = 0.827)\) was observed between \(v\) and \(\gamma\). The above equations show that as the approach speed increased so did the leg plant angle and knee angle at touchdown. The interdependence of the variables shows that the change in approach speed due to the direct intervention also produced systematic changes in the plant angle and the knee angle at touchdown. The high correlation between the approach parameters makes it difficult to establish with any certainty the influence of just one parameter on jump height. For example, the high correlation between the approach speed and the knee angle means that the quadratic regression of jump height against approach speed is also taking into consideration the effect of the knee angle.

It was previously shown that jump height was most strongly correlated with a linear function of the knee angle at touchdown. The influence of the knee angle \(\gamma\) was taken into consideration by linearly detrending the jump height \(h\), the approach speed \(v\) and the plant angle \(\phi\) with respect to \(\gamma\). Residual values \(h_\gamma\) were obtained for each of the 16 jumps by calculating the difference between the recorded value of \(h\) and the predicted value obtained from the linear regression equation relating \(h\) to \(\gamma\). In the same way the residual values for \(v_\gamma\) and \(\phi_\gamma\) were determined.
The relationship between the jump height \( h \) and the approach speed \( v \) was revised, taking into consideration the effect of the knee angle \( \gamma \). This was achieved by regressing the residuals of jump height against the residuals of approach speed. Figure 3.3.14 shows the relationship between the jump height and approach speed when both are detrended against the knee angle.

\[
\text{residuals in jump height [m]} \quad \text{residuals in approach speed [m.s}^{-1}] \\
-0.60 -0.50 -0.40 -0.30 -0.20 -0.10 \quad 0.10 0.20 0.30
\]

![Figure 3.3.14. The relationship between jump height and approach speed linearly detrended for knee angle.](image)

The regression equation relating the residuals of jump height and approach speed suggested no significant linear relationship between jump height and approach speed when the influence of the knee angle was included. This lack of linear correlation is evident from Figure 3.3.14. It was previously shown that without the influence of \( \gamma \) there was a strong \((r^2 = 0.647)\) and significant \((p < 0.001)\) linear relationship between jump height and approach speed (Figure 3.3.8).

Previously, Figure 3.3.9 showed that the quadratic relationship between jump height and approach speed was even stronger \((r^2 = 0.734, p < 0.001)\) than the linear relationship. The regression of the residuals in jump height and approach speed was therefore extended to include a quadratic term.
\[ h_\gamma = 0.6038 \nu_\gamma - 0.04319 \nu^2_\gamma \quad \text{s.e.} = 0.022 \text{ m} \quad r^2 = 0.160 \quad p = 0.295 \]
\[ p_\nu = 0.145 \quad p_{\nu'} = 0.156 \]

However, the \( p \) values for the coefficients in both \( \nu \) and \( \nu' \) remained high suggesting a lack of significance in any quadratic relationship between jump height and approach speed when the effect of the knee angle is taken into consideration. The small coefficient of determination \( (r^2 = 0.160) \) further suggests a lack of any such quadratic relationship between the residuals of jump height and approach speed. This lack of a quadratic relationship is evident in Figure 3.3.14.

The lack of any significant relationship when both \( h \) and \( \nu \) were linearly detrended with respect to \( \gamma \) may have been due to the strong and significant linear relationship between \( \nu \) and \( \gamma \) \( (r^2 = 0.827, p < 0.001) \). Regressing the residuals in knee angle of jump height against approach speed is equivalent to regressing jump height against approach speed and knee angle.

\[ h = -0.501 + 0.003393 \gamma + 0.6038 \nu - 0.04319 \nu^2 \quad \text{s.e.} = 0.024 \text{ m} \quad r^2 = 0.761 \quad p < 0.001 \]
\[ p_c = 0.691 \quad p_\gamma = 0.269 \quad p_\nu = 0.178 \quad p_{\nu'} = 0.191 \]

This equation shows a strong \( (r^2 = 0.761) \) and significant \( (p < 0.001) \) relationship. However the high \( p \) values for each of the coefficient terms indicates a lack of certainty in their values and therefore in the equation.

It was previously shown that there was a fairly strong \( (r^2 = 0.468) \) and significant \( (p = 0.003) \) linear relationship between jump height and the leg plant angle \( \phi \). The inclusion of a quadratic term \( \phi^2 \) further increased the strength of this relationship \( (r^2 = 0.542) \). However the leg plant angle was also found to be correlated \( (r^2 = 0.550, \quad p = 0.001) \) with the knee angle at touchdown. The linear and quadratic relationships obtained between jump height and plant angle may therefore have been influenced by the knee angle, as was shown to be the case when investigating the influence of approach speed on jump height. In order to take into account the influence of the knee angle \( \gamma \), the residuals \( h_\gamma \) and \( \phi_\gamma \) were considered.

\[ h_\gamma = 0.00304 \phi_\gamma \quad \text{s.e.} = 0.024 \text{ m} \quad r^2 = 0.025 \quad p = 0.542 \]

The small coefficient of determination and the high \( p \) value indicate a lack of any linear relationship between jump height and plant angle when the influence of the knee angle is removed. It was previously shown (Figure 3.3.11) that there was no significant quadratic relationship between jump height and plant angle. This quadratic relationship was revised taking into consideration the influence of the knee angle. Figure 3.3.15
shows the relationship between jump height and plant angle when both are detrended against the knee angle.

\[
h_\gamma = 0.16807 \phi_\gamma - 0.00247 \phi_\gamma^2 \hspace{1cm} \text{s.e.} = 0.020 \hspace{1cm} r^2 = 0.288 \hspace{1cm} p = 0.093
\]
\[
p_{\phi} = 0.037 \hspace{1cm} p_{\phi} = 0.039
\]

The standard error of 0.020 m is considerably smaller than the value of 0.032 m obtained from the original quadratic regression of \( h \) against \( \phi \), without detrending for \( \gamma \). The standard error of 0.020 m is also slightly smaller than the value of 0.023 m obtained from the linear regression of \( h_\gamma \) against \( \phi_\gamma \). The strength of the quadratic relationship \((r^2 = 0.288)\) is also stronger than the linear relationship \((r^2 = 0.025)\). The significance of the relationship is also increased when the quadratic term is included in the regression. With some degree of significance in the quadratic relationship between jump height and plant angle this equation can be used to calculate the optimum plant angle. The optimum plant angle calculated from the quadratic relationship between \( h \) and \( \phi \) detrended for \( \gamma \) was 34.0°. Previously an optimum value of 36.8° (Figure 3.3.11) was obtained when the quadratic relationship between \( h \) and \( \phi \) was not detrended for \( \gamma \).

Regressing the residuals in knee angle of jump height against plant angle is equivalent to a regression of jump height against plant angle and knee angle.
This equation shows a strong ($r^2 = 0.797$) and significant ($p < 0.001$) relationship with small $p$ values for the coefficients in $\gamma$, $\phi$ and $\phi^2$. There was however a lack of significance in the constant term ($p_c = 0.227$). It may therefore be concluded that there was some evidence of a quadratic relationship between jump height and plant angle. This equation produced an optimum plant angle value of $35.0^\circ$. In comparison an optimum value of $36.8^\circ$ was obtained when the influence of the knee angle was neglected. The highest recorded value from Table 3.3.5 was $37.3^\circ$.

It was shown previously that of all the linear relationships between the approach parameters the correlation between $\gamma$ and $y$ was the weakest ($r^2 = 0.550$). The strong correlation between $v$ and $y$ however implies that when the plant angle was detrended against $\gamma$ there may also have been some influence from $v$ included in the regression. The relationship between jump height and plant angle was therefore repeated, detrending for both $v$ and $\gamma$. Figure 3.3.16 shows the relationship between jump height and plant angle when both are detrended against approach speed and knee angle.
The small p values for the equation coefficients indicate a significant quadratic relationship between jump height and plant angle, supporting the results of the initial regressions. In this equation the standard error in jump height has been reduced to its smallest value (0.019 m) over all of the regressions. This equation produced an optimum plant angle of 33.6°. This equation can also be considered without the use of residual values by regressing h against $\phi$, $\phi^2$, v and $\gamma$.

$$h = -1.952 + 0.19547 \phi - 0.002908 \phi^2 + 0.03941 v + 0.003456 \gamma$$

s.e. = 0.022 m $r^2 = 0.821$ p < 0.001

$p_v = 0.166$ $p_\phi = 0.034$ $p_\phi^2 = 0.035$ $p_v = 0.258$ $p_\gamma = 0.196$

This equation further supports the quadratic relationship between jump height and plant angle since the p values for the coefficients in both $\phi$ and $\phi^2$ are small. This equation produced an optimum plant angle value of 33.6°. However, the p values for the coefficients in the constant term, v and $\gamma$ were high. It was previously shown that v was very strongly correlated with $\gamma$. The coefficient in v was therefore removed, producing the regression equation obtained previously.

$$h = -1.679 + 0.16807 \phi - 0.00247 \phi^2 + 0.005876 \gamma$$

s.e. = 0.022 m $r^2 = 0.797$ p < 0.001

$p_v = 0.227$ $p_\phi = 0.054$ $p_\phi^2 = 0.057$ $p_\gamma = 0.002$

This equation provides a means of predicting the jump height for given values of the leg plant angle $\phi$ and knee angle $\gamma$. The approach speed v is not represented due to its high correlation with $\gamma$. The strength of the correlation ($r^2 = 0.797$) indicates that this regression equation accounted for 80% of the observed variance in jump height.

### 3.3.7 Using the regression equation to investigate the variation in jump height

The regression equation previously determined was used to calculate the predicted jump height for each of the sixteen jumps analysed using LVC and RCC (Figure 3.3.1). Table 3.3.6 shows the leg plant angle $\phi$ and the knee angle $\gamma$ for each jump, previously presented in Table 3.3.5. Also shown in Table 3.3.6 is the observed jump height and the predicted jump height obtained using the regression equation.

$$h = -1.679 + 0.16807 \phi - 0.00247 \phi^2 + 0.005876 \gamma$$
Table 3.3.6. Using the regression equation to predict jump height for the cine data

<table>
<thead>
<tr>
<th>Trial</th>
<th>Leg plant angle [°]</th>
<th>Knee angle [°]</th>
<th>Actual jump height [m]</th>
<th>Predicted jump height [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>br05</td>
<td>33.8°</td>
<td>158.9°</td>
<td>2.113</td>
<td>2.114</td>
</tr>
<tr>
<td>br06</td>
<td>33.7°</td>
<td>163.7°</td>
<td>2.129</td>
<td>2.142</td>
</tr>
<tr>
<td>br07</td>
<td>33.2°</td>
<td>168.0°</td>
<td>2.172</td>
<td>2.166</td>
</tr>
<tr>
<td>br08</td>
<td>29.7°</td>
<td>159.0°</td>
<td>2.069</td>
<td>2.068</td>
</tr>
<tr>
<td>br09</td>
<td>33.8°</td>
<td>172.9°</td>
<td>2.136</td>
<td>2.196</td>
</tr>
<tr>
<td>br10</td>
<td>34.3°</td>
<td>172.4°</td>
<td>2.207</td>
<td>2.193</td>
</tr>
<tr>
<td>br11</td>
<td>34.0°</td>
<td>171.9°</td>
<td>2.204</td>
<td>2.190</td>
</tr>
<tr>
<td>br12</td>
<td>30.9°</td>
<td>166.6°</td>
<td>2.140</td>
<td>2.135</td>
</tr>
<tr>
<td>br13</td>
<td>34.4°</td>
<td>171.4°</td>
<td>2.208</td>
<td>2.187</td>
</tr>
<tr>
<td>br14</td>
<td>35.5°</td>
<td>174.4°</td>
<td>2.224</td>
<td>2.199</td>
</tr>
<tr>
<td>br16</td>
<td>35.8°</td>
<td>177.2°</td>
<td>2.221</td>
<td>2.213</td>
</tr>
<tr>
<td>br18</td>
<td>35.0°</td>
<td>175.6°</td>
<td>2.215</td>
<td>2.210</td>
</tr>
<tr>
<td>br20</td>
<td>37.3°</td>
<td>176.1°</td>
<td>2.174</td>
<td>2.188</td>
</tr>
<tr>
<td>br22</td>
<td>33.6°</td>
<td>172.6°</td>
<td>2.179</td>
<td>2.194</td>
</tr>
<tr>
<td>br24</td>
<td>33.9°</td>
<td>170.5°</td>
<td>2.182</td>
<td>2.182</td>
</tr>
<tr>
<td>br26</td>
<td>35.9°</td>
<td>173.5°</td>
<td>2.180</td>
<td>2.191</td>
</tr>
</tbody>
</table>

The standard deviation s.d. of the actual jump heights in Table 3.3.6 from their mean value was 0.043 m. The root mean square difference R between the actual jump height and the predicted jump height over the 16 jumps was calculated as 0.019 m. The experimentally determined regression equation was therefore able to account for 80% of the observed variation in jump height over the 16 jumps.

\[
\frac{s.d.^2 - R^2}{s.d.^2} \times 100\% = \frac{0.043^2 - 0.019^2}{0.043^2} \times 100\% = 80.10\%
\]

Table 3.3.3 shows that in addition to the 16 jumps analysed using RCC, an additional 9 jumps (br28 - br43) were analysed using LVC and RVC at a framing rate of 50 Hz. Table 3.3.7 shows the plant angle $\phi_{\text{video}}$ and knee angle $\gamma_{\text{video}}$ values obtained for jumps br28 - br43 from the analysis using LVC and RVC. These data previously presented in Table 3.3.4 were not used in obtaining the previously presented regression equation. Table 3.3.7 also shows the actual jump height $h_{\text{video}}$ for each jump and the predicted jump height $h_{\text{pred}}$ obtained from the regression equation.

The standard deviation s.d. in $h_{\text{video}}$ from the mean value over the nine jumps was 0.033 m. The root mean square difference R between $h_{\text{video}}$ and $h_{\text{pred}}$ was 0.023 m. Using the nine independent jumps the experimentally determined regression equation was therefore able to account for 51% of the observed variation in jump height.

\[
\frac{0.033^2 - 0.023^2}{0.033^2} \times 100\% = 51.42\%
\]
The regression equations presented previously were based on the analysis of 16 jumps (br05 - br26) carried out using LVC and RCC. These 16 jumps were selected on the basis of greater accuracy in the approach parameters and jump height due to the greater framing rate of RCC (198 Hz) in comparison with RVC (50 Hz). These 16 jumps were also analysed using LVC and RVC. For each jump the values obtained for the plant angle $\phi$ and the knee angle $\gamma$ were compared by linearly regressing the values obtained from the analysis using LVC and RCC against the values obtained from the analysis using LVC and RVC. The coefficient terms were averaged over the sixteen jumps to obtain a single equation for predicting the 'cine' angle from the 'video' angle.

\[
\phi_{\text{cine}} = -2.00 + 1.06186 \phi_{\text{video}} \quad \text{s.e.} = 0.545^\circ \quad r^2 = 0.917 \quad p < 0.001 \quad \rho_c = 0.501 \quad \phi < 0.001
\]

\[
\gamma_{\text{cine}} = 25.17 + 0.8762 \gamma_{\text{video}} \quad \text{s.e.} = 3.362^\circ \quad r^2 = 0.668 \quad p < 0.001 \quad \rho_c = 0.373 \quad \gamma < 0.001
\]

As stated previously, in addition to the 16 jumps analysed using RCC an additional 9 jumps (br28 - br43) were analysed at a framing rate of 50 Hz using RVC. The previous equations were used to convert the values obtained for $\phi$ and $\gamma$ into the values that would have been expected if the greater framing rate of 198 Hz had been used. Table 3.3.8 shows the values of plant angle $\phi_{\text{video}}$ and knee angle $\gamma_{\text{video}}$ obtained for jumps br28 - br43 from the analysis using LVC and RVC, and the converted 'cine' values, $\phi_{\text{cine}}$ and $\gamma_{\text{cine}}$. These converted values were then substituted into the regression equation determined previously to obtain a predicted jump height $h_{\text{pred}}$. 

---

### Table 3.3.7. Using the regression equation to predict jump height for the video data

<table>
<thead>
<tr>
<th>Trial</th>
<th>$\phi_{\text{video}}[^\circ]$</th>
<th>$\gamma_{\text{video}}[^\circ]$</th>
<th>$h_{\text{video}}[\text{m}]$</th>
<th>$h_{\text{pred}}[\text{m}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>br28</td>
<td>36.0°</td>
<td>169.8°</td>
<td>2.183</td>
<td>2.168</td>
</tr>
<tr>
<td>br30</td>
<td>32.7°</td>
<td>170.1°</td>
<td>2.176</td>
<td>2.175</td>
</tr>
<tr>
<td>br32</td>
<td>31.7°</td>
<td>167.9°</td>
<td>2.132</td>
<td>2.153</td>
</tr>
<tr>
<td>br34</td>
<td>29.8°</td>
<td>159.7°</td>
<td>2.131</td>
<td>2.074</td>
</tr>
<tr>
<td>br36</td>
<td>33.6°</td>
<td>168.9°</td>
<td>2.206</td>
<td>2.172</td>
</tr>
<tr>
<td>br38</td>
<td>34.3°</td>
<td>171.4°</td>
<td>2.199</td>
<td>2.187</td>
</tr>
<tr>
<td>br40</td>
<td>33.1°</td>
<td>168.3°</td>
<td>2.158</td>
<td>2.167</td>
</tr>
<tr>
<td>br42</td>
<td>34.4°</td>
<td>171.8°</td>
<td>2.177</td>
<td>2.189</td>
</tr>
<tr>
<td>br43</td>
<td>34.8°</td>
<td>169.7°</td>
<td>2.232</td>
<td>2.176</td>
</tr>
</tbody>
</table>
Table 3.3.8. Using the regression equation to predict jump height for the cine converted video data

<table>
<thead>
<tr>
<th>Trial</th>
<th>$\phi_{\text{video}}[^\circ]$</th>
<th>$\gamma_{\text{video}}[^\circ]$</th>
<th>$\phi_{\text{cine}}[^\circ]$</th>
<th>$\gamma_{\text{cine}}[^\circ]$</th>
<th>$h_{\text{video}}[m]$</th>
<th>$h_{\text{pred}}[m]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>br28</td>
<td>36.0°</td>
<td>169.8°</td>
<td>36.2°</td>
<td>173.9°</td>
<td>2.183</td>
<td>2.190</td>
</tr>
<tr>
<td>br30</td>
<td>32.7°</td>
<td>170.1°</td>
<td>32.7°</td>
<td>174.2°</td>
<td>2.176</td>
<td>2.199</td>
</tr>
<tr>
<td>br32</td>
<td>31.7°</td>
<td>167.9°</td>
<td>31.7°</td>
<td>172.3°</td>
<td>2.132</td>
<td>2.179</td>
</tr>
<tr>
<td>br34</td>
<td>29.8°</td>
<td>159.7°</td>
<td>29.6°</td>
<td>165.1°</td>
<td>2.131</td>
<td>2.102</td>
</tr>
<tr>
<td>br36</td>
<td>33.6°</td>
<td>168.9°</td>
<td>33.7°</td>
<td>173.2°</td>
<td>2.206</td>
<td>2.198</td>
</tr>
<tr>
<td>br38</td>
<td>34.3°</td>
<td>171.4°</td>
<td>34.4°</td>
<td>175.4°</td>
<td>2.199</td>
<td>2.210</td>
</tr>
<tr>
<td>br40</td>
<td>33.1°</td>
<td>168.3°</td>
<td>33.1°</td>
<td>172.6°</td>
<td>2.158</td>
<td>2.192</td>
</tr>
<tr>
<td>br42</td>
<td>34.4°</td>
<td>171.8°</td>
<td>34.5°</td>
<td>175.7°</td>
<td>2.177</td>
<td>2.212</td>
</tr>
<tr>
<td>br43</td>
<td>34.8°</td>
<td>169.7°</td>
<td>35.0°</td>
<td>173.9°</td>
<td>2.232</td>
<td>2.200</td>
</tr>
</tbody>
</table>

The standard deviation s.d. in $h_{\text{video}}$ from the mean value over the nine jumps was 0.033 m. The root mean square difference $R$ between $h_{\text{video}}$ and $h_{\text{pred}}$ from Table 3.3.8 was 0.028 m. Over the 9 jumps the experimentally determined regression equation was therefore able to account for 28% of the observed variation in jump height.

\[
\frac{0.033^2 - 0.028^2}{0.033^2} \times 100\% = 28.01\%
\]

3.3.8 Discussion

The present study was carried out in the training environment. The data collection was therefore free from the potential problems imposed by the competition environment. The training environment also provided greater experimental control so that a greater amount and range of data could be collected than observed in a single competition.

Hay (1985) stated that a high jumper may perform differently in the training environment than in competition. It was proposed that these differences may be evident not only in the heights jumped but also in the technique used. The technique observed in training may therefore not be a true representation of the athlete's jump height potential.

The pilot study collected data on ten jumps by Brendan Reilly over two competitions. The jump height varied from 2.22 - 2.35 m in competition B2, and from 2.19 - 2.25 m in competition B3. The corresponding ranges in the approach speed and plant angle were 7.15 - 7.44 m.s$^{-1}$ and 6.95 - 7.29 m.s$^{-1}$ and 36.8 - 38.6° and 37.2 - 41.3° respectively. In the present study a maximum jump height of 2.23 m was achieved. The maximum approach speed was 7.26 m.s$^{-1}$ and the maximum plant angle was 37.3°. These maximum values were comparable with the values observed in competition. Therefore the data collected in the present study were a true representation of the athlete's technique.
Dapena et al. (1990) proposed that a high jumper cannot make many maximum effort jumps in a single day. In the present study a total of 43 jumps were performed by the athlete in a single session. A total of 29 jumps were performed from the athlete's full length competition type approach and may be termed maximum effort. Successful bar clearances were sporadic during these 29 trials, although notably the subject did clear a bar height of 2.20 m on trial number 43. It may therefore be concluded that there was no apparent deterioration of performance during the data collection.

Collecting data in the training environment allowed direct intervention to be used so that some measure of experimental control was obtained. The length of the approach was gradually lengthened producing a range in approach speed of 1.53 m.s\(^{-1}\) over 16 jumps (Table 3.3.2). In contrast 10 competition jumps produced a range of 0.49 m.s\(^{-1}\) in approach speed (Table 3.2.3). It was expected that the changes in approach speed would also induce changes in other approach parameters. In the present study a range of 7.6° in leg plant angle was obtained, compared with a range of 4.5° in competition. The range in knee angle in the present study was 18.3°. No comparable range was presented in the pilot study due to the inaccuracy resulting from the low sampling frequency of 25 Hz.

In the present study 25 jumps were analysed at a sampling rate of 50 Hz. Sixteen of these jumps were also analysed at a sampling rate of 198 Hz. These 16 jumps were used to determine the influence of selected approach parameters on jump height performance, due to the greater accuracy arising from the increased framing rate.

Figure 3.3.8 showed the relationship between approach speed \(v\) and jump height \(h\) for the 16 jumps and gives an indication of the effect of the direct intervention. Generally, as the approach was lengthened the approach speed increased resulting in an increased jump height. A strong \((r^2 = 0.647)\) and significant \((p < 0.001)\) linear correlation was obtained between approach speed and jump height. This relationship implied that the optimum approach speed would be located at the maximum of the experimental data set. The maximum recorded value of approach speed was 7.26 m.s\(^{-1}\). The inclusion of a quadratic term \(v^2\) (Figure 3.3.9) increased the strength of the correlation \((r^2 = 0.734)\). The optimum approach speed was calculated as 7.20 m.s\(^{-1}\). This optimum value must however be treated with some caution. The high \(p\) value \((p = 0.502)\) for the constant term indicated a lack of significance in this coefficient value. The \(p\) values for the coefficients in \(v\) and \(v^2\) showed that both coefficient values were significant at the 90% level.

Figure 3.3.10 showed a fairly strong \((r^2 = 0.468)\) and significant \((p = 0.003)\) linear relationship between leg plant angle \(\phi\) and jump height \(h\), suggesting that the optimum plant angle was located at the maximum of the experimental data. The maximum recorded value was 37.3°. The inclusion of a quadratic term \(\phi^2\) (Figure 3.3.11) resulted in a slight increase in the strength of the correlation \((r^2 = 0.542)\). However the high \(p\) values for each of the coefficient terms in the quadratic regression equation indicated a lack of significance in the optimum plant angle of 36.8°.
The third selected approach parameter was the knee angle $\gamma$ at touchdown, with $180^\circ$ representing a straight leg. Figure 3.3.12 showed the strong ($r^2 = 0.715$) and significant ($p < 0.001$) linear relationship between knee angle and jump height so that a straighter leg at touchdown resulted in a greater jump height. The maximum recorded value was $177.2^\circ$. The inclusion of a quadratic term (Figure 3.3.13) increased the strength of the correlation ($r^2 = 0.730$) with an optimum knee angle of $186.2^\circ$. The large $p$ values for each of the coefficients in the quadratic regression equation suggest a lack of a significant quadratic relationship between knee angle and jump height.

It was expected that the changes in approach speed due to the direct intervention would also bring about changes in the other approach parameters. However the changes in $v$ produced systematic changes in both $\phi$ and $\gamma$ so that the approach parameters were highly correlated with each other. This was particularly true between $v$ and $\gamma$ ($r^2 = 0.827$). The interdependence of the approach parameters made it difficult to establish with any certainty the influence of a single approach parameter on jump height.

The influence of the approach speed on jump height performance was repeated with both parameters linearly detrended for knee angle. Figure 3.3.14 showed a plot of the residual values $h_\gamma$ and $v_\gamma$. Both the linear and quadratic regressions showed no significant relationship due to the strong correlation between $v$ and $\gamma$. Similarly, the relationship between the leg plant angle and jump height was repeated (Figure 3.3.15) using residual values $\phi_\gamma$ and $h_\gamma$. Again no significant linear or quadratic relationship was obtained.

Regressing the residuals in knee angle $\gamma$ of jump height $h$ against plant angle $\phi$ was equivalent to regressing jump height against plant angle and knee angle. The regression equation investigating the quadratic relationship between $h_\gamma$ and $\phi_\gamma$ was reconsidered without using residual values by regressing $h$ against $\phi$, $\phi^2$ and $\gamma$. The resulting regression equation further substantiated the existence of a strong quadratic relationship between plant angle and jump height. The strength of the correlation was high ($r^2 = 0.797$) and the $p$ values for the coefficients in $\phi$, $\phi^2$ and $\gamma$ were significant at the 90% level. The regression equation produced an optimum plant angle of $35.0^\circ$. Previously the quadratic relationship between $h$ and $\phi$, without the inclusion of a term in $\gamma$, produced an optimum plant angle of $36.8^\circ$. The largest recorded plant angle was $37.3^\circ$.

An additional term in $v$ was included in the previous regression resulting in the coefficients in $v$ and $\gamma$ becoming insignificant due to their interdependence. The term in $v$ was therefore removed so that a regression equation was obtained which could predict jump height for given values of leg plant angle and knee angle. The coefficient of determination ($r^2 = 0.797$) showed that the regression equation was able to account for 80% of the observed variance in jump height. The regression equation was used to determine the predicted jump height for each of the 16 jumps used to derive the equation. As expected from the $r^2$ value the regression equation was shown to be able to account for 80% of the observed variation in jump height.
An additional 9 jumps (br28 - br43) analysed at a framing rate of 50 Hz were independent of the regression equation. The plant angle $\phi$ and knee angle $\gamma$ values at touchdown for each of these jumps as determined from the video analysis were substituted into the regression equation. A predicted jump height was subsequently obtained for each jump. Over the 9 independent jumps the regression equation was shown to be able to account for 51% of the observed variation in jump height.

It was speculated that the reduced percentage variation in jump height accounted for by the regression equation using the independent video data was due to systematic differences in the recorded values of $\phi$ and $\gamma$. These systematic differences were proposed to be a result of the different framing rates used. For each of the 16 jumps analysed at both 50 Hz and 198 Hz the plant and knee angle values obtained from the cine analysis were linearly regressed against the corresponding values obtained from the video analysis. Linear regression equations were obtained which allowed the video value $(\phi_{video}, \gamma_{video})$ to be converted to the corresponding cine value $(\phi_{cine}, \gamma_{cine})$.

This conversion was carried out for the 9 independent jumps so that new predicted jump height values were calculated for each jump. Using these data the regression equation was shown to account for 28% of the variation in jump height. The reduced percentage variation accounted for was due in part to the error involved in the conversion of $(\phi, \gamma)_{video}$ into $(\phi, \gamma)_{cine}$. The large p values in the constant coefficients for both $\phi$ and $\gamma$ in the conversion equations indicate a lack of significance which would have produced error in the predicted jump height $h_{pred}$.

The precision errors in $\phi_{video}$ and $\gamma_{video}$ from the analysis using LVC and RVC at 50 Hz were 2.1° and 3.8° respectively. The analysis using LVC and RCC at 198 Hz resulted in precision errors in $\phi_{cine}$ and $\gamma_{cine}$ of 0.6° and 0.8° respectively. The marked reduction in percentage of jump height variation accounted for by the regression equation was therefore due to the reduced accuracy in the values of $\phi$ and $\gamma$ obtained from the video analysis in comparison with the greater framing rate of the cine analysis.

**3.3.9 Conclusions**

There was evidence of linear correlation between jump height and each of the approach parameters within the experimental range of data. For each approach parameter the inclusion of a quadratic term further increased the strength of the correlation. The calculated optimum approach speed and leg plant angle were close to the maximum of the experimental data set. Jump height was further enhanced by planting with a straight leg at touchdown. It may therefore be concluded that the optimum approach was fast, with the leg planted away from the vertical, and with minimum flexion at the knee joint.

A regression equation was obtained for predicting jump height performance from the three approach parameters at touchdown. This regression equation was shown to account for 80% of the observed variation in jump height performance.
3.4 Limitations of the experimental determination of optimum technique

Competition data were shown to be limited by the small range of data observed in approach parameters so that it was difficult to investigate their influence on jump height performance. It was consequently not possible to accurately determine optimum values for the approach parameters. Competition data were further shown to be limited by the amount of data obtained. Data must be amassed over several competitions in order to obtain sufficient data for valid statistical analyses. However the level of performance was shown to vary at different competitions so that it is not valid to carry out statistical analyses of a data set comprising data from different competitions.

The multiple trial study considered many of the methodological problems highlighted by the pilot study. A single data collection session was carried out in the training environment. Direct intervention was successful in obtaining a wider range in approach speed than was observed in competition. Changes in the approach speed caused systematic changes in the leg plant angle and knee angle so that the approach parameters were highly correlated with each other. This interdependence made it difficult to examine the influence of a single approach parameter on jump height.

The experimental determination of optimum technique was therefore limited by a lack of experimental control, since changing one parameter also caused changes in other parameters. Alexander (1990) used a simulation model to show the quadratic relationship between approach speed and jump height. This relationship was obtained at a given value for the leg plant angle and the knee angle. The advantage of the theoretical approach is that it allows a single parameter to be altered, while the others maintain a constant value. This was not possible using an experimental approach.

