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Volatile Public Spending in a Model of Money and Sustainable Growth

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Abstract
In a model where seignorage provides the financing instrument for the government’s budget, public spending volatility has an adverse effect on long-run growth. This negative relationship arises because the incidence of volatility in this type of public policy is responsible for higher average money growth, thus induces individuals to devote less time/effort towards capital accumulation. Another implication of the model is that policy variability provides a possible argument behind the positive correlation between inflation and inflation variability.

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1 Introduction

The purpose of this paper is to investigate and explain the impact of policy variability (specifically, variability in public spending) in macroeconomic performance. The point of departure of the present analysis from existing ones lies on analysing this relationship under a framework whereby public spending is financed via money creation. One implication from this assumption is the following: if spending policies represent the actual generating source of volatility, then the (partially) endogenous mechanism through which money supply is determined raises the possibility that what causes nominal volatility in the first place may also cause an increase on the average rate of money growth. This idea is crucial, not only because it can justify the sign of the correlation between volatile public spending and output growth, but also because it can provide a possible theoretical justification behind the positive correlation between the average rate of inflation and its variance. In addition to the above, the incidence of policy variability allows the present framework to account for the empirically observed sign of the correlation between inflation, inflation variability and economic growth.

It is widely observed that government policies display a certain degree of volatility. Obviously, one can think of a variety of underlying causes leading to this observation. Irrespective of such causes, however, an outcome of significant importance is the possibility that variability in policies can emerge as an additional factor determining the long-term macroeconomic performance, as this is appropriately reflected by the trend of output growth. Such significance is readily understood once we realise that even small changes in trend growth – if sustained – are sufficient to generate substantial changes to the level of real GDP over the medium- and long-term. This has been an issue of concern for both empirical (e.g., Brunetti, 1998; Furceri, 2007) and theoretical analyses (e.g., Aizenman and Marion, 1993; Hopenhayn and Muniaiguria, 1996; Varvarigos, 2007).\footnote{Empirical studies seem to reach a consensus on a negative correlation between various measures of policy volatility and average GDP growth. From the theoretical perspective, existing results seem to be rather mixed (see Footnote 2).}

The aforementioned theoretical studies consider cases where the volatility of government policies is absorbed and – partially or completely – transmitted to the macroeconomic
environment via fiscal instruments such as proportional tax rates and subsidies. Nevertheless, various authors, such as Fischer (1982) and Poterba and Rotemberg (1990), have suggested that seignorage (that is, the revenues a government can generate by increasing the money supply) is a more suitable financing instrument for temporary variations in public spending – perhaps due to the relative ease and speed of money creation as opposed to the more lengthy political and legislative procedure required to adjust various forms of taxation. Support for this view is provided by Click (1998) who reports estimation results showing a statistically significant correlation between measures of seignorage and the standard deviation of government spending for a cross-section of countries. Of course, such arguments imply that, under a framework of inflationary finance, volatile public spending may cause nominal variability as this is reflected by the variability of money supply.

In general, seignorage is considered as a method of expenditure finance appealing to many governments – notably those of developing economies. The commonly accepted arguments in favour of the above statement concern the insufficient revenues from taxation (e.g., due to the low tax base, imperfect tax collection mechanisms or widespread tax evasion), the poor credit ratings that prevent some developing countries from borrowing in world financial markets and the political benefits derived from the fact that the electorate is generally averse to direct taxation and inclined to ‘punish’ those who impose it. Indeed, Fischer (1982) and Click (1998) present data illustrating that, for a large number of countries, a higher degree of variability in the use of seignorage may indeed be accompanied by higher variability in government spending.

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2 Aizenman and Marion (1993) develop a model in which firms are subject to a tax on profits that fluctuates randomly between low and high values, the difference between which is used as a measure of policy variability. It is shown that an increase in such variability may either increase or decrease growth by an amount that depends on the degree of persistence in policies. In a similar vein, Hopenhayn and Muniagurria (1996) present a model in which firms receive randomly either positive or zero subsidies to their investments – positive subsidies being financed by proportional income tax rates. Again, it is shown that policy variability may either increase or decrease growth depending on how variability is actually measured: the former case occurs when they consider the amplitude of the policy variable while the latter case occurs when they consider the frequency of changes in the policy variable. Varvarigos (2007) constructs two models of endogenous growth with variability in productive public spending. In the first model, where public inputs are included in the economy’s production technology, policy volatility may either increase or decrease trend growth depending on whether the parameter measuring the elasticity of output with respect to public inputs is above or below a critical threshold respectively. In the second model, where public inputs complement private inputs in the formation of human capital, policy volatility has a negative impact on trend growth.
developing countries, revenues from money creation account for a significant fraction of total government revenues, while Basu (2001) reports the results of a cross-country study that indicates a positive and statistically significant correlation between the average seignorage rate and the public investment rate. The significance of inflationary finance is even more aptly reflected in an idea brought forward by Agbonyitor (1998): he argues that structural economic reforms in several Sub-Saharan African countries were undermined by reductions in the provision of public services and poor maintenance of public infrastructure as a result of shortages in local funds due to reduced seignorage.

As I hinted earlier, the aforementioned ideas motivate my present analysis. In this paper I construct a model whereby the accumulation of human or efficiency capital provides the source of endogenous productivity improvements and, therefore, sustainable growth. In this framework I assume that the government provides a stream of public spending which is composed of lump-sum (monetary) transfers and (potentially productive) public goods/services. An important feature is that public spending varies overtime – an assumption captured by imposing a stochastic process for the government spending-to-output ratio. The government finances its spending via money creation. The underlying cause that generates a motive to the private sector for holding money is a cash-in-advance constraint on consumption.

Under this framework, the equilibrium optimal capital investment decisions depend on money growth. Assuming that money supply had an independent stochastic process, my model would predict that average money growth reduces long-run output growth while monetary variability increases it. However, the key to my model is that money represents an instrument of public finance through the government’s budget constraint. Consequently, the growth rate of money supply becomes a function of public spending – more importantly, a

\[ \text{(some existing theoretical analyses seem to suggest a positive relationship between nominal variability and long-run growth. Dotsey and Sarte (2000) argue that, in response to nominal volatility, a precautionary increase in saving leads to more physical capital investment and, consequently, growth. Varvarigos (forth.) shows that nominal variability induces an increase in money demand, thus leading to a permanent decrease of the transaction costs associated with consumption. As a result individuals find optimal to devote more time towards both labour and human capital formation - the latter effect ultimately leading to higher growth. Crucial for such results seems to be the fact that, contrary to this paper's framework, both analyses follow the conventional approach of specifying an independent stochastic process for money supply.)} \]
non-linear one. Therefore, its stochastic process is not independent but rather driven by the stochastic process of the public spending variable. This crucial feature is what generates one of the model’s basic results: an increase in policy variability does not only result in an increase in the variability of money growth (leading to a positive growth effect) but it is also associated with an increase of the average rate of money growth (leading to a negative growth effect). As it happens, the second effect apparently dominates and higher variability in policy has, on average, a detrimental effect on output growth. The assumption that public spending is directly productive exemplifies this effect but it does not change it qualitatively.

