Use of conducting crucibles in medium-frequency induction melting of non-ferrous metals

This item was submitted to Loughborough University's Institutional Repository by the/an author.

Additional Information:


Metadata Record: [https://dspace.lboro.ac.uk/2134/28091](https://dspace.lboro.ac.uk/2134/28091)

Publisher: © M.R. Kargahi

Rights: This work is made available according to the conditions of the Creative Commons Attribution-NonCommercial-NoDerivatives 2.5 Generic (CC BY-NC-ND 2.5) licence. Full details of this licence are available at: [http://creativecommons.org/licenses/by-nc-nd/2.5/](http://creativecommons.org/licenses/by-nc-nd/2.5/)

Please cite the published version.
This item was submitted to Loughborough University as a PhD thesis by the author and is made available in the Institutional Repository (https://dspace.lboro.ac.uk/) under the following Creative Commons Licence conditions.

For the full text of this licence, please go to: http://creativecommons.org/licenses/by-nc-nd/2.5/
USE OF CONDUCTING CRUCIBLES IN MEDIUM FREQUENCY INDUCTION MELTING OF NON-FERROUS METALS

by

MOHAMMAD REZA KARGAHI

A DOCTORAL THESIS

Submitted in partial fulfilment of the requirements for the award of Ph.D. of Loughborough University of Technology 1987.

Supervisor: Dr. L. Hobson

Department of Electronic and Electrical Engineering

© by M.R. Kargahi 1987
SUMMARY

Carbon bonded silicon carbide and clay bonded graphite crucibles are used in non-ferrous induction melting furnaces. Silicon carbide crucibles especially have encountered premature failure when used at high power densities and operating frequencies. This is thought to be related to their non-uniform properties. To gain a more thorough understanding of the problem, an equivalent circuit analysis has been applied to the composite load of crucible and metal charge. An explanation of the analysis is put forward, and results given including electrical characteristics, the distribution of power input between charge and crucible and resultant meniscus height for typical industrial installations. Probes are used to measure the current density and magnetic field strength on the surface of the crucibles. Electrical resistivity and power densities at different axial points of the crucibles are estimated from the results.

To assess thermal behaviour of crucibles in service, transient temperature distributions in the crucible and charge during melting cycle are obtained. Heat transfer between crucible and charge, heat losses, non-uniform power densities and non-linear variation of properties with temperature are taken into account. Results are used to correlate different melting configurations with critical modes of crucible failure.

Remedies including good foundry practice and possible changes in crucible manufacture as well as electromagnetic solutions are put forward to enhance (silicon carbide) crucible lifetime.
TO MY FATHER
WHO VALUES EDUCATION ABOVE ALL,
TO MY MOTHER,
AND TO MY LOST ONES
ACKNOWLEDGEMENT

I wish to express my deepest gratitude to my supervisor Dr. L. Hobson for his continual support, advice and encouragement throughout the duration of this research.

I also wish to convey my thanks to the staff of the Electronic and Electrical Engineering Department and those of the Computer Centre at Loughborough for their services.

I am grateful to Dr. A.H. Whitfield of Engineering Mathematics Department for helpful discussions and comments and to Dr. D.S. Coleman of Engineering Materials and Design Department for assisting with SEM examinations.

I am also indebted to my colleagues in the laboratory for their friendship and encouragement, to Mrs P.A. Higgs for typing the thesis and finally to my wife for her patience and understanding.

Lastly, my hearty thanks are due to my family at home for their unconditional love and financial support which has made my studies possible.

M R Kargahi

Loughborough

May 1987
<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>Page No</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE PAGE</td>
<td>(i)</td>
</tr>
<tr>
<td>SUMMARY</td>
<td>(ii)</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>(iii)</td>
</tr>
<tr>
<td>CONTENTS</td>
<td>(iv)</td>
</tr>
<tr>
<td>LIST OF PLATES</td>
<td>(xi)</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>(xii)</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>(xiii)</td>
</tr>
<tr>
<td>CHAPTER 1 - INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>CHAPTER 2 - REVIEW</td>
<td>16</td>
</tr>
<tr>
<td>2.1 Induction Melting</td>
<td>18</td>
</tr>
<tr>
<td>2.2 Coreless Furnace</td>
<td>19</td>
</tr>
<tr>
<td>2.2.1 Skin effect</td>
<td>20</td>
</tr>
<tr>
<td>2.2.2 Stirring</td>
<td>20</td>
</tr>
<tr>
<td>2.2.3 Choice of furnace frequency</td>
<td>25</td>
</tr>
<tr>
<td>2.3 Medium Frequency Coreless Furnaces</td>
<td>27</td>
</tr>
<tr>
<td>2.3.1 Power Sources</td>
<td>28</td>
</tr>
<tr>
<td>2.3.1.1 Thyristor inverter</td>
<td>29</td>
</tr>
<tr>
<td>2.3.1.2 Principles of operation of thyristor inverter</td>
<td>30</td>
</tr>
<tr>
<td>2.3.1.3 Load Circuit</td>
<td>34</td>
</tr>
<tr>
<td>2.3.1.4 Water cooling</td>
<td>35</td>
</tr>
<tr>
<td>2.4 Melting of Non-ferrous Metals</td>
<td>36</td>
</tr>
<tr>
<td>2.5 Crucible Induction Melting</td>
<td>38</td>
</tr>
<tr>
<td>2.5.1 Conducting crucible induction melting</td>
<td>40</td>
</tr>
</tbody>
</table>
2.5.1.1 Clay-graphite crucibles
2.5.1.2 Silicon carbide crucibles
2.5.2 Crucible shapes and furnace types
2.5.2.1 Tilting furnace
2.5.2.2 Push-up furnace
2.5.2.3 Lift-coil and lift-swing furnaces
2.6 Crucibles and Melting Techniques
2.7 Analysis of Conducting Crucible Induction Furnace
2.7.1 Analytical approach - equivalent circuit technique

CHAPTER 3 - ANALYSIS
3.1 Introduction
3.2 Equivalent Circuit of the Conducting Crucible Furnace
3.3 Resistances \( X_0 \), \( X_R \) and \( R_1 \) of the Equivalent Circuit of the Conducting Crucible Furnace
3.3.1 Reactance \( X_R \) of return flow path
3.3.2 Reactance \( X_0 \) of the refractory packing
3.3.3 Resistance \( R_1 \) of the furnace coil
3.4 Impedance \( Z_2 \) and \( Z_3 \) of the conducting crucible and charge
3.4.1 Poynting theorem
3.4.2 Complex power output and impedances \( Z_2 \) and \( Z_3 \)
3.4.3 Solution of field equations in the conducting crucible furnace
3.5 Electrical Values of the Conducting Crucible Furnace
3.5.1 Impedance of the furnace circuit
3.5.2 Currents, power, efficiency and power factor
3.5.3 Distribution of power between charge and conducting crucible

3.5.4 Pressure and meniscus in the charge

3.6 Implementation of the Theory

3.6.1 Distribution of power between charge and crucible

3.6.2 Meniscus height

3.7 Conclusions

| CHAPTER 4 - MEASUREMENTS AND ESTIMATION OF CRUCIBLE MATERIALS PROPERTIES 127 |
|-------------------------|------------------|
| 4.1 Introduction       | 128              |
| 4.2 Measurement of Resistivity by Probes | 129 |
| 4.2.1 Current density probe | 129 |
| 4.2.2 Magnetic field-strength probe | 131 |
| 4.3 Estimation of Crucible Resistivity | 132 |
| 4.4 Microscopic Examination of C/SiC Crucible Material | 141 |
| 4.5 Estimation of Thermal Properties of crucible materials at Various Temperatures | 157 |
| 4.5.1 Specific heat | 158 |
| 4.5.2 Thermal conductivity | 162 |
| 4.6 Other Crucible Properties | 164 |
| 4.6.1 Bulk density | 164 |
| 4.6.2 Coefficient of thermal expansion | 167 |
| 4.6.3 Young's modulus | 167 |
| 4.7 Conclusions | 167 |
CHAPTER 5 - AXIAL POWER DISTRIBUTION

5.1 Introduction 171
5.2 Layers of Composite Load 173
5.3 Normalizing Procedure 176
5.4 Power Distribution 180
5.5 Rates of Temperature Rise 180
5.5.1 Heat balance equation 180
5.6 Conclusions 185

CHAPTER 6 - TEMPERATURE DISTRIBUTION IN CRUCIBLE AND CHARGE 186

6.1 Introduction 187
6.2 Thermal Shock/Stress 188
6.3 Temperature Distribution 193
6.4 Heat Transfers 194
6.5 Stepwise Calculation of Temperature Distribution 197
6.6 Simultaneous Solution of Temperature Distribution 202
6.6.1 Simultaneous solution of temperature derivatives by numerical integration 204
6.6.2 The external subroutine 206
6.6.2.1 The crucible 206
6.6.2.2 The charge 209
6.7 Heat Losses 211
6.7.1 Conduction losses 214
6.7.2 Radiation losses 216
6.7.3 Convection losses 216
6.8 Simultaneous Solution Including Heat Losses 218
6.8.1 External subroutine including losses
   6.8.1.1 The crucible
   6.8.1.2 The charge

6.9 Results of Simultaneous Solution Including Heat Losses

6.10 Conclusions

CHAPTER 7 - MODES OF FAILURE

7.1 Introduction

7.2 Effect of Operating Frequency

7.3 Effect of Power Density

7.4 Effect of Type of Crucible

7.5 Effect of Initial Crucible Temperature

7.6 Effect of Charge

7.7 Effect of Charge Packing Density

7.8 Effect of Preheated Scrap

7.9 Effect of Type of Furnace

7.10 Summary of the Results - Conclusions
   7.10.1 Operating frequency
   7.10.2 Power density
   7.10.3 Crucible type
   7.10.4 Initial Crucible temperature
   7.10.5 Charge Packing density
   7.10.6 Preheated scrap
   7.10.7 Type of furnace
## CHAPTER 8 CONCLUSIONS AND POSSIBLE REMEDIES

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td>292</td>
</tr>
<tr>
<td>8.2</td>
<td>293</td>
</tr>
<tr>
<td>8.2.1</td>
<td>296</td>
</tr>
<tr>
<td>8.2.2</td>
<td>300</td>
</tr>
<tr>
<td>8.2.3</td>
<td>302</td>
</tr>
</tbody>
</table>

### APPENDIXES

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No</th>
</tr>
</thead>
<tbody>
<tr>
<td>A3.1</td>
<td>315</td>
</tr>
<tr>
<td>A3.2</td>
<td>315</td>
</tr>
<tr>
<td>A4.1</td>
<td>330</td>
</tr>
<tr>
<td>A4.1.1</td>
<td>330</td>
</tr>
<tr>
<td>A4.1.2</td>
<td>333</td>
</tr>
<tr>
<td>A4.1.3</td>
<td>336</td>
</tr>
<tr>
<td>A4.2</td>
<td>337</td>
</tr>
<tr>
<td>A4.3</td>
<td>338</td>
</tr>
<tr>
<td>A4.4</td>
<td>339</td>
</tr>
<tr>
<td>A5.1</td>
<td>341</td>
</tr>
<tr>
<td>A6.1</td>
<td>345</td>
</tr>
<tr>
<td>A6.2</td>
<td>349</td>
</tr>
</tbody>
</table>
A6.3 Emissivities of relevant materials 352
A6.4 Determination of the flow regime for convection losses 354
A6.5 Listing of subroutine SUB19L code 355
A7.1 Effect of lower operating frequency on clay/C temperature distribution 358
A7.2 Thermal stresses in crucibles 360
A8.1 Effect of uniform resistivity on clay/C temperature distribution 366

BIBLIOGRAPHY 368
LIST OF PLATES

Fig. 1.5  Cracking near the base of crucibles (Morganite Crucibles Ltd)

Fig. 2.12 Clay-graphite crucible melting copper alloy (Morganite Crucibles Ltd)

Fig. 4.4  Probes attached to the crucible and the experimental set-up

Fig. 4.10 Division of wall section in separate parts

Fig. 4.12 SEM photomicrograph of the bottom section of the carbon-silicon carbide crucible wall (magnification x 40)

Fig. 4.13 SEM photomicrograph of the middle section of the carbon-silicon carbide crucible wall (magnification x 40)

Fig. 4.14 SEM photomicrograph of the top section of the carbon-silicon carbide crucible wall (magnification x 40)

Fig. 4.16 Silicon map of the bottom section of the crucible wall (SEM)

Fig. 4.17 Silicon map of the middle section of the crucible wall (SEM)

Fig. 4.18 Silicon map of the top section of the crucible wall (SEM)
LIST OF TABLES

Table 2.1  Crucible materials

Table 3.1  Electrical operating characteristics of the furnace installation

Table 4.1  Probe results for crucibles

Table 6.1  Thermal stress resistance factors for crucible materials

Table 7.1  Safe operating conditions.
**LIST OF SYMBOLS**

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>QUANTITY</th>
<th>SYMBOL FOR UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>radius</td>
<td>m</td>
</tr>
<tr>
<td>A</td>
<td>area</td>
<td>m²</td>
</tr>
<tr>
<td>A, B, C</td>
<td>complex integrated constants</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>magnetic flux density</td>
<td>T</td>
</tr>
<tr>
<td>C</td>
<td>coefficient (Chap. 6)</td>
<td></td>
</tr>
<tr>
<td>C_p</td>
<td>specific heat</td>
<td>J kg⁻¹ K⁻¹</td>
</tr>
<tr>
<td>d</td>
<td>diameter</td>
<td>m</td>
</tr>
<tr>
<td>D</td>
<td>characteristic dimension</td>
<td>m</td>
</tr>
<tr>
<td>E</td>
<td>electric field strength</td>
<td>V m⁻¹</td>
</tr>
<tr>
<td>E</td>
<td>elastic modulus</td>
<td>Pa</td>
</tr>
<tr>
<td>f</td>
<td>frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>F</td>
<td>force</td>
<td>N</td>
</tr>
<tr>
<td>F</td>
<td>(k.A) (Chap. 6)</td>
<td>W K⁻¹</td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration</td>
<td>m s⁻²</td>
</tr>
<tr>
<td>h₁</td>
<td>height of coil turns</td>
<td>m</td>
</tr>
<tr>
<td>h₂</td>
<td>meniscus height</td>
<td>m</td>
</tr>
<tr>
<td>h</td>
<td>convection heat transfer coefficient</td>
<td>W m⁻² K⁻¹</td>
</tr>
<tr>
<td>H</td>
<td>magnetic field strength</td>
<td>A m⁻¹</td>
</tr>
<tr>
<td>H</td>
<td>height</td>
<td>m</td>
</tr>
<tr>
<td>H_p</td>
<td>height of refractory packing</td>
<td>m</td>
</tr>
<tr>
<td>I</td>
<td>current</td>
<td>A</td>
</tr>
<tr>
<td>J</td>
<td>current density</td>
<td>A m⁻²</td>
</tr>
<tr>
<td>k</td>
<td>coupling factor</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity</td>
<td>W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>k₁</td>
<td>space factor</td>
<td></td>
</tr>
<tr>
<td>k_r</td>
<td>return flux factor</td>
<td></td>
</tr>
<tr>
<td>λ_m</td>
<td>magnetic field length</td>
<td>m</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>L</td>
<td>self inductance</td>
<td>H</td>
</tr>
<tr>
<td>m</td>
<td>mass</td>
<td>kg</td>
</tr>
<tr>
<td>( n_3 )</td>
<td>ratio of charge to charge and crucible power</td>
<td>kg</td>
</tr>
<tr>
<td>( N_l )</td>
<td>number of coil turns</td>
<td>Pa</td>
</tr>
<tr>
<td>p</td>
<td>pressure</td>
<td>W</td>
</tr>
<tr>
<td>P</td>
<td>furnace active power</td>
<td>W</td>
</tr>
<tr>
<td>( P_s )</td>
<td>specific power density</td>
<td>W kg(^{-3})</td>
</tr>
<tr>
<td>( Q_l )</td>
<td>furnace reactive power</td>
<td>VAR</td>
</tr>
<tr>
<td>q</td>
<td>rate of heat flow</td>
<td>W</td>
</tr>
<tr>
<td>Q</td>
<td>heat</td>
<td>J</td>
</tr>
<tr>
<td>r</td>
<td>radius</td>
<td>m</td>
</tr>
<tr>
<td>R</td>
<td>resistance</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>( R_m )</td>
<td>thermal stress resistance factor</td>
<td>K</td>
</tr>
<tr>
<td>( \dot{S}, \ddot{S} )</td>
<td>Poynting vector, complex poynting vector</td>
<td>W m(^{-2})</td>
</tr>
<tr>
<td>( S_t )</td>
<td>tensile strength</td>
<td>Pa</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
<td>s</td>
</tr>
<tr>
<td>t</td>
<td>thickness</td>
<td>m</td>
</tr>
<tr>
<td>T</td>
<td>absolute temperature</td>
<td>( ^{\circ} )K</td>
</tr>
<tr>
<td>U</td>
<td>potential difference</td>
<td>V</td>
</tr>
<tr>
<td>( U_m )</td>
<td>magnetic potential difference</td>
<td>A</td>
</tr>
<tr>
<td>V</td>
<td>potential difference</td>
<td>V</td>
</tr>
<tr>
<td>w</td>
<td>normalized power</td>
<td>W</td>
</tr>
<tr>
<td>W</td>
<td>power</td>
<td>W</td>
</tr>
<tr>
<td>( W_r )</td>
<td>radiated power</td>
<td>W</td>
</tr>
<tr>
<td>x</td>
<td>distance</td>
<td>m</td>
</tr>
<tr>
<td>( x_{22}, x_{23}, x_{33}, x_3 )</td>
<td>Bessel function arguments ( fn (r, \delta) )</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>X</td>
<td>reactance</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>Z</td>
<td>impedance</td>
<td>( \Omega )</td>
</tr>
</tbody>
</table>
Greek Symbols

\( \alpha \): coefficient of thermal expansion  \( K^{-1} \)

\( \beta \): coefficient of cubic thermal expansion  \( K^{-1} \)

\( \gamma \): contact area factor

\( \gamma \): density  \( kg \ m^{-3} \)

\( \gamma_3 \): specific gravity of charge  \( N\ m^{-3} \)

\( \gamma_B \): bulk density  \( kg\ m^{-3} \)

\( \delta \): skin depth  \( m \)

\( \varepsilon \): emissivity of radiating body

\( \eta \): efficiency

\( \theta \): temperature  \( ^\circ C \)

\( \lambda \): dimensionless factor \( (fn(\rho, \mu)) \)

\( \lambda \): fraction of absorbed losses

\( \mu \): permeability  \( H\ m^{-1} \)

\( \mu_r \): relative permeability

\( \mu_0 \): permeability of vacuum  \( H\ m^{-1} \)

\( \nu \): dynamic viscosity  \( kg\ m^{-1}\ s^{-1} \)

\( \nu \): poisson's ratio

\( \rho \): resistivity  \( \Omega m \)

\( \sigma \): conductivity  \( S\ m^{-1} \)

\( \sigma \): Stephan Boltzman constant  \( W\ m^{-2}\ K^{-4} \)

\( \phi \): resistance factor

\( \phi \): flux  \( Wb \)

\( \psi_s \): screening factor

\( \omega \): angular frequency  \( rad\ s^{-1} \)

\( j\omega \): \( \frac{d}{dt} \) for sinusoids
Suffixes and Indices

1  coil
2  conducting crucible
3  charge
23 crucible and charge
0  non-conducting refractory
R  return flux
*  complex conjugate
m  magnetic
P, s primary, secondary
g, s generating, cross sectional
D  crucible disc
i, o inside, outside

Abbreviations

C/SiC; C-SiC carbon bonded silicon carbide
clay/C; Clay-C clay bonded graphite
RHS right hand side
r, \( \theta, z \) cylindrical coordinate system components
Im, Re imaginary part, real part
Gr Grashof number
Pr Prandtle number

Bessel functions
ber, bei
ker, kei

\( I_0, K_0 \) addition of prime denotes differentials

SI units are used throughout this work. complex numbers are designated by underlining.
INTRODUCTION

Modern foundry practice demands melting metals rapidly, efficiently and safely to produce improved quality. Fast melting means profitability while efficient practice ensures optimum fuel usage. To achieve safety at work, robust and reliable equipment with relatively long service life is needed. Good design of such melting equipment requires a complete knowledge of the problems involved.

A crucible may be defined as "a container for molten metal". This could be expanded to say "a container for molten metal in which the material is melted, maintained and/or carried" [1.1]*. Historically they have been made of clay.

The art of metal melting by fuel-firing crucibles is ancient. Since early fifties however, electric furnaces have replaced fuel-fired for both melting and holding in all branches of metal melting. Electric furnaces offer the best conditions for the closest possible control of all aspects of the melting process. In general, when comparing electric melting with fuel-firing, improvements are claimed [1.2] in respect of:

(a) temperature control
(b) reduced melting losses
(c) compositional control
(d) freedom from fuel-borne contamination, and
(e) cleanliness of the melting shop environment.

*Details of references are given at the end of each respective chapter.
Induction furnaces form an important group of electric melting furnaces and have had a profound effect on melting practice in the foundry. An induction furnace operates like a transformer. The primary winding is a water-cooled coil connected to an alternating current supply. The coil surrounds the metal to be melted which acts as the secondary winding.

Induction furnaces melt the charge by inducing eddy current losses inside them. They work on the same principles as induction heating, whereby heating of metallic charge is continued until melting takes place.

In an induction furnace the secondary winding is the metallic charge and/or the crucible. A crucible in this context is the refractory material between the charge and the coil. In the majority of cases it is a rammed monolithic refractory but quite often a preformed crucible is used. These are either conducting or non-conducting.

Induction furnaces are becoming most popular because they generate uniform heat within the charge and develop automatic stirring. There are two distinct types of induction furnaces.

(i) Core type or channel furnace.

(ii) Coreless type.

These furnaces have different applications and also differ in the method of construction. Nevertheless the principle for heat generation is the same for both types of furnace.
Channel induction furnaces heat up only a small portion of the total charge in a channel (core) which is then recirculated to the main chamber of the furnace [1,3]. Because of the shape of channel furnaces they are almost invariably lined with rammed materials and do not utilise preformed crucibles. Channel induction furnaces are most suitable for continuous production processes, i.e. they are mainly used for holding as they need to be operated with a molten heel. The limitations are largely overcome by coreless furnaces.

![Diagram of a coreless induction melting unit](image_url)

**FIG. 1.1 CORELESS INDUCTION MELTING UNIT**

The general features of a coreless furnace are shown in Fig. 1.1. When an alternating current is applied to the furnace coil a magnetic field is
produced which induces eddy currents in the metallic charge placed within this field. The induced currents cause heat generation and metal melting. When the metal is molten, the repulsive forces between the coil and the charge cause movement of the metal. It creates a metal flow pattern shown in the diagram. The refractory lining in large coreless furnaces (500 kg capacity upwards) is usually of rammed type which is installed in situ. A metallic former is used for this purpose which is melted away in the first charge.

Alternating currents produce an alternating field around the axis of the conductor which increases in intensity as a function of distance from the axis. The current density is thus reduced in the core, while the outside (the skin) carries a higher current density. The tendency of concentration near the edge is termed skin effect.

Induction melting is done at mains and medium frequencies depending on the special application and type of melt. Mains frequency furnaces are most suitable for tonnage melting. Medium frequency means higher power density due to smaller skin depths, and thus smaller furnace capacities. So preformed crucibles are used in small medium frequency furnaces. These include tilting, push-up and lift-coil furnaces. When one particular alloy is melted or relatively larger amounts are needed for bigger castings; a tilting medium frequency furnace, shown in Fig. 1.2 is used. When small amounts of variety of alloys are melted and so separate crucibles are required, push-up (Fig. 1.3) or lift-coil furnaces are utilized. Push-up furnaces are built as single and twin units.
FIG. 1.2  TILTING FURNACE WITH A60 CRUCIBLE
The electrical system for all these furnaces is the same as any medium frequency coreless furnace. In the case of push-up and lift-coil furnaces, the coil is less closely coupled to the crucible but still energised by same type of power supply.

Ferrous alloys have high electrical resistance and are extremely magnetic. It is customary to melt these metals in induction units with non-conducting rammed linings. The metal acts as the secondary circuit. In small furnaces melting ferrous alloys, crucibles made of white refractory alumina, silica and magnesia are sometimes used.
This work is concerned with use of medium frequency coreless furnaces in non-ferrous melting. In non-ferrous industries, such as aluminium and copper alloys, small coreless furnaces use prefired refractory crucibles which are useful for quickly melting small quantities (typically less than 200 kg) of a particular alloy. The next melt can be a different alloy using a different crucible. Medium frequency melting furnaces (1 and 3 kHz) are increasingly being used as shown in Fig. 1.4 [1,4].

Fig. 1.4 Growth of medium-frequency induction furnaces in non-ferrous foundries in the UK
Non-ferrous alloys like BS 1400 copper alloys have much lower electrical resistivities than ferrous alloys and are therefore more difficult to melt from cold. In a batch melting operation, where all the metal is emptied after each melt, a long starting period may be necessary. In order to avoid this problem and improve the melting efficiency an electrically conducting crucible can be used.

The crucible can be either clay bonded graphite or carbon bonded silicon carbide. Depending upon the design parameters of the furnace, but principally related to the crucible resistivity and the operating frequency of the coil, the crucible will draw some or nearly all of the induced power. Both clay-graphite and silicon carbide conduct electricity although clay-graphite is not as good a conductor as silicon carbide. There are also crucibles which are said to be made of a blend of both materials.

In a conducting crucible induction furnace, current is induced in the crucible as well as the metallic charge, and consequently heat is generated in the crucible as well as the metal charge. This has the advantages of improved efficiency and less vigorous stirring.

However, use of the more conducting carbon bonded silicon carbide crucibles especially, has encountered problems of premature failure especially at high power densities and operating frequencies. Practical cases reported in private communications with manufacturers indicate that the location of failure is predominantly near the bottom of the crucible walls. Flaws may initiate axially or azimuthally; propagating
Fig. 1.5. Cracking near the base of crucibles (Morganite Crucibles Ltd).
azimuthally round the crucible to meet up and cause failure as shown in Fig. 1.5.

Effective heating requires a coil designed to match approximately the electrical characteristics of the crucible as well as those of charge, taking into account the voltage and frequency of the electrical supply. The problems met in practice are reported as being:

(a) insufficient energy transferred from the coil, resulting in slow melting or failure to achieve melting
(b) overheating of the crucible arising from too much energy being induced directly into the crucible
(c) localized overheating of crucibles
(d) excessive stirring in the molten charge
(e) premature failure of crucibles in service; especially at high power levels and operating frequencies.

The Morganite Crucible Limited Company has been a large manufacturer of such crucibles in sizes which range from the smallest, capable of holding less than a kilogram of brass, to the largest with a capacity of about 200 kg of the same metal. This project was instigated by the company who wished to obtain a larger share of induction heating market, but were hindered by problems experienced in using their crucibles in this type of melting application.

Historically, the company has always manufactured clay-graphite (Super mix) crucibles for fuel-fired furnaces, by the wet method, that is, the
When the induction furnace manufacturers started to use crucibles in the late twenties and early thirties, the Company initially supplied small cylindrical crucibles manufactured from the standard Super mix. This practice resulted in crucible problems due to electrical mismatch which eventually led to crucibles being produced in selected mixes with specific resistivities. During the forties and fifties this allowed the Company to cater for the crucible requirements for the increasing popularity in America, of the lift-coil furnace. In the UK however, induction melting had not increased to the same extent, so crucible manufacture was confined to an ever increasing individual requirements.

During the sixties, the general increase in production of silicon carbide (Suprex mix) crucibles gave the induction furnace manufacturers a new type of crucible to utilize. However the majority of furnace manufacturers decided not to use this type of crucible which required new coil designs etc. The first experiments in the UK using Suprex were carried out by Ajax and Birlec using the TPC980, still the largest Morganite Suprex crucible used in induction furnaces. The furnaces were mains or triple frequency, and used for producing master aluminium alloys, for which rammed linings were not ideally suited.

Since the sixties, a larger number of induction furnaces using Suprex have been installed in the UK. Other firms like Inductotherm and Radyne have commissioned units utilizing Suprex. Both manufacturers have produced
push-up units using Suprex operating on a wide variety of alloys.

In the early seventies crucible manufacture at Battersea had ceased and been transferred to Norton premises. Roller forming techniques were applied to new mixes, developed to match the electrical characteristics of Super clay-graphite. To a large extent, these mixes were unsuccessful as they suffered from thermal shock during use. Some firms were able to convert to using Suprex or Superstar (a blend of Super and Suprex) but problems arose with the remainder. However, these more conducting crucibles are most efficient for melting non-ferrous alloys and the problems associated with their usage remained to be solved in order to exploit the market.

Meanwhile, manufacturing the clay-graphite crucibles continued. In order to retain the basic Super mix, a new manufacturing technique was introduced - Isostatic Pressing. This was because the old method was too labour intensive. This development coincided with an extensive marketing campaign by Inductotherm to launch push-up furnaces. Initially problems were encountered with blistering, thermal shock and perishing but these problems were gradually solved by changing the mix, improving the manufacturing technique and developing a glaze especially for induction melting applications.

As for silicon carbide crucibles, thermal shock cracking has remained a problem and extensive development work has been carried out to establish the reasons for this before they can supply the market with a reliable product.
An important factor affecting the performance is thought to be the electrical resistivity of the crucible and the manufacturers which produce crucibles of different resistivities, have been investigating how this might be increased or decreased to obtain the desired result. However, the problems have not yet been fully resolved and they have been seeking answers to why the crucibles fail prematurely and what are the suitable values for the resistivity of crucible materials. Furthermore, they have been trying to determine what makes crucibles fail prematurely, and what appropriate power levels and operating frequencies will achieve effective melting, and to develop reliable methods of designing furnace coils.

The objective of the project is therefore to investigate the characteristics involved and the effect of their variation on the crucibles performance to narrow down the problem to the reasons for such premature failures. In doing so, an analytical approach is used to the problem because:

(i) Numerical techniques have been applied to induction heating problems. They are powerful techniques for solving particular problems but require exact knowledge of the materials properties without which their application would not be worthwhile.

(ii) The Morganite Crucible Limited Company required a means of assessing and evaluating the effect of variation of furnace parameters that could be handled by their staff as a design tool.

Chapter 2 reviews the related work done on the subject by others. This
1. INTRODUCTION: THE PROBLEM

2. REVIEW OF PAST WORK

3. THEORETICAL ANALYSIS

4. MEASUREMENTS AND ESTIMATION OF PROPERTIES

5. AXIAL POWER DISTRIBUTION

6. CRUCIBLE AND CHARGE TEMPERATURE DISTRIBUTION

7. MODES OF FAILURE

8. CONCLUSIONS AND POSSIBLE REMEDIES

APPENDIXES

BIBLIOGRAPHY

FIG. 1.6  STRUCTURE OF THE THESIS
is to bring the reader up to date with the situation elsewhere and the state of the art. Chapter 3 covers the analysis and the mathematics involved. Chapter 4 is on measurement of crucible resistivities by using probes and estimation of other crucible properties. It should be noted that except in Chapter 4, the higher conductivity crucibles will be referred to as 'silicon carbide crucibles'. In Chapter 4, however, they will be called 'carbon bonded silicon carbide crucibles' in order to distinguish between the crucible type and the silicon carbide (SiC) constituent. Likewise, clay-graphite crucibles will be called 'clay bonded graphite crucibles' in Chapter 4. Chapter 5 uses the results of measurements to predict power distribution and rates of temperature rise. Thermal stresses are discussed in Chapter 6 and the temperature distributions are obtained. This combined with Chapter 7, deals also with modes of failure. Possible solutions to the problem are given in Chapter 8 which contains the conclusions. The appendices follow. The structure of the thesis is outlined by the flow chart in Fig. 1.6.

References


1.2 PUGH, S.: "Electric melting - is this your answer?", Foundry Trade J., September 1976, Vol. 141, No. 3095, pp. 17-42


Previous work on the relevant subjects is reviewed in this chapter to bring the reader up to date with the present state of the art.
2 REVIEW

There exist many different types of electric furnaces. From an electrical viewpoint, it is customary to classify them as:

(i) Arc
(ii) Resistance
(iii) Induction

More detailed classification of electric furnaces is given elsewhere [2.1]. All these types of furnaces can be used for melting of non-ferrous metals. But the arc furnace, however, cannot be used for copper-rich alloys or aluminium as the mode of application of heat is unsuited to these lower melting point metals and excessive volatilization and oxidation would result [2.2].

Arc furnaces are primarily used for the melting of metals such as steels, nickel and its alloys, and for smelting and electrochemical purposes. These are robust devices and utilize the high temperatures evolved when electrodes strike scrap metal. Arc furnaces are used for high power densities and large capacities in melting carbon steels and iron, for production of special steel alloys. More details of arc furnace are given in other literature [2.1, 2.3, 2.4].

Resistance furnaces use heating elements made of nickel or iron based alloys and some refractory materials which are installed on the walls of furnace, leaving a hollow working space or chamber for the charge. The design and configuration varies with the mode of heat transfer used in the furnace; radiation, convection or conduction [2.5]. When compared with induction melting, electric resistance is a relatively slow process
governed by the principles of radiation heat transfer [2.6].

Resistance crucible furnaces have been in use since 1940 and their number is growing in the aluminium industry because of their economic competitiveness as against conventional gas or oil. Electric bale-out furnaces using metallic elements are installed to feed low-pressure aluminium diecasting machines. Some copper alloy foundries have shown interest in the use of resistance furnaces for bronze and gun-metals with non-metallic heating elements [2.7, 2.8].

The main theme of this work is on induction furnaces. It has been known for more than a century that heat can be developed by electromagnetic induction, but it has not been considered as a method of heating until the use of alternating current in iron-cored coils presented serious difficulties due to the heating effect. The background of induction heating was established by contributions of Faraday, Foucault, Heaviside, Thompson and Ewing in the last century. A detailed history of early work and principles of induction heating are given by Stansel [2.9] and Davies and Simpson [2.10].

2.1 Induction Melting

Induction melting works on the same principles as induction heating. When an alternating current is allowed to flow in the primary coil of the induction furnace, eddy currents are induced in the metallic charge. The charge opposes the flow of the current causing heat ($i^2 r$ losses) to be generated. Ultimately, the amount of heat generated by the charge is sufficient to cause melting.
One of the first induction furnaces was made by Kjellin in 1900. Northrup started the development of an induction furnace in 1916 to operate at frequencies higher than mains. In 1927 the Electric Furnace Co. (EFCO) in England built the first medium frequency furnace in Sheffield. Other pioneers include Wyatt and Dreyfus [2.10].

Induction furnaces are divided in two major groups: channel and coreless. Requirements of production decide the type, capacity, frequency and electrical rating of the furnace to be used. Applications of coreless and channel furnaces vary according to actual role; melting, holding or refining, and according to the charge.

Copper base, nickel base, zinc and ferrous alloys are all successfully melted in channel furnaces as the metal oxides are lighter than the metal and float to the top of the molten bath where they can be skimmed off. With aluminium alloys however, there is the disadvantage that the oxide has a density similar to the metal and therefore remains in suspension. As the metal passes through the channel, the non-conducting oxide becomes attached to the channel walls, causing clogging and has to be removed fairly regularly. Channel furnaces are discussed elsewhere [2.11-2.13].

2.2 Coreless Furnace

The coreless furnace consists of a cylindrical refractory container for the charge, surrounded by a hollow water-cooled copper coil of a number of turns which produces a magnetic field. For more details on their construction and general use the reader is referred to published literature [2.10, 2.14-2.17]. Foundry coreless furnaces are built to melt
at mains and medium frequencies. They are mainly used as remelting units.

2.2.1 Skin effect

Skin effect is the phenomenon of concentration of magnetic field strength and thus current density on the outer edge of a conductor. Skin depth or depth of penetration, as it is sometimes called, is defined as the depth (from the surface) over which a uniform current equal to the surface current value would develop the same power as that of the actual current distribution. Skin depth is defined as:

\[ \delta = \left( \frac{\text{resistivity}}{\pi \times \text{frequency} \times \text{permeability}} \right)^\frac{1}{2} \]  

(2.1)

In a cylindrical conductor, when skin depth is small compared with the radius (about one fifth), the current density falls away approximately exponentially from its value at the surface. When the material's resistivity is high or the frequency is low, the current density falls away less rapidly. In order to relate skin depth with melting applications, the variation with temperature of for aluminium and copper are shown in Fig. 2.1 between room temperature and molten state. (See also App. A4.3)

2.2.2 Stirring

In the coreless furnace, when the metal is molten and is fluid, there will be hydrodynamic forces set up by the interaction of the magnetic fluxes and currents which produce vigorous stirring. The turbulence in the charge increases as the frequency drops [2.10]. Stirring is most
FIG. 2.1 VARIATION OF SKIN DEPTH WITH TEMPERATURE FOR ALUMINIUM AND COPPER.
beneficial when melting cast irons where carburization depends upon the mixing in the molten bath. The adjustment of silicon and especially carbon content of the melt depends for its success on having sufficient metal movement at the melt surface [2.18]. On the other hand, excessive stirring is undesirable for melting non-ferrous alloys, which makes medium frequency furnaces suitable for these alloys as generally they produce less stirring than mains frequency furnaces.

The topic of stirring is a complicated subject. More rigorous investigations into stirring are detailed elsewhere [2.19-2.24]. It is related to the repulsive forces between adjacent parts of the molten metal. It depends on furnace geometry, viscosity and specific gravity of the metal as well as the surface velocity of the melt. Generalizations have been given by authors to express approximate relationships between stirring and furnace characteristics.

The following relationship has been given [2.25]:

$$B_s = \frac{P_s}{\sqrt{f}} \quad (2.2)$$

where,

- $B_s$ = bath stirring intensity
- $P_s$ = specific power density (W kg$^{-1}$)
- $f$ = frequency

Therefore, for a given power density or for a given power level and bath size, intensity of stirring reduces with increased frequency as shown in Fig. 2.2 which indicates why medium frequency furnaces generate much lower
FIG. 2.2  SIMPLIFIED VARIATION OF STIRRING INTENSITY WITH FREQUENCY.

FIG. 2.3  SIMPLIFIED VARIATION OF SPECIFIC POWER DENSITY WITH FREQUENCY.
stirring. Expressed in a different way, the relationship shows that for a given bath stirring, power density can be increased by increasing the frequency, as shown in Fig. 2.3. This allows faster melting from cold of non-ferrous alloys. It must be appreciated that Figs. 2.2 and 2.3 are merely aimed to show the general relationships for a typical furnace. Finally, at a given frequency and bath size, intensity of stirring is proportional to the power density. This is shown in Fig. 2.4.

The relationship between meniscus height and the intensity of stirring is
equally difficult. It has been shown \[2.17\] that stirring intensity is proportional to the melt surface velocity, i.e.

\[
B_s \propto V_s \quad (2.3)
\]

It has also been shown \[2.26\] that if the top turn of the furnace coil is at the same height as the surface of the molten bath then for a laminar melt flow,

\[
V_s \propto h_u \quad (2.4)
\]

and for a turbulent melt flow,

\[
V_s \propto \sqrt{h_u} \quad (2.5)
\]

where

\[ h_u = \text{meniscus height in the bath} \]

Wilford \[2.18\] has assumed that eqn. 2.4 holds and that in a medium frequency furnace, the condition of the top coil turn is satisfied. It can therefore be deduced from eqns. 2.3 and 2.4 that

\[
B_s \propto h_u \quad (2.6)
\]

which means that intensity of stirring is approximately proportional to meniscus height.

2.2.3 Choice of furnace frequency

In terms of metal melting, coreless induction furnaces may be classified into

(a) Low frequency - up to 200 Hz and including mains frequency (50 Hz) and triple frequency (150 Hz).

(b) Medium frequency - 200 Hz to 10 kHz. Most popular frequencies in
this range are 1 kHz, 2 kHz and 3 kHz.
(c) High frequency - over 10 kHz. Some small laboratory furnaces use crucibles at these frequencies but this is generally experimental work.

Operating frequency plays an important role in determining the characteristics of a melting unit. There are several basic points to take into account in choosing the correct frequency. These include furnace size, stirring, starting, size of charge, cost and melting specifications. At first sight, it is desirable to use mains frequency equipment wherever possible, as the cost is less than the medium frequency equivalent [2.10]. However, the decision should be based on a careful analysis of all aspects.

Higher operating frequencies enable greater power density to be induced into the charge. This is an advantage of medium frequency furnaces. Larger furnaces are operated at lower frequencies [2.27, 2.28] to cater for the intensive stirring effect. The intensity of the stirring action is governed by the frequency, the power input and the furnace size. At low frequencies the stirring action is more vigorous and this can lead to more rapid wear on the lining. Low frequency furnaces are most efficient when melting large chunks of metal. Hence, the furnace size also becomes large.

For starting from cold, mains frequency furnaces need a solid plug, conforming approximately to the internal bath diameter and occupying one third of its volume. The melting rate is slow until a molten heel is
formed [2.29]. This is overcome in medium frequency furnaces. The size of scrap is also important. Very thin sections of metal tend to favour higher frequencies; although swarf can be dealt with either by using a high enough frequency to enhance ratio of particle size to skin depth, or, more commonly by a low frequency to give sufficient stirring to bring it rapidly into the already molten metal bath [2.10, 2.30].

The advent of solid-state equipment has changed the economics of the frequency conversion appreciably in the last two decades. Medium frequency melting furnaces have been shown [2.18] to offer a method of saving useful amounts of energy in consumption when compared with mains frequency. The total cost of medium frequency installation can be less than the mains frequency alternative. By comparing the melting consumption at reduced output, the significant energy savings of medium frequency are realized. In these conditions the use of the bigger mains frequency furnace which has to retain a molten heel, obviously suffers in comparison with the smaller medium frequency crucible which is completely emptied prior to melting the next charge batch.

2.3 Medium Frequency Coreless Furnaces

The medium frequency induction melting units that are now in operation work at frequencies between 200 Hz up to and including 3 kHz [2.31]. They are powered by solid-state inverters which have provided a great leap forward in making medium frequency melting more economical and reliable.

Medium frequency furnaces are reported [2.32] to be used as increasingly versatile units to cover vast range of melting temperatures, making
increasing inroads into the domain of 50 Hz unit because of its flexibility for handling unexpected shut-down conditions and its ability to start easily from cold and to provide tailored stirring, allowing foundrymen to adapt to the small scale productions. Ferrous alloys, however, which are associated with higher melting temperatures and higher stirring are melted satisfactorily at the lower end of the medium frequency scale; 200 or 250 Hz in typical sizes of up to 1 or 2 tonnes capacity with adequate stirring.

The medium frequency installation can be considered as comprising a number of units [2.31]:

(a) a medium frequency inverter or power source with its associated switchgear and transformers,
(b) a capacitor bank for tuning the furnace to the required operating frequency.
(c) one or more coreless furnaces,
(d) a cooling system feeding the above items.

2.3.1 Power sources

It is not the purpose here to describe the design and construction of power sources in detail. They are, however, briefly discussed in terms of their general features and how these may influence furnace usage. For more details the reader is referred to reference sources [2.10, 2.33].

Electric furnaces have recently become cheaper to run than traditional fuel-fired furnaces when melting more common metals. At the same time, developments in solid-state power supplies have brought about their
capital cost within the reach of the smaller foundry. Frequency conversion equipment have been around since early this century. The use of medium frequency furnaces started with rotary generators in the early thirties. In the late sixties, solid-state generators using thyristors began to appear. They have the advantage of higher efficiency and ease of control and have made significant impact within the metal melting industry by providing high frequency electrical energy.

2.3.1.1 Thyristor inverter [2.34]

Thyristor inverters are now considered as the direct successors to motor alternators because they:

(a) have a conversion efficiency of higher then 90% throughout the normal power range

(b) will automatically compensate for load impedance variations avoiding the need to rematch with capacitor switching to maintain the output power factor

(c) are physically compact, easily installed and require no special foundations

(d) have modern electronic control to provide automatic limits allowing the generator to be used to its full power without fear of tripping but with fast protection under real fault conduction

(e) have no run-up time, no start-up surges of current.

The development of the thyristor has led to a new concept in power sources for induction melting equipment. In the simplest terms the thyristor can be considered as a switch that is controlled by a gate electrode. Current flow in the 'forward' direction is initiated by a positive current
pulse applied to the gate and the thyristor conducts until the current falls to zero. The thyristor has both a current and voltage limitation. The former influences the power dissipation in the unit and the latter controls the behaviour of the device in the sense that at high forward voltages or high rates of rise of voltage the unit may turn-on spontaneously, whilst if the reverse voltage is excessive then voltage breakdown may occur [2.33]. Turn-off of the thyristor is not instantaneous and the finite time taken by this operation limits the frequency performance. Developments in the design of the thyristor are being extended. Medium frequency thyristor sources rated at over 1 MW are available at frequencies up to 3 kHz.

2.3.1.2 Principles of operation of thyristor inverter [2.33]

Although different manufacturers offer generators incorporating different designs, all designs stem from four fundamental types: voltage-fed, current-fed, cycloconverter and variable mark space ratio converter. The first two are mostly used to supply induction heating/melting equipment.

