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PART D

The Measurement of Vehicle Drag
SECTION D1

Introduction

A conclusion of Part B and the saga of the "Midland Red Coach" in Section C11 show that it is essential to know the drag force of a vehicle accurately in order to predict its performance. The usual form of drag force expression is a second order polynomial with vehicle velocity as the independent variable. The second order term is called the "aerodynamic" drag, while the constant and linear terms together are said to be the "rolling resistance". The form of this expression is given in Section B3.

It is necessary to study therefore, the techniques available for drag measurement and the suitability of the usual form of the drag expression. The study of the techniques available in Section D2 is an extension, in rather more detail, of the instrumentation review in Section A2, subsection 8. It is shown in Part D that the "acceleration" test is a suitable technique and has the advantage that it is cheap to conduct and that it is accurate, since it has been used as the basis of assessment of wind tunnel tests (24). Further, it yields the full drag force, both aerodynamic and rolling and caters to some extent for the interaction of the parameters affecting the rolling resistance mentioned in Section B3. The deceleration test may be developed therefore to provide data in order to assess the suitability of the usual form of the drag expression.
Also, the Department of Transport Technology, Loughborough University of Technology, is developing its own vehicle speed measurement and data reduction equipment. The deceleration test forms a convenient means of assessing the accuracy and repeatability of different instrument systems.

Part D therefore, develops the deceleration test as a means of obtaining the drag coefficients of a vehicle by providing answers to the main objections to the use of the deceleration test. These objections are the effect of wind speed and the tedious data reduction. Further, a built-in checking procedure is provided.

Note: The data reduction technique for the deceleration test contained in Part D together with details of the instrumentation under development have been published in a paper by G.C. Lucas and J. Britton, read at the Aerodynamics of Road Vehicles symposium, City University, November, 1969.
SECTION D2

Rolling or tyre resistance

There are two methods in current use for determining the rolling resistance of a tyre. The first is to enclose a test wheel in a box and to tow the box using a load cell in the tow bar. The box is designed to eliminate the aerodynamic drag of the test wheel itself. The test wheel may be loaded by placing weights in panniers fixed to the test wheel frame. Such a rig is described and illustrated in the January 1967 edition of the Automotive Design Engineer (2).

The main disadvantage with this type of rig is that there is little control of road surface or of tyre temperature. Both these factors can have an effect upon tyre rolling resistance (see Part B, section 3).

The second method of measuring tyre rolling resistance is to load the test wheel against a rotating drum of large diameter. Because closer control can be made with this type of rig, it is favoured even though the surface in contact with the tyre is curved rather than flat. Allowance has to be made for the curved surface when interpreting the results.

The usual practice, therefore, is for the tyre manufacturer to supply the vehicle manufacturer with the full characteristics of his product.
There is the difficulty in interpreting these characteristics for vehicle performance work, since tyre temperature, inflation pressure, torque level and road surface should be specified. It is not unnatural, therefore, that vehicle rolling resistance is usually represented by a constant term. Reference to Fig. B3.2 shows that this is approximately true up to the speed at which the standing wave forms in the tyre tread.
Aerodynamic drag

The usual method of obtaining the aerodynamic drag coefficient of a vehicle is to subject either the vehicle itself or a model to a wind tunnel test. These tests pose certain difficulties which may be classified under four headings.

1) Correct representation of the ground
2) Blockage effects
3) Correct representation of the vehicle
4) Reynolds number effects

Points 3) and 4) may be avoided by using a full scale wind tunnel of the type existing at the Motor Industry Research Association, Lindley. Here quite large motor cars can be accommodated and there is the added advantage of a chassis dynamometer, capable of absorbing 250 horsepower, built into the wind tunnel (24). However, such facilities are expensive.

Blockage effects may be measured separately and so allowance may be made.

The correct representation of the ground however, is difficult to deal with. On the road, a vehicle moves relative to the air and the ground. In a wind tunnel, the vehicle is stationary and so may have no velocity relative to the ground. The various methods employed to overcome this problem are given by White and Carr (24). Briefly, there are
a) Fixed platform to represent the ground. This is the most common method but it does mean a thick boundary layer underneath the car where one would not exist in practice.

b) Fixed platform with suction to remove the boundary layer.

c) Image method.
The model is imaged about the ground plane thereby causing symmetrical air flow and correct representation of the ground. The main difficulties, apart from the cost of two models, are the size of tunnel required and the control of the downwash.

d) Semi-image and platform.
Designed to alleviate the extra model required for c) however, a large tunnel is still required. The model is mounted on a platform. Below the platform at right angles to the platform and affixed to it a wall is positioned such that the flow splits symmetrically at the nose of the platform.

e) Endless belt.
The model is arranged in the tunnel with its wheels touching an endless belt moving at the air speed in the working section. Such installations are expensive and are usually confined to small scale tunnels.
Deceleration test

This method has a lot to offer because it requires no expensive wind tunnel installations or special rigs and because it yields both the aerodynamic drag coefficient and the rolling resistance.

The vehicle under test is driven up to a speed not far short of its maximum speed and then allowed to coast in neutral gear. The deceleration against speed history of the vehicle is recorded during this coasting period either by direct measurement of acceleration or from a plot of vehicle speed against time. The test track must be straight and preferably level. If a small gradient exists, it must be known. Several such test tracks at The Motor Industry Research Association proving ground at Linsley meet these requirements.

It is usual to favour the less direct method of measuring the vehicle speed against time characteristic, rather than attempt to measure vehicle deceleration directly. Unless the track is virtually free of all bumps etc., the pitching and vertical motion of the vehicle makes the measurement of vehicle deceleration difficult (43). It is possible to measure the vehicle deceleration accurately using accelerometers positioned at the centre of gravity of the vehicle, but such instrumentation is expensive and time consuming to set up.
Fig. D2.1 is a typical plot of the less direct method of a vehicle speed against time history of a deceleration test. This is used to illustrate the technique of the reduction of the results in order to obtain the drag coefficients. Fig. D2.1 refers to the Honda S800 and was extracted from "The Autocar" (31).

Measure the slope of Fig. D2.1, calculate or otherwise obtain the vehicle deceleration at a number of different vehicle speeds throughout the range. Then, knowing the mass of the vehicle and its rotating parts, obtain the drag force using Newton's second law. Hence obtain a plot of vehicle drag against vehicle speed. This drag force is due to both the aerodynamic and the rolling resistance.

The particular appeal of this method for this vehicle performance work lies in its cheapness, simplicity and not least to the fact that "The Autocar" began publishing the results of deceleration tests as part of their road test report on vehicles. This latter point seemed to offer a very cheap facility for obtaining the full drag characteristics of a large number of vehicles. Such data would be invaluable in future vehicle performance work.

The main barrier seemed to be the large amount of work involved in the reduction of the data. Accordingly, a special study was made of this point and a data reduction procedure designed.
SECTION D3

Deceleration test data reduction

Experimental points plotted on a graph must contain some element of scatter. Any data reduction method therefore, should include some smoothing process. The obvious method of approach is, starting from the accepted drag formula

\[ F_d = W(Ad + BdV) + K.A.V^2 \]

D3.1

to work back and so obtain the mathematical expression

\[ V = V(t) \]

D3.2

which governs the results of the deceleration test. In fitting equation D3.2 to the experimental points, the three unknown drag coefficients \(Ad\), \(Bd\) and \(K\) may be found.

The full derivation of the function in equation D3.2, which includes the possibility of a wind speed, is given in Appendix D.1 and shows that equation D3.2 may take any one of three forms.

Given that \(V = V_0\) when \(t = 0\) and zero wind speed, if

\[ \frac{WAd}{K.A} > \frac{(W.Bd)^2}{(2.K.A)^2} \]

the function in equation D3.2 is

\[ V \text{ mile/h} = f \left[ \frac{(V_0 + n) \cdot \tan (m \cdot f \cdot t)}{1 + (V_0 + n) \cdot \tan (m \cdot f \cdot t)} \right] - n \]

D3.3

\[ f = \alpha \quad V_0 = V_1 \]

\[ \gamma = n \quad m = \omega \beta \]
where

\[ f = \left| \frac{W_{\text{Ad}} - (W_{\text{Bd}})^2}{4m^2(K.A)^2} \right| \]

\[ m = \frac{15K.A.}{22m_3} \]

and \( n = \frac{W_{\text{Bd}}}{2K.A.} \)

If

\[ \frac{W_{\text{Ad}}}{K.A.} \geq \left( \frac{W_{\text{Bd}}}{2K.A.} \right)^2, \] an unlikely event

\[ V \text{ mile/h} = \frac{1}{m.t + \frac{1}{(V_0 + n)}} = n \]

If however,

\[ \left( \frac{W_{\text{Bd}}}{2K.A.} \right)^2 > \frac{W_{\text{Ad}}}{K.A.} \]

\[ V \text{ mile/h} = f \cdot \left[ \frac{\text{Tanh} \left( m.f.t \right) + (V_0 + n)}{1 + \left( \frac{V_0 + n}{\text{Tanh} \left( m.f.t \right)} \right)} \right] - n \]

It is evident, therefore, that certain difficulties exist with the obvious method of approach. The first is that the drag coefficients themselves are involved in the determination of which of the three expressions is applicable. This could be overcome
by trying to evaluate the drag coefficients using all three expressions and then by selecting the appropriate set. A "long-winded" procedure in itself.

The second difficulty is in fitting experimental results to such complex expressions by the method of "least squares" or any other method. Equation D3.7 is linear, and so may be accommodated quite easily, but its application is unlikely. Appendix D2 shows that equation D3.8 may be linearised and so rendered by mathematical treatment. However, there is no simple solution to the treatment of equation D3.3. It is not possible to linearise a tangent function. Numerical methods would have to be employed.

Certain simplifications may be made by setting \( B_d = 0 \) and \( A_d = 0.013 \) for radial ply tyres and \( A_d = 0.018 \) for cross ply tyres (see Part B, section 3). The only unknown then is the aerodynamic drag factor \( K \). However, a great deal of the attraction of the deceleration method is lost because it is capable of giving the rolling resistance which, as is emphasized above, may not be expressed by a constant term only. Even with this short-cut, the actual computation time involved using a digital computer is likely to be greater than the time taken measuring slopes and doing the calculations by hand. Such a situation would be ridiculous.
It was decided, therefore, to drop the idea of fitting the correct law to the experimental points in favour of a technique which uses a method of finding the slope at any point on the vehicle speed against time graph and then proceeding as one would with calculations by hand. Such a method is easy to program for reduction using a digital computer and is likely to be at least as accurate as hand calculations, particularly if the hand calculations involve measuring slopes of graphs. It is possible then to check the accuracy of the drag coefficients by evaluating the appropriate expression (equ. D3.3, D3.7 or D3.8) and comparing the answer with the original plot of vehicle speed against time.

To find the slope of the vehicle speed against time curve at any of the points shown in Fig. D2.1, two methods were considered. The first takes the point under consideration and the two adjacent points, see Fig. D3.1, and constructs a quadratic through the three points. The quadratic is then differentiated to obtain the slope and hence the deceleration.

Considering point \( (V_n, t_n) \) in Fig. D3.1 and the two adjacent points \((V_{n-1}, t_{n-1})\) and \((V_{n+1}, t_{n+1})\), a line through the three points must be satisfied by

\[
\begin{align*}
V_{n-1} &= a + b \cdot t_{n-1} + c \cdot (t_{n-1})^2 \\
V_n &= a + b \cdot t_n + c \cdot (t_n)^2 \\
V_{n+1} &= a + b \cdot t_{n+1} + c \cdot (t_{n+1})^2
\end{align*}
\]

--- D3.9
Solving these three simultaneous equations gives:

\[ b = \frac{\frac{t^2}{n-1}(v_{n+1} - v_n) + \frac{t^2}{n^2}(v_{n-1} - v_{n-1}) + \frac{t^2}{n+1}(v_n - v_{n-1})}{\frac{t^2}{n-1} + \frac{t^2}{n^2} + \frac{t^2}{n+1}} \]  

--- D3.10

and

\[ c = \frac{\frac{t^2}{n-1}(v_n - v_{n-1}) + \frac{t^2}{n^2}(v_{n+1} - v_{n-1}) + \frac{t^2}{n+1}(v_{n-1} - v_n)}{\frac{t^2}{n-1} + \frac{t^2}{n^2} + \frac{t^2}{n+1} + \frac{t^2}{n+1} + \frac{t^2}{n+1}} \]  

--- D3.11

hence the vehicle acceleration at speed \( v_n \) is

\[ a_n = \frac{22}{15} (b + 2ct_n) \text{ ft/s}^2 \]  

--- D3.12

In order to test this method, the slope at each point in Fig. D2.1 was measured by drawing a tangent at every point to a smooth curve drawn through the points. These figures are compared with those calculated above in Table D3.1. The discrepancy is quite small, largely because the data had been smoothed first by drawing the graph. Using raw test results, considerable errors were produced by this method. Table D3.1 shows the calculated acceleration at 10.0 seconds to be higher than that at 7.5 seconds, an impossible situation. This sort of deviation from the true increases very rapidly with the degree of scatter on the results. The method could be made viable by very carefully smoothing out all results first, a tedious procedure which introduces the possibilities of errors in the data. Any point described with the wrong ordinate, an easy mistake when reading graphs and
punching cards, produces a very large error in the calculation of the slope at the point and at the two adjacent points. Accordingly, the method was discarded.

The second method employs a technique used elsewhere in this vehicle performance work, that of fitting a polynomial to the raw vehicle speed against time results. This means that any point recorded wrongly will have little effect on the overall result, a useful feature when handling test results. Also, a polynomial is easy to handle mathematically and can be used to generate new points if necessary. It can be differentiated readily to give the vehicle deceleration at any speed. By using the method of "least squares" to fit the polynomial, the calculated vehicle acceleration at any vehicle speed is likely to be much more accurate than by measuring the slope of the graph, because the method of "least squares" puts the "best" line through the experimental points.

It is necessary, therefore, to fix the order of polynomial required to obtain reasonable accuracy. Table D3.2 lists the result of a polynomial curve fit to the data contained in Fig. D2.1. Program B079, listed in Appendix B1, was used for this work. Polynomials of order number 4, 5 and 6 appear adequate. The accuracy is within the accuracy of the data used. Note the error recorded at time $t = 10$ seconds. The quadratic interpolation
results in Table D3.1 suggest that this point is in error. Table D3.2 confirms this.

The accuracy of a seventh order polynomial is appreciably better in this case, but there is a real danger in using a high order because it would strive to accommodate scattered points. Since the purpose is to obtain the slope of the vehicle speed against time graph, this is not desirable. It was felt, therefore, that a sixth order polynomial should be adequate for the vast majority of test results. However, it was decided also to reserve the provision of using a polynomial of a different order number if found to be desirable.
SECTION D4

Deceleration test data handling

The procedure therefore, for reducing the data from a deceleration test is

1) fit a polynomial to \( V \) against \( t \)

results

2) hence determine the vehicle deceleration versus speed characteristic.

3) using Newton's 2nd law and the equivalent mass of the vehicle, obtain vehicle drag force \( F_d = F_d(V) \).

4) obtain the drag coefficients \( A_d \) and \( K \) assuming that there is no term proportional to velocity (i.e. \( B_d = 0 \)), also

5) assume \( B_d = 0 \) and find \( A_d \), \( B_d \) and \( K \).

6) check that accuracy of the results of both 4) and 5) above by feeding the drag coefficients obtained back into the appropriate \( V = V(t) \) expression, that is either equation A.D.1.9, A.D.1.13 or A.D.1.22 in Appendix D.1. The re-calculated velocities \( (V) \) may then be compared with the deceleration test results.

By adopting this procedure, the accuracy of the \( V = V(t) \) polynomial curve fit may be checked, the test results may be checked against the two usual forms of the drag force expression and finally, an overall check on both the deceleration test itself and the reduction of its results by substituting back into the
expression governing the original data. The check on the deceleration test itself is a very useful feature since it throws light upon the standard of accuracy required in a deceleration test. The examples given below illustrate this point.

The listing of the digital computer program designed to reduce the drag coefficients of a vehicle from the results of a deceleration test is given in Appendix D3. The form of the output from this program is shown in Table D.4.1. After the heading data there follows the input data for reference purposes. This is followed by the results of the sixth order polynomial curve fit to the results of the deceleration test. The next set of figures is the fitting of a second order polynomial to the drag force against speed figures in order to obtain \( A_d, B_d \) and \( K \) (or, in the example shown, \( A_d, B_d \) and \( A \cdot K \)). Then follows a set of figures fitting a first order polynomial to the drag force and the square of the relative air speed data. The final set of figures headed "check on accuracy of results" gives the original read-in time and vehicle speed figures in columns one and two. Columns three and four are the calculated speed figures (see Appendix D1) with \( B_d 
eq 0 \) and \( B_d = 0 \) respectively. These latter columns should be compared with columns two in order to estimate the accuracy of the overall reduction of the
results and of the conduction of the deceleration test itself.

The data used in Table D4.1 is that depicted by the V against t graph in Fig. D2.1, which was taken from "The Autocar" (31). The graph published by "The Autocar" is such that it is difficult to be too precise about the value of each ordinate. Also published is the information that conditions on the day of the test were "blustery" with a wind speed of 15 - 20 mile/h. Such conditions are not conducive to good results from a deceleration test.

These points are borne out in Table D4.1. The fit of the sixth order polynomial is good but those of the two sets of drag figures show some error, particularly at the low vehicle speed of 20 mile/h. Also, the rolling resistance coefficient of $\text{Ad} = 0.016119$ is rather higher than one would expect from radial ply tyres.

Accordingly, it was decided to investigate the accuracy of the deceleration test and the reduction of its results further in Section D5.
SECTION D5

Accuracy of the deceleration test

It is shown in Section 4 that the reduction of the HONDA S800 deceleration test data published in "Autocar" (31) highlighted some inaccuracies. Some of these inaccuracies undoubtedly arose when reading points off the small, thick-lined graph given in "Autocar". It was decided, therefore, to obtain the actual test data from "Autocar". The Author is indebted to Mr. Geoffrey P. Howard, Assistant Technical Editor of "Autocar" for the information contained in Table D.5.1 (37).

The figures in Table D.5.1 show the results of two tests, one in each direction. The wind speed is quoted at 10 to 15 mile/h. at an angle of approximately 30° to the test track. Table D.5.1 does not contain sufficient points in order to fit a sixth order polynomial. Accordingly, the information contained is plotted as Fig. D.5.1. This graph shows that the figures given as 44.4 and 47.9 in Table D.5.1 must be misprints and should read 24.4 and 37.9 respectively.

A comparison between Figs. D.5.1 and D.2.1 reveals the published deceleration curve for the HONDA S800 as a line approximately mid-way between the two lines shown in Fig. D.5.1. This cannot be an accurate procedure, the wind speed is high and is not known precisely. Also a wind direction of 30° approximately to the track must mean a considerable yaw drag component.
Taking the wind speed as 12.5 mile/h. and -12.5 mile/h.,
at 30° to the head-on direction and submitting the data shown
in Fig. D.5.1 to program B032 resulted in Tables D.5.2 and D.5.3
These tables when compared with Table D.4.1, show a little
improvement in the accuracy of the reduction. But with only five
or six points given on the deceleration curves and such a high
wind speed at such a high angle to the direction of the vehicle,
no credence can be placed upon the drag coefficients given.

The instrumentation, designed within the Department of
Transport Technology, Loughborough University of Technology,
specifically for vehicle performance work was not built during
the writing of this thesis, nor was it considered likely to be in
a reasonable state of development for this current program of work.
Accordingly, an approach was made to The Motor Industry Research
Association for the results of a deceleration test carried out
with care and using reasonable instrumentation.

Now M.G.S. White of M.I.R.A. had completed a series of tests
on a large number of vehicles. This series included both wind
tunnel and deceleration tests and very good agreement was found
between the two. The data given in Table D.5.4 relates to the
M.I.R.A. deceleration test on the SIMCA 1000 and the Author is
indebted to Mr. White and M.I.R.A. for this information.

Table D.5.4 quotes the vehicle speed at 2 second intervals
for two runs. The results for the first run given are for a
high speed to a medium speed. The second set given relates to the medium to slow speed range. Some phasing therefore is necessary to obtain a single graph. This phasing is best done by plotting a graph, see Fig. D.5.2. The "join" is between 15 and 18 seconds.

Other information given by Mr. White includes the vehicle weight, projected frontal area, ambient conditions on the day of the test and the measured aerodynamic drag coefficient of the SIMCA 1000 in the full scale wind tunnel of 0.408.

Using the M.I.R.A. data in conjunction with program B032 results in Table D.5.5. This gives $C_D = 0.4052$, which agrees very closely with the wind tunnel result. The rolling resistance from the deceleration test of $Ad = 0.0219$ seemed a little high for cross ply tyres until, checking back with M.I.R.A., it was learned that the deceleration test was conducted on the one-mile straight which has a slight gradient. The results given refer to the test in the up-hill direction, hence the rolling resistance figure quoted includes this slight gradient. The rolling resistance coefficient obtained by M.I.R.A. was $Ad = 0.02055$.

The column of error figures for the sixth order polynomial curve fit to the test data shows very little error in the fit except perhaps at 15.2 and 17.2 seconds. That is at the "join" referred to above. The indication here is that the "join" could be better.
The aerodynamic drag coefficient given in Table D.5.5 for the condition when \( \text{Bd} \neq 0 \) is not really relevant, since the Bd coefficient includes both aerodynamic and rolling resistance effects. The comparison with wind tunnel work must be made with the \( \text{Bd} = 0 \) results.

The "Check on Accuracy of Results" figures show the test itself and the reduction of the results to be accurate and that the drag coefficients may be quoted with confidence. Once again however, the "join" between the two sets of results is shown up. This feature of the program is important. Any errors in the initial handling of the data become apparent in the reduction using the program. Had this data been reduced by hand, the small error in the join of these two curves would not have been noticed. Consequently, the resulting drag coefficients could not have been quoted with the same confidence.

Table D.5.5 would suggest that the drag expression involving three non-zero coefficients, that is \( \text{Ad}, \text{Bd} \) and \( K \), is more accurate than the drag expression using \( \text{Ad} \) and \( K \) only. This means that the drag of a vehicle is not composed only of a constant rolling resistance term and an aerodynamic drag term proportional to \( v^2 \). The situation is more complex. As vehicle performance calculations become more exact, account will have to be taken of not only the \( V \) term, but the \( v^3, v^4, v^5 \) etc. terms also, since these terms undoubtedly exist.
The deceleration test therefore, is capable of yielding the total drag of a vehicle and not just one particular coefficient. The total drag that is, except for the component of the tyre rolling resistance due to the torque level transmitted through the wheels. This component may be supplied by the tyre manufacturer and added in separately.
SECTION D6.

Concluding remarks on the measurement of drag

A small scale wind tunnel can be useful in establishing the aerodynamic drag coefficient of a vehicle. The main difficulty is the correct simulation of vehicle shape, particularly the cooling system and the underside of the vehicle. An accurate model of a vehicle however may cost as much as the acquisition of the production vehicle itself. Such a facility therefore may be used to good advantage in prototype work.

The full scale wind tunnel at the Motor Industry Research Association will afford an accurate indication of the aerodynamic drag coefficient of a vehicle. When dealing with the vehicle itself, rather than a model, the built-in chassis dynamometer is an added attraction. The main disadvantages are the cost of hiring this facility and the small error due to the lack of a moving ground plane.

The tyre characteristics, and hence the rolling resistance of the vehicle, are available from the tyre manufacturers and may be expressed as a function of inflation pressure, tyre temperature, torque transmitted, etc.

If the above facilities are not available, or if their cost is prohibitive, the simple deceleration test is capable of affording the full drag characteristics of the vehicle. That is both the aerodynamic drag and the rolling resistance of the tyres. In connection with the latter, it is not necessary to assume this
to be a constant term, the deceleration test is capable of
giving the full tyre resistance, less that due to the torque
level in the tyre. Again, this torque level component can be
supplied by the tyre manufacturer and added on separately.

The main disadvantages in using the deceleration test are
climatic conditions on the day of the test, particularly wind
speed, and the large amount of work involved in extracting the
drag coefficients from the test data. Both these points are
answered to a very large extent by the use of the digital computer
program B032. It is not necessary to handle the vehicle speed
against time data at all from the deceleration test except to
punch it onto cards. Program B032 caters for a small wind speed
on the day of the test provided that it is measured during the
test and is at a small angle only to the direction of the vehicle.

This extends considerably the number of days during the year
suitable for conducting a deceleration test.

The positive attractions in using B032 are that a check is
made automatically of the accuracy of the reduction and of the
test itself, and that two sets of drag coefficients are given
dependent upon upon whether Bd is considered non-zero or zero.
The interaction of the tyre pressure and temperature on rolling resistance is to a large extent, accommodated in the deceleration test. This is not the case with the rolling resistance obtained from separate tyre rig tests.

The deceleration test, together with the data reduction procedure developed in Part D, may be used to assess vehicle speed instrumentation, since the resulting drag coefficients form the basis of an assessment of accuracy and repeatability.

The little application of the technique conducted to date suggests that accuracy of the drag expression could be improved by the consideration of other, higher order, terms in the polynomial.
PART E

Fuel Consumption
SECTION A.1

Introduction

There are a number of approaches to the estimation of the fuel consumption of a proposed design of motor vehicle. The first, used by Fourquet (62), is to calculate the fuel consumption of a vehicle during a full throttle acceleration run. There appears to be little merit in this exercise however, other than in the investigation of fuel consumption during vehicle acceleration. The results obtained have little meaning during normal vehicle usage since, if a driver selects full power from the engine, he is saying that he requires a high vehicle acceleration and that fuel consumption is secondary. If he requires a low fuel consumption, the technique he must follow is to maintain a steady, low, vehicle speed. In fairness to Fourquet, it should be stated that his paper (62) is devoted largely to the technique of using an analogue computer in vehicle performance calculations. He does little with the results he obtains.

The second approach is to describe a typical route or journey (79) (75) and to estimate the fuel consumed. From this the expected average fuel consumption in miles per gallon of the projected vehicle design may be obtained. It is understandable that this technique should be of interest to a vehicle manufacturer. His interest, during the development phase of a new vehicle, must centre on the fuel consumption which will be returned by the customer. Hence the typical test route
and the basing of design calculations on this test route. Cornell (57) has pointed out however, that routes are changing. Road vehicles are spending more time on motorways (freeways). This leads to longer trips, higher speeds and nearer steady state driving. It is difficult therefore, to describe a test route which represents fairly the journey behaviour of a typical driver. Further, as has been pointed out by Forster (48), the fuel consumption of a vehicle is affected greatly by the manner in which it is driven. Scheffler and Niepoth (56) have shown that the "warming-up" period in the operation of a vehicle has a devastating effect on fuel consumption.

The third, and more fundamental, approach should be adopted. That of estimating the fuel consumption of a vehicle when running at a steady speed. This then enables a more fundamental study to be made of the effects of engine type and size, gearing, drag etc. The effects of transients, such as an acceleration to overtake another vehicle, the warming up period etc., may be added onto the results of a steady-state study later. Always assuming that the effects of these transients are known accurately.

So many factors can affect the fuel consumed during a transient. Factors such as carburettor design on a petrol engine, design of manifold, temperature of manifold etc. The current interest in exhaust pollution has highlighted this point and efforts have been made to reduce the fuel consumed during an acceleration by better carburation.
A steady-state match study therefore between the engine and the vehicle should be the aim. This may be verified by accurately conducted steady state road tests on the vehicle. Further, a full parametric study is possible for the designer to enable him to obtain the best design possible. Lastly, steady-state knowledge of fuel consumption sets a convenient norm which can be used to assess the many and varied transient conditions.

A number of techniques are available for the measurement of fuel consumption during a road test. For strictly steady-state measurements, the "petrometa" manufactured by M.G.A. Industries Ltd. is suitable. This is a displacement type of instrument mounted in the fuel line which gives an electrical signal every time a measured quantity (say 1/250 pint) of fuel is passed. This signal is used to operate a counter in the vehicle. There are more sophisticated versions of this technique aimed at coping with the fuel consumed during acceleration runs. Such devices rely on a very small measured quantity of fuel per electrical signal in order to reduce the error of measurement. The reader is referred to a Russian paper (51) the Fiat system (96) and a S.A.E. paper by Sturm (76). The intermittent operation of the lift pump on both a petrol and a compression ignition engine makes the measurement of the true instantaneous flow rate of fuel difficult.

Part E of this thesis therefore describes how a match study should be conducted, the effects of a poor match, a fundamental study
of the different engine types particularly petrol and compression ignition (66) the effect of engine size and finally, the result of a parametric study.
Match between engine and vehicle

The usual way of studying the match between two connected components is to combine the characteristics of both into one graph. A match study between an engine and a vehicle is no exception. The characteristics of the vehicle are made up from the drag data and the gear ratios. The engine characteristics must give data on output in conjunction with efficiency data throughout the engine speed range from low load to full load.

There are two parameters in use to assess engine efficiency. These are the brake thermal efficiency and the brake specific fuel consumption (rigorous definitions are given by the Author elsewhere (1)). Of the two, the brake specific fuel consumption, or s.f.c., is preferable since it relates the fuel consumed and the power output directly without involving the calorific value of the fuel or other constants. The most fundamental output parameter of an engine is its torque or brake mean effective pressure. Power output is secondary since it is the product of torque and engine speed. However, the use of engine power as the output parameter is preferred to the use of torque or brake mean effective pressure since, on matching an engine to a vehicle, the power throughout the transmission system stays substantially constant, factored only by the transmission efficiency. The torque however is affected by gear ratios. Also, the thermal efficiency, whether expressed as brake thermal efficiency or brake
specific fuel consumption, is expressed in terms of the engine power output.

The engine characteristics, comprising power, speed, efficiency data throughout the load range may be arranged in a number of ways. A plot of s.f.c. against power for different engine speeds has been used (65) (39). Fig. E2.1 shows such a plot for the Cummins NH-250 compression ignition engine taken from Joyner (65). This plot favours a close, fundamental, study of the engine only since it is made up from, and related closely to, the engine consumption loops (see Section E4 and Ref. (1) ). Hence the influence of the intake system may be seen with clarity. A better form of expressing the engine characteristics for matching purposes is shown in Fig. E2.2. This is a plot of engine brake horsepower, corrected for ambient conditions, against engine speed for different load settings. Superimposed onto this plot are lines of constant brake specific fuel consumption.

The abscissa is the usual engine performance independent variable, speed, and the ordinate axis the power output. These two quantities are related directly to the power absorbed versus vehicle speed characteristic of the vehicle.

Fig. E2.2 is composed by combining two separate engine characteristics. The first is a plot of the power versus speed lines for constant throttle opening and the second consists of a plot of s.f.c. and engine speed for the same constant throttle angles. These plots are shown as Figs. E.2.3 and E2.4 respectively. The procedure for combining these two plots is to draw a horizontal line on Fig. E2.4
representing a constant s.f.c. of, say 0.80 lb/(bhp h). Read off the corresponding engine speeds and throttle angles and to cross plot these data onto Fig. E2.3. Repeat for other lines of constant s.f.c. and so form Fig. E2.2.

It should be mentioned here that the use of throttle angle is not the best parameter to represent load. It is shown below that, for a petrol engine, inlet manifold depression is to be preferred. The point is of little importance in discussing the technique of conducting a match, particularly since lines of constant inlet manifold depression are best obtained by holding the throttle angle constant and measuring the depression and then cross-plotting the results to obtain the lines of constant manifold depression. The technique of expressing the match is not altered in character by omitting this step.

The vehicle is described by a plot of the power required at the road wheels to propel the vehicle at constant speed against vehicle speed. The power required is obtained from

\[
\text{required horsepower} = \frac{F d \times V \times 22}{150} \quad \text{E2.1}
\]

where \( V \) represents vehicle speed mile/h.

The vehicle drag force being described in Section B3 of this thesis.
Table E2.1 shows how the vehicle characteristic is factored by the transmission efficiency (see Section B4) and the overall gear ratio to relate to conditions at the engine flywheel. Hence Fig. E2.5 may be formed, being a combination of the full engine characteristics, both power and thermal efficiency, and the vehicle characteristics. Fig. E2.5 therefore describes fully the steady state operation of the engine and vehicle combination.

A study of Fig. E2.5 shows immediately the degree of undergearing, as defined in Section B10. If the load line crosses the full throttle engine power curve at the maximum brake horsepower speed of the engine the degree of undergearing is said to be unity.

The vertical distance between points A and B on Fig. E2.5 represents the power available for acceleration. This shows that undergearing increases considerably the power available for acceleration for a particular gear number.

The load line in relation to the lines of constant s.f.c. lines shows the efficiency of the combination. Fig. E2.5 represents a match study of vehicle A (the 7 cwt. van) and the load line corresponds to a vehicle weight of 2128 lbf running on a level road with zero wind speed.

Table E2.2 traces out the intermediate calculations from Fig. E2.5 in order to obtain the steady state fuel consumption (mile/gallon) against vehicle speed shown in Fig. E2.6. The intermediate calculations
require the specific gravity (or density) of the fuel during the engine test. The fuel was petrol of specific gravity 0.740. It is perhaps worth mentioning here that had the s.f.c. been expressed as volume flow rate per b.h.p., instead of mass flow rate per b.h.p. it would not be necessary to use the specific gravity figure.

Fig. E2.6 suggests that there is a maximum economy operating point at a low vehicle speed when running in top gear. The trend of the curve (the full line in Fig. E2.6) shows the fuel consumption to increase markedly with vehicle speed.

Superimposed onto Fig. E2.6 (the dotted line) is the measured fuel consumption of vehicle A as reported by "The Autocar" (11) when testing the Estate version of Vehicle A of the same weight of 2128 lbf. As mentioned previously, these vehicles are of identical specification except for the shape of the rear end.

The difference highlights the main weakness in a match study. In order to define the shape of a constant specific fuel consumption line with reasonable accuracy, a large number of engine test readings must be taken. Also, in cross plotting the s.f.c. figures, some extrapolation is necessary. Such errors however, are usually small scale and the resulting scatter may be ironed out when plotting the fuel consumption curve Fig. E2.6. Not all the difference between the measured and calculated curves in Fig. E2.6 may be attributed to errors in the constant specific fuel consumption contours.
The data for Figs. 12.2, 12.3 and 12.4 were obtained by the Author from an identical engine to the engine in Vehicle A. The engine was in full vehicle trim with the exception of the exhaust system and a very large number of readings were taken. The results were reduced using a digital computer program in order to eliminate arithmetical errors and were corrected for ambient conditions to 29.53" Hg pressure and 60°F temperature.

In view of the errors possible in defining the specific fuel consumption contours, it was decided that the match study should always be conducted by "hand". In principle, it should be possible to devise a computer program to accept the raw engine test results and to produce the full engine characteristics. Such a program would have to conduct a large number of curve fits and may be thrown very much off course by a single false reading or a mis-punched data card. It was felt that processing by hand would be much more accurate.

Section 15 shows the effect on fuel consumption of changing to another gear number. If however, an infinitely variable ratio transmission were available and viable, say of the hydrostatic type, such that engine speed is completely independent of vehicle speed, then one would wish to control the engine to run on "the optimum control line" shown "dashed" in Fig. 12.5. This gives the minimum fuel consumption for any particular power demand of the vehicle. Should one wish a high acceleration, the control could be arranged to run the engine at its maximum brake horsepower speed so affording the greatest possible
power available for acceleration. Many such infinitely variable transmission designs have been investigated or are under development (39), (40), (61), (82), (87), (88), (91), (92). Most suffer from a poor mechanical efficiency, noise and high initial cost. Nevertheless Fig. E2.5 shows the incentive for the development of a suitable design.