The single subject study used direct intervention to obtain a wider range in approach speed and other approach parameters than was observed in competition. This was obtained by using data from sub-optimal approaches. The approach was gradually lengthened so that the approach speed increased from a comparatively small value from a five stride approach to a value close to optimum from the full length approach. However, no data were obtained at approach speeds in excess of the optimum value due to the greater potential risk of injury to the subject. With no data in excess of the optimum value there is less confidence in fitting a quadratic function through the data points to extrapolate outside of the experimental range.

In contrast the theoretical approach allows an unlimited amount and range of data to be collected. The theoretical approach predicts the result of a hypothetical situation. There is therefore no direct interaction with a real athlete. Subsequently there is no limitation on the number of jumps performed by the model or the selected values for the approach parameters. Experimental control is therefore much greater using the theoretical approach to determine optimum technique.
CHAPTER 4

THEORETICAL DETERMINATION OF OPTIMUM TECHNIQUE

4.1 Introduction

This chapter details the development of a two segment simulation model of the high jump takeoff. The model will be used to determine the optimum conditions at touchdown which will result in maximising the peak height reached by the mass centre after takeoff. The optimum conditions at touchdown were investigated experimentally in Chapter 3.

The high jump takeoff is performed from a single leg. The takeoff phase is defined as the period of time between the instant when the takeoff foot first touches the ground (touchdown) and the instant when it loses contact with the ground (toe-off). Figure 4.1.1 shows the differences in body configuration between touchdown and toe-off.

![Figure 4.1.1. Changes in body configuration from touchdown to toe-off; (adapted from Dyson, 1986).](image)

At touchdown the body has a backward lean so that the whole body mass centre is located close to the hip of the takeoff leg. During the takeoff phase the mass centre rotates forward so that at toe-off the mass centre is almost directly above the support foot. During the latter part of the takeoff phase the support leg extends at the hip, knee and ankle joints and the free limbs are actively raised. This serves to increase the height of the whole body mass centre. The mass centre therefore also rises upward during the takeoff phase so that at toe-off the mass centre is located a greater distance above the hip joint than at touchdown. This represents a limitation of the simulation model of Alexander (1990) which assumed that the mass centre of the whole body was located at the hip joint.
Figure 4.1.2 shows how the location of the whole body mass centre relative to the hip of the takeoff leg varied during the takeoff phase in an actual jump. The data presented in Figure 4.1.2 were obtained from the analysis of jump br14 conducted in the experimental study presented in Chapter 3.3. The high jumper is represented as a link system joining the ankle, knee, hip and shoulder joints on the takeoff side of the body. The data are presented at a frequency of 50 Hz.

![Diagram of high jump](image)

Figure 4.1.2. Location of the mass centre during the takeoff phase.

Figure 4.1.2 shows the forward and upward movement of the whole body mass centre relative to the hip of the support leg during the takeoff phase. Also evident is the increase in height of the ankle, knee and hip joints of the takeoff leg. This increase in height is due to the extension at these joints during the concentric phase of the takeoff.

### 4.2 Development of a rigid two segment simulation model

The simulation model (Figure 4.2.1) is based largely upon the model of Alexander (1990). The model is two-dimensional in nature, aiming to determine how the conditions just after touchdown influence the peak height reached by the mass centre in flight. No concern is afforded to the body rotation that is evident during the takeoff phase and bar clearance.
Figure 4.2.1. A rigid two segment simulation model of the high jump takeoff.

The model consists of two rigid rods of equal length a connected at a frictionless hinge knee joint. A single torque generator is located at the knee joint, representing the net muscular activity at the knee joint during the takeoff phase. The distal segment represents the shank of the takeoff leg. The shank was defined as a straight line joining the ankle to the knee of the takeoff leg. The proximal segment represents the thigh of the takeoff leg, defined as a straight line joining the knee to the hip of the takeoff leg. The segments are massless, with the entire mass m of the athlete modelled as a point mass located at the hip joint of the takeoff leg.

There is no foot segment. The foot is represented by a point at the distal end of the shank segment. Immediately after touchdown the foot is located at the origin of a Cartesian reference system whose y axis is set to coincide with the ground. Motion is restricted to the yz plane. Movement in the y direction, as defined by the model, is analogous to the direction of travel of the whole body mass centre during the final stride of the approach. The Cartesian coordinates (y, z) of the mass centre allow the determination of the leg plant angle $\phi$. In contrast to the model of Alexander (1990) the plant angle is defined from the vertical, so that the loading on the takeoff leg increases as the initial plant angle is increased.

Throughout the period of ground contact the extensor muscles of the takeoff leg exert a torque at the knee joint. The knee extensor muscles are assumed to be maximally activated throughout ground contact. The torque exerted at the knee is dependent on the rate of change of the knee angle $\gamma$. Additional nomenclature is provided in the following section. The nomenclature includes a description of each of the terms used in the
equations of motion used to drive the simulation model. The analogy for each term as
defined in the computer listing is also included in parentheses. The computer program
modAlex is shown in Appendix 2.

4.2.1 Nomenclature

• Optimisation criterion

\[ z_{h2} \]  \[ \text{Vertical distance that the mass centre is raised in flight after takeoff} \]

• Component locations, velocities and accelerations of whole body mass centre

\[ y \]  \[ \text{Horizontal coordinate of mass centre} \]
\[ \dot{y} \]  \[ \text{Horizontal velocity of mass centre} \]
\[ \ddot{y} \]  \[ \text{Horizontal acceleration of mass centre} \]
\[ z \]  \[ \text{Vertical coordinate of mass centre} \]
\[ \dot{z} \]  \[ \text{Vertical velocity of mass centre} \]
\[ \ddot{z} \]  \[ \text{Vertical acceleration of mass centre} \]

• Segment lengths and model angles

\[ a \]  \[ \text{Segment length} \]
\[ \phi \]  \[ \text{Leg plant angle} \]
\[ \gamma \]  \[ \text{Knee angle} \]

\[ \pi \]  \[ 3.14159265358 \]
\[ [\text{rtd}] \]  \[ \text{Radians-to-degrees} \ 3.14159265358 \]
\[ [\text{dtr}] \]  \[ \text{Degrees-to-radians} \ \pi \]

• Knee torque

\[ T \]  \[ \text{Torque exerted at the knee joint} \]
\[ T_{\text{max}} \]  \[ \text{Maximum torque exerted at the knee joint} \]
\[ \dot{\gamma} \]  \[ \text{Knee angular velocity} \]
\[ \dot{\gamma}_{\text{max}} \]  \[ \text{Maximum knee angular velocity} \]
\[ c \]  \[ \text{Compliance of series elastic muscle component} \]
\[ h \]  \[ \text{Hill's muscle constant} \]
\[ t \]  \[ \text{Time} \]
• **Ground reaction forces**

\[ G \quad [G] \quad \text{Ground reaction force} \]

\[ F \quad [F] \quad \text{Horizontal ground reaction force} \]

\[ R \quad [R] \quad \text{Vertical ground reaction force} \]

\[ m \quad \text{[mass]} \quad \text{Whole body mass} \]

\[ g \quad \text{[grav]} \quad \text{Acceleration due to gravity} \]

### 4.2.2 Model inputs

The aim of the simulation model was to optimise the three approach parameters investigated experimentally in Chapter 3.3. These were the approach speed, the leg plant angle, and the knee angle at touchdown. These approach parameters were input to the simulation model to specify the initial conditions at touchdown.

The approach speed was defined as the horizontal velocity of the whole body mass centre over the final approach stride. The leg plant angle was defined as the angle between the vertical and a straight line joining the ankle and hip joints of the takeoff leg (Figure 3.3.7). The knee angle was defined as the angle between the shank segment and the thigh segment of the takeoff leg (Figure 3.3.7). The approach parameters in the simulation model were therefore as defined in the experimental study of Chapter 3.3. In order to obtain realistic inputs for the simulation model the results presented in Table 3.3.5 from the single subject experimental study were used.

### 4.2.3 Model outputs

• **Optimisation criterion**

The primary output required from the simulation model was the peak height reached by the mass centre, since this was defined as the optimisation criterion. This height was obtained as the sum of two separate heights; the height \((H_1 \text{ in Figure 2.1.1})\) of the mass centre at the instant of toe-off and the height \((H_2 \text{ in Figure 2.1.1})\) that the mass centre was raised in flight. The height of the mass centre at the instant of toe-off was determined by calculating the vertical coordinate \(z_g\) of the mass centre. The height \(z_{h2}\) that the mass centre is raised in flight is dependent on the vertical velocity at toe-off. The jump height or peak height reached by the mass centre was calculated as the sum of \((z_g + z_{h2})\).

• **Time histories**

The simulation model is driven by the single muscle located at the knee joint. The torque exerted at the knee is calculated as a function of the knee angular velocity. The time histories of both the knee angular velocity and the knee torque are output from the model throughout the period of ground contact.
A knowledge of the knee torque at any instant in time allows the calculation of the ground reaction force. The horizontal and vertical components of the ground reaction force are output throughout the takeoff phase. These forces allow the calculation of the component accelerations of the whole body mass centre. Numerical integration over a time step subsequently allows the determination of the component mass centre location and velocity. The new mass centre coordinates allow the determination of the new leg plant angle and knee angle. The time histories of the leg plant angle and knee angle are output by the model throughout the period of ground contact.

The takeoff phase ends when the vertical ground reaction force falls to zero. At the instant of toe-off the vertical coordinate of the mass centre velocity is used to determine the height reached by the mass centre in flight. The total jump height requires that the vertical location of the mass centre at toe-off is also determined.

4.2.4 The equations of motion

From Figure 4.2.1 it can be seen that the component locations of the mass centre are related to the segment length $a$, the leg plant angle $\phi$ and the knee angle $\gamma$.

\[(y^2 + z^2)^{\frac{3}{2}} = 2a\sin(\gamma/2)\]

\[\tan \phi = -\frac{y}{z}\] ... (1)

The input of the plant angle $\phi$ and the knee angle $\gamma$ to the simulation model allows the determination of the mass centre coordinates at touchdown by solving equation (1).

\[y = -z\tan \phi\]
\[z = 2a\sin(\gamma/2)\cos \phi\] ... (2)

The torque generator located at the knee joint is assumed to be fully activated throughout the period of ground contact. During the takeoff the torque exerted at the knee joint depends on the rate of change in length of the contractile component of the knee muscles. Any change $\delta \gamma$ in the knee angle corresponds to a change in length of both the contractile component and the series elastic component of the knee extensor muscles. In the following expression changes in the contractile component are denoted $\delta \gamma_c$. Changes due to the series elastic component are equal to the product of the angular compliance $c$ and the change in knee torque $\delta T$.

\[\delta \gamma = \delta \gamma_c + c.\delta T\] ... (3)
In the absence of series compliance, the rate of change in length of the contractile component of the muscle is therefore equivalent to the rate of change of the knee angle. The assumed relationship between the rate of change in knee angle and the torque exerted at the knee joint is shown in Figure 4.2.2.

![Torque T vs Knee Angular Velocity](image)

**Figure 4.2.2.** The assumed relationship between the knee angular velocity and the torque exerted at the knee.

Figure 4.2.2 defines the following expressions for calculating the knee torque $T$ as a function of the rate of change in knee angle.

\[
\begin{align*}
\text{if } \dot{\gamma} &< 0 & T &= T_{\text{max}} \\
\text{if } \dot{\gamma} &> 0 & T &= T_{\text{max}} \left[ \frac{\dot{\gamma}_{\text{max}} - \dot{\gamma}}{\dot{\gamma}_{\text{max}} + h \dot{\gamma}} \right]
\end{align*}
\]  

... (4)

The torque exerted at the knee joint is therefore maximal throughout the eccentric phase of the takeoff. The second equation, for calculating torque during the concentric phase of the takeoff, is analogous to the muscle equation of Hill (1970).

The ground reaction force $G$ is calculated from the knee torque $T$. The line of action of the ground reaction force is aligned with the point mass centre at the hip. Figure 4.2.1 allows the determination of the perpendicular distance between the line of action of the ground reaction force and the knee joint. The horizontal component $F$ and the vertical component $R$ of the ground reaction force $G$ can then be determined by resolving $G$ given the leg plant angle $\phi$. 
\[ G = \frac{T}{a \cos(Y/2)} \quad F = G \sin \phi \]
\[ R = G \cos \phi \] ... (5)

The component ground reaction forces allow the calculation of the component mass centre accelerations using Newton's Second Law. Given the time iteration step \( \delta t \) the component mass centre velocities and new coordinates can subsequently be obtained.

\[ \ddot{y} = \frac{F}{m} \]
\[ \ddot{z} = \left( \frac{R}{m} \right) - g \]
\[ \dot{y}_1 = y_0 + \dot{y} \delta t \]
\[ \dot{z}_1 = z_0 + \dot{z} \delta t \]
\[ y_1 = y_0 + \dot{y}_0 \delta t + \frac{1}{2} (\ddot{y}_0 \delta t^2) \]
\[ z_1 = z_0 + \dot{z}_0 \delta t + \frac{1}{2} (\ddot{z}_0 \delta t^2) \] ... (6)

The new plant angle \( \phi \) and knee angle \( \gamma \) can then be determined by rearranging equation (1).

\[ \gamma = 2 \sin^{-1} \left( \frac{y^2 + z^2 \delta^2}{2 \cdot a} \right) \]
\[ \phi = - \tan^{-1} \frac{y}{z} \] ... (7)

Differentiating equation (1) in \( \gamma \) with respect to time gives the knee angular velocity.

\[ \dot{\gamma} = \frac{y \dot{y} + z \dot{z}}{a^2 \sin \gamma} \] ... (8)

Equation (3) calculates the rate of shortening of the contractile component of the muscle from the knee angular velocity. Subsequently equation (4) calculates the torque exerted at the knee joint at the next time iteration. Equations (3) to (8) are repeated until the instant of toe-off.

Toe-off occurs when the torque exerted at the knee becomes zero, since at zero torque there is zero ground reaction force. At the instant of toe-off the vertical coordinate of the mass centre represents the height \( H_1 \) in Figure 2.1.1. In the simulation model the mass centre is assumed to be located at the hip joint. In reality however, Figure 4.1.2 shows that the mass centre of the body is located somewhat higher than the hip. Figure 4.1.2 also shows that the mass centre is elevated relative to the hip during the takeoff phase as a result of the upward motion of the free limbs. Extension at the ankle joint also serves to raise the mass centre, whereas in the simulation model the ankle is assumed to be fixed at ground level throughout the period of ground contact.
In order to obtain a realistic value for the height $H_1$ of the mass centre at toe-off a constant term $k_1$ must therefore be added to the vertical coordinate of the mass centre output from the simulation model. The height $H_1$ is calculated as:

$$H_1 = z + k_1 \quad \ldots (9)$$

The peak mass centre height is equal to the sum of the height $H_1$ and the height $z_{h2}$ ($H_2$ in Figure 2.1.1) that the mass centre is raised in flight. The height $z_{h2}$ is calculated as a function of the vertical velocity of the mass centre at the instant of toe-off.

$$z_{h2} = \frac{\dot{z}^2}{2g} \quad \ldots (10)$$

### 4.2.5 Assignment of model parameter values

- **Segment length**

  The two rigid segments forming the model were assumed to be of equal length. The length of the shank segment (ankle-knee) and of the thigh segment (knee-hip) were obtained from anthropometric measurements taken directly from the same elite high jumper that performed the jumps in Chapter 3.3. The full set of anthropometric measurements is shown in Appendix 1a.

  A summary of the anthropometric measurements required for calculating the model segment length $a$ is shown in Table 4.2.1.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right shank</td>
<td>0.473</td>
</tr>
<tr>
<td>Right thigh</td>
<td>0.455</td>
</tr>
</tbody>
</table>

It can be seen that the two segments were of similar length, the shank being 0.018 m longer than the thigh. In order to obtain a single value for the model segment length $a$ the two segment lengths were averaged, so that:

$$a = 0.464 \text{ m}$$
• Muscle parameters

Equation (4), relating the knee torque to the rate of change in knee angle, included a constant term \( h \) and a maximum angular velocity. The constant term \( h \) is equivalent to the parameter \( P_0/a \) in Hill's muscle equation as defined by Woledge et al. (1985).

\[
(P + a)(v + h) = (P_0 + a)b
\]

The relation between the muscle force \( P \) produced at a shortening velocity \( v \) was described as part of a rectangular hyperbola with asymptotes at \(-a\) and \(-b\). In the previous equation \( P_0 \) defines the force exerted during an isometric contraction \((v = 0)\). For this parameter Alexander (1990) stated that a value of 3 was typical for fast mammalian muscle. In the simulation model it was therefore assumed that \( h = 3.0 \).

Alexander cited Bobbert and van Ingen Schenau (1988) in stating that the maximum knee angular velocity during takeoff for standing jumps was about 17 rad.s\(^{-1}\). It was further stated that the unloaded rate of shortening of the quadriceps muscles was likely to be higher than this. Alexander (1990) assumed a value of 35 rad.s\(^{-1}\).

In the simulation model a value of 35 rad.s\(^{-1}\) was assumed for the maximum rate of shortening of the contractile muscle component. In the absence of series compliance this was equal to the maximum knee angular velocity. Alexander used values of 0.0 N.m\(^{-1}\) and 0.0003 N.m\(^{-1}\) for the angular compliance \( c \). Table 4.2.2 shows a summary of the muscle parameters used in the simulation model.

Table 4.2.2. Summary of muscle parameters in model

<table>
<thead>
<tr>
<th>parameter</th>
<th>model value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>3.0</td>
</tr>
<tr>
<td>( \dot{\gamma}_{c, max} )</td>
<td>35.0 rad.s(^{-1})</td>
</tr>
<tr>
<td>( c )</td>
<td>0.0 N.m(^{-1})</td>
</tr>
</tbody>
</table>

• Maximum knee torque

The maximum knee torque was selected so that the model produced jump heights equivalent to the values observed in the single-subject experimental study. The data presented in Table 3.3.5 were used to obtain realistic approach parameters for input to the simulation model, and a realistic jump height for these initial conditions.

The peak jump height \( H_p \) was subdivided into the contributions from the height \( H_1 \) of the mass centre at toe-off and the height \( H_2 \) that the mass centre was raised in flight. Jumps br16 - br26 were used since they were performed from a full length approach and therefore likely to be close to optimum performance. Table 4.2.3 shows the approach
speed, plant angle, knee angle, and the heights H1, H2 and Hp for each jump. The average value over the six jumps is also included in Table 4.2.3.

Table 4.2.3. Approach parameters and jump height performance averaged over jumps br16 to br26

<table>
<thead>
<tr>
<th>jump</th>
<th>approach speed [m.s⁻¹]</th>
<th>plant angle [°]</th>
<th>knee angle [°]</th>
<th>H1 [m]</th>
<th>Hp [m]</th>
<th>H2 [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>br16</td>
<td>7.22</td>
<td>35.8°</td>
<td>177.2°</td>
<td>1.393</td>
<td>2.221</td>
<td>0.828</td>
</tr>
<tr>
<td>br18</td>
<td>7.16</td>
<td>35.0°</td>
<td>175.6°</td>
<td>1.420</td>
<td>2.215</td>
<td>0.795</td>
</tr>
<tr>
<td>br20</td>
<td>7.26</td>
<td>37.3°</td>
<td>176.1°</td>
<td>1.394</td>
<td>2.174</td>
<td>0.780</td>
</tr>
<tr>
<td>br22</td>
<td>6.88</td>
<td>33.6°</td>
<td>172.6°</td>
<td>1.388</td>
<td>2.179</td>
<td>0.791</td>
</tr>
<tr>
<td>br24</td>
<td>6.77</td>
<td>33.9°</td>
<td>170.5°</td>
<td>1.406</td>
<td>2.182</td>
<td>0.776</td>
</tr>
<tr>
<td>br26</td>
<td>6.81</td>
<td>35.9°</td>
<td>173.5°</td>
<td>1.401</td>
<td>2.180</td>
<td>0.779</td>
</tr>
<tr>
<td>mean</td>
<td>7.02</td>
<td>35.3°</td>
<td>174.3°</td>
<td>1.400</td>
<td>2.191</td>
<td>0.791</td>
</tr>
</tbody>
</table>

The mean values of approach speed, plant angle and knee angle from Table 4.2.3 were input to the simulation model. Figure 4.2.3 shows the observed relationship between the maximum torque $T_{\text{max}}$ in the simulation model and the height $z_{h2}$ that the mass centre was raised in flight. The mass centre was assumed to have zero vertical velocity just after impact in all simulations. It was found that a maximum knee torque of 1735 N.m was required raise the mass centre 0.791 m in flight.

![Figure 4.2.3. The relationship between maximum knee torque and the height the mass centre is raised in flight.](image)
• The constant \( k_1 \)

The output from the previous simulation run with \( T_{\text{max}} = 1735 \text{ N.m} \) is included in Appendix 3a. In this simulation the height of the mass centre (hip) at toe-off was 0.836 m. Table 4.2.3 showed that the height \( H_1 \) of the mass centre at takeoff, averaged over six real jumps, was 1.400 m. From equation (9) the value of \( k_1 \) was therefore calculated as 0.564 m. This value of \( k_1 \) was added to the mass centre height at toe-off in all subsequent simulations. The simulation in Appendix 3a therefore produced a peak mass centre height of 2.191 m (0.836 m + 0.564 m + 0.791 m).

4.2.6 Comparison of model and actual performance

Table 3.3.5 showed a summary of initial conditions and jump height performance for sixteen jumps analysed in Chapter 3.3 at the greater framing rate of 198 Hz. The approach speed, leg plant angle and knee angle at touchdown were input to the simulation model for each separate jump. The mass centre was assumed to have zero vertical velocity at touchdown in all simulations. Table 4.2.4 shows the actual jump height performance and the simulated jump height \( (z + 0.564 + z_{h2}) \) output from the simulation model for each jump. Appendix 3b includes the simulation output and ground reaction force time histories for trial br26 as an example.

Table 4.2.4. A comparison of simulated and actual jump height performance

<table>
<thead>
<tr>
<th>Trial</th>
<th>Approach speed [m.s(^{-1})]</th>
<th>Leg plant angle [°]</th>
<th>Knee angle [°]</th>
<th>Actual jump height [m]</th>
<th>Simulated jump height [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>br05</td>
<td>5.73</td>
<td>33.8°</td>
<td>158.9°</td>
<td>2.113</td>
<td>2.070</td>
</tr>
<tr>
<td>br06</td>
<td>6.29</td>
<td>33.7°</td>
<td>163.7°</td>
<td>2.129</td>
<td>2.115</td>
</tr>
<tr>
<td>br07</td>
<td>6.50</td>
<td>33.2°</td>
<td>168.0°</td>
<td>2.172</td>
<td>2.113</td>
</tr>
<tr>
<td>br08</td>
<td>5.87</td>
<td>29.7°</td>
<td>159.0°</td>
<td>2.069</td>
<td>2.031</td>
</tr>
<tr>
<td>br09</td>
<td>6.28</td>
<td>33.8°</td>
<td>172.9°</td>
<td>2.136</td>
<td>2.056</td>
</tr>
<tr>
<td>br10</td>
<td>6.68</td>
<td>34.3°</td>
<td>172.4°</td>
<td>2.207</td>
<td>2.131</td>
</tr>
<tr>
<td>br11</td>
<td>6.62</td>
<td>34.0°</td>
<td>171.9°</td>
<td>2.204</td>
<td>2.120</td>
</tr>
<tr>
<td>br12</td>
<td>6.13</td>
<td>30.9°</td>
<td>166.6°</td>
<td>2.140</td>
<td>2.035</td>
</tr>
<tr>
<td>br13</td>
<td>6.63</td>
<td>34.4°</td>
<td>171.4°</td>
<td>2.208</td>
<td>2.131</td>
</tr>
<tr>
<td>br14</td>
<td>6.73</td>
<td>35.5°</td>
<td>174.4°</td>
<td>2.224</td>
<td>2.145</td>
</tr>
<tr>
<td>br16</td>
<td>7.22</td>
<td>35.8°</td>
<td>177.2°</td>
<td>2.221</td>
<td>2.215</td>
</tr>
<tr>
<td>br18</td>
<td>7.16</td>
<td>35.0°</td>
<td>175.6°</td>
<td>2.215</td>
<td>2.201</td>
</tr>
<tr>
<td>br20</td>
<td>7.26</td>
<td>37.3°</td>
<td>176.1°</td>
<td>2.174</td>
<td>2.251</td>
</tr>
<tr>
<td>br22</td>
<td>6.88</td>
<td>33.6°</td>
<td>172.6°</td>
<td>2.179</td>
<td>2.150</td>
</tr>
<tr>
<td>br24</td>
<td>6.77</td>
<td>33.9°</td>
<td>170.5°</td>
<td>2.182</td>
<td>2.149</td>
</tr>
<tr>
<td>br26</td>
<td>6.81</td>
<td>35.9°</td>
<td>173.5°</td>
<td>2.180</td>
<td>2.170</td>
</tr>
</tbody>
</table>
With the exception of jump br20 the actual jump height was greater than the simulated jump height. From Table 4.2.4 the standard deviation of the actual jump height from the mean value was 0.044 m. The root mean square difference between the actual jump height and the simulated jump height over the 16 jumps was 0.060 m. The rigid two segment model therefore accounted for none of the observed variation in jump height.

4.2.7 Optimum conditions at touchdown

With the model parameters determined previously the simulation model was used to determine the optimum touchdown conditions. A Simulated Annealing optimisation algorithm (Goffe et al., 1994) was used, with the peak mass centre height \((z + k_1 + z_{h2})\) defined as the score function to be maximised. Initially the mean touchdown conditions from Table 4.2.3 were input to the optimisation algorithm. In order to check that the obtained optimum was a global solution the optimisation procedure was repeated from different starting points, i.e. with varying initial conditions. The global optimum solution is included in Appendix 3c and produced the following result:

\[
\begin{align*}
\text{Optimum touchdown conditions} & \quad \ddot{y} = 10.19 \text{ m.s}^{-1} \quad \phi = 51.58^\circ \quad \gamma = 179.0^\circ \\
\text{Resulting jump height} \quad & \quad z_{h2} = 1.257 \text{ m} \quad z_p = 2.704 \text{ m}
\end{align*}
\]

In order to determine the behaviour of the simulation model around optimum performance each of the parameters was varied in turn. Initially the approach speed was varied in multiple simulations with the knee angle at touchdown set to 179.0° and the leg plant angle constant at 51.58°. Figure 4.2.4 shows the non-linear relationship obtained between jump height and approach speed at optimum knee angle at plant angle values. The peak of the curve in Figure 4.2.4 defines the optimum approach speed, previously determined as 10.19 m.s\(^{-1}\).

Similarly, Figure 4.2.5 shows the model determined relationship between jump height and leg plant angle at optimum approach speed and knee angle at touchdown. It can be seen that a non-linear relationship was obtained between jump height and plant angle, with the peak of the curve defining the optimum plant angle of 51.58°.

In Figure 4.2.6 the model determined relationship between jump height and knee angle at touchdown is shown at optimum approach speed and leg plant angle. It can be seen that jump height increased gradually up to a maximum knee angle of 179°, suggesting that jump height was optimised by planting with a straight leg at touchdown.
Figure 4.2.4. The relationship between jump height and approach speed at the model determined optimum knee angle and plant angle.

Figure 4.2.5. The relationship between jump height and leg plant angle at the model determined optimum approach speed and knee angle.
Figure 4.2.6. The relationship between jump height and knee angle at the model determined optimum approach speed and plant angle.

Figure 4.2.7 shows the time history of the component ground reaction forces for the optimum solution. The forces are expressed in body weights.

\[ T_{\text{max}} = 1735 \text{ N.m} \]
\[ \dot{y} = 10.19 \text{ m.s}^{-1} \quad \phi = 51.58^\circ \]
\[ \ddot{z} = 0.00 \text{ m.s}^{-1} \quad \gamma = 179.0^\circ \]

Figure 4.2.7. Time histories of the component ground reaction forces for the optimum touchdown conditions.
4.2.8 Discussion

Both the structure and the equations of motion driving the simulation model developed in this Chapter were equivalent to the model of Alexander (1990). Many of the limitations of the rigid two segment simulation model are therefore as discussed by Alexander.

The aim of the model was to determine the touchdown conditions that maximise the peak height that the mass centre reaches in flight. The rotation and changes in body configuration that occur during the flight phase of the high jump were therefore ignored.

The model was driven by a single torque generator located at the knee joint. This muscle was assumed to be fully active throughout the duration of the takeoff phase. The torque generated by this muscle was calculated as a function of the knee angular velocity, as defined by equation (4). This torque-angular velocity relationship is analogous to the much documented force-velocity relation of muscle (Hill, 1970). However no account was made for the force-length relation of muscle (Woledge et al., 1985). The torque generated at the knee was assumed to be independent of the knee angle. Alexander attempted to validate the omission of the torque-angle relationship on the basis that the knee moved through a small angular range during the takeoff phase. Appendix 3b shows that for the simulation of jump br26 the knee angle at touchdown was 173.50°. The knee angle then decreased to a minimum value of 154.49° before increasing again to a value of 167.87° at toe-off.

Section 4.2.5 described the assignment of the various model parameter values. Table 4.2.2 shows the muscle parameters used in the model. These parameter values were equivalent to those used by Alexander. While Alexander stated that these values were little better than estimates it was also noted that it would be difficult to obtain accurate parameter values. Further, the parameter values used by Alexander were shown to be fairly insensitive in altering the conclusions regarding optimum technique.

Further limitations of the rigid two segment simulation model relate to the inertial characteristics of the model. The segments were assumed to have zero mass. Alexander noted that non-zero segmental mass would have further increased the initial ground reaction force after touchdown, since additional force would have been required to decelerate the segments after impact. The entire body mass is concentrated at the hip joint and is treated as a single point mass. However when a person stands with legs straight and vertical the whole body mass centre is located above the hip joint. At toe-off in the high jump takeoff the mass centre is a greater distance above the hip joint due to the elevation of the free limbs. Figure 4.1.2 shows the time history of the mass centre position relative to the hip joint throughout the takeoff phase of a real high jump.

The lengths of the shank (knee to ankle) and thigh (knee to hip) segments of the takeoff leg were obtained using anthropometric measurements taken directly from the
subject. The thigh segment was 0.018 m longer than the shank segment. Since both segments were of similar length the assumption of two equal length segments seemed reasonable. The anthropometric segment lengths were averaged to obtain a single value of 0.464 m. In comparison Alexander (1990) used a segment length of 0.50 m.

The point of contact with the ground is a point on the distal end of the shank segment. There is no foot segment and subsequently zero foot compliance so that no account is made for the elastic properties of the shod foot. Further, during the high jump takeoff the toe remains in contact with the ground after the heel has lost contact. Analysis of the toe and heel coordinates during the takeoff phase of jump br26 from the film analysis of Chapter 3.3 showed that the heel maintained ground contact for 0.08 s. The total time of takeoff was 0.14 s.

Appendix 3b shows the time histories of the component ground reaction forces throughout the simulated takeoff phase. In this simulation the touchdown conditions from jump br26 were used as input and produced a realistic jump height of 2.170 m. It can be seen that the vertical ground reaction force became negative after 0.029 s. It was stated previously that the actual takeoff time for jump br26 was 0.14 s. This discrepancy must be due in part to the lack of a foot segment. Appendix 3b also shows that the ground reaction forces rose unrealistically rapidly to peak initial values. This must be due in part to the model making no account for the elastic properties of the shod foot.