Some additional implications of the model are related to the link between the average inflation rate, the variance of inflation and long-run output growth. In this framework, policy variability generates inflation variability as well – the reason being that actual inflation varies in response to the resulting fluctuations in output growth and money supply. Furthermore, due to the non-linear effect that policy shocks exert on the actual inflation rate, increased policy variability results in higher inflation on average. Finally, given the responses of both inflation and its variance to more volatile policies, the model implies that both inflation and its variability are negatively correlated with the rate of output growth.

The rest of the paper is organised as follows: Section 2 outlines the fundamental characteristics of the model economy. In Section 3 I define and derive the competitive, dynamic general equilibrium and in Section 4 I illustrate the effect of public spending volatility on long-run growth. Section 5 identifies how the impact of policy variability on capital accumulation is channelled through the effects of public spending on average money growth and its variability, and it shows the correlations between average inflation, inflation volatility and trend growth. Section 6 presents the welfare implications from the incidence of volatile public spending and Section 7 concludes.

2 The Structure of the Economy

Consider an artificial economy populated by a large mass (normalised to unity) of identical individuals. Every individual acts both as a consumer and producer of a single, perishable commodity. Time is discrete, indexed by $t$ (which belongs to the set of nonnegative integers $\mathbb{N}$) and measured from 0 to $\infty$ with $t = 0$ being the initial period. Each agent is assumed to be infinitely lived and there is no population growth. Every period an agent carries a stock
of – previously accumulated – human or ‘efficiency’ capital, denoted by $a_t$, which represents the agent’s skill, ability and expertise in transforming her labour input into consumable output. For brevity, the remaining analysis will refer to the variable $a_t$ simply as ‘capital’. The agent begins her lifetime with an initial endowment of capital equal to $a_0 > 0$. In addition, I assume the presence of a government whose activities involve the issuing and printing of the economy’s medium of exchange, henceforth called ‘money’, the provision of lump-sum income transfers (in monetary terms) to the private sector and the provision of public goods. Concerning the timing of events, I employ the widely used assumption that all uncertainty relating to current events is resolved at the beginning of each period, prior to an agent’s formation of optimal decisions.

2.1 The Private Sector

Each period, an individual is endowed with one unit of labour and one unit of potential leisure time. The unit of labour is supplied inelastically, and is combined with the agent’s capital as to produce $y_t$ units of output according to

$$y_t = f(a_t).$$

I assume that the function $f(\cdot)$ satisfies $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$. I also assume that the marginal product of capital is supported by a lower bound, therefore I impose the condition $\lim_{a_t \to \infty} f'(a_t) = A > 0$.

The agent accumulates capital by combining units of time or effort, denoted by $e_t$, her previously accumulated stock of capital and the provision of public goods and services, denoted by $g_t$, according to

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4 To the extent that capital goods require some human resources (like time or effort) to be manufactured, my definition of capital can be broadened to include tangible characteristics as well.

5 This is an innocuous assumption for the model’s implications and employed here simply to save on notation. Introducing elastic labour supply would not change any of the model’s results qualitatively because the solution for labour would be stationary, independent of current realisations of the random shock, and with qualitatively identical response to volatility compared with the optimal time spent on capital formation (see Footnote 9).

6 Later it will become clear that this assumption, combined with the linear homogeneity of the capital formation technology, will allow the derivation of an equilibrium with sustained growth even when public goods contribute to the process of capital accumulation.
\[ a_{t+1} = Z(e_t)\Psi(a_t, g_t) + (1 - \delta)a_t, \]

where \(\delta \in [0,1]\) is the capital's depreciation rate. I assume \(Z(0) = 0\), \(Z'(e_t) > 0\) and \(Z''(e_t) \leq 0\). The function \(\Psi()\) is twice continuously differentiable on \(\mathbb{R}^{2+}\), strictly increasing and concave in both arguments, has positive cross derivatives and assumed to be homogeneous of degree one. It satisfies \(\Psi(a_t, 0) = \Psi(0, g_t) = \Psi(0, 0) = 0\) and the Inada conditions \(\lim_{x_t \to 0} \Psi'(x_t) \to \infty\) and \(\lim_{x_t \to \infty} \Psi'(x_t) = 0\) for \(x_t = \{a_t, g_t\}\). To facilitate the derivation of analytical solutions – and without any loss of generality – for the remaining analysis I impose full depreciation of capital, that is \(\delta = 1\).

The presence of \(e_t\) captures all the activities that the individual undertakes as to promote the formation of capital and, therefore, productivity. These activities may include formal education, manufacturing of capital goods, training, research, learning and adoption of new technologies, updating with respect to advanced production techniques etc. The contribution of the public sector, signified by the presence of the variable \(g_t\), can be explained in terms of the beneficial aspects from the provision of such goods and services as basic infrastructure (e.g., on transportation and telecommunication), public schooling, national health services, the legal framework that establishes property rights and the efficiency of the judicial system, national defence etc. – all being important factors on supporting the evolution of capital and productivity.

By assumption, the individual is endowed each period with one unit of potential leisure time. Given that she spends \(e_t\) of this amount in accumulating capital, actual leisure time is given by the residual

\[ l_t = 1 - e_t. \]

The agent’s well-being is described by an additively separable, lifetime utility function which depends on the consumption of goods, denoted by \(c_t\), and leisure, \(l_t\), and given by

\[ V = E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + \xi l_t], \]

7 The assumptions \(Z(0) = 0\) and \(\Psi(g_t = 0) = 0\) can be relaxed without causing any considerable alteration to the model's results. However, the restriction \(\Psi(a_t = 0) = 0\) is important as it facilitates the analytical solution of the model significantly.
where $E_0$ is the conditional expectations operator, $\zeta > 0$ is a parameter indicating the weight assigned to the utility derived from leisure activities and $\beta$ is a psychological discount factor, related to the rate of time preference $\varrho > 0$ according to the inverse relationship $\beta = 1/(1 + \varrho)$. Both functions $u(\cdot)$ and $v(\cdot)$ are assumed to be twice continuously differentiable on $\mathbb{R}_{++}$, strictly increasing and concave. They also satisfy the Inada conditions $\lim_{x_i \to 0} f'(x_i) \to \infty$ and $\lim_{x_i \to \infty} f'(x_i) = 0$ for $x_i = \{c_i, l_i\}$ and $f(\cdot) = \{u(\cdot), v(\cdot)\}$ respectively.

The per-period budget constraint for an individual is given by
\[
e_i + \frac{m_{t+1}}{p_t} + \frac{b_{t+1}}{p_t} \leq y_i + \frac{m_t}{p_t} + \frac{\tau_t}{p_t} + \frac{i_t b_t}{p_t},
\]
where $m_t$ denotes nominal money balances carried from the previous period, $m_{t+1}$ denotes nominal money holdings chosen to be carried forward to next period, $p_t$ is the aggregate price level and $\tau_t$ is a monetary transfer distributed by the government to the agent. In addition, $b_t$ denotes privately issued bonds (in nominal terms) and $i_t > 1$ denotes the gross interest rate on private debt. The variable $b_t$ can be either positive or negative depending on whether an agent is a net lender or a net borrower respectively. Aggregation across all agents implies that the total amount of equilibrium debt will be zero.

