Vehicle performance calculations

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VEHICLE PERFORMANCE CALCULATIONS

by

G.G. LUCAS

Thesis submitted in fulfillment of the requirements for the award of Doctor of Philosophy of Loughborough University of Technology.

March 1970
Summary

This work is largely a theoretical study of vehicle performance and transmission design to supplement the meagre existing information. The theories and computer programs developed are already in use by industry and by the staff at Loughborough for the teaching of vehicle performance to undergraduate and post graduate students.

The term "vehicle performance" includes fuel consumption together with acceleration and maximum speed in a straight line. The computer programs developed deal with the performance calculations of all classes of road vehicle with manual, and with automatic transmission. The ability to conduct parametric studies is a special feature of these programs and is one of the main assets. It is shown that, while the accuracy of prediction is similar to that of current experimental measurement, a great deal of work has yet to be done, particularly on understanding transmission efficiency.

A special study has been made into the "deceleration test" as a means to obtaining the vehicle drag coefficients. This has been developed into a self-checking data reduction system.

A rational theory is developed for the design of the gear ratios of a manual transmission. The main computer program dealing with manual transmissions includes this, for use if desired, together with a number of other features, one of which is the treatment of the gear change points. These are considered to be dependent, rather than independent variables.
The main computer program dealing with automatic transmissions has been arranged in a form most suitable to the designer. The results of a full match study between the engine and the torque converter are arranged to be available, keeping the time-to-speed computation small and in a form suitable for a parametric study. A special feature is the treatment of the effects of the accelerating engine.

The parametric studies include the effect of weight, rolling resistance, tyre growth, aerodynamic drag, wind speed, ambient pressure and temperature, underbonnet temperature, wheel inertia, drive axle ratio, engine inertia, gear ratios and gradient.

Fuel consumption is considered separately as a steady-state study only. This is achieved by a match study between engine and vehicle. The fuel consumption of motor cars and commercial vehicles are investigated together with the features characterising the various types of power unit available. Parametric studies include the effect of weight, engine size and type, aerodynamic drag and overall gear ratio.

The main conclusions from these parametric studies are that ambient conditions and "underbonnet" temperature can have a significant effect upon performance, aerodynamic drag has a marked effect on maximum vehicle speed and fuel consumption, vehicle weight has more effect on acceleration than on maximum speed. Road wheel inertia was found to have little effect on acceleration and the relationship was found to be approximately linear. The fuel consumption of a vehicle
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SECTION A1

INTRODUCTION
Introduction

Interest in vehicle performance work started some six years ago with the teaching of engine and transmission design. It soon became apparent that little or no rational approaches had been developed for, say, the choice of gear ratios. The theories which did exist in the textbooks, for example the theory that states that the gear ratios should be in geometric progression, were obviously inadequate and were not followed by the motor car manufacturers. Little data exists for the designer on transmission efficiency and other factors affecting the design of a vehicle. Even the often quoted expression for the "equivalent mass" of a vehicle was found to be incorrect. This expression must include the transmission efficiency. Theories had to be developed for the clutch engagement period during the "take-off" of a vehicle on a full throttle acceleration run and for the prediction and effect of "wheel-spin". This interest was intensified some three years ago, when it was decided to offer a post-graduate short course on "vehicle performance".

Vehicle performance, in the context of this thesis, is limited to straight-line behaviour; and includes maximum vehicle speed, the time-to-speed of an accelerating vehicle and fuel consumption. The time-to-speed is always taken at full throttle, but this is not an essential condition. Braking behaviour is not included. This is very much a subject on its own.
This thesis therefore develops a rational approach to the calculation of vehicle performance and is intended, primarily, as an aid to the Designer. Digital computers play an important role and the programs have been arranged such that a parametric design study is possible. That is that the effect of one design parameter only may be studied in considerable detail, all other design data remaining constant.

The timing of this work seems appropriate. Many of the engineering departments of the vehicle manufacturers now have access to digital computers. This means that much of the rule-of-thumb type of design can be replaced by a more analytical approach. Also, the speed of a modern computer makes a full parametric study possible. Further, information is accumulating rapidly on such topics as vehicle drag. It is now possible to estimate the aerodynamic drag coefficient of a new design. The increased use of tape recorders in the instrumentation of vehicles has facilitated data reduction and has helped the drive for better, more accurate instrumentation. Thus, better data is becoming available to the Designer. Finally, a point which gives considerable impetus to the manufacturer to conduct design calculations, particularly parametric studies, the shapes of new vehicles seem very similar. A number of mass produced, medium size cars appear very similar in design. The same can be said of commercial vehicles also. The features which characterised a particular manufacturer in the past are disappearing. The designs are converging towards a common
optimum. This means therefore that the manufacturer who "has the edge" on his fellows will prosper. Hence, the incentive for vehicle performance calculations.

A search of the literature (see Section A2) and, perhaps more importantly, a survey of the unpublished performance work of the vehicle manufacturers in this country has revealed a number of these vehicle performance computer programs. Concerning the prediction of the time-to-speed of a manual gear change vehicle, the most common program, none have been found to treat the gear change points as dependent variables. All, without exception, have required that this information, or the engine speed immediately prior to a gear change, be read in with the input data. If the design under investigation is not a radical departure from previous experience, it may well be possible to pre-judge the gear change points. On a full throttle acceleration run however, where the object is to obtain a given high speed in the shortest possible time, the gear engaged at any one time must be that giving the greatest vehicle acceleration. Hence, the gear change points are dependent variables and are functions of the other design variables. This is one essential difference between the manual time-to-speed program described in this thesis and others. Another is that this program contains also a rational acceptable method of fixing the gear ratios, if necessary. A number of vehicle manufacturers in this country are benefitting directly from this work in that they have, and are using, this time-to-speed program. This program accepts engine torque data, vehicle
design parameters together with ambient and environmental data
and evaluates the speed, time and distance characteristic of
the accelerating vehicle and the maximum vehicle speed. The
development of this program and the relevant theories are described
in Part B of this thesis.

A sister program is described in Part F. This concerns the
prediction of the time-to-speed of an automatic transmissioned
vehicle. Such a vehicle has to be treated in a different manner
because there is no straight forward relationship between the
engine and the vehicle speeds. The program and technique have
been developed to give as much information as possible to the
Designer with minimal effort on his part. Further, computer time
has been kept low and the possibility of a full parametric study
in very little extra computer time has been made possible by
isolating the torque converter and engine match study and making
it a "once only" operation. This technique is original and caters
for changing relationship between the engine and torque converter
without iterative procedures. Further, it provides the Designer with
full match details between the engine and the torque converter to
enable him to make a wise choice. Part F contains also an analysis
of a three element torque converter which puts on a formal footing
the often used capacity or K-factor to describe its performance.
This analysis has since been published by the Author in conjunction
with Mr. A. Rayner, a colleague, as a technical report (44).
Part D of this thesis resurrects the "deceleration" or "coasting" test as a means for measuring the drag coefficients of an existing vehicle. This work supplements other work on vehicle instrumentation in the Department of Transport Technology at Loughborough and enables the deceleration test to be used accurately and effectively without the expense of large wind tunnels and tyre rigs. The contribution of the work of this thesis here lies in the handling of the data and the use of a digital computer in the data reduction. Built-in accuracy checks are a feature of this work.

Part C is devoted to a vehicle parametric study. A number of published documents concerning vehicle performance computer programs advocate their use in parametric studies. Surprisingly therefore, very little information is published on such parametric studies. It is of considerable interest therefore to study the effect on vehicle acceleration and maximum speed of such design parameters as vehicle weight, drag coefficients, gear ratios and the number of ratios, drive axle ratio and rolling radius of drive wheels, engine inertia, transmission efficiency, tyre growth and road wheel inertia. Also, such environmental conditions as ambient pressure and temperature, wind speed and gradient, together with "under-bonnet" temperature. Some of these results, particularly those on the effect of changing the drive axle ratio on vehicle time-to-speed, have caused considerable surprise. Part C contains also an analysis of what went wrong in the design of the "Midland Red Motorway Coach". This concerns a well known paper by Pearson (25) in which his design
calculations did not match up with subsequent vehicle performance.

Part E concerns the fuel consumption of vehicles. The analytical treatment here is rather different and divorced from the rest of the work because it is not related to the full or fixed throttle engine condition. Calculation of the expected fuel consumption at such a condition has little value in practice because the emphasis is invariably on power output, acceleration and high speed. Part E therefore concerns part load operation and the factors affecting fuel economy. An appropriate parametric study is included also in Part E.

The scope of the work in this thesis includes motor cars and commercial vehicles having petrol, compression ignition or gas turbine engines. Some mention is made also of specialist earth-moving, agricultural and military cross-country vehicles. The special problems and conditions associated with these vehicles are outlined, reviewed and discussed. The techniques for vehicle performance calculations developed however, would not cater satisfactorily for them. Time-to-speed or high vehicle speed usually has little meaning with such vehicles. Their requirements are peculiar to themselves and strongly influenced by their environment, particularly the terrain they are traversing. It is felt therefore, that such vehicles should be treated on an independent basis and that there is no point in attempting an "all embracing" computer program to cater for every such contingency. A few such "specialist" programs have been
published and these are reviewed in the Literature Survey (Section A2).

Another point which must be mentioned is that this thesis contains little collaboration of the theories developed with practical tests, other than the results of tests published by the semi-technical press. Those which are included are just adequate to show that the calculation technique is fundamentally sound, but no more. They are not sufficient to throw light on the validity of the expression used for transmission efficiency for instance. The vehicle instrumentation work has lagged behind the theoretical work. The next phase therefore, beyond the scope of this thesis, must be a very close study of such matters as transmission efficiency, engine torque output under dynamic conditions, tyre growth etc. by both separate rig test and the comparison of measured and calculated performance tests.
SECTION A2

Literature Survey
1. **Introductory Remarks**

Existing papers, dealing with vehicle performance calculations, have been analysed and are reported on in the seven sub-sections given below. Sub-section 2 deals with existing methods of calculating time-to-speed and maximum vehicle speed and the related theoretical parametric study is covered by sub-section 3. The special case of the time-to-speed of automatic transmissioned vehicles is given in sub-section 6. The calculation and the parametric study of the fuel consumption of a vehicle requires a different technique to time-to-speed calculations and this is covered in sub-section 5. Sub-section 4 deals with the measurement of vehicle drag by the "deceleration" test. This is a feature of the work in this thesis. Test instrumentation and the performance prediction of special vehicles, such as those employed by military authorities, are dealt with in sub-sections 8 and 7 respectively.
2. **Acceleration and maximum speed**

Taborek (4), in a series of articles in "Machine Design" in 1957, set the stage for vehicle performance calculations. He dealt with fundamental considerations such as the forces acting upon a rolling wheel, either elastic or rigid, wheel spin and the coefficient of friction between tyre and ground. After a number of articles dealing with the forces acting on the vehicle, performance on a gradient, the drag forces, transmission efficiency and traction limits, he developed a technique for the calculation of the time-to-speed of an accelerating vehicle. This, he achieved by effecting the integral

\[
\text{time-to-speed} = \int_{V_1}^{V_2} \frac{\delta V}{f}
\]

Since the vehicle acceleration (f) is a function of the vehicle speed (V), the integration is not straightforward. Taborek advocates a graphical technique in this solution. This procedure he outlines in some detail.

Much of Taborek's work has been overtaken by time, largely because the now widespread use of the digital computer has rendered redundant the rule-of-thumb expressions developed to avoid difficult analytical procedures. However, fundamentally, Taborek's work is correct and forms a good foundation for vehicle performance work.

Without access to a computer, graphical techniques can be effective. Two such techniques were described by Ellis (120) in 1958.
The next real milestone is provided by Sets (78) in 1960. His comprehensive S.A.E. paper emphasizes the importance of using digital computers in vehicle performance work particularly, as he mentions, for parametric studies. He specifies three criteria to be assessed, without stating specifically their significance. The criteria are:

1. 0 - 4 second start-up distance using full throttle and including the time for the initial depression of the accelerator pedal.

2. 0 - 10 second start-up distance, otherwise as above.

3. 50 mile/h passing time. The initial speed is said to be steady at 50 mile/h, the throttle is fully depressed and the time and distance required to overtake a 50 mile/h car is calculated. The calculation ends when the 190 ft exists between the two cars.

The emphasis of the paper is on automatic transmissioned vehicles. Presumably, this is why Sets expresses the equivalent mass of the vehicle as a function of the ratio engine speed/vehicle speed in a graph, rather than use a simple algebraic expression. The start-up response of the engine and vehicle to the initial depression of the accelerator pedal is given empirically by a plot of time against a quantity \(K\) defined as
\[ K = \frac{\text{drive torque} \times \text{drive axle ratio}}{\text{vehicle weight} \times \text{rolling radius}} \]

This plot is the average of data from tests on many automatic transmissioned vehicles. Flow charts and full details of his computer program are given, together with typical results. Prediction accuracy is claimed to be within \( \pm 5\% \).

The time-to-speed is effected by employing a simple numerical method, programmed for a digital computer, to deal with Taberek's integral. Gear change points are specified by the program user by giving the engine speed just before change. The engine torque versus engine speed data are read in as a table.

Apart from an over-simplification in the performance prediction of automatics, which will be discussed later in this literature survey in the Section dealing with automatics, Setz's program lacks generality. It is possible to obtain the 0 - 4 second start up distance from the 0 - 10 second calculation. Indeed, it is possible to obtain all three of his criteria and a great deal more also by a straightforward, full throttle time-to-speed calculation. This would give both time and distance throughout the vehicle speed range, affording considerable saving in programming and computer time.

Setz's paper is useful however, in stressing the need to employ digital computers in this work. To underline this, he gives a table giving the time and cost for both manual transmissions and automatics calculations conducted by hand and by computer. A calculation for the
former by hand takes 180 minutes and costs 22.5 dollars, whereas, by computer, the time is 3 minutes and the cost 3 dollars. This difference should be greater with a later design of computer.

Louden and Lukey (79) also in 1960, outlined the Buick technique of using a digital computer in vehicle performance calculations. Again their work was concerned largely with automatic transmissioned vehicles. Much of their input data was in tabular form from which they developed "library tapes" in order to minimise the possibility of input data errors. Of interest is their technique for employing the computer to give a graphical output. The calculations afford the time-to-speed and distance travelled during an acceleration run. Again, the low cost and the ease with which parametric studies may be made are stressed. The predictions are generally said to be within production tolerances. Small, second order effects, such as tyre growth and weight transfer due to aerodynamic lift had been removed since their effect was found to be small. There is no provision for gear changes in the Louden and Lukey program since all the torque multiplication is gained by the torque converter in conjunction with the drive axle ratio.

Noon (75), in 1962, also published a paper concerning the use of a digital computer to predict vehicle performance. His main concern however, was with route analysis, rather than time-to-speed. Fourquet (62), in 1964, in an article in "Ingenieurs de l'Automobile", demonstrated how an analogue computer could be used to calculate the time-to-speed of an accelerating vehicle. Function generators were
used to describe the engine torque curve, the transmission efficiency and the rate at which the engine consumed fuel. The output was graphs of vehicle speed, vehicle acceleration and distance travelled against time. Again the importance of vehicle performance calculation and prediction was emphasized.

In 1966, Slivar and Desoyer (55) set out in detail and in a fundamental manner the external forces acting upon a vehicle. They tended to be critical of the use of ill-defined terms, such as "rolling resistance", "reduced mass" and "tractive force". Their equations of motion for the vehicle take account of tyre creep and of the offset of the reaction force on the tyre as a result of its torque transmission. Their fundamental considerations result in a set of equations which, they suggest, may be solved by numerical methods if a number of unknown functions are first established. These functions concern their tyre creep and eccentricity parameters which may be obtained from dynamometer tests. The design of these tests is outlined and described in their paper.

This valuable paper is a little ahead of its time. Insufficient information is at hand on the more mundane aspects of vehicle performance to justify too rigid a definition of rolling resistance. For instance, the term "transmission efficiency" is slipped into the derivation of the equations without the rigid definition accorded to other parameters. After the formation of the equations of motion, it is implied that adequate knowledge exists on the transmission efficiency, engine torque under dynamic conditions, bearing friction
torques and wheel air resistance. The paper is valuable, if
only because it advocates a fundamental approach and outlines how
this may be achieved. It is thought however, that a number of
aspects in the paper require closer study. In particular the
definition of transmission efficiency which, it is thought, should
not cover the entire transmission system, but should be split up
to cover each separate portion of the transmission. Only such a
consideration would justify the attention to detail elsewhere.

Fiala (74), at the other end of the analytical scale, published
an article in the journal ATZ in 1962 outlining a rule-of-thumb
method of obtaining an approximation to a vehicle's full throttle
time-to-speed speed characteristic. He starts with the simplifying
assumption that the force available for acceleration is linear with
speed. From this he develops an expression for vehicle speed as an
exponential function of time. The constants in this expression are
the maximum speed of the vehicle and the initial acceleration.
Since the latter is not readily available, this expression is re-
arranged in terms of the "half life (th)". That is the time to reach
half the maximum vehicle speed. The expressions then become

\[ v = v_{\text{max}} \left( 1 - e^{-0.69 \frac{t}{th}} \right) \]

The half life may be determined from acceleration tests but, for
all private cars, it is said to be approximately 10 seconds. The
better performance cars having a slightly less half life. If the
half life and the maximum vehicle speed are known, the above equation enables one to plot an approximation to the time-to-speed curve of a vehicle. With some experience, it should be possible to specify the half life of a projected vehicle. Fiela claims that this technique automatically takes account of the fact that the gear change points are not independent variables, but are functions of the vehicle design. The method provides a useful and ready means of comparison. It must be applied to a great deal of test data before it can be used with confidence on performance prediction work.

While little has been published on the time-to-speed prediction of an accelerating vehicle, it is known that the vehicle manufacturers have access to, and use, vehicle performance computer programs of one sort or another.

Concerning other aspects of performance work, the choice of gear ratios, "wheel spin", etc., Forster (64) in an excellent paper published in 1963 by Automobil-Industrie outlined a rational technique for the choice of bottom gear ratio. He shows that a basis of maximum gradeability and maximum possible acceleration amount to one and the same condition and advocates that this be used. He lists the gradients of some of the mountain passes on the continent of Europe and points out and gives some indication of the drop in engine power at the high altitude of these passes. Further, he suggests that, even in flat country, there are garage ramps, entrances to ferries etc. of considerable gradient. This philosophy he develops both analytically and with practical examples taking account of clutch slip. His treatment of top gear ratio however, is not quite so rigorous and the intermediate gear ratios are covered in a paragraph.
3. Theoretical parametric studies

Many vehicle design and environmental parameters are known to affect vehicle time-to-speed and maximum speed. The more important parameters are thought to be:

1. vehicle weight
2. gradient
3. drag (both rolling resistance and aerodynamic drag)
4. engine torque curve shape
5. number of gear ratios
6. the spacing of the gear ratios
7. the gear ratios and the drive axle ratio
8. rolling radius of drive wheels
9. engine inertia
10. wind velocity
11. coefficient of friction between wheel and ground
12. road wheel inertia

A literature search has yielded two documents only dealing specifically with the calculation of the effect of any of these parameters. The first is Webb's paper (113) published by the Institution of Mechanical Engineers in its Proceedings of 1952-3 concerning the calculated effect of gradient on the maximum speed of a motor car. The calculated points which he uses to plot his curve showing this effect are reproduced in Fig. A2.1, together
with an indication of the gear number engaged. This information is given in Webb's table 1. Webb has drawn a smooth curve through his points, whereas it is thought that there should be a discontinuity in the slope as a gear change has to be made from a higher to a lower gear as the gradient is increased. He does not comment on the shape of his curve, since his pre-occupation is with the effect of gradient upon fuel consumption. In fact, he uses these results for this purpose.

Surprisingly, no direct study has been found on the effect of vehicle weight on either the full throttle time-to-speed or the maximum speed of a vehicle. Clearly, since the effect of a gradient is closely allied to the effect of vehicle weight, some conclusions could be drawn concerning the latter from Webb's calculations. Joyner (65) produces graphs showing the large saving in the power required to propel large commercial vehicles at differing high speeds. These graphs could be translated to give the effect of gross vehicle weight upon the maximum speed of a commercial vehicle.

The second document concerns the effect of the aerodynamic drag coefficient on engine power required, and hence on the maximum vehicle speed, by White and Carr(24). This shows large power reductions by reducing the aerodynamic drag coefficient and hence significant gains in vehicle maximum speed. The Author, in his discussion of the paper, showed the calculated increase in maximum vehicle speed to be some 8% for a 25% reduction in the aerodynamic drag coefficient and very little change in the time-to-accelerate to about three-quarters of the maximum vehicle speed.
4. **Determination of vehicle drag by the "Deceleration", or "Coasting" test.**

This test is probably too mundane to be described in detail in the literature, since it is simply the application of Newton's second law to a coasting vehicle. Measurement of the deceleration affords the drag force causing it. Hertz and Ukrainetz (43) used this method to obtain the aerodynamic drag coefficient only of a vehicle. They achieved this by inflating the tyres to a high pressure. This has the effect of rendering the rolling resistance to a near constant. The measurement of vehicle deceleration was achieved indirectly from a plot of vehicle velocity against time. A self-cocking 35 mm camera being used to photograph simultaneously an accurately calibrated speed-meter and a timer. Such a technique is not affected by changes in attitude of a vehicle.

A plot of the deceleration against the square of vehicle speed yielded a straight line from which the aerodynamic drag was obtained. The projected frontal area of the vehicle was determined from a photograph taken from 180 feet using a 200 mm telephoto lens on a 35 mm camera.

It is claimed that the deceleration of the vehicle was obtained to within ± 3% provided that the wind speed was below 2 mile/h.

Experiments were conducted with all vents closed, with vents and windows open and with some attempt at streamlining the vehicle. These show a 3% increase in $C_D$ by opening both the vents and the
windows. This correlates well with White's (9) 6 to 7% increase for engine cooling. Their crude (their term) attempt at streamlining produced a 17% decrease in the aerodynamic drag coefficient from which they conclude, like White and Carr (24) that there are potential gains from a study of the aerodynamic drag of a vehicle.

Pearson (25) too employs the deceleration test to determine the drag coefficients of his motorway coach. His Institution of Mechanical Engineers' paper is a classic in that he reports a large discrepancy between the calculated power required to propel his coach at 80 mile/h and that actually required. He could not account for this discrepancy, although it is probably a result of his drag measurement.
5. Fuel consumption

a) Experimental measurement of vehicle fuel consumption (steady-state and "overall")

There is an abundant source of experimental information on the steady-state fuel consumption of vehicles, particularly motor cars, in the technical press. Journals such as "Motor", "Autocar" and "Car Supplement" publish such experimental data as part of their road test reports on vehicles. Such data is not corrected in any way for ambient pressure and temperature, or for wind speed. These reports could be used to provide evidence on any car-to-car variation in fuel consumption, since the road test on a particular model is frequently duplicated by the different publishers. Apart from this possibility, no other source of evidence on car-to-car variation has been found.

These road test reports also provide evidence on the "overall" fuel consumption of motor vehicles. This is said to take some account of transient operation, such as acceleration, braking, gear changing etc. The term "overall" is ill-defined and inevitably includes a high element of driver behaviour (65). Basically, the overall fuel consumption figure is derived by noting the fuel consumed and the distance travelled throughout a long period of the road test and by dividing the latter by the former. It is admitted by some of the publishers that the figures produced tend to be too low to represent the expected overall fuel consumption of the vehicle in the hands of a typical driver. Some modify the measured overall fuel consumption to allow for the arduous use of the accelerator, brakes etc. of
their test drivers. Joyner (65) points out the importance of driver influence on the overall fuel consumption. Smith et al (107) outline a scheme to record typical journeys in order that a statistical approach might be used to study driver habits. They advocate this approach particularly for use in exhaust emission work.

Everall (109), in his tests on a series of vehicles, has tried to relate the overall fuel consumption to the vehicle speed. He has found that a motor car has an optimum speed at approximately 35 mile/h and that the fuel consumption is double that at this optimum speed when the motor car is operated at only 10 mile/h. Warren (58) supports this conclusion in his studies on overall fuel consumption using six motor cars of different sizes. He shows a large deviation between steady-state results and overall results at low average vehicle speeds. This deviation is abated somewhat in the mid speed range and is very small at high speed. His conclusions are not confined to petrol engined vehicles, since one of his motor cars was powered by a compression ignition engine. These differences he attributes to the increase in the transient operation of the vehicle which increases as prevailing conditions force the average vehicle speed down.

A startling transient effect on fuel consumption is shown by Warren (58) and by Scheffler and Niepeth (56) to be the "warm-up period". Warren suggests that 0.08 to 0.09 gallon of fuel is consumed in starting up from cold. Scheffler and Niepeth concentrated on the
effect of the warm-up period in order to show that there need be nothing wrong with a new motor car when it returned a poor fuel consumption on short trips, particularly in the winter months. Their tests were conducted on the General Motors proving ground and showed that the warm-up effect was not due solely to the use of the choke, since the same trend was shown for summer start-ups when no choke was used. However, more fuel was consumed during start-up on a cold day than on a hot day. They suggest that a 60 deg F change in ambient temperature corresponds to a 1 mile/gall change in the overall fuel consumption for fully warmed up city travel. Their conclusions are that, on a cold day, the warm-up period extends to some ten miles and that it has a devastating effect on the overall fuel consumption. Warren conclusions follow the same trend, but he considered also the evaporation loss from the carburettor when the vehicle is left for a short time between trips. Sturm (76) produces results of warm-up tests to illustrate the usefulness of his fuel measurement equipment. His conclusions are very similar.

The many transient conditions have not been analysed in detail in an attempt to relate steady-state fuel consumption tests to overall fuel consumption figures. Evattall (109) has conducted overall fuel consumption tests using his six vehicles on both ordinary roads and motorways at the same average speed and suggests that motorway operation results in 10–20% improvement in fuel consumption.
It would seem that work should be directed towards the relationship between the steady-state and overall fuel consumption of vehicles. A statistical approach, similar to that of Smith et al (107) must be considered as a possible technique.
b) **Theoretical prediction of vehicle fuel consumption**
(steady-state and "overall")

A great deal of the credit for the development of the engine-vehicle match study (as outlined in Part E) must go to Dr. J.G. Giles (39). The calculation of the vehicle steady-state fuel consumption from this match study has been spelt out in detail by the Author elsewhere (1). Few other workers have used this technique. Setz (78) mentions briefly the need to calculate the part throttle steady-state fuel consumption, but he does not show how this is to be done. Joyner (65) produces engine characteristics in the form of brake specific fuel consumption versus brake horsepower with lines of constant engine speed and advocates that this be used for a match study. The Giles (39) and Greene and Lucas (1) engine characteristic plot is a graph of engine brake horsepower versus engine speed with the superimposition of lines of constant brake specific fuel consumption. Forster (64), in his mammoth article on automatic transmissions describes and advocates a steady-state match study. Also Webb (113) uses the technique in his study on the effect of gradient on fuel consumption. In general however, there is a distinct dearth of references to engine and vehicle match studies and the calculation of the steady state fuel consumption in the literature, presumably because the results obtained are thought to be tied to one particular engine or vehicle such that it hardly seems worth the work involved.
The engine characteristic plots have however been used for other purposes. Macmillan (92) uses them in order to assess different types of transmission. S intertw (105) uses them in order to derive an index which is said to characterise the overall fuel economy of the engine, rather than the engine and vehicle combination.

Techniques, other than a full vehicle match study, have been used to calculate the fuel consumption of a vehicle. Fourquet employed an analogue computer with function generators and calculated the full throttle fuel consumption during an acceleration run. The function generators supplying the basic engine full throttle torque and fuel consumption curves. He seems to sense that the fuel consumption curve produced is of little use by implying that a part throttle curve would produce a lower fuel consumption. His main interest however was to demonstrate the use of an analogue computer in vehicle performance work, rather than in the results he produced.

Others (75), (79) attached to manufacturing organisations and concerned at the reported overall fuel consumptions of their vehicles, tackle the problem of the calculation of the expected fuel consumption by describing typical routes. Loudan and Lukey's (79) route is based upon their actual test route and is obtained by recording gradient, vehicle speed etc. at 2 second intervals during an actual test run with a vehicle. The velocity data is differentiated to give acceleration data and the total information obtained has been used to
compile the theoretical route used in the calculation of the expected overall fuel consumption of a vehicle. They then express acceleration phases as equivalent constant velocity phases on a gradient and proceed to evaluate the engine torque and speed required throughout the defined route. Reference is then made to engine fuel consumption charts of fuel flow rate versus engine speed for different throttle angles (load). From this the fuel consumed is totalled and, from knowledge of the route length, the overall fuel consumption obtained. Correlation of the calculated with actual test results is said to be good.

Moon (75) uses a similar technique for the calculation of the overall fuel consumption of trucks and coaches. He specifies in detail a typical route defining speed, gradients, driver behaviour, such as gear change points etc. His calculated figure is said to be within 5% of the actual.

The obvious weak link in such exercises is in relating the actual test route fuel consumption to that obtained by an owner/driver of a motor vehicle. Such figures are known to depend largely upon the technique used by the driver (65), particularly in respect to his use of the accelerator and brake pedals. Louden and Lukey do not specify how their quasi steady-state treatment of the acceleration phases deals with the accelerator pedal in the carburettor. Also, a 2 second interval of velocity data seems rather large to obtain realistic acceleration data. A further consideration, which must
be faced, is that the pattern of actual journeys is constantly changing (57) in terms of distance, speed and number of journeys per year. Thus, even if a typical route can be defined, it is not likely to remain typical for a long period of time. Perhaps a better approach is the "centre of operation" suggestion of Giles (61) in which he notes the centre of the most often used portion of the load curve and calculates the overall fuel consumption from this. Certainly this approach is easier in terms of calculation and could prove to be just as valid as a means of comparing and assessing vehicle fuel consumption.

A number of specialist typical routes have been devised for the calculation of vehicle range. This is related closely to fuel consumption. McKenzie et al. (47) report thirteen typical routes for military vehicles. Their analysis of expected vehicle behaviour and range is detailed in that they consider the driver to be an integral part of the system. Basing their simulation of the driver on recorded responses from aircraft simulators, they have represented a driver by

1. a lag between input and reaction
2. a magnitude and rate of change judgement
3. a limb-neuromuscular lag
4. a threshold of indifference in that a driver (or pilot) is found to deliberately ignore small errors.

Both digital and analogue computers are used for this work and the writers conclude that the results so far obtained are very promising.
Other theoretical studies consider the range of electric vehicles, a significant factor when contemplating the future of such vehicles. Tenniswood and Crastseij (49) conduct such an analysis in order to make certain recommendations to designers. Gačić (110) has developed a simulation of an electric vehicle in order to study its range. This simulation consists of a vehicle having a conventional engine with the acceleration characteristics of an electric vehicle and a meter (termed a computer) which logs the power used. The meter reading is analogous to the state of charge of the batteries in an electric vehicle.
c) Effect of vehicle design parameters on fuel consumption

The importance of the fuel costs in the operation of commercial vehicles has been emphasized by Minervini (99) and by Joyner (65). The cost of private motoring is dealt with periodically by the motoring press, but is not treated with the same degree of analysis. One senses that the private motorist dare not face up to the real cost of the convenience of ownership. Joyner conducted his cost analysis for a fleet of 250 horsepower, compression ignition trucks, basing his figures on a life of 500,000 miles. He showed that the fuel costs were four times the first cost and four times the maintenance costs. Clearly therefore, a detailed parametric study is likely to pay dividends.

There appears to be no glaring anomalies in the type of power plant used. Muller (60) and Annand (66) both proclaim the compression ignition engine master of the commercial vehicle field. Annand suggests that the light commercial market could be lost to the spark ignition engine if a stratified charge engine is successfully developed. At the other end of the commercial field, the gas turbine engine is making inroads.

The use of the spark ignition engine in motor cars emphasizes the motorists apparent apathy regarding cost and his concern for comfort and convenience. This situation could change dramatically if legislative authorities continue their enthusiasm for a pollution free environment (99).
Very little has been published on the effect of the size of the engine on vehicle fuel consumption. Perhaps this is because engines tend to be thought of as having very individualistic characteristics. Cornell (57) however shows steady-state fuel consumption curves for an American motor car powered by a small 6-cylinder engine, a large 6-cylinder engine and an 8-cylinder engine respectively. These curves show a 15% difference at 30 mile/h, but very little difference at 80 mile/h. For the effect of engine size or output on the overall, city travel, fully warmed-up fuel economy, Scheffler and Niepoth (56) suggest, as a rule-of-thumb guide, 1 mile/gall change for every 120 horsepower change in engine output. This, of course, is for American motor cars where 1 mile/gall represents quite a high percentage change. Joyner (65) suggests an engine size for large American trucks to be capable of powering the vehicle at 5 - 10 mile/h only above the cruise speed.