The advantage of the voltage-fed or swept frequency system is the ability to match full power into loads which vary in impedance. In the current-fed or load-resonant system, the output from the load is tapped and fed back as a commutation circuit. Current fed inverters have high efficiencies (97%) than voltage-fed systems (90%) especially at reduced power output. The current-fed inverters are most commonly utilized in the UK. The existing tilting furnace (Section 2.5.2.1) used for experiments described in Chapter 4, was also supplied by a 3 kHz 100 kW thyristor inverter of this type, the principles of operation of which are
The basic circuit is shown in Fig. 2.5. The inverter employs a controlled three phase full wave bridge wherein three phase line frequency is rectified to direct current, smoothed by a filter circuit, and passed on to a parallel bridge inverter. The inverter alternatively switches the direct current through the load via thyristor pair 1-2 and thyristor pair 3-4 respectively. Fig. 2.6 shows the appropriate waveforms. The alternating load current \( i_L \) takes the form of a rectangular wave with its peak value equal to the direct current \( I_{DC} \) at the frequency of switching of the thyristors. The load is driven at a leading power factor to permit the current to commute from one pair of inverter thyristors to the other at the completion of each half cycle of currents. Thus \( \phi \) is the load angle by which the rectangular load current leads the sinusoidal load voltage.

When thyristors 1 and 2 are conducting, commutation is brought about by firing 3 and 4. During the commutation, all four thyristors conduct for a finite time \( \delta \) and there is zero voltage across the inverter. During this time the load capacitor discharges backwards into the inverter making thyristors 1 and 2 reverse biased. At the end of the interval, \( i_L \) has reversed, thyristors 3 and 4 are conducting the full load current. The commutation must take place before the end of a half cycle or otherwise the polarity of the voltage on the load capacitor is not such that the outgoing arms will be turned off, and a fire through would follow. Not only the commutation process has to be complete before the end of a half cycle, but, there must be a remaining interval \( \rho \) to provide a reverse
FIG.-2.5 CURRENT FED THYRISTOR INVERTER
Fig. 2.6 Waveforms of the inverter and load
voltage on the outgoing arms as shown on the inverter d.c. voltage waveform.

The inverter is not self starting. To start it, the starting contactor is closed, and the capacitor of the starting circuit is primed with the polarity shown. Then thyristors 1 and 4 are fired. This puts a short circuit on the controlled rectifier and builds up the current through the series reactor to 50% of its full load. Thyristors 5 and 3 are then fired, thyristor 1 turns off, and the current in the series reactor then flows via thyristor 3 into the starting capacitor, reversing its charge, and into the load circuit starting it ringing. Next, thyristors 1, 2 and 6 are fired and the load voltage oscillation builds up. After approximately 10 cycles of medium frequency, 5 and 6 are no longer fired and the starting process is complete. The starting contactor is then opened and the inverter is operating normally.

The inverter always operates at a frequency slightly higher than the resonant frequency of the load circuit and changes automatically with any changes in the load circuit resonant frequency. In this case, however, power control is obtained by varying the d.c. voltage fed to the inverter and the load circuit can be used to switch or commutate the thyristors of the inverter.

2.3.1.3 Load circuit

The output terminals of the generator are connected to the furnace coil by water-cooled coaxial copper cables. They are supported away from the inverter cubicle so as to avoid heating of the steel casing.
Depending on the impedance of the load and type and output rating of the inverter, a matching transformer may be required between the inverter and the furnace coil. The current-fed inverter used had an output transformer which was double wound and had two identical secondaries each rated at 400 V. Output medium frequency voltage was controlled on the two isolated secondaries by using copper links to connect these secondaries with their capacitors either in series or in parallel to obtain maximum output voltages of 400 or 800 V.

The output capacitors comprised of four upright cans, two connected across each secondary. The capacitors used were of medium frequency polypropylene type. They were rated at 500 volts, 3 kHz and each tapped at 20+ 40+ 80+ 80+ 80+ 80+ kVAR. A total of 1364 kVAR was required at 800 V output.

With smaller units, the capacitors are placed inside the cubicle. With larger units, a capacitor bank is positioned behind the mounting platform of the furnace. It is, however, good practice to site the tuning capacitors outside the cubicle when possible, in order to gain easy access for changing of tappings on the capacitors for matching purposes.

2.3.1.4 Water cooling [2.34]

Although air cooled generators are available, the majority of manufacturers use water cooling both to avoid the need for air cleaners and to utilize the cooling water which is necessary for cooling capacitors, busbars and coils. It is usual to feed the water firstly through the capacitors and then through the furnace coil which raises the water temperature more.
As the thyristor mounting plates are at different potentials, the cooling circuits must be arranged to minimize leakage currents between them. This is usually achieved by interconnecting such components with non-conducting rubber or plastic hoses. The required hose lengths may be excessive, particularly in high power high voltage units. In this case the generator is provided with its own water circuit using pure high resistivity (minimum 2Ω m) water. Closed cooling systems avoiding oxygen and light to prevent corrosion and algae, and connected to a cooling tower of main cooling system through a water/water heat exchanger will rarely require special treatments. Individual flow switches and adjustable valves will provide early warning of blockage and permit melting to continue.

Future developments in solid state frequency conversion can be seen as extension of the boundaries of frequency, both upwards and downwards. Developments include single thyristor inverter which can become economical in the low power (50-100 kW) units. At the lower end of the frequency scale, inverters are being built economically for high powers at frequencies of 150-200 Hz providing tailored frequency and good efficiency [2.35]. Control reliability should increase further by the use of microprocessors reducing number of control circuit boards and terminations. Cheaper units will allow small foundries to benefit from the use of medium frequency furnaces.

2.4 Melting of Non-ferrous Metals
Non-ferrous melting covers production of aluminium, copper, zinc and their alloys, all kinds of brasses, bronzes, cupronickels and gun metals. Non-
ferrous melting is characterized by low temperatures, low electrical resistivities and low stirring required.

Non-ferrous alloys have relatively low electrical resistivities. Furnace coil efficiency can be defined simply as the ratio of charge resistance to the sum of the charge and furnace coil resistances. Because of the associated lower melting temperatures of these alloys, charge resistivities which normally rise with temperature, remain relatively lower. The efficiency is therefore reduced. This is compensated for by using medium frequencies which decrease the skin depth which in turn increase the charge resistance and the furnace coil efficiency.

Vigorous stirring can be harmful especially for some of BS1400 copper alloys like aluminium bronzes. Increased frequency reduces stirring in a given bath size and for a given power level. Combination of the lower stirring and higher power density is ideal for melting of these alloys especially when using low grade scrap which is lighter and more susceptible to oxidation. This then improves the metal loss [2.36, 2.37]. Of all the operating costs involved in the production of non-ferrous alloys, metal loss is the most important and in general the most neglected. The institute of British Foundrymen committee survey of melting losses in 1965 quoted melting losses from 1.8% to no less than 13.9%. Actual improvements can be quite impressive when using medium frequency. Considering the present costs of raw material, very large savings can result [2.38, 2.39]. In aluminium melting, for example, metal loss arises as a result of oxidation and subsequent dross formation; every 1% saving being equivalent to roughly £10/tonne.
Use of medium frequency as opposed to mains frequency for non-ferrous alloys, provides faster melting from cold through greater penetration of power due to smaller skin depth in charge. This is particularly useful with low grade scrap where ratio of charge size to skin depth is poor. Furthermore, larger power densities are achieved which means smaller units without the restriction of excessive stirring [2.25] and lower consumption rate at output levels below maximum enabling the furnace to be emptied totally after each melt [2.18].

In short, the following advantages can be claimed by using medium frequency for non-ferrous melting:
(a) rapid cold start
(b) smaller starting pieces
(c) increased power density
(d) tailored stir
(e) higher efficiency.

Typical jobbing work, melting principally copper based alloys, can be carried out successfully in a medium frequency furnace. Such furnaces utilize free standing crucibles to contain the metal charge.

2.5 Crucible Induction Melting

Coreless melting furnaces come in a wide variety of forms but from design point of view, they can be divided into furnaces which are tilted, either hydraulically or by some other method, or those in which there is a separate crucible for the metal, where either the crucible is moved or the coil surrounding it is moved, and some specialist types of furnace such as roll-over furnaces, used for investment casting, or fixed furnaces for
bale out or bottom pouring, etc. [2.30].

Tilt furnaces cover the widest range. The larger tilt furnaces (above 500 kg steel capacity) are made of steel frame and often use lamination packs which direct the stray flux from passing through the frame and heating it. These furnaces are tilted hydraulically and use rammed lining and are for tonnage melting.

The smaller free standing tilter units are constructed of asbestos cement board known in the UK as Sindanyo and in the USA as Transite. Now with the availability of high density ceramic fibre board the possible hazards of asbestos can be avoided and the advantages of this construction retained. These furnaces are hoist tilted or lifted from the rear by a crane and use prefired crucibles.

Direct melting of metal in a portable crucible, as distinct from pouring molten metal from a furnace into such a crucible, is well suited to situations where relatively small amounts of different metals or alloys are melted at different times.

Push-up, lift-coil and lift-swing furnaces as well use prefired crucibles. The small tilting furnace together with the push-up furnace form a group of compact furnaces which are becoming most popular in the small jobbing foundries.

The following advantages are claimed when using crucible furnaces.

(a) easy crucible change means that alloy changes to avoid contamination
also quicker

(b) no need for melt-out formers
(c) no need for wash-out melt if contamination is a problem
(d) prefired liner with backing material can be a better safeguard against leakage
(e) less problem with unskilled labour producing a poor rammed lining
(f) quicker turn round time generally 4-5 hours as opposed to 2-3 days.

Crucibles may be divided in three groups as shown in Table 2.1 [2.1]. Electrical conductivity of these portable crucibles is an important factor as pointed out in the Table.

Table 2.1: CRUCIBLE MATERIALS

<table>
<thead>
<tr>
<th>Conducting</th>
<th>Semi-conducting</th>
<th>Non-conducting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphite/SiC</td>
<td>Mixture of clay and graphite</td>
<td>Basic</td>
</tr>
<tr>
<td>(carbon-bonded)</td>
<td>or clay and graphite and silicon carbide</td>
<td>Neut.ral</td>
</tr>
<tr>
<td></td>
<td>sintered magnesia with small bonding</td>
<td>Zirconium</td>
</tr>
</tbody>
</table>

2.5.1 Conducting crucible induction melting

Conducting crucibles are essentially preformed high temperature refractory pots made in different shapes and sizes, which have relatively low electrical resistivities ranging typically from $1000 \times 10^{-8} \Omega \text{ m}$ to $60,000 \times 10^{-8} \Omega \text{ m}$ at room temperature. Conducting crucibles made of carbon bonded silicon carbide have resistivities from $1000 \times 10^{-8}$ to $10,000 \times 10^{-8} \Omega \text{ m}$, whereas the resistivity of clay bonded graphite crucibles is up to $60,000 \times 10^{-8} \Omega \text{ m}$ at room temperature [2.40]. They are often called 'hot' and 'cold' pots respectively. Depending on the mix constituents, the resistivity of crucibles vary. Clay-graphite crucibles with highest
resistivities can behave like the non-conducting crucibles referred to in Table 2.1. This will be illustrated in Chapter 3.

The replacement of clay-graphite crucibles by the more conducting silicon carbide type in induction furnace installations has been advocated \[2.41\] on the grounds of:

(a) greater furnace coil efficiency, especially with highly conducting alloys. The greater coil efficiency is achieved because the conducting crucibles that absorb some of the induced power, act as an extension to the charge, thereby increasing the total charge resistance

(b) reduced stirring forces at a given power level and operating frequency

(c) faster melting of swarf from cold

(d) reduced chilling of the metal in transit from the furnace to the casting station

(e) reduced tendency of melt to freeze at the lip when pouring.

2.5.1.1 Clay-graphite crucibles \[2.42\]

Many furnace manufacturers favour the use of clay-graphite crucibles with a low conductivity so that the majority of the power is induced in the metal. Little heat is generated in the walls of the crucible, resulting in relatively longer life. An obvious disadvantage is the necessity to heat the metal higher than its normal casting temperature to account for heat losses into the crucible and chilling whilst pouring over the cold crucible wall. In addition, the stirring action of the furnace tends to throw some heavy alloying elements on to the cooler walls of the crucible where they oxidize and cause hard dross build-up. This problem will also
apply to the higher temperature copper base alloys. Hence a higher conductivity crucible is required which will generate more heat in the walls. A mix change will alter the electrical characteristics of the crucible but the same effect can be achieved by altering the coil design or alternatively silicon carbide crucibles can be used.

2.5.1.2 Silicon carbide crucibles

With some non-ferrous metals, mainly copper base and silver alloys the low electrical resistance of the metal means a lower electrical melting efficiency for the primary coil and the resulting heat losses are carried away by the water. This loss of efficiency can be largely overcome by generating the power in the walls of a silicon carbide crucible. Initially the crucible draws most of the power, transmitting the heat to the charge by conduction and radiation. As the temperature of the charge rises, the resistivity of the metal also increases enabling the charge itself to draw more power. When the charge is fully molten, more of the power is transmitted to the molten metal. The crucible at this stage draws less power. The hotter crucible allows more time for metal treatment and alloying additions. Heat losses to the coil during melting can be substantially reduced by the use of ceramic fibre insulation between the crucible and the coil in lift-out furnaces.

The arguments regarding the use of 'cold' (clay-graphite) or 'hot' (silicon carbide) crucibles are summarized below [2.37, 2.43, 2.44].

The cold (or cooler) crucible approach has the following advantages:
(a) Almost all the heat is generated in the charge which is particularly
useful when melting material of poor packing density such as returns.

(b) The relative coolness of the crucible contributes to operator comfort when metal is taken from the furnace for casting.

(c) Crucible cost is less than hot crucibles for induction melting.

The disadvantages of cooler crucible are as follows:

(a) For a given power and frequency, stirring is more than in hot crucible.

(b) Excessive stirring causes dross formation. Glass or sand cover fluxes swept beneath melt surface cause glaze build-up.

(c) Superheating is needed to avoid chilling in transit.

(d) Cold objects touching will give the cold pot thermal shocks.

The advantages of hot crucibles are as follows:

(a) much less stirring at a given power level and frequency than with cold crucible

(b) faster melting of fine charge like swarf

(c) less chilling of metal in transit.

(d) reduced energy consumption.

Against these there are the disadvantages of hot crucibles:

(a) Bulky materials such as ingots and returns have a poor packing density and little contact with the pot thus reducing efficiency of heat transfer.

(b) Depending on power and frequency, heat losses from the outside may be very high.

(c) difficult to handle and susceptible to thermal shock.
2.5.2 Crucible shapes and furnace types

Crucible shapes are shown in Fig. 2.7. When considering crucibles for induction units, the maximum possible capacity is obtained with a cylindrical crucible. It is also the ideal shape when considering the electrical design [2.42]. Due to its shape however, a cylindrical crucible is difficult to lift with conventional handling equipment (Fig. 2.7). The vast majority of cylindrical or 'E' shape crucibles are used in tilting furnaces with a supporting backing material (Fig. 2.8).

The standard British crucible for the fuel-fired lift-out furnace has always been the 'C' shape [2.41]. In recent years some foundries have converted to the European 'A' shape which, because of its design, is more stable in free standing position within the furnace. The 'A' shape is the crucible which has become popular in the push-up and lift-coil furnace.

Comparison between different shapes of crucible related to lifetime were carried out [2.45] using three crucible shapes similar to 'E', 'C' and 'A' shown in Fig. 2.7. The largest number of melts were obtained by the 'A' shape crucible.

The most popular size of crucible in push-up furnaces is the A60 (nominal capacity 60 kg bronze) but a range of sizes is available.

2.5.2.1 Tilting furnace

This is shown in Fig. 2.8. When a foundry specialises in one particular alloy or alloys which are chemically compatible or requires large
Fig. 2.7 Comparison of A, C, and E shape crucibles
Fig. 2.8  CORELESS TILTING FURNACE SHOWING CYLINDRICAL 'E'
SHAPE CRUCIBLE SUPPORTED BY RAMMED REFRACTORY
BACKING (Morganite Crucible Ltd.)
quantities of metal for bigger castings, a tilting type furnace is used. In addition to the 'E' shape, standard shapes of crucibles can also be used.

The crucible is installed on the earth leakage probe and on a refractory base and a powdered backing rammed between the crucible and coil. The top is sealed with a refractory cement to prevent crucible movement during tilting, to form the spout and prevent metal leakage to the coil. The crucible remains in position until a reline is required. The use of a crucible permits quick and easy replacement of the lining. A prefired crucible also reduces the possibilities of errors caused by poor ramming, drying out and fritting of refractory linings. The need for melt out formers and wash-out melts of non-compatible materials is also eliminated. Tilting units of this type with capacity of up to 250 kg are now in operation.

For precision casting, double axis tilt furnace can be used. This allows pouring directly into shell moulds. The furnace starts to tilt on one axis (about half way up its front face) and then reaches forward to tilt on a second axis near the lip.

2.5.2.2 Push-up furnace

Here non-compatible alloys are melted down in the crucible which is used as a casting ladle too (Fig. 2.9). In North America the barrel shaped crucible or 'F' shape is very popular in these furnaces. In the UK however, the 'A' shape (Fig. 2.7) has become more popular. The maximum size depends on the operatives but up to 200 kg capacity are currently
Fig. 2.9 Double push-up furnace using 'A' shaped crucibles (Morganite crucibles LTD)
being used.

The crucible sits on a refractory support mounted on top of a stainless steel tube which is at the head of the hydraulic lifting ram. Ceramic fibre insulation is used to fill the gap between the crucible and coil to reduce heat losses from the crucible, and to protect coil from damage. The crucible is moved up and out of the furnace coil enclosure by the lifting mechanism when the melt is ready for pouring. Metal spillage, in the event of total crucible failure, is contained by a split refractory tray which surrounds the crucible and is possible to withdraw.

These units come in single or twin form and their attraction for the non-ferrous user is that a wide variety of differing metals can be melted, simply by changing the crucibles to avoid cross-contamination. The main reasons for choosing a twin unit are to use different sized crucibles on either side of the push-up unit or consistent use of two different metals. The push-up has the disadvantage of requiring provisions for the lifting ram underneath.

2.5.2.3 Lift-coil and lift-swing furnaces
Lift-coil furnace is very similar to the push-up but the coil is lifted off the crucible (Fig. 2.10). The crucible stands on the floor or on a concrete pillar. The other scheme, called lift-swing employs one coil which can be lowered over two working positions. Whilst one pot is being heated, the other is being prepared. When the first is melted, the coil is lifted, swung through 90° horizontally, and lowered over the second crucible (Fig. 2.11).
Fig. 2.10 Lift-coil furnace

Fig. 2.11 Lift-swing furnace
Because the coil has to be designed to go over the larger diameter of the crucible, this results in a poor electrical coupling and these furnaces have been superseded by the push-up.

A more recent type of furnace is the "drop-coil" or "drop-down" furnace which is a development of the push-up; whereby the coil is moved up and down rather than the crucible. As the coil 'drops down' to expose the crucible after melting, it can also be more closely coupled to the crucible. The furnace needs no foundation or holes in the foundry floor. In addition, the lifting mechanism is at the rear, well out of the way of molten metal making it safer and easier to service.

2.6 Crucibles and Melting Techniques [2.42, 2.43]

A considerable amount of time has been devoted by manufacturers in producing crucibles which will withstand most of the abuse that foundrymen tend to give them. The result of these efforts is a product which should give maximum resistance to thermal shock, mechanical damage and chemical attack from fluxes, slags and metals. However, with induction melting there are a few points where crucible melting practice is different from fuel fired furnaces.

(a) In tilting furnaces the earth leakage probe should be sited to give adequate warning of metal leakage following crucible failure [2.46].
(b) Some allowance must be made for the expansion of the crucible when it is heated up to operating temperature. The backing material must not be rammed too tightly.
(c) When the crucible has been sealed in position with a refractory cement, the normal precautions for slow warm-up should be taken. If
the moisture is forced out too quickly, steam is produced and can cause bumping and cracking of the crucible. This may even damage the coil inter-turn insulation.

(d) After installation of the crucible it is advisable to gradually bring it up to operating temperature. This is not so critical in low powered furnaces or when using clay-graphite crucibles which do not absorb most of the power. Maximum power causes a hot band to develop very quickly in the mid-wall (Fig. 2.12) and subjects the crucible to thermal stress [2.42].

Fig. 2.12 Clay-graphite crucible melting copper (Morganite Crucible Ltd)
(e) In furnaces where the power is rapidly induced directly into the charge, care has to be taken with the charging of the metal. Small scrap should be placed in the bottom of the crucible to act as a buffer for large scrap or heavy ingot. The ingot should be placed carefully in the middle of the crucible in order to allow for expansion of the metal and thus avoid wedging [2.47].

(f) When the lower half of the charge has become molten, the liquid heel will preferentially absorb the power due to the increased resistivity of the molten metal. A particular hazard exists when cold scrap is dumped over the liquid metal bath. The possibility at such a time of charge bridging, that is, sticking to the crucible wall during its descent, can be dangerous and should be avoided because it can result in rapid superheating of the molten pool formed underneath the hanging solid charge. The superheating coupled with the stirring action will cause excessive internal attack in the lower half of the crucible. In the region where the bottom of the wall joins the base of the crucible, compositional mismatch may sometimes occur due to the method of manufacture. This effect can lead to even greater local wear in that region of the crucible.

(g) Special attention must also be paid when melting small charges, in heat absorbing silicon carbide crucibles. Once the metal is molten and falls below the influence of the power coil, the crucible absorbs the heat and can cause a thermal runaway.

(h) Clay-graphite crucibles of 60 kg capacity (A60 size) can be used satisfactorily at 3 kHz at power levels below 125 kW. At higher powers, a larger clay-graphite crucible would need to be energized at 1 kHz to reduce crucible heating. As a rough guide, a power density
2 kW/kg capacity can be taken as the maximum for clay-graphite crucibles at 3 kHz \([2.37]\).

(k) Although the hot crucibles absorb the power efficiently, they can't transfer the power to charge effectively unless careful charging is practiced, that is, fine materials first plus tightly packed ingots followed by returns when a molten heel has been established.

(m) For a given power and frequency more stirring will occur in a clay-graphite than with silicon carbide crucible, though with careful furnace design this should be overcome. With excessive stirring in this type of crucible dross formation is increased.

Crucible manufacture is also not easy. Minor changes in production such as the replacement of an old mixer for the materials by a new one, have resulted in the production of inferior crucibles with voids etc in them. Where the crucible is expected to generate some of the heat in the furnace, uniformity of mixing is also very important to ensure uniform heating. Both types of defect can cause premature crucible failure due to thermal stress.

In conclusion, the main concern of any foundryman is to get maximum number of melts from any one crucible. This should be done rapidly and efficiently without the risk of nasty accidents with crucible breakage and molten metal thrown all over the melting shop. In a bronze foundry, fifty melts per crucible is typical.

2.7 Analysis of Conducting Crucible Induction Furnace

Whilst it is recognized that greater accuracy is obtainable by using
numerical techniques in solving specific induction heating/melting problems, it is argued that orthodox analytical methods provide sufficiently accurate results and can be used more easily as a tool for designing conducting crucible induction furnaces. Most furnace installations available have been designed without the aid of sophisticated numerical simulation. Large sectors in industry are still reluctant to adopt numerical techniques primarily because the simple analytical methods have worked satisfactorily. However, for completeness a brief review of the relevant work done by numerical techniques has been included.

Past work on induction heating analysis by various numerical methods are reviewed first. These are followed by coreless furnace references. Finally, previous applications of numerical techniques to the calculation of conducting crucible induction furnaces with composite loads are reviewed.

Holmdahl and Sundberg in 1963 [2.48] applied the finite difference method to the induction heating of ferromagnetic cylinders. In this method, the workpiece is divided into sub-regions. The region under consideration is divided up into a grid and the relevant differential equations are solved at every point on the grid by using simple algebraic expressions obtained from Taylor's power series to replace differential quotients with difference quotients. The method is simple and suitable for two-dimensional electro-thermal and also turbulent electromagnetic problems. The temperature rise in long cylindrical workpieces were considered with reference made to the change of permeability (magnetic properties) at
curie temperature. The permeability was calculated as a function of magnetic field strength. The resistivity was obtained as function of temperature. The problem involved solving two differential equations governing radial variation of magnetic field strength and temperature. Their work is useful in predicting the temperature rise of surface to core of solid cylindrical billets in through heating.

Reichert in 1968 [2.49] compiled a method of calculating the characteristics of induction heating installations and temperature distribution in the load. The method comprised solving Maxwell's equations by finite difference technique to determine the electrical relations in induction heating installations. It is a powerful program which, if modified, may be applied to various electroheat installations including furnaces. It covers asymmetrical and non-homogeneous charges. The program (in ALGOL) is lengthy and requires considerable computation time and needs to be modified for specific application. In the example given, it is applied to a coreless furnace with molten iron charge. The magnetic flux distributions in the furnace crucible were shown for cases of totally or partially filled bath, worn lining as well as with lamination packs. The furnace characteristics calculated were shown to be within 5% agreement with measured values.

Gibson in 1976 [2.50] also developed a finite difference computer package for obtaining temperatures in different regions of an inductively heated workpiece. The variation of magnetic permeability was obtained using Frohlich formula. The package is general but needs substantial computing time and should be greatly modified for specific applications.
Coggon in 1971 [2.51] applied the finite element technique to modelling of various electromagnetic fields. He explained the principles of finite element. The technique is particularly useful for induction heating as opposed to stirring problems. Sabonnadire in 1984 [2.52] reported the use of interactive software for the design of induction heating coils. The mathematical basis of the software was the finite element method applied to the Maxwell's equations. The paper illustrates magnetic flux distribution within the workpiece.

The finite element method is concerned with the solution of mathematical or physical problems which are usually defined in a continuous domain by local differential equations. To render the problem amenable to numerical treatment, the infinite degrees of freedom of the system are discretized or replaced by a finite number of unknown parameters [2.53]. The object to be analyzed is divided into small segments called elements and the relations among them represented by simultaneous partial differential equations.

Nemoto and Tabuchi in 1984 [2.54] applied the finite element method to prediction of temperature distribution in inductively heated cylindrical workloads. The technique was illustrated with examples given for surface hardening of rolls used in steel making, seam annealing of longitudinally welded pipes and bar end heating for forging application. The analysis consisted of simultaneous solution of Maxwell's diffusion equation and two-dimensional non-steady heat conduction equation in cylindrical coordinates. For the roll heater example, the model with 25 rectangular elements representing a quarter of the axisymmetrical assembly; with
finer mesh near the surface of the roll's middle section, has been shown as well as magnetic flux and isothermal plots generated by the program. The surface to centre temperature-time curve results were shown for the workpiece with change of initial temperature. For the seam annealer example involving a welded pipe moving through four separate inductors, different routines have been used in the program to cater for repeated heating and cooling of the work load through specified temperatures. Properties except permeability at curie point have been assumed to remain unchanged.

Kolbe and Reiss in 1967 [2.55] investigated the variation of current density and temperature distribution in axisymmetrical workpiece and inductor assembly of an induction heating installation by solving Maxwell's differential equations using a type of mutually coupled circuit method. A square shaped element was used in the grid. Using an example, the work was developed in three stages: determination of current density distribution; of temperature distribution, and finally of temperature distribution coupled with current density variation as a function of temperature. The example given was for soldering a brass tube to a brass flange by using a single turn inductor placed around the tube, at 10V and 8 kHz. In each stage, the current density and temperature distribution were shown in radial and axial directions. The effect of close coupling to generate maximum heat in the desired area was demonstrated. They concluded that if localized heating was a practical problem, lower frequency or lower power could be applied to allow diffusion of heat to take place. Their work is applicable to inductor design for soldering or brazing by induction heating. Approximations
include assumption of constant thermal properties. The method also requires the use of powerful computers.

Dudley and Burke in 1972 [2.56] applied the mutually coupled equivalent circuit method to obtain current distributions in three dimensional geometries of axisymmetric bodies. In this method, the conducting material to be investigated is divided along current streamlines into a finite number of segments in each of which the current may be considered constant. Coupled circuit equations may then be written linking the currents in these subconductors which may be found by solving the resulting system of linear complex equations. Using expressions for inductance calculation [2.57], the resistance and self-inductance of each subconductor, together with the mutual inductance between them are obtained. All terms are collected in an impedance matrix equation which is solved to obtain the unknown currents. Three cases were examined. For the wound coil and billet, the segment sizes were chosen to be small near the outside surface of the billet where the current density was expected to vary more rapidly in radial direction. The matrix equation describing the equivalent circuit consisted of square and column matrices containing billet impedances and unknown currents; a square matrix containing all coil and billet mutual impedances, and the given coil currents. Current densities at 10 kHz were predicted for three typical cases with circular symmetry about the coil axis. The results were shown and were verified by current density probe measurements. The method cannot be used for magnetic materials, and compared with other numerical methods requires greater number of subdivisions which make it more costly in terms of computation.
Fikus and Sajdak in 1978 [2.58] attempted to analyse the characteristics of a steel crucible induction furnace. They applied Fourier integral method to the solution of Maxwell's differential equations. They assumed an infinitely long two-layer cylindrical load inside a sectionalized furnace coil with and without lamination backing. Power density was obtained using Poynting's vector leading to distribution of power between charge and crucible. The worked example given was for a large (1.5 tonne) furnace with steel crucible; and close agreement between the mathematical model and measured results were claimed. The paper highlights an alternative method of solving Maxwell's differential equations. The results are poorly illustrated and are few and far between.

Based on earlier work [2.59], Lupi et al in 1975 [2.60] developed a method for the calculation of inductors with spaced coils. The method can be applied to sectionalized inductors consisting of a number of coils of equal diameter and uneven spacings. Imaginary coils with no current through them were assumed in the free spaces between the real inductors. Equivalent magnetic circuits for all coils sections were established and the relationship between magnetic fluxes, mmf's and reluctances; and between applied voltages, currents and admittances were expressed in matrix form with admittances of all coil sections included in the matrix. Depending on the series or parallel connections between the coils, the voltages or currents of the imaginary coils sections were expressed in terms of voltages and admittances of the real coils. Using computer to solve the matrix system of equations, electrical parameters of inductors with a number of coils were determined when the applied voltages or
currents were known. Results have been given of resistance and impedance of the load and inductor plotted against spacing between sections, at various frequencies up to 10 kHz and compared with previous analytical and experimental values. They include solid and thin hollow cylindrical loads with load diameter to coil diameter ratio of 0.57 and coils having height equal to diameter. The results show increasing the distance between the coils reduces electrical parameters initially by about 20% and then remaining approximately unchanged beyond the distance of one diameter of the coil. The method is useful in the analysis and calculation of installations for continuous induction heating of rods and billets.

Lupi in 1977 [2.61] studied the design of inductors for conducting crucible induction furnaces using analytical methods of periodic field and sectionalized inductors, as well as numerical method of mutually coupled circuits.

The first method was periodic fields method previously used by Lavers and Bringer [2.62, 2.63]. Assuming a long two layer cylindrical load of crucible and molten charge within a short furnace coil shown in Fig. 2.12, the complex furnace impedance was given as

\[ Z_e = X_{10} \left\{ \left( \frac{D}{D_1} \right) G + j \left[ \frac{K_N}{\left( \frac{D}{D_1} \right)^2} Q \right] \right\} \quad (2.7) \]

where \( X_{10} \) = reactance of the inductor coil in the absence of the load and without end effects

\( G, Q \) = coefficients for calculating the real and imaginary parts of \( Z_e \); fn (dimensions, resistivities \( m_2 = \sqrt{\frac{D}{\delta_2}} \))
FIG. 2.13 SHORT INDUCTOR AND LONG TWO-LAYER LOAD

(1) THE CHARGE. (2) THE CRUCIBLE
$K_N = \text{Nagaoka's coefficient of the coil.}$

The results of calculation have been presented as plots of the product $mG$ and $Q$ against dimensionless ratio of crucible wall thickness to skin depth, at various combinations of coil crucible and charge diameter and inductor length, as well as for different resistivity ratios. The influence of air gap and length to diameter of the coil over the whole range of abscissa have been shown. The parameters used in the graphical results are dimensionless. Whilst they are aimed to be used on general purposes, they need to be interpreted according to one's requirement for any given variable. Range of frequencies and air-gap dimension for maximum power input into the crucible have also been indicated by showing the variation of power in crucible to power in crucible and charge with thickness to skin depth ratio.

In the second analytical method referred to earlier [2.60] the furnace characteristics were obtained by the sectionalized inductor method; taking into account approximately the end effect of the load and the effect of partially filled crucible. The inductor coil was divided into three sections touching one another, and terminating in a further imaginary coil with no driving current, corresponding to the end effects. In each section, the load was considered as an infinitely long cylinder of crucible and metal, or air, and coefficients of active and reactive power were obtained for each section. Then neglecting coil resistance, a matrix equation expressing currents in terms of admittances, voltage and currents was set up from the equivalent magnetic circuit. Admittances of all sections in terms of furnace parameters were obtained from the matrix
equation. The furnace impedance was then found in terms of admittances. Using a computer program and further equations (not given in the paper), electrical parameters of the inductor crucible load system have been calculated and compared with experimental results. Calculated and measured values for the ratio of crucible and charge resistance to coil reactance were plotted against percentage filling of metal at different values of crucible thickness to skin depth. These were also plotted against crucible diameter at frequencies of 2, 4 and 8 kHz and said to be in close agreement with measured values. The results showed that at high frequencies, the percentage filling had smaller effect on power absorbed.

The limitations of the first two analytical methods to cater for curvature of crucible walls and to show distribution of current density were partly resolved by employing numerical methods. The mutually coupled circuit method developed by Dudley and Burke [2.56] was therefore used. The inductor, crucible and load were divided into a set of elementary circular circuits for which the impedance matrix equation was applied.

In this way, the problem was reduced to the solution of a system of n linear algebraic equations with complex coefficients whose unknowns were the induced currents in the elements. The system of equations were solved and current and power density distribution in the crucible and metal as well as other characteristics were determined. The results of the computer program have been given as current density and voltage in the inductor turns with reference to the geometrical configurations for straight and curved empty crucibles and also for crucibles partially filled with metal. The results showed the increase in the values of
parameters corresponding to the end of crucible inside the inductor and the decrease due to the crucible top emerging outside the coil; for various values of thickness to skin depth ratios of crucible.

The effect of metal inside the crucible was shown not to alter voltage distribution in the turns of the inductor and reduce current density in the crucible markedly at the surface of the melt in partially filled crucible. The current density in the charge obtained were qualitatively similar to those with non-conducting crucibles.

The paper shows three methods of calculation for conducting crucible furnaces. The results of first two methods were similar to those obtained by Reichert [2.74] and Sundberg [2.73]. The results obtained by the third method were similar to those obtained by Reichert [2.49] using finite difference method.

The periodic field method is useful for choosing the operating frequency. The sectionalized inductor method enables the impedance of the furnace coil to be calculated fairly accurately. Both these analytical methods have limitations which can be overcome by the use of numerical methods. The coupled circuit technique provided more information regarding geometry but it required considerable calculation time and modification for applying to the conducting crucible induction furnace.

Numerical techniques are not useful unless they can be applied with relative ease by the engineer, and without the need for large computers. Furthermore, the use of numerical methods for the purpose of this work is
not considered worthwhile because exact knowledge of materials properties are not available. As pointed out earlier, most of the materials properties have been put together from different sources (this is explained in Chapter 4) and the Morganite Company who were concerned with their staff ability to cope with numerical techniques did want an analytical approach towards the problem under investigation, to be used as a design tool.

2.7.1 Analytical approach - equivalent circuit technique

The analytical method essentially consists of applying the theory of induction heating to the two layer cylindrical load, and requires the solution of diffusion equation in cylindrical form for the sinusoidally varying magnetic field strength.

\[
\frac{d^2 H}{dr^2} + \frac{1}{r} \frac{dH}{dr} -j \frac{2}{\sigma^2} H = 0
\]  

(2.8)

This is one of Bessel's equations and is sometimes referred to as skin effect equation. Calculation of furnace characteristics including allotment of induced power between the crucible and charge are subsequently used to investigate the thermal behaviour of crucibles in service.

Firstly, previous works on the induction heating of single loads as in billet heating or coreless furnaces are reviewed. These are then followed by reviewing the past work on induction heating of two layer loads which are related to conducting crucible furnaces.
There is close analogy between the theories of induction heating (or melting) and transformers. By considering magnetic flux paths in different parts of the furnace, reluctances in the magnetic circuit are calculated and are referred to the primary side using Faraday's law of induction to obtain the equivalent electrical circuit. The impedances and reactances in the equivalent circuit are finally calculated in order to obtain the furnace characteristics. This constitutes the equivalent circuit technique and has been detailed and applied to the design and calculation of induction heating coils [2.68, 2.69].

Based on earlier work [2.64], Dwight and Bagai in 1935 [2.65] applied the theory of induction heating to the calculation of coreless furnaces. The charge was treated as a solid cylinder. They derived the standard differential Bessel equation (eqn. 2.8) for magnetic field strength in the charge from first principles and calculated electrical characteristics of the furnace including impedance, efficiency and power factor. The coil was assumed to be infinitely long with negligible radial thickness and known number of turns. No equivalent circuit was used and no end effects were considered. Their work included expressions for current density at any radius \( r \) in the charge of radius \( R \):

\[
J_r = \frac{\sqrt{2}}{\delta} \cdot H_r \cdot \frac{\text{ber}'(\sqrt{2} r/\delta) + j \cdot \text{bei}'(\sqrt{2} r/\delta)}{\text{ber}(\sqrt{2} R/\delta) + j \cdot \text{bei}(\sqrt{2} R/\delta)} \tag{2.9}
\]

The general theory of eddy current heating of solid and single hollow cylinders was given by McLachlan [2.66a, 266b] who derived the Bessel equation in cylindrical form from first principles for magnetic field strength and current density. Skin effect in a circular tube was also
considered. Using Poyting's theorem, he obtained eddy current loss in the core of a long solenoid, coaxial with the coil. The power loss in the solid cylindrical core was expressed in terms of loss functions $W$ and $II$ as Bessel functions with argument $x = \frac{\sqrt{2}}{\delta} \cdot \frac{R}{\delta}$ of the core, shown in Fig. 2.14, later to be adopted as flux factors $q$ and $p$ [2.69].

![Graph showing dimensionless loss functions for solid cylinder](image)

Fig. 2.14 Dimensionless loss functions for solid cylinder

\[
W = \frac{\sqrt{2} \cdot \delta}{R} \cdot \frac{\text{ber} x \cdot \text{ber}' x + \text{bei} x \cdot \text{bei}' x}{\text{ber}^2 x + \text{bei}^2 x} \tag{2.10}
\]

\[
\Pi = \frac{\sqrt{2} \cdot \delta}{R} \cdot \frac{\text{ber} x \cdot \text{bei}' x - \text{bei} x \cdot \text{ber}' x}{\text{ber}^2 x + \text{bei}^2 x} \tag{2.11}
\]
He did not consider end effects. He discussed the application of the analysis to induction furnaces using non-conducting crucibles.

Coil design for induction heating of billets by application of equivalent circuit technique was given amongst others, by Baker in 1944 [2.67] for induction heating of non-magnetic conductors with long work coils. He also gave an equation for the power input to thin-walled \( t \leq 0.1 \text{ dia} \) hollow cylinders when heated at frequencies at which skin depth was several times the wall thickness \( t \). Vaughan and Williamson in 1945 [2.68] developed a similar method of coil design for non-magnetic solid and hollow cylindrical loads of different thicknesses. They used Nagaoka's constant \( K_5 \) in the calculation of the reactance of the air gap \( (X_0') \) to represent the coil shortness \( (X_0 = K_1X_0') \) where, \( K_1 = f_n(K_5, \text{radii}) \). \( K_5 \) was also used to account for the large air-gap between the coil and work load \( (X_0 = K_5X_0') \).

Baker in 1957 [2.69] published a more complete version of his earlier work and gave detailed formulation of coil design procedure for billet heating by equivalent circuit technique. Using earlier work involving derivation of flux factors \( p \) and \( q \) from skin effect equation for solid cylindrical workpiece, he expressed the magnetic fluxes through the three regions, coil, workpiece and air-gap in terms of magnetic field strength tangential to the workpiece. The equivalent circuit was given for the long coil and work load as shown in Fig. 2.15.

\[
I_c = \frac{H_0 \cdot \frac{V_c}{N_c}}{\sqrt{2}} \quad (2.12)
\]
FIG. 2.15 EQUIVALENT CIRCUIT WITH LONG COIL

FIG. 2.16 EQUIVALENT CIRCUIT WITH SHORT COIL
In the calculations he introduced an empirical correction factor $k_T$ to account for the spacing between turns and other imperfections encountered in practical work coils such as spiral effect, taper, dents, etc. $k_T$ was called coil resistance factor as it was used in the formula to calculate a.c. resistance of the coil. An average value of 1.15 was allocated to it for a single layer coil. $k_T$ corresponds to resistance factor $\phi_1$, of Reichert [2.74]. From the equivalent circuit calculations, all electrical characteristics of the billet heater were obtained.

For the electrically short coil, he introduced an end effect shunt reactance $X_e$ where $X_e$ is the reactance due to the reluctance $R_e$ of the external flux path. A larger coil current would be required to supply the external drop $R_e \phi_0$; $\phi_0$ being the flux outside. The equivalent circuit with the short coil is shown in Fig 2.16. $R_e$ corresponds to $R_{mR}$ of Reichert [2.74].

$R_e$ was assumed to be approximately inversely proportional to the coil perimeter $p_c$ and was given as

$$R_e = \frac{1.8}{p_c} \quad (2.13)$$

Work done by Baker and Vaughan and Williamson are powerful techniques which are readily available, well established and easily understood. They are often used for induction heating coil designs for single cylindrical loads.
Attempts have been made to improve the accuracy of empirical correction factors. The assumption of uniform field strength across the air-gap between the coil and charge has been discussed by Lavers and Biringer [2.70]. They have introduced a correction factor as a power series in terms of the furnace geometry parameters. But it is expected that only minor improvements can be achieved if used in the case of conducting crucible furnaces where the air gap is partly replaced by a conducting media, in which the drop of magnetic field strength is considered.

Having briefly reviewed induction heating of single loads, background work on the analysis of induction heating composite bodies are now reviewed here.

Wright in 1937 [2.71] developed the theory of eddy current heating of composite loads. This was based on McLachlan's work. The composite load consisted of a long hollow cylindrical tube representing a valve envelope and inside, a coaxial cylinder; hollow or solid, representing a valve electrode. He combined the theory of induction heating of solid cylinders and single hollow cylinders and modified it by consideration of the inner boundary conditions for the envelope which were affected by the presence of the inner charge.

For the coil, he assumed a long single layer solenoid. He also assumed constant magnetic field strength over the gap between the outer and inner charge. With the aid of Poynting's theorem, expressions for the induced power in both the inner and outer charge were developed in terms of Bessel functions. His work is versatile and adaptable to different materials.
No account was taken of the coil's finite length. The analysis was not applied to furnaces and did not include design of the work coil. It does, however, form the basis of later works done on analysis of conducting crucible induction furnace.

Subsequent works have been carried out based on the theory of induction heating of composite loads for the analysis of conducting crucible induction furnace by Reichert [2.74] followed by Toropov in 1969 [2.72] who introduced tabulated correction factors in terms of magnetic field strength, in the calculation of coil impedance and of leakage reactance, to cater for short coil effect. Other works have either involved reapplication of the theory to the calculation of induction heating of various composite bodies in industry [2.73] partly by numerical techniques, or have concerned improving the accuracy of results for conducting crucible furnaces through the use of numerical techniques [2.61] received earlier.

Sundberg in 1965 [2.73] studied induction heating of bodies inside shells. Starting with the solution of Maxwell's equation in cylindrical form and using Poynting's law, he showed how to increase the power input to the inner body and applied the theory to heating of molten metal inside ladles by placing inductor coils on the outside. Relative permeability for the inner body was assumed both constant and a function of field strength and the effect shown in both cases. He calculated the power densities in the metallic shell and the inner body using both analytical and a computerized version of the analytical method. In the latter, he assumed both the shell and the inner body to be made up of m vertical layers of
thickness $\Delta r$. By considering the radial drop in magnetic field strength $H$ over $\Delta r$ from one layer to another, the first derivative $\frac{dH}{dr}$ was obtained in terms of $H$ and $r$. The second derivative $\frac{d^2H}{dr^2}$ was found by considering the change of $\frac{dH}{dr}$ over the same two layers. Hence by elementary differentiation, he collected general expressions for the terms of the diffusion equation in cylindrical form and solved it using matrices to represent $m$ simultaneous equations. The results were then compared with those obtained by the manual method.

The skin depths in the metallic ladles are much smaller than those in conducting crucibles. The work has useful applications: screening of steel constructions with copper sheets against alternating fields, and power input calculation required to maintain hot molten metal inside ladles but is not directly applicable to the analysis of conducting crucible induction furnaces and no coil design procedure has been included.

Lupi [2.61] has applied three methods to the problem of induction heating composite bodies and has shown some results as Reichert. By applying a numerical technique to the conducting crucible induction furnace, effect of furnace parameters on the distribution of current density over the crucible axial geometry have been shown.

Based on Wright's work, Reichert in 1965 [2.74] developed the analysis of the two layer furnaces, that is the composite load of charge and conducting crucible. The finite length of the coil was accounted for by using an empirical return flux factor $k_r$ whose value depends on the type
of backing: iron lamination or air. He developed a coil design method
by the equivalent circuit technique. He also introduced the calculation
of meniscus height in the molten bath. This makes his work directly
relevant to the analysis of furnaces. The application of his work,
however, involved use of charts calibrated graphs and several factors.
Modifications have been made as shown in Chapter 3.

