The development and experimental assessment of an analytical technique for the determination of stresses in mine winding drums

Additional Information:


Metadata Record: https://dspace.lib.ac.uk/2134/28310

Publisher: © R.S. de Andrade

Rights: This work is made available according to the conditions of the Creative Commons Attribution-NonCommercial-NoDerivatives 2.5 Generic (CC BY-NC-ND 2.5) licence. Full details of this licence are available at: http://creativecommons.org/licenses/by-nc-nd/2.5/

Please cite the published version.
This item was submitted to Loughborough University as a PhD thesis by the author and is made available in the Institutional Repository (https://dspace.lboro.ac.uk/) under the following Creative Commons Licence conditions.

For the full text of this licence, please go to: http://creativecommons.org/licenses/by-nc-nd/2.5/
<table>
<thead>
<tr>
<th>VOL. NO</th>
<th>CLASS MARK</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Date due for return:**

13 SEP. 985

**Loan copy:**

25/5/91

**Loan 1 mth + 2 unless recalled**

**Date due:**

17 MAY 1991

**Loan 3 wks. + 3 unless recalled**
THE DEVELOPMENT AND EXPERIMENTAL ASSESSMENT OF
AN ANALYTICAL TECHNIQUE FOR THE DETERMINATION
OF STRESSES IN MINE WINDING DRUMS

by

RONALDO SOARES DE ANDRADE
B.Sc., M.Sc., M.Tech.

A Doctoral Thesis

Submitted in partial fulfilment of the requirement for the award of
Doctor of Philosophy of the Loughborough University of Technology

February 1982

© by R.S. de Andrade, 1982
To Joana, Mariana, Daniel and Gabriel
I wish to thank the Brazilian Government who through the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and the Universidade Federal do Rio de Janeiro provided me with the necessary financial support to carry out this study.

My thanks also to M.B. Wild and Company Limited, Birmingham, for entirely financing the experimental part of this work; for allowing me to include FIGs. 2.3, 3.3 and 3.4 and the specifications of the winding drum described in section 2.3 and also for the assistance given by its personnel.

I am most thankful to my supervisor, Mr. D.G. Smith, for his support, advice and patience.

I wish to extend my thanks to all those who contributed for the execution of this work and acknowledge the help and assistance given by:

Professor M. Graneek and Mr. C. Rodwell who made this project possible.

Mr. T. Kirk, technician of the Engineering Design Centre for the advice and assistance with the drafting equipment and with the computer used in this work.

Mrs. C. Woodiwiss, secretary of the Engineering Design Centre for her assistance and advice when I was typing this thesis.

The shopfloor personnel of M.B. Wild and Co.Ltd. for their assistance during the experimental part of the work.
SUMMARY

This work presents an analytical technique for the determination of stresses in mine winding drums based on classical theories for thin shells and flat plates. Comparison between experimental results and predictions applying the technique showed good agreement demonstrating that it can provide design engineers with a powerful tool for the stress analysis of cylindrical mine winding drums. The flexibility of the technique in allowing the examination of different winding drum configurations also makes it a means of rapidly determining the effect of parameter changes at the design stage.
CONTENTS

AKNOWLEDGEMENTS i
DECLARATION ii
SUMMARY iii
CONTENTS iv
INTRODUCTION

CHAPTER 1
THE MINE WINDING DRUM 3
1.1 Mine Winders 3
1.2 The Winding Operation 9
1.3 Winding Drum Construction 10
1.4 Loads Acting on a Winding Drum 10
    1.4.1 Self-weight 11
    1.4.2 Rope pull 11
    1.4.3 Rope compression 12
    1.4.4 Load due to shaft bending 16
    1.4.5 Driving torque 16
    1.4.6 Loads due to the application of brakes 16
    1.4.7 Loads due to drum movement 16

CHAPTER 2
STRESSES IN WINDING DRUMS 18
2.1 Effects of the Rope Compression 18
2.2 Determination of Stresses on Drums 24
2.3 An Actual Drum 29
2.4 Approximate Stresses on the Actual Drum 32
    2.4.1 Self-weight 33
    2.4.2 Rope pull 35
    2.4.3 Rope compression 36
    2.4.4 Shaft bending 40
    2.4.5 Driving torque 42
    2.4.6 Application of brakes 43
    2.4.7 Drum movement 44
    2.4.8 Summary 45
CHAPTER 3
THE SCALE MODEL

3.1 Considerations on Scaling
3.2 Choice of the Scale
  3.2.1 Availability of materials and parts
  3.2.2 Feasibility of manufacture
  3.2.3 Feasibility of assembly
  3.2.4 Construction of the rig
  3.2.5 Operation of the rig
  3.2.6 Instrumentation of the rig
  3.2.7 Manufacturing costs
3.3 Model and Rig Design and Construction
3.4 Deviations on the Model
  3.4.1 Shaft
  3.4.2 Bearings
  3.4.3 Brake rings
  3.4.4 Rope
  3.4.5 Loading
  3.4.6 Rope storage reel
3.5 Model Loading and Rig Operation
3.6 Instrumentation of the Model

CHAPTER 4
THE THEORETICAL MODELS

4.1 Drum Roll
  4.1.1 Equations of equilibrium
  4.1.2 Deformations
  4.1.3 Solution of the problem
  4.1.4 Roll with simply supported or built in edges
  4.1.5 Roll with stiffening rings
4.2 Side Plates
  4.2.1 Bending moment applied by the drum roll
    4.2.1.1 Equations of equilibrium
    4.2.1.2 Deformations
    4.2.1.3 Solution of the problem
  4.2.2 Compressive force applied by the drum roll
    4.2.2.1 Equations of equilibrium
    4.2.2.2 Deformations
    4.2.2.3 Solution of the problem
4.3 Combination of the Theoretical Models for the Drum Roll and Side Plates 110
4.4 Action of the Bent Shaft on the Side Plates 115
4.5 Summary of Equations 119
  4.5.1 Drum roll 119
  4.5.2 Side plates 121
4.6 Theoretical Representation of the Scale Model 124
  4.6.1 Effect of the rope pressure 124
  4.6.2 Effect of the bent shaft 129

CHAPTER 5
TEST RESULTS 132
5.1 Layout of the Strain Gauges 132
5.2 Test Conditions 136
  5.2.1 Loading conditions 136
  5.2.2 Loads 139
  5.2.3 Drum configuration 139
  5.2.4 Drum position 139
5.3 Test Procedures 140
5.4 Tests Performed 141
5.5 Data Manipulation 143
5.6 Analysis of the Results 143
  5.6.1 Variation of the measurements 143
  5.6.2 Effect of the drum position 146
  5.6.3 Isolation of the effect of each loading 149
  5.6.4 Presentation of the results 149
  5.6.5 Control gauges 155
  5.6.6 Effects of the self-weights and vertical loads 156
  5.6.7 Comparison between the first and second groups of tests 156
  5.6.8 Effect of different loads 163
  5.6.9 Effect of the brake rings 163
  5.6.10 Effect of the drum split 172
  5.6.11 Effect of the position of the loading 182
  5.6.12 Exploratory gauges 185
5.7 Extrapolation of the Results to the Full Size Drum 186
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHAPTER 6</td>
<td></td>
</tr>
<tr>
<td>VALIDATION OF THE THEORETICAL MODELS</td>
<td></td>
</tr>
<tr>
<td>6.1 Predictions for the Second Group of Tests</td>
<td></td>
</tr>
<tr>
<td>6.1.1 Drum roll</td>
<td>189</td>
</tr>
<tr>
<td>6.1.2 Side plates</td>
<td>189</td>
</tr>
<tr>
<td>6.2 Predictions for the Fifth Group of Tests</td>
<td>201</td>
</tr>
<tr>
<td>6.3 Predictions for the Strains Induced by the Bent Shaft</td>
<td>216</td>
</tr>
<tr>
<td>6.4 Summary</td>
<td>217</td>
</tr>
<tr>
<td>CONCLUSIONS</td>
<td>220</td>
</tr>
<tr>
<td>SUGGESTIONS FOR FURTHER WORK</td>
<td>221</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>222</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>225</td>
</tr>
<tr>
<td>APPENDIX 1</td>
<td></td>
</tr>
<tr>
<td>MODEL DRUM FABRICATION DRAWINGS</td>
<td>227</td>
</tr>
<tr>
<td>APPENDIX 2</td>
<td></td>
</tr>
<tr>
<td>STRAIN GAUGE CIRCUIT ANALYSIS</td>
<td>236</td>
</tr>
<tr>
<td>APPENDIX 3</td>
<td></td>
</tr>
<tr>
<td>A STRUCTURE FOR THE THEORETICAL MODELS FOR WINDING DRUMS</td>
<td>242</td>
</tr>
<tr>
<td>APPENDIX 4</td>
<td></td>
</tr>
<tr>
<td>COMPUTER PROGRAMS</td>
<td>250</td>
</tr>
<tr>
<td>APPENDIX 5</td>
<td></td>
</tr>
<tr>
<td>RECORDED DATA</td>
<td>271</td>
</tr>
<tr>
<td>APPENDIX 6</td>
<td></td>
</tr>
<tr>
<td>PRINCIPAL STRAINS AT GAUGES NOS. 46, 47 AND 48</td>
<td>283</td>
</tr>
</tbody>
</table>
INTRODUCTION

This thesis is the result of a co-operative project between M.B. Wild and Company Limited, a leading British manufacturer of mining equipment and the Engineering Design Centre of Loughborough University of Technology.

Amongst other products, the Company manufactures mine winding drums which are basically comprised of a thin cylindrical roll supported by two circular side plates mounted on a shaft. Their main function is to raise or lower a conveyance in a mine shaft with a rope which is wound onto or off the drum.

The design of such drums using elementary stress analysis techniques has been acceptable in the past but the Company is interested in producing more economical designs. In order to do so, more accurate techniques were required to determine stress levels in the drums. In order to validate the techniques it was considered necessary to obtain data from experiments on actual winding drums. This was not practical because this equipment operates in very demanding production systems where any stoppages or delays are expensive.

The Company and the Engineering Design Centre had worked together in the past on another project also related to winding drums when a small scale model had been built, instrumented with strain gauges and tested with very satisfactory results. This exercise opened up the potentialities of scale model testing. Having in mind that a small model of a winding drum would be cheaper than a full size one and permit investigation and modifications without the constraints of installed equipment, the decision was made to approach this new investigation in a similar manner.

When this research work began two new large drums of 6 metres in diameter were being designed by the Company. One of these drums was selected as the basis for the model. After deciding on the scale for the model a specification for the test rig was prepared incorporating the instrumentation
together with the layout of the strain gauges. The gauges were positioned at points where high stresses were expected and also arranged to permit delineation of the pattern of mechanical behaviour of the drum.

While the test rig was being manufactured the literature on winding drum design was studied in search for analytical methods to predict their mechanical behaviour. At this stage it became clear that most of the authors who had studied the problem analytically applied the same thin shell theory to determine stresses in the drum. This theory could only represent the drum roll. For a complete representation it had to be combined with a plate theory to represent the side plates. A method to apply both theories was then developed and, although it was simple in its structure, it required the manipulation of large systems of equations for which the aid of a computer was needed. Hence, a computer program was prepared which provided a tool for the theoretical prediction of stress levels.

After the test rig had been commissioned a comprehensive series of tests were carried out generating approximately 10,000 measurements. The test results were fed into a computer to be analysed and compared with the theoretical predictions. The analysis of the results comprised the examination of the effects of various loading conditions and drum configurations. The comparison between the predicted and the observed values generally showed good agreement.

With the exception of the design of the test rig structure, the manufacture and assembly of the test rig and the design and installation of the instrumentation all the work described in this thesis was carried out by the author.

Unless otherwise stated the units in the dissertation follow the International System of Units (SI), with the linear dimensions given in millimetres.

For consistency with the notation used in the text the formulae or equations taken from the references do not necessarily follow the notation of the original work.
CHAPTER 1
THE MINE WINDING DRUM

1.1 Mine Winders

One of the main handling operations in underground mines is the transportation of men and materials in vertical shafts to and from depths that in many cases exceed 2,000 metres.

The principal means of transportation are conveyances suspended from wire ropes and hoisted by drum winders. These can be grouped into two basic types:

(a) Friction winders or Koepe pulleys
(b) Winding drums

The first type, shown in FIG. 1.1a, consists of a grooved pulley with less than one turn of rope around it. When the drum rotates the movement is transmitted to the rope purely by friction. To prevent slippage the following methods are commonly adopted:

i- The pulley is lined with an adequate material, wood being traditionally used but some plastics have been tried recently.

ii- An auxiliary sheave is introduced to increase the contact angle of the rope.

iii- A balance rope is fitted so the out of balance weight is only the payload.

Friction winders are also built to operate with more than one rope, as shown in FIG. 1.1b. This is advantageous in very deep shafts and high payloads because ropes and drums of smaller diameter can be used giving a more economical system.

The second type of winder is the winding drum which is basically a winch as represented in FIG. 1.2a. In this type, known as a single drum,
FIG. 1.1 FRICTION WINDERS
the movement of the rope is achieved by winding it on or off the drum. A drum can operate two conveyances (FIG. 1.2b) and in this arrangement when one of the conveyances is being raised the other one is being lowered. This kind of winding drum, known as a double drum, normally operates with a balance rope so that the torque required to maintain a constant winding speed is kept constant.

A common arrangement shown in FIG. 1.2c, is when two single drums are mounted on the same shaft with one or both of them clutched to the shaft. This allows one to be rotated while the other is held stationary by unclutching and braking it thus permitting a relative adjustment of the conveyances to wind from different levels.

Winding drums themselves can be classified into five types, shown in FIG. 1.3, namely:

a- Cylindrical
b- Conical
c- Cylindro-conical
d- Bi-cylindro-conical
e- Multi-rope cylindrical

Among these the cylindrical drum is the most widely used.

Winding drums are driven from a power source through the shaft on which they are mounted. Steam engines once largely used as the driving unit have been almost completely replaced by either AC or DC electric motors directly coupled or geared to the shaft. Some typical arrangements being shown in FIG. 1.4. Both the acceleration or retardation of a winding drum are achieved with the driving motor. The brakes fitted on the drums are used only for parking or emergency purposes. The types of brakes and their usual arrangements are shown in FIG. 1.5.

The study presented in this thesis has been restricted to cylindrical drums which are described in more detail in the next section.
FIG.13 TYPES OF WINDING DRUMS
FIG. 14  TYPICAL DRIVING ARRANGEMENTS

FIG. 15  BASIC BRAKE TYPES
A comprehensive discussion on the design, construction, application and operation of winding drums and also of friction winders is presented by Broughton. Multi-rope winding drums which are not discussed by Broughton have been analysed by Tudhope.

1.2 The Winding Operation

To facilitate the presentation of the problem, all the discussion and analysis in this work has been done for the double cylindrical drum. However, the analysis and results can be extended to single or multi-rope drums.

The winding operation basically consists of transporting a conveyance between two different levels in a vertical, or sometimes inclined, mine shaft. The whole weight of the conveyance plus the payload is supported by a wire rope whose other end is attached to the winding drum. In a double drum there are two such ropes each supporting one conveyance. At the beginning of a winding operation one of the conveyances is at the lower level and the other at the upper level. The side of the drum with the upper conveyance has coiled onto it a length of rope equal to the distance between the two levels together with a number of "dead coils". The other side of the drum supporting the lower conveyance has only "dead coils", their purpose being to avoid subjecting the drum rope attachment to high loads. In order to move the lower conveyance upwards the drum is rotated thus winding the rope onto the empty side. At the same time the rope on the other side, which is coiled in the same direction, winds off and the upper conveyance moves downwards reaching the lower level when the other reaches the upper level. In some cases more than one layer of rope is wound onto the drum so that long travels can be achieved with relatively smaller drums.

The winding operation is performed repeatedly for the whole life of the drum. Commonly, when hoisting materials, the drum is operated continuously except during loading and unloading or when inspection, maintenance and repairs are required. In these conditions the speed of winding depends on the hourly output of materials required, the weight hoisted
per trip and the arrangement for loading and unloading. Conveyance speeds of 15m/s are common and some mines operate with speeds in excess of 22m/s. Drum angular speeds of up to 7.2rad/s (70rpm) are not uncommon. The acceleration and retardation rates of the conveyances are usually the same and normally range between 0.6 and 0.9m/s² but rates of up to 3.6m/s² are known. Emergency braking when transporting people is subjected to regulation in most countries and is usually restricted to deceleration rates not exceeding 5.0m/s².

1.3 Winding Drum Construction

Cylindrical drums are made as iron or steel castings, steel fabrications or a combination of these processes. They consist mainly of a shell, or roll, supported by side plates, or cheeks, which are bolted or keyed to a shaft. An actual drum is presented in section 2.3. Brake paths are either cast integral with or bolted to the drum sides. Ring stiffeners are cast, bolted or welded inside the drum to provide reinforcement. The shell can be externally grooved to accomodate the rope. When operating with more than one layer of rope, flanges are required at the sides and at the middle to support the rope. The wire rope is attached to the drum sides or to the shaft and is passed through a tunnel on the roll where it is coiled. Shafts are steel forgings and rest on two or more bearings. To facilitate transportation and assembly, large drums are made in sections. In the most used construction the whole drum is split into halves across a diameter.

In this work the use of the word "drum" will be in reference to a structure similar to that shown in FIG. 2.3 but excluding the shaft and bearings.

1.4 Loads Acting on a Winding Drum

The relevant loads acting on a drum are identified in this section and
those considered in this research work are appropriately indicated. The
determination of the stresses induced by each load is discussed in the
following chapter.

The loads are:

1- Self-weight
2- Rope pull
3- Rope compression
4- Load due to shaft bending
5- Driving torque
6- Loads due to the application of brakes
7- Loads due to drum movement

1.4.1 Self-weight

This comprises the weight of the drum, the weight of the length of
rope coiled on it and the weight of the shaft. The effect of this load
on the drum is usually neglected but is considered in the design of the
shaft.

1.4.2 Rope pull

This load (FIG. 1.6) which acts tangentially to the drum comprises the
weight of the conveyance plus the payload and plus the weight of the suspended
rope. When a balance rope is fitted the latter weight remains constant
throughout the winding cycle whilst without it that weight increases on the
side of the descending conveyance and decreases on the other side as the
wind proceeds. When the drum rotates dynamic friction forces generated
by the conveyance and rope guiding systems adds to the rope pull.

The rope pull has two direct effects on the drum:

i- A bending imposed on it as if the drum were acting as
   a beam.
ii- A torsion applied to the drum roll by the pull acting tangentially to it.

These effects are shown to be small compared with the compression of the rope on the drum and are usually neglected in their design.

1.4.3 Rope compression

This is an indirect effect of the rope pull. The tensioned rope coiled around the drum exerts a compressive force on it and provides the major contribution to the stresses to which the drum is subjected. To appreciate this force consider a drum of radius \( R \) with one coil of tensioned rope around it and assume that the tension \( T \) is constant along the rope as shown in FIG. 1.7. Consider an element of the rope embracing an arc \( d\theta \) on the drum and assume that the drum resists the action of the rope with a uniform force \( F \) per unit of length as shown in FIG. 1.7b. Considering that the system is in equilibrium, and that the reaction of the drum is \( F.R.d\theta \), the sum of forces projected on the y axis is

\[
F.R.d\theta - 2.T\sin(d\theta/2) = 0
\]

For a very small angle \( d\theta \), \( \sin(d\theta/2) \approx d\theta/2 \) and then

\[
F = \frac{T}{R} \tag{1.1}
\]

The compressive forces due to the rope not only deform the drum roll but also the side plates supporting it as shown in FIG. 1.8.

When winding more than one layer of rope the drum is fitted with flanges at the sides and at the middle to support the rope. Such flanges are also subjected to the action of the tensioned coils of the rope exerting forces as indicated in FIG. 1.9.
FIG. 1.6  ROPE PULL

FIG. 1.7  FORCE APPLIED BY TENSIONED ROPE
FIG. 1.8 DEFORMATIONS INDUCED BY THE ROPE COMPRESSION
FIG. 1.9 FORCES ON THE DRUM FLANGES

FIG. 1.10 DEFORMATIONS INDUCED BY THE SHAFT BENDING
1.4.4 Load due to shaft bending

The rope pull plus the drum and shaft self weights are totally supported by the shaft which on bending induces deformations on the drum particularly affecting the side plates as shown in FIG. 1.10.

1.4.5 Driving torque

The drum movement is achieved by the application of a torque through the shaft which is transmitted to the drum via the side plates. Its maximum value in normal operation is at the beginning of the wind with the fully loaded conveyance at the lower position.

1.4.6 Loads due to the application of brakes

Regardless of the type of brake shown in FIG. 1.5 they induce a torque either to stop the drum or to maintain it stationary. Shoe brakes also apply a compressive load to the brake path which is transmitted to the drum itself.

The heat generated by the braking action gives rise to stresses due to thermal gradients.

1.4.7 Loads due to drum movement

These comprises the centrifugal forces due to the drum rotation, those caused by the acceleration or deceleration at the beginning and end of the winding cycle respectively and deceleration during emergency braking. Also, when coiling several layers of rope, an acceleration or deceleration of the conveyance occurs every time the rope changes from one layer to another. The moving rope and drum produce vibrations which also contribute to the induction of loads.
The loads discussed are present in most winding drums but in practice designers tend to stress a drum to resist the rope compression and then check its strength for the other loads. An important consideration in the drum design is in regard to fatigue. Winding drums are subjected to cyclic stresses not only during each rotation but also during the winding cycle which in some applications is repeated for nearly 24 hours a day all the year round.

The work described in this thesis deals mainly with the effects of the compression applied by the rope on both the drum roll and drum sides. The effects of the shaft bending on the side plates are also considered.
CHAPTER 2
STRESSES IN WINDING DRUMS

Available methods for stressing winding drums are discussed in this chapter which also includes the determination of the stress levels induced on an actual drum by the loads described in section 1.4. Firstly, the effects of the rope compression, which is the more important load is considered in more detail.

2.1 Effects of the Rope Compression

It has been shown in section 1.4.3 that each coil of rope applies on the drum roll a force per unit of length equal to the coil tension divided by the drum radius as given by Eq. 1.1. Most designers work with the pressure applied by the rope rather than the force. This pressure $p$, is calculated assuming that the force is distributed across the rope diameter $d$ and is given by

$$p = \frac{F}{d} = \frac{T}{Rd}$$  \hspace{1cm} 2.1

Some designers substitute the rope diameter by the rope pitch, the distance between the centres of two successive coils.

The reason for obtaining the pressure applied by the rope is so that existing theories for pressure vessels and shells subjected to external pressure can be used to assist in the stress analysis of the drums. To simplify the problem further the pressure is assumed to be uniform along and around the drum. This uniformity implies that all the coils are wound under the same tension maintaining it throughout the wind. Of course, when a balance rope is not fitted the coils are wound under varying tension and in this situation designers calculate the pressure by either considering the pull at the beginning of the wind, with the loaded conveyance at the lower position, or using an average pressure with the conveyance at mid-position.
On account of the drum deformation the rope coils do not maintain the tension with which they are coiled regardless of whether or not a balance rope is fitted. As the wind proceeds each additional coil compresses the drum and induces a reduction in its diameter so that as a consequence the previously wound coils have their tension relaxed. The amount of relaxation depends on the elasticities of the rope and drum and also depends on the position of the coil along the drum shell. The deformations of a drum near the roll edges are usually very small because of the strength given by the sides and the coils close to them relax very little. At the middle of the roll the deformation is more pronounced and the coils there relax more.

If the drum is fitted with stiffening rings its deformation is reduced and the rope relaxes less.

The process of rope relaxation is the same when winding one or more layers. With more than one layer each additional coil wound on not only induces a deformation on the drum but also on all the previously wound layers.

It's apparent that with coil relaxation the actual pressure on the drum will be lower than that calculated with Eq. 2.1 but designers tend to ignore this fact and use the value given by the equation when designing drums to operate with only one layer of rope; the error being on the safe side. The same procedure is sometimes adopted when designing for more than one layer. The design pressure given by the value from Eq. 2.1 multiplied by the number of layers, overestimates the pressure leading to designs of drums which are unnecessarily strong and heavy.

The precise analytical determination of the tension relaxation within each coil is complex because of the difficulty in describing the mechanical interactions between the rope coils within each layer and with the drum. Most of the authors who deal with the problem work towards establishing equations to determine factors to be applied to Eq. 2.1 depending on the number of layers of rope. The rope pressure is given by the value calculated with the equation multiplied by the appropriate factor.
ATKINSON and TAYLOR report a method to determine the relaxation of each coil along a drum roll but, unfortunately without revealing it. They also present the development of a simple formula to calculate rope factors considering the rope diameter and cross-sectional area, the drum roll thickness and the moduli of elasticity of the roll and rope.

Since the ropes used in winding drums are made with wire strands as shown in FIG. 2.1, rather than as a solid section, the modulus of elasticity considered is not that of the strands but an apparent modulus which is determined experimentally by the rope manufacturers. In order to obtain this figure a given length of rope is tensioned and the modulus calculated by dividing the applied tension by the rope cross-sectional area and by the observed strain. The rope cross-sectional area, given by the sum of the cross-sectional areas of the individual wires is known as the rope metallic area. Further, on account of the rope construction its apparent modulus of elasticity in the radial direction, or the transverse modulus of elasticity, is smaller than that in the direction of the length. This modulus is not usually supplied by the rope manufacturers.

FIG. 2.1 WINDING ROPES
In an extensive work on the mechanics of ropes on cylindrical hoists, DIETZ\textsuperscript{11} discusses a method for the determination of the relaxation of any coil at any layer considering the same parameters as the previous authors and in addition, the transverse modulus of elasticity of the rope. In the method, the mechanical relationships among the coils are represented by a set of simultaneous equations from whose solution the relaxation can be determined. He also presents a series of charts for the determination of factors for each layer together with the results of a series of experiments to determine the transverse modulus of elasticity of several ropes. The experiments showed that the modulus depend not only on the type of the rope but also on the number of layers, on the tension on the rope and on the existence or not of grooves on the drum. Experiments comparing predicted factors of relaxation for up to eight layers with observed values generally showed that the actual relaxations were more than predicted.

Methods proposed by other authors give factors relative to the pressure applied by the first layer, the factor for this layer being taken as unity. WATERS\textsuperscript{12}, one of the first investigators to consider the problem, presents a table of rope factors for up to six layers without showing how they have been arrived at. The use of his table is restricted to a limited range of drum configurations. EGAWA and TANEDA\textsuperscript{13} developed an iterative procedure to determine the factors considering the same parameters as Dietz and also the geometrical arrangement of the coils around the drum. Their procedure was verified experimentally. In an extensive investigation on data collected from more than 460 winders DOLAN\textsuperscript{14} discusses his own formula and another by P.Hendry to calculate factors of relaxation considering the rope diameter, the drum roll thickness and the moduli of elasticity of the roll and rope.

Considering the same parameters TORRANCE\textsuperscript{15} proposed an alternative formula and, using the experimental results from Egawa and Taneda, compared his method with those discussed by Dolan showing that his own approach gives better agreement although predicting higher values than those of Egawa and Taneda for a large number of layers.
In all methods presented in the foregoing it is assumed that there is no slippage between layers of rope and between the first layer and the drum. It is also assumed that the drum is not radially restrained by stiffening rings, although Atkinson and Taylor and also Dietz suggest that such restriction can be considered in their methods. When stiffening rings are present the common practice is to add their cross-sectional areas to the cross-sectional area of the drum roll, determine an equivalent thickness for the roll and use this equivalent roll without stiffening rings to calculate the factors.

In order to evaluate the various methods discussed, the drum used in the experimental part of this work has been chosen as an example. Its characteristics were as follows:

- Drum radius \( R = 610 \text{mm} \)
- Drum equivalent thickness \( t_e = 21 \text{mm} \)
- Modulus of elasticity of the drum \( E = 206 \text{kN/mm}^2 \)
- Rope diameter \( d = 10.22 \text{mm} \)
- Rope metallic area \( A_R = 67.02 \text{mm}^2 \)
- Longitudinal modulus of elasticity of rope \( E_R = 147 \text{kN/mm}^2 \)
- Number of layers \( n = 6 \)

The transverse modulus of elasticity of the rope used was not known. Experimental results from Dietz give a transverse modulus of elasticity of 200 to 300N/mm\(^2\) for a rope having longitudinal modulus of elasticity of 118kN/mm\(^2\) and a ratio of 0.44 between the metallic area and the area enclosed by a circle having the same diameter as the rope. For the experimental rope of the example the ratio was 0.82, hence its transverse modulus of elasticity should be higher than that determined by Dietz.

In order to make a comparison of the methods the transverse modulus was initially assumed to be 500N/mm\(^2\) in order that the charts given by Dietz could be used. The factors determined with the different methods, relative to the pressure on the first layer are shown in Table 2.1, where the factors determined with Water's method had to be approximated because the tables given by this author do not comprise factors for drums with the
### Table 2.1
Factors of rope relaxation

<table>
<thead>
<tr>
<th>Number of layers</th>
<th>Egawa &amp; Taneda</th>
<th>Dietz</th>
<th>Waters*</th>
<th>Atkinson &amp; Taylor</th>
<th>Ede</th>
<th>Hendry</th>
<th>Torrance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>1.80</td>
<td>1.91</td>
<td>1.70</td>
<td>1.84</td>
<td>1.74</td>
<td>1.65</td>
<td>1.74</td>
</tr>
<tr>
<td>3</td>
<td>2.44</td>
<td>2.55</td>
<td>2.02</td>
<td>2.55</td>
<td>2.33</td>
<td>2.71</td>
<td>2.33</td>
</tr>
<tr>
<td>4</td>
<td>2.95</td>
<td>2.98</td>
<td>2.16</td>
<td>3.17</td>
<td>2.82</td>
<td>3.57</td>
<td>2.82</td>
</tr>
<tr>
<td>5</td>
<td>3.37</td>
<td>3.26</td>
<td>2.22</td>
<td>3.73</td>
<td>3.24</td>
<td>4.42</td>
<td>3.24</td>
</tr>
<tr>
<td>6</td>
<td>3.71</td>
<td>3.49</td>
<td>2.23</td>
<td>4.23</td>
<td>3.60</td>
<td>5.28</td>
<td>3.60</td>
</tr>
</tbody>
</table>

* approximate

### Table 2.2
Rope relaxation factors for different transverse moduli of elasticity
( using Egawa and Taneda method )

<table>
<thead>
<tr>
<th>Number of layers</th>
<th>Transverse modulus of elasticity - N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>250</td>
</tr>
<tr>
<td>1</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>1.78</td>
</tr>
<tr>
<td>3</td>
<td>2.37</td>
</tr>
<tr>
<td>4</td>
<td>2.81</td>
</tr>
<tr>
<td>5</td>
<td>3.14</td>
</tr>
<tr>
<td>6</td>
<td>3.39</td>
</tr>
</tbody>
</table>
parameters of the example.

The methods given by Egawa and Taneda and by Dietz have been verified experimentally and the former will be taken as reference, their results showing good agreement for all layers. The assessment of the factors determined with the other methods is best made by comparing them with factors calculated for different transverse moduli of elasticity using Egawa and Taneda's method as shown in Table 2.2.

Comparing Tables 2.1 and 2.2 it can be seen that the factors from Waters are low while those from Hendry are high. Factors from Dolan and Torrance show good agreement with the factors for a low modulus of elasticity and are low for the higher moduli. The factors given by Atkinson and Taylor are high for low moduli and show good agreement for the highest modulus of elasticity considered.

It will be observed in Table 2.2 that the factors tend to stabilise as the modulus of elasticity increases and the figures from Atkinson and Taylor's method appear to give limiting values. When the transverse modulus of elasticity is not known the use of these limits seems to be the safest proposition when designing a winding drum.

The remaining fact about the effect of the rope compression on winding drums is that there is still much to be understood in order to make an adequate analytical representation of the mechanics of the problem. In the meantime, purely experimental work relating factors of relaxation to rope diameter, drum thickness and tension on the rope for the types of rope most commonly used would be of great value to drum designers.

2.2 Determination of Stresses on Drums

Most methods for stressing winding drums concentrate on the drum roll itself and the one most commonly used assumes that the net pressure applied by the rope is uniformly distributed along and around the drum. The drum
roll can then be considered as a thin-walled cylinder and the maximum hoop stress, \( \sigma_1 \), calculated using the expression

\[
\sigma_1 = \frac{pR}{t}
\]

where

- \( p \) - pressure applied by the rope
- \( R \) - radius of drum roll
- \( t \) - thickness of drum roll

This expression is derived by considering an element removed from a thin-walled open-ended cylinder subjected to a uniform external pressure as shown in FIG. 2.2a. Considering the external force and the stress resultants acting on the element as shown in FIG. 2.2b the equations of equilibrium in the directions \( x \) and \( y \) are

\[
\begin{align*}
\sigma_2 t R \theta \delta & - \sigma_2 t R \delta \theta = 0 \\
p R \delta \theta dx &= \sigma_1 t \sin(\delta \theta/2) dx + \sigma_1 t \sin(\delta \theta/2) dx
\end{align*}
\]

where

- \( \sigma_2 \) - axial stress
- \( \delta \theta \) - angle subtending the element
- \( dx \) - length of the element

For a very small value of \( \delta \theta \), \( \sin(\delta \theta/2) = \delta \theta/2 \), and cancelling the common factors

\[
\sigma_1 = \frac{pR}{t} \quad \text{and} \quad \sigma_2 = 0
\]

If the cylinder has radial restraints such as side plates or stiffening rings the above expressions are not valid. Stresses will vary along the cylinder and their determination requires more advanced theories. However, if the distance between the restraints is sufficiently long their effect will not be transmitted to the central part of the cylinder where the
FIG. 2.2  FORCES ON A THIN-WALLED OPEN-ENDED CYLINDER
stresses will be practically the same as if there was no restraint. The distance \( L \) for which this occurs is given by

\[
L > 5\sqrt{\frac{t}{E}}
\]

for materials with Poisson ratio \( \nu = 0.3 \) (see, e.g., TIMOSHENKO, ch. 3).

The value calculated using Eq. 2.2 gives the order of magnitude of the hoop stress on a long cylinder, the maximum stress being about 8% higher.

The Eq. 2.2 is also commonly used when the drum is fitted with stiffening rings. In this case the actual drum roll thickness is substituted by an equivalent thickness taking into account the area of the rings as described in section 2.1. Although this procedure greatly simplifies the problem the errors involved in the estimation of the hoop stress can be large if a small number of rings is used or if the cross-sectional area of the drum roll is significantly smaller than those of the rings. Also the axial stresses induced on a drum fitted with stiffening rings can be higher than the hoop stresses calculated using Eq. 2.2.

WATERS superimposes on the stress given by Eq. 2.2 the stresses determined by assuming that the drum roll also acts as a beam simply supported at the ends and subjected to a vertical force due to the rope pull. The effect of side plates or stiffening rings on the bending of the roll is not considered but he derives an expression to calculate the force applied by multi-layers of tensioned rope on the drum flanges from which he calculates the axial stress induced by that force on the roll.

A general equation to calculate drum deflections and stresses was developed by TORRANCE based on an unpublished work by P. Hendry who considered hoop stresses together with the axial stresses induced by the bending of the drum roll along its axis. Torrance solves the general equation for a semi-infinite cylinder with built-in edges but without stiffening rings and
determines expressions for maximum hoop stresses and shear forces at the edges. There is an implicit assumption in the solution that the roll is sufficiently long for the effect of the bending at one edge not to interact with that at the other. The theory used by Torrance is a simplification of the more exact theory of bending of thin shells which is presented in chapter 4. The results obtained with the latter theory are higher than those obtained with Torrance's theory by a factor of $1/(1-v^2)$, $v$ being the Poisson ratio.

From the general solution of the more exact theory given in chapter 4 CRAWFORD\textsuperscript{17} developed a method to determine the drum roll thickness and the cross-sectional area of the stiffening rings assuming them to be similar in area and equally spaced along the roll. He also assumes that the roll edges are supported by rings with the same area as the stiffening rings and proceeds considering that all rings are radially elastic but otherwise perfectly rigid. Despite the eventual limitations in cases of drums with rings not equally spaced or with different areas Crawford's work was a major contribution towards a more analytical treatment of the problem of stressing winding drums.

ATKINSON and TAYLOR\textsuperscript{3 to 10} report, in a series of papers, the development of a computer aided method to calculate deformations and stresses in winding drums considering the stiffening rings, the side plates and also the variation in rope tension due both to the relaxation caused by the drum deformation and the change in rope weight which occurs when the drum operates without a balance rope. The authors, who discuss several aspects of the design of winding drums, use the same theory as Crawford to analyse the drum roll but unfortunately do not disclose the techniques applied to model the drums nor the numerical methods used in the computer programs developed. A further discussion of the method was presented by ATKINSON and PREATTER\textsuperscript{18} where results from an example are analysed.

In his work on the mechanics of ropes on hoists DIETZ\textsuperscript{11} suggests a technique to analyse drum rolls based on the same shell theory as used by Crawford, and Atkinson and Taylor, using a numerical method known as transfer
matrix (refer to PESTEL and LECKIE\textsuperscript{19}). He discusses how the effects of stiffening rings, flanges and side plates can be considered.

The stresses induced on the flanges when winding several layers of rope on the drum, previously discussed in section 1.4.3, have been considered by WATERS\textsuperscript{12} who developed an expression to calculate the total force applied by the rope on the flange and from that, the stresses. BELLAMY and PHILIPS\textsuperscript{20} who also investigated in this area through a series of experiments, present a set of flange design curves giving flange forces for four different ropes. DIETZ\textsuperscript{11} also considered the problem comparing three different theories with experimental results. He also gives a set of curves for the determination of stresses at the junction of the flange with the drum roll.

Stresses induced by the loads discussed in section 1.4 other than the rope compression are commonly assessed by general methods available in the literature.

The majority of the analytical work on winding drums deals only with the determination of the stresses induced by the rope compression on the drum roll and most authors applied the same theory of thin shells to solve the problem. The stresses induced on the side plates by the loads transmitted by the drum roll were considered by Atkinson and Taylor who applied a circular plate theory to analyse the sides. In their method the authors consider the drum roll and side plates as separate entities and utilize the results obtained to study the drum as a complete structure.

The thin shell theory applied by most authors and a circular flat plate theory are presented in chapter 4 where it is shown how they can be combined together to represent a winding drum as a complete structure.

2.3 An Actual Drum

The study in this work was based on a drum designed by M.B.Wild and
Co. Ltd. to lift men and materials and whose cross-section is shown in FIG. 2.3.

It is a double drum fabricated in steel with a grooved roll internally reinforced by stiffening rings and supported on two side plates; an external ring bolted onto the centre of the roll divides it into two sections. Each section winds one rope which on erection is introduced into the drum through holes at the roll edges and attached to the side plates after having a length coiled around a storage reel. This length of rope is necessary in order to comply with safety regulations which require a piece of rope to be cut from the end attached to the conveyance every six months for inspection. The original suspended length of the rope is made up again with rope from the storage reel. The function of the ring is to provide a separation for the ropes when alternatively wound in two layers. Cast iron rings of spherical graphite act as brake disks and are fastened to each side plate. The plates are bolted to a forged steel shaft which is supported by two self-aligning spherical seated journal bearings, and directly coupled to an electric motor (not shown). The drum roll, stiffening rings, brake rings and side plates are split into halves across a diameter. The two halves of the assembly are bolted together at the sides, at the stiffening rings and at the brake rings. In addition the roll halves are butted together and bolted by means of internal strapping plates. The drum is to operate in a system with the same configuration as shown in FIG. 1.2b with the ropes at an angle of about 30° with the horizontal. The main parameters of the system are:

<table>
<thead>
<tr>
<th>Performance</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-depth of wind</td>
<td>615m</td>
</tr>
<tr>
<td>-payload</td>
<td>78kN</td>
</tr>
<tr>
<td>-duty</td>
<td>34winds/h</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Drum</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-diameter(nominal)</td>
<td>6100mm</td>
</tr>
<tr>
<td>-length</td>
<td>3150mm</td>
</tr>
<tr>
<td>-roll thickness(nominal)</td>
<td>70mm</td>
</tr>
<tr>
<td>-side thickness(nominal)</td>
<td>65mm</td>
</tr>
</tbody>
</table>
- shaft diameter 650mm
- total cross-sectional area of stiffening rings and external ring 95000mm²
- weight (without shaft) 912kN
- shaft weight 224kN
- inertia 772500kgm²
- angular speed 4rad/s (38.2rpm)

Rope
- diameter 51mm
- weight 144N/m
- number of layers (maximum) 2
- number of dead coils 4

Balance rope
- weight 144N/m

Conveyance
- weight 113kN
- speed 12.2m/s
- acceleration and retardation 0.61m/s²

Drive
- type electric DC motor
- power 1600kW
- weight of armature 176kN

The actual drum, also referred to in this work as the prototype or full size drum, has been studied through a scale model described in chapter 3 but an estimation of the stresses induced in it is presented in the next section to illustrate the effect of each type of load described in section 1.4.

2.4 Approximate Stresses on the Actual Drum

The relevant loads acting on a winding drum were described in section 1.4 where it was stated that the rope compression is predominant. In order to demonstrate this, approximate calculations of the stresses induced by some of the loads on the prototype drum are presented. In order to use
standard formulae and methods available in the literature the following simplifications were made:

i- The brake rings, rope storage reel and motor armature were omitted.

ii- The stiffened roll was substituted by an equivalent plain roll whose thickness \( t_e \) was given by

\[
t_e = \text{roll thickness} + \left( \frac{\text{area of the rings}}{\text{roll length}} \right)
\]

\[
t_e = 70 + \left( \frac{95000}{3150} \right) = 100 \text{mm}
\]

iii- The drum structure was considered as having the configuration represented in FIG. 2.4 where the shaft was simply supported, the diameter of the side plates was the same as the roll and the shaft flanges were considered to be perfectly rigid hubs that follow the shaft without themselves deforming. The drum was also considered to be homogeneous, without a diametral split.

iv- The stresses were calculated for each load acting separately.

