Numerical quadrature and its applications

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Numerical Quadrature

and its Applications

by

G. A. Evans
Acknowledgements

In a period of eighteen years many people influence one's academic development, especially remembering that in the early years one is setting off on a career as a junior lecturer and at the end one has enjoyed senior status for almost a decade.

I have been fortunate to have had continuing support from Professor Storey who has been my head of department throughout almost the whole period of this work. He has provided an environment to allow research development despite at times departmental pressure caused by high teaching loads.

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It is the personal contacts which keep work progressing on a day to day basis, and result in considerable influence. One man stands out in this respect. Dr. John Hyslop became a friend and colleague and a provider of superb problems in his area of quantum mechanics in the early stages of this work. He died tragically in February, 1985.
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