In order to generate a motive for holding money balances, I emphasise its role as a ‘medium of exchange’ by imposing the requirement that money is a requisite to purchase consumption goods. This assumption is illustrated by a cash-in advance constraint of the form
\[
e_i \leq \frac{m_t}{p_t} + \frac{\tau_t}{p_t}.
\]

The private sector’s description is completed by assuming that in the initial period, $t = 0$, every agent is endowed with an amount of nominal money equal to $m_0 > 0$.

2.2 The Public Sector

As indicated before, the government’s role in the economy is twofold. On the one hand, the government is entrusted with the legal authorisation of issuing and printing money. On the other hand, it is assigned with the provision of lump-sum nominal transfers as well as the
provision of (potentially productive) public goods. Although printing money is intrinsically costless, the provision of public inputs is not as it requires real resources in order to be financed. For this purpose, the government exploits its advantageous position on issuing money bills as to generate the required revenues. It achieves this by using part of the increase in money supply with the aim of acquiring real resources which are, subsequently, transformed into public goods in an one-to-one basis. This is possible as long as the government satisfies only a part of the private sector’s demand for cash via the newly printed money (i.e., by providing a fraction of its seignorage revenues back to the private sector in the form of lump-sum monetary transfers). The remaining part is satisfied in exchange for real resources.

Formally, each period the government prints a quantity of money equal to \( \omega_t \). Therefore, denoting money supply at time \( t \) by \( m_t' \), its evolution can be written as

\[
m_{t+1}' = m_t' + \omega_t. \tag{7}
\]

I will assume that \( \omega_t \) is proportional to \( m_t' \) according to

\[
\omega_t = \mu_t m_t', \tag{8}
\]

where \( \mu_t \in (0,1) \) denotes the net growth rate for the supply of money. Consequently, we can write (7) as

\[
m_{t+1}' = (1 + \mu_t)m_t'. \tag{9}
\]

As indicated above, the government can use the revenues from money creation as to provide a stream of public spending equal to

\[
\Gamma_t = m_{t+1}' - m_t' = \frac{\omega_t}{p_t}. \tag{10}
\]

A fixed fraction \( q \in (0,1) \) of spending is provided back to the private sector in the form of nominal transfers. That is

\[
\tau_t = q p_t \Gamma_t. \tag{11}
\]

However, the remaining fraction \( (1-q) \in (0,1) \) is kept in the form of real resource revenues that are used for the provision of public goods according to

\[
g_t = (1-q) \Gamma_t. \tag{12}
\]
Following others (e.g., Barro and Sala-i-Martin, 1992), I assume that the quantity of public spending is directly proportional to total output as to ensure the derivation of an equilibrium with sustainable output growth. It means that we can write
\[ \Gamma_t = \gamma_t y_t, \quad (13) \]
where \( \gamma_t \in (0,1) \).

In this framework, policy variability corresponds to variability in public spending. In the context of this model, this is associated with random changes in the indicator of ‘government size’, therefore it is captured by the assumption that \( \{y_t\}_{t=0}^{\infty} \) is a sequence of identically and independently distributed random variables with constant mean \( \bar{y} \) and constant variance \( \sigma_y^2 \), \( \forall t \). In terms of intuition, \( y_t \) may be thought as a type of productivity shock for the public sector’s operation technology. Alternatively, various aspects that may contribute to this type of volatility are, among other possible, political considerations (e.g., according to the prevailing political situation, the government’s priorities may vary), economic considerations (e.g., various aspects of budgetary policy), political instability or even situations that are not under the control of the government, like natural disasters or differences in the overall characteristics (e.g., ability and experience) of public servants or contractors who are assigned with the task of delivering public goods, services, transfers etc.

3 Equilibrium

The previous Section provided the characteristics of the economic environment under consideration. Now, we can use this framework as to derive the dynamic general equilibrium of this economy, described by the following

**Definition.** Given the initial values \( a_0, m_0 > 0 \), the competitive, stochastic dynamic general equilibrium is a sequence of quantities \( \{c_t, y_t, e_t, l_t, g_t, \omega_t, \mu_t, \gamma_t, p_t, i_t, m_{t+1}, m^i_{t+1}, a_{t+1}, b_{t+1}\}_{t=0}^{\infty} \) such that:

(i) Given \( \{g_t, \omega_t, \mu_t, \gamma_t, p_t, m^i_{t+1}\}_{t=0}^{\infty} \), the quantities \( \{c_t, e_t, l_t, m_{t+1}, a_{t+1}, b_{t+1}\}_{t=0}^{\infty} \) solve an agent’s optimisation problem.

(ii) The solutions for \( e_t \) and \( l_t \) are stationary.

(iii) The goods market clears every period, i.e., \( y_t = e_t + g_t \), \( \forall t \geq 0 \).
The money market clears every period, i.e., \( m_t = m^*_t \ \forall t \geq 0 \).

The private bond market clears every period, i.e., \( b_t = 0 \ \forall t \geq 0 \).

The government’s budget constraint is satisfied every period, i.e.,
\[
\Gamma_t = \frac{\tau_t}{\bar{p}_t} + g_t = \frac{m^*_{t+1} - m^*_t}{\bar{p}_t}
\]
\( \forall t \geq 0 \).

### 3.1 Functional Forms

Prior to the derivation of the equilibrium defined above, I will assign specific functional forms to the private sector’s preferences and technologies which I generically described previously. The reason for doing so is because my purpose is to provide an analytically tractable framework that admits closed-form solutions and allows a precise characterisation of the intuition and the mechanisms involved in this model economy. With this in mind, my choices of functional forms are those that have been commonly used in stochastic, dynamic general equilibrium models.

For the output production technology in (1), I employ a simple linear function. Therefore, (1) becomes
\[
y_t = A a_t .
\]
(14)

Moving to the technology describing the accumulation of capital, I choose \( Z(e_t) = \Phi e_t^\phi \), with \( \Phi > 0 \) and \( \phi \in (0,1] \), and \( \Psi(a_t, g_t) = a_t^\psi g_t^{1-\psi} \), with \( \psi \in (0,1] \). Combined with the already imposed restriction of full capital depreciation, (2) becomes
\[
a_{t+1} = \Phi e_t^\phi a_t^\psi g_t^{1-\psi}.
\]
(15)

Finally, the functions \( u(\cdot) \) and \( v(\cdot) \) are both chosen to be natural logarithms of the arguments of lifetime utility. It means that (4) becomes
\[
V = E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \xi \ln(l_t)].
\]
(16)

As argued above, the above specifications will allow the derivation of closed-form solutions in equilibrium. My next step is the actual derivation of this equilibrium.
3.2 Dynamic Optimisation

Denote the Lagrange multipliers associated with (2), (5) and (6) by \( \zeta_i \), \( \lambda_i \) and \( \theta_i \) respectively. Taking account of the functional forms used in equation (14)-(16), the first order conditions for an agent’s maximisation problem are given by

\[
\frac{1}{e_i} = \lambda_i + \theta_i ,
\]

\[
\frac{\zeta}{1 - e_i} = \zeta_i \Phi e_i^{-1} a_i^\phi \gamma_i^{1-\phi} ,
\]

\[
\lambda_i = \beta E_i \left[ \frac{p_i}{p_{i+1}^e} (\lambda_{i+1} + \theta_{i+1}) \right] ,
\]

\[
\lambda_i = \beta E_i \left( \frac{p_i}{p_{i+1}^e} \lambda_{i+1} \gamma_{i+1} \right) ,
\]

\[
\zeta_i = \beta \Phi \phi E_i (\zeta_{i+1} e_i^{\phi} a_i^{\theta i+1} g_i^{1-\phi}) + \beta A E_i (\lambda_{i+1}) .
\]

The condition in (17) equates the marginal benefit and the marginal cost of additional consumption: the marginal benefit is equal to the marginal utility of consumption while the marginal cost exceeds the shadow value of foregone wealth because of the additional costs of consumption associated with the cash-in-advance requirement.