In most cases the only stresses considered for the drum roll were the hoop and axial stresses \( \sigma_1 \) and \( \sigma_2 \) respectively, as shown in FIG. 2.5a and for the drum sides the radial and tangential stresses \( \sigma_3 \) and \( \sigma_4 \) respectively, as shown in FIG. 2.5b. In few cases the shear stresses \( \tau_r \) on the roll and \( \tau_p \) on the side plates have also been considered.

2.4.1 Self-weight

Only the effect of the roll self-weight on the roll itself is considered here. From PFLÜGER\(^2\), the maxima stresses at the midspan of the drum roll are given by

\[
\sigma_{11} = -pR
\]

\[
\sigma_{21} = \frac{\pm pL^2}{4R}
\]

where

\[L - \text{roll length}\]

\[\rho - \text{specific weight of the drum material, } 7.65 \times 10^{-5} \text{N/mm}^3\]

The values of the stresses are
FIG. 24 SIMPLIFIED DRUM CONFIGURATION
The additional subscript in the notation for stress is to distinguish the various stresses that are to be calculated in this section. A negative sign indicates a compressive stress while a positive a tensile stress.

The above expressions for the stresses can be arrived at if the roll is considered as a beam simply supported at the edges and under its self weight.

2.4.2 Rope pull

In considering this load the pull on both rope ends was assumed to be the same and with the conveyance fully loaded. The tension $T$ in the rope was then

$$T = \text{payload} + \text{weight of conveyance} + \text{weight of rope}$$

$$T = 78,000 + 113,000 + 615 \times 144 = 279,560 \text{N}$$

To account for the bending imposed by the rope pull on the roll it has been assumed that it acts as a beam simply supported at the edges with a load of $2xT$ applied at its midspan. The maximum axial stress $\sigma_{22}$ occurs at the centre and is given by

$$\sigma_{22} = \frac{2TL}{4 \pi R^2 t_e}$$

which is the expression also used by WATERS$^{12}$. Hence

$$\sigma_{22} = \pm 0.15 \text{N/mm}^2$$

Another effect of the rope pull on the roll is to apply torsion since the pull acts tangentially to it. The shear stress $\tau_{r1}$ induced by this action is given by

$$\tau_{r1} = \frac{TR}{2 \pi R^2 t_e}$$
which after substitution gives
\[ \tau_{r1} = 0.14 \text{N/mm}^2 \]

### 2.4.3 Rope compression

For the determination of the stresses induced by this load the pressure \( p \) applied by the rope was obtained from Eq. 2.1 where
\[ p = \frac{F}{R_d} = 1.80 \text{N/mm}^2 \]

The length of 3150\( \text{mm} \) of the drum roll is greater than the value given by the expression \( 5\sqrt{R_e} = 2761 \text{mm} \) (see Eq. 2.3). Assuming the rope pressure \( p \) to be uniformly distributed along the drum and neglecting the rope relaxation, the hoop stress \( \sigma_{12} \) at the middle of the roll is approximately
\[ \sigma_{12} = \frac{pR}{t_e} \]
\[ \sigma_{12} = -54.9 \text{N/mm}^2 \]

The side plates restrain the drum roll radially developing axial stresses along it but, before considering this effect it is necessary to establish the interaction between the plates and the roll. In order to do so the plates are assumed to be rigidly connected to the roll and the joint allowed only to rotate without displacement as shown in FIG. 2.6. Because of the rigidity at the joint, the slopes \( \theta_r \) and \( \theta_p \) of the roll and the plate res-
pectively will be the same. The action of the rope pressure creates a moment $M$ at the joint which is equally experienced by the roll and the side plate. Thus, the configuration in FIG. 2.6 can be represented as in FIG. 2.7 where the moment is shown applied at the simple supports.

![Diagram](image)

**FIG. 2.7 EQUIVALENT ROLL SIDE JOINT**

It has already been shown that the drum roll is long enough, hence the bending at the edges does not interact with each other. For this case the slope $\theta_r$ and the moment $M$ on the roll are related by the following expression (TIMOSHENKO\textsuperscript{16}, ch.15)

$$\theta_r = -\frac{M}{2\beta_e D_e} + \frac{pR^2\beta_e}{Et_e}$$

where

$$\beta_e = \frac{4}{3(1-\nu^2)} \frac{R^2t_e^2}{Et_e}$$

$$D_e = \frac{Et_e^3}{12(1-\nu^2)}$$

$E$ - elasticity modulus
\v - Poisson ratio

From ROARK\textsuperscript{22}, table 24, case 5d, the relationship between $\theta_p$ and $M$ for the side plate is
\[ \theta_p = -0.232 \frac{RM}{D_p} \]  

where \[ D_p = \frac{Et^3}{12(1-\nu^2)} \]

\[ t_p \text{ - side plate thickness} \]

Considering that \( \theta_r = -\theta_p \) because of the different sign convention used in each expression, the solution of Eqs. 2.4 and 2.5 gives

\[ \theta_0 = -\theta_p = 1.75 \times 10^{-3} \]

\[ M = 12875 \text{N/mm/mm} \]

The axial stress \( \sigma_{23} \) induced by the moment \( M \) on the roll at the joint is given by\(^2^2\)

\[ \sigma_{23} = \pm \frac{6M}{t_e} \]

\[ \sigma_{23} = \pm 7.67 \text{N/mm}^2 \]

Stresses on the side plates can be calculated knowing the values of \( \theta \) and \( M \). The radial stress \( \sigma_{31} \) induced by the moment \( M \) on the side at the joint with the roll is given by\(^2^2\)

\[ \sigma_{31} = \pm \frac{6M}{t_p} \]

\[ \sigma_{31} = \pm 18.16 \text{N/mm}^2 \]

The tangential stress \( \sigma_{41} \) at the joint is given by\(^2^2\)

\[ \sigma_{41} = \pm \frac{6}{t_p} \left( D_p (1-\nu^2) \theta_p + \nu M \right) \]

\[ \sigma_{41} = \pm 1.60 \text{N/mm}^2 \]
At the hub the moment $M$ induces a radial stress $\sigma_{32}$ on the side plates given by\textsuperscript{22}

$$\sigma_{32} = \frac{6}{t_p^2} \frac{1.29M}{\sqrt{r}}$$

$$\sigma_{32} = \pm 23.43 \text{N/mm}^2$$

The tangential stress $\sigma_{42}$ at the hub is given by\textsuperscript{22}

$$\sigma_{42} = \nu \sigma_{32}$$

$$\sigma_{42} = \pm 7.03 \text{N/mm}^2$$

Another effect of the rope pressure on the side plates is the compression transmitted by the roll edges. In order to evaluate this it is considered that the side plate supports the roll without deflecting radially and the force $Fr$ applied by the roll on the plate is equal to the reaction of the support with the sign changed. Such force is given by\textsuperscript{16}

$$Fr = -2\beta e_D e_r - 2\beta e M$$

$$Fr = -416.94 \text{N/mm}$$

In order to determine the stresses induced by the force $Fr$ on the side plates they have been assumed to be disks under external pressure with the inner edges prevented from being displaced radially by perfectly rigid hubs. By combining cases la and lc from table 32 of ROARK\textsuperscript{22} the following expressions for the radial stress $\sigma_3$ and the tangential stress $\sigma_4$ at a radius $r$ of the plate have been developed.

$$\sigma_3 = \left[ \frac{R^2 + Kr_i^2 - (1+K)R^2r_i^2}{r^2} \right] \frac{Fr}{t_p(R^2-r_i^2)}$$

$$\sigma_4 = \left[ \frac{R^2 + Kr_i^2 + (1+K)R^2r_i^2}{r^2} \right] \frac{Fr}{t_p(R^2-r_i^2)}$$
where

\[ K = -\frac{2R^2}{(R^2 + r_i^2) + \nu(R^2 - r_i^2)} \]

\[ r_i \] - radius of the hub

At the joint, \( r = R \) and the radial and tangential stresses \( \sigma_{33} \) and \( \sigma_{43} \), respectively, are

\[ \sigma_{33} = -6.10 \text{N/mm}^2 \]
\[ \sigma_{43} = -6.41 \text{N/mm}^2 \]

At the hub, \( r = r_i \) and the radial and tangential stresses \( \sigma_{34} \) and \( \sigma_{44} \), respectively, are

\[ \sigma_{34} = -2.90 \text{N/mm}^2 \]
\[ \sigma_{44} = -9.61 \text{N/mm}^2 \]

2.4.4 Shaft bending

The bending of the shaft is due to the rope pull plus the drum and shaft self-weights. In order to simplify the calculation of the stresses the shaft-weight has been neglected and the other loads assumed to be equally transmitted to the shaft by the side plates in a loading configuration as shown in FIG 2.8. The force \( F_t \) was determined as the resultant

FIG. 2.8 SHAFT LOADING
of the drum self-weight and the rope pull as shown in FIG. 2.9.

When the shaft bends, the hubs connecting it to the side plates suffer an angular displacement which is transmitted to the side plates as shown in FIG. 2.10. If this displacement is known, the stresses on the side plates can be calculated. Assuming that each hub is totally rigid and also that the side plates add no stiffness to the shaft, the angular displacement transmitted to the side is equal to the slope $\psi$ of the shaft at the hub.
hub. This slope is given by (see ROARK$^{22}$, table 3)

$$\psi = \frac{F_t(a \times l - 2a^2)}{4EI}$$

where

- $l$ - length of the shaft
- $a$ - distance of load to support
- $I$ - moment of area of the shaft

Hence,

$$\psi = 5.00 \times 10^{-4}$$

Assuming that the loading configuration for the side plate can be represented as in FIG. 2.10, where, for simplification, the side is assumed to be built into a rigid roll, the maximum radial stress $\sigma_{35}$, induced by $\psi$ is given by (ROARK$^{22}$, table 24, case 21)

$$\sigma_{35} = \frac{\beta \varepsilon t_p}{a R}$$

where $\alpha$ and $\beta$ are constants whose values depend on the ratio $r_1/R$ and which are approximately 0.595 and 4.41 in this case. Hence,

$$\sigma_{35} = \pm 16.27 \text{N/mm}^2$$

2.4.5 Driving torque

The maximum torque in normal operation is applied at the beginning of the wind when the drum is accelerated. Ignoring friction, this torque $T_{0n}$ is given by (BROUGHTON$^1$)

$$T_{0n} = \left[ I_d + (2W_c + P + 2W_R L_w R^2 \frac{R^2}{g}) \frac{a}{R} \right] + PR$$

where

- $I_d$ - inertia of the drum
- $W_c$ - weight of the conveyance
- $P$ - payload
\[ w_R - \text{weight of the rope per unit of length} \]
\[ L_w - \text{depth of the wind} \]
\[ g - \text{acceleration of gravity} \]
\[ a - \text{acceleration of the conveyance} \]

Hence,
\[ T_{0n} = 483646\text{Nm} \]

The total torque is transmitted through the shaft to the side plates where the maximum stress is induced at the joint with the hub. Assuming that the worst case is when only one side takes the whole torque, the shear stress \( \tau_{p1} \) at the hub is

\[ \tau_{p1} = \frac{T_{0n}}{2\pi r_1^2 t_p} \]
\[ \tau_{p1} = 2.80\text{N/mm}^2 \]

The torque is transmitted to the drum roll where it induces a stress \( \tau_{r2} \) given by

\[ \tau_{r2} = \frac{T_{0n}}{2\pi R^2 t_e} \]
\[ \tau_{r2} = 0.08\text{N/mm}^2 \]

2.4.6 Application of brakes

Only the stresses induced by the braking torque will be considered. Such torque is maximum in an emergency and is given by

\[ T_{0b} = \left[ I_d + (2W_c + P + 2w_R L_w) \frac{R^2}{g} \right] B + PR \]

where \( b \) is the braking deceleration taken as 4.9m/s\(^2\). Hence,
\[ T_{0b} = 2211925\text{Nm} \]
If the braking is done by the brakes only, the torque applied will induce a stress $\tau_{r3}$ on the drum roll given by

$$\tau_{r3} = \frac{TO_b}{2nR^2t_e}$$

$\tau_{r3} = 0.36 \text{N/mm}^2$

If the braking is done by the motor, the highest stress will be $\tau_{p2}$ on the side plates at the hub and is given by

$$\tau_{p2} = \frac{TO_b}{2n^2r^2t_p}$$

$\tau_{p2} = 12.80 \text{N/mm}^2$

### 2.4.7 Drum movement

The hoop stress $\sigma_{13}$ caused by the centrifugal force at the middle of the drum roll is given by

$$\sigma_{13} = \mu R^2 \omega^2$$

where

$\mu$ - specific mass of the drum material, $7.80 \times 10^{-6} \text{kg/mm}^3$

$\omega$ - drum angular speed, $4 \text{rad/s}$

Hence,

$$\sigma_{13} = 1.16 \text{N/mm}^2$$

The stresses on the side plates may be evaluated by assuming them to be built into the hub, where the radial displacement is zero, and free at the periphery. Considering these assumptions and the dimensions of the plate, the radial and tangential stresses $\sigma_3$ and $\sigma_4$ at a radius $r$ are, by reference to BUDYNAS, given by

$$\sigma_3 = \frac{(37473 + \frac{8.368 \times 10^9}{r^2} - \frac{3tyr^2}{800})\mu \omega^2}{r^2}$$
\[
\sigma_4 = (37473 - \frac{8.368 \times 10^9}{r^2} - \frac{1+3v_r^2}{800})u_0^2
\]

At the hub \(r=r_1\) and the radial and tangential stresses \(\sigma_3\) and \(\sigma_4\), respectively, are

\[
\sigma_3 = 6.93\text{N/mm}^2 \\
\sigma_4 = 2.08\text{N/mm}^2
\]

Other effects of the drum movement are the variation caused by the acceleration or retardation of the conveyances. During normal operation these are, by design, 6% of the gravitational pull and the additional loads they induce on the drum can be neglected in the presence of other loads. During emergency braking at a retardation rate of 4.9m/s\(^2\), the suspended loads would be increased by 50%. However, the rope pressure, which induces the highest stresses in the drum, would not be affected significantly. Indeed, if the drum is braked to a halt when the conveyances are at the maximum speed of 12.2m/s, the time required to stop will be approximately 2.5s and the distance travelled by the conveyance approximately 15m which corresponds to 0.8 coils of rope. The stresses due to the rope compression, as calculated in section 2.4.3, were caused by a pressure of approximately 62 coils, so that one coil having a tension 50% greater will cause little variation in the total stress.

The increase in the suspended loads will have more effect on the stresses induced by the shaft bending which will increase by about 10%.

2.4.8 Summary

The various stresses calculated in the foregoing may be summarized as follows:

**Self-weight**

roll - at the middle  \(\sigma_{11} = 0.24\) \(\text{N/mm}^2\)
Rope pull

roll - at the middle \[ \sigma_{21} = \pm 6.22 \times 10^{-2} \text{ N/mm}^2 \]
\[ \tau_{r1} = 0.14 \text{ N/mm}^2 \]

Rope compression

roll - at the middle \[ \sigma_{12} = -54.90 \text{ N/mm}^2 \]
roll - at the edge \[ \sigma_{23} = \pm 7.67 \text{ N/mm}^2 \]
side - at the periphery (due to bending) \[ \sigma_{31} = \pm 18.16 \text{ N/mm}^2 \]
\[ \sigma_{41} = \pm 1.60 \text{ N/mm}^2 \]
side - at the hub (due to bending) \[ \sigma_{32} = \pm 24.43 \text{ N/mm}^2 \]
\[ \sigma_{42} = \pm 7.03 \text{ N/mm}^2 \]
side - at the periphery (due to compression) \[ \sigma_{33} = -6.10 \text{ N/mm}^2 \]
\[ \sigma_{43} = -6.41 \text{ N/mm}^2 \]
side - at the hub (due to compression) \[ \sigma_{34} = -2.90 \text{ N/mm}^2 \]
\[ \sigma_{44} = -9.61 \text{ N/mm}^2 \]

Shaft bending

side - at the hub \[ \sigma_{35} = \pm 16.27 \text{ N/mm}^2 \]

Driving torque

roll \[ \tau_{r2} = 0.08 \text{ N/mm}^2 \]
side - at the hub \[ \tau_{p1} = 2.80 \text{ N/mm}^2 \]

Application of brakes (emergency)

roll \[ \tau_{r3} = 0.36 \text{ N/mm}^2 \]
side - at the hub \[ \tau_{p2} = 12.80 \text{ N/mm}^2 \]

Drum movement

roll - at the middle \[ \sigma_{13} = 1.16 \text{ N/mm}^2 \]
side - at the hub \[ \sigma_{36} = 6.93 \text{ N/mm}^2 \]
\[ \sigma_{45} = 2.08 \text{ N/mm}^2 \]

It is evident from the foregoing approximate calculations that the highest stresses which occur in the drum roll are due to the rope compression. These stresses vary during the winding cycle and in practice, in the drum considered, they will be even greater due to the fact that two layers of rope can be wound on either half of the drum.

Other significant stresses occur in the side plates due both to the
rope compression and the bending of the shaft. These stresses not only vary during the winding cycle but those caused by the shaft bending change in direction for each revolution of the drum. It is towards all those stresses that this work is directed. Stresses due to self-weight, rope pull, driving torque, application of brakes and drum movement were not considered in the analytical models discussed in chapter 4.

For sufficiently long drums (see Eq. 2.3) without stiffening rings the methods used in the preceding calculations would give information precise enough for design purposes. For short drums or drums with stiffening rings the effects of the side plates and of the stiffeners on the deformation of the roll, only partially taken into account by assuming an equivalent thickness, could make the actual stresses on the roll significantly different from those calculated. Unless such effects are known or accounted for, the results obtained from the methods have to be treated with caution.
CHAPTER 3
THE SCALE MODEL

In order to study the mechanical behaviour of the actual drum presented in section 2.3 it was necessary to build a model as the prototype was not available for testing and also because it would be cheaper and more practical to do so. The type of model sought was such that a complete similarity with the prototype, concerning the phenomena in study, should be obtained. These phenomena were the stresses and deflections induced by non-dynamic forces and moments. In order to achieve the similarity, the relationship between the model and the prototype were analysed through the theory of models and dimensional analysis.

3.1 Considerations on Scaling

In order to be similar to the prototype a model has to maintain the relationships among all the variables involved in the phenomena. It is necessary therefore to know which variables are involved and which transformations are required from the prototype to the model.

For the problem in question stresses $\sigma$ and deflections $u$ can be generically described by the following functions:

$$\sigma = f_1(L, F, E, v, \mu, g)$$  \hspace{1cm} 3.1
$$u = f_2(L, F, E, v, \mu, g)$$  \hspace{1cm} 3.2

where the variables represent

$\text{L - length}$
$\text{F - force}$
$\text{E - modulus of elasticity}$
$\text{v - Poisson ratio}$
$\text{\mu - specific mass}$
$\text{g - gravitational pull}$

Eqs. 3.1 and 3.2 can be transformed and written as:
Provided that the functions $f_1$ and $f_2$ are dimensionally homogeneous they can be reduced to a relationship among a complete set of dimensionless products. This is Buckingham's theorem as given by Langhaar who also states that "a set of dimensionless products is complete if each product in the set is independent from the others, and every other dimensionless products of the variables is a product of the powers of dimensionless products in the set". Although complete sets of dimensionless products can be systematically calculated as shown by Langhaar, in the present case it was easier to determine them by inspection and the reduced functions written as

\[ \psi_1 \left( \frac{\sigma L^2}{F}, \frac{F}{E L^2}, \nu, \frac{g L^3}{F} \right) = 0 \]  
\[ \psi_2 \left( \frac{u LE}{F}, \frac{F}{E L^2}, \nu, \frac{g L^3}{F} \right) = 0 \]

Functions $\psi_1$ and $\psi_2$ describe the relationships among the variables involved in the phenomena in study and, if the model is to be similar to the prototype those relationships have to be maintained (refer to Buckingham). Considering that Eqs. 3.3 and 3.4 describe the prototype, a similar model is described by

\[ \psi_1 \left( \frac{\sigma_{m} L^2}{F_m}, \frac{F_m}{E_m L^2_m}, \nu_m, \frac{g_{m} L^3}{F_m} \right) = 0 \]  
\[ \psi_2 \left( \frac{u_m L E_m}{F_m}, \frac{F_m}{E_m L^2_m}, \nu_m, \frac{g_{m} L^3}{F_m} \right) = 0 \]

where the subscript m means a variable in the model. The functions $\psi_1$ and $\psi_2$, which are zero for the variables in the prototype, will also be zero for the variables in the model if the corresponding parameters in each function are the same. Then if

\[ \frac{\sigma_{m} L^2}{F_m} = \frac{\sigma L^2}{F} \]
the prototype and the model are similar. Defining the ratio between the variables in the model and the prototype as the scale factor of this variable and denoting it by \( S \) subscripted by the symbol of the respective variable, the last set of equations can be rewritten as

\[
\frac{\sigma_{m}}{E_{m}} = \frac{\sigma}{E}
\]

\[
\nu_{m} = \nu
\]

\[
\frac{g_{m} v_{m} L_{m}^{3}}{F_{m}} = \frac{g_{L} L_{m}^{3}}{F}
\]

\[
\frac{u_{m} L_{m} E_{m}}{F_{m}} = \frac{u L E}{F}
\]

The scale factors have to satisfy the above equations.

In defining the scales for the model of the actual winding drum the first aspect that had to be considered was the decision taken in the early stages of the project to manufacture the model in the sponsoring Company workshop. By so doing, costs would be minimized and also it would be possible to take full advantage of the Company's technical expertise and manu-
facturing facilities. It followed that the model should be made in the same materials as the prototype if the requirements for similarity permitted. Indeed, in problems of stress and strain where the properties of materials play an important role, this is a reasonable decision particularly when considering that tailoring material properties to satisfy modelling requirements can be a very complex, if not an impossible task. With the same materials, \( S_E = 1 \), \( S_v = 1 \), \( S_u = 1 \) and the equations relating the various factors are reduced to

\[
\begin{align*}
S_\sigma &= \frac{S_F}{S_L^2} \quad &3.7 \\
S_F &= S_L^2 \quad &3.8 \\
S_g &= \frac{S_L^3}{S_F} \quad &3.9 \\
S_u &= \frac{S_L}{S_F} \quad &3.10
\end{align*}
\]

According to Eqs. 3.8 and 3.9 the scale factor for the gravitational pull has to be the same as for the length. Although it can be admitted that scaling the gravitational pull is feasible, it was not viable for the problem in question and \( S_g \) had to be made equal to unity, implying that all the other scale factors would also have to be unity if Eqs. 3.7 to 3.10 were to be satisfied, meaning that no small scale model was possible. In order to alleviate this situation, the common procedure is to relax one or more of the requirements for complete similarity and to build a non similar, or distorted, model. If the departure from similarity is known and can be accounted for, the model can be used in almost the same way as if it was similar. In the present case the requirement relaxed was that given by Eq. 3.9, which was omitted, so reducing the conditions to be satisfied to

\[
\begin{align*}
S_\sigma &= 1 \\
S_F &= S_L^2 \\
S_u &= \frac{S_L}{S_F} = S_L
\end{align*}
\]
Remembering that the stresses and the displacements were the variables to be measured, the scale of the model would be totally defined by choosing either the scale factor for the forces or for the lengths. In this work the choice has been referred to the lengths.

The relaxation of the condition in Eq. 3.9 affects only the self-weight of the parts in the model drum which will be different by a factor of $S_L$ from a truly similar model. In section 2.4 it was seen that the effect of self-weight was only important when considering the stresses due to the shaft bending. The measurements of such stresses, obtained from the model, would have to be corrected for self-weight to give the results that would be obtained with a truly similar model.

3.2 Choice of the Scale

The main factor considered in the choice of the scale was that, in order to keep costs to a minimum, standard parts should be used in the model. Some materials and parts specified for the prototype drum, such as steel plates and bolts, were standard sizes and, being metric, scale factors such as 1/2, 1/2.5, 1/4, 1/5 or 1/10 would almost certainly yield sizes for which standard parts would be available. The scales of 1/2 and 1/2.5 were rejected because the model would be too large and expensive. In order to decide among the other scales, several aspects were considered as discussed in the following sections.

3.2.1 Availability of materials and parts

The parts considered were:

i- Steel plates; in order to get the right standard thickness to avoid unnecessary machining.

ii- Bolts; to avoid machining special sizes.

iii- Rope; to ensure a transmission of pressure similar to the prototype.
The plates specified for the prototype had thicknesses of 25, 40, 50, 65 and 70mm. The scaled thicknesses and their nearest standards are presented in Table 3.1 which shows that, with the scales of 1/5 or 1/10 a greater percentage of standard plates could be used.

Table 3.1
Scaled and standard plate thicknesses

<table>
<thead>
<tr>
<th>Scale</th>
<th>25</th>
<th>40</th>
<th>50</th>
<th>65</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4</td>
<td>6.25</td>
<td>10.0</td>
<td>12.5</td>
<td>16.25</td>
<td>17.5</td>
</tr>
<tr>
<td></td>
<td>(6.0)</td>
<td>(10.0)</td>
<td>(15.0)</td>
<td>(16.0)</td>
<td>(16.0)</td>
</tr>
<tr>
<td>1/5</td>
<td>5.0</td>
<td>6.0</td>
<td>10.0</td>
<td>13.0</td>
<td>14.0</td>
</tr>
<tr>
<td></td>
<td>(5.0)</td>
<td>(6.0)</td>
<td>(10.0)</td>
<td>(15.0)</td>
<td>(15.0)</td>
</tr>
<tr>
<td>1/10</td>
<td>2.5</td>
<td>4.0</td>
<td>5.0</td>
<td>6.5</td>
<td>7.0</td>
</tr>
<tr>
<td></td>
<td>(2.5)</td>
<td>(4.0)</td>
<td>(5.0)</td>
<td>(6.0)</td>
<td>(8.0)</td>
</tr>
</tbody>
</table>

The bolts used in the prototype were specified as M30, M36, M56 and M64. The scaled diameters are shown in Table 3.2 together with their nearest standards. It will be noticed that, for any chosen scale at least two scaled

Table 3.2
Scaled and standard bolt sizes

<table>
<thead>
<tr>
<th>Scale</th>
<th>M30</th>
<th>M36</th>
<th>M56</th>
<th>M64</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4</td>
<td>7.5</td>
<td>9.0</td>
<td>14.0</td>
<td>16.0</td>
</tr>
<tr>
<td></td>
<td>(7/8&quot;)</td>
<td>(3/8&quot;)</td>
<td>(7/16&quot;)</td>
<td>(9/16&quot;)</td>
</tr>
<tr>
<td>1/5</td>
<td>6.0</td>
<td>7.2</td>
<td>11.2</td>
<td>12.8</td>
</tr>
<tr>
<td></td>
<td>(5/16&quot;)</td>
<td>(7/16&quot;)</td>
<td>(1/2&quot;)</td>
<td>(3/8&quot;)</td>
</tr>
<tr>
<td>1/10</td>
<td>3.0</td>
<td>3.6</td>
<td>5.6</td>
<td>6.4</td>
</tr>
<tr>
<td></td>
<td>(3/16&quot;)</td>
<td>(5/32&quot;)</td>
<td>(1/4&quot;)</td>
<td>(1/4&quot;)</td>
</tr>
</tbody>
</table>
diameters could not be obtained as standards. It was then decided that the bolts were not a good factor to consider in the choice of the scale. In order to avoid the use of specially made bolts, they had to be selected from different types of standard bolts after deciding on the scale of the model. They were specified not only to approximate to the scaled diameters but also, to the scaled strength areas. Such selection is shown in Table 3.3 for the scale of 1/5 only.

Table 3.3
Selected bolts

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Scaled</th>
<th>Standard bolt selected</th>
</tr>
</thead>
<tbody>
<tr>
<td>type</td>
<td>(mm)</td>
<td>area (mm²)</td>
</tr>
<tr>
<td>M30</td>
<td>30.0</td>
<td>561.0</td>
</tr>
<tr>
<td>M36</td>
<td>36.0</td>
<td>817.0</td>
</tr>
<tr>
<td>M56</td>
<td>56.0</td>
<td>2030.0</td>
</tr>
<tr>
<td>M64</td>
<td>64.0</td>
<td>2680.0</td>
</tr>
</tbody>
</table>

The rope for the prototype was of the locked coil type with a diameter of 51mm, a metallic area of 1840mm² and a stretch modulus of 1.37x10⁵N/mm². The smallest rope obtainable of the same type, was a 16mm diameter corresponding to a scale factor of about 1/3. In order to specify an equivalent scaled rope of another type, not only the diameter but also the metallic area had to be considered. An approach was made to the Technical Services of British Ropes Limited were it was recommended that it is acceptable to consider ropes to be mechanically equivalent if the product of the stretch modulus and the metallic area is the same. Table 3.4 shows the values of the properties of the prototype rope and those corresponding to each scale. The values given in bracket are for a spiral strand, a type of rope suggested by British Ropes and whose behaviour would approximate the locked coil type.

Although for the scale 1/4 there was a very good equivalence, the discrepancies for the other scales were very small and, in practical terms, no preference could be established among the scale factors.
Table 3.4
Scaled ropes

<table>
<thead>
<tr>
<th>Scale</th>
<th>diam. (mm)</th>
<th>area, (A_R) (mm²)</th>
<th>mod., (E_R) (10⁵ N/mm²)</th>
<th>(E_R \times A_R) (10⁶ N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>prototype</td>
<td>51.0</td>
<td>1840.0</td>
<td>1.37</td>
<td>2520.1</td>
</tr>
<tr>
<td>1</td>
<td>12.75</td>
<td>115.0</td>
<td>1.37</td>
<td>157.5</td>
</tr>
<tr>
<td>4</td>
<td>(12.89)</td>
<td>(107.1)</td>
<td>(1.47)</td>
<td>(157.4)</td>
</tr>
<tr>
<td>5</td>
<td>(10.2)</td>
<td>(67.0)</td>
<td>(1.47)</td>
<td>(98.5)</td>
</tr>
<tr>
<td>10</td>
<td>(5.1)</td>
<td>(16.5)</td>
<td>(1.47)</td>
<td>(24.3)</td>
</tr>
</tbody>
</table>

3.2.2 Feasibility of manufacture

No significant difficulties were foreseen in the manufacturing processes or procedures of the model compared with the prototype. The shaft, which is forged on the prototype, would have to be build up for the model in the interest of minimizing costs, but with this modification no technical difficulties were expected in manufacturing the model drum in any of the scales. However, the drum had a large number of welded parts and since in most of the Company's products the minimum thickness is 8mm, the welding of thinner plates would require extra care. On these grounds the scales of 1/4 and 1/5 were preferred.

3.2.3 Feasibility of assembly

The Company was used to dealing with large and heavy drums so the model assembly would present no handling problem. The main point considered was that, to tighten most of the bolts in the model it would be necessary to have access to the inside. A small scale would make this difficult and again, the scales 1/4 and 1/5 were preferred.
3.2.4 Construction of the rig

The final scheme for the model rig, which was agreed with the Company in the early stages of the project, is shown in FIG. 3.1 where the drum is supported above the floor by a structure made of steel sections and the load simulated by weights in a carrier hung from the rope. In this scheme the loading is different from the prototype, the implications of which are discussed in section 3.4.6. From the point of view of the construction of the rig, the 1/10 scale was the best being the smallest and lightest.

3.2.5 Operation of the rig

In order to simulate the action of the rope, the drum had to be rotated so that the rope could be progressively wound under tension. The drum had to be rotated with a hand crank system and the smaller the drum the less mass to be moved. Again preference was given to the 1/10 scale.

3.2.6 Instrumentation of the rig

The model was to be instrumented with strain gauges, the majority being placed inside the drum roll between the stiffening rings. To accommodate the envisaged number, special gauges would have to be used if a small scale was chosen. The time and care required to fit them would contribute to raise the cost of instrumentation. The scales of 1/4 and 1/5 would permit the use of standard strain gauges of 5mm in length.

3.2.7 Manufacturing costs

The costs of manufacture were not quantified but, in a qualitative comparison, the Company decided that the scale of 1/10 was not desirable. Such a scale would make the drum too small compared with those normally made by the Company thus increasing the labour involvement. Considering the remaining scales of 1/4 and 1/5, the latter was preferred as the cost of materials and components would be less.
FIG. 3.1  MODEL RIG SCHEME

Drive mechanism
manual or
motorised

Model drum

Support structure

Load
After considering the aspects presented in the foregoing, the scale factor of 1/5 was chosen and hence, the relevant variables in the model would have the following relationships with those in the prototype:

i- Linear dimensions would be 1/5th.
ii- Forces would be 1/25th in magnitude.
iii- Self-weight would be 1/125th, i.e., 1/5th of that required for a similar model.
iv- Deflections would be 1/5th.
v- Stresses would be the same.

The relationship in iv and v would be valid only if the self-weight was not one of the forces inducing them. In that case, the deformations or stresses measured on the model would have to be corrected for the self-weight in order to obtain the correct correspondence with the prototype.

3.3 Model and Rig Design and Construction

The cross-section of the model drum is shown in FIG. 3.2. During design the main task was to scale the existing prototype. Some modifications in specifications, such as weld preparation and tolerancing, were necessary to make them realistic for manufacturing purposes. The brake rings on the prototype are cast in two halves and, the experience of the Company was that, in order to minimize distortion, the process of casting was expensive. Also, the process of casting each ring in one piece and then dividing into halves, could result in distortion even after heat treatment. It was then decided to cast the rings and leave as one piece.

In the prototype the shaft is a forging, the hubs being integral to it. For the model the shaft was machined from a bar and the hubs, machined from plates, fitted by shrinking onto it.

The prototype has a drive motor coupled to the drum shaft. On the model, this was simulated by a gear fixed to the model shaft at a position corresponding to the centre of gravity of the motor armature. This gear
was also used for manual rotation of the drum.

The prototype bearings are self-aligning journal bearings and the model was fitted with self-aligning roller bearings; their width being shorter than that defined by the scaling factors. To compensate, the model shaft diameter was slightly increased along the scaled length of the bearing in an attempt to reproduce the same stiffness as a scaled bearing.

The fabrication drawings for the model drum are shown in Appendix 1.

The rig structure to support the drum was designed by subcontract, the author's task being the preparation of its specification and the checking of its strength. The layout of the whole rig is shown in FIG. 3.3 and the actual rig is shown in FIG. 3.4.

3.4 Deviations of the Model

In addition to the deviations from the intended similarity already discussed for the model, others were necessary for technical and practical reasons. The relevant deviations and their implications are given in the following sections.

3.4.1 Shaft

The stresses which would be significantly affected by the shaft were those induced on the side plates when the shaft bends. The side plates were less stiff than the hubs and it was reasonable to assume that, as far as the sides were concerned, the action of the shrink fitted hubs would be approximately that of hubs integral with the shaft. Clearly, the state of stresses in the hubs and in the shaft, at the region of the fit, would be different from the prototype but, this was outside the scope of the investigation.
FIG. 3.3  MODEL RIG LAYOUT

1. drum
2. rope
3. pulley
4. weight carrier
5. weights
6. drum divider
7. driving gear
8. hand crank
9. instrumentation cable reel
10. instrumentation cable pulleys
FIG. 3.4 MODEL RIG
3.4.2 Bearings

The bearings in the model, although narrower than the scaled width, were also self-aligning and as such, were expected to behave as simple supports like those on the prototype. The increase in the model shaft diameter to reproduce the bending stiffness of the correctly scaled bearings, was a reasonable approximation and any differences introduced by it were neglected.

3.4.3 Brake rings

The use of brake rings as one piece castings was a major deviation because they were expected to increase the bending stiffness of the side plates to which they were to be bolted. Thus, the increase provided by the model rings would be different than if they were in halves. Simple analytical procedures to evaluate the two cases could not be found and, as tests were to be carried out with and without the brake rings fitted, it was decided to await actual results. Should the changes in the levels of stresses between the two conditions be significant, the rings would be split, corrected for any deformation and fitted to the drum.

3.4.4 Rope

The cross-sections of the ropes for the prototype and for the model are shown in FIG 3.5. It seems that because of its construction the locked coil type of rope, used in the prototype, has a proportionally larger area of con-

![FIG. 3.5 CROSS-SECTION OF THE ROPES (not to scale)](Image)
tact with the drum roll than the spiral strand used in the model. Consequently, the rope on the model would induce somewhat greater bearing stresses on the roll than if the proper type of rope was used. On the other hand, because of its construction, the spiral strand can "flatten" somewhat more than the locked coil thus increasing the area of contact with the model roll. After these considerations, it seemed reasonable to assume that for the model, the transmission of pressure by the spiral strand would be practically similar to that of a locked coil rope.

3.4.5 Loading

On the prototype drum the rope pull acts at an angle of $30^\circ$ to the horizontal. On the model however, it was more practical to apply this pull downwards together with the self-weight. A procedure was then devised to isolate the effects of both the self weight and the vertical pull when manipulating the results of the tests so, a correction for the load direction could be made if necessary. This procedure will be discussed in chapter 5.

3.4.6 Rope storage reel

The rope storage reels shown in FIG. 2.3 were omitted on the model. These elements were considered to contribute little to strength of the side plates and it was more economical not to model them.

Summarizing the foregoing, the only deviation which might prove to be significant was that due to the brake rings which could be modified if necessary. Otherwise, the model was a reasonable representation of the prototype in that the results obtained from tests could be extrapolated to predict its performance. The model could also be considered as a winding drum in its own right and, as such, any theoretical or empirical analysis developed for winding drums would apply.
3.5 Model Loading and Rig Operation

The model drum, as shown in FIG. 3.3, was loaded by the weights in the carrier. As the drum was rotated by the hand crank, the rope was wound on one side and wound off the other in a similar fashion to the normal operation of the prototype. The rope pull was equal at both sides and maintained constant. This situation corresponds to a winding system fitted with a balance rope hoisting conveyances equally loaded. The length of rope wound off one side was wound onto the other. The length of rope was sufficient to have both sides fully coiled with a single layer of 26 coils each or one side with 26 coils on the first layer and 21 coils on a second layer. This would leave the other side with 4 coils representing the dead coils in exactly the same arrangement of the prototype.

The rig operation consisted in rotating the drum with the hand crank in one direction or the other to wind the desired number of coils on each side. With the rope tensioned, the coils formed neatly on the drum grooves or on top of the first layer without the need for guidance. When the rope was not tensioned guidance was necessary. The second layer would start forming by itself when the rope reached the drum divider where it would climb on the last coil of the first layer. The drum could be left stationary in any position by holding the driving gear with a plunger mechanism. The rope ends were clamped inside the drum where they were introduced through holes on the drum roll edges.

The weights simulating the loads were slotted steel disks which could slide on or off the carrier to make up the required load. Disks weighing 720 and 360N were used and were handled manually.

The weight carrier could at any time be lifted by a pair of hand-chain hoists to release the tension on the rope. The drum could be freely rotated when the carrier was in this position allowing the rope to be wound on or off without tension. This procedure was used during tests to return the drum to an unloaded condition after being loaded.
3.6 Instrumentation of the Model

The stresses and deflections induced by the different loads on the model were assessed from the strains measured with 61 single, electrical resistance, precision foil, strain gauges manufactured by Micro Measurements Limited. They were type EA-06-250BG-350 with a resistance of 350Ω and temperature self-compensating for steel. The layout of the gauges on the drum is shown and discussed in chapter 5.

Each strain gauge was connected to an arm of a Wheatstone bridge from which the variations of the strains could be measured as variations in voltage. The fundamental gauge circuit is analysed in Appendix 2 where it is shown that the variation in voltage measured across the Wheatstone bridge is directly proportional to the variation in strain on the gauge. The scheme for the gauge circuits used in the model is shown in FIG. 3.6. Each gauge was connected to a precision resistor with the same resistance forming half of a Wheatstone bridge. A potentiometer across them provided an adjustment to set the voltage across the bridge to an adequate value to be used as reference. The other half of the bridge was common to all gauges. For convenience and simplicity, each gauge was connected to its half of the bridge by two wires rather than three as is common practice in strain gauging. This type of connection is sensitive to temperature changes introducing a spurious component in the measured strain variation. This component could be significant at low levels of strain so a correction was applied to all results from the tests to compensate for this fact. The effect of temperature changes on the measured strains and how it was corrected is presented in Appendix 2. The temperature was monitored by means of a thermometer hanging from the rig structure near the drum.

To allow for eventual measurements of strains with the drum rotating, the signals from the gauges were transmitted by a cable brought through the drum shaft, coiled on a reel shown in FIG. 3.3, passed around a system of pulleys and fixed at the end. The system of pulleys was to maintain the cable in a stretched condition when wound off the reel in order to avoid tangling. The cable could only transmit signals from up to 30 gauges at once, so that the gauges were grouped into two sets, each connected to a different
FIG. 3.6 GAUGES CIRCUIT SCHEME
circuit board fixed to one of the drum sides, as shown in FIG. 3.7, where a connector at the end of the cable could be alternatively plugged in. The other cable end was fitted to a box shown in FIG. 3.8, containing the common half of the Wheatstone bridge, the connector for the power supply, a resistor to calibrate the factor of proportionality between voltage and strain (this calibration is explained in Appendix 2) and another connector to which a shorter length of cable could be connected. The circuit diagram of the components in box is shown in FIG 3.6. At the other end of the short cable was an output board with pins corresponding to each gauge on the set to which the main cable was connected and a pin corresponding to the common half of the bridge. To know the voltage across any gauge bridge, it was only necessary to measure at the output board the voltage between the common pin and the pin corresponding to the gauge.

The main operation with the instrumentation basically consisted in plugging the main cable into the board of the chosen set of gauges and then measuring, at the output board with a voltmeter, the voltage across the bridge of any selected gauge. In order to obtain a strain measurement, a reference voltage had to be obtained and recorded, the drum loading conditions changed and the voltage read again. The voltage variation multiplied by the factor of proportionality, would give the variation in strain between the two loading conditions. During the tests, the voltages were initially measured with a FLUKE 8050A digital multimeter shown in FIG. 3.8, and their values manually recorded. Later on, they were measured with a SOLARTRON 3430BD data logger which printed the values on a strip of paper.