Reichert's work is a direct application of the equivalent circuit
technique to the conducting crucible furnace to develop a coil design
method. Because of the importance that the analysis in this work depends
on it; Reichert's work is explained in greater detail in a separate
chapter; Chapter 3.

References

2.1 PASHKIS, V., and PERRSON, B.S.: 'Industrial electric furnaces and

2.2 ROBIETT, A.G.: "Metal melting from the small user's point of view",
industrial monograph H9, The Electricity Council EDA Division, 1968

2.3 PALMER, L.W.: "Recent trends in electric arc furnace techniques",

2.4 SAKULIN, M.: "Simulation of electric arcs in melting furnaces",
Paper 1.4, ibid.

2.5 OTTO, C.A.: 'Electric Furnaces' (George Newnes, London, 1958)

2.6 RAMSELL, P.G.: "Metal melting using resistance furnaces", IEE Conf.
on Electricity for Materials Processing and Conservation (EMPAC),

2.7 DAVIES, I.: 'Recent electric-furnace developments for non-ferrous

2.8 ATKINS, R.: 'Recent developments in electric melting and holding for


2.15 EDGERLEY, C.J.: "Improvements in the melting efficiency of coreless furnaces", IEE Conf. on EMPAC, March 1977, IEE Conf. Publ. No. 149, pp. 28-33


2.17 EDGERLEY, C.J. and LANGMAN, R.D., "Progress in the design of furnace control and power supply equipment for induction furnaces", Electricity Council Research Centre, ECRC/M1305, Dec. 1979


2.19 HODGKINS, W.R.: "Mathematical calculations on electromagnetic stirring", ECRC/M12, 1972


2.25 ARELMANN, F.: "Features and main applications of medium frequency and mains frequency coreless induction furnaces", 10th Int. Congress on Electroheat, Stockholm, June 1984


2.29 PUGH, S.: "Electric melting - is this your answer?", ibid., 1976, Vol. 141, (3095), pp. 17-42


2.31 BAINES, A.P.: "The choice of medium frequency for non-ferrous melting", 10th Int. Congress on Electroheat, Stockholm, 18-22 June 1984

2.32 DAVIES, I.: "Electric melting - the key to new markets for small foundries", The British Foundryman, 1985, Vol. 78, Pt. 1, pp. 11-12


2.34 HOBSON, L.: 'Guide to induction heating equipment' (British National Committee for Electroheat, 1984)


2.37 DAVIES, I.: Private communications, 1980


2.39 SMITH, L.: "Medium frequency induction melting of non-ferrous metals", IEE Colloq. on 'Electricity in non-ferrous industries',


2.42 ADCOCK, F.: Private communications, 1981


2.44 SMITH, L.: "The influence of lining selection and charge material characteristics on copper melting in a medium frequency coreless induction furnace", Inst. of Metals Conference, Birmingham, Apr. 1987


2.50 GIBSON, R.C.: "A computer method of calculating the eddy current heating of magnetic materials with a comparison between predicted and measured results in a 2 MVA induction furnace" COMPUMAG Conference on computation of magnetic fields, Oxford, March/April 1976


2.52 SABONNADIERE, J.C.: "Méthodes interactives de calcul des systèmes de chauffage par induction", 10th Int. Congress on Electroheat, Stockholm, 18-22 June 1984

2.53 CHARI, M.V.K. and SILVESTER, P.P.: 'Finite elements in electrical and magnetic field problems' (John Wiley & Sons, 1980)

2.54 NEMOTO, K. and TABUCHI, M.: "Thermal analysis of induction heating by the finite element using a computer", 10th Int. Congress on
Electroheat, Stockholm, June 1984


2.57 GROVER, F.W.: 'Inductance Calculations - working formulas and tables' (Van Nostrand, New York 1946)


2.61 LUPI, S.: "Design of inductors for induction furnaces with conducting crucible", World Elektrotech. Congress, Moscow, June 1977


2.66a McLACHLAN, N.W.: 'Bessel functions for engineers' (Oxford University Press, 1934)


2.73 SUNDBERG, Y.: 'Induction Heating, with special reference to bodies inside metallic shells' (Vastra Aros Tryckeri Aktiebolag, Vasteras, 1965)

The mathematical theory involved in the calculation of conducting crucible induction furnace is presented here. An equivalent circuit method is applied to the composite load of crucible and charge. Electrical characteristics, distribution of power, and meniscus height are evaluated. The theory has been adapted to be applied with more flexibility and without the need for approximations and use of charts.
3 ANALYSIS

3.1 Introduction

The basic concept of induction heating is similar to the transformer theory (Fig. 3.1(a)). If the secondary is considered as a single-turn short-circuited winding, the secondary current will be high and losses will develop (Fig. 3.1 (b)). In an induction furnace, the losses generate heat in the charge and ultimately cause melting. More explanation has been given in earlier Chapters and elsewhere [3.1].

Equivalent circuit techniques have been applied to the design of induction heating and melting installations for many years [3.2, 3.3]. They represent an industrially well accepted method of designing induction heating systems in which the effect of parameter changes can readily be assessed and in which the incorporation of empirical factors taking into account industrial practice is facilitated.

When conducting crucibles are used in place of rammed refractory lining in coreless induction furnaces, the magnetic field strength and current density are further distributed over crucible and charge. The application of equivalent circuit techniques to the analysis of coreless induction furnaces with electrically conducting crucibles is therefore complicated by the distribution of magnetic flux between the crucible and metal, which together constitute a composite load. The equivalent circuit must then include components representing the allotment of induced currents in the crucible as well as the charge [3.4]. The arrangement of the conducting crucible furnace is shown in Fig. 3.2.
FIG. 3.1 BASIC CONCEPT OF INDUCTION-HEATING COIL AND LOAD.
Fig. 3.2 CONDUCTING CRUCIBLE FURNACE:
1. Furnace Coil  
2. Conducting Crucible  
3. Charge
Fig. 3.2 shows the design of the conducting crucible furnace. It comprises the cylindrical furnace coil 1 fitted round the refractory ceramic 0, the conducting crucible 2 and the charge 3. The alternating current $I_1$ flowing through the coil, sets up a magnetic field with the field densities $H_0$, $H_2$ and $H_3$ in the refractory crucible and charge. Eddy currents with densities $J_2$ and $J_3$ are thus produced by $H_2$ and $H_3$.

3.2 Equivalent Circuit of the Conducting Crucible Furnace

The method has been applied to the calculation of induction furnaces [3.5] with rammed lining using approximations of the magnetic field and its interconnection with the current flowing through the furnace coil.

Application of the equivalent circuit method to the conducting crucible furnace is aimed to evaluate furnace characteristics for design, and the distribution of power between charge and crucible. It is necessary to make approximations concerning the pattern of field inside the coil. The leakage flux which links with the coil but not the crucible, is neglected. Return flux outside the finite coil is expressed in terms of empirical factors [3.6]. It is assumed that the coil-turns and the crucible have no inclination, and that the metal forms a solid homogeneous core. It is further assumed that the crucible is either entirely filled or entirely empty. The theory allows consideration of radial distribution only, of magnetic field and current density in crucible and charge; the magnetic field acts entirely axially and an average magnetic field strength is used [3.3]. The field is assumed to vary sinusoidally with time, which can be written as:
FIG. 3.3 FLOWCHART SHOWING THE EQUIVALENT CIRCUIT TECHNIQUE

1. Magnetic flux paths
2. Calculation of reluctance in magnetic circuit
3. Equivalent circuit
4. Calculation of circuit components \( X_R, X_0 \) and \( R_1 \)
5. Poisson vector
6. Complex power
7. Complex resistances obtained in terms of \( J_{(p)} \) and \( H_{(p)} \)
8. Maxwell's diffusion equation
9. Boundary conditions
10. Solution of \( J_{(p)} \) and \( H_{(p)} \) in terms of Bessel functions
11. Calculations of other circuit components \( Z_2 \) and \( Z_3 \) in terms of Bessel functions
12. Calculation of total furnace impedance
   - Reactive power
   - Efficiency
   - Power factor
   - Number of coil turns
   - Meniscus height
The sequence of calculations is shown in Fig. 3.3. The total magnetic flux linking the turns of the work coil is expressed in terms of flux linking different parts of the furnace. Magnetic Resistances are obtained and converted into electrical resistances using induction laws. Expressions for the equivalent circuit components, \( R_1 \), \( X_0 \) and \( X_R \) are obtained in terms of operating frequency, permeability and geometry of the installation.

The complex impedance of the conducting crucible and metal charge, i.e. \( Z_2 \) and \( Z_3 \) are derived from Poynting's vector and Maxwell's diffusion equation. Furnace characteristics are then obtained using circuit analysis.

The magnetic circuit is shown in Fig. 3.4. Axial lines of flux \( \Phi_1 \), \( \Phi_0 \), \( \Phi_2 \) and \( \Phi_3 \) pass through the furnace coil 1, the refractory 0, crucible 2 and the charge 3. The sum, \( \Phi_R = \Phi_0 + \Phi_1 + \Phi_2 + \Phi_3 \) closes around the outside area of the furnace.

\[
\Phi_R = \Phi_0 + \Phi_1 + \Phi_2 + \Phi_3
\]  

The paths of these fluxes have the reluctances \( R_{WO} \) to \( R_{MR} \). They form the magnetic circuit shown on the right of Fig. 3.4. \( \Phi_1 \) and the corresponding reluctance is neglected because it is small compared with \( \Phi_R \). The reactance is allowed for later by using a complex resistance for the coil.
FIG. 3.4 MAGNETIC CIRCUIT OF THE CONDUCTING CRUCIBLE FURNACE WITH MAGNETIC FLUX PATHS IN DIFFERENT REGIONS

Flux is shown by symbol $\Phi$

The suffixes are:
1 furnace coil
0 airgap or nonconducting refractory
2 conducting crucible
3 metal charge
R return flow
When the current $I_1$ amperes flows through the coil of $N_1$ turns, the mmf is the total current linked with the magnetic circuit, namely $I_1 N_1$ amperes. The mmf in a magnetic circuit is responsible for pushing the flux round and is thus analogous to the emf in an electric circuit. This driving force can be represented by a magnetic potential difference in the magnetic circuit. The furnace coil which provides the mmf is shown in Fig. 3.5. The magnetic circuit of Fig. 3.4 is redrawn symbolically in Fig. 3.6.

Referring to Fig. 3.6(a), the magnetic voltage $U_{ml}$, produced by the current flow $I_1 N_1$ through the coil, which is:

$$U_{ml} = I_1 N_1 = (U_{mR} + U_{me}) = \Phi_R R_{mR} + \Phi_0 R_{m0} \quad (3.4)$$

is the force that pushes the flux round. $U_{ml}$ is to be divided into $U_{mR}$ and $U_{me}$ according to Fig. 3.6(a) and eqn. 3.4.

The resulting reluctance of the magnetic circuit (Fig. 3.6 (b)) becomes:

$$R_m = R_{mR} + \frac{1}{1/R_{m0} + 1/R_{m2} + 1/R_{m3}} \quad (3.5)$$

In Fig. 3.7, $R_m$ has been referred to the primary side; multiplied by $\frac{2 N_p^2}{N_1 N_s}$.
Fig. 3.5 Furnace Coil Circuit

Fig. 3.6 Equivalent magnetic circuit of the conducting crucible furnace
(a) individual reluctances
(b) resultant reluctance
The connection between $Z$ and $R_m$ is obtained from Faraday's law and Ohm's law for a magnetic circuit.

Voltage across $N_1Z = U_1 -$ voltage across $R_1$

$$2I_1N_1Z = U_1 - R_1I_1 = j\omega N_1\Phi_R$$

(3.6)

and

$$\Phi_R = \frac{I_1N_1}{R_m}$$

(3.7)

thus obtaining

$$Z = j\omega \frac{1}{R_m}$$

(3.8)

or with eqn. 3.5;

$$Z = \frac{j\omega}{R_{mR} + \frac{1}{\frac{1}{R_{m0}} + \frac{1}{R_{m2}} + \frac{1}{R_{m3}}}}$$

(3.9)

or

$$Z = \frac{1}{\frac{1}{j\omega R_{mR}} + \frac{1}{\frac{j\omega}{R_{m0}} + \frac{j\omega}{R_{m2}} + \frac{j\omega}{R_{m3}}}}$$

(3.10)

The unit of $\frac{\omega}{R_m}$ is ohm and the individual corresponding terms are:

$$X_R = \frac{\omega}{R_{mR}}$$

the reactance of the return path.
\( X_0 = \frac{\omega}{R_m0} \), the reactance of the refractory

and

\( Z_2 = \frac{j\omega}{R_m2} \), the complex resistance of the conducting crucible

\( Z_3 = \frac{j\omega}{R_m3} \), the complex resistance of the charge.

Equation 3.10 represents the equivalent circuit for a conducting crucible furnace shown in Fig. 3.8.

The magnetic voltages \( U_{me} \) and \( U_{mR} \) in Fig. 3.6 (a) correspond to currents \( I_e \) and \( I_R \) in Fig. 3.8.

\[
I_e N_1 = U_{me} - H_0 H_2 \quad (3.11)
\]

\[
I_R N_1 = U_{mR} \quad (3.12)
\]

The calculation of circuit parameters of Fig. 3.8 now follow. \( R_1 \), \( X_0 \) and \( X_R \) are first obtained in the following section. \( Z_2 \) and \( Z_3 \) are found using Poynting vector and complex power. All the furnace characteristics like current, power, power distribution, etc. can then be found from the equivalent circuit of Fig. 3.8.

3.3 Resistance \( X_R \), \( X_0 \) and \( R_1 \) of the Equivalent Circuit of the Conducting Crucible Furnace [3.5]

3.3.1 Reactance \( X_R \) of return flow path

The magnetic flux completes its path outside the coil for which an additional mmf is required. The reactance of this flow path is
Fig. 3.7 Equivalent impedance

Fig. 3.8 Equivalent circuit of a conducting crucible induction furnace
Expression in eqn. 3.13 is analogous to that of electrical resistance where \( \mu \) has replaced conductivity.

The reactance \( X_R \) of return flow path is therefore

\[
X_R = \omega \cdot \frac{1}{R_{mR}} = \omega \cdot \mu_0 \frac{\pi r_1^2}{H_1 - H_2 + 2(0.45 + k_r) r_1}
\]  \hspace{1cm} (3.14)

where

- \( r_1 \) = coil radius (m)
- \( H_1 \) = coil height (m)
- \( H_2 \) = crucible height (m)
- \( H_1 - H_2 + 2(0.45 + k_r) r_1 \) = effective length of the field lines outside the cylindrical coil.

The value of factor \( k_r \) is between 0 and 0.1 for the return flow through air [3.6].

### 3.3.2 Reactance \( X_0 \) of the refractory packing

From eqn. 3.14

\[
X_0 = \omega \frac{1}{R_{m0}} = \omega \cdot \mu_0 \frac{\pi (r_1^2 - r_2^2)}{H_2}
\]  \hspace{1cm} (3.15)
The effective length of field lines is $H_2$ and the flow cross section is the annulus area.

3.3.3 Resistance $R_1$ of the furnace coil [3.7]

The resistance of the furnace coil is

$$R_1 = \rho \frac{2\pi r}{k_1 H_1 \delta_1} N_1^2 \phi_1$$  \hspace{1cm} (3.16)

where

$$k_1 = \text{coil space factor} = \frac{N_1 h_1}{H_1}$$ \hspace{1cm} (3.17)

$h_1$ = axial thickness of coil turns

$$\delta_1 = \sqrt{\frac{2\rho_1}{\omega \mu_0 k_1}}$$ \hspace{1cm} (3.18)

and

$$\phi_1 = (1+j) \frac{\sinh (2t_1/\delta_1) - j \sin (2t_1/\delta_1)}{\cosh (2t_1/\delta_1) - \cos (2t_1/\delta_1)}$$ \hspace{1cm} (3.19)

where $t_1$ = radial thickness of the coil.

3.4 Impedance $Z_2$ and $Z_3$ of the Conducting Crucible and Charge

3.4.1 Poynting theorem [3.8]

Eddy currents involve ohmic losses. Poynting's theorem is the expression of the law of conservation of energy applied to electromagnetic fields. The Poynting vector is the vector product
which can be regarded as the instantaneous power density flow at a point. In terms of complex vectors, which are used when the vector components vary sinusoidaly with time, the complex Poynting vector

\[ \mathbf{\mathcal{S}} = \frac{1}{2} (\mathbf{E} \times \mathbf{H}^*) \]  

is the time average power density flow. \( \mathbf{H}^* \) is the complex conjugate of \( \mathbf{H} \) which is related to \( \mathbf{H} \) by eqn.3.22.

\[ \mathbf{H} = \text{Re} \left[ \mathbf{H} e^{j\omega t} \right] - \frac{1}{2} (\mathbf{H} e^{j\omega t} + \mathbf{H}^* e^{-j\omega t}) \]  

### 3.4.2 Complex power output and impedances \( Z_2 \) and \( Z_3 \)

Complex power is consumed by the \( Z_2 \) and \( Z_3 \) of Fig. 3.8 according to

\[ \mathcal{W}_2 = N_2^2 \cdot Z_2 \cdot I_e \cdot I_e^* \]  
\[ \mathcal{W}_3 = N_1 \cdot Z_3 \cdot I_e \cdot I_e^* \]  

The sum of the ohmic losses and the power absorbed by the magnetic field is given by the net inflow of power [3.8].

\[ W = -\int \mathcal{S} \, ds \]

which as explained for this case

\[ -\int \mathcal{S} \, ds \]
\[ -\frac{1}{2} \int (\mathbf{E} \times \mathbf{H}^*) \, ds \]
The surface integral is worked out by substituting for $\mathbf{E}$ and $\mathbf{H}^*$ from eqns. 3.1 and 3.2 and writing $2\pi r H$ for $ds$. The complex power consumed in the circular cylinder of a conducting crucible furnace with radius $r$ and height $H$ therefore becomes

$$W_r = -\rho \oint \mathbf{J}(r) \cdot \mathbf{H}^*(r) \cdot 2\pi r H$$

(3.25)

By inserting skin depth $\delta$ and rearranging:

$$W_r = -\rho \frac{2\pi r}{H \cdot \delta} \cdot H^2 \cdot \delta \cdot \frac{\mathbf{J}(r) \cdot \mathbf{H}^*(r)}{H \cdot \delta}$$

From eqn. 3.11

$$H = \frac{I_e \cdot N_1}{H_0}$$

and thus

$$W_r = -\rho \frac{2\pi r}{H \cdot \delta} \cdot \delta \cdot I_e \cdot I_e^* \cdot N_1 \cdot \frac{2}{H_0 \cdot H_0^*} \cdot \frac{\mathbf{J}(r) \cdot \mathbf{H}^*(r)}{H(r)} \cdot \frac{H(r)}{H_0 \cdot H_0^*}$$

Rearranging

$$W_r = -\rho \frac{2\pi r}{H \cdot \delta} \cdot \delta \cdot \frac{\mathbf{J}(r)}{H(r)} \cdot \frac{H(r)}{H_0} \cdot \frac{H^*(r)}{H_0^*} \cdot I_e \cdot I_e^* \cdot N_1^2$$

(3.26)

where,

$$\delta = \sqrt{\frac{2\rho}{\omega \mu}}$$
The corresponding general form of eqns. 3.23 and 3.24 is also

\[ 2 \dot{W}_r = N_1 \cdot Z_r \cdot I_e \cdot I_e^* \]  \hspace{1cm} (3.27)

From eqns. 3.26 and 3.27, the impedance \( Z_r \) of the circular cylinder becomes

\[ Z_r = -\rho \frac{2\pi r}{H \cdot \delta} \cdot J(r) \cdot \frac{H(r)}{H_0} \cdot \frac{H^*(r)}{H_0} \]  \hspace{1cm} (3.28)

For the charge, \( r = r_3, \delta = \delta_3, \rho = \rho_3, H = H_2 \) and \( \mu = \mu_3 \) and so impedance \( Z_3 \) from eqn. 3.28 becomes

\[ Z_3 = -\rho_3 \frac{2\pi r_3}{H_2 \cdot \delta_3} \cdot J_3(r_3) \cdot \frac{H_3(r_3)}{H_0} \cdot \frac{H_3^*(r_3)}{H_0} \]  \hspace{1cm} (3.29)

The \( J \) and \( H \) terms are obtained in the next section. For simplicity, some of the terms may be grouped together [3.4] as:

- the A.C. resistance of the charge

\[ R_3 = \rho_3 \cdot \frac{2\pi r_3}{H_2 \cdot \delta_3} \]  \hspace{1cm} (3.30)

- the dimensionless resistance factor

\[ \delta_3 = -\delta_3 \]  \hspace{1cm} (3.31)

- and the dimensionless screening factor [3.8].

\[ \phi_s \cdot \phi_s^* = \frac{H_3(r_3)}{H_0} \cdot \frac{H_3^*(r_3)}{H_0^*} \]  \hspace{1cm} (3.32)
The factors are obtained in terms of Bessel functions from the solution of diffusion equation as explained in the next section. The Bessel function expressions thus obtained are shown in Appendix A3.1; as used in the computer code.

For the conducting crucible and the charge, the combined impedance $Z_2 + Z_3$ from eqn. 3.28 becomes

$$Z_2 + Z_3 = -\rho_2 \cdot \frac{2\pi r_2}{H_2 \delta_2} \cdot \delta_2 \cdot \frac{J_2(r_2)}{H_0}$$  \hspace{1cm} (3.33)$$

where again, the A.C. resistance of the conducting crucible is

$$R_2 = \rho_2 \cdot \frac{2\pi r_2}{H_2 \delta_2}$$  \hspace{1cm} (3.34)$$

and the dimensionless resistance factor for charge and crucible is the

$$\phi_{23} = -\delta_2 \cdot \frac{J_2(r_2)}{H_0}$$  \hspace{1cm} (3.35)$$

The expression for $\phi_{23}$ in terms of Bessel functions is also included in Appendix A3.1.

3.4.3 Solution of field equations in the conducting crucible furnace

In order to evaluate $Z_3$ and $Z_2 + Z_3$ in eqns. 3.29 and 3.33, it is required to obtain $J(r)$ and $H(r)$ at boundaries $r_2$ and $r_3$ from solution of field equations.

For the field inside the conducting crucible furnace [3.8]
\[ \nabla^2 \mathbf{H} = \sigma \mu_0 \mu_r \frac{\partial \mathbf{H}}{\partial t} \]

\[ \frac{\mu}{\rho} \frac{\partial \mathbf{H}}{\partial t} \]

which for sinusoidally varying field can be written as

\[ \nabla^2 \mathbf{H} = \frac{\mu}{\rho} j \omega \mathbf{H} - \frac{\mu}{\rho} \mathbf{H} \]

The current density \( \mathbf{J}(r) \) at any point in the field is related to \( \mathbf{H} \) by

\[ \mathbf{J}(r) = \text{curl} \mathbf{H} \]

As mentioned in Section 3.2, the magnetic field is axial and varies only in the radial direction. Therefore the Maxwell's diffusion equation in circular-cylindrical coordinates, for radial direction is

\[ \nabla^2 \mathbf{H}(r) = \frac{d^2 \mathbf{H}(r)}{dr^2} + \frac{1}{r} \frac{d \mathbf{H}(r)}{dr} \]

and

\[ \text{curl} \mathbf{H} = - \frac{d \mathbf{H}(r)}{dr} \]

Combining eqn. 3.36 with 3.38 and 3.37 with 3.39, the field equations become

\[ \frac{d^2 \mathbf{H}(r)}{dr^2} + \frac{1}{r} \frac{d \mathbf{H}(r)}{dr} - j \omega \frac{\mu}{\rho} \mathbf{H}(r) = 0 \]
and

\[ J(r) = - \frac{dM(r)}{dr} \quad (3.41) \]

and the Ohm's law linking electric field strength to the current density expressed as:

\[ E = \rho \cdot J(r) \quad (3.42) \]

Eqn. 3.40 is a differential Bessel equation of first kind, the general solution of which (eqn. 3.47) is in terms of modified Bessel functions of cylindrical coordinates

\[ I_0 \left( \frac{r}{\delta} \right) \text{ and } K_0 \left( \frac{r}{\delta} \right) \]

Setting \( \sqrt{2} \cdot \frac{r}{\delta} = x \),

\[ I_0 \left( \frac{x}{\delta} \right) = \text{ber} x + j \text{bei} x \quad (3.43) \]

\[ K_0 \left( \frac{x}{\delta} \right) = \text{ker} x + j \text{kei} x \quad (3.44) \]

\[ I_0' \left( \frac{x}{\delta} \right) = \text{ber}' x + j \text{bei}' x \quad (3.45) \]

\[ K_0' \left( \frac{x}{\delta} \right) = \text{ker}' x + j \text{kei}' x \quad (3.46) \]

The Bessel functions used in this work are included in Appendix A3.2. The derivation and various mathematical tables are covered elsewhere [3.9, 3.10].

The general solution to eqn. 3.40 is

\[ M(r) = \Delta \cdot I_0 \left( \frac{r}{\delta} \right) + B \cdot K_0 \left( \frac{r}{\delta} \right) \quad (3.47) \]
which is applied to different regions of the conducting crucible furnace. In the non-conducting refractory \((r_2 < r < r_1)\), conduction is zero and thus \(J(r) = 0\) which means that field strength is constant.

\[
H_3(r) = H_0 = \frac{I_0 N_1}{H_2} \quad ((3.11))
\]

In the conducting crucible \((r_3 < r < r_2)\), the solution of eqn. 3.40 is

\[
H_2(r) = A_2 I_0 \left(\frac{r}{\delta_2}\right) + B_2 K_0 \left(\frac{r}{\delta_2}\right) \quad (3.48)
\]

and according to eqn 3.41.

\[
J_2(r) = \frac{\sqrt{2}}{\delta_2} \left[ A_2 I_0' \left(\frac{r}{\delta_2}\right) + B_2 K_0' \left(\frac{r}{\delta_2}\right) \right] \quad (3.49)
\]

In the charge \(3 (0 < r < r_3)\), the solution of eqn. 3.40 consists only of the \(I_0\) term because \(K_0 \to \infty\) as \(r \to 0\); necessitating \(B_2\) to be zero (Appendix A3.2). Therefore

\[
H_3(r) = A_3 I_0 \left(\frac{r}{\delta_3}\right) \quad (3.50)
\]

and

\[
J_3(r) = -\frac{\sqrt{2}}{\delta_3} A_3 I_0' \left(\frac{r}{\delta_3}\right) \quad (3.51)
\]
To find the coefficients $A_2$, $B_2$ and $A_3$, boundary conditions are applied. At $r = r_2$, $H_2 = H_0$. This is used in eqn. 3.48. At the common boundary of charge and crucible, i.e. at $r = r_3$, $H_3(r_3) = H_2(r_3)$. This is used to equate eqns. 3.50 and 3.48. Also at $r = r_3$, $E_3(r_3) = E_2(r_3)$ and so from eqn. 3.42, $\rho_3 J_3(r_3) = \rho_2 J_2(r_3)$. This is used to equate $\rho_3 \times \text{RHS of eqn 3.51}$ to $\rho_2 \times \text{RHS of eqn. 3.49}$. From these, the following equations are obtained:

\[
H_0 = A_2 I_0 \left( J_2 x_{22} \right) + B_2 K_0 \left( J_2 x_{22} \right) \tag{3.52}
\]

\[
\Delta_2 I_0 \left( J_2 x_{23} \right) + B_2 K_0 \left( J_2 x_{23} \right) = A_3 I_0 \left( J_3 x_{33} \right) \tag{3.53}
\]

\[
\frac{\rho_3}{\delta_3} \cdot A_3 I_0' \left( J_3 x_{33} \right) = \frac{\rho_2}{\delta_2} \left[ A_2 I_0' \left( J_2 x_{23} \right) + B_2 K_0' \left( J_2 x_{23} \right) \right] \tag{3.54}
\]

where:

\[
x_{22} = \sqrt{2} \frac{r_2}{\delta_2} \tag{3.55}
\]

\[
x_{23} = \sqrt{2} \frac{r_3}{\delta_2} \tag{3.56}
\]

\[
x_{33} = \sqrt{2} \frac{r_3}{\delta_3} \tag{3.57}
\]
Coefficients $A_2$, $B_2$ and $A_3$ are then obtained from the solution of eqns. 3.52 to 3.54:

$$A_2 = H_0 \frac{-K_0(J J x_{23}).I_0'(J J x_{33}).\lambda + K_0'(J J x_{23}).I_0(J J x_{33})}{\Delta}$$  \hspace{1cm} (3.58)

$$B_2 = H_0 \frac{-I_0'(J J x_{23}).I_0(J J x_{33}) + \lambda.I_0(J J x_{23}).I_0'(J J x_{33})}{\Delta}$$  \hspace{1cm} (3.59)

$$A_3 = H_0 \frac{I_0(J J x_{23}).K_0'(J J x_{23}) - I_0'(J J x_{23}).K_0(J J x_{23})}{\Delta}$$  \hspace{1cm} (3.60)

where

$$\Delta = I_0(J J x_{22}) \left[ K_0'(J J x_{23}).I_0(J J x_{33}) - K_0(J J x_{23}).\lambda.I_0'(J J x_{33}) \right]$$

$$+ K_0(J J x_{22}) \left[ I_0(J J x_{23}).\lambda.I_0'(J J x_{33}) - I_0(J J x_{33}).I_0'(J J x_{23}) \right]$$  \hspace{1cm} (3.61)

and

$$\lambda = \frac{\rho_3}{\delta_3} \cdot \frac{\delta_2}{\rho_2} = \sqrt{\frac{\rho_3.\mu_3}{\rho_2.\mu_2}}$$  \hspace{1cm} (3.62)

Having obtained the coefficients, they are substituted back into solutions. To obtain $Z_3$, coefficient $A_3$ is substituted into eqns. 3.50 and 3.51 to get $J_3(r)$ and $H_3(r)$ which are then used in eqn. 3.29. To obtain $Z_2 + Z_3$, coefficients $A_2$ and $B_2$ are substituted in eqn. 3.48 to get $H_2(r)$ which is used in eqn 3.33. The $H_0$ terms cancel out.
3.5 Electrical Values of the Conducting Crucible Furnace

3.5.1 Impedance of the furnace circuit

Referring to Fig. 3.8, the furnace impedance is

\[ Z = \frac{B_1}{N_1^2} + \frac{j X_R (j X_0 + Z_2 + Z_3)}{j X_R + j X_0 + Z_2 + Z_3} \]

\[ = \frac{B_1}{N_1^2} + \frac{j X_R (j X_0 + Z_2 + Z_3)}{j (X_R + X_0 + \text{Im}(Z_2 + Z_3)) + \text{Re}(Z_2 + Z_3)} \]  \hspace{1cm} (3.63)

It is assumed that

\[ \text{Re}(Z_2 + Z_3) \ll X_R + X_0 + \text{Im}(Z_2 + Z_3) \]

Therefore

\[ Z = \frac{B_1}{N_1^2} + \frac{X_R (j X_0 + Z_2 + Z_3)}{X_R + X_0 + \text{Im}(Z_2 + Z_3)} \]

\[ = \frac{B_1}{N_1^2} + k \left[ j(X_0 + \text{Im}(Z_2 + Z_3)) + \text{Re}(Z_2 + Z_3) \right] \]

\[ = \frac{B_1}{N_1^2} + k \cdot \text{Re}(Z_2 + Z_3) + j k \left[ X_0 + \text{Im}(Z_2 + Z_3) \right] \]  \hspace{1cm} (3.64)

where

\[ k = \frac{X_R}{X_R + X_0 + \text{Im}(Z_2 + Z_3)} \]  \hspace{1cm} (3.65)

is called the coupling factor.
3.5.2 Currents, Power, Efficiency and Power Factor of the conducting crucible furnace

Currents $I_1$ and $I_e$ (Fig. 3.8) are:

$$I_1 = \frac{U_1}{N_1^2 \cdot |Z|} \quad (3.66)$$

$$I_e = \frac{U_1(1-B_1/N_1^2 \cdot Z)}{N_1^2 \cdot (jX_0+Z_2+Z_3)} \quad (3.67)$$

Power input into the furnace is therefore:

$$W = P_1 + jQ_1 = Z \cdot N_1^2 \cdot I_1^2 \quad (3.68)$$

where

$$P_1 = N_1^2 \cdot I_1^2 \cdot \text{Re}(Z)$$

$$Q_1 = N_1^2 \cdot I_1^2 \cdot \text{Im}(Z)$$

Substituting for $Z$ from eqn. 3.64,

$$W = N_1^2 \cdot I_1^2 \left\{ R_1/N_1^2 + k \cdot \text{Re}(Z_2+Z_3) + jk[X_0 + \text{Im}(Z_2+Z_3)] \right\}$$

$$= N_1^2 \cdot I_1^2 \left\{ \text{Re}(R_1/N_1^2) + k \cdot \text{Re}(Z_2+Z_3) \\
+ j[\text{Im}(R_1/N_1^2) + k(X_0 + \text{Im}(Z_2+Z_3))] \right\} \quad (3.69)$$

The electrical efficiency of the conducting crucible furnace is

$$\eta = \frac{\text{Power in (charge + crucible)}}{\text{Power in (charge + crucible + coil)}}$$

which obtained from eqn. 3.69 is
\[ \eta = \frac{\text{resistance of (charge + crucible)}}{\text{resistance of (charge + crucible + coil)}} \cdot \frac{I_1 N_1^2}{I_1 N_1^2} \]

\[ \eta = \frac{k \cdot \text{Re}(Z_2 + Z_3)}{\text{Re}(\text{Re}(R_{1/N_1^2}) + k \cdot \text{Re}(Z_2 + Z_3))} \]  

The power factor

\[ \cos \phi_1 = \frac{2}{\text{Re}(\text{Re}(R_{1/N_1}) + k \cdot \text{Re}(Z_2 + Z_3))} \]

The number of furnace coil turns is obtained from eqns. 3.66 and 3.68 by equating \( I_1^2 \):

\[ N_1 = \frac{U_1}{|Z|} \cdot \sqrt{\frac{\text{Re}(Z)}{\text{Re}(W)}} \]  

3.5.3 **Distribution of power between charge and conducting crucible**

Ratio of power in charge to power in charge and crucible is an important characteristic of conducting crucible furnace. Too much power produced in the crucible makes the operation uneconomical and results in poor stirring in the bath. From eqns. 3.27, 3.29 and 3.33, the power produced in the charge and the power produced in the charge and crucible are:
The ratio is therefore

\[
\frac{\text{Re} \, H_3}{\text{Re}(H_2 + H_3)} = \frac{R_3}{R_2} \cdot \frac{\Psi_s \cdot \Psi_s^* \cdot \text{Re} \, \Phi_3}{\text{Re} \, \Phi_23}\]  

(3.75)

### 3.5.4 Pressure and meniscus in the charge

The magnetic force alone is responsible for the pressure in the charge in an induction furnace [3.4].

The average pressure \( p_3 \) in the charge is

\[
\int_{r_3}^{R} d \rho_3 - \mu_3 \cdot \int_{r_3}^{R} \text{Re} \, J_3 \cdot H_3 \, d \rho
\]

and from eqns. 3.50 and 3.51.

\[
H_3(r) = H_3(r_3) \cdot \frac{I_0(\sqrt{2}j r/\delta_3)}{I_0(\sqrt{2}j r_3/\delta_3)}
\]

(3.77)

and

\[
J_3(r) = J_3(r_3) \cdot \frac{\sqrt{2}j}{\delta_3} \cdot \frac{I_0'(/2j r/\delta_3)}{I_0(/2j r_3/\delta_3)}
\]

(3.78)

Inserting eqns. 3.77 and 3.78 into 3.76, pressure \( p_3 \) is obtained.
The maximum pressure is at the crucible centre.

\[
p_3(0) = \frac{1}{2} \mu_3 H_3(r_3) H_3^*(r_3) \left[ 1 - \frac{1}{\text{ber}^2 x_3 + \text{bei}^2 x_3} \right]
\]

(3.80)

At the crucible edge \((r = r_3)\), pressure \(p_3(r_3) = 0\).

Introducing power input \(\mathcal{W}_3\) from eqns. 3.25 and 3.29 into eqn. 3.80,

\[
\mathcal{W}_3 = \frac{\rho_3^3}{\delta_3} \cdot 2\pi r_3 H_2 H_3(r_3) H_3^*(r_3).
\]

(3.81)

The maximum bath pressure is therefore obtained by eliminating \(H_3(r_3) H_3^*(r_3)\) between eqns 3.80 and 3.81.

\[
p_3(0) = \frac{\mu_3}{2\rho_3} \cdot \frac{\mathcal{W}_3}{2\pi r_3 H_2} \cdot \text{Re} \left\{ \frac{H_3(r_3)}{J_3(r_3)} \right\} \left[ 1 - \frac{1}{\text{ber}^2 x_3 + \text{bei}^2 x_3} \right]
\]

(3.82)

Using eqns. 3.50 and 3.51 the meniscus height is obtained from maximum pressure.
where $\gamma_3$ is the charge specific gravity ($N m^{-3}$).

### 3.6 The implementation of the Theory

To establish effect of different parameters, the theory was applied to conducting crucibles of clay bonded graphite and carbon bonded silicon carbide.

The computer code written for use of the theory is included in Appendix A3.1. The flow chart is shown in Fig. 3.9.

The results obtained are based on data given by the crucible manufacturers and those of existing furnace installation. The crucible resistivities at different temperatures were taken from Figs. 4.19 and 4.20.

Applying the analysis to the design of a clay-graphite crucible induction furnace with aluminium or copper as the charge, gave sensible results in line with know industrial-furnace installations. The results of the analysis are given in Table 3.1.

Pure copper and pure aluminium are not widely melted in furnaces of this type. Most foundries using tilting or push-up furnaces melt the
Table 3.1: Electrical operating characteristics of furnace installation

<table>
<thead>
<tr>
<th>Charge/crucible</th>
<th>f, ( f ), Hz</th>
<th>V, ( V ), V</th>
<th>I, ( I ), A</th>
<th>Q, ( Q ), kVAR</th>
<th>( \frac{W_3}{W_2+W_3} ), N</th>
<th>h, ( h ), mm</th>
<th>( \eta ), ( \eta ), %</th>
<th>W, ( W ), kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium/clay-graphite</td>
<td>3000</td>
<td>800</td>
<td>2591</td>
<td>1920</td>
<td>12</td>
<td>0.82</td>
<td>132</td>
<td>67</td>
</tr>
<tr>
<td>Copper/clay - graphite</td>
<td>3000</td>
<td>800</td>
<td>2364</td>
<td>1845</td>
<td>12</td>
<td>0.82</td>
<td>37</td>
<td>64.8</td>
</tr>
</tbody>
</table>

\( f \) = operating frequency, Hz

\( V \) = work coil voltage, V

\( I \) = work coil current, A

\( Q \) = reactive power requirements, kVAR

\( \frac{W_3}{W_2+W_3} \) = ratio of power in charge to power in crucible and charge

\( h \) = meniscus height, mm

\( \eta \) = overall efficiency

\( W \) = furnace input power, kW

\( W_3 \) = power induced in charge, W

\( W_2 \) = power induced in crucible, W
various BS1400 alloys of copper, none of which have conductivities as good as pure copper and many have conductivities closer to that of aluminium. In the absence of adequate data on their properties, it is necessary to choose pure metals which are comparable. The analysis assumed an A60 size crucible full of metal, a 100 kW furnace operating frequency of 3 kHz and a crucible temperature of approximately that of the molten metal.

3.6.1 Distribution of Power between charge and crucible

Variations of the power distribution between charge and crucible with frequency, crucible thickness and crucible resistivity are shown in Figs 3.10 - 3.13.

In each Figure, the nominal values relate to the existing furnace installation with a total power capability of 100 kW, operating frequency 3 kHz and a clay-graphite or silicon carbide crucible of A60 size. As expected, a decrease in frequency decreases the power input directly to the crucible under all circumstances investigated. However, the relationship was more pronounced for carbon-bonded silicon carbide crucibles with an aluminium rather than a copper charge. Using clay-graphite crucibles with either metal charge and the existing furnace installation approximately 20% of the furnace power is generated within the crucible itself. To obtain similar results using carbon-bonded silicon carbide crucibles, a reduction in operating frequency to approximately 2 kHz with a copper charge is suggested whereas with an aluminium charge a frequency reduction to well below 1 kHz would be required.

The distribution of furnace power with crucible resistivity at an operating
Fig. 3.10 Variation of power distribution between charge and crucible with the operating frequency, for clay-graphite and silicon carbide crucibles
Nominal value refers to 3 kHz operating frequency
Fig. 3.11 Variation of power distribution between charge and crucible with the crucible wall thickness for clay-graphite and silicon carbide crucibles
Nominal value refers to size A60 crucible
Fig. 3.12 Variation of power distribution between charge and crucible with the crucible resistivity at the molten charge (aluminium) temperature.
Fig. 3.13 Variation of power distribution between charge and crucible with the crucible resistivity at the molten charge (copper) temperature

Crucible Resistivity $\rho_z (\Omega \cdot m)$

@ 1200°C

Distribution of Furnace Power $\frac{W_3}{W_2 + W_3}$

S.C

C.G.
frequency of 3 kHz is shown in Figs. 3.12 and 3.13. Assuming crucible resistivities at the temperature of the molten metal, the power generated within a silicon carbide crucible with aluminium charge is approximately twice that within an equivalent clay-graphite crucible. More importantly the rate of change of power distribution is over four times greater when silicon carbide crucibles are used, and hence the production tolerance for a successful life is substantially reduced.

Using the work coil and metal charge diameter associated with the furnace installation, the effect of changing the crucible thickness, and hence the airgap/refractory thickness, was evaluated and the results shown in Fig (3.11). For a clay-graphite crucible approximately 20% of the furnace power for either an aluminium or copper charge is generated within the crucible itself at the nominal frequency, resistivity and crucible thickness etc. In comparison, 33% of the furnace power would be generated directly within a silicon carbide crucible with a copper charge, and as much as 62% with an aluminium charge. To reduce the loss within the silicon carbide crucibles to approximately that of clay-graphite, the thickness would have to be reduced by just over 5 mm for a copper charge, which may be practicable, but for an aluminium charge a reduction of approximately 50% would be required.

3.6.2 Meniscus height

A basic phenomenon associated with the melting of metals in coreless induction furnaces is the stirring forces produced and the resultant turbulence within the molten metal. Although a vigorous stirring action can be beneficial when melting iron or steel, it is usually undesirable in nonferrous applications. The turbulence destroys the protective surface
Fig. 3.14 Variation of meniscus height in the charge with surface power density of crucible and charge for 3 kHz induction furnace

Nominal value refers to size A60 crucible, 100 kW furnace and 3 kHz frequency
layer of flux used to prevent oxidization or hydrogen pick-up.

The magnitude of the stirring action is directly proportional to the power dissipated within the metal itself and inversely proportional to the operating frequency [9]. Having calculated the power generated within the charge using the equivalent circuit technique described in Section 3.6.2 the height of the meniscus formed h, which is related to the magnitude of the stirring forces, can be found from eqn. 3.83. Variation of the height of the meniscus with furnace power density, frequency, crucible resistivity and thickness are shown in Figs. 3.14 - 3.18.

The meniscus height is larger within an aluminium charge rather than a copper charge and when clay-graphite crucibles rather than silicon-carbide crucibles are used for a particular operating frequency, furnace power input and crucible geometry.

The height of the meniscus varies linearly with the furnace power density (see Fig. 3.14) which is in agreement with previous work on induction coreless furnaces [3.11]. It should be noted that in this case the furnace power input has been kept constant and the meniscus height is not directly related to the inverse of the frequency, as in the case of induction furnaces with nonconducting crucibles or refractories [3.11]. Any variation in the frequency will change the distribution of furnace power between the charge and crucible, and hence the magnitude of the power input to the metal, which, of course is itself directly related to the meniscus height.

The higher conductivity of carbon bonded silicon-carbide crucibles means that
Fig. 3.15 Variation of meniscus height in the charge with the operating frequency for a 100 kW furnace using clay graphite and silicon carbide crucibles.

Nominal value refers to 3 kHz frequency.
Fig. 3.16 Variation of meniscus height in the charge with the crucible resistivity at the molten charge temperature.
Fig. 3.17 Variation of meniscus height in the charge with the crucible resistivity at the molten charge temperature
Fig. 3.18 Variation of meniscus height in the charge with the crucible wall thickness for a 100 kW, 3 kHz induction furnace.
the electrical energy is shared between crucible and charge. (Figs 3.12 and 3.13). For a very small change in the resistivity of the crucible, the ratio of power in the charge to total power is changed drastically. This is more pronounced for carbon-bonded silicon carbide crucibles and with aluminium rather than a copper charge. It is therefore required to be able to measure the resistivity of these crucibles.

Although the composition of the silicon carbide mix is said to be the same, crucible lifetime varies from crucible and from batch to batch. The variation in lifetime must be associated with the crucible resistivity whose measurement is vital. Unfortunately, direct resistance measurement is not possible as the glaze would have to be removed to make proper contact with the crucible material. The development of a nondestructive method of measurement of crucible resistivity was therefore of paramount importance.

3.7 Conclusion
The basic theory has been explained and implemented to show the important parameters and their effect on furnace performance. It will be reapplied in later chapters to estimate the effect of variation of these parameters in a given crucible and to explain thermal stresses.