Surprisingly, little work has been published on the effect of vehicle weight, although many recognise its importance. Joyner (65) lists it as an important effect and gives tables and graphs showing the power required to propel large trucks at various steady speeds. Bland (59), in an article in "Bus and Coach" entitled "Saving weight means saving money" advocates a saving of 0.56 to 1.0 mile/gall for every ton of weight reduction for a coach in normal service. For a double-decker 'bus in town service, he quotes 0.75 mile/gall per ton. Scheffler and Niepoth (56) suggest 1 mile/gall change for every 400 lb change in the weight of an American motor car. This is said
to be for normal city travel with the car in its fully warmed-up condition. Clearly, therefore, these rule-of-thumb figures are overall fuel consumption figures rather than steady-state.

Allied to the effect of weight is the effect of gradient on fuel consumption. Webb (113), as early as 1952, conducted a theoretical analysis based on a match study between a 10 hp car and its engine. This study was essentially steady-state and specifically excludes accelerations. However, negative gradients with the engine on overrun, with and without the application of brakes, were considered. His conclusions were that a level road is the most economical condition, but that gradients up to 4% can be tolerated. However, gradients of above 4% on a route affect the fuel consumption markedly.

Perhaps the vehicle design parameter which has received most attention is the aerodynamic drag coefficient. Gate and Neak (31) studied the effect of several design changes to the exterior of a Ford Anglia saloon. Their initial survey showed little differences in the projected frontal areas of vehicles in the same class, but significant differences in the drag coefficient. Hence their attention to the detail shape of the exterior of the vehicle by wind tunnel studies. Their findings are that a wing mirror has a drag of 1 lbf at 65 mile/h and they suggest that this is equivalent to 0.05 mile/gall. An advertising board on the roof, similar to that used for taxis, was thought to be equivalent to 1.5 mile/gall at 30 mile/h, whilst a roof rack fitted with luggage decreased the fuel economy.
by 3 mile/gall at 60 mile/h. Full scale wind tunnel tests on the Anglia showed that streamlined bubbles over the headlights did not affect vehicle drag and that a "fast-back" insert actually increased the drag by 1%. A false windsreen however, reduced the drag by 16%. This was thought to be equivalent to 1 mile/gall at 40 mile/h.

Tendiswood and Graetzal (49) also advocated a reduction in the aerodynamic drag coefficient of electric cars as a means to increase their range. They too felt that little could be done by reducing the projected frontal area and suggested a minimum of 18 sq. ft, representing two people sitting abreast.

White (9) and White and Carr (24) have conducted a campaign aimed at the reduction of the aerodynamic drag coefficient of British motor cars. They have shown that this is, in general, significantly higher than that of motor cars of continental manufacture. They use as evidence the high reduction in horsepower necessary to power a car at high speed by reducing the drag coefficient. In the discussion to White and Carr's paper (discussion of points raised by the Author) they suggest, from steady-state studies on a 1 litre car, that reducing the drag coefficient from 0.44 to the very low value of 0.25 improves the fuel consumption from 31 to 45 mile/gall - a saving of 1 gall per 100 miles.

Joyner (65), concerning heavy commercial vehicles, advocates that attention be paid to the shape of the load and urges that the frontal area of a van type trailer be no larger than that required
to carry the load. He quotes experiments on the streamlining of a super transport truck which resulted in a change in fuel consumption from 1.77 to 2.03 mile/gal at 70 mile/hr with a change in the horse-power required from 203 to 119 hp. Minervini (98), on the other hand, advocates an increase in the projected frontal area of commercial vehicles on the grounds that it is a reasonably efficient way of carrying more load. He notes however that commercial vehicles of the same class have an almost constant projected frontal area and makes the point that the effect of the projected frontal area on operating cost decreases as the size of the vehicle increases. His costing takes account of first cost, depreciation, fuel costs, maintenance etc. and is expressed in terms of "lire per kilometer per square meter of projected frontal area". The results of his studies are given in table A2.1 and show a substantial gain in both operating costs and the power required by reducing the aerodynamic drag coefficient when operating commercial vehicles on motorways.

Waters (111) makes some suggestions on how the aerodynamic drag coefficient of tractor-trailer type vehicles particularly can be reduced. They are that the leading edges should be curved, that forward facing surfaces should be inclined rearwards, that, in plan-form, the front edges should be curved and the rear tapered (guide vanes down the rear edges may assist this tapering) and that the gap between tractor and trailer be reduced to a minimum. If the gap cannot be reduced significantly, Waters recommends that it be
increased considerably.

The effect of wind speed has not been studied in any detail. Cornell (57) suggests that, for American motor cars, a 18 mile/h wind results in a 5 mile/gall change in fuel consumption.

Knoroz (54) has studied theoretically the effect of tyre rolling resistance on vehicle fuel consumption and suggests that a 1% change in rolling resistance results in 0.25 to 0.3\% change in the fuel consumption. He feels that the rolling resistance of a tyre could be reduced by as much as 1% if necessary. Tenniswood and Graetzl (49) however feel that, for electric cars, the rolling resistance could be decreased to 50\% of its current value.

The effect of ambient temperature on fuel consumption has been given a brief mention by Schaffler and Riepold (56) who give a 60 deg F change in ambient temperature as equivalent to 1 mile/gall change in the fuel consumption of American cars.

Also, Petevshov (52) and Joyner (65) have looked at the effect of axle layout and transmission arrangements of very large commercial vehicles. Joyner gives a 4\% increase in fuel consumption as a result of employing dual axles. Petevshov looks at such devices as inter-axle differentials, unlocking mechanisms, free wheels etc. and develops empirical, rule-of-thumb expressions.

Finally, Joyner (65) suggests that a watch be kept on the number and type of accessory used on trucks. He lists the air compressor, alternator, cooling fan, power steering and the cab air conditioning system. He feels that much could be done to improve the efficiency
of these accessory drives.

As a tool in the study of the effect of vehicle design parameters upon fuel consumption and performance generally, Gambor et al. describe their rig consisting of the power train of a typical motor vehicle. It has been so designed that many of the parameters may be altered and their effect recorded experimentally. Such a rig should prove to be a useful complement to theoretical studies.
6. **Automatic Transmissions - Performance Calculations**

The distinguishing feature of an automatic transmissioned vehicle is that the engine speed is not related in a simple manner to the vehicle speed. There are many designs of transmission ranging from hydrostatic to hydrodynamic. The equations governing a number of practical hydrostatic arrangements are given by Macmillan and Davies (91). Also, White and Cistic (36) analyse parallel transmission systems coupled by a differential unit. Their main concern however, is a drive to a machine tool. Molly (88), in an article in ATZ in October 1966, gives the arrangement and design of a number of practical hydrostatic vehicle transmissions and Worn and Walker (87) give details of the Dowty hydrostatic transmission. A survey of such transmissions is given by Giles (61).

In general however, hydrostatic transmissions are of interest because of their potential, after considerable design development. The vast majority of automatic transmissions are of the hydrodynamic type (40), employing a torque converter or fluid coupling in series with a conventional stepped ratio gear box. Such systems are described by Giles (39) and by Mitchell (40) and are typified by the Borg-Warner design.

Not unnaturally therefore, performance calculation techniques for automatic transmissioned vehicles are based upon a power train comprising an engine, torque converter or fluid flywheel, gearbox and drive axle in series. Such techniques must, of necessity, take
account of the torque converter or fluid flywheel characteristics.

Lucas and Rayner (44) have very recently (1968) analysed in some detail the characteristics of a three element torque converter and have shown that there is good reason for the popular description of torque converter characteristics by two graphs only. One of torque multiplication ratio against speed ratio and the other of a quantity generally termed the K-factor against speed ratio. The K-factor is defined as the quotient engine speed divided by the square root of engine output torque. This analysis is a direct result of the work contained in this thesis. Similarly, for a fluid flywheel, Rayner (38) has developed the basic equations which may be simply re-arranged to give a unique relationship between the K-factor and the speed ratio. The torque ratio of a fluid flywheel is, of course, unity. Another analysis of a torque converter, treating the unit as a vibration damper operating under dynamic conditions, is given by Ishihara and Emori (85). This work is useful because it gives a basis for using the two characteristic curves under transient conditions. That is to say that quasi-steady state consideration is permissible provided that the transient or perturbation frequency is not greater than half the impeller speed. This condition is met in the case of an accelerating vehicle.

Typical torque converter characteristics are given by Ordozica (83) in the January 1966 edition of the S.A.E. Journal. He describes, in a rather confusing manner, the basic technique of matching a
torque converter to an engine and goes on to suggest how this may be used to determine the time-to-speed of an automatic transmissioned vehicle. However, no account is taken of the fact that the engine torque from an accelerating engine is less than that returned by a steady-state test by the amount required to accelerate the engine inertia. This is a common error in automatic performance calculations. He does use his intermediate results however to show how to fix the gear change point such that a high acceleration is maintained. Also, he shows that the effect of this decision on the performance of the vehicle on a gradient.

Ott (42) adopts a more analytical approach in his technique for the calculation of the time-to-speed of an automatic transmissioned vehicle. He derives the equations of motion for the engine driving the impeller of the torque converter and for the torque converter turbine driving the vehicle. These he solves using a numerical method. This technique allows automatically for the change in engine torque output due to the accelerating engine inertia. Ott's paper is valuable also because he conducted rig tests and shows excellent correlation between the results of these tests and his calculations. This again confirms that it is permissible to treat the performance characteristics of the torque converter in a quasi-steady manner.

While not subtracting from the value of his work, it should be pointed out that Ott failed to realise the significance and the
uniqueness of the K-factor in describing the performance of the torque converter. Instead, he uses a family of curves of input torque against input speed for a range of speed ratio values. To these he fitted curves of the form

\[ y = \frac{l - a_2 x - a_3 x^2}{a_2 + a_4 x} \]

by the method of least squares for use with his computer program.

The gear change points would normally provide a discontinuity in the integration process used by Ott. He overcame this by specifying in considerable detail the gear change process. This involved the behaviour and inertia of the gear wheels and clutches in the gear box and added considerable complexity to the analysis but, by so doing, he rendered the integration process continuous throughout.

From an Engineers view-point, Ott's technique is lengthy, in computational time and does not give a good insight into the performance of the individual components. For instance, it affords little information on the match between the engine and the torque converter. Also, Ott does not attempt to allow for the possibility of "wheel spin" in his technique. For conventional automatic transmissioned vehicle performance calculations, it is thought that the more straight-forward approaches are preferable. However, for complex transmission
systems involving units in parallel, this technique and those described by Macmillan and Davies (91), Macmillan (92) and White and Christie (86) are indispensable.

An example of a more straightforward approach is given by Sets (73). This paper is mentioned above in the section dealing with time-to-speed calculations. The concern here however, is with that part of it dealing with the calculations peculiar to automatics. He expresses the torque characteristics by the two curves E-factor and torque ratio versus speed ratio and illustrates the shape of these curves. This then he used to match the engine and torque converter together to give the converter output shaft torque against speed characteristic. This is then used, without allowance for the decrease in torque due to the accelerating engine, to calculate the vehicle time-to-speed using Taborok's (4) integral. Speed is the independent variable with a step length of 1/10 mile/h for the integration process using a digital computer, 5 mile/h when using a hand calculator. It is not clear, from a study of the paper, to what extent the allowance for engine inertia has been omitted. Certainly there is no provision for the shift in the match between engine and torque converter resulting from the drop in engine torque and there is doubt whether there is any allowance for the general drop in torque level. The engine inertia itself is not read in specifically and is not mentioned in the descriptions of the computational processes, although it could be included in the term "inertia of rotating parts."
A similar procedure is described by Louden and Lukey (79). They too use Taborek's integral and omit to cater adequately for the accelerating engine. In that part of their paper dealing with the match between the engine and the torque converter, they did not use the speed ratio across the torque converter as the independent variable. As a consequence, the match can only be completed by trial and error. Of interest, but of no importance, is their torque converter parameter which they term the "capacity factor". This is the square of the K-factor defined above and is used for the same purpose. Louden and Lukey say that it is assumed, for simple representation, that this definition is true for all values of converter input torque. This emphasizes the widespread use of the capacity or K-factor and the assumption that it is unique. The Lucas and Haynes paper (44) now formalises this procedure.
7. **Miscellany**

A number of papers may be found dealing with specialised vehicle performance calculations for such applications as military, agricultural and earth moving vehicles. Such vehicles tend to have their own special problems. Time-to-speed calculations tend to be of little importance. The treatment of tracked vehicles may be different from that of wheeled vehicles. Also, a particular type of vehicle, say the Jeep, may be analysed in a different manner for agricultural or civil use than by the military.

It is usual, in the calculation of the performance of such vehicles, to specify in detail the behaviour of the vehicle and perhaps of the driver also. Further, the ground or terrain influences the performance considerably and must be specified. This latter point has led to an interest in soil mechanics. A number of theories exist in the prediction of the force required to move a wheel or track through soil. These have been reviewed by Firth (50) and tend to be based upon the work required to displace the soil. Reese (117) points out that the theories work quite well for frictionless clay soils and for lightly loaded vehicles in firm soils where sinkage is small. For soft soils however the theories do not agree very well with experimental evidence. The complex stress patterns in the soil caused by the passing vehicle are important and cannot be neglected. These theories are developed by Little (5), Sekar (6), Reese (7) and Payne et al.
A data sheet giving the basic details has been published by "The Automotive Design Engineering" (90). The basic procedure for the calculation of the rolling resistance of a wheel on soil is contained in Section 3, Part B of this thesis. The special problems of tracked vehicles has been covered by Cleare (73). Not only is the soil mechanics problem rather different but the relatively high power losses in the track itself requires attention. This latter point is influenced by track tension.

Turning now to computer programs designed to predict the performance of such vehicles, McKenzie et al. (47), in an excellent and detailed S.A.E. paper published in 1967, lay down a rational procedure for the prediction of the performance of military vehicles. They point out the value of such programs for parametric study purposes and that continuous development is required in order to incorporate the latest information. On a cross-country run they maintain that there is a limit to the maximum speed of the vehicle caused by one or more of the following factors.

1. The maximum pitch and bounce accelerations tolerated by the personnel
2. obstacles
3. power and traction supply
4. driver ability

They state that, in the bounce mode, the vertical acceleration limit is 0.27 ± 0.03g. In the pitch mode the limit is 5.77 ± 0.8 rad/s².
while in the roll mode it is rather higher at $9.6 \pm 1.6 \text{ rad/s}^2$.

Of interest is the fact that the driver is considered an integral part of the system. His behaviour, specified in some detail, is outlined above in the section dealing with Fuel Consumption. The forces resisting motion are taken to be

1. air resistance (usually negligible)
2. vertical displacement of the soil
3. forward displacement of the soil
4. gradient of slope
5. inertia
6. vegetation resistance

Such forces are described in analytical form in the paper.

Another paper of interest here, by Barrett and Nicholson (116) describes a computer program for the prediction of farm vehicle performance. Again the value of a parametric study is emphasized.

To describe the tractor, the front and rear weight, wheel base, rear tyre rolling radius and drawbar height are read in. The engine is described by its torque curve, fuel consumption curve and its governed speed. The transmission by gear ratios etc. and the field test course by the average drawbar pull together with a percentage range of pull. The operator (driver) is specified by stating the engine speed limits between which he will operate the tractor. The output from their program gives the time to complete a 25 mile run, average tractor speed, total fuel consumed, total gear changes etc.
and is said to predict this information accurately.

Ogorziewicz's paper (80) is rather critical of the presentation of the report by Bischoff and Arno upon which it is based. However, this report is not given in his list of references. The work concerns the development of a computer program to predict the performance of military vehicles. The Jeep is mentioned, and its design history given in some detail. A number of available standard axle ratios have been used in the evaluation of the acceleration of a particular military vehicle. Of these, the 6.90 to 1 ratio was selected, since it best met a rather complex specification which stated, amongst other items, that the vehicle must be capable of 30 mile/h speed on a 3% grade, that it must be capable of climbing a 60% grade when fully laden and that it must be capable of at least 50 mile/h speed on the level. The program itself is not given in detail, but it is said to be based on Taborek's (4) work.
3. Instrumentation for the performance of vehicles

a) Wind tunnels

Wind tunnels are a useful tool in "straight-line" vehicle performance work in that they enable the aerodynamic drag coefficient of a vehicle to be determined. Apart from the many small scale wind tunnels, the main facilities in this country are invested at the Motor Industry's Research Association at Lindley, near Hunsleton. Their full size and quarter scale wind tunnels are adequately described by White and Carr (24). A notable feature of their full size tunnel is its inbuilt chassis dynamometer and the balance arrangement.

This balance copes ingeniously with the measurement of the relatively small vertical aerodynamic forces in relation to the large force from the weight of the vehicle by using two springs in parallel. One spring is used to take out the vehicle weight and is locked in position before the test commences. Cato and Meek (81) describe their balance arrangement as being a portable device rather than a permanent fixture in an American full size wind tunnel facility. They employ "levapads", that is air bearing pads, under each of the four vehicle wheels. A complex system of strain gauged links constraining the levapads enables the main aerodynamic forces to be obtained.
White and Carr discuss the use of the MIRA quarter scale tunnel and the manufacture of models. These models are said to be expensive since, in order to adequately represent a vehicle, dimensions must be within $\pm 1/16"$ and good representation must be made of all external features. These include the number plates, bumpers, lamps, door handles, rain gutters, window frames etc. since these items can account for 15% of the vehicle drag. A further 10% of the drag and 20% of the lift forces are accounted for by the underbody details. These must be reproduced accurately.

Air flow through the coolant system need not be represented, since this factor is usually known from accumulated data on the subject and no may be added on later. Hence, by following this advice, good correlation may be obtained between model and full size wind tunnel tests. Also, White and Carr (24) report 1 to 1 correlation between full size wind tunnel and actual road tests in a large and detailed series of vehicle drag tests.
b. Road Tests

The type of road test relevant to this work is confined to travel in a straight line. It does not include braking, but does include acceleration tests, maximum speed tests, steady-state fuel consumption tests and deceleration tests in order to establish the vehicle drag coefficients. Such tests may or may not be on a level load.

Some use may be made of such an instrument as a chassis dynamometer (24). The car is placed with its drive wheels on one or two large diameter rollers. The rollers are connected to a dynamometer which affords a reaction to and measures the drive force. An alternative device is the trailer dynamometer (112). This is a trailer hitched to the rear of and pulled by the test vehicle. The wheels of the trailer are connected to some power absorption device. This may be a fan or electrical generator and a bank of resistors. Such devices are of use in obtaining the power at the drive wheel characteristics and of measuring the performance of the vehicle on a simulated gradient.

Alexander (77) of the Road Research Laboratories has described their standard, all purpose, vehicle instrumentation. A "fifth wheel" device is used to measure vehicle speed. This is a small wheel towed behind the vehicle, the translational speed of the centre of which is known accurately in relation to the rotational speed.
Also, presumably as a check, the Road Research Laboratories use two parallel beams of light spaced 24° apart across the test track. Further instrumentation, not of direct interest here, concerns the brake line pressure, a decelerometer, a gyroscope to give the heading of the vehicle, pitch and roll transducers and steering wheel angle. A ten channel recorder is used to record the signals.

The fifth wheel device is common in vehicle performance tests. The speed signal obtained may be an analogue signal, that is a steady voltage or current level proportional to a steady speed, as obtained from a tacho-generator connected to and driven by the fifth wheel. Or the signal may be digital in character, as obtained from a photo or inductive transducer "looking" at a toothed wheel connected to the fifth wheel. The road tests conducted by the motoring press are made using an analogue type of fifth wheel in conjunction with a stop watch.

A more sophisticated set-up is described by Sturm(76). He employs a digital type of fifth wheel and records the pulses on magnetic tape. This is said to be accurate to within 1%, automatic and continuous in recording, adjustable to test conditions and procedures, capable of minute data increments and finally, of great importance, adaptable to easy data reduction. In the case, say, of an acceleration run, a two track tape recorder is used. Sturm records the fifth wheel pulses and a time signal from a 1000 Hz
tuning fork oscillator. Such signals are then in a form suitable for data reduction. A gated circuit is tripped by the time signals and the distance pulses in between are counted. By double differentiation, first the velocity, and then the acceleration of the vehicle is obtained. Sturm did not, at the time of his paper (1962), use a digital computer for this work, but such signals on magnetic tape are in a very convenient form for employing a data logger and processing by computer. Recording a pulse, rather than an analogue signal, means that the quality of the recording is not important, just the number of pulses.

Sturm considered other systems and rejected the stop watch technique by saying that its shortcomings are well known to those working in the field. Measurement techniques, where the results are obtained in graphical form, have definite drawbacks when further data processing has to be done as, for instance, when the results of several runs in each direction have to be averaged. Sturm ruled out all systems which had an inherent time lag or mechanical inertial problems without giving examples of such systems. Conventional accelerometers were ruled out because they may be influenced by the attitude of the car.

Another practical method of measuring a vehicle speed and acceleration is described by Ardoino et al (96) of Fiat. This system is geared to a test track, rather than a vehicle. White lines are painted across the track. These are "sensed" by a photo
transistor carried by the vehicle. The Fiat unit carrying the photo transistor has an automatic sensitivity control which operates according to the different lighting conditions. It is important that, in intense daylight, the blank bands in the road are not seen as white strips. Also, the transistor should not respond to white stones, paper etc. Such a system is said to be inexpensive to operate and is stated as being suitable for use on roads open to normal traffic! A variation on this technique, also in use at Fiats, is to place blocks of two photo transistors spaced 10 meters apart at intervals along a test track. Such a system is expensive and difficult to set up. Power has to be supplied to each block and lamps have to be positioned across the track facing each photo transistor. Such a system has been used on part of the Turin - Piacenza motorway. Care has to be taken to ensure that the photo transistors are not triggered by other cars or by other spurious means. Rain can upset the system by interfering with the light beam and because it is difficult to make the long lengths of cable, with all the control and junction boxes, waterproof. Fiat have a fifth wheel system also. This is said to be accurate to within 1%, but is thought to be vulnerable to rough treatment.

Hughes, of the Consumers Association, in the Discussion of the Ardoiño et al paper, describes the radar system used by "Motorng - Which?" This was installed after considerable difficulties had been experienced with photo transistors mounted along a test track. Apart from reliability troubles, a major problem encountered with one type
of test, designed to assess the overtaking ability of a vehicle, was that the driver had to arrive at the first photo transistor at precisely the correct speed and to start his acceleration at this point. The radar device consists of a 6 ft diameter dish-shaped radar transmitter and receiver arranged to look along the \( \frac{4}{3} \) mile long test track. A balanced land line is arranged to carry the Doppler pulse back to the laboratory. The system required some development, particularly with regard to the long length of test track and the automatic level control of the narrow beam signal strength. Associated with the system is a small computer which processes the results. Using this, it is possible to simulate a constant speed vehicle to overtake. The results from a full throttle run on the test vehicle can, therefore, be expressed automatically as the distance required to overtake such a vehicle. Accuracy is said to be one hundred times better than the old (photo-transistor) system. Distances can be measured to better than \( \pm 3 \) cm and time to within \( \pm 0.001 \) s.

A Doppler radar system is under development at Loughborough. This system, which will be the subject of a paper in November, 1969, dispenses with the need for a large dish-shaped transmitter-receiver. Instead, a small torch-shaped device is clamped onto the front of the test vehicle and made to look obliquely down at the road.

Another device to measure the instantaneous speed of a vehicle is described by Loudon and Lukey (79). This consists of a magnetic
transducer looking at the periphery of a gear wheel drive by the
transmission output shaft. This system is similar to the fifth
wheel system and is described by Ardioni et al (96) as not very
accurate since tyre growth, and any slip or creep between tyre and
road, can cause 3 to 4% error.

Magnetic tape appears to be the general recording medium because
tape recorders are small and robust and because of the flexibility of
data reduction in the laboratory. Fletcher (114) however gives
details of the application of telemetry to the testing of motor
vehicles. His conclusions are that this system is quite versatile,
but costly.

The measurement of fuel flow rate is generally on a volume basis,
rather than a mass basis for the small engines used in automotive
applications. The devices described in the literature are all on
the basis of the time taken for the engine to consume a specified
volume of fuel. Ardoino et al (96) use a mercury piston in glass
vessels to displace a known quantity of fuel. An inductive device
senses the ends of the mercury piston stroke and causes the piston
to reverse its direction. The volume displaced by the piston may
be 10 ml or 25 ml. Although this system is continuous in operation,
it is essentially a device for the measurement of steady-state fuel
consumption.

Sturm (76) attempts to adapt this type of system for transient
measurement by making the displaced volume very small. His device
consists of a "leakless" piston which, although it displaces \( \frac{1}{2} \) gall.
of fuel, has a rod attached to it with a large number of notches cut in it. A photo-transistor senses the passing of these notches. At the end of piston travel, the flow is automatically reversed in the meter. Each space between the notches represents 1/10,000 gallon of fuel. This method reduces the significance of the "switching" error. The 133 notches/inch necessary for the calibration were achieved by the use of "Ronchi Rulings".
c) **Rigs**

Genbon et al (108) have described their rig designed specifically for vehicle performance work. This consists of a laboratory layout of a vehicle comprising engine, transmission, drive wheels and a dynamometer. The instrumentation of the rig is comprehensive and sophisticated and, to some limited extent, design parameters may be modified on the rig. Gau (110) describes a rig representing an electric vehicle. This is a conventional motor car, with a conventional engine, modified to give the accelerative performance of an electric car.

Other rigs are of a specialist nature to study some aspect of performance work. These include the many and varied dynamometer rigs to study tyre performance and transmission efficiency. Of those dealing with the former, the elaborate rig of Seki et al (95) must be mentioned. This consists of a tyred wheel rolling on a large drum with a complex force transmission linkage in order to measure the rolling resistance, load on the tyre etc. The rig has been used to study such matters as the effect vehicle speed, load, inflation pressure and temperature, rim width, road surface, wheel alignment, size of tyre, material of tyre, tyre and tread design on rolling resistance. A number of such rigs may be found described in the literature (45), (46), (71).

The dynamometers themselves and basic engine test bed layouts are adequately described in text books, such as that of Greene and Lucas (1).
### List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Projected frontal area of a vehicle</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>$A_d$</td>
<td>Rolling resistance coefficient</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>Longitudinal position of centre of gravity of a vehicle from the centre line of front wheels</td>
<td>ft</td>
</tr>
<tr>
<td>$B_d$</td>
<td>Rolling resistance coefficient</td>
<td>h/mile</td>
</tr>
<tr>
<td>$b$</td>
<td>Longitudinal position of centre of gravity of a vehicle from centre-line of rear wheels</td>
<td>ft</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of rut</td>
<td>ft</td>
</tr>
<tr>
<td>$C_{amb}$</td>
<td>Correction factor for ambient conditions</td>
<td></td>
</tr>
<tr>
<td>$C_b$</td>
<td>Aerodynamic drag coefficient</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>A distance</td>
<td>ft</td>
</tr>
<tr>
<td>$q_m$</td>
<td>Meridional component of fluid velocity</td>
<td>ft/s</td>
</tr>
<tr>
<td>DAR</td>
<td>Drive axle ratio</td>
<td></td>
</tr>
<tr>
<td>Det</td>
<td>Determinant</td>
<td></td>
</tr>
<tr>
<td>DUG</td>
<td>Degree of under/over-gearing</td>
<td></td>
</tr>
<tr>
<td>$DV$</td>
<td>Speed increment</td>
<td>mile/h</td>
</tr>
<tr>
<td>$F_d$</td>
<td>Drag force</td>
<td>lbf</td>
</tr>
<tr>
<td>$F_p$</td>
<td>Normal reaction between front wheel and ground</td>
<td>lbf</td>
</tr>
<tr>
<td>$F_l$</td>
<td>Limiting tractive force</td>
<td>lbf</td>
</tr>
<tr>
<td>$F_p$</td>
<td>Propulsive force</td>
<td>lbf</td>
</tr>
<tr>
<td>$F_r$</td>
<td>Normal reaction between rear wheel and ground</td>
<td>lbf</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>f</td>
<td>Acceleration</td>
<td>ft/s²</td>
</tr>
<tr>
<td>GR</td>
<td>Gear ratio</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>Height of the centre of gravity of a vehicle</td>
<td>ft</td>
</tr>
<tr>
<td>I</td>
<td>A particular gear number</td>
<td></td>
</tr>
<tr>
<td>Iₑ</td>
<td>Engine inertia</td>
<td>slug ft²</td>
</tr>
<tr>
<td>Iᵥ</td>
<td>Total vehicle wheel inertia</td>
<td>slug ft²</td>
</tr>
<tr>
<td>i</td>
<td>Gradient</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>Torque converter K-factor = $N/\sqrt{T}$</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>Aerodynamic drag constant = $\frac{1}{2} \rho C_D (v^{1/2})^3$</td>
<td></td>
</tr>
<tr>
<td>mₑ</td>
<td>Equivalent mass of vehicle</td>
<td>slug</td>
</tr>
<tr>
<td>mᵥ</td>
<td>Mass of vehicle</td>
<td>slug</td>
</tr>
<tr>
<td>N</td>
<td>Rotational speed</td>
<td>rev/min</td>
</tr>
<tr>
<td>NG</td>
<td>Number of forward gear ratios</td>
<td></td>
</tr>
<tr>
<td>N_L</td>
<td>Lower limit to gearbox input speed</td>
<td>rev/min</td>
</tr>
<tr>
<td>N_max</td>
<td>Maximum engine speed</td>
<td>rev/min</td>
</tr>
<tr>
<td>N_min</td>
<td>Minimum engine speed</td>
<td>rev/min</td>
</tr>
<tr>
<td>N_c</td>
<td>Torque converter output shaft speed</td>
<td>rev/min</td>
</tr>
<tr>
<td>N_p</td>
<td>Engine maximum power speed</td>
<td>rev/min</td>
</tr>
<tr>
<td>N_T</td>
<td>Engine maximum torque speed</td>
<td>rev/min</td>
</tr>
<tr>
<td>N_u</td>
<td>Upper limit to gear input speed</td>
<td>rev/min</td>
</tr>
<tr>
<td>OGR</td>
<td>Overall gear ratio</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>Pressure</td>
<td></td>
</tr>
<tr>
<td>Pamb</td>
<td>Ambient pressure</td>
<td>in Hg</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Standard ambient pressure</td>
<td>in Hg</td>
</tr>
<tr>
<td>$P_v$</td>
<td>Water vapour pressure</td>
<td>in Hg</td>
</tr>
<tr>
<td>$R$</td>
<td>Gas constant</td>
<td>ft lbf/lb deg F</td>
</tr>
<tr>
<td>$r_r$</td>
<td>Rolling radius</td>
<td>ft</td>
</tr>
<tr>
<td>$SR$</td>
<td>Speed ratio</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>distance travelled</td>
<td>ft</td>
</tr>
<tr>
<td>$T$</td>
<td>Torque</td>
<td>lbf ft</td>
</tr>
<tr>
<td>$T_a$</td>
<td>Torque required to accelerate the engine</td>
<td>lbf ft</td>
</tr>
<tr>
<td>$T_{max}$</td>
<td>Maximum engine torque</td>
<td>lbf ft</td>
</tr>
<tr>
<td>$T_o$</td>
<td>Torque converter output torque</td>
<td>lbf ft</td>
</tr>
<tr>
<td>$TR$</td>
<td>Torque ratio</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>$t_{amb}$</td>
<td>Ambient temperature</td>
<td>°F</td>
</tr>
<tr>
<td>$t_{gc}$</td>
<td>Gear charge time</td>
<td>s</td>
</tr>
<tr>
<td>$V$</td>
<td>Vehicle speed</td>
<td>mile/h</td>
</tr>
<tr>
<td>$V_f$</td>
<td>Final vehicle speed</td>
<td>mile/h</td>
</tr>
<tr>
<td>$V_i$</td>
<td>Initial vehicle speed</td>
<td>mile/h</td>
</tr>
<tr>
<td>$V_{max}$</td>
<td>Maximum vehicle speed</td>
<td>mile/h</td>
</tr>
<tr>
<td>$V_w$</td>
<td>Wind speed</td>
<td>mile/h</td>
</tr>
<tr>
<td>$V_{v_0}$</td>
<td>Component of wind speed to head-on direction</td>
<td>mile/h</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity</td>
<td>ft/s</td>
</tr>
<tr>
<td>$W$</td>
<td>Weight (vehicle)</td>
<td>lbf</td>
</tr>
<tr>
<td>$X$</td>
<td>Dependent variable</td>
<td></td>
</tr>
</tbody>
</table>
Independent variable

Depth of sinkage of vehicle in rut ft

Drive axle mechanical efficiency

Gearbox mechanical efficiency

Transmission efficiency = $\eta_{DA} \times \eta_{G}$

Angle of gradient = $\sin^{-1}$ rad or deg

Coefficient of friction between tyre and ground

Air density slug/ft$^3$
PART B

Manual transmission accelerative and maximum speed performance techniques.
SECTION BL

Introductory Remarks

Part B is concerned primarily with the time-to-speed calculations of vehicles having "manual" gearboxes. The main performance computer program (B001) is contained and outlined in Section B12. This program accepts engine torque and speed range data, design details of the vehicle and its transmission, such as drag coefficients and gear ratios, together with environmental data such as wind speed, gradient etc. The main function is to evaluate the Taborek (4) integral between any two specified vehicle speeds. This calculates in a printed table of time and distance travelled figures against vehicle speed. Also, the program gives the designer information on gear "overlap", the possibility of the vehicle overturning on a gradient or of slipping on a gradient when parked. The program will, if desired, fix the gear ratios for the designer.