The expected gain in fuel economy, assuming a transmission efficiency for the ideal transmission to be identical to that of the manual gearbox outlined in Section B4, is shown by the "dashed" line in Fig. E2.6. The intermediate calculations are given in Table E2.2. Fig. E2.6 shows a significant gain in fuel economy at low speed. This gain diminishes as vehicle speed is increased since the power required approaches the maximum power of the engine.

It is mentioned earlier in this section that the engine characteristics of a match study should be expressed as lines of constant inlet manifold depression for a petrol engine, rather than as lines of constant throttle angle as in Fig. E2.5. There are two main reasons for this. The first is that a practical infinitely variable transmission control would almost certainly employ inlet manifold depression for a speed-load parameter. The second reason is that the "charge-weight" law (1) suggests a linear relationship between air mass flow rate into the engine and inlet manifold pressure. Hence the power output of an engine should have a linear relationship with inlet manifold pressure (1). This simplifies the design of the control system considerably.
Fig. E2.5 emphasizes that, if a wide engine speed range is required during operation, as exists in vehicles, a full match study between engine and vehicle is desirable in order to study the efficiency of the engine-vehicle combination. It is meaningless to quote such engine performance figures as maximum brake horsepower and minimum specific fuel consumption. The performance of the combination is the criterion and only a match study can predict this.
SECTION E3

Conversion of Torque - speed to Power - speed Engine Characteristics

The engine reports issued by The Motor Industry Research Association are an abundant source of engine characteristic data. M.I.R.A. test and report upon a large number of engines of foreign manufacture. The engine types are not restricted to petrol and compression ignition. Special engines, such as the N.S.U. Wankel rotary engine, are tested. These engine reports are very detailed in that they contain information on the materials used for the engine components, assembly data, peculiarities and special features. Also included in each report is a graph giving the full, measured engine characteristics.

Unfortunately, these characteristics are presented as a graph of the more fundamental output parameter, torque against engine speed, with lines of constant s.f.c. rather than engine power as advocated in Section E2 and shown in Fig. E2.2. The test reports on compression ignition engines use brake mean effective pressure as the ordinate. This is directly proportional to torque output and of more interest to compression ignition engine workers.

Fig. E3.1 depicts a typical M.I.R.A. engine characteristic for the Peugeot 404 of engine torque plotted against engine speed with lines of constant specific fuel consumption. Also shown are lines of constant throttle angles. It is by no means apparent just where
the optimum control line lines. Fig. E3.2 shows the same characteristics with lines of constant power superimposed and the optimum control line sketched through the points of minimum s.f.c. on each constant power line.

In order to sketch the optimum control line onto the M.I.R.A. graphs to facilitate analysis, two approaches are possible.

1) Sketch lines of constant pressure directly onto the M.I.R.A. graphs as shown in Fig. E3.2

2) Replot the M.I.R.A. graph with corrected engine power as ordinate.

Of the two, the second approach is likely to be of more use in an engine-vehicle match study, but is likely also to involve more labour.

Both approaches have been studied. Approach 1) can be made very easy by the manufacturer of the drawing aid shown in Fig. E3.3. This has been designed to help in the sketching of a hyperbola and so afford lines of constant power using the expression

\[
\text{Torque} = \frac{\text{Power}}{\text{rotational speed}}
\]

The instrument has not been manufactured since attention was focussed on the second approach. It is felt that it should be possible to improve this design considerably in order to provide a captive
pencil which sketches out the curve on moving another suitable control, rather than having to move the control through discrete distances before marking the graph with a pencil point. A short analysis of the design of this instrument is given in Appendix E1.

In order to reduce the considerable amount of labour involved in converting a plot of torque (or b.m.e.p.) against speed to power against speed, it was decided to use the graph plotter facility of the Loughborough I.C.L. 1905 digital computer. Program B184, listed in Appendix E2, was prepared to this end. This program is fed first with a number of torque (or b.m.e.p.) and speed points along the full throttle (or rack) line and then with torque-speed points around the constant s.f.o. lines. The program converts the torque (or b.m.e.p.) figures to power figures and re-plots each curve. Each curve produced by the graph plotter is numbered consecutively and so may be identified by the line printer print-out. An example of a portion of this print-out is given in Table E3.1 for the Peugeot 404. This gives the power-speed figures used by the graph plotter and the specific fuel consumption figure for the curve or the information that the curve is the full throttle line (curve 1).

Appendix E3 contains a number of converted engine characteristics.
Type of Engine

1) Petrol and compression ignition

A study of the characteristics given in Appendix E3 reveals a fundamental difference between the curves of a petrol engine compared with those of a compression ignition engine. The optimum control line of the petrol engine lies very much closer to the full throttle line and the s.f.c. contours present a steeper face on both sides of the optimum control line.

This reflects the basic difference between the two engine types, as explained elsewhere by the Author (1). The petrol engine, in its present form, must be throttled in order to control the load. Fig. E4.1 shows the typical consumption loop of a petrol engine in which the engine is run at a constant speed and the mixture strength varied from very rich (point A) to very weak (point B). At point B the mixture strength is such that the maximum power or torque output is obtained, whereas at D the minimum fuel consumption is returned. Point C in between represents stoichiometric conditions. At point E the mixture strength is so weak, and the flame speed so low that a flame is in evidence throughout the power stroke and the ensuing exhaust stroke. On opening the inlet valve for the induction stroke, the new charge is lit by the flame and the characteristic "spitting back" into the carburettor is experienced.
It should be noted however, that the power returned at point E is still some 85% of the maximum possible. Hence it is impossible to control the conventional petrol engine by mixture quality as with the compression ignition engine. As the fuel intake is reduced to cater for a low load, the air flow rate must be reduced also to maintain a near stoichiometric, and hence a near constant fuel/air ratio. The air intake therefore, must be throttled.

It has been shown by the Author (1) that throttling the intake to an engine results in a high loss to the coolant at low load. The temperature of the charge around the engine cycle remains of substantially the same form and magnitude, irrespective of load. This is a result of the near stoichiometric fuel/air ratio. Hence the heat flow rate to coolant is substantially constant and, at low load, represents a high proportion of the energy intake to the engine. The s.f.c. of the petrol engine, therefore, rises rapidly as load is removed.

Fig. E4.2 shows a series of consumption loops throughout the load range of a petrol engine, all at the same constant speed. A carburettor is usually designed to follow as closely as possible the minima in the curves in order to produce the lowest part load s.f.c. and to richen at full throttle in order to give the maximum torque output. The "dashed" line of Fig. E4.2.
The poor part load efficiency can be reflected in the petrol engine characteristics shown in Fig. E2.2 by the s.f.c. values along a vertical line representing a constant engine speed. Starting from the full throttle line, the s.f.c. values rapidly pass through a minimum (optimum control line) and then rise as the load is removed. This is the condition shown by the "dashed" line in Fig. E4.2.

The air intake of a compression ignition engine is not throttled (in general) and the load of the engine is controlled by fuel/air ratio. At light load, therefore, the mixture strength is weak and the cycle temperature pattern shows lower temperatures. The energy loss to coolant therefore, is considerably less than that for the petrol engine at light loads. The dotted line in Fig. E4.2 shows the measured consumption loop of a compression ignition engine. For the purposes of comparison, both the petrol and the compression ignition engine were run at their respective maximum torque speeds.

The consumption loop of the compression ignition engine is appreciably lower at full load, reflecting the higher compression ratio of the compression ignition engine (16:1, as opposed to the very low value of 4.7:1 for the petrol engine). However, the efficiency of the compression ignition engine stays high as load is removed.

Again, this can be seen in any of the compression ignition engine characteristics in Appendix E3 by drawing in a vertical line representing a constant engine speed. The s.f.c. values change
little along the line and the minimum (optimum control line) is
some way from the full rack line.

The optimum control line of the compression ignition engine
therefore, is much nearer the load line than is the case with the
petrol engine. Also, the part load thermal efficiency of the
compression ignition engine is appreciably higher. Vehicles
generally spend the majority of their working lives running at
part load. Hence the establishment of the compression ignition
engine in the commercial vehicle field. The use of this engine
would be very much wider if other problems, such as noise, smell,
weight, initial cost could be overcome (66).

An obvious conclusion to draw from this study is that encourage-
ment should be given to research and development of the "stratified
charge" engine. This is a petrol engine in which the load is
controlled by mixture quality, as in the compression ignition
engine. The charge in the combustion chamber is arranged to be
heterogeneous, being rich in fuel in that part of the chamber where
combustion is initiated and weak elsewhere. This conclusion must be
endorsed also from considerations of exhaust pollution. A
stratified charge engine is of considerable benefit in reducing
the noxious components in an engine exhaust (99).

Another fruitful line of study must be into the control of
hydrostatic transmissions (91), (92) and their development.
2) The Rotary Engine

The rotary engine is epitomized by the N.S.U. Wankel engine. The characteristics of the N.S.U. KKM 502 engine, as fitted in the N.S.U. "Spider" sports car, are shown as Fig. A.E.3.4 in Appendix E3. Superimposed onto Fig. A.E.3.4 is the normal load line of the Spider taken from Fig. 7 of a paper by Dr. ING. W Frode (100). Fig. A.E.3.4 shows the Spider to be of neutral gearing (degree of undergearing = 1).

The overall top gear ratio of the Spider is 16.0 mile/h per 1000 rev/min engine speed (103) giving a maximum vehicle speed calculated from Fig. A.E.3.4 of 92.7 mile/h. This agrees well with the 92 mile/h measured by Autocar (103).

Fig. E4.3 shows the fuel consumption of the Spider, as measured by Autocar (103), in conjunction with the calculated fuel consumption using Fig. A.E.3.4. The specific gravity of the petrol was assumed to be 0.740. Again, the agreement is very good, engendering confidence in the match plot.

The fuel consumption of the Spider is not good for a small sports car. Fig. A.E.3.4 shows the characteristics of the N.S.U. Wankel engine to be very similar to those of a petrol engine. This is to be expected, since the intake of the rotary engine is throttled, just as on a conventional petrol engine. If anything, the optimum control line of the KKM 502 is even nearer the full throttle power line than most petrol engines. Also, the general level of the specific
fuel consumption figures is slightly higher. In conclusion, therefore, it may be said that the engine characteristics of the N.S.U. Wankel are those of a slightly inferior petrol engine.
3) The Gas Turbine

Some difficulty was experienced in obtaining the characteristics of a gas turbine engine until "The Automobile Engineer" (101) published an article on the Leyland 25/350/R. Fig. E4.4 is a copy of a graph from this article. It is possible to obtain the engine characteristics in the required form (see Fig. E2.2) by noting speed and b.h.p. data along a particular fuel flow rate line on Fig. E4.4. The specific fuel consumption figures may be obtained from these data. Table E4.1 outlines the arithmetic and the steps taken. Then, plotting the s.f.c. figures against engine speed for constant fuel flow rates, as shown in Fig. E4.5, facilitates the cross-plotting to obtain the required form, Fig. E4.6. Shown also on Fig. E4.6 is the load line for a typical 38 ton truck (not the Leyland truck, since the drag figures were not available at the time of writing).

The most striking feature of Fig. E4.6 is the close proximity between the load line and the optimum control line. The two lines are coincident for all but the very low output shaft speed range. This emphasizes the folly in assessing an engine for a particular duty by means other than a full match study. A cursory study of the thermal efficiency along the "full throttle" line would show that between full speed and half engine speed the s.f.c. is less than 0.500 lb/(bhp h) and that below half engine speed, the thermal efficiency drops rapidly. However, a full match study shows that,
for level road running, the efficiency of the match is quite good
even down to a very low output shaft speed. A characteristic and
redeeming feature of the gas turbine engine is therefore, the
position of the optimum control line.

The optimum control line of a free power turbine machine must
take a path through the maxima of the power curves. Ideally, such
an engine is a "constant power" machine. For a constant gas generator
shaft speed, the air mass flow rate through the engine is substantially
constant. It follows therefore, that the power input to the free
power turbine is constant under such conditions, irrespective of
power turbine speed. The output torque therefore, should ideally,
approach infinity as the speed of the free power turbine is reduced.

A turbine with fixed geometry blading is capable of operating
efficiently over a narrow speed range only. The optimum speed is
designed to be about 80% of the maximum speed possible of the turbine
in order to obtain as wide a speed range as possible at high efficiency.
At low speed, therefore, (less than 1/3 maximum speed) the efficiency
falls off rapidly to zero at turbine stall.

The power output curve therefore, instead of returning constant
power output, follows the shape of the efficiency curve. The maximum
in a power curve, therefore, coincides with the maximum in the
appropriate efficiency curve.

Fig. E4.4 shows lines of constant fuel consumption, rather than
lines of constant gas generator speed. The original figure in
"Automobile Engineer" however makes it clear that the fuel flow rate and the speed of the gas generator are geared together since the fuel flow rate has a "scheduling control". A fuel flow rate of 139 lb/h represents 100% compressor speed, 117 lb/h represents 95%. Similarly, the other fuel flow rates shown in Figs. E4.4, E4.5 and E4.6 represent 90%, 80%, 70%, 50% and idle compressor speeds respectively.

The torque output of the free power turbine machine therefore, rises to 2 or 3 times its full speed torque at stall speed when running at the maximum allowable gas generator speed. A desirable feature in the power unit of a vehicle.

The extent to which the position of the power curve maxima move towards the lower output speed range as the gas generator speed is reduced is related closely to a separate match study of the components comprising the engine. By paying careful attention to the characteristics of the engine components and the way they are matched together, it is possible to make the load line and the optimum control line coincide. This means therefore, that the engine designer should have the drag coefficients and full details of the vehicle when he designs the engine. Such a component match study must be conducted anyway in order to ensure that the compressor will not run into surge during operation. This cannot be left to chance, since the surge line on the compressor characteristics lies close to the locus of
maximum isentropic efficiency of the compressor. Hence, for a
good match, the match line, or the "equilibrium running line" must
be quite close to the surge line. Appendix E4 outlines the procedure
for the component match in conjunction with the load characteristics.

In conclusion, therefore, while bench tests show the gas

turbine to be inefficient at low output shaft speeds, a full match
study between engine and load shows that the true situation is rather
better. Further, there is the possibility of altering the blade
angles etc. of the engine components in order to obtain the best
overall match possible. This point is worth pursuing further to
consider the effect of variable turbine inlet guide vane geometry
on the overall match, particularly when it is remembered that the
output torque curve of the free power turbine machine is excellent
for automotive purposes.

It would appear from this short analysis that the gas turbine
has a future in the automotive field, particularly if a small design
complication can improve the thermal efficiency. The fact that a
gas turbine is a "multi-fuel" engine and so can run on a very wide
range of fuels, together with the excellent torque curve, must ensure
its introduction into heavy commercial and, perhaps, military
vehicles.
4) **Increase in engine power output**

An existing engine may be modified in order to increase the maximum power output by supercharging, turbocharging, tuning of inlet and/or exhaust systems or by decreasing the inlet pressure drop by fitting two carburetters in place of one. It is of interest therefore, to study the effect of such modifications upon the match between the engine and the vehicle.

Fig. E4.7 shows the characteristics of the engine fitted to vehicle A, the standard single Solex carburetter however being replaced by twin s.u. carburetters. This engine test was conducted by the Author in his Engine Laboratory using the same engine from which Fig. E2.2 was produced. A comparison between Fig. E4.7 and the corresponding single Solex plot, Fig. E2.5, shows an increase in the maximum engine efficiency. Presumably, this is a result of the better mixing and distribution of the twin s.u. system. Also, the maximum corrected power output of the engine has risen from 34 bhp to 39 bhp as a result of the lower pressure drop across the inlet system. The general shape of the two sets of characteristics however, is very similar.

Superimposed onto Fig. E4.7 is the same load line shown in Fig. E2.5, that of vehicle A in normal trim. No alteration has been made to the overall top gear ratio. A comparison of the two match
studies shows that the Vehicle A has near unity degree of under-
gearing with the twin s.u. carburettor engine, whereas it is some
what overgeared with the standard engine. This is a direct result
of the power increase.

A significant difference is shown in Fig. E4.8, a comparison
of the steady-state fuel consumption of Vehicle A with the engine
in standard trim and with the twin s.u. carburettors fitted. It can
be seen that Vehicle A is better matched to the twin s.u. carburettor
version of its engine than the standard single Solex carburettor
version. The two curves in Fig. E4.8 are both of calculated fuel
consumptions, using the two match studies. It is of interest to note
however, that the "optimum control line" fuel consumption is practically
identical for both forms of the engine. The two curves in Fig. E4.8
emphasize the desirability of conducting a match study between the
engine and the vehicle.

Another way to increase the power output of an engine is by
turbo-charging. It is expected that legislation will be introduced
soon to increase the maximum allowable gross vehicle weight of a
commercial vehicle from 32 tons to 38 or 45 tons. This legislation
may well be accompanied by a law governing the minimum allowable
power/weight of a heavy commercial vehicle. This means that more
powerful engines will be necessary. It is possible to achieve this
additional power by turbo-charging an existing large engine.
Figs. E4.9 and E4.10 are the engine characteristics of a large automotive compression ignition engine, normally aspirated and turbo-charged respectively. The material for these figures was supplied by A. Bowbottom (104). A comparison shows at a glance that turbo-charging increases the maximum power output substantially, from 183 to 230 bhp. A more detailed comparison is made difficult however, because the power output has increased. It is not possible to draw conclusions without taking some account of the distortion of the power scale due to the turbo-charging.

This problem is general in engine assessment work. Giles (61) advocates a "centre of operation" criterion for the assessment of the running time of a particular transmission. This centre of operation is the intersection of a horizontal line on the engine characteristic plot representing, say, 50% maximum engine power and the load line. This idea could be extended to form an assessment of the fuel economy of the engine and vehicle.

Macmillan (92) notes that the torque against speed curves of an engine with constant throttle angle are approximately linear. This he uses as a basis for the non-dimensional representation of engine power. His aim was to produce a relatively simple expression representing the engine in order that he might analyse a range of automatic transmission systems. This too could be extended to form a non-dimensional engine characteristic plot for the purpose of comparison.
Szirtes (105) has developed a rather ingenious index in order to assess the overall fuel consumption of an engine. He notes that a plot of the area enclosed by a specific fuel consumption contour is approximately linear with the specific fuel consumption. The slope of this plot ($\tan \phi$) and the intercept ($b_o$) on the $x$-axis (i.e. the minimum possible s.f.c.) are used to form the index

$$
\mathcal{L} = b_0 \sqrt{\frac{P_{\text{max}}}{\tan \phi}},
$$

where $P_{\text{max}}$ is the maximum power of the engine and $N_p$ the engine speed at which it occurs.

This gives a number which is said to be a measure of the overall efficiency of the engine. It assumes equal probability of operation within a particular s.f.c. contour and hence takes no account of the load requirements or of the shape and position of the s.f.c. contours.

It has been emphasized in this Section, and shown in the analysis of the gas turbine particularly, that account must be taken of the load. Rowbottom compared Figs. E4.9 and E4.10 simply by looking at the curves and concluded that the optimum control line of the turbo-charged engine was nearer the maximum power line and, as a consequence, the additional power had been obtained at the expense of fuel economy. It is not practicable to compare the steady state fuel economy of the engines powering a particular vehicle directly, since the engines serve different sizes of vehicle.
Rowbottom's conclusion is by no means obvious from a casual study of Figs. E4.9 and E4.10.

The technique employed by Prof. Macmillan is attractive in that it could culminate in a plot of non-dimensional power versus non-dimensional engine speed. Comparison of the characteristics of different engines could be effected by laying one plot on top of the other and drawing conclusions from the slope and position of the specific fuel consumption contours.

To obtain a numerical assessment of a match, a standard, non-dimensional, vehicle is required. The "centre of operation" technique of Giles could be used as a rough and ready guide, but it is suggested that a better approach is to standardise on a simple relationship between vehicle weight and vehicle projected frontal area. A survey would suggest a typical relationship. Let this be

\[ A = A(w) \]  \hspace{1cm} \text{E4.2}

It is possible to put a numerical value on a typical drag coefficient \( C_d \) and rolling resistance coefficient \( A_q \). The expression for the required power becomes

\[
\text{required power} = \frac{1}{\sqrt{T}} \left\{ W.A_d + \frac{1}{2} \cdot \rho \cdot C_D \cdot A(w) \cdot V^2 \cdot (22)^2 \right\} \frac{22}{15} V \]

\hspace{1cm} \text{E4.3}

assuming that \( B_d = 0 \).
By specifying a typical power/weight ratio for the class of vehicle it is possible to calculate the maximum speed of the typical vehicle using the maximum brake horsepower of the engine in equation E4.3. The transmission efficiency may be assumed to be 0.90. From knowledge of the maximum vehicle speed and the engine speed at maximum brake horsepower, the overall gear ratio is calculable by specifying unity degree of undergearing. Hence, the load line of a typical vehicle can be superimposed onto the engine characteristics.

The two versions of the compression ignition engine in Figs. E4.9 and E4.10 are intended for use in large commercial vehicles. It is suggested, therefore, that the cab size of such vehicles is substantially constant. Therefore, the projected frontal area (A) of a typical vehicle for these engine characteristics may be fixed. A figure of $A = 60 \text{ ft}^2$ is suggested. Taking a power/weight ratio of 8 horsepower per ton, $A_d = 0.01$ and $C_D = 0.72$ enables the typical load lines to be put on Figs. E4.9 and E4.10.

A study of the engine specific fuel consumption figures along these typical load lines forms the basis for a numerical comparison. A tentative study of these two typical load lines suggests a slight increase in engine efficiency of the turbo-charged version at high engine speed and a slight decrease at low engine speed.
The comparison technique is not perfect, since it depends upon ill-defined "typical" parameters. It is felt intuitively that a simple relationship exists between vehicle weight and projected frontal area, perhaps of the form

\[ A = k w + c \]  \hspace{1cm} \text{--- Eq. 4}

However, this relationship may be valid for a class of vehicles only and perhaps the vehicles of one manufacturer only within that class.

Nevertheless, a basis of comparison now exists where there was none before. This basis takes account of the load requirements, an essential requirement in the assessment and comparison of similar engines. Also, the basis of comparison proposed is capable of development. It could, for instance, be combined with a non-dimensional technique similar to that proposed by Prof. Macmillan (92).
SECTION E5

Parametric Study

1) Effect of vehicle weight

Vehicle A is considered first, being representative of a small car powered by a petrol engine. It was decided to use the twin s.u. carburettor version of its engine for purposes of comparison, since this results in near unity degree of undergearing with the standard load line and standard overall top gear ratio. A 20% increase in vehicle weight above the standard test figure of 2128 lbf represents two additional passengers plus some luggage. The sort of addition encountered during the holiday period. Also, 20% increase represents the sort of weight penalty a manufacturer could incur if he were careless in the design of his vehicle.

These two examples represent two separate conditions in the investigation of the effect of a 20% increase in vehicle weight. The first suggests that the weight is added to the vehicle with no change in overall top gear ratio. Consequently, the vehicle becomes over­ geared. The second implies a different overall top gear ratio in order to maintain unity degree of undergearing.

Vehicle A in standard trim, having an engine with the characteristics depicted in Fig. E4.7, has a maximum speed of

\[
5000 \text{ (rev/min)} \times 0.0152 \text{ (mile/h per rev/min)} = 76.0 \text{ mile/h}
\]
Increasing the vehicle weight by 20% to 2552 lbf and assuming a transmission efficiency of 0.90 at maximum vehicle speed results in a maximum possible vehicle speed of 73.25 mile/h at unity degree of undergearing. This entails an overall top gear ratio change from 0.0152 to 0.0148 mile/h per rev/min. These figures were derived by following the calculation procedure outlined in Section E4 for the comparison of engine characteristics.

The drag force expression of vehicle A is given by

\[ F_d = 0.0179 \cdot W + 0.025 \cdot V^2 \text{ lbf} \]

This enables the juxtaposition of the two new load lines to be seen in relation to the standard load line on the engine characteristics (Fig. E5.1). Simply increasing the vehicle weight without modifying the overall gear ratio overgears the vehicle. The engine speed at maximum vehicle speed is shown to be 4800 rev/min. This represents a maximum vehicle speed of

\[ 4800 \times 0.0152 = 73 \text{ mile/h} \]

This is little different from the maximum speed given above as a result of changing the overall top gear ratio to maintain unity degree of undergearing.

The increased weight load line maintaining unity degree of undergearing is seen to be very close to the standard line at high speed and to deviate from it a little at low engine speeds.

Fig. E5.2 shows the calculated steady state fuel consumptions
for the two 20% increased weight conditions as compared with the standard case (see also Fig. E4.8). Table E5.1 outlines the calculations in the compilation of Figs E5.1 and E5.2.

Fig. E5.2 shows little difference in the steady-state fuel consumption between the two 20% increased weight conditions. Modifying the overall top gear ratio to maintain unity degree of understoichiometry slightly worsens the fuel consumption in the mid-speed range. Fig. E5.2 shows also that the weight of a small petrol engined vehicle has little effect on the steady-state fuel consumption. In fact a small increase in weight is shown to better the fuel consumption very slightly in the mid-speed range. This accords with experience.

The fuel consumption returned from a long holiday run is rarely worse than that for normal usage, and may be considerably better. This may be partly as a result of decrease in the proportion of "warm-up" time to journey time (56). However, this benefit is maintained on a long holiday run, even though the weight of the vehicle has increased considerably from that of normal usage and the average speed of the run is probably higher (57).

The conclusion to be drawn from Fig. E5.2 is that a small change in the weight of a petrol engined vehicle causes very little change in the steady-state fuel consumption. Because the load line of such a vehicle is far removed from the optimum control line and because the slope of the specific fuel consumption contours is steep near the
load line, a small increase in vehicle weight may actually result in a better match between engine and vehicle and a better steady-state fuel consumption. However, an increase in vehicle weight must increase the fuel consumption during transit conditions, since more force is required to accelerate and retard the vehicle. Thus the overall fuel consumption can be expected to increase, with vehicle weight. In relating steady-state fuel consumption figures to the expected overall figures therefore, the vehicle weight must appear as a factor.

The Author has shown elsewhere (1) that a massive increase in the weight of a petrol engined vehicle results in an increase in fuel consumption. Fig. E5.3, taken from Chapter 4 of "The Testing of Internal Combustion Engines" (1) depicts the calculated fuel consumption of Vehicle A in its unladen and laden conditions. This difference is 7 cwt. and represents a weight increase of some 40%. Fig. E5.3 suggests a maximum in the laden fuel consumption (mile/gall) curve at a vehicle speed of 25 mile/h approximately. Also, the laden and unladen curves become closer together as maximum vehicle speed is approached, emphasizing the predominance of aerodynamic drag at high vehicle speed.

Fig. E5.3 was produced on the assumption of Vehicle A in standard trim having its engine in the standard condition with one carburettor and with its normal overall top gear ratio of 0.0152 mile/h per Rev/min.
It is of interest now to study the effect of the weight of a large commercial vehicle on the steady-state fuel consumption. The vehicle chosen is the 23 ton vehicle represented by Fig. E4.9. Fig. E5.4 is a reproduction of Fig. E4.9 with two additional load lines representing a decrease in vehicle weight of 25%. Unity degree of undergearing is assumed for one condition and the standard overall gear ratio assumed for the other.

Fig. E5.5 shows the calculated steady-state fuel consumption for these two conditions in relation to the standard condition. This shows that the effect of weight is marked. A 25% reduction in vehicle weight results in a decrease in fuel consumption of some 20%. This decrease is a result of the form of the compression ignition engine characteristics. The optimum control line being closer to the load line and the slope of the specific fuel consumption contours being less steep than that of the petrol engine.

Again, there is a small difference only in the steady-state fuel consumption between the two "25% decrease in weight" conditions. The line corresponding to an overall gear ratio change, in order to maintain unity degree of undergearing shows a slight improvement. This is to be expected, since the other condition is undergeared.

The general conclusion here is broadly in line with Bland's (59) conclusion published in "Bus and Coach" concerning the fuel economy of coaches entitled "Saving weight means saving money".
He suggests a saving of 0.56 to 1.00 mile/gall for every ton weight reduced. His rule of thumb guide for the fuel consumption of a double-decker bus on about-town use is 0.75 mile/gall per ton weight, thus tying the fuel consumption exclusively to weight.
2) **Effect of overall gear ratio**

Two studies are made, as with the previous sub-section. The first is representative of a small petrol engined vehicle. Again, Vehicle A with the twin s.u. version of its engine is used. The second is again the 23 ton commercial vehicle.

Fig. E5.6 depicts the Vehicle A engine characteristics with its normal load line (shown as a full line and marked $0GR = 0.01520$) and with 5\% changes in overall gear ration between 15\% high and 10\% low. At an overall gear ratio of 0.01748 (15\% high), the maximum speed of the vehicle is

\[
4160 \text{ (rev/min)} \times 0.01748 = 72.8 \text{ mile/h}
\]

a decrease of some 4.2\% from the optimum. At the other extreme taken, the maximum speed of the vehicle at an overall gear ratio of 0.01368 mile/h per rev/min may be shown to be 74.5 mile/h, a decrease of some 2\% only.

Fig. E5.7 depicts the corresponding steady-state fuel consumption curves and table E5.2 outlines the intermediate calculations. The curves in Fig. E5.7 show that overgearing gives a small but significant gain in steady-state fuel consumption but that the rate of gain diminishes as the overgearing is increased. Undergearing, on the other hand, results in quite a high loss in economy.
Fig. B5.8 is the heavy commercial vehicle equivalent of Fig. B5.6. Table B5.3 outlines the intermediate calculations for the commercial vehicle and Fig. B5.9 plots out the corresponding steady-state fuel consumption curves.

Fig. B5.9 produces a very similar result to that of Fig. B5.7 namely, that overgearing operates on the law of diminishing returns and that undergearing is costly in terms of fuel.

The overall conclusion to be drawn from this study is that a careful compromise must be made between steady-state fuel consumption and acceleration in top gear. Alternatively, consideration should be given to special gear ratios to cater for one or both of these performance parameters.
3. Effect of aerodynamic drag coefficient

Again, Vehicle A with its twin s.u. engine and the 23 ton commercial vehicle are taken as being representative examples of a small petrol and a heavy compression ignition vehicles respectively.

White and Carr (24) show that the aerodynamic drag coefficient of a large number of motor cars lie between 0.33 and 0.56. Vehicle A is near the mean of this range with an aerodynamic drag coefficient of 0.495. By considering, therefore, three cases for the small petrol vehicle, the normal aerodynamic drag, a 25% reduction and a 25% increase, most of the range quoted by White and Carr is covered. These three cases are considered twice. First with the overall gearing altered also to maintain near unity degree of undergearing. This would be the case if the manufacturer were to alter the aerodynamic drag of his vehicle. Secondly, assuming no change in overall gear ratio from normal, as would be the case if a vehicle owner were to add a roof rack or "fair-in" the bodywork.

Fig. E5.10 shows the engine characteristics for the first consideration with the degree of undergearing fixed at the normal value of near unity. Little difference is discernable between the three curves, however, the "25% low" line is nearer the optimum control line. Fig. 5.11 shows the corresponding calculated steady-state fuel consumption curves. This shows substantial gains in fuel economy throughout the running range as a result of reducing the aerodynamic drag coefficient. This gain is approximately 20% at low speed rising
to 30% at high speed. This explains the good fuel consumption of some continental cars which, in general, have lower aerodynamic drag coefficients than British cars (9).

To obtain the normal degree of undergearing, the overall gear ratio is 0.01468 mile/h per rev/min for the high $C_D$ case and 0.01640 mile/h per rev/min for the low $C_D$ case. These figures were fixed using the method outlined in Section 4 for fixing a typical load line. Although Fig. 55.10 shows similar specific fuel consumption figures throughout the engine speed range, the real gain by reducing $C_D$ is in the lower power required to propel the vehicle. Hence the product ($s.f.o. \times b.h.p.$) to give the fuel flow rate is much lower in the case of the low $C_D$. A further, secondary benefit is the increase in overall gear ratio necessary for the low $C_D$ case in order to maintain unity degree of undergearing.

Fig. 55.12 depicts the engine characteristic plot for the second consideration. The three load lines are shown and the overall gear ratio is maintained at its normal value of 0.0152 mile/h per rev/min. These load lines are much wider apart than those of Fig. 55.10.

The corresponding steady-state fuel consumption curves are shown in Fig. 55.13. These are of the same trend as those of Fig. 55.11, but the effect of change in the aerodynamic drag coefficient is not quite so dramatic.

From the point of view of steady-state fuel consumption therefore, it is well worth paying attention to the aerodynamic shape of a motor
car. This conclusion applies both to the manufacturer when considering the styling and the owner when contemplating the addition of appendages. Dawley (70) and Cato; and Meek (81) show the effect of components and of minor changes in aerodynamic shape.

Turning now to the heavy commercial vehicle, Fig. E5.14 shows the 25% change in the aerodynamic drag coefficient lines on the engine characteristic plot. This plot assumes constant and unity degree of undergearing. Using the method outlined in Section 4 the overall gear ratios become respectively for the high drag case 0.02681 and for the low drag case, 0.03038 mile/h per rev/min. The corresponding steady-state fuel consumption plots are shown in Fig. E5.15. This shows a large saving in fuel costs as a result of reducing the aerodynamic drag coefficient, but not quite as large a saving as for the small motor car. This is to be expected because the aerodynamic drag of a large, heavy commercial vehicle is not so high in relation to the rolling resistance as that of a small motor car.

Figs. E5.16 and E5.17 are the corresponding plots for the heavy commercial vehicle assuming that the change in aerodynamic drag coefficient is made with no change in overall gear ratio. Fig. E5.17, when compared with Fig. E5.15, shows virtually no difference in the change in fuel consumption when maintaining a constant degree of
undergearing or when maintaining a constant overall gear ratio.

This conclusion could be implied also from a study of Fig. 55.9.

While the saving is shown to be less for the heavy commercial vehicle when expressed as a percentage, it is, however, just as important to study the aerodynamic shape of such a vehicle. The actual cost involved itself is greater because the vehicle is greater. Also, the vehicle has to operate commercially as an economical proposition. This requirement need not be true for a private motor car. Joyner (65) shows that the fuel cost of a heavy commercial vehicle is by far the highest cost when expressed as cost per mile and is four times that of the first cost.

The overall conclusion, therefore, is that the aerodynamic drag coefficient has a marked effect on the steady-state fuel consumption of a vehicle. It is unfortunate that the actual fuel consumption of a vehicle in normal operation is appreciably higher than that predicted by steady-state considerations. A great deal of the apparent benefit to be gained by reducing the aerodynamic drag coefficient is therefore lost. Warren (58) shows a measured 16-18 mile/gall for a car of European manufacture in the 20 to 30 mile/h speed range described as "normal urban driving." This compares with a measured steady-state figure of 35 mile/gall. In the 40-45 speed range described as "fast traffic driving" the measured overall fuel consumption is 23-26 mile/gall compared with a steady-state figure of 30 mile/gall. For "60-65 mile/h thruway driving" the measured figures are very similar to the
steady-state, 23–25 compared with 25 mile/gall. One test on a car in a heavily congested area, where the driver was making 32–37 traffic stops per 10 miles, each with an average time of 15s, showed a fuel consumption of 16–18 mile/gall.

As the building of motorways and high speed roads continues and as leisure time to make long trips increases, the importance of the aerodynamic drag of a motor car increases (57). The extension of the motorway network benefits also the commercial vehicle operators. Hence, it is becoming important also to pay attention to the aerodynamic drag of commercial vehicles (see also Joyner (65)).