The pattern of the ground reaction force time histories in Appendix 3b is similar to that presented by Alexander. However the magnitudes of the forces in Appendix 3b are greater than the forces presented by Alexander who showed a peak vertical ground reaction force of 10.0 body weights, followed by a plateau at about 4.5 body weights. Similarly, Aura and Viitasalo (1989) presented a vertical force time history for the takeoff by a high jumper which showed a peak of 8.4 body weights followed by a plateau at about 4.9 body weights. The simulation of jump br26 in Appendix 3b shows a plateau at about 19.0 body weights.

The component ground reaction forces were calculated from the knee torque using equation (5). Alexander used a maximum knee torque of 858 N.m, selected to obtain "reasonably realistic ground forces". In the present simulation model the knee torque was selected so that the model performed realistically in terms of jump height. Over six jumps performed from a full length approach (Chapter 3.3) the average height \( z_{h2} \) that mass centre was raised in flight was 0.791 m (Table 4.2.3). A maximum knee torque of 1735 N.m was required to raise the mass centre 0.791 m in flight given realistic touchdown conditions. It was therefore to be expected that the ground reaction forces output in the present simulation model would be of greater magnitude than the forces presented by Alexander.

Alexander (1990) stated that 858 N.m was likely to be greater than the torques that act at the knees of athletes during the high jump takeoff. It was proposed that an
unrealistically large knee moment was required to produce a realistic ground force. This in turn was proposed to be due to the point of force application being located at the distal end of the shank segment, whereas in reality the ground force probably acts further forward on the foot. The ground reaction forces output by the present simulation model would therefore be expected to exceed the forces observed during real high jumping.

As stated previously a maximum knee torque of \( T_{\text{max}} = 1735 \text{ N.m} \) was required to produce \( z_{h2} = 0.791 \text{ m} \). Figure 4.2.3 shows the relationship between \( T_{\text{max}} \) and \( z_{h2} \). Jump height performance increased almost linearly up to a \( T_{\text{max}} \) of about 1000 N.m. Thereafter the rate of increase in \( z_{h2} \) decreased with increasing \( T_{\text{max}} \). Jump height performance was shown to plateau at about 0.85 m. Increasing \( T_{\text{max}} \) above 1500 N.m was shown to produce little increase in \( z_{h2} \). This is an interesting result since one might assume that increased strength would continue to result in greater jump height performance.

From Figure 4.2.3 it appears that jump height performance may depend not only on the maximum torque but also on the profile of the torque-angular velocity relationship (Figure 4.2.2). As the maximum knee torque was increased in the model the duration of the ground contact decreased. As the maximum torque was increased the takeoff became more like an instantaneous rebound. Jump height performance therefore plateaued so that increasing strength had little effect on performance.

Appendix 3a shows the resulting performance given touchdown conditions as obtained from real performances (Table 4.2.3) with \( T_{\text{max}} \) set to 1735 N.m. The simulation required that the mass centre was raised 0.791 m in flight. At the instant of toe-off the vertical coordinate of the mass centre (hip) was 0.836 m. A constant term \( k_1 = 0.564 \text{ m} \) was added to the simulated mass centre height to produce a mass centre height at toe-off of 1.400 m (Table 4.2.3) and a total jump height of 2.191 m. The magnitude of \( k_1 \) quantifies the limitation of the assumption that the whole body mass centre is located at the hip joint in the model.

The performance of the model was evaluated against real performances using the touchdown conditions for 16 jumps from Chapter 3.3 as input. For each jump the vertical mass centre velocity at touchdown was assumed to be zero. Table 4.2.4 shows that the actual jump height was generally greater than the simulated jump height. The exception was jump br20. It should be noted that this jump had the fastest approach speed, the greatest plant angle and a relatively straight leg at touchdown. Jump br20 was therefore most likely to be closest to optimum performance.

Over the 16 jumps the standard deviation of the actual jump height from the mean value was 0.044 m. The root mean square difference between the actual jump height and the simulated jump height over the 16 jumps was calculated as 0.060 m. The model was therefore able to account for none of the observed variation in jump height. The jump height prediction using the model was therefore less accurate than using a constant value over the 16 jumps. Previously the regression equation determined in Chapter 3.3 relating
jump height to the approach parameters was shown to account for 80% of the observed variation in jump height performance.

Appendix 3b shows the simulated performance of jump br26. The total takeoff time of 0.029 s was unrealistically short due to the limitation discussed previously. As a consequence of the short takeoff time the plant angle at toe-off remained high at 26°. During a real high jump takeoff the mass centre rotates forward to a position over the takeoff foot at toe-off. A plant angle of near zero would therefore have been expected. The knee angle decreased to a realistic minimum value of 154°. However the value of 168° at toe-off was lower than observed in real performances.

The model was used to investigate optimum technique using a Simulated Annealing (Goffe et al., 1994) optimisation algorithm to obtain a global optimum solution. The optimisation procedure was carried out with the vertical mass centre touchdown velocity set to zero in all simulations. The optimum solution, shown in Appendix 3c, produced a jump height of 2.704 m. The mass centre height at toe-off was 1.447 m, slightly greater than any of the values presented in Table 4.2.3 for real performances. However, the mass centre was raised 1.257 m in flight, a marked increase on the previously determined value of 0.791 m. This discrepancy is explained by the optimum touchdown conditions.

The knee angle at touchdown was found to be optimum at 179.0° (Figure 4.2.6). In the optimisation procedure the knee angle was constrained to a maximum value of 179.0°, since a perfectly straight leg would have resulted in infinitely large ground reaction forces. The optimum knee angle was therefore equivalent to the maximum permitted value which supports the results of Chapter 3.3. However the optimum approach speed of 10.19 m.s⁻¹ (Figure 4.2.4) and the optimum leg plant angle of 51.58° (Figure 4.2.5) exceeded the range of experimental data presented in Chapter 3.3. The fastest recorded approach speed was 7.26 m.s⁻¹ and the greatest plant angle was 37.3°.

The unrealistically large values for the optimum approach speed and leg plant angle were a direct consequence of the large maximum torque used in the model. Many coaches advocate a fast and low approach to the high jump bar. The model was strong enough to accommodate an unrealistically fast approach speed and shallow plant angle and was subsequently able to jump unrealistically high.

Figures 4.2.4 to 4.2.6 show the behaviour of the model around optimum performance. Alexander (1990) showed that jump height was maximised at intermediate values of approach speed and plant angle. Although the optimum approach speed and leg plant angle were unrealistically high, both parameters showed the expected non-linear relationship with jump height. Figure 4.2.6 showed that jump height performance increased almost linearly with increasing knee angle at touchdown.
4.2.9 Conclusions

The rigid two segment simulation model required an unrealistically large maximum knee torque of 1735 N.m to produce realistic jump height performance. The large maximum knee torque produced an unrealistic optimum solution. The jump height was 2.704 m with the mass centre raised 1.257 m in flight. The optimum approach speed of 10.19 m.s\(^{-1}\) and the optimum plant angle of 51.58° were greater than the upper limit of the experimental data presented in Chapter 3.3. Jump height was therefore shown to be maximised using a fast approach with the leg planted away from the vertical. Jump height performance was further enhanced by a straight leg at touchdown. The large maximum torque produced unrealistically large ground reaction forces and subsequently a short takeoff time. The model was not able to account for any of the observed variation in jump height over 16 real jumps.

4.3 Modifying the rigid two segment model

The basic form of the modified simulation model was unchanged from the model described previously. Modifications were made with the aim of reducing the maximum knee torque required to produce realistic jump height performance. Subsequently it was to be expected that the optimum solution would become more realistic.

The structure of the model was as described in Chapter 4.2 and shown diagrammatically in Figure 4.2.1. The nomenclature was the same as presented in Section 4.2.1. The model inputs were as defined in Section 4.2.2 with an additional input of the vertical mass centre velocity at touchdown. Previously this parameter was assigned a zero value. In the present model realistic values were obtained from further analysis of the jumps performed in Chapter 3.3. The optimisation procedure therefore included four touchdown parameters; the component mass centre velocities, the leg plant angle and the knee angle. The equations of motion remained unchanged from Section 4.2.4.

4.3.1 Assignment of model parameter values

- Segment length

The assumption of two equal length segments was maintained. The definitions of the shank and thigh segments were varied however. The shank was defined as the distance between the point of foot contact with the ground and the knee. The thigh was defined as the distance between the knee and the mass centre of the whole body less the shank of the takeoff leg. The respective lengths of the shank and thigh segments were obtained from the experimental film analysis of Chapter 3.3. A summary of the average segmental lengths at touchdown obtained from jumps br16 - br26 is shown in Table 4.3.1.
Table 4.3.1. Summary of segmental lengths obtained from film analysis

<table>
<thead>
<tr>
<th>segment</th>
<th>length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>right shank</td>
<td>0.595</td>
</tr>
<tr>
<td>right thigh</td>
<td>0.625</td>
</tr>
</tbody>
</table>

It can be seen that the average thigh segment length was 0.03 m longer than the average shank segment length at touchdown. Since the two segments were of similar length it was deemed valid to obtain a single shank length \( a \) by averaging the two values in Table 4.3.1. The shank segment length was therefore calculated as:

\[ a = 0.610 \text{ m} \]

• **Muscle parameters**

  The muscle parameter values required in the modified simulation model were the same as in the model defined in Chapter 4.2. The assigned parameter values were the same for both models. Performance of the two models could therefore be compared directly. The muscle parameter values used in the modified simulation model were those values presented previously in Table 4.2.2.

• **Maximum knee torque**

  As in Chapter 4.2 the maximum knee torque was selected so that the model produced a realistic mass centre velocity at toe-off, and subsequently that the mass centre was raised an appropriate height in flight.

  The data presented in Table 3.3.5 were used to obtain realistic parameter values for approach speed, plant angle and knee angle for input to the simulation model. Table 4.2.3 showed the approach speed, plant angle, knee angle, and the heights \( H_1, H_2 \) and \( H_p \) for each jump. Jumps br16 - br26 were used since they were likely to be close to optimum performance. The simulation model developed in Chapter 4.2 assumed that the mass centre had zero vertical velocity at touchdown. From the film analysis of jumps br16 - br26 the mass centre coordinates were used to determine the vertical mass centre velocity at touchdown. Table 4.3.2 shows a summary of the approach parameters including the vertical mass centre velocity at touchdown for each jump.
Table 4.3.2. Approach parameters averaged over jumps br16 to br26

<table>
<thead>
<tr>
<th>jump</th>
<th>mass centre velocity [m.s⁻¹]</th>
<th>plant angle [°]</th>
<th>knee angle [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>horizontal  vertical</td>
<td></td>
<td></td>
</tr>
<tr>
<td>br16</td>
<td>7.22 -0.55</td>
<td>35.8°</td>
<td>177.2°</td>
</tr>
<tr>
<td>br18</td>
<td>7.31 -0.45</td>
<td>35.0°</td>
<td>175.6°</td>
</tr>
<tr>
<td>br20</td>
<td>7.26 0.00</td>
<td>37.3°</td>
<td>176.1°</td>
</tr>
<tr>
<td>br22</td>
<td>6.88 -0.25</td>
<td>33.6°</td>
<td>172.6°</td>
</tr>
<tr>
<td>br24</td>
<td>6.77 -0.45</td>
<td>33.9°</td>
<td>170.5°</td>
</tr>
<tr>
<td>br26</td>
<td>6.81 0.15</td>
<td>35.9°</td>
<td>173.5°</td>
</tr>
<tr>
<td>mean</td>
<td>7.02 -0.26</td>
<td>35.3°</td>
<td>174.3°</td>
</tr>
</tbody>
</table>

The mean values of the component mass centre velocities, approach speed, plant angle and knee angle from Table 4.3.2 were used as input to the simulation model. The value of the maximum torque $T_{max}$ in the model was varied until a value of $z_{h2} = 0.791$ m was obtained (Table 4.2.3). Figure 4.3.1 shows the observed relationship between the maximum torque $T_{max}$ in the simulation model and the height $z_{h2}$ that the mass centre was raised in flight. From further simulations it was found that a value of $T_{max} = 1550$ N.m was required to produce a value of $z_{h2} = 0.791$ m in the modified simulation model. Also shown in Figure 4.3.1 is the same relationship for a segment length of $a = 0.464$ m and zero vertical velocity at touchdown, as presented in Figure 4.2.3.

Figure 4.3.1. The modified relationship between the maximum knee torque and the height the mass centre is raised in flight.
The maximum knee torque of 1550 N.m required to raise the mass centre 0.791 m in flight was not dissimilar to the previous value of 1735 N.m derived in Section 4.2.5. Increasing the segment length in the simulation model served to reduce the value of $T_{\text{max}}$ required to produce $z_{h2} = 0.791$ m. However the concurrent assignment of a negative vertical velocity at touchdown from Table 4.3.2 required an increase in $T_{\text{max}}$ to maintain jump height performance. The net effect of the increased segment length and the decreased vertical mass centre touchdown velocity was a minimal change in the maximum knee torque required to raise the mass centre 0.791 m in flight.

It was previously stated that the aim of the modifications to the simulation model was to reduce the assigned value of $T_{\text{max}}$, and thereby obtain a more realistic optimum solution. This was achieved by altering the definition of the shank and thigh segments which served to increase the segment length. Table 4.3.2 showed that the average vertical mass centre velocity at touchdown was $-0.26$ m.s$^{-1}$. Previously a zero value was assumed. Maintaining the increased segment length, but reducing the vertical velocity at touchdown to zero, the relationship between $T_{\text{max}}$ and $z_{h2}$ was reconsidered. The maximum knee torque required to raise the mass centre 0.791 m in flight was calculated as 1455 N.m.

Table 4.3.2 showed that the maximum vertical mass centre velocity at touchdown over the six jumps was 0.15 m.s$^{-1}$. Further increasing the assigned value of the vertical mass centre velocity input to the simulation model to 0.15 m.s$^{-1}$ resulted in a required maximum knee torque of 1400 N.m. Figure 4.3.2 shows the relationship between the maximum knee torque $T_{\text{max}}$ and the height that the mass centre was raised in flight for each assigned value of the vertical mass centre velocity at touchdown.

![Figure 4.3.2](image.png)

**Figure 4.3.2.** The influence of vertical mass centre touchdown velocity on the relationship between $T_{\text{max}}$ and $z_{h2}$.
The influence of vertical mass centre velocity at touchdown on the maximum knee torque required to raise the mass centre 0.791 m in flight is summarised in Table 4.3.3. In each case the horizontal mass centre velocity, the knee angle and the leg plant angle were set to the mean values presented in Table 4.3.2.

Table 4.3.3. The influence of the vertical mass centre touchdown velocity on the assigned value of maximum knee torque

<table>
<thead>
<tr>
<th>( \dot{z} ) [m.s(^{-1})]</th>
<th>( T_{max} ) [N.m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.26</td>
<td>1550</td>
</tr>
<tr>
<td>0.00</td>
<td>1455</td>
</tr>
<tr>
<td>0.15</td>
<td>1400</td>
</tr>
</tbody>
</table>

- The constant \( k_1 \)

The output from the previous simulation run with \( T_{max} \) set to 1550 N.m is included in Appendix 4a. The peak mass centre height in Appendix 4a is the sum of the simulated mass centre height at toe-off, the constant \( k_1 \) and the height \( z_{h2} \) that the mass centre was raised in flight. Table 4.3.4 summarises the contributions to the peak mass centre height \( z_p \) for each simulation in Table 4.3.3. The constant \( k_1 \) was calculated for each simulation using equation (9) derived in Chapter 4.2 to produce a realistic peak mass centre height.

Table 4.3.4. Determining the constant \( k_1 \) for varying vertical mass centre touchdown velocity

<table>
<thead>
<tr>
<th>( T_{max} ) [N.m]</th>
<th>( z ) [m]</th>
<th>( k_1 ) [m]</th>
<th>( z_{h2} ) [m]</th>
<th>( z_p ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1550</td>
<td>1.127</td>
<td>0.273</td>
<td>0.791</td>
<td>2.191</td>
</tr>
<tr>
<td>1455</td>
<td>1.126</td>
<td>0.274</td>
<td>0.791</td>
<td>2.191</td>
</tr>
<tr>
<td>1400</td>
<td>1.126</td>
<td>0.274</td>
<td>0.791</td>
<td>2.191</td>
</tr>
</tbody>
</table>

4.3.2 Comparison of model and actual performance

Table 3.3.5 showed a summary of touchdown conditions and jump height performance for 16 jumps analysed in Chapter 3.3 at the greater framing rate of 198 Hz. The approach speed, leg plant angle and knee angle at touchdown were input to the modified simulation model for each separate jump.
The vertical mass centre velocity was also determined for each jump and input to the modified simulation model. Table 4.3.5 shows the initial touchdown conditions and jump height performance for each of the sixteen real jumps.

Table 4.3.5. A summary of the touchdown conditions input to the modified simulation model

<table>
<thead>
<tr>
<th>Trial</th>
<th>$\dot{y}_g$ [m.s$^{-1}$]</th>
<th>$\dot{z}_g$ [m.s$^{-1}$]</th>
<th>$\phi$ [$^\circ$]</th>
<th>$\gamma$ [$^\circ$]</th>
<th>Actual jump height [m]</th>
<th>Simulated jump height [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>br05</td>
<td>5.73</td>
<td>-0.55</td>
<td>33.8$^\circ$</td>
<td>158.9$^\circ$</td>
<td>2.113</td>
<td>2.077</td>
</tr>
<tr>
<td>br06</td>
<td>6.29</td>
<td>-0.40</td>
<td>33.7$^\circ$</td>
<td>163.7$^\circ$</td>
<td>2.129</td>
<td>2.122</td>
</tr>
<tr>
<td>br07</td>
<td>6.50</td>
<td>-0.20</td>
<td>33.2$^\circ$</td>
<td>168.0$^\circ$</td>
<td>2.172</td>
<td>2.128</td>
</tr>
<tr>
<td>br08</td>
<td>5.87</td>
<td>-0.05</td>
<td>29.7$^\circ$</td>
<td>159.0$^\circ$</td>
<td>2.069</td>
<td>2.057</td>
</tr>
<tr>
<td>br09</td>
<td>6.28</td>
<td>-0.40</td>
<td>33.8$^\circ$</td>
<td>172.9$^\circ$</td>
<td>2.136</td>
<td>2.063</td>
</tr>
<tr>
<td>br10</td>
<td>6.68</td>
<td>-0.60</td>
<td>34.3$^\circ$</td>
<td>172.4$^\circ$</td>
<td>2.207</td>
<td>2.128</td>
</tr>
<tr>
<td>br11</td>
<td>6.62</td>
<td>-0.50</td>
<td>34.0$^\circ$</td>
<td>171.9$^\circ$</td>
<td>2.204</td>
<td>2.122</td>
</tr>
<tr>
<td>br12</td>
<td>6.13</td>
<td>-0.40</td>
<td>30.9$^\circ$</td>
<td>166.6$^\circ$</td>
<td>2.140</td>
<td>2.052</td>
</tr>
<tr>
<td>br13</td>
<td>6.63</td>
<td>-0.50</td>
<td>34.4$^\circ$</td>
<td>171.4$^\circ$</td>
<td>2.208</td>
<td>2.131</td>
</tr>
<tr>
<td>br14</td>
<td>6.73</td>
<td>-0.50</td>
<td>35.5$^\circ$</td>
<td>174.4$^\circ$</td>
<td>2.224</td>
<td>2.136</td>
</tr>
<tr>
<td>br16</td>
<td>7.22</td>
<td>-0.55</td>
<td>35.8$^\circ$</td>
<td>177.2$^\circ$</td>
<td>2.221</td>
<td>2.174</td>
</tr>
<tr>
<td>br18</td>
<td>7.16</td>
<td>-0.45</td>
<td>35.0$^\circ$</td>
<td>175.6$^\circ$</td>
<td>2.215</td>
<td>2.188</td>
</tr>
<tr>
<td>br20</td>
<td>7.26</td>
<td>0.00</td>
<td>37.3$^\circ$</td>
<td>176.1$^\circ$</td>
<td>2.174</td>
<td>2.253</td>
</tr>
<tr>
<td>br22</td>
<td>6.88</td>
<td>-0.25</td>
<td>33.6$^\circ$</td>
<td>172.6$^\circ$</td>
<td>2.179</td>
<td>2.159</td>
</tr>
<tr>
<td>br24</td>
<td>6.77</td>
<td>-0.45</td>
<td>33.9$^\circ$</td>
<td>170.5$^\circ$</td>
<td>2.182</td>
<td>2.150</td>
</tr>
<tr>
<td>br26</td>
<td>6.81</td>
<td>0.15</td>
<td>35.9$^\circ$</td>
<td>173.5$^\circ$</td>
<td>2.180</td>
<td>2.194</td>
</tr>
</tbody>
</table>

The touchdown conditions for each jump from Table 4.3.5 were input to the simulation model. For each jump three scenarios were considered. Each of the maximum knee torque values presented in Table 4.3.4 were used. For each case the peak mass centre height ($z + k_1 + z_{h2}$) output from the simulation model is shown in Table 4.3.6. Table 4.3.6 also shows the actual jump height performance. Appendix 4b includes the simulation for trial br26 with $T_{max} = 1550$ N.m as an example.

It can be seen that the actual jump height was generally greater than the simulated jump height. Exceptions were trial br20 with a fast approach speed and shallow leg plant angle, and trial br26 with a relatively large vertical touchdown velocity.
Table 4.3.6. The influence of maximum knee torque on simulated jump height performance

<table>
<thead>
<tr>
<th>Trial</th>
<th>Actual jump height [m]</th>
<th>Simulated jump height [m]</th>
<th>T_{max} = 1550 N.m</th>
<th>1455 N.m</th>
<th>1400 N.m</th>
</tr>
</thead>
<tbody>
<tr>
<td>br05</td>
<td>2.113</td>
<td>2.071</td>
<td>2.056</td>
<td>2.046</td>
<td></td>
</tr>
<tr>
<td>br06</td>
<td>2.129</td>
<td>2.116</td>
<td>2.100</td>
<td>2.088</td>
<td></td>
</tr>
<tr>
<td>br07</td>
<td>2.172</td>
<td>2.121</td>
<td>2.107</td>
<td>2.098</td>
<td></td>
</tr>
<tr>
<td>br08</td>
<td>2.069</td>
<td>2.049</td>
<td>2.034</td>
<td>2.024</td>
<td></td>
</tr>
<tr>
<td>br09</td>
<td>2.136</td>
<td>2.060</td>
<td>2.054</td>
<td>2.049</td>
<td></td>
</tr>
<tr>
<td>br10</td>
<td>2.207</td>
<td>2.125</td>
<td>2.112</td>
<td>2.103</td>
<td></td>
</tr>
<tr>
<td>br11</td>
<td>2.204</td>
<td>2.118</td>
<td>2.106</td>
<td>2.097</td>
<td></td>
</tr>
<tr>
<td>br12</td>
<td>2.140</td>
<td>2.044</td>
<td>2.032</td>
<td>2.024</td>
<td></td>
</tr>
<tr>
<td>br13</td>
<td>2.208</td>
<td>2.128</td>
<td>2.115</td>
<td>2.105</td>
<td></td>
</tr>
<tr>
<td>br14</td>
<td>2.224</td>
<td>2.139</td>
<td>2.129</td>
<td>2.121</td>
<td></td>
</tr>
<tr>
<td>br16</td>
<td>2.221</td>
<td>2.200</td>
<td>2.186</td>
<td>2.174</td>
<td></td>
</tr>
<tr>
<td>br18</td>
<td>2.215</td>
<td>2.192</td>
<td>2.177</td>
<td>2.166</td>
<td></td>
</tr>
<tr>
<td>br20</td>
<td>2.174</td>
<td>2.263</td>
<td>2.249</td>
<td>2.238</td>
<td></td>
</tr>
<tr>
<td>br22</td>
<td>2.179</td>
<td>2.154</td>
<td>2.140</td>
<td>2.130</td>
<td></td>
</tr>
<tr>
<td>br24</td>
<td>2.182</td>
<td>2.145</td>
<td>2.130</td>
<td>2.118</td>
<td></td>
</tr>
<tr>
<td>br26</td>
<td>2.180</td>
<td>2.190</td>
<td>2.180</td>
<td>2.172</td>
<td></td>
</tr>
</tbody>
</table>

The standard deviation in the actual jump height from the mean value over the 16 jumps was 0.044 m. The root mean square difference between the actual jump height and the simulated over the 16 jumps was determined for each value of T_{max}. The root mean square values for maximum knee torque values of 1550 N.m, 1455 N.m and 1400 N.m were 0.061 m, 0.069 m and 0.076 m respectively.

The root mean square difference between the actual jump height and the simulated jump height was therefore greater than the standard deviation in the actual jump height for each value of T_{max}. The model was therefore able to account for none of the observed variation in jump height over the 16 jumps.

In the present model the vertical mass centre velocity at touchdown was obtained from the film analysis of Chapter 3.3 for each jump. In Chapter 4.2 the model assumed zero vertical touchdown velocity in all simulations. The modified model was used to repeat the simulations presented in Table 4.3.6 with zero vertical touchdown velocity for each jump. The results are presented in Table 4.3.7.
Table 4.3.7. The influence of maximum knee torque on simulated jump height performance with zero vertical touchdown velocity

<table>
<thead>
<tr>
<th>Trial</th>
<th>Actual jump height [m]</th>
<th>Simulated jump height [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$T_{\text{max}} = 1550$ N.m</td>
</tr>
<tr>
<td>br05</td>
<td>2.113</td>
<td>2.087</td>
</tr>
<tr>
<td>br06</td>
<td>2.129</td>
<td>2.131</td>
</tr>
<tr>
<td>br07</td>
<td>2.172</td>
<td>2.128</td>
</tr>
<tr>
<td>br08</td>
<td>2.069</td>
<td>2.051</td>
</tr>
<tr>
<td>br09</td>
<td>2.136</td>
<td>2.070</td>
</tr>
<tr>
<td>br10</td>
<td>2.207</td>
<td>2.146</td>
</tr>
<tr>
<td>br11</td>
<td>2.204</td>
<td>2.135</td>
</tr>
<tr>
<td>br12</td>
<td>2.140</td>
<td>2.052</td>
</tr>
<tr>
<td>br13</td>
<td>2.208</td>
<td>2.145</td>
</tr>
<tr>
<td>br14</td>
<td>2.224</td>
<td>2.158</td>
</tr>
<tr>
<td>br16</td>
<td>2.221</td>
<td>2.228</td>
</tr>
<tr>
<td>br18</td>
<td>2.215</td>
<td>2.215</td>
</tr>
<tr>
<td>br20</td>
<td>2.174</td>
<td>2.263</td>
</tr>
<tr>
<td>br22</td>
<td>2.179</td>
<td>2.164</td>
</tr>
<tr>
<td>br24</td>
<td>2.182</td>
<td>2.164</td>
</tr>
<tr>
<td>br26</td>
<td>2.180</td>
<td>2.183</td>
</tr>
</tbody>
</table>

The root mean square values for maximum knee torque values of 1550 N.m, 1455 N.m and 1400 N.m were 0.050 m, 0.056 m and 0.062 m respectively over the 16 jumps. The standard deviation in the actual jump height from the mean value over the 16 jumps was 0.044 m.

The root mean square difference between the actual jump height and the simulated jump height was therefore greater than the standard deviation in the actual jump height for each value of $T_{\text{max}}$. The model was therefore able to account for none of the observed variation in jump height over the 16 jumps with the assumption of zero vertical touchdown velocity.

4.3.3 Optimum conditions at touchdown

With the increased segment length and modified maximum knee torque the simulation model was used to determine the optimum touchdown conditions. The same Simulated Annealing optimisation algorithm was used as in Chapter 4.2, with the vertical mass centre velocity at touchdown also optimised. The score function to be optimised was again the peak mass centre height ($z + k_1 + z_{K2}$). The optimisation procedure was carried out for each value of $T_{\text{max}}$ in Table 4.3.4.
For each value of $T_{\text{max}}$ the vertical mass centre touchdown velocity was constrained to an upper limit of zero. This procedure was adopted since preliminary optimisations showed that this parameter was optimised at the maximum permitted level. Upper bounds were therefore set in an attempt to obtain a realistic optimum solution. Constraining the vertical mass centre touchdown velocity to zero produced results directly comparable with the optimum solution obtained in Chapter 4.2 for the model with a shorter segment length.

In order to check that the obtained optimum was a global solution the optimisation procedure was repeated from different starting points, i.e. with varying initial conditions. The global optimum solution for each value of $T_{\text{max}}$ with the vertical touchdown velocity constrained to zero is shown in Table 4.3.8.

Table 4.3.2 showed that the average vertical touchdown velocity over six real jumps was $-0.26 \text{ m.s}^{-1}$. The optimisation procedure was repeated with the vertical touchdown velocity constrained to a maximum value of $-0.26 \text{ m.s}^{-1}$. The global optimum solution for $T_{\text{max}} = 1550 \text{ N.m}$ is shown in Table 4.3.8 and is included in Appendix 4c.

<table>
<thead>
<tr>
<th>$T_{\text{max}}$</th>
<th>$\dot{z}_{\text{max}}$</th>
<th>$\dot{y}$</th>
<th>$\dot{z}$</th>
<th>$\phi$</th>
<th>$\gamma$</th>
<th>$z_p$</th>
<th>$z_k$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[N.m]</td>
<td>[m.s$^{-1}$]</td>
<td>[m.s$^{-1}$]</td>
<td>[°]</td>
<td>[°]</td>
<td>[m]</td>
<td>[m]</td>
<td>[m]</td>
<td></td>
</tr>
<tr>
<td>1550</td>
<td>0.00</td>
<td>9.51</td>
<td>0.00</td>
<td>51.11°</td>
<td>179.0°</td>
<td>2.608</td>
<td>1.156</td>
<td>1.452</td>
</tr>
<tr>
<td>1455</td>
<td>0.00</td>
<td>9.21</td>
<td>0.00</td>
<td>50.59°</td>
<td>179.0°</td>
<td>2.533</td>
<td>1.078</td>
<td>1.455</td>
</tr>
<tr>
<td>1400</td>
<td>0.00</td>
<td>9.02</td>
<td>0.00</td>
<td>50.24°</td>
<td>179.0°</td>
<td>2.488</td>
<td>1.033</td>
<td>1.455</td>
</tr>
<tr>
<td>1550</td>
<td>-0.26</td>
<td>9.37</td>
<td>-0.26</td>
<td>51.71°</td>
<td>179.0°</td>
<td>2.556</td>
<td>1.101</td>
<td>1.455</td>
</tr>
</tbody>
</table>

With $T_{\text{max}}$ set to 1550 N.m and a vertical mass centre touchdown velocity of $-0.26 \text{ m.s}^{-1}$, the behaviour of the model around optimum performance was examined by varying each of the parameters in turn.

Figure 4.3.3 shows the relationship between jump height and approach speed obtained from the simulation model at optimum vertical mass centre velocity, plant angle and knee angle at touchdown. It can be seen that there is a non-linear relationship between approach speed and jump height. The peak of the curve in Figure 4.3.3 defines the optimum approach speed, previously determined as 9.37 m.s$^{-1}$ (Table 4.3.8). Also shown in Figure 4.3.3 is the relationship between jump height and approach speed from the original model developed in Chapter 4.2.
Figure 4.3.3. The relationship between jump height and approach speed at optimum vertical velocity, knee angle and leg plant angle.