Program B001 has been written with the designer in mind. It incorporates a switching device enabling any design parameter to be altered at will so affording the designer with the results of a full parametric study, a valuable feature of such computer programs (78) (79). Typical parametric studies are contained in Part C of this work.

Program B001 calculates also the maximum speed of a vehicle in each of the gear ratios. However, the accuracy of this calculation is to within 0.5 mile/h only. Section B13 therefore, outlines a separate maximum speed computer program (B051) for use when more accurate maximum speed calculations are necessary. This was found to be desirable in some parametric studies.
Section B2 outlines the reasons and the technique developed for expressing the engine torque curve as a polynomial expression. i.e.

Section B3 deals with the vehicle drag expression, while Sections B4 and B5 outline expressions for the transmission efficiency and equivalent mass of the vehicle respectively. Section B6 defines gradient (i).

Sections B7 and B8 develop theories for dealing with the behaviour of the vehicle during "take-off" from rest and during the condition unknown as "wheel-spin". Section B9 deals with certain limitations on the vehicle design imposed by a gradient.

Section B10 develops the theories used in program E001 for the fixing of the gear ratios of a manual gear box. Part B11 outlines optimisation techniques suitable for use in fixing the intermediate gear ratios.

The theories developed in Part B are assessed and the results from their use are compared with those of published road tests. Two vehicles primarily are used in this assessment, the engine torque and the drag characteristics of which are known accurately. The first, Vehicle A, is the small 7 cwt van owned by the Department of Transport Technology, Loughborough University of Technology. The second, Vehicle B is a medium size saloon car of quite high performance. Design details of Vehicle B were supplied by its Manufacturer.

The time-to-speed calculations of vehicles having automatic transmissions are treated in a different manner to those having manual transmissions. Appropriate theories for the performance calculations of such vehicles are contained in Part F of this work.
The Engine Torque Curve

Table B2.1 lists the results of a test bed run on a 1000 cm\(^3\) petrol engine. For these results to be of use in conjunction with a digital computer program on vehicle performance calculations, they must be presented in such a way that the computer can be fed with the engine torque corresponding to a particular engine speed. This particular engine speed may not be one of the discrete engine speeds listed in Table B2.1. There are four methods of presenting this information. In order of sophistication they are :-

1. Manual
2. Use of array storage and interpolative routine
3. Use of a curve fitting technique
4. Use of peripheral equipment capable of reading a graph

The first is to plot a graph of engine torque against engine speed and to write the program such that the computer pauses and then asks for engine torque at the particular engine speed under consideration. This information is read off the graph and then fed into the computer. With modern, fast computers operating on the "closed shop" principle, this method is not practical. It is not a good method even with slow machines because of the possibility of an error in ascertaining and transporting the information into the computer store and because of the time wasted during this operation. This method may be justified when used with an "on-line"
machine.

The second method is to feed the data, similar to that contained in Table B2.1, into a two dimensional array within the computer store and to use an interpolative routine to determine the engine torque at any engine speed. It is preferable to smooth out experimental errors from the test data before storing.

The third method is to fit a mathematical law to a plot of engine torque against engine speed. In the absence of the known mathematical relationship between torque and engine speed, a polynomial expression of the form

\[ T = s_T + b_T \frac{N}{1000} + c_T \left( \frac{N}{1000} \right)^2 + d_T \left( \frac{N}{1000} \right)^3 \text{ etc.} \quad \text{B2.1} \]

is probably the most suitable, since it is very versatile and the accuracy of representation can usually be controlled by the number of terms used in the polynomial. It is then required to feed into the computer the polynomial coefficients \( s_T, b_T, c_T \) etc. only, and to program it to generate the engine torque at the particular speed under consideration from the polynomial expression.

The fourth method involves using advanced peripheral equipment not available on the I.C.T. 1905 machine. Basically, a graph of engine torque against engine speed together with the scales of the torque and speed axes are fed into an auxiliary unit attached to the computer. A light sensing device or other suitable pick-up follows the curve drawn. When the computer asks for the torque at a particular engine speed, the pick-up takes up the appropriate position on the graph enabling the ordinate value to be read into the computer store.
Now there exists on an engine torque curve two important and often quoted points. The first is the value of the maximum torque and the engine speed at which it occurs. The second is the point of maximum engine power. Since a polynomial is amenable to mathematical treatment and may be readily differentiated to find a maximum point, and since it is likely that the number of polynomial coefficients necessary to describe a torque curve is rather less than the number of values stored in a two dimensional array necessary to define the curve adequately, the polynomial curve fitting technique was chosen. Because a polynomial is a continuous mathematical expression, changes in the engine torque characteristic of the type caused by changes in ambient pressure and temperature can be accommodated simply by factoring each of the polynomial coefficients.

The decision to use a polynomial curve fitting technique avoided the necessity to develop complicated peripheral equipment or the supplying of a suitable interpolative subroutine. However, a polynomial curve fitting technique had to be developed.

An adaptation of a standard polynomial curve fitting program by the method of "least squares" was used. This being well tried and readily available. A write-up on the program is contained in Appendix B1.

In order to avoid very small polynomial coefficients and the manipulation of very large numbers by the computer during the intermediate calculation of the coefficients, it was decided to use the term \( (N/1000) \) as the independent variable in the polynomial
expression, rather than engine speed \( (N) \).

Fig. B2.1 shows a plot of the data contained in Table B2.1 and may be regarded as a typical engine torque curve, superimposed onto this plot are fitted polynomials of the order 2, 3, 4 and 6. The higher orders represent the torque curve adequately since they are within experimental accuracy. It was decided, therefore, to standardise on the sixth order polynomial in order to reserve some degree of versatility which might be desirable with an unusually shaped torque curve. On the occasions when a lower order polynomial fits a particular torque curve better than a sixth order, the coefficients of the lower order polynomial can be used, the remaining coefficients up to and including the sixth order term being taken as zero.

In developing the technique of expressing engine torque curves as polynomial expressions, two points arose.

The first is that the data used to fit a polynomial of order as high as the sixth must cover the complete speed range of the engine. The reason for this is that the fitted curve will lie close to the points specified, within the range of the points specified. Outside this range the curve may take up a shape quite unrepresentative of an engine torque curve. If the physical extremities of an engine's speed range are 3000 and 7500 rev/min, points near to these values should be included in the data used for the curve fit. In the absence of experimental points at the extremes of engine speed, it is preferable to sketch in the remainder of the torque curve rather than allow the curve fit program to fix a law.
Fig. B2.2 depicts the polynomial expression

\[ T = 1798.29 - 2809.125 \frac{N}{1000} + 1653.0912 \left( \frac{N}{1000} \right)^2 - 478.0614 \left( \frac{N}{1000} \right)^3 + 73.888 \left( \frac{N}{1000} \right)^4 - 5.87582 \left( \frac{N}{1000} \right)^5 + .1892811 \left( \frac{N}{1000} \right)^6 \]

fitted to the torque curve of a 1500 cm³ racing engine. The crosses denote the experimental points used for the curve fit and the continuous line is a plot of the polynomial expression. If it is desirable to run this engine above 7500 rev/min, this particular polynomial expression is not representative since the expression rises sharply above 7500 rev/min. While Fig. B2.2 demonstrates the necessity of using data throughout the full engine speed range, it is not desirable to extrapolate the torque curve too far into speed regions where it is just not possible to run the engine, since this subtracts from the accuracy of the curve fit in the real speed range of the engine.

The second point to note is that the number of data points used in the curve fit should be high in relation to the order of the polynomial. In the case of a sixth order expression, twenty to thirty points are appropriate. The absolute minimum number of points is the order of the polynomial plus one, i.e. seven in the case of a sixth order polynomial, since it has seven unknown coefficients. If the minimum number is used, then the fitted curve will pass through all the points and the curve fit may be said to be exact. While the curve may pass through all the points, the route it takes in between points may not represent an engine torque curve. This danger is avoided by using a large number of data points.
If seven only experimental points are taken and there is a lot of scatter on the points and no attempt is made to smooth, then something like Fig B2.3 may result. The dotted line represents the true torque curve, the crosses the experimental points and the full line the polynomial expression which passes through all seven experimental points, but does not represent the true torque curve. If the scatter is reduced by smoothing, then the seriousness of this second point diminishes. However, if one takes the trouble to smooth out scatter, it is very little extra work to take 25 points off the smoothed curve rather than seven.

The evaluation of the polynomial expressions fitted to Figs. B2.2 and B2.3 was executed using an "evaluation of a polynomial expression" program especially designed for the purpose. Its listing is contained in Appendix B1 also.
Experimental determination of an engine torque curve

The engine should be installed on an appropriate test bed and should be in full vehicle trim. The fan, dynamo, etc. must be in situ and care must be taken to ensure that the appropriate air cleaner is fitted to the engine. If possible, the exhaust system also should be that appropriate to the vehicle under consideration.

Fig B2.4 and B2.5 show the measured torque curve of a 1000 cm³ and a 1500 cm³ engine respectively, first with the air cleaner removed, and then with the air cleaner in place. Vehicle performance calculations using the wrong curve will result in acceleration times very much in error, particularly in the case of the smaller engine. However, maximum vehicle speeds calculations will result in similar answers.

Care must be taken to ensure that the oil temperature is similar to that of the engine installed in a vehicle. Fig. B2.6 depicts the results of an engine test. The encircled dots are torque measurements with engine speed progressively increasing, the crosses with engine speed decreasing. Other test results with the oil temperature carefully maintained at a constant value show the torque curve when the engine speed is decreasing to be lower than that when the engine speed is increasing.

The conclusion is, therefore, that if oil temperature is maintained constant, the lower torque curve when the engine speed is decreasing is due to the increasing heat transfer to the air entering the engine. The engine and its surroundings being hotter.
If, however, the oil temperature is not controlled, the effect of the low oil viscosity in the hot engine when the speed is being reduced, may result in a higher torque curve. The oil temperature was not controlled during the engine test culminating in Fig. B2.6.

The effect of oil viscosity and heat pick-up on vehicle performance may be significant. A vehicle accelerating up through the gears from a standing start may be expected to have a cooler engine than the same vehicle undergoing a prolonged test to determine its maximum speed.

A reasonable procedure, therefore, is to ensure a representative air flow over the engine test bed and to take readings both with engine speed increasing and decreasing. The oil temperature being uncontrolled. The curve taken from the results should then lie close to the points with engine speed increasing for the majority of the engine speed range blending into those with engine speed decreasing near maximum engine speed.

This curve, when used in vehicle performance calculations, should give both accurate time to speed and maximum vehicle speed figures.

This procedure produces results which agree well with experimentally determined vehicle performance results, but it does bring into question the procedure of using a torque curve obtained from a steady running experiment in calculations where it is implicit that the engine is accelerating.

The procedure is justified by saying that the engine acceleration, during a full throttle run up through the gears on a vehicle, is small. However, as knowledge of vehicle drag and transmission losses
increases and vehicle performance calculations become more exact, it will be necessary to take account of the changes in torque output with rate of change in engine speed.

There are two methods of achieving this. One is to fit the engine into several vehicles, the drag characteristics of which are known accurately and to subject these vehicles to carefully controlled acceleration experiments. By working backwards through the calculations, it is possible to obtain the engine torque curves relative to the different vehicles and hence to differing vehicle acceleration. The second is to use an engine test bed with an eddy current type of dynamometer. This type of dynamometer has, by its very nature, a servo-control on engine speed which can be modified quite simply to programme the engine to a particular acceleration characteristic. By adding flywheels to simulate vehicle inertia and by arranging the instrumentation to give a print-out of torque versus speed, the effect of engine acceleration on torque output can be studied.

The second method is undoubtedly the more versatile, but it does suffer from the disadvantage that it is very difficult to simulate full vehicle conditions upon an engine test bed. Matters such as oil temperature and the design of the exhaust system are difficult to simulate exactly. The approach should be, therefore, to conduct both sets of experiments since a comparison between the results of the two is likely to throw new light onto the problem. However, for the present work, steady state measurements of torque are considered adequate.

To conclude these remarks on the experimental determination of engine torque it must be stated that all experimental results
should be corrected for the effects of ambient pressure and temperature. The accepted method of correction is to factor the observed torque to give the corrected torque of a petrol engine or of a compression ignition engine at full rack opening according to

\[ T = T_0 \times \left( \frac{P_s}{P_{amb} - P_v} \right) \times \sqrt{\frac{(t_{amb} + 460)}{(t_s + 460)}} \]

where \( T_0 \) is the observed torque, \( P_{amb} \) in Hg and \( t_{amb} \) °F the observed ambient pressure and temperature.

\( P_v \) in Hg is the water temperature vapour pressure on the day of the test and \( P_s \) in Hg and \( t_s \) °F are the arbitrarily chosen standard ambient pressure and temperature.

In the case of a compression ignition engine on part load, use

\[ T = T_0 \times \left( \frac{P_s}{P_{amb} - P_v} \right) \times \frac{t_{amb} + 460}{t_s + 460} \]

The difference being that there is no square root sign over the temperature term.

Reasons and a full justification for these expressions are given by the Author in Chapter 7 of "The testing of Internal Combustion engines" (1).
**SECTION B3**

**Expression of vehicle drag**

In vehicle performance calculations, it is essential to know with some degree of accuracy the forces resisting vehicle motion. These forces need to be evaluated for the straight-ahead vehicle motion only, it is not necessary for this work to consider the additional complication involved with cornering or with braking. The forces resisting motion can be considered under three broad headings.

1. deformation of the wheel
2. deformation of the ground
3. air flow over the vehicle

Items 1 and 2 together are usually termed the rolling resistance of a vehicle and item 3 the aerodynamic drag.

The special case of power losses in the transmission system is considered later in Section B4.

**1. Deformation of the wheel**

The pneumatic tyre is particularly suitable for use on road vehicles because of its contribution to comfort, its excellent adhesion properties and because it does not break up the road surface to the extent of a more rigid wheel. However, the vehicle load and the tractive effort are not carried without deformation. In the case of a pneumatic tyre on the hard surface of the modern road, the deformation of the tyre accounts for some 90–95% of the
rolling resistance of a vehicle. Wheel windage and slippage losses are small in comparison (2). The distortion of the tread as it passes through the contact area results in a hysteresis loss which manifests itself as heat, raising the temperature of the tyre.

It is necessary now to differentiate between the two types of tyre construction. That is between radial and cross ply tyres. A radial ply tyre lays its tread squarely onto the road, with very little lateral constraint offered by the walls of the tyre. This means that there is virtually no lateral slippage between tread and road. However, the tread does bend as it passes through the contact area. In the case of a cross ply tyre, the constraints offered by the tyre walls cause distortion as the tread passes through the contact area. This results in hysteresis losses and some loss due to slippage between tread and road. This accounts for the lower rolling resistance of the radial ply tyre compared with the cross ply and its superior wear and fuel consumption characteristics.

Experience shows that the other factors affecting rolling resistance of a pneumatic tyre on a hard surface such that deformation of the hard surface can be ignored are:— tyre temperature, inflation pressure, vehicle speed, load and deflection, tread thickness, number of plies and mix of rubber and the torque transmitted (2).

The deflection of a tyre bears a direct relationship with the load which causes it. The greater the deflection, the greater the
hysteresis loss and hence the greater the rolling resistance. It is usual, therefore, to express the rolling resistance of a tyre as a non-dimensional rolling resistance coefficient

$$A_u = \frac{\text{Rolling resistance}}{\text{Load on tyre}}$$

The same units of force being used for rolling resistance and load on tyre. When dealing with the total rolling resistance of a vehicle, the sum of the loads on the tyres is applicable, that is the vehicle weight.

Dealing first with the effect of transmitting torque upon rolling resistance. Conversion of torque to tractive force causes distortion of the tread. This results in areas of micro-slip and areas of "sticking" within the contact area. The greater the torque transmitted, the greater the area of micro-slip and hence the greater the rolling resistance. A fuller description of the mechanism of micro-slip and stick is contained in an article by Halling and Brothers (3). Fig. B3.1 reproduced from Automotive Design Engineering (2) depicts the result of experiments and suggests that the rolling resistance of a tyre can increase by a factor of 2 to 3 if transmitting torque.

Now, vehicle performance calculations are usually conducted for the full throttle engine condition. Here the torque level is high and fairly constant throughout the lower vehicle speed range where the rolling resistance is important. The rolling resistance coefficient used therefore, should be at the appropriate torque level. As the effect of torque on rolling
resistance becomes better understood and as vehicle performance calculations become more sophisticated, it will be necessary to make a continuous allowance.

Temperature has a marked effect on rolling resistance. The resistance decreasing as tyre temperature increases, due to the more supple behaviour of the rubber and the decrease in hysteresis. Increase in inflation pressure also decreases rolling resistance because the contact area between tyre and road is reduced.

If the effect of vehicle speed upon rolling resistance is assessed independently of the other factors, a steady rising rolling resistance results with increase in speed because of the increased hysteresis loss. However, this increased loss causes an increase in tyre temperature which results in an increase in inflation pressure. These two secondary effects tend to keep the rolling resistance down for the reasons given above. The net result is shown typically in Fig. B3.2 taken from Automotive Design Engineering (2). This shows a near constant rolling resistance for much of the speed range, rising very suddenly in the radial ply case and not so suddenly in the case of the cross ply, at a high speed. The sudden increase in rolling resistance is due to the formation of a standing wave setting up in the tread in the wake of the contact area. If the tyre is allowed to run for long in this condition, the hysteresis loss will result in such an increase in tyre temperature that the tyre may fail.
It is usual, therefore, for a particular type of tyre construction, number of plies, mix of rubber, tread thickness etc. to express the rolling resistance as a constant coefficient, with perhaps a linear function in vehicle velocity. In general, therefore, for a complete vehicle it can be written that

\[
\text{rolling resistance} = W (A_g + B_d \cdot V) \quad \text{lbf} \quad \text{B3.1}
\]

where \( W \) is vehicle weight lbf, \( V \) is vehicle speed mile/h and \( A_g \) and \( B_d \) the rolling resistance coefficients.

In the absence of good experimental data, it is usual to take \( B_d = 0 \) and the value of \( A_g \) from figures contained in "Automotive Design Engineering" (2) and reproduced as Table B3.1.
2. **Deformation of the ground**

Consider the wheel itself to be rigid and initially, the ground to be elastic. A vertical load on the wheel causes deformation of the ground. The ground in front of the moving wheel is heaped up due to the increased compression as shown in Fig. B3.3. The result is a flow of ground material from the front, under and around the wheel to the rear culminating in energy loss or rolling resistance.

The effect of this upon the wheel is to move the resultant of the normal reaction forces \( F \) to some point \( A \). Resolving force \( F \) into its vertical and horizontal components, the vertical component must equal the load on the wheel \( W \) and the horizontal component is the rolling resistance force. Taking moments about the centre of the wheel \( O \) produces the result

\[
\text{rolling resistance} = W \cdot \frac{AB}{OB} = W \cdot A_d \quad \text{----- B3.2}
\]

where again, \( A_d \) is the rolling resistance coefficient, found from experimental data. However, the experimental data is not nearly so profuse as in the case of an elastic wheel on hard ground and there is the added difficulty that there is far greater variation in elastic ground characteristics than in elastic tyre characteristics.

Fortunately, the deformation of the ground is insignificant for a normal vehicle tyre on a hard road and only comes in prominence in certain cross country cases. In such cases,
special experiments must be made to evaluate the rolling resistance coefficient.

Consider now the case of a rigid wheel on ground which is not elastic. The ground deforms but does not return to its original condition after the wheel has passed. A rut is left by the wheel.

If the ground is assumed wholly plastic, the rolling resistance may be expressed by the simple relationship (5).

Rolling resistance = b \cdot p \cdot a \quad B3.3

where \( b \) is the width of the rut

\( p \) is the normal pressure

and \( a \) is the depth of sinkage

There is much experimental evidence to support the use of this expression (5)

Bokker (6) has developed a relationship between the pressure (\( p \)) under a flat plate and the vertical penetration or sinkage (\( a \)) it causes in a given soil

\[ p = \left( \frac{k_c}{b} + k_f \right) \cdot a^n \quad B3.4 \]

where \( n \) = sinkage exponent

\( k_c = \) cohesive sinkage modulus

and \( k_f = \) friction sinkage modulus

all properties of the given soil.

\( b \) denotes width of ground contact area.

This expression has been made to fit the experimental data of a wheel in soil better and the terms non-dimensionalized by
Reece (7). Bekker's and Reece's work is commented upon and reported by Payne et al (8).

If the relationship between p and s is known and can be expressed in a manner similar to the equation of Bekker or Reece, then it is possible to find both the pressure under the wheel (p) and the sinkage (s), because the two terms are also related by

\[ \text{vehicle weight} = p \times \text{projected contact area under wheels}. \]

The projected contact area under the wheels is a function of the sinkage (s).

Hence, using the expression B3.3 the rolling resistance can be found.

This means that the case of a rigid wheel in plastic ground has to be treated differently to that of a rigid wheel in elastic ground or that of an elastic tyre on a hard road. It is no longer sufficient to say that rolling resistance is directly proportional to vehicle weight. The relationship between rolling resistance and vehicle weight is more complex and involves a detailed knowledge of the soil or ground material.
3. **Air flow over the vehicle**

The moving vehicle, in displacing the surrounding air, has a resulting resisting force, termed the aerodynamic drag, imposed upon it.

It is usual to express this drag non-dimensionally using the aerodynamic drag coefficient

\[
C_D = \frac{\text{Drag}}{\frac{1}{2} \rho v^2 \times \text{characteristic area}}
\]

The problem is now to find a suitable characteristic area. In studying this problem further, the conventional procedure of considering air movement relative to the vehicle will be adopted.

The total aerodynamic drag is the sum of three separate types of aerodynamic effects.

1. **Air flow in the boundary layer resulting in loss of momentum of main stream.** The resulting drag is termed the **skin friction drag**.

2. **Air flow in the trailing vortex resulting in the induced drag.**

3. **Integration of (normal pressure \(x\) area) around the vehicle results in a net force opposing direction of motion.** This is termed the **normal pressure drag**.

Expressing the skin friction drag non-dimensionally results in

\[
\text{skin friction drag} = C_{Df} \times \frac{1}{2} \rho v_1^2 \times \text{surface area}
\]
where \( C_{Df} \) is the appropriate friction drag coefficient \( v_1 \) is air velocity outside boundary layer in ft/sec. This may be greater than the vehicle velocity.

The surface area is the appropriate area of the vehicle subjected to skin friction. A distinction should be made here between skin friction caused by laminar air flow compared with that caused by turbulent flow. The friction drag coefficients differ, which means that the two effects should be assessed separately.

The skin friction drag of a vehicle is usually small, however, it can reach significant proportions on a long coach.

The drag component caused by the trailing vortex downwash is small also. It is related to the lift, which is small on most vehicles. The appropriate surface area in the expression for the trailing vortex drag coefficient is that causing lift.

The majority of the aerodynamic drag on a vehicle, therefore, is caused by the pressure difference between front and rear facing surfaces. Separation of flow results in a wake behind the vehicle which in turn results in less pressure on the rear facing surfaces than on the front facing surfaces.

Accordingly, it is usual to express the aerodynamic drag of a vehicle as

\[
\text{aerodynamic drag} = C_D \times \frac{1}{2} \rho v^2 \times A
\]

where \( v \) = vehicle speed ft/sec

\( A \) = Projected frontal Area ft\(^2\)
The aerodynamic drag coefficient $C_D$ may be obtained by experiment. Strictly, it includes the surface friction drag and the induced drag coefficients, duly factored from their respective surface areas to the projected frontal area. This latter point should be remembered in any theoretical attempt to establish the total $C_D$ of a vehicle or in the comparison of drag coefficients.

The projected frontal area of a vehicle is that area enclosed by projection of the front elevation of the vehicle on a drawing or the area enclosed by a photograph of the front of the vehicle taken from a great distance from the vehicle. The latter is best achieved using a telephoto lens on the camera. Fig. B3.4 depicts the frontal view of vehicle A taken from a distance of 250 feet.

The aerodynamic drag coefficient can be measured sufficiently accurately to necessitate the accurate determination of the projected frontal area. It used to be considered sufficient to estimate the projected frontal area using the expression (4),

$$A = 0.9 \times \text{vehicle height} \times \text{track width} \quad \text{B3.8}$$

It is doubtful however, if such an expression can now be considered a substitute for actual measurement except in the most approximate of calculations.
Concluding Remarks

The total drag of a vehicle may be expressed by

\[ F_d = W(ad + Bd \cdot V) + C_D \times \frac{1}{2} \rho (V \times \frac{22}{15})^2 \times A \text{lbf} \quad \text{--- B3.9} \]

where \( V \) is road speed of vehicle mile/h, except in the case of a vehicle progressing through plastic ground leaving a rut behind it. Such a case requires a detailed knowledge of the soil characteristics and has to be treated in the manner outlined above.

For the usual case of a vehicle having pneumatic tyres running on hard ground, the constants in the drag equation are well understood and, for a particular case, can be evaluated.

The drag of a rigid wheeled vehicle running on elastic ground may be expressed by the drag equation given above, but the rolling resistance constants \( ad \) and \( Bd \) are not nearly so well understood as in the case of the pneumatic tyred vehicle.

One final point on the consideration of drag data concerns wind velocity and its effect. If the wind velocity is low and at a small angle only to the straight ahead direction it is sufficient to say that total vehicle drag of a vehicle is

\[ F_d = W(ad + Bd \cdot V) + C_D \times \frac{1}{2} \rho \left( (V + V_{w_o}) \times \frac{22}{15} \right)^2 \times A \text{lbf} \quad \text{--- B3.10} \]

where \( V_{w_o} \) mile/h is the component of the wind velocity in the "head-on" direction. If however, the wind velocity is high and/or at a large angle relative to the straight ahead direction
of the vehicle then special consideration must be given to the addition yaw drag. Typical yaw drag coefficients and indeed typical drag coefficients of motor vehicles are contained in a confidential Motor Industry Research Association report (9).

Part D of this thesis discusses the measurement of road vehicle drag.
SECTION B4

Transmission Efficiency ($T$)

The term "transmission efficiency" is used to describe the mechanical efficiency of the entire transmission system of a vehicle. That is of the gearbox, complete with bearings and gears, also of the drive axle with its wheel bearings and the gears and bearings of the differential unit.

A literature survey has produced no authoritative data on transmission efficiency values. Each manufacturer uses has own figures, usually evolved some time ago and rarely re-appraised. His justification for their use is that they appear sensible and that they seem to produce the right answers.

Certainly it is difficult to measure transmission efficiency because one is looking for a small torque difference between absolute torque values that are high. A back-to-back gearbox rig has been built in the Department of Transport Technology, Loughborough University of Technology and at present work is being done perfecting the instrumentation of the rig. This work will take some time, so that conclusive results will not be available for these vehicle performance calculations.

For a particular design of gearbox or drive axle, the efficiency will be a function of (input or output torque, input or output speed and ratio). This function will depend upon whether the gearbox is of direct or indirect type and whether the drive axle contains a hypoid bevel or a worm and wheel.
The transmission efficiency should increase as input torque increases because gear and bearing drag become less significant. Also, the transmission efficiency might be expected to decrease as input speed increases and as gear ratio increases.

When dealing with vehicle performance calculations, it is usual to consider the full throttle engine torque. Therefore, the torque level is high in relation to the maximum allowable torque envisaged by the transmission designer. It would be reasonable to say, therefore, that the effect of torque variation on transmission efficiency is small for the full throttle performance case. Accordingly, this effect is neglected.

Now one motor car manufacturer assumes that his gearbox efficiencies are not only independent of torque level, but of speed also. The top gear efficiency of a 4-speed gearbox is taken as 97.5% and the other three ratios as 95.0%. Drive axle efficiency is taken as 93% throughout.

Another manufacturer, in his vehicle performance calculations assumes the transmission efficiency figures contained in Table B4.1. The figures for the gear ratios other than top are 94% approximately of the top gear ratio efficiency. These figures include the drive axle efficiency.

Yet another motor car manufacturer works to the data contained in Table B4.2. These figures are said to be the total transmission efficiencies, that is inclusive of drive axle efficiency, and
to account also for the fact that the engine torque data is obtained from a test bed having an exhaust system different to that fitted to the vehicle. The data equivalent to a three speed gearbox is contained in Table B4.3. Both sets of data have been used for many years. A study of Tables B4.2 and B4.3 shows that the efficiency of gear 2 for the three speed gearbox is the mean of those of gears 2 and 3 for the four speed gearbox. This rather suggests that intuition was used to compile the data, and not much experimental evidence. However, the figures seem reasonable.

It was decided therefore, to generalise the figures contained in Tables B4.2 and B4.3 for use in vehicle performance calculations. The first task therefore, was to relate drive axle efficiency to vehicle speed, rather than to engine speed. Relating the drive axle efficiency to engine speed must mean that it is expressed as a function of gearbox ratio. This is not a realistic situation. The gearbox has nothing to do with drive axle efficiency. Relating the drive axle efficiency to vehicle speed avoids this. Now a note with Tables B4.2 and B4.3 states that 97% of the transmission losses are attributable to the drive axle, and that the figures are suitable for use in a small car (1000 cm$^3$ to 1700 cm$^3$ capacity) performance calculations.
Hence,

\[ 0.97 \left( 100 - \gamma_T \right) = 100 - \gamma_{DA} \]  \quad --- B4.1

giving \[ \gamma_{DA} = 3 + 0.97 \gamma_T \]  \quad --- B4.2

and \[ \gamma_G = \gamma_T / \gamma_{DA} \]  \quad --- B4.3

where \( \gamma_G \) denotes efficiency of the gearbox

\( \gamma_{DA} \) denotes efficiency of the drive axle

and \( \gamma_T = \gamma_G \times \gamma_{DA} \) is the transmission efficiency

Assuming a vehicle having an overall gear ratio of

0.016 mile/h per engine rev/min and possessing a four speed
gearbox having ratios

1st = 3.6 : 1  
2nd = 2.4 : 1  
3rd = 1.4 : 1  
Top = 1.0 : 1

results in the curves shown in Fig. B4.1. It should be noted

that the gearbox efficiency is now related to vehicle speed also
rather than engine speed. In view of the doubtful origin of this
data and because the gearbox efficiency is small in comparison
with that of the drive axle, it is not a serious error to suppose
that the gearbox efficiency is in some way dependent on the drive
axle ratio since drive axle ratios do not vary very much anyway.

However, by expressing the gearbox efficiency also in terms of
vehicle speed does enable one comprehensive expression for trans-
misison efficiency to be formed.
The next step was to evolve an expression to give the transmission efficiency at any road speed in any gear. It is more satisfactory to express the information contained in Fig. B4.1 as a mathematical expression when programming a digital computer to do the work. Information in graphical form cannot easily be fed to a computer for the reasons given in Section B2.

Now a study of Fig. B4.1 shows the gearbox efficiencies in the various gears to be approximately straight lines, the level of efficiency dropping with the lower gears and also falling off more steeply with speed with the lower gears. Fitting linear relationships to the four gearbox efficiencies results in

\[
\begin{align*}
\gamma_g(\text{top}) &= 99.7576 - 0.008792. V \% \quad \text{B4.4} \\
\gamma_g(3rd) &= 98.9659 - 0.020678. V \% \quad \text{B4.5} \\
\gamma_g(2nd) &= 98.16812 - 0.040921. V \% \quad \text{B4.6} \\
\gamma_g(\text{bottom}) &= 97.50447 - 0.077314. V \% \quad \text{B4.7}
\end{align*}
\]

where \( V \) denotes vehicle speed mile/h.