There are a number of suggestions as to where savings may be made in the aerodynamic drag of a vehicle. Cato and Meek (81) suggest from a series of wind tunnel tests that a wing mirror costs seven cents in fuel per 1000 miles at 65 mile/h. An advertising board, similar to that fitted to taxis, can reduce the fuel consumption figures by 1.5 mile/gall. They suggest however, that attention should be paid to reducing the projected frontal area of a vehicle. Their conclusions are based upon steady-state considerations.

Perhaps the first component on a motor car worthy of study is that suggested by the Author in the discussion of a paper by White and Carr (24), namely the engine cooling system. Dawley (70) states that the cooling system of a conventional layout accounts for about 10% of the aerodynamic drag. White shows elsewhere (9) that the cooling drag
is quite significant. Careful attention to the design here must prove beneficial since, from basic thermodynamics, the addition of heat to a ducted system is capable of returning a nett thrust. Effort in this direction should result in a better cooling system, less drag without upsetting the styling of the vehicle. Other components, such as windscrean, rear window, wings etc. may be subjected to a more long term programme since production, styling, handling and possibly safety are involved.

Joyner (65) suggests that for a commercial vehicle having a van type trailer, the frontal area should be no longer than that required for the loads to be carried. He suggests further that the engine driven accessories should be chosen carefully and that an engine having a good match with the vehicle be chosen.
4. **Effect of engine size**

It is of interest now to look at the effect of engine size on the steady-state fuel consumption. Earlier studies (see Section E4) have shown that an increased engine output does not necessarily result in a worsening of the steady-state fuel consumption. The match may be better.

Vehicle A is again chosen as being representative of a small petrol engine vehicle. However, since the characteristics of similar engines to that of vehicle A but of differing size were not available for this study, it was decided to use three engines of the Fiat range, the characteristics of which are available through the M.I.R.A. reports on foreign vehicles. The three engines chosen are the engine fitted in the 1964 Fiat 850 S of 843 cc capacity, the engine fitted in the Autobianchi Primula of 1221 cc capacity, and the engine fitted in the 1960 Fiat 2100 saloon of 2100 cc capacity. The characteristics of these engines are presented as plots of engine torque against engine speed. These have been translated to plots of engine power against engine speed in the manner described in Section E3 using computer program BL84. Figs. E5.18, E5.19 and E5.20 respectively show these engine characteristics together with the load line of vehicle A. A degree of undergearing of unity is assumed for each of the match studies for comparison purposes. Hence, the overall gear ratios are respectively, 0.01383, 0.01618 and 0.01936 mile/h per rev/min. These three characteristic plots may be seen
to be very similar. Perhaps the general efficiency level of the large 2100 cc engine is slightly higher than that of the other two. The bump in the full throttle line of the large engine near the maximum power speed is discernable also in the original MIRA curves, as also is the deviation in the 0.500 specific fuel consumption line shown in Fig. 85.19.

Fig. B5.21 shows the corresponding, calculated steady-state fuel consumption of Vehicle A when fitted with these engines. This shows a relatively small, but noticeable difference between the 1221 cc and the 2100 cc engine, the larger engine returning the poorer fuel consumption. These two curves may be seen to draw together at high vehicle speed.

The curve of the small 843 cc engine shows large gains at low vehicle speeds. These initial gains diminish rapidly as the vehicle speed increases such that, at 65 mile/h, the larger 1221 cc engine returns a better fuel consumption. The maximum power of this small engine is similar to the normal engine fitted to Vehicle A. The steady-state fuel consumption curve is generally similar, perhaps a little better than that of the normal engine.

It would be unsafe to draw firm, general conclusions from this study. Its scope is not sufficiently wide. However, the suggestion is that a small engine produces a very good fuel consumption at low vehicle speeds. An increase in the size of engine rapidly reduces the benefits to almost a common level. At high vehicle speeds, the
suggestion is the match is better with the larger engines. Cornell (57) supports this conclusion in his study of three engines powering an American motor car. The engines were a small 6-cylinder, a large 6-cylinder and an 8-cylinder respectively. His graphs show gains with the smaller engines at low speed with very little difference at high vehicle speed.

It is important therefore, to study the effect of engine size very carefully during the design stage of a vehicle. The economy benefits of a small engine have to be weighed very carefully against the better performance of the large engine.

Turning now to the 23 ton commercial vehicle powered by a compression ignition engine. Fig. 65.22 depicts the engine characteristics of a smaller engine to that previously considered, (see Fig. 64.19) but of having the same manufacturer and of similar characteristics. Fig. 65.23 shows the characteristics of a larger compression ignition engine, the manufacturer of which is not common to the other two and the engine is a turbo-charged two-stroke. However, the characteristics look very similar to those of the other two. The load lines of the 23 ton truck are shown on these two plots again assuming unity degree of undergearing for the purposes of comparison. Fig. 65.22 shows the position of the load line on the small engine characteristics close to the optimum control line.

The Author is indebted to A. Rowbottom (104) for the supply of the characteristics depicted in Figs. 65.22 and 65.23.
Fig. E5.24 shows the steady-state fuel consumption curves of the large engine and the small engine in relation to the "normal" engine considered previously (see the "9.0 hp/ton line" on Fig. E5.5).

The suggestion to be made is similar to that for the petrol engines that a small engine shows large gains in fuel economy at low vehicle speed with the position reversing at high vehicle speeds.

It would be of help in furthering this study to have a mathematical model of a set of typical engine characteristics. Perhaps in non-dimensional form which may be distorted at will to represent say a petrol engine or a compression ignition engine. Such a model would be of help generally in a parametric study since it would eliminate the peculiarities of a particular engine and allow very small changes in a parameter, rather than the large steps in considering say three different engines. It is intended to carry out this work in the near future.
5. **Automatic transmissions**

The distinguishing feature of a conventional automatic transmission is its use of a torque converter. This unit, by its very nature, introduces a degree of "slip" in the drive line, hence a degree of power loss. A full steady-state, match study may be conducted between a vehicle and its automatic transmission and engine in just the same way as outlined above by first conducting a separate match study between the engine and the torque converter, as outlined in Section F3, using computer program B071. This separate match study affords the characteristics at the output shaft from the torque converter. The engine characteristics may then be related to this output shaft and the vehicle-power unit match conducted as above.

The engine-torque converter match allows for the power absorbed by any "front pump" in the transmission. The vehicle-power unit match study must allow for any "rear pump".

Studies of this nature yield information on the effect of different torque converters on the steady-state fuel consumption. This effect is likely to be small since Section F3 shows that the coupling point of the torque converter is reached at quite a low vehicle speed. A typical vehicle speed at the coupling point is half the maximum vehicle speed for steady-state running.

Warren (58) has conducted road tests on two large cars of American manufacturer having automatic transmissions in conjunction
with a number of European manufacture; and concludes that the poorer fuel consumption of the former is not primarily due to the automatic transmission, but that it may be a contributing factor. Forster (93) has conducted tests specifically to evaluate the effect of an automatic transmission. He used two cars (presumably similar, although he does not say) and regularly exchanged drivers and transmissions. He concludes that an automatic transmission causes a 6% - 10% increase in fuel consumption and that it is responsible for more fuel being consumed during transient conditions. The overall increase in the fuel consumption he dismisses as being within the tolerance of vehicles and driver habits. This conclusion is fairly representative of the conclusion of road test reports on manual and automatic vehicles.

Cornell (57) suggests that the steady-state fuel consumption of an automatic vehicle could be better than a manual at high vehicle speed because the drive axle ratio is generally higher geared (lower numerical value of gearing) and because the engine mixture strength could be leaner. His tests on otherwise identical cars suggests a saving of 1 mile/gall by employing an automatic transmission under steady-state, high speed conditions.
SECTION 56

Conclusions

1. The engine characteristics and the vehicle characteristics should not be assessed independently. A full match study is important and necessary at the design state.

2. A full match study is capable of predicting the steady-state fuel consumption of a vehicle.

3. The technique of reducing the "raw" engine test bed results to produce the engine power characteristic plot and the techniques for converting from torque to power require further development.

4. A full study is required of the effect of transient behaviour on fuel consumption. Here the statistical analysis technique of Smith et al (107) and/or the laboratory rig simulating a complete vehicle of Genhool et al (108) may be of use.

5. It is thought that vehicle mass has an important effect on the transient fuel consumption.

6. The development of the "stratified charge" engine should be studied carefully, since it affords a better match with a conventional vehicle than does a petrol engine.
7. The gas turbine study emphasizes the validity of conclusion 1. above. The steady-state fuel consumption of a vehicle powered by a gas turbine is appreciably better than a cursory glance at the full load characteristics of the engine would suggest.

8. It may be possible to modify the components of a gas turbine engine to produce a better match between the engine and the vehicle.

9. The good match between engine and vehicle of the gas turbine should ensure its place in the heavy commercial field at least. This position could be extended by the introduction of variable geometry inlet guide vanes to the turbine and/or some other sophistication to enhance the part load efficiency while still retaining the excellent torque characteristics. An important consideration here is the multi-fuel capabilities of the gas turbine engine.

10. The basis for the comparison of engine characteristic plots outlined in Section E4 is reasonable in that it takes account of a typical load. This feature is essential in any comparison. However, further development and proving is required.

11. A small increase in the weight of a petrol engined vehicle has little effect on the steady-state fuel consumption. This conclusion is irrespective of whether the overall gear ratio is modified to maintain constant degree of undergearing. A large increase in vehicle weight however does have a pronounced effect.
Weight is shown to be important with a vehicle powered by a compression ignition engine. Again, this conclusion is valid irrespective of whether constant degree of undergearing is maintained.

A further detailed study is required on the effect of vehicle weight. This should be conducted by very small changes in vehicle weight over a wide range.

12. Overgearing a vehicle gives significant gains in the steady-state fuel consumption, but the rate of gain diminishes as the degree of undergearing decreases. Undergearing results in a significant worsening of the steady-state fuel consumption. This conclusion is valid both for petrol and compression ignition engined vehicles.

The degree of undergearing of a vehicle and/or the fitting of an overdrive unit require careful consideration at the design stage in order to provide a good compromise between top gear acceleration and fuel consumption.

13. Changes in the aerodynamic drag coefficient of a vehicle produce significant changes in the steady-state fuel consumption. It is confirmed that an effort should be made to reduce the aerodynamic drag of future designs and that the cooling system warrants the first attention, since its modification is likely to have little effect on styling, sales appeal or safety.
14. The effect of engine size in a vehicle is not fully conclusive. Studies suggest however, that a small engine produces a very low steady-state fuel consumption at low vehicle speeds and that this situation worsens rapidly to a common level as engine size is increased.

At high vehicle speeds, a large engine may return a significantly better steady-state fuel consumption than a small engine.

This means that the size of engine requires very careful consideration at the design stage of the vehicle in order to optimise on fuel consumption, maximum vehicle speed and accelerative performance. Also, since the effect of engine size may vary dramatically with vehicle speed, the effect of vehicle transient behaviour must be considered. Further study may enable each factor to be counted and a full optimisation study made on the effect of engine size.

15. A mathematical model is required of the typical engine characteristics of, say, a petrol engine and of a compression ignition engine. This could be produced by initially describing the specific fuel consumption contours by simple ellipses. Such a model could be easily distorted to simulate different effects. It would be a relatively simple matter to program it for a digital computer and should prove an invaluable tool in the study of the effect of engine size, vehicle weight and other important parameters.
16. In conjunction with conclusion 15. above, work must continue in an attempt to "non-dimensionalize" the match study plots in order to assist studies of a theoretical nature and as an aid in comparison (see also conclusion 10 above).

17. If, as Joyner (65) suggests, the fuel bill of commercial vehicle operation is so high, a concentrated effort is due to reduce it. Much can be done at the design stage by match studies.

18. The use of an automatic transmission in a motor car slightly worsens the fuel consumption due, in part, to the effect of an automatic transmission during transients. By running the mixture strength of the engine weak (lean), the steady-state fuel consumption at high vehicle speed can be better than the manual transmission version.
PART F

Performance of Vehicles having

Automatic Transmissions
SECTION F

Introduction

A vehicle having a torque converter in the transmission line, or any device which removes the simple, direct relationship between engine speed and vehicle speed, should be treated differently from the performance calculation point of view. The computer program BOOL developed for manual transmissions is not suitable. It is desirable, however, to develop a technique for dealing with the performance calculation of automatic transmission vehicles because their popularity is increasing.

It is outside the scope of this work to describe the many types and designs of automatic transmissions and torque converters. A full survey and history of development is given by Giles (39) and, more recently, a catalogue of the different types in use today is given by Mitchell (40). This latter reference shows that practically all of the designs in use today in Europe and Japan consist of a torque converter in series with some form of stepped ratio gearbox. None incorporate a drive line by passing this system. The torque converter is used to give extra and a variable torque multiplication, some degree of "cushioning" in the drive and for its ability to give a smooth take-off to the vehicle.

Part F, therefore, is devoted to the development of a suitable technique for calculating the performance of a vehicle having a torque
converter in series with a stepped ratio gearbox type of transmission. A typical example being the Borg-Warner 35, the essential components of which, from the performance calculation point of view, are outlined in Fig. Fl.1.

The input shaft to the torque converter is driven, either directly or through gears, by the engine. To this input shaft is fastened the torque converter impeller. The drive is then taken through the fluid to the torque converter turbine and thence to the gearbox. The reaction member is arranged on a free-wheel device such that it can turn in one direction only. When the reaction member is stationary, a degree of torque multiplication is provided by the torque converter. When the reaction member is free-wheeling, the drive through the torque converter is similar to that through a fluid flywheel and there is no torque multiplication. The point at which the reaction member starts to free wheel is termed the "coupling point".

The standard Borg-Warner 35 transmission incorporates two oil pumps in the gearbox. Both pumps supply fluid to the control system within the gearbox. The front pump is driven by the engine and is the larger. The rear pump is driven by the transmission and may be used to activate the control system when the vehicle is being "push-started," that is when the engine is out of action. It is becoming the practice, however, to dispense with the rear pump.
The gearbox provides a number of stepped ratios which are automatically changed at pre-determined vehicle speeds (and load conditions). Hence the gear change speeds with an automatic transmission are independent variables which must be specified. This constitutes an important difference between the performance calculations for automatic transmissions and manual transmissions. The other essential difference is that there is no simple direct relationship between engine speed and vehicle speed. In order to calculate the engine speed at a particular vehicle speed, the characteristics of the torque converter must be known.
Characteristics of the torque converter

In order to effect the match between an engine and its torque converter it is necessary to marry together the characteristics of both. The relevant engine characteristic is its torque against speed curve for the throttle angle or fuel pump rack position under consideration. In this work, full throttle or full rack opening.

The relevant torque converter characteristics are the curve of

\[ K\text{-factor} = \frac{\text{input speed } (N_e)}{\sqrt{\text{input torque } (T_e)}} \quad \text{against} \quad \frac{\text{output shaft speed } (N_o)}{\text{input shaft speed } (N_e)} \]

and

\[ \text{Torque ratio} = \frac{\text{output shaft torque } (T_o)}{\text{input shaft torque } (T_e)} \quad \text{against} \quad \frac{\text{output shaft speed } (N_o)}{\text{input shaft speed } (N_e)} \]

It is shown below that both these curves are unique for a particular torque converter.

Rayner (38) has analysed the fluid flow in a torque converter and has set up the three basic equations governing the performance
of a torque converter. In simplified form, they are:

**Power equation**

\[
\begin{align*}
T_I = & \ a \ \left( \frac{N_E}{C_m} \right)^2 + b \ \frac{N_E}{C_m} + c \ \frac{N_o}{C_m}^2 + d \ \frac{N_o}{C_m} + e = 0 \\
\end{align*}
\]

Input torque equation

\[
\frac{T_I}{C_m^2} = f \ \frac{N_E}{C_m} + g
\]

Output torque equation

\[
\frac{T_o}{C_m} = f \ \frac{N_E}{C_m} + l \ \frac{N_o}{C_m} + m
\]

The terms \(a, b, c, d, e, f, g, l, m\) are constants for a particular torque converter and are functions of the geometry of the torque converter only. They are made up from blade radii, angles, etc. These terms are developed in full by Raymer (38). The term \(C_m\) is the "meridional component of fluid velocity" within the torque converter. This term is of interest to the torque converter designer but of little interest to the Performance Engineer. Accordingly, it is desirable to eliminate it from the above equations.

Defining

\[
\text{speed ratio } SR = \frac{N_o}{N_E}
\]

and using the identity
\[
\frac{N_E}{C_m} = \frac{N_E}{C_m} \times N_E = \frac{N_E}{C_m} \times SR
\]  \hspace{1cm} \text{--- F2.5}

in equation F2.1 in order to form

\[
(a + c \cdot SR^2) \left(\frac{N_E}{C_m}\right)^2 + (b + d \cdot SR) \left(\frac{N_E}{C_m}\right) + e = 0
\]  \hspace{1cm} \text{--- F2.6}

The term \(C_m\) may be isolated from equation F2.2, giving

\[
C_m = \frac{-f \cdot N_E \pm \sqrt{f^2 \cdot N_E^2 + 4 \cdot \frac{T_{B+G}}{2g}}}{2g}
\]  \hspace{1cm} \text{--- F2.7}

whence

\[
\frac{N_E}{C_m} = \frac{2g}{-f \pm \sqrt{f^2 + 4 \cdot \frac{T_{B+G}}{N_E^2}}}
\]  \hspace{1cm} \text{--- F2.8}

substituting equation F2.8 into F2.6 affords a unique relationship

between the term \(\left\{\frac{N_E}{C_m}\right\}\) and SR for the particular torque converter

\[
\left\{\sqrt{\frac{N_E}{C_m}}\right\}
\]

under consideration having design constants \(a, b, c, d, \) etc. This

relationship therefore, links together input torque and shaft speed

and output shaft speed. It is now necessary to find another relation-

ship to give the output torque in order to describe the full

characteristics of the torque converter.

This second relationship may be obtained by re-writing equation

F2.3 as

\[
\frac{T_2}{C_m^2} = \frac{N_E}{C_m} \left(\frac{\sqrt{E \times SR}}{C_m}\right) - m
\]  \hspace{1cm} \text{--- F2.9}
and dividing equation F2.9 by F2.2 to give

\[
\frac{T_o}{T_e} = f\left(\frac{N_B}{C_m}\right) + 1 \cdot \frac{(N_B \times SR)}{(C_m)} + m
\]

\[
\frac{f\left(\frac{N_B}{C_m}\right) + g}{f\left(\frac{N_B}{C_m}\right)}
\]

- F2.10

Again, substituting expression F2.7 into F2.10 yields a relationship between torque ratio \( \frac{T_o}{T_e} \), SR and the K factor \( \frac{N_B}{C_m} \).

Since the K factor against speed ratio curve is unique, there exists a unique relationship between torque ratio and speed ratio for any particular torque converter.

The above relationships are given in full, together with a comprehensive torque converter design procedure in a tech. note by Lucas and Rayner (44).

A typical set of torque converter curves is shown in Fig. F2.1.
SECTION F3

Match between engine and torque converter

Fig. F3.1 illustrates the problem to be solved of an accelerating vehicle and an accelerating engine coupled by a torque converter. \( T_E \) is the steady state torque output from the engine and \( T_i \) the input torque to the torque converter. It has been demonstrated by Ott (42) that it is permissible to assume quasi-steady conditions for the operation of the torque converter. Hence

\[
T_i = \left( \text{impeller, or pump speed} \right)^2 \left( \frac{\text{K-factor}}{} \right)
\]

using the steady state torque converter K-factor defined and described in Section F2. The equation of motion for the engine becomes therefore

\[
T_E - T_i = I_g \cdot \frac{d w_E}{dt}
\]

where \( w_E \) is engine speed in rad/s.

The equation of motion for the vehicle itself is

\[
T_o \times \frac{DAR \times GR}{R_F} \times \sqrt{T} = F \quad \frac{d}{dt} \left( E \times \frac{v}{E} \times \frac{22}{15} \right)
\]

where the output torque from the turbine of the torque converter is given by

\[
T_o = T_i \times TR
\]
and \( V \) is the vehicle speed (mile/h).

Section F2 shows both \( T_i \) and \( T_o \) to be functions of engine speed \( (\omega_e) \) and vehicle speed \( (V) \) only (that is of speed ratio). Hence equations F3.1 and F3.2 represent two non-linear differential equations having time \((t)\) as the independent variable and dependent variables \( \omega_e \) and \( V \). It is more convenient, however, to consider vehicle speed \( (V) \) as the independent variable, since the gear change points on a full throttle time-to-speed test of an automatic transmission vehicle are fixed at pre-determined speeds. Putting equation F3.2 into F3.1 and re-arranging yields therefore

\[
\frac{d \omega_e}{dV} = \left( \frac{T_e - T_i}{T_o} \right) \times \frac{22 \, \eta_e}{15} \left( \frac{T_o \times \text{DAR} \times \text{GR} \times \gamma_f}{T_i} \right) - F_d \quad \text{--- F3.4}
\]

Equations F3.4 and F3.2 may be solved independently using a standard mathematical technique to yield the engine speed at a particular vehicle speed during an acceleration run and the time-to-speed.

The integration process may be commenced by finding the engine "stall speed". That is the steady state, full throttle engine speed corresponding to a torque converter speed ratio of zero. This may be determined from a previous match study between the engine and the torque converter or by setting
in equation F3.4 and using an iterative technique to find the engine speed at which the steady state engine torque \( T_E \) equals the torque converter input torque \( T_i \).

The gear changes present a problem since they represent a discontinuity in the integration process. Ott (42) attempted to overcome the problem by specifying in some detail the gear change itself, thereby making the integration continuous, but necessitating the specification of the gear change clutches, individual gear wheel inertias etc. Perhaps a more reasonable approach to the problem is to assume that the engine acceleration becomes zero at a gear change. Hence the integration process continues again from a new start, the new boundary condition being the steady state engine speed corresponding to the gear change vehicle speed. Again, this steady state engine speed is found from a previous match study or by the iterative procedure outlined above.

A vehicle performance program (B167) was devised using the above philosophy and the Runge-Kutta-Gill technique for the solution of the differential equations. This program is listed and described in Appendix F5.

Now an important function of a vehicle performance program, such as B167, is that of a parametric study as an aid to the Design
Engineers. It is shown later in Section F6, and in particular in Table F6.1, that the torque converter itself has little effect on the time-to-speed of a vehicle. The choice of a particular torque converter is not made from time-to-speed or maximum speed considerations, but from engine noise, engine stall speed and engine response behaviour considerations. It is of greater benefit therefore, to study separately, and in some detail, the match between a particular engine and a particular torque converter. Once this match has been declared satisfactory, it is possible to consider its performance in a vehicle.

Adopting this approach reduces considerably the computational time involved, particularly when conducting a vehicle parametric study, since the calculations involved in the torque converter - engine match are settled, and need not be repeated for each parameter change.

Also, the time-to-speed integration technique may be the simple technique outlined in Section B12 for manual transmission vehicles. This, in itself introduces a considerable saving in computational time since a vehicle speed step length of 2 mile/h. may be used with the same order of accuracy to that shown in Section B12. Using a step length greater than 0.1 mile/h. with the Runge-Kutta-Gill process could result in an unstable situation arising during the calculation, particularly as maximum vehicle speed is approached.
Several other difficulties were found with the Runge-Kutta-Gill process. The first being the difficulty in interferring with the integration procedure in order to accommodate phenomena such as "wheel-spin" and the possibility that the specified final speed of the integration process may be greater than the maximum speed of the vehicle. Also, in considering the effect of engine inertia on vehicle performance, equation F3.4 shows that it is not possible to consider the interesting and ultimate condition of zero engine inertia. An indeterminacy exists which shows the specified equations of motion to be no longer applicable. The problem is different.

It was decided, therefore, to devise a steady-state matching procedure between the engine and its torque converter and to evolve a technique whereby the results of this match study might be used in vehicle performance calculations, due allowance being made for the change in the match caused by the accelerating engine.

It is expected that the technique of using the Runge-Kutta-Gill integration process, as outlined in Appendix F5, may prove a more beneficial technique when dealing with the performance calculations of automatic vehicles having shunt transmissions or other complications.

Before dealing with the match study proper between engine and torque converter, a means has to be devised to deal with the torque
required to drive the fluid pumps, particularly the front pump. The front pump torque is subtracted from the engine torque before passing to the torque converter.

Since the majority of automatic transmissions is of the type Borg-Warner 35, it was decided to build into the digital computer programs the Borg-Warner pump figures and to make provision for the substitution of any others, should that become necessary. It was decided also to ignore the fact that the control pressures within the Borg-Warner 35 alter during a gear change, thus altering the torque necessary to drive the pumps. This change is quite small and the level of torque necessary to drive the pumps is small also in relation to the torque output from the engine.

Figs. F3.2 and F3.3, kindly supplied by Borg-Warner Ltd., depict the horsepower necessary to drive the front and rear pumps respectively. A high proportion of the power required in both cases constitutes mechanical loss which is largely independent of the pressure head on the pump. The total horsepower absorbed figures were converted to torque figures to which fourth order polynomials were fitted.

Listed below is the matching study digital computer program B071 designed to effect the match. Once again, use is made of the very versatile polynomial curve fit in order to describe the match mathematically.
Storage spaces PUMP(1) to PUMP(5) give the fourth order polynomial for the front pump torque. This is followed by "Heading" cards. Statement number 2 reads in four fixed point numbers. NT is the number of points on the engine torque curve, NK is the number of speed ratio points used to describe the torque converter characteristics. NPUMP is any positive or negative integer. If, however, NPUMP is set at zero, the program will ignore the torque required to drive the front pump. If a non-zero integer is assigned to the fourth term NCARD, cards are punched out giving the polynomial coefficients of the relevant curve fits described below ready for use in the subsequent vehicle performance program. If these cards are not required, NCARD should be set at zero.

If NT is read in as zero or a negative integer, the program will accept the coefficients of a previous sixth order polynomial curve fit to the engine torque curve, together with the minimum and maximum allowable engine speeds. Between statement number 3 and 7 therefore the program deals with these coefficients. Fifty points on the engine torque curve are generated from which are subtracted the front pump torque figures.

If, however, NT is a positive integer, the engine speed and torque figures are read in. To these is fitted a sixth order polynomial because it will be required in the subsequent vehicle performance program. The coefficients of this sixth order polynomial are
not used in this matching study program. The front pump torque figures are then subtracted from the engine torque. Hence the computer store holds NT points of engine torque less front pump torque figures, NT being 50 if the original torque curve was read in as a polynomial.

Statement number 9 reads in 50 sets of torque converter speed ratio (SR), K-factor (AK) and torque ratio (TR) figures. These are carried to the NT engine figures in the following manner.

First the engine K-factors are evaluated. That is engine output speed divided by the square root of net engine output torque. The engine speed is then expressed as a sixth order polynomial function of the engine K-factor. Hence, for each of the torque converter speed ratio points, the engine speed is known also. Since the speed ratio is specified and the torque ratio of the torque converter known, the output torque from the torque converter versus speed characteristic is known. Thus the match is complete.

Statement 12+6 down to statement number 11 evaluates the horsepower into and the horsepower out from the torque converter. These figures and their difference are printed out. The difference is the power loss from the torque converter dissipated as heat energy.

The program then proceeds to fit an eighth order polynomial to the output torque against output shaft speed figures for subsequent use in the vehicle performance program. This is discussed further below.
Also, a sixth order polynomial is fitted to the engine speed and torque converter output shaft speed figures, again for subsequent use in the vehicle performance program.

Finally, a card, either blank or bearing an integer is read in. If blank, the program ends. If an integer, the program switches back to the appropriate statement number.

The polynomial curve fit sub routine is that listed in Appendix B1.

Table F3.1 is a typical output from program B071. It represents a match study between the engine of vehicle B and a Borg-Warner 225K torque converter. As may be seen, the curve fits are very good. The curve fit of interest however is the eighth order polynomial describing the torque converter output torque characteristic.

The crosses in Fig. F3.4 denote the match study points. Note the "coupling point" at 2500 rev/min. approximately, at which the reaction member in the torque converter commences to free-wheel. At output shaft speeds above the coupling point, there is no torque multiplication and the shape of the torque curve is very similar to that of the engine torque curve. The dots in Fig. F3.4 denote the evaluation of the eighth order polynomial at every 100 rev/min. interval from zero speed to 6000 rev/min. of the torque converter output shaft. The program used for this evaluation was B054 listed in Appendix B1.
The curve fit generally is good. The coupling point should not be quite so rounded, but this is of little consequence. Consideration was given to describing the output torque curve as two lower order polynomials meeting at the coupling point. The amount of work and computer time involved would be similar. This would produce the discontinuity in the slope of the output torque assumed to exist at the coupling point.

However, since this point is of little importance and since it may be desirable to cater for torque converters having more than one reaction member and hence more than one "coupling point," it was decided to retain the eighth order polynomial.

The asset of an eighth order polynomial is its versatility, and this can be its weakness also. The full speed range of the engine must be specified for the reasons given in Part B, section 2. Similarly, the torque converter data must be extrapolated to include a K-factor at least as high as the maximum possible engine K-factor. If this is not done, the highest output shaft speed considered may be less than the speed obtainable during a performance run. Now Fig. F3.4 shows that an eighth order polynomial can veer rapidly off course outside the data range. In fact the output torque at 6000 rev/min given by the polynomial used in Fig. F3.4 is -99.442 lbf ft. If it were possible for the output shaft of the torque converter in vehicle B to reach
6000 rev/min, the vehicle performance calculations would be in error. However, since a high torque converter K-factor was included in the match data, the resulting eighth order polynomial is adequate, and it was found in the later vehicle performance calculations that the torque converter output shaft speed at maximum vehicle speed was 5000 rev/min, approximately.

The figures giving the horsepower into and out of the torque converter emphasize the poorer fuel consumption of the automatic vehicle compared with its manual counterpart.

Program B071, therefore, enables the Designer to study in considerable detail the match between a particular engine and a particular torque converter before committing himself to vehicle performance calculations. The error columns in the curve fit print-outs confirm that there is little or no error in the original data used.

The engine torque against engine speed, the output shaft torque against output shaft speed and the engine speed against output shaft speed polynomials fully describe the match mathematically and are used subsequently on the vehicle performance program.
Listing of matching study

digital computer program

BOZ1
characteristics of the system.

In the context of engine torque analysis, the system's performance is evaluated based on attributes such as RPM, which stands for the number of revolutions per minute of the engine, and torque, which is the measure of the twisting force applied on the engine's shaft. The engine speed in RPM, along with other parameters like air charge and fuel flow, influence the engine torque output. The calculations related to these parameters are carried out using mathematical algorithms to determine the engine's efficiency and output under different conditions.

The program described in the text snippet includes a series of instructions to calculate engine torque using a sixth-order polynomial. The polynomial is defined in terms of coefficients and variables that represent the system's characteristics. These coefficients and variables are integrated into the program to calculate the engine torque at various RPM values. The program reads input data from external sources and performs calculations to provide the desired output, which is the engine torque at a given RPM.
JJ=J+1
CALL POLY (N,N,NT,J,0)
WRITE(2,117)
WRITE(2,116)(1,C(I),I=1,JJ)
IF(NCARD,N'T,0) WRITE(3,118)JJ,(C(I),I=1,JJ)
WRITE(2,108)
IF(NPUMP)3,25,0
DO 23 J=1,NT
FPUMP=PUMP(1)
DO 24 I=2,5
FPUMP=FPUMP*PUMP(I)*EK(J)**(I-1)
R(J)=R(J)-FPUMP
23 CONTINUE
24 FCONTINUE
DO 25 I=1,NT
T(I)=R(I)
R(I)=EK(I)
25 CONTINUE
CR NOW DENOTES (ENGINE SPEED RPM)/1000
CT NOW DENOTES ENGINE TORQUE LBF FT
READ(1,104)(SR(I),AK(I),TR(I),I=1,NK)
CS=SPED RATIO OF TC AK=K-FACTOR OF TC TR=TORQUE RATIO OF TC
LAST=6
LA=LAST+1
DO 10 I=1,NT
EK(I)=R(I)*10./SORT(T(I))
C EK DENOTES (ENGINE K-FACTOR)/100
10 CONTINUE
WRITE(2,110)
CALL POLY (N,N,NT,LAST,0)
WRITE(2,111)
WRITE(2,106)(1,C(I),I=1,LA)
WRITE(2,108)
WRITE(2,114)
DO 11 I=1,NK
EN(I)=C(I)
11 EN(I)=EN(I)+C(J)*(AK(I)/100.)*(J-1)
EN(I)=EN(I)*1000.
TCLOSS=0.
ET=(EN(I)/AK(I))**2-TCLOSS
EK(I)=SR(I)*EN(I)/1000.
C EK IS NOW OUTPUT SPEED FROM TORQUE CONVERTER DIVIDED BY 1000
R(I)=ET*TR(I)
CR IS NOW OUTPUT TORQUE FROM TORQUE CONVERTER  LBF FT
PIN=2.*3.14159265*EN(I)*ET/33000.
POUT=2.*3.14159265*EK(I)*R(I)/33.
PDIF = PIN - PO_1
WRITE (2, 115) EN(I), SR(I), PIN, POUT, PDIF
11 CONTINUE
WRITE (2, 105)
J = 8
JJ = J + 1
CALL POLY (J, 0, NK, J, 0)
WRITE (2, 109)
WRITE (2, 106) (I, C(I), I = 1, JJ)
IF (NCARD, GT, 0) WRITE (3, 118) JJ, (C(I), I = 1, JJ)
DO 14 I = 1, NK
14 R(I) = EN(I) / 1000.
C R IS NOW (ENGINE RPM) / 1000
C E K IS STILL TC (OUTPUT SPEED REV/MIN) / 1000
WRITE (2, 112)
J = 6
JJ = J + 1
CALL POLY (J, 0, NK, J, 0)
WRITE (2, 113)
WRITE (2, 106) (I, C(I), I = 1, JJ)
IF (NCARD, GT, 0) WRITE (3, 118) JJ, (C(I), I = 1, JJ)
READ (1, 107)
IF (J) 13, 13,
WRITE (2, 108)
GO TO (1, 2, 3, 4, 7, 8, 6, 9)
13 CONTINUE
STOP
100 FORMAT (30X 42 ENGINE AND TORQUE CONVERTER CHARACTERISTIC//)
101 FORMAT (80H)
102 FORMAT (1H0)
103 FORMAT (410)
104 FORMAT (160F2.0)
105 FORMAT (1HI I x20HPOLYNOMIAL CURVE FIT/5X73H(ENGINE RPM) / 1000 AS A
  FUNCTION OF (TC OUTPUT TORQUE LBF FT. / 1) AGAINST ((TC OUTPUT SPEED (REV/MIN)) / 1000 ) = X//)
106 FORMAT (4(13.2X, G16.10, 2X))
107 FORMAT (12)
108 FORMAT (1H1)
109 FORMAT (109H POLYNOMIAL COEFFICIENTS GIVING TC OUTPUT TORQUE LBF FT
  AS A FUNCTION OF (TC OUTPUT SPEED (REV/MIN)) / 1000 ARE//)
110 FORMAT (1H015X20HPOLYNOMIAL CURVE FIT/5X59H (ENGINE SPEED) / 1000 ) = Y
  AGAINST ((ENGINE K-FACTOR) / 100 ) = X//)
111 FORMAT (92H POLYNOMIAL COEFFICIENTS GIVING (ENGINE RPM) / 1000 AS A
  FUNCTION OF (ENGINE K-FACTOR) / 100 ARE//)
112 FORMAT (1HI15X20HPOLYNOMIAL CURVE FIT/5X55H (ENGINE RPM) / 1000 ) = Y
  AGAINST ((TC OUTPUT RPM) / 1000 ) = X//)
113 FORMAT (91H POLYNOMIAL COEFFICIENTS GIVING (ENGINE RPM) / 1000 AS A
  FUNCTION OF (TC OUTPUT RPM) / 1000 ARE//)
114 FORMAT(20H ENGINE SPEED RPM 14HTC SPEED RAT106X13WHORSEPOWER INT
1X14WHORSEPOWER OUT6Y25WHORSEPOWER LOSS FROM TC/)
115 FORMAT(6(21.G12.6,6X))
116 FORMAT(1H01)X20HPOLYNOMIAL CURVE FIT/5X61H(ENGINE TORQUE LBF FT)=Y
1 AGAINST (ENGINE SPEED RPM)/1000)=X/)
117 FORMAT(86W0=POLYNOMIAL COEFFICIENTS GIVING ENGINE TORQUE AS A FUNCT
1ION OF (ENGINE SPEED)/1000 ARE/) 118 FORMAT(12/4(F16.10,2X))
END

END OF SEGMENT, LENGTH 934, NAME 8071
SECTION F4

Allowance for engine inertia

By using the results of the match study outlined in Appendix F1 in the calculation of the performance of automatic transmissioned vehicles, some procedure for allowing for the mismatch caused by the accelerating engine must be devised. The use of polynomials to describe the match makes this adjustment relatively simple.