REFERENCES
3.4 REICHERT, K.: "The calculation of coreles furnaces with electrically


3.6 SIEGERT H.: "Inductive heating", Techn. Rundschau, 1961, Nos. 13 and 38


3.9 DWIGHT, H.B.: "Bessel functions for A-C problems", AIEE Trans., 1929, pp. 812-820

3.10 McLACHLAN, N.W.: 'Bessel function for engineers' (Oxford University Press, 1934)

MEASUREMENTS AND ESTIMATION OF CRUCIBLE MATERIAL PROPERTIES

Probes are used to measure the current density and magnetic field strength on the surface of carbon-silicon carbide and clay-graphite crucibles. From the results, electrical resistivities at different axial parts of the crucibles are estimated. Surfaces of crucible segments are examined under microscope. Thermal properties of crucible materials are estimated at various temperatures.
4 MEASUREMENTS AND ESTIMATION OF CRUCIBLE MATERIALS PROPERTIES

4.1 Introduction

Reports of crucible failures indicate localized heating patterns in preferred regions of some crucibles more than others. This suggests that crucible properties are not uniform. The importance of electrical resistivity in determining power distribution, has been pointed out in the previous chapter. A simple non-destructive method was therefore sought to measure the resistivities at different heights. Electrical resistivity of crucibles is important in the azimuthal direction because of the flow of induced currents.

Theory of current density and magnetic field strength probes is explained and shown to be applicable. The probes are used to measure the current density and magnetic field strength on the surface of crucibles. From the results, resistivities are estimated. Finally, samples of crucible surface are prepared and examined under the microscope to see the surface structure.

Although the composition of the carbon-silicon carbide mix is said to be uniform, crucible lifetime varies from crucible to crucible and from batch to batch. The variation in lifetime must be associated with the crucible resistivity whose measurement is not possible as the glaze would have to be removed to make proper contact with the crucible material. This would result in the mixture falling apart by flaking. The development of a non-destructive method of measurement was therefore important.
4.2 Measurement of Resistivity by Probes

A practical method was devised to measure the electrical resistivity of the crucibles at various axial positions by using current density and magnetic field strength H-probes. Tests were carried out at 50°C and all instruments were shielded against the effect of the high electromagnetic fields.

4.2.1 Current density probe [4.1]

These transducers, shown in Fig. 4.1, were attached to both the inside and outside of the crucible wall [4.2]. The probe consisted of a length of thin insulated wire held firmly against the surface of the crucible wall. The wire used was 0.3 mm diameter enamelled copper. The ends of the wire were twisted together so as to experience the same electromagnetic field. The output from the probe was measured using a high-impedance voltmeter. From the voltage reading the ρJ product was obtained for the inside and outside of the crucible wall according to eqn. 4.1.

\[ E = \frac{V_J}{l_J} = \rho J \]  

(4.1)

where

\[ \rho = \text{crucible resistivity (Ωm)} \]
\[ J = \text{surface current density (A m}^{-2}) \]
\[ V = \text{voltage across probe terminals (V)} \]
\[ l_J = \text{length of probe in contact with the surface (m)} \]
Fig. 4.1 Filament-type current density probe

Fig. 4.2 Magnetic-field-strength probe
The proof on theory of current density probes leading to eqn. 4.1 is included in Appendix A4.1. It also contains the theory of H-probes.

4.2.2 Magnetic field-strength probe

The magnetic field-strength probe was essentially a search coil and consisted of a length of wire (0.3 mm diameter) wrapped closely round a thin strip of insulating material of 2 mm thickness shown in Fig. 4.2. The air-gap between the outside surface of the crucible and the inside surface of the work coil varied along the length of the crucible, because of its taper; from 8 mm at the top to 20 mm at the bottom. Probes were positioned and held firmly on the inside and outside surfaces of the crucible wall. The two ends of the wire were then twisted and voltage across them measured. The voltage across the probe terminals is

\[
V_H = N_H \cdot A \cdot \mu_r \cdot \mu_0 \cdot 2\pi f \cdot H \quad (4.2)
\]

where \( A \) is the area of the insulation

\[
A = 2\pi r_0 t
\]

Hence the magnitude of the magnetic field strength is

\[
H = \frac{V_H}{N_H \cdot 2\pi r_0 t \cdot 2\pi f \cdot \mu_r \cdot \mu_0} \quad (4.3)
\]

where
$H$ = magnitude of magnetic field (A m$^{-1}$)
$V_H$ = voltage reading from the probe (V)
$N_H$ = number of turns of wire round the insulating strip
$r_o$ = outer radius of the crucible (m)
$t$ = radial thickness of the insulation (m)
$\mu_r$ = relative permeability of the insulation
$\mu_o$ = permeability of free space (H m$^{-1}$)

The $H$-probe results showed that different parts of the crucibles experience different values of magnetic field strength. Variation of magnetic field strength along the length of crucibles is shown in Fig. 4.3.

Values of magnetic field strength measured by the $H$-probes agreed within 10% with theoretical values calculated [4.3] for the coil, at the same radius and heights. (Appendix A4.2)

The problem is associated mainly with A60 crucibles which are recommended by manufacturers as being typical middle-sized crucibles. The laboratory-scale installation available at the time used A8 crucibles, and it was on these that the probe measurements were carried out. Current density and magnetic field-strength probes attached to the crucible, and the experimental set-up are shown in Fig. 4.4.

4.3 Estimation of Crucible Resistivity

Variation of current density and magnetic field strength from outside to the inside of the crucible walls are shown in Figs. 4.5 - 4.8. These are
Fig. 4.3 Axial variation of magnetic field strength about the length of the crucible
Fig. 4.4. Probes attached to the crucible and the experimental set-up.
theoretical curves. The current densities are in fact derivatives of magnetic field strengths across the crucible wall and are obtained by using eqn. (4.5). The field-strength curves were produced by applying the analysis to the two crucibles and obtaining variation over their thickness according to Bessel function solutions given in Apps. A3.1 and A3.2. Typical midrange resistivities were assumed for each crucible material. The ratios of the outer to inner values of magnetic field strength approach unity with higher crucible resistivities.

Figs. 4.5 and 4.6 show that for any given coil current, the variation of current density is approximately linear and has small gradients in both carbon-silicon carbide and clay-graphite crucibles of the size used. Thus, assuming that the resistivity of the crucible does not change radially, an average value of $\rho J$ can be obtained from the inside and outside values of $\rho J$, as shown in eqn. 4.4.

\[
(\rho J) = \frac{1}{2} \left[ (\rho J)_i + (\rho J)_o \right]
\]  

(4.4.)

where the i and o suffixes refer to inside and outside.

Similarly, Figs. 4.7 and 4.8 show that the drops of the magnetic field strength for carbon-silicon carbide and clay-graphite crucibles have small gradients. These magnetic field-strength curves are also assumed to be linear. From the solution of the diffusion equation in cylindrical form,

\[
J = - \frac{\partial H}{\partial r}
\]  

(4.5)
Fig. 4.5 Theoretical variation of current density across the wall of the silicon carbide crucible
Fig. 4.6 Theoretical variation of current density across the wall of the clay-graphite crucible
Fig. 4.7 Theoretical variation of magnetic field strength across the wall of the silicon carbide crucible
Fig. 4.8 Theoretical variation of magnetic field strength across the wall of the clay-graphite crucible.
where the RHS represents radial variation of \( H \). As \( H \) is assumed to vary linearly,

\[
J = \frac{H_0 - H_i}{r_o - r_i}
\]  

(4.6)

where \( r_i \) = inner radius of the crucible (m)

\( r_o \) = outer radius of the crucible (m)

Combining eqns. 4.4 and 4.6,

\[
\rho = \left( \frac{\rho J}{J} \right)
\]

\[
\rho = \frac{1}{2} \left[ (\rho J)_i + (\rho J)_o \right]
\]

\[
\frac{H_0 - H_i}{r_o - r_i}
\]

(4.7)

To verify the method, an aluminium tube of known resistivity and of similar thickness/diameter and diameter/skin depth ratios was tested and the resistivity obtained agreed with the actual value to within 10%.

For the two types of crucibles, probes were attached to three axially equidistant positions inside and outside so that any axial variations of resistivity could be obtained. The values obtained from top to bottom are shown in Table 4.1. The results show that the resistivity of the crucibles vary axially. Hence, power density which is a function of \( \rho J^2 \), is expected to vary axially also.
Table 4.1: Probe results for crucibles

<table>
<thead>
<tr>
<th>Position</th>
<th>Measured resistivities at 50°C $\times 10^{-8} \Omega \text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Silicon Carbide</td>
</tr>
<tr>
<td>Top</td>
<td>4600</td>
</tr>
<tr>
<td>Middle</td>
<td>2150</td>
</tr>
<tr>
<td>Bottom</td>
<td>5300</td>
</tr>
</tbody>
</table>

4.4 Microscopic Examination of C/SiC Crucible Material

Tar is the bonding matrix in silicon carbide crucibles used in this work. It is supposed to be the dominant low resistivity constituent in determining overall resistivity of the crucible. This means that the C/SiC crucibles are expected to have uniform resistivity.

However, probe measurements have shown otherwise. So it was decided to examine the surfaces of different sections of these crucibles under microscope. This was done in 4 stages:

(1) Cutting the crucible, (2) Mounting the specimens, (3) Preparation for microscope, (4) Examination and results.

(1) Cutting the crucible: Induced eddy currents flow azimuthally in the wall of conducting crucibles. Thus, the path of the current must be viewed from a direction at right angles to a radial plane, that is looking down onto a ring or part of a ring, sliced out of the wall.

Fig. 4.9 shows how this was done. A hacksaw was used to slice out a
Fig. 4.9 'U'-shape section cut out of the crucible in the axial plane
U-shaped piece of the crucible along its axial plane. One side of the U was marked out in three parts, top, middle and bottom and then cut into nine smaller pieces (Fig. 4.10).

Fig. 4.11 shows the small piece of the material used as the specimen. The thickness is that of the crucible wall. Its height and depth were reduced suitably for mounting. The path of the current is shown by the arrow I. The path was therefore viewed from top.

(2) Mounting the specimen: Sections 2, 5 and 8 were mounted with their face A uppermost and horizontal. The mounting was done as follows:

(a) Place a piece of cardboard on the desk.
(b) Place a numbered metal ring holder on the cardboard.
(c) Secure the ring in place by putting plasticene around it.
(d) Prepare the resin mixture. This was in the ratio 10 parts Araldite MY750 to one part Hardener HY951. The mixture was then poured into the ring. A total of 2.2 gm was used.
(e) The specimen was then placed inside the ring in a manner described above.
(f) Before letting the resin to harden up completely, it was necessary to evacuate the air by a small vacuum pump. The specimen was therefore placed inside a glass jar. The outlet was connected to a vacuum pump by a rubber tube. Air was pumped out for 30 seconds and released for 10 second intervals, a few times. Then the pump was switched off. The specimens, being porous refractory materials, needed more time because of the porosity.
FIG. 4.10 Division of wall section in separate parts

FIG. 4.11 Specimen before mounting.
After this, they were placed on top of an electric furnace to receive gentle heat and set gradually without undue expansion. They were ready for the next stage 24 hours later.

(3) Preparation for microscope: When specimens were firmly held by the resin inside the ring, it was necessary to produce a flat surface for viewing.

Parts of the specimen were still covered with the resin which would show as a greyish area under the microscope. But, the extra resin was removed by rough grinding on a linisher. Then, specimens were ground in stages from rough emery paper grade 240 to 600, and to fine paper grade 1200. This was done manually on strips of paper wetted by water flowing down them. Between each stage of grinding, specimens were washed and dried.

Unfortunately, the grinding operation was hindered by the fact that the material was neither soft and ductile, nor hard and brittle but a mixture of both. The carbon matrix was a powdery and very soft material that would grind away readily. The carbide particles on the other hand, were hard and difficult to grind. As a result of this, carbon matrix surrounding the carbide particles would flake off by grinding, leaving the carbides to simply fall off.

The final surface would thus be full of scratches. One way to overcome this was to grind them only on the very fine paper. But this would take ages. So, the specimens were transferred onto a
mechanical rotary polisher where the wheel rotated the specimens by friction.

The wheel was padded and diamond paste of 6 micron (1200 emery) was spread on it. By frequent lubrication, the specimens were polished as best as possible. Ideally, they should have been done on a vibratory polisher. But, neither time nor enough units were available. The advantage of the latter is that the specimens could be left overnight.

(4) Examination and results: The polished specimens were washed with alcohol and wiped with cotton wool soaked in alcohol to remove any drying stains. Ordinary microscope examination would not produce clear results. They were therefore examined under scanning electron microscope (SEM) at different magnifications and modes.

This sophisticated machine would first remove the air out of the specimens and the chamber inside which they were held. It would then scan the surface of the specimen at different magnifications. The view was seen on the TV screen and photographed for record. This was done for each specimen so that the corresponding photographs could be compared.

The negatives obtained were printed on 100 mm square papers and the photomicrographs are shown in Figs. 4.12 - 4.14.

Fig. 4.15 shows a schematic drawing of the material's structure with
Fig. 4.12 SEM photomicrograph of the bottom section of the carbon silicon carbide crucible wall (magnification x40)
Fig. 4.13 SEM photomicrograph of the middle section of the carbon silicon carbide crucible wall (magnification x40)
Fig. 4.14  SEM photomicrograph of the top section of the carbon silicon carbide crucible wall (magnification x40)
low resistivity carbon matrix and higher resistivity SiC particles embedded in the matrix. For a current flowing from left to right, the carbide particles act as resistances shown in the diagram as R1, R2, R3, etc.

To obtain the equivalent of Fig. 4.15 under the microscope, the specimens were mapped. This meant that the surfaces were charged. Since carbide particles charge up to a different degree compared with the carbon matrix, by virtue of different atomic structure, the resulting view showed the carbide quite distinct from the carbon. The negatives obtained were printed and are shown in Figs. 4.16 - 4.18.

The probe measurements earlier gave resistivities for the crucible sections as shown in Table 4.1. The microscopic examination results have shown that there is a difference in the amount of silicon carbide from section to section. The silicon mapping showed a variation in silicon carbide content in line with the measurement results given in Table 4.1.

Variation of electrical resistivity with temperature for both types of crucible are shown in Figs. 4.19 and 4.20. The curves were used for reading crucible resistivities at different temperatures which were required in the calculations. The measured resistivities at 50°C for different sections of the crucibles were combined with the nominal curves of Figs. 4.19 and 4.20 to obtain variations of resistivity with temperature for each section; following the same trend shown in Figs.
Carbon matrix

SiC Particle

Low resistivity C
High resistivity SiC \{ C/SiC

ΣR High

FIG. 4.15 Schematic diagram of material's structure
Fig. 4.16 Silicon map of the bottom section of the crucible wall
Fig. 4.17 Silicon map of the middle section of the crucible wall
Fig. 4.18 Silicon map of the top section of the crucible wall
FIG. 4.19 VARIATION OF ELECTRICAL RESISTIVITY WITH TEMPERATURE FOR CARBON BONDED SILICON CARBIDE CRUCIBLE MATERIAL (MORGANITE THERMAL DESIGNS LTD.)
FIG. 4.20  VARIATION OF ELECTRICAL RESISTIVITY WITH TEMPERATURE FOR CLAY BONDED GRAPHITE CRUCIBLE MATERIAL. (MORGANITE THERMAL DESIGNS LTD)
4.19 and 4.20. Skin depths calculated at various temperatures for both crucible types are shown in Appendix A4.3.

4.5 Estimation of Thermal Properties of Crucible Materials at Various Temperatures

The crucible manufacturers were not in a position to provide some of the data required for the calculations in the remaining chapters. These data had to be put together independently and used in conjunction with what information was available.

Based on the brief chemical analysis given, additional data were collected to estimate crucible properties and their variation with temperature through the melt cycle. Where exact information was lacking, properties of similar materials have been used.

The crucibles are manufactured in a variety of mixes depending on the application. They are referred to by their bonding matrix and secondary phase. For the purpose of this work, two mixes have been chosen: carbon bonded silicon carbide (C/SiC) and clay bonded graphite (clay/C). The particular mixes considered are commercially known as "Suprex" ware and "Super" ware respectively. "Suprex" are carbon bonded silicon carbide crucibles containing mineral flake graphite and silicon carbide plus small quantities of other refractory materials. "Super" crucibles are made from mineral flake graphite and refractory materials, with a clay bond [4.4].

The main constituents in both mixes are: carbon, silicon carbide, silica
and alumina. The chemical analyses [4.4] are typically as follows:

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>PERCENTAGE OF COMPONENTS IN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>carbon/silicon carbide</td>
</tr>
<tr>
<td>carbon</td>
<td>35</td>
</tr>
<tr>
<td>silicon carbide</td>
<td>35</td>
</tr>
<tr>
<td>silica</td>
<td>18</td>
</tr>
<tr>
<td>alumina</td>
<td>4</td>
</tr>
<tr>
<td>iron oxide ($Fe_2O_3$)</td>
<td>3</td>
</tr>
<tr>
<td>alkalis</td>
<td>-</td>
</tr>
<tr>
<td>alkalis and other metal oxides</td>
<td>balance</td>
</tr>
</tbody>
</table>

4.5.1 Specific heat

This is dependent on the microstructure and may be estimated by proportional addition [4.5, 4.6, 4.7] of specific heats using known fractions of components. Variation with temperature of specific heat $C_p$ of the components is shown in Figs. 4.21 to 4.23.

The silica containing fraction of the materials is combined with the alumina as a clay-based bond, and the specific heat characteristics of fired clay are very similar to that of alumina, except near room temperature [4.8]. So values of $C_p$ of either silica (Fig. 4.21) or of alumina (Fig. 4.22), could be used. In the calculations, however, values corresponding to that of silica (Fig. 4.21) were used.

The iron content is somewhat more difficult to assess because it will
FIG. 4.21  VARIATION OF SPECIFIC HEAT OF CARBON AND SILICA WITH TEMPERATURE
FIG. 4.22  VARIATION OF SPECIFIC HEAT OF THE COMPONENTS: SILICON CARBIDE AND ALUMINA WITH TEMPERATURE [Ref 4.9].
Fig. 4.23 VARIATION OF SPECIFIC HEAT OF IRON OXIDES WITH TEMPERATURE.
depend on its valency state, as illustrated in Fig. 4.23. If as the manufacturers data suggest, it is in the form of Fe$_2$O$_3$ (curve 1), this will have a slightly higher specific heat than that in the form of FeO (curve 2). But since the crucibles are likely to have been fired under reducing conditions, FeO is more likely while Fe$_2$O$_3$ is normally quoted in the analyses. The curves in Fig. 4.23 have been prepared on the basis that FeO is likely to reside in a glassy or crystalline state combined with silica [4.8]. The curves also show the specific heat of fayalite (2 FeO. SiO$_2$) which contains 70.51% wt FeO (curve 3). On this basis the values for FeO content in the combined state (curve 4) can be compared with those in the free state (curve 2). The two are quite similar (within 10%) and either one may be used. In the calculations, however, values corresponding to curve 2 [4.9] were actually used.

The constituents covered in Figs. 4.21 to 4.23 make up only 95% and 99% of carbon/silicon carbide and clay/graphite mixes respectively. Due to the uncertainty in the type of alkalis and metal oxides, the compositions were considered adequate for the purposes required. The $C_p$ value for each component was multiplied by the fraction of that component in the material. The sum of these products gave the bulk specific heats. As an example, for C/SiC at 500°C, $C_p$ was obtained by multiplying 0.35 by 1590 for carbon; 0.35 by 1120 for silicon carbide, and so on to give 1215 J kg$^{-1}$ K$^{-1}$. Variation with temperature of bulk specific heats of the two crucible materials are shown in Fig. 4.24.

4.5.2 Thermal conductivity

This depends on the ratio of the components and is expected to drop
FIG. 4.24 \text{VARIATION WITH TEMPERATURE OF SPECIFIC HEAT OF CLAY BONDED GRAPHITE AND CARBON BONDED SILICON CARBIDE CRUCIBLE MATERIALS}
significantly with increasing temperature. It also depends on the grain orientation \[4.4, 4.7\] and on porosity. Effect of porosity is not dealt with here. The effect of grain orientation, due to possible misalignment of grains in clay/graphite mix, is discussed in Appendix A4.4. Bulk thermal conductivities for the two mixes were obtained by proportional addition \[4.7\] of thermal conductivities \(k\) of the components, in the same manner as was done for specific heat estimation.

Variation with temperature of \(k\) values of the components \[4.10, 4.11, 4.12, 4.13\] are shown in Fig. 4.25. For \(\text{Fe}_2\text{O}_3\), the values of \(k\) of \((\text{SiO}_2 + \text{Fe}_2\text{O}_3)\) were used instead \[4.11\]. The resulting variation of bulk thermal conductivities with temperature for both crucible materials are shown in Fig. 4.26. These values will be used in the calculations of Chapters 6 and 7. Silicon carbide particles take up a more random orientation as was shown in Section 4.4 and so direction of grains has little effect on thermal conductivity of carbon bonded silicon carbide crucibles.

4.6 Other Crucible Properties

4.6.1 Bulk density

A range of values were provided by the manufacturers as:

\[
1800 - 2000 \text{ kg m}^{-3} \text{ for C/SiC}
\]

and

\[
1600 - 2000 \text{ kg m}^{-3} \text{ for clay/C}
\]

The measured values corresponded to lower side of each range, i.e. approximately 1900 kg m\(^{-3}\) and 1615 kg m\(^{-3}\) respectively at room
Fig. 4.25. VARIATION OF THERMAL CONDUCTIVITY OF VARIOUS COMPONENTS WITH TEMPERATURE. (ADAPTED FROM RICHERSON AND TOLUKIAN).
Fig. 4. 26. VARIATION OF THERMAL CONDUCTIVITY OF CARBON BONDED SILICON CARBIDE AND CLAY BONDED GRAPHITE (WITH THE GRAIN).
4.6.2 Coefficient of thermal expansion

Mean values were given by the manufacturers as

\[ 4.0 \times 10^{-6} \text{ K}^{-1} \text{ for C/SiC} \]

and

\[ 3.5 \times 10^{-6} \text{ K}^{-1} \text{ for clay/C} \]

4.6.3 Young's modulus

Values were estimated [4.10] as:

\[ 158 \text{ G Pa for C/SiC} \]

and

\[ 112 \text{ G Pa for clay/C.} \]

4.7 Conclusions

A measurement method was sought to investigate variation in crucible resistivities which are thought to cause non-uniform power distribution. Theory and use of current density and magnetic field strength probes were explained. The probes were used to measure current density and magnetic field strength on the surfaces of crucibles. From the results, resistivities of carbon-silicon carbide and clay-graphite pots were estimated at 50°C. A non-destructive method of measurement of crucible resistivities has therefore been developed. It has illustrated the extent to which variation in properties occurred.

Surfaces of different sections of carbon-silicon carbide crucible were cut...
out and prepared for examination under scanning microscope at appropriate magnifications, and by silicon mapping. The results showed that variation of silicon carbide phase exists between the different sections, in line with the values measured by probes.

Variation of thermal properties of crucibles materials with temperature were estimated from the available manufacturers data. Other crucible material properties were also found. The results will be used in the forthcoming chapters.

References


4.4 Morganite Crucibles Co. Ltd: Private communication on physical and chemical properties of crucible ware, 1984

4.5 MORRELL, R.: Private communication, Jan. 1985

4.6 MOSS, J.B.: 'Properties of engineering materials' (Butterworth Group, 1971)

4.7 VAN VLACK, L.H.: 'Physical ceramics for engineers' (Addison-Wesley, 1964)

4.8 MORRELL, R.: Private communication, July 1986


4.11 TOULIKIAN, Y.S., POWELL, R.W., HO, C.Y. and KLEMENS, P.G.: 'Thermophysical properties of matter', Thermal conductivity,
nonmetallic solids, Vol. 2 (IFI/PLENUM, 1970)


CHAPTER 5

AXIAL POWER DISTRIBUTION

The axial power distribution within the composite load is obtained incorporating the axial variation in electrical resistivity of the crucibles found in the previous chapter.
5 AXIAL POWER DISTRIBUTION

5.1 Introduction

Explanation and results of the analytical approach to the problem have been discussed in Chapter 3, where it was shown that crucible resistivity has a major effect on distribution of induced power. Last chapter dealt with measurements of crucible resistivity showing it is non-uniform along the height of the crucibles. In order to use the results here, the basic theory is extended and reapplied to the composite load divided up into imaginary layers. The axial power distribution is obtained. Effect of crucible geometry is also included in the calculation.

The calculation of power distribution is simplified at this stage by neglecting heat transfers. It is therefore a crude attempt to establish whether axial variation of resistivity can explain crucible cracking. Once this is done, the method needs to be taken further to relate to temperatures by including heat losses and the heat exchanges that occur between neighbouring segments within the furnace structure. These are covered in the next chapter.

5.2 Layers of Composite Load

The equivalent circuit calculated power distribution in the crucible and charge. The method was modified to calculate power distribution in different regions of crucible itself. The composite load of crucible and charge was thus visualized as being made up of layers of charge and segments of crucible stacked upon each other as shown in Fig. 5.1.

The method of equivalent circuit described in Chapter 3 was then applied
Fig. 5.1 The three layers of crucible and charge
to each layer of crucible and charge and power distribution between crucible and charge was calculated, along with efficiency. Then power share in the crucible was obtained using eqn. 5.1 [5.1].

\[ W_2 = (1 - n_3) \cdot W \cdot \eta \]  

(5.1)

where

\begin{align*}
W_2 & = \text{share of power in crucible (W)} \\
n_3 & = \text{power in charge/power in charge and crucible} \\
W & = \text{furnace power (W)} \\
\eta & = \text{efficiency}
\end{align*}

The results were then normalized for compatibility.

5.3 Normalizing Procedure

The calculation applied to each layer gave values of efficiency, power ratio between charge and crucible, number of turns \( N_1 \) and coil current \( I_1 \). The normalizing of the results was carried out in steps.

Step (1)

The number of turns were equalized by dividing the sum by a random number and equating all \( N_1 \)'s to the value obtained. Therefore, power, ampere-turns and volt per turn remained the same and,

\[ \text{new current} = \frac{\text{ampere-turns}}{\text{new turns}} \]  

(5.2)

\[ \text{new voltage} = \frac{\text{volt per turn}}{\text{new turns}} \]  

(5.3)
impedance/(turns)² = \frac{\text{impedance}}{(\text{new turns})²} \quad (5.4)

\textbf{Step (2)}

The currents were equalized by dividing the sum by a random number and equating all \(I_1\)'s to the value obtained. Number of turns and ampere-turns remained the same and,

\[
\text{new voltage} = \text{voltage} \times \text{current ratio} \quad (5.6)
\]

where

\[
\text{current ratio} = \frac{\text{new current}}{\text{old current}}
\]

\[
\text{new volt per turn} = \frac{\text{new voltage}}{\text{number of turns}} \quad (5.7)
\]

\[
\text{new ampere-turns} = \text{new current} \times \text{number of turns} \quad (5.8)
\]

\[
\text{impedance/(turns)²} = \frac{\text{volt per turn}}{\text{ampere-turns}} \quad (5.9)
\]

and

\[
\text{new power} = \text{old power} \times \text{current ratio} \quad (5.10)
\]

because

\[
w = k I^2
\]
or

\[
\frac{W_{\text{new}}}{I_{\text{new}}^2} = \frac{W_{\text{old}}}{I_{\text{old}}^2} \tag{5.11}
\]

**Step (3)**

The sum of the powers was reduced back to initial furnace power; by dividing through by the sum. Number of turns remained the same but,

new power = \( \frac{\text{old power}}{\Sigma \text{power}} \tag{5.12} \)

new current = old current \( \times \sqrt{\left(\frac{\text{power}}{\Sigma \text{power}}\right)} \) \( \tag{5.13} \)

new voltage = old voltage \( \times \sqrt{\left(\frac{\text{power}}{\Sigma \text{power}}\right)} \) \( \tag{5.14} \)

new ampere-turns = new current \( \times \text{number of turns} \) \( \tag{5.15} \)

new volt per turn = \( \frac{\text{new voltage}}{\text{number of turns}} \) \( \tag{5.16} \)

The next step involved checking if sum of the powers equalled furnace power and sum of voltages equalled furnace voltage.

**Step (4)**

To reduce sum of the voltages back to initial supply voltage, voltages and number of turns were multiplied by \( \frac{\text{voltage}}{\Sigma \text{voltage}} \) and the currents by \( \frac{\text{voltage}}{\Sigma \text{voltage}} \)'

Powers remained the same whose sum checked with furnace supply power. Ampere-turns, volt per turn and impedance/(turns) remained the same.
Finally, from the normalized power terms, individual powers in each segment were calculated using eqn. 5.1.

\[ W_{2j} = (1 - n_{3j}) \cdot n_j \cdot W_j \]  \( j \) denotes different layers  \( (5.17) \)

where

\[ W \] = normalized power \( (W) \).

Hence, powers for each segment were obtained. The code for this section is included in Appendix A5.1.

5.4 Power Distribution

The power in each portion of crucible was therefore calculated in the manner described above; which could take into account the variation of crucible resistivity from one segment to the other. The axial power distributions in silicon carbide crucibles used at 100 kW, 3 kHz are shown in Fig. 5.2. The values of resistivity for different sections are those shown in Table 4.1.

Most crucibles are tapered. To take into account the effect of this, radii of crucible and charge were changed accordingly from layer to layer in the calculations. The result of adding taper is shown in Fig. 5.3.

When the disc at the bottom of the crucible was also considered (Fig. 5.4) and the power induced in it was calculated, there was a marked effect on
Fig. 5.2 Axial variation of power due to variation of resistivity along the height of the silicon carbide crucible at 100 kW, 3 kHz
Fig. 5.3 Axial variation of power due to taper and variation of resistivity, along the height of silicon carbide crucible at 100 kW, 3 kHz
FIG. 5.4 A60 CRUCIBLE WITH CHARGE SHOWING THE FOUR LAYERS OF THE COMPOSITE LOAD;
power in the disc, the charge was assumed to be negligible in dimensions. The axial power distribution with the disc is shown in Fig. 5.5.

5.5 Rates of Temperature Rise
Variation in axial power distribution gives rise to different rates of temperature rise between the crucible segments.

5.5.1 Heat balance equation [5.2]
Neglecting all heat transfer, rates of temperature rise were obtained from calculated power values, by applying the heat balance eqn. 5.18.

Heat gained by crucible (J) = mass x specific heat x temperature rise

\[ Q_2 = Y_2 \cdot \pi (R_2^2 - R_3^2) \cdot H_2 \cdot C_{P_2} \cdot \Delta \theta_2 \] (5.18)

where

- \( Y_2 \) = crucible bulk density (kg m\(^{-3}\))
- \( C_{P_2} \) = crucible specific heat capacity (J Kg\(^{-1}\) K\(^{-1}\))
- \( \Delta \theta_2 \) = temperature rise (K)
- \( R_2 \) and \( R_3 \) and \( H_2 \) are radii of crucible and charge and height of crucible segment (m)

Therefore, the induced power (W) is

\[ W_2 = Y_2 \cdot \pi (R_2^2 - R_3^2) \cdot H_2 \cdot C_{P_2} \cdot \frac{d\theta_2}{dt} \] (5.19)
Fig. 5.5 Axial variation of induced power in the silicon carbide crucible due to change of resistivity, taper and power in disc
The rate of temperature rise \((Ks^{-1})\) is

\[
\frac{d\theta}{dt} = \frac{w_2}{\gamma_2 \pi (R_2^2 - R_3^2) H_2 C_p}
\]  

(5.20)

Thus, rates of temperature rise were obtained for the four segments of the crucible from normalized powers. This was done without considering the heat transfer between charge and crucible or losses to outside. The results are shown in Fig. 5.6.

The results with the disc included in the calculations are shown in Fig. 5.7. The three curves drawn are an attempt to represent the cold stage, the heel formation and the fully molten stages of the melt cycles. They were obtained by using the properties at ambient, 350°C and 720°C. Bearing in mind the approximations used, the curves indicate that at different stages through the melt cycle, the middle region of the crucible wall has the highest rate of temperature rise; compared with other portions of the wall. But the maximum difference exists between the disc and bottom regions of the wall making that region a prime stress raiser.

In practice, crucibles fail mainly by cracking around the base between the middle and bottom sections. The results show that greater variations in surface power density are associated with silicon carbide crucibles [5.3] within the region prone to failure. Methods aimed at reducing these variations should therefore enhance crucible lifetime.
Fig. 5.6 Axial variation of rates of temperature rise due to non-uniform resistivity in silicon carbide crucible at 100 kW, 3 kHz (a) no taper (b) taper included
Fig. 5.7 Axial variation of rates of temperature rise due to taper, variation of resistivity and power in the disc of silicon carbide crucible at 100 kW, 3 kHz. Curve (1) refers to cold stage; (2) to heel formation stage and (3) to fully molten stage of the melt cycle.
5.6 Conclusions

The two-dimensional equivalent circuit method was extended by dividing the composite load into seven segments. This enabled power distribution to be calculated in different segments along the height, taking into account variation in resistivity and geometry. Power in the crucible disc at the base was also calculated. No heat transfer was considered. The method of calculating axial power distribution suffers from limitations mainly because heat exchanges have not been included and because smaller segments should be considered. Nevertheless, the results seem to explain the failure.

Rates of temperature rise for each segment were obtained using heat balance equation. Because of the simplified approach, the values estimated were considered too high but they provided a means of comparing different regions' behaviour. The highest rate of temperature rise along the crucible wall was in the middle, and the maximum overall difference was between the disc and bottom wall, indicating a possible failure zone. Refinements to the method follow in the next chapter.

References

5.1 REICHERT, K.: Private communication, 1985


The purpose of this chapter is to devise a method that can predict the transient temperature distribution in the crucibles, which can be used to explain reasons for crucibles cracking in service.
6 TEMPERATURE DISTRIBUTION IN CRUCIBLE AND CHARGE

6.1 Introduction

Thermal stresses are a form of internal stressing arising from restricted thermal expansion or contraction of a body. Such restriction arises when a temperature gradient causes one part of a body to expand or contract more than another part. Thermal effects can be either transient or steady, and in the former case can involve many cycles of heating and cooling. The dilation (i.e. the expansion or contraction) depends upon temperature, but so also do the elastic constants such as Young's modulus, so that the thermal stresses are related to temperature distribution [6.1] and it is therefore necessary to produce a temperature profile of the crucibles.

A material can experience thermal stresses during heating or cooling. Since most cases of crucible failure that have been reported, refer to the heating stage, emphasis will be put on the heating period. Ideally, values of stresses involved should be estimated but it is recognized that to enter into a detailed analysis to quantify thermal stresses is itself a new project and beyond the scope of this work.

In the absence of sufficiently accurate data on the crucible materials' properties and for reasons outlined later in the chapter, the following are done:

(1) A comparison is made between the two materials (clay-graphite and silicon carbide) to establish their ability to withstand thermal stresses. This will take into account their properties and is
(ii) After a brief review of previous works about the subject, it will be attempted to obtain the temperature distribution within the crucibles and to correlate this with thermal stresses.

To produce a temperature profile of the crucible various factors must be taken into account, i.e. heat transfer, heat losses and temperature dependance of the material's properties. This task is presented in progressive stages: (a) Stepwise heat transfer, (b) Simultaneous heat transfer. In each stage the solution is gradually refined further to build up a more accurate picture that is close to practical situation.

Fig. 6.1 shows a flow chart which describes the progress of the work from relation of thermal stresses to temperature distribution, onwards through the analysis of the temperature distribution with verification and finally results of 3 kHz, 100 kW tilting furnace using both silicon carbide and clay-graphite crucibles with aluminium and copper charges.

6.2 Thermal Shock/Stress

Thermal shock is defined as rapid temperature changes, causing sudden build-up of thermal stresses, which may result in distortion, spalling, rupture and/or failure of an object. A good deal of work has been done on thermal shock and refractory materials' resistance to it; mainly by Kingery [6.2] and Hasselman [6.3].

Resistance parameters are a theoretical measure of a material's ability to withstand thermal stresses and strains. Hasselman has developed several
FIG. 6.1 STRUCTURE OF THE CHAPTER
INPUT DATA: PROPERTIES, INITIAL AND FINAL CONDITIONS

\[ t = 0; \quad \theta = \theta_0 \]

HOTPOT: \( I_p, \eta, n_3 \) FOR LAYERS

SUBNORM: NORMALISED POWER DISTRIBUTION, TEMPERATURE DERIVATIVES

INTEGRATION OF TEMPERATURE DERIVATIVES

TEMPERATURE PROFILE OUTPUT

\[ t = t + \Delta t \]

\( \theta < \theta_p \)?

YES

END

FIG. 6.1 (a) FLOWCHART OF COMPUTER PROGRAM SHOWING OVERALL METHOD OF CALCULATION OF MELTING CYCLE
expressions in terms of the material's elastic constants. They deal with resistance to crack nucleation and growth or propagation and damage. Assuming fracture occurs by tension, and thus by setting the maximum thermal stress equal to the material's tensile strength, the expressions can be used to obtain maximum allowable temperature difference, heat flux and temperature gradient, to which a ceramic body can be subjected prior to catastrophic thermal stress failure [6.3]. These are called fracture resistance parameters. Other theories give solutions for the extent of crack propagation rather than initiation; and the degree of damage expected to result. From these, the thermal stress 'damage' resistance parameters are obtained. The parameters can be used for preliminary selection and screening of materials for a given thermal environment.

The criterion of lack of fracture by thermal stress dictates high strength and low modulus of elasticity. For a low degree of damage, selection of materials should be based on low strength and high modulus of elasticity [6.4]. Since the avoidance of fracture is always desired, high fracture resistance parameters should be pursued first.

The important factors influencing thermal shock resistance are:

- coefficient of thermal expansion \( \alpha \) (K\(^{-1}\))
- thermal conductivity \( k \) (W m\(^{-1}\)K\(^{-1}\))
- elastic modulus \( E \) (Pa)
- strength (tensile) \( S_t \) (Pa)

The peak stress that occurs on the surface as a result of a thermal shock, is given by eqn. 6.1 [6.5].
\[
\sigma_{th} = \frac{E \cdot a \cdot \Delta \theta}{(1-\nu)} \tag{6.1}
\]

where

\[
\begin{align*}
\sigma_{th} & = \text{thermal stress (Pa)} \\
\Delta \theta & = \text{temperature difference in the body (K)} \\
\nu & = \text{Poisson's ratio}
\end{align*}
\]

Equation 6.1 shows that the thermal stress can be reduced by decreasing \( \Delta \theta \) between two regions. The critical maximum temperature difference at fracture can therefore be obtained for a material by substituting its tensile strength \( S_t \) for \( \sigma_{th} \) [6.6], and re-arranging eqn. 6.1.

\[
\Delta \theta = \frac{S_t \cdot (1-\nu)}{E \cdot a} \tag{6.2}
\]

The maximum allowable temperature difference in eqn. 6.2 has been called resistance factor [6.2] or thermal stress resistance parameter \( R \) [6.3].

Values of \( R \) were calculated for both silicon carbide and clay-graphite and are shown in Table 6.1. The properties that were needed, were estimated from data provided by the manufacturers and from reference sources [6.7, 6.8, 6.9, 6.10].
Table 6.1

<table>
<thead>
<tr>
<th>Material</th>
<th>C-Silicon carbide</th>
<th>Clay-graphite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength (MPa)</td>
<td>11.28</td>
<td>11.77</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.19</td>
<td>0.175</td>
</tr>
<tr>
<td>Thermal expansion coefficient (K⁻¹)</td>
<td>4.0 x 10⁻⁶</td>
<td>3.5 x 10⁻⁶</td>
</tr>
<tr>
<td>Elastic modulus (GPa)</td>
<td>158</td>
<td>112</td>
</tr>
<tr>
<td>( R = \frac{E}{E \cdot \alpha} ) (K)</td>
<td>14.1</td>
<td>24.8</td>
</tr>
</tbody>
</table>

The figures calculated must be dealt with cautiously. They are dependent on the accuracy of the material's properties. No heat transfer is taken into account in the expression for \( R \) and it therefore gives the limiting thermal shock resistance parameters. However, it provides a theoretical means of comparing the two crucible materials. The values in Table 6.1 indicate that clay graphite is slightly more shock resistant.

6.3 Temperature Distribution

Analytical solutions for the temperatures in solid and hollow cylinders for steady and non-steady heat conduction have been covered by Carslaw and Jaeger [6.11]. Although the mathematics are complicated, their work forms the basis for most later works on temperature distribution. Baker [6.12] has modified their work and applied it to induction billet heating. Davies and Simpson [6.13] have also used the analysis for induction heating of cylindrical billets and have included the radiation losses from surface of billets. They have not covered hollow cylinders. The solutions [6.11, 6.12, 6.13] are only useful for induction heating of
homogeneous (non-composite) bodies with regular shapes, and where uniform mode of heating with no phase changes are involved.

Newman [6.14] developed a fundamental equation for non-steady heat conduction in internally heated non-metallic/ceramic cylinders. By generating graphs and tables, he calculated temperature distribution in internally heated hollow cylinders of known thermal properties. The annulus was heated from an initial temperature all along its inner surface at a uniform and known rate (thermal power per unit length). The axis of cylinder was assumed sufficiently long so that heat flow outwards could be considered radial without a component in the axial direction. The method, however, may only be applied to the case of an empty crucible, neglecting losses and the disc at the bottom.

6.4 Heat Transfers

The crucible exchanges heat with the charge and with the surroundings, be it air, muffle or some kind of refractory. The pieces of charge also exchange heat with neighbouring pieces.

Because the power is not equally distributed between charge and crucible, the resulting temperature difference will cause heat transfer between them. The direction of the heat flow depends mainly on the crucible material (as discussed in Chapter 3). Often the charge consists of a mixture of swarf and borings with no proper geometry (Fig. 6.2(a)). It is however assumed to have regular shape and the fact that it does not, is covered by introducing a packing factor $\beta$ for packing density and contact areas.
FIG. 6.2 (a) CRUCIBLE AND CHARGE
(b) FOUR LAYERS OF COMPOSITE LOAD
As regards modes of failure, the differences between push-up and tilter are few. The shape and dimensions of work-coil and crucible are the same. The magnetic field strength would have the same non-uniformities in both cases. They both operate within the same range of frequencies.

The method of pouring is different. In the case of tilter, the whole furnace is swung round an axis in line with pouring lip. This is done by overhead cranes with these smaller units. In the push-up, the crucible which sits inside an overflow receptacle, moves up and out of the furnace coil by a ram from underneath when metal is ready for pouring. The crucible is then lifted by means of tongs or shanks and emptied into moulds. This mechanical handling of hot crucibles by cold objects make them susceptible to thermal shock. But this is not discussed here as the interest is confined to the effects during heating up period.

The other difference between the two furnaces concerns the refractory material between the coil and crucible. Push-up furnaces use ceramic fibre insulation to fill the gap. This is packed in the sides as well as possible, by the manufacturer, which makes it similar to the rammed refractory around the crucible in the tilter. In both cases the air gap is negligible and so heat losses from crucible surface through the packed refractory may be assumed to occur by conduction alone. Nevertheless, allowance was made in the calculations for any difference by using different values of thermal conductivity for the packings. For the packing in the tilter a value of $1.0 \text{ Wm}^{-1}\text{K}^{-1}$ was used and for the push-up a value of $0.5 \text{ Wm}^{-1}\text{K}^{-1}$ was used to represent the fibre packing [7.2] round the crucible.
The heat transfers are considered to be solely by conduction. The Fourier equation of conduction states that

\[ q = -k \cdot A \frac{\Delta \Theta}{\Delta x} \]  \hspace{1cm} (6.4)

where
- \( q \) is rate of conducted heat (W)
- \( k \) is thermal conductivity (W m\(^{-1}\) K\(^{-1}\))
- \( A \) is area through which heat flows (m\(^2\))
- \( \Delta \Theta \) is the temperature difference (K)
- \( \Delta x \) is the distance between two points (m)

The negative sign is conventionally used to represent a positive heat flow in the direction of a negative temperature gradient. For the case of a silicon carbide crucible, the negative temperature gradient is radially inwards from crucible to charge.

The charge segment, say \( M_3 \) in Fig 6.2(b), is considered to be subjected to a heat flux and therefore its conductivity \( k_3 \) is used in the conduction equation.

The total power in \( M_3 \) = induced power + \( q_{23} \)

\[ q_{23} = \frac{k_3 A_3}{R_3} (\Theta_2 - \Theta_3) \]  \hspace{1cm} (6.5)
The resultant rate of temperature rise is therefore

\[ \dot{\theta}_3 = \frac{d\theta_3}{dt} = \frac{1}{(m \cdot C_p)_3} \cdot (\text{induced power in } m_3 + q_{23}) \] (6.6)

The corresponding resultant value for \( \dot{\theta}_2 \) is

\[ \dot{\theta}_2 = \frac{d\theta_2}{dt} = \frac{1}{(m \cdot C_p)_2} \cdot (\text{induced power in } m_2 - q_{23}) \] (6.7)

where

- \( m \) = mass (kg)
- \( C_p \) = specific heat (J kg\(^{-1}\) K\(^{-1}\))

The heat conduction between segments of crucible (Section 6.6.2) are also obtained in the same manner using eqn. (6.4). These and heat transfers between crucible and charge are considered first. Inter-charge heat transfer and heat losses to outside are taken into account in the final calculations.