Figure 4.3.4 shows the model determined relationship between jump height and leg plant angle at optimum approach speed and knee angle at touchdown. The peak of the curve defines the optimum plant angle of 51.71°. Also shown in Figure 4.3.4 is the relationship between jump height and plant angle from the original model developed in Chapter 4.2.

Figure 4.3.4. The relationship between jump height and leg plant angle at optimum approach speed, vertical velocity and knee angle.
In Figure 4.3.5 the model determined relationship between jump height and knee angle at touchdown is shown at optimum approach speed and leg plant angle. Also shown in Figure 4.3.5 is the relationship between jump height and knee angle from the original model developed in Chapter 4.2.

![Graph showing relationship between jump height and knee angle](image)

Figure 4.3.5. The relationship between jump height and knee angle at optimum approach speed, vertical velocity and leg plant angle.

The optimum solution for $T_{\text{max}} = 1550$ N.m with the vertical mass centre touchdown velocity constrained to a maximum value of $-0.26$ m.s$^{-1}$ is included in Appendix 4c. Figure 4.3.6 shows the time history of the component ground reaction forces for the optimum solution. The forces are expressed in units of body weight. Also shown in Figure 4.3.6 are the ground reaction force time histories for the optimum solution derived from the original model developed in Chapter 4.2.
The aim of the modifications to the simulation model was to use a more realistic value for the segment length with the expectation that this would reduce the maximum knee torque required to produce realistic jump height performance. The components of the model and the equations of motion driving the model were unchanged. The limitations previously discussed in Section 4.2.8 therefore still apply to the modified model. The sole structural change to the model was an increase in the segment length \( a \).

The previous anthropometrically measured shank length made no account of the distance that the ankle is raised off the ground during the takeoff phase (Figure 4.1.2). In the modified model the shank length was determined from the film analysis of Chapter 3.3 as the distance between the knee and the ground at touchdown. Similarly, the anthropometrically measured thigh length in the original model made no account for the fact that the mass centre is located above the hip joint and its location relative to the hip changes during the takeoff phase (Figure 4.1.2). In the modified model the proximal end of the thigh segment was defined as the location of the mass centre of the whole body less the shank of the takeoff leg. This mass centre location was obtained from the film analysis of Chapter 3.3. Table 4.3.1 showed the shank and thigh segment lengths at touchdown averaged over six real jumps. The modified segment length was calculated as 0.610 m, compared with the anthropometrically determined value of 0.464 m.
The muscle parameter values presented in Table 4.2.2 were maintained to allow a direct investigation of the influence on performance of the change in segment length. The criteria for the assignment of the maximum knee torque remained as requiring the mass centre to be raised 0.791 m in flight given realistic touchdown conditions. The modified model considered the influence on performance of the vertical mass centre velocity at touchdown. In the original model a zero value was assumed for this parameter. Table 4.3.2 showed that over six real jumps performed from a full length approach the average vertical mass centre velocity at touchdown was -0.26 m.s\(^{-1}\). With the mean touchdown conditions in Table 4.3.2 used as input the modified model required a maximum knee torque of 1550 N.m to raise the mass centre 0.791 m in flight. The output from this simulation is shown in Appendix 4a.

Appendix 4a shows that the total takeoff time for this simulation was 0.050 s. This unrealistically short takeoff time is a result of the large maximum knee torque which produces large ground reaction forces. As discussed for the original model the short takeoff time produced an unrealistic plant angle value at toe-off. In the performance of a real high jump the mass centre rotates forward during the takeoff so that at toe-off the mass centre is over the takeoff foot, giving a near zero plant angle. In Appendix 4c the plant angle was 22° at toe-off.

Figure 4.3.1 shows the relationship between knee torque \( T_{\text{max}} \) and the height \( z_{h2} \) that the mass centre is raised in flight for the modified model. Also shown is the corresponding relationship for the original model with \( T_{\text{max}} = 1735 \) N.m. It can be seen that the relationship between \( T_{\text{max}} \) and \( z_{h2} \) was very similar to that discussed previously for the original model. Jump height performance was shown to plateau at about 0.90 m with increased strength greater than a maximum knee torque of about 1750 N.m producing little improvement in performance.

In the original model the vertical mass centre touchdown velocity was assumed to be zero. With zero vertical mass centre velocity in the modified model a maximum knee torque of 1455 N.m was required to raise the mass centre 0.791 m in flight. Relative to the original model with \( T_{\text{max}} = 1735 \) N.m, the increase in the segment length therefore resulted in a marked decreased in the required maximum knee torque. Table 4.3.2 showed that over six real jumps the maximum recorded mass centre velocity was 0.15 m.s\(^{-1}\). With this touchdown condition input to the modified model a maximum knee torque of 1400 N.m was required to raise the mass centre 0.791 m in flight. Table 4.3.3 summarises the influence of the vertical mass centre touchdown velocity on the required maximum knee torque.

Figure 4.3.2 showed that the relationship between \( T_{\text{max}} \) and \( z_{h2} \) for the three considered vertical mass centre touchdown velocities was very similar in each case. For a given \( T_{\text{max}} \) a greater vertical touchdown velocity was shown to increase jump height performance. In each case jump height performance was shown to plateau at about
0.90 m. This phenomenon, discussed in Section 4.2.8 for the original model, is due to the large maximum torque values. As the muscle is made stronger the takeoff becomes increasingly like an instantaneous rebound.

Table 4.3.4 showed that the value of $k_1$ required to produce a realistic mass centre height at toe-off ranged from 0.273 - 0.274 m over the three values of $T_{\text{max}}$. In comparison the original model required a $k_1$ value of 0.564 m. The increased segment length in the modified model therefore produced a more realistic simulated mass centre height at toe-off, and subsequently a marked reduction in the assigned value of $k_1$.

The modified model was evaluated against real performances using the touchdown conditions in Table 4.3.5 as input. These touchdown conditions, determined from the film analysis of Chapter 3.3, included the vertical mass centre touchdown velocities. Each of the three values of $T_{\text{max}}$ were considered. In each case the root mean square difference between the actual jump height and the simulated jump height was greater than the standard deviation of the actual jump height from the mean value over the 16 jumps. As with the previously developed model, the modified model was therefore not able to account for any of the observed variation in jump height.

In the original model the vertical mass centre touchdown velocity was assumed to be zero for each jump. The evaluation of the modified model was repeated assuming zero vertical touchdown velocity in each simulation (Table 4.3.7). With zero vertical touchdown velocity the modified model was still not able to account for any of the observed variation in jump height performance. The predicted jump height from the simulation model was therefore less accurate than using a constant jump height value over the 16 jumps.

Appendix 4b shows the simulation of jump br26 with $T_{\text{max}}$ set to 1550 N.m and a vertical touchdown velocity of 0.15 m.s\(^{-1}\). As in Appendix 4a the time of takeoff is unrealistically short as a result of the large maximum knee torque and subsequently the large ground reaction forces. The short takeoff time resulted in a large plant angle of approximately $25^\circ$ at toe-off.

The optimum touchdown conditions were investigated for each of the three values of $T_{\text{max}}$ in the modified simulation model. As in Chapter 4.2 the optimisation procedure was carried out using a Simulated Annealing (Goffe et al., 1994) optimisation routine. Preliminary simulations showed that both the knee angle and vertical mass centre velocity at touchdown were optimised at the maximum permitted level.

The knee angle at touchdown was therefore constrained to a maximum value of $179.0^\circ$, since a perfectly straight leg would have resulted in infinitely large ground reaction forces. Optimisations were carried out for $T_{\text{max}}$ with the vertical touchdown velocity constrained to a maximum value of zero to allow direct comparison with the results obtained from the original model. Table 4.3.8 shows a summary of the optimum solutions.
In the original model a $T_{\text{max}}$ of 1735 N.m produced an optimum approach speed of 10.19 m.s$^{-1}$ and an optimum plant angle of 51.58°, with the mass centre raised 1.257 m in flight. Table 4.3.8 shows that reducing $T_{\text{max}}$ by increasing the segment length resulted in a slight decrease in the optimum approach speed (9.02 - 9.51 m.s$^{-1}$), and an optimum plant angle that was slightly closer to the vertical (50.24° - 51.11°). This resulted in a decrease in jump height performance relative to the original model, with the mass centre raised from 1.033 m to 1.156 m in flight. The increased segment length therefore produced a more realistic optimum performance. However, the optimum approach parameters remained outside the upper bound of the experimental data (Chapter 3.3).

The average vertical mass centre touchdown velocity was -0.26 m.s$^{-1}$. Also shown in Table 4.3.8 is the optimum solution with $T_{\text{max}}$ set to 1550 N.m and the vertical touchdown velocity constrained to a maximum value of -0.26 m.s$^{-1}$. This simulation is included in Appendix 4c. The optimum approach speed was 9.37 m.s$^{-1}$ and the optimum leg plant angle was 51.71°, with the mass centre raised 1.101 m in flight. Appendix 4c shows that the total time of takeoff for this simulation was 0.135 s. The increased takeoff time relative to Appendix 4b resulted in a more realistic plant angle of approximately 10° at toe-off. However the knee angle decreased to a minimum value of 116°. This knee flexion is much greater than is observed during the performance of a real high jump. Analysis of the knee angle time histories for the jumps presented in Chapter 3.3 showed a typical minimum knee angle of 155°. The great knee flexion evident in the optimum solution is further evidence that the assigned maximum knee torque is too great. In Appendix 4c the knee extends to only 159° at toe-off. In comparison, knee angles in excess of 170° were evident in the real jumps performed.

Figure 4.3.6 shows the time histories of the component ground reaction forces throughout the takeoff phase for the optimum solution. Also shown are the corresponding time histories for the original model. It can be seen that the takeoff time of 0.135 s was more realistic in the modified model. In the real jumps performed in Chapter 3.3 the takeoff time was typically around 0.14 s. This increased takeoff time relative to the original model was due to a decrease in the magnitude of the ground reaction forces. The decreased ground reaction forces were a direct consequence of the reduced maximum torque in the modified model.

Figures 4.3.3 to 4.3.5 showed the behaviour of the model around optimum performance. Figure 4.3.3 shows the non-linear relationship between approach speed and jump height. It can be seen that the reduced maximum knee torque produced a decrease in the optimum approach speed relative to the original model, with decreased jump height for a given approach speed. Similarly Figure 4.3.4 shows the non-linear relationship between plant angle and jump height. The reduced maximum knee torque produced a slight increase in the optimum leg plant angle at 51.71° relative to 51.58° in the original model. Jump height was shown to be decreased for a given plant angle in the modified
model. Figure 4.3.5 showed that jump height increased almost linearly with increasing knee angle at touchdown, up to a maximum value of 179.0°. This near linear relationship was also observed for the original model, however the smaller maximum torque in the modified model resulted in decreased jump height for a given knee angle.

4.3.5 Conclusions

Increasing the segment length in the rigid two segment model was effective in reducing the maximum knee torque required to produce realistic jump height performance. With a vertical mass centre touchdown velocity of -0.26 m.s\(^{-1}\) a maximum torque of 1550 N.m was required. The reduction in maximum knee torque resulted in decreased magnitude of the ground reaction forces and consequently a more realistic takeoff time relative to the original model.

The reduction in maximum knee torque also produced a slightly more realistic optimum solution relative to the original model. However the optimum approach speed of 9.37 m.s\(^{-1}\) and the optimum plant angle of 51.71° remained outside of the upper bound of the experimental data. The mass centre was raised 1.101 m in flight, reaching a peak height of 2.556 m. The maximum knee torque used in the two segment simulation model was therefore shown to be too large, producing an unrealistic optimum solution. The modified model was unable to account for any of the observed variation in jump height performance over 16 jumps.

The large maximum knee torque required to produce realistic performance is due to the simplicity of the rigid two segment model. This simplicity produces limitations as previously discussed. One of the limitations of the present rigid model is that it is initiated with the distal end of the shank segment already in contact with the ground. It may be speculated that the required maximum joint torque and subsequently the optimum touchdown conditions would become more realistic if the model were to incorporate the initial impact phase of the high jump takeoff.

The results of the optimisation using the rigid two segment simulation model show that jump height performance is maximised by using a fast approach with the leg planted away from the vertical. Jump height was shown to be further enhanced by planting with a straight leg at touchdown.
4.4 Development of an elastic two segment simulation model

The model remains two-dimensional in nature, aiming to determine how the conditions at touchdown influence the peak height reached by the mass centre in flight. No concern is afforded to the body rotation that is evident during the takeoff phase and bar clearance. The takeoff is therefore modelled as a running one legged planar takeoff. This is not dissimilar to the high jump approach. Analysis of the jumps presented in Chapter 3.3 showed that the approach angle over the final approach stride is very similar to the direction of travel of the mass centre during the flight phase.

The simulation model, shown in Figure 4.4.1, consists of two rigid rods of fixed length connected at a frictionless hinge knee joint. A single torque generator is located at the knee joint, representing the net muscular activity at the knee joint during the takeoff phase. In these respects the model is equivalent in structure to the models developed previously. In the present model however the assumptions of equal length and massless segments were removed. The whole body mass is therefore no longer represented as a single point mass located at the proximal end of the thigh segment.

The elastic nature of the takeoff phase is considered in the present model. Attached to the distal end of the distal segment, representing the foot, is a double spring system. This allows the initial impact to be considered, whereas previously the model required the foot to be stationary and already in contact with the ground.

Figure 4.4.1. An elastic two segment simulation model of the high jump takeoff.
The distal segment, hereafter referred to as the shank segment, actually represents the shank-plus-foot of the takeoff leg. During the eccentric phase of the takeoff the length of this segment equates to the ankle-to-knee distance plus the distance of the ankle from the ground. During the concentric phase the ankle joint extends so that the ankle is raised as shown in Figure 4.1.2. The distal segment length is then equivalent to the toe-to-knee distance.

The proximal segment, hereafter referred to as the thigh segment, joins the knee to the mass centre of the whole body less the shank of the takeoff leg. Figure 4.1.2 showed that the mass centre of the whole body rotates forward and upward during the takeoff phase. This is due in part to the elevation of the arms and free leg along with extension at the hip joint during the takeoff phase. The length of the thigh segment therefore varies during the takeoff.

The extension at the ankle and hip joints, and the actions of the free limbs contribute to raising the mass centre during the takeoff phase. In the simulation model the contribution from the extension at the knee joint of the takeoff leg is considered. The muscular activity at the knee joint is represented by the torque generator located at the knee. The torque exerted is calculated as a function of the knee angular velocity. A schematic representation and free body diagram of the simulation model is shown in Figure 4.4.2. Motion is restricted to the yz plane. Movement in the y direction, as defined by the model, is analogous to the direction of travel of the whole body mass centre during the final stride of the approach.

Figure 4.4.2. Schematic representation and free body diagram of the elastic model.
The shank segment has a length $c$ and mass $m_a$. The mass centre of this segment is located a distance $a$ from the distal end of the segment. The thigh segment has a length $b$ equivalent to the distance between the knee joint and the mass centre of the whole body less the shank of the takeoff leg. The thigh segment has a mass $m_b$, equivalent to the whole body mass $m$ minus the shank segment mass $m_a$. The configuration of the model is specified by the segmental lengths and orientation angles. The point of contact with the ground is at the origin $O$. Additional nomenclature is provided in the following section. The nomenclature includes a description of each of the terms used in the equations of motion used to drive the simulation model. The analogy for each term as defined in the computer listing is also included in parentheses. The computer program 2seg is shown in Appendix 5.

4.4.1 Nomenclature

- **Optimisation criterion**
  
  $z_{h2}$ \([zh2]\) Vertical distance that the mass centre is raised in flight after takeoff

- **Location and velocity of whole body mass centre**
  
  $y_g$ \([yg]\) Horizontal coordinate of whole body mass centre
  $\dot{y}_g$ \([ygd]\) Horizontal velocity of whole body mass centre
  $z_g$ \([zg]\) Vertical coordinate of whole body mass centre
  $\dot{z}_g$ \([zgd]\) Vertical velocity of whole body mass centre

- **Locations, velocities and accelerations of segmental mass centres**
  
  $y_a$ \([ya]\) Horizontal coordinate of shank segment mass centre
  $\dot{y}_a$ \([yad]\) Horizontal velocity of shank segment mass centre
  $\ddot{y}_a$ \([yadd]\) Horizontal acceleration of shank segment mass centre
  $z_a$ \([za]\) Vertical coordinate of shank segment mass centre
  $\dot{z}_a$ \([zad]\) Vertical velocity of shank segment mass centre
  $\ddot{z}_a$ \([zadd]\) Vertical acceleration of shank segment mass centre

  $y_b$ \([yb]\) Horizontal coordinate of thigh segment mass centre
  $\dot{y}_b$ \([ybd]\) Horizontal velocity of thigh segment mass centre
  $\ddot{y}_b$ \([ybdd]\) Horizontal acceleration of thigh segment mass centre
  $z_b$ \([zb]\) Vertical coordinate of thigh segment mass centre
  $\dot{z}_b$ \([zbd]\) Vertical velocity of thigh segment mass centre
  $\ddot{z}_b$ \([zbdd]\) Vertical acceleration of thigh segment mass centre
• **Locations, velocities and accelerations of the foot**

\[ y_s \] Horizontal coordinate of foot
\[ \dot{y}_s \] Horizontal velocity of foot
\[ \ddot{y}_s \] Horizontal acceleration of foot
\[ z_s \] Vertical coordinate of foot
\[ \dot{z}_s \] Vertical velocity of foot
\[ \ddot{z}_s \] Vertical acceleration of foot

• **Segmental lengths**

| \( a \) | Length from distal end of shank segment to shank segment mass centre |
| \( b \) | Length from knee to whole-body-less-shank mass centre |
| \( c \) | Length from distal end of shank segment to knee |

• **Segment orientation angles**

\[ \theta \] Angle between the shank segment and the horizontal
\[ \dot{\theta} \] Angular velocity of \( \theta \)
\[ \ddot{\theta} \] Angular acceleration of \( \theta \)
\[ \sigma \] Angle between the thigh segment and the horizontal
\[ \dot{\sigma} \] Angular velocity of \( \sigma \)
\[ \ddot{\sigma} \] Angular acceleration of \( \sigma \)

• **Leg plant angle and knee angle**

\[ \phi \] Leg plant angle
\[ \phi_f \] Leg plant angle as defined in Chapter 3.3
\[ \gamma \] Knee angle
\[ \gamma_{\text{min}} \] Minimum knee angle
\[ \gamma' \] Knee angular velocity
\[ \dot{\gamma} \] Knee angular acceleration
\[ \gamma_f \] Knee angle as defined in Chapter 3.3

\[ \pi \] 3.14159265358
\[ \text{[rtd]} \] Radians-to-degrees \([= 180/\pi]\)
\[ \text{[dtr]} \] Degrees-to-radians \([= \pi/180]\)

• **Spring parameters**

\[ k_{sy} \] Spring stiffness in the horizontal direction
\[ k_{sz} \] Spring stiffness in the vertical direction
\[ k_{vy} \] Spring damping in the horizontal direction
\[ k_{vz} \] Spring damping in the vertical direction
• Knee torque

\( T_k \) [tork] Torque exerted at the knee joint
\( T_o \) Moment of force about the origin O
\( k_f \) Constant scaling factor for muscle torque

\( h_k \) Angular momentum about the knee joint
\( \dot{h}_k \) Rate of change of \( h_k \)
\( h_o \) Angular momentum about the origin
\( \dot{h}_o \) Rate of change of \( h_o \)

\( I_a \) [ia] Moment of inertia of the shank segment about its mass centre
\( I_b \) [ib] Moment of inertia of the thigh segment about its mass centre

• Ground reaction forces

\( F \) [F] Horizontal ground reaction force
\( R \) [R] Vertical ground reaction force

\( m_a \) [ma] Mass of the shank segment
\( m_b \) [mb] Mass of the thigh segment
\( m \) [mass] Whole body mass \([= m_a + m_b]\)
\( g \) [grav] Acceleration due to gravity

4.4.2 Model inputs

In order to obtain realistic inputs for the simulation model the results presented in Chapter 3.3 from the single-subject experimental study were used. The model inputs were those parameters which were to be optimised. The aim of the simulation model was to optimise the three approach parameters investigated experimentally in Chapter 3.3. These were the horizontal velocity of the athlete’s whole body mass centre over the final approach stride, the leg plant angle, and the knee angle at touchdown. The input values for these touchdown parameters were as defined in Chapter 3.3. This allowed direct comparison of the simulated performance with the actual performance.

Although not considered in the experimental determination of optimum technique, the vertical velocity of the mass centre just before touchdown was also input to the simulation model. Although not presented in Chapter 3.3, the vertical velocity of the whole body mass centre just before touchdown was obtained from the analysis of each jump so that a realistic value could be input to the simulation model.

Further analysis of the real jumps performed in Chapter 3.3 was required in order to determine the angular velocities in both the leg plant angle and the knee angle just prior to touchdown. These angular velocities were determined from the angle time histories.
4.4.3 Model outputs

- **Optimisation criterion**
  The primary output required from the simulation model was the peak height reached by the mass centre, since this was defined as the optimisation criterion. This height was obtained as the sum of two separate heights; the height of the mass centre at the instant of toe-off and the height that the mass centre was raised in flight.

  The height (H1 in Figure 2.1.1) of the mass centre at the instant of toe-off was determined by calculating the vertical coordinate \( z_g \) of the mass centre. The location \((y_g, z_g)\) of the mass centre relative to the origin was calculated throughout the takeoff as a function of the segmental lengths and orientations, with an additional contribution from the spring system.

  The height \( z_{h2} \) (H2 in Figure 2.1.1) that the mass centre is raised in flight was dependent on the vertical velocity at toe-off. The time histories of the component mass centre velocities were output throughout the takeoff phase. The jump height or peak height reached by the mass centre was calculated as the sum of \((z_g + z_{h2})\).

- **Time histories**

  It was previously described how the time history of the mass centre location was calculated from the segmental lengths and orientations. In order to calculate the height \( z_g \) of the mass centre at toe-off the actual instant of toe-off had to be determined in the model. This was done by considering the time histories of the ground reaction forces during the takeoff.

  The horizontal component \( F \) and the vertical component \( R \) of the ground reaction force were calculated using the component stiffness \((k_{sy}, k_{sz})\) and damping \((k_{vy}, k_{vz})\) parameters of the spring system. Also used in the calculation of the ground reaction forces \( F \) and \( R \) were the time histories of the component locations and velocities of the spring system.

  The instant of toe-off was defined as the instant when the vertical ground reaction force \( R \) became zero. At the instant of toe-off the knee angle and leg plant angle were determined. These angles were calculated from the segmental lengths and orientation angles \( \theta \) and \( \sigma \). Time histories of the segmental lengths and orientation angles were therefore output from the model. This allowed the calculation of the time histories of the knee angle \( \gamma \) and the plant angle \( \phi \) throughout the takeoff phase. The knee angle \( \gamma_{\text{min}} \) at maximum knee flexion could therefore be determined.

  The time histories of \( \theta \) and \( \sigma \) allowed the determination of the changes in the segmental angular velocities and accelerations throughout the takeoff. The knee angular velocity and acceleration could therefore be determined at any instant during the takeoff phase. The angular velocity at the knee joint was required in order to calculate the torque
acting at the knee joint throughout the period of ground contact. The time history of the knee joint torque was output from the simulation model.

The time histories of the segmental lengths and orientation angles also allowed the calculation of the component locations, velocities and accelerations of the segmental mass centres throughout the takeoff. Subsequently the time histories of the component locations and velocities of the whole body mass centre could be output from the model. As described previously, the vertical mass centre coordinate and vertical velocity at toe-off were used to determine the peak mass centre height.

4.4.4 Transformation of the plant and knee angles

The leg plant angle $\phi_l$ and knee angle $\gamma_l$ at touchdown were input to the simulation model, as stated previously. The values of $\phi$ and $\gamma$ input to the simulation model were determined from the film analysis of Chapter 3.3. In the experimental determination of optimum technique the leg plant angle was defined as the angle between the vertical and a straight line joining the ankle and hip joints of the takeoff leg. In the simulation model the definition was changed so that the plant angle was measured as the angle between the vertical and a straight line joining the point of contact with the ground to the mass centre of the whole body less the shank of the takeoff leg.

The knee angle was defined as the angle between the shank and thigh segments of the takeoff leg. In the film analysis the shank was defined as joining the ankle to the knee, and the thigh as joining the knee to the hip of the takeoff leg. In the simulation model the shank joins the point of contact with the ground to the knee, and the thigh joins the knee to the mass centre of the whole body less shank.

The plant and knee angle values calculated in the simulation model were subsequently not directly comparable with the values presented in the film analysis of Chapter 3.3. Figure 4.4.3 illustrates the varying definitions between the plant angle $\phi_l$ from the experimental study and the angle $\phi_m$ in the model, and between the knee angle $\gamma_l$ from the experimental study and the model angle $\gamma_m$. It was therefore necessary to convert the input film values of $\phi$ and $\gamma$ into the corresponding model values.

In order to determine the required conversion factor the location of the whole-body less-shank mass centre was determined for each of the jumps analysed in Chapter 3.3. The revised definitions for the plant angle $\phi_m$ and knee angle $\gamma_m$ from Figure 4.4.3 were used to obtain time histories of the model defined plant and knee angles for each jump. These time histories were directly comparable with those originally obtained for the film defined plant and knee angles in Chapter 3.3.
Figure 4.4.3. Differences in the definition of the leg plant angle and knee angle between the experimental film analysis and the simulation model.

Figure 4.4.4 shows the respective time histories of $\gamma_m$ and $\gamma_t$ throughout the takeoff phase for jump br20. Time zero represents the instant of touchdown.

Figure 4.4.4. The knee angle time history as defined in the film analysis and as defined in the simulation model for jump br20.
Figure 4.4.5 shows the respective time histories of $\phi_m$ and $\phi_t$ for jump br20. Time zero represents the instant of touchdown.

Figure 4.4.5. The plant angle time history as defined in the film analysis and as defined in the simulation model for jump br20.

Figures 4.4.4 and 4.4.5 showed that the pattern of the time histories of both $\gamma$ and $\phi$ were similar between the film values and the model values for jump br26. The model knee angle and plant angle were output throughout the takeoff phase in each simulation. For the purpose of evaluation against real performances the model was also required to output film values of the knee angle at touchdown, maximum flexion and toe-off. The model was also required to output film values of the plant angle at touchdown and toe-off.

Figure 4.4.4 showed that the knee angle was similar between film and model definitions at touchdown. However, thereafter the model knee angle was greater than the film knee angle. This would have been expected from the relative definitions shown in Figure 4.4.3. The difference between the model and film knee angles was greater at maximum knee flexion than at toe-off. It was therefore not valid to fit a single linear regression for determining the difference between the film and model knee angles for the entire takeoff phase.

Similarly, Figure 4.4.5 showed that the plant angle was very similar between film and model definitions at touchdown. Thereafter the film plant angle was consistently greater than the model value, as would have been expected from Figure 4.4.3. The relationship between the film and model defined plant angles was therefore different at touchdown and toe-off.
A total of 16 jumps were analysed at the greater framing rate of 198 Hz in the experimental study of Chapter 3.3. Linear regression was used to obtain relationships between film and model angles over the 16 jumps for the knee angle and leg plant angle at touchdown.

- **Knee angle $\gamma$ at touchdown**

  At touchdown the film knee angle $\gamma_f$ was input to the simulation model. This value was then converted to the model defined angle $\gamma_m$. The model angle $\gamma_m$ was therefore linearly regressed against the film angle $\gamma_f$ over the 16 jumps.

  \[
  y_m = 14.820 + 0.933 \gamma_f \quad \text{s.e.} = 1.858^\circ \quad r^2 = 0.867 \quad p_c = 0.380 \quad p_\gamma < 0.001
  \]

  The resulting regression equation showed a strong linear correlation ($r^2 = 0.867$). However the significance of the coefficient in the constant term was low ($p = 0.380$). The regression was therefore forced through the origin in order to remove the uncertainty in the constant term.

  \[
  y_m = 1.02202 \gamma_f \quad \text{s.e.} = 1.846^\circ \quad r^2 = 0.867 \quad p_\gamma < 0.001
  \]

- **Minimum knee angle $\gamma$**

  The minimum knee angle $\gamma_{m,\min}$ was calculated in the simulation model from the output joint angle time history. In order to evaluate the performance of the simulation model with real jumps the film value $\gamma_{f,\min}$ was calculated from the film angle $\gamma_{m,\min}$.

  \[
  \gamma_{f,\min} = -36.74 + 1.129 \gamma_{m,\min} \quad \text{s.e.} = 1.187^\circ \quad r^2 = 0.898 \quad p_c = 0.037 \quad p_\gamma < 0.001
  \]

- **Knee angle $\gamma$ at toe-off**

  The knee angle at toe-off was also used to evaluate the performance of the model. The model angle $\gamma_m$ at toe-off was subsequently required to be converted into the film angle $\gamma_f$. The film angle $\gamma_f$ was therefore linearly regressed against the model angle $\gamma_m$.

  \[
  \gamma_f = 215.43 - 0.231 \gamma_m \quad \text{s.e.} = 3.188^\circ \quad r^2 = 0.046 \quad p_c = 0.001 \quad p_\gamma = 0.423
  \]

  The regression equation produced a weak linear correlation ($r^2 = 0.046$) and uncertainty in the coefficient term in $\gamma$. The mean value of $\gamma_m$ over the 16 jumps was 175.81°. The corresponding value of $\gamma_f$ was 174.89°. It was therefore assumed that at toe-off:

  \[
  \gamma_f = \gamma_m - 0.92^\circ
  \]
• **Plant angle \( \phi \) at touchdown**

At touchdown the film plant angle \( \phi_f \) determined from Chapter 3.3 was input to the model. This value was then converted to the model defined angle \( \phi_m \). The model angle \( \phi_m \) was therefore linearly regressed against the film angle \( \phi_f \) over the 16 jumps.

\[
\phi_m = -1.144 + 1.003 \phi_f \\
\text{s.e.} = 1.001^\circ \\
\text{r}^2 = 0.812 \\
P_c = 0.799 \\
P_\phi < 0.001
\]

The resulting regression equation showed a strong linear correlation \( (r^2 = 0.812) \). However the significance of the coefficient in the constant term was low \( (p = 0.799) \). The regression was therefore forced through the origin in order to remove the uncertainty in the constant term.

\[
\phi_m = 0.96957 \phi_f \\
\text{s.e.} = 0.970^\circ \\
\text{r}^2 = 0.867 \\
P_{\phi} < 0.001
\]

• **Plant angle \( \phi \) at toe-off**

The plant angle at toe-off was used to evaluate the performance of the model. The model angle \( \phi_m \) at toe-off was subsequently required to be converted into the film angle \( \phi_f \). The film angle \( \phi_f \) was therefore linearly regressed against the model angle \( \phi_m \).

\[
\phi_f = 2.238 + 0.431 \phi_m \\
\text{s.e.} = 1.657^\circ \\
\text{r}^2 = 0.373 \\
P_c < 0.001 \\
P_\phi = 0.012
\]

4.4.5 **Equations of motion**

Having converted the input plant and knee angles into the model defined plant and knee angles at touchdown the orientation angles of each segment could be determined. Figure 4.4.6 shows the definition of the segment orientation angles in the simulation model. The shank segment angle relative to the horizontal is defined \( \theta \). The thigh segment angle relative to the horizontal is defined \( \sigma \).