Before studying the problem in detail, it was considered advisable to simulate a change in the match between an accelerating engine and a torque converter. This was achieved by using program B071 in the normal way using the normal steady state torque curve of the engine and then by repeating the run with 10 lbf ft subtracted from the engine torque curve. The resulting torque converter output torque curves are shown as Fig. F4.1 and the engine speed against output shaft speed curves as Fig. F4.2.

A point corresponding to a particular speed ratio moves down and to the left as a result of the decrease in engine torque. Appendix F2 shows that the percentage change in output speed in Fig. F4.1 is half the percentage change in output torque. That is that

\[
\frac{\Delta N_o}{N_o I = 0} = \frac{\Delta T_o I}{2 \cdot T_{oI} = 0}
\]

--- F4.1
Also, considering the shift of a particular point in Fig. F4.2, the percentage change in engine speed must equal the percentage change in output speed because the speed ratio is constant. A study of Figs. F4.1 and F4.2 shows these relationships to be true.

In order to consider in detail the change in output torque resulting from a decrease in engine torque, Fig. F4.3 is shown. This is a "close-up" of a portion of Fig. F4.1. Consider an accelerating vehicle at a particular vehicle speed V. If the engine has zero inertia, then match point A is applicable, and the torque converter output shaft speed is \( N_{0I} = 0 \). Since however, a real engine has inertia, the match line shifts. The new point corresponding to the same torque converter output shaft speed is marked in Fig. F4.3 as point C. A study of Fig. F4.3 shows that the output torque from the converter is now

\[
T_{OC} = T_{OA} - \Delta T_{OI} - \Delta T_{OM}
\]  

--- F4.2

\( T_{OA} \) is the steady running torque output
\( \Delta T_{OI} \) is the change in torque due to inertia
\( \Delta T_{OM} \) is the change in torque due to mis-matching

Now, \( T_{OA} \) is known from the steady state matching study and it can be shown that
\[ \Delta T_{oI} = \text{function (1) (vehicle acceleration } f) \] and that

\[ \Delta T_{oM} = \text{function (2) } (\Delta T_{oI}) = \text{function (3) } (f) \]

The full derivation of these three functions is given in Appendix F3.

It follows, therefore, that a relationship exists between the known steady running output torque \( T_{oA} \), the actual output torque \( T_{oG} \) and the acceleration of the vehicle \( f \).

Appendix F3 shows this relationship to be

\[
\begin{align*}
\Delta f &= \frac{(T_{oA} - T_{oC}) T_{E} I_{-c}}{\left( \frac{T_{oA}}{N_{o}} \right) \left( \frac{T_{oG}}{N_{o}} \right) - \left( \frac{E_{o}}{N_{o}} \right)^2} \times \frac{I_{c}}{E_{o} x (\frac{R_{o}}{N_{o}}) x \text{DAR} x \text{uR}} \\
\end{align*}
\]

Applying Newton's second law to the accelerating vehicle as a whole produces the relationship

\[
\begin{align*}
\Delta f &= \frac{T_{oG} x \text{DAR} x \text{uR} x \gamma_{r}}{R_{r} x E_{r} x m_{E}} - \frac{F_{d}}{m_{E}} \\
\end{align*}
\]

where the equivalent mass of the vehicle is given by
\[ m_E = \frac{W}{g} + I_v + \text{prop. shaft inertia} \times \left( \frac{\text{DAR}}{r_p} \right)^2 \]

\[ + \gamma_T \left\{ \text{turbine inertia} \times \left( \frac{GR \times \text{DAR}}{r_p} \right)^2 \right\} \text{ slug} \quad \text{--- F4.5} \]

Equating relationships F4.3 and F4.4 and re-arranging produces the expression

\[ T_{oc} = \frac{T_{oa} - \left( \frac{3}{2} \frac{d}{N_0} \right) \times N_0}{(I_e - \frac{d}{N_0})} \times \left[ \frac{T_{e_{x=0}}}{(\frac{F_d}{N_0} \times \frac{\text{DAR} \times GR}{r_p})} + \frac{F_d \times \left( \frac{\text{DAR} \times GR}{r_p} \right)}{\frac{F_d \times \left( \frac{\text{DAR} \times GR}{r_p} \right)}{}} \times \left( \frac{1}{(I_e - \frac{d}{N_0})} \right) \right] \quad \text{--- F4.6} \]

Every term on the right hand side of this expression is either specified or calculable. Having obtained \( T_{oc} \), the vehicle acceleration \( (f) \) may be determined using equation F4.4 and hence the performance of the vehicle.

The term \( \left( \frac{\text{DAR} \times GR}{r_p} \right) \) in expression F4.6 should be noted. This is a very common grouping in vehicle performance work.

An analysis of expression F4.6 shows that, if the engine inertia and/or the engine acceleration is set at zero, the output torque from the torque converter is the steady state output torque, as one would expect. Also, if the slope of the output torque curve is zero, as it may be near the coupling point, \( \Delta T_{om} = 0 \) and there is no mismatch.
It follows, therefore, that given the result of a matching study between engine and torque converter, the performance of the vehicle may be calculated directly and in a straightforward manner without resort to iteration. This has been made possible because the match is fully described mathematically.

It is possible to proceed further and calculate the speed of the accelerating engine at a particular vehicle speed \( V \).

Since \( T_{0c} \) is now known, the vehicle acceleration (\( f \)) is known, \( \Delta T_{0c} \) is known and the percentage change in output torque \( \left( \Delta T_{0c}/T_{0c} = 0 \right) \) is known. Now the output shaft speed is related to the engine speed for a particular point on the torque converter characteristics, that is for a particular speed ratio, as follows

\[
N_{0I} = 0 = SR \times N_{E} = 0
\]

and \( N_{0I} = SR \times N_{E} \)

The value of \( SR \) being common to both. It follows, therefore, that

\[
\Delta N_{0} = SR \times \Delta N_{E}
\]  \( \text{--- F4.7} \)

and using equation F4.1, that

\[
\frac{\Delta N_{E}}{N_{0I}} = 0 = \frac{\Delta N_{E}}{N_{E}} = \frac{\Delta T_{0I}}{2T_{0I}} = 0
\]  \( \text{--- F4.8} \)
This means that the percentage change in engine speed of a particular point in Fig. F4.2 as a result of a drop in engine torque output equals the percentage change in output shaft speed. Fig. F4.2 shows this very clearly. At high output shaft speeds, when the speed ratio approaches unity, the two curves merge into one with a slope of 45°. Actual values taken from Fig. F4.2 confirm also that the percentage change is the same for both engine speed and output shaft speed.

Fig. F4.4 is a close-up of a portion of Fig. F4.2. The engine speed corresponding to a particular output shaft speed ($N_0$) with an engine having no inertia is known (point A). It is required therefore, to find the engine speed corresponding to output shaft speed ($N_0$) when the engine inertia is not zero (point C).

This is given by

$$N_E C = N_E I = 0 \left\{ 1 - \frac{\Delta N_E}{N_E I = 0} \right\} + \Delta N_0 \times \left( \frac{\Delta N_E}{\Delta N_0} \right)_{N_0 I} \quad -- F4.9$$

where the slope term is the slope of the lower curve at point B.

Appendix F4 shows that the slope of the upper curve at A is the same as that of the lower curve at B. Using this and equation F4.8 produces
Again, every term on the right hand side is known and so the speed of an accelerating engine is calculable.

The assumption likely to cause the largest error in the calculation of the output torque is the use of \((\delta N_2/\delta N_0)\) at point B, rather than at point C. Appendix F3 shows that this assumption causes an error in the time up to 90 mile/h. of a vehicle of less than 0.04%, a negligible amount. Nevertheless, the error is calculated at the end of a step length and the next step length calculations are modified accordingly. The final error therefore, is likely to be very small indeed.

This section therefore lays down a rational theory to enable the output shaft torque, the vehicle acceleration, the engine speed and acceleration and hence the performance of the vehicle to be calculated in a straightforward manner.
SECTION F5

Performance Program - Automatic transmissions

Listed below is the digital computer program (B058) designed to calculate the performance of a vehicle having an automatic transmission. The program accepts the results of the previous match study between engine and torque converter (B071) and the vehicle design parameters, such as weight, position of the centre of gravity, wheel sizes, gear ratios etc. Unlike the manual transmission programs (B001 and B033), the gear change speeds are read in as independent variables.

The layout of the main program is similar to B001. The input data is printed out for reference. From statement number 32 to the STOR card, the maximum horsepower in the torque converter output shaft, the speed at which it occurs and the maximum speed of the vehicle are evaluated in a manner similar to B001.

Statement number 29+1 down to statement number 53+5 stores the coefficients of two fourth order polynomials describing the torque absorbed by the fluid pumps. Storage spaces PUMP (1) to PUMP (5) are ascribed to the rear pump, PUMP (6) to PUMP (10) to the front pump. To delete the rear pump, read N PUMP as zero.

The subroutine used to evaluate the time to speed integral
\[ t = \int_{V_i}^{V_f} \frac{1}{k} \, dv \]
is called T I M A and is described as follows.

After setting up the initial values of the variable quantities, the step length (DV) is chosen such that there are at least 25 steps in the integration. The main DO LOOP down to statement number 3 evaluates the vehicle acceleration at each step. DO LOOP down to statement number 4 and then down to statement number 5 arranges the gear changes at the specified speeds. From there down to statement number 9+5 the theory described in Section F4 is applied. The DO LOOP down to statement number 26 calculates the error in using \( \frac{\partial N_G}{\partial N_c} \) instead of \( \frac{\partial N_G}{\partial N_b} \) as described in Section F4. This error is stored and applied as a correction during the calculation of the next step length.

From there down to the end of the subroutine the calculations are similar to those in subroutine T I M B (S001). The test applied for wheel spin is identical and follows the theory laid down in Part B.

There is one important difference in that the theory of take-off with an automatic transmission is more simple than for a manual transmission. It is assumed that a test driver places one foot on the throttle pedal and the other on the brake pedal before a test. At the instant of time \( t = 0 \), he takes his foot off the brake. Hence there is no transition after \( t = 0 \). The engine is already at its stall speed and is already connected to the transmission through a torque converter. There is no clutch engagement to worry about.
Table F5.1 gives a typical output from program B058. It represents the calculated performance of Vehicle B fitted with a 225K torque converter. The results of the matching study example in Section F3 were used. The layout of the time to speed table is very similar to that of B001 except that the engine speed is now listed, since it is not tied directly to vehicle speed.

Note that the performance of the automatic version of Vehicle B is better than the manual version up to about 25 mile/h, and that thereafter it is poorer (see Table B12.2). The calculated maximum speed of the automatic version is lower, as one would expect. The gearbox assumed fitted to Vehicle B was the Borg-Warner 35.

Fig. F5.1 is a plot of the engine speed against time information given in Table F5.1. This shows the engine acceleration at take-off to be low rising quite rapidly to a near constant value in first gear. This near constant engine acceleration towards the end of the first gear phase is the highest level of engine acceleration reached during the performance run, and is approximately 50 rad/s² for Vehicle B. Assuming an engine inertia of 0.15 slug ft², this represents an inertia torque of 7.5 lbf ft, or some 7% of maximum engine torque. This supports the use of the assumptions in the mathematical procedure developed for dealing with engine inertia.

The second gear engine acceleration is lower and virtually constant. The top gear engine acceleration decreases as the vehicle approaches its maximum speed.
Listing of Performance Program (Automation)

8058
*FORTRAN 8058, G.G.LUCAS

NG : TITLE
MASTER 8058

DIMENSION ET(10), ES(10), TCT(10), X(10), GCS(10), PUMP(10)
COMMON ET, ES, TCT, X, GCT, DAR, RR, AD, BD, AK, WTV, AIW, AIE, KA, A, B, K, XK, NG
PI = 3.14159265359

20 CONTINUE
16 CONTINUE
J=25
WRITE(2,100)
READ(1,101)
WRITE(2,102)
1 READ(1,103) NET
2 READ(1,104)(ET(I),I=1,NET),VBS,AIE
WRITE(2,106)
WRITE(2,128)(ET(I),I=1,NET)
WRITE(2,129)
WRITE(2,107)VBS,AIE
WRITE(2,129)
IF(J-2)26,25,0
3 READ(1,103) NES
4 READ(1,104)(ES(I),I=1,NES)
WRITE(2,108)
WRITE(2,128)(ES(I),I=1,NES)
WRITE(2,129)
IF(J-4)26,25,0
5 READ(1,103) NTCT
6 READ(1,104)(TCT(I),I=1,NTCT)
WRITE(2,109)
WRITE(2,128)(TCT(I),I=1,NTCT)
WRITE(2,129)
IF(J-6)26,25,0
9 CONTINUE
7 READ(1,103) NG, NPUMP
10 CONTINUE
8 READ(1,104)(X(I),I=1,NG)
WRITE(2,110)(X(I),I=1,NG)
WRITE(2,129)
IF(J-8)26,25,0
READ(1,104)(GCS(I),I=1,NG-1)
WRITE(2,111)(GCS(I),I=1,NG-1)
WRITE(2,129)
IF(J-11)26,25,0
11 READ(1,104) DAR, RR, AIW, PSI, TI
AIW=AIW+PSI*DAR*DAR
IF(J-11)26,25,0
12 READ(1,104)A,B,H,KA
IF(J-12)26,25,0
13 READ(1,104)AD, BD, AK, XK
XK=XK/100000000.
IF(J-13)26,25,0
14 READ(1,104)G,CF,GCT
IF(J-14)26,25,0
15 READ(1,104)V1,V2,WV
   IF(J=15)26,26,27
27 CONTINUE
28 WRITE(2,112)DAK,PR,AIM,WK
   WRITE(2,130)11
   WRITE(2,113)WA,A,H
   WRITE(2,114)AD,BD,AK,G,CF,GCT
   WRITE(2,115)V1,V2,WV
   IF(KA)28,29
   WRITE(2,116)
   GO TO 29
29 CONTINUE
   IF(NPUMP)54,52,0
   PUMP(1)=1.51595803
   PUMP(2)=.933208837
   PUMP(3)=.357206661
   PUMP(4)=.0565263932
   PUMP(5)=.00373440272
   GO TO 53
52 DO 54 I=1,5
54 PUMP(I)=0.
53 CONTINUE
   PUMP(6)=3.07241256
   PUMP(7)=-1.44482083
   PUMP(8)=.91072275
   PUMP(9)=-.173875627
   PUMP(10)=.011072038
   CC=SQRT(1.-5.*G)
   C=B*CC+H*G
   BW=1.-C/CC/(A+B)
   IF(G-BW*CF*CC)30,30,0
   WRITE(2,118)
30 CONTINUE
   IF(KA)31,0,0
   C=B*CC-G*(H-RR)
   IF(TCT(1)*DAK*X(I)-W*C)31,0,0
   WRITE(2,119)
31 CONTINUE
   CALL TIMA(TS,VI,V2,G,CF)
   WRITE(2,102)
   WRITE(2,120)TS
   J=0
   R=ES(I)
32 CONTINUE
   IF(R-.5,GE,UBS/1000.0,OR.R.LE.-.1)GO TO 38
   DTD=TCT(I)
   DO 33 I=-2,NTCT
33 DTD=DTD+I*TCT(1)*R**(I-1)
   IF(J=1)0,34
   IF(DTD)34,0,0
   R=R+.5
   J=1
   GO TO 32
34 CONTINUE
   IF(J=10)0,0,35
   IF(DTD)0,0,35
   R=R+.05
   J=10
GO TO 32
35 CONTINUE
   IF (DTD .GE. 36.0) THEN
      R = R + 0.05
      J = 100
   GO TO 32
36 DTD = DTD + TCT(1)
   DO 37 I = 2, NTCT
37 DTD = DTD + TCT(I) * R**(I-1)
   DTD = 2.0 * PI * R * DTD / 33.
   RPM = R * 1000.
   WRITE (2, 121) DTD, RPM
   VM = 0.
   J = 0
38 CONTINUE
   DF = W*(AD + VM + BD) + AK*VM*VM
   RX = RR*(1.0 + VM*VM - 900.0)
   OGR = PI * RX / 660.0 * DAR * X(NG)
   IF (VM .GE. 11.0 - VM*OGR) 45.0, 0, 0
   GRAO = VB5 + OGR
   WRITE (2, 122) GRAO
   GO TO 46
45 CONTINUE
   TEF = 98*(1.99758 - .000000875*VM)*1.96 - .0000316*VM - .0000058*VM*VM)
   R = VM / OGR / 1000.
   T = TCT(1)
   RPUMP = PUMP(1)
   DO 39 I = 2, NTCT
39 T = T + TCT(I) * R**(I-1)
   DO 51 I = 2, 3
51 RPUMP = RPUMP + PUMP(1)*(R/X(NG))*R**(I-1)
   T = T - RPUMP
   TF = T*X(NG)*TEF*DAR/RX
   IF (J .GE. 10.0, 0, 0)
   IF (TF .LE. DF) 41.0, 0, 0
   VM = VM + 10.
   J = 1
   GO TO 38
40 CONTINUE
   IF (DF .LE. TF) 42.43, 44
41 CONTINUE
   IF (VM .GE. 45.0, 0, 0)
   VM = VM + 10.
   GO TO 38
44 VM = VM - 1.
   J = 10
   GO TO 38
42 VM = VM + 5.
43 CONTINUE
   WRITE (2, 123) VM
46 CONTINUE
   V = RPM * OGR
   GRAD = VM/V
   WRITE (2, 124) GRAD
47 CONTINUE
   J = 0
   READ (1, 126)
   IF (J .GE. 45, 0, 0)
   WRITE (2, 126)
   WRITE (2, 105)
EN: 10 11 12 13 14 15 161, J
48 WRITE( 2, 127 )
STOP
100 FORMAT( 20X34H, G, LUCAS DEPT OF TRANSPORT TECHNOLOGY/20X37WLOUGHBOR
100 UNIVERSITY OF TECHNOLOGY/18X39WVEHICLE PERFORMANCE - AUTOMA
2TIC GEARBOX/)
101 FORMAT( 80W )
102 FORMAT( 1HO/ )
103 FORMAT( 1010 )
104 FORMAT( 20F0.0 )
105 FORMAT( 1HO/5X7HNEW RUN/ )
106 FORMAT( 68W ENGINE TORQUE LBF FT = POLY((ENGINE SPEED)/1000) COEF
106 107 FORMAT( 34H MAXIMUM ALLOWABLE ENGINE SPEED = F9.1,9H REV/MIN/23H F
107 108 FORMAT( 68W ENGINE SPEED/1000 = POLY((TC OUTPUT SPEED)/1000) COEF
108 109 FORMAT( 73W TC OUTPUT TORQUE LBF FT = POLY((TC OUTPUT SPEED)/1000
109 110 FORMAT( 16H GEAR RATIOS ARE/1X5G17.8 )
110 111 FORMAT( 42H SPECIFIED GEAR CHANGE SPEEDS (MILE/H) ARE/1X5G17.8 )
111 112 FORMAT( 20W DRIVE AXLE RATIO = F10.5,5X24HROLLING RADIUS (FEET) = F
112 19.5/40H TOTAL ROAD WHEEL INERTIA (SLUG SQFT) = F10.5,5X21HTYRE GRD
113 114 FORMAT( 15H VEHICLE WT LBF20X36WPOSITION OF CENTRE OF GRAVITY (FEET
114 115 FORMAT( 31W INITIAL TEST SPEED (MILE/H) = F9.2/29H FINAL TEST SPEED
115 116 FORMAT( 30X25HFRONT WHEEL DRIVE VEHICLE/ )
116 117 FORMAT( 30X24HREAR WHEEL DRIVE VEHICLE/ )
117 118 FORMAT( 20X61H* * VEHICLE MAY SLIP DOWNHILL WITH HANDBRAKE ONLY S
118 119 FORMAT( 20X49H* * VEHICLE MAY OVERTURN IN FIRST GEAR * * * )/
119 120 FORMAT( 10X43HW-tank TO SPEED ON SPECIFIED GRADIENT (SECONDS) = F12.4
120 121 FORMAT( 37W MAXIMUM BHP FROM TORQUE CONVERTER = F11.3,34W. AT TC OU
121 122 FORMAT( 53W ENGINE MAXIMUM SPEED LIMITS MAXIMUM VEHICLE SPEED TOF10
122 123 FORMAT( 34W MAXIMUM VEHICLE SPEED (MILE/H) = FIT0.2/ )
123 124 FORMAT( 26W DEGREE OF UNDERGEARING = FIT0.5/ )
124 125 FORMAT( 12 )
125 126 FORMAT( 1H1 )
126 127 FORMAT( 10X13HEND OF CALCULATIONS )
127 128 FORMAT( 1X5G17.9 )
128 129 FORMAT( 1HO )
129 130 FORMAT( 30W TURBINE INERTIA SLUG SQFT = F9.4/ )
130 END

END OF SEGMENT, LENGTH 1045, NAME B058
SUBROUTINE TIMA (TS, V1, V2, G, CF)

DIMENSION ET(10), ES(10), TCT(10), X(10), GCS(10), DS(6), AZ(6), PUMP(10)
COMMON ET, ES, TCT, X, GCT, DAR, RR, AD, BD, AK, WV, W, AIW, AIE, KA, A, B, H, XK, NG

1, NES, NTCY, NET, PI, VBS, GCS, TI, PUMP, D1, D2, D3, D4, D5

DATA DS(1), DS(2), DS(3), DS(4), DS(5)/1320., 1640., 423., 2640., 3280., 846., 5280., /, AZ(1), AZ(2), AZ(3), AZ(4), AZ(5)/8HI/4 MILE, 8HI/2 KILO, 8HI/2 KILO, 8H, 1 KILO , 8H 1

MILE/

DV=2.
NGEAR=1
NDIS=1
AA1=0.
F=0.
TSL=0.
DISFL=0.
GY=0.
DISF=0.
TS=V.
CDEDR=0.
GCS(NG)=V2=1.2
V=V1
CC=SQR(1.-G*G)
BW=CF*X/(A*B)
IF(KA)3, 13, 13
AB=B
HH=-H
GO TO 14

13 CONTINUE.
AB=A
HH=H

14=C=AB*CC+HH*H.
WRITE(2,100)

2 CONTINUE.
IF((V2-V1)/AV-25.)0, 1,1
DV=DV/2.
GO TO 2

1 CONTINUE.
KV=(V2-V1)/AV+1.
DO 3 KK=1, KV
RX=RR*(1.-X*K*(V+V-900.))
VWW=ABS(V+VW)
DF=V*(G+AD+V*BD)+AK*(V+VW)*VWW
AN=0.
GR=X(1)
DO 4 I=2, NG
IF(V.GE.GCS(I-1),AND.V.LT.GCS(I)) GR=X(I)

4 CONTINUE.
IF(GY)5, 5, 0
IF(GY=GR)0, 9, 0
AN=GCT
NGEAR=NGEAR+1
WRITE(2,101)GR, GY

5 GY=GR

TYZ=DAR+GH/RX
OGR=PI*15./660./TYZ
IF(V-GCS(1))0, 24, 24
AI=NG-1
GO TO 23

24 CONTINUE
DO 22 I=2, N
IF(V.GE.GCS(I-1).AND.V.LT.GCS(I)) AI=NG-I
22 CONTINUE
23 CONTINUE

.TEF= .98*( .95- .000316*V-.000058*V*V)*(.9978*(1-.007*A1)-.0000879
1-V+2.08*A1)
EM=V/32.2+AI+V/R+R+I*T=E(F+DAR/RX)*2
R=V/GR/1000.
T=T+CT(I)
TC=CDR=0.
RPUMPP=PUMP(I)
IF(R)=0,0.25
DCD=TC(I)
GO TO 17
25 CONTINUE
DO 6 I=2, NTCT
DCD=CDR*(I-1)*TC(I)*R**2=I-2
6 T=T+CT(I)*R**2(I-1)
DO 20 I=2,5
20 RPUMPP=PUMP(I)*R/GR**2(I-1)
17 CONTINUE
T=T-TPUMP
ER=ES(I)
DER=0.
IF(R)=0,0.18
DER=ES(E)
GO TO 19
18 CONTINUE
DO 7 I=2, NES
DER=DER*DER*(I-1)*ES(I)*R**2(I-1)
7 ER=ER+ES(I)*R**2(I-1)
19 CONTINUE
IF(ER-VBS/1000) B8, 8
WRITE(2,102) VBS, V, TS
GO TO 15
8 CONTINUE
FPUMP=PUMP(I)
DO 21 I=7, 10
21 FPUMP=PUMP(I)*ER**2(I-6)
TE=ET(I)
DO 9 I=2, NET
9 TE=TE*ET(I)*ER**2(I-1)
TF=FPPUMP
TZ=AI*(DER*CDR)/1
TQ=DCD*R**2.*T
TA=(T/TQ+DF*TVZ*TVZ/EM)/(TZ*TEF*TVZ*TVZ/EM+1./T)
TRA=(1.+DF*TVZ**2/EM)/(TZ*TEF*TVZ**2/EM+1./T)
DN=1-8, 8
DER=ER*(1.-DN)+DER*1.+DN
DNSX=ES(E)
DO 26 I=3, NES
26 DNSX=DNSX(I-1)*ES(I)*R*(1.+DN)**2(I-2)
CD=CDR*DNSX+M*1
T=TA
TF=T*TEF*TVZ
PF=T*DF
IF(PF)=0,0,10
WRITE(2,103) TS
RETURN
10 CONTINUE
F = PF / EW
TFF = BW * (C + HW * F / 32.2)
FMAX = (5W = (Aa + CC + HH * G) - DF + W / 2.0 * (AD + V * BD)) / (W / 32.2 + AIW / 2.0 / RR / RR - BW = W / 32.2)
IF (F = FMAX) 11, 11.0
WRITE (2, 104)
F = FMAX
PF = F * EW
TF = PF + DF
11 CONTINUE
AA2 = (AA + DV * 22. / 15. / F) / 2.
IF (V = V1) 12, 12.0
TS = TS + AA2 + AN
12 CONTINUE
AA1 = DV * 22. / 15. / F
VS = VS + 22. / 15.
ER = ER * 1000.
RF = 1. / F
IF (V = V1) 0.27, 0
DISF = DIS + (V = DV / 2.0) * 22. / 15. * (AA2 + AN)
27 CONTINUE
IF (NOIS = -6.0) 15, 15
IF (DISF - DS(NDIS)) 15.0, 0
DS = (DISF - DS(NDIS)) / (DISF - DISFL)
TSD = TS - NO * (TS - TSL)
VD = V - DD * DV
WRITE (2, 107)
WRITE (2, 106) AZ(NDIS), VS, TS
WRITE (2, 107)
NDIS = NDIS + 1
15 CONTINUE
DISFL = DISF
TSL = TS
WRITE (2, 105) V, F, RF, VS, DF, NGEAR, TS, DISF, TF, PF, ER
3 = V + DV
16 CONTINUE
RETURN
100 FORMAT (1HO//30X46H* * * TABLE OF TIME TO SPEED CALCULATION * * */
1113H SPEED ACCEL 1 ACCEL SPEED DRAG GEAR TIME
2 clistance tract, and propul. force engine speed/91H MPH PT/
3SEC2 SEC2/FT FT/SEC LBF. NO. SEC FEET LBF
4F 
101 FORMAT (1HO//2X29H* * * GEAR CHANGE * * * 10X14WRATIO IS NOW F10.4, 1
13H IT WAS F10.4/)
102 FORMAT (1HO//1X31H ENGINE SPEED ABOVE MAXIMUM OF P9.1, 9W REV/MIN/9Y
123H VEHICLE SPEED MILE/H = F9.2, 10X15H TIME SO FAR IS F10.3, 3HSEC/)
103 FORMAT (1HO//29X22HSET SPEED UNOBTAINABLE//5X20H TIME SO FAR SEC = F10.3/)
104 FORMAT (1HO//3X22H* * * WHEEL SPIN * * * )
106 FORMAT (10X, A8.2) MARK PASSED SPEED = F8.2, 31W MILE/H, TIME APP
107 FORMAT (1HO)
END
END OF SEGMENT, LENGTH 961, NAME TIMA
SECTION F6

Investigation

The investigation into parametric changes with automatics is not covered as extensively as with the manual gearbox in Part C. There are three main reasons for this. The first is that many of the conclusions of Part C apply in broad outline to automatics also. The second is that the torque converter introduces a new dimension which extends the range of the parameters to be investigated quite considerably. The third reason is that, since the theory outlined in Section F contains a large proportion of new and original material, it was felt that the emphasis should be on the results of the theory.

The first point to be studied is the growing practice of deleting the rear fluid pump from the Borg-Warner 35 gearbox. This pump supplies fluid to the control system within the gearbox as a supplement to the front pump. The particular and special function of the rear pump is to supply the control system when the engine is stationary. Thus it is possible to "push-start" a car fitted with a Borg-Warner 35 having a rear pump. A useful facility when the battery is flat or the starter system defective.

Repeating the performance calculations on Vehicle B, this time with the rear pump deleted results in Fig. F6.1. This shows the
percentage gain in the time to speed against vehicle speed to be less than 1% for the important low speed end of the range and a low percentage gain at the high speed end. Certainly it is difficult to justify the removal of the rear pump on performance grounds, although doubtless it may be justified by economic considerations. Some provision should be made for starting a car in an emergency and it should be noted that a starting handle is not always successful or desirable when used with an engine having a torque converter.

The next investigation in this Section concerns a particular engine, having the full throttle torque curve shown in Fig. F6.2, fitted with several torque converters.

Torque converter GS/1A has a relatively high K-factor and a low torque ratio at stall. Conversely, SS11 has a low K-factor and a high torque ratio. It had been intended to use G7/1A as a third torque converter since it has a stall K-factor about identical to SS11 and a stall torque ratio about identical to GS/1A. However, it was considered more fruitful to use a composite torque converter having the actual K-factor characteristics of the SS11 and the torque ratio characteristics of the GS/1A. It may well be that it is not possible to manufacture such a torque converter, but the use of this hybrid does enable certain conclusions to be drawn. The torque converter characteristics are shown in Fig. F6.3.
Matching each of the torque converters in turn to the engine using program B071 produces the steady running output torque curves shown in Fig. F6.4, and the engine speed against output shaft speed curves of Fig. F6.5.

The latter set of curves confirms that the steady running engine speed curve is linked to the torque converter K-factor and is not affected by the torque ratio characteristic. The engine speed for the composite torque converter is identical to that of the SSll. A low stall K-factor resulting in a low engine speed at stall.

The engine speed curve for the GS/1A rises steadily up to the coupling point after which it is nearly straight at a slope of 45° approximately. The engine speed curve for the SSll and the composite torque converter remains almost constant until just before the coupling point. It then steepens considerably to form the near unity speed ratio after coupling. This "dwell" in the curve is a direct result of a flat torque converter K-factor characteristic. Because,

\[ \frac{N_E}{T_E} = \text{constant} \]

and hence \( N_E \rightarrow \text{constant} \) at low values of torque converter speed ratio.
Considering now the torque curves in Fig. F6.4. The curves for the G8/1A and the SS11 are as expected. The high stall torque of the SS11 and its low coupling speed are in evidence. The composite torque converter however, shows that the output torque is a function of both torque ratio and K-factor. At low speed ratios, the torque curve follows closely the G8/1A but, as the coupling point is approached, it is seen to follow the SS11 torque curve. The conclusion is therefore, that the stall torque is governed almost exclusively by the torque ratio but that at the coupling point, the output torque is determined by the K-factor characteristic. Furthermore, the output speed at coupling is shown to be largely a function of the K-factor characteristic. This is shown by the small change in output shaft speed at coupling between the SS11 and the composite curves in Fig. F6.4.

The difference in the effects of the three torque converters upon vehicle performance is small. Table F6.1 summarises the results of using the Vehicle performance program B058 as a means of comparing the performance of the three torque converters. The vehicle weight used was 2799 lbf, drive axle ratio 3.538 in conjunction with the Borg-Warner 35 automatic gearbox having ratios 2.39, 1.45 and 1.00.

This Table shows that all three torque converters result in wheel-spin at take-off, but that the SS11 wheel-spin persists a little longer than that of the other two. This is a direct result
of the high torque ratio at stall. The coefficient of friction between tyre and road was taken as 1.0 in all three cases.

Torque converter GS/1A produces the best time to speed, followed by the composite torque converter. The differences, however, are quite small.

The engine speed with the composite is slightly different to that with the SS.11, particularly in regions of high engine acceleration. However, this difference is small. The engine speed with the GS/1A is higher, particularly at take-off.

A study of Table F6.1 in conjunction with Fig. F6.5 reveals that the coupling point is reached at 19 mile/h. in first gear with the composite torque converter and that the engine speed is above the coupling point in second and top. Using SS11 results in a similar coupling point at 17 mile/h. in first gear only. With GS/1A the vehicle speed at the coupling point is high at 25 mile/h. in first, 42 mile/h. in second and 60 mile/h. in top. The efficiency of the GS/1A therefore, is lower at the lower vehicle speeds than the other two torque converters.

It must be appreciated that the performance criterion plays a small part only in the choice of a torque converter. Considerations of noise and of "engine "fussiness" are more important. Nevertheless the performance calculations should be performed.
The matching study theory is well known. Using polynomial curve fits in order to effect a match with a digital computer does not detract from the accuracy of a match. It is now desirable therefore, to study the accuracy of the new theory. This uses the polynomial curve fits to calculate the effect of engine inertia upon a match and to make due allowance in vehicle performance calculations. The new theory makes two corrections. The first is to deduct the inertia torque from the engine torque and to calculate its effect at the torque converter output shaft without change in match. The second is to recognise that a drop in engine torque level must result in a change in the match between engine and torque converter and to correct accordingly.

The accuracy of these two corrections may be assessed individually as follows. First, by setting engine inertia $I_o = 0$ causing no drop in the steady state engine torque and no mismatch. Hence the accuracy of the two corrections combined may be assessed. Secondly, by setting $\Delta T_o / \Delta N_o = 0$, thus assuming no mismatch only.