6.5 Stepwise Calculation of Temperature Distribution

The equivalent circuit of chapter 3 calculated the power distribution between the crucible and charge. In Chapter 5, the analysis was re-applied to the segmented composite load to find the effect of non-uniform properties, in terms of axial power distribution and rates of temperature rise. Then the disc at the bottom and the tapering of crucible were included in the calculation. No attempt was made to calculate heat transfer; losses or actual temperature distribution.
As a first attempt, the method of stepwise heat transfer by averaging temperature gradients is carried out to estimate the temperature distribution in the crucible and charge.

Starting with all segments at 20°C, the power distributions are calculated to obtain rates of temperature rise for each segment. Then a time interval $t$ of say one minute, assumed to be short enough for linear rise of temperature, is chosen. Temperature rises are obtained after the time interval by 

$$\Delta \theta = t \frac{d\theta}{dt}$$

and added to initial temperatures. Average heat exchanges are calculated between the crucible and charge, and crucible to crucible segments to get resultant powers. The rises are then doubled, added to initial temperatures and used as temperatures at which properties for next interval are calculated. This provides a better estimate of temperature rise between the two operating times because further increase of temperature would be expected. The procedure is then repeated over the next minute. This method is thus aimed to simulate the non-steady heat flow characteristics involved.

The method is illustrated in Fig. 6.3 for temperatures of the silicon carbide disc. At the end of the first minute, the temperature rise of 208.4K was added to initial temperature of 20°C, giving a temperature of 228.4°C at $t = 1$ minute. Properties for second minute were obtained at temperature of 437°C, that is $(20 + 2 \times 208.4)^\circ C$. This was repeated for all seven segments. The calculation was carried out by subroutine HT. The code is included in Appendix A6.1. Figs. 6.4 and 6.5 show the results obtained by the same procedure for crucible wall, with aluminium
FIG. 6.3 Temperature-time graph for disc portion of S.C crucible with Al charge, showing temperature-rises after each minute to obtain actual temperatures; and the temperature at which properties were calculated for the next minute.
FIG. 6.4. Temperature-time curves for the disc and walls of silicon carbide crucible, obtained by singular heat transfer between the crucible and aluminum charge. Highest temperature of charge is 540° at t=10 mins.
FIG. 6.5 Temperature-time curves for the disc and walls of silicon-carbide crucible, obtained by singular heat transfer between the crucible and copper charge. Highest temperature of charge is 750°C at t=15 mins.
and copper charge.

The temperatures predicted for the disc were very high. However, the results showed improvement over those considered with no heat transfer in chapter 5, and so provided a rough insight to the temperature distribution within the charge as well as the crucible. It is recognized that the method suffered from deficiencies. The charge did not approach melting temperature within realistic times. The order of heat transfer was based on hottest segment, predicted by power distribution, first losing heat to neighbouring parts. In fact, all heat exchanges occurred at the same time. It was tried to emulate this by changing values of contact area factor $\beta$, but the method proved to be too sensitive to this parameter; which was the biggest drawback. Hence, the second attempt was concentrated on simultaneous heat transfer solution.

6.6 Simultaneous Solution of Temperature Distribution

To investigate transients in the solutions to the equations expressing rates of temperature rise for various segments, the equations should be solved simultaneously.

Furthermore, the seven segments used so far, were only adequate as a means of dividing the composite load into some large portions, to differentiate one end of the crucible from the other. The number of segments had to be increased and their sizes reduced for a finer analysis. So it was decided to increase the number of layers to 10. It is realised that unless the size of each segment is infinitesimally small, there exists temperature gradient within it. It was assumed however, that any one
FIG. 6.6 TEN LAYERS OF COMPOSITE LOAD
segment was at one uniform temperature at any one time.

Until steady state is reached, the heat transfer direction was assumed to be from crucible to charge in the case of silicon carbide crucibles and vice versa for clay-graphite. The temperature derivatives for all nineteen segments (Fig. 6.6) were derived as explained in Section 6.6.2. They were solved simultaneously using numerical integration. The system of equations was in the general form:

\[
\dot{\theta}_{ij} = \frac{1}{C_{ij}} \left[ w_{ij} - \left( \sum_{k=1}^{3} F_{ikj} \cdot \theta_{kj} \right) + \sum_{k=2}^{3} (1 - \delta_{ik}) \cdot F_{ikj} \cdot \theta_{kj} \right] + F_{ijl} \cdot \delta_{i(j-1)} + F_{iij} \cdot \theta_{i(j+1)}
\]

where

\[
\delta_{ik} = 1 \text{ if } i = k
\]

and

\[
\delta_{ik} = 0 \text{ if } i \neq k
\]

6.6.1 Simultaneous solution of temperature derivatives by numerical integration

The numerical integration was done using the NAG routines. An external subroutine FCN, containing the derivatives had to be prepared. This would automatically be involked when the NAG routine was called from within the main program [6.15]. The subroutine had to evaluate the function F (derivate \( \dot{\theta}_i \)) for given values of arguments \( T, \theta_1, \ldots, \theta_n \). T refers to time.

NAG routine DO2YAF was chosen. The routine advances the solution of a system of first order ordinary differential equations.
FIG. 6.7 (a) SEGMENTATION OF CRUCIBLE AND CHARGE
(b) HEAT TRANSFER BETWEEN SEGMENTS
\[ \dot{Y}_i = F [T, Y_1, Y_2, ..., Y_n], \quad i=1,2,...,n \]  

(6.8)

from \( T = X \) to \( T = X + H \) using a single step of Merson’s form of Runge-Kutta method. The system is defined by the subroutine FCN which evaluated the derivatives \( F \) in terms of \( T \) and \( Y_1, Y_2, ..., Y_n \). \( H \) refers to step size which should be less than or equal to the time constant; roughly one fifth of the time taken to reach steady state condition. It is advisable to use a small step size, otherwise unstable response occurs in the solution.

6.6.2 The external subroutine

The segments are shown in Fig. 6.7(a). Because of symmetry only one half of the cross section is shown. Assuming that each segment of crucible exchanged heat with its neighbouring charge and crucible segments (Fig. 6.7(b)), the overall net or gross power distributions were calculated for all nineteen segments to determine the temperature derivatives.

6.6.2.1 The crucible

\[
\text{Resultant power} = \text{Induced power} - \text{heat transfer to charge in the same layer} + \text{heat transfer from lower layer} - \text{heat transfer to upper layer} - \text{heat loss}
\]

Thus,
\[
\begin{align*}
W'_{2j} &= W_{2j} \\
&= q_{2j\rightarrow 3j} \\
&\quad + q_{2(j-1)\rightarrow 2j} \\
&\quad - q_{2j\rightarrow 2(j+1)} \\
&\quad - \text{heat loss}
\end{align*}
\]  \hspace{2cm} (6.9)

where \( j \) denotes position of layers.

Heat transfer from lower layer is zero for the disc. Similarly, the heat transfer to upper layer is zero for segment 18. The heat exchanges with the surroundings are covered in Section 6.7.

The charge could be anything ranging from low grade swarf and borings to high grade pieces of casting returns and ingots. The condition of these might be greasy and oxidized or relatively clean and shiny. So depending on these factors and whether molten or not, the heat transfer between charge and crucible would be by convection as well as conduction. In the absence of any data on convection heat transfer coefficients between these surfaces, however, it was decided to treat all heat transfers as conduction.

Work has been done [6.16] on heat transfer and effect of contact area between different materials' surfaces. In the foregoing analysis, however, the packing factor \( \beta \) (eqn. (5.3)) is used as a coefficient of the area \( A \) in the Fourier's conduction equation. This factor is also assumed to represent the packing density of the charge in the crucible. The resultant power of eqn. (6.9) was therefore expressed as:
\[ w_{2j} = w_{2j} - \frac{k_{3j} \cdot \beta_{j} \cdot As_{3j}}{x_{3j}} \cdot (\theta_{3j} - \theta_{3j}) + \frac{k_{2j} \cdot As_{2j}}{0.5 \cdot H_{2j}} \cdot (\theta_{2(j-1)} - \theta_{2j}) - \frac{k_{2(j+1)} \cdot As_{2(j+1)}}{0.5 \cdot H_{2(j+1)}} \cdot (\theta_{2j} - \theta_{2(j+1)}) \] (6.10)

where

\( As = \) cross-sectional area \((m^2)\)

\( Ag = \) generating area \((m^2)\)

For any segment, the heat capacity \( C \) \((J K^{-1})\) is

\[ C = m \cdot Cp \] (6.11)

Therefore, dividing eqn. (6.10) through by heat capacity, the derivatives for rates of temperature rise are obtained:

\[ \dot{\theta}_{2j} = \frac{1}{C_{2j}} \cdot w'_{2j} \] (6.12)

Let

\[ F_{21j} = \frac{k_{3j} \cdot \beta_{j} \cdot As_{3j}}{x_{3j}} \] (6.13)

\[ F_{22j} = \frac{k_{2j} \cdot As_{2j}}{0.5 \cdot H_{2j}} \] (6.14)

\[ F_{23j} = \frac{k_{2(j+1)} \cdot As_{2(j+1)}}{0.5 \cdot H_{2(j+1)}} \] (6.15)
Substituting eqns. (6.13) to (6.15) into eqn. (6.10) and collecting like 
6-terms on the RHS, eq. (6.10) becomes

\[
\dot{\theta}_{2j} = \frac{1}{C_{2j}} \left[ w_{2j} - \left( F_{21j} + F_{22j} + F_{23j} \right) \cdot \theta_{2j} \right.
\]

\[
+ F_{21j} \cdot \theta_{3j}
\]

\[
+ F_{22j} \cdot \theta_{2(j-1)}
\]

\[
+ F_{23j} \cdot \theta_{2(j+1)} \right]
\]

(6.16)

The general expression (6.16) for the derivatives applies only to layers 3
to 9. Some terms disappear from the expression for layers 1, 2 and 10.

6.6.2.2 The charge

Heat transferred from crucible to charge segments was added to the induced power in charge. Similarly, heat exchanges occurred between segments of charge themselves; by the virtue of small temperature differences. For a homogeneous charge, these differences should be zero. But when effect of crucible disc and taper are included with crucible-charge heat transfer, temperature differences between charge segments ought to be taken into account. Inter-charge heat transfer occurs by convection as well as conduction but the latter was assumed, because specific convection heat transfer coefficients were not available. The resultant power in the charge segments were evaluated in a similar manner to that of crucible. For the case of silicon carbide crucible,

Resultant power in charge = induced power
Thus,

\[ w'_{3j} = w_{3j} + q_{2j\rightarrow3j} + q_{3(j-1)\rightarrow3j} - q_{3j\rightarrow3(j+1)} \]  \hspace{1cm} (6.17)

For segment 3, the heat transfer from the lower layer, is that from the disc. The resultant power was therefore expressed as:

\[ w'_{3j} = w_{3j} + \frac{k_{3j}}{x_{3j}} \cdot \beta_j \cdot (\theta_{2j} - \theta_{3j}) + \frac{k_{3j}}{0.5 \cdot H_{3j}} \cdot \left( \theta_{3(j-1)} - \theta_{3j} \right) - \frac{k_{3(j+1)}}{0.5 \cdot H_{3(j+1)}} \cdot \left( \theta_{3j} - \theta_{3(j+1)} \right) \]  \hspace{1cm} (6.18)

Let,

\[ F_{31j} = \frac{k_{3j} \cdot \beta_j \cdot A_{3j}}{x_{3j}} \]  \hspace{1cm} (6.19)

\[ F_{32j} = \frac{k_{3j} \cdot A_{3j}}{0.5 \cdot H_{3j}} \]  \hspace{1cm} (6.20)

\[ F_{33j} = \frac{k_{3(j+1)} \cdot A_{3(j+1)}}{0.5 \cdot H_{3(j+1)}} \]  \hspace{1cm} (6.21)

Dividing through by heat capacity, the derivatives become:

\[ \dot{\theta}_{3j} = \frac{1}{C_{3j}} \cdot w'_{3j} \]  \hspace{1cm} (6.22)

Substituting eqns. (6.19) to (6.21) into eqn. (6.18) and collecting like
\[ \dot{\theta}_{3j} = \frac{1}{c_{3j}} \cdot \left[ w_{3j} \right. \]
\[ - (F_{31j} + F_{32j} + F_{33j}) \cdot \theta_{3j} \]
\[ + F_{31j} \cdot \theta_{2j} \]
\[ + F_{32j} \cdot \theta_{3(j-1)} \]
\[ + F_{33j} \cdot \theta_{3(j+1)} \]  
(6.23)

The general expression (6.23) for charge temperature derivatives applies only to layers 3 to 9. Some of the terms disappear from the expression for layers 2 and 10. For segment 19, heat transfer to upper layer is zero.

The subroutine code is included in SUB19 in Appendix A6.2. The solutions to the derivatives, with losses to outside excluded, were thus obtained using the NAG routine D02YAF with a step size of 10 seconds. Typical industrial installations quoted melt times of 22 minutes for copper and 9 minutes for aluminium in size A60 crucibles at 100 kW [6.17]. So a step size of 10 seconds was considered short enough. The results are shown in Figs. 6.8 and 6.9 for copper and aluminium charges in silicon carbide crucible. The charge temperature approached disc temperature more quickly with the simultaneous solution.

6.7 Heat losses

The energy losses from a metal melting furnace are [6.18]:

(1) The heat absorbed by the structure of the furnace. This is absorbed largely by the refractory when the furnace is allowed to cool. This was
FIG. 6.8 TEMPERATURE-TIME CURVES FOR THE DISC AND THE WALLS OF THE SILICON CARBIDE CRUCIBLE, OBTAINED BY SIMULTANEOUS HEAT TRANSFER BETWEEN THE CRUCIBLE AND COPPER CHARGE
FIG. 6.9 TEMPERATURE TIME CURVES FOR THE DISC AND WALLS OF SILICON CARBIDE CRUCIBLE, OBTAINED BY SIMULTANEOUS HEAT TRANSFER BETWEEN THE CRUCIBLE AND ALUMINIUM CHARGE
considered as a percentage of the total losses.

(2) The heat flow through the furnace wall. This is removed by the water in
the coil. This was considered by conduction.

(3) The heat dissipated from the surface of the furnace, mainly by radiation,
lost to the atmosphere. This was estimated by radiation.

Referring to Fig 6.10 showing a tilting induction furnace, going from the axis
of the crucible radially outwards, there is the charge, the crucible wall,
the packing which holds the crucible in place, a thin (1.2mm) refractory
paper on the inside of the furnace coil, the coil itself and finally the
circulating water are the different media through which heat would travel.
Beyond these is the furnace structure which is normally made of thick
asbestos or transite boards (about 20mm thick). The charge surface and rim
of the crucible are open to atmosphere and lose heat by radiation and
convection. The heat from the crucible is conducted outwards as well as
downwards, through the rammed refractory material around it.

Heat losses have been expressed [6.18, 6.19] as percentages of input power
for typical steel melting installations. This approach is adequate for
estimating overall performance and efficiency.

6.7.1 Conduction losses

The heat lost from crucible was assumed to be removed by the cooling water in
the coil, after being conducted through the packing. This was calculated by
eqn. 6.4. For the wall of crucible, \( \Delta x \) was taken as the difference between
coil and crucible radii, and \( k \) as thermal conductivity of the packing
material. For the disc, \( \Delta x \) was taken as thickness of the packing under
FIG. 6.10 CONDUCTING CRUCIBLE INDUCTION FURNACE
crucible, $H_p$. The conduction losses were up to 6.2% and 3.7% of furnace power for silicon carbide crucible with copper and aluminium charge respectively.

### 6.7.2 Radiation Losses

The heat dissipated from the surface of the charge and rim of the crucible were partly by radiation from grey bodies. The radiation losses from a grey body are determined by Stefan-Boltzman Law:

$$ w_r = \varepsilon \sigma A (T^4 - T_a^4) $$

(6.24)

where:

- $w_r$ = radiated power (W)
- $\varepsilon$ = emissivity of the radiating body
- $\sigma$ = Stephen-Boltzman constant $= 5.669 \times 10^{-8}$ (W m$^{-2}$ K$^{-4}$)
- $A$ = surface area of radiating body (m$^2$)
- $T$ = Temperature of the radiating body in absolute
- $T_a$ = ambient temperature in absolute

The values of $\varepsilon$ were obtained from various sources [6.8, 6.20, 6.21], detailed in Appendix A6.3. The following values were used in the calculations: SiC-0.8; C.G. - 0.5; Al - 0.11; Cu - 0.65. Ambient temperature was taken as 20°C. The combined radiation losses from silicon carbide crucible and charge were up to 10% and 1.1% of furnace power, for copper and aluminium respectively.

### 6.7.3 Convection losses

The convection losses from tilting furnace, occur at the top. These are
from the crucible rim and charge surface.

Newton's equation for convection heat transfer is

\[ q_c = h \cdot A \cdot (\theta_1 - \theta_2) \]  \hspace{1cm} (6.25)

where

- \( q_c \) = heat transfer (W)
- \( h \) = convective heat transfer coefficient (W m\(^{-2}\) K\(^{-1}\))
- \( \theta_1 \) = temperature of hot body (°C)
- \( \theta_2 \) = temperature of fluid at lower temperature (°C)

The values of \( h \) depend on flow regime, fluid properties and the temperature difference [6.21] and whether free or forced convection occurs. Some empirical data are available giving values of \( h \) for laminar and turbulent flow across various geometries [6.22, 6.23]. In a general form, \( h \) is given by the expression:

\[ h = C \cdot \left( \frac{\Delta \theta}{D} \right)^{25} \]  \hspace{1cm} (6.26)

where

- \( C \) is a coefficient depending on configuration
- \( D \) is a characteristic dimension (m)
- \( \Delta \theta = \theta_1 - \theta_2 \) (K)

Therefore, eqn. 6.25 becomes

\[ q_c = C \cdot \left( \frac{\Delta \theta}{D} \right)^{25} \cdot A \cdot (\Delta \theta) \]  \hspace{1cm} (6.27)
6.7.3.1 Determination of flow regime

To determine whether flow was laminar or turbulent, it was required to find the approximate value of Grashoff Prandtle number (GrPr) [6.22]. The procedure is included in Appendix A6.4, showing the flow was laminar. The expressions for \( h \) (eqn. 6.26) corresponding to free convection in laminar flow were then obtained [6.22, 6.23] as follows for:

- Top surface of the charge: 
  \[ h = 1.32 \frac{(\Delta \theta)^{2.25}}{2 \times r_3} \]  
  (6.28)

- Rim of the crucible: 
  \[ h = 1.32 \frac{(\Delta \theta)^{2.25}}{2x(r_2-r_3)} \]  
  (6.29)

- Side of the crucible: 
  \[ h = 1.42 \frac{(\Delta \theta)^{2.25}}{H_2} \]  
  (6.30)

- Underneath the crucible: 
  \[ h = 0.61 \frac{(\Delta \theta)^{2.25}}{2xr_2} \]  
  (6.31)

Equations 6.30 and 6.31 are for free standing crucible; not the packed crucible in tilting furnace. The actual heat lost by convection is calculated by eqn. 6.27. The sum of the convection losses from silicon carbide crucible rim and charge surface were up to 8% and 5.1% of furnace power for copper and aluminium respectively.

6.8 Simultaneous Solution Including Heat Losses

All heat losses were included in the calculations to complete the analysis. The heat loss terms were subtracted from resultant powers expressed in eqns. 6.10 and 6.18 for crucible and charge segments respectively. The overall resultant rates of temperature rise were then obtained.
Fig. 6.11 Inside and outside heat transfer mechanism of crucible furnace
The resulting system of equations were solved simultaneously by numerical integration, using NAG routines. The single step Runge-Kutta method could no longer be used to solve the equations; because the power terms in the heat loss expressions would produce a non-linear or "stiff" system of ordinary differential equations. Therefore, it was decided to use the NAG routine D02EAF instead. The routine integrates a stiff system of first order differential equations

\[ \dot{Y}_i = F_i [T, Y_1, Y_2, \ldots, Y_n], \quad i = 1, 2, \ldots, n \]

from \( T = X \) to \( T = X_{END} \) using a variable-order variable step Gear method. The system was defined by an external subroutine FCN.

6.8.1 External subroutine including losses

The subroutine is basically the same as the one derived in Section 6.6.2, only completed by including heat loss expressions. Various heat losses to outside are shown schematically in Fig. 6.11. Charge loses heat to atmosphere only at the top surface whereas crucible loses heat to outside in all directions.

By the same procedure as in Section 6.6.2, the new derivatives containing all heat transfers and heat losses were obtained.

6.8.1.1 The crucible

Starting with eqn. 6.10, the heat loss terms were tagged on. The resultant power in crucible segment is therefore
\[ w_{2j}^\prime = w_{2j} - (1+\lambda) \cdot \frac{k_o \cdot A_{2j}}{r_1 - r_{2j}} \cdot (\theta_{2j} - \theta_a) \text{; side loss} \]

\[- (1+\lambda) \cdot \frac{k_o \cdot A_{2(1)}}{H_p} \cdot (\theta_{2(1)} - \theta_a) \text{; only for disc} \]

\[- h^2 e_2 A_{2(10)} \cdot (\theta_{2(10)} - \theta_a) \text{; only for crucible rim} \]

\[- e_2 A_{2(10)} \cdot \sigma (T_{2(10)}^4 - T_a^4) \text{; only for crucible rim} \]

(6.32)

where

\[ \lambda = \text{percentage of losses absorbed by furnace structure} \]

\[ k = \text{thermal conductivity of packing material (W m}^{-1} \text{ K}^{-1}) \]

\[ r = \text{radius of furnace coil (m)} \]

\[ \theta_a = \text{ambient temperature (°C)} \]

\[ Hp = \text{thickness of refractory packing under crucible (m)} \]

\[ T_2 = \theta_2 + 273 \text{ (°K)} \]

\[ T_a = \theta_a + 273 \text{ (°K)} \]

Let

\[ F_{24j} = \frac{k_o \cdot A_{2j}}{r_1 - r_{2j}} \] \hspace{1cm} (6.33)

\[ F_{DS} = \frac{k_o \cdot A_{2(1)}}{H_p} \] \hspace{1cm} (6.34)

Substituting eqn. (6.33) into eqn. (6.32), the resultant power for wall segments becomes

\[ w_{2j}^\prime = w_{2j} - (1+\lambda) \cdot F_{24j} \cdot (\theta_{2j} - \theta_a) \] \hspace{1cm} (6.35)
Temperature derivatives are obtained from resultant powers by eqn. (6.12). Therefore, dividing through by heat capacity, substituting eqn. (6.16) and collecting like terms on the RHS, eqn. (6.35) gives

\[
\dot{\theta}_{2j} = \frac{1}{C_{2j}} \{ w_{2j} - \left[ F_{21j} + F_{22j} + F_{23j} + (1+\lambda) \cdot F_{24j} \right] \cdot \theta_{2j} + F_{21j} \cdot \theta_{3j} + F_{22j} \cdot \theta_{2(j-1)} + F_{23j} \cdot \theta_{2(j+1)} + F_{24j} \cdot (1+\lambda) \cdot \theta_a \} \tag{6.36}
\]

Resultant temperature derivatives including all losses and heat transfers were calculated by eqn. (6.36). The equation, as it stands, is complete for layers 3 to 9 only and requires modifications for layers 1, 2 and 10: for the disc, \( F_{22} \) is zero but the extra loss term \( F_{D5} \) should be brought in. Similarly, for segment 18, \( F_{23} \) is zero but convection and radiation losses, represented by the last two sentences of eqn. (6.32), are added on (cf. eqn. 6.16).

6.8.1.2 The charge
Heat escapes from top surface of the charge. The expressions for convection and radiation losses were added on to eqn.(6.18) to give resultant power in the top segment as
\[ w''_{3(10)} = w'_{3(10)} \]

\[ - h_3 \cdot \varepsilon_3 \cdot A_{3(10)} \cdot (\theta_{3(10)} - \theta_a) \]

\[ - \varepsilon_3 \cdot A_{3(10)} \cdot \sigma \cdot (T_{3(10)}^4 - T_a^4) \quad (6.37) \]

where

\[ T_3 = \theta_3 + 274 \, (^{\circ}K) \]

Substituting for \( w'_{3(10)} \), dividing through by heat capacity and collecting like terms on the RHS, the resultant temperature derivative for the top segment of the charge is found. The subroutine code is included in SUB19L shown in Appendix A6.5.

6.9 Results of Simultaneous Solution Including Heat Losses

To verify the method it was necessary to show that certain predictions agree with known practical data. The method was thus applied to a well-packed charge heated from cold in an A60 size crucible at 3kHz and 100kW tilter furnace. The temperature time curves for silicon carbide crucible with aluminium and copper charge are shown in Figs. 6.12 and 6.13.

The results show that all the aluminium charge (Fig. 6.12) reaches pouring temperature of 720\(^{\circ}C\) in 11 minutes and the copper charge (Fig. 6.13) is all at 1200\(^{\circ}C\) in about 29 minutes which are in line with manufacturers' data.

The axial temperature gradients are used to correlate thermal stresses at different modes which are likely to cause failure. The results may be thus reproduced in such a manner as to depict the transient temperature
Fig. 6.12 Temperature-time curves for the disc and walls of silicon carbide crucible with aluminium charge, obtained by simultaneous heat transfer solution including heat losses.
Fig. 6.13 Temperature time curves for the disc and wall of silicon carbide crucible with copper charge, obtained by simultaneous heat transfer solution including heat losses.
Fig. 6.14 Transient temperature distributions in crucible and charge during the melting cycle.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 C
CHARGE: aluminium PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 11 mins.
Fig. 6.15 Transient temperature distributions in crucible and charge during the melting cycle.

FURNACE:
- Tilter
- Frequency: 3 kHz
- Power Input: 100 kW

CRUCIBLE:
- C-silicon carbide
- Axial Resistivity: non-uniform
- Initial Temperature: 20°C

CHARGE:
- Copper
- Packing: well-packed
- Initial Temperature: 20°C
- Time to Reach Pouring Temperature: 30 mins.
Fig. 6.16 Transient temperature distributions in crucible and charge during the melting cycle.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20°C
CHARGE: aluminium PACKING: well-packed INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 19 mins.
Fig. 6.17 Transient temperature distributions in crucible and charge during the melting cycle.

FURNACE: tilt filler FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 C
CHARGE: copper PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 47 mins.
distribution in crucible and charge axially. The results of temperature distribution are shown in Figs. 6.14 to 6.17 heated to pouring temperatures of charge.

6.10 Conclusions

A method was devised to obtain the transient temperature distribution in the crucibles successfully, taking into account the heat transfers and detailed heat losses involved as well as the variation of relevant properties with temperature. The results showed that the times elapsed to reach melting/pouring temperatures correspond to those quoted by manufacturers for similar practical cases. The technique can be used to predict temperature distribution for detail configurations and this is done in Chapter 7 under the explanation of the modes of failure. The stepwise method was found to be good enough as a first approximation but required modification by simultaneous solution.

References


6.9 International Critical Tables (McGraw-Hill, New York, 1929)

6.10 BUDNIKOV, P.P.: 'The technology of ceramics and refractories', translated by Scripta Technica, (Edward Arnold Pub., 1964)


6.17 MORGANITE THERMAL DESIGNS LTD: Data sheet, Ref. No.FD 88/9-79


CHAPTER 7

MODES OF FAILURE

The method devised in the previous chapter is used here to predict temperature distribution within the crucibles for various configurations of furnace, crucible and charge in order to explain the crucible failure.
7.1 Introduction

It has been established that temperature gradients as well as actual temperatures cause thermal stresses which can lead to failure in the crucibles. A method has been detailed in the previous chapter that can predict transient temperature distribution within the conducting crucibles used in induction melting furnaces. It is used to assess the likelihood of failure under various operating conditions. In doing so, several variables representing various modes of operation need to be looked at. They include: the type of furnace, operating frequency and power input; type of crucible, properties and initial temperatures; type of charge, packing density and starting temperature. The design of induction furnace should be considered, whether tilter, push-up or lift-swing as the construction affects heat losses. As for the crucible, the two kinds are considered: clay-graphite and silicon carbide. Emphasis is put on the latter type. The initial crucible temperatures may also influence the results, whether cold or preheated which may be from the previous heat. The dimensions and chemical compositions also ought to be looked at although these are fixed for any given crucible by the manufacturers.

Finally, for the charge, the alloy should be specified. Pure aluminium and copper are used to represent the range of copper alloys normally melted in such crucibles. The grade and quality of scrap determine how it can be packed in the crucible; loosely or tightly. One needs to distinguish also between charging from cold, preheated or molten state although the latter is not common practice because these furnaces are not often used for holding.
In total, 1500 or so sets of results would be required to cover the above possibilities. For brevity however, combination of the more important variables are only considered as shown in Fig. 7.1, and different modes of failure are featured by showing the effect of these on the temperature distributions. In Fig. 7.1 dotted lines indicate areas which are less prone to failure. The wordings on the left refer to the particular variables considered. Parameters like crucible dimensions and composition as well as power inputs below 100 kW have been omitted from the Figure.

A set of nominal values representing typical installations were chosen so the effect of different parameters could be shown relatively by departing from these nominal values. Unless otherwise stated, the results were based on 100 kW, 3 kHz tilting furnace using A60 size crucibles with well-packed aluminium or copper charge heating up from cold.

The assessment of the likelihood of failure is based on comparisons made of the particular crucible temperatures and local temperature gradients against various datum situations. Practical failure tests on push-up furnaces of similar size, rating etc have been carried out [7.1] which have shown acceptable lifetimes for silicon carbide crucibles when operating at 1 kHz, 100 kW power input. Similarly acceptable lifetimes for clay-graphite crucibles have been achieved at 3 kHz, 100 kW. Figures 7.2 - 7.5 show the corresponding temperature distributions which have been used as an example of the limit of the safe operating range, against which other results may be compared.

It was considered that assessment of the likelihood of failure based solely on thermal resistance parameters was not satisfactory because of the poor data on material properties available. Extensive practical testing has been carried out which has established acceptable lifetimes for these crucibles and a comparison with these results was thought to be the most conclusive. Full details are given in reference 7.1.
Fig. 7.1. Combination of important variables. (a) furnace types and characteristics, (b) crucible, (c) charge. Solid lines represent main problem areas.
The temperature distribution results show that temperature gradients increase towards the end of the melting cycle. They also show that there are typically two important areas of concern, i.e. the mid-wall region and the lower wall region just above the crucible bottom. This is in agreement with reported cases of failure. The large temperature gradients apparent at the top of the crucibles when melting copper charge, are due to heat losses from the top surface of the crucibles associated with longer melting cycles of copper. Details of the temperature and typical temperature gradients relevant to the safe operating conditions of conducting crucibles are given in Table 7.1.

7.2 Effect of Operating Frequency

Operating frequencies of 1 kHz, 2 kHz and 3 kHz are commonly associated with conducting crucible furnaces. Acceptable crucible life-times have been achieved with silicon carbide crucibles operating at 1 kHz [7.1] and certain manufacturers do recommend 2 kHz operation.

Higher frequency reduces stirring which in turn reduces the heat transfers between the charge and crucible but this is not discussed here. The proportion of power induced in the crucible increases with the frequency as shown in Chapter 3, and so larger temperature gradients can be expected in the pot.

Figures 7.6 - 7.9 show the temperature distribution within silicon carbide crucibles when melting aluminium and copper respectively. The operating frequencies are 2 kHz and 3 kHz. Comparing Figs. 7.2, 7.6 and 7.7 with each other and Figs. 7.3, 7.8, 7.9 together, it can be seen that the
### Table 7.1: safe operating conditions

<table>
<thead>
<tr>
<th>CRUCIBLE</th>
<th>Silicon carbide @ 1 kHz, 100 kW</th>
<th>Clay - graphite @ 3 kHz, 100 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRUCIBLE</td>
<td>aluminium</td>
<td>copper</td>
</tr>
<tr>
<td>CHARGE</td>
<td>mid-wall</td>
<td>Bottom</td>
</tr>
<tr>
<td>CRITICAL ZONE</td>
<td>mid-wall</td>
<td>Bottom</td>
</tr>
<tr>
<td>MAXIMUM TEMPERATURE (°C)</td>
<td>754.5</td>
<td>791.5</td>
</tr>
<tr>
<td>CRITICAL TEMPERATURE GRADIENT (K)</td>
<td>18.1</td>
<td>20</td>
</tr>
</tbody>
</table>
increased frequency increases the maximum temperature attained by the crucible and the local temperature gradients within the critical areas of the crucible especially with the lower electrical conductivity (aluminium) charge. For example, when the aluminium charge has reached its pouring temperature (720°C) the crucible bottom is over 950°C and with a local temperature difference between segments of 38K which is more than twice that in the safe operating case of Fig. 7.2. The analysis suggests silicon carbide crucibles can be operated successfully at 2 kHz if the conductivity of alloy is close to that of pure copper. Operation with alloys of conductivities close to that of aluminium is, however, not advisable even at 2 kHz. See also Appendix A7.1.

7.3 Effect of Power Density

Typically total furnace power inputs of between 75 kW and 150 kW are associated with A60 size silicon carbide crucibles. With larger sizes, 200 kW are also common. Figures 7.10-7.13 show the temperature distributions in silicon carbide crucibles with aluminium and copper charges at 75 kW and 150 kW power levels. These Figures should be compared with Figs. 7.7 and 7.9 which are for power levels of 100 kW at the same frequency etc and they should be compared with Figs. 7.2 and 7.3 for safe operating conditions.

The results suggest that silicon carbide crucibles will fail in 3 kHz operation even at power levels as low as 75 kW. Crucible temperatures around 900°C when melting aluminium charges are nearly 100 K greater than those associated with the safe operating range of Fig. 7.2. Temperature gradients are also increased by 70%. The results also suggest that with
higher conductivity alloys, acceptable life-times may be possible although crucible temperatures of 1290°C are achieved even at 75 kW power levels which approaches the limiting crucible values [7.1].

Applying the analysis to clay-graphite crucibles did show greater sensitivity to power density levels. The resulting temperature distributions with aluminium and copper charges are shown in Figs. 7.14 and 7.15. Comparison of these with Figs. 7.4 and 7.5 indicates that operation at 3 kHz and power levels in excess of 150 kW would decrease the expected life-time appreciably.

7.4 Effect of Type of Crucible
Carbon bonded silicon carbide crucibles have higher electrical conductivities than clay bonded graphite crucibles and a larger proportion of energy is induced directly into the crucible. The resulting effect on power allotment and stirring forces within the melt were discussed in Chapter 3.

Temperature distributions for silicon carbide crucibles at 3 kHz and 100 kW are shown in Figs. 7.7 and 7.9. These should be compared with Figs. 7.4 and 7.5 for effect of crucible material and with Figs. 7.2 and 7.3 for safe operating conditions.

The results show faster melting when silicon carbide crucibles are used with even as much as a 40% reduction in melt time from cold. The industrially used combinations of 1 kHz for silicon carbide and 3 kHz for clay-graphite give very similar melting times. However, the maximum
crucible temperatures and local temperature gradients are more severe in
the silicon carbide case even at 1 kHz. It is also quite obvious from
Figs. 7.7 and 7.9 that the silicon carbide crucibles are more susceptible
to failure than the clay-graphite especially when melting lower
conductivity alloys.

7.5 Effect of Initial Crucible Temperature

Most jobbing foundries using such crucibles tend to charge the scrap into
cold crucibles. They are seldom preheated although it is recommended
practice aimed to lengthen crucible life-time; especially when pots are
new. Nevertheless, crucibles tend to be preheated indirectly from the
previous heat. It is difficult to estimate to what temperature they drop
to before the next charge but in this analysis half the pouring
temperature was assumed. As for directly preheated pots, one third of
the charge pouring temperature was used in the calculations.

Temperature distributions for silicon carbide crucibles with initial
temperatures above ambient are shown in Figs. 7.16-7.19. Comparisons of
these Figures with Figs. 7.7 and 7.9 for crucibles starting at room
temperature, shows that higher initial crucible temperatures have little
improving effect on the likelihood of failure. Similar results were
obtained for clay-graphite crucibles as shown in Figs. 7.20-7.23 which
should be compared with Figs. 7.4 and 7.5 that are for crucibles starting
from cold. With 1% reduction in critical temperature gradients and no
reduction in maximum temperatures attained, the results therefore indicate
that preheating pots has little effect on improving life-time expectancy
and cannot be recommended as being worthwhile. The melting cycles were,
however, decreased as expected.

7.6 Effect of charge
Throughout this work aluminium and copper have been used as typical charge materials representing the range of BS1400 copper alloys. The crucible temperature distribution with each charge material have been covered by the results already given in the previous sections. It is clear that the worst effects in crucible temperature distributions occur when the lower electrical conductivity (aluminium) charges are used.

7.7 Effect of Charge Packing Density
Medium frequency induction melting furnaces are praised for reducing metal losses when melting low grade scrap. They can be started without a molten heel but it is good foundry practice to have high packing of scrap which would enhance the heat transfer between crucible and charge that could be beneficial when silicon carbide crucibles are used. The charge packing and grade of scrap are represented by a contact area factor $\beta$ (Section 6.4) which is given a value of 1.0 for good packing of stampings or swarf, and a value of 0.1 for loosely-packed returns and ingots. An average value of 0.5 is used to represent a mixture.

The effect of charge packing density on the temperature distribution for silicon carbide crucibles is shown in Figs. 7.24-7.27 by comparison with Figs. 7.7 and 7.9. With loosely-packed charge of aluminium the critical temperature gradients and maximum crucible temperature are increased by up to 25%. With loosely-packed copper charge, the critical temperature gradients are doubled but the maximum temperatures are only increased by
about 4% due to the longer melting cycle. The results suggest that the packing density can have a major influence on crucible life-time, and well-packed charges must be considered compulsory if long crucible life-times are to be achieved.

The effect of charge packing density on the temperature distribution for clay-graphite crucibles is shown in Figs. 7.28-7.31 by comparison with Figs. 7.2 and 7.3. The results suggest that a loosely-packed charge can be the cause of failure of even clay-graphite crucibles especially when melting lower conductivity alloys.

In this simplified analysis, $\beta$ is assumed constant throughout the melting cycle. In practice as the molten heel forms the contact area between charge and crucible at the bottom increases. This also applies to other regions as melting occurs. Therefore, if for example, the value of $\beta$ is changed from an initial value of 0.1 to 1.0 as melting commences, the results would be closer to practical situation. This phenomenon is shown for mid-wall temperature of silicon carbide crucible in Fig. 7.32 with aluminium charge. It can be seen that the peak temperature of the crucible was reached just before heel formation after which it dropped to a lower value. This was then followed by a second rise as the charge reached its pouring temperature. In doing so, the crucible and charge temperature shown in Fig. 7.33 stayed very close to each other and manifested a more uniform temperature profile. The results with copper charge are shown in Figs. 7.34 and 7.35. Superheat for pouring temperature of copper is almost twice that for aluminium.
7.8 **Effect of Preheated Scrap**

It is possible to preheat the scrap to, say, 300°C or to one third of its pouring temperature before charging in order to burn off any grease or oil on the surface.

Temperature distributions for silicon carbide crucibles with preheated scrap are shown in Figs. 7.36-7.39. Comparison of these Figures with Figs. 7.7 and 7.9 for cold scrap, showed that preheating the charge changed the critical temperature gradients by 1%. Similar results were obtained for clay-graphite crucibles as shown in Figs. 7.40-7.43 which should be compared with Figs. 7.4 and 7.5 that are for cold scrap. Preheating of scrap cannot therefore be recommended on the grounds that it does not appreciably enhance crucible life-times. The melting cycles were, however, decreased as expected.

7.9 **Effect of Type of Furnace**

Types of furnace were discussed in Chapters 1 and 2. The tilting induction furnace was shown in Figs. 1.2 and 2.8. In the calculations so far, all dimensions refer to the furnace and coil installation using size 460 crucibles. The small tilting furnace, although using prefired crucibles, is similar to the rammed crucible type tilters because the prefired crucibles are secured in place by refractory powder. This makes push-up furnaces more popular as they offer the desired flexibility through changing crucibles. Lift-swing induction furnaces are rarely used compared with push-up and tilter partly because of their poor electrical coupling.
Fig. 7.2 Crucible and charge transient temperature distribution during melting considered to be safe operating conditions.

FURNACE: tilter FREQUENCY: 1 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform INITIAL TEMPERATURE: 20°C
CHARGE: aluminium PACKING: well-packed INITIAL TEMPERATURE: 20°C TIME TO REACH POURING TEMPERATURE: 19.5 mins.
Fig. 7.3 Crucible and charge transient temperature distribution during melting considered to be safe operating conditions.

FURNACE: 
- tilting FREQUENCY: 1 kHz
- POWER INPUT: 100 kW

CRUCIBLE: 
- C-silicon carbide AXIAL RESISTIVITY: non-uniform
- INITIAL TEMPERATURE: 20°C

CHARGE: 
- copper PACKING: well-packed
- INITIAL TEMPERATURE: 20°C
- TIME TO REACH POURING TEMPERATURE: 59 mins.
Fig. 7.4 Crucible and charge transient temperature distribution during melting considered to be safe operating conditions.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: non-uniform INITIAL TEMPERATURE: 20°C
CHARGE: aluminium PACKING: well-packed INITIAL TEMPERATURE: 20°C TIME TO REACH POURING TEMPERATURE: 19 mins.
Fig. 7.5  Crucible and charge transient temperature distribution during melting considered to be safe operating conditions.

FURNACE:  filler FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE:  clay-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 C
CHARGE:  copper PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 47 mins.
Effect of frequency on temperature distribution.

**FURNACE:**  
- **FREQUENCY:** 2 kHz  
- **POWER INPUT:** 100 kW  
- **C-silicon carbide**  

**CRUCIBLE:**  
- **AXIAL RESISTIVITY:** non-uniform  
- **INITIAL TEMPERATURE:** 20°C  

**CHARGE:**  
- **aluminium**  
- **PACKING:** well-packed  
- **INITIAL TEMPERATURE:** 20°C  
- **TIME TO REACH POURING TEMPERATURE:** 13.5 mins.
Fig. 7.7  Effect of frequency on temperature distribution.

FURNACE: tilter  FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide  AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20°C
CHARGE: aluminium  PACKING: well-packed  INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 11 mins.
Fig. 7.8 Effect of frequency on temperature distribution.

FURNACE: tiltter FREQUENCY: 2 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform INITIAL TEMPERATURE: 20°C
CHARGE: copper PACKING: well-packed INITIAL TEMPERATURE: 20°C TIME TO REACH POURING TEMPERATURE: 33 mins.
Fig. 7.9 Effect of frequency on temperature distribution.

Furnace: Tilted FREQUENCY: 3 kHz POWER INPUT: 100 kW
Crucible: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20°C
Charge: Copper PACKING: well-packed INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 30 mins.
Fig. 7.10 Effect of power density on temperature distribution.

FURNACE: tilter  FREQUENCY: 3 kHz  POWER INPUT: 75 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 C
CHARGE: aluminium  PACKING: well-packed  INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 15 mins.
Fig. 7.11 Effect of power density on temperature distribution.

FURNACE: tilter
FREQUENCY: 3 kHz
POWER INPUT: 150 kW

CRUCIBLE: C-silicon carbide
AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20°C

CHARGE: aluminium
PACKING: well-packed
INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 7.5 mins.
Fig. 7.12  Effect of power density on temperature distribution.

FURNACE:  tilt frequency: 3 kHz  power input: 75 kW
CRUCIBLE:  C-silicon carbide axial resistivity: non-uniform
            initial temperature: 20 C
CHARGE:    copper packing: well-packed initial temperature: 20 C
            time to reach pouring temperature: 46 mins.
**Fig. 7.13** Effect of power density on temperature distribution.

**FURNACE:**
- Tilt: 3 kHz
- Power Input: 150 kW

**CRUCIBLE:**
- C-silicon carbide
- Axial Resistivity: non-uniform
- Initial Temperature: 20°C

**CHARGE:**
- Copper
- Packing: well-packed
- Initial Temperature: 20°C
- Time to Reach Pouring Temperature: 18.5 mins.
Fig. 7.14 Effect of power density on temperature distribution.

FURNACE: tilt arrangement FREQUENCY: 3 kHz POWER INPUT: 150 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20°C
CHARGE: aluminium PACKING: well-packed INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 12.5 mins.
Fig. 7.15 Effect of power density on temperature distribution.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 150 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 C
CHARGE: copper PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 29 mins.
Fig. 7.16 Effect of crucible preheating on temperature distribution.

FURNACE:
- Tiitter
- FREQUENCY: 3 kHz
- POWER INPUT: 100 kW

CRUCIBLE:
- C-silicon carbide
- AXIAL RESISTIVITY: non-uniform
- INITIAL TEMPERATURE: 240°C (preheated)

CHARGE:
- Aluminium
- PACKING: well-packed
- INITIAL TEMPERATURE: 20°C
- TIME TO REACH POURING TEMPERATURE: 11 mins.
Fig. 7.17 Effect of initial crucible temperature on temperature distribution.

FURNACE: tiltter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 360 C (hot from previous heat)
CHARGE: aluminium PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 10.5 mins.
Fig. 7.18 Effect of crucible preheating on temperature distribution.

FURNACE:  
- Frequency: 3 kHz  
- Power Input: 100 kW

CRUCIBLE:  
- C-silicon carbide  
- Axial Resistivity: non-uniform  
- Initial Temperature: 400°C (preheated)

CHARGE:  
- Copper  
- Packing: well-packed  
- Initial Temperature: 20°C  
- Time to Reach Pouring Temperature: 20 mins.
Fig. 7.19  Effect of initial crucible temperature on temperature distribution.