Similarly, fitting a quadratic to the drive axle efficiency resulted in

\[
\gamma_{DA} = 96.0 - 0.0316. V - 0.00058. V^2 \% \quad \text{B4.8}
\]

A study of the gearbox efficiency expressions for the various gears showed that the constant term decreases in an arithmetic progressing approximately. The amount deducted each time being roughly 0.7. The vehicle speed term however, was not so easily
dealt with. Various forms of fit were tried. The closest being a power law which put the vehicle speed coefficient equal to

\[ v = 0.00879 \times 2.08^{(NG - 1)} \]  

B4.9

where NG is the number of gear ratios and

1 is a particular gear ratio number
(i.e. 1 for 1st, 2 for 2nd etc.)

The resulting expression for transmission efficiency therefore is

\[ \eta = (96.0 - 0.0316 \times V - 0.00058 \times V^2) \]

\[ \times (99.758 \times (1 - 0.007 \times (NG - 1))) \]

\[ - 0.00879 \times V \times 2.08^{(NG - 1)} \]  

B4.10

V being vehicle speed mile/h.

Fig. B4.2 shows the overall transmission efficiencies with the various gear ratio numbers engaged calculated from the information contained in Fig. B4.1 as full lines. The dotted lines are the transmission efficiencies resulting from equation B4.10. The agreement is far closer than the origin of the data warrants.
SECTION B5

Equivalent Mass of Vehicle

The resultant propulsive force on the vehicle, after subtraction of the drag force, accelerates the total mass of the vehicle, the inertia of the road wheels and the inertia of the engine and transmission. It is convenient therefore, in performance calculations of a vehicle with a fixed ratio gearbox, to embrace the three inertias into one "equivalent mass of vehicle" expression.

Appendix B2 shows that the equivalent mass of a vehicle is expressed as

\[ M_E = M_V + \frac{I_W}{(r_T)^2} + I_e \times \frac{?}{r_T} \times \left( \frac{\text{DAR} \times \text{GR}}{r_T} \right)^2 \text{slug} \quad \text{--- B5.1} \]

where

- \( M_V \) = mass of vehicle, slug
- \( I_W \) = total inertia of all road wheels, slug ft\(^2\)
- \( I_e \) = engine inertia, slug ft\(^2\)

Note that the engine inertia \( I_e \) should be multiplied by the appropriate value of the transmission efficiency \( ? \) in addition to being factored by the square of (the drive-axle ratio times gear ratio divided by rolling radius of drive wheels) quotient. This point is rarely appreciated, and yet it can make a significant difference to the performance of a vehicle in the lower gears.

A second point worthy of note concerns tyre growth with vehicle speed. The often used "Dunlop" formula for rolling radius as a function of speed is

\[ r_T = (r_T)_{30} \times (1 + K \times (v^2 - 900)) \quad \text{--- B5.2} \]
where \((r_r)_{30}\) is the rolling radius at 30 mile/h
and \(V\) is vehicle speed mile/h.

Factor \(K\) is zero for radial ply tyres, \(1/210000\) for ordinary
cross ply tyres and \(1/300000\) for high speed cross ply tyres
(RS5 tyres).

Now if the rolling radius alters, the overall gear ratio
will alter also. Hence the engine inertia is factored by the
square of the reciprocal of rolling radius as a function of speed.
This does not apply in the case of the road wheels however, because
tyre growth alters the wheel inertia also. The extent of this
alteration to wheel inertia is difficult to estimate because a
wheel in contact with the ground is not circular and will grow
less at the point of contact than elsewhere. It might be argued,
therefore, that the majority of the tyre will grow more than the
figure indicated by the Dunlop formula. On the other hand, the
radius of gyration of the wheel will be less than the rolling
radius. So that a fair assumption is that the quotient \(I_p/(r_r)^2\)
remains constant, irrespective of vehicle speed.

At a particular vehicle speed, therefore, the "tractive force"
is obtained by multiplying the steady state engine torque by the
transmission efficiency and the overall gear ratio

\[ F_T = \frac{T \times \eta_T \times \frac{DAR \times GE}{r_r}} \]

--- B5.3

The "propulsive force" is given by subtracting the drag
force from the tractive force.
Finally, vehicle acceleration is given by a formulation of Newton's second law

\[ F_p = F_T - F_d \quad \text{--- B5.4} \]

\[ F_p = M_E \times f \quad \text{--- B5.5} \]
SECTION B6

Gradient

For the purpose of this vehicle performance work, gradient is defined as

\[ i = \sin \Theta \]  \quad \text{B6.1}

where angle of gradient \( \Theta \) is defined in Fig B6.1

It is more usual in the automotive industry to express a gradient as \( l \) in \( N \), where \( N \) is the reciprocal of the definition of (i) given above. However, use of this notation would mean that the gradient of a level road would be \( l \) in infinity, and a digital computer cannot deal with such numbers as infinity.

Using the definition chosen gives a gradient of zero for a level road and a gradient of unity for a perpendicular wall.

The component of the vehicle weight acting down the slope is

\[ W \sin \Theta = W \cdot i \]  \quad \text{B6.2}

hence the total resistance to motion of a vehicle on a gradient is given by an extension of the drag expression

\[ F_d = W(A_d + i + B_d V) + C_D \frac{x}{2} \sqrt{(V + V_{in}) x \frac{22}{15}} \times A \text{ lb} \]  \quad \text{B6.3}
SECTION B7

Vehicle Take-off

The precise mechanism of vehicle take-off is not fully understood; particularly if it is the intention of the driver to attain a high speed in the shortest possible time. It has been found unrealistic simply to say that the clutch becomes fully engaged at zero vehicle speed, because this must mean that engine speed is zero also at that instant. In practice this cannot be so. For the purpose of vehicle performance calculations, it is possible to produce a synthetic engine torque characteristic which extends down to zero engine speed, but the resulting polynomial curve fit is necessarily constrained to suit this condition resulting in a deterioration in the fit along the remainder of the torque curve. This technique has been tried and found to require some trial and error manipulation of the extrapolation of the torque curve in order to obtain a good fit. Also the resulting take-off times in subsequent vehicle performance calculations are far too slow. Accordingly, the simple theory of instantaneous clutch engagement at zero speed was abandoned and a more realistic appraisal of the situation made.

Just before take-off the vehicle is stationary, the engine is rotating at a high speed and the driver ready to start engaging the clutch. A few seconds later the vehicle is travelling at speed with the clutch fully home. During those few seconds the
engine has decelerated to match the vehicle speed whilst, at the same time, the vehicle has accelerated. It follows, therefore, that the clutch must slip considerably during take-off, and that the decelerating engine inertia must add to the torque supplied by the engine.

The question is "what part does engine inertia play?" and indeed "what part does the driver play?" "Do different drivers produce different take-off characteristics?" This must be so to some extent. But if the case of a professional vehicle test driver is considered rather than that of an amateur driver, the problem may be simplified by the assumption that a professional will take-off in the shortest time possible. It follows, therefore, that all test drivers will return identical take-off times in the same car and that, provided test data obtained by a test driver is used, the results will be reproducible and free from driver influence.

How the clutch of a motor vehicle is designed to transmit 125% approximately of maximum steady state, engine torque. The additional 25% is to allow for additional engine torque, clutch wear, some contamination of the plates etc. Much more than 25% would produce little slip and hence a harsh take-off. The clutch, therefore, is capable of transmitting little more than maximum engine torque.
The usual take-off procedure, when obtaining the vehicle time to speed characteristic is to press the accelerator pedal right down before the time keepers' signal to start. On receipt of the signal, the left foot is taken off the clutch pedal and the vehicle moves forward. The right foot is kept right down on the accelerator pedal throughout the test.

At the very high engine speed at the start of take-off, the steady state torque delivered by the engine will be low because the engine speed is above maximum brake horsepower speed, and near or at valve bounce speed. However, the potential to be gained from the decelerating engine is high.

If the engine speed were set at maximum torque speed, there would be little or no torque from the decelerating engine but the steady state torque would be high.

Bearing in mind the above remarks, particularly those relating to the maximum capacity of the clutch when fully home, the realisation that the torque transmission process through the clutch, even if it were fully understood, can never be perfect; it would seem reasonable to assume that until the clutch ceases to slip the torque transmitted through it is equivalent to the maximum steady state torque of the engine. Above a vehicle speed equivalent to maximum torque speed the torque delivered by the engine is that dictated by the engine torque curve and the clutch does not slip.
It might be thought from the above argument that more than maximum engine torque is transmitted during take-off. In an ideal system, this might be so, but because of the transient nature of the take-off, the engine is unlikely to deliver its full throttle, steady state torque. Also, the pressure on the clutch pedal cannot be realised instantaneously, which means that the full torque transmitting potential of the clutch cannot be wholly realised.

Now, since the time taken for the take-off period is small, any error in the above will be of small consequence also. It is of interest therefore, to see how the theory accords with practice.

Fig. B7.1 is a graph of the calculated vehicle speed against time take-off characteristic of a 2000 cm$^3$ saloon car. The graph depicted in the insert is the measured performance of the car extracted from Auto car (10). As can be seen, the first gear performances agree very well.

It must be admitted that the agreement is above average, but the take-off period is so short that some error can be tolerated. As vehicle performance calculations become more exact it will be necessary to delve into the question of take-off much more thoroughly. At the moment, however, the above theory is adequate.

Note:

It is shown in Part G of this thesis that lack of knowledge on transmission efficiency renders the small inaccuracy of the above, simplified, model insignificant compared to that which may be caused by using an inadequate formulation of transmission efficiency.

A more accurate model of clutch take-up is being prepared, outside the work of this thesis, in order that a careful study may be made of the optimum rate of clutch engagement and vehicle take-off on a gradient.
Wheel Spin

Wheel spin often occurs when a high performance car accelerates from rest. The term is used to describe that characteristic lack of adhesion which causes slippage between driving tyres and the road. If too much torque is supplied to the drive wheels, slippage or wheel spin will occur. Alternatively, wheel spin can occur on a slippery road, that is when the coefficient of friction between tyre and road is low. In this case, the torque supplied to the drive wheels may be low also.

If wheel spin occurs when a vehicle is undergoing a full throttle performance test up through the gears, time is lost. From the performance calculation point of view, the questions are "when will wheel spin occur" and "how much time is lost if wheel spin occurs?"

The first question may be answered in simple terms by saying that wheel spin will occur when the drive force between tyre and road exceeds the product (coefficient of friction x normal reaction between tyre and road).

The estimation of the drive force between tyre and road is best tackled by considering the case of a conventional motor car having four wheels, two of which are drive wheels. The torque delivered by the engine has subtracted from it the torque necessary to accelerate the engine inertia. It is then multiplied by the
gear ratio and the drive axle ratio and factored by the transmission efficiency. This then gives the torque in the drive axle shafts which, before it is translated into the drive force, has subtracted from it the torque necessary to accelerate the two drive wheels only and the tyre losses of those two drive wheels only.

The second question is not so easy to answer because, once there is relative motion at the point of contact between tyre and road, the so-called coefficient of friction between tyre and road diminishes. Also, the engine speed and the rate of engine acceleration are higher by unknown factors, hence the actual torque produced by the engine during wheel spin is not known. There is also the driver element. The driver may decide to take his foot off the accelerator and to start again at a lower torque level, or he may decide to keep his foot down until wheel spin ceases using the driving force developed by the spinning wheels to accelerate his vehicle.

Again, for the purpose of these performance calculations, it will be assumed that a professional driver is at the helm and that the driving force at the drive wheels is the maximum possible. That is that the torque supplied to the drive wheels is regulated by the driver to maintain them just on the point of slipping throughout the wheel spin period. This policy eliminates the driver element, so simplifying the calculations and rendering possible comparisons of sets of performance calculations. By
saying that the wheels are on the point of slipping means that the engine speed is known and that the static coefficient of friction between tyres and road can be used. If follows therefore, that the calculated time up through the gears when wheel spin occurs will be the best possible.

Fig. B8.1 shows the general case of a vehicle upon a gradient. The product (coefficient of friction x normal reaction between road and drive wheels), termed "The limiting tractive force", is given by

\[ F_L = \mu \frac{W}{(a+b)} \left( a \cos \Theta + h \left( \sin \Theta + \frac{g}{f} \right) \right) \text{ lb f} \]  

for a rear wheel drive vehicle and by

\[ F_L = \mu \frac{W}{(a+b)} \left( b \cos \Theta - h \left( \sin \Theta + \frac{g}{f} \right) \right) \text{ lb f} \]  

for a front wheel drive vehicle.

The maximum vehicle acceleration possible therefore, occurs when the drive force at the drive wheels equal this limiting tractive force. Appendix B3 shows that, for a rear wheel drive vehicle, the maximum possible acceleration of the vehicle is

\[ a_{\text{max}} = \frac{\mu \frac{W}{(a+b)} \left( a \cos \Theta + h \sin \Theta \right) - F_d + \frac{W}{a+b} (A_d + B_d \cdot V)}{g + \frac{I_m}{2 \cdot (r_r)^2} - \frac{\mu W \cdot h}{g(a+b)}} \]  

and for a front wheel drive vehicle, the maximum possible vehicle acceleration is
\[ f_{\text{max}} = \frac{W (b \cos \theta - a \sin \theta)}{(a+b)} - F_d + \frac{W}{2} (A_d + B_d V) \]

\[ \frac{W + l_v}{g} + \frac{W h}{2 (r_p)^2} + \frac{\mu W a h}{g(a+b)} \quad \text{(B3.4)} \]

Fig. B3.2 is a carpet plot of equation B3.1 with the maximum acceleration expression superimposed for a typical rear wheel drive vehicle. Particulars of the vehicle are that:

- vehicle weight: \( W = 2128 \text{ lbf} \)
- position of centre of gravity:
  - \( a = 3.7 \text{ ft} \)
  - \( b = 3.85 \text{ ft} \)
  - \( h = 1.66 \text{ ft} \)
- rolling radius of wheels: \( r_p = 0.94 \text{ ft} \)
- total wheel inertia: \( I_v = 1.96 \text{ slug ft}^2 \)
- rolling resistance drag coefficients:
  - \( A_d = 0.018 \)
  - \( B_d = 0 \)

Fig. B3.2 is plotted for a gradient \( i = 0 \) and shows the coefficient of friction between drive wheels and road varying from 0 to 1 in steps of 0.1.

The first point to note is that the limiting tractive force increases with increase in acceleration due to the weight transfer onto the rear wheels. Fig. B3.2 shows this effect decreasing as the coefficient of friction is decreased.

When the coefficient of friction between drive wheels and road \( \mu = 1.0 \), the maximum possible acceleration of the vehicle is \( 19.44 \text{ ft/sec}^2 \). The normal engine fitted to this vehicle cannot
accelerate the vehicle at anything like this value, even in first gear. In fact it produces wheel spin in first gear when the coefficient of friction between drive wheels and road drops to about 0.4 to 0.5.

The locus of maximum vehicle acceleration is near linear, bowing slightly in the direction to delay the onset of wheel spin as the coefficient of friction is decreased.

Fig. B8.3 is the corresponding plot for a front wheel drive vehicle, using equations B8.2 and B8.4. The vehicle particulars are identical to those in Fig. B8.2 and the gradient is again zero.

The first point to note in comparison is the effect of weight transfer off the front wheels as vehicle acceleration is increased. This effect diminishes as the coefficient of friction decreases.

The maximum possible vehicle acceleration for the front wheel drive vehicle is very much lower than that of the rear wheel drive vehicle, 13.04 ft/sec² compared with 19.44 ft/sec². Also the locus of maximum vehicle acceleration is bowed slightly in the direction to increase the onset of wheel spin as the coefficient of friction is decreased.

The comparison here may not be strictly fair since the position of the centre of gravity of the front wheel drive vehicle could move forward in arranging the final drive at the front. Nevertheless, a vehicle having its centre of gravity well forward was chosen in order to test the wheel spin theory and its results, favouring as they do the rear wheel drive vehicle, accord with practice.
Figs. B8.4 and B8.5 show the limiting tractive force equations plotted as carpet plots for the rear wheel drive and front wheel drive vehicle respectively. Again, the data is as before but this time the coefficient of friction between drive wheels and road is maintained at $\mu = 1.0$ and the gradient varied from $i = 0$ to the very steep gradient of $i = \sin^{-1}20^\circ$. Both figures show that the weight transfer due to acceleration to be independent of gradient and that this effect is detrimental to wheel spin in the case of the front wheel drive vehicle.

The locus of maximum vehicle acceleration rapidly approaches zero acceleration as the gradient is increased for the front wheel drive vehicle whereas, for the rear wheel drive vehicle, it would still be possible to attain an acceleration of 7.46 ft/sec$^2$ on a slope of $20^\circ$ if the engine was large enough.

The calculated values from which Figs. B8.2, B8.3, B8.4 and B8.5 are plotted are given in Tables 1 to 6 of Appendix B3.

It is difficult to be too dogmatic that the theory developed above for wheel spin is adequate without a great deal of experimental data. Certainly it produces answers which seem to agree with practice and for this reason is used in the ensuing vehicle performance program.

The computer program tests for wheel spin at every appropriate stage by evaluating the drive force between tyres and road and comparing it with the limiting tractive force. If it is greater, the acceleration is set at 99% of the maximum possible and the
corresponding drive force calculated. The factor of 99% can be used as a "time lost factor", but it is really there to ensure stability in the calculations. That is that, on the second time through, with the new drive force, this drive force cannot exceed the limiting tractive force. This could happen if the factor were 100% because the binary translation of a decimal number is not exact and because the computer "rounds off" numbers anyway. The time lost factor, therefore, may prove useful in making the theory fit experimental data, if and when experimental data accumulates.

Note:

While the technique employed for allowing for the effects of "wheel-spin" is adequate in the context of this work, it is appreciated that the reaction between tyre and road may be dynamic in character.

That is that the transmission of the forces through the elastic vehicle and tyre may result in a reaction between tyre and ground that is oscillatory in character. Such a reaction may have a significant influence on the gradeability of a vehicle.
Limitations imposed by a gradient

Use of a computer program to assess vehicle performance enables the whole range of each design parameter to be investigated. Convention can be "thrown to the winds" in designing a vehicle for a particular job, since the design parameters can be optimised to do that job.

Occasionally, however, one is brought down to earth by some unavoidable realism. If performance calculations are being conducted for a proposed vehicle on a gradient, then the vehicle should be capable of holding on the handbrake on that gradient. The assumption here is that the handbrake mechanism itself is perfect and that the wheels it operates upon, taken to be the rear wheels, are locked. There are a number of instances when a driver may wish to hold his vehicle on the handbrake when on a gradient. The vehicle should not slip downhill therefore, when the rear wheels are locked. Alternatively, the possibility of slippage should be known at the design stage. It may be that some other handbrake system is possible, such as one operating on all wheels, rather than just the rear.

Fig. B9.1 depicts the worst case of a vehicle facing downhill parked upon a gradient. If the vehicle is not to slip downhill, the product (coefficient of friction x normal reaction at the contact area between rear wheels and ground) must be greater than
the downhill component of the vehicle weight. That is that

\[ \frac{\mu}{(a+b)} (a \cos \theta - h \sin \theta) > 1 \quad \text{--- B9.1} \]

must be true.

Another troublesome limitation is that the vehicle may overturn backwards when climbing the hill due to the high torque reaction from the drive axle. In the case of an agricultural tractor, the condition may be realised on the level.

For reasons of safety, if for no other reason, this possibility should be known at the design stage.

Fig B9.2 shows that a rear wheel drive vehicle will pitch up if the torque reaction is greater than the product \((w \times c)\). The criterion to adopt therefore, is to test the case of the lowest gear ratio with the engine delivering maximum torque. If the answer to this criterion is satisfactory, then the vehicle must be free from the possibility of overturning on the slope considered.

Therefore, the test is whether or not

\[ T_{\text{max}} \times \text{DR} \times \text{GR}(1) > W \left( b \cos \theta - (h - r_1) \sin \theta \right) \quad \text{--- B9.2} \]
SECTION 810

The gear ratios

Part of any procedure for calculating vehicle performance should include a rational method of designing the gear ratios of a proposed vehicle. Either every gear ratio should be designed to fulfil a particular function or all the gear ratios together optimised to produce the best overall effect, or some philosophy in between should be followed.

The procedure outlined below is to fix bottom and top gear ratios to carry out particular jobs and to optimise the intermediate gear ratios against a performance criterion.

Top gear

If a "straight-through" gearbox is used, that is a gearbox so designed that the input shaft is connected direct to the output shaft in the often used top gear case in order to avoid the transference of torque through gears, the design of top gear ratio is really a means of fixing the drive axle ratio and/or the rolling radius of the drive wheels.

The product of the gearing effects of the rolling radius, the drive axle ratio and the gear box in top gear is termed the "overall top gear ratio". This overall top gear ratio is fixed by considerations of fuel economy, top gear acceleration, engine life and engine noise. One number, termed "the degree of under-gearing", can be used to establish a balance between these
conflicting effects. This term is defined below.

Fig. B10.1 is a plot of engine full throttle brake horse power at the flywheel against engine speed. Superimposed onto this plot is the power required at the flywheel by the vehicle in top gear on zero gradient with no wind velocity.

The intersection between the two curves gives the engine speed at maximum vehicle speed. Varying the overall top gear ratio alters the position of the intersection. The greatest value possible of maximum vehicle speed is obtained if the intersection occurs at maximum brake horsepower. If the intersection is to the right of maximum brake horsepower, as depicted in Fig. B10.1 the vehicle is said to be "undergeared". If the intersection is to the left, the vehicle is said to be "overgeared".

By undergearing a vehicle, the power available for top gear acceleration is increased. This power is represented by the difference between the two curves. By overgearing a vehicle, the top gear acceleration is reduced markedly, but engine speed is lower and hence there is less engine noise and wear. Also, as is shown in Part E, top gear fuel economy may be better.

It is simply a question of choosing a reasonable position of the intersection for the duty envisaged of the vehicle.

Denoting maximum brake horsepower engine speed by \( N_p \) rev/min, maximum vehicle speed by \( V_{\text{max}} \) and defining the overall top gear as

\[
\text{OGR} = \frac{\pi}{30} \times \frac{\frac{F_{\text{p}}}{\text{GR} \times \text{BAR}}}{22} \times 15 \text{ mile/h per rev/min of engine speed}
\]
the degree of undergearing may be expressed as

\[
\text{DUG} = \frac{V_{\text{max}}}{N_p \times \text{OCR}}
\]

This quotient gives a non-dimensional number of unity for neutral gearing, that is for the greatest possible maximum vehicle speed case, a number greater than unity for the undergeared case and a number less than unity for the overgeared case.

The vehicle designer specifies the degree of undergearing to suit the vehicle application, the rolling radius of the drive wheels and either the drive axle ratio or that the gearbox is of the straight-through type. At maximum vehicle speed, the horsepower required by the vehicle at the engine flywheel is given by

\[
drag \text{ h.p.} = \left\{W(A_d + B_d \cdot V_{\text{max}}) + \frac{1}{2} \rho \frac{V_{\text{max}}^2}{(22)^2} \right\} 
\times \frac{V_{\text{max}} \times 22}{550 \times 15} \times \frac{1}{T}
\]

and the horsepower supplied by the engine is given by

\[
\text{engine h.p.} = \frac{T(N_p \times \text{DU}) \times 2 \pi \times (N_p \times \text{DU})}{60 \times 550}
\]

Equating the drag h.p to the engine h.p and solving for \(V_{\text{max}}\) yields the maximum vehicle speed on the level with no wind velocity. Substitution of \(V_{\text{max}}\) into the expression defining the degree of undergearing gives the overall top gear ratio. If the gearbox top gear ratio is 1 : 1, the drive axle ratio is determined from
If the gearbox is not of straight-through design and the drive axle ratio is specified, then gearbox top gear ratio is given by

\[
\text{top gear ratio} = \frac{r_f \cdot V_{\text{max}}}{30 \cdot OGR \cdot DAB \cdot \frac{15}{22}}
\]

--- B10.6

Note that the rolling radius \((r_f)\) should be that appropriate to maximum vehicle speed.

Equating expressions B10.3 and B10.4 produces a cubic expression in \(V_{\text{max}}\). Such an expression can be solved analytically, but it was decided to use a simple iterative technique in order to evaluate \(V_{\text{max}}\). The reason for using an iterative technique rather than say, Cardan's solution of a cubic, is not because it is quicker, but rather that an analytical solution would produce three roots. The vehicle performance computer program would then have to be so designed to pick out the appropriate root. Such a procedure may not always be a simple matter. Whether the approach is basically analytical or iterative, some iteration is necessary because transmission efficiency is a function of vehicle speed.
The philosophy for fixing bottom gear is not nearly so straightforward as that for top gear ratio. Several theories exist. The most common theory is that the vehicle should be capable of climbing a specified gradient, say 1 in 4 or 1 in 3. Some say that the vehicle should be capable of moving off from rest in its laden state on a specified gradient, usually 1 in 4.

The latter philosophy is probably influenced by the performance tests carried out by the semi-technical press on behalf of the public. It is usual to subject the vehicle to a take-off on a 1 in 4 gradient. It was seen earlier in Section B7 that to cater for the take-off condition at the design stage is difficult, and this applies particularly to take-off on a gradient. By specifying that the vehicle must be capable of a take-off is usually taken to mean that the vehicle must be capable of climbing the gradient with some torque in reserve. But how much in reserve? Suppose there is too much in reserve such that wheel-spin results? Suppose that the vehicle under design is a special purpose vehicle which will never be expected to climb a 1 in 4 gradient or alternatively, suppose it is a special purpose vehicle which is expected to climb gradients much steeper than 1 in 3?

The foregoing questions serve to show that a general philosophy is required. The one suggested is that "the vehicle shall be capable of climbing the maximum gradient possible, without
wheel-spin. There is a limit to the gradient a particular vehicle can climb irrespective of engine size and gear ratio. This limit is reached when the downhill component of the vehicle weight equals the limiting tractive force defined in Section B3. It is physically impossible for the particular vehicle to scale a gradient greater than this by relying on adhesion between tyre and road. Bottom gear therefore, should be so arranged to provide sufficient force at the drive wheels to meet this condition.

Reference to Section B3. and Fig. B3.1 shows that the limiting tractive force for a rear wheel drive vehicle is

$$F_t = \frac{\mu W}{(a+b)} (a \cos \theta + h \sin \theta) \text{ lbf} \quad B10.7$$

and for a front wheel drive vehicle

$$F_t = \frac{\mu W}{(a+b)} (b \cos \theta - h \sin \theta) \text{ lbf} \quad B10.8$$

It is assumed that vehicle acceleration is zero. Putting the coefficient of friction between ground and tyres $\mu = 1$, equating the limiting tractive force to the downhill component of the vehicle weight

$$F_t = W \sin \theta \quad B10.9$$

and solving for $\theta$ gives

$$\theta = \tan^{-1} \left( \frac{a}{a+b-h} \right) \quad \text{for a rear wheel drive vehicle} \quad B10.10$$

and

$$\theta = \tan^{-1} \left( \frac{b}{a+b+h} \right) \quad \text{for a front wheel drive vehicle} \quad B10.11$$
So that the maximum gradient that a particular vehicle is capable of climbing is a function of the position of the centre of gravity of the vehicle only. By putting $\mu = 1$, the function is simple and affords the steepest gradient possible, no provision being made for the possibility of positive "keying" between tyre and ground.

It is perhaps worth noting that, for an approximately 50/50 weight distribution, the rear wheel vehicle is capable of climbing a steeper gradient than a front wheel drive vehicle.

If the above exercise is carried out for a four-wheel drive vehicle, the resulting value for the angle of the gradient is that

$$\theta = 45^\circ$$

which is a gradient of 1 in 1.414 or $i = 0.7071$.

Bottom gear ratio therefore, is fixed by the expression

$$GR_{(1st)} = W \times \frac{\sin \theta \times \frac{F_r}{F_{max} \times \tau_{(1st)} \times DAR}}{H0.12}$$

That is by translating the maximum engine torque into a force at the drive wheels equal to $W \sin \theta$. An appropriate value being used for the first gear transmission efficiency. The vehicle weight ($W$) in the expression need not necessarily be the fully laden value, but it should be more than the kerb weight. It should include the driver's weight and that of a passenger or a small amount of luggage. It must be remembered that a conventional motor car may be very heavily laden particularly during the holiday season, in fact it may be towing a caravan. Such eventualities should be catered for.
So much for the theory. How does it accord with practice? Table E10.1 lists the actual products of (bottom gear ratio x drive axle ratio) and the values calculated according to the above theory for vehicles A and B.

**TABLE E10.1**

Bottom gear ratio x drive axle ratio

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Actual</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle A</td>
<td>18.30</td>
<td>22.76</td>
</tr>
<tr>
<td>Vehicle B</td>
<td>12.83</td>
<td>14.72</td>
</tr>
</tbody>
</table>

In both cases the calculated values give a lower overall bottom gear ratio. It is of interest therefore, to study test reports on these two vehicles conducted by the "Consumer Association" in their "Car Supplement" to the publication "Which?"

The relevant report on Vehicle A (12) on page 4 under the heading "Climbing Hills" states that vehicle A would only just start on a hill of 1 in 4 with the engine near to stalling and carrying less than half the standard test load. Other vehicles tested at the same time were very much better at hill climbing. The Austin A40 and the Morris Minor (improved) started on a 1 in 4 hill with the full test load of 690 lbf with no apparent difficulty.
The relevant report on Vehicle B (13), page 101, again under the heading "Climbing Hills" states that the other vehicles tested in the group climbed the 1 in 4 hill with the full test load. The report continues by stating that vehicle B "had difficulty in starting - the clutch had to be let in very slowly to get the car moving - and did not accelerate well, though it completed six runs. By the end of the test the clutch was slipping badly."

The conclusion to be gained from the Consumers' Association reports is that the bottom gear of Vehicle B should be lowered slightly and that of Vehicle A lowered considerably. In fact lowered to values similar to those suggested by the theory.

Lowering bottom gear ratio would increase manoeuvrability at very low vehicle speeds, an important point with the increasing traffic density. Take-off would be smoother with less clutch slip and therefore, less excitation of transmission torsional vibrations and other body vibrations.

Fig. B10.2 shows the calculated take-off of Vehicle A with the actual bottom gear ratio of 4.118 and again with the ratio fixed by the theory of 5.120. The interesting point to note is that there is little discernable difference in the time it takes to move off from rest. The reason for this is the over-riding influence of the engine inertia. Repeating the calculations with zero engine inertia produces greatly improved take-off times, with the bottom gear ratio fixed by the theory rather better than that
for the actual bottom gear ratio.

Thus lowering bottom gear ratio on these vehicles would not be detrimental to performance in any respect. Before drawing any final conclusions concerning the theory, it would be of advantage to apply the theory to say a vehicle fitted with a high performance engine, to see if the resulting bottom gear ratio accords with that usually associated with a high powered vehicle.

Accordingly, the bottom gear ratio was calculated for Vehicle A fitted with a 1500 cm$^3$ racing engine having the torque curve shown in Fig. B10.3. This engine is of $\frac{1}{2}$ times the capacity of the engine normally fitted to Vehicle A and is tuned for racing purposes, as may be seen by the rather high maximum torque speed of 5000 rev/min. The theory gives a bottom gear ratio of 2.738 in conjunction with a drive axle ratio of 4.099. These are the sort of figures usually associated with high performance vehicles.

It may be stated, therefore, that the theory proposed for fixing bottom gear ratio is such as to cater for all eventualities. The vehicle so designed can climb the maximum gradient possible without the bottom gear so low as to produce wheel spin. Also the theory is rational and not geared to the rather artificial requirement of a gradient of 1 in 4 or 1 in 3. The position of the centre of gravity of the conventional motor car is such that a bottom gear so designed will enable the vehicle to climb a 1 in 3 hill with ease, and there are eight hills listed in the RAC Guide and Handbook having a maximum gradient of 1 in 3 and one having
a maximum gradient of 1 in 2\frac{1}{2} in Great Britain. Presumably there are hills as least as steep elsewhere in the world and a motor car should be able to negotiate such hills to the limit fixed only by the position of its centre of gravity. There is no logical reason to limit any vehicle to a 1 in 4 or a 1 in 3 hill if it is capable of steeper gradients. There is no loss in performance in catering for the steepest gradient possible. This latter point is investigated more fully in Part C.
The intermediate gear ratios.