Fig. F6.6 shows the calculated percentage change in time to speed and engine speed of vehicle B fitted with a 225R torque converter as a result of setting the engine inertia at zero. This suggests an error of some 6% in the time up to the mid-speed range. This error decreases considerably as maximum vehicle speed is approached. The error in engine speed is about 2% at low speed decreasing rapidly to a negligible error.
Hence, making no allowance whatever for engine inertia and taking the steady state output torque from the match study without modification, results in quite a small error in the time to speed. The error is less than that incurred by an unrealistic assumption regarding transmission efficiency.

Setting the slope of the output torque curve at zero and repeating the performance calculations results in Fig. F6.7. This shows that making no allowance for the mismatch caused by the engine inertia torque produces very little error in the time to speed. The maximum error is only 0.2%.

Fig. F4.4 in Section F4 shows that the calculated engine speed is lower if no allowance is made for the mismatch. Fig. F6.7 suggests that this error is 5% approximately at the end of the first gear range. The error decreases rapidly as vehicle speed increases.

Since the new theory makes ample provision for both engine inertia itself and the change in the match between engine and torque converter caused by engine inertia, it may be used with confidence for vehicle performance calculations.

Finally, it should be mentioned that no comparison between calculated and test performance has been made for five reasons.

1) Few automatics are tested by the semi-technical press.
2) Part B, Appendix B8 shows discrepancies in the tests carried out by the semi-technical press.
3) the Department of Transport Technology, University of Technology, Loughborough, has not yet developed the instrumentation in order to carry out performance tests.

4) the automatic version of Vehicle B has been tested by one of the Motoring magazines, but it is apparent that the take-off procedure is different to the more obvious technique assumed in the theory. That is of depressing both throttle pedal and brake pedal and releasing the brake at $t = 0$. Hence, no real comparison is possible.

5) it must be confessed that the expression used for transmission efficiency in the performance calculations of automatics is simply 98% of that used for manual transmissions. The 98% has been inserted into the expression in order to take some account of the higher oil churning losses in the automatic gearbox. It has no experimental backing.

In the absence of any experimental evidence, the assumptions regarding transmission efficiency seem reasonable. But Fig. F6.8, a plot of time to speed of vehicle B assuming 100% transmission efficiency in comparison with the efficiency given above, emphasises the importance of transmission efficiency in these calculations. The maximum speed of Vehicle B assuming 100%
transmission efficiency is 101.5 mile/h. compared with 97.5 mile/h. assuming 98% of the manual transmission efficiency. The decrease in time to speed may be as much as 30%.

This, therefore, endorses the conclusion reached in connection with manual transmissions, that research efforts must be channelled into evolving a satisfactory expression for transmission efficiency in order to increase the accuracy of performance calculations.
Conclusions

1) The match between an engine and a torque converter may be described adequately using polynomials.

2) This match study is best conducted separately from the main vehicle performance calculations.

3) Performance itself is a criterion in choosing a torque converter for a particular engine, albeit, not a very important criterion. Considerations of noise and engine speed are more important. The matching program B071 is capable of yielding engine speed data. In this connection, it may be of advantage to repeat the match study at differing throttle angles throughout the engine load range. This endorses conclusion (2) above.

4) The results of the steady state matching study may be used in vehicle performance calculations in conjunction with the theory developed to allow for engine inertia. The theory is proved and is adequate.

5) A full parametric study of vehicle performance is possible with the minimum of computation. This is because a separate vehicle performance program (B058) is used which accepts t/
results of a previous match study between engine and torque converter. The new theory avoids the necessity of repeating this match study.

6) Research is required in order to provide an accurate expression for transmission efficiency.
PART 0

Future work and References
SECTION 91

Future Work

a) Transmission efficiency

It is thought that the greatest barrier to more accurate vehicle performance calculations is the lack of knowledge on transmission efficiency. It is not surprising that this lack exists because the efficiency of a transmission is difficult to measure. One is faced with the measurement of a small difference between large quantities.

The basic expression used throughout this work looks reasonable, but its foundations are shaky. It is shown in Section 76, Fig. 76.8, that a small change in the expression for transmission efficiency has a marked effect on the calculated time-to-speed of a high performance automatic transmissioned car. Fig. 11.1 shows the corresponding curves for the manual version of vehicle B, while Figs. 11.2 and 11.3 show the curves for a low performance van (vehicle A) and for a commercial vehicle (the Midland Red Coach), respectively. These curves emphasize the importance of the need for an accurate expression for transmission efficiency in vehicle performance calculations.

It is thought that separate expressions should be sought for gearboxes and drive-axles and that these should cover both motor cars and commercial vehicles with, and without, automatic transmissions.
Such expressions should be more than a simple function of speed and gear number, as has been used throughout this work. It should at least be of the form

\[
\eta_T = \eta_T(\text{input speed, input torque, gear ratio and viscosity of the oil})
\]

\[\text{Eq. 1}\]

Until this work has been done, it is not worth worrying too much about proving tests to establish the accuracy or otherwise of the performance programs except in so far as the comparison between road tests and theory could be used to provide information on the transmission efficiency. Embellishments to the computer programs to include the effects of the change in the reaction forces at the wheels due to aerodynamic lift and pitching moment or the refinements of Slubar and Desoyer (55) cannot yet be considered. Their inclusion would be meaningless at this stage.

Some work has been done already in the Department of Transport Technology, Loughborough University to measure gearbox transmission efficiency using a "four-square" or "back-to-back" rig (118). This has been hampered by the lack of accurate torque measuring devices. It is proposed to continue this work and to extend the field to cover other types of transmission.
b) **Vehicle instrumentation**

Work is to continue on the development of the two systems employed by the Department of Transport Technology, Loughborough University of Technology. One is a "fifth wheel" device of the digital type. The other works on the Dobbler Radar principle and consists of a small torch-like device which clips onto the front of a vehicle and shines obliquely down at the road. The immediate task is to use the deceleration test to develop the technique and the data handling of these methods. The long term aim is to employ these methods for measurement of vehicle speed during road tests, particularly the difficult condition of braking.
c) **Allowance for Ambient conditions**

Part C shows that wind speed, ambient pressure and temperature and under bonnet temperature can have a significant effect on vehicle performance. It is proposed to use the vehicle performance computer programs to develop a technique to correct the results of vehicle performance tests in much the same sort of way that engine test-bed results are corrected for ambient conditions.

It is thought that this can best be done by using the computer programs developed in this thesis for a range of vehicle types, rather than wholly by actual vehicle tests. This would avoid spurious results. The developed correction procedures must, however, be subjected to practical test before they can be accepted.

Similarly, it is proposed to extend the work of Part E to consider the effect of ambient conditions on the fuel consumption of vehicles. It may be that this work cannot be complete before a mathematical model is developed describing the engine characteristics.
Engine

Work is in hand to develop a data handling system for engine test bed work. This facility is in an advanced stage of development and will afford engine torque, engine speed, air flow rate and fuel flow rate readings on paper tape. This means therefore, that many readings can be taken and a digital computer used to process and plot out the engine characteristics. It is hoped to use this facility to investigate the effect of a small engine acceleration on the torque output curve by slowly running the engine up through its speed range and by logging torque and speed continuously. Also, it is hoped to study in more detail the effect of oil temperature on the torque output curve.

Effort must be directed to providing a mathematical model of typical engine characteristics. It is thought that this will prove a very useful tool in the effect of parameters, such as engine size, engine type, engine modifications, ambient conditions etc., on vehicle fuel consumption. Coupled with this, it is proposed to study procedures for the use of non-dimensional parameters to describe the engine.

Part E shows that reducing the aerodynamic drag coefficient of a vehicle can have a significant effect on fuel consumption and points to the engine cooling system as an obvious and unwarranted
source of drag. Attention must be paid therefore to a ducted cooling system as a means to reduce or eliminate this drag.

Other work, which may not be conducted at Loughborough, should be directed into devices to improve the characteristics of gas turbine engines, since Part E shows that they have a future with very large commercial vehicles which could be extended to small vehicles.

Part E shows also that work should be directed to the development of a stratified charge engine, or to a quiet, light weight, compression ignition engine for use in motor cars.
e) Drag

There is evidence that a second order polynomial expression may not be sufficient to describe the drag of a motor vehicle. It is shown in Part B that it is an approximation only to describe the rolling resistance by a constant term or by an expression linear with vehicle velocity. Further, by treating the aerodynamic drag as though it were all normal pressure drag, introduces some small error. It is proposed to study other possible drag expressions, say, higher order polynomials, using the deceleration test and vehicle instrumentation developed at Loughborough.

Detailed work on rolling resistance is being conducted by the tyre manufacturers and others. It is expected that this, using tyre rigs such as that described by Saki et al (95), will lead to the need to define much more closely the term "rolling resistance". Perhaps on the lines suggested by Slibar and Desoyer (55). The most pressing aspect of this work, from the point of view of vehicle performance calculations, is the effect of torque transmission through the tyre on the so-called rolling resistance. The time is approaching when this must be included in any calculation technique, particularly for conditions other than the "full throttle" condition.

An investigation should be conducted into the Dunlop expression for tyre growth. Using the Slibar and Desoyer very fundamental approach, such an expression becomes unnecessary. It being replaced
by the stiffness coefficient etc. Section C3 shows the effect
of tyre growth to be very small, but the dependence of the Dunlop
formula on the datum speed of 30 mile/h appears arbitrary and
artificial. This caused some small difficulties in Section C3
(see Fig. C3.2). Pogosbekov (63) shows that the change of the
rolling radius of a tyre with speed is inevitably tied up with
slip. For the sake of completeness therefore, this matter, together
with the effect of torque transmission, should be reviewed.
f) Vehicle take-off

The mechanism of torque transmission through a slipping clutch during take-off, wheel-spin and hill starts is not fully understood. This matter requires closer study backed by experimental measurements. Such a study should yield a better technique for the calculation of vehicle performance during clutch slip. However, such a study is desirable and opportune in its own right because the time taken for a vehicle to cover a short distance in the initial stages of a full throttle acceleration run is likely to become an important performance parameter. Such a parameter is used by Setz (78) and should increase in prominence as traffic density increases and speed limits decrease. The emphasis in this country seems to be moving quite rapidly from maximum speed to "nippiness".
g) **Off-the-road-vehicles**

This aspect of vehicle performance calculations appears to be too specialised for any general treatment. Others (Reece, Bekker etc) are conducting detailed studies into the effect of plastic ground on vehicle drag. It is proposed therefore, to keep abreast of new work in the field and, possibly, to seek to investigate the performance of a particular type or class of vehicle under contract.
h) **Effect of transients on fuel consumption**

The prediction of steady-state fuel consumption is reasonably clear and straightforward. Some work is required to facilitate parametric studies (see Sub-section d) above). The problem now arises on how to use the steady-state fuel consumption to give the fuel consumption returned by a particular driver over a particular route. This could be achieved approximately by using the "centre of operation" technique advocated by Dr. Giles (61), but it is thought that a much closer individual study should first be made into effects of the fuel injection/carburation system, driver technique, terrain and vehicle weight. Section 25 suggests that this latter point, vehicle weight, may have considerable effect and, therefore, should not be ignored.
1) **Automatics**

Since the development of hydrostatic transmission systems will continue and may soon become a reality, work should commence on classifying the types of hydrostatic systems and on designing suitable vehicle performance calculating techniques. It is expected that such techniques will employ the works of Macmillan (92), Ott (42), White and Christie (86), Ishihara and Emori (85) and Macmillan and Davies (91).

The technique of torque converter design outlined in this thesis and published elsewhere (44) may be improved by reference to the results of rig tests. It is thought that some attempt should be made to base design calculations on a realistic three-dimensional flow through the torque converter, rather than assuming a "mean path" flow.

Also, on the subject of torque converters, work is required on the study of the "over-run" condition, when the vehicle drives the engine through the torque converter. As far as is known, no attempt is made to predict the behaviour of the torque converter in this condition. The torque converter design is modified if it is found unsuitable in the over-run condition during development. A realistic theory, which could be an extension of the work of Lucas and Rayner (44), would enable a better optimisation of the torque converter design to be made at the design stage. Experience has shown that to attempt to avoid this condition, by providing a free-wheel between engine and transmission which "locks-up" on over-run, does not afford a satisfactory solution. The large change in engine speed between the over-run and the subsequent normal drive condition is generally unacceptable.
SECTION G2

List of References


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   with reference to torque converter and power changing"

43. Hertz, P.B. and Ukrainez. "Auto. - Aerodynamic drag -
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44. Lucas, G.G. and Rayner, A. 1968. "Torque converter design
   calculations". Tech. note T.T. 6803 Dept. of
   Transport Technology, Loughborough University of
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The above list of references is not intended as a "bibliography". All have been used in this study of "Vehicle performance calculations". The normal process of editing this thesis may, however have resulted in a few redundancies. The following references have come to hand during the final stages of the compilation of this thesis. They are relevant and will be employed in the continuation of this work.


(a rough translation exists at Loughborough).

APPENDIX E1

Polynomial Curve Fit (E079)
The program listed below uses the "method of least squares" to fit a curve, first of order 1 and successively increasing the order by 1, until a curve of the specified order is fitted to the data supplied. The specified order must not exceed 15 and should not exceed 10 if any of the data is outside the range .01 to 100. This is because accuracy is impaired if large or small numbers are raised to high powers.

Input data
Card I in free fixed point format ISW2, ISW3, NCASES
ISW2 set at 1 if the polynomial coefficients for each of the orders fitted up to and including the specified order are to be printed. If the coefficients of the specified order only are required, set ISW2 = 0
ISW3 set at 1 if the table of observed versus calculated values are required for each order up to and including the specified order. Setting ISW3 = 0 produces the table for the specified order only.
NCASES Number of separate sets of data to which curves are to be fitted.
Card 2 Heading card, omit column 1.
Anything contained on Card 2 will be printed out as a title in the output.
Card 3 Free fixed point format N, LAST.

N is the number of data points,
LAST is the specified order

Card 4 Free floating point format

X_1, Y_1, X_2, Y_2, X_3, Y_3, X_4, Y_4 etc. up to X_N, Y_N

If NCASES is greater than 1, repeat from Card 2.

**Output (on line printer)**

After the heading, the polynomial coefficients are printed in E-format to 9 significant figures. This is followed by a Table containing the read-in X-values (independent variable), the read-in Y-values (dependent variable), the calculated Y-values using the fitted polynomial and finally the difference between the calculated and the read-in X-values.
*FORTRAN Compilation by #Xfas MK 1C  Date  20/01/68  Time  19/48/54*

*FORTRAN B079, G.G. Lucas Curve Fit*

MASTER B079

DIMENSION X(100), Y(100), COF(18)

COMMON X, Y, COF

READ(1,101) ISW2, ISW3, NCASES

DO 1 ICASE = 1, NCASES

WRITE(2,100)

READ(1,102)

WRITE(2,102)

WRITE(2,103)

READ(1,101) Y, LAST

READ(1,104) (X(I), Y(I), I = 1, N)

CALL POLY (ISW2, ISW3, N, LAST, 0)

1 CONTINUE

STOP

100 FORMAT (1H145X20H POLYNOMIAL CURVE FIT//)

101 FORMAT (6I0)

102 FORMAT (80H)

103 FORMAT (1H5//)

104 FORMAT (200F0.0)

END

END OF SEGMENT, LENGTH 95, NAME B079
SUBROUTINE POLY (ISH2, ISW3, N, LAST, IH)
DIMENSION X(100), Y(100), CNP(1A), A(17, 17), SUMX(31), SUMY(17)
COMMON X, Y, CNP
70 SUMX(1)=0.
SUMX(2)=0.
SUMX(3)=0.
SUMY(1)=0.
SUMY(2)=0.
DO 90 I=1, N
CX=SUMX(I)
CY=SUMY(I)
SUMX(1)=SUMY(1)+1.
SUMX(2)=SUMX(2)+CX
SUMX(3)=SUMX(3)+CX*CX
SUMY(1)=SUMY(1)+CY
90 SUMY(2)=SUMY(2)+CX*CY
NORD=1
91 L=NORD+1
KK=L+1
DO 101 I=1, L
DO 100 J=1, L
IK=J-1+1
100 A(I, J)=SUMX(IK)
101 A(I, KK)=SUMY(1)
DO 140 I=1, L
A(KK, I)=1.
KKK=K+1
DO 110 J=KKK, KK
110 A(KK, J)=0.
C=1./A(I, I)
DO 120 I=2, KK
DO 120 J=KKK, KK
120 A(I, J)=A(I, J)-A(1, J)*A(I, I)*C
DO 140 I=1, L
DO 140 J=KKK, KK
140 A(I, J)=A(I+1, J)
S2=0.
DO 160 J=1, N
YJ=Y(J)
XJ=X(J)
S1=0.
S1=S1+A(I, KK)
DO 150 I=1, NORD
150 S1=S1+A(I+1, KK)*XJ**2
160 S2=S2+(S1-YJ)**2
S2=SORT(S2/C)
IF (IS#2) 151, 161, 163
161 IF (NORD-LAST) 171, 163, 171
163 WRITE(2, 1020) NORD, S2
WRITE(2, 1090)
DO 164 I=1, L
J=I-1
CNP(I)=A(I, KK)
164 WRITE(2, 1030) J, A(I, KK)
IF (IS#3) 165, 165, 167
165 IF (NORD-LAST) 171, 167, 171
167 WRITE(2, 1100)
DO 169 I=1, N
S1=0.
S1=A(1, KK)
DO 168 J=1, NORD
168 S1=S1+A(J+1, KK)*X(I)**J
S3=Y(I)-S1
WRITE(2,1040)X(I),Y(I),S1,S3
IF (NORD-LAST) 171, 173, 173
171 NORD=NORD+1
J=2*NORD
SUMX(J)=0.
SUMX(J+1)=0.
SUMY(NORD+1)=0.
DO 172 I=1, N
CX=X(I)
CY=Y(I)
SUMX(J)=SUMX(J)+CX**(J-1)
SUMX(J+1)=SUMX(J+1)+CX**J
172 SUMY(NORD+1)=SUMY(NORD+1)+CY*CX**NORD
GO TO 91
173 CONTINUE
1020 FORMAT(10X,4ORDER = 13.5X25XROOT MEAN SQUARE ERROR = E10.6)
1090 FORMAT(1X,10X,10X,11X,COEFFICIENT)
1030 FORMAT(13,E23.9)
1100 FORMAT (11X,19X19X16X9X4CALC19X5ERROR)
1040 FORMAT (1X,4E20.8)
RETURN
END

END OF SEGMENT, LENGTH 801, NAME POLY
Evaluation of a Polynomial (8054)

The program listed below accepts a set of polynomial coefficients and evaluates the polynomial function between two values of the independent variable in specified steps.

Input data

Card 1. Heading card, omit column 1.

Any data on card 1 (except column 1) will be printed out as a title.

Card 2. Free fixed point format NC, NP

NC is the number of polynomial coefficients (i.e. polynomial order number plus 1)
NP is the number of points to be evaluated.

Card 3. Free floating point format STEP, XNIT

STEP is the step length between the value of the independent variable at each evaluation.

XNIT is the initial value of the independent variable.

Card 4. Free floating point format

The polynomial coefficients starting with the constant coefficient.

Card 5. If program is to be used again immediately with fresh data, put a positive or negative integer in columns 1 and 2. If evaluation is to end, leave Card 5 blank.
*FORTAN B054, G.G. LUCAS EVALUATION OF A POLYNOMIAL
MASTER B054
DIMENSION C(11)
1 CONTINUE
WRITE(2,100)
100 FORMAT(10x37HEVALUATION OF A POLYNOMIAL EXPRESSION//)
READ(1,109)
WRITE(2,109)
WRITE(2,110)
READ(1,110)NC,NP
101 FORMAT(510)
C NC IS NUMBER OF COEFFICIENTS (ORDER + 1), NP IS NUMBER OF POINTS P
READ(1,102)STEP,XNIT
102 FORMAT(11F0.0)
C STEP=STEP LENGTH,XNIT=INITIAL VALUE OF INDEPENDENT VARIABLE
READ(1,102)(C(I),I=1,NC)
WRITE(2,103)
103 FORMAT(28HPOLYNOMIAL COEFFICIENTS ARE/)
DO 2 I=1,NC
2 WRITE(2,104)C(I)
104 FORMAT(E19.8)
WRITE(2,103)
105 FORMAT(14010X20HINDEPENDENT VARIABLE10X18HINDEPENDENT VARIABLE/)
X=XNIT
DO 3 J=1,1NP
Y=C(J)
3 X=X+STEP
READ(1,111)JJ
111 FORMAT(I2)
IF(JJ).EQ.5,0
WRITE(2,107)
107 FORMAT(13H1 NEW RUN//)
GO TO 1
5 CONTINUE
WRITE(2,110)
WRITE(2,108)
108 FORMAT(21HO END OF EVALUATION)
109 FORMAT(80H)
1
110 FORMAT(1HO)
STOP
END

END OF SEGMENT, LENGTH 205, NAME B054
APPENDIX B2

Equivalent Mass of a Vehicle

At a particular engine speed during an acceleration run, the torque delivered by the engine is, neglecting transient effects, the measured steady running torque less that required to accelerate the engine. Therefore, it can be written that

\[ \text{force on vehicle} = (T - T_e) \times \frac{\gamma_T \times \text{DAR} \times \text{GR}}{r_T} \text{lbf} \quad --- 1 \]

\( T_e \) is the torque required to accelerate the engine and is given by

\[ T_e = I_e \times \frac{\text{DAR} \times \text{GR} \times f}{r_T} \quad --- 2 \]

\( I_e = \) engine inertia slug ft\(^2\)

\( \text{DAR} = \) drive-axle ratio

\( \text{GR} = \) gear ratio

\( r_T = \) rolling radius of drive wheels ft

\( f = \) vehicle acceleration ft/sec\(^2\)

equation 1 may then be re-written as

\[ \text{force on vehicle} = F_T - I_e \times \frac{\gamma_T \times (\text{DAR} \times \text{GR})^2 \times f}{r_T} \quad --- 3 \]

where \( F_T = T - T_e \) and may be termed the "tractive force" --- 4

This force on the vehicle is required to accelerate the vehicle mass itself, the inertia of the road wheels and to overcome the drag force \( F_d \) whence,

\[ F_T = I_e \times \frac{\gamma_T \times (\text{DAR} \times \text{GR})^2 \times f}{(\frac{r_T}{r_T^2})} = (\frac{m_v + I_v}{r_T^2}) f + F_d \quad --- 5 \]
therefore the propulsive force on the vehicle is

\[ P_p = F_T - F_d = f_0 \left( \frac{m_v + I_w + I_o}{r_r^2} \right) \left( \frac{\gamma T (DA \times GH)^2}{r_r^2} \right) \]  \hspace{1cm} (6)

Comparison with

\[ P_p = f_0 \cdot m_E \]  \hspace{1cm} (7)

yields that the equivalent mass of the vehicle is

\[ m_E = m_v + I_w + I_o \times \frac{\gamma T (DA \times GH)^2}{r_r^2} \text{ slug} \]  \hspace{1cm} (8)
In this appendix, the equations governing the theory of wheel spin contained in section 8, part B are developed.

Wheel spin is said to occur when the drive force at the drive wheels is greater than the limiting tractive force.

The limiting tractive force, for a rear wheel drive vehicle is

\[ F_1 = \frac{W(a \cos \theta + h(\sin \theta + \frac{g}{k}))}{(a+b)} \text{ lbf} \]  \hspace{1cm} (1)

and for a front wheel drive vehicle

\[ F_1 = \frac{W(b \cos \theta - h(\sin \theta + \frac{g}{k}))}{(a+b)} \text{ lbf} \]  \hspace{1cm} (2)

(These expressions can be derived from Fig. B3.1 section 8, part B.)

The drive force at the drive wheels is obtained by subtracting the accelerating inertia torque of the engine from the steady running torque, factoring this by the transmission efficiency and the product of (gear ratio x drive axle ratio), subtracting the accelerating inertia torque of the drive wheels only, dividing this torque by the rolling radius, from which is subtracted the rolling resistance of the drive wheels only.

This accelerates the vehicle mass and that of the free road wheels, overcomes the aerodynamic drag and the gradient term in the drag equation and finally caters for the rolling resistance of the free road wheels.
Thus, for a conventional type of vehicle having half its road wheels as drive wheels and no slippage between drive wheels and road,

\[
\text{drive force} = T \left( \frac{W + I_w}{g} \right) + \text{Aerodynamic drag} + \frac{W \cdot l}{2 \times (r_\tau)^2} + \frac{W}{2} (A_d + B_d \cdot V)
\]

which is equivalent to

\[
\text{drive force} = T \left( \frac{W + I_w}{g} \right) + F_d - \frac{W}{2} (A_d + B_d \cdot V)
\]

The maximum possible vehicle acceleration is obtained by equating the drive force to the limiting tractive force producing, in the case of the rear wheel drive vehicle

\[
f_{\text{max}} = \frac{\frac{W}{g} (a \cos \Theta + h \sin \Theta) - F_d - \frac{W}{2} (A_d + B_d \cdot V)}{\frac{W}{g} + \frac{I_w}{2 \times (r_\tau)^2} + \frac{W \cdot h}{g(a + b)}}
\]

and for a front wheel drive vehicle

\[
f_{\text{max}} = \frac{\frac{W}{g} (b \cos \Theta - h \sin \Theta) - F_d + \frac{W}{2} (A_d + B_d \cdot V)}{\frac{W}{g} + \frac{I_w}{2 \times (r_\tau)^2} + \frac{W \cdot h}{g(a + b)}}
\]

The Tables A.B3.1 to A.B3.4 given below list a proportion of the calculated values of limiting tractive force.

The Tables A.B3.5 and A.B3.6 list some results of maximum vehicle acceleration calculations.
The vehicle particulars used in the calculations are that:

- Weight: \( W = 21.28 \text{ lbf} \)
- Rolling radius: \((r_r)_{30} = 0.94 \text{ ft} \)
- Total wheel inertia: \( I_w = 1.96 \text{ slug ft}^2 \)
- Position of centre of gravity:
  - \( a = 3.70 \text{ ft} \)
  - \( b = 3.85 \text{ ft} \)
  - \( h = 1.66 \text{ ft} \)
- Drag coefficients:
  - \( A_d = 0.018 \)
  - \( B_d = 0. \)

These Tables are used to construct the graphs in Figs. B8.2 to B8.5 given in Section 8, part B.

The listing of the computer program (B065) used in the evaluation of the figures in the Tables is given below.
LIST(LD)  
PROGRAM(ROADS)  
INPUT1=0.0  
OUTPUT4=120  
END  
MAPIA ROAD  
WRITE(4,100)  
100 FORMAT(50X10MMWHEEL BFIN//)  
WRITE(4,101)  
101 FORMAT(40X16MMPAR WHEEL DRIVE//)  
W=2128.  
A=3.7  
B=3.85  
H=1.66  
RR=.94  
AIW=1.96  
AD=.018  
BD=0.  
V=0.  
M=0.  
CONTINUE  
CF=0.  
DO 1 I=1,19  
WRITE(4,102)CF  
102 FORMAT(29HW COEFFICIENT OF FRICTION ISF6.5)  
THETA=0.  
DO 2 J=1,20  
F=0.  
DO 3 K=1,6  
T=THETA*3.14159265/180.  
TF=CF*W/(A+R)*(A+cos(T)+H*sin(T)+F/32.2))  
WRITE(4,103)THETA,F,TF  
103 FORMAT(F7.3,F10.2,F15.5)  
3 F=F+.4.  
FM=(CF*W/(A+R)*(A+cos(T)+H*sin(T))=W/2.+(AD+BD*V+2.*sin(T))/W/32.2  
1.2+AIW/2.)/RR/RR-CFOW*H/32.2/(A+B))  
WRITE(4,104)THETA,FM  
104 FORMAT(F7.2,F15.4)  
2 THETA=THETA+.2.  
1 CF=CF+.1  
IF(M)=0.0.5  
105 FORMAT(1I9)  
WRITE(4,105)  
WRITE(4,106)  
106 FORMAT(40X17MMREAR WHEEL DRIVE//)  
M=1  
B=3.7  
A=3.85  
H=1.66  
GO TO 5  
CONTINUE  
STOP  
END  
FINISH
The classical method of designing intermediate gear ratios

The basic assumption behind this method is that the engine operates between two speeds. The lower limit is near the maximum torque speed when the engine throttle or fuel pump rack is fully open. The upper limit is below maximum allowable engine speed but may be above maximum brake horsepower speed. A gear change with an accelerating vehicle changes the engine speed from the upper limit to the lower limit.

Fig. A.B4.1 depicts this condition in a plot of input to gearbox speed (related to engine speed) against output from gearbox speed (related to vehicle speed). For a particular gear ratio, the relationship between input and output speeds is represented by a straight line passing through the origin, the slope of the line being the gear ratio. The lower and upper limits on the input to gearbox speed are denoted by \( N_L \) and \( N_U \) respectively. When the upper speed limit is reached in first gear, a change is made to gear 2, the input speed dropping to the lower limit \( N_L \) and the output speed remaining at \( N_1 \). This process is repeated for successive gear changes throughout the vehicle acceleration period until top gear ratio is engaged.

By definition, the relationship between first gear ratio and the gearbox input and output speeds is
This relationship is shown in Fig. A.B4.1

Also, by definition,

\[
GR(2) = \frac{N_u}{N_2} \quad (2)
\]

and, as can be seen in Fig. A.B4.1

\[
GR(2) = \frac{N_L}{N_1} = \frac{N_L}{N_u} \cdot \frac{N_u}{N_1} = \frac{N_L}{N_u} \cdot GR(1) \quad (3)
\]

similarly,

\[
GR(3) = \frac{N_L}{N_2} = \frac{N_L}{N_u} \cdot \frac{N_u}{N_2} = \frac{N_L}{N_u} \cdot GR(2) \quad (4)
\]

\[
GR(4) = \left(\frac{N_L}{N_u}\right)^3 \cdot GR(1) \quad (5)
\]

and

\[
GR(HG) = \left(\frac{N_L}{N_u}\right)^{NG-1} \cdot GR(1) \quad (6)
\]

This is a geometric progression

Now top gear ratio and bottom gear ratio, \(GR(HG)\) and \(GR(1)\) are fixed by other considerations.
Thus the number of gear ratios ($N_G$) required is obtained by satisfying

$$N_L = \frac{\sqrt{\text{top gear ratio} \times N_G}}{\text{bottom gear ratio}}$$

where $N_G$ must be an integer greater than 1.

If the maximum torque speed of the engine is low, the engine is said to be "flexible". $N_L$ can be low and so necessitating a small number of gear ratios only.

If $N_L$ is fixed at a figure considerably greater than that equivalent to engine maximum torque speed, a larger number of gear ratios results, but there will be more "overlap" between the gear ratios. That is to say that, near a vehicle gear change speed, either of two gear ratios may be engaged. This is an important consideration in performance work on gradients.
APPENDIX B5

Tailor-made Optimisation Technique

It is not necessary to optimise the intermediate gear ratios to a high degree of accuracy, because the transmission designer must be allowed some latitude. Accordingly, a method was devised to keep computer time to a minimum. Basically, the tailor-made technique optimises each intermediate gear ratio separately, and in turn. This cycle is repeated four times.

Fig. A.B5.1 depicts the expected shape of the time to speed variation against the value of a particular gear ratio (GR₁).

Point 1 on the curve denotes the initial guess. Point 2 denotes an evaluation of the function when the particular gear ratio has been increased by 0.1. Point 3 when the gear ratio has been increased by a further 0.1. At every evaluation, a check is made to see if the new point produces a lower or higher time to speed. The gear ratios continue to be increased by 0.1 until an evaluation (point 5) proves worse than the previous value. The procedure then is to continue subtracting 0.01 from the gear ratio until the optimum point has been passed. 0.005 is then added to the gear ratio and it declared to be the optimum value.

If the second evaluation (point 2 on Fig. A.B5.2) should prove worse than the initial guess, then 0.2 is subtracted from the gear ratio at point 2 and the method continues to subtract 0.1 until the optimum is passed. The method then adds 0.005 is then subtracted.
ratio declared the optimum.

The first difficulty envisaged is shown in Fig. A.B5.3. Successive trials with gear ratio increasing in steps of 0.1 produce better answers, even though point 4 has passed the optimum. To increase the gear ratio beyond this before starting the subtraction of 0.01 would mean up to twenty evaluations in steps of 0.01. Accordingly, the method stores the current evaluation, the previous evaluation, and the penultimate evaluation. By comparing the current evaluation with the penultimate evaluation it is possible to assess whether the previous value has passed the optimum. If the current evaluation returns a greater time to speed than the penultimate, the technique is to jump back to the previous evaluation and to start re-tracing in steps of 0.01. Similarly, if 0.1 is successively subtracted from the gear ratio under consideration and the current evaluation proves worse than the penultimate, the method returns to the previous value and 0.01 is successively added onto the gear ratio until the optimum is found.

The initial comparison involves comparing the second evaluation of the time to speed with the first. In order to prevent the program reaching premature conclusions based upon the device described in the previous paragraph, the penultimate evaluation is deliberately set at a very high value. In fact to 1000 times the first evaluation. On the next run through, the penultimate
evaluation assumes the value of the first and the previous the value of the second, and so on.

Fig. A.B5.4 is a picture of the tailor-made optimisation program. \( T_3 \) denotes the current evaluation of the function, \( T_2 \) the previous evaluation and \( T_{SI} \) the penultimate evaluation. \( J \) is a marker or tracer used to keep track of the previous path through the program at any given time. The circuit on the left of Fig. A.B5.4, coloured green, is the main circuit. Initially, \( J \) is set at zero. If successive trials continue to produce better times to speed, \( J \) is re-set at 10 and 0.1 is added on to the gear ratio each time. If the initial direction of increasing the gear ratio is found to be wrong, \( J \) is set at 1 and 0.1 subtracted from the gear ratio each time.

This continues until \( TS \) is no longer less than \( T_{SI} \). A test is then made to see if \( TS \) is less than the penultimate evaluation \( T_{SI} \). If it is, 0.01 is successively added to or subtracted from the gear ratio as appropriate, the marker \( J \) being used to decide which path to take. If, however, \( TS \) is greater than \( T_{SI} \), the upper red circuit is used. This causes the program to jump back to the previous trial and to start the small scale search for the optimum, through the lower red circuit on the left.

Much of the lower part of Fig. A.B5.4 is the small scale search of adding or subtracting 0.01 onto the gear ratio. The red circuit to the extreme right, however, is to cater for the unlikely event of the current evaluation proving equal to the penultimate evaluation. In this case it is taken that the previous value of
gear ratio is the optimum.

Each gear ratio is subjected to this program in turn, and the whole procedure repeated in order to cover the routine four times.

The various possible routes are depicted in Figs. A.B5.5 to A.B5.11. The initial guess is shown as point 1 in each case and the current, previous and penultimate function evaluation, together with the value of the tracer J, is shown in the adjoining tables for each step. Fig. A.B5.5 shows the process of optimisation when the initial guess is some way to the left of the optimum. Fig. A.B5.6 depicts a similar set of circumstances using the penultimate evaluation to shorten the procedure. Fig. A.B5.7 is when the initial guess is to the right of the optimum. Fig. A.B5.8 shows the condition when the initial guess is to the left of the optimum and the second step returns a better result, but is past the optimum. In Fig. A.B5.9 and Fig. A.B5.10 the initial guess is very close to the optimum, in the one case, just to the left and in the other, just to the right. Fig. A.B5.11 shows the initial guess at or very close to the optimum such that the evaluations either side are equal. The tables trace out the path followed until the small scale search is started.

