Optimum placement of UAV as relays

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Abstract—Unmanned aerial vehicles (UAVs) as aerial base stations or relays are becoming increasingly important in communications. In this letter, the optimum placement of a relaying UAV for maximum reliability is studied. The total power loss, the overall outage, and the overall bit error rate are derived as reliability measures. The optimum altitude is investigated for both static and mobile UAVs. Numerical results show that different reliability measures have slightly different optimum altitudes and that decode-and-forward is better than amplify-and-forward.

Index Terms—Bit error rate, outage, placement, power loss, unmanned aerial vehicles.

I. INTRODUCTION

Unmanned aerial vehicles (UAVs) are becoming more and more attractive as aerial base stations or relays to provide network coverage [1], [2]. One challenge in UAV communications is the flight time constraint of UAV [3]. Another important issue is the placement of the UAV [4].

Several researchers have worked on the optimum placement of UAVs as aerial base stations. In the seminal paper [5], the authors proposed a path loss model that accommodates both LOS and NLOS conditions. Alzenad et al. [6] extended the result to a 3D space. Mozaffari et al. [7] considered the optimum placement of UAV in device-to-device communications. References [8]–[11] further explored the use of multiple UAVs to cover a certain area. All these works have provided very useful insights on the placement of UAV as an aerial base station. However, an important issue that has been largely ignored in these works is that UAV may have limited storage and processing capabilities so that the data it receives from the ground user will have to be relayed to a remote ground station for further processing.

Works on the optimum placement of UAVs as relays have also been conducted. For example, reference [12] proposed a variable-rate approach to optimize the achievable rate for a relaying UAV. Reference [13] studied the placement of a relaying UAV in a multi-rate network. Similarly, in [14], the flying path of the UAV was optimized. These works have mainly focused on the relaying distance of UAV, not the altitude. Also, they did not consider the fact that the ground user may be more power-limited than the remote station such that the power loss in the hop from the ground user to UAV needs to be minimized.

In this work, we study the optimum altitude of the UAV as a relaying station using realistic UAV channel models and numerical search by focusing on the reliability metrics in terms of power loss, outage probability and bit error rate (BER). Both static and mobile UAVs are considered. Numerical results show that the altitude that optimizes the relaying performance is significantly different from the altitude that optimizes the hop from the ground user to UAV. They also show that different performance measures have slightly different optimum altitudes and that decode-and-forward (DF) performs better than amplify-and-forward (AF).

Compared with [5]–[11], this work considers both hops from the ground user to UAV and from UAV to the remote station in the placement optimization, while [5]–[11] only considered the hop from the ground user to UAV. Also, compared with [12]–[14], this work uses realistic UAV channel models and fixes the UAV on top of the ground user to minimize the power loss for the ground user, while [12]–[14] used standard wireless channel models and the UAV was placed between the ground user and the remote station. Moreover, this work focuses on the reliability, while the previous works focused on the capacity.

II. SYSTEM MODEL

Consider a UAV communications system as shown in Fig. 1. The ground user is located in a circle with radius $r_A$ and angle $\alpha_A$ in polar coordinates. If the UAV is static, it is fixed on top of the center of the circle with an altitude of $h$, as shown in Fig. 1(b). If the UAV is mobile, it flies in a circle with radius $r_U$ and angle $\alpha_U$ at some time, as shown in Fig. 1(c). The remote station is located $d$ meters away.

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from the center of the coverage area with a height of $h_B$. The ground user, the UAV and the remote station form a three-node relaying system, where the ground user acts as the source or destination node and the remote station acts as the destination or source node, respectively, depending on the direction of communications, while the UAV acts as relay in both directions.

The UAV can be some heavy-duty drone with enough payload to carry the wireless equipment. For example, the DJI Agras MG-1 drone can carry a payload of up to 10 kg, while small cell modules Cisco USC 8718/8818 weigh less than 1 kg and Nokia mini 4G base station weighs between 2 and 5 kg. The payload and flight control of drones can affect the system design in some applications or terrains. For the static UAV case, rotary-wing drones can be used, while for the mobile UAV case, fixed-wing drones can be used. Also, the UAV is expected to provide coverage for users within the area to minimize the distance between the UA V and the remote station. The ground user may be battery-powered with low transmission power, so it is necessary for the UAV to stay on top of it to minimize the power loss, as the relaying performance is determined by the weaker hop. For multiple users, we assume that the cell radius is $r_A$ so that the obtained result is the worst-case scenario to guarantee a minimum performance at the edge of the cell, similar to [5]–[11]. Users inside the cell are expected to have better performances. In the case of multiple users, orthogonal channels can be used to avoid co-channel interference.