FURNACE:  
- Tilt Frequency: 3 kHz  
- Power input: 100 kW

CRUCIBLE:  
- C-silicon carbide  
  - Axial Resistivity: non-uniform
  - Initial Temperature: 600°C (hot from previous heat)

CHARGE:  
- Copper  
- Packing: well-packed  
  - Initial Temperature: 20°C

TIME TO REACH POURING TEMPERATURE: 29 min.
Fig. 7.20 Effect of initial crucible temperature on temperature distribution.

FURNACE: tilt  FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE: clay-graphite  AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 240 C (preheated)
CHARGE: aluminium  PACKING: well-packed  INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 18 mins.
Fig. 7.21 Effect of initial crucible temperature on temperature distribution.

FURNACE: tilter  FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE: clay-graphite  AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 360 C (hot from previous heat)
CHARGE: aluminium  PACKING: well-packed  INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 17.5 mins.
Fig. 7.22 Effect of initial crucible temperature on temperature distribution.

FURNACE: tiltter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 400 C (preheated)
CHARGE: copper PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 44 mins.
Fig 7.23 Effect of initial crucible temperature on temperature distribution.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 600 C (hot from previous heat)
CHARGE: copper PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 41 mins.
Fig. 7.24 Effect of charge packing on temperature distribution.

Furnace: tiltcr frequency: 3 kHz power input: 100 kW
Crucible: C-silicon carbide axial resistivity: non-uniform
Initial temperature: 20 C
Charge: aluminium packing: medium-packed initial temperature: 20 C
Time to reach pouring temperature: 12 mins.
Fig. 7.25 Effect of charge packing on temperature distribution.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform INITIAL TEMPERATURE: 20 C
CHARGE: aluminium PACKING: loosely-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 13 mins.
Fig. 7.26 Effect of charge packing on temperature distribution.

FURNACE: tiltler  FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide  AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20°C
CHARGE: copper  PACKING: medium-packed  INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 31 mins.
Fig. 7.27 Effect of charge packing on temperature distribution.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20°C
CHARGE: copper PACKING: loosely-packed INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 32.5 mins.
Furnace: tilt, frequency: 3 kHz, power input: 100 kW
Crucible: clay-graphite, axial resistivity: non-uniform, initial temperature: 20°C
Charge: aluminium, packing: medium-packed, initial temperature: 20°C
Time to reach pouring temperature: 19 min

Fig. 7.28 Effect of charge packing on temperature distribution.
Fig. 7.29 Effect of charge packing on temperature distribution.

FURNACE: tilt for FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20°C
CHARGE: aluminium PACKING: loosely-packed INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 19 min
Fig. 7.30 Effect of charge packing on temperature distribution.

**FURNACE:** tiltler  **FREQUENCY:** 3 kHz  **POWER INPUT:** 100 kW

**CRUCIBLE:** clay-graphite  **AXIAL RESISTIVITY:** non-uniform  **INITIAL TEMPERATURE:** 20°C

**CHARGE:** copper  **PACKING:** medium-packed  **INITIAL TEMPERATURE:** 20°C  **TIME TO REACH POURING TEMPERATURE:** 47 min
Fig. 7.31 Effect of charge packing on temperature distribution.

FURNACE: tilting FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 °C
CHARGE: copper PACKING: loosely-packed INITIAL TEMPERATURE: 20 °C
TIME TO REACH POURING TEMPERATURE: 47 min
Fig. 7.32 Temperature-time curve showing effect of increased contact area factor B on the crucible temperature when molten aluminium heel forms
Fig. 7.33 Effect of increased contact area factor on temperature distribution at the formation of the molten heel.

**FURNACE:** tilting  
**FREQUENCY:** 3 kHz  
**POWER INPUT:** 100 kW  

**CRUCIBLE:** C-silicon carbide  
**AXIAL RESISTIVITY:** non-uniform  
**INITIAL TEMPERATURE:** 20 °C  

**CHARGE:** aluminium  
**PACKING:** loosely-packed  
**INITIAL TEMPERATURE:** 20 °C  
**TIME TO REACH POURING TEMPERATURE:** 12 mins.
Fig. 7.34 Temperature-time curve showing effect of increased contact area factor $\beta$ on the crucible temperature when molten copper heel forms.
Fig. 7.35 Effect of increased contact area factor on temperature distribution at the formation of the molten heel.

FURNACE: tilting FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 °C
CHARGE: copper PACKING: loosely-packed INITIAL TEMPERATURE: 20 °C
TIME TO REACH POURING TEMPERATURE: 32.5 mins.
Fig. 7.36  Effect of charge preheating on temperature distribution.

FURNACE:  tilt  FREQUENCY:  3 kHz  POWER INPUT:  100 kW
CRUCIBLE:  C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE:  240 C
CHARGE:  aluminium  PACKING: well-packed  PREHEATED TO:  240 C
TIME TO REACH POURING TEMPERATURE:  9 mins.
Fig. 7.37 Effect of charge preheating on temperature distribution.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 300 C
CHARGE: aluminium PACKING: well-packed PREHEATED TO: 300 C
TIME TO REACH POURING TEMPERATURE: 8 mins.
Fig. 7.38 Effect of charge preheating on temperature distribution.

FURNACE:  tilter FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 400°C
CHARGE:  copper PACKING: well-packed PREHEATED TO: 400°C
TIME TO REACH POURING TEMPERATURE: 24 mins.
Fig. 7.39 Effect of charge preheating on temperature distribution.

FURNACE: tiltter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 300 C
CHARGE: copper PACKING: well-packed PREHEATED TO: 300 C
TIME TO REACH POURING TEMPERATURE: 26 mins.
Fig. 7.40 Effect of charge preheating on temperature distribution.

**FURNACE:** tiltler  
**FREQUENCY:** 3 kHz  
**POWER INPUT:** 100 kW  
**CRUCIBLE:** cly-graphite  
**AXIAL RESISTIVITY:** non-uniform  
**INITIAL TEMPERATURE:** 240°C  
**CHARGE:** aluminium  
**PACKING:** well-packed  
**PREHEATED TO:** 240°C  
**TIME TO REACH POURING TEMPERATURE:** 14 mins.
Fig. 7.41  Effect of charge preheating on temperature distribution.

FURNACE: tilter  FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE: cly-graphite  AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 300 C
CHARGE: aluminium  PACKING: well-packed  PREHEATED TO: 300 C
TIME TO REACH POURING TEMPERATURE: 12 mins.
Fig. 7.42 Effect of charge preheating on temperature distribution.

FURNACE: t i l t e r FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: c y - g r a p h i t e AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 400 C
CHARGE: c o p p e r PACKING: well-packed PREHEATED TO: 400 C
TIME TO REACH POURING TEMPERATURE: 33 mins.
Fig. 7.43 Effect of charge preheating on temperature distribution.

FURNACE: tilting FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: cly-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 300 C
CHARGE: copper PACKING: well-packed PREHEATED TO: 300 C
TIME TO REACH POURING TEMPERATURE: 37 mins.
Fig. 7.44 Effect of furnace type on temperature distribution.

FURNACE: push-up  FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide  AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 C
CHARGE: aluminium  PACKING: well-packed  INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 10.5 mins.
Fig. 7.45 Effect of furnace type on temperature distribution.

FURNACE: push-up FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform  INITIAL TEMPERATURE: 20 °C
CHARGE: copper  PACKING: well-packed INITIAL TEMPERATURE: 20 °C  TIME TO REACH POURING TEMPERATURE: 28.5 mins.
Fig. 7.46 Effect of furnace type on temperature distribution.

FURNACE: push-up  FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE: clay-graphite  AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20°C
CHARGE: aluminium  PACKING: well-packed  INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 18 mins.
Fig. 7.47 Effect of furnace type on temperature distribution.

FURNACE: push-up FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 °C
CHARGE: copper PACKING: well-packed INITIAL TEMPERATURE: 20 °C
TIME TO REACH POURING TEMPERATURE: 43.5 mins.
The effect of furnace type on temperature distributions for silicon carbide is shown in Figs. 7.44 and 7.45 by comparison with Figs. 7.7 and 7.9 which are for tilting furnace. The results show that maximum crucible temperature and critical temperature gradients are increased by up to 7% as a result of reduced heat losses. However, the crucible will have more freedom to expand in the push-up furnace and will be subjected to lower stresses overall. Similarly, the effect of furnace type for clay-graphite is shown in Figs. 7.46 and 7.47 by comparison with Figs. 7.4 and 7.5 giving about 5% rise in the maximum temperature and critical temperature gradients. The results therefore indicate that push-up type of furnace does not affect the crucible life-time significantly.

Thermal stresses in crucibles were calculated using the temperature profiles obtained, to confirm comparison of results of typical modes of operation (Appendix A7.2).

7.10 Summary of the Results - Conclusions
Temperature distributions in carbon bonded silicon carbide and clay bonded graphite crucibles used in tilting and push-up induction furnaces for melting copper alloys, were obtained. The following observations were made from the point of view of increasing crucible life-time.

7.10.1 Operating frequency: Longer crucible life-times may be obtained at lower operating frequencies. Silicon carbide crucibles may be safely used at 2 kHz, 100 kW with high conductivity alloys.
7.10.2 Power density: Silicon carbide crucibles can be used safely with higher conductivity alloys at below 75 kW power level and 3 kHz. Clay-graphite crucibles will fail at power levels in excess of 150 kW, 3 kHz.

7.10.3 Crucible type: Silicon carbide crucibles are more prone to failure than clay-graphite even at 1 kHz operating frequency, especially when lower conductivity alloys are melted.

7.10.4 Initial crucible temperature: Preheating crucibles has little effect in the improvement in the criterion for failure and does not enhance the life-time expectancy.

7.10.5 Charge packing density: Loosely-packed charge can be the cause of failure of even clay-graphite crucibles. Packing density can have a major influence on crucible life-time and well-packed charges are advocated.

7.10.6 Preheated scrap: Preheating scrap does not have any significant effect on crucible life-time and is not strongly recommended.

7.10.7 Type of furnace: Crucibles behaved similarly in both push-up and tilting induction furnaces. Type of furnace has little effect on life-time expectancy of crucibles.

References


CONCLUSIONS AND POSSIBLE REMEDIES

The results and their implications are summarized and suggestions are put forward to implement the theory to reduce the likelihood of premature failure of conducting crucibles in service.
CONCLUSIONS AND POSSIBLE REMEDIES

The prime objective of this project that was to establish the reasons for premature crucible failure, has been fulfilled. A number of papers have been produced based on the results of different chapters. In order to emphasize the main accomplishments on route to the final objective, a summary of results and what has been done, is given in the following Section in chronological order.

Possible remedies are described in Section 8.2 aimed at making the use of silicon carbide crucibles feasible.

8.1 Summary of the Research

(1) Past work related to all aspects of induction melting of non-ferrous metals and analysis of conducting crucible furnace were reviewed to assess the state of the art.

(2) The mathematical analysis of induction heating of composite loads based on equivalent circuit technique was provided.

(3) The analysis was adapted to be applied in the form of a computer program with greater flexibility and without the need for approximations.

(4) The analysis was applied to induction furnaces using electrically conducting crucibles of carbon bonded silicon carbide and clay bonded graphite to assess importance of various furnace parameters.
(5) Effect of operating frequency, power density, electrical resistivity of crucibles and crucible wall thickness upon the furnace power distribution and meniscous height in the molten charge were investigated. The importance of crucible electrical resistivity in furnace performance was highlighted.

(6) Probes were made to measure current density and magnetic field strength on the surface of silicon carbide and clay-graphite crucibles to estimate their electrical resistivities by a non-destructive method. Axial variation of crucible resistivities were obtained.

(7) The probe measurements were justified by surface examination of segments of silicon carbide crucible material under scanning electron microscope.

(8) Other temperature dependent properties of both types of crucibles such as specific heat and thermal conductivity were estimated between ambient and melting temperature of copper based on available manufacturers' data on the composition of crucibles.

(9) The analysis was refined to cater for three dimensional application and for axial variation of crucible properties.

(10) Axial variation of power density was obtained leading to axial variation of rates of temperature rise in the crucibles.

(11) To assess thermal behaviour of crucibles throughout the melting cycle,
Transient temperature distributions were obtained for crucible and charge using numerical integration of temperature derivatives derived by heat balance equation; taking into account heat transfers between crucible and charge, heat losses and non-linear variation of crucible and charge properties during the melting cycle.

(12) The method was applied to predict temperature distributions for various configurations of melting copper alloys in conducting crucible induction furnaces to establish critical modes of failure.

(13) Effect of operating frequency, power density, crucible type, initial temperature, type of charge, packing density, preheating scrap and furnace type on crucible failure or otherwise were investigated by comparison of maximum temperatures and local temperature gradients.

(14) From an overall picture of the melting cycle, safe operating conditions were determined.
8.2 Possible Remedies

Finding the source of the problem and the cause of crucible failure at different modes of operation, which was the main task of the work, has been carried out. It is timely to put forward ways of alleviating the problem. Based on the findings so far, some methods of enhancing crucible life-time can therefore be outlined here.

There are many courses of action which this work suggests may help remedy the problem of premature failure of silicon carbide crucibles when used at high frequencies and high power densities. Essentially the possible remedies fall into two categories: firstly those associated with good foundry practice and secondly those associated with the electromagnetic coupling between the furnace coil and the crucible.

General preventative measures may be taken depending on individual foundry discipline. Crucibles should be stored in a clean and dry environment and preferably in an upright position. Crucibles should be kept clean otherwise layers of dross form which act as strong heat barriers. Dross also expands more than the crucible. Thick dross layers can therefore burst the crucible when heated.

The crucibles should be handled with care especially in transportation to prevent cracking or chipping which could promote crucible failure. Crucibles should be transported in trolleys and not be rolled like beer barrels. The easily damaged glaze cracks off if the crucible is rolled and the exposed ceramic will become more prone to failure through embrittlement.
When charging, pieces of metal should not be thrown into the crucible. Tongs should be used to prevent brakage, otherwise liquid metal will penetrate into the cracks formed which will cause the crucible to burst when it is heated up again. Fluxes should be applied shortly before pouring metal. If mixed in too long before the metal is poured, chemical attack to the crucible ware will occur which will shorten crucible lifetime.

After melting a charge, the crucible should be emptied as soon as possible, leaving no residue in the pot. If overheated metal is left standing in the crucible, grooves are eaten into the wall of the pot at melt level which weakens the wall. If this persists, metal level should be varied from charge to charge.

The analysis has shown that charge packing is of great importance if premature failure is to be avoided. Ideally a high packing density charge is required with low grade scrap tightly packed at the bottom of crucible and the larger pieces on top; taking care not to allow bridging to occur.

Non-uniformity of crucible resistivity and the subsequent power density and temperature distribution established in the crucible is the main cause of crucible failure. Any change in the method of manufacture of the crucibles which would decrease the non-uniformity of resistivity would be a major step forward. Figures 8.1 and 8.2 show the resulting temperature-time variations for a uniform resistivity silicon carbide crucible at 3 kHz, 100 kW with aluminium and copper charges. These
Fig. 8.1 Effect of uniform resistivity on temperature distribution.

- **FURNACE:** Tilted FREQUENCY: 3 kHz  POWER INPUT: 100 kW
- **CRUCIBLE:** C-silicon carbide AXIAL RESISTIVITY: uniform
  INITIAL TEMPERATURE: 20°C
- **CHARGE:** Aluminium PACKING: well-packed INITIAL TEMPERATURE: 20°C
  TIME TO REACH POURING TEMPERATURE: 12 mins.
Fig. 8.2 Effect of uniform resistivity on temperature distribution.

FURNACE: tiltter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: uniform
INITIAL TEMPERATURE: 20°C
CHARGE: copper PACKING: well-packed INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 32 mins.
results should be compared with Figs. 7.2 and 7.3 of last chapter. It is obvious from the results that localized temperature gradients are reduced when uniform resistivity crucibles are used. See also Appendix A8.1.

Unfortunately those possible remedies associated with good foundry practice or crucible manufacture are very often beyond any direct control from the furnace manufacturer. However, there are a number of possible modifications to the furnace design itself which could help reduce the problems of premature crucible failure.

8.2.1 Changes in furnace design

The analysis has shown that a reduction in the operating frequency of silicon carbide crucible furnaces can extend the crucible life-time at the expense of increased melting times. The results suggest that acceptable crucible life-times can be achieved when operating at 2 KHz, 100 kW especially with high conductivity copper alloys. Similarly, if 3 kHz is used then the total power input must be reduced below 75 kW.

Furnace design, i.e.tilter, push-up or lift-swing, has little effect on the life-time expectancy of crucibles. Nevertheless, the magnitude of the thermal stresses in the crucible could be reduced by using a more pliable or lower thermal expansion type of refractory backing material, particularly in tilting furnaces to allow for crucible expansion. This would pack the crucible in firmly but at the same time leave room for play.
Fig. 8.3 Schematically drawn positions of coil and crucible relative to each other
There are however other methods of reducing the non-uniformity of temperature distribution within the crucible, i.e. by changing the magnetic coupling between the crucible and the furnace coil.

8.2.2 Axial position of crucible relative to furnace coil

Figure 8.3 shows schematically some of the various configurations of furnace coil and crucible. It is common practice to position the crucible in the centre of the furnace coil (Fig. 8.3(a)) or such that the bottom of the crucible and coil are at the same level (Fig. 8.3(b)). In both cases the analysis has shown that maximum temperatures and critical temperature gradients occur at the wall just above the very hot base or at the crucible mid-wall. If the crucible is lowered or the furnace coil is raised so that the crucible bottom or disc falls just outside the coil (Fig. 8.3(c)) then it will be subjected to a lower magnetic field strength. The top of the crucible wall is then subjected to higher magnetic field strength when lowered axially into the coil. The resultant lower temperature shown in Fig. 8.4 and lower thermal stresses produced near the bottom of the crucible wall will enhance crucible lifetime. The improvement in the local temperature gradients is obvious when compared with Fig. 7.3, and longer crucible lifetime is predicted but the melt time has been increased by 18% if a 50% reduction of power input to crucible bottom is assumed. The case of lower conductivity alloys is shown in Fig. 8.5 which when compared with Fig. 7.2 shows an improved profile with an increase of 36% in melt-time.

8.2.3 Tapering of crucible and furnace coil

Whilst the manufacturers of crucibles may not wish to change the
Fig. 8.4 Effect of positioning crucible disc outside coil on temperature distribution.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform INITIAL TEMPERATURE: 20°C
CHARGE: copper PACKING: well-packed INITIAL TEMPERATURE: 20°C TIME TO REACH POURING TEMPERATURE: 36 mins.
Fig. 8.5 Effect of positioning crucible disc outside coil on temperature distribution.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 C
CHARGE: aluminium PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 12 mins.
constituents of the silicon carbide mix, they may be able to vary the taper at the bottom of the crucible (Fig. 8.3 (d)). In Fig. 8.6 the results are shown when the crucibles have a taper of approximately 10°. Commercially available 'A' shape crucibles upon which this study is based have a taper of approximately 7°. From Fig. 8.6 it can be seen that increasing the crucible taper will decrease the magnetic coupling to the hot disc of the crucible bottom. The excessive temperature and temperature gradients are therefore reduced and longer crucible life-times can be expected. The incorporation of a 10° taper increases the melt time by 4%. With lower conductivity charge, the melt time is increased by 8% (Fig. 8.7).

It is more likely that a taper between the crucible and coil would be achieved in practice by an increase in the furnace coil diameter (Fig. 8.3 (e)). In Fig. 8.8 the reduction in the local temperature gradients at the bottom of the crucible by incorporating a taper of 15° in the furnace coil are shown. There is also a decrease in maximum crucible temperature. The profile for aluminium charge is shown in Fig. 8.9.

The critical temperature gradients within the crucible occur at the midpoint as well as the bottom of the crucible. It would be far less practical to introduce a varying taper angle along the length of the coil, and a variable coil pitch solution is another possibility. Fig. 8.10 shows the effect of increasing the coil pitch in the middle area of the crucible and should be compared with Fig. 7.3. It can be seen that the temperature distribution has been improved but at the expense of longer melt times, i.e. an increase of less than 4% if a reduction in local power
Fig. 8.6 Effect of crucible taper of 10 degrees on temperature distribution.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 C
CHARGE: copper PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 31.5 mins.
Fig. 8.7 Effect of crucible taper of 10 degrees on temperature distribution.

FURNACE: tilter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform 
INITIAL TEMPERATURE: 20°C
CHARGE: aluminium PACKING: well-packed INITIAL TEMPERATURE: 20°C TIME TO REACH POURING TEMPERATURE: 12 mins.
Fig. 8.8 Effect of tapering furnace coil by 15 degrees on temperature distribution.

**FURNACE:**
- Tilt angle FREQUENCY: 3 kHz
- POWER INPUT: 100 kW

**CRUCIBLE:**
- C-silicon carbide AXIAL RESISTIVITY: non-uniform
- INITIAL TEMPERATURE: 20°C

**CHARGE:**
- Copper PACKING: well-packed
- INITIAL TEMPERATURE: 20°C
- TIME TO REACH POURING TEMPERATURE: 43 mins.
Fig. 8.9 Effect of tapering furnace coil by 15 degrees on temperature distribution.

FURNACE: tilt type  
FREQUENCY: 3 kHz  
POWER INPUT: 100 kW  

CRUCIBLE: C-silicon carbide  
AXIAL RESISTIVITY: non-uniform  
INITIAL TEMPERATURE: 20 C  

CHARGE: aluminium  
PACKING: well-packed  
INITIAL TEMPERATURE: 20 C  
TIME TO REACH POURING TEMPERATURE: 14.5 mins.
Fig. 8.10  Effect of larger coil pitch on temperature distribution.

FURNACE:  Tiltter FREQUENCY: 3 kHz  POWER INPUT: 100 kW
CRUCIBLE:  C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 C
CHARGE:  Copper PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 32.5 mins.
Fig. 8.11 Effect of larger coil pitch on temperature distribution.

FURNACE: tilting
FREQUENCY: 3 kHz
POWER INPUT: 100 kW

CRUCIBLE: C-silicon carbide
AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 °C

CHARGE: aluminium
PACKING: well-packed
INITIAL TEMPERATURE: 20 °C
TIME TO REACH POURING TEMPERATURE: 12 mins.
**Fig. 8.12** Temperature distribution with combined electromagnetic remedies to illustrate flexibility of analysis.

- **Furnace:** tilter, Frequency: 3 kHz, Power Input: 100 kW
- **Crucible:** C-silicon carbide, Axial Resistivity: non-uniform
  Initial Temperature: 20°C
- **Charge:** Copper, Packing: well-packed
  Initial Temperature: 20°C
  Time to reach pouring temperature: 33.5 min
Fig. 8.13 Temperature distribution with combined electromagnetic remedies to illustrate flexibility of analysis.

FURNACE: tilt position FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: C-silicon carbide AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20 C
CHARGE: aluminium PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 13 mins.
input of 10% is assumed. Similarly the profile of Fig. 8.11 is produced with aluminium charge by assuming 20% reduction in local power input. This should be compared with Fig. 7.2 and shows an increase of 8% in melt time.

In order to show the flexibility of this analysis an attempt was made to combine the various possible electromagnetic remedies to produce an improved temperature distribution, Fig. 8.12. The profile of Fig. 8.12 has been produced by reducing the coil pitch in the mid-portion of the crucible and positioning the bottom of the crucible below that of the furnace coil. With a coil of this type the longest crucible life-time can be expected and operation with silicon carbide crucibles at 3 kHz and 100 kW could be envisaged albeit with longer melt-times. Assuming a 30% reduction in crucible disc power input and a 10% reduction in power input to mid-portion of crucible wall increases melt time by approximately 10%. Flatter temperature distributions are obtained even with lower conductivity alloys when combined remedies are applied as shown in Fig. 8.13. The profile was produced assuming a 50% reduction in crucible disc power input and a 20% reduction to the power input in the mid-portion of the crucible wall which increased the melt time by less than 25% when compared with Fig. 7.2.
APPENDIXES
APPENDIX A3.1

*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-

* Listing of HOTPOT program code *

*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-*-

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION U1,F,W,RO1,RO2,RO3,R1,R2,R3,R,
1 H1,H2,H3,S1,S2,S3,K1,KR,TCN,
2 MUR1,MUR2,MUR3,MU1,MU2,MU3,
3 GAMAI,GAMA2,GAMA3,SC1,SC2,SC3

DOUBLE PRECISION MC2,MC3
DOUBLE PRECISION C,T,PI,UO
COMPLEX AJ
DOUBLE PRECISION RAJ,GAJ
DOUBLE PRECISION OMEG
DOUBLE PRECISION DEL1,DEL2,DEL3
DOUBLE PRECISION A,A1,A2,A3
DOUBLE PRECISION XR,AM,LM
DOUBLE PRECISION XO
DOUBLE PRECISION RES2
DOUBLE PRECISION LAMDA
DOUBLE PRECISION X22,X23,X33, X1,X2,X3
DOUBLE PRECISION DD2,X22RT
COMPLEX F3
COMPLEX Z3
DOUBLE PRECISION RZ3,GZ3
COMPLEX F23
DOUBLE PRECISION RF23,GF23
COMPLEX ALF3,ALBTF3,F661,F662,F663,F664,F665,F666
DOUBLE PRECISION AL,BL,A22
DOUBLE PRECISION SR1,SR2,SR3
COMPLEX CA,CB,CC,CD
DOUBLE PRECISION DEL24,D
COMPLEX QC
DOUBLE PRECISION MQC,RQC,GQC
COMPLEX F1
DOUBLE PRECISION RF1,GF1
DOUBLE PRECISION A12
COMPLEX AA
COMPLEX ZF
DOUBLE PRECISION ZFT,ZFTNU
DOUBLE PRECISION N1,N1NU
COMPLEX RES1,RES1NU
DOUBLE PRECISION RR1,GR1,RR1NU,GR1NU
DOUBLE PRECISION Z1,Z1NU
DOUBLE PRECISION FI1,FI1NU
DOUBLE PRECISION FI,FINU
DOUBLE PRECISION I1,I1NU
DOUBLE PRECISION Q1,Q1NU
DOUBLE PRECISION ETA,ETANU
COMPLEX SYS
DOUBLE PRECISION EN3
DOUBLE PRECISION W1, W2, W3, W1NU, W2NU, W3NU
DOUBLE PRECISION WN1, WN2, WN3, WN1NU, WN2NU, WN3NU

DOUBLE PRECISION SW2, SW2NU
DOUBLE PRECISION SW3, SW3NU

c surface power density of pot by w2, w2nu
DOUBLE PRECISION SPD2, P2NU

c surface power density of pot by w2+w3, w2nu+w3nu
DOUBLE PRECISION SPD23, P23NU

c dteta/min by w2, w2nu --pot
DOUBLE PRECISION DT2, DT2NU

c dteta/min by w2+w3, w2nu+w3nu
DOUBLE PRECISION DT23, DT23NU

c dteta/min by w3, w3nu --charge
DOUBLE PRECISION DT3, DT3NU

DOUBLE PRECISION HU, HNU, P03, P03NU
COMPLEX Z2
DOUBLE PRECISION XA2

REAL U1A(25), FA(25), WA(25)
REAL R01A(25), R02A(25), R03A(25)
REAL R1A(25), R2A(25), R3A(25), RA(25)
REAL H1A(25), H2A(25), H3A(25)
REAL HH2A(25), HA(25)
REAL S1A(25), S2A(25), S3A(25)
REAL K1A(25), KRA(25), TCNA(25)
REAL MUR1A(25), MUR2A(25), MUR3A(25)
REAL GAM1A(25), GAM2A(25), GAM3A(25)
REAL SC1A(25), SC2A(25), SC3A(25)
REAL MC2A(25), MC3A(25)
REAL DEL1A(25), DEL2A(25), DEL3A(25)
REAL XDEL2A(25)
REAL A1A(25), A2A(25), A3A(25)
REAL LAMDA(25)
REAL X22A(25), X23A(25), X33A(25)
REAL X1A(25), X2A(25), X3A(25)
REAL X22RTA(25), DD2A(25)
REAL AF3(25), ARF3(25), AGF3(25)
REAL AZ3(25), ARZ3(25), AGZ3(25)
REAL AF23(25), ARF23(25), AGF23(25)
REAL AZ23(25), ARZ23(25), AGZ23(25)
REAL AQC(25), ARQC(25), AGQC(25), AMQC(25)
REAL AF1(25), ARF1(25), AGF1(25)
REAL AZF(25), ARZF(25), AGZF(25)
REAL AZFNU(25), ARZFNU(25), AGZFNU(25)
REAL AN1(25), AN1NU(25)
REAL ARE1(25), ARR1(25), AGR1(25)
REAL ARS1NU(25), ARR1NU(25), AGR1NU(25)
REAL AZ1(25), AZ1NU(25)
REAL AFI1(25), AFI1NU(25)
REAL AFI(25), AFINU(25)
REAL AQF(25), AQFNU(25)
REAL AZFT(25), AZFTNU(25)
REAL A1(25), A1NU(25)
REAL AQ1(25), AQ1NU(25)
REAL AETA(25), AETANU(25)
REAL ASYS(25)
REAL AEN3(25)
REAL AW1(25),AW1NU(25)
REAL AW2(25),AW2NU(25)
REAL AW3(25),AW3NU(25)

c sum of all w2nu, \( w2nu \)
REAL A2NUPC(25)

c sum of all w3nu, \( w3nu \)
REAL A3NUPC(25)

c surface power density of pot by \( w2, w2nu \)
REAL ASPD2(25), AP2NU(25)

c surface power density of pot by \( w2+w3, w2nu+w3nu \)
REAL ASPD23(25), AP23NU(25)

c dteta/min by \( w2, w2nu \)
REAL AT2(25), AT2NU(25)

c dteta/min by \( w3, w3nu \)
REAL AT3(25), AT3NU(25)

c dteta/min by \( w2+w3, w2nu+w3nu \)
REAL AT23(25), AT23NU(25)

REAL AP03(25), AP03NU(25)
REAL AHU(25), AHUNU(25)
REAL ASBD2(25)
REAL AZ2(25), ARZ2(25), AGZ2(25)
REAL W2BWA(25)
REAL XA2A(25)
REAL X2T(25), XQF(25), XFI(25), XFR(25)
REAL HOA(25), H0NUA(25)
REAL GOA(25)
REAL RH2RA(25), GH2RA(25), RG2RA(25), GG2RA(25)
REAL H2RA(25), G2RA(25), H3RA(25), G3RA(25),
1 T0A(15), T2A(15), T3A(15), DT0A(15), DT2A(15), DT3A(15)
REAL RC1A(25), GC1A(25), RC2A(25), GC2A(25),
1 RC3A(25), GC3A(25), C1A(25), C2A(25), C3A(25)
COMPLEX DLTA11, DLTA12, DLTA21,
1 DLTA22, C11, C12, C21, C22, C31, C32
COMPLEX DELTA, DELTA1, DELTA2, C1, C2, C3,
1 W52, E52, W53, E53, W54, E54
COMPLEX ATLED
COMPLEX H2R, G2R, H3R, G3R, HOR, COR
COMPLEX E2
REAL E2A(25), RE2A(25), GE2A(25)

c dclr_common /blks/
C DOUBLE PRECISION U1, W, PI, GAMMA2, GAMMA3, SC2, SC3
COMMON /BLK1/ U1, W, PI, GAMMA2, GAMMA3, SC2, SC3
C REAL GAMMA2A(25), GAMMA3A(25), SC2A(25), SC3A(25)
COMMON /BLK2/ GAMMA2A, GAMMA3A, SC2A, SC3A
C REAL U1A(25), WA(25), R1A(25), R2A(25), R3A(25), H2A(25)
COMMON /BLK3/ U1A, WA, R1A, R2A, R3A, H2A, H3A
C REAL AN1(25), A11(25), AETA(25), AEN3(25)
COMMON /BLK4/ AN1, A11, AETA, AEN3
REAL BW24(25), BW24PC(25), BSPD2(25), BSPD23(25)
COMMON /BLK5/ BW24, BW24PC, BSPD2, BSPD23
REAL BEN3(25), BETA(25), BEN34(25), BETA4(25)
COMMON /BLK6/ BEN3, BETA, BEN34, BETA4
REAL BW34(25), BW34PC(25), BH0(25)
COMMON /BLK7/ BW34, BW34PC, BH0
REAL BT2(25), TSEC2(25), BT23(25), TSEC32(25), BT3(25)
C COMMON /BLK8/ BT2,TSEC2, BT23,TSEC23, BT3
C INTEGER NL(25),LAYERS
C COMMON /BLK9/ NL,LAYERS
C DOUBLE PRECISION KON2, KON3
C COMMON /BLK10/ KON2,KON3
C REAL KON2A(25), KON3A(25)
C COMMON /BLK11/ KON2A, KON3A
C REAL AG2A(25),AG3A(25),AS2A(25),AS3A(25)
C COMMON /BLK12/ AG2A,AG3A,AS2A,AS3A
C COMMON /BLK13/ MC2A,MC3A
C REAL BTAD3, BTA23B,BTA23M,BTA23T, BTA23A(10)
C COMMON /BLK14/ BTAD3, BTA23B,BTA23M,BTA23T, BTA23A

REAL NT,NM,NB,ND,
1 IT,IM,IB,ID,
2 NIT,NIM,NIB,NID
REAL NNORM1, NT1, NM1, NB1, ND1,
1 IT1, IM1, IB1, ID1,
2 NIT1, NIM1, NIB1, NID1
REAL INORM2, IT2, IM2, IB2, ID2,
1 NT2, NM2, NB2, ND2,
2 NIT2, NIM2, NIB2, NID2
REAL SU13, NT4, NM4, NB4, ND4,
1 IT4, IM4, IB4, ID4,
2 NIT4, NIM4, NIB4, NID4
INTEGER NLC251,DATEC161,LABEL(301,TIMEC161,
1 TITLE(6,67),IWRITEC671,LAYERS

$INSERT BESSELFUNCS
READ(5,*)U1,F,W,R01,R02,R03,R1,R2,R3,
* H1,H2,H3,T1,T2,K1,KR,MUR1,MUR2,MUR3,
* GAMMA2,GAMMA3,
C=1.E-7
AJ=CMPLX(0.0,1.0)
GAJ=REAL(AJ)
RAJ=AIMAG(AJ)
T=1.4142135620D0
PI=3.1415926540D0
U0=4.*PI*C
OMEG=2.*PI*F
MU1=MUR1*U0
MU2=MUR2*U0
MU3=MUR3*U0
DEL1=DSQRT(2.*R01/(OMEG*MU1*K1))
A1=T1/DEL1
C REACTANCE OF RETURN FLOW PATH
XR=OMEG*U0*PI*R1**2/(H1-H2+2.*C*(0.45+KR)*R1)
C REACTANCE OF REFRACTORY
AM=PI*(R1**2-R2**2)
LM=H2
X0=(OMEG*U0*AM)/LM
C CHARGE AND POT RESISTANCE
DEL2=DSQRT(2.*R02/(OMEG*MU2))
A2=T2/DEL2
RES2=R02*2.*PI*R2/(H2*DEL2)
RES3=R03*2.*PI*R3/(H2*DEL3)
LAMDA=DSQRT((R03/R02)*(MU3/MU2))
DEL3=DSQRT(2.*R03/(OMEG*MU3))
T3=R3
A3=T3/DEL3
X22 = T*R2/DEL2
X23 = T*R3/DEL2
X33 = T*R3/DEL3
F3 = T*(YOD33/Y033)
RF3 = REAL(F3)
GF3 = AIMAG(F3)

C
ALF3 = LAMDA*F3
ALBTF3 = (LAMDA/T)*F3
F231 = (Q0D22*Y023) - (Q023*Y0D22)
F232 = Y0D22*Q0D23
F233 = Q0D22*Y0D23
F234 = (Q022*Y023) - (Q023*Y022)
F235 = Y022*Q0D23
F236 = Q022*Y0D23
F23 = (ALF3*F231 + T*F232 - T*F233) / (ALBTF3*F234 + F235 - F236)
RF23 = REAL(F23)
GF23 = AIMAG(F23)

C
Z23 = RES2*F23
RZ23 = REAL(Z23)
GZ23 = AIMAG(Z23)

C
RKC = XR / (XR + X0 + AIMAG(Z23))
GKC = 0.0D0
KC = CMPLX(RKC, GKC)
MKC = CABS(KC)
RKC = REAL(KC)
GKC = AIMAG(KC)

C
A12 = 2.*A1
F1 = (1 + AJ)*(SINH(A12) - AJ*DSIN(A12))
1 / (COSH(A12) - DCOS(A12))
RF1 = REAL(F1)
GF1 = AIMAG(F1)

C
AA = R01*2.*PI*R1*F1 / (K1*H1*DEL1)
FURNACE IMPEDANCE
ZF = AA + MKC*REAL(Z23) + AJ*MKC*(X0 + AIMAG(Z23))
RZF = REAL(ZF)
GZF = AIMAG(ZF)
N1 = (U1/CABS(ZF))*DSQRT(DABS(REAL(ZF)/W))

C
COIL RES.
RES1 = R01*2.*PI*R1*N1**2*F1 / (K1*H1*DEL1)
RR1 = REAL(RES1)
GR1 = AIMAG(RES1)
Z1 = CABS(RES1)

C
PHASE ANGLE FI1
FI1 = (180./PI)*ATAN(AIMAG(ZF)/REAL(ZF))
PF = DCOS(FI1)

C
Q-FACTOR
QF = AIMAG(ZF)/REAL(ZF)
I1 = U1 / ((N1)**2*CABS(ZF))

C
Q1-kVar
Q1 = ((I1)**2*N1**2*AIMAG(ZF))/1000
ETA = (MKC*REAL(Z23)) / (MKC*REAL(Z23) + REAL(AA))

C
SYS = (Y023*Q0D23 - Y0D23*Q023) / ((LAMDA/T)*F3*(Q022*
1 *Y023 - Q023*Y022) + Y022*Q0D23 - Q022*Y0D23)

C
RSYS = SYS*CONJG(SYS)
RSYS = (CABS(SYS)**2
SYS = RSYS
Z3 = RES3*F3*SYS

C POWER DISTRIBUTION
EN3 = (R3/R2)*((LAMDA*SYS*REAL(F3))/REAL(F23)
WN3 = EN3*ETA
W3 = WN3*W
WN2 = (1.-EN3)*ETA
W2 = WN2*W
WN1 = 1.-ETA
W1 = WN1*W
SPD23 = (W2+W3)/(2.*PI*R2*H2)
SPD2 = (W2)/(2.*PI*R2*H2)

C
PO3 = (MU3/(2.*R03))**DEL3*(W3/(2.*PI*R3*H2))
C
1 * ((BR33**2+BI33**2-1.)/(T*(BR33*BRD33+BI33*BID33))
HU = PO3/(9.81*GAMA3)

C DELTA1 = Y022*(QOD23*Y033-Q023*LAMDA*YOD33)
DELTA2 = Q022*(Y023*LAMDA*YOD33-Y033*Y023)
DELTA = DELTA1 + DELTA2

C atled = conjugate(delta)/[square(absolute(delta)**2]
C
ATLED = CONJG(DELTA)/(CABS(DELTA)**2
C
CBSL = CABS(DELTA)
C2BSD = CBSL**2
ATLED = CONJG(DELTA)/C2BSD

C
C1 = H0*(ATLED)*((-Q023*YOD33*LAMDA)+(QOD23*Y033))
C11 = LAMDA*Q023*YOD33
C12 = QOD23*Y033
C1 = H0*(ATLED)*(-C11+C12)

C
C2 = H0*(ATLED)*((-YOD23*Y033)+(LAMDA*Y023*YOD33))
C21 = YOD23*Y033
C22 = LAMDA*Y023*YOD33
C2 = H0*(ATLED)*(-C21+C22)

C
C3 = H0*(ATLED)*((Y023*QOD23)-(YOD23*Q023))
C31 = Y023*QOD23
C32 = YOD23*Q023
C3 = H0*(ATLED)*(C31-C32)

C
H2R = C1*Y0X2+C2*Q0X2
G2R = -(T/DEL2)*((C1*Y0DX2)+(C2*Q0DX2))
H3R = C3*Y0X3
G3R = -(T/DEL3)*C3*Y0DX3
E2 = R02*G2R
RE2 = REAL(E2)

C @ boundary:
H2R2 = C1*Y022+C2*Q022
G2R2 = -(T/DEL2)*((C1*Y0D22)+(C2*Q0D22))
H3R3 = C3*Y033
G3R3 = -(T/DEL3)*C3*Y0D33

C
CALL EXIT
END
APPENDIX A3.2

*---------------------------------------------------------------------*
* Listing of BESSELFUNCS program code                              *
*---------------------------------------------------------------------*

C arguments 1.0-10.0

DOUBLE PRECISION A0, A022
DOUBLE PRECISION B0, B0D, B10, BIDO, Q0, QRO, QRD0, Q10, QIDO
DOUBLE PRECISION B022, B0D022, B1022, BIDO22, Q022, QRD022,
  1 QI022, QIDO22
DOUBLE PRECISION B023, B0D023, B1023, BIDO23, Q023, QRD023,
  1 QI023, QIDO23
DOUBLE PRECISION B033, B0D033, B1033, BIDO33, Q033, QRD033,
  1 QI033, QIDO33

COMPLEX Y00, Y0D0, Q00, QD0
DOUBLE PRECISION RY00, GY00,
  1 RY00, GY00,
  2 RQ00, GQ00,
  3 RQD0, GQDO
COMPLEX Y022, Y0D22, Q022, QD22
COMPLEX Y023, Y0D23, Q023, QD23
COMPLEX Y033, Y0D33, Q033, QD33

C arguments greater than 10.0

DOUBLE PRECISION B1, B11, B1D1, Q1, QRD1, QI1, QID1
DOUBLE PRECISION B122, B1D122, B1D122, Q122, QRD122
DOUBLE PRECISION B123, B1D123, B1D123, Q123, QRD123
DOUBLE PRECISION B133, B1D133, B1D133, Q133, QRD133
DOUBLE PRECISION Q1122, Q1123, Q1133
DOUBLE PRECISION QID122, QID123, QID133
COMPLEX Y101, Y1D1, Q101, QD1
DOUBLE PRECISION RY101, GY101,
  1 RY101, GY101,
  2 RQ101, GQ101,
  3 RQD1, GQDO1
COMPLEX Y122, Y1D122, Q122, QD122
COMPLEX Y123, Y1D123, Q123, QD123
COMPLEX Y133, Y1D133, Q133, QD133

C arguments 0.0-1.0

DOUBLE PRECISION B0, B0D0, B10, BIDO, Q0, QRO, QRD0, Q10, QIDO
DOUBLE PRECISION B022, B0D022, B1022, BIDO22, Q022, QRD022,
  1 QI022, QIDO22
DOUBLE PRECISION B023, B0D023, B1023, BIDO23, Q023, QRD023,
  1 QI023, QIDO23
DOUBLE PRECISION B033, B0D033, B1033, BIDO33, Q033, QRD033,
  1 QI033, QIDO33
COMPLEX Y00, Y0D0, Q00, QD0
DOUBLE PRECISION RY00, GY00,
  1 RY00, GY00,
  2 RQ00, GQ00,
  3 RQD0, GQDO0
COMPLEX Y022, Y0D022, Q022, QD22
COMPLEX Y023, Y0D023, Q023, QD23
COMPLEX Y033, Y0D033, Q033, QD33

C

C arguments 1.0-10.0

*---------------------------------------------------------------------*
* List of BESSELFUNCS program code                                   *
*---------------------------------------------------------------------*
DOUBLE PRECISION FUNCTION BR(Z)
BR(Z)=1.-(1./64.)*Z**4
1  +(1./147456.)*Z**8
2  -(1./2.1233D9)*Z**12
3  +(1./1.06542D14)*Z**16
4  -(1./1.38078D19)*Z**20
5  +(1./3.8494069D24)*Z**24
6  -(1./2.04012D30)*Z**28
C  7  +(1./1.880178D36)*Z**32
C  8  -(1./2.816838D42)*Z**36

DOUBLE PRECISION FUNCTION BRD(Z)
BRD(Z)=-(4./64.)*Z**3
1  +(8./147456.)*Z**7
2  -(12./2.1233D9)*Z**11
3  +(16./1.06542D14)*Z**15
4  -(20./1.38078D19)*Z**19
5  +(24./3.8494069D24)*Z**23
6  -(28./2.04012D30)*Z**27
C  7  +(32./1.880178D36)*Z**31
C  8  -(36./2.816838D42)*Z**35

DOUBLE PRECISION FUNCTION BI(Z)
BI(Z)=(1./4.)*Z**2
1  -(1./2304.)*Z**6
2  +(10./14.7456D6)*Z**10
3  -(14./4.16179B11)*Z**14
4  +(18./3.45196D16)*Z**18
5  -(22./6.683D21)*Z**22
6  +(26./2.6022D27)*Z**26
7  -(30./1.836D33)*Z**30
C  8  +(34./2.173486D39)*Z**34
C  9  -(38./4.06754D45)*Z**38

DOUBLE PRECISION FUNCTION BID(Z)
BID(Z)=(2./4.)*Z**1
1  -(6./2304.)*Z**5
2  +(10./14.7456D6)*Z**9
3  -(14./4.16179B11)*Z**13
4  +(18./3.45196D16)*Z**17
5  -(22./6.683D21)*Z**21
6  +(26./2.6022D27)*Z**25
7  -(30./1.836D33)*Z**29
C  8  +(34./2.173486D39)*Z**33
C  9  -(38./4.06754D45)*Z**37