From the foregoing, it will be seen that the tolerance or the search for a particular optimum gear ratio is ± 0.005. This is usually adequate for practical purposes. The real advantage of the method lies in the elimination of unnecessary searches.
of the function.

The listing of the optimisation procedure is contained within the listing of the main vehicle performance program (B001) in Section B12.
APPENDIX B6

The Rosenbrock Optimisation Subroutine

The subroutine listed below finds the maximum or the minimum of a function of up to 16 independent variables subject to up to 24 constraints.

The variables X(I), I = 1, 2, 3 etc. must be set up initially in the MASTER segment of the program.

The first subroutine (FXS68) is the control subroutine.

The arguments are as follows

- N = Number of independent variables
- M = Number of constraints

\[ N \leq M \leq 24 \]

- RMAX = The number of iterations per variable
- IPRINT, if set at zero, extra print out is obtained.
- BJ Put at +1. if a maximum is sought, -1. for a minimum.
- F is the function evaluation to be maximised or minimised.

Reservation of 32 storage spaces in DIMENSION W (32) carries the 32 quantities contained in the MASTER program necessary to evaluate the function through the Rosenbrock subroutine and into the special subroutine CALLGH which generates the function to be maximised or minimised.

G (I) and H (I) are the upper and lower bounds respectively of variable X (I). These bounds are set up in the MASTER program.
If no constraints are needed, put

\[
\begin{align*}
G(I) &= x(I) - 1 \\
H(I) &= x(I) + 1
\end{align*}
\]

and reset these values each time in the subroutine CALIGH.

The subroutine CALIGH is identical to the subroutine TIME contained in the main vehicle performance program (B001) listed in Section B12.

The structure of a program using the Rosenbrock subroutines therefore is

```
MASTER PROGRAM

ROSENBROCK SUBROUTINES

SUBROUTINE GENERATING FUNCTION
```
SUBROUTINE PXS68(N,M,KMAX,PRINT,BJ,F)
DIMENSION ARRAY(308),G(48),W(48),X(24),W(32)
COMMON ARRAY,G,H,X,W
CALL PXS61A(N,M,L,IT,ICOUNT,NA)
201 CALL CALXGH(N,M,IT,F)
CALL PXS63A(N,M,L,IT,ICOUNT,INDIC,F,BJ,KMAX,NA,NG)
GO TO (202,204,201),INDIC
202 CALL PXS62A(N,M,L,NA)
204 CALL PXS64A(N,M,L,IT,ICOUNT,PRINT,INDIC,KMAX,BJ,NA,NG)
GO TO (201,203),INDIC
203 RETURN
END

END OF SEGMENT, LENGTH 95, NAME PXS68
SUBROUTINE PXS61A(N,M,L,IT,ICOUNT,NA)
DIMENSION B(2),U(2),A(272),D(16),E(16)
COMMON R,U,A,D,E
EXPF(Y) = EXP(Y)
LOGF(Y) = ALOG(Y)
SINF(X) = SIN(X)
COSF(Y) = COS(X)
ATANF(X) = ATAN(X)
SORTF(X) = SORT(X)
ABSF(Y) = ABS(Y)
B(1)=0,
R(2)=0,
ICOUNT=0
DO 1 L=1,N
A(L)=0.1
E(L)=0.
K=L
DO 3 K=1,N
K=K+N
R(K)=0.
IF(K-KR)1,3,1
3 A(K)=1.
1 CONTINUE
L=N
IT=1
WRITE(4,2)
2 FORMAT(50H: PXS6, MAXIMUM OR MINIMUM OF A CONSTRAINED FUNCTION)
RETURN
END
SUBROUTINE PXS61A(N,M,L,IT,ICOUNT,NA)
DIMENSION B(2),U(2),A(272),D(16),E(16)
COMMON A,J,A,D,E

EXPF(X) = EXP(X)
LOGF(X) = ALOG(X)
SINF(X) = SIN(X)
COSF(X) = COS(X)
ATANF(X) = ATAN(X)
SORTF(X) = SORT(X)
ABSF(X) = ABS(X)

B(1) = 0.
B(2) = 0.
ICOUNT = 0
DO 1I=1,N
A(L) = .1
E(L) = 0.
K = L
DO 1 K=1,N
K = K + N
A(K) = 0.
IF (I = K) 1,3,1
3 A(K) = 1.
1 CONTINUE
L = N
WRITE (4,2)
2 FORMAT (50H: PXS61A, MAXIMUM OR MINIMUM OF A CONSTRAINED FUNCTION)
RETURN
END
SUBROUTINE PY562A(N,M,L,NA)
SORTF(X)=SORTF(X)
DIMENSION B(2),U(2),A(272),D(16)
COMMON B,U,A,D

4 L=N-1
  J0=1
106 K=N+J0+N
  A(K)=D(N)*A(K)
  K=K+1
104 K=N+J0+K
41 A(K)=D(KP)*A(K)+A(K+1)
  KR=K-1
103 J0=J0+1
  IF(N=J0)105,103,104
105 DO 29 L=1,2
  B(L)=0
  K=L
  ON 31 JT=1,4
  K=K+N
31 B(L)=A(K)*A(K)+B(L)
29 B(L)=SORTF(R(I))
  R(2)=R(2)/B(1)
  J0=1

5 L=1
6 IF(L=J0)45,7,43
43 R0=0
  K=J0
  DO 44 KR=1,N
  K=K+N
  JS=K-L
44 BO=A(K)*A(JS)+BO
  K=J0
  DO 45 KR=1,N
  K=K+N
  JS=K-L
45 A(K)=A(JS)*BO+A(K)
  L=L+1
  GO TO 7
43 R0=0
  K=1
  DO 46 JT=1,N
  K=K+N
46 BO=A(K)*A(K)+BO
  BO=SORTF(BO)
  K=J0
  AD=1./BO
  DO 47 JT=1,N
K = K + N
47 A(K) = ANS(A(K))
J0 = J0 + 1
IF (N = J0) 9.5
8 RETURN
END

END OF SEGMENT, LENGTH 396, NAME PXS82A
SURROUNTE PY53A(N, M, L, IT, ICOUNT, INDIC, F, BJ, KMAX, NA, NG)
ARSF(Y) =ABS(Y)

DIMENSION B(2), U(2), A(272), D(16), E(16), G(48), H(48), X(24)
COMMON B, U, A, D, E, G, H, X
U(IT)=F*BJ
IS=1
102 IF(G(IS)>Y(IS)) 61, 22, 22
61 IF(Y(IS)-H(IS)) 62, 22, 22
62 IF(U(IT)-U(IT)) 63, 63, 16
63 K=K+15
GO=0.9999*G(IS)+0.0001*H(IS)
H0=G(IS)+H(IS)-GO
IF(GD=Y(IS)) 44, 64, 24
64 IF(Y(IS)>HO) 65, 65, 26
65 G(KR)=U(1)
H(KR)=U(1)
98 IS=IS+1
98 IF(IS=M) 102, 102, 21
21 IF(IT-2) 166, 14, 166
166 IT=2
66 INDIC=2
GO TO 101
22 IF(IT-2) 23, 16, 23
23 WRITE(4, 99)
99 FORMAT(43H INITIAL VALUES OF X NOT WITHIN CONSTRAINTS)
GO TO 68
24 IF(IT-1) 88, 24, 88
68 GO=H0-X(IS)/(GO-G(IS))
H0=U(IT)=G(KR)
25 BD=-2.0*GD+4.0*GO-3.0
U(IT)=BD*GO*HO*U(IT)
GO TO 98
26 IF(IT-1) 167, 23, 67
67 GO=(Y(IS)-HO)/(H(IS)-HO)
H0=U(IT)=H(KR)
GO TO 25
14 IF(U(IS)-U(2)) 94, 54, 16
54 GO=ABS(F(E(L))
IF(GO-1.1) 55, 55, 15
55 E(L)=1.5
15 D(L)=D(L)+A(L)
U(IS)=U(2)
A(L)=A(L)+A(L)
GO TO 17
16 K=K
DO 56 IS=1, N
K=K+1
56 X(IS)=-A(KR)+A(L)+Y(IS)
A(I) = 0.5*A(L)
IF (F(L)) 17, 57, 47
57 E(L) = E(L)
17 IF (ICOUNT = N-KMAX) 58, 58, 66
58 DO 59 IS = 1, 4
18 IF (F(IS)) 59, 59, 18
60 CONTINUE
INDIC = 1
GO TO 101
59 CONTINUE
END

END OF SEGMENT, LENGTH 645, NAME PX565A
SUBROUTINE PXS64A(N,M,L,IT,ICOUNT,IPRINT,INDIC,KMAX,BJ,NA,NG)
  DIMENSION B(2),U(2),A(272),D(15),E(15),G(48),H(48),I(28)
  COMMON B,U,A,D,E,G,H,I
  B0=BJ+U(1)
  IF(IPRINT)35,48,33
  48 WRITE(4,102)ICOUNT,B0,B(1),B(2)
    DO 49 L=1,M
  49 WRITE(4,103)Y(L)
  50 IF(IT-1)11,9,9
  9 INDIC=2
    GO TO 105
  102 FORMAT(15,2(A,E12.5),F20.5)
  103 FORMAT(A,E12.5)
  11 DO 52 L=1,N
    N(L)=0.
  52 L=1.
    K=L
    DO 53 KR=1,N
      K=K+N
    53 Y(KR)=A(L)*A(K)*X(KR)
    ICOUNT=ICOUNT+1
    IT=2
    INDIC=1
  105 RETURN
END

END OF SEGMENT, LENGTH  177, NAME  PXS64A
APPENDIX B7

More detailed optimisation

The program listed below (B080) uses the Rosenbrock optimisation subroutines of Appendix B6 in order to find the intermediate gear ratios which give the minimum sum of the time up to

$\frac{1}{v_2}, \frac{1}{v_2}, \frac{1}{v_2}, \frac{1}{v_2}$ and $v_2$

where $v_2$ mi/a/h is some specified final speed.

This is an attempt at emphasizing the importance of the vehicle acceleration at the lower speeds.

The read-in data to B080 is identical to that for B001 described in Section B1.2. Program B080 carries on to evaluate engine maximum torque speed and maximum brake horsepower speed. Statement number 60 sets the initial values of the intermediate gear ratios such that their reciprocal values are in arithmetic progression. Between statement numbers 402 and 401 the Rosenbrock constraints $G(k)$ and $H(k)$ are set. Statement number 401 + 1 sets the test gradient $(G00$) at zero and statement number 401 + 2 fixes the degree of printout required. Statement number 401 + 3 calls up the Rosenbrock subroutines. Forty iterations per variable are specified.

B080 continues by evaluating the time to accelerate between two speed limits on the specified gradient if it is greater than zero. Finally, B080 evaluates maximum vehicle speed in a manner
identical to BOOl.

Following the listing of BO80 is a typical printout. This consists of the headings, title and the read-in data, followed by the results of the engine maximum torque and brake horsepower speed evaluations. The legend "PESO, MAXIMUM OR MINIMUM OF A CONSTRAINED FUNCTION" denotes entry into the Rosenbrock subroutines. The numbers following show the progress of the optimisation procedure. Column 1 is the number of iterations. Column 2 gives the current evaluation of the function. Column 3 and 4 concern the mechanism of the procedure, and give the current step length and degree of rotation of the axes of the evaluation respectively. After printing out the current data in the four columns, the value of the variables are listed. In this example there are two.

The final table of the print out gives the gear ratios and the maximum vehicle speed on the level in each of the gears.
MASTER BOLO

DIMENSION ARRAY(308), G(48), H(48), X(24), W(26), GR(10), F(10)

COMMON ARRAY, G, H, X, GR, CT, DAR, RR, AT, BT, CT, DT, ET, AD, BD, AK, WW, AIW, AIE

CF, KA, A, R, H, NG, NO, VBS, V1, V2, GGG, RM, FT, GT, RMIN, DTD, Wv, RX

EXPF(X) = EXP(X)

LOGF(X) = ALOG(X)

SINF(X) = SIN(X)

COSF(X) = COS(X)

ATANF(X) = ATAN(X)

SORTF(X) = SORT(X)

ABSF(X) = ABS(X)

CONTINUE

J=25

WRITE(4,100)

READ(I,1001)

WRITE(4,1001)

WRITE(4,1002)

READ(I,102)AT, BT, CT, DT, ET, FT, GT, VBS, NO

IF(J=1)2,26,2

CONTINUE

3 CONTINUE

2 READ(I,102)NG, NB, NBOT

IF(NB)5,6,6

CONTINUE

READ(I,108)(GR(I), I=1, NG)

IF(J=6)7,26,7

READ(I,108)DAR, RR

IF(J=7)8,26,8

4 READ(I,108)UG, RR

NO=6

IF(J=5)6,6,8

READ(I,108)AIW

IF(J=8)9,26,9

9 READ(I,108)AIW, RMIN

IF(J=9)10,26,10

READ(I,108)WW, A, B, HH, KA

IF(J=10)11,26,11

READ(I,108)AD, BD, AK, XK

XK = XK / 1000000.

IF(J=11)12,26,12

12 READ(I,108)GG

IF(J=12)13,26,13

READ(I,108)GCT, CF

IF(J=13)14,26,14

READ(I,108)V1, V2, Wv

IF(J=14)15,26,15

26 WRITE(4,115)

WRITE(4,114)AT, BT, CT, DT, ET, FT, GT, VBS, AIW
WRITE(4,115)
WRITE(4,116)AD, BD, AK, GG, GCT, CF, NG
WRITE(4,131)
WRITE(4,132)WW, A, B, HH, RR, AIW
WRITE(4,117)V1
WRITE(4,118)V2
WRITE(4,1003)VW
IF(VA)27,28,26
27 WRITE(4,119)
   GO TO 29
28 WRITE(4,120)
   GO TO 29
29 IF(J=25)252,250,250
252 IF(J=12)556,251,251
556 IF(J=3)250,557,251
251 J=55
   GO TO 49
250 J=0
   RM=RMN
30 DTOB=*2, CT*RM/1000.+3, DT*(RM/1000.)**2+4.*ET*(RM/1000.)**3+5.*
   IFT*(RM/1000.)**4+6.*GT*(RM/1000.)**5
   IF(J=1)31,31,32
31 IF(DT0)32,33,33
33 RM=RM+500.
   J=1
   GO TO 30
32 IF(J=10)34,34,35
34 IF(DT0)36,36,35
36 RM=RM-50.
   J=10
   GO TO 30
35 IF(DT0)37,38,38
38 RM=RM+50.
   J=100
   GO TO 30
37 DTOA=AT+BT*RM/1000.+CT*RM*RM/1000000.+DT*(RM/1000.)**2+4.*ET*(RM/1000. )**3+5.*
   IFT*(RM/1000.)**4+6.*GT*(RM/1000.)**5
   WRITE(4,121)DTO, RM
   J=0
   RMP=RM
39 DPA=AT+2.*AT*RMP/1000.+3.*CT*(RMP/1000.)**2+4.*DT*(RMP/1000.)**3+5.*
   IFT*(RMP/1000.)**4+6.*GT6.*(RMP/1000.)**5+GT7.*(RMP/1000.)**6
   IF(J=1)40,41,42
40 IF(DP)41,42,42
42 RMP=RMP+500.
   J=1
   GO TO 39
41 IF(J=10)43,43,44
43 IF(DP)45,45,44
45 RMP=RMP-50.
    J=10
    GO TO 39
44 IF(DP)<46,47,47
47 RMP=RMP+5.
    J=100
    GO TO 39
46 DP=2.1415926/33000.*(AT+BT*RMP/1000.+CT*(RMP/1000.))**2+DT*(RMP/11000.)*3+ET*(RMP/1000.)*4+FT*(RMP/1000.)*5+GT*(RMP/1000.)*6)*R
    2MP
    WRITE(4,122)DP,RMP
557 CONTINUE
    IF(INB)4A,49,49
48 CONTINUE
    IF(RMP>DUG-VBS)256,256,257
257 DUG=VBS/RMP
    WRITE(4,151)
256 CONTINUE
    IF(KA)50,51,51
51 GRAD=ATAN(A/(A+B-H/H))
    BOT=WH*SIN(Grad)*RR/DTDR
    IT=(4,151)G
52 CONTINUE
    IF(I(A))50,51,51
50 GRAD=ATAN(B/(A+B-H/H))
    BOT=WH*SIN(Grad)*RR/DTDR
52 CONTINUE
    GRAD=1./SIN(F(Grad))
    WRITE(4,124)GRAD
    GRAD=RMP*DUG/1000.
      GRAD=15.14159/660.*RMP*(AT+BT*GRAD+CT*GRAD**2+DT*GRAD**3+ET*GRAD**4+FT*GRAD**5+GT*GRAD**6)*DUG
    J=0
    VM=0.
54 TEF=1.99758-.0000879*VM+1.96-.000316*VM-.000058*VM*VM
    TEF=VM*(WH*(AD+VM*BD)+AK*VM*VM)/TEF
    IF(J)55,55,58
55 IF(Grad-TEF)58,59,57
57 VM=VM+10.
    J=1
    GO TO 54
58 IF(TEF-Grad)59,407,56
56 VM=VM-1.
    J=10
    GO TO 54
59 CONTINUE
    VM=VM+.5
407 CONTINUE
WRITE(4,125) VM,DUG
IF(NBOT.0,1100,0
READ(1,108) NBT, DAR
GO TO 1101
1100 CONTINUE
RX=RR*(1.+X*(VM*VM-900.))
DAR=RMP*DG*RR*3.14159*15./VM/660.
BOT=BOT*DAR
1101 CONTINUE
WRITE(4,126)
WRITE(4,127) DAR
WRITE(4,128) BOT
ANG=NG-1
DO 60 1=1,NG
AI=I-1
60 GR(I)=1./(1./BOT*(AI*(1.-1./BOT))/ANG)
GRAD=0.
JJ=NG-1
IF(JJ)84,405,400
400 CONTINUE
JJJ=NG-2
IF(JJJ)61,62,62
61 CONTINUE
IPRINT=0
DO 63 I=2, JJ
63 WRITE(4,129) I, GR(I)
GO TO 402
62 CONTINUE
IPRINT=1
402 CONTINUE
DO 401 I=2, JJ
401 K=1-1
G(K)=1.
H(K)=BOT
X(K)=GR(I)
GGG=0.
IPRINT=0
CALL PX6B (JJJ, JJJ, 40, IPRINT, -1., TS)
DO 76 I=2, JJ
76 K=I-1
GR(I)=X(K)
GRAD=R*=3.14159*RR*15./GR(I)/DAR/660.
TEF=GRAD*VBS/RM
76 WRITE(4,140) I, GR(I), GRAD, TEF
WRITE(4,134) TS
IF(NG)84,85,84
84 CONTINUE
GGG=GG
GRAD=VBS+GR
WRITE(4,138)GR(1),GRAD,1
GO TO 88
90 CONTINUE
WRITE(4,139)GR(1),VM
88 CONTINUE
V=RMP*GR
GRAD=VM/V
WRITE(4,142)GRAD
WRITE(4,127)GR
87 J=0
READ (1,102)J
IF(J)203,203,99
99 CONTINUE
WRITE(4,1000)
GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15),J
203 WRITE(4,150)
STOP 01
100 FORMAT(10X19H,G,LUCAS AUTO DEPT/8X37HLOUGHBOROUGH UNIVERSITY OF T
1ECHNOLOGY/10X45HVEHICLE PERFORMANCE - ROSENROCK OPTIMISATION/)
101 FORMAT(10X49HVEHICLE MAY OVERTURN IN FIRST GEAR ****/)
102 FORMAT(312)
103 FORMAT(2F8.5)
105 FORMAT(F7.1,3F6.2,12)
106 FORMAT(F3F8.6)
107 FORMAT(F7.5)
108 FORMAT(10F0.0)
111 FORMAT(2F6.4)
112 FORMAT(2F4.0)
113 FORMAT(/5X7HNEW RUN/)
114 FORMAT(15X29HENGINE TORQUE CHARACTERISTICS27X20H MAX. ENGINE ENGI
1NE/74X18HSPEED INERTIA/7F10.5,F11.1,F11.3/)
115 FORMAT(5X17HGR & COEFFICIENTS8X42HGRADIENT GEAR CHANGE COEFF
1 NO OF42X29HTIME OF FRICTION GEARS)
116 FORMAT(3F9.5,3F11.5,110/)
117 FORMAT(21H INITIAL SPEED MPH =F5.0)
118 FORMAT(18H FINAL SPEED MPH =F5.0/)
131 FORMAT(65H WEIGHT POSITION OF C OF G ROLLING RAD FT INERTIA
10F WHEELS)
132 FORMAT(F7.1,3F7.2,F15.4,F16.3/)
119 FORMAT(26H FRONT WHEEL DRIVE VEHICLE/)
120 FORMAT(25H REAR WHEEL DRIVE VEHICLE/)
121 FORMAT(30H MAXIMUM ENGINE TORQUE LB.FT =F8.3/27H MAXIMUM TORQUE SP
1 EED RPM =F6.0/)
122 FORMAT(27H MAXIMUM BRAKE HORSEPOWER =F8.3/17H MAX. BHP SPEED =F6.0
1/)
123 FORMAT(10X66HVEHICLE MAY OVERTURN ON MAX. GRADIENT IN FIRS
1 T GEAR ****/)

...
124 FORMAT(41H MAX. GRADIENT VEHICLE CAN CLIMB IS 1 IN F6.2/)
125 FORMAT(28H MAX. SPEED OF VEHICLE MPH = F6.1/25H DEGREE OF UNDERGEARING = F7.4/)
126 FORMAT(3RH TOP GEAR RATIO TAKEN AS 1 TO 1, GIVING)
127 FORMAT(19H DRIVE AXLE RATIO = F7.3)
128 FORMAT(20H BOTTOM GEAR RATIO = F7.3/)
129 FORMAT(12H GEAR RATIO 13,3H = F7.3/14H INITIAL GUESS)
130 FORMAT(12H GEAR RATIO 13,3H = F7.3/5X21H TIME TO SPEED SECS = F8.2/)
131 FORMAT(34H SUMMATION OF TIMES TO SPEED SEC = F9.2/)
132 FORMAT(/30H TIME TO SPEED ON LEVEL SECS = F8.2/)
133 FORMAT(/15X23H MAXIMUM SPEED ON LEVEL/27H GEAR RATIO SPEED MPH)
134 FORMAT(/12H GEAR RATIO 13,3H = F7.3/5X6.1,4H MPH TO LIMITATION SET BY VALVE BOUNCE OFF6.1,4H MPH)
135 FORMAT(/22H END OF CALCULATIONS)
136 FORMAT(/76H DUE TO ENGINE SPEED LIMIT DEGREE OF UNDERGEARING IS REDUCED TO FIGURE BELOW/)
1000 FORMAT(1H1)
1001 FORMAT(8OH)
1002 FORMAT(1HO/)
1003 FORMAT(32H HEAD-ON WIND VELOCITY MILE/H = F8.2/)
END
END OF SEGMENT, LENGTH 1570, NAME BO80
SUBROUTINE C4LXGH (JJ, JJ, IT, TMX)
DIMENSION ARRAY(308), G(48), H(48), X(24), W(26), GR(10), F(10)
COMMON ARRAY, G, H, X, GR, GCT, DAR, PR, AT, BT, CT, DT, ET, AD, BD, AK, WW, AIW, AIE
V2=VMAX/2.
TMX=0.
DO 70 MAX=1, 5
TS=0.
V=V1
AA1=0.
DV=2.
NDIS=0.
DISF=0.
L=0.
CC=SQRT(1.-GGG*GGG)
J3=NG-1.
DO 43 I=2, J3
K=I-1.
43 GR(I)=X(K).
IF(V=V1)42, 42, 3.
42 CONTINUE.
C=B*CC+HH*GGG.
BW=1.-C/CC/(A+B).
IF (GGG-BW*CF*CC) 3, 3.
WRITE(4,,100)
3 CONTINUE.
BW=CF*W1U/(A+B).
IF((V2-V)/DV-25.) 45, 5.
DO 4 DV=DV/2.
GO TO 3.
5 KV=(V2-V)/DV+1.
DO 6 K=1, KV
DF=WW*(GGG+AO+U*BD)+AK*(V+WV)**2.
DO 7 I=1, NG.
AI=NG-I.
GRR=3.1415926*15,.RR/660./DAR/GR(I).
R=V/6RR.
IF(VBS=R)27, 28, 28.
27 F(I)=0.
GO TO 7.
28 CONTINUE.
IF(RM=R)66, 66, 0.
T=DTR.
GO TO 57.
66 CONTINUE.
T=AT+BT*R/1000.+CT*(R/1000.)*4+DT*(R/1000.)*3+ET*(R/1000.)*4+FT
1*(R/1000.)*5+GT*(R/1000.)*6.
67 CONTINUE.
TF = T * TEF * GR(1) * DAR / RR
31 CONTINUE
PF = TF - DF
IF (PF < 14.15) GO TO 7
14 F(I) = 0.
GO TO 7
15 EM = WW / 32.2 + AIW / RR / RR + AIE * TEF * (GR(1) * DAR / RR) ** 2
F(I) = PF / EM
IF (KA) 10, 11, 11
10 C = B * CC - HH * GGG
TFF = BW * (C - HH + F(I) / 32.2)
GO TO 30
11 C = A * CC - HH * GGG
TFF = BW * (C - HH + F(I) / 32.2)
30 CONTINUE
IF (F(I) * (WW / 32.2 + AIW / RR / RR / 2.) - TFF + DF - WW / 2. * (AD + V * BD)) 12, 12, 13
13 CONTINUE
WRITE (4, 104) V, I, TFF, TF
IF (KA) 147, 48, 48
47 CONTINUE
TF = .99 * ((BW * C - DF + WW * (AD + BD + V) / 2.) * EM / (WW / 32.2 + AIW / RR / RR / 2.) + BW * HH / 3
12.21 + DF)
GO TO 31
48 CONTINUE
TF = .99 * ((BW * C - DF + WW * (AD + BD + V) / 2.) * EM / (WW / 32.2 + AIW / RR / RR / 2.) - BW * HH / 3
12.21 + DF)
GO TO 31
12 CONTINUE
7 CONTINUE
FG = 0.
J = L
DO 16 I = 1, NG
IF (F(I) = FG) 16, 16, 18
18 FG = F(I)
J = I
16 CONTINUE
IF (FG) 19, 19, 20
19 CONTINUE
OGR = 3.1415926 + 15. * RR / 660. / DAR / GR(1)
IF (V = RM + OGR) 6.6, 62
62 CONTINUE
WRITE (4, 106) J, V, V2
WRITE (4, 111)
GO TO 50
20 CONTINUE
IF (J = L) 21, 22, 21
21 CONTINUE
   IF(NO=0,6A,68
      WRITE(1,101)J,V
68 CONTINUE
   IF(L=23,24,23
23 AN=gCT
   GO TO 17
22 CONTINUE
24 AN=0.
17 L=J
   AA2=(AA1+DV*22./15./FG)/2.
   IF(V-V1)60,61,60
60 CONTINUE
   TS=TS+AA2+AN
61 AA1=DV*22./15./FG
   DISF=(V+DV/2.)*22./15.*((AA2+AN)
   IF(DISF-1320.)65,0,0
   NOIS=1
65 CONTINUE
6 V=V+DV
50 CONTINUE
   IF(NO=44,45,45
44 WRITE(A,109)TS
45 CONTINUE
   TMAX=TMAX+TS
   V2=V2+VMAX/8.
70 CONTINUE
RETURN
100 FORMAT(10X65H ***** VEHICLE WILL SLIP DOWNHILL WITH HAND BRAKE ONL
1Y SET *****/)
101 FORMAT(21H GEAR CHANGE TO RATIO13,20H GEAR CHANGE SPEED =F7.2/)
104 FORMAT(F8.2,18X18HWHEEL SPIN IN GEAR14,13H FRIC1, LBF =F10.2,F11.2'
1)
106 FORMAT(30W SET SPEED UNOBTAINABLE GEAR=13,13H SPEED MPH =F7.2,23
1H SET UPPER SPEED MPH =F7.2/)
107 FORMAT(55W SPEED ACCEL 1/ACCEL SPEED DRAG GEAR RATIO/
140H MPH FT/SEC2 SEC2/FT FT/SEC LBF)
108 FORMAT(2F7.2,F11.5,F8.2,F10.2,18)
109 FORMAT(29H INTERMEDIATE TIME TO SPEED =E14.6)
111 FORMAT(27H TIME SO FAR IS GIVEN BELOW)
112 FORMAT(5X33H QUARTER MILE MARK PASSED SPEED =F7.2,18MILE/HOUR, T
1IME =F8.1,15HSECONDS APPROX.)
END

END OF SEGMENT, LENGTH 752, NAME CALXGH
G.G. LUCAS AUTO DEPT
LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY

VEHICLE PERFORMANCE - ROSENBRORCK OPTIMISE

MORE DETAILED OPTIMISATION: VEHICLE A - 6 DEC 1967

ENGINE TORQUE CHARACTERISTICS

<table>
<thead>
<tr>
<th>ENGINE SPEED</th>
<th>ENGINE INERTIA</th>
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<tbody>
<tr>
<td>0.18596</td>
<td>0.140</td>
</tr>
<tr>
<td>47.89786</td>
<td>5500.0</td>
</tr>
<tr>
<td>-17.13362</td>
<td>0.140</td>
</tr>
<tr>
<td>2.87567</td>
<td>0.017900</td>
</tr>
<tr>
<td>-0.42059</td>
<td>0.000000</td>
</tr>
<tr>
<td>0.06966</td>
<td>0.025000</td>
</tr>
<tr>
<td>-0.00570</td>
<td>1.000000</td>
</tr>
<tr>
<td>6500.0</td>
<td>1.000000</td>
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</table>

MAX. ENGINE SPEED: 5400 RPM
MAX. ENGINE INERTIA: 0.140 lb-ft/s^2

DRAG COEFFICIENTS

<table>
<thead>
<tr>
<th>COEFF</th>
<th>NO OF</th>
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<tbody>
<tr>
<td>0.017900</td>
<td>4</td>
</tr>
<tr>
<td>0.000000</td>
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<tr>
<td>0.025000</td>
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GRADIENT GEAR CHANGE COEFF

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<th>COEFF</th>
<th>NO OF</th>
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<tbody>
<tr>
<td>0.000000</td>
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<tr>
<td>1.000000</td>
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</tbody>
</table>

WEIGHT POSITION OF C-OF G

<table>
<thead>
<tr>
<th>POSITION</th>
<th>ROLLING RAD FT</th>
<th>INERTIA OF WHEELS</th>
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</thead>
<tbody>
<tr>
<td>2128.0</td>
<td>3.70</td>
<td>5.16</td>
</tr>
<tr>
<td>3.70</td>
<td>3.85</td>
<td>1.66</td>
</tr>
<tr>
<td>3.85</td>
<td>1.66</td>
<td>0.9400</td>
</tr>
<tr>
<td>1.66</td>
<td>0.9400</td>
<td>1.98</td>
</tr>
</tbody>
</table>

IN INITIAL SPEED MPH = 0
FINAL SPEED MPH = 54
HEAD-ON WIND VELOCITY MILE/H = 0.00
REAR WHEEL DRIVE VEHICLE
MAXIMUM ENGINE TORQUE LB-FT = 46.761
MAXIMUM TORQUE SPEED RPM = 2520
MAXIMUM BRAKE HORSEPOWER = 34.817
MAX. BHP SPEED = 5010
MAX. GRADIENT VEHICLE CAN CLIMB IS 1 IN 1.88
MAX. SPEED OF VEHICLE MPH = 70.5
DEGREE OF UNDERGEARING = 0.9300
TOP GEAR RATIO TAKEN AS 1 TO 1, GIVING
DRIVE AXLE RATIO = 4.444
BOTTOM GEAR RATIO = 4.118
<table>
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<th>PX56, Maximum or Minimum of a Constrained Function</th>
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</thead>
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<td>0</td>
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</tr>
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</tr>
<tr>
<td>81</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

**Gear Ratio:**

2 = 2.356

16.2 MPH to Limitation Set by Valve Bounce of 41.7 MPH

**Gear Ratio:**

3 = 1.953

24.9 MPH to Limitation Set by Valve Bounce of 63.2 MPH

**Time to Speed on Level Secs:** 67.94
<table>
<thead>
<tr>
<th>Gear Ratio</th>
<th>Speed (MPH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1180</td>
<td>Engine Max.</td>
</tr>
<tr>
<td>2.3562</td>
<td>40.9</td>
</tr>
<tr>
<td>1.5534</td>
<td>59.2</td>
</tr>
<tr>
<td>1.0000</td>
<td>70.6</td>
</tr>
</tbody>
</table>

Degree of Undergearing = 0.9332

Drive Axle Ratio = 4.444
APPENDIX B8

Accuracy of Road Tests

The Road Tests carried out by the semi-technical press are conducted by skilled operators using a stop watch to measure time and an analogue type of fifth wheel to measure vehicle speed. The venue of the tests is invariably the Motor Industry Research Association proving ground at Lindley.

The type of instrumentation is not the best possible. The industry and M.I.R.A. prefer a digital type of fifth wheel and no stop watch. The pulses from the fifth wheel are recorded on magnetic tape and analysed later by a special data reduction technique. This gives a more accurate record of vehicle speed against time. However, stop watches in the hands of experts can give acceptable results.

The difference between a stop watch test and a test conducted in the most accurate manner possible is likely to be quite small. So small in fact that it may be swamped by the effect of ambient conditions at the time of the test or by small production differences between the test vehicles.

It was considered preferable therefore, to compare one stop watch road test against another for a number of motor cars. The tests chosen for this purpose were those of "The Autocar" and "The Motor". Both magazines test a new model soon after introduction and publish Road Test reports.
Sometimes the same vehicle is used by the two magazines. From these tests, the accuracy of the tests themselves may be assessed. More usually, however, different examples of the same model are tested, from which some assessment may be made of the production differences between cars of the same design.

Figs A.B.8.1 to A.B.8.5 are the "Autocar" and "Motor" road test results using the same vehicle in each case. Fig. A.B.8.6 to A.B.8.9 are a selection of road test results by both "Autocar" and "Motor" on different examples of the same design in each case.

In Fig. A.B.8.1 the "Motor" acceleration times are considerably poorer than the "Autocar" figures. It is doubtful if all the discrepancy can be attributed to vehicle test weight difference because Fig. A.B.8.2 shows the "Motor" times poorer again, however this time the "Motor" test weight is less.

Turning now to road tests on different examples of the same design, Fig. A.B.8.6 shows that there can be appreciable differences. In general, however, the road test results contained in Figs. A.B.8.6 to A.B.8.9 show a similar order of discrepancy to those in Figs. A.B.8.1 to A.B.8.5. This means that the technique of the road tests conducted by the semi-technical press could be improved. Conducting road tests when the wind speed is appreciable does not help. The recorded maximum vehicle speeds are given also in Fig. A.B.8.1 to A.B.8.9. These again show a significant difference in the testing by the two authorities.
The data used to construct Figs. A.B.8.1 to A.B.8.9 was taken from References (29) to (35).

In order to compare the accuracy of the calculated performance of a vehicle and the corresponding road test, the performance of the saloon version of vehicle A was chosen. The reason for the choice is that much of the design data is common to vehicle A, the drag data is known (9) and the vehicle has been road tested (28), (36). The test weight for both the Autocar and the Motor road tests was 2016 lbf (18 cwt). Weather conditions on the day of the Autocar road test are described as "dry, still" with an air temperature of 52°F (ambient pressure not quoted). The weather prevailing on the day of the Motor road test is described as "hot and dry with slight wind" (temperature 70 - 82°F, pressure 30.2 - 30.3 in Hg).