The classical method of fixing the intermediate gear ratios, described in Appendix B4, is to arrange all the ratios in geometric progression. This method is frequently used in designing the intermediate gear ratios in the gearboxes of large commercial vehicles. Such gearboxes have a large number of ratios relative to the number of ratios in a motor car gearbox. Also, there may be a two speed axle in operation. The large variation in the all-up-weight of a commercial vehicle necessitates the large number of ratios. These should be well spaced with the two speed axle providing no duplication. By arranging the ratios of the main gearbox in geometric progression, the two speed axle can be so arranged to provide a further set of gear ratios midway between the originals. Thus ensuring that a ratio is not duplicated.

A cursory glance at the gear ratios listed in the brochures will suffice to show that the geometric progression theory does not hold in the design of motor car gearboxes. In fact, the reciprocals of the gear ratios are closer to an arithmetric progression than the gear ratios themselves to a geometric progression. This means that the vehicle speed range for each gear ratio is approximately constant.

The reason for the gear ratios being closer as top gear is approached is due to the rapidly increasing power requirement at the higher vehicle speeds. The aerodynamic drag power requirement increases as the cube of vehicle speed and constitutes the major
proportion of the power requirement of a motor car at the higher vehicle speeds. The power requirement of a large commercial vehicle however, is formed largely by the high rolling resistance. Thus, the rate of increase of the drag power with vehicle speed is not nearly so high with the commercial vehicle and so the higher gear ratios are spaced wider apart.

Since the commercial vehicle is reasonably well catered for, it is now necessary to devise a suitable design procedure for fixing the intermediate gear ratios and the number of intermediate gear ratios in a motor car gearbox.

The ratios must have sufficient "overlap". This means that when a full throttle gear change is made from a low gear to a higher gear, the engine speed before the change being near the maximum allowable; the engine speed after the change should be above a minimum engine speed. The extent that it is above is a measure of the overlap. The importance of overlap manifests itself when the vehicle is driven in hilly terrain and full throttle is used. This overlap requirement is used initially to fix the number of gear ratios.

The values of the intermediate gear ratios have no effect on maximum vehicle speed; nor do they have any large effect on fuel economy since their use is during a transient phase only. However, they do influence the time to speed from a standing start and account should be taken of this influence in fixing these intermediate gear ratios. The real test, therefore, is the ability of a motor
car to leave other similar cars behind when accelerating away from a standing start. How the test of this ability should not extend right up to maximum vehicle speed, because the higher vehicle speeds are the domain of top gear ratio. But the test should extend to something like three-quarters of maximum vehicle speed.

The suggestion is therefore, that the intermediate gear ratios should be fixed by saying that the standing start time up to say, three-quarters of maximum vehicle speed, should be a minimum.

The three-quarters of maximum vehicle speed is purely arbitrary and stems really from the fact that the majority of motor cars have a four speed gear box, the last quarter of the speed range being the province of top gear. It is as well therefore, to test the effect of this upper speed limit on the assessment, and indeed to test the effect of the philosophy generally.

Tables B10.2 and B10.3 give the actual gear ratios of vehicles A and B and the calculated values assuming a number of upper speed limits to the optimisation procedure. For the purpose of comparison the bottom gear ratios, top gear ratios and the drive axle ratios are the actual values.

Considering first vehicle A. Table B10.2 shows that optimising on the time to speed up to a final speed between 50 and 60 mile/h will produce a set of gear ratios similar to the actual ratios of vehicle A. Now the maximum speed of vehicle A is 71 mile/h approximately. Three quarters of this is 54 mile/h approximately.
Optimising again with a final speed of 54 mile/h results in gear ratios 4.118, 2.2293, 1.5276 and 1.0000. These figures are close to the actual. Calculated second gear is slightly higher, third gear slightly lower than actual. Lowering bottom gear ratio to 5.12, the value fixed by the theory contained earlier in Section B:10 repeating the optimisation on the time up to 54 mile/h, produces a second gear ratio almost identical to the actual, 2.386 compared with 2.396. Third gear however, is still lower at 1.514 compared with 1.412.

The corresponding analysis for vehicle B contained in Table B10.3 produces a similar result to that of vehicle A. Optimising on 0-76 mile/h, which is three quarters of the maximum speed of vehicle B results in a second gear ratio slightly higher than actual and a third gear ratio slightly lower than actual.

Now the human brain is very good at optimisation, provided that it has some past experience to work on. A designer, experienced in a particular class of motor car, can simply write down a good set of intermediate gear ratios. These will work very well and perhaps draw minor criticism only from the semi-technical press in their road test reports. The fact that a rationally based theory produces similar answers is interesting. In part C of this thesis, the vehicle performance program is used to compare the various calculated gear ratios with the actual in order that a detailed assessment of the theory might be made.
Tables B10.4 and B10.5 give the gear overlap figures for the optimized ratios based upon the time up to three quarters of maximum vehicle speed for vehicles A and B. Bottom gear is said to extend over a vehicle speed range zero to that corresponding to engine maximum (or valve bounce) speed. Second and subsequent gear ratios extend between vehicle speeds corresponding to engine maximum torque speed and valve bounce speed. These overlap figures are adequate and should not produce problems in service, any more than the actual gear ratios do.

It should perhaps be mentioned here that, in evaluating time to speed figures for this optimisation study, it is assumed that a gear change is made in zero time. This gives a reasonable basis for comparison and approximates closely to the gear change times made by a professional test driver when going up through the gears.

This then is the technique adopted for fixing the intermediate gear ratios. A further method was considered, it being an extension of the method above. It is based upon the argument that the times up to the lower speeds are more important than the times up to the higher speeds. In starting off from rest, any from the traffic lights, it is more important to be first across and to make gains in the initial stages. Car owners judge cars on the results of such tests. Rarely do they subject cars to a full race since traffic conditions and the law of the country forbid it. A car's reputation for good performance can be made on its ability to be consistently first across the traffic lights.
Now this may not be in the best interests of road safety, but this is what happens, and the car designer has to recognise the fact and design accordingly.

To cater for this argument the intermediate gears for Vehicle A were optimised using the criterion that the sum of the times up to

\[
\frac{1}{2} \times 0.9V_{\text{max}}, \quad \frac{1}{3} \times 0.9V_{\text{max}}, \quad \frac{1}{4} \times 0.9V_{\text{max}}, \quad \frac{1}{6} \times 0.9V_{\text{max}}, \quad \frac{1}{12} \times 0.9V_{\text{max}}, \quad 0.9V_{\text{max}}
\]

should be a minimum.

For vehicle A, this means optimising the sum of the times up to

32, 40, 48, 56 and 64 mile/h.

This then should put more emphasis on the acceleration during the lower speeds because the 0 - 32 mile/h time is included in the other times. By starting at 0 - 32 mile/h, rather than a lower speed, avoids putting excessive emphasis on the speed range solely the province of bottom gear. By including a speed range approaching maximum speed should mean that some account is taken of this speed range in the optimisation procedure. In fact the whole speed range applicable to the intermediate gear ratios should be covered, the lower speed range more than the upper speed range.

Using this criterion, the resulting gear ratios for vehicle A are disappointing. They are:

<table>
<thead>
<tr>
<th>Bottom</th>
<th>2nd</th>
<th>3rd</th>
<th>Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.118</td>
<td>2.0822</td>
<td>1.2874</td>
<td>1.000</td>
</tr>
</tbody>
</table>
compared with the actual ratios

<table>
<thead>
<tr>
<th>Bottom</th>
<th>2nd</th>
<th>3rd</th>
<th>Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.118</td>
<td>2.396</td>
<td>1.412</td>
<td>1.000</td>
</tr>
</tbody>
</table>

A study of the ratios given in Table H10.2 optimising on a 0 - 64 mile/h speed range only, shows that some emphasis has indeed been given to the lower speed range. The more detailed optimisation procedure has produced a lower second and a lower third gear ratio. However, the result is not nearly so good as optimising on 0 to \( \frac{1}{4} \) maximum speed.

The reason for the poor results of the more detailed optimisation procedure lies in the fact that the interval of one second in time is far more important at the lower speeds than at the upper speeds. In using the upper speed range at all results in the situation that it is far easier to save time at the higher speeds. Vehicle A takes as long to accelerate between 60 and 68 mile/h and covers a similar distance as between 0 and 60 mile/h. Thus, if the upper speed range is included, excessive emphasis is given to it, even when an attempt is made to emphasise the lower speed range.

Repeating the more detailed optimisation procedure optimising on the sum of the times up to

\[ \frac{1}{8} \times \frac{2v_{\text{max}}}{T^2}, \frac{1}{8} \times \frac{2v_{\text{max}}}{T^3}, \frac{1}{8} \times \frac{2v_{\text{max}}}{T^4}, \frac{1}{8} \times \frac{2v_{\text{max}}}{T^5} \]

in order to eliminate altogether the higher speed range and to maintain the emphasis on the lower speed range produces Vehicle
A gear ratios of 4.118, 2.3562, 1.5534 and 1.000. These figures are very much better, but they are no real improvement upon the figures obtained by simply optimising on the time up to three quarters of maximum speed.

The results of this latter evaluation using the more detailed optimisation technique are given at the end of Appendix B7.

Since optimising upon three quarters of maximum vehicle speed seems to produce reasonable gear ratios, this method was adopted. A listing of the more detailed optimisation procedure is contained in Appendix B7.
Optimisation of intermediate gear ratios

Optimisation

Optimisation is a mathematical procedure for juggling the variables of a problem in order to produce an optimum result. There are many optimisation techniques, so that it is wise to spend some time choosing a technique suitable to the problem in hand.

In optimising the intermediate gear ratios, to produce a best time to speed, the intermediate gear ratios must certainly be listed as variables. If it is intended to specify arbitrarily the time spent in each gear ratio, then these times also are variables and must be juggled along with the gear ratios. This may produce a large number of variables, accordingly a short cut was sought. The short cut used was to make the sub-routine generating the time up to speed fix the appropriate gear number at any particular speed. It does this by evaluating the acceleration in all the specified gear ratios and choosing the one giving the greatest acceleration. If this gear number is different from the one chosen an increment of speed earlier, then a gear change is made and the gear change time added onto the accumulated time to speed. This then eliminates the time spent in each of the gear ratios as variables.

The function formed by the time to speed sub-routine, therefore, is quite complicated, but it should be well behaved near the optimum
point. That is to say that altering a ratio to move the time to speed from the optimum is unlikely to cause a violent change in time to speed. Fig. B11.1 is a plot of time to speed against the one intermediate gear ratio of a vehicle having a three speed gearbox. The calculated times up to 54 mile/h, that is three quarters approximately of the maximum vehicle speed, and details of the vehicle are contained in Table B11.1. Fig. B11.1 shows that, for this vehicle at least the function is very well behaved. There is a gentle curve through the one minimum value with near linear relationships on either side of the optimum. The main point to note is that there is no other minimum value to confuse an optimisation technique. The small scale scatter of the plotted points reflects the numerical method of effecting the integration in the time to speed calculation.
The Method

The function may be well behaved, but it is complicated, and is not amenable to a direct analytical treatment. It was decided, therefore, to devise a special optimisation technique for this work. This task seemed easy at first, but a number of complications soon appeared. Nevertheless, the task was finished and it works very well indeed. Basically, it deals with each intermediate gear ratio in turn, repeating the whole process four times in order to allow the system to settle down. From the mathematicians point of view, this is not a good technique. But the method is quite accurate, does the job quicker than a proprietary technique and occupies less storage space in the computer. The reason for its apparent success over other methods lies in the fact that the function is well behaved and that the method cannot be applied directly to other problems. It was tailor made for this job. The method is described in Appendix B5.

In order to test the method and to have an alternative to meet criticisms on the grounds of lack of generality, a study of the proprietary techniques was made and a suitable alternative method was sought.

An alternative method must be one of the gradient or direct search techniques because of the complexity of the function. The technique of dividing the range of each of the intermediate gear ratios into a large number of incremental steps and to evaluate the function at all combinations and then to pick out the best is a thoroughly bad method. It is inelegant mathematically, and takes far too long.
The "Method of steepest ascent" would work with this problem, but it can be subjected to the same sort of criticism as the tailor made technique in that it will fail under certain conditions. In this case the certain conditions are that, if there are "ridges" in the plot of function against the variables, the method of steepest ascent is liable to get stuck on a ridge. There is a variant to this method, called Booth's modification (17), which prevents the optimisation procedure going all the way up a hill, in fact it only goes 90% of the way. This variant certainly helps with ridges.

Analytical gradient methods, which employ the partial derivatives of a function, are difficult to use because of the amount of calculation involved in finding the derivatives at each step. Such methods include the Newton - Raphson method, which fits a quadratic to the function at each step. Also the Davidon (14), (16) method which uses a variable matrix technique, starting with a gradient method which gradually changes to an analytical technique. This latter method is very good if the first derivatives can be evaluated. If however, a numerical method has to be employed to obtain the first derivatives as in this case, the second derivatives should also be evaluated in order to ensure that the first derivatives are of sufficient accuracy. This then rules out the Davidon method because of the large amount of computation involved.
A direct search method, therefore, has to be used. A common direct search method is the "Simplex" method. In two dimensions, it evaluates the function at three points of a triangle, rejects the worst and sets up another triangle, the new point being a reflection of the rejected point. This technique is illustrated in Fig. Ill.2. In a three dimensional problem, i.e., three variables, a tetrahedron is used. Thus for \( n \) variables, a regular simplex shape is set up having \( (n+1) \) vertices. At each operation the worst vertex is rejected and is replaced by its reflection about the centroid of the remaining vertices. So the simplex shape progresses towards the optimum. There are techniques for dealing with such matters as the simplex shape going off course. This can happen in certain circumstances because the reflected evaluation can prove worse than the one rejected. The greatest snag with the simplex method, however, lies in its behaviour when near to the optimum. The simplex shape tends to rotate around the optimum.

For this reason, more than any other, the simplex method was rejected. It is possible to put the variables quite close to the optimum initially with a good guess. The simplex shape would have to be very small to improve on the initial guess and home onto the optimum value. The smallness of the shape would, in all probability, frustrate the simplex method since the function itself is evaluated using a numerical method and hence has some small error.
A well-favoured direct search method is the Rosenbrock method (15), (18), (19). This does not rely on climbing over the surface of a multi-dimensional hill. A search is made around the initial guess and then the method bores through the hill in the direction of the optimum. If the initial guess is a long way from the optimum, this method will move to the vicinity of the optimum very quickly, but once there it finds it more difficult to move in the optimum direction.

Another particularly good direct search method is the Powell method (20). This method scores if the initial guess is close to the optimum. In two dimensions, the method searches and finds the best point on a straight line. It repeats the process for another adjacent line and then searches along a line through the two best points. The Powell method is illustrated in Fig. B1.3.

The Rosenbrock method was chosen largely because a computer program existed for this at Loughborough and was known to be working. The Powell method does not cater directly for constraints. The intermediate gear ratios must lie between bottom and top gear ratios and anyway, they cannot be negative. This deficiency could be overcome by squaring the variables to eliminate negative values and by the use of cosine functions to keep the intermediate gear ratios between bottom and top gear ratios. However, since the Rosenbrock program caters directly for constraints and because it was readily available, it was used as an alternative to the tailor-made technique.
To compare the Rosenbrock method with the tailor-made method, the gear ratios of vehicle B were optimised on the time up to 75 mile/h. Table B1.2 gives the results of the two methods. The agreement is quite close producing no discernable difference in the time up to 75 mile/h. The program for the tailor-made calculations is far shorter than the Rosenbrock, hence compilation time and computer storage space are less. Also the tailor-made program required 69 passes through the subroutine generating the function time up to 75 mile/h, compared with 30 for the Rosenbrock. Hence execution time is less using the tailor-made technique.

Provided that the function (time to speed) is well behaved, the tailor-made technique is adequate. It does not produce gear ratios accurate to a large number of decimal points as the proprietary methods are capable of doing, but then great accuracy is not required. The detail designer has to accommodate hunting teeth on the gear wheels and to ensure that the sum of the mating gears on layshaft and mainshaft is constant for all meshing pairs in the gearbox. He has to have some room to manoeuvre. Furthermore, Fig. B1.1 shows that a small difference between an intermediate gear ratio and its optimum value results in a small loss only in time up to speed, making due allowance for the very fine ordinate scale when reading Fig. B1.1.

Applying the tailor-made and the Rosenbrock techniques to the vehicle data used to construct Fig. B1.1 results in an optimum intermediate gear ratio of 1.5782 for the tailor-made compared
with a figure of 1.5684 for the Rosenbrock. Reference to Fig. Bl1.1 shows the tailor-made to be rather nearer the optimum. The Rosenbrock is capable of much better accuracy, but the number of passes through the subroutine generating the function would have to be stepped up from 40 passes per variable to at least 200.

One final point concerning general, proprietary techniques such as the Rosenbrock. On several occasions, when optimising the intermediate gear ratios of a 4 or more speed gearbox, the Rosenbrock technique has mixed up the gear numbers. For instance, it has put the second gear higher than the third. In fact it has changed them over. This does not matter, because the gear number is just a name, but it does emphasize the need for constraints. The constraints on the use of the Rosenbrock technique are simply that all the intermediate gear ratios must lie in between bottom and top gear ratios. Tightening up these constraints would eliminate the tendency of the Rosenbrock program to interchange gear numbers.

In conclusion, therefore, two methods of optimisation are available at present and both work. Work will continue to include the Powell method with pseudo-constraints, since this method is likely to prove the most suitable. A listing of the Rosenbrock subroutine is contained in Appendix B6 with some notes on its use.
SECTION 8.12

Main Vehicle Performance Program (B001)

B001 is primarily an aid to the Design Engineer. Basically, it will calculate the time to speed up through the gears and the maximum speed of a vehicle. The gear change points are fixed by the program at the best speeds.

The program will, if asked, fix the gear ratios. Top gear is fixed by specifying the degree of undergearing defined as:

\[
DUG = \frac{\text{maximum speed of vehicle on the level}}{\text{vehicle speed corresponding to max. bhp in top gear}}
\]

Bottom gear is fixed by the criterion that the vehicle shall be capable of climbing the maximum gradient possible, without wheel-spin.

The intermediate gear ratios are fixed by optimising the criterion that the time up to a specified speed should be a minimum. It is suggested that the specified speed be \(3/4\) of the maximum speed of the vehicle on the level.

The facility is provided also for reading in the values of top and bottom gear ratios and using the optimisation procedure for fixing the intermediate gears only.

Certain other useful data accrue from use of the program, such as engine speed values of maximum engine torque and power, gear overlap, time to cover 1/4 mile, whether the vehicle will hold on the handbrake on the gradient specified, whether the vehicle will
overturn on the gradient specified, amount of wheel-spin and maximum vehicle speeds in the lower gears.

The first part of the program listed below down to and including statement number 14 concerns the reading in of the data. This is on punched cards in the following manner.

**Card 1**  (Format statement number 1001)
Title, or other relevant information, such as date, model of vehicle, etc. Column 1 of card 1 is left blank. Whatever is put in the other 79 columns is reproduced in the print-out. It is not used in the program.

**Card 2**  AT, BT, CT, DT, ET, FT, CT, VBS, NO
Coefficients of sixth order polynomial curve fit to engine torque characteristic, engine maximum speed rev/min and a number which, if negative, will result in extra print-out.
Card 2 is in free floating point format.

**Card 3**  NG, NB, NBOT in format 112
NG is the number of gear ratios (should not exceed 10)
NB is a number which, if negative, means that the program will fix all the gear ratios.
NBOT is a number which, if negative and if NB is negative also, means that the program will fix the intermediate gear ratios only. The drive axle ratio and bottom gear ratio are then read-in separately, see below.
Continuing with the case that NB is zero or positive

**Card 4**  GR(I)
Gear ratios, starting with 1st or bottom. Free floating point format.
Drive axle ratio, rolling radius of drive wheels at 30 mile/h
\((r_r)_{30}\) in feet. Free floating point format.

Total road wheel inertia (slug ft\(^2\)) in free format

Engine inertia (slug ft\(^2\)), minimum engine speed (rev/min)
Free floating point format.

\(W, A, B, H, KA\) in free floating point format

\[ W = \text{vehicle weight (lbf)} \]
\[ A = \text{position of cg aft of front wheel centre (ft)} \]
\[ B = \text{position of cg forward of rear wheel centre (ft)} \]
\[ H = \text{height of cg above ground level (ft)} \]
\[ KA = \text{is a number which if set positive denotes a rear wheel drive vehicle, if negative a front wheel drive vehicle.} \]

Drag coefficients and tyre growth factor.

\[ \text{Drag lbf} = W(AB + BD.V) + AK.V^2 \]
where \(V\) is vehicle speed (mile/h)

Rolling radius at \(V\) mile/h
\[ = (r_r)_{30} \times \left( 1 + \frac{XX}{100}(V^2 - 900) \right) \]
This is the Dunlop formula for tyre growth with speed.

for cross plies
\[ CG1 \quad XX = 4.76 \]
\[ RS5 \quad XX = 3.04 \]

for radial ply tyres take \(XX = 0\).
Card 10  G in free floating point format
Gradient (i) as defined in Section B6.

Card 11  GOT, GP in free floating point format
Gear change time (sec) and the coefficient of friction
between drive wheels and ground.

Card 12  V1, V2, WV in free floating point format
V1 = initial vehicle speed from which calculations start
(mile/h)
V2 = final vehicle speed (mile/h)
WV = head-on component of wind velocity (mile/h)
If the switch device at the end of the program (described
below) is not to be used and it is desired to end the
calculations,
Card 13  BLANK.

If NB is negative

Card 4  DUG, RR in free floating point format.
DUG = degree of gearing as defined by equation
H10.2 and Fig. H10.1 in Section H10.
RR = (r_f)30 explained above.
Cards 5 to 11 as per cards 6 to 12 above.

If NBOT is negative also

Card 12  BOT, DAR in free floating point format.
BOT = 1st gear ratio (GR(1))
DAR = drive axle ratio
Again, unless switch device is to be used, the last card is blank.

Switch device
(at the end of program 0001)

Read J in fixed point format I2

If \( J = 0 \), i.e. a blank card, calculations end

If \( J = 1 \) Read in another torque polynomial expression, maximum engine speed rev/min, a number (WO) as per Card 2 above. Program then repeats calculations with new engine data, all other data remaining as before.

If \( J = 2 \) Read NG, NB, NBOT. Format 312.

and on a new card, either gear ratios or Degree of undergearing and Rolling radius (ft) in free format depending whether NB is positive (or zero), or negative. Program will then repeat in full.

If \( J = 3 \) As for \( J = 2 \), except that program will not recalculate engine maximum torque and maximum BHP values and speeds.

If \( J = 4 \) For use with optimisation. Read in new values of Degree of Undergearing. Rolling radius (ft) in free format.

If \( J = 5 \) Read NG only. Format 12 and, on a new card, the gear ratios in free format. Program will not work out engine max torque and max BHP values and speeds and will skip the optimisation routine.

If \( J = 6 \) Read new gear ratios only, in free format. It is assumed that the number of gears remain as before and that the
optimisation routine is not to be used.

If $J = 7$ Read new Drive axle ratio, Rolling radius (ft). In free format.

If $J = 8$ Read new Total road wheel inertia slug $ft^2$. Free format.

If $J = 9$ Read new Engine inertia (slug $ft^2$), minimum speed of engine (rev/min). Free format.

If $J = 10$ Read new $W, A, B, H, KA$. Free format.

If $J = 11$ Read new drag coefficients $AD, BD, AK$ in free format.

If $J = 12$ Read new gradient ($G$). Free format.

If $J = 13$ Read new Gear change time (sec.), coefficient of friction between tyres and road. Free format.

If $J = 14$ Read new $V_1, V_2, PV$ mile/h. Free format.

If $J = 15$ Go right back to the beginning and read in values for all the parameters.

The switching device can be used repeatedly and is useful to study the change in the performance of a vehicle by altering one parameter only.

The last card of any set of data should be a blank card.

Statement numbers 410 down to and including 28 prints out the read-in data for reference purposes. The IF statements numbers 29, 252 and 556 are to prevent duplication in the calculations after the switch device has been used.
The maximum engine torque and the engine speed at which it occurs are evaluated between statement numbers 250 to 37. This is achieved by setting the engine speed at the minimum engine speed (MIN) and evaluating the first derivative of the engine torque polynomial. This is repeated at intervals of 500 rev/min. added onto the engine speed until the derivative is found to be negative. A search is then made by deducting 50 rev/min. successively from the engine speed until the first derivative of the torque polynomial is again positive. A small scale search is then made by successively adding 5 rev/min. onto the engine speed until the derivative is found to be negative again. This then is said to be maximum engine torque speed and is printed out, together with the value of maximum engine torque, to format statement number 121.

The process is repeated down to and including statement number 46 with the product of engine speed and torque in order to find maximum brake horsepower and the speed at which it occurs. Again a search is made in three parts in order to locate the engine speed at which the first derivative of the product of engine speed and engine torque is zero. Maximum brake horsepower at the engine speed at which it occurs are printed out to format statement number 122.

The IF statement at statement number 557+1 is the division between use of the program to determine the gear ratios and predetermined, read-in values. If (NB) is negative, that is that
the program is to determine the gear ratios, the control passes to statement number 48.

The IF statement at statement number 48+1 checks that the degree of undergearing read in on data card 4 is compatible with the value of the maximum allowable engine speed read-in on data card 2. If not, the degree of undergearing is re-set at its highest value possible and the message contained in format statement number 151 is printed out.

Statement number 51 and the following statement fixes the overall first gear ratio for the rear wheel drive vehicle. The following IF statement checks that the vehicle will not overturn nose-up in first gear on the steepest gradient that the vehicle can climb without wheel-spin. If it is possible to overturn the vehicle, the message contained in format statement number 123 is printed out. Statement number 50 and the following statement fixes the overall bottom gear ratio for the front wheel drive vehicle.

Format statement number 124 prints out the maximum gradient the vehicle can climb without wheel-spin in the more usual form of

\[ 1 \text{ in} \frac{1}{\sin \theta} \text{ (i.e. 1 in 1)} \]

The next part of the program, down to statement number 407, uses the specified degree of undergearing to find the maximum speed of the vehicle on the level by means of an iterative process. This is in order to fix the drive axle ratio, since top gear is assumed to be straight-through. Hence, first gear ratio is fixed by dividing the overall first gear ratio by the drive axle ratio.
Alternatively, if NBOT is read in as a negative value, the statement at 407+3 reads in predetermined values of first gear ratio and drive axle ratio.

The DO loop down to statement number 60 places the reciprocals of the intermediate gear ratios in arithmetic progression as an initial guess before optimisation.

The IF statement at number 60+3 ensures that if there are only three gear ratios, that is one only intermediate gear ratio, the tailor-made optimisation procedure is traversed once only. If there are more than three gear ratios, it is traversed four times in order that the optimisation procedure may settle down. If there are no intermediate gear ratios, the optimisation procedure is by-passed.

As a guide, the initial guess at the intermediate gear ratios is printed out to format statement number 129.

The large DO loop down to statement number 64 is the tailor-made optimisation procedure described in Appendix B5.

DO loop down to statement number 76 calculates and prints out the "overlap" figures for the gear ratios. This is followed by a print-out of time up to specified speed on the level to format statement number 134.

Should, however, the specified gradient be non-zero, the time to speed on the specified gradient is evaluated by calling up the sub-routine TIME at statement number 84.
Here the program branches to the last part if the optimisation procedure has been used. If the optimisation procedure has not been used, the program carries on to check that the vehicle will not overturn on the specified gradient in first gear, to calculate the gear ratio "overlap" figures and to call up sub-routine TIME in order to calculate the time up to the specified speed on the specified gradient.

The final part of the program concerns the calculation of the maximum speed of the vehicle on the level in each of the gears. This is done by an iterative process in much the same way as engine maximum torque and b.h.p. values were evaluated above. The process starts by setting vehicle speed at that which is equivalent to engine minimum speed. The procedure then is to calculate the tractive force and to compare this with the drag force. If it is larger, ten mile/h is added onto the vehicle speed successively until the tractive force is found to be less than the drag force. The vehicle speed is successively reduced in steps of one mile/h until again the tractive force is greater than the drag force. A rounding-off figure of 0.5 mile/h is added onto the vehicle speed which is then declared to be the maximum vehicle speed on the level in that particular gear ratio. This is then repeated for the other gear ratios. It follows, therefore, that the maximum vehicle speed is only accurate to \(\pm 0.5\) mile/h. This is adequate for most purposes. If, however, a more accurate estimation is required, say for studying the effect of a parameter upon vehicle maximum
speed, then a one dimensional version of the Rosenbrock
optimisation procedure may be used. This is described in
Section B13.

After printing out the maximum vehicle speed in the various
gear ratios, the degree of undergearing is evaluated. This may
be slightly different to that specified if the optimisation
procedure is used, because of the tolerance on the calculation
of maximum vehicle speed.