The results of the tests performed with the model are discussed in chapter 5.
FIG. 3.8 INSTRUMENTS

- Voltmeter
- Main cable box
- Power supply
- Output board
CHAPTER 4
THE THEORETICAL MODELS

Winding drums are basically cylindrical shells supported at the edges by circular plates centrally attached to a shaft. In this chapter, theories for the shell and the plates are firstly considered separately and then combined to represent a winding drum as a whole structure. Subsequently, the effect of the shaft on the side plates is also considered. Later in the chapter, a theoretical representation of the scale drum is presented.

The purpose of developing the theories in full, is to provide winding drum designers with a specific reference giving the complete theoretical basis for the analytical procedures proposed.

4.1 Drum Roll

In the development of the theoretical model, the drum roll is taken initially as a plain hollow cylinder and the effects of the action of external forces discussed. Later on, a technique for the inclusion of stiffening rings is presented. First, the roll is taken as the cylinder shown in FIG. 4.1.

Any point on the cylinder can be defined by a system of coordinates \( x \), \( \phi \) and \( z \) where

- \( x \) - is the distance to a reference plane normal to the cylinder axis.
- \( \phi \) - is the angle of the arc on the cylinder circumference from the point to a reference plane containing the cylinder axis.
- \( z \) - is the distance from the point to a reference plane normal to a radius of the cylinder.

Consider an element of the cylinder, isolated as if it were a free body, with the forces and moments acting on it as shown in FIGs. 4.2 and 4.3, respectively, and where the symbols represent
$$p_x, p_{\phi}, p_z$$ - components of the external loads per unit of area in the directions $x$, $\phi$ and $z$ respectively.

$$N_x, N_{\phi}$$ - normal forces along the directions $x$ and $\phi$.

$$N_x \phi$$ - shearing force acting on a surface normal to direction $x$ and along direction $\phi$.

$$N_{\phi} x$$ - shearing force acting on a surface normal to direction $\phi$ and along direction $x$.

$$Q_x, Q_{\phi}$$ - shearing forces acting on surfaces normal to directions $x$ and $\phi$ and along direction $z$.

$$M_x$$ - bending moment on a section normal to direction $x$.

$$M_{\phi}$$ - bending moment on a section normal to direction $\phi$.

$$M_{x \phi}$$ - twisting moment acting on a surface normal to direction $x$ and in the $\phi$ direction.

$$M_{x \phi}$$ - twisting moment acting on a surface normal to direction $\phi$ and in the $x$ direction.

All the forces and moments are per unit of length.
FIG. 4.2. FORCES ON A CYLINDER ELEMENT

FIG. 4.3. MOMENTS ON A CYLINDER ELEMENT
Some simplifications can be made in this system of moments and forces if it is assumed first, that the load on the roll is a uniformly distributed pressure $p$ around and along it and secondly, that the roll itself is a uniform cylinder. The former assumption is not strictly true as the pressure applied by the rope varies as the wind proceeds, as described in section 2.1. The latter assumption is only strictly true for drums without a split at the centre line. However, for drum rolls whose edges at the split are butted and securely fastened together, this assumption is reasonable. Both assumptions allow great simplification in the analysis of the problem.

Then, due to the symmetry of the load $N_{x}$, $N_{x}$, $Q_{x}$, $M_{x}$ and $M_{x}$ vanish and $N$ and $M$ are constants. The simplified system of forces and moments thus becomes as shown in FIG. 4.4.

In order to establish relationships for the solution of the problem, the equations for the equilibrium of the element are considered first.

4.1.1 Equations of equilibrium

The equations of equilibrium are obtained from FIG. 4.4 by projecting the forces onto the axes $x$, $\phi$ and $z$ and taking moments relative to these axes. A moment is taken as positive if it acts clockwise to an observer placed at infinity and otherwise negative. The equations are:

Forces on $x$

$$-N_{x}R_{x} \phi + (N_{x} + \frac{3N_{x}}{3x} dx)R_{x} \phi = 0$$

$$\frac{3N_{x}}{3x} dx = 0$$

$$\therefore N_{x} = \text{constant}$$

4.1

Forces on $\phi$

$$-N_{\phi}cos(d\phi/2)dx + N_{\phi}cos(d\phi/2)dx = 0$$
FIG. 4.4 ELEMENT OF CYLINDER UNDER SYMMETRIC LOAD

\[ M_x + \frac{\partial M_x}{\partial x} dx \]

\[ N_x + \frac{\partial N_x}{\partial x} dx \]

\[ N_{\phi} \]

\[ M_{\phi} \]

\[ \phi \]

\[ x \]

\[ z \]
Forces on z

\[-N_\phi \sin(d\phi/2)\,dx - N_\phi \sin(d\phi/2)\,dx + Q_x R\,d\phi - \left(Q_x + \frac{\partial Q_x}{\partial x}\right)R \, d\phi - pR \, d\phi \, dx = 0\]

If $d\phi$ is very small, $\sin(d\phi/2) = d\phi/2$ and hence

\[-N_\phi - R\frac{\partial Q_x}{\partial x} - pR = 0 \quad 4.2\]

Moments relative to $x$

\[-M_\phi \, dx + M_\phi \, dx - N_\phi \sin(d\phi/2)R \, dx\,d\phi/2 + N_\phi \sin(d\phi/2)R \, dx\,d\phi/2 = 0\]

Moments relative to $\phi$

\[M_x R\,d\phi - (M_x + \frac{\partial M_x}{\partial x} \, dx)R \, d\phi + Q_x R \, dx \, d\phi/2 + (Q_x + \frac{\partial Q_x}{\partial x})R \, dx \, d\phi/2 = 0\]

\[\therefore - \frac{\partial M_x}{\partial x} R + Q_x R + \frac{\partial Q_x}{\partial x} R \, dx/2 = 0\]

Neglecting the second order term

\[Q_x = \frac{\partial M_x}{\partial x} \quad 4.3\]

As there are no components of moments relative to $z$, the equations of equilibrium are reduced to Eqs. 4.1, 4.2 and 4.3. Since there are three equations and four unknowns, $N_x$, $N_\phi$, $Q_x$ and $M_x$, the problem is indeterminate and hence is necessary to consider the deformations of the element.

**4.1.2 Deformations**

The discussion of deformations is based on FLÜGGE\(^26\) and it starts by defining as "middle surface" the surface in the cylinder whose radius is the mean between the outer and the inner radii of the cylinder.
Consider an arbitrary point A of the cylinder (refer to FIGs. 4.5 and 4.6) with coordinates $x$, $\phi$ and $z$, where $z$ is measured from the middle surface, being positive outwards. The displacement of point A can be described by its components in the directions $x$, $\phi$ and $z$ using the following notation:

- $u_A$ - displacement along $x$
- $v_A$ - displacement along $\phi$
- $w_A$ - displacement along $z$ or radial displacement

The difficulties encountered in determining $u_A$, $v_A$ and $w_A$ in the three dimensional state of stress to which the cylinder element is subjected, can be greatly reduced by making certain assumptions and approximations. The first of these is that, straight lines normal to the middle surface remain straight, normal to it and do not change their length after the cylinder deformation. This is a fundamental assumption, known as Love-Kirchhoff approximation, used in the development of classical theories for beams, plates and shells. DONNEL\textsuperscript{27} discusses these theories and the implications of the approximation. This assumption means that, if the initial and final positions of points on the middle surface are known, the position of points laying on straight lines passing through the points on the middle surface will also be known. Consequently, the deformation of any point in the cylinder is known. Thus, a three dimensional problem is reduced to two dimensions.

The implication of the assumption is that, the effects on deformation due to stresses in the $z$ direction are neglected.

A second assumption is that, the normal stresses $\sigma_z$ in the direction $z$ can be neglected when compared with the normal stresses $\sigma_x$ and $\sigma_\phi$ in the directions $x$ and $\phi$, and the cylinder element considered to be in a state of plane stress.

Both assumptions introduce negligible errors if the cylinder shell is thin. Comparing the shell theory developed with these assumptions with more exact theories, DONNEL\textsuperscript{27} shows that, for a cylinder whose thickness is
10% of its radius, the error is about 5%. In practical winding drums the thicknesses are normally less than 3% of the radii so that, the errors due to the assumption are acceptable.

A further assumption, due to LOVE\textsuperscript{28}, is that, all displacements can be neglected in comparison with the cylinder radius \( R \) and also their first derivatives neglected in comparison with unity. With this assumption the equations of the problem will be linear and the theories developed with them are known as linear or first order theories.

Using the three foregoing assumptions, the relationships between the displacements \( u_A, v_A \) and \( w_A \) of an arbitrary point \( A \) and the displacements \( u, v \) and \( w \) of the point \( A_0 \) on the middle surface which lies on the normal from \( A \) can be established. Those displacements are shown in FIGs. 4.5 and 4.6 where \( A' \) and \( A'_0 \) are the positions of \( A \) and \( A_0 \) after the deformation.

FIG. 4.5a shows the displacements on a section transverse to the cylinder generator for a cylinder under arbitrary load. The assumption of a uniform axisymmetric load implies that displacements on the transverse sections are also axisymmetric. Hence, \( v_A \) and \( v \) have to be zero and the displacements can be represented as in FIG 4.5b from where it will be seen that

\[
\omega_A = w
\]

From FIG 4.6

\[
u_A = u - z \sin \omega A
\]

or

\[
u_A = u - \frac{2 \omega}{\omega_A}
\]

The normal strains \( \epsilon_x, \epsilon_\phi \) and the shear strain \( \gamma_{x\phi} \) at point \( A \) can be established as functions of the displacements given by Eqs. 4.4 and 4.5. In order to do so, it is necessary to consider the cylinder element shown in FIG 4.7a where an element under arbitrary load is represented. As before, because of symmetry, \( v_A \) vanishes and also the displacements are independent of \( \phi \) so, the deformations can be represented more simply as in
FIG. 4.5 DISPLACEMENTS ON A SECTION TRANSVERSE TO THE CYLINDER GENERATOR

FIG. 4.6 DISPLACEMENTS ON A SECTION ALONG THE CYLINDER GENERATOR
FIG. 4.7  DEFORMATIONS ON A CYLINDER ELEMENT
FIG. 4.7b. It will be seen that the strain $\varepsilon_x$ is given by

$$
\varepsilon_x = \left( u_A + \frac{\partial u_A}{\partial x} dx - u_A \right) = \frac{\partial u_A}{\partial x} \tag{4.6}
$$

The strain $\varepsilon_\phi$ is arrived at by noting in FIG. 4.7c that, the original length of the element $(R + z)d\phi$ changes to $(R + z + w_A)d\phi$ after the deformation. The strain is then

$$
\varepsilon_\phi = \frac{(R + z + w_A)d\phi - (R + z)d\phi}{(R + z)d\phi} = \frac{w_A}{R + z} \tag{4.7}
$$

The shear strain, given by the angular displacements of the element sides, is zero in this case.

By combining Eqs. 4.4 to 4.7, the normal strains $\varepsilon_x$ and $\varepsilon_\phi$ can be expressed as functions of the displacements of point $A_0$ on the middle surface. Then

$$
\varepsilon_x = \frac{3u}{3x} - \frac{2w}{3x^2} \tag{4.8}
$$

$$
\varepsilon_\phi = \frac{w}{R + z} \tag{4.9}
$$

The normal stresses $\sigma_x$ and $\sigma_\phi$, acting on the cylinder element, can be related to the strains $\varepsilon_x$ and $\varepsilon_\phi$ by Hooke's law. Then

$$
\sigma_x = \frac{E}{(1 - \nu^2)}(\varepsilon_x + \nu \varepsilon_\phi) \tag{4.10}
$$

$$
\sigma_\phi = \frac{E}{(1 - \nu^2)}(\varepsilon_\phi + \nu \varepsilon_x) \tag{4.11}
$$

Combining the expressions above with Eqs. 4.8 and 4.9 gives

$$
\sigma_x = \frac{E}{(1 - \nu^2)}\left( \frac{3u}{3x} - \frac{2w}{3x^2} + \frac{vw}{R + z} \right) \tag{4.10}
$$

$$
\sigma_\phi = \frac{E}{(1 - \nu^2)}\left( \frac{w}{R + z} + \nu \frac{\partial u}{\partial x} - \nu^2 \frac{\partial w}{\partial x^2} \right) \tag{4.11}
$$
Considering again the cylinder element, the stresses acting on a slice of it are shown in FIG. 4.8. The stress resultant acting on the elemental slice, in the direction $\phi$, is given by

$$\sigma_{\phi} \, dxdz$$

The total normal force $N_{\phi}$ acting on the element is given by

$$N_{\phi} = \int_{-t/2}^{t/2} \sigma_{\phi} \, dxdz \quad 4.12$$

The stress resultant also applies a moment to the middle surface given by

$$z\sigma_{\phi} \, dxdz$$

The total moment $M_{\phi}$ is given by

$$M_{\phi} = -\int_{-t/2}^{t/2} z\sigma_{\phi} \, dz \quad 4.13$$

where the negative sign is to be consistent with the directions of the moments represented in FIG. 4.4.

The force acting on the slice in the direction $x$ is given by

$$\sigma_{x}(R + z) d\phi dz$$

The total normal force $N_{x}$ is given by

$$N_{xRd\phi} = \int_{-t/2}^{t/2} (R + z)\sigma_{x} \, d\phi dz$$
FIG. 4.8 STRESSES ON CYLINDER ELEMENT
and the total moment $M_x$ is given by

$$M_x = - \int_{-t/2}^{t/2} \frac{z(R + z)\sigma_x dz}{R}$$  \hspace{1cm} (4.15)$$

Combining Eqs. 4.12 to 4.15, for the forces and moments, with the expressions in Eqs. 4.10 and 4.11 for the stresses, yields

$$N_x = \int_{-t/2}^{t/2} \frac{E}{(1 - \nu^2)} \left[ \frac{R + z}{R} \frac{\partial w}{\partial x} + \nu \frac{\partial u}{\partial x} - \nu z \frac{\partial^2 w}{\partial x^2} \right] dz$$

$$M_x = - \int_{-t/2}^{t/2} \frac{E}{(1 - \nu^2)} \left[ \frac{R + z}{R} \frac{\partial w}{\partial x} + \nu \frac{\partial u}{\partial x} - \nu z \frac{\partial^2 w}{\partial x^2} \right] \frac{R + z}{R} dz$$

$$N_x = \int_{-t/2}^{t/2} \frac{E}{(1 - \nu^2)} \left[ \frac{\partial u}{\partial x} - \nu \frac{\partial^2 w}{\partial x^2} \right] (R + z) dz$$

$$M_x = - \int_{-t/2}^{t/2} \frac{E}{(1 - \nu^2)} \left[ \frac{\partial u}{\partial x} - \nu \frac{\partial^2 w}{\partial x^2} \right] (R + z) \frac{R + z}{R} dz$$

A further approximation adopted in the analysis of thin cylindrical shells and introduced by DONNEL \textsuperscript{29}, assumes that the distance $z$ can be neglec-
ted in the presence of the radius \( R \), i.e., \( R+z = R \). Introducing the approximation and integrating the expressions for the forces and moments, gives

\[
N_\phi = \frac{Et}{(1 - \nu^2)}(\frac{w}{R} + \frac{\partial u}{\partial x}) \quad 4.16
\]

\[
M_\phi = \frac{Et^3}{12(1 - \nu^2)} \frac{\partial^2 w}{\partial x^2} \quad 4.17
\]

\[
N_x = \frac{Et}{(1 - \nu^2)}(\frac{\partial u}{\partial x} + \frac{w}{R}) \quad 4.18
\]

\[
M_x = \frac{Et^3}{12(1 - \nu^2)} \frac{\partial^2 w}{\partial x^2} \quad 4.19
\]

Eqs. 4.16 to 4.19 represent the elastic law for a cylindrical shell under axisymmetric load. These equations combined with the equilibrium equations will allow the solution of the problem of the action of the rope pressure on the drum roll.

4.1.3 Solution of the problem

Combining Eqs. 4.16 to 4.19 with Eqs. 4.1 to 4.3, yields:

\[
N_x = K(\frac{\partial u}{\partial x} + \frac{w}{R}) = \text{constant} \quad 4.20
\]

\[
K(\frac{\partial w}{\partial R}) + \frac{\partial^2 u}{\partial x^2} + \frac{2}{3}R \frac{\partial^2 w}{\partial x^2} = -pR \quad 4.21
\]

where

\[
K = \frac{Et}{(1 - \nu^2)}
\]

\[
D = \frac{Et^3}{12(1 - \nu^2)}
\]

The quantities \( K \) and \( D \) are generally called the "extensional" and the
"flexural stiffness", respectively, of the shell.

Considering that the cylinder has constant thickness and combining Eqs. 4.20 and 4.21 gives

\[
\frac{4}{\pi} \frac{\partial^4 w}{\partial x^4} + 4 \beta^4 w = -\frac{1}{R^2} (p + \frac{v N_x}{R}) \quad 4.22
\]

where

\[
\beta^4 = \frac{Et}{4R^2D} = \frac{3(1 - \nu^2)}{R^2t^2}
\]

The problem of circular cylindrical shells subjected to axisymmetric load is reduced to the solution of the differential equation in Eq. 4.22. The solution, whose development can be found in reference 21, is

\[
w(x) = C_1 \sin \beta x \sinh \beta x + C_2 \sin \beta x \cosh \beta x + C_3 \cos \beta x \sinh \beta x + C_4 \cos \beta x \cosh \beta x - \frac{R^2}{Et} (p + \frac{v N_x}{R}) \quad 4.23
\]

where

\[
w(x) = \text{radial displacement at } x
\]

\[C_1, C_2, C_3, C_4 = \text{constants of integration}\]

The axial force \(N_x\), appearing in the solution, represents in the winding drum problem, the resistance offered by the side plates to displacements of the roll edges and also the force applied by the rope to the side flanges in the multi-layering condition. Although the drum being studied is wound with two layers of rope, the second layer does not apply forces to the side plates so that the axial force is reduced to the reactions due to the plates. This axial reaction, commonly neglected in the discussion of pressurized cylindrical shells \(^{16}\), will be taken as zero in the remainder of this discussion.

The four constants of integration in Eq. 4.23 are determined by considering the conditions at the edges of the cylinder as boundary conditions. In order to illustrate the application of the solution, the cases for simply supported and built in edges are considered in the next section.
4.1.4 Roll with simply supported or built in edges

Consider first, a roll under external pressure $p$ with the edges simply supported, as shown in FIG. 4.9.

For convenience, the origin of the $x$ axis is taken as midway along the length $L$. At the edges, $x=\pm L/2$, the deflections and the moments are zero. Then,

$$w(-L/2) = 0, \quad w(L/2) = 0$$
$$M_x(-L/2) = 0, \quad M_x(L/2) = 0$$

Considering that, $M_x = D\frac{\partial w}{\partial x^2}$, the equation for the moment $M_x(x)$ at point $x$ is obtained by twice differentiating Eq. 4.23 and multiplying by $D$.

The four constants of integration are determined by solving simultaneously the four equations given by the boundary conditions. The solution is

$$C_2 = C_3 = 0$$
$$C_1 = \frac{pR^2}{Et} \frac{2 \sin \alpha \sinh \alpha}{\cos 2\alpha + \cosh 2\alpha}$$
$$C_4 = \frac{pR^2}{Et} \frac{2 \cos \alpha \cosh \alpha}{\cos 2\alpha + \cosh 2\alpha}$$

where

$$\alpha = \frac{\beta L}{2}$$
A roll with built in edges can be analysed in a similar way, and in this case, the radial deflections and the slopes \( w' \) are zero at the edges. The boundary conditions are represented by the equations

\[
\begin{align*}
    w(-L/2) &= 0, & w(L/2) &= 0, \\
    w'(-L/2) &= 0, & w'(L/2) &= 0.
\end{align*}
\]

With the equations for the slopes obtained by differentiating once Eq. 4.23, the solution for the system of equations is

\[
\begin{align*}
    C_2 &= C_3 = 0, \\
    C_1 &= \frac{pR^2}{E_t} \left( \frac{\sin \alpha \cosh \alpha - \cos \alpha \sinh \alpha}{\sin 2\alpha + \sinh 2\alpha} \right), \\
    C_4 &= \frac{pR^2}{E_t} \left( \frac{\sin \alpha \cosh \alpha + \cos \alpha \sinh \alpha}{\sin 2\alpha + \sinh 2\alpha} \right).
\end{align*}
\]

Different boundary conditions can be analised in a similar way. HETÉNYI\textsuperscript{30} shows that the equation obtained for the radial deflection of a cylinder under axisymmetric load is the same as that for the deflection of a beam on elastic foundations. His analyses of several cases of beams under different loadings and boundary conditions are a good reference for the study of cylinders under axisymmetric load.

The solution in Eq. 4.23 can be directly applied to describe the behaviour of a winding drum if there are no stiffening rings; if the drum sides can be assumed perfectly rigid and if the roll is considered either simply supported or built in to the sides. A more accurate analysis requires the consideration of the elasticity of the sides and the effects of the stiffening rings when these are present. The introduction of elastic sides is discussed in section 4.3.

4.1.5 Roll with stiffening rings

To demonstrate how the effects of stiffening rings can be taken into account, assume a drum roll simply supported at the edges and with a stiff-
fening ring at \( x = 0 \) as shown in FIG. 4.10.

![Stiffening Ring Diagram](image)

**FIG. 4.10** ROLL WITH STIFFENING RING

At the edges, the deflections and moments are zero. However, these conditions are not sufficient to fully describe the stiffened cylinder.

One technique is to assume that the ring acts on a circle, split the cylinder roll at this circle and represent the two resulting sub-cylinders as isolated free bodies. This is shown in FIG. 4.11 where \( w_i(0), w'_i(0), M_{x_i}(0) \) and \( Q_{x_i}(0) \) are, respectively, the deflection, slope, moment and shear force of the sub-cylinder \( i \) at the position of the ring. \( w_j(0), w'_j(0), M_{x_j}(0) \) and \( Q_{x_j}(0) \) are the corresponding variables for sub-cylinder \( j \).

Eq. 4.23 and its derivatives may now be applied to each sub-cylinder resulting in a set of four constants of integration to be determined for each sub-cylinder. The boundary conditions at \( x = \pm L/2 \) provide four equations
and four more equations are obtained from the conditions at the ring where, because of the continuity of the cylinder and of the equilibrium it can be written that

\[ w_i(0) = w_j(0) \]
\[ w_i'(0) = w_j'(0) \]
\[ M_{x_i}(0) - M_{x_j}(0) = -M_s \]
\[ Q_{x_i}(0) - Q_{x_j}(0) = -Q_s \]

where \( M_s \) is the moment reaction and \( Q_s \) the force reaction offered by the ring which are both unknown. However, if it is assumed that the ring is thin, i.e., the dimensions of its cross-section are small compared with the radius; that the ring axis passes through the centroids of the cross-sections; that the loads are applied at the axis and the ring radius is approximately equal to the radius of the roll, then, from FLÜGGE²⁶

\[ Q_s = \frac{E_s A_s}{R^2} \]
\[ M_s = \frac{E_s I_s}{R^2} \]

where

\( \delta \) - radial deflection experienced by the ring
\( \gamma \) - rotation experienced by the ring cross-section
\( A_s \) - area of the ring cross-section
\( I_s \) - moment of area of the ring cross-section in relation to an axis passing through the ring centre and the centroid of the cross-section
\( E_s \) - modulus of elasticity of the ring

Supposing that the ring perfectly follows the deformations of the roll it can be written that

\[ \delta = w_i(0) = w_j(0) \]
\[ \gamma = w_i'(0) = w_j'(0) \]
and then

\[ Q_s = w_1(0) \frac{E_A}{R^2} \]
\[ M_s = w_1'(0) \frac{E_I}{R^2} \]

The above expressions combined with the equations representing the continuity and equilibrium of the roll at the ring and with the equations for the roll boundary conditions, form a system of simultaneous equations whose solution gives two sets of constants of integration, each set being valid for only one of the sub-cylinders.

Generalizing, if the roll had N stiffening rings, it could be split into N+1 sub-cylinders and the solution of a system of 4x(N+1) equations would be required to describe the whole cylinder.

Several authors have dealt with the problem of stiffened circular shells under uniform pressure, trying to establish a solution in a single equation. NOVOZHILOV\textsuperscript{31} approximates the stiffened shell with an equivalent, without rings, exhibiting different elastic properties in the axial and circumferential directions and remarks that the method involves small errors if the rings are sufficiently closely spaced and in sufficiently large numbers. FLÜGGE\textsuperscript{26} and WILSON\textsuperscript{32} consider equally spaced rings and assume that the deformation of the cylinder, between the rings, is symmetric in respect to a plane passing midway between the rings.

The advantage of the method presented here over those of the referred authors is that there are no restrictions on the spacing of the rings and few on their shapes. Furthermore, it can be extended to account for conditions between the cylinder edges other than stiffening rings, provided they are axisymmetric. For example, consider a simply supported roll partially loaded as shown in FIG. 4.12. Splitting the roll at x=0, where the load discontinuity occurs, the equations representing the conditions at the edges and at the split are
The condition that $p=0$ for the sub-cylinder $j$ is implicit in Eq. 4.24.

As a rule, a cylinder can be split into sub-cylinders at any axisymmetric geometrical or loading discontinuity. However, it must be kept in mind that for a geometrical discontinuity such as a stiffening ring, the method is an approximation for it cannot fully show the mechanical interaction between the cylinder and the ring at that point. Nevertheless, the experimental results discussed in chapter 5 show that the approximation is valid.

Theoretically, there is no limit to the number of sub-cylinders into which a cylinder can be split. The only restriction is related to the manipulation of the simultaneous equations created. While with two sub-cylin-
ders the task is possible by manual means, as the number of splits increases manual techniques become infeasible and computer aided techniques become necessary.

The theoretical model for the winding drum problem, considering stiffening rings and also side plates, is presented in Appendix 3 in a structure suitable for implementation into a computer.

4.2 Side Plates

The drum side may be assumed to be a thin uniform circular plate as shown in FIG. 4.13 and any point in it defined by a system of coordinates $r$, $\theta$, and $y$ where

- $r$ - is the distance to the axis of the plate.
- $\theta$ - is the angle that the diameter passing through the point forms with a reference diameter.
- $y$ - is the distance to the middle surface along a normal to the plate.
The middle surface is defined as the surface equidistant to the two faces of the plate.

An element of the plate isolated as a free body is represented in FIGs. 4.14 and 4.15 with the forces and moments acting on it. The notation in the figures is

\[ P_r, P_\theta, P_y \] components of the external loads per unit of area in the directions \( r \), \( \theta \) and \( y \) respectively.

\[ N_r, N_\theta \] normal forces along the directions \( r \) and \( \theta \).

\[ N_r \] shearing force acting on a surface normal to direction \( r \) and along direction \( \theta \).

\[ N_{er} \] shearing force acting on a surface normal to direction \( \theta \) and along direction \( r \).

\[ Q_r, Q_\theta \] shearing forces acting on surfaces normal to directions \( r \) and \( \theta \) and along direction \( y \).

\[ M_r \] bending moment on a section normal to direction \( r \).

\[ M_\theta \] bending moment on a section normal to direction \( \theta \).

\[ M_{er} \] twisting moment acting on a surface normal to direction \( r \) and in the \( \theta \) direction.

\[ M_{or} \] twisting moment acting on a surface normal to direction \( \theta \) and in the \( r \) direction.

All forces and moments are per unit of length.

The assumption of uniformity of the side plate is not strictly true for drums split diametrically. However, split side plates are commonly securely fastened to the drum roll and shaft and also butted and bolted together. In such a case, it seems reasonable to consider that the effects of the discontinuity are localized near the split and that the split plate overall behaves similarly to a uniform plate. Allowing the use of the assumption results in a more simplified approach than would otherwise be the case.

In a similar procedure to the one adopted for the analysis of the drum roll, the system of forces and moments on the plate element can be simplified. This can be done by considering the symmetry of the loading which, in this case, comprises a bending moment around the plate edge and a compressive force acting in the plane of the plate; both transmitted by the drum roll.
FIG. 4.14 FORCES ON A PLATE ELEMENT

FIG. 4.15 MOMENTS ON A PLATE ELEMENT
Solutions for the problem of circular plates subjected to the combined action of these types of loads have been discussed by several authors as, for example, MANSFIELD\textsuperscript{33}, ch. 3. They involve four constants of integration which can be readily determined if the applied moment and force are known. In the winding drum problem however, they are not known but can be represented as expressions derived from Eq. 4.23. In this case they will be functions of the constants of integration appearing in that equation which are also unknown. The moment and force expressed in this manner, when combined with the solution for the plate, produce a system of equations where the constants of integration are related non-linearly, making their determination very complex. To simplify the analysis, the effects of the moment and compressive force will be considered separately and their combined effect obtained by superposition. This is an approximation that for small deflections, which is the case for winding drums, introduces only small errors.

4.2.1 Bending moment applied by the drum roll

4.2.1.1 Equations of equilibrium

Considering only a bending moment symmetrically applied around the periphery of the plate, the system of forces and moments acting on an element of the plate is reduced to that shown in FIG. 4.16 where, because of the symmetry, $N_{re}$, $N_{er}$, $Q_{e}$, $M_{re}$ and $M_{er}$ do not apply and $M_{e}$ is constant. The forces $N_{r}$ and $N_{e}$ are commonly taken as zero in small deflections problems not involving in plane loads (refer, e.g., to DONNEL\textsuperscript{27}, ch. 4). Also, because of the nature of the load, the components $p_{r}$, $p_{\theta}$ and $p_{y}$ are zero.

For the element to be in equilibrium, the sum of forces in the direction $y$ has to be

$$-Q_{r} r \, d\theta + \left( Q_{r} + r \frac{a Q_{r}}{a r} \right) (r + \, d r) \, d \theta = 0$$

or, ignoring the small terms of higher order

$$Q_{r} + r \frac{a Q_{r}}{a r} = 0 \quad 4.25$$
FIG. 4.16 ELEMENT OF PLATE UNDER SYMMETRIC MOMENT
The sum of moments relative to the $\theta$ direction is given by

$$\left(M_r + \frac{3M_r}{r}dr\right)(r + dr)d\theta - \left(Q_r + \frac{3Q_r}{r}dr\right)(r + dr)d\theta dr/2 - M_r r d\theta - Q_r r d\theta dr/2 - 2M_\theta dr \sin(d\theta/2) = 0$$

Neglecting the terms of higher order, the small difference between the shear forces on the two opposite curved sides of the element and also taking $\sin(d\theta/2) = d\theta/2$, then

$$r \frac{3M_r}{ar} + M_r - rQ_r - M_\theta = 0$$

Differentiating in respect to $r$ and substituting in Eq. 4.25 gives

$$r \frac{3Q_r}{ar^2} + 2 \frac{3M_r}{ar} - \frac{3M_\theta}{ar} = 0 \quad 4.26$$

The consideration of the equilibrium of the element does not provide other equations and, in order to determine the unknowns $M_r$ and $M_\theta$ it is necessary to consider the deformations.

4.2.1.2 Deformations

Assuming that the plate is thin in relation to its radius and using the Love-Kirchhoff approximation discussed in section 4.1.2, the stresses and strains in the $y$ direction can be neglected such that is only necessary to examine the normal strains $\epsilon_r$ and $\epsilon_\theta$ in the directions $r$ and $\theta$ respectively.

In order to establish the expression for $\epsilon_r$, the deformed plate may be represented by a cross-section through a diameter as shown in FIG. 4.17a.

It will be seen in the figure that the length $dr$, before deformation, of a lamina taken at a distance $y$ from the middle surface, as shown in FIG. 4.17b, changes to a length

$$(\rho_r + y)d\phi = (1 + \frac{y}{\rho_r})ds$$
FIG. 4.17 CROSS-SECTION OF DEFORMED PLATE

FIG. 4.18 TANGENTIAL CURVATURE
after deformation. For small deflections \( ds = dr \) and the radial strain \( \varepsilon_r \) is given by

\[
\varepsilon_r = \frac{(1 + y/\rho_r)dr - dr}{dr} = \frac{y}{\rho_r}
\]

The curvature \( 1/\rho_r \) of the deformed surface in the direction \( r \) is then

\[
\frac{1}{\rho_r} = \frac{d\phi}{ds} = \frac{d\phi}{dr}
\]

Because of the small deflections, it will be seen from FIG. 4.17c that

\[
\phi = \tan\phi = \frac{dw}{P} = \frac{d\phi}{dr}
\]

where

\( w_p \) - deformation of the plate in the direction \( y \)

The expression for the radial strain \( \varepsilon_r \) can thus take the form

\[
\varepsilon_r = \frac{2}{P} \frac{d^2 w}{dr^2}
\]

which is written as partial differential equation in order to be consistent with the mathematical development to follow.

The tangential strain \( \varepsilon_\theta \) of a point in the plate at a distance \( y \) from the middle surface, is given by an expression similar to Eq. 4.27, i.e., it is the product of the curvature \( 1/\rho_\theta \) of the deformed surface in the direction \( \theta \) and the distance \( y \) as given, e.g., by TIMOSHENKO16, ch. 3. In order to determine the curvature, reference should be made to FIG. 4.18a where, because of symmetry, normals taken at any point on a circle of radius \( r \) on the deformed plate intercept at a point \( 0 \) on the axis of the plate. It follows that the distance from \( 0 \) to any of the points is the radius of curvature \( \rho_\theta \) in the direction \( \theta \). Assuming small deflections and from FIG. 4.18b

\[
\rho_\theta \sin\phi = \rho_\theta \phi = r
\]
As already seen \( \phi = \frac{aw}{\partial r} \), from which it follows that

\[
\frac{1}{\rho_\theta} = \frac{aw}{\partial \ar}
\]

Thus, the tangential strain is given by

\[
\epsilon_\theta = \frac{yaw}{\partial \ar}
\] 4.29

In order to relate the radial and tangential strains \( \epsilon_r \) and \( \epsilon_\theta \) to the moments \( M_r \) and \( M_\theta \), it is necessary to investigate the stresses acting on an element of the plate as shown in FIG. 4.19.

On a side of the element normal to direction \( e \), a lamina of thickness \( dy \) and at a distance \( y \) from the middle surface, is subjected to a stress resultant

\[
\sigma_\theta \, dy \, dr
\]

This resultant applies a moment to the plate, in relation to its middle surface equal to

\[
\sigma_\theta \, y \, dr \, dy
\]

The total moment \( M_\theta \), applied by the resultants at all laminas is, by reference to FIG. 4.16, given by

\[
M_\theta \, dr = \int_{-t_p/2}^{t_p/2} \sigma_\theta \, y \, dr \, dy
\] 4.30

Applying the same reasoning, the moment \( M_r \), on a section normal to the \( r \) direction is given by

\[
M_r \, rd\theta = \int_{-t_p/2}^{t_p/2} \sigma_r \, y \, rd\theta \, dy
\] 4.31
FIG. 4.19 STRESSES ON A PLATE ELEMENT
The radial and tangential stresses $\sigma_r$ and $\sigma_\theta$, are related to the radial and tangential strains $\varepsilon_r$ and $\varepsilon_\theta$, by Hooke's law as

$$\sigma_r = \frac{E_p}{(1 - \nu_p^2)}(\varepsilon_r + \nu_p \varepsilon_\theta)$$

$$\sigma_\theta = \frac{E_p}{(1 - \nu_p^2)}(\varepsilon_\theta + \nu_p \varepsilon_r)$$

where

- $E_p$ - modulus of elasticity for the plate
- $\nu_p$ - Poisson ratio for the plate

Using Eqs. 4.28 and 4.29, the expressions for the stresses $\sigma_r$ and $\sigma_\theta$ can be rewritten as

$$\sigma_r = \frac{E_p}{(1 - \nu_p^2)}(\frac{\partial^2 w}{\partial r^2} + \frac{\nu_p}{r} \frac{\partial w}{\partial r})$$

$$\sigma_\theta = \frac{E_p}{(1 - \nu_p^2)}(\frac{1}{r^2} \frac{\partial w}{\partial r} + \frac{\nu_p}{r} \frac{\partial^2 w}{\partial r^2})$$

Substituting the above equations into Eqs. 4.30 and 4.31 and integrating, yields the values of the moments $M_r$ and $M_\theta$ as

$$M_r = D_p \left(\frac{\partial^2 w}{\partial r^2} + \frac{\nu_p}{r} \frac{\partial w}{\partial r}\right)$$

$$M_\theta = D_p \left(\frac{1}{r^2} \frac{\partial w}{\partial r} + \frac{\nu_p}{r} \frac{\partial^2 w}{\partial r^2}\right)$$

where

$$D_p = \frac{E_p t^3}{12(1 - \nu_p^2)}$$
4.2.1.3 Solution of the problem

The expressions for $M_r$ and $M_\theta$ in Eqs. 4.32 and 4.33, substituted into Eq. 4.26 result in

$$\frac{2w_p}{ar^4} + 2\frac{aw_p}{ar^3} - \frac{1}{r^3} \frac{aw_p}{ar} + \frac{1}{r^2} \frac{aw_p}{ar} = 0$$

or, more conveniently

$$\frac{1}{r^3} \frac{a}{ar} \left\{ r^3 \frac{a}{ar} \right\} = 0$$

This differential equation represents the behaviour of a circular plate under axisymmetrical load. Its general solution, obtained by successive integrations, is

$$w_p(r) = C_{p1} + C_{p2} \log r + C_{p3} r^2 + C_{p4} r^2 \log r \quad \text{4.34}$$

giving the plate deflection $w_p(r)$, at a radius $r$. The constants of integration, $C_{p1}, C_{p2}, C_{p3}$ and $C_{p4}$, are determined in each case by examining the boundary conditions at the edges of the plate.

4.2.2 Compressive force applied by the drum roll

4.2.2.1 Equations of equilibrium

The system of forces and moments acting on a element of the plate, shown in FIGs. 4.14 and 4.15, is reduced to that shown in FIG. 4.20 if the plate is subjected only to the action of axisymmetric forces in its plane.

Resolving the forces in the direction $r$ results in the expression

$$(N_r + \frac{3N_r}{ar} dr)(r + dr) d\theta - N_r r d\theta - 2N_\theta \sin(d\theta/2) dr = 0$$
FIG. 4.20 ELEMENT OF PLATE UNDER SYMMETRIC IN PLANE FORCE
Since \( \theta \) is small and neglecting terms of higher order

\[
N_r + r \frac{\partial N_r}{\partial r} - N_\theta = 0
\]  

4.35

Once again, the conditions of equilibrium are not sufficient for the solution of the problem and the deformations have to be considered.

4.2.2.2 Deformations

In order to consider the deformations, reference has to be made to FIG. 4.21 where \( u_r \) is the displacement in the radial direction. The radial strain \( e_r \) is then

\[
e_r = \frac{1}{dr} (u_r + \frac{\partial u_r}{\partial r} dr - u_r) = \frac{\partial u_r}{\partial r}
\]  

4.36

In the tangential direction, an elemental fiber having an initial length \( r d\theta \) is stretched to a length \( (r + u_r) d\theta \) and the tangential strain \( e_\theta \) is

\[
e_\theta = \frac{(r + u_r - r) d\theta}{r d\theta} = \frac{u_r}{r}
\]  

4.37

The radial stress \( \sigma_r \) and the tangential stress \( \sigma_\theta \) can be related to the radial strain \( e_r \) and the tangential strain \( e_\theta \) by Hooke's law. Hence,
By differentiating Eq. 4.39 and combining with Eq. 4.38 gives

\[
r \frac{\partial \sigma_{\theta}}{\partial r} - r \nu \frac{\partial \sigma_{r}}{\partial r} + (1 + \nu_p)(\sigma_{\theta} - \sigma_r) = 0
\]

4.50

In the element, the radial stress \( \sigma_r \) and the tangential stress \( \sigma_{\theta} \) are given by

\[
\sigma_r = \frac{N_r r \delta \theta}{r \delta \theta t_p} = \frac{N_r}{t_p}
\]

\[
\sigma_{\theta} = \frac{N_{\theta} dr}{d \theta t_p} = \frac{N_{\theta}}{t_p}
\]

Substituting these expressions into Eq. 4.50

\[
r \frac{\partial^2 N_{\theta}}{\partial r^2} - r \nu \frac{\partial N_r}{\partial r} + (1 + \nu_p)(N_{\theta} - N_r) = 0
\]

4.51

This equation combined with the equilibrium equation in Eq. 4.35 would allow the solution of the problem.

4.2.2.3 Solution of the problem

Taking Eq. 4.35 and its first differential in respect to \( r \) and substituting into Eq. 4.51 gives

\[
r \frac{\partial^2 N_r}{\partial r^2} + 3 \frac{\partial N_r}{\partial r} = 0
\]
A general solution for this equation is

\[ N_r(r) = C_{p5} + \frac{C_{p6}}{r^2} \]  \hspace{1cm} 4.52

And, from Eq. 4.35, it can be established that

\[ N_\theta(r) = C_{p5} - \frac{C_{p6}}{r^2} \]  \hspace{1cm} 4.53

where

- \( N_r(r) \) - normal force at a radius \( r \) in the radial direction
- \( N_\theta(r) \) - normal force at a radius \( r \) in the tangential direction
- \( C_{p5}, C_{p6} \) - constants of integration to be determined by the examination of the boundary conditions

As an illustration, consider a plate with a concentric hole and subjected to forces \( F_0 \) and \( F_i \), per unit of length, acting on the outer and inner edges, respectively, as shown in FIG. 4.22.