From Fig. 1, one has the coordinates for the ground user as $(r_A \cos(\alpha_A), r_A \sin(\alpha_A), 0)$ and the coordinates for the remote ground station as $(d, 0, h_B)$. For static UAV in Fig. 1(b), the coordinates for the UAV is $(r_U \cos(\alpha_U), r_U \sin(\alpha_U), h)$. The time variance has been included in $\alpha_U$. Using these coordinates, for static UAV, the distance between the ground user and the UAV is

$$d_1 = \sqrt{r_A^2 + h^2},$$

(1)
and the distance between the UAV and the remote station is

$$d_2 = \sqrt{(h - h_B)^2 + d^2}.$$  

(2)

For mobile UAV, the distance between the ground user and the UAV is

$$d_1 = \sqrt{h^2 + r_U^2 + r_A^2 - 2r_A r_U \cos(\alpha_U - \alpha_A)},$$  

(3)
and the distance between the UAV and the remote station is

$$d_2 = \sqrt{(h - h_B)^2 + r_U^2 + d^2 - 2d r_U \cos(\alpha_U)},$$

(4)

where $\alpha_U$ is the angle between the UA V and the x axis denoted in Fig. 1(c). It is determined by $\alpha_U = \omega t + \alpha_0$, where $\omega$ is the angular velocity of the UA V and $\alpha_0$ is the initial angle. Using these distances, the path loss in the hop from the ground user to the UAV is given by [5]

$$PL^A = \frac{A_1}{1 + a_1 e^{-b_1 (\theta_A - a_1) + B_1}},$$

(5)

where $A_1 = \eta_{lOS} - \eta_{NLOS}$, $B_1 = 20 \log_{10}(d_1) + 20 \log_{10}(4\pi f/c) + \eta_{NLOS}$, $f$ is the carrier frequency, $c$ is the speed of light, $\eta_{lOS}$, $\eta_{NLOS}$, $a_1$ and $b_1$ are constants related to the propagation environments in this link, and $\theta_A = \frac{180}{\pi} \arctan(\frac{r_A}{h})$. The path loss in the hop from the UA V to the remote station follows the same model as

$$PL^B = \frac{A_2}{1 + a_2 e^{-b_2 (\theta_B - a_2) + B_2}},$$

(6)

where $A_2 = \eta_{lOS} - \eta_{NLOS}$, $B_2 = 20 \log_{10}(d_2) + 20 \log_{10}(4\pi f/c) + \eta_{NLOS}$, $\eta_{lOS}$, $\eta_{NLOS}$, $a_1$ and $b_1$ are constants related to the propagation environments in this link, and $\theta_B = \frac{180}{\pi} \arctan(\frac{h - h_B}{d})$. In [5]–[11], only $PL^A$ was considered in the optimization of $h$. Next, we will derive the overall outage probability and overall BER in a separate section to improve presentation.

### III. OUTAGE AND BER ANALYSIS

The absolute power loss is given by

$$Q_A = 10 \frac{PL^A}{10},$$

(7)

$$Q_B = 10 \frac{PL^B}{10}.$$  

(7)

If AF is used, the UAV receives the information from the ground user and forwards it to the remote station without any further processing or the other way around if the remote station transmits data. Then, the end-to-end signal-to-noise ratio (SNR) can be shown as

$$\gamma_{AF} = \frac{\gamma_A + \gamma_B}{\gamma_A + \gamma_B + 1},$$

(8)

where $\gamma_A = \frac{|g_A|^2}{2Q_A}$ and $\gamma_B = \frac{|g_B|^2}{2Q_B}$ are the hop SNRs, $g_A$ and $g_B$ are the fading coefficients, and $2\sigma^2$ is the noise variance at both the ground user and the remote station. Assume Nakagami-$m$ fading channels so that one has

$$f_{|g_A|^2}(x) = \frac{2}{\Gamma(m_A)} \left( \frac{m_A}{\Omega_A} \right)^{m_A} x^{2m_A - 1} e^{-\frac{m_A x}{\Omega_A}},$$

(9)

$$f_{|g_B|^2}(x) = \frac{2}{\Gamma(m_B)} \left( \frac{m_B}{\Omega_B} \right)^{m_B} x^{2m_B - 1} e^{-\frac{m_B x}{\Omega_B}},$$

(9)

where $\Gamma(\cdot)$ is the Gamma function [15, eq. (8.310.1)], $m_A$ and $m_B$ are the Nakagami $m$ parameters and $\Omega_A = E(|g_A|^2)$ and $\Omega_B = E(|g_B|^2)$ are the average fading powers.

Using (8) and (9), the cumulative distribution function (CDF) of $\gamma_{AF}$ can be derived as [16]

$$F_{AF}(x) = 1 - 2 e^{-\left( \frac{|h|}{\beta_{1,2}} \right)^2} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} C_1(n, k, m) \frac{K_n - m + 1}{\beta_{1,2}} \left( \frac{\sinh(x + 1)}{x} \right)^{\frac{x + m + 1}{2}} x^{\alpha_1 + k},$$