The program ends with the switch arrangement described
above. By setting \( J \) at any integer up to and including 15, it
is possible to end the calculations \( (J = 0) \) or to branch back to
the beginning and alter one particular parameter only. This
means that a full study may be made of one parameter at the one
compilation.
FORTRAN COMPILATION BY #XFAS MK 1B    DATE 03/01/68 TIME 12/56/43

*FORTRAN 0001, G.G.LUCAS
MASTER 0001
DIMENSION GR(10), F(10)
COMMON GCT, DAR, RR, AT, BT, CT, DT, ET, AD, BD, AK, W, AIW, AIE, CF, KA, A, B, H;
ING, NO, VBS, RM, FT, GT, RMIN, DTDR, WV, XK
EXPF(X) = EXP(X)
LOGF(X) = ALOG(X)
SINF(X) = SIN(X)
COSF(X) = COS(X)
ATANF(X) = ATAN(X)
SORTF(X) = SORT(X)
ABSF(X) = ABS(X)
15 CONTINUE
  J=25
  WRITE(2,100) 1 READ(1,1001)
  WRITE(2,1002) READ(1,100)
1 READ(1,108) AT, BT, CT, DT, ET, FT, GT, VBS, NO
  IF(J=1) 2,26,2
5 CONTINUE
3 CONTINUE
2 READ(1,102) NG, NB, NBOT
  IF(NB)4,6,6
6 CONTINUE
  READ(1,108) GR(I), I=1, NG
  IF(J=6)26,26,7
7 READ(1,108) DAR, RR
  IF(J=7)8,26,8
4 READ(1,108) DUG, RR
  NO=6
  IF(J=9)26,26,8
8 READ(1,108) AIW
  IF(J=8)9,26,9
9 READ(1,108) AIE, RMIN
  IF(J=9)10,26,10
10 READ(1,108) AN, B, H, KA
  IF(J=10)11,26,11
11 READ(1,108) AD, BD, AK, XK
  XK=KK/1000000.
  IF(J=11)12,26,12
12 READ(1,108) G
  IF(J=12)13,26,13
13 READ(1,108) GCT, CF


IF(J=13)14,26,14
14 READ(1,108)V1,V2,WV
IF(J=14)410;26,410
26 WRITE(2,113)
410 WRITE(2,114)AT,BT,CT,DT,ET,FT,GT,VBS,AIE
WRITE(2,115)
WRITE(2,116)AD,BD,AK,G,GCT,CF,NG
WRITE(2,131)
WRITE(2,132)W,A,B,H,RR,AIW
WRITE(2,117)V1
WRITE(2,118)V2
WRITE(2,1003)WV
IF(KA=27,26,28
27 WRITE(2,119)
GO TO 29
28 WRITE(2,120)
29 IF(J=25)25;25,250
252 IF(J=12)556;251,251
556 IF(J=5)250,557,251
251 J=55
GO TO 49
250 J=0
RM=RMIN
30 DTDR=BT+2.*CT+RM/1000.*3.*DT+(RM/1000.)**2.*ET+(RM/1000.)**3.*ET**4.*ET**5.*ET**6.*ET**7.*ET**8.*ET**9.*ET**10.*ET**11.*ET**12.*ET
31 IF(DTDR)32,33,33
32 RM=RM+500.
J=1
GO TO 30
33 IF(DTDR)37,38,38
34 IF(DTDR)36,36,35
35 IF(DTDR)37,38,38
36 RM=RM+50.
J=10
GO TO 30
37 DTDR=AT+BT+RM/1000.*CT+RM+RM/1000000,*DT+(RM/1000.)**3.*ET+(RM/1000.)**4.*ET**5.*ET**6.*ET**7.*ET**8.*ET**9.*ET**10.*ET**11.*ET**12.*ET
38 RM=RM+5.
J=100
GO TO 30
39 DP=AT+BT+RM/1000.*CT+(RM/1000.)**2.*DT+(RM/1000.)**3.*ET**4.*ET**5.*ET**6.*ET**7.*ET**8.*ET**9.*ET**10.*ET**11.*ET**12.*ET
40 IF(DP)41,42,42
42 RMP=RMP+500.
   J=1
   GO TO 39
41 IF(J=10)43,43,44
43 IF(DP)45,45,44
45 RMP=RMP+50.
   J=10
   GO TO 39
44 IF(DP)46,47,47
47 RMP=RMP+5.
   J=100
   GO TO 39
46 OP=2*5.1415926/33000. *(AT+BT*RMP/1000.+CT*(RMP/1000.))**2+DT*(RMP/1000.)**3+ET*(RMP/1000.))**4+FT*(RMP/1000.))**5+GT*(RMP/1000.))**6)*R
2MP
   WRITE(2,122)DP,RMP
557 CONTINUE
557 IF(NB)48,49,49
49 CONTINUE
   IF(RMP>DUG=VBS)256,256,257
257 DUG=VBS/RMP
   WRITE(2,151)
260 CONTINUE
517 IF(KA)50,51,51
51 CONTINUE
   IF(15,51,51)
52 CONTINUE
   IF(TEF=GRAD-TEF)58,59,56
56 TEF=(9.9758-.0000879*VM)*(.96-.000316+VM-.000058*VM*VM)
   TEF=VM*(W*(AD+VM*BD)*AX*VM*VM)/TEF
   IF(J=1)55,55,58
55 IF(TEF=GRAD)59,59,57
57 VM=VM+10.
   J=1
   GO TO 54
58 IF(TEF=GRAD)59,407,56
56 VM=VM+1.
J=10
GO TO 54
59 CONTINUE
VM=VM+.5
407 CONTINUE
WRITE(2,125)VM,DUG
IF(NBOT).GE.1100,0
READ(1,108)BOT,DAR
GO TO 1101
1100 CONTINUE
RX=RR*(1.+.X**3*(VM*VM-900.)
DAR=RMP*DUG*RX*3.14159265*15./VM/660.
BOT=BOT/DAR
1101 CONTINUE
WRITE(2,126)
WRITE(2,127)DAR
WRITE(2,128)BOT
ANG=NG=1
DO 60 I=1,NG
AI=I-1
60 GR(I)=1./(1./BOT+(AI*(1.-1./BOT))/ANG)
GRAD=0.
JJ=NG=1
IF(NG=3)84,315,316
315 KK=1
GO TO 317
316 KK=4
317 CONTINUE
DO 63 I=2,JJ
63 WRITE(2,129)I,GR(I)
62 CONTINUE
DO 64 K=1,KK
DO 65 I=2,JJ
CALL TIME(TTS,V2,V1,GRAD)
TSX=TTS+1000.
J=0
66 GR(I)=GR(I)+.1
67 CONTINUE
CALL TIME(TS,V2,V1,GRAD)
420 WRITE(2,104)I,J,GR(I),TS,TTS,TSX
421 CONTINUE
270 CONTINUE
IF(TS-TTS>.78,69
68 CONTINUE
IF(J=1)72,70,72
72 J=10
TSX=TTS
TTS = TS
GO TO 66
70 GR(I) = GR(I) + 1
J = 1
TSX = TTS
TTS = TS
GO TO 67
69 CONTINUE
IF(TSX = TS) 257, 260, 269
269 CONTINUE
IF(J = 1) 75, 73, 71
75 GR(I) = GR(I) + 1
TSX = TTS
TTS = TS
TS = TSX
J = 1
GO TO 270
73 GR(I) = GR(I) + 1
TTS = TS
CALL TIME (TS, V2, V1, GRAD)
422 WRITE(2, 104) I, J, GR(I), TS, TTS, TSX
423 CONTINUE
IF(TS = TTS) 73, 74, 330
71 GR(I) = GR(I) + 1
TTS = TS
CALL TIME (TS, V2, V1, GRAD)
424 WRITE(2, 104) I, J, GR(I), TS, TTS, TSX
425 CONTINUE
IF(TS = TTS) 73, 74, 331
78 CONTINUE
IF(J = 1) 71, 73, 71
267 CONTINUE
IF(J = 1) 312, 311, 312
311 GR(I) = GR(I) + 1
TS = TTS
GO TO 270
312 GR(I) = GR(I) + 1
TS = TTS
GO TO 270
268 TS = TTS
IF(J = 1) 313, 314, 313
313 GR(I) = GR(I) + 1
GO TO 74
314 GR(I) = GR(I) + 1
GO TO 74
330 CONTINUE
GR(I) = GR(I) + 0.005
GO TO 74
331 CONTINUE
GR(I)=GR(I)+.005
74 CONTINUE
77 WRITE(2,130)I,GR(I),TS
65 CONTINUE
64 CONTINUE
DO 76 I=2,JJ
GRAD=RM*3,14159*RR*15./GR(I)/DAR/660,
TEF=GRAD+VBS/RM
76 WRITE(2,140)I,GR(I),GRAD,TEF
WRITE(2,134)TS
IF(G)84,85,84
84 CALL TIME (TS,V2,V1,G)
WRITE(2,133)TS
85 CONTINUE
GO TO 255
49 IF(KA)80,81,81
81 CC=SQRTF(1,-G*R)
C=B*C*G*(H*RR)
IF(DTDR+DAR+GR(I)=W+C)80,80,82
82 WRITE(2,101)
80 CONTINUE
DO 86 I=1,NG
GRAD=RM*3,14159*RR*15./GR(I)/DAR/660,
TEF=GRAD+VBS/RM
86 WRITE(2,140)I,GR(I),GRAD,TEF
CALL TIME(TS,V2,V1,G)
WRITE(2,133)TS
255 CONTINUE
WRITE(2,137)
DO 88 I=1,NG
OGR=3,1419265*15,*RR/660./DAR/GR(I)
VM=OGR*RMIN
J=0
89 DF=W*(AD+VM+BD)+AK*VM+VM
AI=NG=I
TEF=196.,000316*VM=.000008*VM+VM*.99798*(1.007*AI)=.0000879*
1VM=2.08*AI
RX=RR*(1.4*VM+900.)
OGR=3,1419265*15,*RX/660./DAR/GR(I)
R=VM/GR
T=AT+BT*(R/1000.)*CT*(R/1000.)**2+DT*(R/1000.)**3+ET*(R/1000.)**4+FT
*(R/1000.)**5+GT*(R/1000.)**6
TP=TR+GR(I)+TEF+DAR/RX
IF(J=1)93,95,94
93 IF(TF=DF)95,96,96
96 VM=VM+10.
J=1
GO TO 89
93 IF(DF=TF)97,406,98
95 CONTINUE
96 IF(VM)90,405,98
97 VM=VM+10.
GO TO 89
98 VM=VM+10.
J=10
GO TO 89
97 VM=VM+5.
96 CONTINUE
IF(VM=VBS*OGR)90,91,91
91 CONTINUE
GRAD=VBS*OGR
WRITE(2,138)GRAD,1
GO TO 88
90 CONTINUE
WRITE(2,139)GRAD,VM
88 CONTINUE
V=RMP*OGR
GRAD=VM/V
WRITE(2,142)GRAD
WRITE(2,127)DAR
87 J=0
READ (1,102)J
IF(J)203,205,99
99 CONTINUE
WRITE(2,100)
GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15),J
203 WRITE(2,150)
STOP
100 FORMAT(10X19HG,G,LUCAS AUTO DEPT/8X37HLOUGHBOROUGH UNIVERSITY OF T
ECHNOLOGY/10X40HVEHICLE PERFORMANCE = LUCAS OPTIMISATION/)
101 FORMAT(10X49H••••• VEHICLE MAY OVERTURN IN FIRST GEAR •••••)
102 FORMAT(312)
104 FORMAT(10H0 GEAR NO=13,4H J=14,12H GEAR RATIO=F8.3,6X6HTIMES=3E14
1.5//)
108 FORMAT(10FO:0)
113 FORMAT(//5X7HNEW RUN//)
114 FORMAT(15X29HENGINE TORQUE CHARACTERISTICS27X20H MAX; ENGINE ENGI
INE/74X18HSPEED = INERTIA/7F10.5,F11.1,F11.5/)  
115 FORMAT(5X17HDRAG COEFFICIENTS6X42HGRADIENT GEAR CHANGE COEFF
1 NO OF/12X29HTIME OF FRICTION GEARS)
116 FORMAT(3F9.5,3F11.5,110/)
117 FORMAT(21H INITIAL SPEED MPH =F5.0)
118 FORMAT(18H FINAL SPEED MPH =F5.0/)
131 FORMAT(65H WEIGHT POSITION OF C OF G ROLLING RAD FT INERTIA
10F WHEELS)
132 FORMAT(F8.1,3F7.2,F15.4,F10.3/)
119 FORMAT(26H FRONT WHEEL DRIVE VEHICLE/)
120 FORMAT(25H REAR WHEEL DRIVE VEHICLE/)
121 FORMAT(30H MAXIMUM ENGINE TORQUE LB-FT =F8.3/27H MAXIMUM TORQUE SPEED RPM =F5.0/)
122 FORMAT(27H MAXIMUM BRAKE HORSEPOWER =F8.3/17H MAX. BHP SPEED =F6.0/)
123 FORMAT(10X65H ****** VEHICLE MAY OVERTURN ON MAX. GRADIENT IN FIRST GEAR ******/)
124 FORMAT(41H MAX. GRADIENT VEHICLE CAN CLIMB IS 1 IN F6.2/)
125 FORMAT(28H MAX. SPEED OF VEHICLE MPH =F6.1/25H MAXIMUM TORQUE RPM =F7.4/)
126 FORMAT(38H TOP GEAR RATIO TAKEN AS 1 TO 1 GIVING/)
127 FORMAT(19H DRIVE AXLE RATIO =F7.3/)
128 FORMAT(20H BOTTOM GEAR RATIO =F7.3/)
129 FORMAT(12H GEAR RATIO 13,3H =F7.3,14H INITIAL GUESS/)
130 FORMAT(12H GEAR RATIO 13,3H =F7.3,9H TIME TO SPEED SECS =F8.2/)
131 FORMAT(133H TIME TO SPEED ON GRADIENT SECS =F8.2/)
132 FORMAT(134H TIME TO SPEED ON LEVEL SECS =F8.2/)
137 FORMAT(15X23H MAXIMUM SPEED ON LEVEL/27H GEAR RATIO = SPEED MPH/)
138 FORMAT(F10.1,5X41H ENGINE MAX. SPEED LIMITS VEHICLE SPEED TO F6.1/12H MPH IN GEAR 13/)
139 FORMAT(10H 15H 11/)
142 FORMAT(25H DEGREE OF UNDERGEARING =F7.4/)
140 FORMAT(12H GEAR RATIO 13,3H =F7.3,5X6.174H MPH TO LIMITATION SET BY VALVE BOUNCE OFF6.1,4H MPH/)
150 FORMAT(122H END OF CALCULATIONS/)
151 FORMAT(76H DUE TO ENGINE SPEED LIMIT DEGREE OF UNDERGEARING IS REDUCED TO FIGURE BELOW/)
1000 FORMAT(1H1)
1001 FORMAT(80H)
1002 FORMAT(1HO/)
1003 FORMAT(32H HEAD-ON WIND VELOCITY MILE/H = F8.2/)
END

END OF SEGMENT, LENGTH 2406, NAME 8001
SUBROUTINE TIME (TS,V2,V1,G)
SORTF(X) = SQRT(X)
DIMENSION GR(10),F(10)
COMMON GR,GCT,DAK,RR,AT,BT,CT,DT,ET,AD,BD,AK,W,AIW,AIE,CF,KA,A,B,H,
IT,NO,VBW,RT,GT,GRM,DTDR,WV,XK
TS=C;
V=V1; initial val: v1
DV=2; initial -disp: 2 rph
DISFL=0;
TSL=0; 
AAI=0;
L=0;
DISF=0;
CC=SQR(TF (1.,G*G))
IF(V<V1)42,42,0
42
CONTINUE
IF(NO)0,41,41
WRITE(2,107)
41
CONTINUE
C=B*CC*H*G
BW=1.0*C/CC/(A+B)
IF(G*W*CF*CC)3,3,2
WRITE(2,100)
3
CONTINUE
BW=CF*W/(A+B)
IF((V2=V)/DV-25.)4,5,5
4
DV=DV/2:
GO TO 3
5
KV=(V2=V)/DV+1.
DO 6 KM=1,KV
RX=RR*(1.,+X*(-(V-V=900.,))
DF=W*(G+AD+W*BD)+AK*(V+W)*2
DO 7 1M1,NG
AMI=NG
DG=3.;1459265*15.,*RX/600.,/DAR/GR*F1)
R=V/OGD
IF(VBS=R)27,28,28
27 F(1)=0.
GO TO 7
28
CONTINUE
IF(RM>R)66,66,0
T=DTDR
GO TO 67
66
CONTINUE
T=AT+BT*(R/1000.)*CT*(R/1000.)*2+DT*(R/1000.)*3+ET*(R/1000.)*4+FT
1*(R/1000.)*5+GT*(R/1000.)*6
67
CONTINUE
TF = TF*GR(I)*TEF*DA/R*RX
31 CONTINUE
PF = TF*DF
IF(PF) = 14, 15, 15
14 F(I) = 0,
GO TO 7
15 EM = W/32; 2 + AIW/RR/RR + AIE*TEF*(GR(I)*DA/R)*2
F(I) = PF/EM
IF(KA) = 10, 11, 11
10 C = B + C = H*G
TFF = BW*(C + H*F(I)/32; 2)
GO TO 30
11 C = A + C = H*G
TFF = BW*(C + H*F(I)/32; 2)
30 CONTINUE
IF(F(I) = W/32; 2 + AIW/RR/RR/2) = TFF*DF = W/2. *(AD*V*BD)) = 12, 12, 13
13 CONTINUE
WRITE(2, 104) V, I, TFF, TF
IF(KA) = 47, 48, 48
47 CONTINUE
TF = *E* ((B*W*DF + W*(AD*BD*V)/2)) * EM/(W/32; 2 + AIW/RR/RR/2. * BW + H/32; 2)
1*DF)
GO TO 31
48 CONTINUE
TF = *E* ((B*W*DF + W*(AD*BD*V)/2)) * EM/(W/32; 2 + AIW/RR/RR/2. * BW + H/32; 2)
1*DF)
GO TO 31
12 CONTINUE
7 CONTINUE
FG = 0;
J = L
DO 16 I = 1, NG
IF(F(I) = FG) = 16, 16, 16
18 FG = F(I)
J = I
16 CONTINUE
IF(FG) = 19, 19, 20
19 CONTINUE
OGR = 3; 14159265*15. *RX/660./DA/R/GR(I)
IF(V = RM*OGR) = 15, 6, 62
62 CONTINUE
WRITE(2; 106) J, V, V2
WRITE(2; 111)
GO TO 50
20 CONTINUE
IF(J = L) = 21, 22, 21
21 CONTINUE
   IF(NO)0,68,68
   WRITE(2,101)J,V
68 CONTINUE
   IF(L)23,24,23
   AN=GET
   GO TO 17
22 CONTINUE
24 AN=0;
17 L=J
   AA2=(AA1+DV*22./15./FG)/2,
   IF(V=V1)60,61,60
60 CONTINUE
   TS=TS+AA2+AN
61 AA1=DV*22./15./FG
   IF(NO)25,26,26
25 RF=1./FG
   VS=V*22./15.
   EM=W/32.24*A1W/RR/RR*AIE*TEF*(GR(J)*DAR/RX)*2
   PP=FG*EM
   TF=PF DF
   IF(V=V1)0,45,0
   DISF(DISF+(V-DV/2.)*22./15.*(AA2+AN)
46 CONTINUE
   IF(NDIS)0,0,65
   IF(DISF=1320.)65,0,0
   DD=(DISF-1320.)/(DISF-DisFL)
   TSD=TS-DD*(TS-TSL)
   VD=V-DD*DV
   WRITE(2,112)VD,TSD
   NDIS=1
65 CONTINUE
   DISFL=DISF
   TSL=TS
   WRITE(2,108)V,FG,RF,VS,DF,J,TS,DISF,TF,PF
26 CONTINUE
6 V=V+DV
90 CONTINUE
RETURN
100 FORMAT(10X65H***** VEHICLE WILL SLIP DOWNHILL WITH HANDBRAKE ONL
   1Y SET *****/)
101 FORMAT(21HGEAR CHANGE TO RATIO 13; 20H GEAR CHANGE SPEED =F7.2/)
104 FORMAT(F8.2;18X16H WHEEL SPIN IN GEAR 14, 13H FRICT. LBF =F10.2,F11.2
   1)
106 FORMAT(30H SET SPEED UNOBTAINABLE. GEAR=13,13H SPEED MPH =F7.2;25
   1H SET UPPER SPEED MPH =F7.2/)
107 FORMAT(97H0 SPEED ACCEL 1/ACCEL SPEED, DRAG GEAR TIM
   1E DISTANCE TRACT, AND PROPUL. FORCE/91H MPH PT/SEC2 SEC2
2/FT   FT/SEC  LBF  NO.  SEC  FEET  LBF
 3 LBF)
 111 FORMAT(27HOTIME SO FAR IS GIVEN BELOW)
 112 FORMAT(5X33HQUARTER MILE MARK PASSED SPEED =F7.2, 18HMILE/HOUR, T
TIME =F8.1, 15HSECONDS APPROX.)
  END

END OF SEGMENT, LENGTH 1027, NAME TIME
Sub-routine TIME

By specifying the initial speed, the final speed, the
gradient and the coefficient of friction between drive wheels
and ground, sub-routine TIME evaluates the integral

\[
time = \int_{V_1}^{V_2} \frac{1}{f} \, dV
\]

using a simple numerical method.

Other parameters required in the evaluation are carried over
from the main program by storing them in a COMMON area.

The numerical method is to divide the speed range into at
least 25 steps, each of length DV. The reciprocal of the vehicle
acceleration is evaluated at the beginning and at the end of each
step length. The time to cover a particular speed step length
therefore, is the product (DV x the average of the acceleration
reciprocals at the beginning and end of step length). The sum of
such times is the time to speed.

A feature of sub-routine TIME is that it evaluates the vehicle
acceleration in each of the gears and chooses the gear giving the
greatest vehicle acceleration. If this gear is different from that
of an increment of speed earlier, a gear change is said to be made
and the gear change time is added onto the accumulated time to speed.
This means that the gear change points are at the best possible
speeds. The speeds which would be used by a professional test driver.
After setting up the initial values, sub-routine TIME starts by checking that the vehicle will hold on the specified gradient with a perfect handbrake only. This check is outlined in Section B9.

Initially, DV is set at 2 mile/h. The IF statement at statement number 3+2 checks that there are at least 25 increments. If not, DV is successively halved until there are at least 25 speed increments. The DO loop down to statement number 6 is the main integration DO loop.

After calculating the dynamic rolling radius of the drive wheels (RI) and the drag force (DF) since they are functions of vehicle speed only, and not the particular gear ratio, the DO loop down to statement number 7 is entered, this evaluates the vehicle acceleration in each of the gears.

For each of the gear ratios, DO loop 7 evaluates the overall gear ratio and hence the speed of the engine. If this is greater than the maximum allowable, a branch is made to the end of the DO loop. If the engine speed is less than maximum torque speed, the engine torque is set at the maximum for the reasons contained in Section B7. Otherwise, the engine torque is evaluated from its sixth order polynomial expression. Next, the overall transmission efficiency as a function of vehicle speed and particular gear number is evaluated (see Section B4.). From here, the tractive
force and then the propulsive force are calculated. If the propulsive force is less than zero, a branch is made to the end of DO loop 7. Finally, after working out the equivalent mass of the vehicle (see Section B5.), the vehicle acceleration is given by

\[ f = \frac{F_p}{M_E} \]

at statement number 15+1.

There then follows a check for wheel-spin in accordance with the theory laid down in Section B8. If wheel-spin occurs, the appropriate message is printed out to format statement number 104 and a correction made to the calculated vehicle acceleration.

Having calculated the vehicle acceleration in each of the gears, DO loop down to statement number 16 selects the gear giving the greatest acceleration (FG). If this is zero or negative, then the vehicle is not capable of the speed under consideration and the message contained in format statement number 106 is printed out.

That portion of the program between statement numbers 20 and 17 checks whether a gear change has been made. If so, the gear change time is accounted for in storage space AN. Statement number 17+1 evaluates the incremental time by treating the incremental area under the l/f against V curve as a trapezium.

This is followed by a calculation of the distance travelled by the vehicle during the increment using the relationship

\[ \text{incremental distance} = v \cdot \Delta t \]

where vehicle speed \( v \) ft/s is the mean of the increment.
Immediately after the accumulated distance has passed the quarter-mile mark, an estimation of time and speed at the quarter mile mark is made by linear interpolation across the last increment. This information is printed out with the message contained in format statement number 112.

Then follows the main print-out of items of interest, if required. Vehicle speed in both mile/h and ft/s units, vehicle acceleration (ft/s\(^2\)) and its reciprocal, drag force lbf, gear number, accumulated time and distance etc.
Tables B12.1 and B12.2 are typical print-outs from use of program BB01. Table B12.1 is a straight forward calculation of time to speed of vehicle A, while Table B12.2 is a similar calculation for vehicle B. It is of use therefore, to compare the calculated performance with actual road test performance figures.

No actual road test figures could be found for vehicle A itself, since it is a van, but three authorities have published figures for the Estate car version of vehicle A. The Estate car has an identical front and underneath, the shape generally is similar but the rear is different. The same three authorities have tested vehicle B and published figures. The three authorities are "The Autocar", "The Motor" and "The Consumers' Association".

The vehicle weights used in the calculations were the actual test weights quoted by "The Autocar" (11), (10), that is 2128 lbf and 3140 lbf respectively. The vehicle weights when tested by "The Motor" were slightly higher at 2240 lbf and 3192 lbf respectively (21), (22). The actual vehicle test weights are not quoted by "The Consumers' Association" in either of the relevant "Motoring Which?" magazines, although the kerb weights are given. Reference to an earlier edition (23) suggests that 450 lbf (the equivalent of three persons) is added to the kerb weight when testing saloon cars and 300 lbf when testing sports cars. This puts the vehicle weights as tested by the Consumers' Association
at the even greater values of 2291 lbf for the Estate version of vehicle A and 3222 lbf for vehicle B.

The drag coefficients for vehicles A and B are known. An example of vehicle A, owned by the Department of Transport Technology, Loughborough University of Technology, has been subjected to a number of drag tests. The actual drag coefficients used in this work were obtained by the Motor Industry Research Association's Staff in their full scale wind tunnel at Lindley.

The engine and drag data for vehicle B were obtained from the manufacturers.

Figs. B12.1 and B12.2 show the calculated times up to speed of vehicles A and B respectively from the figures contained in Tables B12.1 and B12.2. Superimposed upon these graphs is the road test information published by "The Autocar" and "The Motor". Now the agreement between the calculated and the actual test results is good in the low speed areas. There is some divergence in the middle speed ranges, particularly on vehicle B between the calculated and the test results of "The Autocar". Again, the agreement is quite good in the upper speed ranges.

A point of interest is that "The Motor" returns better times to speed for both vehicles, even though their vehicle test weight were higher. A closer study of "The Autocar" road test report (11), (10) reveal a wind speed of up to 15 mile/h during of both vehicles. "The Motor" (21), (22) say that their
little or no wind when they tested the vehicles. It is shown in
Part C that averaging out the results from runs in opposite
directions will not compensate for a wind speed of 15 mile/h.,
particularly at the higher vehicle speeds.

Another point of interest is that both "The Autocar" and
"The Motor" publish times over several 20 mile/h. speed ranges
in the upper gears. These figures are reproduced in Tables B12.3,
B12.4, B12.5 and B12.6. "The Motor" do not state their gear change
points but "The Autocar" (11) show clearly that in their standing
start acceleration test, the speed range 40 - 60 mile for vehicle
A is traversed in third gear. The time for this is given as 24.2
seconds, whereas reference to Table B12.3 shows the separate 40 -
60 mile/h. time in third gear to be the lower time of 21.5 seconds.

Now it is reasonable to expect a difference between these
test results, indeed it would be suspicious if there was no
discrepancy. But one would expect the standing start time between
40 - 60 mile/h. to be less than the time accelerating from a
steady vehicle speed of 40 mile/h. up to 60 mile/h., because of
the transition loss from steady running to acceleration in the
latter case. A discrepancy of over 10% does however cause one to
question the road test itself.

The road test results on these vehicles conducted by the
Consumers' Association are presented in a different manner. For
each vehicle, one standing start time only is given and this is
the time to cover 1760 feet (1/3 mile) in the case of vehicle A
and 1800 feet in the case of vehicle B. These figures (21), (22) are 30.2 seconds for vehicle A and 24.6 seconds for vehicle B.

In order to afford a comparison between the calculated performance and the times measured by the Consumers' Association, Fig. B12.3 was prepared to extract the corresponding calculated times from Tables B12.1 and B12.2.

Fig. B12.3 shows the calculated standing start time for vehicle A to cover 1/3 mile to be 30.2 seconds, and for vehicle B to cover 1800 feet to be 24.2 seconds.

The agreement here is very good. The calculated time being identical for vehicle A and 0.4 seconds faster in the case of vehicle B. In practical terms therefore, because the vehicle weights in the Consumers' Association test were greater, the times returned by calculation are probably a little slower than the measured times of the Consumers' Association.

Summing up, therefore, it may be stated that the calculated times to speed of these two vehicles agree quite closely with those measured by "The Motor", are more favourable than those measured by "The Autocar", and may be slightly less favourable than those measured by the Consumers' Association.

An assessment of the accuracy of the road tests carried out by the semi-technical press is contained in Appendix B3. This confirms also that there can be considerable differences between two vehicles of the same design.
Also contained in Appendix B8 is the calculated performance of the saloon version of vehicle A. The drag coefficients for the saloon version were obtained from a confidential report issued by the Motor Industry Research Association (9). This calculated performance is compared with the Autocar Road Test report (28). Again the agreement between measured and calculated is good.

Turning now to the speeds at which the gear changes are made; only the Autocar gives this information. The actual gear change speeds for the Estate car version of vehicle A are given as 22 mile/h. for 1st to 2nd, 39 mile/h. for 2nd to 3rd and 60 mile/h. for 3rd to top. The calculated gear change speeds can be seen in Table B12.1 to be 20, 38 and 62 mile/h. respectively.

Likewise for vehicle B. The actual Autocar gear change speeds are 29, 50 and 80 mile/h. compared with the figures in Table B12.2 of 32, 54 and 82 mile/h.

The agreement is very good and is well within experimental repeatability. The calculated gear change speeds have more uniform corresponding engine speeds than the actual values given by Autocar. The engine speeds are not printed out, but may be calculated from the known overall gear ratios. The engine speed immediately before a gear change is above maximum brake horsepower speed but is below the maximum allowable engine speed. This emphasizes the desirability of making the vehicle performance program determine the gear change speeds, rather than to try to fix them arbitrarily before the calculations begin.
Fig. B12.3 shows also that the linear interpolation used to estimate the time and the vehicle speed at the quarter mile mark is accurate.

Table B12.7 lists the measured maximum speeds of vehicles A and B in the four gears together with the calculated maximum vehicle speeds. It is not certain that the maximum speeds in the lower gear ratios quoted by "Autocar" and "The Motor" are the measured values. Those for vehicle B listed by "The Motor" are stated as being the vehicle speeds corresponding to an engine speed of 6000 rev/min, rather than actual maximum vehicle speeds. A study of Table B12.2 shows that the maximum allowable engine speed does indeed limit the vehicle speed in the lower gears. The maximum allowable engine speed assumed in the calculations for vehicle B was 6200 rev/min.

The interesting figures however, are the maximum vehicle speeds in top gear. A study of Table B12.7 would suggest that the calculated values are low, being about 1 mile/h. below the Autocar's figures for both vehicles and rather more below those of "The Motor" and The Consumers' Association.

"The Motor" publishes several "maximum" speeds per vehicle and these may hold the key to the explanation of the differences shown in Table B12.7. The "Maximile" is a timed quarter mile after one mile accelerating from rest. Essentially a short duration test. The higher maximum speeds are returned by running the vehicle around the high speed test track at the Motor Industry Research Association's establishment at Lindley. A test of much longer duration in which
oil temperatures in the engine and transmission will rise to produce less viscous drag on the moving parts.

This point was noticed during performance tests on engines and was commented upon in Section B2. If an engine is left running at high speed on a test bed, the power output will creep up over quite a considerable period of time. "The Motor" therefore, is quite right in drawing a distinction between a maximum vehicle speed obtained from a sustained run and one obtained from a short duration run.

The calculated maximum vehicle speeds are within experimental accuracy. For evidence of this statement, see Appendix B8.

One final point to check concerning the calculation of vehicle performance is that the speed step length used in the integration of the time to speed is small enough. Table B12.8 is a re-calculation of the information contained in Table B12.2, this time with a step length of 1 mile/h instead of 2 mile/h. As may be seen, the error in using the larger step length is less than 0.2% in both the time and distance calculations right up to 94 mile/h. This is well within the accuracy of engine torque measurement and the data used for transmission efficiencies. Therefore, the arrangement of at least 25 steps is considered adequate.

In connection with step length and its effect upon accuracy, however, it is relevant to note that a typical curve of the reciprocal of vehicle acceleration against vehicle speed is "concave upwards".
This means that the error in assuming the strip of area to be a trapezium is accumulative. This error can be minimised by recognising that the form of the time to speed against vehicle speed curve follows approximately a logarithmic function. Details of this function were supplied by P. Stubbs (41) and may be found in Appendix B.

It was considered unnecessary however, to incorporate the small additional complication involved, since a step length of 2 mile/h was thought to be the maximum one should use in order to define the gear change points to a reasonable tolerance. The accuracy with this step length is shown above to be adequate.

In conclusion, therefore, it may be stated that the calculation of the performance of a vehicle using program 8001 produces answers within the range of experimental results. The main benefit, however, to be derived from such a program is that of a parametric study of vehicle performance, rather than a straight-forward prediction of the performance of a projected vehicle. It is intended as an aid to the Designer.

It will be appreciated that if the program is to be used for the accurate assessment of vehicle performance, the engine torque curve and the drag coefficients of the vehicle must be known accurately. The problems associated with engine torque curves are discussed in Section 2 of Part B. The measurement of vehicle drag is dealt with in Part D of this Thesis.
SECTION B13

Maximum speed of vehicle

The estimation of maximum vehicle speed within the vehicle performance programs B001 and B033 is accurate to ±0.5 mile/h. only. This is considered adequate for most purposes. Occasionally, however, it is necessary to consider the effect of a vehicle design parameter upon maximum vehicle speed. Then it is necessary to reduce the tolerance of the calculation in order to ensure that it does not swamp the effect of the change in the parameter. Accordingly, a separate maximum vehicle speed program (B051) was prepared.

Program B051 is listed below. The read-in data on cards is as follows.

Card 1

Title card, leave column 1 blank. The information contained on card 1 is simply printed out as a title, it is not used in the program.

Cards 2 - 5 below are in free floating point format.

Card 2

Engine torque polynomial coefficients, maximum engine speed, minimum engine speed.

Card 3

Vehicle weight lbf, gradient (1), head-on component of wind speed mile/h.
Card 4

Vehicle drag coefficients, as given in Section B12.

Card 5

Gearbox ratio, drive axle ratio, rolling radius of drive wheels at 30 miles/h (ft), $\Delta I$, tyre growth factor defined in Section B12.

$$\Delta I = (N^g - 1)$$

(i.e. if top gear, $\Delta I = 0$. If bottom gear, $\Delta I = N^g - 1$)

Card 6

Fixed point format (12)

Blank card if it is required to finish calculations. If it is required to change a parameter, use the switch device by reading in the appropriate number on Card 6 in a similar manner to that described in Section B12.

The program prints out the read-in information for reference purposes and then proceeds to set up the boundary speeds for the maximum speed calculation. These are the vehicle speeds corresponding to engine minimum and maximum speeds. The program uses a one-dimensional version of the Rosenbrock direct search routine described in Section B11 and Appendix B6. This is labelled sub-routine PIMask2 in the listing.
The initial guess at the maximum speed of the vehicle is set at $x = 0.6$ of the vehicle speed corresponding to maximum engine speed. The Rosenbrock subroutine then calls up the special subprogram generating the function of $X$ upon which it operates to find the minimum.

The special function of $X, (\text{FUN}(X))$ is the square of the propulsive force on the vehicle. At maximum vehicle speed, this function will be zero. At vehicle speeds above and below the maximum, the subprogram will generate a positive value for the function. It can never be negative, hence a clearly defined minimum exists in the function.