The angles \( \alpha \) and \( \beta \) are used to describe a geometrical approximation which simplifies the calculation of the segment orientation angles. From Figure 4.4.6 it can be seen that:

\[
\theta = 90^\circ - (\phi - \beta) \\
\sigma = 90^\circ - (\phi + \alpha)
\]

A geometrical approximation was used which assumed that the segments were of equal length. This simplified the geometry so that:

\[
\alpha = \beta = 90^\circ - \frac{\gamma}{2}
\]
The segment orientation angles at touchdown were then calculated as follows:

\[ \theta = 90° - \phi + \beta \]
\[ \sigma = 90° - \phi - \beta \]
\[ = 180° - \frac{\gamma}{2} - \phi \]
\[ = \frac{\gamma}{2} - \phi \]

Differentiating these expressions in \( \theta \) and \( \sigma \) with respect to time produced the segmental angular velocities.

\[ \dot{\theta} = -\frac{\dot{\gamma}}{2} - \dot{\phi} \]
\[ \dot{\sigma} = \frac{\dot{\gamma}}{2} - \dot{\phi} \]

From Figure 4.4.2 it was then possible to determine the horizontal and vertical coordinates \( (y_a, z_a) \) of the mass centre of the shank segment relative to the point of impact. The location coordinates of the shank mass centre relative to the distal end of the shank segment are a function of the segment length and orientation angle. In addition, there is a contribution from the location coordinates of the spring system.

By differentiating the location coordinates with respect to time the component velocities of the shank segment mass centre were determined. Differentiation of the component velocities enabled the component accelerations of the shank segment mass centre to be determined.
\[
\begin{align*}
  y_a &= -a\cos\theta + y_s \\
  \dot{y}_a &= a\dot{\theta}\sin\theta + \dot{y}_s \\
  \ddot{y}_a &= a\ddot{\theta}\sin\theta - a\dot{\theta}\cos\theta + \ddot{y}_s \\
  z_a &= a\sin\theta + z_s \\
  \dot{z}_a &= a\dot{\theta}\cos\theta + \dot{z}_s \\
  \ddot{z}_a &= a\ddot{\theta}\cos\theta - a\dot{\theta}\sin\theta + \ddot{z}_s
\end{align*}
\]

The location coordinates \((y_b, z_b)\) of the mass centre of the thigh segment relative to the origin were also determined from the free body diagram. These expressions were used to determine the component velocities and accelerations of the mass centre of the thigh segment. The equations used to determine each of these terms are shown below.

\[
\begin{align*}
  y_b &= -c\cos\theta - b\cos\sigma + y_s \\
  \dot{y}_b &= c\dot{\theta}\sin\theta + b\dot{\sigma}\sin\sigma + \dot{y}_s \\
  \ddot{y}_b &= c\ddot{\theta}\cos\theta + c\dddot{\theta}\sin\theta + b\dot{\sigma}\cos\sigma + b\dot{\sigma}\sin\sigma + \ddot{y}_s \\
  z_b &= c\sin\theta + b\sin\sigma + z_s \\
  \dot{z}_b &= c\dot{\theta}\cos\theta + b\dot{\sigma}\cos\sigma + \dot{z}_s \\
  \ddot{z}_b &= -c\ddot{\theta}\sin\theta + c\dddot{\theta}\cos\theta - b\dot{\sigma}^2\sin\sigma + b\ddot{\sigma}\cos\sigma + \ddot{z}_s
\end{align*}
\]

Using equations (1) and (2) it is possible to determine an expression for calculating the component velocities of the whole body mass centre from the segmental velocities.

\[
\begin{align*}
  (m_a + m_b)\dot{y}_g &= m_a\dot{y}_a + m_b\dot{y}_b \\
  &= m_a a\sin\theta\dot{\theta} + m_b (c\sin\theta\dot{\theta} + b\sin\sigma\dot{\sigma}) + (m_a + m_b)\dot{y}_s \\
  (m_a + m_b)\dot{z}_g &= m_a\dot{z}_a + m_b\dot{z}_b \\
  &= m_a a\cos\theta\dot{\theta} + m_b (c\cos\theta\dot{\theta} + b\cos\sigma\dot{\sigma}) + (m_a + m_b)\dot{z}_s
\end{align*}
\]

Rearranging these equations allowed the calculation of the component velocities of the spring system, representing the foot segment.

\[
\begin{align*}
  \dot{y}_s &= \dot{y}_g - \frac{m_a}{m_a + m_b} a\sin\theta\dot{\theta} - \frac{m_b}{m_a + m_b} (c\sin\theta\dot{\theta} + b\sin\sigma\dot{\sigma}) \\
  \dot{z}_s &= \dot{z}_g - \frac{m_a}{m_a + m_b} a\cos\theta\dot{\theta} - \frac{m_b}{m_a + m_b} (c\cos\theta\dot{\theta} + b\cos\sigma\dot{\sigma})
\end{align*}
\]
Rearranging these equations produces two simultaneous equations in the segmental angular velocities.

\[
\dot{\theta}(m_a a \sin \theta + m_b c \sin \theta) + \dot{\sigma}(m_b b \sin \sigma) = (m_a + m_b)(\dot{\gamma}_g - \dot{\gamma}_s)
\]

\[
\dot{\theta}(m_a a \cos \theta + m_b c \cos \theta) + \dot{\sigma}(m_b b \cos \sigma) = (m_a + m_b)(\dot{\gamma}_g - \dot{\gamma}_s)
\]

These simultaneous equations were solved to give the angular velocity of both segments.

let ...

\[
k_1 = (m_a a + m_b c) \sin \theta
\]

\[
k_2 = m_b b \sin \sigma
\]

\[
k_3 = (m_a a + m_b c) \cos \theta
\]

\[
k_4 = m_b b \cos \sigma
\]

so that ...

\[
\dot{\theta} k_1 + \dot{\sigma} k_2 = (m_a + m_b)(\dot{\gamma}_g - \dot{\gamma}_s)
\]

\[
\dot{\theta} k_3 + \dot{\sigma} k_4 = (m_a + m_b)(\dot{\gamma}_s - \dot{\gamma}_s)
\]

Rearranging equation (4) gives an expression for the thigh angular velocity.

\[
\dot{\sigma} = \frac{(m_a + m_b)(\dot{\gamma}_g - \dot{\gamma}_s)}{k_2} \dot{\theta} k_1
\]

Substitution into equation (5) allows the determination of the angular velocity of the shank segment.

\[
\dot{\theta} k_3 + \frac{(m_a + m_b)(\dot{\gamma}_s - \dot{\gamma}_s) k_4}{k_2} - \dot{\theta} k_1 k_4 = (m_a + m_b)(\dot{\gamma}_g - \dot{\gamma}_s)
\]

\[
\dot{\theta} k_2 k_3 - \dot{\theta} k_4 = (m_a + m_b)(\dot{\gamma}_g - \dot{\gamma}_s) k_2 - (m_a + m_b)(\dot{\gamma}_s - \dot{\gamma}_s) k_4
\]

\[
\dot{\theta} = \frac{(m_a + m_b)((\dot{\gamma}_g - \dot{\gamma}_s) k_2 - (\dot{\gamma}_s - \dot{\gamma}_s) k_4)]}{k_2 k_3 - k_1 k_4}
\]

Similarly, the equation (4) may be rearranged to give an expression for the shank angular velocity. Substitution into equation (5) then allows the determination of the angular velocity of the thigh segment.
\[ \dot{\theta} = \frac{(m_a + m_b)\ddot{y}_g - \ddot{y}_s}{k_1} - \dot{\sigma}k_2 \]

\[ \frac{(m_a + m_b)(\ddot{y}_g - \ddot{y}_s)k_3}{k_1} - \dot{\sigma}k_2k_3 + \dot{\sigma}k_4 = (m_a + m_b)(\ddot{y}_g - \ddot{y}_s)k_1 \]

\[ \dot{\sigma}k_1k_4 - \dot{\sigma}k_2k_3 = (m_a + m_b)(\ddot{y}_g - \ddot{y}_s)k_1 - (m_a + m_b)(\ddot{y}_g - \ddot{y}_s)k_3 \]

\[ \dot{\sigma} = \frac{(m_a + m_b)(\ddot{y}_g - \ddot{y}_s)k_1 - (\ddot{y}_g - \ddot{y}_s)k_3}{k_1k_4 - k_2k_3} \quad \ldots (7) \]

Having determined the segmental angular velocities it is possible to determine the angular velocity, and subsequently the angular acceleration of the knee joint.

\[ \gamma = 180^{\circ} + \sigma - \theta \]

\[ \dot{\gamma} = \dot{\sigma} - \dot{\theta} \]

\[ \ddot{\gamma} = \ddot{\sigma} - \ddot{\theta} \quad \ldots (8) \]

In order to determine the torque acting about the point of contact with the ground it was necessary to derive an expression for the angular momentum \( h_o \) of both segments.

\[ h_o = I_a\dot{\theta} + m_b\ddot{y}z_a - m_b\ddot{y}z_a \\
+ I_b\ddot{\sigma} + m_b\ddot{y}z_b - m_b\ddot{y}z_b \]

The torque \( T_o \) about the point of contact with the ground due to the weights of the two segments is equal to the rate of change of the angular momentum about the origin.

\[ T_o = (m_a\ddot{y}_a + m_b\ddot{y}_b)g = h_o \]

\[ = I_a\ddot{\theta} + m_b\ddot{y}z_a - m_b\ddot{y}z_a \\
+ I_b\ddot{\sigma} + m_b\ddot{y}z_b - m_b\ddot{y}z_b \quad \ldots (9) \]

Similarly, in order to determine the torque about the knee joint an expression for the angular momentum \( h_k \) of the thigh segment about the knee was derived.

\[ h_k = I_b\ddot{\sigma} + m_b\ddot{z}_b(z_b - z_k) + m_b\ddot{z}_b(y_b - y_k) \]

where \( z_k \) is the fixed vertical coordinate of the knee

\[ (z_b - z_k) = b\sin\sigma \quad (y_b - y_k) = -b\cos\sigma \]
The moment about the knee joint due to the weight of the thigh segment and the knee extensor torque $T_k$ is equal to the rate of change of the angular momentum of the thigh segment about the knee joint.

$$T_k - m_b g b \cos \sigma = \dot{h}_k$$

$$= I_b \ddot{\theta} + m_b y_b \sin \sigma + m_b z_b \cos \sigma \quad \ldots \quad (10)$$

As described previously, the horizontal ground reaction force $F$ and the vertical ground reaction force $R$ can be calculated using the spring parameters. The ground reaction forces can also be calculated using Newton's Second Law.

$$F = -(k_{sy} y_s + k_{vy} \dot{y}_s) = m_a \ddot{y}_a + m_b \ddot{y}_b$$

$$R = -(k_{sz} z_s + k_{vz} \dot{z}_s) = m_a \ddot{z}_a + m_b \ddot{z}_b + mg$$

$$\ldots \quad (11) \quad \ldots \quad (12)$$

Equations (1) and (2) defined the component segmental mass centre locations, velocities and accelerations. These terms were substituted into equations (9) to (12).

$$T_o = (m_a y_a + m_b y_b) g = I_a \ddot{\theta} + m_a y_a z_a - m_a y_a z_a$$

$$= I_a \ddot{\theta} + m_a \ddot{y}_a + m_b \ddot{y}_b$$

$$= I_a \ddot{\theta} + I_b \ddot{\sigma}$$

$$+ m_a (a \ddot{\theta} \sin \theta - a \dot{\theta} \cos \theta + \ddot{y}_s)(a \sin \theta + z_s)$$

$$- m_a (a \ddot{\theta} \cos \theta - a \dot{\theta} \sin \theta + \ddot{z}_s)(-a \cos \theta + \ddot{y}_s)$$

$$+ m_b (c \ddot{\theta} \cos \theta + c \ddot{\sigma} \sin \theta + b \ddot{\sigma} \cos \sigma + \ddot{y}_s)(c \sin \theta + b \sin \sigma + \ddot{z}_s)$$

$$- m_b (-c \ddot{\sigma} \sin \theta + c \ddot{\sigma} \cos \theta - b \ddot{\sigma} \sin \sigma + \ddot{z}_s)(-c \cos \theta - b \cos \sigma + \ddot{y}_s)$$

$$\ldots \quad (13)$$

$$T_k = -m_b g b \cos \sigma = I_b \ddot{\sigma} + m_b \ddot{y}_b \sin \sigma + m_b \ddot{z}_b \cos \sigma$$

$$= I_b \ddot{\sigma} + m_b \ddot{y}_b \sin \theta(c \ddot{\theta} \cos \theta + c \ddot{\sigma} \sin \theta + b \ddot{\sigma} \cos \sigma + b \ddot{\sigma} \sin \sigma + \ddot{y}_s)$$

$$+ m_b \ddot{z}_b \cos \theta(-c \ddot{\sigma} \sin \theta + c \ddot{\sigma} \cos \theta - b \ddot{\sigma} \sin \sigma + b \ddot{\sigma} \cos \sigma + \ddot{z}_s)$$

$$\ldots \quad (14)$$
\[ F = -(k_{sy} y_s + k_{vy} y_s) = m_a \ddot{y}_a + m_b \ddot{y}_b \]
\[ = m_a (a \ddot{\theta} \sin \theta - a \dot{\theta} \cos \theta + \ddot{y}_s) \]
\[ + m_b (c \ddot{\theta} \cos \theta + c \dot{\theta} \sin \theta + b \dot{\sigma}^2 \cos \sigma + b \dot{\sigma} \sin \sigma + \ddot{y}_s) \]
\[ = m_a (a \ddot{\theta} \cos \theta - a \dot{\theta} \sin \theta + \ddot{y}_s) \]
\[ + m_b (-c \ddot{\theta} \sin \theta + c \dot{\theta} \cos \theta - b \dot{\sigma}^2 \sin \sigma + b \dot{\sigma} \cos \sigma + \ddot{y}_s) \]
\[ R = -(k_{sz} z_s + k_{vz} z_s) - mg = m_a \ddot{z}_a + m_b \ddot{z}_b \]
\[ = m_a (a \ddot{\theta} \cos \theta - a \dot{\theta} \sin \theta + \ddot{z}_s) \]
\[ + m_b (-c \ddot{\theta} \sin \theta + c \dot{\theta} \cos \theta - b \dot{\sigma}^2 \sin \sigma + b \dot{\sigma} \cos \sigma + \ddot{z}_s) \]

Equations (13) to (16) were rearranged to be of the form:

\[ a_{ij} \ddot{\theta} + a_{ij} \dot{\theta} + a_{ij} \ddot{y} + a_{ij} \ddot{z} = b_i \]

The coefficients \( a_{ij} \) and \( b_i \) were determined for each of the equations (13) to (16).

\( T_0: \)
\[ a_{11} = I_a + m_a a^2 + m_b c^2 + m_b b_c \cos (\theta - \sigma) \]
\[ + m_a a \sin \theta z_s - m_a a \cos \theta y_s + m_c c \sin \theta z_s - m_b b c \cos \theta y_s \]
\[ a_{12} = I_b + m_b b^2 + m_b b c \cos (\theta - \sigma) + m_b b c \sin \sigma z_s - m_b b c \cos \sigma y_s \]
\[ a_{13} = m_a a \sin \theta + m_b c \sin \theta + m_b b \sin \sigma + m_a z_s + m_b z_s \]
\[ a_{14} = m_a a \cos \theta + m_b c \cos \theta + m_b b \cos \sigma - m_a y_s - m_b y_s \]
\[ b_1 = m_a (y_s g - a \cos \theta g - a \dot{\theta}^2 \sin \theta y_s - a \dot{\theta} \cos \theta z_s) \]
\[ + m_b (y_s g - c \cos \theta g - b \cos \sigma g + b \dot{\sigma}^2 \sin (\theta - \sigma) - b \dot{\sigma} \cos \sigma (\theta - \sigma)) \]
\[ - b \dot{\sigma}^2 \sin \sigma y_s - b \dot{\sigma}^2 \cos \sigma z_s - c \dot{\theta} \sin \sigma y_s - c \dot{\theta} \cos \sigma z_s \]

\( T_k: \)
\[ a_{21} = m_b b c \cos (\theta - \sigma) \]
\[ a_{22} = I_b + m_b b^2 \]
\[ a_{23} = m_b b \sin \sigma \]
\[ a_{24} = m_b b \cos \sigma \]
\[ b_2 = m_a (0) + m_b (-b \cos \sigma g - b \dot{\theta}^2 \sin (\theta - \sigma)) \]
The rearranged form of equations (13) to (16) produced four simultaneous equations in four unknowns. In order to determine the values of the accelerations in $\theta$, $\sigma$, $y_s$ and $z_s$ a matrix equation was formed using the coefficients $a_{ij}$ and $b_i$. The least squares solution to this matrix equation was obtained using the method of Stewart (1973). Given the accelerations in $\theta$, $\sigma$, $y_s$ and $z_s$, and the iterative time interval it was possible to determine the angular velocities in $\theta$ and $\sigma$, and the component linear velocities of the spring system. Further integration gave the orientation angles $\theta$ and $\sigma$, and the component locations $(y_s, z_s)$ of the spring system.

Equations (1) and (2) were then used to determine the component locations, velocities and accelerations of the segmental mass centres. Subsequently the component locations and velocities of the whole body mass centre were calculated. The solution to equations (1) and (2) also enabled the calculation of the component ground reaction forces using equations (11) and (12). The segment orientation angles $\theta$ and $\sigma$ enabled the determination of the knee angle $\gamma$ and the leg plant angle $\phi$.

Given the angular accelerations in $\theta$ and $\sigma$ the angular velocities were determined using numerical integration. Using Taylor’s series approximations around $t = t_0$ gives:

$$\theta_i = \theta_0 + \dot{\theta}_0 t + \frac{1}{2} \ddot{\theta}_0 t^2$$

$$\dot{\theta}_i = \dot{\theta}_0 + \ddot{\theta}_0 t$$

$$\sigma_i = \sigma_0 + \dot{\sigma}_0 t + \frac{1}{2} \ddot{\sigma}_0 t^2$$

$$\dot{\sigma}_i = \dot{\sigma}_0 + \ddot{\sigma}_0 t$$
These equations comprise Euler's method of direct integration. This method has only first-order accuracy and assumes that the acceleration determined at \( t_0 \) is constant throughout the integration step. In the simulation model second-order accuracy was obtained by using the initial second derivative in \( \theta_0 \) to give initial estimates of \( \theta_1 \) and its first derivative. The second derivative of the mean value \( \theta_m = \frac{1}{2}(\theta_0 + \theta_1) \) was then used to calculate \( \theta_1 \) and its first derivative using the equations shown below. This modified Euler method is a second-order Runge-Kutta method.

\[
\begin{align*}
\theta_1 &= \theta_0 + \dot{\theta}_0 t + \frac{1}{2} \ddot{\theta}_m t^2 \\
\sigma_1 &= \sigma_0 + \dot{\sigma}_0 t + \frac{1}{2} \ddot{\sigma}_m t^2 \\
\dot{\theta}_1 &= \dot{\theta}_0 + \ddot{\theta}_m t \\
\dot{\sigma}_1 &= \dot{\sigma}_0 + \ddot{\sigma}_m t
\end{align*}
\]

4.4.6 Determination of the inertia parameters

The lengths of the shank and thigh segments as defined in the simulation model were determined from the film analysis of Chapter 3.3. Six jumps (br16 - br26) were used to determine the average length of both the shank segment and thigh segment of the takeoff leg at the instant of touchdown. The segmental lengths obtained from the film analysis were presented in Table 4.3.1. In Figure 4.2.2 the shank segment length is denoted \( c \) and the thigh segment length is denoted \( b \).

Figure 4.2.2 showed that it was also necessary to determine the location of the shank segment mass centre. In order to determine the length \( c \) in Figure 4.2.2 anthropometric measurements were recorded directly from the subject used in the experimental study of Chapter 3.3. The anthropometric data recorded were required as input into the inertia model of Yeadon (1990). The full set of anthropometric measurements recorded from the subject are provided in Appendix 1a.

Also input into the inertia model were the cadaver density values of Dempster et al. (1955), Clauser et al. (1969) and Chandler et al. (1975). The predicted whole body mass from each data set was compared with the actual body mass of Brendan Reilly. The recorded body mass was 78.70 kg. The three data sets produced whole body mass estimates of 86.60 kg, 88.83 kg and 80.77 kg respectively. These respective values equate to approximately 110%, 113% and 103% of the actual body weight.

The cadaver data set of Chandler et al. (1975) was therefore used as it provided the best estimate of the whole body mass. The segmental inertia data output from the Yeadon inertia model using this data set were scaled to correspond to the actual body weight.
The inertia model output is shown in Appendix 1b and comprises the segmental masses, the segmental moments of inertia about the principal axes and the locations of the segmental mass centres relative to the proximal joint. The moment of inertia of each segment was taken about the sagittal plane given the two-dimensional nature of the model and the point of observation. It was assumed in the simulation model that the segmental masses and moments of inertia remained constant throughout the takeoff. For the shank segment the location of the segmental mass centre relative to the knee joint also remained constant. The location of the thigh mass centre was coincident with the most proximal point of the segment. Table 4.4.1 shows a summary of the output from the Yeadon (1990) inertia model which was used in the simulation model.

Table 4.4.1. Summary of the inertia characteristics of the athlete

<table>
<thead>
<tr>
<th>segment</th>
<th>mass [kg]</th>
<th>length [m]</th>
<th>distance of c.m. from distal joint [m]</th>
<th>moment of inertia [kg.m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>shank</td>
<td>mₕ = 4.317</td>
<td>c = 0.595</td>
<td>a = 0.412</td>
<td>Iₕ = 0.074</td>
</tr>
<tr>
<td>thigh</td>
<td>mₗ = 74.383</td>
<td>b = 0.625</td>
<td>b = 0.625</td>
<td>Iₗ = 13.220</td>
</tr>
</tbody>
</table>

4.4.7 Determination of the muscle parameters

- Introduction

The simulation model comprises a single torque generator located at the knee joint of the takeoff leg. The torque generator enables the quantification of the maximum torque that can be produced by the net muscular action around a joint. In the simulation model the single torque generator represents the muscular activity at the knee joint of the support leg throughout the takeoff phase of the high jump. The torque generated was calculated as a function of the joint angular velocity using the method of King (1998), whereby the torque is represented by a function defined by a number of muscle parameters. These parameters are calculated from experimental data collected directly from the subject using an isokinetic dynamometer.

Isokinetic dynamometers allow the quantification of the maximum torque that can be generated over a range of joint angles and angular velocities. Dynamometers are however typically limited by a maximum angular velocity of around ± 200 - 300°.s⁻¹. A method is therefore required so that torque production may be extrapolated over a much wider range of angular velocities.
• The isokinetic dynamometer

A Kin-Com 125E isokinetic dynamometer was used which enables joint isolation by positioning the subject relative to the crank of the machine in such a way that the movement of extraneous limbs is minimised. This dynamometer model allowed the angular velocity of the crank arm to be controlled during maximal joint flexion (eccentric) and extension (concentric) within a range of ± 250°.s\(^{-1}\). The crank angle and angular velocity along with the magnitude of the force produced during the contraction were output from the computer controlled dynamometer at a frequency of 100 Hz.

The force produced by the subject tangential to the axis of rotation of the crank arm was measured by a load cell attached to the crank. The force was measured at a resolution of 1 N up to a maximum of 2000 N. The crank angle was output in degrees via a conversion from a voltage reading measured by a potentiometer. The crank angular velocity was output using an internal tachometer which measured the rotational speed of the dynamometer's motor. The crank angle and angular velocity were measured to resolutions of 0.01° and 0.25°.s\(^{-1}\) respectively.

• Data collection

A limitation of many muscle driven simulation models has been the failure to determine muscle parameters from direct measurements on an individual. In the present study all measurements were taken directly from a single subject and were performed in a single session. The subject was the same elite male high jumper from whom the inertia parameters were determined and whom performed the jumps analysed in Chapter 3.3.

The knee joint of the takeoff (right) leg was tested using isokinetic trials at a range of controlled angular velocities throughout a specified range of motion. The controlled velocities for both eccentric and concentric muscular work ranged from 50°.s\(^{-1}\) to the maximum velocity of 250°.s\(^{-1}\) as constrained by the dynamometer. Specific values were obtained for the angular range of motion at the knee joint from video data of real high jump performances by the subject. These jumps were analysed in Chapter 3.2.

Prior to collecting joint force data from the subject the dynamometer was calibrated using a number of procedures as outlined in the operators manual. The calibration involved checking the measurements of the force and the crank angle and angular velocity. Additional independent tests were also carried using a spirit level to validate the crank angle measurement in the horizontal and vertical positions.

Immediately prior to testing the subject was appropriately positioned and secured on the dynamometer so that the rotational axes of the joint and the crank arm were aligned. This minimised movement of the longitudinal axis of the leg segment relative to the crank arm. The specified range of angular motion was constrained using mechanical stops in the interests of personal safety. The subject was in possession of a patient interrupt switch throughout the testing which enabled the subject to stop a trial at any time.
The subject was permitted "warm-up" trials in which to become accustomed to the exercise protocol and prepare for the data collection. When the subject declared himself ready to commence a trial the data collection was initiated. The subject was then instructed to commence the trial. Throughout each trial the subject was given verbal encouragement in an effort to maintain maximal effort. Between trials the subject remained secured to the dynamometer but was given complete autonomy as to when each trial began.

For each isokinetic trial the protocol involved the subject performing two repetitions of concentric and eccentric exercise. The subject was instructed to contract the muscles maximally throughout both repetitions of concentric-eccentric work. Details of each test performed in sequential order are shown in Table 4.4.2.

Table 4.4.2. Sequential order of muscle strength testing at the knee

<table>
<thead>
<tr>
<th>File</th>
<th>Angular velocity [°/s]</th>
<th>Range of motion [°]</th>
<th>Moment arm length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>brk01</td>
<td>50°/sec</td>
<td>129° - 165°</td>
<td>0.24</td>
</tr>
<tr>
<td>brk02</td>
<td>50°/sec</td>
<td>129° - 165°</td>
<td>0.24</td>
</tr>
<tr>
<td>brk03</td>
<td>100°/sec</td>
<td>129° - 165°</td>
<td>0.24</td>
</tr>
<tr>
<td>brk04</td>
<td>100°/sec</td>
<td>129° - 165°</td>
<td>0.24</td>
</tr>
<tr>
<td>brk05</td>
<td>150°/sec</td>
<td>129° - 165°</td>
<td>0.24</td>
</tr>
<tr>
<td>brk06</td>
<td>150°/sec</td>
<td>129° - 165°</td>
<td>0.24</td>
</tr>
<tr>
<td>brk07</td>
<td>200°/sec</td>
<td>129° - 165°</td>
<td>0.24</td>
</tr>
<tr>
<td>brk08</td>
<td>250°/sec</td>
<td>129° - 165°</td>
<td>0.24</td>
</tr>
<tr>
<td>brk09</td>
<td>250°/sec</td>
<td>129° - 165°</td>
<td>0.24</td>
</tr>
</tbody>
</table>

*Data analysis*

The action of the extensor muscles of the takeoff leg during the high jump takeoff is eccentric-concentric. However, the isokinetic dynamometer operated in the reverse sense, i.e. concentric-eccentric. Therefore in order to obtain an exercise protocol in which the subject performed eccentric work followed by concentric work, two repetitions of eccentric-concentric work were performed, as stated previously. The data of interest were then extracted as the middle portion of the test. This testing protocol also ensured that the muscles were fully activated throughout the eccentric-concentric phase of interest. For example, in trial brk01 the protocol would have been as follows:

concentric 50°/sec → eccentric 50°/sec → concentric 50°/sec → eccentric 50°/sec

129° → 165° → 129° → 165° → 129°
This protocol would have resulted in the joint angle and joint angular velocity time histories shown in Figures 4.4.7 and 4.4.8 respectively. In these figures the horizontal axis represents the time from the start of data collection, rather than the start of muscular exertion. For each trial data capture began before the subject commenced the sequence of muscular contraction so that no data were excluded. From the original data set the middle eccentric-concentric phase was selected so that the original data were edited to include only the data between the dashed lines in Figures 4.4.7 and 4.4.8.

**Figure 4.4.7.** Joint angle time history for dynamometer trial brk01.

**Figure 4.4.8.** Joint angular velocity time history for dynamometer trial brk01.
The dynamometer output ASCII data files for each trial containing time histories of the crank angle and angular velocity, and the force measured throughout the trial. Appendix 6a contains a portion of the output file from trial brk01. The torque generated at any instant during a trial was calculated as the product of the measured force and the moment arm. The moment arm, presented in Table 4.4.2 was measured as the distance between the point of attachment of the force transducer and the centre of the axis of joint rotation. Appendix 6b shows in graphical form the time histories of the crank angle and angular velocity, and the joint torque exerted for trial brk01.

- The muscle model

In the simulation model the torque generated is calculated as a function of the joint angular velocity. The torque data obtained from the dynamometer were constrained to an angular velocity of ±250°.s⁻¹. However, the knee joint angle time histories for the jumps analysed in Chapter 3.3 showed that angular velocities in excess of 600°.s⁻¹ are observed at the knee joint during the high jump takeoff. A method was therefore required so that the torque data obtained from the dynamometer may be extrapolated to the greater angular velocities that are observed during the high jump takeoff.

Hill (1970) fitted a hyperbolic function to the force-velocity relationship of whole muscle. This function, shown in Figure 4.4.9, was applicable only to concentric muscular activity. Figure 4.4.9 also shows the double hyperbolic relationship observed by Edman (1988) in single fibres during both eccentric and concentric muscular activity. The double hyperbolic force-velocity relationship of Edman shows a plateauing of the force generated at high eccentric velocities and low concentric velocities. There is a marked drop in force at approximately zero velocity and a second less pronounced drop at high concentric velocities.

![Figure 4.4.9. The force-velocity relationships of Hill (1970) and Edman (1988).](image-url)
The data collected directly from the athlete using the dynamometer showed a plateau in the torque values at low concentric velocities. The force-velocity relationship of Edman (1988) was therefore a closer representation of the experimental data obtained. A function with the general characteristics of the double hyperbolic relationship was therefore required to fit the experimental data over the whole range of angular velocities.

King (1998) initially derived a three parameter exponential function to extend the force-velocity relationship of Hill to include eccentric muscular contractions. In order to incorporate the double hyperbolic nature of the relationship observed by Edman the three parameter function was extended by combining two exponential functions. A four parameter function was derived which tended to an asymptote at \( \frac{b}{c} \) at high concentric velocities. A second function defined by two parameters was also derived. This function tended to unity at high eccentric velocities and zero at high concentric velocities. The four parameter and two parameter functions are shown in Figure 4.4.10.

\[
T = \frac{a + be^{p\omega}}{1 + ce^{p\omega}} \quad \text{four parameter function}
\]

\[
T = \frac{1}{1 + de^{q\omega}} \quad \text{two parameter function}
\]

where \( a, b, c, d, p \) and \( q \) are positive constants
\( \omega = \) angular velocity (radians)
\( T = \) torque (N.m)

Figure 4.4.10. General shape of the four and two parameter functions; from King (1998).
The four and two parameter functions were combined to obtain a function defined by six parameters. The six parameter exponential function was defined by the following equation, which is the product of the four and two parameter functions. This function is shown in Figure 4.4.11.

\[
T = \frac{a + be^{p\omega}}{(1 + ce^{p\omega})(1 + de^{q\omega})}
\]

six parameter function

\begin{figure}
\centering
\includegraphics[width=0.6\textwidth]{six-parameter-function.png}
\caption{General shape of the six parameter function; from King (1998).}
\end{figure}

It can be seen from Figure 4.4.11 that this six parameter function has the same general shape as Edman's force-velocity relationship. The function has a plateau in torque at both high eccentric and low concentric velocities. There is a marked drop in the function close to zero velocity and a second less pronounced drop at high concentric velocities before the function tends to an asymptote at zero torque.