DOUBLE PRECISION FUNCTION QR(Z)
QR(Z)=(AO(Z))*BR(Z)
1  +(PY/4.)*BI(Z)
2  -(1.5/64.)*Z**4
3  +(2.08/147456.)*Z**8
4  -(2.45/2.1233D9)*Z**12
5  +(2.72/1.06542D14)*Z**16
6  -(2.93/1.38078D19)*Z**20
7  +(3.1/3.8494069D24)*Z**24
8  -(3.25/2.0412D30)*Z**28
C  9  +(3.38/1.880178D36)*Z**32
DOUBLE PRECISION FUNCTION QRD(Z)
QRD(Z) = (AO(Z))*BRD(Z) - BR(Z)*(1./Z)
1  + (PY/4.)*BID(Z)
2  - (6./64.)*Z**3
3  + (2.08*8./147456.)*Z**7
4  - (2.45*12./2.1233D9)*Z**11
5  + (2.72*16./1.065421D14)*Z**15
6  - (2.93*20./1.380781D19)*Z**19
7  + (3.12*24./3.849406D24)*Z**23
8  - (3.25*28./2.0412D30)*Z**27

DOUBLE PRECISION FUNCTION QI(Z)
QI(Z) = (AO(Z))*BI(Z)
1  - (PY/4.)*BR(Z)
2  + (1./4.)*Z**2
3  - (1.63/2304.)*Z**6
4  + (2.26/14.7456D6)*Z**10
5  - (2.593/4.161798D11)*Z**14
6  + (2.83/3.45196D16)*Z**18
7  - (3.02/6.683D21)*Z**22
8  + (3.18/2.6022D27)*Z**26
C 9  - (3.31/1.836D33)*Z**30
C 10  + (3.44/2.173486D39)*Z**34

DOUBLE PRECISION FUNCTION QID(Z)
QID(Z) = (AO(Z))*BID(Z) - BI(Z)*(1./Z)
1  - (PY/4.)*BRD(Z)
2  + (2./4.)*Z
3  - (1.63*6./2304.)*Z**5
4  + (2.26*10./14.7456D6)*Z**9
5  - (2.593*14./4.161798D11)*Z**13
6  + (2.83*18./3.45196D16)*Z**17
7  - (3.02*22./6.683D21)*Z**21
8  + (3.18*26./2.6022D27)*Z**25
C 9  - (3.31*30./1.836D33)*Z**29
C 10  + (3.44*34./2.173486D39)*Z**33

COMPLEX FUNCTION YO(Z)
YO(Z) = CMPLX(BR(Z), BI(Z))
RYO(Z) = BR(Z)
GYO(Z) = BI(Z)
YO(Z) = CMPLX(RYO(Z), GYO(Z))

COMPLEX FUNCTION YOD(Z)
YOD(Z) = CMPLX(BRD(Z), BID(Z))
RYOD(Z) = BRD(Z)
GYOD(Z) = BID(Z)
YOD(Z) = CMPLX(RYOD(Z), GYOD(Z))

COMPLEX FUNCTION QO(Z)
QO(Z) = CMPLX(QR(Z), QI(Z))
RQO(Z) = QR(Z)
GQO(Z) = QI(Z)
QO(Z) = CMPLX(RQO(Z), GQO(Z))

COMPLEX FUNCTION QOD(Z)
C \[ \text{QOD}(Z) = \text{CMPLX}(\text{QRD}(Z), \text{QID}(Z)) \]
\[ \text{RQOD}(Z) = \text{QRD}(Z) \]
\[ \text{GQOD}(Z) = \text{QID}(Z) \]
\[ \text{QOD}(Z) = \text{CMPLX}(\text{RQOD}(Z), \text{GQOD}(Z)) \]

C Arguments greater than 10.0

**DOUBLE PRECISION FUNCTION BR1(Z)**
\[ \text{BR1}(Z) = (0.3989 \times \text{DEXP}(Z/\text{RT})/\text{DSQRT}(Z)) \]
\[ 1 \times (\text{DSIN}(40.514 \times Z+67.5)/\text{RD}) \]
\[ 2 + (1/(8.2Z)) \times \text{DSIN}(40.514 \times Z+22.5)/\text{RD}) \]

**DOUBLE PRECISION FUNCTION BRD1(Z)**
\[ \text{BRD1}(Z) = (0.3989 \times \text{DEXP}(Z/\text{RT})/\text{DSQRT}(Z)) \]
\[ 1 \times (\text{DSIN}(40.514 \times Z+112.5)/\text{RD}) \]
\[ 2 - (3/(8.2Z)) \times \text{DSIN}(40.514 \times Z+67.5)/\text{RD}) \]

**DOUBLE PRECISION FUNCTION BI1(Z)**
\[ \text{BI1}(Z) = (0.3989 \times \text{DEXP}(Z/\text{RT})/\text{DSQRT}(Z)) \]
\[ 1 \times (\text{SIN}(40.514 \times Z-22.5)/\text{RD}) \]
\[ 2 + (1/(8.2Z)) \times \text{SIN}(40.514 \times Z-67.5)/\text{RD}) \]

**DOUBLE PRECISION FUNCTION BID1(Z)**
\[ \text{BID1}(Z) = (0.3989 \times \text{DEXP}(Z/\text{RT})/\text{DSQRT}(Z)) \]
\[ 1 \times (\text{SIN}(40.514 \times Z-22.5)/\text{RD}) \]
\[ 2 - (3/(8.2Z)) \times \text{SIN}(40.514 \times Z-67.5)/\text{RD}) \]

**DOUBLE PRECISION FUNCTION QRI(Z)**
\[ \text{QRI}(Z) = (1.2533 \times \text{DEXP}(-Z/\text{RT})/\text{DSQRT}(Z)) \]
\[ 1 \times (\text{SIN}(40.514 \times Z+112.5)/\text{RD}) \]
\[ 2 + (1/(8.2Z)) \times \text{SIN}(40.514 \times Z-22.5)/\text{RD}) \]

**DOUBLE PRECISION FUNCTION QRD1(Z)**
\[ \text{QRD1}(Z) = (-1.2533 \times \text{DEXP}(-Z/\text{RT})/\text{DSQRT}(Z)) \]
\[ 1 \times (\text{SIN}(40.514 \times Z+22.5)/\text{RD}) \]
\[ 2 + (3/(8.2Z)) \times \text{SIN}(40.514 \times Z+67.5)/\text{RD}) \]

**DOUBLE PRECISION FUNCTION QI1(Z)**
\[ \text{QI1}(Z) = (1.2533 \times \text{DEXP}(-Z/\text{RT})/\text{DSQRT}(Z)) \]
\[ 1 \times (\text{SIN}(40.514 \times Z+22.5)/\text{RD}) \]
\[ 2 + (1/(8.2Z)) \times \text{SIN}(40.514 \times Z+67.5)/\text{RD}) \]

**DOUBLE PRECISION FUNCTION QID1(Z)**
\[ \text{QID1}(Z) = (1.2533 \times \text{DEXP}(-Z/\text{RT})/\text{DSQRT}(Z)) \]
\[ 1 \times (\text{SIN}(40.514 \times Z-22.5)/\text{RD}) \]
\[ 2 + (3/(8.2Z)) \times \text{SIN}(40.514 \times Z+22.5)/\text{RD}) \]

**COMPLEX FUNCTION Y01(Z)**
\[ \text{Y01}(Z) = \text{CMPLX}(\text{BR1}(Z), \text{BI1}(Z)) \]
\[ \text{RY01}(Z) = \text{BR1}(Z) \]
\[ \text{GY01}(Z) = \text{BI1}(Z) \]
\[ \text{Y01}(Z) = \text{CMPLX}(\text{RY01}(Z), \text{GY01}(Z)) \]

**COMPLEX FUNCTION YOD1(Z)**
\[ \text{YOD1}(Z) = \text{CMPLX}(\text{BRD1}(Z), \text{BID1}(Z)) \]
\[ \text{RYOD1}(Z) = \text{BRD1}(Z) \]
\[ \text{GYOD1}(Z) = \text{BID1}(Z) \]
\[ \text{YOD1}(Z) = \text{CMPLX}(\text{RYOD1}(Z), \text{GYOD1}(Z)) \]

**COMPLEX FUNCTION Q01(Z)**
\[ \text{Q01}(Z) = \text{CMPLX}(\text{QRI}(Z), \text{QI1}(Z)) \]
RQ01(Z) = QR1(Z)
GQ01(Z) = QI1(Z)
Q01(Z) = CMPLX(RQ01(Z), GQ01(Z))

C COMPLEX FUNCTION QOD1(Z)
QOD1(Z) = CMPLX(QRD1(Z), QID1(Z))
RQD01(Z) = QRD1(Z)
GQD01(Z) = QID1(Z)
QD1(Z) = CMPLX(RQD01(Z), GQD01(Z))

C arguments 0.0-1.0
C DOUBLE PRECISION FUNCTION V(Z)
V(Z) = 0.5*Z

C DOUBLE PRECISION FUNCTION BRO(Z)
BRO(Z) = BR(Z)

C DOUBLE PRECISION FUNCTION BRD0(Z)
BRD0(Z) = BRD(Z)

C DOUBLE PRECISION FUNCTION BLO(Z)
BLO(Z) = BI(Z)

C DOUBLE PRECISION FUNCTION BIDO(Z)
BIDO(Z) = BID(Z)

C DOUBLE PRECISION FUNCTION QRO(Z)
QRO(Z) = AO(Z)
1 + .25*PY*V(Z)**2
2 + .25*(A0(Z)+1.5)*V(Z)**4
3 - (1./144.)*PY*V(Z)**6
4 + (1./576.)*((A0(Z)+(25./12.))*V(Z)**8

C DOUBLE PRECISION FUNCTION QRDO(Z)
QRD0(Z) = -1./(2.*V(Z))
1 + (1./4.)*PY*V(Z)
2 + .5*(A0(Z)+(5./4.))*V(Z)**3
3 - (1./48.)*PY*V(Z)**5
4 + (1./144.)*(A0(Z)+(47./24.))*V(Z)**7

C DOUBLE PRECISION FUNCTION QIO(Z)
QIO(Z) = (-.25)*PY
1 + (A0(Z)+1.)*V(Z)**2
2 + (1./16.)*PY*V(Z)**4
3 - (1./36.)*(A0(Z)+(11./6.))*V(Z)**6
4 - (1./2304.)*PY*V(Z)**8

C DOUBLE PRECISION FUNCTION QIDO(Z)
QIDO(Z) = (A0(Z)+.5)*V(Z)
1 + (1./8.)*PY*V(Z)**3
2 - (1./12.)*(A0(Z)+(5./3.))*V(Z)**5
3 - (1./576.)*PY*V(Z)**7

C COMPLEX FUNCTION YOO(Z)
YOO(Z) = CMPLX(BRO(Z), BLO(Z))
RYOO(Z) = BRO(Z)
GYOO(Z) = BLO(Z)
YOO(Z) = CMPLX(RYOO(Z), GYOO(Z))

C COMPLEX FUNCTION YODO(Z)
C YODO(Z) = CMPLX(BRDO(Z), BIDO(Z))
RYODO(Z) = BRDO(Z)
GYODO(Z) = BIDO(Z)
YODO(Z) = CMPLX(RYODO(Z), GYODO(Z))

CC
COMPLEX FUNCTION Q00(Z)
C
Q00(Z) = CMPLX(QR0(Z), QI0(Z))
Q00(Z) = QI0(Z)
Q00(Z) = CMPLX(Q00(Z), GQ00(Z))

CC
COMPLEX FUNCTION QODO(Z)
C
QODO(Z) = CMPLX(QRDO(Z), QIDO(Z))
QRDO(Z) = QRDO(Z)
QIDO(Z) = QIDO(Z)
QODO(Z) = CMPLX(QRDO(Z), GQIDO(Z))

RT = DSQRT(2.0DO)
PY = 3.1415926540DO
RD = 180.0DO / PY

CCC
C use of bessel functions according to limit of argument
BR22 = BR(X22)
BRD22 = BRD(X22)
BI22 = BI(X22)
BID22 = BID(X22)
QR22 = QR(X22)
A022 = A0(X22)
QRD22 = QRD(X22)
QI22 = QI(X22)
QID22 = QID(X22)
Y022 = Y0(X22)
YOD22 = YOD(X22)
Q022 = Q0(X22)
QOD22 = QOD(X22)
Y022 = Y0(X22)

C X >10
BR122 = BR1(X22)
BRD122 = BRD1(X22)
BI122 = BI1(X22)
BID122 = BID1(X22)
QR122 = QR1(X22)
QRD122 = QRD1(X22)
QI122 = QI1(X22)
QID122 = QID1(X22)
Y0122 = Y01(X22)
YOD122 = YOD1(X22)
Q0122 = Q01(X22)
QOD122 = QOD1(X22)
IF(X22.GT.10.)GO TO 1223
GO TO 1224

1223 BR22 = BR122
BRD22 = BRD122
BI22 = BI122
BID22 = BID122
QR22 = QR122
QRD22 = QRD122
QI22 = QI122
QID22 = QID122
Y022 = Y0122
Y0D22 = Y0D122
Q022 = Q0122
Q0D22 = QOD122

1224 CONTINUE
BR022 = BRO(X22)
BRD022 = BRD0(X22)
BI022 = BIO(X22)
BID022 = BID0(X22)
QR022 = QR0(X22)
QRD022 = QRD0(X22)
QI022 = QI0(X22)
QID022 = QID0(X22)
Y0022 = Y00(X22)
Y0D022 = Y0D0(X22)
Q0022 = Q00(X22)
Q0D022 = QOD0(X22)

IF(X22.GE.0.AND.X22.LT.1.) GO TO 1225
GO TO 1226

1225 BR22 = BRO22
BRD22 = BRD022
BI22 = BI022
BID22 = BID022
QR22 = QR022
QRD22 = QRD022
QI22 = QI022
QID22 = QID022
Y022 = Y0022
Y0D22 = Y0D022
Q022 = Q0022
Q0D22 = QOD022

1226 CONTINUE
BR23 = BR(X23)
BRD23 = BRD(X23)
BI23 = BI(X23)
BID23 = BID(X23)
QR23 = QR(X23)
QRD23 = QRD(X23)
QI23 = QI(X23)
QID23 = QID(X23)
Y023 = Y0(X23)
Y0D23 = Y0D(X23)
Q023 = Q0(X23)
Q0D23 = QOD(X23)

C
BR123 = BR1(X23)
BRD123 = BRD1(X23)
BI123 = BI1(X23)
BID123 = BID1(X23)
QR123 = QR1(X23)
QRD123 = QRD1(X23)
QI123 = QI1(X23)
QID123 = QID1(X23)
Y0123 = Y01(X23)
Y0D123 = Y0D1(X23)
Q0123 = Q01(X23)
Q0D123 = QOD1(X23)

IF(X23.GT.10.) GO TO 1233
GO TO 1234

1233 BR23 = BR123
BRD23 = BRD123
BI23 = BI123
BID23 = BID123
QR23 = QR123
QRD23 = QRD123
QI23 = QI123
QID23 = QID123
Y023 = Y0123
YOD23 = YOD123
Q023 = Q0123
QOD23 = QOD123

1234 CONTINUE
BR023 = BR0(X23)
BRD023 = BRD0(X23)
B1023 = B10(X23)
BID023 = BID0(X23)
QR023 = QR0(X23)
QRD023 = QRD0(X23)
QI023 = QI0(X23)
QID023 = QID0(X23)
Y0023 = Y00(X23)
YOD023 = YOD0(X23)
Q0023 = Q00(X23)
QOD023 = QOD0(X23)

IF(X23.GE.0.AND.X23.LT.1) GO TO 1235
GO TO 1236

1235 BR23 = BR023
BRD23 = BRD023
B123 = B1023
BID23 = BID023
QR23 = QR023
QRD23 = QRD023
QI23 = QI023
QID23 = QID023
Y023 = Y0023
YOD23 = YOD023
Q023 = Q0023
QOD23 = QOD023

1236 CONTINUE
BR33 = BR(X33)
BRD33 = BRD(X33)
B133 = B1(X33)
BID33 = BID(X33)
QR33 = QR(X33)
QRD33 = QRD(X33)
QI33 = QI(X33)
QID33 = QID(X33)
Y033 = Y0(X33)
YOD33 = YOD(X33)
Q033 = Q0(X33)
QOD33 = QOD(X33)

C
BR133 = BR1(X33)
BRD133 = BRD1(X33)
B1133 = B11(X33)
BID133 = BID1(X33)
QR133 = QR1(X33)
QRD133 = QRD1(X33)
QI133 = QI1(X33)
QID133 = QID1(X33)
Y0133 = Y01(X33)
YOD133=YOD1(X33)
Q0133=Q01(X33)
QOD133=QOD1(X33)
IF(X33.GT.10.)GO TO 1333
GO TO 1334

1333 BR33=BR133
BRD33=BRD133
BI33=BI133
BID33=BID133
QR33=QR133
QRD33=QRD133
Q133=Q1133
QID33=QID133
Y033=Y0133
YOD33=YOD133
Q033=Q0133
QOD33=QOD133

1334 CONTINUE
BR033=BR0(X33)
BRD033=BRD0(X33)
BI033=BI0(X33)
BID033=BID0(X33)
QR033=QR0(X33)
QRD033=QRD0(X33)
Q1033=Q10(X33)
QID033=QID0(X33)
Y0033=Y00(X33)
YOD033=YOD0(X33)
Q0033=Q00(X33)
QOD033=QOD0(X33)
IF(X33.GE.0.AND.X33.LT.1.)GO TO 1335
GO TO 1336

1335 BR33=BR033
BRD33=BRD033
BI33=BI033
BID33=BID033
QR33=QR033
QRD33=QRD033
Q133=Q1033
QID33=QID033
Y033=Y0033
YOD33=YOD033
Q033=Q0033
QOD33=QOD033

1336 CONTINUE
A4.1 Theory of the Current Density Probe

This theory is an extension of that proposed by Burke and Alden [4.1]. The probe can be considered as shown in Fig. A4.1.1. From Burke and Alden's paper:

\[ V = \phi \int_{\text{ADG}} E \, dl - \phi \int_{\text{CDE}} pJ \, dl \]  

(A4.1.1)

where

- \( \phi \int \) = line integral around the closed path ABCDEFGA
- \( \int_{\text{ADG}} \) = line integral around the path ABCDEFG
- \( E \) = electric field intensity in V m\(^{-1}\)
- \( p \) = medium resistivity in \( \Omega \)m
- \( J \) = current density in A m\(^{-2}\)

It is assumed that the voltmeter used to measure the voltage across the probe draws negligible current, and therefore the current density (in the probe) in the paths ABC and GFE is small enough to be ignored.

Hence:

\[ \phi \int_{\text{ADG}} pJ \, dl = \phi \int_{\text{CDE}} pJ \, dl \]  

(A4.1.2)

Paths AB and GF experience the same field since they are coaxial or tightly twisted together and terminals G and A are assumed to be in a field free region.

Therefore:
Fig. A4.1.1 Schematic representation of current density probe
\[ \oint E \cdot dl = \int_{EF} E \cdot dl + \int_{BC} E \cdot dl + \int_{CDE} E \cdot dl \quad (A4.1.3) \]

Since \( E = \rho J \),

\[ \oint E \cdot dl = \int_{EF} \rho J dl + \int_{BC} \rho J dl + \int_{CDE} \rho J dl \quad (A4.1.4) \]

Therefore by using eqns. A4.1.2 and A4.1.4 in eqn. A4.1.1:

\[ V_{GA} = \int_{EF} \rho J dl + \int_{BC} \rho J dl \quad (A4.1.5) \]

If, as in the circular probe case, points C, D and E are all connected electrically by the probe wire, eqn. A4.1.5 becomes:

\[ V_{GA} = \int_{EF} \rho J dl \]

\[ = \left[ \rho J dl \right]_0^{2\pi r} = \left[ \rho J dl \right]_0^{2\pi R} \]

where \( 2\pi r \) is the length of the probe and \( r \) is the radius of the probe, and to give the surface current density this is made equal to \( r_2 \), the radius of the crucible.

\[ V_{GA} = \rho J 2\pi R \quad (A4.1.6) \]

or

\[ |V_{GA}| = 2\pi R |J| \quad (A4.1.7) \]

Thus the magnitude of the surface current density of crucible can be found from the magnitude of the voltage across the probe.
A4.1.2 Theory of the Magnetic Field Strength Probe

The magnetic field strength H-probe consists of a wire wrapped around the crucible, with a layer of insulation as shown in Fig. A4.1.2.

Magnetic field strength, \( H \), is directly related to the flux \( \phi \) in the insulation area, and the induced voltage across the probe is proportional to the rate of change of this flux, and so the voltage across the probe can be used to find the magnetic field strength in the insulation. This is now proved formally.

Faraday's second law states that, 'The magnitude of an induced e.m.f. is proportional to the rate of change of flux linkage', and this is sometimes expressed as Newmann's Equation:

\[
|V| = N \frac{d|\phi|}{dt} \quad (A4.1.8)
\]

\[
|\phi| = |B|A \quad (A4.1.9)
\]

where

- \( N \) = number of turns, in this case one
- \( |\phi| \) = the magnitude of the flux
- \( |V| \) = the magnitude of the induced voltage
- \( |B| \) = the magnitude of the flux density
- \( A \) = the insulation area
- \( N \) and \( A \) are both constant

Putting eqns. A4.1.9 into eqn. A4.1.8 gives:
\[ |V| = \frac{NA|B|}{dt} \quad (A4.1.10) \]

since
\[ |B| = \mu_0\mu_r|H| \quad (A4.1.11) \]

and also \( N \) and \( A \) are both constant

(where \( |H| = \) the magnitude of the magnetic field strength)

this gives
\[ |V| = NA \mu_0\mu_r \frac{d|H|}{dt} \quad (A4.1.12) \]

The magnetic field strength varies sinusoidally with time and so \( \frac{dH}{dt} = j\omega H \)

where \( H \) is the magnetic field strength in the insulation. Thus,
\[ |V| = NA \mu_0\mu_r \omega |H| \quad (A4.1.13) \]

Hence the magnitude of the magnetic field strength can be found directly from the induced voltage in the probe.

It is wanted to measure in the magnetic field on the surface of the crucible, but it is necessary to have an area, \( A \), in which the flux is present. If the area is too small, insufficient induced voltage is present to be measured easily and accurately, and if the area is too large the magnetic field strength measured will be that strength some distance from the surface and not the actual surface value. (This becomes even more of a problem with small airgap heating coils because it is possible to have the insulation almost as thick as the airgap, which is an obviously undesirable situation. Thus the insulation is made as thin as possible while maintaining a voltage which can be accurately measured.
Fig. A4.1.2  H-Probe
A4.1.3 Note on Induced Voltages in Both Current Density and Magnetic Field Strength Probes

If the measured induced voltage is the r.m.s. value, then the current density and the magnetic field strength calculated from this voltage will also be the r.m.s. value.
Appendix A4.2

COMPARISON BETWEEN MEASURED AND CALCULATED VALUES OF FIELD STRENGTH [4.3]

FIG A4.2.1  CALCULATED [4.3] AND MEASURED AXIAL MAGNETIC FIELD STRENGTH INSIDE THE FURNACE COIL  \( r/a = 0.94, L/a = 3 \)
Appendix A4.3

SKIN DEPTHS IN CARBON - SILICON CARBIDE AND CLAY-GRAPHITE CRUCIBLES

Fig. A4.3.1 Variation of skin depth with temperature in carbon-silicon carbide and clay-graphite crucibles through the melting cycle
APPENDIX A4.4

EFFECT OF GRAIN ORIENTATION ON THERMAL CONDUCTIVITY IN CLAY-GRAFITE CRUCIBLES

Due to the method of manufacture, grain misorientation is likely to occur in some regions of these crucibles. The direction of these graphite grains which are more conductive than the clay matrix can therefore affect the thermal conductivity of the crucible material.

The bulk thermal conductivity $k_B$, perpendicular to the grain is obtained [4.6] as:

$$\frac{1}{k_B} = \sum \frac{\text{Percentage of component}}{k \text{ of component}}$$  \hspace{1cm} (A4.4.1)

and the variation with temperature is shown in Fig. A4.4.1.

These values are shown for the purpose of illustration and do not enter calculations in Chapters 6 and 7 because heat transfers are considered in direction along the grain and in regions where misorientation of the grains is least expected to occur.
FIG. A.4.4.1 VARIATION OF THERMAL CONDUCTIVITY OF CLAY BONDED GRAPHITE WITH TEMPERATURE (PERPENDICULAR TO THE GRAIN)
APPENDIX A5.1

Listing of SUBNORM subroutine code

DOUBLE PRECISION U1, W, PI, GAM2, GAM3, SC2, SC3
COMMON /BLK1/ U1, W, PI, GAM2, GAM3, SC2, SC3
REAL GAM2A(25), GAM3A(25), SC2A(25), SC3A(25)
COMMON /BLK2/ GAM2A, GAM3A, SC2A, SC3A
REAL U1A(25), WA(25), R1A(25), R2A(25), R3A(25), H2A(25), H3A(25)
COMMON /BLK3/ U1A, WA, R1A, R2A, R3A, H2A, H3A
REAL AN1(25), AI1(25), ETA(25), EN3(25)
COMMON /BLK4/ AN1, AI1, ETA, EN3
REAL U1A(25), WA(25), R1A(25), R2A(25), R3A(25), H2A(25), H3A(25)
COMMON /BLK5/ U1A, WA, R1A, R2A, R3A, H2A, H3A
REAL BW24(25), BW24PC(25), BSDP2(25), BSDP23(25)
COMMON /BLK6/ BW24, BW24PC, BSDP2, BSDP23
REAL BEN3(25), ETA(25), EN34(25), ETA4(25)
COMMON /BLK7/ BEN3, ETA, EN34, ETA4
REAL BW34(25), BW34PC(25), H0(25)
COMMON /BLK8/ BW34, BW34PC, H0
REAL BT2(25), TSEC2(25), BT23(25), TSEC23(25), BT3(25)
COMMON /BLK9/ BT2, TSEC2, BT23, TSEC23, BT3
INTEGER NL(25), LAYERS
COMMON /BLK9/ NL, LAYERS

 INTEGER LN
REAL WB1(25), WB2(25), WB3(25), WB4(25)
REAL SWB, SWB1, SWB2, SWB3, SWB4
REAL BN(25), BI(25)
REAL BW2(25), BW2BYS(25), SBW2
REAL BNI(25)
REAL U1BN(25), ZBNN(25)
REAL NNORM1
REAL BNI(25), BNI1(25), U1BN1(25), BI1(25)
REAL U1(25), ZBNN1(25)
REAL INORM2
REAL B12(25), BN2(25), BNI2(25)
REAL U12(25), U1BN2(25), ZBNN2(25)
REAL BN3(25), B13(25)
REAL U13(25), BNI3(25), U1BN3(25), SU13
REAL U14(25), BN4(25), B14(25)
REAL BNI4(25), ZBNN4(25), U1BN4(25)
REAL SBN
REAL SBI1
REAL SBW24

DO 15 LN=1, LAYERS
BN(LN)=AN1(LN)
BI(LN)=AI1(LN)
ETA(LN)=ETA(LN)
EN3(LN)=EN3(LN)

B2(LN)=(1.-BEN3(LN))*BETA(LN)*WA(LN)
BW2(LN) = (1 - BEN3(LN))
BW2(LN) = BW2(LN) * BETA(LN)
BW2(LN) = BW2(LN) * WA(LN)

15 CONTINUE

c
SBW2 = 0.0
DO 16 I = 1, LAYERS
SBW2 = SBW2 + BW2(I)
16 CONTINUE

DO 20 LN = 1, LAYERS
BW2BYS(LN) = BW2(LN) / SBW2

BNI(LN) = BN(LN) * BI(LN)
U1BN(LN) = U1A(LN) / BN(LN)
ZBNN(LN) = U1BN(LN) / BNI(LN)
20 CONTINUE

C
CC
C
Step 1.
C Make all N the same
C Change them to a fraction of the sum, say, 1/3 or 1/4, and call it NNORM1.
SBN = 0.0
DO 21 I = 1, LAYERS
SBN = SBN + BN(I)
21 CONTINUE

NNORM1 = SBN / (LAYERS + 11.1111)

DO 25 LN = 1, LAYERS
n = nnorm1
BN1(LN) = NNORM1
C power remains the same
WB1(LN) = WA(LN)
C n must remain the same
BNI1(LN) = BNI(LN)
C u1/n remains the same
U1BN1(LN) = U1BN(LN)
C new current
I1 = n1/n1
BI1(LN) = BNI1(LN) / BN1(LN)
C new voltage
U1 = (u1/n) * n1
U11(LN) = U1BN1(LN) * BN1(LN)
C (z/nn)1 = (u1/n) * n1
= (u1/n) * (1/n1) = (1/nn) * z1 * (1(nn) = z/nn, and remains the same
ZBN11(LN) = U1BN1(LN) / BNI1(LN)
25 CONTINUE

C
CC
C
Step 2
SBI1 = 0.0
DO 26 I = 1, LAYERS
SBI1 = SBI1 + BI1(I)
26 CONTINUE

INORM2 = SBI1 / (LAYERS + 11.1111)

DO 30 LN = 1, LAYERS
BI2(LN) = INORM2
BN2(LN) = BN1(LN)
\[ \begin{align*}
BN12(LN) &= BN2(LN) \cdot BI2(LN) \\
U12(LN) &= U11(LN) \cdot BI2(LN) / BI1(LN) \\
U1BN2(LN) &= U12(LN) / BN2(LN) \\
ZBN2(LN) &= U1BN2(LN) / BN12(LN) \\
WB2(LN) &= WBI2(LN) \cdot (BI2(LN) \cdot BI1(LN) / BI1(LN)^2 \cdot BI1(LN) / BI1(LN)^2) \\
BN12(LN) &= BN2(LN) \cdot BI2(LN) \\
U1BN2(LN) &= U12(LN) / BN2(LN) \\
ZBN2(LN) &= U1BN2(LN) / BN12(LN) \\
\end{align*} \]

30 CONTINUE

\[ \begin{align*}
C \quad & \quad SWB2 = 0.0 & \quad \text{DO 31 I = 1, LAYERS} & \quad SWB2 = SWB2 + WB2(I) \\
31 \quad & \quad \text{CONTINUE} \\
\end{align*} \]

C step3

DO 32 LN = 1, LAYERS

\[ \begin{align*}
WB3(LN) &= WB2(LN) \cdot (WA(LN) / SWB2) \\
\end{align*} \]

32 CONTINUE

\[ \begin{align*}
SWB3 &= 0.0 & \quad \text{DO 33 I = 1, LAYERS} & \quad SWB3 = SWB3 + WB3(I) \\
33 \quad & \quad \text{CONTINUE} \\
\end{align*} \]

DO 35 LN = 1, LAYERS

\[ \begin{align*}
BN3(LN) &= BN2(LN) \\
BI3(LN) &= BI2(LN) \cdot (\sqrt{WA(LN) / SWB2}) \\
U13(LN) &= U12(LN) \cdot (\sqrt{WA(LN) / SWB2}) \\
BN3(LN) &= BN3(LN) \cdot BI3(LN) \\
U1BN3(LN) &= U13(LN) / BN3(LN) \\
\end{align*} \]

35 CONTINUE

\[ \begin{align*}
SWB3 &= 0.0 & \quad \text{DO 36 I = 1, LAYERS} & \quad SWB3 = SWB3 + WB3(I) \\
36 \quad & \quad \text{CONTINUE} \\
\end{align*} \]

SU13 = 0.0

DO 37 I = 1, LAYERS

SU13 = SU13 + U13(I)

37 CONTINUE

C step 4

DO 38 LN = 1, LAYERS

\[ \begin{align*}
U14(LN) &= U13(LN) \cdot (UA(LN) / SU13) \\
\end{align*} \]

38 CONTINUE

\[ \begin{align*}
SU14 &= 0.0 & \quad \text{DO 39 I = 1, LAYERS} & \quad SU14 = SU14 + U14(I) \\
39 \quad & \quad \text{CONTINUE} \\
\end{align*} \]

DO 40 LN = 1, LAYERS

\[ \begin{align*}
BN4(LN) &= BN3(LN) \cdot (UA(LN) / SU13) \\
BI4(LN) &= BI3(LN) \cdot (SU13 / UA(LN)) \\
\end{align*} \]
C

WB4(LN)=WB3(LN)
40 CONTINUE

SWB4=0.0
DO 41 I=1,LAYERS
SWB4=SWB4+WB4(I)
41 CONTINUE

DO 45 LN=1,LAYERS
BNI4(LN)=BNI3(LN)
U1BN4(LN)=U14(LN)/BN4(LN)
ZBNN4(LN)=U1BN4(LN)/BNI4(LN)
U1BN4(LN)=U14(LN)/BN4(LN)
ZBNN4(LN)=U1BN4(LN)/BNI4(LN)

BW24(LN)=(1.-BEN3(LN))*BETA(LN)*WB4(LN)

SWB24=0.0
DO 46 I=1,LAYERS
SWB24=SWB24+BW24(I)
46 CONTINUE

SWB34=0.0
DO 47 I=1,LAYERS
SWB34=SWB34+BW34(I)
47 CONTINUE

DO 50 LN=1,LAYERS
BW24PC(LN)=100.*BW24(LN)/SBW24
BW34PC(LN)=100.*BW34(LN)/SBW34

C Surface power density
BSPD2(LN)=BW24(LN)/(2.*PI*R2A(LN)*H2A(LN))
BSPD23(LN)=(BW24(LN)+BW34(LN))/(2.*PI*R2A(LN)*H2A(LN))

C or:
BSPD23(LN)=(BSPD2(LN)/BW24(LN))*(BW24(LN)+BW34(LN))

c rate of temp. rise of pot per second:
TSEC2(LN)=BSPD2(LN)/(.5*GAMA2A(LN)*SC2A(LN)
1*{(R2A(LN)**2-R3A(LN)**2)/R2A(LN))}

C or per minute:
BT2(LN)=60.*BW24(LN)/(GAMA2A(LN)**2)*SC2A(LN)
1*PI*R2A(LN)**2-R3A(LN)**2)

BT23(LN)=(BT2(LN)/BW24(LN))*BW24(LN)+BW34(LN))

c rate of temp. rise of charge per min.:
BT3(LN)=60.*BW34(LN)/(GAMA3A(LN)**2)*SC3A(LN)
1*PI*R3A(LN)**2)

BEN34(LN)=BW34(LN)/(BW34(LN)+BW24(LN))

BETA4(LN)=(BW34(LN)+BW24(LN))/WB4(LN)
50 CONTINUE

C

100 RETURN
END
APPENDIX A6.1

________________________________________________________________________
* Listing of HT subroutine code *

________________________________________________________________________

C seven segments, 7 eqns., no losses considered.
C h.t. between pot & charge included.

DOUBLE PRECISION U1, W, PI, GAMMA2, GAMMA3, SC2, SC3
COMMON /BLK1/ U1, W, PI, GAMMA2, GAMMA3, SC2, SC3
REAL U1A(25), WA(25), R1A(25), R2A(25), R3A(25), H2A(25), H3A(25)
COMMON /BLK3/ U1A, WA, R1A, R2A, R3A, H2A, H3A
REAL BW24(25), BW24PC(25), BSPD2(25), BSPD23(25)
COMMON /BLK5/ BW24, BW24PC, BSPD2, BSPD23
REAL BEN3(25), BETA(25), BEN34(25), BETA4(25)
COMMON /BLK6/ BEN3, BETA, BEN34, BETA4
REAL BW34(25), BW34PC(25), BHO(25)
COMMON /BLK7/ BW34, BW34PC, BHO
REAL BT2(25), TSEC2(25), BT23(25), TSEC23(25), BT3(25)
COMMON /BLK8/ BT2, TSEC2, BT23, TSEC23, BT3
INTEGER NL(25), LAYERS
COMMON /BLK9/ NL, LAYERS
DOUBLE PRECISION KON2, KON3
COMMON /BLK10/ KON2, KON3
REAL AG2A(25), AG3A(25), AS2A(25), AS3A(25)
COMMON /BLK12/ AG2A, AG3A, AS2A, AS3A
REAL MC2A(25), MC3A(25)
COMMON /BLK13/ MC2A, MC3A
REAL BTAD3, BTAD2B, BTAD3M, BTAD2T, BTAD2A(10)
COMMON /BLK14/ BTAD3, BTAD2B, BTAD3M, BTAD2T, BTAD2A
DOUBLE PRECISION T2D, T2B, T2M, T2T
COMMON /BLK400/ T2D, T2B, T2M, T2T
DOUBLE PRECISION T3B, T3M, T3T
COMMON /BLK410/ T3B, T3M, T3T
REAL W22C(25), W23C(25)
INTEGER LC

CCC
C disc to charge @ bottom
BTAD3 = .2
WD3B = KON3*AS3B*BTAD3*0.5*(BT2(1)-BT3B)*H3B
WRITE(*,30) WD3B, BTAD3
30 FORMAT('h.t from disc to charge @ bottom=',F6.2,1X,'with contact area coeff. = ',F6.2)

C Hence, power in disc after h.t. with bott-charge is
W24D = BW24(1) - WD3B
C power in bottom charge after h.t. with disc is
W34B = BW3B + WD3B
WRITE(*,45)
45 FORMAT('Set #',5X,'W22C, h.t. pot to pot'/)
WRITE(1,46)(NL(I),W22C(I),I=1,9)
46 FORMAT(2X,I2,9X,F8.3)
C
CCC
C
C h.t. from disc to bottom-wall
WD2B=KON2*AS2B*0.5*(BT2(1)-BT2B)*H2B

WRITE(1,50) WD2B
50 FORMAT(//1H ,'h.t. from disc to bottom wall= ',F8.2/)
c hence, power in disc after further h.t. with bott-wall
W24D=W24D - WD2B
c Resultant temp-rise in disc after h.t.
T2D=(W24D*60.)/MC2A(1)

WRITE(1,51) W24D,T2D
51 FORMAT(//1H ,'power in disc after h.t. = ',F8.2, 
1 /1H ,'Resultant temp-rise in disc after h.t.=',F8.2//)
CC
c power in bottom wall after h.t.
W24B=BW24B+WD2B
C
CCC
C
C bottom wall to middle wall
W2B2M=KON2*AS2M*0.5*(BT2B-BT2M)/H2M

WRITE(1,55) W2B2M
55 FORMAT(//1H ,'h.t. from bottom wall to mid. wall= ',F8.2/)
c hence, power in mid-wall after h.t.
W24M=BW24M+W2B2M
C
CCC
C
C middle wall to top wall
W2M2T=KON2*AS2T*0.5*(BT2M-BT2T)/H2T

WRITE(1,56) W2M2T
56 FORMAT(//1H ,'h.t. from mid-wall to top-wall= ',F8.2//)
c hence, power in top-wall after h.t.
W24T=BW24T+W2M2T
C
CCC
C
C bottom-wall to bottom-charge
BTA23B=1
W23B=KON3*AG3B*BTA23B*0.5*(BT2B-BT3B)/R3B

WRITE(1,66) W23B,BTA23B
66 FORMAT(//1H ,'h.t. from bottom wall to bottom charge= ',F8.2, 
1 /1H ,'with contact area coeff. = ',F8.2)
CC
c power in bottom-charge after further h.t.
W34B=W34B + W23B
c Resultant temp.-rise in bottom charge after h.t.
T3B=(W34B*60.)/(MC3A(2)+MC3A(3)+MC3A(4))

WRITE(1,67) W34B,T3B
67 FORMAT(1'H ','power in bottom-charge after h.t. = ', F8.2,
1 /1'H ', Resultant temp-rise in bottom-charge after h.t. = ', F8.2//
CC
c power in bottom-wall after further h.t.
W24B=W24B - W23B - W2B2M
c Resultant temp-rise in bottom-wall after h.t.
T2B=(W24B*60.)/MC2B

WRITE(1, 69) W24B, T2B
69 FORMAT(1'H ','power in bottom-wall after h.t. = ', F8.2,
1 /1'H ', Resultant temp-rise in bottom-wall after h.t. = ', F8.2//
C CCCC
c mid-wall to mid-charge
BTA23M= 1
W23M=KON3*AG3M*BTA23M*0.5*(BT2M-BT3M)/R3M

WRITE(1, 70) W23M, BTA23M
70 FORMAT(1'H ','h.t from mid-wall to mid-charge = ', F8.2,
1 /1'H ', with contact area coeff. = ', F8.2)
CC
c power in mid-charge after h.t.
W34M=BW34M+W23M
c Resultant temp-rise in mid.-charge after h.t.
T3M=(W34M*60.)/MC3M

WRITE(1, 71) W34M, T3M
71 FORMAT(1'H ','power in mid-charge after h.t. = ', F8.2,
1 /1'H ', Resultant temp-rise in mid-charge after h.t. = ', F8.2//
CC CCCC
c power in mid-wall after h.t. with top-wall
W24M=W24M - W2M2T
c power in mid-wall after further h.t. with mid-charge
W24M=W24M - W23M
c Resulting temp-rise in mid-wall after h.t.
T2M=(W24M*60.)/MC2M

WRITE(1, 73) W24M, T2M
73 FORMAT(1'H ',’power in mid-wall after h.t. = ', F8.2,
1 /1'H ', Resultant temp-rise in mid-wall after h.t. = ', F8.2//
C CCCC
c top-wall to top-charge
BTA23T= 1
W23T= KON3*AG3T*BTA23T*0.5*(BT2T-BT3T)/R3T

WRITE(1, 75) W23T, BTA23T
75 FORMAT(1'H ',’h.t from top-wall to top-charge = ', F8.2,
1 /1'H ', with contact area coeff. = ', F8.2)
c power in top-charge after h.t.
W34T=BW34T+W23T
c Resultant temp-rise in top-charge after h.t.
T3T=(W34T*60.)/MC3T

WRITE(1, 85) W34T, T3T
85 FORMAT(1'H ',’power in top-charge after h.t. = ', F8.2,
1 /1'H ', Resultant temp-rise in top-charge after h.t. = ', F8.2//
c power in top-wall after h.t.
W24T=W24T - W23T

c Resulting temp-rise in top-wall after h.t.
T2T=(W24T*60.)/MC2T

WRITE(1,87) W24T,T2T

87 FORMAT(//1H , 'power in top-wall after h.t.=',F8.2,
1 /1H , 'Resultant temp-rise in top-wall after h.t.=',F8.2//)

CC
BT2D=BT2(1)

WRITE(1,89)

89 FORMAT(1H , 'PROFILE OF TEMPERATURE-RISES WITH NO H.T. :'/)
WRITE(1,91) BT2T,BT3T,BT2M,BT3M,BT2B,BT3B,BT2D

91 FORMAT(//1H , 'T2T= ',F8.2,' T3T= ',F8.2,
1 /1H , ' T2M=',F8.2,' T3M=',F8.2,
2 /1H , ' T2B=',F8.2,' T3B=',F8.2,
3 /1H , ' T2D=',F8.2//)

CC

WRITE(1,93) BTAD3,BTAD23M

93 FORMAT(1H , 'PROFILE OF TEMPERATURE-RISES AFTER H.T. :'
1 /1H , 'with contact area coeff. with disc = ',F8.2,
2 /1H , 'with contact area coeff. with walls = ',F8.2)
WRITE(1,95) T2T,T3T,T2M,T3M,T2B,T3B,T2D

95 FORMAT(//1H , 'T2T= ',F8.2,' T3T= ',F8.2,
1 /1H , ' T2M=',F8.2,' T3M=',F8.2,
2 /1H , ' T2B=',F8.2,' T3B=',F8.2,
3 /1H , ' T2D=',F8.2//)

CC

T2T2=2.0*T2T
T3T2=2.0*T3T
T2M2=2.0*T2M
T3M2=2.0*T3M
T2B2=2.0*T2B
T3B2=2.0*T3B
T2D2=2.0*T2D

WRITE(1,97)

97 FORMAT(1H , 'DOUBLED TEMP-RISES AFTER H.T.',
1 /1H , ' GIVE TEMPS. FOR NEXT TIME-INCREMENT PROPERTIES :'/)
WRITE(1,99) T2T2,T3T2,T2M2,T3M2,T2B2,T3B2,T2D2

99 FORMAT(//1H , ' T= ',F8.2,' M= ',F8.2,
1 /1H , ' B= ',F8.2,' D= ',F8.2,
2 /1H , ' B= ',F8.2,' D= ',F8.2,
3 /1H , ' B= ',F8.2,' D= ',F8.2//)

100 RETURN
END
APPENDIX A6.2

Listing of SUB19 subroutine code

---

c 19 segments, 19 eqns. inter-charge h.t. no losses

SUBROUTINE FCN(T,Z,D)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION U1,W,PI,GAMA2,GAMA3,SC2,SC3
COMMON /BLK1/ U1,W,PI,GAMA2,GAMA3,SC2,SC3
REAL U1A(25),WA(25),R1A(25),R2A(25),R3A(25),H2A(25),H3A(25)
COMMON /BLK3/ U1A,WA,R1A,R2A,R3A,H2A,H3A
REAL BW24(25),BW24PC(25),BSPD2(25),BSPD23(25)
COMMON /BLK5/ BW24,BW24PC, BSPD2, BSPD23
REAL BEN3(25),BETA(25), BEN34(25),BETA4(25)
COMMON /BLK6/BEN3,BETA, BEN34,BETA4
REAL BW34(25),BW34PC(25),BHO(25)
COMMON /BLK7/ BW34,BW34PC,BHO
REAL BT2(25),TSEC2(25), BT23(25),TSEC23(25), BT3(25)
COMMON /BLK8/ BT2,TSEC2, BT23,TSEC23, BT3
INTEGER NL(25),LAYERS
COMMON /BLK9/ NL,LAYERS
DOUBLE PRECISION KON2, KON3
COMMON /BLK10/ KON2,KON3
REAL KON2A(25), KON3A(25)
COMMON /BLK11/ KON2A, KON3A
REAL AG2A(25),AG3A(25),AS2A(25),AS3A(25)
COMMON /BLK12/ AG2A,AG3A,AS2A,AS3A
REAL MC2A(25),MC3A(25)
COMMON /BLK13/ MC2A,MC3A
REAL BTAD3, BTA23B,BTA23M,BTA23T, BTA23A(10)
COMMON /BLK14/ BTAD3, BTA23B,BTA23M,BTA23T, BTA23A