An initial step length of 5 mile/h. is made from the initial guess which increases to $(1.5 \times 5)$ if successful. The tolerance on the maximum vehicle speed calculation is governed by the value given to (EPS). In this case, the tolerance is set at $\pm 0.05$ mile/h. That is an order of accuracy greater than that of the main vehicle performance programs.

A typical print-out is given below. After the titles, there follows the print-out of the read-in data. The overall gear ratio printed out actually corresponds to that at 30 mile/h. It does not take account of tyre growth, since it is used simply to set up the boundaries to vehicle speed.

There then follows three columns of figures. The first is the number of iterations, that is the number of times the function of
vehicle speed has been generated. The second column contains the evaluation of the function in E-format, while the third column gives the corresponding value of vehicle speed (X), again in E-format.

The three columns are followed by a print-out of maximum vehicle speed in F-format, followed by the corresponding value of engine speed, again in F-format.
Listing of Program B02

Maximum Vehicle Speed Program
*FORTRAN B051, G.G. LUCAS TRANS TECH,
MASTER B051
COMMON AT, BT, CT, DT, ET, FT, GT, VRS, AD, BD, AK, OGR, WV, GRAD, WV1, AI, R, RR, XK
5 CONTINUE
WRITE(2,100)
100 FORMAT(10X19HG,G,LUCAS AUTO DEPT/PX37HLOUGHBOURGH UNIVERSITY OF T
TECHNOLOGY/15X21HMAXIMUM VEHICLE SPEED/)
READ(1,101)
101 FORMAT(60H
1)
WRITE(2,101)
WRITE(2,102)
102 FORMAT(1HO/)
1 READ(1,103)AT,BT,CT,DT,ET,FT,GT,VRS,RMIN
103 FORMAT(10FO,0)
IF(J=1)21,12
2 READ(1,103)WV,GRAD,WV
IF(J=2)3,13
3 READ(1,103)AD,BD,AK
IF(J=3)4,14
4 READ(1,103)GR,DAI,RR,Al,XK
X=XX/1000000
11 CONTINUE
OGR=3.1514926+15.*RR/660./DAR/GR
WRITE(2,104)AT,BT,CT,DT,ET,FT,GT,VRS
104 FORMAT(15X29HENGINE TORQUE CHARACTERISTICS927X18H MAX ENGINE SPEED
1/7F10,5,F18.1)
WRITE(2,112)
112 FORMAT(1H=8X7HREV/MIN/)
WRITE(2,105)AD,BD,AK,WV,GRAD,WV
105 FORMAT(5X17HDRAG COEFFICIENT8X18HVEHICLE WEIGHT LBF,8X8HGRADIENT8
1X17HIND SPEED MILE/H /3F9.6,F14.1,F22.5,F20.2/)
WRITE(2,106)GR,DAR,RR,OG,GR
106 FORMAT(17H GEAR BOX RATIO =F9.4/19H DRIVE AXLE RATIO =F9.4/30H ROL
1LING RADIUS OF WHEELS FT =F8.4/49H OVERALL GEAR RATIO MILE/H PER
2ENGINE REV/MIN =F1.1,7/21H TYRE GROWTH FACTOR =F14.10/)
PRINT=1
H=OGR*VBS
G=OGR*GRMIN
X=H,.6
A=5.
S=1.5
EPS=.08
CALL PXMEW2 (F,X,A,.EPS,1,PRINT,G,H,S)
WRITE(2,102)
WRITE(2,107)X
107 FORMAT(37H MAXIMUM VEHICLE SPEED MILE/H =F9.3)
WRITE(2,111)R
READ(1,108)J
108 FORMAT(12)
   IF(J)12,12,0
   WRITE(2,109)
109 FORMAT(1H1)
   GO TO (1,2,3,4,5),J
12 CONTINUE
   WRITE(2,110)
   STOP
110 FORMAT(23HO END OF CALCULATIONS)
111 FORMAT(30HO ENGINE SPEED REV/MIN =F9.1/
   END
   FUNCTION FUNC(X)
   COMMON AT, BT, CT, DT, ET, FT, G, AT, BT, AD, BD, AK, OGR, WW, GRAD, WV, AI, R, RR, XK
   DF=WW*(AD+G*AD*X+BD)*AK*(X+WV)**2
   TF=(.96-.00316*X+.000058*X*X)*(,99758*(1,-.007*AI)-,0000879*X**2
   1,08**AI)
   RX=RR*(1,+.X=(X=900,))
   R=RX/OGR/1000,/(RX*RR)
   T=AT+BT*R+CT*R+DT*R**3+ET*R**4+FT*R**5+GT*R**6
   TF=T*TF+15**3,14159265/660,/(OGR/RX*RR)
   R=R+1000,.
   FUNCTION(TF=DF)**2
   RETURN
   END
SUBROUTINE XMEX2(F, X, A, B, EPS, N, X, G, H, S)
   ICO=10
   WRITE(2,200)ICO,F,P,X
   J=2
   F=B*FP
17 FKP=F
   1 X=X+A
   IF(N)3,2,3
   2 ICO=ICO+1
   F=B*FP
   GO TO 9
   3 IF(G=X)14,4,5
   4 IF(X=W)12,2,6
   5 I=G
   WRITE(2,201)
   201 FORMAT(15H AT LOWER BOUNDARY)
   GO TO 7
   6 X=H
   WRITE(2,202)
   202 FORMAT(15H AT UPPER BOUNDARY)
   7 IF(N)18,8,8
   8 N=1
   GO TO 2
   9 IF(K)11,10,11
   10 FP=BS*PI
WRITE(2,200)IC0,FP,X
11 IF(N+1)=12,20,12
12 IF(FI=F)14,13,13
13 F=F1
   A=S+A
   J=2
   GO TO 1
14 X=X-A
15 A=-5*A
   J=J-1
   IF(J)=16,1
16 FP=S+F
   WRITE(2,200)IC0,FP,X
   J=2
   GO TO 23
18 CONTINUE
   FP=FUNC(X)
   WRITE(2,200)IC0,FP,X
   GO TO 21
23 IF(ABS(A)=EPS)21,21,17
21 F=FP
   RETURN
200 FORMAT(I5,2E20,8)
20 N=2
   F=F1
   GO TO 15
END
+DATA
Typical Print-Out From Program B051
MAXIMUM VEHICLE SPEED

7 CWT VAN STRAIGHT RUN RR=0(V)

ENGINE TORQUE CHARACTERISTICS

MAX. ENGINE SPEED

DRAG COEFFICIENTS

VEHICLE WEIGHT LBF

VEHICLE WEIGHT

VEHICLE WEIGHT LBF

VEHICLE WEIGHT

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SECTION BI2

Concluding Remarks

Section BI2 and Appendix ABS show that the accuracy of the manual time to speed program EOOI to be within the experimental accuracy of the road tests conducted by the semi-technical press. The accuracy may be less adequate when compared against the results of professional tests. However, more sophisticated instruments and a better technique cannot eliminate the differences between vehicles of the same design and the effects of the environment. This latter point is studied in Part C while Part D develops a technique of obtaining reliably and accurately the drag coefficients of a vehicle from a deceleration test. The repeatability of such tests can be used therefore, to assess the instrumentation used for vehicle performance work.

A study of Part B reveals that the accuracy of prediction may be improved by further and detailed research into the provision of a more realistic expression for transmission efficiency particularly. This point is explored further in Part D, Section 1(a).

Part C, Section 3 shows that significant errors can be introduced by inadequate vehicle drag data. The literature survey (Part A, Section 2 (8a)) emphasizes the care required with wind tunnel tests using models. The use of a full scale wind tunnel avoids the difficulties with models, but it is expensive. Part D therefore, sets out to overcome the main disadvantages of the deceleration test, a simple and acknowledgedly accurate method of obtaining the drag coefficients (24). The deceleration test will cater not only for the aerodynamic drag, but the rolling resistance also. An aspect of
vehicle drag which requires more investigation is the effect of
torque transmission through the tyre on rolling resistance.

Other, less important, areas requiring a more detailed study
are the effect of transient operation on engine torque output,
the mechanics of vehicle "take-off" and of "wheel-spin".

The main use envisaged however, for such performance prediction
programs is in design or parametric studies. The degree of accuracy
therefore, is not of paramount importance, as long as the program will
predict faithfully the effect of a particular design parameter. Part
C is devoted largely to such parametric studies.

Parametric studies usually require a realistic set of gear ratios.
If a particular design parameter is varied outside the normal range
experienced by the designer, such that he can no longer use his
experience in fixing the gear ratios, the use of the design procedure
outlined in Section B10 should prove invaluable.

Another innovation in performance program B001, which has not
been found in other programs surveyed, is the treatment of the gear
change points as dependent, rather than independent, variables. The
parametric studies in Part C and the optimisation procedures used in
fixing the intermediate gear ratios in Sections B10 and B11 emphasize
that the gear change points must be left to the program. An unnecessary
and undesirable constraint is imposed by their predetermination.
PART C

Parametric study of vehicle

accelerative and maximum

speed performance
SECTION C1

Introductory Remarks

Having developed the theory for the manual transmission performance computer programs in Part B, Part C illustrates some of their uses. The aim here is to study the effect of the main vehicle design parameters on acceleration and maximum speed and to demonstrate the versatility of the programs as design tools.

In consideration of some of the design parameters, such as vehicle weight, the effect is well known qualitatively. An increase in weight will decrease both the acceleration and the maximum speed of a vehicle. Often however, the Designer does not know quantitatively the change in performance for a particular case. This lack of knowledge may place an unwarranted constraint on his design.

The general trends of the parametric studies in Part C, with two exceptions, may be seen to be the expected trends. The point of interest is that numerical values are placed on the effect of gear ratio, vehicle weight, gradient, ambient temperature etc. for the cases considered.

It is shown the effect of drive axle ratio on time to speed does not follow entirely the expected trend. The other exception is the effect of ambient temperature on accelerative performance.

Included also in Part C is an appraisal of Pearson's I.Mech.E. paper (25) on the "Midland Red Coach" in an attempt to discover why his calculated performance did not correspond with later measured results.
SECTION C2

Effect of vehicle weight

It is perhaps relevant to start the analysis of the many vehicle design parameters affecting performance by studying vehicle weight. It can be said immediately that reduction in all-up-weight will improve performance, but there comes a point when it becomes expensive to reduce weight because vehicle design and production become more complex in order to maintain strength and body rigidity. What, therefore, is the incentive to reduce weight?

Table C2.1 lists the results of time to speed calculations of Vehicle A using program SOL. The vehicle weight was varied from 30% below normal to 30% above. Listed are the resulting times up to 20, 30, 40, 50, 60 and 64 mile/h.

Fig. C2.1 plots the information in Table C2.1 on a percentage basis in an attempt to generalise the conclusions to be drawn. The percentile datum being the normal vehicle weight of 2128 lbf.

When the final vehicle speed is low, say up to 40 mile/h, with Vehicle A, a near linear relationship exists between time to speed and vehicle weight. This is because the aerodynamic drag is low, hence the force opposing vehicle motion is approximately proportional to vehicle weight. As the final speed is increased, say to 64 mile/h with vehicle A, the relationship is less linear. Increase in vehicle
weight producing an over increasing time to speed. If vehicle weight is progressively reduced, the gain in the time to a high final speed becomes less and less.

The important point to note is the effect of weight upon the initial stages of a standing start. The reputation of a vehicle for good performance ("nippiness") rests with the 0 - 20 mile/h. time, largely because the public roads are such that it is not practical to test a vehicle over a greater speed range. The traffic lights form a useful starting point and the performance of other vehicles a useful assessment reference. Fig. 02.1 shows that the effect of change in vehicle weight is progressively more effective in the early stages of a standing start.

A rough rule-of-thumb guide seems to be, therefore, that a 10% reduction in vehicle weight produces a similar reduction in the time to cruise speed, the higher the final speed, the greater the effect of change in weight.

Turning now to the effect of vehicle weight upon maximum vehicle speed. Fig. 02.2 shows a plot of percentage change in all-up-weight for Vehicle A. The maximum speed computer program B051 was used for this work. The small scale scatter of the graph reflects the ± 0.05 mile/h. accuracy in the determination of maximum vehicle speed. The reason for this may be seen in Fig. 02.3. This is a sketch of the full throttle power available curve and the load

The relationship is virtually linear and suggests that a 10% increase in all-up-weight produces a 1% only decrease in maximum vehicle speed.
requirement curves plotted against vehicle speed. Maximum vehicle speed is at the intersection of the power available and the appropriate load curve. Increasing the vehicle weight results in the dotted load line which crosses the power available curve at a slightly lower speed only. The predominant force resisting motion of a motor car at maximum speed is that of aerodynamic drag. Rolling resistance, and hence vehicle weight, have little effect.
SECTION C3

Effect of vehicle drag

1) Rolling resistance

Fig. C3.1 shows the calculated time to speed of vehicle B fitted with
(a) cross ply tyres of zero growth factor
   \( Ad = 0.018 \)
(b) radial ply tyres of zero growth factor
   \( Ad = 0.013 \)
(c) idealised tyres of zero growth factor and zero rolling resistance as a basis for comparison.

Vehicle B is fitted with radial ply tyres as standard. Using the cheaper cross ply tyres produces little deterioration in acceleration until 60 mile/h is reached. From then until maximum vehicle speed, the difference is measurable.

Repeating the calculation for zero rolling resistance emphasises the loss in performance caused by rolling resistance.

Table C3.1 gives the corresponding maximum vehicle speeds on the level. Here there is a small but significant difference between radial and cross ply tyres. 106 mile/h represents the ultimate maximum speed in terms of reduction in rolling resistance.

If the research into the reduction of rolling resistance has reached an advanced stage, such that it is expensive to effect a
measurable reduction, then it may well be more profitable to channel development energies elsewhere from the point of view of vehicle performance. Say into producing slightly larger, low cost power units which would give the same performance benefits. This is almost certainly true if the research into lower rolling resistance tyres has to be paid for by higher tyre prices.

2) **Effect of tyre growth**

The "Dunlop" tyre growth expression given in Part B, section 5 was added to the vehicle performance programmes at quite a late stage. Previously, it had been assumed that tyre growth had little effect. It is of interest, therefore, to assess the influence upon the time to speed and upon maximum speed of the vehicle.

Calculations were based upon Vehicle B, which has radial ply tyres as standard. It is a reasonable assumption to say that the rolling radius of a radial ply tyre remains constant with vehicle speed. If, however, a growth rate equal to that of a cross ply tyre is assumed and the performance re-calculated, an assessment of the effect of tyre growth may be made. Such an effect is not possible to measure in practice since a cross ply tyre has a higher growth rate and a higher rolling resistance. It is not possible to divorce the one from the other. However, no such
limitation exists with theoretical calculations.

Fig. C3.2 shows the calculated time to speed of the high growth rate radial ply tyre expressed as a percentage of that for the standard, zero growth rate radial ply tyre plotted against vehicle speed.

The initial small gain in favour of the high growth rate tyre is artificial because the Dunlop formula is based upon the rolling radius at 30 mile/h. Since the same "static" rolling radius of 0.9707 feet was used in both cases, the high growth rate tyre was interpreted as having a smaller rolling radius below 30 mile/h. The initial, slightly greater acceleration of the high growth tyre reflects this effect.

The gear change speeds are shown on Fig. C3.2 because they are slightly different in the two cases and because the gear changes themselves have some effect. A deflection from the general trend of the graph towards a greater acceleration with the high growth rate tyre may be seen at the second to third and, more particularly, at the third to top gear change points. This effect is studied in detail in Section C7 which deals with the effect of drive axle ratio.

The general trend, therefore, is to a poorer time to speed. It is less than 2% and there is the mitigation of the local better time to speed at the gear change points.
The effect of tyre growth alters the degree of undergearing of Vehicle B from very slightly undergeared (1.0093) to very slightly overgeared (0.9669). The effect of tyre growth, therefore, on maximum vehicle speed is very small indeed in the case of Vehicle B. Certainly it is less than the ± 0.5 mile/h. accuracy of program BOO1, since the same figure of 101.1 mile/h. is returned in both cases.

In the early stages of the development of the vehicle performance programs it was noticed that if a good agreement was obtained between calculated and measured time to speed, the calculated maximum speed of the vehicle was less than the measured. This, as explained in Part B, Section 12 is thought to be due to an oil temperature increase and hence less drag in both the engine and the transmission and is a direct result of the long sustained run necessary to obtain the highest possible vehicle speed. The correlation is much better if a definition of maximum speed similar to the "maximile" of "The Motor" is adopted. It had been suggested, however, that the lower calculated maximum speed was because allowance for tyre growth had not been incorporated in the calculations. The foregoing shows that this is not the case.

It is agreed, however, some allowance for tyre growth, where it exists, should be made and that the Dunlop formula is as good as any other for this purpose. Accordingly, it is now an integral part of the vehicle performance programs.
Effect of Aerodynamic Drag

Reduction in the aerodynamic drag coefficient produces a significant drop in the horsepower required to move the vehicle at a speed near its maximum speed (24). One is tempted to think, therefore, that an all-out campaign for the reduction in aerodynamic drag is required. White, R.G.S. of The Motor Industry Research Association, in his survey of the aerodynamic drag coefficients of modern vehicles (9), shows that a typical drag coefficient is \( C_D = 0.45 \). Some cars with poor aerodynamic shapes approach 0.56 while other cars (often of continental manufacture) have a drag coefficient less than 0.4. A notable car of French manufacture is listed as having a coefficient \( C_D = 0.33 \). White concludes from this work that it is not too difficult to obtain a drag coefficient approaching 0.3 and urges (24) that this should be done.

Now the saving in horsepower as a result of a reduction in drag coefficient has a dramatic effect on maximum vehicle speed. Performance calculations on Vehicle B show that by reducing \( C_D \) from its normal 0.428 to 0.320, that is in the ratio 4 to 3, increases the maximum speed of the vehicle from 102 mile/h approximately to 109 mile/h, approximately. A significant increase. Vehicle B is not unlike the example chosen by White in his paper (24) to illustrate this point. Extrapolation of his Fig. 8.1 to cater for the greater power of Vehicle B produces almost
identical maximum vehicle speeds for the two aerodynamic drag coefficients.

However, the importance of maximum vehicle speed as a performance parameter is declining. Consideration must, therefore, be given to the effect of aerodynamic drag coefficient upon the full throttle acceleration of the vehicle up through the gears.

Fig. C3.3 depicts the results of performance calculations on Vehicle B. Decreasing the aerodynamic drag coefficient from its normal value of \( C_D = 0.428 \) to 0.32 produces no discernable difference in the time up to at least 60 mile/h. This seems surprising at first, until one recalls that a car having a roof rack heavily laden with bulky holiday luggage is at no real disadvantage compared to a similar car carrying a similar load having no roof rack. The addition of luggage on a roof rack can double the aerodynamic drag of a vehicle. Not only is \( C_D \) increased but the projected frontal area also.

It was suggested, however, that a car having a low power to weight ratio would be much more sensitive to changes in the aerodynamic drag coefficient. Fig. C3.4 shows the calculations repeated for Vehicle B with its normal 2 litre engine replaced by the 1 litre engine of Vehicle A. Again, a change in the aerodynamic drag coefficient from \( C_D = 0.428 \) to 0.32 produces a significant change in maximum vehicle speed, increasing it from 74 mile/h to
78 mile/h. Approximately. However, the time to speed characteristic is very similar to that of a high power to weight ratio vehicle. There is very little difference until a speed approaching maximum vehicle speed is reached. The gear ratios used in this latter exercise were those of Vehicle A, the drive axle ratio being modified to 4.789 in order to maintain a similar degree of undergearing value to the normal value for Vehicle B.

The reason for the small difference in time to speed is that in the lower speed range and particularly in the lower gears, there is plenty of force available for acceleration. A small change in drag is insignificant compared with the force necessary to accelerate the vehicle and the engine.

The above time to speed studies assume no change in drive axle ratio in order to maintain a constant degree of undergearing as the aerodynamic drag is changed. Reducing the drag coefficient effectively undergears the vehicle, which should therefore produce a more favourable time to speed. Thus a change in drive axle ratio in order to produce a constant degree of undergearing would result in even less difference in time to speed than is shown in Figs. C3.3 and C3.4.

When contemplating a change to the external shape of a vehicle for the purposes of reducing aerodynamic drag, other factors have to be considered. The more important are those of market appeal (styling) and production. Changes to the external shape can have profound effects here. Other factors include the desirability of the ons
ensuring that the driver of the vehicle may see the extremities of vehicle when manoeuvring. This latter point increases in importance as traffic density and parking problems increase.

Certainly the aerodynamic drag coefficients of vehicles of British manufacture should be decreased, but Figs. C3.3 and C3.4 show that the problem is not as urgent as might appear at first sight. The first attack should be made upon the cooling drag. Modification to the cooling system of the Citreon DS 19 reduced the overall aerodynamic drag of the car dramatically (9). Such a modification must benefit the engine cooling also as well as reducing the aerodynamic drag. A further advantage is that the external shape of the vehicle is little affected.

Such action should provide adequate breathing space to allow a detailed study of the external shape of the vehicle.
SECTION C4

Effect of Wind Speed

Wind speed is such an unavoidable feature of the practical measurement of vehicle performance that it is desirable to assess its effect.

Fig. C4.1 shows the results of time to speed calculations on Vehicle B in standard trim with a variety of head-on wind speeds, both positive and negative. Once again, an attempt has been made to generalise the results by plotting the percentage change in time to speed against wind speed, using zero wind speed as datum. The point to note is that a head wind is far more effective than a tail wind and that this effect increases rapidly as the vehicle speed increases. If wind speed is negative (tail wind) the term \((V + V_w)^2\) in the drag equation changes less rapidly than if \(V_w\) is positive. This explains the greater effect of the head wind. This effect is even more marked on the time to the higher vehicle speeds because of the influence of a significant change in vehicle drag when the vehicle acceleration is falling off rapidly anyway.

Fig. C4.2 depicts the effect of wind speed on the percentage change in maximum vehicle speed and serves to emphasize that a head wind depresses maximum vehicle speed much more than a tail wind elevates it. The curve itself is not linear, although it would seem reasonable to assume a linear relationship in the wind speed range ±10 mile/h.
It is unfortunate that the Motar Industry Research Association testing ground at Lindley is so exposed such that there are few days only throughout a year when wind speed is negligible. This is true particularly of the new twin tank, one mile straight which lies in a cutting. This twin track is used for time to speed measurements.

Road test reports compiled by the semi-technical press contain a general indication of wind speed on the day of the test, but not of its direction relative to the vehicle. The test technique includes testing the vehicle in both directions of the track and averaging out the results of both the time to speed and maximum vehicle speed measurements.

Figs. C4.1 and C4.2 show that this averaging may be reasonable in the case of maximum speed if the wind speed is low. It certainly cannot be justified in measuring the time to speed of a car with a high maximum speed, even if the wind speed is low.

The above assumes a wind speed in the straight ahead direction only. If, however, there is an appreciable cross wind component also, a yaw drag component results.

One should not put too much reliance on the time to speed figures in a road test report which contains words such as "Test Conditions, 15 to 20 mile/h. wind." A head wind of 20 mile/h causes a 30% increase in the time to 80 mile/h. with Vehicle B. A tail wind results in a 12% reduction only.
It should be possible, therefore, to devise an empirical relationship in order to allow for the effects of wind speed on an acceleration test. Fig. C4.3 shows the percentage change in time to speed, for differing final speeds, plotted against wind speed for Vehicle A. A comparison between Figs. C4.1 and C4.3 shows that the curves are very similar in form. The 0 - 60 mile/h. curve of Vehicle A is almost identical to the 0 - 90 mile/h. of the faster Vehicle B. Non-dimensionalising the final speed by dividing by the vehicle maximum speed gives

\[
\frac{\text{60}}{\text{71}} = 0.845 \text{ for Vehicle A}
\]

and

\[
\frac{\text{90}}{\text{102}} = 0.865 \text{ for Vehicle B}
\]

The suggestion is therefore, that the measured time to speed with a small head or tail wind speed can be corrected to give the time to speed on a windless day by using a correction factor which is a function of wind speed and non-dimensional speed.

i.e.

\[
\text{time}_{\text{corrected}} = \text{time}_{\text{measured}} \times \left(1 + f\left(V_w, \frac{V}{V_{\text{max}}}\right)\right)
\]

--- C4.1

By testing the vehicle, therefore, in both directions and correcting for wind speed in each case it should be sufficient then to average out any small differences in the corrected values to give a representative result.

A preliminary survey of the problem suggests that the function may not be simple.
Simplifying the dynamics of an accelerating vehicle by assuming constant engine torque and fixed gear ratio produces the expression for vehicle acceleration

\[ f = \frac{g}{k} \left( \frac{2 \pi \cdot GR \cdot DAR \cdot \sqrt{r}}{r} - W \cdot Ad - k \cdot A \cdot (V + V_w)^2 \right) \tag{C4.2} \]

which may be written

\[ f = a^2 - b^2 \frac{(V + V_w)^2}{(V + V_{w_0})^2} \tag{C4.3} \]

Thus, the time to speed becomes

\[ t = \frac{1}{b^2} \int_{V_i}^{V_f} \frac{dV}{(A)^2 - (V + V_w)^2} \tag{C4.4} \]

i.e.

\[ t = \frac{1}{a \cdot b} \tan^{-1} \left( \frac{(V + V_w)}{(A/B)} \right) \tag{C4.5} \]

This suggests that one should look to an exponential empirical function in order to correct for wind speed. A vehicle performance program, such as B001, would be invaluable in the derivation of such a formula. It would be very difficult to collect sufficient reliable data experimentally, or to deal with a sufficiently large range of vehicles.

Included in this work also should be an allowance for ambient pressure and temperature.
Effect of ambient pressure and temperature

In considering the effect of ambient pressure and temperature upon vehicle performance, there are two points to note. The first is that change in ambient conditions affects engine performance. This is mentioned in Section B2 and is dealt with more fully by the Author in chapter 7 of a book entitled "The Testing of Internal Combustion Engines" (1).

At full throttle on a spark ignition engine and at full fuel pump rack opening on a compression ignition engine, the corrected torque output is related to the measured or observed torque by

$$ T = T_0 \times \left\{ \left[ \frac{P_s}{P_{amb}} \right] \times \sqrt{\frac{t_{amb} + 460}{t_0 + 460}} \right\} $$

where $T_0$ is the observed torque output. Suffices (s) and (amb) denote standard and ambient respectively, (p) represents pressure and ($t^\circ F$) represents temperature.

Thus

$$ T = T_0 \times C_{amb} $$

where $C_{amb}$ is the correction factor for ambient conditions given by the function of pressure and temperature in the square brackets in expression C5.1

The second point to note is that a change in the density of the ambient air affects the aerodynamic drag of a vehicle.
Section B3 shows that the aerodynamic drag of a vehicle is given by

\[ F_d(aero) = C_D \times \frac{1}{2} \rho v^2 A \]  
--- C5.3

Using the characteristic equation for a perfect gas to replace air density yields

\[ F_d(aero) = C_D \times \frac{1}{2} \times \left( \frac{P_0}{R(t_0 + 460)} \right) \times v^2 A \]  
--- C5.4

where \( R \) is the gas constant for air.

Introducing the correction factor for ambient conditions affords

\[ F_d(aero) = C_D \times \frac{1}{2} \times \frac{P_0}{R \sqrt{(t_0 + 460)}} \times v^2 A \times \frac{1}{C_{amb} \sqrt{(t_0 + 460)}} \]

\[ = \text{constant} \times \frac{v^2}{C_{amb} \sqrt{(t_0 + 460)}} \]  
--- C5.5

Table C5.1 shows the percentage change in pressure and temperature, and the consequential change in aerodynamic drag, in order to effect a 5% change in the correction factor for ambient conditions.

Fig. C5.1 shows the results of time to speed calculations on vehicle A assuming a ± 5% change in the correction factor, first at constant temperature by altering the ambient pressure only and secondly at constant pressure by altering the ambient temperature.
Constant temperature change in $C_{amb}$

An increase in ambient pressure results in an increase in torque output from the engine and a similar increase in aerodynamic drag in the vehicle. Because the aerodynamic drag is of little importance in the low speed range, there is a significant decrease in the time to speed in the early stages. This advantage decreases as the vehicle speed increases until, at the higher speeds, the curve runs almost parallel to the datum curve. However, the gains in the initial stages remain. The curve is labelled A in Fig. 05.1.

If, however, the ambient pressure is decreased, producing a drop in the torque output of the engine, the effect on vehicle time to speed is more marked. The loss in torque at low speed produces a large loss in time which is not compensated for by the decrease in aerodynamic drag at the higher speeds. This curve is labelled B in Fig. 05.1.

Constant pressure change in $C_{amb}$

A decrease in ambient temperature produces an increase in engine torque. Hence the gain in time to speed is similar to that for an increase in pressure. However, decreasing ambient temperature produces such a large increase in aerodynamic drag that, at about 60 mile/h. with Vehicle A, the initial advantage is lost and thereafter an INCREASE in time to speed results. The relevant curve is labelled C in Fig. 05.1.
Increasing ambient temperature causes a reduction in engine torque output. This causes a considerable increase in time to speed in the early stages. The corresponding large decrease in aerodynamic drag begins to manifest itself at the higher speeds. The curve (labelled D in Fig. C5.1) approaches the datum curve, but does not cross it in the speed range of Vehicle A considered.

Table C5.2 lists the maximum speed of Vehicle A with changes in ambient conditions. The interesting point is that increasing the ambient temperature increases the maximum speed of Vehicle A, albeit by 0.3% only.

Change in "Under-bonnet" temperature

The temperature of the air entering the intake of a vehicle engine may be appreciably higher than ambient because it has passed over the hot parts of the engine installation. A ±10% change in intake temperature results in approximately ±5% change in engine torque output. It is relevant, therefore, to assess the change in time to speed with a ±5% change in $C_{amb}$ and no corresponding change in aerodynamic drag. Fig. C5.2 shows the results of such calculations on Vehicle A.

Increasing the "under-bonnet" temperature shows a pronounced increase in time to speed. More so than line D in Fig. C5.1 indicates where there is a compensating decrease in drag. Decreasing the under-bonnet temperature produces significant gains in time to speed.
Increasing the under-bonnet temperature to give $C_{amb} = 1.05$ results in a $-2.7\%$ change in maximum vehicle speed.

Decreasing the under-bonnet temperature to give $C_{amb} = 0.95$ results in a $+3.9\%$ change in maximum vehicle speed.
SECTION C6

Effect of Total Road Wheel Inertia

The total road wheel inertia includes also the inertia of the transmission shaft, factored by the square of the drive axle ratio.

Fig. C6.1 depicts the results of calculations on Vehicle A. Again, an attempt is made to generalise the results by plotting the percentage change in time to speed against the percentage change in total road wheel inertia.

The interesting point to note is that the relationship is linear within the practical range \( \pm 30\% \) change in total road wheel inertia. The time to a high speed is affected slightly more than the time to a low speed. However, even a \( 30\% \) change in total road wheel inertia produces less than \( 1\% \) change in the time to a speed just short of the maximum speed of vehicle A.

Since the road wheel inertia can have no effect on maximum vehicle speed, there is no point in attempting to reduce it to achieve better performance. The gains would be small compared with the effort put into the exercise.
SECTION C7

Effect of drive axle ratio

Introductory Remarks

It was thought initially that altering the rolling radius of the drive wheels was equivalent to altering the drive axle ratio. This former course was favoured because the higher the numerical value of the rolling radius, the higher the overall gearing of the vehicle. Whereas the higher the drive axle ratio, the lower the overall gearing.

Altering the rolling radius of the drive wheels produced an effect on the time to speed that was not expected. In attempting to explain this, it was realised that altering the rolling radius would modify also the term $Iw/(r_t)^2$ in the expression for the equivalent mass of the vehicle. Accordingly, the calculations were repeated, this time altering the drive axle ratio. The general form of the curves produced however, were very similar to those obtained by altering the rolling radius. The peculiarities, therefore, cannot be blamed upon any difference between altering rolling radius and altering drive axle ratio.

This type of investigation emphasizes the benefits to be derived from a vehicle performance computer program. Had the peculiarities referred to above been obtained in practical tests, one may have been very tempted to blame them upon the test drivers, or upon some change in vehicle or weather conditions.
The investigation

Fig. C7.1 depicts the percentage change from normal in time to several final speeds against the percentage change from the normal drive axle ratio of Vehicle A.

It was expected that such a graph would show a pronounced minimum on the under-g geared side. This is shown clearly in the 0-60 mile/h. and the 0-64 mile/h. lines. It was not expected, however, that another minimum would exist on the over-g geared side. Over-gearing and high acceleration are not generally considered to be synonymous.