Equation (18) balances the marginal cost and the marginal benefit of spending more time/effort to the accumulation of capital: the former is associated with the marginal utility cost of foregone leisure and the latter corresponds to the value of the higher stock of available capital. Given the condition in (21), the higher stock of capital is valuable not only because it increases the discounted expected value of future consumption (resulting from the corresponding increase of future output) but also because it allows by itself a further evolution of the capital stock in the future. The above arguments become apparent if we substitute (15) in (18) and (21) and use (14) to get

\[
\frac{\zeta}{1 - e_i} = \frac{\phi \zeta e_i a_{i+1}}{e_i} ,
\]

\[
\zeta_i a_{i+1} = \beta \Phi \phi E_i (\zeta_{i+1} a_{i+2}^e) + \beta A E_i (\lambda_{i+1} y_{i+1}) .
\]

The condition in (19) equates the marginal cost and the marginal benefit of additional holdings of nominal money: the marginal cost relates to the shadow value of wealth while the marginal benefit is derived by the expected discounted utility value of future
consumption (made possible through the availability of more cash), appropriately measured in real terms by taking account of expected future inflation. Finally, equation (20) balances the marginal cost and marginal benefit from borrowing/lending: for a borrower, the former corresponds to the discounted utility value (in real terms, after taking account of expected inflation) of future foregone wealth due to debt repayment while the latter corresponds to the shadow value of additional current wealth; for a lender, the shadow value of wealth is the marginal cost of nominal bond holdings and the marginal benefit is associated with the discounted, expected increase in wealth (and, therefore, utility from consumption) due to interest repayments, always in real terms after accounting for expected future inflation.

A result that will facilitate the derivation of analytical solutions is given as

**Lemma 1.** In equilibrium, the cash-in-advance constraint is binding.

**Proof.** Suppose that the cash-in-advance constraint does not bind. Given the complementary slackness condition, this implies that $\theta_t = 0 \ \forall t \geq 0$. Substituting this in (19) and observing that its left hand side is equal to the left hand side of equation (20), we get

$$E_t \left( \frac{p_t}{p_{t+1}} \lambda_{t+1} i_{t+1} \right) = E_t \left( \frac{p_t}{p_{t+1}} \lambda_{t+1} \right).$$

Of course, this cannot be true given that $i_{t+1} > 1$ by assumption. Consequently, the multiplier $\theta_t$ is strictly positive for every $t$ and the complementary slackness condition requires $c_t = \frac{m_t + \epsilon_t}{p_t}.$

Effectively, Lemma 1 states that as long as interest-bearing assets can provide the function of real resource transfers intertemporally then agents will never hold money as a store of value. Instead, they hold money only to acquire consumption goods – that is, they use cash purely as a medium of exchange.

### 3.3 Policy Variability and Monetary Randomness

In this part I will show how and why policy variability causes variability in the growth rate of money supply. Given Lemma 1, the money market equilibrium condition, and equations (8), (10) and (11), we can write (6) as
\[ \epsilon_t = \frac{m_t}{\hat{p}_t} (1 + q\mu_t). \quad (24) \]

Substituting (17) in (19) yields
\[ \lambda_t = \beta E_t \left( \frac{p_t}{\hat{p}_{t+1}} \frac{1}{\epsilon_{t+1}} \right). \quad (25) \]

Utilising the money market equilibrium condition, we can rewrite (9) as
\[ \frac{m_{t+1}}{p_t} = (1 + \mu_t) \frac{m_t}{p_t}. \quad (26) \]

Next, we can simultaneously multiply and divide the left hand side of (26) by \( p_{t+1} \), substitute (24) and solve the resulting expression for \( \frac{p_t}{p_{t+1}} \). Eventually, it yields
\[ \frac{p_t}{p_{t+1}} = \frac{\epsilon_{t+1}}{\epsilon_t} \frac{1 + q\mu_t}{1 + \mu_t} \frac{1}{1 + q\mu_{t+1}}. \quad (27) \]

The above expression can be substituted back in (25). Taking account that all uncertainty regarding variables chosen at period \( t \) is resolved after the realisation of the state of nature at the beginning of \( t \), this substitution yields
\[ \lambda_t = \frac{\beta I_t}{\epsilon_t}, \quad (28) \]

where \( I_t = \frac{1 + q\mu_t}{1 + \mu_t} E_t \left( \frac{1}{1 + q\mu_{t+1}} \right) \).

The next step is to combine equations (8), (13) and (24), together with the money market equilibrium, to get
\[ \gamma_t \sigma_t = \frac{\mu_t}{1 + q\mu_t} \epsilon_t. \quad (29) \]

The goods market equilibrium condition is given by \( \epsilon_t = y_t - g_t \). Using (12) and (13) in this condition, substituting the resulting expression for \( \epsilon_t \) back in equation (29) and solving for \( \mu_t \) results in

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\[ \text{Notice that part of the government’s total spending involves transfers back to the private sector. Only the part of resources used for the provision of public goods (productive or not) represents a resource transfer from the private to the public sector.} \]
\[
\mu_i = \frac{\gamma_i}{1 - \gamma_i} = M(\gamma_i), \quad (30)
\]

where \( M'(\cdot) > 0, \quad M''(\cdot) > 0 \).

Equation (30) lays out the characteristic that distinguishes this model from existing theoretical analyses on the nominal variability-economic growth nexus. Similarly to these analyses, it shows that the money growth rate is stochastic. The major difference, however, stems from the fact that money growth does not follow an independent stochastic process. Instead, having taken account of the role of money supply as an instrument of government policy (in that case, as an instrument of revenue extraction), money growth relies on the distributional properties governing public spending as this is reflected by the random variable \( \gamma_i \). More importantly, we can notice from (30) that money growth is a non-linear function of the random policy variable \( \gamma_i \), contrary to scenarios where public spending is financed by proportional income taxation – scenarios which result in a linear proportionality between the tax rate and the government spending-to-output ratio. The non-linearity in the present framework arises from the fact that aggregate money demand represents the base on which the public sector relies to extract seignorage revenues. Due to its ‘medium of exchange’ attribute, aggregate money demand is proportional to aggregate consumption. However, the latter is reduced by a rise in government spending as this requires the government to reduce the private sector’s disposable income. Therefore, by trying to increase public revenues (as to finance spending) the public sector reduces aggregate consumption which reduces the aggregate monetary base and thus inhibits the potential revenues that the government can extract via money creation. Later, it will become clear that this non-linearity is an important determinant not only for the effects of policy volatility on growth but also for the implications arising for the relationship between inflation, inflation variability and economic growth.

### 3.4 Optimal Allocation of Time/Effort

The model’s basic structure reveals the importance of privately committed inputs towards capital accumulation (captured by \( e_i \)) for the economy’s growth rate. This part of the analysis is devoted to the analytical derivation of the equilibrium value for this variable.