\textbf{Results}

The muscle model calculates the torque \( T \) at a given angular velocity \( \omega \) using the previously derived function. This function requires the determination of six parameters; \( a, b, c, d, p \) and \( q \). These parameters were determined using the torque-angular velocity data obtained using the isokinetic dynamometer. Figure 4.4.12 shows the function defined by the six parameters. The equation defining the function in Figure 4.4.12 is also given.
In the torque-angular velocity relationship shown in Figure 4.4.12 there is zero torque exerted beyond a knee angular velocity of \(352^\circ \cdot s^{-1}\). This is a consequence of being able to collect data up to a maximum angular velocity of only \(250^\circ \cdot s^{-1}\). However, during the high jump takeoff knee angular velocities in excess of \(350^\circ \cdot s^{-1}\) are observed.

The torque-angular velocity relationship in the simulation model was therefore modified so that torque was exerted at greater angular velocities. The relationship shown in Figure 4.4.12 was modified so that 1% of the maximum eccentric torque was exerted at a concentric angular velocity of \(800^\circ \cdot s^{-1}\). The modified torque-angular velocity relationship used in the simulation model is shown in Figure 4.4.13.

This modification required that angular velocities in excess of \(250^\circ \cdot s^{-1}\) in the simulation model were transformed. This transformation was performed as follows:

\[
\text{if } \dot{\gamma} > 250^\circ \cdot s^{-1} \text{ then } \\
\dot{\gamma}_{\text{mod}} = (352 - 250) \cdot \frac{\dot{\gamma} - 250}{800 - 250} + 250
\]
4.4.8 Determination of the spring parameters and muscle scaling factor

Four spring parameters were defined in the nomenclature; the spring stiffness coefficients \((k_{sy}, k_{sz})\) in the horizontal and vertical directions, and the spring damping coefficients \((k_{vy}, k_{vz})\) in the horizontal and vertical directions. The component stiffness parameters relate to the resistance offered by the spring to alterations in its length. The component damping parameters relate to the progressive decrease in the amplitude of vibration of the oscillating spring system. The nomenclature also defined a constant \(k_f\) for scaling the muscle torque obtained from the modified dynamometer torque-angular velocity relationship. This scaling factor allowed the strength of the muscle to be varied.

In order to determine each of the spring parameter values and the muscle scaling factor an optimisation procedure was used. The simulation model was incorporated into a Simulated Annealing optimisation algorithm (Goffe et al., 1994). The four spring parameters and the muscle scaling factor were read as input to the simulation model. The optimisation of the spring parameters and muscle factor required that a score or cost function be defined. The aim of the optimisation procedure was to obtain spring parameter values and a muscle scaling factor that would result in realistic jump height performance given realistic initial conditions.

Section 4.4.2 described the model inputs. The film analysis of Chapter 3.3 was used to obtain realistic initial conditions. Jumps br16 - br26 were used to determine the
component velocities of the whole body mass centre just before touchdown. The leg plant angle and the knee angle, and their respective angular velocities at touchdown were also determined for each jump. These approach parameters were averaged over the six jumps to obtain a single value for each. The average value of each touchdown parameter was then input to the simulation model. Table 4.4.3 summarises the approach parameter values input to the simulation model for optimising the spring parameters.

Table 4.4.3. Approach parameter values used to optimise the spring parameters

<table>
<thead>
<tr>
<th>Trial</th>
<th>$\dot{y}$ [m.s$^{-1}$]</th>
<th>$\dot{z}$ [m.s$^{-1}$]</th>
<th>$\phi$ [$^\circ$]</th>
<th>$\gamma$ [$^\circ$]</th>
<th>$\dot{\phi}$ [$^\circ$.s$^{-1}$]</th>
<th>$\dot{\gamma}$ [$^\circ$.s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>br16</td>
<td>7.22</td>
<td>-0.55</td>
<td>35.8$^\circ$</td>
<td>177.2$^\circ$</td>
<td>-240</td>
<td>-305</td>
</tr>
<tr>
<td>br18</td>
<td>7.16</td>
<td>-0.45</td>
<td>35.0$^\circ$</td>
<td>175.6$^\circ$</td>
<td>-335</td>
<td>-110</td>
</tr>
<tr>
<td>br20</td>
<td>7.26</td>
<td>0.00</td>
<td>37.3$^\circ$</td>
<td>176.1$^\circ$</td>
<td>-135</td>
<td>20</td>
</tr>
<tr>
<td>br22</td>
<td>6.88</td>
<td>-0.25</td>
<td>33.6$^\circ$</td>
<td>172.6$^\circ$</td>
<td>-335</td>
<td>-210</td>
</tr>
<tr>
<td>br24</td>
<td>6.77</td>
<td>-0.45</td>
<td>33.9$^\circ$</td>
<td>170.5$^\circ$</td>
<td>-180</td>
<td>-290</td>
</tr>
<tr>
<td>br26</td>
<td>6.81</td>
<td>0.15</td>
<td>35.9$^\circ$</td>
<td>173.5$^\circ$</td>
<td>-155</td>
<td>-305</td>
</tr>
<tr>
<td>mean</td>
<td>7.02</td>
<td>-0.26</td>
<td>35.3$^\circ$</td>
<td>174.3$^\circ$</td>
<td>-230</td>
<td>-200</td>
</tr>
</tbody>
</table>

The same jumps were used to obtain realistic output values related to performance. The peak mass centre height and the knee angle at toe-off were determined for each jump. These values were then averaged over the six jumps to obtain a single value for each. Table 4.4.4 summarises the output parameter values used in optimising the spring parameters.

Table 4.4.4. Output parameter values used to optimise the spring parameters

<table>
<thead>
<tr>
<th>Trial</th>
<th>$z_{12}$ [m]</th>
<th>$\gamma$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>br16</td>
<td>0.828</td>
<td>176.3$^\circ$</td>
</tr>
<tr>
<td>br18</td>
<td>0.795</td>
<td>178.9$^\circ$</td>
</tr>
<tr>
<td>br20</td>
<td>0.780</td>
<td>170.8$^\circ$</td>
</tr>
<tr>
<td>br22</td>
<td>0.791</td>
<td>176.6$^\circ$</td>
</tr>
<tr>
<td>br24</td>
<td>0.776</td>
<td>176.5$^\circ$</td>
</tr>
<tr>
<td>br26</td>
<td>0.779</td>
<td>169.6$^\circ$</td>
</tr>
<tr>
<td>mean</td>
<td>0.791</td>
<td>174.8$^\circ$</td>
</tr>
</tbody>
</table>
The output parameters in Table 4.4.4 were used to define the cost function of the optimisation algorithm. For each of the output parameters the difference between the mean recorded value from Table 4.4.4 and the simulated value was to be minimised. This would ensure that the optimum spring parameters and muscle scale factor would produce realistic performance given realistic initial conditions, evaluated using the real data from Chapter 3.3. The cost function to be minimised was therefore defined as follows:

$$\text{cost} = \left( \frac{z_{k2} - 0.791}{0.001} \right)^2 + \left( \frac{\gamma_t - 174.8}{1.0} \right)^2$$

A global optimum was obtained and checked by repeating the optimisation from different starting points. The optimum spring parameters and muscle scale factor are presented in Table 4.4.5. A time iteration of 0.0001 s was used in each simulation in the optimisation procedure.

Table 4.4.5. Optimum spring parameters and muscle scale factor

<table>
<thead>
<tr>
<th>stiffness coefficient</th>
<th>damping coefficient</th>
<th>muscle scale factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{sy}[\text{N.m}^{-1}]$</td>
<td>$k_{sz}[\text{N.m}^{-1}]$</td>
<td>$k_{vy}[\text{N.m}^{-1}\text{s}]$</td>
</tr>
<tr>
<td>363,970</td>
<td>100,420</td>
<td>320</td>
</tr>
</tbody>
</table>

The output from the optimum solution with the spring parameters and muscle scale factor set to the values in Table 4.4.5 is included in Appendix 7a. In this simulation the mass centre height at toe-off was 1.091 m. Table 4.2.3 showed that the mean mass centre height at toe-off from the analysis of real jumps was 1.400 m. A constant term (previously defined as $k_1$ in Chapter 4.2) of 0.309 m was therefore added to the simulated mass centre height at toe-off in each subsequent simulation.

4.4.9. Comparison of model and actual performance

Table 4.3.5 showed a summary of touchdown conditions and jump height performance for 16 jumps analysed in Chapter 3.3 at the greater framing rate of 198 Hz. The approach speed, vertical mass centre velocity, leg plant angle and knee angle at touchdown were input to the simulation model for each of the sixteen jumps. Table 4.4.6 shows the initial touchdown conditions input to the simulation model and the resulting simulated jump height performance. Also shown in Table 4.4.6 is the actual jump height performance. For each simulation a time iteration of 0.0001 s was used.
Table 4.4.6. A comparison of simulated and actual jump height performance

<table>
<thead>
<tr>
<th>Trial</th>
<th>$\dot{y}_g$ [m.s(^{-1})]</th>
<th>$\ddot{y}_g$ [m.s(^{-2})]</th>
<th>$\phi_l$ [°]</th>
<th>$\gamma_l$ [°]</th>
<th>Actual jump height [m]</th>
<th>Simulated jump height [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>br05</td>
<td>5.73</td>
<td>-0.55</td>
<td>33.8°</td>
<td>158.9°</td>
<td>2.113</td>
<td>1.794</td>
</tr>
<tr>
<td>br06</td>
<td>6.29</td>
<td>-0.40</td>
<td>33.7°</td>
<td>163.7°</td>
<td>2.129</td>
<td>1.885</td>
</tr>
<tr>
<td>br07</td>
<td>6.50</td>
<td>-0.20</td>
<td>33.2°</td>
<td>168.0°</td>
<td>2.172</td>
<td>1.938</td>
</tr>
<tr>
<td>br08</td>
<td>5.87</td>
<td>-0.05</td>
<td>29.7°</td>
<td>159.0°</td>
<td>2.069</td>
<td>1.674</td>
</tr>
<tr>
<td>br09</td>
<td>6.28</td>
<td>-0.40</td>
<td>33.8°</td>
<td>172.9°</td>
<td>2.136</td>
<td>2.030</td>
</tr>
<tr>
<td>br10</td>
<td>6.68</td>
<td>-0.60</td>
<td>34.3°</td>
<td>172.4°</td>
<td>2.207</td>
<td>2.132</td>
</tr>
<tr>
<td>br11</td>
<td>6.62</td>
<td>-0.50</td>
<td>34.0°</td>
<td>171.9°</td>
<td>2.204</td>
<td>2.089</td>
</tr>
<tr>
<td>br12</td>
<td>6.13</td>
<td>-0.40</td>
<td>30.9°</td>
<td>166.6°</td>
<td>2.140</td>
<td>2.045</td>
</tr>
<tr>
<td>br13</td>
<td>6.63</td>
<td>-0.50</td>
<td>34.4°</td>
<td>171.4°</td>
<td>2.208</td>
<td>2.100</td>
</tr>
<tr>
<td>br14</td>
<td>6.73</td>
<td>-0.50</td>
<td>35.5°</td>
<td>174.4°</td>
<td>2.224</td>
<td>2.178</td>
</tr>
<tr>
<td>br16</td>
<td>7.22</td>
<td>-0.55</td>
<td>35.8°</td>
<td>177.2°</td>
<td>2.221</td>
<td>2.283</td>
</tr>
<tr>
<td>br18</td>
<td>7.16</td>
<td>-0.45</td>
<td>35.0°</td>
<td>175.6°</td>
<td>2.215</td>
<td>2.216</td>
</tr>
<tr>
<td>br20</td>
<td>7.26</td>
<td>0.00</td>
<td>37.3°</td>
<td>176.1°</td>
<td>2.174</td>
<td>2.252</td>
</tr>
<tr>
<td>br22</td>
<td>6.88</td>
<td>-0.25</td>
<td>33.6°</td>
<td>172.6°</td>
<td>2.179</td>
<td>2.055</td>
</tr>
<tr>
<td>br24</td>
<td>6.77</td>
<td>-0.45</td>
<td>33.9°</td>
<td>170.5°</td>
<td>2.182</td>
<td>2.064</td>
</tr>
<tr>
<td>br26</td>
<td>6.81</td>
<td>0.15</td>
<td>35.9°</td>
<td>173.5°</td>
<td>2.180</td>
<td>2.082</td>
</tr>
</tbody>
</table>

The simulated output for jump br26 is included in Appendix 7b as an example. It can be seen that the actual jump height was generally greater than the simulated jump height. The exceptions were jumps br16, br18 and br20. In these jumps it should be noted that the approach speed was greater than the average value of 7.02 m.s\(^{-1}\) used in determining the spring parameters and muscle scale factor. The plant angle and knee angle values at touchdown in these jumps were also large relative to the average values in Table 4.4.3. These jumps were therefore likely to be closest to optimum performance.

The root mean square difference between the actual jump height and the simulated jump height over the 16 jumps in Table 4.4.6 was 0.172 m. The standard deviation of the actual jump height from the mean value over the 16 jumps was 0.044 m. The elastic two segment simulation model was therefore not able to account for any of the observed variation in jump height over the 16 jumps.
4.4.10. Optimum conditions at touchdown

In order to determine the optimum touchdown conditions programme 2seg was incorporated into the same Simulated Annealing (Goffe et al., 1994) optimisation algorithm used in Chapters 4.2 and 4.3. The algorithm was used to optimise the horizontal and vertical mass centre touchdown velocities, the leg plant angle and the knee angle at touchdown. The vertical mass centre velocity was constrained to a maximum value of -0.26 m.s\(^{-1}\). The peak mass centre height was defined as the cost function to be maximised. In order to ensure that the obtained optimum solution was global the optimisation procedure was repeated from varying starting points, i.e. with varying initial estimates of the approach parameters. The global optimum solution is included in Appendix 7c and produced the following result:

\[
\begin{align*}
\text{Optimum touchdown conditions} & \quad \dot{y}_g = 12.23 \text{m.s}^{-1} \quad \dot{z}_g = -0.26 \text{m.s}^{-1} \\
& \quad \phi_l = 61.49^\circ \quad \gamma_l = 179.0^\circ \\
\text{Resulting jump height} & \quad z_{h2} = 3.748 \text{m} \quad z_p = 4.914 \text{m}
\end{align*}
\]

In order to determine the behaviour of the simulation model around optimum performance each of the approach parameters were varied in turn. Figure 4.4.14 shows the non-linear relationship obtained between jump height and approach speed at optimum vertical mass centre velocity, plant angle and knee angle at touchdown. The peak of the curve defines the optimum approach speed of 12.23 m.s\(^{-1}\).

![Figure 4.4.14. Behaviour of the elastic model around optimum approach speed.](image-url)
Similarly, Figure 4.4.15 shows the model determined non-linear relationship between plant angle and jump height at optimum approach speed, vertical mass centre velocity and knee angle at touchdown.

![Graph showing the relationship between plant angle and jump height](image)

**Figure 4.4.15.** Behaviour of the elastic model around optimum plant angle.

Figure 4.4.16 shows the model determined non-linear relationship between knee angle and jump height at optimum approach speed, vertical mass centre velocity and leg plant angle. Although the knee angle was constrained to a maximum value of 179°, Figure 4.4.16 shows the influence on jump height of knee angles up to 185°.

![Graph showing the relationship between knee angle and jump height](image)

**Figure 4.4.16.** Behaviour of the elastic model around optimum knee angle.
Figure 4.4.17 shows the time history of the component ground reaction forces for the optimum solution. The forces are expressed in body weights.

![Graph showing force and time history](image)

Figure 4.4.17. Ground reaction force time histories for the optimum solution.

From Figure 4.4.17 it can be seen that the total time of takeoff was 0.061 s. The minimum knee angle \( \gamma_{f,\text{min}} \) was 146.6° and the knee angle \( \gamma_f \) at toe-off was 163.5°. The performance from the optimum solution was unrealistic in each of these factors.

In order to produce a realistic performance the optimisation was constrained to produce an optimum solution which displayed a realistic range of motion in knee angle. These constraints were determined from the film analysis of the real jumps in Chapter 3.3. The cost function was set to zero if the minimum knee angle was less than 150° or if the final knee angle was less than 170°. These knee angles were as defined in the film analysis so that \( \gamma_f \) and \( \gamma_{f,\text{min}} \) were used in setting the constraints. The total takeoff time in the optimum solution was 0.061 s. A further constraint was placed on the optimisation procedure so that the cost function was set to zero if the total takeoff time was less than 0.065 s. These constraints produced the following optimum solution which is included in Appendix 7d.

**Optimum touchdown conditions**

\[
\begin{align*}
\dot{z}_g &= 7.58 \text{ m.s}^{-1} \\
\phi_l &= 43.09^\circ \\
\gamma_f &= 176.07^\circ \\
\dot{z}_g &= -0.26 \text{ m.s}^{-1}
\end{align*}
\]

**Resulting jump height**

\[
\begin{align*}
z_{h2} &= 1.265 \text{ m} \\
z_p &= 2.609 \text{ m}
\end{align*}
\]
4.4.11. The influence of the spring parameters on optimum technique

The takeoff time for the optimum solution in Appendix 7c was 0.061 s. In comparison to a takeoff time of around 0.140 s for real jumps this simulated takeoff time is unrealistically short. Previously the short time of takeoff for the rigid models developed in Chapters 4.2 and 4.3 was attributed to the large maximum knee torque. In comparison with the maximum torques of 1735 N.m and 1550 N.m used in the rigid models, Appendix 7c shows a maximum knee torque of 753 N.m. While this torque may still exceed the torques that act at the knee joint in the performance of a real high jump, it may be further speculated that the short takeoff time was due to the assigned spring parameter values.

It was speculated that softer springs would result in increased damping of the movement and subsequently prolong the time of takeoff. The optimisation procedure described in Section 4.4.8 was therefore repeated with a cost function that incorporated the observed contact time for the mean of the six full approach jumps. The new optimum spring parameters and muscle scale factor are presented in Table 4.4.7. The new optimum solution is shown below and is included in Appendix 7e.

Table 4.4.7. New optimum spring parameters and muscle scale factor

<table>
<thead>
<tr>
<th>Stiffness coefficient</th>
<th>Damping coefficient</th>
<th>Muscle scale factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{sy}$ [N.m⁻¹]</td>
<td>$k_{sz}$ [N.m⁻¹]</td>
<td>$k_{vy}$ [N.m⁻¹.s]</td>
</tr>
<tr>
<td>25,001</td>
<td>52,302</td>
<td>71</td>
</tr>
</tbody>
</table>

The procedure outlined in Section 4.4.10 was repeated to determine the optimum touchdown conditions for the new spring parameters and muscle scale factor. The global optimum touchdown conditions are shown below and the resulting performance for the optimum solution is included in Appendix 7f.

Optimum touchdown conditions ...

$\dot{y}_g = 9.32 \text{ m.s}^{-1}$
$\dot{z}_e = -0.26 \text{ m.s}^{-1}$
$\phi_i = 63.67^\circ$
$\gamma_i = 178.44^\circ$

Resulting jump height ...

$z_{h2} = 3.734 \text{ m}$
$z_p = 5.019 \text{ m}$
4.4.12. Discussion

It was speculated that the great knee torque required to produce realistic performance in the previously developed rigid model was partly due to the model failing to represent the initial impact phase of the takeoff. In the present model a spring system was attached to the distal end of the shank segment so that the elastic nature of the initial impact could be considered.

Spring stiffness and damping parameters were required in both the horizontal and vertical directions. A muscle scale factor was also defined which increased the torque obtained from the torque-angular velocity relationship. The spring parameters and muscle scale factor were determined using an optimisation procedure. Touchdown conditions obtained from real jumps were input to the simulation model. A cost function was defined which specified that the optimum spring parameters and muscle scale factor (Table 4.4.5) produced a realistic jump height and knee angle at toe-off. The resulting simulation is shown in Appendix 7a.

From Appendix 7a it can be seen that the mass centre was raised 0.764 m in flight. The knee angle, as defined in the film analysis of Chapter 3.3, decreased from a touchdown value of 174.30° to a minimum value of 154.76°. The knee angle at toe-off was 173.20°. The corresponding plant angle was 8.23°. In each of these factors the simulated performance was realistic. However the total takeoff time of 0.0713 s was unrealistically short compared with around 0.145 s for a real jump.

Having determined the spring parameters and muscle scale factor the model was used to determine the simulated jump height for the touchdown conditions of 16 real jumps (Table 4.4.6). Generally the real jump height was greater than the simulated jump height. Exceptions were jumps br16 to br20, which notably had relatively fast approach speeds and large plant and knee angles at touchdown. Given the results of the experimental study and the rigid simulation model, these jumps were therefore most likely to be close to optimum performance.

The root mean square difference between the actual jump height and the simulated jump height was determined over the 16 jumps. The root mean square difference (0.172 m) was greater than the standard deviation (0.044 m) of the actual jump height from the mean value. The elastic model therefore accounted for none of the observed variation in jump height over the 16 jumps. The predicted jump height using the elastic simulation model was therefore less accurate than using a constant jump height value over the 16 jumps.

Appendix 7b shows the simulated performance of jump br26. Realistic values were obtained for the minimum knee angle and the plant angle, knee angle and mass centre height at toe-off. However the jump height was approximately 0.10 m less than the actual jump height and the simulated takeoff time of 0.07 s was unrealistically short.
The elastic model was used to determine the touchdown conditions which maximised the peak mass centre height. The optimum solution is shown in Appendix 7c. The optimum approach speed was determined as $12.23 \text{ m.s}^{-1}$. The optimum approach speed for the modified rigid model was $9.37 \text{ m.s}^{-1}$. However these values are not directly comparable since the rigid model failed to consider the initial impact phase. In the rigid model the movement was initiated with the foot already on the ground, by which time the horizontal mass centre velocity has reduced from its value just before touchdown.

In Appendix 7c the initial approach speed was $12.23 \text{ m.s}^{-1}$. The corresponding horizontal velocity of the foot was calculated as $10.00 \text{ m.s}^{-1}$. From the optimum solution it was found that the horizontal foot velocity was reduced to zero after $0.0176 \text{ s}$. At this time the approach speed was $7.77 \text{ m.s}^{-1}$, a decrease of $4.46 \text{ m.s}^{-1}$ from the initial value. The optimum approach speed of $9.37 \text{ m.s}^{-1}$ obtained using the rigid model is therefore equivalent to an approach speed of $13.83 \text{ m.s}^{-1}$ in the elastic model. The optimum approach speeds from both models exceed the fastest recorded approach speed of $7.26 \text{ m.s}^{-1}$ from the experimental study.

The optimum plant angle was determined as $61.49^\circ$. The largest recorded plant angle in the experimental study was $37.3^\circ$. Jump height was shown to be maximised at a knee angle of $179.0^\circ$ at touchdown. The elastic model therefore produced an optimum approach that supported the conclusions of both the rigid model and the experimental study. However the optimum touchdown conditions were unrealistic relative to the capabilities of the athlete. Appendix 7c shows that the optimised touchdown conditions resulted in a jump height of $4.914 \text{ m}$.

Figures 4.4.14 to 4.4.16 show the behaviour of the model around optimum performance. It can be seen that a non-linear relationship was obtained between approach speed and jump height. However the pattern of the relationship showed that an approach speed of around $8 \text{ m.s}^{-1}$ was required to produce a jump height of $2.00 \text{ m}$, and that jump height performance decreased rapidly at approach speeds in excess of the optimum value. From the simulations performed to produce Figure 4.4.14 it was found that for approach speeds less than $8 \text{ m.s}^{-1}$ the knee angle collapsed so that there was no concentric phase to the takeoff. This phenomenon was due to the straight leg being planted more than $60^\circ$ from the vertical. As the approach speed increased in excess of the optimum value the takeoff became increasingly like an instantaneous rebound. The reduced takeoff time resulted in decreased vertical mass centre velocity at toe-off.

Figure 4.4.15 shows the non-linear relationship obtained between plant angle and jump height. This relationship showed a similar pattern to that discussed between approach speed and jump height. As a result of the fast optimum approach speed and straight leg plant a plant angle of $50^\circ$ was required to obtain a concentric phase and thereby prevent buckling of the takeoff leg. As the plant angle increased beyond the optimum value of $61.49^\circ$ the reduced takeoff time resulted in decreased jump height.
Figure 4.4.16 shows the relationship obtained between the knee angle at touchdown and jump height. Previously the rigid model showed that jump height increased almost linearly up to a maximum knee angle of 179.0°. The multiple trial experimental study in Chapter 3.3 showed that there was a strong linear relationship between knee angle and jump height. Figure 4.4.16 shows that there is an optimum knee angle of 180°. Jump height was shown to increase gradually with increasing knee angle at touchdown. A knee angle of 170° was required to prevent the knee collapsing during the takeoff phase, as a result of the fast approach speed and shallow plant angle.

The elastic model therefore produced an optimum technique that was beyond the physical capabilities of the athlete. The optimum solution in Appendix 7c shows that the total time of takeoff was 0.0608 s. The knee reached a minimum angle of 146.55° before extending to 163.53° at toe-off. In comparison with real jumps the angular range of motion of the knee was unrealistic. The optimisation procedure was repeated with constraints imposed relating to performance. The time of takeoff was constrained to a minimum value of 0.065 s. The minimum knee angle was constrained to 150° to prevent the knee collapsing, and the optimisation required a knee angle of at least 170° at toe-off. These constraints produced realistic optimum touchdown conditions in comparison with the experimental data of Chapter 3.3. The constrained optimum solution is included in Appendix 7d. The optimum approach speed was 7.58 m.s⁻¹. The optimum plant angle was 43.09° with a knee angle at plant of 176.07°. The resulting jump height was 2.609 m with a takeoff time of 0.069 s.

Figure 4.4.17 showed the time histories of the component ground reaction forces for the unconstrained optimum solution. It can be seen that the horizontal and vertical reaction forces reached maximum values at 32.8 and 24.4 bodyweights respectively. In contrast with the rigid model the elastic model was able to show the gradual increase in the ground reaction forces up to their peak values. Since the rigid model made no account for the initial impact phase the initial ground reaction forces were unrealistically high. The initial peak reaction forces in the rigid model were greater than the peak reaction forces in the elastic model. In the rigid model a maximum knee torque of 1550 N.m was used. In Appendix 7a where the muscle scale factor in the elastic model was optimised to produce realistic jump height performance the maximum torque value was 753 N.m.

The inclusion of an elastic element in the model therefore reduced the maximum knee torque required to produce realistic performance. However the muscle scale factor of 4.81 resulted in a model that was still too strong relative to the athlete's capabilities, thereby producing an unrealistic optimum technique.

The takeoff time for the optimum technique was 0.0608 s. In the performance of real jumps from Chapter 3.3 a takeoff time of 0.145 s was typical. It was speculated that the short simulated takeoff time was due to the assigned spring parameters. The spring parameter values represent the elastic properties of both the shod foot and the athletic
track surface. It was proposed that a softer spring would increase the damping of the movement and subsequently prolong the takeoff time. A new cost function was therefore used to prolong the contact time in the optimisation of the spring parameters and muscle scale factor. Appendix 7e shows the simulated performance with the new spring parameters and muscle scale factor for realistic touchdown conditions. The reduced spring stiffness and damping coefficients produced a takeoff time of 0.1119 s. However there was a very short concentric phase to the takeoff and the vertical depression of the spring increased considerably.

The new spring parameters also resulted in a decreased muscle scale factor. The reduced strength of the muscle produced a slower optimum approach speed. However the optimum technique with the new spring parameters remained unrealistic. Appendix 7f shows that the takeoff time was 0.1383 s. However the minimum knee angle was 169.07°, with the knee in hyper-extension at toe-off. The maximum vertical depression of the spring was 0.20 m. The resulting jump height was 5.019 m.

The modifications to the assigned spring parameter values were therefore shown to have a positive influence on the simulated takeoff time. However the optimum technique with the softer springs produced an unrealistic jump. It may therefore be speculated that in order to prolong the takeoff time the simulation model must account for the actions of the free limbs during the takeoff. The phenomenon of wobbling mass during the initial impact phase and the subsequent vigorous upward drive of the arms and free leg serve to maintain ground contact.

4.4.13. Conclusions

The inclusion of an elastic element in the two segment simulation model enabled the initial impact phase to be considered. Relative to the rigid model a decreased knee torque was required to produce realistic performance. The torque obtained from the experimentally determined torque-angular velocity relationship was increased by a scale factor of 4.81.

The strength of the model remained in excess of the athlete's capabilities. Subsequently an unrealistic optimum technique was obtained, resulting in a jump height of 4.914 m. The optimum approach speed of 12.23 m.s\(^{-1}\) and the optimum plant angle of 61.49° were far greater than the maxima of the experimental data. The elastic model was shown to be able to account for none of the observed variation in jump height over 16 jumps.

The technical implications from the elastic model further suggest that the optimum approach is fast with a straight leg planted away from the vertical.
4.5 Summary of the theoretical determination of optimum technique

Yeadon and Challis (1994) stated that a simulation model should be evaluated in the area in which it is to be applied and that simulations should be based on modifications to data acquired in the relevant environment. The simulation models developed in this chapter were evaluated using the experimental data presented in Chapter 3.3.

Each of the simulation models developed represented the actions of the support leg during the high jump takeoff. The models were two-dimensional in nature, aiming to determine how the conditions at touchdown influence the peak mass centre height. Each model comprised two segments connected at a frictionless knee joint. A single torque generator was located at the knee, representing the net muscular activity around the knee joint throughout the takeoff phase.

Initially a rigid two segment model was developed, based largely on the simulation model of Alexander (1990). The segments were assumed to be of equal length and to have zero mass. The segment length was initially determined as 0.464 m by averaging the anthropometrically measured shank (ankle to knee) and thigh (knee to hip) lengths of the subject. The body mass was represented as a single point mass located at the hip joint. This assumption resulted in an unrealistically low simulated mass centre height at toe-off. A constant value of 0.564 m was added to the simulated mass centre height at toe-off in all simulations so that a realistic mass centre height was obtained.

Six real jumps (Chapter 3.3) were used to obtain realistic touchdown conditions and subsequent jump height performance. In order to replicate jump height performance for the given touchdown conditions a maximum knee torque of 1735 N.m was required. The touchdown conditions for 16 real jumps (Chapter 3.3) were input to the simulation model. In each case the vertical mass centre velocity at touchdown was assumed to be zero. The root mean square difference between the actual and simulated jump heights was greater than the standard deviation in the actual jump height over the 16 jumps. The model was therefore able to account for none of the observed variation in jump height performance.