![Plate with a hole diagram](image)

**FIG. 4.22 PLATE WITH A HOLE**

Adopting the convention that radial forces acting outwards are positive, the boundary conditions in this case are:

at \( r=R \)

\[ N_r(R) = -F_0 \]
at \( r=r_i \)

\[ N_r(r_i) = F_i \]

Substituting these conditions in Eq. 4.52 gives

\[ -F_o = C_{p5} + \frac{C_{p6}}{r_{i}^2} \]

\[ F_i = C_{p5} + \frac{C_{p6}}{r_{i}^2} \]

Solving for the constants of integration \( C_{p5} \) and \( C_{p6} \) gives

\[ C_{p5} = -\frac{F_o R^2 + F_i r_{i}^2}{(R^2 - r_{i}^2)} \quad 4.54 \]

\[ C_{p6} = \frac{r_{i}^2 R^2 (F_i + F_o)}{(R^2 - r_{i}^2)} \quad 4.55 \]

In the winding drum problem, the plate is fixed to a hub on the shaft, at the edge of the central hole and, the radial reaction \( F_i \) of the hub is not known. However, this force can be related to the force \( F_o \) applied at the outer edge of the plate if it is assumed that, the hub is perfectly rigid and the plate built into it. In this case, the radial deflection of the plate at the hub is zero. The radial deflection \( u_r(r) \) at a radius \( r \) of the plate can be derived from Eq. 4.39 and, after the proper substitution, written as

\[ u_r(r) = \frac{r}{E_p t_p (R^2 - r_{i}^2)} [-(1 - \nu_p)(F_o R^2 + F_i r_{i}^2) - (1 + \nu_p)(F_o + F_i) \frac{r_{i}^2 R^2}{(R^2 - r_{i}^2)}] \quad 4.56 \]

By making Eq. 4.56 equal zero at \( r=r_i \) and solving for \( F_i \) yields

\[ F_i = \frac{2R^2}{(R^2 + r_{i}^2) + \nu_p (R^2 - r_{i}^2) F_o} \quad 4.57 \]
Hence, a circular plate built in at the edge of a central hole and subjected to an in-plane force at the outer edge, would suffer a reaction at the inner edge, proportional to the applied force and given by Eq. 4.57. Therefore, forces, stresses, strains and deflections at any point of the plate can be expressed as functions of the force $F_0$ only.

4.3 Combination of the Theoretical Models for the Drum Roll and Side plates

Consider a drum roll without stiffening rings, supported by two side plates identified as side plate 1 and side plate 2, centrally built into perfectly rigid hubs attached to a perfectly rigid shaft. Such configuration can be represented in the manner shown in FIG. 4.23

Further consider that the roll and the side plates have the same thickness, are made of the same material and that the drum roll is subjected to a uniform external pressure $p$.

By reference to Eqs. 4.23 and 4.34, it can be seen that the solution of
such a problem requires the determination of twelve constants of integration, four for the roll and four for each side plate. It is necessary, therefore, to identify twelve independent equations relating the constants. Four of these equations can be established from the conditions of each side plate at the hub where, because of the built in support, the deflections and slopes are zero. Other equations can be established from the conditions at the joints of the roll with the side plates by assuming that each joint is perfectly rigid in the sense that the roll and the plate experience no displacement relative to each other. Thus, at the joint, the slopes of the roll and of the side plate will be the same as also will be the axial moment on the roll and the radial moment on the plate. These conditions provide a further four equations. The remaining equations can be obtained by assuming that, at each joint the side plate suffers no deflection normal to its plane and that the roll moves inwards by \( \delta \) as shown in FIG. 4.24.

![FIG. 4.24 DEFLECTION AT THE JOINT](image)

The conventions adopted in setting up the twelve equations are:

i- The origin of the coordinate \( x \) for the roll is taken at the middle surface of the side plate 1 and positive to the right.

ii- The moments are positive if acting as shown in FIG. 4.25.

iii- The notation is the same used throughout sections 4.1 and 4.2 with the variables for the side plate 1 and side plate 2 being identified by subscripts 1 and 2 respectively.
With reference to FIG. 4.23 the twelve equations describing the problem are:

at point a
\[ w_1(r_1) = 0 \]
\[ w_1'(r_1) = 0 \]

at point b
\[ w_1(R) = 0 \]
\[ w(0) = \delta_b \] \hspace{1cm} 4.58
\[ w_1'(R) = -w'(0) \]
\[ M_{r1}(R) = -M_x(0) \] \hspace{1cm} 4.59

at point c
\[ w_2(R) = 0 \]
\[ w(L) = \delta_c \]
\[ w_2'(R) = w'(L) \]
\[ M_{r2}(R) = M_x(L) \]

at point d
\[ w_2(r_1) = 0 \]
\[ w_2'(r_1) = 0 \]

where
\[ \delta_b, \delta_c \] - radial deflections of the roll at points b and c respectively.
The negative signs in the equations involving moments and slopes of the roll and the plates are for consistency with the system of coordinates and with the direction of the moments.

All the expressions in the equations may be obtained from Eqs. 4.23 and 4.34 and their derivatives. The deflections $\delta_b$ and $\delta_c$ are obtained by assuming that they are equal to radial displacements of the side plates caused only by in plane forces equal to the shear forces $Q_x(0)$ and $Q_x(L)$ on the roll at points $b$ and $c$.

From Eqs. 4.3 and 4.19 the shear force $Q_x$ is

$$Q_x = \frac{2}{2} \frac{d^2 M_x}{dx^2} = D \frac{3}{2} \frac{d^3 W}{dx^3}$$

By combining Eqs. 4.56 and 4.57, the radial deflections of the side plates at points $b$ and $c$, where $r=R$, can be expressed as

$$\delta_b = -K_p Q_x(0)$$

$$\delta_c = -K_p Q_x(L)$$

where

$$K_p = \frac{R}{E(R^2-r_1^2)} \left\{ (v-1)R^2 \left[ 1 - \frac{2r_1^2}{(R^2+r_1^2)(vR^2)} \right] - (v+1)r_1^2 \left[ 1 - \frac{2R^2}{(R^2+r_1^2)(vR^2)} \right] \right\}$$

The negative signs in Eqs. 4.60 and 4.61 are to indicate that the forces are compressive. These equations, combined with the previous set of twelve equations, permit the determination of the constants of integration to give the solution of the problem of a drum roll supported by elastic side plates.

For drums with stiffening rings, the number of constants of integration would be increased by four for each ring. The additional equations required would be established from the conditions at the rings as shown in section 4.1.5.

Some winding drums are fitted with brake rings, as is the case of the drum being studied. To illustrate how these rings may be included in the
theoretical representation, suppose that the drum in FIG. 4.23 is fitted with one brake ring at the joint b as shown in FIG. 4.26.

\[ M_b = w'(0) \frac{I_b E_b}{R^2} \]

\[ Q_b = \delta_b \frac{A_b E_b}{R^2} \]

where
- \( I_b \) - moment of area of the brake ring cross-section in relation to an axis passing through the ring centre and the centroid of the cross-section
- \( A_b \) - area of the brake ring cross-section
- \( E_b \) - modulus of elasticity for the brake ring

These two reactions have to be considered in the equations representing the conditions at point b. The moment reaction is introduced into Eq. 4.59 considering the equilibrium of moments at the point and modifying the equation relating the moments to

\[ M_{r1}(R) = -M_x(0) + M_b \]
The force reaction is introduced considering the equilibrium of forces at the point b. The force \(-Q_x(0)\) applied by the drum roll to the side plate and brake ring is reacted by a force \(\delta_b/K_p\) from the plate, and, by a force \(Q_b\) from the ring such that

\[
\frac{\delta_b}{K_p} + Q_b = -Q_x(0)
\]

(By reference to Eq. 4.60, the ratio \(\delta_b/K_p\) gives the force necessary to produce a deflection \(\delta_b\) on the plate)

Substituting the expression of \(Q_b\) and solving the equation above for \(\delta_b\) yields

\[
\delta_b = \left(\frac{R^2K_p}{P_b} + \frac{K_pA_E}{b}\right)Q_x(0)
\]

The substitution of this expression for \(\delta_b\) into Eq. 4.58 introduces the force reaction of the brake ring into the conditions at point b.

A brake ring fitted to the joint c would be considered in a similar manner.

### 4.4 Action of the Bent Shaft on the Side Plates

In order to simplify the analysis of the problem, it is assumed firstly, that the effect of the action of the bent shaft on the plates can be determined separately and then, superposed on the effects of other loads. Secondly, in addition to the plates being built to perfectly rigid hubs, these follow exactly the slope of the shaft. Thirdly, it is assumed that the side plates are built into a perfectly rigid drum roll.

Suppose that the bent shaft acts on a side plate in the manner shown in Fig. 4.27 where \(\psi\) is the slope of the shaft at the hub and \(M\) is the moment applied by the shaft to the plate. Such a moment is asymmetric in relation to the axis of the plate and hence, the solution for the bending of plates, derived in section 4.2 cannot be applied.
FIG. 4.27  ACTION OF BENT SHAFT
The establishment of a solution applicable to the present case involves the development of a theory for the general bending of circular plates. The full development of this theory is beyond the scope of this work. Instead, the main steps of it will be discussed and only the final results presented. The full development of the theory can be found, e.g., in TIMOSHENKO\textsuperscript{16}, ch. 9.

In the first step of the development, the forces and moments acting on a plate are considered as in FIGs. 4.14 and 4.15. Assuming that there are no forces acting in the plane of the plate and, examining the equilibrium and also the deformation of the element in the plate, the following relationship can be arrived at:

\[
\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2 \sin^2 \theta} \left( \frac{\partial^2}{\partial \theta^2} \right) \frac{\partial^2 w_p}{\partial r^2} + \frac{1}{r} \frac{\partial w_p}{\partial r} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 w_p}{\partial \theta^2} = \frac{p_y}{p}
\]

where the notation is the same as in section 4.2.

A solution for this differential equation is the solution of the problem. In the present case, the load component \( p_y \) is zero and a solution is

\[
w_p(r, \theta) = (C_p7 + C_p8r^3 + C_p9r^{-1} + C_p10 \log r) \cos \theta
\]

where

- \( w_p(r, \theta) \) - plate deflection at a radius \( r \) and an angle \( \theta \)
- \( C_p7, C_p8, C_p9, C_p10 \) - constants of integration
- \( \rho = \frac{r}{R} \)

Using Eq. 4.62, REISSNER\textsuperscript{34} solved the problem represented in FIG. 4.27 taking the directions \( r \) and \( y \) as shown in the figure; the origin of the angle \( \theta \) in the plane of the figure and the boundary conditions as follows:

at the hub

\[
\begin{align*}
    w_p(r_1, \theta) &= 0 \\
    w_p'(r_1, \theta) &= 0
\end{align*}
\]
at the roll
\[ w_p(R, \theta) = R \sin \psi \cos \theta \]
\[ w'_p(R, \theta) = R \tan \psi \cos \theta \]
considering small deflections, these become
\[ w_p(R, \theta) = R \psi \cos \theta \]
\[ w'_p(R, \theta) = R \psi \cos \theta \]

The constants of integration arrived at by Reissner can be written in the following form:

\[ C_{p7} = c_1 R \psi = \frac{R \psi}{2} \left[ 1 - \rho_i^2 + 2(1 + \rho_i^2) \log \rho_i \right] \]
\[ C_{p8} = c_2 R \psi = \frac{R \psi}{2} \left[ 1 - \rho_i^2 + \frac{1}{(1 + \rho_i^2) \log \rho_i} \right] \]
\[ C_{p9} = c_3 R \psi = 1 - C_{p7} - C_{p8} \]
\[ C_{p10} = c_4 R \psi = 2(1 - C_{p7} - 2C_{p8}) \]

where
\[ c_1, c_2, c_3, c_4 \] - auxiliary constants
\[ \rho_i = \frac{r_i}{R} \]

With Eq. 4.62, its derivatives and the constants of integration given above, the variables at any point of the plate can be determined if the slope \( \psi \) is known. Expressions for some of the variables are presented in the next section. The slope \( \psi \) may be related to the moment \( M \) applied by the shaft by consideration of the equilibrium at the outer edge of the plate where the sum of moments along it has to be equal to \( M \), which is represented by

\[ M = \int_0^{2\pi} (M_r \cos \theta + M_0 \cos \theta - Q_r R^2 \cos \theta) d\theta \]
Substituting the expressions for $M_r$, $M_{r\theta}$ and $Q_r$, by reference to section 4.5.1, and integrating gives

$$M = 4\pi D \rho c_w \psi$$  \hspace{1cm} 4.63

Hence, the variables on the plate can also be determined if the moment $M$ is known.

In order to determine either the slope or the moment, it is necessary to analyse the deflection of the shaft combined with the plate. This analysis is illustrated in section 4.6.2 where the shaft of the scale model is considered.

The formula given by ROARK\textsuperscript{22} and used in section 2.4.4 for the determination of the stresses induced by the bent shaft on the side plate, is derived from the solution given by Reissner.

4.5 Summary of Equations

Before describing the scale model theoretically, it is advantageous to summarize the relevant equations obtained in the previous sections of the chapter. Where applicable, the equation number previously used is repeated. References in brackets indicate the equations, section or bibliographic reference from which the expression was obtained. The notation is the same as that used throughout the chapter.

4.5.1 Drum roll

Radial deflection

$$w(x) = C_1 \sin x \sinh bx + C_2 \sin x \cosh bx + C_3 \cos x \sinh bx + C_4 \cos x \cosh bx - \frac{R^2}{6 \pi D}$$  \hspace{1cm} (4.23)

Slope

$$w'(x) = \frac{3w}{\alpha x}$$
Axial moment
\[ M_x(x) = \frac{2}{a^2} \frac{d^2w}{dx^2} \] (4.19)

Shear force
\[ Q_x(x) = \frac{3}{a^2} \frac{d^2w}{dx^3} \] (4.3, 4.19)

Axial strain
\[ \varepsilon_x(x) = \frac{w(x)}{R^2} - \frac{a^2}{a^2} \frac{d^2w}{dx^2} \] (4.8, 4.20)

Circumferential strain
\[ \varepsilon_\phi(x) = \frac{w(x)}{R} \] (4.9)

Axial stress
\[ \sigma_x(x) = \frac{E}{(1 - \nu^2)} \left( \frac{d^2w}{dx^2} \right)^2 \] (4.10, 4.20)

Circumferential stress
\[ \sigma_\phi(x) = \frac{E}{(1 - \nu^2)} \left[ \left( \frac{d^2w}{dx^2} \right)^2 \frac{w(x)}{R} - \frac{a^2}{a^2} \frac{d^2w}{dx^2} \right] \] (4.11, 4.20)

Moment reaction of stiffening ring at \( x=a \)
\[ M_s = \frac{E}{S} \frac{I_s}{R^2} w'(a) \] (sec. 4.1.5)

Force reaction of stiffening ring at \( x=a \)
\[ Q_s = \frac{E}{S} \frac{A_s}{R^2} w(a) \] (sec. 4.1.5)

Moment reaction of brake ring at \( x=b \)
\[ M_b = \frac{E}{S} \frac{I_b}{R^2} w'(b) \] (sec. 4.3)

Force reaction of brake ring at \( x=b \)
\[ Q_b = -\frac{E}{b} A_b \frac{R^2 K_b}{R^2 + K_b E A_b} Q_x(b) \] (sec. 4.3)
4.5.2 Side plates

i- Radial bending only

Deflection
\[ w_p(r) = C_{p1} + C_{p2} \log r + C_{p3} r^2 + C_{p4} r^2 \log r \] 4.34

Slope
\[ w'_p(r) = \frac{aw_p}{\bar{r}} \]

Radial moment
\[ M_r(r) = D_p \left( \frac{aw_p}{\bar{r}^2} + \frac{v \frac{aw_p}{\bar{r}}}{\bar{r}} \right) \] 4.32

Tangential moment
\[ M_\theta(r) = D_p \left( \frac{1}{r^2 \bar{r}} \frac{aw_p}{\bar{r}} + \frac{2}{r^2 \bar{r}^2} \frac{aw_p}{\bar{r}} \right) \] 4.33

Shear force
\[ Q_r(r) = D_p \left( \frac{3}{r^2 \bar{r}} \frac{aw_p}{\bar{r}^2} + \frac{2}{r^2 \bar{r}^2} \frac{aw_p}{\bar{r}} - \frac{1}{r^2 \bar{r}} \frac{aw_p}{\bar{r}} \right) \] (sec. 4.2.1.1)

Radial strain
\[ \epsilon_r(r) = \frac{2}{3} \frac{aw_p}{\bar{r}^3} \] 4.28

Tangential strain
\[ \epsilon_\theta(r) = \frac{2}{r^2 \bar{r}} \frac{aw_p}{\bar{r}} \] 4.29

Radial stress
\[ \sigma_r(r) = \frac{12y_{M}}{t_p} s(r) \] (sec. 4.2.1.2)

Tangential stress
\[ \sigma_\theta(r) = \frac{12y_{M}}{t_p} s_\theta(r) \] (sec. 4.2.1.2)
ii- In plane force only and plate with a central hole

Constants

\[
C_{ps} = \frac{F_0 R^2 + F_0 r_1^2}{(R^2 - r_1^2)}
\]

4.54

\[
C_{p6} = \frac{r_1^2 R^2 (F_4 + F_0)}{(R^2 - r_1^2)}
\]

4.55

Reaction of built in hub

\[
F_i = \frac{2R^2}{(R^2 + r_1^2) + \nu_p (R^2 - r_1^2)} F_0
\]

4.57

Radial strain

\[
\varepsilon_r(r) = \frac{1}{E_p t_p} \left[ (1 - \nu_p)C_{ps} + (1 + \nu_p) \frac{C_{p6}}{r_2^2} \right]
\]

(sec. 4.2.2.2)

Tangential strain

\[
\varepsilon_\theta(r) = \frac{1}{E_p t_p} \left[ (1 - \nu_p)C_{ps} - (1 + \nu_p) \frac{C_{p6}}{r_2^2} \right]
\]

(sec. 4.2.2.2)

Radial stress

\[
\sigma_r(r) = \frac{1}{t_p} (C_{ps} + \frac{C_{p6}}{r_2^2})
\]

(4.52)

Tangential stress

\[
\sigma_\theta(r) = \frac{1}{t_p} (C_{ps} - \frac{C_{p6}}{r_2^2})
\]

(4.53)

Radial displacement

\[
u_r(r) = r \varepsilon_\theta(r)
\]

4.37

iii- Action of bent shaft on plate with built in inner and outer edges

Constants

\[
C_{p7} = C_1 R_\psi = \frac{R_\psi}{2} \left[ 1 - \rho_1^2 + 2(1 + \rho_1^2) \log \rho_1 \right]
\]

(sec. 4.4)
\[ C_{p8} = c_2R \psi = \frac{R \psi}{2} \left[ 1 - \rho_1^2 + \left( 1 + \rho_1^2 \right) \log \rho_1 \right] \]  
(sect. 4.4)

\[ C_{p9} = c_3R \psi = 1 - C_{p7} - C_{p8} \]  
(sect. 4.4)

\[ C_{p10} = c_4R \psi = 2(1 - C_{p7} - 2C_{p8}) \]  
(sect. 4.4)

**Deflection**

\[ w_p(r, \theta) = (C_{p7} + C_{p8} \rho^3 + C_{p9} \rho^{-1} + C_{p10} \rho \log \rho) \cos \theta \]

**Radial moment**

\[ M_r(r, \theta) = \frac{D \psi}{R} \left[ 2(3 + \nu_p) c_2 \rho + 2(1 - \nu_p) c_3 \rho^{-3} + (1 + \nu_p) c_4 \rho^{-1} \right] \cos \theta \]  
(ref. 34)

**Tangential moment**

\[ M_\theta(r, \theta) = \frac{D \psi}{R} \left[ 2(3 \nu_p + 1) c_2 \rho + 2(\nu_p - 1) c_3 \rho^{-3} + (1 + \nu_p) c_4 \rho^{-1} \right] \cos \theta \]  
(ref. 34)

**Twisting moment**

\[ M_{r\theta}(r, \theta) = -M_{\theta r}(r, \theta) = \frac{D \psi}{R} (1 - \nu_p)(2c_2 \rho - 2c_3 \rho^{-3} + c_4 \rho^{-1}) \sin \theta \]  
(ref. 34)

**Shear force**

\[ Q_r(r, \theta) = \frac{D \psi}{R}(8c_2 - 2c_3 \rho^{-2}) \cos \theta \]  
(ref. 34)

**Radial stress**

\[ \sigma_r(r, \theta) = \frac{12}{\xi_2} \frac{M_r(r, \theta)}{R} \]  
(ref. 34)

**Tangential stress**

\[ \sigma_\theta(r, \theta) = \frac{12}{\xi_3} \frac{M_\theta(r, \theta)}{R} \]  
(ref. 34)
Shear stress

\[ \tau(r, \theta) = \frac{12}{3\pi} M (r, \theta) \quad \text{(ref. 34)} \]

Moment applied by the shaft

\[ M = 4 \pi D \cdot c_4 \psi \]

4.6 Theoretical Representation of the Scale Model

4.6.1 Effect of the rope pressure

The following, is a general procedure to represent theoretically the effect of the rope pressure on a winding drum:

1 - Identify all the relevant geometrical and loading discontinuities on the roll and build the equations to represent the conditions at each of them.

2 - Examine the joints between the roll and the side plates and write the equations to describe them considering the effects of brake rings if present.

3 - Consider the conditions of support of the side plates at the hubs and formulate their equations.

4 - Put the equations in a form where the only unknowns are the constants of integration for either the drum roll or the side plates.

5 - Solve the resulting system of simultaneous equations.

At this stage, the problem is completely solved. Variables such as deflections, stresses, strains, etc... can be determined by numerical substitution into the equations expressing them.
Following the steps of the procedure, the theoretical representation of the scale model started with the identification in step 1, of the discontinuities on the drum roll, marked from 1 to 13 in FIG. 4.28.

![Diagram of discontinuities on the roll](image)

**FIG. 4.28 DISCONTINUITIES ON THE ROLL**

Considering initially the stiffening rings, it was assumed that their force reactions, which were actually distributed along the rim, could be substituted by forces localized at the edges and at the centre of the rim. The force reactions of a ring were included in the theoretical model through the consideration of the cross-section of the ring as in section 4.1.5.

In order to represent the stiffening rings of the scale model, the areas of their cross-sections were assumed to be concentrated at the points of the localized force reactions and distributed as shown in FIG. 4.29. As for

![Diagram of stiffening rings](image)

**FIG. 4.29 REPRESENTATION OF STIFFENING RINGS**
the moment reactions it was assumed that they were localized at the centre of the rim and included into the theoretical model through the consideration of the moments of area of the rings concentrated as shown in FIG. 4.29. In the illustrations in FIGs. 4.29 to 4.31, the various $A_s$ and $I_s$ represent, respectively, the cross-sectional areas and the moments of area of the rings as indicated. The subscripts are only to differentiate between the various parts and rings.

The stiffening ring at the centre of the drum and the drum dividing ring lie on the same centre line and the edges of their rims were almost on the same vertical lines. These rings were represented as shown in FIG. 4.30.

![Actual representation](image_url)

**FIG.4.30 STIFFENING RING AND DIVIDING RING**

The discontinuities marked as 1 and 13, defined the position of the side ledges. These were reinforced by gussets, making difficult the simulation of their force and moment reactions. This was overcome by assuming that the ledges were rings attached to the roll and not to the side plates. The cross-sectional areas and moments of areas were assumed to be concentrated at the edges. The edges of the roll were then assumed to be directly connected to the sides as shown in FIG. 4.31, which, in anticipation to step 2 of the procedure, also shows how the areas $A_b$ and the moments of area $I_b$
The positions marked as 2 and 12 in FIG. 4.28 indicate the last of the dead coils on each side of the drum roll. It was assumed that the pressure transmitted by the dead coils was zero and hence, between positions 2 and 12 and each side plate there was no rope pressure acting on the drum roll.

The examination of the conditions on the drum roll was concluded with the consideration of the pressure applied by the rope, between positions 2 and 6 and 8 and 12, in the following manner:

i - With a single layer of rope on both sides of the drum, the applied pressure \( p_1 \) was calculated with Eq. 2.1 where the rope diameter was substituted by the rope pitch, hence

\[
p_1 = \frac{T}{R \times \text{pitch}}
\]

ii - With two layers acting on one side, the pressure there was taken as \( p_1 = 2 \times p_2 \) while the pressure on the other side was taken as zero.

This determination of the pressures did not take into account the effect of the rope relaxation as discussed in section 2.1. In order to consider this effect, factors of rope relaxation were calculated with the following formula derived by ATKINSON and TAYLOR\(^6\) which gives the factor of relaxation...
FR\textsubscript{n} for n layers of rope and where the parameters and its values were the same as given in section 2.1:

\[
FR\textsubscript{n} = \prod_{i=1}^{n} \left[ \frac{1 + \frac{0.2513K}{1 + (i - 1)K}}{1 + \frac{0.7566K}{1 + (i - 1)K}} \right]\
\]

where

\[
K = \frac{A_R E_R}{Ed t_e}
\]

The net pressure \( p'\) applied by \( n \) layers of rope was given by

\[
p'\textsubscript{n} = FR\textsubscript{n} p_1
\]

The expression has been derived for winding drums without stiffening rings and, to account for them, an equivalent thickness \( t_e \) for the drum roll, obtained as described in section 2.1, was used.

For the scale model the factors of relaxation were \( FR_1 = 0.90 \) and \( FR_2 = 1.65 \).

The equations representing the conditions at each discontinuity point on the scale drum roll are not presented here.

Part of step 2 of the procedure was implemented earlier when the sides and the brake rings were considered. In order to complete this step, it was necessary to represent the side plates which extended beyond the radius \( R \) of the drum roll to a radius \( R_0 \) as shown in FIG. 4.31. In order to do so, each plate was considered as comprising an inner plate of radius \( R \) to whose periphery an outer plate, the drum roll and a brake disk were rigidly joined. The conditions at the joint were described by examination of the equilibrium and continuity between each element, similarly to that developed in section 4.3. However, the inclusion of an outer plate meant that four additional constants of integration had to be determined to describe its behaviour. The conditions at the joint provided only two additional equations, namely, that the inner and outer plates would have the same deflections and same
slopes at that point. Two more equations were obtained by considering that the radial moment and the shear force at the free edge of the outer plate were zero.

The representation of the conditions on the side plates was completed with step 3 where the plates were assumed to be built into perfectly rigid supports at the pitch circle diameter of the bolts.

The side plates of the scale model had a ring of six holes symmetrically placed, as shown in drawing no.79V009/XD108 19 in Appendix 1. In order to overcome the computational complexity of a more precise method of analysis, such as that presented by KRAUS\textsuperscript{35}, it was assumed that the side plates would behave, as a whole, like solid ones.

The theoretical model built with the foregoing assumptions, reduced the scale model problem to the solution of a system of seventy two simultaneous equations interrelating the unknown constants of integration, which would permit the determination of deflections, strains, stresses, etc... at any point on the drum. In order to manipulate such a system of equations, a computer program written in enhanced \textit{BASIC} language was prepared for a Hewlett-Packard 9845B desktop computer available at the Engineering Design Centre. The program builds the whole system of equations arranging the coefficients of the constants of integration in a matrix and the independent term of each equation into a vector. The matrix is then inverted and multiplied by the vector, resulting in another vector whose elements are the constants sought. The program can also provide the display of the values of any variable along the roll or the side plates in form of a graph on a visual display screen. Although written specifically for the scale model, the program structure is such that it can be generalized to enable the analysis of other winding drum configurations. A program listing for the scale model is presented in Appendix 4.

4.6.2 Effect of the bent shaft

The procedure to represent theoretically the effect of the bent shaft on
the side plates requires the following steps:

1 - Consider the shaft as a beam and define its loading configuration representing the influence of the side plates as moment reactions applied at the position of the hubs.

2 - Write the system of equations representing the conditions of the shaft at each geometrical and loading discontinuity.

The solution of this system of equations would give the values of the moment reactions of the side plates. These reactions are equal and opposite in sign to the moments applied by the shaft to the side plates with which the deflections, stresses, strains, etc... at any point on the side plates can be determined as discussed in section 4.4.

In the first part of the procedure to represent the scale model, the shaft loading configuration was considered as shown in FIG. 4.32 where the notation is

![FIG. 4.32 MODEL SHAFT LOADING](image)

As the moment reactions \( M_1 \) and \( M_2 \) were unknown, the system of equations written in the second part of the procedure was indeterminate. However, from Eq. 4.63, these moments could be related to the slopes \( \psi_1 \) and \( \psi_2 \), respectively at points 1 and 2 of the shaft, by the expressions
These two additional expressions permitted the determination of the system of equations.

The solution of the system was obtained with the program presented in Appendix 4.

As before, the side plates have been assumed to be solid.
CHAPTER 5
TEST RESULTS

The main objectives of the tests with the model drum were:

i - To observe the mechanical behaviour of the drum under different conditions and investigate how this behaviour was affected by factors such as the drum position, the loads, the brake rings and the drum split.

ii - To compare the results with the predictions made using the theoretical models discussed in chapter 4.

This chapter deals not only with the tests performed on the drum but also with the fulfilment of the first objective; the second objective is the subject of the next chapter.

The information required for the accomplishment of both objectives was obtained through the measurements of strains at several points on the model drum. The test rig and the instrumentation were described in chapter 3, however, it is necessary to discuss the layout of the strain gauges in more detail.

5.1 Layout of the Strain Gauges

The strain gauges, whose layout is shown in FIG. 5.1, were placed either on a plane AA away from the drum split or on a plane BB near the split. The purpose of those on plane AA was to show the pattern of strains in the drum without the influence of the discontinuity due to the split; those on plane BB were intended to show the effect of its influence.

The gauges nos. 1 to 18 and 38 to 45 were placed on the inside of the drum roll on either the circumferential or axial directions. Results will not be given for gauge no. 1 as it was damaged during the drum assembly. Gauge no. 19, which was to be fitted to the outside of the drum roll was
Instrumented drum side viewed from inside

Strain gauges numbered in brackets were outside the drum

FIG. 5.1 LAYOUT OF STRAIN GAUGES
FIG. 5.1 LAYOUT OF STRAIN GAUGES (continued)
The gauges on the side plates, nos. 20 to 37, were placed in pairs, one on the outside and the other on the inside as mirror images of each other, in either the radial or tangential directions.

During a first group of tests referred to in section 5.4, gauge no. 31 was inoperative but was in operation for the other groups of tests.

Three gauges forming a rectangular rosette, nos. 46 to 48, were fitted on one of the plates used for strapping the roll halves together.

Six bolts in all were instrumented. Two were placed at the joint between the roll and the side support; one of them, no. 49, being away from the split and the other, no. 50, near the split. Two other bolts, nos. 51 and 52, were at the strapping plates. The remaining two, nos. 53 and 54, were at the joint of the side plate halves.

The gauges on the instrumented bolts were fitted diametrically opposite to each other, as shown in FIG. 5.2, and connected in series in an arrangement which cancels the strains due to bending showing only those due either to axial tension or compression. In order to provide room for the gauges, the instrumented bolts nos. 49 to 52 were bigger than the scaled bolts on the model but the grooves on them were machined to leave a cross-sectional area approximately equal to that of the scaled bolts. The instrumented
bolts nos. 53 and 54 were the same as the scaled bolts so that the machining of the grooves left them with a cross-sectional area only one half their original area.

After all tests were performed it was found that the strain gauges on the instrumented bolt no. 50 were faulty; for this reason, the results from this bolt have been omitted.

Strain gauges nos. 46 to 54 were exploratory gauges for which no theoretical analysis was done in order to predict the strains at them.

Two strain gauges, nos. 55 and 56, were placed on the shaft in the axial direction midway between the hubs. These gauges were fitted mainly for control purposes as the behaviour of the shaft could be predicted theoretically without difficulty.

For purposes of identification, the side of the drum with the majority of gauges will be referred as the "instrumented side" while the side with the minority of gauges will be referred as the "non-instrumented side" as indicated in FIG. 5.1b.

5.2 Test Conditions

5.2.1 Loading conditions

Tests were carried out for the following five static loading conditions:

1 - No load condition

For this case the untensioned rope was wound as shown in FIG. 5.3b; the weight carrier, referred in FIG. 3.3, being supported by means of two chain hoists. In this situation, the drum was subjected to its self-weight plus the rope self-weight.

The observations at this loading condition were necessary in order to
FIG. 5.3 LOADING CONFIGURATIONS
set the datum for the effects of the other loading configurations. It also served as a means of assessing the effects of the drum and rope self-weight.

2 - Vertical load condition
With the untensioned rope as in the configuration for the previous case, the carrier was loaded and the hoists removed. The drum was then subjected to a vertical load in addition to the self-weight due to the drum plus rope.

Tests with this loading configuration were required in order to observe the effects on the drum of the vertical load alone.

3 - One layer of rope on both sides of the drum
This loading condition had the configuration shown in FIG. 5.3b where the whole rope, with the exception of the dead coils, was tensioned.

This loading condition will also be referred as "one layer of rope".

4 - Two layers of rope on the instrumented side
For this situation, shown in FIG. 5.3a, both layers of rope were wound under tension on the instrumented side of the drum, the dead coils remaining untensioned.

5 - Two layers of rope on the non-instrumented side
This configuration, shown in FIG. 5.3c, is a reflection of the previous condition. Both layers of rope were wound under tension on the non-instrumented side, the dead coils remaining untensioned.

In the last three loading configurations the drum was simultaneously subjected to its self-weight, to the rope self-weight, to the vertical load and to the rope compression. The fourth and fifth configurations represented the most severe loading conditions enforced on the drum, which occur at the beginning or at the end of the winding cycle. The third configuration
represented the loading condition occurring at the middle of the winding cycle.

5.2.2 Loads

The self-weight of the drum was 7260N, excluding the shaft, and the weight of the rope was 1080N.

The vertical load applied in most of the tests was 21.0kN which, for the full size drum, was equivalent to the weight of the suspended rope plus the weight of both conveyances loaded with 80% of the design payload.

To evaluate the effect of different loads tests were also carried out with loads of 27.8kN and 14.4kN.

5.2.3 Drum configuration

Most of the tests were performed without the brake rings bolted in position but hanging on the drum shaft. Later, they were bolted in position; this configuration will be referred to as the "complete model".

5.2.4 Drum position

One of the aspects to be observed in the tests was the distribution of strains around the drum circumference. Since the main loads were acting either vertically or radially, it was considered sufficient to measure strains with the gauges at four positions, namely, at the top and bottom of a vertical plane containing the drum axis and on the right and left hand sides of a horizontal plane also containing the drum axis.

This choice of positions was based on the assumption that the whole drum would behave like a beam, hence, a gauge positioned in the horizontal plane would not show strains caused by vertical loads, whilst a gauge at the top or bottom positions would show the effect of these loads. The effect of
radial loads would remain constant at all positions.

The four drum positions will be referred as the "reading positions".

5.3 Test Procedures

For each strain gauge a "reading" is defined as the voltage across its Wheatstone bridge, recorded at a given instant. A "set of readings" is defined as the readings for all strain gauges at all four drum positions for a given loading condition.

The main operation carried out during the tests was the setting of the drum into a loading condition and the recording of a set of readings. The reading positions were marked from 1 to 4 on the drum and the readings of all sets were recorded after moving each mark, successively, to the top position starting with position 1.

All the tests carried out with the various drum configurations and loads followed a similar general procedure. They were commenced by putting the drum in the desired configuration, setting it in the no load condition and taking a set of readings. The drum was rotated through the four positions and finally back to the first position. The carrier was loaded with the required weight and the chain hoists removed; the drum was then in the vertical loading condition. After taking readings at the first drum position, the carrier was lifted by the hoists, the drum rotated to the next position, where the hoists were again removed and the readings for this position taken. These steps were repeated until the readings at the fourth position had been taken thus completing the set of readings. This operation was to prevent the rope becoming tensioned around the drum. In actual fact, it was impossible to prevent the first two to four coils of the rope being tensioned. However, the method adopted helped to minimize the effect.

The next action in the procedure was the tensioning of the rope which was achieved by rotating the loaded drum. First, the layer of rope on the instrumented side of the drum was transferred as the second layer on the
The non-instrumented side. Both layers were then transferred, under tension, back to the instrumented side where a set of readings would be taken. At this stage, the loading conditions could be alternatively changed to a single layer of rope on both sides of the drum or two layers on either side. Where it was necessary to repeat a set of readings, the procedure adopted was to move the drum to another condition and then return to the loading condition to be repeated.

When changing the magnitude of the vertical load, the weight carrier had to be hoisted up, the tension on the rope relaxed by rotating the drum in the same manner as to tension it and the whole procedure repeated with the new load.

As part of the general procedure, the environment temperature was monitored at the beginning and at the end of each set of readings and their average recorded.

5.4 Tests Performed

Before describing the tests in more detail, it is necessary to define a "group of tests" as the collection of all sets of readings taken with a given load and a given drum configuration.

The sequence of tests started with a preliminary group being carried out to check various aspects of the experiment such as the performance of the instrumentation, the consistency of readings, the temperature effect and test procedures. The results of these tests are not presented in this dissertation.

Following this, five groups of tests were performed generating approximately 10,000 recorded values which are listed in Appendix 5. The tests have been classified by sets of readings numbered from 1 to 46. A summary of the tests is presented in table 5.1 where the number of the sets taken for each condition are shown.
### Table 5.1
Sets of readings taken in each group of tests

<table>
<thead>
<tr>
<th>Group of tests</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>brake rings not fitted</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>brake rings fitted</td>
</tr>
<tr>
<td>digital multimeter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21.0 kN</td>
<td>21.0 kN</td>
<td>14.4 kN</td>
<td>27.5 kN</td>
<td>21.0 kN</td>
<td></td>
</tr>
<tr>
<td>no load</td>
<td>1</td>
<td>18</td>
<td>29</td>
<td>35</td>
<td>41</td>
</tr>
<tr>
<td>vertical load</td>
<td>2</td>
<td>19</td>
<td>30</td>
<td>36</td>
<td>42</td>
</tr>
<tr>
<td>one layer of rope</td>
<td>3, 4, 6, 8, 9, 11, 12, 14, 15, 17</td>
<td>21, 22, 24</td>
<td>32, 34</td>
<td>36, 40</td>
<td>44, 46</td>
</tr>
<tr>
<td>two layers on the instrumented side</td>
<td>5, 7, 10, 13</td>
<td>23, 27</td>
<td>31</td>
<td>37</td>
<td>43</td>
</tr>
<tr>
<td>two layers on the non-instrumented side</td>
<td>16</td>
<td>20, 25</td>
<td>33</td>
<td>39</td>
<td>45</td>
</tr>
<tr>
<td>total of sets</td>
<td>17</td>
<td>11</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>
The first group of tests was carried out using a multimeter as the read-out equipment. For the other groups, carried out a few weeks later, a data logger was used as the read-out equipment.

The second group of tests was a repetition of the first group.

5.5 Data Manipulation

All the test data, recorded in micro-volts, was keyed into a Hewlett-Packard 9845B desktop computer, checked and stored on a magnetic tape.

The first step on the data manipulation was the correction for temperature using the method described in Appendix 2 and adopting a temperature of 20°C as datum.

Secondly, the readings were converted from micro-volts to strains by multiplying them by the factor of proportionality, \(2.506 \times 10^{-7} \, \text{c/\mu V}\), between strain and voltage established in Appendix 2. The symbol \(\varepsilon\) will be used to represent strain for the remainder of this dissertation.

After these two steps, the results were analysed as described in the following sections.

5.6 Analysis of the Results

5.6.1 Variation of the measurements

In order to assess the degree of variation in the measurements, the sets of readings taken in the first group of tests with one layer of rope on both sides of the drum were used as a basis because they were the sets with the greatest number of repetitions. Estimates of the standard deviations of the measurements for each drum position and each gauge were calculated using the following expression given by DAVIES\(^{36}\), ch. 3:
\[ s_{ij} = \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} (\bar{x}_{ij} - x_{ijk})^2} \]

where

- \( s_{ij} \) - standard deviation for gauge \( i \) at position \( j \)
- \( x_{ijk} \) - readings of gauge \( i \) at position \( j \) in set \( k \)
- \( n \) - number of sets considered

**In calculating the standard deviations one of the ten sets of readings, no. 17, was not included as it showed a discrepancy in relation to the others identified as a settlement in the drum structure which is discussed in section 5.6.7.**

The calculated standard deviations are shown in Table 5.2 where the position of the gauge on the drum is identified. The direction of some of the gauges is indicated by the letters c and a respectively for the circumferential and axial directions on the drum and by the letters t and r respectively for the tangential and radial directions on the side plates.

From the table, it can be seen that for most gauges the deviations for each of the four positions are very close, indicating a common source of error; possibly the accuracy of the read-out equipment which, in this case was the multimeter with an accuracy of \( \pm 20 \mu V \). The differences in the deviations between gauges is possibly due to an imperfect correction for temperature effects, resulting in a bias in the readings. The temperature during tests was measured outside the drum when, ideally, it should have been taken at each strain gauge wire but, this was impracticable.