DOUBLE PRECISION KONR
COMMON /BLK601/ KONR
DOUBLE PRECISION HBSE
COMMON /BLK602/ HBSE
DOUBLE PRECISION LDA
COMMON /BLK603/ LDA
DOUBLE PRECISION EPS2
COMMON /BLK604/ EPS2
DOUBLE PRECISION EPS3
COMMON /BLK605/ EPS3

C .. SCALAR ARGUMENTS ..
DOUBLE PRECISION T

C .. ARRAY ARGUMENTS ..
DOUBLE PRECISION D(19), Z(19)

DOUBLE PRECISION C2A(10), C3A(10)
DOUBLE PRECISION F21A(10),F22A(10),F23A(10)
DOUBLE PRECISION F31A(10),F32A(10),F33A(10)

TA=20.0D
TK=273.000

\[ C1=1./MC2A(1)\]
\[ F12=KON2A(2)\times AS2A(2)/(.5\times H2A(2))\]
\[ F13=KON3A(2)\times BTAD3\times AS3A(2)/(.5\times H3A(2))\]
\[ D(1)=C1\times BW24(1)\]
\[ * -C1\times (F12+F13)\times Z(1)\]
\[ * +C1\times F12\times Z(2)\]
\[ * +C1\times F13\times Z(3)\]
\[ * +C1\times TA\]

\[ C2=1./MC2A(2)\]
\[ F21=KON3A(2)\times BTA23A(2)\times AG3A(2)/R3A(2)\]
\[ F22=KON2A(2)\times AS2A(2)/(.5\times H2A(2))\]
\[ F23=KON2A(3)\times AS2A(3)/(.5\times H2A(3))\]
\[ D(2)=C2\times BW24(2)\]
\[ * -C2\times (F21+F22+F23)\times Z(2)\]
\[ * +C2\times F21\times Z(3)\]
\[ * +C2\times F22\times Z(1)\]
\[ * +C2\times F23\times Z(4)\]
\[ * +C2\times TA\]

\[ C3=1./MC3A(2)\]
\[ F31=KON3A(2)\times BTA23A(2)\times AG3A(2)/R3A(2)\]
\[ F32=KON2A(2)\times AS2A(2)/(.5\times H2A(2))\]
\[ F33=KON3A(3)\times AS3A(3)/(.5\times H3A(3))\]
\[ D(3)=C3\times BW34(2)\]
\[ * -C3\times (F31+F32+F33)\times Z(3)\]
\[ * +C3\times F31\times Z(2)\]
\[ * +C3\times F32\times Z(1)\]
\[ * +C3\times F33\times Z(5)\]

DO 99 I=3,9

\[ C2A(I)=1./MC2A(I)\]
\[ F21A(I)=KON3A(I)\times BTA23A(I)\times AG3A(I)/R3A(I)\]
\[ F22A(I)=KON2A(I)\times AS2A(I)/(.5\times H2A(I))\]
\[ F23A(I)=KON2A(I+1)\times AS2A(I+1)/(.5\times H2A(I+1))\]
\[ D(2*I-2)=C2A(I)\times BW24(I)\]
\[ * -C2A(I)\times (F21A(I)+F22A(I)+F23A(I))\times Z(2*I-2)\]
\[ * +C2A(I)\times F21A(I)\times Z(2*I-1)\]
\[ * +C2A(I)\times F22A(I)\times Z(2*I-4)\]
\[ * +C2A(I)\times F23A(I)\times Z(2*I)\]
\[ * +C2A(I)\times TA\]

\[ C3A(I)=1./MC3A(I)\]
\[ F31A(I)=KON3A(I)\times BTA23A(I)\times AG3A(I)/R3A(I)\]
\[ F32A(I)=KON3A(I)\times AS3A(I)/(.5\times H3A(I))\]
\[ F33A(I)=KON3A(I+1)\times AS3A(I+1)/(.5\times H3A(I+1))\]
\[ D(2*I-1)=C3A(I)\times BW34(I)\]
\[ * -C3A(I)\times (F31A(I)+F32A(I)+F33A(I))\times Z(2*I-1)\]
\[ * +C3A(I)\times F31A(I)\times Z(2*I-2)\]
\[ * +C3A(I)\times F32A(I)\times Z(2*I-3)\]
\[ * +C3A(I)\times F33A(I)\times Z(2*I+1)\]

99 CONTINUE

\[ C18=1./MC2A(10)\]
\[ F181=KON3A(10)\times BTA23A(10)\times AG3A(10)/R3A(10)\]
\[ F182=KON2A(10)\times AS2A(10)/(.5\times H2A(10))\]
CL18 = 1. / (2. * (R2A(10) - R3A(10)))
D(18) = C18 * BW24(10)
* -C18 * (F181 + F182) * Z(18)
* +C18 * F181 * Z(19)
* +C18 * F182 * Z(16)

C19 = 1. / MC3A(10)
F191 = K0N3A(10) * BTA23A(10) * AG3A(10) / R3A(10)
F192 = KON3A(10) * AS3A(10) / (0.5 * H3A(10))
CL19 = 1. / (2. * R3A(10))
D(19) = C19 * BW34(10)
* -C19 * (F191 + F192) * Z(19)
* +C19 * F191 * Z(18)
* +C19 * F192 * Z(17)

100 RETURN
END
Appendix A6.3: Emissivities of relevant materials

Table A6.3.1 Emissivity of refractory materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Condition</th>
<th>Temperature</th>
<th>Emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>silica brick</td>
<td></td>
<td>1000</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1100</td>
<td>0.85</td>
</tr>
<tr>
<td>graphite</td>
<td></td>
<td>0-3600</td>
<td>0.7 - 0.8</td>
</tr>
<tr>
<td>asbestos paper</td>
<td></td>
<td>20</td>
<td>0.95</td>
</tr>
<tr>
<td>asbestos board</td>
<td></td>
<td>20</td>
<td>0.96</td>
</tr>
<tr>
<td>alumina sintered</td>
<td></td>
<td>1000-1500</td>
<td>0.4</td>
</tr>
<tr>
<td>fire brick</td>
<td></td>
<td>1000</td>
<td>0.78</td>
</tr>
<tr>
<td>fire clay</td>
<td></td>
<td>1000</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1500</td>
<td>0.7</td>
</tr>
<tr>
<td>corundum emery</td>
<td></td>
<td>80</td>
<td>0.855</td>
</tr>
<tr>
<td>clay fired</td>
<td></td>
<td>67</td>
<td>0.91</td>
</tr>
<tr>
<td>refractory brick</td>
<td>ordinary</td>
<td>1100</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>white</td>
<td>1100</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>dark chrome</td>
<td>1100</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Table A6.3.2. Emissivity of aluminium and copper [6.7,6.21]

<table>
<thead>
<tr>
<th>Material</th>
<th>Condition</th>
<th>Temperature</th>
<th>emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>aluminium</td>
<td>polished</td>
<td>50-500</td>
<td>0.04-0.06</td>
</tr>
<tr>
<td></td>
<td>rough surface</td>
<td>20.50</td>
<td>0.06-0.07</td>
</tr>
<tr>
<td></td>
<td>strongly oxidized</td>
<td>25</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td></td>
<td>500</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>oxidized</td>
<td>200</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>600</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>bright rolled</td>
<td>170</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td></td>
<td>500</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>polished, 98% pure</td>
<td>93</td>
<td>0.05</td>
</tr>
<tr>
<td>copper</td>
<td>polished</td>
<td>20</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>heavily oxidized</td>
<td>20</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>oxidized</td>
<td>50</td>
<td>0.6-0.7</td>
</tr>
<tr>
<td></td>
<td>unoxidized liquid</td>
<td>-</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>lightly oxidized</td>
<td>20</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>scraped</td>
<td>20</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>black oxidized</td>
<td>20</td>
<td>0.078</td>
</tr>
</tbody>
</table>

After Weast, R.C. and Eckert, G.R.G.
APPENDIX A6.4

TO DETERMINE THE FLOW REGIME FOR CONVECTION LOSSES

To determine whether the flow is laminar or turbulent, it is necessary to evaluate the approximate value of \((Gr \cdot Pr)\), the Rayleigh number. For this purpose it may be assumed that the value of \((\beta \cdot g \cdot \rho \cdot C_p / \mu \cdot k)\) is \(6.4 \times 10^2\) \([6.22]\). This has then to be multiplied by a (characteristic linear dimension)\(^3\) \(\Delta \theta\) \((m^3 \cdot K)\); to determine the dimensionless product \((Gr \cdot Pr)\).

Fluid properties are \(\beta, \rho, C_p, \mu\) and \(k\).

where \(\beta\) = cubic expansion coefficient \((K^{-1})\)

\(\rho\) = density of warmer air \((kg \cdot m^{-3})\)

\(\mu\) = dynamic viscosity \((kg \cdot m^{-1} \cdot s^{-1})\)

\(g\) = gravitational acceleration \((m \cdot s^{-2})\)

For the A60 size crucible, the linear dimension for the top rim is

\[
2 \times (R_2 - R_3) = 2 \times t_2 = 0.051 \text{ m.}
\]

\(\Delta \theta\) may range from about 35K to about 1180K depending on the melting stage, charge, etc. Therefore for initial stage:

\[
(Gr \cdot Pr) = 0.05^3 \times 35 \times 6.4 \times 10^7 = 28 \times 10^4
\]

In final stages:

\[
(Gr \cdot Pr) = 28 \times 10^4 \times \frac{1180}{35} = 9.44 \times 10^6
\]

In both cases,

\(10^4 < (Gr \cdot Pr) < 10^9\)

which indicates the flow is laminar \([6.22, 6.23]\) and consequently justifies use of equations 6.28 and 6.29.
APPENDIX A6.5

Listing of SUB19L subroutine code

c 19 segments, 19 eqns. all losses & int-charge h.t. included
SUBROUTINE FCN(T,Z,D)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

DOUBLE PRECISION T,Z,D
REAL U1(25),WA(25),R1A(25),R2A(25),R3A(25),H2A(25),H3A(25)
REAL BW24(25),BW24PC(25),BSPD2(25),BSPD23(25)
REAL BEN3(25),BETA(25),BEN34(25),BETA4(25)
REAL BW34(25),BW34PC(25),BH0(25)
REAL BT2(25),TSEC2(25),BT23(25),TSEC23(25),BT3(25)
INTEGER NL(25),LAYERS

DOUBLE PRECISION KON2(KON3)
REAL KON2A(25),KON3A(25)
REAL MC2A(25),MC3A(25)
REAL BTAD3,BTA23B,BTA23M,BTA23T,BTA23A(10)

DOUBLE PRECISION KONR
DOUBLE PRECISION T
DOUBLE PRECISION D(19),Z(19)
DOUBLE PRECISION C2A(10),C3A(10)
DOUBLE PRECISION F21A(10), F22A(10), F23A(10), F24A(10)
DOUBLE PRECISION F31A(10), F32A(10), F33A(10)

TA=20.D0
TK=273.0D0

C1=1./MC2A(I)
F12=KON2A(2)*AS2A(2)/(.5*H2A(2))
F13=KON3A(2)*BTAD3*AS3A(2)/(.5*H3A(2))
F14=KONR*AG2A(1)/(R1A(1)-R2A(1))
FD5=KONR*AS2A(1)/HBSE
D(1)=C1*BW24(1)
* -C1*(F12+F13+(1.+LDA)*(F14+FD5))*Z(1)
  +C1*F12*Z(2)
  +C1*F13*Z(3)
  +C1*(1.+LDA)*(F14+FD5)*TA

C2=1./MC2A(2)
F21=KON3A(2)*BTA23A(2)*AG3A(2)/R3A(2)
F22=KON2A(2)*AS2A(2)/(.5*H2A(2))
F23=KON2A(3)*AS2A(3)/(.5*H2A(3))
F24=KONR*AG2A(2)/(R1A(2)-R2A(2))
D(2)=C2*BW24(2)
* -C2*(F21+F22+F23+(1.+LDA)*F24)*Z(2)
  +C2*F21*Z(3)
  +C2*F22*Z(1)
  +C2*F23*Z(4)
  +C2*(1.+LDA)*F24*TA

C3=1./MC3A(2)
F31=KON3A(2)*BTA23A(2)*AG3A(2)/R3A(2)
F32=KON3A(2)*BTAD3*AS3A(2)/(.5*H3A(2))
F33=KON3A(3)*AS3A(3)/(.5*H3A(3))
D(3)=C3*BW34(2)
* -C3*(F31+F32+F33)*Z(3)
  +C3*F31*Z(2)
  +C3*F32*Z(1)
  +C3*F33*Z(5)

DO 99 I=3,9
C2A(I)=1./MC2A(I)
F21A(I)=KON3A(I)*BTA23A(I)*AG3A(I)/R3A(I)
F22A(I)=KON2A(I)*AS2A(I)/(.5*H2A(I))
F23A(I)=KON2A(I+1)*AS2A(I+1)/(.5*H2A(I+1))
F24A(I)=KONR*AG2A(I)/(R1A(I)-R2A(I))
D(2*I-2)=C2A(I)*BW24(I)
* -C2A(I)*(F21A(I)+F22A(I)+F23A(I)+(1.+LDA)*F24A(I))*Z(2*I-2)
  +C2A(I)*F21A(I)*Z(2*I-1)
  +C2A(I)*F22A(I)*Z(2*I-4)
  +C2A(I)*F23A(I)*Z(2*I)
  +C2A(I)*(1.+LDA)*F24A(I)*TA

C3A(I)=1./MC3A(I)
F31A(I)=KON3A(I)*BTA23A(I)*AG3A(I)/R3A(I)
F32A(I)=KON3A(I)*AS3A(I)/(.5*H3A(I))
F33A(I) = KON3A(I+1) * AS3A(I+1) / (.5 * H3A(I+1))

D(2*I-1) = C3A(I) * BW34(I)
* - C3A(I) * (F31A(I) + F32A(I) + F33A(I)) * Z(2*I-1)
* + C3A(I) * F31A(I) * Z(2*I-2)
* + C3A(I) * F32A(I) * Z(2*I-3)
* + C3A(I) * F33A(I) * Z(2*I+1)

CONTINUE

C18 = 1. / MC2A(10)
F181 = KON3A(10) * BTA23A(10) * AG3A(10) / R3A(10)
F182 = KON2A(10) * AS2A(10) / (.5 * H2A(10))
F184 = KONR * AG2A(10) / (R1A(10) - R2A(10))
CL18 = 1. / (2. * (R2A(10) - R3A(10)))
D(18) = C18 * BW34(10)
* - C18 * (F181 + F182 + (1. + LDA) * F184) * Z(18)
* + C18 * F181 * Z(19)

+ C18 * F182 * Z(16)
* - C18 * 1.32 * (CL18 * (Z(18) - TA)) * .025
**AS2A(10) * (Z(18) - TA)
* - C18 * EPS2 * AS2A(10) * 5.669D-8
*( (Z(18) + TK) - (TA + TK) )
*( (Z(18) + TK) + (TA + TK) )
*( (Z(18) + TK) ** 2 + (TA + TK) ** 2 )
* + C18 * (1. + LDA) * F184 * TA

C19 = 1. / MC3A(10)
F191 = KON3A(10) * BTA23A(10) * AG3A(10) / R3A(10)
F192 = KON3A(10) * AS3A(10) / (.5 * H3A(10))
CL19 = 1. / (2. * R3A(10))
D(19) = C19 * BW34(10)
* - C19 * (F191 + F192) * Z(19)
* + C19 * F191 * Z(18)

+ C19 * F192 * Z(17)
* - C19 * 1.32 * (CL19 * (Z(19) - TA)) * .025
**AS3A(10) * (Z(19) - TA)
* - C19 * EPS3 * AS3A(10) * 5.669D-8
*( (Z(19) + TK) - (TA + TK) )
*( (Z(19) + TK) + (TA + TK) )
*( (Z(19) + TK) ** 2 + (TA + TK) ** 2 )

100 RETURN
END
Effect of lower operating frequency on temperature distribution.

- Furnace: Tilter
- Frequency: 2 kHz
- Power Input: 100 kW
- Crucible: Clay-graphite axial resistivity: non-uniform
- Initial Temperature: 20°C
- Charge: Aluminium packing: well-packed
- Initial Temperature: 20°C
- Time to reach pouring temperature: 27.5 mins.
Fig. A7.1.2 Effect of lower operating frequency on temperature distribution.

FURNACE: tilter FREQUENCY: 2 kHz  POWER INPUT: 100 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: non-uniform
INITIAL TEMPERATURE: 20°C
CHARGE: copper PACKING: well-packed INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 61 mins.
MAXIMUM PRINCIPAL (MOST POSITIVE) STRESS (MIDDLE SURFACE)
MULTIPLY BY 18

-78.29
-69.29
-58.29
-49.29
-39.29
-29.38
-19.38
-8.18
1.8
11.88

LOAD CASE = 1
WHOLE STRUCTURE DRAWN AS DEFINED IN FRONT. ORDER

PRODUCED MRKARG ON 12/06/87 AT 1059 HOURS

DRAWING NO. 2
SCALE = 0.5256 11
DRAWING TYPE = 38

THERMAL STRESSES IN CRUCIBLES

VIEW FROM X = 0.0000
Y = 0.0000
Z = 1.000

Z TOWARDS VIEWER

360
TITLE Fig A7.2.2 Thermal stresses in crucible & pouring temp. Fig 7.3

VIEW FROM X=0.0000
Y=0.0000
Z=1.000

Z TOWARDS VIEWER

MAXIMUM
PRINCIPAL
(DEST
POSITIVE)
STRESS
(MIDDLE SURFACE)
MULTIPLY
BY 10^8
-99.10
-65.50
-72.90
-68.20
-71.50
-34.90
-22.90
-9.660
2.600
15.80

LOAD CASE = 1

WHOLE STRUCTURE: DRAWN
AS DEFINED IN FRONT. ORDER

PRODUCED MRKARG
ON 16/06/67 AT 2240 HOURS

DRAWING NO. 2
SCALE = 0.0256
DRAWING TYPE= 30
Fig A7.2.3 Thermal stresses in crucible & pouring temp for Fig 7.4

Load Case = 1
Whole Structure drawn as defined in front. order

Maximum principal (most positive) stress (middle surface) multiply by 10^8

-15.00
-19.80
-23.20
-20.10
-22.20
-16.40
-18.60
-4.200
1.000
6.270

View from
X = 0.0000
Y = 0.0000
Z = 1.000

Z towards viewer

Principal (positive) stress (middle surface) multiply by 10^8

Scale = 0.0250
Drawing type = 30
Fig A7.2.4 Thermal stresses in crucible & pouring temp for Fig 7.5

Maximum principal stress (host positive)

Stress (middle surface) multiply by 10^6

-75.50
-65.90
-56.30
-46.70
-37.10
-27.50
-17.90
-7.30
2.00
11.70

Load Case = 1
Whole structure drawn as defined in front order

Produced by MKARG on 16/06/97 at 2049 hours

Drawing No. 2
Scale = 0.8256
Drawing Type = 30
Fig A7.2.5 Thermal stresses in crucible & pouring temp for Fig 7.7

View from X = 0.0000
Y = 0.0000
Z = 1.000

Maximum principal (most positive) stress (middle surface) multiply by 10^6:
-107.8
-94.06
-88.20
-87.20
-53.68
-40.28
-25.40
-12.80
6.2500
14.30

Load case = 1
Whole structure drawn as defined in front, order

Produced by MKARG
On 18/06/97 at 2135 hours

Drawing no. 2
Scale = 0.625
Drawing type = 30

Title: Fig A7.2.5  Thermal stresses in crucible & pouring temp for Fig 7.7

Legend:
- Z towards viewer

Drawings:
- Original drawing
- Copy 1

Structural units:
- Multiply by 10^2

Dimensions:
- 1.0, 2.0, 3.0, 4.0, 5.0

Notes:
- Principal stresses
- Maximum stress
- Load case
- Drawing type

Date:
- 18/06/97
TITLE Fig A7.2.4 Thermal stresses in crucible & pouring temp for Fig 7.8

VIEW FROM X = 0.0000
Y = 0.0000
Z = 1.000

Z TOWARDS VIEWER

MAXIMUM
PRINCIPAL
(MOST
POSITIVE)
STRESS
(MIDDLE SURFACE)
MULTIPLY
BY 10^6
-145.0
-126.0
-98.8
-92.5
-76.6
-52.2
-33.5
-14.8
3.8
22.5

LOAD CASE = 1
WHOLE STRUCTURE DRAWN
AS DEFINED IN FRONT. ORDER

PRODUCED NRKARG
ON 16/06/97 AT 22.9 HOURS

DRAWING NO. 2
SCALE = 0.6250
DRAWING TYPE = 30
TITLE Fig A7.2.7 Thermal stresses in crucible & pouring temp For Fig 7.25

VIEW FROM X = 0.0000
Y = 0.0000
Z = 1.00

MAXIMUM
PRINCIPAL
(MOST
POSITIVE)
STRESS
(MIDDLE SURFACE)
MULTIPLY
BY 10^6

-113.8
-99.18
-84.76
-70.40
-56.18
-41.80
-27.20
-12.80
1.480
15.80

LOAD CASE = 1
WHOLE STRUCTURE DRAWN AS DEFINED IN FRONT ORDER

PRODUCED MRKARG ON 16-06-97 AT 22 9 HOURS

MULTIPLY BY 10^6

1.0 2.0 3.0 4.0 5.0 FT.

MULTIPLY BY 10^2

1.0 2.0 3.0 4.0 5.0 IN.

DRAWING NO. 2
SCALE = 0.0250
DRAWING TYPE = 30
Fig 1.1.2.9 Thermal stresses in crucible & pouring temp for Fig 7.46

Title: Fig 1.1.2.9 Thermal stresses in crucible & pouring temp for Fig 7.46

View from:
X = 0.0000
Y = 0.0000
Z = 1.000

Toward viewer:
Z

Maximum principal (most positive) stress (middle surface) multiply by 10^6
-9.97480E-4
0.00520
0.00623
0.00531
0.00747
0.00950
1.000
1.037
1.053
1.073

Load case = 1
Whole structure drawn as defined in front. order

Produced by MRKARG on 10/06/87 at 1859 Hours

Drawing type: 38
Drawing number: 2
Scale: 0.6250
Structural units: 11
Fig. A8.1.1 Effect of uniform resistivity on temperature distribution.

FURNACE: tilt
FREQUENCY: 3 kHz
POWER INPUT: 100 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: uniform
INITIAL TEMPERATURE: 20°C
CHARGE: aluminium
PACKING: well-packed
INITIAL TEMPERATURE: 20°C
TIME TO REACH POURING TEMPERATURE: 23.5 mins.
Fig. A8.1.2 Effect of uniform resistivity on temperature distribution.

FURNACE: tiltter FREQUENCY: 3 kHz POWER INPUT: 100 kW
CRUCIBLE: clay-graphite AXIAL RESISTIVITY: uniform
INITIAL TEMPERATURE: 20 C
CHARGE: copper PACKING: well-packed INITIAL TEMPERATURE: 20 C
TIME TO REACH POURING TEMPERATURE: 61.5 mins.
BIBLIOGRAPHY


FORD, W.F.: 'The effect of heat on ceramics' (Maclaren and Sons Ltd, 1967)


JOHNS, D.J.: 'Thermal stress analyses' (Pergamon Press, 1965)


Theoretical investigations into the use of conducting crucibles in medium-frequency metal melting


Indexing term: Power utilization and industrial application

Abstract: Clay-graphite and carbon-bonded silicon-carbide crucibles are used in nonferrous induction melting furnaces. Silicon-carbide crucibles especially have encountered premature failure when used at high power densities and operating frequencies. This is thought to be related to their nonuniform properties. To gain a more thorough understanding of this problem, an equivalent circuit analysis has been applied to the composite load of crucible and metal charge. An explanation of the analysis is put forward and results giving include electrical characteristics, the distribution of power input between charge and crucible, and resultant meniscus height for typical industrial installations. The paper also explains the use of probes to measure the current density and magnetic field strength on the surface of these crucibles. The results are used to estimate electrical resistivity and power densities at different axial points of the crucible.

1 Introduction

The induction heating process has characteristics similar to those of a transformer with a short-circuited secondary winding. The primary winding is a water-cooled copper coil connected to an AC supply. The coil surrounds the material to be heated and, in an induction furnace, the short-circuited secondary winding is in fact the metallic charge. When an alternating current is allowed to flow in the primary coil, eddy currents are induced in the metallic work piece. The resistance of the charge opposes the flow of the current causing heat to be generated. Ultimately, the amount of heat generated by the charge is sufficient to cause melting.

Refractory material is required between the charge and the induction heating work coil. This can be either a rammed monolithic refractory or of a preformed-crucible construction.

Conducting crucibles are essentially preformed high-temperature refractory pots made in different shapes and sizes, which have relatively-low electrical resistivities ranging typically from \(10^{-5}\) \(\Omega\)m to \(6 \times 10^{-4}\) \(\Omega\)m at room temperature. Conducting crucibles made of carbon-bonded silicon carbide have resistivities from \(10^{-5}\) \(\Omega\)m to \(10^{-4}\) \(\Omega\)m, whereas the resistivity of clay-bonded graphite crucibles is up to \(6 \times 10^{-4}\) \(\Omega\)m at room temperature.

The flexibility of production associated with the melting of nonferrous metals in portable crucibles or tilting furnaces with preformed crucible linings is well suited to foundries where relatively small amounts (typically less than 200 kg) of different metals or alloys are required. The heating of the crucible and its charge can be carried out in either a fuel-fired furnace, an electrical-resistance furnace or a medium-frequency induction furnace. In recent years there has been rapid growth in the number of industrial installations using induction furnaces [1], arising from the use of solid-state technology for frequency conversion and improvements in associated foundry practice.

The choice of frequency is important in these furnaces. According to the size and the charge, the operating frequencies range between 50 Hz and 10 kHz. Generally speaking, lower operating frequencies are used for larger furnaces. Medium-frequency melting furnaces (1 and 3 kHz) are increasingly being used as shown in Fig. 1 [2].

![Fig. 1. Growth of medium-frequency induction furnaces in nonferrous foundries in the UK.](image)

The replacement of clay graphite crucibles by carbon-bonded silicon-carbide crucibles in induction furnace installations has been advocated [3] on the grounds of:

(a) greater furnace coil efficiency, especially with high conductivity metals
(b) reduced stirring forces at a given power level and operating frequency
(c) faster melting of swarf from cold
(d) reducing chilling of the metal in transit from the furnace to the casting position.

However, the use of carbon-bonded silicon-carbide crucibles has encountered problems of premature failure especially at high power densities and operating frequencies. Practical cases reported in private communications with manufacturers indicate that the location of failure is predominantly near the bottom of crucible walls. Flaws may initiate axially or azimuthally; propagating azimuthally round the crucible to meet up and cause failure.
To gain a more thorough understanding of this problem, an equivalent circuit analysis has been used to evaluate the operational parameters associated with typical industrial melting furnaces using conducting crucibles and to assess their effect on the crucible lifetime. Practical arrangements of a tilting medium frequency furnace with an A60 crucible are shown in Fig. 2.

It was decided to use an analytical model of the coil, crucible and charge to obtain theoretical predictions of the furnace performance. Sections 2 and 3 are theoretical. They describe the model used and the results of the analysis.

Equivalent circuit techniques have been applied to the design of induction heating and melting installations for many years \[4, 5\]. They represent an industrially well accepted method of designing induction heating systems in which the effect of parameter changes can readily be assessed and in which the incorporation of empirical factors taking into account industrial practice is facilitated.

In an equivalent circuit analysis the total magnetic flux linking the turns of the work coil is expressed in terms of the magnetic field strength at the surface of the load and hence related to the work coil current and voltage required. The equivalent circuit model thus derived can be solved by circuit analysis to give the power input to the workpiece, work coil turns, current carrying capacity, efficiency, reactive voltampere requirements etc. Fig. 3 shows the magnetic flux paths associated with a conducting crucible induction furnace.

Expressions for \(Z_r\), \(X_R\) and \(X_0\) can readily be obtained in terms of the operating frequency, magnetic permeability and geometry of the installation and detailed explanations are given elsewhere \[5, 7, 8\].

The complex impedance of the conducting crucible and metal charge, i.e. \(Z_2\) and \(Z_3\) of Fig. 4, are derived \[6\] from Maxwell’s equations and the use of Poynting’s theorem.

In a generalised form the complex impedance \(Z_r\) of any circular cylinder can be expressed as:

\[
Z_r = -\rho \frac{2\pi \delta}{l} \cdot \frac{J(r)}{H(r)} \cdot \frac{H (r) }{H_0} \cdot \frac{H(r)}{H_0} \]

where

- \(\rho\) = resistivity of load at radius \(r\) \(\Omega\m\)
- \(l\) = length of cylindrical load, \(m\)
- \(\delta\) = \(\sqrt{\frac{2\rho}{\omega\mu_0 l}}\) = skin depth in material, \(m\)
- \(\omega\) = angular frequency, rad/s
- \(J(r)\) = current density at radius \(r\), \(A/m^2\)
- \(H(r)\) = magnetic field strength at radius \(r\), \(A/m\)
- \(H_0\) = magnetic field strength on outer surface, \(A/m\)
Using eqn. 1, the complex impedances of the metal charge \( Z_3 \) and of the composite load \( Z_2 + Z_3 \) can be written as

\[
Z_3 = -\rho_3 \cdot \frac{2nr_3}{1 - \delta_3} \cdot \frac{J(r_3)}{H(r_3)} \cdot \frac{H_0}{H_0}
\]

(2)

\[
Z_2 + Z_3 = -\rho_2 \cdot \frac{2nr_2}{1 - \delta_2} \cdot \frac{J(r_2)}{H(r_2)}
\]

(3)

To evaluate the complex impedance of any form of composite load the values of the current density and magnetic field strength at any radius \( r \) must be obtained from the solution of the diffusion equation (See Appendix equivalent circuit analysis can therefore lead to the calculation of circuit parameter efficiencies etc., and the power distribution between the metal and the conducting crucible.

Applying the analysis to the design of a clay-graphite crucible induction furnace with aluminium or copper as the charge, gave sensible results in line with known industrial-furnace installations. The results of the analysis are given in Table 1.

Pure copper and pure aluminium are not widely melted in furnaces of this type. Most foundries using tilting or push-up furnaces melt the various BS 1400 alloys of copper, none of which have conductivities as good as that of pure copper and many have conductivities closer to that of aluminium. In the absence of adequate data on their resistivity are shown in Figs. 5-8.

Variations of the power distribution between charge and crucible under all circumstances investigated. However, the relationship was more pronounced for carbon-bonded silicon-carbide crucibles with an aluminium rather than a copper charge. Using clay-graphite crucibles with either metal charge and the existing furnace installation, approximately 20% of the furnace power is generated within the crucible itself. To obtain similar results using carbon-bonded silicon-carbide crucibles, a reduction in operating frequency to approximately 2 kHz with a copper charge is suggested whereas with an aluminium charge a frequency reduction to well below 1 kHz would be required.

The distribution of furnace power with crucible resistivity at an operating frequency of 3 kHz is shown in Figs. 5-8.

---

**Table 1: Electrical operating characteristics of furnace installation**

<table>
<thead>
<tr>
<th>Charge/crucible</th>
<th>( f )</th>
<th>( V )</th>
<th>( I )</th>
<th>( Q )</th>
<th>( N )</th>
<th>( W_1 )</th>
<th>( h )</th>
<th>( \eta )</th>
<th>( W_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium/clay-graphite</td>
<td>3000</td>
<td>800</td>
<td>2591</td>
<td>1920</td>
<td>12</td>
<td>0.82</td>
<td>122</td>
<td>67</td>
<td>100</td>
</tr>
<tr>
<td>Copper/clay-graphite</td>
<td>3000</td>
<td>800</td>
<td>2384</td>
<td>1845</td>
<td>12</td>
<td>0.82</td>
<td>37</td>
<td>64.8</td>
<td>100</td>
</tr>
</tbody>
</table>

\( f \) = operating frequency, Hz  
\( V \) = work coil voltage, V  
\( I \) = work coil current, A  
\( Q \) = reactive power requirements, kVAR  
\( W_1 \) = power induced in charge, W  
\( W_2 \) = power induced in crucible, W  
\( h \) = meniscus height, mm  
\( \eta \) = overall efficiency  
\( W_1 \) = furnace input power, kW  
\( W_1 + W_2 \) = ratio of power in charge to power in crucible and charge
7 and 8. Assuming crucible resistivities at the temperature of the molten metal, the power generated within a silicon-carbide crucible with aluminium charge is approximately

\[
\text{Power} \approx 10^{0.9} \text{ W/m}^2
\]

are used, and hence the production tolerance for a successful life is substantially reduced.

Using the work coil and metal charge diameter associated with the furnace installation, the effect of changing the crucible thickness, and hence the airgap/refractory thickness, was evaluated and the results shown in Fig. 6. For a clay-graphite crucible approximately 20% of the furnace power for either an aluminium or copper charge is generated within the crucible itself at the nominal frequency, resistivity and crucible thickness etc. In comparison, 33% of the furnace power would be generated directly within a silicon-carbide crucible with a copper charge, and as much as 62% with an aluminium charge. To reduce the loss within the silicon-carbide crucibles to approximately that of clay-graphite, the thickness would have to be reduced by just over 5 mm for a copper charge, which may be practicable, but for an aluminium charge a reduction of approximately 50% would be required.

3 Meniscus height

A basic phenomenon associated with the melting of metals in coreless induction furnaces is the stirring forces produced and the resultant turbulence within the molten metal. Although a vigorous stirring action can be beneficial when melting iron or steel, it is usually undesirable in nonferrous applications. The turbulence destroys the protective surface layer of flux used to prevent oxidation or hydrogen pick-up.

The magnitude of the stirring action is directly proportional to the power dissipated within the metal itself and inversely proportional to the operating frequency [9]. Having calculated the power generated within the charge twice that within an equivalent clay-graphite crucible. More importantly the rate of change of power distribution is over four times greater when silicon-carbide crucibles

\[
\text{Meniscus height} \approx 200 \text{ mm}
\]

\[
\text{Surface power density} \approx 200 \text{ kW/m}^2
\]

Fig. 6 Variation of meniscus height in the charge with surface power density of crucible and charge for 3 kHz induction furnace

Nominal value refers to size 500 crucible, 300 kW furnace and 3 kHz frequency

using the equivalent circuit technique described in Section 2, the height of the meniscus formed \( h \), which is related to

\[
h = \frac{\mu_0 \rho \delta_0 \lambda_0}{2 \rho_3 \gamma_3} \frac{W_j}{2 \pi r_3} \frac{1}{l} \quad (14)
\]

where

- \( h \) = meniscus height, m
- \( \mu_0 \) = relative permeability of charge
- \( \rho_3 \) = specific gravity of charge, N/m³
- \( W_j \) = power input to metal, W

Variation of the height of the meniscus with furnace power density, frequency, crucible resistivity and thickness are shown in Figs. 9–13.

The meniscus height is larger within an aluminium charge rather than a copper charge and when clay-graphite crucibles rather than silicon-carbide crucibles are used for a particular operating frequency, furnace power input and crucible geometry.

The height of the meniscus varies linearly with the furnace power density (see Fig. 9) which is in agreement
should be noted that in this case the furnace power input
with previous work on induction coreless furnaces [9]. It
has been kept constant and the meniscus height is not
directly related to the inverse of the frequency, as in the
case of induction furnaces with nonconducting crucibles or
refractories [9]. Any variation in the frequency will change
the distribution of furnace power between the charge and
refractories [9]. Any variation in the frequency will change
the ratio of furnace power in the charge to total power is changed drastically. (Figs. 7 and 8). For a very
small change in the resistivity of the crucible, the ratio of
power in the charge to total power is changed drastically.
This is more pronounced for carbon-bonded silicon­
carbide crucibles and with aluminium rather than a copper
charge. It is therefore required to be able to measure the
resistivity of these crucibles.

Although the composition of the silicon carbide mix is
said to be the same, crucible lifetime varies from crucible
and from batch to batch. The variation in lifetime must be
associated with the crucible resistivity whose measurement
is vital. Unfortunately, direct resistance measurement is
not possible as the glaze would have to be removed to
make proper contact with the crucible material. The
development of a nondestructive method of measurement of
crucible resistivity was therefore of paramount importance.

4 Measurement of resistivity by probes

A practical method was devised to measure the electrical
resistivity of the crucibles at various axial positions by
using current density and magnetic field strength H-
probes. The results also produced a clear picture of the
axial distribution of the surface power density induced
within the crucible. Tests were carried out at 30°C and all
instruments were shielded against the effects of the high
electromagnetic fields.

4.1 Current density probe [10]

These transducers were attached to both the inside and
outside of the crucible wall (Fig. 14). The probe consisted
of a length of thin insulated wire held firmly against the
surface of the crucible wall. The wire was used with 0.3 mm-
diameter enamelled copper. The ends of the wire were
twisted together so as to experience the same electromagnetic field. The output from the probe was measured using
a high-impedance voltmeter. From the voltage reading the
\( pJ \) product was obtained for the inside and outside of the

crucible wall according to eqn. 5:

\[
E = \frac{V}{I} = \rho J
\]

where

- \( \rho \) is crucible resistivity, \( \Omega \cdot \text{m} \)
- \( J \) is surface current density, \( \text{A/m}^2 \)
- \( V \) is voltage across probe terminals, \( \text{V} \)
- \( I \) is length of the probe in contact with the surface, \( \text{m} \)

4.2 Magnetic-field-strength probe

The magnetic-field-strength probe was essentially a search
coil and consisted of a length of wire (0.3 mm diameter)
wrapped closely round a thin strip of insulating material of
2 mm thickness (Fig. 15). The airgap between the outside
surface of the crucible and the inside surface of the work
coil varied along the length of the crucible, because of its
taper; from 8 mm at the top to 20 mm at the bottom.
Probes were positioned and held firmly on the inside and
outside surfaces of the crucible wall. The two ends of the
wire were then twisted together and voltage across them
measured. The voltage across the probe terminals is

\[
V_H = N_H j_0 I, 2\pi f H
\]
where $A$ is the area of the insulation:

$$A = 2\pi r_0 t$$

Hence the magnitude of the magnetic field strength is

$$H = \frac{V_H}{N_H 2\pi r_0 (2\pi/\mu_0 \mu_r)}$$

where

- $H$ = magnitude of magnetic field, A/m
- $V_H$ = voltage reading from the probe, V
- $N_H$ = number of turns of wire round the insulating strip
- $r_0$ = outer radius of the crucible, m
- $t$ = thickness of the insulation, m
- $\mu_r$ = relative permeability of the insulation
- $\mu_0$ = permeability of free space

The $H$-probe results showed that different parts of the crucibles experience different values of magnetic field strength. Variation of magnetic field strength along the length of crucibles is shown in Fig. 16.

The problem is associated with A60 crucibles which are recommended by manufacturers as being typical middle-sized crucibles. The laboratory-scale installation available at the time used A8 crucibles, and it was on these that the probe measurements were carried out. Current-density and magnetic-field-strength probes attached to the crucible are shown in Fig. 17.

### 4.3 Estimation of crucible resistivity

Variations of current density and magnetic field strength from the outside to the inside of the crucible walls are shown in Figs. 18–21. These are theoretical curves. The current densities are in fact derivatives of magnetic field strengths across the crucible wall and are obtained by using eqn. 8. The field-strength curves were produced by...
applying the analysis to the two crucibles and obtaining variation over their thickness according to Bessel function

\[-20 N! \cdot 10^{3} \cdot 10^{-9}S \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \sim \si
inside and outside so that any axial variations of resistivity may be obtained. The values obtained from top to bottom are shown in Table 2. The results show that the resistivities of the crucibles vary axially. Hence power density, which is a function of $\rho J^2$, is expected to vary axially also.

5 Power distribution

5.1 Percentage power distribution

Using the crucible resistivities from Table 2 and taking into account the crucible taper, percentage power distribution in the three sections of the crucibles were obtained [11] and are shown in Table 3.

Table 3: Percentage power distribution

<table>
<thead>
<tr>
<th>Position</th>
<th>Percentage power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>32.2</td>
</tr>
<tr>
<td>Middle</td>
<td>51.6</td>
</tr>
<tr>
<td>Bottom</td>
<td>16.2</td>
</tr>
</tbody>
</table>

Since the range of resistivities associated with both crucibles, especially silicon-carbide crucibles, was largely dependent on the variation in crucible properties, it suggested that the reason for the premature failure of certain crucibles, especially silicon-carbide crucibles, was largely dependent on the variation in crucible properties.

5.2 Surface power density

The corresponding surface power densities induced within the three sections of the crucibles are shown in Table 4.

Table 4: Surface power density

<table>
<thead>
<tr>
<th>Position</th>
<th>Surface power density, kW/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>75.7</td>
</tr>
<tr>
<td>Middle</td>
<td>127.7</td>
</tr>
<tr>
<td>Bottom</td>
<td>45.0</td>
</tr>
</tbody>
</table>

6 Conclusions

Modern foundry practice demands faster metal melting rates with a subsequent increase in the power densities and operating frequencies associated with industrial units. A nondestructive method of measurement of crucible resistivities was therefore developed. It was used to illustrate the range of resistivities associated with both silicon-carbide and clay-graphite based crucibles and the extent to which axial variation in properties occurred.

The finite length of the induction metal melting coil and the effect of the presence of the crucible, i.e. its position and taper, were also investigated in order that an estimate of the surface power density variations could be made.

In practice, crucibles fail mainly by cracking around the base of the crucible between the middle and bottom sections. The results show that far greater variations in surface power densities are associated with silicon-carbide crucibles within the region prone to failure. Methods aimed at reducing these variations should therefore enhance crucible lifetime.

Further work is now being carried out on larger crucibles taking into account the crucible properties, sizes, position in work coil and the effects of different situations within the melting cycle; i.e. the variation of packing density in a cold charge and various intermediate states up to a fully molten charge.

7 References

5 DAVIES, J.E., and SIMPSON, P.G.: 'Induction heating handbook' (McGraw-Hill, Maidenhead 1979)
12 MCLAUGHLIN, N.W.: 'Bessel functions for engineers' (Oxford University Press, 1934)

8 Appendix: Solution of diffusion equation

Assuming a cylindrical load and a radial variation in magnetic field strength only, then the diffusion equation simplifies to

$$\frac{d^2 H(r)}{dr^2} + \frac{1}{r} \frac{d H(r)}{dr} = \frac{\mu_0}{\rho} \frac{d H_0}{dr} H(r) \tag{11}$$

Solutions to eqn. 11 are of the form

$$H(r) = C_0 J_0 \left( \sqrt{2} \frac{r}{\delta} \right) + C_{+1} K_0 \left( \sqrt{2} \frac{r}{\delta} \right) \tag{12}$$

and

$$J(r) = -\frac{d H(r)}{dr} \tag{13}$$

where $J_0$ and $K_0$ are Bessel functions, the prime denotes the first derivative and $C$ represents the constants to be evaluated [12]. Using the appropriate boundary conditions associated with a composite load of conducting crucible and molten metal charge the following relationships are obtained from which the constants can be evaluated.

At the outer surface of the crucible, $H_2 (r_2) = H_0$; hence

$$H_0 = C_0 J_0 \left( \sqrt{2} \frac{r_2}{\delta_2} \right) + C_{+1} K_0 \left( \sqrt{2} \frac{r_2}{\delta_2} \right) \tag{14}$$

and at the boundary between the charge and the crucible,

$$H_3 (r_3) = H_2 (r_3)$$

$$\rho_2 J_3 (r_3) = \rho_3 J_2 (r_3)$$
thus
\[ C_1 I_0 \left( \sqrt{2} \frac{r_1}{\delta_2} \right) = C_1 I_0 \left( \sqrt{2} \frac{r_1}{\delta_2} \right) + C_2 K_0 \left( \sqrt{2} \frac{r_2}{\delta_2} \right) \] (15)

and
\[ \frac{\partial}{\partial \delta_2} C_3 I_0 \left( \sqrt{2} \frac{r_3}{\delta_2} \right) = \frac{\partial}{\partial \delta_2} \left[ C_1 I_0 \left( \sqrt{2} \frac{r_1}{\delta_2} \right) + C_2 K_0 \left( \sqrt{2} \frac{r_2}{\delta_2} \right) \right] \] (16)

There are therefore three equations from which relationships for the three constants \( C_1, C_2 \) and \( C_3 \) in terms of \( H_0 \), the magnetic field strength at the outer surface of the crucible, may be obtained. Expressions for \( J_2(r_2), J_3(r_3) \) and \( H_3(r_3) \) can then be found and substituted in eqns. 2 and 3 to evaluate the complex impedance associated with the metal charge \( (Z_1) \) and the composite load of crucible and metal charge \( (Z_1 + Z_2) \).