Table A.B.8.1 records the input data used in the calculated performance and the results of the calculation. The calculated maximum vehicle speed of the saloon version of vehicle A is 75.1 mile/h compared with 75.5 mile/h recorded by "The Motor" (36) and 76.8 mile/h by "The Autocar" (28).

Fig. A.B.8.10 compares the calculated time to speed with the figures given by the "Motor" and the "Autocar". The discrepancy between the two actual road tests is very similar to that shown above. The calculated time to speed agrees closely with that of the "Autocar" in the higher speed range and is somewhat worse in the lower speed range.
APPENDIX B2

Time to speed integral

The integral
\[ t = \int_{V_1}^{V_2} \frac{1}{f} \, dV \quad \text{--- A.B.9.1} \]

has to be evaluated using numerical methods since the function

\[ f = f(V) \]

is not known.

Reference to Fig. C7.2 in Part C suggests that it is a reasonable supposition to state that vehicle acceleration varies linearly with vehicle speed. Hence, at speed \( V \), where \( V_1 < V < V_2 \),

\[ f = f_1 + \frac{V - V_1}{V_2 - V_1} (f_2 - f_1) \quad \text{--- A.B.9.2} \]

Putting A.B.9.2 into A.B.9.1 and effecting the integration results in the time to speed between vehicle speeds \( V_1 \) and \( V_2 \) as

\[ t = \frac{(V_2 - V_1)}{(f_2 - f_1)} \left\{ \log \left( \frac{f_2}{f_1} \right) \right\} \quad \text{--- A.B.9.3} \]

Using expression A.B.9.3 therefore, to calculate the area of a thin strip under the \( 1/f \) vs. \( V \) curve is likely to be more accurate than simply assuming the strip to be trapezoidal in shape.

Expression A.B.9.3 shows however, an indeterminacy to exist when \( f_2 = f_1 \). This may be investigated further by writing
\[
\frac{t_2}{t_1} = 1 + \frac{t_2 - t_1}{t_1}
\]

and expanding the logarithmic term in expression A.B.9.3 to give

\[t = \frac{(v_2 - v_1)}{f_1} - \frac{(v_2 - v_1)(t_2 - t_1)}{2f_1^2}\]

etc. \hspace{1cm} \text{A.B.9.4}

As \(t_2 \to t_1 \to t\), this becomes

\[t = \frac{v_2 - v_1}{t}\]

which assumes constant acceleration during the speed interval \((v_2 - v_1)\).

If, however, \(|(t_2 - t)|\) is small, the expression A.B.9.4 may be replaced by the approximate relationship

\[t = \frac{(v_2 - v_1)}{(t_2 + t_1)/2} \hspace{1cm} \text{A.B.9.5}\]

Equation A.B.9.6 therefore assumes that each thin strip under the \(1/f\) vs. \(V\) curve is a trapezium.

This investigation may be continued further to calculate the distance \((s)\) covered during speed interval \((v_2 - v_1)\). Noting that

\[s = \int_{v_1}^{v_2} \frac{V}{t} dV \hspace{1cm} \text{A.B.9.7}\]

and incorporating equation A.B.9.2 results in
\[ s = \frac{(v_2 - v_1)^2}{f_2 - f_1} - \frac{(v_2 - v_1) (f_1 \cdot v_2 - f_2 \cdot v_1) \cdot \log \left( \frac{f_2}{f_1} \right)}{(f_2 - f_1)^2} \ldots A. B. 9. 8 \]

Again, this may be expanded to give the relationship

\[ s = \frac{v_2 - v_1}{f_1} \left\{ v_1 \left(1 - \frac{(f_2 \cdot f_1)}{2} \right) + \frac{v_2}{2} - \frac{(f_1 \cdot v_2 - f_2 \cdot v_1) (f_2 - f_1)}{3 \cdot f_1^2} \right\} \]

etc. \}

\]

As \( f_2 \rightarrow f_1 \rightarrow f \),

\[ s \rightarrow \frac{(v_2^2 - v_1^2)}{2f} \]

which assumes constant acceleration during the speed interval \( (v_2 - v_1) \).

The assumption that vehicle acceleration is a linear function of vehicle speed is closer to the actual function than that assuming the reciprocal of vehicle acceleration is linear with speed (the trapezoidal strip). Use of equations A.B.9.3 and A.B.9.8, in order to calculate the time elapsed and the distance covered during the speed interval \( V_1 \) to \( V_2 \), will be more accurate than simply taking the mean acceleration reciprocal during the speed interval. The advantage diminishes, however, as the speed interval diminishes in size.

Should it ever become necessary to question the accuracy of relationships A.B.9.3 and A.B.9.8, it will be necessary to develop new expressions for time and distance starting with an assumption similar to that implicit in equation C4.5, section C, developed in connection with wind speed corrections.
APPENDIX D1

Derivation of function $V = V(t)$ governing the results of deceleration tests

Vehicle drag is usually expressed in the form

$$F_d = W(Ad + BdV) + K.A.(V + V_w \cos \Theta_w)^2 \text{ lbf} \quad --- A.D.1.1$$

where $W =$ vehicle weight lbf

$Ad$, $Bd$ and $K$ are drag coefficients

$A =$ projected frontal area of vehicle ft$^2$

$V =$ vehicle speed mile/h

$V_w =$ wind speed mile/h

$\Theta_w =$ direction of wind relative to head-on direction

Equation A.D.1.1 reduces to

$$F_d = WAd + K.A.V_w^2 \cos^2 \Theta_w$$

$$+ (W.Bd + 2.K.A. V_w \cos \Theta_w)V$$

$$+ K.A.V^2 \text{ lbf} \quad --- A.D.1.2$$

which is a second order polynomial in $V$ of the form

$$F_d = a + bV + cV^2 \text{ lbf} \quad --- A.D.1.3$$

During a deceleration test, the force causing the deceleration is the vehicle drag force. Hence, from Newton's Second Law

$$F_d = a + bV + cV^2 = -M_g \cdot \frac{dV}{dt} \cdot \frac{22}{15} \quad --- A.D.1.4$$
\[
\frac{dV}{a + bV + cV^2} = -\frac{dt}{M_B} \frac{15}{42}
\]
giving
\[
\frac{1}{c} \int \frac{dV}{(V + \frac{b}{2c})^2 + \frac{a - \frac{b^2}{4c^2}}{c}} = -\int \frac{15}{22M_B} \cdot dt \quad \text{--- A.D.1.5}
\]

There are three possible forms to solution of equation A.D.1.5 dependent upon whether
\[
a - \frac{b^2}{4c^2} \quad \text{is negative, zero or positive.}
\]

If \( a > \frac{b^2}{4c^2} \)

using the substitution that \( u = V + \frac{b}{2c} \)

therefore, \( \frac{du}{dv} = 1 \)

equation A.D.1.5 becomes
\[
\frac{1}{c} \int \frac{du}{u^2 + \frac{4a - \frac{b^2}{4c^2}}{c}} = -\frac{15t}{22M_B} \quad \text{const.} \quad \text{--- A.D.1.6}
\]

This is a standard integral and reduces to
isolating $V$ yields

$$V = \left( \frac{a - b^2}{4c^2} \right)^{\frac{1}{2}} \tan \left\{ \frac{-15t \times c \left( \frac{a - b^2}{4c^2} \right)^{\frac{1}{2}}}{22M} \right\} + \left\{ \text{const.} \times c \left( \frac{a - b^2}{4c^2} \right)^{\frac{1}{2}} \right\} = \frac{b}{2c}$$  \hspace{5cm} (A.D.1.8)

The constant of integration may be found by specifying the initial velocity of the deceleration test. That is by saying that

$$V = V_0 \text{ when } t = 0$$

Hence, the final expression is

$$V \text{ mile/h} = f \left\{ \frac{V_0 + n}{1 + \left( \frac{V_0 + n}{1} \right) \tan (\pi f t)} \right\} = n$$  \hspace{5cm} (A.D.1.9)

where

$$f = \left| \frac{a - b^2}{4c^2} \right|^{\frac{1}{2}}$$  \hspace{5cm} (A.D.1.10)
and \( n = \frac{b}{2a} \)  \[ \text{A.D.1.12} \]

also

\[
a = W_Ad + X_AV_u^2 \cos^2 \theta_u \quad \text{A.D.1.13}
\]

\[
b = W_Bd + 2X_AV_u \cos \theta_u \quad \text{A.D.1.14}
\]

and \( c = X_A \)  \[ \text{A.D.1.15} \]

If \( a = \frac{b^2}{4c^2} \), again making the substitution that

\[
u = \sqrt[2]{\frac{b}{c}}
\]

equation A.D.1.5 becomes

\[
\frac{1}{c} \int \frac{du}{u^2} = - \int \frac{15}{22M_B} \cdot dt
\]  \[ \text{A.D.1.16} \]

integrating gives

\[
\frac{1}{c(v + \frac{b}{2c})} = \frac{15t}{22M_B} + \text{const.}
\]

from which

\[
V = \frac{1}{m \cdot t + c \cdot \text{const.}} - n
\]  \[ \text{A.D.1.17} \]

where \( m \) and \( n \) are given by equations A.D.1.11, A.D.1.12, A.D.1.14 and A.D.1.15
putting in the boundary condition that

\[ V = V_0 \text{ when } t = 0 \]

yields

\[ \frac{\text{V} \text{miles/h}}{\text{m.t} + \left( \frac{1}{V_0 + n} \right)} \]  

--- A.D.1.18

If \( \frac{b^2}{4c^2} > a \) again making the substitution

that

\[ u = V + \frac{1}{c} \cdot \frac{h}{c} \]  

--- A.D.1.19

equation A.D.1.5 now becomes

\[ \frac{1}{c} \int \frac{du}{u^2 - f^2} = - \left\{ \frac{15t + \text{const.}}{22Mc} \right\} \]  

--- A.D.1.20

the solution is that

\[ \log_{e} \frac{1+y}{1-y} = 2 \cdot m.f \cdot (t + \text{const.}) \]  

--- A.D.1.21

re-arrangement and substitution of the boundary condition that

\[ V = V_0 \text{ when } t = 0 \]

yields
\[ \text{mile/h} = f \left[ \frac{\text{Tanh} \left( m \cdot f \cdot t \right) + \left( V_0 + n \right)}{1 + \left( \frac{V_0 + n}{f} \right) \text{Tanh} \left( m \cdot f \cdot t \right)} \right] - n \quad \text{A.D.1.22} \]

again, \( f, n \) and \( n \) are defined by equations A.D.1.10 to A.D.1.15.
APPENDIX D.2

Linearisation of a TANH function

Appendix D.1 shows that the experimental results of a deceleration test upon a vehicle should obey one of the equations A.D.1.9, A.D.1.18 or A.D.1.22 dependent upon the drag coefficients of the vehicle. Since the purpose of the deceleration test is to find the drag coefficients, the equations should first be linearised to enable a curve fitting procedure, such as the method of "least squares" to be used.

Equation A.D.1.18 is relatively simple to deal with. Equation A.D.1.9 is virtually impossible since it is a tangent function.

Equation A.D.1.22 may, however, be linearised.

Equation A.D.1.21 in Appendix D.1 is given as

\[
\text{Loge} \frac{1+\frac{y}{k}}{1-\frac{y}{k}} = 2.m.f. (t+\text{const.}) \tag{A.D.1.21}
\]

this may be re-written in the form

\[
y = 2.k. (t+\text{const.}) \tag{A.D.2.1}
\]

where

\[
y = \text{Loge} \frac{1+\frac{y}{k}}{1-\frac{y}{k}} \tag{A.D.2.2}
\]
It may be shown that

\[ u = f \cdot \text{Tanh} \left( k(t + \text{const.}) \right) \] —— A.D.2.3

where \( u \) is defined by equation A.D.1.19 as

\[ u = V + \frac{1}{2} \frac{b}{c} \] —— A.D.1.19

If \( B \delta = 0 \) and there was zero wind speed during the test, then \( b = 0 \)

Hence, \( u = V \)

Now a feature of a Tanh function is that it is asymptotic to unity and that initially, the function approaches unity very rapidly.

Note that

\[ \tanh 2 = .96 \]
\[ \tanh 3 = .995 \]
\[ \tanh 4 = .999 \]

Thus a curve of the experimental results \( V = V(t) \) will reveal \( f \) in equation A.D.2.3 as the value of \( V \) at a large time \( t \).

Having found \( f \), it is possible, by using a "least squares" computer program or by plotting \( y \) against \( t \), to find a preliminary value of \( k \).

This preliminary value of \( k \) may be improved upon by subsequent iteration until the desired accuracy is reached.
APPENDIX D.3

Digital computer program to reduce the vehicle drag coefficients from the results of a deceleration test.

ICT 1905 program B032

This program, listed below, accepts time against speed points from a deceleration test on a vehicle. This data is reduced to the drag coefficients, first assuming that there is a drag term proportional to velocity and then assuming that vehicle drag consists of a rolling resistance term and an aerodynamic drag term only. Finally, the program gives a check on the accuracy of the overall test and reduction of results, also a comparison can be made of the accuracy of one set of drag coefficients against the other.

The program starts by smoothing out the input data by fitting a sixth order polynomial to the raw results. The accuracy of this fit can be assessed because the full "read-in" values with the respective curve fit values are printed out in the form of a table.

The output data is so arranged to give adequate information on the accuracy of the reduction and the actual test at every stage. Allowance is made for wind speed, but it should be realised that, for accuracy, the wind speed should be low and at a small angle only to the head-on direction of the vehicle.
Input Read-in data

Card 1 Heading data, vehicle model, date etc. Leave column 1 blank.

Card 2 \( n \), \( K \) Free fixed point format

\[ n = \text{No. of data points} \]
\[ K = \text{Order of polynomial curve fit to input data} \]

If zero, the program will use sixth order.

Card 3 Weight of vehicle \( \text{1bf} \)

Sum total of wheel inertias \( \text{slug ft}^2 \)
rolling radius of wheels \( \text{ft} \)
ambient pressure \( \text{in Hg} \)
ambient temperature \( \text{OF} \)
projected frontal area of vehicle \( \text{ft}^2 \)

(if the projected frontal area is not known, read is zero)

wind speed \( \text{mile/h} \)
wind direction relative to head-on \( \text{deg} \)
All in free floating point format

Card 4 and subsequent cards

input data in free floating point format consisting of \( n \) pairs of time and speed points. Put time (s) first followed by speed (mile/h).
For a repeat run with new data, put any positive integer in columns 1 and 2 of the next card.
Repeat from Card 1.
Otherwise, leave the last card BLANK.

An example of the output from this program is given in Part D, section 4.

Statement numbers 32 to 102 + 1 of the listed program concern the "read-in" data given above. Statement numbers 102 + 2 to 1 set the projected frontal area of the vehicle at unity if the actual value is not known, that is if it has been read in as zero. Statement number 1 + 1 evaluated the equivalent mass of the vehicle and the following DO LOOP down to statement 2 sets up the input data for the polynomial curve fit subroutine POLY, listed in Appendix B.1 From there down to statement number 106 the input data is printed out for reference.

The coefficients of the sixth order polynomial curve fit at statement number 106 + 1 are given as COF(1), COF(2) ——COF(7).

The DO LOOP down to statement number 3 sets X as the vehicle speed, differentiates the polynomial expression to obtain the vehicle deceleration which is then multiplied by the equivalent mass of the vehicle to give the drag force. This drag force is then given the symbol Y. Statement 107 + 1 fits a second order
polynomial to these X and Y values. The resulting coefficients COF(1), COF(2) and COF(3) are then reduced, making due allowance for the head-on wind speed during the test, to give the drag coefficients Ad, Bd and K. These values are printed-out. If the projected frontal area of the vehicle is not known, the product (K·A) is printed-out instead of K. Also printed-out here is the aerodynamic drag coefficient Cd and the air density on the day of the test.

Statement numbers 15+1 down to 19 test the coefficients to see which of the expressions A.D.1.9, A.D.1.18 and A.D.1.22 in Appendix D.1 is applicable to the original data. The appropriate expression is partially reduced and stored as AL, GC and AM.

The DO LOOP to statement number 7 gives the symbol X to the square of the vehicle's relative air speed. Symbol Y is, as before, the drag force. Statement number 7+1 fits a straight line through the X and Y points from which the drag coefficients, assuming Bd = 0, are found. These drag coefficients are printed-out.

The program between statement numbers 13 and 117 concerns the check on the accuracy of the results and of the test itself. Both sets of calculated drag coefficients are fed back into the appropriate V = V(t) expression governing the deceleration test (see Appendix D.1). The original vehicle speed against time
results are compared with the two sets of calculated results using the two sets of reduced drag coefficients.

Statement number 117+1 reads in a card which, if blank, signals the end of the calculations. If, however, a positive integer is found in either Column 1 or Column 2 or both, the program switches to the beginning and another set of data may be read-in.
FORTRAN-B032, G.A. LUCAS - DRAG COEFFICIENTS FROM DECELERATION TESTS
Masters B032

DIMENSION V(100),T(100),X(100),Y(100),COF(6)
COMMON Y,COF

32 CONTINUE
READ(1,100)
100 FORMAT (8H)

READ(17,101)N,K
IF(K10,0.29
K=6

29 CONTINUE
101 FORMAT (510)
READ(1,102) W, AIW, RR, PRESS, TEMP, PA, WV, WD
102 FORMAT (100/RD, 0)
READ(1,102)(T(I),VI),I=1,N
IF(PA< .00001),10,0

PA=1
1 CONTINUE
EM=V/32.2+AIW/RR/RR
10 CONTINUE
DO 2 I=1, N
T(I)=T(I-1)

V(I)=V(I-1)
2 CONTINUE
WRITE(2,103)
103 FORMAT (8X48HREDUCTION OF DRAG DATA FROM DECELERATION TESTS//)
WRITE(2,104)
104 FORMAT (19H VEHICLE TESTED WAS)
WRITE(2,122)
122 FORMAT (1H0//)
IF(PA< 1.00001),133, 33.0
WRITE(2,121)PA
121 FORMAT (38X31HPROJECTED FRONTAL AREA SQ.FT = F9.3)
33 CONTINUE
WRITE(2,105)W, AIW, RR, PRESS, TEMP, WV, WD
105 FORMAT (21H VEHICLE WEIGHT LBF = F9.1/33W TOTAL WHEEL INERTIA SLUG = S
1.0FT = RF.3.6X19 ROLLING RADIUS FT = RF.4.123W AMBIENT PRESS. IN HG =
2RF.3.6X21H AMBIENT TEMP. DEG F = RF.3/23W WIND VELOCITY MILE H = F6.17
3.6X40W DIRECTION RELATIVE TO HEAD-ON DEG = F6.17/)
WRITE(2,106)
106 FORMAT (78H POLYNOMIAL CURVE FIT OF INPUT DATA, X=TIME SECONDS AND
Y=VEHICLE SPEED MILE/H/)
CALL POLY (n,0,N,K,0)
AV=COF(I)
K=K+1
DO 3 I=1,N
Y(I)=Y(I-1)
V(I)=COF(I)
3 CONTINUE
WRITE(2,118)
108 FORMAT(68H) VEHICLE-DRAG EQUATION IN THE FORM \( \text{DRAG} = \pi \cdot \frac{(A + B \cdot V)}{\text{AREA} \cdot K \cdot \text{VEHICLE SPEED} + \text{WIND SPEED})} \)

109 FORMAT(10H) AREA X K=F10.6

110 FORMAT(10H) K=F10.7, 8X27 THE PROJECTED FRONTAL AREA PT=F7.2

111 FORMAT(68N) VEHICLE-DRAG EQUATION IN THE FORM \( \text{DRAG} = \pi \cdot \frac{(A + B \cdot V)}{\text{AREA} \cdot K \cdot \text{VEHICLE SPEED} + \text{WIND SPEED})} \)
WRITE(2, 110) AK, PA
9 CONTINUE
WRITE(2, 112)
112 FORMAT(54H WHERE W=VEHICLE WEIGHT LBF AND V=VEHICLE SPEED MILE/H/)
CD=AK*2./RHO/2.1911121
!PA=-1.011, 0, 12
WRITE(2, 116) CD, RHO
GO TO 13
12 WRITE(2, 119) CD, RHO
13 CONTINUE
WRITE(2, 115)
115 CONTINUE
WRITE(2, 32H1)
' CHECK ON ACCURACY OF RESULTS/16X10 ORIGINAL 4X19MH'
17A=V/AREA.K.VXXV9X14MH; A=AREA.K.VXXV3X8H TRIME-SEC3X12HSPEED MILE/
2H7X12HSPEED MILE/H14X12HSPEED MILE/H/)
115 CONTINUE
DRAG COEFFICIENT CD=0.4, 10X23HAIR DENSITY SLUG/CU.FT'
1'=111.7/)
S1=0.
S2=0.
KK=U
!PD(GD-AN+AN)25, 20, 21
25 CONTINUE
DO 10 I=1, N
TAM=TANH(AN*T(I))
Y(I)=AL*(TAM+GC)/(1.+GC*TAM)=AN
S2=S2+(V(I)*Y(I))**2
10 CONTINUE
GO TO 22
20 CONTINUE
DO 25 I=1, N
Y(I)=1./TAM*(T(I)+GC)=AN
S2=S2+(V(I)**2)*Y(I))**2
25 CONTINUE
GO TO 22
21 CONTINUE
DO 24 I=1, N
TAM=TANH(AN*T(I))
Y(I)=AL*(GC-TAM)/(1.+GC*TAM)=AN
S2=S2+(V(I)**2)*Y(I))**2
24 CONTINUE
22 CONTINUE
S2=S2/N
!FP(KK-1)0.26'
DO 31 I=1, N
31 Y(I)=Y(I)
S1=S2
S2=J.
KK=10
AD=AD+PA*AK/W*(WV*COS(WD))**2
BD=PA*AK*2.*WV*COS(WD)/W
AN=AN+BD/2./PA/AK
!FOR(W+AD/PA/AN=AN*AN0.27, 26
AL=SQRT(AN*AN-W+AD/PA/AK)
GC=(AV+AN)/AL
AM=PA*AK+AL*15./22./EM
GO TO 25
27 CONTINUE
GC=1./(AV+AN)
2M=PA*AK*15./22./EM
GO TO 20
28 CONTINUE
AL = SORT(W+AN/PA/AK-AN*AN)
GC = (AV*AN)/AL
AM = PA*AK*AL+15.622/EM
GO TO 21
26 CONTINUE
DO 11 I=1,N
WRITE(2,114)T(I),V(I),X(I),Y(I)
11 CONTINUE
S1 = SORT(S1)
S2 = SORT(S2)
WRITE(2,117)S1, S2
117 FORMAT(26H0ROOT MEAN SQUARE ERROR=E20.6,E20.6)
READ(11,120)LLL,
120 FORMAT(12)
IF(LLL)30,30,0
WRITE(2,118)
GO TO 32
30 CONTINUE
114 FORMAT(F11.4,F13.3,F19.3,F26.3)
116 FORMAT(55H DRAG COEFFICIENT X AREA/CDXAREA=F10.4,8X23HAIR DENSIT
1Y SLUG/CU:FT=F11.7)
END

END-OF-SEGMENT, LENGTH 1225, NAME B052
APPENDIX E1

Constant power curves

This appendix concerns the design of a drawing instrument suitable for sketching the constant power curves on an engine torque-speed graph (Fig. E3.1).

Since, for a constant power curve

\[ P = \text{constant} = T \times N \quad \text{--- A.E.1.1} \]

the relationship between ordinate (T) and abscissa (N) is

\[ T = \frac{\text{const} (P)}{N} \quad \text{--- A.E.1.2} \]

which describes a hyperbola.

Mable and Ovirk (102), in a chapter of their book entitled "computing mechanisms", describes the device shown in Fig. A.E.1.1 as an "inversion" mechanism. This has been re-arranged to form Fig. E3.3. A study of the similar triangles (pdb) and (acp) in Fig. E3.3 shows that the instrument will produce the relationship between torque (T) and engine speed (N) given in equation A.E.1.2.

The product \((A.B)\) being proportional to the constant power \((P)\). It is not necessary to know the value of a constant power line in Fig. E3.2 in order to sketch in the optimum control line.

The geometry of the mechanism shown in Fig. E3.3 may be modified at nut \((0)\) in order to provide the different constant power curves. These curves are produced by placing the instrument onto a torque-speed plot with the nut \((0)\) locked. A pencil point is placed in \((q)\).
Continuing in this fashion marks out a constant power curve.
Unlocking nut (o) and re-positioning fulcrum point (p) before re-locking and repeating the operation produces a constant power curve of a different value.

It is envisaged that the instrument could be produced in plastic and that it would be of considerable benefit if a large number of torque-speed plots required lines of constant power.
TABLE A.E.2.I lists computer program BL84. The first "call" statement, after the DIMENSION AND DATA statements, calls up the clock built into the ICL 1905. Storage space II therefore contains the time. The second "call" statement opens up the graph plotter attached to the ICL 1905, ready for use. The program then arranges to execute NCASE different sets of engine characteristics. For each set, the engine capacity, expected maximum brake horsepower, whether the engine curves are expressed as torque or brake mean effective pressure curves and finally, whether the engine is a two or a four-stroke, must be supplied. The value of the expected maximum brake horsepower is used to set up the scale of the ordinate axis. It is not used in the calculations and so need not be accurate. The figure supplied by the engine manufacturer is adequate.

After printing the information read in for reference purposes, the program calculates "C", \( C \) being the torque or b.m.e.p. conversion factor in order to obtain horsepower. Next, the minimum and maximum engine speed values on the existing engine torque (or b.m.e.p.) plot are read in.

The program arranges the characteristics to be plotted suitable for a report of A4 size. That is that YINS = 10" and XINS = 6".
The Y-scale is arranged to read from zero to a round figure appropriate to PMAX. Similarly, the X-scale is arranged to read from XMIN to a round figure appropriate to XMAX. The value of XMIN is then modified to give a round figure at every inch of the X-scale, i.e. the step length. If XMIN becomes negative in the process, the whole X-scale is then moved along until XMIN is zero. The program then calls UTP44 which draws in the X and Y scales using the graph plotter.

First, the full throttle (or full rack) power curve is dealt with. The number of points (NP) in order to describe the curve is read in. This is followed by the full throttle speed and torque (or b.m.e.p.) points. These are then converted to power-speed points using the conversion factor "C". These points are then supplied to the graph plotter (UTP4B) and the full throttle power line is plotted.

Next, the number of s.f.c. lines to be plotted is read in (NLINE). Then, for each s.f.c. line, the procedure is the same as for the full throttle line.

After the last s.f.c. line has been plotted, the clock is again consulted and the time taken, in seconds, is printed out. When all the engine characteristics have been processed, the program closes down the graph-plotter by calling UTPCL and stops.

A typical time for the execution of one set of engine characteristics is in the region of 180 to 220 seconds.
APPENDIX E3

Engine characteristics

This Appendix contains a number of engine characteristics converted from those in the Motor Industry's Research Association publications by using the computer program B184 listed in Appendix E2. Below are some notes on each of the plots.

Fig. A.E.3.1

Peugeot 404, petrol engine, 1961 model

capacity $93.77 \text{ in}^3$ 1618 cc
bore 3.307 in 84 mm
stroke 2.874 in 73 mm

4 - cylinder, overhead valve, engine of compression ratio 7.3 to 1 having a single solex carburettor. The engine is water cooled.

Fig. A.E.3.1 should be compared with the original torque - speed plot re-produced as Figs. E3.1 and E3.2.

Fig. A.E.3.2

1961, Volkswagen VW 1500, air cooled, petrol engine

capacity $91.1 \text{ in}^3$ 1493 cc
bore 3.27 in 83 mm
stroke 2.72 in 69 mm

four cylinders, 4-stroke, in horizontally opposed pairs.
Fig. A.E.3.3

1964, Simca 1500, 4 stroke, petrol engine

capacity 90 in$^3$ 1475 cc
bore 2.96 in 75.2 mm
stroke 3.27 in 83.0 mm

Four cylinder, in line, vertical engine.

Fig. A.E.3.4

N.S.U. Wankel rotary engine (petrol) KRM 502

This engine is fitted into the N.S.U. "spider" sports car. The compression ratio of the engine is 8.6 to 1. The load line on Fig. A.E.3.4 is that of the spider, taken from Prode (100).

Fig. A.E.3.5

M.A.N. Multi-fuel engine, type M246 MV3A. The engine is designed to run on both diesel fuel and petrol and employs the "M" combustion system designed by Dr. J.S. Meurer. The curves in Fig. A.E.3.5 are a result of running the engine on diesel fuel.

The engine characteristics are unusual in that they resemble those of a compression ignition engine at low speed and those of a petrol engine at high engine speed. Nevertheless, the very low s.f.c. figures (0.400 maximum) and the very shallow gradient of the s.f.c. contour denotes the curves as basically those of a compression ignition engine.
Fig. A.E.3.6

Deutz F6L 514, compression ignition engine

capacity 487 in³ 7983 cc
bore 4.33 in 110 mm
stroke 5.51 in 140 mm

Six cylinder, in-line, air-cooled, four-stroke, overhead valve engine.

Fig. A.E.3.7

G.M.C. 6-71E, 2-stroke, compression ignition engine

capacity 425.6 in³ 6974 cc
bore 4.25 in 108 mm
stroke 5.00 in 127 mm

compression ratio 17 : 1

Six cylinders in-line, water cooled.
**APPENDIX E4**

*Gas Turbine component match (free power turbine)*

Fig. A.E.4.1 shows schematically the arrangement of the free power turbine machine and the number convention of the stations used in this Appendix. Fig. A.E.4.2 depicts the general shape of the pressure and temperature characteristics of the compressor, of the compressor turbine and of the free power turbine. An explanation for the general shape of these curves and the use of the non-dimensional parameters may be found in textbooks on gas turbine engines.

The component match study is conducted by first superimposing the compressor-turbine characteristics onto the compressor characteristics in order to study the gas generator match, and then by superimposing the load and the free power turbine characteristics onto the compressor characteristics to obtain the full component match study.

Dealing first with the gas generator component match.

**Gas generator match**

The compatibility equations to be satisfied are:

rotational speed

\[
\frac{H}{\sqrt{T_{t1}}} = \frac{H}{\sqrt{T_{t3}}} \times \sqrt{\frac{T_{t2}}{T_{t1}}} \quad - A.E.4.1
\]
mass flow rate

\[ \frac{m_1 \sqrt{T_{t1}}}{P_{t1}} = \frac{m_3 \sqrt{T_{t3}}}{P_{t3}} \times \sqrt{\frac{T_{t2}}{T_{t3}}} \cdot \frac{P_{t3}}{P_{t1}} \quad \text{--- A.E.4.2} \]

defined as

\[ \frac{T_{t2} - T_{t1}}{T_{t1}} = \frac{\gamma_m}{\gamma_m} \frac{cp34}{cp12} \frac{T_{t3} - T_{t4}}{T_{t3}} \frac{T_{t3}}{T_{t1}} \quad \text{--- A.E.4.3} \]

where

- \( T_t = \) gas total temperature
- \( \gamma_m = \) mechanical efficiency
- \( cp = \) specific heat at constant pressure
- \( P_t = \) total pressure
- \( N = \) rotational speed
- \( \dot{m} = \) air mass flow rate

It is assumed that the mass flow rate of fuel equals the air mass flow rate necessary to cool the turbine bearings and rotors.

The procedure now is to search along a particular \( N/\sqrt{T_{t1}} \) line on the compressor characteristics by trial and error methods in order to find the point which satisfies the three compatibility equations for a specified value of \( T_{t2}/T_{t1} \). Repeat for other values of \( T_{t2}/T_{t1} \) on the compressor characteristic.

The resulting graph, Fig. A.E.4.3, represents the gas generator.
component match. This gives the all important inlet to turbine temperature for high compressor efficiency without compressor surge.

The second stage is to deal with the free power turbine and the load.

**Power Turbine Match**

The procedure is, from a plot of the required load horsepower against power turbine output shaft speed, to obtain the equilibrium running line on the gas generator characteristics.

It is usual to neglect any "ram" pressure in the engine air inlet and to assume that the air is expanded down to ambient pressure over the free power turbine (i.e. $P_{t3} = P_{t1}$)

Again, the three compatibility equations are those of speed, mass flow rate and power.

The procedure from here is to search along a $T_{t3}/T_{t1}$ line to find the equilibrium running point. In detail, this may be achieved by

a) guess a point on a particular $T_{t3}/T_{t1}$ line on the gas generator characteristics.
b) read off $P_{t3}/P_{t1}$, $(\frac{m}{\sqrt{T_{t1}}})/P_{t1}$ and $N/\sqrt{T_{t1}}$
c) from equations A.5.4.1 and A.5.4.2 evaluate $N/\sqrt{T_{t3}}$ and $(\frac{m}{\sqrt{T_{t3}}})/P_{t3}$
d) hence fix the point on the compressor-turbine characteristics and obtain $\frac{P_{t3}}{P_{t4}}$ and $\frac{T_{t4}}{T_{t3}}$ from $\frac{T_{t3} - T_{t4}}{T_{t3}}$

e) now obtain

$$\frac{P_{t4}}{P_{t5}} = \frac{P_{t4}}{P_{t3}} \frac{P_{t3}}{P_{t1}}$$

and

$$\frac{\sqrt{T_{t4}}}{P_{t4}} = \frac{\sqrt{T_{t3}}}{P_{t3}} \times \frac{\sqrt{T_{t4}}}{P_{t4}}$$

f) the guessed point may now be located on the power turbine characteristics, from which the speed of the output shaft ($N_p$) may be found from

$$N_p = \frac{N_p}{\sqrt{T_{t4}}} \sqrt{\frac{T_{t3}}{T_{t1}}} \sqrt{\frac{T_{t3}}{T_{t1}}} \cdot \sqrt{\frac{T_{t3}}{T_{t1}}}$$

g) and the power output from

$$\text{power output} = \sqrt{\frac{T_{t4}}{P_{t1}}} \cdot 0.45 \cdot \frac{P_{t1}}{\sqrt{T_{t1}}} \cdot \frac{(T_{t3} - T_{t4})}{T_{t4}}$$

$$\frac{T_{t4}}{T_{t3}} \frac{T_{t3}}{T_{t1}}$$
b) If this does not equal the power required by the load at output shaft speed ($P_o$), try another point on the same $T_{t_2}/T_{t_1}$ line until equilibrium point is found.

i) Repeat with other $T_{t_2}/T_{t_1}$ lines until, finally, the "equilibrium running line" is determined on the gas generator characteristics as shown in Fig. A.E.4.4.

Fig. A.E.4.4 shows at a glance the overall component match in relation to the load and the risk of compressor surge during operation.
APPENDIX F2

Engine - torque converter match procedure

Introductory Remarks

The engine speed is not related directly to vehicle speed with an automatic transmission. It may however, be calculated using the torque converter characteristics and the engine characteristics sketched in Fig. A.F.1.1. The K-factor and torque ratio (TR) characteristics are unique for a particular torque converter and describe fully its performance. The engine torque curve is shown and, for convenience, a development of the torque curve known as the engine K-factor against speed curve is shown. The engine K-factor is defined as

\[
\text{engine K-factor} = \frac{\text{engine speed rev/min}}{\sqrt{\text{net engine output torque lbf ft}}}
\]

and forms the link between the engine and the torque converter characteristics. The torque converter K-factor and torque ratio terms are defined in Part F, section 2.
The "Direct" Method

The engine and torque converter are considered as a combined unit, called the power unit. The torque against speed characteristic of the output shaft from the power unit is calculated as follows.