Table 5.2 also shows the standard deviations, \( s_i \), for each gauge \( i \), which were calculated with the expression

\[ s_i = \sqrt{\frac{1}{4} \sum_{j=1}^{4} s_{ij}^2} \]
### Table 5.2

**Standard deviations (micro-strains)**

First group of tests – readings for one layer of rope

<table>
<thead>
<tr>
<th>Gauge</th>
<th>For the drum position</th>
<th>For the gauge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top</td>
<td>Right</td>
</tr>
<tr>
<td>-c-</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>a</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>a</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>a</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>a</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>a</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>a</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>c</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>a</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>c</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>a</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>c</td>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td>a</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>c</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>a</td>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>c</td>
<td>17</td>
<td>7</td>
</tr>
<tr>
<td>-a-</td>
<td>18</td>
<td>15</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-t-</td>
<td>20</td>
<td>6</td>
</tr>
<tr>
<td>r</td>
<td>21</td>
<td>6</td>
</tr>
<tr>
<td>r</td>
<td>22</td>
<td>2</td>
</tr>
<tr>
<td>r</td>
<td>23</td>
<td>6</td>
</tr>
<tr>
<td>t</td>
<td>24</td>
<td>5</td>
</tr>
<tr>
<td>t</td>
<td>25</td>
<td>3</td>
</tr>
<tr>
<td>r</td>
<td>26</td>
<td>7</td>
</tr>
<tr>
<td>r</td>
<td>27</td>
<td>3</td>
</tr>
<tr>
<td>t</td>
<td>28</td>
<td>6</td>
</tr>
<tr>
<td>t</td>
<td>29</td>
<td>5</td>
</tr>
<tr>
<td>r</td>
<td>30</td>
<td>7</td>
</tr>
<tr>
<td>r</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>r</td>
<td>32</td>
<td>7</td>
</tr>
<tr>
<td>t</td>
<td>33</td>
<td>8</td>
</tr>
<tr>
<td>t</td>
<td>34</td>
<td>3</td>
</tr>
<tr>
<td>r</td>
<td>35</td>
<td>4</td>
</tr>
<tr>
<td>r</td>
<td>36</td>
<td>8</td>
</tr>
<tr>
<td>-t-</td>
<td>37</td>
<td>14</td>
</tr>
<tr>
<td>-c-</td>
<td>38</td>
<td>5</td>
</tr>
<tr>
<td>a</td>
<td>39</td>
<td>7</td>
</tr>
<tr>
<td>c</td>
<td>40</td>
<td>5</td>
</tr>
<tr>
<td>c</td>
<td>41</td>
<td>9</td>
</tr>
<tr>
<td>a</td>
<td>42</td>
<td>14</td>
</tr>
<tr>
<td>c</td>
<td>43</td>
<td>3</td>
</tr>
<tr>
<td>c</td>
<td>44</td>
<td>4</td>
</tr>
<tr>
<td>-c-</td>
<td>45</td>
<td>3</td>
</tr>
<tr>
<td>---</td>
<td>46</td>
<td>6</td>
</tr>
<tr>
<td>r</td>
<td>47</td>
<td>13</td>
</tr>
<tr>
<td>---</td>
<td>48</td>
<td>4</td>
</tr>
<tr>
<td>---</td>
<td>49</td>
<td>9</td>
</tr>
<tr>
<td>a</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>r</td>
<td>51</td>
<td>9</td>
</tr>
<tr>
<td>r</td>
<td>52</td>
<td>5</td>
</tr>
<tr>
<td>---</td>
<td>53</td>
<td>22</td>
</tr>
<tr>
<td>---</td>
<td>54</td>
<td>11</td>
</tr>
<tr>
<td>---</td>
<td>55</td>
<td>11</td>
</tr>
<tr>
<td>---</td>
<td>56</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In calculating these deviations it was assumed that, for each gauge, the distributions of the errors in the readings for each position were the same, though, the averages were not necessarily so. By further assuming that the distributions were the same for any position and any gauge, the standard deviation $s$, for all nine sets of readings, was calculated using the expression

$$s = \sqrt{\frac{\sum_{i,j} s^2_{ij}}{4 \times 52}}$$

The standard deviation of $\mu_e$ calculated as above can be taken as the standard deviation for the whole experiment by further extending the assumption of the same distribution to any group of tests.

For the second to the fifth group of tests, the error is likely to be less since their readings were taken with a data logger whose accuracy, ±10μV, was greater than that of the multimeter used for the readings in the first group of tests.

5.6.2 Effect of the drum position

One of the factors to be examined in the test results, concerned the extent to which the drum position influenced the levels of strains on each gauge. In order to investigate this effect, the same nine sets of readings used in the previous section were taken as a reference. Then, the average $\bar{x}_{ik}$ of the strains at the reading positions was calculated for each gauge $i$ in each set $k$ with the expression

$$\bar{x}_{ik} = \frac{\sum_{j=1,2,3,4} x_{ijk}}{4}$$

Subsequently, the average for each gauge was subtracted from the strains at each reading position in the expression

$$e_{ijk} = x_{ijk} - \bar{x}_{ik} \quad (j=1,2,3,4)$$
The four residues $e_{ijk}$ obtained, should give an indication of the effect of the drum position. If they were all zero or very small it would mean that the values of the strains at each position were close to each other and, the influence of the drum position would be zero or very small. The greater the difference in the values of the strains the greater the residues and the effect of the drum position.

Considering the residues as indicators of such an effect, a measure could be the average $\overline{e_i}$ of the absolute values of all residues associated with a gauge and given by

$$\overline{e_i} = \frac{\sum |e_{ijk}|}{k_j}$$

Table 5.3 shows the averages for all gauges and, it will be noted that, gauges nos. 55 and 56 are those with the highest deviations. Indeed, these were the gauges on the shaft where the strains clearly depend on the position of the drum.

The gauges with the next greatest deviations were nos. 23, 26 and 35, positioned in the radial direction on the side plate near the hub where the effect of the bent shaft was the greatest. Oddly, gauge no. 32 which is in the same position of the other three did not show the same effect.

Gauges nos. 53 and 54, on the instrumented bolts at the side joint, also showed some of the effect of the drum position but, for the other gauges, it was difficult to know whether the results on the table were due to this effect or purely due to the variations in the measurements.

A conclusion that could be drawn from this analysis was that, the effect of the drum position was most significant at the shaft and at the side plate near the hub. At the other points, it was not great enough to be detected within the level of accuracy present in the readings.
Table 5.3
Average influence of the drum position (in micro-strains)

<table>
<thead>
<tr>
<th>Gauge</th>
<th>-c-</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>-a-</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-t-</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>t</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>-t-</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>-c-</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>-c-</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>56</td>
</tr>
</tbody>
</table>
5.6.3 Isolation of the effect of each loading

For purposes of analysis, it was necessary to isolate the effects of the rope pressure, vertical load and drum and rope self-weights.

The effect of rope pressure was present in the sets of readings obtained for the loading conditions with one layer of rope on both sides of the drum and with two layers on either side. These readings also contained the effects of drum and rope self-weights and of the vertical load. These two effects together were present in the set of readings obtained for the vertical loading condition. Thus, to isolate the effect of the rope pressure, the readings from the vertical loading condition set at the top, right-hand side, bottom and left-hand side positions of each gauge were subtracted from the readings at the same positions in the sets containing the rope pressure effect.

The vertical load effect was isolated by subtracting from the readings in the vertical loading condition set those obtained with the no load condition which, comprised only the effects of drum and rope self-weights.

The effect of drum and rope self-weights could not be isolated as the set of readings from the no load condition was used as datum. However, assuming that the drum behaved like a beam, the maximum strain induced by the self weight on each gauge could be assessed by taking the average of the difference between the absolute values of the readings at the top and bottom positions.

5.6.4 Presentation of the results

The results for each group of tests are presented in Tables 5.4 to 5.8 showing the isolated effect of each loading condition. In order to obtain the figures given in columns (C), (D) and (E) the values for all drum positions and for all sets in the same loading condition were averaged. A positive value indicates tensile strain and a negative value, compressive strain.
### Table 5.4
FIRST GROUP OF TESTS
Micro-strains induced by a load of 21.0 kN

<table>
<thead>
<tr>
<th>Gauge</th>
<th>Loading conditions</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-c- 1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>a 2</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>-1</td>
<td>15</td>
<td>-10</td>
</tr>
<tr>
<td>c 3</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>-47</td>
<td>-136</td>
<td>-7</td>
</tr>
<tr>
<td>a 4</td>
<td>-</td>
<td>3</td>
<td>0</td>
<td>54</td>
<td>152</td>
<td>32</td>
</tr>
<tr>
<td>c 5</td>
<td>-</td>
<td>3</td>
<td>0</td>
<td>-135</td>
<td>-300</td>
<td>-16</td>
</tr>
<tr>
<td>a 6</td>
<td>-</td>
<td>5</td>
<td>0</td>
<td>85</td>
<td>237</td>
<td>-14</td>
</tr>
<tr>
<td>c 7</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>-164</td>
<td>-333</td>
<td>-7</td>
</tr>
<tr>
<td>a 8</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>113</td>
<td>219</td>
<td>29</td>
</tr>
<tr>
<td>c 9</td>
<td>-</td>
<td>3</td>
<td>0</td>
<td>-160</td>
<td>-320</td>
<td>19</td>
</tr>
<tr>
<td>a 10</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>95</td>
<td>222</td>
<td>-42</td>
</tr>
<tr>
<td>c 11</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>-149</td>
<td>-280</td>
<td>15</td>
</tr>
<tr>
<td>a 12</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>79</td>
<td>238</td>
<td>-102</td>
</tr>
<tr>
<td>c 13</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>-87</td>
<td>-143</td>
<td>-12</td>
</tr>
<tr>
<td>a 14</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>-46</td>
<td>19</td>
<td>-117</td>
</tr>
<tr>
<td>c 15</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>-51</td>
<td>-51</td>
<td>-41</td>
</tr>
<tr>
<td>a 16</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>-120</td>
<td>-101</td>
<td>-116</td>
</tr>
<tr>
<td>c 17</td>
<td>-</td>
<td>3</td>
<td>5</td>
<td>-128</td>
<td>13</td>
<td>-255</td>
</tr>
<tr>
<td>-a- 18</td>
<td>-</td>
<td>0</td>
<td>10</td>
<td>64</td>
<td>-104</td>
<td>203</td>
</tr>
<tr>
<td>a 19</td>
<td>-</td>
<td>5</td>
<td>3</td>
<td>39</td>
<td>44</td>
<td>18</td>
</tr>
<tr>
<td>-t- 20</td>
<td>-</td>
<td>3</td>
<td>5</td>
<td>40</td>
<td>36</td>
<td>33</td>
</tr>
<tr>
<td>r 21</td>
<td>5</td>
<td>3</td>
<td>10</td>
<td>56</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>r 22</td>
<td>-</td>
<td>3</td>
<td>5</td>
<td>41</td>
<td>38</td>
<td>85</td>
</tr>
<tr>
<td>r 23</td>
<td>10</td>
<td>3</td>
<td>5</td>
<td>60</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>r 24</td>
<td>-</td>
<td>3</td>
<td>5</td>
<td>60</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>r 25</td>
<td>-</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>r 26</td>
<td>10</td>
<td>3</td>
<td>5</td>
<td>-78</td>
<td>-48</td>
<td>-94</td>
</tr>
<tr>
<td>r 27</td>
<td>3</td>
<td>8</td>
<td>37</td>
<td>31</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>r 28</td>
<td>3</td>
<td>3</td>
<td>12</td>
<td>15</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>r 29</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>20</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>r 30</td>
<td>3</td>
<td>8</td>
<td>-56</td>
<td>-65</td>
<td>-64</td>
<td></td>
</tr>
<tr>
<td>r 31</td>
<td>5</td>
<td>10</td>
<td>12</td>
<td>15</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>r 32</td>
<td>3</td>
<td>3</td>
<td>-160</td>
<td>-14</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>r 33</td>
<td>3</td>
<td>8</td>
<td>-56</td>
<td>-65</td>
<td>-64</td>
<td></td>
</tr>
<tr>
<td>r 34</td>
<td>3</td>
<td>3</td>
<td>-27</td>
<td>-67</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>r 35</td>
<td>8</td>
<td>18</td>
<td>40</td>
<td>10</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>r 36</td>
<td>3</td>
<td>12</td>
<td>26</td>
<td>4</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>r 37</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>2</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>r 38</td>
<td>8</td>
<td>18</td>
<td>40</td>
<td>10</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>-t- 39</td>
<td>-</td>
<td>3</td>
<td>5</td>
<td>-18</td>
<td>-36</td>
<td>-37</td>
</tr>
<tr>
<td>-c- 40</td>
<td>-</td>
<td>3</td>
<td>3</td>
<td>-27</td>
<td>-67</td>
<td>7</td>
</tr>
<tr>
<td>a 41</td>
<td>0</td>
<td>3</td>
<td>19</td>
<td>46</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>c 42</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-40</td>
<td>-132</td>
<td>6</td>
</tr>
<tr>
<td>a 43</td>
<td>3</td>
<td>0</td>
<td>-212</td>
<td>-314</td>
<td>-18</td>
<td></td>
</tr>
<tr>
<td>c 44</td>
<td>0</td>
<td>3</td>
<td>-160</td>
<td>-11</td>
<td>-309</td>
<td></td>
</tr>
<tr>
<td>-c- 45</td>
<td>0</td>
<td>3</td>
<td>-160</td>
<td>-12</td>
<td>-299</td>
<td></td>
</tr>
<tr>
<td>-t- 46</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>66</td>
<td>-56</td>
<td></td>
</tr>
<tr>
<td>-c- 47</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>66</td>
<td>-56</td>
<td></td>
</tr>
<tr>
<td>-t- 48</td>
<td>3</td>
<td>0</td>
<td>-48</td>
<td>-60</td>
<td>-46</td>
<td></td>
</tr>
<tr>
<td>-c- 49</td>
<td>3</td>
<td>0</td>
<td>-40</td>
<td>-40</td>
<td>-46</td>
<td></td>
</tr>
<tr>
<td>-t- 50</td>
<td>3</td>
<td>3</td>
<td>-40</td>
<td>-40</td>
<td>-46</td>
<td></td>
</tr>
<tr>
<td>-c- 51</td>
<td>3</td>
<td>0</td>
<td>-40</td>
<td>-40</td>
<td>-46</td>
<td></td>
</tr>
<tr>
<td>-t- 52</td>
<td>3</td>
<td>3</td>
<td>-40</td>
<td>-40</td>
<td>-46</td>
<td></td>
</tr>
<tr>
<td>-c- 53</td>
<td>3</td>
<td>3</td>
<td>-40</td>
<td>-40</td>
<td>-46</td>
<td></td>
</tr>
<tr>
<td>-t- 54</td>
<td>3</td>
<td>3</td>
<td>-40</td>
<td>-40</td>
<td>-46</td>
<td></td>
</tr>
<tr>
<td>-c- 55</td>
<td>3</td>
<td>3</td>
<td>-40</td>
<td>-40</td>
<td>-46</td>
<td></td>
</tr>
<tr>
<td>-t- 56</td>
<td>3</td>
<td>3</td>
<td>-40</td>
<td>-40</td>
<td>-46</td>
<td></td>
</tr>
</tbody>
</table>

**Where:**

- **(A)** No load
- **(B)** Vertical load
- **(C)** One layer of rope on both sides
- **(D)** Two layers of rope on the instrumented side
- **(E)** Two layers of rope on the non-instrumented side
Table 5.5
SECOND GROUP OF TESTS
Micro-strains induced by a load of 21.0 kN

<table>
<thead>
<tr>
<th>Gauge</th>
<th>Loading conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A)</td>
</tr>
<tr>
<td>(A) No load</td>
<td></td>
</tr>
<tr>
<td>(B) Vertical load</td>
<td></td>
</tr>
<tr>
<td>(C) One layer of rope on both sides</td>
<td></td>
</tr>
<tr>
<td>(D) Two layers of rope on the instrumented side</td>
<td></td>
</tr>
<tr>
<td>(E) Two layers of rope on the non-instrumented side</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Roll</th>
<th>Side plate</th>
<th>Roll</th>
<th>Strap plate</th>
<th>Bolts shaft</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>B</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>C</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>D</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>E</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Roll</th>
<th>Side plate</th>
<th>Roll</th>
<th>Strap plate</th>
<th>Bolts shaft</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>B</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>C</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>D</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>E</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Roll</th>
<th>Side plate</th>
<th>Roll</th>
<th>Strap plate</th>
<th>Bolts shaft</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>B</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>C</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>D</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>E</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Where:

(A) No load
(B) Vertical load
(C) One layer of rope on both sides
(D) Two layers of rope on the instrumented side
(E) Two layers of rope on the non-instrumented side
### Table 5.6

**THIRD GROUP OF TESTS**

Micro-strains induced by a load of 14.4 kN

<table>
<thead>
<tr>
<th>Gauge</th>
<th>Loading conditions</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>16</td>
<td>27</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>5</td>
<td>-62</td>
<td>-120</td>
<td>-9</td>
</tr>
<tr>
<td>-c-</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>75</td>
<td>138</td>
<td>31</td>
</tr>
<tr>
<td>-c-</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>-124</td>
<td>-229</td>
<td>-3</td>
</tr>
<tr>
<td>-c-</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>111</td>
<td>208</td>
<td>18</td>
</tr>
<tr>
<td>-c-</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>91</td>
<td>161</td>
<td>13</td>
</tr>
<tr>
<td>-c-</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>5</td>
<td>-104</td>
<td>-214</td>
<td>21</td>
</tr>
<tr>
<td>-c-</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>5</td>
<td>77</td>
<td>157</td>
<td>-25</td>
</tr>
<tr>
<td>-c-</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>-92</td>
<td>-176</td>
<td>23</td>
</tr>
<tr>
<td>-c-</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>13</td>
<td>42</td>
<td>139</td>
<td>-79</td>
</tr>
<tr>
<td>-c-</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>8</td>
<td>-50</td>
<td>-83</td>
<td>3</td>
</tr>
<tr>
<td>-c-</td>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>-52</td>
<td>-17</td>
<td>-90</td>
</tr>
<tr>
<td>-c-</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>8</td>
<td>-37</td>
<td>-37</td>
<td>-22</td>
</tr>
<tr>
<td>-c-</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>-83</td>
<td>-74</td>
<td>-77</td>
</tr>
<tr>
<td>-c-</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>10</td>
<td>-93</td>
<td>3</td>
<td>-162</td>
</tr>
<tr>
<td>-c-</td>
<td>17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>5</td>
<td>53</td>
<td>-53</td>
<td>145</td>
</tr>
<tr>
<td>-t-</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>16</td>
<td>29</td>
<td>7</td>
</tr>
<tr>
<td>-t-</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0</td>
<td>24</td>
<td>23</td>
<td>7</td>
</tr>
<tr>
<td>-t-</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>3</td>
<td>40</td>
<td>44</td>
<td>32</td>
</tr>
<tr>
<td>-t-</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>18</td>
<td>-39</td>
<td>-18</td>
<td>-51</td>
</tr>
<tr>
<td>-t-</td>
<td>22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>5</td>
<td>-24</td>
<td>-19</td>
<td>-24</td>
</tr>
<tr>
<td>-t-</td>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>-t-</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>13</td>
<td>18</td>
<td>-49</td>
<td>-29</td>
<td>-53</td>
</tr>
<tr>
<td>-t-</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>3</td>
<td>16</td>
<td>16</td>
<td>19</td>
</tr>
<tr>
<td>-t-</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>8</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>-t-</td>
<td>27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>-7</td>
<td>3</td>
</tr>
<tr>
<td>-t-</td>
<td>28</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>8</td>
<td>-38</td>
<td>-38</td>
<td>-33</td>
</tr>
<tr>
<td>-t-</td>
<td>29</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>8</td>
<td>-51</td>
<td>-58</td>
<td>-37</td>
</tr>
<tr>
<td>-t-</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>8</td>
<td>-81</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>-t-</td>
<td>31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>25</td>
<td>4</td>
<td>45</td>
</tr>
<tr>
<td>-t-</td>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>-t-</td>
<td>33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>10</td>
<td>23</td>
<td>-6</td>
<td>43</td>
</tr>
<tr>
<td>-t-</td>
<td>34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>8</td>
<td>-24</td>
<td>-19</td>
<td>-20</td>
</tr>
<tr>
<td>-t-</td>
<td>35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>3</td>
<td>-9</td>
<td>-21</td>
<td>-3</td>
</tr>
<tr>
<td>-c-</td>
<td>36</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>3</td>
<td>-13</td>
<td>-49</td>
<td>6</td>
</tr>
<tr>
<td>-c-</td>
<td>37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>29</td>
<td>45</td>
<td>18</td>
</tr>
<tr>
<td>-c-</td>
<td>38</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>-37</td>
<td>-113</td>
<td>4</td>
</tr>
<tr>
<td>-c-</td>
<td>39</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0</td>
<td>-160</td>
<td>-220</td>
<td>-11</td>
</tr>
<tr>
<td>-c-</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>5</td>
<td>19</td>
<td>136</td>
<td>-91</td>
</tr>
<tr>
<td>-c-</td>
<td>41</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>-104</td>
<td>0</td>
<td>-198</td>
</tr>
<tr>
<td>-c-</td>
<td>42</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0</td>
<td>-110</td>
<td>-1</td>
<td>-204</td>
</tr>
<tr>
<td>-c-</td>
<td>43</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>-107</td>
<td>-1</td>
<td>-200</td>
</tr>
<tr>
<td>-c-</td>
<td>44</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>-30</td>
<td>34</td>
<td>-51</td>
</tr>
<tr>
<td>-c-</td>
<td>45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>-30</td>
<td>34</td>
<td>-51</td>
</tr>
<tr>
<td>-c-</td>
<td>46</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>-6</td>
<td>7</td>
<td>-11</td>
</tr>
<tr>
<td>-c-</td>
<td>47</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>-6</td>
<td>7</td>
<td>-11</td>
</tr>
<tr>
<td>-c-</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>-5</td>
<td>1</td>
</tr>
<tr>
<td>-c-</td>
<td>49</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>-c-</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>22</td>
<td>-11</td>
</tr>
<tr>
<td>-c-</td>
<td>51</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>23</td>
<td>-225</td>
<td>-277</td>
<td>-60</td>
</tr>
<tr>
<td>-c-</td>
<td>52</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>38</td>
<td>-5</td>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>-c-</td>
<td>53</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>40</td>
<td>-9</td>
<td>-9</td>
<td>-6</td>
</tr>
</tbody>
</table>

Where:

(A) No load
(B) Vertical load
(C) One layer of rope on both sides
(D) Two layers of rope on the instrumented side
(E) Two layers of rope on the non-instrumented side
Table 5.7
FOURTH GROUP OF TESTS
Micro-strains induced by a load of 27.8kN

<table>
<thead>
<tr>
<th>Gauge</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a 2</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>17</td>
<td>5</td>
</tr>
<tr>
<td>c 3</td>
<td>3</td>
<td>3</td>
<td>-128</td>
<td>-244</td>
<td>-23</td>
</tr>
<tr>
<td>a 4</td>
<td>0</td>
<td>3</td>
<td>117</td>
<td>216</td>
<td>45</td>
</tr>
<tr>
<td>c 5</td>
<td>3</td>
<td>3</td>
<td>-256</td>
<td>-471</td>
<td>-17</td>
</tr>
<tr>
<td>a 6</td>
<td>3</td>
<td>3</td>
<td>207</td>
<td>378</td>
<td>30</td>
</tr>
<tr>
<td>c 7</td>
<td>3</td>
<td>3</td>
<td>-270</td>
<td>-515</td>
<td>-1</td>
</tr>
<tr>
<td>a 8</td>
<td>3</td>
<td>5</td>
<td>173</td>
<td>305</td>
<td>10</td>
</tr>
<tr>
<td>c 9</td>
<td>0</td>
<td>10</td>
<td>-236</td>
<td>-469</td>
<td>29</td>
</tr>
<tr>
<td>a 10</td>
<td>3</td>
<td>20</td>
<td>-181</td>
<td>-380</td>
<td>44</td>
</tr>
<tr>
<td>a 12</td>
<td>3</td>
<td>20</td>
<td>86</td>
<td>281</td>
<td>-157</td>
</tr>
<tr>
<td>c 13</td>
<td>3</td>
<td>20</td>
<td>-112</td>
<td>-199</td>
<td>-1</td>
</tr>
<tr>
<td>a 14</td>
<td>3</td>
<td>10</td>
<td>-108</td>
<td>-31</td>
<td>-182</td>
</tr>
<tr>
<td>c 15</td>
<td>3</td>
<td>15</td>
<td>-69</td>
<td>-94</td>
<td>-59</td>
</tr>
<tr>
<td>a 16</td>
<td>0</td>
<td>5</td>
<td>-180</td>
<td>-164</td>
<td>-161</td>
</tr>
<tr>
<td>c 17</td>
<td>0</td>
<td>8</td>
<td>-164</td>
<td>14</td>
<td>-309</td>
</tr>
<tr>
<td>a 18</td>
<td>3</td>
<td>5</td>
<td>83</td>
<td>-141</td>
<td>271</td>
</tr>
<tr>
<td>a 19</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>r 21</td>
<td>3</td>
<td>3</td>
<td>41</td>
<td>34</td>
<td>37</td>
</tr>
<tr>
<td>r 22</td>
<td>3</td>
<td>3</td>
<td>82</td>
<td>91</td>
<td>60</td>
</tr>
<tr>
<td>r 23</td>
<td>8</td>
<td>35</td>
<td>-86</td>
<td>-49</td>
<td>-106</td>
</tr>
<tr>
<td>t 24</td>
<td>3</td>
<td>8</td>
<td>-56</td>
<td>-47</td>
<td>-58</td>
</tr>
<tr>
<td>t 25</td>
<td>0</td>
<td>0</td>
<td>-7</td>
<td>-8</td>
<td>-5</td>
</tr>
<tr>
<td>r 26</td>
<td>13</td>
<td>28</td>
<td>-88</td>
<td>-59</td>
<td>-99</td>
</tr>
<tr>
<td>r 27</td>
<td>3</td>
<td>8</td>
<td>26</td>
<td>35</td>
<td>24</td>
</tr>
<tr>
<td>t 28</td>
<td>0</td>
<td>5</td>
<td>13</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>t 29</td>
<td>3</td>
<td>5</td>
<td>-23</td>
<td>-43</td>
<td>-12</td>
</tr>
<tr>
<td>r 30</td>
<td>0</td>
<td>13</td>
<td>-75</td>
<td>-90</td>
<td>-63</td>
</tr>
<tr>
<td>r 31</td>
<td>0</td>
<td>13</td>
<td>-107</td>
<td>-130</td>
<td>-78</td>
</tr>
<tr>
<td>r 32</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>-11</td>
<td>22</td>
</tr>
<tr>
<td>t 33</td>
<td>0</td>
<td>3</td>
<td>45</td>
<td>3</td>
<td>75</td>
</tr>
<tr>
<td>r 34</td>
<td>0</td>
<td>8</td>
<td>-9</td>
<td>-16</td>
<td>-6</td>
</tr>
<tr>
<td>r 35</td>
<td>8</td>
<td>23</td>
<td>40</td>
<td>4</td>
<td>71</td>
</tr>
<tr>
<td>r 36</td>
<td>3</td>
<td>15</td>
<td>-39</td>
<td>-51</td>
<td>-40</td>
</tr>
<tr>
<td>t 37</td>
<td>0</td>
<td>3</td>
<td>-23</td>
<td>-49</td>
<td>-4</td>
</tr>
<tr>
<td>-c- 38</td>
<td>3</td>
<td>10</td>
<td>-54</td>
<td>-117</td>
<td>-3</td>
</tr>
<tr>
<td>a 39</td>
<td>0</td>
<td>0</td>
<td>23</td>
<td>52</td>
<td>15</td>
</tr>
<tr>
<td>c 40</td>
<td>0</td>
<td>8</td>
<td>-124</td>
<td>-251</td>
<td>-17</td>
</tr>
<tr>
<td>c 41</td>
<td>0</td>
<td>0</td>
<td>-305</td>
<td>-413</td>
<td>-22</td>
</tr>
<tr>
<td>a 42</td>
<td>0</td>
<td>3</td>
<td>75</td>
<td>298</td>
<td>-158</td>
</tr>
<tr>
<td>c 43</td>
<td>3</td>
<td>3</td>
<td>-222</td>
<td>-14</td>
<td>-411</td>
</tr>
<tr>
<td>c 44</td>
<td>0</td>
<td>3</td>
<td>-228</td>
<td>-15</td>
<td>-417</td>
</tr>
<tr>
<td>-c- 45</td>
<td>0</td>
<td>5</td>
<td>-219</td>
<td>-19</td>
<td>-406</td>
</tr>
<tr>
<td>--- 46</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>68</td>
<td>-66</td>
</tr>
<tr>
<td>--- 47</td>
<td>0</td>
<td>3</td>
<td>10</td>
<td>39</td>
<td>-29</td>
</tr>
<tr>
<td>--- 48</td>
<td>0</td>
<td>0</td>
<td>-12</td>
<td>4</td>
<td>-23</td>
</tr>
<tr>
<td>--- 49</td>
<td>3</td>
<td>3</td>
<td>-24</td>
<td>-17</td>
<td>-25</td>
</tr>
<tr>
<td>a 50</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>a 51</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>20</td>
<td>-11</td>
</tr>
<tr>
<td>a 52</td>
<td>0</td>
<td>0</td>
<td>14</td>
<td>61</td>
<td>-22</td>
</tr>
<tr>
<td>a 53</td>
<td>8</td>
<td>58</td>
<td>-353</td>
<td>-377</td>
<td>-206</td>
</tr>
<tr>
<td>--- 54</td>
<td>5</td>
<td>28</td>
<td>-107</td>
<td>-127</td>
<td>-76</td>
</tr>
<tr>
<td>--- 55</td>
<td>25</td>
<td>78</td>
<td>4</td>
<td>-3</td>
<td>6</td>
</tr>
<tr>
<td>--- 56</td>
<td>23</td>
<td>78</td>
<td>9</td>
<td>-1</td>
<td>7</td>
</tr>
</tbody>
</table>

where:

(A) No load
(B) Vertical load
(C) One layer of rope on both sides
(D) Two layers of rope on the instrumented side
(E) Two layers of rope on the non-instrumented side


Table 5.8
Micro-strains induced by a load of 21.0kN (complete model)

<table>
<thead>
<tr>
<th>Gauge</th>
<th>Loading conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A)</td>
</tr>
<tr>
<td>-c-</td>
<td>-</td>
</tr>
<tr>
<td>a</td>
<td>2</td>
</tr>
<tr>
<td>c</td>
<td>3</td>
</tr>
<tr>
<td>a</td>
<td>4</td>
</tr>
<tr>
<td>c</td>
<td>5</td>
</tr>
<tr>
<td>a</td>
<td>6</td>
</tr>
<tr>
<td>c</td>
<td>7</td>
</tr>
<tr>
<td>a</td>
<td>8</td>
</tr>
<tr>
<td>c</td>
<td>9</td>
</tr>
<tr>
<td>a</td>
<td>10</td>
</tr>
<tr>
<td>c</td>
<td>11</td>
</tr>
<tr>
<td>a</td>
<td>12</td>
</tr>
<tr>
<td>c</td>
<td>13</td>
</tr>
<tr>
<td>a</td>
<td>14</td>
</tr>
<tr>
<td>c</td>
<td>15</td>
</tr>
<tr>
<td>a</td>
<td>16</td>
</tr>
<tr>
<td>c</td>
<td>17</td>
</tr>
<tr>
<td>-a-</td>
<td>18</td>
</tr>
<tr>
<td>a</td>
<td>19</td>
</tr>
<tr>
<td>t</td>
<td>20</td>
</tr>
<tr>
<td>r</td>
<td>21</td>
</tr>
<tr>
<td>r</td>
<td>22</td>
</tr>
<tr>
<td>r</td>
<td>23</td>
</tr>
<tr>
<td>t</td>
<td>24</td>
</tr>
<tr>
<td>t</td>
<td>25</td>
</tr>
<tr>
<td>r</td>
<td>26</td>
</tr>
<tr>
<td>r</td>
<td>27</td>
</tr>
<tr>
<td>t</td>
<td>28</td>
</tr>
<tr>
<td>t</td>
<td>29</td>
</tr>
<tr>
<td>r</td>
<td>30</td>
</tr>
<tr>
<td>r</td>
<td>31</td>
</tr>
<tr>
<td>r</td>
<td>32</td>
</tr>
<tr>
<td>t</td>
<td>33</td>
</tr>
<tr>
<td>t</td>
<td>34</td>
</tr>
<tr>
<td>r</td>
<td>35</td>
</tr>
<tr>
<td>r</td>
<td>36</td>
</tr>
<tr>
<td>-t-</td>
<td>37</td>
</tr>
<tr>
<td>-c-</td>
<td>38</td>
</tr>
<tr>
<td>a</td>
<td>39</td>
</tr>
<tr>
<td>c</td>
<td>40</td>
</tr>
<tr>
<td>c</td>
<td>41</td>
</tr>
<tr>
<td>a</td>
<td>42</td>
</tr>
<tr>
<td>c</td>
<td>43</td>
</tr>
<tr>
<td>c</td>
<td>44</td>
</tr>
<tr>
<td>-c-</td>
<td>45</td>
</tr>
<tr>
<td>c</td>
<td>46</td>
</tr>
<tr>
<td>c</td>
<td>47</td>
</tr>
<tr>
<td>c</td>
<td>48</td>
</tr>
<tr>
<td>c</td>
<td>49</td>
</tr>
<tr>
<td>a</td>
<td>50</td>
</tr>
<tr>
<td>b</td>
<td>51</td>
</tr>
<tr>
<td>b</td>
<td>52</td>
</tr>
<tr>
<td>b</td>
<td>53</td>
</tr>
<tr>
<td>b</td>
<td>54</td>
</tr>
<tr>
<td>b</td>
<td>55</td>
</tr>
<tr>
<td>b</td>
<td>56</td>
</tr>
</tbody>
</table>

Where:

(A) No load
(B) Vertical load
(C) One layer of rope on both sides
(D) Two layers of rope on the instrumented side
(E) Two layers of rope on the non-instrumented side.
For the vertical load loading condition, the values given in columns (B) of the tables are the averages of the absolute values of the effects of this loading condition at the top and bottom positions.

For the drum and rope self-weights, the no load condition, columns (A), the figures in the tables are the absolute values of the maxima effects determined as discussed in the previous section.

The discussions presented in the following sections will always refer to the results in Tables 5.4 to 5.8.

5.6:5 Control gauges

For the control gauges, nos. 55 and 56 on the shaft, the strains induced by the self weights were approximately the same for all groups of tests as indeed they should be.

For purposes of comparison, the strains calculated using standard methods for stressing beams, and neglecting the restraint of the side plates, was 23με.

The effect of the vertical load on the control gauges was almost the same for the first, second and fifth group of tests, performed with the same load. The calculated value for this case was 63με.

For the third and fourth group of tests, the results also compared well with the calculated values of 43με and 85με respectively.

In all groups of tests, the strains induced on the control gauges by the pressure due to one or two layers of rope should, theoretically, be zero. The non zero values shown in columns (C), (D) and (E) can be attributed to the accuracy of the instrumentation.

The closeness of values predicted for the control gauges with those obtained in the tests was an indication that the test results could be conside-
red as an adequate representation of the behaviour of the scale model.

The control gauges will not be considered again in the remaining discussion.

5.6.6 Effects of the self-weights and vertical loads

In all groups of tests, the strains induced by the drum and rope self-weights, columns (A), and by the vertical load, column (B), were consistently higher for gauges nos. 23, 26 and 35, on the side plate near the hub, and for gauges nos. 53 and 54, on bolts on the joint of the side halves, thus confirming the analysis presented in section 5.6.2.

The strains on gauge no. 32 were expected to be close to those on gauge no. 23, being fitted opposite to it, but were significantly lower in all groups of tests. The reason for this could not be determined.

In some groups of tests a few gauges on the drum roll, nos. 11 to 15, showed relatively high strains induced by the vertical load. Those gauges were near to the central part of the drum where the load was applied in that loading condition.

Generally, the strains induced by the self weights and vertical load were much lower than those induced by the rope pressure. However, the results indicate that the effects of the self-weights and vertical load on the side plates near the hub require particular attention in the design of the drum.

5.6.7 Comparison between the first and second groups of tests

The first and second group of tests were carried out with the same load and same drum conditions, hence, they should give similar results.
In each group of tests, the sets of readings comprising the largest number of repetitions were for the loading condition of one layer of rope. The results obtained from these sets, given in columns (C) of the tables, were then taken as the reference.

In order to facilitate the comparison, only the gauges on the roll and side plate, away from the drum split, namely those on plane AA in FIG. 5.3, were considered. They were grouped according to their directions on the roll or on the side plate and their strains, for both groups of tests, are shown in FIGs. 5.4 to 5.7.

Clear discrepancies between the results in the two groups of tests can be observed for some gauges. One possible cause for the discrepancies could have been the different read-out equipment used in each group of tests, but, that would have been likely to affect the results of all gauges. In order to investigate the causes of the discrepancies, the results of the sets of readings in the first group of tests were examined individually. The results are illustrated in FIG. 5.8 where the average of the strains in all four reading positions for each gauge in the circumferential direction is plotted. The numbers on the lines represent the order in which the tests have been carried out.

The results of the last test, no. 10, although having a similar pattern, showed a marked variation in relation to all the others. One possible explanation was that the drum could have been plastically deformed by the load, but all the strains observed during all tests were well below the yielding point of the drum and this hypothesis was discounted. The most plausible cause for the differences was that the drum structure had settled after a few load cycles. However, due to the complexity of the structure with several bolted joints, it was impossible to identify where and what sort of accommodation had occurred. The fact is that, all sets or readings taken subsequently to the settlement maintained the pattern even after the drum had been unloaded four times and also after the brake rings had been bolted in position.
FIG. 5.4  CIRCUMFERENTIAL MICRO-STRAINS FOR ONE LAYER OF ROPE (comparison between the first and second groups of tests)
FIG. 5.5  AXIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
( comparison between the first and second groups of tests )
FIG. 5.6 RADIAl MICRO-STRAINS FOR ONE LAYER OF ROPE
(comparison between the first and second groups of tests)
Fig. 5.7 Tangential Micro-Strains for One Layer of Rope
(comparison between the first and second groups of tests)
FIG. 5.8 AVERAGE CIRCUMFERENTIAL MICRO-STRAINS
FOR EACH SET OF READINGS WITH ONE LAYER
OF ROPE IN THE FIRST GROUP OF TESTS
(load = 21.0kN)
The first group of tests has been, therefore, excluded from the remainder of the discussion.

Apart from the extraneous results of the tenth set of readings, the variations in the other sets seemed to be only caused by the inherent errors in measurement. The box in FIG. 5.8, showing the results for gauge no. 11, illustrates that they followed no logical sequence. Therefore, the analyses done in sections 5.6.1 and 5.6.2, with the results of the first group of tests, appear to be valid.

5.6.8 Effect of different loads

The effects of different loads on the drum were examined by comparing the results from the second, third and fourth groups of tests and carried out with loads of 21.0kN, 14.4kN and 27.8kN.

The comparison is illustrated in FIGs. 5.9 to 5.12 where are plotted the results of the strains induced by one layer of rope on the gauges away from the drum split.

The ratios between the results for each load agree well with the ratios between the loads. The same applies to other loadings as can be observed from the Tables 5.5 to 5.7.

5.6.9 Effect of the brake rings.

The brake rings were bolted in position for the fifth group of tests. Their effect can be observed by comparing the results with the second group of tests which were done with the same load but without the rings.

The strains induced on the gauges away from the drum split by one layer of rope are shown in FIGs. 5.13 to 5.16 which reveal that the rings had a
FIG. 5.9  CIRCUMFERENTIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
    (effect of different loads)
FIG. 5.10 AXIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
( effect of different loads )

Δ second group of tests
( load = 21.0kN )

○ third group of tests
( load = 14.4kN )

□ fourth group of tests
( load = 27.8kN )

Scale:
Horizontal: 1 division = 25 millimetres
Vertical: 1 division = 25 micro-strains
FIG. 5.11 RADIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
(effect of different loads)
FIG. 5.12  TANGENTIAL MICRO-STRAINS FOR ONE LAYER OF ROPE  
(effect of different loads)
FIG. 5.13  CIRCUMFERENCE MICRO-STRAINS FOR ONE LAYER OF ROPE
( effect of brake rings )
FIG. 5.14 AXIAL MICRO-STRAINS FOR ONE LAYER OF ROPE  
( effect of brake rings )
FIG. 5.15  RADIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
  (effect of brake rings)
FIG. 5.16  TANGENTIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
   (effect of brake rings)
localized effect near the edges of the roll but no influence farther from them. Hence, the brake rings do not significantly contribute to the strength of the drum roll. However, they do assist in strengthening the side plates as the graphs for the radial and tangential strains on the side plates, shown in FIGs. 5.15 and 5.16, reveal.

5.6.10 Effect of the drum split

This effect was investigated by considering the results from the fourth group of tests as they showed the highest strains.

The effect was assessed by comparing the strains induced by one layer of rope and two layers on the instrumented side on the gauges near and away from the drum split.

Examining initially the gauges on the drum roll, their strains are plotted in FIGs. 5.17 to 5.20 where those corresponding to the gauges near the split are identified by the gauge number.

Generally, the strains shown by the gauges near the split were approximately the same as those shown by the gauges away from the split and on the same circumference; the exception being gauge no. 41 whose strains were higher than those on gauge no. 11 as shown in FIGs. 5.17 and 5.18.

Although there was no gauge on the same circumference as gauge no. 38, the results for this gauge seem to follow the same pattern as gauge no. 40 which showed results similar to gauge no. 3 on the same circumference but away from the drum split. Thus, it seems reasonable to assume that, a gauge on the same circumference as gauge no. 38 but away from the split would have shown the same results as the latter.

Examining the results shown in FIG. 5.17 for gauge no. 41, it was observed that for one layer of rope the difference relative to the result of gauge no. 11 was greater than for two layers, as shown in FIG. 5.18, although the pressure applied in this loading condition was nearly twice as much as
FIG. 5.17 CIRCUMFERENTIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
( effect of the drum split )

Scale:
Horizontal 1 division = 25 millimetres
Vertical 1 division = 25 micro-strains

○ observed values
(fourth group of tests)
(load = 27.8kN)
FIG. 5.18 CIRCUMFERENTIAL MICRO-STRAINS FOR TWO LAYERS OF ROPE ON THE INSTRUMENTED SIDE (effect of the drum split)

- observed values
  (fourth group of tests)
  (load = 27.8kN)

Scale:
Horizontal 1 division= 25 millimetres
Vertical 1 division= 25 micro-strains
FIG. 5.19  AXIAL MICRO-STRAINS FOR ONE LAYER OF ROPE

( effect of the drum split )

Scale:
Horizontal 1 division = 25 millimetres
Vertical 1 division = 25 micro-strains

• observed values

( fourth group of tests )
( load = 27.8 kN )
FIG. 5.20  AXIAL MICRO-STRAINS FOR TWO LAYERS OF ROPE ON THE INSTRUMENTED SIDE (effect of the drum split)

- observed values
  (fourth group of tests)
  (load = 27.8kN)

Scale:
Horizontal 1 division = 25 millimetres
Vertical 1 division = 25 micro-strains
in the former condition.

The most probable cause for this effect was that, due to the manufacturing process, namely cold rolling, it is not possible to make perfectly semi-cylindrical roll halves. The regions near the straight edge of the drum roll are not cylindrical but flat. On the other hand, the stiffening rings can be made almost circular and it is not uncommon for a gap to be found between a ring and a drum roll at the split after the assembly.

When the drum is loaded, the rope pressure forces the flat regions of the roll to follow the shape of the ring and induces bending moments in the circumferential direction whose strains add to the circumferential strains induced by the rope compression.

The results from the scale model suggested that those bending moments were proportionally greater for smaller loads possibly because the displacements of the straight edges were less restricted. As the external pressure was increased, the two halves of the drum were pressed against each other more firmly, the bending of the flat regions was reduced and hence, the strains shown by gauge no. 41 approached those on gauge no. 11.