Upon substitution of (29) and (30) in equation (28) we get
\[\lambda_t = \frac{\beta}{y_t} E_t \left( \frac{1}{1 + q\mu_{t+1}} \right). \]  

(31)

Now, we can substitute (31) in (23) and use the law of iterated expectations to get

\[\xi, a_{t+1} = \beta \phi E_t (\xi, a_{t+1}) + \beta^2 \Lambda, \]

(32)

where

\[\Lambda = E_t \left( \frac{1}{1 + q\mu_{t+1}} \right) E_t \left( \frac{1}{1 + q\mu_{t+2}} \right) \forall t \geq 1. \]

(33)

The constancy of this expectation term emerges from the assumption that the stochastic process of this model (i.e., that of \(\gamma_t\)) – and to which money growth is related through equation (30) – generates constant mean and constant variance.

Notice that the expression in (32) is a stochastic difference equation with solution

\[\xi, a_{t+1} = \frac{\beta^2 \Lambda}{1 - \beta \phi}. \]

(34)

The solution in (34) satisfies the transversality condition \(\lim_{t \to \infty} \beta^t E_t (\xi, a_{t+1}) = 0\) and can be verified by direct substitution back in (32).

We can obtain the optimal solution for \(e_t\) after substituting (34) in (22) and rearranging the resulting expression. Eventually, we get

\[e_t = \frac{\varphi \beta^2 \Lambda}{\varphi \beta^2 \Lambda + (1 - \beta \phi)^{\xi}} = \bar{e}. \]

(35)

The optimal decision concerning time/effort spent on the formation of capital displays intuitive responses to the model’s structural parameters of preferences and technologies. Specifically, an increase in the discount factor (corresponding to a reduction in the rate of time preference \(\rho\)), and an increase in the elasticity of \(a_{t+1}\) with respect to either the time spent on accumulating capital (i.e., an increase in \(\varphi\)) or the current (previously accumulated) stock of capital (i.e., an increase in \(\psi\)) induce individuals to reduce their leisure and spend more time on augmenting their capital stock. The opposite occurs with a higher value for \(\xi\), i.e., the leisure’s assigned weight for the agent’s felicity.9

9 To reinforce my claim in Footnote 6, consider a scenario where the structure of the economy includes elastic labour supply, denoted by \(n\). Obviously, (14) and (3) change to \(y_t = An, h_t\) and \(l_t = 1 - e_t - n\), respectively. It
More important for my analysis is the optimal response of $\epsilon_t$ to the parameters of the distribution of the policy variable $\gamma_t$. Given (30) and (33), this optimal response will be identified through the presence of the term $\Lambda$ in (35). It is of significant importance because this will determine, to a large extent, the effect of policy variability on the trend of output growth. This is an issue to which I now turn.

4 Policy Variability and Long-Run Economic Growth

We can derive the equilibrium growth rate of output by combining equations (12)-(15) and (35). Eventually, we get

$$\frac{y_{t+1}}{y_t} = \Phi[A(1-q)]^{1-\psi} \tau^\gamma y_{t+1}^{1-\psi} \equiv \Omega(\tau, \gamma_t).$$

(36)

The expression in (36) yields the short-run rate of output growth. I refer to ‘short-run’ growth in the sense that $\frac{y_{t+1}}{y_t}$ is state-dependent as it varies over time because of the variability in the policy variable $\gamma_t$. The long-run rate of output growth can be obtained by taking the mean value of $\frac{y_{t+1}}{y_t}$ from (36). In general, there are two distinct channels through which policy variability impinges on long-run (or trend) output growth. On the one hand, the direct growth effect of $\gamma_t$ is non-linear. On the other hand, policy variability will affect $\tau$ through its effect on the statistical properties of the growth rate of money supply $\mu_{t+1}$, $\forall s > 0$.

With the purpose of clarifying the aforementioned ideas, I shall use the following

is straightforward to show that the equilibrium solutions are $n_t = \frac{\beta \Lambda (1-\beta \phi)}{\varphi \beta^2 \Lambda + \beta \Lambda (1-\beta \phi) + \xi (1-\beta \phi)}$ and $\epsilon_t = \frac{\varphi \beta^2 \Lambda}{\varphi \beta^2 \Lambda + \beta \Lambda (1-\beta \phi) + \xi (1-\beta \phi)}$, while the solutions for (30) and (33) still apply. Consequently, it is evident that the inclusion of elastic labour supply will leave all the model’s results qualitatively unaffected.
**Theorem.** Let $X$ be some random variable generating a constant mean $\mu$ and a constant variance $\sigma^2$. Also, let $F(X)$ be some continuous function. Then $\text{Mean}[F(\cdot)] \approx \tilde{F}(\sigma^2)$ such that $\tilde{F}'(\cdot) > 0$ iff $F_{XX}(\cdot) > 0$ ($< 0$).

**Proof.** Take a second order Taylor series approximation for $F(\cdot)$ around $\mu$ as to get

$$F(X) \approx F(\mu) + F_X(\mu)(X - \mu) + \frac{1}{2} F_{XX}(\mu)(X - \mu)^2.$$  
Taking expectations in both sides and using $E(X) = \mu$, $E(X - \mu)^2 = \sigma^2$ yields

$$\text{Mean}[F(X)] \approx F(\mu) + \frac{1}{2} F_{XX}(\mu) \sigma^2 \equiv \tilde{F}(\sigma^2).$$

Obviously, the sign of the second derivative $F_{XX}(\cdot)$ determines the qualitative effects of $\sigma^2$ on $\text{Mean}[F(X)]$. ■

In terms of intuition, when $F(\cdot)$ is convex (concave) the positive effect from an exogenous shock is more (less) pronounced than the negative effect generated by an exogenous shock of equal magnitude but of opposite direction. Hence, the variance of $X$ has, on average, a positive (negative) effect on the function $F(\cdot)$.

An important result emerging from the analysis so far can be stated as

**Proposition 1.** The optimal allocation of time/effort towards the accumulation of capital is inversely related to policy variability.

**Proof.** Substitute (30) in (33) to get

$$\Lambda = E_t \left( \frac{1 - \gamma_{t+2}}{1 - (1 - q)\gamma_{t+2}} \right).$$

The term inside expectations is concave in the random policy variable. Therefore, following appropriate application of the Theorem, we can deduce that $\Lambda \equiv \tilde{\Lambda}(\sigma^2)$ such that $\tilde{\Lambda}'(\cdot) < 0$. Observing (36), we can check that $\tau$ is monotonically increasing in $\Lambda$. Consequently, we conclude that $\tau \equiv \varepsilon(\sigma^2)$ and $\varepsilon'(\cdot) < 0$. ■
Subsequently, we are ready to identify one of the main results of the paper – i.e., the result concerning the impact of policy variability on the long-run growth rate of output. This will be illustrated through

**Proposition 2.** The long-run growth rate is inversely related to policy variability irrespective on whether public goods are productive or not.