The rigid two segment model was used to determine optimum technique. The optimum approach speed was determined as 10.19 m.s⁻¹, the optimum leg plant angle as 51.58°, and the optimum knee angle at touchdown as 179.0°. Both approach speed and leg plant angle showed a non-linear relationship with jump height, supporting the results of Alexander. However the optimum values for both parameters exceeded the upper bounds of the experimental data presented in Chapter 3.3. From the experimental study the maximum recorded values for the approach speed and plant angle were 7.26 m.s⁻¹ and 37.3° respectively. Jump height was shown to increase almost linearly with increasing knee angle. This finding supported the results of the experimental study in Chapter 3.3.

The optimum touchdown conditions from the simulation model supported the conclusions of Chapter 3.3, indicating that an optimum approach is fast with the takeoff
leg straight and planted away from the vertical. However the unrealistically fast and low optimum approach produced an unrealistically large jump height of 2.704 m. It was concluded that the optimum touchdown conditions were unrealistic as a result of the large maximum knee torque used in the model. Further consequences of the large maximum knee torque, coupled with a virtually straight leg at plant, were unrealistically large ground reaction forces. Subsequently the takeoff time was unrealistically short.

In an attempt to reduce the maximum knee torque and thereby obtain a more realistic optimum solution the rigid two segment model was modified. The limitation of locating the whole body mass centre at the hip joint was addressed by changing the definition of the leg segments. Previously the anthropometrically measured segment lengths made no account for the height that the ankle is raised off the ground throughout the takeoff phase or the fact that the mass centre is located above the hip. In the modified model the segment length was determined from the film analysis of Chapter 3.3. Real jumps were used to determine the knee to ground distance at touchdown and the distance between the knee and the mass centre of the whole body less the shank of the takeoff leg. An average segment length of 0.610 m was obtained, compared with the original value of 0.464 m. With the increased segment length a reduced constant term of 0.273 m was required to produce a realistic mass centre height at toe-off.

With an increased segment length the required maximum torque was reduced from 1735 N.m to 1455 N.m, assuming a zero vertical mass centre touchdown velocity. Analysis of the real jumps (Chapter 3.3) showed that the average vertical touchdown velocity over six jumps was -0.26 m.s\(^{-1}\). With this value input to the simulation model a maximum knee torque of 1550 N.m was required. As with the original model, the modified simulation model was shown to be able to account for none of the observed variation in jump height performance. In comparison the regression equation formulated in Chapter 3.3 from the experimental analysis was able to account for 80% of the variation in jump height performance over the same 16 jumps.

As expected the reduced maximum knee torque produced a more realistic optimum technique. The knee angle remained optimised at 179.0°. The optimum approach speed decreased from 10.19 m.s\(^{-1}\) to 9.37 m.s\(^{-1}\), while the optimum plant angle increased slightly from 51.58° to 51.71°. The resulting jump height in the optimum solution decreased from 2.704 m to 2.556 m. Therefore, despite the reduced maximum torque as a result of the increased segment length, the optimum approach speed and leg plant angle remained outside of the upper bound of the experimental data. Subsequently the optimum jump height performance was unrealistically large.

It was therefore concluded that an unrealistically strong muscle was required to produce realistic jump height performance for a rigid two segment model of the high jump takeoff. Subsequently the model was able to accommodate approach speeds and plant angle values that exceeded the actual capabilities of the athlete. As a result the
model was able to produce jump height performances in excess of those observed in real performances. The great muscular strength also resulted in unrealistically short takeoff times.

It was speculated that the large maximum knee torque, and subsequently the unrealistic optimum touchdown conditions were a consequence of the model's simplicity. In the rigid model the movement was initiated with the takeoff foot already on the ground. The rigid model therefore made no account for the initial impact phase of the high jump takeoff. An elastic element was incorporated into the two segment model so that the impact phase immediately after touchdown could be modelled. It was proposed that this elastic element would reduce the maximum knee torque in the model.

In the elastic model the knee torque was calculated from an experimentally determined torque-angular velocity relationship. This value was then scaled to produce realistic performance. In the elastic model the maximum recorded knee torque was 753 N.m, compared with 1550 N.m for the rigid model.

The muscle scale factor was 4.81, so that the model produced a torque 4.81 times greater than the torque produced experimentally by the athlete at a given angular velocity. Despite the reduction in $T_{\text{max}}$ relative to the rigid models the muscular strength of the model remained too great so that an unrealistic optimum performance was obtained. The optimum approach speed of 12.23 m.s$^{-1}$ and the optimum plant angle of 61.49° were far in excess of the experimentally determined maxima. Jump height was shown to be maximised with a straight leg at touchdown. The large approach speed, straight leg plant and shallow plant angle produced interesting results when the behaviour of the model away from optimum performance was examined. For example, at the optimum approach speed and plant angle a knee angle of 170° at touchdown was required in order to prevent the knee collapsing. As with the rigid models the elastic model was not able to account for any of the observed variation in jump height performance over 16 jumps.

The optimum solution using the elastic model produced a takeoff time of 0.0608 s. The spring stiffness and damping parameters were reduced and the softer spring produced a realistic takeoff time. However the performance of the model in terms of jump height and the angular range of motion at the knee were unrealistic. It was therefore concluded that the short takeoff time using the elastic model was due to the model making no account for the actions of the free limbs. The downward swing of the arms and the phenomenon of wobbling mass during the initial phase of the takeoff, and the subsequent vigorous upward drive of the arms and free leg serve to prolong the period of ground contact. Such factors were not accounted for using the two segment models developed.

The technical implications resulting from the theoretical determination of optimum technique are that in order to maximise jump height the athlete should approach fast, with the leg planted away from the vertical and with minimum knee flexion. In order to further improve jump height the athlete must increase the strength of the takeoff leg.
CHAPTER 5

SUMMARY AND DISCUSSION

In this study two related aims have been pursued, viz:

(a) To determine the influence of approach parameters on jump height performance for an elite male high jumper.
(b) To determine the optimum technique for the athlete.

In order to address these aims both an experimental approach and a theoretical modelling approach were used. The optimum technique for an elite male high jumper was thereby investigated both experimentally and theoretically. Seven questions were raised in the Introduction chapter. The extent to which these questions have been answered will now be considered.

5.1 Summary of findings

1. What is the optimum approach speed for an elite male high jumper?

The approach speed was defined as the horizontal velocity of the athlete's mass centre over the final approach stride. The optimum approach speed was defined as that which resulted in the greatest peak mass centre height.

The multiple trial experimental study in Chapter 3.3 showed that there was a fairly strong \((r^2 = 0.647)\) and significant \((p < 0.001)\) linear relationship between approach speed and jump height. This relationship implied that the faster the approach the greater the jump height. The maximum recorded approach speed was 7.26 m.s\(^{-1}\). The inclusion of a quadratic term in the regression analysis increased the strength of the correlation \((r^2 = 0.734, p < 0.001)\) between approach speed and jump height. The quadratic function defining the relationship between approach speed and jump height produced an optimum approach speed of 7.20 m.s\(^{-1}\). The optimum approach speed was therefore located towards the maximum of the experimental data set.

The calculation of the optimum approach speed from the experimental study was limited by the interdependence of the approach parameters. It was therefore difficult to determine the influence on jump height of changes in the approach speed alone. The development of a simulation model allowed the approach speed to be varied for constant values of knee angle and plant angle. In addition the use of a simulation model allowed an unlimited amount and range of data to be collected, including data in excess of the optimum approach speed.
The rigid two segment model developed in Chapter 4.2 produced an optimum approach speed of 10.19 m.s\(^{-1}\). The modifications made to the model in Chapter 4.3 resulted in an optimum approach speed of 9.37 m.s\(^{-1}\). These optimum values far exceed the maximum of the experimental data set and produced unrealistically large jump heights as a result of the large maximum knee torque used in the rigid model. The elastic model developed in Chapter 4.4 produced an optimum approach speed of 12.23 m.s\(^{-1}\). When the performance of the model was constrained to produce a realistic knee angle pattern an optimum approach speed of 7.58 m.s\(^{-1}\) was obtained.

It was difficult to determine accurately the optimum approach speed for the elite high jumper. There were limitations in both the experimental and theoretical approaches. The experimental data does indicate, however, that the athlete used approach speeds close to optimum in his full length approaches.

2. **What is the optimum leg plant angle for the same athlete?**

In the experimental study the leg plant angle was defined at touchdown as the angle between the vertical and a straight line joining the ankle to the hip of the takeoff leg. In the simulation models the plant angle was defined as the angle between the vertical and a straight line joining the point of contact with the ground to the mass centre of the whole body less the shank of the takeoff leg. The optimum plant angle was defined as that which resulted in the greatest jump height.

The multiple trial study in Chapter 3.3 showed that there was a fairly strong \((r^2 = 0.468)\) and significant \((p = 0.003)\) linear relationship between plant angle and jump height. This relationship implied that the greater the angle of leg plant away from the vertical the greater the jump height. The maximum recorded plant angle was 37.3°. The inclusion of a quadratic term in the regression analysis increased the strength of the correlation \((r^2 = 0.542, \ p = 0.006)\) between plant angle and jump height. The quadratic function produced an optimum plant angle of 36.8°. The optimum plant angle was therefore located towards the maximum of the experimental data set.

The calculation of the optimum plant angle from the experimental study was limited by the interdependence of the approach parameters. It was therefore difficult to determine the influence on jump height of changes in the plant angle alone. The use of a simulation model allowed the plant angle to be varied independently and enabled data in excess of the optimum plant angle to be collected.

The rigid two segment model developed in Chapter 4.2 produced an optimum plant angle of 51.58°. The modifications made to the model in Chapter 4.3 resulted in an optimum plant angle of 51.71°. These optimum values exceed the maximum of the experimental data set and produced unrealistically large jump heights as a result of the
large maximum knee torque used in the rigid model. The elastic model developed in Chapter 4.4 produced an optimum plant angle of 61.49°. When the performance of the model was constrained an optimum plant angle of 43.09° was obtained.

It was difficult to determine accurately the optimum leg plant angle for the elite high jumper, with limitations in both the experimental and theoretical approaches. The experimental data does indicate, however, that the athlete used plant angles close to optimum in his full length approaches.

3. What is the optimum knee angle at plant for the same athlete?

In the experimental study the knee angle was defined as the angle between the shank and thigh segments of the takeoff leg at the instant of touchdown. The optimum knee angle was defined as that which resulted in the greatest jump height.

The multiple trial study in Chapter 3.3 showed that there was a strong (r² = 0.715) and significant (p < 0.001) linear relationship between knee angle and jump height. This relationship implied that the straighter the leg at touchdown the greater the jump height. The maximum recorded knee angle was 177.2°. The inclusion of a quadratic term resulted in a strong (r² = 0.730) and significant (p < 0.001) quadratic relationship between knee angle and jump height from the case study. The quadratic function defining this relationship produced an optimum knee angle of 186.2°. This result further suggested that the jump height is optimised by having a straight leg at touchdown.

The calculation of the optimum knee angle from the experimental study was limited by the interdependence of the approach parameters. It was therefore difficult to determine the influence on jump height of changes in the knee angle alone. The development of a simulation model allowed the knee angle to be varied for constant values of approach speed and plant angle.

The rigid two segment models developed in Chapters 4.2 and 4.3 produced an optimum knee angle in excess of 179.0°. This was the maximum value permitted in the model, since a perfectly straight leg would have resulted in infinitesimally large ground reaction forces. The rigid model therefore supported the results of the experimental study in showing that jump height is maximised by having a straight leg at touchdown. The elastic model developed in Chapter 4.4 also produced an optimum knee angle of 179.0°. When the performance of the model was constrained to produce a realistic knee angle pattern an optimum knee angle of 176.07° was obtained.

The results of both the experimental and theoretical studies therefore suggest that jump height is maximised by having a straight leg at touchdown, i.e. a knee angle of 180°. The maximum knee angle recorded in the experimental study was 177.2°. A certain degree of knee flexion may be required at touchdown to reduce injury potential.
4. How does jump height performance vary away from the optimum technique?

The experimental study of Chapter 3.3 produced both linear and quadratic regression equations which showed how jump height performance varied away from the optimum value of each approach parameter. These relationships were shown graphically in Figures 3.3.8 to 3.3.13. Similarly, the simulation models developed in Chapter 4 showed how jump height performance varied away from the optimum approach parameter values using multiple simulations.

In contrast to the experimental study the theoretical simulations enabled a single approach parameter to be varied while the others maintained a fixed initial value. Parameter values in excess of the experimentally determined optimum values could also be used. It was therefore possible to show how jump height performance varied at approach speeds faster than the optimum value, for example.

The multiple trial experimental study showed that the inclusion of a quadratic term increased the strength of the correlation between approach speed and jump height and produced a realistic optimum approach speed. It was therefore concluded that there was evidence of some curvature in the relationship between approach speed and jump height. Figure 3.3.9 showed that the quadratic curve was fairly flat near the peak, so that a small reduction in approach speed had little influence on jump height. This has important implications for competitive high jumping since approach speeds slower than optimum may be used to successfully clear early bar heights.

Similarly both the rigid and the elastic simulation models showed a non-linear relationship between approach speed and jump height so that jump height is maximised at an intermediate value of approach speed. However, each of the simulation models produced an unrealistic optimum approach speed. Jump height performance was shown to decrease markedly away from the optimum approach speed. However the approach speeds recorded in the experimental study show that the athlete would not be performing near the theoretically determined optimum approach speed.

The experimental study showed that the inclusion of a quadratic term increased the strength of the correlation between plant angle and jump height and produced a realistic optimum plant angle. There was therefore evidence of some curvature in the relationship between plant angle and jump height. Figure 3.3.11 showed that jump height was more sensitive to changes in plant angle than changes in approach speed.

Similarly both the rigid and elastic simulation models showed a non-linear relationship between plant angle and jump height so that jump height is maximised at an intermediate value of plant angle. However, each of the simulation models produced an optimum plant angle that was far greater than the maxima of the experimental data set. The rigid link model showed that near optimum the jump height was fairly insensitive to changes in the plant angle. In contrast the elastic model showed that jump height
decreased markedly away from the optimum plant angle. However it should be noted that in the elastic model the knee was shown to collapse at plant angles less than $50^\circ$ at the optimum approach speed of 12.23 m.s$^{-1}$.

The experimental study showed that the inclusion of a quadratic term only slightly increased the strength of the correlation between the knee angle at touchdown and jump height. However, this quadratic function produced an optimum knee angle of $186.2^\circ$ and there was little confidence in the regression equation’s coefficients. There was greater confidence in the linear function between knee angle and jump height which suggested that jump height was maximised by having a straight leg at touchdown. This result was supported by the rigid model which showed that jump height increased linearly with knee angle up to the maximum permitted value of $179.0^\circ$. The elastic model also showed that jump height was maximised with a straight leg at touchdown.

5. *How sensitive is the optimum technique to changes in muscular strength?*

The results of the experimental study showed that the optimum approach was fast with a straight leg planted away from the vertical. Many coaches advocate a fast and low approach to the bar although Dapena (1984) considered that the athlete may be too fast and too low. If the takeoff leg is not strong enough the knee will then be forced to flex excessively during the takeoff phase.

Many coaches describe a "buckling" of the takeoff leg which may reduce the vertical mass centre velocity at toe-off. The speed of the approach, the shallowness of the plant and the extent to which the knee can be kept straight at touchdown will vary between athletes. Optimum technique is therefore specific to a given individual and may depend largely on the strength of the athlete.

The simulation models developed in Chapter 4 showed that with an unrealistically large maximum knee torque the model was able to jump unrealistically high. The great muscle strength enabled the model to use an approach speed that was faster than the maximum of the recorded data set. Combined with the fast approach speed was a plant angle which also exceeded the maximum of the experimental data set. Further the leg was planted with a knee angle of $179^\circ$. Increased strength was shown to enable a faster and lower approach, and also to produce greater jump heights for given touchdown conditions.

The technical implications of this result are that in order to increase jump height performance the takeoff leg must be strengthened to enable a faster approach and a greater plant angle.
6. How sensitive is the optimum technique to changes in the inertial characteristics of the body?

The rigid simulation model developed in Chapters 4.2 and 4.3 showed that an increase in segment length produced slightly greater jump heights for given touchdown conditions. An increased segment length therefore reduced the muscle strength required to produce a specified jump height performance. Subsequently the optimum approach speed and plant angle were reduced as a result of the decrease in muscle strength. An increased segment length was also shown to produce a greater mass centre height at toe-off, giving an inherent advantage to taller athletes.

7. What other parameters may influence jump height performance?

The present study considered the influence on jump height performance of the approach speed, the leg plant angle and the knee angle at touchdown. The multiple trial experimental study of Chapter 3.3 produced a regression equation relating jump height to the approach parameters. This regression equation was shown to be able to account for 80% of the observed variation in jump height performance over 16 real jumps. The touchdown conditions for each of the 16 jumps were input into both the rigid and the elastic two segment simulation models developed in Chapter 4. The resulting simulated jump height was linearly regressed against the actual jump height over the 16 jumps. The rigid and elastic models were shown to be able to account for none of the observed variation in jump height.

There were other variables that also influenced jump height. The simulation models were less successful than the experimental study in accounting for the variation in jump height. It may therefore be speculated that some of the additional factors that influenced jump height were not taken into account in the simulation model. Although not analysed directly some of these additional factors may have been included unintentionally in the experimental study. The three approach parameters were shown to be interdependent. It may be that additional performance variables also changed systematically with the changes in approach speed.

Additional touchdown conditions which may need to be included to account for more of the observed variation in jump height include the vertical mass centre velocity at touchdown and the angle of body lean, for example. The actions of the free limbs throughout the takeoff phase also contribute to the generation of vertical mass centre velocity. The flexion at the ankle and hip joints that occur during the high jump takeoff may also influence jump height performance. In order to include such factors a multi-segment simulation model would be required. Future directions are discussed in more detail in Chapter 5.3.
5.2 To the future

It is proposed that the inability of the two segment simulation model to produce a realistic optimum technique is a direct result of the model's simplicity. A two segment model driven by a single torque generator can make no account for many aspects of high jumping technique. Immediately after impact the phenomenon of wobbling mass and the initial downward swing of the arms serves to decrease the magnitude of the vertical ground reaction force. Later in the takeoff phase the vigorous upward movement of the arms and free leg would serve to increase the vertical reaction force, thereby prolonging the period of ground contact. It was previously mentioned that the toe of the takeoff foot maintains ground contact after the heel is raised from the ground. A two segment model can take no account of these actions, and notably a common limitation of the models developed in the present study was an unrealistically short takeoff time.

Another example of a limitation of the two segment models developed in the present study was the representation of segment length. In the rigid model the segments were assumed to be of equal length, an obvious limitation. However even with the more realistic representation of the segmental inertial characteristics in the elastic model the segmental length was assumed to stay constant throughout the takeoff phase. This assumption will be expanded upon to show how future studies may improve the realism of a simulation model of the high jump takeoff.

The location coordinates of the segmental mass centres during the takeoff were calculated as a function of the segmental lengths and orientations. The calculation of the component segmental mass centre velocities and accelerations during the takeoff phase must therefore consider the rate of change of the segment length. The film analysis of jumps br16 - br26 from Chapter 3.3 was used to determine the time histories of the segment lengths.

In order to model the varying length of the shank segment throughout the period of ground contact the takeoff phase may be initially separated into the eccentric and concentric phases. For the eccentric phase the ground-to-knee distance was used to represent the length of the shank segment. During the concentric phase of the takeoff the extension at the ankle joint gradually increases the length of the shank segment. For the concentric phase of the takeoff the toe-to-knee length obtained from the film analysis was used to represent the length of the shank segment.

Figure 5.2.1 shows the time history of the shank segment length throughout the takeoff phase for jumps br16 - br26. Time zero represents the instant of touchdown. It can be seen that the shank segment length was fairly constant for the first 0.09 s of the takeoff phase for each jump. This time period comprises the whole of the eccentric phase of the takeoff. From 0.09 s the shank length progressively increased until the instant of toe-off. This increase in segment length was due to extension at the ankle joint.
Figure 5.3.1. Time history of the shank segment length throughout the takeoff phase.

It may therefore be assumed that the shank maintains a constant length throughout the eccentric phase of the high jump. During the concentric phase of the high jump the shank angle may be modelled as a quadratic function in knee angle since there was evidence of some curvature in the increase in length as a function of time. In order to remove any discontinuities in the modelled shank length time history the quadratic function should be given a constant term equivalent to the shank length during the eccentric phase. The shank segment length throughout the takeoff phase may therefore be represented using the function shown in Figure 5.2.2.

Figure 5.3.2. Modelling shank segment length as a function of time.
In order to model the varying length of the thigh segment throughout the period of ground contact the knee-to-mass centre (whole body less shank) distance may be used. Figure 5.2.3 shows the time history of the thigh segment length throughout the takeoff phase for jumps br16 - br26. Time zero represents the instant of touchdown.

Figure 5.3.3. Time history of the thigh segment length throughout the takeoff phase.

It can be seen that the thigh segment length initially decreased after touchdown in each jump. This initial decrease in the thigh segment length is partly due to the phenomenon of wobbling mass during the initial impact phase of the takeoff. The thigh segment reached a minimum length at approximately 0.04 s after touchdown. At this time the knee is still flexing so that the thigh segment length begins to decrease during the eccentric phase of the takeoff.

From $t = 0.04$ s the thigh segment length progressively increased up to the instant of toe-off. The general shape of the thigh segment length time history for each jump in Figure 5.2.3 suggests that a polynomial may be used to express length as a function of time. Figure 5.2.4 shows how the changing thigh segment length may be modelled as a function of time throughout the takeoff phase.
The modelling of the changing segment lengths during the takeoff phase illustrates one limitation of the simulation models developed in the present study. It is proposed that in order to more accurately simulate the high jump takeoff a multi-segment model is required. This model should incorporate a foot segment to improve the realism of the simulated period of ground contact. The actions of the free limbs should also be accounted for since they influence mass centre location and ground reaction forces, and also perform an integral role in generating vertical mass centre velocity. A multi-segment model would also allow the location of the whole body mass centre to be represented accurately. The model should be driven by multiple joint torques, with both the muscle and inertial characteristics personalised to a single subject. It is proposed that a model with such complexity may clarify a number of issues regarding optimum high jumping technique.
REFERENCES


APPENDICES

Appendix 1a  Anthropometric measurements for segmental inertia parameters
Appendix 1b  Subject specific inertia parameters
Appendix 2  Computer program modAlex
Appendix 3a  Determination of $t_{max}$ for program modAlex
Appendix 3b  Simulated output of jump br26 using modAlex
Appendix 3c  Optimum simulation for two-segment model
Appendix 4a  Determination of $t_{max}$ for modified program modAlex
Appendix 4b  Simulated output of jump br26 using modified modAlex
Appendix 4c  Optimum simulation for modified model
Appendix 5  Computer program 2seg
Appendix 6a  Sample of Kin-Com output for trial brk01
Appendix 6b  Time histories of joint angle, angular velocity and torque output for Kin-Com trial brk01
Appendix 7a  Simulated output for optimum spring parameters and muscle scale factor
Appendix 7b  Simulated output of jump br26
Appendix 7c  Optimum simulation
Appendix 7d  Constrained optimum simulation
Appendix 7e  Simulated output for new spring parameters and muscle scale factor
Appendix 7f  Optimum simulation for new spring parameters and muscle scale factor
APPENDIX 1a

ANTHROPOMETRIC MEASUREMENTS FOR SEGMENTAL INERTIA PARAMETERS

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All measurements in millimetres

**TORSO**

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**LEFT ARM**

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<th>wrist</th>
<th>thumb</th>
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**RIGHT ARM**

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**LEFT LEG**

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<th>heel</th>
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**RIGHT LEG**

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</table>
APPENDIX 1b

SUBJECT SPECIFIC INERTIA PARAMETERS

Format and sequence of data presentation

*segment name*

*mass (kg), distance of mass centre from proximal joint (m), length (m)*

*principal moments of inertia (kg.m²)*

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<th>Distance</th>
<th>Length</th>
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<th>Moment of Inertia 2 (kg.m²)</th>
<th>Moment of Inertia 3 (kg.m²)</th>
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<td>-0.004</td>
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</table>

*Whole body:*

*Mass = 80.77 kg*

*Density = 0.946 kg.m²*
APPENDIX 2

COMPUTER PROGRAM \textit{modAlex}

\texttt{modAlex} calculates the height of a jump given the inputs of approach velocity and leg plant angle at touchdown, taking into account compliance.

Plant angle is calculated from the vertical parameter ($nn = 100000$)

\begin{verbatim}

character*15 input, output

integer j

print *, 'state input filename'
read(*,15)input

print *, 'state output filename'
read(*,15)output

15 format(3a)

open(16, file=input, form='formatted')
open(20, file=output, form='formatted')

\end{verbatim}
read the initial plant angle, knee angle, mass centre component velocities and the muscle factor.

read(16,*) phi0, ga0, yd0, zd0, tmax

a = 0.464
a = 0.610
grav = 9.81
mass = 78.7
h = 3
tork0 = tmax
gadmax = 35.00
gad0 = 0.00
dt = 0.0001
n = 100000

convert the plant and knee angles to radians

pi = 3.14159265358
rtd = 180.0/pi
dtr = pi/180.0

phi0 = phi0*dtr
ga0 = ga0*dtr
calculate the initial mass centre coordinates

\[ z_0 = 2a \cdot (\sin(gaO/2)) \cdot (\cos(phiO)) \]
\[ y_0 = -z_0 \cdot \tan(phiO) \]

initiate the do loop, but first set the output titles

write (20,*)' TOUCHDOWN CONDITIONS
write (20,*)' horizontal vertical plant
write (20,*') knee'
write (20,*') c.m. velocity c.m. velocity angle
write (20,*') angle'
write (20,104) ydO, zdO, phiO*rtd, gaO*rtd
104 format(2X,f10.3,9X,f10.3,9X,f10.2,9X,f10.2)
write (20,*)'
write (20,*)'
write (20,*)'
write (20,*') n ct F R plant knee
write (20,*') angvel Tk(bw)'
do 100, j=1,n

calculate the ground reaction force.
if (gad0.le.0.0) then
  tork(j) = tmax
else
  tork(j) = gadmax - gad0
  tork(j) = tork(j)/(gadmax+(h*gad0))
  tork(j) = tork(j)*tmax
end if

c
G(j)=tork(j)/(a*cos(ga0/2))
G(j)=G(j)/(mass*grav)

calculate the components of this force
c
F(j) = G(j)*(sin(phi0))
R(j) = G(j)*(cos(phi0))

c
if (j.le.11) go to 290
if (j.ge.11) go to 289

289 if (mod(j-1,10).eq.0) then
290 write(20,108) j-l, (j-I)*dt, F(j), R(j), phi0*rtd,
* ga0*rtd, gad0, tork0/(mass*grav*a)
108 format(i6,1f10.4,6f9.2)
end if

c
305 if (tork(j).le.0.0) go to 200

calculate the component mass centre accelerations
c
ydd0 = -F(j)*grav
zdd0 = (R(j) - 1)*grav
calculate the new component mass centre velocities

\[
yd(j) = yd0 + ydd0*dt \\
zd(j) = zd0 + zdd0*dt
\]

calculate the new mass centre coordinates

\[
y(j) = y0 + yd0*dt + 0.5*ydd0*dt*dt \\
z(j) = z0 + zd0*dt + 0.5*zdd0*dt*dt
\]

calculate the new plant angle and knee angle

\[
\phi(j) = (\text{atan}((-y(j))/z(j))) \\
g(a(j)) = \sqrt{(y(j)**2) + (z(j)**2)} \\
g(a(j)) = g(a(j))/(2*a)
\]

if (g(a(j)).gt.1.0) go to 485
if (g(a(j)).le.1.0) go to 486

485 \quad g(a(j)) = 1.0
486 \quad g(a(j)) = (\text{asin}(g(a(j)))) \\
\quad g(a(j)) = g(a(j))*2
calculate the knee angular velocity

500 \quad \text{gad}(j) = y(j) \cdot yd(j) + z(j) \cdot zd(j)
\quad \text{gad}(j) = \text{gad}(j)/(a^a \cdot \sin(ga(j)))

reset the velocity, coordinate and angle values for the
next step of the loop

\text{yO} = y(j)
\text{zO} = z(j)
\text{ydO} = yd(j)
\text{zdO} = zd(j)
\text{torkO} = \text{tork}(j)
\text{phiO} = \text{phi}(j)
\text{gaO} = \text{ga}(j)
\text{gadO} = \text{gad}(j)

100 \text{ continue}

close all files

do 800 \text{ j} = 1, 20
\text{close}(j)
800 \text{ continue}
200 \[ zh2 = \frac{z_d0^2}{2 \cdot grav} \]

```
c write (20,*)  
write (20,*)  
write (20,*) Height c.m is raised(m) Maximum height o
* f c.m. (m)
write (20,*) 

c write (20,106) zh2, zh2 + z0 + 0.564

c write (20,106) zh2, zh2 + z0
106 format(13Xf10.3,19Xf10.3)
c
   stop
end
```
**APPENDIX 3a**

**DETERMINATION OF $t_{\text{max}}$ FOR PROGRAM *modAlex***

*Touchdown conditions:* \( \dot{y}_g = 7.02 \, \text{m.s}^{-1} \) \( \dot{z}_g = 0.00 \, \text{m.s}^{-1} \) \( \phi = 35.30^\circ \) \( \gamma = 174.30^\circ \)

*Output:*

<table>
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<tr>
<th>n</th>
<th>time [s]</th>
<th>F [mg]</th>
<th>R [mg]</th>
<th>$\phi$ [$^\circ$]</th>
<th>$\gamma$ [$^\circ$]</th>
<th>$\dot{\gamma}$ [rad/s]</th>
<th>T [mga]</th>
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*Height c.m. is raised (m):* 0.791

*Maximum height of c.m. (m):* 2.191
APPENDIX 3b

SIMULATED OUTPUT OF JUMP br26 USING \textit{modAlex}

\textit{Touchdown conditions: } \( \dot{y}_g = 6.81 \text{ m.s}^{-1} \quad \ddot{z}_g = 0.00 \text{ m.s}^{-1} \quad \Phi = 35.90^\circ \quad \Upsilon = 173.50^\circ \)

\textit{Output: }

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\textit{Height c.m. is raised (m) Maximum height of c.m. (m)}

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0.775 & 2.170
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APPENDIX 3c

OPTIMUM SIMULATION FOR TWO-SEGMENT MODEL

**Touchdown conditions:**

\[ \gamma_g = 10.19 \text{ m.s}^{-1} \quad \dot{\gamma}_g = 0.00 \text{ m.s}^{-1} \quad \phi = 51.58^\circ \quad \gamma = 179.00^\circ \]

**Output:**

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*Height c.m. is raised (m)  Maximum height of c.m. (m)*

1.257  2.704
APPENDIX 4a

DETERMINATION OF $t_{\text{max}}$ FOR MODIFIED PROGRAM modAlex

**Touchdown conditions:** $\dot{y}_g = 7.02 \text{ m.s}^{-1}$ \quad $z_\dot{\gamma} = -0.26 \text{ m.s}^{-1}$ \quad $\phi = 35.30^\circ$ \quad $\gamma = 174.30^\circ$

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**APPENDIX 4b**

**SIMULATED OUTPUT OF JUMP br26 USING MODIFIED modAlex**

**Touchdown conditions:**  
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APPENDIX 4c

OPTIMUM SIMULATION FOR MODIFIED MODEL

Touchdown conditions: \( \dot{y}_g = 9.37 \text{ m.s}^{-1} \), \( \ddot{z}_g = -0.26 \text{ m.s}^{-1} \), \( \phi = 51.71^\circ \), \( \gamma = 179.00^\circ \)

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APPENDIX 5

COMPUTER PROGRAM 2seg

2seg calculates the jump height for a 2 segment model of the high jump takeoff.
Input component mass centre velocities, plant angle, knee angle, plant angular velocity and knee angular velocity just prior to touchdown.

double precision
* ys, ysd, ysdd, yso, ysd0, ysdd0, ksy, kvy,
* zs, zsd, zsdd, zso, zsd0, zsdd0, ksz, kvz,
* the, thed, thedd, the0, thed0, thedd0,
* sig, sigd, sigdd, sig0, sigd0, sigdd0,
* temp1, temp2, k1, k2, k3, k4,
* aa(6,4), bb(4), x(4),
* ga, gad, gadd, ga0, gad0, gadd0, gamin,
* gaf, gafd, gafdd,
* phi, phid, phif, phifd,
* yg, zg, ygd, zgdd, zh2,
* a, ad, add,
* b, bd, bdd,
* c, cd, cdd

double precision
* ya, yad, yadd,
* za, zad, zadd,
* yb, ybd, ybdd,
* zb, zbd, zbdd,
* ma, mb, mass, ia, ib,
* dt, grav,
* time, pi, rtd, dr,
* R, F, tork, kf, tk, tkn,
* wk, kact, pka, pkb, pkc, pkd, pkp, pkq

c character*15 input, output
c integer j, mm, m, n, ni, np, one, toggle
c
print *, 'state input filename'
read(*,15)input
print *,'state output filename'
read(*,15)output

15 format(3a)
open(16,file=input,form='formatted')
open(20,file=output,form='formatted')

Read phif = Leg plant angle (film definition)
gaf = knee angle (film definition)
ygd = horizontal velocity of mass centre
zgd = vertical velocity of mass centre just before
the instant of impact

Set the plant and knee angular velocities (deg/s) from film

phifd = -230.0
gafd = -200.0
dt = 0.0001
ni = 5000
np = 10

Print the title and column headings in the output file

write (20,*)'TOUCHDOWN CONDITIONS'
write (20,*)' phif  gaf  phifd  gafd  ygd  zgd'
write (20,101) phif, gaf, phifd, gafd, ygd, zgd
101 format(6f10.3)
write (20,*)'OUTPUT FILE: FORMAT'
write (20,*)'
write(20,*),' time(s)'
write(20,*), phi(deg) ga(deg) F(mg) R(mg)'
write(20,*), the(deg) sig(deg) thed(rad) sigd(rad)'
write(20,*), yg(m) zg(m) ygd(m/s) zgd(m/s)'
write(20,*), ys(m) zs(m) ysd(m/s) zsd(m/s)'
write(20,*), gad(rad/s) tork(Nm)'
write (20,*), ' OUTPUT FILE: RESULTS '

Convert the input film knee and plant angles and angular velocities from degrees to radians

pi = 3.14159265358
rtd = 180.0 / pi
dtr = pi / 180.0

phif = phif*dtr
gaf = gaf*dtr

phifd = phifd*dtr
gafd = gafd*dtr

Convert the plant and knee angles and angular velocities into the equivalent values for the model definition.

phi = 0.969572*phif
ga = 1.02202*gaf

phid = 0.969572*phifd
gad = 1.02202*gafd

Set the segment angular accelerations, and thereby calculate the angular acceleration at the knee joint.