It is worthwhile mentioning that the prototype drum was designed to have a gap between the roll halves which is filled with shims during assembly to reduce the effect of the discontinuity of the drum roll at the split. This procedure, also adopted in the model drum, seems to be effective as the test results indicate.

In order to consider the gauges on the side plates, their strains were plotted in FIGs. 5.21 to 5.24 where the result for each gauge has been identified by the gauge number. The results suggest that the strains near the split are either the same or lower than away from it and hence, do not require special consideration. The results from gauge no. 32 were the exception in that they did not follow the same trend. However, because the strains showed by that gauge in all groups of tests varied within a range of 33 με as compared to the ranges of 88 με, 70 με and 103 με for the gauges nos. 23, 26 and 35 in a similar position, it is reasonable to assume that the gauge no. 32 had not been functioning correctly.
FIG. 5.21 RADIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
(effect of the drum split)

Gauges near the split:
26, 27, 35, 36

○ observed values

(fourth group of tests)
(load = 27.8kN)

Scale:
Horizontal 1 division = 25 micro-strains
Vertical 1 division = 25 millimetres
Fig. 5.22  Radial micro-strains for two layers of rope on the instrumented side
(effect of the drum split)

Gauges near the split:
26, 27, 35, 36

Observation values
(Fourth group of tests)
(load = 27.8 kN)

Scale:
Horizontal 1 division = 25 micro-strains
Vertical 1 division = 25 milimetres
Gauges near the split
25, 28, 34, 37

△ observed values
(fourth group of tests)
(load - 27.8kN)

Scale:
Horizontal 1 division = 25 micro-strains
Vertical 1 division = 25 millimetres

FIG. 5.23 TANGENTIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
(effect of the drum split)
FIG. 5.24 TANGENTIAL MICRO-STRAINS FOR TWO LAYERS OF ROPE ON THE INSTRUMENTED SIDE (effect of the drum split)
5.6.11 Effect of the position of the loading

This effect was observed on gauges on the side plate and is illustrated in FIG. 5.25. It shows the radial strains induced on gauges away from the drum split by one layer of rope and by two layers on either side of the drum. Once again, the results were from the fourth group of tests.

For the instrumented side, the loading conditions in order of increasing pressure were two layers of rope on the non-instrumented side and one layer of rope on both sides and two layers of rope on the instrumented side. This increase in pressure was reflected by the results of all gauges except those near the hub, namely nos. 23 and 32. These apparently contradictory results from these gauges can be understood by examining the action of the rope pressure.

When the rope compressed the drum roll, each side plate was subjected to a radial moment which tended not only to rotate it at the joint with the roll but also to displace it towards the other plate. If both side plates were subjected to the same moment, which was the case when one layer of rope was compressing both sides of the drum, the displacements, being transmitted by the roll, would cancel each other and the moments would only cause rotations at the joints. However, if the moments were different on each side plate, as was the case when two layers of rope were compressing only one of the drum sides, a displacement would take place towards the side not compressed and the pattern of bending on each plate would be different. This change in pattern can be observed in FIG. 5.25 if the distribution of the strains induced by one layer of rope is considered to be the normal pattern. The strains induced by two layers of rope on the instrumented side should be greater than those induced by one layer by a factor of about two which was the ratio between the pressure applied by each loading, but this was not the case. The reason seems to be the freedom of displacement afforded by the plate which allowed part of the load action to be transmitted to the other plate, increasing the strains there. This also seems to be the reason why for the loading condition of two layers of rope on the non-instrumented side, which transmitted much less pressure to the instrumented side plate, the strains were in the same order of magnitude as those for one layer of rope.
FIG. 5.25  RADIAL MICRO-STRAINS FOR ONE AND TWO LAYERS OF ROPE
( effect of the position of the loading )
C, D, E observed values

( fourth group of tests
  load = 21.8 kN )

Scales:
Horizontal 1 division = 25 micro-strains
Vertical 1 division = 25 millimetres

C one layer of rope on both sides of the drum
D two layers of rope on the instrumented side
E two layers of rope on the non-instrumented side

FIG. 5.26 TANGENTIAL MICRO-STRAINS FOR ONE AND TWO LAYERS OF ROPE
  ( effect of the position of the loading )
The effect of the loading position could be also observed in the tangential strains, shown in FIG. 5.26, despite the very small variation shown by most gauges.

5.6.12 Exploratory gauges

These gauges, nos. 46 to 54, were introduced in an attempt to observe the behaviour of the drum structure at points where little information was available.

The gauges to be considered first are those on the strapping plate, namely nos. 46 to 48. They were placed in an arrangement known as a rectangular rosette which is used when the directions of the principal strains are not known. The values of these principal strains, determined with the results from the gauges as described in Appendix 6, are shown in Table 5.9 for each group of tests and each loading. The results did not follow any consistent pattern and the only conclusion that could be drawn from them was that the level of strains on the strapping plate did not exceed the levels on the drum roll.

One possible explanation for the inconsistency of the results was that, although the strapping plates were fastened to the drum by six bolts, there was no guarantee that they would exactly follow the deformations of the roll.
Any relative movements between the strapping plate and the roll would change the pattern of strains on the plate. Those relative movements were observed with the results from the exploratory gauges nos. 51 and 52, positioned on bolts on the strapping plates, which showed positive strains in most tests. This indicated that the strapping plates were forced to move away from the roll thus tensioning the bolts.

The remaining exploratory gauges, nos. 49, 53 and 54, also placed on bolts, generally showed compressive strains indicating a reduction in their assembling tensions. The levels of strains on these bolts did not suggest they required special attention during design. However, a more adequate assessment could only be made if the strains due to bending had also been measured.

5.7 Extrapolation of the Results to the Full Size Drum

The deviations in the scale model drum from complete similarity with the prototype generally made the conditions in the model worse than they actually would be and, consequently, the strains observed on the model would represent upper limits for the strains on the prototype. The exceptions were the strains induced by self-weights which on the model were a fifth of those to be induced on the prototype. Further, the brake rings on the model were continuous rather than split as on the prototype drum, which possibly contributed to lower the strains on the roll and side plates. However, the results of the tests without brake rings showed that the diametral split on the side plate did not have the effect of significantly raising the strains. Therefore, it is reasonable to assume that, if the model had been fitted with split brake rings the results would not be higher than those obtained.

The strains on the full size drum were extrapolated from the results of the fifth group of tests with the model. The maximum expected strains induced by the combined effects of the loadings are shown in Table 5.10. The values in column (A) are the absolute values of the strains induced by the self-weights on the model, multiplied by 5 to account for the deviation from the model similarity due to the scaling of gravity as discussed in section 3.1.
### Table 5.10
Maxima micro-strains expected to be induced on the full size drum by a load of 25x21.0kN

<table>
<thead>
<tr>
<th>Gauge</th>
<th>Loading conditions</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-c- 1</td>
<td>No load</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>a 2</td>
<td>Vertical load+(A)</td>
<td>15</td>
<td>23</td>
<td>-45</td>
<td>-74</td>
<td>37</td>
</tr>
<tr>
<td>c 3</td>
<td>One layer of rope on both sides+(B)</td>
<td>15</td>
<td>20</td>
<td>-46</td>
<td>-94</td>
<td>26</td>
</tr>
<tr>
<td>a 4</td>
<td>Two layers of rope on the instrumented side+(B)</td>
<td>15</td>
<td>28</td>
<td>-80</td>
<td>-84</td>
<td>-56</td>
</tr>
<tr>
<td>c .5</td>
<td>Two layers of rope on the non-instrumented side+(B)</td>
<td>25</td>
<td>28</td>
<td>-155</td>
<td>-141</td>
<td>-144</td>
</tr>
<tr>
<td>a 6</td>
<td>(A) No load</td>
<td>15</td>
<td>23</td>
<td>184</td>
<td>310</td>
<td>55</td>
</tr>
<tr>
<td>c 7</td>
<td>(B) Vertical load+(A)</td>
<td>15</td>
<td>20</td>
<td>-209</td>
<td>-379</td>
<td>31</td>
</tr>
<tr>
<td>a 8</td>
<td>(C) One layer of rope on both sides+(B)</td>
<td>15</td>
<td>28</td>
<td>159</td>
<td>269</td>
<td>38</td>
</tr>
<tr>
<td>c 9</td>
<td>(D) Two layers of rope on the instrumented side+(B)</td>
<td>15</td>
<td>25</td>
<td>-193</td>
<td>-362</td>
<td>61</td>
</tr>
<tr>
<td>a 10</td>
<td>(E) Two layers of rope on the non-instrumented side+(B)</td>
<td>3</td>
<td>127</td>
<td>251</td>
<td>-38</td>
<td></td>
</tr>
<tr>
<td>c 11</td>
<td>(A) No load</td>
<td>15</td>
<td>18</td>
<td>-160</td>
<td>-306</td>
<td>58</td>
</tr>
<tr>
<td>a 12</td>
<td>(B) Vertical load+(A)</td>
<td>15</td>
<td>33</td>
<td>102</td>
<td>254</td>
<td>-149</td>
</tr>
<tr>
<td>c 13</td>
<td>(C) One layer of rope on both sides+(B)</td>
<td>15</td>
<td>28</td>
<td>-104</td>
<td>-167</td>
<td>42</td>
</tr>
<tr>
<td>a 14</td>
<td>(D) Two layers of rope on the instrumented side+(B)</td>
<td>25</td>
<td>28</td>
<td>-94</td>
<td>-34</td>
<td>-153</td>
</tr>
<tr>
<td>c 15</td>
<td>(E) Two layers of rope on the non-instrumented side+(B)</td>
<td>15</td>
<td>28</td>
<td>-80</td>
<td>-84</td>
<td>-56</td>
</tr>
<tr>
<td>a 16</td>
<td>-155</td>
<td>-141</td>
<td>-144</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c 17</td>
<td>-152</td>
<td>43</td>
<td>-264</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-a- 18</td>
<td>25</td>
<td>28</td>
<td>109</td>
<td>-116</td>
<td>255</td>
<td></td>
</tr>
<tr>
<td>-t- 19</td>
<td>25</td>
<td>33</td>
<td>102</td>
<td>254</td>
<td>-149</td>
<td></td>
</tr>
<tr>
<td>r 21</td>
<td>-104</td>
<td>-167</td>
<td>42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r 22</td>
<td>55</td>
<td>53</td>
<td>56</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r 23</td>
<td>-100</td>
<td>-95</td>
<td>-107</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t 24</td>
<td>8</td>
<td>-36</td>
<td>-41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t 25</td>
<td>0</td>
<td>3</td>
<td>12</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r 26</td>
<td>65</td>
<td>83</td>
<td>-126</td>
<td>-124</td>
<td>-118</td>
<td></td>
</tr>
<tr>
<td>r 27</td>
<td>0</td>
<td>5</td>
<td>22</td>
<td>45</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>t 28</td>
<td>0</td>
<td>8</td>
<td>21</td>
<td>8</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>r 29</td>
<td>25</td>
<td>25</td>
<td>-29</td>
<td>-48</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>r 30</td>
<td>0</td>
<td>13</td>
<td>-65</td>
<td>-91</td>
<td>-42</td>
<td></td>
</tr>
<tr>
<td>r 31</td>
<td>0</td>
<td>10</td>
<td>-64</td>
<td>-83</td>
<td>-43</td>
<td></td>
</tr>
<tr>
<td>r 32</td>
<td>40</td>
<td>43</td>
<td>55</td>
<td>47</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>t 33</td>
<td>25</td>
<td>30</td>
<td>39</td>
<td>-49</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>t 34</td>
<td>0</td>
<td>5</td>
<td>13</td>
<td>6</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>r 35</td>
<td>15</td>
<td>38</td>
<td>-46</td>
<td>-70</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>r 36</td>
<td>25</td>
<td>28</td>
<td>-51</td>
<td>-67</td>
<td>-45</td>
<td></td>
</tr>
<tr>
<td>t 37</td>
<td>25</td>
<td>30</td>
<td>-42</td>
<td>-62</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>-c- 38</td>
<td>15</td>
<td>23</td>
<td>23</td>
<td>-48</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>a 39</td>
<td>15</td>
<td>23</td>
<td>67</td>
<td>87</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>c 40</td>
<td>15</td>
<td>20</td>
<td>-21</td>
<td>-42</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>c 41</td>
<td>0</td>
<td>5</td>
<td>-230</td>
<td>-320</td>
<td>-13</td>
<td></td>
</tr>
<tr>
<td>a 42</td>
<td>15</td>
<td>23</td>
<td>77</td>
<td>251</td>
<td>-156</td>
<td></td>
</tr>
<tr>
<td>c 43</td>
<td>25</td>
<td>28</td>
<td>-174</td>
<td>-30</td>
<td>-304</td>
<td></td>
</tr>
<tr>
<td>c 44</td>
<td>15</td>
<td>18</td>
<td>-164</td>
<td>18</td>
<td>-293</td>
<td></td>
</tr>
<tr>
<td>-c- 45</td>
<td>15</td>
<td>18</td>
<td>-163</td>
<td>-23</td>
<td>-287</td>
<td></td>
</tr>
<tr>
<td>--- 46</td>
<td>40</td>
<td>43</td>
<td>-83</td>
<td>69</td>
<td>-139</td>
<td></td>
</tr>
<tr>
<td>--- 47</td>
<td>15</td>
<td>15</td>
<td>18</td>
<td>38</td>
<td>-40</td>
<td></td>
</tr>
<tr>
<td>--- 48</td>
<td>25</td>
<td>30</td>
<td>-36</td>
<td>39</td>
<td>-47</td>
<td></td>
</tr>
<tr>
<td>--- 49</td>
<td>15</td>
<td>15</td>
<td>-33</td>
<td>-30</td>
<td>-27</td>
<td></td>
</tr>
<tr>
<td>--- 50</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>--- 51</td>
<td>0</td>
<td>3</td>
<td>13</td>
<td>17</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>--- 52</td>
<td>0</td>
<td>3</td>
<td>15</td>
<td>50</td>
<td>-13</td>
<td></td>
</tr>
<tr>
<td>--- 53</td>
<td>15</td>
<td>28</td>
<td>-107</td>
<td>-127</td>
<td>-32</td>
<td></td>
</tr>
<tr>
<td>--- 54</td>
<td>0</td>
<td>10</td>
<td>-55</td>
<td>-81</td>
<td>-17</td>
<td></td>
</tr>
<tr>
<td>--- 55</td>
<td>125</td>
<td>183</td>
<td>183</td>
<td>183</td>
<td>185</td>
<td></td>
</tr>
<tr>
<td>--- 56</td>
<td>115</td>
<td>180</td>
<td>183</td>
<td>184</td>
<td>183</td>
<td></td>
</tr>
</tbody>
</table>
Column (B) shows the absolute values of the strains induced by the vertical load, which would be the same on the prototype and on the model, added to the values in column (A). The values on columns (C), (D) and (E) are the strains due to one layer of rope on both sides and two layers on either side of the drum, which would be the same on the prototype as on the model, added to the values from column (B) taking into account the sign of the strains.

For all columns, the values of the strains at the bolts with gauges nos. 53 and 54 have been calculated taking into account the strength area of the scaled bolts as discussed in section 5.1.

The maximum strain, in absolute value, observed in the whole table was 379µε for gauge no. 7, column (D), which is well below the yield strain for the roll, being 900µε minimum.

The test results indicated that the strength of the full size drum is adequate to cope with the loads applied by the rope in conditions similar to those in the tests.
CHAPTER 6
VALIDATION OF THE THEORETICAL MODELS

In order to validate the theoretical models discussed in chapter 4, predictions made with them have been compared with the results from the tests presented in chapter 5. The comparison was initially done with the results from the second group of tests where the load was 21.0kN and without the brake rings. Subsequently, the comparison was done with the results from the fifth group of tests which was carried out with the same load but with the brake rings bolted in position.

The theoretical predictions, made using the computer programs listed in Appendix 4, were plotted in graphical form showing the micro-strains induced along the drum roll or side plates. The test results considered in the comparison were those from the gauges away from the drum split.

The test results shown in sections 6.1 and 6.2 are from columns (C) or (D) of Tables 5.5 to 5.8.

6.1 Predictions for the Second Group of Tests

6.1.1 Drum roll

The predictions of the strains induced on the roll by one layer of rope and two layers on the instrumented side are shown in FIGs. 6.1 to 6.4. Each figure shows the predictions with and without the consideration of rope relaxation. In both cases there was a fair correlation with the actual results both in the pattern and also in magnitude. However, it is self-evident that the predictions considering rope relaxation are in closer agreement. It can be concluded that, the rope relaxation is an effect to be considered although, if neglected, predicted strains will be higher than the actual strains and therefore will be on the "safe side" for design purposes.
FIG. 6.1  CIRCUMFERENTIAL MICRO-STRAINS FOR ONE LAYER OF ROPE  
(predicted vs. observed)
FIG. 6.2 AXIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
(predicted vs. observed)
FIG. 6.3  CIRCUMFERENTIAL MICRO-STRAINS FOR THE LAYERS OF ROPE  
( predicted vs. observed )
FIG. 6.4  AXIAL MICRO-STRAINS FOR THE LAYERS OF ROPE
(predicted vs. observed)
It is interesting to observe that the rope factors of relaxation calculated with the formula developed by Atkinson and Taylor, presented in section 4.6.1, appear to provide satisfactory correction despite the fact that the formula was developed for non-stiffened drums.

The other predictions presented in the remainder of this chapter had been made taking into consideration the effect of rope relaxation.

One final point to be made about the predictions on the drum roll, concern gauges nos. 2 and 3 placed under the side plate ledge. The test results for these gauges have been plotted together with those for the gauges on the roll to give an idea of the levels of strains near the joint with the side plates. The comparison between the predictions and the results for gauges nos. 2 and 3 has to be made with caution because of the number of assumptions made to simplify the theoretical representation of the joint.

6.1.2 Side plate

The predictions of the strains induced by one layer of rope are shown in FIGs 6.5 and 6.6.

The radial strains predicted for gauges nos. 22 and 31 were lower than the test results as shown in FIG. 6.5. This was an expected outcome since the theoretical model did not take into account the ring of holes in the side plates. Using the predicted values, an estimation of the actual results can be made if it is assumed that the average stresses and strains between the holes are proportional to the stresses and strains on the unperforated plates at the same points. It is also assumed that the factor of proportionality is the ratio between the total cylindrical area delimited by the circle passing through the points where the gauges were (FIG. 6.7) and the area left after the holes were cut. By reference to FIG. 6.7, this ratio is approximately equal to

\[
\frac{2\pi\times430}{6\times220} = 2.05
\]
FIG. 6.5  RADIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
(predicted vs. observed)
(predictions considering rope relaxation)
FIG. 6.6 TANGENTIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
(predicted vs. observed)
(predictions considering rope relaxation)
The predicted strain for the unperforated plate was $33 \mu \varepsilon$, hence, the average strain between the holes becomes $2.05 \times 33 = 66 \mu \varepsilon$ which compares with the observed absolute values of $76 \mu \varepsilon$ and $80 \mu \varepsilon$.

The radial strains predicted for the other gauges showed good agreement despite their proximity to the joints.

For the tangential strains shown in FIG. 6.5, the predictions for gauges nos. 24 and 33 were low. The differences can be accounted for on the assumption that the side plate was built into a perfectly rigid hub. In practice, not only does the hub deform but also the bolts which fix the plate to it allow some displacement making tangential strains near the hub different from predicted. The observed tangential strains were generally low in magnitude and, consequently, more sensitive to small disturbances in the drum structure than the strains shown by other gauges. Such disturbances could significantly change the pattern of strains and hence, observed values for tangential strains should be regarded with caution.

Considering the predictions for the strains induced on the side plate by the action of two layers of rope on the instrumented side, as shown in FIGs. 6.8 and 6.9, it will be noted that the predicted radial strains were significantly different from the test results. This was due to the assumption made
FIG. 6.8  RADIAL MICRO-STRAINS FOR TWO LAYERS OF ROPE
(predicted vs. observed)
(predictions considering rope relaxation)
FIG. 6.9  TANGENTIAL MICRO-STRAINS FOR TWO LAYERS OF ROPE
(predicted vs. observed)
(predictions considering rope relaxation)
in the theoretical representation of the model drum that the points at the junctions of the side plates with the roll would not suffer any displacement in the direction of the drum axis. However, as seen in section 5.6.11, such displacement occurs when two layers of rope are compressing the drum sides. The predictions therefore, were for a worse case where the prevention of the axial displacements did not allow a distribution of the load between the two sides and, in consequence, the calculated strains were higher than those actually occurring.

In an attempt to improve the predictions, the theoretical representation was modified to allow axial displacements of the junctions. This was done by replacing the assumption that both joints would not move, by the assumption that both joints would move axially by the same amount as illustrated in FIG. 6.10. This new assumption implied that the drum roll would transmit the load from one plate to another without itself deforming.

![FIG. 6.10 DISPLACEMENTS OF THE JUNCTIONS](image)

The initial assumption was described in the theoretical representation of the model drum by two separate expressions equating to zero the deflection of each side plate at the junction with the drum roll. In order to include the new assumption, those two expressions were replaced by one equating the deflections of each side plate at the junction with the drum roll and by another equating the shear forces acting on each plate also at the junction.
The predictions made with the new assumption are shown in FIGs. 6.11 and 6.12, the effect on the radial strains being self-evident.

Strains predicted for the third and fourth group of tests showed the same degree of agreement with the actual results as for the second group of tests. This is illustrated in FIGs. 6.13 to 6.16 where the predicted strains, taking into account the axial displacements of the side plates, are shown together with the test results for one layer of rope and two layers on the instrumented side of the drum.

6.2 Predictions for the Fifth Group of Tests

The predictions and the test results of the strains induced by one layer of rope and two layers on the instrumented side, for the fifth group of tests, are shown in FIGs. 6.17 to 6.24. The strains predicted include rope relaxation and axial movement of the side plates.

There was, generally, the same sort of agreement between the predictions and observations as obtained for the second group of tests without the brake rings. However, it is necessary to comment about the differences shown by some gauges measuring radial strains. It will be noted in FIGs. 6.19 and 6.23 that the strains predicted for gauges nos. 21 and 30 were low. The main cause for this seemed to be the assumption in the theoretical representation of the model that each brake ring acted as an integral part of the side plates and their action concentrated at the junction between the side plates and the drum roll. Actually, that action was distributed over an annulus of each side plate. Furthermore, the brake rings were fastened to the side plates by bolts which, because of their elasticity, would allow some relative displacement between the side plates and the rings changing the distribution of strains.

Further in FIGs. 6.19 and 6.23, it can be seen that the test results for gauge no. 22 were, in absolute values, lower than those for gauge no. 31 at the same radial position. The results for gauge no. 22 were practically the same irrespective of whether the loading condition was one layer of rope or
FIG. 6.11 RADIAL MICRO-STRAINS FOR TWO LAYERS OF ROPE
(predicted vs. observed)
(predictions considering rope relaxation)
(and axial displacement of the drum roll)

Scale:
Horizontal 1 division = 25 micro-strains
Vertical 1 division = 25 millimetres
FIG. 6.12  TANGENTIAL MICRO-STRAINS FOR TWO LAYERS OF ROPE  
( predicted vs. observed )  
( predictions considering rope relaxation )  
( and axial displacement of the drum roll )
FIG. 6.14  THIRD GROUP OF TESTS (load = 14.4 kN)
MICRO-STRAINS FOR TWO LAYERS OF ROPE
(predicted vs. observed)
(considering rope relaxation)
(axial displacement of the drum roll)

(a) Circumferential
(b) Axial
(c) Radial
(d) Tangential
FIG. 6.15  FOURTH GROUP OF TESTS ( load = 27.8kN )
MICRO-STRAINS FOR ONE LAYER OF ROPE
(predicted vs. observed)
(predictions considering rope relaxation)
(and axial displacement of the drum roll)

(a) Circumferential
(b) Axial
(c) Radial
(d) Tangential

observed values
FIG. 6.16  FOURTH GROUP OF TESTS  (load = 27.8kN)
MICRO-STRAINS FOR TWO LAYERS OF ROPE
(predicted vs. observed)
(predictions considering rope relaxation)
(and axial displacement of the drum roll)

(a) Circumferential
(b) Axial
(c) Radial
(d) Tangential
FIG. 6.17  CIRCUMFERENTIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
( predicted vs. observed ) - Complete Model
( predictions considering rope relaxation )
( and axial displacement of the drum roll )
FIG. 6.18 AXIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
(predicted vs. observed) - Complete Model
(predictions considering rope relaxation)
(and axial displacement of the drum roll)

Scale:
Horizontal 1 division = 25 millimetres
Vertical 1 division = 25 micro-strains

- predictions

observed values
(fifth group of tests)
(load = 21.0 kN)
(brake rings fitted)
FIG. 6.19 RADIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
(predicted vs. observed) - Complete Model
(predictions considering rope relaxation)
(and axial displacement of the drum roll)
FIG. 6.20 TANGENTIAL MICRO-STRAINS FOR ONE LAYER OF ROPE
(predicated vs. observed) - Complete Model
(predictions considering rope relaxation)
(and axial displacement of the drum roll)

Predictions

Observed values
(fifth group of tests)
(load = 21.0 kN)
(brazen rings fitted)

Scale:
Horizontal 1 division = 25 micro-strains
Vertical 1 division = 25 millimetres
FIG. 6.21 - CIRCUMFERENTIAL MICRO-STRAINS FOR TWO LAYERS OF ROPE
( predicted vs. observed ) - Complete Model
( predictions considering rope relaxation )
( and axial displacement of the drum roll )
FIG. 6.22  AXIAL MICRO-STRAINS FOR TWO LAYERS OF ROPE
( predicted vs. observed ) - Complete Model
( predictions considering rope relaxation )
( and axial displacement of the drum roll )
FIG. 6.23  RADIAL MICRO-STRAINS FOR TWO LAYERS OF ROPE
(predicted vs. observed) - Complete Model
(predictions considering rope relaxation)
(and axial displacement of the drum roll)
FIG. 6.24  TANGENTIAL MICRO-STRAINS FOR TWO LAYERS OF ROPE
(predicted vs. observed) - Complete Model
(predictions considering rope relaxation)
(and axial displacement of the drum roll)

- predictions

observation values
(fifth group of tests)
(load = 21.0 kN)
(brake rings fitted)

Scale:
Horizontal 1 division = 25 micro-strains
Vertical 1 division = 25 millimetres
two layers on either side of the drum. This suggests that for some reason the gauge was working incorrectly.

6.3 Predictions for the Strains Induced by the Bent Shaft

The strains induced on the side plates by the bent shaft were predicted separately using the theory discussed in section 4.4 and with the aid of another computer program listed in Appendix 4.

The comparison between the predictions and the test results was made by examining the strains shown by gauges nos. 23 and 24 near the hub but away from the drum split. These gauges being chosen because they generally showed higher strains than the other gauges fitted in the same relative position on the side plate.

The test results were those in columns (A) and (B) of Tables 5.5 to 5.8 indicating the strains induced by the self-weights and vertical load respectively. The results and predictions are shown on Table 6.1 where good agreement will be noted.

Table 6.1
Micro-strains induced by the bent shaft on gauges near the hub

<table>
<thead>
<tr>
<th>Load</th>
<th>gauge number</th>
<th>micro-strains observed</th>
<th>micro-strains predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>self-weights</td>
<td>23</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>(all groups of tests)</td>
<td>24</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>21.0 kN (second group of tests)</td>
<td>23</td>
<td>23</td>
<td>25</td>
</tr>
<tr>
<td>14.4 kN (third group of tests)</td>
<td>24</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>27.8 kN (fourth group of tests)</td>
<td>23</td>
<td>35</td>
<td>34</td>
</tr>
<tr>
<td>21.0 kN (fifth group of tests)</td>
<td>24</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>


6.4 Summary

From the discussion in the previous sections, the assessment of the theoretical models can be summarized as follows:

1 - The predictions for the strains on the drum roll showed good agreement with the results both in pattern and magnitude.

2 - The observation of rope relaxation considerably improved the predictions and, although in the actual rope relaxation the tension on each coil of rope is thought to be relieved by a different factor, the assumption that they are relieved by the same factor proved to be an adequate approximation.

3 - The predictions for the strains on the side plates also showed good agreement for one layer of rope because actual strains were not affected by the position of the loading.

4 - When the actual strains were affected by the position of the loading, i.e., for two layers of rope, the predictions were very conservative. However, a great improvement was achieved when the theoretical representation was modified to allow axial displacements of the side plates.

5 - The ring of holes on the side plates seemed to have only affected the strains local to them and, predictions considering a factor of proportionality based on the areas between the holes agreed well with the actual results.

6 - The predictions made for the strains induced by the bent shaft on the side plates also agreed well with the test results.

From the assessment, it can be concluded that the theoretical representation of the model drum as discussed in chapter 4 is adequate for design purposes.
In order to appreciate the accuracy of the theoretical model, the predictions made for the strains induced by one layer of rope and two layers on the instrumented side have been compared with the test results from the second to the fifth groups of tests for the gauges on the drum roll and on the side plates and away from the drum split. Gauges nos. 2 and 3 were not included in the comparison.

A histogram of the frequency of the errors is shown in FIG. 6.25 where it can be seen that they follow a Gaussian distribution, the average being 2.6\textmu \varepsilon.

It will be noted from the histogram that the maximum absolute error was 75\textmu \varepsilon. If this is compared with the maximum strain observed among the results, namely -515\textmu \varepsilon for gauge no. 7 in the fourth group of tests, it will be seen that the error is 14.6% of the result. This degree of accuracy being satisfactory in this type of design work.

This level of accuracy of the theoretical model was confirmed when predictions were compared with results of further tests performed with the scale model by M.B.Wild and Co.Ltd. during the preparation of this thesis but without the participation by the author. For these tests the drum was modified and wound with up to seven layers of rope on one side. Using the information made available by the Company, the computer program was modified to cater for the modifications done on the drum. The average percentage error of the predictions for the new tests was less than 8%. The model drum has been loaded up to yielding point and, for a maximum observed strain of 940\textmu \varepsilon the prediction was 10% higher.
FIG. 6.25  FREQUENCY OF THE RELATIVE ERRORS OF THE PREDICTIONS
( Error = prediction - observation )
CONCLUSIONS

The research showed that classical theories for thin cylindrical shells and for circular flat plates combined together can provide a very satisfactory theoretical model for the analysis of cylindrical mine winding drums.

Such a theoretical model will not only allow the determination of stresses on the drum as a whole structure but also give information about deflections, slopes, shear forces and moments. The possibility of introducing different stiffening rings or loads at any position and of rapidly changing geometrical parameters such as drum diameter, drum length, drum roll and drum sides thicknesses makes it extremely flexible and a powerful tool for the investigation of alternative drum configurations.

The theoretical model can also be used as a simulator of a winding drum operation to examine the variations in stresses and deflections induced in a drum as each coil is wound.

The author hopes that this work will contribute to confirm the belief of several authors and design engineers in the field that, winding drums behave like thin shells under uniformly distributed axisymmetric pressure and further, that drums can be analysed as a complete structure by combining flat plate theory with the shell theory.

The author also hopes that the work will provide design engineers with an effective and reliable technique for the analysis of drum structures allowing the stresses and deflections to be predicted with much greater accuracy than at present. The application of the technique should, therefore, result in a better understanding of the behaviour of drum structures and in more economical designs.
SUGGESTIONS FOR FURTHER WORK

There is still a fair amount of research that could be done in regard to mine winding drum analysis in order to further improve the theoretical models presented in this thesis and also to generate information to support its application. The author would like to suggest the following topics for further work:

1 - Rope relaxation. The expression used to determine the rope relaxation given by Eq. 4.64 produced results showing good agreement with actual test figures for the drum studied. Its advantages in relation to other methods are its simple structure, requiring very little computational effort, coupled with the fact that it can be used to calculate factors for only one layer of rope. Further work to examine the delimitations of this approach would be useful.

2 - Effect of the drum split. Although the results from the tests suggested that the effect of the drum split was minimized by filling the gap between the drum roll halves with shims, the split is a "weakness" in the drum structure and more specific investigation to establish its influence would be valuable.

3 - Drum stability. There is very little experimental data and very little known about the stability of winding drums. Experimental work examining, for instance, the influence of the drum split and of the stiffening rings on the drum stability, could provide valuable information to assist drum designers in defining design criteria concerning the stressing of the drums.
REFERENCES

1 - BROUGHTON, H.H. 

2 - TUDHOPE, I.S.D. 
   "Multi-rope winders for high capacity hoisting". Paper 12, Interna-
   tional Conference on Hoisting Men, Materials and Minerals;

3 - ATKINSON, L.T.J. and TAYLOR, G.L. 
   "Winding drums", part I. Colliery Engineering, vol 43, Dec 1966,
   pg. 524-530.

4 - ATKINSON, L.T.J. and TAYLOR, G.L. 
   "Winding drums", part II. Colliery Engineering, vol 44, Jan 1967,
   pg. 32-39.

5 - ATKINSON, L.T.J. and TAYLOR, G.L. 
   "Winding drums", part III. Colliery Engineering, vol 44, Feb 1967,
   pg. 79-84.

6 - ATKINSON, L.T.J. and TAYLOR, G.L. 
   "Winding drums", part IV. Colliery Engineering, vol 44, Mar 1967,
   pg. 115-121.

7 - ATKINSON, L.T.J. and TAYLOR, G.L. 
   "Winding drums", part V. Colliery Engineering, vol 44, Apr 1967,
   pg. 158-164.

8 - ATKINSON, L.T.J. and TAYLOR, G.L. 
   "Winding drums", part VI. Colliery Engineering, vol 44, May 1967,
   pg. 201-206.

9 - ATKINSON, L.T.J. and TAYLOR, G.L. 
   "Winding drums", part VII. Colliery Engineering, vol 44, Jun 1967,
   pg. 236-243.

10 - ATKINSON, L.T.J. and TAYLOR, G.L. 
    "Winding drums", part VIII. Colliery Engineering, vol 44, Aug 1967,
    pg. 315-321.

11 - DIETZ, P. 
   "Ein Verfahren zur Berechnung ein-und mehrlagig bewickelter
   Seiltrommeln". Fortschritt-Berichte Der VDI Zeitschriften, part 3,
   n 12, Jul 1972.

12 - WATERS, E.O. 
   "Rational design of hoisting drums". Transactions of the ASME, vol 42,
   1920, pg. 463-485.
13 - EGAWA, T. and TANEDA, M.

14 - DOLAN, J.

15 - TORRANCE, B.McK.

16 - TIMOSHENKO, S.P. and WOINOVSKY-KRIEGER, S.

17 - CRAWFORD, W.R.

18 - ATKINSON, L.T.J. and PREATTER, R.W.T.

19 - PESTEL, E.C. and LECKIE, F.A.

20 - BELLAMY, N.W. and PHILLIPS, B.D.A.

21 - PFLÜGER, A.

22 - ROARK, R.J. and YOUNG, W.C.

23 - BUDYNAS, R.G.

24 - BUCKINGHAM, E.

25 - LANGHAAR, H.L.

26 - FLÜGGE, W.
27 - DONNEL, L.H.

28 - LOVE, A.E.H.
Cambridge University Press, 1952.

29 - DONNEL, L.H.

30 - HETÉNYI, M.

31 - NOVOZHILOV, V.V.

32 - WILSON, L.B.
"The deformation under uniform pressure of a circular cylindrical shell supported by equally spaced circular ring frames". Naval Construction Research Establishment, Reports NCRE/R337A, B and C, Dec 1956.

33 - MANSFIELD, E.H.

34 - REISSNER, H.
"Über die unsymmetrische Biegung dünner Kreisringplatten". Ingenieur-Archiv, vol 1, Sep 1929, pg. 72-83.

35 - KRAUS, H.

36 - DAVIES, O.L.
BIBLIOGRAPHY

List of books and papers also consulted but not referred to in the text.

ADINI, A.
Analysis of Shell Structures by the Finite Element Method.

AMEME (The Association of Mining Electrical and Mechanical Engineers)
"The transportation of men and materials in shafts and underground".

BAKER, E.H.; KOVALEVSKY, L. and RISH, F.L.

BEERKIRCHER, G.

BEHR, H.C.

BURFFIT, A.J.

FUNG, Y.C. and SECHLER, E.E.

GILL, S.S.

HALLET, A.W. and PLUMSTEAD, E.R.A.
"Some thoughts on deep level hoisting". Mining Magazine, Jun 1964, pg. 374-381.

HETENYI, M.

IMM (Institution of Mining and Metallurgy)
"Wire ropes in mines". Proceedings of a Conference held at Lemington Spa, Sep 1951.

IPSEN, D.C.
JAEGGER, L.

KRAUS, H.

KRAUS, H.

LATHAM, R.W.

MEYER, M.L.

MURPHY, G.
"Models with incomplete correspondence with the prototype". Journal of the Franklin Institute, vol 292, no 6, Dec 1971, pg. 513-518.

PALACIOS, J.

PIGGOT, B.R.

PRICE, A.B.
Winding Engine Calculation for Mining Engineers. GEC, Kent, 1957.

ROARK, R.J.
"Stresses produced in a circular plate by eccentric loading and by transverse couple". Bulletin of the University of Wisconsin, Jan 1932.

SCHURING, D.J.

TAYLOR, E.S.
APPENDIX 1
MODEL DRUM FABRICATION DRAWINGS
FOR INSTRUMENTED DRUM SIDE POSITION

SEE Dwg. 790011/10/B

NOTE:
1. STRAIN GAUGES MOUNTED BY BOLTS ARE LOCATED AT 300 DEGREES TO each other.
2. TWO GAUGES, 135 AND 225, TO BE PLACED ON THE SHAWTA MIDWAY BETWEEN THE PLATES AND AMONG THE AXIAL.

SECTION AA (INSTRUMENTATION PLANE D1)

SECTION BB (INSTRUMENTATION PLANE D2)

DETAIL E (FULL SIZE)

DETAIL F (FULL SIZE)

DETAIL G (FULL SIZE)

ENGINEERING DESIGN CENTRE — LOUGHBOROUGH UNIVERSITY of TECHNOLOGY

Title: 1/5 SCALE MODEL DRUM

STRAIN GAUGE LAYOUT

Material: Unspecified

Scale: 1:25

Surface Finish:

Drawn by RSA

Checked:

Date: 7-2-79

Drawing No.: 79001B

A0

Dimensions in: mm

Tolerances:
Linear ±
Angular ±

Unit: mmo
3 STIFFENING RINGS REQUIRED (10 MILLIES)

NOTE: ALL WELD FILLERS TO BE SOWN WARM

CLEANSANCE FOR INSTRUMENTATION

ONLY ONE STIFFENING RING TO BE MACHINED AS SHOWN

SET DIAL TOOLS FOR POSITIONS
OF MACHINED HOLE

ENGINEERING DESIGN CENTRE - LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY

Material: STEEL PLATES GRADE 43A BS 5442
Scale: 1:25
Drawn by: RSA

Unspecified Tolerances - Linear ± 0.2 mm
Angular ± Surface Finish

Date: 30-6-79

REFERENCE: CP 

DIMENSIONS IN: MM

THIRD ANGLE PROJECTION

Drawing No. 70V004 XD108 16
3D DRUMS REQUIRED (4 HAVES)
ONE DRUMSIDE COVERS ACROSS TOP TUNNEL HOLES

TWO DRUMSIDES REQUIRED (4 HAVES)
ONE DRUMSIDE MOVED FROM INSIDE

DIMENSIONS IN: mm

ENGINEERING DESIGN CENTRE – Loughborough UNIVERSITY of TECHNOLOGY

Title 1/5 SCALE MODEL DRUM FABRICATED DRUM SIDE

Material STEEL PLATES GRADE 43A BS 4360
Scale 1:2.5
Unspecified Tolerances Linear ± 0.2 mm
Angular ±
Surface Finish

Drawn RSA

Date 11-5-79

Checked

Drawing No. 79V/009 XD/08 19

Issue Modification Date
APPENDIX 2
STRAIN GAUGE CIRCUIT ANALYSIS

The basic circuit for the strain gauges used in the scale model drum was a Wheatstone bridge with one strain gauge forming one of the arms. This is represented in FIG. A2.1 where the notation is

![Diagram of a Wheatstone bridge](image)

**FIG. A2.1 BASIC CIRCUIT**

- $r_g$ - gauge resistance
- $r_d$ - "dummy gauge" resistance, equal to $r_g$
- $r_w$ - resistance of the wires connecting the gauges to the bridge
- $r_b$ - bridge completion resistance
- $r_{cal}$ - calibration resistance

A voltage $V$ applied between $a$ and $d$ induces currents $i_1$ and $i_2$ in the arms of the bridge. Since the voltage drop $v$ between any two points in the bridge is given by Ohm's law, it follows that:
The voltage $v_{ab}$ across the bridge is given by

$$v_{ab} = (r_g + r_w)i_2$$

The voltage $v_{bd}$ is

$$v_{bd} = r_d i_2 = r_g i_2$$

The voltage $v_{ac}$ is

$$v_{ac} = r_b i_1$$

The voltage $v_{cd}$ is

$$v_{cd} = r_b i_1$$

The voltage $v_{ad}$ is

$$v_{ad} = v_{ab} + v_{bd} = (2r_g + r_w)i_2 = V$$

The voltage $v_{ad}$ is also

$$v_{ad} = v_{ac} + v_{cd} = 2r_b i_1 = V$$

The voltage $v_{bc}$ is

$$v_{bc} = v_{ab} - v_{ac} = (r_g + r_w)i_2 - r_b i_1$$

and the voltage $v_{bc}$ across the bridge is given by

$$v_{bc} = \frac{(r_g + r_w)}{(2r_g + r_w)}V - \frac{V}{2} \quad \text{A2.1}$$

The application of electrical resistance strain gauges for measurement is based on the property that electrical conductors change their resistance when subjected to mechanical strain. Such changes can be monitored through the variation in voltage across $b$ and $c$ in the bridge. Indeed, if a gauge is strained and its resistance changed to $r_g + \Delta r_g$, the voltage $v_{bc}$ will change to $v_{bc} + \Delta v$ given by

$$v_{bc} + \Delta v = \frac{(r_g + \Delta r_g + r_w)}{(2r_g + \Delta r_g + r_w)}V - \frac{V}{2} \quad \text{A2.2}$$

Subtracting Eq. A2.1 from A2.2 and manipulating the resulting expression, yields

$$\Delta v = \frac{r_g \Delta r_g}{(2r_g + \Delta r_g + r_w)(r_w + 2r_g)}V$$
In the actual circuit \( 2r_g \) is equal to 700\( \Omega \); \( r_w \) is equal to 0.9\( \Omega \) for the strain gauge with the longest wire and \( \Delta r_g \) would be 0.7\( \Omega \) if the scale model drum reached yielding point. Then, since \( r_w \) and \( \Delta r_g \) are small in comparison with \( 2r_g \) it can be written without significant error that

\[
\Delta v = \frac{\Delta r_g}{4r_g} V
\]

Since the voltage \( V \) is maintained constant

\[
\frac{\Delta r_g}{r_g} = k\Delta v
\]

where \( k \) is a factor of proportionality.