*Proof.* The growth rate, as obtained in equation (36), is monotonically increasing in $\bar{\tau}$ and concave in $\gamma_t$ (as long as $\psi < 1$). Applying the results of the Theorem and Proposition 1 we can deduce that

$$Mean\left(\frac{y_{t+1}}{y_t}\right) = Mean(\Omega(e(\sigma^2_t), y_t)) = \tilde{\Omega}(\sigma^2_t),$$

where $\tilde{\Omega}(\cdot) < 0$.  ■

As long as $\psi < 1$, the available technology allows public goods to be productive since they complement private inputs in the formation of capital – this being evident by the presence of $\gamma_t$ to which the growth rate is concave according to (36). This represents one of the channels through which $\sigma^2_t$ inhibits trend growth. The other channel works through the effect of policy variability on private inputs towards capital formation, $\bar{\tau}$, as this was identified in Proposition 1. The growth effect of volatile policy through this term is the most interesting one as it is not direct but works through the impact of the parameters of money supply on optimal investment decisions, due to the fact that money is non-neutral in this economy.

At this point it is important to shed some light on how volatile policies may impinge on the statistical properties of money growth, ultimately determining (to a large extent) their impact on economic growth. This is the purpose of the following Section.

### 5 Money, Inflation, its Variance and Long-Run Growth

Let us revisit the expression in (33) – i.e., the one determining the effect of money supply and, therefore, public spending policy on the private sector’s decisions concerning capital
accumulation. Observing the term inside expectations, we can see that it is convex in $\mu_{t+2}$. Effectively, this reveals that had this model considered an independent stochastic process for money growth, nominal variability would display a positive correlation with income growth. However, although $\gamma$ is ultimately responsible for generating variability in $\mu$, the former is inversely related with growth as we have already established.

A better understanding of the rationale behind the previous observation can be made possible through

**Lemma 2.** Policy variability, $\sigma_{\gamma}^2$, generates (i) variability in money growth, and (ii) higher money growth on average.

*Proof.* Part (i) of Lemma 2 is obvious after observation of (30) which reveals that money growth is, ultimately, a function of the random variable $\gamma$. The same equation, combined with the previous Theorem, can be utilised for establishing Part (ii) given that that $M(\gamma)$ is convex. ■

The idea behind Lemma 2 becomes even more transparent once we consider a specific probability distribution for $\gamma$, and utilise it to obtain the first and second moments of the probability distribution for $\mu$. Let us assume, for example, that the distribution for $\gamma$ satisfies

$$
\text{prob}\{\gamma = \overline{\gamma} - \sigma_{\gamma}\} = \text{prob}\{\gamma = \overline{\gamma} + \sigma_{\gamma}\} = \frac{1}{2} \quad \forall t \in [0, \infty),
$$

where $\overline{\gamma} > \sigma_{\gamma}$, $\overline{\gamma} + \sigma_{\gamma} < 1$, $E(\gamma_t) = \overline{\gamma}$ and $\text{Var}(\gamma_t) = \sigma_{\gamma}^2$. Denoting $E(\mu_t) = \overline{\mu}$ and $\text{Var}(\mu_t) = \sigma_{\mu}^2$, it is straightforward to establish that

$$
\overline{\mu} = \frac{\overline{\gamma} - (\overline{\gamma}^2 - \sigma_{\gamma}^2)}{(1 - \overline{\gamma})^2 - \sigma_{\gamma}^2},
$$

$$
\sigma_{\mu}^2 = \frac{\sigma_{\gamma}^2}{[(1 - \overline{\gamma})^2 - \sigma_{\gamma}^2]^2}.
$$

The calculations are relegated to an Appendix.
Obviously, $\sigma_\gamma^2 > 0$ if and only if $\sigma_\gamma^2 > 0$ and $\partial \sigma_\gamma^2 / \partial \sigma_\gamma^2 > 0$. Naturally, nominal variability emerges because of variability in public spending policy and results from the role of money as a financing instrument for the government’s budget. In addition, we can see that $\partial \pi / \partial \sigma_\gamma^2 > 0$ meaning that an increase in policy volatility results in higher money growth on average. The latter effect is what ultimately determines the sign of the correlation between policy variability and capital investment decisions by the private sector. On the one hand, by generating variability in money growth, it stimulates the optimal value for $\gamma$ as can be seen from (33) and (35). On the other hand, by increasing average money growth, policy variability impedes $\gamma$ — once more this being evident by observing (33) and (35). Eventually, the latter effect dominates and determines, to a large extent, the impact of $\sigma_\gamma^2$ on long-run output growth.

Effectively, the model argues that nominal variability is not harmful for growth per se. Instead, what causes it is responsible for higher average money growth as well and it is the latter effect that impedes the economy’s long-run macroeconomic performance. In the context of this model, this result can be summarised in the form of

**Corollary 1.** There is a positive correlation between average money growth and its volatility and it results from the incidence of variability in public spending.

For many years, the idea that there is a strong positive correlation between inflation and inflation variability has been suggested by numerous empirical analyses (e.g., Logue and Willett, 1976; Barro, 1995; Judson and Orphanides, 1999; Fischer et al., 2002; Wilson, 2006; Thornton, forth.) to such an extent that it is, nowadays, widely recognised as a stylised fact. This model provides a possible theoretical rationalisation for this empirical fact by arguing that the one of the links between the first and second moments of the stochastic process for the endogenously determined money growth derives from variability in public spending. In this respect, it formalises an argument first brought forward by Friedman (1977), who

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11 In this respect, this model is not at odds with the results obtained by Dotsey and Sarte (2000) and Varvarigos (forth.). However, contrary to these analyses, it identifies the positive link between average inflation and inflation variability. It does this by relaxing the assumption of an independent stochastic process for money supply.
explicitly referred to various public policies (other than monetary policy) as responsible for the observation that high (on average) inflation tends to be more volatile as well.\footnote{A slightly alternative justification is provided by Deveraux (1989): he argues that the variability from real shocks is responsible for both an increase in nominal variability and the reduction of the degree of wage indexation by workers. The latter outcome allows the government to use discretionary policy more effectively through the incidence of surprise inflation.}

Of course, one has to remember that, in a growing economy, money growth is just a proxy for inflation, contrary to economies with stationary output in which, along a steady-state equilibrium, money growth and inflation are coincident. Despite this fact, it is possible to show formally that the implications for the positive correlation between average money growth and its variance carry forward to the correlation between the average gross rate and the variability of inflation, i.e., $\frac{p_{t+1}}{p_t}$.

An expression for the actual, equilibrium rate of inflation is given in equation (27). Combining the goods market equilibrium condition together with (12) and (13) we get

\[ \epsilon_t = [1 - (1 - q)\gamma_t]\gamma_y. \]

Substituting in (27) and solving for $\frac{p_{t+1}}{p_t}$ yields

\[ \frac{p_{t+1}}{p_t} = (1 + \mu_t) \frac{\gamma_t[1 - (1 - q)\gamma_t](1 + q\gamma_{t+1})} {\gamma_{t+1}[1 - (1 - q)\gamma_{t+1}](1 + q\gamma_t)}. \]

(39)

In the absence of stochastic elements, (39) would be reduced to the familiar Fisher-type equation linking real and nominal growth. However, the implications here are richer. To see this, substitute (30) and (36) in (39) to get

\[ \frac{p_{t+1}}{p_t} = \Phi[A(1 - q)]^{1 - \psi} [\varepsilon(\sigma_\gamma^2)]^{-\psi} \frac{\gamma_t^{\phi - 1}}{(1 - \gamma_{t+1})} \equiv P(\sigma_\gamma^2, \gamma_t, \gamma_{t+1}). \]