Both minimum move to the left and diminish as the final speed is reduced, but at a final speed of 50 - 60 mile/h, yet another minimum appears on the over-g geared side. As an aid in the study of the phenomenon, graphs of the vehicle acceleration against vehicle speed (Fig. C7.2) and the corresponding propulsive force against vehicle speed (Fig. C7.3) were prepared.

These graphs are of similar form of course because acceleration is linked with force by Newton's second law, but the high propulsive force at zero speed afforded by the low gearing does not manifest itself in a correspondingly high acceleration. The reason for this is the increase in equivalent vehicle mass due to the engine inertia factored by the square of the overall gear ratio. In a low gear, the effective engine inertia is great and is of similar magnitude to the inertia of the vehicle itself. This has a depressing effect on any increase in propulsive force at low speed.
Since the effective engine inertia is altered markedly by change in overall gear ratio such that it can suppress the increase in propulsive force caused by a lower overall gear ratio, it was felt that the minima on the over-geared side were caused by engine inertia.

This was shown to be untrue by repeating the time to speed calculations for Vehicle A, this time setting the engine inertia at zero. These results are shown in Fig. C7.4. The minima on the over-geared side are still there, the curves are slightly different to Fig. C7.1, but they are essentially of similar form.

The reason for the minima on the over-geared side is shown in Fig. C7.2. The vehicle acceleration drops in the vicinity of a gear change point. As a gear change point is approached, the vehicle acceleration gets progressively lower as the engine speed increases away from maximum torque speed. This continues until it is found more beneficial to lose the torque multiplication effect of a low gear in favour of bringing the engine speed nearer its maximum torque speed. Accordingly, a gear change is made.

When a gear change from third to top becomes desirable to achieve the final speed specified, a "4" is marked at the appropriate point in Fig. C7.1. To the right of this point, the overall gear ratio is such that no gear change is necessary and that the final speed specified can best be achieved by remaining in the lower gear. To the left of this point a change becomes necessary just before the final speed is reached.
It must be emphasized here that in all these calculations, the gear change time itself is considered zero for comparison purposes. It is true that, in a practical test, one would avoid making a gear change just before the specified final speed is reached. This is because some time must be lost in the change, however small. However, this argument does not apply in this study because there is no loss in time in making the gear change itself. The loss in time occurs because of the rapidly falling acceleration just prior to a gear change.

Fig. C7.5 is a sketch of the type of curve in Fig. C7.1. The dotted line shows the expected trend of the curve if an infinitely variable gearbox were available. Because a stepped gearbox is used and it becomes necessary to engage top gear before the final speed is reached, the actual curve departs from the ideal. As the overall gear ratio is made higher (more over-geared) and it no longer becomes necessary to engage top gear, the actual curve returns to the ideal. The effect is to produce a secondary minimum in the actual curve on the over-geared side of the main minimum.

At the lower final speeds, the same argument applies to a lesser extent to the second to third gear change. This is marked by a "3" on Fig. C7.1. A figure "4" or "3" can be seen therefore just to the left of every secondary minima in Fig. C7.1. The additional minima appearing on the over-geared side at the lower final speeds are caused by the avoidance of having to engage third gear.

It is difficult to take advantage of this phenomenon in the design of a vehicle, because the secondary minima move so rapidly
as the final speed is changed. If the time to a particular final speed, say 60 mile/h is of paramount importance in a particular vehicle design, such that the times to 55 mile/h and 65 mile/h are of little consequence in comparison, then it may well be worth fixing the overall gear ratio at the secondary minima in order to gain all the benefits of over-gearing as well as a low time to speed. This could perhaps be applied at the lower speeds because the time to cross traffic lights from a standing start is important. Unfortunately, (or perhaps fortunately) the secondary minima diminish as the final speed is lowered. It may be of advantage to study the effect however, if it is intended to employ a three speed gearbox in conjunction with an inflexible engine. The secondary minima should be more marked with a three speed gearbox.

Fig. 07.6 shows the effect of drive axle ratio upon maximum vehicle speed. Again, the parameters are taken as percentages from normal. The normal drive axle ratio of Vehicle A is such that it is some 6% over-g geared. As would be expected, the highest maximum vehicle speed is obtained with unity degree of undergearing. The curve falls off steadily and symmetrically either side of the maximum.
SECTION C8

Effect of Engine Inertia

In Section C7, mention was made of the crippling effect of engine inertia upon the initial acceleration of the vehicle. It is relevant, therefore, to study in some detail the effect of engine inertia upon vehicle performance.

Table C8.1 lists the calculated time to speed of Vehicle A in its normal trim, first with the engine inertia of 0.14 slug ft$^2$, assumed throughout this work as being its normal engine inertia, and then with other engine inertias from zero to 0.6 slug ft$^2$. The assumed figure of 0.14 slug ft$^2$ was calculated from known dimensions and mass of clutch and flywheel and is of sufficient accuracy for comparative studies.

It is perhaps worthy of mention here that it is not necessary to remove and strip the engine from the vehicle in order to measure and so obtain a more accurate figure for the normal engine inertia. It is sufficient to measure the stiffness of the transmission shaft of the vehicle in situ and then to set the engine-transmission shaft system in free vibration and measure the frequency of the vibration. The rear end of the transmission shaft may be clamped or fitted with a large inertia. The engine inertia is then calculable from simple vibration theory.

Fig. C8.1 shows the information contained in Table C8.1 plotted as the percentage change in time to speed from normal against engine
inertia for several final vehicle speeds. The effect of engine inertia upon the percentage change in time to speed is linear within the practical range of engine inertia change, that is from zero to twice its normal value. If the engine inertia is increased to four times its normal value, the effect of engine inertia becomes less and less important.

Perhaps the most important conclusion to be drawn from Fig. 68.1 is that engine inertia has far greater effect during the initial stages of an acceleration run than during the latter stages. Reducing the engine inertia to 50% of its normal value causes a 20% decrease in the time to 10 mile/h. However, the gain in the time to 68 mile/h. is 5% only. The greater influence of engine inertia during the initial stages may be attributed to the effect of the lower gear ratios.

It is not suggested that the engine inertia of Vehicle A should be reduced in order to improve its take-off, because Vehicle A is not a high performance vehicle. The ability of the vehicle to crawl smoothly at low speed is more important than any gain in take-off time resulting from a smaller flywheel. However, this argument does not apply to a racing vehicle.

In order to assess the effect of engine inertia on a high performance vehicle, the calculations were repeated on Vehicle A, this time fitted with a 1500 cm³ racing engine. The results of these calculations are depicted in Fig. 68.2. Again the curves
are linear, but the effect of engine inertia is not nearly so great. Reducing the engine inertia by 50% gives a 10% only decrease in the time to 10 mile/h, compared with the 20% gain with the normal engine fitted to vehicle A.

Nevertheless, the gain is significant. It is worth reducing the engine inertia to as low a value as possible, particularly since the greatest gain is during take-off. The reason for the less spectacular percentage gain in take-off time with a high performance vehicle is due largely to the higher gear ratios. A five speed gearbox was assumed for Vehicle A fitted with the racing engine, having ratios 2.738, 2.225, 1.759, 1.421 and 1.00 in conjunction with a drive axle ratio of 4.099. This is in place of the normal four speed gearbox of Vehicle A having gear ratios 4.118, 2.396, 1.412 and 1.000 in conjunction with a drive axle ratio of 4.444. The ratios of the five speed gearbox were designed using the optimisation technique described in Part B, section 10. The higher bottom gear ratio of the five speed gearbox reduces the effective engine inertia considerably. The effect of engine inertia on the time to the higher speeds of Vehicle A is almost independent of the power unit fitted. This is because the higher gear ratios are similar.
SECTION C9

Investigation into gear ratios

Introductory Remarks

In Part B, certain theories were advocated for the rational design of the gear ratios in the transmission system of a vehicle. The selection of bottom and top gear ratios is applicable to any vehicle. The procedure for the intermediate gear ratios using an optimisation procedure is applicable largely to the design of motor car gearboxes.

Some justification for the use of these theories was made in Part B. The intention here is to carry the investigation further. This section deals with bottom gear ratios and the intermediate gear ratios only. The effect of the design of top gear ratio largely concerns the fuel consumption of the vehicle.
**Bottom gear ratio**

It was shown in Part B, Section 10 that using the theory "bottom gear should be fixed by the criterion that a vehicle shall climb the maximum gradient possible without wheel spin" produced a workable bottom gear ratio. The bottom gear ratio so designed for Vehicle A was somewhat lower than the actual bottom gear ratio, but Fig. B10.2 showed that the take-off time was almost identical for the two ratios. In fact, Fig. B10.2 showed that if the engine inertia of Vehicle A was reduced, the take-off time for the lower bottom gear ratio would be appreciably better.

Further, Part B, section 10 showed that the bottom gear ratio fixed for a high performance, racing vehicle using the above criterion was very reasonable.

Similar calculations on Vehicle B illustrated in Part B, section 10, produced a bottom gear ratio closer to actual than had been the case with Vehicle A, but that the ratio was still lower than actual. Now both Vehicles A and B have been criticised for their high bottom gear ratios (12), (13). It is perhaps relevant, therefore, to look at some other vehicles. In fact to the vehicle praised by the Consumer Association for its ability to start on a 1 in 4 gradient, the Austin A40 (12).

This vehicle has a centre of gravity position

\[
a = 3.22 \text{ ft} \\
b = 4.03 \text{ ft} \\
h = 1.7 \text{ ft}
\]
giving the maximum gradient that it is possible to climb of

\[ i_{\text{max}} = 0.5018 \]

since the vehicle has a rear wheel drive.

The test weight of the vehicle is 2050 lbf, the rolling radius of the drive wheels 0.902 ft, and the published maximum torque of the engine 60 lbf ft. This results in a calculated product of (bottom gear ratio x drive axle ratio) of 15.5, compared with 15.27 actual.

Repeating the exercise for a front wheel drive vehicle by the same manufacturer results in a calculated product of (bottom gear ratio x drive axle ratio) of 14.4 for the Morris 1100 compared with 14.99 actual.

In conclusion, therefore, the bottom gear ratio of a motor car has little effect on vehicle acceleration or upon the take-off time. This is due to the depressing effect of engine inertia. A low bottom gear ratio may cost more to produce, but manoeuvrability should be easier and smoother. There is little point in fixing bottom gear ratio lower than that advocated by the criterion above. Conversely, bottom gear ratio should not be appreciably higher than that set by this criterion.
Intermediate gear ratios

It is relevant first to investigate the result of using the optimisation technique described in Part B section 10 for fixing the intermediate gear ratios. Table C9.1 lists five sets of gear ratios for Vehicle A as follows.

1) The actual set of gear ratios. This set is used as datum.
2) Obtained by specifying the actual drive axle and bottom gear ratio and optimising on the time to \( \frac{3}{4} \) of maximum vehicle speed (54 mile/h.)
3) Obtained by allowing the vehicle performance program to fix bottom gear ratio. The intermediate gear ratios were again fixed by optimising the time to 54 mile/h.
4) Obtained by using the more detailed optimisation technique mentioned in Part B section 10. Bottom gear and drive axle ratio were specified as the actual values and the intermediate gear ratios optimised to give a minimum of the sum of the times to

\[
\frac{1}{4} \times 0.9 \; V_{\text{max}}, \frac{1}{8} \times 0.9 \; V_{\text{max}}, \frac{1}{4} \times 0.9 \; V_{\text{max}}, \frac{1}{8} \times 0.9 \; V_{\text{max}}, 0.9 \; V_{\text{max}}
\]

5) Obtained by using the variation on the more detailed optimisation technique which emphasises still further the lower speed ratio.

The sum of the times to speed optimised are

\[
\frac{1}{4} \times \frac{3}{8} \; V_{\text{max}}, \frac{1}{8} \times \frac{3}{8} \; V_{\text{max}}, \frac{1}{4} \times \frac{3}{8} \; V_{\text{max}}, \frac{1}{8} \times \frac{3}{8} \; V_{\text{max}}, \frac{3}{8} \; V_{\text{max}}
\]

Again, actual drive axle and bottom gear ratios were used.
Vehicle performance program BOOI was used to evaluate the time to 68 mile/h. for each set of gear ratios for Vehicle A in standard trim. The results of those calculations are shown in Fig. C9.1 as a plot of percentage change in time to speed from actual against vehicle speed.

A study of Fig. C9.1 shows that it would appear impossible to score throughout the speed range on the actual gear ratios. A gain in the time to speed at one particular speed is accompanied by a loss at another speed.

Set 2 produces substantial gains in the range 40 to 55 mile/h. and a small loss in the 20 - 35 mile/h. range. Above 55 mile/h set 2 is rather worst than actual. The speed range 0 - 20 mile/h. results in the same time to speed as the actual set because the same bottom gear ratio was used. The result for Set 2 emphasizes how much easier it is for the optimisation procedure to gain time at the higher speeds. This point was discussed in Part B Section 10.

The curve for the first of the more detailed optimisation, labelled Set 4, shows that the attempt to cater for the majority of the speed range of the vehicle and to emphasize the lower speed range failed because of the point mentioned above. The optimisation procedure finds it much easier to save a second in the higher speed range than in the more important low speed range. At 30 mile/h. with Set 4, the time to speed is some 2½% worse than actual. However, substantial gains are made towards maximum speed.
Set 5 was produced by cutting out the higher speed range altogether and so placing more emphasis on the lower speed range. This resulted in a significant improvement in the 35 to 55 mile/h. speed range with a small loss only in the 20 to 30 mile/h. speed range.

Gear ratio Set 3 was produced by optimising on the time to \( \frac{3}{4} \) maximum vehicle speed and allowing the performance program to choose bottom gear ratio in accordance with the criterion laid down in Part B section 10. This puts bottom gear ratio rather low at 5.12 compared with the actual ratio 4.118. Fig. C9.1 shows that Set 3 produces the desired initial gain (albeit a very small gain) of some 1\( \frac{1}{2} \) in the time to 10 mile/h. This changes to a 2\% loss at 20 mile/h. and does not show any further gains until a vehicle speed of 40 mile/h. is reached.

Before drawing general conclusions, it is desirable to study a set of higher intermediate gear ratios and a set of low ratios. The ratios listed in Table B10.3 are convenient for this purpose. These ratios were evaluated for vehicle B by optimising the time to several final speeds. When the specified final speed was low, a set of low intermediate gear ratios resulted.

All six sets of gear ratios in Table B10.3 have the drive axle ratio fixed at 3.54 and bottom gear ratio fixed at 3.624. Again, the actual set of gear ratios is taken as datum and Fig. C9.2 shows the percentage change in the time to speed against vehicle
speed for each of the sets.

The intermediate gear ratios resulting from optimising the time to \( \frac{3}{4} \) maximum speed of Vehicle B, 2.059 and 1.434, produce a time to speed characteristic similar to that of the actual set within \( \pm 1\% \). It is similar in shape to Set 2 in Fig. 69.1 in that there is a slight loss in time at low speeds and a gain as the \( \frac{3}{4} \) of maximum vehicle speed is approached.

As the intermediate gear ratios become lower (larger numerically) there is an initial gain in time to speed followed by a larger loss as vehicle speed increases.

A high set of intermediate gear ratios, 1.863 and 1.258 produces a significant loss in time to speed throughout most of the running range. Only at 85 mile/h. does it start to show a gain.

The general conclusion may be drawn that optimising the intermediate gear ratios on the time to \( \frac{3}{4} \) maximum speed produces a reasonable set of gear ratios. The intermediate gear ratios for the two cases evaluated tend to be slightly high in that it would be of benefit to gain a little more time to speed in the lower speed range. In the absence of a large amount of experience in the design of gear boxes and in fixing gear ratios, the use of the optimisation procedure is viable. It is not necessary at this stage to resort to the more detailed optimisation procedure because the results may not be better. The additional computation time involved is quite considerable.
An important consideration in the design of a gearbox is the number of gear ratios to use. Table C9.2 lists the gear ratios produced by the vehicle performance program for Vehicle A assuming a 3-speed, 4-speed and then a 5-speed gearbox. Bottom gear and the intermediate gears were fixed by the criteria laid down in Part B, Section 10 and mentioned above. Drive axle ratio was taken as 4.444 throughout.

Fig. C9.3 depicts the percentage change in time to speed against vehicle speed for the 3 and 5 speed gearboxes, the 4 speed gearbox being taken as datum.

The initial gain in time with the 3 speed gearbox and the initial loss with the 5 speed gearbox is false. It stems from the expression of overall transmission efficiency used. The expression is a function of gear number, rather than gear ratio. This means that the transmission efficiency for bottom gear of a 5 speed gearbox is set lower than that for bottom gear of a 4 or a 3 speed gearbox. This may not be true.

Above the bottom gear regime the 3 speed gearbox is markedly inferior. The loss in acceleration in the mid speed range is high. The point to note is that the 5 speed gearbox is only slightly better than the 4 speed gearbox. It is doubtful if the margin would justify the additional expense of the 5 speed gearbox.

Table C9.3 lists the "overlap" speeds for the three gearboxes. The lower figure for each gear corresponds to engine maximum torque
speed and the upper figure to the maximum allowable engine speed. Table C9.3 shows that the 3 speed gearbox must be ruled out anyway because there is no overlap between first and second gear ratios. The 4 and 5 speed gearboxes are satisfactory in this respect. Some overlap is essential for hill climbing and for general flexibility considerations.
Concluding Remarks

A low bottom gear ratio is of advantage for a smooth take-off and in manoeuvring. There is no disadvantage in the initial acceleration, in fact there is a small gain. One British motor car manufacturer fixes bottom gear ratio at a figure very similar to that given by the theory in Part B, section 10 and is praised by the semi-technical press for the gradeability of his motor cars. Other manufacturers advocating a high bottom gear are criticised. For the little extra cost involved it is worth designing bottom gear ratio of a motor car to cater for the maximum gradient that it is possible to climb, without wheelspin.

Using the theory developed in Part B, section 10 for the design of the intermediate gear ratios of a motor car produces a sensible, well balanced set of gear ratios. These can be improved by an extensive further investigation or by considerable experience in the design of gearboxes for similar motor cars.

Top gear and/or drive axle ratio are usually fixed in conjunction with fuel consumption considerations. This is investigated in Part E.
SECTION C10

Gradeability

Any theoretical performance assessment of a vehicle should include an analysis of its performance on a gradient. It should be established that there is reasonable "overlap" on the gear ratios to cope with hilly terrain. It can be very disconcerting to find that neither the gear ratio engaged at the moment or the one lower will cope with a particular gradient. The engine speed in the existing gear ratio being too low and the engine speed in the lower gear ratio too high. This is even more marked when it is remembered that no hill is of constant gradient for long and that the driver has to cater for changing traffic conditions.

The time up through the gears on a gradient is not very important. Such a condition is not an established performance criterion. The characteristics of the time up through the gears will be very similar to those of rolling resistance coefficient $A_d$ because the two terms are added together in the drag equation and hence are mutually interchangeable numerically.

$$F_d = W(A_d + B_d \cdot V) + \text{constant} \times V^2$$

Hence reference should be made to Fig. C3.1 and perhaps the graph showing the effect of the percentage change in vehicle weight, Fig. C2.1.

The point to study, therefore, is the maximum speed of the vehicle on a specified gradient and the appropriate gear ratio.
This was achieved by using the main vehicle performance program $BOOl$, specifying the normal particulars of the vehicle, the gradient and an impossibly high final speed $V_F$. Vehicle $B$ was used for this work and final speed $V_F$ was set at 200 mile/h. This means that the speed at which "set speed unobtainable" is printed out is the maximum speed approximately on the specified gradient. The gear ratio engaged at the time therefore is the appropriate gear ratio. This work could have been carried out just as well using the maximum vehicle speed program $BO51$.

Fig. C10.1 is a plot of the maximum speed of vehicle $B$ in normal trim against gradient $(i)$. The numbers used to define the points are the appropriate gear ratio numbers.

It was thought desirable to mark on Fig. C10.1 the point corresponding to engine maximum torque speed in each gear and the maximum vehicle speed on the level in each gear. The information for the latter is printed out in the results. The former is readily calculable because the vehicle speed corresponding to engine maximum torque speed is

$$V = \frac{N_T \times \text{overall gear ratio in top gear}}{\text{gear ratio}}$$

Since the engine maximum torque speed of Vehicle $B$ shown in the printed results is 2770 rev/min and since the overall top gear ratio is 0.0196 mile/h. per rev/min, the vehicle speeds corresponding to engine maximum torque speed are as listed in Table C10.1 for each of the four gear ratios.
The gradient which renders this speed the maximum speed of the vehicle is calculable from

\[
\left( \frac{T \times GR \times DAR \times \gamma_T}{F_F} \right) - W (Ad + 1 + Bd \cdot V) - \text{Const.} \times V^2 = 0
\]

hence

\[
i = \frac{1}{W} \left( \frac{T \times GR \times DAR \times \gamma_T}{F_F} \right) - \frac{(\text{Drag force on level})}{W}
\]

The drag force on the level is obtainable for the appropriate speed from the results of gradient equals zero calculation.

Fig. C10.1 and Table C10.1 show that the maximum gradient that it is possible for Vehicle B to climb theoretically is

\[i = 0.401\]

which is a gradient of 1 in 2.5.

Fig. C10.1 shows also that there is sufficient overlap between the gear ratios for satisfactory performance and that this situation should not change as the engine wears and is no longer capable of delivering its scheduled 107.8 lbf ft maximum torque.
SECTION C11

"The Midland Red Coach"

Whenever vehicle performance engineers meet, the conversation invariably includes discussion of a paper by J. Pearson entitled the "Design and Operation of Motroway Coaches" (25). This paper was included in the Proceedings of The Automobile Division of the Institution of Mechanical Engineers (1961-2) and has become a classic example in the vehicle performance field.

Briefly, with the advent of the Motorways and, in particular, the building of the M1, the Birmingham and Midland Motor Omnibus Co. Ltd. set out to design a coach to travel between London and Birmingham at cruising speeds up to 80 mile/h. This was a departure from the usual practice since, hitherto, the law had limited the speed of coaches to 30 mile/h.

A major worry of the designers seems to have been the size of the engine. From engine tests and deceleration tests (see Part D of this thesis) the calculated horsepower required by the vehicle at 80 mile/h. was 189 hp. However, either because such a size of engine was not available or because the engineering intuition of the designers told them that something must be wrong with the calculations, a smaller 135 hp engine was fitted. A study of the discussion to the paper seems to indicate that a larger engine was not available.
This engine, after some little development, proved adequate. Pearson (25) states "No satisfactory answer can be given to explain the difference between these two (horsepower) figures."

The performance interest in the coach was not so much upon acceleration as upon maximum vehicle speed, that is cruising speed. What was wrong with the calculations? Why were the designers some 50% out in their calculation of the size of engine?

Sufficient figures are given in Pearson's paper to provide, either directly or by deduction, the data for the vehicle performance program. Measured times to certain speeds and maximum coach speeds also are quoted in the paper. It is of interest, therefore, to repeat his calculations in an attempt to pinpoint the source of the discrepancy.

Pearson's Fig. 15 entitled "314 K.L. engine turbocharged" gives a plot of the torque curve of the engine. The "dashed" line was taken since this was the torque curve after some service development of the fuel pump.

The coach was fitted with a five speed gearbox, the quoted ratios being 4.968, 2.74, 1.617, 1.0, 0.73 in conjunction with a drive axle ratio of 4.44. Tyre sizes are given from which the rolling radius of 1.66 ft is known (27) and the total inertia of the road wheels, 42.0 slug ft², may be deduced.

Wind speed and gradient were set at zero and the coefficient between tyres and ground at unity.
The fully laden weight of the coach is given as 21560 lbf together with the fully laden axle loads. From this information and dimensioned outline drawings of the coach, the position of the centre of gravity was fixed. The height of the position of the centre of gravity was estimated.

The coach was fitted with a manual gearbox and, since the inertia of the moving parts was large, it was not thought reasonable to assume a gear change time of zero as had been the case with motor cars. It is difficult to estimate the gear change time of a fully laden coach when the driver is striving to obtain the lowest time to speed. After some thought, the figure of 1 second was assumed for the gear change time.

Estimating the drag coefficients of the coach was not quite so straightforward. Adopting the usual practice for rolling resistance of setting $B_d = 0$ and taking $A_d$ from Table B3.1 gives the rolling resistance coefficient $A_d = .009$. The projected frontal area of the coach may be estimated from the dimensioned outline drawings shown in the paper at 72.0 ft$^2$. It was thought preferable to estimate from the dimensions given rather than to measure the area of the front elevation drawing of the coach because the drawings may be "not to scale".

The aerodynamic drag coefficient presented the greatest difficulty. Two pieces of information in the paper enabled a calculation of aerodynamic drag coefficient. The first is Pearsens Fig. 14 entitled "CM5 tractive resistance" in which is
plotted air drag horsepower against coach speed. Converting this data to air drag force against coach speed produces Fig. C11.1. The slope of a reasonable straight line through the points being 0.0859 lbf/(mile/h)^2. This produces an aerodynamic drag coefficient of

$$C_D = 0.465$$

The other line of attack stems from the authors reply to a question contained on page 290 in the discussion of the paper in which he says that from the coasting deceleration tests a value for K in the air resistance formula was found to be 0.00126. This puts the aerodynamic drag coefficient at

$$C_D = 0.493$$

Now these figures, whilst consistent, seem rather too high for a coach, even bearing in mind that this coach had a "lantern" type or built up windscreen rather than the "wrap-round" type of a modern coach. No importance can be attached to the consistency of the two figures since they both must stem from the same source. That is from the deceleration tests. If the tests were not carried out correctly, then both figures are wrong.

Hoerner (26) quotes an aerodynamic drag coefficient of 0.45 for a 1940 "Standard" omnibus with protuberances and a rough under side. The omnibus depicted is of a much earlier design than the Midland Red Coach under consideration. It is shown to have a long bonnet typical of the American coaches of that period.
With a faired underside and no protuberances the aerodynamic drag coefficient is reduced to 0.27. Modifying the standard omnibus still further by giving it a blunt rear end results in an aerodynamic drag coefficient of 0.25. Hoerner states also that a "box" on wheels having rounded edges equal in radius to 0.1 times the height of the box has an aerodynamic drag coefficient of 0.46.

It may well be, therefore, that the aerodynamic drag coefficient of the Midland Red Coach is too high. Certainly this is the obvious figure to question because, as was shown in Section 3, changes in the aerodynamic drag coefficient results in large changes in maximum vehicle speed.

Table C11.1 lists the results of calculations using the vehicle performance program B001 with a number of different aerodynamic drag coefficients ranging from 0.28 to 0.465. Other data is as described above. Also given in Table C11.1 for comparison are the measured times to speed and the maximum coach speed quoted.

The figures given in Table C11.1 show that an aerodynamic drag coefficient of 0.28 fits both the measured acceleration figures and the measured maximum coach speed almost exactly. However, it must be admitted that it is difficult to believe that this coach has an aerodynamic drag coefficient as low as 0.28. A private conversation with Mr. R.G.S. White, who is in charge of the aerodynamics section at The Motor Industry Research
Association, revealed that such a low aerodynamic drag coefficient could just be possible.

Pearson, on page 289 of his paper (25), implies that the deceleration tests were carried out on an earlier design of coach having an identical front to the projected coach. This means that the initial speed of the deceleration test would be limited to some 55 mile/h. He goes on to say that shape of the new coach was good aerodynamically and that the wind noise was extremely low even at speeds of 80 mile/h.

Fig. C11.1 shows a moderate degree of scatter at the low speed points. This almost certainly occurred in extracting the information from Pearson's Fig. 14. However, had the deceleration test been carried out carelessly (see Part D), a similar degree of inaccuracy may well have occurred. Extrapolation of the results from a low speed deceleration test to 80 mile/h. may well result in very inaccurate drag coefficients. The dashed line on Fig. C11.1 through the low speed points corresponds approximately to $C_D = 0.28$ emphasizing this possibility. The 55 mile/h. point is marked on Fig. C11.1 for guidance.

There are other acceleration figures and another maximum speed figure quoted in Pearson's paper. These were said to be returned after the coach had been running for some time and the moving parts of engine and transmission "bedded in". The new
acceleration times are given as 0-30 mile/h. 15s, 0-50 mile/h. 41s, 0-70 mile/h. 70s and a maximum coach speed of 85 mile/h.

A study of these new figures in conjunction with those of Table C11.1 shows that there is more than a discrepancy in aerodynamic drag coefficient here. An interesting point is that the quoted governed speed of the compression ignition engine is 2200 rev/min. The print-out of the computed results showed that this is equivalent to a vehicle speed of 80.4 mile/h. in top gear. A measured speed of 85 mile/h. shows either careless measurement or, more likely, a large degree of unrecorded modification. Usually the governor on a compression ignition engine is such that it is impossible to persuade the engine to running very much faster than its governed speed.

One final point on these results concern transmission efficiency. It may be that the transmission efficiency of this coach is greater than the vehicle performance program would suggest. This is because of the high torque level in the transmission. Certainly one would expect some benefit from "bedding-in". However, the new figures recorded above would suggest more changes than this.

In conclusion therefore, it may be stated that the aerodynamic drag coefficient is almost certainly in error. The deceleration tests on this vehicle were conducted on the one
mile straight at The Motor Industry Research Association's testing ground at Lindley. Did the people conducting the test realise that there is a slight gradient on this track? Was there any wind on the day of the test? It is almost certain that time measurements were made using a stop watch. This is now recognised as being only just adequate, in the hands of an expert. Finally, how was the speed measurement made? These and other questions must be answered before one can get to the bottom of the problem. Because the accurate measurement of drag is so important, Part D of this thesis is devoted to the description of suitable methods, the pitfalls and the effect of errors.

If such a design study were to be made now it is almost certain that the aerodynamic drag would be tied down much more closely during the design stage. There are wind tunnels available including the full size tunnel at M.I.R.A. Also, the industry's experience now is so much greater. If the engine is known and the drag coefficients fixed at the design stage, considerable benefits may be obtained from a vehicle performance program such as BLUE. A study of the discussion of Pearson's paper reveals that the designers of the Midland Red Coach G35 were sufficiently worried by the results of their performance calculations that they would have fitted a larger engine had one been available. The cost of the coach, therefore, would have been greater. On
finding the actual coach performance adequate at the prototype stage, it is inconceivable that the designers would have reverted to the smaller engine. Pearson mentioned being pressed for development time in the discussion to his paper.

A good vehicle performance computer program would have been invaluable in the design of C315 and, because of the speed of a modern computer, could have saved the manufacturers both time and money in developing the design of the Midland Red Coach.
SECTION C12

Concluding Remarks

Part C of this thesis is devoted to a study of some of the more important design parameters of a motor vehicle. It emphasises the importance of a vehicle performance program as a design tool.

Some of the more important conclusions reached in the parametric study are summarised as follows:

1) A 10% reduction in vehicle weight produces a 10% approximately reduction on the time to cruise speed and a 1% approximately increase on maximum vehicle speed.

2) Decreasing the aerodynamic drag of a vehicle has little effect on the time to cruise speed but has a significant effect on maximum vehicle speed.

3) A small gain in performance is possible by reducing the rolling resistance of the wheels.

4) The effect of appreciable wind speed during an acceleration test is such that it is not reliable to test in two opposite directions and average the results. A method of dealing with such results is suggested. This requires further study in which a program such as B001 should prove invaluable.

However, it is reasonable to average the results of
opposite direction runs in the case of maximum vehicle speed tests provided that the wind speed is low.

5) decreasing the "under-bonnet" temperature produces significant gains in vehicle performance.

6) an increase in ambient pressure produces a decrease in time to speed.

7) a decrease in ambient temperature may lead to an increase in time to speed even though the engine torque output is greater.

8) decreasing road wheel inertia has little effect on the time to speed and no effect on maximum vehicle speed.

9) arranging the drive axle ratio such that the vehicle is undergeared generally reduces the time to speed. However, it is possible to produce other minima in the graph on the overgeared side. The greatest maximum vehicle speed is obtained at Degree of Undergearing = 1.

10) the percentage gain in time to speed is almost linear with percentage decrease in engine inertia and shows the greatest gain to be in the initial stages of an acceleration test.

11) the theory outlined in Part B for the determination of bottom gear ratio is reasonable.
the theory outlined in Part B for the determination of the intermediate gear ratios is viable and, in the absence of previous experience, may be used.

It is demonstrated also that program BO01 may be used in gradeability studies and is a useful design tool.

Detailed studies of off road vehicles has been omitted from Part C because the scope of such studies is so wide. Preliminary investigations into the use of BO01 with agricultural tractor studies produced promising results however.