If any other resistance in the bridge was varied, the corresponding variation in voltage between \( b \) and \( c \) would be related to the relative resistance variation by the factor \( k \). Comparing Eqs. A2.3 and A2.4 it can be seen that \( k = 4/V \). If \( V \) is known \( k \) can be immediately calculated. However, in order to determine \( k \), the common practice is to induce a known relative variation to one of the resistances in the bridge, measure the variation in voltage between \( b \) and \( c \) and then use Eq. A2.4. This procedure is known as bridge calibration.

If, for example, a resistance \( r_{cal} \) is connected in parallel with one of the resistors \( r_b \) shown in FIG. A2.1, its resistance will change to \( r_b + \Delta r_b \) where

\[
\Delta r_b = \frac{r_b^2}{r_b + r_{cal}}
\]

Calling \( \Delta v_{cal} \) the voltage variation caused by that change in resistance, from Eq. A2.4

\[
k = -\frac{r_b}{(r_b + r_{cal})\Delta v_{cal}}
\]
In the instrumentation circuit of the scale model drum $r_b = 1000\Omega$; $r_{cal} = 2.2 \times 10^6 \Omega$ and the measured voltage variation was $\Delta v_{cal} = -880 \mu V$, then

$$k = 5.163 \times 10^{-7} \Omega/\mu V$$

A2.1 Relationship Between Voltage and Strain in the Scale Model Drum

The relative change in resistance of a gauge subjected to a strain $\varepsilon$ is

$$\frac{\Delta r_g}{r_g} = GF \varepsilon$$  \hspace{1cm} \text{(A2.5)}$$

where GF is known as gauge factor or sensitivity and is specified by the gauge manufacturer. For the gauges in the scale model GF = 2.06.

Combining Eqs. A2.4 and A2.5

$$\frac{\varepsilon}{\Delta V} = \frac{k}{GF}$$

Then, the factor of proportionality between strain and voltage for the scale model drum is

$$\frac{\varepsilon}{\Delta V} = 2.506 \times 10^{-7} m/m/\mu V$$

A2.2 Temperature Effect and Correction

Returning to FIG A2.1 and assuming that together with a change in the gauge resistance there is also a change in the resistance of the wires to $r_w + \Delta r_w$ which changes the voltage $v_{bc}$ to $v_{bc} + \Delta v_1$. Then,

$$v_{bc} + \Delta v_1 = \frac{\left(r_g + \Delta r_g + r_w + \Delta r_w\right)}{\left(2r_g + \Delta r_g + r_w + \Delta r_w\right)} V - \frac{V}{2}$$  \hspace{1cm} \text{(A2.6)}$$
By subtracting Eq. A2.1 from A2.6 and manipulating

$$\Delta V_1 = \frac{(\Delta r_g + \Delta r_w)r_g}{(2r_g + \Delta r_g + r_w + \Delta r_w)(2r_g + r_w)V}$$

Again, since $r_w$, $\Delta r_g$ and $\Delta r_w$ are small compared with $r_g$, then

$$\Delta V_1 = \frac{\Delta r_g + \Delta r_w}{4r_g} V = \frac{\Delta r_g + \Delta r_w}{kr_g}$$

In order to isolate the variation induced by the strain from that induced by the change in resistance of the wire, the component $\Delta r_w$ has to be obtained. The only important source of variation in the wire resistance is that due to temperature. If the temperature change $\Delta \theta$ is known, $\Delta r_w$ can be calculated using

$$\Delta r_w = L r a \Delta \theta$$

where

- $L$ - length of the gauge wire
- $r$ - wire resistance per unit of length; 0.214$\Omega$/m for the wires in the scale model.
- $a$ - temperature coefficient of resistance; $4.27\times10^{-3}\Omega/\Omega/^\circ C$, for copper at 20$^\circ$C.

In order to correct the measured voltage $\Delta V_1$ for the effect of temperature, it is necessary to subtract from it the value given by

$$\frac{L r a \Delta \theta}{kr_g}$$

The result will be the variation induced by the strain applied to the gauge.

Table A2.1 shows the wire lengths of each gauge on the scale model drum together with the variation induced by one degree of change in temperature.
Table A2.1
Strain variation induced by one degree of change in temperature

<table>
<thead>
<tr>
<th>Gauge Number</th>
<th>Wire Length (m)</th>
<th>Variation (micro-strains)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>a 2</td>
<td>4.20</td>
<td>5</td>
</tr>
<tr>
<td>c 3</td>
<td>4.10</td>
<td>5</td>
</tr>
<tr>
<td>a 4</td>
<td>4.10</td>
<td>5</td>
</tr>
<tr>
<td>c 5</td>
<td>3.95</td>
<td>5</td>
</tr>
<tr>
<td>a 6</td>
<td>3.95</td>
<td>5</td>
</tr>
<tr>
<td>c 7</td>
<td>3.90</td>
<td>5</td>
</tr>
<tr>
<td>a 8</td>
<td>3.90</td>
<td>5</td>
</tr>
<tr>
<td>c 9</td>
<td>3.50</td>
<td>4</td>
</tr>
<tr>
<td>a 10</td>
<td>3.50</td>
<td>4</td>
</tr>
<tr>
<td>c 11</td>
<td>3.40</td>
<td>4</td>
</tr>
<tr>
<td>a 12</td>
<td>3.40</td>
<td>4</td>
</tr>
<tr>
<td>c 13</td>
<td>3.35</td>
<td>4</td>
</tr>
<tr>
<td>a 14</td>
<td>3.35</td>
<td>4</td>
</tr>
<tr>
<td>c 15</td>
<td>3.00</td>
<td>4</td>
</tr>
<tr>
<td>a 16</td>
<td>3.00</td>
<td>4</td>
</tr>
<tr>
<td>c 17</td>
<td>2.90</td>
<td>4</td>
</tr>
<tr>
<td>a 18</td>
<td>2.90</td>
<td>4</td>
</tr>
<tr>
<td>c 19</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>t 20</td>
<td>4.20</td>
<td>5</td>
</tr>
<tr>
<td>r 21</td>
<td>4.20</td>
<td>5</td>
</tr>
<tr>
<td>r 22</td>
<td>4.50</td>
<td>6</td>
</tr>
<tr>
<td>r 23</td>
<td>4.40</td>
<td>6</td>
</tr>
<tr>
<td>t 24</td>
<td>4.40</td>
<td>6</td>
</tr>
<tr>
<td>t 25</td>
<td>4.40</td>
<td>6</td>
</tr>
<tr>
<td>r 26</td>
<td>4.40</td>
<td>6</td>
</tr>
<tr>
<td>r 27</td>
<td>2.75</td>
<td>3</td>
</tr>
<tr>
<td>t 28</td>
<td>2.75</td>
<td>3</td>
</tr>
<tr>
<td>t 29</td>
<td>4.20</td>
<td>5</td>
</tr>
<tr>
<td>r 30</td>
<td>4.20</td>
<td>5</td>
</tr>
<tr>
<td>r 31</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>r 32</td>
<td>4.40</td>
<td>6</td>
</tr>
<tr>
<td>t 33</td>
<td>4.40</td>
<td>6</td>
</tr>
<tr>
<td>t 34</td>
<td>3.80</td>
<td>5</td>
</tr>
<tr>
<td>r 35</td>
<td>3.80</td>
<td>5</td>
</tr>
<tr>
<td>r 36</td>
<td>2.75</td>
<td>3</td>
</tr>
<tr>
<td>r 37</td>
<td>2.75</td>
<td>3</td>
</tr>
<tr>
<td>c 38</td>
<td>2.65</td>
<td>3</td>
</tr>
<tr>
<td>a 39</td>
<td>2.65</td>
<td>3</td>
</tr>
<tr>
<td>c 40</td>
<td>2.60</td>
<td>3</td>
</tr>
<tr>
<td>c 41</td>
<td>1.95</td>
<td>2</td>
</tr>
<tr>
<td>a 42</td>
<td>1.95</td>
<td>2</td>
</tr>
<tr>
<td>c 43</td>
<td>2.40</td>
<td>3</td>
</tr>
<tr>
<td>c 44</td>
<td>2.40</td>
<td>3</td>
</tr>
<tr>
<td>c 45</td>
<td>2.40</td>
<td>3</td>
</tr>
<tr>
<td>c 46</td>
<td>2.10</td>
<td>3</td>
</tr>
<tr>
<td>---</td>
<td>2.10</td>
<td>3</td>
</tr>
<tr>
<td>---</td>
<td>2.20</td>
<td>1</td>
</tr>
<tr>
<td>---</td>
<td>1.60</td>
<td>1</td>
</tr>
<tr>
<td>---</td>
<td>1.60</td>
<td>1</td>
</tr>
<tr>
<td>---</td>
<td>1.60</td>
<td>1</td>
</tr>
<tr>
<td>---</td>
<td>1.40</td>
<td>2</td>
</tr>
<tr>
<td>---</td>
<td>1.40</td>
<td>2</td>
</tr>
<tr>
<td>---</td>
<td>1.40</td>
<td>2</td>
</tr>
<tr>
<td>---</td>
<td>1.40</td>
<td>2</td>
</tr>
<tr>
<td>---</td>
<td>1.40</td>
<td>2</td>
</tr>
</tbody>
</table>
APPENDIX 3
A STRUCTURE FOR THE THEORETICAL MODEL FOR WINDING DRUMS

The aim of this Appendix is to show how the theoretical model which describes the effect of the rope compression on a winding drum and which was discussed in chapter 4, can be structured to facilitate its manipulation on a digital computer. Unless otherwise stated, the notation is the same as that in chapter 4.

As an illustration, consider in FIG. A3.1 a drum roll under an external pressure \( p \) supported by side plates built into rigid hubs at positions 1 and N.

Assume that the roll and plates are made of the same material. Further, consider that the roll is stiffened at positions 3 to N-2 by rings with cross-sectional areas \( A_s \), moments of area \( I_s \) and elasticity moduli \( E_s \). As discussed in chapter 4, a set of four constants of integration is associated with each region between any two adjacent discontinuities, defining a total of \( 4 \times N \) constants of integration and requiring \( 4 \times N \) simultaneous equations to determine them. Suppose \( C_{m1}, C_{m2}, C_{m3} \) and \( C_{m4} \) constitute the set of constants for the region between positions \( m \) and \( m+1 \) for \( 1 \leq m < N \). The equations involving the
constants of integration represent the conditions of the drum at each discontinuity point from 1 to N. Writing the equations for each point successively and starting at point 1 gives:

1 - Conditions at point 1, as discussed in section 4.3.

\[ w_p(r_1) = 0 \]
\[ w'_p(r_1) = 0 \]

Using Eq. 4.34 and its first derivative

\[ \frac{1}{r_i} C_{12} + 2r_i C_{13} + (r_i + 2r_i \log r_i) C_{14} = 0 \]
\[ C_{11} + \log r_i C_{12} + r_i^2 C_{13} + r_i^2 \log r_i C_{14} = 0 \]

These equations can be represented in matricial form as

\[ A_1 C_1 = B_1 \]

where

\[ A_1 = \begin{bmatrix} 1 & \log r_i & r_i^2 & r_i^2 \log r_i \\ 0 & \frac{1}{r_i} & 2r_i & (r_i + 2r_i \log r_i) \end{bmatrix} \]

\[ C_1 = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \end{bmatrix} \]

\[ B_1 = \begin{bmatrix} 0 & 0 \end{bmatrix} \]

As \( C_1 \) and \( B_1 \), all vectors in this Appendix will be written as row vectors.

Before writing the equations for point 2 it is necessary to define

\[ V_m = \sin \alpha_m \sinh \beta_m \]
\[ X_m = \sin \beta_m \cosh \beta_m \]
\[ Y_m = \cos \beta x_m \sinh \beta x_m \]
\[ Z_m = \cos \beta x_m \cosh \beta x_m \]

where
\[ x_m \] - coordinate of point \( m \)

2. Conditions at point 2, as discussed in section 4.3.

\[ w_p(R) = 0 \]
\[ w(x_1) + K_p Q(x_1) = 0 \]
\[ w'(R) + w'(x_1) = 0 \]
\[ M_r(R) + M_x(x_1) = 0 \]

Using Eqs. 4.23 and 4.34 and their derivatives and referring to section 4.5, the equations can be written as

\[
C_{11} + \log R C_{12} + R^2 C_{13} + R^2 \log R C_{14} = 0
\]

\[
V_1 C_{21} + X_1 C_{22} + Y_1 C_{23} + Z_1 C_{24} + 2K_p D B^3 [(-X_1 + Y_1) C_{21} +
+ (-V_1 + Z_1) C_{22} - (V_1 + Z_1) C_{23} - (X_1 + Y_1) C_{24}] = \frac{p R^2}{E E}
\]

\[
\frac{1}{R} C_{12} + 2 R C_{13} + (R + 2 R \log R) C_{14} + 6 [(X_1 + Y_1) C_{21} +
+ (V_1 + Z_1) C_{22} + (-V_1 + Z_1) C_{23} + (-X_1 + Y_1) C_{24}] = 0
\]

\[
D_p [\left(\frac{v - 1}{R^2}\right) C_{12} + 2(v + 1) C_{13} + (3 + v + 2 \log R + 2 v \log R) C_{14}] +
+ 2 D B^2 (-Z_1 C_{21} - Y_1 C_{22} + X_1 C_{23} + V_1 C_{24}) = 0
\]

Representing these equations in matrix form

\[ A_2 C_2 = B_2 \]

where
\[ A_2 \] - is given on page 245
\[ A_2 = \begin{pmatrix} 1 & \log R & R^2 & R^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1 + 2K_p D\theta^3 (-X_1 + Y_1) & X_1 + 2K_p D\theta^3 (-V_1 + Z_1) & Y_1 - 2K_p D\theta^3 (V_1 + Z_1) & Z_1 - 2K_p D\theta^3 (X_1 + Y_1) \\ 0 & \frac{1}{R} & 2R & R + 2R & \log R & \beta (X_1 + Y_1) & \beta (V_1 + Z_1) & \beta (-V_1 + Z_1) & \beta (-X_1 + Y_1) \\ 0 & D_p \frac{(n-1)}{R^2} & 2D_p (n+1) & D_p (3n+2\log R + 2n\log R) & -2D\theta^2 Z_1 & -2D\theta^2 Y_1 & 2D\theta^2 X_1 & 2D\theta^2 V_1 \end{pmatrix} \]

\[ A_m = \begin{pmatrix} V_m & X_m & Y_m & Z_m & -V_m & -X_m & -Y_m & -Z_m \\ \beta (X_m + Y_m) & \beta (V_m + Z_m) & \beta (-V_m + Z_m) & \beta (-X_m + Y_m) & -\beta (X_m + Y_m) & -\beta (V_m + Z_m) & -\beta (-V_m + Z_m) & -\beta (-X_m + Y_m) \\ -aZ_m - b(X_m + Y_m) & -aY_m - b(V_m + Z_m) & aX_m - b(-V_m + Z_m) & aV_m - b(-X_m + Y_m) & aZ_m & aY_m & -aX_m & -aV_m \\ c(-X_m + Y_m) - dV_m & c(-V_m + Z_m) - dX_m & c(-V_m - Z_m) - dY_m & c(-X_m - Z_m) - dZ_m & -c(-X_m + Y_m) & -c(-V_m + Z_m) & -c(-V_m - Z_m) & -c(-X_m - V_m) \end{pmatrix} \]

Where \( a = 2D\theta^2 \), \( b = \beta \frac{E_s I_s}{R^2} \), \( c = 2D\theta^3 \), \( d = \frac{E_s A_s}{R^2} \).
3. Conditions at points 3 to N-2, as discussed in section 4.1.5.

At any point m, 3 ≤ m ≤ N-2, the equations have the form

\[ w_i(x_m) - w_j(x_m) = 0 \]
\[ w_i'(x_m) - w_j'(x_m) = 0 \]
\[ M_{xi}(x_m) - M_{xj}(x_m) + \frac{E_s I_s}{R^2} w_i'(x_m) = 0 \]
\[ Q_{xi}(x_m) - Q_{xj}(x_m) + \frac{E_s A_s}{R^2} w_i(x_m) = 0 \]

The subscripts \( i \) and \( j \) represent, respectively, the sub-cylinder at the left-hand side and the sub-cylinder at the right-hand side of the point \( m \).

For brevity, these equations will not be rewritten explicitly, their matricial representation being

\[ A_m C_m = B_m \]

where

\( A_m \) is given on page 245

\[ C_m = \begin{bmatrix} C_{(m-1)1} & C_{(m-1)2} & C_{(m-1)3} & C_{(m-1)4} & C_m & C_m & C_m & C_m \end{bmatrix} \]

\[ B_m = \begin{bmatrix} 0 & 0 & 0 & \frac{E_s A_s}{Et} \end{bmatrix} \]
4 - Conditions at point N-1.

The equations representing the conditions at this point have the same form as those representing the conditions at point 2. Their matricial representation is

\[ A_{N-1}C_{N-1} = B_{N-1} \]

where

\[ C_{N-1} = \begin{bmatrix} C_{(N-2)1} & C_{(N-2)2} & C_{(N-2)3} & C_{(N-2)4} & C_{(N-1)1} & C_{(N-1)2} & C_{(N-1)3} & C_{(N-1)4} \end{bmatrix} \]

\[ B_{N-1} = \begin{bmatrix} 0 & \frac{pR^2}{Et} & 0 & 0 \end{bmatrix} \]

The matrix \( A_{N-1} \) is as the matrix \( A_2 \) with \( V_1, X_1, Y_1 \) and \( Z_1 \) substituted by \( V_{N-1}, X_{N-1}, Y_{N-1} \) and \( Z_{N-1} \) and with the sign of the elements of the last four columns of the last row changed.

5 - Conditions at point N.

The matricial representation of the equations representing the conditions at this point is

\[ A_N C_N = B_N \]

where

\[ A_N = A_1 \], given in the conditions at point 1.

\[ C_N = \begin{bmatrix} C_{N1} & C_{N2} & C_{N3} & C_{N4} \end{bmatrix} \]

\[ B_N = \begin{bmatrix} 0 & 0 \end{bmatrix} \]

At this stage, the whole set of \( 4 \times N \) simultaneous equations has been established. This whole set can also be written in matricial form as

\[ AC = B \]

The matrix A is shown in FIG. A3.2 where the sub-matrices in the boxes
FIG. A3.2 MATRIX A
are those presented in the foregoing and the numbers in brackets represent their dimensions. All elements of matrix A not in the boxes are zero. The vectors C and B are

\[
C = \begin{bmatrix} C_1 & C_2 & \ldots & C_N \end{bmatrix}
\]

\[
B = \begin{bmatrix} B_1 & B_2 & \ldots & B_N \end{bmatrix}
\]

where the elements are the vectors also presented in the foregoing.

The matrix A is a square matrix and the expression in Eq. A3.1 can be solved for C and written as

\[
C = A^{-1}B
\]

where

\[
A^{-1} - \text{inverse of } A
\]

Thus the constants of integration can be determined by forming the matrix A, finding its inverse and multiplying the vector B by it. The elements of the resulting vector C are the constants of integration sought.

The winding drum problem when put in the form discussed above can be readily programmed into most digital computers operating with a high level language such as FORTRAN or BASIC. Both the matrix A and the vector B can be totally generated in the computer and subroutines for matrix inversion are provided for most of them. The maximum number of discontinuity points that the program can accept will be limited by the size of the computer memory. However, it seems that the size of memory required can be reduced if the matrix A is transformed into a diagonal matrix of submatrices. In this case, it will only be necessary to operate with the submatrices rather than with the whole of the matrix A. Another alternative is the use of the matrix transfer method. 

APPENDIX 4
COMPUTER PROGRAMS
Program to calculate the constants of integration for the scale model problem. All the parameters or constants are specified in the program itself. The program also allows the calculation of deflections, slopes, moments, shears, strains and stresses at any point of the drum roll or sides. Graphs for any of those variables can also be displayed.

Copyright 1981 - R.S. de Andrade

Parameters and constants

Layers=1
Ropefac=.90
IF Layers=2 THEN Ropefac=1.65
Pull=2150/2*9.81
Length=630
T=14 ! Thickness of the roll
Tp=13 ! Thickness of the side plates
Rro=660 ! Radius of the outer edge of the plate
Radius=610
Rri=130 ! Radius of the plate at the hub
Ropepitch=11
Ab=(86+58+61)*10*1 ! Area of the brake ring
Ib=1*(86*10^3/12+86*10*22^2+58*10^3/12+58*10*49^2+10*6
1^3/12+61*10*14^2) ! Moment of inertia of the brake ring
Poisson=.3
E=206000 ! Elasticity modulus for the roll and plate
Eb=176500 ! Elasticity modulus for the brake rings
Pressure=Pull/(Radius*Ropepitch)*Ropefac
D=E*T^3/(12*(1-Poisson^2))
Dp=E*Tp^3/(12*(1-Poisson^2))
Beta=(3*(1-Poisson^2)/(Radius*T)^2)^.25
Position of the discontinuity points on the roll

READ L(*)

Data: DATA 0,40,44,140.0,165.0,190.0,290.0,315.0,340.0, 440.0,465.0,490.0,566,590,630

Generation of the matrix of coefficients

Conditions at the roll between the joints with sides

Deflections

Slopes

Moments

Introduction of the inertia of the stiffening ring

Iner=60*8^3/12+10*50^3/12

IS=Iner

IF (L(Mu)=40) OR (L(Mu)=590) THEN IS=12*40^3/12
IF L(M) = 315.0 THEN Is = 1nEr + 8*64^3/12

IF L(M) = 290.0 OR L(M) = 340.0 THEN Is = 46*10^3/12

Mom = E*ls*Fac1/Radius^2

A(I+3, J+1) = -FNZ(P) * (2*D*Beta^3) - Beta*(FHX(P) + FNY(P)) * M

A(I+3, J+2) = -FNY(P) * (2*D*Beta^3) - Beta*(FHX(P) + FNY(P)) * M

A(I+3, J+3) = FNX(P) * (2*D*Beta^3) - Beta*(-FNY(P) + FNZ(P)) * M

A(I+3, J+4) = FNY(P) * (2*D*Beta^3) - Beta*(-FNX(P) + FNY(P)) * M

A(I+3, J+5) = FNX(P) * (2*D*Beta^3)

A(I+3, J+6) = FNY(P) * (2*D*Beta^3)

A(I+3, J+7) = -FNX(P) * (2*D*Beta^3)

A(I+3, J+8) = FNY(P) * (2*D*Beta^3)

Shears

Introduction of the areas of the stiffening ring

Area = 50*10

As = Area

IF L(M) = 40 OR L(M) = 590 THEN As = 12*40

IF L(M) = 315.0 THEN As = Area + 10*64 / 3 + 60*B

IF L(M) = 140.0 OR L(M) = 190.0 OR L(M) = 440.0 OR L(M) = 490.0 THEN As = Area / 3

IF L(M) = 165.0 OR L(M) = 465.0 THEN As = Area / 3 + 8.60

Qt = Fac1*(As*E)/Radius^2 + Fac2*(As*E)/Radius^2

A(I+4, J+1) = -FNYCP) + FHZ(P» * C2*D*Beta A 3) - FHYCP)* Qt

A(I+4, J+2) = (-FNYCP) + FHZ(P» * C2*D*Beta A 3) - FNX(P)* Qt

A(I+4, J+3) = C-FNZ(P) - FNZCP» * C2*D*Beta A 3) - FNY(P)* Qt

A(I+4, J+4) = C-FNXCP) - FNY(P».C2*D*Beta A 3) - FHZ(P)* Qt

A(I+4, J+5) = C-FNXCP) + FNYCP»*(2*D*Beta A 3)

A(I+4, J+6) = (-FNVCP) + FNZCP»*C2*D*BetaA3)

A(I+4, J+7) = (-FNYCP)-FNZCP»*C2*D*BetaA3)

A(I+4, J+8) = (-FNXCP)-FNYCP»*(2*D*BetaA3)

NEXT Mu

Conditions at the hubs and at the roll-side joints

FOR M = 0 TO 1

P = Beta*L(14)*M

I = (4*14+6)*M

J1 = (4*14+4)*M

Deflection at hub

J = J1 + 8*M

A(I+1, J+1) = 1

A(I+1, J+2) = LOG(Rri)

A(I+1, J+3) = Rri^2

A(I+1, J+4) = LOG(Rri)*Rri^2

NEXT Mu
A(I+2, J+1) = 0
A(I+2, J+2) = 1/Rri
A(I+2, J+3) = 2*Rri
A(I+2, J+4) = Rri + 2*Rri * LOG(Rri)

Deflections of side at joint

A(I+3, J+1) = 1
A(I+3, J+2) = LOG(Radius)
A(I+3, J+3) = Radius^2
A(I+3, J+4) = LOG(Radius) * Radius^2

Deflection of roll at joint

J = J1 + 8*(1-M)

Ko = -2*Radius^2/(Radius^2 + Rri^2 + Poisson*(Radius^2 - Rri^2))
Kp = Radius*(Poisson-1)*(Radius^2 + Ko*Rri^2) - (Poisson+1) * Rri^2*(1+Ko)/(E*Tp*(Radius^2 - Rri^2))
Kq = Kp * Radius^2/(Radius^2 + Ab*Eb*Kp)

Slope of roll at joint equal slope of side

Moment at side equal moment at roll
radius))
2040 |
2050 | Moment at the outerpart of the side
2060 |
2070 J=J1+4
2080 R(I+6,J+1)=0
2090 A(I+6,J+2)=-Dp*(Poisson-1)/Radius^2
2100 R(I+6,J+3)=-2*Dp*(1+Poisson)
2110 A(I+6,J+4)=-Dp*(3+Poisson+2*LOG(Radius)+2*Poisson*LOG(Radius))
2120 |
2130 | Roll moment
2140 |
2150 J=J1+8*(1-M)
2160 Mo=Eb*lb/Radius^2
2170 A(I+6,J+1)=2*D*Beta^2*FNY(P)*(-1)^M-Beta*(FHX(P)+FNY(P))
2180 A(I+6,J+2)=2*D*Beta^2*FNY(P)*(-1)^M-Beta*(FHX(P)+FNY(P))
2190 A(I+6,J+3)=-2*D*Beta^2*FHX(P)*(-1)^M-Beta*(-FNY(P)+FHX(P))
2200 A(I+6,J+4)=-2*D*Beta^2*FNY(P)*(-1)^M-Beta*(-FNY(P)+FHX(P))
2210 |
2220 | Deflection of innerpart of side equal deflection of outerpart
2230 |
2240 | Innerpart
2250 |
2260 J=J1+8*M
2270 R(I+7,J+1)=1
2280 A(I+7,J+2)=LOG(Radius)
2290 A(I+7,J+3)=Radius^2
2300 A(I+7,J+4)=LOG(Radius)*Radius^2
2310 |
2320 | Outerpart
2330 J=J1+4
2340 A(I+7,J+1)=-1
2350 A(I+7,J+2)=-LOG(Radius)
2360 A(I+7,J+3)=-Radius^2
2370 A(I+7,J+4)=-LOG(Radius)*Radius^2
2380 |
2390 | Slope of innerpart of side equal slope of outerpart
2400 |
2410 | Innerpart
2420 |
2430 J=J1+8*M
2440 A(I+8,J+1)=0
2450 A(I+8,J+2)=1/Radius
2460 A(I+8,J+3)=2*Radius
2470 A(I+8,J+4)=Radius+2*Radius*LOG(Radius)
2480 |
2490 | Outerpart
2500 |
2510 J=J1+4
2520 A(I+8,J+1)=0
2530 A(I+8,J+2)=-1/Radius
\[ A(I+8, J+3) = -2 \times \text{Radius} \]
\[ A(I+8, J+4) = -(\text{Radius} + 2 \times \text{Radius} \times \log(\text{Radius})) \]

Moment at outer edge of side

\[ A(I+9, J+1) = 0 \]
\[ A(I+9, J+2) = Dp \times ((\text{Poisson} - 1) / Rro^2) \]
\[ A(I+9, J+3) = 2 \times Dp \times (1 + \text{Poisson}) \]
\[ A(I+9, J+4) = Dp \times (3 + \text{Poisson} + 2 \times \log(Rro) + 2 \times \text{Poisson} \times \log(Rro)) \]

Shear at outer edge of side

\[ A(I+10, J+3) = 0 \]
\[ A(I+10, J+2) = Dp \times (I / Rro^3) \]
\[ A(I+10, J+3) = Dp \times (4 / Rro) \]
\[ A(I+10, J+4) = 0 \]

Next

Generation of the vector of independent terms

\[ B = 2 \times \text{E} \times \text{R} \]
\[ Bb = 2 \times \text{E} \times \text{R} \]
\[ Bq = 2 \times \text{E} \times \text{R} \]

FOR \( S = 1 \) TO \( 13 \)

\[ \text{Fac1} = \text{Fac2} = 1 \]
\[ \text{Fac1} = 1 \]
\[ \text{IF} \ (S = 1) \ \text{OR} \ (S = 7) \ \text{OR} \ (S = 13) \ \text{THEN} \ \text{Fac1} = \text{Fac2} = \text{Fac1} = 0 \]
\[ \text{IF} \ S = 2 \ \text{THEN} \ \text{Fac1} = \text{Fac2} = 0 \]
\[ \text{IF} \ S = 6 \ \text{THEN} \ \text{Fac2} = 0 \]
\[ \text{IF} \ S = 8 \ \text{THEN} \ \text{Fac1} = \text{Fac2} = 0 \]
\[ \text{Area} = 50 \times 10 \]
\[ \text{As} = \text{Area} \]
\[ \text{IF} \ (S = 3) \ \text{OR} \ (S = 4) \ \text{OR} \ (S = 5) \ \text{OR} \ (S = 9) \ \text{OR} \ (S = 10) \ \text{OR} \ (S = 11) \ \text{THEN} \ \text{As} = \text{Area} / 3 \]
\[ \text{IF} \ (S = 4) \ \text{OR} \ (S = 10) \ \text{THEN} \ \text{As} = \text{As} / 3 \times 60 	imes 8 \]
\[ \text{IF} \ (S = 6) \ \text{OR} \ (S = 8) \ \text{THEN} \ \text{As} = (\text{As} + 10 \times 64) / 3 + 46 \times 10 \]
\[ \text{IF} \ (\text{Layers} = 2) \ \text{AND} \ (S = 8) \ \text{THEN} \ \text{Fac1} = \text{Fac2} = \text{Fac1} = 0 \]
\[ \text{Next} \ S \]

\[ \text{Comeco} = 11 \]
\[ \text{Fim} = 4 \times 14 + 3 \]
\[ S = 0 \]

FOR \( I = \text{Comeco} \) TO \( \text{Fim} \) STEP 4

\[ B(I) = Bb(S) \]
\[ B(I+3) = Bq(S) \]

NEXT \( I \)

Inversion of the matrix of coefficients
DISP "MATRIX INVERTING"
MAT A=INV(A)
DISP ""

Determination of the constants of integration
MAT C=ZER
MAT C=A*B

Tape recording of the constants
ASSIGN #7 TO "CONS00"
PRINT #7;C(*)

Determination or display of the variables

PRINTER IS 16
PRINT TAB(8),"VARIABLE CODES",LIN(2)," 1 - Roll deflection"," 2 - Roll slope"
PRINT " 3 - Roll moment",LIN(1)," 4 - Roll shear",LIN(1)," 5 - Roll longitudinal strain"
PRINT " 6 - Roll longitudinal stress",LIN(1)," 7 - Roll circumferential strain",LIN(1)," 8 - Roll circumferential stress",LIN(1)," 9 - Side deflection"
PRINT "10 - Side slope",LIN(1),"11 - Side radial moment",LIN(1),"12 - Side tangential moment",LIN(1),"13 - Side radial strain"
PRINT "14 - Side radial stress",LIN(1),"15 - Side tangential strain",LIN(1),"16 - Side tangential stress",LIN(1),"17 - Side shear"

PRINTER IS 16
A$(1)="Roll deflection at 
A$(2)="Roll slope at 
A$(3)="Roll moment at 
A$(4)="Roll shear at 
A$(5)="Roll axial strain at 
A$(6)="Roll axial stress at 
A$(7)="Roll circumferential strain at 
A$(8)="Roll circumferential stress at 
A$(9)="Side deflection at 
A$(10)="Side slope at 
A$(11)="Side radial moment at 
A$(12)="Side tangential moment at 
A$(13)="Side radial strain at 
A$(14)="Side radial stress at 
A$(15)="Side tangential strain at 
A$(16)="Side tangential stress at 
A$(17)="Side shear at 
B$(1)="mm"
B$(2)="rad"
B$(3)="Hmm/mm"
B$(4)="N/mm"
New condition: INPUT "VALUES OR GRAPHS OF THE VARIABLES ? (answer V or G and press CONT)",O$
3750 INPUT "Press the variable code and CONT",W
3760 
3770 IF O$="G" THEN GOTO 4590
3780 
3790 ! Determination of the values
3800 
3810 IF W)=9 THEN GOTO 4210
3820 !
3830 ! Values on the roll
3840 !
3850 INPUT "Input the distance of the point to the left hand edge",Dx
3860 FOR I=1 TO 14
3870 IF Dx<L(I) THEN GOTO 3910
3880 NEXT I
3890 PRINT "ERROR - The point";Dx;" is not on the roll"
3900 GOTO 3850
3910 J=4*I+4
3920 FOR M=I TO 4
3930 Cr(M)=C(J+M)
3940 NEXT M
3950 Fac=1
3960 IF (I=1) OR (I=2) OR (I=7) OR (I=8) OR (I=13) OR (I=14)
3970 THEN Fac=0
3980 IF (Layers=2) AND ((I=1) OR (I=2) OR (I)=7)) THEN Fac=0
3990 H=Beta*Dx
4000 ! Deflection
4010 ! Slope
4020 Value(2)=Wp=Beta*(Cr(2)-Cr(3))*FNY(H)+(Cr(I)-Cr(4))*FNX(H)+(Cr(2)+Cr(3)*FNZ(H))
4030 ! Moment
4040 Value(3)=Mx=2*D*Beta*(Cr(3)+Cr(2)*FNY(H)-(Cr(4)+Cr(I)*FNX(H)+(Cr(1)-Cr(4)*FNY(H))
4050 ! Shear
4060 Value(4)=Qx=2*D*Beta^2*(-(Cr(4)*FNY(H)-Cr(3)*FNX(H)+Cr(2)*Cr(1))*FNZ(H))
4070 ! Axial strain
4080 Value(5)=Strainx=(T/2*Mx/D-Poisson*Wx/Radius)*1E6
4090 ! Axial stress
4100 Value(6)=Stressx=6*Mx/T^2
4110 ! Circumferential strain
4120 Value(7)=Straino=Wx/Radius*1E6
4130 ! Circumferential stress
4140 Value(8)=Stresso=6*Poisson*Mx/T^2+E*Wx/Radius
4150 !
4160 PRINT A$(W);Dx; "mm is ";Value(W);B$(W)
4170 GOTO 3740
4180 !
4190 ! Values on the side
4200 !
4210 INPUT "Input the distance of the point to the center of the plate",Dx
4220 J=0
4230 IF Dx>Radius THEN J=4
4240 FOR M=1 TO 4
4250 Cp(M)=C(M+J)
4260 NEXT M
4270 Z=Dx
4280 Qx=-(2*D*Beta^3)*C(10)-C(11)*FNZ(0)
4290 Pe=Qx/Tp
4300 ! Radial stress due in-plane force
4310 Sir=Pe*(-Ko*Rri^2-Radius^2+(Radius*Rri/Z)^2+(1+Ko))/(Radius^2-Rri^2)
4320 ! Tangential stress due in-plane force
4330 Sit=Pe*(-Ko*Rri^2-Radius^2-(Radius*Rri/Z)^2+(1+Ko))/(Radius^2-Rri^2)
4340 ! Deflection
4350 Value(9)=Wp=Cp(1)+Cp(2)*LOG(Z)+Cp(3)*Z^2+Cp(4)*LOG(Z)+Z^2
4360 ! Slope
4370 Value(10)=Wpsl=Cp(2)/Z+2*Z*Cp(3)+(Z+2*Z*LOG(Z))*Cp(4)
4380 ! Radial moment
4390 Value(11)=Mr=Dp*(Cp(2)*(Poisson-1)/Z^2+2*(1+Poisson)*Cp(3)+(3+Poisson)*LOG(Z)+2*Poisson*LOG(Z))*Cp(4)
4400 ! Tangential moment
4410 Value(12)=Mt=Dp*((1-Poisson)*Cp(2)/Z^2+(2+2*Poisson)*Cp(3)+(3+Poisson+2*LOG(Z)+2*Poisson*LOG(Z))*Cp(4))
4420 ! Radial strain
4430 Value(13)=Strr=(-Tp*(-Cp(2)/Z^2+2*Cp(3)+(2*LOG(Z)+3)*Cp(4)
4440 ! Tangential strain
4450 Value(14)=Stsr=-6*Mr/Tp^2+Sir
4460 ! Tangential stress
4470 Value(15)=Sttr=(-Tp*(Cp(2)/Z^2+2*Z*Cp(3)+(2*Z*LOG(Z)+Z)*Cp(4))/(2*Z)+(Sit-Poisson*Sir)/E)*1E6
4480 ! Tangential stress
4490 Value(16)=Stst=-6*Mt/Tp^2+Sit
4500 ! Shear
4510 Value(17)=Qr=Dp*(Cp(2)/Z^3+4*Cp(3)/Z)
4520 !
4530 PRINT A$(W);Dx; "mm is ";Value(W);B$(W)
4540 GOTO 3740
4550 !
4560 !
4570 ! Graph plotting
4580 !
4590 IF W>=9 THEN GOTO Side
4600  J=1:  Ji=0
4610  MOVE 0,0
4620  FOR I=1 TO 14
4630  Fac=1
4640  IF (I=1) OR (I=2) OR (I=7) OR (I=8) OR (I=13) OR (I=14) THEN Fac=0
4650  !
4660  !
4670  IF (Layers=2) AND ((I=1) OR (I=2) OR (I=7)) THEN Fac=0
4680  !
4690  !
4700  J=4*I+4
4710  FOR M=1 TO 4
4720  Cr(M)=C(J+M)
4730  NEXT M
4740  Lem=L(I-1)
4750  L=L(I)
4760  ON W GOTO 4770,4790,4810,4830,4850,4870,4890,4910
4770  SCALE -20,630,-.85,.1
4780  GOTO 4920
4790  SCALE -20,630,-4E-3,4E-3
4800  GOTO 4920
4810  SCALE -20,630,-9500,9500
4820  GOTO 4920
4830  SCALE -20,630,-200,200
4840  GOTO 4920
4850  SCALE -20,630,-5.5E-4,5.5E-4
4860  GOTO 4920
4870  SCALE -20,630,-100,100
4880  GOTO 4920
4890  SCALE -20,630,-5.5E-4,5.5E-4
4900  GOTO 4920
4910  SCALE -20,630,-100,100
4920  GRAPHICS
4930  !
4940  !
4950  Tot=(630/50
4960  !
4970  !
4980  ON W GOTO 5040,5110,5240,5370,5500,5650,5790,5930
4990  !
5000  !
5010  !
5020  Roll deflection
5030  !
5040  IF I=1 THEN MOVE 0,0
5050  FOR Z=Lem TO Le STEP Tot
5060  H=Beta*Z
5070  Wx=Cr(1)*FNY(H)+Cr(2)*FXN(H)+Cr(3)*FNY(H)+Cr(4)*FNZ(H)
- Fac*Pressure*Radius^2/(E*T)
5080  DRAW Z,Wx
5090  NEXT Z
5100  GOTO Mnu
5110  !
5120  !
5130  !
5140  Roll slope
! IF I=1 THEN MOVE 0,0
5170 FOR Z=Lem TO Le STEP Tot
5180 H=Beta*Z
5190 Wpl=Beta*({Cr(2)-Cr(3)}*FNY(H)+{Cr(1)-Cr(4)}*FNX(H)+{Cr(1)+Cr(4)}*FNY(H)+{Cr(2)+Cr(3)}*FNZ(H))
5200 IF Z=0 THEN MOVE Z,Wpl
5210 DRAW Z,Wpl
5220 NEXT Z
5230 GOTO Mun
5240 ! Roll moment
5250 ! Roll shear
5260 ! Roll axial strain
5270 ! IF I=1 THEN MOVE 0,0
5300 FOR Z=Lem TO Le STEP Tot
5310 H=Beta*Z
5320 Hx=2*D*Beta*(Cr(1)*FNY(H)+Cr(2)*FNX(H)+Cr(3)*FNY(H)+Cr(4)*FNZ(H))
5330 IF Z=0 THEN MOVE Z,Hx
5340 DRAW Z,Hx
5350 NEXT Z
5360 GOTO Mun
5370 !
5380 !
5390 !
5400 ! Roll shear
5410 !
5420 IF I=1 THEN MOVE 0,0
5430 FOR Z=Lem TO Le STEP Tot
5440 H=Beta*Z
5450 Qx=2*D*Beta*(Cr(1)*FNY(H)+Cr(2)*FNX(H)+Cr(3)*FNY(H)+Cr(4)*FNZ(H))
5460 IF Z=0 THEN MOVE Z,Qx
5470 DRAW Z,Qx
5480 NEXT Z
5490 GOTO Mun
5500 !
5510 !
5520 ! Roll axial strain
5530 ! IF I=1 THEN MOVE 0,0
5550 FOR Z=Lem TO Le STEP Tot
5570 H=Beta*Z
5580 Wx=Cr(1)*FNY(H)+Cr(2)*FNX(H)+Cr(3)*FNY(H)+Cr(4)*FNZ(H)
5590 Mx=2*D*Beta*(Cr(1)*FNY(H)+Cr(2)*FNX(H)+Cr(3)*FNY(H)+Cr(4)*FNZ(H))
5600 Strainx=T/2*Mx/D-Poisson*Wx/Radius
5610 IF Z=0 THEN MOVE Z,Strainx
5620 DRAW Z,Strainx
5630 NEXT Z
5640 GOTO Mun
5650 !
5660 !
5670 !
5660 ! Roll axial stress
5670 !
5680 IF I=1 THEN MOVE 0,0
5690 FOR Z=Zem TO Ze STEP Tot
5700 H=Beta*Z
5710 Mx=2*D*Beta^2*(-Cr(4)*FNV(H)-Cr(3)*FNX(H)+Cr(2)*FNY(H)
+Cr(1)*FNZ(H))
5720 Stressx=6*Mx/T^2
5730 IF Z=0 THEN MOVE Z,Stressx
5740 DRAW Z,Stressx
5750 NEXT Z
5760 GOTO Mun
5770 !
5780 ! Roll circumferential strain
5790 !
5800 IF I=1 THEN MOVE 0,0
5810 FOR Z=Zem TO Ze STEP Tot
5820 H=Beta*Z
5830 Wx=Cr(1)*FNV(H)+Cr(2)*FNX(H)+Cr(3)*FNY(H)+Cr(4)*FNZ(H)
-Fac*Pressure*Radius^2/(E*T)
5840 Straino=Wx/Radius
5850 IF Z=0 THEN MOVE Z,Straino
5860 DRAW Z,Straino
5870 NEXT Z
5880 GOTO Mun
5890 !
5900 ! Roll circumferential stress
5910 !
5920 IF I=1 THEN MOVE 0,0
5930 FOR Z=Zem TO Ze STEP Tot
5940 H=Beta*Z
5950 Wx=Cr(1)*FNV(H)+Cr(2)*FNX(H)+Cr(3)*FNY(H)+Cr(4)*FNZ(H)
-Fac*Pressure*Radius^2/(E*T)
5960 Mx=2*D*Beta^2*(-Cr(4)*FNV(H)-Cr(3)*FNX(H)+Cr(2)*FNY(H)
+Cr(1)*FNZ(H))
5970 Stresso=6*Poisson*Mx/T^2+E*Wx/Radius
5980 IF Z=0 THEN MOVE Z,Stresso
5990 DRAW Z,Stresso
6000 NEXT Z
6010 !
6020 !
6030 !
6040 MUN: NEXT I
6050 BEEP
6060 PAUSE
6070 GOTO New_condition
6080 !
6090 !
6100 !
6110 Side:
6120 !
6130 !
6140 !
6150 !
6160 !
6170 !
6180 Side:
6190 !
6200 !
6210 FOR Sus=1 TO 2
FOR I=1 TO 2
IF I=1 THEN In=0
IF I=1 THEN Fim=Radius
IF I=2 THEN In=Radius
IF I=2 THEN Fim=Rro
Tot=5
FOR M=1 TO 4
Cp(M)=C(4*(I-1)+M)
IF Sus=2 THEN Cp(M)=C(72-4*I+M)
NEXT M
Determination of the stresses due the in-plane forces
IF Sus=2 THEN O=52
IF Sus=2 THEN P=9
IF Sus=2 THEN P=Beta*L(14)
0x=2*D*Beta^3*(-(C(11+0)+C(19+0))*FNY(P)-(C(12+0)+C(9+0)))*FNX(P)+(C(10+0)-C(11+0))*FZ(P)
Pe=Qx/Tp*Radius^2/(Radius^2+Kp*Ab+Eb)
IF Sus=2 THEN Pe=Pe
!
GRAPHICS
ON W COTO 11379,11379,11379,11379,11379,11379,11379,11390
379,6460,6590,6720,6850,6980,7150,7300,7470,7620
!
!
!
Side_deflection:
SCALE -.85,.85,0,660
MOVE Wp,In
FOR Z=ln TO Fim STEP Tot
Wp=Cp(1)+Cp(2)*LOG(Z)+Cp(3)*Z^2+Cp(4)*LOG(Z)*Z^2
IF Z<Rri THEN Wp=0
DRAW Wp,Z
NEXT Z
COTO Moc
!
!
!
!
Side_slope:
SCALE -4E-3,4E-3,0,660
MOVE Wps1,In
FOR Z=ln TO Fim STEP Tot
Wps1=Cp(2)/Z+2*Z*Cp(3)+(Z+2*Z*LOG(Z))*Cp(4)
IF Z<Rri THEN Wps1=0
DRAW Wps1,Z
NEXT Z
COTO Moc
!
!
!
!
Side_rad_moment:
SCALE -9500,9500,0,660
MOVE Mr,In
FOR Z=ln TO Fim STEP Tot
6770 Mr=Ip*(Cp(2)*((Poisson-1)/Z^2)+2+(1+Poisson)*Cp(3)+3*Poisson+2*log(Z)+2*Poisson*log(Z)*Cp(4))
6780 IF Z<Rri THEN Mr=0
6790 DRAW Mr,Z
6800 NEXT Z
6810 GOTO Moc
6820 !
6830 !
6840 !
6850 Side_tan_moment: !
6860 !
6870 SCALE -9500,9500,0,660
6880 MOVE Mt,In
6890 FOR Z=In TO Fim STEP Tot
6900 Mt=Ip*((1-Poisson)*Cp(2)/Z^2+(2+2*Poisson)*Cp(3)+3*Poisson+2*log(Z)+2*Poisson*log(Z)*Cp(4))
6910 IF Z<Rri THEN Mt=0
6920 DRAW Mt,Z
6930 NEXT Z
6940 GOTO Moc
6950 !
6960 !
6970 !
6980 Side_rad_strain: !
6990 !
7000 SCALE -5.5E-4,5.5E-4,0,660
7010 MOVE Strr,In
7020 FOR Z=In TO Fim STEP Tot
7030 Strr=Pe*(-Ko*Rri^2-Radius^2+(Radius*Rri)/Z^2+(1+Ko))/(Radius^2-Rri^2)
7040 Mr=Ip*(Cp(2)*((Poisson-1)/Z^2)+2+(1+Poisson)*Cp(3)+3*Poisson+2*log(Z)+2*Poisson*log(Z)*Cp(4))
7050 Strr=-6*Mr/TP^2+Strr
7060 IF Z<Rri THEN Strr=0
7070 DRAW Strr,Z
7080 NEXT Z
7090 !
7100 !
7110 GOTO Moc
7120 !
7130 !
7140 !
7150 Side_rad_stress: !
7160 !
7170 SCALE -100,100,0,660
7180 MOVE Stsr,In
7190 FOR Z=In TO Fim STEP Tot
7200 Stsr=Pe*(-Ko*Rri^2-Radius^2/(Radius*Rri)/Z^2+(1+Ko))/(Radius^2-Rri^2)
7210 Mr=Ip*(Cp(2)*((Poisson-1)/Z^2)+2+(1+Poisson)*Cp(3)+3*Poisson+2*log(Z)+2*Poisson*log(Z)*Cp(4))
7220 Stsr=-6*Mr/Typ^2+Stsr
7230 IF Z<Rri THEN Stsr=0
7240 DRAW Stsr,Z
7250 NEXT Z
SCALE -5.5E-4,5.5E-4,0,660
FOR Z=In TO Fin STEP Tot
Sir=Pe*(-Ko*Rri^2-Radius^2+(Radius*Rri/Z)^2*(1+Ko))/(Radius^2-Rri^2)
Sit=Pe*(-Ko*Rri^2-Radius^2-(Radius*Rri/Z)^2*(1+Ko))/(Radius^2-Rri^2)
Mr=Dp*(Cp(2)*(1-Poisson)*Z^2+2*(1+Poisson)*Cp(3)+(3+Poisson+1+2*LOG(Z)))
Mt=Dp*(Cp(2)/Z^2+2*Z*Cp(3)+(2*Z+LOG(Z)))
Strt=-Tp*(Cp(2)/Z^2+2*Z*Cp(3)+(2*Z+LOG(Z))+Z)*E
IF Z<Rri THEN Strt=0
DRAW Strt,Z
NEXT Z
GOTO Moc