(40)

Given (40) we can derive a result in the form of

**Lemma 3.** Policy variability, $\sigma_\gamma^2$, generates variability in the inflation rate.

**Proof.** Evidently, equation (40) reveals that if $\sigma_\gamma^2 = 0$, i.e., if $\gamma_t = \gamma \forall t$, then

\[ \text{Variance} \left( \frac{p_{t+1}}{p_t} \right) = 0. \]
\[ \text{Variance} \left( \frac{\ddot{p}_{t+1}}{\ddot{p}_t} \right) = \ddot{P}(\sigma^2_t), \]

where \( \ddot{P}(0) = 0 \). ■

In addition to the above, we can take account of the fact that the policy shocks have non-linear effects to the actual inflation rate to derive

**Proposition 3.** Both the inflation rate (on average) and its variance are positively related to policy variability.

**Proof.** From (40) observe that \( \ddot{P}(\cdot) \) is convex to realisations of both \( \gamma_t \) and \( \gamma_{t+1} \). Subsequently, apply the Theorem in (40), taking account of the result in Proposition 1 as well. The conclusion is that

\[ \text{Mean} \left( \frac{\ddot{p}_{t+1}}{\ddot{p}_t} \right) = \ddot{P}(\sigma^2_t), \]

where \( \ddot{P}'(\cdot) > 0 \). Given Proposition 1 and Lemma 3, we also conclude that \( \ddot{P}'(\cdot) > 0 \). ■

Intuitively, by stimulating money growth (on average) and its variance, more volatile policies increase the average inflation rate and the variance of inflation respectively. These effects are actually reinforced by the fact that policy variability is detrimental to trend growth, thus imposing an additional upward pressure to both inflation and its variance.

Combining Lemma 3 and Proposition 3, we can draw another result in the form of

**Corollary 2.** There is a positive correlation between the variance of inflation and its average rate, and it results from the incidence of variability in public spending.

Finally, by blending the results from Propositions 2 and 3, we can make inferences on issues relevant to the inflation-economic growth nexus. These are summarised in

**Corollary 3.** The incidence of policy variability generates a negative correlation between (i) output growth and inflation, and (ii) output growth and inflation variability.
There is a consensus among existing empirical analyses, relating to the argument that output growth is negatively related to both inflation and its variance (e.g., Grier and Tullock, 1989; Judson and Orphanides, 1999; Grier and Perry, 2000; Fountas and Karanasos, 2007). The previous discussion shows that my model can account for both these empirically observed correlations.

6 Social Welfare

The idea that variability affects all major real variables in the model economy implies that it has significant repercussions for social welfare. In order to identify them, we will combine previously obtained results together with the lifetime utility function from equation (16).

The aggregate equilibrium condition in the goods market is:

$$\epsilon_t = y_t - g_t = [1 - (1 - q)y_t] - Aa_t.$$  

Using (12), (13), (15), the result from Proposition 1 and substituting recursively yields

$$\epsilon_t = a_o[1 - (1 - q)y_t] - A\Phi^t \left( \prod_{i=0}^{t-1} \gamma_i \right)^{1-\psi}.$$  

where $\Phi \equiv [A(1-q)]^{1-\psi}[\epsilon(\sigma_t^2)]^\nu$. Next, we can combine (3) with the result from Proposition 1 to derive $l_t = 1 - \epsilon(\sigma_t^2) \equiv \delta(\sigma_t^2)$ such that $\delta'(\cdot) > 0$. Substitution of these results in (16) yields

$$V = E_0 \sum_{r=0}^\infty \beta^r \left\{ \ln(Aa_o) + \ln[1 - (1 - q)y_t] + \tau \ln \Phi + (1 - \phi) \ln \left( \prod_{i=0}^{t-1} \gamma_i \right) + \xi \ln[\delta(\sigma_t^2)] \right\},$$

which can be manipulated algebraically to derive

$$V \approx \frac{\ln(Aa_o)}{1 - \beta} + \frac{\Xi_1(\sigma_t^2)}{1 - \beta} + \frac{\beta \Xi_2(\sigma_t^2)}{1 - \beta^2} + \frac{(1 - \psi) \beta \Xi_3(\sigma_t^2)}{(1 - \beta)^2} + \frac{\Xi_4(\sigma_t^2)}{1 - \beta},$$

where $\Xi_1(\sigma_t^2) \equiv E_o[\ln[1 - (1 - q)y_t]]$, $\Xi_2(\sigma_t^2) \equiv \ln\{A^{1-\psi}[\epsilon(\sigma_t^2)]^\nu\}$, $\Xi_3(\sigma_t^2) \equiv E_o[\ln(y_t)]$ for $i \geq 0$ and $\Xi_4(\sigma_t^2) = \ln[\delta(\sigma_t^2)]$ are all constants and related to policy variability according to previous results.

The welfare implications from policy variability can be illustrated through
Proposition 4. There are two conflicting effects of policy variability on social welfare. On the one hand, by reducing the growth rate, variability has a cumulative, negative impact on aggregate consumption. On the other hand, by reducing the time spent on capital accumulation, variability increases the available time for leisure activities.

Proof. Straightforward differentiation of (43) yields

\[
\frac{\partial V}{\partial \sigma_y^2} = \frac{\xi'()}{1-\beta} + \frac{\beta \xi'_y()}{(1-\beta)^2} + \frac{(1-\psi)\beta \xi'_y()}{(1-\beta)^3} + \frac{\xi'_y()}{1-\beta}.
\]

Application of the Theorem reveals that the first three terms are negative (capturing the cumulative effect on aggregate consumption due to reduced output growth) and the last term is positive (capturing the increase in leisure).

Although volatility in public spending enhances leisure activities and contributes to an improvement of social welfare, a casual glance at equation (42) indicates that the decline in social welfare due to the negative effect of volatility on long-run growth seems to be stronger and probably dominant for reasonable combinations of parameter values. As a result, it is highly likely that policy variability contributes to reduced well-being for the economy’s private sector as a whole.\(^{13}\)

7 Conclusions

The novelty of this paper is the idea of using seignorage (i.e., money creation) as a means of financing volatile public spending within a stochastic, dynamic general equilibrium model of endogenously sustainable growth. Although nominal variability emerges solely because of policy variability (corresponding to variability in public spending), and despite the fact that money growth volatility by itself is beneficial for growth, the model predicts a negative correlation between long-run output growth and policy variability because the latter results in higher money growth as well. Therefore, given that the sign of the correlation between

\(^{13}\) Although numerical examples could be utilised here, the qualitative focus of the paper does not justify such a task. In any case, even without undertaking such simulations, it is safe to conjecture that the higher are the parameters of the capital formation technology or the lower is the assigned weight of utility from leisure activities, then the greater is the scope for higher volatility to induce a reduction in welfare.
volatility and growth proves negative, it remains in accordance with existing empirical analyses on the issue.

Equally important are the other implication of the model. As both inflation variance and average inflation are elevated by volatility in public spending, the model provides a possible account for the strong positive correlation between inflation and its variability, as well as their negative correlation with output growth.