(10)
where \( a_1 = m_A, \beta_1 = \frac{\Omega_A}{n_{m_A} Q_A}, a_2 = m_B, \beta_2 = \frac{\Omega_B}{n_{m_B} Q_B}, \)
\[ C_1(n, k, m) = \frac{\beta_1^{n-k-m} \beta_2^{m-1} \Gamma(n-k-m) \Gamma(m-k)}{m \Gamma(m-k-n+1)}, \]
and \( K_{n-m+1}(\cdot) \) is the \((n-m+1)\)-th modified Bessel function of the second type [15, eq. (8.432.1)]. Using (10), the outage probability can be derived as
\[ P_o^{AF} = P_{F} \{ \gamma_{AF} < \gamma_{th} \} = F_{AF}(\gamma_{th}). \] (11)

The exact bit error rate (BER) could be calculated by using the exact end-to-end SNR in (8), but this does not lead to a simple expression for optimization [17]. Thus, an approximate BER can be calculated by approximating the exact end-to-end SNR in (8) with the harmonic mean as \( \gamma_{AF} \approx \frac{\gamma_{AF}^B}{\gamma_{AF}^B + mB} \) and using the harmonic mean for binary phase shift keying (BPSK) as
\[ P_e^{AF} \approx \frac{1}{2} - \sum_{n=0}^{\infty} \prod_{k=0}^{\infty} \frac{C_1(n, k, m)\left( \frac{4}{\sqrt{\beta_1 \beta_2}} \right)^{n-m+1}}{\Gamma(\alpha_1 + k + n - m + 1.5)\Gamma(\alpha_2 + k + n - m + 1.5)} \times \frac{\Gamma(\alpha_1 + k + 1)(\frac{1}{\beta_1} + \frac{1}{\beta_2} + 1)^{\alpha_1 + k + n - m + 1}}{\Gamma(\frac{1}{\beta_1} + \frac{1}{\beta_2} + 1)}, \] (12)

where \( F(\cdot, \cdot; \cdot) \) is the hypergeometric function [15, eq. (9.100)], and we have replaced \( x + 1 \) with \( x \) in (10) following the results in [16] and used it in (11). To calculate the BER with [15, eq. (6.621.3)]. From (12), one can obtain the results for Rayleigh fading by letting \( a_1 = a_2 = 1 \). Also, in the asymptotic case when \( \beta_A \) and \( \beta_B \) are very large due to either small path loss or large transmission power, \( P_e \propto \frac{1}{\beta_1 \beta_2} \). Thus, \( P_e \) changes with \( \beta_1 \) at a rate of \( a_1 \), and with \( \beta_2 \) at a rate of 1.

If DF is used, the UAV decodes the information sent by the ground user and then sends the decoded information to the remote station or the other way around if the remote station sends data. In this case, the end-to-end SNR can be shown as
\[ \gamma_{DF} = \min \{ \gamma_A, \gamma_B \}. \] (13)

Using (13), the outage probability can be derived as
\[ P_o^{DF} = 1 - [1 - F_{\gamma_{A}}(\gamma_{th})][1 - F_{\gamma_{B}}(\gamma_{th})], \] (14)
where \( F_{\gamma_{A}}(x) = P(\alpha_1, \frac{\beta_1}{x}), \gamma_{DF}(x) = P(\alpha_1, \frac{\beta_1}{x}), \) and \( P(\cdot, \cdot) \) is the incomplete Gamma function [15, eq. (8.350.1)]. The BER for BPSK can be derived by averaging the instantaneous BER over the joint probability density function of \( \gamma_A \) and \( \gamma_B \) as
\[ P_e^{DF} = H(\beta_1, a_1) + H(\beta_2, a_2) - 2H(\beta_1, a_1)H(\beta_2, a_2), \] (15)
where \( H(x, y) = \frac{1}{2} - \frac{1}{2} \sqrt{\frac{1}{ \frac{1}{x} + \frac{1}{y} } } \sum_{n=0}^{\infty} \frac{1}{(n+1+x+y)^2} \).

Our goal is to find the optimum altitude \( h \) that minimizes the total power loss of \( Q_A + Q_B \), the overall outage of \( P_o^{AF} \) or \( P_o^{DF} \), and the overall BER of \( P_e^{AF} \) or \( P_e^{DF} \) for maximum reliability using numerical search.
The BER performance improves and the optimum altitudes increase as the altitude increases. The BER performance improves and the optimum altitudes increase as the altitude increases.

**Fig. 5. BER vs. the altitude $h$ for static UAV.**

The BER performance improves and the optimum altitudes increase as the $m$ parameter increases.

**V. CONCLUSION**

The optimum altitude of UAV as a relay has been studied. The total power loss, the overall outage and the overall BER have been derived and numerically optimized for both AF and DF. Numerical results have shown that different performance measures have slightly different optimum altitudes and that DF performs better than AF. Our study considers a single user at the cell edge. It could be extended to a group of users. In this case, cyclical multiple access can be used [18], [19]. The system-level performance metrics, such as the sum rate or the minimum rate of all users can be maximized. Also, our study does not consider practical factors, such as heading, gyro or acceleration. They can change the distance between transmitter and receiver and hence, affect the performance. However, they are beyond the scope of this letter item.

**REFERENCES**


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