SCALE -100,100,0,660
FOR Z=In TO Fin STEP Tot
Sit=Pe*(-Ko*Rri^2-Radius^2-(Radius*Rri/Z)^2*(1+Ko))/(Radius^2-Rri^2)
Mt=Dp*(Cp(2)/Z^2+2*Z*Cp(3)+(2*Z+LOG(Z)))
Stst=6*Mt/Tp^2+Sit
IF Z<Rri THEN Stst=0
DRAW Stst,Z
NEXT Z
GOTO Moc

SIDE shearr
SCALE -200,200,0,660
FOR Z=In TO Fin STEP Tot
Qr=Dp*(Cp(2)/Z^2+4*Cp(3)/Z)
IF Z<Rri THEN Qr=0
DRAW Qr,Z
NEXT Z
GOTO Moc
END
This program calculates deflections, slopes and moments on the model drum shaft taking into account the rest of the side plates. It also calculates strains on the plates.

```
10 ! SHAFT2
20 ! This program calculates deflections, slopes and moments
30 ! on the model drum shaft taking into account the rest
40 ! of the side plates. It also calculates strains on the plates
50 !
60 OPTION BASE 1
70 DIM Ash(18,18),Bsh(18),Csh(18),L(0:7)
80 !
90 G=1720 ! Weight of the driving gear
100 S=1.236 ! Weight of the shaft per unit of length
110 Self_weight=6340*0! Drum plus rope self-weight
120 Load=21000
130 Ft=Self_weight+Load
140 P=Ft/2
150 Tp=13 ! Plate thickness
160 E=206000
170 !
180 ! Position of the discontinuity points on the shaft
190 L(0)=0
200 L(1)=350
210 L(2)=470
220 L(3)=630
230 L(4)=1250
240 L(5)=1410
250 L(6)=1530
260 L(7)=1660
270 !
280 ! Determination of the parameters of the plate
290 !
300 Radius=610
310 Rri=130
320 Gauge_position=160
330 Rho=Gauge_position/Radius
340 Rhoi=Rri/Radius
350 F2=(1+Rhoi^2)*LOG(Rhoi)
360 F1=1-Rhoi^2
370 Cc1=.500*(F1+2*F2)/(F1+F2)
380 Cc2=.500*1/(F1+F2)
390 Cc3=1-Cc1-Cc2
400 Cc4=2*(1-Cc1-2*Cc2)
410 F3=2*(3+.3)*Cc2*Rho+2*(1-.3)*Cc3*Rho*(-3)+(1+.3)*Cc4*Rho*(-1)
420 F4=2*(3*.3+1)*Cc2*Rho+2*(.3-1)*Cc3*Rho*(-3)+(1+.3)*Cc4*Rho*(-1)
430 !
440 !
450 Alfa=12*(1-.3^2)/(4*PI*Cc4)
460 T=Tp^3/Alfa
470 I1=PI*140^4/64
480 I2=PI*130^4/64
490 T=T/12
500 U=(P*(2*(120+160)+620)-G*350-S*350^2/2+S*1310^2/2)/1180
510 Q=2*P+G+S*1660-U
520 !
```
Building up of the equations for the determination of the shaft deflection line:

MAT \( Ash = ZER \)

MAT \( Bsh = ZER \)

MAT \( Csh = ZER \)

\[
\begin{align*}
\text{Bsh(3)} &= (11 - 12) * (G * L(2)^2 / 2 + S * L(2)^3 / 6) \\
\text{Bsh(4)} &= (11 - 12) * (G * L(2)^3 / 6 - S * L(2)^4 / 24) \\
\text{Bsh(5)} &= -P * L(3)^2 / 2 \\
\text{Bsh(6)} &= -P * L(3)^3 / 3 \\
\text{Bsh(7)} &= -P * L(4)^2 / 2 \\
\text{Bsh(8)} &= -P * L(4)^3 / 3 \\
\end{align*}
\]

\[
\begin{align*}
\text{Ash(1,1)} &= 1 \\
\text{Ash(1,3)} &= -1 \\
\text{Ash(1,17)} &= -L(1)^2 / 2 \\
\text{Ash(2,2)} &= 1 \\
\text{Ash(2,4)} &= -1 \\
\text{Ash(2,17)} &= L(1)^3 / 6 \\
\text{Ash(3,3)} &= 12 \\
\text{Ash(3,5)} &= -11 \\
\text{Ash(3,17)} &= (11 - 12) * (L(2)^2 / 2 - L(1) * L(2)) \\
\text{Ash(4,3)} &= 12 * L(2) \\
\text{Ash(4,4)} &= 12 \\
\text{Ash(4,5)} &= -11 * L(2) \\
\text{Ash(4,6)} &= -11 \\
\text{Ash(4,17)} &= (11 - 12) * (L(2)^3 / 6 - L(1) * L(2)^2 / 2) \\
\text{Ash(5,5)} &= 1 \\
\text{Ash(5,7)} &= -1 \\
\text{Ash(5,15)} &= -L(3) \\
\text{Ash(5,9)} &= -L(3) \\
\text{Ash(5,10)} &= -L(3)^2 / 2 \\
\text{Bsh(5)} &= -P * L(3)^2 / 2 \\
\text{Bsh(6)} &= -P * L(3)^3 / 3 \\
\text{Bsh(7)} &= -P * L(4)^2 / 2 \\
\text{Bsh(8)} &= -P * L(4)^3 / 3 \\
\text{Ash(6,5)} &= L(3) \\
\text{Ash(6,6)} &= 1 \\
\text{Ash(6,7)} &= -L(3) \\
\text{Ash(6,8)} &= -1 \\
\text{Ash(6,15)} &= -L(3)^2 / 2 \\
\text{Ash(7,7)} &= 1 \\
\text{Ash(7,9)} &= -1 \\
\text{Ash(7,16)} &= L(4) \\
\text{Bsh(7)} &= -P * L(4)^2 / 2 \\
\text{Bsh(8)} &= -P * L(4)^3 / 3 \\
\text{Ash(8,7)} &= L(4) \\
\text{Ash(8,8)} &= 1 \\
\text{Ash(8,9)} &= -L(4) \\
\text{Ash(8,10)} &= -1 \\
\text{Ash(8,16)} &= L(4)^2 / 2 \\
\text{Bsh(8)} &= -P * L(4)^3 / 3 \\
\end{align*}
\]
1105
1106
1107
1108
1109
1110
1111
1112
1113
Ash(9, 9) = I1
1114
Ash(9, 11) = -I2
1115
Ash(9, 15) = -(12 - 11) * L(5)
1116
Ash(9, 16) = (12 - 11) * L(5)
1117
Ash(9, 17) = (12 - 11) * (L(5)^2 / 2 - L(1) * L(5))
1118
Bsh(9) = (12 - 11) * (G * L(5)^2 / 2 + S * L(5)^3 / 6 + P * (L(5)^2 - (L(3) +
1119
L(4)) * L(5)))
1120
1121
1122
Ash(10, 9) = I1 + L(5)
1123
Ash(10, 10) = I1
1124
Ash(10, 11) = -12 * L(5)
1125
Ash(10, 12) = -12
1126
Ash(10, 15) = -(12 - 11) * (L(5)^2 / 2)
1127
Ash(10, 16) = (12 - 11) * (L(5)^2 / 2)
1128
Ash(10, 17) = (12 - 11) * (L(5)^3 / 2 - L(1) * L(5)^2 / 2)
1129
Bsh(10) = (12 - 11) * (G * L(5)^3 / 6 + S * L(5)^4 / 24 + P * (L(5)^3 / 3 - (L
1130
(3) + L(4)) * L(5)^2 / 2))
1131
1132
1133
1134
Ash(11, 11) = I1
1135
Ash(11, 13) = -1
1136
Ash(11, 18) = -L(6)^2 / 2
1137
Ash(11, 18) = L(6)^3 / 6
1138
1139
1140
1141
1142
Bsh(11) = G * L(6)^3 / 6 - S * L(1)^3 / 4 / 24
1143
1144
1145
Ash(12, 11) = L(6)
1146
Ash(12, 12) = 1
1147
Ash(12, 14) = -1
1148
Ash(12, 18) = L(6)^3 / 6
1149
Ash(12, 18) = L(6)^3 / 2
1150
1151
1152
1153
Ash(13, 1) = L(1)
1154
Ash(13, 2) = 1
1155
Bsh(13) = -G * L(1)^3 / 6 - S * L(1)^3 / 24
1156
1157
1158
1159
Ash(14, 11) = L(6)
1160
Ash(14, 12) = 1
1161
Ash(14, 14) = -L(6)^2 / 2
1162
Ash(14, 16) = -L(6)^2 / 2
1163
Ash(14, 17) = -L(6)^3 / 2 - L(1) * L(6)^2 / 2
1164
Bsh(14) = G * L(6)^3 / 6 - S * L(6)^3 / 24 + P * (2 * L(6)^3 / 3 - (L(3) +
1165
L(4)) * L(6)^2 / 2 - L(4) * L(6)^2 / 2)
1166
1167
1168
1169
Ash(15, 5) = T
1170
Ash(15, 15) = -1
1171
Ash(15, 17) = -(12 - 11) * (L(3)^2 / 2 - L(1) * L(3))
1172
Bsh(15) = T * (G * L(3)^2 / 2 + S * L(3)^3 / 6)
1173
1174
1175
1176
Ash(16, 7) = T
1177
Ash(16, 15) = T * L(4)
1178
Ash(16, 16) = I1
1179
Ash(16, 17) = -T * (L(4)^2 / 2 - L(1) * L(4))
1180
Bsh(16) = -T * (G * L(4)^2 / 2 + S * L(4)^3 / 6 + P * (L(4)^2 - 2 * L(3) * L(4)
1181
}))
1182
1640 | 
1650 | 
1660 | Ash(17,15)=-1/1180 
1670 | Ash(17,16)=1/1180 
1680 | Ash(17,17)=1 
1690 | Bsh(17)=0 
1700 | 
1710 | 
1720 | Ash(18,15)=1/1180 
1730 | Ash(18,16)=-1/1180 
1740 | Ash(18,18)=1 
1750 | Bsh(18)=U 
1760 | 
1770 | 
1780 | MAT.Ash=INV(Ash) 
1790 | MAT Csh=Ash*Bsh 
1800 | 
1810 | M1=Csh(15) 
1820 | M2=Csh(16) 
1830 | Ra=Csh(17) 
1840 | Rb=Csh(18) 
1850 | IF Tp=0 THEN GOTO 2050 
1860 | 
1870 | Calculation of the strains on the plate 
1880 | 
1890 | Sr=3*F3/(Radius*Tp^2*2*PI*Cc^4)*M1 
1900 | St=3*F4/(Radius*Tp^2*2*PI*Cc^4)*M1 
1910 | Strainr=(Sr-.3*St)/E*1000000 
1920 | Straint=(St-.3*Sr)/E*1000000 
1930 | PRINT "Strains at the gauges near the hub" 
1940 | PRINT "Radial strain=",Strainr 
1950 | PRINT "Tangential strain=",Straint 
1960 | 
1970 | Display of the deflection, slope and moment 
1980 | 
1990 | CODES 
2000 | Deflection=1 
2010 | Slope=2 
2020 | Moment=3 
2030 | Shear=4 
2040 | 
2050 | Again: INPUT "GRAPH CODE",Gc 
2060 | ON Gc GOTO 2070,2090,2110,2130 
2070 | SCALE 0,L(7),-1,1 
2080 | GOTO 2140 
2090 | SCALE 0,L(7),-4E-3,4E-3 
2100 | GOTO 2140 
2110 | SCALE 0,L(7),-6E6,6E6 
2120 | GOTO 2140 
2130 | SCALE 0,L(7),-2E4,2E4 
2140 | FRAME 
2150 | GRAPHICS 
2160 | MOVE 0,0 
2170 | LINE TYPE 9 
2180 | DRAW L(1),0 
2190 | DRAW L(3),0 
2200 | DRAW L(4),0 
2210 | DRAW L(6),0
2220 DRAW L(7),0
2230 LINE TYPE 1
2240 MOVE 0,0
2250 FOR Z=0 TO L(7) STEP L(7)/100
2260 E1=E2=E3=E4=E5=E6=0
2270 F0=F1=F2=F3=F4=F5=F6=0
2280 IF Z<L(1) THEN F0=1
2290 IF (Z)=L(1) AND (Z<L(2)) THEN F1=1
2300 IF (Z)=L(2) AND (Z<L(3)) THEN F2=1
2310 IF (Z)=L(3) AND (Z<L(4)) THEN F3=1
2320 IF (Z)=L(4) AND (Z<L(5)) THEN F4=1
2330 IF (Z)=L(5) AND (Z<L(6)) THEN F5=1
2340 IF Z>=L(6) THEN F6=1
2350 IF Z>=L(1) THEN E1=1
2360 IF Z>=L(2) THEN E2=1
2370 IF Z>=L(3) THEN E3=1
2380 IF Z>=L(4) THEN E4=1
2390 IF Z>=L(5) THEN E5=1
2400 IF Z>=L(6) THEN E6=1
2410 El=-E*<(F0+F1+F5+F6)*11+(F2+F3+F4)*12)
2420 !
2430 ! Determination of the variables
2440 !
2450 !
2460 ! Shear
2470 Qr=G*Z+S+3*P+E4*P-E1*Ra-E6*Rb
2480 ! Mmoment
2490 Mr=G*Z+S*(Z-L(3))*1+E4*P*(Z-L(4))+E1*Ra*(Z-L(1)-E6*Rb*(Z-L(6)))+M1*E3-M2*E4
2500 !
2510 Kk=Csh(1)*F0+Csh(3)*F1+Csh(5)*F2+Csh(7)*F3+Csh(9)*F4+Csh(11)*F5+Csh(13)*F6
2520 Kkk=Csh(2)*F0+Csh(4)*F1+Csh(6)*F2+Csh(8)*F3+Csh(10)*F4+Csh(12)*F5+Csh(14)*F6+Kk*Z
2530 ! Slope
2540 Wpsl=(G*Z^2/2+Z^3/6+S*E3*P+(Z-L(2)+E4*P+(Z^2/2-Z*L(3)))-E1*Ra*Z^2/2-Z*L(1))-E6*Rb*(Z^2/2-Z*L(6))-Kkk
2550 ! Deflection
2570 Wp=(Wp+(M1*E3-M2*E4)*Z^2/2)/Ei
2580 !
2590 ! Determination of the strain on the shaft
2600 Stf=Mr*130/2/12/E*1000000
2610 IF Z=946.2 THEN PRINT "Strain at the middle of the shaft";Stf
2620 ON Gc GOTO 2630,2650,2670,2690
2630 DRAW Z,WP
2640 GOTO 2700
2650 DRAW Z,Wpsl
2660 GOTO 2700
2670 DRAW Z,Mr
2680 GOTO 2700
2690 DRAW Z,Qr
2700 NEXT Z
2710 BEEP
2720 PAUSE
2730 GOTO Again
2740 STOP
APPENDIX 5
RECORDED DATA

The readings recorded during the tests with the scale model are presented in Table A5.1. The units are micro-Volts and for each set of readings the values in the 1st, 2nd, 3rd and 4th columns are, respectively, the readings with the gauges at the top, right-hand side, bottom and left-hand side positions as defined in section 5.2.4.

The environment temperature recorded during each set of readings is shown in Table A5.2.
<table>
<thead>
<tr>
<th>Gauge</th>
<th>SET 1</th>
<th>SET 2</th>
<th>SET 3</th>
<th>SET 4</th>
<th>SET 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>11</td>
<td>10</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>15</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-493</td>
<td>-492</td>
<td>-491</td>
<td>-491</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>-5</td>
<td>-6</td>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>-3</td>
<td>8</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>-107</td>
<td>-107</td>
<td>-107</td>
<td>-106</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>-1389</td>
<td>-1389</td>
<td>-1388</td>
<td>-1387</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>402</td>
<td>403</td>
<td>402</td>
<td>403</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>6</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>9</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>-1</td>
<td>-4</td>
<td>-71</td>
<td>-11</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>109</td>
<td>108</td>
<td>109</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>9</td>
<td>10</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>9</td>
<td>10</td>
<td>9</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>10</td>
<td>8</td>
<td>5</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>-2</td>
<td>-1</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>182</td>
<td>182</td>
<td>182</td>
<td>181</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>-2</td>
<td>-9</td>
<td>-10</td>
<td>-9</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>508</td>
<td>508</td>
<td>508</td>
<td>506</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>2365</td>
<td>2356</td>
<td>2354</td>
<td>2349</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>187</td>
<td>184</td>
<td>183</td>
<td>183</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>-216</td>
<td>-217</td>
<td>-217</td>
<td>-218</td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>823</td>
<td>818</td>
<td>809</td>
<td>810</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>931</td>
<td>928</td>
<td>925</td>
<td>927</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>-2</td>
<td>6</td>
<td>15</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>-806</td>
<td>-796</td>
<td>-785</td>
<td>-795</td>
<td></td>
</tr>
<tr>
<td>Gauge</td>
<td>SET 6</td>
<td>SET 7</td>
<td>SET 8</td>
<td>SET 9</td>
<td>SET 10</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
<td>16</td>
<td>15</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>16</td>
<td>15</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>-45</td>
<td>-44</td>
<td>-44</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>57</td>
<td>-57</td>
<td>-57</td>
<td>-57</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>61</td>
<td>63</td>
<td>61</td>
<td>62</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>59</td>
<td>59</td>
<td>58</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-15</td>
<td>-14</td>
<td>-13</td>
<td>-12</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-18</td>
<td>-17</td>
<td>-16</td>
<td>-15</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-12</td>
<td>-11</td>
<td>-10</td>
<td>-9</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>-8</td>
<td>-7</td>
<td>-6</td>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>17</td>
<td>16</td>
<td>15</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>12</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>18</td>
<td>17</td>
<td>16</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>16</td>
<td>15</td>
<td>14</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>14</td>
<td>13</td>
<td>12</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>12</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
</tbody>
</table>

Table A5.1 (continued)
<table>
<thead>
<tr>
<th>Gauge</th>
<th>SLT 11</th>
<th>SET 12</th>
<th>SET 13</th>
<th>SET 14</th>
<th>SET 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A5.1 (continued)
<table>
<thead>
<tr>
<th>Gauge</th>
<th>SET 16</th>
<th>SET 17</th>
<th>SET 18</th>
<th>SET 19</th>
<th>SET 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>8</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>9</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>10</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>11</td>
<td>-404</td>
<td>-404</td>
<td>-404</td>
<td>-404</td>
<td>-404</td>
</tr>
<tr>
<td>12</td>
<td>-46</td>
<td>-46</td>
<td>-46</td>
<td>-46</td>
<td>-46</td>
</tr>
<tr>
<td>13</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>14</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>15</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>16</td>
<td>-165</td>
<td>-165</td>
<td>-165</td>
<td>-165</td>
<td>-165</td>
</tr>
<tr>
<td>17</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>18</td>
<td>490</td>
<td>490</td>
<td>490</td>
<td>490</td>
<td>490</td>
</tr>
<tr>
<td>19</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>20</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>21</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>22</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>23</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>24</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>25</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>26</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>27</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>28</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>29</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>30</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>31</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>32</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>33</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>34</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>35</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>36</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>37</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>38</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>39</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>40</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>41</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>42</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>43</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>44</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>45</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>46</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>47</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>48</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>49</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>50</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>51</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>52</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>53</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>54</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>55</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>56</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
</tr>
</tbody>
</table>

*Table A5.1 (continued)*
<table>
<thead>
<tr>
<th>Gauge</th>
<th>SET 21</th>
<th>SET 22</th>
<th>SET 23</th>
<th>SET 24</th>
<th>SET 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>17</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>19</td>
<td>19</td>
<td>19</td>
<td>19</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>23</td>
<td>23</td>
<td>23</td>
<td>23</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
</tr>
<tr>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>29</td>
<td>29</td>
<td>29</td>
<td>29</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>31</td>
<td>31</td>
<td>31</td>
<td>31</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>33</td>
<td>33</td>
<td>33</td>
<td>33</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>35</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>37</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td>39</td>
<td>39</td>
<td>39</td>
<td>39</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>41</td>
<td>41</td>
<td>41</td>
<td>41</td>
<td>41</td>
<td>41</td>
</tr>
<tr>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>43</td>
<td>43</td>
<td>43</td>
<td>43</td>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
</tr>
<tr>
<td>47</td>
<td>47</td>
<td>47</td>
<td>47</td>
<td>47</td>
<td>47</td>
</tr>
<tr>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>49</td>
<td>49</td>
<td>49</td>
<td>49</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
</tr>
<tr>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
<td>53</td>
<td>53</td>
<td>53</td>
<td>53</td>
<td>53</td>
<td>53</td>
</tr>
<tr>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
<td>54</td>
</tr>
<tr>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>56</td>
</tr>
<tr>
<td>57</td>
<td>57</td>
<td>57</td>
<td>57</td>
<td>57</td>
<td>57</td>
</tr>
<tr>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
</tr>
<tr>
<td>59</td>
<td>59</td>
<td>59</td>
<td>59</td>
<td>59</td>
<td>59</td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>61</td>
<td>61</td>
<td>61</td>
<td>61</td>
<td>61</td>
<td>61</td>
</tr>
<tr>
<td>62</td>
<td>62</td>
<td>62</td>
<td>62</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
<td>63</td>
</tr>
<tr>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>65</td>
<td>65</td>
<td>65</td>
<td>65</td>
<td>65</td>
<td>65</td>
</tr>
<tr>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>66</td>
</tr>
<tr>
<td>67</td>
<td>67</td>
<td>67</td>
<td>67</td>
<td>67</td>
<td>67</td>
</tr>
<tr>
<td>68</td>
<td>68</td>
<td>68</td>
<td>68</td>
<td>68</td>
<td>68</td>
</tr>
<tr>
<td>69</td>
<td>69</td>
<td>69</td>
<td>69</td>
<td>69</td>
<td>69</td>
</tr>
<tr>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>71</td>
<td>71</td>
<td>71</td>
<td>71</td>
<td>71</td>
<td>71</td>
</tr>
<tr>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
<td>72</td>
</tr>
<tr>
<td>73</td>
<td>73</td>
<td>73</td>
<td>73</td>
<td>73</td>
<td>73</td>
</tr>
<tr>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>76</td>
<td>76</td>
<td>76</td>
<td>76</td>
<td>76</td>
<td>76</td>
</tr>
<tr>
<td>77</td>
<td>77</td>
<td>77</td>
<td>77</td>
<td>77</td>
<td>77</td>
</tr>
<tr>
<td>78</td>
<td>78</td>
<td>78</td>
<td>78</td>
<td>78</td>
<td>78</td>
</tr>
<tr>
<td>79</td>
<td>79</td>
<td>79</td>
<td>79</td>
<td>79</td>
<td>79</td>
</tr>
<tr>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>81</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>81</td>
</tr>
<tr>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
</tr>
</tbody>
</table>

Table A5.1 (continued)
<table>
<thead>
<tr>
<th>Gauge</th>
<th>SET 31</th>
<th>SET 32</th>
<th>SET 33</th>
<th>SET 34</th>
<th>SET 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A5.1 (continued)
<table>
<thead>
<tr>
<th>Gauge</th>
<th>SET 36</th>
<th>SET 37</th>
<th>SET 38</th>
<th>SET 39</th>
<th>SET 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>33</td>
<td>33</td>
<td>29</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>-3</td>
<td>-2</td>
<td>-3</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-7</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>25</td>
<td>20</td>
<td>20</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>-512</td>
<td>-498</td>
<td>-498</td>
<td>-506</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>22</td>
<td>6</td>
<td>4</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>-7</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>12</td>
<td>11</td>
<td>4</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>-6</td>
<td>2</td>
<td>4</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>-105</td>
<td>-107</td>
<td>-108</td>
<td>-108</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>-1405</td>
<td>-1402</td>
<td>-1400</td>
<td>-1403</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>418</td>
<td>421</td>
<td>413</td>
<td>414</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>13</td>
<td>10</td>
<td>8</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>16</td>
<td>13</td>
<td>14</td>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>22</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>-11</td>
<td>8</td>
<td>23</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>10</td>
<td>12</td>
<td>18</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>13</td>
<td>13</td>
<td>18</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>-12</td>
<td>2</td>
<td>20</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>22</td>
<td>20</td>
<td>14</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>18</td>
<td>18</td>
<td>16</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>12</td>
<td>12</td>
<td>32</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>13</td>
<td>19</td>
<td>23</td>
<td>19</td>
<td>17</td>
</tr>
<tr>
<td>31</td>
<td>246</td>
<td>256</td>
<td>256</td>
<td>249</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>31</td>
<td>24</td>
<td>20</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>36</td>
<td>26</td>
<td>21</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>38</td>
<td>32</td>
<td>15</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>15</td>
<td>21</td>
<td>27</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>22</td>
<td>22</td>
<td>21</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>23</td>
<td>19</td>
<td>20</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>12</td>
<td>9</td>
<td>6</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>-188</td>
<td>-186</td>
<td>-186</td>
<td>-196</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>-254</td>
<td>-255</td>
<td>-254</td>
<td>-254</td>
<td>-254</td>
</tr>
<tr>
<td>42</td>
<td>9</td>
<td>5</td>
<td>11</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>13</td>
<td>12</td>
<td>11</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>14</td>
<td>14</td>
<td>13</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>65</td>
<td>65</td>
<td>73</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>12</td>
<td>14</td>
<td>14</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>23</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>494</td>
<td>485</td>
<td>484</td>
<td>484</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>133</td>
<td>150</td>
<td>1504</td>
<td>1500</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>403</td>
<td>403</td>
<td>403</td>
<td>403</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>-241</td>
<td>-243</td>
<td>-241</td>
<td>-236</td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>822</td>
<td>810</td>
<td>771</td>
<td>776</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>3</td>
<td>30</td>
<td>32</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>8</td>
<td>9</td>
<td>62</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>-837</td>
<td>-810</td>
<td>-758</td>
<td>-784</td>
<td></td>
</tr>
</tbody>
</table>

Table A5.1 (continued)
<table>
<thead>
<tr>
<th>Gauge</th>
<th>SET 46</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>49</td>
</tr>
<tr>
<td>3</td>
<td>-15</td>
</tr>
<tr>
<td>4</td>
<td>73</td>
</tr>
<tr>
<td>5</td>
<td>-73</td>
</tr>
<tr>
<td>6</td>
<td>78</td>
</tr>
<tr>
<td>7</td>
<td>-94</td>
</tr>
<tr>
<td>8</td>
<td>99</td>
</tr>
<tr>
<td>9</td>
<td>-73</td>
</tr>
<tr>
<td>10</td>
<td>71</td>
</tr>
<tr>
<td>11</td>
<td>-556</td>
</tr>
<tr>
<td>12</td>
<td>40</td>
</tr>
<tr>
<td>13</td>
<td>-27</td>
</tr>
<tr>
<td>14</td>
<td>-18</td>
</tr>
<tr>
<td>15</td>
<td>-23</td>
</tr>
<tr>
<td>16</td>
<td>-155</td>
</tr>
<tr>
<td>17</td>
<td>-1449</td>
</tr>
<tr>
<td>18</td>
<td>445</td>
</tr>
<tr>
<td>19</td>
<td>---</td>
</tr>
<tr>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>21</td>
<td>25</td>
</tr>
<tr>
<td>22</td>
<td>44</td>
</tr>
<tr>
<td>23</td>
<td>-25</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
</tr>
<tr>
<td>25</td>
<td>14</td>
</tr>
<tr>
<td>26</td>
<td>-31</td>
</tr>
<tr>
<td>27</td>
<td>-2</td>
</tr>
<tr>
<td>28</td>
<td>39</td>
</tr>
<tr>
<td>29</td>
<td>128</td>
</tr>
<tr>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>31</td>
<td>218</td>
</tr>
<tr>
<td>32</td>
<td>38</td>
</tr>
<tr>
<td>33</td>
<td>57</td>
</tr>
<tr>
<td>34</td>
<td>24</td>
</tr>
<tr>
<td>35</td>
<td>54</td>
</tr>
<tr>
<td>36</td>
<td>32</td>
</tr>
<tr>
<td>37</td>
<td>26</td>
</tr>
<tr>
<td>38</td>
<td>11</td>
</tr>
<tr>
<td>39</td>
<td>29</td>
</tr>
<tr>
<td>40</td>
<td>-82</td>
</tr>
<tr>
<td>41</td>
<td>-274</td>
</tr>
<tr>
<td>42</td>
<td>30</td>
</tr>
<tr>
<td>43</td>
<td>-55</td>
</tr>
<tr>
<td>44</td>
<td>-51</td>
</tr>
<tr>
<td>45</td>
<td>-49</td>
</tr>
<tr>
<td>46</td>
<td>56</td>
</tr>
<tr>
<td>47</td>
<td>17</td>
</tr>
<tr>
<td>48</td>
<td>23</td>
</tr>
<tr>
<td>49</td>
<td>512</td>
</tr>
<tr>
<td>50</td>
<td>1609</td>
</tr>
<tr>
<td>51</td>
<td>392</td>
</tr>
<tr>
<td>52</td>
<td>-241</td>
</tr>
<tr>
<td>53</td>
<td>695</td>
</tr>
<tr>
<td>54</td>
<td>16</td>
</tr>
<tr>
<td>55</td>
<td>-10</td>
</tr>
<tr>
<td>56</td>
<td>-827</td>
</tr>
</tbody>
</table>

Table A5.1 (continued)
Table A5.2
Average temperature during each set of readings

<table>
<thead>
<tr>
<th>Set Number</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.00</td>
</tr>
<tr>
<td>2</td>
<td>20.00</td>
</tr>
<tr>
<td>3</td>
<td>21.50</td>
</tr>
<tr>
<td>4</td>
<td>19.25</td>
</tr>
<tr>
<td>5</td>
<td>19.75</td>
</tr>
<tr>
<td>6</td>
<td>20.00</td>
</tr>
<tr>
<td>7</td>
<td>20.25</td>
</tr>
<tr>
<td>8</td>
<td>21.25</td>
</tr>
<tr>
<td>9</td>
<td>24.00</td>
</tr>
<tr>
<td>10</td>
<td>24.25</td>
</tr>
<tr>
<td>11</td>
<td>24.50</td>
</tr>
<tr>
<td>12</td>
<td>19.50</td>
</tr>
<tr>
<td>13</td>
<td>19.00</td>
</tr>
<tr>
<td>14</td>
<td>19.25</td>
</tr>
<tr>
<td>15</td>
<td>16.00</td>
</tr>
<tr>
<td>16</td>
<td>16.25</td>
</tr>
<tr>
<td>17</td>
<td>17.00</td>
</tr>
<tr>
<td>18</td>
<td>19.00</td>
</tr>
<tr>
<td>19</td>
<td>18.50</td>
</tr>
<tr>
<td>20</td>
<td>17.00</td>
</tr>
<tr>
<td>21</td>
<td>16.50</td>
</tr>
<tr>
<td>22</td>
<td>21.50</td>
</tr>
<tr>
<td>23</td>
<td>19.00</td>
</tr>
<tr>
<td>24</td>
<td>20.00</td>
</tr>
<tr>
<td>25</td>
<td>20.00</td>
</tr>
<tr>
<td>26</td>
<td>19.50</td>
</tr>
<tr>
<td>27</td>
<td>20.00</td>
</tr>
<tr>
<td>28</td>
<td>19.50</td>
</tr>
<tr>
<td>29</td>
<td>18.50</td>
</tr>
<tr>
<td>30</td>
<td>20.00</td>
</tr>
<tr>
<td>31</td>
<td>17.50</td>
</tr>
<tr>
<td>32</td>
<td>17.00</td>
</tr>
<tr>
<td>33</td>
<td>17.00</td>
</tr>
<tr>
<td>34</td>
<td>17.00</td>
</tr>
<tr>
<td>35</td>
<td>19.50</td>
</tr>
<tr>
<td>36</td>
<td>19.50</td>
</tr>
<tr>
<td>37</td>
<td>19.00</td>
</tr>
<tr>
<td>38</td>
<td>20.00</td>
</tr>
<tr>
<td>39</td>
<td>19.50</td>
</tr>
<tr>
<td>40</td>
<td>20.00</td>
</tr>
<tr>
<td>41</td>
<td>21.00</td>
</tr>
<tr>
<td>42</td>
<td>23.00</td>
</tr>
<tr>
<td>43</td>
<td>20.00</td>
</tr>
<tr>
<td>44</td>
<td>19.50</td>
</tr>
<tr>
<td>45</td>
<td>18.50</td>
</tr>
<tr>
<td>46</td>
<td>18.50</td>
</tr>
</tbody>
</table>
APPENDIX 6
PRINCIPAL STRAINS AT GAUGES NOS. 46, 47 AND 48

The principal strains at point 0, shown in FIG. A6.1, can be calculated from the strains shown by gauges nos. 46, 47 and 48 using the following expressions by reference, e.g., to BUDYNAS:

$$
\epsilon_{\text{min}} = \frac{1}{2}(\epsilon_4 + \epsilon_8) - \frac{1}{2}\sqrt{2((\epsilon_4 - \epsilon_6)^2 + (\epsilon_6 - \epsilon_8)^2)}
$$

$$
\epsilon_{\text{max}} = \frac{1}{2}(\epsilon_4 + \epsilon_8) + \frac{1}{2}\sqrt{2((\epsilon_4 - \epsilon_6)^2 + (\epsilon_6 - \epsilon_8)^2)}
$$

where

- $\epsilon_{\text{min}}$ - minimum principal strain
- $\epsilon_{\text{max}}$ - maximum principal strain
- $\epsilon_4, \epsilon_4, \epsilon_8$ - strains shown by the gauges

FIG. A6.1 GAUGES ON STRAP PLATE