The theoretical framework’s specifications for technologies, preferences, and stochastic processes are carefully chosen to allow analytical solutions through which the analysis benefits from clarity of both the intuition and the mechanisms involved. As always, the model can be enriched with additional assumptions that may broaden its implications. For example, the stochastic process for the policy (public spending) variable could be generalised to allow for persistence. Furthermore, a similar framework could pursue a more conventional approach to capital accumulation by allowing physical output as an input to the formation of capital – perhaps with the added characteristic of a cash-in-advance constraint to the purchase of capital goods. As long as these additions can expand our understanding of the issues mentioned and discussed in this paper, such approaches may constitute fruitful and worth undertaking avenues for future research.

References


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**Appendix**

**Derivation of (37) and (38)**

Given equation (30) and the specified stochastic process, we have

\[
E(\mu_t) = 0.5 \left[ \frac{\bar{\sigma}_r}{1-(\bar{\gamma} - \sigma_r)} \right] + 0.5 \left[ \frac{\bar{\sigma}_r}{1-(\bar{\gamma} + \sigma_r)} \right].
\]

Factorising the above and manipulating algebraically yields

\[
E(\mu_t) = 0.5 \frac{(\bar{\gamma} - \sigma_r)[1-(\bar{\gamma} + \sigma_r)] + (\bar{\gamma} + \sigma_r)[1-(\bar{\gamma} - \sigma_r)]}{[1-(\bar{\gamma} - \sigma_r)][1-(\bar{\gamma} + \sigma_r)]} = ...
\]

\[
= 0.5 \frac{\bar{\gamma} - \sigma_r - (\bar{\gamma} - \sigma_r)(\bar{\gamma} + \sigma_r) + \bar{\gamma} + \sigma_r - (\bar{\gamma} + \sigma_r)(\bar{\gamma} - \sigma_r)}{[(1-\bar{\gamma}) - \sigma_r][1-\bar{\gamma} + \sigma_r]} = ...
\]

\[
= 0.5 \frac{2\bar{\gamma} - 2(\bar{\gamma}^2 - \sigma_r^2)}{(1-\bar{\gamma})^2 - \sigma_r^2} = ...
\]

\[
= \frac{\bar{\gamma} - (\bar{\gamma}^2 - \sigma_r^2)}{(1-\bar{\gamma})^2 - \sigma_r^2} \equiv \bar{\mu},
\]

which is equation (37) in the main text.

By definition, the variance of money growth is equal to \( E(\mu_t - \bar{\mu})^2 \). Using equation (30) and the specified stochastic process we get

\[
Var(\mu_t) = 0.5 \left[ \frac{\bar{\gamma} - \sigma_r}{1-(\bar{\gamma} - \sigma_r)} - \frac{\bar{\gamma} - (\bar{\gamma}^2 - \sigma_r^2)}{(1-\bar{\gamma})^2 - \sigma_r^2} \right]^2 + 0.5 \left[ \frac{\bar{\gamma} + \sigma_r}{1-(\bar{\gamma} + \sigma_r)} - \frac{\bar{\gamma} - (\bar{\gamma}^2 - \sigma_r^2)}{(1-\bar{\gamma})^2 - \sigma_r^2} \right]^2. \tag{A1}
\]

The first term inside brackets can be manipulated to yield
\[
\frac{\bar{y} - \sigma_y}{1 - (\bar{y} - \sigma_y)} - \frac{\bar{y} - (\bar{y}^2 - \sigma_y^2)}{(1 - \bar{y})^2 - \sigma_y^2} = \ldots
\]

\[
= \frac{1}{1 - (\bar{y} - \sigma_y)} \left[ \bar{y} - \sigma_y - \frac{\bar{y} - (\bar{y}^2 - \sigma_y^2)}{1 - (\bar{y} + \sigma_y)} \right] = \ldots
\]

\[
= \frac{1}{1 - (\bar{y} - \sigma_y)} \left[ (\bar{y} - \sigma_y)[1 - (\bar{y} + \sigma_y)] - \bar{y} + (\bar{y}^2 - \sigma_y^2) \right] = \ldots
\]

\[
= \frac{1}{1 - (\bar{y} - \sigma_y)} \left[ (\bar{y} - \sigma_y)(\bar{y}^2 - \sigma_y^2) - \bar{y} + \bar{y}^2 - \sigma_y^2 \right] = \ldots
\]

\[
\frac{\bar{y} + \sigma_y}{1 - (\bar{y} + \sigma_y)} - \frac{\bar{y} - (\bar{y}^2 - \sigma_y^2)}{(1 - \bar{y})^2 - \sigma_y^2} = \ldots
\]

\[
= \frac{1}{1 - (\bar{y} + \sigma_y)} \left[ \bar{y} + \sigma_y - \frac{\bar{y} - (\bar{y}^2 - \sigma_y^2)}{1 - (\bar{y} - \sigma_y)} \right] = \ldots
\]

\[
= \frac{1}{1 - (\bar{y} + \sigma_y)} \left[ (\bar{y} + \sigma_y)[1 - (\bar{y} - \sigma_y)] - \bar{y} + (\bar{y}^2 - \sigma_y^2) \right] = \ldots
\]

\[
= \frac{1}{1 - (\bar{y} + \sigma_y)} \left[ (\bar{y} + \sigma_y)(\bar{y}^2 - \sigma_y^2) - \bar{y} + \bar{y}^2 - \sigma_y^2 \right] = \ldots
\]

\[
\frac{-\sigma_y}{(1 - \bar{y})^2 - \sigma_y^2}.
\]

A similar procedure for the second term inside brackets yields

\[
\frac{\bar{y} - \sigma_y}{1 - (\bar{y} - \sigma_y)} - \frac{\bar{y} - (\bar{y}^2 - \sigma_y^2)}{(1 - \bar{y})^2 - \sigma_y^2} = \ldots
\]

\[
= \frac{1}{1 - (\bar{y} - \sigma_y)} \left[ \bar{y} - \sigma_y - \frac{\bar{y} - (\bar{y}^2 - \sigma_y^2)}{1 - (\bar{y} + \sigma_y)} \right] = \ldots
\]

\[
= \frac{1}{1 - (\bar{y} - \sigma_y)} \left[ (\bar{y} - \sigma_y)[1 - (\bar{y} + \sigma_y)] - \bar{y} + (\bar{y}^2 - \sigma_y^2) \right] = \ldots
\]

\[
= \frac{1}{1 - (\bar{y} - \sigma_y)} \left[ (\bar{y} - \sigma_y)(\bar{y}^2 - \sigma_y^2) - \bar{y} + \bar{y}^2 - \sigma_y^2 \right] = \ldots
\]

\[
\frac{\sigma_y}{(1 - \bar{y})^2 - \sigma_y^2}.
\]

It is a matter of substituting (A2) and (A3) in (A1) to get
\[ Var(\mu_t) = 0.5 \left[ \frac{-\sigma_y}{(1-\bar{y})^2 - \sigma_y^2} \right]^2 + 0.5 \left[ \frac{\sigma_y}{(1-\bar{y})^2 - \sigma_y^2} \right]^2 = \ldots \]

\[ = 0.5 \frac{2\sigma_y^2}{[(1-\bar{y})^2 - \sigma_y^2]^2} = \ldots \]

\[ = \frac{\sigma_y^2}{[(1-\bar{y})^2 - \sigma_y^2]^2} \equiv \sigma_y^2, \]

which is equation (38) in the main text.