Assuming steady state running,

1) taking speed ratio (SR) as the independent variable, split the speed ratio range (i.e. zero to unity) into small steps.

2) for each speed ratio, consult the torque converter characteristics and read off the K-factor.

3) transfer this to the engine K-factor curve and read off engine speed ($N_E$)

4) hence calculate engine torque

5) consult the torque converter characteristics and read off torque ratio (TR)

6) multiply engine torque and torque ratio to give output shaft torque

7) multiply engine speed by speed ratio to give output shaft speed ($N_O$)
8) hence obtain the steady state output shaft torque against speed characteristic of the power unit.

The vehicle performance may then be calculated in the normal way provided that some allowance is made for the mis-match between engine and torque converter caused by the accelerating engine.

Other Methods

Methods, other than the Runge-Kutta-Gill and the "Direct" method described above, may be employed in the vehicle performance calculation of automatic transmissioned vehicles. Two such methods are described below as "Iterative Approach No. 1" and "Iterative Approach No. 2". As their description implies, they are trial and error type solutions. Their inclusion here is largely for completeness, they have little to recommend them since the engine and torque converter matching procedure cannot be separated from the vehicle performance calculations.
Iterative Approach No. 1

The engine and the torque converter are treated as separate and distinct units. If the vehicle is accelerating (under full throttle), at a particular speed $V$ mile/h. and hence with a known, specified gear ratio,

1) calculate torque converter output shaft speed ($N_o$) from $V$, gear ratio, drive axle ratio and rolling radius of tyres.

2) guess engine speed ($N_E$) and engine acceleration ($dN_E/dt$)

3) calculate speed ratio across torque converter ($N_o/N_E$) and net engine output torque (after allowing for engine inertia, front pump etc.)

4) evaluate engine $K$-factor

5) putting engine $K$-factor = torque converter $K$-factor, read off speed ratio (SR) from torque converter characteristic.

6) from (SR) and $N_o$, evaluate engine speed $N_E$. If $N_E$ is different from the value assumed above at (2), assume a new value and go back to (2)
7) having obtained \( N_E \) and speed ratio (SR) to the required accuracy, read off the torque ratio (TR) from the other torque converter characteristic.

8) hence calculate torque converter output torque and so calculate the vehicle acceleration and time during speed interval \( dV \).

9) knowing the time interval and the engine speed at the previous vehicle speed \((V-dV)\), calculate engine acceleration.

10) if engine acceleration is different to that assumed at (2) above, assume a new value and go back to (2).

11) if the engine acceleration is within a specified tolerance, routine ends and the vehicle acceleration calculated above is declared the true value.
Iterative Approach No. 2
This is a variation on Iterative Approaches No. 1. The speed ratio of the torque converter is taken as the independent variable and the speed ratio range split up into steps.

1) for each speed ratio, consult the torque converter curves and read off the K-factor and the torque ratio (TR)

2) guess engine acceleration and so calculate the engine inertia torque.

3) construct the engine K-factor against speed curve from the engine torque curve and the engine inertia torque.

4) from this curve and the torque converter K-factor, calculate engine speed.

5) hence calculate engine torque output from the engine.

6) multiply this by torque ratio (TR) to give the output torque from the torque converter.

7) calculate vehicle acceleration

8) hence re-calculate engine acceleration and go back to (2) until the required accuracy is obtained.
APPENDIX F2

Relationship between percentage change in output shaft speed and torque due to change in engine torque

Consider a particular speed ratio of the torque converter. Hence (SR) and torque ratio (TR) are known and are fixed. Let the engine torque drop by a small proportion (a), thus

\[ T_{E,I} = T_{E,I} = 0 (1 - a) \]

where \( a = \frac{\Delta T_E}{T_{E,I} = 0} \)

From the two torque converter characteristics are obtained the two sets of relationships

\[ T_{O,I} = 0 = TR \times T_{E,I} = 0 \]

\[ T_{O,I} = TR \times T_{E,I} \]

and

\[ N_{O,I} = 0 = SR \times N_{E,I} = 0 \]

\[ N_{O,I} = SR \times N_{E,I} \]

Now, if equilibrium running is assumed such that the torque converter K-factor characteristic is unique, then the torque converter K-factor = the engine K-factor.
Hence,
\[
\frac{N_{EI} = 0}{\sqrt{T_{EI}} = 0} = \frac{N_{E}}{T_{EI}} \quad \text{--- A.F.2.6}
\]

therefore, using equation A.F.2.1
\[
N_{EI} = \left( N_{E} \right) \frac{x (1 - a)^{2}}{\sqrt{T_{EI}} = 0} \quad \text{--- A.F.2.7}
\]

since \(a \leq 1\), the first term of the binomial expression may be used, hence.
\[
N_{EI} = N_{E} \left( 1 - \frac{a}{2} \right) \quad \text{--- A.F.2.7}
\]

putting equation A.F.2.7 into A.F.2.5 yields
\[
N_{O} = SR \times R_{E} \left( 1 - \frac{a}{2} \right) \quad \text{--- A.F.2.4}
\]

Using equation A.F.2.4 gives
\[
N_{O} = \left( N_{O} \right) \left( 1 - \frac{a}{2} \right) \quad \text{--- A.F.2.8}
\]

Also, putting equation A.F.2.1 into A.F.2.3 yields
\[
T_{O} = TR \times T_{E} \left( 1 - a \right) \quad \text{--- A.F.2.2}
\]

Using equation A.F.2.2 gives
\[ T_{oi} = T_{oi} = 0 \times (1 - a) \]  
--- A.F.2.9

Equation A.F.2.9 shows that the percentage change in torque converter output torque equals that of the engine torque, that is that

\[ \Delta T_{oi} = \Delta T_{ei} = 0 \]  
--- A.F.2.10

Equation A.F.2.8 shows that the percentage change in torque converter output speed is half that of the engine torque, that is that

\[ \frac{\Delta N_o}{N_{oi} = 0} = \frac{\Delta T_E}{2T_{EI} = 0} \]  
--- A.F.2.11

It follows therefore that

\[ \frac{\Delta N_o}{N_{oi} = 0} = \frac{\Delta T_{oi}}{2T_{oi} = 0} \]  
--- A.F.2.12
APPENDIX F3

Calculation of the change in output torque due to an accelerating engine

Referring to Fig. F4.3 in the main text, a drop in engine torque causes the curve to move down and to the left. The relationships between these two points A and B is known (see Appendix F2). However, it is required to find an expression for the torque at point C. Thus

\[ T_{OC} = T_{OA} - \Delta T_{O1} - \Delta T_{OM} \]  \hspace{1cm} \text{(F.3.1)}

The terms in this expression are defined in Fig. F4.3 and section F4.

Now, Fig. F4.3 shows that, if the change is small,

\[ \Delta T_{OM} = - \left( \frac{\partial T_o}{\partial N_o} \right)_B \times \Delta N_o \]

Note that \( \left( \frac{\partial T_o}{\partial N_o} \right)_B \) is negative, usually.

i.e.

\[ \Delta T_{OM} = - \left( \frac{\partial T_o}{\partial N_o} \right)_B \times N_{0I} = 0 \times \Delta N_o \]

\[ \frac{N_{0I}}{N_{0I}} = 0 \]
therefore, using equation A.F.2.12 in Appendix F2

\[
\Delta T_{OM} = - \left( \frac{\Delta T_o}{\Delta N_0} \right) \times N_0 I = 0 \times \frac{\Delta T_o}{2T_o I} = 0
\]

Now, the slope of the curve at point B is not known directly. It is not exactly equal to the slope of the curve through A, which is known, because the percentage change in speed is half that of torque. Using reasoning identical to that in Appendix F4, it can be shown that the slope of the curve through A equals that of a matching study curve through point D in Fig. F4.3. A small error only is incurred in using the known slope through A because the correction for mis-match \( \Delta T_{OM} \) is small anyway.

Therefore, it may be said that

\[
\Delta T_{OM} = - \left( \frac{\Delta T_o}{\Delta N_0} \right) \times N_0 I = 0 \times \frac{\Delta T_o}{2T_o I} = 0
\]

It is now necessary to establish a relationship between the vehicle acceleration \( f \) and the decrease in engine torque output due to its own acceleration.

The angular acceleration of the torque converter output shaft is related to vehicle acceleration by
\[
\frac{\Delta N_E}{\Delta t} = \frac{f \times \text{DAR} \times \text{GR}}{r_T}
\]  
--- A.F.3.4

Now the engine acceleration is given by

\[
\frac{\Delta N_E}{\Delta t} = \frac{\Delta N_E}{\Delta t} \times \frac{\Delta N_0}{\Delta t}
\]  
--- A.F.3.5

The engine inertia torque is given by

\[
\Delta T_E = I_e \times \frac{\Delta N_E}{\Delta t}
\]  
--- A.F.3.6

Combining equations A.F.3.4, A.F.3.5 and A.F.3.6 yields the relationship

\[
\Delta T_E = I_e \times \frac{\Delta N_E}{\Delta t} \times \frac{f \times \text{DAR} \times \text{GR}}{r_T}
\]  
--- A.F.3.7

Finally, using equations A.F.2.2 and A.F.2.3 in Appendix F2,

\[
\Delta T_{\text{v}} = T_R \times \left\{ \left( I_e \times \left( \frac{\Delta N_E}{\Delta t} \right) \times \frac{f \times \text{DAR} \times \text{GR}}{r_T} \right) \right\}
\]  
--- A.F.3.8

Hence \( \Delta T_{\text{v}} \) is expressed in terms of vehicle acceleration \( f \).

Since the change in torque due to mis-match \( \Delta T_{\text{om}} \) is related to \( \Delta T_{\text{v}} \), see equation A.F.3.3 above, that too may be expressed in terms of vehicle acceleration.
The term \( \frac{\partial N_E}{\partial N_0} \) in expression A.F.3.8 should be the slope of the curve at point C in Fig. P4.4.

It is now possible to proceed further and obtain the output torque from the torque converter making due allowance for the accelerating engine.

Substituting equation A.F.3.8 first into A.F.3.3 and then into A.F.3.1 yields

\[
T_{o_c} = T_{o_A} + \left( I_e \times \left( \frac{\partial N_E}{\partial N_0} \right) \times \frac{f \times DAR \times GR}{r_x} \right) \left( \frac{T_{o_A}}{2N_0} - \frac{T_{o_A} - T_{o_0}}{2T_{E_{I-0}}} \right)
\]

\[
\text{A.F.3.9}
\]

This may be re-arranged to isolate the vehicle acceleration

\[
f = \left( \frac{T_{o_A} - T_{o_0}}{T_{o_A}} \right) T_{E_{I-0}} \times \frac{r_x}{I_e \times \left( \frac{\partial N_E}{\partial N_0} \right) \times DAR \times GR}
\]

\[
\text{A.F.3.10}
\]

Now the term \( \frac{\partial N_E}{\partial N_0} \) in equation A.F.3.10 is known only for point A in Fig. P4.4 Appendix P4 shows that the slope at point A equals the slope at point B. However, the term \( \frac{\partial N_E}{\partial N_0} \) refers to the slope at
point C. Using the known slope introduces a small error. The error is small because Fig. F4.4 shows that when the slope is large at high engine speeds, there is very little difference in the slopes of the two curves. When the engine speed is low, the slopes are small anyway.

However, it is possible to make some allowance for this error when calculating the vehicle performance. Having calculated the output torque from the torque converter \( T_o \) as described in section F4, using the approximate slope \( \frac{\delta N_o}{\delta N_i} \), the term

\[
\alpha = \frac{\Delta T_o}{T_o i=0}
\]

is known.

Using equation A.F.2.8 in Appendix F2 and the known polynomial expression governing the slope of the curve, the slope at C may be found from

\[
\left( \frac{\delta N_o}{\delta N_i} \right)_C = C_2 + 2C_3 \left( N_o i=0 \frac{1+g}{1+2} \right) + 3C_4 \left( N_o i=0 \left( \frac{1+g}{1+2} \right)^2 \right) + \text{etc.}
\]

It is not necessary to iterate, since the effect of the difference is very small. It is sufficient to note the difference and to add it onto the calculated slope at the next vehicle speed.
Thus the error in one step length is applied to the next. Inclusion of this procedure altered the calculated time to 90 mile/h. of a vehicle from 53.0597 seconds to 53.0733 seconds, a negligible difference.
APPENDIX F4

Rate of change of engine speed with output shaft speed:

with reference to Fig. F4.4 in Section 4, the purpose of this appendix is to show that the slope of the curve through B is the same as the slope of the curve through A.

The known polynomial expression linking engine speed and torque converter output shaft speed is

\[ N_{E_I} = b_1 + b_2 \cdot N_{o_{I=0}} + b_3 \cdot N_{o_{I=0}}^2 + b_4 \cdot N_{o_{I=0}}^3 + \text{etc.} \quad \text{--- A.F.4.1} \]

thus

\[ \begin{pmatrix} N_{E_I} \\ N_{o_I} \end{pmatrix}_A = b_2 + 2 \cdot b_3 \cdot N_{o_{I=0}} + 3 \cdot b_4 \cdot N_{o_{I=0}}^2 + \text{etc.} \quad \text{--- A.F.4.2} \]

now, from Appendix F2, equations A.F.2.7 and A.F.2.8

\[ N_{E_I} = N_{E_{I=0}} \times \left(1 - \frac{a}{2}\right) \]

and

\[ N_{o_I} = N_{o_{I=0}} \times \left(1 - \frac{a}{2}\right) \]

hence, combining A.F.2.7 and A.F.4.1 and using A.F.2.8

\[ N_{E_I} = \left(1 - \frac{a}{2}\right) \left(b_1 + b_2 \cdot \frac{N_{o_I}}{(1 - \frac{a}{2})} + b_3 \cdot \frac{N_{o_I}^2}{(1 - \frac{a}{2})^2} + \text{etc.} \right) \quad \text{--- A.F.4.3} \]

thus, the slope at B
\[
\frac{d N_i}{d N_{0i}} = N_2 + 2b_3 \left( \frac{N_{0i}}{1 - \frac{\theta}{2}} \right) + 3b_4 \left( \frac{N_{0i}}{1 - \frac{\theta}{2}} \right)^2 + \text{etc.}
\]

that is that

\[
\begin{align*}
\left\{ \frac{d N_i}{d N_{0i}} \right\} & = b_2 + 2b_3 \left( N_{0i} \right)_{i=0} + 3b_4 \left( N_{0i} \right)_{i=0}^2 + \text{etc.} \\
\left\{ \frac{d N_i}{d N_{0i}} \right\} & = A.F.4.4
\end{align*}
\]

Comparing equation A.F.4.4 with A.F.4.2 shows the right hand sides to be identical. Hence the slope at \( B = \) slope at \( A \).
APPENDIX E5

Listing of computer program B167

Automatic vehicle performance program using Runge-Kutta-Cull.

together with sample output
MA~;ER B167
DIMENSION X(10), GCS(10), PUMP(10), ET(10), TK(10), TR(10), ESPED(2000)
COMMON ET, TK, TR, X, GCS, PUMP, DAR, RR, AD, BD, AK, WV, AIW, AIE, KA, A, B, H, X

K, NG, NET, NTK, NTR, PT, VBS, GCT, TI, GR, JJJ, RMIN, ESPED, K

PI = 3.14159265359

CALL TIME (11)
WRITE (2, 139) 11

20 CONTINUE
16 CONTINUE
J = 25
WRITE (2, 100)
READ (1, 104)
WRITE (2, 101)
WRITE (2, 102)
1 READ (1, 103) NET
2 READ (1, 104) (ET(I), I = 1, NET), VBS, ATE, RMIN
WRITE (2, 106)
WRITE (2, 128) (ET(I), I = 1, NET)
WRITE (2, 129)
WRITE (2, 107) VBS, AIE
WRITE (2, 129)
IF (J = 2) 26, 26.0
3 READ (1, 103) NTK
4 READ (1, 104) (TK(I), I = 1, NTK)
WRITE (2, 108)
WRITE (2, 128) (TK(I), I = 1, NTK)
WRITE (2, 129)
IF (J = 4) 26, 26.0
5 READ (1, 103) NTR
6 READ (1, 104) (TR(I), I = 1, NTR)
WRITE (2, 109)
WRITE (2, 128) (TR(I), I = 1, NTR)
WRITE (2, 129)
IF (J = 6) 26, 26.0
9 CONTINUE
7 READ (1, 103) NG, NPUMP
10 CONTINUE
8 READ (1, 104) (X(I), I = 1, NG)
WRITE (2, 110) (X(I), I = 1, NG)
WRITE (2, 129)
IF (J = 8) 26, 26.0
READ (1, 104) (GCS(I), I = 1, NG - 1)
WRITE (2, 111) (GCS(I), I = 1, NG - 1)
WRITE (2, 129)
IF (J = 10) 26, 26.0
11 READ (1, 104) DAR, RR, AIW, PSI, TI
AIW = AIW - PSI * DAR * DAR
IF (J = 11) 26, 26.0
```plaintext
12 READ(1,104)W,A,B,H,KX
   IF(J-12)26,26,0
13 READ(1,104)AD,BD,AK,KX
   IF(J-13)26,26,0
14 READ(1,104)3,CF,GCT
   IF(J-14)26,26,0
15 READ(1,104)V1,V2,WV
   IF(J-15)26,26,27
27 CONTINUE
26 CONTINUE
   WRITE(2,112)DAR,RR,AIL,W,XK
   WRITE(2,130)TI
   WRITE(2,113)W,A,B,H
   WRITE(2,114)AD,BD,AK,G,CF,GCT
   WRITE(2,115)V1,V2,WV
   IF(KA)0,28,28
   WRITE(2,116)
   GO TO 29
28 WRITE(2,117)
29 CONTINUE
   IF(NPUMP)0,52,0
   PUMP(1)=1.64595803
   PUMP(2)=-9.33208837
   PUMP(3)=3.67206682
   PUMP(4)=0.0565263932
   PUMP(5)=0.00373440272
   GO TO 53
52 DO 54 T=1,5
54 PUMP(1)=0.
53 CONTINUE
   PUMP(6)=3.07241256
   PUMP(7)=-1.44482083
   PUMP(8)=0.91072275
   PUMP(9)=-0.73875627
   PUMP(10)=0.010702038
   CC=SQR(1.+G*G)
   C=G*CC+H*G
   BW=1.-C/CC/(A+B)
   TFG=BW*CF+CC)30.30:0
   WRITE(2,118)
30 CONTINUE
   CALL ITIME (12)
   T3=12-11
   WRITE(2,131)12.13
   CALL TIM (TS,V1,V2,G,CF)
   CALL ITIME (14)
   T5=14-12
```

16=14-11
WRITE(2,131)14,15,16
WRITE(2,102)
WRITE(2,120)TS
J=0
READ(1,125)J
IF(J)48,48,0
WRITE(2,126)
WRITE(2,105)
GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,J

48 WRITE(2,127)
STOP
100 FORMAT(20X38H G.LUCAS DEPT OF TRANSPORT TECHNOLOGY/20X37HLOUGHBOURGH UNIVERSITY OF TECHNOLOGY/77/18X39HVEHICLE PERFORMANCE = AUTOMATIC GEARBOX/25X22H MUNING RUNGE-KUTTA-GILL/77)
101 FORMAT(40H)
102 FORMAT(YOH/77)
103 FORMAT(T0T0)
104 FORMAT(20F0.0)
105 FORMAT(Y0H/5X7HNEW RUN/77)
106 FORMAT(68H ENGINE TORQUE LB FT = POLY(ENGINE SPEED)/1000) COEFFICIENTS ARE
107 FORMAT(34H MAXIMUM ALLOWABLE ENGINE SPEED = F9.1/9H REV/MIN/29H ENGINE INERTIA SLUG SQFT = FR.4/77)
108 FORMAT(63H TORQUE CONVERTER K-FACTOR = POLY(SPEED RATIO) COEFFICIENTS ARE)
109 FORMAT(47H TORQUE CONVERTER TORQUE RATIO = POLY(SPEED RATIO) COEFFICIENTS ARE)
110 FORMAT(16H GEAR RATIOS ARE/1X5G17.8)
111 FORMAT(42H SPECIFIED GEAR CHANGE SPEEDS (MILE/H) ARE/1X5G17.6)
112 FORMAT(20H DRIVE AXLE RATIO = F10.5/5X24H ROLLING RADIUS (FEET) = F9.5/40H TOTAL ROAD WHEEL INERTIA (SLUG SQFT) = F10.5,5X21H TYRE GR0 24TH FACTOR = F14.10/77)
113 FORMAT(15H VEHICLE WT LBF20X36HPOSITION OF CENTRE OF GRAVITY (FEET) = F12.3/25X30F10.3/77)
114 FORMAT(30H VEHICLE DRAUGHT COEFFICIENTS ARE3F10.5/19H SPECIFIED GRADIENT 10X23H COEFFICIENT OF FRICTION10X23H GEAR CHANGE TIME (SEC) = F1 29.4/12X77)
115 FORMAT(51H INITIAL TEST SPEED (MILE/H) = F9.2/29H FINAL TEST SPEED 1 (MILE/H) = F10.2/23H WIND SPEED (MILE/H) = F9.2/77)
116 FORMAT(30X25H FRONT WHEEL DRIVE VEHICLE/77)
117 FORMAT(30X24H REAR WHEEL DRIVE VEHICLE/77)
118 FORMAT(20X6H** Vehicle MAY SLIP DOWNHILL WITH MABRAKE ONLY S 1 ET ** */77)
119 FORMAT(20X46H** Vehicle MAY OVERTURN IN FIRST GEAR ** */77)
120 FORMAT(10X48H TIME TO SPEED ON SPECIFIED GRADIENT (SECONDS) = F12.4/77)
121 FORMAT('37H MAXIMUM BHP FROM TORQUE CONVERTER = F11.3,34H AT TC OUTPUT SPEED (REV/MIN) OF F12.2/1')
122 FORMAT('53H ENGINE MAXIMUM SPEED LIMITS MAXIMUM VEHICLE SPEED TOP10.3,12H MILE/H/1')
123 FORMAT('34H MAXIMUM VEHICLE SPEED (MILE/H) = F10.2/1')
124 FORMAT('26H DEGREE OF UNDERGEARING' = 'F10.5/1')
125 FORMAT('12')
126 FORMAT('1H1')
127 FORMAT('10X1HEND OF CALCULATIONS')
128 FORMAT('1X5G17.9')
129 FORMAT('1H0')
130 FORMAT('30H TURBINE INERTIA SLUG SQ.FT = F9.4/1')
131 FORMAT('41B')
500 FORMAT('10E12.5')
END

END OF SEGMENT; LENGTH = 714, NAME = B167
SUBROUTINE TIM (TS, V1, V2, G, CF)
DIMENSION X(10), GCS(10), PUMP(10), ET(10), TK(10), TR(10), ESPED(2000)
DIMENSION Y(2), DY(2), Q(2)
DIMENSION DS(6), AZ(6)
NC, NG, NET, NTK, NTR, PI, VBS, OCT, TI, GR, JJ, RMN, ESPED, KKK
DATA DS(1), DS(2), DS(13), DS(4), DS(5), /1320, 1640, 423, 2640, 3280, 846, 5280, /, AZ(1), AZ(2), AZ
2(3), AZ(4), AZ(5), /8H1/4 MILE, 8H1/2 KILO, 8H1/2 MILE, 8H - KILO, 8H - 1
3MILE/
NDIS=1
DISFL=0;
GDI=0;
DISF=0.
TS=0.
GR=X(1)
M=2
JPRINT=0
AH=.1
INIT=0
KKK=0
GCS(NG)=V2*12
CC=SORT(1, G, 6)
NW=CF/W'(A+B)
IF(KA)=0, 13, 13
AB=B
HH=H
GO TO 14
13 CONTINUE
AB=A
HH=H
14 C=AB*CC+HH*G
DO 70 I=1, 200
70 ESPED(I)=0
V(1)=V1
WRITE(2, 100)
51=RMN/1000
TF(R, EQ, 0.1 R, 1)
J=0
50 CONTINUE
TE=ET(1)
DO 52 T=2, NET
52 TE=TE+ET(I)+R*(I-1)
TPK=TK(1)
RX=RR*(1, +XX*(Y(1)+Y(1)-900,-))
OGR=GR+DAR/RX
SR=Y(1)+22.0GR/15./R/1000./PI*30.
DO 53 T=2, NTK
53  TPK=TPK+TK(1)*SR**((1=1)
FPUMP=PUMP(4)
DO 54 1=7,10
54  FPUMP=PUMP(1)*R**((1-6)
TPK=(R+1000./TPK)**2
IF(J=1)0,0,55
IF(TE-TPUMP-TPK)56,0,0
R=R+.2
J=1
GO TO 50
55  CONTINUE
IF(TPK+FPUMP-TE)57,58,59
56  CONTINUE
IF(R-RMIN/1000.)50,0,59
R=R+.2
GO TO 50
59  R=R-.01
J=10
GO TO 50
57  R=R-.005
58  CONTINUE
Y(2)=R+1000.*PI/30.
60  CONTINUE
T(LAST)=Y(2)
JJ=INIT+1
CALL RKG (M, INIT, AH, Y, DY, Q)
INIT=INIT+1
IF(KK=1)0,51,62
EPED(INIT)=Y(2)*30./PI
DO 63 J=2,NG
IF(Y(1),GE,GCS(I-1),AND,Y(1),LT,GCS(I))GR=X(I)
63  CONTINUE
IF(GY)0,64,0
IF(GY=GR)0,64,0
GY=GR
GO TO 51
64  CONTINUE
GY=GR
IF(Y(1)-V2)60,0,0
KKK=1
INIT=0
RR=X(1)
GY=0.
V(1)=V1
V(2)=0.
GO TO 60
61  CONTINUE
62  CONTINUE
DO 65 I=2,NG
IF(Y(1).GE.GCS(I-1).AND.Y(1).LT.GCS(I)) GR=X(I)
   65 CONTINUE
   AN=0.
   IF(GY).GT.66,0
   IF(GY=0) AN=66,0
   WRITE(2,101) BR,GY
   66-GY=GR
   Y(2)=Y(2)-AN
   DISF=DISF+(Y(1)-AN/2.)*22./15.-(Y(2)-TLAST)
   IF(NDIS=6) DISF=DISF+67,67
   IF(DISF=DS(NDIS)) DISF=DISF+67,0
   DS=(DISF-DS(NDIS))/DISP
   TSD=Y(2)-D*(Y(2)-TLAST)
   VD=Y(1)-D*AN
   WRITE(2,107)
   WRITE(2,106) AZ(NDIS),VD,TSD
   WRITE(2,107)
   NDIS=NDIS+1
   67 CONTINUE
   DSFL=DISF
   IF(INIT=1)/10-JPRINT)68,0,68
   JPRINT=JPRINT+1
   WRITE(2,105) Y(1),Y(2),DISF,ESPED(INIT)
   68 CONTINUE
   IF(Y(1)=Y2) 60,0,0
   TS=Y(2)
   RETURN
100 FORMAT(1HO/130X46H*---TABLE OF TIME TO SPEED CALCULATION---*/
110 ---SPEED MILE/H9X9H TIME SECS11X11 HDISTANCE FT5X18H ENG. SPEED R
120 ---EV/MIN/F)
101 FORMAT(1HO20X23H*---GEAR CHANGE ---*10X14HRATIO 18 NOW 100.4.1
130 ---IT WAS 100.4/) 102 FORMAT(1HO10X31HENGINE SPEED ABOVE MAXIMUM OF 99.1.9H REV/MIN/5X
120 ---VEHICLE SPEED MILE/H = 99.2.10X15H TIME SO FAR IS 100.3.3HSEC/)
103 FORMAT(1HO/25X22HSET SPEED UNOBTAINABLE 5X20H TIME SO FAR SEC =
120 ---1F12.5/) 104 FORMAT(1HO35X22H*---WHEEL SPIN ---**/
105 FORMAT(1HO10X51H9X9H 21H MARK PASSED SPEED 58.2.31H MILE/H, 5X20H TIME APP
120 ---1ROX. SEC = 10.2)
107 FORMAT(1HO)
500 FORMAT(10E12.5)
END

END OF SEGMENT, LENGTH 754, NAME TIM
SUBROUTINE DERY (M, V, DV)
DIMENSION X(10), GCS(10), PUMP(10), ET(10), TK(10), TR(10), ESPED(2000)
DIMENSION Y(M), DY(M)

COMMON ET, TK, TR, X, GCS, PUMP, DAR, R, AD, BD, AK, W, W, AIW, AIE, KA, A, B, M,
                        NK, NG, NET, NT, NTR, PI, VBS, GCT, TI, GR, JJJ, RMIN, ESPED, KKK

RX=RR*(1.0+XK*Y(1)+Y(1)+900.)
OCR=GR*DAR/RX
DF=(G=AD*Y(1)+BD)*AK*(Y(1)+WV)*ABS(Y(1)+WV)

IF(Y(1)-GCS(1))O.2;2

CONTINUE
DO 4 I=2, NG
TF/Y(1)-GCS(1), AND;Y(1);LT_GCS(1) AING=I

CONTINUE
TF=M;.98<.96-.000316*Y(1)-.000058*Y(1)-Y(A)>(.99758*(1-.007*AI)

EMW/32.2+AI/W/RAR/RAR*TF*OCR**2
IF(KKK-10.1, 1
R=Y(2)/3.0.*PI/1000.,

CONTINUE
DO 5 I=2, NET

CONTINUE
TF=T/Y(1)-R***(I-1)
FPUMP=PUMP(6)

CONTINUE
DO 6 I=7, 10

CONTINUE
FPUMP=PUMP+PUMP(I)-R***(I-6)
SR=Y(1)/Y(2)+22.*OCR/15.

CONTINUE
TPK=TK(I)

CONTINUE
DO 7 I=2, NTK

CONTINUE
TPK=TPK+TK(I)*SR**(I-1)
TPK=(R+/1000, /TPK)**2
TRK=TR(I)

CONTINUE
DO 8 I=2, NTR

CONTINUE
TRK=TRK-TR(I)*SR**(I-1)
IF(TRK, TF, I), TRK=1.
TF=TRK*TPK*OCR*TF

CONTINUE
DO 9 I=2, 5

CONTINUE
R=RPUMP+RUMP(I)-S***(I-1)
DY(2)=(TF-FPUMP-TPK)/AIE+EM+22./15.(TF-DF)
RETURN

CONTINUE
S=Y(1)+22.*/15.*DAR/RX/1000.
S=S+5U./Pl

CONTINUE
R=RPUMP+RUMP(I)
DO 10 I=2,5
10 PUMP=R*PUMP*PUMP(I)*S**(I-1)
   RSP=(J(JJJ)+2.0*GR+30.15.1I)
   TPK=TRK(I)
DO 12 I=2,NTR
12 TRK=TRK+TR(I)*SR**(I-1)
   TRK=(ESPED(JJJ)/TPK)**2
   TRK=TR(I)
   TF(THK.LT.1.) TRK=1.
   TFT=TRK*TK+GR+TEF
   OY(I)=EM=22.15./(TFT-DF)
RETURN
500 FORMAT(12E10.3)
END

END OF SEGMENT: LENGTH 562: NAME DERY
SUBROUTINE RKG (M, INIT, H, Y, DY, Q)
DIMENSION Y(M), DY(M), Q(M)

IF (INIT .NE. 0) GO TO 4

DY(1) = H
DO 3 J = 1, M
3 G(O) = U,

20 CALL DERY (M, Y, DY)
RETURN

4 J = 1

16 CONTINUE
DO 14 I = 2, M
14 DY(I) = DY(I) - H
DO 2 I = 1, M
GO TO (4, 5, 6, 7) J

4 R = 5 * DY(I) - Q(I)
GO TO 8

5 R = 29289321881 * (DY(I) - Q(I))
GO TO 8

6 R = 1.7071067812 * (DY(I) - Q(I))
GO TO 8

7 R = (DY(I) - 2 * Q(I)) / 6.
8 V(I) = Y(I) + R
SQ(I) = 3 * R
GO TO (9, 10, 11, 9) J
9 Q(I) = SQ(I) * V(I)
GO TO 2

10 Q(I) = SQ(I) * 29289321881 * DY(I)
GO TO 2

11 Q(I) = SQ(I) * 1.7071067812 * DY(I)
2 CONTINUE

J = J + 1
IF (J = 4) GO TO 20,

CALL DERY (M, Y, DY)
GO TO 16

END OF SEGMENT; LENGTH 266, NAME RKG
**VEHICLE PERFORMANCE — AUTOMATIC GEARBOX**

**USING RUNGE-KUTTA-GILL**

**VEHICLE TEST RUN — 21 NOV 1968**

<table>
<thead>
<tr>
<th>ENGINE TORQUE LBF-FT = POLY(ENGINE SPEED)/1000</th>
<th>COEFFICIENTS ARE</th>
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<tr>
<td>74.2772539</td>
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<td>-0.200829666E-01</td>
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**MAXIMUM ALLOWABLE ENGINE SPEED = 5600.0 REV/MIN**

**ENGINE INERTIA: SLUG SQFT = 0.1500**

<table>
<thead>
<tr>
<th>TORQUE CONVERTER $c$-FACTOR = POLY(SPEED RATIO)</th>
<th>COEFFICIENTS ARE</th>
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<tr>
<td>217.936537</td>
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<td>-174006.916</td>
<td>65510.7737</td>
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<table>
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<tr>
<th>TORQUE CONVERTER $k$-FACTOR = POLY(SPEED RATIO)</th>
<th>COEFFICIENTS ARE</th>
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<tr>
<td>1.88998302</td>
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<td>-22.7994081</td>
<td>-0.36707637</td>
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<tr>
<td>6.22309931</td>
<td>19.3487414</td>
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</tbody>
</table>
GEAR RATIOS ARE
2.3900000  1.4500000  1.0000000

SPECIFIED GEAR CHANGE SPEEDS (MILE/H) ARE
38,000     58,000

DRIVE AXLE RATIO = 3.54000
ROLLING RADIUS (FEET) = 0.95000
TOTAL ROAD WHEEL INERTIA (SLUG SQ.FT) = 2.10000
TYRE GROWTH FACTOR = 0.000000000
TURBINE INERTIA SLUG SQ.FT = 0.0000

VEHICLE WT-LBF
3140.0

POSITION OF CENTRE OF GRAVITY (FEET)
4.140  4.480  1.830

VEHICLE DRAG COEFFICIENTS ARE
0.01300  0.00000  0.02330

SPECIFIED GRADIENT
0.00000

COEFFICIENT OF FRICTION
1.00000

GEAR CHANGE TIME (SEC)
0.00000

INITIAL TEST SPEED (MILE/H) = 0.00
FINAL TEST SPEED (MILE/H) = 80.00
WIND SPEED (MILE/H) = 0.00

REAR WHEEL DRIVE VEHICLE

42322     4
## TABLE OF TIME-TO-SPEED CALCULATION

<table>
<thead>
<tr>
<th>Speed Mile/H</th>
<th>Time Secs</th>
<th>Distance Ft</th>
<th>Eng. Speed Rev/Min</th>
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</thead>
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<tr>
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<td>0.0000</td>
<td>0.0000</td>
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<td>1.00</td>
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<td>RPM</td>
<td>Gear Ratio</td>
<td>Ratio is Now</td>
<td>It Was</td>
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<td>------------</td>
<td>--------------</td>
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***GEAR CHANGE***

Ratio is Now: 1.4500  It Was: 2.3900

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<th>RPM</th>
<th>Gear Ratio</th>
<th>Ratio is Now</th>
<th>It Was</th>
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<td>40.00</td>
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<td>229.6671</td>
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***GEAR CHANGE***

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