Set the component locations and accelerations of the foot.
Calculate the segment orientation angles and angular velocities from the plant and knee angles and angular velocities.

\[ \text{the} = \pi - \frac{\text{ga}}{2.0} - \phi \]
\[ \text{thed} = -\frac{\text{gad}}{2.0} - \phi \]

Set the parameter values required to calculate the angular knee velocity: They are segmental masses, moments of inertia, lengths and angular velocities.

\[ \text{ma} = 4.317 \]
\[ \text{mb} = 75.183 \]
\[ \text{mass} = \text{ma} + \text{mb} \]
\[ \text{ia} = 0.074 \]
\[ \text{ib} = 13.22 \]
\[ a = 0.4116 \]
\[ b = 0.5836 \]
\[ c = 0.5996 \]
\[ \text{grav} = 9.81 \]
Calculate the component foot velocities

\[ ysd = ygd - (\frac{ma}{mass}) \cdot (a \cdot \sin(\theta) \cdot \theta_{ed}) \]
\[ ysd = ysd - (\frac{mb}{mass}) \cdot (c \cdot \sin(\theta) \cdot \theta_{ed}) \]
\[ ysd = ysd - (\frac{mb}{mass}) \cdot (b \cdot \sin(\sigma) \cdot \sigma_{d}) \]

\[ zsd = zgd - (\frac{ma}{mass}) \cdot (a \cdot \cos(\theta) \cdot \theta_{ed}) \]
\[ zsd = zsd - (\frac{mb}{mass}) \cdot (c \cdot \cos(\theta) \cdot \theta_{ed}) \]
\[ zsd = zsd - (\frac{mb}{mass}) \cdot (b \cdot \cos(\sigma) \cdot \sigma_{d}) \]

**THE TAKEOFF PHASE**

\[ \text{one} = 1 \]

Loop starts here

\[ \text{do 100, j=1, ni} \]
\[ \text{time} = \text{dt}\cdot(j - 1) \]
\[ \text{if (mod(j-1, np).eq.0) then} \]
\[ \text{write (20,*),} \]
\[ \text{write(20,700) time} \]
\[ \text{700 format(1f8.3)} \]
\[ \text{endif} \]
\[ \text{gad} = \text{sigd} - \text{thed} \]

\[ \text{gamin is minimum knee angle} \]

\[ \text{if (one.eq.1.and.gad.gt.0.0) then} \]
\[ \text{gamin = ga} \]
\[ \text{one = 2} \]
\[ \text{endif} \]
if (gad.gt.0.0) then
  ga0 = ga
  gad0 = gad
  gadd0 = gadd
if (ga.gt.pi) then
  ga0 = pi
  gad0 = 0.0
  gadd0 = 0.0
endif

The segment lengths remain constant.

Calculate the plant and knee angles.

phi = c*cos(the) + b*cos(sig)
phi = phi/(c*sin(the) + b*sin(sig))
phi = atan(phi)

ga = pi + sig - the

Calculate the new coordinates and component velocities of the segmental and whole body mass centres.

ya = ys - a*cos(the)
yad = ysd - ad*cos(the) + a*thed*sin(the)
yadd = ysd - ad*cos(the) + 2.0*ad*thed*sin(the) + a*thedd*sin(the)
Given the component segmental accelerations the forces acting on the segments and subsequently the ground reaction forces can be calculated.

\[
\begin{align*}
za &= zs + a \cdot \sin(\text{the}) \\
zad &= zsd + ad \cdot \sin(\text{the}) + a \cdot \text{thedd} \cdot \cos(\text{the}) \\
zadd &= zsdd + ad \cdot \sin(\text{the}) + 2.0 \cdot ad \cdot \text{thedd} \cdot \cos(\text{the}) \\
&\quad - a \cdot \text{thedd} \cdot \text{thedd} \cdot \sin(\text{the}) \\
&\quad + a \cdot \text{thedd} \cdot \cos(\text{the})
\end{align*}
\]

\[
\begin{align*}
yb &= ys - c \cdot \cos(\text{the}) - b \cdot \cos(\text{sig}) \\
ybd &= ysd - cd \cdot \cos(\text{the}) + c \cdot \text{thedd} \cdot \sin(\text{the}) \\
&\quad - bd \cdot \cos(\text{sig}) + b \cdot \text{sigd} \cdot \sin(\text{sig}) \\
ybdd &= ysdd - cdd \cdot \cos(\text{the}) \\
&\quad + 2.0 \cdot cd \cdot \text{thedd} \cdot \sin(\text{the}) \\
&\quad + c \cdot \text{thedd} \cdot \text{thedd} \cdot \cos(\text{the}) \\
&\quad + c \cdot \text{thedd} \cdot \sin(\text{the}) \\
&\quad - bdd \cdot \cos(\text{sig}) \\
&\quad + 2.0 \cdot bd \cdot \text{sigd} \cdot \sin(\text{sig}) \\
&\quad + b \cdot \text{sigd} \cdot \text{sigd} \cdot \cos(\text{sig}) \\
&\quad + b \cdot \text{sigdd} \cdot \sin(\text{sig})
\end{align*}
\]

\[
\begin{align*}
zb &= zs + c \cdot \sin(\text{the}) + b \cdot \sin(\text{sig}) \\
zbd &= zsd + cd \cdot \sin(\text{the}) + c \cdot \text{thedd} \cdot \cos(\text{the}) \\
zbdd &= zsdd + cdd \cdot \sin(\text{the}) \\
&\quad + 2.0 \cdot cdd \cdot \text{thedd} \cdot \cos(\text{the}) \\
&\quad + ad \cdot \text{thedd} \cdot \sin(\text{the}) \\
&\quad + c \cdot \text{thedd} \cdot \cos(\text{the}) \\
&\quad + bdd \cdot \sin(\text{sig}) \\
&\quad + 2.0 \cdot bdd \cdot \text{sigd} \cdot \cos(\text{sig}) \\
&\quad - b \cdot \text{sigd} \cdot \text{sigd} \cdot \sin(\text{sig}) \\
&\quad + b \cdot \text{sigdd} \cdot \cos(\text{sig})
\end{align*}
\]

\[
\begin{align*}
F &= -k sy \cdot ys - kvy \cdot ysd \\
R &= -ksz \cdot zs - kvz \cdot zsd
\end{align*}
\]

if (mod(j-1,np).eq.0) then
write(20,708) phi*57.296, ga*57.296, F/(mass*grav),
* R/(mass*grav)
write(20,708)the*57.296, sig*57.296, thed, sigd
708 format(8X,4f10.3)
endif
Given the component velocities of both segments the component velocities of the whole body mass centre can be calculated.

\[ y_{gd} = \frac{ma \cdot y_{ad} + mb \cdot y_{bd}}{mass} \]
\[ z_{gd} = \frac{ma \cdot z_{ad} + mb \cdot z_{bd}}{mass} \]
\[ y_{g} = \frac{ma \cdot y_{a} + mb \cdot y_{b}}{mass} \]
\[ z_{g} = \frac{ma \cdot z_{a} + mb \cdot z_{b}}{mass} \]

if (mod(G-I,np).eq.0) then
  write (20,714) yg,zg,ygd,zgd
write(20,714) ys, zs, ysd, zsd
endif

714 format(8X,4f10.3)
endif

toggle = 1

Given the angular velocity at the knee the muscle subroutine calculates the torque acting at the knee joint.

Modify torque angular velocity relationship to give 1% maximum eccentric torque at concentric angular velocity of 800 deg/sec.

if (gad.gt.250*dtr) then
  gad = (gad*rtd - 250)/(800 - 250)
  gad = gad*(352 - 250)
  gad = gad + 250
  gad = gad*dtr
endif

500 continue

wk = gad
\[ p_{ka} = 156.4931818 \]
\[ p_{kb} = 503.9582434 \]
\[ p_{kc} = 4.486176356 \]
\[ p_{kd} = 0.0000102091 \]
\[ p_{kp} = 2.419882841 \]
\[ p_{kq} = 2.569673685 \]

\[
\text{if}(w_k \leq 0.0) \text{ then} \\
\ k_{act} = 1.0 \\
\ t_k = (p_{ka}+p_{kb} \cdot dexp(p_{kp} \cdot w_k)) \cdot k_{act}/ \left( (1+p_{kc} \cdot dexp(p_{kp} \cdot w_k)) \cdot (1+p_{kd} \cdot dexp(p_{kq} \cdot w_k)) \right) \\
\text{else} \\
\ k_{act} = 1.0 \\
\ t_{kn} = (p_{ka} \cdot dexp(-(p_{kp}+p_{kq}) \cdot w_k)+p_{kb} \cdot dexp(-p_{kq} \cdot w_k)) \cdot k_{act}/ \left( (p_{kc}+dexp(-p_{kp} \cdot w_k)) \cdot (p_{kd}+dexp(-p_{kq} \cdot w_k)) \right) \\
\ t_k = t_{kn} \\
\text{endif} \\
\]

\[
\text{tork} = k_f \cdot t_k \\
\]

\[
\text{if} \ (\text{toggle.eq.1.and.mod(j-1,np).eq.0}) \text{ then} \\
\text{write}(20,705) \text{ gad, tork} \\
\text{705 format}(8X,2f10.3) \\
\text{endif} \\
\]

\[
\text{cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
\[ aa(2,4) = mb*b*\cos(sig) \]
\[ aa(3,1) = ma*a*\sin(the) + mb*c*\sin(the) \]
\[ aa(3,2) = mb*b*\sin(sig) \]
\[ aa(3,3) = \text{mass} \]
\[ aa(3,4) = 0.0 \]
\[ aa(4,1) = ma*a*\cos(the) + mb*c*\cos(the) \]
\[ aa(4,2) = mb*c*\cos(sig) \]
\[ aa(4,3) = 0.0 \]
\[ aa(4,4) = \text{mass} \]

\[ \text{temp1} = \text{ys}^{*}\text{grav} - a^{*}\text{grav}^{*}\cos(the) - 2*a^{*}\text{ad}^{*}\text{thed} + \]
* \[ a^{*}\text{thed}^{*}2^{*}\sin(the)^{*}\text{ys} - a^{*}\text{thed}^{*}2^{*}\cos(the)^{*}\text{zs} + \]
* \[ 2*a^{*}\text{ad}^{*}\text{thed}^{*}\cos(the)^{*}\text{ys} - 2*a^{*}\text{ad}^{*}\text{thed}^{*}\sin(the)^{*}\text{zs} \]

\[ \text{temp2} = \text{ys}^{*}\text{grav} - c^{*}\text{grav}^{*}\cos(the) - b^{*}\text{grav}^{*}\cos(sig) + \]
* \[ b^{*}\text{sigd}^{*}2^{*}\sin(sig)^{*}\text{ys} - b^{*}\text{sigd}^{*}2^{*}\cos(sig)^{*}\text{zs} - \]
* \[ c^{*}\text{thed}^{*}2^{*}\sin(the)^{*}\text{ys} - c^{*}\text{thed}^{*}2^{*}\cos(the)^{*}\text{zs} + \]
* \[ 2*b^{*}\text{bd}^{*}\text{sigd}^{*}\cos(sig)^{*}\text{ys} - 2*b^{*}\text{bd}^{*}\text{sigd}^{*}\sin(sig)^{*}\text{zs} + \]
* \[ 2*b^{*}\text{cd}^{*}\text{thed}^{*}\cos(the)^{*}\text{ys} - 2*b^{*}\text{cd}^{*}\text{thed}^{*}\sin(the)^{*}\text{zs} - \]
* \[ 2*b^{*}\text{bd}^{*}\text{sigd} - 2*c^{*}\text{cd}^{*}\text{thed} + \]
* \[ b^{*}\text{cd}^{*}\text{sigd}^{*}\cos(the-sig) - b^{*}\text{cd}^{*}\text{sigd}^{*}\sin(the-sig) - \]
* \[ b^{*}\text{cd}^{*}\text{thed}^{*}\cos(the-sig) - 2*b^{*}\text{cd}^{*}\text{sigd}^{*}\cos(the-sig) + \]
* \[ b^{*}\text{cd}^{*}\text{thed}^{*}2^{*}\sin(the-sig) - b^{*}\text{cd}^{*}\text{sigd}^{*}2^{*}\sin(the-sig) \]

\[ \text{bb}(1) = ma^{*}\text{temp1} + mb^{*}\text{temp2} \]

\[ \text{temp1} = 0.0 \]

\[ \text{temp2} = -b^{*}\text{grav}^{*}\cos(sig) - 2*b^{*}\text{bd}^{*}\text{sigd} - \]
* \[ 2*b^{*}\text{cd}^{*}\text{thed}^{*}\cos(the-sig) + b^{*}\text{cd}^{*}\text{thed}^{*}2^{*}\sin(the-sig) - \]
* \[ b^{*}\text{cd}^{*}\text{sigd}^{*}\sin(the-sig) \]

\[ \text{bb}(2) = ma^{*}\text{temp1} + mb^{*}\text{temp2} + \text{tork} \]

\[ \text{temp1} = \text{add}^{*}\cos(the) - 2*a^{*}\text{thed}^{*}\sin(the) - a^{*}\text{thed}^{*}2^{*}\cos(the) \]
temp2 = cdd*cos(\(\theta\)) - 2*cd*\(\theta^2\)*cos(\(\theta\)) + bdd*cos(\(\sigma\)) - 2*bd*\(\sigma^2\)*sin(\(\sigma\)) - b*\(\sigma^2\)*cos(\(\sigma\))

bb(3) = ma*temp1 + mb*temp2 - ksy*ys - kvy*ysd

\[
\begin{align*}
\text{temp1} &= -add\sin(\theta) - 2*ad*\theta^2\cos(\theta) + a*\theta^2\sin(\theta) \\
\text{temp2} &= -cdd\sin(\theta) - 2*cd*\theta^2\cos(\theta) + c*\theta^2\sin(\theta)
\end{align*}
\]

bb(4) = ma*temp1 + mb*temp2 - ksz*zs - kvz*zsd - mass*grav

\[
\begin{align*}
\text{mm} &= 6 \\
\text{m} &= 4 \\
\text{n} &= 4
\end{align*}
\]

call solve( x, aa, bb, mm, m, n )

\[
\begin{align*}
\text{thedd} &= x(1) \\
\text{sigdd} &= x(2) \\
\text{ysdd} &= x(3) \\
\text{zsdd} &= x(4)
\end{align*}
\]

gadd = sigdd - thedd

c
The angles theta and sigma and their respective velocities can be determined at the next instant in time by the process of direct integration. Use two passes to estimate the accelerations over the integration step. In the first pass use the acceleration at the start of the interval to estimate theta, sigma, ys and zs and their first derivatives at the end of the interval. In the second pass average the accelerations at the beginning and end of the interval.

c
if (toggle.eq.1) then

\[
\begin{align*}
\text{the0} &= \text{the} \\
\text{thed0} &= \text{thed} \\
\text{thedd0} &= \text{thedd}
\end{align*}
\]

c
\[
\begin{align*}
\text{sig0} &= \text{sig} \\
\text{sigd0} &= \text{sigd} \\
\text{sigdd0} &= \text{sigdd}
\end{align*}
\]
ys0 = ys
ysd0 = ysd
ysdd0 = ysdd
c
zs0 = zs
zsd0 = zsd
zsd0 = zsd

c
endif

thedd = 0.5*thedd + 0.5*thedd0
sigdd = 0.5*sigdd + 0.5*sigdd0
ysdd = 0.5*ysdd + 0.5*ysdd0
zsd = 0.5*zsd + 0.5*zsd0
c
the = the0 + the0*dt + 0.5*thedd*dt*dt
thed = the0 + thedd*dt
c
sig = sig0 + sig0*dt + 0.5*sigdd*dt*dt
sigd = sigd0 + sigdd*dt
c
ys = ys0 + ysd0*dt + 0.5*ysdd*dt*dt
ysd = ysd0 + ysd*dt
c
zs = zs0 + zsd0*dt + 0.5*zsd*dt*dt
zsd = zsd0 + zsd*dt
c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

c
if (R .le. -1.0 .And. time .ge. 0.030) go to 200
toggle = -toggle

if (toggle .eq. -1) go to 500
c
100 continue
c
c
ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

c
c
Close all files
c
c
cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
do 800 j = 1, 20
  close(j)
800 continue

zh2 = zgd ** 2
zh2 = zh2 / (2 * grav)

write (20, *) '     
write (20, *) '     
write (20, *) ' Time of takeoff (s)'
write (20, 104) time
write (20, *) '     
write (20, *) ' Height of c.m. at t.o. (m)'
write (20, 103) zg + 0.309
write (20, *) '     
write (20, *) ' Maximum height of c.m. (m)'
write (20, 103) zh2 + zg + 0.309

103 format (f10.3)
104 format (Xf10.4)

phi = phi * rtd
phif = 2.2384 + 0.4313 * phi

ga = ga * rtd
gaf = ga - 0.92

write (20, *) '     
write (20, *) ' Plant angle at toe-off (deg)'
write (20, 1001) phif
write (20, *) '     
write (20, *) ' Knee angle at toe-off (deg)'
write (20, 1002) gaf

Convert the model values for the plant angle and knee angle
at toe-off into values comparable with the film data.

Convert gamin to film value for output

Convert gamin to film value for output
write (20,*), 'Minimum knee angle (deg)'
write (20,1002) gamin
1001 format (f10.2)
1002 format (Xf10.2)

stop
end

c subroutine solve( x, a, b, mm, m, n ) obtains the least squares
solution to the matrix equation \( ax = b \).

c parameters:
\( mm \) : first dimension of array a as declared in the
Calling program
\( m \) : number of equations
\( n \) : number of unknowns
\( a \) : dp array of dimension (mm,nn) where:
\( mm > m+1 \) and \( nn > n \)
\( a \) is overwritten on exit
\( b \) : dp array of dimension at least \( n \)
on entry \( b \) contains the rhs of the equation
on exit \( b \) contains the residuals
\( x \) : dp array of dimension at least \( n \)
on exit \( x \) contains the solution vector

c The least squares calculation. The algorithm used is based
on Algorithms 3.8 and 3.10 in: G. W. Stewart's "Introduction to
**APPENDIX 6a**

**SAMPLE OF KIN-COM OUTPUT FOR TRIAL brk01**

KIN-COM TEST RESULT
ASCII DUMP

Name : REILLY, BRENDAN  
Test Date : 26.10.94  
Joint : KNEE  
Side : RIGHT  
Muscle Group : EXT/FLEX  
Lever Arm : 24 cm  
Parameters : SPEED mode  
Movement Speed: Forth = 50 deg/sec, Back = 50 deg/sec  
Start Force : Forth = 50 N, Back = 50 N  
Force Limits : Low = 20 N, High = 2000 N  
Start Angle : 129 deg  
Stop Angle : 165 deg  
Acceleration : LOW  
Deceleration : LOW

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Angle (deg)</th>
<th>Velocity (deg/sec)</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.35</td>
<td>129.00</td>
<td>-0.25</td>
<td>-129</td>
</tr>
<tr>
<td>4.36</td>
<td>128.75</td>
<td>-0.25</td>
<td>-105</td>
</tr>
<tr>
<td>4.37</td>
<td>129.00</td>
<td>0.00</td>
<td>-75</td>
</tr>
<tr>
<td>4.38</td>
<td>129.00</td>
<td>0.25</td>
<td>-32</td>
</tr>
<tr>
<td>4.39</td>
<td>129.00</td>
<td>0.25</td>
<td>21</td>
</tr>
<tr>
<td>4.40</td>
<td>129.00</td>
<td>0.75</td>
<td>112</td>
</tr>
<tr>
<td>4.41</td>
<td>129.00</td>
<td>1.75</td>
<td>214</td>
</tr>
<tr>
<td>4.42</td>
<td>129.00</td>
<td>3.50</td>
<td>344</td>
</tr>
<tr>
<td>4.43</td>
<td>129.25</td>
<td>8.50</td>
<td>457</td>
</tr>
<tr>
<td>4.44</td>
<td>129.25</td>
<td>14.25</td>
<td>520</td>
</tr>
<tr>
<td>4.45</td>
<td>129.50</td>
<td>20.00</td>
<td>604</td>
</tr>
<tr>
<td>4.46</td>
<td>130.00</td>
<td>25.50</td>
<td>674</td>
</tr>
<tr>
<td>4.47</td>
<td>130.25</td>
<td>30.00</td>
<td>688</td>
</tr>
<tr>
<td>4.48</td>
<td>130.50</td>
<td>35.00</td>
<td>698</td>
</tr>
<tr>
<td>4.49</td>
<td>131.00</td>
<td>40.25</td>
<td>717</td>
</tr>
<tr>
<td>4.50</td>
<td>131.25</td>
<td>45.00</td>
<td>713</td>
</tr>
<tr>
<td>4.51</td>
<td>131.75</td>
<td>49.75</td>
<td>703</td>
</tr>
<tr>
<td>4.52</td>
<td>132.25</td>
<td>54.50</td>
<td>698</td>
</tr>
<tr>
<td>4.53</td>
<td>133.00</td>
<td>52.25</td>
<td>686</td>
</tr>
<tr>
<td>4.54</td>
<td>133.50</td>
<td>50.75</td>
<td>691</td>
</tr>
</tbody>
</table>
APPENDIX 6b

TIME HISTORIES OF JOINT ANGLE, ANGULAR VELOCITY AND TORQUE OUTPUT FOR KIN-COM TRIAL brk01

angle [°]

angular velocity [°/s]

torque [N.m]
APPENDIX 7a

SIMULATED OUTPUT FOR OPTIMUM SPRING PARAMETERS AND MUSCLE SCALE FACTOR

TOUCHDOWN CONDITIONS

\[
\begin{array}{cccccc}
\text{phif} & \text{gaf} & \text{phid} & \text{gad} & \text{ygd} & \text{zgd} \\
35.300 & 174.300 & -230.000 & -200.000 & 7.020 & -0.260 \\
\end{array}
\]

OUTPUT FILE : FORMAT

<table>
<thead>
<tr>
<th>time(s)</th>
<th>phi(deg)</th>
<th>ga(deg)</th>
<th>F(mg)</th>
<th>R(mg)</th>
<th>the(deg)</th>
<th>sig(deg)</th>
<th>thed(rad)</th>
<th>sigd(rad)</th>
<th>yg(m)</th>
<th>zg(m)</th>
<th>ygd(m/s)</th>
<th>zgd(m/s)</th>
<th>ys(m)</th>
<th>zs(m)</th>
<th>ysd(m/s)</th>
<th>zsd(m/s)</th>
<th>gad(rad/s)</th>
<th>tork(Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>34.213</td>
<td>178.139</td>
<td>-1.345</td>
<td>2.121</td>
<td>56.705</td>
<td>54.843</td>
<td>5.676</td>
<td>2.108</td>
<td>-0.641</td>
<td>0.944</td>
<td>7.020</td>
<td>-0.260</td>
<td>0.000</td>
<td>0.000</td>
<td>3.272</td>
<td>-2.766</td>
<td>-3.568</td>
<td>752.688</td>
</tr>
<tr>
<td>0.010</td>
<td>30.958</td>
<td>174.573</td>
<td>-5.777</td>
<td>7.174</td>
<td>61.720</td>
<td>56.292</td>
<td>9.281</td>
<td>2.963</td>
<td>-0.573</td>
<td>0.943</td>
<td>6.519</td>
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<td>0.012</td>
<td>-0.036</td>
<td>0.341</td>
<td>-3.390</td>
<td>-6.318</td>
<td>752.857</td>
</tr>
<tr>
<td>0.020</td>
<td>27.550</td>
<td>171.653</td>
<td>-6.420</td>
<td>9.644</td>
<td>66.567</td>
<td>58.219</td>
<td>7.672</td>
<td>3.735</td>
<td>-0.511</td>
<td>0.947</td>
<td>5.902</td>
<td>0.844</td>
<td>0.014</td>
<td>-0.063</td>
<td>0.001</td>
<td>-2.040</td>
<td>-3.937</td>
<td>752.788</td>
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<tr>
<td>0.030</td>
<td>24.413</td>
<td>170.052</td>
<td>-5.901</td>
<td>10.298</td>
<td>70.495</td>
<td>60.546</td>
<td>6.080</td>
<td>4.368</td>
<td>-0.455</td>
<td>0.960</td>
<td>5.291</td>
<td>1.733</td>
<td>0.013</td>
<td>-0.076</td>
<td>-0.186</td>
<td>-0.649</td>
<td>-1.712</td>
<td>738.739</td>
</tr>
<tr>
<td>Time of takeoff (s)</td>
<td>0.0713</td>
<td></td>
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<tr>
<td>Height of c.m. at t.o. (m)</td>
<td>1.091</td>
<td></td>
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<tr>
<td>Maximum height of c.m. (m)</td>
<td>1.855</td>
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</tr>
<tr>
<td>Plant angle at toe-off (deg)</td>
<td>8.23</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Knee angle at toe-off (deg)</td>
<td>173.20</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Minimum knee angle (deg)</td>
<td>154.76</td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Time of takeoff</th>
<th>Height of c.m. at t.o.</th>
<th>Maximum height of c.m.</th>
<th>Plant angle at toe-off</th>
<th>Knee angle at toe-off</th>
<th>Minimum knee angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.040</td>
<td>21.519</td>
<td>169.571</td>
<td>-4.465</td>
<td>9.364</td>
<td>0.040</td>
<td>21.519</td>
</tr>
<tr>
<td></td>
<td>73.626</td>
<td>63.196</td>
<td>4.839</td>
<td>4.846</td>
<td>-0.405</td>
<td>169.571</td>
</tr>
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<td></td>
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<td>0.599</td>
<td>0.007</td>
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<td>18.902</td>
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<tr>
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# APPENDIX 7b

## SIMULATED OUTPUT OF JUMP br26

### TOUCHDOWN CONDITIONS

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Maximum height of c.m. (m)
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Plant angle at toe-off (deg)
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Knee angle at toe-off (deg)
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Minimum knee angle (deg)
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APPENDIX 7c

OPTIMUM SIMULATION

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the (deg)  sig (deg)  thed (rad)  sigd (rad)  
yg (m)    zg (m)     ygd (m/s)  zgd (m/s)  
ys (m)    zs (m)     ysd (m/s)  zsd (m/s)  
gad (rad/s)  tork (Nm)  

OUTPUT FILE : RESULTS (np = 100)

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**Height of c.m. at t.o. (m)**

*1.166*

**Maximum height of c.m. (m)**

*4.914*

**Plant angle at toe-off (deg)**

*18.47*

**Knee angle at toe-off (deg)**

*163.53*

**Minimum knee angle (deg)**

*146.55*
APPENDIX 7d

CONSTRAINED OPTIMUM SIMULATION

TOUCHDOWN CONDITIONS

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### APPENDIX 7e

**SIMULATED OUTPUT FOR NEW SPRING PARAMETERS AND MUSCLE SCALE FACTOR**

**TOUCHDOWN CONDITIONS**

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APPENDIX 7f

OPTIMUM SIMULATION FOR NEW SPRING PARAMETERS AND MUSCLE SCALE FACTOR

TOUCHDOWN CONDITIONS

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63.670 & 178.440 & -230.000 & -200.000 & 9.320 & -0.260 \\
\end{array}
\]

OUTPUT FILE : FORMAT

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OUTPUT FILE : RESULTS (np = 100)

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*Time of takeoff (s)*

0.1383

*Height of c.m. at t.o. (m)*

1.285

*Maximum height of c.m. (m)*

5.019

*Plant angle at toe-off (deg)*

13.11

*Knee angle at toe-off (deg)*

190.35

*Minimum knee angle (deg)*

169.07